University of Cincinnati

Date: 3/9/2016

I, Andrew St. George, hereby submit this original work as part of the requirements for the degree of Doctor of Philosophy in Aerospace Engineering.

It is entitled:
Development and Testing of Pulsed and Rotating Detonation Combustors

Student’s name: Andrew St. George

This work and its defense approved by:

Committee chair: Ephraim Gutmark, Ph.D., D.Sc.

Committee member: Shaaban Abdallah, Ph.D.

Committee member: David Munday, Ph.D.

Committee member: Mark Turner, Sc.D.

19906
Development and Testing of Pulsed and Rotating Detonation Combustors

A Dissertation Submitted to the Graduate School of the University of Cincinnati in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in the Department of Aerospace Engineering of the College of Engineering and Applied Sciences

2016

by

Andrew C. St. George
B.S. Aerospace Engineering
University of Cincinnati
2010

Committee Chair: Dr. Ephraim Gutmark
Abstract

Detonation is a self-sustaining, supersonic, shock-driven, exothermic reaction. Detonation combustion can theoretically provide significant improvements in thermodynamic efficiency over constant pressure combustion when incorporated into existing cycles. To harness this potential performance benefit, countless studies have worked to develop detonation combustors and integrate these devices into existing systems. This dissertation consists of a series of investigations on two types of detonation combustors: the pulse detonation combustor (PDC) and the rotating detonation combustor (RDC).

In the first two investigations, an array of air-breathing PDCs is integrated with an axial power turbine. The system is initially operated with steady and pulsed cold air flow to determine the effect of pulsed flow on turbine performance. Various averaging approaches are employed to calculate turbine efficiency, but only flow-weighted (e.g., mass or work averaging) definitions have physical significance. Pulsed flow turbine efficiency is comparable to steady flow efficiency at high corrected flow rates and low rotor speeds. At these conditions, the pulse duty cycle expands and the variation of the rotor incidence angle is constrained to a favorable range. The system is operated with pulsed detonating flow to determine the effect of frequency, fill fraction, and rotor speed on turbine performance. For some conditions, output power exceeds the maximum attainable value from steady constant pressure combustion due to a significant increase in available power from the detonation products. However, the turbine component efficiency estimated from classical thermodynamic analysis is four times lower than the steady design point efficiency. Analysis of blade angles shows a significant penalty due to the detonation, fill, and purge processes simultaneously imposed on the rotor.

The latter six investigations focus on fundamental research of the RDC concept. A specially-tailored RDC data analysis approach is developed, which employs cross-correlations to detect the combustor operating state as it evolves during a test. This method enables expedient detection of the operating state from sensors placed outside the combustor, and can also identify and quantify instabilities. An
investigation is conducted on a tangentially-injecting initiator tube to characterize the RDC ignition process. Maximum energy deposition for this ignition method is an order of magnitude lower than the required energy for direct initiation, and detonation develops via a deflagration-to-detonation transition process. Stable rotating detonation is preceded by a transitory onset phase with a stochastic duration, which appears to be a function of the reactant injection pressure ratio.

Hydrogen-ethylene fuel blends are explored as an interim strategy to transition to stable detonation in ethylene-air mixtures. While moderate hydrogen addition enables stable operation, removal of the supplemental hydrogen triggers instability and failure. Chemical kinetic analysis indicates that elevated reactant pressure is far more significant than hydrogen addition, and suggests that the stabilizing effect of hydrogen is physical, rather than kinetic. The role of kinetic effects (e.g., cell width) is also assessed, using H2-O2-N2 mixtures. Detonation is observed when the normalized channel width exceeds the classical limit of $w_{cn}/\lambda > 0.5$, and the number of detonations increases predictably when the detonation perimeter exceeds a critical value.
Acknowledgments

The following work represents six of the most stressful, demanding, and formative years of my life. I have undoubtedly sprouted numerous gray hairs on account of the myriad of financial, technical, and bureaucratic challenges that have sprung up over the course of this research. This work would not be possible without the significant and commendable contributions of my colleagues. I am forever indebted to Robbie Driscoll and Will Stoddard, who first invited me to join the UC Detonation Team, and to Vijay Anand, whose competitive drive has reminded me that there’s always more to learn. Many technical discussions (occasionally bordering on intense verbal altercations) with my teammates have forced me to adapt to constructive criticism and have greatly improved the quality of this research. To Brad Romanchuk, Steve Randall, and Ethan Knight, I extend overwhelming praise for helping me to build and/or resurrect the troublesome (and at times maddeningly problematic) PDC turbine system.

I am grateful to Bhupa Malla for introducing me to cross-correlations—I had no idea that aeroacoustics data processing would have so much relevance to detonation research. I applaud the patience of Matt Pinchak for allowing Robbie and I to basically commandeer our shared office and turn it into a makeshift conference room on a regular basis. I am thankful to all my fellow researchers in the GDPL, who have made this an exciting and rewarding place to work. We’ve shared a number of beers on a number of porches over the years, commiserated about our mutual plight, and celebrated our hard-earned victories. The strong sense of community (and the occasional rock climbing expedition) has helped me retain my sanity over the years when things weren’t going my way.

I extend my gratitude to a number of laboratory and university staff members, past and present: Russ DiMicco, who has enthusiastically helped me with lab-related questions and difficulties, long after he left our laboratory; Curt Fox, whose inexhaustible humor falls somewhere between award-winning puns and bad dad jokes, for sharing his impressive repository of technical and historical knowledge and teaching me that “RTFM” is valuable piece of advice in itself; Mark Grooms, for contributing his
incredible mechanical talents to help maintain the laboratory, even as the ceiling literally began to collapse; Brenda Smith, Teresa Meyer, and Shelly Tipton for their considerable help keeping all my paperwork in order and always extending kind encouragement and support for my research; Leva Wilson, whose laudable accounting skills have kept my projects and my paycheck in the black over the years; Julie Muenchen, for personally saving me on more than one occasion, as I completely neglected to fill out important paperwork and register for important things (perhaps most notably, forgetting to register for graduate school).

I am grateful to my advisor, Dr. Ephraim Gutmark, for his mentoring and guidance over the years, as I slowly evolved from a clueless novice researcher into a senior member of the laboratory. I thank him for his unwavering support of my research over the years. I thank Dr. Dave Munday for being an instrumental mentor in my early years of graduate school, and for encouraging me to take responsibility and ownership over my research. Special thanks to Dr. John Hoke and ISSI/AFRL for their financial and technical support in the development of the PDC-turbine system, and to Chris Mathias and Logan Stevenson of GE for their support in the development of the RDC system. I would also like to thank Dr. Shaaban Abdallah and Dr. Mark Turner for being a part of my dissertation committee.

Last, I would like to thank my lovely girlfriend and the love of my life, Amy. Over many long days and many late nights of research, she has been a steadfast pillar of emotional support. She has been tremendously patient and understanding with me on my long journey to completing this dissertation.
# Table of Contents

Abstract ................................................................................................................................. iii
Acknowledgments .................................................................................................................. v
Table of Contents .................................................................................................................. vii
List of Symbols ...................................................................................................................... x
List of Figures .........................................................................................................................xiv
List of Tables ........................................................................................................................ xx

Chapter 1 – Introduction ........................................................................................................ 1
  1.1 The Detonation .............................................................................................................. 1
  1.2 Three-Dimensional Detonation Structure ........................................................................ 6
  1.3 Detonation Initiation ..................................................................................................... 9
  1.4 The Pulse Detonation Combustor .................................................................................. 15
  1.5 Unsteady Turbine Behavior .......................................................................................... 18
  1.6 Unsteady Turbine Efficiency Definitions ...................................................................... 22
  1.7 The Rotating Detonation Combustor .......................................................................... 26
  1.8 Objectives of the Present Research .............................................................................. 43

Chapter 2 – Experimental Facility ....................................................................................... 46
  2.1 Overview of the Detonation Test Facility ...................................................................... 46
  2.2 Pulse Detonation Combustor-Turbine System .............................................................. 53
  2.3 Rotating Detonation Combustor System ...................................................................... 64

Chapter 3 – Axial Turbine Performance under Steady Air and Pulsed Air Conditions ........... 73
  3.1 Experimental Approach .............................................................................................. 73
  3.2 Steady Air Flow Performance Mapping ...................................................................... 76
  3.3 Pulsed Air Flow Profiles ............................................................................................. 78
  3.4 Pulsed Air Performance Mapping .............................................................................. 82
  3.5 Conclusion .................................................................................................................... 88

Chapter 4 – Axial Turbine Performance under Pulsed Detonating Conditions ..................... 91
  4.1 Experimental Approach .............................................................................................. 92
  4.2 Verification of Detonating Flow .................................................................................... 95
  4.3 Analysis of Pressure Profiles ....................................................................................... 97
  4.4 Analysis of Output Power ........................................................................................... 103
4.5 Estimation of Component Efficiency ................................................................. 107
4.6 Rotor Incidence Angle Considerations ............................................................ 113
4.7 Conclusion ........................................................................................................... 116

Chapter 5 – Correlation-Based Detection of the Operating State in a Rotating Detonation Combustor 118
5.1 Experimental Approach ................................................................................... 119
5.2 Signal Processing Methodology ....................................................................... 121
5.3 Application of the Algorithm ......................................................................... 132
5.4 Conclusion ........................................................................................................... 140

Chapter 6 – Characterization of Initiator Dynamics in a Rotating Detonation Combustor ........ 141
6.1 Experimental Approach ................................................................................... 141
6.2 Blast Wave Trajectory ...................................................................................... 145
6.3 Energy Estimation ............................................................................................. 147
6.4 Effect of Initiator Mixture ................................................................................ 152
6.5 Effect of RDC Parameters ................................................................................ 154
6.6 Initiation in the Reacting RDC Environment .................................................... 157
6.7 Conclusion ........................................................................................................... 161

Chapter 7 – Starting Transients and Detonation Onset Behavior in a Rotating Detonation Combustor. 163
7.1 Experimental Approach ................................................................................... 163
7.2 Onset Phase Behavior for Selected Cases ......................................................... 165
7.3 Onset Analysis for Baseline Operating Map ..................................................... 177
7.4 Conclusion ........................................................................................................... 178

Chapter 8 – Fuel Blending as a Means to Achieve Initiation in a Rotating Detonation Combustor .... 180
8.1 Experimental Approach ................................................................................... 180
8.2 Baseline Ethylene-Air Operation ..................................................................... 183
8.3 Blended Ethylene-Hydrogen-Air Operation ..................................................... 187
8.4 Hydrogen-to-Ethylene Transition .................................................................... 193
8.5 Conclusion ........................................................................................................... 196

Chapter 9 – Chemical Kinetic Analysis of Detonability-Enhancement for Ethylene-Oxidizer Mixtures ... 198
9.1 Numerical Method ............................................................................................. 199
9.2 Baseline Detonation Sensitivity ....................................................................... 203
9.3 Effect of Pressure ............................................................................................. 205
9.4 Effect of Temperature ...................................................................................... 208
9.5 Effect of Oxygen ........................................................................................................ 210
9.6 Effect of Hydrogen Substitution .............................................................................. 212
9.7 Conclusion .................................................................................................................. 218

Chapter 10 – On the Existence and Multiplicity of Rotating Detonations ..................... 220
10.1 Experimental Approach ...................................................................................... 220
10.2 Determination of Detonation Regime ................................................................. 224
10.3 Analysis of Detonation Wave Speed .................................................................... 226
10.4 Analysis of Geometric Scaling Parameters ........................................................ 228
10.5 Conclusion ............................................................................................................. 230

Chapter 11 – Conclusions and Recommendations ...................................................... 232
11.1 PDC-Turbine Investigations .................................................................................. 232
11.2 RDC Investigations .............................................................................................. 233
11.3 Closing Remarks .................................................................................................. 237

Publications ...................................................................................................................... 240
References ....................................................................................................................... 243
List of Symbols

Acronyms/Initialisms

CJ  Chapman-Jouguet
CTAP capillary tube averaged pressure
DDT deflagration to detonation transition
FA  flame acceleration
FJ  Fickett-Jacobs
ID  inner diameter
ITP infinite tube pressure
LFI  low frequency instability
LHV lower heating value
LPD longitudinal pulsed detonation
LPM longitudinal pulsed mode
MA  mass-averaged
MFP mass flow parameter
OD  outer diameter
PCB piezoelectric pressure transducer
PDC pulse detonation combustor
PR  pressure ratio
RDC rotating detonation combustor
TA  time-averaged
WA  work-averaged
VN  von Neumann
ZND Zel’dovich-von Neumann-Döring

Variables

a  speed of sound
(a/f) air-fuel ratio (by mass)
A  area, cell width proportionality constant
c absolute velocity at the stator exit
$C_p$ specific heat
$C_{ch}$ numerical constant
d diameter
E energy
$E_a$ activation energy
f frequency
ff fill fraction
h detonation height
$H_0$ total enthalpy
i rotor incidence angle
$J_0$ integration constant
j geometric scaling exponent
$L_{ch}$ RDC channel length
m pressure exponent for cell size scaling
$m$ mass flow rate
M Mach number
n number of detonation waves
N rotor rotational speed
$N'$ corrected rotor speed
p detonation wave perimeter
P pressure
$pf$ purge fraction
q specific heat addition
$q\bar{q}$ dimensionless heat addition
$Q_{add}$ total heat addition
r radius
$r_0$ explosion length
R specific gas constant
\( R_{xx} \) normalized auto-correlation magnitude
\( R_{xy} \) normalized cross-correlation magnitude
\( R<#> \) RDC sensor row #
\( s \) specific entropy
\( s_x \) source window standard deviation
\( t \) time
\( t_b \) burn time
\( T \) temperature
\( u \) velocity
\( U \) rotor linear velocity
\( \mathcal{V} \) volume
\( w \) relative velocity at the rotor inlet
\( w_{ch} \) RDC channel width
\( W \) detonation wave speed
\( x \) RDC axial coordinate
\( X \) arbitrary vector
\( y_i \) species mass fraction
\( Y \) arbitrary vector
\( z \) RDC azimuthal (linear) coordinate

**Greek Symbols**
\( \alpha \) absolute velocity flow angle (PDC-Turbine); phase delay (RDC)
\( \beta \) rotor relative flow angle (PDC-Turbine); nitrogen dilution ratio (RDC)
\( \gamma \) ratio of specific heats
\( \Delta_i \) induction length
\( \Delta_r \) exothermic reaction length
\( \eta \) efficiency
\( \theta \) mid-plane probe rotational angle (PDC-Turbine); azimuthal (angular) coordinate (RDC)
\( \hat{\theta} \) reduced activation energy
\( \lambda \)  
Detonation cell width

\( \xi \)  
Initiator blast wave trajectory angular coordinate

\( \rho \)  
Density

\( \dot{\sigma} \)  
Thermicity rate

\( \tau \)  
Shaft torque (PDC-Turbine); auto-correlation time shift (RDC)

\( \tau' \)  
Cross-correlation time shift (RDC)

\( \phi \)  
Equivalence ratio

\( \chi \)  
Stability parameter

\( \chi_{<\text{species}>} \)  
Molar fraction of \(<\text{species}>\)

\( \psi \)  
Detonation pre-compression temperature ratio

\( \Omega_i \)  
Species production rates

**Subscripts**

0  total

c  channel

crit  critical

des  design condition

e  exit

f  fuel

i  initial/inlet

\( o \)  outer

\( \text{ox} \)  oxidizer

s  shock

th  thermal

turb  turbine
List of Figures

Figure 1.1. Chapman Jouguet detonation and deflagration solutions (adapted from Lee 2008) ………… 2
Figure 1.2. ZND detonation structure (adapted from Kuo 2002) ................................................................. 3
Figure 1.3. Ideal cycle analysis (adapted from Heiser and Pratt 2002) .......................................................... 5
Figure 1.4. Thermal efficiency comparison of ideal and real cycles (Heiser and Pratt 2002) ......................... 5
Figure 1.5. Triple point schematic (adapted from Lee and Radulescu 2005) .................................................. 6
Figure 1.6. Cellular detonation structure (adapted from Lee and Radulescu 2005) ................................. 7
Figure 1.7. Detonation initiation regimes (Bach et al. 1969) ...................................................................... 11
Figure 1.8. Detonation initiator concepts (Touse 2003) .......................................................................... 12
Figure 1.9. Detonation diffraction at an area change (Edwards et al. 1979) .................................................... 13
Figure 1.10. Onset of detonation due to a localized explosion (Urtiew and Oppenheim 1966) .................... 15
Figure 1.11. General pulse detonation combustor (PDC) cycle ............................................................... 16
Figure 1.12. Simplified 1-D modeling of PDC pressure profile (Bussing and Pappas 1994) ...................... 17
Figure 1.13. Typical turbine steady performance map (Saravanamutto et al. 2009) ................................. 19
Figure 1.14. Pulsed flow characteristics (adapted from Abidat et al. 1998) .................................................. 20
Figure 1.15. Variation of flow angles over a pulse cycle (Karamanis et al. 2001) ........................................ 21
Figure 1.16. Detonation exhaust interaction with a stator (Van Zante et al. 2007) ....................................... 22
Figure 1.16. RDC flowfield (adapted from Schwer and Kailasanath 2014) .................................................. 27
Figure 1.17. Performance comparison of rotating and pulsed detonation concepts (Yi et al. 2011) .... 28
Figure 1.18. Initiator tube injecting a detonation into the RDC channel ...................................................... 30
Figure 1.19. Subcritical initiation in a 2-D “unwrapped” RDC channel (adapted from Miller et al. 2013) 31
Figure 1.21. Pressure traces of the onset phase (Kindracki et al. 2011, CH₄-O₂) .......................................... 32
Figure 1.22. Comparison of onset behavior for three ignition sources (Yang et al. 2016, H₂-air) .............. 33
Figure 1.23. Hydrogen-assisted kerosene-air operation in an RDC (Lukasik et al. 2013) ......................... 34
Figure 1.24. Stable detonation wave rotation with a single or multiple wave fronts (Driscoll 2016) ........ 35
Figure 1.25. Example of chaotic detonation propagation in an RDC (Anand et al. 2015b) ....................... 35
Figure 1.26. Example of low frequency instability (LFI) in an RDC (Anand et al. 2015b) ....................... 36
Figure 1.27. Example of mode switching in an RDC (Anand et al. 2015b) ................................................. 37
Figure 1.28. Longitudinal pulsed detonation in an RDC (Anand et al. 2015b) ................................................. 37
Figure 1.29. Measured and computed heat flux into the outer wall of an RDC (Theuerkauf et al. 2016) .............. 38
Figure 1.30. Thermally-induced distortion and signal loss from PCB transducers within the RDC .......... 39
Figure 1.31. Effect of channel width on RDC operation (Kindracki et al. 2011) ................................................. 40
Figure 1.32. RDC flow structure for three channel widths (Zhou and Wang 2013) ............................................. 41
Figure 1.33. Instantaneous chemiluminescence of the detonation front (Rankin et al. 2015a) ..................... 41
Figure 1.34. Evolution in the number of detonation waves from \( n = 3 \to 1 \) (Bykovskii et al. 2006b) .............. 42
Figure 2.1. UC Detonation Systems: (a) PDC-turbine system, and (b) RDC system ........................................ 46
Figure 2.2. Schematic of the detonation test facility ......................................................................................... 47
Figure 2.3. Schematic of the air system and high capacity fuel system ............................................................. 48
Figure 2.4. Schematic of the low capacity fuel system ..................................................................................... 50
Figure 2.5. Flow rate regulation and metering assembly .................................................................................. 51
Figure 2.6. High speed automotive valving system .......................................................................................... 54
Figure 2.7. Reactant and purge air delivery scheme ......................................................................................... 55
Figure 2.8. Schematic of ignition module ....................................................................................................... 56
Figure 2.9. PDC array linked with axial turbine via adapter hardware ............................................................ 57
Figure 2.10. PDC pulse progression before and after adapter weldment ....................................................... 58
Figure 2.11. Cutaway of flow path through the axial turbine .......................................................................... 59
Figure 2.12. Schematic of Garrett JFS-100-13A power generation turbine unit (from user manual) .......... 60
Figure 2.13. a) Stationary and b) radially-traversable probe locations within the turbine assembly .......... 61
Figure 2.14. Pitot probe design and dynamic calibration ................................................................................. 62
Figure 2.15. Schematic of RDC ..................................................................................................................... 64
Figure 2.16. Schematic of RDC initiator tube .................................................................................................. 67
Figure 2.17. Flush-mounted PCB sensors used to capture RDC pressure evolution ...................................... 68
Figure 2.18. ITP (PCB) transducers used to capture RDC pressure evolution ................................................ 69
Figure 2.19. Oxidizer injection slot transducers used to capture RDC pressure evolution .......................... 70
Figure 2.20. Capillary tube averaged pressure evolution for RDC with a converging nozzle .................. 70
Figure 2.21. Kulite pressure evolution for RDC with a converging nozzle ................................................. 71
Figure 2.22. Ionization probes used to capture RDC ionization evolution ................................................................. 72
Figure 3.1. Schematic of traversable probe locations at stage boundaries ................................................................. 74
Figure 3.2: Steady, full and 50% partial admission performance maps for N'/N'_DES characteristics ........... 77
Figure 3.3: Total pressure evolution at the turbine inlet plane at 10 Hz ................................................................. 78
Figure 3.4: Pulse shape sensitivity to frequency for 0.78 kg/s and 0.40 kg/s .............................................................. 79
Figure 3.5: Evolution of swirl at the turbine inlet at 10 Hz, 0.60 kg/s for varying pitot probe angles .......... 80
Figure 3.6: Evolution of swirl at the turbine exit at 10 Hz, 0.60 kg/s for varying pitot probe angles .......... 81
Figure 3.7: Comparison of efficiency calculation methods for 10 Hz, 53% design corrected speed ...... 83
Figure 3.8: Corrected mass flow versus pressure ratio for all pulsation frequencies ...................... 84
Figure 3.9: Flow rate evolution for a turbine inlet sector at 20 Hz, for maximum turbine loading ....... 85
Figure 3.10: Flow angle evolution over a pulse cycle for nominal and reduced rotor speeds .......... 86
Figure 3.11: Effect of flow angles on work-averaged efficiencies for 10 Hz pulsations ................. 87
Figure 3.12: Work-averaged efficiency maps at the lowest and highest pulse frequencies .......... 88
Figure 4.1: Schematic of the PDC-Turbine system with sensor port locations .................. 93
Figure 4.2: Ionization record for the first probe in each combustor (f = 10 Hz, ff = 0.8, pf = 0.8) .......... 96
Figure 4.3: Ionization record for both probes in tube 1 (f = 10 Hz, ff = 0.8, pf = 0.8) ...................... 97
Figure 4.4: Kulite turbine inlet pressure record for a typical test (f = 5 Hz, ff = 0.43, pf = 0.43) .......... 98
Figure 4.5: Pressure comparison from multiple tubes for a typical test (f = 5 Hz, ff = 0.43, pf = 0.43) ..... 99
Figure 4.6: Effect of fill fraction on inlet and exit pressure profiles during pulse active period (f = 5 Hz)100
Figure 4.7: Effect of rotor speed on headwall ITP-PCB pressure profiles (f = 10 Hz, ff = 0.65, pf = 0.83) 101
Figure 4.8: Effect of frequency on headwall ITP pressure profiles (ff = 0.65, pf = 0.80) .................. 102
Figure 4.9: Output power and rotor speed evolution for a typical test (f = 10 Hz, ff = 0.73, pf = 0.73) ... 103
Figure 4.10: Temperature evolution for a typical test (f = 10 Hz, ff = 0.73, pf = 0.73) ..................... 104
Figure 4.11: Effect of rotor speed and inlet condition on turbine output power ..................... 106
Figure 4.12: Overview of the Fickett-Jacobs cycle (Wintenberger and Shepherd 2006) .......... 107
Figure 4.13: Comparison of Detonation (FJ) and Brayton cycles (γ = 1.2, qadd = 6, ψ = 1.5) .......... 109
Figure 4.14: Comparison of experimental data to predicted cycle efficiencies (γ = 1.2, q = 6) ............. 112
Figure 4.15: Velocity triangles of the rotor inflow at low rotor speed condition ............ 114
Figure 4.16: Velocity triangles of the rotor inflow at high rotor speed condition................................. 115
Figure 5.1: Schematic of various RDC sensor types and installation locations........................................ 119
Figure 5.2: High-pass filtering of flush-mounted PCB transducer .......................................................... 122
Figure 5.3: Flowchart of the correlation-based algorithm........................................................................ 125
Figure 5.4: Auto-correlation process for a single instance of detonation wave passage ....................... 127
Figure 5.5: Pressure evolution for entire example case for the three azimuthal sensor locations.......... 128
Figure 5.6: Auto-correlation results for the three azimuthal sensor stations ....................................... 129
Figure 5.7: Cross-correlation process for a single instance for the station I → station II sensor pair..... 130
Figure 5.8: Cross-correlation results for the three azimuthal sensor pairs .......................................... 131
Figure 5.9: Comparison of sensor response for various installations at three azimuthal stations ......... 133
Figure 5.10: Comparison of time scales and phase shift for various installations (m = 0.4 kg/s, φ = 1.23) .......................................................... 134
Figure 5.11: Detonation wave direction reversal and mode transition (m = 0.5 kg/s, φ = 1.57) ............ 136
Figure 5.12: Measurement data and correlation output for air slot transducers (m = 0.2 kg/s, φ = 1.05) .............................................................................................................. 138
Figure 6.1: Schematic of sensor clustering in the vicinity of the initiator tube entry region .......... 142
Figure 6.2: Schematic of PCB pressure transducer locations in the RDC ............................................ 143
Figure 6.3: Phase-locked pressure evolution at a) forward and b) perpendicular sensor locations ....... 146
Figure 6.4: Blast wave trajectory for H₂-O₂, φ = 2.05, wₘₐₖ = 7.6 mm, Pᵢ = 1 bar .......................... 147
Figure 6.5: Blast wave trajectory analysis for H₂-O₂, φ = 2.05, wₘₐₖ = 7.6 mm, Pᵢ = 1 bar .............. 149
Figure 6.6: Comparison of initiator performance for a range of equivalence ratios .......................... 152
Figure 6.7: Comparison of H₂-O₂ and C₂H₄-O₂ blast trajectories for maximum energy deposition cases 153
Figure 6.8: Comparison of H₂-O₂ and C₂H₄-O₂ peak pressures for maximum energy deposition cases .... 154
Figure 6.9: Comparison of blast wave trajectories for varying wch (C₂H₄-O₂, φ = 3.0, Pᵢ = 1 bar) ........... 154
Figure 6.10: Comparison of blast wave peak pressures for varying wℓₙ (C₂H₄-O₂, φ = 3.0, Pᵢ = 1 bar) .... 155
Figure 6.11: Estimated energy deposition for a range of channel widths and pressures .................. 156
Figure 6.12: Blast wave propagation for a reacting medium in the RDC channel (RDC: H₂-air, φ = 1.03, Initiator: H₂-O₂, φ = 2.00).......................................................... 158
Figure 6.13: Phase-locked pressure evolution at three circumferential locations capturing the initial blast wave propagation in the RDC channel (RDC: H$_2$-air, $\phi = 1.03$, Initiator: H$_2$-O$_2$, $\phi = 2.00$) ........................................ 159

Figure 6.14: RDC pressure evolution at three azimuthal sensor locations (RDC: H$_2$-air, $\phi = 1.03$, Initiator: H$_2$-O$_2$, $\phi = 2.00$) .......................................................... 160

Figure 7.1: RDC schematic indicating position of injection slot and exit plane transducers .............. 163

Figure 7.2: Spectrogram for exit plane PCB ($\theta = 60^\circ$) for Case A .......................................................... 165

Figure 7.3: Wave speed and phase delay analysis for exit plane transducers for Case A .............. 166

Figure 7.4: Pressure evolution in the exit plane shortly after RDC ignition for Case A .............. 167

Figure 7.5: Spectrogram for exit plane PCB ($\theta = 60^\circ$) for Case B .......................................................... 168

Figure 7.6: Wave speed and phase delay analysis for exit plane transducers for Case B .............. 169

Figure 7.7: Pressure evolution in the exit plane shortly after RDC ignition for Case B .............. 170

Figure 7.8: Spectrogram for exit plane PCB ($\theta = 60^\circ$) for Case C .......................................................... 171

Figure 7.9: Wave speed and phase delay analysis for exit plane transducers for Case C .............. 172

Figure 7.10: Pressure evolution in the exit plane shortly after RDC ignition for Case C .............. 173

Figure 7.11: Spectrogram for air injection slot PCB ($\theta = 40^\circ$) for Case D ........................................ 174

Figure 7.12: Frequency and phase delay analysis for air slot transducers for Case D .............. 175

Figure 7.13: Pressure evolution in the air injection slot shortly after RDC ignition for Case D .............. 175

Figure 7.14: Comparison of onset time definitions for stable detonation cases at two flow rates ........ 177

Figure 8.1: RDC schematic indicating reactant injection scheme and instrumentation layout ........... 180

Figure 8.2: Examples of typical lightoff behavior for ethylene-air mixtures ($m_{\text{AIR}} = 0.3 \text{ kg/s}$) ............ 185

Figure 8.3: Instability and appearance of burst phenomenon ($m_{\text{AIR}} = 0.5 \text{ kg/s}, \phi = 0.89$) ............ 186

Figure 8.4: Baseline ethylene-air operating range .......................................................... 187

Figure 8.5: Fully-assisted hydrogen-ethylene fuel blend operating maps ........................................ 188

Figure 8.6: Fully stable detonation case ($\phi_{\text{TOTAL}} = 1.18, \phi_{H_2} = 0.18$) ........................................ 190

Figure 8.7: Moderately unstable detonation case ($\phi_{\text{TOTAL}} = 1.00, \phi_{H_2} = 0.18$) ........................................ 191

Figure 8.8: Severely unstable detonation case ($\phi_{\text{TOTAL}} = 0.86, \phi_{H_2} = 0.09$) ........................................ 191

Figure 8.9: Typical extinction and recovery of rotation ($m_{\text{AIR}} = 0.5 \text{ kg/s}, \phi_{\text{TOTAL}} = 0.94, \phi_{H_2} = 0.03$) ......... 192

Figure 8.10: Transition between rotation and longitudinal pulsing modes ($m_{\text{AIR}} = 0.5 \text{ kg/s}, \phi_{\text{TOTAL}} = 0.93, \phi_{H_2} = 0.13$) ................................................................. 193
Figure 8.11: Onset of instability and subsequent failure for extended hydrogen-to-ethylene transition ($m_{\text{AIR}} = 0.5 \text{ kg/s}, \phi_{\text{TOTAL}} = 1.01, \phi_{\text{H}_2} = 0.18$) .......................................................................................................................... 195

Figure 9.1: Comparison of the four kinetic mechanisms with experimental shock tube data ............ 200

Figure 9.2: Baseline cell width estimations using the Ng (single-parameter) and Gavrikov et al. (two-parameter) cell width models with the four kinetic mechanisms ........................................................................ 204

Figure 9.3: Thermicity profiles for stoichiometric ($\phi = 1$) and rich ($\phi = 2$) C$_2$H$_4$-air mixtures ............ 205

Figure 9.4: Cell width variation with initial pressure for stoichiometric C$_2$H$_4$-air mixtures .............. 206

Figure 9.5: Cell width variation with equivalence ratio for three initial pressures ......................... 207

Figure 9.6: Cell width variation with initial temperature for stoichiometric C$_2$H$_4$-air mixtures ........ 208

Figure 9.7: Cell width variation with equivalence ratio for four initial temperatures ....................... 209

Figure 9.8: Cell width variation with nitrogen dilution ratio for stoichiometric C$_2$H$_4$-O$_2$-N$_2$ mixtures ..... 211

Figure 9.9: Cell width variation with equivalence ratio for a range of nitrogen dilution ratios ........ 212

Figure 9.10: Induction length comparison with the results of Lu et al. (2003) ...................................... 213

Figure 9.11: Cell width comparison of C$_2$H$_4$-H$_2$-air mixtures ($\phi = 1, P_i = 1 \text{ bar}, T_i = 300 \text{ K}$) ........ 214

Figure 9.12: Cell width variation with equivalence ratio of C$_2$H$_4$-H$_2$-air mixtures ........................ 215

Figure 9.13: Cell width variation with equivalence ratio of C$_2$H$_4$-H$_2$-air mixtures ......................... 217

Figure 10.1: Rotating detonation combustor schematic with sensor layout ........................................ 222

Figure 10.2: Polynomial fit of experimentally measured cell width for H$_2$-O$_2$-N$_2$ mixtures ............ 223

Figure 10.3: Detail of exit plane pressure evolution for selected cases .............................................. 224

Figure 10.4: Determination of number of waves from maximum ideal rotation frequency ............. 225

Figure 10.5: Detonation regimes, partitioned by number of waves .................................................. 226

Figure 10.6: Detonation wave speed normalized to ideal Chapman-Jouguet value ......................... 227

Figure 10.7: Detonation regimes demarcated by lines of maximum normalized detonation perimeter 229
List of Tables

Table 1.1. Typical cell sizes for fuel-O₂ and fuel-air mixtures (Desbordes and Presles 2012) ..................... 8
Table 2.1. Turbine Design Parameters ............................................................................................................. 60
Table 2.2. RDC geometric parameters ............................................................................................................ 65
Table 2.3. Initiator tube geometric parameters ............................................................................................... 67
Table 4.1. Summary of PDC-turbine test cases .............................................................................................. 105
Table 5.1. Instrumentation overview ............................................................................................................. 120
Table 5.2. Correlation magnitude interpretation ............................................................................................. 123
Table 6.1. Initiator settings for initiator blast wave decay tests ................................................................. 144
Table 6.2. Components of Uncertainty in Energy Estimation ........................................................................ 152
Table 7.1: Overview of onset test cases .......................................................................................................... 164
Table 7.2: Summary of onset time results ...................................................................................................... 176
Table 8.1: Selected baseline ethylene-air cases for \( \dot{m}_{\text{AIR}} = 0.5 \text{ kg/s} \) ............................................. 184
Table 8.2: Selected hydrogen-assisted operating cases for \( \dot{m}_{\text{AIR}} = 0.5 \text{ kg/s}, \ \phi_{\text{H2}} = 0.18 \) ............ 189
Table 8.3: Summary of extended hydrogen-to-ethylene transition cases .................................................... 194
Table 9.1: Summary of ZND Simulation Conditions ..................................................................................... 203
Table 10.1: Overview of test parameters ......................................................................................................... 221
Table 10.2: Instrumentation Overview ......................................................................................................... 223
Chapter 1 – Introduction

1.1 The Detonation

A detonation is a supersonic mode of combustion consisting of a shock wave coupled to a reaction zone. The shock wave is sufficiently strong to initiate chemical reactions due to the high temperature and pressure in the post-shock environment. Precursor, chain-branching reactions drive radical production and culminate in exothermic, termination reactions. The heat release chokes the flow relative to the shock front, forming the necessary boundary condition to sustain the propagation speed of the detonation structure.

This process was first modeled by Mikelson, and later Chapman and Jouguet as a zero-dimensional jump condition, assuming simultaneous shock compression and heat release across a discontinuity (Lee 2008). The Chapman-Jouguet (CJ) detonation solution can be calculated for a mixture by solving the 1-D inviscid conservation equations with a heat release term \( (q) \) and the equation of state (Eqs. 1.1a-d, adapted from Turns 2006). State 1 corresponds to unburned reactants, state 2 corresponds to detonation products, and the heat release term in Eq. 1.1c denotes the heat of combustion per mass of mixture, and is a function of the chemical composition of the reactants.

\[
\begin{align*}
\rho_1 u_1 &= \rho_2 u_2 & (1.1a) \\
P_1 + \rho_1 u_1^2 &= P_2 + \rho_2 u_2^2 & (1.1b) \\
c_p T_1 + \frac{u_1^2}{2} + q &= c_p T_2 + \frac{u_2^2}{2} & (1.1c) \\
P &= \rho RT & (1.1d)
\end{align*}
\]

This set of equations is essentially the normal shock relations with a source term \( (q) \), and it yields two valid solutions (Figure 1.1), corresponding to subsonic and supersonic propagation of the discontinuity.
relative to the unburned gas (state 1). The supersonic solution \( (u_1/a_1 > 1) \) is defined as the detonation solution, which also satisfies sonic flow of the products (state 2) relative to the discontinuity \( (u_2/a_2 = 1) \).

CJ detonation velocities are typically on the order of kilometers per second (typical Mach numbers from 5-10 and pressure ratios of 13-55) (Friedman 1953), such that reactants are consumed at approximately constant volume, and yield a significant rise in pressure (i.e. pressure “gain” combustion).

![Figure 1.1. Chapman Jouguet detonation and deflagration solutions (adapted from Lee 2008)](image)

Several decades later, Zel’dovich, von Neumann, and Döring developed a higher fidelity, 1-D detonation model with a finite-thickness reaction zone behind the leading shock. The ZND detonation model uses the CJ detonation velocity as a starting point to determine the post-shock conditions. In the post-shock region, the flow field is evaluated from chemically reacting, 1-D, inviscid, conservation equations. The problem can be solved in the wave-fixed frame and simplified into a system of ordinary differential equations (Fickett and Davis 1979). Behind the shock, relative flow velocity \( (u) \), density \( (\rho) \), pressure \( (P) \), and species concentrations \( (y_i) \) are expressed as functions of time (Eqs. 1.2a-d). Solution of this system of equations is greatly simplified by introducing a variable known as thermicity \( (\sigma) \), which is essentially a
dimensionless measure of heat release (Eqs. 1.2e-f). Species production rates ($\Omega_i$) and thermicity coefficients ($\sigma_i$) are evaluated from a chemical kinetic mechanism.

\[
\frac{Dw}{Dt} = \frac{u\dot{\sigma}}{1 - M^2} \quad (1.2a)
\]

\[
\frac{Dp}{Dt} = -\frac{\dot{\rho}\dot{\sigma}}{1 - M^2} \quad (1.2b)
\]

\[
\frac{DP}{Dt} = -\frac{\rho u^2 \dot{\sigma}}{1 - M^2} \quad (1.2c)
\]

\[
\frac{DY_i}{Dt} = \Omega_i \quad (1.2d)
\]

\[
\dot{\sigma} = \sum_{i=1}^{K} \sigma_i \Omega_i \quad (1.2e)
\]

\[
\sigma_i = \frac{1}{\rho a^2} \frac{\partial p}{\partial y_i} \bigg|_{p,\rho,y_{j\neq i}} \quad (1.2f)
\]

The resulting ZND structure downstream of the shock consists of an induction zone ($\Delta_i$), followed by a strongly exothermic reaction zone ($\Delta_r$), as depicted in Figure 1.2. The boundary between these two regions is denoted by the location of peak thermicity (Kao and Shepherd 2008). The induction zone begins at the post-shock, or “von Neumann” plane, and chain-branching reactions, excited at these elevated temperatures, allow radical species to proliferate.

![Figure 1.2. ZND detonation structure (adapted from Kuo 2002)](image)

Transitioning into the reaction zone, this gradual buildup of intermediate species culminates with recombination reactions and intense exothermic heat release, which is coupled with acceleration of the
products away from the leading shock. Upon achieving chemical equilibrium, the products are thermally choked relative to the leading shock, and converge to CJ conditions. For many stoichiometric fuel-air mixtures, the thickness of the ZND detonation structure is on the order of a millimeter (Schultz and Shepherd 2000).

The Fickett-Jacobs detonation cycle was first conceived by Jacobs (1956), and models the detonation as a one-dimensional process by coupling the normal shock relations to Rayleigh heat addition, then expanding the products back to their initial, quiescent state. Heiser and Pratt (2002) adopted this detonation structure to model the thermodynamic cycle of an idealized detonation combustor. Assuming an isentropic compressor and turbine, the ideal detonation cycle yields less entropy and simultaneously provides greater potential power extraction than a traditional constant pressure (Brayton) cycle, or a constant volume (Humphrey) cycle (Figure 1.3). The leading shock of the detonation compresses and heats the reactants from state 3 to state 3a, which yields a higher effective initial pressure prior to heat release. As the detonation products accelerate to sonic velocity relative to the detonation front due to heat release, the portion of the cycle between states 3a and 4 strongly resembles Rayleigh flow (idealized 1-D flow with heat addition).

For a fixed initial pressure ratio, the ideal detonation cycle always outperforms the ideal Brayton cycle due to the additional compression provided by the detonation (Figure 1.4). However, by implementing realistic component efficiencies for the compressor and turbine ($\eta = 90\%$), and a dimensionless heat release of $\bar{q} = 5$, the detonation cycle only retains its superiority for moderate amounts of pre-compression ($P_3/P_0 < 40-50$). The divergence in performance between ideal and real detonation cycles highlights the need for a detailed investigation of component efficiencies under detonating flow conditions. In the limit of zero pre-compression, the Brayton cycle no longer produces useful power output, while the detonation cycle continues to generate its own compression and achieves thermal
efficiencies of $\eta_{\text{thermal}} = 20$-50\% (dependent on the dimensionless heat release and component efficiencies). The thermodynamic performance of the detonation cycle in this regime provides a strong incentive to develop detonation-based combustors and integrate them into existing cycles.

Figure 1.3. Ideal cycle analysis (adapted from Heiser and Pratt 2002)

Figure 1.4. Thermal efficiency comparison of ideal and real cycles (Heiser and Pratt 2002)
1.2 Three-Dimensional Detonation Structure

While the concept of a one-dimensional detonation structure is expedient for reduced-order modeling, experimentally-observed detonations are highly three-dimensional and contain a complex system of shock waves (Lee and Radulescu 2005). Behind the leading shock front, the exothermic reaction time scales depend strongly on the post-shock conditions (i.e. relative degree of shock heating and compression), while the shock strength is dependent on the local rate of heat release. This circular dependence yields a nonlinear and generally unstable coupling between the leading shock and the reaction zone. This latent instability of the detonation can manifest as a simple, oscillatory response in the leading shock velocity and induction length, or as very chaotic, multi-frequency oscillations (Short 2001, Ng 2005). For an initially planar detonation front, small perturbations in the transverse direction excite an unstable response with a characteristic frequency related to the temperature sensitivity of the reaction (Gamezo et al. 1999). Well-defined transverse shock waves form behind the leading shock front, providing additional compression and periodically colliding with one another. The intersection of a transverse shock wave with the leading shock is referred to as a triple point (Figure 1.5).

![Triple point schematic](image)

**Figure 1.5. Triple point schematic (adapted from Lee and Radulescu 2005)**

As transverse waves and their associated triple points propagate and collide, a cellular structure emerges (Figure 1.6). The spacing of the transverse waves determines the width of the detonation cell ($\lambda$), which is intrinsically linked to a characteristic reaction length (Shchelkin and Troshin 1963). While the local transverse wave spacing can be highly irregular, especially for fuel-air mixtures, the published data typically report average cell width from these patterns. The cell width, $\lambda$, is the most important
dynamic parameter of detonations, and is a direct measure of the detonation sensitivity of a mixture (Lee 1984, Vasil’ev 2006). It is a function of the mixture composition and thermodynamic state, with smaller cell width indicating higher detonability (e.g. detonation sensitivity). It correlates strongly with macroscopic detonation behavior: detonation propagation limits, transmission limits, and critical initiation energy requirements can typically be defined in terms of λ.

Transverse wave collisions are necessary to sustain detonation propagation for most mixtures. To successfully propagate a detonation through a confining geometry (i.e. tube, channel, etc.), the characteristic length of this confinement must exceed a minimum size for which these transverse features are supported. For successful detonation onset and propagation, the channel height must exceed one cell width for rectangular geometries (h > λ) (Vasil’ev 1982), or the tube diameter must exceed one cell width for circular geometries (d > λ) (Moen et al. 1981). For a given geometric confinement, detonations will fail if the cell width exceeds these size limits, thereby restricting the successful operating range. Enhancing the detonation sensitivity of the mixture (i.e., reducing λ) extends the detonability limits for a given confinement. Cell sizes are on the order of a millimeter for typical fuel-oxygen mixtures, and an order of magnitude higher for most fuel-air mixtures (Table 1.1).
Table 1.1. Typical cell sizes for fuel-O$_2$ and fuel-air mixtures (Desbordes and Presles 2012)

<table>
<thead>
<tr>
<th>Fuel</th>
<th>$\lambda$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(fuel-O$_2$)</td>
</tr>
<tr>
<td>H$_2$</td>
<td>1.4</td>
</tr>
<tr>
<td>C$_2$H$_2$</td>
<td>0.1</td>
</tr>
<tr>
<td>C$_2$H$_4$</td>
<td>0.5</td>
</tr>
<tr>
<td>C$_3$H$_6$</td>
<td>0.9</td>
</tr>
<tr>
<td>C$_3$H$_8$</td>
<td>0.9</td>
</tr>
<tr>
<td>CH$_4$</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Due to the intrinsic link between chemical kinetics and the generation of the cellular structure, Shchelkin and Troshin (1963) first postulated that the cell width is proportional to a representative chemical induction length ($\Delta_i$), (e.g., $\lambda = A \cdot \Delta_i$, where $A$ is a proportionality constant). The induction length can be estimated numerically by employing the ZND detonation model with an appropriate chemical kinetic mechanism to evaluate the detonation structure. Utilizing this approach, Westbrook and Urtiew (1982) calculated the induction length for a wide range of mixtures and related it to a number of macroscopic detonation properties, including cell width. Utilizing the experimental data of Manzhalei et al. (1974), the proportionality constant was evaluated for fuel-oxygen mixtures as $\lambda = 29 \cdot \Delta_i$. However, Shepherd et al. (1986) reported significant discrepancies when using a single proportionality constant for a wide range of mixtures, suggesting that this relation was not universally valid. Recent studies by Gavrikov et al. (2000) and Ng (2005) proposed more complex forms of the original relation to achieve better agreement with experimental data. These studies express the proportionality constant ($A$) as a function of one or more variables calculated from the ZND solution. While these relations are particularly promising, Schultz and Shepherd (2000) illustrate the significant impact of the chemical kinetic mechanism on the computed induction length (which varies by a factor of two, depending on the mechanism).
1.3 Detonation Initiation

Direct detonation initiation for fuel-air mixtures requires an immense amount of energy deposition (on the order of 100 kJ) at relatively high power (Kaneshige and Shepherd 1997). As discussed by Lee, initiation by a strong blast requires energy deposition of sufficient magnitude to maintain the blast strength above a threshold value for a minimum duration of time (Lee 1977). Induction reactions occur within the blast radius at the elevated post-shock temperatures, but heat release behind the shock is not immediate due to finite induction time of the reactants. If the blast energy is below the critical value, the heat release is not sufficiently rapid and the reaction front decouples from the leading shock. The strong blast wave then decays monotonically to a sound wave ($M_s \to 1$) as the leading shock progressively separates from the reaction front. In a successful initiation scenario, the decay of the initiating blast wave is halted by the rapid onset of heat release, and the reaction front couples with the leading shock. Zel’dovich hypothesized that the critical initiation energy of a mixture was proportional to the induction length scale raised to a power (Eq. 1.3), dependent on the geometric confinement of the initiation, where $j = 0, 1, 2$ correspond to planar, cylindrical, and spherical initiation respectively (Lee 2008). This relationship between a detonation length scale and the critical energy is almost universally adopted in expressions for the critical energy.

$$E_{\text{Critical}} \propto \Delta_{\text{Induction}}^{j+1}$$

(1.3)

In the absence of heat release, as in the case of a blast wave propagating into an inert mixture, the blast strength at a given radius is a function of the total energy contained within the blast wave (Taylor 1950a). Theoretical similarity solutions for blast wave propagation assume that the density, velocity, and pressure profiles behind the leading shock are of the same shape as the blast propagates outward. When scaled appropriately by a similarity variable, these profiles can be normalized and collapsed, yielding an approximate solution for the blast propagation. In the absence of heat release, a strong
blast decays monotonically, as modeled by the similarity solution of Taylor (1950a). The total energy contained within the blast can be estimated by collapsing the experimentally observed trajectory onto the theoretical curve (Taylor 1950b).

The general form of the similarity solution (Eq. 1.4) was extended to all geometries by Sakurai (1953), where constants $J_0$ and $C_i$ are functions of specific heat ratio and geometry. This solution defines a characteristic length, referred to as the explosion length ($r_0$), from the total energy deposition and the initial pressure of the medium (Eq. 1.5). For the first order approximation, the Mach number of the leading shock ($M_s$) at a given radius is directly proportional to the total blast energy.

\[
M^2 \left( \frac{r}{r_0} \right)^{j+1} = \frac{1}{J_0} \left[ 1 + \frac{C_1}{M^2} + \frac{C_2}{2 \cdot M^4} + \cdots \right]^{-1}
\]

(1.4)

\[
r_0 = \left( \frac{E_j}{P_l} \right)^{\frac{1}{j+1}}
\]

(1.5)

The historical approximate solution developed by Taylor (1950a) is valid for very strong blast waves ($M_s \gg 1$), but breaks down at intermediate blast wave strengths ($M_s < 3$), necessitating the inclusion of additional terms to improve the validity of the approximation (Sakurai 1954). The quasi-similarity approach of Oshima (1960) improves upon this approach, extending the accuracy of the solution to weak blast waves at larger shock wave radii ($M_s \sim 1$).

From Eq. 1.4, the blast strength at a given radius is related to the initial energy input, which determines the outcome of the initiation process. The experimental work of Bach et al. (1969) demonstrates the existence of a limiting, “critical” energy which produces successful initiation (as in Zel’'dovich et al. 1957).

Energy deposition less than the critical level ($E_{crit}$) produces “subcritical” initiation (Figure 1.7a), where the blast is too weak, chemical induction times are too long and heat release occurs too late to forestall the blast decay. Likewise, energy deposition in excess of the critical value ($E > E_{crit}$) is sufficiently powerful to minimize induction times, attain rapid heat release, halt the blast decay, and stabilize to a
detonating state. The near-limit, critical initiation case exhibits highly unstable, spatially irregular behavior (Figure 1.7b) as the shock front and reaction zone are coupled, but initially propagate in an unstable manner at sub-CJ ($M_s < M_{CJ}$) velocities. The growth of instabilities in the expanding front generates localized explosions and eventually culminates in asymmetric detonation development.

![Detonation initiation regimes](image)

a) Subcritical initiation ($E < E_{crit}$)

b) Critical initiation ($E = E_{crit}$)

Figure 1.7. Detonation initiation regimes (Bach et al. 1969)
There are several practical approaches to circumvent these relatively large energy requirements and achieve detonation in fuel-air mixtures. The pre-detonator concept utilizes a secondary, highly-reactive initiator mixture (i.e. fuel-oxygen) to start a detonation, which is then transmitted into the less-reactive fuel-air mixture (Figure 1.8). Detonation transmission studies indicate regimes similar to those for direct initiation. At the interface between initiator and target mixtures, the detonation experiences an abrupt transition in its structure due to the changing reactant composition. Successful transmission requires a sufficiently strong initiator charge to maintain the blast beyond the interface. If the blast remains above critical strength for the reaction length of the target mixture, the detonation stabilizes, and successful transmission is achieved (Kuznetsov et al. 1997).

In practice, a small initiator volume is used to limit the quantity of initiator mixture (Touse 2003), which then diffracts into a larger volume. This approach is limited by the simultaneous destabilizing effects of rapid expansion and mixture composition change, which can result in failure of the front (Figure 1.9). Expansion waves propagate laterally into the detonation front, inducing curvature and weakening the post-shock state. For moderate expansion, the reaction time scales are lengthened, producing locally

![Figure 1.8. Detonation initiator concepts (Touse 2003)](悲歡離合)
enlarged detonation cells (Thomas et al. 1986). At the boundaries, the detonation front can fail locally due to the inward propagation of expansion waves, and the blast wave decouples from the reaction zone. If there are a sufficient number of transverse waves at the initial exit plane, global failure is averted, and the detonation front is transmitted into the larger volume. For channels of circular cross-section expanding into unconfined space, the critical diameter for successful transmission is $d_{\text{crit}} = 13 \lambda$ (Knystautas et al. 1982).

![Figure 1.9. Detonation diffraction at an area change (Edwards et al. 1979)](image)

In cases where direct initiation or transmission is impractical, a detonation can be achieved from a low-speed deflagration through a deflagration-to-detonation transition (DDT) process. The advantages to this approach are the use of a low-energy ignition source and elimination of high-energy initiator reactants and their associated complexity. Drawbacks include the need for additional transition length in a confining channel, and loss-inducing obstacles, which are generally required to reduce this DDT
length to practical values for most detonation applications. The DDT process is categorized into two basic phases, a preliminary phase of creation of conditions for the onset of detonation, and a final phase of onset of detonation (Ciccarelli and Dorofeev 2008). This classification captures a broad range of configuration-dependent flame acceleration (FA) phenomena within the first phase, while almost all DDT scenarios exhibit similar detonation onset behavior in the second phase.

The flame acceleration (FA) phase may use a number of turbulence-generating mechanisms to distort and elongate the flame front to increase the propagation speed. Turbulence-inducing obstacles include spiral inserts such as the Shchelkin spiral (Meyer et al. 2002), arrays of constricting orifices (Matsukov et al. 1999), swept ramps (Brophy et al. 2010), as well as fluidic jets (Knox et al. 2011). Deformation of the flame front increases the flame area, augmenting the burning rate. In a confined duct, products expand in the direction of flame propagation, pushing the flame downstream and inducing a velocity in the unburned reactants. As the induced velocity grows, the boundary layer in the induced reactant flow becomes turbulent, enhancing the flame acceleration. The flame structure eventually achieves supersonic speeds relative to the reactants, and compression waves coalesce into a leading shock, followed by a turbulent flame brush (Shepherd and Lee 1992).

This coupled shock-flame structure differs from a detonation in that it relies on turbulent diffusion, rather than shock heating as the key mechanism to sustain the exothermic reaction (Chao and Lee 2003). Typical propagation speeds for this fast deflagration structure are on the order of 50% of the ideal CJ detonation speed. Upon reaching this choked flame regime, onset of detonation is imminent, provided that the geometry is sufficiently large (d/Λ ≥ 1) to support the detonation structure (Peraldi et al. 1986). The onset phase typically relies on a strong, localized explosion within the shock-flame structure, which propagates as an overdriven detonation and overtakes the leading shock (Urtiew and Oppenheim 1966). This localized explosion can be induced by a number of mechanisms, including
shock-obstacle reflections, shock-shock interactions, shock-flame interactions, and instabilities in the flame (Ciccarelli and Dorofeev 2008). The pre-compressed, preheated reactants behind the leading shock are rapidly consumed, and the detonation front stabilizes (Figure 1.10).

![Image of detonation front](image)

**Figure 1.10. Onset of detonation due to a localized explosion (Urtiew and Oppenheim 1966)**

### 1.4 The Pulse Detonation Combustor

Motivated by the thermodynamic advantage of detonation, early researchers attempted to incorporate detonation combustion into a propulsion cycle. The first of these practical combustors, the pulse detonation combustor (PDC), was first developed in the 1940s (Kailasanath 2000), following years of active research on intermittent combustion concepts such as the pulsejet (Coleman 2001). The PDC is a device which *periodically* consumes reactants using a detonation. It typically consists of a tube or channel to provide confinement, valving to inject reactants, an ignition source to initiate combustion, and optionally, specialized obstacles to aid the DDT process for less detonable mixtures.
The pulse detonation cycle is typically comprised of five basic phases (Figure 1.11). During the fill phase, fresh reactants are injected into the detonation tube via mechanical or fluidic valving. When filling has completed, the mixture is ignited, typically by a low-power source, such as an automotive spark, or a gaseous pre-detonator device. Depending on the mixture and the initiation method, direct detonation initiation may be achieved; for most practical fuel-air mixtures, a deflagration-to-detonation-transition (DDT) process occurs with the aid of transition-enhancing obstacles. In the third phase, a detonation is formed and propagates downstream through the reactants at supersonic velocities. The products then exhaust and expand downstream in a process sometimes referred to as blowdown, and the detonation tube eventually relaxes to ambient pressure. As the residual gas temperatures within the tube are typically high enough to induce autoignition of reactants in the next cycle, the tube is typically purged with a nonreactive buffer mixture. This eliminates undesirable deflagration of the freshly injected reactants and ensures periodic operation.

Figure 1.11. General pulse detonation combustor (PDC) cycle

Key advantages of the PDC concept are low complexity and high controllability. There are few moving parts, and these are typically associated with mechanical valving. The design is simple, with bench-scale devices typically comprised of commercially-available pipe parts and automotive spark ignition systems. Precise control of the initial conditions is maintained by timed and regulated injection of reactant flow.
Placement of the ignition source and timing of the spark signal allows repeatable initiation of the combustion process. Careful design of DDT obstacles for a given reactant mixture and tube geometry can provide repeatable transition to detonation between consecutive cycles (Kailasanath 2003). Provided that the tube geometry is adequate to support stable detonation, propagation velocities are approximately constant near the CJ value (Lee et al. 2013). The detonation front is generally treated as planar, allowing the PDC cycle to be described with simplified, unsteady 1-D analysis (Figure 1.11), as employed by Bussing and Pappas (1994). The simple, cost-effective, PDC provides tight regulation of the detonation process and is the preferred cycle when repeatability and stability are desired.

Unfortunately, there are many practical challenges that impede efficient integration of pulse detonation combustors into existing engine cycles. Robust valving of fresh reactants generally requires moving parts and adds weight and complexity. The unsteady admission of fresh reactants and unsteady expulsion of exhaust products generates considerable unsteadiness at the compressor exit and turbine inlet, which can severely degrade performance of those components. Direct detonation initiation is difficult to achieve for less-detonable, commercially-viable hydrocarbon fuels, requiring the use of loss-inducing DDT obstacles (Kailasanath 2003). The length of tube required to achieve DDT increases the combustor size, and the time required for the transition process limits the maximum operating
frequency. Compact designs are problematic for less-detonable fuels due to channel size requirements for stable detonation propagation.

1.5 Unsteady Turbine Behavior

The optimization of a turbine under pulsating inflow conditions is paramount to the successful integration of a detonation-based combustor. Though detonation combustion has the potential for higher thermal efficiency than conventional, constant pressure combustion, efficient work extraction from the highly unsteady combustion process must be achieved. Turbines operating under pulsating conditions experience substantial variations in thermodynamic quantities, flow angles, and efficiency. As the flow field rapidly evolves over a single pulse cycle, unique features and losses are generated within the device. Attempting to optimize turbine performance for a wide range of unsteady conditions is problematic, and tradeoffs are likely necessary to balance the various losses.

In turbine design and analysis, parameters are typically scaled to a quasi-dimensionless form to collapse data from different operating conditions (Saravanamuttoo et al. 2009). For a fixed turbine geometry, and fixed gas composition, these scaled forms of mass flow (ṁ) (Eq. 1.6) and rotor speed (N) (Eq. 1.7) are functions of local Mach number. The mass flow parameter is linked to the Mach number distribution in the stator, while the corrected speed is linked to the Mach number experienced by the rotor (Mattingly and von Ohain 2006). For fixed MFP and N’, the velocity triangles are fixed, thereby fixing the rotor incidence angle, i.e. the “angle of attack” of the rotor blades relative to the rotor inflow. As the stator Mach number reaches unity, the turbine is choked and MFP reaches a maximum. Steady turbine efficiency is defined as the ratio of actual power extraction to ideal power extraction (Eq. 1.8).

The ideal power term assumes isentropic expansion through the turbine to determine the total enthalpy change for a device without losses. The actual power term is expressed as shaft power, the product of the shaft torque (τ) and speed (N) extracted through the rotor.
Mass Flow Parameter = \( \text{MFP} = \frac{\dot{m}\sqrt{T_0}}{P_0} = \sqrt{\frac{\gamma}{R}} \cdot A \cdot f(M_{\text{stator}}) \) \hspace{1cm} (1.6)

Corrected Speed = \( N' = \frac{N}{\sqrt{T_0}} = \sqrt{\frac{\gamma}{R}} \cdot f(M_{\text{rotor}}) \) \hspace{1cm} (1.7)

\( \eta_{\text{turbine}} = \frac{\text{Actual Power}}{\text{Ideal Power}} = \frac{N \cdot \tau}{\dot{m} \cdot c_p \cdot T_{\text{inlet}} \left( 1 - \left(\frac{P_{\text{exit}}}{P_{\text{inlet}}}\right)^{\frac{\gamma - 1}{\gamma}} \right)} \) \hspace{1cm} (1.8)

Turbine performance is characterized by a performance map (Figure 1.13), where MFP and \( N' \) are normalized to design values. Lines of constant \( N' \), referred to as speed characteristics, form the basis for steady performance maps. The turbine loading is varied with flow rate to produce a fixed corrected rotor speed across a range of MFP. The turbine operating condition is fully defined by two independent parameters and occupies a single, unique point on the map. Efficiency curves accompany the MFP characteristics to describe the performance for each condition.

Figure 1.13. Typical turbine steady performance map (Saravanamutto et al. 2009)
By contrast, a turbine operating under pulsating flow experiences a diverse range of flow conditions for each pulse cycle, sometimes represented by a locus of points. This pulsed flow characteristic, or orbit (Figure 1.14) may span a large portion of the steady performance map, in contrast to a single point for steady flow. The shape, size, and complexity of the orbit are strong functions of pulse amplitude, shape, and frequency (Copeland et al. 2011). These orbits often deviate from the N’ characteristics of the steady map due to hysteresis between MFP and pressure ratio (PR) (Palfreyman et al. 2005). Mass flow and efficiency estimations from the steady performance curve are subject to this hysteresis error and may not accurately represent the instantaneous condition.

![Figure 1.14. Pulsed flow characteristics (adapted from Abidat et al. 1998)](image)

Large variations in turbine inlet conditions generate unique flow features and loss mechanisms, including reversed flow, large variations in blade angles, and negative power (i.e. the turbine does work on the flow), as explored by Karamanis et al. (2001), Palfreyman and Martinez-Botas (2005), and
Copeland et al. (2011). The direction of the relative velocity vector \( W \) can vary considerably, even for constant rotor speed \( U \), due to the fluctuation of the stator exit velocity \( C \) (Figure 1.15). Large deviations from the design flow angles can generate significant losses; negative incidence is associated with a diminished capacity to turn the flow and extract work, while very positive incidence may cause separation losses. Depending on the pulse shape and amplitude, substantial portions of the cycle may be spent at off-design flow angles, and rotor designs with poor off-design performance will suffer accordingly (Karamanis et al. 2001, Hellström 2008). If the pulse cycle includes periods of nearly quiescent flow, the rotor experiences “windage” losses due to friction at the rotor boundaries (Daneshyar et al. 1969). Multiple admission sites with out-of-phase pulses as employed in Copeland et al. (2012) can mitigate periods of windage, and increase the percentage of pulse-active time per cycle (i.e. pulse duty cycle), but generate complex interactions between inlets.

Recent studies of coupled PDC-turbine systems highlight some additional aspects of detonating inflow. A visualization study of detonation interaction with a cascade (Rasheed et al. 2004) indicates strong shock reflection off the stator vanes, and establishment of choked flow at the stator exit. Laser-based diagnostics employed by Rouser et al. (2014) indicate unsteady mass storage and expulsion, highly concentrated pulses of total enthalpy, and periods of reversed flow possibly associated with shock

![Figure 1.15. Variation of flow angles over a pulse cycle (Karamanis et al. 2001)](image-url)
reflections. The simulations of Nango et al. (2008) indicate peak blade forces an order of magnitude greater than time averaged values, which can accelerate the rotor and produce oscillatory rotor speeds as in Caldwell and Gutmark (2008) and Xiaofeng et al. (2013). For multiple PDC admission sites, complicated interactions can occur for out-of-phase, sequential detonations. The simulations of Van Zante et al. (2007) indicate high velocity, transverse flow between adjacent, inactive admission sites (Figure 1.16). For the valveless PDC array of Rasheed et al. (2011), sequential detonations create interference between adjacent tubes, disrupting combustor operation. For the valved PDC array of Caldwell et al. (2008), out-of-phase interaction between two detonation plumes has a strong attenuating effect on the initial blast.

![Figure 1.16. Detonation exhaust interaction with a stator (Van Zante et al. 2007)](image)

### 1.6 Unsteady Turbine Efficiency Definitions

Assessing performance and defining appropriate figures of merit for turbomachinery operating under highly unsteady conditions is not yet standardized in the propulsion community. A clear, relative comparison between steady and pulsed flow efficiency is difficult to obtain, given the flexibility in the definition of ideal power extraction for unsteady conditions and the choice of an appropriate averaging technique (Rouser 2012, Paxson and Kaemming 2012, Van Zante et al. 2007, and Suresh et al. 2012). As such, this study considers five variants: the integrated unsteady definition (Eq. 1.9) and four “bulk” definitions (Eq. 1.10) using various averaging approaches. Time averaging (Eq. 1.11), “analog” capillary
tube averaging (using constant capillary tube averaged pressure values as bulk quantities), mass averaging (Eq. 1.12), and work averaging (Eqs. 1.13a,b) are considered in this study.

The integrated unsteady definition (Eq. 1.9) substitutes time-variant quantities into the standard, steady isentropic efficiency equation in place of constant values. The actual power term (numerator) and ideal power term (denominator) are integrated separately. The ideal power integrand contains an instantaneous enthalpy flux term and is strongly influenced by the flow distribution over the cycle. The bulk efficiency definitions (Eq. 1.10) are formulated using scalars for each of the thermodynamic quantities, where different averaging techniques are used to calculate bulk pressures. Time-averaging (Eq. 1.11) is the simplest approach, where the arithmetic mean of the pressure signal is used as the bulk value, and the instantaneous mass flux is not considered. The capillary tube average pressure (CTAP) method of bulk pressure calculation is based on heavily damped measurements, which are time-averaged over the cycle at the turbine inlet and exit. These are notionally similar to the time-averaging employed in Eq. 1.11, but are calculated from approximately steady capillary measurements. Mass-averaging (Eq. 1.12) is more complex, as it weights the total pressure by the instantaneous mass flux, neglecting the contribution of total pressure during periods of the cycle where flow has ceased.

\[
\bar{\eta}_{unsteady} = \frac{\int_0^{t_{cycle}} N(t) \cdot \tau(t) dt}{\int_0^{t_{cycle}} \dot{m}_{inlet}(t) \cdot c_p \cdot T_{0inlet}(t) \left( 1 - \left( \frac{P_{0exit}(t)}{p_{0inlet}(t)} \right)^{\gamma-1} \right) dt} \quad (1.9)
\]

\[
\bar{\eta}_{average} = \frac{\bar{N} \cdot \bar{\tau}}{\bar{\dot{m}}_{inlet} \cdot c_p \bar{T}_{0inlet} \left( 1 - \left( \frac{p_{0bulk}}{p_{0inlet}} \right)^{\gamma-1} \right)} \quad (1.10)
\]

\[
P_0^{TA} = \frac{1}{t_{cycle}} \int_0^{t_{cycle}} P_0(t) dt \quad (1.11) \quad P_0^{MA} = \frac{\int_0^{t_{cycle}} \dot{m}(t) P_0(t) dt}{\int_0^{t_{cycle}} \dot{m}(t) dt} \quad (1.12)
\]
Work-averaging (Cumpsty and Horlock 2006, Pianko and Wazelt 1982) is devised to find bulk total pressure values that provide equivalent ideal work extraction between steady and unsteady flows. By holding the outflow state constant and extracting equal work from both the unsteady inflow and an equivalent flow with fixed inlet total pressure, the work-averaged inlet total pressure can be calculated (Eq. 1.13a). Then, by holding the inflow state constant, work extraction is held constant between the unsteady outflow and an equivalent flow with fixed exit total pressure to find the work-averaged exit total pressure (Eq. 1.13b). While the work-averaged definition is the most complex of the three averaging types, it is the most physical meaningful for efficiency calculation, as it preserves the total power available for extraction from each infinitesimal unit of mass. It has been proposed by Cumpsty and Horlock (2006) as the most appropriate definition when dealing with non-uniform flow for flow through turbomachines. Unlike the pressures, the only appropriate method for averaging the enthalpy is mass-averaging (Eq. 1.14), as it conserves the total energy flux into the system.

\[
p_{\text{wa} \, \text{in}} = \left( \frac{\int_0^{t_{\text{cycle}}} \dot{m}_{\text{inlet}}(t) \cdot c_p \cdot T_{\text{inlet}}(t) \, dt}{\int_0^{t_{\text{cycle}}} \dot{m}_{\text{inlet}}(t) \cdot c_p \cdot \frac{T_{\text{inlet}}(t)}{(P_{\text{inlet}}(t))^{\gamma-1}} \, dt} \right)^{\frac{\gamma}{\gamma-1}} \tag{1.13a}
\]

\[
p_{\text{wa} \, \text{out}} = \left( \frac{1}{t_{\text{cycle}}} \int_0^{t_{\text{cycle}}} \left( \frac{P_{\text{outlet}}(t)}{P_{\text{exit}}(t)} \right)^{\gamma-1} \, dt \right)^{\frac{1}{\gamma-1}} \tag{1.13b}
\]

In cases where thermodynamic quantities cannot be accurately measured due to lack of instrumentation access or survivability, thermal efficiency (Eq. 1.15) can be used as an alternative approach to characterize performance. Ideal power is defined by the chemical energy content within the fuel (i.e. lower heating value), while the actual power is directly measured. This expression requires
less information to evaluate, but does not account for the availability prior to combustion. It is also somewhat misleading, as ideal detonation cycle thermal efficiencies are 30-50% for small amounts of pre-compression prior to detonation (Heiser and Pratt 2002).

\[
\eta_{\text{thermal}} = \frac{\text{Actual Power}}{\text{Chemically Available Power}} = \frac{N \cdot \tau}{\dot{m}_{\text{fuel}} \cdot LHV_{\text{fuel}}} \tag{1.15}
\]

Due to differences in boundary conditions and pulse generation techniques, there is also disagreement over the effect of pulse frequency on turbine performance. Ni et al. (2013) and Hellström (2008) claim that turbine efficiency drops with increased pulse frequency, Caldwell and Gutmark (2008), Copeland et al. (2011), and Fernelius et al. (2013) claim that performance improves, and Wallace et al. (1969) proposes the concept of an optimal frequency range.

Chen et al. (1996) hypothesize that performance is more sensitive to pulse amplitude, than to frequency. Copeland et al. (2011) further suggest a strong link between frequency and unsteady mass storage in the upstream geometry. This has been supported by the studies of Rajoo et al. (2010) and Fernelius et al. (2013), which observe lower amplitude fluctuations and larger duty cycles at increased pulse frequencies. The instantaneous mass flow through the turbine is largely dependent on geometry, and the pulse blowdown period will be lengthened at higher flow rates, promoting greater mass storage in upstream volumes. In the case of Ni et al. (2013), the upstream boundary conditions do not allow for sufficient mass storage and retain the poor performance associated with high amplitude, low duty cycle pulse operation. However, in most of the experimental studies, the upstream geometry is sufficient to support unsteady mass storage, and smaller inlet fluctuations are typically correlated with higher frequency operation. The overall flow variation at the rotor inlet is reduced, potentially improving or reducing performance. The net effect depends on whether or not the reduced range of inlet conditions is more favorable, especially the range of rotor incidence angle (Hellström 2008).
For PDC-turbine systems, pulse amplitudes are a result of the detonation process, rather than mechanical pulse generation, and are determined by the initial reactant state. The blockage imposed by the turbine results in elevated initial reactant pressures during the PDC fill phase. The presence of the turbine also prolongs the PDC exhaust phase (Schauer et al. 2003), extending the effective pulse duty cycle. Increasing the PDC cycle frequency mitigates unfavorable quiescent periods and further extends the duty cycle, improving performance (Rouser et al. 2013). To reduce the relatively large pressure amplitudes, Rouser et al. (2014) recommend partially filling the PDC tubes to allow the detonation to decouple and decay prior to turbine interaction. The simulations of Nango et al. (2009) achieve peak PDC work extraction at 150% of the design rotor speed, potentially due to favorable flow angles during the initial, high-enthalpy-rate exhaust period. To date, thermal efficiencies reported for single-admission PDC-radial turbine systems are typically below 10% (Schauer et al. 2003, Sakurai et al. 2005). Efficiencies of multiple-admission PDC-axial turbine systems are more promising, as Rasheed et al. (2011) and Glaser (2007) demonstrate efficiencies comparable to steady flow when PDC exhaust is mixed with a steady bypass flow.

**1.7 The Rotating Detonation Combustor**

While significant progress has been made in overcoming PDC technical challenges, a new type of detonation combustor, the rotating detonation combustor (RDC), has emerged as a popular alternative. An RDC is usually comprised of an annular channel, supplied continuously with reactants, which supports an azimuthally-propagating detonation wave (Figure 1.17). The channel is continuously fed with fresh reactants, which are consumed with each successive pass of the rotating detonation wave, and products are ejected axially downstream (Schwer and Kailasanath 2010). The azimuthal detonation velocity for a RDC is quite large (typically in excess of 1 km/s), yielding operating frequencies in the kilohertz regime (Lu and Braun 2014).
While the pulsed detonation cycle is typically modeled as an unsteady 1-D process (as in Figure 1.12), the rotating detonation cycle is commonly considered as a steady 2-D process in the wave-fixed frame (as in the temperature contour of Figure 1.17). By neglecting the radial dimension of the combustor and considering the flow field in the detonation frame of reference, the flow is approximately steady. There are several prominent features in the RDC flow field. Fresh reactants are injected at the base of the combustor, and products flow axially downstream and exit the combustor. In Figure 1.17, the detonation wave propagates from left to right into a wedge of unburned reactants. Above the unburned wedge of reactants are products from the previous instance of detonation passage, and deflagration burning can occur at the contact surface where these two regions meet. The detonation front terminates at the top of the reactant wedge, and creates an oblique shock in the products. Behind the detonation front, the newly-generated products and old products form a shear layer. Near the upstream end of the combustor, the reactant injection is temporarily halted by high pressure products, but gradually recovers as the detonation products expand and drop in pressure. Other flow features (not pictured in the present example), may occur for various inlet and exit conditions, such as secondary, reflected shocks, or reversed flow and complex shock structures in the reactant plenums (Schwer and Kailasanath 2011).
While RDCs were first developed and tested more than half a century ago (Voitsekhovski 1959), RDC research has broadened significantly in the last decade, partially due to the potential benefits over the historically-dominant pulse detonation combustors. The continuous, valveless injection of reactants in RDCs is mechanically superior to valved PDC designs, as it results in reduced weight and complexity, and reduces the unsteadiness imposed on upstream components. Furthermore, precise timing and control is generally limited to a single RDC initiation event, compared to the repeated cyclic initiation required in PDC operation. As the RDC operates in the kilohertz regime, exhaust pulsations are more easily mixed out, reducing the unsteadiness on downstream components (Yi et al. 2011), as seen in Figure 1.18. While initiation is still difficult to achieve for less-detonable fuel-air mixtures, transition to detonation in an RDC is only required upon startup, as detonation can be continuously maintained. Geometrically, the RDC is more compact than the PDC, and its annular shape may improve ease of integration with annular engine components. These favorable aspects have spurred renewed worldwide interest in RDC research at institutions in Russia (LIH), Poland (Warsaw Univ. of Tech.), France (ENSMA, MBDA), Japan (Nagoya Univ., JAXA), China (Peking Univ., Nanjing Univ., National Univ. of Def. Tech.), and the United States (AFRL, NPS, NRL, DoE, Aerojet-Rocketdyne, Univ. of Texas, Arlington, Univ. of Cincinnati, Univ. of Washington, Seattle, Purdue Univ.).

![Figure 1.18. Performance comparison of rotating and pulsed detonation concepts (Yi et al. 2011)](image-url)
Despite these advantages, there are significant technical challenges for RDCs that impede widespread adoption. Achieving adequate mixing of reactants remains a challenge for non-premixed designs (Bykovskii et al. 2006a, Nordeen et al. 2013, Cocks et al. 2016), while flashback remains a considerable problem for premixed designs as discovered in the UC facility, and as addressed in detail by the premixed design concept of Andrus et al. 2016a. Initiation and stable operation of less-detonable fuels remains difficult, especially when using air as an oxidizer (Bykovskii et al. 2006b, Dyer et al. 2012, Kindracki 2015). RDCs are subject to a range of various instabilities (Anand et al. 2015b), which can induce partial or complete failure of the detonation. The number of detonation waves (n) within the combustor and the propagation direction of these waves are difficult to predict a priori for a given geometry and flow condition. Excessive heat loading can lead to failure or reduced lifespan of the combustor and downstream components without the addition of active cooling or secondary flow (Tellefsen 2012, Naples et al. 2014, Theuerkauf 2013). These challenges highlight the critical need for RDC research to unlock the capabilities of detonation-based cycles.

1.7.1. RDC: Initiation
Tangentially-injecting pre-detonator initiator tubes are commonly used for initiation in RDCs (Dyer et al. 2012, Suchocki et al. 2012, Frolov et al. 2015, Liu et al. 2012, Wang et al. 2015, Kindracki et al. 2011). A detonation is established in the initiator tube using a highly detonable fuel-oxygen mixture, and propagates into the RDC annulus (Figure 1.19). For successful transmission of the detonation wave into the annulus, the detonation must simultaneously adapt to diffraction into a larger volume and changing mixture composition (Touse 2003). Considering the diffraction process, critical initiator tube diameters for successful transmission into an unconfined space are on the order of 10 mm for fuel-oxygen mixtures and 250-1000 mm for fuel-air mixtures (Schultz and Shepherd 2000). For diffraction into a confined volume, such as a thin channel, reflection of the emerging detonation with the confining walls is the main mechanism of initiation (Murray and Lee 1983). Considering detonation behavior at an abrupt
change in mixture composition, the initial fuel-oxygen detonation transmits a strong blast wave (i.e. leading shock followed by an expansion) into the target mixture as it reaches the interface (Kuznetsov et al. 1997). For a sufficiently strong blast wave and high target mixture reactivity, an exothermic reaction couples with the leading shock and stabilizes to a detonation.

![Diagram of Initiator tube injecting a detonation into the RDC channel](image)

**Figure 1.19. Initiator tube injecting a detonation into the RDC channel**

For many practical fuel-air mixtures, energy deposition from initiator tubes may not attain the critical value for direct initiation (i.e. subcritical energy). A delay is observed between ignition process and the onset of rotation within the RDC for hydrogen-air (Frolov et al. 2015, Liu et al. 2012), hydrogen-vitiated air (Wang et al. 2015), and methane-oxygen (Kindracki et al. 2011) mixtures. Frolov et al. (2015) report that the duration of the starting transient depends on combustor geometry, the initiating detonation, and the reactivity of the mixture. Miller et al. (2013) developed a linear, “unwrapped” RDC apparatus to visualize the initiation process for H₂-air mixtures with a gaseous H₂-O₂ initiator tube. The leading shock decouples significantly from the combustion front, indicating subcritical initiation. The resulting
deflagration combustion front must undergo a deflagration-to-detonation transition (DDT) process to reestablish the detonation.

Figure 1.20. Subcritical initiation in a 2-D “unwrapped” RDC channel (adapted from Miller et al. 2013)

In addition to initiator tubes, spark ignition has been used for mixtures with lower initiation energy (Nicholls et al. 1966), and charges of electrically-detonated high explosive have been used for mixtures with high initiation energy (Bykovskii et al. 2006b). More recently, Bykovskii et al. (2014) utilized multiple initiation strategies to achieve detonation of fuel-air mixtures in an RDC, including a low-power heat pulse, injection of a product jet, and direct transmission of a detonation. For the detonation transmission case (highest power level ignition source), none of the experiments resulted in a direct transition of the initiating wave into a stable rotating detonation.

1.7.2. RDC: Starting Transients

The transitory period, which follows the RDC ignition event and concludes with the onset of a stable, periodic operating mode, is hereafter referred to as the onset phase. This onset phase is not unique to a single facility or combustor geometry and has been observed and discussed in several studies (Bykovskii et al. 2014, Frolov et al. 2015, Liu et al. 2012, Kindracki et al. 2011). All the cases explored by Bykovskii et al. (2014) exhibited a transitional process of 4-80 ms duration, which is associated with the recovery of uniform air injection for detonation transmission, or the development of tangential instability and
subsequent DDT when using the other ignition sources. Frolov et al. (2015) report similar behavior, and a chaotic, aperiodic phase is evident in the ionization probe traces after the ignition event. In the pressure traces of Kindracki et al. (2011), the onset phase appears as a period of low activity (i.e. no discernible high-amplitude pressure oscillations, Figure 1.21). This concludes within several milliseconds with the development of rotation, and a marked increase in the amplitude of pressure oscillations.

![Figure 1.21. Pressure traces of the onset phase (Kindracki et al. 2011, CH₄-O₂)](image)

Peng et al. (2015) used a 30mJ spark plug to initiate an RDC, and report onset time (reported in that work as “DDT time”) of 1-7 ms between the ignition event and the onset of detonation (Figure 1.22a). The duration is relatively stochastic, but appears to depend partially on the operating condition. This appears to loosely support the hypothesis put forth by Bykovskii et al. (2014) that the reactant injection pressure ratio has a governing effect on the duration of the onset phase. A continuation of that study (Yang et al. 2016) compared various high and low energy ignition sources, including a pre-detonator (i.e. H₂-O₂ “thermojet”), and indicates that the combustor converges to an identical end (terminal) state, regardless of the ignition energy (Figure 1.22b). While there is still a large degree of variability in the onset time for the same case (±50%), the higher energy ignition source yields lower average onset time.
1.7.3. RDC: Reactivity Enhancement

For less-detonable hydrocarbon fuels, such as propane and kerosene, tested by Bykovskii et al. (2006b), detonation could not be established in an RDC, even with three initiators. However, significant oxygenation of the oxidizer (β = N₂/O₂ = 1) coupled with the use of high explosive was sufficient to improve the mixture detonability and achieve successful initiation within the combustor. Another RDC developed by Dyer et al. (2012) experienced intermittent initiation for ethylene-air mixtures, but achieved successful operation for a slightly oxygenated mixture (24.8% O₂ content oxidizer, i.e. β ≈ 3).

Other researchers have considered the use of hydrogen as a fuel additive to improve ignition characteristics and increase detonability (Zhang et al. 2012, Lu et al. 2003). For methane-hydrogen fuel blends, significantly faster ignition was observed only for large fractions of hydrogen (composing 80% of the fuel by volume), with the added effect of complex pressure dependence on the ignition behavior (Zhang et al. 2012). The complex pressure dependence of hydrogen ignition is due to the second explosion limit for hydrogen, where chain-branching reactions necessary for rapid radical production are suppressed at high pressures by chain-termination reactions due to an increase in third-body collisions (Lu et al. 2003). Hydrogen was also implemented as a fuel additive in an RDC to gradually transition from hydrogen-air mixtures to kerosene-air mixtures (Lukasik et al. 2013). Sustained detonation for
kerosene-air mixtures for hydrogen-assisted lightoff was only demonstrated with the addition of 20% isopropyl nitrate to the kerosene. Significant preheating of the reactants ($T_{\text{air}} = 400$ K, $T_{\text{fuel}} = 350$ K) allowed for successful detonation of kerosene-air mixtures without the need for additives at relatively rich conditions ($\phi = 1.4$). For many of these cases, pressure amplitudes appear unstable and necking is observed in the pressure evolution, indicating occasional extinction of the detonation (Figure 1.23).

![Figure 1.23. Hydrogen-assisted kerosene-air operation in an RDC (Lukasik et al. 2013)](image)

1.7.4. RDC: Stability

In the present facility, stable rotating detonation can be established with a single rotating wave ($n = 1$), or multiple co-rotating waves ($n = 2, 3$, etc.) which propagate in the same azimuthal direction (Figure 1.24). Multiple, counter-rotating waves can hypothetically form, but are not recognizable as a stable periodic operating mode in the present facility. However, for many combustor configurations and operating conditions, rotating detonation is temporarily established, but exhibits some degree of instability. Four commonly observed instability types have been described in detail by Anand et al. (2015b) as: 1.) chaotic instability, 2.) waxing and waning (low frequency instability), 3.) mode switching, and 4.) longitudinal pulsed detonation (LPD). In addition to these unstable behaviors, the detonation can also fail entirely and reignite/re-establish itself, and the propagation direction can spontaneously change (e.g., from clockwise to counterclockwise).
The chaotic instability is marked by the appearance of detonation-magnitude pressure oscillations which are relatively aperiodic (Figure 1.25a). There is a period of low activity in the pressure signal, bordered by two relatively large pressure peaks (indicated by arrows). While periodic behavior does begin to develop at $t = 0.235s$, the oscillation amplitudes are not consistent, and the spacing between peaks is somewhat erratic. For this instability type, brief “packets” of rotation are typically observed, interspersed by primarily chaotic behavior. This is reflected in the FFT of the pressure evolution (Figure 1.25b), which indicates a broad dominant frequency “plateau” between 2.5 and 3 kHz. Some of the individual packets of detonation rotation occur at higher or lower speeds, which smear the effective operating frequency across a wide band.

---

**Figure 1.24. Stable detonation wave rotation with a single or multiple wave fronts (Driscoll 2016)**

---

**Figure 1.25. Example of chaotic detonation propagation in an RDC (Anand et al. 2015b)**
The waxing and waning instability is marked by relatively sinusoidal oscillations in the amplitude of pressure fluctuations (Figure 1.26a). This may correspond to a waxing and waning in the perceived strength of the detonation wave, though the simulations of Wu et al. (2014) suggest that this may reflect oscillations in the height of the detonation wave. The waxing and waning frequency essentially modulates the relatively constant rotation frequency and appears in the FFT as a small peak at ≈235 Hz (Figure 1.26b). As this roughly corresponds to the Helmholtz frequency of the air plenum upstream of the combustor, Anand et al. (2016a) attribute this instability to resonant coupling between the combustor and the reactant feed system.

![Graph of pressure evolution](image1)

**a)** Detail of pressure evolution  **b)** FFT of pressure evolution

*Figure 1.26. Example of low frequency instability (LFI) in an RDC (Anand et al. 2015b)*

The mode switching instability is marked by a sudden change in the operating frequency, which corresponds to a change in the number of detonation waves in the combustor (e.g., \( n = 1 \rightarrow 2 \)), or a sudden change from rotating detonation to longitudinal pulsed detonation. This instability can occur in transition regions where the reactant mass flow and mixture detonability are sufficiently high to spontaneously form additional detonation waves. A mode switching event is typically discernible in the pressure trace from a change in the spacing between consecutive peaks (Figure 1.27a), but is easier to detect in a spectrogram (Figure 1.27b) as a rapid shift in the fundamental frequency. For this particular
case, the combustor appears to be more stable in the multiple wave, $n = 2$ state, than the single wave $n = 1$ state, as the peak frequency is sharply defined in the latter portion of the test.

![Graphs of pressure evolution and spectrogram](image)

**Figure 1.27. Example of mode switching in an RDC (Anand et al. 2015b)**

Longitudinal pulsed detonation is a unique instability marked by the presence of repeated, axially-propagating detonation waves in an RDC (Figure 1.28a). The entire process is self-sustaining and extremely periodic, making it difficult to distinguish from rotating detonation without a set of azimuthally-distributed sensors. When the combustor experiences LPD, all azimuthal sensors become phase-locked (Figure 1.28b), and the system behaves as a high-frequency, valveless pulse detonation combustor. In the present facility, this instability is only observed for cases utilizing a converging exit nozzle, which correspond to instances of subsonic air injection.

![Graphs of sensor placements](image)

**Figure 1.28. Longitudinal pulsed detonation in an RDC (Anand et al. 2015b)**
1.7.5. RDC: Heat Transfer

Under typical stable operation, heat loading into the combustor walls is severe, and can damage the combustor hardware, downstream hardware, or sensors located in the flow path. Time-averaged fluxes are on the order of ≈ 1 MW/m² as reported by Bykovskii and Vedernikov (2009), while instantaneous heat fluxes of ≈ 20 MW/m² have been reported by Theuerkauf et al. (2016). Numerical simulations and experiments suggest that the highest peak heat fluxes occur at the detonation front (Figure 1.29), though injection of fresh reactants provides a cooling effect (negative heat flux). Axially downstream of the detonation, the walls are exposed to continuous positive heat flux.

![Image of heat flux into the outer wall of an RDC](image)

**Figure 1.29. Measured and computed heat flux into the outer wall of an RDC (Theuerkauf et al. 2016)**

This has impeded efforts to integrate a turbine downstream of an RDC, and triggered catastrophic failure of the turbine unit within 2s after combustor ignition (Tellefsen 2012). While thermally-managed devices have been developed for continuous operation, such as the water-cooled RDC developed by Theuerkauf (2013), most RDCs under active study are uncooled. In the present facility, the extreme thermal environment limits the combustor operating time to several seconds. Furthermore, excessive heat transfer creates problems for sensors: PCB sensors exposed to high temperatures will experience severe signal distortion (Walter 2004), eventual signal loss (Figure 1.30).
1.7.6. Detonation Limits and Scaling

The regime in which an RDC can successfully sustain a detonation is a function of the reactant mixture, and the minimum required size of the combustor is a function of the mixture cell width. Bykovskii et al. (2006a) propose minimum, critical values of combustor geometric parameters, including the width of the channel \((w_{ch})\) and the height of the reactant mixture \((h)\) ahead of the detonation wave (i.e. the detonation wave height). The domain of detonation existence can be defined with respect to these parameters, normalized by the cell size \((w_{ch}/\lambda, h/\lambda)\). For the regime where detonation is possible, the number of waves within the combustor is primarily determined by the volumetric flow rate of reactants compared to the critical value of reactant height required to sustain a detonation (Wolanski 2013).

Considering the effect of channel width on the RDC behavior, Bykovskii and Zhdan (2015) note that the minimum channel width in RDC is subject to the same constraints as for classical detonation propagation in plane channels. Applying the scaling limits of Vasil’ev (1982), the width of a rectangular channel must be on the order of the cell width for successful onset and propagation of detonation, with marginal propagation possible for as low as \(w_{ch}/\lambda \geq 0.5\). Below this threshold, the detonation will fail, despite possibly supporting multiple transverse waves in the height dimension \((h >> \lambda)\) (Vasil’ev 1987). The study of Kindracki et al. (2011) appears to substantiate this limit, as the stability of detonation...
propagation for what appears to be a stoichiometric H₂-air mixture ($\lambda \approx 12$ mm) is considerably improved when the channel width is increased from 5 mm ($w_{ch}/\lambda \approx 0.4$) to 10 mm ($w_{ch}/\lambda \approx 0.8$). The scatter in detonation wave speed is considerably reduced and the peak pressures for consecutive waves are consistently higher and more uniform.

![Graphs showing pressure and velocity over time for different channel widths](image)

\[ w_{ch} = 5 \text{ mm} \quad \text{and} \quad w_{ch} = 10 \text{ mm} \]

**Figure 1.31. Effect of channel width on RDC operation (Kindracki et al. 2011)**

For channel widths in excess of the critical value, significantly increasing the channel width has a complex impact on the flow field. When employing a wide channel, Bykovskii *et al.* (2006b) report a change in the structure behind the detonation front from a single concentrated heat release region, into a distributed, oscillatory heat release region. The numerical study of Zhou and Wang (2013) shows a marked increase in non-uniformity and curvature of the detonation front when the channel width is increased (Figure 1.32). Powerful, secondary shock wave structures reflect between the inner and outer walls of the channel, yielding complex, secondary heat release regions. While large channel widths may widen the range of mixtures which can successfully detonate within the channel, it is unclear if the complex radial variations in the flow field have a negative impact on the detonation process.
Bykovskii et al. (2006a) suggest that the minimum wave height to sustain detonation is $h/\lambda = 12\pm 5$, when neglecting physical processes, such as mixing, that increase this value. Frolov et al. (2015) report wave height $h = 100-130$ mm for an H$_2$-air mixture in a combustor with no exit blockage, which yields $h/\lambda = 8$-11, assuming a cell width of $\lambda = 12$mm. Rankin et al. (2015a) have obtained high speed chemiluminescence images of rotating detonations with stoichiometric H$_2$-air (Figure 1.33), which indicate detonation heights on the order of 20-50mm ($h/\lambda \approx 2-5$). Anand et al. (2015a) note detonations supported for the range $h/\lambda \approx 0.5-7$ and note the appearance of multiple waves for $h/\lambda > 7$ for choked reactant injection. Considering the disparate range of $h/\lambda$ between these studies, it is possible that the combustor geometry and mixing quality strongly influences the critical value of normalized wave height.

Figure 1.32. RDC flow structure for three channel widths (Zhou and Wang 2013)

Figure 1.33. Instantaneous chemiluminescence of the detonation front (Rankin et al. 2015a)
An increase in the number of detonation waves ($n$) has been correlated to surpassing an integer multiple of a critical fill height of fresh mixture ($h > n\cdot h_{\text{critical}}$) (Wolanski 2013, Bykovskii et al. 2006a). The number of fronts is governed by a self-correcting mechanism, where formation of an additional front beyond the equilibrium number causes at least one of the fronts to be of insufficient size and fail (Voitsekhovskii 1959). Likewise, for fewer than the equilibrium number of fronts, the height of the unburned mixture reaches the critical size prior to the arrival of one of the existing detonation waves and can be spontaneously ignited and form a new wave. Bykovskii et al. (2006b) observe $n = 3 \rightarrow 1$ for H$_2$-air mixtures as the equivalence ratio and mass flux varies (Figure 1.34).

**Figure 1.34. Evolution in the number of detonation waves from $n = 3 \rightarrow 1$ (Bykovskii et al. 2006b)**

Bykovskii et al. (2006b) attribute this evolution to changes in the fill height relative to the critical value, though only a single front ($n = 1$) is observed for unchoked air injection. Zhdan and Syryamin (2013) consider the number of detonation waves as an eigenvalue problem, and numerically determine that the stable state of the solution is for the shortest possible domain length which supports detonation. This implies that the combustor reaches a state of equilibrium for the highest number of possible waves (i.e. $n$ should naturally maximize itself).
1.8 Objectives of the Present Research

This dissertation spans a series of investigations on detonation-based combustors, grouped by combustor type into two categories: pulsed detonation and rotating detonation. As such, the present research can be considered as two, independent sections. The first two investigations focus on the integration of an array of pulsed detonation combustors with a turbine. The latter six investigations focus on characterizing the fundamental processes and operating behavior of a rotating detonation combustor.

1.8.1. Objectives of the PDC-Turbine Research

The principal objective of this research is to determine the component efficiency of a turbine operating under pulsed, detonating inflow conditions. As diagnostic options are significantly limited in this harsh measurement environment (high instantaneous heat fluxes, shock waves, extreme unsteadiness, etc.), the pulsed detonating flow investigation is preceded by a pulsed air flow investigation. The objective of the pulsed air investigation is to characterize the turbine efficiency under the surrogate case of highly unsteady pulsed air inflow. The turbine behavior under these conditions can be applied to the more severe pulsed detonating flow investigation. The objective of the pulsed detonating flow investigation is to characterize the turbine performance, primarily using global measurements of reactant flow rates and output power. This study attempts to assess the impact of various PDC parameters and turbine loading on power extraction.

1.8.2. Objectives of the RDC Research

The principal objective of this research is to identify the necessary conditions to establish and sustain stable rotating detonation for a given combustor configuration and reactant mixture. In support of the other RDC investigations, the first investigation develops a new technique to improve interpretation of RDC experimental data. A correlation-based approach is developed to detect the combustor operating
state. This method provides robustness (useful for noisy measurements) and flexibility (effective for multiple sensor types and locations).

The second investigation focuses on the initiation of an RDC by a tangentially-injecting initiator tube. Its objective is to quantify the energy deposition for this type of initiation method and assess whether direct initiation is possible for fuel-air mixtures. Building upon those results, the third investigation explores the starting transients following the initiation event, which culminate in detonation onset. The goal of the onset investigation is to assess whether the onset duration and behavior can be predicted for a given combustor operating condition.

The fourth investigation uses detonability enhancement of reactants to attempt to achieve stable operation of a less-detonable fuel-air mixture. Stable rotating detonation could not be achieved for ethylene-air mixtures in the present RDC geometry. Moderate addition of hydrogen is used to achieve stable operation with ethylene-hydrogen-air mixtures. The primary goal of this investigation is to determine the efficacy of establishing stable operation with a highly detonable mixture, then transitioning to a less detonable mixture. Building upon this experimental investigation, the fifth investigation employs numerical analysis and detailed chemical kinetics to estimate the detonation cell width of various reactant mixtures. The objective of this study is to evaluate the relative improvement in detonability for fuel-oxidizer mixtures for varying amounts of pressurization, heating, H₂ substitution in the fuel, and O₂ substitution in the oxidizer. These numerical estimates indicate the optimal detonability-enhancing strategy for this class of mixtures, which can be implemented within the RDC to achieve successful operation with ethylene.

The previous investigations highlight the dominant role of detonation cell width on the regime of successful RDC operation. The objective of the sixth investigation is to quantify the effect of the normalized detonation height \((h/\lambda)\) and normalized channel width \((w_{ch}/\lambda)\) on the existence and
multiplicity \((n)\) of rotating detonation. This investigation attempts to isolate the kinetic effects (i.e. \(\lambda\)) from fluidic effects (reactant injection pressure ratio) by holding the oxidizer flow rate constant and varying the oxidizer composition from pure air \((\beta = 3.76)\) to oxygen-enriched air \((\beta = 2)\).
Chapter 2 – Experimental Facility

2.1 Overview of the Detonation Test Facility

2.1.1. Overview of Detonation Systems

All the present experimental investigations are conducted within the Detonation Test Facility, a subset of the Gas Dynamics and Propulsion Laboratory (GDPL) at the University of Cincinnati. This work utilizes two detonation systems (Figure 2.1): the pulse detonation combustor (PDC) turbine system, and the rotating detonation combustor (RDC) system, addressed in detail by Munday et al. (2013), and St. George et al. (2015) respectively. For the PDC-turbine investigations, the PDC-turbine system is operated in pulsed-air and pulsed-detonation modes. Both systems are contained within a remotely-located test bunker, partitioned off of the main GDPL facility.

![Figure 2.1. UC Detonation Systems: (a) PDC-turbine system, and (b) RDC system](image)

2.1.2. Test Bunker

The test bunker is located underground to the east of the GDPL, and is connected to the main laboratory by an outdoor walkway (colored dark gray in Figure 2.2). The foundation of the outdoor walkway is several meters below the ground level, and this walkway accommodates two fuel storage and delivery systems (colored red). The test bunker measures 5.3m x 4.1m, and offers ample space to contain and operate both the detonation systems. The bunker walls are comprised of 30 cm thick reinforced
concrete to fully contain any potential catastrophic failures and provide protection from fires. During experiments, operators are stationed at the control area (blue), located within the main laboratory building. In addition to protecting operators from system failures, this layout also shields the operators from dangerously high noise levels (>160 dB measured by Glaser 2007) which occur during typical detonation experiments. The isolated outdoor location of the test bunker also isolates combustion exhaust products from the main laboratory building and simplifies exhaust/ventilation.

![Figure 2.2. Schematic of the detonation test facility](image)

**2.1.3. Air and Fuel Supply**

High pressure compressed air is supplied to the test bunker via a two inch schedule 40 pipe, and provides up to $\dot{m}_{\text{air}} = 1.2 \text{ kg/s}$ at pressures up to approximately 12 bar (denoted in blue in Figure 2.3). This supply is isolated from the test bunker by a pneumatically-driven isolation valve within the workshop area, which is manually controlled by the operators from the control area. After entering the
bunker, this supply line splits into two independent air streams, which are isolated from the flow regulation and metering system via manual ball valves.

Figure 2.3. Schematic of the air system and high capacity fuel system

Two reactant delivery systems supply gaseous reactants to the test bunker: the high capacity fuel system and the low capacity fuel system. The high capacity fuel system is located within the outdoor walkway, approximately 20 m from the detonation combustors (Figure 2.3). This system supplies the primary fuel source to both detonation systems and is compatible with a range of gaseous fuels. It consists of four compressed gaseous fuel cylinders, independently regulated and linked to a common fuel manifold. Gaseous fuel (e.g. ethylene or hydrogen) is regulated from cylinder pressures of 20-150 bar down to a working pressure of approximately 15 bar. The system can achieve a maximum flow rate of $m_{H_2} = 28 \text{ g/s}$ for hydrogen, and $m_{C_2H_4} = 105 \text{ g/s}$ for ethylene. This gaseous fuel supply is isolated from the test bunker via a manually-controlled ball valve and a pneumatically-operated angle seat valve.
After the isolation valves, the stainless steel supply line (1.9 cm outer diameter) extends directly from the high capacity fuel system into the test bunker.

A cylinder of high pressure gaseous nitrogen is located alongside the fuel cylinders, which is regulated from a cylinder pressure of up to 140 bar down to a working pressure of 12 bar. This serves as an inert pilot gas to control pneumatically-driven safety valves, as well as electronic proportional control valves for gas regulation. A system of normally-open and normally-closed solenoid valves supplies and vents the nitrogen supply from the pneumatic safety loop. These solenoid valves are powered from an independent 12V DC voltage source as part of the emergency stop system, allowing the operator to manually terminate gas flow in the event of a computer failure.

The low capacity fuel system is also located within the outdoor walkway, approximately 10 m from the detonation combustors (Figure 2.4). This system serves as the reactant source for the RDC initiator, and can also be used as a secondary reactant source for the RDC. When employed for the latter purpose, this system can provide supplemental fuel to blend with the RDC primary fuel (i.e. dual fuel capability), or supplemental oxygen to blend with the RDC air supply (i.e. oxygen enrichment). This system is typically configured to use three compressed gas cylinders (fuel, oxygen, and nitrogen), but can also be configured to simultaneously use two fuel cylinders, or two oxygen cylinders to increase the reactant capacity. These gases are regulated at the fuel system from cylinder pressures of 20-150 bar down to a supply pressure of approximately 18 bar prior to interfacing with the delivery cart. The delivery cart contains a series of pneumatically-driven normally-open and normally-closed valves to isolate the reactants from the test bunker, and vent these gases out of the bunker in the event of an emergency. These pneumatic valves are controlled by a laboratory “shop” air supply (7 bar), administered to the delivery cart via the gaseous control station in the control area. Downstream of the isolation valves, reactants are fed into the bunker via three supply lines (stainless steel tubing for oxygen and fuel,
copper tubing for nitrogen). Within the test bunker, reactants are isolated from the experiment by a set of manually-operated ball valves. Downstream of these ball valves, reactants are regulated from supply pressures of 18 bar down to working pressures of up to 10 bar prior to the experiment.

![Diagram of the low capacity fuel system](image)

**Figure 2.4. Schematic of the low capacity fuel system**

Air flow rates and primary fuel flow rates for both systems are controlled through interchangeable Flowmaxx sonic nozzles and pilot-operated Norgren regulators (Figure 2.5). The nitrogen supply from the high capacity fuel system (Figure 2.3) is supplied to a set of Norgren I/P electro-proportional control valves. These I/P valves receive an analog voltage signal from the control computer, and supply a set pressure to the dome-loaded slave regulators. The slave regulators (indicated with a white box in the figure) are directly in the flow path, and directly control the gas pressure upstream of the sonic nozzles. The sonic nozzle assemblies consist of three parts: an approach tube, a (converging-diverging) sonic nozzle, and an exit tube. These are designed for the gas supply to choke across the sonic nozzle,
allowing the flow rate through the assembly to be determined entirely from upstream flow properties. Each assembly accommodates several sonic nozzle sizes, which can be switched to vary the range of achievable flow rates for a fixed range of upstream supply pressures. Under nominal operation, the diverging portion of the C-D nozzle allows for significant total pressure recovery downstream of the throat, and choked flow is established for a pressure ratio of 1.2 or higher between the approach tube and exit tube. GE Unik 5000 pressure transducers are used to measure pressure upstream and downstream of the sonic nozzles as well as the barometric pressure, and Omega type-T thermocouples are used to measure upstream temperature. The advantage of this approach over orifice plates and other flow metering techniques is significantly lower uncertainty in the flow rate measurement (<±1%) and the ability to isolate flow rate control from unsteady processes occurring downstream.

![Flow rate regulation and metering assembly](image)

**Figure 2.5. Flow rate regulation and metering assembly**

### 2.1.4. Safety System

A series of safety features are integrated into the detonation facility to ensure the safety of the operators and fully contain system failures. The reinforced concrete bunker constitutes a passive safety
system and shields operators from the experiment. A high flow rate exhaust ventilation system (7000 CFM) is installed within the test bunker to remove exhaust products or unburned reactants. This system uses a 3.7 kW Greenheck explosion-proof fan to provide six air changes per minute (change rate of 360/hr). The ventilation ducts simultaneously entrain flow from floor level, ceiling level, and experiment level to prevent accumulation of light and heavy gases. Additionally, potentially harmful gaseous levels are monitored with an RKI Instruments gas detection system. Monitored gases include combustible gases (i.e., hydrogen, ethylene, etc.), oxygen, and carbon monoxide.

The test bunker is continuously monitored by a closed-circuit television (CCTV) system with a set of four cameras, providing multiple viewing angles and digital video recording (DVR) for playback of footage. Two cameras are located at ground level, above the outdoor walkway, to monitor for foot traffic in the corridor. This ensures that non-laboratory personnel are at a safe distance from the facility during hot fire testing and are not exposed to dangerous noise levels or system failures. These cameras are equipped with infrared functionality, enabling visual monitoring of the test bunker and outdoor areas during low light conditions, typical of the detonation tests (conducted during night hours).

An integrated 12VDC emergency stop circuit provides instant shutdown capability for the ignition circuit and all fuel and oxidizer delivery systems. The E-stop circuit is powered by a 12V automotive battery connected to a 15A battery charger. The positive lead from the battery proceeds through a series of four manual E-stop buttons, located at the control area, the workshop entrance, the walkway entrance, and on the data acquisition cart. After the final E-stop button, the circuit is gated by a relay, which requires a digital signal from the control computer to open and administer power to the detonation systems. When powered, the E-stop circuit controls:

- injection of reactants from the low capacity fuel system,
- the solenoids which control inert gas flow to pilot regulators of the high capacity fuel system
• the ignition module for both detonation systems
• the interlock for the electric motor (used for valving of the PDC-turbine system)

The state of the E-stop loop is also monitored by the data acquisition system, resulting in a complete interlock. A manually-triggered E-stop event will be followed by automatic shutdown of the system from the control computer (and a computer-driven E-stop event will disrupt power to the circuit).

2.1.5. Data Acquisition (DAQ) and Control System

Data acquisition (DAQ) and control of the facility are achieved using a high bandwidth National Instruments control and acquisition system. Located within the test bunker is an 18-slot, NI PXIe-1065 system, remotely operated through a fiber optic cable, allowing for up to 3 GB/s communication. System control and data acquisition includes:

• 16 channels of low speed analog output (up to 10 kHz)
• 14 channels of high speed counter/digital output (up to 10 MHz)
• 32 channels of low speed analog input (up to 10 kHz)
• 24 channels of high speed analog input (up to 5 MHz)

2.2 Pulse Detonation Combustor-Turbine System

2.2.1. Description of Pulse Detonation Combustor Array

A Chevrolet Vortec 4200 six-cylinder, automotive engine head (Figure 2.6) has been repurposed as an array of high-speed valving for pulse detonation combustors. The valve train is driven by a 37kW (50 hp), three-phase electric motor, with speed controlled by a variable frequency drive. A series of gears and timing belts connect the motor to the crank gear, which rotates at 2:1 relative to the motor. Within the engine head, the rotation speed is geared back down 1:2 between the camshafts and the crank gear, such that the rotation speed of the camshafts is equal to that of the electric motor. The valve pulse frequency is varied from 5 to 20 Hz, and is controlled in a closed feedback loop using the stock, Hall effect camshaft position sensor. Using the stock camshaft arrangement, the valves actuate sequentially
with 60 deg. phase separation between pulses in the stock firing order (1-5-3-6-2-4). The valves are open for approximately one third of the valve cycle (33% duty cycle), yielding roughly 50% overlap between adjacent pulses.

![Crank Gear, Mating Interface, Electric Motor, Vortec 4200 Engine Head, Gear Assembly, Oil Pump](image.png)

**Figure 2.6. High speed automotive valving system**

The valve train is lubricated by an electrically-driven oil system, consisting of a Graco Apex oil pump, an aluminum oil tank, and supply and return manifolds. Automotive oil is siphoned from the oil tank and passes through a filter prior to entering the oil pump. Downstream of the pump, a bypass line regulates the supply pressure, and oil is distributed to the valve galleries and crank gear box. As the engine head is installed horizontally (in contrast to its usual vertical installation in an automobile), ports have been drilled in the bottom of the engine head to allow the oil to return to the supply tank.

The stock intake and exhaust manifolds have been replaced by a custom intake manifold for premixed reactant injection and a custom exhaust manifold for purge air injection (the exhaust valves operate in reverse from their intended automotive application) as highlighted in Figure 2.7. The PDC array utilizes two air supply streams: one air stream for combustion (i.e. “combustion air”) and the other for purging products from the previous detonation cycle and acting as an inert buffer gas (i.e. “purge air”). Both air
streams and the fuel stream are separated from the sonic nozzle assemblies by surge tanks, which damp out the unsteadiness generated by the valving and maintain choked flow at the flow metering. The combustion air mixes with the fuel stream in a mixing junction approximately one meter upstream of the premixed reactant manifold. The premixed reactant manifold consists of a cylindrical plenum, a blocker plate interface (which can restrict reactant flow to fewer than six combustor tubes), and six, equal length runners which mate to the engine head. The purge manifold consists of a cylindrical plenum and six manual ball valves, which can restrict purge flow to fewer than six combustor tubes. When operating the system in the pulsed air mode, the premixed reactant stream does not contain fuel and becomes the primary air stream, and the purge stream is not used.

![Figure 2.7. Reactant and purge air delivery scheme](image)

When operating the system in pulsed detonation mode, ignition of the combustors is achieved with an ignition module. This module is constructed primarily from aftermarket automotive ignition components, and consists of two aftermarket MSD DIS-4 ignition units, six MSD Blaster coils, and a custom box of Crydom solid state relays for triggering (Figure 2.8). The stock Hall effect sensor monitors
the camshaft position and sends a 0-5 VDC signal to the DAQ system. For each rotation of the camshaft, a single 5VDC pulse occurs, and the rising edge is used as the trigger signal for the ignition timing. Within the DAQ system, counter circuits are used to generate a single retriggerable pulse output for each of the six combustors. The time delay of the counter-based ignition output relative to the camshaft trigger signal is calculated using the firing order of the engine and the camshaft rotation frequency. The counter ignition command signals are transmitted from the DAQ cart to the ignition module through a bundle of BNC coaxial cables and terminate into the solid state relay box.

![Figure 2.8. Schematic of ignition module](image)

The wiring harnesses from the dual MSD DIS-4 modules are routed into the solid state relay box, and contain trigger wires (inputs) and coil wires (outputs) for each spark circuit. When this ignition system is installed in a typical automotive application, the spark is triggered by grounding the trigger wire to the
automobile chassis. In the present application, a set of normally-closed solid state relays are used to ground the trigger wires at appropriate times with a 0-5 VDC command signal from the control computer. The outgoing coil wires are routed out of the relay box via a pair of MIL-SPEC electrical connectors and extend to a row of six MSD Blaster SS Coils. The coils are connected to stock automotive M14 spark plugs (installed within the engine head) via high voltage MSD spark plug wires.

The PDC array mates with the engine head approximately 3 cm downstream of the valving. This array consists of a weldment of six detonation tubes, 50 mm in diameter, constructed out of schedule 40 carbon steel (Figure 2.9). This diameter is sufficient to support ethylene-air detonations ($\lambda \approx 25$ mm at $\phi = 1$), and hydrogen-air detonations ($\lambda \approx 10$ mm at $\phi = 1$). The detonation tubes are equipped with removable Shchelkin-type spiral devices, approximately 1m in length, to promote rapid transition to detonation. The linear weldment links with an adapter weldment prior to the turbine inlet, yielding a total combustor length of approximately 2m.

![Figure 2.9. PDC array linked with axial turbine via adapter hardware](image)

Instrumentation ports are located above and below each tube in the linear weldment at three axial locations: adjacent to the headwall, 1 m downstream of the headwall, and 1.1 m downstream of the headwall. The headwall location can be used to determine the pressure profile at the upstream end of the combustor, while the downstream ports can be used to verify detonation wave speeds after the
spiral obstacle. The adapter weldment is also constructed of Schedule 40 carbon steel pipe (50 mm diameter), and converts the linear array of tubes to a circular array via equal-length runners. This weldment is designed such that the stock firing order of the tubes (1-5-3-6-2-4) is converted to a counterclockwise progression of pulses entering the turbine (Figure 2.10). After the adapter weldment converts the flow path from a linear array to a circular array, an additional adapter converts the circular array into a sectored annulus just prior to the turbine inlet plane.

**Firing Order (facing turbine inlet):**

![Diagram showing firing order]

To maximize the strength of the pulsations experienced by the turbine and restrict mixing between adjacent PDC tubes, the six streams are fully partitioned up to approximately 2.5 cm upstream of the turbine stator vanes. This close-coupled system is intended to maximize the fluctuations in turbine inlet quantities and flow variations between adjacent admission sectors. Figure 2.11 depicts the flow path...
from the end of the adapter weldment (orange) through the turbine exhaust duct (yellow), and the corresponding evolution of cross-sectional area. The area of each tube is contracted down from \( A \approx 130 \) cm\(^2\) to \( A \approx 55 \) cm\(^2\) as the flow path is converted from a circular cross section to a sectored annular cross section (within the turquoise weldment). The area is held constant through a series of mating plates (red), and expands significantly as the flow enters the stator (purple). The flow then enters the rotor (green) and expands through the turbine exhaust duct (yellow), exiting downward.

![Diagram of flow path through the axial turbine](image)

**Figure 2.11. Cutaway of flow path through the axial turbine**

### 2.2.2. Description of Turbine

The power generation section from a Garrett JFS-100-13A gas turbine engine has been adopted as the turbine (Figure 2.12). Originally designed to serve as a secondary turbine downstream of a gas generator section, the turbine is employed as a primary turbine in the present study, and is directly exposed to high temperature combustor exhaust gases. This single stage, axial flow turbine is equipped with an integrated gear box providing 18.3:1 reduction in shaft speed. The rotation direction of the rotor is counterclockwise facing the turbine inlet, matching the direction of the pulse progression illustrated in Figure 2.10. The design parameters of the turbine are listed in Table 2.1. All turbine quantities in the following investigations are normalized to the equivalent design values.
Figure 2.12. Schematic of Garrett JFS-100-13A power generation turbine unit (from user manual)

As the system was designed for combusting, detonation inflow into the turbine at high total temperatures, the maximum flow rate for pulsed air operation is approximately 80% of the equivalent turbine design corrected mass flow.

Table 2.1. Turbine Design Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PR</td>
<td>1.66</td>
</tr>
<tr>
<td>$T_0$</td>
<td>1055 K</td>
</tr>
<tr>
<td>N</td>
<td>60400 RPM</td>
</tr>
<tr>
<td>$\dot{m_0}/\sqrt{P_0}$</td>
<td>0.000149 kg·K$^{1/2}$/Pa</td>
</tr>
<tr>
<td>Rated Output</td>
<td>67.1 kW</td>
</tr>
<tr>
<td>$\eta_{\text{isentropic}}$</td>
<td>61.6%</td>
</tr>
<tr>
<td>Stator Vanes</td>
<td>22</td>
</tr>
<tr>
<td>Rotor Blades</td>
<td>36</td>
</tr>
<tr>
<td>Mean Inlet Radius</td>
<td>5.18 cm</td>
</tr>
<tr>
<td>Mean Exit Radius</td>
<td>6.55 cm</td>
</tr>
<tr>
<td>Stator Inlet Area</td>
<td>58 cm$^2$</td>
</tr>
<tr>
<td>Rotor Exit Area</td>
<td>121 cm$^2$</td>
</tr>
</tbody>
</table>

Turbine loading is provided by a Stuska XS-111 water brake dynamometer connected to the turbine output shaft. Loading is varied using a manual control valve that governs the rate of water flow into the
dynamometer, with a maximum rate of approximately 35 L/min. Torque and shaft speed are measured by a Lebow 1604-2K Torque Sensor and Rotary Transformer located on the output shaft, between the turbine assembly and the dynamometer.

2.2.3. Turbine Instrumentation

The turbine is instrumented with a variety of fixed sensors and radially traversing probes to monitor conditions at the stator inlet and rotor exit (Figure 2.13). For pulsed air flow testing, the measurement environment is relatively favorable due to low temperatures and pressures, allowing the use of uncooled, miniature transducers and thermocouples. Surface-mount Kulite LE-125 thin-line pressure transducers are embedded at the leading edge of the stator vanes, in the stator-rotor interspace, and at the rotor exit to capture static pressure. Pairs of Omega Type-E thermocouples are also installed in these locations to capture static temperature. Inlet and exit total pressures are measured using traversable high-speed pitot probes, based off the probe designs of Brandner et al. (2004).

![Figure 2.13. a) Stationary and b) radially-traversable probe locations within the turbine assembly](image)

This high-speed pitot probe contains a Kulite XCQ-062 ultra-miniature pressure transducer, embedded approximately 1 cm from the desired sensing location (Figure 2.14). The probe can be traversed radially across the entire inlet and exit span of the stage, as well as fully rotated to measure different flow angles. Using a hot film sensor and a high frequency pulse generator (fast-acting solenoid valve) for
dynamic calibration, the probe time response was verified at $\tau = 200\mu$s, providing an effective time resolution of 5 kHz. Signal conditioning and amplification for all Kulite sensors is provided by Validyne SG297A signal conditioner modules.

![Probe design with cutaway and Calibration via high speed pulse generator](image)

**Figure 2.14. Pitot probe design and dynamic calibration**

In addition to time-resolved pressure measurements, the turbine inlet and exit are instrumented with four Omega PX209 pressure transducers in a capillary tube averaged pressure (CTAP) configuration (as employed by Naples *et al*. 2014) with a normalized length of $L/D = 1000$ for the capillary tubes to ensure steadiness in the measurement. In principle, a CTAP sensor provides a repeatable analog-averaged measurement, and is useful for harsh measurement environments where the transducer will not survive direct exposure to the working gas.

Relatively high response static inlet temperature is achieved using the embedded dual-type-E thermocouple setup by employing the methodology and signal filtering techniques developed by Kar *et al*. (2004,2009). The principle of this approach is to reconstruct the actual temperature from two phase-lagged temperature signals of differing time responses. Cold junction compensation and amplification are provided by a Michigan Scientific Miniature Thermocouple Amplifier with 19 kHz custom bandwidth.
For detonation testing, inlet and exit total pressures are measured using a reinforced pitot tube design with an embedded, water-cooled, fast-response pressure transducer (Kulite WCT-312M), similar to the design of the pulsed air flow pitot probe. Unfortunately, due to the extremely short duration of the detonation event and the complex shock interactions with the probe, it remains difficult to properly recover time-resolved total pressure in a detonating flow. This is consistent with the difficulties reported by Stevens et al. (2015), which indicate that a Kiel-type total pressure probe may record lower pressures than static transducers when used to measure detonating flow. High speed PCB piezoelectric pressure transducers (113B24) are used to measure static pressure at the turbine inlet. These are installed in both flush-mounted and infinite tube pressure (ITP) configurations. The flush-mounted configuration yields exceptional time response, but with the penalty of excessive heat loading, which eventually distorts the sensor response. The ITP configuration, discussed by Naples et al. (2014) and Stevens et al. (2015), utilizes a short standoff tube to shield the transducer from thermal loading, but sacrifices some temporal response and may experience shock reflections within the standoff tube.

While type-E thermocouples are rated for operation up to $T \approx 1200\text{K}$, the bead diameters of the thermocouples used in the pulsed air study are relatively small, and have limited survivability in the detonation environment. While the phase-lagged thermocouple measurements can still resolve temperature fluctuations of several hundred Kelvin during pulsed detonation tests, the lifespan of these thermocouple probes is typically 2 to 3 tests. To reduce hardware replacement costs, time-averaged type-K thermocouple measurements are used for detonating flow cases in lieu of the delicate, time-resolved measurements. CTAP transducers are also employed to measure average pressures at the stage boundaries.
2.3 Rotating Detonation Combustor System

2.3.1. Description of RDC

The present RDC (Figure 2.15) is based on the Shank et al. (2012) design developed at the Air Force Research Laboratory (AFRL). The baseline components in direct contact with combustion are machined from stainless steel, while aluminum is used for upstream components. The combustor consists of a center body and outer body which comprise the walls of the channel, with reactant injection occurring on the upstream end at the base of the RDC channel. As in the Shank baseline design, oxidizer is injected radially-inward from an oxidizer plenum through an oxidizer injection slot. Fuel is injected axially from a fuel plenum through circular orifices, and enters the combustor perpendicular to the oxidizer stream to generate crossflow mixing.

![Figure 2.15. Schematic of RDC](image-url)
The center body is mounted onto the fuel injection plate, while the outer body bolts into the outer wall of the oxidizer plenum. The oxidizer plenum is fed radially by five, equal-length runners from an upstream manifold, while the fuel plenum is fed axially by a single, central inlet. Initiation is achieved with a tangentially-injecting initiator tube, which enters the RDC channel just above the fuel injection plate. To increase the chamber pressure above ambient conditions, a converging nozzle was developed, which mounts directly to the center body. The entire RDC assembly is mounted horizontally on a modular test stand constructed of steel strut channel and extruded aluminum rail.

As the baseline Shank design was originally designed to be modular, many interchangeable components were developed to modify the combustor performance. The dimensions of the baseline, nozzle-less configuration described in Anand et al. (2015a) and Driscoll et al. (2015) and are reproduced in Table 2.2. The combustor channel width ($w_{ch}$) can be varied by changing the center body component. The fuel injection area and arrangement of fuel injection orifices can be varied by changing the fuel injection plate. The oxidizer injection area (i.e., width of the oxidizer slot) can be varied by changing the oxidizer spacer ring. The combustor exit area can be adjusted by installing the converging nozzle in tandem with a range of nozzle shims. These shims can control the axial position of the nozzle in increments of 0.25 mm, yielding precise control of the exit area (and the pre-ignition channel pressure at a given flow rate). This design enables significant, low-cost alteration of the combustor geometry, allowing many configurations to be tested efficiently.

<table>
<thead>
<tr>
<th>Table 2.2. RDC geometric parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer Diameter</td>
</tr>
<tr>
<td>Channel Width</td>
</tr>
<tr>
<td>Channel Length</td>
</tr>
<tr>
<td>Oxidizer Injection Area</td>
</tr>
<tr>
<td>Fuel Injection Area</td>
</tr>
</tbody>
</table>
Primary fuel flow, supplied by the high capacity fuel system, is administered to the rig through a Bi-Torq pneumatically-actuated isolation ball valve. This valve is located just upstream of the fuel plenum, allowing fuel flow rates to stabilize within 2-3 seconds of fuel introduction. To improve fuel flow response and reduce stabilization times, the primary fuel sonic nozzle assembly was relocated directly onto the test stand to minimize the volume downstream of the sonic nozzle. Oxidizer flow is ducted from the sonic nozzle assembly (located on south wall of the test bunker) into an upstream manifold, located on the test stand, before proceeding to the oxidizer plenum. Typical RDC oxidizer flow rates are between $m_{OX} = 0.2$-0.5 kg/s for most test conditions.

The low capacity fuel system provides the reactant supply for the initiator, in addition to enabling blended-fuel operation when coupled with the primary fuel system, or oxygen-enriched operation when coupled with the air supply system. Gaseous oxygen, fuel, and nitrogen are manually regulated within the test bunker to the desired experiment pressure. Fuel and oxygen are delivered to the experiment in stainless steel tubing, while plastic tubing is used for nitrogen delivery. When operated as a secondary fuel supply, this system can sustain flow rates of up to 3 g/s for hydrogen (10 g/s for ethylene), or approximately 60 g/s of oxygen when configured for oxygen enrichment.

The initiator tube (Figure 2.16) consists of a stainless steel tube, approximately 210 mm in length, which enters the RDC tangentially at a height of approximately 21.5 mm from the base of the combustor. Approximately 50 mm prior to entering the RDC channel, the initiator tube inner diameter contracts from a smooth section down to a threaded section (ID = 4.8 $\rightarrow$ 3.2 mm). This threaded section is identical to the initiator tube entry region employed by Shank et al. (2012), and is meant to approximate a Shchelkin spiral (Miller 2013). The dimensions of the RDC initiator tube are summarized in Table 2.3.

The initiator was originally designed to use a hydrogen-oxygen mixture, but has been successfully operated with ethylene-mixtures as well. Fuel and oxidizer for the initiator are supplied through a pair
of Parker solenoid valves and impinge in a mixing junction. Flow rates for the initiator fuel and oxidizer supply are calculated from supply pressures upstream of the solenoid valves using flow coefficients determined from choked flow valve calibration. A nitrogen purge valve is located upstream of the mixing junction on the fuel supply line to expunge any residual gases after the conclusion of each test to improve repeatability. Ignition within the initiator tube is provided by an automotive spark plug, powered by the MSD ignition module. Ignition energy is on the order of 0.1 J, which is sufficient to directly initiate detonation or yield almost instantaneous DDT for highly detonable fuel-oxygen mixtures.

<table>
<thead>
<tr>
<th>Table 2.3. Initiator tube geometric parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
</tr>
<tr>
<td>Outer Diameter</td>
</tr>
<tr>
<td>Inner Diameter</td>
</tr>
<tr>
<td>Entrance Diameter</td>
</tr>
<tr>
<td>Volume</td>
</tr>
</tbody>
</table>

![Figure 2.16. Schematic of RDC initiator tube](image)

2.3.2. Instrumentation

Instrumentation ports are located on the outer wall of the combustor and within the oxidizer injection slot. The RDC channel contains four axially-spaced rows of ports (R1,R2,R3,R4), with the first row
located approximately 3 cm from the base of the combustor. The oxidizer injection slot ports are recessed radially outward from the combustor outer wall by approximately 22 mm, and are azimuthally shifted from the flush-mounted ports by 20°. Sensors can be installed at three azimuthal stations (I, II, III), with 120° spacing between stations.

PCB transducers are used in the RDC to capture detonation wave passage and pressure evolution (Figure 2.17). Model 113B24 ICP® pressure transducers provide fast response times (< 1μs) on account of the piezoelectric nature of the sensing element. This also restricts usage of the sensor for measuring very low frequency or steady-state pressure signals, as these are generally filtered out by the compensating circuit. These sensors are supported by a PCB 498A 8-channel signal conditioner, which filters and amplifies the pressure signal prior to the 1 MHz DAQ. While these piezoelectric sensors are exceptionally rugged and provide excellent survivability under pulse detonating conditions, the continuous, severe heat loading of the RDC environment can significantly degrade the lifespan of these sensors for flush-mounted installations.

In this facility, the flush-mounted installation was initially used due to the inherently faster response rate and clarity of signal. The main drawback of this configuration is excessive heat soak of the transducer, followed by signal loss within fractions of a second after the onset of rotating detonation.
To prolong sensor lifespan at the expense of response rate, indirect installations are used to reduce heat loading on the sensor face. The infinite tube pressure (ITP) configuration (Figure 2.18), reduces signal reflection by allowing pressure fluctuations to propagate outward from the combustor and attenuate in a long, smooth-walled tube (ID ≈ 3 mm), and minimizes the resonance frequency of the standoff. The sensor is mounted perpendicular to the standoff tube, at a standoff distance of approximately 42 mm from the outer wall of the combustor. While frequency response is reduced, and reflection noise is incurred, PCB signal loss is effectively prevented and signal shift due to heat soak is minimal, even under long-duration tests of several seconds.

As reported by Fotia et al. (2014) and Rankin et al. (2015b), the passage of a detonation wave strongly disrupts the reactant injection, and this disruption can be recorded by pressure transducers installed within the oxidizer injection slot (Figure 2.19). The onset of periodic behavior within the oxidizer injection slot correlates well with onset of stable detonation rotation within the combustor and captures the same fundamental frequency (Anand et al. 2016a). Recent simulations by Cocks et al. (2016) reveal that the detonation produces a strong oblique shock, which propagates upstream into the oxidizer plenum at a slight phase delay from the detonation. As these installations are upstream of the combustion process, they are well isolated from high temperature combustion products, and typically

![Figure 2.18. ITP (PCB) transducers used to capture RDC pressure evolution](image-url)
exhibit less reflection noise and greater consistency between azimuthal locations compared to ITP installations.

Figure 2.19. Oxidizer injection slot transducers used to capture RDC pressure evolution

To capture steady state and low frequency changes in pressure, GE Unik 5000 series and Omega PX 309 series low speed pressure transducers are installed in a CTAP configuration. The smooth-walled tubes from the PCB ITP configuration serve a dual use as capillary tubes to damp out high frequency content and provide a relatively stable low frequency measurement at the end of the tube. For approximately 1m of capillary tube, the transducer response is free of high frequency noise and provides a steady, repeatable measurement (Figure 2.20).

Figure 2.20. Capillary tube averaged pressure evolution for RDC with a converging nozzle
These are used to capture oxidizer and fuel plenum and RDC channel steady-state pressures to characterize injection pressure ratios and combustor conditions at lightoff and at limit cycle conditions. When coupled with the dynamic, high frequency response of the PCB transducers, steady state and low frequency CTAP measurements provide the means to reconstruct the time-resolved absolute pressure evolution.

To capture the complete time-accurate absolute pressure evolution within the combustor with a single sensor, Kulite transducers are installed in an ITP configuration. Both water-cooled (WCT-312-35barA) and non-cooled (XTEL-190-100G) variants have been successfully used with Validyne SG297A Signal Conditioning Modules to provide amplified transducer output. While Kulite subminiature transducers exhibit reduced frequency response compared to piezoelectric-based transducers, these sensors exhibit improved noise immunity and the ability to simultaneously capture high frequency, low frequency, and steady state dynamics. This is best illustrated by Kulite-recorded pressure evolution for RDE operation with a back-pressurizing nozzle (Figure 2.21).

Figure 2.21. Kulite pressure evolution for RDC with a converging nozzle

Ionization probes are used as a secondary high frequency diagnostic to capture the transit of the unsteady combustion front. While pressure evolution is captured by PCB sensors, wave passage is not
always correlated to the presence of active combustion, and may indicate a decaying blast wave or shock reflection. Ionization probes provide a cost-effective, robust measurement of combustion wave passage and immunity to non-combusting acoustic and reflection-based noise. These probes detect large fluctuations in fluid electrical resistance when integrated with a compensating circuit (Figure 2.22). A voltage source charges a capacitor, and as the fluid resistance drops sharply in the presence of ionization, the capacitor discharges, producing a strong induced current. Automotive spark plugs are a cost-effective replaceable component for the sensing portion of the circuit exposed to the combustor. The sensor bandwidth can be adjusted by changing the resistance of the circuit. To improve the signal strength, the sensing element must be installed in a location experiencing large fluctuations in fluid electrical resistance. The fill region of the combustor is ideal, as this location is periodically exposed to highly ionized flow (detonation) and inert flow (fresh reactants).

![Circuit diagram and ionization evolution](image)

**Figure 2.22.** Ionization probes used to capture RDC ionization evolution
Chapter 3 – Axial Turbine Performance under Steady Air and Pulsed Air Conditions

The present study simulates detonating combustion inflow to the turbine with high gradient, high amplitude, non-combusting, pressure pulsations without including temperature effects. Beginning with a cold-flow study allows a broader range of sensitive, minimally-invasive instrumentation to be employed within the turbine, since probe survivability is limited under the high temperature conditions of pulsed detonation combustion. In a departure from previous experiments implementing a single admission site (Rouser 2012), or a plenum-based inflow (Glaser et al. 2007, Caldwell et al. 2008, Fernelius et al. 2013), this study partitions the inflow into six admission sectors which experience out-of-phase, or sequential pulsations. Compared to previous studies with sequential pulsations (Rasheed et al. 2011, Caldwell et al. 2008), there is no steady bypass-air flow, and mixing between the inlet sectors prior to the stator vanes is minimized to intensify the effects of extreme unsteadiness. This setup provides improved approximation to a realistic annular combustor layout with multiple out-of-phase pulse detonation combustors compared to existing studies. Measurements have been taken within the turbine at the stage boundaries, rather than upstream or downstream in volutes, to clearly capture the unsteady inlet and exit quantities, while providing a thermodynamically rigorous treatment of the turbine stage. Thermodynamic conditions are formulated under a variety of pulse conditions to determine the utility of performance metrics and efficiency definitions in correlating pulsed flow with steady flow by providing a comparison between the two regimes of operation.

3.1 Experimental Approach

Steady air flow tests are conducted on the turbine to establish the baseline performance. Cases of full admission and 50% partial admission are investigated. For the partial admission tests, every other inlet sector is blocked off to produce steady, 50% partial admission conditions. This is intended to mimic turbine behavior for out-of-phase pulsations. Performance is mapped for a range of flow rates at six corrected speed (N') characteristics ranging from 28-100% of the rotor design speed (N'_{DES}). For the
steady flow tests, the intake valves in the engine head are removed from the system, preserving most of the geometry upstream of the turbine for comparable flow paths between all cases. For the partial admission case, a plate is installed in the intake manifold upstream of the valving to eliminate flow from every other inlet sector.

To examine the flow features within the turbine over a pulse cycle, the distribution of total pressure is measured in detail for a limited number of cases to identify variations in flow direction and magnitude at the stage boundaries. The turbine loading is held constant, and three air flow rates (0.40 kg/s, 0.60 kg/s, and 0.78 kg/s) are studied at four pulse frequencies (5, 10, 15, and 20 Hz). At the stage boundaries, the traversable total pressure probes are rotated (θ) in increments of 45° at 50% span to capture the flow angle evolution (Figure 3.1). Additional refinement (±22.5°) is conducted at upstream angles for the two lower flow rate cases. To map the radial variation, the probes are pointed both upstream and downstream (θ = 0° and 180°) and traversed across five locations between the hub and tip. These span locations included 10% (hub), 30%, 50%, 70%, and 90% (tip).

![Diagram of traversable probe locations at stage boundaries](image)

**Figure 3.1. Schematic of traversable probe locations at stage boundaries**

After the detailed examination of the air flow field, the traversable total pressure probes are fixed at the mean radii (50% span) measurement locations and pointed upstream (θ = 0°) for the remainder of the
Pulsed air tests are conducted over a range of time-averaged inflow conditions similar to the steady flow tests for pulse frequencies of 5, 10, 15, and 20 Hz. Flow rate and loading are varied for each frequency to generate corrected turbine speed characteristics across the operating range from 25-100% corrected design rotor speed ($N'_{\text{DES}}$) and 25-80% design corrected mass flow ($\text{MFP}_{\text{DES}}$). At each stabilized setpoint, data is recorded at 1 MHz for 10 seconds, yielding a minimum of 50 pulse cycles for phase averaging. To improve the signal-to-noise ratio, the data is broken into consecutive sets of 100 data points and each of these is averaged to produce a single point, reducing the effective sample rate to 10 kHz. Turbine loading is more difficult to stabilize to a desired corrected speed under pulsed flow conditions, and to maximize setpoints, the map is constructed by holding loading fixed and varying speed over the achievable range rather than matching specific speed characteristics. Smooth surfaces are fitted to the data for each frequency to interpolate corrected speed characteristics for comparison to the steady flow cases.

For all cycle calculations, each of the six admission sectors is assumed to have matching, but phase-shifted flow. Instantaneous inlet mass flow profiles are generated by phase-shifting measurements from neighboring inlet sectors to calculate time-resolved density and velocity. Because of the axial proximity of the two traversable probes, negligible mass storage is assumed between them, and the instantaneous mass flow profile at the turbine inlet is also used in calculations as the profile at the turbine exit. When the stator flow velocity is nonzero, the stator blade geometry and rotor speed are used to estimate the flow direction at the rotor inlet at the mean radius. The design relative flow angle ($\beta_{\text{Des}}$) is subtracted from the rotor inflow angle to find the rotor incidence angle. Mass averaging is employed to calculate the mass-averaged rotor incidence angle for the cycle. As incidence losses are expected to scale with flow rate, this averaging approach is expected to yield the most meaningful correlation with cycle efficiency.
Uncertainty analysis for the experiment has been performed by the method of sequential perturbations (Moffat 1988). Uncertainties for each of the measurements are determined in situ based on cycle-to-cycle variations. These individual measurement uncertainties are then applied one at a time to the raw measurements to determine the sensitivity of the result. For complex expressions this approach is much simpler than analytically derived sensitivity functions, as these are complicated expressions, comprising dozens of integrated terms, many of which have singularities as instantaneous flow rate approached zero during the cycle. In cases experiencing a steady flow component, the analytically-derived sensitivities converged to those calculated by the method of sequential perturbations. The sensitivity function for each measured quantity is multiplied by a deviation equal to three standard errors for that measurement, to generate a 99.7% confidence interval. Assuming the measurement errors are uncorrelated, the root of the sum of squares is taken to be the total deviation.

At low flow rates, sensitivities are very high for the static and total inlet pressures, as small errors in static-to-total pressure ratio can generate large errors in Mach number and alter the instantaneous flow rate. At higher flow rates, these pressure sensitivities are diminished, but sensitivity to the inlet static temperature is increased. Large fluctuations in the Mach number produce significant total temperature variation over the cycle. Cycle efficiencies for the highest flow rate cases ($\dot{m} = 1.2$ kg/s) are accurate to within ±1%, while the accuracy in the efficiency calculation drops to ±5% at the lowest flow rates.

### 3.2 Steady Air Flow Performance Mapping

Turbine performance under non-combusting, steady, full admission flow is in good agreement with the rated design point (depicted by a star), with corrected mass flow nearly converging to the design value (Figure 3.2a). Due to facility air flow rate limitations ($\dot{m}_{\text{MAX}} = 1.2$ kg/s), the highest corrected speed characteristics do not extend out to peak efficiency values (Figure 3.2b), but appear in good agreement with the rated design value if extrapolated out to the design point. The peak efficiency rise between the
first two characteristics suggests improved performance at higher pressure ratios, corresponding to higher corrected flow rates. While the efficiency characteristics tend to plateau at higher pressure ratios, sharp declines are observed for excessively low pressure ratios. This efficiency behavior can be attributed to the rotor incidence angle, as higher pressure ratio operation corresponds to higher MFP. High MFP and low $N'$ yield strongly positive rotor incidence ($\beta > \beta_{\text{des}}$), while low MFP and high $N'$ generate negative rotor incidence ($\beta < \beta_{\text{des}}$). The expansion of flow through a turbine generates a favorable pressure gradient, making it less sensitive to incidence than a compressor. While this allows efficient operation over a wide range of rotor incidence angles, the performance will suffer for excessively positive or negative rotor incidence. For strongly negative incidence, the degree to which the rotor can turn the flow is reduced, diminishing the potential work extraction. Likewise, for strongly positive incidence, separation losses dominate.

![Graph showing MFP and Efficiency for Full and Partial Admissions](image)

*Figure 3.2: Steady, full and 50% partial admission performance maps for $N'/N'_{\text{DES}}$ characteristics*

Speed characteristics for 50% partial admission inflow are constructed for the same range of flow rates as for the full admission case. Corrected mass flow drops significantly for the 50% partial admission case due to increased inlet total pressure, despite increased mass flux in active sectors. Though the flow path allows for some spillage of flow from active sectors into inactive sectors, a fraction of the stator
passages are likely deprived of flow, such that compared to the steady flow case, less of the stator inlet experiences active flow. Mass-averaged inlet total pressures are much higher under partial admission, implying more work is available for extraction. However, the efficiency decays considerably under partial admission, with peak values dropping by nearly 20% relative to the full admission values. This large drop in peak efficiency and corrected mass flow under steady, partial admission conditions is in good agreement with prior published work (Copeland et al. 2011, Benson and Scrimshaw 1965, Craig et al. 1968, Ni et al. 2013).

3.3 Pulsed Air Flow Profiles

Detailed study of flow profiles at the stage boundaries uncovers several trends that highlight the effects of frequency and flow rate on the pulse cycle. Total pressure probes at the stator inlet plane and the rotor exit plane are traversed radially across the passage from the hub to the tip. Probes are oriented through a range of angles from θ = 0° (i.e., facing upstream) to 180° (i.e., facing downstream), with clockwise probe rotation denoted as positive to coincide with the rotor rotational direction. The inlet total pressure profile (Figure 3.3) is relatively uniform from the hub radius (10% span) to the mean radius (50% span), but begins to diminish approaching the tip (90% span). The extent of the decline is dependent on flow rate, with the intermediate flow case experiencing a moderate deficit of approximately 10%.

Figure 3.3: Total pressure evolution at the turbine inlet plane at 10 Hz
The lowest flow case experiences a large separation region at the tip location, as seen by the overlap of upstream and downstream pressure traces, with reversed flow occurring in the tip region after pulse blowdown. At the mean radius, flow reversal is never observed for any of the cases, making a full flow reversal of the entire sector unlikely, even during inactive pulse periods.

Stator inlet total pressure histories for all four pulse frequencies are shown in Figure 3.4 for two of the flow rate conditions. Probes are oriented upstream ($\theta = 0^\circ$) at the mean radius location of the stator inlet plane. The effect of frequency on the inlet pulse shape is sharply magnified as the flow rate is increased. At the lower flow rate, the pulse amplitude is nearly identical for all frequencies, and the duty cycle is minimally expanded at higher frequencies, only slightly stretching out the pulse shape. When the flow rate is increased, the pulse shape is more sensitive to frequency, and as pulse frequency is increased, pulse amplitude decreases as the duty cycle expanded.

![Figure 3.4: Pulse shape sensitivity to frequency for 0.78 kg/s and 0.40 kg/s](image)

For the higher flow rate, a small positive component of flow is observed during pulse inactive periods for 15 and 20 Hz pulsations, indicating the onset of continuous flow through the stator passages. This suggests that there is sufficient volume between the pulse generation apparatus and the turbine for mass storage effects to manifest themselves, producing a strong correlation between frequency and the
pulse amplitude and duty cycle as in previous experimental studies (Benson and Woods 1960, Rajoo and Martinez-Botas 2010, Copeland et al. 2011, Fernelius et al. 2013).

When the pressure histories for all probe angular orientations are considered, the inflow exhibits a small component of time-dependent swirl (Figure 3.5). While the θ = 0° probe orientation measures the highest total pressure levels compared to all other orientations, each angle pair (i.e. ± 22.5° ± 45°, etc.) reaches peak pressures for the positive and negative angles at different times. Peak total pressure for negative angles consistently occurs before peak pressure for positive angles, indicating that a small, alternating component of swirl exists in the inflow. Because the θ = 0° orientation is dominant throughout the cycle compared to the other traces, it can be concluded that the swirl component is of sufficiently small to fall within the range of ± 22.5°. The negative-to-positive evolution of the dominant flow angle suggests that the variation is due to the counter-clockwise progression of pulses, and implies that flow from adjacent pulse sectors slightly modifies the flow direction. While strongest at low frequency and low flow rate, the swirl becomes less pronounced at higher flow rates, particularly at high frequencies.

Figure 3.5: Evolution of swirl at the turbine inlet at 10 Hz, 0.60 kg/s for varying pitot probe angles
The rotor outflow (Figure 3.6) exhibits similar trends in that the outflow duty cycle is similar to the inflow duty cycle and expands with flow rate and frequency. This agreement between the inflow and outflow suggests there is negligible mass storage, or damping, between the probe locations at the stage boundaries. While pulse amplitudes are much lower at the rotor exit plane, the outflow pressure histories are similar in shape to the inflow and are highly unsteady. Pulse amplitudes at the rotor exit increase with flow rate, indicating there is substantial energy content within the outflow still available for work extraction. A steady treatment of the exit would inaccurately neglect this additional total pressure and suggest lower apparent stage efficiencies.

![Figure 3.6: Evolution of swirl at the turbine exit at 10 Hz, 0.60 kg/s for varying pitot probe angles](image)

There is no reversed flow observed at the outflow mean radius (50% Span<sub>Exit</sub>) for any of the cases, though there are strong variations in exit swirl over the pulse cycle. During periods of active flow, the
exit swirl angle rapidly evolves between idle and active states. The idle state is dominated by strongly negative outflow angles ($P_{\max}$ occurs for $\theta = -67^\circ$), while the active state experiences moderately positive outflow angle ($P_{\max}$ occurs for $\theta = 22.5^\circ$, then $\theta = 45^\circ$). The first 40% of the cycle is spent in the idle state, followed by an abrupt transition to the active state, which then smoothly transitions back to the idle state over the remainder of the cycle, as seen in Figure 3.6. This evolution of exit swirl is caused by variation of rotor incidence over the pulse cycle (Karamanis et al. 2001). Rotor inlet velocities are high for the duration of the pulse active period and incidence is generally positive (Figure 3.10), generating slightly positive rotor exit flow angles. As the flow rate decreases at blowdown, the rotor incidence shifts to a negative value, and for fixed rotor speed, the rotor outflow angle also becomes highly negative.

At higher frequencies and flow rates, the transition period of the rotor outflow angle and the subsequent blowdown flow state is lengthened, mirroring the increased blowdown experienced at the stator inlet in Figure 3.4. Because the transition time between active-to-idle states is lengthened at high frequency, periods of negative incidence are reduced, improving the efficiency. Likewise, when turbine loading is increased the rotor speed is reduced, so that rotor incidence is strongly positive during active periods and is near the design value during idle periods. This scenario potentially mitigates the negative incidence losses.

3.4 Pulsed Air Performance Mapping

Bulk cycle pressure ratios for unsteady performance maps are formulated using five averaging techniques: CTAP, Time-Averaged (TA), Mass-Averaged (MA), Work-Averaged (WA), and Reverse-Instantaneous. The methods that weight the active flow periods are a better representation of the available work contained in the turbine inflow when used to formulate efficiencies (Figure 3.7). Two of the averaging methods produce extremely low values of inlet total pressure and isentropic power (CTAP and time-averaged), and these efficiencies are disproportionately high and in many cases, significantly
exceed the maximum rated steady turbine efficiency of 61.6% listed in Table 2.1. The integrated instantaneous method, work-averaged method, and mass-averaged method yield similar efficiency values across the performance map, though the instantaneous efficiencies are consistently the highest while mass-averaged efficiencies tend to be the lowest.

![Diagram](image)

**Figure 3.7: Comparison of efficiency calculation methods for 10 Hz, 53% design corrected speed**

Work-averaged bulk total pressure definitions are used for the following turbine maps as a good compromise between the mass-averaged and reverse-instantaneous definitions. While the work-averaging method is regarded by Cumpsty and Horlock (2006) as the most thermodynamically rigorous approach, it is also practical to implement. The unique treatment of the exit requires less measurements or assumptions about the instantaneous flow rate at the turbine exit plane. The reverse-instantaneous definition can incur phase delay errors for sharp-gradient high amplitude pressure pulses due to the finite travel time of the pulse, while work-averaging is immune to this problem.

The MFP is plotted against pressure ratio for all the frequencies in Figure 3.8, with steady data included for reference. While the steady full admission (solid black) and 50% partial admission (solid gray) characteristics smoothly approach their respective maximum values, the pulsed flow data exhibits
entirely different, frequency-dependent behavior. Initially, all pulsed cases behave analogously to a steady, approximately 33% partial admission case, with all curves collapsing to a monotonic curve below the existing steady, 50% partial admission data. When the MFP is low, there is ample time for the turbine to absorb the pulse, as described by Copeland et al. (2011). Low Mach number and low backpressure upstream of the stator allow for short blowdown periods and minimal impedance to the pulses. As the flow rate increases, and the duty cycle is extended, the MFP curves begin to deviate from the initial trend, with deviation occurring more quickly at higher frequencies. Each of the frequencies eventually diverges upwards, experiencing large gains in MFP for relatively small changes in pressure ratio. All of the frequencies surpass the steady, 50% partial admission values and continue to rise toward the steady full admission design value. Similar work by Fernelius et al. (2013) demonstrates that these frequency curves eventually converge with the steady data at higher pressure ratios.

![Diagram](image)

**Figure 3.8:** Corrected mass flow versus pressure ratio for all pulsation frequencies

The duty cycle expansion initially occurs at high frequency and eventually results in a continuous flow component, which continues to grow at higher flow rates. The reduction of the pulse amplitude and expansion of the blowdown period is linked to increased frequency, and biases the higher frequency
setpoints to lower total pressure ratios due to the decrease in pulse inlet total pressure. As the flow rate is increased, choked flow occurs in portions of the stator during the active pulse period, and pulse amplitudes are damped relative to a system with less blockage (Benson and Woods 1960). For a fixed flow rate, increasing the frequency produces smaller mass content per pulse, which further reduces pulse amplitudes and lengthens system blowdown times. Together, these frequency and flow rate effects increase the corrected mass flow.

![Figure 3.9: Flow rate evolution for a turbine inlet sector at 20 Hz, for maximum turbine loading](image)

Instantaneous mass flow evolution at the stator inlet is reported in Figure 3.9 for varying cycle MFP and maximum turbine loading for 20 Hz pulsations. During the pulse idle state, a steady flow component first appears at moderate flow rates and higher frequencies and begins to grow, admitting more flow to the turbine at a lower total pressure. This results in active flow over the stator inlet at all times, effectively augmenting the MFP. As the flow rate increases and the blowdown lengthens, a substantial component of the useful mass flow is expended during the idle state. Consequently, the turbine inflow exhibits increasingly steady behavior and is less dominated by the pulse active period. The Mach number continues to rise, until choking occurs at the stator exit during the pulse active period of the
cycle. Projecting to a large enough pressure ratio and frequency, behavior analogous to the steady full admission curve will occur as the instantaneous mass flow profiles begin to stabilize, causing the fluctuating component to vanish in favor of a dominant, steady flow component.

The notional evolution of the rotor incidence angle over the cycle is depicted in Figure 3.10 by a pair of velocity triangles representing the active and idle flow states. As the stator outflow velocity $C$ is reduced, with the turbine speed $U$ held constant, the relative flow angle shifts from an optimal, active value, to a suboptimal, idle value. If the change in flow direction is as severe as illustrated in Figure 3.10 (left), flow separation and rotor deceleration will occur. For significantly reduced rotor speeds (Figure 3.10, right), the range of angles is minimized, eliminating periods of negative incidence in favor of continuous positive incidence.

![Figure 3.10: Flow angle evolution over a pulse cycle for nominal and reduced rotor speeds](image)

To explore this relationship, work-averaged cycle efficiency contours are plotted with respect to normalized MFP and $N'$ (Figure 3.11a). A diagonal line indicates the design flow angle, separating regions of dominant positive and negative rotor incidence. To the right, efficiencies are plotted against
mass-averaged rotor incidence angle for curves of fixed corrected mass flow. For a given MFP, efficiency is increased as the rotor incidence becomes less negative, but tends to stabilize for positive values of incidence. This asymmetric sensitivity to rotor incidence is in agreement with the steady flow performance. The relationship between incidence and efficiency also suggests that maintaining the highest possible loading under pulsed flow, or appropriately matching components to operate the turbine at lower corrected speeds will augment the turbine performance.

![Diagram](image)

**Figure 3.11: Effect of flow angles on work-averaged efficiencies for 10 Hz pulsations**

Work-averaged efficiency maps are shown for the entire turbine operating range at the highest and lowest frequencies in Figure 3.12 (with the steady, full admission data illustrated with dashed black lines). Speed characteristics are interpolated from a surface composed of all data points acquired at each frequency. The major difference between the efficiency maps for the two frequencies is the shift in total pressure ratio, which reflects the reduced work-averaged total pressure at the turbine inlet experienced at higher frequencies. Peak efficiency values are very similar between frequencies, and lower speed characteristics reached peak values comparable to those measured under steady, full admission conditions. However, higher speed characteristics do not reach their maximum values, likely because the flow rates are low compared to the turbine speed, yielding a suboptimal range of rotor
incidence angles over the pulse cycle. Despite this, the efficiency values for these characteristics at all frequencies are still in close agreement with each other, and with the steady data.

![Graphs showing work-averaged efficiency maps at 5 Hz (Lowest Frequency) and 20 Hz (Highest Frequency)](image)

**Figure 3.12: Work-averaged efficiency maps at the lowest and highest pulse frequencies**

This similarity between efficiency maps for the frequencies indicates that despite the increased duty cycle, and the deviation of MFP for increased frequency, the efficiency of the device may not be a strong function of frequency. This may be due partially to the effective incidence angle generated by the steady flow component during the pulse idle state. When the stator exit velocity is low compared to the rotor speed, the idle flow component may impact the rotor at strongly negative incidence angles. The early onset of a steady flow component may provide no additional benefit under these conditions. Therefore, the efficiency becomes a much stronger function of MFP than frequency.

### 3.5 Conclusion

An axial turbine is supplied with high gradient, high amplitude pressure pulsations to study the effects on turbine performance of closely-coupled, multiple-admission, out-of-phase pulsed air flow. The steady tests reveal a significant decrease in both corrected flow and performance for steady, 50% partial admission operation compared to the steady, full admission case.
A detailed examination of the inflow and outflow profiles conducted at four frequencies for three operating points provides insight to the effect of frequency and flow rate on the characteristics of the system. Increased frequency distorts the pulse shape by reducing the amplitude and expanding the duty cycle, and this effect is magnified with increases in flow rate. This behavior is highly dependent on the upstream geometry, which govern the filling and emptying characteristics, and this must be taken into account when designing an unsteady turbine system, as well as simulating realistic inlet conditions upstream of the turbine. The outflow exhibits significant evolution of exit swirl, implying substantial variations in rotor incidence angle, as well as large residual amplitudes in total pressure, revealing potential for additional work extraction. This poses a unique challenge for the consistent measurement of time-accurate exit total pressure for strongly varying flow angles. Also, this indicates that a steady treatment of the exit will neglect the opportunity for additional work extraction and arbitrarily combine the mixing losses into the stage efficiency calculation. This approach is likely to underestimate the performance capabilities of a multi-stage axial turbine under these flow conditions.

An expanded investigation conducted over the turbine operating range for pulsed air flow at four frequencies reveals unique corrected mass flow characteristics. While the corrected flow rate of the turbine exhibits partial admission behavior at low flow rates, a frequency-dependent divergence begins to occur at higher flow rates, and the corrected flow trends toward steady, full admission behavior for high frequencies and flow rates. The blowdown following the active pulse period grows with flow rate, culminating in the appearance of a steady flow component.

Of the five methods used to generate pulsed flow efficiencies for the turbine, those based on time-averaging were found to be a poor representation of the physics, implying much higher efficiencies under pulsed flow than under steady conditions. While similar in magnitude to the other flow-weighted efficiency formulations, work-averaged efficiency is selected as the preferred performance metric, due
to both its thermodynamically rigorous formulation, and its reduced experimental uncertainty due to fewer required flow measurements. Peak efficiencies are within a few percent of those observed for the steady full admission case, and also indicate very similar performance for all frequencies. Pulsed flow performance does not appear to incur the same severe losses observed for the steady, 50% partial admission case, and this effect is likely attributed to the increased system blowdown resulting from the filling and emptying characteristics encountered for this system. Stability of the stator exit condition may mitigate a large degree of the rotor losses by providing favorable rotor incidence conditions throughout a large portion of the cycle. The data suggest that performance is strongly dependent on corrected flow rate and mass-averaged incidence but only weakly dependent on frequency.
Chapter 4 – Axial Turbine Performance under Pulsed Detonating Conditions

While the previous pulsed-air investigation provides insight on the efficient extraction of power from a pulsing, unsteady flow, there are some key differences between pulsed-air and pulsed-detonating inflow conditions that motivate additional experimental treatment. Turbine performance for pulsed-air conditions is a strong function of corrected flow rate and the range of flow angles experienced by the rotor, and is only weakly dependent on frequency. However, for the case of detonating flow, there is a maximum volumetric flow rate for a given pulse frequency, equal to the product of the combustor volume and the firing frequency (exceeding this volumetric flow rate will result in detonation propagating beyond the combustor and through the turbine). While the combustor can operate in a partially-filled state, finite DDT length imposes a lower bound to the degree of partial filling. These restrictions establish an intrinsic link between frequency and flow rate, unique to the detonating flow case. Attempting to optimize the range of rotor flow angles also poses a challenge for detonating flow, as the turbine must simultaneously accommodate a wide range of inlet conditions produced by high temperature detonation exhaust and relatively cool purge gases.

Previous integration studies at the UC facility (Glaser et al. 2007), and other facilities (Rasheed et al. 2011, Rouser et al. 2011) achieved significant power extraction despite the considerable high amplitude unsteadiness of the detonation process. However, previous studies with multiple combustors used considerable fractions of bypass air, or mixing plenums to damp out fluctuations. Others (Rouser et al. 2014, Hoke et al. 2002) utilized a single combustor, resulting in spatially uniform, but highly unsteady turbine inlet flow. All previous studies span a limited number of turbine operating conditions, rather than the entirety of the operating map. This study characterizes power extraction in a multiple PDC-axial turbine system for a wide range of PDC and turbine operating conditions. In a departure from a plenum-based inflow, the PDCs remain partitioned near the turbine inlet. This allows turbine
performance to be characterized under realistic, high-amplitude PDC exhaust conditions.

4.1 Experimental Approach

A reduced portion of the turbine pulsed-air operating map is revisited with pulsed detonating flow, as the allowable flow rates for a given frequency are significantly restricted. All six combustors in the PDC array are utilized with the stock firing order, yielding a sequential, counterclockwise progression of pulsed detonations entering the turbine. Stoichiometric ethylene-air ($\phi = 1$) is used as the reactant mixture for the PDCs, air is used as the purge gas, and all gases enter the combustor at approximately ambient temperature ($T_i = 290-300$ K). Tests are primarily conducted at two firing frequencies (5 and 10 Hz), for several fill fractions ($ff$) and purge fractions ($pf$). The fill fraction (Eq. 4.1) is defined as the volumetric fraction of the PDC tube filled with reactants during each cycle, and can be varied from $0.4 \leq ff \leq 1.0$ in the present combustor. The purge fraction (Eq. 4.2) is the volumetric fraction of the tube filled with purge gases during each cycle, and can be varied arbitrarily (as there are no DDT or “over-purge” constraints), but is varied from $0.4 \leq pf \leq 1.0$ in the present investigation. The turbine loading is varied for each inflow condition (fixed $f$, $ff$ and $pf$) to produce a range of turbine rotor speeds and determine the dependence of output power on the rotor speed.

$$ff = \frac{f_{fill}}{V_{tube}} = \frac{m_{fill}}{V_{tube}} = \frac{m_{fill}}{f_V_{tube}} = \frac{1}{P_{fill}}$$ (4.1)

$$pf = \frac{m_{purge}}{f_V_{tube}} = \frac{T_{purge}}{p_{purge}}$$ (4.2)

The two water-cooled Kulite pitot probes developed for this investigation have a larger diameter than the uncooled design used in the pulsed-air investigation, and could not be installed directly at the turbine inlet plane or rotor exit plane with the present hardware. One of these probes is installed upstream of the turbine within PDC Tube 1, in the second turbine inlet port (Figure 4.1, 1.78m axially downstream of the PDC headwall). The downstream probe is installed within the turbine exhaust duct,
approximately downstream of the inlet sector corresponding to PDC Tube 1. While the Kulite pitot probes provided superior signal quality for the initial testing, these sensors were tragically destroyed by an electrical power surge and could not be implemented in all the tests. PCB transducers are installed in the flush-mounted configuration in Tubes 1-4 in the first set of turbine inlet ports (Figure 4.1, 1.68m axially downstream of the PDC headwall). While these were able to resolve the initial detonation pressure peak with exceptional clarity, the heat loading was too severe to resolve pressure dynamics over the remainder of the pulse cycle and gradually degraded the sensors. Later in the testing campaign, an ITP-type installation was adopted, and the transducer is located in the Tube 1 headwall port. Ionization probes are installed within the dual ionization probe ports within all six PDC tubes to capture the detonation wave speed using the time of flight technique.

**Figure 4.1: Schematic of the PDC-Turbine system with sensor port locations**

CTAP sensors are also installed at headwall ports in an attempt to estimate the reactant fill density from the average headwall pressure \( P_{\text{fill}} = P_{\text{purge}} = P_{\text{CTAP}} \). Type-K exposed thermocouples are embedded directly into the headwall of each tube, and provide a low-fidelity measure of whether all the combustors are actively firing. Two additional type-K grounded thermocouples are located at the
second turbine inlet port and in the turbine exhaust duct to estimate the time-averaged turbine inlet and exit temperatures. Torque and shaft speed are measured from the output shaft connecting the turbine to the dynamometer.

The system is not run to thermal equilibrium, as the power turbine used in this study is not rated for continuous use at these high temperature inlet conditions. The system starts at a relatively cold condition \((T_i \leq 350 \text{ K})\) and is run for a minimum 10 seconds prior to data acquisition to stabilize the turbine operating point. Because of the unpredictable starting transient of the system, the relatively brief run times, and the large disparity between initial and final rotor speed, it is extremely difficult to predict the terminal state of the system a priori. As the turbine loading is fixed prior to ignition of the system, this should notionally yield a unique end condition (terminal rotor speed and output power), but this is seldom achieved in practice. An adequately resolved range of rotor speeds for a given inlet condition is achieved through trial and error. At each stabilized set point, data is recorded at 1 MHz for 5 seconds, yielding a minimum of 25 pulse cycles for phase averaging. Test cases containing more than 15% drift in the set point are discarded.

For all cycle calculations, each of the six admission sectors is assumed to have matching, but phase-shifted flow. Measurements from adjacent PDC tubes are compared by phase-shifting measurements from neighboring inlet sectors by the appropriate time delay based off the firing order. The PCB-based pressure measurements only capture the fluctuating pressure component and cannot resolve a mean pressure component. The average PCB pressure over the test is subtracted from the signal, preserving the fluctuating component, and added to CTAP measurements to calculate time-resolved pressure. Without accurate, time-resolved measurements of temperature and velocity, it is problematic to implement any of the flow-based weighting techniques to generate bulk quantities and directly evaluate component efficiency. Instead, the thermal efficiency (Eq. 1.15) is calculated from the steady quantities
by dividing the shaft power output by the product of the fuel flow rate and lower heating value. Component efficiencies can be estimated from thermal efficiencies by making several assumptions regarding the efficiency of the detonation process and applying classical thermodynamic analysis.

Due to the difficulty of accurately measuring temperature in this environment, and the spatial and temporal nonuniformity in temperature, a simple energy-balance method is used to calculate a representative bulk temperature for the flow entering the turbine. As there is no appropriate way to accurately account for the unsteady effects with the present instrumentation, these are completely neglected. Assuming constant specific heat for the reacting and purge flow \( c_p = 1.004 \text{ kJ/kg K} \) and using the lower heating value of ethylene provided by Domalski (1972) \( \text{LHV}_{C_2H_4} = 48.4 \text{ MJ/kg} \), the final enthalpy is calculated as the sum of initial enthalpy and the added heat:

\[
H_{02} = H_{01} + Q_{add} \tag{4.3a}
\]

\[
T_{02} \cdot c_p \cdot \dot{m}_{tot} = T_{01} \cdot c_p \cdot \dot{m}_{tot} + \text{LHV}_{C_2H_4} \cdot \dot{m}_{C_2H_4} \tag{4.3b}
\]

\[
T_{02} = T_{01} + \frac{\text{LHV}_{C_2H_4} \cdot \dot{m}_{C_2H_4}}{c_p \cdot \dot{m}_{tot}} \tag{4.3c}
\]

This bulk estimate of the total temperature entering the turbine can be used to formulate corrected turbine speed (Eq. 1.7). However, because of the disparity in total specific enthalpy between the product and purge flows, this may not correlate to the average relative blade angles experienced by either flow.

4.2 Verification of Detonating Flow

While low-speed monitoring of CTAP magnitudes and headwall temperatures serves as a basic method to verify combustion in each of the PDC tubes, the ionization records are used to calculate combustion wave speed and directly verify the presence of detonation. The ionization record for a typical test consists of sequential ionization events for each tube, consistent with the firing order (1-5-3-6-2-4), with
six total detonation pulses for each cycle (Figure 4.2). As the present ionization probes are automotive 
spark plugs, there is some variation in the quality of each probe due to erosion of the ceramic insulator 
around the central electrode, and damage or removal of the side electrode. Gradual degradation can 
compromise the effectiveness of the probe in producing a consistent voltage drop. In the case depicted 
in Figure 4.2, all tubes consistently produce \( \Delta V \geq 4V \) in the presence of detonation, which is sufficient to 
detect the combustion wave and apply time-of-flight calculation.

![Ionization Record](image)

**Figure 4.2: Ionization record for the first probe in each combustor \((f = 10 \text{ Hz}, ff = 0.8, pf = 0.8)\)**

Each of the combustors is equipped with a pair of ionization probes, spaced approximately 100 mm 
apart, which can resolve arrival time of the combustion wave within \( \pm 1 \mu s \). Because of the speed of the 
detonation front, a pair of ionization traces in a single PDC tube appear to be coincident from a global 
perspective (Figure 4.3a), and have a very regular, periodic response. A detailed view of the ionization 
traces (Figure 4.3b), shows the phase lag between the two sensors, equal to \( \Delta t = 53 \mu s \) for the depicted 
detonation event. This corresponds to a wave speed of \( W = 1.89 \text{ km/s} \), close to the theoretical value for 
a stoichiometric ethylene-air mixture (at \( P = 1 \text{ bar}, T = 300K \), \( W_{CJ} = 1.82 \text{ km/s} \). Extending this calculation 
to the rest of the pulses, the wave speed is consistently near the ideal detonation value (Figure 4.3), 
verifying that detonation is periodically achieved in the combustor. While there are many instances of
wave speed exceeding the CJ value, this overdrive \( W > W_{CJ} \) is commonly observed by Knystautas et al. (1986) and Driscoll et al. (2015b), when detonation propagates beyond a DDT obstacle into smooth section of tube. As the DDT obstacle extends to the first ionization sensor in the present investigation, this may be responsible for the slight overdrive.

![Graphs showing wave speed and ionization records](image)

**Figure 4.3: Ionization record for both probes in tube 1 \( (f = 10 \text{ Hz}, \, ff = 0.8, \, pf = 0.8) \)**

### 4.3 Analysis of Pressure Profiles

The evolution of the turbine inlet pressure for a typical test case is depicted in Figure 4.4, as measured by the water-cooled pitot probe. The peak pressure magnitude \( P_{\text{MAX}} \) for a detonation cycle is quite repeatable, with all peak pressures collapsing within ±6% of the mean peak pressure (average \( P_{\text{MAX}} \) for
All detonation cycles). For this mixture, assuming ambient initial conditions \( (P_i = 1 \text{ bar}, \ T_i = 300 \text{K}) \), the post-detonation pressure calculated from the NASA CEA code is \( P_{\text{CJ}} = 18.2 \text{ bar} \). The discrepancy between the measured pressure and the CJ pressure can be attributed to two factors. The fill fraction for this case is significantly less than \( ff = 1 \), and the transducer (located near the turbine inlet) experiences a decaying blast wave instead of a detonation. Also, the pitot probe configuration has slower response (\( \approx 20 \mu s \)) than a flush-mount installation and cannot fully resolve the peak pressure of a transient event. The phase average (Figure 4.4b) provides a clearer view of the pressure evolution in a combustor during a single detonation cycle. The window for the averaging has been deliberately selected to fix the peak pulse pressure at the midpoint of the window. The pulse active period, where the pressure exceeds its initial value \( (P > P_i \approx 1 \text{ bar}) \), is relatively brief and persists for approximately 5-6 ms (2.5-3.0% of the total cycle). This is in significant contrast to the pulse shape observed in the pulsed-air investigation, where the pulse active period typically exceeds 33% of the cycle time. There is a slight sub-atmospheric component observed after blowdown, where the measured pressure descends below \( P = 1 \text{ bar} \), followed by a recovery to the initial, pre-detonation pressure. Extremely faint fluctuations are evident during the pulse idle period at regular intervals, corresponding to feedback from adjacent PDC tubes.

Figure 4.4: Kulite turbine inlet pressure record for a typical test \( (f = 5 \text{ Hz}, ff = 0.43, pf = 0.43) \)

The Kulite inlet pressure evolution is compared against flush-mounted PCB pressure evolution from the
four PCB transducer locations (Figure 4.5a). The phase-shifted, phase averaged PCB pressure evolution agrees well between the four detonation tubes containing transducers. Shortly after the pulse active period, heat loading on the PCB sensors produces a strongly negative $(P < 0 \text{ bar})$, nonphysical drift in the signal, which persists for 50 ms before recovering to the initial pre-detonation pressure. The PCB transducers also detect the faint feedback ($\Delta P \approx 0.1 \text{ bar}$) from adjacent detonation tubes. A detailed view of the pulse active period (Figure 4.5b) highlights the fast response and sharp definition of the PCB pressure profile prior to heat soak. Following the incident shock event ($t = 0.1s$), the Kulite transducer detects a secondary pressure peak, with an elapsed time of $\Delta t \approx 60\mu s$ between events. The PCB transducers also capture this secondary shock, with significantly better resolution of the gradient, and with an elapsed time of approximate elapsed time of $\Delta t \approx 85\mu s$ between events. From the arrangement of sensors, this appears to be an upstream-traveling shock wave, possibly generated by the interaction of the downstream travelling blast wave and the area constriction prior to the turbine inlet. While the reflected shock produces a temporary pressure rise, the entire blowdown period persists for only 5-6 ms with the present geometry. After a brief delay corresponding to the shock transit time, a large pressure pulse is recorded by the Kulite sensor in the turbine exhaust duct, indicating that some of the available power within the detonation pulse is exiting the turbine and attenuating within the exhaust flow.

![Figure 4.5: Pressure comparison from multiple tubes for a typical test ($f = 5 \text{ Hz}, f_{f} = 0.43, p_{f} = 0.43$)](image)
The phase-averaged pitot pressure evolution for the turbine inlet and exit sensors is compared for four fill fractions at \( f = 5 \) Hz (Figure 4.6). Pulse amplitude increases monotonically with fill fraction, as the detonation propagates farther down the tube before degenerating into a blast wave and decaying. The pressure profile upstream of the turbine (Figure 4.6a) contains both the primary and secondary shocks, and the elapsed time between the shocks is reduced with increasing fill fraction. This is presumably due to the increased velocity (and strength) of the downstream-propagating wave, which yields a stronger, faster reflected shock. As in the previous figure, there is a consistent delay in the pressure pulse between upstream and downstream locations, which corresponds to the finite travel time of the pulse through the turbine. The downstream pulse amplitude is relatively large (up to \( P = 5 \) bar) and also scales with fill fraction, indicating that the availability lost through the exit plane scales with the availability entering the turbine. Following the arrival of the detonation pulse, the downstream pressure rapidly decays within \( \approx 1 \) ms, and then continues to gradually attenuate back to its initial condition. There are secondary fluctuations in the downstream pressure, but these are likely influenced by the rotor dynamics and difficult to repeat for varying fill fractions due to the difficulty of achieving repeatable rotor speed between tests.

Figure 4.6: Effect of fill fraction on inlet and exit pressure profiles during pulse active period (\( f = 5 \) Hz)
While the flush-mounted PCB transducers provide superb resolution of the pressure during the pulse active period, the heat loading and corresponding signal drift is increasingly problematic for higher frequencies and fill fractions. To address this issue, a PCB in an ITP installation was implemented at the combustor headwall port of Tube 1 in lieu of the damaged Kulite pitot sensors. This sensor is capable of resolving pressure fluctuations during the idle state and for the entirety of the detonation blowdown period, but experiences a heavily attenuated pulse and reduced response time on account of the standoff tube.

The phase-averaged ITP evolution for the same turbine inlet condition at four rotor speeds are compared (Figure 4.7a). There is almost no discernible difference between the four profiles, indicating that rotor speed has little to no effect on the flow behavior upstream of the turbine. The pressure profile contains the same secondary peaks observed in Figure 4.5, but with significantly higher amplitude. This feedback phenomenon appears to increase with total flow rate through the combustor.

![Figure 4.7: Effect of rotor speed on headwall ITP-PCB pressure profiles (f = 10 Hz, ff = 0.65, pf = 0.83)](image)

A detailed view of the pulse active period demonstrates the severe reflection-based noise occurring after the detonation event (Figure 4.7b). There are initially intense fluctuations, possibly due to pressure buildup prior to DDT. Then the signal experiences a gradual rise over several hundred
microseconds, then decays with a series of smaller reflections. However, despite these reflections, the four cases collapse almost perfectly, indicating that rotor speed has no effect on the upstream behavior during blowdown. This is sensible, as the detonation products are choked at the constriction prior to the turbine inlet and disturbances cannot propagate upstream.

The effect of frequency is explored by comparing the phase-averaged ITP profiles of three cases ($f = 5$, 10, 15 Hz) with comparable fill fraction ($ff = 0.65$). The pulse time scales for the three frequencies are normalized to collapse the profiles (Figure 4.8a). As the frequency increases, the amplitude of the secondary pressure peaks increases. The blowdown period, when expressed as a fraction of the cycle time, is significantly lengthened, and appears to blend with the secondary peak for the $f = 15$ Hz case. The pulse active period is depicted without normalizing the time scale (Figure 4.8b), and the start of the detonation pulses are synchronized at $t = 0$. From this view, it appears that the blowdown process is comparable in duration for the three frequencies. The principal advantage of higher frequencies is to reduce the cycle time relative to a finite blowdown period. This expands the effective duty cycle and provides more continuous flow to the turbine, as reported by Rouser et al. (2013).

![Figure 4.8: Effect of frequency on headwall ITP pressure profiles ($ff = 0.65, pf = 0.80$)](image)
4.4 Analysis of Output Power

The evolution of rotor speed and output power for a typical test case is depicted in Figure 4.9. The power and speed typically stabilize to a terminal value within 5-10s of fuel introduction. Prior to the beginning of detonation, there are fluctuations in output power due to the unsteady torque generated by pulsed air flow through the turbine. Under detonating operation, the intensity of these torque fluctuations increases, yielding ± 10% variation in output power. The large fluctuations in power output may be responsible for the difficulty of achieving repeatable turbine loading and rotor speed for almost identical dynamometer settings. This also generates significant vibrations in the test apparatus, commonly shaking sensor wires and bolts loose. In three instances, the vibrations were severe enough to induce catastrophic failure of the drive train connecting the turbine to the dynamometer. A detailed analysis of the vibrations and potential coupling of the unsteady torque with the structural modes of the test setup is beyond the scope of the present work, but certainly merits an additional investigation.

Figure 4.9: Output power and rotor speed evolution for a typical test ($f = 10$ Hz, $ff = 0.73$, $pf = 0.73$)

The type-K thermocouples installed at the turbine inlet and within the turbine exhaust duct are not sufficiently responsive to reach a stable temperature for the present test length (Figure 4.10). These thermocouples are processed by a Newport Electronics panel meter, which provides cold junction compensation and amplification, at the cost of a significant response penalty. The thermocouple
installed at the Tube 1 headwall has an exposed junction and stabilizes quickly to a limit cycle temperature. This temperature is significantly lower than the turbine inlet temperature, as the aluminum headwall acts as a heat sink during the test, and the entry region of the combustor is cooled by the injection of fresh reactants and purge gases. Downstream, the thermocouples are grounded, as exposed thermocouples experience rapid failure (typically within several tests) due to cracking of the junction in the detonation environment. These sensors typically do not stabilize by the end of the test due to their response time, and cannot be used to accurately estimate the turbine inlet temperature.

![Temperature evolution for a typical test](image)

**Figure 4.10: Temperature evolution for a typical test ($f = 10\ Hz, ff \approx 0.73, pf \approx 0.73$)**

The output power, and corresponding thermal efficiencies are summarized in Table 4.1 for the 5 Hz and 10 Hz cases which reached an adequate degree of set point stability. The normalized corrected speed is calculated from the rotor speed, using Eqs. 4.3a-c to estimate the bulk turbine inlet temperature. Average headwall pressure, as recorded by the CTAP sensors, is also reported, as it is used in the calculation of fill and purge fractions. As this time-averaged pressure measurement cannot distinguish the pressure during the fill phase apart from the pressure during the pulse active phase, it may underestimate the true fill and purge fractions by assuming higher gas density. Using this pressure measurement to estimate the initial reactant state, the analysis method of Heiser and Pratt (2002) is employed to evaluate ideal thermal efficiencies for the Brayton and Fickett-Jacobs (detonation) cycles.
Table 4.1. Summary of PDC-turbine test cases

<table>
<thead>
<tr>
<th>$f$</th>
<th>$P_{\text{CLAP}}$</th>
<th>$ff$</th>
<th>$\rho_f$</th>
<th>$N/N_{\text{DES}}$</th>
<th>$N$</th>
<th>Power ($10^3$ RPM)</th>
<th>$\eta_{\text{in}}$ (exp)</th>
<th>$\eta_{\text{in}}/\eta_{\text{ideal}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hz</td>
<td>bar</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>kW</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.24</td>
<td>0.73</td>
<td>0.73</td>
<td>0.23</td>
<td>18.2</td>
<td>18.3</td>
<td>2.4%</td>
<td>6.0%</td>
<td>26.9%</td>
</tr>
<tr>
<td>1.24</td>
<td>0.72</td>
<td>0.72</td>
<td>0.24</td>
<td>18.9</td>
<td>18.6</td>
<td>2.4%</td>
<td>6.1%</td>
<td>26.9%</td>
</tr>
<tr>
<td>1.25</td>
<td>0.71</td>
<td>0.71</td>
<td>0.27</td>
<td>21.3</td>
<td>19.3</td>
<td>2.6%</td>
<td>6.4%</td>
<td>27.1%</td>
</tr>
<tr>
<td>1.25</td>
<td>0.71</td>
<td>0.71</td>
<td>0.29</td>
<td>22.8</td>
<td>20.4</td>
<td>2.7%</td>
<td>6.5%</td>
<td>27.1%</td>
</tr>
<tr>
<td>1.25</td>
<td>0.72</td>
<td>0.72</td>
<td>0.31</td>
<td>24.5</td>
<td>21.3</td>
<td>2.8%</td>
<td>6.4%</td>
<td>27.1%</td>
</tr>
<tr>
<td>1.27</td>
<td>0.70</td>
<td>0.87</td>
<td>0.38</td>
<td>29.1</td>
<td>22.8</td>
<td>3.0%</td>
<td>6.9%</td>
<td>27.4%</td>
</tr>
<tr>
<td>1.27</td>
<td>0.59</td>
<td>0.86</td>
<td>0.22</td>
<td>15.8</td>
<td>16.8</td>
<td>2.6%</td>
<td>6.8%</td>
<td>27.4%</td>
</tr>
<tr>
<td>1.31</td>
<td>0.65</td>
<td>0.83</td>
<td>0.45</td>
<td>33.8</td>
<td>16.0</td>
<td>2.2%</td>
<td>7.7%</td>
<td>28.0%</td>
</tr>
<tr>
<td>1.27</td>
<td>0.65</td>
<td>0.94</td>
<td>0.55</td>
<td>40.4</td>
<td>7.7</td>
<td>1.1%</td>
<td>6.8%</td>
<td>27.4%</td>
</tr>
<tr>
<td>1.28</td>
<td>0.64</td>
<td>0.93</td>
<td>0.58</td>
<td>42.7</td>
<td>7.8</td>
<td>1.1%</td>
<td>6.9%</td>
<td>27.5%</td>
</tr>
<tr>
<td>1.23</td>
<td>0.60</td>
<td>0.62</td>
<td>0.21</td>
<td>16.8</td>
<td>10.3</td>
<td>1.6%</td>
<td>6.0%</td>
<td>26.8%</td>
</tr>
<tr>
<td>1.23</td>
<td>0.61</td>
<td>0.62</td>
<td>0.23</td>
<td>18.5</td>
<td>10.9</td>
<td>1.7%</td>
<td>6.0%</td>
<td>26.8%</td>
</tr>
<tr>
<td>1.23</td>
<td>0.60</td>
<td>0.59</td>
<td>0.38</td>
<td>29.9</td>
<td>10.9</td>
<td>1.7%</td>
<td>5.8%</td>
<td>26.7%</td>
</tr>
<tr>
<td>1.18</td>
<td>0.54</td>
<td>0.56</td>
<td>0.20</td>
<td>15.3</td>
<td>8.2</td>
<td>1.5%</td>
<td>4.9%</td>
<td>26.1%</td>
</tr>
<tr>
<td>1.18</td>
<td>0.54</td>
<td>0.56</td>
<td>0.21</td>
<td>16.3</td>
<td>8.4</td>
<td>1.6%</td>
<td>4.9%</td>
<td>26.1%</td>
</tr>
<tr>
<td>1.18</td>
<td>0.54</td>
<td>0.56</td>
<td>0.27</td>
<td>21.2</td>
<td>9.4</td>
<td>1.7%</td>
<td>4.8%</td>
<td>26.0%</td>
</tr>
<tr>
<td>1.13</td>
<td>0.47</td>
<td>0.49</td>
<td>0.18</td>
<td>14.1</td>
<td>5.9</td>
<td>1.3%</td>
<td>3.7%</td>
<td>25.2%</td>
</tr>
<tr>
<td>1.13</td>
<td>0.47</td>
<td>0.49</td>
<td>0.21</td>
<td>16.4</td>
<td>6.4</td>
<td>1.4%</td>
<td>3.6%</td>
<td>25.2%</td>
</tr>
<tr>
<td>1.12</td>
<td>0.48</td>
<td>0.50</td>
<td>0.44</td>
<td>34.5</td>
<td>4.5</td>
<td>1.0%</td>
<td>3.3%</td>
<td>25.0%</td>
</tr>
<tr>
<td>1.12</td>
<td>0.47</td>
<td>0.49</td>
<td>0.44</td>
<td>35.1</td>
<td>4.9</td>
<td>1.1%</td>
<td>3.4%</td>
<td>25.0%</td>
</tr>
<tr>
<td>1.08</td>
<td>0.80</td>
<td>1.01</td>
<td>0.26</td>
<td>20.0</td>
<td>1.6</td>
<td>0.4%</td>
<td>2.3%</td>
<td>24.3%</td>
</tr>
<tr>
<td>1.08</td>
<td>0.80</td>
<td>1.01</td>
<td>0.26</td>
<td>20.0</td>
<td>1.6</td>
<td>0.4%</td>
<td>2.3%</td>
<td>24.3%</td>
</tr>
<tr>
<td>1.08</td>
<td>0.80</td>
<td>1.00</td>
<td>0.17</td>
<td>13.1</td>
<td>4.5</td>
<td>1.2%</td>
<td>2.5%</td>
<td>24.4%</td>
</tr>
<tr>
<td>1.08</td>
<td>0.80</td>
<td>1.01</td>
<td>0.19</td>
<td>14.1</td>
<td>2.8</td>
<td>0.8%</td>
<td>2.4%</td>
<td>24.3%</td>
</tr>
<tr>
<td>1.08</td>
<td>0.80</td>
<td>1.01</td>
<td>0.21</td>
<td>15.7</td>
<td>5.1</td>
<td>1.4%</td>
<td>2.4%</td>
<td>24.4%</td>
</tr>
<tr>
<td>1.08</td>
<td>0.80</td>
<td>1.00</td>
<td>0.21</td>
<td>15.9</td>
<td>3.8</td>
<td>1.0%</td>
<td>2.5%</td>
<td>24.4%</td>
</tr>
<tr>
<td>1.09</td>
<td>0.80</td>
<td>1.00</td>
<td>0.24</td>
<td>17.8</td>
<td>1.1</td>
<td>0.3%</td>
<td>2.5%</td>
<td>24.4%</td>
</tr>
<tr>
<td>1.09</td>
<td>0.80</td>
<td>1.00</td>
<td>0.26</td>
<td>20.0</td>
<td>1.6</td>
<td>0.4%</td>
<td>2.3%</td>
<td>24.3%</td>
</tr>
<tr>
<td>1.08</td>
<td>0.74</td>
<td>0.78</td>
<td>0.16</td>
<td>12.8</td>
<td>3.7</td>
<td>1.1%</td>
<td>2.4%</td>
<td>24.3%</td>
</tr>
<tr>
<td>1.07</td>
<td>0.75</td>
<td>0.79</td>
<td>0.29</td>
<td>22.8</td>
<td>4.3</td>
<td>1.3%</td>
<td>2.2%</td>
<td>24.2%</td>
</tr>
<tr>
<td>1.03</td>
<td>0.64</td>
<td>0.67</td>
<td>0.44</td>
<td>34.6</td>
<td>0.9</td>
<td>0.3%</td>
<td>1.2%</td>
<td>23.5%</td>
</tr>
<tr>
<td>1.04</td>
<td>0.50</td>
<td>0.51</td>
<td>0.11</td>
<td>8.6</td>
<td>1.7</td>
<td>0.8%</td>
<td>1.3%</td>
<td>23.6%</td>
</tr>
<tr>
<td>1.04</td>
<td>0.50</td>
<td>0.51</td>
<td>0.11</td>
<td>8.6</td>
<td>1.7</td>
<td>0.8%</td>
<td>1.3%</td>
<td>23.6%</td>
</tr>
<tr>
<td>1.04</td>
<td>0.50</td>
<td>0.51</td>
<td>0.12</td>
<td>9.4</td>
<td>1.8</td>
<td>0.8%</td>
<td>1.3%</td>
<td>23.6%</td>
</tr>
<tr>
<td>1.02</td>
<td>0.50</td>
<td>0.51</td>
<td>0.36</td>
<td>28.4</td>
<td>1.2</td>
<td>0.6%</td>
<td>0.8%</td>
<td>23.3%</td>
</tr>
<tr>
<td>1.03</td>
<td>0.42</td>
<td>0.43</td>
<td>0.09</td>
<td>7.4</td>
<td>1.2</td>
<td>0.7%</td>
<td>1.0%</td>
<td>23.4%</td>
</tr>
<tr>
<td>1.03</td>
<td>0.42</td>
<td>0.43</td>
<td>0.09</td>
<td>7.5</td>
<td>1.2</td>
<td>0.6%</td>
<td>1.0%</td>
<td>23.4%</td>
</tr>
<tr>
<td>1.03</td>
<td>0.42</td>
<td>0.43</td>
<td>0.09</td>
<td>7.5</td>
<td>1.2</td>
<td>0.7%</td>
<td>1.0%</td>
<td>23.4%</td>
</tr>
<tr>
<td>1.01</td>
<td>0.42</td>
<td>0.43</td>
<td>0.29</td>
<td>23.1</td>
<td>1.0</td>
<td>0.6%</td>
<td>0.6%</td>
<td>23.1%</td>
</tr>
</tbody>
</table>
The output power is plotted against the normalized corrected speed for both frequencies, for various fill and purge fractions (Figure 4.11). The power output for the 10 Hz cases is significantly higher than the that observed for the 5 Hz cases, such that the minimum 10 Hz power output is on the same order as the maximum 5 Hz power output. For the 5 Hz data, the power output appears to scale qualitatively with fill and purge fraction, but there is a large degree of scatter in the data due to poor set point stability. For corrected turbine speeds above 30% $N_{DES}$, power output falls considerably, and for several of the inlet conditions, higher rotor speeds could not be achieved, even for the lowest allowable dynamometer loading. This implies that at higher rotor speeds, the turbine efficiency drops to zero or is negative under some conditions and cannot extract a net power output.

The 10 Hz data exhibits two well-defined trends: the output power scales monotonically with fill and purge fraction, and there is an optimal rotor speed for peak power extraction. The first trend is sensible, as the available power should also scale monotonically with fill and purge fraction, and if the turbine efficiency is comparable, the output power should increase. Considering that the power increases by a factor of two (Power = 10 kW $\rightarrow$ 20 kW) while the fill fraction only increases moderately ($ff = 0.55 \rightarrow 0.70$), the specific power also increases with flow rate. The rotor speed trend is most pronounced for the $ff = 0.7, pf = 0.7$ curve, which experiences a steep penalty to output power away from the optimal speed.

![Figure 4.11: Effect of rotor speed and inlet condition on turbine output power](image-url)
4.5 Estimation of Component Efficiency

By making some assumptions to the problem (neglecting real gas effects and constructing zeroth-order thermodynamic modeling), the Fickett-Jacobs (FJ) detonation cycle can be applied, much like the analyses of Heiser and Pratt (2002) and Wintenberger and Shepherd (2006). The ideal FJ detonation cycle consists of isentropic compression (1→2), detonation (modeled as a normal shock followed by Rayleigh heat addition), isentropic expansion (5→6), and constant-pressure heat rejection.

![Diagram of the Fickett-Jacobs cycle](image)

**Figure 4.12: Overview of the Fickett-Jacobs cycle (Wintenberger and Shepherd 2006)**
The key unique component of this analysis is the detonation step, which employs moving pistons to emphasize the fundamental limitations of extracting work from the post-detonation CJ state. As first discussed by Zel'dovich (1940), the post-detonation state seemingly contains high availability, but the detonation structure requires expansion of the products to sustain itself, and this kinetic energy is not available as useful work. This is demonstrated in Figure 4.12, by preserving the post-detonation kinetic energy with a moving piston, instead of allowing the products to expand away from the detonation (thus doing work on the detonation). In place of the products, the piston does work on the detonation, which is imparted back to the piston when the flow is isentropically brought to rest in step (4→5).

Attempting to formulate the detonation cycle without accounting for this step and instead extracting work directly from the post-detonation state will result in optimistic, but nonphysical, estimates of available power output.

The ideal detonation cycle is compared to an ideal Brayton cycle with equal heat addition, using a T-s diagram to illustrate the key features of the cycles (Figure 4.13a). The nomenclature of Heiser and Pratt (2002) is merged with the nomenclature employed by Wintenberger and Shepherd (2006) to describe the thermodynamic states. Reactants are isentropically compressed from the initial state (1) to the pre-combustion state (2). For the Brayton cycle, heat is added at constant pressure to arrive at the post-combustion state. In the detonation cycle, reactants are shock-compressed from state 2 to state 3a, then undergo Rayleigh heat addition (in the wave-relative reference frame) to arrive at state 4. The detonation products expand from state 4 to the quiescent, post-detonation state 5. The products then expand isentropically between state 5 to state 6, providing useful work, some of which must be diverted for the compression process.

The degree of pre-compression is expressed here in terms of the temperature rise during 1→2 (ψ = T₂/T₁ = 1.5). Typical pressure ratios recorded by CTAP sensors fall in the range of P₂/P₁ ≤ 1.31, and correspond
to $\psi < 1.05$, which is comparatively lower than the present example. As discussed by Heiser and Pratt (2002), the Brayton cycle is extremely ineffective in this region, converging to $\eta_{th} = 0$ at $\psi = 0$. By contrast, the detonation cycle retains a finite amount of available power ($\Delta T_{5\rightarrow6} > 0$), even at $\psi = 0$, due to the compression provided by the detonation.

![Figure 4.13: Comparison of Detonation (FJ) and Brayton cycles ($y = 1.2, q_{add} = 6, \psi = 1.5$)](image)

Implementing realistic component efficiencies for each portion of the cycle, Heiser and Pratt (2002) developed three component efficiencies: compression ($\eta_c$), burner ($\eta_b$), and expansion ($\eta_e$). In their analysis, the expansion between $4 \rightarrow 5$ is considered together with the expansion between $5 \rightarrow 6$, and a single efficiency is applied to both processes. In the present analysis, which is concerned specifically with the turbine efficiency ($5 \rightarrow 6$), the two processes are decomposed, with separate efficiencies applied to each expansion. As the loss mechanisms within the turbine are independent from losses in the initial expansion process ($4 \rightarrow 5$), this approach is justified. In the cycle depiction of Figure 4.13b, and in the following cycle analysis, all component efficiencies, with the exception of the turbine are unity ($\eta_c = \eta_b = \eta_e(4\rightarrow5) = 1$).
To apply detonation cycle analysis to the experimental cases, state 5 is determined from the reactant pre-compression $\psi$, and normalized heat addition $\bar{q}_{add}$. As noted by Wintenberger and Shepherd (2006), the specific heat ratio for a detonation cycle should fall within the range $\gamma = 1.1 - 1.2$ to be representative of the detonation products. Heiser and Pratt (2002) demonstrate the calculation method of the detonation Mach number as a function of $\psi$ and $\bar{q}$ (Eq. 4.4). A normalized heat addition of $\bar{q}_{add} = 6$ is selected to produce the appropriate detonation Mach number for a stoichiometric ethylene-air mixture.

$$M_{CJ}^2 = (\gamma + 1)(\bar{q}_{add}/\psi) + 1 + \sqrt{[(\gamma + 1)(\bar{q}_{add}/\psi)]^2 - 1}$$  \hspace{1cm} (4.4)

The post-detonation state (4) can be determined relative to the initial state by decomposing the process into its constituent parts: pre-compression, shock compression, and Rayleigh heat addition (Eq. 4.5):

$$\frac{T_4}{T_1} = \frac{T_2}{T_1} \cdot \frac{T_{3a}}{T_2} \cdot \frac{T_4}{T_{3a}} = \psi \cdot \left(\frac{T_{3a}}{T_2}\right) \cdot \left(\frac{T_4}{T_{3a}}\right)$$ \hspace{1cm} (4.5)

The normal shock relations are applied determine the temperature ratio (Eq. 4.6a) and entropy generation (Eq. 4.6b) across the shock $2 \to 3a$. The post-shock Mach number relative to the wave is also calculated to provide the initial condition for the Rayleigh relations (Eq. 4.6c):

$$\frac{T_{3a}}{T_2} = \frac{\Delta s_{2 \to 3a}}{c_p} = \ln \left(\frac{2}{(\gamma + 1)M_{CJ}^2 - (\gamma - 1)}\right) + 1 + \ln \left(\frac{2\gamma}{(\gamma + 1)}\right)$$ \hspace{1cm} (4.6a)

$$\Delta s_{2 \to 3a} = \frac{2}{(\gamma + 1)M_{CJ}^2 - (\gamma - 1)} + \frac{1}{\gamma} \ln \left(\frac{2\gamma}{(\gamma + 1)}M_{CJ}^2 - (\gamma - 1)\right)$$ \hspace{1cm} (4.6b)

$$M_{3a}^2 = \frac{(\gamma - 1)M_{CJ}^2 + 2}{2\gamma M_{CJ}^2 - (\gamma - 1)}$$ \hspace{1cm} (4.6c)

The Rayleigh flow relations are applied determine the temperature ratio (Eq. 4.7a) and entropy generation (Eq. 4.7b) during the heat addition process $3a \to 4$.
\[ \frac{T_4}{T_{3a}} = \left[ 1 + \frac{\gamma M^2_{3a}}{(1 + \gamma)M^2_{3a}} \right]^\frac{\gamma+1}{\gamma} \]  
(4.7a)

\[ \frac{\Delta s_{3a \rightarrow 4}}{c_p} = \ln \left[ M^2_{3a} \left( \frac{\gamma + 1}{1 + \gamma M^2_{3a}} \right) \right] \]  
(4.7b)

State 5 can be determined from state 4 by assuming an isentropic process and expanding the detonation products relative to the detonation, up to the detonation velocity (\( u = W_{CJ} \)), thereby achieving quiescent flow in the laboratory frame of reference:

\[ T_4 \left[ 1 + \frac{\gamma - 1}{2} M^2_4 \right] = T_5 \left[ 1 + \frac{\gamma - 1}{2} M^2_5 \right] \]  
(4.8a)

\[ M^2_4 \equiv 1 \]  
(4.8b)

\[ M^2_5 = \frac{W^2_{CJ}}{\gamma RT_5} = \frac{W^2_{CJ}}{\gamma RT_2 \cdot \left( \frac{T_2}{T_5} \right)} = M^2_{CJ} \cdot \left( \frac{T_2}{T_5} \right) = M^2_{CJ} \cdot \psi \cdot \left( \frac{T_1}{T_5} \right) \]  
(4.8c)

\[ \frac{T_4}{T_1} \left( \frac{\gamma + 1}{2} \right) = \frac{T_5}{T_1} \left[ 1 + \left( \frac{\gamma - 1}{2} \right) M^2_{CJ} \cdot \psi \cdot \left( \frac{T_1}{T_5} \right) \right] = \frac{T_5}{T_1} + \left( \frac{\gamma - 1}{2} \right) M^2_{CJ} \cdot \psi \]  
(4.8d)

\[ \frac{T_5}{T_1} = \frac{T_4}{T_1} \left( \frac{\gamma + 1}{2} \right) - \psi \cdot M^2_{CJ} \left( \frac{\gamma - 1}{2} \right) \]  
(4.8e)

The sum of the entropy generated by the shock and heat addition processes determines the total entropy generated by the ideal detonation cycle. In the present analysis, the turbine provides an additional entropy source between states 5\( \rightarrow \)6 for the real detonation cycle:

\[ \frac{\Delta s_{5 \rightarrow 6}}{c_p} = \ln \left[ \frac{T_6}{T_5} \right] - \frac{\gamma - 1}{\gamma} \ln \left[ \frac{P_6}{P_5} \right] = \ln \left[ \frac{T_6/T_5}{T_{6i}/T_5} \right] \]  
(4.9)

The normalized heat rejection \( \tilde{q}_{rej} \) occurs at constant pressure, and is calculated from the total entropy:

\[ \tilde{q}_{rej} = \frac{q_{rej}}{c_p T_1} = \frac{c_p (T_6 - T_1)}{c_p T_1} = \frac{c_p T_1}{c_p T_1} \left( \frac{T_6}{T_1} - 1 \right) = \left( e^{\Delta s_{5 \rightarrow 6}/c_p} - 1 \right) \]  
(4.10)
The thermal efficiency is defined as net work extracted from the system, divided by the heat addition into the system. As the work out of the system is equal to the difference between the heat addition \( \ddot{q}_{\text{add}} \) and heat rejection \( \ddot{q}_{\text{rej}} \), the thermal efficiency can also be expressed in terms of these quantities, as implemented by Heiser and Pratt (2002):

\[
\eta_{\text{thermal}} = \frac{W_{\text{net}}}{\ddot{q}_{\text{add}}} = \frac{\ddot{q}_{\text{add}} - \ddot{q}_{\text{rej}}}{\ddot{q}_{\text{add}}} = 1 - \frac{\ddot{q}_{\text{rej}}}{\ddot{q}_{\text{add}}} \quad (4.11)
\]

Cycle thermal efficiencies are computed for the ideal and real detonation and Brayton cycles and compared to the experimentally-measured thermal efficiencies (Figure 4.14). Thermal efficiency \( \eta_{\text{th}} \) is plotted against the initial reactant pressure ratio, which is evaluated from the CTAP measurement for the experimental data. As the design point efficiency of the turbine is \( \eta_{\text{turb}} = 61.6\% \), the ideal cycles provide extremely optimistic estimates of \( \eta_{\text{th}} \). Assuming a turbine efficiency of \( \eta_{\text{turb}} = 61.6\% \), the real detonation cycle efficiency still exceeds the experimental data by a considerable margin (Figure 4.14a). However, many of the cases exceed the real Brayton cycle \( (\eta_{\text{turb}} = 61.6\%) \) efficiency, with some points even exceeding the ideal Brayton efficiency. For these cases, the detonation cycle provides a tangible overall improvement to the cycle by utilizing detonation combustion.

![Figure 4.14: Comparison of experimental data to predicted cycle efficiencies (\( \gamma = 1.2, \ddot{q} = 6 \))](image-url)
However, this overall improvement to the cycle output does not equate to comparable performance of the turbine work extraction process. Thermal efficiency curves are imposed upon a detailed view of the experimental data for varying turbine efficiencies (Figure 4.14b). The 10 Hz experimental data fall between the range $\eta_{turb} = 5$-15%, while the 5 Hz cases are considerably less efficient with $\eta_{turb} = 2$-7%. Compared to the turbine efficiencies measured under pulsed-air conditions, the turbine efficiencies for pulsed detonating flow are extremely poor. The Brayton cycle efficiencies are included to highlight that a relatively inefficient turbine with a detonation combustor can outperform a relatively efficient turbine with a conventional combustor at these low cycle pressure ratios.

4.6 Rotor Incidence Angle Considerations

The severe performance penalty imposed on the turbine motivates an idealized analysis of the flow angles within the turbine. While the flow condition cannot be directly measured at the stator exit, some basic assumptions allow for estimation of the rotor and stator flow angles. These assumptions are applied to two $f = 10$ Hz cases with comparable flow rates, but two disparate rotor speeds. The turbine inflow consists of sequential, out-of-phase detonations, which means that while one inlet sector experiences detonating flow, another sector is simultaneously experiencing purge flow. For the detonating sector, the transient high pressures produce choked flow at the stator exit ($c_{det} = a_{det}$), such that the exit velocity (i.e. speed of sound) can be predicted from temperature. Employing the state 5 properties from the previous thermodynamic analysis ($T_5 \approx 2100K$), the stator exit temperature is ($T_i \approx 1800K$) and the exit velocity is computed as $c_{det} \approx 830$ m/s. The purge sector flow velocity is estimated from the total purge flow rate, assuming a purge duty cycle of 33% (as determined from the pulsed-air study investigation), and using the CTAP sensors to estimate density. Heat transfer between the purge flow and the walls is neglected, and the stator exit temperature is assumed as $T_i \approx 300$ K. The stator and rotor profiles from the mean radii are employed as representative turbine geometry to define the design blade angles.
Considering first the low rotor speed case, velocity triangles are generated for both the detonating sector and purge sector, relative to a constant rotor speed of $U \approx 168 \text{ m/s}$ (Figure 4.15). There is a significant disparity in the rotor inflow angle between the two sectors, with a relatively positive incidence angle of $i_{\text{det}} = 39^\circ$ in the detonating sector, and an extremely negative incidence angle of $i_{\text{purge}} = -81^\circ$ in the purge sector. For the detonating sector, $i_{\text{det}} >> 0^\circ$ is likely to generate separation losses on the suction side of the rotor blade, but the rotor is still hypothetically able to turn the detonating flow and extract a significant amount of angular momentum. For the purge sector, $i_{\text{purge}} << 0^\circ$ is even more problematic, as the purge flow cannot be effectively turned, and provides no work. In this instance, $\beta_2 = -61^\circ$ and $\beta_3 = -59^\circ$, such that the turbine does a small amount of work on the purge flow.

Employing the same analysis to a case with significantly higher rotor speed (Figure 4.16), there is a moderate shift in the flow angles. The purge fraction for this case is slightly larger, yielding a modest increase in $c_{\text{purge}}$, though the purge sector incidence ($i_{\text{purge}} = -91.8^\circ$) is comparable to the previous case. However, the magnitude of the relative velocity ($w_{\text{purge}}$) is notably larger, which forces the turbine to impart a greater quantity of angular momentum into the purge flow to turn it from $\beta_2 = -72^\circ$ to $\beta_3 = -59^\circ$. 

**Figure 4.15: Velocity triangles of the rotor inflow at low rotor speed condition**

$f = 10 \text{ Hz}$  
$ff = 0.72$  
$pf = 0.72$  
$N = 24.5 \cdot 10^3 \text{ RPM}$  
$\eta_{\text{th}} = 2.8\%$  
$T_i = 1800 \text{ K}$,  
$c_{\text{det}} \approx 830 \text{ m/s}$  
$\beta_2 = 20^\circ$  
$\alpha_2 = 65^\circ$  
$\beta_3 = 59^\circ$  
$U = 168 \text{ m/s}$  
$U$  
$w_{\text{det}}$  
$C_{\text{det}}$  
$c_{\text{purge}}$  
$w_{\text{purge}}$  
$T_i = 300 \text{ K}$,  
$c_{\text{purge}} \approx 100 \text{ m/s}$
At this condition, the purge flow is actively decelerating the turbine and subtracting work from the system ("purge braking"). This deficiency in the purge flow angle is partially offset by the moderately favorable detonating sector incidence angle ($i_{det} = 32.6^\circ$).

Upon first glance, it appears that reducing the rotor speed will improve performance to some extent by limiting or eliminating the negative effects of purge braking. This approach is at least partially constrained by the separation losses incurred for $i_{det} >> 0^\circ$, which is likely responsible for the appearance of an optimal rotor speed in Figure 4.11b. However, there is an additional penalty that must be taken into account when reducing the rotor speed, as demonstrated by the Euler turbomachinery equation:

$$\text{Power} = \text{Torque} \cdot \dot{\omega} = \dot{m} (w_{t1} \cdot r_1 - w_{t2} \cdot r_2) \omega = \dot{m} \cdot U \cdot (w_{t1} - w_{t2})$$

(4.12)

The power extracted from the turbine is proportional to both the change in tangential velocity and the rotor speed. Considering the first case, ($U \approx 168 \text{ m/s}$), the change in tangential velocity for the detonating sector is $\Delta w_{t1 \vert_{det}} = 1167 \text{ m/s}$, yielding a specific power of $\text{Power}_{\text{specific}} = 196 \text{ kW/(kg/s)}$. The corresponding purge sector calculation yields $\Delta w_{t1 \vert_{purge}} = -6 \text{ m/s}$ and $\text{Power}_{\text{specific}} = -1 \text{ kW/(kg/s)}$, which
has a very slight negative contribution to the total power output. Multiplying the specific power by the flow rates of each component, the detonating flow contributes 48.7 kW of power, while the purge flow removes -0.25 kW, for a net power of 48.4 kW. This is significantly higher than the experimentally-measured power of 21.3 kW, as this simplified analysis does not account for flow separation, shock losses, or the period of detonation blowdown when the stator exit is unchoked.

Now considering the second case (U ≈ 290 m/s), there is a slight decrease in the tangential velocity change for the detonating sector (Δw_t |det = 1042 m/s), but this is completely offset by the large increase in rotor speed, yielding Power_{specific} = 305 kW/(kg/s). However, the purge sector produces a larger negative power component, with Δw_t |purge = -78 m/s and Power_{specific} = -23 kW/(kg/s). Scaling these values by the appropriate flow rates, the detonating flow power of 69.2 kW and the purge flow power of -7.5 kW combine for a net power of 61.7 kW. However, this estimate significantly exceeds the experimentally-measured power of 7.7 kW, possibly because the analysis greatly underestimates the losses imposed by the extreme angle and relative velocity of the purge flow. Despite the limitations of the present analysis, it reveals a distinct tradeoff between the benefit of high rotor speed on power extraction from the detonating flow (Eq. 4.12), and the drawback of the purge braking effect.

4.7 Conclusion

An axial turbine is explored under pulsed detonating inflow conditions to determine the effect of frequency, fill fraction, and rotor speed on turbine performance. Several fill and purge fractions are studied at two frequencies (f = 5, 10 Hz) for a range of rotor speeds. Analysis of the pressure profiles reveals that increasing fill fraction produces a monotonic increase in the pulse amplitude entering the turbine and lengthens the detonation blowdown time. Increasing the pulse frequency has the effect of increasing the duty cycle by reducing the cycle time, but does not significantly extend the blowdown duration. Secondary pressure peaks are observed at the combustor headwall at six times the firing
frequency, most likely corresponding to feedback from adjacent tubes. These fluctuations intensify with increasing frequency, and could possibly disrupt the detonation cycle at significantly higher flow rates. By contrast, the rotor speed has no discernible effect on the pressure evolution upstream of the turbine.

When operating under detonating flow, the turbine experiences significant fluctuations in torque, which produce ± 10% oscillations in power at a stabilized flow condition. This hinders the repeatability of set points, and produces intense vibrations, which can induce catastrophic failure of the system. Successful measurement of time-accurate temperature and other flow quantities is impeded by the destructive nature of the detonation environment and the limited robustness of available diagnostics.

Examining the variation of output power with fill/purge fraction and rotor speed, the 5 Hz data contains significantly more scatter than the 10 Hz data, but power generally increases with fill/purge fraction and reaches a maximum at an optimal rotor speed. Classical, detonation cycle analysis is applied to estimate the turbine efficiency from the cycle thermal efficiency. Experimentally-measured thermal efficiencies occasionally exceed the theoretical maximum Brayton cycle efficiencies, demonstrating an overall cycle improvement by using detonative combustion. However, this is in spite of a substantial reduction in the turbine efficiency ($\eta_{turb} |_{MAX} < 15\%$), which is four times lower than the design rated efficiency. Analysis of the rotor flow angles reveals significant disparity between adjacent admission sectors simultaneously undergoing different portions of the cycle. This produces a “purge braking” phenomenon, where an inlet sector expelling purge gases actively extracts power from the turbine as a neighboring detonating sector imparts power to the turbine. These competing flow components demonstrate a deficiency unique to a multiple-admission PDC-turbine system. As the purge and fill processes are typically regarded as necessary components to a practical PDC cycle, this poses a significant challenge for the future design of an efficient PDC-turbine system.
Chapter 5 – Correlation-Based Detection of the Operating State in a Rotating Detonation Combustor

The RDC operating state is commonly characterized using high frequency pressure transducers to detect the periodic passage of detonation. These signals can be processed with time-of-flight algorithms to determine wave speed, as discussed extensively by Russo (2011). This method typically involves scanning the pressure history for peaks above a user-specified threshold, and calculating the distance between adjacent peaks to determine the rotation time scale. As discussed in Anand et al. (2015b), this may yield erroneous values of wave speed if there are instabilities or instances of wave reversal. Time-of-flight algorithms can be modified to filter out spurious pressure fluctuations and reject nonphysical wave speeds in an attempt to mitigate this problem. However, for pressure signals with reflections, noise/ringing, or excessive attenuation, as observed by Anand et al. (2015b, 2016a), it may be difficult to detect the physically appropriate peak. For these signals, and tuning of the algorithm is somewhat arbitrary and subjective. Lu et al. (2009) demonstrate the efficacy of cross-correlation based techniques for evaluating wave propagation time in a more objective fashion. They compute the non-stationary cross-correlation (essentially a sliding dot product) of two detonation pressure traces to determine the time shift which achieves the best alignment between the two traces. The optimal value of time shift (which yields the maximum cross-correlation) corresponds to the time delay between the two events. While this approach has been applied to linear detonation propagation in a shock tube, it can be modified and recursively applied to track periodic detonation propagation within the RDC.

The present study develops a correlation-based method specifically tailored for use with RDC measurement data to extract the rotation time scale and direction as it evolves through a test. A data processing algorithm is applied to RDC pressure evolution from PCB transducers installed at various locations, and ionization evolution from flush-mounted ionization probes. The central objective of this work is to determine the efficacy and robustness of a correlation-based method for detailed RDC data
analysis. This approach may improve flexibility of sensor installations, while enabling reliable extraction of RDC operating statistics.

5.1 Experimental Approach

For this study, the combustor geometry is fixed, and the relevant geometric parameters for this configuration are identical to those provided in Table 2.2. For this investigation, the RDC is configured for operation with H₂-air mixtures. Air is constantly supplied to the combustor, while hydrogen is pneumatically valved upstream and supplied intermittently for each test. Each test consists of a one second fuel dump, followed by an ignition event and a controlled test length of approximately 0.3s. The test concludes with termination of fuel flow and eventual extinction of combustion. As flush-mounted transducers are installed within the combustor wall, the test length cannot exceed this value without damaging the sensors. Ignition is achieved with a tangentially-injecting initiator tube containing a C₂H₄-O₂ mixture (ϕ = 2.0), which enters the combustor approximately 22 mm axially downstream of the base of the combustor. This initiates combustion within the RDC channel, and detonation develops through a deflagration-to-detonation transition (DDT) process.

Figure 5.1: Schematic of various RDC sensor types and installation locations
To capture the pressure evolution following ignition, PCB pressure transducers and ionization probes are installed in various locations within the combustor (Table 5.1). Each set of sensors is installed at three azimuthal locations, (stations I, II, and III in Figure 5.1), with 120° spacing between sensors. A set of three ionization probes are flush-mounted within the first instrumentation row in the combustor to achieve close proximity to the reactant injection. These probes detect large fluctuations in the electrical resistance of the flow, and must be periodically exposed to highly ionized flow (combustion) and inert flow (fresh reactants) to properly capture detonation propagation. A set of three transducers are flush-mounted in the second instrumentation row to directly measure the detonation pressure within the combustor. A corresponding set of transducers is installed in an ITP configuration in the third instrumentation row. The ITP installation consists of a sensor mounted perpendicular to the axis of a standoff tube, which extends radially outward from the combustor. This tube has an inner diameter of approximately 3mm, and the sensor is located at a distance of 42 mm from the outer wall of the combustor. Three transducers are located within the air injection slot at a slight azimuthal offset from the other instrumentation locations at a distance of 22 mm from the outer wall of the combustor. The passage of a detonation wave strongly disrupts the air injection (Rankin et al. 2015b, Anand et al. 2016a), and this disruption can be recorded by pressure transducers installed within the air injection slot. Onset of periodic behavior within the air injection slot correlates well with onset of stable rotation within the combustor and captures the same fundamental frequency (Anand et al. 2016a).

Table 5.1. Instrumentation overview

<table>
<thead>
<tr>
<th>Sensor Type</th>
<th>Installation Type</th>
<th>Location Row</th>
<th>Location Θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ionization Probe</td>
<td>Flush-mounted</td>
<td>1</td>
<td>60°, 180°, 300°</td>
</tr>
<tr>
<td>PCB Transducer</td>
<td>Flush-mounted</td>
<td>2</td>
<td>60°, 180°, 300°</td>
</tr>
<tr>
<td>PCB Transducer</td>
<td>ITP</td>
<td>3</td>
<td>60°, 180°, 300°</td>
</tr>
<tr>
<td>PCB Transducer</td>
<td>Air injection slot</td>
<td>-</td>
<td>40°, 160°, 280°</td>
</tr>
</tbody>
</table>
This study reproduces select test conditions previously investigated for this RDC configuration by Driscoll et al. (2015a) and Anand et al. (2015a). The combustor operating modes explored in the present study are a single rotating detonation wave ($n = 1$), and two rotating detonation waves ($n = 2$). The combustor exhibits varying degrees of stability, ranging from fully stable, periodic behavior to completely incoherent, aperiodic behavior. Commonly observed instabilities of interest, as reported by Anand et al. (2015b), include chaotic instability, waxing and waning instability, and mode switching instability. The chaotic instability is punctuated by large fluctuations in the elapsed time between consecutive waves as well as the corresponding peak pressures of these waves. For highly chaotic cases, lack of well-defined periodicity precludes the calculation of a consistent operating frequency. Coherent, periodic behavior may emerge briefly for several cycles before disintegrating into chaos. The waxing and waning instability is observed to some degree for most conditions, and manifests as a low frequency oscillation in the peak pressure amplitude between consecutive detonation waves. In regions of the operating map where two stable modes overlap (i.e. boundary between $n = 1$ and $n = 2$), an instantaneous change in the operating mode can occur (Suchocki et al. 2012). This mode switching instability is characterized by a sudden shift in the number of detonation waves ($n = 1 \leftrightarrow 2$), which manifests as an abrupt change in the operating frequency (Frolov et al. 2015, Anand et al. 2015b). To assess the effectiveness of the proposed correlation-based method, pressure and ionization measurements are acquired for these rotational operating modes and instabilities.

5.2 Signal Processing Methodology

The time-resolved signals are processed with a correlation-based algorithm to extract detailed, quantitative measures of the detonation time scales, stability, and mode of operation. First, the data is preconditioned with a high-pass filter with a low frequency bound of 100 Hz to remove any low frequency components in the signal (Figure 5.2). The primary role of the preconditioning is to remove the spurious thermal drift commonly observed for flush mounted PCBs, as in Wang et al. (2015).
Excessive heating of flush-mounted PCB transducers typically produces a non-physical, low frequency, negative response in the pressure signal due to thermal stresses within the sensor body, as discussed by Walter (2004). This preconditioning step is not required for ionization sensors, or for PCB sensors installed in locations with minimal heat loading, (e.g., air slot and ITP transducers).

A cross-correlation is essentially a sliding dot product of two vectors (i.e. $X(t)$ and $Y(t)$), where one of the vectors is shifted relative to the other by $\tau$ (i.e. $X(t) \cdot Y(t+\tau)$). Auto-correlation is the special case of cross-correlation where a signal is shifted relative to itself (i.e. $X(t) \cdot X(t+\tau)$). The normalized cross-correlation is computed by subtracting the mean component from each vector, and normalizing each vector by its standard deviation prior to calculating the dot product between the vectors (Eq. 5.1). This normalizes the magnitude of each of the vectors, such that the maximum attainable normalized cross-correlation is unity ($R_{XY}|_{MAX} = 1$), which occurs when the vectors are identical ($X = Y$). For most practical applications, where these vectors correspond to measurement signals (such as combustor pressure fluctuations in the present case), there is some level of noise within one or both of the signals, and $R_{XY} < 1$. However, for the case of auto-correlation, the vectors are identical when the shift is zero ($\tau = 0$) and
the signal is aligned with itself, which yields \( R_{xx} = 1 \). When a signal is periodic, the auto-correlation is also periodic with the same frequency (Lee et al. 1950), as the shifted signal periodically realigns with the original signal.

\[
R_{XY}(\tau) = \frac{1}{n} \sum_{t=0}^{n} \frac{(X(t) - \bar{X})(Y(t + \tau) - \bar{Y})}{s_X \cdot s_Y}
\]

A correlation magnitude of \( R_{xy} = 1 \) indicates that two signals are perfectly correlated, and a magnitude of \( R_{xy} = 0 \) indicates no correlation (as would be observed for two vectors of white noise). Between these two values, the interpretation of the correlation magnitude depends extensively on the amount of random variance contained within signal (i.e. the signal-to-noise ratio). Evans (1996) suggests the following interpretation for the correlation magnitude:

<table>
<thead>
<tr>
<th>Correlation magnitude interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00 &lt; ( R_{xy} &lt; 0.40 )</td>
</tr>
<tr>
<td>0.40 &lt; ( R_{xy} &lt; 0.59 )</td>
</tr>
<tr>
<td>0.60 &lt; ( R_{xy} &lt; 0.79 )</td>
</tr>
<tr>
<td>0.80 &lt; ( R_{xy} &lt; 1.00 )</td>
</tr>
</tbody>
</table>

The proposed algorithm processes the measurement time-traces in two loops: an auto-correlation loop and a cross-correlation loop (Figure 5.3). The auto-correlation loop is applied to each measurement location (i.e. azimuthal stations I, II, III) to capture the elapsed time between consecutive detonation waves. A source window is selected from the measurement time-trace, which contains a minimum of one instance of detonation wave passage. A search window is specified, which must include at least one additional instance of detonation, corresponding to the next wave passage event. The default search window length used in the present study is three times the source window length, and this window begins at the same index as the source window (i.e. the search window contains the full source
The source window is shifted relative to the search window by a time shift ($\tau$), and the normalized auto-correlation ($R_{xx}$) is computed for each value of $\tau$. The initial correlation peak at $\tau = 0$ corresponds to alignment of the source window with itself, and the second correlation peak corresponds to alignment with the next instance of detonation. The value of $\tau$ corresponding to the second peak is the elapsed time between the two detonation events (i.e. time scale for rotation), and the peak auto-correlation magnitude $R_{xx}(\tau)$ represents the degree of similarity between the two waves. These values are stored in an output vector, and $\tau$ is used to initialize the source window of the following iteration to contain the newly-identified detonation wave.

The cross-correlation loop is applied to each pair of azimuthally-spaced measurement locations to extract phase information (i.e. direction of wave rotation). For a wave rotating through the RDC at a constant speed, a consistent time delay should exist between wave passage events for a pair of azimuthally-spaced sensors. Three measurement locations are available for each installation type in the present instrumentation scheme (stations I, II, III), yielding up to three potential sensor pairs for cross-correlation (i.e. I→II, II→III, III→I). The first sensor is designated as the source sensor, and the latter as the search sensor. A window is specified for the measurement time-traces, which must contain a minimum of one instance of detonation wave passage for both sensor locations. Analogous to the auto-correlation loop, the source window is shifted relative to the search window by a time shift ($\tau'$), and the normalized cross-correlation ($R_{xy}$) is computed for each value of $\tau'$. The elapsed time between detonation waves ($\tau$), as determined in the auto-correlation loop, is used to specify the bounds of the time shift $\tau'$, such that the source window is shifted in either direction by half the rotation time ($\pm \tau/2$). The time shift $\tau'$ is converted to an equivalent phase angle $\alpha$ by normalizing by $\tau$ and converting to degrees (Eq. 5.2).

$$\alpha = \left(\frac{\tau'}{\tau}\right) \cdot 360^\circ \quad (5.2)$$
Within this range (-180° ≤ α ≤ +180°), a single peak is observed in the cross-correlation vector, corresponding to the phase lag between the two detonation events. The peak cross-correlation magnitude $R_{XY}(α)$ represents the degree of similarity between the detonation wave as observed by the two sensor locations. These values ($α, R_{XY}(α)$) are stored in an output vector, and the process is repeated for the next cross-correlation window.

**Figure 5.3: Flowchart of the correlation-based algorithm**

This process is demonstrated for an example case ($\dot{m} = 0.4$ kg/s, $\phi = 1.23$), utilizing the three azimuthal stations of flush-mounted PCB sensors:
5.2.1. Auto-correlation Loop

The pre-filtered pressure evolution recorded by the station III transducer is illustrated in Figure 5.4a. A gray, vertical line signifies the subset of the pressure evolution depicted in Figure 5.4b. The source window begins at \( t = 162.50 \) ms and extends to \( t = 162.86 \) ms, and contains a single detonation event. The search window, which fully contains the source window, also begins at \( t = 162.5 \) ms, and is three times the size of the source window (≈1 ms in length). The search window contains four detonation events (labeled A-D), which includes the initial wave captured in the source window (wave A). Initially, when the source window is aligned with itself (\( \tau = 0 \), wave A matched to wave A), the normalized auto-correlation begins at its maximum (\( R_{XX} = 1 \)), as seen in Figure 5.4c. As the source is shifted through the search window and the waves shift out of alignment, the correlation magnitude drops off rapidly, and fluctuates around zero (\( R_{XX} = 0.058 \) at \( \tau = 0.15 \) ms). When the source window aligns with the next wave in the search window (wave A matched to wave B), the normalized auto-correlation reaches a local maximum (\( R_{XX} = 0.793 \)), and the corresponding value of \( \tau = 0.26 \) ms is the elapsed time between detonation waves. Due to fluctuations in the signal, wave A is not identical to wave B, and \( R_{XX} < 1 \) at each local maxima as wave A aligns with subsequent waves.

The normalized auto-correlation vector is processed to detect the local maximum corresponding to the next detonation wave (wave A matched to wave B). As seen in Figure 5.4c, the correlation vector contains three well-defined local maxima (A, B, and C), and may contain more if the search window is sufficiently large. To avoid erroneously selecting the initial peak (\( \tau = 0 \)), where the wave A is matched to itself, the initial portion of the correlation vector is truncated. As wave A shifts out of alignment with itself, the correlation magnitude drops below zero, and this threshold of \( R_{XX} < 0 \) is used to determine the truncation length (\( \tau < 29 \) µs is omitted in the example of Figure 5.4c).

At the conclusion of this iteration of the auto-correlation loop, the elapsed time between waves (\( \tau = 260 \) µs) and the peak correlation value (\( R_{XX} = 0.793 \)) are stored in output vectors. The newly identified wave
(B), which has been successfully located in the search window, is assigned as the source wave for the following iteration. The source window length for the new iteration is set to 150% of the lap time from the previous iteration, which allows this window length to adapt to changes in wave spacing. As before, the search window length is set to a default length of three times the source window length. This iterating process continues for the entire test length, until the proposed search window overlaps with the end of the data vector.

![Graphs and diagrams showing pressure evolution, data subset, and normalized auto-correlation vector.]

Figure 5.4: Auto-correlation process for a single instance of detonation wave passage

The transducer data from all three azimuthal stations is depicted in Figure 5.5a for the entire test length of the present example case. The pressure fluctuations for stations I and III are comparable, with 2.5 bar $P > -1$ bar, while station II sees consistently higher pressure fluctuations (5 bar $P > -1.5$ bar). This
enhanced amplitude may correspond to higher detonation strength at station II, or may possibly be associated with sensor inaccuracy under high heat loading. In the application of the auto-correlation algorithm, the standard deviation \( (s_x) \) of the source window is calculated for each wave and implemented in Eq. 5.1. Incidentally, as the pressure signal has been high-pass filtered to remove low frequency fluctuations (including the mean component), the mean pressure is zero, and the standard deviation is equal to the root-mean-squared pressure. The standard deviation of the source window can be stored in an output vector and used as a measure of the signal fluctuation amplitude, as illustrated in Figure 5.5b. In the present case, \( s_x \) converges rapidly to a stable value for each signal, with stations I and III having comparable magnitude and station II having significantly larger magnitude. However, the standard deviation of the station II transducer drops rapidly at \( t \approx 0.25 \text{s} \) and converges to \( s_x = 0 \), corresponding to signal loss. This signal attenuation is observed for flush-mounted transducers and is associated with the extreme thermal environment.

![Graphs showing pressure evolution and source window standard deviation](image)

**Figure 5.5: Pressure evolution for entire example case for the three azimuthal sensor locations**

Rotation times (Figure 5.6a) and associated auto-correlation magnitudes (Figure 5.6b) identify erratic behavior following the ignition event, which evolves into stable, periodic wave propagation by \( t \approx 36 \text{ ms} \). Aperiodic pressure fluctuations are commonly observed in RDCs after ignition before the eventual onset of stable detonation rotation (Bykovskii *et al.* 2014). Following this onset, there is a gradual decrease in
the elapsed time between detonations from $t = 0.036-0.1s$, which converges to $\tau = 260\pm 2\ \mu s$ (Figure 5.6a). This converged rotation time corresponds to an operating frequency of $f = 3.85 \ kHz$, which yields a rotational speed of $W = 1.86 \ km/s$ in the present combustor. This is close to the ideal, Chapman-Jouguet wave speed (≈92% $W_{CJ}$). The auto-correlation magnitudes also converge to a stable range during this period ($R_{XX} = 0.7-0.9$), which correspond to strong or very strong correlation between consecutive waves. This stable periodic state persists until $t = 0.35s$, when the detonation begins to decelerate ($\tau$ increases). As the fuel flow to the combustor is interrupted at $t \approx 0.3s$, the equivalence ratio gradually decreases, resulting in a breakdown of periodicity at $t = 0.37s$ and complete extinction of combustion by $t \approx 0.385s$.

![Graph showing onset and conclusion of periodic behavior](image1)

**Figure 5.6: Auto-correlation results for the three azimuthal sensor stations**

**5.2.2. Cross-correlation Loop**

As evident from the slight decrease in $\tau$ between $t = 0.05-0.10s$, and the slight increase in $\tau$ for $t > 0.35s$ (Figure 5.6a), the rotational speed can evolve during typical RDC operation, and in some cases, the direction of rotation can vary (Russo et al. 2011). This yields a change in the time delay ($\tau'$) between detonation events at two azimuthal stations. The time shift between each sensor pair is calculated at regular intervals throughout the test to capture the evolution of $\tau'$. Using a cross-correlation of the sensor pair, $\tau'$ can only be accurately determined with the same time resolution as $\tau$, as each vector in
the cross-correlation must contain a minimum of one wave. The present example employs a cross-correlation source window size of 1ms, which yields 3-4 waves per source window when \( n = 1 \) (Figure 5.7a), and 7-8 waves per window when \( n = 2 \). Analogous to the source and search windows established in the auto-correlation loop, the first sensor of the sensor pair is designated as the source (e.g., station I) and the latter sensor is used as the search (e.g., station II). The search window is extended by half the lap time before and after the source window (\( \pm \tau/2 \)) to ensure that it fully overlaps the source window for the full range of the cross-correlation. At \( t = 163 \) ms after the RDC ignition event, the lap time is stable at \( \tau = 260 \) µs, so the search window (station II) is extended by \( \pm 130 \) µs. The source window is then shifted relative to the search window to generate the cross-correlation vector (Figure 5.7b). For the range of \( \pm \tau/2 \), which corresponds to a complete cycle of detonation rotation around the combustor (\( \tau = 260 \) µs at \( t = 0.163s \) for the present example, as determined from the auto-correlation loop), the peak cross-correlation value (\( R_{XY} = 0.59 \)) occurs at \( \tau' = 88 \) µs. Using Eq. 5.2 to convert the delay time to an equivalent phase angle, this corresponds to \( \alpha = 120^\circ \). The convention for the station pairs (source → search) is counterclockwise, or consistent with ascending values of \( \theta \). This means that, the rotation proceeds in the positive \( \theta \) (counterclockwise) direction for positive phase shift (\( \alpha > 0^\circ \)).

![Figure 5.7: Cross-correlation process for a single instance for the station I → station II sensor pair](image)

a) Overlay of source and search windows  

b) Corresponding normalized cross-correlation
The phase delay (Figure 5.8a) and corresponding peak cross-correlation magnitudes (Figure 5.8b) are calculated for the three station pairs for the entire test case. Azimuthal cross-correlations and corresponding phase delays exhibit the same general features as identified in the auto-correlation loop. Shortly after RDC ignition, the cross-correlation magnitude between the sensor pairs is initially low ($R_{XY} < 0.4$ intermittently for all pairs), but rises by approximately $t = 36$ ms, coincident with the onset of periodic behavior. For $t > 0.25s$, the signal loss at station II results in a decline in the cross-correlations involving this signal ($I \rightarrow II, II \rightarrow III$). For the unaffected signal pair ($III \rightarrow I$), the phase shift is consistent at $\alpha \approx 120^\circ$ until $t = 0.37s$, which coincides with the breakdown of periodicity in Figure 5.6a. For the stable portion of the test, the phase shift is consistent and positive, indicating sustained rotation in the $+\theta$ (counterclockwise) direction. Prior to the station II signal loss, the cross-correlation magnitudes converge to a stable range ($R_{XY} = 0.5-0.7$), corresponding to moderate to strong correlation between azimuthal stations.

![Cross-correlation results for the three azimuthal sensor pairs](image)

**Figure 5.8: Cross-correlation results for the three azimuthal sensor pairs**

The auto-correlation and cross-correlation output can be used to detect the appearance of unstable behavior and quantify the operating state. The present example case reaches a single stable time scale ($\tau = 260$ $\mu$s), which corresponds to a single rotating detonation ($n = 1$) propagating at a speed of $W = 1.86$ km/s. As there are not multiple stable time scales (i.e. step changes in $\tau$ from one stable magnitude
to another), there is no rapid transition in the operating state (i.e. no mode switch). Extinction of the detonation appears as a discontinuity in $\tau$ for all three azimuthal stations, and is occasionally preceded by deceleration of the detonation wave. For this example case, there are no extinction events while the reactant flow rates are stable, but extinction occurs after the termination of fuel flow. A change in the detonation propagation direction (wave reversal) appears as a discontinuity in $\alpha$, and is not observed for the present case. Mean auto-correlation and cross-correlation magnitudes are a measure of the temporal and spatial variation of the detonation wave. Using the classification from Table 5.2, the consistency of the detonation wave shape is strong-to-very strong between consecutive detonation events at a fixed sensor location ($R_{xx} = 0.7-0.9$). Preservation of the wave shape is moderate-to-strong between sensor locations ($R_{xy} = 0.5-0.7$), which implies that the detonation varies slightly as it propagates azimuthally.

5.3 Application of the Algorithm

5.3.1. Application to Various Sensor Installations

The present correlation-based algorithm is advantageous for extracting the RDC operating state from noisy, distorted, or indirect measurements. Signals from the various transducer installations and the ionization probe installation are compared in Figure 5.9 for the example case ($\dot{m} = 0.4$ kg/s, $\phi = 1.23$). For sensors not installed directly within the combustor, repeated passage of a detonation generates a periodic disturbance, which may differ in shape from the pressure signal within the combustor, and may lack an easily identifiable peak (Figure 5.9b,c). For the ITP installation, large variations in wave shape are observed between azimuthal stations, with especially high pressure fluctuations at station II. As these fluctuations are occasionally in excess of those observed for the flush-mounted sensors, which are directly exposed to detonation, this behavior is likely nonphysical. The pressure amplitudes may be due to resonance within the standoff tube, similar to the response observed by Stevens et al. (2015). For the other two stations (I and III), the ITP signals are weaker than the flush-mounted and air injection slot
signals. It is possible that each station experiences a slightly different set of reflections as the detonation-induced shock diffracts into the standoff tube. Despite the presence of strong reflections, the ITP signals are still periodic, which allows the use of auto-correlations to extract the lap time. For the ionization probes, the signal is entirely different in shape and magnitude from the pressure evolution, and detonation events appear as a smooth reduction in voltage (Figure 5.9d). The periodicity is very pronounced, and the signal is free of the high frequency fluctuations observed in the transducer signals.

![Graphs of sensor response for various installations at three azimuthal stations](image)

**Figure 5.9: Comparison of sensor response for various installations at three azimuthal stations**

The correlation algorithm is applied to all four sets of measurement installations to determine if the RDC operating state can be effectively extracted. Figure 5.10 compares \( \tau \) and \( \alpha \) evolution for a single azimuthal location (station II), and a single pair of azimuthal locations (stations I → II). As previously
discussed, the PCB in the flush-mounted installation experiences signal loss due to heat soak, which highlights the difficulty of this installation, despite the well-defined wave shape that it yields. The air injection slot transducer, ITP transducer, and the ionization probe recover the same stable time scale ($\tau = 260 \, \mu s$), but exhibit slightly different behavior for the initial onset period ($t < 36 \, ms$) when the detonation propagation is chaotic. As these installation types do not suffer from signal loss due to heat soak, the complete extent of the test is detected, up to the breakdown in periodic behavior at $t \approx 0.38s$.

During the stable period, the ionization probe exhibits slightly larger fluctuations in the lap time than the PCB transducers, possibly due to fluctuations in the combustion that cannot be readily observed in the pressure measurement. Also, the mean auto-correlation and cross-correlation for the ionization probes is exceptionally high, possibly because this sensor type is immune to shock reflection-based noise.

Figure 5.10: Comparison of time scales and phase shift for various installations ($m = 0.4 \, kg/s, \phi = 1.23$)
The air injection slot PCBs and ionization probes are able to resolve the same phase shift between the two azimuthal locations, but the ITP transducers exhibit significantly weaker mean cross-correlation and aberrant phase information for $t = 0.04-0.10s$. This is likely a result of variations in the shock reflection patterns in the standoff tubes, and the aberrant ringing behavior of station II. Differences in the reflection patterns for the ITP sensors obstruct the use of cross-correlation to properly extract the phase information. In spite of this difficulty, the ITP transducer phase shift converges to the correct value by $t = 0.1s$. While the air injection slot transducers and ionization probes can fully recover the phase information, the ITP transducers are less reliable for this purpose.

5.3.2. Detection of Wave Reversal and Mode Switching

The algorithm is efficient at detecting sudden changes in rotation direction (wave reversal) as well as rotation time scale (mode switching). A sudden, discontinuous change in $\tau$ may indicate a change in the number of detonation waves within the combustor, while a sign change in $\alpha$ indicates a reversal in the rotation direction. Figure 5.11 illustrates an example case ($\dot{m} = 0.5 \text{ kg/s}, \phi = 1.57$), which features both a wave direction reversal and a mode switch ($n = 1 \rightarrow 2$), as detected by PCB transducers within the air injection slot. A change in the detonation behavior is evident from the pressure evolution (Figure 5.11a), as the pressure fluctuations are significantly lower for $t > 0.215s$. Stable, periodic rotation is observed shortly after RDC ignition ($t \approx 2 \text{ ms}$) and converges to a lap time of $\tau = 255 \mu s$ ($f = 3.92 \text{ kHz}$). This time scale corresponds to the existence of a single wave ($n = 1$) rotating at a speed of $W = 1.90 \text{ km/s}$ (90.5% $W_{\text{CJ}}$). For the air injection geometry used in this study, the flow is nominally choked through the air injection slot at this flow rate, which weakens the fluctuations induced by the detonation. The reduced amplitude of the pressure fluctuations yields some aberrant instances of lap times for the station II sensor location. These deviations are often a multiple of two of the correct lap time, as the auto-correlation algorithm skips over the appropriate wave due to poor correlation and instead aligns with the following wave.
Figure 5.11: Detonation wave direction reversal and mode transition ($\dot{m} = 0.5 \text{ kg/s}, \phi = 1.57$)

A series of wave deceleration events are observed in the rotation time evolution, starting at $t \approx 0.12s$ and continuing up to the mode transition event at $t = 0.215s$ (Figure 5.11b). These events are identified
by a consistent, gradual increase in \( \tau \) for all three azimuthal stations. This deceleration is followed by recovery of the detonation wave to its original velocity, as evidenced by convergence of \( \tau \rightarrow 255 \mu s \). The event at \( t = 0.166s \) coincides with a reversal of the wave direction, as evidenced by a change in \( \alpha \approx +120^\circ \rightarrow \alpha \approx -120^\circ \) (Figure 5.11c). A rapid drop in the rotation time is observed at \( t = 0.215s \), from an average value of \( \tau = 255 \mu s \) to an average value of \( \tau = 138 \mu s \) (\( f = 7.24 \) kHz). If assuming \( n = 1 \), this frequency corresponds to a wave speed of \( W = 3.51 \) km/s, which is significantly higher than the ideal CJ speed for this mixture (\( W_{CJ} = 2.095 \) km/s). This precludes the possibility of a single wave (\( n \neq 1 \)) and indicates that two waves are rotating within the combustor (\( n = 2 \)) with a mean wave speed of \( W = 1.75 \) km/s (\( \approx 84\% \) \( W_{CJ} \)). While there are two detonation waves within the combustor (i.e. waves A and B), the correlation algorithm does not distinguish between the two fronts, but rather auto-correlates each instance of wave passage. The wave passage events recorded by a fixed sensor location will alternate between wave A and wave B, such that each wave front is correlated to its neighboring detonation wave (A\( \rightarrow \)B, B\( \rightarrow \)A), rather than to itself. The auto-correlation magnitude is dependent on the similarity of the two detonation waves. The phase information is also distorted by the existence of two waves, and may yield an erroneous assessment of the propagation direction. For instance, assuming both waves are propagating in the +\( \theta \) direction, if wave A arrives at station I, and shortly afterward, wave B arrives at station III, it appears that the propagation direction is -\( \theta \). If three waves exist in the combustor with three azimuthal sensor stations, wave passage occurs at the same time for all sensors, yielding \( \alpha = 0^\circ \). Thus, phase information must be interpreted carefully for instances of multiple waves.

5.3.3. Detection of Waxing and Waning Instability

The algorithm can also detect the existence of the waxing and waning instability (Anand et al. 2015b). Figure 5.12a depicts the pressure evolution for an example case (\( \bar{m} = 0.2, \phi = 1.05 \)) featuring the waxing and waning instability, as detected by PCB transducers within the air injection slot. The amplitude of the pressure fluctuations varies between high (\( P \approx \pm 1.5 \) bar) and low states (\( P \approx \pm 0.5 \) bar).
138

Figure 5.12: Measurement data and correlation output for air slot transducers ($\dot{m} = 0.2 \, \text{kg/s}, \phi = 1.05$)
The source window standard deviation captures the gradual, low frequency variation in the pressure fluctuation amplitudes, and reveals that these oscillations are out of phase between the three azimuthal stations. In contrast to the relatively constant profile observed for the stable case (Figure 5.12b), there are large fluctuations in standard deviation about a mean value ($s_X = 0.35 \pm 0.2$ bar). The size of the mean component (0.35 bar) relative to the fluctuating component (0.2 bar) can be employed as a metric for the strength of this instability ($\approx 57\%$ for the present case).

The rotation times from the auto-correlation loop are given in Figure 5.12c, and indicate immediate onset of rotation within a few milliseconds of RDC ignition. A mean rotation time of $\tau = 339$ $\mu$s is observed for the test, which corresponds to a mean rotation frequency of $f = 2.95$ kHz ($n = 1$, $W = 1.43$ km/s). However, there are considerable fluctuations in $\tau$ for all three sensor locations for the entirety of the test, which contrasts with the stable $\tau$ evolution of the case in Figure 5.6a. While the detonation propagation is continuous, the velocity varies considerably between consecutive rotations. Overall, the mean auto-correlations are relatively high, ($R_{XX} > 0.7$), indicating that the general shape of the wave is consistent between consecutive laps, despite variations in $\tau$ and $s_X$. Furthermore, the mean cross-correlation coefficients indicate moderate-to-strong ($R_{XY} \approx 0.7$) correlation between sensor locations, indicating that the wave shape is mostly preserved as it propagates azimuthally.

The phase shift evolution in Figure 5.12d indicates several instances of wave direction reversal between the RDC ignition event up to $t = 65$ms. After this transitory initial period, the wave direction remains consistent in the +$\theta$ direction, but undergoes significant oscillations for the remainder of the test. The period of these oscillations is consistent with the period of waxing and waning. While equal phase shifts of $\alpha = 120^\circ$ indicate stable, azimuthally uniform rotation (as in Figure 5.8a), differences in the magnitude of the phase shift between the three sensor pairs indicates that the wave speed varies azimuthally. Considering an instance where the first sensor pair (station I→II) has $\alpha < 120^\circ$, the detonation time of
flight between the arc $\theta = 60^\circ \rightarrow 180^\circ$ is less than one third of the overall rotation time. This means the wave propagation is faster for this sector of the combustor, and slower for the remaining portion of the combustor. The oscillations in phase shift occur out of phase for all three sensor pairs, and each of the pairs temporarily has $\alpha < 120^\circ$, indicating that the sector experiencing faster-than-average wave propagation is not constant, and shifts continuously during the test.

5.4 Conclusion

A correlation-based method is developed to characterize the operating state in a rotating detonation combustor from a set of azimuthally-spaced sensors. This method uses auto-correlations to detect the rotation time scale of the detonation at a fixed sensor location, and cross-correlations between adjacent locations to detect the propagation direction. The functionality of the algorithm is demonstrated using RDC experimental data collected from PCB transducers and ionization probes.

This method of data extraction enables new options for RDC instrumentation, as the operating state can be readily identified from remotely-located or noisy sensors. The method can be efficiently scaled for a larger number of azimuthal sensors, which may provide more fidelity to the combustor diagnosis, or improve characterization of the operating state with multiple co-rotating waves. The algorithm can also be modified to allow information fusion from different sensor types and installation types to provide improved accuracy and redundancy.

Application of this method to RDC pressure evolution allows for efficient detection and quantification of instabilities. This algorithm can readily detect instances of detonation extinction, wave reversal, and mode switching, allowing the occurrence of these features to be efficiently computed for an RDC test case. Threshold values of mean correlations and percentage fluctuations in the rotation time scale can be integrated with the rate of wave reversals, extinction events, and mode switches to yield a global stability classification.
Chapter 6 – Characterization of Initiator Dynamics in a Rotating Detonation Combustor

A series of experiments are conducted to characterize the strength and directivity of the initiator blast wave as it propagates into the RDC channel. To properly isolate the blast strength, the blast wave must propagate in an inert medium, and the RDC channel is supplied only with air. Two initiator mixture types, H₂-O₂ and C₂H₄-O₂, are explored for a range of equivalence ratios from near-stoichiometric to rich. The blast wave trajectory and strength is collapsed to a theoretical model to estimate the blast energy deposition from the initiator tube into the RDC annulus. The optimal mixture equivalence ratio is selected for each fuel, and the RDC channel width and pressure are varied to assess the effects on initiator performance. Estimates of energy deposition are used to evaluate the feasibility of achieving direct initiation with tangentially-injecting initiators for typical RDC fuel-air mixtures. The study concludes with experimental characterization of RDC initiation behavior for a stoichiometric H₂-air mixture for optimized initiator tube settings.

6.1 Experimental Approach

This investigation employs several geometric variations on the baseline configuration introduced earlier (Table 2.2). The channel width (w_{ch}) is varied between 50-100% of the baseline value (w_{ch} = 3.8, 5.7, 7.6 mm), while the exit area is varied between 25-100% of the baseline value using a converging nozzle (A_e = 8.5, 11.8, 35.0 cm²). The combustor outer diameter, injection geometry, and initiator tube geometry remain fixed. In contrast to the usual instrumentation arrangement, instrumentation is clustered in the vicinity of the initiator entry point to capture the propagation of the blast wave as it enters the RDC channel and expands and attenuates (Figure 6.1).

The test procedure begins by establishing a constant, steady air flow rate through the RDC channel. Gaseous oxygen and gaseous fuel are injected into the initiator tube for 20 ms to completely fill the tube with reactants. At the completion of the fill phase, the solenoid valves are shut, terminating the flow of
fuel and oxygen. Simultaneously, the spark ignition is triggered, and combustion begins within the initiator tube. A detonation propagates through the tube and is recorded with the ionization probes before entering the RDC channel. The initiator blast propagates into the RDC channel and attenuates, concluding the test. Approximately 1s after conclusion of the test, nitrogen is administered through the purge supply valve for 0.5s, clearing out the exhaust gases from the initiator tube.

Figure 6.1: Schematic of sensor clustering in the vicinity of the initiator tube entry region

To properly capture the blast wave trajectory and peak pressure, an array of twelve PCB pressure transducers (model 113B24) are flush mounted within the outer wall of the RDC channel. The baseline RDC instrumentation layouts employed in this study consist of five azimuthal stations in four axial rows, clustered around the initiator tube entry region. The axial spacing between instrumentation rows is approximately 25 mm, while the azimuthal spacing is 30° (Figure 6.2). The station closest to the initiator entry plane is designated θ = 0°, with positive azimuthal angles corresponding to the initiator injection direction. The four axial instrumentation rows are referred to as R1-R4, in ascending order in the axial (downstream) direction. Two sensor layouts are used in this study: the first layout (denoted in Figure 6.2 by gray symbols), with greater clustering in the azimuthal direction, and the second layout (red outlined symbols) with greater clustering in the θ ≥ 0° (forward) region. The first layout is used during the initial characterization of the blast trajectory to capture the asymmetric blast propagation behavior.
reported by Miller et al. (2013). The second layout is employed in the latter portion of the study to improve blast wave energy estimates due to the increased blast energy content in the forward \((z > 0)\) direction.

The test matrix to characterize the initiator performance is summarized in Table 6.1. To isolate the energy content within the initiator blast wave, the RDC channel is not supplied with fuel, such that the propagation medium for the blast wave is inert. To determine the effect of equivalence ratio, the channel width is held constant at \(w_{ch} = 7.6\) mm, and the RDC channel pressure is maintained at ambient conditions \((P_i = 1\) bar) without an exit nozzle. The RDC primary air flow is maintained at \(m_{\text{air}} = 0.1\) kg/s for the duration of all equivalence ratio tests to flush out reactant overfill from the initiator. For both initiator fuel types \((\text{H}_2\text{-O}_2\) and \(\text{C}_2\text{H}_4\text{-O}_2)\), the equivalence ratio \((\phi)\) is varied in approximate increments of

![Figure 6.2: Schematic of PCB pressure transducer locations in the RDC](image-url)
Δφ = 0.15 from stoichiometric (ϕ = 1), to rich (ϕ = 3) conditions. For fuel-oxygen mixtures, peak detonation Mach number occurs near stoichiometric conditions for H₂-O₂ and at highly rich conditions (ϕ = 2.4) for C₂H₄-O₂ (Schultz and Shepherd 2000).

The optimal initiator equivalence ratio as determined from estimated blast energy deposition is selected for both initiator fuel types. This optimal initiator mixture is held constant as channel pressure and width are varied. As the initiator tube is directly connected to the RDC channel, and no diaphragm or blockage is used to partition the two volumes, the pressure in the RDC channel is equal to the pressure in the initiator tube. A converging nozzle is attached to the RDC channel to restrict the exit area and increase the channel pressure. Two converging nozzle settings with different exit areas (Aₑ = 11.8 cm² and Aₑ = 8.5 cm²) are used to achieve a range of channel pressures. The RDC air flow rate is throttled between 0.10-0.45 kg/s for these two converging nozzle settings, yielding RDC channel pressures of Pᵢ = 1.0-1.5 bar for the first setting and Pᵢ = 1.5-2.0 bar for the second setting. The Pᵢ = 1.5 bar case is recorded for both converging nozzle settings to provide overlap between the two exit area geometries.

### Table 6.1. Initiator settings for initiator blast wave decay tests

<table>
<thead>
<tr>
<th>Initiator Reactant Composition</th>
<th>H₂-O₂, C₂H₄-O₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalence Ratio</td>
<td>1.0 – 3.0</td>
</tr>
<tr>
<td>Channel Width</td>
<td>7.6, 5.7, 3.8 mm</td>
</tr>
<tr>
<td>Channel Pressure</td>
<td>1.00, 1.25, 1.50, 1.75, 2.00 bar</td>
</tr>
</tbody>
</table>

For each initiator setting, test points are repeated a minimum of twenty times to ensure statistical accuracy. Phase averages are calculated for each of the pressure histories with uncertainties in peak pressure and shock arrival time calculated prior to averaging. The average arrival time of the shock for each sensor location is measured relative to the first ionization probe in the initiator tube. This value is shifted by a time delay calculated from the initiator wave speed and the distance between the ionization
probe and the initiator entry plane. As piezoelectric transducers cannot capture steady-state pressure, the PCB peak pressures are added to the low speed chamber pressure measurement to calculate the total peak pressure of the leading shock.

The optimal initiator fuel type is selected from the initiator blast wave decay tests as determined from estimated blast energy deposition. This optimized initiator mixture is tested under reacting RDC conditions to assess the performance in the actual RDC environment. In addition to flush-mounted transducers located near the initiator tube entry region, three flush-mounted transducers are installed within the first row (R1) around the circumference of the combustor (θ = 60°, -60°, and 180°) to capture detonation rotation. Ignition is preceded by a 1s fuel dump to allow the lightoff condition to stabilize. To minimize risk to instrumentation due to excessive heat loading, the test length is limited to 0.2s, followed by shutoff of the fuel supply.

6.2 Blast Wave Trajectory

The PCB transducer array exhibits strong repeatability between successive tests at a given initiator setting, as depicted by the phase-locked pressure evolution from twenty consecutive tests at two sensor locations (Figure 6.3). Prior to the phase-locking procedure, scatter in the arrival time ranges from ± 1 μs for transducers near the initiator entrance region, to ± 4 μs for the outermost sensor locations. In the entrance region, the blast pressure rises monotonically to a peak value and persists above ambient conditions for over 100 μs. Scatter in the peak pressure measurement is more severe (± 20%), possibly due to the size of the transducer diaphragm (5.5 mm) relative to the speed of the blast wave. For a blast wave traveling at $M_s = 2$ in air, the propagation time across the transducer diaphragm is approximately 8 μs, and the corresponding peak pressure is a representative average over the surface of the diaphragm. Due to the finite thickness of the transducers and the rapid expansion behind the leading shock, the true peak pressure of the leading shock may not be fully captured (Walter 2004). Mach number estimates
calculated from the peak pressure measurement are lower than those calculated from the slope of the blast trajectory (=30% lower at the R1, θ = 30° measurement location).

The blast wave is significantly stronger in the forward direction (z > 0) along the initiator tube axis (Figure 6.3a), compared to the perpendicular axis (Figure 6.3b), as depicted by the pressure evolution. For sensors located rearward of the initiator entry plane (z < 0), peak pressures are consistently weak ($P_{\text{max}} < 2$ bar) and poorly-defined compared to the sharply-captured pressures forward of the entry plane (z > 0). For the limiting rearward sensor location (R1, θ = -60°), the peak pressure signal is extremely low amplitude ($P_{\text{max}} < 1.1$ bar), potentially indicating that the blast wave has degenerated to a sound wave ($M_s \approx 1$) in the rearward direction.

![Figure 6.3: Phase-locked pressure evolution at a) forward and b) perpendicular sensor locations](image)

The average blast wave arrival time is calculated for each sensor location, and expressed as an elapsed time relative to the entry of the blast in the RDC channel. The blast arrival time at all spatial locations can be estimated from a finite number of sensors by using a 2-dimensional interpolation technique. To facilitate the interpolation of the blast trajectory, sensor locations are converted into polar coordinates. The radial coordinate ($r$) is defined as the distance of each sensor from the entry point of the initiator tube, while the angular coordinate ($\xi$) is defined as the angle between the initiator tube axis and the
radial vector. The blast arrival time, by definition, is equal to zero for \( r = 0 \) at all angles \( 0^\circ \leq \xi \leq 180^\circ \), providing additional control points for the interpolation. Biharmonic spline interpolation is used in MATLAB to generate an interpolated arrival time surface from the scattered arrival time data (Figure 6.4a). The blast arrival time increases monotonically with radial distance from the entry point as the blast propagates outward. For clarity, the interpolated surface is transformed back to a Cartesian representation and plotted in axial and azimuthal coordinates relative to the initiator tube entry point (Figure 6.4b). The blast wave trajectory is strongly asymmetric, with bias in the forward direction \((z > 0)\). The interpolated blast wave profiles are highly skewed, and approximately elliptical in shape.

![Figure 6.4: Blast wave trajectory for \( \text{H}_2\text{-O}_2 \), \( \phi = 2.05 \), \( w_{in} = 7.6 \text{ mm} \), \( P_i = 1 \text{ bar} \)](image)

6.3 Energy Estimation

The energy contained within the initiator blast wave can be estimated by collapsing the trajectory onto the theoretical cylindrical blast solution of Oshima (1960). This study assumes that a thin annular channel, as in the case of the RDC geometry, approximates parallel plates and behaves analogous to cylindrical blast initiation, such that this theory is applicable. Blast wave variation across the channel width and the effects of channel curvature are neglected. The theoretical solution is expressed in terms of dimensionless quantities for arrival time \( t \) and radial distance \( r \) (Eq. 6.1). A unique characteristic length \( r_0 \), referred to as the explosion length, is defined in the theoretical solution from the blast energy
$E$ and initial pressure $P_i$ (Eq. 6.2). The theoretical solution for the Mach number is given by differentiating the trajectory solution with respect to the dimensionless radius $r/r_0$ (Eq. 6.3).

\[
\frac{a \cdot t}{r_0} = f\left(\frac{r}{r_0}\right)
\]

\[
r_0 = \sqrt{\frac{E}{2\pi P_i}}
\]

\[
\frac{\partial}{\partial(r/r_0)}\left(\frac{a \cdot t}{r_0}\right) = \left(\frac{r_0}{a} \frac{\partial t}{\partial r}ight) = \frac{a}{u} = \frac{1}{M} = f'(\frac{r}{r_0})
\]

The experimental trajectory is collapsed onto the theoretical trajectory by finding the optimal choice of the explosion length $r_0$ which minimizes the squared error between the two. The speed of sound is assumed constant in the RDC channel at $a = 343$ m/s, as the medium of propagation is air. With an assumed value for the sound speed, the Mach number of the leading shock can be calculated from the slope of the experimentally measured trajectory (Eq. 6.4). The experimentally-determined Mach number (Eq. 6.4) is fitted to the theoretical Mach number solution by varying the explosion length $r_0$. This rescales the experimental Mach number in the radial direction to achieve the best agreement with the theoretical solution (Figure 6.5a). The least-squares solution for $r_0$ is then used to calculate the blast energy from Eq. 6.2.

\[
M = \frac{u}{a} = \left(\frac{\partial r}{\partial t}\right) = \left[\frac{a}{\partial t}\right]^{-1}
\]

Oshima’s theoretical solution was developed for symmetric blasts, propagating uniformly in all directions (i.e. $r_0$ = constant for all $\xi$), rather than the asymmetric blast profile observed in Figure 6.4. Chiu et al. (1977) modeled the propagation of an asymmetric, ellipsoidal blast wave, which attenuates at different rates along the major and minor axis of the blast wave. However, away from the explosion source, the blast overpressure ($\Delta P/P_i$) decays at nearly the same slope with dimensionless radius ($r/r_0$) as
the symmetric, spherical case. This motivates the piece-wise application of Oshima’s symmetric blast solution as a means to compensate for the asymmetry. As the blast energy is not uniformly distributed, the energy content in each sector of the blast is calculated separately. By taking a radial slice of the trajectory and locally fitting to the theoretical solution, the local explosion length $r_0(\xi)$ is determined.

For the trajectory in Figure 6.4, the locally-fit solution is provided in Figure 6.5a for $\xi = 25^\circ$, which yields a local explosion length of $r_0 = 80.5$ mm, corresponding to a local energy content of 0.086 J/degree for this local sector (Eq. 6.5). As the theoretical solution is developed for cylindrical blasts, the energy $E$ in Eq. 6.2 is expressed in J/m, and is converted to J by multiplying by the channel width $w_{ch}$.

$$E_{local} = 2\pi \cdot r_0^2 \cdot P_i$$

The explosion length distribution (in Figure 6.5b) is highly asymmetric, indicating that most of the blast energy is deposited in the forward direction, along the initiator tube axis. This distribution has an approximately elliptical shape. Based off the significant skew, the tangential entry of the initiator tube is very effective at biasing the energy in one direction.

![Blast Wave Trajectory](image1)

![Blast Wave Strength](image2)

**Figure 6.5:** Blast wave trajectory analysis for H$_2$-O$_2$, $\phi = 2.05$, $w_{ch} = 7.6$ mm, $P_i = 1$ bar
The local values of explosion length are used to calculate the local blast energy in each sector, and then numerically integrated to determine the total blast energy (Eq. 6.6). As all sensors are located axially downstream of the initiator tube entry point \((x > 0)\), the experimental trajectory is not defined for \(x < 0\).

The visualization of Miller et al. (2013) suggests that the blast is symmetric across the \(z\) axis, such that energy deposition above and below the \(z\) axis is equal, and the integrated energy for \(0^\circ \leq \xi \leq 180^\circ\) is doubled to estimate the total blast energy. The total estimated energy deposition is \(E = 10.3\ \text{J}\) for the case depicted in Figure 6.5, which equates to approximately 35% of the chemical energy available in the initiator tube for this \(\text{H}_2-\text{O}_2\) mixture.

\[
E_{\text{total}} = w_{\text{ch}} \cdot \int_0^{2\pi} E_{\text{local}}\, d\xi = w_{\text{ch}} \cdot 2 \cdot \int_0^\pi E_{\text{local}}\, d\xi = w_{\text{ch}} \cdot 2 \cdot \sum_{i=0}^n E_{\text{local},i} \cdot \Delta\xi \quad (6.6)
\]

The sensitivity of this energy estimation procedure is evaluated for various components of experimental uncertainty, assuming the uncertainties are uncorrelated. The energy estimation procedure is carried out for the same data set while altering a single parameter \((x_i)\) in the calculation to determine the sensitivity \((\partial E/\partial x_i)\) (Eq. 6.7). The key parameters contributing to the uncertainty are the placement and inclusion of measurement locations, the calculation of detonation wave speed in the initiator tube, the assumption of the geometric origin of the blast wave, the variation in channel width, and the variation in speed of sound in the channel. Assuming these components of uncertainty are uncorrelated, the net effect of these sources on the overall uncertainty of the energy deposition is up to 24.5% for \(\text{H}_2-\text{O}_2\) mixtures, and up to 18.5% for \(\text{C}_2\text{H}_4-\text{O}_2\) mixtures (Table 6.2). Uncertainty is greater for \(\text{H}_2-\text{O}_2\) mixtures due to the impact of sound speed variations on the energy estimation.

\[
\Delta E = \sqrt{\sum \left( \frac{\partial E}{\partial x_i} \Delta x_i \right)^2} \quad (6.7)
\]

Exclusion of a measurement location from the calculation procedure slightly alters the interpolated blast trajectory contour, with the probe nearest the blast origin \((R1, \theta = 30^\circ)\) exhibiting the highest sensitivity.
to the overall energy result (6.5% uncertainty). Most of the blast energy is contained within the forward direction \( z > 0 \) (i.e. \( \xi \leq 90^\circ \)), and exclusion of all sensors in the \( \theta = -30^\circ \) and \( \theta = -60^\circ \) locations produces only minor changes in the total energy calculation (2.7% uncertainty). As interpolation of the trajectory contour is more sensitive for sensors in the \( \theta = 0^\circ, 30^\circ, 60^\circ \) locations, the second sensor layout described in Figure 6.2 is used in the remainder of the study to improve energy estimates. For sensor layout 2, the blast trajectory is poorly resolved for \( \xi > 90^\circ \), though estimates calculated from sensor layout 1 indicate that the energy content in this region is relatively low. To reduce uncertainty and provide better comparison between various initiator tube and RDC settings, energy deposition is calculated exclusively for \( z > 0 \) (\( 0^\circ \leq \xi \leq 90^\circ \)), with the rearward component (\( \xi > 90^\circ \)) neglected.

Uncertainty in the initiator tube detonation wave speed alters the estimated arrival time of the blast in the RDC channel and contributes an uncertainty of 8.0% to the energy estimation. As the blast is highly three-dimensional as it diffracts into the RDC channel, the true origin of the blast may be shifted slightly from the estimated geometric origin. Perturbing the assumed location of the blast origin by the diameter of the initiator tube contributes 11.6% uncertainty to the energy estimation. Due to alignment error of RDC components, local variation in the channel width contributes an additional 10% uncertainty.

For cases using \( \text{H}_2-\text{O}_2 \) mixtures in the initiator tube, reactant leakage may alter the speed of sound in the RDC channel, as the small molecular weight of \( \text{H}_2 \) yields very high sonic velocities (\( a = 660 \text{ m/s} \) for \( \text{H}_2-\text{O}_2 \) at \( \phi = 2 \)). This effect is significantly weaker for \( \text{C}_2\text{H}_4-\text{O}_2 \) mixtures, as the molecular weight of ethylene is similar to that of air and the sonic velocities are comparable (\( a = 339 \text{ m/s} \) for \( \text{H}_2-\text{C}_2\text{H}_4 \) at \( \phi = 2 \)). Comparing the initiator mixture flow rates to the air flow rate in the RDC channel, the global speed of sound may be several percent higher (\( a = 363 \text{ m/s} \)) for the \( \text{H}_2-\text{O}_2 \) initiator mixture. For a larger assumed speed of sound, the calculated energy deposition is lower by approximately 16%.
Table 6.2: Components of Uncertainty in Energy Estimation

<table>
<thead>
<tr>
<th>Component</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor Placement/Trajectory Interpolation</td>
<td>6.5%</td>
</tr>
<tr>
<td>Initiator Tube Detonation Wave Speed</td>
<td>8.0%</td>
</tr>
<tr>
<td>Variation in Blast Wave Origin</td>
<td>11.6%</td>
</tr>
<tr>
<td>Variation in Channel Width</td>
<td>10.0%</td>
</tr>
<tr>
<td>Variation in Speed of Sound*</td>
<td>16.1%</td>
</tr>
<tr>
<td>Effective Uncertainty of Total Blast Energy</td>
<td>24.5%</td>
</tr>
</tbody>
</table>

6.4 Effect of Initiator Mixture

Considering the entire range of equivalence ratios for the two initiator fuels, initiator wave speeds exhibit a deficit from the ideal Chapman-Jouguet (CJ) values (Figure 6.6a), potentially due to the small diameter of the initiator tube relative to the detonation cell size, as discussed by Murray (2008). The estimated energy deposition is considerably higher for the H₂-O₂ mixture compared to the C₂H₄-O₂ mixture for 3.0 > φ > 1.0 (Figure 6.6b). The energy deposition of the H₂-O₂ mixture achieves a maximum for φ = 2.5, while the energy deposition of the C₂H₄-O₂ mixture rises steadily up to the highest equivalence ratio tested (φ = 3.0).

![Figure 6.6: Comparison of initiator performance for a range of equivalence ratios](image-url)
Despite the general improvement with increasing $\phi$, the H$_2$-O$_2$ cases begin to exhibit an increase in initiator tube misfires and aberrant, non-detonating behavior for $\phi > 2.2$. While these misfire test cases were not used in the blast trajectory reconstruction and energy estimation, they indicate undesirable behavior, and the optimal setting for subsequent H$_2$-O$_2$ tests is selected as $\phi = 2.0$. The C$_2$H$_4$-O$_2$ mixtures do not exhibit abnormal behavior at high $\phi$, and the optimal setting for subsequent C$_2$H$_4$ tests is selected as $\phi = 3.0$. While it is possible that increasing $\phi$ may yield improved energy deposition for C$_2$H$_4$-O$_2$ mixtures, the potential improvement appears to be minor.

The blast wave trajectories and peak pressure histories for the highest stable energy deposition cases for each fuel type ($\phi = 2.0$ for H$_2$-O$_2$ and $\phi = 3.0$ for C$_2$H$_4$-O$_2$) are provided in Figure 6.7 and Figure 6.8, respectively. Considering the trajectories in Figure 6.7, the blast propagation for the H$_2$-O$_2$ case is significantly faster than the C$_2$H$_4$-O$_2$ case in the forward azimuthal direction ($z$), but has almost identical propagation in the axial direction ($x$).

However, peak pressures recorded in the forward direction ($z > 0$) are comparable for both cases (Figure 6.8), suggesting similar leading shock strength. This mismatch in arrival time for comparable peak pressures implies variation in the local speed of sound. While the sound speed for the C$_2$H$_4$-O$_2$ mixture ($a = 328$ m/s) is comparable to that of air, the sound speed in the H$_2$-O$_2$ mixture ($a = 661$ m/s) is
approximately twice that of air. Residual concentrations of initiator mixture from the initiator tube filling process can locally alter the speed of sound, enhancing blast propagation speed.

Figure 6.8: Comparison of H$_2$-O$_2$ and C$_2$H$_4$-O$_2$ peak pressures for maximum energy deposition cases

6.5 Effect of RDC Parameters

Comparing the effect of channel width ($w_{ch}$) on the blast, a 50% reduction in the width significantly boosts the propagation speed (Figure 6.9), as the blast energy is distributed into a smaller volume.

Figure 6.9: Comparison of blast wave trajectories for varying $w_{ch}$ (C$_2$H$_4$-O$_2$, $\phi = 3.0$, $P_i = 1$ bar)
This additional confinement also results in higher peak pressures for the reduced channel width (Figure 6.10). By limiting the expansion of the blast wave in the RDC channel, strong post-shock conditions are preserved over a greater distance.

![Comparison of blast wave peak pressures for varying channel widths.](image)

**Figure 6.10: Comparison of blast wave peak pressures for varying \( w_{ch} \) (\( \text{C}_2\text{H}_4\text{-O}_2, \phi = 3.0, P_i = 1 \text{ bar} \))**

While the leading shock strength and propagation speed are enhanced for the smaller channel (\( w_{ch} = 3.8 \text{ mm} \)), the overall energy deposition does not vary significantly for the various channel widths across a range of initial channel pressures (Figure 6.11). Greater scatter is observed for the \( \text{H}_2\text{-O}_2 \) mixture, possibly due to the distorting effect of sound speed variations on the energy estimation. This has interesting implications for initiator design, as additional confinement in the entry region will enhance initiation for a fixed amount of energy deposition.

The effect of initial channel pressure on the energy deposition highlights another difference in performance between the two fuel types. The \( \text{H}_2\text{-O}_2 \) mixture exhibits a complex pressure dependence and yields reduced or comparable energy deposition at elevated pressures compared to the ambient condition. The relatively neutral behavior of \( \text{H}_2\text{-O}_2 \) may be attributed to second explosion limit effects.
discussed in detail by Ng et al. (2007), which suppress hydrogen detonability at elevated pressure conditions due to an increase in chain termination reactions. This contrasts sharply with the C\textsubscript{2}H\textsubscript{4}-O\textsubscript{2} mixture, which shows consistent, linear improvement at elevated pressures. Doubling the initial channel pressure approximately doubles the energy deposition for C\textsubscript{2}H\textsubscript{4}-O\textsubscript{2}, resulting in comparable performance for the two fuel types at the elevated pressure condition.

![Figure 6.11: Estimated energy deposition for a range of channel widths and pressures](image)

When accounting for the pressure scaling in (Eq. 6.2), the explosion length drops with increasing pressure for fixed blast wave energy. This results in weaker blast propagation at elevated pressures due to the increase in density of the pre-shocked mixture. The blast energy must increase proportionally with pressure to yield the same blast wave strength at a given radius. If the target fuel-air mixture in the RDC exhibits increasing detonability with pressure, as in the case of hydrocarbon-air mixtures, this may compensate for the drop in initiator effectiveness. However, for the case of constant or decreasing detonability with pressure, the likelihood of direct initiation will diminish at elevated pressures.

The likelihood of direct initiation is assessed for H\textsubscript{2}-air mixtures for the maximum recorded energy deposition at ambient conditions ($E \approx 13.6$ J at $P_i = 1$ bar). The critical energy requirement for spherical,
unconfined detonation of a stoichiometric H₂-air mixture is reported by Kaneshige and Shepherd (1997) as $E_{\text{crit,sph}}|_{\phi=1} = 4,300$ J. This can be converted to an equivalent critical energy of $E_{\text{crit,cyl}}|_{\phi=1} = 12.3$ kJ/m for cylindrical initiation from the scaling suggested by Lee (1977). For the largest channel width ($w_{ch} = 7.6$ mm), and assuming equal energy distribution across the annulus depth, this corresponds to a critical energy of approximately $E_{\text{crit}} = 93$ J. This critical energy value is almost an order of magnitude larger than the maximum energy deposition ($E/E_{\text{crit}} \approx 15\%$). As the peak detonability for H₂-air mixtures occurs near stoichiometric conditions (Knystautas et al. 1984), this represents a minimum requirement for the target mixture, and it is highly unlikely that direct initiation can be achieved with the current initiator tube design.

### 6.6 Initiation in the Reacting RDC Environment

The initiation process is characterized for near-stoichiometric ($\phi \approx 1$) H₂-air conditions at the baseline RDC air flow rate ($\dot{m}_{\text{air}} = 0.1$ kg/s) to confirm the absence of direct initiation. The RDC channel pressure is maintained at ambient conditions ($P_i = 1$ bar), and the largest channel width ($w_{ch} = 7.6$ mm) is used, as it has demonstrated successful detonation in prior RDC experiments (Anand et al. 2015a). The highest performing initiator mixture (H₂-O₂, $\phi = 2.0$) is used to maximize the likelihood of direct initiation. This reacting initiation case is repeated a total of five times to ensure repeatability. For each of the tests, stable rotation is observed within the chamber prior to the conclusion of the test at $t = 0.2$s. This verifies that the combustor is capable of supporting stable rotating detonation at this condition. However, for all cases, there exists a delay between the initiating blast and clearly defined detonation rotation.

The blast wave trajectory and peak pressures for the reacting case are examined in Figure 6.12, and are qualitatively similar to the non-reacting case explored in Figure 6.7 and Figure 6.8. As in the non-reacting case, the trajectory is significantly skewed in the forward direction ($z > 0$), compared to the
perpendicular direction ($x > 0$). Despite a slight increase in the speed of sound ($a = 410$ m/s), blast arrival times are not significantly different from those observed in Figure 6.7, indicating that the blast wave maintains a similar trajectory. The peak pressure evolution Figure 6.12b depicts monotonically decreasing peak pressures as the blast wave propagates away from the source, though resolution is limited in the perpendicular direction ($x > 0$). The similarity of the blast trajectory to the non-reacting case and the decaying peak pressures indicate that direct initiation has not been achieved.

Examine the three transducers (located in the first row, R1) installed around the RDC circumference (Figure 6.13), the blast wave propagation around the annular channel is consistent between the five tests. The $\theta = +60^\circ$ sensor location detects the forward traveling blast wave at $t = 53$ $\mu$s, followed closely by a secondary shock. As in the non-reacting case, the blast propagates more slowly in the rearward direction, and the rearward traveling blast wave reaches the $\theta = -60^\circ$ sensor location at $t \approx 190$ $\mu$s. The $\theta = 180^\circ$ sensor location is situated on the opposite side of the annulus from the initiator entry point, and is the last of the three to register the arrival of the blast wave at $t \approx 300$ $\mu$s. Calculating an average wave speed between the $\theta = +60^\circ$ and $\theta = 180^\circ$ sensor locations, the blast wave propagates at 0.65 km/s, or

![Figure 6.12: Blast wave propagation for a reacting medium in the RDC channel (RDC: H₂-air, $\phi = 1.03$, Initiator: H₂-O₂, $\phi = 2.00$)]
$M_s \approx 1.6$ between the two sensors. Lee (1977) notes that for the limiting case of direct initiation, the blast wave can decay partially to a sub-detonation state ($M_s \approx 4$) before stabilizing into a detonation. However, the measured blast wave is significantly weaker than this critical threshold, reinforcing that direct initiation is not achieved.

Figure 6.13: Phase-locked pressure evolution at three circumferential locations capturing the initial blast wave propagation in the RDC channel (RDC: $H_2$-air, $\phi = 1.03$, Initiator: $H_2$-O$_2$, $\phi = 2.00$)

Figure 6.14 depicts the long-term behavior within the RDC channel following the subcritical initiation event for a single test. While the initial blast wave fails to directly initiate a detonation in the vicinity of the initiator tube entry region, activity persists in the combustor as evidenced by high amplitude pressure peaks occurring after $t > 0.5$ ms. At first, repeatable phase differences between the sensor locations are not discernible, and peak pressures for a given sensor are not periodic. The behavior is somewhat complex until $t \approx 2.7$ ms, when consistent, periodic waveforms develop at all three sensor locations, separated in phase by $\approx 120^\circ$. This indicates consistent wave rotation in the combustor. The rotational frequency for this case is approximately $2.6$ kHz, corresponding to a detonation wave speed of approximately $1.25$ km/s, consistent with the wave speeds observed by Bykovskii et al. (2006b).
While direct initiation is not realized, detonation forms within the RDC channel after a short transitory period of several milliseconds, consistent with the observations of Frolov et al. (2015), Liu et al. (2012), Wang et al. (2015), and Kindracki et al. (2011). This transition period is approximately equal in length between consecutive tests, but consists of highly irregular pressure evolution. The inconsistency in the phase of the signals at the three sensor locations suggests complex interaction between multiple, counter-rotating wave fronts. The numerical study of Dubrovskii et al. (2015) explores the supercritical initiation process in an RDC by generating two counter-rotating detonation waves in the notional vicinity of an initiator tube source. Due to circumferential variations in the reactant injection, one of the waves becomes slightly stronger than the other. After a series of counter-rotating collisions, the stronger wave becomes dominant and the weaker wave attenuates, yielding a single detonation wave in the RDC. In the present study, the initiator tube produces two counter-rotating blast waves, likely followed by decoupled combustion fronts, rather than fully-coupled detonation waves. However, it is possible that a similar mechanism occurs for subcritical initiation and allows one wave to gradually overtake and dominate the other, culminating in a single rotating detonation wave.
While the exact mechanism of detonation onset is not readily identified, direct initiation is not a prerequisite for achieving detonation within an RDC. Bykovskii et al. (2007) studied the initiation of H₂-air mixtures in a plane-radial vortex chamber using a blasting wire ignition source. Rapid transition to detonation for stoichiometric H₂-air was reported for energy deposition of $E > 0.1$ J, which is an order of magnitude smaller than the lowest energy deposition recorded in this study. Furthermore, the experimental study of Peng et al. (2015) studies the feasibility of a low energy ignition source in an RDC, and reports transition to detonation in H₂-air mixtures for $E = 30$ mJ (a factor of three lower than the low energy bound reported by Bykovskii et al. (2007)) at success rates of 75-94%. In both studies, the successful onset of detonation is dependent on reactant composition and flow rate, and may therefore be related to reactant mixing as well. However, these studies imply that the least-optimal initiator setting explored in this study should be more than sufficient to guarantee development of H₂-air detonation in an RDC. Accurate determination of the low energy bound for transition to detonation requires the use of a significantly weaker ignition source.

6.7 Conclusion

Initiator blast wave propagation is recorded for two initiator fuel-oxidizer mixtures for a range of equivalence ratios. Optimal fuel-oxidizer mixtures are selected to explore the effects of varying RDC initial pressure and channel width on the blast propagation. The blast trajectory and strength are consistent between successive tests, but exhibit highly asymmetric bias in the initiator injection (forward) direction. The energy content of the blast is estimated by locally scaling the blast trajectory to the theoretical blast wave solution of Oshima (1960).

The blast energy distribution is extremely biased in the forward direction, and the blast front propagating in the rearward direction is relatively weak and degenerates rapidly. Maximum initiator energy deposition is achieved for both fuel types at relatively rich conditions ($\phi > 2$). The optimal H₂-O₂ mixture yields almost twice the energy deposition of the highest performing C₂H₆-O₂ mixture, and
achieves ≈13.6 J at ambient RDC channel pressure. Reducing the channel width does not significantly alter the total energy deposition, but enhances the propagation speed and blast strength due to the restricted expansion. Elevated initial pressures improve energy deposition for C₂H₄-O₂ mixtures, but have a more complex effect on H₂-O₂ mixtures ranging from neutral to negative. Energy deposition for the highest performing initiator setting is almost an order of magnitude smaller than the required critical energy for direct initiation.

The RDC initiation process is explored for a near-stoichiometric H₂-air mixture using the highest performing H₂-O₂ initiator setting. Direct detonation initiation is not observed, but detonation is established within the channel after a brief, transitory period. This irregular, transitory behavior persists for several milliseconds prior to the onset of clearly defined rotation. This demonstrates that direct initiation is not a prerequisite for establishing detonation in an RDC. Considering previous studies of subcritical initiation in the RDC environment, the lowest energy deposition initiator tube configuration is likely to achieve transition to detonation.
Chapter 7 – Starting Transients and Detonation Onset Behavior in a Rotating Detonation Combustor

The transitory period, which follows the RDC ignition event and concludes with the onset of a stable, periodic operating mode, is hereafter referred to as the onset phase. A series of experiments are conducted to characterize the behavior and duration of the onset phase for H$_2$-air mixtures. As the process is generally quite complex and stochastic, the repeatability of the onset behavior and duration is assessed for four combustor conditions. The effects of flow rate, equivalence ratio, and the presence of a converging nozzle on the onset phase are explored.

7.1 Experimental Approach

For this study, the channel width and injection geometry is fixed to the baseline values in Table 2.2, and two exit areas are considered. The baseline exit area of the RDC is equal to the cross sectional area of the channel ($A_e = 35$ cm$^2$ for the baseline channel width $w_{ch} = 7.6$ mm). The area is reduced to approximately one third of its baseline value ($A_e = 11.8$ cm$^2$) with the addition of a converging nozzle.

Figure 7.1: RDC schematic indicating position of injection slot and exit plane transducers

To capture the pressure evolution within the combustor, a total of six PCB pressure transducers (model 113B24) are employed (Figure 7.1), with three installed the air injection slot ($\theta = 40^\circ, 160^\circ, 280^\circ$) and
three mounted in the exit plane of the combustor ($\theta = 60^\circ, 180^\circ, 300^\circ$). While flush mounted installations provide the best pressure response and accurately recover the transient behavior, these installation locations typically result in rapid sensor degradation for extended test lengths and high flow rates. Large pressure fluctuations are observed in numerical simulations of the RDC exhaust flow (Schwer and Kailasanath 2012), and transducers in the combustor exit plane can also detect the detonation-induced unsteadiness. The exit plane transducers are recessed from the outer wall of the combustor by 25mm to avoid damage due to heating by the exhaust flow. All transducers are sampled at an acquisition rate of 1 MHz.

For this study, the RDC is configured for operation with H$_2$-air mixtures. Air is constantly supplied to the combustor, while hydrogen is temporarily administered for each test, starting 1s prior to RDC ignition to allow the flow rate and fuel plenum pressure to stabilize. Initiation is achieved with a tangentially-injecting initiator tube containing a C$_2$H$_{2}$-O$_2$ mixture ($\phi = 2.0$). Four experimental cases are considered in this study (Table 7.1). The post-onset (terminal) behavior of these cases has been previously characterized by Anand et al. (2015a) and Driscoll et al. (2015a), where the combustor operated at these conditions for a test length of 1s. In the present study, the test length is restricted to approximately 0.3s, to focus primarily on the onset phase. Each case is repeated for a total of ten tests per case to determine whether the onset behavior is comparable between subsequent tests, and whether the duration of the onset phase is repeatable.

<table>
<thead>
<tr>
<th>Case</th>
<th>$m_{\text{air}}$ (kg/s)</th>
<th>$\phi$</th>
<th>Nozzle</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.50</td>
<td>1.03</td>
<td>no</td>
</tr>
<tr>
<td>B</td>
<td>0.40</td>
<td>1.43</td>
<td>no</td>
</tr>
<tr>
<td>C</td>
<td>0.40</td>
<td>1.04</td>
<td>no</td>
</tr>
<tr>
<td>D</td>
<td>0.30</td>
<td>0.93</td>
<td>yes</td>
</tr>
</tbody>
</table>
7.2 Onset Phase Behavior for Selected Cases

For cases A, B, and C, the predominant operating mode is a single, stable rotating detonation, which is observed for all but one test. For case D, the predominant operating mode is longitudinal pulsed detonation (LPD), where the detonation propagates axially through the combustor, rather than azimuthally. The LPD instability is observed for lean cases ($\phi < 1$) when a converging nozzle is installed onto the combustor exit (Anand et al. 2015b). For all but one test, the combustor stabilizes to a periodic operating condition prior to the end of the test. The onset time is defined as the duration between the initiation event and the appearance of stable, periodic behavior.

7.2.1. Case A: $\dot{m}_{\text{air}} = 0.50 \text{ kg/s}, \phi = 1.03$, No Nozzle

For case A, stable detonation rotation is achieved for all tests with a median onset time of $t_{\text{onset}} = 17 \text{ ms}$. Onset of rotation occurs rapidly, with $t_{\text{onset}} = 25 \text{ ms}$ for the longest onset duration. The onset behavior is very repeatable between tests, and the fifth test in the series is employed for detailed analysis. A spectrogram is generated from the pressure evolution of the exit plane PCB at $\theta = 60^\circ$ (Figure 7.2). For most of the test length ($t > 35 \text{ ms}$), a well-defined line appears at $f = 3.78 \text{ kHz}$, which indicates that the test rapidly converges to a stable operating frequency. There is a clear harmonic band at twice the fundamental frequency (7.56 kHz).

![Spectrogram for exit plane PCB ($\theta = 60^\circ$) for Case A](image)

Figure 7.2: Spectrogram for exit plane PCB ($\theta = 60^\circ$) for Case A
RDC pressure traces contain sharp gradients, and are generally not sinusoidal, which yields harmonic distortion when using an FFT, and generates high amplitude harmonic peaks. The transducer data is high pass filtered with a cutoff frequency of 100 Hz to remove any low frequency distortion due to heat loading on the transducers. This attenuates the lower region of the spectrogram for $f < 1$ kHz, and no peaks are observed there. While ignition occurs at $t = 0$, there is no well-defined frequency band. Instead, a series of light-colored horizontal bands exist across the entire frequency range between $t = 0$-25 ms, and chaotic noise dominates the signal.

The three exit plane PCB transducers are used to determine the rotation time scale (Figure 7.3a). Each transducer signal is auto-correlated to itself to determine the elapsed time between detonation passage. Using all three transducers, the mean rotation time scale is calculated and converted into a detonation wave speed ($W_s$). Cross-correlations are also computed between each transducer pair ($\theta = 60^\circ \rightarrow 180^\circ$, $\theta = 180^\circ \rightarrow 300^\circ$, $\theta = 300^\circ \rightarrow 60^\circ$) to calculate the time delay between azimuthal stations. The time delay between two azimuthally-spaced sensors is normalized by the rotation time to convert to a phase delay (Figure 7.3b).

![Graphs showing wave speed and phase delay analysis for exit plane transducers for Case A](image.png)

**Figure 7.3**: Wave speed and phase delay analysis for exit plane transducers for Case A

Mean wave speed is relatively erratic for the first 15ms, due to the irregular spacing between
detonation peaks during the onset phase. After the onset event, the wave speed gradually rises from $W_s = 1.6 \rightarrow 1.75$ km/s, indicating that the detonation accelerates after the initial formation of a stable operating condition. After the onset, the mean phase delay between the three sensor pairs is constant at approximately -120°. As the sensors are azimuthally spaced at 120° angular increments, this indicates that rotation is in the $-\theta$ direction (clockwise). Following this onset event, the phase delay and wave speed stabilize to constant values, and are free of discontinuities, indicating that detonation propagates continuously for the duration of the test.

The detailed pressure evolution in the exit plane confirms the existence of the chaotic instability after RDC ignition (Figure 7.4). High amplitude, aperiodic pressure peaks continue up to the onset of rotation at $t = 16$ ms. This chaotic unstable behavior is commonly observed for combustor configurations utilizing subsonic reactant injection, and is associated with detonation pressure feedback into the reactant plenums (Anand et al. 2015b).

![Figure 7.4: Pressure evolution in the exit plane shortly after RDC ignition for Case A](image)

As Bykovskii et al. (2014) attribute the onset phase to disruption of the air injection, it is reasonable to assume that this instability manifests due to occlusion of the reactant injection. At the moment of initiation, the combustor is filled entirely with reactants, and the initial rapid consumption of this
reactant volume may be extremely disruptive, compared to the detonation-induced fluctuations at the stable, post-onset condition. If this is the primary mechanism behind the onset phase, the combustor should stabilize more easily with highly choked injection.

7.2.2. Case B: $\dot{m}_{\text{air}} = 0.40 \text{ kg/s}, \phi = 1.43, \text{ No Nozzle}$

For case B, stable, single detonation wave rotation is achieved for all tests with a median onset time of $t_{\text{onset}} = 168 \text{ ms}$. The onset phase persists significantly longer than for case A, and has a large degree of scatter, with the duration ranging from $t_{\text{onset}} = 136$-184 ms. The tenth test in the series, which has close to the median onset value for this case, is employed for detailed analysis. A spectrogram is generated from the pressure evolution of the exit plane PCB at $\theta = 60^\circ$ (Figure 7.5). A well-defined line, representing the combustor operating frequency, extends from $t \approx 5$ms (almost immediately after ignition) until the end of the test. For the range $t = 0$-0.2s, this line is slightly wider than for the end portion of the test ($t = 0.2$-0.3s), and exhibits several discontinuities and variations in intensity. This suggests that there is unstable, predominantly rotating detonation within the combustor, which undergoes repeated extinction and re-initiation. At $t \approx 0.2$s, the line becomes relatively thin and demarcates the stable rotation frequency of $f = 3.86$ kHz. As before, a prominent harmonic band extends across the spectrogram at twice the combustor operating frequency.

![Figure 7.5: Spectrogram for exit plane PCB ($\theta = 60^\circ$) for Case B](image-url)
As before, mean wave speed is erratic during the onset phase, and fluctuations persist up to \( t = 195 \) ms (Figure 7.6a). A sudden, sharp drop in the wave speed suggests a detonation extinction event (protracted elapsed time between two detonation peaks). There are numerous instances of extinction, especially for \( t = 12-36 \) ms, but the fluctuations are less severe for \( t > 115 \) ms. Despite these fluctuations, wave speed gradually increases during the onset phase, and eventually stabilizes to a terminal value of \( W_s = 1.77 \) km/s. When the detonation fails and re-initiates, the rotation direction may change, as depicted by the discontinuities in the mean phase delay in (Figure 7.6b). Conversely, each instance of direction switching is accompanied by a corresponding extinction event which enables the switch. Comparing the period of continuous \(+\theta\) rotation from \( t = 36-68 \) ms in Figure 7.6b to the same interval in Figure 7.6a, this region is relatively stable, but culminates in an extinction event. The detonation alternates between \(+\theta\) (counterclockwise) and \(-\theta\) (clockwise) rotation, as the sign of the phase delay changes and finally stabilizes to a terminal state of \(+\theta\) rotation. The final extinction event occurs at \( t = 164 \) ms, and is followed by a re-initiation event at \( t = 165 \) ms and continuous detonation propagation for the remainder of the test.

![Graph](image1.png)

**Figure 7.6: Wave speed and phase delay analysis for exit plane transducers for Case B**

The exit plane pressure evolution following the initiation event is provided in Figure 7.7. In comparison
to Case A, rotation is also achieved within the first 25ms of the test for Case B. However, while the
detonation rotation in Case A converges to a stable, terminal state, the rotation in Case B is unstable
and suffers from a series of extinction events at \( t = 5 \text{ ms} \), \( t = 14 \text{ ms} \), \( t = 18 \text{ ms} \), and \( t = 22 \text{ ms} \). For these
interrruptive periods of rotation, periodic behavior (regular spacing of pressure peaks) is observed for all
three sensor locations, and there is a discernible phase difference between sensor locations.

![Figure 7.7: Pressure evolution in the exit plane shortly after RDC ignition for Case B](image)

In contrast to Case A, the air flow rate is 20% lower, while the fuel flow rate is about 10% higher, so the
air injection pressure ratio is reduced and the fuel injection pressure ratio is increased. The change in
onset behavior may be attributed to the reduction in the air injection pressure ratio, which allows the
initiation event to exert a stronger influence on the air plenum and hinder its recovery. While rotating
detonation temporarily forms within the combustor, it is possible that the initiation event triggers low
frequency fluctuations within the plenums (as observed by Anand et al. 2016a) and that these persist
and destabilize the detonation.

**7.2.3. Case C: \( \dot{m}_{\text{air}} = 0.40 \text{ kg/s}, \phi = 1.04 \), No Nozzle**

For Case C, stable, single detonation wave rotation is achieved for 90% of the tests, as one test remained
unstable up to \( t = 0.3 \text{s} \) and did not develop into a stable, periodic mode. Omitting this test, the median
onset time is $t_{\text{onset}} = 182$ ms, which is comparable to Case B. However, there is an even greater degree of scatter between the tests, with the onset time ranging from $t_{\text{onset}} = 87-207$ ms. Considering this observed scatter, it is likely that the unstable test had the potential to transition to stable rotation if the test length was sufficiently extended. The sixth test in the series, which is close to the median onset value for this case, is employed in the detailed analysis. As for the other cases, a spectrogram is generated from the pressure evolution of the exit plane PCB at $\theta = 60^\circ$ (Figure 7.8). In contrast to Case B, the fundamental frequency is poorly defined in the onset region between $t = 0.05-0.18s$, and despite its weak amplitude is discernible as $f \approx 3$ kHz. There is a clear demarcation between this noisy region and the sharp line extending from $t = 0.18-0.3s$, which represents the stable operating mode ($f = 3.61$ kHz). As the pressure peaks in the onset region have lower amplitude, the harmonic band is less pronounced. Similar to the onset phase of Case A (Figure 7.2), the entire onset region is punctuated by many thin lines extending across the entire frequency domain, indicating the presence of sustained chaotic instability. Within this unstable region, the faint presence of the $f \approx 3$ kHz band suggests that intermittent periods of rotation occur within the combustor.

![Spectrogram for exit plane PCB (θ = 60°) for Case C](image)

Figure 7.8: Spectrogram for exit plane PCB ($\theta = 60^\circ$) for Case C

Due to the presence of chaotic instability, the wave speed is relatively noisy, and exhibits a lower average value of $W_s = 1.4$ km/s during the onset phase, compared to its stable, terminal value of $W_s =$
1.66 km/s (Figure 7.9a). There is no discernible gradual growth in $W_s$ during the onset phase as with Case B, though the large fluctuations may obscure this behavior. The erratic evolution of the phase delay in Figure 7.9b does not contain extended periods of constant phase at ±120°. While rotation may intermittently form, the duration of these events is sufficiently brief to preclude accurate determination of the rotation direction. This behavior contrasts sharply with the behavior of Case B, where instances of continuous rotation extend for several milliseconds prior to failure (Figure 7.6b). The onset phase ends abruptly at $t = 183$ ms, when stable $-\theta$ rotation spontaneously develops.

![Graphs showing wave speed and phase delay analysis for Case C](image)

**Figure 7.9: Wave speed and phase delay analysis for exit plane transducers for Case C**

The exit plane pressure evolution for Case C depicts the chaotic instability following the initiation event (Figure 7.10). High amplitude pressure peaks are observed at irregular intervals, and occasionally decay into extended periods of weak pressure fluctuations, as evidenced by the interval $t = 4-7.5$ ms. The pressure evolution of Case C is similar to the chaotic pressure evolution observed for Case A in the interval $t = 0-15$ ms in Figure 7.4, but this chaotic behavior extends for the entirety of the onset phase. While this case has the same air injection pressure ratio as Case B, the fuel flow rate is 27% lower, yielding a lower fuel injection pressure ratio.
Figure 7.10: Pressure evolution in the exit plane shortly after RDC ignition for Case C

Though the mutually reduced injection pressure ratios compared to Case A may enhance destabilization of both plenums, it is possible that the relative pressure ratios of the air and fuel injection play a role.

For Case C, the fuel and air injection pressure ratios are approximately equal, which should yield equal recovery times for the two plenums. As Case A has approximately the same equivalence ratio, it also has nearly equal fuel and air injection pressure ratios (though both are increased relative to Case C), notionally resulting in equal plenum recovery. By contrast, Case B has higher fuel injection pressure ratio than air injection pressure ratio, and the fuel plenum should recover from the initiation event prior to the air plenum. Comparing the onset behavior, Case A and Case C both experience chaotic instability, while Case B facilitates extended periods of continuous rotation, suggesting that the relative pressure ratio of the air and fuel plenums plays a role.

7.2.4. Case D: $\dot{m}_{\text{air}} = 0.30 \text{ kg/s}, \phi = 0.93$, With Nozzle

Case D, which utilizes a converging nozzle, stabilizes to longitudinal pulsed detonation (LPD) for all tests. As the installation of the converging nozzle distorts pressure measurements in the combustor exit plane, the air injection slot transducers are used to characterize the onset process. The onset phase is extremely short compared to the previous cases (median onset time of $t_{\text{onset}} = 6 \text{ ms}$). With the exception
of one test, (which develops into single wave rotation for \( t = 5-70 \) ms, and then stabilizes to LPD) the onset phase concludes by \( t = 7 \) ms. The third test in the series is employed for detailed analysis. A spectrogram is generated from the pressure evolution of the air injection slot PCB at \( \theta = 40^\circ \) (Figure 7.11). A well-defined line at \( f = 3.83 \) kHz extends from \( t = 6 \) ms to the conclusion of the test, indicating that the combustor rapidly converges to a stable operating frequency. There is a lower frequency band at \( f \approx 2 \) kHz, which appears due to slight oscillation in the strength of the LPD, which varies regularly between every other wave. Strong harmonics are observed for both the fundamental LPD frequency as well as the lower frequency band.

![Figure 7.11: Spectrogram for air injection slot PCB (\( \theta = 40^\circ \)) for Case D](image)

As detonation does not propagate azimuthally for the case of LPD, wave speed is ill-defined for this operating state, and only the LPD frequency is given in Figure 7.12a. The initial LPD frequency is 10-15% lower than the terminal frequency, and rises gradually to its terminal value after the onset phase. In contrast to the previous three cases, there is no rotation for case D, and hence no phase delay between the azimuthal stations, as seen in Figure 7.12b. While the LPD phenomenon is approximately uniform about the combustor circumference, there are slight phase delay fluctuations between \( \pm 15^\circ \), indicating slight azimuthal distortions.
The air injection slot pressure evolution for Case D depicts the onset of LPD following the initiation event (Figure 7.13). Pressure fluctuations are observed after initiation, which eventually develop into rotation at $t = 3\text{ms}$. The phase difference between the three transducers gradually disappears by $t = 6\text{ms}$, indicating that azimuthal rotation has evolved into axial pulsation.

Despite having lower injection pressure ratios for both air and fuel than the previous three cases, the onset for case D is both rapid and repeatable. This suggests that there is a fundamental difference in
the onset process of LPD, relative to the rotating cases. The result is somewhat counterintuitive, as the addition of a converging nozzle on the combustor exit reduces the reactant pressure ratios. As the nozzle restricts the flow of exhaust gases, it should intensify the effect of the initiation event on the plenums and elongate the recovery period. However, the elevated combustor pressures after initiation may improve the detonation sensitivity of the incoming reactants and facilitate more rapid development of detonation. It is possible that this rapid onset mechanism also extends to rotating cases which utilize a converging nozzle.

7.2.5. Summary of Selected Cases
The onset times for each test for all four cases are summarized in Table 7.2. For each case, the onset times vary randomly with the test number, indicating that there is not an underlying systematic onset trend for a fixed operating condition.

<table>
<thead>
<tr>
<th>Case</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>ṁ</td>
<td>0.5</td>
<td>0.4</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>φ</td>
<td>1.03</td>
<td>1.43</td>
<td>1.04</td>
<td>0.93</td>
</tr>
<tr>
<td>Nozzle</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test</th>
<th>Onset Time (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18 161 105 6</td>
</tr>
<tr>
<td>2</td>
<td>20 175 87 6</td>
</tr>
<tr>
<td>3</td>
<td>12 155 - 6</td>
</tr>
<tr>
<td>4</td>
<td>12 170 182 5</td>
</tr>
<tr>
<td>5</td>
<td>16 184 204 6</td>
</tr>
<tr>
<td>6</td>
<td>25 174 183 5</td>
</tr>
<tr>
<td>7</td>
<td>12 136 143 7</td>
</tr>
<tr>
<td>8</td>
<td>13 150 207 77</td>
</tr>
<tr>
<td>9</td>
<td>23 173 108 6</td>
</tr>
<tr>
<td>10</td>
<td>21 165 196 6</td>
</tr>
</tbody>
</table>
While the onset phase has a highly stochastic duration which warrants a statistical approach, the underlying behavior of the four cases is highly repeatable. All tests in Case A contain a brief period of chaotic instability, followed by a rapid onset of stable rotation and continue without extinctions or wave reversals. Likewise, all tests in case B rapidly develop into rotation within several milliseconds, but contain several extinction events and wave reversals between periods of continuous rotation. All tests in Case C exhibit the same sustained chaotic instability, and with the exception of the Test 3, all these tests abruptly transition to stable rotation. With the exception of Test 8, all tests in Case D rapidly develop into LPD within several milliseconds. The pressure evolution reveals the same features between tests, and the underlying mechanisms governing the onset phase appear to be the same, despite the stochastic variations in duration.

### 7.3 Onset Analysis for Baseline Operating Map

The data processing methodology applied in the previous section is applied to the RDC data collected by Driscoll et al. (2015a) for the baseline combustor configuration. Terminally stable operation is achieved for flow rates of $\dot{m}_{\text{air}} \geq 0.40 \text{ kg/s}$ for the entire equivalence ratio range between lean and rich operating limits. Onset times, as calculated in the previous section, are reported in Figure 7.14a for these cases.

![Figure 7.14: Comparison of onset time definitions for stable detonation cases at two flow rates](image)
Cases are categorized by their onset behavior as either periodic with extinctions (e.g., case B), or chaotic (e.g., case C). Chaotic, aperiodic onset behavior occurs for the lean cases, while periodic onset behavior with extinctions occurs for stoichiometric to rich cases. The onset duration clearly decreases with increasing equivalence ratio and increasing air flow rate for the chaotic onset cases. Unfortunately, no clear pattern emerges for the periodic onset cases, which vary randomly by an order of magnitude and have no clear dependence on equivalence ratio or air flow rate. However, by redefining onset time as the delay between ignition and the appearance of “locally” stable behavior (e.g., continuous detonation rotation for a minimum number of consecutive laps), a global trend begins to emerge (Figure 7.14b). Under this revised definition of onset time, the period $t = 36$-$68$ ms for case B (Figure 7.6b) qualifies as locally stable, such that $t_{\text{onset}} = 36$ ms. Applying this to the remainder of the periodic onset cases in the operating map, the onset duration decreases with increasing equivalence ratio, reinforcing and continuing the trend observed for the chaotic onset cases.

7.4 Conclusion

The onset phase between RDC initiation and the appearance of a stable, periodic operating state is characterized for four combustor conditions. The first three cases explore the onset behavior for varying flow rate and equivalence ratio for rotating cases without an exit nozzle. The fourth case explores onset behavior for the case of longitudinal pulsed detonation with a converging nozzle installed at the combustor exit. For each case, frequency domain analysis is employed in the form of spectrograms, and time domain analysis is employed in the form of wave speed, phase delay, and pressure evolution.

The median duration and scatter of the onset phase is a function of flow rate and the combustor exit geometry. The onset phase concludes rapidly for the higher flow rate (Case A) and for the case utilizing a converging nozzle exit (Case D). For the first instance (rapid onset for Case A), this dependence on
flow rate may be more appropriately attributed to a dependence on the air injection pressure ratio. Cases B and C have lower air injection pressure ratios and cannot quickly recover and stabilize after the initiation event. For the latter instance (rapid onset for Case D), the presence of an exit nozzle may encourage rapid development of LPD via strong shock reflections due to the exit constriction, and elevated pressures in the combustor, despite a reduction in the injection pressure ratios.

The duration of the onset phase is very stochastic, but the combustor behavior during this transitory period is very repeatable. For the three cases without a converging nozzle exit, the onset duration varies by a factor of two for the current sample size of ten tests, which is comparable to the scatter observed by Peng et al. (2015). While it may be difficult to accurately predict the duration, it may be feasible to predict the onset behavior. While a detailed analysis of all ten tests for each case is beyond the scope of what can be included in the present paper, the authors find that the frequency and time domain behavior is extremely consistent for the tests in each case. A comparison of onset behavior between the three rotating cases implies that the relative injection pressure ratios of air and fuel may govern the behavior. For comparable injection pressure ratios, chaotic instability is observed during the onset phase. For fuel injection pressure ratio in excess of the air injection pressure ratio, periods of stable rotation are observed with intermittent extinction events and wave reversals.
Chapter 8 – Fuel Blending as a Means to Achieve Initiation in a Rotating Detonation Combustor

The following investigation explores the efficacy of using hydrogen-ethylene fuel blends to circumvent the difficulties with achieving rotating detonation in ethylene-air mixtures. By achieving detonation in a relatively detonable hydrogen-ethylene fuel blend, consisting primarily of ethylene, the fuel can then gradually be converted to pure ethylene. The gradual transition of the mixture to a target composition may enable the detonation to stabilize to a terminal state which does not develop naturally using the existing traditional initiation approach.

8.1 Experimental Approach

For this investigation, the combustor is configured for premixed injection of ethylene and air to circumvent the difficulties of reactant mixing and create favorable conditions for detonation. Prior unpublished investigations in this facility failed to establish rotating detonation for non-premixed ethylene-air mixtures for the baseline combustor configuration. The premixed ethylene-air is supplied into the combustor through the injection slot from the premixed plenum (Figure 8.1). Initiation is achieved with an H2-O2 initiator tube, and a converging nozzle is used to increase the lightoff pressure.

Figure 8.1: RDC schematic indicating reactant injection scheme and instrumentation layout
The fuel plenum, typically used for primary fuel delivery, is used for secondary fuel (H₂) delivery. Prior to this investigation, hydrogen was added directly into the premixed plenum, greatly increasing the prevalence of flashback. While it is desirable to have all reactants premixed, flashback difficulties motivated the segregation of the hydrogen delivery from the ethylene-air delivery. For testing of pure ethylene-air mixtures, the fuel plenum is not used and ostensibly remains empty. For cases utilizing fuel blends of hydrogen and ethylene, the plenum is used for continuous hydrogen delivery. For cases transitioning from a fuel blend to pure ethylene, the plenum is initially used for hydrogen delivery, and then becomes inactive.

For cases utilizing hydrogen blending, a secondary fuel line supplies the baseline fuel plenum with hydrogen. A pneumatically-actuated isolation valve controls the timing of H₂ flow relative to the primary C₂H₄ flow. An orifice plate is installed upstream of the isolation valve to measure the hydrogen flow rate to the combustor. Supply pressures are manually regulated between test blocks, such that the hydrogen flow rate is fixed for a range of ethylene flow rates. The ignition sequence is preceded by a 3s fuel dump for both fuels to allow fuel flow rates to stabilize. The test length is held constant at 1s for all baseline and blended cases with the exception of the extended transition cases, which are 3s in length.

For the fuel blends explored, the primary fuel is ethylene (C₂H₄), and the secondary fuel is hydrogen (H₂), with the hydrogen contribution expressed by its stoichiometric contribution. Equivalence ratios are calculated based on the amount of required air to stoichiometrically consume the fuel, and so it is useful to decompose the equivalence ratio into H₂ and C₂H₄ components (Eqs. 8.1-8.2). A relatively large volume fraction of hydrogen is required to produce a noticeable stoichiometric impact, as 1 mole of C₂H₄ is the stoichiometric equivalent of 6 moles of H₂, as explored by Lu et al. (2003). The crossover point at which \( \phi_{H_2} = \phi_{C_2H_4} \) occurs for a volumetric fraction of \( X_{H_2} = 0.857 \) (85.7% H₂ by volume).
For this study, the rig is instrumented with PCB transducers, installed in an infinite tube pressure configuration, with approximately 42 mm standoff distance from the outer diameter of the combustor. The ITP assemblies are located in the third row of instrumentation at the three azimuthal stations. For the extended transition cases, a fourth transducer is installed within the air injection slot at azimuthal station II to capture the disruption of reactant injection by the passage of the detonation wave. Three ionization probes are flush-mounted in the first instrumentation row at the three azimuthal stations to provide a secondary measurement of rotation and frequency content, as well as verify the existence of combustion in the RDC channel. Burn time, or \( t_b \), is defined as the length of time after lightoff for which ionization continues to occur within the channel. When \( t_b \) is less than the intended test length, the combustion ceases within the channel, generally resulting in external burning, referred to as a "popout" failure. When ionization does not occur within the channel after lightoff, or \( t_b = 0 \), this is regarded as a failure to establish rotation, or a "no rotation" failure.

A set of baseline, ethylene-air tests is conducted to determine whether detonation rotation can be established and maintained for ethylene-air mixtures. The investigation is primarily focused on an air flow rate of \( \dot{m}_{\text{air}} = 0.5 \text{ kg/s} \), where behavior is less stochastic. With the converging nozzle installed (\( A_e = 11.8 \text{ cm}^2 \)) this air flow rate yields an initial pressure of \( P_i = 1.9\pm0.05 \text{ bar} \) prior to ignition, while the reactant temperature is subject to changes in ambient conditions and is approximately \( T_i = 285\pm10 \text{ K} \).

After establishing the baseline operating range for ethylene-air mixtures, blended fuel tests are conducted to explore the effect of \( \text{H}_2 \) addition on the operating range and detonation behavior. These tests are fully-assisted, meaning that the supplemental \( \text{H}_2 \) flow rate is maintained through the entire
test. The geometry remains identical to the baseline ethylene cases, except for the addition of H\textsubscript{2} flow through the previously vacant fuel plenum. A total of four H\textsubscript{2} flow rates are explored for the baseline air flow rate of $\dot{m}_{\text{air}} = 0.5$ kg/s. Hydrogen flow rate is fixed for a range of ethylene flow rates, resulting in lines of constant $\phi_{\text{H2}}$ for a given supply pressure.

Following the fully-assisted blended fuel tests, cases exhibiting repeatable fully-assisted detonation are used as the starting point for extended transition cases. For these cases, hydrogen is retracted 0.5s after ignition, allowing for transition to an unassisted, ethylene-air mixture. A preliminary investigation indicates that H\textsubscript{2} is fully depleted within less than a second after the secondary fuel line is closed, such that a test length of 3s is sufficient to guarantee absence of H\textsubscript{2} from the system.

### 8.2 Baseline Ethylene-Air Operation

For the premixed combustor configuration at the baseline air flow rate ($\dot{m}_{\text{air}} = 0.5$ kg/s), onset of coherent rotation is observed over a fairly wide range of equivalence ratios (Table 8.1). Despite experiencing successful lightoff, almost all cases exhibit burn times $t_b < 1.0$, indicating combustion does not occur within the RDC for the entire test, but prematurely fails. This failure mode is termed a “popout,” as combustion is no longer contained within the combustor, and deteriorates into an external turbulent flame, anchored to the combustor exit. Most of these cases exhibit instability, consisting of brief periods of rotation followed by extinction and recovery of the detonation front. This instability is qualitatively described for each case from “severe” to “none” depending on the prevalence of these extinction and recovery events and the stability of the pressure amplitude between consecutive detonation laps. In addition to instability in the rotational behavior, there exists a mode for which the PCB sensors experience the same pressure evolution, devoid of phase shift, at frequencies near 1 kHz. This large amplitude pulsing phenomenon is similar to the regime of operation described by Bykovskii et al. (2011), and is referred to as the longitudinal pulsed mode (LPM). For lower equivalence ratios, the
pressure histories lose coherence and the waveforms are not as well-defined. Fundamental frequencies are relatively low for a rotating detonation and indicate that the dominant mode may be pulsed rather than rotational. For higher equivalence ratios, clear, single-wave rotation is observed, and wave speeds calculated from rotational frequencies exhibit a deficit from the ideal CJ values ($W_{CJ} = 1.75\text{--}1.85 \text{ km/s}$). These deficits are on the order of 50% of the ideal values ($W \approx 0.5W_{CJ}$), indicating operation in the quasi-detonation regime, possibly due to losses incurred by the instability.

### Table 8.1: Selected baseline ethylene-air cases for $\dot{m}_{\text{Air}} = 0.5 \text{ kg/s}$

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>Frequency (kHz)</th>
<th>Wave Speed (km/s)</th>
<th>Burn Time (s)</th>
<th>Instability</th>
<th>Operational Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>1.20</td>
<td>0.55</td>
<td>0.69</td>
<td>Moderate</td>
<td>Incoherent Waveforms</td>
</tr>
<tr>
<td>0.79</td>
<td>1.25</td>
<td>0.58</td>
<td>1.00</td>
<td>Moderate</td>
<td>Incoherent Waveforms</td>
</tr>
<tr>
<td>0.79</td>
<td>1.28</td>
<td>0.59</td>
<td>0.15</td>
<td>Moderate</td>
<td>Incoherent Waveforms</td>
</tr>
<tr>
<td>0.84</td>
<td>1.85</td>
<td>0.85</td>
<td>0.21</td>
<td>Severe</td>
<td>Single Wave ($n = 1$)</td>
</tr>
<tr>
<td>0.89</td>
<td>1.78</td>
<td>0.82</td>
<td>0.05</td>
<td>Severe</td>
<td>Single Wave ($n = 1$)</td>
</tr>
<tr>
<td>0.89</td>
<td>1.76</td>
<td>0.81</td>
<td>0.10</td>
<td>Severe</td>
<td>Single Wave ($n = 1$)</td>
</tr>
<tr>
<td>0.94</td>
<td>1.78</td>
<td>0.82</td>
<td>0.06</td>
<td>Severe</td>
<td>Single Wave ($n = 1$)</td>
</tr>
<tr>
<td>0.99</td>
<td>1.70</td>
<td>0.78</td>
<td>0.05</td>
<td>Severe</td>
<td>Single Wave ($n = 1$)</td>
</tr>
<tr>
<td>1.04</td>
<td>1.83</td>
<td>0.84</td>
<td>0.02</td>
<td>Severe</td>
<td>Single Wave ($n = 1$)</td>
</tr>
<tr>
<td>1.08</td>
<td>1.82</td>
<td>0.83</td>
<td>0.04</td>
<td>Severe</td>
<td>Single Wave ($n = 1$)</td>
</tr>
</tbody>
</table>

Pressure evolution from a preliminary test campaign for $\dot{m}_{\text{Air}} = 0.3 \text{ kg/s}$ highlights some of the typical post-ignition behavior observed for ethylene-air mixtures. Some cases experience immediate onset of rotation upon ignition, followed by semi-stable rotation interspersed with LPM (Figure 8.2a). For some cases, ignition is followed immediately by LPM behavior, consisting of a strong, nonrotating, pressure oscillation with no phase shift between azimuthal stations (Figure 8.2b). While this appears to briefly transition into noisy rotation, the rotation is relatively weak and promptly decays, resulting in a popout failure. This type of behavior is characteristic of many cases with $t_b < 30 \text{ ms}$, and constitutes unstable detonation ($n = 1$). Another type of ignition behavior typical of very rich or lean mixtures, or mixtures with low initial pressure, is the failure to establish rotation (Figure 8.2c). The pressure evolution shows a
large spike from the initiating blast, followed by secondary reflections, but no phase shift is observed, and no coherent, rotating structures appear.

Figure 8.2: Examples of typical lightoff behavior for ethylene-air mixtures ($\dot{m}_{\text{Air}} = 0.3 \text{ kg/s}$)
For cases with sustained instability, wave rotation decays, resulting in an extinction event, followed by the spontaneous ignition and recovery of detonation. This process exhibits a periodic behavior that appears as a burst pattern in the PCB pressure history (Figure 8.3a). Between the extinction and subsequent recovery events, flame holding or very low speed, low pressure flame rotation maintains active combustion within the channel. For each burst, recovery may be possible for certain conditions, but if the combustion is entirely quenched or flame holding is disrupted, the ignition and recovery may not be possible.

For the case in Figure 8.3, three bursts are evident in the time-resolved PCB pressure history, and the third burst corresponds to an irrecoverable extinction and concludes the test in a popout failure. Capillary tube averaged pressures (Figure 8.3b) identify a clear low frequency fluctuation (on the order
of 20 Hz) with a time scale similar to the occurrence of the burst phenomenon, with an amplitude on the order of $\Delta P \approx 1$ bar. Also, the ostensibly empty fuel plenum experiences a strong spike at the same approximate time of the irrecoverable extinction event. The slight phase delay between the fuel plenum and combustor pressures may indicate coupling between the plenums, allowing reversed flow and potential accumulation of premixed reactants or products into the fuel plenum.

Rotation could not be successfully achieved without the addition of the converging nozzle for any of the flow rates or equivalence ratios tested. The baseline ethylene-air operating range with a converging nozzle ($A_e = 11.8 \text{ cm}^2$) is summarized in Figure 8.4. For fixed combustor exit area, initial pressure at ignition scales monotonically with flow rate, and the highest initial pressures are observed at $\dot{m}_{\text{AIR}} = 0.5$ kg/s, corresponding to $P_i \approx 1.75$ bar. While all the rotating detonation cases eventually culminate in popout failure, burn times generally increase with flow rate.

![Figure 8.4: Baseline ethylene-air operating range](image)

### 8.3 Blended Ethylene-Hydrogen-Air Operation

In an attempt to improve operability and attain successful rotating detonation, hydrogen is blended into the combustor to supplement the ethylene-air reactants and provide fully-assisted operation. While the
initial intent was to parametrically study a range of flow rates, behavior is stochastic for lower flow rates and exhibits a strong dependence on initial reactant pressure. Many cases conducted at lower flow rates fail to produce rotation, even for large degrees of hydrogen blending (60% H₂ by volume). For a small number of test cases, highly unstable rotation is maintained for the duration of the test. Some of the \( \dot{m}_{\text{Air}} = 0.4 \text{ kg/s} \) test cases are summarized in an abridged operating map (Figure 8.5a), to highlight the dependence of the operating range on initial pressure (\( P_i = 1.57\pm0.03 \text{ bar} \) for \( \dot{m}_{\text{Air}} = 0.4 \text{ kg/s} \)). The primary focus of this fully-assisted fuel blend investigation is the expansion of operability at the highest flow rate (\( \dot{m}_{\text{Air}} = 0.5 \text{ kg/s} \)).

![Diagram](image)

**Figure 8.5: Fully-assisted hydrogen-ethylene fuel blend operating maps**

Operating maps for both air flow rates are provided in Figure 8.5, with operation classified into three categories. Note that the designation of “rotation” used here does not imply absence of burst-like instability, or periodic occurrence of extinction and recovery events, but only denotes that rotation is present, and combustion is maintained for the duration of the intended test length. Cases experiencing rotation followed by a popout failure, generally due to an irrecoverable extinction event are classified as “popout/transitory,” while cases failing to establish rotation are simply classified as “no rotation.” While pure ethylene-air cases are dominated by the popout failure mode, prevalence of “rotation” cases increases with increasing hydrogen content (Figure 8.5b). With the exception of a small number of
slightly rich cases, successful rotation is achieved for almost all cases at $\dot{m}_{\text{air}} = 0.5 \text{ kg/s}$. Furthermore, with the exception of the leanest case ($\phi_{\text{TOTAL}} = 0.80$), repeatable rotation is achieved and sustained for the entire test length for the $\phi_{\text{H2}} = 0.18$ cases.

The fully-assisted blended cases with the highest $\text{H}_2$ flow rate ($\phi_{\text{H2}} = 0.18$) are summarized in Table 8.2 to highlight the effect of hydrogen addition. Several key features distinguish hydrogen-assisted behavior from baseline ethylene behavior. In general, rotation frequencies (i.e., wave speeds) are higher than those seen for ethylene-air mixtures, corresponding to lower CJ deficits. Single wave ($n = 1$) rotation is exclusively observed for this configuration due to insufficient mass flux to support multiple waves, but all cases exhibit coherent, rotating wave structures. The severity of the instability is effectively reduced or eliminated for all cases, with fewer observed extinction events, and reduction in peak pressure variation between detonation wave laps. While the addition of hydrogen does not necessarily augment the operating range, it strongly stabilizes the rotation and promotes a slight increase in the wave speed.

**Table 8.2: Selected hydrogen-assisted operating cases for $\dot{m}_{\text{air}} = 0.5 \text{ kg/s, } \phi_{\text{H2}} = 0.18$**

<table>
<thead>
<tr>
<th>$\phi_{\text{TOTAL}}$</th>
<th>$\phi_{\text{H2}}$</th>
<th>% $\text{H}_2$ (by vol.)</th>
<th>Frequency (kHz)</th>
<th>Wave Speed (km/s)</th>
<th>Burn Time (s)</th>
<th>Instability</th>
<th>Operational Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.21</td>
<td>0.18</td>
<td>52%</td>
<td>1.87</td>
<td>0.86</td>
<td>1</td>
<td>None</td>
<td>Single Wave</td>
</tr>
<tr>
<td>1.18</td>
<td>0.18</td>
<td>52%</td>
<td>1.92</td>
<td>0.88</td>
<td>1</td>
<td>None</td>
<td>Single Wave</td>
</tr>
<tr>
<td>1.18</td>
<td>0.17</td>
<td>50%</td>
<td>1.94</td>
<td>0.89</td>
<td>1</td>
<td>None</td>
<td>Single Wave</td>
</tr>
<tr>
<td>1.11</td>
<td>0.18</td>
<td>54%</td>
<td>1.92</td>
<td>0.88</td>
<td>1</td>
<td>Minimal</td>
<td>Single Wave</td>
</tr>
<tr>
<td>1.09</td>
<td>0.18</td>
<td>54%</td>
<td>1.94</td>
<td>0.89</td>
<td>1</td>
<td>Low</td>
<td>Single Wave</td>
</tr>
<tr>
<td>1.05</td>
<td>0.18</td>
<td>56%</td>
<td>2.11</td>
<td>0.97</td>
<td>1</td>
<td>Low</td>
<td>Single Wave</td>
</tr>
<tr>
<td>1.00</td>
<td>0.18</td>
<td>57%</td>
<td>2.05</td>
<td>0.94</td>
<td>1</td>
<td>Moderate</td>
<td>Single Wave</td>
</tr>
<tr>
<td>1.00</td>
<td>0.18</td>
<td>57%</td>
<td>2.08</td>
<td>0.95</td>
<td>1</td>
<td>Moderate</td>
<td>Single Wave</td>
</tr>
<tr>
<td>0.93</td>
<td>0.18</td>
<td>57%</td>
<td>2.08</td>
<td>0.96</td>
<td>1</td>
<td>Low</td>
<td>Single Wave</td>
</tr>
<tr>
<td>0.93</td>
<td>0.18</td>
<td>59%</td>
<td>2.04</td>
<td>0.94</td>
<td>1</td>
<td>Low</td>
<td>Single Wave</td>
</tr>
<tr>
<td>0.90</td>
<td>0.18</td>
<td>60%</td>
<td>2.08</td>
<td>0.96</td>
<td>1</td>
<td>Moderate</td>
<td>Single Wave</td>
</tr>
<tr>
<td>0.89</td>
<td>0.17</td>
<td>60%</td>
<td>2.06</td>
<td>0.95</td>
<td>1</td>
<td>Low</td>
<td>Single Wave</td>
</tr>
<tr>
<td>0.86</td>
<td>0.18</td>
<td>61%</td>
<td>2.08</td>
<td>0.96</td>
<td>1</td>
<td>Moderate</td>
<td>Single Wave</td>
</tr>
<tr>
<td>0.86</td>
<td>0.18</td>
<td>61%</td>
<td>2.06</td>
<td>0.95</td>
<td>1</td>
<td>Moderate</td>
<td>Single Wave</td>
</tr>
<tr>
<td>0.84</td>
<td>0.17</td>
<td>61%</td>
<td>2.04</td>
<td>0.94</td>
<td>1</td>
<td>Low</td>
<td>Single Wave</td>
</tr>
<tr>
<td>0.81</td>
<td>0.18</td>
<td>63%</td>
<td>2.02</td>
<td>0.93</td>
<td>1</td>
<td>Low</td>
<td>Single Wave</td>
</tr>
</tbody>
</table>
To further explore the effect of H\(_2\) addition on the existence and stability of detonation, the pressure evolution from three selected cases is processed with a peak-detection time-of-flight algorithm to track the wave speed evolution between consecutive detonation laps. The instability for the three cases of interest is qualitatively described as “none,” “moderate,” and “severe.” The first case (\(\dot{m}_{\text{AIR}} = 0.5\) kg/s, \(\phi_{\text{TOTAL}} = 1.18\), \(\phi_{\text{H2}} = 0.18\)), exhibiting no instability, has relatively constant wave speed with good lap-to-lap repeatability, averaging \(W \approx 0.9\) km/s (Figure 8.6). The detected pressure peaks are denoted in the PCB pressure evolution as black circles, and indicate stable, constant peak amplitudes.

![Pressure evolution and Wave speed evolution graphs](image)

**Figure 8.6: Fully stable detonation case \((\phi_{\text{TOTAL}} = 1.18, \phi_{\text{H2}} = 0.18)\)**

The second, moderately unstable case (\(\dot{m}_{\text{AIR}} = 0.5\) kg/s, \(\phi_{\text{TOTAL}} = 1.00\), \(\phi_{\text{H2}} = 0.18\)), initially behaves as a stable case until approximately \(t = 0.6\) s after lightoff, when a strong instability develops (Figure 8.7). After the onset of instability, the distinctive burst pattern appears in the pressure history due to large fluctuations in peak pressures. The wave speed calculation is very noisy due to loss of coherence in the wave structure, but contains faint traces of sinusoidal wave speed fluctuation between \(W = 0.6-1.0\) km/s at a similar frequency to the large scale pressure oscillation (i.e., the 20-30 Hz “burst” frequency).
The third case ($\dot{m}_{\text{AIR}} = 0.5 \text{ kg/s}, \phi_{\text{TOTAL}} = 0.86, \phi_{\text{H2}} = 0.09$), exhibiting severe instability, experiences the onset of instability almost immediately following lightoff (Figure 8.8). The burst pattern is very periodic and the wave speed and pressure experience large, sustained, low frequency variations. This case constitutes the near-limit behavior for instability, as many similar cases experience an irrecoverable extinction event following a burst, and abruptly end in popout failure.
As hydrogen addition promotes the sustained existence of combustion within the combustor, even for cases exhibiting severe instability, hydrogen may promote the recovery of detonation following an extinction event. A typical extinction event is presented in Figure 8.9, to highlight the main components of the extinction and recovery process. Rotation is observed from $t = 0.2s$ to $t = 0.21s$, but temporally sinusoidal variations in peak pressure are evident, as well as spatial variations in peak pressures between the three azimuthal stations. Near $t = 0.21s$, the pressure amplitudes at stations I and II suddenly grow, and a lower frequency disturbance at approximately 25% of the rotational frequency manifests at all three azimuthal stations. Peak pressures quickly diminish, and the pressure traces collapse onto each other by $t = 0.23s$, signifying extinction of detonation. At $t = 0.234s$, a sharp rise in pressure occurs simultaneously at all three sensor locations, indicating the onset of LPM. Within several milliseconds, this quickly develops into rotation, which again begins to exhibit sinusoidal peak pressure fluctuations. Hydrogen may promote the development of the LPM mode responsible for recovery, or may support faster transition from the LPM mode to a single wave rotational mode.

![Figure 8.9: Typical extinction and recovery of rotation (\(\dot{m}_{\text{AIR}} = 0.5\ \text{kg/s, } \phi_{\text{TOTAL}} = 0.94, \phi_{\text{H}_2} = 0.03\))](image)

A typical transition event between the LPM (axial pulsing) mode and the single wave ($n = 1$) rotation mode is depicted in Figure 8.10. Two key features allow the LPM mode to be easily distinguished from the usual rotational mode: lower characteristic frequency and lack of phase shift between circumferentially distributed sensors. LPM onset begins at $t =0.495s$ and becomes especially
pronounced at $t = 0.525\text{s}$, until transitioning back to a rotational mode at approximately $t = 0.540\text{s}$. If the LPM mode propagates as an axially-oriented oscillation, reactant injection will be uniformly disrupted for the entire annulus simultaneously, and ignition of the next wave of fresh reactants is contingent on exhaust gas recirculation. If the combustor geometry does not promote some degree of flame holding or recirculation zone, realization of the LPM may not be possible, and onset of an LPM mode may result in immediate extinction and popout failure. Furthermore, the LPM mode requires some degree of channel pressurization or confinement to manifest itself, as it is not observed for previous $\text{H}_2$ tests conducted without a converging nozzle. The relationship between the LPM regime, channel pressurization, and the instability cannot be fully verified with the present instrumentation.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8.10.png}
\caption{Transition between rotation and longitudinal pulsing modes ($\dot{m}_{\text{AIR}} = 0.5 \text{ kg/s, } \phi_{\text{TOTAL}} = 0.93, \phi_{\text{H}_2} = 0.13$)}
\end{figure}

8.4 Hydrogen-to-Ethylene Transition

Building upon the success of the blended fully-assisted cases, which exhibit sustained, relatively stable rotation for the highest hydrogen flow rate ($\phi_{\text{H}_2} = 0.18$), extended tests are conducted to transition a stable blended fuel mixture to an ethylene-air mixture. All extended transition tests are 3.0s in length, with the first 0.5s of the test utilizing full hydrogen assist to ensure lightoff and stable rotation. These are conducted for the same range of equivalence ratios listed in Table 8.2 to reproduce relatively stable initial conditions. The results of the extended transition cases are summarized in Table 8.3.
Table 8.3: Summary of extended hydrogen-to-ethylene transition cases

<table>
<thead>
<tr>
<th>$\phi_{\text{TOTAL}}$</th>
<th>$\phi_{\text{H}_2}$ (by vol.)</th>
<th>% $\text{H}_2$</th>
<th>Frequency</th>
<th>Wave Speed</th>
<th>Burn Time</th>
<th>Instability</th>
<th>Operational Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.91</td>
<td>0.18</td>
<td>55%</td>
<td>2.08</td>
<td>0.95</td>
<td>3</td>
<td>Severe</td>
<td>Single Wave</td>
</tr>
<tr>
<td>0.98</td>
<td>0.18</td>
<td>53%</td>
<td>1.97</td>
<td>0.91</td>
<td>3</td>
<td>Severe</td>
<td>Single Wave</td>
</tr>
<tr>
<td>1.01</td>
<td>0.18</td>
<td>52%</td>
<td>1.98</td>
<td>0.91</td>
<td>1.75</td>
<td>Severe</td>
<td>Single Wave</td>
</tr>
<tr>
<td>1.07</td>
<td>0.18</td>
<td>50%</td>
<td>1.92</td>
<td>0.88</td>
<td>0.72</td>
<td>Severe</td>
<td>Flashback</td>
</tr>
<tr>
<td>1.11</td>
<td>0.18</td>
<td>49%</td>
<td>1.87</td>
<td>0.86</td>
<td>0.75</td>
<td>Severe</td>
<td>Flashback</td>
</tr>
<tr>
<td>1.17</td>
<td>0.18</td>
<td>48%</td>
<td>1.82</td>
<td>0.83</td>
<td>0.885</td>
<td>Severe</td>
<td>Flashback</td>
</tr>
<tr>
<td>1.21</td>
<td>0.18</td>
<td>47%</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>No Rotation</td>
</tr>
</tbody>
</table>

Characteristic frequencies are similar to those produced by the fully-assisted tests, but all cases exhibit severe instability. Furthermore, a new “flashback” failure mode is induced for some of the moderately rich cases, where after the retraction and depletion of hydrogen, combustion travels upstream and stabilizes within the air plenum. The lean cases, despite experiencing severe instability, manage to remain within the channel for the entirety of the test. For the richest case of $\phi_{\text{TOTAL}} = 1.21$, ignition is not achieved in the mixture and no rotation is observed.

To understand the role of hydrogen in the suppression of instability, combustor PCB pressure evolution (Figure 8.11a) and capillary tube averaged pressure evolution (Figure 8.11b) are plotted over the active test length ($t_b = 1.75s$) for the $\phi_{\text{TOTAL}} = 1.01$ case. It is apparent that after the removal of $\text{H}_2$ at $t = 0.5s$, instability develops almost immediately. CTAP traces indicate equalization of the fuel plenum pressure with the channel pressure at approximately $t = 0.6s$, indicating depletion of $\text{H}_2$ within the plenum. Shortly afterwards, a low-frequency oscillation ($\approx 20 \text{ Hz}$) with an amplitude of $\Delta P = 1 \text{ bar}$ develops within the channel and this disturbance is transmitted to both the air plenum and the ostensibly “empty” fuel plenum. For every 4-5 periods of the low frequency oscillation, CTAP spikes indicate a large rise in fuel plenum pressure, indicating potential combustion within this plenum. It is unclear whether the PCB
burst pattern related to extinction and recovery is a direct result of the low frequency oscillation, or the cause of the oscillation, but the two may be linked via a feedback mechanism.

**Figure 8.11:** Onset of instability and subsequent failure for extended hydrogen-to-ethylene transition ($\dot{m}_{\text{AIR}} = 0.5 \text{ kg/s, } \phi_{\text{TOTAL}} = 1.01, \phi_{\text{H2}} = 0.18$)
The onset of instability is also captured in the wave speed evolution, which indicates relatively stable rotation for over 1000 laps, followed by a large increase in noise due to loss of coherence in the wave form (Figure 8.11c). For the unstable portion of the test, the peak-capturing algorithm lacks the robustness to track the relatively erratic pressure evolution, but faintly indicates a slight decrease in average wave speed.

It is possible that the main mechanism governing the successful development of detonation in ethylene-hydrogen-air blends is channel pressurization, and that hydrogen provides only a secondary effect on ignition. Furthermore, the positive stability characteristics may not be chemical in nature, but may be the result of a physical mechanism in the RDC. The large spikes in fuel plenum pressure are observed in the absence of hydrogen usage, but are of sufficiently high magnitude to imply combustion within the plenum. This requires the deposition of fresh reactants into the fuel plenum, which is made possible in the absence of a pressure gradient across the fuel injectors, such as when the fuel plenum is not in active use. The presence of the converging nozzle and the resulting channel pressurization may enhance this plenum interaction. Following a plenum combustion event, products may be vented out through the fuel injection plate into the RDC channel, potentially destabilizing the detonation and disrupting reactant injection. If this fuel plenum interaction is the source of instability, the role of hydrogen may be to flush products and air from the plenum to inhibit plenum-based combustion. For high channel pressurization, this may require a relatively high injection pressure ratio to be effective, which may explain the dependence of detonation stability on hydrogen flow rate.

8.5 Conclusion

A series of tests are conducted to determine the efficacy of achieving detonation in hydrogen-ethylene fuel blends, then transitioning to ethylene-air mixtures. Baseline detonation behavior is characterized for ethylene-air mixtures, then hydrogen-ethylene-air mixtures, before attempting to gradually
transition from a hydrogen-assisted condition to a pure ethylene condition. Ethylene-air mixtures exhibit moderate-to-extreme “bursting” instability in the present combustor configuration. This unstable combustion is rarely maintained for the entire test length and typically culminates in popout failure. Addition of moderate quantities of hydrogen ($\phi_{H_2} = 0.18$) is sufficient to suppress instability, enabling detonation rotation to persist for the entire test length, but at a significant wave speed deficit from the ideal CJ condition ($W \approx 0.5W_{CJ}$).

While stable detonation is possible with hydrogen addition, retraction of the supplementary hydrogen triggers the appearance of instability. It is reasonable to assume that the baseline combustor geometry does not support rotating detonation for this mixture, and attempting to transition to a pure ethylene-air mixture will yield unstable behavior. The necessity of the converging exit nozzle implies that initial reactant pressure is also of considerable importance in determining whether detonation is possible. As both initial pressure and hydrogen content affect the detonation cell width of the reactants, it is possible that many of the unstable cases occur for mixtures near the detonability limit in the present geometry. Analysis of cell width relative to the combustor dimensions, and dependence of cell width on reactant composition and thermodynamic state may elucidate the experimentally-observed trends.
Chapter 9 – Chemical Kinetic Analysis of Detonability-Enhancement for Ethylene-Oxidizer Mixtures

Ethylene (C\textsubscript{2}H\textsubscript{4}) is commonly employed in detonation-based combustors as evidenced by the literature survey of Roy et al. (2004), and serves as a surrogate fuel for larger, less detonable hydrocarbons. Previous studies have explored the effects of composition and thermodynamic state on the detonability of C\textsubscript{2}H\textsubscript{4}-air mixtures. Elevated initial pressure (\(P_i\)) significantly boosts detonability (Bauer et al. 1986), though experimental data is somewhat sparse, and numerical data is limited to stoichiometric mixtures at pressures of up to \(P_i \leq 2\) bar (Schultz and Shepherd 2000). The effect of elevated initial temperature (\(T_i\)) is less clear and has only received detailed treatment for stoichiometric C\textsubscript{2}H\textsubscript{4}-O\textsubscript{2}-Ar mixtures (Auffret et al. 1999), and stoichiometric C\textsubscript{2}H\textsubscript{4}-air mixtures of up to \(T_i \leq 373\)K (100°C) (Tieszen et al. 1991). O\textsubscript{2} addition to the oxidizer (reduced N\textsubscript{2} concentration) has a potent detonability-enhancing effect (Moen et al. 1981, Bauer et al. 1986, and Vasil’ev 1998), yielding reductions in cell width of up to two orders of magnitude for stoichiometric mixtures when pure O\textsubscript{2} is used as the oxidizer. The effect of H\textsubscript{2} as a fuel additive (substitution of various fractions of C\textsubscript{2}H\textsubscript{4} with a stoichiometric equivalent of H\textsubscript{2}) has only received numerical treatment (Lu et al. 2003). For stoichiometric mixtures at ambient initial conditions, a significant effect is achieved for modest H\textsubscript{2} substitution (50% reduction in the induction length for 10% replacement of the fuel with H\textsubscript{2}).

The present investigation builds upon previous studies to fully characterize the effects of initial pressure and temperature, oxygen concentration, and hydrogen addition on the detonability of baseline, ethylene-air mixtures. Applying the recently developed empirical approaches of Gavrikov et al. (2000) and Ng (2005) and four detailed chemical kinetic mechanisms, the ZND detonation model is used to estimate cell width. While previous studies report these effects for stoichiometric mixtures, this study explores the impact of these parameters for lean and rich mixtures as well. The effect of elevated initial temperature is explored for an extended range (up to \(T_i = 600\)K) to resolve the impact on detonability.
The effect of elevated $O_2$ content is explored for oxidizers ranging from pure $O_2$ to air to explore the benefit of moderate oxygen addition. The effect of hydrogen substitution is explored for a range of initial pressures to determine whether the significant benefit reported by Lu et al. (2003) is preserved at these conditions. The central objective of this work is to assess the effectiveness of these detonability enhancement strategies for $C_2H_4$-oxidizer mixtures. This information can be used to augment the operating range of existing detonation combustors, or improve scaling capabilities for future combustors. Conversely, some approaches can be used in reverse to reduce the detonation sensitivity in environments where detonation is undesirable or hazardous.

9.1 Numerical Method

Solutions are computed for the steady, one-dimensional ZND detonation equations in the form developed by Fickett and Davis (1979). This stiff system of ordinary differential equations is solved using MATLAB with a prescribed relative error tolerance of $1e^{-10}$. These computations employ the Shock and Detonation Toolbox developed by Kao and Shepherd (2008), and the open-source chemical kinetics software Cantera (Goodwin et al. 2014). The reaction rates and thermodynamic information are evaluated from a detailed chemical kinetic mechanism.

Four chemical mechanisms are employed in the present study: GRI Mech 3.0$^1$ (Smith et al.), San Diego Mech$^2$ (UCSD 2014), AramcoMech 1.3$^3$ (Metcalfe et al. 2013), and USC Mech II$^4$ (Wang et al. 2007). These detailed mechanisms have been developed for hydrocarbon oxidation and validated against a range of flame and shock tube data. Recently published ethylene-air shock tube data at elevated temperatures and pressures (Kopp et al. 2014, Penyazkov et al. 2009) allows for additional validation of these mechanisms at conditions representative of detonations. Because of the test procedure

---

1 Available for download at http://combustion.berkeley.edu/gri-mech/version30/text30.html
2 Available for download at http://web.eng.ucsd.edu/mae/groups/combustion/mechanism.html
3 Available for download at http://c3.nuigalway.ie/Mechanism_release/download.html
employed in these studies, the post-shock pressure within the shock tube varies over the range of temperature ($P = 10.7-16.5$ atm). An average pressure ($P_{\text{avg}}$) is used to describe the post-shock condition.

Figure 9.1: Comparison of the four kinetic mechanisms with experimental shock tube data
for each data set. Ignition delay time data from Kopp et al. (2014) is determined simultaneously from OH emission trace and pressure history. Ignition delay time from Penyazkov et al. (2009) is reported for a number of methods, and the OH emission results are employed in the present comparison. Constant volume simulations are performed for the four mechanisms, using the peak in the OH emission profile to calculate ignition delay time. Three equivalence ratios are simulated for a range of reciprocal temperature (1000/T = 0.65-0.95 K⁻¹) at an average post-shock pressure of $P_{\text{avg}} = 13.5$ atm.

The two sets of experimental data, despite slight differences in pressure, exhibit good agreement. The GRI 3.0 mechanism was developed and optimized primarily for methane combustion, but also includes a submechanism for ethylene oxidation. However, it deviates significantly from the other mechanisms at low temperatures, and does not adequately capture the general trend of the data. USC Mech II overpredicts the ignition delay for lean, high temperature conditions, but shows better agreement for rich, high temperature conditions. The slope of the curve, which is proportional to global activation energy (Schultz and Shepherd 2000), is lower than the slope indicated by the experimental data. San Diego Mech and AramcoMech 1.3 show comparable behavior over this range, capturing the general trend of the data, but underestimating ignition delay for stoichiometric and rich conditions. Approaching the high temperature limit of the simulated range (the minimum value of reciprocal temperature), ignition delays for the four mechanisms agree within a factor of two, but the general behavior of San Diego Mech and AramcoMech 1.3 offer the best agreement with the experimentally observed trends.

Thermicity ($\sigma$), as defined by Fickett and Davis (1979), is a non-dimensional parameter that “represents transformation of energy obtained from breaking chemical bonds and from the change in the number of particles or molecular sizes by chemical reaction, into energy of heat and motion.” Thermicity coefficients corresponding to each species are multiplied by species production rates and added to
arrive at the thermicity parameter ($\dot{\sigma}$), which denotes non-dimensional energy release rate. Following the method of Ng (2005), the induction length ($\Delta_i$) is defined as the distance between the shock and the location of peak thermicity. The representative reaction length is defined as the post-reaction (Chapman-Jouguet) particle velocity divided by the peak thermicity magnitude ($\Delta_r = u'_{CJ}/\dot{\sigma}_{max}$). This follows the recommendation by Schultz and Shepherd (2000) that induction length should be defined relative to the point of maximum heat release rather than a maximum species concentration (such as OH), which may vary considerably with reactant composition.

To estimate detonation cell width from the computed induction length, two empirical models are used: the single parameter model developed by Ng (2005), and the two parameter model developed by Gavrikov et al. (2000). These models build off the proportionality approach of Shchelkin and Troshin (1963) by defining the proportionality constant $A$ as a function of various parameters calculated from the ZND solution. Ng proposes that $A$ is a function of the stability parameter $\chi = \Delta_i/\Delta_r\hat{\theta}$, where $\hat{\theta}$ denotes the reduced activation energy, normalized by the post-shock, von Neumann conditions ($\hat{\theta} = E_a/RT_{VN}$). Gavrikov et al. propose that $A$ is a function of $\hat{\theta}$ and the temperature ratio across the leading shock ($T_{VN}/T_i$). Ng utilizes the Chapman-Jouguet velocity ($W_{CJ}$) to evaluate the post-shock conditions, and a ±1% perturbation in shock velocity to evaluate the reduced activation energy. In contrast, Gavrikov et al. use an overdriven shock (1.3 $W_{CJ}$) to evaluate the post-shock state, and a significantly larger perturbation in shock velocity (1.0 $W_{CJ}$ and 1.6 $W_{CJ}$) to evaluate the reduced activation energy.

This study assesses the efficacy of four strategies for increasing the detonation sensitivity of a baseline, ethylene-air mixture: pressurization, heating, oxygen enrichment, and hydrogen enrichment. To address these strategies, solutions are computed for a range of initial pressure, initial temperature, oxygen concentration (relative to N$_2$ in the oxidizer), and hydrogen concentration (relative to C$_2$H$_4$ in the fuel). Following the convention of Bauer et al. (1986), the oxygen concentration in the oxidizer is
expressed in terms of the nitrogen dilution ratio ($\beta = N_2:O_2$), where $\beta = 3.76$ for air. Likewise, following the convention of Lu et al. (2003), hydrogen concentration in the fuel is expressed as a fraction of the stoichiometric contribution ($\phi_{H_2}/\phi_{Total}$). The molar fuel-air ratio at stoichiometric conditions is six times higher for ethylene than for hydrogen, such that the equivalent molar concentration of fuel is six times higher when ethylene is fully replaced with hydrogen. The simulation conditions are provided below:

Table 9.1: Summary of ZND Simulation Conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>0.5-2.0</td>
</tr>
<tr>
<td>$P_i$</td>
<td>1-10 bar</td>
</tr>
<tr>
<td>$T_i$</td>
<td>300-600 K</td>
</tr>
<tr>
<td>$\beta$ ($N_2:O_2$)</td>
<td>3.76-0.0</td>
</tr>
<tr>
<td>% $H_2$ (by stoich.)</td>
<td>0-100%</td>
</tr>
</tbody>
</table>

9.2 Baseline Detonation Sensitivity

The baseline detonation cell width is computed for $P_i = 1$ bar, $T_i = 300$K for the equivalence ratio range of $\phi = 0.5-2.0$ (Figure 9.2). Uncertainty bounds for the experimental data represent a factor of two in the measured cell width (i.e., 50%-200% $\lambda_{exp}$), as Lee (2008) notes that smoked foil measurements can vary by a factor of two due to irregularity in the cellular patterns. The estimates predicted by the Gavrikov et al. model are approximately an order of magnitude lower than experimental values. Imbert et al. (2005) report similar discrepancies for argon-diluted n-heptane-oxygen mixtures, and attribute this error to extrapolation of reaction rates to the extreme temperatures and pressures imposed by the overdriven (1.3 $W_{CJ}$) post-shock conditions. Due to this significant deviation, consistent between all four mechanisms, the two parameter Gavrikov et al. model is not suited for this set of mixtures, and the single parameter Ng model is used for the remainder of the study.
The single parameter model yields significantly better agreement, and all four mechanisms fall within the experimental uncertainty bounds for stoichiometric conditions. Predictions at lean conditions are also in adequate agreement, with the exception of the GRI 3.0 mechanism, but are less valid for rich conditions ($\phi > 1.25$). Experimental data is comparatively sparse in this region, as Knystautas et al. (1982) are the only available source for cell width of moderately rich $C_2H_4$-air mixtures. Analogous to the shock tube data (Figure 9.1), USC Mech II overestimates cell width at lean conditions ($\phi = 0.5$), and slightly underestimates cell width at rich conditions ($\phi = 2$), while San Diego Mech and AramcoMech agree well at lean conditions and underestimate significantly at rich conditions.

![Figure 9.2: Baseline cell width estimations using the Ng (single-parameter) and Gavrikov et al. (two-parameter) cell width models with the four kinetic mechanisms](image)

San Diego Mech deviates from the typical concave cell width profile for $\phi \geq 1.9$ due to the unique thermicity profile of $C_2H_4$-air and the adopted definition of induction length. Thermicity profiles for the four mechanisms are illustrated in Figure 9.3 for two equivalence ratios ($\phi = 1, \phi = 2$). At stoichiometric conditions, the thermicity profile is asymmetric, with USC Mech II and AramcoMech predicting a distinct initial heat release stage with a local thermicity peak. This asymmetry is less pronounced for San Diego Mech and GRI 3.0, which do not exhibit a distinct initial peak, but contain a slight leftward bulge in the
thermicity profile. This unique thermicity profile is similar to the multi-stage heat release simulated by Sugiyama and Matsuo (2011), and may be responsible for the development of cellular variation or irregular cellular substructure, as observed in the smoked foil results of Moen et al. (1982).

![Figure 9.3: Thermicity profiles for stoichiometric ($\phi = 1$) and rich ($\phi = 2$) C$_2$H$_4$-air mixtures](image)

At the rich condition, this initial heat release peak is significantly more pronounced for all mechanisms but GRI 3.0, and becomes the dominant thermicity peak for San Diego Mech. For AramcoMech and USC Mech II, the initial peak is comparable in magnitude to the secondary peak. As the induction length is calculated from the distance of the peak thermicity to the leading shock, a shift in the dominant peak yields an apparent reduction in cell width. With the exception of GRI 3.0, peak thermicity values are significantly lower for the rich condition and the thermicity profile is wider, indicating that the heat release process is more gradual and spatially distributed.

### 9.3 Effect of Pressure

The effect of initial pressure is explored for all mechanisms from $P_i = 1$-10 bar for stoichiometric conditions (Figure 9.4). All mechanisms predict a monotonic decrease in cell width with increasing pressure, though the effect is weaker for GRI 3.0 and USC Mech II. These predictions are plotted against...
experimental data from Bauer et al. (1986), which have been converted from cell lengths to cell widths by assuming a cellular aspect ratio of 2 as proposed by that study. AramcoMech and San Diego Mech provide reasonable agreement with these data, while the others overestimate cell width by a factor of two at the highest pressure. On logarithmic axes, AramcoMech and San Diego Mech are roughly linear over the computed range, suggesting that the behavior observed by Schultz and Shepherd (2000) for $P_i = 0.2\text{-}2.0$ bar extends to higher pressures. This qualitatively matches the induction length predictions of Westbrook and Urtiew (1982), which are well modeled by a power law ($\Delta \propto P^m$). This is analogous to the power law cell width relation proposed by Auffret et al. (1999) for argon-diluted C$_2$H$_4$-O$_2$ mixtures. For stoichiometric C$_2$H$_4$-air mixtures up to $P_i = 10$ bar, the calculated cell widths conform well to $\lambda \propto P^m$, with exponents of $m = -0.62$ for AramcoMech and $m = -0.69$ for San Diego Mech. This yields an overall cell width reduction of 75-80% between the highest pressure case and the baseline case at $\phi = 1$. This pressure sensitivity weakens slightly for lean mixtures ($\phi = 0.75$: $m_{\text{Aramco}} = -0.43$, $m_{\text{SanDiego}} = -0.50$), but remains uniform for rich mixtures ($\phi = 1.5$: $m_{\text{Aramco}} = -0.62$, $m_{\text{SanDiego}} = -0.69$).

Figure 9.4: Cell width variation with initial pressure for stoichiometric C$_2$H$_4$-air mixtures
The combined effect of pressure and equivalence ratio is explored in Figure 9.5 using San Diego Mech. While both AramcoMech and San Diego Mech yield comparable trends, the number of reactions in the former (1542) significantly exceeds the latter (297). This generates a considerable disparity in computation time, and San Diego Mech is employed here due to its computational efficiency. Experimental cell width data are from Bauer et al. (1986) and are converted from cell length using a cellular aspect ratio of 2. Elevated initial pressure yields a consistent reduction in cell width for the entire range of equivalence ratio, though the reduction is more pronounced for $\phi \geq 1$. A shift in the dominant thermicity peak, previously identified in the baseline for $\phi \geq 1.9$, is also evident at elevated pressures. This manifests as a slight apparent decrease in the computed cell width due to the position of the two thermicity peaks relative to the shock front. The transition point occurs at $\phi \approx 1.8$ for $P_i = 3$ bar, and at $\phi \approx 1.7$ for $P_i = 10$ bar, indicating that elevated pressures have a complex impact on the heat release profile.

![Figure 9.5: Cell width variation with equivalence ratio for three initial pressures](image-url)
9.4 Effect of Temperature

The effect of initial temperature is explored for all mechanisms from $T_i = 300$-600 K for stoichiometric conditions at an initial pressure of $P_i = 1$ bar (Figure 9.6). The initial pressure is held constant rather than the initial density, as this is the more physically relevant case when heating of reactants is employed in a laboratory setting. The numerical study of Vasil’ev (2006) compares the case of constant initial pressure against constant initial density for a number of fuel-oxygen mixtures, and the latter case strongly emulates the effect of elevated initial pressure at constant initial temperature, as explored in the previous section. For the $C_2H_4$-air mixture employed here, minimal variation is observed over the simulated temperature range. The temperature dependence is weak enough that a logarithmic scale is not necessary, with a maximum cell width reduction of only 17%, observed for GRI 3.0. USC Mech II predicts a similar modest reduction of 16%, while the remaining two mechanisms yield negligible improvement.

![Figure 9.6: Cell width variation with initial temperature for stoichiometric $C_2H_4$-air mixtures](image)

Figure 9.6: Cell width variation with initial temperature for stoichiometric $C_2H_4$-air mixtures
While experimental data is relatively sparse for this mixture and cannot be used to validate this trend directly, all mechanisms but USC Mech II show good agreement with the results of Tiezsen et al. (1991). Auffret et al. (1991) report a power law increase in cell width with initial temperature in argon-diluted C₂H₂-O₂ mixtures, but use relatively low diluent fractions and mixtures with high detonation sensitivity (λ ≈ 1-10 mm). It is possible that elevated initial temperatures are ineffective at improving the sensitivity of mixtures with high initial detonability (fuel-O₂), but more effective for those with low initial detonability (fuel-air).

Expanding the investigation to non-stoichiometric mixtures, the effect of initial temperature is assessed over a range of equivalence ratios in Figure 9.7 using San Diego Mech (Pᵢ = 1 bar). There is almost no discernible effect for stoichiometric to moderately rich (ϕ = 1.5) conditions, and modest to moderate reductions at the lean and rich boundaries. This strongly resembles the behavior of H₂-air mixtures at elevated initial temperatures as reported in the experiments of Ciccarelli et al. (1994), which shows considerably enhanced sensitivity for lean conditions, but a reduced impact elsewhere.

Figure 9.7: Cell width variation with equivalence ratio for four initial temperatures
For the \( \text{C}_2\text{H}_4\)-air mixtures explored in Figure 9.7, the cell width is reduced by 83% for \( \phi = 0.5 \) at \( T_i = 600\text{K} \). The overall effect of elevated initial temperature is to significantly boost detonation sensitivity for lean (and to a lesser extent, rich) mixtures, without greatly affecting the stoichiometric condition where peak baseline detonability is observed.

Complex behavior is observed for \( \phi \geq 1.8 \), similar to the rich behavior at elevated initial pressures encountered in the previous section. The peak switching phenomenon in the thermicity profile occurs for all simulated temperatures near the rich boundary, and the transition point shifts to higher equivalence ratios as the temperature is increased. This is in contrast to the elevated initial pressure trend, where the transition point shifts to lower equivalence ratios as the pressure is increased. Despite this irregular phenomenon, the underlying trend suggests a modest reduction in cell width for \( \phi \geq 1.5 \), which is clearly identifiable when the second thermicity peak is dominant in the heat release profile.

### 9.5 Effect of Oxygen

The effect of oxygen concentration in the oxidizer (i.e. amount of nitrogen dilution) is explored in Figure 9.8 for stoichiometric \( \text{C}_2\text{H}_4\)-oxidizer mixtures. The dilution ratio (\( \beta = \text{N}_2:\text{O}_2 \)) is varied to simulate oxidizers ranging from oxygen (\( \beta = 0 \)) to air (\( \beta = 3.76 \)). The initial temperature is \( T_i = 300\text{K} \) and the initial pressure is \( P_i = 1 \text{ bar} \). For all four mechanisms, cell width drops nearly two orders of magnitude when transitioning from \( \text{C}_2\text{H}_4\)-air to \( \text{C}_2\text{H}_4\text{-O}_2 \), reproducing the trend reported by Bauer et al. (1986) and Knystautas et al. (1982). Despite differences in magnitude, the general shape of the curve is consistent between the mechanisms. Experimental cell width data from smoked foil measurements are taken from Knystautas et al. (1982), and critical diameter measurements from Moen et al. (1981) are converted to cell width measurements by using the relation \( d_{\text{crit}} = 13\lambda \). While agreement is satisfactory for the four mechanisms, the USC Mech II appears to overpredict cell width at the \( \text{O}_2 \) limit (\( \beta \to 0 \)), while the others underpredict at the air limit (\( \beta \to 3.76 \)), though the deviation is within the uncertainty bound.
To highlight this potent impact over a range of equivalence ratios, several nitrogen dilution ratios are explored in Figure 9.9 for San Diego Mech at the baseline pressure and temperature ($P_i = 1$ bar, $T_i = 300$K). The reductions observed for stoichiometric mixtures extend uniformly to lean and rich conditions, analogous to the trend observed for elevated initial pressure. For the rich condition ($\phi = 2$), the thermicity peak shift phenomenon disappears for $\beta \leq 3$, as the second thermicity peak becomes dominant with reduced nitrogen dilution ratio. While detonation sensitivity is maximized (i.e. cell width is minimized) near $\phi = 1.3$ for a fuel-air mixture, the point of peak detonability gradually shifts to higher $\phi$ as the oxidizer transitions to $O_2$. This enhances the cell width reduction for rich conditions as $\beta$ is reduced.

Given this extreme improvement in detonation sensitivity, supplementing a detonation combustor with additional $O_2$ is an attractive strategy, as implemented by Bykovskii et al. (2006b) and Dyer et al. (2012).

Considering the case of $\beta = 3$ (oxidizer is 25% $O_2$ by vol.), as employed by Dyer et al., this oxidizer mixture can be achieved by replacing a modest fraction of the air supply with supplemental $O_2$. For this
condition, the required supplemental O\textsubscript{2} is 5.9\% by mass relative to the air supply, such that each kilogram of air is supplemented with 59 g of O\textsubscript{2}. This quantity of O\textsubscript{2} supplementation is easily implemented in an experimental setting and provides an approximate cell width reduction of 40\% at stoichiometric conditions. The reduction is consistent at rich conditions (47\% reduction at $\phi = 1.7$), and considerably more pronounced at lean conditions (75\% reduction at $\phi = 0.5$). Oxygen enrichment, even in modest quantities, yields a dramatic performance benefit for the entire range of equivalence ratio.

![Diagram](image)

**Figure 9.9:** Cell width variation with equivalence ratio for a range of nitrogen dilution ratios

### 9.6 Effect of Hydrogen Substitution

The effect of H\textsubscript{2} substitution on the detonability of C\textsubscript{2}H\textsubscript{4}-(H\textsubscript{2})-air mixtures is explored for stoichiometric mixtures at baseline conditions ($P_i = 1$ bar, $T_i = 300$K) in Figure 9.10. Varying fractions of C\textsubscript{2}H\textsubscript{4} are replaced by a stoichiometrically-equivalent quantity of H\textsubscript{2} to simulate the use of hydrogen as a fuel additive. For reference, a stoichiometric fraction of 50\% is composed of C\textsubscript{2}H\textsubscript{4} + 6H\textsubscript{2}, as one mole of ethylene is the stoichiometric equivalent of six moles of hydrogen. For comparison to the numerical results of Lu et al. (2003), the induction length, rather than cell width, is plotted against the H\textsubscript{2} fraction by stoichiometry.
Figure 9.10: Induction length comparison with the results of Lu et al. (2003)

The computed induction length for the four current mechanisms is plotted against the two mechanisms employed by Lu et al. for comparison. All four mechanisms predict significantly smaller induction length for the baseline, 0% H₂ case (i.e. C₂H₄-air mixture) than the two mechanisms used previously, by a factor of two or more. This discrepancy may be attributed to the optimization used in the development of the mechanisms of Qin et al. (2000) and Marinov et al. (1996). While components of the Qin et al. mechanism were originally developed for smaller hydrocarbon oxidation (C₃), the optimization of the entire mechanism for propane oxidation required the modification of C₃ rate parameters. Optimization targets did not include shock tube ignition delays for C₂H₄, and a detailed analysis of the mechanism accuracy for C₂H₄ oxidation was not conducted. Likewise, the Marinov et al. mechanism was developed to model rich, sooting, premixed ethane and methane combustion, and was validated with atmospheric pressure combustion experiments. The mechanism is not optimized for stoichiometric C₂H₄ oxidation and its validity may not be preserved for higher pressures and stoichiometric conditions. Using the four mechanisms employed in the present study, the benefit of H₂ substitution is less pronounced than previously reported.
Cell width predictions for the four mechanisms are provided for the same range of stoichiometric \( \text{C}_2\text{H}_4-(\text{H}_2) \)-air mixtures (Figure 9.11). While there is considerable disparity in magnitude for the \( \text{C}_2\text{H}_4 \)-air mixture, all mechanisms converge to approximately the same value for the \( \text{H}_2 \)-air mixture, due to similarities in the underlying \( \text{H}_2 \) oxidation submechanisms. Consistent with the previous computations, USC Mech II predicts larger cell widths than the other mechanisms, implying a greater overall cell width reduction for \( \text{H}_2 \) substitution in the fuel. Relative to the mechanisms employed by Lu et al., where over half the overall reduction is achieved by 10% \( \text{H}_2 \) substitution, the reduction for USC Mech II is far more gradual. GRI 3.0 yields a sharp initial reduction for small \( \text{H}_2 \) concentrations, but is less sensitive to \( \text{H}_2 \) substitution beyond 10% \( \text{H}_2 \). AramcoMech 1.3 and San Diego Mech predict comparable monotonic reductions, culminating in an overall reduction of approximately 50% as the fuel composition approaches pure \( \text{H}_2 \).

![Graph](image)

**Figure 9.11: Cell width comparison of \( \text{C}_2\text{H}_4-\text{H}_2 \)-air mixtures (\( \phi = 1 \), \( P_i = 1 \) bar, \( T_i = 300 \) K)**

The effect of \( \text{H}_2 \) substitution is explored over a range of equivalence ratios in Figure 9.12 using San Diego Mech at the baseline pressure and temperature (\( P_i = 1 \) bar, \( T_i = 300 \)K). An increase in cell width is
observed for moderate amounts of H\textsubscript{2} substitution at the lean condition (\(\phi = 0.5\)), contrary to the improvement for stoichiometric conditions. While the effect of H\textsubscript{2} substitution is unclear for \(\phi > 1.7\) due to the thermicity peak switching phenomenon in the baseline case, 25\% H\textsubscript{2} substitution slightly increases the cell width at \(\phi = 1.5\). However, this aberrant, peak switching behavior is suppressed with H\textsubscript{2} addition and is not observed for any of the cases containing at least 25\% H\textsubscript{2} by stoichiometry. There is a gradual shift in the equivalence ratio for which the cell width reaches a minimum (\(\phi = 1.3\) for 0\% H\textsubscript{2} vs. \(\phi = 1.1\) for 100\% H\textsubscript{2}), further limiting the benefit for rich conditions. Detonability improvement is more consistent for mildly lean (\(\phi = 0.75\)) and stoichiometric conditions, where 50-60\% reductions in cell width are observed.

![Cell width variation with equivalence ratio of C\textsubscript{2}H\textsubscript{4}-H\textsubscript{2}-air mixtures](image)

**Figure 9.12: Cell width variation with equivalence ratio of C\textsubscript{2}H\textsubscript{4}-H\textsubscript{2}-air mixtures**

Hydrogen detonability exhibits unique and complex pressure dependence, as reported by Westbrook and Urtiliew (1982), and is attributed to the competition between a chain branching reaction and a pressure-dependent recombination reaction. Lu et al. (2003) explore this effect for a range of equivalence ratios (\(\phi = 0.5-4.0\)) and report non-monotonic trends in the induction length, which reaches
a pressure-dependent minimum for each equivalence ratio for $P_i = 0-3.5$ atm. As this behavior is observed for H$_2$-air mixtures, it is possible that similar pressure dependence may manifest for C$_2$H$_4$-H$_2$-air mixtures, compromising the cell width reductions observed at the baseline pressure ($P_i = 1$ bar). The effect of initial pressure on the cell width of C$_2$H$_4$-H$_2$-air mixtures at the baseline temperature ($T_i = 300K$) is explored for three equivalence ratios using the San Diego Mech (Figure 9.13).

While significant cell width reductions are achieved for the lean condition ($\phi = 0.75$) at the baseline pressure, there is a well-defined crossover point in the trend for $P_i = 2.9$ bar. Above this threshold, hydrogen addition to the fuel reduces the cell width (i.e. H$_2$ behaves as a detonation inhibitor). Cell width monotonically decreases with increasing initial pressure up to 25% H$_2$ by stoichiometry, and for greater H$_2$ concentrations, local minima and maxima appear. For the stoichiometric condition ($\phi = 1.0$), this inhibiting behavior is less pronounced, and the crossover point shifts to $P_i \approx 4.5$ bar. Mild amounts of H$_2$ addition (25% H$_2$ by stoich.) only weakly increase cell width above the crossover pressure, and monotonic reduction of cell width with pressure extends up to 50% H$_2$ by stoichiometry. The rich condition ($\phi = 1.5$) exhibits a slightly different trend, as the smallest amount of H$_2$ addition (25%) has a mildly inhibiting effect for $P_i > 1.4$ bar. A moderate benefit is observed at the baseline pressure for greater amounts of H$_2$ addition, but the crossover point occurs at different pressures for each mixture (i.e. 50% H$_2$ acts as an inhibitor for $P_i > 2.6$ bar, while 75% H$_2$ acts as an inhibitor for $P_i > 3.5$ bar). The cell width increase observed for H$_2$ addition at the highest initial pressure ($P_i = 10$ bar) is comparable in percentage to the increase for the lean condition ($\phi = 0.75$). Thus, the benefit of H$_2$ addition observed for the baseline pressure does not successfully extend to higher pressures. Any H$_2$ concentration sufficient to yield significant cell width reductions at $P_i = 1$ bar exhibits inhibiting behavior at elevated pressures, limiting the utility of this approach.
Figure 9.13: Cell width variation with equivalence ratio of C$_2$H$_4$-H$_2$-air mixtures
9.7 Conclusion

Four chemical kinetic mechanisms are used in conjunction with two empirical modeling approaches to determine the effect of various parameters on the detonation cell width of C<sub>2</sub>H<sub>4</sub>-oxidizer mixtures. The cell width (λ) is used as a metric for the detonation sensitivity of these mixtures, where reduction in λ denotes increased sensitivity. Of the four kinetic mechanisms, GRI 3.0 yields the worst match to the available C<sub>2</sub>H<sub>4</sub>-air shock tube data. USC Mech II generally overestimates ignition delay times, while San Diego Mech and AramcoMech 1.3 underestimate ignition delay times. Applying these mechanisms in tandem with the empirical models, the Gavrikov et al. approach greatly underestimates cell width for C<sub>2</sub>H<sub>4</sub>-air mixtures, while the Ng approach yields more reasonable estimates. Comparing the computational estimates to experimental data at baseline pressure and temperature (P<sub>i</sub> = 1 bar, T<sub>i</sub> = 300K), San Diego Mech and AramcoMech 1.3 yield an adequate match for lean to stoichiometric conditions (φ = 0.5-1.0), while USC Mech II gives the best agreement at rich conditions (φ = 1.0-2.0). However, over a wider range of initial pressures and temperatures, San Diego Mech and AramcoMech 1.3 provide a better match to the experimental data, while USC Mech II tends to overestimate cell width.

Elevated initial pressures greatly reduce cell width for the entire range of equivalence ratios explored in this study, though the benefit is weaker for lean conditions (φ < 1.0). A tenfold increase in initial pressure yields approximately an 80% reduction in cell width for a stoichiometric C<sub>2</sub>H<sub>4</sub>-air mixture. Elevated initial temperatures provide little to no improvement at stoichiometric conditions, but offer some benefit for lean and relatively rich (φ ≥ 1.5) mixtures. Reducing the nitrogen content in the oxidizer (i.e., enriching the air with O<sub>2</sub>) significantly reduces cell width for all equivalence ratios, even for relatively small quantity of O<sub>2</sub> (40% reduction at stoichiometric conditions for β = 3). Introducing H<sub>2</sub> as an additive to the fuel yields a moderate reduction in cell width for φ = 1.0, though the improvement is not as substantial as previously reported in the literature. The benefit is somewhat weaker for lean and
rich conditions, and a cell width increase is observed for small amounts of H\textsubscript{2} substitution (25\% by stoichiometry) for lean (\(\phi < 0.6\)) and rich (\(\phi > 1.5\)) conditions. Furthermore, H\textsubscript{2} acts as a detonation inhibitor and significantly increases cell width at elevated initial pressures.

In summary, pressurizing the reactants to elevated initial pressures, and addition of supplemental oxygen are highly effective methods of enhancing the detonation sensitivity. Heating the reactants to elevated initial temperatures is a relatively ineffective detonability enhancement strategy, and provides only modest enhancement for C\textsubscript{2}H\textsubscript{4}-air mixtures. Substituting a portion of the fuel with H\textsubscript{2} provides a moderate benefit, but requires relatively large H\textsubscript{2} concentrations, and is not effective at elevated pressures.
Chapter 10 – On the Existence and Multiplicity of Rotating Detonations

While continuous detonation propagation has been achieved for a number of mixtures for varying combustor geometries (Bykovskii and Zhdan 2015), there is an incomplete understanding of which mixtures and geometries will fail to support detonation. Also, while the existence of multiple detonation waves within an RDC has been experimentally observed many times (Voitsekhovskii 1959, Lu and Braun 2014, Wolański 2013, Frolov et al. 2015), it is unclear if the number of waves can be accurately predicted a priori (Anand et al. 2015b). This study explores the relationship between wave height, channel width, and cell width by varying the channel width of the combustor and the composition of the oxidizer. While maintaining a constant oxidizer flow rate and equivalence ratio, the cell width of the mixture is altered by varying the nitrogen dilution ratio ($\beta = N_2:O_2$) of the oxidizer. This allows the cell width of the reactant mixture to be varied independently, while maintaining an approximately constant injection pressure ratio, mass flux, and mixing conditions. By employing four channel widths, the ratio of fill height to channel width can be altered independently of the cell width. This study examines the role of cell width on RDC operation, including its relationship to the combustor geometric parameters, detonation propagation limits, and number of waves.

10.1 Experimental Approach

The present investigation utilizes the baseline reactant injection scheme with fixed oxidizer and fuel injection area as listed in Table 2.2. Fuel is injected axially through a uniformly distributed series of circular orifices. These orifices are located near the oxidizer injection slot to produce high velocity crossflow mixing of the fuel and oxidizer. A comparable injection scheme has been characterized with acetone PLIF (Rankin et al. 2016), revealing uniform reactant mixing within several millimeters of the base of the combustor. The combustor accommodates adjustments to $w_{ch}$ by varying the inner diameter with a fixed outer diameter for a total of four channel widths. While Bykovskii et al. (2006a)
note that variation of $w_{ch}$ affects mixing to a certain extent, this may be less applicable to the present
injection scheme, as only the inner diameter of the combustor is varied. As the fuel and oxidizer are
injected at choked velocities at the combustor outer diameter, there should be little to no effect on the
mixing process.

For this investigation, the RDC operates on stoichiometric ($\phi = 1$) $\text{H}_2$-$\text{O}_2$-$\text{N}_2$ mixtures. The oxidizer is
administered to the experiment through two supply lines, which deliver air and $\text{O}_2$. The two lines mix at
a junction 2m upstream of the combustor, resulting in oxygen-enriched air. The nitrogen dilution ratio
($\beta = \text{N}_2:\text{O}_2$) varies from $\beta = 3.76$ (air) to $\beta = 2$, as the air supply is substituted with increasing fractions of
$\text{O}_2$. For this study, the total oxidizer flow rate remains fixed, and the fuel flow rate is varied to maintain
a constant equivalence ratio as the oxygen concentration increases.

<table>
<thead>
<tr>
<th>Table 10.1: Overview of test parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry</strong></td>
</tr>
<tr>
<td>Outer Diameter</td>
</tr>
<tr>
<td>Channel Width</td>
</tr>
<tr>
<td>Channel Length</td>
</tr>
<tr>
<td>Oxidizer Injection Area</td>
</tr>
<tr>
<td>Fuel Injection Area</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reactants ($2\text{H}_2+\text{O}_2+\beta\text{N}_2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{\text{oxidizer}}$</td>
</tr>
<tr>
<td>$\phi$</td>
</tr>
<tr>
<td>$\beta$</td>
</tr>
</tbody>
</table>

Prior to and following each test, the air portion of the oxidizer flows continuously. The $\text{O}_2$ portion of the
oxidizer and $\text{H}_2$ are pneumatically valved upstream of the combustor and are supplied 2s prior to RDC
ignition to allow flow rates to stabilize. Ignition is achieved with a tangentially-injecting pre-detonator
tube containing a $\text{C}_2\text{H}_4$-$\text{O}_2$ mixture ($\phi = 2$), which enters the combustor approximately 22 mm axially
downstream of the base of the combustor (Figure 10.1). The test length is fixed at 1s, and concludes with the termination of the fuel and oxygen supply.

To capture the passage of detonation at the combustor inlet, three PCB pressure transducers are installed within the oxidizer injection slot at three azimuthal locations (Table 10.2). A set of three ionization probes are flush-mounted within the first instrumentation row to detect detonation passage in the channel. A total of six PCB pressure transducers are placed in the exit plane of the combustor (approximately 150mm downstream of the base of the RDC channel), facing radially inward. Large pressure fluctuations are observed in numerical simulations of the RDC exhaust plane as reported by Schwer and Kailasanath (2012). Transducers located in the RDC exit plane can recover the detonation-induced unsteadiness and detect the rotational frequency. To avoid damage due to excessive heating, these exit plane transducers are recessed from the outer wall of the combustor by ≈25mm. All instrumentation is sampled at an acquisition rate of 1 MHz.
Table 10.2: Instrumentation Overview

<table>
<thead>
<tr>
<th>Sensor Type</th>
<th>Installation Type</th>
<th>Location</th>
<th>θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ionization Probe</td>
<td>Flush-Mounted</td>
<td>1</td>
<td>60°, 180°, 300°</td>
</tr>
<tr>
<td>PCB Transducer</td>
<td>Oxidizer Injection Slot</td>
<td>n/a</td>
<td>40°, 160°, 280°</td>
</tr>
<tr>
<td>PCB Transducer</td>
<td>Exit Plane</td>
<td>n/a</td>
<td>15°, 60°, 150°, 195°, 240°, 330°</td>
</tr>
</tbody>
</table>

Detonation cell width of each mixture is estimated by using an empirical fit to existing experimental data (Figure 10.2). The effect of nitrogen dilution on cell width for stoichiometric H₂-O₂-N₂ mixtures at ambient initial temperature and pressure (1 bar, 288 K) has been thoroughly documented (Liu et al. 1984, Knystautas et al. 1982, Vasil’ev 1998). The data are modeled over the range 0 ≤ β ≤ 3.76 by expressing log₁₀(λ) as a second order polynomial of β (Eq. 10.1). The dashed lines represent ±25% bounds of the model, which enclose the experimental data.

\[
\log_{10}(\lambda) = -0.048 \cdot \beta^2 + 0.46 \cdot \beta + 0.047
\]  

\[
2H_2 + O_2 + \beta N_2
\]

Figure 10.2: Polynomial fit of experimentally measured cell width for H₂-O₂-N₂ mixtures

\[
\log_{10} \lambda = -0.048 \cdot \beta^2 + 0.46 \cdot \beta + 0.047 \tag{10.1}
\]
10.2 Determination of Detonation Regime

Continuous rotating detonation is achieved for each of the four channel widths, though the two smallest channel widths \( w_{ch} = 3.8 \text{ mm} \) and \( 5.7 \text{ mm} \) exhibit failures for higher values of \( \beta \). Rotation is verified from the distinct phase lag between pressure peaks between the azimuthally-distributed exit plane transducers (Figure 10.3). As the depicted sensors are within a 90° arc of the combustor circumference, the pressure peaks become increasingly clustered together as the number of waves within the channel increases.

![Figure 10.3: Detail of exit plane pressure evolution for selected cases](image)

For \( \beta > 2.5 \), the smallest channel width \( w_{ch} = 3.8 \text{ mm} \) exhibits no signs of rotation following initiation, and a turbulent flame anchors itself external to the combustor exit. For \( w_{ch} = 5.7 \text{ mm} \), at \( \beta > 3 \) chaotic pressure peaks are observed briefly after ignition, and aperiodic combustion persists within the channel for 20-50 ms. This unstable combustion decays, is ejected from the combustor, and stabilizes as an external turbulent flame.

To determine the number of waves \( (n) \) within the combustor, the maximum frequency for one or more waves is defined as an integer multiple of the ideal Chapman-Jouguet speed divided by the outer circumference of the combustor (Eq. 10.2). The elapsed time between successive waves in the pressure
evolution is used to calculate the effective rotation frequency \( f \). Rotating detonations typically propagate at velocities less than the ideal Chapman-Jouguet detonation speed (Bykovskii et al. 2006a), and excessively high rotation frequency \( f > f_{\text{max}} \) i.e., \( W > W_{\text{ CJ}} \) indicates the presence of multiple waves within the combustor \( n > 1 \).

\[
 f_{\text{max}} = \frac{n \cdot W_{\text{ CJ}}}{\pi \cdot d_{\text{outer}}} \tag{10.2}
\]

These frequency bounds allow the operating regimes to be neatly partitioned by the number of waves (Figure 10.4). The detonation wave speed is then calculated as the product of the operating frequency and combustor outer circumference divided by the number of waves.

Figure 10.4: Determination of number of waves from maximum ideal rotation frequency

The operating range of the combustor is depicted for nitrogen dilution ratios ranging from \( 2 \leq \beta \leq 3.76 \) for the four channel widths. As evident from Figure 10.5, the number of waves increases as \( \beta \) is reduced, though there is also a dependence on channel width. While \( w_{\text{ ch}} = 3.8 \text{ mm} \) only supports \( n = 3 \), the largest channel width \( (w_{\text{ ch}} = 13.1 \text{ mm}) \) remains in the \( n = 1 \) regime for most of the operating range.
It should be noted that the operating range given in Figure 10.5 represents the “dominant” mode, as cases near the boundary of \( n = 1 \leftrightarrow 2 \) and \( n = 2 \leftrightarrow 3 \) can stabilize to two distinct values of \( n \). For these transitory cases, the less dominant secondary mode typically comprises less than 10% of the test length.

Figure 10.5: Detonation regimes, partitioned by number of waves

### 10.3 Analysis of Detonation Wave Speed

Detonation wave speeds for the four channel widths are normalized by the ideal CJ speed over the operating range and provided in Figure 10.6. Black-filled shapes denote \( n = 1 \), gray-filled shapes denote \( n = 2 \), and hollow shapes denote \( n = 3 \). For the smallest channel width (\( w_{\text{ch}} = 3.8 \) mm), \( n = 2 \) is briefly observed as a secondary mode, which rapidly transitions to the dominant \( n = 3 \) mode. This transition, which increases the total number of waves, is accompanied by a loss in propagation speed (\( W/W_{\text{CJ}} = 0.9 \rightarrow 0.85 \)). A similar, but reversed transition is observed for \( w_{\text{ch}} = 7.6 \) mm, where the secondary mode \( n = 2 \) occurs briefly at the beginning of the test and is followed by the dominant, \( n = 1 \) mode, yielding an increase in propagation speed (\( W/W_{\text{CJ}} = 0.69 \rightarrow 0.94 \)). The abrupt change in detonation wave velocity which accompanies a change in \( n \) may be due to the dependence of wave speed on detonation wave
height. For constant reactant flow rate, as the number of detonation fronts increases, the height of each individual front must decrease. If the waves propagate at a reduced velocity, the effective reactant fill time (and thus the detonation height) will increase as there will be greater elapsed time between successive waves. If each detonation is constrained by a minimum allowable height, this reduction in velocity may increase the stability of the detonation. The multiple detonation fronts may then achieve an equilibrium which simultaneously optimizes the size and propagation velocity of each of the fronts.

![Graphs showing relationship between W/W_CJ and β for different w_ch values](image)

**Figure 10.6: Detonation wave speed normalized to ideal Chapman-Jouguet value**

For a fixed number of waves (n = constant), the normalized speed increases as β decreases. For a fixed value of β, the normalized speed decreases as n increases. There is also a complex dependence on channel width, as w_ch = 13.1 mm exhibits lower normalized speeds than the other channel widths for comparable n. This increased deficit relative to the ideal CJ velocity may be related to a change in the detonation structure for increased w_ch as observed numerically by Zhou and Wang (2013). For a large
channel width \((w_{ch} = 38\text{mm})\), Bykovskii and Zhdan (2015) report degeneration of detonation waves and the appearance of a shock wave within the combustor, which decelerates the products.

### 10.4 Analysis of Geometric Scaling Parameters

As the appearance of multiple waves \((n > 1)\) within the combustor is typically associated with reaching a threshold height of reactant mixture, the wave height is calculated for each of the cases. Using conservation of mass and the assumption that all reactants are consumed by the detonation yields Eq. 10.3, which can be readily rearranged to solve for \(h\) (Eq. 10.4). The total mass flow rate \(\dot{m}\) is known from flow metering of fuel, air, and oxygen, while the wave speed \(W\) is calculated from the operating frequency. Density in the fill region is calculated using ideal gas law and assuming expansion to atmospheric pressure in the channel (1 bar). This wave height represents the total height assuming a single detonation wave \((n = 1)\), and can be divided by the number of waves to evaluate the height of an individual detonation wave for the case that \(n > 1\).

\[
\dot{m} = \rho \cdot v \cdot A = \rho_{fill} \cdot W \cdot \left(h \cdot w_{ch}\right)
\]

\[
h_{total} = \frac{\dot{m}}{\rho_{fill} \cdot W \cdot w_{ch}}
\]

In the study of Bykovskii et al. (2006b), the evolution of \(n\) is discussed primarily in terms of the mass flux of reactants through the combustor for simultaneous variation of the equivalence ratio. However, in the present study, the total mass flow rate is approximately constant, such that the mass flux is nearly fixed for a given channel width, and the equivalence ratio is constant. While the detonation wave speed increases moderately with decreasing \(\beta\), this effect is partially offset by the reduction of density with \(\beta\) over this range \((\approx 12\% \text{ decrease})\). This yields a weak dependence of total wave height with respect to \(\beta\) for a fixed channel width. However, a reduction in \(\beta\) from 3.76→2 yields \(\approx 50\% \text{ reduction}\) in the cell width \((\lambda)\), such that \(h/\lambda\) changes appreciably with \(\beta\). As there is considerable variation in \(n\) over the
operating range, it can be inferred that $\lambda$ is the principal parameter governing the operating mode.

The normalized total wave height ($h/\lambda$) is plotted against the normalized channel width ($w_{ch}/\lambda$) in Figure 10.7 for all cases. Successful operation occurs for $w_{ch}/\lambda \geq 0.5$, which agrees with the limit proposed by Vasil’ev (1987) and the failure regime in Figure 10.5 falls below this threshold. For the present study, all cases have $h/\lambda \geq 1.6$, and there is not a distinct failure regime due to insufficient $h/\lambda$.

For the largest channel width ($w_{ch} = 13.1$ mm), a transition from $n = 1 \rightarrow 2$ occurs at $h/\lambda \approx 2$, followed by a transition from $n = 2 \rightarrow 3$ at $h/\lambda \approx 3$. While this implies that the critical wave height is approximately one cell width, this trend does not hold for the other channel widths. The second largest channel width ($w_{ch} = 7.6$ mm) transitions from $n = 1 \rightarrow 2$ at $h/\lambda \approx 3$, indicating a significantly larger (+50%) critical wave height. Furthermore, despite reaching $h/\lambda \approx 4.5$, there is no transition beyond $n = 2$. The smallest channel width does not transition beyond $n = 3$, despite reaching a normalized wave height of $h/\lambda \approx 9.6$. Considering this complex behavior, transitions in $n$ cannot be fully described in terms of a threshold value of $h/\lambda$, but are clearly influenced by both $h/\lambda$ and $w_{ch}/\lambda$.

Figure 10.7: Detonation regimes demarcated by lines of maximum normalized detonation perimeter

For the largest channel width ($w_{ch} = 13.1$ mm), a transition from $n = 1 \rightarrow 2$ occurs at $h/\lambda \approx 2$, followed by a transition from $n = 2 \rightarrow 3$ at $h/\lambda \approx 3$. While this implies that the critical wave height is approximately one cell width, this trend does not hold for the other channel widths. The second largest channel width ($w_{ch} = 7.6$ mm) transitions from $n = 1 \rightarrow 2$ at $h/\lambda \approx 3$, indicating a significantly larger (+50%) critical wave height. Furthermore, despite reaching $h/\lambda \approx 4.5$, there is no transition beyond $n = 2$. The smallest channel width does not transition beyond $n = 3$, despite reaching a normalized wave height of $h/\lambda \approx 9.6$. Considering this complex behavior, transitions in $n$ cannot be fully described in terms of a threshold value of $h/\lambda$, but are clearly influenced by both $h/\lambda$ and $w_{ch}/\lambda$.  

229
The regions of \( n = 1-3 \) in Figure 10.7 appear to be neatly partitioned by linear boundaries, indicating that a relationship for the transition is of the following form, where \( C_1 \) and \( C_2 \) are constants:

\[
\frac{h}{\lambda} = C_1 \frac{w_{ch}}{\lambda} + C_2
\] (10.5)

Fortuitously, the line demarcating the \( n = 1 \rightarrow 2 \) boundary has a slope of -1, and the relation can be rearranged, revealing that this line denotes a constant value of the normalized perimeter of the individual detonation wave, assuming it to be a rectangle \((p/\lambda)\):

\[
\frac{h}{\lambda} + \frac{w_{ch}}{\lambda} = C_2 = \frac{1}{2} \frac{p}{\lambda}
\] (10.6)

The intercept \((C_2)\) can be fitted to the experimentally-observed division between the \( n = 1 \rightarrow 2 \) boundary, yielding \( p/\lambda = 7.4 \). Thus, there exists a critical value of the normalized perimeter \( p/\lambda = 7.4 \), and above this threshold, the detonation splits into two \( (n \rightarrow 2) \), such that the normalized perimeter satisfies \( p/\lambda < 7.4 \) for each of the two resultant waves. As \( \beta \) (and therefore \( \lambda \)) is progressively reduced, the normalized perimeter of each of the two waves approaches the threshold. The slope and intercept of the second boundary line are two times that of the first, such that it represents the threshold where both the detonation waves \( (n = 2) \) reach the limit of \( p/\lambda = 7.4 \) and trigger a transition to \( n = 3 \). The momentary appearance of secondary modes, correspond to cases where the normalized detonation wave perimeter exceeds the critical value. These cases promptly transition to a primary, dominant mode satisfying \( p/\lambda \leq 7.4 \), indicating that this is the most stable state of operation.

**10.5 Conclusion**

The operating behavior of a rotating detonation combustor is explored for stoichiometric \( \text{H}_2\text{-O}_2\text{-N}_2 \) mixtures for four channel widths to determine the role of cell width and combustor geometry on the
existence of detonation and number of waves. The ratio of nitrogen dilution ratio \((\beta = N_2:O_2)\) of the oxidizer is varied from \(\beta = 3.76\) (air) to \(\beta = 2\), yielding a factor of two reduction in the cell width. The regime of successful detonation is bounded by a minimum normalized channel width of \(w_{ch}/\lambda > 0.5\), in close agreement with the classical limit for detonation propagation in a rectangular channel. While in the present study, a distinct critical value is only observed for \(w_{ch}/\lambda\), it is reasonable to assume that this limit extends to the normalized wave height (i.e. \(h/\lambda > 0.5\)). Implementation of significantly larger \(w_{ch}\), or reduction of the reactant flow rate may elucidate the limiting behavior of \(h/\lambda\).

An increase in the number of detonation waves within the combustor can be predicted by a critical value of the normalized perimeter of the detonation wave \((p/\lambda > 7.4)\). For the range of channel widths explored in this study, a single critical value of \(h/\lambda\) is insufficient to predict the number of waves. Considering previous studies utilizing a single channel width, the perimeter scales almost proportionally to the wave height when \(h >> w_{ch}\), so the effect of the channel width on the number of waves may have been overlooked. While the present data are well-modeled by this approach, it is interesting to consider the case of a very large normalized channel width \((w_{ch} >> \lambda)\). For this case, the normalized perimeter will exceed the proposed critical value, even for limiting values of wave height \((h/\lambda \approx 0.5)\) that cannot be subdivided further to form new waves. Given the reactant injection scheme employed in the present combustor, it may be possible to support detonation waves on the combustor outer diameter which do not fully extend across the entirety of the channel.

From the present work, it appears viable to predict the existence and multiplicity of rotating detonations from the combustor geometry and mixture cell width. Investigation of additional mixtures (for which cell width data is readily available) is warranted to determine whether these trends extend universally.
Chapter 11 – Conclusions and Recommendations

11.1 PDC-Turbine Investigations

As explored in the pulsed air flow investigation, pulsed flow efficiencies must incorporate flow-weighted averaging to have physical significance, as time-averaging does not properly account for the availability within the flow. This poses a problem for efficiency estimates calculated directly from experimental measurements when these quantities are difficult to capture (i.e., detonating flow). Pulsed flow contributes a loss to performance, but this loss can be effectively mitigated for cold flow pulsations by increasing the flow rate (and thus, the duty cycle or pulse active period). There is a tradeoff between increasing the rotor speed (which yields greater work extraction as evident from Eq. 4.12) and avoiding unfavorable incidence losses when the stator exit velocity is low.

These factors are somewhat applicable to pulsed detonating flow, though several unsteady effects are magnified: The excursions in thermodynamic quantities (e.g., pressure and temperature) are severe for detonating flow, and time-resolved mass fluxes at the turbine inlet and exit are difficult to estimate with the available measurement techniques without making several sweeping assumptions. This prevents the application of many of the flow-weighted efficiency definitions employed for the pulsed air investigation. Also, due to limits on the degree of partial filling of the combustors, the reactant flow is intrinsically tied to frequency, and increasing total flow rate typically requires increasing the firing frequency. While the duty cycle is increased at higher firing frequency, higher reactant flow rates generate stronger feedback between adjacent combustors. It is possible that this feedback can produce combustor misfires, and impose a practical limit on the firing frequency. The sequential firing pattern employed in this study has intrinsic limitations, as purge-braking inevitably occurs due to the necessity of the fill, detonation, and purge processes occurring simultaneously for different inlet sectors. There is a ruinous tradeoff between effective power extraction from detonating flow and reduction of power imparted on the purge flow. Significant reduction of the purge fraction may eliminate this issue, but will
compromise the turbine survivability, as the average turbine inlet temperature will likely exceed the
design temperature by a factor of two. A simultaneous firing pattern may be possible with alteration of
the valving, though the same purge-braking losses will be incurred without allowing large excursions in
rotor speed between each pulse.

Given the outcome of the present investigation, several future investigations are recommended:

- Develop improved diagnostics to capture flow quantities within the combustors and turbine,
  and improve measurements of turbine speed and torque.
- Characterize the effect of purge fraction on turbine performance for fixed fill fraction to provide
  a complete understanding of the losses imposed by purge-braking.
- Adopt of a turbine with significantly higher design inlet temperature to enable operation with
  lower purge fraction (may require significant redesign of the integration hardware).
- Utilize lean fuel-air mixtures with lower peak temperature to enable operation with lower purge
  fraction (may require larger combustor size due to cell size limitations).
- Operate the system at thermal equilibrium (which requires a reduction in turbine inlet
  temperature or an improvement in the turbine design temperature).
- Estimate heat transfer during starting transient and steady operation, and incorporation of the
  heat transfer into a diabatic efficiency definition for better comparison to an ideal cycle.

11.2 RDC Investigations

A correlation-based algorithm is developed to detect the operating state in a rotating detonation
combustor. It resolves wave speed and propagation direction, while simultaneously detecting several
instability types, such as mode switching, LPD (from phase-locking of signals), waxing and waning of
detonation peak pressures, and chaotic detonation propagation. The following areas merit additional
investigation:
• Develop and adopt of a global stability classification, using the quantitative output from the correlation algorithm. Many instabilities are almost always present to a certain degree (e.g., waxing and waning), or present for all cases prior to onset (e.g., chaotic instability). Boolean-type classification of an operating point as “unstable” may be misleading or shortsighted, compared to a quantitative stability rating based off the observed instability types.

• Improve the algorithm to explicitly identify and track the speed and direction of multiple waves. While this was considered to be unnecessarily complex for the present combustor conditions, scaled-up combustors will likely implement multiple wave operation.

• Scale a similar two-dimensional version of the algorithm to apply a similar analysis approach to high-speed video data. This will enable improved extraction and analysis of the wave structure, especially when this structure is complex (e.g., a hollow combustor).

RDC initiation with a tangentially-injecting initiator tube is explored in two phases. The energy deposition (in a non-reacting combustor flow) is characterized for a range of initiator mixtures and combustor conditions, and the resulting initiation event (in a reacting H₂-air combustor flow) is observed with optimized initiator tube settings. The optimized initiator tube does not provide sufficient energy deposition for direct initiation for fuel-air mixtures, and subcritical initiation is observed. While it is likely that subcritical initiation is typical for all fuel-air RDCs, it may be possible to excite direct initiation with a significantly larger initiator tube, or for chemically sensitized reactant mixtures. The following additional investigations are recommended:

• Characterize the leading shock trajectory and decay for an unconfined, three-dimensional expansion (using shadowgraph or Schlieren visualization techniques) to provide validation for the energy estimation method employed within the RDC.
• Extend this three-dimensional attenuation approach to characterize energy content of larger initiator tubes, or effect of varying initiator tube geometry (e.g., diameter, total volume, exit area). This may reveal additional scaling laws for energy deposition as a function of initiator tube geometry and improve future initiator tube design.

• However, as it appears that direct initiation is not a prerequisite for attaining detonation in an RDC, it may be interesting to determine the ignition limit of the combustor by using an extremely low energy ignition source (e.g., automotive spark plug).

A complex transition process is observed after the subcritical ignition event, which eventually culminates in rotating detonation. The duration of this onset phase appears to be a function of the reactant injection pressure ratio and is somewhat stochastic, varying by a factor of two for the same initial condition. While the duration of the onset phase varies, the combustor operating state during this phase is comparable between successive tests. The present investigation is limited to a single combustor configuration, and motivates the following additional investigations:

• Characterize the onset phase for multiple combustor configurations, varying the channel width and injection area (preferably with high spatial resolution of sensors in axial and azimuthal directions, or with high speed video). This should ideally include variation of the reactant injection area for comparable reactant flow rates to isolate and determine the effect of injection pressure ratio on the onset behavior.

• Determine the effect of the ignition process on the onset phase. It is possible that a strong ignition event may be more disruptive to the reactant injection and delay the onset of rotating detonation. Furthermore, it may be of interest to determine whether synchronized or delayed ignition at multiple spatial locations can induce rapid detonation onset.
Addition of supplementary H₂ (i.e., fuel blending) is employed to achieve successful development of detonation in C₂H₄-H₂-air mixtures. While stable operation is established for these fuel blends, it is only maintained by continuously supplying the additional H₂. Removal of the H₂ triggers the resurgence of bursting instability, and eventual failure of the unstable C₂H₄-air detonation. Chemical kinetic analysis of these mixtures indicates that elevated reactant pressure is far more significant than H₂ addition, and it is likely that the stabilizing effects observed in the present study are physical, rather than kinetic. A recent investigation by Wilhite et al. (2016) demonstrates successful (non-premixed) C₂H₄-air operation in a combustor with larger channel width (w_{ch} = 13.1 mm), and it appears that the configuration used in the present study is below the critical size required to support C₂H₄-air detonation. The following additional investigations may supplement the current understanding of the results:

- Revisit this investigation for a larger channel width (w_{ch} = 13.1 mm) to determine whether H₂ addition provides a stabilizing impact for geometry which already supports C₂H₄-air detonation.
- As premixing may be responsible for the bursting instability, it may be interesting to revisit this investigation with non-premixed reactants (for w_{ch} = 13.1 mm).
- As the difference in pre-ignition and terminal pressure may be responsible for the bursting instability, it may be interesting to explore detonation behavior for a spring-loaded, variable-area nozzle. This investigation should assess the efficacy of a variable-area nozzle in maintaining constant combustor pressure during a test, and then utilize this concept for C₂H₄-air mixtures.

Detonation scaling behavior is explored for a range of stoichiometric H₂-O₂-N₂ mixtures to determine the role of detonation cell size on rotating detonation existence and multiplicity. Detonation is supported within the combustor when the normalized channel width exceeds the classical limit of w_{ch}/λ > 0.5, and the number of detonations increases predictably when the detonation perimeter exceeds a critical value. However, the study is constrained to a single oxidizer flow rate (300 g/s), a single fuel (H₂), and a
single injection scheme, and does not explore the contribution of physical factors, such as mixing. The following future investigations are recommended:

- Explore combustor operation at larger channel widths, including hollow combustor geometry (no center body) to determine if the proposed scaling laws hold
- Extend combustor operation to lower $\beta$ by increasing the oxygen enrichment
- Revisit this investigation at additional oxidizer flow rates
- Revisit this investigation for a different fuel ($C_2H_4$)
- Revisit this investigation for a different injection scheme
- Conduct high speed visualization of the transition process in the number of waves to provide better understanding of the transition mechanism, and possibly identify why secondary modes occur temporarily within the combustor

11.3 **Closing Remarks**

Forecasting the future of PDCs, it appears that several fundamental impediments will ultimately thwart integration into a practical engine cycle. The high amplitude, low frequency unsteadiness of the traditional PDC is difficult to efficiently process with a turbine, and multiple out-of-phase combustors produces undesirable purge braking losses. Simultaneous firing arrangements may improve performance, but will undoubtedly result in larger excursions in rotor torque and speed. Implementation of an elastic shaft, or fluid/hydraulic coupling may damp out these fluctuations and prevent vibration-induced failure of the system. If the turbine is optimized for power extraction from detonating flow, the purge flow should be minimized or eliminated, though this will significantly increase thermal loading on the turbine. While the present work employs stoichiometric $C_2H_4$-air detonations, a significantly scaled-up combustor can operate for lean fuel-air mixtures, thereby reducing the temperature of the detonation products. Conversely, the purge flow may be retained if a suitable
geometry can be developed to partition hot and cold fluid streams and duct these streams through separate, optimized portions of the rotor (e.g., as in a Ranque-Hilsch vortex tube).

Even if these mechanical, fluidic, and thermal difficulties are resolved, the PDC concept is limited by poor scaling in several key areas: finite DDT length, finite fill, purge, and blowdown times (limits maximum frequency), valving complexity and weight, and excessive combustor surface area. Several strategies such as specialized obstacles, shock initiation, or shock-flame interactions could be implemented to reduce DDT length. Passive fluidic valving may circumvent some of the mechanical difficulties of high frequency operation and may enable faster fill and purge processes. If out-of-phase firing is cast aside in favor of sequential firing, combustor surface area can be reduced by implementing a single, large combustor. However, future investigators will likely conclude that a passively-valved, single combustor PDC arrangement devoid of DDT obstacles is essentially an RDC. Furthermore, it is interesting to consider the intentional implementation of the longitudinal pulsed detonation instability to produce high frequency PDC-like behavior with RDC geometry.

The RDC concept, though promising, is not without its own unique technical challenges, the most critical of which are: excessive inlet losses, adequate mixing of reactants and mitigation of reactant-product mixing, detonation stability, and extreme heat transfer. While detonation provides a net rise in pressure during the combustion process, choked reactant injection (as employed in the present study to improve detonation stability and reactant mixing) imposes overwhelming losses in total pressure and completely negates any thermodynamic benefit. Any viable future RDC concept must overcome this inlet loss problem without sacrificing mixing quality and detonation stability. Unique injection geometries, such as diode-style Tesla valves, or swirled injection offer significant prospects. However, injection geometry, reactant mixing, and detonation stability are intrinsically coupled. This highlights the most distressing facet of RDC research: detonation behavior is acutely sensitive to very subtle changes in combustor
geometry. Fortunately, the recent use of optical diagnostics in RDC experiments, coupled with high-fidelity LES simulations might shed light on some of the fundamental interactions unique to non-premixed, heterogeneous detonations in these geometries. While heat transfer rates are quite high, exhaust gas temperatures are comparable to that of PDCs with similar reactant mixtures. Scaling up the combustor geometry to successfully operate with lean fuel-air mixtures may reduce exhaust gas temperatures to reasonable levels. Alternatively, the injection geometry can be arranged to take advantage of faster recovery of air relative to fuel (i.e., “stratification”) to reduce the global equivalence ratio and hence, the average exhaust gas temperature.

In summary, there are many unanswered questions regarding the complex physical and chemical processes occurring within RDCs. While a myriad of RDC technical challenges remain under investigation, the number of active RDC research facilities (and the number of well-funded, enthusiastic investigators) is rapidly growing. Thus, it is the opinion of the author that these issues will soon be resolved and a significant thermodynamic benefit will be demonstrated for RDCs within a decade.
Publications

(Primary Author)

**Peer-Reviewed Journals (Published)**


**Peer-Reviewed Journals (Currently in Review)**


**Conference Papers**


240
(Secondary Author)

**Peer-Reviewed Journals**


**Conference Papers**


References


Fickett, W., and Davis, W.C., (1979) *Detonation*, University of California Press.


Oshima, K., (1960) “Blast waves produced by exploding wire,” Report No. 358, Aeronautical Research Institute, University of Tokyo, Tokyo, Japan.


detonation engines with subcritical and supercritical CO₂,” The 4th International Symposium –
Supercritical CO₂ Power Cycles, Pittsburgh, PA.

Rotating Detonation Engine Facility at the University of Cincinnati,” 53rd AIAA Aerospace Sciences

measurement techniques with application to detonation waves,” 53rd AIAA Aerospace Sciences


Engine Operation,” 50th AIAA Aerospace Sciences Meeting including the New Horizons Forum and

detonations with two successive reactions model,” Proceedings of the Combustion Institute, Vol. 33,
pp. 2227-2233.


Taylor, G., (1950a) “The formation of a blast wave by a very intense explosion. I. Theoretical discussion,”
Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences, Vol. 201,
No. 1065, pp. 159-174.

Taylor, G., (1950b) “The formation of a blast wave by a very intense explosion. II. The Atomic Explosion
of 1945,” Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences,
Vol. 201, No. 1065, pp. 175-186.

Tellefsen, J.R., (2012) Build up and operation of an axial turbine driven by a rotary detonation engine,
(Master’s Thesis) Air Force Institute of Technology, WPAFB, OH.

Theuerkauf, S.W. (2013) Heat exchanger design and testing for a 6-inch rotating detonation engine,
(Master’s Thesis) Air Force Institute of Technology, WPAFB, OH.

of simulated and measured instantaneous heat flux in a rotating detonation engine,” 54th AIAA

divergent channels,” In: Bowen et al. (Eds.) AIAA Progress in Astronautics and Aeronautics. AIAA, NY,

SAND85-1263, NUREG/CR-4905, Sandia National Laboratories, Albuquerque, NM.


Boston, MA.

University of California at San Diego, (2014) Chemical-kinetic mechanisms for combustion applications,
Mechanical and Aerospace Engineering (Combustion Research), University of California at San Diego


