I, Shrirang Deshpande, hereby submit this original work as part of the requirements for the degree of Master of Science in Mechanical Engineering.

It is entitled: Improving observability in experimental analysis of rotating systems

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Improving observability in experimental analysis of rotating systems

A thesis submitted to the Graduate School of
the University of Cincinnati
in partial fulfilment of requirements for the degree of

Master of Science

in the Department of Mechanical and Materials Engineering
of the College of Engineering & Applied Sciences

September 2014

by

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May 2010

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Abstract

The vast field of rotational systems – including measurement capabilities, analytical tools and observability – is still evolving. Spectral maps and order tracks are the most popular tools for analyzing the behavior of various components subject to a rotational characteristic. Although various forms of these tools are well researched and implemented, they are still susceptible to improper sensor location on the structure and to measurement noise. This thesis attempts to bridge the gap between properly located sensors and effective analysis based on their measurements. The concept of singular value decomposition (SVD), which is already well used in modal analysis, forms the basis of observability improvement. By using response data acquired from multiple sensors on a structure, it is possible to calculate and plot the singular values obtained from the entire frequency domain response data at each point in the spectral map graph. The resulting singular value plots will depict the magnitude of contribution of the sensor assembly which can form a noise-free and reliable basis for further analytical tools.

Generating an accurate estimate of the instantaneous RPM from the raw tachometer data is as important as the ensuing analysis. Although most of the work published in the area of rotational systems touches on the topic of constructing an estimate of instantaneous RPM, a properly structured reference is needed which can be implemented easily in a software environment like MATLAB™.
Acknowledgements

It is tough to fit all of the people who have helped me complete my Master’s degree, but I will try nevertheless!

I would like to thank Dr. David L Brown for being an inspiration throughout my time at SDRL. Even though there were only a few instances when we met, his vast amount of previous work provided tremendous help to my learning process. I am immensely grateful and thankful to Dr. Allyn W Phillips, for helping me out at any time of the day, regardless of the level of difficulty, throughout this research. His friendly and open manner allowed me to never hesitate in seeking his expert advice on mathematical and software related issues.

I cannot thank Dr. Randall J Allemang enough. Firstly, for agreeing to be my thesis advisor! His constant support in the midst of his busy schedule made sure I was on the right track. I also thank for him placing his confidence in me and allowing me to freely work for this Thesis with his vital and timely guidelines. The basic conceptual idea of this work of research was his input and I am glad to have been a part of it and learn a great deal. I would also like to again specially thank Dr. Allemang and Dr. Phillips for providing me financial support throughout the academic year of 2013-14. It was truly a humbling experience to study and work in the presence of these minds.

Being away from my home country wasn’t easy. Although it was made to feel like home by the constant unwavering backing of my cousin Sourabh, his wife Komal and their beautiful daughter Riya, in Pittsburgh, PA. It was complete fun while working at SDRL and I owe a lot of that to my lab-mates – Murali, Rahul, Karan, Hasan, Vikrant, Rajeev and Scott Phillips! My time at Cincinnati turned out to be more fun than expected because of my friends (roommates and “virtual” roommates)!
And lastly, I would like to thank my parents. Right from the day I decided to come to the United States to this day, all I have received from them is solid support. It must be tough, when both of their children are out of the country and still to be supportive to this level. I cannot ever thank them enough for that.
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# Nomenclature

## Abbreviations

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<th>Description</th>
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<tbody>
<tr>
<td>RPM</td>
<td>Revolutions per minute</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>SVD</td>
<td>Singular Value Decomposition</td>
</tr>
<tr>
<td>DSP</td>
<td>Digital Signal Processing</td>
</tr>
<tr>
<td>DSA</td>
<td>Digital Signal Analyzer</td>
</tr>
<tr>
<td>TVDFT</td>
<td>Time Variant Discrete Fourier Transform</td>
</tr>
<tr>
<td>COT</td>
<td>Computed Order Tracking</td>
</tr>
<tr>
<td>MPE</td>
<td>Modal Parameter Estimation</td>
</tr>
<tr>
<td>CMIF</td>
<td>Complex Mode Indicator Function</td>
</tr>
<tr>
<td>FRF</td>
<td>Frequency Response Function</td>
</tr>
<tr>
<td>DOF</td>
<td>Degree of Freedom</td>
</tr>
<tr>
<td>EVD</td>
<td>Eigen Value Decomposition</td>
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</tbody>
</table>

## Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>Δt</td>
<td>Discrete sampling time interval</td>
</tr>
<tr>
<td>F&lt;sub&gt;samp&lt;/sub&gt;</td>
<td>Sampling frequency of the DSA</td>
</tr>
<tr>
<td>F&lt;sub&gt;nyq&lt;/sub&gt;</td>
<td>Nyquist frequency</td>
</tr>
<tr>
<td>F&lt;sub&gt;max&lt;/sub&gt;</td>
<td>Maximum frequency that can be observed in a spectrum</td>
</tr>
<tr>
<td>O&lt;sub&gt;samp&lt;/sub&gt;</td>
<td>Sampling order</td>
</tr>
<tr>
<td>O&lt;sub&gt;nyq&lt;/sub&gt;</td>
<td>Nyquist order</td>
</tr>
<tr>
<td>O&lt;sub&gt;max&lt;/sub&gt;</td>
<td>Maximum order that can be observed in a spectrum</td>
</tr>
<tr>
<td>Δo</td>
<td>Delta Order – order resolution</td>
</tr>
<tr>
<td>t</td>
<td>Time variable</td>
</tr>
<tr>
<td>T</td>
<td>Total sample time analyzed</td>
</tr>
<tr>
<td>N</td>
<td>Data block size/total number of spectral lines</td>
</tr>
<tr>
<td>Δf</td>
<td>Delta frequency – frequency resolution</td>
</tr>
<tr>
<td>[A]</td>
<td>Base matrix</td>
</tr>
<tr>
<td>[U]</td>
<td>Left singular matrix</td>
</tr>
<tr>
<td>[V]</td>
<td>Right singular matrix</td>
</tr>
<tr>
<td>[Σ]</td>
<td>Singular value matrix</td>
</tr>
<tr>
<td>[I]</td>
<td>Identity matrix</td>
</tr>
<tr>
<td>[Q]</td>
<td>Eigen vector matrix</td>
</tr>
<tr>
<td>[Λ]</td>
<td>Eigen value matrix</td>
</tr>
<tr>
<td>σ</td>
<td>Singular value</td>
</tr>
</tbody>
</table>
1 Introduction

For the purposes of health monitoring, diagnosis, damage detection, product testing and modal analysis, rotational systems are required to be analyzed using several numerical techniques. Every technique has its own particular characteristics which form the basis of an accurate extraction of relevant data, on the basis of which further decisions can be made in the design stages as well as maintenance procedures.

But, all of these techniques and procedures analyze data which has been collected from various sensors on the component or the structure – many times without any prior knowledge whether the sensor location is active in the experiment or not. Here, looms large a vulnerability of further analysis, purely based on sensor location. Unfortunately, none of the order tracking techniques, consider the physical relevance of the sensor location. The data recorded from a sensor is ultimately used in any order tracking method. A misrepresentation of a particular dynamic characteristic, due to improper sensor location on a complex structure, can lead to wrong interpretations; leading to a cascading effect which can magnify the possibility of a catastrophic failure.

Hence, a new numerical technique is needed, which can inform the user of the physical relevance of a particular sensor assembly on the test-structure, so that irrelevant data acquisition is avoided and only the major/active sources are in consideration.

1.1 Need to Improve Observability

Rotating systems inherently pose a difficulty in sensor placement. Although, the response of relatively simple machines like, multi-plane rotor rigs and rigid rotor shafts can be observed by placing piezo-electric transducers like accelerometers, on their bearings. In these cases, often, analyzing the bearing response is sufficient in determining the dynamic nature during operation of the machine. For example,
for multi-plane balancing, balancing calculations can be performed using data acquired for 3 different cases only on the two support bearings\textsuperscript{[20]}. In other cases, like small sized high speed rotors, often the casing or the housing is sufficient for sensor placement to get a fair picture on the behavior of the machine.

When rotating systems involving coupling characteristics or having multiple moving components are to be analyzed, a considerable number of sensors are required to observe the structure. These systems are often relatively bigger in size than, say, simple rotating systems. Due to their size and the difference in dynamic characteristics at separate points, it is necessary to have a valid sensor assembly, which would provide reliable data to be analyzed further.

If, a sensor is placed on a relatively inactive point, the data acquired by the sensor would not contain the true dynamic nature of the machine. This can occur within three principal directions for a particular point, or within different locations on a test-structure.

For example, consider a point on an engine test-cell (corresponding to a point on the gearbox on the actual vehicle) which is chosen such that the maximum vibration or response is inherently in the \(+z\) direction, and a tri-axial accelerometer is not chosen to observe it. Instead, a uni-axial accelerometer is placed on that location in the \(+y\) direction.
If, the same data, in the +y direction is carried forward in order tracking analysis, the corresponding order history of order 12 is shown to be in the following figure. It can be said, observing the order history at this point, that there is no significant contribution from the 12th order of this machine’s vibration.

But if, the order tracks from +x and +z direction (acquired from data provided by a tri-axial accelerometer) are observed together with the one from +y, it can be said that Order 12 is very much active in the RPM run-up in the case of sensor in the +z direction.
Figure 1-3: Comparison of 12\textsuperscript{th} order amplitudes vs RPM for 3 sensors (+x, +y and +z)

Suppose, two different sensor locations are considered on a structure, as shown in following figures.
Visible from Figure 1-4 and Figure 1-5, there are more active orders in Location 2 than in the first. If data which is acquired from the sensor showing low order activity is used as a baseline case, it can be interpreted that there is no need for further action to correct for the cause of vibration for a particular order.

Cases similar to these are abundantly possible on a complex structure where there is limited prior knowledge of dynamically active points. Thus, catastrophic failures or damages are possible when moving ahead with the interpretations made on acquired data with “faulty location” sensors.
1.2 New Method – Virtual Dominant sensor

As discussed before, a new numerical technique is needed, which can inform the user of the physical relevance of a particular sensor assembly on the test-structure, so that irrelevant data acquisition is avoided and only the major/active sources are in consideration. The gap between relevant analysis based on measurements and sensor location needs to be bridged.

Using, the concept of singular value decomposition (SVD), on the entire response cross-power spectrum matrix, a relatively new method has been developed in this research. Another technique based on singular value decomposition applicable to rotating system analysis was developed by Blough et al in 1999 which enabled the independent operating shape determination based on order track measurements \[^9\].

In the technique developed in this research, using the response data acquired from multiple sensors on the structure, it is possible to calculate and plot the singular values obtained from the correlation of entire frequency domain response data at each 2-dimensional point (with a magnitude) in the spectral map graph. The resulting singular value plots will depict the relative magnitude of contribution of the sensor assembly, forming a reliable basis for further analytical tools. The proposed method can produce results which can be interpreted as a quantification of the sensor contribution at each location. Singular values with relatively higher magnitudes will contain and show information from major sources of vibration/faults/mechanical-signatures.

The SVD approach to experimental rotational systems analysis also yields an improved reduction of noise, since the dominant response (achieved by the SVD process) from multiple sensors acts similarly to averaging in modal analysis measurements. Often, in the experimental analysis of rotating systems, averaging is not possible due to rapid speed changes in the test structure.
In essence, the new method will enable the user to observe the entire structure as a whole, based on the sensor assembly used. Virtually, one dominant single sensor will be observing the structure. There are a multitude of possibilities which can be extended using this method, example – reduction of measurement noise. By blending with existing order tracking methods, useful tools in real-time and post-processing applications can also be devised. These will be discussed in a further sub-section.
2 Literature Review

2.1 Spectral Maps

Spectral maps are a popular tool in analyzing data from rotating machines at various speeds. The vibration spectrum of the measured data (Z-axis) is plotted against the frequency axis (X-axis) with instantaneous speed (RPM) usually on the Y-axis. Apart from speed (RPM), parameters like Temperature, Time and Load can also be plotted as the third dimension in a spectral map.

Analyzing response spectrum or the auto-power spectrum at different speeds is important for rotating systems. This is because often times, the mechanical signature of a defect may not appear at all speeds and better understanding of the machine can be gained by analyzing the response spectrum over a speed range. [13]

A simple order track based on fast fourier transform is based on the accuracy of the RPM spectral map. Often, observing a spectral map can give immediate insight on the behavior of the machine at the measured point across a speed range which can be beneficial in further analytical procedures like order tracking.

2.2 Tachometer Signals

The most important parameter in experimental analysis of rotating systems is the instantaneous speed. Every analytical procedure, is based on the accurate estimation of the instantaneous speed. This is due to the fact that the instantaneous frequency of the order(s) to be analyzed, is a direct multiple of the fundamental machine speed, which is the instantaneous speed measured from the tachometer signal. Hence, the most important channel during data acquisition is the tachometer channel. Accuracy of the
estimation of the RPM is directly based on the quality of the tachometer signal processing tools employed.

Various signal processing tools have been developed after the historical method of conditioning tachometer channels with tracking ratio tuners. These developments have been primarily due to the sweep rate limitations posed by the tracking ratio tuner \[^6\]. The variety of signal processing tools is directly due to the different types of Tachometer measurement techniques used in the industry.

The different types of tachometers used are shaft encoders, infrared light type, laser type and variable and conditional-variable reluctance type \[^20\].

The output of the tachometer can be handled generally in two ways \[^13\]. One method is to feed the tachometer output through one channel of a multi-channel digital signal analyzer (DSA) which will produce a pulse train proportional to the rotational frequency of the shaft. Alternatively, through specialized hardware integrated to the DSA’s external sample control, it is possible to determine the rotating speed as well.

Regardless of the type of sensor used, it is important to ensure that the sensor used to generate a pulse train is not affected by transients and contains little noise \[^18\]. There are a variety of issues related to tachometer signals depending on the type of sensors used. For example, light based sensors have a susceptibility to ambient light or backlight. Often, unwanted reflective surfaces (due to scratches, or cracks) can be present on the shaft which can trigger a pulse resulting in an inaccurate pulse train. On the other hand, when dealing tachometers whose pulse train, and the trigger is based on a missing tooth on the wheel, problems can develop when an unwanted crack or shiny surface is present on a position other than the missing tooth. This can trigger another pulse which would be interpreted by the DSA as one complete revolution and can cause problems in the ensuing analysis or processes. For example, this type of problem is often associated with engine flywheels, where a proximity sensor is
used as a tachometer, whose output is directly fed into the engine control unit. An inaccurate RPM estimate would be interpreted along with other ambient conditions, which can trigger problems in the fuel systems unit and generate misfires and other dangerous phenomena.

Poorly spaced toothed-wheel pulses, close clearances, misalignment can cause problems when dealing with the tachometer signal. Eccentricity of the wheel mounting (on which the tachometer signal is generated from) can cause an amplitude modulated signal in the time domain. A similar effect can be observed if instead of wheel eccentricity, the tachometer probe mounting is unstable or loose and has a relative displacement with the shaft surface as the wheel rotates.

2.3 RPM Estimation

Most of the physically triggered problems associated with tachometer signals are avoidable. Problems which can’t be avoided can be dealt with during the signal processing of the tachometer channel output. For example, it is important to not use AC coupling when dealing with Infra-red or Laser based tachometers. Application of windows on the signal is not advisable. Often, the tachometer processing involves estimating the zero-crossings of the pulse in the time domain. It is possible that due to the discretized nature of the sampling of the tachometer signal, it might be that the zero crossing time instant is not a multiple of the sampling time interval \((\Delta t = 1/F_{\text{sampling}})\). In these cases, it is advantageous to use numerical interpolation between two sampled time values to estimate the zero-crossing instant.

A particular signal processing technique will be employed depending on the type of tachometer and its associated pulse train. For example, for a signal which is characterized by one pulse per revolution, a simple pulse detection technique combined with interpolation to estimate zero crossings would be enough. For signals characterized by multiple pulses per revolution, a similar technique can be employed. But the number of pulses per revolution must accounted for in the RPM estimation. In general, for this type of signal, RPM can be estimated by the following equation.
\[
RPM = \frac{60}{(t_{k+1} - t_k) \times P} \quad 2.1
\]

Where,

\( t_k \) is the \( k \)th zero crossing time-instant or time-instant of the \( k \)th pulse detected,

\( P \) is the number of pulses per revolution

A detailed description of estimating instantaneous speed from raw tachometer signals – one/multiple pulse(s) per revolution and missing tooth tachometer pulse train, will be explained in the next section. Other techniques which are possible when dealing with tachometer signals are: a) FFT based technique b) Hilbert transform techniques \[^{[20]}\].

The FFT based technique is effective if implemented correctly. The digitized time domain tachometer signal can be divided into subsets of data depending a particular block size (512, 1024, 2048 etc.). These blocks can be then fast fourier transformed (and scaled appropriately) and the frequency of the primary peak can be captured. The primary peak frequency multiplied by 60 yields the corresponding RPM value.

The complex nature of the phasor created by hilbert transforming a time domain signal, is used in calculating instantaneous frequency. The real part of the output is the measured data and the imaginary part of the output is the hilbert transform of the data. The unwrapped instantaneous phase of this complex valued output can be differentiated to estimate instantaneous frequency (proportional to RPM).
2.3.1 Spline Fitting

Commonly, spline fitting is used to estimate instantaneous frequency. After estimating the first RPM estimate from the tachometer pulses in the time domain, it is often necessary to curve-fit the data so that a smooth curve is obtained.

![Figure 2-1: Example of first estimate RPM curve](image)

A smooth RPM curve which accurately fits the RPM trace is required to account for the actual inertial characteristic of the machine under investigation. This has many implications on the ensuing data processing and analysis.

For example, a shaft can accelerate (angularly) proportional to the torque applied on it, limited by its rotational inertia. If the RPM estimate from the tachometer pulses has considerable variation from one sample time to the next (a difference of Δt), which cannot be deemed viable according to the physical property (rotational inertia) of the machine, that estimated value cannot be considered in further analysis. This phenomena can also account for errors during data processing when an inaccurate RPM value is assigned to a Fast Fourier Transformed time block during spectral mapping.

The numerical processing which is used to curve-fit the data uses the concept of cubic splines [23]. A cubic spline is a piecewise defined, polynomial function which has a high degree of smoothness at knots.
after the polynomial pieces connect\textsuperscript{[15]}. This technique was developed by Vold and Leuridian to work in conjunction with their work on Kalman filtering\textsuperscript{[23]}.

The entire RPM time history – estimated from tachometer pulse arrival times – is divided into a number of segments depending on their slew rate. Each segment is then fitted with a least squares spline with specific boundary conditions enforced to ensure continuity and smoothness. The boundary conditions are governed by end conditions of the splines and the derivatives of the spline equation \textsuperscript{[6]}. A step-by-step numerical procedure is explained in further sections. An advanced spline-fit algorithm, which is necessary for data having a high degree of variance is also briefly explained.

The importance of spline-fitting the RPM estimates lies majorly in the fact that it provides a smooth RPM curve which is often physically viable with the machine, and a solid base for estimation of instantaneous speed.

\section{2.4 Order Tracking}

An order of vibration of a rotating system is a time varying phasor with an instantaneous rotating frequency, proportional to the rotating frequency of the fundamental (or reference) shaft \textsuperscript{[6]}.

\begin{equation}
Order Frequency = \frac{Reference\ RPM \times k}{60}
\end{equation}

Where, \(k\) = the order being analyzed. '\(k\)' can be any positive integer value (direct harmonics) or non-integer value (fractional harmonics).

A proper mathematical definition of the time varying phasor can be defined as follows \textsuperscript{[6]}.

\begin{equation}
X(t) = A(k,t) \sin(2\pi i \left(\frac{k}{p}\right) t + \phi_k)
\end{equation}

Where,
A(k,t) is the amplitude of the order ‘k’ as a function of time,

Φₖ is the phase angle of order ‘k’,

‘p’ is the period of the primary order in seconds and ‘t’ is the time variable.

When analyzing rotating systems for purposes such as fault diagnosis, preventive maintenance or general health monitoring, it is of vital importance to understand the nature of the mechanical signatures that various faults produce. Some characteristics which are speed related, can be tracked by using order tracking procedures. For example, rotor imbalance is a 1x characteristic (1ˢᵗ order) and shaft misalignment is usually a 2x characteristic (2ⁿᵈ order) [13]. Characteristic bearing frequencies like the ball pass outer and inner frequencies, ball spin frequency and the fundamental train frequencies, have all a particular relationship with the shaft RPM [12][13]. Accordingly, the order related information with respect to these frequencies will be showing up in the RPM spectral maps. In cases, of fluid-film bearings, the destabilization force due to the circulation of entrained fluid, is present at a characteristic speed of the average of shaft and housing speeds, i.e about 0.43-0.48 of shaft rotational speed. More information on vibration characteristics of common machinery faults can be found in reference 13.

On the other hand, while the dominant response of a fault may be a particular order, higher order harmonics of this response are common, since the originating signals are rarely purely sinusoidal.

To get a clear picture of the dynamics of various faults and characteristics, it is hence, essential to have a noise-free order track from the response data acquired in the time domain.

**2.4.1 FFT Based Order Tracking**

Fast fourier transform based order tracking is the simplest and oldest method of order analysis. It is simple and fast to implement. The basis of FFT based order tracking is to divide the time history, according to a data block size, perform a FFT on each block, (choose overlap if desired) and then capture
the amplitude and frequency information associated with the frequency of the order being analyzed.

The transform equations are the FFT transform equations.

\[ a_m = \frac{1}{N} \sum_{n=1}^{N} x(n\Delta t) \cos(2\pi f_m n\Delta t) \]  
\[ b_m = \frac{1}{N} \sum_{n=1}^{N} x(n\Delta t) \sin(2\pi f_m n\Delta t) \]

Instead of analyzing just one spectral line in the frequency domain, a number of lines are summed around the order frequency to get a generalized estimate. The summation is based on either constant frequency bandwidth, constant order bandwidth or constant percentage bandwidth. Further details can be well understood from reference 6.

Where,

\[ x(n\Delta t) \] is the \( n \)th discrete data sample,

\[ f_m \] is the frequency of the sine/cosine terms,

\( a_m \) and \( b_m \) are the Fourier coefficients.

The time domain data, on which the FFT operation is performed, is acquired according to the values based on Shannon’s sampling theorem.

\[ \Delta f = \frac{1}{T} = \frac{1}{N \cdot \Delta t} \]

\[ F_{nyquist} = F_{max} = \frac{F_{sample}}{2} \]

\[ F_{sample} = \frac{1}{\Delta t} \]

Where,
\[ T = \text{total sample time which is analyzed,} \]

\[ N = \text{Total number of time points over which the transform is performed; equal to the size of the data block which is chosen,} \]

\[ \Delta t = \text{Time spacing of the samples,} \]

\[ \Delta f = \text{Frequency resolution of the resulting frequency spectrum,} \]

\[ F_{\text{amp}} = \text{Sample frequency, } F_{\text{nyq}} = \text{Nyquist frequency.} \]

For measured data, which is associated with a moderate to high slew rate, the amplitude information of various orders (especially the higher) will be smeared in the frequency spectra because, every FFT spectrum is assigned to a RPM value \[ ^5 \]. For high slew rates, often, the assignment is not proper due to the nature of the time window and the data block size. Based upon severe limitations associated with leakage and ability to handle fast slew rates, FFT based order analysis methods are generally not preferred in the industry.

**2.4.2 Resampling Based Computed Order Tracking**

Digital resampling based order tracking, uses an extensive numerical procedure to first estimate the arrival times of constant angle increment intervals, resample the time domain data at constant \( \Delta \theta \) intervals and then interpolate to estimate amplitudes at the resampled times using the data sampled with a constant \( \Delta t \). The angle domain data, when fourier transformed, provides a leakage-free order estimate according to the digital signal processing (DSP) parameters set by the angle-order relationships which are analogous to the parameters in the time-frequency relationships \[ ^6[^1]^[18] \]. The parameters in the angle-order domain are related by the following equations.
\[ \Delta \phi = \frac{1}{R} = \frac{1}{N + \Delta \theta} \]

\[ O_{\text{nyquist}} = O_{\text{max}} = \frac{O_{\text{sample}}}{2} \]

\[ O_{\text{sample}} = \frac{1}{\Delta \theta} \]

Where,

\( \Delta \phi \) is the order spacing of the resulting order spectrum,

\( R \) is the number of revolutions analyzed in a block,

\( N \) is the number of time points over which the transform is performed,

\( \Delta \theta \) is the angular spacing of the resampled samples,

\( O_{\text{max}} \) is the maximum order to be analyzed,

\( O_{\text{sample}} \) is the angular sampling rate and \( O_{\text{nyquist}} \) is the Nyquist order.

The advantages of this procedure are mainly in the leakage free estimates in the order domain due to resampling at constant angle increments in the angle domain \([6][11][18]\). But due to its computational complexity and possible requirement of special hardware for real-time applications, it has its limitations. A complete and well documented analysis of computed order tracking (COT) methods can be found in references 6, 11 and 18.

### 2.4.3 Vold-Kalman Filter Based Order Tracking

Vold and Leuridian developed an order tracking technique based on the Kalman Filter offering a variety of advantages like the ability to handle rapid slew rates in order tracking, de-coupling of close and crossing orders and ability to analyze multiple orders at once \([22]\).
A vast amount of literature has been published on Vold-Kalman order tracking and its mathematical characteristics and usage in real-world applications. References 5, 7, 17, 22 and 23 provide an in-depth knowledge of the Vold-Kalman order tracking filter formulation and application.

The Kalman tracking filter is formulated on the basis of two equations. The structural equation - which describes the mathematical characteristics of the filter (by modeling the order as a perfect sine wave and taking into consideration, the variation of the amplitude over consecutive time points) and the data equation – which describes the relationship between the order information and the measured data.

Vold in 1997, further modified the equations in order to simplify the application resulting in an equation set as in the following equations [21]. These are the basis of the Vold-Kalman filter.

\[ x(n) = x(n + 1) + e(n) \]  \hspace{1cm} 2.12

\[ y(n) = x(n) + y(n) \]  \hspace{1cm} 2.13

Where, \( x(n) \) is the signal to be extracted, ‘\( n \)’ is the time point being analyzed, \( e(n) \) is the amplitude change of the order from one sampled time point to the next and \( y \) is the nuisance component, which is the signal containing the non-tracked orders and random noise.

In the B&K application note (B&K, 1991) the analogy of orders as an amplitude/phase modulated radio signal, might be of help in understanding the dynamics of this concept. It is mentioned that, the underlying sine wave (order), the frequency of which is a multiple or harmonic of the fundamental component (instantaneous speed) would be the carrier wave, while the varying amplitude and phase function that modulates this carrier wave is the radio program, or in this case, the complex amplitude envelope. Vold-Kalman filtering provides the luxury of estimating both of these characteristics.
Various references provide enough detail on how to formulate and solve the equation set in a least-squares sense as well as how to choose the parameters which influence filter behavior\(^5\)\(^{17}\).

2.4.4 Time-Variant Discrete Fourier Transform (TVDFT)

The TVDFT based order tracking technique is relatively new to rotational systems analysis. Developed in 1997, it is based upon a discrete fourier transform, in which the frequency of the kernel matches the frequency of the order of interest at each instant in time. This technique also possesses good characteristics in separating close or crossing orders through a post processing calculation with an orthogonality compensation matrix (OCM).

The TVDFT combines the desirable characteristics of the FFT based order tracking and re-sampling based techniques, at the same time avoiding the computational complexity of the latter. It is defined as a DFT whose kernel varies as a function of time defined by the rpm of the machine, but the damping\(^1\) does not vary as a function of time\(^6\)\(^{10}\). It is based on a constant delta-t sampled data. The transform equation of the TVDFT are as follows.

\[
a_m = \frac{1}{N} \sum_{n=1}^{N} x(n\Delta t) \cos(2\pi \int_{0}^{n\Delta t} (o_m \times \Delta t \times \frac{rpm}{60}) \, dt) \tag{2.14}
\]

\[
b_m = \frac{1}{N} \sum_{n=1}^{N} x(n\Delta t) \sin(2\pi \int_{0}^{n\Delta t} (o_m \times \Delta t \times \frac{rpm}{60}) \, dt) \tag{2.15}
\]

Where, \(o_m\) is the order being analyzed, \(a_m\) is the fourier coefficient for the cosine term for \(o_m\), \(b_m\) is the fourier coefficient for the sine term for \(o_m\), \(rpm\) is the instantaneous RPM of the machine.

\(^1\) Damping is mentioned here based on a comparison between TVDFT and chirp-z transform. The former can be considered a special case of the latter\(^10\). Chirp-z transform is defined as a fourier transform with a kernel whose frequency and damping vary as a function of time.

Damping is defined as the property of a mechanical system that restrains the amplitude of motion with each successive cycle\(^13\).
To minimize leakage, the following relationships are necessary.

\[ \frac{1}{\Delta o} = integer \]  \hspace{1cm} 2.16

\[ \frac{\text{order tracked}}{\Delta o} = integer \]  \hspace{1cm} 2.17

The application of OCM on the orders of interest tracked using TVDFT, allows closely spaced and crossing orders to be estimated accurately and faster sweep rates to be handled easily. Using the TVDFT with the application of OCM, it is possible to de-couple very closely spaced orders which otherwise are very difficult to separate using standard FFT or re-sampling techniques because they may beat with one another.

A comprehensive analysis and mathematical formulation of this method along with the orthogonality compensation matrix formulation can be found in references 6 and 10.
3 Instantaneous Speed Estimation and Spectral Mapping

In rotation systems vibrational analysis, the most important parameter to measure and accurately estimate is the instantaneous RPM. An accurate estimation and curve-fit of the processed raw tachometer data would provide with a smooth RPM curve which will form the basis of further analysis in the field of rotation systems vibrations.

Methods of analyzing different components of this area - including Kalman filtering, computed order tracking (COT), spectral mapping and the time-variant discrete fourier transform (TVDFT) are all based on an accurate estimate of instantaneous RPM. An inaccurate curve-fit through the initial rpm estimate would be a major source of processing error in several techniques. For example, if the instantaneous RPM at a certain time point in the response history does not closely match the actual RPM of the machine under test, the response spectra in the frequency domain and the auto power spectrum calculated for the estimated RPM would not be assigned to the correct RPM value – as the estimated value differs from the actual one. This inaccurate assignment of spectra would give an inaccurate estimation of further analytical components which use the instantaneous speed as their foundation.

For the purpose stated above and many more, it is essential to have an RPM estimate which closely matches its actual RPM curve. The following sections describe a step-by-step procedure of the same.

3.1 Tachometer Signal Processing

A raw tachometer signal is a series of pulses generated by the tachometer depending on the type of sensor used. Ideally, a square wave pulse train should be observed in the time history. But due to the
digitized nature of the signal; which is sampled at the sampling frequency as well, the signal has some irregularities associated with it and is often not centered as the signal comes through a DC coupling.

Centering of the signal is important because, it provides better accuracy in estimating the zero-crossings of the signal (which ultimately provides a better RPM estimate) than without centering. Without centering, the methods to estimate zero-crossings include storing the pulse detection counter as either the start of a pulse or the end of a pulse, neither of which do not provide a better estimate of RPM as will be discussed briefly.

![Raw vs Centered Tachometer Signal](image)

*Figure 3-1: Raw vs Centered Tachometer Signal*

The centered tachometer signal is then used for estimating the zero-level (threshold for pulse detection) crossing time instants. These instants in time are stored once a pulse is detected. A pulse can be detected using the difference in sign values of the tachometer signal (+ for tach values above the threshold and – for tach values below the threshold) between two consecutive sampled time points. Detection of a sign change from -1 to +1 is analogous to the start of a new pulse. Once a pulse rise is detected, the zero-level crossing time instant is stored. Due to the nature of the sampling parameters used during data acquisition, it is not always necessary that a crossing time instant will be
an integer multiple of the $\Delta t$ of the sampled data. Hence, the zero-level crossing time instant is calculated by an interpolation formula.

$$x_{crossing} = \left( \frac{(0 - y_1) \times (x_2 - x_1)}{y_2 - y_1} \right) + x_1$$

3.1

**Figure 3-2 : Interpolation for estimate zero-crossings of Tachometer pulses**

Without interpolation, zero-crossings are estimate by – 1) storing time instant at start of pulse or 2) storing time instant at end of pulse. In cases where the machine does not rotate above 10,000 RPM, or when the slew rates are considerable, there will not be any considerable difference between estimates from interpolation or no interpolation. But, for machines which generally rotate above 10,000 or 15,000 RPM, their high speed generally causes a considerable error in the tachometer signal for a single-pulse-per-rotation signal. (Having a signal which can have more number of pulses per revolution might result in a better initial RPM estimate). The following two figures show an example of the RPM estimates for a rotor rotating at high speeds.
This error is visible from Figure 3-3 where there is a sizeable variation in the RPM estimate which cannot exist physically due to the inertia of the system. Also, an estimate like Figure 3-4 is easier to curve-fit.

### 3.2 Generating the First RPM Estimate

Using the time instants at each zero-level crossing, an estimate of the RPM between 2 consecutive pulses is calculated as follows.

\[
RPM_i = \frac{60}{t_{n+1} - t_n}
\]  

Where, \( t_n \) = time instant of the \( n^{th} \) pulse detected, \( i = i^{th} \) RPM calculated. The time point corresponding to a particular RPM estimate is calculated by:

\[
t_i = \frac{(t_{n+1} + t_n)}{2}
\]

The following figure shows the estimated zero crossings and corresponding time points for the RPM estimation of a tachometer signal.
Figure 3-5: Estimated zero crossings and corresponding time points of a tachometer signal

A sample plot of the RPM estimate against the corresponding time points is shown below.

Figure 3-6: Sample RPM estimate curve – without considerable variation

For cases when the tachometer pulses are well separated in time, the initial rpm estimate might be used further for advanced analysis provided the variation of RPM is well within physical inertial limits of the machine under test. But in most cases, a smooth curve is not obtained and especially at higher RPM values, the estimate has a considerable amount of variation associated, which cannot be accepted. An example plot is shown below.
Consider for example, Figure 3-7, where the variation of RPM at higher values is considerable and cannot be accepted. Also, the system which was under test in this particular example, did not have such low inertia to produce a variation of rpm in this magnitude. It is hence necessary for an accurate curve-fitting of the RPM trace to get a smooth curve which is advantageous for further processing and which is physically viable for the structure as well.

### 3.3 Cubic Spline Fitting

Once, an RPM estimate is made, the entire RPM data is divided into a number of segments and cubic splines are fit onto each segment with boundary conditions enforced between the sections to make the RPM estimate smoother. A cubic spline is a piecewise defined smooth polynomial function which has a high degree of smoothness at knots where polynomial pieces connect\textsuperscript{[15]}.

The boundary conditions are based on end conditions and derivatives of the spline equation. The end conditions enforced are that the value of the spline at the end of each section is equal to the value of the spline at the start of the next section. This ensures that there are no discontinuities in the curve.
The first derivatives are forced to be equal at the end of each section and the start of the next section. Forcing the first derivatives to be equal ensures that the curve will be smooth, with a constant slope at each transition [6].

The RPM data is divided into a number of segments, depending on the slew-rate and then a cubic spline interpolation curve-fit is performed on each segment. This smoothens out the variations in the RPM curve trace which enables the user to have a better and accurate estimate of Instantaneous RPM which is explained in the following sections. For an RPM profile which is varying rapidly, probably the number of segments to be fit would be greater than 50 or 60, so that the RPM profile is tracked accurately. For a profile which is just a linear sweep, even 10 segments will suffice. Examples of both the types are shown below.

![Figure 3-8: Examples of Spline Fits of Linear Sweep RPM and Rapid Sweep RPM](image)

Frequently, there are cases when the first Spline fit estimate would produce a smooth enough curve without considerable RPM variations. These spline fit estimates might then be used further in the estimation of Instantaneous RPM. But, in cases where the raw RPM profile itself has high variations, either due to the high speed ranges (in case of high speed rotors) or due to rapid slew rates, the
spline fit estimate might not be smooth enough to be acceptable. An advanced spline fit method is needed in these types of cases.

3.3.1 Advanced Spline Based Tachometer Analysis

The following figure shows an example of the raw RPM estimate against a spline fit estimate on it.

![Figure 3-9: High Variation at high RPM after Spline Fitting](image)

As seen in Figure 3-9, some areas of the spline-fit RPM trace have considerable variance. The structure on which the plotted data is based on, has enough inertia to deem that estimate of the RPM unacceptable. In some other industry cases, there are instances of – a missed tachometer pulse, extra pulses due to a tiny crack on the flywheel etc. To effectively curve-fit the data with these characteristics, Vold et al developed an enhanced spline-fit technique \(^{[23]}\). The developments in the presented technique more accurately fit the spline functions.

The effects of missed or extra tachometer pulses is effectively eliminated through first of these developments. The method is called *shaving*. The first step is to fit the original spline to the
estimated periods. The second step compares each originally estimated rpm value to the rpm value of the spline. The estimated RPM value is removed from the dataset, if the difference between estimated and the spline rpm is higher than a pre-determined value. After this procedure is completed on the entire data set, the splines are fit again to the reduced set of estimated values. This multi-iterative spline fit effectively removes all effects of the missed or extra tachometer pulses as well as sections where the variance was at an unacceptable range \(^6\).

The following figure shows this iterative process.

![Figure 3-10: Iterative Shaving Process](image)

The second development that Vold included in the enhanced spline fitting algorithm was the ability to relax the first derivative constraints. The user has to accurately pick the points to relax the derivatives depending on the RPM profile. This development allows the algorithm to accurately follow rapidly changing rpm profiles as well as profiles which include sudden increase or decrease of rpm (example gearboxes). The details of this algorithm are available in reference 23.
It is to be noted that, the pre-determined limit which decides whether to keep the estimated RPM value should be based upon the examination of the structure under test. Depending on the rotational inertia of the structure, a maximum RPM variation value or a variation band has to be calculated and used in the algorithm. Any estimate which falls outside this band will be discarded by the algorithm thus incorporating the true physical nature of the structure.

3.4 Instantaneous Speed and RPM Spectral Mapping

The process of estimation of instantaneous RPM runs parallel with generating RPM (or time) spectral maps. For spectral map generation, the response signal in the time domain is Fast Fourier transformed in a series of sliding block windows across the entire time domain data. An instantaneous RPM value which would be corresponding to the response spectrum in the frequency domain, is assigned during this process. Several factors affect the accuracy of the output. As discussed earlier, an inappropriately assigned RPM value to each spectrum would result in errors in further analytical procedures.

The factors which affect the accuracy of this process are:

- Data block size (historically powers of 2 for computational efficiency)
- Method of assigning an RPM value as the instantaneous RPM
- Amount of overlap
- Knowledge of slew rate which forms the basis of data block size and instantaneous RPM value assignment

The parameters stated above can yield a number of combinations that produce a good output. But it is imperative to choose the right combination based on the slew rate in order to avoid incorrect
assignments to the RPM value. A new and simple method to avoid such situations is described in the last sub-section which can be used to make an informed choice on data block size and method of assigning an RPM value to the block.

### 3.4.1 Spectral Mapping

The response signal in the time domain is fast fourier transformed (and scaled) in a series of sliding block windows across the entire data range. Each data block of the response signal is applied with a hanning window. Once the factors which govern the parameters to be used while performing the FFT are decided, a progressive algorithm can be used to store the spectral data. There are as many spectrums as permitted by the number of slides of the data block window and the amount of overlap.

The fourier transform is based on the parameters calculated from shannon’s sampling theorem. The number of slides in the time domain depends on the size of the data block and the amount of overlap used.

\[
\text{Number of sliding windows} \propto \frac{1}{\text{Data block size}}
\]

\[
\text{Number of sliding windows} \propto \text{Amount of overlap}
\]

The data block size is independent of the amount of overlap used. The following figure shows a sample response signal and the sliding windows over which an FFT is performed. The amount of overlap used is 50%.
3.4.2 Instantaneous RPM

Similar to the process described above, the instantaneous RPM of the machine can be estimated from the sliding windows on the spline-fit RPM trace estimated earlier. The user has to make a choice in order to assign a RPM value within the block. The possibilities are:

- First RPM value within the block
- Mean RPM value within the block
- Last RPM value within the block
- Median RPM value within the block

Depending on the slew rate, the data block size and the overlap can be adjusted and a logical choice be made for the assignment. For example, for better frequency resolution, if the size of the data block is 4096, and the speed variation within the data block between the maximum and the minimum value is over 50%, it would not be appropriate to choose the first or the last value within the block as the
instantaneous RPM. The mean RPM value might be a better choice in this case. To generate more estimates of the instantaneous RPM for better resolution in the Y direction of the spectral map, and to follow the RPM curve closely, a high amount of overlap can be used as well.

Figure 3-12: Sample Instantaneous RPM curve against initial RPM estimate

3.4.3 Output

Now, for each response window the size of the data block, for which there is a fourier transform (and/or auto power spectrum), an instantaneous RPM value can be assigned. Each of these spectra are then plotted in a waterfall format to analyze the entire response signal in the frequency domain. A sample plot is shown below. The data shown was acquired on a 2-plane unbalance rig.
The colormap above is viewed from the +Z direction that is directly overhead the map.

### 3.5 RPM Variation Criterion

To prevent wrong assignments of the estimated instantaneous RPM to a response spectrum, especially for cases with high slew rates, a new criterion is developed in the work of this research.

It is obvious that there is a trade-off between the data block size and the frequency resolution in the frequency domain during the process of spectral mapping. Data block window overlapping might be used somewhat as a counter to this but only for moderately varying slew rates. For rapid changes in speed, it is necessary to avoid using large data block sizes in order to prevent wrong assignment of the RPM.
The RPM variation criterion enables the user to observe the variation of each RPM in a particular data block, with respect to the RPM value which is chosen to be the instantaneous speed for that block.

A spline-fit RPM estimate is needed first in any procedure. A loop which needs user input for data block size, overlap to be used and value of RPM to be assigned can be used to calculate the maximum variation of each data block from the chosen RPM value. This maximum variation can be then plotted and the user can decide whether to go ahead with the combination chosen or to repeat the procedure until acceptable range of variation is observed.

There are various choices for each parameter to be decided:

- Data block size – 256/512/1024/2048 and so on
- Overlap used – 25% / 50% / 75% / 87.5% / 93.75%
- RPM value to be chosen – First value, Last value, Mean value or Median value

Depending on the slew rate observed after initial RPM estimate and the spline-fit estimate, the user can make a choice of parameters. For example, for the RPM profile shown in Figure 3-14, the following plots show RPM variations, for different parameters chosen.
Figure 3-14: RPM Variation Criterion Plots for 4 different block sizes

The 4 plots in the set show the RPM variation for 4 different block sizes – 256, 512, 1024 and 2048.

Figure 3-14 shows the plots of the block-wise maximum RPM variation for each block size. It is clear that BS=2048 cannot be used in this procedure. Although for better frequency resolution, a small block-size won’t suffice. Hence, BS=1024, which shows slightly satisfactory results can be used. But, the method of choosing RPM can be changed to mean value. The comparison is shown in Figure 3-15.
However, if a choice was made which had RPM variation corresponding to BS=2048, there would be severe smearing in the corresponding spectral map due to the variation shown in Figure 3-14. With the help of this criterion, a logical combination of choices can be made for RPM spectral mapping and estimation of instantaneous RPM.

### 3.6 Missing-Tooth Tachometer Signal Processing

So far, the tachometer signal processing associated with a single pulse per revolution pulse train has been discussed. If the reference shaft used for recording tachometer pulses, by the use of multiple pulses per revolution, the only change in the processing is to divide the RPM estimate between two pulses by the number of pulses per rev.

---

2 Before progressing, please note that the 100% variation spikes seen in the figures are not indicating that the chosen value deviates maximum at this value from the data block. The spike is due to the fact the machine starts rotating at a particular time instant in that data block and until then, there was no rotation. Therefore, the maximum variation within that data block reaches 100 percent as the difference is between the RPM value and zero.
Where, \( P \) is the number of pulses per revolution.

But for a pulse train, where one complete revolution is characterized by a missing pulse in an otherwise continuous pulse-train, the procedure is slightly modified. In this case, either the reference shaft is toothed, or the tachometer pulses are acquired on a gear mounted on a shaft having a missing tooth. The engine flywheel is an example of the latter.

The pulse train characteristics are dependent on the type of tachometer used – infra-red, laser etc. The following figure shows a synthesized missing-tooth tachometer signal. The seven pulses that are shown in each revolution are similar in nature to that generated by infra-red type tachometers, which detect a pulse when a reflective strip passes by the probe.

![Sample missing tooth tachometer signal](image)

*Figure 3-16: Sample missing tooth tachometer signal*\(^{[20]}\)

The RPM estimate is calculated by the equation described above for any 2 consecutive teeth.
However, whenever the missing-tooth pulse passes by, the RPM estimate would be nearly half the previous RPM estimate. For example, if RPM$_i$ was the immediate previous estimate before the missing tooth passed by, then, RPM$_i$ would be roughly in the range of 1.5 to 2.5 times RPM$_{i+1}$. It is unlikely that RPM$_{i+1}$ would be equal to RPM$_i$ as it would physically mean that the machine slowed down half to its original speed within one tooth-pass. In most cases where the machine shaft has considerably inertia, this is not possible. Hence it is safe to assume this range to detect a missing tooth.

When this is detected, the associated RPM$_{i+1}$ estimated can then be multiplied by 2, to compensate for a missing tooth between the two pulses. This would ensure a smooth and logical curve to the RPM profile. The following plot shows the RPM estimate for the missing-tooth tachometer profile.

![Figure 3-17: RPM profile for missing-tooth tachometer signal](image)

The remaining procedure to estimate the instantaneous RPM from the first RPM estimate remain the same as described above.
4 Singular Value Decomposition

The relatively new numerical procedure to be discussed further is based on the concept of singular value decomposition. The SVD approach to experimental analysis of rotational systems was first identified by Dr. Jason Blough in his PhD dissertation\[^6\] in 1998, and has not been pursued ever since. Before discussing the usage of the new tool, it is apt to briefly touch on the popular topic of singular value decomposition, its interpretation and existing use in modal and rotational systems analysis.

4.1 SVD Overview

Singular value decomposition (SVD) is a factorization technique similar (and related) to eigen-value decomposition. Its equation is as follows.

\[
[A] = [U] \times [\Sigma] \times [V]^T 
\]

Where, \([A]\) is any matrix of size \(mxn\), \([U]\) is a square matrix of size \(mxm\), \([\Sigma]\) is a rectangular diagonal matrix of size \(mxn\) and \([V]\) is a square matrix of size \(nxn\). The columns of \([U]\) are called left singular vectors and the columns of \([V]\) are called right singular vectors. The diagonal entries in the diagonal matrix \([\Sigma]\), are scalar quantities called “singular values”. They are denoted by \(\sigma_1, \sigma_2...\sigma_m\) and so on. The matrix is called, the singular value matrix.

\([U]\) and \([V]\) matrices are orthonormal matrices. That is, \([U] \times [U]^T = [I]\) and \([V] \times [V]^T = [I]\)

The singular values, are arranged in descending order. Hence, the first few singular values, which are significantly larger in magnitude than the rest of the singular values, reveal the rank of the base \([A]\) matrix. They also denote the effective number of independent rows and columns of the base matrix.
When discussing SVD, it is imperative to understand its relationship with another important factorization method called the eigen value decomposition (EVD). EVD is an important numerical technique, which is also used in principal component analysis \(^{[19]}\).

The relationship between EVD and SVD is as follows.

\[
\]

4.2

Hence, it can be observed that the singular values on the diagonal of the \([\Sigma]\) matrix are the square roots of the non-zero eigenvalues of both \([A][A]^T\) and \([A]^T[A]\) matrices. They are always real, scalar constants. Another notable point is that, the columns of the left singular matrix \([U]\) are eigenvectors of \([A][A]^T\) and the columns of right singular matrix \([V]\) are eigenvectors of \([A]^T[A]\).

For complex matrices, the ‘T’ operator, denoting transpose of the matrix is replaced by ‘H’ operator, denoting the hermitian of the complex matrix. \([U]\) and \([V]\) matrices, in case of complex matrices, become unitary. That is, \([U] \times [U]^H = [I]\) and \([V] \times [V]^H = [I]\).

The singular value matrix indicates the magnitude of important information in the base \([A]\) matrix. To further illustrate the rank determination by the number of significant singular values, it can be said that, the number of first few significant singular values in the \([\Sigma]\) matrix, denotes the number of linearly independent characteristics present in the base \([A]\) matrix \(^{[21]}\). The remaining singular values have a considerable lower order of magnitude than the significant ones.

With respect to vibration response analysis, the dominant response is captured by the largest (first) singular value, since the right and left vectors are unitary, i.e they are normalized to orthogonal with unit length.
Singular value decomposition is an increasingly popular tool in modal analysis and rotational systems analysis. The singular value matrix, and the left and right singular matrices have a significant amount of information which can be used to analyze various aspects of modal parameter estimation and consequently be extended to rotational systems analysis.

4.2 Use in Modal Analysis

There are various aspects in modal analysis and modal parameter estimation (MPE) which are related to the SVD technique.

The concept of forced response decomposition is based on the singular value decomposition of the data contributed by entire forced response vectors in a data space of size $N_v \times N_v$. The number of significant values in the data space, $N$, is an indication of number of contributing modal vectors in the data [2].

Equation condensation methods, which are used to reduce the number of equations based upon measured data to match the number of unknowns in the modal parameter estimation algorithms more closely, make use of the SVD [2]. The transformation matrix required to perform the equation condensation, is based on the numerically precise results of the SVD.

Similar to equation condensation, SVD is used in coefficient condensation techniques (in low order modal identification algorithms) as well, to reduce the number of physical co-ordinates ($N_o$) to an approximate number of effective modal frequencies ($N_e$) [2]. The transformation matrix, which is used for the condensation of measured data, contains left singular vectors corresponding to the significant singular values ($N_v$).

Another very important application of SVD in MPE, is in the Complex Mode Indicator Function (CMIF) tool. CMIF indicates the existence of real normal or complex modes and the relative magnitude of each mode. The SVD of the FRF (Frequency Response Function) matrix at each frequency (or at each spectral
line), provides a set of singular values. The peaking singular values, indicating presence of a mode at that frequency, denote the magnitude by which a particular mode is excited \(^\text{[2]}\) \(^\text{[3]}\).

The rate of change of singular values is used as an indicator of rank of the measured data matrix. For theoretical data, singular values approach zero when the rank of the matrix is reached. But, for measured data, this is not the case. Instead, the magnitude of the singular values drops considerably as compared to the significant values. Hence, this technique is used for rank estimation of the matrix which is based on acquired data \(^\text{[2]}\). There are a couple different techniques using the successive ratios of singular values or normalized ratios of singular values, which detect the rapid change in ratio to determine the rank or model order of the system \(^\text{[2]}\).

### 4.3 Use in Rotational Systems Analysis

A technique based on singular value decomposition and the CMIF application was developed by Blough et al in 1999 which enabled the independent operating shape determination\(^3\) based on order track measurements\(^9\).

The operating shape decomposition is done by performing a singular value decomposition of the operating shapes estimated at each order at each rpm. The following equation shows the same.

\[
[U] \times [\Sigma] \times [V]^H = SVD([O_1] \quad [O_2] \quad \ldots \quad [O_m])
\]

Where, \([U]\) is the left singular vector matrix, \([\Sigma]\) is the singular value diagonal matrix, \([V]\) is the right singular vector matrix, \([O_i]\) is the operating shape vector of order \(i\) at the analysis rpm.

---

\(^3\) Operating shapes are calculated from order track measurements at each measured degree of freedom, at each rpm. Hence, for each analysis rpm, the test-machine, having \(m\) sensors located at different locations, will have an operating shape, having \(m\) points, corresponding to an order. The amplitudes of the operating shape at each measured degree of freedom, and at each rpm, will be the order track estimate (for that order and that sensor) calculated from the TVDFT procedure.
The left singular vectors in the \([U]\) matrix, represent the set of linearly independent operating shapes corresponding to the singular values. In most cases, these linearly independent operating shapes will approximate the actual mode shapes of the system. There are as many independent operating shapes as there are modes in the system plus forcing vectors which are excited at the given rpm or the number of orders evaluated – the minimum of the two.

The singular values, at each rpm, will identify how well excited each linearly independent operating shape is at that rpm value. All of the singular values can be plotted against RPM, just like the CMIF plot is formulated in modal analysis (in CMIF, singular values are plotted against frequency of vibration). There will be, obviously, as many singular value curves, as there are orders included in the analysis, the highest curve at any rpm value, being the best excited shape at that rpm.

To get a picture of how many independent shapes are excited at any rpm value, the number of singular value curves which are peaking, can be observed at that value.

This technique is based on the order tracking results obtained from the time-variant discrete fourier transform (TVDFT). The orders tracked are orthogonal to each other, which forms a basis to decompose each operating shape at each rpm value into a set of linearly independent operating shapes \([9]\).

The singular vectors which are obtained from the SVD procedure above, can be plugged in to MAC (modal assurance criterion)\(^4\) calculations, to find the rpm values at which similar shape is excited. The singular vector auto-power can indicate at each RPM value how well a singular vector (or if applicable, mode shape) is excited \([9]\). These concepts are well documented in reference 9.

\(^4\) In modal analysis, the modal assurance criterion is a scalar constant relating the causal relationship between two modal vectors \([2]\). The mathematical definition can be found in reference 2.
The concepts discussed illustrate how to reduce a large number of response points to a condensed measurement. Some of these techniques can be used on the results generated by the new numerical method which will be discussed in the next sub-section and is the focus of this research.

4.4 Singular Value Maps – Mathematical Background

As discussed in the previous sections, through the concept of RPM (or time) spectral mapping, a frequency domain response of the particular degree of freedom on a rotating machinery can be estimated at each RPM value. The various parameters associated with the accurate estimation of instantaneous speed (RPM), clear spectral maps and accurate assignment of RPM to frequency domain response, have been already explained in the previous sections.

Using the above mentioned concepts as a base, it is possible to further enhance the “observability” of rotating systems using singular value decomposition on the cross-power matrix of the frequency response vector at each frequency of vibration and RPM.

As estimated from the spectral map calculations, a typical response vector in the frequency domain, for m sensors used, would be a column vector of size mx1, containing a term from each degree of freedom. This vector would be complex valued, as each term is estimated via an FFT, and would exist at each frequency of vibration and each RPM.

(Note: The frequency response vector is the vector containing the Fourier transform coefficients – magnitude of sine and cosine term – of the time domain response of m degrees of freedom, at each frequency of vibration and each RPM. It is not a FRF column.)

\[
R_{k,n} = \begin{bmatrix}
a_1 + i b_1 \\
a_2 + i b_2 \\
\vdots \\
\alpha_m + i b_m
\end{bmatrix}_{m \times 1}
\]

4.4
Where, $R_{k,n}$ is the response vector at the $k^{th}$ RPM and $n^{th}$ frequency of vibration being evaluated, $m$ is the number of sensors being evaluated, $a_j$ and $b_j$ are the cosine and sine fourier-coefficients of the time domain response for sensor $j$, $i$ is the imaginary number.

The cross-power of this column vector is a square matrix of size $m \times m$, having the auto-power terms in the diagonal and cross-correlation terms in the off-diagonal positions.

\[
C_{k,n} = R_{k,n} \times R_{k,n}^H
= \begin{bmatrix}
(a_1 + ib_1) \times (a_1 - ib_1) & \cdots & (a_1 + ib_1) \times (a_m - ib_m) \\
\vdots & \ddots & \vdots \\
(a_m + ib_m) \times (a_1 - ib_1) & \cdots & (a_m + ib_m) \times (a_m - ib_m)
\end{bmatrix}_{m \times m}
\]

As indicated in the equations, the cross power matrix $C_{k,n}$ exists at each RPM and each frequency of vibration. Hence, for each matrix $C_{k,n}$, a singular value decomposition can be performed.

\[
[U]_{k,n} \times [\Sigma]_{k,n} \times [V]^H_{k,n} = SVD([C]_{k,n})
\]

As known from an SVD operation, a left singular matrix $[U]$, a right singular matrix $[V]$ and a diagonal singular value matrix $[\Sigma]$, is produced at each RPM and frequency of vibration.

\[
[[\Sigma]_{k,n}]_{m \times m} = \begin{bmatrix}
\sigma_1 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \sigma_m
\end{bmatrix}_{m \times m}
\]

The singular values $\sigma_1$, $\sigma_2$, $\sigma_3$ and so on, will hence exist at each point in the RPM vs frequency map, and will have a magnitude associated with them. All of the singular values are real scalar values. Similar to a color map, or a spectral map, all $m$ sets of singular values can be plotted in $m$ different maps. These can be called singular value maps.
It can be noted that any random noise collectively present in the response vector (generated from response of all sensors) will not contribute to the dominant singular values that will be further used in the analysis. Thus, the SVD approach can result in noise-reduction in rotating system analysis when common DSP averaging methods cannot be used due to the machine’s slew rate.

The number of significant singular values will be equal to the number of linearly independent responses in the data. The rank of the singular value matrices, as depicted from the number of significant singular values, will reflect upon the redundancy in the data.

Sample singular value maps, and their interpretations are discussed in the next section.

### 4.5 Interpretations of the SV Maps

To illustrate the application of the singular value maps, the following example is used. The data is acquired on a leaf blower – which was driven by a variable AC motor, driving the single crank-piston assembly which drove the multi-blade rotor. A total of 8 sensors were placed on different locations.

(Figures in Appendix)

The RPM spectral maps at each degree of freedom are depicted in the figures below. All of the maps are viewed from a point of view in the +z direction, i.e directly overhead.

PCB Accelerometers with a sensitivity of 100 mv/g were used in recording the response data for this example. Magnitude scale in each RPM spectral map is a logarithmic scale in the units of milli-volts (mV).
Figure 4-1: RPM Spectral Maps at Locations 1-4 on Leaf Blower

Figure 4-2: RPM Spectral Maps at Locations 5-8 on Leaf Blower
As visible from the spectral maps, smearing of frequency content, consistencies of order trails and amplitudes of different orders – differ from sensor to sensor. Sensor 8 seems to be on an inappropriate location due to the absences of major dynamic characteristics of the machine.

Using, the concept described above, it is possible to estimate all 8 singular values (8 virtual sensors) at each RPM and each frequency of vibration.

### 4.5.1 Magnitudes of Singular Value – Rank

The first two significant singular values can be plotted in a 3-dimensional map, or “singular value maps” and are shown below.

![4.3: σ1 and σ2 plots for plots shown in Figure 4-1 and Figure 4-2](image)

The maximum magnitude of the $\sigma_2$ graph can be seen to be around $10^{-38}$. The maximum magnitude of the remaining significant singular values were found to be around this range as well. In total, it was shown from the singular value maps, that $\sigma_1$ map had a maximum value of the order of $10^{-1}$ while $\sigma_2$ to $\sigma_5$ had an order of magnitude around -38 to -40. The remaining three singular value plots contained...
information to the order of -60 and beyond and hence can be considered insignificant. Only $\sigma_1$ and $\sigma_2$ are plotted above, as the remaining three plots were similar to the second singular value plot.

In this case, it can be said that the rank is 5 due to the presence of 5 significant singular values. However, the difference in magnitude between the first and second singular value is about 30 orders of magnitude. Hence, the effective rank is closer to 1.

### 4.5.2 Noise Reduction

Due to the large difference in orders of magnitude between the $\sigma_1$ plot and the remaining maps, it can be interpreted that the first singular value map depicts only the major sources of vibration/dynamic characteristic of the machine in this test condition.

Except for the smearing effect which is observed at higher frequencies and higher RPMs in FFT based spectral maps (which is due to the spreading of the energy content of orders or harmonics over more spectral lines, because of their rapid change in frequency, $^{[10]}$), the smearing of the frequency content at different areas of the maps for different locations, is not visible in the singular value plot. Also, there is a considerable noise-reduction due to the retention of only important or major information by the singular value decomposition procedure on the cross-power matrices.

Noise reduction is possible because noise is not consistently present at the same frequency content in all sensors. One degree of freedom may have different susceptibility to noise at different conditions as compared with another measurement degree of freedom. This difference becomes advantageous during the singular value decomposition procedure, as the same noise information is not present in all sensors. On the other hand, the true mechanical signature of the machine, like harmonics related to number of blades, firing order or mechanical faults, will be present throughout, if not at a constant amplitude. The contribution from these characteristics will hence be present at a significant magnitude in the cross
power matrix and eventually would be depicted in the singular value plot. While, noise is “averaged” out to produce a virtually noise-free spectral map.

4.5.3 Order Amplitudes Compensated

Several order lines are visible throughout the spectral maps which have been plotted above (Figure 4-1 and Figure 4-2). It is also clear that the magnitudes associated with the same order, differ from location to location. This can be attributed to the fact that different points on a big enough structure have a different response with respect to rpm. But if, only a few sensors, which do not show a considerable amplitude of a particular harmonic in their corresponding order histories, are considered as baseline, it might prove to be disadvantageous to say the least. This attribute of observability was discussed in a previous section.

Therefore, to gauge a structure’s response as a whole, it would be apt to observe a map like the singular value map, which shows an “averaged” order history as can be seen in the Figure 4-3. This can be advantageous in understanding whether there is any significant information at all, which is related to an order of interest, which can be observed from a sensor within the sensor assembly. As discussed above, if a particular order is active (say, 2\textsuperscript{nd}) within the RPM range of the test run and is detected by say, more than half of the sensors, its frequency content will show up in the singular value map, despite it being inactive in other locations.

4.5.4 Sensor Assembly Quantification

Depending on the structure being analyzed, the singular value map concept can be applied on either: 1) Response data from one location, 3 principal directions or 2) Response data from different locations.

If the first case is analyzed, the single “virtual sensor” will be depicting the major contributions from the directional sensors (which acquire data via a tri-axial accelerometer). It can be also said that the virtual
sensor will be “observing” the corresponding node in a complete sense, taking into consideration the major sources of vibration and dynamics from all three directions and filtering out, or “averaging” out the noise. Data which is significantly present for 2 or all sensors, but not in a considerable magnitude, will eventually be depicted in $\sigma^2$ plots or even the remaining ones.

In the second case, the virtual dominant sensor concept will be justifiable only if the structure or component is small enough, for it to be able to be characterized by a single virtual degree of freedom rather than the entire set. The procedure will be similar to equation or coefficient condensation in modal analysis. In this case, the spatial domain is being condensed to the number of significant singular values obtained from the SVD procedure. The interpretation would be that, the virtual dominant sensor is “observing” the behavior of the entire component as a whole, retaining important information and leaving out the noise. The successive singular value plots are observed to understand the primary and secondary (and further), operating characteristics of the machine under test.

For a large rotating machinery assembly, the above two concepts can be blended. Each sub-component can be “observed” as one. Similarly, the nodes of each sub-component, can be virtual single nodes achieved through the single virtual sensor concept.
5 Results

5.1 Data Acquisition

Every data set which would be subsequently discussed further is composed of multiple outputs at different locations, and a tachometer signal. The following is a general overview of the hardware used to acquire data, the rotating systems used and digital signal processing (DSP) parameters.

Multiple sets of data were acquired on 4 sets of rotating systems, including different types of slew rates. To keep this thesis report concise, the vast number of plots from each data set are not included. Hence, only the plots which are pertinent to the discussion will be shown.

5.1.1 Overview

1. Interface – VTI Technology, CT-310 A Mainframe, 8 channels per card
2. Software – X-Modal III, UC-SDRL
3. Sensors – PCB Tear-drop 100 mv/g accelerometers
4. Tachometer – Infrared type, Ono Sokki
5. Algorithm generation and results – MATLAB™

5.1.2 Rotating Systems Used

1. Toro Leaf Blower –
   - The spark plug and fuel systems were disconnected.
   - Single cylinder crank-piston mechanism coupled with multi-blade rotor, was driven by an AC motor rated 115/230 V, 60 Hz.
   - Slew rate was controlled by a variable voltage Variac governing the AC motor output.
• 8 sensors – 2 on cylinder head, 6 on rotor housing, 1 tachometer.

2. Small size high speed rotor with unbalance and damaged blades
   • Slew rate controlled by a Variac as described above, governing the output of the integrated AC motor driving the rotor.
   • 4 sensors on housing, 1 tachometer.

3. 2-plane unbalance rigid rotor rig.
   • Slew rate controlled by integrated Variac on the AC motor driving the rigid rotor.
   • 2 sensors on bearings, 1 tachometer.

4. 6-cylinder 4-stroke diesel engine and drive. This data was acquired and provided by ACS Inc[25].

5.1.3 Digital Signal Processing Parameters

• The maximum frequency was set to 2000 Hz for each case.
• The maximum frequency of the system was rated to be about 82% of the Nyquist frequency.
• Period of observation for each case, except for the leaf blower, was 30 seconds. The leaf blower was observed for 20 seconds for each data set.
• Auto-ranging was performed for each channel before every data set acquisition to prevent overloading of the sensors.

Along with the systems described above, the multiple response and single-channel tachometer data from a 6-cylinder 4-stroke diesel engine test-cell was also used in validating the new concept. This data was acquired and provided by ACS Inc[25]. The inclusion of this data turned out helpful in the analysis as it was acquired on a much more complex system than the ones described above.
5.2 Results and Interpretations

To illustrate the advantage of the new procedure on different type of data sets, the results can be classified into the following sections.

**Note:** All of the RPM spectral maps and the singular value maps plotted are at a view point of 90 degree elevation and 0 degree orientation. That is, they are viewed from a point which is directly overhead the map. **Magnitude scale in each RPM spectral map is a logarithmic scale in the units of milli-volts (mV).**

5.2.1 Same DOF, 3 Principal Directions

Often, the forced response of a structure under non-stationary conditions is not similar throughout the 3 principal directions. Due to this, testing procedure requires tri-axial accelerometers on various points on the structure. The following plots show RPM spectral maps of the response spectrum calculated on data acquired on a node on an Engine test-cell. Each directional point: +x, +y and +z has a distinct characteristic in the spectral maps.

![RPM Spectral Maps](image)

*Figure 5-1: RPM profile and RPM Spectral Maps at same DOF, 3 different directions*
The differences in the three maps are visible. When the order lines of the 12\textsuperscript{th}, 24\textsuperscript{th} and 36\textsuperscript{th} orders are compared only from the spectral map view, it is evident that there are amplitude differences. Observing spectral maps of +x and +y direction, it can be said that there is considerable amount of frequency content smear, albeit, the areas of smearing are different. Spectral map in the +z direction shows relatively less smearing.

If the singular value map concept is applied on this particular sensor assembly, 2 significant singular plots are observed.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{sigma_1_2.png}
\caption{\(\sigma_1\) and \(\sigma_2\) plots for data shown in Figure 5-1}
\end{figure}

As was discussed in the previous section, the first singular value map retains the important and the most significant contribution from the sensors. In this case, the 12\textsuperscript{th}, 24\textsuperscript{th} and 36\textsuperscript{th} orders were quite visible in all three spectral maps. Hence, the first singular value plot, the \(\sigma_1\) plot, retained this information and is therefore distinctly visible. The \(\sigma_1\) plot also has very low amount of smearing across the map, thus depicting a virtually noise-free “averaged” estimate.
On the other hand, all three maps displayed some hint of remaining orders, like 10th, 11th, 22nd, 23rd, 34th and 35th. Some small levels of the fractional orders were also visible. Please note that the amplitude ranges of these orders varied in all three maps. Due to this variation in amplitude, the cross contributions of the order information from the 3 sources, averaged out. These characteristics are visible in the second singular value plot, the $\sigma_2$ plot and also in the $\sigma_1$ plot at lower amplitudes.

Hence, the $\sigma_2$ plot, in this case, can be considered as the RPM spectral map of a virtual dominant sensor, enveloping the entire node and showing the primary major characteristics of the node. The $\sigma_2$ plot would then be depicting a sensor showing the secondary major characteristics of the node.

### 5.2.2 Different DOFs on Same Component

In this case, a small high speed rotor, with 4 accelerometers on the housing, would be considered. The following plots show the instantaneous speed and RPM spectral maps of 4 sensors on the housing.

![Figure 5-3: RPM profile for the test-run on high speed rotor](image-url)
Although their physical locations aren’t considerably apart, there are a few differences that are visible clearly, after comparing all maps. For example, the most dominant order, 5.5\textsuperscript{th}, has different amplitudes when the 4 maps are compared around the region covering 10,000 RPM and 16,000 RPM. On the other hand, the 0.75\textsuperscript{th} order amplitude profile looks consistent throughout the sensor assembly. The profiles of the 9\textsuperscript{th}, 9.5\textsuperscript{th} and 10\textsuperscript{th} orders are also dissimilar within the 4 maps. It is seen that their peak behavior region (around, 900 Hz and 1800 Hz) has different amplitudes in the 4 maps.

All 4 plots also have faintly visible profiles of the 1.75\textsuperscript{th} and 2.25\textsuperscript{th} orders, and also the modulated order profiles of the 0.75 order with the 3\textsuperscript{rd} natural frequency at about 280 Hz. These profiles are visible after 15,000 RPM and 200 to 800 Hz, and are present at different amplitude levels in each plot.

The following plots show the first two significant singular value plots obtained after plugging the response data of all 4 sensors into the SVD algorithm.
The $\sigma_1$ plot clearly shows a well-defined order trail of the most dominant order, the 5.5$^{th}$. The dips in amplitude, especially visible in Sensor 1 and 4, are not to be seen in the Singular Value plot, because the other 2 sensors have a significant contribution at those regions. The 9$^{th}$, 9.5$^{th}$ and 10$^{th}$ order profiles are also very distinct and are clearly showing the region of interest where these orders have large amplitudes.

The faint profiles of the 1.75$^{th}$ and 2.25$^{th}$ orders, as well as the modulated orders discussed above, are visible through consistent amplitude trails, thus depicting that there is some, behavior associated with these profiles within the sensor assembly.

The $\sigma_1$ plot, is thus serving to be a good indicator of the major overall behavior of this system under consideration. For a bigger system containing multiple high speed rotors at different locations, this concept of a virtual dominant sensor observing an entire structure (or sub-structure, in this example), would be helpful in observing the behavior of the entire system as a whole.

The $\sigma_2$ plot does not show an extraordinarily different profile than the first singular value plot and hence can be discarded along with the remaining two SV plots.
5.2.3 Entire Structure

For a complex structural assembly, to characterize the dynamic characteristics in any particular direction, it would require a multitude of uni-axial and tri-axial accelerometers placed at various nodes. Observing each sensor’s spectral map, one by one, direction by direction, proves to be tedious and severely time consuming. It is also possible for a large enough assembly that, the behavior projected by one set of sensors on a component will not be the same as a sensor set on another component.

With the developed singular value map procedure, it is now possible to observe such a machine by combining the contribution from all sensors in a particular direction. The resultant singular value map will then characterize the entire assembly’s behavior in that direction.

For example, the following plots show the RPM spectral maps in the +x direction on 11 different nodes on an engine, driveshaft and housing assembly. Sensors on points 132, 140 and 142 constitute the first group of sensors. Sensors on points 300, 301, 302 and 304 form the second set of sensors. Points 600, 602, 620 and 622 form a 3rd set of sensors. The grouping is done according to their placements on different components in the assembly.
Figure 5-6: RPM Profile and RPM spectral maps at first set of sensors

Figure 5-7: RPM spectral maps at second set of sensors
The difference is starkly visible in each sensor set. The first group has only the 12th and 24th orders as major contributors and 36th and 48th orders with moderate contributions. There is considerable frequency smear content at higher frequencies as well. The second group has almost all orders active in one or the other sensor at a considerable level. Sensor at 302 has a considerable smearing effect that is visible. The third group has active contribution from the first 24 harmonics (0.5th, 1st, 1.5th to 12th order), with some smearing associated with the same region. The higher orders are not that dominant in the third group.

The singular value maps from the data plugged in from all these 11 sensors are shown below.
Figure 5-9: σ1 and σ2 plots for data shown in Figure 5-6, Figure 5-7 and Figure 5-8

Figure 5-10: σ1 and σ8 plots for data shown in Figure 5-6, Figure 5-7 and Figure 5-8

The σ1 plot shows the most dominant characteristic (from levels $10^2$ to $10^5$) in the sensor assembly to be around $10^{th}$ to $20^{th}$ orders. This is because that region in the spectral map has significant contributions from 8 of the 11 sensors (groups 2 and 3). The secondary dominant behavior (from levels $10^{-2}$ to $10^2$) is observed around $1^{st}$ to $10^{th}$ orders and orders above $24^{th}$. This is due to the fact that only 1 of the 3 group contributes majorly around these regions in the map. For example, group 1 only showed major
trails at orders which are multiples of 12, while group 3 did not show major trails after the 18th order, leaving only group 2 to be the dominant contributor in that region.

It is also seen that in the $\sigma_1$ plot, the smearing effect is extremely low as compared to the original spectral maps.

Looking at the relative magnitudes of various order trials, it is possible to extract the most dominant characteristics from a sensor assembly and then progressively, the second most and so on. Although the secondary characteristics in the $\sigma_1$ plot might be related to primary behavior in a group of sensors; because the entire assembly is in consideration, that particular behavior is not consistent throughout. Hence, that particular behavior, say for example, the region above 24th order, is averaged accordingly and only comes up in an amplitude range (from levels $10^{-2}$ to $10^3$) which deems it secondary.

While, the region between 10th and 20th orders is visible in almost all sensors and hence comes up as the primary characteristic in the $\sigma_1$ plot.

The $\sigma_2$ and $\sigma_8$ plots are shown to reveal that although the trails remain at similar relative levels, the smearing effect from $\sigma_2$ to $\sigma_8$ increases gradually, from singular value to singular value. This signifies that smearing is present at different frequency regions along different maps.

5.2.4 Sensor on Improper Location

If during pre-testing, a particular channel or sensor is found to be faulty, then that test component can be either removed or replaced. But, if the channel data is to be at an acceptable voltage range (for piezo-electric transducers) which is within that of other channels, it is difficult to determine the sensor location’s reliability, just by observing the acquired time domain data. If, during testing, the software and the hardware interface do not possess an ability to plot a waterfall map in real-time, it wouldn’t be
until post-processing that the user finds out whether a particular channel gathered reliable data which is true to the structure or not.

When arisen, this case might produce a possibility of a re-test to re-acquire better data. The singular value map concept can alleviate this problem as well. For example, the following plots show RPM spectral maps from 4 sensors on a leaf-blower rotor housing.

*Figure 5-11: RPM Profile for test-run on Leaf Blower*

*Figure 5-12: RPM Spectral Maps of data from 4 sensors on Leaf Blower*
In the spectral map for sensor 8, it is clear that the location on the housing is poor as compared to the data being shown in other maps. Whether further analysis can be undertaken without a re-test can be understood by observing 2 different first singular value maps (σ₁ maps).

The first map is from data plugged in from all 4 sensors – while, the other from data plugged in from only the first 3, ‘good’ sensors. If there is no significant difference between the two, it can be safely said that a re-test is unnecessary and the major dynamic behavior of the system is being projected by 3 sensors (the first 3) and the 4th one is simply to be excluded. If there is a significant difference, this would mean that the 4th sensor is either not in a bad location and is indeed showing a dynamic behavior related to that particular area of the system, or a re-test is necessary.

![Figure 5-13: σ₁ plot comparison of Sensor assembly having all sensors and assembly having 3 good sensors](image)

Both the σ₁ plots depict the same data. There is no difference in the maps and hence it can be said that the first 3 sensors are enough to project the major behavior of the system and the 4th is to be excluded from further analysis. There is no need to perform a re-test in this case.
5.3 Uses

5.3.1 SV Maps

The possible interpretations and usage of the “virtual dominant sensor” tool which have been touched upon in sections 4.5 and 5.2 can be summarized as follows.

- **Rank**: As previously discussed, the number of significant singular values from a data set represents the number of linearly independent rows or columns in the base matrix, or simply put, the rank of the matrix. Therefore, the number of significant observable singular plots will be denoting the number of linearly independent physical contributions from each sensor in the cross-power matrix.

If the number of significant singular values (and similarly plots) are less than the number of outputs ‘m’ used in the calculation, it implies that there is a level of redundancy in the data.

Hence, if rank \( r < m \), a productive use of this information can be to try out different positions of the sensors on the test-structure. If the rank increases, this will result in a better sensor combination on the test-structure which verifies that physically, the entire structure is being observed without any redundancy.

Further, if more number of sensors are used on a structure than required, singular value maps can be developed by dropping out a sensor or a combination of sensors (according to location) and observing the data after each calculation. This approach to data processing is a statistical approach known as jackknife resampling (similar to bootstrap statistics) \(^{[14]}\). A possible use of this technique is presented in section 5.3.3.

- **Noise reduction**: It has been discussed before that, the noise is “averaged” out by the Singular Value Decomposition procedure on each cross-power matrix, because it’s frequency content is not consistent though-out every measurement degree of freedom.
As, due to the limitation posed by the Fast Fourier Transform method that the signal should be periodic within the observation window, averaging cannot be performed for a non-stationary signal from a rotating machinery. This method can be used effectively to reduce noise and observe the structure with only the major characteristics distinctly visible.

- Sensor assembly quantification: In Section 5, it has been illustrated by the use of some sample results, that the virtual dominant sensor can be applied to a node, which is being observed in 3 principal directions, or to a component having multiple sensors on it. It has also been discussed how the virtual dominant sensor concept can be used to determine the primary and secondary behavior of an entire test structure by using the data from all the sensors in the SVD calculations.

The results and these interpretations throw light on the advantages and uses of this process.

**5.3.2 Blending With Existing Order Tracking Methods**

Similar to creating a singular value map by using the cross-power matrix at each frequency of vibration and each RPM, the singular values can be calculated by using the cross-power matrix at each order of interest and each RPM. The order information can be from one of the order tracking methods like TVDFT, Kalman Filtering etc.

To make it computationally less time consuming, the Virtual dominant sensor concept can be applied to only the order of interest. The resultant significant singular value curve can then be viewed against RPM on the x-axis.

For example, instead of using the entire order spectrum, consider just an order ‘O’. (The order spectrum can be considered as well, with each order spaced Δo apart, according to the corresponding angle-order
DSP relationships. In this case then, a singular value map can be plotted which would look similar to an order spectrum in a waterfall format, or an order map).

For order ‘O’, at each RPM, the response would be a column, similar to as described in Section 4. The cross-power of this vector would be a square matrix.

\[ R_{k,O} = \begin{bmatrix} a_1 + i b_1 \\ a_2 + i b_2 \\ \vdots \\ a_m + i b_m \end{bmatrix}_{k,O} \]

5.1

\[ C_{k,O} = R_{k,O} \times R_{k,O}^H \]

5.2

SVD procedure on this cross-power matrix would result in the following, at each RPM for this order estimate.

\[ [U]_{k,O} \times [\Sigma]_{k,O} \times [V]^H_{k,O} = SVD([C]_{k,O}) \]

5.3

The significant singular values, in the singular value matrix [\Sigma], can be then plotted against RPM. In case of an order-RPM spectral map, a similar singular value order map can be plotted.

The significance of the singular value curve for any order being considered, would have the same interpretations as have been explained earlier. For example, if a singular value order curve is plotted for a particular sensor assembly, the SV curve will be denoting the combined behavior for the order being considered as a function of RPM.

The following examples can illustrate these results further.

5.3.2.1 Same DOF, 3 Principal Directions

Suppose, the example from 5.2.1 is considered. The comparison of the order tracks for the 12th order in each direction is shown in the figure below. The procedure used is TVDFT.
Evidently the region of 1500-1600 RPM is where the order has the maximum amplitude for +x and +z direction. But, for +y direction, there is a dip in amplitude. Had there not been a tri-axial accelerometer at the node and instead only +y direction was considered, the results would have indicated that the 12th order is not active in this RPM region.

If the virtual dominant sensor concept is applied on the order tracks in all three directions, the $\sigma_1$ singular value curve can be scaled and plotted against RPM and be compared with the order tracks as shown in the next figure.
The solid-line curve suggests that there is considerable amplitude range for the 12th order in the RPM region discussed above. The curve compensates for the drops in amplitude from one direction by considering the peaks in amplitude from the other two, hence showing a more complete behavior of the node.

In order tracking procedures, both – magnitude and phase are of importance. But, singular values are real scalar quantities without phase and hence the singular value curve does not have any phase related information. Although, elements of the cross-power matrix have phase associated with them and so do the singular vectors. Accurately extracting the phase of the singular value order track remains an open topic for further research in this area.

**Note:** The scaling factor used to bring the singular value curve at a comparable level to the order tracks, is arbitrarily selected in this example (mean of the maximum values of each order track). An appropriate scaling factor determination algorithm is a topic of future research.

### 5.3.2.2 Different DOFs on Same Component

The case of a leaf blower tested with 7 good sensors (Sensors 1, 2, 5 and 6 in the +y direction, Sensors 3 and 4 in +z, Sensor 7 in +x) and the following RPM profile is considered.

![Figure 5-16: RPM Profile for test-run on Leaf Blower](image)
Using the TVDFT method, the first order is tracked for all 7 sensors, and is compared between each sensor as shown in the following figure.

![First order track comparison for 7 sensors](image)

**Figure 5-17: 1st Order Track comparison for 7 sensors**

Barring some common peaks and valleys in the order amplitude vs RPM plot, the 1\textsuperscript{st} order behavior differs within this sensor assembly. For example, only 3 sensors show a dip in the order amplitude in the RPM range of 600 to 800, while sensors 4, 6 and 7 show a different behavior in regions where there are otherwise similar characteristics from the remaining sensors.

This difference in amplitudes can be understood within the structure by also observing the operating shapes. But to quantify the dynamic behavior of the machine as a whole, the virtual dominant sensor concept is applied.

It is to be noted again that only the magnitude information is available for singular value curves while extracting phase related information is still a topic of further research. The possibilities of phase extraction from the base cross-power matrix or the singular vectors has been discussed in the previous example.
As noted earlier, the scaling factor value is not entirely accurate. Though, by observing the $\sigma_1$ singular value curve across the RPM range, a general profile for the 1st order can be gauged. The common peaks and valleys are retained. The singular value curve amplitudes at regions which are dominated by few sensors rather than all, like the ones discussed above, are averaged and hence show a combined response of the structure.

5.3.3. Real-time tool

The real-time tool implementation can also be useful for determining the best sensor combination on a rotating machine. For a large structure having many active points, choosing the right sensor combination to observe it fully, is often a challenging task. Unless the dynamic characteristics of the structure are well understood, it is hard to identify “active” points.

One plausible use of the newly developed virtual dominant sensor tool, is to identify the best sensor combination on a complex structure. To avoid time-consuming re-testing using different combinations, it is better to have a real-time application with more number of sensors than required.
Having more number of sensors than really required enables the user to pick any combination and observe the “virtual dominant sensor” output of that combination only after one test run. Depending on the slew rate, loading conditions, geometry and the structural dynamics of the system, a predicted response can be looked for within a sensor assembly. The user can then identify the points required to characterize the whole system according to the dominant sensor output.

The real-time application of this tool can help in determining exactly which sensors are to be used in the further analysis, thus eliminating the need of trial-and-error testing with different sensor assemblies. This aspect would be advantageous especially for rotating systems which are tested for their modal parameters without prior theoretical or computer simulated results.

As mentioned before, jackknife resampling can be utilized to pick an optimal sensor set. Jackknife resampling is a linear approximation of the bootstrap\(^5\) technique in statistics\(^{[14]}\). In modal analysis, a similar approach is used in identifying system poles and modal frequencies from computational poles\(^{[4]}\). Large number of solution sets – acquired from test data, using different data subsets, model orders or solution methods – are compared visually and/or numerically, resulting in estimation of consistent underlying structural behavior (system poles) leaving out the inconsistent noise and computational poles\(^{[4]}\).

\(^5\) In jackknife resampling, the estimator of a parameter is found by systematically leaving out each observation from a data-set and calculating the estimate and then finding the average of these calculations. Bootstrapping is a method for assigning measures of accuracy to sample estimates allowing estimation of the sampling distribution of almost any statistic\(^{[14]}\).
6 Summary

6.1 Conclusions

Accurately estimating instantaneous RPM is the basic and the most important aspect of rotational systems analysis. The included detailed procedure provides the necessary steps to undertake this analytical tool. A new method of assigning a response spectrum accurately to an instantaneous RPM value was presented and applied successfully on acquired data.

The virtual dominant sensor concept using the singular value decomposition technique on the cross-power matrix at each vibration frequency (or order of vibration) and each RPM provided results which appear cleaner than RPM spectral maps. The singular value maps also have the advantage of enhancing the major dynamic characteristics of a rotating machine.

The practicality of using the virtual dominant sensor concept - by observing the $\sigma_1$ plot and further progressive plots and so on - was illustrated by results from data acquired on actual rotating systems. The ability of this concept to quantify the overall dynamic behavior of a node (in all 3 directions) or a component (with different sensor locations) or an entire structure, makes it a new development in the field of experimental analysis of rotating systems.

The interpretations of the results of the singular value maps are directly related to the results provided by the SVD procedure on any square matrix. The primary and secondary behavior of a rotating machine as observed by a particular sensor assembly can be observed by the $\sigma_1$ plot and the relative magnitudes of the characteristics present.
There can be now possibilities of blending this concept with existing order tracking methods, as illustrated with examples on acquired data. Visualizing the order domain for an entire node, component or structure can prove to be useful in understanding the behavior of various harmonic related aspects.

A method is needed which provides results which are noise-free and compensate the limitations posed by the FFT in rotational systems analysis. The virtual dominant sensor concept results are “averaged” in the spatial domain due to the advantage of using the SVD procedure and hence provide a noise-free estimate of the overall dynamic behavior of the machine.

### 6.2 Recommendations for Future Work

The real-time tool application discussed in 5.3.3 using the virtual dominant sensor concept can be developed in a software like MATLAB™, which can allow online visualization of the singular value plots. It can be tested on an actual rotating machine. Multiple sensors, more than the minimum required can be placed on a moderately large structure and the virtual dominant sensor output from different combinations of sensors can be observed to determine the best possible sensor assembly.

The various advantages and limitations of the real-time application can only be investigated during and after actual testing.

As discussed in Section 5.3.2, the Virtual dominant sensor concept was applied to the order domain data obtained from an order tracking method. The singular value curve was then plotted against RPM and compared with the order tracks of that particular order for all sensors. As mentioned in the section, the scaling factor was chosen arbitrarily and hence, there is a need for further investigation in choosing the right scaling factor for this process.

An alternate method of using singular values for virtual dominant sensor order tracks can be, converting the singular values in the frequency domain, to time domain via the inverse fast fourier transform.
method, the IFFT. This time domain data, then can be plugged in to various order tracking methods to achieve singular value order tracks. Different methods can be tried out to determine the best possible one.

But to convert the frequency domain values to time domain values by the IFFT, a phase characteristic must be associated with the singular values in the frequency domain. The singular values are real scalar quantities and hence do not have any phase information. During the SVD procedure at each vibration frequency and each RPM, the base matrix i.e the cross-power matrix is calculated from the response column vector, a complex quantity. Each term in the column vector has a phase associated with it. Also, the singular vectors in the singular vector matrix resulting from the SVD procedure, have phase information as they are complex quantities. Exactly which term, or terms are to be used to extract phase information and use it for the IFFT method, is a topic of further research.

One usage which is proposed in this material involved the condensation of data from 3 principal directions on a node, to one virtual dominant sensor. This concept can be extended for large structures with multiple and multi-directional sensor orientations in calculating condensed operating shapes.

For example, if there are 6 types of directional sensors used on a structure, +x,+y,+z,-x,-y and –z. If each node on the structure has at-least 3 directional sensors, they can be condensed into one Virtual Dominant sensor by the procedure explained in this material. Once each node has a dominant sensor output associated with it, the operating shapes for different orders can be calculated by using the virtual dominant sensor order track data. These operating shapes can be called the condensed operating shapes.

The condensed operating shapes, when observed at different RPMs, will provide an overall big picture of the amplitudes of the response at each node. This again, can be extended to a real-time application.
Various concepts discussed in the reference 9 describe the procedure to find out linearly independent operating shapes. There are mathematical explanations which describe concepts like finding the number of independent shapes excited at each rpm value, finding whether a given shape is excited at any other rpm, finding the order with the highest amplitude at a given rpm and finding the mode enhanced order track, similar to an enhanced FRF in modal analysis\textsuperscript{[9]}. 

The condensed operating shapes information can then be plugged in these procedures as described in the paper. There will be relatively less noise present in each plot, because of the ability of the SVD to retain only the major contributions. These “cleaner” plots can prove to be more effective in finding out the dynamic aspects of the machine which are to be tackled for a noise or vibration problem.
7 References


[8] Blough, J. R. Multi-Tachometer Order Tracking and Operating Shape Extraction. ISMA.


[25] ACS Inc - Order data from Engine test cell
Appendices

Appendix A – MATLAB scripts

The script variables are used the same throughout unless specified in separate function.

A.1 First RPM estimation with raw tachometer data

%Basic formulation : Reference [17]
load('C:\RotorData\LBCase_1.mat'); %Load the MAT file

%When imported from X-Modal III, the file is in a "struct" format named "%vacq_DATA. The variables after the '.' symbol, like DeltaFreq are vacq_DATA variable names."

T=1/(vacq_DATA.DeltaFreq); % Calculating total time T
N=length(vacq_DATA.InputTime); % Extracting total number of data points
time=linspace(0,T,N); %defining "time" vector

[nchans,ndata]=size(vacq_DATA.OutputTime);
nresps=nchans; %storing the number of response channels

tachsig=vacq_DATA.InputTime; %the raw tachometer data
response=vacq_DATA.OutputTime(1:nresps,:);

deltaT=time(2)-time(1);
Fmax=vacq_DATA.SpanFreq;

counter=0; %initiating pulse detection counter

tachsig=tachsig-(max(tachsig)/2); %centering the tach pulse

%Detecting pulse by finding the start of pulse

for ii=2:length(tachsig)
    if sign(tachsig(1,ii))==1 && sign(tachsig(1,ii-1))==-1 %start of pulse
        counter=counter+1;
        tn(1,counter)=((0-tachsig(1,ii-1))/(tachsig(1,ii)-tachsig(1,ii-1)))/(time(1,ii)-time(1,ii-1)))+time(1,ii-1); %Zero level crossing of the pulse
    end
end

npul=counter; %number of pulses counted

for kk=1:npul-2
rpm(1,kk)=60/(tn(1,kk+1) - tn(1,kk)) ; %calculating RPM
ti(1,kk)=(tn(1,kk+1) + tn(1,kk))/2; %time points for rpm values
end

%plotting the initial RPM estimate
figure(1);
plot(ti,rpm);
xlabel('Time');
ylabel('RPM estimate');
grid;
title('RPM estimate vs time');

A.2 Spline fitting algorithm with shaving

%Basic formulation - References [6], [19], [22] and [23]
%Spline fit (advanced algorithm - shaving applied)
deci=2; %counter for shaving
rpm2=rpm; %defining duplicate rpm variable for shaving loop
ti2=ti; %defining duplicate time points variable for shaving loop

while deci==2 %Spline fitting loop will run until user stops it (deci==1)
clearvars RPMS;

iteration=2; %shaving loop initiation counter

while iteration==2 %shaving loop will run until "iteration ==1"
seg=input('How many segments of the RPM estimate graph for the spline fit?');

clearvars RPMSa;
time2=linspace(ti2(1),ti2(end),seg);
RPMSa=spline(ti2,rpm2,time2); %spline fit

figure(2);
plot(ti2,rpm2,time2,RPMSa,'r-');grid;
title('Comparison of Previous RPM estimate and Spline fit');
legend('Previous RPM estimate','Current Spline fit','Location','SouthEast');
iteration=menu('Shaving iteration?','No','Yes'); %asking user for a shaving iteration
if iteration==2
    RPMSa=spline(time2,RPMSa,ti2); %making RPMSa vector length equal to rpm2
end

if iteration==2
    limit=input('Maximum rpm variation ?');
    reducedset=rpm2(abs(rpm2-RPMSa)<limit); %Shaving
    reducedti=ti2(find(abs(rpm2-RPMSa)<limit)); %Shaving
    rpm2=reducedset;
    ti2=reducedti;
end
A.3 RPM variation criterion

%RPM variation criterion
%Developed by Shrirang Deshpande, December 2013, Structural Dynamics Research Lab, University of Cincinnati.

while deci2==1

clearvars rpmvari InstRpmtemp;
res=0;
opt=menu('Size of data block?','64','128','256','512','1024','2048');
if opt==1
    res=64;
elseif opt==2
    res=128;
elseif opt==3
    res=256;
elseif opt==4
    res=512;
elseif opt==5
    res=1024;
elseif opt==6
    res=2048;
end

ov=0;valu=0;
lap=menu('Amount of overlap for the data block when sweeping?','50%','75%','87.5%','93.75%','Increment slide by 1 data point');
if lap==1
    ov=2; valu=50;
elseif lap==2
    ov=4; valu=75;
elseif lap==3
    ov=8; valu=87.5;
elseif lap==4
    ov=16; valu=93.75;
elseif lap==5
    ov=res; valu=99.99;
end

cho=menu('RPM estimate method within the block?', 'Mean value', 'First value', 'Median', 'Last value');
nslidestemp=0;
for ss=0:(res/ov):(length(RPMS)-res)
    nslidestemp=nslidestemp+1;
    if cho==1
        InstRpmtemp(1,nslidestemp)=mean(RPMS(1,(1+(ss)) : (res+(ss)))); %mean rpm
        stx=(["Mean RPM chosen"]) ;
    elseif cho==2
        InstRpmtemp(1,nslidestemp)=RPMS(1,(1+(ss)));
        %first rpm value of the block
        stx=(["First RPM value chosen"]) ;
    elseif cho==3
        InstRpmtemp(1,nslidestemp)=median(RPMS(1,(1+(ss)) : (res+(ss)))); %median rpm value of the block
        stx=(["Median RPM value chosen"]) ;
    elseif cho==4
        InstRpmtemp(1,nslidestemp)=RPMS(1,(res+ss));
        %last rpm value of the block
        stx=(["Last RPM value chosen"]) ;
    end

    rpmvari(1,nslidestemp)=max(abs((InstRpmtemp(1,nslidestemp) - RPMS(1,(1+(ss)) : (res+(ss))))*100/InstRpmtemp(1,nslidestemp)))); %storing only the maximum value of RPM variation within the block
end
rpmvari(rpmvari>100)=100;
st=(["Maximum %age variation of RPM estimate block-wise when '"]) ;
st2=([" and datablock size = '"]) ;
figure(3);
plot(linspace(0,nslidestemp,length(rpmvari)),rpmvari); %plotting maximum RPM variation block wise
xlabel('Data blocks');
ylabel('Maximum Percentage variation');
grid;
title([st stx st2 num2str(res)]);
deci2=menu('Is this acceptable?','No','Yes');
end
A.4 RPM spectral mapping

```matlab
% RPM Spectral Mapping
% Basic formulation - References [1] and [13]

winc=menu('Choose window','Hanning','Flat-top');
if winc==1
    window=hann(res);
else
    window=flatopwin(res);
end

freq=linspace(0,Fmax,res/2); % defining frequency vector, 'res' is the block size
RESP=zeros(1,res);
nslides=0;

for ss=0:(res/ov):(length(RPMS)-res)% Increments according to overlap and block size.
    responseblock=zeros(nresps,res);
    responseblock=response(:,((1+ss):(res+ss)));
    RFFT=zeros(nresps,res);

    for tt=1:nresps
        responseblock(tt,:)=responseblock(tt,:).*window';
        RFFT(tt,:)=fft(responseblock(tt,:));
    end

    RFFT=RFFT*(2/res); % Scaling
    nslides=nslides+1; % incrementing the number of slides counter
    Rpmresp(nslides,:)=RFFT; % storing the responseblock's FFT in the nslides counter

    % storing the user-defined value within the RPM block as the
    % Instantaneous RPM
    if cho==1
        InstRpm(1,nslides)=mean(RPMS(1,(1+(ss)):(res+(ss)))); % mean rpm value of the block
    elseif cho==2
        InstRpm(1,nslides)=RPMS(1,(1+(ss)));
    elseif cho==3
        InstRpm(1,nslides)=median(RPMS(1,(1+(ss)):(res+(ss)))); % median rpm value of the block
    elseif cho==4
        InstRpm(1,nslides)=RPMS(1,(res+ss)); % last rpm value of the block
    end
end
```

tl=[{'RPM Spectral map at response loc'}];
tla=[{'Time spectral map at response loc'}];
t2=[{'Window slides in time domain'}];
t3=[{'& block size'}];
The lowest order of magnitude is 

disp([rnda num2str(min(min(min((log(abs(Rpmresp)))))))]);
LL=input('Enter lower limit');

for uu=1:nresps  
  \textit{Spectral Map plotting} 
  Zdata=log(abs(Rpmresp(:,uu,1:(res/2)))) ; \%discarding 2nd half of each 
  FFTd block 
  Zdata(Zdata<LL)=NaN; \%cutting out portion below limit 
  figure; 
  waterfall(freq,InstRpm,squeeze(Zdata)); 
  colorbar('Location','EastOutside'); 
  xlabel('Freq uptil Fmax'); 
  ylabel('Inst RPM'); 
  zlabel('Log mag absolute response'); 
  title([t1 num2str(uu) t2 num2str(nslides) t3 num2str(res)]); 
  view(0,90);
end

t0=(['\textit{Instantaneous RPM estimate with }%overlap=']);
figure; 
plot(linspace(0,max(time),length(InstRpm)),InstRpm); \%plotting the InstRpm 
grid; 
title([t0 num2str(valu)]);
xlabel('Time'); 
ylabel('RPM');

\textbf{A.5 Singular value maps}

%Concept by Dr Randall J. Allemang. Script developed by Shrirang Deshpande.

nslid=0;
[A,B,C]=size(Rpmresp);

for aa=1:length(InstRpm)
  for bb=1:length(freq)
    R=Rpmresp(aa,:,bb); \%extracting the response vector for the 
    particular rpm and frequency 
    R=R.'; \%converting into a column vector 
    Rh=R'; 
    matrix=zeros(length(R),length(R)); 
    matrix=R*Rh; \%Cross-power matrix 
    values=zeros(length(R),1); 
    values=svd(matrix); \%SVD of the cross power matrix 
    svalues(:,aa,bb)=values; \%storing the singular values according to 
    rpm and frequency 
  end
end

t0=(['\textit{sigma }']);
for cc=1:length(R)
  figure;
  waterfall(freq,InstRpm,squeeze(log(svalues(cc,:,1:length(freq)))))); 
  xlabel('Frequency (Hz)');

A.6 Whitening out low magnitude data

%whitening out low magnitude data (ink-saver)
figure();
newmap=colormap; %colormap is a 64x3 matrix, where in each row the RGB level
of a color is defined
newmap(1:32,:)=1; %the first 32 rows (or colors) are dark blue to green.
These can be set to white(R=1,G=1,B=1) to only see colors above green to red
colormap(newmap); %setting colormap according to colors defined by newmap

A.7 Time-variant discrete fourier transform (TVDFT)

function orderspec = tvdft(responseblock,orders,InstRpm,deltaT)

%Formulation -
"The Time Variant Discrete Fourier Transform as An Order Tracking Method" by
Jason Blough
%SAE Journal of Sound and Vibration, Paper #972006, Reference [10]
tvdft (responseblock,orders,InstRpm,deltaT)
tvdft function estimates the Time-Variant Discrete Fourier transform of
the time domain responseblock for the orders specified in the "orders"
vector for the Instantaneous RPM value given

dtvec=deltaT:deltaT:deltaT*length(responseblock);

for aa=1:length(orders)
    term=zeros(1,length(responseblock));
    an=zeros(1,length(responseblock));
    bn=zeros(1,length(responseblock));
    suman=0;sumbn=0;
    term=orders(1,aa).*InstRpm.*dtvec/60;
    an=responseblock.*cos(2*pi.*term);
    bn=responseblock.*sin(2*pi.*term);
    suman=sum(an);
    sumbn=sum(bn);
    An(1,aa)=suman/length(responseblock);
    Bn(1,aa)=sumbn/length(responseblock);
end

uncomporders=An + j.*Bn; %uncompensated orders
%orthogonality compensation matrix
wind=hann(length(responseblock));
OCM=zeros(length(orders),length(orders));
for ii=1:length(orders)
    for jj=1:length(orders)
        term2=orders(1,ii)*InstRpm.*dtvec/60;
        term3=orders(1,jj)*InstRpm.*dtvec/60;
        term4=(exp(j*2*pi.*term2).*wind.')*((exp(j*2*pi.*term3)').');
        sumterm4=sum(term4);
        OCM(ii,jj)=sumterm4/length(responseblock);
    end
end

ordersist=inv(OCM)*(uncomporders.');
end
Appendix B – Rotating systems images

B.1 Toro leaf blower

Figure 0-1 – Shaft connection from AC motor to leaf blower crank.

Figure 0-2 – Infra-red tachometer and reflective strip.
B.2 Vacuum cleaner rotor (Small sized rotor)

Figure 0-8 – Sensors on rotor housing

Figure 0-9 - Tachometer
B.3 2-plane unbalance rig

*Figure 0-10 – 2-plane unbalance rig test set up*
B.4 Engine test-cell

(Note: Data and images provided by ACS Inc)

Figure 0-11 – Wireframe and actual structure – provided by ACS Inc [25]

Figure 0-12 – Wireframe model – provided by ACS Inc [25]