I, Deepak Ganga Dharan, hereby submit this original work as part of the requirements for the degree of Master of Science in Mechanical Engineering.

It is entitled:
Numerical Analysis of End-Sealed Squeeze-Film Damper Bearings using Moving Reference Frame Formulation

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Numerical Analysis of End-Sealed Squeeze-Film Damper Bearings using Moving Reference Frame Formulation

A thesis submitted to the Graduate School of the University of Cincinnati in partial fulfillment of the requirements for the degree of Master of Science in the Department of Mechanical and Materials Engineering of the College of Engineering and Applied Science by

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ABSTRACT

Squeeze-film dampers (SFDs) are commonly used in aircraft engines because of their high damping coefficient and the associated ability to dissipate vibrations and improve the dynamic stability of rotor-bearing systems. The goal of the present work is to perform Computational Fluid Dynamics (CFD) simulation of uncavitated, end-sealed SFDs with axially centered or offset circumferential groove, and compare the simulation results for the tangential and radial forces with experimental data. For circular whirling motion of the SFD, a moving reference frame (MRF) formulation is used to transform the otherwise time-varying fluid domain to a time-independent domain, and enable an efficient steady-state simulation. The approach is first verified by comparing the simulation results with the Reynolds analytical solutions for long SFD (wherein pressure is a function of only the azimuthal direction, θ) and short SFD (wherein pressure is a function of both θ and the axial direction, z) bearings. The Reynolds solutions neglect inertia (Reynolds number Re = 0), whereas the computational model requires a non-zero Re, however small. With Re = 0.96, the computed tangential force (directly proportional to damping) yields just 0.09% and 0.66% difference for the long and short SFD Reynolds solutions, respectively, thereby verifying the correct implementation of the MRF formulation. Next, the mathematical model used is validated by comparing the CFD results with available experimental data. The computed tangential force for the centered- and offset-groove configurations compares within
6.55% and 6.72%, respectively, with the data, suggesting that the simulation methodology developed can be used, within this tolerance, to predict the forces for end-sealed SFDs. Finally, a parametric study is conducted by varying geometrical parameters (radial clearance and groove position) and operating parameters (feeding pressure, feeding temperature, and whirling speed) for this flow. Groove position and feeding temperature are observed to affect the damping force significantly, whereas feeding pressure has a negligible effect on the forces, for an uncavitated bearing. In addition, the tangential force is seen to vary directly with whirl velocity, and inversely with radial clearance. Similar behavior is seen for the Reynolds solutions. Therefore, the present computational approach can be used to aid optimization of the design/operating conditions to achieve maximum damping for end-sealed SFDs.
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NOMENCLATURE

Latin Letters:

$\bar{\alpha}, \bar{\beta}, \bar{\gamma}, \bar{\delta}$ integration constants (in Eqs. (16) and (17))

$b_o$ characteristic length in z direction

c radial clearance, $r_o - r_j$

$\bar{E}, \bar{F}$ integration constants (in Eqs. (56) and (58))

e eccentricity of bearing

$\bar{G}, \bar{H}$ integration constants (in Eqs. (70) and (71))

g acceleration due to gravity

h film thickness

$h_o$ characteristic length in y direction

L axial length (same as $b_o$)

$L_{ref}$ reference length

$l_o$ characteristic length in x direction

$Mod_R$ modified radial coordinate for the grid points (in Eq. 86)

$Mod_X$ modified x-coordinate for the grid points (in Eq. 88)

$Mod_Y$ modified y-coordinate for the grid points (in Eq. 89)

$O'$ center of journal

O center of housing

$\bar{p}$ dimensionless pressure

p pressure

q volumetric flow rate

$q'$ volumetric flow rate per unit axial length
\( q_x' \) volumetric flow rate per unit axial length in x-direction

\( q_y' \) volumetric flow rate per unit axial length in y-direction

\( R \) radial coordinates for the grid points (in Eq. 85)

\( \text{Re} \) Reynolds number

\( \text{Re}^* \) modified Reynolds number (in Eq. (5))

\( r_b \) radius of bearing

\( r_j \) radius of journal

\( T \) azimuthal coordinate for the grid points (in Eq. 87)

\( \bar{t} \) dimensionless time

\( t \) time

\( t_0 \) characteristic time

\( u, v, w \) velocity components along x-, y-, and z-direction, respectively

\( u_o, v_o, w_o \) characteristic velocities along x-, y-, and z-direction, respectively

\( \bar{u}, \bar{v}, \bar{w} \) dimensionless velocity components along x-, y-, and z-direction, respectively

\( u_A, v_A, w_A \) velocity components for surface A

\( u_B, v_B, w_B \) velocity components for surface B

\( u_{O,B}, v_{O,B} \) relative velocity of \( O' \) w.r.t. point B

\( U_{\text{ref}} \) reference velocity

\( \bar{u}_r \) velocity of MRF relative to absolute frame

\( \bar{V} \) absolute velocity in inertial frame

\( \bar{v}_r \) relative velocity viewed from moving frame

\( \bar{v}_t \) frame translational velocity

\( x, y, z \) cartesian coordinates

\( \bar{x}, \bar{y}, \bar{z} \) dimensionless coordinates

Greek Letters:

\( \gamma \) Sommerfeld variable

\( \varepsilon \) eccentricity ratio, \( e/c \)

\( \theta \) azimuthal coordinate in cylindrical polar system

\( \mu \) absolute viscosity
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<td>$\mu_0$</td>
<td>characteristic absolute viscosity</td>
</tr>
<tr>
<td>$\bar{\mu}$</td>
<td>dimensionless absolute viscosity</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>characteristic density</td>
</tr>
<tr>
<td>$\bar{\rho}$</td>
<td>dimensionless density</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>squeeze number (in Eq. (6))</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>rotational coordinate</td>
</tr>
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<td>$\phi$</td>
<td>whirl angle</td>
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<td>$\omega$</td>
<td>whirling velocity</td>
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<td>$\omega_j$</td>
<td>journal rotation speed</td>
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<tr>
<td>$\omega_b$</td>
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1. INTRODUCTION

Squeeze-film dampers (SFDs), commonly used in aircraft engines, add damping to rolling-element support bearings. This allows effective mechanical isolation of components, damps excessive journal motion amplitudes, and subdues rotordynamic instabilities. Generally, a hydrodynamic bearing consists of a stationary housing and a rotating element, called journal, mounted on a shaft. However, in an SFD, an anti-rotation pin is utilized to prevent rotating motion of the journal. Hence, the rotating shaft imparts a whirling motion to the journal of an SFD bearing, as compared to a rotational and a whirling motion in case of a journal bearing. When a lubricant is filled in the annular clearance between the journal and the bearing housing, a thin film is formed. During operation, the journal whirling motion squeezes the lubricant film and, hence, the name squeeze-film damper. The resulting pressure reaction forces damp excessive journal whirl motion amplitudes.

Generally for an SFD, the damping coefficient is considered as a benchmark of good performance. The damping coefficient is the ratio of the magnitude of tangential force to the magnitude of velocity of the damper. The higher the tangential force for a given operating speed, the greater is the damping coefficient and, hence, better the bearing performance.

Earlier studies of SFDs have relied on both analytical and experimental analyses. However, both these methods have their limitations, because of which effects of geometrical and operating
parameters have not been comprehensively studied. An analytical solution makes various assumptions to simplify the problem, and experimental tests are time consuming, complex and very expensive. With evolving computational capabilities, it is important that a methodology is developed to simulate SFDs, so that dependence on experimental results is reduced and to minimize the assumptions used to obtain the analytical solutions.

The goal of the present study is to investigate the feasibility of application of a Moving Reference Frame (MRF) (steady-state solver) approach, which is computationally less demanding than a traditional dynamic-mesh approach, and to determine the tangential force generated in an SFD.

1.1. Background

Squeeze-Film damper technology has become widespread during the last four decades. A comprehensive review of past research conducted between 1963 and 2002 has been documented by Della Pietra and Adiletta [2][3]. They discuss in detail all the significant analytical and experimental work conducted on SFDs till date. Zeidan et al. [4] state the continued effort at developing an approach for designing an optimum squeeze-film geometry. Prior theoretical investigations neglected the relative importance of fluid inertia, lubricant dynamic cavitation, and boundary conditions such as oil feeding system and end seals. Present-day researchers are not satisfied with the predictive capabilities of the analytical models that were developed under these simplified assumptions. Real SFD design and operating conditions are sometimes more complex for these theoretical models to be considered. The present study utilizes the full Navier-Stokes equations, solved using a commercially available CFD software, eliminating the simplifying assumptions of negligible inertial effect as has been done in the theoretical models.
San Andres [5] has published comprehensive notes detailing various aspects of hydrodynamic bearings including squeeze-film dampers. The contents of these notes range from introducing hydrodynamic lubrication to new readers, to dynamics of fluid-film bearings in various conditions such as static loading, to dynamic loading, to cavitation for advanced readers. San Andres [6] has identified the damping and inertia coefficients for open-ended SFDs with center groove and compared the coefficients with the predicted values from the bulk flow model [7]. The predicted damping and inertia coefficients from the bulk-flow model agreed well with that from the experiments. He was able to conclude that the center groove does not divide the film land into two effective dampers, but the groove actually contributes to the force components. The bulk flow model is an improvement over traditional analytical models, as it takes into account the flow interaction between the lands and the groove. Boppa [8] studied the effect of center groove on the dynamic pressure distribution. She conducted simulations using MRF and found the computational results agree closely with the experiments performed by Delgado [7]. Even though, the bearing was open ended, Boppa [8] was able to validate her MRF model with experimental results. This prompted us to use MRF approach in our numerical simulation as well. Khandare [9] studied the effect of cavitation on the pressure distribution using adaptive-mesh approach and compared the pressures at the mid-span of the SFD land. He found the computational results in close agreement with those obtained experimentally by Delgado [7].

Due to space or flow limitations, end seals are implemented in bearings to reduce the axial flow of lubricant and, thereby, increase the bearing’s damping capability. End seals also provide protection against air entrapment or ingestion, [10][11]. The end seal design is highly empirical and, so far, only experience dictates the best seal type to be employed in various applications [5]. End seals may be of various types: O-rings, piston rings, and end plates. O-rings and piston rings
have an elliptical and a rectangular cross-section, respectively, and require end-grooves for fitting. The end plate, on the other hand, is more like a solid plate between the journal and the housing. O-rings are implemented in many compressor designs due to their simple design and good sealing capability. However, creep due to long-term use has been a constant issue for larger loads, which can prove to be fatal for the compressor. Piston rings are preferred over end plates due to their ability to dissipate the heat generated because of through-flow. Hence, for high-temperature applications, such as turbine shafts used in aircraft engines, piston rings are generally preferred.

All computational studies performed till date were for either open-ended or partially-open dampers, even though some experimental data is available for end-sealed dampers. San Andres et al. [12] identified the damping and inertia coefficients for end-ended SFDs with centered groove and compared the coefficients with the predicted values from the bulk flow model [7]. The predicted damping and inertia coefficients from the bulk-flow model agreed well with that from the experiments for short dampers. In case of long dampers, the model under-predicted the inertia coefficient by ~ 25%. They also compared the damping and inertia coefficients between the end-sealed and the open-ended dampers. The damping was higher for the end-sealed damper compared to open-ended by 3.8 times and 2.7 times for short and long bearings, respectively. They also found end-sealed dampers to predict 2 times to 2.7 times more inertia coefficient compared to open-ended dampers for short and long bearings, respectively. Defaye et al. [1][13] performed extensive experiments on end-sealed dampers to capture the radial and tangential force components. They not only compared the force coefficients between the centered-groove, offset-groove, and the non-grooved damper, but also presented the influence of the inertial force, cavitation, and turbulence regime on the pressure distribution, indicating the restrictions of the analytical models for detailed studies.
With evolving computational capabilities, it is important that a methodology is developed to simulate SFDs so that dependence on experimental results is reduced. The present study investigates the feasibility of application of a Moving Reference Frame (MRF) (steady-state solver) approach, which is computationally less demanding, to determine the tangential force generated in an SFD. This work is unique because it includes end-seals, and hence, will represent the first documented work of simulating an end-sealed SFD bearings.

As discussed earlier, an SFD bearing is essentially a journal bearing with the journal rotation prevented by using an anti-rotation pin. Figure 1 shows a typical SFD configuration. The rotating shaft is typically the component, like a crankshaft, an axle, or a gas turbine shaft, which needs damping. A journal is mounted on the shaft using ball bearings. Lubricant is filled between the journal and the stationary housing. Due to the rotation of the shaft, a combination of rotational and whirling motion is imparted to the journal. Using an anti-rotation pin eliminates the journal rotation; hence, the only motion that the journal exhibits is whirling. The elimination of the journal rotation doubles the magnitude of the tangential force, as will be apparent from the analytical results presented in Chapter 3.

![Figure 1. SFD Configuration](image)
Next, we will discuss in detail the Reynolds equation, derived by simplifying the full Navier-Stokes equations, and the resulting Reynolds solutions for long and short bearings. We will use these analytical solutions to verify our simulation methodology.

1.2. Objectives

The present study pursues the following objectives in order to successfully achieve the goals stated:

1. *Simulation Methodology and its Verification*

A simulation methodology is developed and is verified by comparing the computational results with the analytical results of long and short SFD bearings.

A long SFD bearing is an analytical case, and has no axial variation in pressure, i.e., pressure is only a function of the azimuthal direction, \( \theta \). In a short SFD bearing, flow occurs in the axial direction, in addition to flow along \( \theta \); hence, \( p = p(\theta, z) \).

The correct implementation of MRF was tested first by comparing the simulation results of long and short SFD bearings with their analytical solutions, where Reynolds number is zero (\( Re = 0 \)).

2. *Simulation Model for End-Sealed Dampers and its Validation*

After establishing the correct implementation of the MRF formulation, the approach is used to simulate both centered- and offset-groove configurations. For the two configurations used in the present study, the bearing geometry and flow parameters correspond to those provided in detail in the experimental work of Defaye et al. [1]. The experimental data of Defaye et al. [1] is then used for validation of the present mathematical model.

3. *Parametric Study for Offset-Groove Configuration*
After validation with experimental data, a parametric study was conducted with the offset-groove configuration, by changing the groove location, supply pressure, supply temperature, radial clearance and whirl speed, and the effect of these parameters was examined on the tangential and radial force components.

ANSYS-Fluent v14.5, a commercially available fluid dynamics solver, was used for the numerical simulation of all the cases in this study.

1.3. Thesis Organization

This thesis document is organized as follows:

Chapter 2 contains background information related to this study. It includes details of the SFD geometry and previous work done on the topic. Chapter 3 offers insight into the numerical approach used to model analytical long and short bearing cases. This includes detailed derivation of Reynolds solution, problem set-up for computations, and details of MRF and its implementation in the code. Finally, we discuss the computational results and compare them with the Reynolds solutions. After confirming good comparison using the MRF approach, the approach is used to analyze end-sealed dampers, the results for which are presented in Chapter 4. Also presented is a parametric study for various geometrical and operating parameters. Chapter 5 discusses the grid-convergence study, and Chapter 6 lists the conclusions drawn from the study and some suggestions for further work on this problem.
1.4. Overview of the Simulated/Studied Cases

Table 1 presents the cases simulated in this present study.

Table 1. Cases Studied/Simulated

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<td>n/a</td>
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<tr>
<td>Solution</td>
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<td>Analytical/Reynolds</td>
<td>Re = 0.96 (Fig. 11)</td>
<td>n/a</td>
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<td>Solution</td>
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2. Long and Short SFD Bearings

Long bearing is an analytical case, where we assume that the bearing is infinitely long. Hence, a long SFD bearing has no axial variation in pressure, i.e., pressure is only a function of the azimuthal direction, $\theta$. Therefore, computationally, this constitutes a 2D geometry.

For short SFD bearings, we assume a finite length, Hence, flow occurs in the axial direction in addition to $\theta$; therefore, $p = p(\theta, z)$. Thus, a short bearing constitutes a 3D geometry where the $z$-axis represents the axial direction. The 2D geometry of a long bearing is extruded in the $z$-direction to create a 3D model for the short bearing.

2.1. Mathematical Formulation and Solution of Reynolds Equation

In this section, we start with the general mass and momentum conservation equations. Then, simplifying them using non-dimensional analysis, we get the reduced conservation equations. Combining the mass and momentum conservation equations into one single equation gives us the Reynolds equation.

2.1.1. General Form of Reynolds Equation

Assumptions made during the derivation of Reynolds Solution:

1. Body forces (gravity), inertial accelerations, electric field acceleration in the fluid film are neglected;
2. Pressure change across the film thickness is negligible since the film thickness is small;
3. Fluid is assumed to be incompressible (constant density) and furthermore, viscosity is
   assumed constant;
4. Fluid inertia in the Navier-Stokes equations is neglected;
5. Zero slip assumed at fluid-solid interfaces.

**Reynolds Equation Derivation**

Hamrock et al. [14] have published the details of the derivation of the Reynolds equation, and
further summarized the derivation of pressure and tangential force expressions for long and short
bearings. This section contains the full details of these derivations. The Reynolds equation is
derived from the Navier-Stokes and continuity equations. For the purpose of this derivation, the
Cartesian coordinate system used is defined such that the bearing face lies in the x-y plane, and
the z coordinate is directed along the axial length of the bearing.

**Continuity Equation:**

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) = 0. \tag{1a}
\]

**Navier-Stokes Equations:**

\[
\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right), \tag{1b}
\]

\[
\rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right), \tag{1c}
\]

\[
\rho \frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right). \tag{1d}
\]
Due to the difference in the length scales in the x-, y-, and z-direction, the coordinates are nondimensionalized by different characteristic quantities. Similar approach is adopted for the velocity variables.

The dimensionless variables are defined as:

\[
\begin{align*}
\tilde{x} &= x/l_o; \quad \tilde{y} = y/h_o; \quad \tilde{z} = z/b_o; \quad \tilde{t} = t/t_o, \\
\tilde{u} &= u/u_o; \quad \tilde{v} = v/v_o; \quad \tilde{w} = w/w_o, \\
\tilde{\mu} &= \mu/\mu_o; \quad \tilde{\rho} = \rho/\rho_o; \quad \tilde{p} = h_o^2 p/\mu u_l.
\end{align*}
\]

The derivation starts with the dimensionless Navier-Stokes equations representing conservation of linear momentum. The x-component of the equation is expressed as:

\[
\frac{\tilde{u}}{t_o} \frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\tilde{u}^2}{l_o} \frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\tilde{u} \tilde{v}}{h_o} \frac{\partial \tilde{u}}{\partial \tilde{y}} + \frac{\tilde{u} \tilde{w}}{b_o} \frac{\partial \tilde{u}}{\partial \tilde{z}} = -\frac{\tilde{\mu}}{t_o h_o^2} \frac{\partial \tilde{p}}{\partial \tilde{x}} + \tilde{\mu} \frac{\partial}{\partial \tilde{t}} \left( \frac{\partial \tilde{u}}{\partial \tilde{x}} \right)
\]

\[
+ \frac{\tilde{\mu} u_o}{\rho_o b_o^2} \frac{\partial}{\partial \tilde{z}} \left[ \frac{\partial \tilde{u}}{\partial \tilde{z}} \right].
\]

Similarly the y- and the z-components of the equation are expressed as:

\[
\begin{align*}
\frac{\tilde{v}}{t_o} \frac{\partial \tilde{v}}{\partial \tilde{x}} + \frac{\tilde{u} \tilde{v}}{h_o} \frac{\partial \tilde{v}}{\partial \tilde{y}} + \frac{\tilde{w} \tilde{v}}{b_o} \frac{\partial \tilde{v}}{\partial \tilde{z}} &= -\frac{\tilde{\mu}}{t_o h_o^2} \frac{\partial \tilde{p}}{\partial \tilde{y}} + \frac{\tilde{\mu} v_o}{\rho_o} \frac{\partial}{\partial \tilde{t}} \left( \frac{\partial \tilde{v}}{\partial \tilde{x}} \right)
\end{align*}
\]

\[
+ \frac{\tilde{\mu} v_o}{\rho_o b_o^2} \frac{\partial}{\partial \tilde{z}} \left[ \frac{\partial \tilde{v}}{\partial \tilde{z}} \right],
\]

and

\[
\begin{align*}
\frac{\tilde{w}}{t_o} \frac{\partial \tilde{w}}{\partial \tilde{x}} + \frac{\tilde{u} \tilde{w}}{h_o} \frac{\partial \tilde{w}}{\partial \tilde{y}} + \frac{\tilde{w}^2}{b_o} \frac{\partial \tilde{w}}{\partial \tilde{z}} &= -\frac{\tilde{\mu}}{t_o h_o^2} \frac{\partial \tilde{p}}{\partial \tilde{x}} + \frac{\tilde{\mu} w_o}{\rho_o b_o^2} \frac{\partial}{\partial \tilde{t}} \left( \frac{\partial \tilde{w}}{\partial \tilde{x}} \right)
\end{align*}
\]

\[
+ \frac{\tilde{\mu} w_o}{\rho_o b_o^2} \frac{\partial}{\partial \tilde{z}} \left[ \frac{\partial \tilde{w}}{\partial \tilde{z}} \right].
\]

The Reynolds number is a dimensionless number, defined as the ratio of inertial and viscous forces:
\[ \text{Re} = \frac{\rho_o U_{ref} L_{ref}}{\mu_o} \]  

(4)

where \( U_{\text{ref}} \) is the reference velocity and \( L_{\text{ref}} \) is the reference length.

In fluid film lubrication, due to the dominance of the viscous term, \( \frac{\partial^2 u}{\partial y^2} \), a modified Reynolds number \( \text{Re}^* \) is used.

\[ \text{Re}^* = \frac{\rho_o u_o h_o^2}{\mu_o l_o} \]  

(5)

Another relevant dimensionless quantity is the squeeze number which is defined as:

\[ \sigma_s = \frac{\rho_o h_o^2}{\mu_o t_o} \]  

(6)

Both \( \text{Re}^* \) and \( \sigma_s \) are dimensionless, and of the order \( \frac{h_o}{l_o} \).

Multiplying the \( x \)-component of the dimensionless Navier-Stokes equations (Eq. (2)) by \( \frac{\rho_o u_o h_o^2}{\mu_o u_o} \) and using Eqs (5) and (6), yields:

\[ \sigma_s \frac{\partial \vec{u}}{\partial t} + \text{Re}^* \frac{\partial \vec{u}}{\partial x} + \text{Re}^* \frac{l_o}{h_o} \frac{v_o}{u_o} \frac{\partial \vec{u}}{\partial y} + \text{Re}^* \frac{l_o}{b_o} \frac{w_o}{u_o} \frac{\partial \vec{u}}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \left( \frac{h_o}{l_o} \right)^2 \frac{\partial}{\partial x} \left( \mu \frac{\partial \vec{u}}{\partial x} \right) \\
+ \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial \vec{u}}{\partial y} \right) + \left( \frac{h_o}{b_o} \right)^2 \frac{1}{\rho} \frac{\partial}{\partial z} \left[ \mu \left( \frac{\partial \vec{u}}{\partial z} \right) \right] \]  

(7a)

Similarly, the \( y \)- and the \( z \)-components of the Navier-Stokes equations (Eqs. (3a) and (3b)) are non-dimensionalized and multiplied by \( \frac{\rho_o u_o h_o^2}{\mu_o v_o} \) and \( \frac{\rho_o u_o h_o^2}{\mu_o w_o} \), respectively to get:
In Eqs. (7a), (7b), and (7c), it can be observed that pressure gradient and second viscous term are of order unity, and the remaining viscous terms are of order \((\frac{h_o}{l_o})^2\) or \((\frac{h_o}{b_o})^2\).

For a hydrodynamic bearing, \(h_o \propto\) radial clearance, \(l_o \propto 2\pi r_j\), and \(b_o \propto L\). Since the radial clearance is of a smaller magnitude than the circumference of the journal or the length of the bearing, \((\frac{h_o}{l_o})^2\) or \((\frac{h_o}{b_o})^2\) are of relatively small magnitudes, and hence, can be neglected. Eq. (7a) then reduces to:

\[
\sigma_o \frac{\partial \tilde{v}}{\partial t} + Re* \frac{\partial \tilde{u}}{\partial x} + Re* \frac{l_o}{h_o} u_o \frac{\partial \tilde{v}}{\partial y} + Re* \frac{l_o}{b_o} u_o \frac{\partial \tilde{v}}{\partial z} = - \frac{u_o l_o}{\rho} \frac{\partial \tilde{p}}{\partial x} + \frac{h_o}{\rho} \frac{\partial}{\partial x} \left( \mu \frac{\partial \tilde{u}}{\partial x} \right) + \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial \tilde{u}}{\partial y} \right) \]

\[
\sigma_o \frac{\partial \tilde{w}}{\partial t} + Re* \frac{\partial \tilde{w}}{\partial x} + Re* \frac{l_o}{h_o} u_o \frac{\partial \tilde{w}}{\partial y} + Re* \frac{l_o}{b_o} u_o \frac{\partial \tilde{w}}{\partial z} = - \frac{u_o l_o}{\rho} \frac{\partial \tilde{p}}{\partial x} + \frac{h_o}{\rho} \frac{\partial}{\partial x} \left( \mu \frac{\partial \tilde{u}}{\partial x} \right) + \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial \tilde{w}}{\partial y} \right) \]

Similarly, the second and third components (y- and z-components, respectively) of the Navier-Stokes equations are non-dimensionalized and neglecting the terms of the order \((\frac{h_o}{l_o})^2\) and \((\frac{h_o}{b_o})^2\), yields:

\[
\sigma_o \frac{\partial \tilde{v}}{\partial t} + Re* \frac{\partial \tilde{v}}{\partial x} + Re* \frac{l_o}{h_o} u_o \frac{\partial \tilde{v}}{\partial y} + Re* \frac{l_o}{b_o} u_o \frac{\partial \tilde{v}}{\partial z} = - \frac{u_o l_o}{h_o \rho} \frac{\partial \tilde{p}}{\partial y} + \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial \tilde{v}}{\partial y} \right) \]

\[
\sigma_o \frac{\partial \tilde{w}}{\partial t} + Re* \frac{\partial \tilde{w}}{\partial x} + Re* \frac{l_o}{h_o} u_o \frac{\partial \tilde{w}}{\partial y} + Re* \frac{l_o}{b_o} u_o \frac{\partial \tilde{w}}{\partial z} = - \frac{u_o l_o}{h_o \rho} \frac{\partial \tilde{p}}{\partial z} + \frac{1}{\rho} \frac{\partial}{\partial z} \left( \mu \frac{\partial \tilde{w}}{\partial z} \right) \]
Moreover, the terms with $\text{Re}^*$ and $\sigma$ can be neglected because of their small magnitudes compared to unity. Also, since $u_0 \propto l_0$, $v_0 \propto h_0$, and $w_0 \propto b_0$, we can write Eqs. (8), (9a) and (9b) as:

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right)$$  \hspace{1cm} (10a)$$

$$0 = -\frac{l_0^2}{h_0^2} \frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial v}{\partial y} \right)$$  \hspace{1cm} (10b)$$

$$0 = -\frac{l_0 h_0}{b_0^2} \frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial w}{\partial y} \right)$$  \hspace{1cm} (10c)$$

Since, $\frac{h_0^2}{l_0^2}$ is a small quantity, Eq. (10b) reduces to:

$$\frac{\partial p}{\partial y} = 0 \rightarrow \bar{p} = f(\bar{x}, \bar{z}, \bar{t})$$  \hspace{1cm} (11)$$

Now that Eqs (10a) and (10c) have been reduced to only order unity, they can be changed back to dimensional form to get:

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right)$$  \hspace{1cm} (12)$$

$$\frac{\partial p}{\partial z} = \frac{\partial}{\partial y} \left( \mu \frac{\partial w}{\partial y} \right)$$  \hspace{1cm} (13)$$
Since $\frac{\partial p}{\partial y} = 0$, it can be concluded that pressure variation across the fluid film is negligible. Hence, pressure is only a function of $x$ and $z$, and independent of $y$. Hence, Eqs. (12) and (13) can be integrated with respect to $y$ to give the following expressions for the velocity gradients in the $y$-direction.

$$\frac{\partial u}{\partial y} = \frac{y}{\mu} \frac{\partial p}{\partial x} + \frac{A}{\mu}, \quad (14)$$

$$\frac{\partial w}{\partial y} = \frac{y}{\mu} \frac{\partial p}{\partial z} + \frac{C}{\mu}. \quad (15)$$

The velocity components can then be obtained by integrating Eqs. (14) and (15) with respect to $y$ to yield:

$$u = \frac{y^2}{2\mu} \frac{\partial p}{\partial x} + \frac{A}{\mu} y + B, \quad (16)$$

$$w = \frac{y^2}{2\mu} \frac{\partial p}{\partial z} + \frac{C}{\mu} y + D. \quad (17)$$

Assuming zero slip at the fluid-solid boundaries, the boundary conditions for velocity are:

$$y = 0, \ u = u_B, \ v = w_B, \quad (18)$$

$$y = h, \ u = u_A, \ v = w_A, \quad (19)$$

where subscripts $B$ and $A$ refer to the surfaces of the journal and the housing, respectively. Applying these conditions to Eqs. (16) and (17), we get the values for the integration constants which are substituted back into Eqs. (14) and (15), to yield:

$$\frac{\partial u}{\partial y} = \left(\frac{2y-h}{2\mu}\right) \frac{\partial p}{\partial x} - \frac{u_B - u_A}{h}, \quad (20)$$
\[
\frac{\partial \bar{w}}{\partial y} = \left( \frac{2y - h}{2\mu} \right) \frac{\partial \bar{p}}{\partial z} - \frac{w_B - w_A}{h},
\]  

(21)

and integrated again to get:

\[
u = -y \left( \frac{h - y}{2\mu} \right) \frac{\partial \bar{p}}{\partial x} + u_B \frac{h - y}{h} + u_A \frac{y}{h},
\]

(22)

\[
w = -y \left( \frac{h - y}{2\mu} \right) \frac{\partial \bar{p}}{\partial z} + w_B \frac{h - y}{h} + w_A \frac{y}{h}.
\]

(23)

Next, we will combine these velocity expressions with the continuity equation to get the Reynolds Equation.

The integral form of the continuity equation (Eq. (1a)) can be expressed as

\[
\int_{A_{y=0}}^{A_{y=h}} \left( \frac{\partial \bar{p}}{\partial t} + \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) \right) dy = 0.
\]

(24)

According to Leibnitz Rule of Integration,

\[
\int_{A_{y=0}}^{A_{y=h}} \frac{\partial}{\partial x} \left[ f(x,y,z) \right] dy = -f(x,y,z) \frac{\partial h}{\partial x} + \frac{\partial}{\partial x} \left( \int_{A_{y=0}}^{A_{y=h}} f(x,y,z) dy \right).
\]

(25)

For the integral form of the continuity equation, \( \rho \) is assumed to be the mean density of the fluid.

The \( u \) term in Eq. (24) integrates to

\[
\int_{A_{y=0}}^{A_{y=h}} \frac{\partial}{\partial x} (\rho u) dy = -(\rho u)_{y=h} \frac{\partial h}{\partial x} + \frac{\partial}{\partial x} \left( \int_{A_{y=0}}^{A_{y=h}} (\rho u) dy \right) = -\rho u_A \frac{\partial h}{\partial x} + \frac{\partial}{\partial x} (\rho y)
\]

(26)

The \( w \) term and the \( v \) term integrate to
The integrated continuity equation (Eq. (24)) then becomes

\[
\int_0^h \frac{\partial}{\partial z} (\rho w) \, dy = -\rho w A \frac{\partial h}{\partial z} + \frac{\partial}{\partial z} \left( \rho \int_0^h w \, dy \right),
\]  

(27(i))

\[
\int_0^h \frac{\partial}{\partial y} (\rho v) \, dy = \rho (v_A - v_B).
\]

(27(ii))

The integrated continuity equation (Eq. (24)) then becomes

\[
\frac{h}{c} \frac{\partial p}{\partial t} - \rho u_A \frac{\partial h}{\partial x} + \frac{\partial}{\partial x} \left( \rho \int_0^h u \, dy \right) - \rho v_A \frac{\partial h}{\partial z} + \frac{\partial}{\partial z} \left( \rho \int_0^h v \, dy \right) + \rho (w_A - w_B) = 0.
\]

(28)

The integrals in Eq. (28) represent the volume flow rates per unit width of the bearing. The volume flow rates per unit width in the x and z directions can be expressed as

\[
q'_x = \int_0^h u \, dy,
\]

(29)

\[
q'_z = \int_0^h w \, dy.
\]

(30)

Substituting for the velocities from Eqs. (22) and (23), into the volume flow rate equations, Eqs. (29) and (30), and then integrating, yields the volume flow rates in the x- and z-directions, respectively:

\[
q'_x = -\frac{h^3}{12\mu} \frac{\partial p}{\partial x} + \frac{u_A + u_B}{2} h,
\]

(31)

\[
q'_z = -\frac{h^3}{12\mu} \frac{\partial p}{\partial z} + \frac{w_A + w_B}{2} h.
\]

(32)

These flow-rate expressions are then introduced into the integrated continuity equation, Eq. (28). This then yields the general Reynolds equation as
\[
0 = \frac{\partial}{\partial x} \left( -\frac{\rho h^3}{12\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( -\frac{\rho h^3}{12\mu} \frac{\partial p}{\partial z} \right) + \frac{\partial}{\partial x} \left( \frac{\rho h (u_A + u_B)}{2} \right) + \frac{\partial}{\partial z} \left( \frac{\rho h (w_A + w_B)}{2} \right) \\
+ \rho (v_A - v_B) - \rho u_A \frac{\partial h}{\partial x} - \rho w_A \frac{\partial h}{\partial z} + h \frac{\partial p}{\partial t}
\]  
(33)

This is the final form of the Reynolds equation.

Next, we will solve this equation to get the analytical solution for the pressure and force components for long and short bearings.

2.1.2. Analytical Solution of Reynolds Equation for Long Bearing

Treating a bearing as infinitely long allows the assumption that the pressure does not experience any variation in the axial direction. This is generally accepted to be a valid assumption for bearings with a length to diameter ratio greater than 2.

In order to determine the forces acting on the journal, the pressure distribution in the fluid film is first calculated. This requires the solution of the Reynolds Equation, Eq. (33), which is re-written here:

\[
\frac{\partial}{\partial x} \left( \frac{\rho h^3}{12\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{\rho h^3}{12\mu} \frac{\partial p}{\partial z} \right) = \frac{\partial}{\partial x} \left( \frac{\rho h (u_A + u_B)}{2} \right) + \frac{\partial}{\partial z} \left( \frac{\rho h (w_A + w_B)}{2} \right) + \rho (v_A - v_B) \\
- \rho u_A \frac{\partial h}{\partial x} - \rho w_A \frac{\partial h}{\partial z} + h \frac{\partial p}{\partial t}
\]  
(33)

Before moving forward with obtaining the solution, the values of \(u_A, u_B, v_A, v_B, w_A, w_B\) are obtained based on the geometry of the setup which is shown in Fig. 2. Figure 2 shows velocities of the surfaces, and how the velocities of two arbitrary points A and B are related to them and to the whirling speed.
The fluid film thickness in the bearing is approximated by neglecting the higher order terms of $e/r_j$ [14], and is represented by

\[ h = c \left( 1 + \frac{e}{c} \cos \phi \right) = c(1 + \epsilon \cos \phi), \quad (34) \]

or, \[ h = c + e \cos \phi. \]

Also, from Fig. 2, we get the following relationships:

\[ \phi = \theta - \Phi, \quad (35) \]

\[ r_j \cos \alpha' = e \cos \Phi + |O'A| \cos \theta, \quad (36) \]

\[ r_j \sin \alpha' = e \sin \Phi + |O'A| \sin \theta. \quad (37) \]

Using Eq. (35) and re-arranging, the film thickness relationship represented by Eq. (34) can be rewritten as
h = c + e\cos(\theta - \Phi). \quad (38)

The surface velocities (represented by subscripts b, and j, respectively) of the housing and journal (arbitrary points on the surface represented by subscripts A, and B, respectively) are given by the following equations obtained from the geometry relationships shown in Fig. 2.

\[ u_A = \omega_b r_b, \quad (39) \]

\[ u_B = \omega_j r_j \cos(\theta - \alpha') + u_{O,B} \sin \theta - v_{O,B} \cos \theta, \quad (40) \]

\[ w_A = w_B = 0, \quad (41) \]

\[ v_A = 0, \quad (42) \]

\[ v_B = -\omega_j r_j \sin(\theta - \alpha') + u_{O,B} \cos \theta + v_{O,B}. \quad (43) \]

where \( u_{O',B} \) and \( v_{O',B} \) can be represented by velocity components of the point \( O' \left( \frac{de}{dt} \text{ and } e \frac{d\Phi}{dt} \right) \) by the following equations:

\[ u_{O',B} = \frac{de}{dt} \cos \Phi - e \frac{d\Phi}{dt} \sin \Phi, \quad (44) \]

\[ v_{O',B} = \frac{de}{dt} \sin \Phi + e \frac{d\Phi}{dt} \cos \Phi. \quad (45) \]

Multiplying both sides of Eq. (36) by \( \sin \theta \), and Eq. (37) by \( \cos \theta \), and then subtracting, we get,

\[ r_j \sin(\theta - \alpha') = e \sin(\theta - \Phi). \quad (46) \]

Substituting Eqs. (39), (44) and (45) into Eqs. (41) and (44), and using Eq. (46) to get the values of \( \sin(\theta - \alpha') \) and \( \cos(\theta - \alpha') \), yields the following equations for \( u_B \) and \( v_B \):
\[ u_n = \omega_j r_j \left[ 1 - \frac{e^2}{r_j^2} \sin^2(\theta - \Phi) \right]^{1/2} + \frac{de}{dt} \sin(\theta - \Phi) - e \frac{d\Phi}{dt} \cos(\theta - \Phi), \quad (47) \]

\[ v_n = -\omega_j e \sin(\theta - \Phi) + \frac{de}{dt} \cos(\theta + \Phi) + e \frac{d\Phi}{dt} \sin(\theta - \Phi). \quad (48) \]

Since, \( e \ll r_j \), the term of order \( \frac{e^2}{r_j^2} \) can be neglected due to its relatively small magnitude in Eq. (47), yielding:

\[ u_n = \omega_j r_j + \frac{de}{dt} \sin(\theta - \Phi) - e \frac{d\Phi}{dt} \cos(\theta - \Phi). \quad (49) \]

Since the thickness of the fluid film is relatively small compared to the radius of curvature of the fluid film, the curvature can be neglected, yielding the x coordinate of unwrapped film can be expressed as:

\[ x = r_j \varphi, \quad (50) \]

so that,

\[ dx = r_j d\varphi. \quad (51) \]

Now, substituting Eqs. (40), (42), (43), (48), (49), and (51) into the general Reynolds equation, Eq. (33), while assuming an incompressible fluid, yields:

\[ \frac{1}{r_j^2} \frac{\partial}{\partial \Phi} \left( h^3 \frac{\partial p}{\partial \Phi} \right) + \frac{\partial}{\partial z} \left( h^3 \frac{\partial p}{\partial z} \right) = \frac{1}{2r_j} \frac{\partial}{\partial \Phi} \left( h \left[ \omega_j r_j + \frac{de}{dt} \sin(\theta - \Phi) - e \frac{d\Phi}{dt} \cos(\theta - \Phi) + \omega_j r_j \right] \right) \]

\[ + \omega_j e \sin(\theta - \Phi) - \frac{de}{dt} \cos(\theta - \Phi) - e \frac{d\Phi}{dt} \sin(\theta - \Phi) - \omega_j e \sin(\theta - \Phi) \quad (52) \]
Since, \( \sin \phi \) and \( \cos \phi \) are proportional to \( h \), second and third term are of the order \( h^2 \) and can be neglected. Retaining only first-order terms on the right side of Eq. (52), yields:

\[
\frac{1}{r_j^2} \frac{\partial}{\partial \phi} \left( \frac{h^3}{12\mu} \frac{\partial p}{\partial \phi} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{12\mu} \frac{\partial p}{\partial z} \right) = \frac{1}{2} (\omega_j + \omega_b) \frac{\partial h}{\partial \phi} - \frac{de}{dt} \frac{\partial}{\partial \phi} \cos (\theta - \Phi) - e \frac{d\Phi}{dt} \sin (\theta - \Phi). \tag{53}
\]

Substituting \( \frac{d\Phi}{dt} = \omega \) we get:

\[
\frac{1}{r_j^2} \frac{\partial}{\partial \phi} \left( \frac{h^3}{12\mu} \frac{\partial p}{\partial \phi} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{12\mu} \frac{\partial p}{\partial z} \right) = \left[ \frac{1}{2} (\omega_j + \omega_b) - \omega \right] \frac{\partial h}{\partial \phi} - \frac{de}{dt} \cos (\theta - \Phi). \tag{54}
\]

2.1.2.1. Pressure Distribution along the Journal for Long Bearing

For a long journal bearing, due to the infinite length, \( \frac{\partial p}{\partial z} \) is negligible compared to \( \frac{\partial p}{\partial \phi} \). Taking this into account, along with the fluid film relationship (Eq. 34) and Eq. (35), Eq. (54) reduces to:

\[
\frac{\partial}{\partial \phi} \left( \frac{h^3}{\mu} \frac{\partial p}{\partial \phi} \right) = 12r_j^2 \left[ \left( \frac{1}{2} (\omega_j + \omega_b) - \omega \right) (-e \sin \phi) - \frac{de}{dt} \cos \phi \right]. \tag{55}
\]

Assuming that the eccentricity ratio remains constant in time \( \frac{de}{dt} = 0 \), using the value of \( h \) from Eq. (34), and integrating Eq. (55) with respect to \( \phi \) and solving for \( \frac{\partial p}{\partial \phi} \), yields:

\[
\frac{\partial p}{\partial \phi} = \frac{12\mu \left( \frac{r_j}{c} \right)^2}{(1 + e \cos \phi)^3} \left[ -e \cos \phi \left( \omega - \frac{\omega_j + \omega_b}{2} \right) - \bar{E} \right], \tag{56}
\]

where \( \bar{E} \) is the integration constant.

Integrating again with respect to \( \phi \), while assuming that viscosity does not vary in the circumferential direction, results in
\[
\frac{\partial p}{\partial \phi} = 12\mu \left( \frac{r_j}{c} \right)^2 \int \left[ -\epsilon \cos \phi \left( \frac{\omega - \frac{\omega_j + \omega_b}{2}}{1 + \epsilon \cos \phi} \right) - \bar{E} \right] d\phi .
\] (57)

To solve this integral, one has to use the Sommerfeld substitution \((\gamma = \tan (\phi/2))\) \([19]\) which then yields:

\[
p = 12\mu \left( \frac{r_j}{c} \right)^2 \left[ -\left( \frac{\omega - \frac{\omega_j + \omega_b}{2}}{1 - \epsilon^2} \right) \gamma - \epsilon \sin \gamma \right] - \frac{1}{2} \left( 2 + \epsilon^2 \right) \gamma - 2\epsilon \sin \gamma + \frac{1}{4} \epsilon^2 \sin 2\gamma \right] \left[ -\frac{1}{2} \left( 2 + \epsilon^2 \right) \gamma - 2\epsilon \sin \gamma \right] + \bar{F} \right]
\] (58)

where \(\bar{F}\) is the integration constant.

In order to solve for the integration constants, \(\bar{E}\) and \(\bar{F}\), the relationship that \(p(\phi) = p(\phi + 2\pi)\) is utilized. The other condition used is \(\phi = 0\), hence, \(\gamma = 0\) and \(\phi = 2\pi\), hence, \(\gamma = 2\pi\).

\[
p(\phi = 0, \gamma = 0) = 12\mu \left( \frac{r_j}{c} \right)^2 \bar{F} \] (59)

\[
p(\phi = 2\pi, \gamma = 2\pi) = 12\mu \left( \frac{r_j}{c} \right)^2 \left[ \frac{3\pi\epsilon^2 \left( \frac{\omega - \frac{\omega_j + \omega_b}{2}}{1 - \epsilon^2} \right)}{\pi \left( 2 + \epsilon^2 \right) \bar{E}} \right] \] (60)

Eqs. (59) and (60) are then set equal to each other, and the constants are evaluated to be:

\[
\bar{E} = \frac{3\epsilon^2}{2 + \epsilon^2} \left( \omega - \frac{\omega_j + \omega_b}{2} \right),
\] (61)
Substituting these values into Eq. (58) for \( p \), then yields:

\[
\begin{align*}
\bar{F} &= \frac{p_o}{12\mu \left( \frac{r_j}{c} \right)^2}. \\
\bar{p} - p_o &= 6\mu \left( \frac{r_j}{c} \right)^2 \left[ \frac{\varepsilon \sin \gamma \left( \varepsilon \cos \gamma - 2 + \varepsilon^2 \right)}{(2 + \varepsilon^2)(1 - \varepsilon^3)^2} \right] \left[ 2\omega - \left( \omega_j + \omega_b \right) \right].
\end{align*}
\] (63)

Then converting from the Sommerfeld variable back to \( \phi \) gives the equation for the pressure distribution for an infinitely long bearing as

\[
\begin{align*}
\bar{p} - p_o &= 6\mu \left( \frac{r_j}{c} \right)^2 \left[ \frac{\varepsilon \sin \phi \left( 2 + \varepsilon \cos \phi \right)}{(2 + \varepsilon^2)(1 + \varepsilon \cos \phi)^2} \right].
\end{align*}
\] (64)

2.1.2.2. Tangential and Radial Force Components on the Journal for Long Bearing

Integrating pressure, \( p \), along the circumference of the journal, we get the force components as:

\[
\begin{align*}
F_r &= \int_0^{2\pi} p_r \, d\phi \cos \phi \\
F_t &= \int_0^{2\pi} p_t \, d\phi \sin \phi
\end{align*}
\] (65a, 65b)

Using the Sommerfeld variable again to integrate Eqs. (65a) and (65b), we get:

\[
\begin{align*}
F_r &= 0, \\
F_t &= \frac{12\pi \mu (\omega_j + \omega_b - 2\omega)(r_j^3)\varepsilon}{c^2 (2 + \varepsilon^2)(1 - \varepsilon^3)^{3/2}}.
\end{align*}
\] (66, 67)
The radial force on the journal is zero. Equation (67) represents an expression for the tangential force on the journal due to the oil squeeze for a long bearing.

Now we will derive the pressure distribution and tangential force expressions for short bearings.

### 2.1.3. Analytical Solution of Reynolds Equation for Short Bearing

The steps in the derivation of the short SFD from the general form of the Reynolds equation are the same as those for the infinitely long bearing, up to the development of Eq. (54).

#### 2.1.3.1. Pressure Distribution along the Journal for Short Bearing

For easy reference, Eq. (54) is re-stated here as:

\[
\frac{1}{r_j^2} \frac{\partial}{\partial \phi} \left( \frac{h^3}{12 \mu} \frac{\partial p}{\partial \phi} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{12 \mu} \frac{\partial p}{\partial z} \right) = \left[ \frac{1}{2} (\omega_j + \omega_b) - \omega \right] \frac{\partial h}{\partial \phi} + \frac{de}{dt} \cos(\theta - \Phi) \tag{54}
\]

For a long bearing, \( \frac{\partial p}{\partial z} \) was neglected. However, for a short bearing, it is assumed that axial flow is more significant than circumferential flow, i.e., \( \frac{\partial}{\partial z} \left( \frac{h^3}{12 \mu} \frac{\partial p}{\partial z} \right) \gg \frac{1}{r_j} \frac{\partial}{\partial \phi} \left( \frac{h^3}{12 \mu} \frac{\partial p}{\partial \phi} \right) \).

This reduces Eq. (54) to

\[
\frac{\partial}{\partial z} \left( \frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = \left[ \frac{1}{2} (\omega_j + \omega_b) - \omega \right] \frac{\partial h}{\partial \phi} + \frac{de}{dt} \cos \phi. \tag{68}
\]

Again for the present study, the eccentricity ratio is assumed, so \( \frac{de}{dt} = 0 \). This, along with the fluid film relationship, Eq. (34), reduces Eq. (68) to

\[
\frac{\partial}{\partial z} \left( \frac{h^3}{12 \mu} \frac{\partial p}{\partial z} \right) = \left[ \frac{1}{2} (\omega_j + \omega_b) - \omega \right] (-e \sin \phi). \tag{69}
\]
Integrating once with respect to \( z \), and then solving for \( \frac{\partial p}{\partial z} \) yields the expression for the pressure gradient as:

\[
\frac{\partial p}{\partial z} = \frac{6\mu}{h^3} \left[ 2\omega - (\omega_j + \omega_b) \right] (e\sin \phi)z + \frac{12\mu}{h^3} \bar{G} ,
\]

where \( \bar{G} \) is an integration constant.

Integrating once again with respect to \( z \) then yields the pressure distribution as:

\[
p = \frac{3\mu}{h^3} \left[ 2\omega - (\omega_j + \omega_b) \right] (e\sin \phi)z^2 + \frac{12\mu}{h^3} \bar{G}z + \bar{H} .
\]

where \( \bar{H} \) is an integration constant.

The integration constants \( \bar{G} \) and \( \bar{H} \) can then be determined by using the boundary conditions, \( p(0) = p(L) = 0 \). Substituting \( z = 0 \) into Eq. (71) leads to:

\[
\bar{H} = 0
\]

Substituting Eq. (72) into Eq. (71) and setting \( z = L \) yields

\[
0 = \frac{3\mu}{h^3} \left[ 2\omega - (\omega_j + \omega_b) \right] (e\sin \phi)L^2 + \frac{12\mu}{h^3} \bar{G}L .
\]

Solving for \( \bar{G} \) yields

\[
\bar{G} = -\left[ 2\omega - (\omega_j + \omega_b) \right] \frac{eL\sin \phi}{h^3} .
\]

Substituting Eqs. (72) and (74) into Eq. (71) gives the pressure distribution for a short bearing as

\[
p = \frac{3\mu(e\sin \phi)}{c^2(1 + \varepsilon \cos \phi)^3} \left[ \omega_j + \omega_b - 2\omega \right] \left( z^2 - Lz \right) .
\]
2.1.3.2. Tangential and Radial Force Components on the Journal for Short Bearing

Integrating pressure, \( p \), along the circumference of the journal, we get the force components as:

\[
F_r = \int_0^{2\pi} p r_j d\phi \cos \phi, \tag{76a}
\]

\[
F_t = \int_0^{2\pi} p r_j d\phi \sin \phi. \tag{76b}
\]

Using the Sommerfeld variable again to integrate Eqs. (76a) and (76b), we get:

\[
F_r = 0, \tag{77}
\]

\[
F_t = \frac{\pi \mu r_j L^2 (\omega_j + \omega_b - 2\omega)}{2 e^2 ((1 - e^2)^{3/2})}. \tag{78}
\]

The radial force on the journal is zero. Equation (78) represents an expression for the tangential force on the journal due to the oil squeeze for a short bearing.

Now the pressure distribution and the tangential force expressions for long and short bearings can be used to verify our computational model.

2.2. Numerical Formulation and Set-up

The eccentricity of the journal relative to the housing leads to temporal deformation of the region occupied by the fluid in an SFD. This would require the use of a computational mesh that also deforms in time, i.e., adapts to the deforming fluid domain in order to always remain boundary aligned.
For an SFD journal in circular whirling motion, use of a coordinate system whirling with the journal, i.e., a moving reference frame (MRF) attached to the journal, leads to a fluid domain that retains its shape in time. This permits a steady-state formulation of this flow problem. The formulation is straightforward for a journal in a circular whirling motion. For non-circular journal whirl, the formulation presently requires the use of an adaptive computational mesh. The present work considers circular whirling motion, and hence, uses an MRF formulation, considerably reducing the computational resources required.

### 2.2.1. Fluid Flow Equations

Fluid flow problems are governed by the following basic equations:

#### 2.2.1.1. Flow Equations in Inertial Reference Frame

- **Mass Conservation:**
  \[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \]  
  \( (79) \)

- **Momentum Conservation:**
  \[ \frac{\partial}{\partial t} (\rho \mathbf{V}) + \nabla \cdot (\rho \mathbf{V} \mathbf{V}) = -\nabla p + \nabla \cdot \mathbf{\tau} + \mathbf{F}. \]  
  \( (80) \)

#### 2.2.1.2. Flow Equations in Moving Reference Frame

Equations (79) and (80) are modified to take into account the change in reference frame from inertial to moving, by introducing relative velocity components \[15].

- **Mass Conservation:**
  \[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}_r) = 0, \]  
  \( (81) \)

- **Momentum Conservation:**
  \[ \frac{\partial}{\partial t} (\rho \mathbf{v}_r) + \nabla \cdot (\rho \mathbf{v}_r \mathbf{v}_r) + \rho (2(\omega \times \mathbf{v}_r) + (\omega \times \omega) \times \mathbf{r} + a \times \mathbf{r} + a) = -\nabla p + \nabla \cdot \mathbf{\tau}_r + \mathbf{F}. \]  
  \( (82) \)

where,

\[ \mathbf{u}_r = \mathbf{v}_t + (\omega \times \mathbf{r}), \]  
\( (83) \)
\[ \vec{v}_r = \vec{V} - \vec{u}_r. \]  

(84)

The momentum equation contains four additional acceleration terms. The first two terms represent the Coriolis acceleration \( (2\vec{\omega} \times \vec{v}_r) \) and the centripetal acceleration \( (\vec{\omega} \times \vec{\omega} \times \vec{r}) \), respectively. These terms appear for both steadily moving reference frames (and are constant) and accelerating reference frames (and/or are functions of time). The third term, \( (\vec{a} \times \vec{r}) \), and the fourth term, \( (\vec{a}) \), are due to the unsteady change of the rotational speed and linear velocity, respectively. These terms vanish for constant translation and/or rotational speed, as happens for the current problem.

### 2.2.2. Bearing Dimensions

A long-bearing geometry is simply two concentric circles with the journal center offset from the origin \((0, 0)\) with the same distance as the value of the journal eccentricity. The journal radius, \(r_j\), is 3 inches, and the outer bearing radius, \(r_o\), is 3.004 inches. The distance between the centers \(OO'\) is 0.0016 inches, the radial clearance is 0.004 inches, and the eccentricity ratio \((\varepsilon)\) is 0.4.

A short bearing geometry is simply a long bearing geometry with a finite axial length. The axial length, \(L\), of 1 inch, was used for the short bearing simulated.

### 2.2.3. Lubricant Properties

Mobil Jet 2 oil was used as the fluid. This lubricant has a density \((\rho)\) of 949.42 kg/m\(^3\) and dynamic viscosity \((\mu)\) of 5.35e-03 Pa.sec at 80°C.

### 2.2.4. Computational Set-up

The computational solutions were obtained using a commercially available finite-volume solver (ANSYS-Fluent 14.5). The details of the underlying finite-volume algorithm were published by
Since the flow is incompressible, a pressure-based solver is used. For pressure–velocity coupling, SIMPLE (semi-implicit method for pressure-linked equations) algorithm developed by Patankar and Spalding [17] was used. The convergence criterion, for both the continuity and the momentum equations, was set at $10^{-12}$.

### 2.2.5. Boundary Conditions

Both the journal and the bearing surfaces were assigned a wall boundary condition. The journal is moving relative to the lubricant with $e\omega$ translational velocity, whereas the bearing is stationary in the absolute frame. For the current case, whirl speed ($\omega$) of 5000 r.p.m. (523.6 rad/sec) was used. In a short bearing, the two axial-end faces were given zero gauge pressure conditions.

![Figure 3. Long and Short SFD Bearings](image)

### 2.2.6. MRF Application in Hydrodynamic Bearings

Figure 4 shows that, for any given position of the journal inside the housing, the journal whirl velocity is always aligned with the y-axis of the MRF. Also, we choose to start our simulation when the journal is in position A, where the minimum film thickness is aligned with the x-axis. This implies that the coordinates of the journal center $O\,'$ are $(e, 0)$. 

![Figure 4](image)
2.2.7. Implementation of MRF in ANSYS-Fluent

Figure 5 shows the implementation of MRF in ANSYS-Fluent. Once the “frame motion” option is checked, the following inputs are given to the moving frame:

1. Rotation axis origin, typically the eccentricity (e),
2. Rotational velocity, and
3. Translational velocity.
Figures 6 and 7 show the boundary conditions on the journal and bearing walls, respectively. Hence, the net journal velocity is calculated as follows:

Through MRF:

Velocity of fluid with respect to journal = \( e\omega + r_j\omega \).

Boundary Condition on the journal:

Velocity of journal with respect to fluid = - \( r_j\alpha \).

Hence, the net journal velocity in the stationary frame = \(( e\omega + r_j\omega) + (-r_j\alpha)\).

= \( e\omega + r_j(\omega - \alpha) \).

Using this approach, one can simulate different types of bearings by changing the value of \( \alpha \).

For Journal Bearing, \( \alpha = 0 \)
For SFD, \( \alpha = \omega \).
For Non-Synchronous Bearing, \( \alpha \neq \omega \).

Figure 6. Boundary Conditions for SFD Journal
For the present study, using $\alpha = \omega$ would give the net journal velocity of $e\omega$. These boundary conditions will be used for simulating the long and short bearings of $Re = 0.96$. For $Re = 0.096$, a whirl velocity of 52.36 rad/s will be used. Now we will look at the specifics of the computational grid generated for this study.

### 3.2.8. Computational Grid

Grid generation is a significant part of any CFD simulation. The mesh has a substantial impact on the convergence rate, solution accuracy and the CPU time needed to get the solution. All the computational grids were created with structured meshes. For a long bearing, a grid with 360 grid points in the circumferential direction, and 30 grid points in the radial direction was created using POINTWISE. These numbers were selected so that each of the 360 points would cover 1 degree of journal rotation in the azimuthal direction. We also performed a grid-convergence study to see the effect of increasing the number of grid points in the radial direction. Increasing the grid points...
beyond 30 had less than 1% impact on the pressure distribution and the magnitude of the force components.

For a short bearing, a similar study was conducted and 50 grid points in the axial direction were found to be optimum.

Now that the computational grid was finalized, along with the boundary conditions mentioned in section 3.2.7, simulations were conducted for both long and short SFD bearings.

### 2.3. Results for Long and Short SFD Bearing

The computational results are obtained using a commercial software, that solves the full Navier-Stokes equations (with non-zero Reynolds number), taking into account both viscous and inertial terms. These results are compared with the Reynolds analytical solutions, given by Eqs. (64), (67), (75), and (78), which consider Re = 0 (inertial terms are neglected).

The Reynolds number for SFDs is defined as \( Re = \frac{\rho \omega c^2}{\mu} \). Hence, for the given geometry and operating conditions, the Reynolds number is 0.96. For Re = 0.96, the difference in the computational result and the analytical solution is expected to be small, as discussed in section 2.3.1. As the Reynolds number increases, the inertial terms contribute increasingly to the solution and the percent difference between the analytical and computational forces increases.

Hence, keeping the Reynolds number as close to zero as possible, we will look at the computational results for Re = 0.96 and compare them with the corresponding Reynolds solution.

#### 2.3.1. Results for Long SFD

First, computational results for Re = 0.96 are discussed. Then, to see the effect of inertial terms on the pressure distribution and force components, the Reynolds number is reduced to 0.096. Also, a
comparison is presented between the pressure distributions and force components for the following Reynolds numbers: 0.096, 0.96, 4.8, and 9.6.

For Re = 0.96, the computational and analytical values of pressure on the journal surface are compared in Fig. 8. From Eq. (64), the analytical pressure becomes zero at 180°. As seen in Fig. 8, for the analytical pressure curve, the pressure values are positive for \( \phi \leq 180^\circ \) and are negative thereafter. But for the computational results, the point of transition from positive to negative pressure variation occurs at 175.50°, this reflects the effect of the inertial terms included in the computational solutions.

![Figure 8. Circumferential Pressure Distribution on Long SFD Journal for Re = 0.96](image)

Table 2 shows a comparison of the analytical and computational values of the r- and \( \theta \)-components of the forces on the long SFD bearing. The percentage difference between the computational and analytical values of tangential force \( (F_t) \) is 0.09%. Therefore, the computational and analytical forces are judged to compare well. Analytically, the radial force \( (F_r) \) is zero, whereas
computationally, we get a value of $1.28 \times 10^5$ N/m that is not zero, but is, at least an order of magnitude smaller as compared to the magnitude of $F_t$.

Table 2. Comparison of analytical and computational forces for Re = 0.96 for Long SFD

<table>
<thead>
<tr>
<th></th>
<th>Analytical (N/m)</th>
<th>Computational (N/m)</th>
<th>% Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_r$</td>
<td>0</td>
<td>$1.28 \times 10^5$</td>
<td>n/a</td>
</tr>
<tr>
<td>$F_t$</td>
<td>$-1.829 \times 10^6$</td>
<td>$-1.828 \times 10^6$</td>
<td>0.09%</td>
</tr>
</tbody>
</table>

To confirm that inertia effects are indeed the cause of a non-zero computational $F_t$ and the pressure curve transition-point shift, the Reynolds number was further reduced to 0.096 by changing the whirl speed to 52.36 rad/sec. Figure 9 compares the analytical and computational pressures on the bearing surface for this reduced whirl speed of 52.36 rad/sec. Computationally, the point of transition from positive to negative pressure value occurs at 179.50°. Analytically, the point of transition should be at 180°. Compared to the computational results for Re = 0.96, the point of transition is much closer to 180° for the case of Re = 0.096, as expected. The lower value of Reynolds number reduces the effect of inertia terms in the computational results.
Figure 9. Circumferential pressure distribution on Long SFD Journal for Re = 0.096

Table 3 compares the simulation and analytical results for the forces. In this case of reduced whirl velocity, \( F_r \) is \( 1.28 \times 10^3 \) N/m computationally, which is 2 orders of magnitude smaller than that obtained for Re = 0.96, and 2 orders of magnitude smaller than \( F_t \) at this lower Re. Thus, as the Reynolds number is reduced, the computed solution for pressure and forces approaches the analytical solution.

Table 3. Comparison of analytical and computational forces for Re = 0.096 for Long SFD

<table>
<thead>
<tr>
<th></th>
<th>Analytical (N/m)</th>
<th>Computational (N/m)</th>
<th>% Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_r )</td>
<td>0</td>
<td>1.28e+03</td>
<td>n/a</td>
</tr>
<tr>
<td>( F_t )</td>
<td>-1.829e+05</td>
<td>-1.828e+05</td>
<td>0.09%</td>
</tr>
</tbody>
</table>

As we saw an improvement in the results for Re = 0.096 case, in terms of reduced order of \( F_r \) and a better match for the pressure distribution, a study was conducted to see the trend of the pressure curve and the force components for different values of Reynolds number. The purpose of this study
was to confirm that, as Reynolds number (Re) is reduced, the computational results for non-zero Re approach the analytical results for Re = 0, and the inverse is observed if Re is increased.

Figure 10 shows the computational results for pressure distribution for Reynolds numbers of 0.096, 0.96, 4.8, and 9.6. Since the increase in Re increases the effect of inertia in the computations, we see an increasing departure from symmetry as the Reynolds number increases beyond unity.

The scale on the left vertical axis corresponds to the results for Reynolds number of 4.8 and 9.6. As Reynolds number is increased to 4.8 and 9.6, the point of transition from negative to positive pressure shifts increasingly further from 180°, to 158.50° & 140.50°, respectively.

The scale on the right ordinate is for pressure values for Reynolds numbers of 0.096 and 0.96. For a Reynolds number of 0.096, the point of transition from negative to positive values of pressure is at 179.50°, i.e., very close to the 180° expected for Re = 0, and this transition point moves away from 180° to 175.50° for Re = 0.96.

Figure 10. Comparison of Pressure Distribution for Long SFD for Various Reynolds Numbers
Table 4 compares the ratio of minimum and maximum pressure values for increasing Reynolds numbers. We observe that, as Reynolds number is increased, the ratio deviates away from unity, hence making the pressure curve unsymmetrical.

Table 4. Comparison of Ratio of Minimum to Maximum Pressure for Various Reynolds Numbers

<table>
<thead>
<tr>
<th>Reynolds Number</th>
<th>0.096</th>
<th>0.96</th>
<th>4.8</th>
<th>9.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{\text{min}}/p_{\text{max}}$</td>
<td>1.004</td>
<td>1.12</td>
<td>1.84</td>
<td>3.20</td>
</tr>
</tbody>
</table>

Table 5 details the percentage difference between the computational and analytical tangential forces and the ratio of magnitudes of the radial and tangential forces. As observed, not only does the percentage difference in $F_t$ increase as Reynolds number increases, but the ratio of radial to tangential forces also increases, confirming that the inertia effect cannot be ignored for higher Reynolds numbers.

Table 5. Comparison of percent difference in $F_t$ and ratio of $F_r/F_t$ for various Reynolds Numbers

| Reynolds Number | Computational $F_r$ (N/m) | Computational $F_t$ (N/m) | Analytical $F_t$ (N/m) | % Diff. $F_t$ | $|F_r/F_t|$ |
|-----------------|---------------------------|--------------------------|----------------------|--------------|------------|
| 0.096           | 1.28e+03                  | -1.828e+05               | -1.829e+05           | 0.09%        | 7e-03      |
| 0.96            | 1.28e+05                  | -1.828e+06               | -1.829e+06           | 0.09%        | 7e-01      |
| 4.8             | 3.19e+06                  | -9.157e+06               | -9.145e+06           | 0.13%        | 0.35       |
| 9.6             | 1.28e+07                  | -1.84e+07                | -1.829e+07           | 0.6%         | 0.7        |
Hence, we can conclude that the effect of inertia is indeed the reason for the difference observed in the computational pressure and forces versus the analytical Reynolds solution, which assumes zero inertia.

### 2.3.2. Results for Short SFD

Next, the computational results for a short SFD are compared with the analytical results obtained from the expressions given in Eqs. (75) and (78). A short bearing is simply a long bearing geometry with a finite axial length. As discussed later, due to the finite axial length, we will see a pressure variation in the axial direction, which was not present in the long bearing computational results.

Figure 11 shows the variation of computed and analytical pressures along the short bearing surface at the axial location of \( z = L/2 \). The computational pressure changes from negative to a positive value at 187.75°, as compared to the analytical point of transition at 180°.

![Figure 11. Circumferential Pressure Distribution on Short SFD Journal for Re = 0.96](image)

When comparing the pressure distributions between the short bearing (Fig. 11) and long bearing (Fig. 8) for the same Reynolds number, we see a considerable difference. The different shape of
the curve can be explained by comparing the analytical expressions (Eqs. (64) and (75)) from the Reynolds solutions. In the case of a long bearing, the curve shape is governed by the expression: \( \sin \phi (2 + \varepsilon \cos \phi) \), whereas for short bearing, the expression is: \( \sin \phi \). We can also see a considerable decrease in the order of pressure magnitudes in the case of the short bearing. This is attributed to different constants preceding the sine and cosine terms. For long bearing, the constant is given by:

\[
6\mu \frac{r_c}{c} \left[ \omega_j + \omega_b - 2\omega \right] \varepsilon \frac{(2 + \varepsilon^2)}{2}
\]

whereas, for the short bearing it is:

\[
\frac{3\mu}{h} e \left[ 2\omega - (\omega_j + \omega_b) \right] (z^2 - Lz),
\]

where the terms \( (z^2 - Lz) \) for \( z = L/2 \) will become \(-L^2/4\).

Table 5 compares the analytical and the computational values of the r- and \( \theta \)-components of forces on the short bearing. The percentage difference of 0.66 % between the computational and analytical values of \( F_t \) is considered small. Analytically, \( F_r \) is zero, however, computational we get a \( F_r \) of 4.06e-01 N.

Hence, we get a similar order for the radial force for short bearing as in the case of long bearing. This can be attributed to the higher Re of 0.96. Reducing Re to a smaller value will reduce the order of \( F_r \), as observed for long bearing.

Table 6. Comparison of Analytical and Computational Forces for Re = 0.96 for Short SFD

<table>
<thead>
<tr>
<th></th>
<th>Analytical (N)</th>
<th>Computational (N)</th>
<th>% Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_r )</td>
<td>0</td>
<td>4.06e+01</td>
<td>n/a</td>
</tr>
<tr>
<td>( F_t )</td>
<td>-5.534e+02</td>
<td>-5.57e+02</td>
<td>0.66%</td>
</tr>
</tbody>
</table>

For a short SFD bearing, the pressure also varies in the axial (z) direction. Figures 12 and 13 show the pressure variation on the bearing surface in the axial direction (z-axis) at two selected azimuthal
locations, $\varphi = 90^\circ$ and $\varphi = 270^\circ$. The analytical solution for the pressure in Eq. (75) shows a quadratic variation in $z$. The computed pressure curves also exhibit a quadratic (parabolic) variation with respect to the axial ($z$) direction. Both the computed and the analytical curves agree well, concluding our study of short bearing.

Figure 12. Axial variation of pressure along journal at $90^\circ$ azimuthal position

Figure 13. Axial variation of pressure along journal at $270^\circ$ azimuthal position

Thus we saw a good agreement between our computational results and the Reynolds solutions for both long and short bearings. Now we will discuss the advantages of using MRF formulation.
2.4. Comparison of Computational Time between MRF and Dynamic-Mesh Formulation

As we can see, the MRF approach was quite successful for simulating long and short SFD bearings. The reason for using MRF was to save computational resources and improve the efficiency of the overall simulation process. If a conventional dynamic-mesh approach had been used, during which the problem formulation is transient and the mesh is adapted after every time step of the journal rotation, the computational time would have been significant.

Table 7 compares the computational time for an MRF formulation and a dynamic-mesh approach for both long and short bearing simulations. While the long bearing (two-dimensional case) simulation using MRF takes only one-third the time compared to dynamic-mesh, this difference increases by a substantial amount for short bearing (three-dimensional case). This difference increases further for end-sealed configurations.

Table 7. Comparison of Computational Time between MRF and Dynamic-Mesh approaches

<table>
<thead>
<tr>
<th></th>
<th>MRF</th>
<th>Dynamic Mesh (1 Rotation of Journal)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Long Bearing</strong></td>
<td>~ 5 mins</td>
<td>~ 15 mins</td>
</tr>
<tr>
<td><strong>Short Bearing</strong></td>
<td>~ 20 mins</td>
<td>~ 10-12 hrs</td>
</tr>
</tbody>
</table>

Thus, the MRF approach developed in the present study was used to determine the computational results for long and short bearings and compare them with the corresponding analytical solutions. We found that the results compare well for both these configurations. As the goal is to simulate end-sealed dampers, this approach will now be used for end-sealed configurations, both centered- and offset-groove configurations, generally used in industry.
3. END-SEALED SFD CONFIGURATION

Having established the correct implementation of the MRF formulation, the approach was then used to simulate both end-sealed centered- and offset-groove configurations, which are two of the common configurations used in aircraft jet engines. Defaye et al. [1] have provided the bearing geometry and the experimental data.

Figure 14. End-Sealed Configuration with Centered Groove
3.1. Description of Experimental Test Rig

The experimental rig consists of a stationary journal mounted on a shaft, and a rotating housing, for ease of data measurement and application of load. The bearing dimensions are given in Section 4.2.1. Two hydraulic shakers mounted 90° apart, provide the whirling motion. Oil enters the bearing from 3 inlets on the journal and exits through the leakage area on the piston rings. Various measurement devices, namely, stress gauges, accelerometers, thermocouples, and gear-wheel flowmeter, are used to collect force, inertial force, inlet temperature, and mass flow rate, respectively.

First, the set-up is filled with oil, and the total force is measured. The total force is the sum of the pressure force due to the oil squeeze and the inertial force due to whirling. Then a dry run test is
performed, wherein the oil is drained, to measure the inertial force. Finally, the inertial force is subtracted from the total force to get the force due to the oil squeeze. For each damper, the pressure and forces are averaged over 1000 rotations. Details of the experimental test-rig have been provided by Defaye. [1]

3.2. Numerical Formulation and Set-up

Since the MRF formulation for the end-sealed configurations is the same as that used for long and short bearings, the computational set-up also remains essentially unchanged.

3.2.1. Bearing Description and Dimensions

The SFD journal diameter is 152 mm (~6 inches). The bearing active length (L) is defined as the distance between the end piston rings. The active length does not include the length with the piston rings. The active length was measured to be 35 mm (1.38 inches), and the radial clearance (c) for the bearing is 0.1 mm (0.004 inches). For the offset groove geometry, the feeding groove depth and width are 50 times the radial clearance. The feeding groove is located at 0.7L of the active length, axially.

The bearing also has 3 oil inlets, 2mm (~0.08 inches) in diameter, located 120° apart. The piston rings, located at the axial ends, have a gap which serves as oil outlet. However, Defaye et al. [1] did not provide dimensions for the piston rings or for the piston ring gaps; consequently, the outlet or the leakage area is not available. Hence, the procedure used in the present study to attain the appropriate leakage area/outlet is described in the next section.
3.2.2. Lubricant Properties

Mobil Jet 2 oil was the lubricant used in the experiments performed by Defaye et al. [1] and the same was used for the present simulations as well. This oil has a dynamic viscosity $\mu$ of $1.76 \times 10^{-2}$ Pa•s and $3.47 \times 10^{-3}$ Pa•s, at temperatures of 50°C and 120°C, respectively. The corresponding densities ($\rho$) are 979 kg/m$^3$ and 938 kg/m$^3$ [1]. Due to a small variation in the oil temperature during the bearing’s operation, the effect of temperature on the lubricant viscosity was neglected in the present study. Hence, the simulations were done assuming constant lubricant viscosity keeping calculations simple and quick.

3.2.3. Boundary Conditions

Both the journal and the bearing surfaces were assigned a wall boundary condition. The journal wall moves relative to the lubricant with $e\omega$ translational velocity, and the bearing is stationary in the absolute frame.

Supply pressure of 10 bar was assigned to the three inlets and the piston ring gap was given atmospheric pressure ($p_{gauge} = 0$).

3.2.4. Computational Grid

A structured grid was created, with 360 grid points in the circumferential direction, 30 grid points in the radial direction, and 60 points in the axial direction. Additional grid points are used near the groove to take into account the change in the direction of flow, from radial at the inlet to tangential inside the bearing. A grid-convergence study was conducted and is presented in Chapter 5.
3.3. Results for End-Sealed Configuration

This section presents the results of the CFD analysis for centered-groove and offset-groove geometries, and their comparison with the experimental results of Defaye et al. [1].

As discussed in section 4.2.1, the dimensions of the piston rings are not known, hence, during the simulations, the oil leakage area is adjusted so as to achieve the mass flow rate, $\dot{m}$, used in the experiment. For example, in the centered-groove case with an eccentricity ratio of $\varepsilon = 0.3$, the experimental mass flow rate recorded was 14 lit/hr or 0.0038 kg/s. In the first simulation, an oil leakage area of $20c \times (c-e)$ was used, and which resulted in a mass flow rate of 0.00076 kg/sec (Table 8). The leakage area was then increased by a factor of 2, i.e., to $40c \times (c-e)$, and further to $100c \times (c-e)$, until the computational mass flow rate reached 0.0032 kg/s which was deemed satisfactorily close to the experimental value of 0.0038 kg/sec.
Table 8. Variation of Forces with Increasing Leakage Area

<table>
<thead>
<tr>
<th>Oil leakage area/ (c-e)</th>
<th>Mass flow rate (kg/s)</th>
<th>Computational F_r (N)</th>
<th>Computational F_t (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20c</td>
<td>0.00076</td>
<td>297.65</td>
<td>-669.64</td>
</tr>
<tr>
<td>40c</td>
<td>0.0015</td>
<td>318.30</td>
<td>-666.61</td>
</tr>
<tr>
<td>60c</td>
<td>0.0021</td>
<td>336.76</td>
<td>-664.90</td>
</tr>
<tr>
<td>100c</td>
<td>0.0032</td>
<td>370.46</td>
<td>-660.61</td>
</tr>
</tbody>
</table>

As observed in Table 8, as the mass flow rate increases, a considerable change is seen in F_r, but the corresponding F_t changes minimally. The considerable change in F_r can be attributed to the hydrostatic effect due to the localized leakage.

In Table 8, the outlet was considered stationary for each simulation since, during operation, the piston ring is pushed towards the housing by the oil pressure. To emulate the rotation of the oil ring, a series of simulations, wherein the location of the leakage area is changed, are performed, to determine whether the rotation of the piston ring (change in the location of oil leakage area) has an impact on the values of the force components. The leakage area considered was (c-e) x 60c.

Table 9. Average mass flow rate and Forces for various leakage locations

<table>
<thead>
<tr>
<th>Deg.</th>
<th>0</th>
<th>40</th>
<th>80</th>
<th>120</th>
<th>160</th>
<th>200</th>
<th>240</th>
<th>280</th>
<th>320</th>
<th>360</th>
<th>Avg.</th>
<th>Result for 100c</th>
</tr>
</thead>
<tbody>
<tr>
<td>ṁ  (kg/s)</td>
<td>0.0021</td>
<td>0.0023</td>
<td>0.0038</td>
<td>0.0046</td>
<td>0.0053</td>
<td>0.0051</td>
<td>0.0041</td>
<td>0.0030</td>
<td>0.0017</td>
<td>0.0021</td>
<td>0.0034</td>
<td>0.0032</td>
</tr>
<tr>
<td>F_r (N)</td>
<td>336.76</td>
<td>322.44</td>
<td>266.82</td>
<td>213.61</td>
<td>197.82</td>
<td>224.58</td>
<td>265.44</td>
<td>301.22</td>
<td>322.3</td>
<td>336.76</td>
<td>272.33</td>
<td>370.46</td>
</tr>
<tr>
<td>F_t (N)</td>
<td>-664.9</td>
<td>-615.02</td>
<td>-596.25</td>
<td>-626.09</td>
<td>-686.73</td>
<td>-780.17</td>
<td>-740.06</td>
<td>-725.67</td>
<td>-699.14</td>
<td>-664.9</td>
<td>-681.56</td>
<td>-660.61</td>
</tr>
</tbody>
</table>
The average value of $F_t$ and mass flow rate is very close to the corresponding values from the previous simulation of oil leakage area of 100c, with percent difference of 3.2% and 6.2% respectively. But $F_r$ changes by $\sim 36\%$. The lower value of $F_r$ can be attributed to the change in the location of the oil inlets relative to the journal. Since there is a negligible change in the tangential force, which is responsible for damping, we can ignore the oil ring rotation.

### 3.3.1. Results for Centered- and Offset-Groove Configurations

In this section, we will compare the results obtained from the centered-groove configuration with that obtained for the offset-groove configuration. The comparison will be made for pressure distributions along the circumference of the journal, and pressure and velocity contours at the axial location of the groove. Because the thickness of the domain is small, this thickness needs to be stretched, in order for the flow results to be visible within this thin region. Hence, the domain is first transformed into radial coordinate using TECPlot 360 by employing the following equations:

\[
R = \sqrt{X^2+Y^2} \tag{85}
\]

\[
\text{Mod}_R = R - 0.07 \tag{86}
\]

\[
T = \text{atan2}(Y,X) \tag{87}
\]

\[
\text{Mod}_X = \text{Mod}_R \cos(T) \tag{88}
\]

\[
\text{Mod}_Y = \text{Mod}_R \sin(T) \tag{89}
\]

Equation (85) gets the radial coordinate values for each mesh point in the domain. Then, this radial coordinate is reduced by 0.07 (approximately, the journal radius in meters), such that the domain in the radial direction is stretched. Equation (87) calculates the theta coordinate values for each
grid point. And finally, Eqs. (88) and (89), calculate back the x-y coordinate values, using the same theta values and the modified r values. Figure 18 shows the effect of coordinate transformation on the grid for the centered-groove configuration.

![Computational Grid](image)

Figure 18. Computational Grid a) Before and b) After Coordinate Transformation

### 3.3.1.1. Circumferential Pressure Distribution for Centered- and Offset-Groove Configurations

Figure 19 shows the pressure distribution along the journal for both centered- and offset-groove configurations. As observed, the offset-groove configuration has a lower peak pressure as compared to the centered-groove configuration. But the mean pressure value for the offset case is $9.96e+05$, compared to $9.89e+05$ for the centered-groove case. Hence, the higher damping force for offset-groove, as discussed in the next section.
The three local regions of disturbances in the curves, in Fig. 19, correspond to the three inlet locations. Recall that the supply pressure was $1\times10^6$ Pa (10 bar).

For oil inlets at $30^\circ$ and $270^\circ$, i.e., Inlet 1 and Inlet 3, $p_{\text{bearing}} < p_{\text{supply}}$, hence, oil flows from the inlet into the domain. For oil inlet at $150^\circ$, i.e., Inlet 2, $p_{\text{bearing}} > p_{\text{supply}}$, hence, oil flows from domain to the inlet. Therefore, the mass-flow rates for inlets at one and three, which are below the supply pressure, will be positive (flow into the domain), whereas, inlet 2 will have a negative mass-flow rate due to the pressure being higher than the supply pressure.

3.3.1.2. Axial Pressure Distribution for Centered- and Offset-Groove Configurations

Pressure variation in the axial direction is shown in Fig. 20 when the SFD is fed into a circumferential groove. The centered-groove configuration has lower pressure values compared to the offset-groove case. We can also observe the location along the axial length where there is no pressure variation. This position corresponds to the groove location.
3.3.1.3. Pressure Contours for Centered- and Offset-Groove Configurations

Figure 21 compares the pressure contours at the axial mid-plane of the groove for the two configurations. As discussed before, the domain has been transformed to see the variation clearly.

We observe the higher pressure at the inlet location corresponding to 150° in both the configurations, confirming the pressure distribution plot in Fig. 19.
3.3.1.4. Streamlines and Velocity Contours for Centered- and Offset-Groove Configurations

Figures 22 and 23 show the velocity contours and streamlines at the axial mid-plane of the groove, for the centered- and offset-groove configurations, respectively. Again, the domain has been radially expanded to show the flow details clearly.
Figure 22. Velocity Contours and Streamlines for Centered-Groove SFD at (a) Axial Mid-Plane of Groove (b) Azimuthal Locations 90 deg. and 270 deg.

The streamlines and velocity contours for the centered-groove case (Fig. 22a) show that the flow is bi-directional near the azimuthal locations of 90 and 270 deg. The 90 deg. location corresponds to the area of the squeeze, and hence, the flow is squeezed out of the plane axially. The 270 deg. location corresponds to the expanding portion of the film, and hence, the flow is drawn into the groove mid-plane from the neighboring axial locations. This flow squeezing and expanding
phenomenon is supported by the streamlines shown in Fig. 22b at the azimuthal locations of 90 deg. and 270 deg.

Figure 23. Velocity Contours and Streamlines for Offset-Groove SFD at Azimuthal Locations 90 deg. and 270 deg.

Figure 23 shows the streamlines and velocity contours for the offset-groove case at the azimuthal locations of 90 deg. and 270 deg. The flow pattern in this figure clearly shows enhanced recirculation within the offset groove as compared to the centered-groove case.
3.3.1.5. **Tangential Velocity versus Film Thickness at Various Azimuthal Locations for Centered-Groove Configuration**

Figure 24. Comparison of Tangential Velocity at Various Azimuthal Locations

Figure 24 shows the tangential velocity distribution along the film thickness at various azimuthal locations. As expected, the curves show a similar pattern, with the magnitude of the velocity decreasing at different azimuthal locations around the journal until the velocity magnitude at 180° is similar that at 0°. We also observe that the velocity magnitude satisfies the boundary conditions at the housing and the journal, i.e., zero at the housing for all locations, and \( \omega (0.011 \text{ m/s}) \) at the journal.

3.3.1.6. **Radial Velocity along the Housing Circumference for Centered-Groove Configuration**

Figure 25 shows the radial velocity along the housing circumference. As expected from Fig. 19 for the pressure distribution, we observe a negative value of radial velocity at the 30° and 270° inlet locations due to flow coming into the domain, whereas, the 150° inlet records a positive value of radial velocity due to flow going out of the domain.
In this section (3.3.1), we studied the details of the flow for both the centered- and the offset-groove configurations in detail. Now, we will validate our computational solution by comparing them with the experimental data.

**3.3.2. Comparison with Experimental Data**

Figures 25 and 26 compare the experimental results with the simulation results for both centered- and offset-groove configurations for various eccentricity ratios, $\varepsilon$. The experimental, as well as the simulation results, show that the damping force values for the offset-groove are around 1.7 times those for the centered-groove configuration. As observed in Fig. 19 and Fig. 20, the mean circumferential pressure and the axial pressure are higher for offset-groove configuration compared to centered groove, hence the increased damping for offset-groove configuration. The simulated tangential force values compare well with experimental values, with an average difference of 6.55% and 6.72% for centered- and offset-groove cases, respectively.
On the other hand, the radial force values are considerably lower in both cases when compared with experimental values. This difference in the radial forces might be due to the inertia of the journal affecting the radial force during the experiments. Another reason can be the non-circularity of the journal path, causing higher harmonics in otherwise a circular path of the journal, leading to higher harmonics in the pressure variation, which affect the radial force.

Figure 26. Comparison of Forces for Centered-Groove SFD
3.3.3. Parametric Study for Offset-Groove Configuration

Because of its higher damping, the offset-groove damper configuration is examined further to study the variation of the force components as a function of the following geometric and/or operating parameters:

- Groove location,
- Supply pressure,
- Supply temperature, and
- Radial clearance and whirl speed.

3.3.3.1. Influence of Groove Location

The location of the groove was varied to study the corresponding effect on the force components. As Table 10 shows, the greater the offset of the groove, the higher is the damping (tangential force). But the groove location has to be optimized, as an increase in the offset of the groove can
also hasten the onset of cavitation, as can be observed from Figs. 5 and 8 in the experimental work of Defaye [1]. As the eccentricity is increased, the critical value of eccentricity where cavitation starts is higher for the centered-groove configuration as compared to the offset-groove configuration. Therefore, for similar operating conditions, as the eccentricity is increased for both configurations, the offset-groove damper cavitates before the centered-groove damper does.

Table 10. Force comparison for various axial locations of groove

<table>
<thead>
<tr>
<th></th>
<th>$F_r$</th>
<th>$F_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Centered Groove</strong></td>
<td>370.46</td>
<td>-660.61</td>
</tr>
<tr>
<td><strong>70% Offset Groove</strong></td>
<td>266.88</td>
<td>-1080.49</td>
</tr>
<tr>
<td><strong>80% Offset Groove</strong></td>
<td>354.66</td>
<td>-1958.78</td>
</tr>
</tbody>
</table>

Hence, the offset of the groove has to be optimized for a set of particular operating conditions. This study is not included and has to be further planned.

3.3.3.2. **Influence of Supply Pressure**

Next, simulations were performed using two different values (2 bar and 10 bar) of supply pressure to study the effect of pressure on the force components. Figure 27 shows the influence of the supply pressure on the tangential and radial force components. Both components of forces show negligible change for our case of uncavitated bearings.
Hence, the supply pressure should be chosen such that, even if cavitation occurs in the bearing, the minimum pressure is above the cavitation pressure.

3.3.3.3. Influence of Supply Temperature

In Fig. 28, the effect of feeding temperature on the tangential force, $F_t$, is investigated, for various values of $\epsilon$, by changing the oil viscosity to correspond to the temperatures of 50°C and 120°C. Dynamic viscosity decreases as the feeding temperature is increased. Consequently, the magnitude of the viscous (tangential) component of the force decreases. The inertial (radial) component, $F_r$, is not affected.
Hence, the oil supply temperature should be kept in mind during operation as a high oil temperature will result in lower damping.

3.3.3.4. Influence of Radial Clearance and Whirl Speed

Examining Eqs. (67) and (78), we observe that the tangential force component, $F_t$, is directly proportional to whirl velocity, $\omega$, and inversely proportional to radial clearance, $c$. Therefore, as seen in Fig. 29, as the clearance is increased from 100µm to 200 µm, the magnitude of the tangential force decreases. Similarly, change in whirl velocity from 17Hz to 99Hz increases the tangential force.
Hence, a parametric study was conducted to see the effect of various operating and geometrical parameters on the force components. This study is not complete and further examination needs to be done. Now, we will look at the grid-convergence study conducted on a centered-groove damper.
4. Grid-Convergence Study

A grid-convergence study is very important for any numerical simulation, as it examines how the computational solution varies with grid refinement and establishes whether, or not, these solutions approach asymptotic values as the grid spacing approaches zero. The centered-groove configuration with an eccentricity ratio of $\varepsilon = 0.3$ is used for this study. Three different grid densities are chosen, as shown in Table 11.

Table 11. Grid Resolution used for Grid-Convergence Study

<table>
<thead>
<tr>
<th>Grid</th>
<th>Number of cells (N)</th>
<th>Grid Resolution (CircumferentialxRadialxAxial)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fine</td>
<td>2,986,904</td>
<td>486x41x81</td>
</tr>
<tr>
<td>Base</td>
<td>1,178,000</td>
<td>360x30x60</td>
</tr>
<tr>
<td>Coarse</td>
<td>500,760</td>
<td>267x23x45</td>
</tr>
</tbody>
</table>

The base grid was refined and coarsened by a factor of 1.35, and the flow was simulated. A minimum grid refinement factor of 1.3 was suggested by Roache [20]. The three computed quantities examined were mass flow rate, radial force, and tangential force, and the discretization error was calculated using the method described by Celik et al. [18].

The number of grid cells in the fine, base, and coarse grids are denoted as $N_1$, $N_2$, and $N_3$, respectively. Refinement factors, $r_{21}$ and $r_{32}$, represent the ratios of the grid spacings, $N_1/N_2$ and
N2/N3, respectively. Values of the quantities examined for the three grid resolutions are represented by $\phi_1$, $\phi_2$, and $\phi_3$. The apparent order of accuracy (Celik et al. [18]) is denoted by $p$, and is calculated as 1.33, 1.61 and 1.88, for $F_r$, $F_t$, and $\dot{m}$, respectively, indicating that the grid used is in the asymptotic range.

Table 12. Grid-Convergence Study

<table>
<thead>
<tr>
<th>N1, N2, N3</th>
<th>$F_r$ (N)</th>
<th>$F_t$ (N)</th>
<th>Mass flow (kg/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1, N2, N3</td>
<td>2986904, 1178000, 500760</td>
<td></td>
<td></td>
</tr>
<tr>
<td>r21</td>
<td>1.36</td>
<td>1.36</td>
<td>1.36</td>
</tr>
<tr>
<td>r32</td>
<td>1.33</td>
<td>1.33</td>
<td>1.33</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>373.1</td>
<td>-659.78</td>
<td>0.00318</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>370.46</td>
<td>-660.61</td>
<td>0.00316</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>368.88</td>
<td>-661.84</td>
<td>0.00315</td>
</tr>
<tr>
<td>$p$</td>
<td>1.33</td>
<td>1.61</td>
<td>1.88</td>
</tr>
<tr>
<td>$\phi_{23}^{ext}$</td>
<td>373.9</td>
<td>-658.5</td>
<td>0.0032</td>
</tr>
<tr>
<td>$e_{23}^{21}$</td>
<td>0.71%</td>
<td>0.13%</td>
<td>0.63%</td>
</tr>
<tr>
<td>$e_{23}^{23}$</td>
<td>0.43%</td>
<td>0.19%</td>
<td>0.32%</td>
</tr>
<tr>
<td>$e_{23}^{ext}$</td>
<td>0.92%</td>
<td>0.32%</td>
<td>0.44%</td>
</tr>
<tr>
<td>$G_{CI}^{23}_{base}$</td>
<td>1.16%</td>
<td>0.40%</td>
<td>0.56%</td>
</tr>
</tbody>
</table>

Two kinds of errors are calculated. An approximate relative error $e_{a}^{23}$, calculated using the value $\phi_2$ and $\phi_3$ obtained from the simulation results, and the extrapolated relative error $e_{ext}^{23}$, calculated using the extrapolated value, $\phi_{ext}^{23}$, of the variable.

Since the difference in the quantities examined, given by $e_{a}^{21}$, are small for the base and fine grids, we can conclude that our base grid has a good resolution and therefore, this grid is used for the present computations and the results reported in this study.
The GCI base values estimate the numerical uncertainties for the base grid solution, and are 1.16%, 0.40%, and 0.56%, for \( F_r \), \( F_t \), and \( \dot{m} \), respectively, indicating a small uncertainty in the solution.

Hence, through this present study, we were able to develop a model to numerically simulate end-sealed SFD’s. This was accompanied by in-depth analysis of the results and parametric study on end-sealed dampers. Finally, we presented the results of grid-convergence study and established the proper use of a CFD solver and the numerical model.
5. CONCLUSION

The fluid flow in Squeeze-Film Dampers (SFDs) is analyzed using Computational Fluid Dynamics (CFD) analysis, to determine the tangential force, $F_t$, responsible for the damping provided by the SFDs. To take advantage of the geometry of the problem and the circular whirling motion of the SFD journal, the CFD model was developed using a Moving-Reference Frame (MRF) formulation. This leads to a steady-state formulation, and hence, to a considerable saving in the computational time required to obtain the numerical solution. On the other hand, the original problem needs to deal with a temporally deforming flow domain, requiring a transient formulation, and a conventional dynamic-mesh approach with ten-fold larger computational time to obtain a solution.

The MRF formulation and the corresponding computational model were confirmed by comparing the results with the analytical Reynolds solutions for long and short SFD bearings. Next, the computational solution was verified by conducting a Grid-Convergence Index (GCI) study. Further, the computational model was validated for both centered- and offset-groove end-sealed dampers by comparing the computed results with the experimental data. Overall, the CFD model worked well, with under 7% difference between the experimental and computed results. Finally, a preliminary parametric study was conducted to investigate the effect of various geometric and operating parameters of the bearing.
5.1. Summary

A summary of the conclusions drawn is presented here:

- Use of MRF, versus a dynamic-mesh approach, reduces computational time requirement significantly. The computational time decreased by a factor of 3 for a long bearing, and by a factor of almost 30 for a short bearing.

- Consistent with experimental data, the computational results show that, when a circumferential oil groove is used, the SFD with an axially offset-groove provides better performance, i.e., larger tangential (damping) force, than the SFD with a groove at the center.

- Increase in the feeding pressure was observed to lead to a minimal effect on the radial and the tangential force components for the uncavitated bearing. For bearings susceptible to cavitation, the feeding pressure needs to be appropriately increased such that the pressure inside the bearing does not fall below the vaporization pressure of the lubricant or the partial pressure of the dissolved gases, to avoid cavitation and the associated loss in damping as well as to minimize material erosion.

- The tangential force, and hence the damping, decreases with an increase in the oil feeding temperature, but the temperature increase has a negligible effect on the radial force component. During operation, the oil temperature can increase, resulting in a reduction of viscosity, and hence, reduced damping. Therefore, oil is generally cooled before being supplied to the dampers.

- The magnitude of the tangential force decreases as clearance is increased while keeping $\omega$ constant, and increases as $\omega$ is increased for constant clearance.
To conclude, a methodology has been developed to simulate end-sealed SFDs, and this approach can be used to study various design and operating parameters to establish the best possible combination of the design parameters that will provide maximum damping.

5.2. Future Work

Some recommendations for further study are presented next:

1. End-sealed dampers without a circumferential groove for various eccentricity ratios should be studied, and their damping compared with that of centered- and offset-groove configurations. This is suggested because Defaye et al. [1] have shown that bearings without a groove provide greater damping than do bearings with a groove. They have also stated that non-grooved dampers tend to cavitate at lower eccentricity ratios compared to grooved dampers operating at the same speed. Hence, further study needs to be conducted in this regard to quantify these observations.

2. SFDs with non-circular orbits should be studied to see the corresponding effect on the force components. This is suggested because, during actual SFD operation, the journal does not follow a circular path, but exhibits a non-circular trajectory. Even though good agreement was observed for the tangential force between the present simulations and the experimental results, the radial force values were not in good agreement. This is partially attributed to the non-circular orbit of the journal in the experiments, and the corresponding presence of higher harmonics in the pressure distribution. Hence, one needs to quantify the effect of non-circularity of the journal path on the force components, and whether a circular path approximation is acceptable so that the MRF formulation can be used to advantage with respect to the computational time requirements.
3. The next phase of the study should consider cavitation, both vapor and gaseous, determine the range of parameters that lead to cavitation, and examine the corresponding effects on the damping coefficient.

4. Finally, piston rings should be included in the SFD geometry and their size optimized, so the geometrical configuration better represents actual SFDs used in engine applications.
REFERENCES


[7]. Delgado, A., “A Linear Fluid Inertia Model for Improved Prediction of Force Coefficients in Grooved Squeeze Film Dampers and Grooved Oil Seal Rings,” Ph.D. dissertation, Texas A&M University, College Station, TX, 2008.


