I, Vipin V. Iyer, hereby submit this original work as part of the requirements for the degree of Master of Science in Mechanical Engineering.

It is entitled:
Face Milling Simulation to Correlate and Predict the Effects of Machine Tool Geometric Errors on Part Flatness Tolerance

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Face Milling Simulation to Correlate and Predict The Effects of Machine Tool Geometric Errors on Part Flatness Tolerance

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of the University of Cincinnati
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By

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Abstract

Face milling using a three or five axis machining center is one of the most common forms of metal manufacturing processes in the world. The simplicity of the process and the accuracy of the part it produces makes it the default machining process to use for a roughing cut and finishing a surface. This research focuses on theoretically simulating a surface cut by the intermittent cutting action of a multi-point face mill cutting tool by partitioning the surface into grids. The points on the surface of each grid that have the lowest depth of cut value represent the imprint of the cutting tool on the finished surface. Machine tool geometric errors are typically present in any milling machine in a cold start condition and tend to progressively get worse over time. This finished surface is virtually generated while taking into account machine tool geometric errors from each of the machining center’s slides as well as the spindle errors by virtual modeling of the machining process. The finished surface is then virtually inspected to determine flatness error, a form tolerance, which is one of the metrics for part accuracy.

A statistical treatment of error combinations is used to determine which machine tool geometric error components of a particular slide have a significant and practical impact on part accuracy defined by the flatness error. This could help improve accuracy during machining since the operator can avoid setting the slides of the machining center to locations that have higher calculated error values for these significant error terms. The study of the effect of these machine tool geometric errors of each machine slide on flatness error can be used to determine parameters of error compensation to help improve part accuracy at end of machine life. Selective error compensation might be cheaper at the end of machine life than complete machine overhaul.
Acknowledgements

This thesis is dedicated to my mother and my brother. Their sacrifices have enabled me to be where I am today.

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1. Introduction

The current competitive world manufacturing environment demands higher machined part accuracy at lower costs with infinite machine life, low upkeep and maintenance, and minimal error compensation. A brand new machining center is, typically, accurate with lower overheads for maintenance. Companies are constantly trying to extract the most out of older machines which have part accuracy issues attributed to machine degradation due to wear of the guide ways between slides and other moving components. This improvement in accuracy is often performed with observational and empirical models which are usually operator dependent. There is an emphasis in the industry to develop predictive analytical models of machining processes using all process parameters. These models are then compared with results from the actual process to understand how well the model works in terms of correlation and accuracy of predicted results. This process helps companies leverage the strength of these well correlated analytical models to understand output of similar products on the basis of the knowledge of the simulated results.

In the long run, the analytical model can result in substantial cost savings if it can accurately predict machined part accuracy (in terms of flatness or other error values) given process parameters, insert geometry and quantified machine tool geometric error values of the machine slides and spindle [1-7]. A non-exhaustive list of factors that affect part accuracy includes tool wear, machine wear, vibrations, effect of heat on cutting action, variation in insert and cutter geometry, spindle errors and other similar systematic, random, controllable and uncontrollable errors [3].

A number of operations can be performed on a vertical machining center which is not restricted to but includes:
• Face milling to create a machined flat surface from raw stock
• Profile milling
• Drilling
• Slotting and plunging

Face milling is a machining process where a cutting tool that has multiple cutting edges rotates about an axis and performs an intermittent cutting action on a work piece. An image of a three axis vertical machining center can be seen in Figure 1.

![Figure 1: Three axis vertical machining center [8]](image)

The number of cutting edges (also known as inserts) on the cutting tool are variable and can be anywhere between 3 and 200 per tool [9]. Figure 2 is an image of a face milling tool. During cutting operation the angle of entry of the cutting tool has a bearing on the cutting forces, vibrations, and chip thickness. There are controllable, uncontrollable and inevitable factors that affect machined part accuracy; this research is an attempt to counter some of the uncontrollable and inevitable factors using virtual machining and statistical analysis. This research also tries to
reduce the effect of machine tool geometric errors (classified as unavoidable errors) while assuming that all other avoidable errors are mitigated by the right parameter selection.

![Figure 2: Face milling cutter [9]](image)

Tool wear, effect of heat on the machining process, vibrations parallax are classified as controllable errors [3]. Effects of tool wear and thermal errors are reduced by controlling the heat flow into the machine tool environment by higher volume of coolant or by redesigning machine tool to lower heat flow sensitivity by using materials with low expansion coefficient [3]. Mechanical vibration errors are reduced through proper design of the machine vibration isolation system [3]. Parallax, failure to calibrate, check zero of instrument and other such user errors are eliminated by better failure mode effects and analysis during design stage of the machine tool. Machine tool geometric errors creep up on a machine over time and have been traditionally compensated by using volumetric error compensation [4, 5].

### 1.1. Motivation For This Research

This research aims develop a robust face milling surface modeling approach and implement it using MATLAB® as the simulation tool. The virtually generated surface will account for machine tool geometric error terms pertaining to the slides, machining parameters-feed, width of
cut, depth cut and the spindle motion. The methodology developed uses brute force to determine tool nose imprint on machined surface. Wholesale error compensation techniques to account for machine flaws have been researched earlier but implementing these compensation techniques are time consuming and expensive [4, 5, 8, 10]. This research aims to determine statistically significant machine tool geometric error terms for each of the machine slides and provide guidance for selective slide error compensation for these machine tool geometric errors. Another benefit of the statistical treatment is that if a particular slide has only a single high impact error term, recommendation to avoid areas on the machine bed where higher values of this error term occur can be made. Thus, non-conducive spots on the machine bed which affect part accuracy adversely are determined.

1.2. Objectives and Impact of this Research

The objectives of this research are to develop a methodology for virtually modeling the face milling process by superimposing the machine tool slide geometric error and spindle geometric error model using homogeneous transformation matrices (HTMs). The virtually machined surface is then virtually inspected to determine flatness error values and analyze these process parameters to determine statistically and practically significant machine tool geometric errors. This information is useful to determine if certain error prone areas of each of the slides and spindle could either be avoided completely or selectively compensated for. A new and simple method to determine surface topography has been developed to simulate the face milling process by sorting the machined surface into small grids and searching for lowest depth of cut values at each of these grids. This research fills the void in literature of the availability of a methodology for simulation of the face milling process and a machining advisor for selective error compensation for particular slides and avoidance of others.
1.3. Outline of the Thesis

This thesis is divided into five chapters. Chapter 1 provides an insight about motivation for this research and a statement of the problem. This chapter also discusses methods that are available for part accuracy improvement, objectives and impact of the research. Chapter 2 presents a detailed literature review of face milling surface generation methodology, machine tool geometric error measurement, homogeneous transformation matrices that can be used to include machine tool geometric errors in surface generation, virtual machining and virtual inspection. Chapter 3 describes the research methodology used to achieve the objectives in this research and discusses machining process parameters, a brief description of machine tool geometric errors of the slides and the spindle, homogeneous transformation matrices used for modeling the surface generation process, sources and measurement of machine tool geometric errors. Chapter 4 describes surface generation approach and DOE runs for statistical treatment of flatness error value and machine tool geometric error terms. Chapter 5 discusses conclusions and future work.
2. Review of Relevant Literature

There has been prior reported research on predictive modeling of face milling as a machining process but most of this work has been geared towards determination of cutter force, chip geometry and its impact on surface finish in terms of surface roughness [11-16].

2.1. Face Milling Process

Face milling is a machining process where the cutter is mounted on a spindle and rotates perpendicular to the work surface. The material removal process occurs as shown in Figure 3. The cutter and the work piece can move relative to each other [17]. The cutting tool consists of a number of removable cutting edges, called inserts, which leave feed marks on the machined surface due to the relative motion between the cutter and the work piece [6]. The geometry of the insert has a large bearing on the surface finish, cutting forces and chip thickness generated during the cutting motion [6].

![Figure 3](image-url)  
Figure 3: Material removal process in face milling, adapted from [17]
Effectively, a face milling tool is a series of single point cutting tool attached to a machining spindle and has similar geometric characteristics as a single point cutting tool [17]. Studies have also shown that there are no fundamental differences between metal forming processes involved in forming chips using single or multi point cutting tools [18]. The basic tool angles and their functions remain the same. The different angles of a milling insert on a milling tool are seen in Figure 4.

![Figure 4: Insert and tool angles adapted from [17]](image)

### 2.2. Tool Geometry

Figure 4 is an image of an insert on a face mill tool and the various tool angles. The back rake and side rake angles influence cutting edge strength, cutting forces and influence chip flow [3, 17-19]. The end relief and the side relief angle prevent the end flank of the tool from rubbing into work piece, the side relief angle in particular prevents the tool from advancing too much into the work piece before the material is machined away [3, 17, 18]. The nose radius connects the two most important parts of the cutting tool- the side cutting edge and the end cutting edge. The tool nose radius has a significant impact on tool life, radial force and machined surface finish.
The end cutting edge angle prevents rubbing between the tool edge and the work piece [3, 17-19]. The lead angle or the side cutting edge angle is the angle between the straight cutting edge of the side of the tool and the side of the tool shank [1, 9, 17, 18]. This is the primary cutting edge and should be sharp at all times. For a given feed rate, increasing this angle widens and thins the chip out, changes the chip flow direction and reduces the chip thickness by cosine of the angle [1, 3, 17-19]. Increasing this angle from 0-45° causes the tool entry into the work piece to move to a fully supported part of the tool usually increasing tool life [18]. A tool nose radius of up to 1.6 mm results in an improvement in tool life and surface finish [1, 3, 17-19]. Too high of a lead angle or tool nose radius can cause tool chatter and poor surface finish, so seeking an optimum value is essential [18]. The effects of increasing tool nose radius, lead angle and end cutting edge angle are very similar [18]. To reduce the impact of high cutting force on the tool, the recommendation is that the ratio of the cutter diameter to the width of cut should be at least 3:2 [1, 6, 9, 17-19]. The desirable cutter and work piece relative positions are seen in Figure 5.

![Figure 5: Desirable cutter-work piece relative positions](image_url)

Figure 5: Desirable cutter-work piece relative positions [17]
2.3. Face Milling Simulation

The face milling cutting process involves simultaneous rotation of the cutter and translation of the cutter or work piece. Several attempts have been made to simulate the milling process numerically. Kim et al. [20] proposed a texture superposition method to evaluate surface asperity of milled surfaces. Chung et al [21] used an approach which reduced tool path generation to a root finding problem for a quartic polynomial (when the cutter bottom surface contains a toroidal surface). Roth et al. [22] determined the surface swept by a cutter on a five-axis machining center by using silhouettes to create imprint curves for varying tool positions. Jang et al. [23] used a voxel based method for simulating the milling process in which the work piece is generated by successively subtracting tool swept volumes from work piece. Tapoglou [24] et al. used a 3D spline and Boolean algebra to create a machined surface although it is not clear how these Boolean operations occur.

In this work the face milling process is simulated based on a first principles approach. The final machined surface is the result of cutting edges removing material from a stock work piece. The imprint that is left on the finished surface is the lowest depth of cut value from each simultaneous cutting action. As the cutter rotates by an angle of $\theta$ from an initial position, the path that an individual cutting edge traces out is a cycloidal curve (a general trochoid) [1, 2, 25]. This means that the edge path can be represented as a circular arc that moves by a discrete amount which is a fraction of the feed per tooth per revolution $S_z$ [1, 2, 25]. The equations that determine positioning of any point on the face milling tool inserts at any given instant are explained further in Chapter 3. In a three axis vertical machining center there are three slides and a spindle that are used to set location of machine tool with respect to work piece.
• The X slide (or the feed slide) is used to feed the work piece into the tool during machining,
• The Y slide (or the width of cut slide) determines how wide the tool is set inside the work piece,
• The Z slide (or the depth of cut slide) determines how deep the inserts are fed into the work piece.

The Z slide also has the head for the spindle where the cutting tool is attached. During the machining process the width of cut and the depth of cut are fixed for one pass, the only slide that is moved is the feed slide while the cutter is rotating in the spindle.

2.4. Factors Affecting Face Milling Accuracy

There are several factors that affect part accuracy of a work piece machined by the face milling process. There are systematic, random, controllable and uncontrollable factors that alter machine performance constantly. Ultimately the accuracy of the machining process hinges on the fact that the cutting edge positioning with respect to the work piece is consistent and repeatable. Ramesh et al [4, 5] investigated and characterized the various types of errors that affect accuracy of a machine tool. Mekid [3] studied and characterized these factors afflicting machining accuracy into several categories.

Errors can be generally classified as systematic, random or systematic and random as seen in Table 1. Research has revealed that machine tool geometric errors make up a major part of this machined surface inaccuracy [3-5]. These errors occur in the machine during the design stage and due to inaccuracies that are created during the machine assembly as a result of tolerance
stack up of machine parts and wear of the machine components over time. These errors tend to be smooth and continuous, can show hysteresis and have random behavior [3].

Geometric errors primarily depend on straightness, surface roughness, machine component misalignment and bearing preloads. Thermal errors account for about 40% of the total dimensional and shape errors in precision machines [3]. These errors exhibit a quasi-static behavior and it is more cost beneficial to compensate for these than eliminate them by using high precision components.

Table 1: General classification of errors, adapted from [3]

<table>
<thead>
<tr>
<th>Error Type</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systematic</td>
<td>Missing factor, Calibration, Failure to check zero, Instrument drift, Hysteresis and lag time</td>
</tr>
<tr>
<td>Random</td>
<td>Instrument resolution, Physical variations</td>
</tr>
<tr>
<td>Random or Systematic errors</td>
<td>Environmental factors, Parallax factor, user errors</td>
</tr>
</tbody>
</table>

Cutting force induced errors are an important source of part distortion and accuracy degradation [3]. These errors are induced by vibration, material instability, instrumentation, loading and tool deflection and as such are easier to control and compensate for. Tables 2, 3 and 4 depict the severity classification of errors of machine tools.

These errors can be eliminated by either eliminating them at the source or by machine tool error compensation through coordinate transformation approaches [3]. Error avoidance by machine design improvement is expensive and the cost rises exponentially with the level of improvement desired [3].
Table 2: General classification of severity & measurability of geometric errors adapted from [3]

<table>
<thead>
<tr>
<th>Geometric and Kinematic Errors</th>
<th>Severity</th>
<th>Measurability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positioning</td>
<td>High</td>
<td>Easy</td>
</tr>
<tr>
<td>Straightness</td>
<td>High</td>
<td>Easy</td>
</tr>
<tr>
<td>Pitch</td>
<td>Medium</td>
<td>Easy</td>
</tr>
<tr>
<td>Yaw</td>
<td>Medium</td>
<td>Easy</td>
</tr>
<tr>
<td>Roll</td>
<td>Medium</td>
<td>Easy</td>
</tr>
<tr>
<td>Abbé</td>
<td>Medium</td>
<td>Easy</td>
</tr>
</tbody>
</table>

Table 3: General Classification of severity & measurability of thermal errors, adapted from [3]

<table>
<thead>
<tr>
<th>Thermal Errors</th>
<th>Severity</th>
<th>Measurability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spindle axial growth</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>Spindle radial drift</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>Spindle thermal deflection</td>
<td>Medium</td>
<td>High</td>
</tr>
<tr>
<td>Distortion of spindle region</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>Expansion of components (machine column, machine base, work piece, bed etc.)</td>
<td>High</td>
<td>High</td>
</tr>
</tbody>
</table>

Table 4: General Classification of severity and measurability of cutting force-induced errors, adapted from [3]

<table>
<thead>
<tr>
<th>Cutting Force- Induced Errors</th>
<th>Severity</th>
<th>Measurability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material instability errors</td>
<td>Medium</td>
<td>Difficult</td>
</tr>
<tr>
<td>Vibration</td>
<td>Medium</td>
<td>Difficult</td>
</tr>
<tr>
<td>Work piece and bed elastic deflection</td>
<td>High</td>
<td>Medium</td>
</tr>
<tr>
<td>Instrumentation errors</td>
<td>Low</td>
<td>Difficult</td>
</tr>
<tr>
<td>Tool wear</td>
<td>Low</td>
<td>Medium</td>
</tr>
<tr>
<td>Spindle elastic deflection</td>
<td>High</td>
<td>Difficult</td>
</tr>
</tbody>
</table>

Error compensation is a widely used technique to deliver accurate machines at lower price. The errors are measured by a very accurate device and then compensated for. Real time error compensation techniques are used for correcting kinematic, thermal and cutting force induced
errors which tends to be expensive. The industry desires active error compensation where errors are monitored during machining process and changes are made online. This method is more cost effective and extends equipment life [4, 5]. Thermal errors are more manageable because they can be controlled by controlling the heat flow into the machine tool environment by coolant volume and controlling ambient air temperature around the machine, design optimization before compensation and real-time thermal compensation [3].

2.5. Machine Tool Geometric Error Compensation

Error compensation with geometric errors has an all or nothing aspect to it. Volumetric error compensation usually occurs for all the slides simultaneously altering the position of the machine tool in space [10]. Several researchers have developed virtual milling systems to optimize material removal rate and reduce effects of cutting force [26, 27]. However these models tend to be limited to determining volumetric error compensation and do not target their improvements in GD&T callout quantification of inaccuracy which tends to be the ultimate yardstick of manufacturing accurate parts. A proper data analysis of the impact of machine tool geometric errors on produced part accuracy should yield information about significant error terms. This research focuses on machine tool geometric error terms that affect flatness error in order to give the end user guidance in terms of accuracy improvement that can be achieved using selective error compensation and identification of areas on the machine bed to avoid in order to achieve certain GD&T callouts.

2.6. Machine Tool Geometric Errors

As noted earlier, the measurability of machine tool geometric errors and the severity of the effect of these errors on machine accuracy tend to be relatively high. Researchers have reported that the
machine tool geometric errors are one of the primary contributors to machining inaccuracy [1, 3-5, 18]. Machine tool geometric errors occur due to the degradation of moving components of machining centers over time, thermal and mechanical deterioration of kinematic links, guide ways, slides etc. These errors tend to get worse over time but inherently have a quasi-static nature of variation because of the slow nature of their change. Several researchers [7, 10, 28, 29] have reported general modeling of geometric errors using homogeneous transformation matrices for multi axis linkage based machine tools. The paradigm for these models tends to be geared towards volumetric error calculation and compensation while the specific focus of this research is towards assessment of the effect of machine tool geometric errors on flatness tolerance (a GD&T parameter).

**2.7. Virtual Inspection**

As seen in figure 6, flatness tolerance defines a zone between two parallel planes within which a surface must lie [36]. The flatness error is equal to the minimum distance between the parallel planes [36]. The tolerance zone for flatness is three dimensional [36] and flatness control is always applied to a planar surface [36]. The algorithm used to inspect the virtually machined surface is described in section 3.6.

![Figure 6: Flatness Tolerance Zone](image)
3. Research Methodology

This chapter describes in detail the methodology used to achieve the objectives of this research. This research addresses challenges in the simulation of the face milling cutting process by accounting for machine tool geometric errors using Homogeneous Transformation Matrices and creating a machining advisor for the machining process. The machined surface is generated for different combinations of machine tool geometric errors for each slide. This simulated surface is inspected for flatness error which is used to determine statistically significant machine tool geometric error terms and interactions for each slide. This research aims to be a framework by which any 3 axis milling machine can be analyzed to determine significant error terms for selective error avoidance and compensation.

3.1 Face Milling Cutter and Insert Geometry

For the purpose of this research a 3 insert face milling cutter with clamp-style geometry inserts is used for virtual face milling [30]. It is a square insert with \( \frac{1}{2} \) inch sides and a clearance angle of 11°. Figure 7 and 8, show the dimensions of the cutter that is assumed in this research. The effective cutter diameter is 150mm and the lead angle after the insert is attached to the cutter is 15°.

![Figure 7: Insert geometry [9]](image-url)
The cutter geometry is assumed to be constant in this research for different simulation runs.

3.2 Process Parameters

In the face milling process the cutting action takes place by rotation of the cutter as it is being fed into the work piece. The important process parameters selected include feed per tooth per revolution, lead angle, depth of cut and rotating speed of the cutting tool. Higher feed per tooth per revolution values are used for roughing cuts while lower values are used for finishing cuts. An appropriate lead angle is necessary in balancing the need for forces produced, maximum depth of cut possible, shock load at entry [17]. 15° and 20° lead angle cutters are good for general milling applications with relatively rigid conditions [6, 18]. In this research a 15° lead angle cutter is chosen as seen in Figure 8. Lower feed per tooth per revolution (fpt) are used for finishing cuts. A value of 0.3-0.8mm/tooth/revolution is fairly typical [6, 9, 18]. In this research, a fpt of 0.3mm/revolution is used which means the tool advances 0.9 mm for every revolution of the cutter.
3.3 Surface Generation

In this section, equations for X and Y co-ordinates of any point on the cutter during cutter motion have been derived. To implement surface generation using these equations and chosen insert geometry, following steps are taken:

i.) Discretize cutting edge and tool nose into smaller points.

ii.) Assuming that the center of the cutter is at (0,0,0) translate the discretized cutting edge by a value equal to the effective diameter (150mm) and rotate it by a value equal to the lead angle (15°). Refer figure 9 for this configuration, the red, green and blue lines represent positioning of the cutting edge on the cutter.

![Figure 9: Cutter start position](image)

iii.) Use the following equations [1, 2, 25] per tooth to generate the tool path during cutting motion. The trochoidal motion of the cutter is described by:

\[
x(\theta) = \left( \frac{S_2+N_t+\theta}{2\pi} \right) + \left( \frac{D+\cos(\theta+\theta_0)}{2} \right) \quad \ldots (1)
\]
\[ y(\theta) = \left( \frac{D \sin (\theta + \theta_0)}{2} \right) \]  

\[ z(\theta) = \text{individual depths from cutting edge} \]  ... (2)

Where

\( S_z \) – Feed per tooth per rotation = 0.3 mm/tooth/rotation

\( N_t \) – Number of teeth per cutter = 3

\( D \) – Effective cutter diameter = 150mm

\( \theta_0 \) – Original location of the insert tooth = 0°, 120°, 240°

Refer Figure 3 for images showing how the cutter and the work piece interface.

Figure 10 and 11 depict the cutting tooth tool path through 5 rotations of the cutter.

Figure 10: Cutter trajectory through five rotations
All avoidable factors that cause inaccuracies in the machined surface such as back cutting are assumed to have no effect on the final finished surface. Figures 12, 13 shows the cutter trajectory with back cutting eliminated.
iv.) Once the feed and depth of cut have been established, the procedure outlined in step iii) is repeated for a predetermined length of cut (in the X direction). The cutter positions, shown in Figure 14, have a feed per tooth per rotation of 1mm and the target length of cut is 30mm, therefore, the cutter rotates 10 times to achieve the desired cut length.

Figure 14: Machined surface, depth of cut 5mm, length of cut 30mm
To generate the topography of the final machined surface the procedure outlined in steps v-vii below is followed.

v.) The tool locations from Figure 14 are sliced into a grid in ascending order of the Y direction as seen in Figure 15. All points in a particular slice are assumed to have the same Y co-ordinate. The smaller the slice width, the more accurate is the result. These slices are numbered as GRIDY(y_coord) where y_coord = 1,2,3,...,y_max. A slice thickness of 0.1 mm in the Y direction is assumed in this research.

Each GRIDY_ycoord from step v.) is sliced in the X direction to create a 3-D grid small box section called GRIDX_xcoord_ycoord where xcoord = 1,2,3..., maximum X for each corresponding Y slice and ycoord = 1,2,3...,maximum Yslice. Again, the smaller the width of the slice in X direction, the more accurate is the result. Figure 17 shows the GRIDY_1 and GRIDX_1_1 in bold with a zoomed in view of the same
shown in Figure 16. A slice thickness of 0.001 mm in the X direction is assumed in this research.

Figure 16: GRIDY and GRIDX 3-D Grid

Figure 17: Zoomed view of bottom left corner from Figure 16

vi.) All the points enclosed in the quadrilateral formed by the lines \(X_1, X_2, Y_1\) and \(Y_2\) (seen in figure 17) are assumed to have the same X and Y coordinates. From this
isolated section, the point with the lowest Z coordinate is saved as an imprint of the surface. Figure 18 shows an isometric view of the final generated surface and an isometric of the original surface is seen in Figure 19.

Figure 18: Generated surface using steps i.) through vii.)

Figure 19: Isometric view of original tool path from Figure 10
Figures 20 and 21 show an orthogonal view of two sections of the same surface before and after the algorithm is applied.

Figure 20: Orthographic view of a section of the uncut surface

Figure 21: Orthographic view of the cut surface with final surface topography from Figure 17

Figure 22 is an image of a sample cut surface generated by using the method outlined in steps i.) through vii.) above.

Figure 22: Sample cut surface
3.4 Machine Tool Geometric Errors

A body has 6 degrees of freedom in space as seen in Figure 23, translation along the three axes and roll, pitch and yaw respectively about X, Y and Z. If this body were to move linearly along only X axis, the three angular errors become roll, pitch and yaw along X, Y and Z axes respectively. Unintended motion along X axis is the positional error; movement along Y and Z axes is horizontal and vertical straightness errors [29]. Thus, there are six geometric errors for each body moving along an axis: one for each degree of freedom. If a machine has three axes, adding the squareness error between the axes brings the total number of errors to twenty one. [10, 27]

![Figure 23: Error motions along any axis](image)

To determine the effects of the machine tool geometric errors of each of the slides of a 3 axis machining center and the spindle, they need to be mapped with respect to a pre-determined reference co-ordinate system. A 4x4 matrix called the homogeneous transformation matrix (HTM) as seen in equation 5 is used for transforming a rigid body into a given co-ordinate system [29]. The first three columns of this HTM represent the orientation of the rigid body’s X, Y, Z axes with respect to the reference co-ordinate system X, Y, Z. The fourth column
represents the position of the origin of the rigid body co-ordinate system with respect to reference co-ordinate system [10, 29].

\[
T^R_N = \begin{bmatrix}
O_{ix} & O_{iy} & O_{ij} & P_x \\
O_{jx} & O_{jy} & O_{jz} & P_y \\
O_{kx} & O_{ky} & O_{kz} & P_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\] ... (4)

The equivalent points with respect to reference frame R represented in the original frame N [10, 29] is

\[
\begin{bmatrix}
X_R \\
Y_R \\
Z_R \\
1
\end{bmatrix} = T^R_N \begin{bmatrix}
X_N \\
Y_N \\
Z_N \\
1
\end{bmatrix}
\] ... (5)

So if \(X_2Y_2Z_2\) co-ordinate system is translated by distances \(x, y, z\) along the \(X, Y\) and \(Z\) axes respectively, the HTMs that will transforms the co-ordinates of a point in the \(X_2Y_2Z_2\) frame into the reference \(X_1Y_1Z_1\) frame is given by the following [10, 29]:

\[
T_{X_2Y_2Z_2}^{X_1Y_1Z_1} = \begin{bmatrix}
1 & 0 & 0 & x \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

If \(X_2Y_2Z_2\) co-ordinate system is rotated by an amount \(\theta_x, \theta_y, \theta_z\) about the \(X, Y\) and \(Z\) axes, the HTMs that transforms the co-ordinates of a point in the \(X_2Y_2Z_2\) frame into the reference \(X_1Y_1Z_1\) frame is given by the following three equations respectively [10, 29]:

\[
T_{X_2Y_2Z_2}^{X_1Y_1Z_1} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \theta_x & -\sin \theta_x & 0 \\
0 & \sin \theta_x & \cos \theta_x & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] ... (7)
Figure 24 explains the homogeneous transformation matrix approach of error transformation. An ideal linear motion carriage is seen and the assumed offset values are $a$, $b$, $c$ in the X, Y, Z directions from the reference frame. In the absence of errors, to transform this carriage into the reference coordinate frame, only pure translation is needed which means the homogeneous transformation matrix $[10, 29]$ is:

$$
T_{X_2Y_2Z_2}^{X_1Y_1Z_1} = \begin{bmatrix}
\cos \theta_y & 0 & \sin \theta_y & 0 \\
0 & 1 & 0 & 0 \\
-\sin \theta_y & 0 & \cos \theta_y & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$ … (8)

$$
T_{X_2Y_2Z_2}^{X_1Y_1Z_1} = \begin{bmatrix}
\cos \theta_z & -\sin \theta_z & 0 & 0 \\
\sin \theta_z & \cos \theta_z & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$ … (9)

As seen in Figure 23, every rigid body has rotational and translational errors associated with their motion. These errors are derived from the 6 degrees of freedom any free body in space has which
occur about and along the reference co-ordinate system axes. These errors are usually a function of where the rigid body is with respect to the reference frame. To obtain the homogeneous transformation matrix to transform all points from the rigid frame to the reference frame, the equations for error terms for X, Y, Z, θx, θy, θz are multiplied serially and higher order terms are neglected [10, 29] which gives:

\[
T_N^R = \begin{bmatrix}
1 & -\varepsilon_z & \varepsilon_y & \delta_x \\
\varepsilon_z & 1 & -\varepsilon_x & \delta_y \\
-\varepsilon_y & \varepsilon_x & 1 & \delta_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\] ...

(11)

If a rigid frame has offsets a, b, c in the X, Y, Z direction as described earlier, then \(T_N^R\) becomes [10, 29]:

\[
T_N^R = \begin{bmatrix}
1 & -\varepsilon_z & \varepsilon_y & a + \delta_x \\
\varepsilon_z & 1 & -\varepsilon_x & b + \delta_y \\
-\varepsilon_y & \varepsilon_x & 1 & c + \delta_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\] ...

(12)

As seen in Figure 24, if the rigid carriage has motion in the X direction by an amount of x, the HTM becomes [10, 29]:

\[
T_N^R = \begin{bmatrix}
1 & -\varepsilon_z & \varepsilon_y & a + \delta_x + x \\
\varepsilon_z & 1 & -\varepsilon_x & b + \delta_y \\
-\varepsilon_y & \varepsilon_x & 1 & c + \delta_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\] ...

(13)

Okafor et. al. [10] refined equation 13 by adding a component accounting for orthogonality errors between axes for multi axis machine tools. If \(\alpha_{xy}\) is the orthogonality error between the X and Y slide and \(\alpha_{xz}\) is the orthogonality error between the X and the Z slide then [10]:

28
\[ \delta_y = \delta'_y + \alpha_{sy}x \]  
\[ \delta_z = \delta'_z + \alpha_{xz}x \]

Where

\[ \delta'_y = Y \text{ straightness of } X\text{-axis as it moves in the } Y \text{ direction} \]
\[ \delta'_z = Z \text{ straightness of } X\text{-axis as it moves in the } Z \text{ direction} \]

### 3.4.1 Machine Tool Geometric Errors: Spindle

Using the same principles from equations 5 to 15, a similar error matrix can be derived for a spindle as seen in Figure 25. In this case, there are no orthogonality errors between the machine tool and the slide since there is no displacement of the tool unlike relative motion between slides during machining.

Multiplying equations 7-10 gives [29],

\[
T_{spindle}^R = \begin{bmatrix}
\cos \varepsilon_y \cos \theta_z & -\cos \varepsilon_y \sin \theta_z & \sin \varepsilon_y & \delta_x \\
\sin \varepsilon_x \sin \varepsilon_y \cos \theta_z + \cos \varepsilon_x \sin \theta_z & \cos \varepsilon_x \cos \theta_z - \sin \varepsilon_x \sin \varepsilon_y \sin \theta_z & -\sin \varepsilon_x \cos \varepsilon_y & \delta_y \\
-\cos \varepsilon_x \varepsilon_y \cos \theta_z + \sin \varepsilon_x \sin \theta_z & \sin \varepsilon_x \cos \theta_z + \cos \varepsilon_x \varepsilon_y \sin \theta_z & \cos \varepsilon_x \cos \varepsilon_y & \delta_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

...(16)

Ignoring second order terms and using small-angle approximations [29] and setting \( \cos \theta = 1 \) and \( \sin \theta = 0 \), this matrix becomes [29]:

\[
T_{spindle}^R = \begin{bmatrix}
1 & -\varepsilon_z & \varepsilon_y & \delta_x \\
\varepsilon_z & 1 & -\varepsilon_x & \delta_y \\
-\varepsilon_y & \varepsilon_x & 1 & \delta_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

...(17)
Figure 25: Figure showing spindle error terms [29]

For a machining center with $N$ rigid links connected in series, if the relative homogenous matrices between the connecting axes are known, the position of the tip (the $N$th axis) relative to the reference coordinate system is a sequential product of the individual homogeneous transformation matrices [29] given by:

$$T_N^R = \sum_{m=1}^{N} m^{-1}T_m = T_1^R T_2^R T_3^R \ldots$$

... (18)

Where:

$R$ = Reference co-ordinate frame into which the points on the tool insert are being transformed to

$N$ = Co-ordinate frame of the $N$th axis (machine tool tip)

$m$ = Number of rigid links in a machining center (1, 2, 3: machining slides, 4: spindle)

For a three axis vertical machining center as seen in figure 23, this equation becomes [29, 37, 8]:

$$T_N^R = T_1^R E_1 T_2^R E_2 T_3^R E_3 T_{tool}^R E_4$$

... (19)

Where values of $N$ range from 1 to 4,

1,2,3,4: Co-ordinate system for $X$ slide, $Y$ slide, $Z$ slide and spindle respectively
To transform any point on the cutting tool insert (co-ordinate axis 4) into reference coordinate system R (O₀ in figure 26) the sequence followed is as described below:

The table is set into position on the saddle represented by co-ordinate frame O₁. This is followed by the saddle being set into position on the bed shown as co-ordinate frame O₂ which is followed by setting the machine tool head relative to the bed seen here as co-ordinate frame O₃. This is followed by setting the insert on the tool in the spindle represented by co-ordinate frame O₄ as seen in figure 26.

\[
\begin{bmatrix}
X_R \\
Y_R \\
Z_R \\
1
\end{bmatrix} = T_1^R E_1 T_2^1 E_2 T_3^2 E_3 T_{tool}^3 E_4 \begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix}
\]  \hspace{1cm} \text{...(20)}

Figure 26: Three axes machining center considered for this research [8]

Any point on the insert can be expressed in the reference co-ordinate system using the following equation [29, 37]:
Where:

\[ T_1^R E_1 : \] HTM and Error matrix associated with the X slide

\[ T_2^1 E_2 : \] HTM and Error matrix associated with the Y slide

\[ T_3^2 E_3 : \] HTM and Error matrix associated with the Z slide

\[ T_{tool}^3 E_4 : \] HTM and Error matrix associated with the spindle

This method of serial multiplication of homogeneous transformation matrices to transform the tool insert with respect to the reference coordinate system has been reported by several researchers [10, 28, 29]. This particular form reported and used in this research has been adapted from the work of Choi et al [8].

### 3.5 Machine Tool Geometric Error Values

Lee et al [28] used a laser interferometer and an electronic level to measure roll and nineteen other error components on a 3-axis Bridgeport CNC milling machine. These error values can be used to theoretically locate slide positions in conjunction with the slide error HTMs to determine tool position at any instant with respect to reference co-ordinate frame [28]. Suh et al. [7] presented research involving five-axis machine tools to measure, calibrate and compensate for table error using auto collimators, polygon mirrors and LVDTs. There has been very little research reported on spindle geometric error values for 3 axis machine tools. This research assumes that the order of magnitude of errors of a circular rotating table is similar to a rotating spindle. The errors measured by Suh et al. [7] are used for computing spindle location using homogeneous transformation matrices. In addition, this research uses a cubic spline to fit and
interpolate the error values on the slides and the spindle. All the error values are shown in tables in the appendix.

### 3.6 Virtual Inspection of Flatness

The virtually machined surface generated by accounting for geometric errors of the machining slides and spindle has a flatness error value which is evaluated by virtual inspection. Form tolerances (of which flatness error is a type) can be evaluated using either Least Squares or Minimum Zone approach [31]. In least squares method, the square of the distances of all points in a dataset is minimized from a feature of perfect form. In the case of flatness tolerance that feature is a plane [31]. Flatness error is the distance between the points on both sides that are farthest away from this feature of perfect form [31, 36].

In the minimum zone method, two planes of perfect form which enclose all the points in the dataset are created in such a way that the distance between these features is minimal. This minimal value is called minimum zone flatness error [31].

There is an extensive amount of research reported on both methods [32-35]. In this research a simplex search approach to measure minimum zone flatness error proposed by Damodarsamy and Anand [31] is used to calculate flatness error of the virtually machined work piece. This method uses an unconstrained minimization approach to calculate flatness tolerance.

If cos $\alpha$, cos $\beta$, cos $\gamma$ are the direction cosines from the origin of the normal vector to a plane intersecting a point (x, y, z) as seen in Figure 27, the equation of the plane in normal form is [31]

$$p = xcos \alpha + ycos \beta + zcos \gamma$$  \hspace{1cm} ... (21)

where $p =$ perpendicular distance of the plane from the origin [31]. Also, the direction cosines are governed by equation 22 [31]
\[ \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \quad \text{... (22)} \]

If any two of the three angles \((\alpha, \beta, \gamma)\) are known, the third can be easily determined using equation 22.

\[ \cos \gamma = \sqrt{1 - \cos^2 \alpha - \cos^2 \beta} \quad \text{... (23)} \]

![Diagram showing direction cosines of a plane with point P(x, y, z).](image)

**Figure 27**: Direction cosines of a plane with point P(x, y, z) [31].

The MZ flatness computation proposed by Damodarsamy and Anand [31] is as follows

\[ p_i(\alpha, \beta) = x_i \cos \alpha + y_i \cos \beta + z_i \sqrt{1 - \cos^2 \alpha - \cos^2 \beta} \quad \text{... (24)} \]

where \(i = 1, 2, \ldots n\) represent all the points in the dataset.

\[ p_{max} = \max \{p_i(t)\} \quad \text{... (25)} \]

\[ p_{min} = \min \{p_i(t)\} \quad \text{... (26)} \]

\[ p_{diff} = p_{max} - p_{min} \quad \text{... (27)} \]

\(p_{max}\) and \(p_{min}\) are computed with parameters [31]

\[ t = [\alpha, \beta] \quad 0^\circ \leq \alpha \leq \beta, \cos^{-1}(-\sin \alpha) \leq \beta \leq \cos^{-1}(\sin \alpha) \quad \text{... (28)} \]
and the parallel planes formed for \( t = [\alpha_0, \beta_0] \) which encloses all points in the dataset determine minimum \( pdiff \) which gives minimum zone flatness as

\[
\text{minimum zone flatness} = \min pdiff (t)
\] \hfill (29)

The parameters \( t = [\alpha_0, \beta_0] \) are determined by using fminsearch in MATLAB which uses the modified simplex search procedure developed by Nedler and Mead and outlined by Damodarsamy and Anand in their research.

### 3.7 Machined Surface Generation Including Errors

The algorithm for surface generation which includes error terms for each of the X, Y, Z slides and the spindle is presented below:

i. Define cutter geometry: cutter diameter, lead angle, number of inserts.

ii. Define slide positions for cutting: Y, Z slide location, X slide start and finish location.

iii. Define cutting parameters: depth of cut, feed per tooth per revolution.

iv. Generate cutter insert motion for single pass machining with constant error values for width of cut machine slide (Y), depth of cut machine slide (Z), varying the tool feed machine slide (X) in very small increments and applying homogeneous transformation matrices to account for machine tool geometric errors using equation 21 [29].

\[
\begin{bmatrix}
X_R \\
Y_R \\
Z_R \\
1
\end{bmatrix} = T_1^R E_1 T_2^E E_2 T_3^E E_3 T_{tool}^E E_4 
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix} \quad \ldots (30)
\]

v. Sort generated points in ascending order of Y co-ordinate to create slices of data in increments of a small value (assumed in this research as 0.1 mm).
vi. Sort each slice from step v. in ascending order of X co-ordinate and partition this sorted data in buckets of a very small value (assumed in this research as 0.001 mm). The points in these small buckets are assumed to have the same X and Y co-ordinate.

vii. Find the point with the lowest Z value from the double sorted data from step vi.

viii. Repeat steps v-vii until the entire set of generated insert locations is traversed.

ix. Determine the minimum zone flatness error value of entire dataset by using the approach outlined by Damodarsamy and Anand [31].
4. Face Milling Simulation and Design of Experiments

The focus of the Design of Experiments (DOE) approach is to identify and isolate statistically and practically significant factors that contribute to flatness errors.

During machining, the Y and Z machine slides which represent width and depth of cut are set into position only once and the X machine slide which represents machine tool feed is constantly moved for each pass. For selective error compensation and error avoidance, significant machine tool geometric error terms for every slide and the spindle that affect flatness error value need to be isolated. This research assumes no interaction between machine tool geometric error terms on each slide during machining. This assumption helps to determine practically significant geometric error terms from each slide and the spindle independently. The practically significant error terms isolated from each slide and spindle are then used for a 2 level ANOVA to determine a regression equation which predicts flatness error value as a function of slide and spindle location.

4.1 X Slide DOE

Due to continuous nature of motion of the X slide, it is hypothesized that the X slide has a larger effect on flatness error than the other slides. For this, a 2 level ANOVA is run for all errors on the X slide. For these simulation runs the machine tool geometric errors on the Y, Z and the spindle are set to 0 since only the effect of machine tool geometric errors on flatness error due to X slide need to be determined. This statistical analysis will eliminate the non-significant machine tool geometric error terms from the X slide.

Figure 28 shows a plot of distance along the slide versus straightness and positioning machine tool geometric errors for the X slide. Figure 29 shows a plot of distance versus angular errors for
the X slide. The data for machine tool geometric errors plotted in figures 28 and 29 are obtained from [28]. The two levels chosen for each of these machine tool geometric errors for machined surface generation represent the highest and lowest values that occur along the machine slide length. Table 5 shows the two levels (high and low) for each error that is chosen for surface generation for ANOVA.

Figure 28: Slide location plotted against positioning and straightness error values for X slide (data from obtained [28])

Figure 29: Slide location plotted against angular error values for X slide (data obtained from [28])
Table 5: 2 level ANOVA factor locations for X slide

<table>
<thead>
<tr>
<th></th>
<th>Positioning for X (xpos)</th>
<th>Straightness in Y (xdely1)</th>
<th>Straightness in Z (xdelz1)</th>
<th>Pitch error for X (xepsx1)</th>
<th>Yaw error for X (xepsz1)</th>
<th>Roll error for X (xepsx1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X Slide Low</td>
<td>0</td>
<td>220</td>
<td>275</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>X Slide High</td>
<td>535</td>
<td>535</td>
<td>0</td>
<td>535</td>
<td>495</td>
<td>535</td>
</tr>
</tbody>
</table>

4.1.1 Results of X slide ANOVA

The ANOVA table as seen in Figure 30 indicates that significant factors (low P value) for the X slide affecting the dependent variable (flatness error) are: Straightness in Z direction (xdelz1), Pitch (xepsy1), Roll (xepsz1).

The range of flatness error values measured is 13.42 micron to 13.54 micron which means that there is not much variation in flatness error value due to X slide for this particular error map, tool geometry, feed and depth of cut.

Figure 30: ANOVA results for only X slide
Not all statistically significant factors have the same amount of practical significance on the dependent variable. The main effects plot (Figure 31) shows that the effect of pitch (xepsy1) on flatness error is a much higher than other statistically significant factors.

As seen in Figure 32, ignoring the higher order of interactions, about 97% of the model can be explained by the three statistically significant factor: straightness in z direction, pitch and roll.

![Main Effects Plot](image)

**Figure 31: Main Effects plot for X slide factors**

**Analysis of Variance for f1, using Adjusted SS for Tests**

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>xdel1</td>
<td>1</td>
<td>0.003636</td>
<td>0.003636</td>
<td>0.003636</td>
<td>61361.26</td>
<td>0.000</td>
</tr>
<tr>
<td>xepsy1</td>
<td>1</td>
<td>0.133141</td>
<td>0.133141</td>
<td>0.133141</td>
<td>2246994.02</td>
<td>0.000</td>
</tr>
<tr>
<td>xpsx1</td>
<td>1</td>
<td>0.004236</td>
<td>0.004236</td>
<td>0.004236</td>
<td>71498.38</td>
<td>0.000</td>
</tr>
<tr>
<td>Error</td>
<td>60</td>
<td>0.000004</td>
<td>0.000004</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>63</td>
<td>0.141017</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ S = 0.000243419 \quad R-Sq = 100.00\% \quad R-Sq(adj) = 100.00\% \]

**Figure 32: Results table from X slide DOE**

Practical significance (epsilon-squared) is a calculation of the percentage of total variance of the dependent variable that is explained by each factor and can be calculated by this formula [38]:

\[
\epsilon^2 = \frac{SS \text{ for each effect}}{Total \ SS}
\]
Table 6 shows the practical significance for each of the statistically significant factors:

Table 6: Practical significance for X slide machine tool geometric errors

<table>
<thead>
<tr>
<th>Source</th>
<th>Seq S</th>
<th>Epsilon %</th>
</tr>
</thead>
<tbody>
<tr>
<td>xdelz1</td>
<td>0.003636</td>
<td>2.5478</td>
</tr>
<tr>
<td>xepsy1</td>
<td>0.133141</td>
<td>94.415</td>
</tr>
<tr>
<td>xepsx1</td>
<td>0.004236</td>
<td>3.004</td>
</tr>
<tr>
<td>Error</td>
<td>0.004240</td>
<td>0.003</td>
</tr>
<tr>
<td>Total</td>
<td>0.141017</td>
<td>100.000</td>
</tr>
</tbody>
</table>

The results from Table 6 show that about 94% of the total variance in flatness error can be explained by the error term defining pitch of the X slide (xepsy1). For this particular combination of machining parameters this machine tool geometric error term is statistically and practically the most significant for flatness errors.

### 4.2 Y Slide ANOVA

Following the same approach as the X slide, a second ANOVA for the Y slide is run. The Y slide sets the width of cut during machining. For this 2 level ANOVA, high and low values are set for all 6 errors of the Y slide while the errors on the X, Z slide and the spindle are set to 0 since only the effect of errors on flatness error due to Y slide needs to be determined. This statistical analysis will eliminate the non-significant error terms from the Y slide.

Figure 33 shows a plot of distance along the slide versus straightness and positioning errors for the Y slide. Figure 34 shows a plot of distance versus angular errors for the Y slide. Data for machine tool geometric errors plotted in figures 33 and 34 are obtained from [28]. Table 7 shows the two levels (high and low) for each error that is chosen for surface generation for ANOVA.
Table 7: 2 level ANOVA factor locations for Y slide

<table>
<thead>
<tr>
<th></th>
<th>Positioning for Y (dely2)</th>
<th>Straightness in X (delx2)</th>
<th>Straightness in Z (delz2)</th>
<th>Pitch error for Y (epsx2)</th>
<th>Yaw error for Y (epsz2)</th>
<th>Roll error for Y (epsy2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y Slide Low</td>
<td>112</td>
<td>196</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Y Slide High</td>
<td>252</td>
<td>224</td>
<td>84</td>
<td>270</td>
<td>180</td>
<td>270</td>
</tr>
</tbody>
</table>
4.2.1 Results of Y slide ANOVA

The ANOVA table as seen in Figure 35 shows significant factors (low P value) for the Y slide affecting the dependent variable (flatness error) are: Straightness in X direction (delx2), Positioning in Y direction (dely2), Pitch, roll and yaw errors for Y axis.

The range of flatness error values measured is 13.43 micron to 13.45 micron is even lower than the range of values observed with X slide. This indicates that there is not much variation in flatness error value due to Y slide for this particular error map, tool geometry, feed and depth of cut.

![Analysis of Variance for f1, using Adjusted SS for Tests](image)

Figure 35: ANOVA results for only Y slide

The main effects plot (Figure 36) shows that the effect of roll (epsy2) on flatness error is much higher than other statistically significant factors. The R-Sq value of 100% shows that the DOE is adequate to explain variance in the flatness error (Figure 37). Practical significance table 8 shows that about 97% of the variance in flatness error is explained by the roll error of Y slide (epsy2).

For this particular combination of machining parameters this machine tool geometric error term (roll error) is statistically and practically most significant.
Following the same approach as seen in sections 4.1 and 4.2, a third ANOVA for the Z slide is run. The Z slide sets the depth cut during machining and as such should have an effect on flatness error. Since this slide is in a constant position for single pass machining, the impact of
error terms and their interactions can be assessed by using ANOVA, DOE and p value based model reduction.

Table 8: Practical significance for Y slide errors

<table>
<thead>
<tr>
<th>Source</th>
<th>Seq S</th>
<th>Epsilon² %</th>
</tr>
</thead>
<tbody>
<tr>
<td>delx2</td>
<td>0.208</td>
<td>0.210</td>
</tr>
<tr>
<td>dely2</td>
<td>0.184</td>
<td>0.186</td>
</tr>
<tr>
<td>delz2</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>epsx2</td>
<td>1.588</td>
<td>1.603</td>
</tr>
<tr>
<td>epsy2</td>
<td>97.094</td>
<td>98.001</td>
</tr>
<tr>
<td>epsz2</td>
<td>0.000</td>
<td>0.0</td>
</tr>
<tr>
<td>Error</td>
<td>0.000</td>
<td>0.0</td>
</tr>
<tr>
<td>Total</td>
<td>99.075</td>
<td>100.00</td>
</tr>
</tbody>
</table>

For this 2 level ANOVA, high and low values are set for all 5 errors of the Z slide while the errors on the X, Y and the spindle are set to 0 since only the effect of machine tool geometric errors on flatness error due to Z slide needs to be determined. This statistical analysis will eliminate the non-significant error terms from the Z slide.

Figure 38 shows a plot of distance along the slide versus straightness and positioning errors for the Z slide. Figure 39 shows a plot of distance versus angular errors for the Z slide. Data for the machine tool geometric errors plotted in figures 38 and 39 are obtained from [28]. Table 9 shows the two levels (high and low) for each error that is chosen for surface generation for ANOVA.
4.3.1 Results of Z slide ANOVA

The main effects and interaction plots seen in figure 40 and figure 41 respectively show that all 5 factors have some level of effect on flatness error. Adding 2, 3, 4 way interactions between the factors gives a 99% R-square value with a very proportional spread of variance explained by the factors and their 2 way, 3 way and 4 way interactions as seen in the practical significance table.

Figure 38: Slide location plotted against positioning and straightness error values for Z slide (data obtained from [28])

Figure 39: Slide location plotted against angular error values for Z slide (data obtained from [28])

46
Table 9: 2 level ANOVA factor locations for Z slide

<table>
<thead>
<tr>
<th>Positioning for Z (delz3)</th>
<th>Straightness in X (delx3)</th>
<th>Straightness in Y (dely3)</th>
<th>Pitch error for Z (epsx3)</th>
<th>Yaw error for Z (epsy3)</th>
<th>Roll error for Z (epsz3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z Slide Low</td>
<td>22</td>
<td>0</td>
<td>11</td>
<td>0</td>
<td>22</td>
</tr>
<tr>
<td>Slide High</td>
<td>44</td>
<td>33</td>
<td>33</td>
<td>99</td>
<td>44</td>
</tr>
</tbody>
</table>

The range of flatness error values measured is 13.46 micron to 264 micron is a much wider range compared to effects from the Y and Z slide. The spread of the flatness error values and the significance of the variance explained by factors and their interactions means that the statistical analysis for the errors of the Z slide needs to be further analyzed to assess and isolate the significant error terms.

![Main Effects Plot for flatness](image)

Figure 40: Main effects plot for Z slide factors
Figure 41: Interaction plot for Z slide factors

Table 10: Practical significance of Z slide errors

<table>
<thead>
<tr>
<th>Source</th>
<th>Seq S</th>
<th>Epsilon$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main effects</td>
<td>0.0023998</td>
<td>21.6</td>
</tr>
<tr>
<td>2-Way Interactions</td>
<td>0.0027033</td>
<td>24.33</td>
</tr>
<tr>
<td>3-Way Interactions</td>
<td>0.0034005</td>
<td>30.61</td>
</tr>
<tr>
<td>4-Way Interactions</td>
<td>0.0025781</td>
<td>23.20</td>
</tr>
<tr>
<td>Residual</td>
<td>0.000029</td>
<td>0.26</td>
</tr>
<tr>
<td>Total</td>
<td>0.0111107</td>
<td>100</td>
</tr>
</tbody>
</table>

4.4  Spindle ANOVA

Following the same approach as seen in sections 4.1 - 4.3, a fourth ANOVA for the spindle is performed. Coupled with the Z slide, the spindle rotates the tool into the work piece and errors in the spindle motion affect the depth cut during machining which should have an effect on flatness.
error. The impact of error terms and their interactions can be assessed by using ANOVA, DOE and p value based model reduction.

For this 2 level ANOVA, high and low values are set for all 6 errors of the spindle and the errors on the X, Y, Z slide are set to 0 since only the effect of errors on flatness error due to spindle error terms needs to be determined. This statistical analysis will eliminate the non-significant error terms from the spindle.

Figure 42 shows a plot of distance along the slide versus straightness and positioning errors for the Z slide. Figure 43 shows a plot of distance versus angular errors for the Z slide. The data for machine tool geometric errors plotted in figures 42 and 43 are obtained from [7]. Table 11 shows the two levels (high and low) for each error that is chosen for surface generation for ANOVA.

Figure 42: Slide location plotted against positioning and straightness error values for Z slide (data obtained from [7])
Figure 43: Slide location plotted against angular error values for the spindle (data obtained from [7])

Table 11: 2 level ANOVA factor locations for the spindle

<table>
<thead>
<tr>
<th>Positioning for Spindle (Lx)</th>
<th>Straightness in Y for Spindle (Ly)</th>
<th>Straightness in Z for Spindle (Lz)</th>
<th>Pitch error for Spindle (Rx)</th>
<th>Yaw error for Spindle (Ry)</th>
<th>Roll error for Spindle (Rz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y Slide Low</td>
<td>180</td>
<td>180</td>
<td>120</td>
<td>180</td>
<td>300</td>
</tr>
<tr>
<td>Y Slide High</td>
<td>330</td>
<td>60</td>
<td>270</td>
<td>0</td>
<td>90</td>
</tr>
</tbody>
</table>

4.4.1 Results of Spindle ANOVA

The main effects and interaction plots seen in figure 44 and figure 45 respectively show that all 5 factors have some level of effect on flatness error. Adding 2, 3, 4 way interactions between the
factors gives a 94% R-square value with a very proportional spread of variance explained by the factors and their 2 way, 3 way and 4 way interactions as seen in the practical significance table. The range of flatness error values measured is 14.3 micron to 56.8 micron is a wider range compared to effects from the X, Y slides but smaller than the Z slide.

The spread of the flatness error values and the significance of the variance explained by factors and their interactions means that the statistical analysis of the errors of the spindle needs to be analyzed further. This will help understand impact of error terms in conjunction with practically significant error terms on the X, Y, Z slides.

Figure 44: Main effects plot for the spindle factors
Figure 45: Interaction plot for the spindle factors

Table 12: Practical significance of spindle errors

<table>
<thead>
<tr>
<th>Source</th>
<th>Seq S</th>
<th>Epsilon$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Main effects</strong></td>
<td>0.001449</td>
<td>14.55</td>
</tr>
<tr>
<td><strong>2-Way Interactions</strong></td>
<td>0.003235</td>
<td>32.5</td>
</tr>
<tr>
<td><strong>3-Way Interactions</strong></td>
<td>0.003213</td>
<td>32.28</td>
</tr>
<tr>
<td><strong>Residuals</strong></td>
<td>0.002099</td>
<td>21.09</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>0.009995</td>
<td>100</td>
</tr>
</tbody>
</table>
4.5 ANOVA with statistically significant terms

Sections 4.1 – 4.4 describe the process to isolate statistically significant error terms for each individual slide and the spindle. When the machining process is underway, only the X slide moves for cutting to take place. To understand the impact of individual errors on each slide and perhaps even assess the influence of interactions among these error terms, a DOE approach with all statistically significant error terms from sections 4.1-.4.4 is created. These error terms are:

X Slide- xepsy1 from the X slide (Pitch error for the X slide)

Y Slide- yepsy2 from the Y slide (Roll error for the Y slide)

Z Slide- Positioning, Straightness in X, Straightness in Z, Pitch error, Yaw error

Spindle- Straightness in X, Straightness in Y, Rotational errors in X, Y, Z directions

This DOE uses a 2 level ANOVA without replicates for the 12 error terms and needs $2^{12} = 4096$ runs. The high-low values for each of these errors are kept the same as in the earlier sections.

4.5.1 Results of overall DOE

The main effects plot, seen in figure 46, shows that one particular factor, rotational error in the Y direction of the spindle, has a much larger effect on flatness error compared to other error terms.

The interaction plot, as seen in figure 47, also confirms that all factors interact with rotational error of the spindle in the y direction to have a much larger effect on flatness error compared to other error term interactions.
Figure 46: Main effects plot for statistically significant factors

Figure 48 shows the ANOVA table for this study reduced to show the statistically significant error terms only. Table 13 shows the calculations for practical significance which indicate that the main effects contribute to about 99.93% of variance in the flatness error values.

Figure 47: Interaction plot for statistically significant factors
The low p values means that the interactions are statistically significant, however the low $\epsilon^2$ values from table indicate that they are not practically significant. Figure 49 shows the regression fit for the model.

Table 13: Practical significance

<table>
<thead>
<tr>
<th>Source</th>
<th>Seq S</th>
<th>% Epsilon^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main Effects</td>
<td>78461</td>
<td>99.9303324</td>
</tr>
<tr>
<td>xepsy1</td>
<td>4.1</td>
<td>0.00522</td>
</tr>
<tr>
<td>yepsy2</td>
<td>0.3</td>
<td>0.000382</td>
</tr>
<tr>
<td>zepsy3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>theta_pos</td>
<td>44</td>
<td>0.05604</td>
</tr>
<tr>
<td>thetapos_epx</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>thetapos_epy</td>
<td>78412.6</td>
<td>99.86869</td>
</tr>
</tbody>
</table>

2-Way Interactions

Residual Error 1 0.001274
Lack of Fit 0.9 0.001146
Total 78515.7 100

Analysis of Variance for flatness (coded units)

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main Effects</td>
<td>6</td>
<td>78461.0</td>
<td>78461.0</td>
<td>13076.8</td>
<td>54313.04</td>
<td>0.000</td>
</tr>
<tr>
<td>xepsy1</td>
<td>1</td>
<td>4.1</td>
<td>4.1</td>
<td>4.1</td>
<td>16875.01</td>
<td>0.000</td>
</tr>
<tr>
<td>yepsy2</td>
<td>1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>1597.62</td>
<td>0.000</td>
</tr>
<tr>
<td>xepsy3</td>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>109.33</td>
<td>0.000</td>
</tr>
<tr>
<td>theta_pos</td>
<td>1</td>
<td>44.0</td>
<td>44.0</td>
<td>44.0</td>
<td>152574.04</td>
<td>0.000</td>
</tr>
<tr>
<td>thetapos_epx</td>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>70.35</td>
<td>0.000</td>
</tr>
<tr>
<td>thetapos_epy</td>
<td>1</td>
<td>78412.6</td>
<td>78412.6</td>
<td>78412.6</td>
<td>3.256E+08</td>
<td>0.000</td>
</tr>
<tr>
<td>2-Way Interactions</td>
<td>12</td>
<td>53.7</td>
<td>53.7</td>
<td>4.5</td>
<td>18571.57</td>
<td>0.000</td>
</tr>
<tr>
<td>xepsy1*thetapos</td>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>148.53</td>
<td>0.000</td>
</tr>
<tr>
<td>xepsy1*thetapos_epx</td>
<td>1</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>1588.82</td>
<td>0.000</td>
</tr>
<tr>
<td>xepsy1*thetapos_epy</td>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>22.69</td>
<td>0.000</td>
</tr>
<tr>
<td>xepsy2*thetapos</td>
<td>1</td>
<td>6.3</td>
<td>6.3</td>
<td>6.3</td>
<td>26073.17</td>
<td>0.000</td>
</tr>
<tr>
<td>xepsy2*thetapos_epx</td>
<td>1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>353.99</td>
<td>0.000</td>
</tr>
<tr>
<td>xepsy2*thetapos_epy</td>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>14.53</td>
<td>0.000</td>
</tr>
<tr>
<td>yepsy2*thetapos</td>
<td>1</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>2451.03</td>
<td>0.000</td>
</tr>
<tr>
<td>yepsy2*thetapos_epx</td>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>8.43</td>
<td>0.004</td>
</tr>
<tr>
<td>yepsy2*thetapos_epy</td>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>170.49</td>
<td>0.000</td>
</tr>
<tr>
<td>thetapos*thetapos_epx</td>
<td>1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>1443.70</td>
<td>0.000</td>
</tr>
<tr>
<td>thetapos*thetapos_epy</td>
<td>1</td>
<td>44.2</td>
<td>44.2</td>
<td>44.2</td>
<td>283522.24</td>
<td>0.000</td>
</tr>
<tr>
<td>thetapos*thetapos</td>
<td>1</td>
<td>1.6</td>
<td>1.6</td>
<td>1.6</td>
<td>6799.45</td>
<td>0.000</td>
</tr>
<tr>
<td>Residual Error</td>
<td>6077</td>
<td>1.0</td>
<td>1.0</td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lack of Fit</td>
<td>45</td>
<td>0.9</td>
<td>0.9</td>
<td>0.0</td>
<td>2654.26</td>
<td>0.000</td>
</tr>
<tr>
<td>Pure Error</td>
<td>4032</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>4095</td>
<td>78515.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 48: ANOVA table for DOE
Figure 49: Regression fit

For this particular combination of error values, this study shows that in the presence of other error terms and interactions, the rotational error of the spindle in the Y direction is statistically and practically significant error term. It also contributes most to the explanation of variance of the dependent variable (flatness) in the presence of other error terms.

This observation is reflected in the regression equation for this model:

\[
\text{Flatness} = 18.7198 - 0.0315\times \text{epsy1} - 0.0079\times \text{epsy2} + 0.0032\times \text{zepsy3} - 0.1037\times \theta_{\text{pos}} + 0.002\times \theta_{\text{pos}}\cdot \text{epx} + 4.3754\times \theta_{\text{pos}}\cdot \text{epy} + 0.0029\times \text{epsy1}\cdot \text{epsy2} + 0.0104\times \text{epsy1}\cdot \theta_{\text{pos}} - 0.0012\times \text{epsy1}\cdot \theta_{\text{pos}}\cdot \text{epx} + 0.0391\times \text{epsy1}\cdot \theta_{\text{pos}}\cdot \text{epy} + 0.0046\times \text{epsy2}\cdot \theta_{\text{pos}}\cdot \text{epx} - 0.009\times \text{epsy2}\cdot \theta_{\text{pos}}\cdot \text{epy} + 0.0120\times \text{epsy2}\cdot \theta_{\text{pos}}\cdot \text{epy} - 0.007\times \text{zepsy3}\cdot \theta_{\text{pos}} - 0.0032\times \text{zepsy3}\cdot \theta_{\text{pos}}\cdot \text{epy} - 0.0092\times \theta_{\text{pos}}\cdot \text{epx}\cdot \text{epy} - 0.01039\times \theta_{\text{pos}}\cdot \text{epx}\cdot \text{epy} - 0.0092\times \theta_{\text{pos}}\cdot \theta_{\text{pos}}\cdot \text{epx}\cdot \text{epy} - 0.01039\times \theta_{\text{pos}}\cdot \theta_{\text{pos}}\cdot \text{epy}\cdot \text{epx}\cdot \text{epy} - 0.02\times \theta_{\text{pos}}\cdot \theta_{\text{pos}}\cdot \text{epy}\cdot \text{epx}\cdot \text{epy} \quad \ldots(22)
\]
For verification of the regression equation and to evaluate goodness of fit of the predicted model, five samples were virtually machined with varying slide error values for verification. These machine tool geometric error values are chosen to encompass the entire range of possible slide motion for each slide and the spindle. These five samples were virtually machined using the method described earlier and the flatness errors were calculated. The regression equation 22 was then used to predict the flatness error values for these five sample. The results from table 14 show a good agreement between predicted and measured flatness error for the assumed slide error values with a total difference between measured and predicted flatness error values between 1 and 5.5%

Table 14: Regression Verification

<table>
<thead>
<tr>
<th>No.</th>
<th>Type</th>
<th>xpesy1</th>
<th>ypesy2</th>
<th>zpos</th>
<th>zdelx3</th>
<th>zdelx3</th>
<th>zepx3</th>
<th>zepx3</th>
<th>theta_pos</th>
<th>thetaapos _dy</th>
<th>thetaapos _epx</th>
<th>thetaapos _epy</th>
<th>thetaapos _epz</th>
<th>MZ Flatness (micron)</th>
<th>Regression equation flatness (micron)</th>
<th>% variation from measured value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Coded</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<td>0</td>
<td>1</td>
<td>1</td>
<td>23.0921</td>
<td>23.3229</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Uncoded</td>
<td>0</td>
<td>0</td>
<td>22</td>
<td>0</td>
<td>11</td>
<td>0</td>
<td>44</td>
<td>180</td>
<td>180</td>
<td>0</td>
<td>180</td>
<td>300</td>
<td>23.0921</td>
<td>23.3229</td>
<td>0.98</td>
</tr>
<tr>
<td>2</td>
<td>Coded</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
<td>0.5</td>
<td>0.55</td>
<td>0.4</td>
<td>0.7</td>
<td>20.3053</td>
<td>20.697</td>
<td>1.89</td>
</tr>
<tr>
<td></td>
<td>Uncoded</td>
<td>53.5</td>
<td>54</td>
<td>28.6</td>
<td>13.2</td>
<td>22</td>
<td>59.4</td>
<td>37.4</td>
<td>300</td>
<td>120</td>
<td>202.5</td>
<td>72</td>
<td>237</td>
<td>20.3053</td>
<td>20.697</td>
<td>1.89</td>
</tr>
<tr>
<td>3</td>
<td>Coded</td>
<td>0.25</td>
<td>0.7</td>
<td>0.4</td>
<td>0.5</td>
<td>0.55</td>
<td>0.75</td>
<td>0.6</td>
<td>0.4</td>
<td>0.3</td>
<td>0.45</td>
<td>0.6</td>
<td></td>
<td>20.56</td>
<td>19.522</td>
<td>-5.3</td>
</tr>
<tr>
<td></td>
<td>Uncoded</td>
<td>133.75</td>
<td>189</td>
<td>30.8</td>
<td>16.5</td>
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5. Conclusions and future work

A new method for virtually simulating the face milling process and investigating the effect of machine tool geometric error values on flatness error has been presented in this research. Given information about error terms of individual slides and spindle, a method to selectively isolate statistically significant error terms and their interactions has been demonstrated. This method will aid in avoiding and compensating for specific error terms during machining instead of the blanket volumetric compensation method that is prevalent.

Future investigation could be focused on validating this model’s findings by measuring machine tool geometric error values on a 3 axis machining center, and running similar analysis using a DOE and ANOVA based approach to verify the findings. Future work could also involve similar analysis for other milling processes such as end milling and multi axes milling machines. Other part errors such as surface roughness and other form errors could also be considered and coupled with multivariate techniques such as MANOVA and canonical correlation. Future work can also focus on optimizing several part geometric errors including dimensional and GD&T errors on a single machining center with known machine tool geometric error values.
References


10. Okafor, A.C. and Y.M. Ertekin, *Derivation of machine tool error models and error compensation procedure for three axes vertical machining center using rigid body...*


Appendix

1. Machine tool geometric error values for three axis machining center [28]

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