I, Paramita Das, hereby submit this original work as part of the requirements for the degree of Master of Science in Mechanical Engineering.

It is entitled:
Optimum Part Build Orientation in Additive Manufacturing for Minimizing Part Errors and Build Time

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Optimum Part Build Orientation in Additive Manufacturing for Minimizing Part Errors and Build Time

A Thesis submitted to the Graduate School of the University of Cincinnati in partial fulfilment of the requirements of the Degree of

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In the Department of Mechanical and Material Science Engineering (ME) of the College of Engineering and Applied Science (CEAS)

By

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Committee Chair: Dr. Sam Anand
ABSTRACT

Additive Manufacturing (AM) is a process where an initially conceptualized 3D CAD model is fabricated by adding successive layers of material on top of each other while eliminating the need for any process planning. Metal powder based AM processes are gaining popularity in several industries such as aerospace, health care, architecture, industrial design, automotive and consumer products due to the ease with which complex and intricate parts can be manufactured. However, achieving part quality and meeting the design tolerances is one of the most crucial challenges faced by AM among various others such as minimizing support structures, build time, build cost, energy expenditure and support structures removal.

The primary cause for not achieving the design tolerances can be assigned to the staircase effect, which is inevitable in AM processes. The leading factor that affects the staircase error and in turn the part quality is the part build orientation. Apart from part quality, build orientation also influences build time, which is another vital aspect since it directly affects the manufacturing cost of the part.

The objective of this thesis is to provide an approach to identify an optimal part build orientation which will satisfy all the Geometric Dimensioning and Tolerancing (GD&T) of the part while minimizing its build time. Siemens PLM NX API is used to extract the GD&T callouts and associated geometric information of the CAD model. This is used later to verify if the design tolerances are met. Next, geometric correlation between build orientation, slice thickness and GD&T errors are established. A non-linear constrained weighted optimization model is also developed to identify the best build orientation for meeting the design tolerances and minimizing part errors along with build time.
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1. INTRODUCTION

Additive Manufacturing (AM) or Layered Manufacturing (LM) is the process of manufacturing a product by depositing material layer by layer. The basic stages involved in any metal powder based AM process are as follows [1]:

1. Generate a 3D CAD model of the part to be fabricated.

2. Convert the CAD model to Stereolithography (STL) / AMF / 3MF [2] file. The STL file is obtained by approximating the CAD surface with planar triangular facets which is the standard input to AM machines. This STL representation of the CAD model is sliced into layers and given as an input to the AM machine for building the part.

3. Set up the AM Machine. The user feeds the build parameters such as slice thickness, material constraints, energy source and part build orientation into the machine.

4. Build the part. This is an automated process where material is deposited layer upon layer to manufacture the part.

5. Remove the part from the AM machine and post process it. Post processing includes finishing of the part, removal of support structures and any extra deposited material.

Figure 1 shows a schematic of the general process flow followed in Additive Manufacturing processes. Once the STL model is obtained, it is sliced by a slicing plane perpendicular to the build axis (usually the ‘z’ axis) to obtain the 2D contour of the part at each slicing height. The distance between two successive slicing planes forms the slice thickness. Starting from the base 2D contour, a layer of material with a pre-defined layer thickness is deposited on top of each other to obtain the manufactured part as shown in [3]. Depending on the technology used to manufacture the part, AM can be classified into different processes as shown in Table 1.
Additive Manufacturing is beginning to enter mainstream manufacturing due to its various benefits such as the ability to manufacture customized and complex shapes or parts, design flexibility, decreased material wastage, reduction in tooling and labor costs and decreased production time as the need for process planning is eliminated. However, there are challenges faced by it and this research addresses one of the key limitations, which is part quality.
Table 1: Additive Manufacturing Processes

<table>
<thead>
<tr>
<th>AM Process</th>
<th>Description of process</th>
<th>Material used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stereolithography (SLA)</td>
<td>A photopolymer liquid is cured by laser power to produce the part</td>
<td>Liquid photopolymers, Composites</td>
</tr>
<tr>
<td>Multi-jet modeling (MJM)</td>
<td>An ink jet printing process that uses piezo print-head technology to deposit either photo-curable plastic resin or casting wax materials layer by layer</td>
<td>Photopolymers, Wax</td>
</tr>
<tr>
<td>Fused Deposition Modeling (FDM)</td>
<td>Partially molten material is extruded and deposited to form the layers</td>
<td>Thermoplastics</td>
</tr>
<tr>
<td>Selective Laser Sintering (SLS)</td>
<td>Metallic, ceramic or polymeric powder is sintered using laser power</td>
<td>Plastic, Metal, Glass, Ceramic, Composites</td>
</tr>
<tr>
<td>Direct Metal Laser Sintering (DMLS)</td>
<td>Metal powder is fused into a solid part by melting it locally using focused laser beam</td>
<td>Metal powder – stainless steel, cobalt, chrome, nickel alloy</td>
</tr>
<tr>
<td>Laminated Object Manufacturing (LMD)</td>
<td>Layers of adhesive coated paper, plastic or metal laminates are successively glued together and a laser cuts into the required shape</td>
<td>Paper, Plastic, Metal laminates, Ceramics, Composites</td>
</tr>
</tbody>
</table>

1.1 MOTIVATION FOR RESEARCH

Achieving part or dimensional accuracy is currently one of the biggest constraints in producing high precision functional parts using Additive Manufacturing. Since the layers that are deposited are of finite thickness, their stacking leads to an error known as the staircase error as shown in Figure 2. The staircase effect diminishes the surface finish of the part produced. Depending on the part geometry, layer thickness and build orientation, the effect of staircase error varies as shown in Figure 3.

Two major controllable process parameters that affect part accuracy are slice thickness and part build orientation. Minimizing the slice thickness would result in replicating the original CAD
model better but it would also lead to increased build time and cost. According to industrial standards, approximately 0.03 to 0.04 mm is the lowest slice thickness that is usually used.

Assuming that the slice thickness is maintained constant for a part, the part build orientation would still affect the part quality and build time. Currently, determining the build orientation is solely dependent on the expertise of the operator. With the increase in the complexity of the part, with features along multiple axes or with tight design tolerances to be met, determining an optimal
build orientation by judgment becomes inefficient. Thus, there is a need to develop a method to determine the optimal build angle which will not only meet the GD&T criteria of the part and minimize build time simultaneously but also provide the user with a choice of prioritizing critical features in the part.

1.2 RESEARCH OBJECTIVE AND IMPACT

The primary objective of this research is to identify the best build orientation such that the design tolerances are met and build time is minimized simultaneously. As discussed earlier, part build orientation plays a significant role in defining the part quality of a manufactured product, its build time and cost and also affects the productivity of the manufacturing unit. Thus it is crucial to have an optimal part build orientation.

The dimensional accuracy issue in AM processes can be solved to a large extent by developing correlations between the process parameters and part errors. Paul and Anand [4, 5] addressed cylindricity and flatness errors. This research extends their work and establishes correlation between perpendicularity, parallelism, angularity, total runout, circular runout and conicity tolerance errors and part build orientation.

In any manufacturing process the Product Manufacturing Information (PMI) plays a very important role. One of the important piece of information contained in the PMI is concerning the GD&T of the part. Typically GD&T information is embedded in the CAD file of the part. Extracting the GD&T information from the CAD model for Additive Manufacturing process planning has not been addressed before. In this research, Siemens PLM NX API is used to extract the GD&T callouts and the associated geometric information of the CAD model, which in turn is utilized later to verify if the design tolerances for the Additive Manufactured part has been satisfied or not.
A weighted optimization model is developed to determine the build angle which will minimize the part errors in a part with multiple critical features along with build time. To determine the overall build time of the part, it is assumed that the cumulative hatching volume of the part is a direct representation of the build time.

Some related work has been performed by Canellidis et al. [6] where they found an optimal build angle using Genetic Algorithm while considering build time, surface roughness and post-processing time as optimization criteria. This research provides a detailed methodology to find an optimal build orientation with minimum build time while satisfying the form and orientation tolerance callouts. The results of this research have been presented using three test parts. Using this approach would prove beneficial to the manufacturer since the optimal build orientation and relative build time will be known prior to actual manufacturing of the part.

1.3 THESIS OUTLINE

This thesis has been categorized into five chapters. Chapter 1 commences with the introduction to AM technologies, its benefits and drawbacks, classification of the AM processes and the basic stages in an AM process. It also describes the motivation of this research, its desired objectives and the impact of it. Chapter 2 discusses the past research conducted in this area with emphasis on optimal part build orientation and its effect on the GD&T errors of a manufactured part and build time in the “Literature Review” section. The methodology and the approach used in this research to obtain the desired results are elaborated in Chapter 3. In Chapter 4 the methodology is applied on three sample parts and the results are discussed. Chapter 5 concludes with inferences and conclusions drawn from the results along with the scope for future research in this field.
2. LITERATURE REVIEW

In Additive Manufacturing, various process parameters such as slice thickness, build orientation, hatching pattern and raster angle affect the part quality in AM. Prior work published on optimal part build orientation, slice thickness, build time and GD&T errors such as form and orientation tolerance errors and their evaluation is presented in this chapter.

2.1 EFFECT OF SLICING ON PART ERRORS

Slice thickness is an important parameter which directly affects the staircase error in a manufactured part. The staircase error leads to cusp height error [7], volumetric error [8], and form errors [4]. The current AM machines depend on uniform slicing of the part but several adaptive slicing algorithms [9, 10] have also been developed over the years. In the present work, slice thickness is assumed to be uniform and constant for building a part and only the effect of build orientation on part quality and build time is considered.

2.2 EFFECT OF BUILD ORIENTATION ON PART ERRORS

The influence of build orientation on part GD&T errors has not been studied in detail and a limited number of papers have been published in this area. Kulkarni et al. [11] discussed the various critical parameters of AM and mentioned part build orientation as one of the significant factors as it affects many other aspects like part quality, build time, volume of support structures required, the mechanical properties of the built part and the distortion or curl produced in the part. They also listed the various approaches to solve the build orientation problem depending on the user’s criteria. Arni and Gupta [12] investigated the effect of build orientation on flatness error and analyzed the feasibility of manufacturing the part with flatness callout. They concluded that
the staircase error formed due to slice thickness and build orientation is the cause of the flatness error on the manufactured part and also established mathematical relation between flatness error, build orientation and slice thickness. Paul and Anand [4, 5] analyzed the effect of build orientation on cylindricity error and developed a correlation between cylindricity error, slice thickness and build orientation. They also introduced the graphical approach to find the optimal build orientation. Lynn-Charney and Rosen [13] used Response Surface Methodology (RSM) to study the effect of process parameters on flatness, parallelism, perpendicularity, concentricity, circularity and positional tolerance. They correlated part errors with slice thickness, wait time, sweep period and overcure by using design of experiments. Ramaswami et al. [14] presented a methodology for evaluating runout tolerance for flat, tapered and cylindrical features using discrete data. They demonstrated that using minimum circumscribing cylinder technique is the preferred way to evaluate datum axis. Prakasvudhisarn and Raman [15] studied the tolerances of conical features using linear least squares method and linear optimization. They also approached it using normalized linear squares and non-linear optimization methods. Wen et al. [16] in their paper used particle swarm optimization to evaluate conicity and cylindricity error. Ollison and Berisso [17] used ANOVA technique to perform an experimental study on cylindricity errors on parts built by 3D Printing. Hanumaiah and Ravi [18] estimated straightness, flatness and circularity errors by conducting an experimental study on eight types of parts built by SLA and DMLS processes. They used an adaptive sampling procedure to calculate the errors. Sood et al. [19] experimentally investigated the influence of process parameters including layer thickness, part orientation, raster angle, air gap and raster width on dimensional accuracy in FDM processes. They used gray Taguchi method to attain optimal level of process parameters to minimize shrinkage and maintain part accuracy. Das et al. [20] developed an algorithm to obtain an optimal build orientation while
minimizing part errors and support structures. Panhalkar et al. [10] used a k-d tree approach to define the adaptive slicing structure with a focus to reduce part geometric errors. Masood et al. [8] calculated the volumetric errors of the CAD part at different orientations by assuming that a complex part is constructed by combining basic primitive volumes. They recommended the best build orientation to be the one with the least volumetric error.

2.3 EFFECT OF BUILD ORIENTATION ON BUILD TIME

Build orientation also plays a significant role in deciding the build time of the part. Cheng et al. [21] developed a multi-objective algorithm to find the optimal part build orientation. After obtaining the value of the primary objective function, which was part accuracy, build time for the corresponding orientation was found and the stability was checked. The process was repeated until the orientation which satisfied the part accuracy with the minimum build time was found. Thompson and Crawford [22] verified the significance of build orientation on parameters like, build time, surface finish and part strength on SLS and Selective Area Laser Deposition (SALD) processes using a design of experiments test and analyzing it by ANOVA. Xu et al. [23] found the optimal build orientation with adaptive slicing while considering other factors such as build time, accuracy and part stability. They used Genetic Algorithm to find the minimum layer thickness allowed at a height with a predefined cusp height tolerance. Byun and Kwan [24] used Simple Additive Weighting (SAW) method of multi criteria decision to determine the optimal build orientation while keeping surface roughness, build time and part cost as the three criteria. Chen and Sullivan [25] presented their build-time prediction by combining the total laser scan time and total recoating time on a layer-by layer basis in a SLA process. Giannatsis et al. [26] carried out an extensive experimental investigation of the laser beam scanning mechanism of SLA to check the validity of the simple relations used in literature for build-time estimation. They found that a
hatching delay is involved in estimating the build time of a part which is related to the total number of hatching vectors, hatching length and velocity. Several others, namely, Alexander et al. [27] Campbell et al. [28] and Ruffo et al. [29] used build time estimation as the basis for their production cost models.
3. METHODOLOGY

This chapter explains in detail the methodology followed in this research to identify the optimal build orientation which will meet the tolerance callouts and also minimize build time. The first section explains the extraction of GD&T information from a CAD part designed in Siemens NX software which is later used in the optimization routine to verify if the design tolerances are met by the manufactured part. Establishing relationship between tolerance errors and part build orientation is discussed in the next section followed by the methodology used to calculate build time. Lastly, the optimization module used in this research is discussed.

An overview of the steps followed in this research is:

- Extract the GD&T callouts and the geometric information from the CAD model using Siemens PLM NX Application Programming Interface (API)
- Establish geometric relations between build orientation and tolerance errors
- Develop 1D mapping of the tolerance errors
- Create an algorithm for calculating the build time
- Extract GD&T callouts from the CAD model which serve as an input to the optimization model
- Execute optimization model to determine best build angle with minimum part errors and build time
3.1 GD&T EXTRACTION FROM THE CAD MODEL

Product Manufacturing Information (PMI) is the way of representing the non-geometric attributes in a 3D CAD model essential for manufacturing the part. It consists of GD&T, annotations, surface finish, material specifications and process notes [30]. In Additive Manufacturing, most of the PMI information is embedded into the CAD model. Thus extracting the GD&T information from the CAD model will help in verifying if the tolerance criteria have been met or not in the manufactured part.

The GD&T callouts of the given part are extracted along with the normal of the face or axis of the non-planar feature associated with each GD&T specification using Siemens PLM NX Application Programming Interface (API). Figure 4 displays the flow of the algorithm used in this process.

![Flowchart for GD&T Extraction from CAD part](image)

Figure 4: Flowchart for GD&T Extraction from CAD part

At any particular orientation, the tolerance errors in the manufactured part are computed and compared with the extracted GD&T callouts. If the errors are less than the callouts, the design criteria has been fulfilled.
The GD&T extraction is explained using an example part. A CAD model with perpendicularity, parallelism, angularity, cylindricity and flatness tolerance associated with it is considered as shown in Figure 5.

![Figure 5: Sample CAD part with GD&T callouts](image)

Information is extracted only from the faces or the features associated with the GD&T callouts. First, the type of surface is determined using the UF functions used in NX API as shown in Figure 7. The different numbers correspond to different surface types. Next, depending on the type of the surface, either the normal information (for a planar surface) or axis information (for any non-planar surface) is obtained using the “dir” command as shown in Figure 6.

Figure 8 displays the result obtained for the CAD model shown above. The “type” in Figure 8 represents the type of surface, e.g. 22 represents planar surface, 16 represents cylindrical feature. Depending on the “type”, the “dir” gives either the normal vector of the face or the axis vector of the cylinder with [0], [1] and [2] as the x, y and z components of it. The angle made by the normal of the face or the axis of the cylinder with the build axis (z-axis) is stored in the variable “degree”. The variable “characteristic” stores the name of the tolerance associated with the face or feature, e.g. characteristic1 is flatness and characteristic2 is cylindricity. The tolerance value associated with each of the tolerance is stored in “Item Value”, e.g. 0.42 for flatness. Similarly, the cylinder is
represented by type 16 with \([0, 0, -1]\) as its axis, making an angle of \(0^\circ\) with the \(z\)-axis and having a cylindricity tolerance of 0.057. Similar information is obtained for all the surfaces of the CAD part associated with any GD&T callout. This extracted information is later utilized in the optimization function to calculate the tolerance errors and obtain the optimal build angle.

Figure 7: UF Function for determining surface type

<table>
<thead>
<tr>
<th>type</th>
<th>Output</th>
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<tbody>
<tr>
<td>16</td>
<td>cylinder</td>
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<tr>
<td>17</td>
<td>cone</td>
</tr>
<tr>
<td>18</td>
<td>sphere</td>
</tr>
<tr>
<td>19</td>
<td>revolved (toroidal)</td>
</tr>
<tr>
<td>20</td>
<td>extruded</td>
</tr>
<tr>
<td>22</td>
<td>bounded plane</td>
</tr>
<tr>
<td>23</td>
<td>fillet (blend)</td>
</tr>
<tr>
<td>43</td>
<td>b-surface</td>
</tr>
<tr>
<td>65</td>
<td>offset surface</td>
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<tr>
<td>66</td>
<td>foreign surface</td>
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Figure 6: UF Function for determining surface normal/axis

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<th>dir []</th>
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<td></td>
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</tr>
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<td>0.00000000000022d3c8</td>
</tr>
<tr>
<td>[0]</td>
<td>0.000000000000000000</td>
</tr>
<tr>
<td>[1]</td>
<td>-1.000000000000000000</td>
</tr>
<tr>
<td>[2]</td>
<td>0.000000000000000000</td>
</tr>
<tr>
<td>characteristic4</td>
<td>FcFCharacteristicAngularity</td>
</tr>
<tr>
<td>tolerance4</td>
<td>(ItemValue=0.035000000000000000 ValueExpression=0x00000000000000000 ValuePrecision=3 )</td>
</tr>
<tr>
<td>lbcd4</td>
<td>(native_text=0x00000007Fbb3c7d40 &quot;A&quot; utf8_text=0x0000000000000000 mode=Locale)</td>
</tr>
</tbody>
</table>

Figure 8: Part information extracted using Siemens PLM NX API
3.2 CORRELATION BETWEEN TOLERANCE ERRORS AND BUILD ORIENTATION

Arni and Gupta [12] in their paper established the relation between flatness error ($\varepsilon_f$) for a nominal flat feature and build orientation as given in equation 1,

$$
\varepsilon_f = \Delta z \cos (\Theta_f) 
$$

where, $\Delta z$ is the slice thickness, $\Theta_f$ is the angle between the normal of the face with flatness tolerance and z axis. Relationship between cylindricity error ($\varepsilon_{cyl}$) and build orientation was addressed by Paul and Anand [4] in their paper as shown in equation 2,

$$
\varepsilon_{cyl} = \Delta z \sin (\Theta_{cyl})
$$

where $\Theta_{cyl}$ is the angle between the axis of the cylinder with cylindricity tolerance and build direction.

In this work, the relations established by Arni and Gupta [12] and Paul and Anand [4] are extended for the orientation tolerances (perpendicularity, parallelism and angularity), runout tolerances (total runout and circular runout), straightness and conicity. Each of the tolerances and their correlation with part build orientation is elaborated in the following section.
The *perpendicularity error* ($\varepsilon_{\text{per}}$) by definition is the minimum tolerance zone defined by two parallel planes perpendicular to a datum plane or axis, within which all the points sampled from the manufactured part must lie [31]. When a part is manufactured in a particular orientation in slices, additional material gets deposited on each layer, producing a staircase effect. This generates two parallel planes within which all the manufactured points of the part lie. An added constraint is that these parallel planes should also be perpendicular to the datum plane. As seen from Figure 10, the perpendicularity tolerance is given by equation 3,

$$
\varepsilon_{\text{per}} = \Delta z \cos (\Theta_{\text{per}}) \tag{3}
$$

where $\Theta_{\text{per}}$ is the angle between the normal of the face with perpendicularity tolerance and z axis.

![Figure 10: Perpendicularity Error](image)

Similarly, *parallelism error* ($\varepsilon_{\text{para}}$) as defined by ASME is the minimum tolerance zone defined by two offset planes parallel to a datum plane or axis, within which all the points sampled from the manufactured part must lie [31]. As shown in Figure 11, the parallelism error ($\varepsilon_{\text{para}}$) is,

$$
\varepsilon_{\text{para}} = \Delta z \cos (\Theta_{\text{para}}) \tag{4}
$$
where $\Theta_{\text{para}}$ is the angle between the normal of the face with parallelism tolerance and z axis.

$\Theta_{\text{para}}$ is the angle between the normal of the face with parallelism tolerance and z axis.

Parallel planes containing manufactured points and parallel to the datum plane

$\Delta z$ cos ($\Theta_{\text{ang}}$)……………………………………. (5)

where $\Theta_{\text{ang}}$ = angle between the normal of the face with angularity tolerance and z axis.

Angularity error ($\varepsilon_{\text{ang}}$) is defined by ASME as the minimum tolerance zone between two offset planes which completely enclose the points sampled from the manufactured feature and are at a specified angle (other than 90° or 0°) to the datum plane [31]. Thus, as shown in Figure 12, angularity error ($\varepsilon_{\text{ang}}$) is given by equation 5,

$\varepsilon_{\text{ang}} = \Delta z \cos (\Theta_{\text{ang}})$……………………………………. (5)

where $\Theta_{\text{ang}}$ = angle between the normal of the face with angularity tolerance and z axis.

Conicity error ($\varepsilon_{\text{con}}$) is defined as the radial distance between two cones having the same basic angle and containing all the manufactured points [16, 31]. This radial distance is illustrated in Figure 13. Conicity can thus be written as,

$\varepsilon_{\text{con}} = \Delta z \sin (\Theta_{\text{cone}} + \alpha)$ …………………………... (6)

where $\Theta_{\text{cone}}$ is the angle between the axis of the cone and z axis, $\alpha$ is the basic angle of the cone.
Runout Tolerances can be applied to only those surfaces which are constructed around a datum axis or at right angles to a datum axis. For a feature constructed around a datum axis, the total runout error ($\varepsilon_{\text{tot\_runout}}$) is the measure of the total deviation of the entire surface for that particular feature from the datum axis [14, 31]. Figure 14 shows the total runout error. It can be calculated by finding the distance between the farthest manufactured point from the datum axis to the nearest manufactured point from the datum axis and can be given by the following relation,
\[ \varepsilon_{\text{tot\_runout}} = r_1 + \Delta z \sin (\Theta_{\text{tot\_runout}}) - r_1 \]

\[ \Rightarrow \varepsilon_{\text{tot\_runout}} = \Delta z \sin (\Theta_{\text{tot\_runout}}) \quad \text{............... (7)} \]

where \( \Theta_{\text{tot\_runout}} = \Theta_4 \) is the angle between the datum axis and \( z \) axis, \( r_1 \) is the distance of the nearest manufactured point from the datum axis.

Circular Runout error \( (\varepsilon_{\text{cir\_runout}}) \) determines the deviation of a feature at a certain cross section, normal to the datum axis [14] as seen in Figure 15. The cross section is assumed to be of very small thickness.

\[ \varepsilon_{\text{cir\_runout}} = \Delta z \sin (\Theta_{\text{cir\_runout}}) \quad \text{............... (8)} \]

where \( \Theta_{\text{cir\_runout}} \) is the angle between the datum axis and \( z \) axis.
Once the mathematical relationship between part build orientation and the tolerance errors are established, the next section discusses the mapping of build orientation for the errors.
3.3 MAPPING OF BUILD ORIENTATIONS FOR THE TOLERANCE ERRORS

In this section, the approach used by Paul and Anand [4] is extended to the orientation tolerances, runout and conicity tolerances. As discussed before, since all the tolerance errors has either a sinusoidal or cosine relationship with the part build orientation, a graphical approach can be established as seen in Figure 16. This is based on the extension of flatness error by Arni and Gupta [12] and cylindricity error by Paul and Anand [4]. To explain it further, only one tolerance (parallelism) is considered in this graph for easier understanding. Assuming that the part has a tolerance callout specified by $\varepsilon_{\text{sp,para}}$, there can be two critical build angles, $\Theta_{\text{cr1}}$ and $\Theta_{\text{cr2}}$ within which if the part is built, the tolerance error will be lower than the tolerance callout and thus it will satisfy the tolerance criteria. A similar approach can be established for the other tolerance errors as well.

![Graphical representation of Parallelism Error vs. Orientation](image)

Figure 16: Graphical representation of tolerance error and build orientation (adapted from Paul and Anand [4])
The above graphical representation can be mathematically written as shown in equations 9 to 14 and is an extension of the work by Paul and Anand [4] for flatness and cylindricity. Assuming that a given AM part feature has a perpendicularity tolerance specification ($\varepsilon_{sp\_per}$), parallelism tolerance specification ($\varepsilon_{sp\_para}$), angularity tolerance specification of ($\varepsilon_{sp\_ang}$), total runout tolerance specification of ($\varepsilon_{sp\_tot}$), circular runout tolerance specification of ($\varepsilon_{sp\_cir}$) and a conicity tolerance specification of ($\varepsilon_{sp\_conicity}$), then for a fixed slice thickness, there exists two orientations where the tolerance errors will be exactly same as the specified callout, as depicted in equation 9 to 14.

$$
\theta_{cr1} = \cos^{-1}\left(\frac{\varepsilon_{sp\_per}}{\Delta z}\right), \quad \theta_{cr2} = 180 - \theta_{cr1} \quad \text{.................................. (9)}
$$

$$
\theta_{cr3} = \cos^{-1}\left(\frac{\varepsilon_{sp\_para}}{\Delta z}\right), \quad \theta_{cr4} = 180 - \theta_{cr3} \quad \text{.................................. (10)}
$$

$$
\theta_{cr5} = \cos^{-1}\left(\frac{\varepsilon_{sp\_ang}}{\Delta z}\right), \quad \theta_{cr6} = 180 - \theta_{cr5} \quad \text{.................................. (11)}
$$

$$
\theta_{cr7} = \sin^{-1}\left(\frac{\varepsilon_{sp\_tot}}{\Delta z}\right), \quad \theta_{cr8} = 180 - \theta_{cr7} \quad \text{.................................. (12)}
$$

$$
\theta_{cr9} = \sin^{-1}\left(\frac{\varepsilon_{sp\_cir}}{\Delta z}\right), \quad \theta_{cr10} = 180 - \theta_{cr9} \quad \text{.................................. (13)}
$$

$$
\gamma_{cr11} = \sin^{-1}\left(\frac{\varepsilon_{sp\_conicity}}{\Delta z}\right), \quad \gamma_{cr12} = 180 - \gamma_{cr11} \quad \text{.................................. (14)}
$$

where, $\gamma = \theta$ (angle between cone axis and z) + $\alpha$ (basic angle)

As seen from the graph in Figure 16 and the equations above, the tolerances of the manufactured part will be met provided the normal of the face or datum axis of the feature associated with the particular tolerance lies within the $\theta_{cr}$ range as shown in equations 15 to 20.

$$
\theta_{cr1} \leq \theta_{per} \leq \theta_{cr2} \quad \text{......................................................... (15)}
$$
\[ \theta_{cr3} \leq \theta_{para} \leq \theta_{cr4} \] .................................................... (16)

\[ \theta_{cr5} \leq \theta_{ang} \leq \theta_{cr6} \] .................................................... (17)

\[ 0 \leq \theta_{tot} \leq \theta_{cr7} \text{ and } \theta_{cr8} \leq \theta_{tot} \leq 180 \] ......................... (18)

\[ 0 \leq \theta_{cir} \leq \theta_{cr9} \text{ and } \theta_{cr10} \leq \theta_{cir} \leq 180 \] ......................... (19)

\[ 0 \leq \gamma_{con} \leq \gamma_{cr11} \text{ and } \gamma_{cr12} \leq \gamma_{con} \leq 180 \] ................. (20)

The critical range of \([\Theta_{cr1} - \Theta_{cr2}], [\Theta_{cr3} - \Theta_{cr4}], [\Theta_{cr5} - \Theta_{cr6}], [0 - \Theta_{cr7}], [\Theta_{cr8} - 180], [0 - \Theta_{cr9}], [\Theta_{cr10} - 180]\) and \([\gamma_{cr11} - \gamma_{cr12}]\) is called the feasibility range or feasibility zone.

Equations 15 to 20 could be solved using equations 9 to 14 respectively [4]. Substituting equation 9 in equation 15, the following can be written:

\[ \cos \theta_{cr1} \leq \cos \theta_{per} \leq \cos(180 - \theta_{cr1}) \] ......................... (21)

\[ \rightarrow \cos \theta_{cr1} \leq \cos \theta_{per} \leq -\cos \theta_{cr1} \] ......................... (22)

\[ \rightarrow \cos^2 \theta_{cr1} \leq \cos^2 \theta_{per} \leq \cos^2 \theta_{cr1} \] ............ (23)

\[ \rightarrow 0 \leq \cos^2 \theta_{per} \leq \cos^2 \theta_{cr1} \] ............ (24)

\[ \rightarrow 0 \leq (c_z^2)_{per} \leq (c_{cr}^2)_{per} \] ............ (25)

where, \(c = \cos (\Theta)\) and \((c_z^2)_{per}\) is the z-component of the normal of the face with perpendicularly tolerance. Thus, for tolerances to be satisfied, the square of the z-component of the normal of the planar feature should lie between 0 and the square of the cosine of the lower critical angle for that feature. Following similar steps, the inequalities for the other tolerances can be also be obtained as shown in equations 26 to 30.
$0 \leq (c_z^2)_{\text{para}} \leq (c_{cr}^2)_{\text{para}}$ .................................. (26)

$0 \leq (c_z^2)_{\text{ang}} \leq (c_{cr}^2)_{\text{ang}}$ .................................. (27)

$(c_{cr}^2)_{\text{tot-run}} \leq (c_z^2)_{\text{tot-run}} \leq 1$ ............................... (28)

$(c_{cr}^2)_{\text{cir-run}} \leq (c_z^2)_{\text{cir-run}} \leq 1$ ............................... (29)

$(c_{cr}^2)_{\text{conicity}} \leq (c_z^2)_{\text{conicity}} \leq 1$ ............................... (30)

Following a similar approach as Paul and Anand [4], the above inequalities can be mapped in a 1D line space. Figure 18 and Figure 17 represent the 1D mapping of perpendicularly tolerance and runout tolerances respectively. The mapping for parallelism and angularity tolerance is the same as perpendicularly since they follow similar cosine relationship.

As seen from the 1D mapping, if the $z$-component of the normal of the face with the tolerance $(c_z^2)$ falls within the infeasible region, a cost could be assigned to quantify the error. The
cost for cylindricity error and flatness error [4] associated with the m\textsuperscript{th} feature with cylindricity tolerance and n\textsuperscript{th} feature with flatness tolerance of a part is given by equation 21 and 22.

\[ p_{cyl} = 1 - (c_{2}^{2})_{cyl}^{m} \] ...................................................(21)

\[ p_{flat} = (c_{2}^{2})_{flat}^{n} \] .............................................................(22)

where, \( p_{cyl} \) and \( p_{flat} \) is the cost for cylindricity error and flatness error respectively. Extending this approach, the cost for perpendicularity and runout tolerances are shown in Figure 19 and Figure 20 respectively. The cost \( (p_{per}, p_{para}, p_{ang}, p_{tot\_runout} \text{ and } p_{cir\_runout}) \) associated with the perpendicularity, parallelism, angularity, total runout and circular runout error is given by equations 21 to 25.

\[ p_{per} = (c_{2}^{2})_{per} \] .............................................................(21)

\[ p_{para} = (c_{2}^{2})_{para} \] .............................................................(22)

\[ p_{ang} = (c_{2}^{2})_{ang} \] .............................................................(23)

\[ p_{tot\_runout} = 1 - (c_{2}^{2})_{tot\_runout}^{l} \] ...................................................(24)

\[ p_{cir\_runout} = 1 - (c_{2}^{2})_{cir\_runout}^{l} \] ...................................................(25)

![Figure 19: Cost for perpendicularity tolerance (adapted from Paul and Anand [4])](image-url)
The cost formulation obtained from the above relations are one of the inputs to the combined optimization model that will be presented in section 3.5. The next section elaborates on the methodology used to calculate the build time at an orientation which is also a parameter in the optimization model.
3.4 PART BUILD TIME CALCULATION

Build time is a crucial parameter which varies with the build orientation and affects the cost of manufacturing the part. It also affects productivity in large scale manufacturing. A relative measure of build time can be estimated by summing the volume of each layer at that orientation. A flowchart explaining the steps followed in determining the build time at any given orientation is shown in Figure 21.

Figure 21: Flowchart for calculating Build Time
A test part as displayed in Figure 22 is considered to explain the algorithm for build time calculation. For the sample part, an STL file is generated which approximates the CAD model by triangulating it. This STL file contains the facet numbers, vertices and the normal information of each facet. For a particular orientation, the STL model is then sliced by a slicing plane which intersects the appropriate planar triangular facets and generates intersection points as shown in Figure 23.

Figure 22: Test Part 1

Figure 23: STL Part and intersection points after slicing the part

The distance between two consecutive slicing planes is determined by the slice thickness. The joining of the intersection points at a slicing height results in the contour of the part at that height as shown in Figure 24. For simplification purposes, contours in a cylinder are shown in Figure 25 and Figure 26.
Once the contour is obtained, the area enclosed by it is calculated using the “Area of Polygon” method [32]. This is a general method used to determine the area enclosed by a set of points on the same plane. Consider \( P_1, P_2, P_3 \ldots P_m \), a set of points in the same plane as shown in
Figure 27. One of the points is selected as the pivot point, (m-1) pivot vectors are formed and the area of polygon is calculated as shown by equations 26 to 28.

\[ P_m = \text{Pivot point} \]

\[ \vec{a}_1 = p_1 - P_m, \quad \vec{a}_2 = p_2 - P_m, \quad \ldots \ldots \quad \vec{a}_{m-1} = p_{m-1} - P_m \quad \ldots \ldots \quad (26) \]

\[ SA_i = 0.5 \times n \times (\vec{a}_i \times \vec{a}_{i+1}) \quad \text{where, n = unit normal} \quad \ldots \ldots \quad (27) \]

\[ \text{Total Area} = |\sum_{i=1}^{m-2} SA_i| \quad \ldots \ldots \quad (28) \]

This process of slicing the part, finding the intersection points, joining the intersection points to obtain the contour and finally calculating the hatching area of each slice is continued until the entire height of the part is covered at that orientation. The hatching area of each layer is then multiplied with the slice thickness to obtain the volume of each deposited layer. The cumulative layer volume is a relative measure of the build time at that orientation. In the optimization model, all the above steps are followed and the build time is calculated at each randomized orientation.
3.5 COMBINED OPTIMIZATION MODEL FOR MINIMIZING TOLERANCE ERRORS AND BUILD TIME

A constrained non-linear weighted optimization model, including the form errors, orientation errors and runout errors is developed to calculate the optimal build angle $(\alpha_{\text{opt}}, \beta_{\text{opt}})$ which would meet the design tolerances while minimizing build time. The part is rotated by the build angles selected randomly by the optimization module and a consolidated error function that takes into account part tolerances and build time is calculated and minimized. At each of the orientations the model constraints ensure that the design tolerance are met. This process continues until at some orientation, a minimum is reached where, both, the constraints are met and the total error function is also minimized.

$$
\min E(\alpha, \beta) = \sum_{i=1}^{n_{\text{per}}} p_i \omega_i + \sum_{j=1}^{n_{\text{para}}} p_j \omega_j + \sum_{k=1}^{n_{\text{ang}}} p_k \omega_k + \sum_{l=1}^{n_{\text{cyl}}} p_l \omega_l + \sum_{m=1}^{n_{\text{flat}}} p_m \omega_m + \sum_{n=1}^{n_{\text{tot\_run}}} p_n \omega_n
$$

$$
+ \sum_{p=1}^{n_{\text{cir\_run}}} p_p \omega_p + \omega_{bt} BT_{\text{norm}}
$$

$$
0 \leq (c_{z2})_{\perp \text{per}} \leq (c_{cr1})_{\perp \text{per}} , \ i = 1, 2, 3 \ldots n_{\text{per}}
$$

$$
0 \leq (c_{z2})_{\perp \text{para}} \leq (c_{cr3})_{\perp \text{para}} , \ j = 1, 2, 3 \ldots n_{\text{para}}
$$

$$
0 \leq (c_{z2})_{\perp \text{ang}} \leq (c_{cr5})_{\perp \text{ang}} , \ k = 1, 2, 3 \ldots n_{\text{ang}}
$$

$$
(c_{cr5})_{\perp \text{cyl}} \leq (c_{z2})_{\perp \text{cyl}} \leq 1, 1 = 1, 2, 3 \ldots n_{\text{cyl}}
$$

$$
0 \leq (c_{z2})_{\perp \text{flat}} \leq (c_{cr5})_{\perp \text{flat}} , \ m = 1, 2, 3 \ldots n_{\text{flat}}
$$

$$
(c_{cr5})_{\perp \text{tot\_run}} \leq (c_{z2})_{\perp \text{tot\_run}} \leq 1 , \ n = 1, 2, 3 \ldots n_{\text{tot\_run}}
$$

$$
(c_{cr5})_{\perp \text{cir\_run}} \leq (c_{z2})_{\perp \text{cir\_run}} \leq 1 , \ p = 1, 2, 3 \ldots n_{\text{cir\_run}}
$$
0° ≤ α ≤ 360°, 0° ≤ β ≤ 360°

\[\sum_{i=1}^{n_{per}} \omega_i + \sum_{j=1}^{n_{para}} \omega_j + \sum_{k=1}^{n_{ang}} \omega_k + \sum_{l=1}^{n_{cyl}} \omega_l + \sum_{m=1}^{n_{flat}} \omega_m + \sum_{n=1}^{n_{tot\_run}} \omega_n + \sum_{p=1}^{n_{cir\_run}} \omega_p + \omega_{bt} = 1\]

\[BT_{norm}(\alpha, \beta) = \frac{BT(\alpha, \beta) - BT_{min}}{BT_{max} - BT_{min}}\]

where, \(\omega_i, \omega_j, \omega_k, \omega_m, \omega_n,\) and \(\omega_p\) are the weights assigned to perpendicularity, parallelism, angularity, cylindricity, flatness, total runout, circular runout tolerance respectively, \(\omega_{bt}\) is weight assigned for build time, \(BT_{norm}\) is the relative build time in terms of hatching volume at an orientation and \(p_i, p_j, p_k, p_m, p_n, p_p\) are the costs for the respective tolerances that were developed in section 3.3. The weights are user defined and they ensure that the most critical features have the least error.

The optimization is formulated as a multivariable minimization problem which has been solved using the fmincon routine available in MATLAB (2013) using the interior-point algorithm [33]. It should be noted that fmincon may provide a Pareto optimum solution rather than a global optimum depending on the user defined starting value. A Pareto optimal solution, also known as non-inferior solution is one in which an improvement in one objective requires a degradation of another [34]. To explain it further, consider a feasible region, \(\Omega\), in the parameter space. \(x\) being a real number belonging to the feasible region, \(\Omega\) that satisfies all the constraints. The corresponding feasible region for the objective function space, denoted by \(\Lambda\) can be mapped as shown in Figure 29. A Pareto optimal solution point can now be defined as \(x^*\) belonging to \(\Omega\) if there does not exist a \(\Delta x\) in the neighborhood of \(x\) such that \((x^* + \Delta x) \in \Omega\) and

\[F_i\ (x^* + \Delta x) \leq F_i (x^*), \ i = 1,2,..m\ and\]

\[F_j (x^* + \Delta x) \leq F_j (x^*) \text{ for atleast one } j.\]
C and D are the points on the curve between which the set of Pareto optimal solutions lie as shown in Figure 28. Points A and B are specific Pareto optimal points because an improvement in F1, requires a deterioration in F2, i.e. $F_{1B} < F_{1A}$, $F_{2B} > F_{2A}$.

If no optimal orientation in which all the GD&T callouts are satisfied is found, either some tolerances have to be relaxed or the weights need to be changed. The PMI information extracted from the NX part as discussed in section 3.1 is utilized in the optimization model to obtain $(\alpha_{opt}, \beta_{opt})$, the optimal build angle by which the part has to be rotated to satisfy all the GD&T callouts.
4. RESULTS

This section presents the results obtained for three sample test cases. The best build orientations are identified for each of the test parts taking into account that the required design tolerances are met and that the build time and tolerance errors are minimized.

4.1 TEST PART 1

Figure 30 represents the first test part that was considered. The STL approximation of this CAD model was used as the main input file in the optimization model. The non-linear, constrained, weighted optimization model discussed in section 3.5 is adopted to obtain the results. Weightages are assigned based on user’s priority. Uniform slice thickness of 0.06 mm (60 µm) was assumed for all the three test parts.

![Figure 30: Test part 1](image)

Table 2 displays the tolerance callouts and weightages associated with each of the tolerances for the test part 1. Four tolerances, namely parallelism, perpendicularity, angularity and cylindricity were considered in this test part. In the first iteration, a weightage of 10% was given
to build time ($w_{bt}$) while the tolerances were assigned between 20% and 25% each ($w$). Based on the optimization model the part should be rotated by $9^\circ$ ($\alpha$) about x-axis and $40^\circ$ ($\beta$) about y-axis to obtain the best build orientation for the given tolerance callouts and weightages assigned as shown in Table 3. The part is rotated by the angles obtained from the optimization module and the final build orientation is shown in Figure 31.

Table 2: Weightages assigned to Test Part 1 for first iteration

<table>
<thead>
<tr>
<th>Run #</th>
<th>Tolerance</th>
<th>Tol Callout (mm)</th>
<th>Weightage given to tols ($w$)</th>
<th>Weightage given to build time ($w_{bt}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Parallelism</td>
<td>0.058</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>Perpendicularity</td>
<td>0.039</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Angularity</td>
<td>0.025</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cylindricity</td>
<td>0.045</td>
<td>0.25</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Optimization result of iteration 1 for Test Part 1

<table>
<thead>
<tr>
<th>Run #</th>
<th>Tolerance</th>
<th>Tol Callout</th>
<th>Rotation about x axis ($\alpha^\circ$)</th>
<th>Rotation about y axis ($\beta^\circ$)</th>
<th>Rotation about z axis ($\gamma^\circ$)</th>
<th>Normalized Build time</th>
<th>Tol Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Parallelism</td>
<td>0.058</td>
<td>9.2°</td>
<td>40.5°</td>
<td>180°</td>
<td>3.13E-04</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>Perpendicularity</td>
<td>0.039</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.039</td>
</tr>
<tr>
<td></td>
<td>Angularity</td>
<td>0.025</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5.75E-04</td>
</tr>
<tr>
<td></td>
<td>Cylindricity</td>
<td>0.045</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0397</td>
</tr>
</tbody>
</table>

Figure 31: Optimal orientation of Test part 1
Table 5 displays the results from the second iteration of test part 1 where minimization of build time is prioritized and given maximum weightage of 55%. The weightage given to angularity tolerance was reduced from 25% in iteration 1 to 5% for this iteration as shown in Table 4. The part has to be rotated by 20° about x-axis and 36° about y-axis to obtain the optimal build orientation. As seen from the result, the relative build time and perpendicularity tolerance error has reduced by 37% and 8% respectively while the angularity tolerance error has increased significantly by 93% as compared to Table 3 although all the tolerance limits are satisfied. Figure 32 displays the optimal build orientation of test part 1 based on the second iteration.

Table 4: Weightages assigned to Test Part 1 for second iteration

<table>
<thead>
<tr>
<th>Run #</th>
<th>Tolerance</th>
<th>Tol Callout (mm)</th>
<th>Weightage given to tols (w)</th>
<th>Weightage given to build time (w_bt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Parallelism</td>
<td>0.058</td>
<td>0.15</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>Perpendicularity</td>
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<td>0.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Angularity</td>
<td>0.025</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cylindricity</td>
<td>0.045</td>
<td>0.15</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Optimization result of iteration 2 for Test Part 1

<table>
<thead>
<tr>
<th>Run #</th>
<th>Tolerance</th>
<th>Tol Callout</th>
<th>Rotation about x axis (α°)</th>
<th>Rotation about y axis (β°)</th>
<th>Rotation about z axis (γ°)</th>
<th>Normalized Build time</th>
<th>Tol Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Parallelism</td>
<td>0.058</td>
<td>20.8°</td>
<td>36.6°</td>
<td>88.3°</td>
<td>1.97E-04</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>Perpendicularity</td>
<td>0.039</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0358</td>
</tr>
<tr>
<td></td>
<td>Angularity</td>
<td>0.025</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0091</td>
</tr>
<tr>
<td></td>
<td>Cylindricity</td>
<td>0.045</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0397</td>
</tr>
</tbody>
</table>
4.2 TEST PART 2

The optimization model is tested on a second test part as shown in Figure 33. Four tolerances were considered in this test part, namely cylindricity (for both the hole and the cylinder), parallelism and angularity with their respective tolerance callouts.

Table 6 and Table 7 summarizes the weightages given to each of the tolerances and the results obtained respectively. 60% weightage is given to build time in this iteration and the rest is divided among the tolerances. As seen from Table 7 the optimal orientation will be achieved
by rotating the part by $\alpha = 39^\circ$ and $\beta = 28^\circ$ about x and y axis respectively. All the tolerance errors are less than the tolerance callouts and the relative build time is 0.013. Figure 34 represents the optimal build orientation for Test part 2.

Table 6: Weightages assigned to Test Part 2 for first iteration

<table>
<thead>
<tr>
<th>Run #</th>
<th>Tolerance</th>
<th>Tol Callout (mm)</th>
<th>Weightage given to tols (w)</th>
<th>Weightage given to build time (w_bt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cylindricity (Hole)</td>
<td>0.06</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cylindricity</td>
<td>0.05</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Parallelism</td>
<td>0.041</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Angularity</td>
<td>0.048</td>
<td>0.1</td>
<td></td>
</tr>
</tbody>
</table>

Table 7: Optimization result of iteration 1 for Test Part 2

<table>
<thead>
<tr>
<th>Run #</th>
<th>Tolerance</th>
<th>Tol Callout</th>
<th>Rotation about x axis ((\alpha^\circ))</th>
<th>Rotation about y axis ((\beta^\circ))</th>
<th>Rotation about z axis ((\gamma^\circ))</th>
<th>Normalized Build time</th>
<th>Tol Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cylindricity (Hole)</td>
<td>0.06</td>
<td>39.3°</td>
<td>27.8°</td>
<td>10°</td>
<td>0.013</td>
<td>0.0438</td>
</tr>
<tr>
<td></td>
<td>Cylindricity</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0497</td>
</tr>
<tr>
<td></td>
<td>Parallelism</td>
<td>0.041</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.041</td>
</tr>
<tr>
<td></td>
<td>Angularity</td>
<td>0.048</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0189</td>
</tr>
</tbody>
</table>

Figure 34: Optimal orientation for Test part 2
In the next iteration, equal weightages of 20% each were assigned to the tolerances and the build time as shown in Table 8. Based on the optimization results, the part should be rotated by 90° about x-axis and 10° about z-axis to obtain the optimal orientation for the given callouts and weightages assigned as shown in Table 9. The part GD&T errors have decreased but simultaneously the build time has increased by 43% as compared to Table 7 because the weightage given to build time was decreased from 60% to 20%. However, with equal distribution of weights to tolerances, the cylindricity tolerance error for the hole has increased by 27% while the cylindricity, parallelism and angularity tolerance errors have decreased by 98%, 98% and 97% respectively. Figure 35 displays the final part orientation as per the optimization results.

Table 8: Weightages assigned to Test Part 2 for second iteration

<table>
<thead>
<tr>
<th>Run #</th>
<th>Tolerance</th>
<th>Tol Callout (mm)</th>
<th>Weightage given to tols (w)</th>
<th>Weightage given to build time (w_bt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Cylindricity (Hole)</td>
<td>0.06</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>Cylindricity</td>
<td>0.05</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Parallelism</td>
<td>0.041</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Angularity</td>
<td>0.048</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

Table 9: Optimization result of iteration 2 for Test Part 2

<table>
<thead>
<tr>
<th>Run #</th>
<th>Tolerance</th>
<th>Tol Callout (mm)</th>
<th>Rotation about x axis (α°)</th>
<th>Rotation about y axis (β°)</th>
<th>Rotation about z axis (Ƴ°)</th>
<th>Normalized Build time</th>
<th>Tol Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Cylindricity (Hole)</td>
<td>0.06</td>
<td>90.6°</td>
<td>0°</td>
<td>10°</td>
<td>0.0229</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>Cylindricity</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6.71E-04</td>
</tr>
<tr>
<td></td>
<td>Parallelism</td>
<td>0.041</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6.71E-04</td>
</tr>
<tr>
<td></td>
<td>Angularity</td>
<td>0.048</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5.57E-04</td>
</tr>
</tbody>
</table>
4.3 TEST PART 3

A third test part as shown in Figure 36 was also analyzed to identify the optimal build orientation to meet the given tolerances and minimize build time.

In the first iteration, 30% weightage was given to angularity, while perpendicularity, parallelism and cylindricity were assigned 20% each as shown in Table 10. 10% weightage was given to build time. Table 11 demonstrates the result from the optimization routine, according to
which the part should be rotated by 74° about x-axis and 90° about y-axis to obtain the optimal build orientation. Figure 37 represents the part after rotating it by the above mentioned angles.

Table 10: Weightages assigned to Test Part 3 for first iteration

<table>
<thead>
<tr>
<th>Run #</th>
<th>Tolerance</th>
<th>Tol Callout (mm)</th>
<th>Weightage given to tols (w)</th>
<th>Weightage given to build time (w_{bt})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Perpendicularity</td>
<td>0.039</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>Parallelism</td>
<td>0.042</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Angularity</td>
<td>0.05</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cylindricity</td>
<td>0.06</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

Table 11: Optimization result of iteration 1 for Test Part 3

<table>
<thead>
<tr>
<th>Run #</th>
<th>Tolerance</th>
<th>Tol Callout</th>
<th>Rotation about x axis (α°)</th>
<th>Rotation about y axis (β°)</th>
<th>Rotation about z axis (Ƴ°)</th>
<th>Normalized Build time</th>
<th>Tol Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Perpendicularity</td>
<td>0.039</td>
<td>73.7°</td>
<td>89.8°</td>
<td>104.27°</td>
<td>0.0179</td>
<td>1.08E-04</td>
</tr>
<tr>
<td></td>
<td>Parallelism</td>
<td>0.042</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.14E-05</td>
</tr>
<tr>
<td></td>
<td>Angularity</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.09E-04</td>
</tr>
<tr>
<td></td>
<td>Cylindricity</td>
<td>0.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.06</td>
</tr>
</tbody>
</table>

Figure 37: Optimized build orientation for Test part 3

Table 13 displays the results obtained from the optimization function when the weightage given to build time was increased from 10% in Table 11 to 80% in Table 12. To obtain the optimal build orientation, the part should be rotated by 47° and 71° about x and y axis respectively. The relative build time is decreased by 20% with this optimal build angle. With reduced weightages of
5% each, the tolerance errors have increased when compared to the previous table but they are within the tolerance callouts due to the tolerance constraints. Figure 38 shows the part at optimal build orientation for the given weightages.

Table 12: Weightages assigned to Test Part 3 for second iteration

<table>
<thead>
<tr>
<th>Run #</th>
<th>Tolerance</th>
<th>Tol Callout (mm)</th>
<th>Weightage given to tols (w)</th>
<th>Weightage given to build time (w_bt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Perpendicularity</td>
<td>0.039</td>
<td>0.05</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>Parallelism</td>
<td>0.042</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Angularity</td>
<td>0.05</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cylindricity</td>
<td>0.06</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>

Table 13: Optimization result of iteration 2 for Test Part 3

<table>
<thead>
<tr>
<th>Run #</th>
<th>Tolerance</th>
<th>Tol Callout</th>
<th>Rotation about x axis (α°)</th>
<th>Rotation about y axis (β°)</th>
<th>Rotation about z axis (Ƴ°)</th>
<th>Normalized Build time</th>
<th>Tol Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Perpendicularity</td>
<td>0.039</td>
<td>47°</td>
<td>71.2°</td>
<td>92.2°</td>
<td>0.0143</td>
<td>0.0141</td>
</tr>
<tr>
<td></td>
<td>Parallelism</td>
<td>0.042</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0131</td>
</tr>
<tr>
<td></td>
<td>Angularity</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0188</td>
</tr>
<tr>
<td></td>
<td>Cylindricity</td>
<td>0.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.057</td>
</tr>
</tbody>
</table>

Figure 38: Optimal orientation for Test part 3
5. CONCLUSIONS AND FUTURE SCOPE

A methodology to identify the optimal part build orientation that focusses on meeting part GD&T errors while minimizing build time in AM is presented in this work. Not much research has been conducted in addressing the geometric correlation between the GD&T part error and build orientation. This research aims to determine the optimal build orientation while considering the staircase error and its effect on the final part geometry and build time. To accomplish the above mentioned goal, a weighted non-linear constrained optimization model was used. The effects of varying the weightage for individual tolerances on the part errors and the build time were analyzed using three test cases. The optimization model estimates and predicts the GD&T errors and the relative build time in the best build orientation. Thus it can act as a predictive tool for AM users prior to actual building the part.

This research focusses on correlating the input parameter of build orientation with geometric tolerances based on staircase error. The methodology used to calculate the relative build time considers only contiguous area of each layer. It does not include parts with internal cavities or pockets. Further, correlation between build orientation and other tolerances, such as position, concentricity, symmetry and profile tolerances are not considered and can be included in the optimization routine. In addition, support structure minimization can be considered along with minimizing build time and achieving design geometric tolerances.
6. REFERENCES


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