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Student's name: Daniel R Cuppoletti

This work and its defense approved by:

Committee chair: Ephraim Gutmark, Ph.D., D.Sc.

Committee member: Steve Martens, Ph.D.

Committee member: Awatef Ilamed, Ph.D.

Committee member: Jeffrey Kastner, Ph.D.

Committee member: David Munday, Ph.D.

Committee member: Mark Turner, Sc.D.
Supersonic Jet Noise Reduction with Novel Fluidic Injection Techniques

A dissertation submitted to the Graduate School of the University of Cincinnati in partial fulfillment of the requirements for the degree of

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by

Daniel Roman Cuppoletti

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Abstract

Supersonic jets provide unique challenges in the aeroacoustic field due to very high jet velocities, shock associated noise components, flow dependence on jet expansion, and stringent performance requirements. Current noise suppression technology for commercial and military jet engines revolves around using chevrons or mechanical vortex generators to increase mixing near the nozzle exit, subsequently reducing peak turbulence levels in the mixing region. Passive noise control methods such as mechanical chevrons cause thrust loss throughout the flight envelope and performance can vary with the engine operating condition. Development of active noise control methods have the potential of improved performance throughout the flight envelope and the benefit of being deactivated when noise control is unnecessary. Fluidic injection of air into a supersonic jet is studied as an active control method with an emphasis on understanding the physics of the problem and identifying the controlling parameters.

An experimental investigation with computational collaboration was conducted to understand the effect of nozzle design on supersonic jet noise and to develop various fluidic injection techniques to control noise from a supersonic jet with a design Mach number of 1.56. The jet was studied at overexpanded, ideally expanded, and underexpanded conditions to evaluate the effects throughout the operational envelope. As a passive noise control method, the internal contour of a realistic nozzle was modified to investigate the effect on acoustics and performance. Thrust was improved up to 10% with no acoustic penalties through nozzle design, however it was found that the shock noise components were highly sensitive to the shock structure in the jet. Steady fluidic injection was used to generate vorticity at the trailing edge of the nozzle showing that noise reduction is achieved through vorticity generation, modification
of the shock structure, and interference with the screech feedback mechanism by decoupling the phase relationship between jet turbulence and shock spacing. Reduction of shock noise was found to be optimum at an intermediate injection pressure due to shock weakening from the fluidic injectors and injector interactions with the jet shock-expansion structure. Large-scale mixing noise reduction was shown to depend on the vorticity strength and circulation. Unprecedented reduction of OASPL up to -8.5 dB were achieved at the peak noise direction through strong jet mixing and rapid collapse of the potential core. Pulsed fluidic injection was investigated to understand the acoustic benefits and drawbacks of unsteady injection. Valve frequency response up to 500 Hz was achieved but noise reduction dropped off above 100 Hz due to poor flow response as verified by hot-wire and dynamic pressure measurements. At low pulse frequencies it was found that moderate noise reduction could be achieved with less flow than steady injection, but in general the mixing noise reduction scaled with the time integrated mass flow injection. It was discovered that the different components of supersonic jet noise had different characteristic response times to unsteady injection. Analysis of high speed shadowgraph images and acoustic spectra was used to identify time response of the jet during the unsteady injection cycle. Development of a code to apply the Effective Perceived Noise Level standard to laboratory jets was used to evaluate the various noise reduction methods. It was shown that ΔEPNL was affected most strongly by the sideline spectrum and the jet shock noise since EPNL is dominated by the maximum noise during a flight event, which almost always occurs when the aircraft passes nearest to the observer. The methods of noise reduction show potential for reduction of EPNL up to -4 dB and 50% reduction in the noise footprint area near the jet take-off location.
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This work is dedicated foremost to my parents whom instilled in me an profound love of learning and taught me the value of perseverance through the life of Leonardo da Vinci. And to my dearest love Kasey for supporting and encouraging me every waking moment in all personal, professional, and cognitive endeavors; you truly are my soul’s counterpart.
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Nomenclature

\( \dot{m} \) mass flow rate  \( T \) temperature or pulse period

\( \dot{Q} \) volume flow rate  \( t \) time

\( A \) area  \( TR \) temperature ratio \( T_o/T_a \)

\( C_d \) Discharge Coefficient \( \dot{m}/\dot{m}_i \)  \( u, v, w \) velocity component

\( D \) diameter  \( U_j \) fully expanded jet velocity

\( DC \) duty cycle  \( U_{norm} \) actual duty cycle

\( F \) thrust force  \( x, y, z \) cartesian coordinates

\( f \) frequency  CAA computational aeroacoustics

\( J \) momentum flux ratio \( \rho u^2/\rho_j u_j^2 \)  EPNL effective perceived noise level

\( K \) acoustic power coefficient  LES large eddy simulation

\( l \) length  NB narrowband

\( M \) Mach number  OASPL overall sound pressure level

\( NPR \) nozzle pressure ratio \( p_o/p_a \)  PIV particle image velocimetry

\( p \) pressure  RANS Reynolds Averaged Navier Stokes

\( R \) Radius  TKE turbulence kinetic energy

Subscripts

xxviii
mean condition
freestream condition
ambient or acoustic condition
convective or cutoff
design condition
exit condition
ground location
ideal, initial, or injection condition
jet condition
momentum component
total condition
pressure component
secondary flow condition
throat condition

Symbols

$\delta_{ij}$  Kronecker delta function
$\gamma$  specific heat ratio
$\phi$  Kirchoff integrated variable
$\phi$  yaw or phase angle
$\psi$  observation (microphone) angle
$\rho$  density
$\sigma$  standard deviation
$\tau_r$  retarded time
$\theta_{inj}$  injection angle or momentum thickness

Superscripts

$'$  fluctuating component
Dissertation Overview

This dissertation combines various investigations aimed at furthering the understanding and reduction of supersonic jet noise. This work served as a continuation of jet noise reduction work that started at UC over a decade ago. Chevrons are the current state of the art technology for jet engine noise reduction that are currently in use. Future noise regulations press for further reduction in aviation noise and current technologies are not sufficient to meet the demands. This dissertation work is primarily experimental although it includes collaborations with a computational group from Chalmers University. The bulk of the research is organized into three sections: 1) Baseline supersonic jet flow & acoustics and geometric nozzle redesign for performance and noise reduction, 2) steady fluidic injection for noise reduction with many novel injector configurations, and 3) pulsed fluidic injection for noise reduction to investigate benefits of pulsing the injection flow. The remaining chapters present near-field pressure results comparing the various noise reduction methods and development of a method to apply the Effective Perceived Noise Level standard to laboratory jet measurements for analyzing the numerous noise reduction methods. The findings presented shed new light on how nozzle design and fluidic injection affect the various components of jet noise and jet performance. Great effort was made to clarify many of the unclear results that exist in the literature due to the numerous nozzle designs and fluidic injection configurations that have been studied. Fluidic injection has the potential to provide greater noise reduction than passive technologies and has the added benefit of being applied only when necessary, reducing performance loss concerns that must be contrasted with system impacts.
Chapter 1: Introduction & Background

A background of classical jet noise research, jet instability theory, aeroacoustic theory, and more is covered in this chapter. The reader is provided with a fundamental understanding of turbulent jet flows and the mechanisms by which aerodynamic flows generate noise. Lighthill’s ground-breaking work on the aerodynamic generation of sound is the foundation of aeroacoustics and Lighthill’s acoustic analogy paved the way for computational fluid dynamics to predict sound generation. An overview of subsonic and supersonic jet noise research is presented along with an overview of noise reduction methods currently in use on production aircraft. The most successful noise reduction technology was the invention of the turbofan engine which decreased turbojet noise by -20 EPNdB by moving a higher mass flow at lower velocity and reducing jet shear velocities. Chevrons have become commonplace on new commercial engines but have been less successful on tactical nozzles with variable expansion levels. Existing literature on fluidic injection for noise control is reviewed along with some of the physics involved. Much of the existing research has been conducted on subsonic jets with up to -2.7 dB reduction in OASPL. Results on supersonic jets has been mixed, with some investigations reporting increased noise and others reporting noise reduction up to -2 dB. Lastly, a review of unsteady fluidic injection on jets is covered and it is found that many studies only report on the flow and mixing characteristics with unsteady injection, few results on acoustics exist. The acoustic results that exist span an array of jet velocities, actuation frequencies, and actuation methods. Acoustic results show that unsteady actuation near the preferred jet frequency results in tonal response of the jet and noise increases. Studies on unsteady actuation frequencies below the preferred jet frequency report inconclusive acoustic results or no acoustic benefit.

Chapter 2: Facility and Methods

An overview of the experiment test facility and experimental methods is provided in this chapter. Operation and control of the jet rig is covered along with details on the common
operating conditions for the jet and fluidic injectors. An overview of the principal measurement techniques is given, including far-field acoustics, near-field pressure, particle image velocimetry, and high-speed shadowgraph. Data processing and experimental uncertainty are addressed for all measurement techniques. This chapter also includes details on the large eddy simulations and computational aeroacoustics conducted at Chalmers University. Details of the collaboration between the computational groups and the experimental group at UC is provided.

Chapter 3: Baseline Supersonic Jet

In order to study noise reduction technologies, it was necessary to characterize and describe the flow field and acoustics for the baseline supersonic jet at design and imperfectly expanded conditions. Optimization of the converging section and throat of the baseline nozzle was conducted to understand the effect of nozzle design on flow, acoustics, and thrust. Shadowgraph was used to characterize the double shock structure of the baseline jets at all operating conditions. Mean velocity and turbulence for the baseline nozzles is presented to show how nozzle contour affects the jet flowfield. Complete thrust analysis, including momentum and pressure thrust, is presented for both nozzle designs. The experimental measurements are complemented by computational results from Chalmers University. Additionally, the effect of faceted surfaces of realistic nozzles was studied to determine the sensitivity of flow and acoustics to azimuthal variation in the nozzle contour.

Chapter 4: Steady Fluidic Injection

This chapter focuses on the use of steady fluidic injection on supersonic jets for noise reduction and accounts for the bulk of the dissertation research. The research is presented in two sections including two different fluidic injection approaches. In the first section a trailing-edge fluidic injection nozzle with twelve pairs of slotted injectors, twenty-four total, is studied and compared to the baseline and a chevron nozzle. Jet spreading and vorticity are in agreement
with other researchers and provides insight into the benefits and drawbacks of fluidic injection compared to chevrons. The second section investigates external fluidic injection by mounting injectors directly at the nozzle exit at four injection angles relative to the jet axis. An extensive acoustic survey shows the effect of momentum flux ratio for 6 and 12 injector configuration on the supersonic jet noise components. Clear trends with injection parameters in mixing noise and shock noise components are identified. Flow measurements including shadowgraph and PIV show that mixing noise reduction is due to strong jet spreading and rapid collapse of the potential core due to strong circulation introduced from streamwise vorticity. The shock noise components are reduced by shock weakening, and at certain injection conditions shock noise can increase above baseline values. A comparison of the measurements with current theory indicates that the shift in shock noise frequency is due to a combination of decrease in shock cell spacing and decrease in convective velocity of the large-scale coherent structures. Unprecedented noise reduction of OASPL up to -8.5 dB in large-scale mixing noise and -6 dB in shock noise is achieved which is far greater than any results currently published in the open literature.

Chapter 5: Pulsed Fluidic Injection

Unsteady control of jet flows has been studied extensively in the past century, often by excitation with speakers or acoustic sources and measurement of jet response. Excitation by acoustic sources was most successful on low speed jets due to the low energy required to excite the jet, but is less applicable for high Reynolds number jets. Considerable efforts to develop novel actuators including mechanical, piezoelectric, and plasma actuators have shown some interesting results, although often the studies focus on jet mixing and neglect acoustics. The acoustic investigations showed that excitation near the jet preferred frequency resulted in strong tonal noise radiation to the far-field which is impractical for noise reduction. In this chapter a pulsed fluidic injection system was developed with available compact valves that could provide mass flows on the order of the steady injection conditions at frequencies
up to 500 Hz, which is $St_{Dj} = 0.06$ at the jet design Mach number. The concept behind the pulsed injection was to two fold. Primarily, it was desired to investigate the effect of frequency and duty cycle on supersonic jet acoustics while reducing the required mass flow rate through unsteady mass addition. Secondly, an effort was made to provide conclusive results of pulsed fluidic injection on supersonic jet acoustics and identification of clear trends with frequency and duty cycle. Large-scale turbulent mixing noise was found to scale with the time integrated mass flow addition to the jet, while 30 Hz was able to provide equivalent noise reduction as steady injection with less flow. Findings on the shock wave response time and screech response time due to pulsation were exposed. Fluidic injection stabilized shock motion in the jet during injection and destabilized again in between pulse cycles. The shocks were found to have a characteristic response time with screech instability growth having the longest response time to reach saturation.

Chapter 6: Internal Fluidic Injection

A novel application of fluidic injection was developed to investigate and understand the effects of discrete fluidic injection internal to the jet nozzle. Various injection locations, angles, and conditions were studied resulting in unique acoustic behavior and flow field modifications. For most conditions the acoustics are relatively unaffected or increased, but for very specific conditions noise was drastically decreased. For optimized conditions the shock noise was completely eliminated and in other cases a jet instability was generated that significantly decreased high frequency noise. Measurements of the velocity field indicated that shock interaction due to shocks from the injection jet interact with the primary jet shocks and significantly reduce the shock strength, attributing to the shock noise reduction. Validation of the unconventional experimental results was achieved with LES computations which provided additional insight into the shock suppression. Optimization of the injection parameters resulted in reduction of OASPL near -7 dB at the upstream and downstream angles simultaneously through a combination of shock disruption and streamwise vorticity introduction.
Chapter 7: Near-Field Pressure

Near-field pressure measurements provide valuable information on peak sound pressure levels near the jet, noise directivity, and noise footprint near the jet. The near-field region contains both hydrodynamic and acoustic pressure fluctuations and provides insight into the dominant noise sources of the jet. The near-field measurements were conducted over a grid extending 28 jet diameters along the jet shear layer and 9 jet diameters radially outwards. Comparison of the baseline nozzle, splined nozzle, trailing-edge injection, and $\theta_{inj} = 90^\circ$ are presented for the the supersonic noise components. Fluidic injection decreased the peak noise levels for large-scale mixing noise, shock noise, and screech. Increases in fine-scale mixing noise were observed near the nozzle exit with decreases downstream beyond $x/Dj = 10$. Fluidic injection suppressed screech noise by -20 dB in the near nozzle region while up to -10 dB for shock noise was achieved. The reductions in near-field pressure correlate well with reductions in the far-field and implies that drastic reduction in the sound levels near an aircraft can be achieved, benefiting the health and safety of personnel that work in close proximity to supersonic jet flows.

Chapter 8: EPNL Calculation

Effective perceived noise level is an aircraft certification measure used by the FAA and global agencies to regulate aircraft noise levels. The measurement standard is unique in that it includes weighting for frequency sensitivity of human hearing, corrections for tonal components, and a duration factor for the length of a noise event. Weighting an acoustic measurement for human hearing is relatively simple, but tonal and duration factors are often disregarded. Aircraft tonal noise is dominated by the turbomachinery noise, however, the corrections prove useful for the tonal noise that is present in supersonic jet noise. A procedure and code was developed to apply the EPNL measurement standard to laboratory measurements by simulating a flight path and scaling the acoustic measurements for each time interval in the flight path. The value of EPNL is an approximation since we are only considering the
contribution from jet noise. However, the change in EPNL, $\Delta$EPNL is useful for evaluating the various noise reduction technologies with one representative metric. The EPNL metric chooses the peak corrected sound level and adds penalties for tone and duration. A take-off flight path is detailed and simulated and a ground mapping of 12 km by 2 km near the runway is simulated. The results showed that for the specific simulated conditions, EPNL is dominated by shock noise which dominates the sideline spectra. EPNL reduction of up to -4 dB can be achieved using the most successful shock noise reducing method for the simulated flight path and ground mapping. The conclusions could change based on the flight path and jet operating condition, especially if flight conditions are accounted for. The developed code is modular and can be applied to any far-field acoustic measurement and flight path.
Aeroacoustics is the scientific field concerned with noise generation by turbulent fluid flows, unsteady flow phenomena, or aerodynamic interaction with surfaces. Jet noise is a research subset of the aeroacoustics field primarily involving noise generation from high Reynolds number jets, characteristic of modern commercial and military propulsion systems. The high gas velocities associated with propulsion systems of modern aircraft result in noise generation from turbulence that occurs when the jet mixes with the surrounding fluid. For subsonic jets, with jet velocities below the local speed of sound, turbulent mixing noise is the dominant noise generation mechanism. Turbulent mixing noise is a broadband noise source related to the wide range of time and length scales in a turbulent flow with a broad range of frequencies and wavelengths. For supersonic jets, with jet velocities above the local speed of sound, additional noise components arise from shock waves in the jet stream. The turbulence interaction with the shock waves radiates sound, contributing to strong broadband and tonal noise in addition to turbulent mixing noise. The shock-turbulence interactions are dominated by the large-scale coherent structures containing the most energy and at certain conditions a feedback mechanism can generate eddy shedding at the nozzle lip that phase locks with the upstream traveling acoustic waves.

The complexities of supersonic jet noise phenomena provide unique research challenges with practical applications of concern to the engineering community. Supersonic jet noise is
of research interest to academic and applied fields alike with issues such as further understanding of noise generation physics, developing noise reduction technologies, environmental noise pollution, and the health of workers in the civilian and military aviation sectors. The arena of supersonic flight is currently only reserved for military flight and supersonic flight over land is prohibited in the United States, especially near populated areas, due to the strong sonic boom generated from the aircraft. If the vision of supersonic civilian transport is to be realized, it is without a doubt that supersonic jet noise will be a limiting factor as noise intensity scales approximately with the 8\textsuperscript{th} power of jet velocity\cite{1}. Research in jet noise reduction is focused on developing solutions to reduce the high noise levels of modern propulsion devices and on furthering the understanding of how noise is generated from aerodynamic flows. This dissertation focuses on reducing supersonic jet noise through passive and active control techniques while aiming to provide further insight into noise generation mechanisms.

1.1 Jet Instability Theory

Noise generation from aerodynamic flows is inherently linked to flow disturbances that manifest as acoustic fluctuations. In free shear flows, these disturbances are mostly due to turbulence generated in high shear regions of the flow. Classical turbulence theories described turbulent flow as “having irregular motion with random variation in time and space with statistical properties and a wide range of scales”\cite{2}. The work of Brown and Roshko\cite{3} redirected the classic view of completely random turbulence by observing large-scale coherent structures in incompressible turbulent mixing layers. Mixing layers, shown schematically in Figure 1.1, can be used to study mixing of two flows and are generated by separating flows of differing density and/or velocity with a splitter plate. The concept of large-scale turbulent structures has come to dominate turbulence and aeroacoustic theories for all types of turbulent flows. The large-scale structures are important for global mixing of flows and contain the largest amount of energy in the flow. Winant and Browand\cite{4} noted a phenomenon in a turbulent shear layer such as vortex ‘pairing’ or ‘amalgamation’ of the turbulent structures similar to
that observed by Freymuth\cite{Freymuth5} in laminar mixing layers. Comparisons of laminar and turbulent mixing layers are shown in Figure 1.2. These large-scale coherent structures develop in the jet from the initial instability waves which roll up into vortex structures. The vortex pairing causes vortex growth and entrainment which result in spreading of the shear layer. The laminar and turbulent mixing layers are both dominated by the large-scale vortex structure that forms although a wide range of vortex sizes are apparent in the turbulent mixing layer. The large-scale vortex structures grow in the flow direction while the fine-scale structures have a more uniform size throughout the mixing layer. The initial instability begins as a Kelvin-Helmholtz instability and has been shown experimentally\cite{Freymuth5} and theoretically\cite{Freymuth6} to range from $0 < St_{\theta_i} = f_i \theta_i / U_0 < 0.04$ where $f_i$ is the initial instability frequency and $\theta_i$ is the initial momentum thickness of the shear layer. The initial instability frequency decreases in the flow direction as vortex pairing occurs, resulting in a decrease in peak frequency along the shear layer.

An axisymmetric jet is a rotational version of a 2D mixing layer, but with marked differences in flow structures due to the additional length scale of the jet diameter $D_j$. The vortex structures that form in the initial shear layer of a laminar jet are toroidal vortices that are ring-shaped around the axis of the jet, as seen in Figure 1.3. The toroidal vortices undergo pairing and tearing just as they do in a mixing layer, eventually resulting in 3D turbulent structures that are strongly rotational. The length of this transitional region becomes shorter with increasing Reynolds number. Therefore in high Reynolds number jets the shear layer is fully turbulent immediately downstream of the nozzle lip as illustrated in Figure 1.4. The rotational flow in the jet shear layer eventually spreads and coalesces at a distance downstream, collapsing the irrotational region of the jet known as the jet ‘potential core’. In the region near the end of the potential core, a range of preferred frequencies based on $D_j$ have been observed by researchers. A review article by Ho & Huerre\cite{Ho7} established that the preferred Strouhal number ranged from $0.25 < St_{D_j} = f_j D_j / U_j < 0.5$ in different experiments. Gutmark & Ho\cite{Gutmark8} postulated that the spread in preferred frequency was due to low amplitude
Figure 1.1: A 2D mixing layer showing large turbulent structures from Tam[10].

disturbances depending on various facilities. Crow and Champagne[9] forced a jet at 2% rms of the mean jet velocity with a loudspeaker at various frequencies and found that the most amplified, or preferred frequency, as determined by the maximum response of the jet column was $St_{Dj} = 0.3$.

At this point, the discussion was focused on work in incompressible mixing layers and shear layers. The convection velocity, $U_c$, or convective Mach number, $M_c$, are terms referring to the speed at which large-scale coherent structures travel in the flow. The turbulent structures in compressible shear layers have a convection velocity lower than the jet velocity as observed by many researchers. Although in actuality the convection velocities of turbulent structures of various length scales are different, the convective velocity concept is related to velocity of the dominant large-scale coherent structures containing the greatest energy. Bogdanoff [13] showed that by equating the dynamic pressures across the shear layer, a convective Mach number can be defined by Eq. 1.1 related to the speed at which the large scale structures travel in the shear layer. Papamoschou & Roshko[14] expanded on Bogdanoff’s derivation to account for the possibility of different specific heat ratios for the two streams. This led to Eq. 1.2 for the convective velocity

$$M_c = \frac{u_1 - u_2}{c_1 + c_2}$$  \hspace{1cm} (1.1)
Figure 1.2: Shadowgraph visualizations of large-scale coherent structures in laminar and turbulent mixing layers.

Figure 1.3: Jet instabilities for a low Reynolds number axisymmetric jet $u_j = 2\,m/s$ visualized with smoke tracer, adapted from Wille[11] quoted in Michalke[12].
Figure 1.4: Illustration of jet instability with increasing Reynolds number (a) to (d) from Crow & Champagne [9].

\[ u_c = \frac{c_2u_1 + c_1u_2}{c_1 + c_2} \]  

(1.2)

where subscripts 1 and 2 denote each stream. For the case of a single jet exhausting into quiescent air relevant to the experiments in this dissertation, where the convective Mach number is \( M_c = u_c/c_\infty \), it is easily shown that both of these definitions simplify to Eq. 1.3 (i.e. \( u_1 = u_j, c_1 = c_j, u_2 = u_\infty \approx 0 \) and \( c_2 = c_\infty \)).

\[ M_c = \frac{u_c}{c_\infty} = \frac{c_\infty u_j}{c_\infty (c_j + c_\infty)} = \frac{u_j}{c_j + c_\infty} \]  

(1.3)

The spreading rate of compressible shear layers is important for mixing which directly affects turbulent mixing noise. Papamoschou & Roshko [14] showed that the compressible spreading rate for a planar shear layer decreases to 0.4 of an incompressible spreading layer as determined by vorticity thickness, shown in Figure 1.5a. Schadow & Gutmark reported similar behavior for compressible shear layers of circular and rectangular jets as illustrated in Figure 1.5b. Reduction in spread rate to 0.2 of the incompressible spread rate was observed
indicating an even greater level of stability of jet shear layers. The convective Mach number is convenient for discussing phenomena related to noise generation in high Reynolds number jets since it relates to the propagation velocity of the turbulent structures. The convective Mach number and jet spread rate are fundamental in understanding the various noise characteristics including the peak frequency of broadband shock noise, screech frequency, and when Mach wave radiation will occur, as discussed in section 1.6.

1.2 Lighthill’s Theory of Aerodynamic Noise

Since the advent of the jet engine there have been concerns and issues with the high sound pressure levels generated from high-speed jet engine exhausts. Within a decade of Sir Frank Whittle and Dr. Hans von Ohain independently inventing the turbojet engine around 1940, researchers began seriously pondering aerodynamic noise generation from high velocity gas streams. By 1950, issues surrounding jet noise prompted Sir James Lighthill to formulate a theory on the aerodynamic generation of sound, which has become the foundation of the aeroacoustics field. In a two-part paper, Lighthill developed and validated a theory of sound generation for turbulent jets in a static surrounding medium[1, 16]. Lighthill showed that the sound intensity was proportional to \( U_j^8 \), the eighth power of the jet velocity. His theory also adequately explained observed phenomena such as convective amplification that is primarily
responsible for the directivity of jet noise and other aerodynamic noise generation. Lighthill’s pioneering work still serves as the foundation of jet noise research, albeit with many variations and modifications that have extended it into theoretical, experimental, and computational arenas of aeroacoustics.

Lighthill mathematically approached the jet noise problem by reformulating the Navier-Stokes equations for compressible fluid flow as a wave equation with density as the operand equated to a source term. The derivation begins with the viscous, compressible forms of continuity (Eq. 1.4) and momentum (Eq. 1.5) equations

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0 \tag{1.4}
\]

\[
\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} - \frac{\partial}{\partial x_j}(p\delta_{ij} - \tau_{ij}) = 0 \tag{1.5}
\]

where \(\delta_{ij}\) is the Kronecker delta function and \(\tau_{ij}\) is the viscous stress tensor.

Taking the difference between the temporal derivative of continuity and the spatial derivative of momentum, the combined equation becomes

\[
\frac{\partial^2 \rho}{\partial t^2} - \frac{\partial^2 \rho u_i u_j}{\partial x_i x_j} - \frac{\partial^2}{\partial x_i x_j}(p\delta_{ij} - \tau_{ij}) = 0 \tag{1.6}
\]

This can be formulated as a wave equation by adding the terms \(c_0^2 \frac{\partial^2 \rho}{\partial x_i^2} - \frac{\partial^2}{\partial x_i x_j} (c_0^2 \rho \cdot \delta_{ij})\), neglecting viscous stresses which are minimal in high Reynolds number flows, and rearranging all remaining terms to the RHS to obtain

\[
\frac{\partial^2 \rho}{\partial t^2} - c_0^2 \frac{\partial^2 \rho}{\partial x_i^2} = \frac{\partial^2}{\partial x_i x_j} [\rho u_i u_j + (p - c_0^2 \rho)\delta_{ij}] = 0 \tag{1.7}
\]

Since we are mainly concerned with flow variable fluctuations that contribute to sound generation, the flow variables are decomposed into their mean and fluctuating components \(p = p_0 + p', \rho = \rho_0 + \rho', \text{ and } u_i = u_{0i} + u_i'\), which are substituted into Eq. 1.7. Removing zero
quantities and neglecting small products of fluctuating variables, Lighthill’s wave equation becomes

\[
\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \frac{\partial^2 \rho'}{\partial x_i^2} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \tag{1.8}
\]

where the term \( T_{ij} \) is the Lighthill stress tensor and the second spatial derivative of \( T_{ij} \) represents the acoustic sources in the flow. The stress tensor is given as

\[
T_{ij} = \rho_0 u'_i u'_j + \left( p' - \rho' c_0^2 \right) \delta_{ij} \tag{1.9}
\]

where \( \rho' \), \( p' \) and \( u' \) are the fluctuating density, pressure, and velocity. The first term represents the turbulent velocity fluctuations and the second term represents the entropy fluctuations, which are generally orders of magnitude less than the turbulent fluctuations unless the flow has significant viscosity or heat conduction phenomena to consider[1].

Although Lighthill’s equation is derived from the exact formulation of the Navier-Stokes equations it has several limitations. The acoustic source term \( T_{ij} \) requires knowledge of the unsteady flow variables at all locations in the flow. Lighthill realized that this could be very difficult to obtain experimentally, so he introduced what has become known as Lighthill’s acoustic analogy, which replaces the unsteady flow with a volume distribution of acoustic sources. Lighthill’s acoustic analogy states that the acoustic sources may move even though the fluid containing them may not. The fluid containing the acoustic sources has fluid properties equal to the ambient conditions or fluid surrounding the flow. The use of Lighthill’s acoustic analogy is therefore only as accurate as the knowledge of the unsteady flow variables. Lighthill’s equation also does not account for flow-acoustic or flow-surface interactions. For cases such as these, other researchers have developed additional methods expanding upon on Lighthill’s theory such as Ffowcs-Williams Hawkings formulation which is better suited to handle these additional source terms. Lighthill’s equation has limited application in the transonic and supersonic flow regimes due to the linearity of the equations. With the evolution of
computational fluid dynamics (CFD) and increases in computing power, direct calculations of the unsteady flow have lead to explosive growth in the aeroacoustics field which has furthered our understanding of high Reynolds number jet flows.

Another major result of Lighthill’s ground breaking work was an approximation of the acoustic sound power radiated by jet flows. The sound radiation field, or density field, for distributed sources in an unbounded medium can be expressed as

\[ \rho - \rho_0 = \frac{1}{4\pi c_0^2} \int \frac{\partial}{\partial t} Q \left( y, t - \frac{|x - y|}{c_0} \right) \frac{dy}{|x - y|} \]  \hspace{1cm} (1.10)

For locations far enough in the radiation field to be considered in the acoustic far-field, at a distance much larger than a given wavelength, Eq. 1.10 can be related to Lighthill’s stress tensor as

\[ \rho - \rho_0 = \frac{1}{4\pi c_0^2} \int \left( \frac{x_i - y_i)(x_j - y_j)}{|x - y|^3} \right) \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} T_{ij} \left( y, t - \frac{|x - y|}{c_0} \right) dy \]  \hspace{1cm} (1.11)

Lighthill was able to show through a frequency analysis and some empirical knowledge of the frequency dependence of turbulent flows, that if one performs a dimensional analysis on Eq. 1.11 the density fluctuations are proportional to the 4th power of velocity as shown in Eq. 1.12

\[ \rho - \rho_0 \sim \frac{1}{c_0^2 \frac{1}{x} c_0^2} \left( \frac{U}{l} \right)^2 \rho_0 U^2 l^3 = \rho_0 \left( \frac{U}{c_0} \right)^4 \frac{l}{x} \]  \hspace{1cm} (1.12)

Combining Eq. 1.12 with the equation for sound intensity it can be shown that the sound intensity (acoustic power per unit area) scales with the velocity to the 8th power.

\[ I(\mathbf{x}) = \frac{P_{ac}}{A} = \frac{c_0^3}{\rho_0} \sigma^2 \left\{ \rho(\mathbf{x}, t) \right\} = \rho_0 U^{8} c_0 \frac{l^2}{x} \]  \hspace{1cm} (1.13)

This shows that the acoustic power \( P_{ac} \) radiating from a high velocity jet flow is mostly dependent on the jet velocity \( U^8_j \). For a characteristic length scale of a circular jet with
diameter \( D_j \), the acoustic power can be rewritten as

\[
P_{ac} = \frac{K \rho_0 U_j^8 D_j^2}{c_0^2} \tag{1.14}
\]

where \( K \) is an acoustic power coefficient to account for the approximate analysis employed to obtain the 8\(^{th} \) power relation. Lighthill noted that in application the approximations would vary with Mach number, Reynolds number, and temperature of the jet flow. Figure 1.6 provides recent experimental evidence for \( U_j^8 \) scaling for jets with temperature ratios up to \( T_{o,j}/T_a = 3.2 \). Good agreement is shown although more careful examination indicates that a \( U_j^n \) fitting with variation in the exponent \( n \) is more appropriate for a wide range of jets.

![Figure 1.6: Experimental evidence of sound power level variation with \( U_j^8 \).[17]](image)

### 1.3 Subsonic Jet Noise Research

The first jet-powered aircraft had turbojet propulsion systems with a single high velocity gas stream that provided thrust with a low mass flow of air accelerated to high velocity. Air enters the engine and the pressure is increased through the compressor stages. Energy is added to the high pressure flow by burning fuel in the combustion chamber. The high
pressure, high temperature air flows through a turbine that extracts energy from the gas to drive the compressor stages. These stages comprise the ‘gas generator’ of the turbojet engine. The excess energy in the gas is accelerated through a nozzle to propel the aircraft. A schematic of the flow in a turbojet is shown in Figure 1.7a with coloring representative of the gas temperatures through the engine. These early turbojet engines were very loud and required many engines to provide thrust for large aircraft. From Lighthill’s derivation that jet noise scales with $U^8_j$, it quickly became evident that the jet velocity was the prime culprit of the large sound pressure levels. In military applications, an afterburner, or augmentor, is commonly required on tactical jets to provide additional thrust to aid in short takeoffs or in evasive maneuvers. Figure 1.7b shows a turbojet engine with an afterburner. Large amounts of fuel are injected from the spraybars downstream of the turbine and combusted to provide additional temperature for thrust. In many cases the gas temperature more than doubles, and since velocity increases proportional to $\sqrt{T}$, velocity $U_j$ can be increased up to a factor of 1.5, resulting in significantly higher sound levels. A variable area nozzle is utilized on tactical fighter jet engines to accommodate the gas expansion during an augmented cycle, or for pressure matching throughout the flight envelope.

Arguably the largest advancement to date in noise reduction of aircraft engines is attributed to the invention of the turbofan engine which powers nearly all modern commercial aircraft. Momentum transfer due to the engine air mass flow and jet velocity provide most of the thrust for a jet engine. The principle of the turbofan engine, shown in Figure 1.7c is to use the gas generator to provide only a fraction of the overall thrust and utilize an additional turbine stage to power a large fan passing increased mass flow at lower velocity. Increasing the air mass flow and reducing peak velocity reduces noise and increases engine cycle efficiency. A time history of commercial aircraft engine noise levels through the last half of the 20th century is shown in Figure 1.8 courtesy of GE[20]. The evolution from turbojets to second generation turbofans resulted in up to -20 EPNdB reduction for the 1,500 feet sideline measurement. The bypass ratio is the ratio of the fan mass flow to the core mass flow BPR=$\dot{m}_f/\dot{m}_c$. Increasing
(a) Turbojet engine showing flows coloring for gas temperature. From FAA[18]

(b) Turbojet engine with afterburner. From Mattingly [19]

(c) Turbofan engine. From Mattingly [19]

Figure 1.7: Schematics of a turbojet engine with an afterburner and a turbofan engine with common station nomenclature.
BPR reduces noise levels, and modern commercial engines have reached BPR=10 and higher. The sound levels decrease with increasing BPR, but there are size constraints for turbofans since they usually hang underneath the wings, are near the aircraft body, or contained within the aircraft body. Further noise reduction are needed for commercial engines to comply with future international noise regulations. Supersonic commercial transport and military aircraft require even more aggressive reductions in jet noise for future viability, which is a major focus of this research.

1.4 Supersonic Jet Noise Research

Jet noise research has focused primarily on subsonic jets, resulting in the development of many passive noise control technologies, most notably chevrons, to reduce turbulent mixing noise. Chevrons are serrations in the trailing edge of the nozzle that protrude slightly into the jet flow to generate streamwise vorticity. Vorticity production enhances mixing of the jet flow and redistributes the turbulent energy to reduce the peak large-scale turbulent mixing noise. Figure 1.9a shows chevrons installed on a production Boeing 787 on the fan nozzle. Supersonic jets provide unique challenges for chevrons as the nozzle geometry is variable to account for various pressure ratios and jet temperatures. Jet velocities are much higher with sound pressure levels up to 40 dB louder than commercial engines. Supersonic jet noise contains shock related components which vary strongly depending on the operating condition and nozzle design. Optimizing chevrons on supersonic jets has been shown to alter the shock structure, reduce shock strength (shock noise), and interrupt the screech feedback mechanism[21, 22, 23, 24]. Figure 1.9b shows chevrons applied to a F404 engine as a retrofit on a static engine test. This is an afterthought to the engine development, with limitations such as the number of movable slats constraining the geometric design, number, and placement of the chevrons. This results in less than optimal noise control, and noise reduction should be integral to the system development of future engine designs. Continued growth of the airline industry and increasing reliance upon high performance aircraft for military operations has
resulted in the need for additional noise reduction technologies. Noise regulations are becoming increasingly strict in order to minimize the impact aviation has on our environment. The financial obligations of the FAA and the military to insulate homes, provide health care, and settle claims is mounting. According to a 2009 NRAC Report, the U.S Department of Veterans Affairs is spending upwards of $1 billion per year on hearing loss claims alone[25]. To allow further innovation in the aviation industry and to reduce the impact of jet noise on the environment and human health, significant noise reduction technologies must be developed concurrently.

In contrast to commercial engines, which have transitioned to high bypass ratio turbofans, military engines used in fighter jets and low profile aircraft have geometrical constraints that have kept bypass ratios low. Most of the bypass flow is used for cooling and mixed in the augmentor duct, essentially resulting in a turbojet engine. Figure 1.10 shows peak SPL for several aircraft that have been in service by the U.S. military since the 1970’s at maximum power with (A/B Power) and without (Mil Power) afterburner. The data was collected by the Joint Strike Fighter (JSF) vibroacoustics team during development and reported by the Navy in 2009[25]. Sound levels are provided for the downstream angle, $\psi = 135^\circ$, measured from the engine inlet direction. One of the most notable features is that over nearly a half-century the peak SPL has hardly changed. For both power settings, current generation aircraft have not reduced peak SPL in comparison to previous generation aircraft. Secondly, these aircraft have SPL vastly higher than commercial engines. Some of these aircraft are capable of launching from the decks of aircraft carriers, presenting a unique situation in which the flight deck personnel work in very close proximity to the aircraft and are subjected to noise levels much higher than personnel in the civilian sector. Cutting-edge hearing protection provides up to -30 dB of noise attenuation when worn correctly which allows for only 9 seconds of exposure to 150 dB noise in an 8-hour period according to OSHA standards[25]. Exposure to sound levels of this magnitude results in hearing loss and other issues that require medical treatment, resulting in financial obligations of the government to treat veterans for the detrimental health
effects.

Figure 1.8: Historical trend illustrating noise levels of commercial aircraft engines over 50 years. EPNdB is the Effective Perceived Noise Level measured in decibels. From Martens (GE) [20]

Figure 1.9: Chevrons on (a) a Boeing 787 and (b) a F404 engine (powers F/A-18 E/F). From U.S. Navy [25].
Figure 1.10: Peak sound pressure levels for modern tactical fighter jets at peak radiation angle ($\psi = 135^\circ$) for maximum (MIL) power and full afterburner (A/B) power. From US Navy [25]
1.5 Supersonic Jet Flows

The supersonic jet flow structures and dynamics govern sound generation and depend strongly on the nozzle design. The simplest illustration of a supersonic nozzle flow is an isentropic, shock-free, quasi-one-dimensional flow inside the nozzle. A decrease in area is required to accelerate a subsonic flow, whereas for supersonic flow an increase in area is required. This stems from the area-velocity relation in Eq. 1.15 that is derived from the differential form of the conservation equations. Examination of this equation shows that for $M < 1$ an increase in velocity requires a decrease in area ($dA < 0$), while for a $M > 1$ an increase in velocity requires an increase in area ($dA < 0$). This relationship is the reason that supersonic nozzles have convergent-divergent designs. Isentropic flow in the nozzle is governed by the Area-Mach number relation given in Eq. 1.16 illustrating that the Mach number is implicitly defined as a function of the local area to the throat (minimum) area at any location in the nozzle. The solution to Eq. 1.16 for a given area ratio provides two Mach numbers, a subsonic and supersonic solution.

\[
\frac{dA}{A} = (M^2 - 1) \frac{du}{u} \tag{1.15}
\]

\[
\left( \frac{A}{A^*} \right)^2 = \frac{1}{M^2} \left[ \frac{2}{\gamma + 1} \left( 1 + \frac{\gamma - 1}{2} M^2 \right) \right]^{(\gamma+1)/(\gamma-1)} \tag{1.16}
\]

Isentropic shock-free flow through a supersonic nozzle is illustrated in Figure 1.11 showing the variation of Mach number, pressure, and temperature through the nozzle, as illustrated in Anderson[26]. Mach number increases to sonic at the nozzle throat and supersonic in the diverging section. The static pressure and static temperature decrease to the critical ratios ($p/p_o = 0.528$ and $T/T_o = 0.833$) at the nozzle throat and lower in the supersonic regime. For a given nozzle design, one isentropic supersonic flow regime exists to provide perfect expansion at the nozzle exit and the isentropic equations hold true in this case. If the pressure ratio across a nozzle exhausting to atmosphere $p_o/p_a$ does not match the design pressure ratio, the
nozzle will be either overexpanded or underexpanded at the exit. It should also be noted that isentropic flow does not account for viscous and boundary layer effects or heat transfer, which can be significant in realistic jet engine flows.

In application, the operating condition and pressure ratio tend to vary. Near isentropic flow only exists in special situations in which the operating conditions are constant or variable area nozzles can adjust to the operating conditions accordingly. For rocket engines, the nozzle geometry is usually fixed, and the nozzle is designed to be fully expanded at a certain point in the flight envelope. The nozzle exit pressure matches atmospheric pressure at only one point during flight as atmospheric pressure decreases with altitude. In jet engine applications, variable area nozzles have been employed to vary the area in flight to match the operating condition as closely as possible. Figure 1.12a is a simplified schematic of a realistic fixed geometry nozzle with geometric definitions. Figure 1.12b is a complex schematic of a realistic variable area nozzle with a number of mechanical components required to expand and contract the nozzle. The nozzles in this study are fixed geometry and the operating condition is varied to study imperfect expansion.

An understanding of the fundamental behavior of overexpanded and underexpanded jet flows is essential for analyzing noise reduction methods on supersonic jets. Pressure or boundary mismatch generates shock or expansion waves at the nozzle exit. Figure 1.13 shows an isentropic nozzle in which the flow is overexpanded at the exit. Schematics of the flow structures associated with noise generation are sketched including representations of the sound waves generated. The shock and expansion waves in the potential core are shown in red and blue, bounded by the shear layer. The shock and expansion waves reflect from the jet centerline in a like manner and from the free boundary in an opposite manner. The shocks are three dimensional and conical in a round jet. This continues within the region of supersonic flow until the shocks can no longer be physically sustained in the jet flow. Fine-scale turbulent structures in the thin initial shear layer generate high frequency noise that dominates the upstream and sideline acoustic spectra. The turbulent length scales increase in the jet flow.
Figure 1.11: Isentropic supersonic flow through a converging-diverging nozzle[26].
direction as discussed in Section 1.1. The turbulent structures interact with the shock waves, perturbing the shocks and the turbulent structures, giving rise to a broadband spectral peak in the acoustic spectra and an increase in fine-scale mixing noise at the sideline and upstream angles[27, 28]. The large-scale structures are convected downstream in the shear layer and radiate high amplitude sound at a preferred angle downstream. For very high-speed jets intense Mach wave radiation can be generated from the large-scale turbulent structures. Intense Mach wave radiation was not studied in this work since the convective velocity $u_c < 1$ for most conditions in this study. Figure 1.14 shows photos of a screeching Mach 1.4 underexpanded jet and and intense Mach wave radiation for a Mach 2 underexpanded jet. The jet shock structures, wide range of turbulent length scales, and intense acoustic waves are all apparent in these images.
Large-scale coherent structures are clearly apparent in low-speed flows as discussed in Section 1.1 on jet instability theory. There is evidence indicating that these structures are important in high Reynolds number flows. It is much more difficult to identify the large-scale structures when embedded in a strongly turbulent flow containing a wide range of turbulent length scales as seen in Figure 1.14. Lighthill’s aeroacoustic theory showed that for a simplified turbulent jet the acoustic sound power scales with $U^8_j$ for a turbulent jet flow. Viswanathan[17] and Tam[31] presented experimental data and arguments supporting a $U^n_j$ power law for practical jets in which the exponent can be empirically determined for jets at very high velocity, at elevated temperature ratios, and at different observation angles. Tam et al.[32] reported experimental evidence from multiple investigators that supports the $n^{th}$ power law, and two figures illustrating this are shown in Figure 1.15. The exponent is very near $n = 8$ at sideline angles for a $T_{o,j}/T_a = 1$ jet, but at angles above $120^\circ$ the exponent increases up to $n = 10$. This suggests the existence of two very different source mechanisms that differ in efficiency of radiating noise. For a heated jet $T_{o,j}/T_a = 2.7$ the exponent varies from $n = 6.5$ upstream to $n = 9$ downstream, indicating that Lighthill’s $U^8_j$ law is invalid at elevated temperatures.

Empirical evidence supports theories that turbulent mixing noise is related to two distinctly different noise sources. Tam[33] empirically developed two similarity spectra, fine-scale ($G$)
and large-scale ($F$) similarity spectra, that fit most jet noise data that are free from shock noise or additional noise sources. The $G$ spectra compares well with spectra up to $\psi = 120^\circ$ and has a more broad peak. The $F$ spectra compares well with the spectra onward from $\psi = 140^\circ$ and has a sharper peak and steeper drop off around the peak frequency. Intermediate spectra can be fit with a combination of both spectra. For comparisons of the similarity spectra with the current jet noise rig data, see Section 4.1.3. Two similarity spectra that fit a wide array of jet noise data again supports a two-source model for turbulent mixing noise. Panda et al.\cite{34, 35} reported the first successful direct correlations of $p'$ with turbulent fluctuations in subsonic and supersonic jets. Figure 1.16 taken from Tam\cite{32} shows variation of $<\rho'uu, p'>$ and $<\rho', p'>$ correlations with observer angle, denoted by $\theta$, which is equivalent to $\psi$ here. There is very little correlation at the sideline angles below $\psi = 120^\circ$ and a strong increase in correlation above $\psi = 120^\circ$. A 20% correlation is quite high and indicative of large-scale turbulence noise having very coherent behavior and efficiently generating noise, similar to Mach wave radiation.

![Figure 1.15: Variation in the $U^n_j$ exponent for cold and heated jets\cite{32}](image)

Figure 1.15: Variation in the $U^n_j$ exponent for cold and heated jets\cite{32}. 

\begin{align*} 
T_o/T_a &= 1.0 \\
T_o/T_a &= 2.7 
\end{align*}
Figure 25. filtered to remove dust particles. In addition, a low-speed (facility. This facility was used to measure the bulk of the jet-flow turbulence far-field facility used for high-subsonic and supersonic jets. The second one is a heated jet measured normalized correlations. These figures are basically figures 12(9).

The direct correlation data measured by Panda and coworkers are remarkable and quite unexpected. Figures 25 and 26 show directivity plots of the maximum of the measured variables. Since the Rayleigh scattering measurements were thoroughly checked using independent means.

For all the data presented below, the positive maximum of the correlation function is used. In the work of Panda was moved at 10°. In this way, the normalized data are used to identify directivity of normalized correlation. Laser probe locations are at (a) and (b) were presented here in figures 25 and 26. In measuring these data, the correct data are presented here in figures 25 and 26. In measuring the turbulence fluctuations in a jet by the Rayleigh-scattering technique, the sound field radiated in the downstream direction. This is true regardless of which turbulence related fluctuation is used. Since the Rayleigh scattering measurements are concentrated in a very localized volume in the jet, a 20% correlation is a huge.

Figure 1.16: Turbulence-pressure and density-pressure correlations from Panda[35].

1.6 Supersonic Jet Acoustics

Supersonic jet noise contains various components of broadband and tonal noise and many of the theories on source mechanisms are still incomplete. Since the acoustics are intrinsically related to the flow structures, many of the various source mechanisms regarding turbulent mixing noise were discussed in the previous section. Figure 1.17 shows narrowband acoustic spectra for a supersonic jet from a smoothly contoured nozzle with a design Mach number $M_d = 1.5$. Just below the jet design Mach number the noise spectrum is dominated by turbulent mixing noise similar to a subsonic jet. At underexpanded conditions, the shock waves in the jet potential core result in broadband shock-associated noise (BBSN) and tonal components that are generally referred to as screech tones. Harper-Bourne and Fisher [36] provided the first comprehensive model explaining BBSN with the assumption that the points at which the shock-expansion waves reflect from the shear layer represent acoustic sources. They realized that the eddy convection time between the shock cells was responsible for the relative phasing of the shock-associated noise and developed a model to predict the peak frequency and amplitude of BBSN. The shock noise components are linked to large-scale coherent structures interacting with the quasi-periodic shock cells in the jet[36, 10]. For
imperfectly expanded jets, Tam[37] showed that shock cell spacing increases proportionally with jet Mach number, resulting in a decrease of the peak BBSN frequency. Through various models Tam was able to show that the peak frequency of the BBSN is related to the convection velocity of the large-scale structures $u_c$ and the shock cell spacing $L_s$ and the observation angle as shown in Eq. 1.17. This result is very simple although it does not provide information on the amplitude of the shock noise and assumes that the shock cell spacing is constant in the jet. More sophisticated models have been developed to predict BBSN amplitude but is beyond the scope of this work.

Screech tones are discrete tones that always appear at a lower frequency than BBSN[10]. Screech tones arise when a feedback loop is created between perturbations of the shear layer and upstream traveling acoustic waves. The feedback loop is initiated by an acoustic disturbance at the nozzle lip that propagates disturbances along the shear layer. These disturbances create acoustic fluctuations that propagate upstream and further perturb the shear layer at the nozzle lip where it is very thin and susceptible to disturbances[10], thereby closing the feedback loop. A schematic of the feedback mechanism from Tam[27] and an illustration of the feedback loop are presented in Figure 1.18. The peak screech frequency is given by the limiting case for $\psi = 0^\circ$ in Eq. 1.18 as screech tends to primarily radiate in the upstream direction. There has been extensive work on screech[29] but there are currently no methods for predicting screech amplitude. The physics governing screech are still not entirely clear and there is a large variation in screech behavior dependent upon nozzle design, jet operating conditions, and jet scale. Screech tones are highly sensitive to the boundary conditions at the nozzle exit. Chevrons and fluidic injection have both been shown to effectively reduce or eliminate screech tones.

$$f_p = \frac{u_c}{L_s (1 + M_c \cos \psi)}$$  \hspace{1cm} (1.17)  

$$f_s = \frac{u_c}{L_s (1 + M_c)}$$  \hspace{1cm} (1.18)
Figure 1.17: Narrowband acoustics for a $M_d = 1.5$ smoothly contoured nozzle operating near design $M_j = 1.49$, and underexpanded $M_j = 1.67$. Adapted from Norum and Seiner [38] as quoted in Tam [10].

Figure 1.18: Typical upstream acoustic spectra illustrating supersonic jet noise components and schematic of the screech feedback theory[27].
1.7 Fluidic Injection

Fluidic injection is the injection of secondary jets of fluid, in this case air, into the primary jet flow to modify the flowfield and subsequently the acoustics of the primary jet. Fluidic injection has shown promise as a noise control method for high Reynolds number jets\cite{39, 40, 41, 42, 43, 44, 45, 46}. Much of the early studies on fluidic injection, prior to 1995, focused on water injection and showed limited acoustic results in the range of air pressures considered practical for flight application\cite{47}. Fluidic injection has been referred to by many names including microjets, fluidevrons, fluid vortex generators, fluidic chevrons, and blowing. The injectors in the present study are larger than micro-scale and therefore are referred to generally as fluidic injectors. An attractive feature of fluidic injection is that it can be conditionally implemented when required as a noise control method in contrast to chevrons that are permanently in the jet flow. In many cases, fluidic injection has been shown to reduce BBSN sound pressure levels without shifting the peak frequency, indicating that mass injection can reduce the strength of the large-scale coherent structures without significantly modifying the shock structures. The classical fluid mechanical problem of a jet in crossflow is the fundamental application behind fluidic injection. The flow structures associated with a supersonic jet in supersonic crossflow is shown in Figure 1.19 from Dickmann\cite{48} and Rana\cite{49}. The jet forms a pair of counter-rotating vortices indicated as “wake vortices” which are referred to as streamwise vorticity in fluidic injection for jet noise applications. A bow shock forms upstream of the injector due to the blockage of the injector in the supersonic crossflow. This bow shock can reflect throughout the primary jet or interact with the existing shocks in the jet. The other features such as the horseshoe vortex, secondary shock, and lambda shock are features that only exist in an internal flow with a surface and are not observed in the present studies.

One of the difficulties that has been encountered thus far is that the design space for fluidic injection is very large. Fluidic injection flow parameters include mass flow, pressure, velocity, and momentum ratio. Injector configuration parameters include injector diameter ratio, pitch and yaw angle with respect to the jet centerline, alignment of non-symmetric injectors, and
injector shape. Two configurations of microjet injection are shown in Figure 1.20. The setup by Krothapalli[39] utilized external injectors individually supplied from a plenum at 60° to the jet. Zaman[50] utilized injectors embedded in the nozzle lip with only one pressure supply port. This is just one example of the variation in fluidic injection configurations. The injection angle relative to the jet axis has been studied with injection angles between 30° and 90° being the most common. However, few conclusions have been reported in the open literature regarding optimal configurations on high Reynolds number jets. Laurendeau et. al.[51, 52] studied injectors external to the nozzle (low-speed side) on a Mach 0.3 jet with injection angles of 12° to the jet axis and convergence angle of 60°. Reduction of -1 dB in OASPL was achieved with up to 10% reduction in turbulence intensities. Basara et. al.[53] studied one injection angle on a coaxial jet with Mach numbers below 0.5 and showed up to -2 dB reduction at the peak jet noise angle, although strong high frequency increases were observed. Maury et. al.[54] conducted an optimization study on injection mass flowrate for 60° injection using the injector designs of Laurendeau et. al.[51]. Up to -2.5 dB reduction was achieved with 1.4% mass flowrate.

Henderson[47] conducted an extensive review of the fluidic injection studies that have been published in the past 50 years regarding jet noise acoustics. Parameters studied include mass flow ratio of the injection mass flow to jet mass flow $\dot{m}_i/\dot{m}_j$, pressure ratio $p_{o,i}/p_a$, injection velocity, momentum flux ratio $J = \sqrt{\rho_i V_i^2/\rho_j V_j^2}$, injection angle to the jet, swirl angle, injector arrangement, and more. Most injectors have been placed at the nozzle exit injecting inwards through the outside of the jet shear layer. Injector shaping plays an important role in the penetration and generation of vorticity as detailed by numerous studies of jets in cross flow[55, 56]. For subsonic jets, increasing injection pressure while decreasing injector size resulted in greater reduction of noise radiated in the peak direction. Callender et al.[43] demonstrated up to reduction of up to -2.4 dB on a cold subsonic jet. For supersonic jets, circular and slotted injectors have different performance characteristics but most injector shapes were effective at eliminating screech from supersonic jets. OASPL reduction of up to
-2.7 dB in cold and hot jets has been observed\[57, 58, 59\]. Fluidic injection has been effective at reducing shock noise narrowband SPL up to -8 dB in a cold supersonic jet with a 1.2% mass injection\[60, 61\].

Fluidic injection of air into the shear layer of jets has been shown to reduce noise primarily through introduction of streamwise vorticity in the jet. Figure 1.21 shows a three-dimensional representation of the mean axial velocity at one plane near the nozzle exit. The penetration of the fluidic injectors into the primary jet results in a crenulation of the jet surface where the streamwise vorticity is generated. In subsonic jets, it has been shown that streamwise vorticity enhances the mixing of the high speed jet with surrounding flow, reducing the peak turbulence levels and subsequently the large-scale mixing noise. Fine-scale mixing noise can increase from the fine-scale turbulence introduced by the injectors\[43, 39\] and overall noise reduction is a balance between these effects. Fluidic injection in supersonic jets has had varying results, especially in regards to the shock noise components. Greska & Krothapalli\[58\] observed increases in shock noise and a shift in the peak frequency with six injectors on a heated Mach 1.8 jet. Krothapalli\[39\] showed suppression of screech and high frequencies at $\psi = 40^\circ$ and suppression of screech and shock noise with no shift in frequency at $\psi = 90^\circ$ shown in Figure 1.22. Henderson & Norum\[59\] observed reduction in shock noise with no shift in the peak frequency with embedded slot injectors on a slightly underexpanded sonic nozzle. Perrino et. al.\[46\] and Cuppoletti et. al.\[61\] observed strong suppression of shock noise without shifting the peak frequency with an embedded injector design on a $M_d = 1.56$ jet. In all studies, suppression of the screech feedback mechanism, when present, is quite effective even at low injection flow rates. These studies only represent a subset of the work on fluidic injection for supersonic jet noise but illustrate the variation in the results for differing jets and injector designs.

Despite the numerous studies on fluidic injection over the past half-century, there is still a lack of carefully conducted experiments to isolate the effect of injection parameters on acoustics. More importantly there is a lack of understanding regarding the physical mechanisms
responsible for noise reduction that has been achieved. A more thorough understanding of the physical mechanisms will allow for further advancements in noise reduction technology. For example, reduction in turbulent mixing noise has been attributed to jet spreading, jet mixing, streamwise vorticity, and reduced peak turbulence. For all the explanations, researchers have not been able to increase the amount of noise reduction significantly with streamwise vorticity generating mechanisms. Much of the research in presented in this dissertation is concerned with further understanding the physical mechanisms responsible for changes in the jet acoustics, especially related to supersonic jets.

Figure 1.19: Supersonic jet in crossflow schematic from Dickmann[48] and (b) LES simulation of a supersonic jet in crossflow from Rana[49].

Figure 1.20: Microjet injection setups from (a) Krothapalli[39] and (b) Zaman[50].
(a) Schematic showing microjet angle and injection location.
(b) Mean axial velocity field with microjet injection.

Figure 1.21: Microjets configuration and mean axial velocity from Alkislar [42].

Figure 1.22: Narrowband spectra with and without noise microjet injection [39]. Note that $\theta = 180 - \psi$. 
1.8 Unsteady Fluidic Injection

The core body of research on fluidic injection into jets for noise reduction has focused on steady flow injection. Commonly studied parameters include mass flow ratio of the injection mass flow to jet mass flow $\dot{m}_i/\dot{m}_j$, pressure ratio $p_{o,i}/p_a$, momentum flux ratio $\sqrt{\rho_i U_i^2/\rho_j U_j^2}$, injection angle to the jet $\theta_i$, swirl angle, and arrangement of injectors around the jet. Work has also been conducted on unsteady excitation of jets using acoustic excitation, plasma actuators, and fluidic actuators primarily to excite natural jet instabilities and enhance jet mixing. The many methods used for actuation are illustrated in Figure 1.23 and an example from Raman[62, 63] is shown in Figure 1.24. It has been shown that the non-dimensional preferred jet frequency, Strouhal number, is between $0.2 \leq St_{Dj} \leq 0.4$, and is commonly around $St_{Dj} = 0.3$. For laboratory supersonic jets $St_{Dj} = 0.3$ is upwards of 1 kHz and higher. Higher frequencies and adequate energy input make it difficult to excite higher Reynolds number jets through conventional means. The available literature on pulsed fluidics as a method for jet noise control is relatively sparse and conclusions on the effectiveness are unclear. Most studies on pulsed fluidics have been performed on low Reynolds number jets[62, 64, 65]. A low Reynolds number makes it easier to excite fundamental jet instabilities ($St_{Dj} = 0.2$ to 0.4) within the capability of low frequency actuators. Pulsed fluidic injection for noise control of high Reynolds number jets has received little attention as most studies have focused on jet mixing and largely overlooked the practical issues with tonal resonance in the acoustic spectra.

Parekh et al.[64] showed that forcing a low speed jet with two opposing slotted pulse jets at $St = 0.2$ enhances turbulent mixing and decreases the potential core length by a factor of two. Parekh also demonstrated that the peak time-averaged Mach number was halved. Freund & Moin[66] conducted a DNS computation on a Mach 0.9 jet with a similar pulse jet arrangement as Parekh et al.[64]. They also showed a large decrease in potential core length due to a significant increase of entrainment into the jet from the large scale jet motions incurred from excitation shown in Figure 1.25. It was pointed out that the asymmetry induced around
the jet axis increased the level of mixing in the jet. Practical tests of pulsed fluidic injection have been carried out on an engine test with the Boeing ACE program\textsuperscript{[67, 68]}. Two opposing arrays of slotted injectors installed near the exit plane of a JT8D-15 converging-diverging nozzle were used to excite the jet. Measurements of the centerline temperature decay of the jet showed a rapid decrease in potential core length. Far field acoustic measurements showed a near 10 dB noise increase in sideline \textit{OASPL} and up to 8 dB increase at the downstream angles. Although the increase in jet mixing results can be desirable for plume signature in tactical aircraft, the harmonic excitation of jets results in resonance of the excitation frequency that is highly undesirable acoustically. The Boeing ACE tests note that the “flapping” of the jet plume is very audible and similar to the sound generated from a propeller aircraft. This resulted in noise increases proportional to the intensity of unsteady injection shown in Figure 1.26. Unsteady fluidic injection conducted by Raman\textsuperscript{[62, 63]} on subsonic jets, and Ibrahim \textit{et al.}\textsuperscript{[69]} on supersonic jets show fundamental and subharmonic tones become prevalent in the acoustic spectra even when \textit{OASPL} is reduced as shown in Figure 1.27. Hafsteinsson \textit{et al.}\textsuperscript{[70]} conducted a study on pulsed injection with a trailing edge injector configuration at frequencies on the order of $St = 0.2 - 0.4$. The results of that study showed significant tonal noise from jet excitation which has been shown to be effective for jet mixing but acoustically undesirable. These findings directed us to conduct a detailed investigation on lower frequency injection to attempt to identify desirable acoustic effects with unsteady injection. Ragaller\textsuperscript{[71]} provided data for low frequency pulsed injection on a Mach 1.8 high temperature jet, but the data was inconclusive and did not identify clear trends on frequency or duty cycle due to limited data.

Steady fluidic injection can provide noise reduction without the addition of tones in the far field acoustics. Few investigations have been conducted on pulsed fluidic injection that have not focused on excitation of fundamental jet instabilities ($St = 0.2 - 0.4$). Most investigations have focused on jet mixing while not presenting acoustic results. In the present investigation a wide range of injection frequencies, duty cycles, and injection angles have been studied. The
investigation has been approached with the goal of determining which configurations provide optimum noise reduction with reduced injection mass flow. An emphasis is placed on understanding the jet dynamics during the unsteady pulse cycle and relating the findings to the acoustic results. LES and CAA are evaluated for capability of predicting the flow and acoustics and are complementary to understanding the measurements since LES can provide time resolved flowfield information which is otherwise difficult to achieve experimentally. Pulsed fluidics have the potential of improving noise reduction while only being activated at times in the flight envelope when noise reduction is desired. Pulsed injection also has the potential to achieve equivalent or better acoustic results than steady injection while reducing the amount of injection mass flow required. This would increase viability of the technology since any air diverted for noise control can impact cycle performance and efficiency.

Figure 1.23: Different actuator types for flow control[64]
Figure 1.24: Setup for unsteady mass addition from two slotted injectors from Raman[63].

Figure 1.25: Vorticity contours from DNS of a Mach 0.9 jet with slotted pulse jets for control[66].
Kibens (1999)

Figure 1.26: Acoustic results for unsteady injection from the Boeing ACE program[67].

Kibens (1999)

Figure 1.27: Acoustic results for unsteady injection from Ibrahim[69].
Chapter 2

Facility and Methodology

2.1 Aeroacoustic Test Facility

The experimental investigations were conducted at the Aeroacoustic Test Facility (ATF) in the Gas Dynamics and Propulsion Laboratory (GDPL) at the University of Cincinnati. The facility features a coaxial jet rig that can simulate scale nozzles at realistic operating conditions in terms of nozzle pressure ratio ($NPR$), velocity, and temperature ratios up to 1.3. The jet rig is in an anechoic test chamber measuring 7.3 m x 7.6 m x 3.4 m with a lower cutoff frequency of $f_{c,\text{lower}} = 350$ Hz. The walls and ceiling are treated with 6” thick fiberglass insulation held in place between wall studs with wire mesh and porous acoustic cloth. Acoustic wedges are used in many anechoic facilities although they are only necessary if lower frequencies ($< 350$ Hz) are of interest. The floor of the anechoic facility is poured concrete equipped with a trench that extends into the control room for running instrumentation, cables, and air supply lines. Modular floor panels (0.4 m x 0.3 m) with the same acoustic treatment as the walls and ceiling cover the floor for acoustic measurements. The modular floor panels allow easy access to the nozzle and are removed for flow measurements. A picture of the ATF with the floor installed is shown in Figure 2.1a and a schematic layout is shown in Figure 2.1b. A constant diameter far-field measurement arc is permanently installed in the facility at 3.75m from the jet rig with the origin at the nozzle exit, corresponding to $65D_j$ for the nozzles used in this study.
This distance is sufficiently in the far-field which is around $40D_j$ for the lowest frequencies of interest\[72\]. A near-field array of microphones can also be placed near the jet on a three-axis linear traverse which can map a grid of near-field pressure. Measurement techniques such as pressure probes and PIV setups can also be attached to the traverse for spatial measurements.

The primary air for the jet rig is supplied by four high pressure tanks capable of storing air at 11,000 kPa ($\approx$100 atm). The air can be heated up to 400 K with a steam heat exchanger and provide up to 2 kg/s air flow when operated in the “low mass” configuration which supplies air from the 4” high pressure air line in the GDPL. To achieve higher mass flows the high pressure air system can be operated in a “high mass” configuration which bypasses the high pressure air into the 6” line and the coaxial line is connected to the core. This configuration was not used in these experiments. The secondary coaxial stream is supplied by a low pressure air tank capable of storing air at 1,000 kPa ($\approx$10 atm) and providing high mass flows. The secondary flow was operated at low flow rates to facilitate seeding the outside of the shear layer for PIV measurements. For further details on the facility and design refer to Callender\[73, 74\]. Pressure and temperature is monitored in the supply manifold and in the plenums of the primary and secondary air stream. The control parameter for operating the jet is the nozzle pressure and temperature which allows for calculation of Mach number, mass flow rate, and other parameters using Eq. 2.1 to 2.3. The mass flow rate for a choked nozzle with sonic flow at the throat ($M_t = 1$) only depends on the throat area, pressure, and temperature. This is for an ideal nozzle however, and the biconical nozzle in the present studies has a discharge coefficient of $C_d = \dot{m}_{actual}/\dot{m}_{ideal} = 0.93$ as determined from CFD studies. Discussion and conditions for other nozzle designs are discussed in each relevant chapter. National Instruments (NI) data acquisition hardware and LabVIEW are used to monitor instrumentation and record facility parameters, details of which are discussed for each measurement specification.

$$\frac{p_o}{p_a} = (1 + \frac{\gamma - 1}{2}M^2)^\frac{\gamma}{\gamma - 1}$$  \hspace{1cm} (2.1)
\begin{equation}
\frac{T_o}{T_a} = 1 + \frac{\gamma - 1}{2} M^2 \tag{2.2}
\end{equation}

\begin{equation}
\dot{m} = \frac{A p_o \sqrt{\gamma}}{\sqrt{R T_o}} \left[ \frac{M}{(1 + \frac{\gamma - 1}{2} M^2)} \right]^{\frac{\gamma + 1}{2(\gamma - 1)}} \tag{2.3}
\end{equation}

\section*{2.2 Baseline Nozzle and Jet Conditions}

The baseline nozzle used in these studies is a biconical converging-diverging nozzle, also referred to as the “sharp throat” nozzle. The biconical design is formed by the intersection of two truncated conical sections, simulating current supersonic engine geometries. The biconical nozzle in Figure 2.2 has a jet diameter $D_j = 57.53$ mm, an area ratio $A_e/A_t = 1.23$ with a design $NPR_d = 4.0$ corresponding to $M_d = 1.56$. Figure 2.2b defines the coordinate system for this nozzle including cartesian and cylindrical coordinates that are commonly used for jets. Cartesian coordinates are presented in this study since PIV is a planar measurement. The jet flow was studied at the design condition along with underexpanded and overexpanded conditions in which the jet is imperfectly expanded at the nozzle exit. The jet operating conditions investigated are shown in Table 2.1. All experiments are run at a temperature...
ratio $TR = 1.25$. Coaxial secondary flow around the primary stream is at $M_s = 0.1$, which is used for PIV ambient flow seeding. The secondary flow was used for acoustics measurements and in LES to allow for direct acoustics and flow field comparison.

![Figure 2.2: Biconical nozzle schematic and coordinates and photo showing nozzle installed on jet rig.](image)

**Table 2.1: Jet operating conditions including the NPR most studied.**

<table>
<thead>
<tr>
<th>$M_j$</th>
<th>$p_a/p_s$</th>
<th>$m_j$ [kg/s]</th>
<th>$U_j$ [m/s]</th>
<th>$T_{ao}/T_{ao}$</th>
<th>$c_j$ [m/s]</th>
<th>$Re$</th>
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<td>1.64</td>
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<td>1.72</td>
<td>508</td>
<td>1.25</td>
<td>313</td>
<td>2.62E+06</td>
</tr>
<tr>
<td>1.56</td>
<td>4.0</td>
<td>1.65</td>
<td>491</td>
<td>1.25</td>
<td>318</td>
<td>2.46E+06</td>
</tr>
<tr>
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<td>3.5</td>
<td>1.44</td>
<td>471</td>
<td>1.25</td>
<td>324</td>
<td>2.14E+06</td>
</tr>
<tr>
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<td>3.0</td>
<td>1.23</td>
<td>446</td>
<td>1.25</td>
<td>332</td>
<td>1.87E+06</td>
</tr>
<tr>
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<td>2.5</td>
<td>1.03</td>
<td>412</td>
<td>1.25</td>
<td>341</td>
<td>1.57E+06</td>
</tr>
</tbody>
</table>

### 2.3 Acoustic Measurements

The GDPL has thirty-two 1/4” Bruel & Kjaer model 4954 free-field condenser microphones with built in preamplifiers. The microphones connect to NI PXI-4498 Dynamic Signal Analyzers (DSA) data acquisition modules (DAQ). The DSA’s are AC-coupled which provides only the fluctuating signal, in this case the fluctuating component of pressure, $p'$. The modules are responsible for analog to digital conversion of the microphone signals and are capable of sampling 16 simultaneous channels at 204.8 kHz with 24-bit resolution. The DSA modules have features including 4 mA excitation current supplied to the microphones, programmatic gain setting, anti-aliasing filters, and TEDS read/write capability (sensor information).
2.3.1 Far-field acoustics

The far-field arc contains thirteen 1/4” Bruel & Kjaer model 4954 free-field condenser microphones with frequency sensitivity up to 100 kHz. The microphones are spaced 10° apart from ψ = 40° to 150° and one microphone at ψ = 35° which is the furthest upstream angle achievable in the facility without interference from the rig. The microphones are sampled at 204.8 kHz and 15 seconds of data are acquired as three 5 seconds sets for ensemble averaging. The data was bandpass filtered with a 5th order Butterworth filter with $f_{c,\text{lower}} = 350$ Hz and $f_{c,\text{upper}} = 100$ kHz. Narrowband frequency spectra were calculated using FFT’s with blocks of 4,096 samples, resulting in frequency spectra averaged over 750 occurrences and $\Delta f = 50$ Hz. For further details on the acoustic data processing refer to section 2.3.3.

2.3.2 Near-field pressure

Near-field pressure measurements are conducted in the region near the jet which contains hydrodynamic and acoustic waves. The technique utilizes an array of microphones to measure pressure over a large spatial distribution and is used to understand noise directivity, noise footprint, and source location. A thirty-two microphone linear array with 1” spacing was affixed to a three-axis traverse and was used to map a plane near the jet as shown in Figure 2.3a. The measurement plane spans $28D_j$ axially and $10D_j$ radially aligned along the jet shear layer. A picture of the microphone array is shown in Figure 2.3b showing the 10° spread angle of the array. The data acquisition and data processing are the same as the far-field except that only one 5 second data set is acquired for each microphone due to the large amounts of data when sampling thirty-two microphones at 40 measurement locations. The processed near-field data contains narrowband spectra and OASPL at every measurement location. Near-field pressure measurements are presented in Chapter 7 for comparison of nozzle designs and noise reduction technologies.
2.3.3 Acoustics Data Processing

To analyze the acoustic data there are many techniques and methods that can be used. LabVIEW has excellent sound processing VI’s however they are difficult to troubleshoot and verify since they use prebuilt functions. Acoustic data processing was done in Matlab to carefully control all processing methods. The most fundamental mathematical tool is the Fourier transform and the discretized version, the Discrete Fourier Transform (DFT). The finite Fourier transform for a continuous function \( p(t) \) is defined in Eq. 2.4 for a signal of length \( T \). For real signals which are not readily described by smooth or continuous mathematical functions, it is much easier to compute the DFT using a Fast Fourier Transform (FFT) algorithm. For a discrete signal \( p_n \) containing \( N \) points the DFT is defined in Eq. 2.5.

\[
P(f) = \int_0^T p(t)e^{-j2\pi ft}dt
\]  
(2.4)

\[
P(f_n) = \Delta t \sum_{n=0}^{N-1} p_n e^{-j2\pi fn/N}
\]  
(2.5)

where \( p_n \) is the discretized pressure signal defined as \( p_n = p(n\Delta t) \) and \( f_n \) are the discretized frequencies \( f_n = n/N\Delta t \) and \( n = 0, 1, 2, ..., N - 1 \).
To compute a statistically averaged frequency spectra the total signal length is separated into a number of discrete blocks $n_d$ of length $L$. For accuracy it is best to use block lengths that are a power of two, but not necessary in Matlab. The DFT is applied to each data block and results in $n_d$ frequency spectra. To present the symmetric two-sided spectra as a single-sided spectrum and account for all of the energy in the signal, the transformed signal $P_n$ is multiplied by its complex conjugate and multiplied by a factor of two. If a window is used, the spectra must be corrected by dividing by the $rms$ of the window function to account for energy loss. The resulting ensemble averaged FFT $X_n$ can be corrected for microphone alignment and frequency response if needed.

$$X_n(f_n) = 2 \cdot \langle |P_n(1:N/2)|^2 \rangle$$ (2.6)

The spectra $X_n$ has units of [pressure$^2$] and is equivalent to the spectral distribution of $p_{rms}^2$. The narrowband spectra are presented as sound pressure level (SPL) or a power spectral density (PSD). The equation for SPL and PSD are shown in Eq. 2.7 and 2.8 and have units of [dB] and [dB/Hz], respectively. It is convenient to present data as PSD since this accounts for the frequency bin width $\Delta f$ allowing for comparisons of data processed with different FFT parameters. Figure 2.4 compares narrowband and third-octave band data as SPL spectra and PSD spectra. The third-octave band (TOB) is a proportional band spectra which approximates human hearing while the narrowband spectra has a constant band width. The third octave band has larger SPL values since it includes more frequencies per band, but the spectra collapse when presented as PSD. Unfortunately, much of these lessons were learned as the research progressed, so the majority of the data is presented as SPL, which is common in literature. Since the data is all processed with the same FFT parameters, SPL serves the purpose of evaluating noise reduction methods. Throughout this dissertation the acoustic spectra are presented in SPL and noise reduction is discussed in terms of SPL or OASPL. Subsonic jet noise data is often presented as TOB since the spectra are often smooth broadband curves with a lack of tonal or narrowband components as a result of the absence
of shock waves. Supersonic jet noise data is commonly presented as narrowband in terms of SPL to resolve the shock noise components clearly. The EPNL calculations also utilize TOB spectra. Although most of the acoustic data is presented as SPL for noise reduction comparison, PSD is valuable when comparing data sets processed with different parameters. Overall sound pressure level (OASPL) is used to quantify noise intensity over all frequencies and can be computed with the \( \text{rms} \) of the fluctuating pressure signal as in Eq. (2.9), or by integrating the narrowband spectra \( X_n(f_n) \). These methods provide the same answer, and the former method was used in this work since microphone corrections were not applied. Agreement of these two methods can be used to validate that the narrowband spectra were computed correctly.

\[
SPL = 10 \log \frac{X_n(f_n)}{p_{ref}^2} \quad (2.7)
\]

\[
PSD = 10 \log \frac{X_n(f_n)/\Delta f}{p_{ref}^2} \quad (2.8)
\]

\[
OASPL = 10 \log \left( \frac{p_{rms}}{p_{ref}} \right)^2 \quad (2.9)
\]

where, \( p_{ref} = 20 \mu Pa \).

### 2.4 Particle Image Velocimetry

#### 2.4.1 Streamwise (2D) PIV

Particle image velocimetry (PIV) is used to measure the jet velocity throughout this work. The LaVision PIV system includes a double-pulsed Nd:YAG laser (New wave Research Solo-PIV), and two Imager Intense CCD cameras with a resolution of 1367 x 1040 pixels. The primary and secondary flows are seeded width atomized olive oil particles on the order of 1 \( \mu \)m. The laser sheet is formed using a cylindrical lens to spread the beam into a sheet and
Figure 2.4: Comparison of narrowband (NB), narrowband PSD (NB PSD), third-octave band (TOB), and third-octave band PSD (TOB PSD) for $\psi = 150^\circ$ spectra.

Figure 2.5a. For axisymmetric flows the laser sheet bisects the centerline plane of symmetry. For fluidic injection data is taken in-plane and out-of-plane with the injectors. Data is acquired with a double frame exposure method and the laser pulse time separation $\Delta t = 4\text{--}5 \, \mu s$. The $\Delta t$ is chosen to provide a $\Delta x$ of 4-6 pixels at the jet centerline and $\Delta x$ of 2-3 pixels in the shear region to minimize freestream velocity uncertainty and reduce uncertainty in the shear layer. Near hardware surfaces laser reflections can cause distortion and measurement error. To alleviate this issue the hardware is treated with rhodamine-B to shift the wavelength of the light scattered from surfaces and 532 nm wavelength centered bandpass filters are used on the cameras. To provide reliable mean and rms velocities, 500 image pairs are acquired. The field of view was chosen to have high resolution near the nozzle for shock resolution and vector post-processing is done in LaVision Davis 8.1 with 16 x 16 windows with 50% overlap for particle image cross-correlations. The streamwise PIV measurement resolves two components of velocity, therefore turbulent kinetic energy (TKE) is computed with twice the radial ($u'v'$) Reynolds stress from experiments shown in Eq. (2.10). This assumption is valid for an axisymmetric jet, but would be less accurate for a jet with
azimuthal variation in the flow field. Examination of the individual Reynolds stresses for the Reynolds Stress Models[75] and LES results validated this assumption. It was found that there is less than 10% difference in the maximum $v'v'$ and $w'w'$. Turbulence is presented in the form of turbulence intensity normalized by the jet velocity for each condition $\sqrt{TKE}/U_j$.

### 2.4.2 Stereoscopic (3D) PIV

The stereoscopic (stereo) PIV measurement is used to resolve three velocity components by utilizing two or more cameras that can extract the out of plane velocity component through a mathematical relationship of the displacements visualized by both cameras. This is the same principle as how human eyesight resolves depth. The laser sheet for stereo PIV is formed using a cylindrical lens to spread the beam into a sheet that illuminates a cross-section of the jet. A spherical focusing lens is not used, resulting in a 1-2 mm laser sheet thickness allowing for measurement of the axial velocity component. Figure 2.5b schematically illustrates the stereo PIV setup with the laser sheet bisecting the jet cross-section. Data is acquired with a double frame exposure method and the laser pulse time separation $\Delta t = 1-1.5 \, \mu s$ depending on the flow velocity. The shorter $\Delta t$ for stereo PIV was chosen to provide accurate resolution of the out-of-plane velocity component, which is 5 to 10 times larger than the in-plane velocity components and has the shortest residence time in the laser plane. The hardware was treated with rhodamine-B to shift the wavelength of the light scattered from surfaces and 532 nm wavelength centered bandpass filters are used on the cameras. Scheimpflug adapters on the camera lens achieve focus across the image plane for off-axis imaging. The primary and secondary flows are seeded with atomized olive oil particles on the order of 1$\mu$m. To provide reliable mean and rms velocities, 500 image pairs are taken at each measurement location. Post processing the images to vector fields is done in LaVision Davis 7.2 using 16 x 16 windows with 50% overlap. The instantaneous vector fields are exported to Matlab to calculate the mean and rms velocity fields. The turbulence kinetic energy is calculated using Eq. 2.11 for all three velocity components and is presented as the square root of turbulent kinetic energy.
normalized to the fully-expanded jet velocity $\sqrt{TKE/U_j}$.

$$TKE_{2D} = \frac{1}{2} (w'u' + 2v'v')$$

(2.10)

$$TKE_{3D} = \frac{1}{2} (w'u' + v'v' + w'w')$$

(2.11)

### 2.5 High-Speed Shadowgraph Technique

Shadowgraph and schlieren are line-of-sight measurement techniques which visualize gradients in index of refraction $n = k\rho - 1$, which is directly proportional to the gas density with $k$ being the Gladstone-Dale coefficient ($k = 0.23$ cm$^3$/g) for air[76]. Schlieren visualizes the first spatial derivative of refractive index $\partial n/\partial x$ and shadowgraph visualizes the second spatial derivative $\partial^2 n/\partial x^2$, which are proportional to first and second spatial derivatives of density. Although schlieren is generally more sensitive, shadowgraph has been shown to be more sensitive for turbulent flows and in flows containing shock waves[76, 77]. The density gradients refract the collimated light rays, causing increases and decreases in light intensity. Shock waves appear as strong dark lines in the shadowgraph proportional to the shock.
Figure 2.6: Schematic setup for a Z-type Schlieren setup[76]. Shadowgraph is achieved without using a knife edge.

strength. The shadowgraph system consisted of an Oriel arc lamp and power supply, two 12” first-surface parabolic mirrors with 6’ focal lengths, and a Photron Fastcam SA4 high-speed camera. The shadowgraph was arranged in a Z-type configuration shown in Figure 2.6, as discussed in Settles[76]. The schematic depicts a schlieren setup which uses a knife edge at the focal point to block light rays deflected towards that direction. This is what provides the light and dark regions corresponding to the direction of refraction in the schliere. The shadowgraph setup is identical although it does not utilize the knife edge. Figure 2.7 is a photo of the shadowgraph setup with the high-speed camera in the ATF. For a distortion-free measurement, any horizontal or vertical angle of the light source must be removed by placing the imaging device at the opposite angles. Shadowgraph recordings were taken between 10,000 fps and 30,000 fps, depending on the phenomenon being investigated. Shadowgraph and schlieren are generally qualitative measurement techniques without extremely precise calibration. It can however be used for accurate spatial measurements and in combination with the high-speed camera some temporal information can be extracted. The present investigation mostly utilized the shadowgraph technique. Time-averaged shadowgraph was used to visualize the mean flow structures, especially for shock waves. Instantaneous shadowgraph was used to extract some length and time scales associated with the supersonic jet dynamics including screech onset and growth, response of the jet to unsteady fluidic actuation, and shock dynamics.
2.6 Fluidic Injection Control and Measurement

Fluidic injection supply air was controlled and regulated inside the control room from a separate control panel. The control panel is pictured in Figure 2.8 showing two supply lines on the left side of the white control panel. The top supply line provided air to the olive oil seeders for PIV with a pressure regulator/air filter with high pressure drop. The lower supply line provides air for fluidic injection and is regulated with a low pressure drop regulator. The flow passes through a pressure indicator and rotameter before exiting the control panel to the fluidic injection system. Mass flow rate, $\dot{m}$, was computed by correcting the volumetric flow rate, $\dot{Q}$, for the supply pressure and temperature using Eq. 2.12, where $SG$ is the specific gravity, which for air is unity, and $std$ denotes standard conditions.

The present investigation employed three methods of supplying air to the injectors at the jet rig. The fluidic injection studies begin with a trailing-edge injection nozzle which supplies air through a small port in the side of the jet rig plenum and partitions into twelve (12) injection tubes with 3.75mm diameter. The twelve tubes run along the outer diameter of the core pipe inside the fan plenum and embed through the back of the nozzle body into a plenum as shown in Figure 2.9a. The tubes exhaust into the plenum which contains flow mixers for
flow steadiness and even pressure in the plenum. The pressurized nozzle plenum supplies air to the injectors in the nozzle. This configuration has up to 30% pressure loss due to the tubing and plenum design, and is not sufficient for pulsed flow. The trailing edge nozzle has 24 slotted injectors measuring 1.6 mm x 1.6 mm. For all other injector configurations the injectors are circular with $D_i = 2.7$ mm, which have the equivalent area of one pair of the trailing edge injectors. For pulsed fluidic injection a new injection method was designed to provide less pressure loss in the supply tubes and to pulse the flow near to the injection location to avoid pulse damping and phase shifting. Figure 2.9b shows the external plenum which is pressurized with four 25.4 mm supply lines, minimizing the supply line pressure loss. The plenum contains twenty-four $1/4''$ ports which connect to the valve inlets. The valve exit reduces the injector diameter to $1/8''$ tube with $D_i = 2.7$ mm diameter flow path. The injectors are mounted in a casing to control the injection angle ($\theta_{inj} = 30^\circ$, 45°, 60°, and 90°). The Bosch NG12 valves are designed for natural gas flow and they were selected since they passed the highest mass flow rate with the lowest pressure drop of all valves evaluated. A design constraint was the valves had to be small enough to be mounted at the jet exit to maximize pulse strength. The valves have up to 50% pressure loss at the highest flow rates with pulse frequencies up to 500 Hz. For steady fluidic injection the valves were removed and replaced with a $1/4''$ to $1/8''$ bushing which provided the lowest pressure drop of all configurations, allowing for the highest injection momentum flux ratios. It was critical that pressure and mass flow rate be recorded for all tests and configurations since the pressure drops were drastically different. Mass flow rate is conserved in the system and actual injection pressures could be back calculated for if the Mach number was known (choked in most cases). This was only fully understood after differences were noticed between CFD and experiments, therefore for some earlier results pressure ratios are quoted on the figures, but mass flow ratios and momentum flux ratios are reported in tabular format for each section. The majority of fluidic injection conditions that are studied are operated at choked conditions. The supply air is always room temperature ($T_o = 293$ K), therefore, Eq. 2.3 can be used to calculate mass flow rate using the supplied injector area. If
only the pressure $p_o$ is quoted than the pressure ratio is approximately $p_o/p_a$ for all cases in which the injectors are external to the nozzle.

![Figure 2.8: Pressure regulator, pressure gauge, and rotameter for fluidics control and measurement.](image)

$$\dot{m} = \dot{Q} \cdot \sqrt{\frac{SG \cdot P \cdot T_{std}}{P_{std} \cdot T}}$$  \hspace{1cm} (2.12)

## 2.7 Pulsed flow measurements

Hot-wire velocity measurements and dynamic pressure sensors were used to measure the quality and amplitude of the air pulses from the Bosch NG12 valves. A Dantec Dynamics hot-film probe, shown in Figure 2.10, was placed at the valve exit to measure flow velocity during the pulse cycle for different conditions. An A.A. Lab Systems Ltd. AN-1005 anemometer was used to balance the bridge for the sensor and calibration was done daily to account for small variations. The control signal was recorded simultaneously to identify any delays and phase shifts in the pulse cycle with respect to the input control signal. The hot-wire signal was low pass filtered at 1 kHz to suppress high frequencies associated with turbulent fluctuations so that the bulk velocity was measured. The hot-wire was calibrated with a calibration jet up to a Mach number of 0.6, after which the hot-wires would break. The relationship between voltage, $E$, and velocity, $u$, was found using $E^2 = A + Bu^{0.5}$ and was used to calculate velocities into
the supersonic flow regime. The measurements were used primarily to determine the valve performance limits in terms of frequency and pulse strength.

2.8 Pressure Probe Measurement

Direct measurement of thrust is not a trivial endeavor when working with laboratory jets since they are usually attached to a rigid piping system or there are measurement difficulties depending on jet conditions. The static thrust equation for a quasi-1D flow is given as $F = \dot{m}u_j + (p_e - p_a)A_j$ where the jet velocity ($u_j$) and jet exit pressure ($p_e$) are assumed constant across the nozzle exit plane. In practical applications there is variation of these mean quantities.
across the exit plane. However, if details of the velocity and pressure are known in the exit plane, integration can be used to compute total thrust.

Conical probe total pressure measurements with isentropic flow relations and velocity data from PIV was used to calculate static pressure at the nozzle exit plane. Figure 2.11 illustrates the assumption of a normal shock upstream of the measurement port of a pitot probe in a supersonic flow. Pressure loss incurred across a normal shock is given by Eq. 2.13 and only depends on the upstream Mach number $M_1$. By measuring $p_{o,2}$ and knowing velocity upstream of the shock $u_1$, $M_1$ can be calculated using isentropic relations with $\pm 5\%$ uncertainty, resulting in $\pm 2.2\%$ uncertainty in $M_1$.

\[
\frac{p_{o,2}}{p_{o,1}} = \left[ \frac{(\gamma + 1)M_1^2}{(\gamma - 1)M_1^2 + 2} \right]^\frac{\gamma}{\gamma - 1} \left[ \frac{\gamma + 1}{2\gamma M_1^2 - (\gamma - 1)} \right]^\frac{1}{\gamma - 1} \tag{2.13}
\]

Figure 2.11: Schematic of pitot probe in supersonic flow.

### 2.9 Experimental Uncertainty Analysis

It is of interest to the scientific community and the experimentalist to accurately quantify experimental uncertainties in order to develop a confidence level in the measurements. This is often overlooked in experimental studies due to the complexities of certain experimental methods, but mostly due to complacency with the sophistication of modern instrumentation.
Simple bias errors can be compensated for with careful calibration, for example, the accuracy of a pressure sensor over its dynamic range. However, complex measurement techniques such as PIV or other laser diagnostics require much further depth of understanding and analysis to accurately quantify uncertainty. The use of digital acquisition systems adds an additional layer of uncertainty, mainly of resolution and dynamic range. This section aims to report some of the experimental measurement uncertainty in this investigation, namely the largest sources of uncertainty.

2.9.1 Facility Uncertainty

The facility operating conditions are monitored through pressure and temperature measurements which allow for calculation of all other variables such as density, pressure ratio, Mach number, velocity, and mass flow rate with the ideal gas law and isentropic relations. The NI PXI-6221 DAQ card has 16-bit resolution which results in $\varepsilon_r(p) = \pm 0.005$ kPa for the core pressure transducer and $\varepsilon_r(T) = \pm 0.0007$ K for the core thermocouple. The resolution uncertainty is orders of magnitude lower than the bias error due to transducer calibration and ability to hold constant $p_o$ and $T_o$ during an experiment, and therefore neglected. The total maximum bias error is $\varepsilon_b(p) = \pm 0.7$ kPa and $\varepsilon_b(T) = \pm 5.5$ K as measured from the standard deviation of the recorded variables during the experiments. This results in maximum derived uncertainties in Mach number $\varepsilon(M) = \pm 0.01$ and velocity $\varepsilon(u_j) = \pm 5$ m/s. These are stated as the maximum uncertainties for a well-controlled experiment and on average the experimental error is within these limits.

2.9.2 Acoustics Uncertainty

Uncertainty in acoustic pressure has historically been higher due to the limits of digital acquisition systems. The B&K 4954 microphones have sensitivity in the range of 2.65 mV/Pa, however signal gain is supplied programatically in the DAQ system. The NI PXI-4498 cards not only can set the gain programatically (0, +10, +20, +30 dB), but the cards have 24-bit
resolution. Far-field acoustic data was acquired with +20 dB (±1 V range) gain and near-field pressure with 0 dB (±10 V range) gain. This results in $\varepsilon_{FF}(p') = 2.2 \times 10^{-5}$ Pa and $\varepsilon_{NF}(p') = 2.2 \times 10^{-4}$ Pa corresponding to a maximum error of ±0.001 dB at 100 dB SPL. Again, the maximum error in the acoustics measurement will be due to calibration and repeatability. Analysis of the large database of acoustic results showed maximum repeatability errors of ±1 dB for narrowband acoustic spectra and ±0.5 dB in OASPL. The largest discrepancies are in the high frequency region due to variation in rig noise throughout the years. Rig noise is a well known issue in the jet noise community and has been addressed by many authors including Viswanathan[78, 72] and others. In the ATF the variation in the internal rig noise is thought to be due to the gasket on the steam heat exchanger and alignment of pipe flanges, however it is not fully understood. This noise varies slowly over time, therefore recording an accurate baseline for each testing phase results in small errors in ΔOASPL and noise reduction results. Examples of repeatability for narrowband acoustics and OASPL is shown in Figure 2.12. For comparisons of the narrowband acoustic spectra with the fine-scale similarity (FSS) and large-scale similarity (LSS) spectra developed by Tam[33], see section 4.1.3.

![Figure 2.12: Example of the repeatability of measurements for various experiments.](image-url)
2.9.3 Screech Effects on Acoustics

Screech instabilities can have a profound effect on the acoustic spectra including one or more tones appearing above the turbulent mixing noise. This is mostly seen in the OASPL measurement which is increased strongly for certain jet operating conditions with screech. Large scale heated jets tend to not have screech instability amplitudes as high as laboratory jets so there is an interest in knowing how screech distorts OASPL. Figures 2.13 and 2.14 show the effect of removing screech from the acoustic spectra for moderate and strong screech by smoothing the screech tones out of the spectra. Smoothing was recursively and locally applied to smooth the tones from the spectra without removing energy from the broadband regions of the spectra. The effect on OASPL is minimal for moderate screech, defined as fundamental screech frequency amplitude that is 15 dB or less above the broadband noise. For strong screech above 15 dB the effect on the OASPL measurement can become extreme. For a 30 dB screech amplitude the OASPL is 11 dB higher at $\psi = 35^\circ$ and 4 dB higher at $\psi = 90^\circ$ which are the primary and secondary propagation directions for this screech mode. This should be kept in mind when evaluating noise reduction performance on screeching jets since large scale heated jets may not have screech instabilities like the laboratory jets. An approach to physically suppressing screech non-intrusively is reported and discussed in 3.6.

Figure 2.13: Effect of screech on $\psi = 35^\circ$, $\psi = 90^\circ$, and OASPL for moderate screech (15 dB).
Figure 2.14: Effect of screech on $\psi = 35^\circ$, $\psi = 90^\circ$, and OASPL for strong screech (30 dB).

2.9.4 Particle Image Velocimetry Uncertainty

PIV uncertainty is an extensive topic that is strongly dependent on the flow physics and measurement parameters. The largest sources of uncertainty in the current study include calibration (equipment), particle dynamics, sampling (number of images), and image processing. Wilson and Smith\cite{79} provided a systematic evaluation of the uncertainty in a rectangular laminar jet through comparisons of hot-wire and PIV data. They showed that particle displacement ($\Delta x$) and velocity gradients are the largest sources of uncertainty. Uncertainty in mean velocity $u$ was less than $\pm 2\%$ for $\Delta x > 2$ pixels and uncertainty in Reynolds normal stress $u'u'$ was minimized for the shear region for $\Delta x > 2.5$ pixels and jet centerline for $\Delta x > 4$ pixels. They also showed that the uncertainty in $u'u'$ due to strong velocity gradients (shear) is highest ($\sim 20\%$) near the nozzle exit and less downstream where the velocity gradients are lower. The effect of particle image diameter, depending on lens f#, was less sensitive, however uncertainty was minimal with a lower f#. The smallest f# used in Wilson and Smith\cite{79} was f5.6, and in this study f2.4 was used to achieve the smallest particle image diameter while still achieving sufficient particle intensity.

Particle dynamics refers to the accuracy with which the particles represent the actual flow velocity. The ability of the particles to follow the flow, respond after passing through shock waves, and represent the turbulent eddies is critical to accurately measuring mean velocity and turbulence quantities. Callender\cite{80} used Phase Doppler Anemometry (PDA) to measure the distribution of particle diameters for the olive oil seeders used in this study. A normal dis-
tribution of particle diameters around 1µm was measured which provides a frequency response of up to 10 kHz[81] resulting in 5% uncertainty in the measured turbulence intensity and less that 1% uncertainty in mean flow velocity. Particle lag through shock waves has been shown to be a large source of error and can take up to 12mm downstream of the shock to decelerate to the gas velocity for 1µm particles[81]. Lazar et.al.[82] analyzed the uncertainties on mean velocity in a supersonic flow with a bow shock and Mach disk, features which are present in the jet flow in this study. It was shown that particle lag through shocks could be up to 20%. A study by Bridges and Wernet[83] analyzed a large database of PIV measurements with various anemometry techniques from literature. It was found that most turbulence intensities agreed within 12% bias error for high subsonic jets. A combination of all of the uncertainties discussed provides confidence in the current measurements of ±5% in mean velocity in and ±15% in turbulence.

2.10 LES & CAA

The research in this dissertation was part of a research grant that included collaboration between the University of Cincinnati (UC), Chalmers University of Technology (CTH) in Gothenburg, Sweden, KTH Royal Institute of Technology (KTH) in Stockholm, Sweden, and GKN Aerospace (GKN - formerly Volvo Aero). This dissertation includes some computational results and discussion from CTH and GKN for the sake of completeness, however it stands to be explicitly stated that none of the computational work was conducted by the author. Haukur Hafsteinsson, under the supervision of his advisor, Lars-Erik Eriksson at Chalmers University, conducted Large Eddy Simulation (LES) and Computational Aeroacoustics (CAA) for many instances of the work reported in this dissertation. The collaboration was a joint supplementation of experiments and computations to provide valuable insight into the experimental results and discoveries, validation and verification of LES and CAA, and high fidelity comparisons of experiments with computations. Comparisons are reported throughout this dissertation and the author is grateful to have had the opportunity to learn from and work
with our colleagues on this research.

2.10.1 Comments on Workflow for Collaboration

The workflow for the computational studies included many discussions between the experimental group at UC and the computational group. The workflow was iterative and a general description is given here. In general, all of the comparison figures in this dissertation were generated by the author from raw data sets provided by the computational groups. This includes the profile comparison figures, contour figures, and all subsequent analysis of the computational data. Bernhard Gustafsson at GKN Aero conducted the nozzle optimization procedure using RANS computations to iteratively modify the nozzle design parameters to achieve the final splined nozzle. The optimization was conducted from input and guidance from Dr. Ephraim Gutmark and the author at UC. Converged RANS solutions were provided to the author. The author post-processed the data for comparison with experimental data. The collaboration with CTH was strongly intertwined in order to improve both the experimental and computational results. Many in-depth discussions shed light on issues with both experimental measurements and implementation of LES. Initial setup of the LES simulation geometry and boundary conditions was highly collaborative between the author and CTH. The converged LES solutions were provided to the author and all of the analysis and figures presented in this dissertation were conducted by the author. This statement on workflow is included to clarify any ambiguity of the work conducted by the author. All statements given on the LES computations are from the authors understanding of the simulations and from personal correspondence between the author and Haukur Hafsteinsson at CTH. Nevertheless, it should be kept in mind that the research project was highly collaborative and the experiments and computations both complemented each other. Much was learned about the way the experiments are conducted, values are measured, and where additional uncertainty can arise. Conversely, much was learned about the sensitivity of the LES approach to accurate geometry, boundary conditions, and initial conditions to name a few. The quality of the experiments
and computations were both drastically improved through the collaboration and a great deal was learned that improved communication and fidelity of the research.

2.10.2 Numerical Approach

The flow field is obtained by solving the compressible form of the Navier-Stokes equations, with the viscous stress defined using Newton’s law and the heat flux with Fourier’s heat law. The system of governing equations is closed by two assumptions of the thermodynamics of gas. First, the gas is considered to thermally follow the ideal gas law. Second, the gas is calorically perfect, implying that internal energy and enthalpy are linear functions of temperature. The Favre-filtered Navier-Stokes equations are solved with a finite volume solver belonging to the G3D family of codes developed by Eriksson[84]. The code solves the compressible flow equations in a conservative form on a boundary-fitted, curvilinear non-orthogonal multi-block mesh. The convective fluxes are solved with a low-dissipation third-order upwind-biased scheme, a second-order centered difference scheme is used for the diffusive fluxes and a second-order three-stage Runge-Kutta technique for time marching. The Smagorinsky part of the model proposed by Erlebacher et al.[85] is used as the subgrid-scale model. Wall functions are used at the nozzle boundaries to model the near wall behavior of the flow. Another important item is the use of a specially modified pressure sensor which ensures that the extra added numerical dissipation needed for shocks is applied only locally and dynamically. The code has been implemented for parallel computations using domain decomposition and a message passing interface (MPI). Detailed description of the numerical scheme and implementation of boundary conditions are given in Eriksson[84], Andersson[86] and Burak[87].

2.10.3 Sound Propagation

Kirchhoff surface integral formulation is used to propagate the noise to the far-field observers for a property $\Phi$ governed by the wave equation, in this case the pressure. The surface $S$ encloses all noise generating structures from the jet flow and by performing the integration
for a specified time interval the pressure fluctuations, e.g. the noise, at a given point outside the surface is obtained. A schematic of the various sizes of Kirchoff surfaces are shown in Figure 2.15 along with the far-field observer locations. Pressure is monitored on the Kirchoff surface and propagated to each far-field observer location. The Kirchoff surface integration method only requires information on the surface which lowers the computational burden. The integral follows as

$$\Phi(y, t) = \frac{1}{4\pi} \int S \left[ \frac{\Phi}{r^2} \frac{\partial r}{\partial n} - \frac{1}{r} \frac{\partial \Phi}{\partial n} + \frac{1}{c_\infty r} \frac{\partial r}{\partial n} \frac{\partial \Phi}{\partial t} \right] dS(x)$$

(2.14)

where $y$ is the observer location in the far-field and $x$ is a location on the surface. The retarded time $\tau_r$ is related to the observer evaluation of time $t$, the distance from observer and surface location, $r = |y - x|$, and the speed of sound in the far-field $c_\infty$ as

$$\tau_r = t - \frac{r}{c_\infty}$$

(2.15)

and the expression within the brackets in Eq. (2.14) is therefore evaluated at retarded time, representing the emission time.
2.10.4 Computational Domain

The same meshing strategy was used for both the sharp and splined nozzles. The computational domain was discretized using a hexahedral block-structured boundary-fitted mesh with 249 mesh blocks and approximately 19M nodes. The domain is divided into three parts: a high-resolution region near the nozzle exit, a medium-resolution region further downstream, and a 2D entrainment region as shown in Figure 2.16. The mesh uses a combination of Cartesian and polar mesh blocks to ensure mesh homogeneity in radial direction throughout the domain. Furthermore a smoothing routine is swept through the domain in order to achieve as orthogonal cells as possible and to ensure numerical accuracy. Sections of the computational mesh are shown in Figure 2.17 and 2.18. Total pressure and total temperature are specified at the nozzle inlet. At the co-flow inlet, the density, velocity and total temperature are set to match conditions in the experiments. The 2D entrainment region has periodical boundary condition in the azimuthal direction. The purpose is to achieve correct mass flow in and out of the domain in order to avoid back-flow. Ambient pressure is specified at the outlet and a damping zone is added at the end of the domain to minimize reflections from the outlet.

![Figure 2.16: LES computational domain.](image)
Figure 2.17: Axial sections of the mesh showing inside the nozzle and a zoomed view of the mesh.

Figure 2.18: A slice through LES domain at (a) nozzle exit, $x = 0$, in the $yz$-plane and (b) at $y = 0$, in the $xz$-plane. Every other node shown, thick lines denote block boundaries.
Chapter 3

Baseline Supersonic Jets

Characterization of the acoustics and flow field for baseline supersonic jets are covered in detail in this chapter. The ‘baseline’ term refers to a circular converging-diverging (C-D) nozzle without modifications or noise reducing technology applied. In depth characterization of the acoustics and fundamental flow features is presented for various jet operating conditions. The detailed study provides a large database of the acoustic and flow properties for the jet studied throughout this dissertation for use in understanding and applying fluidic injection for noise reduction. The bulk of this chapter deals with the effect of optimizing the nozzle contour on flow, acoustics, and performance of an axisymmetric C-D nozzle. The final section details the effects of faceted (polygonal) nozzles simulating realistic nozzle geometries.

3.1 Background

Supersonic jet noise has been a significant topic of research for the past 50 years as flight vehicles have operated in the supersonic flight regime. Classical research on supersonic jet noise has focused on studying smoothly varying “method of characteristic” type converging-diverging nozzles. At fully expanded conditions these nozzles are shock-free and the noise spectrum is dominated by turbulent mixing noise and Mach wave radiation[10, 27]. At imperfectly expanded conditions the supersonic jet stream contains a quasi-periodic shock-expansion...
structure which gives rise to broadband shock associated noise (BBSN) and discrete screech tones. These smooth nozzle contours are ideal for nozzle performance, but in practical applications, especially on modern supersonic jet engines, the nozzle contours are formed with conical sections to allow for area variation throughout the flight envelope and to reduce length, weight, and cost. Practical nozzles tend to have relatively sharp throats at the intersection of two conical sections. This sharp throat can cause strong internal shocks to form which adversely affect nozzle performance and generate shock noise components across the jet operating range. A series of technical notes by Migdal & Landis\cite{88}, Darwell & Badham\cite{89}, and Migdal & Kossen\cite{90} showed that conical nozzles can have shock formation near the centerline due to intersection of Mach lines from flow overturning. The authors showed that slight modifications to the curvature at the throat could reduce or eliminate shocks by accommodating the overturning of the flow. Additionally it was suggested that a conical divergent section could be attached after this point to achieve a shock free flow with a simpler design than proposed by Rao\cite{91}. This suggested that jet thrust could be improved with geometry modification, but the question remained, how would the components of supersonic jet noise depend on elimination of internal shocks?

Limited acoustic data is available for practical nozzle geometries and although the effect of nozzle contour on performance (thrust) is understood, it is not quite clear how the nozzle contour affects the complex supersonic noise components. Tam & Tanna\cite{37} and Norum & Seiner\cite{92} studied the shock noise from convergent and convergent-divergent nozzles. At supersonic jet velocities the convergent nozzle is always underexpanded and shock noise increases with jet Mach number. Convergent-divergent nozzles have a pronounced dip in shock noise in an envelope around the nozzle design condition. Seiner & Norum\cite{93} also studied broadband shock associated noise of a nozzle representative of an F-15. This nozzle has higher shock noise at all conditions than a nozzle with a smoothly varying contour that is shock-free at the design condition. Flow and acoustic measurements by Kuo et al.\cite{94}, Munday et al.\cite{95}, and Bridges et al.\cite{23} have shown that practical nozzle geometries require additional consid-
erations including double shock structures, shock noise across all operating conditions, and different screech characteristics in comparison with ideal nozzles.

The present experimental and computational investigation analyzes the differences in acoustics, flowfield, and thrust for biconical (sharp) and optimized (splined) axisymmetric nozzles. The experimental and computational study was conducted in three phases. The first phase included optimizing the nozzle converging section and throat contour to reduce internal shocks and maximize thrust using RANS computations near the nozzle design condition. The second phase included experimental measurements and LES computations of the flowfield and acoustics, and comparison of the results. The third phase analyzed the jet performance by developing a method to measure the momentum and pressure thrust and compare with the computations. The latter two phases also include analysis at one underexpanded and two overexpanded conditions.

3.2 Nozzle Design Procedure

The optimization of the converging section and throat of a biconical nozzle was conducted computationally with a Reynolds Averaged Navier Stokes (RANS) turbulent flow solver in ANSYS CFX v12.1 using the $SST k-\omega$ turbulence model. The optimization was conducted by Bernhard Gustafsson at GKN Aerospace (formerly Volvo Aero), accompanied by discussions with the author on design issues related to the experiments. The writing and figures included in this dissertation were produced by the author from the converged RANS solutions provided by GKN. Machine drawings and all experiments were conducted by the author. The goal was to minimize internal shock formation and maximize thrust. The final nozzle design has a smoothly varying (5th-order spline) convergent section up to the throat which transitions smoothly to a conical diverging section identical to that of the biconical nozzle. The biconical nozzle is alternately referred to as “sharp” and the optimized nozzle is referred to as the “splined” nozzle. ANSYS DesignModeler was used for optimization of the nozzle with throat corner radius $R_c$ and convergence angle $\theta_{conv}$ as the design variables, shown in Figure 3.1a.
Response surfaces from the initial RANS simulations were used in evaluating 3000 virtual designs. Thrust and velocity profile deviations from the mean were used as objectives for the optimization, determined by integrating over the nozzle exit plane \( \frac{1}{A} \int_A (u - \bar{u}) dA \). Figure 3.1b highlights three nozzle designs on a scatter plot of velocity profile uniformity, colored by thrust. The ‘Short Optimized Nozzle’ had the same overall convergent section length, \( L_c \), as the biconical nozzle. The ‘Long Nozzle’ had an elongated convergent section that increased thrust and velocity profile uniformity although it had some recirculation in the convergent section from the small radius that transitioned from the jet pipe into the nozzle. The recirculation was reduced for the ‘Long Nozzle’ by defining the convergence contour with a 5th-order polynomial, resulting in the final splined nozzle design. The splined nozzle is more efficient and can produce the same thrust as the biconical nozzle at a lower \( NPR \). This is the result of increased mass flow at a given \( NPR \), from the higher discharge coefficient \( (C_{d,\text{sharp}} = 0.93, C_{d,\text{splined}} = 0.99) \). Further details on the nozzle design optimization procedure and detailed comparison between the computed and measured flow field are reported by Gustafsson et al. \([75]\). The splined nozzle throat shock strength is significantly reduced and nearly eliminated as seen in the uniform exit Mach number profile in Figure 3.2a. The peak Mach number is lower and the slipline that is present from the shock interaction in the sharp throat nozzle is eliminated. Evidence of a weak shock near the nozzle lip of the contoured throat nozzle is seen in the exit pressure profile in Figure 3.2b. The radial pressure variation at the nozzle exit is strongly reduced for the splined nozzle, similar to the Mach number profile. Schematic cross-sections of the nozzles in this investigation are shown in Figure 3.3 for comparison. Two thrust-matched and two overexpanded \( NPR \) are primarily studied in this investigation for the sharp and splined nozzle detailed in Table 3.1. The thrust matched condition were determined from RANS and verified with LES and experimental measurements, but knowledge of thrust at overexpanded conditions was not known a priori. It should be noted that the \( U_8^8 \) scaling for acoustics is a simplification for one-dimensional flow and this study quantifies both the acoustics and jet thrust.
Figure 3.1: Design optimization parameters $R_c$ and $L_c$. The scatter plot illustrates the uniformity of the exit velocity profile (lower value is more uniform) and colored by thrust with red being highest.

Figure 3.2: Comparison of RANS exit profiles of Mach number and pressure for the biconical throat and splined throat nozzles. “Baseline” is the biconical nozzle, “Splined” is the optimized nozzle.

Figure 3.3: Supersonic nozzle cross sections used in this study.
Table 3.1: Operating conditions for this investigation calculated from isentropic flow relations.

<table>
<thead>
<tr>
<th>NPR</th>
<th>$M_j$</th>
<th>$V_j$ (m/s)</th>
<th>NPR</th>
<th>$M_j$</th>
<th>$V_j$ (m/s)</th>
</tr>
</thead>
<tbody>
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<td>Thrust Matched</td>
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<td>1.64</td>
<td>507.5</td>
<td>4.27</td>
<td>1.60</td>
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<td>1.56</td>
<td>491.1</td>
<td>3.83</td>
<td>1.53</td>
</tr>
<tr>
<td>NPR Matched</td>
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<td>1.36</td>
<td>445.7</td>
<td>3.00</td>
<td>1.36</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>1.22</td>
<td>412.1</td>
<td>2.50</td>
<td>1.22</td>
</tr>
</tbody>
</table>

3.3 Baseline Jet Flow Fields

Time-averaged shadowgraph images of the two nozzle designs operating at various $NPR$ are shown in Figure 3.4. The sharp throat nozzle is on the left and the splined nozzle on the right. The design condition is $NPR_d = 4.0$ ($M_j = 1.56$) for the sharp throat nozzle and $NPR_d = 4.33$ ($M_j = 1.61$) for the splined nozzle. It should be noted that the jet flows are not quasi-one-dimensional, they are at the least two-dimensional axisymmetric, and with fluidic injection or considering turbulent structures, become highly three dimensional. The ‘design’ condition in the classical sense is only an approximation. The nozzles are thrust-matched at the $NPR_{sharp} = 4.5/NPR_{splined} = 4.27$ and $NPR_{sharp} = 4.0/NPR_{splined} = 3.83$ as determined from RANS and the optimization procedure. The sharp throat nozzle has a pronounced dual shock structure at the higher $NPR$. At the lower $NPR$ the double shock system dissipates more quickly while at the higher $NPR$ it persists further downstream. The splined nozzle has a singular shock structure that is very similar to an isentropic nozzle operating at off-design conditions. At $NPR = 4.27$ a weak secondary shock is seen in the jet plume. The shadowgraph provides nice qualitative visualization of the jet flow, but quantitative information is not easily extracted since the image intensities are relative to the background values. Spatial information such as shock cell spacing can be extracted for the first few shock cells. However, the data was acquired with a high-speed camera, so quantitative temporal information can be extracted for analysis of the unsteady phenomenon as presented in later sections.
Figure 3.4: Mean shadowgraph for the biconical and splined nozzles at various operating conditions.
3.4 Mean Velocity and Turbulence

The key difference in the nozzle designs is the converging section and throat contour which alters internal shock formation. With the splined nozzle design, $C_d$ is increased from 0.93 to 0.99 providing higher mass flow at a given $NPR$. The initial hypothesis was that for thrust matched conditions, the reduction in throat shock strength would reduce BBSN generated by the dual shock system. As seen in the previous section, the acoustics remain relatively unchanged with exception of the screech characteristics.

Streamwise PIV measurements near the nozzle exit show the velocity field, shock structure, and turbulence in the first $3D_j$. Figure 3.5 to 3.12 show contours of normalized mean axial velocity and mean turbulence intensity for both nozzles at all operating conditions. All figures are shown on the same contour range allowing for comparison of the shock structures, velocity magnitudes, and visualization of the shear layer near the nozzle. LES provides the added benefit of visualizing the flow structure internal to the nozzle. Detailed axial and radial profiles are shown in Figure 3.13 to 3.18. Measurements and computational results are shown using the same scales and with the same contour levels for direct comparisons.

At the thrust matched underexpanded condition in Figure 3.5, the biconical nozzle has a very pronounced dual shock structure while the splined nozzle has a strong single shock structure with a weak secondary shock emanating from the nozzle throat. The turbulence levels are similar with slightly stronger near nozzle turbulence for the biconical nozzle contributing to the slightly increased high frequency mixing noise in Figure 3.19a. This is due to the internal shock impinging on the shear layer directly after the nozzle exit. Figure 3.6 shows how the shock structure forms internal to the nozzle as the flow is overexpanded by the sharp change in direction generating a strong internal shock. This shock is significantly weakened in the splined nozzle indicated by less variation in velocity and uniform velocity at the exit plane. At the thrust matched design condition in Figure 3.7 & 3.8 it is apparent that the biconical nozzle still has a pronounced double shock structure while the weak second shock of the splined nozzle has merged with the primary shock creating a single shock train.
This correlates with a growth in screech shown in Figure 3.19b. The internal shock in the biconical nozzle seems to have a stabilizing effect on the jet, preventing growth of the screech instability.

For \( NPR = 3.0 \) in Figure 3.9, the velocity fields begin to look similar. Within \( 0.4D_j \) the nozzle lip shock forms a Mach disk and a slipline, generating a single shock train. For the biconical nozzle this suppresses the secondary shock after the point where the Mach disk forms. This particular shock structure is very susceptible to screech and results in strong screech amplitudes and many harmonics. At this condition the turbulence in the jet potential core is higher around \( x/D_j = 2.5 \) signaling large amplitude oscillations of the jet from strong screech. There seems to be correlation between the level of periodicity of the shock structure and the susceptibility of the jet to screech instability. Finer resolution of how the shocks change with operating condition could quantify this effect further. LES predicts slight separation for the biconical nozzle in Figure 3.10 which generates a shift in the shock cell system. See section 3.4.1 for further discussion of separation in LES. At \( NPR = 2.5 \), Figure 3.11, the Mach disk is further upstream, but still results in a single shock train in the jet potential core. The biconical nozzle shocks are coherent while the splined nozzle has a slipline that passes through a majority of the shock cells. Overall the acoustics are identical (see Figure 3.19d) at this condition and appear to be independent of the presence of the slipline. It is apparent in Figure 3.12 that LES predicts separation in the divergent section of the nozzle for both the biconical and splined nozzles at this highly overexpanded condition. This had minimal effect on the acoustics (Figure 3.21d and 3.22d) indicating that the flow structures were captured accurately with the sources shifted slightly upstream.

Figure 3.13 and 3.14 show axial profiles of axial velocity at \( y/D_j = 0.05 \) just off the centerline from experiments and LES. The off-axis location was chosen to avoid comparison along the slipline. The biconical nozzle clearly shows the double shock-expansion structure at design and underexpanded conditions indicated by the closely spaced variations in velocity between \( x/D_j = 0.5 \) to 1 and \( x/D_j = 1.5 \) to 2. The shock strength and velocity gradient are higher for
LES and less steep as measured by PIV. A lower velocity gradient through the shocks could be an artifact of particle lag as discussed in section 2.9.4 and better agreement is seen for cases with lower shock strengths. The splined nozzle shows a single shock-expansion structure and smoother variations in velocity. Better comparison of the PIV and LES is observed for the splined nozzle with a slight overprediction of velocity for the LES. Interestingly the ‘global’ shock strength, or the difference between maximum and minimum velocity, for the biconical and splined nozzles is equivalent, for example $0.9 \leq u/U_j \leq 1.1$ for the thrust matched operating conditions. This indicates that reducing the internal shocks actually strengthens, not weakens, the overall shock strength in the jet. This is the reason for the negligible changes in shock noise seen in the narrowband acoustics. At overexpanded conditions the shapes of the axial velocity profiles match the experiments well but the LES captures a few small velocity variations that are not present in the experiments. This could be an issue with the numerics handling a strongly screeching jet which requires computational time to develop the instability cycle fully. For $NPR = 3.0$ the shock strength is 25% stronger for the splined nozzle which may account for the increased screech amplitude observed in the acoustics. Evidence of separation in the LES is indicated by the shift in the shock-expansion structure in Figures 3.13c, 3.13d, and 3.14d. The separation causes the entire shock-expansion system to shift axially upstream and the shocks are artificially high for the splined nozzle at $NPR = 2.5$ due to the separation. Reexamining Figure 3.11 and 3.12 indicates this increased shock strength is due to LES not developing as strong of a slipline because of the separation. Figures 3.15 and 3.16 show radial profiles of mean axial velocity. The experiment profiles depict every fourth data point for clarity, but the line is constructed using all of the data points to avoid inaccuracies in regions of high velocity gradients. Excellent agreement is observed between experiments and LES with the exception of $NPR = 2.5$, where LES shows a stronger slipline at $r/D_j = 0$ and slightly thicker shear layer for the biconical nozzle at $NPR = 4.5$ and 4.0. The separation in LES is indicated by the velocity deficits in the $x/D_j = 0.05$ profiles at $NPR = 2.5$ and the upstream shift of the jet accounts for the discrepancies in the velocity.
profiles at the downstream locations.

Turbulence intensity comparisons in Figures 3.17 and 3.18 have discrepancies in magnitude up to $x/D_j = 1$ but match very well at $x/D_j = 3$. Near the nozzle exit the shear layer thickness approaches the spatial resolution of the PIV, resulting in higher uncertainty in this region, and making it likely that PIV underpredicts turbulence in the shear layer near the nozzle. Conversely an initially laminar shear layer in the LES due to use of wall functions internal to the nozzle could also overpredict the turbulence near the nozzle from strong gradients near the nozzle lip. The discrepancies seen near the nozzle could be a combination of both of these errors. Downstream at $x/D_j = 3$ the turbulence profile magnitudes and shear layer thickness match very well as the LES begins to develop a fully turbulent shear layer and the shear layer thickness better matches the PIV interrogation window size. Ideally, variation in the PIV interrogation window size to dynamically match the dominant turbulent length scales would reduce uncertainty in the turbulence intensity. Overall, negligible differences in the turbulence for the biconical and splined nozzles are observed in agreement with the minimal changes in turbulent mixing noise. The development of near nozzle turbulence is relatively independent of the geometrical changes of the converging section and throat and dominated by the jet velocity and design of the divergent section.

3.4.1 Comments on separation

The following statement is from personal discussion between the author and Haukur Hafsteinsson at CTH and is an excerpt from Cuppolett et al.\cite{96}. Comparisons between the predicted and measured flow fields for the highly over-expanded case ($NPR = 2.5$) show that the LES method fails to predict the axial location of the shocks correctly. The reason for this is that the LES predicts an interior separation near the nozzle exit which is not observed in the experiments. Such interior separations usually appear when $p_e/p_a$ is in the range $0.30 - 0.45$\cite{97}. In this case the ratio is near 0.6 which implies that there should not be any separation, in accordance with the experimental data. A possible cause for this discrepancy is
the wall functions applied in the LES solver. This works well for attached flows with fully turbulent boundary layers and with a mesh resolution of $y^+$ in the range of 30-100. The main function of the wall model is to supply a realistic momentum loss due to friction. However, for flows rather close to separation the predicted shear stress is underestimated and this may lead to premature separation. Once separation has occurred, the wall model applied cannot ‘reverse’ the process. There are two remedies to this problem: a) use a highly refined grid close to the wall and an improved wall model, and b) remove the wall function approach entirely, i.e. use a slip-wall condition. In case (a) a more accurate shear stress could be obtained but the computational cost would be significantly higher. In case (b) the slip conditions would ensure a non-separating flow due to the lack of any wall shear stress at all. However, there would be some overestimation of the nozzle thrust due to the lack of any momentum loss inside the nozzle. Other hybrid URANS/LES approaches[98] have also performed well for high Reynolds number flows where separation is an issue.
Figure 3.5: Axial velocity and turbulence contours comparing the mean flow features of the biconical and splined nozzles. $NPR_{\text{sharp}} = 4.5$ (top), $NPR_{\text{splined}} = 4.27$ (bottom)

Figure 3.6: LES axial velocity and turbulence contours comparing the mean flow features of the biconical and splined nozzles. $NPR_{\text{sharp}} = 4.5$ (top), $NPR_{\text{splined}} = 4.27$ (bottom)
Figure 3.7: Axial velocity and turbulence contours comparing the mean flow features of the biconical and splined nozzles. \( NPR_{\text{sharp}} = 4.0 \) (top), \( NPR_{\text{splined}} = 3.83 \) (bottom).

Figure 3.8: LES axial velocity and turbulence contours comparing the mean flow features of the biconical and splined nozzles. \( NPR_{\text{sharp}} = 4.0 \) (top), \( NPR_{\text{splined}} = 3.83 \) (bottom).
Figure 3.9: Axial velocity and turbulence contours comparing the mean flow features of the biconical and splined nozzles. $NPR_{\text{sharp}} = 3.0$ (top), $NPR_{\text{splined}} = 3.0$ (bottom).

Figure 3.10: LES axial velocity and turbulence contours comparing the mean flow features of the biconical and splined nozzles. $NPR_{\text{sharp}} = 3.0$ (top), $NPR_{\text{splined}} = 3.0$ (bottom).
Figure 3.11: Axial velocity and turbulence contours comparing the mean flow features of the biconical and splined nozzles. $NPR_{\text{sharp}} = 2.5$(top), $NPR_{\text{splined}} = 2.5$(bottom)

Figure 3.12: LES axial velocity and turbulence contours comparing the mean flow features of the biconical and splined nozzles. $NPR_{\text{sharp}} = 2.5$(top), $NPR_{\text{splined}} = 2.5$(bottom)
Figure 3.13: Axial profiles of axial velocity comparing PIV and LES for the biconical nozzle.

Figure 3.14: Axial profiles of axial velocity comparing PIV and LES for the splined nozzle.
Figure 3.15: Radial profiles of axial velocity comparing PIV and LES for the biconical nozzle.
Figure 3.16: Radial profiles of axial velocity comparing PIV and LES for the splined nozzle.
Figure 3.17: Radial profiles of turbulent kinetic energy comparing PIV and LES for the biconical nozzle.
Figure 3.18: Radial profiles of turbulent kinetic energy comparing PIV and LES for the splined nozzle.
3.5 Far-field Acoustic Measurements

Far-field acoustic measurements of the biconical and splined nozzles are compared to identify how the supersonic noise components are affected by the throat contour and reduction of throat shock strength. For visualization purposes, all narrowband figures are shown staggered by adding 30 dB iteratively to the actual $SPL$ value for the $\psi = 150^\circ$ (i.e. the $\psi = 35^\circ$ spectra is $SPL_{150^\circ} + 120$ dB). Figure 3.19 shows narrowband spectra for the biconical and splined nozzles at five observation angles for the two thrust matched conditions and two overexpanded conditions. At the underexpanded thrust matched condition the spectra are virtually identical at all observation angles with the splined nozzle having slightly less high frequency noise. At the design condition the biconical nozzle exhibits a low amplitude ($\leq 15$ dB above background) screech apparent only at the upstream angle while the splined nozzle exhibits 15 dB higher screech amplitude at the upstream angle. The primary screech frequency appears at all angles, and two harmonics of the primary screech frequency appear at $\psi = 90^\circ$. At $NPR = 3.0$ very strong screech occurs at all observation angles for both nozzle designs. Many harmonics are apparent in the spectrum, although the splined nozzle screech is still stronger with more harmonics. Minor differences between the biconical and splined nozzles are observed with exception of the high frequencies which could be attributed to the effect of screech on the mixing noise components. Norum et al.\cite{99} and Cuppoletti et al.\cite{96}, among others, have noted that screech can suppress shock noise and alter the fine-scale mixing noise spectra. At $NPR = 2.5$ the acoustic spectra are identical again with slightly different screech amplitudes. Figure 3.20 shows OASPL for both nozzles at the various operating conditions. The directional nature of BBSN and screech is emphasized in OASPL by very sharp rises at the dominant propagation angles. For $NPR = 3.0$ the splined nozzle has higher OASPL at all angles but the trend is similar to the biconical nozzle. It is difficult to determine if the higher OASPL is due to the screech amplitude or a difference in thrust level. Strong screech amplitudes can increase or decrease the other noise components. With no screech at $NPR = 2.5$ the OASPL are identical while the conditions are not thrust-matched. The splined nozzle is more susceptible to
screech across the operating envelope. The same acoustics can be achieved at thrust-matched conditions with 4% - 5% less pressure. This shows that nozzle performance can be improved without acoustic penalty at certain operating conditions.

Far-field acoustics measurements are compared to LES computed far-field acoustics for validation of the CAA methods. The biconical and splined nozzle comparisons are shown in Figure 3.21 & 3.22. All operating NPR are shown to illustrate the effectiveness of LES capturing the various components of supersonic jet noise including the large and fine scale mixing noise, BBSN, and screech. Very good comparison is seen overall, especially for the cases without strong screech. The directional characteristics of the mixing noise and BBSN noise are in good agreement with measurements. Discrepancies arise at low frequencies below 2 kHz which is attributed to the short computation time $O(\text{ms})$ compared to measurement time (15 seconds). The premature decay of the high frequency noise above 50 kHz from LES is due to mesh resolution limitations. For both nozzles, LES captures the peak frequency, amplitude, and directional shift of the BBSN very well. It does however slightly under-predict the decay of the spectral peak at the aft angles (i.e. $\psi = 120^\circ$) and the suppression of the spectral peak when strong screech occurs.

The effect of screech on the acoustic spectrum seems to be the strongest source of discrepancy between the measured and predicted acoustics. The primary frequency is very well captured at the dominant radiation angle, $\psi = 35^\circ$, especially when the screech amplitude is of moderate amplitude. For conditions in which screech is more than 10 dB above the background at $\psi = 35^\circ$, harmonics begin to appear in the $\psi = 90^\circ$ spectrum. Once screech is above 15 dB at $\psi = 35^\circ$ many harmonics are apparent at all observer locations. This large amplitude screech is usually coupled with large amplitude motion of the jet depending on the excited mode, and is accompanied by a reduction in the turbulent mixing noise and BBSN. This is not captured by the LES and could be due to the simulation time which is inadequate for screech instability growth to limit cycle oscillation.
Figure 3.19: Narrowband acoustics from experiments comparing sharp and splined nozzles.

(a) $NPR_{\text{sharp}} = 4.5, NPR_{\text{splined}} = 4.27$

(b) $NPR_{\text{sharp}} = 4.0, NPR_{\text{splined}} = 3.83$

(c) $NPR_{\text{sharp}} = 3.0, NPR_{\text{splined}} = 3.0$

(d) $NPR_{\text{sharp}} = 2.5, NPR_{\text{splined}} = 2.5$
Figure 3.20: OASPL from experiments comparing sharp and splined nozzles at thrust matched conditions.
Figure 3.21: Narrowband acoustics comparing experiments and computations for the sharp nozzle.
Figure 3.22: Narrowband acoustics comparing experiments and computations for the splined nozzle.
3.6 Screech Suppression Effect on Acoustics

The splined nozzle has increased performance for a given $NPR$ with the primary difference being the screech characteristics. It is well known that tabs[100, 101], chevrons[21], and fluidic injection[61] at the nozzle trailing edge are effective at reducing screech in jets. However, these intrusive methods also affect the mean jet flow by altering shocks and jet mixing. Norum[99] showed with a convergent nozzle that placing a reflector at $\lambda/4$ locations from the jet exit plane, where $\lambda$ is the wavelength of the primary screech frequency, up to 20 dB of screech suppression can be achieved. In the present study a 12” x 12” reflector with a cardboard backing was placed at the $\lambda/4$ location upstream of the nozzle exit plane as shown in Figure 3.23. The surface of the reflector is a 1” thick cratered acoustic foam.

The foam reflector was effective at suppressing screech at different conditions. When the primary screech tone was greater than 20 dB from the background noise, it could not be completely suppressed. Figure 3.24 shows the narrowband and OASPL for suppression of a strong screech tone ($NPR = 3.0$) and weak screech tone ($NPR = 2.5$). At $NPR = 3.0$ the primary screech tone is suppressed 20 dB at $\psi = 35^\circ$ which suppresses all of the harmonic tones at that angle. The suppression of the screech tone results in a 5-10 dB increase in the BBSN spectral hump. At $NPR = 2.5$ usage of the reflector nearly eliminates the screech, however the BBSN remains unchanged. Norum[99] also noted this inverse relationship for high amplitude screech tones and independence of screech and BBSN for lower amplitude screech. When the screech is suppressed for both nozzles at $NPR = 3.0$, the OASPL is higher at all angles for the splined nozzle indicating that stronger screech is not solely responsible for the increased noise of the splined nozzle, the higher thrust is significant in generating more noise, unlike at $NPR = 2.5$ where higher thrust is achieved with no acoustic penalty.
3.7 Thrust Analysis

The thrust matched conditions (underexpanded and design) were initially determined from the RANS optimization which was conducted prior to the experiments. The overexpanded conditions had interesting acoustic differences when operated at the same NPR, but since $C_d$ is higher for the splined nozzle it is assumed that the splined nozzle has higher thrust at these conditions. To further investigate the thrust of the two nozzles, the momentum thrust was calculated from PIV measurements, pressure thrust was calculated using the method described in section 2.8, and compared with calculations from LES. The total static thrust is defined as

$$ F = F_m + F_p = 2\pi \int_0^r (\tilde{m}_t \cdot u(r)) + [p_s(r) - p_a]) r \cdot dr \quad (3.1) $$

where $F_m$ is the momentum thrust component and $F_p$ is the pressure thrust component. Experimental mass flow rate for the choked nozzle was calculated from the discharge coefficient and nozzle stagnation conditions using the $C_d$ determined from initial RANS simulations ($C_{d,\text{sharp}} = 0.93, C_{d,\text{splined}} = 0.99$) using Eq. (3.2) for choked mass flow.

$$ \tilde{m}_t = C_d \frac{A_t P_o}{\sqrt{RT_o}} \sqrt{\frac{2\gamma \frac{\gamma+1}{\gamma+1}}{\gamma+1}} \quad (3.2) $$

Figure 3.24: Effect of foam reflector on overexpanded conditions.
The experimental mass flowrate is within 0.5% of what was computed in LES for all conditions, therefore comparison of momentum thrust computed from LES and PIV depends only on integration of axial velocity across the nozzle exit. Computed thrust was compared with ideal thrust calculated from the isentropic mass flow and fully expanded jet velocity for the nozzle operating condition. Figures 3.25a & 3.25b show experimental and computed total thrust along with the ideal thrust. Excellent comparison between experiments and LES is seen for both nozzles at the underexpanded and design conditions. At overexpanded conditions LES overpredicts the thrust due to separation (refer to section 3.4.1). The experimental data is shown again in Figure 3.26a with biconical and splined data on the same figure. The biconical nozzle has lower thrust at all setpoints which is a combination of a lower $C_d$ and velocity deficits due to stronger shocks at the nozzle exit. The splined nozzle has thrust closer to ideal at all operating conditions, although significant negative pressure thrust is evident at overexpanded conditions.

The thrust coefficient $C_f = \frac{F}{F_1}$ shown in Figure 3.26b, with $F$ being total thrust and $F_1$ being ideal thrust, depicts the improved performance with the splined nozzle. The total thrust is indicated by the black and dark gray bars, while the light gray bars indicate negative pressure thrust that reduces the momentum thrust. The sum of the dark and light bar indicate the momentum thrust. The splined nozzle has 10% higher thrust at $NPR = 2.5$ and at $NPR = 3.0$. The splined nozzle has equivalent thrust while operated at 4% lower $NPR$ for design and underexpanded conditions. At $NPR = 2.5$ the 10% thrust increase is achieved with no acoustic penalty. This is due to the Mach disk location that creates a similar shock structure for both nozzles downstream in the dominant source region. This finding is significant since tactical jets usually operate at overexpanded conditions[102], especially near ground level where noise is of greater concern. At $NPR = 3.0$ the 10% thrust increase is accompanied by strong screech and higher noise at all angles. Although the debate on whether full-scale hot jets screech is not necessarily settled, it is not commonly observed in practice as it is in the laboratory[103, 29]. Figure 3.27 expresses the pressure thrust as a
fraction of momentum thrust, $\frac{F_p}{F_m}$, with a negative value indicating pressure drag as a result of jet overexpansion. Presentation of the thrust components in this manner illustrates how much of the momentum thrust is lost due to pressure drag. At design condition it is seen that both nozzles are slightly overexpanded, illustrating one issue with assuming quasi-1D flow with conical nozzles. Agreement between experiments and LES pressure thrust is seen at underexpanded and design conditions. At overexpanded conditions the experiments show an increase in negative pressure thrust. The separation in LES results in significant error at $NPR = 3.0$ and a switch to positive pressure thrust at $NPR = 2.5$. As the flow separates and the shock train shifts upstream, the Mach disk of the jet increases the pressure before the exit plane. The separation becomes significant when predicting total thrust even though the acoustics are well captured.

![Comparison of total thrust at different conditions for biconical and splined nozzles.](image)

(a) Biconical nozzle. $PIV$ & $LES$

(b) Splined nozzle. $PIV$ & $LES$

**Figure 3.25:** Comparison of total thrust at different conditions for biconical and splined nozzles.

### 3.8 Faceted Nozzles

Current tactical fighter jets employ variable area nozzles to adapt the area ratio to match the engine conditions throughout the flight envelope and to accommodate an afterburner which requires an increase in nozzle area to pass the same mass flow when reheating the turbine exhaust flow. To achieve area control for an axisymmetric circular nozzle it is constructed
Figure 3.26: Total thrust from experiments and Thrust Coefficient $C_f$ with negative pressure thrust shown in light gray.

Figure 3.27: Comparison of pressure thrust for experiments and LES.
from overlapping slats which are attached to hinge points and control arms. By actuating the control arms the nozzle can change the angles forming the convergent and divergent sections of the nozzle to increase the throat area and change the area ratio. Examples of these nozzles in application are shown in Figure 3.28. The contours of these nozzles is far from an ideal nozzle contour and it has not clearly been understood how, if at all, this affects noise generation. To investigate this question, far-field acoustic measurements of smoothly axisymmetric nozzles and “faceted” nozzles that simulate the flow surfaces of a variable area nozzle were compared to determine the effects on acoustics. Models and cross-sections of the biconical and splined faceted nozzles are shown in Figures 3.29 and 3.30 showing the flat faceted surfaces internal to the nozzle. The design of these nozzles were provided by GKN Aero and contain twenty-four facets. The complexity of the geometry posed challenges for conventional manufacturing techniques, therefore the nozzles were manufactured with a rapid-prototyping method known as stereo lithography. Stereo lithography is an additive manufacturing technique which builds the product geometry in a resin bath using UV radiation from a laser to cure the polymer resin. The nozzles were fabricated using Somos® NanoTool™, a ceramic polymer composite that could withstand the jet temperatures up to 400 K. The facets are difficult to resolve in the photos due to the color of the material, therefore snapshots are accompanied by the solid models.

Stereoscopic PIV measurements near the nozzle exit are presented in Figures 3.31 and 3.32 for $x/D_j = 0.5$ and 1. The biconical axisymmetric nozzle measurements are on the left and the biconical faceted nozzle measurements are on the right. The jet shear layer assumes the shape of the faceted nozzle which becomes a bit more apparent downstream, however the deformation of the shear layer is not extremely significant. Turbulence intensity levels are similar for both nozzles. Further downstream measurements, which are not shown, the faceted nozzle flowfield assumes a circular cross section. Far-field acoustic measurements for the biconical and splined faceted nozzles are shown in Figures 3.33 and 3.34 compared to their respective axisymmetric designs. The narrowband acoustics are essentially identical with the
exception of some differences in screech intensity. Screech intensity depends on the nozzle lip thickness and it is well known that screech intensity has temporal variation, so this is not surprising. At laboratory scale it is evident that facets do not significantly affect the jet acoustics. Realistic engine nozzles have discontinuities, gaps, cooling holes, and other features that may affect the acoustic spectra. Greska[104] compared laboratory measurements with engine data and found very good agreement with the exception of frequencies below their laboratory cutoff frequency. It will be demonstrated in the following sections that significant modification of the jet flowfield is required to modify the acoustics.
Figure 3.28: Variable area nozzles on 4th generation fighter aircraft.

Figure 3.29: Sharp (biconical) faceted nozzle.

Figure 3.30: Splined faceted nozzle.
Figure 3.31: Cross-stream sector of mean axial velocity for biconical nozzle.

Figure 3.32: Cross-stream sector of turbulence intensity for biconical nozzle.
Figure 3.33: Narrowband for biconical faceted nozzle compared with biconical axisymmetric nozzle.

Figure 3.34: Narrowband for splined faceted nozzle compared with splined axisymmetric nozzle.
3.9 Summary

The effect of contouring the converging section and throat of a biconical converging-diverging nozzle was studied to determine the effect on acoustics and thrust at various nozzle pressure ratios. The strength of BBSN was shown to depend primarily on the overall shock strength in the jet. The increased periodicity of the shock train resulted in screech over a broader range of operating conditions and higher amplitudes than the biconical nozzle. The acoustics computed from LES matched the experiments well, with exceptions at the high and low frequencies on the fringe of the computation limits. The primary differences in the acoustic spectra for the two nozzle designs were captured, unless very high amplitude screech was present. Non-intrusive screech suppression demonstrated how turbulent mixing noise and shock noise components can be significantly affected by screech.

The mean velocity and turbulence was measured and compared very well with LES. Mean velocity profiles and complex shock structures agreed well with measurements except when premature separation in LES shifted the shock structure upstream for the highly overexpanded conditions. Discrepancies in turbulence intensity were seen near the nozzle exit due to PIV spatially filtering the thin shear layer and LES initial conditions. Turbulence intensity was in very good agreement with LES downstream \( x/Dj = 3 \) as the turbulent length scales increased. Momentum and pressure thrust were determined experimentally and the method was validated by excellent comparison with LES at the design and overexpanded conditions. LES inaccurately predicted thrust for overexpanded conditions due to separation in the nozzle. The splined nozzle had higher thrust at all conditions due to reduced momentum loss from shocks at a given \( NPR \). At thrust matched conditions it was shown that the splined nozzle provided equivalent acoustics with 4% lower operating pressure. At \( NPR = 2.5 \) the splined nozzle had identical acoustics while operating with 10% higher thrust. At \( NPR = 3.0 \) the splined nozzle had stronger screech and higher noise at all frequencies with 10% higher thrust. It was proven that through improved nozzle design the jet thrust can be increased significantly while avoiding acoustic penalties.
Chapter 4

Steady Fluidic Injection

Fluidic injection of air into the shear layer of a jet has been investigated as a vorticity production method for noise reduction. The concept of introducing streamwise vorticity into the shear layer at the nozzle exit can extract energy from the jet potential core in the near nozzle region, reducing the peak turbulence levels in the mixing region downstream. This results in a lower SPL at the preferred mixing noise frequency with an increase in high frequency noise near the nozzle. Fluidic injection and other vorticity generation methods of noise reduction require trade-offs to balance high frequency noise increase with reduction in mixing noise. The focus of jet noise reduction is usually on reducing the peak noise values along with decrease in overall noise. Higher frequencies attenuate in the atmosphere more quickly. Therefore, a balance in OASPL reduction can be achieved through reduction in peak noise and minimizing high frequency noise increases. Fluidic injection systems are characterized by many design parameters including injection location, angle, mass flow, number of injectors, pressure, momentum flux ratio, and more. The large design space has made it difficult for researchers to make broad conclusions about injector designs, although fluidic injection has shown promise at achieving greater noise reduction than any other method. Requirements of engine cycle modifications and complex system design for an active control system further complicate implementation of fluidic injection. This investigation focuses on injector designs and parameters that have not been thoroughly studied, but more importantly attempts to
identify trends and draw conclusions about fluidic injection parameters in terms of the effect on supersonic jet acoustics and flowfield. Supersonic jets provide unique challenges including effects on shocks and shock noise, screech instabilities, and the deflection of streamlines at the nozzle exit based on the level of jet expansion.

This chapter focuses on steady fluidic injection of air into the shear layer of a supersonic jet for noise reduction. The study is realized as two bodies of work, the first uses an integrated injection system in which twenty-four slotted injectors are embedded in the trailing edge of the nozzle that inject air directly into the shear layer. The second method consists of injecting air through circular injectors mounted external to the nozzle, injecting through the shear layer. The latter part of the investigations are accompanied by LES simulations from Haukur Hafsteinsson at Chalmers University, which aid in understanding the physical mechanisms responsible for observed changes in the flow and acoustics. Various configurations and operating conditions were studied for both cases and the goal was to aid in the understanding of how fluidic injection parameters govern noise control and to develop insight as to how supersonic noise components respond to fluidic injection, and the physical mechanisms responsible.

4.1 Trailing Edge Fluidic Injection

This study focuses on supersonic jet noise reduction, primarily by fluidic injection, on a $M_d = 1.56$ jet operating at over-expanded, ideally expanded, and under-expanded conditions. Measurements of the velocity and turbulence are presented to support and explain the findings of the acoustic study. Few studies have been conducted on fluidic injection effects on acoustics together with detailed flow measurements on supersonic jets. Previous fluidic injection studies have used various injector shapes, sizes, mass flows, and angles. Most existing research on fluidic injection focuses at nozzle design Mach numbers or converging only nozzles, and few conclusions have been drawn on optimal injection parameters and configurations. In the present study, far-field acoustics and stereo particle image velocimetry (SPIV) were used to investigate the performance of the supersonic jet with fluidic injection. Changes in the
flowfield are compared with the noise reduction at different conditions to provide insight into the link between flow and acoustics for fluidic injection and how the performance varies with jet operating conditions.

Noise mitigation of supersonic jets continues to be a challenging issue since current noise reduction technologies such as passive mixing devices have provided only modest noise reduction with drawbacks such as drag penalties and increased noise at higher frequencies. A sample overview of chevron research is provided here while a detailed review of fluidic injection research was presented in Section 1.7. Many methods to reduce jet noise have been proposed and investigated since jet engines have become widely used in the commercial and military aviation sectors. This includes modifying the jet flow field by placing obstructions at the nozzle exit including tabs, lobed mixers, and chevrons[101]. Chevrons have been especially successful at reducing jet noise in subsonic jets by generating streamwise vortices that promote increased mixing of the jet with the surrounding fluid[105, 106, 80, 22]. The increased mixing promotes quicker decay of the jet velocity and reductions in the peak turbulence levels in the jet[80, 107, 43, 108]. It has been shown that by reducing the peak turbulence levels in a jet, significant reduction in noise is achieved in the near-field pressure and far-field acoustic regions[40, 41, 46, 42]. Chevron design optimization in the laboratory and flight tests has resulted in low frequency noise reduction while minimizing high frequency noise increase and thrust penalties that increase with chevron penetration[101, 109, 47]. While much of the focus on chevron designs was aimed toward high subsonic jets for commercial transport aircraft, there still remains a need to develop noise reduction methods for supersonic jets, especially those with variable nozzle geometries. Increased jet mixing for tactical jets not only has the potential for noise reduction, but also for infrared signature reduction by reducing the jet plume temperature. Henderson[47] and Munday[110] studied chevrons on supersonic jets at various operating conditions for tactical nozzle designs. Both authors showed that with a given chevron design, greater degrees of underexpansion are more effective at reducing low frequency noise and broadband shock noise. Greater underexpansion is effectively the same as
increasing chevron penetration as the flow expands outward through the chevrons. However, chevrons are less effective at reducing noise for over-expanded jets since the inward deflection of the jet at the nozzle exit plane reduces the effective penetration of the chevrons, generating less vorticity\[21, 47, 111, 23\]. This is a significant finding since over-expanded operation is much more common at low altitude and sea-level conditions where noise levels are of greatest concern\[47, 23\].

### 4.1.1 Experimental Methods

Operating conditions for the jet are provided in Table 2.1. Since operation of practical military engines occurs primarily at fully expanded or over-expanded conditions due to the variable area nozzle, two over-expanded conditions, $NPR = 2.5$ and $3.5$, were investigated along with the design $NPR = 4.0$. Additionally, an under-expanded condition of $NPR = 4.5$ was investigated to compare fluidic injection to chevrons. Cross-sections of the nozzle designs used in this study are shown in Figure 4.1. Chevrons are usually attached at the trailing edge of a nozzle with a slight angle (5° to 20° to the primary jet) to achieve noise reduction without too much thrust loss or high frequency penalty. It was desired to develop an injection system that would more accurately represent a system that could be aerodynamically incorporated into a supersonic nozzle and inject at the trailing edge of the nozzle. The trailing edge injectors were incorporated into the sharp throat baseline nozzle with a plenum in the body of the nozzle that can be pressurized. The injectors have a square cross-section (1.6 mm x 1.6 mm) and exit at the trailing edge of the nozzle on the inside contour at an angle of 11° to the primary jet. The design is meant to simulate chevrons which led to an array of 24 injectors; 12 pairs of injectors with each injector pair angled inwards toward each other with a 10° half angle (yaw angle). Air is provided to the plenum with 12 tubes embedded in the body of the nozzle and discharging into the plenum. Surfaces across from the supply tubes stagnate the air to provide steady flow and suppress any pressure fluctuations in the supply air. A 30% pressure loss is incurred from the supply pressure to the injector plenum.
This investigation includes results for three fluidic injection conditions with either constant injection mass flow ratio or pressure ratio. The injection parameters are summarized in Table 4.1. For constant mass flow injection, $p_{o,i}/p_a$ ranges from 1.8 to 3.0 for the different operating conditions. Two constant pressure ratio tests were examined at $p_{o,i}/p_a = 3.3$ and $5.2$, $p_{o,i} = 325$ kPa and 518 kPa, respectively. The highest injection pressure ratio $p_{o,i}/p_a = 5.2$ has mass flow ratios from 2.5% to 4.6%. The total pressure was measured in the nozzle plenum just upstream of the injector inlets. The injector jet velocity, $u_i$, is the fully-expanded velocity for the applied pressure ratio (supersonic for $p_{o,i}/p_a > 1.89$). The momentum flux ratio, also known as effective velocity ratio, shown in Eq. 4.1, is commonly used for scaling jet trajectory and penetration in jet in crossflow research\[55\] and ranges from 0.97 to 1.68 for all conditions in this study.

![Figure 4.1: Nozzle cross-sections showing chevrons and trailing edge injection nozzle.](image)

\[ J = \sqrt{\frac{\rho_i U_i^2}{\rho_j U_j^2}} \quad (4.1) \]

### 4.1.2 Jet Flowfield Measurements

In axisymmetric jets there is minimal azimuthal variation in the time-averaged velocity and turbulence characteristics. However, chevrons and fluidic injection the streamwise structures become relevant in time-averaged measurements. Cross-stream stereo PIV measurements were
Table 4.1: Trailing-edge fluidic injection conditions.

<table>
<thead>
<tr>
<th>$m_i = \text{constant}$</th>
<th>$p_i = \text{constant} (325 \text{ kPa})$</th>
<th>$p_i = \text{constant} (518 \text{ kPa})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_0/p_a$</td>
<td>$m_i/m_j$</td>
<td>$p_i/p_a$</td>
</tr>
<tr>
<td>2.5</td>
<td>1.40%</td>
<td>1.1 (1.8)</td>
</tr>
<tr>
<td>3.5</td>
<td>1.40%</td>
<td>1.3 (2.4)</td>
</tr>
<tr>
<td>4.0</td>
<td>1.40%</td>
<td>1.5 (2.7)</td>
</tr>
<tr>
<td>4.5</td>
<td>1.40%</td>
<td>1.6 (3.0)</td>
</tr>
</tbody>
</table>

conducted to capture the three dimensional nature of the jet when streamwise vorticity was generated. The jet was operated at two overexpanded conditions, design, and an underexpanded condition. Figures 4.2 to 4.5 show measurements at $x/D_j = 1$ as 1/3 sectors of the jet combined in a single contour figure with the baseline in the upper right, chevrons on the left, and fluidic injection on the bottom. Radial velocity and turbulence profiles are shown in plane and out of plane with the chevrons (Chev, Tip and Chev, Root) and fluidic injectors (FI, Tip and FI, Root). The fluidic injectors were operated with a pressure ratio $p_{o,i}/p_a = 3.3$ which is the intermediate pressure studied but illustrates the crenulation of the shear layer and effectiveness of the injectors at different jet operating conditions. The chevrons create strong inward deflection of the flow by the chevron inner surface and outward expulsion of flow in between the chevrons at the chevron root. The fluidic injectors create smaller expulsions of fluid in between the injectors in comparison to the chevrons. The baseline jet shows a thin shear layer with turbulence intensity of 7%. Chevrons create a significantly thicker shear layer and increase of turbulence intensity to 10% at $x/D_j = 1$. Fluidic injection also causes an increase in the shear layer thickness, but it is less than with the chevrons. Fluidic injection creates a different distribution of turbulence in the shear layer with lower peak turbulence levels at $x/D_j = 1$. Fluidic injection peak turbulence levels are 10%-15% lower than with chevrons at all NPR. The turbulence for this jet at all Mach numbers and with all configurations is only partially anisotropic. The turbulence energy distribution for the individual component peak values is 55%-65% for $u'$ and 16.5%-22.5% for $v'$ and $w'$.

Radial profiles of mean velocity and turbulence intensity are presented in (c) and (d) of Figures 4.2 to 4.5 at $x/D_j = 1$. The solid line represents all vectors in the profile but only
every fourth data point is shown for clarity of the figures. For chevrons and fluidic injection, the profiles are taken along the dashed lines corresponding to the chevron tip (Chev, Tip), chevron root (Chev, Root), in-line with the injectors (FI, Tip), and in between the injectors (FI, Root) to emphasize the level of shear layer crenulation. At $NPR = 2.5$, the chevrons and fluidic injection have minimal effect on the mean velocity profile and only create a 10% increase in turbulence levels. The jet is strongly over-expanded and the jet deflects inwards due to the strong shocks at the nozzle exit. This is evident in the mean velocity profile showing the inner location of the shear layer at $r/D_j = 0.38$. This keeps the chevrons and fluidic injectors from interacting with the jet flow. With increasing nozzle pressure ratios, the inner location of the shear layer is at $r/D_j = 0.4$, 0.41 and 0.42, respectively. At $NPR = 3.5$ the chevrons and fluidic injection begin to affect the shear layer of the jet flow. For $NPR = 3.5$ to 4.5 the jet is spread strongly by chevrons and fluidic injection, resulting in 30% - 45% higher turbulence levels. This effect is stronger with increasing nozzle pressure ratio. As the jet reaches the under-expanded regime the fluid is deflected further inwards and outwards. Fluidic injection causes mostly outward ejection of flow since they are at a shallower angle than the chevrons and the radial component of the injector fluid is comparatively smaller than what is induced by the chevron inner surface. The stronger interaction of the chevrons with the mean flow creates peak turbulence intensity 10%-20% higher for the chevrons over fluidic injection.

The chevrons and fluidic injectors both induce streamwise vorticity at the nozzle exit to enhance mixing and modify the turbulence distribution in the jet. Figure 4.6 schematically represents the vorticity introduced by both methods. Each chevron induces a pair of counter-rotating vortices with strength that is determined by the inward and outward flow deflection of the jet by the chevrons. The fluidic injector pairs generate similar streamwise vortices, but a small pair of secondary vortices are generated between the gap of the two injectors. These vortices merge with the larger vortices by $x/D_j = 2$ creating a flowfield much like the chevrons. Alkislar[42, 112] showed that vortices from fluidic injection can have different initial trajectories than chevron vortices. However, by $x/D_j = 2$ or 3 the vortex cores are on the
same trajectory. This is not a generalized case, since the fluidic injectors were at a much steeper angle relative to the horizontal than the chevrons tips (60° relative to 12.8°) in that study.

Figure 4.2: Mean flowfield contours and profiles of axial velocity and turbulence intensity for NPR = 4.5 at x/D = 1.

Generation of streamwise (axial) vortices using chevrons and fluidic injection is understood to be the primary mechanism for reducing turbulent mixing noise. Alkislar[42, 112] presented
Figure 4.3: Mean flowfield contours and profiles of axial velocity and turbulence intensity for $NPR = 4.0$ at $x/D = 1$. See Figure 4.2 for annotations.
Figure 4.4: Mean flowfield contours and profiles of axial velocity and turbulence intensity for $NPR = 3.5$ at $x/D = 1$. See Figure 4.2 for annotations.
Figure 4.5: Mean flowfield contours and profiles of axial velocity and turbulence intensity for $NPR = 2.5$ at $x/D = 1$. See Figure 4.2 for annotations.
thorough investigations on the role of streamwise vortices from chevrons and fluidic injection for a $M = 0.9$ jet. Cross-stream PIV measurements of the jet allowed Alkislar to show how the vorticity magnitude and vorticity trajectory differ between chevrons and fluidic injectors. Alkislar noted that chevrons increase the turbulent kinetic energy up to 30% in the initial shear layer region while microjets only increase TKE up to 10% which is in agreement with the measurements above. Figure 4.7a compares the maximum normalized streamwise vorticity measured along the jet at $M_d = 1.56$ with the results from Alkislar[42] at $M = 0.9$ for chevrons and fluidic injectors. Although less axial measurement locations were conducted in this study due to the various setpoints investigated, fairly good agreement is seen with Alkislar’s data. The decay of the vortices matches very well with the chevrons, but the decay rate of the fluidic injectors is quicker in the present study which is attributed to the higher injection angle ($60^\circ$) in Alkislar’s study. The initial vorticity strength is in good agreement for both chevrons and fluidic injectors, but the persistence of the vortices in the jet is clearly related to the injection angle and effective penetration of the injectors. It should be noted however that the vorticity magnitude and trajectory behavior is not a settled matter as noted by comparing data from a second paper by Alkislar[112] in Figure 4.7b. This paper studied the effect of enhancing chevrons with fluidic injectors, but also examined the effect for chevrons and fluidic injectors separately. The primary difference in the second paper is that the number of chevrons and fluidic injectors was doubled. Increasing the number of chevrons or injectors
decreases the spacing between the streamwise vortices which is thought to increase the decay rate of the vortices through quicker vortex interaction[113, 114], although this effect has yet to be quantified. The peak magnitude of normalized streamwise vorticity is shown in Figure 4.8 for all jet operating conditions. Supporting the TKE measurements, it is seen that the initial vortex strength is weak at $NPR = 2.5$ as the chevrons and fluidic injectors have little interaction with the primary jet shear layer. Increasing $NPR$ results in stronger initial vortex strength for chevrons compared to that for fluidic injectors. The initial normalized peak vortex strength saturates around $\omega_x D_j/U_j = 4$ for fluidic injectors and $\omega_x D_j/U_j = 5$ for chevrons at the design and underexpanded conditions based on the chevron penetration and highest momentum flux ratio for the injectors. For all $NPR$ the effect of the vortices has diminished to the baseline values by $x/D_j = 4$.

Another measure of the effect of mixing devices on jets is shown by the spreading rate of the shear layer. The shear layer thickness is defined in Arakeri[41] and Morris[114] as the difference of the radial locations where $u/U_j = 0.95$ and 0.10, respectively. This causes difficulties in the present investigation since the supersonic flow has shocks internal to the nozzle and a secondary flow of $M_s = 0.1$. In this case, the shear layer thickness is defined as the difference in radius where the velocity reaches 0.95 of the jet velocity (shear layer inner edge) and where the velocity reaches 1.05 of the freestream ($U_{fs}$) (shear layer outer edge), $\delta = r_{u=0.95U_j} - r_{u=1.05U_{fs}}$. Figure 4.9 shows the normalized shear layer thickness, $\delta/D_j$, at $x/D_j = 1$ for increasing $NPR$. For the chevron and fluidic injection, the shear layer thickness is measured from the minimum (Tip) to the maximum (Root) shear layer edge. For the baseline nozzle, the shear layer is thinner with increasing $NPR$ at the fixed $x/D_j = 1$ since the jet velocity is higher. Chevrons increase the shear layer thickness approximately twice that of the fluidic injectors since the solid surface of the chevrons causes much more inward flow deflection than fluidic injection.
Figure 4.7: Comparison of maximum streamwise vorticity at various axial locations showing data spread for chevrons and fluidic injectors of similar designs operating with similar jet and injector conditions.

4.1.3 Baseline Acoustics

Figure 4.10 shows far-field acoustic narrowband spectra for the jet operating at the four NPR investigated. At $NPR = 2.5$ the flow is highly over-expanded with a low level of screech and broadband shock noise present in the spectrum. At $NPR = 3.5$ the jet is still over-expanded and exhibits a strong screeching mode with harmonics at most observation angles even in the downstream acoustic spectra. Large-scale motions of the jet and shocks can subsequently alter the broadband shock noise and turbulent mixing noise. When evaluating noise reduction methods on screeching jets, the change in the noise spectrum should not be compared directly with jets that exhibit different levels of screech. Many times when screech is reduced or eliminated in a strongly screeching jet, the BBSN and turbulent mixing noise can be increased significantly. Norum[99] proposed that strongly screeching jets disrupt the phase relationship of the shock structure that is responsible for BBSN. This illustrates how significantly the acoustic spectra can vary for a non-ideal converging-diverging nozzle at various operating conditions. The sensitivity of the various noise components on the shock structure and screech behavior creates difficulties when comparing nozzle designs. Comparison of the measured acoustic spectra for $NPR = 2.5$ and 4.0 with similarity spectra from Tam[33]
Figure 4.8: Maximum normalized streamwise vorticity magnitude.

Figure 4.9: Shear layer thickness and increase in jet spread relative to baseline shear layer thickness.
is shown in Figure 4.11. The fine-scale similarity (FSS) spectrum agrees very well with 90° observer angle and the large-scale similarity (LSS) spectrum agrees very well with the 140° observer angle. Discrepancies in the high frequencies is due to amplification of the fine scale turbulent mixing noise due to the shock noise components as noted by Tam[33]. Also, a comparison of the baseline acoustic results is provided in Cuppoletti et. al.[96] through high fidelity comparisons with Large-Eddy Simulation (LES) and Computational Aeroacoustics (CAA) results.

4.1.4 Effect of Injection Pressure

The narrowband acoustic spectra and ΔOASPL for the fluidic injection are shown in Figure 4.12 to Figure 4.15 for NPR = 4.5 to 2.5. The narrowband spectra are shown at ψ = 35°, 90°, and 140° and ΔOASPL is shown for all observer angles. At underexpanded conditions the exit pressure is higher than the ambient pressure, which not only causes outward deflection of the jet through expansion waves, but also acts to reduce the effective pressure ratio of the injectors. For this reason, it is seen in Figure 4.12 that fluidic injection has minimal effect on the mixing noise components. Despite this, screech is effectively suppressed with the injectors at all injection pressures. Up to 5 dB reduction of BBSN is measured at 90° and again a higher frequency peak at StDj = 0.2 begins to appear. Up to 2 dB noise reduction is achieved at the sideline and upstream angles and up to 1.2 dB reduction at the downstream angles.

The most significant noise reduction is achieved for the NPR = 4.0 jet at the highest injection pressure tested. Figure 4.13 shows that increasing injection pressure results in further suppression of BBSN and screech. At pi/pa = 2.7 and 3.3 the peak BBSN levels are reduced without much alteration at the other frequencies. At pi/pa = 5.2 the peak BBSN levels at StDj = 0.6 are reduced up to 15 dB in the 90° narrowband spectrum although a higher frequency peak is increasing in magnitude at StDj = 0.12. This is most likely a factor of reduction in the shock cell spacing caused from interaction of the injectors with the shocks near the nozzle exit. Further data is needed to fully quantify the physical mechanisms causing this peak BBSN shift.
to higher frequencies. Minimal differences are observed in the turbulent mixing noise at 140°, with the exception of slight suppression at $St_{Dj} > 0.2$. Overall noise reduction of 2.6 dB is achieved at 35° and 1 dB at 140°. This indicates that increasing injection pressure results in further alteration of the jet shock train with minimal effect on the large-scale turbulent structures at this shallow injection angle.

At $NPR = 3.5$ the jet is over-expanded and exhibits very strong screech that is clearly apparent in the upstream and sideline angles. As the screech tone is reduced, an increase in BBSN is observed as the excited jet mode is suppressed. This observed noise is attributed to suppression of the excited jet mode. When the injectors do not interact with the main jet, the injector jets become a high frequency noise source external to the jet shear layer. At the highest injection pressure the jets interact with the main jet and high frequency noise reduction is observed at all observation angles. The combination of all of these competing changes in the acoustic spectra results in an OASPL reduction of 3.2 dB at 35°, and 3 dB at 140°. The sharp changes in $\Delta OASPL$ at various microphone angles arise from the presence of screech and the effect it has on BBSN and turbulent mixing noise. Screech is highly directional, radiating in the upstream and sideline angles as shown in Figure 4.14. The screech tone contributes sharp variation in OASPL at observer angles with the strongest screech. Suppression of screech results in drastic changes in $\Delta OASPL$. It is important to investigate the narrowband spectra or a metric that accounts for tones to identify the balance of reduction in screech amplitude and increase in other noise components. These results illustrate the dependence of the various noise components on the presence and intensity of screech.

At $NPR=2.5$, shown in Figure 4.15, the fluidic injection provides minor changes in the acoustic spectra. The two lowest injection pressures cause an increase in high frequency noise and slight reductions in screech, resulting in an overall increase in OASPL. The increase in noise at frequencies greater than $St_{Dj} = 2$ is attributed to fine scale turbulence from the injectors minimal impingement on the main jet as shown in the flowfield measurements. This high frequency actuator noise has been noted by other researchers\cite{43, 51, 54} and can
present a challenge to achieving overall noise reduction. As seen in the turbulence profiles, the fluidic injection begins impacting the main jet at the highest injection pressure, further reducing screech and negating the high frequency increase, with up to 1 dB reduction in OASPL. The lack of noise reduction at $NPR = 2.5$ with low injection pressure is due to the flow streamlines being directed inward from shocks emanating at the nozzle exit, evident in the velocity profiles in Figure 4.5, also observed by Bridges\cite{Bridges}. This reduces the effective penetration of the fluidic injectors into the jet shear layer, which are at a shallow angle in comparison to other fluid injection studies that inject from outside the shear layer\cite{101, 41, 42, 113}. These results indicate that interaction of the injectors is critical to achieving noise reduction. At overexpanded conditions the jet deflects away from the injectors while at underexpanded conditions the effectiveness of injection is reduced due to reduced pressure ratio.

4.1.5 Comparison with Chevrons

Figure 4.16 compares chevrons with fluidic injection at the highest injection pressure. Comparison of $\Delta$OASPL is shown since the changes in the noise components are similar to fluidic injection, only differing in magnitude of the effect. Fluidic injection and chevrons are effectively equivalent at the highly over-expanded $NPR = 2.5$ since only slight interaction with the jet occurs as shown in Figure 4.8d and Figure 4.9b. At $NPR = 3.5$, the chevrons reduce the screech tone and the harmonics while simultaneously causing an increase in BBSN, very similar to the effect of the fluidic injection. Fluidic injection is more effective at reducing screech and the other noise components. At the design condition $NPR = 4.0$, fluidic injection is more effective at reducing screech and BBSN while chevrons are slightly more effective at reducing the turbulent mixing noise in the aft direction. At the under-expanded condition $NPR = 4.5$, the chevrons are much more effective than fluidic injection. The additional pressure at the nozzle exit forces the flow out through the chevrons creating stronger streamwise vorticity and mixing. This increases the effective penetration of the chevron and generates stronger
streamwise vorticity. Bridges\cite{83} has also investigated this effect on similar tactical nozzle geometry. Conversely, the opposite effect occurs with fluidic injectors. The positive pressure at the nozzle exit reduces the effective injection pressure ratio, and the momentum flux ratio is effectively lower. Provided a fluidic injection system had a high enough momentum flux ratio, it could provide equivalent noise reduction as chevrons at the under-expanded condition.

4.1.6 Effect of Injector Notches

Embedding the fluidic injectors into the nozzle lip causes interaction with the flow that is not present with fluidic injector designs that are external to the nozzle. The slots are effectively small serrations in the end of the nozzle and they interact with the flow similar to small tabs or chevrons. The acoustic effect of the slots has been quantified to ensure that the fluidic injection is creating additional noise reduction. Figure 4.17 shows the $\Delta$OASPL for the injectors without flow and the injectors at the highest injection pressure. At all NPR, fluidic injection provides more noise reduction than the serrations alone. The increased fluidic injection provides up to 1.5 dB reduction over the slots themselves, especially at the aft angles. In Figure 4.17c, injection provides minimal differences in $\Delta$OASPL, as the serrations are quite effective at reducing the screech tone that dominates the upstream spectra. Fluidic injection increases the vorticity strength at the nozzle and creates more turbulent mixing noise reduction and further shock noise reduction.
Figure 4.10: Far-field acoustics for sharp throat nozzle at various operating conditions.

Figure 4.11: Comparison of $90^\circ$ spectra with FSS and $140^\circ$ spectra with LSS for $NPR = 2.5$ and 4.0.
Figure 4.12: Far-field acoustics showing the effect of injection pressure for $NPR = 4.5$. 
Figure 4.13: Far-field acoustics showing the effect of injection pressure for $NPR = 4.0$. 

(a) $\psi = 35^\circ$  
(b) $\psi = 90^\circ$  
(c) $\psi = 140^\circ$  
(d) OASPL
Figure 4.14: Far-field acoustics showing the effect of injection pressure for $NPR = 3.5$. 
Figure 4.15: Far-field acoustics showing the effect of injection pressure for $NPR = 2.5$. 
Figure 4.16: Comparison of highest injection pressure $p_{o,i}/p_a = 5.2$ (red $\triangle$) to chevrons (blue $\blacklozenge$) at various NPR.
Figure 4.17: Comparison of highest injection pressure $p_{oi}/p_a = 5.2$ (red $\triangle$) to no injection (blue $\Box$) at various NPR.
4.1.7 Summary: Trailing Edge Injection

An investigation of the performance of fluidic injection compared to that with chevrons on a converging-diverging supersonic jet at design and off-design conditions was conducted. It was shown that fluidic injection can provide improved noise reduction over chevrons if the injection momentum flux ratio is sufficient to interact with the over-expanded jets. The ability of fluidic injection to adjust to the jet operating condition is very beneficial from a noise reduction point of view as supersonic jets are rarely fully expanded at take-off. The improved noise reduction of fluidic injection is achieved with lower peak turbulence levels near the nozzle than with chevrons. At the design condition BBSN was reduced by 10-15 dB, which is significantly more than obtained by chevrons. OASPL reductions of up to 3.5 dB in the upstream angles and 2 dB in the sideline angles were observed at \( NPR = 3.5 \) due to elimination of screech and suppression of BBSN. The effectiveness of the respective noise reduction devices was strongly dependent on the penetration and vorticity strength achieved in the shear layer as illustrated with the flow field measurements. It was shown that noise reduction is improved at all conditions through increasing the momentum flux ratio of the injectors. Since converging-diverging nozzles often operate at various expansion levels, fluidic injection provides a more flexible system to achieve noise reduction over a wide operating range. Chevrons deflect more flow inwards toward the jet centerline than the fluidic injectors, resulting in higher turbulence levels near the nozzle, yet this does not directly correlate with more noise reduction. Most of the noise reduction observed was due to the fluidic injection reducing BBSN and disrupting the screech tone feedback loop.
4.2 External Fluidic Injection

The current investigation further investigated injection angle, \( \theta_{inj} \), and momentum flux ratio, \( J \), to develop relationships for the response of supersonic jet noise components to fluidic injection parameters. Existing studies are inconsistent for supersonic jets and this investigation aimed to identify the behavior of the flow and acoustics to injector angle and momentum flux ratio. The momentum flux ratio was varied between \( J = 0.7 \) and 1.35 for a \( M_j = 1.56 \) jet. Injector angles of 30°, 45°, 60°, and 90° were studied. Acoustics for six injectors and twelve injectors equally spaced around the primary jet are presented along with a detailed study of the flowfield for six injectors. The effect of fluidic injection on the shock noise and mixing noise components is examined and the underlying physics are analyzed and discussed.

4.2.1 Experimental Methods

Fluidic injectors were mounted in a casing at the nozzle lip at four angles (30°, 45°, 60°, and 90°). Two configurations, six (6) injectors and twelve (12) injectors, equally spaced around the circumference of the nozzle are presented. Figure 4.18 shows the injector setups at \( \theta_{inj} = 60° \). Cross sections and a close-up of the injector in the injector mount is shown in Figure 4.19. The injector mounts were counter-bored with a ledge for mating to the injector tube. The inner diameter of the injector and the injector mount were matched so that there was no change in flow area. The outer injector casing was replaced for various injection angles. This analysis is focused primarily on the fluidic injection parameters with the primary jet operating at the design condition. Table 4.2 shows the jet conditions for this study. The injectors are circular with \( A_{inj} = 5.6 \text{ mm}^2 \) and \( d_{inj}/D_j = 0.046 \) corresponding to the flow area for each injector pair from the trailing-edge injection nozzle. The total injected air mass flow rate was measured and the conditions are shown in Table 4.3. The isentropic pressure ratio is back-calculated for choked conditions and used to calculate Mach number, fully expanded jet velocity, and momentum flux ratio \( J = \sqrt{\rho_i U_i^2 / \rho_j U_j^2} \). The goal was to match injection momentum flux ratios, but due to mass flow limitations the highest injection conditions for 6 injectors were not matched in momentum flux ratio for 12 injectors.
Figure 4.18: Schematics for 6 and 12 injectors at $\theta_{inj} = 60^\circ$.

Figure 4.19: Cross sections of injector configuration at $\theta_{inj} = 60^\circ$.

Table 4.2: Operating conditions for primary jet.

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<th>$m_j$ [kg/s]</th>
<th>$U_j$ [m/s]</th>
<th>$T_{ai}/T_a$</th>
<th>$c_j$ [m/s]</th>
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Table 4.3: Fluidic injection conditions for four momentum flux ratios for 6 injectors and 12 injectors.

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<tr>
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<td>2.34</td>
<td>1.00</td>
<td>287</td>
<td>337</td>
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<td>4.93</td>
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<td>488</td>
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<th>$p_{o,i}/p_a$</th>
<th>$T_{o,i}/T_a$</th>
<th>$c_i$ [m/s]</th>
<th>$U_i$ [m/s]</th>
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4.2.2 Acoustics Results

A parametric study investigating injection angle ($\theta_{inj}$) and injection momentum flux ratio ($J$) was conducted to determine how the supersonic noise components are affected. Figures 4.20 and 4.21 show the narrowband and OASPL for varying injector angle at the highest momentum flux ratio ($J = 1.35$) and varying injector momentum flux ratio at $\theta_{inj} = 60^\circ$ for six injectors. The baseline acoustics are shown without injection hardware on the nozzle since screech can be amplified by surfaces at the nozzle exit. The narrowband spectra are staggered by 30 dB to present the spectra on a single abscissa. Increasing $\theta_{inj}$ for $J = 1.35$ results in reduction of screech and large-scale mixing noise. Shock noise is reduced and shifted to higher frequency for lower $\theta_{inj}$, but at $\theta_{inj} = 90^\circ$ shock noise is increased. Fine-scale mixing noise is increased with increasing injection angle. Similar results are observed at $60^\circ$ injection angle for increasing $J$. Large-scale mixing noise reduction is achieved, although an increase in peak shock noise is not observed as is seen for $90^\circ$ injection angle at $J = 1.35$. It becomes clear that increasing the injection angle is similar to increasing the injection pressure. At $90^\circ$ injection angle the large-scale mixing noise is continually reduced with increasing $J$ indicated by the decrease in OASPL at $\psi = 150^\circ$. At sideline and upstream injection angles, OASPL decreases for $J = 0.7$ and 0.95 and then begins to increase for $J = 1.13$ and 1.35. From the narrowband spectra it is seen that the peak shock noise is shifted to higher frequency which is usually attributed to reduced
shock cell spacing with injection. However, when $J$ increases above a threshold it is observed that shock noise is increased through strengthening of the jet shock train or from the injectors inducing strong additional shocks in the jet. Figure 4.22 shows the $\Delta$OASPL for all four $\theta_{inj}$ and $J$ studied for six injectors. The same trends can be seen in these figures with 2-4 dB reduction in the sideline and upstream angles until noise is increased for the highest $J$ and $\theta_{inj}$. At $\psi = 150^\circ$ OASPL is reduced 4 dB for 30$^\circ$ injection and up to 8.5 dB for 90$^\circ$ injection. Figure 4.23 shows the $\Delta$OASPL for twelve injectors at all $J$. Similar trends are noticed as with the 6 injectors with the exception of less dependence on momentum flux ratio for 30$^\circ$ to 60$^\circ$ injection. At 90$^\circ$ injection, upstream and sideline noise is increased more than the six injector configuration at lower $J$.

The behavior of the shock noise components is fundamentally different from the large scale turbulent mixing noise components. Figure 4.24 illustrates the behavior of these components with $J$ for all four $\theta_{inj}$. The figures show SPL for an individual narrowband frequency representative of the peak value for each noise component. Large-scale mixing noise is shown for $\psi = 150^\circ$ at 1 kHz, shock noise at $\psi = 90^\circ$ at 6 kHz, screech at $\psi = 35^\circ$ at 2.2 kHz, and fine-scale mixing noise at $\psi = 90^\circ$ at 40 kHz. Six injectors shows increasing $\theta_{inj}$ and $J$, large-scale mixing noise is continually reduced from 112 dB to 104 dB without leveling out. This indicates that mixing noise is further reduced with increasing injection penetration generating stronger streamwise vorticity. Twelve injectors show a leveling off of noise reduction and less noise reduction is achieved. This is due to the streamwise vorticity breakdown occurring sooner from vortex interactions. The behavior of shock noise is the most interesting finding shown in Figures 4.24c and 4.24d. Shock noise is reduced with increasing $J$ until an optimum reduction occurs and then increases for higher $J$. The momentum flux at which the inflection point occurs depends on the injection angle and number of injectors. For higher $\theta_{inj}$ the inflection point occurs at lower $J$. The complex interactions of the jet shock train with the injectors results in varying behavior of the shock noise. This explains the variation in observations from previous studies. The shock noise reduction can be optimized based on the injector location, configuration, and injection conditions. Screech is reduced with a low momentum flux ratio and levels off with higher injection angles providing more screech suppression for six injectors. Twelve injectors illustrates the same behavior with the exception of 90$^\circ$ which increases the screech amplitude. For all configurations the fine-scale mixing noise is further increased with higher $J$ and higher $\theta_{inj}$. The
OASPL metric is an integration of all of these varying noise components and can often mask the behavior of the various noise components. An optimal fluidic injection configuration should balance the effect of all of these noise components for the desired noise reduction. For example, Gee et. al.[103] showed with acoustic pressure mapping on a full-scale jet that large-scale mixing noise dominates the far-field while shock noise is significant in the near-field of the jet. An active fluidic injection system could be tailored for select noise reduction in specific scenarios or varied during a takeoff condition for directed noise reduction.

LES and CAA were conducted to verify the experimental findings and provide additional insight into the mechanisms controlling the response of the acoustics. The flowfield and acoustics were computed for the four injection momentum flux ratios for $\theta_{inj} = 60^\circ$ configuration with six injectors. The $\theta_{inj} = 60^\circ$ condition was chosen since it exhibited significant noise reduction, the switching behavior of shock noise, and since many fluidic injection studies have investigated this injection angle. Extensive comparison of CAA with measurements for baseline jet flows are discussed in Cuppoletti et. al.[96] and Hafsteinsson et. al[70]. Figure 4.25a shows comparison of the measured narrowband spectra (Exp) and computed spectra (LES). CAA compares very well with the measured acoustics, accurately capturing the screech frequency and peak shock noise frequency and the large-scale mixing noise. CAA underpredicts the highest frequencies ($f > 50kHz$) due to mesh resolution and subgrid-scale modeling. For the observation angles where shock noise is doppler shifted to higher frequencies ($\psi = 110^\circ$ to $140^\circ$) CAA overpredicts the sound pressure levels which is exacerbated for $J = 1.35$ in which the shock strength is strongly increased. This effect, compounded by the overprediction of the high frequencies with injection, result in higher OASPL values than measured in the experiments. Figures 4.25b and 4.25c show that identical trends are captured in the OASPL and $\Delta$OASPL as measured in Figure 4.22c with an offset of about $+2$ dB due to the high frequencies in the CAA with injection. At the upstream and sideline angles the noise is reduced for $J = 0.7$ and 0.95 then increases for $J = 1.13$ and 1.35. The mixing noise is continually reduced at $\psi = 150^\circ$ for increasing $J$. This shows that LES is accurately capturing the dominant flow dynamics and the effects that fluidic injection has on the flow structures.
Figure 4.20: Narrowband acoustics for $M_j = 1.56$ with 6 injectors.

(a) Effect of injection angle for $J = 1.35$

(b) Effect of $J$ for $60^\circ$ injection angle

Figure 4.21: Effect of injection angles and momentum flux ratio on OASPL with 6 injectors for $M_j = 1.56$. 
Figure 4.22: Effect of momentum flux ratio on $\Delta$OASPL at all angles with 6 injectors for $M_j = 1.56$. 
Figure 4.23: Effect of momentum flux ratio on $\Delta$OASPL at all angles with 12 injectors for $M_j = 1.56$. 

(a) 30° injection angle
(b) 45° injection angle
(c) 60° injection angle
(d) 90° injection angle
Figure 4.24: Effect of momentum flux ratio on supersonic jet noise components for $NPR = 4.0$ jet. SPL is shown from narrowband spectra at single frequency.
4.2.3 Shadowgraph

Shadowgraph provides a useful tool to understand how the changes in acoustics are related to the changes in the jet flowfield, particularly in the shock structure. For an axisymmetric configuration the jet provides a pseudo-two-dimensional view of the jet, but it is difficult to use for quantitative purposes since it is a line-of-sight technique. High-speed shadowgraph videos were ensemble averaged to visualize how the injection penetrates the jet and alters the shocks in the jet. Figure 4.26 shows the shock structure for the baseline jet operating at the design Mach number $M_d = 1.56$. The double shock structure results from the conical converging-diverging nozzle design with shocks emanating at the nozzle throat in addition to the shocks at the nozzle exit. This type of nozzle is discussed extensively in other publications [96, 75, 115, 23]. The instantaneous image shows the thin initial shear layer fine scale turbulence in the initial jet region.
Figure 4.26 shows the mean shock structure with 6 injectors ($J = 1.35$) and instantaneous images at the same condition. The shock structure is further affected by increased $\theta_{inj}$. The shock structure is pulled towards the nozzle due to the effect of the injectors on the initial shock angle and the inward spreading of the jet shear layer. The downstream shocks become weakened and the relative spacing between the internal shock train and the lip shock train is altered. The instantaneous images show how the jet mixing is increased with injection angle as the turbulence increases and the initial jet shear layer is thickened. A schematic of these effects on the shock train for an ideal nozzle is shown in Figure 4.27. There are two causes for the decrease in shock cell spacing: 1.) The initial shock angle increases due to the injectors and 2.) the inward spreading of the shear layer results in reflection of the shock and expansion waves at a lower radius. These two causes result in an upstream shift of the entire lip shock train and a decrease in the shock cell spacing $L_{s,2} < L_{s,1}$ which accounts for the shift in the peak shock noise frequency. The inward spreading of the shear layer also results in a decrease in the potential core length. This hypotheses is supported with the PIV measurements and LES results presented in the following section.

4.2.4 PIV Measurements

Measurements of the jet flowfield up to $15D_j$ are shown in Figures 4.28 to 4.30 for six injectors at $\theta_{inj} = 60^\circ$. The measurement domain was $5D_j$ with two cameras and three measurement locations were stitched together to provide the full domain. The fluidic injection results in increased turbulence in the near-nozzle region ($0 < x/D_j < 3$) with drastic reduction in the peak turbulence levels downstream. Increasing the momentum flux ratio results in further shrinking of the jet potential core and further reduction in peak turbulence levels downstream. Fluidic injection at $J = 1.35$ reduces the peak $\sqrt{\overline{TKE}}/U_j$ from 18% to 13% at $x/D_j > 10$. This accounts for the reduction in the large-scale mixing noise. The increased near nozzle turbulence is mostly fine-scale turbulence which accounts for the increase in fine-scale mixing noise with increased momentum flux ratio.

To present a clear view of the modification of the initial jet region, Figures 4.31 through 4.37 show the first $5D_j$ with reduced contour levels for the mean velocity to emphasize the shock structure in the jet. All measurements shown were taken in-plane with the injectors. It was seen in the shadowgraph measurements that the interaction of the injectors with the jet shock train is very complex in the
Figure 4.26: Shadowgraph for $M_j = 1.56$ showing mean and instantaneous flowfield.
near-nozzle region. The jet shock structure tends to realign to a single shock system downstream ($x/D_j > 3$). The shock strength can be inferred from the contours based on the variation in mean velocity along the jet indicated by the concentrated high-velocity and low-velocity regions and the undulation of the jet along the shear layer. The highest momentum flux ratio for the four injection angles are shown in Figures 4.32 to 4.35. The jet shock strength appears to increase in strength with increasing injection angle and becomes very strong for $\theta_{inj} = 90^\circ$ and resembling the shock structure of an underexpanded jet. The peak magnitude and spatial extent of the turbulence levels in the initial shear layer increase with $\theta_{inj}$. The baseline jet has $\sqrt{\overline{\text{KE}}}/U_j < 15\%$ along the first $5D_j$, while for $\theta_{inj} = 90^\circ$ $\sqrt{\overline{\text{KE}}}/U_j > 20\%$ up to $x/D_j = 4$. The shear layer spreads further inward and outward with increased $\theta_{inj}$ and the jet potential core is nearly collapsed by $x/D_j = 5$ for $\theta_{inj} = 90^\circ$. Figures 4.36 and 4.37 show the optimum shock noise reduction cases at $J = 0.95$ for $\theta_{inj} = 60^\circ$ and $90^\circ$. The velocity field is more homogeneous indicating reduced shock strength and appears to be due to the shock interactions directly downstream of the nozzle exit.

Axial profiles of mean velocity provide a quantitative view of the shock strength and decay of velocity along the jet. Figures 4.38 and 4.39 show the axial velocity profiles at the jet centerline ($r/D_j = 0$) and at half the jet radius ($r/D_j = 0.25$). The line represents every velocity vector while only every twentieth vector point is plotted for clarity. Shock strength is usually determined by the pressure gradient across the shock but can also be discussed by the velocity gradient. The discussion that follows qualitatively discusses shock strength from the maximum variation of velocity through the shocks in the jet. The effect of $\theta_{inj}$ at $J = 1.35$ clearly shows the effect on shock strength indicated by strong variations in velocity. It is seen that that at $\theta_{inj} = 90^\circ$ the velocity gradient due to shock strength is larger than the other $\theta_{inj}$ and the baseline condition. There is however, much
quicker decay of velocity in the jet indicating a shorter potential core and strong jet mixing. For $\theta_{inj} = 60^\circ$, increasing momentum flux results in quicker decay of the mean velocity and the shock strength is minimal for $J = 0.95$ which agrees with the observations in the shock noise. The optimum shock noise and mixing noise reducing momentum flux ratios are shown in Figure 4.40 for $\theta_{inj} = 60^\circ$ and $90^\circ$. These clearly show that for $J = 0.95$ the shock strength is strongly reduced with 5% – 6% variation in velocity at $x/D_j = 5$ in comparison to 10% – 15% for $J = 1.35$. It is evident that the effects of fluidic injection on supersonic jet flows differ from subsonic flows due to the complexities involved in the shock wave interactions. The design and implementation of the injectors will likely have an effect on the interaction of the injectors with the jet shocks. While increasing $\theta_{inj}$ and $J$ further increases streamwise vorticity strength and reduction in turbulent mixing noise, other factors play a role in the acoustic results. All of the flowfield analysis presented was for six injectors while data for twelve injectors is still being processed. Preliminary results show that the vortex interaction with more injectors occurs sooner in the jet which inhibits the growth of streamwise vortices. Figure 4.41 shows the streamwise vorticity for three axial planes along the jet. Although twelve injectors results in larger initial vorticity at $x/D_j = 0.5$ and 1.0, the proximity of the injectors results in less penetration of the vortices into the jet core and quicker breakdown of the vortices by $x/D_j = 2.0$. The shape and size of the induced streamwise vorticity is critical in enhancing the jet mixing and reducing the large-scale mixing noise.
Figure 4.29: Mean velocity and turbulence for 6 injectors, $\theta_{\text{inj}} = 60^\circ$, and $J = 0.7$.

Figure 4.30: Mean velocity and turbulence for 6 injectors, $\theta_{\text{inj}} = 60^\circ$, and $J = 1.35$.

Figure 4.31: Initial region mean velocity and turbulence for the baseline $M_j = 1.56$ jet.
Figure 4.32: Initial region mean velocity and turbulence for 6 injectors, $\theta_{inj} = 30^\circ$, and $J = 1.35$.

Figure 4.33: Initial region mean velocity and turbulence for 6 injectors, $\theta_{inj} = 45^\circ$, and $J = 1.35$.

Figure 4.34: Initial region mean velocity and turbulence for 6 injectors, $\theta_{inj} = 60^\circ$, and $J = 1.35$. 
Figure 4.35: Initial region mean velocity and turbulence for 6 injectors, $\theta_{\text{inj}} = 90^\circ$, and $J = 1.35$.

Figure 4.36: Initial region mean velocity and turbulence for 6 injectors, $\theta_{\text{inj}} = 60^\circ$, and $J = 0.95$.

Figure 4.37: Initial region mean velocity and turbulence for 6 injectors, $\theta_{\text{inj}} = 90^\circ$, and $J = 0.95$. 

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Figure 4.38: Axial velocity profiles for 6 injectors for all injection angles at the highest injection condition $J = 1.35$.

Figure 4.39: Axial velocity profiles for 6 injectors at all injection conditions for $\theta_{inj} = 60^\circ$. 
Figure 4.40: Axial velocity profiles for 6 injectors at the optimal shock noise reduction ($J = 0.95$) and mixing noise reduction ($J = 1.35$) conditions.

### 4.2.5 LES & PIV Comparison

Comparison of the measured flowfield with the computed flowfield is essential in understanding the discrepancies in the acoustics. The experiments and computations complement the understanding of the jet flow with fluidic injection since LES can provide the full three-dimensional flowfield and the experiments can be used as validation. Figure 4.42 show the mean streamwise jet velocity for $\theta_{inj} = 60^\circ$ with PIV measurements presented on the top half with a black border. The PIV measurements are taken slightly downstream of the nozzle exit due to reflection issues. There is excellent comparison between PIV and LES for the baseline jet and all injection cases. The complex near-nozzle shock structure is captured in both PIV and LES and shock locations are very accurate up to $x/D_j = 3$. The shock location begins to deviate in the LES after $x/D_j = 3.5$ due to reduced mesh resolution at this location. In all cases LES predicts shock locations persisting further in the jet than PIV most likely due to slightly low dissipation in the numerical scheme. Axial profiles of mean velocity for the baseline jet and $J = 0.7$ and 1.35 are shown in Figure 4.43 at the jet centerline, half-radius, and full-radius. The complex shock structure is well captured with PIV and LES at all radial locations. LES underpredicts the velocity decay at the centerline but is more accurate at the outer radius. The persistence of the jet shock train is overpredicted with LES. The jet dynamics are strong near the collapse of the potential core where the jet column instability is largest and the
Figure 4.41: Streamwise vorticity contours from LES for $\theta_{inj} = 60^\circ$ at various axial locations. (Top) 6 injectors at $J = 0.7$, (middle) 6 injectors at $J = 1.35$, and (bottom) 12 injectors at $J = 1.12$. 

(a) $x/D_j = 0.5$  
(b) $x/D_j = 1.0$  
(c) $x/D_j = 2.0$
measurements show that the shocks have difficulty sustaining themselves in this dynamic region of the jet.

Figures 4.44 and 4.45 compare radial profiles of mean velocity and turbulence for the baseline jet and $J = 0.7$ and 1.35. Excellent comparison between PIV and LES is shown in mean velocity up to $x/D_j = 8$ including the velocity gradients from shocks and the shear layer spread for all cases. LES underpredicts the decay of the jet velocity in the potential core although it captures the jet spreading quite accurately. Turbulence profiles are also in very good agreement. The turbulence magnitude and profile shape at $x/D_j = 0.5$ and 2 are in excellent agreement. Fluidic injection results in up to 35% increase in near-nozzle turbulence values and rapid thickening of the shear layer. Downstream turbulence levels are reduced by up to 25% for $\theta_{inj} = 60^\circ$ at $J = 1.35$. LES captures finer structures in the flow near the centerline where turbulence is generated from the slipline and shocks. PIV seems to spatially filter these values due to the finite interrogation window size that is larger than the mesh resolution in LES. Turbulence values begin to deviate from the measurements beyond $x/D_j = 4$ although the profile shape is in agreement. These discrepancies could be due to multiple factors including reduced mesh resolution in this region or the turbulence dissipation in the numerical scheme. It is also possible that the PIV measurement is overpredicting the turbulence magnitude based on the post-processing method. The interrogation window size is 16x16 for all locations, but it is possible that a larger window size is needed at the downstream locations where the turbulent length scales are larger. In order to reduce error from spurious vectors, control over the RMS value for neighboring vectors is used to remove incorrect vectors. This can artificially alter the turbulence levels if the RMS values are too high or too low. Further investigation is needed to establish detailed uncertainty in the turbulence levels based on these factors. Overall the measurements and computations are in excellent agreement considering the complexities of the flowfield.
Figure 4.42: Comparison of PIV and LES mean streamwise velocity for 6 injectors, $\theta_{inj} = 60^\circ$. PIV data is shown above (with black border) and LES is shown below.
Figure 4.43: Axial profiles of PIV and LES mean velocity for 6 injectors, $\theta_{inj} = 60^\circ$ at the jet centerline, half-radius, and full radius.
Figure 4.44: Radial profiles of PIV and LES axial velocity for 6 injectors, $\theta_{\text{inj}} = 60^\circ$. 

(a) Baseline $M_j = 1.56$  
(b) $J = 0.7$  
(c) $J = 1.35$
Figure 4.45: Radial profiles of PIV and LES turbulence for 6 injectors, $\theta_{mj} = 60^\circ$. 

(a) Baseline $M_j = 1.56$

(b) $J = 0.7$

(c) $J = 1.35$
4.2.6 Shock Angles and Convective Mach Number

A quantitative analysis to explain some of the changes in shock noise is offered here. The main changes in the shock noise from fluidic injection were 1) shift in the peak frequency $f_p$ of BBSN to higher frequencies and 2) an increase or decrease in peak BBSN amplitude depending on fluidic injection conditions. The PIV measurements clearly illustrated that the global shock strength in the jet is directly related to the amplitude of BBSN and can be nearly eliminated at optimum fluidic injection conditions. An initial qualitative description of the physical mechanisms responsible for the shift in peak BBSN frequency were offered in Figure 4.27. To test the hypothesis that the initial shock angle is altered with fluidic injection, the shadowgraph and PIV data was analyzed to extract the shock angle for the various injection conditions. The following figures use injection pressure instead of momentum flux ratio to illustrate trends, only because some additional data was used here from shadowgraph recordings. Figure 4.46a shows the change in initial shock angle $\beta$ with fluidic injection. The initial angle is measured as $\beta = 39^\circ$ and $\beta$ increases with increasing injection angle $\theta_{inj}$ and injection pressure $p_{o,i}$. At the highest injection pressure at $\theta_{inj} = 90^\circ$ the initial injection angle $\beta = 53^\circ$. Oblique shock theory allows us to predict the Mach number using the $\theta-\beta-M$ relationship. Assuming that the streamline deflection angle is $\theta = 2.78^\circ$ (the divergence angle of the nozzle) and using the shock angle $\beta$, the Mach number upstream of the shock at the nozzle exit, $M_\beta$, can be estimated. From this, the convective Mach number $M_c$ was determined to be $M_c = 0.45 * M_\beta$ to match the baseline jet convective Mach number. Figure 4.46b shows the change in convective Mach number with fluidic injection at the four injection angles. Fluidic injection causes $M_c$ to decrease up to 20% for the highest injection angle and pressure. This reduction in $M_c$ will affect both the shock noise and large-scale turbulent mixing noise. Although this is an indirect method of calculating $M_c$ it provides insight into the physical changes in the flowfield that are causing the changes in the acoustics.

Another method of determining $M_c$ was used to verify the findings. Current shock noise theory shows that the peak shock noise frequency, $f_p$ in Eq. 1.17, depends on shock cell spacing $L_s$ and convective Mach number $M_c$ (implicitly from convective velocity $u_c$). Now, the shock cell spacing $L_s$ is directly related to the initial shock angle $\beta$, following the discussion in section 4.2.3. Comparing acoustic data to shock theory using only the change in shock cell spacing in Eq. 1.17 a discrepancy
was found indicating that the change in shock cell spacing was not the only factor increasing the BBSN peak frequency. The other parameter that determines the peak BBSN frequency is $u_c$. As discussed in the introduction, the convective velocity is usually related to the largest and most energetic turbulent structures, but clearly all of the various turbulent eddy sizes are convected at a range of velocities. There are many theories and methods to determine the convective velocity, but the most obvious method here was to calculate $u_c$ from acoustic data at $\psi = 90^\circ$ where the term in the denominator of Eq. 1.17 is unity and becomes $f_p = u_c/L_s$. Convective Mach number then follows as $M_c = u_c/a_\infty$. Convective Mach number was calculated for each injection condition using this method. Figure 4.47a shows comparison of $f_p$ from acoustic data with predictions from shadowgraph using Eq. 1.17, showing excellent agreement when the appropriate $M_c$ is calculated. Figure 4.47b shows comparison of $M_c$ calculated from both methods with the shock angle method exhibiting stronger variation. The method of calculating $M_c$ from shock cell spacing and acoustic data is more robust but requires more data. Both methods show a decrease in $M_c$ with fluidic injection which is important in predicting the shift in peak BBSN frequency and must also contribute to reduction in the large-scale mixing noise. This analysis could be carried out further with more fluidic injection data and validated with measurements of convective velocity in the shear layer of the jet.

![Graphs showing shock angle and convective Mach number](image)

Figure 4.46: Initial shock angle $\beta$ and convective Mach number $M_c$ at nozzle exit for $NPR = 4.0$ with 6 injectors at four injection angles. Shock angle determined from shadowgraph and $M_c$ calculated using oblique shock theory.
Figure 4.47: (a) Comparison of $f_p$ from data and Eq. 1.17 from shock theory for $\theta_{inj} = 60^\circ$ and (b) $M_c$ calculated from shock theory at $\psi = 90^\circ$ compared with $M_c$ calculated from shock angle for $\theta_{inj} = 60^\circ$.

4.2.7 Streamwise vorticity structure

Streamwise vorticity places a critical role in modification of large-scale mixing, turbulent mixing noise, and the turbulence energy spectrum for many high Reynolds number flows. Reduction in turbulent mixing noise of up to -8 dB in large-scale mixing noise is unprecedented for any other vorticity generating noise reduction methods reported in literature. Analysis of the shock structure in the jet has shown that shock weakening is responsible for shock noise suppression and for each $\theta_{inj}$, a critical $J$ achieves optimal shock suppression. Turbulent mixing noise continues to decrease with increasing $\theta_{inj}$ and $J$. Figure 4.48 shows contours of vorticity and normalized shock strength $\nabla p/|\nabla p|$ for $\theta_{inj} = 60^\circ$ and comparison with the baseline jet from LES. The vorticity isocontours are colored by vorticity magnitude and shock isocontours are colored by pressure. The vorticity isocontours are on the same scale for all cases. Therefore, the relative length of the isocontours indicates the persistence of the vorticities in the jet. The shock strength is normalized to the pressure gradient norm for each case, therefore coloring the shocks by pressure indicates shock strength, with variation from red to blue indicating strong shocks. The shock contours are in agreement with the PIV results showing minimum shock strength with 6 injectors at $J = 0.95$ and increasing the near-nozzle shocks with $J = 1.35$ and 12 injectors at $J = 1.12$. The shock structures are highly three dimensional in the first few nozzle diameters as the vorticities deform the shock into a star-like pattern, which settles into a polygon and finally settles back to circular. The effect is less pronounced with
the 12 injector condition as the jet shocks settle much earlier. The vorticity strength for 6 injectors appears to be relatively independent of \( J \) as they break down and the vorticity structure is very similar. Vorticity for 12 injectors does not persist in the jet as long as 6 injectors, but the vorticies are stretched out radially as they interact with each other. The trajectories of the vortices appear to reach deeper into the jet potential core for the 6 injectors.

While the three-dimensional isocontours show the jet structures with injection very nicely, a more quantitative measure of the physics is needed to draw conclusions. Contours of normalized streamwise vorticity for experiments and LES at \( \theta_{inj} = 60^\circ, J = 1.35 \) are presented in Figure 4.49 to 4.52 for \( x/D_j = 0.25 \) to 2. Vectors of \( v \) and \( w \) are superimposed on the contours. The shape and size of the vortices are in good agreement with LES, with the experiments capturing some of the fine-scale features quite well. The roll up of the shear layer is not captured in the LES as strongly most likely due to lack of realistic initial conditions. At \( x/D_j \geq 0.5 \) the experiments show vorticies which are more triangular in shape, while LES predicts more circular vortices. The shape differences are less downstream, however LES predicts a more rapid decay of vorticity strength than measured in the experiments. This could be partially responsible for the discrepancies seen in the velocity and TKE profiles. The differences are minor and could be improved upon in the LES simulation, but there is no doubt that LES is a highly capable tool for predicting and visualizing these flowfields. The use of both experiments and computations is highly complementary and aids in a deeper understanding of the flow physics involved.

The vorticity shape, size, and strength depends strongly on the injection parameters \( \theta_{inj} \) and \( J \). Figures 4.53 through 4.60 present streamwise vorticity contours for 6 injectors at all four angles at \( J = 0.7 \) and \( J = 1.35 \) for \( x/D_j = 0.25 \) and 1. For all injection angles the initial vorticies are larger at the higher momentum flux ratio. At the downstream locations, the vorticies are larger and spread to cover more of the windowed domain. There is not much discernible difference between the vorticity strength at either axial location or for the different injection angles. The effect of momentum flux ratio seems to be that the counter-rotating vorticies are at greater inclination to each other with higher momentum flux. The penetration of the injected flow into the core of the jet causes more flow to be displaced around the injected jet and expelled in between the injectors. The higher velocity of the ejected fluid results in a lower pressure outside the vortex pair which inclines them away from
the injected jet. This contributes to the entrainment of the streamwise vortices, thereby increasing the vortex size. Injection with 12 injectors at $\theta_{inj} = 60^\circ$, shown in Figures 4.61 and 4.62, results in smaller vortices than observed with 6 injectors at the same angle. The vortices interact almost immediately downstream of the nozzle exit, causing the vortices to spread radially outward at a greater rate than 6 injectors. This has a net effect of reducing the penetration of the injected flow and results in quicker breakdown of the vortex structures. The additional injectors hinder the high energy jet flow from spreading radially outwards and result in less mixing of the jet near the nozzle exit.

An even more quantitative presentation of the streamwise vorticity is required to definitively explain the acoustic observations. The initial hypothesis was that the vorticity magnitude increases strongly with $\theta_{inj}$ and $J$. Alkislar[42, 112] provided some of the most comprehensive results on streamwise vorticity for microjets and chevrons, and he showed that the trajectories and decay rates of microjets and chevrons differed significantly. Alkislar did not however, account for the shape and size of the vorticies, which differed greatly, and only investigated one microjet configuration. Figure 4.63 shows the maximum vorticity magnitude for all injection angles comparing 6 injectors at $J = 0.7$ and $J = 1.35$. The initial vorticity strength is 25% to 50% higher at $J = 1.35$, but quickly decreases to the same vorticity strength as $J = 0.7$ for $x/D_j > 0.5$. Figure 4.64 shows similar behavior for the number of injectors with slightly higher initial vorticity for 12 injectors which quickly equalizes downstream.

These results indicate that maximum vorticity strength does not correlate very strongly with mixing noise reduction as initially hypothesized. To quantify the size and overall vorticity strength, the circulation was calculated as in Eq. 4.2 by integrating over the windowed domain from the figures above. The values are indicative of the circulation strength for one vortex pair. For example, with 6 injectors the equation was applied directly, but for 12 injectors there are two vortex pairs in the domain so the circulation value was divided by 2. Figure 4.65 shows the circulation for all injection angles at $J = 0.7$ and $J = 1.35$. The circulation strength is greater for $J = 1.35$ at all injection angles indicating that the vortices are larger and overall stronger. The vorticies undergo a growth period in the first jet diameter and begin decreasing in circulation strength downstream. For all injection angles the circulation strength continues to be greater up to $x/D_j = 3$. The greatest
circulation strength is with $\theta_{inj} = 90^\circ$ at $J = 1.35$, which correlates with the greatest reduction in large-scale mixing noise. Comparison of the number of injectors with circulation is shown in Figure 4.66. Circulation is stronger for 6 injectors at both momentum flux ratios, although the decay of circulation exhibits interesting behavior. At $J = 0.7$ the decay rate is greater for 6 injectors while the circulation is relatively constant for 12 injectors up to $x/D_j = 1$ at which point the 12 injector circulation decay rate sharply increases due to interaction and breakdown of the streamwise vorticity.

At the highest momentum flux ratio case, the circulation grows slightly up to $x/D_j = 0.5$ for 6 and 12 injectors, at which point both cases have an increase in circulation decay rate that is greater for 12 injectors, again indicating vortex interaction and breakdown. This has been hypothesized by Castelain\[113\] previously, but it is believed these results offer the first definitive proof of this phenomena. The relative spacing and size of streamwise counter-rotating vortex pairs is important in maintaining vortex strength in the flow and achieving greater mixing noise reduction. Figures 4.67 and 4.68 compare the effect of injection angle at $J = 0.7$ and $J = 1.35$ on maximum vorticity and circulation. The results are nearly independent of $\theta_{inj}$ and depend more on $J$, although $\theta_{inj} = 90^\circ$ is much more effective than the other injection angles.

$$\Gamma_x = \int \int |\omega_x| dydz \quad (4.2)$$
Figure 4.48: Visualization from LES data showing shock strength ($\nabla p/\|\nabla p\|$) colored by pressure and isocontours of vorticity ($\omega_x D_j/U_j$) colored by vorticity magnitude. The variation in pressure indicates shock strength and the length of the vorticity isocontours indicate vorticity persistence in the jet.
Figure 4.49: Vorticity contours for 6 injectors at $x/D_j = 0.25$ for $\theta_{inj} = 60^\circ$, $J = 1.35$. Experiments on left, LES on right.

Figure 4.50: Vorticity contours for 6 injectors at $x/D_j = 0.5$ for $\theta_{inj} = 60^\circ$, $J = 1.35$. Experiments on left, LES on right.
Figure 4.51: Vorticity contours for 6 injectors at $x/D_j = 1$ for $\theta_{inj} = 60^\circ$, $J = 1.35$. Experiments on left, LES on right.

Figure 4.52: Vorticity contours for 6 injectors at $x/D_j = 2$ for $\theta_{inj} = 60^\circ$, $J = 1.35$. Experiments on left, LES on right.
Figure 4.53: Vorticity contours for 6 injectors at $x/D_j = 0.25$ for $\theta_{inj} = 30^\circ$ from experiments.

Figure 4.54: Vorticity contours for 6 injectors at $x/D_j = 1$ for $\theta_{inj} = 30^\circ$ from experiments.
Figure 4.55: Vorticity contours for 6 injectors at \( x/D_j = 0.25 \) for \( \theta_{inj} = 45^\circ \) from experiments.

Figure 4.56: Vorticity contours for 6 injectors at \( x/D_j = 1 \) for \( \theta_{inj} = 45^\circ \) from experiments.
Figure 4.57: Vorticity contours for 6 injectors at \( x/D_j = 0.25 \) for \( \theta_{inj} = 60^\circ \) from experiments.

Figure 4.58: Vorticity contours for 6 injectors at \( x/D_j = 1 \) for \( \theta_{inj} = 60^\circ \) from experiments.
Figure 4.59: Vorticity contours for 6 injectors at $x/D_j = 0.25$ for $\theta_{inj} = 90^\circ$ from experiments.

Figure 4.60: Vorticity contours for 6 injectors at $x/D_j = 1$ for $\theta_{inj} = 90^\circ$ from experiments.
Figure 4.61: Vorticity contours for 12 injectors at $x/D_j = 0.25$ for $\theta_{inj} = 60^\circ$.

Figure 4.62: Vorticity contours for 12 injectors at $x/D_j = 1$ for $\theta_{inj} = 60^\circ$.
Figure 4.63: Maximum vorticity magnitude for 6 and 12 injectors at all angles for $J = 0.7$ and $J = 1.35$.

Figure 4.64: Maximum vorticity magnitude for 6 and 12 injectors $\theta_{inj} = 60^\circ$. 
Figure 4.65: Integrated vorticity magnitude at all angles for $J = 0.7$ (black) and $J = 1.35$ (red).

Figure 4.66: Integrated vorticity magnitude for 6 (black) and 12 (red) injectors $\theta_{\text{inj}} = 60^\circ$. 
Figure 4.67: Maximum vorticity magnitude for 6 injectors at all angles for $J = 0.7$ and 1.35.

Figure 4.68: Integrated vorticity magnitude for 6 injectors at all angles for $J = 0.7$ and 1.35.
4.2.8 Summary: External Fluidic Injection

A parametric study on fluidic injection angle ($\theta_{inj}$), momentum flux ratio ($J$), and number of injectors was conducted to evaluate the behavior of supersonic jet noise components for different injection configurations. An acoustic survey revealed that shock noise can be decreased or increased depending on the fluidic injection configuration. An inflection point in shock noise that is dependent on $\theta_{inj}$ and $J$ was shown to be the result of shock weakening or strengthening. For a given injection configuration an optimal $J$ can be identified where maximum shock noise reduction occurs. The behavior of mixing noise was more linear with $\theta_{inj}$ and $J$. Large-scale mixing noise continually decreased with increases in $\theta_{inj}$ and $J$ as a result of increased streamwise vorticity that increases jet mixing and reduces peak turbulence levels in the downstream region of the jet which is dominated by large-scale turbulence structures. Fine-scale mixing noise increases with $\theta_{inj}$ and $J$ due to increased fine-scale turbulence generated in the initial jet region. Reductions in OASPL are a combined result of these changes in noise components. Noise reduction of 3 to 4 dB in $\Delta$OASPL at upstream and sideline angles was achieved with six injectors and up to 8.5 dB in $\Delta$OASPL at $\psi = 150^\circ$. Less noise reduction was achieved with twelve injectors due to inhibition of streamwise vorticity growth from vortex interactions although further data analysis is needed to fully verify this hypothesis. A theory on the reason for shift in peak shock noise frequency was proposed and supported with PIV and LES results.

Experimental measurements with excellent LES comparisons for the flowfield and acoustics with fluidic injection were presented. CAA captures the same trends observed in acoustic measurements although some discrepancies in the high frequencies need to be resolved to adequately predict the absolute sound levels. LES sufficiently captures the flow structures responsible for noise radiation and the effects of fluidic injection on the supersonic jet. Very good agreement in mean velocity and turbulence was observed in the near-nozzle region where the jet flow is strongly modified by fluidic injection. To accurately predict the potential core length and turbulence magnitudes downstream, investigation into the sensitivity of the results to LES setup and PIV processing needs to be conducted.
Chapter 5

Pulsed Fluidic Injection

Investigation of pulsed fluidic injection for four injection angles at the jet design condition was conducted. There have been many studies on unsteady control of jets as reviewed in Chapter 1, and the goal of this investigation was to identify if there were any acoustic or system benefits to pulsing the fluidic injection. The concept behind the pulsed injection was two fold. Primarily, it was desired to investigate the effect of frequency and duty cycle on supersonic jet acoustics while reducing the required mass flow rate through unsteady mass addition. Secondly, an effort was made to provide conclusive results of pulsed fluidic injection on supersonic jet acoustics and to identify clear trends with frequency and duty cycle. Table 5.1 summarizes recent research on active control of jets, excluding the classical work using speakers to excite low Reynolds number jets. The existing research spans a vast combination of nozzle geometries, jet velocities, actuation methods, and frequencies. Most of the studies present flow measurements and the studies with acoustics show minimal noise reduction or noise increase, or present a wide range of results in which no conclusions can be drawn. A major effort of this investigation was to develop a well controlled experiment that could provide insight into effects of the various unsteady injection parameters on a supersonic jet with regards to acoustics. A compact system for pulsing the injection flow was developed to be mounted at the nozzle exit. To control the pulsed flow, six Bosch NGI2 injectors were used to pulse airflow at $J = 0.97$ up to 500 Hz, $St_{Dj} = 0.06$ at the jet design Mach number. The injectors were 2.7 mm diameter and had 50 mm of flow path from valve exit to injection at the nozzle exit. The valves were controlled through LabVIEW and measurements were synchronized with the control signal. Figure 5.1 shows
Table 5.1: Sample of recent research on active control of jets.

<table>
<thead>
<tr>
<th>Author</th>
<th>Year</th>
<th>Actuators</th>
<th>Exp./Sim.</th>
<th>Nozzle</th>
<th>Flow</th>
<th>Acoustics</th>
<th>Mach</th>
<th>Uj [m/s]</th>
<th>Freq. [Hz]</th>
<th>Stj0</th>
<th>Fluid</th>
<th>m_i/m_j</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parekh et al.</td>
<td>1996</td>
<td>Mech.</td>
<td>Exp.</td>
<td>Rectangular</td>
<td>X</td>
<td>1.47</td>
<td>5000</td>
<td>0.08</td>
<td>1.15</td>
<td>30.50</td>
<td>221</td>
<td>N/A</td>
</tr>
<tr>
<td>Raman</td>
<td>1996</td>
<td>Fluidic</td>
<td>Exp.</td>
<td>Circular</td>
<td>X</td>
<td>0.08, 0.15</td>
<td>30, 50</td>
<td>0.20, 0.47</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Ribens &amp; Chenault et al.</td>
<td>1997</td>
<td>Fluidic</td>
<td>Exp.</td>
<td>Rectangular</td>
<td>X</td>
<td>0.13</td>
<td>43</td>
<td>1, 170</td>
<td>0.001-0.15</td>
<td>Air</td>
<td>12%</td>
<td>N/A</td>
</tr>
<tr>
<td>Freund &amp; Moin</td>
<td>2000</td>
<td>Mech/Fluidic</td>
<td>Exp./Sim.</td>
<td>Circular</td>
<td>X</td>
<td>0.1-0.9</td>
<td>30-300</td>
<td>0.05-0.45</td>
<td>Air</td>
<td>1.5%</td>
<td>2%</td>
<td>N/A</td>
</tr>
<tr>
<td>Ibrahim et al.</td>
<td>2002</td>
<td>Fluidic</td>
<td>Exp.</td>
<td>Circular</td>
<td>X</td>
<td>1.0, 1.36</td>
<td>340</td>
<td>7200</td>
<td>0.16</td>
<td>Air</td>
<td>2%</td>
<td>4%, 6%</td>
</tr>
<tr>
<td>Ragallier et al.</td>
<td>2009</td>
<td>Fluidic</td>
<td>Exp.</td>
<td>Circular</td>
<td>X</td>
<td>1.8</td>
<td>700-1000</td>
<td>1.5, 10</td>
<td>&lt; 0.001</td>
<td>Water</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kamran &amp; McQuirk</td>
<td>2011</td>
<td>Fluidic</td>
<td>Exp.</td>
<td>Circular</td>
<td>X</td>
<td>0.9</td>
<td>105</td>
<td>1030</td>
<td>0.22</td>
<td>Air</td>
<td>0.13%</td>
<td></td>
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<tr>
<td>Hafsteinsson et al.</td>
<td>2012</td>
<td>Fluidic</td>
<td>Sim.</td>
<td>Circular</td>
<td>X</td>
<td>1.56</td>
<td>500</td>
<td>300-7000</td>
<td>0.03-0.8</td>
<td>Air</td>
<td>1.60%</td>
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A schematic of the injector configuration at θ_inj = 45° and a picture of the injectors mounted on the nozzle at θ_inj = 30°.

5.1 Hot-wire Measurements

Hot-wire velocity measurements were used to quantify the quality and amplitude of the air pulses as discussed in section 2.7. A Dantec Dynamics hot-film probe was placed at the valve exit to measure flow velocity during the pulse cycle for different conditions. An A.A.Lab Systems LTD. AN-1005 anemometer was used to balance the bridge for the sensor and calibration was done daily to account for variations in the calibration. The control signal was recorded simultaneously to identify any delays and phase shifts in the pulse cycle with respect to the input control signal.

5.2 High-Speed Shadowgraph Technique

High-speed shadowgraph was used to provide time resolved flow visualization to characterize time scales involved with pulsed fluidic injection as discussed in section 2.5. The shadowgraph setup consisted of an Oriel arc lamp and power supply, two 12" first-surface parabolic mirrors with 6’ focal lengths, and a Photron Fastcam SA4 high-speed camera. The shadowgraph was arranged in a Z-type configuration as discussed in Settles[76]. Shadowgraph recordings were taken at 10,000 frames per second (fps) and the recording was triggered relative to the injection control signal in order to capture the pulsed injection adequately.
Figure 5.1: Pulsed injector setup for 6 injectors arranged axisymmetrically around the jet.

Table 5.2: Operating condition for the primary jet and injectors including mass flow ratio $m_i/m_j$ and momentum flux ratio $\rho_i U_i^2 / \rho_j U_j^2$.

<table>
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<tr>
<th></th>
<th>$M_j$</th>
<th>$p_{a,j}/p_a$</th>
<th>$T_{a,j}/T_a$</th>
<th>$c_j$ [m/s]</th>
<th>$U_j$ [m/s]</th>
<th>$m_j$ [kg/s]</th>
<th>$Re$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jet</td>
<td>1.56</td>
<td>4.00</td>
<td>1.25</td>
<td>315</td>
<td>491</td>
<td>1.65</td>
<td>2.46E+06</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$M_i$</th>
<th>$p_{a,i}/p_a$</th>
<th>$T_{a,i}/T_a$</th>
<th>$c_i$ [m/s]</th>
<th>$U_i$ [m/s]</th>
<th>$m_i/m_j$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injectors</td>
<td>1.51</td>
<td>3.75</td>
<td>1.00</td>
<td>287</td>
<td>435</td>
<td>1.50%</td>
<td>0.97</td>
</tr>
</tbody>
</table>
5.3 LES and CAA

The description of the LES solver and CAA approach are discussed in Section 2.10. Details specific to this pulsed injection study are discussed here. Total pressure and total enthalpy were specified at the nozzle inlet as $p = 405300.0\ Pa$ and $368651.5\ J/kg$. A stagnant flow was specified around the nozzle. The 2D entrainment region had a periodic boundary condition in the azimuthal direction with the purpose of achieving correct mass flow balance in and out of the domain. Ambient pressure ($p_{\infty} = 101325\ Pa$) was specified at the outlet and a damping zone was added at the end of the domain to minimize acoustic reflections from the outlet. The injector casing installed at the nozzle exit in the experiment was simplified as a wall boundary condition at the nozzle exit. The wall had an angle perpendicular to the flow axis and the area of the wall was set equal to the cross-sectional area of the casing. Twelve injectors were implemented as a mass-flow boundary condition just upstream of the nozzle exit and each injector has an angle of $60^\circ$ relative to the flow axis, see Figure 5.2. The cross-sectional area of each injector was $A_i \approx 4.9 \cdot 10^{-6}\ m^2$ and the mesh resolution was relatively coarse. For example, each injector spanned two cells in the flow direction and three cells in the azimuthal direction. The pulsed injectors were varied in a sinusoidal manner between zero mass flow and a maximum mass flow equal to the steady-state injection. The total mass flow ratio between the twelve steady-state injectors and the main jet mass flow was $\dot{m}_i/\dot{m}_j \approx 1.1\%$.

Figure 5.2: The nozzle geometry used in the LES simulations. A reflective wall was attached to the nozzle exit with an angle perpendicular to the flow axis. The 12 injectors were evenly distributed around the nozzle exit (marked with red). The injectors were implemented with a mass flow boundary condition with an angle of $\theta_{inj} = 60^\circ$ to the flow axis.
5.3.1 Comments on sampling length

The simulations include pulsed injection results for frequencies of $f_{inj} = 10 \text{ Hz}$, $f_{inj} = 50 \text{ Hz}$ and $f_{inj} = 100 \text{ Hz}$. These frequencies are relatively low in comparison with the timescales of the turbulent eddies in the jet shear layer and therefore act to some extent as a quasi-steady state injection. This means that the flow should have time to stabilize with increasing or decreasing injection. A low pulsed injection frequency was a challenge from a numerical standpoint, since longer sampling time was required compared to the characteristic time scales of the turbulent jet if statistical convergence was to be achieved. Therefore, during the computational time only a few pulse cycles could be practically simulated. More importantly was that the sampling length was restricted to an integer number of injection periods in order to not have an incomplete pulse cycle in the time averaged result. One injection period may be sufficient to get a rough first estimate of the flow and acoustic statistics for the lowest injection frequency ($f_{inj} = 10 \text{ Hz}$).

5.4 Injector Characterization

The injectors were selected to have control over a broad range of frequencies and duty cycles while providing injection mass flows on the order of previous investigations that have demonstrated noise reduction potential [61, 46, 45]. Many considerations were taken into account to select valves that would provide high mass pulsations over a wide operating range. The difficulty of using valves for pulsed flow was the response time of the valve mechanism, which limits the maximum frequency or minimum response time. In general, at higher frequencies the mass flow passed through the valve was reduced. Valves with rotary mechanisms can reach higher frequencies but are usually constrained with a limited operational frequency range, lower mass flow, or large size. Valve size was a major consideration as it was desired to mount the valves as near to the injection location since hot-wire measurements showed that flow pulses are dampened through long passages. Various valves were investigated with time resolved velocity measurements using a hot-wire anemometer to characterize the pulse waveforms the valves provided. This allowed selection of valves with the highest frequency response and best performance at the mass flows required. To provide the control over the phase of each actuator, it was ideal to have an array of valves that each regulated flow for a single injector.
This study presents results for axisymmetric injection with 6 injectors operating in phase with each other.

Characterizing the actuator flow response was imperative to understand how the flow response differed from the control signal and ideal operation. An extensive study of various valves, tubing lengths, tubing diameters, and associated fittings was conducted to ensure strong flow pulse transmission. To maximize transmission of the flow pulses, a few design features were developed for achieving strong pulses. Matching the tubing inner diameter closely with the valve exit diameter (6.3 mm) and removing abrupt area changes, especially in fittings, resulted in minimal pulse dampening. The tubing diameter was decreased (2.7 mm) to achieve the desired injection velocity, but great care was taken to provide a smoothly varying change in area. Increased tubing length also provided significant dampening of flow pulses so the valves were placed very near the jet exit with less than 50 mm of tubing from valve exit to injection location. It was not practical to measure the valve performance with the primary jet operating, however bench measurements were conducted on the exact configuration that was used when installed on the jet including operational pressure and tubing lengths.

![Graphs](image)

Figure 5.3: Normalized injector velocity for 50% duty cycle at various frequencies.

The measured velocity was normalized to the steady flow value $u_{\text{steady}} = 435$ m/s. The duty cycle is the percent “on” time of the flow pulses, for example, 20% duty cycle means the flow was on for 20% of the cycle period for a given frequency. The time was normalized to the cycle period to collapse
waveforms of different frequencies. Time $t/T = 0$ corresponds to the control signal (square-wave) so that flow lag time could be determined. Figure 5.3 shows pulse waveforms for the low (1 - 100 Hz) and high (100 - 500 Hz) frequency range at 50% duty cycle. In the low frequency range the flow reaches the steady flow velocity although the lag time increases with frequency. The spike in flow velocity at the end of the pulse cycle is due to acceleration of flow through the valve mechanism as it closes and decreases area. In the high frequency range the lag time continues to increase with frequency until at 500 Hz the maximum velocity lags behind the control signal by one cycle period. Frequencies above 200 Hz exhibit decay of the flow waveform and peak velocity. The waveform becomes increasingly sinusoidal above 100 Hz and peak velocity was 25% of the steady velocity at 500 Hz. It was apparent that the capability of the injector performance decays significantly at frequencies above 100 Hz.

Figure 5.4 illustrates the effect of duty cycle for 30 Hz and 100 Hz pulsation. At 30 Hz the flow pulses follow the square-wave control signal at all duty cycles. At 100 Hz the flow response decays with reduced duty cycle. When the injector “on” time approaches the time response for the valve then pulse decay occurs and the flow does not reach the steady flow velocity.

Measurements of the flow response to the control signal show that the flow response deviates from the ideal control signal significantly over the operating range. To quantify the actuator output relative to steady injection, a true duty cycle was calculated by integrating the measured velocity for each pulse cycle. The integrated velocity was defined in Eq. 5.1 and Eq. 5.2 where $U_{\text{norm}}$ represents...
the true duty cycle based on the total velocity input over a cycle relative to steady injection. Jet in crossflow studies commonly scale spatial quantities such as penetration and trajectories based on momentum. It was found that using temporal variation of a primitive variable was more conducive for scaling acoustic results. Figure 5.5 shows $U_{norm}$ vs. injection frequency $f_{inj}$ for various duty cycles. At low frequencies $U_{norm}$ closely represents the control duty cycle and as $f_{inj}$ was increased, $U_{norm}$ decreased fairly linearly. For 50% control duty cycle, $U_{norm}$ was 20% at 500 Hz due to degradation of the valve performance. Again, it is imperative to fully characterize an actuator when studying unsteady actuation to fully understand the implications of the results. A lack of actuator characterization contributes to the variation of results in previous unsteady fluidic injection studies.

\[
u_{inj} = \frac{1}{T} \int_0^T u(t) dt \quad (5.1)
\]

\[
U_{norm} = \frac{u_{inj}}{u_{steady}} \quad (5.2)
\]

Figure 5.5: Injector performance across the operating range of frequencies and duty cycles.
5.5 Flowfield Results

High-speed shadowgraph allows clear visualization of the density variations in a supersonic jet including turbulence and shock waves. Time averaged shadowgraph visualizes the pseudo-stationary shock waves by averaging out the spatially varying turbulence. Figure 5.6 shows time averaged shadowgraph images during steady injection at various angles. The $M_j = 1.56$ jet without fluidic injection is shown in Figure 5.6a. The jet potential core has a dual shock train, one set emanating from the sharp throat internal to the nozzle, and one set originating at the nozzle lip. These types of nozzles have been extensively investigated by Cuppoletti et al. [96], Gustafsson et al. [75], Munday et al. [115, 116], Bridges et al. [23], and Kuo et al. [94]. Figure 5.6b shows the six injectors operating without the primary jet flow. The injected jets are underexpanded with a fully expanded Mach number $M_{inj} = 1.51$ and break down at the jet centerline as they impinge on one another. During operation on the primary jet, penetration did not reach the centerline for the momentum flux ratio of the injectors. The circular aberrations are from the zooming lens optics and are minimal when operating the jet.

Fluidic injection into supersonic jets interacts with the shocks in the jet potential core as well as introducing streamwise vorticity. A fluidic injector injected transversely into a supersonic jet is very similar to a jet in supersonic crossflow problem which has been studied extensively as discussed in chapter 1. Each injector has a bow shock that forms upstream of the injector which subsequently reduces the downstream Mach number, increases the shock angle in the jet potential core, and effectively shortens the shock spacing in the jet potential core. This decreased shock spacing results in the shift of the peak BBSN to higher frequency [61, 46, 45]. Figures 5.6c to 5.6f show the effect of injection angle on the shock structures in the jet. Increasing the injection angle creates stronger shocks in the jet which pull the shock train in further and disrupt the downstream shocks. Increasing the injection angle also increased the penetration of the secondary jets as the normal component of velocity (normal to the jet flow) was increased. It was not useful to quantify injector penetration from the shadowgraph images since it is a line-of-sight visualization technique and the injectors are arranged axisymmetrically, so further data is necessary to accurately determine penetration.

While time averaged shadowgraph provided an idea of the pseudo-steady state features of the jet, the reality is that shocks are highly dynamic as they interact with turbulent structures and are set
into motion. The fluidic injectors cause increased jet spreading as streamwise vortices enhance mixing in the shear layer which also becomes averaged out. Figure 5.7 shows instantaneous shadowgraph of the jet as the injection was activated. The contraction of the shock train occurred over 1 ms before reaching a pseudo-steady state condition. When injection was deactivated the shocks took about 1.4 ms to extend back to their original position. The response time was determined by analyzing the moving $rms$ difference of the image intensity to identify where strong changes in the images occur. The downstream shocks lagged the motion of the first shock as the disturbance due to the injection was convected downstream. Therefore, the shocks throughout the entire potential core required more time to reach the pseudo-steady state condition. The same shock dynamics were captured with LES as shown in Figure 5.8, which presents streamwise velocity contours during half of a pulse cycle at 100 Hz. The fluidic injection forced the lip shock upstream which increased the shock angle, resulting in decreased shock spacing. The flow and shocks internal to the nozzle remained unaffected as the disturbances from the fluidic injection do not propagate upstream in supersonic flow. This is significant as it indicates a minimal effect on performance and thrust for a supersonic nozzle. The shock motion was clearly captured even though there were differences between the LES and experiments discussed in Section 5.3. The time averaged flowfield showed no significant effect with regard to jet spreading or enhanced mixing from the pulsed injection as the injection frequencies in this study are below the preferred jet frequencies.

For conditions with strong screech, an associated lag time for the screech instability to develop was observed in the flow. Figure 5.9 shows the development of a jet column instability up to 11 ms after the injection was deactivated. Large scale motion of the jet potential core was not observed until 7 ms after the injection was deactivated. The response time of the screech instability was within the time scales of the pulsed injection frequency range investigated in this study. This indicates the possibility that response time of shocks and the jet column instability could allow enhanced noise suppression with reduced mass flow if the growth of the shock related instabilities and screech instabilities can remain suppressed with unsteady injection. The results of these shadowgraph results highlight the dynamic behavior of the jet with and without injection and more understanding will surface through more quantitative measures of the jet dynamics.
Figure 5.6: Mean shadowgraph images showing shock structure in the jet at various $\theta_{inj}$ with and without fluidic injection.
Figure 5.7: Instantaneous shadowgraph showing the contraction of the shock train as injection was activated for $\theta_{mj} = 90^\circ$. The dashed line references the first lip shock reflection of the lip shock before injection was activated. Time $t=0$ ms was the instant the injectors were activated.
Figure 5.8: Instantaneous velocity from LES showing the contraction of the shock train as injection was activated for $\theta_{inj} = 60^\circ$ for 100 Hz injection. The dashed line references the first lip shock reflection of the lip shock before injection was activated. Phase angle $\phi = 0^\circ$ to $180^\circ$ represents half of the pulse cycle.
Figure 5.9: Instantaneous shadowgraph showing the development of jet column instability during screech feedback for $\theta_{inj} = 90^\circ$. Time $t = 0$ ms was the instant the injectors were activated.
5.6 Acoustics Results

5.6.1 Steady Injection

To provide a benchmark for the effectiveness of unsteady injection it was useful to compare with steady injection. Narrowband acoustics for the baseline jet and steady injection at all $\theta_{inj}$ in Figure 5.10 show changes in the various noise components at $\psi = 35^\circ$, $90^\circ$, and $150^\circ$. Increasing the injection angle was similar to increasing the momentum flux ratio of the injection, as the normal velocity injected into the jet increased. Further reduction of screech and shock noise was achieved with increased injection angle which was also evident in Figure 5.11 in the sideline and upstream angles. Mixing noise reduction does not exhibit such a linear trend with injection angle. Injection at $\theta_{inj} = 30^\circ$ provides $\Delta OASPL = -2.6$ dB while $\theta_{inj} = 60^\circ$ provides $\Delta OASPL = -2.9$ dB. However, $\theta_{inj} = 90^\circ$ provides $\Delta OASPL = -4.2$ dB. This was due to the trade-off between low frequencies and high frequencies as fluidic injection enhances mixing of the large scale turbulence yet introduces fine scale turbulence which causes high frequency lift.

![Figure 5.10: Narrowband acoustics showing the effect of injection angle with steady injection.](image)

5.6.2 Injection Duty Cycle

Unsteady injection has the potential to provide equivalent or enhanced noise reduction with less mass flow required than continuous injection. The duty cycle, $DC = \tau/T$ is a parameter for controlling the mass flow provided during a pulsed injection cycle. It was seen in Section 5.4 that as
frequency was increased and control duty cycle was reduced, the actual duty cycle was reduced as a result of valve response time. Figure 5.12 shows ΔOASPL for varying duty cycles from 20% to 80% at 100 Hz injection and all θ_{inj}. For all θ_{inj} noise at ψ = 150° was further reduced with increasing duty cycle. At θ_{inj} = 30° the upstream and sideline angles were less affected by duty cycle. The more shallow injection angle did not provide enough shock interaction time to disrupt the screech and shock noise mechanisms. At θ_{inj} = 60° and 90° the shock and screech noise seemed to decrease linearly with duty cycles above 50%. This effect was due to the time response of the shock and screech noise mechanisms discussed in Section 5.5. Enough activated injection time must pass for noise reduction to be achieved and after that point, additional time results in more noise reduction on a time averaged basis. Figure 5.13 shows the trend of ΔOASPL with duty cycle for ψ = 35°, 90°, and 150°. The linear trends with duty cycle are clearly shown as well as the ineffectiveness of θ_{inj} = 30° to reduce noise at the upstream and sideline angles. A further look at the narrowband acoustics (not shown) for these cases show there was, in fact, some shock noise reduction for θ_{inj} = 30°, but it was offset by some high frequency lift. The high frequency noise can be generated from too little submersion of the secondary flow into the primary jet flow, which can radiate injector noise to the farfield. Conversely, too steep of an injection angle can generate more streamwise vorticity which can also result in high frequency lift in exchange for reduced low frequency noise. The least high frequency lift occurred for θ_{inj} = 45°, which indicates that this injection angle balances these two effects more than the other angles. The narrowband also showed that harmonics of the 100 Hz
injection appeared in the acoustic spectrum up to 1 kHz which can contribute to higher OASPL.

5.6.3 Injection Frequency

Injection frequencies up to 400 Hz were investigated at the various duty cycles and injection angles. Figure 5.15 presents ΔOASPL at 50% duty cycle for \( f_{\text{inj}} = 1 \) Hz to 400 Hz. For frequencies below 100 Hz there are minimal differences from frequencies below 50 Hz providing slightly more noise reduction. Performance of \( f_{\text{inj}} \) above 100 Hz results in less noise reduction and even increases in ΔOASPL at the upstream angles. Figure 5.16 presents a finer sweep of frequencies for \( f_{\text{inj}} = \)
Figure 5.13: The effect of duty cycle on $\Delta$OASPL at 100 Hz and $\psi = 35^\circ$, 90$^\circ$, and 150$^\circ$.

Figure 5.14: Narrowband showing the effect of injection angle at 100 Hz and 80% duty cycle for $\psi = 35^\circ$, 90$^\circ$, and 150$^\circ$.

1 Hz to 100 Hz at $\theta_{inj} = 60^\circ$ for $DC = 20\%$, 40\%, 60\%, and 80\%. Trends with duty cycle match what was previously shown, but it became apparent that lower $f_{inj}$ such as 5 Hz, 15 Hz, and 30 Hz resulted in more noise reduction at a given duty cycle, especially at $\psi = 150^\circ$. For $f_{inj} = 30$ Hz and $DC = 20\%$ more than 50\% of the steady injection noise reduction was achieved with 20\% of the injected mass flow. Although 30 Hz is well below the preferred jet frequency, the acoustic results seemed to favor this frequency over the others. To provide a clearer picture of the effect of low frequency injection, Figure 5.17 shows noise reduction at single observer locations $\psi = 35^\circ$, 90$^\circ$, and 150$^\circ$ for $\theta_{inj} = 60^\circ$. The dashed line represents the $\Delta$OASPL for steady injection at the given observer angle. It was clear that 30 Hz provided enhanced noise reduction with reduced duty cycle at all observer angles. The goal of this research was to identify any injection conditions that can provide enhanced noise reduction with less mass flow than steady injection. To quantify the noise reduction per unit mass flow, the acoustic results were normalized to $U_{norm}$ for $\theta_{inj} = 60^\circ$ and 90$^\circ$. 

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in Figures 5.18 and 5.19, respectively. It was clearly demonstrated that \( \Delta \text{OASPL} \) can be scaled based on the actual duty cycle of the injection pulses. The normalized acoustic results also highlight the enhanced noise reduction achieved with 30 Hz injection since five times as much noise reduction per unit mass flow was achieved for \( \psi = 150^\circ \) with 20% duty cycle. The results on the shock noise components at upstream and sideline angles are not as significant, indicating that the effect may be related to the turbulent structures in the jet instead of shock dynamics. Figures 5.20 and 5.21 show narrowband acoustics for 30 Hz at \( \theta_{\text{inj}} = 60^\circ \) and \( 90^\circ \). Unlike 100 Hz injection, harmonics of the injection frequency do not proliferate in the narrowband acoustic spectra. The high frequency lift was also not as prolific in the spectra while still providing suppression of screech, shock noise, and mixing noise.
Figure 5.15: The effect of frequency (1 Hz - 400 Hz) at 50% duty cycle on $\Delta$OASPL at all observer angles for all $\theta_{inj}$. 

(a) $\theta_{inj} = 30^\circ$  
(b) $\theta_{inj} = 45^\circ$  
(c) $\theta_{inj} = 60^\circ$  
(d) $\theta_{inj} = 90^\circ$
Figure 5.16: The effect of low frequency (1 Hz - 100 Hz) for $theta_{inj} = 60^\circ$ on $\Delta$OASPL at all observer angles. Duty cycles of 20\%, 40\%, 60\%, and 80\% are shown.
Figure 5.17: The effect of frequency on ΔOASPL at 60° injection angle for ψ = 35°, 90°, and 150°.

Figure 5.18: The effect of frequency on ΔOASPL normalized to $U_{\text{norm}}$ at 60° injection angle for ψ = 35°, 90°, and 150°.

Figure 5.19: The effect of frequency on ΔOASPL normalized to $U_{\text{norm}}$ at 90° injection angle for ψ = 35°, 90°.
Figure 5.20: Narrowband acoustics for 30 Hz at 60° injection angle for ψ = 35°, 90°, and 150°.

Figure 5.21: Narrowband acoustics for 30 Hz at 90° injection angle for ψ = 35°, 90°, and 150°.
5.6.4 CAA Comparisons

Computational aeroacoustics (CAA) has demonstrated the capability of predicting the acoustics for supersonic jets with and without fluidic injection[96, 70, 117]. Utilizing CAA to predict acoustics with unsteady noise control methods was a more formidable task, as difficulties arose such as performing enough iterations to capture the flow dynamics that contributed to the acoustic field, especially at the low frequencies in this study. Figure 5.22 compares measured and CAA narrowband acoustics for the baseline jet and with steady injection. CAA has less statistical convergence of the acoustic spectra since the sample time was on the order of 100 ms while measurements are ensemble averaged over 15 s. CAA captures the acoustic spectra at \( \psi = 35^\circ \) extremely well, including shock noise, screech frequency, and screech amplitude. CAA overpredicts the reduction in shock noise, shift in peak frequency, and high frequency lift with steady injection. CAA slightly overpredicts the turbulent mixing noise spectrum at \( \psi = 150^\circ \) but seems to capture the noise reduction with steady injection.

Figure 5.23 shows experimental and CAA \( \Delta OASPL \) results for 10 Hz, 50 Hz, 100 Hz, and steady injection. For pulsed injection, CAA predicted minimal differences for the three frequencies, in agreement with the overall trend observed in experiments. The largest discrepancies were at the upstream and downstream observation angles. The differences in acoustics could be partially due to the slight differences in implementation of the injectors in LES compared to experiments. Accounting for all the difficulties, the same conclusions can be reached from CAA results for low frequency injection on this high Reynolds number jet. These results illustrate the robustness of CAA even with orders of magnitude difference in time scales involved and CAA will continue to improve with increasing computational resources and understanding gained from these types of investigations.

5.7 Summary

A comprehensive set of data was presented demonstrating the effect of pulsed injection on a \( M_j = 1.56 \) jet. Noise reduction was achieved for frequencies of 100 Hz and below corresponding to adequate valve response. It was shown that noise reduction increases proportional to the duty cycle for most frequencies and, in general, noise reduction was less sensitive to the frequency of injection at
Figure 5.22: Comparison of experimental and CAA narrowband acoustics showing the effect of injection angle with steady injection.

Figure 5.23: Comparison of ΔOASPL for experiments and CAA at 10 Hz, 50 Hz, 100 Hz, and steady injection.
the low frequencies investigated. For turbulent mixing noise it was shown that reduction in $\Delta$OASPL scales with the actual duty cycle for most frequencies. A frequency of $f_{inj} = 30$ Hz provides enhanced noise reduction with significantly less mass flow required although a sufficient explanation has not yet been achieved. The temporal evolution of the pseudo-steady state shocks plays a fundamental role in production of shock associated noise and time scales were identified based on the convection speed of the shocks in the jet during an injection cycle. Reduction of shock and screech noise components are strongly dependent on the time scales required for the shocks to stabilize within the jet and growth of the instability waves during periods when injection was deactivated. Screech has a longer time scale associated with growth and saturation which makes low frequency pulsed injection more viable for suppression. The results presented have extracted some of the physics involved in unsteady injection. Some phenomena that were not fully explained were observed, such as the reason for enhanced noise reduction at 30 Hz. CAA provided moderate comparison with the acoustic measurements and was able to capture trends. Continued efforts to use experiments and computations to complement each other and enhance understanding of the physics involved in these complicated systems is essential moving forward. Further investigation of the effects of pulsed injection on noise suppression will not only further development of active noise control systems and reduce mass flow required for such a system, but could also be used as a diagnostic to further understand the development and propagation of the various supersonic noise components. For suggested future experiments to clarify these findings, refer to Chapter 9.
Chapter 6

Internal Fluidic Injection

This chapter investigates a novel application of fluidic injection that implements fluidic injectors internal to the jet nozzle. There are many parameters governing the design of a fluidic injection system, which prompted investigation of some unconventional injection studies. If the injector design (shape, size, etc.) is disregarded, the main parameters are injector placement in three dimensional space (location and angular alignment) and the amount of injected fluid (mass flow, pressure, etc.).

Most applications of fluidic injection have been conducted with injectors arranged at the exterior of the nozzle injecting inwards towards the jet centerline through the shear layer. The design of the trailing edge injection system elicited the idea of moving the injectors further inside the nozzle and injecting at different locations and angles inside the nozzle. A study of injection of discrete jets in the supersonic flow in the diverging portion of the nozzle was conducted. The initial parameter study on the biconical nozzle for a few internal injection cases included acoustics and flow measurements in order to determine the best conditions. The results from the initial study were extended for many other injector configurations on both the biconical and splined nozzle and a detailed far-field acoustic survey was conducted. Some very unique effects of internal injection on the supersonic jet acoustics and noise components were observed, exhibiting potential for significant noise reduction of supersonic jets. The initial study was relatively exploratory and much time was spent attempting to understand the unique results, including LES computational support for the best configurations.

The findings of the initial study were expanded to some new injector design concepts and far-field acoustics were used to screen the new designs for effectiveness, although much further work is needed.
to fully understand the physical mechanisms involved.

6.1 Comments on Timeline of Results

To the knowledge of the author there was no prior existing literature detailing the acoustic control demonstrated in this study as of May 2011 when this research study was initiated. There has been considerable work on methods of fluidic injection in supersonic nozzles for area control and thrust vectoring. Numerous patents also detail fluidic control devices similar to the one described here. However, all of the existing patents attribute noise reduction to vortex generation or area control which is fundamentally different from the results of the present study. Deere\cite{118} summarizes the research conducted at NASA Langley on area control and fluidic thrust vectoring dating back to 1993. Flamm et. al.\cite{119} studied thrust vectoring experimentally and computationally with a dual throat nozzle and arrays of control jets internal to the jet nozzle. Kuo et. al.\cite{120} demonstrated some acoustic results for a similar configuration deemed ‘fluidic inserts’, although only qualitative insight was given into the physical mechanisms involved in altering the acoustics. It should be noted that the internal fluidic injection concept was developed and tested in May 2011 independently at UC with the author and his advisor more than a year and a half prior to the publication by Kuo. The results were not presented due to a pending patent submission with the project sponsors FMV and GKN Aerospace. This dissertation is the first report of the results of this study and will be embargoed for one year while the patent procedure is completed and a peer-reviewed journal submission is drafted.

6.2 Comments on Pressure Ratio for Internal Injection

Quantifying the injection parameters for internal injection provided some unique issues. Section 2.6 discussed some issues with using pressure to compare injection systems since the system pressure drop changes with any changes in the fluidics system or operating conditions. Measurement of mass flow rate is a much better method of comparing injection configurations, however this was only fully realized in hindsight. Initially the rotameter was calibrated for pressure and mass flow rate on the trailing-edge injection system. The pressure recorded at the fluidics control panel was varied to study the internal injection cases. It was not until LES computations were run and showed the same
results at different injection pressure ratios that it was realized that there was a large pressure drop to the nozzle plenum causing up to 30% pressure loss from the control room to injectors. Another major difficulty with using injection pressure ratio is that normally, \( p_{o,i}/p_a \) is quoted but for an injection location inside the nozzle, the pressure ratio across the injector \( p_{o,i}/p \neq p_{o,i}/p_a \) while the flow expands through the nozzle. Pressure varies along the nozzle and the nozzle was not designed for or instrumented to measure wall pressure. This can be acquired from LES computations, illustrating a situation in which the experiments and computations are complementary.

### 6.3 Internal Injection Parameter Study

To optimize the injection location, beginning from the trailing edge fluidic injector design, the question arose as to what the effect of injection angle and location would have had on the acoustics. Figure 6.1a schematically shows the nomenclature used for the present study. Due to the limited nozzle thickness at the trailing edge, the injection angle was limited to \( \theta_{inj} = 15^\circ \) without having to move the injection location further upstream. A prototype nozzle was designed with injectors located at different axial locations upstream and injection angles of \( \theta_{inj} = 30^\circ, 60^\circ, \) and \( 90^\circ \) as shown in Figure 6.1b. The injector nozzle was an adapter replacing the trailing-edge injector and used the outer sheath that forms a plenum for the injectors as shown in Figure 6.2. Twelve injectors with \( D_i = 2.7 \) mm were equally spaced azimuthally around the nozzle wall. The axial location of the injectors is measured from the nozzle exit \((x/D_j = 0)\). The 30\(^\circ\) and 60\(^\circ\) injectors were located at \( x/D_j = -0.39 \) and \( x/D_j = -0.77 \) with each twelve injector array azimuthally rotated by 15\(^\circ\) for a total of four (4) injector sets. Only one set of twelve injectors was run at any given time, while the unused injectors were filled with drywall compound and sanded smooth and flush with the nozzle wall. This was an effective and cheap method for studying many fluidic injector parameters as the compound withstood the injection pressures and jet temperature (up to 367 K). To verify that the sanded drywall compound had no effect on the acoustics, all of the injectors were filled and acoustics were measured and compared with the baseline nozzle. Additional 90\(^\circ\) injectors just upstream and downstream of the nozzle throat and in the convergent section were tested but had negligible acoustic effects and are not discussed.
(a) Injection angle nomenclature

(b) Injector designs

Figure 6.1: Injection angle nomenclature and injector patterns.

Figure 6.2: Internal injection nozzle assembly.
6.3.1 Acoustic Measurements for Internal Injection

Far-field acoustics were measured for the four injector sets operating at different pressures to determine the change in acoustics with internal injection. This was accomplished by adjusting the injection pressure ($p_{o,i}$) at the fluidics control panel to various injection pressures and measuring the far-field acoustics. Unfortunately, mass flow rate was not recorded for the initial tests as discussed in section 2.6. The injection pressure was incrementally increased up to 120 psig (925 kPa). Many of the injection pressures tested at each location had little to no effect on the far-field acoustics and in some cases additional noise was created. The detailed trends discussed in chapter 4 were not apparent and there was more of a discrete switching behavior observed at first. It was discovered that at some very specific conditions reduction in shock noise could be achieved at $NPR = 4.0$ and for the overexpanded condition, $NPR = 2.5$, reduction in the high frequency mixing noise. The best injection configuration was $\theta_{inj} = 60^\circ$ at $x/D_j = -0.77$. All of the other injection configurations (locations and angles) either had no effect or increased the screech and/or fine-scale mixing noise slightly. All configurations had no effect on the large-scale mixing noise for all injection pressures indicating that internal injection does not cause mixing enhancement by vorticity production. The only other noise reduction condition was for $NPR = 4.5$ with injection at intermediate pressures and $\theta_{inj} = 30^\circ$ at $x/D_j = -0.77$. However, that case was not studied further as it shows noise reduction similar to the best $NPR = 2.5$ case and the underexpanded case is more difficult to study.

To gain a clearer picture of why only specific conditions achieved noise reduction while most injection conditions were ineffective, a finer sweep of $p_{o,i}$ was conducted for the best injection configuration of $\theta_{inj} = 60^\circ$ at $x/D_j = -0.77$. Figure 6.3 shows 3-D surface plots of $\Delta$OASPL for all microphone angles and $p_{o,i}$ from 20 psig to 120 psig for $NPR = 4.0$ and $NPR = 2.5$. The surface magnitude and coloring denotes the amount of noise reduction with red representing noise increase and blue representing the best noise reduction. In both cases the highest noise reduction is at the upstream and sideline angles which are dominated by shock and fine-scale mixing noise. The optimum noise reduction case is around 110 psig for $NPR = 4.0$ and 55 psig for $NPR = 2.5$. For $NPR = 2.5$ there is a clear optimum injection pressure for noise reduction indicated by the lowest $\Delta$OASPL value in the surface plot. For $NPR = 4.0$ it appears that that condition has just about been reached at the maximum injection pressure. This is illustrated more clearly in Figure 6.4a.
for $\psi = 35^\circ$, $90^\circ$, and $150^\circ$ showing up to -5 dB reduction in OASPL at $\psi = 35^\circ$ for the optimum $p_{o,i}$. This optimum condition is similar to the trend seen for BBSN indicating that this is possibly a mechanism that alters the shock pattern in the jet and subsequently the shock noise components. The effect on large-scale turbulent mixing noise is negligible for both cases as shown on the $\psi = 150^\circ$ spectra. Figure 6.4b shows the effect of injection pressure on $\Delta$OASPL for $\theta_{inj} = 30^\circ$ showing 1 dB increase in noise at sideline and upstream angles and a flat -1 dB decrease at $150^\circ$. These results are representative of all of the other acoustically ineffective injection configurations. For $NPR = 2.5$ a maximum noise reduction is achieved with both $\theta_{inj} = 60^\circ$ and $\theta_{inj} = 30^\circ$ at different injection pressures as shown in Figure 6.4c and 6.4d. Again, minimal effect on the large-scale mixing noise at $\psi = 150^\circ$ is observed.

Figure 6.3: Mapping injection pressure effect on $\Delta$OASPL for $60^\circ$ injection at $x/D_j = -0.77$ for all observer angles. Optimum noise reduction at upstream angles. Pressure measured in [psig].

Inspecting the narrowband spectral data sheds some light on the dependence of the noise components on $p_{o,i}$. Figure 6.5 shows surface plots of the narrowband spectra data at $\psi = 35^\circ$ for varying $p_{o,i}$. The top figures show the entire spectra for $NPR = 4.0$ and $NPR = 2.5$ while the bottom figures show a zoomed in region of the shock noise components. For $NPR = 4.0$, increasing $p_{o,i}$ initially amplifies the screech and BBSN until approximately $p_{o,i} = 55$ psig is reached and the screech is suppressed and the BBSN amplitude decreases. For $NPR = 2.5$, increasing $p_{o,i}$ initially decreases BBSN and fine-scale mixing noise but amplifies screech. The BBSN reaches a minimum around
Figure 6.4: Effect of injection pressure on ΔOASPL at $\psi = 35^\circ$, $90^\circ$, and $150^\circ$ at $x/D_j = -0.77$. 
$p_{o,i} = 55$ psig while screech reaches a maximum. Further increase of $p_{o,i}$ results in strengthening of the shock noise and reduction in screech. Figure 6.6 shows narrowband spectra for $NPR = 4.0$ at the optimum injection pressure $p_{o,i} = 110$ psig at $\theta_{inj} = 60^\circ$. The peak of the shock noise is suppressed while the fine-scale mixing noise is relatively unchanged, particularly at $\psi = 90^\circ$ indicating minor effect on the turbulent mixing length scales. Figure 6.7 shows the negligible effect on noise for $\theta_{inj} = 30^\circ$. For $NPR = 2.5$, $\theta_{inj} = 60^\circ$ injection, shown in Figure 6.8, there is suppression of shock noise due to screech amplification, but most notably due to the drastic reduction in fine-scale mixing noise. It appears that the fine-scale mixing noise shifted to lower frequencies indicating a shift in turbulent length scales, possibly due to the large-scale jet motion from the feedback instability or changes in the initial shear layer thickness. Strong screech instabilities are known to affect the mixing noise components as shown by non-intrusive screech suppression as shown by Norum[99] and Cuppoletti[96]. Nearly identical changes in the narrowband spectra are seen in Figure 6.9 for $NPR = 2.5$ with $\theta_{inj} = 30^\circ$ injection at $p_{o,i} = 75$ psig. This indicates that the same mechanism of noise reduction can be achieved with different combinations of injection angle and pressure. The two NPR clearly have different mechanisms of action governing the noise benefits. Flow visualization and measurement with high-speed shadowgraph and PIV assist in understanding the flow physics.

### 6.3.2 Flowfield Measurements for Internal Injection

To investigate the physical mechanisms behind the optimum noise reduction, streamwise PIV measurements were conducted on the jet at $\theta_{inj} = 60^\circ$ and $30^\circ$ at $x/D_j = -0.77$ to quantify changes in the shock structure of the jet and any discernible changes in the velocity field. As discussed previously, the actual pressure ratios for the optimum conditions of $p_{o,i} = 55$ psig at $NPR = 2.5$ is $p_{o,i}/p_a = 3.0$ and of $p_{o,i} = 110$ psig at $NPR = 4.0$ is $p_{o,i}/p_a = 5.2$. The effective pressure ratio across the injector is different depending on the injector location. Figure 6.10 shows mean axial velocity and turbulence for $\theta_{inj} = 30^\circ$ injection at $p_{o,i}/p_a = 5.2$ for four NPR. These contours can be compared directly with the baseline biconical jet contours in section 3.4. Mean velocity indicates strong shocks by variation in the velocity magnitude. For $NPR = 4.5, 3.0$, and $2.5$ there are some effects on the global shock structure but overall the shock strength remains unchanged. A localized turbulence region generated from the internal injection is seen being ejected from the nozzle exit,
Figure 6.5: Mapping injection pressure effect on narrowband at $\psi = 35^\circ$ for $60^\circ$ injection at $x/D_j = -0.77$ for all observer angles.

Figure 6.6: Narrowband spectra for NPR = 4.0 with injection at $p_{o,i} = 110$ psig, $\theta_{inj} = 60^\circ$ at $x/D_j = -0.77$. 
Figure 6.7: Narrowband spectra for $NPR = 4.0$ with injection at $p_{o,i} = 110$ psig, $\theta_{inj} = 30^\circ$ at $x/D_j = -0.77$.

Figure 6.8: Narrowband spectra for $NPR = 2.5$ with injection at $p_{o,i} = 55$ psig, $\theta_{inj} = 60^\circ$ at $x/D_j = -0.77$.

Figure 6.9: Narrowband spectra for $NPR = 2.5$ with injection at $p_{o,i} = 75$ psig, $\theta_{inj} = 30^\circ$ at $x/D_j = -0.77$. 

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although this has no effect on the acoustics, with the exception of $NPR = 2.5$ which shows a slight increase in the high frequencies. This indicates that coherent streamwise vorticity is not aiding in mixing the jet flow. For $NPR = 4.0$ the shock structure appears to be completely disrupted with an additional shock structure directly in between the original shock system. This additional shock generation appears to interact destructively with the original shocks creating a nearly uniform velocity in the jet. This shock weakening and cancellation is responsible for suppression of BBSN. Figure 6.11 shows contours for $\theta_{inj} = 30^\circ$ injection at $p_{o,i} = 5.2$. For $NPR = 4.5, 4.0, \text{ and } 3.0$ the global shock structure remains intact, corresponding to conditions that have minor acoustic differences. There is less axial variation in velocity through the shock cells for $NPR = 2.5$ and a blurring of the measurement which is indicative of a screeching jet causing large scale shock motion, in agreement with the acoustic results. This is not the optimum injection pressure for $NPR = 2.5$, but by re-examining Figure 6.4d there is still up to -4 dB reduction at $\psi = 35^\circ$ for this injection pressure. Minimal changes in the near-nozzle TKE magnitudes in comparison with the baseline jet indicate that the large reduction in fine-scale mixing noise is most likely attributed to the screech amplification, however further investigation is warranted to fully understand this phenomenon.

The $NPR = 4.0, \theta_{inj} = 60^\circ$ injection was also studied with LES computations. Various injection pressure ratios $p_{o,i}/p_a$ were studied to verify the experimental findings and provide information internal to the nozzle, which is not easily measured experimentally. Figure 6.12 presents instantaneous axial velocity contours for the baseline biconical jet at $NPR = 4.0$ and five $p_{o,i}/p_a$ values for $\theta_{inj} = 60^\circ$. At $p_{o,i}/p_a = 5.2$ the additional shock system is apparent in the jet potential core in agreement with the experiments. Increasing $p_{o,i}/p_a = 5.2$ forces the throat shock upstream, changing the shock angle and subsequently shifting the shock system generated from the throat relative to the shock system generated from the nozzle lip. At $p_{o,i}/p_a = 5.2$ an additional oblique shock is generated downstream of the injector and is spaced approximately equidistant between the internal shock cell as shown in Figure 6.13. This additional shock generation is responsible for weakening the initial shocks and provides a more uniform velocity field. At higher injection pressure the injectors back-pressure the throat shocks, generating a bow shock that does not reflect throughout the jet potential core. The shock generated downstream of the injectors is still present and resembles the original throat shocks, only shifted downstream. This is the reason for the acoustics recovering to
the original levels at higher injection pressures as a dual shock system is re-established. This method
of noise reduction by shock interference has not been reported before and has potential for various
applications in supersonic flows.

6.4 Acoustics Survey on Internal Injector Designs

The findings of the initial internal injection studies prompted a study of other injection locations,
angles, and configurations. The design space is very large, but from the initial findings we decided
on further configurations. The faceted nozzle designs presented in chapter 3.8 facilitated the use of
stereo lithography to “3D print” the nozzles. Stereo lithography requires a resin bath for fabrication
of the nozzles which has a fixed volume for fabricating the parts. The cost per nozzle was reduced
significantly by utilizing the entire fabrication volume to fabricate multiple nozzles. Ten nozzles were
fabricated including the two baseline biconical and splined nozzles. The remaining eight injector
nozzles were four biconical and four splined nozzles of identical injector designs. A photo of the
rapid prototyped nozzles is shown in Figure 6.14 with and without injector holes filled and sanded.
The injector nozzle designs parameters are summarized in Table 6.1 and illustrated in Figures 6.15
through 6.18. Nozzle A has $\theta_{inj} = 60^\circ$ and $\theta_{inj} = 90^\circ$ injectors at three axial locations. It was
of interest to study $\theta_{inj} = 90^\circ$ injection at the $x/D_j = -0.77$ location to test if the same acoustic
benefit can be achieved with lower injection mass flow. Nozzle B has $60^\circ$ injectors placed nearer to
the nozzle throat and injectors angled upstream of the flow direction $\theta_{inj} = 120^\circ$. Nozzle C tests
the effect of swirl with injectors that are aligned at $\theta_{inj} = 90^\circ$ with a yaw angle of $\phi_{inj} = \pm 30^\circ$.
Nozzle D is designed with pairs of injectors in tandem with $\theta_{inj} = 90^\circ$. Gutmark et al.[121] have
shown that the penetration of a jet in crossflow is greatly enhanced for the rear jet in a tandem jet
configuration. It was hypothesized that this configuration could enhance the mixing noise reduction
by increasing streamwise vorticity, in particular for the tandem injector pair directly inside the nozzle
lip. Although the injection nozzles are faceted, it was already shown that the facets have negligible
effect on the far-field acoustics.

The optimum conditions for the various nozzle designs are shown to illustrate the best acoustic
results. The shock weakening results for the biconical nozzle at the design condition were achieved
Figure 6.10: Internal injection for $\theta_{inj} = 60^\circ$ at $x/D_j = -0.77$ with $p_{o,i}/p_a = 5.2$ for four NPR. Mean axial velocity (left) and turbulence intensity (right).
Figure 6.11: Internal injection for $\theta_{\text{inj}} = 30^\circ$ at $x/D_j = -0.77$ with $p_{\text{o,i}}/p_a = 5.2$ for four NPR. Mean axial velocity (left) and turbulence intensity (right).
Figure 6.12: Effect of injection pressure for 60° injection at $x/D_j = -0.7$. Instantaneous contours of axial velocity.
Figure 6.13: Internal shocks for the jet at $NPR = 4.0$ with and without internal injection. Optimum injection results in a third shock behind the injector that disrupts the throat and lip shocks.

Figure 6.14: Photo of rapid prototyped nozzle (Nozzle A) and injection holes filled with drywall compound.

<table>
<thead>
<tr>
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<th>Table 6.1: Second generation internal injection designs.</th>
<th>Nozzle B</th>
<th>Nozzle C</th>
<th>Nozzle D</th>
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<tbody>
<tr>
<td></td>
<td>Nozzle A</td>
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<td>A2</td>
<td>A3</td>
</tr>
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<td>-0.39</td>
<td>-0.77</td>
<td>-0.06</td>
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<td>$60^\circ$</td>
<td>$60^\circ$</td>
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<td>$\phi_{inj}$</td>
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Figure 6.15: Injector nozzle with $\theta_{inj} = 60^\circ$ and $90^\circ$ (Nozzle A).

Figure 6.16: Injector nozzle with injection near throat $x/D_j = -0.9$ and upstream injection $\theta_{inj} = 120^\circ$ (Nozzle B).
Figure 6.17: Injector nozzle with yaw (swirling injection (Nozzle C)).

Figure 6.18: Injector nozzle with tandem $\theta_{inj} = 90^\circ$ injectors (Nozzle D).
through interaction of the internal shocks, lip shocks, and shocks generated behind the injector in between the other shocks. It was desired to see if similar shock noise reduction could be achieved with $\theta_{inj} = 90^\circ$ at the same injection location and for the splined nozzle which has weaker internal shocks. Figure 6.19 shows $\Delta$OASPL for the optimum conditions for the biconical and splined nozzle at the design $NPR$ with $\theta_{inj} = 90^\circ$ at $x/D_j = -0.77$ (Nozzle A6). The results for the biconical nozzle are repeatable showing up to -4 dB at $\psi = 35^\circ$ and minimal effect on the large-scale mixing noise at $\psi = 150^\circ$. The splined nozzle shows up to -8 dB reduction at $psi = 35^\circ$ and -2.5 dB at $\psi = 150^\circ$. This is mostly attributed to screech suppression, but examination of the narrowband spectra in Figures 6.20 and 6.21 show that the splined nozzle has complete suppression of BBSN at $\psi = 90^\circ$. This injection condition is even more effective at shock noise elimination on the splined nozzle which was shown to have better thrust performance.

Results from the initial study indicated no acoustic benefit for injection at $x/D_j = -0.39$, although there was indication that moving the injectors slightly closer to the throat to $x/D_j = -0.9$ might achieve shock weakening with less injection pressure. Figure 6.22 shows $\Delta$OASPL for the optimum conditions for the biconical and splined nozzle at the design $NPR$ with $\theta_{inj} = 60^\circ$ at $x/D_j = -0.9$ (Nozzle B1). For the biconical nozzle the acoustic benefit is completely lost with this small injection location change. For the splined nozzle some of the benefit is retained, although it is less effective with only -6 dB reduction at $\psi = 35^\circ$ and -2 dB at $\psi = 150^\circ$. The narrowband spectra in Figures 6.23 and 6.24 indicate the same behavior in shock noise reduction as the previous cases with minimal effect on large-scale mixing noise. This indicates that the additional shock generated behind the injector is not in an optimal location to weaken the throat and lip shocks. A mathematical description of this behavior has not been developed yet; however, a qualitative description is offered. We will discuss the shock formation behind the injector as the ‘induced’ shock. The location of the induced shock should be precisely half the distance from the throat shock and lip shock in order to minimize the velocity variations in between shock waves. The throat shock wave generates a pressure loss and reduces velocity, after which the flow expands to the nozzle exit where a shock exists at the nozzle lip for the overexpanded and design $NPR = 4.0$ or 3.83. When the induced shock is located precisely half way between the throat shock and lip shock, then the flow velocity is decreased again in the divergent section of the nozzle, resulting in less variation in velocity and reduced shock strength.
throughout the jet. This is identical to the approach taken for reducing total pressure loss for a supersonic inlet by using multiple shallow ramps to induce weak oblique shocks instead of one large angle ramp. Further theory development and measurements may be needed to fully understand the phenomenon.

There were some limitations due to nozzle lip thickness of the stainless steel nozzle used in the first section of this study. It was difficult to retroactively fabricate injectors near the nozzle lip. The rapid prototyped nozzles allowed us to design injectors directly inside the nozzle lip at \( x/D_j = -0.06 \). Figure 6.25 shows \( \Delta OASPL \) for \( \theta_{inj} = 90^\circ \) at this location (Nozzle A4). At a lower intermediate pressure than the other cases, \( p_{o,i} = 55 \) psig, large reductions in noise at the upstream and downstream angles is observed. The biconical and splined nozzle show suppression of -5 dB and -7 dB at \( \psi = 35^\circ \) along with up to -6 dB reduction at \( \psi = 150^\circ \). Inspection of the narrowband spectra shows a complete suppression of the broad frequency range associated with BBSN on the biconical nozzle. Detailed changes in the acoustics are shown in the narrowband spectra shown in Figures 6.26 and 6.27. The splined nozzle shows strong suppression of shock noise at \( \psi = 90^\circ \) with and significant weakening of shock noise at \( \psi = 35^\circ \). For both conditions there is considerable decrease in the large-scale mixing noise with up to -6 dB reduction at the peak frequency and a few decibel reduction at higher frequencies. This injection condition not only was the most successful at reducing shock noise, but also generated strong mixing in the jet to reduce large-scale mixing noise. Flow measurements were not conducted for this condition, but it is highly likely that strong vorticity is generated in the jet flow. The internal injection cases further upstream suppress streamwise vorticity because of the strong pressure gradients in the nozzle and the flow is confined by the nozzle walls. It is seen that the shock noise is increased with further injection pressure at this condition similar to the results of the external injection case. The strong shock noise suppression could be due to area control or interaction of the injectors with the lip shocks, but detailed measurements of the flowfield for this condition are necessary to conclusively unravel the physical mechanisms for this injection condition. This injection condition represents the best acoustic results achieved throughout all of the studies with large noise reduction with minimal injection pressure and mass flow.

Results for some of the remaining nozzles are presented in Figure 6.28 for swirling injection (Nozzle C1), Figure 6.29 for tandem injection (Nozzle D1), and Figure 6.30 for \( \theta_{inj} = 120^\circ \) injection.
(Nozzle B2). The other injection cases either did not exhibit new findings or could not be tested as some of the first nozzles failed during testing. The nozzles that failed separated near the attachment point due to the strength of the stereo lithography, but the issue was mitigated by supporting the thinnest part of the nozzle with a layer of epoxy resin. The swirling injection with $\theta_{inj} = 90^\circ$ and $\phi_{inj} = +30^\circ$ at $x/D_j = -0.06$ had identical results to the injection case without swirl. The biconical nozzle is shown for tandem injection showing up to -7 dB reduction in large-scale mixing noise for $NPR = 4.0$ and up to 2 dB in shock noise. Tandem injection was effective on the overexpanded condition $NPR = 2.5$ with up to -4 dB reduction at $\psi = 35$ and 150$^\circ$, although the increase at sideline angles is attributed to strong high frequency noise increase due to the injectors not being fully immersed in the overexpanded jet flow. This high frequency increase could be minimized by injecting slightly further upstream where the injectors could be further immersed in the jet flow. However, detailed flow measurements are necessary to verify this. Increasing the injection angle to $\theta_{inj} = 120^\circ$ at $x/D_j = -0.06$ shows essentially the same behavior as injection at $\theta_{inj} = 90^\circ$. Although jet in crossflow studies have shown greater penetration and stronger vorticity for injecting beyond perpendicular to a flow, those results do not result in greater noise reduction for the range of conditions studied.
Figure 6.19: Internal injection ΔOASPL for θ_{inj} = 90° at \( x/D_j = -0.77 \).

Figure 6.20: Narrowband spectra for biconical nozzle, \( NPR = 4.0 \) with θ_{inj} = 90° at \( x/D_j = -0.77 \).

Figure 6.21: Narrowband spectra for splined nozzle, \( NPR = 3.83 \) with θ_{inj} = 90° at \( x/D_j = -0.77 \).
Figure 6.22: Internal injection ΔOASPL for $\theta_{inj} = 60^\circ$ at $x/D_j = -0.9$.

Figure 6.23: Narrowband spectra for biconical nozzle, $NPR = 4.0$ with $\theta_{inj} = 60^\circ$ at $x/D_j = -0.9$.

Figure 6.24: Narrowband spectra for splined nozzle, $NPR = 3.83$ with $\theta_{inj} = 60^\circ$ at $x/D_j = -0.9$. 
Figure 6.25: Internal injection $\Delta$OASPL for $\theta_{\text{inj}} = 90^\circ$ at $x/D_j = -0.06$.

Figure 6.26: Narrowband spectra for biconical nozzle, $NPR = 4.0$ with $\theta_{\text{inj}} = 90^\circ$ at $x/D_j = -0.06$.

Figure 6.27: Narrowband spectra for splined nozzle, $NPR = 3.83$ with $\theta_{\text{inj}} = 90^\circ$ at $x/D_j = -0.06$. 
Figure 6.28: Swirling internal injection $\Delta$OASPL for $\theta_{inj} = 90^\circ$ and $\phi_{inj} = +30^\circ$ at $x/D_j = -0.06$.

Figure 6.29: Tandem internal injection $\Delta$OASPL with $\theta_{inj} = 90^\circ$ at $x/D_j = -0.06$.

Figure 6.30: Upstream internal injection $\Delta$OASPL with $\theta_{inj} = 120^\circ$ at $x/D_j = -0.1$. 
6.5 Summary

A novel use of fluidic injection was studied and unique acoustic results were achieved and analyzed. Through a combination of various injector designs and conditions it was shown that shock noise could be completely suppressed using internal injectors for deconstructive interference with the existing shocks in the jet. The physical mechanisms uncovered in this study have not been reported in literature for fluidic injection, which has mainly been focused on the effect of streamwise vorticity on jet mixing or area control. It was demonstrated that the injection location, angle, and pressure or momentum flux ratio are all critical parameters in achieving noise reduction. Optimum injection pressures were identified for the various internal injection locations by finely sweeping the injection pressure and identifying the conditions of optimal noise reduction. Injection at $\theta_{inj} = 60^\circ$ at $x/D_j = -0.77$ resulted in shock noise suppression with minimal effect on large-scale mixing noise. Shock noise reduction is achieved through generation of a shock behind the injectors which disrupts the existing shock train. This condition does not generate enough streamwise vorticity to affect mixing noise. Internal injection directly inside the nozzle lip demonstrated complete suppression of shock noise in combination with strong reduction in large-scale turbulent mixing noise due to streamwise vorticity generation. This demonstrates that the shock disruption mechanism can be combined with vorticity generation methods to achieve significant noise reduction in shock and mixing noise. These new findings require much further development of theory and investigation to fully uncover all of the physics involved in these acoustic results. However, these results represent some of the most significant reductions in supersonic jet noise with fluidic injection.
Chapter 7

Near-Field Pressure Measurements

This section presents near-field pressure measurements which are very useful in understanding source locations, noise directionality, and noise footprint. The data is presented in this chapter separately due to the large amount of data that was generated and analyzed over the measurement domain. The purpose of this chapter is to present the near-field pressure data separately and compare the various noise reduction methods and nozzle designs. The near-field measurement technique is described in detail in section 2.3.2. At each measurement location, narrowband spectra are calculated and the near-field pressure maps are presented at single frequencies or as OASPL. The directionality of the noise components can clearly be seen where the highest sound levels persist and lobes are present in the contours.

This section analyzes the baseline (biconical) nozzle, splined nozzle, trailing-edge injection, and 90° external injection. The jet flowfield is superimposed on the near-field pressure results for the baseline jet in Figure 7.1. This gives the reader an idea of the source locations relative to the jet flowfield. The turbulent mixing noise at 1 kHz is shown with turbulence intensity and the shock noise is shown with mean velocity, which are illustrative of the turbulence and shock noise sources, respectively. The turbulent mixing noise shows high SPL along the shear layer with a directivity indicated by the lobe formed by the contours very near the $\psi = 170^\circ$ angle as measured from the nozzle exit. The shock noise shows directivity, indicated by the dashed lines, towards the $\psi = 90^\circ$ and $\psi = 150^\circ$ observer locations. It becomes clear that the source locations for the different noise components can be located at different locations and/or distributed along the jet. The shock noise
Figure 7.1: Directionality of 1 kHz mixing noise and BBSN compared to jet flowfield.

appears to originate between $x/D_j = 10$ and 15 directed radially outwards from the jet while the large-scale turbulent mixing noise is generated along the entire jet shear layer with peak noise directed downstream. The near-field measurements illustrate how the noise reduction methods alter the noise directivity and noise footprint.

### 7.1 Near-Field Pressure Contours

Contours of OASPL are shown in Figure 7.2 for the biconical (sharp) and splined nozzles at all operating conditions. The contours are all on the same scale to emphasize the increase in SPL with increasing nozzle pressure ratio. For example, at the underexpanded $NPR$ it is seen that the 140 dB contour extends to the boundaries of the measurement domain, only covers half of the domain for $NPR = 3.5$, and barely extends more than $r/D_j = 3$ for $NPR = 2.5$. Minimal differences in the amplitudes and directivity for the sharp and splined nozzles are seen, reflecting the far-field acoustics results in section 3.5. There are some differences in the noise footprint for the conditions with strong screech ($NPR = 3.83, 3.5$, and $3.0$). The splined nozzle exhibits a wave pattern along the innermost row of the pressure map which usually indicates strong screech as the acoustic feedback mechanism creates a standing wave along the jet shear layer. The splined nozzle also has stronger lobed structures from the increased screech. Some of the repetitive artifacts in the near-field pressure are attributed to variation in the frequency response for the various microphones. Although careful calibration was conducted, these artifacts could be due to microphone misalignment and/or dynamic response. The data was not smoothed to avoid suppressing some of the discontinuous features that appear with
screech and BBSN. It should also be noted that the near-field pressure measurement contains both hydrodynamic and acoustic data as it is not considered to be in the acoustic far-field. Regardless, the near-field contours provide high quality data within the experimental uncertainty analysis and very helpful visualizations of how the noise reduction methods modify the noise footprint.

To maintain a concise narrative and not overwhelm the reader with figures, the near-field pressure contours presented are for the nozzle design condition and for a strongly screeching condition \((NPR = 3.0)\). Comparisons are made between the biconical nozzle, splined nozzle, trailing-edge injection \(\theta_{inj} = 11^\circ\) at \(p_{o,i}/p_a = 5.2\), and biconical nozzle with \(\theta_{inj} = 90^\circ\) operated with \(p_{o,i}/p_a = 3.75\). Figures 7.3 and 7.4 compare OASPL for these cases at design \(NPR\) and \(NPR = 3.0\). Trailing-edge injection at \(\theta_{inj} = 11^\circ\) provides about 2 dB reduction in the downstream region and reduces OASPL along the shear layer. Injection at \(\theta_{inj} = 90^\circ\) provides up to 5 dB reduction downstream and shifts the high OASPL region upstream with a stronger sideline directivity from the fine-scale turbulence generated near the nozzle. For \(NPR = 3.0\) both injection methods suppress the high OASPL due to screech and the strong sideline lobes. Injection at \(\theta_{inj} = 90^\circ\) again reduces OASPL further downstream and has a more compact noise footprint due to more rapid mixing of the jet and shortening of the potential core. This is a trend seen in all of the near-field measurements. The large scale mixing noise \(f = 1\ kHz\) figures are shown in Figures 7.5 and 7.6 and similar conclusions can be made as the OASPL contours. At design, the 125 dB contour is reduced from \(x/D_j = 25\) to \(15\) for \(\theta_{inj} = 90^\circ\) and SPL in the peak radiation direction is reduced by 2-3 dB. A similar result is observed at \(NPR = 3.0\), although the presence of strong screech can affect the mixing noise components, and therefore screech suppression can increase some of the mixing noise levels. This manifests as less noise reduction in the downstream angles.

Contours of the shock noise components and fine-scale mixing noise show more drastic effects. Figures 7.7 and 7.8 show contours at the fundamental screech frequency for each condition. Strong acoustic radiation occurs in the upstream and downstream directions and a standing wave pattern appears above the shock locations. Fluidic injection is successful at decreasing screech SPL, especially in the upstream direction. The noise footprint at the screech frequency is drastically reduced resulting in 10-20 dB reduction at the sideline and upstream angles. Of course this depends on the initial screech intensity, as the higher the screech amplitude the more noise reduction can be achieved. Once
Figure 7.2: Near-field OASPL for sharp and splined nozzles at all setpoints. Top two rows are $NPR_{sharp} = 4.5$ and 4.0, $NPR_{splined} = 4.27$ and 3.83. Bottom three rows are $NPR = 3.5$, 3.0, and 2.5.
the screech is completely suppressed then no additional reduction can be achieved at this frequency, and results have shown that this can be achieved with very little injection flow. The mechanisms and physics of screeching jets is still a point of contention in the research community. If screech is a phase match between the shock motions and large-scale coherent structures in the shear layer then there are a few theories that could explain screech suppression. The spreading of the jet shear layer changes the characteristic length scale of the turbulent structures and introduces finer turbulence structures which could disrupt the development of the large-scale coherent structures. Another hypothesis is that the modification of the shock structure disrupts the symmetry that supports unstable modes in the jet, decoupling the shock-turbulence feedback. Further measurements of the shear layer instability frequencies are necessary to test these hypotheses further.

Contours for the peak shock noise frequency are shown in Figures 7.9 and 7.10 and it is evident that the peak radiation direction is towards the sideline angle. At the design condition, fluidic injection is very effective at suppressing the sideline lobe up to 15 dB. Injection at $\theta_{inj} = 90^\circ$, $p_{o,i}/p_a = 3.75$ is very near the optimum shock noise reduction from Section 4.2 which explains why the shock noise is nearly completely eliminated. For both the design condition and $NPR = 3.0$ with injection, the peak noise direction for the sideline lobe shifts upstream angles upstream. This occurs from the shortening of the potential core and reduction in shock cell spacing. It should also be noted that the peak frequency may have shifted with injection, but the comparisons are at the same frequency. It is evident that fluidic injection is very successful at suppressing the shock noise components in the jet near-field, which has strong implications for personnel and structures near the aircraft. It is of interest to investigate the high frequencies as well which can increase with fluidic injection. Figures 7.11 and 7.12 show the $f = 50$ kHz contours and there is clearly an increase with fluidic injection. The higher injection angle $\theta_{inj} = 90^\circ$ has a higher penalty although it is 5 dB near the nozzle but only 3 dB at $r/D_j = 9$. These increases are significantly less and have smaller footprints than the large reductions in shock noise. The effect also diminishes with distance since atmospheric attenuation is inversely proportional to frequency. The benefits of injection outweigh the penalties in the near-field if the injection is tailored for shock noise reduction.
Figure 7.3: Near-field OASPL for $NPR_{\text{sharp}} = 4.0$, $NPR_{\text{splined}} = 3.83$.

Figure 7.4: Near-field OASPL for $NPR = 3.0$. 
Figure 7.5: Near-field $f = 1$ kHz for $NPR_{\text{sharp}} = 4.0$, $NPR_{\text{splined}} = 3.83$.

Figure 7.6: Near-field $f = 1$ kHz for $NPR = 3.0$. 238
Figure 7.7: Near-field screech for $NPR_{\text{sharp}} = 4.0$, $NPR_{\text{splined}} = 3.83$.

Figure 7.8: Near-field screech for $NPR = 3.0$. 

(a) Baseline

(b) Splined

(c) $\theta_{\text{inj}} = 11^\circ$, $p_{o,i}/p_a = 5.2$

(d) $\theta_{\text{inj}} = 90^\circ$, $p_{o,i}/p_a = 3.75$
Figure 7.9: Near-field shock noise for $NPR_{\text{sharp}} = 4.0$, $NPR_{\text{splined}} = 3.83$.

Figure 7.10: Near-field shock noise for $NPR = 3.0$. 

\( \theta_{\text{inj}} = 11^\circ, \frac{p_{o,i}}{p_a} = 5.2 \) 

\( \theta_{\text{inj}} = 90^\circ, \frac{p_{o,i}}{p_a} = 3.75 \)
Figure 7.11: Near-field $f = 50$ kHz for $NPR_{\text{sharp}} = 4.0$, $NPR_{\text{splined}} = 3.83$.

Figure 7.12: Near-field $f = 50$ kHz for $NPR = 3.0$. 
7.2 Near-Field Pressure Profiles

While contour plots provide directivity of certain frequencies and semi-quantitative representations of the near-field pressure distribution, profiles provide a more quantitative method of comparing the changes in noise components with fluidic injection technologies. Profiles are presented for the innermost row of microphones termed ‘r0’ and the outermost row of microphones termed ‘r9’. The number does not indicate a constant $r/D_j$ value since the microphone array was parallel to the jet shear layer at $10^\circ$. Figure 7.13 shows that fluidic injection increases OASPL in the first $3D_j$ while decreasing it everywhere else. The high frequency increase is negligible in OASPL further from the jet at r9. Note that at r9, OASPL is relatively constant as the profile is relatively parallel to the contour levels. Profiles at $f = 1$ kHz, in Figure 7.14, show SPL growth along the jet with a shift in the peak location with injection due to increased mixing and potential core decrease. At location r9, up to 5 dB reduction is observed and peak values are located downstream. Profiles at the screech frequency, in Figure 7.15, show SPL magnitudes up to 150 dB for the biconical nozzle and 160 dB for the splined nozzle. Significant reductions of SPL, up to 20 dB, are achieved near the jet with injection. The trailing edge injection nozzle is just as efficient as $\theta_{inj} = 90^\circ$ indicating that disrupting the feedback mechanism can be achieved with a wide range of injection configurations. At the peak shock noise frequency, in Figure 7.16, it is seen that along the jet shear layer SPL reaches a peak between $x/D_j = 5$ and 10 and with $\theta_{inj} = 90^\circ$ the peak is shifted upstream and suppressed by 5 dB downstream. At location r9 the peak radiation direction is apparent from the lobe for the biconical and splined nozzle. This lobe is strongly suppressed with injection and the shift in propagation direction is clearly seen with injection. Fine-scale mixing noise, $f = 50$ kHz, in Figure 7.17, shows a 5 dB increase up to $x/D_j = 5$ with a 5 dB decrease downstream for the design condition. For $NPR = 3.0$ the effect is less prevalent due to the effect of screech on the mixing noise components.

7.3 Summary

Near-field pressure results showed strong directivity of the supersonic jet noise components and changes in the near-field pressure distribution with fluidic injection. Comparison of the baseline nozzle, splined nozzle, trailing-edge injection, and $\theta_{inj} = 90^\circ$ were presented for the supersonic noise
Figure 7.13: Near-field profiles along r0 and r9 for OASPL.
Figure 7.14: Near-field profiles along r0 and r9 for $f = 1 \text{ kHz}$. Legend in previous figure.
Figure 7.15: Near-field profiles along r0 and r9 for screech.
Figure 7.16: Near-field profiles along r0 and r9 for shock noise. Legend in previous figure
Figure 7.17: Near-field profiles along r0 and r9 for f = 50 kHz. Legend in previous figure.
components. Minimal differences were observed between the biconical and splined nozzles, with the exception of screech intensity, in agreement with far-field acoustics. External fluidic injection at $\theta_{inj} = 90^\circ$ provides greater noise reduction and strong suppression of screech, shock noise, and turbulent mixing noise in the near-field. Fluidic injection decreased the peak noise levels for large-scale mixing noise, shock noise, and screech. Increases in fine-scale mixing noise were observed near the nozzle exit, with peak directivity towards $\psi = 90^\circ$, and decreased downstream beyond $x/D_j = 10$. Fluidic injection suppressed screech noise by -20 dB in the near nozzle region while up to -10 dB reduction for shock noise was achieved. The reductions in near-field pressure agreed with reductions in the far-field and implied that drastic reduction in the sound levels near an aircraft can be achieved, benefiting the health and safety of personnel that work in close proximity to supersonic jet flows.
Chapter 8

EPNL Calculation for Flight Noise Prediction

Jet noise is inherently an engineering concern due to the fact that it is physiological disturbance to humans and can cause physical damage to engineering systems. Much of the research so far has focused on understanding the physics involved in supersonic jet acoustics, reduction of the supersonic jet noise components, and assessing the practicality of noise reduction methods. The combination of tonal noise (screech), semi-broadband noise (BBSN), and broadband noise (turbulent mixing noise) complicates the evaluation of all the various noise reduction methods. It was previously shown that supersonic noise components respond differently to various noise reduction methods and it was challenging to identify the most successful technology or configuration. Ideally a noise evaluation criteria that can estimate how the changes in acoustic spectra will be received and interpreted by a human observer should be used to determine successful noise reduction. The method used by the U.S. Federal Aviation Administration (FAA) to certify commercial aircraft is known as effective perceived noise level (EPNL). The FAA standard is detailed in the FAA AC-36-4A[122]. This method evaluates the noise from an entire aircraft during take-off or approach and from a sideline (lateral) location. The EPNL measurement standard is based on decades of psycho-acoustic research studies that evaluated how humans perceive noise, and includes corrections for frequency perception, time-averaging, tone perception, and duration of the noise source[123]. Figure 8.1 illustrates a general noise history during a flyover event with superpositions of aircraft noise components including fan inlet,
fan exhaust, and jet noise. The jet noise component is not relevant until after the aircraft is directly above the observer due to the directional nature of jet noise. The EPNL measurement standard was applied to the laboratory jet noise measurements to investigate the various noise reduction methods. The EPNL calculation was not meant to be a direct computation of EPNL since turbomachinery noise and other aircraft noise sources were not included. The change in EPNL with noise reduction application, or $\Delta$EPNL, is useful for evaluating the success of a noise reduction method. By mapping the EPNL and $\Delta$EPNL values on a ground plane, it is also possible to quantify the reduction in noise footprint. This also allows the complex noise results to be presented as a single value for evaluation of overall noise including tonal and shock noise.

8.1 EPNL Measurement Standard

The EPNL measurement standard is detailed, followed by extrapolation of the measurement technique for the laboratory measurements. For clarity, the description of the EPNL measurement is summarized and complete details of the procedure can be found in the FAA standard[122]. Typical measurement of EPNL begins with recording a time history of acoustic pressure with a microphone during a prescribed flight path. For this analysis, it is of greatest interest to evaluate a take-off condition in which the engine is at or near full-power. A typical take-off profile is illustrated in Figure 8.2 in which the measurement is made at a prescribed location downstream of the runway underneath the flight path. The measurement is made over a period of time that is long enough to capture the $\pm$10 dB down period from the maximum noise level (described later). The steps in calculating EPNL are detailed below.

1. The temporal measurement is converted to 24 one-third octave bands for each $\Delta t = 0.5$ second interval during the measurement. The one-third octave bands are represented as $\text{SPL}(i,k)$ where SPL is in decibels, $i$ indexes the frequency band, and $k$ indexes the time increment.

2. The one-third octave spectra $\text{SPL}(i,k)$ are converted to perceived noisiness $n(i,k)$ with units of [noys]. This is derived from the equal loudness contours (Figure 8.3) which weight the frequency spectrum for human frequency sensitivity. Perceived noisiness is calculated from a table and formula[122] which weight the sound pressure level based on frequency band and
amplitude.

3. Total perceived noisiness $N(k)$ is computed from the perceived noisiness spectra as

$$N(k) = 0.85n(k) + 0.15\sum_{i=1}^{24}n(i,k)$$

where $n(k)$ is the maximum of the 24 values of $n(i,k)$. This can be considered a frequency-weighted equivalent of OASPL with units of [noys].

4. Perceived noise level $PNL(k)$ is computed from $N(k)$ as $PNL(k) = 40.0 + \frac{10}{\log 2} \log N(k)$ with units of [PNdB].

5. The tone-corrected perceived noise level $PNLT(k) = PNL(k) + C(k)$ where $C(k)$ is a tone correction factor. $C(k)$ is determined based on how large the tone amplitude is above the smoothed one-third octave spectra. The calculation procedure is very detailed[122] and $C(k)$ is only computed for the maximum tone amplitude. $C(k)$ can range from $0 - 6\frac{2}{3}$ PNdB.

6. The final step in obtaining EPNL is determining the duration factor and adding it to the maximum $PNLT(k)$ level $EPNL=PNLTM+D$. $PNLTM$ is indicated in Figure 8.4 as the maximum $PNLT(k)$ value during the flyover event. The duration factor is determined by integrating over the 10 dB down period on either side of $PNLTM$ indicated by $d = t(2) - t(1)$. The duration factor is calculated using Equation 8.1.

$$D = 10 \log \left[ \left( \frac{1}{T} \right) \sum_{k=0}^{d/\Delta t} \Delta t \cdot 10^{\frac{PNLT(k)}{10}} \right] - PNLTM \quad (8.1)$$

### 8.2 Simulated EPNL for Laboratory Data

Application of EPNL to static laboratory data is difficult since the noise source is not in motion. During a flyover event the aircraft has a given distance and angle from the observer location. The procedure for calculating EPNL from laboratory data is detailed in Figure 8.5. For a simulated flight profile a distance and angle to any other location can be determined. The layout of the far-field measurement arc in the anechoic chamber provides directional noise measurements. Since the acoustic spectra has small variance between microphones, an interpolated spectra for any angle can be computed using the microphone location between $35^\circ$ and $150^\circ$. Sound level attenuation by atmospheric absorption is added to the measured acoustic spectra resulting in a “lossless” spectra,
Figure 8.1: Noise history during aircraft flyover from ISO 532-1967 taken from Crocker[123].

Figure 8.2: Typical take-off flight path for EPNL measurement[122].

Figure 8.4: Time history of $PNLT(k)$ indicating maximum $PNLT$ ($PNLTM$) and duration time for $h = 10$ dB down period from PNLTM ($d = t(2) - t(1)$).
following the method of Bass et al.[124]. The laboratory spectra are converted to engine frequencies which are inversely proportional to the jet diameter \(D_{\text{act}} / D_{\text{lab}} = 9\). Therefore, the actual engine frequencies are \(1/9^{th}\) lower than laboratory frequencies. Acoustic scaling laws are used to scale the measured acoustic spectra to the observer distance at each time interval in the simulated flight path, following the method detailed in Viswanathan[125], and atmospheric absorption is subtracted from the scaled spectra to obtain the final simulated acoustic spectra for the specified time interval. This is carried out for each time interval throughout the simulated flight path and iteratively computed for as many observer locations as desired. The bulk of the analysis presented here is computed for a 12 km by 2 km area surrounding a runway during a takeoff flight condition. The full code set is presented in Appendix A.

Additional assumptions and considerations were made in developing the EPNL code. For cases where the aircraft was at observation angles \(\psi < 35^\circ\) the observation angle was forced to \(\psi = 35^\circ\) and for \(\psi > 150^\circ\) the observation angle was forced to \(\psi = 150^\circ\). A noise floor limit for the third-octave band SPL was set to 40 dB to avoid atmospheric attenuation that results in SPL values out of the range required for EPNL calculation. In actuality ambient noise would be higher than 40 dB. Any near-field effects were neglected for small observer distances since only far-field acoustic data was used. The doppler shift of frequencies was also not included in the code. This could be taken into account and may change some of the frequency weightings, but it most likely would not affect the EPNL values very drastically and was not included in the EPNL calculation. Flight effects on the acoustic spectra would have a greater effect on the results but data with flight effect information was not available. All of these considerations could be incorporated into the code or accounted for with more realistic data, but the code proved very helpful to identify optimum noise reduction related to the EPNL measurement standard.

8.3 Flight Path

A take-off flight path was used for EPNL analysis as prescribed by FMV and GKN Aero. The aircraft begins at time \(t = 0\) s at \(x_g = 0\) km, \(y_g = 0\) km at rest on the runway \((z=0\) km\) and accelerates at a constant rate \((\approx3.375\) m/s\(^2\)) to 90 m/s at 1200 m down range. The aircraft lifts off
and climbs at $10^\circ$ while continuing to accelerate to 190 m/s with the same acceleration. The flight path was simulated for 150 seconds in 0.5 second increments. For every time iteration the distance to the aircraft and the angle to the aircraft was computed for every point in the ground plane. The ground plane spans from $x = -2$ km to 10 km with 1 km resolution and $y = -1$ km to 1 km with 0.1 km resolution. The ground plane size and resolution was determined after some experience with how the EPNL contours were shaped and were chosen to provide clear results without extensive computational time. For a larger simulated ground plane the total flight time must increase in order to capture the 10 dB down period for all locations on the ground. The ground plane desired depends on the purpose of the EPNL simulation. For example, aircraft certifications are usually performed for a location downstream of the runway beneath the flight path or at a sideline location. If one is interested in the EPNL values that a worker near the aircraft will be subjected to, a grid with resolution around the runway would be most informative. If mapping community noise was of concern, a much larger ground plane and longer flight time must be simulated to determine noise contours in the larger domain. The ground plane used in this study was focused near the runway region to study what factors contribute most to EPNL near the take-off flight path.

Figure 8.5: Procedure for EPNL calculation using static laboratory data.
A sample EPNL contour mapping of the ground plane is shown in Figure 8.6 along with plots of the distance from the ground location to the aircraft ($R$) and the angle between the ground location and the aircraft ($\psi$) which represents the far-field acoustic observation angle. The simulations assume the jet noise source and aircraft are a point source and the Euclidean distance and angle were computed. Three ground locations are shown along the flight path, 2 km behind, 4 km down range, and 10 km down range of the aircraft start location. The acceleration of the aircraft is apparent in the nonlinear portion of the distance during approach. The time at which the aircraft is nearest the ground observer increases with downstream distance, and at the observation angle when the nearest distance is very near $\psi = 90^\circ$. Behind the aircraft the observation angle is between $\psi = 175^\circ$ and $180^\circ$ while for locations down range $0^\circ \leq \psi \leq 180^\circ$. For observation angles below $35^\circ$ or above $150^\circ$ the acoustic spectra was forced to the nearest spectra. It is clear for the 10 km distance figure that longer simulation time is needed to ensure the 10 dB down period is captured for the furthest points from take-off. The distance and observer angles away from the ground plane centerline are similar with slower rate of change in observation angle and distance.

### 8.4 EPNL Results

Contours of EPNL are shown in Figure 8.7 for the biconical nozzle at four $NPR$ with identical contour ranges. Near the aircraft start location EPNL is in excess of 125 EPNdB while the majority of the ground location is dominated by 85 to 95 EPNdB. The expanse of the 95 EPNdB contour increases with increasing jet Mach number as expected. The purpose of these figures is for comparison of the effect of jet Mach number on EPNL, but also to illustrate that it is difficult to visually discern the scope of the contours. EPNL contours for 6 external fluidic injectors at $\theta_{inj} = 90^\circ$ and $J = 0.7$, 0.95, 1.13, and 1.35 are shown in Figure 8.8. Again, when these contours are compared with Figure 8.7a it is difficult to determine how much reduction in EPNL is actually achieved. It is much more valuable to analyze the EPNL ground contours with other methods.

Figure 8.9 plots $\Delta$EPNL at the point of maximum EPNL in the ground contour for external injection with 6 and 12 injectors on $NPR = 4.0$. The $\Delta$EPNL$_{max}$ represents the change in EPNL for the point of maximum EPNL for the baseline case, which is at the start location of the
aircraft. For all injection angles with 6 injectors the maximum ∆EPNL is -2.5 EPNdB with the optimum corresponding to the best shock noise reduction case. For 12 injectors ∆EPNL is worse for all cases with the exception of θ_{inj} = 30° which has the same effect as 6 injectors. These results are in line with the results for optimum shock noise reduction at ψ = 90°, indicating that shock noise reduction may be the most beneficial for EPNL reduction near the aircraft on the ground. The trends are very similar to what was observed in ΔOASPL for shock noise reduction in Section 4.2. The reduction in peak EPNL is of importance in very close proximity to the aircraft. However, it is also of interest to quantify EPNL reduction along the ground domain. Figure 8.10 presents ∆EPNL along the ground centerline y_{g} = 0 km and along the edge of the domain y_{g} = 1 km for external injection at θ_{inj} = 90° for NPR = 4.0 and all four momentum flux ratios. Along the ground centerline, up to -4.5 EPNdB is achieved behind the jet start location with -3 to -4 EPNdB reduction along the ground plane with the optimum injection momentum flux being J = 0.95 in agreement with the best shock noise reduction cases. Along the edge of the ground domain a more constant value of -4.5 EPNdB reduction is achieved as the sideline seems to also be dominated by shock noise reduction. These results vary for larger simulated domains and different flight paths. Since EPNL chooses the peak value of PNLT and adds penalties for duration, it is clear that the simulated EPNL measurement is dominated by the observation angle when the jet is nearest to the observer. This usually happens to be a sideline angle which is dominated by shock noise. If a larger domain was mapped, for example, a neighborhood surrounding an airport, ∆EPNL would mostly depend on the relative location with respect to the flight path.

Another benefit of ground mapping EPNL is that reduction in the area of a constant EPNdB contour level can be computed. This type of value is often used for noise mapping near airports since the expanse of noise pollution to surrounding communities is of concern. Figure 8.11 shows change in area ΔArea/Area for three EPNL values with external injection at θ_{inj} = 90° for NPR = 4.0. The Area is the computed ground area for a constant EPNdB value and ΔArea is the change in the original area with noise reduction applied. The choice of a suitable contour level is important in illustrating noise reduction because a value that is too high or too low on the range will indicate no change in area. For NPR = 4.0, EPNL = 95 EPNdB is a suitable choice, although 105 EPNdB is very similar. External fluidic injection at θ_{inj} = 90° provides up to 50% reduction in 95 EPNdB
contour area with 6 injectors at \( J = 0.7 \) and 0.95. Reduction of up to 40\% is seen with 12 injectors at \( J = 0.7 \) and at higher momentum flux ratios there is an increase in EPNdB contour area due to shock strengthening and increases in fine-scale mixing noise. Figure 8.12 presents the change in the 95 EPNdB contour area for \( \theta_{\text{inj}} = 30^\circ \), 45\(^\circ\), and 60\(^\circ\). In agreement with the results from Section 4.2, less reduction in contour area is achieved with shallower injection angles and the optimum injection momentum flux shifts to a higher value corresponding to shock noise reduction.

### 8.5 Summary

A code was developed for computation of EPNL based on FAA AC-36-4A using a simulated flight path and static laboratory jet noise measurements. The code was structured in a manner that any time history of pressure and flight path can be used as inputs. A take-off flight path was simulated and EPNL was computed on a ground plane to evaluate the effectiveness of fluidic injection on reducing EPNL. The EPNL results were dominated by the sideline acoustic spectra for the flight path considered, since in most cases the aircraft was nearest the observer at an observation angle

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Figure 8.6: Distance and observer angles for three observer locations underneath the take-off flight path.
Figure 8.7: EPNL contours for the biconical nozzle at four \( NPR \).
Figure 8.8: EPNL contours for the biconical nozzle at $NPR = 4.0$ and 6 external injectors at $\theta_{inj} = 90^\circ$. 

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Figure 8.9: ΔEPNL for NPR = 4.0 with θ_{inj} = 90°. ΔEPNL is computed for the maximum value on the grid representative of ΔEPNL at x_g = 0 km, y_g = 0 km.

Figure 8.10: ΔEPNL along the flight path centerline y_g = 0 km and along the edge of the ground plane y_g = 1 km for the biconical nozzle at NPR = 4.0 with θ_{inj} = 90°.
Figure 8.11: Effect of contour level on EPNdB area reduction for NPR = 4.0, external injection θ_{inj} = 90°.
Figure 8.12: Effect of contour level on EPNdB area reduction.
near $\psi = 90^\circ$ as it passed directly by the observer. Fluidic injection conditions that achieved strong suppression of shock noise achieved up to -4 EPNdB along the edge of the ground plane. Peak EPNL values near the aircraft were reduced up to -2 EPNdB with injection. The optimum EPNL reduction conditions achieved up to 50% reduction in 95 EPNdB area on the ground plane. Great focus is placed on reduction of turbulent mixing noise, however, these results indicate that shock noise is more important for supersonic jets in the regions near a take-off location. This finding has implications for noise reduction in communities near supersonic jet aircraft operations.
Chapter 9

Research Contributions and Recommendations

The research included in this dissertation is only a portion of the acquired data, yet demonstrates all of the major findings. Insights into the physical mechanisms responsible for modifying the jet flow and acoustics were presented. Unprecedented noise reduction on a supersonic jet was achieved, laying the foundation for further noise reduction with future expansions of fluidic injection. The fluidic injection research not only achieved significant noise reduction, but the flow measurements and computational collaboration provide insights into many of the physical mechanisms of supersonic jet noise reduction that have gone unexplained for quite some time. The pulsed fluidic injection system also became a tool for studying the complex dynamics associated with supersonic jet noise and has potential for further insights into shock wave dynamics.

9.1 Research Contributions

Contributions of the research include:

- Characterization of the mean flow and acoustics of a biconical nozzle and splined nozzle, optimized for minimal throat shocks.

- Experimental measurement of momentum and pressure thrust and validation with LES.
• Characterization of mean flow and acoustics of a chevron and trailing-edge injection nozzle. Analysis of streamwise flow structures showed that mixing noise reduction correlated with stronger vorticity generation.

• Developed controlled experiments to study the effects of injection angle and momentum flux ratio on supersonic jet flow and acoustics.

• Demonstrated that increased injection angle and momentum flux ratio results in further large-scale mixing noise reduction, while an optimum intermediate momentum flux ratio results in suppression of shock noise.

• Demonstrated up to -8.5 dB reduction in OASPL at the peak mixing noise observation angle and up to -6 dB reduction at upstream and sideline angles.

• Provided an explanation of the physical mechanisms involved in reduction of shock noise and increase in peak shock noise frequency with fluidic injection. Comparisons with existing theory indicated that convective Mach number is reduced with fluidic injection, which also contributes to the shift in peak shock noise frequency.

• Demonstrated that the streamwise vorticity size and extent is more strongly correlated with large-scale mixing noise reduction than peak vorticity magnitude.

• Demonstrated that for pulsed fluidic injection at frequencies below the preferred jet frequency, the large-scale mixing noise scales with the actual duty cycle of the injected flow.

• Demonstrated that the shock noise components have characteristic time scales related to instability growth periods that can be interrupted with unsteady injection.

• Developed of a code to calculate EPNL for any arbitrary flight path and static laboratory test data.

9.2 Recommended Future Research

• Study higher injection momentum flux ratios and higher injection pressures. The injection pressures in this study were limited to 1,100 kPa. The reduction in turbulent mixing noise was
directly correlated with increased momentum flux and no plateau was found.

- Study the effect of momentum flux ratio at various injector diameters to reduce mass flow ratio.

- Study different fluidic injector shapes that generate different streamwise vortex shapes. Reduction in large-scale mixing noise was affected much more by the size and extent of the streamwise vortices.

- Study system and cycle analysis of the effect that fluidic injection has on cycle efficiency when extracting the required flow from various points in the cycle. Future application of fluidic injection depends on developing practical implementations that can be effectively used for noise reduction.

- Further study pulsed fluidic injection, particularly 30 Hz injection, to determine the reason for enhanced noise reduction. The use of different valves could answer the question if there is a performance issue with the valves. Pulsing at frequencies higher than 500 Hz, but lower than the preferred jet frequency, could suppress shock and screech noise with significantly less flow.

- Carry out additional research into the response time for the jet shocks, shock noise, and screech instability using valves with higher frequencies. Additionally, measurements with microphones or hot-wires around the jet could be used to more accurately determine the onset and growth of shock and screech noise. The measurements should be triggered by the injection signal to determine exactly when the valves open and close.

- Expand on the EPNL calculation by studying the effects of ground plane size (community-size), resolution on the ground plane, and different flight paths. Application of the EPNL code to subsonic jet noise data may yield very different results than those for supersonic jets.
References


Appendix A

Matlab Codes for EPNL Calculation

EPNL Calculation Code (CalculateEPNL.m)

```matlab
clc; clear all;

% Load Processed Data and Flight Path to calculate PNL, PNLT, D, & EPNL
startdir = pwd;

% Load Pressure Data
sep = filesep; startdir = pwd;
folder_name = uigetdir(startdir);
matfiles = dir(fullfile(folder_name sep '**.mat'));

% for i = 1:length(matfiles)
% names{i} = matfiles(i).name;
% end

for ii = 1:length(names)
    Pname = names{ii};
    % [Fname,Fdir] = uigetfile('*.*','Select Flight Path File');
    Fname = 'TakeOff_LargeGrid_LowRes_50s';
    Fdir = 'C:\Users\Daniel Cuppoletti\Dropbox\EPNL Calculation\Flight Paths\';
    % [Pname,Pdir] = uigetfile('*.*','Select Processed Acoustics File');
    Pdir = fullfile(Pname sep); load(fullfile(Pname));
end

load(fullfile(Fdir Fname));
load(fullfile(Pdir Pname));
clear PN PNL PNLT TOBact EPNL;
fc_pref = [50 63 80 100 125 160 200 250 315 400 500 630 800 1000 1250 ... 1600 2000 2500 3150 4000 5000 6300 8000 10000]';
Dlab=0.057531; %m
Rlab = 65*Dlab; %m

% Calculate Atmospheric Attenuation Coefficients
ps = 1; %Atm. Pressure, (atm)
T = 300; %305.5 %Atm. Temp, (K)
hr = 20; %21.3 %Rel. Humidity (%)
```
TOBalpha = AtmAttenuation(fc, pref, ps, T, hr); \% alpha in dB/ft
TOBalpha = TOBalpha * 3280.83989501; \% alpha in dB/km
\% Interpolate Spectra and Scale for each Grid Point and Time Point
anglow = zeros(size(R));
for i = 1:size(R,1)
    for j = 1:size(R,2)
        for k = 1:length(t)
            disp('i = '); disp(i);
            disp('j = '); disp(j);
            disp('k = '); disp(k);
            if theta(i, j, k) < 35
                anglow = 1;
            elseif theta(i, j, k) > 150 || isnan(theta(i, j, k)) == 1
                anglow = length(angle) + 1;
            else
                anglow = find(angle < theta(i, j, k), 1, 'last');
            end
            if anglow == 1
                TOBspectra(i, j, k, :) = TOBlossless(:, 1);
            elseif anglow == 14
                TOBspectra(i, j, k, :) = TOBlossless(:, end);
            else
                weight = (angle(anglow) - theta(i, j, k)) / ...
                    (angle(anglow) - angle(anglow - 1));
                TOBspectra(i, j, k, :) = (1 - weight) * TOBlossless(:, anglow) ... 
                    + weight * TOBlossless(:, anglow - 1);
            end
        end
    end
end
TOBatt = TOBalpha * R(i, j, k); \% TOBalpha [dB/km] and R [km]
TOBact(i, j, k, :) = squeeze(TOBspectra(i, j, k, :)) - TOBatt - ...
    10 * log10(R(i, j, k) / (Rlab / 1000));
ind = find(TOBact(i, j, k, :) < 40); \% Set minimum SPL to avoid NaN
TOBact(i, j, k, ind) = 40;
\% \% Invokes TOB2PNL.m
[PN(i, j, k, :), N(i, j, k), PNLact(i, j, k)] = ...
    TOB2PNL(squeeze(TOBact(i, j, k, :)));
\% \% Invokes ToneCorrection.m
[C(i, j, k), SPL2(i, j, k, :)] = ...
    ToneCorrection(fc, pref, squeeze(TOBact(i, j, k, :)));
PNLT(i, j, k) = PNLact(i, j, k) + C(i, j, k);
PNTM(i, j) = max(PNLT(i, j, :), [], 3);
\% 10 dB down period and duration correction
dBdown = find((PNLTM(i, j) - PNLTM(i, j,:)) < 10);
D(i, j) = 10 * log10((1 / size(dBdown, 1)) * ...
        sum(10. ^ squeeze(PNLTM(i, j, dBdown) ./ 10)) - PNLTM(i, j);
EPNL(i, j) = PNLTM(i, j) + D(i, j);
clc;
end
end
saveswitch = 1;
if saveswitch == 1;
    save([Pdir Pname], 'PNLT', 'TOBact', 'EPNL', '-append');
end
Atmospheric Attenuation Coefficients (AtmAttenuation.m)

1 function [alpha] = AtmAttenuation(f,ps,T,hr)
2 % f = 10:10:100000;
3 % ps = 1; % Atm. Pressure, (atm)
4 ps0 = 1; % Ref Pressure, (atm)
5 % T = 300; % Atm. Temp, (K)
6 To = 293.15; % Ref Temp, (K)
7 T0 = 273.16; % Triple Point Isotherm Temp, (K)
8 psat = ps0*10^((10.79584*(1-(To1/T)))-5.02808*log10(T/To1)+... 11 1.50474e+4*(1-10^((8.29692*((T/To1))^-1)))+... 12 2.2195983);
13 % hr = 70:%0:10:100; % Relative humidity
14 h = hr*(psat/pso)/(ps/pso);
15 frN = (ps/pso)*sqrt(T/To)*(9+280*h*exp(-4.17*((To/T)^(1/3))-1));
16 frO = (ps/pso)*(24+4.04e4*h*(0.02+h)*(0.391+h)^(1/4)+... 17 (frN+(f.^2./frN))) ; % Units - Np/m (Nepers per meter)
18 alpha = alpha/3.2808399/.115129254; % dB/ft

Convert TOB to PNL (TOB2PNL.m)

1 function [PN,N,PNL] = TOB2PNL(TOB)
2
3 % Convert TOB to PNL
4 % for k = 1:size(TOBact,3);
5 % TOB = squeeze(TOBact(1,1,k,:));
6 %Initialize Arrays
7 SPL = zeros(size(TOB,1),5); M = zeros(size(TOB,1),4);
8 %Mathematical formulation for noy values
9 SPL(:,1) = [91 85.9 87.3 79.9 79.8 76 74 74.9 94.6 inf(1,13) 44.3 50.7]';
10 SPL(:,2) = [64 60 56 53 51 48 46 44 42 40 40 40 40 40 38 34 32 30 29 29 30 31 37 41]';
11 SPL(:,3) = [52 51 49 47 46 45 43 42 41 40 40 40 40 40 38 34 32 30 29 29 30 31 34 37]';
12 SPL(:,4) = [49 44 39 34 30 27 24 21 18 16 16 16 16 15 12 9 5 4 5 6 10 17 21]';
13 SPL(:,5) = [55 51 46 42 39 36 33 30 27 25 25 25 25 25 23 21 18 15 14 14 15 17 23 29]';
14 M(:,1) = [0.043478 0.040570 0.036831 0.036831 0.035336 0.033333 ... 15 0.033333 0.032051 0.030675 repmat(0.030103,1,6) ... 16 repmat(0.029960,1,7) 0.042285 0.042285]';
17 M(:,2) = [repmat(0.03103,1,9) -inf(1,13) 0.029960 0.029960]';
18 M(:,3) = [0.079520 0.068160 0.068160 0.059640 repmat(0.053013,1,10) ... 19 0.059640 repmat(0.053013,1,2) repmat(0.047712,1,2) ... 20 repmat(0.053013,1,2) 0.068160 0.079520 0.059640]';
21 M(:,4) = [repmat(0.058098,1,2) 0.052288 0.047534 repmat(0.043573,1,2) ...
% Convert TOB spectra to Perceived Noisiness
marker = zeros(size(TOB,1),4);
n = zeros(size(marker));
PN = zeros(size(TOB));
for i = 1:size(TOB,2)
    %
    marker(:,1) = TOB(:,i)>SPL(:,1);
    marker(:,2) = TOB(:,i)>SPL(:,2)&TOB(:,i)<SPL(:,1);
    marker(:,3) = TOB(:,i)>SPL(:,5)&TOB(:,i)<SPL(:,2);
    marker(:,4) = TOB(:,i)>SPL(:,4)&TOB(:,i)<SPL(:,5);
    n(:,1) = 10.ˆ(M(:,2).*(TOB(:,i)-SPL(:,3)));
    n(:,2) = 10.ˆ(M(:,1).*(TOB(:,i)-SPL(:,2)));
    n(:,3) = 0.3*10.ˆ(M(:,4).*(TOB(:,i)-SPL(:,5)));
    n(:,4) = 0.1*10.ˆ(M(:,3).*(TOB(:,i)-SPL(:,4)));
    a = n.*marker; a(isnan(a))=0; %replace NaN with zeros
    PN(:,i) = sum(a,2);
    marker = zeros(size(marker)); n = zeros(size(n));
end
N = max(PN)+0.15*(sum(PN,1)-max(PN));
PNL = 40+(10/log10(2))∗log10(N);
clear i
end

Calculate Tone Correction (ToneCorrection.m)
22 \text{sl} = \text{diff}({\text{SPL1}(3:\text{end},:)};
23 \text{sl} = [\text{sl}(1,:) \; \text{sl} ; \text{sl} (\text{end},:)];
24 \%
25 s = (1/3) * (\text{sl}(1:\text{end}-2,:) + \text{sl}(2:\text{end}-1,:) + \text{sl}(3:\text{end},:));
26 \text{SPL2} = \text{TOB}(1:3,:);
27 \text{for} \ i = 4: \text{size}({\text{TOB},1})
28 \text{SPL2}(i,:) = \text{SPL2}(i-1,:) + s(i-3,:);
29 \text{end}
30 \text{clear} \ i
31 \text{F} = \text{TOB} - \text{SPL2};
32 \%
33 \text{Find1} = \text{find}(\text{fc}\_\text{pref}\geq50 \& \text{fc}\_\text{pref}\leq500);
34 \text{Find2} = \text{find}(\text{fc}\_\text{pref}\geq500 \& \text{fc}\_\text{pref}\leq5000);
35 \text{Find3} = \text{find}(\text{fc}\_\text{pref}\geq5000 \& \text{fc}\_\text{pref}\leq10000);
36 \%
37 \text{Ftone} = \text{find}(\text{F} > 1.5);
38 \%
39 \text{for} \ i = 1: \text{length}({\text{Ftone}})
40 \quad \text{ind} = \text{Ftone}(i);
41 \quad \text{if} \ \text{ind} \leq \text{max}(\text{Find1}) \text{ || ind} \geq \text{min}(\text{Find3})
42 \quad \quad \text{if} \ \text{F}(\text{ind}) < 3
43 \quad \quad \quad \text{C}(i) = \text{F}(\text{ind})/3 - 0.5;
44 \quad \quad \text{elseif} \ \text{F}(\text{ind}) \geq 3 \& \& \text{F}(\text{ind}) < 20
45 \quad \quad \quad \text{C}(i) = \text{F}(\text{ind})/6;
46 \quad \quad \text{elseif} \ \text{F}(\text{ind}) \geq 20
47 \quad \quad \quad \text{C}(i) = 3.333;
48 \quad \quad \text{end}
49 \quad \text{elseif} \ \text{ind} \geq \text{min}(\text{Find2}) \& \& \text{ind} \leq \text{max}(\text{Find2})
50 \quad \quad \text{if} \ \text{F}(\text{ind}) < 3
51 \quad \quad \quad \text{C}(i) = 2 * (\text{F}(\text{ind})/3) - 1;
52 \quad \quad \text{elseif} \ \text{F}(\text{ind}) \geq 3 \& \& \text{F}(\text{ind}) < 20
53 \quad \quad \quad \text{C}(i) = \text{F}(\text{ind})/3;
54 \quad \quad \text{elseif} \ \text{F}(\text{ind}) \geq 20
55 \quad \quad \quad \text{C}(i) = 6.666;
56 \quad \quad \text{end}
57 \quad \text{end}
58 \text{end}
59 \%
60 \text{if} \ \text{exist}('\text{C}') == 0
61 \quad \text{Cmax} = 0;
62 \text{else}
63 \quad \text{Cmax} = \text{max}(\text{C});
64 \text{end}
Flight Path Generation (FlightPlan_TakeOff.m)

```matlab
clc; clear all; close all;
fssep = filesep;

%Constant altitude, constant aircraft speed
xi = 0; yi = 0; zi = 0;
v = 0;

% Vj = 500; % [m/s] Velocity of Aircraft
% Uj = 500; % [m/s] Velocity of Jet Flow
% Zj = 5000; % [m] Altitude of Aircraft
% xi = -50000; % [m] Starting Location of Aircraft

% Ground Mapping Dimensions
xg = -2:0.5:10; %[km]
yg = -1:0.05:1; %[km]
[Xg Yg] = meshgrid(xg,yg);

%Flight Path Calculation
tf = 150; %flight duration [s]
t = 0:0.5:tf;

for k = 1:length(t)-1
    if t(k) < 27
        ax(k) = 3.375; %m/s^2
        ay(k) = 0;
        az(k) = 0;
    elseif t(k) >= 27 && t(k) < 56.5
        ax(k) = 3.32373;
        ay(k) = 0;
        az(k) = 0.586063;
    elseif t(k) >= 56.5
        ax(k) = 0;
        ay(k) = 0;
        az(k) = 0;
    end
    v(1) = 0; vy(1) = 0; vz(1) = 0;
    xj(1) = 0; yj(1) = 0; zj(1) = 0;
    vx(k+1) = vi + vx(k) + ax(k)*(t(k+1)-t(k));
    vy(k+1) = vi + vy(k) + ay(k)*(t(k+1)-t(k));
    vz(k+1) = vi + vz(k) + az(k)*(t(k+1)-t(k));
end

for k = 1:length(t)-1
    if t(k) < 27
        xj(k+1) = xi + xj(k) + vx(k)*(t(k+1)-t(k)) + 0.5*(ax(k).*((t(k+1)-t(k))^2));
        yj(k+1) = vi + yj(k) + vy(k)*(t(k+1)-t(k)) + 0.5*(ay(k).*((t(k+1)-t(k))^2));
        zj(k+1) = vi + zj(k) + vz(k)*(t(k+1)-t(k)) + 0.5*(az(k).*((t(k+1)-t(k))^2));
    end
end

clear k;

R = zeros(length(xg),length(yg),length(t));
theta = zeros(length(xg),length(yg),length(t));
```
for i = 1:length(xg)
    for j = 1:length(yg)
        for k = 1:length(t)
            p1 = [xg(i) yg(j) 0]; p2 = [xj(k) yj(k) zj(k)]/1000;
            % p1 = [1 0 0]; p2=[xj-0 yj-0 zj-0];
            R(i,j,k) = pdist([p1;p2], 'euclidean');
            theta(i,j,k) = acos(dot([1 0 0],p1-p2)/... 
                              (norm([1 0 0]).*norm(p1-p2)))*(180/pi);
        end
    end
end

xj = xj/1000; yj = yj/1000; zj = zj/1000;
Vj = sqrt(vx.^2+vy.^2+vz.^2);

figure; subplot(2,1,1); plot(t,squeeze(R(end,end,:)));
subplot(2,1,2); plot(t,squeeze(theta(end,end,:)));

fname = 'TakeOff_LargeGrid_LowRes_150s.mat';
save([pwd fsep 'Flight Paths' fsep fname],...
    'R','theta','t','xg','yg','Xg','Yg','xj','yj','zj','Vj');