I, Evan C Hilderbrand, hereby submit this original work as part of the requirements for the degree of Master of Science in Electrical Engineering.

It is entitled:
Blood Sample Characterization Using A Co-Axial Transmission Line

Student's name: Evan C Hilderbrand

This work and its defense approved by:

Committee chair: Altan Ferendeci, Ph.D.
Committee member: Punit Boolchand, Ph.D.
Committee member: George Shaw, Ph.D.
Blood Sample Characterization Using A Co-Axial Transmission Line

A thesis submitted to the
Graduate School
of the University of Cincinnati
in partial fulfillment of the
requirements for the degree of

Master of Science

in the Department of Electrical Engineering and Computing Systems
of the College of Engineering and Applied Sciences

by

Evan Hilderbrand

March 2015

B.S. University of Cincinnati

December 2014

Defense Committee:
Altan Ferendeci, Ph.D. (chair)
George Shaw, M.D., Ph.D.
Punit Boolchand, Ph.D.
ABSTRACT

Blood Sample Characterization Using A Co-Axial Transmission Line

By

Evan Hilderbrand

Microwave Tomography (MWT) is being developed as an accurate detection of hemorrhagic strokes using microwaves. A transmission line technique is used to measure the relative permittivity of blood as a function of frequency and is presented to provide insight on an appropriate operating frequency for MWT. A response calibration method is used on a coaxial sample holder to accurately measure the scattering parameters of and characterization of liquid samples. This thesis will demonstrate the ability to accurately measure liquid samples by replacing the coaxial line dielectric with air. Two methods are used to verify the transmission line measurements. Experimental results from water and 99% isopropyl alcohol are compared against known values to provide verification of the coaxial airline to accurately measure blood samples in the microwave region.
ACKNOWLEDGMENTS

I would like to thank Dr. Altan Ferendeci for his guidance and patience in the development of the validation and this thesis. Thanks also to Dr. George Shaw and Dr. Punit Boolchand for their input and advice.

I would also like to thank Peter Kosel for providing me with foundational knowledge in both electronics and electromagnetics, which have been instrumental in pursuing this degree.

I would like to thank my future wife Mallory for her guidance and support throughout this and many other endeavors.

Finally thank you to both of my parents, Lisa and Glen, for their love, inspiration, and guidance over the course of my career.
# TABLE OF CONTENTS

LIST OF FIGURES .............................................................................................................................................. vii

CHAPTER ONE: INTRODUCTION ..................................................................................................................... 1

CHAPTER TWO: MEASUREMENT SYSTEM SELECTION .................................................................................. 3

  Background and Motivation ............................................................................................................................ 3

  Measurement Systems .................................................................................................................................... 7

CHAPTER THREE: MEASUREMENT SETUP AND CALIBRATION .............................................................. 12

  Sample Holder Selection ............................................................................................................................... 12

  Calibration Methods ...................................................................................................................................... 14

  Calibration System Comparisons ................................................................................................................ 19

CHAPTER FOUR: REFERENCE LIQUID DIELECTRIC MEASUREMENTS ..................................................... 26

  System verification with known dielectrics .................................................................................................. 26

  System Verification by Solving For Dielectric .......................................................................................... 34

CHAPTER FIVE: BLOOD SAMPLE DIELECTRIC MEASUREMENTS .......................................................... 44

CHAPTER SIX: CONCLUSION .......................................................................................................................... 52

BIBLIOGRAPHY .................................................................................................................................................. 54

APPENDIX A: RICHMOND MULTILAYER EQUATIONS .............................................................................. 57
APPENDIX B: S-PARAMETER MATLAB SIMULATION CODE ............63

APPENDIX C: MATLAB PERMITTIVITY CALCULATOR ..................70
LIST OF FIGURES

Figure 1: Broadband data gathered on blood covering the 0.5 – 2GHz frequency range [5] ................................................................. 5

Figure 2: Coaxial transmission line resonator [10] ................................................................. 8

Figure 3: APC 7 to Type N(F) adapter (side view and approximate dimensions) ................................................................. 13

Figure 4: General two port network .................................................................................... 14

Figure 5: TRL required measurements (Through, Reflect, Line) ................................. 16

Figure 6: Setup 1 fully assembled (left) and ready for sample placement (right) ........................................................................ 18

Figure 7: Setup 2 fully assembled (left) and ready for sample placement (right) ........................................................................ 18

Figure 8: Measurement of the empty sample holder Setup 1 after TRL calibration ................................................................. 20

Figure 9: Setup 1 water measurement after TRL calibration ................................ ........ 21

Figure 10: Setup 1 water measurement after response calibration ................................ .... 22

Figure 11: Setup 2 water measurement after response calibration ................................ .... 23

Figure 12: 70% Isopropyl Alcohol Complex Permittivity [20] ........................................ 27

Figure 13: Theoretical values for the permittivity of distilled water at 23°C [11] ........ 28

Figure 14: Theoretical values for the loss tangent of distilled water at 23°C [11] ........ 29
**Figure 15:** Prediction using ideal epsilon values vs. measured 3cm distilled water (Setup 2) ................................................................. 31

**Figure 16:** Prediction using ideal epsilon values vs. measured 1.5cm distilled water (Setup 2) ........................................................................................................................................... 32

**Figure 17:** 3.0cm thick distilled water measurement with response calibration. 35

**Figure 18:** Solved epsilon real and imaginary for 3.0cm of distilled water ........ 36

**Figure 19:** 1.6cm thick distilled water measurement with response calibration. 37

**Figure 20:** Solved epsilon real and imaginary for 1.6cm of distilled water ........ 38

**Figure 21:** 2.5cm thick 99% isopropyl alcohol measurement with response calibration ............................................................................................................... 39

**Figure 22:** Solved epsilon real and imaginary for 2.5cm of 99% isopropyl alcohol ........................................................................................................................................... 40

**Figure 23:** 2.3cm thick 99% isopropyl alcohol measurement with response calibration ............................................................................................................... 41

**Figure 24:** Solved epsilon real and imaginary for 2.3cm of 99% isopropyl alcohol ........................................................................................................................................... 42

**Figure 25:** Magnitude and phase of blood at 23°C from 0.1-0.5GHz............... 45

**Figure 26:** Permittivity of blood at 23°C from 0.1-0.5GHz.............................. 46

**Figure 27:** Magnitude and phase of blood at 23°C from 0.5-3GHz............... 47

**Figure 28:** Permittivity of blood at 23°C from 0.5-3GHz.............................. 48

**Figure 29:** Magnitude and phase of blood at 23°C from 3-8GHz............... 49
Figure 30: Permittivity of blood at 23°C from 3-8GHz.......................................... 50

Figure A-1: A redrawing of J. H. Richmond’s multilayer stack from [2] .......... 57

Figure A-2: Transmission Line example ................................................................. 61
CHAPTER ONE: INTRODUCTION

On a Global scale approximately 15 million strokes occur yearly with 5 million of them resulting in death and an additional 5 million resulting in permanent disability [1]. In the United States alone, more than 750,000 people will suffer a stroke annually with the number of stroke related deaths per year reaching 140,000 [1]. In a clinical setting, Microwave Tomography (MWT) has been proven as a feasible method of constant monitoring of at-risk stroke patients [5-7]. MWT has the added benefits of not being prohibitively expensive or using a dangerous radiation source. The ideal operating frequency for the MWT device has not yet been chosen in-part due to the lack of high resolution experimental data on blood within the range of 0.5 – 2GHz. Through the use of a coaxial airline as a sample holder, one has the capability of developing a convenient and relatively inexpensive device that is capable of measuring liquid samples. That device would be capable of measuring liquid samples without being limited to a narrow frequency band, being restricted to resonance frequencies in a broadband, or requiring specialized equipment, or software. Coaxial components are widely available and suitable coaxial airlines can often be found in calibration kits, which require no added expense.

Free – space multilayer sample equations allow for the prediction and verification of water and isopropyl alcohol samples which are used to demonstrate the validity of the measurement device as well as the setup and
calibration procedure [2]. These liquid samples, with known theoretical values, allow verification without relying solely on the blood samples for which there is little available data. It is hoped that these measurements will aid in the selection of an operational frequency for the MWT device by providing better understanding of the frequency response of blood from 0.5 – 2GHz.
CHAPTER TWO: MEASUREMENT SYSTEM SELECTION

Background and Motivation

Currently the only diagnostic tools available to doctors are Ultrasound, Computer Tomography (CT) scans, and Magnetic Resonance Imaging (MRI) scans. MRI’s and CT scanning equipment provide the best data for diagnosis, but are the most expensive to use and require lengthy examination times when compared to the progression of symptoms in a stroke [3]. Ultrasound is typically only used to assess the condition of intracranial and carotid arteries after either an MRI or a CT scan have been completed [4]. Ultrasounds are not used as a method of stroke detection nor are they suited to constant monitoring of the patient for early detection of a stroke [4]. These devices are also used in a wide variety of other cases meaning that immediate patient admittance to either an MRI or CT scan is not always possible and can further complicate diagnosis and treatment. The CT scan is also notable for its use of X-ray radiation in the development of a brain scan, the resulting radiation exposure to the patient means that one can only undergo a limited number of CT scans in a short time period. In an attempt to deal with these issues, Microwave Tomography (MWT) has been researched through multiple studies for feasibility [5-7]. The benefit of this technology is the use of low power, non-ionizing radiation, which provides the ability to constantly monitor the condition of stroke patients, much like a CT scan, but without prohibitively expensive equipment or being limited by radiation exposure limits. Additionally
the MWT can be relatively compact compared to an MRI or CT scanner with the possibility of being used without requiring relocation of the patient. The utility of this device comes from the difference in electrical properties of healthy and affected tissues as well as a noticeable difference between blood and the surrounding grey and white matter in the brain. Through several feasibility studies, it has been demonstrated that the addition of multiple antennas, both transmitting and receiving, have the ability to map an image of the relative scatters and also map the relative permittivity of matter in the head [5-7]. Predicting or reconstructing the location of hemorrhagic strokes is the primary goal of these studies, with ischemic strokes being considered in only the most recent studies. The cause of hemorrhagic strokes is a blood vessel bursting in the brain that allows blood to accumulate in the surrounding tissue, while an ischemic stroke is caused by a sudden decrease in blood supply to part of the brain due to a cardiovascular obstruction, either locally or elsewhere in the body [1]. To predict the behavior of these systems the frequency specific dielectric properties of the various healthy and affected tissues in the head must be known. The frequency range used in these studies generally spans between 0.5GHz and 2GHz. An ideal frequency for this device has not yet been determined. It is known that the limiting factors determining this range are that frequencies higher than 2GHz have excessive attenuation through the head, which makes frequencies greater than 2GHz difficult to use at low power levels, while frequencies significantly lower than 2GHz have poor resolution for imaging [5]. An additional limitation to selecting
the appropriate operational frequency for MWT is the lack of high resolution experimental data on blood from 0.5 – 2GHz as seen in figure 1. One of the more recent experimental studies often cited by feasibility papers is a survey of broadband measurements of biological tissues from 10Hz – 100GHz with very low resolution in the range of 0.5 – 2GHz [8].

Figure 1: Broadband data gathered on blood covering the 0.5 – 2GHz frequency range [5]
A higher resolution measurement would both validate these studies and provide additional insight into an appropriate operation frequency for MWT. Due to the cause of strokes often being blood in the brain, any resonance or variation in the frequency response would allow blood to be detected more easily. The detection of a resonance or variation at a given frequency would be of prime importance in the selection of an operational frequency for MWT.

An additional note when considering measuring blood is the similarity to water in both permittivity and permeability. Both distilled water and blood have a relative permittivity of ~70 and a relative permeability of ~1 in the frequency range of interest, which allows distilled water to be used to verify the measurement method chosen before making any blood sample measurements [8, 10, 11].
Measurement Systems

The primary limitation with blood measurements is the lack of available test equipment suited for measuring liquid biological samples without requiring specialized equipment or software. A free space solution utilizing a patch antenna to measure various biological samples was put forward as a noninvasive method to monitor glucose levels in the body [12]. The method involved manufacturing a phantom, which would mimic the dielectric properties of certain biological tissues under various conditions. The accuracy of the device and the ability to use it as a free space measurement system allows for various methods to test the permittivity of blood using free space measurements. Unfortunately the device is a patch resonator, which would be relatively simple to manufacture, but would only operate properly at certain frequencies. It would not allow for high resolution in the measurement data that is desired to better understand the frequency response of blood.

A semi-free space system utilizing an HP4291A material impedance analyzer is often used by labs to measure both solids and liquids accurately. The most recent paper [10] involving a measured blood sample characterized the blood in the range from 0.1 – 1.8GHz, but with similar resolution to the data in Figure 1. The use of the HP4291A impedance analyzer implies that an open ended coaxial line was used to measure the samples. The literature notes using an equivalent (or lumped) circuit model to calculate the permittivity and conductivity of the blood samples. When compared against a numerical analysis point-matching theory
model an error for the equivalent circuit model was noted of 8-16% increasing non-linearly in the range 1MHz – 1GHz [10]. While the open ended coaxial line is often used to measure liquid samples, the literature indicates that utilizing an equivalent circuit model becomes error prone in the frequency range of interest and recommends the use of an impedance analyzer, which is not be available for use and expensive to acquire.

An alternative broadband measurement device which does use a network analyzer and is capable of measuring liquids is presented as the coaxial transmission line resonator. The device, shown in figure 2, is based on resonant cavity principles involving the measurement of an empty reference and a filled cavity sample so that the resonance frequency and quality factor of the device filled with the liquid sample can be compared to that of free space [13].

![Coaxial transmission line resonator diagram]

*Figure 2: Coaxial transmission line resonator [13]*
This measurement device is often used for the accuracy of the results. Resonant cavity structures tend to provide precise data while requiring less complicated calculations and measurement setup for the data processing. Unfortunately this device, while widely used and well-proven, it is not well suited to the task of measuring blood at a high resolution. The cavity resonator structure of the device means that it will only operate at specific frequency intervals, which are determined by the size of the resonant cavity itself. The potential sizes for the resonant cavities laid for the 0.3-12GHz frequency range have approximate volumes of ~210 mL and ~120 mL [13]. Large sample sizes make such a device infeasible for use in blood sample measurements. A smaller device could be constructed to overcome this problem, but the resonant cavity will still be hindered by the lack of data resolution. Furthermore, machining of such a cavity structure is difficult and acquiring a functioning cavity of a small enough size to have the potential for use in blood sample measurements can be difficult. Also, resonant cavity measurement run times can last a significant amount of time for a single measured frequency. Resonant cavities are unreliable when measuring samples with high loss tangents as it can significantly reduce the quality factor of the cavity and provide the user with inaccurate data [14].

A similar structure to the resonant cavity is a simple air-filled coaxial transmission line [15]. Characterization of materials using coaxial sample-holders is a well-researched field with there being many similarities to rectangular waveguides and free-space measurement systems regarding both the calibrations
and the network analyzer systems used to run the measurements. The primary downside of using a coaxial sample holder over a rectangular waveguide or free space measurement system is the precision required to machine the sample so that it will fit in the coaxial sample holder tightly enough to ensure that the measurement is accurate. This downside is entirely negated when using a liquid sample as the sample holder simply needs to be held in a vertical position to perfectly fill the gap between the inner and outer conductors. An advantage over other waveguides is in the wide bandwidth of coaxial lines when compared to a rectangular waveguide as well as the availability of low cost coaxial components. As mentioned previously, there are many well-known calibration methods for the coaxial transmission line and with the availability of an HP8510C network analyzer, calibration kits, and a multitude of adapters, the coaxial airline would be the simplest, accurate, broadband, high resolution, and cost-effective measurement device for blood sample measurements. The volume of measurement sample needed for filling a 7mm airline similar to a set of LPC 7 coaxial airlines can be as little as 1 mL depending on the length of airline used in the measurement [15].

A set of multilayer sample equations was used to model and simulate the results for this measurement technique based on sample thickness [2]. These methods provided the capability to predict the inevitable standing waves from finite sample sizes visible in the data and accentuated by the high permittivity values of the liquid samples. Appendix A contains a description of the relevant
multilayer equations used to calculate predictions for scattering parameters and solve for permittivity values from sample measurements.
CHAPTER THREE: MEASUREMENT SETUP AND CALIBRATION

Sample Holder Selection

With the selection of the coaxial airline as the sample holder for the upcoming measurements and the decision to use the HP8510C Vector Network Analyzer (VNA) to run the measurements, the overall components, setup, and calibration scheme still need to be chosen to carry out an accurate measurement. Compared to many other pieces of equipment, coaxial airlines can be fairly inexpensive, however the size of the desired airline needs to be small enough to minimize the amount of blood required to operate in the desired frequency range of $0.5 - 2\text{GHz}$, while still being large enough to easily work with. As a result, coaxial airlines of approximately 7mm diameter, commonly known as Type N or APC 7 airlines, were chosen to be used as the appropriate size for the setup. While coaxial components can be fairly inexpensive, small airlines such as the LPC7 airlines are more expensive as the airlines are often manufactured to be used as high precision references in a calibration kit rather than as a simple lossless sample holder. The university possesses a Type N calibration kit, but the kit does not contain a reference airline as would be desired for the measurement, however it does contain two APC 7 to Type N (Female) adapters. The adapters are much longer than appear to be necessary and are not filled with a Teflon dielectric as can be seen in Figure 3.
The selected adapter to be used as a sample holder cannot be disassembled to accurately measure the internals of the device, nor could accurate drawings representing the internals of the adapter be acquired. Therefore to measure the internal volume of the airline it was filled with distilled water and the total amount of water held by the airline was found to be approximately 1.1mL. Using the standard dimensions for Type N given by [16] (internal conductor radius = 0.157cm, outer conductor radius = 0.406cm) the internal length of the airline is approximately 2.7cm, not including the Type N connection point, which has a wider radius for the outer conducting wall. This test indicates that the sample size required for a full airline filled measurement is reasonable and easily repeatable. It also shows that the internals of the adapter prevent any leakage of liquid through the APC 7 adapter end. In order to connect this adapter to the HP8510C VNA, a series of coaxial adapters and cable lengths are necessary. With the equipment available in the lab the APC 7 to Type N adapter is fully capable of being a liquid sample holder for measurements and is immediately available for use with the only task left to consider being the calibration method to be used on the HP8510C VNA.
Calibration Methods

The HP8510C is a two port VNA which measures amplitude and phase of the electrical response of a system to a given frequency input with the response characteristics being recorded as the scattering parameters of the system. On a two port VNA the scattering parameters are $S_{11}$, $S_{21}$, $S_{12}$, and $S_{22}$ with $S_{11}$ and $S_{22}$ being measures of the reflection S parameters on ports 1 and 2 respectively. $S_{21}$ is the measure of the transmission scattering parameter from port 1 to port 2 with $S_{12}$ being the measure of the transmission scattering parameter from port 2 to port 1.

Figure 4: General two port network

If the inputs to a system are $a_1$ and $a_2$ and the response characteristics of a systems are $b_1$ and $b_2$ then the S-parameters are given as the following based on Figure 4

\[
S_{11} = \frac{b_1}{a_1}, \quad S_{12} = \frac{b_1}{a_2} \tag{3.1}
\]

\[
S_{21} = \frac{b_2}{a_1}, \quad S_{22} = \frac{b_2}{a_2} \tag{3.2}
\]

Where $S_{11}$ and $S_{22}$ represent the overall reflection response of the system and $S_{21}$ and $S_{12}$ represent the overall transmission response of the system. In a given frequency range the HP8510C has a resolution of up to 801 samples at each of which
it measures all four S-parameters. The measured S-parameters can then be stored to in a real, imaginary format for further analysis.

The front of the HP8510C has two ports with 3.5mm connections that can be used to connect to the desired test system. To connect the Type N to APC 7 sample holder there will need to be two 3.5mm cables to connect to the VNA, and two 3.5mm adapters to convert from Type N to 3.5mm and to convert from APC 7 to 3.5mm. While the transmission line diameters change along the length of the connected device to the VNA the characteristic impedance of all of the individual components is 50 Ohms so that there are no impedance mismatches in the transmission line for an empty sample holder measurement. However there will be reflection boundaries and phase delays caused by the various adapter and cable connection points along the length of the line. To deal with this a calibration method will need to be implemented to allow for precise, reliable measurements of the various liquid samples.

The selected adapter to be used as a sample holder, while free, carries with it a few problems with respect to calibration procedure. The most commonly used high precision calibration method is the TRL (Through, Reflect, Line) calibration procedure. The calibration kit available for use in the lab is the 85054A Type N calibration kit, which contains open, short, lowband load, and sliding load terminations for Type N connectors as well as the two Type N to APC 7 precision connectors mentioned earlier to be used as sample holders. Lacking a precision
matched broadband load means that similar calibration procedures over a broad range using TRM (Through, Reflect, Match) and LRM (Line, Reflect, Match) are not possible without additional equipment. The same is true for LRL (Line, Reflect, Line) as there are not multiple lengths of adapters that could be used as line standards. Therefore of the TRL family of calibrations only TRL has the potential to be used in this measurement setup. For the TRL calibration to be performed properly there needs to be three sets of measurements made to calibrate out any system response characteristics that are not directly related to the addition of a sample. Figure 5 describes the three required measurements indicating that the two adapters connected to the 3.5mm cables need to be of the same design so that they can mate together for the “Through” measurement.

*Figure 5: TRL required measurements (Through, Reflect, Line)*

The need for the “Through” measurement presents the problem of an additional APC 7 to Type N (M) adapter being added to the measurement setup so that the two connection ends attached to the 3.5mm cables can mate together. While this may add additional and unwanted response characteristics to
the system they may be removable with the TRL calibration. The second measurement requiring the reference shorts is simple as the required equipment is available. The HP85054A calibration kit does not contain a line standard, but the combined Type N (F) to APC 7 and APC 7 to Type N (M) adapters can be used as a standard line length with the appropriate calibration constants programmed into the calibration kit on the HP8510C [17]. The calibration constants were downloaded from Keysight’s website and installed onto the 8510C, but the line length for the two attached adapters needs to be calculated by hand as they are not given in the provided calibration kit file [18]. Once the calibration is performed the transmission and reflection frequency response errors of the fully connected, empty sample holder should be corrected and be approximately zero for $S_{21}$ and $S_{12}$ (indicating full transmission response) and approximately $-\infty$ for $S_{11}$ and $S_{22}$ (indicating no reflection response). There was significant difficulty in achieving response characteristics that approached these expectations given the sensitivity to the line length in the calibration procedure and the fact that the “line standard” being used is actually two sample holders being placed together. As a result a comparison was made using a liquid sample between the TRL calibration already described in depth and a Through-response calibration. The Through-response calibration consists of only two measurements to calibrate a sample and is able to correct the transmission frequency response error for transmission measurements only [19]. For the calculations only the transmission data will be utilized to determine the permittivity of the liquid samples. For the first is an empty
“Through” measurement, which consists of the entire measurement setup fully connected with not sample in place, similar to the line standard measurement used in the TRL calibration [19]. The second measurement is the actual sample measurement, which is normalized using the “Through” measurement of the Through-response calibration. Since a “Through” measurement without the sample holder is no longer necessary, a smaller measurement setup without the second adapter (APC 7 to Type N (M)) is possible. This setup, when fully connected will be referred to as “Setup 2” with “Setup 1” including the additional adapter as shown in Figure 6.

Figure 6: Setup 1 fully assembled (left) and ready for sample placement (right)

Figure 7: Setup 2 fully assembled (left) and ready for sample placement (right)
Calibration System Comparisons

Figure 7 demonstrates the downside of the TRL calibration with the extra adapter present in Setup 1, therefore a comparison was made between a Setup 1 and Setup 2 with the sample holder filled with water in the range of 0.5 – 2GHz with 801 samples. Several sets of measurements were made to compare a TRL calibration with Setup 1, Response calibration with Setup 1 and a Response calibration with Setup 2. An additional measurement was made to show the results of an empty measurement after the TRL calibration. All S-parameter results are given in magnitude (dB) vs. frequency and phase angle (degree) vs. frequency for each measurement. The results are displayed in Figures 9-11.
Figure 8: Measurement of the empty sample holder Setup 1 after TRL calibration
Figure 9: Setup 1 water measurement after TRL calibration
Figure 10: Setup 1 water measurement after response calibration
Figure 11: Setup 2 water measurement after response calibration
The results for the TRL calibration display a sharp change in the data noticeable in all four S-parameters, but it is most notable in the reflection parameters $S_{11}$ and $S_{22}$. This was partially caused by the phase for the sample holder section not being properly calibrated out and it was also likely affected by using the sample holder and the attached adapter as a line standard in the calibration routine for which the line standard parameters used in the calibration kit may not be accurate for the assembled line. Also the results after using the response calibrations with both Setup 1 and Setup 2 are very similar to the results of the TRL calibration, while not being subject to the sharp errors in the data. Since the response calibration method results do not change significantly from the TRL calibration results, it will be used going forward, as this method is more reliable for the test setup. The response calibration method does not calibrate reflection parameters $S_{11}$ and $S_{22}$, but it does correct frequency response error for transmission parameters $S_{21}$ and $S_{12}$, which are the primary focus of the measurement since they are similar to what one would expect to see in a functioning MWT setup. Additionally the use of the response calibration, which is more of a normalization procedure, can be done offline in a program like Microsoft Excel or MATLAB. This means that if multiple empty “Through” measurements are made over the desired frequency ranges, then only a single sample insertion is required. The user can then sweep all of the desired frequency ranges and calibrate the results later using the “Through” measurements that had been taken previously. As a result of this decision, Setup 2 will be used going forward for the
liquid measurements as it removes the additional reflection boundaries presented with the use of the extra adapter in Setup 1.
CHAPTER FOUR: REFERENCE LIQUID DIELECTRIC MEASUREMENTS

System verification with known dielectrics

In order to validate any blood measurement tests, verification of the test system needs to be done with another material with known material properties, preferably one that has similar permittivity values to the expected values for blood. For this reason distilled water was chosen to validate the test setup. In addition to water, 99% isopropyl alcohol was chosen to verify the test system since it has a lower dielectric constant and can be easier to work with than distilled water [20]. In order to validate the measurement system a simulation code was written in MATLAB based on equations for solving multilayer problems [2]. The simulation code used to predict one of the water measurements is laid out in Appendix B for reference. Variations of the simulation code will be used to predict what the measured S-parameters for varying amounts of distilled water should look like in magnitude and phase plots. The known values for frequency specific real and imaginary values of permittivity for distilled water were obtained to compare against measurement results [11]. Because the percentage of isopropyl alcohol used in the measurements is not the same as was used in the reference source the frequency specific values for permittivity will not directly match so the frequency response of 99% isopropyl alcohol is not directly predicted, however it is provided for comparison in Figure 12 [20].
The real and imaginary epsilon values for distilled water were obtained with a set measurement temperature of 23°C (Room temperature) and selecting operation frequencies for measurement [11]. The permittivity values for distilled water were then placed in a table with a change in frequency of 0.025GHz for consecutive points covering the range of 0.5 – 3GHz with 801 permittivity data points to be used for prediction of S-parameters. The epsilon and loss tangent data parameters are shown in Figures 13-14 [11].

Figure 12: 70% Isopropyl Alcohol Complex Permittivity [20]
Figure 13: Theoretical values for the permittivity of distilled water at 23°C [11]
To compare against a measured sample the thickness of the sample needs to be known accurately. One method to ensure the accurate thickness of the sample is to match the phase angle of two separate frequencies and solve for the thickness of the sample based on the different sized wavelengths in the distilled water. For example two frequencies in a measurement set have the same phase angle, the frequencies are 0.53GHz and 1.75GHz, the procedure is as follows

\[ \lambda_{w1} = \frac{\lambda_1}{\sqrt{\varepsilon'}} \quad , \quad \lambda_{w2} = \frac{\lambda_1}{\sqrt{\varepsilon'}} \quad (4.2) \]

Where \( \varepsilon' \) is the real permittivity at \( f_1 \) and \( f_2 \) and \( \lambda_{w1} \) and \( \lambda_{w2} \) are the wavelengths in water. The length of the sample is where

\[ \frac{\text{sample length}}{\lambda_{w1}} = \frac{\text{sample length}}{\lambda_{w2}} \quad (4.3) \]
Using the sample thickness obtained from this method, the S-parameter values for distilled water can be predicted to verify the measurement setup and calibration even though the values to be “predicted” must already be measured to use this method. The results of this particular measurement indicated a sample length of ~3cm. Using the MATLAB code for S-parameter prediction the measurement can be compared to the ideal values for distilled water as shown in Figure 15.
Figure 15: Prediction using ideal epsilon values vs. measured 3cm distilled water (Setup 2)
Figure 16: Prediction using ideal epsilon values vs. measured 1.5cm distilled water (Setup 2)
The second set of measurements, with half as much distilled water, (Figure 16) the same phase matching method in (4.3). However due to the thickness of the sample being smaller, the phase matching condition will need to be the sample thickness at which two frequencies are 180° out of phase after passing through the sample as shown in (4.4) since there are no frequencies in the measured range that have the same phase. Using this method and the code in Appendix B the measurement setup can be considered to be accurately measuring the sample based on the predicted magnitude and phase from ideal values.

\[
0.5 = \left| \frac{\text{sample length}}{\lambda_{w1}} - \frac{\text{sample length}}{\lambda_{w2}} \right| \tag{4.4}
\]
System Verification by Solving For Dielectric

To further verify the validity of the test setup the multilayer sample equations were inverted and written into a MATLAB code detailed in Appendix C, which uses the measured transmission S-parameters as inputs to solve for the real and imaginary parts of epsilon [2]. The solver is used to solve for both distilled water using 3.0cm and 1.6cm sample thicknesses and 99% isopropyl alcohol using 2.5cm and 2.3cm sample thicknesses over the combined range of 1 – 8GHz. The results of attempts to solve for the permittivity using the inverted equations from are compared against the ideal permittivity values used in the earlier calculations to predict the S-parameters.
Figure 17: 3.0cm thick distilled water measurement with response calibration
Figure 18: Solved epsilon real and imaginary for 3.0cm of distilled water
Figure 19: 1.6cm thick distilled water measurement with response calibration
Figure 20: Solved epsilon real and imaginary for 1.6 cm of distilled water
Figure 21: 2.5cm thick 99% isopropyl alcohol measurement with response calibration
Figure 22: Solved epsilon real and imaginary for 2.5cm of 99% isopropyl alcohol
Figure 23: 2.3cm thick 99% isopropyl alcohol measurement with response calibration
Figure 24: Solved epsilon real and imaginary for 2.3cm of 99% isopropyl alcohol
The close tracking of the theoretical values for permittivity of both 99% isopropyl alcohol and distilled water indicate the accuracy of this measurement system. While the permittivity values do not lay directly on top of the theoretical values, the permittivity values are high enough that any minor changes to the measured response characteristics will result in significant \( \varepsilon \) parameter changes. The wide range of values for the \( \varepsilon'' \) of 3 cm of distilled water can be explained by the fact that overall permittivity is a vector quantity defined by

\[
\varepsilon_r = \varepsilon'_r + \varepsilon''_r
\]  

(4. 5)

When the average value for \( \varepsilon'_r \) of water is \(~78\) and the average value for \( \varepsilon''_r \) is \(-7\) a significant change in \( \varepsilon''_r \) is required to significantly change the overall phase angle to match any error in the phase component of the S-parameters. The fact that such large swings in the \( \varepsilon''_r \) permittivity solution for isopropyl alcohol do not exist with \( \varepsilon'_r \) for 99% isopropyl alcohol being approximately \( 1/3 \)rd that of the \( \varepsilon'_r \) for distilled water confirms this. The ability of the measurement setup to both match S-parameter predictions from known permittivity values and closely track the known permittivity using data from the measurements indicates validation of the measurement setup and calibration method as well the liquid sample measurements.
CHAPTER FIVE: BLOOD SAMPLE DIELECTRIC MEASUREMENTS

All blood sample measurements were run using citrated blood at room temperature (23°C). The refrigerated blood samples used were provided by the University Of Cincinnati, College Of Medicine for testing. Prior to testing, the blood was allowed to come to room temperature before making any measurements. The blood samples were then measured in various ranges to determine any peculiarity in the frequency response of blood over the desired frequency range of 0.5 – 2GHz using the HP8510C setup to maximum data resolution. The total frequency range covered by the data measurements is 0.1 – 8GHz with the permittivity values for the blood characterized. The frequency responses for each associated plot is also provided. The measurement setup used is identical to that used for the liquid samples in chapter 4. In order to determine the approximate sample thickness for the calculations, one must use a combination of the phase matching method used in chapter 4 as well as use the measure of the total volume of the sample placed into the transmission line with the known airline dimensions [14].
Figure 25: Magnitude and phase of blood at 23°C from 0.1-0.5GHz
Figure 26: Permittivity of blood at 23°C from 0.1-0.5GHz
Figure 27: Magnitude and phase of blood at 23°C from 0.5-3GHz
Figure 28: Permittivity of blood at 23°C from 0.5-3GHz
**Figure 29: Magnitude and phase of blood at 23°C from 3-8GHz**
Figure 30: Permittivity of blood at $23^\circ$C from 3-8GHz
In the lowest frequency region (0.1-0.5) the $\varepsilon''$ component of the permittivity indicates a flat conductivity response, which matches known conductivity data for blood in the frequency region below 100MHz [8]. The results for the $\varepsilon'$ permittivity of blood fall off at a relatively constant rate over the entire span of 0.1 – 8GHz with no significant findings in the region of 0.5-2GHz, which is consistent with known data [8]. The overall values for the relative permittivity are approximately 60 from 1-3GHz, which is also consistent with the known relative permittivity of blood [5,8]. The most notable parameter, which varies with sample thickness and frequency, is the phase component of the frequency response measurements. This is to be expected as the phase is more sensitive to changes in the electrical thickness of the sample material than the magnitude parameter of a frequency response. Unfortunately when relying on phase change to predict the presence of a stroke, the potential exists for higher frequencies, in the 0.5-2GHz range, to show no phase change after passing through a large hemorrhagic stroke. This can occur when the blood is on the order of the wavelength in the blood, such that, after passing through the sample the wave has undergone a 360° phase shift, resulting in no measurable difference in the phase component. Based on this, it would be recommended for MWT to avoid frequencies at which this problem may occur.
CHAPTER SIX: CONCLUSION

Using known values for the relative permittivity of water and isopropyl alcohol verification for a transmission line calibration method has been made to both test a liquid sample characterization system and to accurately record the frequency response of blood over the desired range for Microwave Tomography, 0.5 – 2GHz, as well as a wider range of 0.1 – 8GHz. The blood measurements have been made to attempt to detect any unexpected changes to the response pattern using a broadband measurement system. The results of the measurements indicate that the overall response pattern for blood is relatively flat in the frequency range considered for MWT. Based on the overall flat response of the blood measurements the most detectable quality of blood compared to surrounding brain matter is likely to be the phase component of a frequency response as there are no noticeable changes in the relative permittivity over the range of 0.5-2GHz. As phase will likely be the best candidate for detection of hemorrhagic strokes, the ideal frequency for detection would not be high enough for the size of the stroke to be approximately the same size as or larger than the wavelength in blood at the chosen frequency. With blood having an index of refraction of approximately 8 in the range of 0.5-2GHz, frequencies higher than 1.3GHz will have wavelengths smaller than 3cm in blood. Once it becomes possible for a full 360° phase shift from passing through a stroke it may become more difficult to detect or estimate the size of a hemorrhagic stroke using MWT. Due to this and the higher attenuation for
frequencies much higher than 1.3GHz it is recommended that a practical frequency range for MWT be restricted to frequencies below 1.3GHz to ensure adequate signal transmission through the brain and to avoid any potential errors caused by strokes on the order of the operational wavelength. The results from these measurements will provide insight to new ideas for either further blood measurements or new methods for detecting hemorrhagic strokes.


APPENDIX A: RICHMOND MULTILAYER EQUATIONS

A summary and explanation of J.H. Richmond’s multilayer equations [2]

Figure A-1 shows the case for a plane wave incident from the left side of the stack, with an incidence angle of $\theta$ on a plane multilayer structure with $N$ homogeneous isotropic layers each having a complex permittivity $\varepsilon_n$, complex permeability $\mu_n$, and thickness $d_n$.

![Figure A-1](image_url)

*Figure A-1: A redrawing of J. H. Richmond’s multilayer stack from [2]*

This multilayer structure can be regarded as having both left and right traveling waves in each layer due to reflection and transmission occurring at each of the finite layer interfaces. The incident wave is coming in from the left side of the multilayer structure (the $A_{N+1}$ layer). For the transverse electric (TE) case the electric field is in the x direction and is given from [2] by

$$E_x^i = E_0 e^{j k_0 y \sin \theta} e^{j k_0 z \cos \theta}$$  \hspace{1cm} (A 1)
where $\theta$ is the angle of incidence, $k_0 = 2\pi/\lambda_0$, and $\lambda_0$ is the free space wavelength. The reflected plane wave is given from [2] by

$$E^r_x = R E_0 e^{j k_0 y \sin \theta} e^{-j k_0 z \cos \theta} \quad \text{(A 2)}$$

where $R$ is the reflection coefficient of the multilayer. The transmitted wave on the right hand side of the multilayer is given from [2] by

$$E^t_x = T E_0 e^{j k_0 y \sin \theta} e^{j k_0 z \cos \theta} \quad \text{(A 3)}$$

where $T$ is the transmission coefficient of the multilayer [2]. Since these equations will be used for a transmission line measurement with an incidence angle of $\theta=0$. This means all the $\sin(\theta)$ terms will go to zero and the $\cos(\theta)$ terms will become one. Therefore the equations A1, A2, and A3 become

$$E^i_x = E_0 e^{j k_0 z} \quad \text{(A 4)}$$

$$E^r_x = R E_0 e^{-j k_0 z} \quad \text{(A 5)}$$

$$E^t_x = T E_0 e^{j k_0 z} \quad \text{(A 6)}$$

Although the field in each layer can be regarded as an infinite sum of plane waves bouncing back and forth in each layer, it is also valid to treat it as two plane waves, one transmitted (right traveling) and the other reflected (left traveling). The complex coefficient $A_n$ represents the right traveling wave and the complex coefficient $B_n$ represents the left traveling waves [2]. Therefore in layer $n$ the total electric field of the wave would be represented by
\[ E_n = A_n e^{\gamma_n z} + B_n e^{-\gamma_n z} \]  \hspace{1cm} (A 7a)

and the n+1 layer would be given by

\[ E_{n+1} = A_{n+1} e^{\gamma_{n+1} z} + B_{n+1} e^{-\gamma_{n+1} z} \]  \hspace{1cm} (A 7b)

where \( \gamma_n \) and \( \gamma_{n+1} \) are the complex propagation respective to each layer and given by

\[ \gamma_n = j\sqrt{\omega^2 \mu_n \varepsilon_n} \quad \text{and} \quad \gamma_{n+1} = j\sqrt{\omega^2 \mu_{n+1} \varepsilon_{n+1}} \]  \hspace{1cm} (A 8a) (A 8b)

With boundary conditions between layers n and n+1 given by

\[ z_n = d_1 + d_2 + \ldots + d_n \]  \hspace{1cm} (A 9)

From the electric field equations given above in (A7a) and (A7b), and using Maxwell’s equation for Faraday’s law

\[ \nabla \times \vec{E} = -j\omega \mu \vec{H} \]  \hspace{1cm} (A 10)

the magnetic field in each layer can be determined [13]. Continuity at each interface is a boundary condition that can also be applied to \( E_x \) and \( H_y \) at the interface \( z = z_n \). Doing all of this results in

\[ A_{n+1} = P_n A_n + Q_n B_n \]  \hspace{1cm} (A 11a)

and

\[ B_{n+1} = R_n A_n + S_n B_n \]  \hspace{1cm} (A 11b)

where
\[ P_n = \left( \frac{1}{2} \right) \left[ 1 + \left( \mu_{n+1} \gamma_n / \mu_n \gamma_{n+1} \right) \right] e^{(\gamma_n - \gamma_{n+1})z_n} \]  
(A 12a)

\[ Q_n = \left( \frac{1}{2} \right) \left[ 1 - \left( \mu_{n+1} \gamma_n / \mu_n \gamma_{n+1} \right) \right] e^{-(\gamma_n + \gamma_{n+1})z_n} \]  
(A 12b)

\[ R_n = \left( \frac{1}{2} \right) \left[ 1 - \left( \mu_{n+1} \gamma_n / \mu_n \gamma_{n+1} \right) \right] e^{(\gamma_n + \gamma_{n+1})z_n} \]  
(A 12c)

\[ S_n = \left( \frac{1}{2} \right) \left[ 1 + \left( \mu_{n+1} \gamma_n / \mu_n \gamma_{n+1} \right) \right] e^{-(\gamma_n - \gamma_{n+1})z_n} \]  
(A 12d)

For a wave incident on the left side of the multistack (figure A-1) the magnitude of the wave traveling to the right in the zeroth layer, \( A_0 \), can be assigned a value of one and since all the components of the transmission line are matched at 50 Ohms, no wave should be reflected back from the right so \( B_0 \) can be assigned a value of zero [2]. Starting with these initial conditions, the coefficients \( A_N \) and \( B_N \) can be calculated recursively in a program such as MATLAB (as is done in Appendices B and C). Once this is done the reflection and transmission coefficients for the total stack can be determined by taking the ratios of the wave coefficients as shown below

\[ R = \frac{B_{N+1}}{A_{N+1}} \quad \text{and} \quad T = \frac{A_0}{A_{N+1}} = \frac{1}{A_{N+1}} \]  
(A 13a) (A 13b)

For the measurements and calculations performed in this thesis the transmission line is a TEM wave so the wavelength and propagation equations in the unfilled airline are the same as free space and as mentioned previously the angle of incidence, \( \theta \), is zero simplifying many of the equations to the form described here from their more general forms [2, 21]. A relation between characteristic
impedances of transmission lines and the free space equations used in the multilayer calculations can be obtained from general characteristic impedance equations [22]. Using Figure A-2 as a general example of a transmission line problem with a dielectric sample placed in the middle of a coaxial sample holder with air on either side.

![Transmission Line example](image)

The characteristic impedance of the various sections of the line can be described using the equation:

\[ Z_0 = \sqrt{\frac{L}{C}} \]  \hspace{1cm} (A 14)

Where \( L \) and \( C \) are

\[ L = \frac{\mu}{2\pi} \ln \left( \frac{b}{a} \right) \]  \hspace{1cm} (A 15a)

\[ C = \frac{2\pi\varepsilon}{\ln \left( \frac{b}{a} \right)} \]  \hspace{1cm} (A 15b)

\( Z_0 \) Becomes
\[ Z_0 = \frac{1}{2\pi} \sqrt{\frac{\mu}{\varepsilon}} \ln \left( \frac{b}{a} \right) \]  

(A 16)

The reflection coefficient for a transmission line is

\[ \Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1} \]  

(A 17)

Substituting in (A 16) for \( Z_0 \) the reflection coefficient becomes

\[ \Gamma = \frac{\frac{1}{2\pi} \sqrt{\frac{\mu_2}{\varepsilon_2}} \ln \left( \frac{b}{a} \right) - \frac{1}{2\pi} \sqrt{\frac{\mu_1}{\varepsilon_1}} \ln \left( \frac{b}{a} \right)}{\frac{1}{2\pi} \sqrt{\frac{\mu_2}{\varepsilon_2}} \ln \left( \frac{b}{a} \right) + \frac{1}{2\pi} \sqrt{\frac{\mu_1}{\varepsilon_1}} \ln \left( \frac{b}{a} \right)} \]  

(A 18)

Which, for lines with a constant inner and outer conductor radii, simplifies to

\[ \Gamma = \frac{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r} - \sqrt{\mu_0 \varepsilon_0}}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r} + \sqrt{\mu_0 \varepsilon_0}} \]  

(A 19)

Which is identical to the reflection coefficient equations for free space equations as are used in the multilayer equations.
APPENDIX B: S-PARAMETER MATLAB SIMULATION CODE

%% Multilayer MATLAB simulation code for distilled water verification

%%%%
%
%
% Calculations based on the paper "Efficient Recursive Solutions for Plane
% and Cylindrical Multilayers" by J. H. Richmond
%
%
% MATLAB code written by Evan Hilderbrand 2/28/2015
%
%
%*********************************************************************
*****
%
%% Multilayer Sample Depiction %%%
%
%
% Layer 1    *Layer 2*      Layer 3     *Layer 4*   Layer 5
% *       *                  *       *       *
% Free Space  *  air  *    water sample  *  air  *  Free Space
% (calibrated  *  gap  *                  *  gap  * (calibrated
% cable to   *       *                  *  gap  *  cable to
% analyzer)  *       *                  *  gap  * analyzer
%

%*********************************************************************
*****
%
%% Clean up any previous calculations %%
clear all
clc
close all

%*********************************************************************
*****
%% Set up Constants %%%
c   =  3e8;       % Speed of Light
eps0=  8.85e-12; % Free-space permittivity
mu0 =  pi*4e-7; % Free-space permeability
Z0  = 377;      % Free space impedance
theta = 0;      % Angle of incidence is 0deg
watermu = 1.0;  % Water is assumed to be non-magnetic

total_airline_length = 0.0362;

z(1) = 1;       % free space thickness placeholder
z(4) = 0.009;   % thickness of APC7 Adapter air layer (m)
z(3) = 0.032;   % thickness of sample (m)
z(2) = total_airline_length - z(3); % thickness of first air layer (m)
z(5) = 1;       % free space thickness placeholder

thickness = z(3)*100; % convert sample thickness to cm and store for title
\[ z(3) = z(2) + z(3); \]
\[ z(4) = z(3) + z(4); \]

% Load accepted epsilon data for water  
%(data columns are: [frequency, \text{eps}' - \text{eps}'')  
load watereps  
freq(:,1) = watereps(:,1)*1e9;

load('Setup2_water_1_Sparams')

SP = S;  
clear S

fstart = 0.5;  
fstop = 3.0;  
numpts = 801;

df = (fstop - fstart)/(numpts-1);  
freq2 = fstart:df:fstop;  
freq2 = freq2' .* 1e9;

lambda(:,1) = c ./ freq(:,);  
% Calculate \(k_0\) and \(w\) for each frequency  
k0 = (2*pi)./lambda;  
w = 2*pi.*freq(:,);
% Loop through each frequency value to calculate their respective frequency
% specific constants for the multilayer problem

i=1;  % Rest frequency counter
while i <= numel(freq)
    \% initial air layer
    gamma(i,1) = 1i*sqrt( (w(i).^2).* (1.0).* (1.0).* mu0*eps0...
        - (k0(i).^2).* ((sin(theta))^2));

    gamma(i,2) = 1i*sqrt( (w(i).^2).* (1.0).* (1.0).* mu0*eps0...
        - (k0(i).^2).* ((sin(theta))^2));

    \% water layer
    gamma(i,3) = 1i*sqrt( (w(i).^2).* watermu.* watereps(i,2)...
        .* mu0*eps0 - (k0(i).^2).* (sin(theta))^2);

    \% final air layer
    gamma(i,4) = 1i*sqrt( (w(i).^2).* (1.0).* (1.0)...
        .* mu0*eps0 - (k0(i).^2).* (sin(theta))^2);
    gamma(i,5) = gamma(i,1);

    A(i,1) = 1;  % Free Space initial condition (layers 1 and 5)
A(i,5) = 1;
B(i,1) = 0; % Free Space initial condition (layers 1 and 5)
B(i,5) = 0;

% Loop through each layer to calculate their respective frequency
% specific constants for the multilayer problem

n=1; % Reset layer counter

while n <= 4

    P(i,n) = 0.5*(1 + (mu0*gamma(i,n))./(mu0*gamma(i,n+1))).*
               exp((gamma(i,n)-gamma(i,n+1)).*z(n));

    Q(i,n) = 0.5*(1 - (mu0*gamma(i,n))./(mu0*gamma(i,n+1))).*
               exp((-gamma(i,n)-gamma(i,n+1)).*z(n));

    R(i,n) = 0.5*(1 - (mu0*gamma(i,n))./(mu0*gamma(i,n+1))).*
               exp((gamma(i,n)+gamma(i,n+1)).*z(n));

    S(i,n) = 0.5*(1 + (mu0*gamma(i,n))./(mu0*gamma(i,n+1))).*
               exp((-gamma(i,n)+gamma(i,n+1)).*z(n));

    A(i,n+1) = P(i,n) * A(i,n) + Q(i,n) * B(i,n);  
    B(i,n+1) = R(i,n) * A(i,n) + S(i,n) * B(i,n); 

end
% Calculate Reflection and Transmission values at each layer
R(i,n) = B(i,n+1) / A(i,n+1);
T(i,n) = 1 / A(i,n+1);

n=n+1; % increment layer counter
end

i=i+1; % increment frequency counter
end

%*********************************************************************
*****
%% Plot and label Transmission an Reflection magnitude and phase %%
h1 = figure;
hold all
plot(freq(:,1)/1e9, 20*log10(abs((R(:,3)))),freq(:,1)/1e9,...
     20*log10(abs(T(:,4))), freq2(:,1)/1e9, 20*log10(abs(SP(:,2,1)))...,
     'Linewidth', 2.0)
legend('Prediction: Reflection', 'Prediction: Transmission',...
       'Measured: Transmission S21', 'Location', 'NorthEast')
title('Reflection and Transmission Magnitude for 3.0cm of Distilled Water',...
      'FontSize', 18)
ylabel('Magnitude (dB)', 'FontSize', 16)
xlabel('Frequency (GHz)', 'FontSize', 16)
grid on
set(gca, 'FontSize', 13) % Increased axis fontsize for clarity
axis([0.5 3 -80 10])

h2 = figure;
hold all

plot( freq(:,1)/1e9, (180/pi)*(angle(R(:,3))), freq(:)/1e9,...
(180/pi)*(angle(T(:,4))), freq2(:)/1e9, (180/pi)*angle(SP(:,2,1)),...
'Linewidth', 2.0)
legend('Prediction: Reflection', 'Prediction: Transmission',...
'Measured: Transmission S21', 'Location', 'NorthEast')
title('Reflection and Transmission Phase for 3.0cm of Distilled Water',...
'FontSize', 18)
ylabel('Phase (deg)', 'FontSize', 16)
xlabel('Frequency (GHz)', 'FontSize', 16)
grid on
set(gca, 'FontSize', 13) % Increased axis fontsize for clarity
axis([0.5 3 -200 200])
APPENDIX C: MATLAB PERMITTIVITY CALCULATOR

%% Multilayer MATLAB epsilon solver code for distilled water verification %

% Calculations based on the paper "Efficient Recursive Solutions for Plane and Cylindrical Multilayers" by J. H. Richmond

% MATLAB code written by Evan Hilderbrand 3/3/2015

%*********************************************************************
*****
%% Multilayer Sample Depiction %%

% Layer 1   *Layer 2*    Layer 3   *Layer 4*   Layer 5
%     *     *       *     *
% Free Space  *  air  *    water sample  *  air  *  Free Space
% (calibrated  *  gap  *       *  gap  * (calibrated
% cable to    *     *       *     * cable to
% analyzer)   *     *       *     * analyzer

%*********************************************************************
*****

%% Clean up any previous calculations %%
clear all
clc

% close all

%************************************************************
****
%% Set up Constants %%%
c   =  3e8; % Speed of Light
eps0=  8.85e-12; % Free-space permittivity
mu0 =  pi*4e-7; % Free-space permeability
Z0  =  377; % Free space impedance
theta = 0; % Angle of incidence is 0deg
watermu = 1.0; % Water is assumed to be non-magnetic

%************************************************************
****
%% Input Frequency, Thicknesses, and Initial guess%%%

% Operational Frequency (GHz)
fstart = 0.5;
fstop  = 3.0;
umpts = 801;

total_airline_length = 0.0362;

z(1) = 1; % free space thickness placeholder
z(2) = 0.009; % thickness of APC7 Adapter air layer (m)
z(3) = 0.030; % thickness of sample (m)
z(4) = total_airline_length - z(3); % thickness of first air layer (m)
z(5) = 1; % free space thickness placeholder

thickness = z(3)*100; % convert sample thickness to cm and store for title

% Input Initial Guesses
guess_eps = 79 - 5j;
mu = 1.0 - 0j; % Assume nonmagnetic
syms eps

df = (fstop - fstart)/(numpts-1);
freq = fstart:df:fstop;
freq = freq .* 1e9;

z(3) = z(2) + z(3);
z(4) = z(3) + z(4);

% eps = eps * ones(numpts,1);
%******************************************************************************
*****
%%% Collect S Parameters

% load('Setup2_meas3_Sparams')
load('Setup2_water_1_Sparams')
% load('Theory_test_Sparams')
load('watereps')

SP = S;
clear S

lambda(:,1) = c ./ freq(:,); % Calculate k0 and w for each frequency
k0 = (2*pi)./lambda;
w = 2*pi.*freq(:,);

%*********************************************************************
*****
%%% Loop through each frequency value to calculate their respective
frequency
% specific constants for the multilayer problem

i=1; % Rest frequency counter
while i <= numel(freq)

% initial air layer
gamma(i,1) = 1i*sqrt((w(i).^2).* (1.0).* (1.0).* (1.0).*...
\[ \mu_0 \varepsilon_0 - (k_0(i) \cdot 2) \cdot ((\sin(\text{theta}))^2) \]

\[ \gamma(i,2) = 1i \cdot \sqrt{\left( w(i) \cdot 2 \right) \cdot (1.0) \cdot (1.0) \cdot \mu_0 \cdot \varepsilon_0 - (k_0(i) \cdot 2) \cdot ((\sin(\text{theta}))^2) } \]

\% isopropyl alcohol layer
\[ \gamma_3 = (1i \cdot \sqrt{ w(i) \cdot 2 \cdot \mu \cdot \varepsilon \cdot \mu_0 \cdot \varepsilon_0 - (k_0(i) \cdot 2) \cdot ((\sin(\text{theta}))^2) } \]

\% final air layer
\[ \gamma(i,4) = 1i \cdot \sqrt{ w(i) \cdot 2 \cdot (1.0) \cdot (1.0) \cdot \mu_0 \cdot \varepsilon_0 - (k_0(i) \cdot 2) \cdot ((\sin(\text{theta}))^2) } \]

\[ \gamma(i,5) = \gamma(i,1) \]

\[ A(i,1) = 1; \quad \% \text{Free Space initial condition (layers 1 and 5)} \]
\[ A(i,5) = 1; \]
\[ B(i,1) = 0; \quad \% \text{Free Space initial condition (layers 1 and 5)} \]
\[ B(i,5) = 0; \]

\% Loop through each layer to calculate their respective frequency
\% specific constants for the multilayer problem

\[ n=1; \quad \% \text{Reset layer counter} \]

\[ P(i,n) = 0.5 \cdot (1 + (\mu_0 \gamma(i,n)) / (\mu_0 \gamma(i,n+1))) \]
\[ *\text{exp}((\gamma(i,n) - \gamma(i,n+1)) \cdot z(n)) ; \]
\[ Q(i,n) = 0.5 \left( 1 - \frac{\mu_0 \gamma(i,n)}{\mu_0 \gamma(i,n+1)} \right) \cdot *\text{exp}((-\gamma(i,n) - \gamma(i,n+1)) \cdot z(n)) ; \]
\[ R(i,n) = 0.5 \left( 1 - \frac{\mu_0 \gamma(i,n)}{\mu_0 \gamma(i,n+1)} \right) \cdot *\text{exp}((\gamma(i,n) + \gamma(i,n+1)) \cdot z(n)) ; \]
\[ S(i,n) = 0.5 \left( 1 + \frac{\mu_0 \gamma(i,n)}{\mu_0 \gamma(i,n+1)} \right) \cdot *\text{exp}((-\gamma(i,n) + \gamma(i,n+1)) \cdot z(n)) ; \]

\[ A(i,n+1) = P(i,n) \cdot A(i,n) + Q(i,n) \cdot B(i,n) ; \]
\[ B(i,n+1) = R(i,n) \cdot A(i,n) + S(i,n) \cdot B(i,n) ; \]

\[ n=2; \]

\% Layer 2 equations for reference
\% % P2 = 0.5 \left( 1 + \frac{\mu_0 \gamma(i,n)}{\mu_0 \gamma_3} \right) \cdot *\text{exp}((\gamma(i,n) - \gamma_3) \cdot z(n)) ; \]
\% % Q2 = 0.5 \left( 1 - \frac{\mu_0 \gamma(i,n)}{\mu_0 \gamma_3} \right) \cdot *\text{exp}((-\gamma(i,n) + \gamma_3) \cdot z(n)) ; \]
\% % R2 = 0.5 \left( 1 - \frac{\mu_0 \gamma(i,n)}{\mu_0 \gamma_3} \right) \cdot *\text{exp}((\gamma(i,n) + \gamma_3) \cdot z(n)) ; \]
\% % S2 = 0.5 \left( 1 + \frac{\mu_0 \gamma(i,n)}{\mu_0 \gamma_3} \right) \cdot *\text{exp}((-\gamma(i,n) - \gamma_3) \cdot z(n)) ; \]
\% % A3 = P2 \cdot A(i,2) + Q2 \cdot B(i,2) ; \]
\% % B3 = R2 \cdot A(i,2) + S2 \cdot B(i,2) ; \]
\[
% P3 = 0.5*(1 + (mu0*gamma3)/(mu0*gamma(i,4)))... \\
% *exp((gamma3-gamma(i,4))*z(3)); \\
% Q3 = 0.5*(1 - (mu0*gamma3)/(mu0*gamma(i,4)))... \\
% *exp(-(gamma3+gamma(i,4))*z(3)); \\
% R3 = 0.5*(1 - (mu0*gamma3)/(mu0*gamma(i,4)))... \\
% *exp((gamma3+gamma(i,4))*z(3)); \\
% S3 = 0.5*(1 + (mu0*gamma3)/(mu0*gamma(i,4)))... \\
% *exp(-(gamma3-gamma(i,4))*z(3)); \\
% \\
% A4 = P3 * A3 + Q3 * B3 ; \\
% B4 = R3 * A3 + S3 * B3 ; \\
\]

\[
% function = real(Reverse multilayer sample equation) - real(S21) \\
% + real(Reverse multilayer sample equation) - imag(S21) \\
fun = @(eps) (real(1/((0.5*(1 + (mu0.*(1i*sqrt( (w(i).^2) ... \\
.* mu.* (eps).* mu0*eps0 - (k0(i).^2).* ((sin(theta))^2))))... \\
./mu0.*gamma(i,4))).*exp(((1i*sqrt( (w(i).^2).* mu.* (eps) ... \\
.* mu0*eps0 - (k0(i).^2).* ((sin(theta))^2))))- \\
gamma(i,4)).*z(3)))... \\
.* ( (0.5*(1 + (mu0.*gamma(i,2)))/(mu0.*1i*sqrt( (w(i).^2)... \\
.* mu.* (eps).* mu0*eps0 - (k0(i).^2).* ((sin(theta))^2)))))... \\
.*exp((gamma(i,2)-(1i*sqrt( (w(i).^2).* mu.* (eps).* mu0*eps0... \\
- (k0(i).^2).* ((sin(theta))^2))))).z(2)))... \\
.* A(i,2) + ( 0.5*(1 - (mu0.*gamma(i,2))/(mu0.*1i... \\
*sqrt( (w(i).^2).* mu.* (eps).* mu0*eps0 - (k0(i).^2)...
\[
\begin{align*}
&\mu \ast ((\sin(\theta))^2))) \ast \exp(-(\text{gamma}(i,2)+(1i\sqrt{w(i)^2}) \dot{\text{mu}} \ast (\text{eps}) \ast \mu_0 \ast \eps_0 - (k_0(i)^2) \ast ((\sin(\theta))^2)))) \\
&\ast z(2) \ast B(i,2) + (0.5 \ast (1 - (\text{mu}_0 \ast (1i\sqrt{w(i)^2}) \dot{\text{mu}} \ast (\text{eps}) \ast \mu_0 \ast \eps_0 - (k_0(i)^2) \ast ((\sin(\theta))^2)))) \\
&\ast \exp(-(\text{gamma}(i,2) \ast (\sin(\theta))^2))) \ast z(3)) \\
&\ast ((\sin(\theta))^2)) + \gamma(i,4) \ast z(3)) \\
&\ast (0.5 \ast (1 - (\text{mu}_0 \ast \gamma(i,2))), (\text{mu}_0 \ast (1i\sqrt{w(i)^2})) \dot{\text{mu}} \ast (\text{eps}) \ast \mu_0 \ast \eps_0 - (k_0(i)^2) \ast ((\sin(\theta))^2))) \\
&\ast \exp((\text{gamma}(i,2) \ast (\sin(\theta))^2))) \ast z(2)) \ast A(i,2) \\
&+ (0.5 \ast (1 + (\text{mu}_0 \ast \gamma(i,2))), (\text{mu}_0 \ast (1i\sqrt{w(i)^2})) \dot{\text{mu}} \ast (\text{eps}) \ast \mu_0 \ast \eps_0 - (k_0(i)^2) \ast ((\sin(\theta))^2))) \\
&\ast \exp((-\gamma(i,2) \ast (1i\sqrt{w(i)^2}) \dot{\text{mu}} \ast (\text{eps}) \ast \mu_0 \ast \eps_0 - (k_0(i)^2) \ast ((\sin(\theta))^2))) \\
&\ast \exp((\gamma(i,2) - (1i\sqrt{w(i)^2}) \dot{\text{mu}} \ast (\text{eps}) \ast \mu_0 \ast \eps_0 - (k_0(i)^2) \ast ((\sin(\theta))^2))) \\
&\ast (0.5 \ast (1 + (\text{mu}_0 \ast \gamma(i,2))), (\text{mu}_0 \ast (1i\sqrt{w(i)^2})) \dot{\text{mu}} \ast (\text{eps}) \ast \mu_0 \ast \eps_0 - (k_0(i)^2) \ast ((\sin(\theta))^2))) \\
\end{align*}
\]
.* ((sin(theta))^2)).*z(2))...  
.* A(i,2) + (0.5*(1 - (mu0.*gamma(i,2))./(mu0.*(1i*sqrt(
(w(i).^2)...  
.* mu.* (eps).* mu0*eps0 - (k0(i).^2).* ((sin(theta))^2)))))...)  
.* exp(-(gamma(i,2)+(1i*sqrt((w(i).^2).* mu.* (eps).* mu0*eps0...  
- (k0(i).^2).* ((sin(theta))^2))).*z(2)) )* B(i,2) ...  
+ (0.5*(1 - (mu0.*(1i*sqrt((w(i).^2).* mu.* (eps).* mu0*eps0...  
- (k0(i).^2).* ((sin(theta))^2))))./(mu0*gamma(i,4))))...  
.* exp(-(1i*sqrt((w(i).^2).* mu.* (eps).* mu0*eps0 -
(k0(i).^2)...  
.* ((sin(theta))^2)))+gamma(i,4).*z(3))...  
.* ( (0.5*(1 - (mu0.*gamma(i,2))./(mu0.*(1i*sqrt((w(i).^2)...  
.* mu.* (eps).* mu0*eps0 - (k0(i).^2).* ((sin(theta))^2)))))...)  
.* exp((gamma(i,2)+(1i*sqrt((w(i).^2).* mu.* (eps).* mu0*eps0...  
- (k0(i).^2).* ((sin(theta))^2))).*z(2)) )* A(i,2) ...  
+ (0.5*(1 + (mu0.*gamma(i,2))./(mu0.*(1i*sqrt((w(i).^2).* mu...  
.* (eps).* mu0*eps0 - (k0(i).^2).* ((sin(theta))^2)))))).*exp(-
(gamma(i,2)...)  
-((1i*sqrt((w(i).^2).* mu.* (eps).* mu0*eps0 - (k0(i).^2)...  
.* ((sin(theta))^2)))).*z(2)) ) * B(i,2)))) -
li*(imag(SP(i,2,1)))) ;

% Use Newton's Method to solve for epsilon
end_eps(i) = newtonraphson(fun,guess_eps);

guess_eps = end_eps(i); % Use last epsilon solution as next guess
i=i+1; % increment frequency counter

end

%%% Plot and label Transmission an Reflection magnitude and phase %%%

h1 = figure;
hold all
plot(freq(:,)/1e9,
real(end_eps),watereps(:,1),real(watereps(:,2)),'Linewidth', 2.0)
title(strrep(strcat('
epsilon\prime for_ ', num2str(thickness), ...
' cm of Distilled Water'),'_',' '), 'FontSize', 18)
legend('Solved', 'Reference', 'Location', 'SouthWest')
ylabel('\epsilon\prime', 'FontSize', 16)
xlabel('Frequency (GHz)', 'FontSize', 16)
grid on
set(gca, 'FontSize', 13) % Increased axis fontsize for clarity
axis([1.0 3 0 100])

%%% Plot and label Transmission an Reflection magnitude and phase %%%

h2 = figure;
hold all
plot(freq(:,)/1e9, imag(end_eps),watereps(:,1),imag(watereps(:,2)),
   'Linewidth', 2.0)
title(strrep(strcat(' \epsilon\prime\prime for_',
   num2str(thickness),...
   'cm of Distilled Water'),'_',' '), 'FontSize', 18)
legend('Solved', 'Reference', 'Location', 'SouthWest')
ylabel('\epsilon\prime\prime', 'FontSize', 16)
xlabel('Frequency (GHz)', 'FontSize', 16)
grid on
set(gca, 'FontSize', 13) % Increased axis fontsize for clarity
axis([1.0 3 -30 0])