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Accurate code phase estimation of LOS GPS signal using Compressive Sensing and multipath mitigation using interpolation/MEDLL

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Accurate code phase estimation of LOS GPS signal using Compressive Sensing and multipath mitigation using interpolation/MEDLL

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Abstract

A wide variety of error sources affect accuracy of the basic GPS measurements of pseudorange (also known as code-phase) and integrated Doppler (also known as carrier-phase). Among these are satellite clock and ephemeris errors, ionospheric delay, tropospheric delay, receiver dynamic tracking error, multipath and thermal noise. Multipath is the dominant error source in high precision GPS applications as others sources can be countered with differential measurements.

Multipath errors result when the receiver receives the direct or line-of-sight (LOS) satellite signal via multiple paths and processes the combined signal as if it were only the direct. This causes anomalies in determining user’s location and velocity. Large time-delay multipath are easily separated by correlators in a GPS receiver due to easy separation of correlation peaks. Medium and small time-delay multipath are more difficult to detect and separate since the correlation peaks are not separated, but rather distorted from the shape of that of a single path.

Many solutions have been proposed to estimate LOS in the presence of multipath, almost all of them require faster sampling than the Nyquist rate. This in turn requires a large bandwidth on the RF frontend, i.e., a large pre-correlation bandwidth. A large bandwidth RF frontend admits more noise, and is also more prone to interference. Therefore a narrowband solution would be more desirable. However, tradeoff is that not enough samples are contained in a correlation peak, making it very difficult, if not impossible, to detect LOS and mitigate multipath.

In this thesis we present a novel method to reconstruct high-sampling rate (super-Nyquist rate) correlation functions from low-sampling rate (Nyquist-rate) correlation functions using the new signal processing paradigm of compressive sampling or compressive sensing (CS). Since the correlation function of GPS signals is sparse, with only a few high peaks, it is suitable to apply the
CS theory so that the above reconstruction can be achieved with almost probability one. We use a narrowband RF frontend, e.g., of a 2 MHz bandwidth as in a typical C/A code GPS receiver. With the aid of the CS, we are able to reconstruct super-Nyquist rate correlation functions of, e.g., 16 MSPS with a CS reconstruction ratio of 4.

Also, an attempt was made to mitigate multipath in GPS applications by using interpolation based methods. A standard interpolation method based on linear or nearest neighbor solution is used to generate super Nyquist rate samples and cross correlation functions are obtained. The output of the interpolation method is passed onto a maximum likelihood estimator based method such as MEDLL that enables us to separate multipath with time separations of only one or two such super-Nyquist rate samples (i.e., at 0.1 chip time or less.). This method works well for estimating the multipath delays and amplitudes even in the presence of short and medium multipath delays.
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First and foremost, I would like to express my humble gratitude to my advisor, my mentor and my committee chair, Dr. Howard Fan for his guidance throughout my thesis work and his support during my course work. His valuable suggestions, work ethic and craving for perfection encouraged me to work well beyond my boundaries and solve all the hurdles that were along the way. His command on varying topics in communication systems and signal processing helped me solve many technical issues and gain valuable domain experience.

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# TABLE OF CONTENTS

1  Introduction .......................................................................................................................... 1

1.1  Problem Statement .............................................................................................................. 3

1.2  Thesis Organization............................................................................................................. 4

2  GPS background .................................................................................................................... 6

2.1  GPS System: ....................................................................................................................... 8

2.2  Structure of GPS signal: .................................................................................................... 11
     2.2.1  C/A code: .................................................................................................................... 13
     2.2.2  P(Y) code or P code: ................................................................................................. 15
     2.2.3  Correlation properties of the C/A code: ................................................................... 15
     2.2.4  Navigation message: ................................................................................................. 18

3  Software Defined GPS receiver ............................................................................................... 19

3.1  Acquisition module: ......................................................................................................... 21
     3.1.1  Serial search in Time domain .................................................................................. 23
     3.1.2  Parallel search in Frequency domain ....................................................................... 24
     3.1.3  Parallel code phase search ...................................................................................... 24

3.2  Tracking module: .............................................................................................................. 24

3.3  Navigation and User position calculation .......................................................................... 25

4  Multipath Mitigation ............................................................................................................... 27

4.1  The Multipath Problem....................................................................................................... 27

4.2  Types of Multipath: .......................................................................................................... 29
     4.2.1  Long delay multipath ............................................................................................... 29
     4.2.2  Medium delay multipath .......................................................................................... 29
LIST OF FIGURES

Figure 1: Structure of the GPS ................................................................. 8
Figure 2: GPS Constellation diagram ..................................................... 9
Figure 3: Generation of GPS at L1 frequency ......................................... 13
Figure 4: C/A code generator .................................................................... 14
Figure 5: GPS Receiver Architecture ..................................................... 19
Figure 6: Block diagram of Acquisition module in a GPS receiver .......... 22
Figure 7: Multipath Propagation ............................................................. 28
Figure 8: Correlation Function distortion due to multipath ..................... 30
Figure 9: Classification of Multipath Mitigation methods ....................... 31
Figure 10: Sampling scheme of compressive sampling ......................... 38
Figure 11: Block diagram of proposed narrowband super-Nyquist rate correlation using CS ............. 42
Figure 12: Transmitter end – 500 samples of transmitted GPS data at -15dB SNR (upper), correlation functions of multipath simulation with 2 reflected paths and 1 LOS (left lower), and a zoomed correlation functions showing all 3 paths separately (right lower). The expected correlation function for each path is shown by colored triangles, which make the overall correlation function shown by the black line. ....................... 50
Figure 13: conventional narrowband acquisition, Estimated codephase (upper) and plot of the acquired satellites (lower). Green bar indicates acquisition of the satellite and blue bar indicates the signal value below threshold. Sampling frequency 4.092 MHz. Threshold used for acquisition is 2.5. SNR -15dB .......... 51
Figure 14: Super-Nyquist sampled wideband acquisition, Estimated codephase (upper) and plot of the acquired satellites (lower). Green bar indicates acquisition of the satellite and blue bar indicates the signal value below threshold. Sampling frequency 16.368 MHz. Threshold used for acquisition is 2.5. SNR -15dB .... 52
Figure 15: CS reconstructed narrow band acquisition, the output after CoSaMp algorithm (upper), reconstructed super-Nyquist correlation function in red (left lower), and zoomed super-Nyquist correlation function showing LOS code phase (right lower). Sampling frequency 4.092 MHz. Reconstructed to 16.368 MSPS. SNR -15dB ................................................................. 53
Figure 16: Transmitter end – 500 samples of transmitted GPS data at -22dB SNR (upper), correlation functions of multipath simulation with 2 reflected paths and 1 LOS (left lower), and zoomed correlation functions
showing all 3 paths separately (right lower). The expected correlation function for each path is shown by colored triangles, which make the overall correlation function shown by the black line. ..................................54

Figure 17: Plot of the acquired satellites. Green bar indicates acquisition of the satellite and blue bar indicates the signal value below threshold. Sampling frequency 16.368 MHz. Threshold used for acquisition is 2.5. SNR -22dB ..........................................................................................................................54

Figure 18: super-Nyquist sampled wideband acquisition, Estimated codephase (upper) and plot of the acquired satellites (lower). Green bar indicates acquisition of the satellite and blue bar indicates the signal value below threshold. Sampling frequency 16.368 MHz. Threshold used for acquisition is 2.5. SNR -22dB....55

Figure 19: CS reconstructed narrow band acquisition, the output after CoSaMp algorithm (upper), reconstructed super-Nyquist correlation function in red (left lower), and zoomed super-Nyquist correlation function showing LOS code phase (right lower). Sampling frequency 4.092 MHz. Reconstructed to 16.368 MSPS. SNR -22dB. ..........................................................................................................................56

Figure 20: Transmitter end – 500 samples of transmitted GPS data at -28dB SNR (upper), correlation functions of multipath simulation with 2 reflected paths and 1 LOS (left lower), and zoomed correlation functions showing all 3 paths separately (right lower). The expected correlation function for each path is shown by colored triangles, which make the overall correlation function shown by the black line. .................57

Figure 21: Power spectral density of PRN10 before and after BPF (upper) and the plot of the acquired satellites (lower). Green bar indicates acquisition of the satellite and blue bar indicates the signal value below threshold. Sampling frequency 16.368 MHz. Threshold used for acquisition is 2.5. SNR -28dB .........................58

Figure 22: CS reconstructed narrow band acquisition, the output after CoSaMp algorithm (upper), reconstructed super-Nyquist correlation function in red (left lower), and zoomed super-Nyquist correlation function showing LOS code phase (right lower). Sampling frequency 4.092 MHz. Reconstructed to 16.368 MSPS. SNR -28dB. ..........................................................................................................................59

Figure 23: Transmitter end – 500 samples of transmitted GPS data at -22dB SNR (upper), correlation functions of multipath simulation with 2 short delay reflected paths and 1 LOS (left lower), and zoomed correlation functions showing all 3 paths separately (right lower). The expected correlation function for each path is shown by colored triangles, which make the overall correlation function shown by the black line. ..........64
Figure 24: Interpolation based acquisition: Short delay multipath added (upper), zoomed in ACF at peak (lower).

Sampling frequency 4.092 MHz, interpolated to 16.368 MHz, SNR = -22dB

Figure 25: Input to and output from MEDLL (upper) and zoomed value at or near the peak (lower), correlation function from interpolation based method is shown in red, LOS – blue, multipath 1 – black and multipath 2 – magenta. Sampling frequency is 4.092MHz, interpolated to 16.368 MHz

Figure 26: Transmitter end – 500 samples of transmitted GPS data at -22dB SNR (upper), correlation functions of multipath simulation with 2 medium delay reflected paths and 1 LOS (left lower), and zoomed correlation functions showing all 3 paths separately (right lower). The expected correlation function for each path is shown by colored triangles, which make the overall correlation function shown by the black line.

Figure 27: Interpolation based acquisition: Medium delay multipath added (upper), zoomed in ACF at peak (lower).

Sampling frequency is 4.092 MHz, interpolated to 16.368 MHz, SNR = -22dB

Figure 28: Input to and output from MEDLL (upper) and zoomed value at or near the peak (lower), correlation function from interpolation based method is shown in red, LOS – blue, multipath 1 – black, multipath 2 – magenta. Sampling frequency is 4.092 MHz, interpolated to 16.368 MHz
LIST OF TABLES

Table 1: Locations of the Control Segment components ................................................................. 10
Table 2: Satellite ID vs code phase assignments .............................................................................. 16
Table 3: Power level and occurrence percentage C/A code correlation .......................................... 18
Table 4: Accuracy of the LOS code phase estimation for different SNRs ...................................... 59
1 Introduction

Global Positioning system (GPS) is a satellite based navigation system that is used for both civilian and military applications related to positioning, timing and navigation. GPS is one of many Global Navigation Satellite Systems (GNSS) such as GLONASS, Galileo, and Beidou. Modulation schemes being used for GNSS signals can be broadly divided into two types, namely binary phase shift keying (BPSK) and binary offset carrier (BOC). The conventional L1 C/A and L1/L2 P(Y) signals are BPSK modulated. In addition to a multitude of hardware based GPS receivers available on the market, software based GPS receivers can also be implemented in general purpose computers for quicker implementation and analysis of new algorithms.

In a GPS receiver, the GPS signal is converted to an Intermediate Frequency (IF) and then passed through an Acquisition block. The acquisition of GPS signals involves estimating code phase and Doppler frequency of the received GPS signal. These parameters are sent to a tracking loop for pseudo range calculation from satellites and thus user position and tracking.

The received GPS signal is highly distorted by atmospheric delays, multipath propagation, ephemeris errors and receiver noise. Many solutions have been proposed to estimate LOS in the presence of multipath, almost all of them require faster sampling than the Nyquist rate. This in turn requires a large bandwidth on the RF frontend, i.e., a large pre-correlation bandwidth. A large bandwidth RF frontend admits more noise, and is also more prone to interference. Therefore a narrowband solution would be more desirable. However, tradeoff is that not enough samples are contained in a correlation peak, making it very difficult, if not impossible, to detect and mitigate multipath [1].
Multipath and shadowing are the major contributors to errors in user position. Multipath is a result of the reception of reflected or diffracted replicas of the desired signal. Since the path travelled by a reflection is always longer than the direct Line of Sight (LOS) path, multipath arrivals are delayed relative to the LOS. When multipath delay is greater than twice of the spreading code period for BPSK modulation, such multipath is effectively resolved by the correlation operation of a receiver since separate peaks appear in a correlator output. Short delay multipath corresponds to reflection from nearby objects that arrive at delays less than twice of the spreading code period after the LOS arrival. Such multipath distorts the correlation function between the received composite (LOS plus multipath) and locally generated reference spreading code at the receiver. They also distort the composite phase of the received signal, introducing errors in pseudo-range and carrier phase measurements that are different among different satellite signals, and thus produce errors in time, position and velocity [2].

Various methods have been proposed to reduce the multipath effects. They can be classified into below categories

• Antenna-based mitigation - This involves improving the gain pattern of antenna to counter the effects of multipath. These antenna-based methods include the use of special antennas, processing in spatial domain with multi-antenna arrays, antenna location strategies and long-term signal observation for inferring multipath parameters [3].

• Improved receiver technology - These methods include all receiver technologies that are used to mitigate multipath. Usage of Narrow Correlators, Multipath Elimination Technique (MET), Edge Correlator, Strobe Correlator, and Multipath Estimation Delay Lock Loop (MEDLL) are some of the examples under this category and they will be discussed in the chapter Multipath Mitigation.
These techniques, however, are not very effective for short delay multipath, due to close-by reflectors. These methods cannot be operated in conjunction with all existing receivers and would need manipulation at the receiver hardware end to work. This remains as one of the major issues with using receiver related techniques. [3].

• Miscellaneous methods – These miscellaneous methods counter multipath by using measurement data and other information generated by the receiver. The day-to-day repeatability of multipath at one location along with SNR measurements is in earlier researches. In a simulated multipath environment, the reflection geometry is used in combination with a special GPS antenna arrangement to detect and track multipath. Researchers also used adaptive filters like Kalman Filters and multiple differential GPS receivers to remove multipath and other errors in final positioning. [3]. Code multipath is calibrated and estimated using spherical harmonics in static applications.

1.1 Problem Statement

In the conventional GPS receivers, the received GPS signal, after converting to an Intermediate Frequency (IF) is sampled at or above the Nyquist frequency to estimate the code phase in the acquisition part of the receiver. In order to improve location accuracy and to separate short delay multipaths, the received GPS signal needs to be sampled at a very high rate. This required high sampling rate in turn requires a large signal bandwidth. However, a large signal bandwidth admits more noise and interference, as the GPS signal is below the noise level, hence lower signal to noise ratio (SNR). Lower SNR degrades the receiver performance, especially in dense urban areas where some GPS satellites may be blocked and multipath is abundant.
Compressive Sensing or Compressive Sampling (CS) is a recent break-through technique in the signal processing area, which relaxes the Nyquist sampling criterion. Signals can be sampled below the Nyquist rate, and yet be almost perfectly reconstructed, since this technique solves an underdetermined problem with a very high probability with almost zero error, provided the solution is sparse in some domain.

It is known that cross correlation function (CCF) between the received composite GPS signal with a locally generated code of a particular satellite results in a peak, based on the properties of GPS pseudorandom code. “The wider the bandwidth of the signal, the sharper will be the peak”. Since the GPS code has a bandwidth of at least 2 MHz (C/A code), the CCF peak is quite sharp and the CCF is a very sparse function. If we somehow compute the CCF with a low sampling frequency, as CCF satisfies the property of sparsity, the CS methodology can be applied to reconstruct the CCF with a higher sampling frequency, thereby to estimate the code phase of CCF more accurately.

1.2 Thesis Organization

Chapter 2 briefly describes the Global Positioning System and explains its different components. We then introduce a few terminologies/concepts that are used in the thesis. These are useful while discussing past research efforts, the current methodology of our approach and our simulation results.

Chapter 3 talks about the software defined GPS receiver, current acquisition and tracking modules and different types of search methods available now.
Chapter 4 talks about various other research works and the current state-of-the-art techniques so far to estimate the LOS in the presence of multipath and to better calculate pseudoranges.

Chapter 5 talks about our approach to the stated problem, and explains the below important steps of the thesis

- Compressive Sensing method to reconstruct higher rate CCF samples from low rate samples.
- Comparing the code phase estimates from the previous step of our methodology with the conventional wideband acquisition and the conventional narrow band acquisition methods.

Chapter 6 talks about the interpolation based methods, and continues with the below steps

- Interpolation based method to reconstruct higher rate samples from low rate samples.
- Estimation of multipath signal parameters using MEDLL.

Chapter 7 concludes the thesis by presenting some of the possible challenges and extended topics for future work.
2  GPS background

Global navigation satellite system (GNSS) refers to satellite based navigation systems providing coverage all over the world, although the number of satellites from any GNSS visible at a given time and location varies. Global positioning system (GPS) is the most widely used GNSS, since it has maintained full operational capability for over two decades. The Global Positioning System (GPS) provides reliable location and time information to any user on the Earth at all times. In addition, the line of sight of the location to be calculated should be accessible to four or more GPS satellites [4]. The triangulation based method is used to determine the position accurately. There are no restrictions on GPS accessibility currently and anyone with a GPS receiver can use it freely right now to calculate their position.

The success of GPS has resulted in the development of new GNSS systems by Russia, India, China and the European Union. Russia developed the GLObal'naya NAvigatsionnaya Sputnikovaya Sistema or GLObal NAvigation Satellite System (GLONASS) in 1976 [5]. It was a fully functional navigation constellation around 1995. But since the collapse of the Soviet Union the GLONASS fell apart leading to coverage gaps and only available partially [5]. The Russian Federation pledged to restore the GLONASS and it achieved full global coverage in the year 2011 [5]. The GLONASS signals are a mixture of Frequency Division Multiple Access (FDMA) and Code Division Multiple Access (CDMA) as multiplexing methods with Direct Sequence Spread Spectrum (DSSS) encoding and binary phase-shift keying (BPSK) modulation scheme.

In March 2002, the European Union (EU) and European Space Agency (ESA) agreed to develop the Galileo positioning system, originally scheduled to be operational in 2010 but now expected to be in full service in 2020 at the earliest. According to European Space Agency, Galileo will
perform better in finding location within 1 meter compared to five meters with NAVSTAR GPS [5]. It also offers greater coverage in dense urban areas and the penetration will improve in northern Europe and high latitude areas. Galileo uses the Code Division Multiple Access (CDMA) multiplexing technique with Binary Offset Carrier (BOC) modulation [5] [6].

China has indicated its intent to globalize regional Navigation System called “Beidou”, which started with three satellites in orbit in 2000, now having 10 satellites in orbit offering navigation services in China and neighboring regions, by 2020. Many other satellite systems called Satellite Based Augmentation Systems (SBAS) have been established to assist the current GNSS systems [5]. They include the U.S. Wide Area Augmentation System (WAAS), the Quasi-Zenith Satellite System (QZSS) covering Japan, the European Geostationary Navigation Overlay Service (EGNOS), the Japanese the Multifunctional Satellite Augmentation System (MSAS) and lastly, the Indian GPS Aided Geo Augmented Navigation (GAGAN) system.

In order to respond to the growing competition from other GNSS systems, GPS has recently added modernized signals L2C, L5 and L1C in addition to the conventional L1 C/A and L1/L2 P(Y) signals [7]. These modernized GPS signals and signals from other GNSS systems vary among themselves in terms of signal structure including modulation schemes, carrier frequencies and bandwidths.

Modulation schemes being used for GNSS signals can be broadly divided into two types, namely binary phase shift keying (BPSK) and binary offset carrier (BOC). The conventional L1 C/A, L2C, and L1/L2 P(Y) signals are BPSK modulated, while L5 has two BPSK signals, one in-phase and the other quadrature phase, implying quadrature phase shift keying (QPSK).
### 2.1 GPS System:

The entire GPS system consists of 3 segments [2] [5] as illustrated in the Figure 1: the Space segment, the Control segment and the User segment.

![Figure 1: Structure of the GPS](image)

#### 1. The Space Segment: This segment consists of 24 satellites revolving the Earth in 6 orbital planes (4 satellites per orbit) each with an inclination of 55 degrees. The satellite elliptic orbit radius is 26,561.75 km and each satellite orbits the Earth twice in a sidereal day (23 hours, 56 minutes and 4.09 seconds). The spacing is such that a minimum of 5 to a maximum of 12 satellites is visible at any point of time from a receiver on the Earth. A GPS receiver calculates its three-dimensional location information (latitude, longitude and altitude) and the current time using the broadcasted signals by the GPS satellites. These satellites also send signals to ground stations that monitor and control GPS operations continuously [4].
2. The **Control Segment** – The control segment is a group of ground stations that monitor, control and operate the GPS satellites. It is composed of

(1) A master control station (MCS)

(2) An alternate master control station (AMCS)

(3) Dedicated and shared ground antennas (GA)

(4) Monitor stations (MS)

The ground antennas receive data about orbits, altitude, location and speed from the visible satellites, track the GPS navigation signal, record local atmospheric data, measure carrier phase and pseudo range from the satellite signals. The collected data is transmitted to the master control station which executes GPS command, monitor and control. With this data, the MCS generates and estimates satellite orbits information and clock corrections, generates the navigation message, monitors and maintains the GPS constellation, verifies the received navigation
messages recorded by the MS and maintains time synchronization. The MCS then corrects the orbital and clock information (also known as ephemeris data), and performs the satellite updates through the use of ground antenna preventing the satellites from drifting away from their orbits [5].

The locations of control segment components are shown below.

<table>
<thead>
<tr>
<th>Component</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Master Control Station</td>
<td>Schriever Air Force Base Colorado Springs, CO</td>
</tr>
<tr>
<td>Alternate Master Control Station</td>
<td>Gaithersburg, MD</td>
</tr>
<tr>
<td>Monitor Stations</td>
<td>Colorado Springs, CO; Cape Canaveral, FL; Hawaii; Ascension Islands; Diego Garcia; Kwajalein</td>
</tr>
<tr>
<td>Ground Antennas</td>
<td>Cape Canaveral, FL; Ascension Islands; Diego Garcia; Kwajalein</td>
</tr>
</tbody>
</table>

Table 1: Locations of the Control Segment components

3. **The User segment:** This segment consists of all receivers that are used to receive GPS signals. It is composed of thousands of U.S. users and military users of the secure GPS (PPS) Precise Positioning Service, and thousands of civil, commercial and scientific users of the Standard Positioning Service (SPS) [5]. Since the user receiver does not transmit any signals, the GPS space segment can provide service to an unlimited number of users [5].

GPS receivers consist of an antenna, tuned to the frequencies transmitted by the satellites, a highly stable clock and receiver-processors that measure and decode the satellite transmissions to provide positioning, velocity, and precise timing information to the user by computing the four dimensions of X, Y, Z (position) and time. They may additionally include a graphical display overlay of maps, providing location, speed, altitude and other information to the user [5]. GPS receivers have
parallel multi-channel/receiver circuit design and are often described by the number of channels, denoting the number of satellite signals it can acquire and track simultaneously [5].

2.2 Structure of GPS signal:

Most of the GPS signals are phase modulated signals using BPSK modulation (Binary Phase Shift Keying). In this communication system, the CDMA (Code Division Multiple Access) spread spectrum scheme is used. The signals are modulated by a set of near orthogonal codes. Correlation needs to be performed at a receiver with a locally generated code to acquire an individual signal. Since all GPS satellites use the same carrier frequency, there is a higher chance of interference among signals even with the CDMA scheme. Since the power levels of different satellite signals vary at the receiver end, distinguishing the peak of a weak signal from an interference peak of a strong signal is very important. Otherwise it can lead to acquiring a different satellite signal, or miss the weak signal all together. Therefore maintaining the receiver input SNR is very important.

There are two types of codes in GPS signals - Precision (P) code and Coarse Acquisition (C/A) code. The P(Y) code is reserved for military use alone, whereas the C/A code is used for civilian applications.

The GPS signals are transmitted on two different frequencies: L1 at 1575.42MHz and L2 at 1227.6 MHz. These frequencies are coherent with a 10.23MHz clock related as follows.

\[
\begin{align*}
L_1 &= 154 \times 10.23 \text{ MHz} = 1575.42 \text{ MHz} \\
L_2 &= 120 \times 10.23 \text{ MHz} = 1227.6 \text{ MHz}
\end{align*}
\]
The L1 frequency carries both C/A coded signals and P(Y) signals. The L2 frequency carries only P(Y) signal. Since civilian applications are the focus of this report, acquisition and tracking of GPS signals at L1 frequency is studied in depth but the same principle applies equally to all GPS signals at any frequency.

The GPS signal at L1 and L2 frequencies for a single satellite can be expressed as follows [5]:

\[
S_{L1} = \sqrt{2} P_{L1} P(t) D(t) \cos(2\pi f_1 t + \phi) + \sqrt{2} P_{C/A} C(t) D(t) \sin(2\pi f_1 t + \phi) \\
S_{L2} = \sqrt{2} P_{L2} P(t) D(t) \cos(2\pi f_2 t + \phi)
\]

where \( S_{L1} \): Signal at L1 frequency

\( S_{L2} \): Signal at L2 frequency

\( P(t) \): the P-code, taking values of ±1

\( D(t) \): the navigation data, taking values of ±1

\( f_1 \): Frequency at L1 (1575.42 MHz)

\( f_2 \): Frequency at L2 (1227.6 MHz)

\( \phi \): Initial phase of the signal

\( C(t) \): the C/A code, taking values of ±1

\( P_{C/A}, P_{L1} \) and \( P_{L2} \): Transmission powers of C/A, P code modulated L1 and P code modulated L2.
Figure 3 shows the generation of GPS at L1 frequency [2].

![Figure 3: Generation of GPS at L1 frequency](image)

### 2.2.1 C/A code:

The C/A code or coarse acquisition code is a pseudo random noise (PRN) sequence of 1023 chips that repeats itself every millisecond. Here “chip” is used instead of “bit” to emphasize that there is no information in the PRN code. The PRN sequences used as C/A code are Gold Code. Each C/A code is generated using a tapped linear feedback shift register. It generates a maximal-length sequence of length $N = 2^n - 1$ elements [8]. For the GPS code, $n = 10$ is used. Each satellite has its own unique and fixed C/A code. The C/A code has a chipping rate of $1.023 \times 10^6$ chips/sec. These chipping rates are larger than actual symbol rate because one symbol is represented by multiple chips.
The C/A code generator contains two shift registers, $G_1$ and $G_2$. Each of them have 10 memory blocks generating sequences of length 1023; modulo-2 addition adds each other to generate a 1023 chip-long C/A code. They reset to all ones after every 1023rd period. These two shift registers are described by the below polynomials.

$$G_1 = 1 + x^3 + x^{10}$$

$$G_2 = 1 + x^2 + x^3 + x^6 + x^8 + x^9 + x^{10}$$

To generate a different C/A code for each satellite, $G_2$ will be selectively delayed by choosing two of its states to a modulo-2 adder [8]. The C/A code is generated by a modulo-2 addition of the result from the phase selector and the $G_1$ code.
In the example shown in Figure 4: C/A code generator, the taps 2 and 6 are selected by Phase select logic and they correspond to satellite number 1 [9].

The code taps defined for each of the 32 satellites and the corresponding code delay in chips is listed in Table 2: Satellite ID vs code phase assignments.

### 2.2.2 P(Y) code or P code:

The actual P code is encrypted by a Y code, so it is often referred to as the P(Y) code. The P(Y) code is currently reserved for military use. The P code is bi-phase modulated at 10.23 MHz and the main lobe of the spectrum is 20.46 MHz wide from null-to-null. The P code is generated from two PRN codes with the same chip rate of 10.23 MHz. The code length generated by these two PRN codes is about $2.35 \times 10^4$ chips which is slightly longer than 38 weeks. However the actual length of the P code is 1 week as the code is reset every week starting at the beginning of the GPS week at Saturday/Sunday midnight.

### 2.2.3 Correlation properties of the C/A code:

One of the most significant properties of the C/A code is its high autocorrelation peak value and low cross-correlation values. In order to detect the presence of a weak signal in presence of strong signals, the peak of autocorrelation of the weak signal must be higher than cross-correlation levels of strong signals. Since Gold code is only near orthogonal and not perfectly orthogonal, cross-correlation values are not zero but small.
<table>
<thead>
<tr>
<th>SV PRN Number</th>
<th>C/A Code Tap Selection</th>
<th>C/A Code Delay (Chips)</th>
<th>P Code Delay (Chips)</th>
<th>First 10 C/A Chips (Octal)</th>
<th>First 12 P Chips (Octal)</th>
</tr>
</thead>
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<tr>
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Table 2: Satellite ID vs code phase assignments
The autocorrelation function (ACF) of C/A code is

\[ R_{ii}(\tau) = \int_{-\infty}^{\infty} A_i(t) A_i(t+\tau) dt \]  

(2)

where \( A_i \) is the C/A code from the \( i \)-th satellite, \( \tau \) is the time shift (also called code phase).

The cross correlation function (CCF) between two satellites is defined as

\[ R_{ij}(\tau) = \int_{-\infty}^{\infty} A_i(t) A_j(t+\tau) dt \]  

(3)

where \( A_i \) is the C/A code from the \( i \)-th satellite, \( A_j \) is the C/A code from the \( j \)-th satellite, and \( \tau \) is the time shift and \( i \neq j \).

The value of the cross-correlation of two Gold codes with an even \( n \) value is one of the three values given as Cross-correlation value:

\[
\begin{align*}
&\begin{cases}
2^{(n+1)/2} + 1 \\
\frac{-1}{P} \\
\frac{2^{(n+1)/2} + 1}{P}
\end{cases} \quad \text{for } P = 2^n - 1 \text{ and } n \text{ is even (} n = 10 \text{ for the GPS signal).}
\end{align*}
\]

The good auto correlation property of the C/A code is used to synchronize the locally generated code with the code of the received composite function to detect the time of arrival, and hence the distance to the satellite (pseudorange).
<table>
<thead>
<tr>
<th>Correlation level</th>
<th>Occurrence percentage (%)</th>
<th>Normalized power in dB</th>
<th>Comparison to ACF peak power</th>
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<td>$10\log_{10} \left( -\frac{65}{1023} \right)^2 = -23.9392$</td>
<td>23.94dB below ACF peak power</td>
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<td>-1</td>
<td>75</td>
<td>$10\log_{10} \left( -\frac{1}{1023} \right)^2 = -60.1975$</td>
<td>60.20dB below ACF peak power</td>
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<td>$\frac{63}{1023}$</td>
<td>12.5</td>
<td>$10\log_{10} \left( \frac{63}{1023} \right)^2 = -24.2107$</td>
<td>24.21dB below ACF peak power</td>
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Table 3: Power level and occurrence percentage C/A code correlation

2.2.4 Navigation message:

The navigation message provides a receiver with the location of the satellite at the time of transmission as well as other information such as the almanac. Each satellite transmits its own position in three dimensional coordinates and time. This navigation message is a 50 bit-per-second data stream superimposed on C/A and P-code via modulo-2 addition. When the receiver matches locally generated code with the code from a particular satellite at the carrier frequency, it can start to decode navigation data message from that satellite.
3 Software Defined GPS receiver

The basic functions of a GNSS receiver are:

- Capture RF signals transmitted by the satellites that are currently in view
- Perform measurements of signal travel time and Doppler shift
- Decoding the navigation message in order to determine the parameters like satellite position, velocity, and clock
- Estimating the user position, velocity, and time.

Modern GPS receivers process the signal in digital means in either hardware or software after the digitization of the received signal. The processing of the signal from antenna reception up to digitization stage is still done by analog hardware [5]. Figure 5 shows a generic GPS receiver. The digitized signal is then processed using digital means, either software or hardware. This digital receiver therefore has more flexibility [10] [6].

![Figure 5: GPS Receiver Architecture](image)

A typical GPS receiver consists of an Antenna, a down-converter to down convert L1 signal to Intermediate Frequency (IF), an ADC to sample and digitize the signal. This digitized signal is passed as input to the digital/software receiver.
There are three chief stages in a digital/software GPS receiver: Acquisition, Tracking and Navigation position calculation. In brief, the Acquisition stage is to acquire the signal from satellites, identify the transmitting satellites, and roughly estimate useful parameters such as Code phase and Doppler frequency shift. The tracking stage performs a demodulation function, then splits up the received signal into its In-phase (I) component and Quadrature (Q) component for code phase tracking and frequency tracking. The navigation data extracted is then passed onto the Navigation calculation block for calculating pseudorange, velocity, and user position [5].

**Pre-correlation Bandwidth**

The RF front-end of most conventional GPS receivers is analog. Usually, the incoming signal enters through the antenna and an immediate amplification is done by the preamplifier. It is then routed to a multi-stage down conversion process. A series of intermediate frequency (IF) mixers and filters, down conversion converts the GPS signal (entered at 1.545GHz for L1 C/A code receivers) to near baseband for correlation, tracking, and subsequent processing [11]. The final intermediate frequency (IF) stage filter bandwidth generally sets the pre-correlation bandwidth (PCB). Narrowband receivers pass only the main lobe of the C/A code power spectrum. All others will be considered wideband [11].

**Wideband Interference**

Wideband interference for example white Gaussian noise, in a theoretical sense has constant energy over all frequencies, f, given by

\[
S(f) = \frac{\eta}{2}, \quad -\infty \leq f \leq \infty
\]  

(4)
where $\eta$ is a constant. Although actual wideband interference sources have finite bandwidth, they effectively raise the thermal noise floor of GPS receivers of all pre correlation bandwidths [11].

**Narrowband Interference**

In general, narrowband interference has a bandwidth $<<1$MHz. This type of interference consists of both intentional and unintentional sources including transmitter harmonics and AM and FM stations. For C/A code receivers these jammers pose a significant threat if they occur at specific frequencies in the C/A power spectrum [11].

**Evil Waveforms**

Satellite signal anomalies or “evil waveforms” result from a failure of the signal generating hardware on one of the GPS space vehicles (SV). These anomalies may cause severe distortions of the autocorrelation peak inside GPS receivers. In local area differential systems, undetected evil waveforms may result in large pseudorange errors, which in general do not cancel. These satellite failures are rare events [11].

**3.1 Acquisition module:**

The purpose of the acquisition stage is to

- Find the satellite signals visible to the receiver
- Estimate the coarse C/A code phase from each satellite
- Estimate the coarse satellite received signal carrier frequency due to Doppler shift for each satellite

The Acquisition module in a conventional GPS receiver is shown in the below block diagram.
First, the received GPS signal at the Intermediate Frequency (IF) is brought down to the base band by multiplying with a locally generated IF/Doppler carrier as part of the carrier phase tracking loop. Based on the number of frequency bins selected over the completer Doppler Frequency shift range (usually ± 10 kHz of IF), the size of frequency bin varies. Usually a frequency bin is about 500 Hz in size (41 bins spanning 10 kHz). There might be residual Doppler in this base band version of the received GPS signal. Then the Nyquist samples (sampling at a frequency that is twice the signal bandwidth) are generated for the top branch of the circular correlator. The local C/A code replica generator generates the C/A code specific to a satellite). These samples are generated at the same Nyquist frequency. This is passed as input at the second branch of the circular correlator.

If a satellite is visible, the acquisition module must determine the following properties of the signal.

**Code phase:**

The code phase denotes the point in the current data block where the C/A code begins. The GPS receiver receives a satellite’s signal after the time it travels through the distance between the
satellite and the receiver, which causes a time lag in the received signal code. A GPS receiver generates an exact copy of the transmitted C/A code for each of the satellites. A correlation is performed between the received composite GPS signal and the locally generated replica of the C/A code. The amount of time delay used in the correlation process to match what is received corresponds to the time lag and is defined as the Code phase. This in turn is used to determine the pseudorange from the transmitting satellite to the receiver.

**Doppler Frequency:**

The frequency of the signal from a specific satellite can differ from its nominal value. In case of down-conversion, the nominal frequency of the GPS signal corresponds to the IF. However, the signals are affected by Doppler Effect (relative motion of the satellite with respect to the receiver). It is important to estimate the Doppler frequency as close as possible in order to properly demodulate the received signal. For a stationary receiver on the Earth, the Doppler frequency shift will not exceed 5 kHz. For a user with very high velocity combined with maximum velocity of the satellite, the Doppler frequency shift can reach as high as 10 kHz.

There are three commonly used Acquisition algorithms based on implementation technique and algorithm performance.

3.1.1 **Serial search in Time domain**

In this method, each possible combination of frequency and the C/A code offset is assessed serially with dedicated correlators to acquiring a particular satellite or the C/A code, one satellite at a time during the process. For faster acquisition rate, multiple correlators can be used where each correlator uses a unique C/A code replica code representing that particular satellite [5].
Though this search method is fairly easy to implement in software and hardware, it is very time consuming for a large number of frequency or code bins.

### 3.1.2 Parallel search in Frequency domain

These methods basically parallelize the frequency search space by utilizing a Fast Fourier Transform (FFT). The incoming IF samples need to be correlated with replica digital versions of the C/A code (phase shifted between 0 and 1022 chips). This process is performed in frequency domain by the faster FFT transform. When the C/A code aligns with the locally generated replica code, a distinct peak in terms of magnitude is observed in the FFT results [5]. The frequency location of this observed peak identifies the frequency of the incoming signal and the shift used in local replica code generator identifies the needed Code phase.

### 3.1.3 Parallel code phase search

These methods parallelize the code phase space therefore disregarding the need to search through all possible code phases. This leaves with only the number of frequency bins that constitute the Doppler shift range of ±10kHz. If the incoming signal samples can be stripped off the intermediate frequency carrier, it will be transformed to baseband leaving only the received C/A code [5]. Acquisition can be performed using Discrete Fourier Transform (DFT) based circular correlation between baseband version of received C/A code and locally generated C/A code. The absolute value of the Inverse Fast Fourier Transform (IFFT) represents the degree of correlation match between the input C/A code and the locally generated replica code [5].

### 3.2 Tracking module:

After the acquisition process is complete, the needed parameters such as code phase and carrier frequency shift are sent to the tracking stage. A delay-locked loop (DLL) is used to track the C/A
code phase and a phase-locked loop (PLL) is used to track the carrier frequency of the incoming signal with Doppler shift.

When the carrier frequency changes, the receiver loses track of the satellite signal. The carrier tracking process can keep phase lock of the satellite signal as long as the frequency shift is within the allowed range. The discrepancy in carrier frequency is sent as a control signal to the numerically controlled oscillator block (NCO) which generates the carrier frequency. In general, a second order Costas loop is used in the tracking process. The original signal is then down-converted to baseband after the frequency tracking loop.

When a code phase shift occurs, the correlation peak will either slide to the left or right of the ideal correlation curve. Therefore an early (E) code and a late (L) code are needed to determine the direction of the shift. The prompt (P) code is moved to the left or right based on the difference between E and L codes. This way the code phase delay can be accurately detected.

When both tracking loops are in lock, it is possible to decode the navigation data message. This is because after the C/A code is stripped off the incoming signal, the data is still modulated by the carrier frequency. For more information on tracking loop practical implementation please refer to [2].

### 3.3 Navigation and User position calculation

Once the tracking loops work properly, the receiver can then estimate exactly when the start of a frame arrives at the receiver, from which a pseudorange can be derived. Precise estimation of the pseudorange from a satellite to a receiver is crucial for a GPS receiver. A pseudorange measurement is computed as the time travel from the satellite to the receiver multiplied by the speed of light in vacuum.
The pseudorange is an estimate and can be erroneous due to receiver clock errors or bias or other errors, including satellite position error on range or atmospheric error. To account for clock offset a GPS receiver needs pseudoranges from at least 4 visible satellites to give a good position estimate. For more detailed explanation on navigation message decoding please refer to [2].
4 Multipath Mitigation

A wide variety of error sources affect accuracy of the basic GPS measurements of pseudorange (also known as code-phase) and integrated Doppler (also known as carrier-phase). Among these are satellite clock and ephemeris errors (known as user range error or URE), ionospheric delay, tropospheric delay, receiver dynamic tracking error, multipath and thermal noise. For the non-differential user, the broadcast ionospheric correction model reduces the ionospheric error by approximately 50% on average [2] [12]. If the user has a dual-frequency receiver, ionospheric delay can be eliminated almost entirely. The standard tropospheric correction model reduces tropospheric error approximately 90% on average [2] [12].

The use of differential techniques theoretically eliminates all error sources which are common to both receivers. For short baselines and fast differential update rates, the common errors are URE, ionospheric and tropospheric delays. Receiver dynamic tracking errors can be controlled through receiver architecture design and thermal noise errors can be reduced through rate-aiding techniques such as carrier-smoothing. The error which remains is multipath. As a result, multipath is the dominant error source in high precision GPS applications.

4.1 The Multipath Problem

For GPS users, Multipath propagation is a phenomenon occurring in wireless telecommunication systems that effects in radio signals reaching the receiving antenna by two or more than two paths. Causes of multipath include ionospheric reflection and refraction, atmospheric ducting, and reflection from nearby objects, water bodies, other reflecting surfaces etc. The reflecting surface may be buildings, hills, ground, water, or any object that happens to be a radio reflector [4]. Multipath errors result when the receiver receives the direct or line-of-sight (LOS) satellite signal
via multiple paths and processes the combined signal as if it were only the direct. A generic multipath propagation diagram is shown in Figure 7.

![Multipath Propagation Diagram](image)

**Figure 7: Multipath Propagation**

Multipath can cause long-term stable error values or it can cause the user’s position to change at fluctuating rates. Multipath can be constructive or destructive. In some cases GPS wanderings arising due to multipath can cause the GPS to jump from one position to another as the multipath signal is unpredictable. It may also cause the GPS to hurdle from using one group of inaccurate signals to another [4]. These jumps can add considerable distances to the pseudo range measurements in GPS receivers.

These multipath errors are particularly difficult to remove since, in general, the following is true:

1) The pseudorange measurement is derived from a code-tracking delay-lock loop (DLL). DLL essentially attempts to derive time-of-arrival measurements from incoming signal amplitudes. This is accomplished by maximizing the signal code autocorrelation function. In the receiver, this translates to using a minimum of 2 correlators to straddle the peak (e.g., such that Early correlator value - Late correlator value=0) [11]. Since the combined LOS and multipath (MP) auto correlation function will have a distorted shape, the distortion introduces a tracking error into the DLL.
2) Pseudorange errors due to multipath, in general are nonlinear functions of multipath amplitude delay, phase and phase rate [11]. In accordance, changes in any of these parameters may significantly change the tracking response of the DLL in the tracking block.

3) Multipath errors do not have zero mean. This is very true for multipath signals with relatively large amplitudes. In consequence, even infinite smoothing of the pseudorange cannot warranty unbiased position errors [11].

4) Multipath is not spatially correlated. Multipath signals affecting a receiver at one location will not affect a receiver at another location in the same way. Hence, processing by using differential methods is not effective against multipath [11].

4.2 Types of Multipath:

Multipath (MP) is classified into three types depending on the delay of MP signal from LOS signal for simulation purposes.

4.2.1 Long delay multipath

If the MP signals are reflected off surfaces far from the GPS receiver and they arrive with more than 1 chip delay after LOS signal, they are considered to be long delay MP signals. For these signals, cumulative MP delay is more than 1 chip compared to LOS. The effect of this multipath is cancelled out when the discriminator function uses half-a-chip delay between early and late correlators in the tracking module.

4.2.2 Medium delay multipath

If the MP signals are reflected off from surfaces and buildings close by to the GPS receiver and they arrive between 0.35 and 1 chip delay, they are considered to be medium delay multipath signals. These MP signals generally occur in urban areas and heavily populated areas.
4.2.3 Short delay multipath

If cumulative MP delay is less than 0.35 chips between MP and LOS signals, the multipath is classified as short delay multipath. These are the signals which are reflected off surfaces very close to the GPS receiver, say a wall next to the receiver or the ground below it. These are very hard to mitigate and contribute a major part to MP error in correlator based methods. Both the medium and short delay multipaths distort the correlation function and a generic impact of multipath on correlation shape (BPSK modulation) is shown in Figure 8.

![Figure 8: Correlation Function distortion due to multipath](image)

In the correlation function distortion due to multipath of Figure 8, the blue dotted line shows the correlation function for LOS signal alone, the green dotted line is a MP signal delayed in time by 0.5 chips and half the amplitude. The red curve shows the distorted correlation function introduced by using constructive multipath.
4.3 Current Multipath Mitigation Methods

Several multipath mitigation methods for GPS signals have been developed in recent past. Among them, the use of special multipath limiting antennas (i.e. choke ring ground antenna setup or multi-beam antennas), post processing techniques to diminish the effect of carrier multipath, carrier smoothing to lessen code multipath, and code tracking algorithms built on receiver internal correlation techniques are some of the most noticeable approaches [13]. All existing receiver internal methods for mitigating multipath can be broadly classified into three different categories as shown in Figure 9.

![Figure 9: Classification of Multipath Mitigation methods](image)

4.3.1 Methods based on correlation

These methods are based on correlation of the received composite GPS signal with locally generated C/A code. They have a delay locked loop (DLL) to track the code phase and a phase locked loop (PLL) to track the carrier frequency. A discriminator function is used in the tracking
block that optimally combines the In-phase and Quadrature measurements to estimate code and phase error and send it back to DLL and PLL.

To avoid the effects of multipath, Narrow Correlator (NC or nEML) was introduced by Novatel. Instead of using a standard correlator with 1 chip spacing between early and late correlators, a spacing of less than 1 (usually 0.1 chips) is used to build up the discriminator function [14]. The choice of correlator spacing in the tracking block depends on the receiver’s available front end bandwidth along with associated sampling frequency [13]. This provides moderate to long delay multipath mitigation and excellent noise and interference mitigation. Double Delta technique (ΔΔ) uses a family of double difference correlators instead of only one in the code discriminator function. A few well-known ΔΔ correlators are High Resolution Correlator (HRC) where the wide pair of correlators has exactly double the chip spacing of the narrow pair [15], Strobe Correlator (SC) which utilizes the linear combination of two correlators as discriminator output and adjusts chip spacing to track signal [16], Pulse Aperture Correlator (PAC) [17] etc. Early/Late slope technique is another multipath mitigation technique that works by determining the slope on both sides of the peak of autocorrelation function and using it to compute the pseudorange correction. Early1/Early2 (E1/E2) tracker is an attempt to track at the point in the autocorrelation function that is not distorted by the presence of the multipath signal [18]. This is achieved by having two correlators on the early slope of the Autocorrelation function separated by a fixed chip spacing. The minimum long delay multipath mitigation of E1/E2 method would not be as good as strobe correlator with a comparable noise level. Since these methods use a correlator, elimination of short delay multipath would require high sampling rates and therefore high pre-correlation bandwidths.
4.3.2 Methods based on estimation

These methods are based on maximum likelihood (ML) estimation. To calculate delay and amplitude estimates of the LOS and the multipath, a ML cost function is defined and its gradient estimated. One of the most satisfactory multipath mitigation techniques is the Multipath Estimating Delay Lock Loop (MEDLL) introduced by Richard van Nee [13] [19] [4]. The MEDLL uses multiple correlators in order to accurately determine the shape of heavy multipath distorted correlation function. Then, a reference function is used to determine the best mixture of LOS and non-LOS components (amplitudes, delays and phases) that would attain the distorted correlation function [13]. The MEDLL is very computationally expensive because of extensive correlator structure per channel but it provides superior multipath mitigation performance compared to narrow correlator. Reduced Search Space Maximum Likelihood Delay Estimator (RSSML) introduced by Bhuiyan et al [13] is another illustration of a maximum likelihood based approach that is capable of mitigating multipath effects with the overhead of higher complexity [20]. The RSSML algorithm attempts to compensate for multipath errors by performing a non-linear curve fitting on the input correlation function from a set of ideal reference correlation functions [13]. In Fast Iterative Maximum Likelihood Algorithm (FIMLA), Newton type technique is used to implement maximum likelihood estimator. By exploiting the GNSS signal model and the code periodicity, the log-likelihood function has been simplified and first and second derivatives are derived [21]. The FIMLA estimator is both effective and of very low computational cost. In all, these methods try to estimate the shape of the multipath corrupted correlation function by using several correlators per channel. These methods also need an accurate reference correlation function for better performance.
4.3.3 Miscellaneous Methods

These methods mitigate multipath based on miscellaneous parameters like Azimuth or Elevation of the satellite or Geometry of the satellite or by modeling of the space channel. These methods depend more on the channel impulse response rather than the correlation function to mitigate multipath. They also try to take advantage by using the GPS data from previous day(s) as GPS data repeats itself every sidereal day.

De-convolution methods [22] and Projection-Onto-Convex-Sets (POCS) [23] take strongest signal into consideration instead of first arriving signal. They use channel impulse response rather than correlation function and are applied for 3 cases – perfect outdoors, typical outdoors and light indoors. The efficiency of these methods highly depends on sampling frequency and pre-correlation bandwidth. Adaptive Filter methods can be used to reduce code and carrier multipath errors in GPS. The filter employed in [24] used a tap-delay line with an adaptive linear neuron (ADALINE) network to estimate the direction and delayed signal parameters. The multipath effect is mitigated by subtracting the estimated multipath effects from processed correlation function. The premise of Multipath Path Invariant (MPI) method [25] was that there exist regions and or properties of auto correlation function that do not vary as a function of multipath parameters [11]. Multiple MPI point locations for a PRN number can be used to check for the bias and then multipath effect can be cancelled out. The major issue with MPI method is that the sections that are supposed to be multipath invariant tend to be affected very much by the level of noise and thus cannot be used as reference points in practice. Also the convergence time of this method is too long which translates to part of initialization time. These methods require modifications to the antenna front end, or very high sampling frequency in case of POCS, or GPS data from previous days. These methods are more suited for either stationary GPS receivers or receivers onto which
additional circuitry can be added. These methods may not be ideal for fast travelling GPS receivers or when there is a limitation on receiver weight.

Most high performance GPS receivers today are wideband. In most cases, they require wider pre-correlation bandwidths for enhanced multipath mitigation capability. Wideband receivers, however, are still significantly more susceptible to electromagnetic interference. In addition, they are more likely to produce unacceptably large pseudorange errors in the presence of satellite signal anomalies. Conversely, narrowband receivers are less vulnerable to narrowband interference and more robust against GPS signal faults [11]. However, they tend to have relatively poor multipath performance.
5 Proposed Method

Compressive Sensing or Compressive Sampling (CS) has recently emerged as a breakthrough technique in the areas of signal processing, wireless communications, information theory, etc. It was introduced by E.J. Candes, J. Romberg, T. Tao and D. L. Donoho, [26, 27] in 2006, and it has been researched heavily since then. With the assumption that the signal to be reconstructed is sparse in some domain, the signal can be sampled with a rate well below the Nyquist rate, and a highly underdetermined system of equations could be efficiently solved using CS reconstruction algorithms with a very high probability and almost no error to recover the sparse signal.

The goal of this chapter is to estimate a high sampling rate cross correlation function using a low sampling rate cross correlation function. The cross correlation function of super-Nyquist rate samples can be estimated using the cross correlation function of the Nyquist rate samples from a narrowband RF frontend. This estimation is possible using CS because the cross correlation function (CCF) of a wideband signal like GPS is sparse. This eliminates the need for a wide band RF front end of 8 or 16 MHz. Elimination of noise or interference spikes in the GPS receiver is possible with narrowband RF front end. Once a higher rate correlation function is estimated, the code phase of CCF can be estimated more accurately. The obtained CCF peak time is needed as an input parameter to the tracking block of a GPS receiver. The better the code phase value, the better is the initialization of the tracking block.

5.1 Compressive Sampling

Consider a vector $x$ of dimension given by $n$, which represents the Nyquist samples of a particular signal of interest. These samples are equivalent to the analog version of the interested signal. Note
that only the analog signal is available to us and the Nyquist rate samples are not available [28].

The representation of the analog signal using \( x \) is only for the convenience of the demonstration.

In CS, a low dimensional measurement vector, say \( y \), is obtained from \( x \) using a sensing matrix \( A \) with the relation \( y = Ax \). The dimension of the measurement vector is considered to be \( m \) where \( m \ll n \) and the sensing matrix \( A \) is \( m \times n \). Since \( m \ll n \), the matrix \( A \) is non-invertible, so that there exists infinitely possible solutions for \( x \) if we are to recover vector \( x \) from \( y \). To find the correct \( x \), the signal of interest has to be sparse in some domain i.e., it should have only a few non-zero entries in at least one of the domains [28]. Mathematically, \( \| x \|_0 = \# \{ k : x_k \neq 0 \} \) where \( k \) is equal to the number of nonzero entries in the signal \( x \) if the vector \( x \) is sparse in the time domain. The vector could also be sparse in some other domain, i.e., let \( x = \Psi s \), where \( \Psi \) is a transformation matrix and \( s \) contains the coefficients of \( x \) in the transformed domain. If \( x \) is sparse in this domain, then \( \| s \|_0 = \# \{ k : s_k \neq 0 \} \) so that \( s \) is sparse. Then \( y = A \Psi s = \Phi s \).

The CS problem essentially is to recover \( s \) from the low dimensional measurement vector \( y \) using the relation \( y = A \Psi s = \Phi s \) and the prior knowledge that the signal of interest is sparse in \( s \). If the signal is sparse in its own domain, i.e. time domain, then \( \Psi = I \), where \( I \) stands for an identity matrix and \( s = x \). To solve \( s \) from such an underdetermined system of equations based on \( y \), the following optimization problem (\( l_1 \)-norm minimization) can be solved

\[
\min_{s} \| s \|_1 \quad \text{subject to} \quad y = A \Psi s \quad (5)
\]

In a noisy setting scheme, this reconstruction optimization problem becomes

\[
\min_{s} \| s \|_1 \quad \text{subject to} \quad \| A \Psi s - y \|_2^2 \leq \varepsilon \quad \text{where} \quad \varepsilon > 0 \quad (6)
\]
Many reconstruction algorithms exist in the CS literature, which solve this linear programming problem efficiently [29]. In this work, the Compressive Sampling Matching Pursuit (CoSaMP) algorithm [30] is used.

One crucial reconstructing condition is that the matrix $\Phi$ needs to satisfy a restricted isometry property (RIP) [27]. It was shown that a random measurement matrix $\mathbf{A}$ satisfies this property [31]. Randomness preserves the most information. In reality, designing such hardware to satisfy above statement is not possible as ADCs sample a signal at a constant rate and not at random time instants [28].

In [32] [33], schemes for practically implementable uniform sampling at a lower than the Nyquist rate are provided, which is illustrated in Figure 10 [28].

In this setting, $x(t)$ is the incoming signal, $p(t)$ is a periodic random sequence with a constant value in one Nyquist time interval i.e.,

$$p(t) = c[n], \quad nT_{NYQ} \leq t \leq (n+1)T_{NYQ}$$  \hspace{1cm} (7)

The random sequence could be generated using a shift register. With today’s technology, such random analog sequences with a very high alternation rate (in GHz range) are achievable. This
random sequence is mixed with the incoming signal by a common analog mixer. The mixer output is then low pass filtered, and then sampled at a rate $M$ times below the Nyquist sampling rate. Each row of the $A$ matrix is then consisted of samples of $c[n]$ in one period. More details are given in Section 5.3.

### 5.2 Existing work On GPS Using CS

Some work using CS in GPS has recently emerged [34] [35]. The main objective of [34] and [35] is to reduce the total number of correlators in a GPS receiver. They completely change the conventional GPS acquisition structure, in that they do not explicitly correlate the received GPS signals with the conventional known GPS's pseudo-random code, which requires many parallel correlators due to multiple satellites and unknown Doppler frequency shifts. Both [34] and [35] use a smaller number of parallel correlation channels compared to a conventional GPS receiver, and correlate with pre-defined periodic waveforms or mixing signals such as the Jacket-Hadamard waveform [35], which are not GPS pseudo-random code as in a conventional GPS receiver. The determination of each satellite's code phase and Doppler is by using a compressive sampling reconstruction method with dictionaries, which is totally different from the conventional correlation and peak detection method. This method must work with multiple parallel channels, as it is simply not applicable to just one channel.

On the other hand, our objective is not to reduce the total number of correlators of a conventional GPS receiver. Instead, our objective is to improve a GPS receiver's ability of multipath mitigation but still use the conventional GPS receiver's narrowband RF frontend. So we do not alter the conventional GPS acquisition structure much. We do not change the conventional GPS code phase and Doppler detection using peak detection on correlator outputs either. In addition, our method
applies to one correlation channel at a time. It can be applied to multiple channels by repeating the same structure.

The methods of [34] and [35] do not change the correlator output resolution explicitly. They could do it implicitly, though, by increasing the dictionary size, but will be at a cost of drastically increased dictionary size and hence increased computational complexity. Our method does not need any dictionary. The matrix $\Phi$ in our method is not a dictionary (see Section 5.3).

5.3 Proposed Narrowband Acquisition Stage Using CS

This work is an application and extension of previous work [28], in which CS is applied to estimate directions of arriving (DOA) wideband signals. The theme of [28] which is also applied here is estimating a high sampling rate cross correlation function using a low sampling rate cross correlation function. The cross correlation function of the Nyquist samples can be estimated using the cross correlation function of sub-Nyquist samples, or to put it another way, the cross correlation function of super-Nyquist rate samples can be estimated using the cross correlation function of the Nyquist rate samples from a narrowband RF frontend. This estimation is possible using CS because a wideband signal’s cross correlation function is sparse, such as that of the GPS signal.

It is known that CCF between the received composite GPS signal and a locally generated code of a particular satellite results in a peak, based on the properties of GPS code. “The wider the bandwidth of the signal, the sharper will be the peak”. Since the GPS code has a bandwidth of at least 2 MHz, the CCF peak is quite sharp and the CCF is a very sparse function. Therefore, the CS methodology can be applied to reconstruct the CCF with a higher sampling frequency from the CCF of an actual low sampling frequency. Once a higher rate correlation function is estimated, the code phase of CCF can be estimated more accurately. The obtained CCF peak time is needed as
an input parameter to the tracking block of a GPS receiver. With this more accurate timing information, the tracking block can be better initialized.

5.3.1 CS Sampling Scheme

The purpose of the RF frontend after antenna reception is to down-convert the received RF GPS signal to an Intermediate Frequency (IF) which is much lower than the input carrier frequency, and eventually further converted to the baseband. Sampling could be done either at the IF, then digitally down-converted to the baseband, or directly at the baseband. Let $x(t)$ denote the down-converted GPS signal. In a conventional GPS receiver, this signal is passed through an ADC at the Nyquist sampling rate $F_S = 4\, \text{MHz}$ to collect the digitized baseband samples $x[n]$. The digitized samples are then passed onto the acquisition part of the digital processing block. In the proposed method, the IF or baseband GPS signal is also sampled at the same Nyquist rate. But with CS sampling so that the correlation function can be reconstructed at a higher rate, the super-Nyquist rate, without having to sample at such a higher rate.

The CS sampling scheme is depicted in Figure 10, but modified from sub-Nyquist vs. Nyquist to Nyquist vs. super-Nyquist, since now the ADC will be working at the Nyquist rate but super-Nyquist correlation samples are to be reconstructed. Let’s assume the incoming signal is $x(t)$, the Nyquist sampling rate is $F_S$ and $M$ is the super-sampling ratio, so that the super-Nyquist sampling frequency is $F = F_SM$. The super-Nyquist sampling interval is then $T = 1/F$. The received signal is first mixed with a periodic random sequence $p(t)$ generated at the desired super-Nyquist sampling rate as described in Compressive Sampling. The mixed analog sequence $\hat{x}(t) = x(t)p(t)$ is then passed through a low-pass filter with a cutoff frequency of $1/(2F_s)$, and then sampled at
the Nyquist rate $F_s$ [28]. Using the notation $x[n] = x(nT)$, the super-Nyquist signal samples that we do not have, the $l$th Nyquist rate CS sample $y[l]$ that we obtain can be written as

$$y[l] = \frac{1}{MT} \int_{lMT}^{(l+1)MT} p(t)x(t)dt$$

$$= \sum_{m=0}^{M-1} c[m] \frac{1}{MT} \int_{(lM+m)T}^{(lM+m+1)T} x(t)dt$$

$$= \frac{1}{M} \sum_{m=0}^{M-1} c[m] x[lM + m]$$

where $x[lM + m]$ are the super-Nyquist rate samples of $x(t)$ defined by

$$x[m] = \frac{1}{T} \int_{mT}^{(m+1)T} x(t)dt$$.

### 5.4 Generation of Super-Nyquist CCF Samples

![Block diagram of proposed narrowband super-Nyquist rate correlation using CS](image)

*Figure 11: Block diagram of proposed narrowband super-Nyquist rate correlation using CS*
Figure 11 illustrates a possible implementation of the proposed scheme. There are two paths in this figure before the Nyquist rate correlation, one path (the first) that comes from CS sampling the received GPS signal, and the other path (the second) that contains internally generated digital C/A code. Shown in Figure 11 are two branches in each path. In actual implementation, however, any number of branches can be used in either path. To save analog hardware, in our simulations illustrated in Section 5.5, the first path has only one branch, and the second path has three branches.

This scheme is an extension of a CS DOA estimation method to GPS application and to circular correlation rather than linear correlation, since the C/A code is time periodic [28]. One time period of the C/A code equaling 1ms of data can be used for acquisition. Using the sampling mechanism of Figure 10, for a periodic super-Nyquist random sequence \( p_{1a}(t) \), the Nyquist samples (factor of \( M \) from the super-Nyquist rate) are obtained for the \( a \)-th branch of the first path, \( y_{1a}[l] \) (the first subscript ‘1’ is used for this first path). The local C/A code replica generator generates the C/A code at the super-Nyquist frequency which is then mixed with a periodic random sequence \( p_{2b}(t) \) at the super-Nyquist frequency. This is LP filtered and decimated by the factor \( M \) to generate \( y_{2b}[l] \) (the first subscript ‘2’ is used for this second path).

Now, the circular correlators in the block “Nyquist rate correlators” calculate the cross-correlation of the samples from two branches ‘1a’ and ‘2b’ as follows

\[
r_{y_{1a},y_{2b}}[q] = \sum_{l=0}^{K-1} y_{1a}[l] y_{2b}^*[l+q]_K
\]

where \( K \) is the number of Nyquist samples acquired and \([\cdot]_K\) denotes the circularly shifted samples with the subscript \( K \) representing the period in circular shift. Because of the CS sampling
scheme [28], $K$ Nyquist samples will contain the information of $L = KM$ super-Nyquist samples as shown by

$$ r_{y_1y_2}[q] = \sum_{l=0}^{K-1} \left( \frac{1}{M} \sum_{m_1=0}^{M-1} c_{1a}[m_1] x_1[lM + m_1] \right) \left( \frac{1}{M} \sum_{m_2=0}^{M-1} c_{2b}[m_2] x_2^*[lM + qM + m_2] \right)_{KM} $$

$$ = \frac{1}{M^2} \sum_{m_1=0}^{M-1} \sum_{m_2=0}^{M-1} c_{1a}[m_1] c_{2b}^*[m_2] \left( \sum_{l=0}^{K-1} x_1[lM + m_1] x_2^*[lM + qM + m_2] \right)_{KM} $$

(10)

Let us analyze the above summation in more detail by expanding over $m_1$ and $m_2$ as

$$ r_{y_1y_2}[q] = \frac{1}{M^2} \left\{ \right. $c_{1a}[M-1]c_{2b}^*[0] \sum_{l=0}^{K-1} x_1[lM + M - 1] x_2^*[lM + qM]_{KM} $$

$$ + c_{1a}[M-2]c_{2b}^*[0] \sum_{l=0}^{K-1} x_1[lM + M - 2] x_2^*[lM + qM]_{KM} $$

$$ + c_{1a}[M-1]c_{2b}^*[1] \sum_{l=0}^{K-1} x_1[lM + M - 1] x_2^*[lM + qM + 1]_{KM} $$

$$ + \cdots + c_{1a}[0]c_{2b}^*[0] \sum_{l=0}^{K-1} x_1[lM] x_2^*[lM + qM]_{KM} $$

$$ + c_{1a}[1]c_{2b}^*[1] \sum_{l=0}^{K-1} x_1[lM + 1] x_2^*[lM + qM + 1]_{KM} + \cdots $$. 

44
\[
+ c_{1a}[M - 1]c_{2h}^*[M - 1] \\
\bullet \sum_{l=0}^{K-1} x_i[lM + M - 1]x_2^*[lM + qM + M - 1]_{KM} \\
+ \ldots \\
+ c_{1a}[0]c_{2h}^*[M - 2] \sum_{l=0}^{K-1} x_i[lM]x_2^*[lM + qM + M - 2]_{KM} \\
+ c_{1a}[1]c_{2h}^*[M - 1] \sum_{l=0}^{K-1} x_i[lM + 1]x_2^*[lM + qM + M - 1]_{KM} \\
+ c_{1a}[0]c_{2h}^*[M - 1] \sum_{l=0}^{K-1} x_i[lM]x_2^*[lM + qM + M - 1]_{KM} \}
\]

Here if we assume

\[
r^{(h)}_{x_i x_2}(qM + n) = \sum_{l=0}^{K-1} x_i[lM + h]x_2^*[lM + qM + h + n]_{KM}, 0 \leq h \leq M - 1, n \in [-M + 1, \ldots, 0, \ldots, M - 1]
\]

so that

\[
\sum_{h=0}^{M-1} r^{(h)}_{x_i x_2}(qM + n) = r_{x_i x_2}(qM + n)
\]

we then have

\[
r_{x_i x_2}(q) = \frac{1}{M^2} \left\{ c_{1a}[M - 1]c_{2h}^*[0]r^{(M-1)}_{x_i x_2}(qM - M + 1) \\
+ c_{1a}[M - 2]c_{2h}^*[0]r^{(M-2)}_{x_i x_2}(qM - M + 2) \\
+ c_{1a}[M - 1]c_{2h}^*[1]r^{(M-1)}_{x_i x_2}(qM - M + 2) + \ldots \\
+ c_{1a}[0]c_{2h}^*[0]r^{(0)}_{x_i x_2}(qM) + c_{1a}[1]c_{2h}^*[1]r^{(1)}_{x_i x_2}(qM) + \ldots \\
+ c_{1a}[M - 1]c_{2h}^*[M - 1]r^{(M-1)}_{x_i x_2}(qM) + \ldots \\
+ c_{1a}[0]c_{2h}^*[M - 2]r^{(0)}_{x_i x_2}(qM + M - 2) \\
+ c_{1a}[1]c_{2h}^*[M - 1]r^{(1)}_{x_i x_2}(qM + M - 2) \\
+ c_{1a}[0]c_{2h}^*[M - 1]r^{(0)}_{x_i x_2}(qM + M - 1) \right\}
\]

45
We can see that each sample $q$ in the Nyquist rate correlation contains $2M - 1$ groups of super-Nyquist rate “correlation” terms. But each group contains $M - |n|$ “correlation” terms of different $h$. In addition, each “correlation” term skips some super-Nyquist samples and is incomplete from the super-Nyquist rate point of view [28]. However, the complete super-Nyquist rate correlation terms as follows can be obtained according to (13)

$$ r_{q,M} [qM + 1], r_{q,M} [qM + 2], \ldots, r_{q,M} [qM + M - 1] $$

(15)

Using $r_{q,M} [q]$ of (14), we now construct a Nyquist rate correlation vector of size $N_1N_2N_R$, where $N_1$ is the number of branches in the first path of Figure 11, $N_2$ is the number of branches in the second path, and $N_R$ is the number of possible code phases for the Nyquist rate correlation (1023x4 for the C/A code with $F_S = 4 \text{ MHz}$). In this simulation we used $N_1 = 1$ and $N_2 = 3$, therefore,

$$ r_y^T = \begin{bmatrix} r_{y_1y_1}[0], & r_{y_1y_2}[0], & r_{y_1y_2}[0], & r_{y_1y_2}[1], \ldots, & r_{y_1y_2}[N_R - 1], & r_{y_1y_2}[N_R - 1], & r_{y_1y_2}[N_R - 1] \end{bmatrix} $$

(16)

Based on Equation (14) we can write $r_y$ as

$$ M^2 r_y = \Phi \hat{r}_x $$

(17)

where the matrix $\Phi$ is of size $N_1N_2N_R \times M^2 N_R$ and is given by

$$ \Phi = \begin{bmatrix} C_1 & C_2 & 0 & \cdots & 0 \\ 0 & C_1 & C_2 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & C_1 & C_2 \\ C_2 & \cdots & \cdots & 0 & C_1 \end{bmatrix}_{N_1N_2N_R \times M^2 N_R} $$

(18)
where $C_1$ is given by

$$
C_1 = \begin{bmatrix}
0_{M-1} & v_{M-1} & 0_{M-2} & v_{M-2} & 0_{M-3} & v_{M-3} & \cdots & 0_1 & v_1 & v_0
\end{bmatrix}
$$
(19)

and $C_2$ is given by

$$
C_2 = \begin{bmatrix}
w_1 & 0_1 & w_2 & 0_2 & w_3 & 0_3 & \cdots & w_{M-1} & 0_{M-1} & 0_M
\end{bmatrix}
$$
(20)

Here $0_j$ is a zero matrix of dimension $N_i N_2 \times I$, and

$$
v_d = \begin{bmatrix}
C_{d,0}^{11} & C_{d+1,0}^{11} & \cdots & C_{M-1,d}^{11} \\
C_{d,0}^{12} & C_{d+1,0}^{12} & \cdots & C_{M-1,d}^{12} \\
\vdots & \vdots & \ddots & \vdots \\
C_{d,0}^{N_1 N_2} & C_{d+1,0}^{N_1 N_2} & \cdots & C_{M-1,d}^{N_1 N_2}
\end{bmatrix}
$$

$$
w_d = \begin{bmatrix}
C_{0,d}^{11} & C_{1,d+1}^{11} & \cdots & C_{M-1,d-1}^{11} \\
C_{0,d}^{12} & C_{1,d+1}^{12} & \cdots & C_{M-1,d-1}^{12} \\
\vdots & \vdots & \ddots & \vdots \\
C_{0,d}^{N_1 N_2} & C_{1,d+1}^{N_1 N_2} & \cdots & C_{M-1,d-1}^{N_1 N_2}
\end{bmatrix}
$$

(21)  (22)

It can be shown that the matrix $\Phi$ will have full rank and will satisfy the restricted isometric property (RIP). The vector to be reconstructed $\hat{x}$ is a sparse vector of size $M^2 N_r$ containing the approximate super-Nyquist rate correlation entries and is given by

$$
\hat{x} = \begin{bmatrix}
\hat{x}_x^T(0) & \hat{x}_x^T(1) & \cdots & \hat{x}_x^T(N_r - 1)
\end{bmatrix}^T
$$
(23)
\[ \mathbf{\hat{r}}_x[g] = \begin{bmatrix} r_{x_1}^{(0)}[gM - M + 1] & \cdots & r_{x_1}^{(M-1)}[gM - M + 1] \\ r_{x_2}^{(0)}[gM - M + 2] & \cdots & r_{x_2}^{(M-1)}[gM - M + 2] \end{bmatrix}^T \]

(24)

5.4.1 Estimation of Code Phase Using Peak of CCF

The model given by \( M^2 \mathbf{r}_y = \Phi \mathbf{\hat{r}}_y = \Phi \mathbf{\hat{r}}_x \) satisfies the traditional compressive sampling framework as the vector to be reconstructed. i.e., \( \mathbf{\hat{r}}_x \) is sparse. Hence, a sparse reconstruction algorithm such as the CoSaMP [30] could be used to solve

\[
\arg \min \| \mathbf{\hat{r}}_x \|_1 \quad \text{subject to} \quad M^2 \mathbf{r}_y = \Phi \mathbf{\hat{r}}_x \tag{25}
\]

The measurement values \( \mathbf{r}_y \), the \( \Phi \) matrix, the expected sparsity, and the allowable error are sent to the CoSaMP algorithm. The output \( \mathbf{\hat{r}}_x \) is used to obtain super-Nyquist rate correlation function \( \mathbf{r}_x \) by overlapping and adding of terms from \( \mathbf{\hat{r}}_x \) according to (13). It can be seen from the Equation (13) that \( M \) terms with each superscript \( h \) need to be added to reconstruct one sample of the super-Nyquist rate correlation function. From Equation (23) and Equation (24), one delay value of \( \mathbf{\hat{r}}_x \) with \( M \) terms is already grouped together. Since they are already sorted in terms of the corresponding delay values, adding up every \( M \) terms starting from the beginning will result in the required super-Nyquist rate correlation function. Then a peak detection can be performed on the final super-Nyquist correlation function to estimate LOS. The position of the highest peak in \( \mathbf{r}_x \) will determine the code phase of the satellite signal.
Clearly the number of rows of the $\mathbf{C}_i$ matrix in (19) and (20) are $N_1N_2$, the product of the number of branches in the two paths of Figure 11, whereas the number of columns is determined by $M$, the up-sampling ratio. For good CS reconstruction quality, the matrix $\Phi$ and hence $\mathbf{C}_i$ cannot be too “flat”. This means that the more branches we put in each path of Figure 11, the larger $M$ as well as better reconstruction quality will be achieved. For the illustration purpose, in this thesis we used $N_1 = 1$, $N_2 = 3$, and $M = 4$. Larger $M$ is achievable with larger $N_1$ and $N_2$, which will certainly increase hardware and software cost, a trade-off for the designer to choose.

5.5 Simulation and Results

In this section, the proposed method is compared with 2 other methods - The conventional narrow band acquisition and the super-Nyquist sampled wideband acquisition.

In all simulations in this section we used $N_1 = 1$, $N_2 = 3$, and the CoSaMP algorithm. The first simulation is to validate the proposed method under a normal GPS SNR condition, -15 dB in a 2 MHz GPS band for the PRN 10. The simulated line of sight (LOS) and two other path GPS signals arrive at 0.061095, 0.061217, 0.061278 milliseconds, respectively, after the beginning of a 1023 chip C/A code. This corresponds to the 1000$^{th}$, 1002$^{nd}$, and 1003$^{rd}$ samples of the code phase when sampled at 16 samples per chip (16,368 SPS), or equivalently at 250$^{th}$, 250.5$^{th}$, and 250.75$^{th}$ samples of the code phase when sampled at 4 samples per chip (i.e., the Nyquist rate samples, 4,092 SPS). The distorted ACF at the transmitter end for satellite 10 is shown in Figure 12.
Figure 12: Transmitter end – 500 samples of transmitted GPS data at -15dB SNR (upper), correlation functions of multipath simulation with 2 reflected paths and 1 LOS (left lower), and zoomed correlation functions showing all 3 paths separately (right lower). The expected correlation function for each path is shown by colored triangles, which make the overall correlation function shown by the black line.

The conventional narrowband correlator works at a sampling frequency of 4.092MHz and the Nyquist rate samples are used to estimate code phase using narrowband acquisition. The narrow band pass filter used is a FIR filter of order 100. This introduces a delay of 50 samples in the acquisition output. From Figure 13, the estimated LOS code phase corresponds to 251\textsuperscript{th} sample when sampled at 4 samples per chip. For this relatively high SNR case of -15dB, narrow band correlator can accurately estimate the code phase of LOS signal. The acquisition threshold used is empirical, which is the ratio of the highest peak to the second highest peak in the correlator output.
Figure 13: conventional narrowband acquisition. Estimated codephase (upper) and plot of the acquired satellites (lower). Green bar indicates acquisition of the satellite and blue bar indicates the signal value below threshold. Sampling frequency 4.092 MHz. Threshold used for acquisition is 2.5, SNR -15dB.

The super-Nyquist sampled wideband correlator works at a sampling frequency of 16.368 MHz and the super-Nyquist rate samples are used to estimate code phase. The wide band filter used is a FIR filter of order 100. This introduces a delay of 50 samples in the acquisition output. From Figure 14, the estimated LOS code phase corresponds to 1001th sample when sampled at 16 samples per chip. The wide band correlator also can estimate the code phase of LOS signal accurately at SNR of -15dB.
The CS method takes input samples at 4 samples per chip, and reconstructs super-Nyquist rate correlation samples at 16 samples per chip. Figure 15 shows the CS reconstructed super-Nyquist rate correlator output. From the maximum value of the reconstructed correlation function, the LOS code phase is correctly estimated to be at sample 1001.
Figure 15: CS reconstructed narrow band acquisition, the output after CoSaMp algorithm (upper), reconstructed super-Nyquist correlation function in red (left lower), and zoomed super-Nyquist correlation function showing LOS code phase (right lower).
Sampling frequency 4.092 MHz. Reconstructed to 16.368 MSPS. SNR -15dB.

The second simulation shows the advantage of the CS reconstruction method over the conventional acquisition in low SNR environments. A BP filter with a bandwidth being half of the corresponding sampling frequency is simulated before sampling. For SNR over a 2 MHz band at or below –20 dB, the conventional narrowband acquisition fails, whereas both the conventional wideband and CS methods still acquire the weak GPS signal until about –25dB SNR. This simulation demonstrates –22dB SNR. Other parameters for this simulation are the same as the first simulation, as shown in Figure 16.
Figure 16: Transmitter end – 500 samples of transmitted GPS data at -22dB SNR (upper), correlation functions of multipath simulation with 2 reflected paths and 1 LOS (left lower), and zoomed correlation functions showing all 3 paths separately (right lower). The expected correlation function for each path is shown by colored triangles, which make the overall correlation function shown by the black line.

Figure 17: Plot of the acquired satellites. Green bar indicates acquisition of the satellite and blue bar indicates the signal value below threshold. Sampling frequency 16.368 MHz. Threshold used for acquisition is 2.5. SNR -22dB
Figure 17 shows that the traditional narrowband acquisition fails to acquire PRN 10 signal for this SNR. The acquisition threshold used is 2.5 and sampling frequency is 4.092 MHz.

Figure 18 shows the conventional wideband correlator output at SNR -22dB. The wide band correlator can estimate the LOS value somewhat accurately to be at sample 1003. The sampling frequency used is 16.368MHz and the acquisition threshold is 2.5.

Figure 18: super-Nyquist sampled wideband acquisition, Estimated codephase (upper) and plot of the acquired satellites (lower). Green bar indicates acquisition of the satellite and blue bar indicates the signal value below threshold. Sampling frequency 16.368 MHz. Threshold used for acquisition is 2.5. SNR -22dB

Figure 19 shows the CS reconstructed super-Nyquist rate correlator output at SNR -22dB. From the maximum value of the reconstructed correlation function, the LOS code phase is accurately
estimated to be at sample 1001. The sampling frequency used is 4.092 MHz (4 samples per chip) but reconstructed to 16 samples per chip.

![Graphs showing CS reconstructed narrow band acquisition and reconstructed super-Nyquist correlation function.]

The third simulation shows the advantage of the CS reconstruction method over the conventional wideband acquisition in very low SNR environments. The SNR is at -28dB now and all other parameters are the same from the previous simulation and this is shown in Figure 20.
Figure 20: Transmitter end – 500 samples of transmitted GPS data at -28dB SNR (upper), correlation functions of multipath simulation with 2 reflected paths and 1 LOS (left lower), and zoomed correlation functions showing all 3 paths separately (right lower). The expected correlation function for each path is shown by colored triangles, which make the overall correlation function shown by the black line.

A BP filter with a bandwidth being half of the corresponding sampling frequency is still simulated before sampling. For SNR over a 2 MHz band below -25 dB, even the conventional wideband correlator is unable to acquire the signal. There are a number of competing peaks and hence the correlator cannot estimate the code phase of the LOS signal.
Figure 21: Power spectral density of PRN10 before and after BPF (upper) and the plot of the acquired satellites (lower). Green bar indicates acquisition of the satellite and blue bar indicates the signal value below threshold. Sampling frequency 16.368 MHz. Threshold used for acquisition is 2.5. SNR = 28 dB.

The CS method, however, continues to work below −25 dB SNR since it admits less noise with a narrow BW RF frontend. Figure 22 shows the CS reconstructed super-Nyquist correlation for −28 dB SNR. We can see that the peak detection is still able to produce somewhat acceptable result, with the LOS identified at sample 1003. Estimation based methods like MEDLL cannot be used in conjunction with CS methods as the correlation shape is not preserved through CS methodology. As long as the peak of LOS is higher than the peak of any of the MP signals, CS methodology would work well and find a solution using peak detection method. This will be discussed in more detail in Section 6.2.
Figure 22: CS reconstructed narrow band acquisition, the output after CoSaMp algorithm (upper), reconstructed super-Nyquist correlation function in red (left lower), and zoomed super-Nyquist correlation function showing LOS code phase (right lower). Sampling frequency 4.092 MHz. Reconstructed to 16.368 MSPS. SNR -28dB.

In summary, the results for these three simulations can be tabulates as shown in Table 4. The original LOS arrival time is 0.061095 milliseconds after the beginning of the C/A code.

<table>
<thead>
<tr>
<th>SNR \ LOS Accuracy</th>
<th>Conventional narrowband acquisition</th>
<th>Super Nyquist wide band acquisition</th>
<th>CS reconstructed narrow band acquisition</th>
</tr>
</thead>
<tbody>
<tr>
<td>-15dB</td>
<td>0.061339 m sec</td>
<td>0.061155 m sec</td>
<td>0.061155 m sec</td>
</tr>
<tr>
<td>-22dB</td>
<td>N/A</td>
<td>0.061278 m sec</td>
<td>0.061155 m sec</td>
</tr>
<tr>
<td>-28dB</td>
<td>N/A</td>
<td>N/A</td>
<td>0.061278 m sec</td>
</tr>
</tbody>
</table>

Table 4: Accuracy of the LOS code phase estimation for different SNRs
Multipath Mitigation using Interpolation/MEDLL

In the mathematical field of statistics, interpolation is a method of constructing new data points between known data points. If the data points to be constructed are known, then decimation and then interpolation can be used to reduce the complexity of needed calculation and thereby increasing the efficiency of such calculation without losing too much accuracy, provided that the Nyquist sampling rate is always satisfied at least with respect to the main part of the signal spectrum. An interpolation error function can thus be computed that determines the percentage of error in the calculation. The interpolation method used may also contribute to such error.

There are many different types of interpolation methods available for statistical models, some of which are listed below.

Linear Interpolation – This is the easiest of all the methods available for interpolation. Linear interpolation between two data points usually calculates the interpolant such that the slope of the line between first point and new data point is the same as the slope of the line between first point and second data point. The linear interpolation method is not very accurate and the error is proportional to the square of the distance between the two points.

Polynomial Interpolation – This type of interpolation is a generalization of linear interpolation, in that the interpolant in linear interpolation is replaced with a higher degree polynomial function. The interpolation error is proportional to the distance between the data points to the power n, where n is the degree of the polynomial and n-1 is the utmost degree of the polynomial passing through all the data points. The common interpolation methods under this type are “PCHIP interpolation” or “cubic interpolation” etc.
Spline Interpolation (piecewise cubic spline interpolation) - Compared to the above methods, spline interpolation produces smoother and better results. Spline interpolation uses low-degree polynomials in each of the intervals and chooses the polynomial functions such that they fit smoothly together. The resulting function is called a spline function. The interpolant is easier to calculate than the higher degree polynomials used in polynomial interpolation.

To summarize, there are varying degrees of complexity involved in interpolation methods and they range from nearest neighbor interpolation to regression interpolation.

6.1 Interpolation in digital signal processing:

In the signal processing domain, interpolation is implemented as the process of converting a sampled digital signal (for example an audio signal) into a higher sampling rate using different digital filtering techniques. Interpolation in terms of digital signal processing contains two steps.

Let the original digital samples be $x[n]$.

a. Create a sequence $x_{L}[n]$ by inserting $L-1$ zero-valued samples between each pair of input samples. This operation is called zero-stuffing or zero-padding.

b. Smooth out the discontinuities by passing through a FIR low pass filter.

The interpolation factor is simply the ratio of the output rate to the input rate. It is usually denoted by $L$. Also, Interpolation can only be done in integer factors as it depends on zero-stuffing. There are workarounds to have fractional factors by using a combination of interpolation and decimation. Just up-sampling adds undesired spectral images to the signal at the multiples of original sampling rate and that’s why they need to be removed by filtering.
This process can be extended to multiple stages to gain computational savings and thus the original samples can be interpolated with no loss of information.

6.2 Interpolation in GPS data

In a GPS receiver using standard correlators, the number of samples contained in the correlation function is limited by the sampling rate used. The local C/A code for a PRN is generated at the same rate as the sampling rate of the incoming GPS signal. To increase the accuracy of the peak detection, more data points are needed in the estimated correlation function. In a software GPS receiver, interpolation methods can be applied to up-sample incoming GPS data before being correlated with C/A code. Based on the properties of the C/A code, it is very easy to generate C/A code at higher rate and store in a memory location. This will help the receiver to work with more number of data samples to an appreciable degree thus increasing the accuracy of the estimated code phase.

From the structure of the GPS signal, it is known that the base band GPS signal has values of ±1, hence a simple interpolation method like the nearest neighbor interpolation or linear interpolation can be used. Higher order polynomial interpolation methods try to fit higher degrees in between data samples thus increasing complexity of the interpolating function. In this thesis, a linear interpolation method is used to up-sample the data from 4 samples per chip to 16 samples per chip. Effectively, the base band signal at Nyquist sample rate of 4.092 MSPS is interpolated to the super Nyquist rate of 16.368 MSPS. The C/A code for the PRN is generated at the super Nyquist rate as well and used in correlation to estimate the correlation function. The interpolation factor of 4 is used in this thesis to be consistent with the previous chapter.
After code phase estimation using interpolation based method, an estimation based method such as MEDLL can be used to estimate the amplitudes and delays of the multipath signals. The MEDLL uses many correlators in order to determine the shape of multipath distorted correlation function accurately [19]. Then, a likelihood function is minimized that determines the best combination of LOS and non-LOS components (amplitudes, delays and phases) [13]. Since the samples are reconstructed to super Nyquist rate now, the MEDLL will better estimate the multipath components present in the distorted correlation function. The tradeoff of this combination is that the MEDLL has to work at super Nyquist rate thus increasing the burden on the correlators.

In summary, the MEDLL is very computationally expensive because of extensive correlator structure per channel but it provides superior multipath mitigation performance compared to other multipath mitigation methods and hence is chosen for this thesis.

The accuracy of the CS reconstructed correlation function at super Nyquist rate depends on the noisy input measurements, the $\Phi$ matrix, an arbitrary sparsity value needed for the greedy pursuit CoSaMP algorithm, the efficiency of the CoSaMP algorithm, the number of iterations run, and the maximum allowable error. The peak value of the correlation function can be estimated accurately but the roll off on either side of the correlation function may not be very smooth out of the CS algorithm, i.e., the correlation function shape is not preserved, as seen in the simulations of the previous Chapter. Thus, unfortunately it is unreliable to estimate accurate values of secondary paths using MEDLL with the CS reconstructed correlation function. For this reason the CS reconstructed super-Nyquist rate correlation functions are not further used with MEDLL.
6.3 Simulation and Results

The first simulation is to validate the interpolation based method under a normal GPS SNR condition at -22 dB and multipath (MP) with short delays. The simulated line of sight (LOS) and two other path GPS signals arrive at 0.061095, 0.061217, 0.061278 milliseconds, respectively, after the beginning of a 1023 chip C/A code. This corresponds to the 1000\textsuperscript{th}, 1002\textsuperscript{nd}, and 1003\textsuperscript{rd} samples of the code phase when sampled at 16 samples per chip (16,368 SPS). The autocorrelation function (ACF) at the transmitter end for satellite 10 and the ACF distorted by short delay multipath is shown in Figure 23.

Figure 23: Transmitter end – 500 samples of transmitted GPS data at -22dB SNR (upper), correlation functions of multipath simulation with 2 short delay reflected paths and 1 LOS (left lower), and zoomed correlation functions showing all 3 paths separately (right lower). The expected correlation function for each path is shown by colored triangles, which make the overall correlation function shown by the black line.
Figure 24 shows the correlation function based on interpolation method for the satellite 10 at SNR -22 dB. As it can be observed that there are many peaks around the main lobe, and it is not as sparse as that from the CS reconstruction technique. The advantage of interpolation based methods is that the interpolated curve is much smoother.

![Correlation Function](image)

**Figure 24**: Interpolation based acquisition: Short delay multipath added (upper), zoomed in ACF at peak (lower). Sampling frequency 4.092 MHz, interpolated to 16.368 MHz, SNR = -22 dB.

The output of the interpolator is fed into a multipath estimation algorithm such as MEDLL to estimate the multipath components. As we can see that the LOS delay estimate of the interpolator based method for short delay multipath 1000\(^{th}\) sample, is exactly equal to the original 1000\(^{th}\)
sample (transmitter side LOS). For the 3-path scenario, the MEDLL accurately estimates the MP1 to 1002\textsuperscript{nd} and MP2 to 1003\textsuperscript{rd} samples as shown in Figure 25.

![Figure 25: Input to and output from MEDLL (upper) and zoomed value at or near the peak (lower), correlation function from interpolation based method is shown in red, LOS – blue, multipath 1 – black and multipath 2 – magenta. Sampling frequency is 4.092MHz, interpolated to 16.368 MHz.]

The second simulation is to validate the interpolation based method under a normal GPS SNR condition at -22 dB and MP at medium delay (between 0.35 and 1 chip delay). The simulated line of sight (LOS) and two other path GPS signals arrive at 0.061095, 0.061461, 0.061644 milliseconds, respectively, after the beginning of a 1023 chip C/A code. This corresponds to the
1000th, 1006th, and 1009th samples of the code phase when interpolated to 16 samples per chip (16,368 SPS). The ACF at the transmitter end for satellite 10 and those distorted by medium delay multipaths are shown in Figure 26.

![Sample of GPS data; SNR at -22 dB](image)

**Figure 26**: Transmitter end – 500 samples of transmitted GPS data at -22dB SNR (upper), correlation functions of multipath simulation with 2 medium delay reflected paths and 1 LOS (left lower), and zoomed correlation functions showing all 3 paths separately (right lower). The expected correlation function for each path is shown by colored triangles, which make the overall correlation function shown by the black line.

Figure 27 shows the correlation function based on the interpolation method for the satellite 10 at SNR -22dB. Even for this case due to low SNR, there are many peaks around the main lobe, and it is not very sparse. The advantages of interpolation based methods are that the interpolated
curve is much smoother, and hence the MP estimation algorithm will yield better results. As we can see that the LOS estimate of the interpolator based method for short delay multipath 1002\textsuperscript{nd} sample, is almost equal to the original 1000\textsuperscript{th} sample (LOS path delay). As can be seen from Figure 27, the fall off of the medium delay ACF curve is much slower than the fall off of the short delay ACF curve depicted in Figure 24.

![Figure 27: Interpolation based acquisition: Medium delay multipath added (upper), zoomed in ACF at peak (lower). Sampling frequency is 4.092 MHz, interpolated to 16.368 MHz, SNR = -22dB.](image)

For this 3-path scenario, the MEDLL estimates the MP1 to 1004\textsuperscript{th} and MP2 to 1005\textsuperscript{th} samples as shown in Figure 28. This is slightly off the original multipath values of 1006\textsuperscript{th} and 1008\textsuperscript{th} samples.
The step accuracy of the estimation algorithm and the roll off factor to the right of peak might have caused this error.

![Estimated and Actual Plots - After Interpolation/MEDLL](image)

**Figure 28:** Input to and output from MEDLL (upper) and zoomed value at or near the peak (lower), correlation function from interpolation based method is shown in red, LOS – blue, multipath 1 – black, multipath 2 – magenta. Sampling frequency is 4.092 MHz, interpolated to 16.368 MHz.

Also, it has to be noted that the original LOS estimation was the 1002\textsuperscript{nd} sample before MEDLL but after MEDLL the LOS value has been corrected to a better estimate of 1000\textsuperscript{th} sample.
7 Conclusion and Future Work

The proposed method using CS provides potential advantages over existing code phase estimation methods. In particular, this method requires smaller RF bandwidth (2 MHz) and hence will admit less noise and interference compared to other fast sampling methods. As the GPS signal is buried under the noise level, a smaller RF bandwidth enables this method to work under lower SNR scenarios than other fast sampling methods.

In terms of the computational burden, the CS sampling and reconstruction requires additional computation. However, this additional computation may be offset by the following computational savings of this method. Since this method works at the Nyquist rate samples of the GPS signal (4.092 MHz), and correlations are performed at the Nyquist rate, this will save much correlation computation compared with wider pre-correlation bandwidths with faster sampling. Furthermore, the locally generated GPS code can be computed beforehand and stored. So the burden of real-time computation is reduced. The actual savings vs. additional computation due to CS varies on a case-by-case basis.

More simulations and analysis are needed to characterize statistical performance of this proposed method using CS, and compare with known theoretical bounds [36] [37] [38]. The above CS method is also equally applicable to other GPS signal waveforms and bandwidths. Aspects of other GNSS constellations like Galileo, Beidou etc. need to be studied before applying CS methodology to them. These are future work and are beyond the scope of this thesis.

The accuracy of the CS reconstructed correlation function at super Nyquist rate depends on the noisy input measurements, the $\Phi$ matrix, an arbitrary sparsity value needed for the greedy pursuit CoSaMP algorithm, the efficiency of the CoSaMP algorithm, the number of iterations run, and the
maximum allowable error. The peak value of the correlation function can be estimated accurately but the roll off on either side of the correlation function may not be very smooth out of the CS algorithm i.e., the correlation function shape is not preserved. Thus, it is not reliable to estimate accurate values of secondary paths using MEDLL with the CS reconstructed correlation function. As long as the value of LOS correlation function is higher than the value of MP correlation function, peak detection method can be used to effectively estimate the correct code phase.

Interpolation based methods are much simpler to implement as they don’t use any dictionaries and hence there is no need of additional storage. These methods preserve the shape of the correlation function, so subsequent MP mitigation methods such as MEDLL can be used. However, the interpolation based methods need to work at super-Nyquist rate to have successful estimation of MP delays and amplitudes. This will put additional burden on the correlators. The interpolation based methods using the standard discriminator will have same issues as the standard correlation based methods. They would have limitations in the presence of dense short delay multipath.

Comparison of CS based and interpolation based methods will depend on a case-by-case basis. The sparsity of the signal, the sampling rate that can be used and the complexity of the system in terms of computational savings will determine which method performs better.
8 References


