I, Michael T Tolston, hereby submit this original work as part of the requirements for the degree of Doctor of Philosophy in Psychology.

It is entitled:
Evaluating the Multi-Scaled Characteristics of Rhythmic Movement

Student's name: Michael T Tolston

This work and its defense approved by:

Committee chair: Kevin Shockley, Ph.D.
Committee member: Michael Richardson, Ph.D.
Committee member: Michael Riley, Ph.D.
Evaluating the Multi-Scaled Characteristics of Rhythmic Movement

A dissertation submitted to the

Division of Graduate Education and Research
of the University of Cincinnati

in partial fulfillment of the
requirements for the degree of

Doctor of Philosophy

in the Department of Psychology
of the McMicken College of Arts and Sciences

by

Michael T. Tolston

M.A., University of Cincinnati
August 2012

Committee Chair: Kevin Shockley, Ph.D.
Committee: Michael A. Riley, Ph.D.

Michael J. Richardson, Ph. D.
Abstract

The objective of the present experiments was to evaluate the multi-scaled nature of human rhythmic movement with respect to simple dynamical systems and phase transitions. Experiment 1 evaluated the statistical nature of the underlying dynamics of human rhythmic movement in a golden-mean (i.e., quasiperiodic) coupling relationship with another oscillator. It was predicted that human rhythmic movement can be shown to belong to the same universality class as a sine-circle map. Participants synchronized their arm movement to a virtual quasiperiodic oscillator. Results showed that participants’ dynamics were of higher dimension than predicted, but within the range of a second order system coupled to two first order systems, and that scaling regions were wider than those predicted. These results indicate that the hypothesis was not supported under the established conditions, though local scaling was shown to have a positive relation with global dimensionality. The second experiment evaluated whether the fluctuations found in behavior during the acquisition of new cognitive structure extend to re-organizations of rhythmic behavior. A coupling was established between a virtual pendulum driven by an independent driver and the movement of each participant. Participants were tasked with coordinating with the pendulum system such that the peak amplitude of the pendulum stayed within a specified range. As the trial progressed, the coupling strength of the driver was increased in small increments to beyond the point where the combined strength of the participants’ movement and the driver was sufficient to push the pendulum out of the directed region. It was predicted that the multifractal analogue of a thermodynamic quantity would be sensitive to qualitative transitions in behavior, and that participants who underwent such transitions spontaneously, i.e., intrinsically, rather than under
direction, i.e., extrinsically, would exhibit critical fluctuations prior to undergoing qualitative changes in their movement. Results showed that participants switched to anti-phase behavior around the critical point, regardless of condition. The proposed multifractal measure was sensitive to the transition, though it did not show differences between groups. The standard deviation of the relative phase between the participants and the driver showed similar patterning to the multifractal measure, but decayed more slowly after the transition point in the intrinsic group than in the extrinsic group. This study succeeded in establishing a link between a measure with conceptual foundations in thermodynamic phase transitions and human behavior, further demonstrating the utility of multi-fractal analyses and corroborating the notion that human beings are softly-assembled, complex systems.
Acknowledgements

I would like to thank everyone involved with this project, from incubation to implementation, especially those who assisted with pilot data collection, including, but not limited to, Patrick Nelepka and Sam Breheim. I thank my peers at the University of Cincinnati for hours of productive conversation and endless support.

I thank my committee, Drs. Michael Riley and Michael Richardson, for an astonishing amount of encouragement and assistance across the spectrum of my graduate experience.

I would like to thank my chair, my advisor, my mentor, and my friend, Dr. Kevin Shockley. Without you, I would have never been where I am today.

Finally, my family, especially my wife, Kayla. For everything. Thank you.
# Table of Contents

List of Figures .............................................................................................................................. viii
List of Tables ................................................................................................................................. x

CHAPTER 1: INTRODUCTION ............................................................................................................. 1
  Phase Transitions and Criticality ................................................................................................. 3
  Multifractal Measurements ........................................................................................................ 8
  Thermodynamics and Multifractals ............................................................................................ 13

CHAPTER 2: General Method ......................................................................................................... 16
  Participants ............................................................................................................................... 16
  Apparatus .............................................................................................................................. 16
  Procedure ............................................................................................................................... 16
  Data Analyses ........................................................................................................................ 17

CHAPTER 3: ..................................................................................................................................... 21
  Experiment 1: Universality of the Circle Map ........................................................................... 21
    Quasiperiodic route to Chaos and the Sine-Circle Map ......................................................... 21
    Method .................................................................................................................................... 26
    Participants ............................................................................................................................ 26
    Apparatus ............................................................................................................................. 26
    Procedure .............................................................................................................................. 27
    Results and Discussion ......................................................................................................... 29

Chapter 4 ..................................................................................................................................... 34
  Experiment 2: Multifractal Analysis of Phase Transitions in Rhythmic Behavior ............... 34
    Method .................................................................................................................................... 38
    Participants ............................................................................................................................ 38
    Apparatus ............................................................................................................................. 38
    Procedure .............................................................................................................................. 40
    Design ..................................................................................................................................... 41
    Results and Discussion ......................................................................................................... 42
    AVR ....................................................................................................................................... 42
Change in AVR within windows ................................................................. 45
Relative Phase ......................................................................................... 46
Standard Deviation of Relative Phase ...................................................... 48
CHAPTER 6: .......................................................................................... 52
GENERAL DISCUSSION .......................................................................... 52
References .............................................................................................. 58
APPENDIX A: INSTRUCTIONS FOR EXPERIMENT 1 ......................... 65
List of Figures

Figure 1. A: Return map of a critically driven oscillator. Note the large cubic inflection around $\theta \sim .25$. For slightly stronger driving forces, the line breaks apart, and the fractal spectrum diverges.

Figure 2. A return map generated from strobing movement data at $\sim 4$ Hz obtained from a person oscillating at the characteristic frequency of the pendulum system at a maximum amplitude of (A) $\sim \pi/2$ or (B) $\sim 7\pi/12$. The increasingly sharp bend around $\theta \sim .45$ was taken as an indication that a cubic inflection (i.e., criticality) might be likely near $5\pi/8$ (the mediant of A and B).

Figure 3. Apparatus (freely spinning wheel with a freely spinning handle) used in experiment 1.

Figure 4. A: Mean multifractal spectra of all participants from Experiment 1, averaged over 180s windows within participants. B: The mean multifractal spectrum obtained by averaging spectra from all participants from Experiment 1. Error bars represent 95% confidence intervals. The box indicates the range of expected values for a critical sine-circle map ($\alpha_{\min} 0.6236$, $\alpha_{\max} 1.8980$, and fractal dimension, maximum $f(\alpha)$, of 1).

Figure 5. Scatter diagram showing the relationship between degree of multifractality (arange) and fractal dimension of rhythmic arm movements measured in experiment 1.

Figure 6. A: Average Change in Area Variation Rate (AVR) as a function of binned epochs. The critical coupling strength occurs in bin 4 (marked by a vertical dashed line). Epochs
one and fifteen were removed from analysis. Error bars represent ±1 standard error. B: Predicted Change in AVR from best fit growth model (centered data; all entered terms included).

Figure 7. A: Average relative phase between participant and driver as a function of binned epochs. The critical coupling strength occurs in bin 4 (marked by a vertical dashed line). Epochs one and fifteen were removed from analysis. Error bars represent ±1 standard error. B: Predicted relative phase from best fit growth model (centered data; all entered terms included).

Figure 8. A: Average standard deviation of relative phase between participant and driver as a function of binned epochs. The critical coupling strength occurs in bin 4 (marked by a vertical dashed line). Epochs one and fifteen were removed from analysis. Error bars represent ±1 standard error. B: Predicted standard deviation of relative phase from best fit growth model (centered data; all entered terms included).
List of Tables

Table 1. Estimated growth curve parameters for AVR.

Table 2. Estimated growth curve parameters for Difference of AVR within Binned Epochs.

Table 3. Estimated growth curve parameters for Mean Relative Phase.

Table 4. Estimated growth curve parameters for Standard Deviation of Relative Phase.
CHAPTER 1: INTRODUCTION

The capacity of individuals to generate rhythmic movement has been evaluated across a range of behaviors, including finger wagging, (Haken, Kelso, & Bunz, 1985) leg swinging (Schmidt, Carello, & Turvey, 1990), chair rocking (Richardson, Marsh, Isenhour, Goodman, & Schmidt, 2007), and pendulum swinging (Kugler & Turvey 1987). In each these studies, the rhythmic behavior under consideration was successfully characterized by the Haken-Kelso-Bunz (HKB) model, a model based on non-linear oscillators. Importantly, this model makes several predictions regarding coordinated action which are consistent with characteristic behaviors of self-organized, complex systems (including phase transitions – qualitative changes in behavior, critical fluctuations – an increase in the variability of an index of the organization of a system, and critical slowing down – the increase in time it takes for a system to recover from perturbation as it approaches a critical point; Haken et al., 1985).

In addition to the preceding observations, a wide range of phenomena associated with rhythmic arm movement, including stability of final conditions despite perturbations (Kelso, Holt, Rubin, & Kugler, 1981) and variations in initial conditions (Kelso, 1977), can be successfully captured by low-dimensional non-linear oscillator models (Beek & Beek, 1988; Kay, Kelso, Saltzman, & Schöner, 1987; Kay, 1988; Kelso, Holt, Kugler, & Turvey, 1980). For instance, Kay and colleagues developed a nonlinear hybrid oscillator model by combining the equations of motion of two well-known systems of equations, the van der Pol and Duffing oscillators. The authors found that this hybrid model accounted for a variety of observed phenomena, including the dynamics of point attractors, which describe behavioral dynamics characterized by the asymptotic approach to a unchanging
and stable equilibrium state (i.e., reaching), as well as periodic attractors, which describe behavioral dynamics characterized by a limit cycle, meaning that all trajectories within a region of attraction converge onto one recurrent orbit (i.e., hand waving). The model also made several theoretical predictions regarding relationships between frequency, amplitude, and movement velocity that were observed in experimental data.

The a priori predictions and subsequent observations of key phenomena across a wide range of experimental settings provides strong support to the claim that human rhythmic behavior is a self-organized, complex phenomenon (Kugler, Kelso, & Turvey, 1980). For instance, critical fluctuations occur when a self-organized complex system is about to undergo a spontaneous qualitative change, and are indicated by an increase in the variability of an order parameter, or key observable that captures system-level organization (Haken, 1975). Such behavior is indicated in coordinated rhythmic behavior by an increase of the variability of relative phase, or variation in the patterns of relative movements of coupled oscillators, and has been observed in a variety of circumstances (Carson, Goodman, Kelso, & Elliott, 1995; Kelso, Scholz, & Schöner, 1986; Schmidt et al., 1990). One advantage of such a self-organized, complex formalization of human behavior is that it relies on general physical principles to understand coordinated movement (Turvey & Shaw, 1999), a move which supports the prediction that observations of the dynamics of self-organization in relatively simple physical systems will generalize across otherwise disparate domains.

The objective of the current experiments was to evaluate the multi-scaled nature of human rhythmic movement. The central hypothesis was that human behavior is softly-
assembled, complex, and constrained by general physical principles. The first specific aim was to evaluate the statistical nature of the underlying dynamics of human rhythmic movement. I predicted that, under appropriate constraints, human rhythmic movement belongs to the same universality class as a simple, one dimensional system (i.e., a sine-circle map). The second specific aim was to evaluate the multi-scaled nature of the re-organization of rhythmic behavior. I predicted that a transition in the dynamics of behavior would spontaneously emerge under appropriate constraints, and would be evident in multi-fractal analyses. Additionally, I predicted that a tell-tale sign of self-organization, critical fluctuations, would presage reorganizations of behavior that happened spontaneously, but not those that that were extrinsically imposed.

**Phase Transitions and Criticality**

Self-organized behavior is characterized by the mutual constraint of many micro-elements in the formation of macroscopic collective variables that extend the spatial and temporal scales of the system beyond those of the individual components (Swenson & Turvey, 1991). A prototypical self-organized system is the Rayleigh-Bénard system, in which viscous fluid under a temperature gradient undergoes a critical phase transition from diffusive heat conduction to extended convection rolls, characterized by oscillatory behavior in which long-range correlations between the micro-elements reduce the functional dimensionality of the system. For example, Rayleigh-Bénard systems have been shown to be governed by dynamics that can be described by the sine-circle-map, a one-dimensional map that captures a variety of interesting phenomenon, including synchronization (e.g., mode-locking) and transitions from periodicity or quasi-periodicity to chaos (Procaccia, 1988). This indicates that the complex behavior of the essentially
infinite degrees of freedom of a dynamic system generally modeled with partial
differential equations can be captured by a very simple low-dimensional model.

Such general concepts may usefully be applied to a principled understanding of
biological organization (Kugler et al., 1980). For example, simple rules applied at a local
level of many interacting agents (i.e., local constraints) can elegantly account for beautiful
global patterns observable in schooling fish and swarming insects (Parrish, Viscido, &
Grünebaum, 2002; Reynolds, 1987). Additionally, the ability of termites to build complex
nests can be succinctly explained in terms of self-organized phase transitions that create
and destroy equilibrium points (Kugler, Shaw, Vincente, & Kinsella-Shaw, 1990). Such
processes are termed self-organized, in that they are spontaneously self-completing, or
autocatalytic (Kugler & Turvey, 1988; Kugler & Shaw, 1990; Ulanowicz, 2009). These
processes are inherent in the competition between energy sources and a dissipative sinks
that are key components of non-linear oscillators (Kugler et al., 1990).

A particular of phase transition, second-order transitions (those that result from
discontinuities of the second derivative in free energy—a key thermodynamic variable that
indexes the amount of usable energy in a system), might be easily found using the
appropriate measures, thanks to the infinite susceptibility of the order parameter around
the critical point, leading to fluctuations whose scaling exponents follow power-law
behavior (i.e., scale-free fluctuations; Kadanoff, 2009). When different phases are
possible, it is useful to distinguish between them using an order parameter. For instance,
the net alignment of magnetic spins defines the order parameter in the well-known Ising
model, while the mass density specifies the order parameter of a boiling liquid (Kadanoff,
In the one-dimensional Ising example, the order parameter jumps suddenly during a first order phase transition; that is, when phase boundaries are crossed at less than the critical temperature, the first derivative of the free energy (net-magnetization) diverges discontinuously and the system spontaneously aligns itself at the macroscopic level in one of the two allowable magnetic orientations (Yeomans, 1992). However, as the system approaches the critical temperature, the magnetic moment continuously increases, but the magnetic susceptibility (the second derivative of the free energy) diverges. In other words, at the critical point, small variations in external perturbations (or random thermal fluctuations) cause the system to spontaneously align in pockets of all sizes, leading to an infinite correlation length that is the hallmark of second order transitions (Wilson, 1979).

A characteristic of thermodynamic phase transitions is the universality of such scaling functions at the critical value of the control parameter, (i.e., an independent variable whose variation results in spontaneous reorganization of the system under consideration), meaning that statistical properties of scaling functions are independent of the particular physical system in which they are observed (Stanley, 1999).

In addition to simple mode-locking behavior predicted by circle-map dynamics, at a critical coupling strength and system frequency (i.e., a golden-mean winding number), there is a cubic inflection of the function underlying the dynamics of the circle-map (e.g., the map loses its monotonic form), resulting in a transition to chaos (Ecke, Mainieri, & Sullivan, 1991). This inflection causes the system to lose its invertibility, or one-one mapping, resulting in the system being disproportionately mapped to particular regions in its phase space. This uneven mapping generates a multifractal structure which has been shown to have a very specific spectrum of singularities, or distribution of local densities,
that characterizes the universality class of circle-maps, and has been observed in Rayleigh-Bénard systems (Ecke et al., 1991; Jensen, Kadanoff, Libchaber, Procaccia, & Stavans, 1985) and electrical circuits (Guerrero & Octavio, 1988).

The generality of the circle-map in explaining rhythmic behavior has been shown to extend to the domain of rhythmic human movement (Treffner & Turvey, 1993), where shifts between resonant frequencies in rhythmic movement has been shown to be likely to follow the most stable path predicted by the sine-circle map. As another example, the tessellation of timing associated with successful juggling can be characterized by the coordinated catch-and-release cycles of the juggler’s hands being constrained by phase locked regions, or Arnold Tongues, of a circle map (Beek, 1989; Turvey, 1990).

Importantly, rhythmic movement of the index finger has been shown to be characterized by a dimension of less than two (Kay, 1988), with one large-scale process (a limit cycle attractor) and a small scale process of non-integer dimension (Turvey, 1990). In evaluating the dimensionality of rhythmic finger movement, Kay utilized the correlation dimension (Grassberger & Procaccia, 2004), a method that evaluates the scaling of recurrent points in a reconstructed phase space (i.e., the multidimensional system in which all the possible configurations of the dynamical system are simultaneously defined) as a function of the distance that defines points as recurrent. However, the correlation dimension is just one out of a (theoretically infinite) number of possible generalized dimensions (Hentschel & Procaccia, 1983) that serve as the basis for generating a distribution of fractal measures to thoroughly characterize sets that have more than one scaling function (Halsey, Jensen, Kadanoff, Procaccia, & Shraiman, 1986).
Calculating generalized dimensions, and multifractal spectra, involves utilizing a variety of statistical moments to weight different densities (indexed by the number of recurrent points) of the phase space, which gives a spectrum of scaling exponents that characterize the intensity of changes in local density of the phase space, or singularities (\(\alpha\)), along with their distributions, \(f(\alpha)\). While the width of the multi-fractal spectrum has been used to quantify the degree of multi-fractality in a variety of experimental situations (Chappell & Scalo, 2001; Stephen & Dixon, 2011; Wu, Li, & Zou, 2015), utilizing the circle map, exact a priori predictions may be made regarding the multi-fractal spectrum of distributions in a phase space at a critical coupling strength under the golden-mean irrational winding number (Jensen et al., 1985). If such distributions are obtained under the appropriate experimental constraints, this would provide strong evidence that the dynamics underlying the observed rhythmic coordination belong to the circle-map universality class. As such, the aim of Experiment 1 is to determine whether the movement dynamics of rhythmic human movement under appropriate constraints belong to the same universality class as the circle map. Such a strong prediction of universality of critical parameters and scale invariant distributions of recurrent points amongst systems as diverse as electric circuits, cells of heated fluid, and human rhythmic behavior allows a deep test of the fundamental ideas behind the self-organized complex systems approach to rhythmic movement. However, such a test is notoriously difficult to conduct, as it relies on both the careful tuning of non-linear coupling parameter and control of the interaction of frequencies of the coupled systems (Fein, Heutmaker, & Gollub, 1985). As such, negative results would have to be interpreted in the light of the strong likelihood of mode-locking in real systems (Bohr, Bak, & Jensen, 1984; Jensen, Bak, & Bohr, 1984), meaning that the
test in all but the most stringent of circumstances loses a good deal of its strength (i.e., falsifiability). This large caveat aside, the present project aims at taking first step towards applying such a protocol to human rhythmic movement. But, before moving further, a more detailed explanation of multifractal measurements must be given.

**Multifractal Measurements**

A central concept in the understanding of dynamical systems and the attractors (e.g., lower dimensional regions in the phase space where the observables of dynamical systems are more likely to be found) that constrain their evolution is the notion of dimension—the number of independent components of the system. Ultimately, the dimension of the system is the number of independent directions in its phase space. But, if the dynamics are governed by dissipative nonlinear interactions (Kugler et al., 1980), an attractor may emerge in which the potentially independent components interactively constrain each other to a lower dimensionality (e.g., the components move less independently).

Another intuitive explanation of dimension is that it is a measure of how the bulk of an object changes with changes in scale (Theiler, 1990). Mathematically,

$$\mu \sim l^d,$$

where $\mu$ is the mass (or bulk), $l$ is the scale of measurement, and $d$ is the dimension. For a single-dimensional system, the bulk scales linearly with length. In two dimensions, bulk scales as $l^2$ and in three dimensions as $l^3$. These latter cases correspond to the familiar notions of area and volume. Perhaps less familiar cases correspond to non-integer dimensions, where the bulk of the set in a sense “bleeds into” the next higher dimension,
but never quite fills it up. The size of the non-integer component of the dimension captures how much of the next higher dimension the set occupies.

To reiterate, the concept of dimension in dynamical systems refers to how many different directions are needed to uniquely specify a location in the phase space (Packard, Crutchfield, Farmer, & Shaw, 1980). Many different dimensions exist. For example, the topological dimension is equal to the minimum set of Euclidean sheets that, when intersected with each other and the attractor, yield a countable number of points (Packard et al., 1980). Another dimension, the fractal dimension, is equal to the size of this supporting set (the total number of dimensions needed to specify a location on the attractor), and is typically the value reported in monofractal analyses. In multifractal analysis, this dimension is captured when the moment \( q \) (to be defined later) is equal to 0, meaning that all observed configurations are counted equally, regardless of the time the system spends in them. During the evolution of a complex dynamical system through its phase space (i.e., possible configurations), mutual interactions among the components generally constrain the evolution to a smaller subset of the phase space, or to an attractor. The mass from Equation 1 can be thought of as the amount of time the system spends in any given region of the attractor. For a mono-fractal, the distribution is the same for all locations, but for dynamical systems with multiplicative interactions (i.e., multifractals), different locations of the attractor are visited with differing probabilities.

A way of measuring the fractality of a dynamical system is to collect a set of samples of the system as it evolves through its phase space (an accurate phase space representation is often obtained by phase space reconstruction of a scalar time series;
Packard et al., 1980), and then quantify how this set of observations changes across varying scales. If there is statistical self-similarity in the set, the number of observed configurations will change reliably as a function of the size of the scale. Critically, even though a single estimate of the dimension of the system, typically the fractal dimension, yields information regarding the geometry of the attractor, it is insufficient to specify the dynamics of the function generating the distribution (Pesin, 2008). For that, a measure of the probabilities of the distributions must be developed, and that is precisely what the multifRACTAL analysis method does.

To begin multifRACTAL analysis of a dynamical system, a partition function is used to divide the space into discrete states. Generally, a box covering method is used that partitions the space into equal-sized volumes, and then counts the number of times the system visits each box (Hentschel & Procaccia, 1983):

\[
D_q = \frac{1}{q-1} \lim_{\epsilon \to 0} \ln \frac{\sum_{i=1}^n p_i^q(\epsilon)}{\ln \epsilon},
\]  

(2)

where \(D_q\) is the generalized dimension, \(p_i^q(\epsilon)\) is the probability that the system can be found in the \(i^{th}\) box, \(n(\epsilon)\) is the total number of boxes visited by the system, with length of the boxes determined by the scaling factor \(\epsilon\), and \(q\) is a real number that weights the probabilities (Wu et al., 2015). Additionally, the function \(\tau\) is the scaling of the partition function (the set of terms in the limit of equation 2) for a given weighting factor, and is given as,

\[
\tau = D_q(q - 1).
\]  

(3)
The spectrum of generalized dimensions in (2) was used by Halsey et al. (1986) to generate two indices, \( \alpha \) and \( f(\alpha) \), that characterize the range of scaling exponents, \( \alpha \), and their densities, \( f(\alpha) \). The term \( \alpha \) is the measure of the local scaling exponent for a given \( \epsilon \) (a covering of the attractor by small \( n \)-spheres, or a dividing of the attractor into \( n \) small boxes; otherwise known as a partition),

\[
\mu_i \propto \epsilon_i^{\alpha_i}. \tag{4}
\]

This number gives the local scaling exponent, \( \alpha \), for at the \( i^{th} \) element of the partition. In other words, this number can be thought of as the local dimension of the attractor, which is free to vary from location to location. The probability of obtaining these local scaling exponents scales as:

\[
n(\alpha, \epsilon) \sim \epsilon^{-f(\alpha)} \Delta \alpha, \tag{5}
\]

where \( n(\alpha, r) \) is the number of boxes that have a scaling exponent falling between \( \alpha + \Delta \alpha \). In other words, \( \alpha \) is the range of scaling exponents (local dimensions), while \( f(\alpha) \) essentially gives their distribution (Chhabra, 1989).

This box-counting method was generalized by Pawelzik & Schuster (1987) based on the correlation integral method of quantifying strange attractors pioneered by Grassberger & Procaccia (1983). By measuring the mass of the attractor based on \( n \)-spheres centered on each point, rather than by arbitrary partitions that may artificially separate points that lie close to each other on the attractor, a more robust measurement is obtained (De Bartolo, Veltri, & Primavera, 2006; Wu et al., 2015) that is more tolerant of the negative \( q \) weights. These negative \( q \) moments obtain measures of the sparsest
regions of the attractor, and are thus sensitive to noise (Chhabra, Meneveau, Jensen, & Sreenivasan, 1989), and can cause problems for box-counting methods (Meisel & Johnson, 1994). This method was later used by Yamaguti and Prado (1995) to generalize the method of direct estimation of the indices $\alpha$ and $f(\alpha)$ developed by Chhabra and Jensen (1989) (the reader is referred to the corresponding papers for the relevant generalized formulas beyond those described here, which are not necessary for the further conceptual development of the ideas outlined above). The method of direct estimation has the benefit of not requiring the Legendre transformation of the generalized dimensions, $D_q$—a transform which smooths $\tau(q)$, and thus would obfuscate any attempt to take meaningful derivatives of this function (Arneodo, Bacry, & Muzy, 1999).

Incidentally, for those familiar with recurrence analysis (Marwan, Carmen Romano, Thiel, & Kurths, 2007; Webber & Zbilut, 1994), this method offers a useful analog to the method of direct estimation described above. The latter amounts to summing the columns of the distance matrix after the Heaviside function has been applied at a given threshold, then dividing each sum by the total number of recurrent points for that threshold. This process yields a set of probabilities that the system is in particular state for a given distribution of states, and these probabilities are then raised to the exponent $q$ to weight the set accordingly.

With respect to Equation 1, the estimated fractal dimension can be understood as the manner in which the mass of the attractor changes as a function of scale. If it is scale-invariant over a sufficient period (a sufficient period is generally considered to be around an order of magnitude on a logarithmic scale), then the mass scales reliably with length,
and the logarithmic rate at which the mass changes gives an estimate of the dimension of the attractor. The number $q$ is an important quantity to understand conceptually. Mathematically, it scales the obtained probabilities by varying factors, allowing locations that are visited more or less frequently to be discriminated by the partition function. Higher, positive values of $q$ weight frequently visited locations on the attractor more heavily than less frequently visited locations. As $q$ increases, all probabilities decrease, but smaller probabilities do so much more rapidly than larger probabilities, minimizing their contribution to the sum and allowing the scaling characteristics of the more frequently visited locations to dominate the rate of change in the limit of the function. The reverse is true for negative $q$ partitions.

**Thermodynamics and Multifractals**

A thermodynamic formalism of the quantities $\alpha$ and $f(\alpha)$ (Jensen, Kadanoff, & Procaccia, 1987) was developed based on the pioneering ideas of Sinai (1972) who had linked dynamical systems to thermodynamics. Conceptually, the variables $q$, $\alpha$, $\tau(q)$, and $f(\alpha)$ can be linked to the thermodynamic quantities of inverse temperature, energy, free energy, and entropy, respectively (Lee & Stanley, 1988). Specifically, $q$ is considered the multifractal analogue of the inverse temperature term in the Boltzmann distribution (Arneodo et al., 1999), where the probability of being in a high energy state for a given average system energy level is always less than being in a low energy state for that same energy level. So, as $q$ gets smaller, states that are visited less frequently are singled out and these relatively rare states are linked to states that have a higher energy level than the higher level $q$ states. The distribution of states for a given energy level is given by $f(\alpha)$, which indexes the number of microstates that a system has for a given temperature, $q$. 
while the term $\alpha$ gives an index of the energy by volume for each of the microstates. Thus, the $f(\alpha)$ curve can be thought of as the multifractal analogue of the entropy versus internal energy relation of a multi-body system (Arneodo et al., 1999).

A conceptual link between discontinuities in the generalized dimensions of an attractor and thermodynamic phase transitions was made by Katzen and Procaccia (1987), and observed in Diffusion-Limited Aggregation by Lee and Stanley (1988). The latter authors demonstrated that sudden jumps in the derivatives in the free energy analogue, $\tau(q)$, can index phase transitions in ways that are intimately linked to second order phase transitions in statistical mechanics (Yeomans, 1992). This idea has been developed by Canessa (2000), who applied the method to financial systems (Canessa, 2001). The concept was later adopted by Da Fonseca, Ferreira, Muruganandam, and Cerdeira (2013), who created a simple summary statistic of the strength of this discontinuity, which the authors termed the area variation rate (AVR).

In summary, multifractal analysis offers a rich array of variables that serve to quantify the processes that constrain dynamical systems. Of those, the variables of most interest in this paper include the fractal dimension, the range of $\alpha$ values, and the second derivative of the $\tau(q)$. Experiment 1 makes use of a known $\alpha$ and $f(\alpha)$ spectrum obtained from a critical, irrationally driven circle map to make specific predictions regarding the multifractal spectrum characterizing human rhythmic movement when it is driven by a quasiperiodic oscillator, while experiment 2 makes predictions regarding discontinuity in the second derivative of $\tau(q)$. With respect to the fractal dimension, weighting all boxes equally (i.e. $q = 0$) yields an estimate of how many different directions are needed to
capture every possible location of the attractor, which in the critical circle map should be equal to one, as the support of the set (i.e., the minimum number of dimensions such that every point on the attractor has a measure, or mass, greater than zero; Munkres, 2000) is a circle, a single dimensional manifold. The $\alpha_{\text{min}}$ and $\alpha_{\text{max}}$ values captures the probabilities that the most dense ($\alpha_{\text{min}}$) and most sparse ($\alpha_{\text{max}}$) regions of the attractor will be mapped to by the action of the sine-circle map, a difference that is brought about by the cubic inflection at the critical point. Further, measuring the second derivative of $\tau(q)$ as a function of time might provide a more sensitive measure of phase transitions than has been used before, which have evaluated either entropy or Hurst exponents, both of which are analogous to the first derivatives of the free energy function.
CHAPTER 2: General Method

Participants

Graduate students from the University of Cincinnati Psychology Department and other members of the community were recruited by word of mouth for Experiment 1. These participants did not receive any benefit from taking part in the study. Undergraduate students enrolled in psychology classes at the University of Cincinnati participated in Experiment 2 on a voluntary basis in return for course credit. All participants were screened for normal or corrected-to-normal vision and reported being free from known neurological, neuromuscular, or skeletal disorders, having no history of balance disorders, no recent injuries that may affect walking or standing, no history of motion sickness or motion-sickness-like symptoms experienced from playing video games or watching movies, and no history of allergies to adhesive tape or to latex or spandex.

Apparatus

A magnetic motion tracking device, set to sample at 120 Hz, was used to collect position data from the participants (Polhemus Fastrak, Polhemus Corporation, Colchester, VT). Computer displays were presented on a 227 × 160 cm projection screen via an Epson EMP-S3 projector (Epson America Inc., Long Beach, CA) set to an 800 × 600 resolution and projecting a display measuring 191 × 144 cm, 80 cm above the ground.

Procedure

Written informed consent was obtained from all participants prior to starting any experimental trial. Following instructions, participants were asked to sit in a padded chair
200 cm from the display and interact with the program according to the instructions they received (unique to each experiment).

**Data Analyses**

All data were separated into non-overlapping epochs (a windowing technique often used in nonlinear analysis; Anastas, Stephen, & Dixon, 2011; Norbert Marwan, Schinkel, & Kurths, 2008) and submitted to multi-fractal analysis (MFA) using an adaptive phase space reconstruction technique developed specifically for this dissertation. To conduct the analysis, the mean delay time for the first minimum in the average-mutual information for all participants in an experiment was calculated and used as the delay in the phase space reconstruction (Marwan et al., 2007; Tolston, Shockley, Riley, & Richardson, 2014). A well-known pitfall in the analysis of phase space dimensionality is sampling a system before it has a chance to evolve beyond the current configuration, a sampling error that causes a downward bias in dimensionality estimates (Theiler, 1986). Thus, a correction equal to the delay (known as a Theiler window) was used in all analyses by not counting these points in the integral.

Another potential issue with the analysis of non-linear data using phase space reconstruction is drift in the parameters affecting motion on the attractor (Hegger, Kantz, Matassini, & Schreiber, 2000). In particular, if the parameters are changing (e.g., the system is non-stationary), Takens’ theorem (the theorem that serves as the motivational basis for phase space reconstruction) does not apply, and the system must be embedded in an $m > 2(D+P)$ space, rather than an $m > 2D$, as is normally required (Hegger et al., 2000), where $m$ is the embedding dimension, $D$ is the dimension of the attractor, and $P$ is
the number of parameters governing motion. It is necessary to note that these concepts are closely related to the categories of dynamics that characterize dynamic systems (Saltzman & Munhall, 1992), which have been used to theorize about differences found between dimensionality estimates of rhythmic behavior using the correlation dimension and those using more sensitive local estimates (Mitra, Riley, Schmidt, & Turvey, 1998).

The number of such terms needed to characterize the behavior of a dynamical system is called the number of active degrees of freedom (adf), and is typically associated with the number of first-order differential equations needed to describe the motion of a system (Riley & Turvey, 2002). For instance, the number of dimensions that are needed to describe a simple oscillator is equal to two, as it is necessary to combine two first-order differential equations to fully characterize the phase space of the system (Riley & Turvey, 2002). However, when such an oscillator is constrained to a limit cycle attractor, the number of independent terms drops to one, as either position or velocity suffice to fully describe the state of the system (Kay, 1988). However, it has also been theorized that the number of adf characterizing the system must also be able to account for any possible changing parameters, which would then act like additional dimensions of the system (Mitra et al., 1998; Riley & Turvey, 2002). In support of this, sensitive measures of local dimensionality, such as local false-nearest neighbors (Abarbanel & Kennel, 1993) have been used to estimate the number of adf necessary to describe the dynamics of rhythmic pendulum swinging, resulting in a range of three to five adf (Mitra et al., 1998; Suvobrata Mitra, Amazeen, & Turvey, 1998; Suvobrata Mitra, Riley, & Turvey, 1997). This estimate is larger than both what is theoretically necessary to describe limit-cycle oscillators and the number of dimensions constraining rhythmic finger wagging estimated using the more
global correlation dimension (Kay, 1988). However, these differences have been partially reconciled via a theorizing of the possibility of evolving parameter dynamics (Mitra et al., 1998). Such notions are closely linked to the method of overembedding used in this project, as well as the notion of multifractality. Specifically, it is interesting to consider the possibility that temporarily changing parameters necessitate the expansion of \( adf \) only during their evolution, whereas the number of autonomous differential equations governing the dynamics of rhythmic movement remains fixed under limit cycle oscillations. Such intermittent activation and deactivation of \( adf \) might be usefully quantified by the sensitivity of multifractal measures to differential densities in the attractors constraining movement dynamics. Specifically, it is likely that individuals who have lower local dimensional variability (i.e., degree of multifractality, or width of the \( \alpha \) spectrum) will also have movements characterized by a lower global dimension (i.e., fractal dimension).

To deal with the potential of changing parameters, Hegger and colleagues (2000) recommend over-embedding data in a phase space, and then reducing the dimensionality by using singular value decomposition (SVD). SVD is a technique that projects the data onto a new coordinate space that maximizes the variability in the data that is captured by the first few principal components, or directions in the multi-dimensional space, in essence compressing covariant signals (e.g., fluctuations in the data which are correlated tend to get mapped to the same component, thereby effectively reducing the dimensionality of the space needed to capture arbitrarily large portions of the variance – arbitrary because any number of components can be used, and when all are utilized, all the variance in the data is accounted for).
Following SVD, components that do not explain a significant portion of the variance are ignored, and the reduced data can then be thought of as a projection of the data onto a lower dimensional manifold that simultaneously maximizes variability accounted for by the projection, and reduces noise (unsystematic variability). Given that 1) coupled oscillator systems with quasiperiodic winding numbers have a tendency to drift in their parameter space (Glazier & Libchaber, 1988) and that 2) the parameter settings in Experiment 2 were explicitly changed during the course of the experiment, one cannot assume stationarity. Hence, following previous work (Hegger et al., 2000; Matassini, Hegger, & Kantz, 2000), the technique of overembedding was employed.

The novel method employed in the current research involves utilizing an adaptive embedding dimension that determined the minimum number of components extracted from principal components analysis (PCA, a special case of SVD that deals with mean-centered data) that have an eigenvalue of at least 1 (a common cut-off in the Factor analysis literature, and one that typically corresponds to the amount of variability that can be accounted for by a single item in a battery; Jolliffe, 2002), and also an algorithm that searches for linear scaling regions in the correlation integral used to evaluate the multifractal characteristics of the data. Specifically, the algorithms searches for the most flat region spanning one full generation on the exponential scale that have at least a 95% coefficient of determination with respect to a line fit through the points (cf. Kelty-Stephen, Palatinus, Saltzman, & Dixon, 2013).
CHAPTER 3:

Experiment 1: Universality of the Circle Map

Quasiperiodic route to Chaos and the Sine-Circle Map

The qualitative character of very complex dynamics that ordinarily require multiple differential equations to model, including frequency locking, period doubling, and transitions to chaos, can be captured by single dimensional iterative maps (Procaccia, 1988). Such mathematical simplification greatly reduces the computational costs involved in the numerical investigation of the properties of the system. An example of one such iterative equation is that of the sine-circle map,

\[
\theta_{i+1} = \theta_i + \Omega - \left( \frac{K}{2\pi} \right) \sin(2\pi) \theta_i,
\]

in which \( \theta \) is the phase of the system, \( \Omega \) is the undressed winding number (i.e., the ratio of frequencies of the individual oscillators, and that which specifies how the system would advance with no interaction) and \( K \) is a nonlinear parameter that captures degree of coupling between the two systems (Cvitanović, 1992). The simple equation above captures a range of phenomenon, including frequency locking (in which coupled oscillators of different intrinsic frequencies synchronize to a common ratio), and transitions from periodic or quasiperiodic motion to chaos, a transition which qualitatively includes period doubling in the periodic case (in which a phase locked system undergoes period-doubling bifurcations) and the breakdown of the surface of the two-torus manifold constraining movement in the latter case, as the map loses invertibility and chaos sets in. In this quasiperiodic route, coupled oscillators with incompatible frequencies undergo a transition to a multistable set of frequency locked
states as the nonlinear parameter, $K$, is increased beyond the critical value, after which the quasiperiodic motion is no longer possible. In particular, circle maps with a golden mean winding number (the average phase advance of the system per iteration of the map, $\theta_{n+1}$ in Equation 1) remain in the quasiperiodic state for all values of $K$ less than or equal to 1. At this critical line, the measure of quasiperiodic winding numbers reduces to 0 (meaning the probability of randomly sampling a quasiperiodic winding number from a random set of systems with very high interactions is null). The reverse case is obtained when the value $K$ is set to zero, where the rational numbers have zero measure, and the irrationals have full measure (this concept of measure, or largeness of a set, is important in the multifractal analyses described earlier). In other words, the probability of mode locking of two independent, non-interacting systems is zero. However, when $K$ is increased, systems with intrinsic frequencies that are close to rational numbers synchronize, mutually constraining their behavior and reducing the dimensionality of the coupled system to a lower dimensional attractor in the shape of a torus with a particular winding number (Ivankov & Kuznetsov, 2001).

The winding number of a circle-map indicates the average phase advance per iteration of the map. During frequency locked behavior, a system of mode-locked, coupled oscillators repeats itself in ways related specifically to the ratio of the faster component of the ratio of their mode-locked frequencies, and also of the slower component, or phase-locked ratio component of the slower system. That is, for a 1:3 resonance, the faster oscillator cycles three times for every one complete cycle of the slower oscillator. Therefore, by strobing the system at the phase locked frequency of the faster oscillator (a
technique known as taking a Poincaré section of the phase space), the system will appear to discretely move to two different locations in the phase space, before returning to the original point. The time it takes for this process to occur is given by the phase-locked period (i.e., the time scale) of the slower oscillator. The winding number of a real system is difficult to determine, as it is a function of both the undressed winding number (independent frequencies of the non-interacting components) and the nonlinear term, as this term causes the components to be pulled to different frequencies from their intrinsic frequencies by their interaction.

One way to estimate the winding number of a pair of coupled oscillators is to strobe the phase space at varying frequencies while investigating the subsequent plots for either periodic motion (discrete points, or clouds of points in systems with noise) in the case of mode locked oscillators, or aperiodic phase-winding (continuous lines), in the case of quasi-periodic oscillators (Jensen et al., 1984). For example, the return map of a pendulum coupled to an oscillator with two incommensurate (i.e., no mode-locking takes place) frequencies at a ratio equal to the golden mean and driven to the critical line corresponding to $K \sim 1$ can be seen in Figure 1. In that figure, a cubic inflection, which causes the fractal spectrum to take on such a particular range of values ($\alpha_{\text{min}} \sim 0.6326$ for the most concentrated regions of the attractor, and $\alpha_{\text{max}} \sim 1.8980$ for the most rarefied) can easily be seen. Figure 2 shows a return map generated by a human participant in a preliminary study that investigated the effect of amplitude on the dynamics of movement when driven by a quasiperiodic (golden-mean) oscillator.
Figure 1. A: Return map of a critically driven oscillator. Note the large cubic inflection around $\theta \sim .25$. For slightly stronger driving forces, the line breaks apart, and the fractal spectrum diverges.

Figure 2. A return map generated from strobing movement data at $\sim 4$ Hz obtained from a person oscillating at the characteristic frequency of the pendulum system at a maximum amplitude of (A) $\sim \pi/2$ or (B) $\sim 7 \pi/12$. The increasingly sharp bend around $\theta \sim .45$ was taken as an indication that a cubic inflection (i.e., criticality) might be likely near $5 \pi/8$ (the mediant of A and B).
It is the dynamics of human movement coupled to a golden-mean oscillator that was of primary interest in this experiment, which attempted to place participants in a critical coupling relationship with an oscillator that was comprised of two super-imposed sine-waves. The relative frequencies of these waves were selected such that they might generate a golden-mean winding number in the rhythmic arm movements of participants who were asked to couple their movements to a virtual oscillator by rotating a freely spinning wheel in synchrony with a visual display of the movement the oscillator. In order to test the hypothesis that human rhythmic behavior can, under appropriate constraints, be characterized by the same universality class as the circle-map, the multi-fractal spectrum \((\alpha, f(\alpha))\) of each participant’s hand movement was calculated using the method of direct estimation (Chhabra, Meneveau, Jensen, & Sreenivasan, 1989; Stephen & Dixon, 2011), and 95% confidence intervals from this distribution was used to test whether the distribution was within the range predicted by the sine-circle-map. Additionally, single sample \(t\)-tests were conducted to evaluate whether obtained values were statistically different from the predicted values. Further, to test for a relation between the degree of multifractality and the number of \(adf\), a Pearson product-moment correlation coefficient was calculated between \(arange\) and fractal dimension. Finally, to evaluate whether the observed dynamics might be explained by the number of components that might be expected to govern the dynamics of a second-order oscillator coupled unidirectionally to two limit-cycle oscillators, a single-sample \(t\)-test was conducted between fractal dimension and the number of components (4) that would characterize such a system (Virgin, 2000).
Method

Participants

Fourteen participants (9 males, 5 females) with ages ranging from 18 to 57 years ($M = 28.29, SD = 9.14$) took part in Experiment 1.

Apparatus

A small plastic wheel with an attached Polhemus motion tracking sensor was used to constrain participants’ movement and to allow them to interact with a virtual oscillator (see Figure 3). A custom openGL program was created that displayed the motion of two super-imposed sinusoidal waves, one set to the characteristic frequency of the apparatus operated by the participants, and the other set to yield a golden mean ratio with the former. The first value was measured empirically by rotating the wheel approximately 20° from the stable equilibrium location (the stable equilibrium state of the wheel assembly is seen in Figure 3) while recording with the attached motion sensor. The wheel was released and allowed to oscillate freely until all motion had fully stopped. A peak picking algorithm was then used to find and measure the distance in samples between peak amplitudes, which was converted to time. Three such measures were taken and the mean frequency of the wheel assembly was calculated to be $\sim 0.954$ Hz ($SD = .002$). Thus, the first frequency of the virtual oscillator was set to 0.95 Hz and the second frequency was set to $\sim 0.587$ Hz (the actual value was calculated to a double precision golden-mean ratio to 0.95 in the openGL program). Movement data from both the participant and the sine wave were projected onto the unit circle, and displayed on a large screen. The driver was set to oscillate at a maximum amplitude of $5\pi/8$ radians.
Procedure

In addition to the general procedure outlined in Chapter 2, participants were asked to couple their arm movements to a virtual oscillator comprised of two super-imposed sine-waves chosen to have a golden-mean frequency ratio. They were given instructions regarding how to interact with the oscillator by manipulating the wheel seen in Figure 3 in response to the displayed oscillator. Participants were instructed to track the driver as if there were a spring or some other force connecting their dot to that of the driver, and to keep their eyes on the driver at all times. Participants were instructed that they were to maintain a constant grip on the handle of the wheel, but minor postural adjustments were permitted during the trial. The display included a green dot, which was linked to the participant’s movement of the wheel in a one-to-one fashion, a light blue dot, representing the independent movement of the oscillator, and a timer in the top left corner of the display (to prevent participants from asking the experimenter how much longer they had...
left in the trial, a common occurrence in the pilot sessions. Participants were asked to not focus on the timer, but to check it briefly if they felt compelled to do so, and asked to focus on the driver and their own movement. Participants sat in an adjustable chair, which they were allowed to manipulate as needed to attain a comfortable seating position. Prior to the full trial, participants performed a practice trial that lasted 90 s and that involved coordinating with a driver that was set to the characteristic frequency of the apparatus. Following this, participants were allowed to readjust themselves to find the most comfortable seating position, and the full 20 minute trial began.

Prior to analysis, the mean values of $a_{\text{min}}, a_{\text{max}}, a_{\text{range}}$ (defined as $a_{\text{max}} - a_{\text{min}}$) and fractal dimension obtained from all participants were submitted to a Grubb's outlier test. No significant outliers were detected ($p > .05$). Data were then evaluated for well-defined multifractal spectra within the region of $\pm 2 q$ by checking for a monotonically decreasing $D_q$ in the specified $q$ region. Epochs that did not meet this criterion were discarded. Data were then averaged over all remaining epochs within participants, and the mean data were submitted to single-sample $t$-tests to evaluate whether $a_{\text{min}}, a_{\text{max}},$ or fractal dimension were significantly different from the values expected from a critical sine-circle map. Additionally, to test for a relation between the degree of multifractality and global dimensionality, a Pearson product-moment correlation coefficient was calculated between $a_{\text{range}}$ and the fractal dimension. Finally, to evaluate whether the observed dynamics might be explained by the number of components that might be expected to govern the dynamics of a second-order oscillator coupled unidirectionally to two limit-cycle oscillators, a single-sample $t$-test was conducted between fractal dimension and the number of components that would characterize such a system.
Results and Discussion

Mean multifractal spectra, obtained by averaging over all epochs within a person, and then by averaging over all participants, can be seen for 180s epochs in Figure 4. The mean fractal dimension was $3.46 (SD = 1.19)$, larger than the expected value of 1, $t(13) = 7.75$, $p < .001$, but not significantly different from that the dynamics of a second-order oscillator coupled unidirectionally to two limit-cycle oscillators, $t(13) = -1.70$, $p = .11$, meaning that participants might have frequently mode-locked to both oscillators independently. The average $a_{\text{min}}$ was $3.06 (SD = 0.76)$, also larger than expected value of 0.6326, $t(12) = 11.91$, $p < .001$, and the mean $a_{\text{max}}$ was $5.35 (SD = 2.48)$, again larger than the expected value of 1.8980, $t(13) = 5.20$, $p < .001$. To test for a relation between the degree of multifractality and the number of $adf$, a Pearson product-moment correlation coefficient was calculated between $a_{\text{range}}$ and fractal dimension. The relationship was significant, $r = .84$, $p < .001$, meaning that as the degree of multifractality in the rhythmic movements of the participants increased, so did their fractal dimension. A scatter diagram of $a_{\text{range}}$ and fractal dimension can be seen in Figure 5.
Results showed that after non-dominant components (e.g., low-power components that may account for parametric drift) were factored out of each participant’s movement, their dynamics were of higher dimension than expected, with scaling regions much wider than those expected from a critical quasiperiodic system governed by the sine-circle map,

Figure 4. A: Mean multifractal spectra of all participants from Experiment 1, averaged over 180 s windows within participants. B: The mean multifractal spectrum obtained by averaging spectra from all participants from Experiment 1. Error bars represent 95% confidence intervals. The box indicates the range of expected values for a critical sine-circle map ($\alpha_{\text{min}}$ 0.6236, $\alpha_{\text{max}}$ 1.8980, and fractal dimension, maximum $f(\alpha)$, of 1).
but still multi-fractal, though with scaling exponents not bound by the expected region. The conclusion is that these results do not support the hypothesis that human rhythmic dynamics in a critical golden-mean relationship with a quasiperiodic oscillator will exhibit the universal scaling characteristics of a critical sine-circle map.

It is interesting to note that, although, the obtained local scaling dimension was significantly higher than the expected fractal dimension of 1, the dimensionality of the participants was not different from the total number of degrees of freedom that could be expected to be operating in a system comprised of an autonomous oscillator coupled to a

Figure 5. Scatter diagram showing the relationship between degree of multifractality ($a_{range}$) and fractal dimension of rhythmic arm movements measured in experiment 1.
driver with two independent frequencies (Virgin, 2000). It is possible that rather than establishing a true golden-mean winding number, where a quasiperiodic relation is obtained in the movements of the participants by following the relative timing of the two oscillators, participants were, on average, mode locking to the frequencies of each oscillator independently. Additionally, the positive relationship between the degree of multifractality in the movements of the participants and the fractal dimension in conjunction with the adaptive embedding technique point to the possibility that participants who were able to maintain relatively stable parameter dynamics were more likely to enter into a lower global dimension.

Together, these results indicate that participants might have been in an intermittent quasiperiodic relationship with the driver, but operating in a region that was rather far from the critical line, with fluctuations in $adf$ resulting from frequent mode locking a likely candidate for the increased dimensionality. Overall, the negative results regarding critical scaling exponents is not entirely surprising, given the precise degree to which parameters corresponding to the non-linear coupling term and the winding number must be tuned to observe quasiperiodic criticality (Glazier & Libchaber, 1988). However, as a first attempt at observing this delicate phenomenon in human rhythmic behavior, the partial successes of the current protocol are encouraging, and future work might easily improve on the methodology established here by observing the width of multifractal spectrum in real-time and adjusting the winding number accordingly (Glazier & Libchaber, 1988), or perhaps by monitoring changes in participants’ $adf$. Specifically, it might be possible to determine the amount of local mode-locking that might be at the root of observed parameter shifts (increases in $adf$), as participants would likely have to re-adjust
their frequency after temporarily mode-locking with one of the components of an irrational system of oscillators.
Chapter 4

Experiment 2: Multifractal Analysis of Phase Transitions in Rhythmic Behavior

Prior research has shown the emergence of new cognitive structures associated with behavioral tasks may be indexed by power-law distributions (Anastas, Stephen, & Dixon, 2011; Stephen, Dixon, & Isenhower, 2009; Stephen et al., 2009). That is, long range correlations in behavioral data are thought to emerge as the system under consideration approaches a critical point, beyond which there is a spontaneous re-organization of the micro-elements bound by the current constraints. Such phase transitions are thought to occur spontaneously as a result of complex, multiplicative, interactions across many scales of the system under investigation (Van Orden, Kloos, & Wallot, 2011). While prior research has evaluated the spontaneous emergence of organizing structures in behavior utilizing a single scaling exponent, multiple scaling regions have also been shown to constrain human behavior in a variety of tasks (Anastas et al., 2011; Harrison, Kelty-Stephen, Vaz, & Michaels, 2014; Stephen & Dixon, 2011). A more complete characterization of such phase transitions in behavior, then, may rely on evaluations of critical fluctuations across the spectrum of singularities that may be obtained via multifractal methods (Dixon, Holden, Mirman, & Stephen, 2012).

The relation between multi-fractal analyses and phase transitions is deeply ingrained in the development of the multi-fractal method (Bohr & Jensen, 1987; Chhabra et al., 1989; Feigenbaum, Jensen, & Procaccia, 1986; Halsey et al., 1986). In particular, phase transitions in time-series that exhibit multi-fractal distributions have been
identified with a critical singularity (i.e., scaling exponent) above which exists an infinite number of phases, and below which a single phase dominates (Canessa, 1993, 2000; Lee & Stanley, 1988; Redelico & Proto, 2012). Recent work has distilled this measure quantifying phase transitions into a single number corresponding to the area underneath a curve, termed the area variation rate (AVR). AVR refers to the analogous free energy—a measure obtained from the spectrum of scaling exponents derived from multifractal analysis (Da Fonseca, Ferreira, Muruganandam, & Cerdeira, 2013).

Da Fonesca and colleagues have demonstrated that stock market crises can be readily identified by AVR, and, perhaps more importantly, that critical fluctuations in time-series data associated with market crashes precede actual crashes by a substantial amount of time, nearly two weeks. Additionally, these critical fluctuations distinguish between crises that are of an intrinsic nature (e.g., the stock market crisis resulting from Lehman Brothers bankruptcy in 2008) and those of an extrinsic nature (e.g., the downgrading of the credit rating of the United States by Standard and Poor in 2011). Furthermore, during quiet periods, this method yields measures that are statistically similar to those obtained from noise, meaning that it may be especially useful for detecting transitions.

In multifractal analyses, the manner in which the partition function (i.e., normalized probability of various states weighted by the factor $q$) is generated naturally leads to a thermodynamic interpretation of the functions of the partition, the role of free energy played by the variable $\tau$. AVR is calculated by finding the second derivative of this variable (e.g., the multifractal analogue of the thermodynamic free energy function), and summing the absolute value of this number over all calculated $q$ moments. This area is
then normalized by the mean value of the area for windows prior to the one under consideration, thus giving an index of the rate of change of the second derivative of \( \tau \) as a function of time.

The present experiment evaluated whether the self-similar fluctuations found in behavior during the acquisition of new cognitive structure observed in previous research (Anastas, et al., 2011; Stephen, et al., 2009) extend to re-organizations of rhythmic behavior. I predicted that phase transitions in cases of spontaneous, intrinsic re-organization would be preceded by critical fluctuations, but not in cases in which the re-organization is externally cued (cf. Anastas et al., 2011). In the study, participants were randomly assigned to either an intrinsic (no cue) or the extrinsic (cued) condition, and were tasked with controlling a virtual pendulum. Participants in the cued condition received instructions regarding anti-phase motion prior to beginning of their trial, and were cued to switch to anti-phase motion half-way through the trial. A virtual pendulum was coupled to the movement of the participant via a motion tracking system and visual feedback presented on a large display. Initially, the external driver weakly interacted with the pendulum system. During this portion of the trial, I predicted that participants would be drawn to an in-phase relationship with the driver (Treffner & Turvey, 1993). As the trial progressed, the coupling strength between the driver and the pendulum was increased in small increments to beyond the point where the combined strength of the participants’ movement and the driver was sufficient to push the pendulum out of the specified region. At this point, perfect in-phase motion with the driver was no longer stable, while anti-phase motion was maximally stable.
In the present experiment, participants were asked to control a virtual pendulum via their hand velocity, which was obtained by differencing contiguous position measurements in the medio-lateral (ML) plane obtained with a motion tracking sensor placed on a small wand controlled by the participants. The equation for the pendulum system was that of the simple damped-driven-pendulum,

\[ ml^2 \ddot{\theta} = -bl^2 \dot{\theta} - mgL \sin \theta + LF(t), \]  

(7)

where \( m \) is the mass of the pendulum bob, \( L \) is the length of the pendulum, \( b \) is the damping coefficient, \( \dot{\theta} \) is the phase velocity of the pendulum, \( \ddot{\theta} \) is the rate of change of the phase velocity (acceleration) of the pendulum, and

\[ F(t) = F_0 \cos(\omega t), \]  

(8)

is the time-dependent force of the driver (Taylor, 2005). This equation can be simplified to:

\[ \ddot{\theta} + 2\beta \dot{\theta} + \omega_0^2 \sin \theta = \gamma \omega_0^2 \cos \omega t, \]  

(9)

where the substituted term \( \beta \) is the mass-normalized damping constant, and \( \gamma \) is the mass normalized driving strength. An additional term was added that accounted for the coupling of the pendulum to the velocity of the participant, yielding the final equation,

\[ \ddot{\theta} + 2\beta \dot{\theta} + \omega_0^2 \sin \theta = \gamma \omega_0^2 \cos \omega t + \gamma_p \dot{\theta}_p, \]  

(10)

where the new term \( \gamma_p \dot{\theta}_p \) adds a force to pendulum proportional to the velocity of the movement generated by the participant. The natural angular velocity of the pendulum, independent of the driver, can be shown to be
\[
\sqrt{\frac{g}{L}} = \omega_0. 
\]  

(11)

Thus, by scaling the force of the driver of virtual pendulum in discrete steps every minute throughout the fifteen minute trial, a critical point was reached at the eight minute mark at which in-phase motion with the pendulum was no longer stable. I predicted that, at this point, participants in both conditions would switch to an anti-phase relation with the driver. Further, I predicted that critical fluctuations, indexed by AVR, would precede the transition in participants who received instructions regarding a switch to anti-phase behavior at the half-way point in the experiment, but would not precede transitions made participants who did not receive such instructions.

**Method**

**Participants**

Seventy-six participants (31 males, 46 females) with ages ranging from 18 to 31 years \((M = 19.66, \ SD = 1.84)\) took part in the first experiment two. All participants were recruited from the University of Cincinnati Psychology Participation pool. Due to participants having difficulty completing the task, the parameter governing the required amplitude of movement of the participants was modified after data from 22 participants were collected, leaving 55 remaining that were included in the reported analyses.

**Apparatus**

A 33 cm wooden drumstick (i.e., a wand; Kidsticks, Vic Firth Company, Boston, MA) with a Polhemus tracker attached was used to allow participants to interact with the pendulum. A more rigorous protocol might use a calibrated pendulum, which would allow
fine-grained control over the eigenfrequency of each participant's movement. However, since the nature of this experiment was to measure qualitative shifts in dynamics rather than quantitative changes in frequency, the simpler setup was deemed appropriate. A custom openGL program was written to display the position of the pendulum, the wand, and also that of the driver. The program determined the position and velocity of the pendulum according to equation 10, and performed the numerical integration in real-time using the Odeint library of the Boost C++ package. The integration was computationally intensive, and caused the program to refresh at 60 Hz, effectively down sampling the data obtained from the motion tracking system. A colored wheel was drawn on the screen, with different colors for varying partitions associated with a yellow neutral zone (between $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$ radians), a green target zone (two partitions, one between $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$ radians, and the other between $\frac{11\pi}{6}$ and $0$ radians and $0$ and $\frac{\pi}{6}$ radians), and a red forbidden zone (between $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ radians) (see Appendix A).

The initial parameters governing the motion of the pendulum were set as follows: $g = 9.8; \beta = \frac{\pi}{4}; \gamma = 0.25; \omega_0 = \omega_d = \pi$; and $kx = 0.75$. $L$ was determined by $\omega_0$ and by $g$ according to equation 11. Every 60 s, the parameter $\gamma$ was increased by 0.107142857, such that at the eight minute mark the force of the driver resulted in in-phase motion by the participant driving the pendulum out of the specified region (this number was calculated numerically by adding a virtual stand in driver for the participant, and calculating the strength required at a velocity and amplitude equal to the driver to push the pendulum out of the specified region), creating a situation where in-phase motion with the pendulum was no longer sufficient to meet the task demands.
Procedure

The University of Cincinnati Institutional Review Board approved all experimental protocols. A written informed consent was administered to all participants prior to starting any experimental trial, to which all participants agreed. After obtaining consent, participants were presented with one of two PowerPoint presentations that they were asked to read at their own pace (see Appendix A). There was no time limit, and participants were allowed to re-read the instructions as many times as they liked. Instructions were identical, except that the participants assigned to the explicit condition received an extra slide indicating that in the beginning of the trial they needed to work with the computer (in-phase), that the force generated by the computer was getting stronger over time, and that once that they would be cued to switch to anti-phase behavior after a period of time had elapsed (the cue was that the dot indicating the location of the driver turned from green to red at the eight minute mark). The program kept track of the location of the pendulum and changes in velocity, and if these changes occurred within the target range, then the program checked that the participant had moved the wand sufficient amount during the interval of that half-cycle (a minimum of 38.1 cm, determined by integrating the calculated velocity of the participants movement of the wand between zero-crossings in the velocity of the pendulum; this value was initially set to 43.18 cm for the first 22 participants, but caused difficulty for many participants, who were confused as to why they were not receiving points). If the conditions were met, then the participant was awarded 5 points, and the total score tracker was updated. If the pendulum reversed direction in the yellow area, no changes in points took place. If the pendulum passed through the red area, the score total flashed red and 1 point was deducted for every 0.1s
spent in the area. Most participants expressed displeasure when this happened, indicating that they were engaged in the task.

Design

The experiment was a single factor (Cue-type) between groups design. Participants were randomly assigned to either the extrinsic or intrinsic Cue-type condition.

Data Reduction

Prior to submission to analysis, data were filtered with a 2nd-order low pass Butterworth filter with the cutoff frequency set to 10 Hz. Data from were then divided into one-minute non-overlapping epochs, and submitted to MFA using the adaptive phase space reconstruction method described previously. Data from these epochs were also analyzed to obtain the instantaneous relative phase between the participant and the driver. Once the measures for all windows were calculated, data were binned in two consecutive windows and the absolute value of the difference between contiguous windows within a bin were used to index the rate of change (e.g., fluctuations) of AVR and the standard deviation of relative phase. Mean AVR was calculated within bins. Mean relative phase was calculated within bins to evaluate whether participants followed the predicted trends (e.g., IP in the first half of the trial, with a switch to AP occurring at the 8 minute mark).

Data were then centered on their grand means, and growth curve modeling was used to determine the effects of nested polynomial combinations of window and condition on AVR, and also to determine if there was a significant epoch² × condition interaction as the final step (cf. Anastas et al., 2011). The higher order terms (e.g., epoch²) capture non-
linear relationships between the predictors and the dependent variable. In particular, 
ePOCH^2, epoch^3 would capture a nonlinear, quadratic relation between epoch and AVR, while 
ePOCH^3 an cubic relation, and so-on. Growth curve modeling, a continuous form of Linear 
Mixed Models, uses nested models to test for the improvement of fit in statistical models 
of the data (amount of variability accounted for by the estimated parameters) for nested 
terms. The Log-likelihood ratio obtained between nested models (-2LL) follows a Chi-
squared distribution, with a degrees-of-freedom parameter equal to the difference in the 
number of parameters between the parent and child model.

For relative phase, as there was not a quadratic curvature predicted around the 
critical region, a linear epoch × condition interaction was added as the final step. 
Polynomial combinations of epoch were added in each model until the model no longer 
impoved in fit, or the epoch^4 term was reached (higher order terms often improved the fit 
of the model, but did not generally add qualitatively meaningful changes above those 
captured by the lower order terms; for parsimony, the fourth-order term was chosen as a 
cutoff).

Results and Discussion

AVR

AVR, or the area variation rate, indexes the degree at which there is change in any 
discontinuity in the multifractal spectrum. Summary statistics of AVR by window and
condition are shown in Figure 5.

![Graphs](image)

**Figure 5** A: Average Area Variation Rate (AVR) as a function of binned epochs. The critical coupling strength occurs in bin 4 (marked by a vertical dashed line). Epochs one and fifteen were removed from analysis. Error bars represent ±1 standard error. B: Predicted AVR from best fit growth model (centered data; all entered terms included).

Adding a linear term for epoch did not significantly improve the model over the null model, \(-2LL \chi^2(1) = 3.73, p = .054\), nor did adding a term for condition, \(-2LL \chi^2(1) = 0.06, p = .79\). Adding random effects for the intercept of AVR by participant did not significantly improve the fit of the model, \(-2LL \chi^2(1) = .259, p = .61\). Inspection of the predicted variance and standard error of the variance showed that the standard error was twice the estimated variance, indicating that AVR did not significantly differ as a function of participant in the first considered window. This term was excluded from the model.

Adding a quadratic term for Epoch\(^2\) did significantly improve the model, \(-2LL \chi^2(1) = 12.18, p = .001\), as did a term for Epoch\(^3\), \(-2LL \chi^2(1) = 10.5, p = .002\). Finally, a term for Epoch\(^4\), was determined to still further improve the fit, \(-2LL \chi^2(1) = 6.88, p = .001\). An
epoch$^2 \times$ condition interaction term was then fit to this final model to determine whether
the rate of growth before and after the critical point differed significantly between
conditions. This term did not improve the previous model, 2LL $\chi^2 (1) = 1.84$, $p = .17$. A
table of coefficients for the final model can be seen in Table 1, while the fitted model can
be seen in Figure 5.

As expected, AVR increased before the critical region around bin four. The
quadratic curvature indicates that the rate of discontinuity in the $F(a)$ curve increased up
to the critical window, then decreased in a trend back to baseline. Adding epoch$^3$ to the
model also significantly improved the fit. Inspection of the predicted data indicated that
this term captures a downward trend in AVR prior to the upward turn captured by the
2nd order term. It appears that participants were trending to a low AVR, but by the
second window (associated with minutes five and six of the trial) the variability in their
multifractal spectrum began to increase. Adding epoch$^4$ appears to capture additional
variability around the middle and end of the curves. There was no effect of Cue-type, nor
was there a Cue-type by epoch interaction; the same trends held statistically for both
groups.

Table 1.
Estimated growth curve parameters for AVR.

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>SE</th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>.17</td>
<td>.07</td>
<td>2.57</td>
<td>.01</td>
</tr>
<tr>
<td>Epoch$^2$</td>
<td>.07</td>
<td>.02</td>
<td>3.79</td>
<td>.00</td>
</tr>
<tr>
<td>Epoch$^3$</td>
<td>.00</td>
<td>.00</td>
<td>-3.28</td>
<td>.00</td>
</tr>
<tr>
<td>Epoch$^4$</td>
<td>.00</td>
<td>.00</td>
<td>2.61</td>
<td>.01</td>
</tr>
<tr>
<td>Cue-type$\times$Epoch$^2$</td>
<td>-.01</td>
<td>.00</td>
<td>-1.36</td>
<td>.17</td>
</tr>
</tbody>
</table>
Change in AVR within windows

The variation of AVR within windows was calculated by taking the absolute value of the difference of AVR within the two minute averaged windows, as a term to measure fluctuations. Summary statistics of AVR by window and Cue-type are shown in Figure 6. Adding a linear term for Epoch significantly improved the model over the null model, $-2LL \chi^2(1) = 11.86, p < .001$, but Cue-type did not, $-2LL \chi^2(1) = 0.93, p = .67$. Adding random effects for the intercept of AVR by participant did not significantly improve the fit of the model, $-2LL \chi^2(1) = 1.87, p = .17$. Adding a quadratic term for epoch$^2$ did significantly improve the model, $-2LL \chi^2(1) = 12.94, p < .001$, but epoch$^3$ did not, $-2LL \chi^2(1) = 0.15, p = .30$. An epoch$^2 \times$Cue-type interaction term was then fit to this final model to determine whether the rate of growth before and after the critical point differed significantly between conditions. This term did not improve the previous model, $-2LL \chi^2(1) = 0.1, p = .75$. A table of coefficients for the final model can be seen in Table 2, while the fitted model can be seen in Figure 6.

Change in AVR increased well before the critical region around bin four, with a significant quadratic curvature being the highest order polynomial that fit (up to order four) throughout the epochs. This partially supports the main hypothesis of Experiment 2, and indicates that the fluctuations expected in AVR for the intrinsic condition occurred for both groups, possibly indicating an increase in instability as the trial progressed.
Table 2.
Estimated growth curve parameters for Difference of AVR within Binned Epochs.

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>SE</th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>.102</td>
<td>.084</td>
<td>1.21</td>
<td>.23</td>
</tr>
<tr>
<td>Epoch</td>
<td>.032</td>
<td>.009</td>
<td>3.63</td>
<td>.00</td>
</tr>
<tr>
<td>Cue-type</td>
<td>.097</td>
<td>.117</td>
<td>.82</td>
<td>.41</td>
</tr>
<tr>
<td>Epoch^2</td>
<td>-.009</td>
<td>.004</td>
<td>-2.33</td>
<td>.02</td>
</tr>
<tr>
<td>Cue-type\times Epoch^2</td>
<td>-.002</td>
<td>.005</td>
<td>-.30</td>
<td>.76</td>
</tr>
</tbody>
</table>

Relative Phase

Relative phase (RP) between the driver and participant was calculated and submitted to growth curve analysis. Summary statistics of RP by epoch and condition are shown in Figure 7. Adding a linear term for epoch significantly improved the model over the null model, -2LL $\chi^2(1) = 522.27$, $p < .001$. Cue-type then further improved the model, -2LL $\chi^2(1) = 4.26$, $p = .04$. Adding random effects for the intercept of AVR $\times$ participant did...
not significantly improve the fit of the model, \(-2LL\chi^2= .72, p = .28\). Adding a quadratic term for epoch\(^2\) did significantly improve the model, \(-2LL\chi^2(1) = 6.86, p = .001\), and epoch\(^3\) further improved the fit, \(-2LL\chi^2(1) = 130.01, p < .001\). Finally, a term for epoch\(^4\), was determined to still further improve the fit, \(-2LL\chi^2 (1) = 8.27, p = .001\). A linear epoch\(\times\) Cue-type interaction term was then fit to this final model to determine whether the rate of growth around the critical point differed significantly between conditions. This term improved the previous model, \(-2LL\chi^2 (1) = 8.46, p = .004\). A table of coefficients for the final model can be seen in Table 3, while the fitted model can be seen in Figure 7. RP began to significantly increase after the second epoch bin, and increased more for the extrinsic group, as indicated by the significant linear interaction between epoch and Cue-type.
Table 3
Estimated growth curve parameters for Mean Relative Phase.

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>SE</th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>77.420</td>
<td>2.962</td>
<td>26.134</td>
<td>0.000</td>
</tr>
<tr>
<td>Epoch</td>
<td>22.435</td>
<td>0.800</td>
<td>28.050</td>
<td>0.000</td>
</tr>
<tr>
<td>Cue-type</td>
<td>-5.981</td>
<td>3.180</td>
<td>-1.881</td>
<td>0.066</td>
</tr>
<tr>
<td>Epoch²</td>
<td>1.231</td>
<td>0.338</td>
<td>3.641</td>
<td>0.000</td>
</tr>
<tr>
<td>Epoch³</td>
<td>-0.325</td>
<td>0.025</td>
<td>-13.080</td>
<td>0.000</td>
</tr>
<tr>
<td>Epoch⁴</td>
<td>-0.025</td>
<td>0.009</td>
<td>-2.934</td>
<td>0.004</td>
</tr>
<tr>
<td>Cue-type × Epoch</td>
<td>-1.618</td>
<td>0.161</td>
<td>-2.929</td>
<td>0.004</td>
</tr>
</tbody>
</table>

**Standard Deviation of Relative Phase**

Summary statistics of standard deviation of RP by window and Cue-type are shown in Figure 8. Adding a linear term for Epoch significantly improved the model over the null
model, -2LL $\chi^2(1) = 43.27, p < .001$. Cue-type then further improved the model, -2LL $\chi^2(1) = 14.25, p < .001$. Adding random effects for the intercept of AVR by participant further improved the fit of the model, -2LL $\chi^2(1) = 15.1, p < .001$. Adding a quadratic term for epoch$^2$ significantly improved the model, -2LL $\chi^2(1) = 44.57, p < .001$, and epoch$^3$ further improved the fit, -2LL $\chi^2(1) = 10.85, p = .001$. Finally, a term for Epoch$^4$, was determined to still further improve the fit, -2LL $\chi^2(1) = 18.44, p < .001$. An epoch$^2$×Cue-type interaction term was then fit to this final model to determine whether the rate of growth before and after the critical point differed significantly between conditions. This term did not improve the previous model, -2LL $\chi^2(1) = 0.05, p = .82$. A table of coefficients for the final model can be seen in Table 4, while the fitted model can be seen in Figure 8. The standard deviation in relative phase showed a similar inverted-U shape as that of AVR, but also was sensitive to a difference in groups: Participants in the intrinsic condition were overall more variable in their relative phase relation with the driver than participants in the extrinsic condition.
Table 4.

Estimated growth curve parameters for Standard Deviation of Relative Phase.

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>SE</th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>28.232</td>
<td>1.374</td>
<td>20.543</td>
<td>0.000</td>
</tr>
<tr>
<td>Epoch</td>
<td>1.860</td>
<td>0.296</td>
<td>6.290</td>
<td>0.000</td>
</tr>
<tr>
<td>Cue-type</td>
<td>3.835</td>
<td>1.759</td>
<td>2.180</td>
<td>0.031</td>
</tr>
<tr>
<td>Epoch²</td>
<td>-0.802</td>
<td>0.137</td>
<td>-5.835</td>
<td>0.000</td>
</tr>
<tr>
<td>Epoch³</td>
<td>-0.034</td>
<td>0.010</td>
<td>-3.426</td>
<td>0.001</td>
</tr>
<tr>
<td>Epoch⁴</td>
<td>0.015</td>
<td>0.003</td>
<td>4.360</td>
<td>0.000</td>
</tr>
<tr>
<td>Cue-type× Epoch²</td>
<td>0.013</td>
<td>0.063</td>
<td>0.214</td>
<td>0.831</td>
</tr>
</tbody>
</table>

The fact that the standard deviation in relative phase showed an inverted-U shape demonstrates that the shift in relative phase was indexed by this variable in the manner that was expected, and which was also consistent with prior work (e.g., Schmidt et al., 1990). The increase in variability of the standard deviation of relative phase in the
intrinsic group indicates they did not enter as stable a phase relation with the driver as the intrinsic group, even well before the critical point in the experiment. This increase in variability, without a main effect of relative phase, indicates that without explicit instruction participants were not as likely to enter a strict phase-locked relation with the driver. This is corroborated by the observed linear interaction between Cue-Type and window, as well as the significant reduction in variability of the growth model of relative phase over the null as a function of Cue-Type.
CHAPTER 6:

GENERAL DISCUSSION

This project evaluated the multi-scaled nature of human movement utilizing key ideas from multifractal analyses and thermodynamics. Two experiments were conducted. The first attempted to answer the question of whether human rhythmic movement in a critical coupling relationship with a quasiperiodic oscillator can be shown to exhibit the same universal scaling characteristics as the sine-circle map. The second experiment looked at the multifractal analogue of thermodynamic free energy for evidence of second-order (continuous) phase transitions during a transition to new behavior as a function of whether participants were instructed to perform a goal-preserving behavioral transition, or were left to their own to discover it.

In the first experiment, multifractal behavior was found that was not bound by the regions expected by a critical sine-circle map. Though the overall dimensionality was higher than expected from experiments studying Rayleigh-Benard convection (Jensen et al., 1985), it was not different from the maximum expected value of a second-order system coupled unidirectionally to two independent oscillators. Further, while ranges of the local scaling exponents were wider than expected, they scaled with the global dimension, indicating a possible link between multifractal measures and categories of dynamics governing participants movement (Mitra et al., 1998; Saltzman & Munhall, 1992). Specifically, changing parameters in the equations of motion that govern nonlinear oscillators has been conceptualized under the framework of three distinct types of dynamics, known as state, parameter, and graph dynamics, respectively (Saltzman & Munhall, 1992). State dynamics captures the effects on dynamics of the variation in the
actual observables constituting the evolution of a system, such as position and velocity, while parameter and graph dynamics capture the fundamental organization of dynamic systems. In particular, graph dynamics encapsulate the form of the interrelated functions that act as the constraints of behavior, that is, the number of equations of motion and parameters that make up the architecture of the system, while parameter dynamics captures the particular quantities that the terms in the equations of motion take. Such concepts are useful in describing the evolving structure of constraints that serve to facilitate rhythmic behavior. In this case, it might be that participants who were able to maintain a relatively stable quasiperiodic relationship with the system-scaled frequency of the two oscillators did not have to readjust their parameters as much as participants who were unable to prevent themselves from mode locking with each frequency independently, and who were thus forced to continuously adjust their frequencies and amplitudes to maintain entrainment with the irrational system. Such local changes in parameter dynamics was characterized by a higher degree of differential densities on the attractor, and also by a higher overall degree of global dimensionality.

Subsequent work will have to look at a range of winding numbers and driving amplitudes to see if this dimensionality is unique to a small range around the utilized parameters. If this number holds, it indicates that participants might have been in a sub-critical, but still quasiperiodic, coupling relationship with the driver. Future experiments might improve the established protocol by careful dynamic tuning of the winding number by variations that might be obtained by keeping track of the close returns in the phase space separated by varying intervals associated with the Fibonacci sequence (Ivankov & Kuznetsov, 2001). That is, rather than tracking in real-time the spectral components of
the system (Ecke, 1991), a method which proves difficult in human subjects, it might be easier to track the probability that close returns in the phase space follow a Fibonacci sequence, and tune the frequency of the driver accordingly. Additionally, the extent of the utility of the method of adaptive phase space reconstruction might be tested by evaluating the change in one-step prediction errors of the return map as a function of the number of dimensions included in both the model and the reconstructed phase space (Hegger et al., 2000). This would provide a strong test for the number of components that should be retained in the data, and would also provide an index of the specific relation to the number of \textit{adf} of the system.

In the second experiment, that the second derivative of the multifractal free energy analogue diverged for negative $q$ around the critical point indicates that a second-order (i.e., continuous) phase transition took place. This happened for both conditions, and for both conditions, the AVR followed closely the standard deviation of relative phase, a well-known index of stability in coupled oscillating systems, while relative phase itself is a likely candidate for an order parameter of such systems (Turvey, 1990). This study succeeded in establishing a link between a measure with conceptual foundations in thermodynamic phase transitions and human behavior, further demonstrating the utility of multi-fractal analyses and corroborating the notion that human beings are softly-assembled, complex systems (Van Orden, Kloos, & Wallot, 2011). The utilization of multifractal analyses leads to the most direct analogy between thermodynamic phase transitions and human behavior of which the experimenter is aware. Additionally, the fact that AVR, which was obtained from the position data of just one of the components in a coupled system, showed similar patterning as the standard deviation of relative phase, a
well-known index of stability (i.e., order parameter) in coupled oscillatory systems, holds promise that such measures might provide meaningful insights into the occurrence of phase transitions in systems in which the order parameter is unknown.

Prior work has shown that the entropy of distributions of recurrent orbits in a phase space fluctuates in a manner predicted by the breakdown of order prior to the emergence of new constraints associated with the acquisition of a novel cognitive structure (Stephen, Boncoddo, Magnuson, & Dixon, 2009), or that the distribution of Hurst exponents (an index of multifractality) change (Anastas et al., 2011; Stephen, Anastas, & Dixon, 2012). In this experiment, the second-derivative of the free-energy analogue was shown to index, and precede, phase transitions in human behavior, both partially replicating and extending prior work in this important area. That fluctuations in AVR preceded the transition in both groups indicates that discontinuities in the distributions of recurrent points in the reconstructed phase spaces occurred in both groups as a function of $q$. This indicates that, as the driver increased in strength, both groups were likely to have a single state begin to dominate the sparsest regions of their phase space, meaning that the distributions did not scale continuously in the negative $q$ partitions. A limitation of the current study is the dependence of the AVR on the most rarefied regions of the attractor, making it a measure that is likely to be very sensitive to sample size (Schertzer & Lovejoy, 1993), though several unreported analyses of the present data over wider windows showed similar patterning in the results.

The nature and origin of fractal patterning in biological data is somewhat in question. Two major candidates include self-organized-criticality (SOC) and multiplicative
cascades of energy between coarser to finer scales of the system, with the two being separated by the assumption that SOC does not produce multifractal distributions (Delignières & Marmelat, 2012). Along this line, some authors have interpreted the multifractal structure of executive-function tasks, such as card sorting, as multiplicative cascades of interaction across scales, rather than SOC (Stephen et al., 2012), but there is a line of thought that supposes the two phenomena are not mutually exclusive (Ihlen & Vereijken, 2010; Marković & Gros, 2014; Schertzer & Lovejoy, 1993). In fact, modified versions of the SOC model have been shown to demonstrate multifractal characteristics (Marković & Gros, 2014; Sinha-Ray, de Água, & Jensen, 2001), though the former group did so at the cost of adding a variable driver to the dynamics, resulting in a non-autonomous system. This concern over this loss of parsimony may be mitigated when viewed against the multiplicative model, which supposes somewhat independent processes operating at different scales. Sinha and colleagues (2001) showed that a SOC forest fire process can account for the multifractal distribution of the ages of trees in the forest, demonstrating an interesting link between the process of SOC and the form of multifractality. Much more work and conceptual development in this area is necessary. In particular, future work may classify movement dynamics of various populations using models of two-scale cantor sets to fit parameters to observed data (Harikrishnan, Misra, Ambika, & Amritkar, 2010) to test the multiplicative model, though a suitable alternative to test a multifractal SOC model would also have to be developed.

In sum, the demonstration of evidence for a second-order phase transition during the restructuring of rhythmic behavior over a critical point provides further evidence that
the link between criticality, thermodynamic processes, psychology, and movement science is one that can perform useful work.
References


APPENDIX A: INSTRUCTIONS FOR EXPERIMENT 1

Below are the slides presented to the participants assigned to the extrinsic condition of the Cue-type factor. Slides for the intrinsic condition were identical, except they did not receive instructions regarding in-phase or anti-phase movement (slide six was omitted).

Game Dynamics

Instructions

Press Space (advance)
Or Backspace (return) to navigate instruction slides

Game Dynamics

• The game:
  – Move the wand to push the virtual object

The object is represented by the Light Dot in the colored circle.
Your wand is represented on the screen by a Yellow Dot. Keep the Yellow Dot on the screen at all times.
The computer, represented by the Green Dot, is also exerting a force on the object.
Your task is to make the object peak in the green area.
Each time it comes to a stop in the green, you receive points.
You have to move the word a sufficient distance each time to accumulate points:
- If the object is peaking in the green, but you are not recording points, you are not moving the yellow dot enough between peaks.
• If object stops in the yellow, no points are awarded.

• Avoid the red area!
  – If the object moves through the red area you to lose points!
Hints

- You and the computer are both exerting force on the object
- *Early on, you need to work with the computer to get points*
  - Move your Yellow Dot with the computer’s Green Dot

- As the game progresses, the computer gets stronger
- *When the computer’s dot turns red, then you need to work against the computer to keep from losing points*
  - Move your yellow Dot in the opposite direction as the Red Dot

That’s all!

The game will last 15 minutes.

Please, pay attention and remain alert to *maximize your score*!

*Your performance is important, try your best!*