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I, Rebecca L Rahschulte, hereby submit this original work as part of the requirements for the degree of Doctor of Philosophy in School Psychology.

It is entitled:
An Examination of the Effectiveness and Efficiency of Detect, Practice, and Repair versus Traditional Cover, Copy, and Compare Procedures: A Component Analysis

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An Examination of the Effectiveness and Efficiency of Detect, Practice, and Repair versus Traditional Cover, Copy, and Compare Procedures: A Component Analysis

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Abstract

This study compared the effects of the Detect, Practice, and Repair (DPR) intervention package versus traditional Cover, Copy, and Compare (CCC) procedures in increasing multiplication math fact accuracy and fluency using an alternating treatments design with a modified control condition. Interventions were administered one-on-one across 4 fourth grade students. Three mutually exclusive multiplication sets were used with one set being assigned to each condition. Effectiveness was assessed through traditional curriculum-based measurement (CBM) procedures and through flashcard card procedures to measure accuracy. In addition, the efficiency of each intervention (i.e., amount of learning per instructional minute) was calculated. Maintenance data were collected to determine if newly learned math facts would be better maintained when taught with the DPR intervention or with the traditional CCC intervention procedures. Social validity data were collected with teachers and students to determine whether one intervention was preferred over another. Although DPR has been examined in five published research studies, it has never been examined through a one-on-one implementation or in a study directly comparing its effectiveness, efficiency, maintenance, and social validity against another intervention. In addition, this study serves as a component analysis since CCC is one component of the DPR package.
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An Examination of the Effectiveness and Efficiency of Detect, Practice, and Repair versus Traditional Cover, Copy, and Compare Procedures: A Component Analysis

The National Mathematics Advisory Panel (NMAP) issued a plea for additional research in the area of mathematics education (NMAP, 2008). The importance of math fact automaticity (i.e., quick and effortless recall of math fact) was stressed by the NMAP due to the foundational nature of these skills, which are needed for higher order operations, such as calculating fractions, geometry and measurement, as well as solving algebraic equations. It is suggested that a lack of automaticity with these skills occur because many math curricula do not adequately allow for frequent and repeated practice opportunities in this area (NMAP, 2008). As a result, supplemental programs and/or interventions that allow for frequent practice opportunities with fast and efficient recall of facts are needed to address this concern for children who have not grasped these important skills.

The intervention research base is replete with effective interventions designed to increase math fact accuracy and fluency. Research-based interventions shown to increase math fact accuracy and fluency include traditional drill and practice (Cooke, Guzaukas, Pressley, & Kerr, 1993), incremental drill (Burns, 2005), taped problems (McCallum, Skinner, & Hutchins, 2004; McCallum, Skinner, Turner, & Saecker, 2006), Cover, Copy, and Compare (CCC; Codding et al., 2007; Grafman & Cates, 2010; Lee & Tingstrom, 1994), and Detect, Practice, and Repair (DPR; Axtell, McCallum, Bell, & Poncy, 2009; Poncy, Fontenelle, & Skinner, 2013; Poncy, Skinner, & Axtell, 2010; Poncy, Skinner, & O’Mara, 2006). However, the majority of these studies have focused primarily on intervention effectiveness without consideration for the amount of instructional time required to bring about the academic gains. The effectiveness of an intervention is determined by the degree to which an intervention program produces academic
growth over baseline (Skinner, 2008). The efficiency of an intervention requires the consideration of academic change over baseline coupled with the instructional time required to bring about that change. Recent research suggests that it is critically important for researchers and practitioners to consider the effectiveness and efficiency of interventions when comparing interventions, as effectiveness data alone could be misleading (Skinner, 2008).

Previous research has conducted comparisons of effectiveness and efficiency using different intervention programs aimed at increasing various academic skills (Bramlett, Cates, Savina, & Lauinger, 2010). This research has included intervention programs designed to improve spelling performance (i.e., traditional drill and practice, interspersal training, and high-p sequencing; Cates et al., 2003), interventions designed to improve sight word recognition (e.g., traditional drill and practice and incremental rehearsal; Volpe, Mule, Briesch, Joseph, & Burns, 2011), interventions designed to improve reading comprehension (e.g., repeated readings; Freeland, Skinner, Jackson, McDaniel, & Smith, 2000), and interventions designed to improve math fact fluency (e.g., written cover, copy, and compare, and verbal cover, copy, and compare; Skinner, Belfiore, Mace, Williams-Wilson, & Johns, 1997). This study seeks to expand upon the current research base that exists on the topic of math fact fluency by further supporting the necessity to examine efficiency in addition to effectiveness when documenting the impact of academic interventions, namely Cover, Copy, and Compare (CCC) and Detect, Practice, and Repair (DPR).

**Cover, Copy, and Compare**

Traditional Cover, Copy, and Compare (CCC) procedures designed to improve accuracy and fluency with math facts involve the following steps: (a) the student views the math fact, (b) the student covers the math fact with their hand or an index card, (c) the student copies the math
fact, (d) the student uncovers the covered math fact, and (e) the student compares the original fact with the written fact (McLaughlin & Skinner, 1996). Variations of interest include the number of times each problem is written, and the incorporation of vocal or subvocal responses independently or in conjunction with written responding as the student works through CCC procedures (e.g., Skinner, Bamberg, Smith, & Powell, 1993).

CCC is a self-managed intervention designed for both students with and without disabilities (Joseph et al., 2012). CCC procedures have been effective in increasing academic achievement in the areas of math (e.g., Grafman & Cates, 2010; Skinner, Turco, Beatty, & Rasavage, 1989), spelling (e.g., Cates et al., 2007; Nies & Belfiore, 2006), science (Smith, Dittmer, & Skinner, 2002), and geography (Skinner & Belfiore, 1992).

As it relates to accuracy and automaticity with math facts, various research studies have found CCC procedures to be effective. Three particular studies of interest focused on the use of CCC procedures designed to improve math fact fluency with elementary students without disabilities, the population of interest in the current study. Grafman and Cates (2010) found evidence that traditional CCC procedures, administered in a classwide format with second grade students (n=47), were effective in increasing both students’ accuracy and fluency with subtraction facts over modified CCC (MCCC) procedures. Pre-test data indicated baseline levels of math fact fluency at 7.75 digits correct per minutes (DCPM) with .69 errors per minute (EPM). Post-test data for the CCC condition found 19.03 DCPM with .59 EPM, and for the MCCC condition 11.28 DCPM with .43 EPM (Grafman & Cates, 2010). Similarly, Lee and Tingstrom (1994) established that CCC procedures implemented in a small group setting with five fifth grade students without disabilities were effective in increasing rate and accuracy with division facts when compared to baseline. The percentage of DCPM during baseline ranged from
71%-97% and following CCC implementation the DCPM percentage ranged from 90%-99% (Lee & Tingstrom, 1994). Codding and colleagues (2007) compared the effects of explicit timing, CCC, and a control condition on the subtraction fact fluency of 98 second and third grade students. The outcome data indicated that the students’ initial level of fluency influenced intervention effectiveness (Codding et al., 2007). The CCC and control conditions were most effective for students whose baseline scores were in the frustrational range whereas explicit timing was more effective for students initially in the instructional range (Codding et al., 2007).

Additional studies have examined the effectiveness of CCC when compared to other math fact accuracy and fluency interventions. One such study by Poncy, Skinner, and Jaspers (2007), examined CCC versus taped problems with a single student diagnosed with moderate mental retardation. This study found that both interventions resulted in increases in the student’s fact accuracy and fluency. However, the taped problems intervention was more effective and required less time to implement in this single case (Poncy et al., 2007).

**Detect, Practice, and Repair**

A single DPR intervention session involves the progression through three intervention phases of detect, practice, and repair. The first phase of the DPR intervention, as outlined by Poncy and Skinner (2006), involves the detection of math facts that are non-automatic. Students complete a math worksheet consisting of 48 multiplication facts. Students are allowed 2 seconds to complete each multiplication problem with the pace per problem being monitored by a metronome set at 30 beats per minute (Poncy & Skinner, 2006). Following the completion of this task, the student circles the first five blank responses or incorrect responses (Poncy & Skinner, 2006).
The next phase involves practice of these non-automatic facts through a Cover, Copy, and Compare (CCC) procedure (Poncy & Skinner, 2006). Students write their first five incomplete or incorrect problems and answers into the “problem and answer” boxes on a CCC worksheet. If needed, the correct answer can be referenced on an answer key displayed in the classroom (Poncy & Skinner, 2006). Following the transfer of the problems and answers to the CCC sheet, students read the first problem and answer (verbal rehearsal using whisper phones or mouth whisper), cover the problem and answer (Cover), write the problem and answer (Copy), and then compare their answer to the original model (Compare). The CCC procedure is repeated five times per problem. CCC has been shown through research to increase math fact fluency across mathematical functions, across different student populations, and across different settings (see Skinner, McLaughlin, & Logan, 1997).

The final phase involves a timed fluency drill followed by feedback provided to the student through self-graphing procedures (Poncy & Skinner, 2006). Using a math probe, derived from the same problem set used in the detect phase, students complete a math sprint involving the completion of as many problems as possible during a 1-minute timed session (Poncy et al., 2013). The students then count the number of total digits correct and self-graph their results. The incorporation of self-graphing procedures into intervention programs has been shown to enhance students’ academic fluency rates (Van Houten, Hill, & Parson, 1975).

There is a compelling research base for Detect, Practice, and Repair as an effective intervention to increase automaticity with math facts across various mathematical functions (Axtell et al., 2009; Poncy et al., 2013; Poncy et al., 2010; Poncy et al., 2006). Poncy and colleagues (2006) utilized DPR procedures in an effort to increase subtraction fact fluency with a group of 14 third-grade students identified as “low achieving” in this skill. Intervention
procedures were implemented for a total of 6 weeks. Using curriculum based measurement (CBM) procedures, this study found weekly increases of 3.2 correct digits (CD) per minute for students participating in the DPR intervention (Poncy et al., 2006). When compared to district norm increases of .5 CD per week, DPR growth rates were found to be statistically significant relative to these district norms (Poncy et al., 2006).

DPR intervention procedures were further examined in an experimental study by Axtell and colleagues (2009) in which students were randomly assigned to experimental or control conditions. The effectiveness of DPR was examined as a classwide intervention (23 students in the experimental condition) for use with middle school students (Axtell et al., 2009). The targeted academic skill was division automaticity. A statistical analysis of the findings documented significantly higher post-test scores ($p=.016$) for the intervention group (adjusted $M=47.53, SD=3.26$) when compared to the control group ($M=33.31, SD=4.39$; Axtell et al., 2009).

Poncy and colleagues (2010) studied the impact of DPR procedures using a multiple-probe-across-problem-sets (tasks) design. Participants included a small group of seven third-grade students in need of math fact fluency remediation in the area of multiplication (Poncy et al., 2010). Results showed large level and trend increases across the three problem sets. Students’ baseline average was 20.9 DCM, intervention average was 33.3 DCM, and maintenance average was 32.9 DCM (Poncy et al., 2010).

Finally, a recent study examined DPR procedures applied to a group (n=11) of third-grade students with a consideration of both effectiveness and efficiency of procedures (Poncy et al., 2013). Operations were differentiated based on the individual students’ skill levels with baseline data indicating an average of 18.4 digits correct per minute (DCPM; Poncy et al., 2013).
Following 11 DPR intervention sessions (approximately 132 minutes), the group average increased to 31.2 DCPM, a 12.8 DCPM increase (Poncy et al., 2013).

Currently, there are no published studies comparing DPR with other math fact accuracy and fluency intervention procedures; however, studies have compared DPR to control conditions through either the use of a multiple baseline design (Poncy et al., 2010; Poncy et al., 2013) or through the integration of a control condition (Axtell et al., 2009). Poncy and colleagues also call for research to determine if DPR enhances learning rate (i.e., intervention efficiency) relative to other interventions.

**CCC and DPR Maintenance**

In addition to the effectiveness and efficiency of intervention procedures, consideration of the maintenance of intervention gains over time must be considered. The gains brought about by intervention implementation and the learning rate is meaningless should the gains not be maintained after the intervention ceases. Previous CCC research studies in the area of math accuracy and fluency have examined maintenance. A study by Lee and Tingstrom (1994) collected “follow-up” data, which confirmed that gains were maintained after CCC implementation had ceased. However, some of the studies that are of the most relevance to current research, neglected to collect maintenance data (e.g., Codding et al., 2007; Grafman & Cates, 2010).

Research on DPR maintenance has revealed only slight declines in math fact fluency following removal of the intervention procedures. Poncy and colleagues (2010) collected maintenance data ranging from 1 to 8 days following completion of the intervention. Math facts were divided into three sets with 4 maintenance points being collected for Set A, 2 points for Set B, and 1 point for Set C. Students maintained most of their increases in computation fluency.
(Poncy et al., 2010). A recent DPR study collected maintenance data in a similar fashion with similar findings (Poncy et al., 2013). Additional DPR studies did not examine or report maintenance (e.g., Axtell et al., 2009; Poncy et al., 2006).

**CCC and DPR Social Validity**

Wolf (1978) suggested that social validity contains three elements: (a) importance of the intervention goals, (b) acceptability of the intervention procedures, and (c) satisfaction with the intervention outcomes. Social validity increases the likelihood that intervention procedures will be delivered as planned (Witt & Elliott, 1985) and that students maintain motivation as it relates to participation in the intervention procedures. Thus, it is vital that interventions possess social validity when assessed by key stakeholders, especially interventionists and student participants. Examination of past research on CCC and DPR has documented variations in the collection of social validity data, including formal data collection, informal data collection, and a lack of consideration of social validity in some studies.

Codding and colleagues (2007) utilized the *Acceptability Assessment – Student Version* (Eckert, 1999) following the last CCC intervention session as a means of measuring student social validity. In this comparison study, students rated the Explicit Timing (ET) intervention higher ($M=3.07$) than the CCC intervention ($M=2.71$) on a scale that ranged from 1 to 4. Grafman and Cates (2010) used less formal procedures asking students to report intervention preference as it relates to traditional CCC versus modified CCC (MCCC) procedures. To examine student preference, a chi-square goodness-of-fit test was conducted, which revealed that students preferred the traditional CCC over the MCCC (Grafman & Cates, 2010). Teachers were asked, “which would you rather use for a) teaching and b) building mathematics skills?”
(Grafman & Cates, 2010, p. 157). Results indicated that teachers preferred the MCCC for both teaching and fluency building (Grafman & Cates, 2010).

As it relates to the collection of social validity data for DPR intervention procedures, of the five published research studies on DPR, none of these studies reported on the social validity of the intervention procedures among key stakeholders.

**Comparing CCC and DPR Intervention Programs**

Although both CCC and DPR have been found to be effective in previous research, it is important to consider which procedure is more effective. Also, the added element of efficiency must be examined and compared across the two intervention procedures. It is hypothesized that the DPR intervention will be more effective than the CCC intervention due to the varied components included within this program (i.e., added detection, repair, and self-graphing components). However, it is predicted that the CCC intervention will be more efficient due to the additional time required to implement all components of DPR and the time lost transitioning between the various DPR intervention components.

In addition, a comparison of the maintenance of academic skills across intervention programs is necessary. Follow-up assessments of accuracy and fluency with math facts will determine whether one intervention program is more successful at maintaining gains over time. Maintenance is vital, as it precedes generalization, which is the ultimate goal of academic and behavioral interventions (Ardion, 2006). When assessing intervention efficiency, maintenance of academic skills should be examined across settings, but are rarely taken into consideration both in research and in practice (Burns & Sterling-Turner, 2010).

Finally, it is important to consider and compare both teacher and student acceptability of the intervention programs to determine if one intervention possesses higher social validity than
the other intervention. As previously described, teacher acceptability increases the probability that intervention fidelity will be high, which in turn will increase the probability of academic or behavioral change (Witt & Elliott, 1985). Limited research has been conducted on the significance of student acceptability; however, it is hypothesized that higher student acceptability will result in higher student engagement in the intervention program and in turn better outcomes.

**Research questions**

This study was designed to answer the following research questions:

1. Is the DPR package or the traditional CCC intervention procedures more effective at increasing multiplication math fact accuracy and fluency?

2. Is the DPR package or the traditional CCC intervention procedures more efficient when considering the learning rate (i.e., amount of learning per instructional minute) for both math fact accuracy and fluency?

3. Are academic improvements (i.e., gains in multiplication math fact accuracy and fluency) maintained better following implementation of the DPR package or the CCC intervention procedures?

4. Are teacher and student social validity ratings (i.e., assessments of the acceptability of intervention procedures, goals, and outcomes) higher for the DPR package or the CCC intervention?

**Method**

**Participants and Setting**

This study was conducted at an elementary school serving students in kindergarten through fifth grades in a small urban school district in the Southern United States. This
elementary school has roughly 483 students based on state report card data from 2012-2013. The racial-ethnic make-up of the school is 43.5% White (non-Hispanic), 29.8% African American, 20.3% Hispanic, and 6.4% Multi-Racial. Students on free lunch represent 92.8% of the school population and another 2.9% are on reduced lunch. Students identified as needing special education services and supports make up 23.4% of the school population. The interventions used within this study were implemented within the school’s afterschool program. The participants in this study included two intervention teachers and four fourth-grade students. Interventions were carried out one-on-one (i.e., one teacher with one student) in a traditional fourth grade classroom setting with the teacher seated directly next to or across from the student participant.

**Intervention Teachers.** Two fourth-grade classroom teachers were recruited and trained to implement the DPR and CCC intervention procedures across all student participants (i.e., two students were assigned to each teacher). Teacher 1 was a certified classroom teacher with three years of teaching experience who self-identified as Caucasian. Teacher 2 was a certified classroom teacher with 25 years of teaching experience who self-identified as Caucasian. The following teacher recruitment procedures were followed: (a) information regarding the research study was shared with all classroom teachers at grade-level team meetings (see recruitment script in Appendix A), and (b) teachers volunteered to participate through verbal notification to the researcher.

**Student Participants.** Student participants included four fourth-grade students (i.e., two male students and two female students) with baseline levels of single-digit multiplication math fact fluency below the mastery criterion of 40 digits correct per minute (Howell, Fox, & Morehead, 1993). Student 1, Student 2, and Student 4 were identified by their parent(s) as Black or African-American and Student 4 was identified as Hispanic/Latino. All students were 9 or 10
years of age when participating in the study, which made them age-appropriate for their assigned grade level. All students spoke English as their first language both at home and at school. All students received free or reduced lunch. Student 1 received speech language services for an identified speech-language impairment, but she did not receive any additional classroom assistance that would be provided through a special education teacher. Student 2, Student 3, and Student 4 did not receive any special education services or supports, and the grade-level math teacher reported that these students did not receive any additional math interventions while participating in this study. Student participants were selected using the following selection procedures: (a) one traditional single-digit multiplication math fact fluency probe was administered to all 66 fourth-grade students and scored by trained school personnel using Curriculum Based Measurement (CBM) procedures (Shapiro, 2010; see administration and scoring procedures in Appendix B), (b) all 66 students demonstrated fluency rates below 20 digits correct per minute and were rescreened by school personnel using one CBM math fact fluency probe to confirm the reliability of the original scores using an alternate form CBM single-digit multiplication math fact fluency probe, and (c) for the 60 students with a confirmed CBM multiplication math fact fluency score below 20 correct digits per minute, a teacher nomination checklist (see Appendix C) was completed by school personnel to identify fourth-grade students who were best matched for participation in this study. The teacher nomination form was used to confirm that CBM data were an accurate representation of students’ classroom performance on math assessments (e.g., tests, work samples). In addition, the form was also used to identify students with disabilities who might already be receiving math supports as outlined on an Individualized Education Program (IEP), which may have made these individuals unsuitable candidates for this program. Finally, the form was designed to identify students who might not
be able to adequately access the intervention for various reasons (e.g., severe visual or motor disabilities, severe attendance concerns, etc.). Fifteen students were considered to be well suited for participation in the study based on their CBM scores combined with appraisals from the teacher nomination checklists. Permission forms went home to students based on priority (i.e., based on the severity of their CBM score). A total of ten permission forms were sent home with six signed consent forms received back from parents. Two of the six students demonstrated increasing baseline data that ended up excluding them from participation in the study.

Research Design

This research study utilized an alternating treatments design with an ongoing control condition on a modified schedule across four student participants to examine DPR effectiveness and efficiency in comparison to traditional CCC procedures. This design allowed for a baseline period followed by the implementation of two interventions alternating with one another to examine the effects on a dependent variable (Kennedy, 2005). In addition, the use of a control condition collected throughout the study (on a modified schedule) allowed for the examination of potential history effects that could have occurred as a result of daily classroom mathematics instruction, thus monitoring the internal validity of the independent variables (Kennedy, 2005; Sindelar, Rosenberg, & Wilson, 1985). The interventions were counterbalanced, which is similar to randomization, but allowed the researcher to sequence the interventions in a planned manner so that the first occurring intervention daily was equally distributed between the two different intervention procedures (Kennedy, 2005). Dependent variables designed to assess intervention effectiveness were administered daily as next day assessments. Dependent variables designed to monitor the control condition were on a modified schedule with assessment occurring every two to three days.
**Materials**

Materials included folders containing the DPR and CCC intervention packets. Additional DPR intervention materials included answer keys containing the multiplication problems and answers for the problem sets, and a metronome. Teachers were supplied with intervention scripts for both the DPR and CCC interventions. Assessment materials included CBM single-digit multiplication math fact fluency probes, flashcards with a known/unknown checklist, intervention fidelity checklists, and social validity rating scales.

**Dependent Variables**

**Intervention Effectiveness – Correct Digits Per Minute.** Students’ single-digit multiplication math fact fluency was assessed using Curriculum Based Measurement (CBM) procedures for administration and scoring (Shapiro, 2010; see Appendix B). CBM probes were developed from three mutually exclusive multiplication problem sets (i.e., Set A, Set B, and Set C; see Appendix D) created by Skinner, Ford, and Yunker (1991). Each set contained 12 different problems with single-digit factors. Multiplication problem sets were created so that each set had the same numeral (2-9) represented as a factor and the total of all correct answers. Each CBM probe included all twelve facts within the set presented four times for a total of 48 problems per probe (six columns and eight rows). Each problem appeared randomly on the probe and the researcher ensured that the same problem was not directly above or below itself or repeated in the problem sequence. Probes created from the three sets were used for baseline, intervention, and maintenance phases. CBM probes designed to assess intervention effectiveness were administered daily as a next day assessments. CBM probes designed to monitor the control condition were on a modified schedule with assessment occurring every two to three days.
**Intervention Effectiveness – Accurate Facts Per Problem Set.** In addition to CBM math fact fluency assessments, a flashcard assessment was also utilized to assess math fact accuracy. Flashcards were developed for each mutually exclusive set of multiplication facts (i.e., Set A, Set B, and Set C; see Appendix D). Administration procedures involved the display of each flashcard by the examiner to the examinee. The examinee had 5 seconds to answer the problem. Cards were placed in one of two piles. If the problem was answered correctly, it was placed in the “known” pile. If the problem was answered incorrectly or the administration of that problem exceeded 5 seconds, the card was placed in the “unknown” pile. Performance feedback was not offered to the students regarding the rightness or wrongness of his/her answers.

Following the assessment, the examiner counted the number of cards in the known pile, divided this number by 12, and multiplied by 100 to record the percentage of known facts within the set. Flashcard sets that were designed to assess intervention effectiveness were administered daily as a next day assessment. The flashcard set designed to monitor the control condition was on a modified schedule with assessment occurring every two to three days.

**Intervention Efficiency – Math Fact Fluency & Accuracy.** Intervention efficiency was calculated using the intervention effectiveness data (i.e., correct digits per minute and percentage of accurate facts per problem set), as well as intervention duration to secure a rate of learning per instructional minute. This was calculated by subtracting the pre-test performance (i.e., median of the three baseline data points) from the post-test performance (i.e., median of the last three intervention data points), and dividing the difference by the total instructional minutes (rounded to the nearest tenth). This formula determined a learning rate for each intervention.

Intervention efficiency was also examined through visual analysis of the graphed data. Both math fact fluency (i.e., correct digits per minute) and math fact accuracy (i.e., percentage of
accurate facts per problem set) were plotted for both the problem set utilized with the DPR intervention and the problem set used with the CCC intervention. The x-axis represented the cumulative instructional minutes for each intervention session utilized throughout the duration of the study. Visual analysis compared the trends in the two data sets (i.e., CCC problem set data and DPR problem set data) for each measure (i.e., correct digits per minute and percentage of accurate facts per problem set. Procedures are similar to those outlined by Skinner, Belfiore, and Watson (2002). In addition, visual analysis was validated through the statistical calculation of the slope of the intervention efficiency data.

**Maintenance – Fluency & Accuracy.** Curriculum Based Measurement (CBM) probes, as designed above, were used to assess maintenance of math fact fluency gains. In addition, accuracy assessments were completed post-intervention as noted above. These maintenance probes were administered at both one week and two weeks post-intervention. Visual analysis of maintenance data included an examination of changes in level and trend of maintenance data points compared to the last three intervention data points for each condition (i.e., CCC, DPR, and control). Visual analysis of trend data was supported through statistical calculations of the slope of the maintenance data for each condition.

**Social validity.** Rating scales were created by the primary researcher to assess the social validity of the CCC and DPR interventions by key stakeholders (i.e., intervention teachers and student participants). Teacher rating scales were designed to measure perceived effectiveness, efficiency, ease of implementation, and intervention acceptability (see Appendix E). These scales consisted of 10 Likert-type response statements each on a 6-point scale (i.e., Strongly Disagree, Disagree, Slightly Disagree, Slightly Agree, Agree, and Strongly Agree). Each teacher completed two rating scales, one for each intervention. Student rating scales were designed to
measure perceived understanding of the intervention, intervention acceptability, and intervention effectiveness (see Appendix E). These scales consisted of four items, each item had three child-friendly response choices (i.e., smiling face, neutral face, frowning face). Each student completed two rating scales, one for each intervention. Social validity ratings for each intervention were compared to the overall effectiveness and efficiency of each intervention program to determine the extent to which teacher and student participants possessed perceptions that aligned with intervention outcomes.

**Inter-Scorer and Inter-Observer Reliability.** To ensure that CBM math fact fluency probes were scored accurately, a random sampling of 30.3% of student worksheets were co-scored following each CBM assessment. Agreement was calculated by dividing the number of correctly scored digits by the number of correctly scored digits plus the number of incorrectly scored digits. The outcome was multiplied by 100% to obtain an inter-scorer reliability percentage. Inter-scorer agreement averaged 99.5% with a range of 96.5% to 100%.

Interobserver agreement data were also collected for intervention session duration data. Intervention duration was recorded by the teacher facilitating the interventions and was co-observed for 36.67% of sessions. Agreement was calculated by examining the teacher’s recording of intervention duration (in seconds) with the observer’s recording of intervention duration (in seconds). The lower duration was dividing by the higher duration and multiplied by 100 to secure an inter-observer reliability estimate. The average agreement was 100% for both Teacher 1 and Teacher2.

**Procedures**

Research phases consisted of baseline, teacher training, intervention implementation, and maintenance. Following baseline, DPR and CCC interventions were implemented within the
afterschool program during each school day (i.e., Monday through Friday) for 15 sessions per student participant. Teachers utilized an intervention script to both teach and facilitate the intervention procedures with each student. Interventions were delivered in a counterbalanced fashion. Intervention durations (i.e., minutes and seconds rounded to the nearest tenth) were documented for both DPR and CCC implementation independently in order for efficiency to be graphed and calculated as previously outlined. Duration data was collected by the teachers through use of stopwatch started at the beginning of the intervention and stopped at the end of the intervention as noted previously. “Next day” CBM and flashcard assessments were conducted during the school day by trained research assistants. Students were pulled individually from daily classroom instruction for 8-12 minutes for CBM and flashcard assessments to be conducted. These “next day” assessments took place on Monday through Friday representing the two intervention problem sets in order to document intervention effectiveness. Therefore, intervention sessions that took place on Monday were assessed on Tuesday, Tuesday intervention sessions were assessed on Wednesday, Wednesday intervention sessions were assessed on Thursday, Thursday intervention sessions were assessed on Friday, and Friday intervention session were assessed on Monday. If a student missed an intervention session, then next day assessments were not conducted. If a student attended the intervention session, but missed the assessment session, then the student was assessed the next day that they returned to school. The fact set used for the control condition was on a modified assessment schedule and was assessed every 2 or 3 days. CBM probes and flashcards created from the three problem sets were administered in random order each day. In addition, during intervention implementation, trained research assistants collected intervention fidelity data 1 to 2 times per week for each teacher to document procedural adherence.
Baseline. During the baseline phase, student participants’ baseline levels of multiplication math fact fluency were assessed using CBM procedures and probes designed using Skinner and colleagues (1991) mutually exclusive multiplication problem sets (i.e., Set A, Set B, and Set C). Alternate forms representing all three problem sets were created as described previously. These probes were administered daily (i.e., Monday through Friday) with each participant until a stable baseline was secured across at least three administrations of each problem set. Following baseline, two problem sets were selected based on near equal performance across the two problem sets and these problem sets were randomly assigned to the CCC intervention or DPR intervention. Near equal performance was important to ensure that valid within-subject conclusions could be made related to the effects of CCC versus DPR interventions. The third problem set was assigned to a “No Treatment” control condition and was assessed throughout the study (on a modified schedule) as a means to ensure internal validity. In addition, flashcard accuracy assessments were conducted to identify known and unknown facts in each set.

Intervention training - teacher. The teachers were trained together on the DPR and CCC intervention procedures using modeling of intervention procedures by the researcher, and opportunities for the teachers to practice DPR and CCC intervention procedures through role-play with performance feedback provided by the researcher. The acquisition of intervention procedures by each teacher was assessed through use of the intervention fidelity checklist utilized during role-play procedures. DPR and CCC implementation began once the teacher achieved 80% accuracy on the intervention fidelity checklist during role-play procedures. The
teachers required two 30-minute training sessions for acquisition to be achieved. Both teachers demonstrated 100% procedural adherence when assessed during role play.

**Detect, Practice, and Repair.** The teacher implemented the DPR intervention daily with student participants one-on-one following the DPR script provided by the researcher (see Appendix F). Through use of a stopwatch, the teacher began timing the duration of the intervention before distributing intervention materials. Then, the teacher distributed the DPR materials to the student including the detect worksheet, the practice worksheet (i.e., CCC worksheet), the repair worksheet, and the student’s graph. The teacher instructed the student on the procedures for the detect worksheet using the wording provided on the script (see Appendix F). The teacher started the metronome set at 30 beats per minute (i.e., one beep every 2 seconds). The student worked through the worksheet completing one problem per beep and skipping any problems that they could not complete within the 2 second time window. If the student lost track of their place on the worksheet, the teacher prompted the student by pointing to the correct problem. Once the student worked through the 48 problems on the detect worksheet, the student would circle the first five problems that were left blank. If there were no blank problems, then the student (using an answer key) identified and circled the first five incorrectly answered items. The five circled problems with correct answers were then transferred to the practice worksheet (i.e., CCC worksheet) under the first column. The teacher provided assistance through prompting and cuing as needed. The student then whispered the first problem and answer to themselves, and then covered the problem and answer with their hand or a provided index card. The student then wrote the problem and answer in the block to the right of the covered problem. Once written, the student picked up their hand or index card to compare the newly written problem with the correctly written problem and answer. If written incorrectly, the student made necessary changes
to the written problem and answer to ensure that it was accurately written. If correctly written, the student repeated the whisper, cover, copy, and compare process five times for the first problem and answer. Once the first row was completed, the student moved onto the second row repeating the same process used in row one. Once the student had completed the entire CCC worksheet, the student moved to the repair worksheet. The repair worksheet had the exact same 48 problems as the detect worksheet. Students were provided one minute to answer as many problems as possible. The teacher said “Begin” and started the stopwatch. Once the one minute had elapsed, the teacher said “Stop” and the student counted the number of problems answered. The student then self-graphed their results on a traditional bar graph depicting each session on the x-axis and the number of problems answered on the y-axis. Students compared their daily performance to previous DPR sessions and reported on any gains made from the previous session. The teacher then stopped the stopwatch and recorded the intervention duration.

**Cover, Copy, and Compare.** The teacher implemented the CCC intervention daily with student participants one-on-one following the CCC script provided by the researcher. Through use of a stopwatch, the teacher began timing the duration of the intervention before distributing intervention materials. Then, the teacher distributed the CCC materials to the student, which included a list of five problems that would be practiced that day and a CCC worksheet. The list of five problems used for the CCC intervention were derived from the daily flashcard assessments (i.e., CCC fact set only) with missed facts being utilized for the CCC intervention. If the student did not have five missed facts when assessed with the flashcard assessment, then missed items on the CBM fluency probe were used (i.e., CCC fact set only). The student was instructed to transfer the problems with correct answers to the first column of the CCC worksheet. The student then whispered the first problem and answer to themselves, and then
covered the problem and answer with their hand or a provided index card. The student then wrote the problem and answer in the block to the right of the covered problem. Once written, the student picked up their hand or index card to compare the newly written problem with the correctly written problem and answer. If written incorrectly, the student made necessary changes to the written problem and answer to ensure that it was accurately written. If correctly written, the student repeated the whisper, cover, copy, and compare process five times for the first problem and answer. Once the first row was completed, the student moved onto the second row repeating the same process used in row one. The teacher then stopped the stopwatch and recorded the intervention duration.

**Maintenance.** To ensure that student participants retained their accuracy and fluency with single-digit multiplication facts following the conclusion of the interventions, follow-up CBM multiplication fact fluency assessments and math fact accuracy assessments were conducted at one week post-intervention and at two weeks post-intervention. CBM and flashcard assessments were conducted representing the three problem sets to document intervention maintenance and maintenance within the control condition. These assessments took place during the school day by trained research assistants. Students were pulled individually from daily classroom instruction for 8-12 minutes for CBM and flashcard assessments to be conducted.

**Reliability and Validity**

**Intervention fidelity.** Intervention fidelity checklists derived from the DPR implementation guidelines outlined by Poncy and Skinner (2006) were used to monitor teacher implementation of the DPR intervention (see checklist in Appendix F). In addition, intervention fidelity checklists were used to monitor teacher implementation of CCC procedures (see
checklist in Appendix G). Intervention fidelity data were collected by members of the research team who were trained to collect these data. Interobserver reliability was calculated as the number of agreements of step occurrence divided by the number of agreements of step occurrence plus the number of disagreements of step occurrence. The outcomes were multiplied by 100% to obtain a reliability percentage. (i.e., observers possessed 100% inter-observer reliability prior to data collection). Adherence data were collected during 40% of the intervention sessions for Teacher 1. Teacher 1’s intervention adherence average was 100% for the CCC intervention and 97.1% for the DPR intervention (range = 87.2% to 100%). Adherence data for Teacher 2 were collected during 33.33% of intervention sessions. Teacher 2 had an intervention adherence average of 100% for the CCC intervention and 100% for the DPR intervention.

Results

The study results are organized by dependent variable, which includes intervention effectiveness data, intervention efficiency data, maintenance data, and social validity data for both the DPR and CCC interventions. Intervention effectiveness was explored through examination of math fact fluency (i.e., digits correct per minute) and math fact accuracy (i.e., accurate facts per problem set) across baseline, intervention, and maintenance conditions. For all areas noted above, visual analysis of data was the primary method of analysis utilized with a focus on trend, level, and variability both within and between conditions. Visual analysis of data is common in single-case designs (Kennedy, 2005). In an effort to examine within-phase and between-phase patterns, trend lines (i.e., best-fit straight lines through the data within a phase) were generated using the Split-Middle Trend Estimation procedures outlined in Kennedy (2005), whereby given seven or more data points within a phase, and each data set is split in half and a
median is determined for each half. A line that intersects the two medians is then plotted. When examining effectiveness data, secondary measures of data analysis included the use of descriptive statistics including means, standard deviations, slopes, percentage of nonoverlapping data, and improvement rate difference, to support visual analysis. Improvement rate difference (IRD) is a calculation of effect size designed for single-case research. It is defined as “the improvement rate (IR) of the treatment phase(s) minus the improvement rate of the baseline phase(s): $IR_T - IR_B = IRD$” (Parker et al., 2009, p. 138). Tentative benchmarks for IRD have been noted in research and suggest that IRD scores of .70 to .75 or higher are deemed large to very large effect sizes, .50 to .70 are deemed moderate effect sizes, and .50 and below are deemed very small effect sizes (Parker et al., 2009).

Analysis of intervention efficiency relied on the calculation of learning rate defined as academic gains per instructional minute (Cates, et al., 2003; Skinner, 2008; Skinner et al., 1997), as a primary analysis tool. Visual analysis was used as a secondary measure of efficiency. Visual analysis of maintenance data included an examination of changes in level and trend of maintenance data points compared to the last three intervention data points for each condition (i.e., CCC, DPR, and control). Visual analysis of trend data was supported through statistical calculations of the slope of the maintenance data for each condition. Descriptive statistics, namely means and range, were used to compare social validity data across the two interventions for all teacher and student participants.

**Intervention Effectiveness – Correct Digits per Minute**

Figures 1 through 4 depict the intervention effectiveness data for Students 1 through 4, and highlight math fact fluency (i.e., correct digits per minute on CBM probes) during baseline, intervention, and maintenance phases. These graphs illustrate the different data trends for the
CCC intervention, the DPR intervention, and the control conditions through use of a trend line for each condition derived using the split-middle technique (Kennedy, 2005). Visual analysis of graphed data examined changes in level and trend for both the CCC and the DPR condition for all four students when compared to baseline and control data, as well as notable differences between interventions (see Figures 1-4).

Student 1 demonstrated between-phase changes in level and trend for both CCC and DPR over baseline, as well as differences in level and trend over the control condition, when examining the effect of these interventions on multiplication math fact fluency (see Figure 1). The trend of the DPR data within the intervention phase, based on examination of trend lines, was steeper than the trend data for the CCC intervention. In addition, DPR data level was higher than CCC data level.

Student 2 demonstrated changes in level and trend of intervention data (i.e., CCC and DPR) over the baseline condition for multiplication math fact fluency, as well as positive level differences and slightly higher trend data than the control condition for both the CCC and DPR conditions (see Figure 2). When examining CCC and DPR data within the intervention phase, CCC level slightly exceeded DPR level based on visual analysis. When examining the mean of the intervention conditions to support visual analysis, DPR intervention average was 21.4 CDM and CCC intervention average was 23.1 CDM. When examining the trend data of CCC and DPR in the intervention condition through visual analysis, the distinctions between the two data sets are inconclusive. Slope data indicates a slightly steeper slope in the DPR data during the intervention condition when compared to the CCC data in the intervention condition (+0.50 and +0.44, respectively; see Table 1).
Student 3 demonstrated positive between-phase changes in both level and trend when examining CCC and DPR data in comparison to baseline data (see Figure 3). Visual analysis of DPR data indicates both positive trend and level differences between this intervention condition and the control condition. Visual analysis of CCC data, in comparison to control data, demonstrates a steeper trend, but level differences are ambiguous. Examination of intervention mean data demonstrates a slightly higher intervention average with 26.7 CDM for the CCC condition and 25.5 CDM for the control condition (see Table 1). Both the level and trend comparisons of CCC and DPR data within the intervention condition, indicate more positive data changes with the DPR condition.

Visual analysis of data for Student 4 found both changes in level and trend for the CCC and DPR conditions over the baseline condition (see Figure 4). DPR demonstrates significantly steeper level and trend data over the control condition through the intervention phase. However, CCC demonstrated similar level data as the control condition, which was supported through intervention mean calculations (21.1 CDM and 21.4 CDM respectively; see Table 1). Visual analysis of trend data shows a steeper slope for the control condition over the CCC intervention condition. When comparing the CCC and DPR intervention conditions, DPR demonstrates a higher level and steeper trend in data than the CCC condition for Student 4.

When comparing CCC and DPR intervention conditions, the average gains over baseline for the CCC condition were 9.2 correct digits per minute and the average gains in the DPR condition were 16.7 correct digits per minute. Gains in the control condition (i.e., 9.1 correct digits per minute) were not significantly different from the gains found in the CCC condition. These gains were calculated by subtracting the average baseline mean from the average intervention mean for each condition. Examining the trend data, steeper trends for the DPR
intervention as compared to the CCC intervention condition and the control condition were found across all four student participants throughout the intervention phase. Visual findings were supported by slope data (see Figures 1-4 and Table 1). The average slope for the control condition was +.50, average slope for CCC condition was +.87, and the average slope for DPR condition was +1.72. DPR produced greater variability in data for three of the four student participants, as noted both through visual analysis of the degree to which data points were dispersed compared to the best-fit line, as well as through calculations of standard deviation (see Figures 1-4 and Table 1).

When comparing the effect size of CCC and DPR, examinations were made based on the IRD and PND of each intervention in comparison to the control condition (see Table 2 and Table 3). The average IRD effect size over control for the DPR intervention was .6667 and the average effect size over control for the CCC intervention was .3340. The average PND for the DPR intervention over control was 68.3% and the average PDN for the CCC intervention over control was 36.7%.
**Figure 1.** Intervention Effectiveness – Math Fact Fluency Data – Student #1

![Student #1 - Multiplication Math Fact Fluency](image1)

**Figure 2.** Intervention Effectiveness – Math Fact Fluency Data – Student #2

![Student #2 - Multiplication Math Fact Fluency](image2)
Figure 3. Intervention Effectiveness – Math Fact Fluency Data – Student #3

Figure 4. Intervention Effectiveness – Math Fact Fluency Data – Student #4
### Table 1. Intervention Effectiveness - Math Fact Fluency (Correct Digits per Minute)

<table>
<thead>
<tr>
<th>Student</th>
<th>Detect, Practice, &amp; Repair</th>
<th>Cover, Copy, &amp; Compare</th>
<th>Control Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline</td>
<td>Intervention</td>
<td>Maintenance</td>
</tr>
<tr>
<td>1</td>
<td>M</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>1.00</td>
<td>7.00</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>0</td>
<td>+1.00</td>
</tr>
<tr>
<td>2</td>
<td>M</td>
<td>13</td>
<td>21.4</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>3.12</td>
<td>3.39</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>-0.75</td>
<td>+0.50</td>
</tr>
<tr>
<td>3</td>
<td>M</td>
<td>13.2</td>
<td>35.2</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>3.75</td>
<td>16.81</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>-3.25</td>
<td>+3.41</td>
</tr>
<tr>
<td>4</td>
<td>M</td>
<td>11.0</td>
<td>31.5</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>0.50</td>
<td>11.35</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>-0.25</td>
<td>+1.98</td>
</tr>
</tbody>
</table>
Table 2. Intervention Effectiveness (Fluency) – Detect, Practice, & Repair compared to Baseline, CCC, and Control Conditions

<table>
<thead>
<tr>
<th>Student</th>
<th>Baseline vs. Detect, Practice, &amp; Repair</th>
<th>Cover, Copy, &amp; Compare vs. Detect, Practice, &amp; Repair</th>
<th>Control Condition vs. Detect, Practice, &amp; Repair</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><strong>IRD</strong> 1.000</td>
<td>.6667</td>
<td>.7333</td>
</tr>
<tr>
<td></td>
<td><strong>PND</strong> 100%</td>
<td>93.3%</td>
<td>80%</td>
</tr>
<tr>
<td>2</td>
<td><strong>IRD</strong> .6667</td>
<td>.2667</td>
<td>.7333</td>
</tr>
<tr>
<td></td>
<td><strong>PND</strong> 86.7%</td>
<td>0%</td>
<td>73.3%</td>
</tr>
<tr>
<td>3</td>
<td><strong>IRD</strong> .6667</td>
<td>.4000</td>
<td>.5333</td>
</tr>
<tr>
<td></td>
<td><strong>PND</strong> 80%</td>
<td>20%</td>
<td>53.3%</td>
</tr>
<tr>
<td>4</td>
<td><strong>IRD</strong> 1.000</td>
<td>.6667</td>
<td>.6667</td>
</tr>
<tr>
<td></td>
<td><strong>PND</strong> 100%</td>
<td>66.7%</td>
<td>66.7%</td>
</tr>
<tr>
<td>M</td>
<td><strong>IRD</strong> .8333</td>
<td>.5000</td>
<td>.6667</td>
</tr>
<tr>
<td></td>
<td><strong>PND</strong> 91.7%</td>
<td>45%</td>
<td>68.3%</td>
</tr>
</tbody>
</table>

*Note.* In column 1, baseline data were collected using the DPR problem set data in comparison to the DPR intervention condition data. Column 2 is comparing the CCC data with the DPR data within the intervention condition. Column 3 is comparing the control data in the intervention phase with the DPR data in the intervention phase.
<table>
<thead>
<tr>
<th>Student</th>
<th>Baseline (A) vs. Cover, Copy, &amp; Compare (B)</th>
<th>Detect, Practice, &amp; Repair (A) vs. Cover, Copy, &amp; Compare (B)</th>
<th>Control Condition (A) vs. Cover, Copy, &amp; Compare (B)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IRD</td>
<td>IRD</td>
<td>IRD</td>
</tr>
<tr>
<td>1</td>
<td>1.000</td>
<td>.6667</td>
<td>.1667</td>
</tr>
<tr>
<td></td>
<td>PND</td>
<td>6.7%</td>
<td>33.3%</td>
</tr>
<tr>
<td>2</td>
<td>IRD</td>
<td>.9333</td>
<td>.7500</td>
</tr>
<tr>
<td></td>
<td>PND</td>
<td>.2667</td>
<td>.80%</td>
</tr>
<tr>
<td>3</td>
<td>IRD</td>
<td>.8667</td>
<td>.3143</td>
</tr>
<tr>
<td></td>
<td>PND</td>
<td>.4000</td>
<td>.33.3%</td>
</tr>
<tr>
<td>4</td>
<td>IRD</td>
<td>.9333</td>
<td>.1048</td>
</tr>
<tr>
<td></td>
<td>PND</td>
<td>.6667</td>
<td>0%</td>
</tr>
<tr>
<td>M</td>
<td>IRD</td>
<td>.9333</td>
<td>.3340</td>
</tr>
<tr>
<td></td>
<td>PND</td>
<td>.5000</td>
<td>36.7%</td>
</tr>
</tbody>
</table>

*Note.* In column 1, baseline data were collected using the CCC problem set data in comparison to the CCC intervention condition data. Column 2 is comparing the DPR data with the CCC data within the intervention condition. Column 3 is comparing the control data in the intervention phase with the CCC data in the intervention phase.
Intervention Effectiveness – Accurate Facts per Problem Set

Intervention effectiveness, as documented through the percentage of accurate facts per problem set, was examined through visual analysis of graphed data for Students 1 through 4 across baseline, intervention, and maintenance phases (see Figures 5-8). Visual analysis included examinations of trend with trend estimation lines generated using split-middle procedures (Kennedy, 2005), changes in level, and variability of data. These findings were then supported through a secondary analysis using descriptive statistics (i.e., mean, standard deviation, slope, IRD, and PND). Examination of CCC and DPR intervention documented both the level and trend data in comparison to baseline and control conditions for all student participants, as well as between intervention distinctions (see Figures 5-8).

Student 1 demonstrated positive level and trend changes over baseline for both the CCC and DPR interventions when examining the percentage of accurate facts per problem set (see Figure 5). In addition, within the intervention phase, higher level and steeper trend differences were noted between the DPR condition and the control condition. Trend data for the CCC intervention was steeper than trend data for the control condition. However, level data was ambiguous through visual analysis. A comparison of intervention condition means indicate a slightly higher CCC mean over control mean (30.0% and 28.0% respectively; see Table 4). When comparing CCC and DPR, trend and level data were both significantly higher for the DPR intervention.

Student 2 demonstrated slight level and trend changes over baseline for the DPR condition (see Figure 6). Level and trend data for Student 2 was ambiguous as it relates to the CCC condition over baseline. Mean data indicates a slight improvement over baseline for the CCC condition (52.8% and 57.2% respectively; see Table 4). Slope data shows a very minor
improvement in trend with the CCC condition over baseline (+0.01 and 0 respectively; see Table 4). When examining intervention data in comparison to the control condition within the intervention phase, DPR and CCC demonstrate positive level differences. DPR demonstrates a steeper data trend over the control condition. CCC data trend data in comparison to control trend data is a bit ambiguous. Calculation of slope indicates a slightly higher trend in CCC data over control data (+0.01 and +0.003 respectively; see Table 4). When comparing the CCC and DPR intervention data, DPR data has a steeper trend and a slightly higher level than CCC data.

CCC and DPR data for Student 3 demonstrates both an improvement in level and trend over baseline (see Figure 7). Within the intervention phase, steeper trends in intervention data for both CCC and DPR were noted over the control condition. DPR level data clearly demonstrated a higher intervention mean over the control condition. However, the CCC level data compared to control data was inconclusive through visual analysis. Calculations of intervention mean for CCC demonstrated a higher average than the control mean (53.3% and 56.0% respectively; see Table 4). When comparing the CCC and DPR interventions through visual analysis, DPR clearly demonstrates steeper trend data and higher level data than the CCC intervention.

Visual analysis of data from Student 4 found both positive trend and level changes for both the CCC and DPR interventions over baseline (see Figure 8). When comparing CCC and DPR interventions to the control condition during the intervention phase, both CCC and DPR demonstrate steeper trends than the control condition. However, level data is not as conclusive due to the high degree of variability with both the CCC and DPR data; variability that was not noted with the control condition. Examination of mean data for the CCC intervention condition, the DPR intervention condition, and the control condition found that CCC possessed the highest data level (i.e., 63.9% accurate facts per problem set), the control condition possessed the second
highest data level (i.e., 56.3% accurate facts per problem set), and the DPR condition possessed
the lowest data level (i.e., 55.6% accurate facts per problem set; see Table 4). When comparing
the CCC and DPR interventions based on trend data, DPR demonstrated a slightly steeper trend
than the CCC intervention.

The average math fact accuracy for the DPR intervention condition was 19.7% of correct
facts per problem set over baseline, with the CCC intervention condition average being 13.6%
over baseline. Gains in the control condition found increases of 9.1% correct facts per problem
set over baseline. When comparing the effect size of CCC and DPR, examinations were made
based on the IRD and PND of each intervention in comparison to the control condition (see
Table 5 and Table 6). The average IRD effect size over control for the DPR intervention was
.5167 and the average effect size over control for the CCC intervention was .3474. The average
PND for the DPR intervention over control was 56.7% and the average PND for the CCC
intervention over control was 33.4%.
**Figure 5.** Intervention Effectiveness – Math Fact Accuracy – Student #1

![Graph showing Student #1's Multiplication Math Fact Accuracy over Sessions.](image)

**Figure 6.** – Intervention Effectiveness – Math Fact Accuracy – Student #2

![Graph showing Student #2's Multiplication Math Fact Accuracy over Sessions.](image)
Figure 7. Intervention Effectiveness – Math Fact Accuracy – Student #3

Figure 8. Intervention Effectiveness – Math Fact Accuracy – Student #4
<table>
<thead>
<tr>
<th>Student</th>
<th>Detect, Practice, &amp; Repair</th>
<th>Cover, Copy, &amp; Compare</th>
<th>Control Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline</td>
<td>Intervention</td>
<td>Maintenance</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>11.1%</td>
<td>44.4%</td>
<td>52.8%</td>
</tr>
<tr>
<td>SD</td>
<td>4.8%</td>
<td>17.4%</td>
<td>5.9%</td>
</tr>
<tr>
<td>M</td>
<td>0</td>
<td>+0.03</td>
<td>-0.08</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>52.8%</td>
<td>60.0%</td>
<td>62.5%</td>
</tr>
<tr>
<td>SD</td>
<td>9.6%</td>
<td>6.5%</td>
<td>5.9%</td>
</tr>
<tr>
<td>M</td>
<td>-0.08</td>
<td>+0.01</td>
<td>-0.08</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>50.0%</td>
<td>68.9%</td>
<td>100.0%</td>
</tr>
<tr>
<td>SD</td>
<td>0.0%</td>
<td>24.1%</td>
<td>0.00%</td>
</tr>
<tr>
<td>M</td>
<td>0</td>
<td>+0.05</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>36.1%</td>
<td>55.6%</td>
<td>79.2%</td>
</tr>
<tr>
<td>SD</td>
<td>12.7%</td>
<td>20.1%</td>
<td>5.9%</td>
</tr>
<tr>
<td>M</td>
<td>+0.04</td>
<td>+0.04</td>
<td>+0.08</td>
</tr>
</tbody>
</table>
Table 5. Intervention Effectiveness (Accuracy) – Detect, Practice, & Repair compared to Baseline, CCC, and Control Conditions

<table>
<thead>
<tr>
<th>Student</th>
<th>IRD Baseline vs. Detect, Practice, &amp; Repair</th>
<th>IRD Cover, Copy, &amp; Compare vs. Detect, Practice, &amp; Repair</th>
<th>IRD Control Condition vs. Detect, Practice, &amp; Repair</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.9333</td>
<td>.4667</td>
<td>.5714</td>
</tr>
<tr>
<td></td>
<td>93.3%</td>
<td>53.3%</td>
<td>53.3%</td>
</tr>
<tr>
<td>2</td>
<td>-.1333</td>
<td>.1333</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>40%</td>
<td>0%</td>
<td>100%</td>
</tr>
<tr>
<td>3</td>
<td>.6667</td>
<td>.4000</td>
<td>.3905</td>
</tr>
<tr>
<td></td>
<td>60%</td>
<td>20%</td>
<td>40%</td>
</tr>
<tr>
<td>4</td>
<td>.5333</td>
<td>.2667</td>
<td>.1048</td>
</tr>
<tr>
<td></td>
<td>46.7%</td>
<td>0%</td>
<td>33.3%</td>
</tr>
<tr>
<td>M</td>
<td>.5000</td>
<td>.3167</td>
<td>.5167</td>
</tr>
<tr>
<td></td>
<td>60%</td>
<td>18.3%</td>
<td>56.7%</td>
</tr>
</tbody>
</table>

Note. In column 1, baseline data were collected using the DPR problem set data in comparison to the DPR intervention condition data. Column 2 is comparing the CCC data with the DPR data within the intervention condition. Column 3 is comparing the control data in the intervention phase with the DPR data in the intervention phase.
<table>
<thead>
<tr>
<th>Student</th>
<th>IRD</th>
<th>PND</th>
<th>IRD</th>
<th>PND</th>
<th>IRD</th>
<th>PND</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>100%</td>
<td>0.4667</td>
<td>0%</td>
<td>0.2468</td>
<td>6.7%</td>
</tr>
<tr>
<td>2</td>
<td>-0.133</td>
<td>20%</td>
<td>0.1333</td>
<td>0%</td>
<td>0.8667</td>
<td>86.7%</td>
</tr>
<tr>
<td>3</td>
<td>0.2667</td>
<td>46.7%</td>
<td>0.4000</td>
<td>0%</td>
<td>0.1048</td>
<td>6.7%</td>
</tr>
<tr>
<td>4</td>
<td>0.4000</td>
<td>60%</td>
<td>0.2667</td>
<td>0%</td>
<td>0.1714</td>
<td>33.3%</td>
</tr>
<tr>
<td>M</td>
<td>0.3667</td>
<td>56.7%</td>
<td>0.3167</td>
<td>0%</td>
<td>0.3474</td>
<td>33.4%</td>
</tr>
</tbody>
</table>

*Note.* In column 1, baseline data were collected using the CCC problem set data in comparison to the CCC intervention condition data. Column 2 is comparing the DPR data with the CCC data within the intervention condition. Column 3 is comparing the control data in the intervention phase with the CCC data in the intervention phase.
**Intervention Efficiency – Correct Digits per Minute**

To examine the intervention efficiency of both the CCC and DPR interventions, a calculation and analysis of learning rate was used as the primary method of analysis. Learning rate is defined as the academic gains per instructional minute, and is calculated by subtracting the median of the last three intervention points from the median of the baseline data and dividing by the instructional minutes required to bring about that change (see Table 7). In addition, as a secondary form of analysis, the efficiency of CCC and DPR interventions in bringing about gains in math fact fluency was illustrated by co-plotting the accumulative instructional minutes for each intervention session with the academic gains (i.e., correct digits per minute) secured. These intervention efficiency results are depicted for Students 1 through 4 (see Figures 9-12).

When comparing the CCC and DPR intervention, learning rate calculations found that the average gains in math fact fluency for the CCC intervention were 0.17 CDM per minute of CCC implementation, whereas the average gains in math fluency of the DPR intervention were 0.16 CDM per minute of DPR implementation (see Table 7). Calculation of learning rate by student found that Student 1 demonstrated a higher learning rate with the DPR intervention than the CCC intervention (.118 and .116 respectively). Student 2 demonstrated a higher learning rate with the CCC intervention than the DPR intervention (.210 and .075 respectively). Student 3 demonstrated a higher learning rate with the DPR intervention than the CCC intervention (.316 and .278 respectively). Finally, Student 4 demonstrated a higher learning rate with the DPR intervention than the CCC intervention (.143 and .062 respectively).

Visual analysis of graphed data indicates positive trend changes when examining the accumulative CCC and DPR implementation duration per intervention session with math fact fluency gains (i.e., correct digits per minute) for all student participants (see Figures 9-12). When
comparing the intervention efficiency of CCC and DPR interventions as it relates to math fact fluency gains, visual analysis shows that the CCC intervention was more efficient for Students 2 and 3 and the DPR intervention was more efficient for Students 1 and 4.
Figure 9. Intervention Efficiency – Math Fact Fluency – Student #1

![Graph showing intervention efficiency for Student #1.](image)

Figure 10. Intervention Efficiency – Math Fact Fluency – Student #2

![Graph showing intervention efficiency for Student #2.](image)
Figure 11. Intervention Efficiency – Math Fact Fluency – Student #3

Figure 12. Intervention Efficiency – Math Fact Fluency – Student #4
<table>
<thead>
<tr>
<th>Condition</th>
<th>Pre-Test*</th>
<th>Post-Test*</th>
<th>Difference</th>
<th>Duration (min.)</th>
<th>Learning Rate</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DPR – Fluency (CDM)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student 1</td>
<td>1.5</td>
<td>25.0</td>
<td>+23.5</td>
<td>198.9</td>
<td>.118</td>
<td>.116</td>
</tr>
<tr>
<td>Student 2</td>
<td>12.0</td>
<td>23.0</td>
<td>+11.0</td>
<td>147.3</td>
<td>.075</td>
<td>.057</td>
</tr>
<tr>
<td>Student 3</td>
<td>11.0</td>
<td>65.5</td>
<td>+54.5</td>
<td>172.4</td>
<td>.316</td>
<td>.318</td>
</tr>
<tr>
<td>Student 4</td>
<td>11.0</td>
<td>34.5</td>
<td>+23.5</td>
<td>164.3</td>
<td>.143</td>
<td>.192</td>
</tr>
<tr>
<td><strong>CCC – Fluency (CDM)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student 1</td>
<td>1.5</td>
<td>14.0</td>
<td>+12.5</td>
<td>107.4</td>
<td>.116</td>
<td>.079</td>
</tr>
<tr>
<td>Student 2</td>
<td>12.5</td>
<td>25.5</td>
<td>+13.0</td>
<td>61.8</td>
<td>.210</td>
<td>.114</td>
</tr>
<tr>
<td>Student 3</td>
<td>16.5</td>
<td>35.5</td>
<td>+19.0</td>
<td>68.4</td>
<td>.278</td>
<td>.452</td>
</tr>
<tr>
<td>Student 4</td>
<td>13.0</td>
<td>17.0</td>
<td>+4.0</td>
<td>64.3</td>
<td>.062</td>
<td>.103</td>
</tr>
</tbody>
</table>

* Pre-test represents the median value of the three baseline data points and post-test represents the median of the last three intervention points.
Intervention Efficiency – Accurate Facts per Problem Set

In addition to examining the intervention efficiency of the CCC intervention and DPR intervention in generating positive changes in math fact fluency, exploration of intervention efficiency on math fact accuracy (i.e., percentage of correct facts per problem set) was also considered using similar previously noted graphing and learning rate calculation procedures. Again, learning rate was used as the primary method of data analysis and visual analysis of efficiency data was a secondary form of analysis.

When comparing the CCC and DPR intervention, learning rate calculations found that the average gains in percentage of accurate facts per problem set for the CCC intervention were 0.21% of accurate facts per problem set per minute of CCC implementation, whereas the average gains in math accuracy of the DPR intervention were 0.32% of accurate facts per problem per minute of DPR implementation (see Table 8). The learning rate for Student 1 with the CCC intervention was 0.39% gain in accurate facts per problem set per minute of CCC implementation, whereas DPR gains were less at 0.25%. Student 2 demonstrated a higher learning rate with the DPR intervention than the CCC intervention (0.05% and 0.00% respectively). Student 3 demonstrated a higher learning rate with the CCC intervention with gains of 0.37% accurate facts per problem set per minute of intervention implementation over DPR gains of 0.29% accurate facts per problem set. Finally, Student 4 demonstrated a higher learning rate with the CCC intervention than the DPR intervention (0.52% and 0.25% respectively).

Visual analysis of graphed data demonstrated positive trend changes when examining the relationship between CCC instructional minutes and percentage of correct facts per problem set for Student 1, Student 3, and Student 4 (see Figure 13 and Figures 15-16). A flat trend estimation
line was found for Student 2 (see Figure 14), but statistical calculation and analysis of slope did find a slight positive trend of +0.28 for this participant. Visual analysis of intervention efficiency data graphically depicting DPR implementation duration with math fact accuracy (i.e., percentage of correct facts per problem set) demonstrates positive trend changes for all student participants (see Figures 13-16).
Figure 13. Intervention Efficiency – Math Fact Accuracy – Student #1

Figure 14. Intervention Efficiency – Math Fact Accuracy – Student #2
Figure 15. Intervention Efficiency – Math Fact Accuracy – Student #3

Figure 16. Intervention Efficiency – Math Fact Accuracy – Student #4
### Table 8. Intervention Efficiency – Percentage of Correct Facts per Set

<table>
<thead>
<tr>
<th>Condition</th>
<th>Pre-Test*</th>
<th>Post-Test*</th>
<th>Difference</th>
<th>Duration (min.)</th>
<th>Learning Rate</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>DPR – Math Fact Accuracy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student 1</td>
<td>8.33%</td>
<td>58.33%</td>
<td>+50.00%</td>
<td>198.9</td>
<td>.251%</td>
<td>.264</td>
</tr>
<tr>
<td>Student 2</td>
<td>58.33%</td>
<td>66.67%</td>
<td>+8.34%</td>
<td>147.3</td>
<td>0.05%</td>
<td>.100</td>
</tr>
<tr>
<td>Student 3</td>
<td>50.00%</td>
<td>100.00%</td>
<td>+50.00%</td>
<td>172.4</td>
<td>.290%</td>
<td>.476</td>
</tr>
<tr>
<td>Student 4</td>
<td>33.33%</td>
<td>75.00%</td>
<td>+41.67%</td>
<td>164.3</td>
<td>.254%</td>
<td>.366</td>
</tr>
<tr>
<td>CCC – Math Fact Accuracy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student 1</td>
<td>0.00%</td>
<td>41.76%</td>
<td>+41.67%</td>
<td>107.4</td>
<td>.388%</td>
<td>.331</td>
</tr>
<tr>
<td>Student 2</td>
<td>58.33%</td>
<td>58.33%</td>
<td>+0.00%</td>
<td>61.8</td>
<td>.000%</td>
<td>.278</td>
</tr>
<tr>
<td>Student 3</td>
<td>41.67%</td>
<td>66.67%</td>
<td>+25.00%</td>
<td>68.4</td>
<td>.365%</td>
<td>.720</td>
</tr>
<tr>
<td>Student 4</td>
<td>50.00%</td>
<td>83.33%</td>
<td>+33.33%</td>
<td>64.3</td>
<td>.518%</td>
<td>.806</td>
</tr>
</tbody>
</table>

* Pre-test represents the median value of the three baseline data points and post-test represents the median of the last three intervention points.
**Maintenance – Correct Digits per Minute**

Maintenance data included two follow-up assessments of math fact fluency and math fact accuracy for all student participants at one week post-intervention and at two weeks post-intervention. Maintenance data were graphed and visually analyzed with an emphasis on trend, as well as level of the maintenance data in comparison to the last three intervention points.

Maintenance data for the CCC intervention found that math fact fluency gains were maintained at both the one week and two week follow-up for Student 2, Student 3, and Student 4 when level was compared to the last three intervention points (see Figures 2-4). Student 2 demonstrated an increasing trend during the maintenance condition with Student 3 and Student 4 demonstrating a slight decreasing trend at the second follow-up (see Figures 2-4). Student 1 demonstrated maintenance at the one week follow-up with a significant decreasing trend in math fact fluency at the two week follow-up (see Figure 1). Statistical calculations of slope in the maintenance phase of the CCC condition ranged from -6.0 to +3.5 with an average slope of -1.13 (see Table 1).

Math fact fluency maintenance data for the DPR intervention found that math fact fluency gains were maintained at both the one week and two week follow-up for Student 2, Student 3, and Student 4 when level was compared to the last three intervention points (see Figures 2-4). Student 3 and Student 4 demonstrated an increasing trend during the maintenance condition with Student 2 demonstrating a slight decreasing trend at the second follow-up. Student 1 demonstrated maintenance at the one week follow-up with a significant decreasing trend in math fact fluency at the two week follow-up (see Figure 1). Statistical calculations of slope ranged from -10.0 to +10.5 with an average slope of -0.13 (see Table 1).
Maintenance – Accurate Facts per Problem Set

Math fact accuracy maintenance data for all student participants was graphically displayed and visually analyzed for all conditions (see Figures 5-8). When examining the degree to which math fact accuracy gains were maintained for the CCC and DPR interventions, an analysis of trend of maintenance data and level of maintenance data in comparison to the last three intervention points was made.

Visual analysis of math fact accuracy maintenance data with the CCC intervention found that gains were maintained at both the one week and two week follow-up for Student 2, Student 3, and Student 4 as evidenced through maintained data level between the last three intervention points and the maintenance points (see Figures 6-8). However, decreasing trends were noted for Student 4 from the one week and two week follow-up (see Figure 8). Student 1 demonstrated a slight level change between the CCC intervention condition and the maintenance condition, as well as a decreasing trend in maintenance data between the one week and two week follow-up (see Figure 5). Statistical calculation of slope for the maintenance phase of the CCC condition demonstrated a range of -0.17 to 0 with an average slope of -0.08 across all participants (see Table 2).

Visual analysis of the math fact accuracy maintenance data generated following implementation of the DPR intervention found that all students maintained the gains achieved through DPR implementation (see Figures 5-8). This was evidenced through maintained data levels between the last three intervention points and both follow-up maintenance data points. However, there was decreasing trends in the maintenance phase for Student 1 and Student 2. Statistical calculation of slope for the maintenance phase of the DPR condition demonstrated a range of -0.08 to +0.08 with an average slope of -0.02 across all participants (see Table 2).
Social Validity

Social validity data were collected via a rating scale administered to all teacher and student participants with results analyzed through descriptive statistics (i.e., mean and range). Table 9 outlines the CCC and DPR social validity results based on the two teacher participant ratings and Table 10 outlines the CCC and DPR social validity results based on the four student participant ratings. On average teacher and student social validity ratings for DPR were higher than ratings for CCC.

The teacher rating scale consisted of 10 items, which were rated on a 6-point Likert scale (i.e., 1=Strongly Disagree, 2=Disagree, 3=Slightly Disagree, 4=Slightly Agree, 5=Agree, 6=Strongly Agree). A mean rating of 6 was assigned to both interventions as it related to the following items: “procedures used in the intervention were easy to implement,” “the training was helpful for accurately implementing the intervention,” and “the duration of each session was appropriate.” The items “this intervention was effective in increasing multiplication math fact fluency in the targeted children” and “this intervention efficiently brought about the desired change in math fact fluency” received a mean score of 5.5 for the DPR intervention and 4.5 for the CCC intervention. Detect, Practice, and Repair was assigned a mean rating of 6 for the following items: “the intervention was effective in increasing multiplication math fact accuracy in the targeted children” and “the intervention efficiently brought about the desired change in math fact accuracy,” whereas CCC received a mean score of 5 and 4.5, respectively. The item “I liked the procedures used in this intervention” received a mean rating of 6 for the DPR intervention and 5.5 for the CCC intervention. Finally, “I would recommend this intervention to other teachers and parents who have children with math fact deficits,” and “overall, this
intervention was beneficial for the targeted children,” received mean ratings of 6 for the DPR intervention and 5 for the CCC intervention.

Overall, student participants also assigned slightly higher ratings on the DPR intervention over the CCC intervention. Student perspectives were secured through a rating scale consisting of four items. Items were rated on a 3-point picture scale (i.e., 1=Frowning Face, 2=Neutral Face, and 3=Smiling Face). An average rating of 3 was assigned to the DPR intervention as it related to the following item “How much did you like the CCC/DPR math activities?,” whereby CCC received an average rating of 2. DPR also secured a higher average rating (i.e., 3) than CCC (i.e., 2.5) on the item “Do you think that the CCC/DPR helped you in learning your math facts?” Additionally, DPR received an average rating of 2.5 on the item stating “Would you recommend CCC/DPR to one of your friends?” whereby CCC received an average rating of 1.75. The only item whereby CCC outscored DPR (mean of 3 and 2.5, respectively) stated “How well did your understand what to do with the CCC/DPR math papers?”
**Table 9. Social Validity Results – Intervention Teachers**

<table>
<thead>
<tr>
<th>Item</th>
<th>CCC Mean*</th>
<th>CCC Range*</th>
<th>DPR Mean*</th>
<th>DPR Range*</th>
</tr>
</thead>
<tbody>
<tr>
<td>This intervention was effective in increasing multiplication math fact fluency in the targeted children.</td>
<td>4.5</td>
<td>4-5</td>
<td>5.5</td>
<td>5-6</td>
</tr>
<tr>
<td>This intervention efficiently brought about the desired change in math fact fluency.</td>
<td>4.5</td>
<td>4-5</td>
<td>5.5</td>
<td>5-6</td>
</tr>
<tr>
<td>This intervention was effective in increasing multiplication math fact accuracy in the targeted children.</td>
<td>5</td>
<td>5-5</td>
<td>6</td>
<td>6-6</td>
</tr>
<tr>
<td>This intervention efficiently brought about the desired change in math fact accuracy.</td>
<td>4.5</td>
<td>4-5</td>
<td>6</td>
<td>6-6</td>
</tr>
<tr>
<td>The procedures involved in the intervention were easy to implement.</td>
<td>6</td>
<td>6-6</td>
<td>6</td>
<td>6-6</td>
</tr>
<tr>
<td>I liked the procedures used in this intervention.</td>
<td>5.5</td>
<td>5-6</td>
<td>6</td>
<td>6-6</td>
</tr>
<tr>
<td>The training was helpful for accurately implementing the intervention.</td>
<td>6</td>
<td>6-6</td>
<td>6</td>
<td>6-6</td>
</tr>
<tr>
<td>The duration of each session was appropriate.</td>
<td>6</td>
<td>6-6</td>
<td>6</td>
<td>6-6</td>
</tr>
<tr>
<td>I would recommend this intervention to other teachers and parents who have children with math fact deficits.</td>
<td>5</td>
<td>5-5</td>
<td>6</td>
<td>6-6</td>
</tr>
<tr>
<td>Overall, this intervention was beneficial for the targeted children.</td>
<td>5</td>
<td>5-5</td>
<td>6</td>
<td>6-6</td>
</tr>
</tbody>
</table>

* Items were rated on a 6-point Likert Scale (i.e., Strongly Disagree – 1, Disagree – 2, Slightly Disagree – 3, Slightly Agree – 4, Agree – 5, and Strongly Agree – 6)
* Scales were completed by the two intervention teachers.
Table 10. Social Validity Results – Student Participants

<table>
<thead>
<tr>
<th></th>
<th>CCC Mean*</th>
<th>CCC Range*</th>
<th>DPR Mean*</th>
<th>DPR Range*</th>
</tr>
</thead>
<tbody>
<tr>
<td>How well did you understand what to do with the CCC/DPR math papers?</td>
<td>3</td>
<td>3-3</td>
<td>2.5</td>
<td>2-3</td>
</tr>
<tr>
<td>How much did you like the CCC/DPR math activities?</td>
<td>2</td>
<td>1-3</td>
<td>3</td>
<td>3-3</td>
</tr>
<tr>
<td>Do you think that the CCC/DPR helped you in learning your math facts?</td>
<td>2.5</td>
<td>2-3</td>
<td>3</td>
<td>3-3</td>
</tr>
<tr>
<td>Would you recommend CCC/DPR to one of your friends?</td>
<td>1.75</td>
<td>1-3</td>
<td>2.5</td>
<td>2-3</td>
</tr>
</tbody>
</table>

* Items were rated on a 3-point picture scale (i.e., Frowning Face – 1, Neutral Face – 2, Smiling Face – 3)
* Scales were completed by the four student participants.

Discussion

Math fact automaticity is vital and research suggests that students are struggling with basic mathematical skills (National Center for Education and Statistics (NCES), 2011), which necessitates the development of and support of research-based interventions to increase math accuracy and fluency. Math interventions have been documented through research, and have been deemed effective. However, more studies are needed to confirm efficiency of these interventions and to devise intervention packages that might enhance learning rate.

The current study was designed to examine four key research questions related to the traditional Cover, Copy, and Compare (CCC) intervention procedures versus a more modern Detect, Practice, and Repair (DPR) intervention package. First, the aim of this research was to determine which intervention was more effective at increasing multiplication math fact accuracy and math fact fluency. Next, the study sought to determine which intervention was more efficient at bringing about these academic gains. In addition, this study sought to examine the maintenance of academic gains over time to determine whether implementation of the CCC
procedures or the DPR package resulted in better preservation of gains. An examination of social validity was a final pursuit of this study with an emphasis on the potential relationship between social validity and effectiveness/efficiency.

**Measuring Intervention Impact: Effectiveness and Efficiency.**

Data analysis in educational research can take many forms, including both visual analysis and statistical analysis, each offering researchers notable advantages and disadvantages, as well as opportunities to convergent and divergent findings. Visual analysis has a rich history with research utilizing single-case design methodology (Kazdin, 1982) and some experts have argued that visual analysis should be the exclusive or at least the primary form of analysis with single-case research (Baer, 1977; Michael, 1974; Parsonson & Baer, 1978; Parsonson & Baer, 1986). At a basic level, reported advantages of visual analysis include its ease of use and its defensibility among researchers (Parker, Cryer, & Burns, 2006). The advantage of visual analysis over statistical techniques includes the ability to simultaneously examine and interpret level changes, the magnitude and direction of data trend, and the variability of data (Kazdin, 1982).

Despite these positive expert appraisals of visual analysis, visual analysis also has disadvantages including concerns with inconsistencies, errors, and subjectivity (DeProspero & Cohen, 1979; Ottenbacher, 1986; Keppel, 1982; Park, Marascuilo, & Gaylord-Ross, 1990). The interrater reliability of visual analysis has produced only low to moderate reliability coefficients (i.e., .46-.60) across several studies (DeProspero & Cohen, 1979; Harbst, Ottenbacher, & Harris, 1991; Ottenbacher, 1990; Park et al., 1990). Yet, recent research has shown that the inclusion of trend lines significantly increases the accuracy of visual analysis (Van Norman, Nelson, Shin, & Christ, 2013). Objectivity in visual analysis can also be improved through the supplemental use
of statistical techniques, especially in cases where treatment effects are ambiguous (Huitema, 1986; Kazdin, 1982).

Statistical analysis used in conjunction with visual analysis could include drawing conclusions through the combined use of graphs with descriptive statistics (e.g., mean, standard deviation, slope), as well as calculations of effect size that include percentage of nonoverlapping data (PND) and improvement rate difference (IRD). Visual analysis combined with effect size produces increased objectivity, precision, increased confidence, and acceptability by the larger research community (Parker et al., 2009). Summary statistics utilizing a non-parametric approach include PND, which examines the degree of change from the baseline to the intervention phase by calculating the percentage of non-overlapping data between the two phases (Mathur, Kavale, Quinn, Forness, & Rutherford, 1998; Scruggs, Mastropieri, Cook, & Escobar, 1986; Scruggs, Mastropieri, Forness, & Kavale, 1988). Strengths of PND includes its nonparametric nature (i.e., free from parametric constraints), its ease of calculating through visual inspection of graphs, and the ease of interpretation (Scruggs et al., 1986). Limitations to PND include its sensitivity to ceiling or floor levels resulting in PND scores of 0% (Wolery, Busick, Reichow, & Barton, 2010) and its inability to tackle variability in data trends (Ma, 2006).

An alternative calculation of effect size, improvement rate difference (IRD), is recommended for summarizing single-case research data (Parker, Vannest, & Brown, 2009). IRD is designed to represent the difference in improvement rates between the baseline and intervention phases (Parker et al., 2009). An advantage of IRD is the ability to calculate confidence intervals with wider confidence intervals making the IRD less trustworthy and narrower confidence intervals indicating that the IRD is more trustworthy (Parker et al., 2009). Also, use of IRD has been more widely accepted by the broader research field due to its
historical use in medical research (Parker et al., 2009). IRD is impacted by ceiling effects, similar to PND, but is not impacted by floor effects.

The examination of intervention efficiency has also included both visual and statistical analysis. Visual analysis can include the graphing of instructional time (in minutes or seconds rather than instructional days) on the horizontal axis, with academic gains plotted on the vertical axis (Skinner, Belifiore, & Watson, 2002). Statistical analysis of intervention efficiency can occur through a calculation of learning rate, which is measured by examining learning level (i.e., academic gains over baseline) divided by instructional time (i.e., duration of intervention implementation; Cates et al., 2003; Skinner, 2008; Skinner et al., 1997). Again, visual and statistical analyzes of intervention efficiency data can produce both consistencies and inconsistencies.

**Intervention Effectiveness**

Data reveals that both the CCC intervention and the DPR intervention were effective at bringing about changes in math fact fluency and math fact accuracy when examining intervention data in comparison to baseline data for all student participants, with the exception of a single student who experienced a similar baseline data trend to DPR intervention data trend when assessing math fact accuracy. In addition, there were notable differences between the intervention conditions and the control conditions with the exception of one student who experienced a steeper trend line for the control condition over the CCC intervention when examining the impact of CCC on math fact fluency. These findings support previous research on the effectiveness of CCC in enhancing math fact fluency (Grafman & Cates, 2010; Lee & Tingstrom, 1994; Poncy et al., 2007) and the effectiveness of DPR in generating math fact fluency gains (Axtell et al., 2009; Poncy et al., 2006; Poncy et al., 2010; Poncy et al., 2013).
When comparing CCC and DPR intervention data, DPR trend estimation lines were steeper for all students for both math fact fluency and math fact accuracy. This suggests that the DPR intervention package was more effective than CCC procedures at bringing about changes in math fact fluency and math fact accuracy. Calculations of improvement rate difference (IRD) and percentage of nonoverlapping data (PND) when compared to the control condition found DPR to be superior to CCC for both math fact fluency and math fact accuracy.

**Intervention Efficiency**

In comparing the intervention efficiency of CCC versus DPR in bringing about gains in math fact fluency, visual display of data (i.e., plotting of instructional minutes with correct digits per minute) did not demonstrate a clear distinction between CCC and DPR as it relates to efficiency. However, calculation of learning rate, which considers improvements over baseline, in addition to instructional time, found DPR to be more efficient for three of the four student participants as it relates specifically to gains in math fact fluency. This violates the original hypothesis, as it was projected that the CCC intervention would be more efficient due to the additional time required to implement all components of DPR and the time lost transitioning between the various DPR intervention components. Intervention efficiency as it relates to math fact accuracy found the CCC intervention to be more efficient than the DPR intervention for three of the four students when examined through visual analysis. These findings were confirmed through learning rate calculations.

**Maintenance**

Math fact fluency maintenance data collected at both one week and two weeks post-intervention found that that both the CCC and DPR interventions were able to adequately maintain gains for three of the four student participants. Decreasing trends in maintenance data
were noted for two of the students following DPR removal and three of the students following CCC removal. Additional maintenance data would be necessary to determine the point at which maintenance data would level off. However, the overall slope of the math fact fluency maintenance data points indicates that declines were occurring more rapidly following CCC termination than DPR termination. Those results should be interpreted with caution given that so few data points are included within this phase.

Math fact accuracy maintenance data indicates that the DPR intervention was successful at maintaining gains for all student participants and that the CCC intervention was able to adequately maintain gains for three of the four student participants. Decreasing trends in maintenance data were noted for two of the students following cessation of the DPR intervention and for three of the students once CCC was removed. Again, the overall slope of the math fact accuracy maintenance data points indicates that declines were occurring more rapidly following CCC cessation than DPR cessation. Again, use caution when interpreting these results given the limited number of data point included within this phase.

**Social Validity**

Social validity rating scales support the finding that teacher participants perceived the DPR intervention as both more effective and more efficient in bringing about gains in both math fact fluency and math fact accuracy. These perceptions were accurate with the exception of CCC more efficiently bringing about gains in math fact accuracy. Both interventions were deemed easy to implement with adequate training provided and an appropriate intervention duration, but preference was given to the DPR intervention regarding likability of the procedures and the perceived benefit for the targeted children. Social validity rating scales indicate that student participants perceived their understanding of the CCC intervention procedures as slightly higher.
than their understanding of the DPR procedures. However, the likeability of DPR procedures was rated higher, as well as the perceived benefit of the DPR intervention over CCC procedures.

Although these findings contribute to the social validity research base tied to these interventions, these findings are especially important to the knowledge base of DPR. Not only has previous research on DPR failed to report on social validity, no previous studies have utilized teachers as implementers of the DPR intervention (i.e., DPR has been implemented by the researcher on all previous studies). Teacher preference for DPR not only supports their acceptance of this intervention, but also their willingness to implement a “complex intervention” that is “labor intensive” (Poncy et al., 2013, p. 224).

**Interpretations and Implications**

Although additional research would be needed to confirm and further support these findings, in general DPR was found to be both more effective and more efficient than CCC in improving math fact fluency. DPR was found to be more effective than CCC in improving math fact accuracy, but CCC was more efficient in bringing about these changes. Maintenance data suggests that DPR may maintain gains better than CCC for both fluency and accuracy, but additional data points would be necessary to confirm. Preference was given to the DPR intervention by key stakeholders.

The Instructional Hierarchy model outlines the stages that an individual progresses through when learning a new skill, as well as the changes in instructional practices that should accompany each stage of learning. This learning hierarchy details the increasing complexity of academic skills displayed as an individual transitions through Acquisition (i.e., Accuracy), Fluency/Proficiency, Generalization, and Application/Adaptation (Haring & Eaton, 1978). The Instructional Hierarchy purposes that students in an the Acquisition stage respond best to
demonstration, modeling and performance feedback, whereby students in the Fluency Building stage respond best to drill, repeated practice, and performance feedback (Haring & Eaton, 1978). Consideration regarding the type of intervention utilized should be dependent upon the individual’s stage in the Instructional Hierarchy (Rivera & Bryant, 1992). An example of the impact of these stage distinctions was highlighted in research conducted by Codding and colleagues (2007). Their results suggested that students’ initial levels of math fact fluency impacted the effectiveness of the researched math fluency interventions with CCC being more effective for students in the frustrational range and Explicit Timing being more effective for students in the instructional range (Codding et al., 2007). Students at the frustrational level, likely in the Acquisition Stage of the Instructional Hierarchy, experienced greater gains from CCC procedures that focus on demonstration, modeling, and performance feedback (Codding et al., 2007). Students in the instructional range, likely in the Fluency Building stage of the Instructional Hierarchy, benefited more from the Explicit Timing techniques focused on drill, repeated practice, and performance feedback (Codding et al., 2007). This research led to a meta-analytic study designed to link skill proficiency with math interventions identified as either acquisition building or fluency building, which found that acquisition-based interventions were most effective with students at the frustrational level with only moderate effects for students at the instructional level (Burns, Codding, Boice, & Lukito, 2010). Unfortunately, a lack of previous studies using fluency-based interventions with students at an instructional level, resulted in an inability to make definitive conclusions as it relates to the impact of math fluency interventions on this population (Burns et al., 2010). Based on the findings of the current study, CCC produced higher efficiency for math accuracy and DPR produced higher efficiency for math fluency, which may add further support to this previous research. If all fluency problems
are not equivalent (meaning some levels of accuracy/fluency respond to certain interventions better than others), then the creation of decision rules to guide intervention selection and intervention change (once a criterion level of performance has been achieved) could result in greater effectiveness and efficiency as it relates to student outcomes.

Haring and Eaton (1978) also tackled the concept of maintenance and suggested that a meaningful level of fluency is needed for retention. This retention can be achieved through “overlearning” and by making the material “meaningful” (Haring & Eaton, 1978). Since DPR was more effective and efficient at achieving math fact fluency, then this may explain why DPR was better able to maintain fluency gains once the intervention ceased. In addition, the procedures involved in DPR create a condition whereby automatic recall of the facts becomes significant to the student. Students make connections between the detect, practice, and repair phases of the intervention, as they develop an understanding of the impact that each phase of the intervention will have on the concluding activity (i.e., the self-graphing component). The self-graphing component appears game-like to students as they compete to beat their score from the previous session. The need to consider maintenance in addition to efficiency has been discussed in the literature (Burns & Sterling-Turner, 2010).

Research has identified elements of strong interventions in school settings, which includes their ecological nature, naturalistic integration, research support, and possession of social validity (Lentz, Allen, & Ehrhardt, 1996). Social validity is needed to promote and insure intervention fidelity, which in turn can produce positive outcomes for students. It is suggested that interventions that are less time-consuming and less complex in nature receive higher social validity ratings than those interventions that are more complex and require more time to implement (Witt, Martens, & Elliott, 1984). However, this study’s finding, that DPR was judged
to be more socially valid than the less complex CCC, is at odds with the conclusions drawn by Witt, Martens, and Elliott (1984). It is possible that a more complex and time-consuming intervention, such as DPR, might be perceived as “doing more” for the student and therefore be rated in a more positive light. In addition, the novelty of an intervention such as DPR might create an increased liking and acceptability of the procedures. The one-on-one implementation may have positively impacted the ease of administration, as well as setting (i.e., afterschool). Higher ratings might have been secured due to financial compensation for afterschool intervention administration.

**Limitations and Future Research**

Although this study contributes to the research knowledge base, there were a number of notable research study limitations/concerns that should be considered when interpreting results and planning for future research. These limitations include potential concerns with data collection methods, internal validity, and external validity limitations.

Despite the fact that improvements in math fact fluency were notable across both interventions in comparison to the control, concerns were noted regarding the potential for artificial improvements in fluency. Most notably, students appeared to increase their rate of responding to CBM probes, but were not necessarily improving their accuracy of responding on these probes (i.e., items were answered incorrectly or unknown items were rapidly crossed out by the student). Considerations were made to additionally measure the percentage of correct problems on math fact fluency probes to capture the potential differences in accuracy of responding that were not captured through the CBM method alone. This too was found to be an imperfect measure, as a student could answer only one item on the probe, answer that one problem correctly, and secure a 100% accuracy rate. New fluency assessment methods should
attempt to address this concern, such procedures might consider limiting the amount of time spend per problem (e.g., metronome paced assessments), which would be a more valid assessment of fluency over current CBM procedures. These procedures would ensure that students do not spend too much or too little time on a problem before moving to the next item and would have the same number of opportunities to respond as their peers during the timed assessment. Probes could generate both a fluency score and an error rate.

In regard to math accuracy assessments, the use of flashcard assessment procedures whereby a student is given five seconds to generate a response muddies the distinction between accuracy and fluency. Limitations regarding math fact accuracy assessment procedures are worthy of discussion, but present a larger debate related to the best means for distinguishing accuracy (i.e., acquisition) from fluency. As noted by Haring and Eaton (1978), “most research on skill acquisition has failed to distinguish that stage [Acquisition] from subsequent levels of development (fluency building, generalization, and adaptation)” (p. 26). Future research might attempt to better define the stages of acquisition and fluency, as well as develop assessment procedures to distinguish between these levels so that the most appropriate interventions can be prescribed.

Although both CCC and DPR interventions achieved IRD effect sizes that were in the large to very large range based on tentative research-based benchmarks (Parker et al., 2009), the average IRD effect size and the average PND for the control condition were higher than the average IRD effect size and average PND points derived from the CCC and DPR interventions as it relates to improvements in math fact fluency. Improvements in the control condition could be an indication of a threat to internal validity. It is possible that the improvements in the control condition could be attributed to history effects such as classroom instruction, which included a
focus on multiplication. In addition, this could be an indication of carry-over effects from the intervention condition(s) to the control condition. Finally, given that data collection occurred on a daily basis, testing effects could have influenced control condition performance. That is, the mere repetition of CBM probes and flashcard assessments could have resulted in math fact fluency and accuracy gains. Future research should attempt to control for these potential threats to internal validity.

This study has limitations with regard to external validity inherent in all single-subject research studies. Systematic replication of intervention procedures, especially for the less documented DPR intervention, should be implemented across student populations and target behaviors (e.g., spelling) to support generalization of these findings. In addition, further research is needed on the sustainability of these intervention procedures, which would produce additional support for the social validity of these intervention procedures.

**Conclusion**

Despite study limitations, Detect, Practice, and Repair (DPR) was shown to be a math fact fluency intervention package that is both effective and efficient in bringing about gains in math fact automaticity over traditional Cover, Copy, and Compare (CCC) procedures. This study contributed to the current scientific knowledge base by examining DPR through a one-on-one implementation by classroom teachers, as previous research studies implemented the intervention in small or large groups. All previous research studies examining DPR have relied on the researcher as interventionist (Poncy et al., 2013). The use of teachers as implementers makes this study an important contribution addressing pleas from previous research (Poncy et al., 2013). In addition, this is the first known study to directly compare DPR’s effectiveness,
efficiency, maintenance, and social validity against another intervention, CCC, which happens to be a component of the DPR package. Previous research recommended that a component analyses, such as this, be conducted to determine which components of the DPR intervention result in its effectiveness, in an effort to devise strategies for continuing to enhance the efficiency of DPR (Poncy et al., 2010; Poncy et al., 2013).
References


Ottenbacher, K. J. (1990). When a picture is worth a thousand $p$ values? A comparison of visual
and quantitative methods to analyze single subject data. *Journal of Special Education*, 23, 436-449.


Appendix A
Teacher Recruitment Script

My name is Rebecca Rahschulte. As a part of my doctoral studies, I’m conducting a research study to determine the impact of two math fact fluency intervention programs on students’ multiplication math fact accuracy and fluency. I’m specifically interested in the degree to which these two programs differ in not only their effectiveness, but also their efficiency. Student outcomes include students’ single-digit multiplication math fact fluency rates and math fact accuracy. Students’ academic gains will be examined with a consideration of the amount of time it took to bring about these gains (i.e., duration of each intervention session).

The math interventions used within this study will be implemented within the Community Learning Center (CLC) afterschool program. Student participants will include six to eight fourth-grade students at John G. Carlisle Elementary School who show deficits in single-digit multiplication math fact fluency.

I’m currently seeking two classroom teachers and/or instructional assistants who might be interested in implementing these interventions in a one-on-one setting with three to four fourth-grade students.

Here is what you would be asked to do:

1. Participate in an afterschool training on the research-based math fact fluency interventions. This will take about 30 minutes (across three separate days).

2. Implement the intervention within the afterschool program for approximately three to five weeks. The program will be implemented five days per week (approximately 30 minutes per one-on-one session for a total of 2 hours daily).

Your involvement in this research study will last no more than six weeks (one week of training on the program, and no more than five weeks of math fluency program implementation).

Your participation is completely voluntary and you can revoke your consent at any time.

Please feel free to contact me if you have any questions about this study or if you are interested in participating. You can contact me via phone at (513) 319-4134 or e-mail me at barhorrl@mail.uc.edu. Thank you for your consideration!
Appendix B
CBM Math Fact Fluency Administration Script and Scoring Guide

CBM Math Fact Fluency Administration Script
- Give the probe to the child(ren)
- Say “Work each problem going from left to right without skipping. If you don’t know how to complete a problem, put an ‘X’ over the problem and go onto the next one. Begin.”
- Begin timing. After 2-minutes say “Stop.”
- Collect the CBM probes.

CBM Math Fact Fluency Scoring Guide
- The evaluator should count the number of correct digits and incorrect digits for each probe.
- If a child completes the worksheet before time is up, the evaluator should divide the number of digits by the total number of second and multiply by 60.
- If a child skips problems on a worksheet, any omitted problems should be scored as errors.
- The total number of correct digits and the total number of incorrect digits are divided by two to obtain the Digits Correct per Minute (DCM).

Math Fact Accuracy Administration & Scoring Guide
Administration procedures will involve the display of each flashcard by the examiner to the examinee. The examinee will have 5 seconds to answer the problem. Cards will be placed in one of two piles. If the problem is answered correctly, it will be placed in the “known” pile. If the problem is answered incorrectly or the administration of that problem exceeds 5 seconds, the card will be placed in the “unknown” pile. Performance feedback will not be offered to the student regarding the rightness or wrongness of his/her answers. Following the assessment, the examiner will count the number of cards in each pile and will record the number of known facts within the set.

Appendix C
Teacher Nomination Checklist

Student Name: _______________________________________

CBM Math Fact Fluency Score: _________________________

Please answer the following questions:

<table>
<thead>
<tr>
<th>Question</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Based on classroom assessments (e.g., tests, work samples), do you feel that this score is an underestimate of this student’s math fact fluency skills?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Is this student receiving special education services that include remediation of math fact fluency deficits?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Does this student have any documented disabilities that might impact his/her ability to adequately access the intervention program (such as being blind or deaf)?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Does this student have documented attendance issues which might impact his/her participation in this intervention program?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If you answered “Yes” to any of the questions above, this might impact the student’s success in the intervention program.

<table>
<thead>
<tr>
<th>Question</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do you feel that the student would benefit from extra practice in the area of multiplication math fact fluency?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Do you think that the student would be a good candidate for this type of extra instruction?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Do you recommend this student for participation?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Additional Comments (Optional):
Appendix D
Mutually Exclusive Multiplication Problem Sets
(Skinner, Ford, & Yunker, 1991)

Set A
5 x 9 = 45
3 x 9 = 27
6 x 6 = 36
3 x 4 = 12
7 x 9 = 63
2 x 2 = 4
6 x 3 = 18
7 x 2 = 14
5 x 4 = 20
4 x 8 = 32
8 x 8 = 64
7 x 5 = 35

Set B
4 x 2 = 8
2 x 6 = 12
4 x 6 = 24
9 x 2 = 18
6 x 8 = 48
7 x 4 = 28
5 x 5 = 25
9 x 9 = 81
3 x 2 = 6
8 x 5 = 40
8 x 7 = 56
7 x 3 = 21

Set C
6 x 9 = 54
7 x 6 = 42
7 x 7 = 49
3 x 5 = 15
4 x 4 = 16
8 x 3 = 24
8 x 2 = 16
3 x 3 = 9
8 x 9 = 72
5 x 6 = 30
9 x 4 = 36
2 x 5 = 10
Appendix E
Social Validity Rating Scales
Please circle the number which best describes your agreement or disagreement with each statement.

<table>
<thead>
<tr>
<th></th>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Slightly Disagree</th>
<th>Slightly Agree</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>This intervention was effective in increasing multiplication math fact <strong>fluency</strong> in the targeted children.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>This intervention efficiently brought about the desired change in math fact <strong>fluency</strong>.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>This intervention was effective in increasing multiplication math fact <strong>accuracy</strong> in the targeted children.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>This intervention efficiently brought about the desired change in math fact <strong>accuracy</strong>.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>The procedures involved in the intervention were easy to implement.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>I liked the procedures used in this intervention.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>The training was helpful for accurately implementing the intervention.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>The duration of each session was appropriate.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>I would recommend this intervention to other teachers and parents who have children with math fact deficits.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Overall, this intervention was beneficial for the targeted children.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>
Student Social Validity Survey

How well did you understand what to do with the CCC/DPR math papers?

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>🙁</td>
<td>😐</td>
<td>😊</td>
<td></td>
</tr>
</tbody>
</table>

How much did you like the CCC/DPR math activities?

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
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</thead>
<tbody>
<tr>
<td>🙁</td>
<td>😐</td>
<td>😊</td>
<td></td>
</tr>
</tbody>
</table>

Do you think that the CCC/DPR helped you in learning your math facts?

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
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<tbody>
<tr>
<td>🙁</td>
<td>😐</td>
<td>😊</td>
<td></td>
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</table>

Would you recommend CCC/DPR to one of your friends?

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>🙁</td>
<td>😐</td>
<td>😊</td>
<td></td>
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</tbody>
</table>
Appendix F
Detect, Practice, and Repair - Intervention Fidelity Checklist

**Instructions for Intervention Fidelity Assessments:** Accurately record the intervention start time (use exact time and do not estimate). Place a checkmark in the “Completed” box for all steps that are accurately completed by the interventionist. If a step is not applicable, then place an “N/A” in the box. Accurately record the intervention end time (use exact time and do not estimate). Calculate the treatment integrity rate.

**START THE TIMER!!!!**

<table>
<thead>
<tr>
<th>Intervention Steps</th>
<th>Completed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass out student folder.</td>
<td></td>
</tr>
<tr>
<td>Say “Please take out your DPR math packet and place it facedown.”</td>
<td></td>
</tr>
<tr>
<td>Say “Today we will be working on increasing how quickly and accurately we can do basic fact problems. Remember, it is important to get the problems right. If you use your fingers or count in your head, you will not have enough time to complete the problems.”</td>
<td></td>
</tr>
<tr>
<td>Say “When I say ‘Begin’ you will complete one problem for every tick that you hear. You will hear a tick every 1-2 seconds. If the problem is not finished by the next tick, then you must go on to the next problem. When I say ‘Stop’ you will put down your pencils.”</td>
<td></td>
</tr>
<tr>
<td>Set metronome at 30 beats per minute. Say “Go.”</td>
<td></td>
</tr>
<tr>
<td>Monitor student performance. If the student is struggling, prompt him/her to complete problems with the tap of the metronome to ensure that the correct items are identified.</td>
<td></td>
</tr>
<tr>
<td>Once the time has surpassed, say “now we are going to find and circle the first five problems that have been left blank, so that we can practice these problems.”</td>
<td></td>
</tr>
<tr>
<td>Help any student that does not have five blank items to identify five missed items (if needed). Refer to the large poster boards, as needed.</td>
<td></td>
</tr>
<tr>
<td>Say, “Turn to the second page of your packet and write the five circled problems down the column called Problem and Answer.”</td>
<td></td>
</tr>
</tbody>
</table>
Monitor student performance. Prompt struggling students and refer them to the large poster board (as needed).

Say “Look at the first problem and say it quietly to yourself. Then, cover this problem with your hand and write the problem and answer. Then, pick up your hand and check to see if you wrote down the correct problem and answer. Do this five times for each problem.”

Praise the student for accurately doing problems and for correctly using procedures. Prompt the student if he/she is struggling with the problems or procedures.

Once the student has completed the CCC procedures, say “Turn to the next page in your packet. You will now complete as many problems as you can in 1 minute. When I say ‘Begin’ start at the top of the page and go across the page. Put an “X” over any problems that you cannot do. When I say ‘Stop’ you will put down your pencils.”

Begin 1-minute timing. When the one minute expires, Say “Stop.”

Say “Count each number that you have written and write the total number at the top of the page.”

Say “Now you will plot that number onto your graph.”

Circulate around the room to make sure that the student’s answers and charting practices are accurate. Help the student if he/she is in need of assistance.

Have the student report if he/she has beaten a previous score so that they can receive praise.


<table>
<thead>
<tr>
<th>Total Steps Completed</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Steps Possible</td>
<td></td>
</tr>
<tr>
<td>Intervention Fidelity Percentage</td>
<td></td>
</tr>
</tbody>
</table>

Total Time (Minutes & Seconds): ______________________
Appendix G

Cover, Copy, and Compare - Intervention Fidelity Checklist

Instructions for Intervention Fidelity Assessments: Accurately record the intervention start time (use exact time and do not estimate). Place a checkmark in the “Completed” box for all steps that are accurately completed by the interventionist. If a step is not applicable, then place an “N/A” in the box. Accurately record the intervention end time (use exact time and do not estimate). Calculate the treatment integrity rate.

START THE TIMER!!!!

<table>
<thead>
<tr>
<th>Intervention Steps</th>
<th>Completed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass out the CCC Worksheet</td>
<td></td>
</tr>
<tr>
<td>Copy down five unknown math facts (problems and answers) from the “unknown” fact pile onto the white board.</td>
<td></td>
</tr>
<tr>
<td>Say, “Write these five problems and answers down the first column called Problem and Answer.”</td>
<td></td>
</tr>
<tr>
<td>Monitor student performance. Prompt struggling students and refer them to the white board (as needed).</td>
<td></td>
</tr>
<tr>
<td>Say “Look at the first problem and say it quietly to yourself. Then, cover this problem with your hand and write the problem and answer. Then, pick up your hand and check to see if you wrote down the correct problem and answer. Do this five times for each problem.”</td>
<td></td>
</tr>
<tr>
<td>Circulate around the room. Praise the student for accurately doing problems and for correctly using procedures. Prompt any students that may be struggling with the problems or procedures.</td>
<td></td>
</tr>
</tbody>
</table>

| Total Steps Completed |           |
| Total Steps Possible  |           |
| Intervention Fidelity Percentage |           |

Time Total (Minutes & Seconds): _____________________