I, Temesgen W Aue, hereby submit this original work as part of the requirements for the degree of Doctor of Philosophy in Civil Engineering.

It is entitled:
Numerical Analysis of Cracking in Concrete Pavements Subjected to Wheel Load and Thermal Curling

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Numerical Analysis of Cracking in Concrete Pavements
Subjected to Wheel Load and Thermal Curling

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DOCTOR OF PHILOSOPHY

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in the School of Advanced Structures
of the College of Engineering and Applied Sciences

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Abstract

The main goal of this research is to implement recent advances in nonlinear fracture mechanics, most notably the introduction of the cohesive zone concept, in investigating the post-cracking behavior of concrete pavements, subjected to wheel load and curling. The cohesive zone is assumed to lie along a specified direction known \textit{a priori}, and cohesive elements are inserted along this path. The study follows a step-by-step approach, beginning with fracture analysis of simply supported beams. In this initial step, experimental and numerical studies available in literature are reproduced and excellent agreement is observed. Furthermore, important fracture parameters and numerical challenges are identified, pertinent to the cohesive zone concept. It is observed that post-crack responses of the beams are sensitive to choices regarding the solution type, the concrete softening curve, and the uncracked region mesh sizes.

Single slabs-on-grade under wheel loads located at the slab edge or interior are considered next. In this phase, it is observed that the increased size of the problem inhibits generating as refined mesh as for the beams, and consequently obtaining a convergent solution poses a significant challenge. This is resolved by using so-called viscous regularization, in which a small viscosity term is introduced. Accordingly, a small deviation of traction stresses beyond the pre-defined material softening curve can be tolerated. Once again, the simulation in this phase is verified by reproducing experimental and numerical results available in literature. The effects of concrete softening curve, cohesive zone mesh, solution method, fracture energy, and tensile strength on the fracture process are investigated. It is observed that the fracture energy is the major parameter that influences the responses.
In a third phase of the study, a single slab-on-grade is subjected to wheel load and curling, individually or in combination. In both cases, it is observed that the diurnal temperature cycle and the shape of its profile through the slab thickness plays a significant role on the post-crack responses of the slab. When the slab top is warmer, unstable cracks form; in contrast, a warmer bottom results in stable cracks, thereby increasing the resistance of the slab and avoiding sudden failure.

The final two phases of the research are devoted to the study of jointed concrete pavements, also subjected to wheel load and temperature variations: the fourth phase encompasses aggregate interlock joints and the fifth phase pertains to dowel bar joints. Linear and nonlinear aggregate interlock mechanisms are simulated and their repercussions on the fracture responses of the slabs are examined. Similarly, the effects of numerical idealization techniques for the dowel-slab interaction, joint size and dowel looseness on the fracture process are examined.

It is concluded that the cohesive zone approach is a very promising tool in the ongoing exploration of fracture behavior in concrete pavements. The techniques can be extended to general loading situations that involve fatigue and crack branching. The results from this study will contribute to the development of a more mechanistic failure analysis of concrete pavements.
To my parents: Ajebush Amante and Wondimu Aure
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List of Symbols and Abbreviations

2D  
   two-dimensional

3D  
   three-dimensional

a  
   notch depth

AASHTO  
   American association of state highway and transportation officials

$A_x, A_y$  
   most probable projected contact areas in x and y-directions

AGG  
   joint stiffness per unit length

b  
   beam width

B  
   brittleness number

CMOD  
   crack mouth opening displacement

CTOD$_c$  
   elastic critical crack tip opening displacement

D  
   dimensionless damage variable, particle size

$D_{max}$  
   maximum aggregate grain size

$D_v$  
   viscous stiffness degradation parameter

$D_{\gamma}$  
   dimensionless unit weight

E  
   Young’s modulus

EB  
   Euler beam

Eff  
   Joint effectiveness

FE  
   finite element

FPZ  
   fracture process zone

FCM  
   fictitious crack model

$f_n, f_t$  
   normal and tangential crack interface stress functions
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$f'_t$</td>
<td>concrete tensile strength</td>
</tr>
<tr>
<td>$f'_c$</td>
<td>concrete compressive strength</td>
</tr>
<tr>
<td>$G$</td>
<td>shear modulus</td>
</tr>
<tr>
<td>$G_f$</td>
<td>initial fracture energy</td>
</tr>
<tr>
<td>$G_F$</td>
<td>total fracture energy</td>
</tr>
<tr>
<td>$h$</td>
<td>beam or slab thickness</td>
</tr>
<tr>
<td>$I$</td>
<td>moment of inertia</td>
</tr>
<tr>
<td>$ITI$</td>
<td>initial time increment</td>
</tr>
<tr>
<td>$k$</td>
<td>modulus of subgrade reaction</td>
</tr>
<tr>
<td>$K_{nn}$, $K_{ss}$, $K_{tt}$</td>
<td>nominal stiffness in normal and two shear directions</td>
</tr>
<tr>
<td>$K_{IC}$</td>
<td>stress intensity factor</td>
</tr>
<tr>
<td>$L$</td>
<td>beam length</td>
</tr>
<tr>
<td>$l$</td>
<td>radius of relative stiffness</td>
</tr>
<tr>
<td>$l_{ch}$</td>
<td>characteristic length of material</td>
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<td>$l_{min}$</td>
<td>minimum arc-length increment</td>
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<tr>
<td>$l_{period}$</td>
<td>total arc-length scale factor</td>
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<tr>
<td>$LEFM$</td>
<td>linear elastic fracture mechanics</td>
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<tr>
<td>$LLD$</td>
<td>load line displacement</td>
</tr>
<tr>
<td>$LTE_O$</td>
<td>load transfer efficiency with respect to crack mouth opening displacement</td>
</tr>
<tr>
<td>$LTE_\delta$</td>
<td>load transfer efficiency with respect to deflection</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>$LTE_\sigma$</td>
<td>load transfer efficiency with respect to stress</td>
</tr>
<tr>
<td>MEPDG</td>
<td>mechanistic-empirical pavement design guide</td>
</tr>
<tr>
<td>$MnTI$</td>
<td>minimum time increment</td>
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<tr>
<td>$MxTI$</td>
<td>maximum time increment</td>
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<td>NLFM</td>
<td>nonlinear fracture mechanics</td>
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<tr>
<td>$N_R$</td>
<td>number of nodal rows</td>
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<td>$P$</td>
<td>applied load</td>
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<td>PCC</td>
<td>Portland cement concrete</td>
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<tr>
<td>$p$</td>
<td>pressure, percent of aggregate passing sieve size $D$</td>
</tr>
<tr>
<td>$p_k$</td>
<td>aggregate volume per unit volume of concrete</td>
</tr>
<tr>
<td>$P_T$</td>
<td>total transferred load</td>
</tr>
<tr>
<td>$P_{max}$</td>
<td>peak load</td>
</tr>
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<td>$q$</td>
<td>aggregate interlock factor</td>
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<td>$S$</td>
<td>beam span</td>
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<td>SEM</td>
<td>size effect method</td>
</tr>
<tr>
<td>$T_0$</td>
<td>initial width of cohesive zone</td>
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<tr>
<td>TB</td>
<td>Timoshenko beam</td>
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<tr>
<td>TE</td>
<td>theory of elasticity</td>
</tr>
<tr>
<td>TLE</td>
<td>transferred load efficiency</td>
</tr>
<tr>
<td>TPFM</td>
<td>two parameter fracture model</td>
</tr>
<tr>
<td>TPS</td>
<td>time period of step</td>
</tr>
</tbody>
</table>
t  traction stress

\( t_{n}, t_{s}, t_{t} \)  nominal stress along normal and two shear directions

\( t'_{i} \)  traction stress corresponding to \( w \) without cracking

\( t'_{s} \)  traction stress for separation \( w \) along the softening curve

\( v \)  deflection

\( v_{\text{max}} \)  maximum deflection

\( UEL \)  user element

\( w \)  crack opening

\( w_{f} \)  crack opening corresponding to initial fracture energy

\( w_{n}, w_{s}, w_{t} \)  elastic cohesive displacements along normal and two shear directions

\( w_{cr} \)  cohesive zone separation at crack initiation

\( w_{e} \)  effective traction displacement

\( w_{f} \)  crack opening at zero traction

\( w_{k} \)  cohesive zone separation at kink point

\( \alpha \)  coefficient of thermal expansion, concrete softening parameter

\( \beta \)  dimensionless peak load

\( \gamma \)  concrete unit weight

\( \Delta \)  vertical displacement at peak load

\( \delta_{L} \)  vertical deflection of loaded slab

\( \Delta l_{in} \)  initial increment in arc-length

\( \delta_{n}, \delta_{s}, \delta_{t} \)  normal and two shear interface displacements

\( \Delta T \)  temperature differential
\( \delta_U \)  
vertical deflection of unloaded slab

\( \varepsilon_n, \varepsilon_s, \varepsilon_t \)  
nominal strain in along normal and two shear directions

\( \zeta \)  
dimensionless load line displacement

\( \eta \)  
dimensionless crack mouth opening displacement

\( \kappa \)  
joint stiffness for corner node

\( \lambda \)  
joint length

\( \mu \)  
Poisson’s ratio, coefficient of friction

\( \sigma_{\text{max}} \)  
maximum bending stress at the bottom of loaded area

\( \sigma_n, \sigma_t \)  
normal and tangential interface stresses

\( \sigma_{pu} \)  
aggregate-paste matrix yielding strength

\( \sigma_{xx} \)  
longitudinal horizontal stress through slab thickness

\( \psi \)  
kink point stress ratio

\( \omega_i \)  
initial crack opening displacement

\( m \)  
viscosity parameter
Chapter 1  Research Overview

1.1  Introduction

Distresses in concrete pavement slabs are caused primarily by traffic loads and thermal curling. The first analytical solution for pavement system responses under such actions was provided by Westergaard (1926, 1927), and was limited by several simplifying assumptions (Ioannides et al., 1985). Due to the complex nature of the problem, many subsequent studies have been devoted to the elimination of these assumptions through the use of computerized numerical techniques, particularly the finite element (FE) method (Huang and Wang, 1973; Tabatabaie and Barenberg, 1978; Ioannides and Korovesis, 1990; Khazanovich, 1994). As a result of this effort, several stand-alone finite element programs for concrete pavement analysis have been developed over the last 40 years, including among others, KENSLABS (Huang and Wang, 1973), ILLI-SLAB (Tabatabaie and Barenberg, 1978), JSLAB (Tayabji and Colley, 1986), FEACONS (Tia et al., 1987), EverFE (Davids et al., 1998) and Pave3D (Nishizawa et al., 2001).

Such analytical efforts have gradually precipitated a major shift in the pavement design paradigm of the American Association of State Highway and Transportation Officials (AASHTO), culminating in the abandonment of the heretofore statistical/empirical serviceability oriented approach (AASHTO, 1993), in favor of a mechanistic-empirical pavement design guide (MEPDG) aimed at distress prevention (AASHTO, 2008). According to the MEPDG, stresses and strains are computed on the basis of engineering mechanics principles using a computer program, and are provided as inputs to statistical/empirical algorithms, usually termed transfer functions, in order to determine anticipated individual distress levels. For concrete pavements, finite element program ISLAB2000 (Khazanovich et al., 2000), a commercial version of ILLI-
**SLAB**, is employed in the calculation of pavement responses. The accuracy of these responses is not particularly critical since the statistical/empirical algorithms into which they are introduced as inputs exhibit coefficients of determination ranging from less than 30% to about 60% (AASHTO, 2008). The rationale of transfer functions for the purpose of converting responses to distresses is founded on the so-called cumulative linear fatigue damage hypothesis (Miner, 1945), which originally pertained to cracking in metals but which has been extended without justification to practically all pavement material distresses. Such phenomenological applications may, therefore, be expected to lead to unrealistic pavement distress level predictions, and statistical/empirical transfer functions remain one of the weakest links in the AASHTO (2008) MEPDG, urgently in need of replacement by a more rational and mechanistic alternative. To accomplish this objective, it is necessary to go beyond a linear elastic material characterization toward a general nonlinear analysis implementing concepts of fracture mechanics, as is done in other branches of engineering (Ioannides, 1997).

With the development of parallel processing, large memory computers and multi-purpose FE commercial software, fracture mechanics promises to enhance our understanding of structural behavior of concrete structures beyond the elastic limit, as cracking is initiated and propagates. Application of fracture mechanics to pavement systems has recently gained momentum as encouraging results are emerging (Ioannides et al., 2006; Song et al., 2006; Roesler et al., 2007; Gaedicke and Roesler, 2009). This study contributes to such ongoing efforts by employing state-of-the-art tools of fracture mechanics in investigating concrete pavement slab cracking as affected by several pertinent input parameters. Results presented elucidate the post-crack
response of single-slab and jointed concrete pavements subjected to wheel load and thermal curling.

1.2 Review of Concrete Fracture

Concrete is a two phase heterogeneous material (cement paste and aggregate particles), whose fracture involves a complex phenomenon comprising “micro-cracking, crack deflection [caused by aggregate particles], aggregate bridging, crack-face friction, crack tip blunting by voids, and crack branching”, usually termed as the fracture process zone (FPZ) (Shah et al., 1995). The size of the FPZ in concrete is relatively larger than in other materials, both brittle and ductile. This phenomenon was first noted by Kaplan (1961), whose experiments demonstrated the formation of unstable cracks preceded by slow and stable crack growth, which indicates that transfer of traction stresses occurs in the FPZ before a complete opening of two cracked faces (see Fig. 1.1). Since linear elastic fracture mechanics (LEFM) assumes linear elastic material behavior and a FPZ of negligible size, it cannot be applied to concrete (Kaplan, 1961; Shah and McGarry, 1971). Instead, nonlinear fracture mechanics (NLFM) has been pursued in concrete fracture analysis. In general, two NLFM approaches are commonly used in the investigation of concrete fracture (Shah et al., 1995). The first is called the “equivalent elastic crack” methodology, and assumes that the energy release rate during cracking is equal to the critical energy release rate consumed during the creation of the two crack surfaces. It effectively represents a modification of LEFM inasmuch as it accounts for the FPZ and the specimen size effect by adjusting the stress intensity factor, $K_{IC}$. The two parameter fracture model (TPFM) by Jenq and Shah (1985) and the size effect method (SEM) by Bažant and Kazemi (1990) are examples of this approach.
The second approach is the fictitious crack model (FCM), proposed by Hillerborg et al. (1976) and representing an application to concrete of the work by Dugdale (1960) and by Barenblatt (1962). The latter had assumed that the energy required to overcome the natural cohesive traction in the material, so as to create a traction-free crack surface, is very large compared to the “energy rate consumed during material fracturing in creating two surfaces” (Shah et al., 1995). Material behavior prior to crack initiation is assumed to be linear elastic. Upon attaining the material tensile strength ($f'_t$), traction stress in the FPZ gradually decreases to zero at a failure crack width ($w_f$), as depicted in Fig. 1.1.

One of the main advantages of the FCM is its suitability for numerical implementation. Different researchers have implemented this approach in FE programs by converting traction stresses along an assumed fracture path to equivalent nodal forces (e.g., Hillerborg et al., 1976; Petersson, 1981; Gustafsson, 1985; Liu, 1994; Ioannides and Sengupta, 2003; Ioannides et al., 2006). Others have created interface elements in accordance with the FCM approach in order to study delamination and progressive damage in composite materials (Reedy et al., 1997; Chen et al., 1999; Feih, 2005). Camanho and Dávila (2002) also created a user element (UEL) for commercial FE package *ABAQUS®* in simulating mixed mode decohesion in composite materials. This element was subsequently incorporated in *ABAQUS®* Version 6.5 “for modeling deformation and damage in finite-thickness adhesive layers between bonded parts.” Similarly, investigators at the University of Illinois, created UELs for *ABAQUS®* in order to simulate crack propagation in concrete beam specimens (Roesler et al., 2007), in asphalt specimens (Song, 2006; Song et al., 2006), and in functionally graded materials (Evangelista et al., 2009; Park et al., 2010). Responses obtained using these UELs are compared in the literature with
experimental measurements and satisfactory agreement is reported (Roesler et al., 2007). Gaedicke and Roesler (2009) recently used cohesive elements that have been incorporated in ABAQUS® in simulating crack propagation in simply supported beams, beams-on-grade, and single slabs-on-grade subjected to wheel loads, and reported very encouraging results. Additional progress is to be expected following the recent formulation of a more general mixed mode cohesive zone idealization using potential energy fields (Park et al., 2009; Paulino et al., 2010).

A major challenge during numerical idealization of the FPZ is achieving a convergent solution, since concrete fracture often involves stiffness degradation and snap-back behavior (Crisfield, 1991). The solution is sensitive to the type of softening curve adopted in describing the crack opening progression once the material strength is attained, as well as the mesh size, penalty stiffness and type of solver used (Chaboche et al., 2001; Camanho and Dávila, 2002; Gao and Bower, 2004). The use of linear softening, moderately low penalty stiffness, and arc-length solvers, along with a very fine mesh may be expected to contribute toward achieving convergent solutions. Refining the mesh in large structural systems, such as concrete pavements, will, of course, be computationally demanding. To overcome such numerical challenges, several researchers have resorted to so-called viscous regularization, according to which a viscosity term is introduced in the constitutive equations of the degrading material so as to control the rate of viscous energy dissipation. In this manner, numerical instability is avoided, and the solution is regularized, i.e., becomes convergent, even if somewhat less accurate (Chaboche et al., 2001; Gao and Bower, 2004; Maimi et al., 2007; Lapczyk and Hurtado, 2007; Hamitouche et al., 2008). ABAQUS® has implemented this technique in preventing convergence problems associated with cohesive elements (ABAQUS, 2009).
Research in this thesis is intended as a contribution in this evolving field of study that aspires to use FCM cohesive fracture simulation with viscous regularization to idealize cracking in single and jointed concrete pavement slabs, subjected to wheel load and thermal curling.

1.3 Research Objective and Significance

The main objective of this dissertation is to implement the built-in cohesive elements that have recently been added to the ABAQUS® element library in order to study crack propagation in concrete pavement slabs subjected to temperature and wheel load. The study seeks to quantify the effect of several variables involved in the fracture process, such as type of concrete softening curve, analysis method, along with cohesive zone width and mesh size. Furthermore, the effects of both linear and nonlinear temperature distributions, with and without a wheel load, on the post-crack response of concrete pavement slabs are investigated.

The post-crack response of jointed concrete pavements with aggregate interlock and dowel bars is also investigated. Both linear and nonlinear aggregate interlock formulations available in the literature are reviewed and implemented in the FE simulation described. The effects of parameters, such as initial joint width and aggregate size on the post-crack response of both the loaded and unloaded slabs are examined. The influence of thermal curling on the post-crack response and on joint efficiency is also investigated.

The study contributes to the ongoing development of rational failure criteria suited as substitutes to the statistical algorithms derived from Miner’s hypothesis that are being used in the current pavement design guides. By identifying important parameters pertinent to material characterization and numerical idealization of plain concrete pavements, this research provides
guidelines for a fracture mechanics determination of distresses in pavement systems. Thus, not only is the state-of-the art in pavement analysis and design enhanced, but also concrete fracture in general is elucidated.

1.4 Research Scope

This study is limited to mode I fracture, which is examined using a discrete crack approach. Accordingly, a crack plane is specified to develop along the anticipated principal stress axis, where FCM cohesive elements are inserted. This stipulation is reasonable for concrete pavement slabs loaded either at the edge or at their interior, and in both these cases the direction of the maximum stress is readily apparent. Furthermore, the effect of shear stress can be reasonably neglected in such structural systems. The intact or uncracked region of the pavement slab is assumed to behave elastically. This study addresses the effect of some of the most important features of a concrete pavement system subjected to static wheel load alone or in combination with temperature curling, notably loading position, load transfer mechanism, and joint characteristics.

1.5 Thesis Organization

The research described in this thesis has been conducted using multi-purpose commercial FE program ABAQUS/STANDARD® version 6.7/9. The Ohio Supercomputer Center's IBM Cluster 1350, which “includes AMD Opteron multi-core technologies”, has been employed in executing input files that were generated for the purposes of the study.

A systematic approach is followed throughout the study, by dividing it into five phases, each of which is presented as an independent journal paper. Phase 1 is devoted to fracture simulation in
simply supported beams, whose geometric and material properties are selected in conformity with earlier studies conducted by independent researchers. The simulation is used to reproduce numerical and experimental studies available in literature, thereby adding to the credibility of the idealization.

In the second phase, the same investigation is extended to individual pavement slabs-on-grade. The geometry and material properties of the slabs is again selected per previously reported investigations conducted by other researchers. Multiple FE runs are carried out to assess the effect of the width of the cohesive zone as well as of the mesh fineness used in this region, the character of the loaded area, the analysis method, the concrete softening curve adopted, the slab size and the fracture parameters. In concrete pavement slabs subjected to wheel loads alone, the critical load position causing maximum bending stress is at the edge. In a concrete pavement slab under both thermal and wheel loads, on the other hand, an interior position may sometimes be critical, depending on the geometry of the slab. Consequently, to simulate crack propagation in a slab loaded at the interior, cohesive elements are inserted along two assumed orthogonal fracture planes, emanating from the slab center and extending parallel to its edges. The FE simulation of the fracture process in phase 2 is also validated by comparisons with numerical and experimental studies reported by other researchers.

In the third phase, fracture of an individual pavement slab-on-grade subjected to temperature curling and wheel load is studied. Linear elastic analysis of the slab under temperature curling alone is first conducted, in order to validate the FE idealization formulated for this purpose. Subsequently, the effect of linear, quadratic, and cubic temperature distributions through the slab thickness is investigated. Both day-and nighttime conditions are examined, focusing on the post-
crack response of the slab under curling alone or when subjected to combined curling-plus-wheel load.

In phase 4, fracture behavior of concrete pavements equipped with aggregate interlock joints is investigated. Linear and nonlinear aggregate interlock formulations available in literature are examined. The effect of aggregate interlock joint characteristics on the post-crack response of the concrete pavement slab under wheel load and curling is studied.

Post-crack responses of dowelled concrete pavements are investigated in phase 5. The effect of joint-related parameters, such as dowel looseness and joint width on the post-crack behavior of the slabs is considered.

### 1.6 References


Figure 1.1 Fracture Process Zone in Concrete
Chapter 2  Simulation of Crack Propagation in Concrete Beams

Abstract

This paper discusses the simulation of crack propagation in concrete beam specimens using commercial finite element package ABAQUS® 6.7-1. Special-purpose cohesive elements are used to simulate the fracture process in accordance with the fictitious crack model. Two- as well as three-dimensional finite element discretizations are carried out. Parameters influencing the responses, such as mesh fineness, cohesive zone width, type of softening curve and analysis technique are studied. The responses are then compared with previous experimental and numerical investigations conducted by various independent researchers. It is demonstrated that cohesive elements can be used in idealizing crack propagation as required in pavement engineering.

2.1 Introduction

The development of a mechanistic-empirical approach for the analysis and design of pavement systems has received increased attention recently, reigniting the debate over the use of statistical/empirical transfer functions, whose experimental verification is questionable, at best (Ioannides, 1997a; Khazanovich and Tomkins, 2005). Following an exhaustive examination of various fracture mechanics options suggested as potential replacements to Miner’s hypothesis (Miner, 1945), Hillerborg’s fictitious crack model (FCM) (Hillerborg et al., 1976) was found to be the most promising for studying crack propagation in Portland cement concrete (PCC) pavements, and a step-by-step approach was outlined for its implementation (Ioannides, 1997a and 1997b). Accordingly, Ioannides and Sengupta (2003) formulated a two-dimensional (2D) numerical procedure to simulate crack propagation on the basis of the FCM for a simply supported beam. The response of the beam over the elastic range was analyzed using commercial finite element (FE) software GTSTRUDL (GTSTRUDL, 1993), while its fracture behavior was studied using a specially coded FORTRAN program, called CRACKIT. To facilitate the generalized application of the concepts implicit in the GTSTRUDL/CRACKIT combination, Ioannides et al., (2005) subsequently implemented their approach by using the general purpose FE package, ABAQUS® (ABAQUS, 2003). They reported at the time that the applicability of the built-in fracture analysis capabilities of ABAQUS® was too limited for pavement engineering, especially since the FCM was not used. Consequently, in their 2D study of simply supported beams, the investigators employed a nonlinear spring element from the ABAQUS® library, JOINTC, to idealize the fracture zone. An alternative approach was developed by Song (2006), who coded a user-defined subroutine in ABAQUS®, thereby creating a user element (UEL); an application of this UEL is described in the 2D study of crack propagation in simply supported
asphalt concrete beams by Song et al. (2006). A series of such 2D 4-noded UEL elements were inserted at the center of the beam to simulate fracture.

A similar approach to that by Song et al. (2006) was followed by Roesler et al. (2007a) who simulated crack propagation in simply supported concrete beam specimens. A UEL for ABAQUS® created by Park (2005) was used to discretize the cohesive zone in accordance with the FCM. The parameters that define the FCM bilinear softening curve were directly determined from the three-point bending test (ASTM D790, 2010). The total fracture energy, $G_F$, was determined from the area of load versus crack mouth opening displacement ($P$-$CMOD$) curve and the initial fracture energy, $G_I$, was computed per either the two parameter fracture model (TPFM) (Jenq and Shah, 1985) or the size effect method (SEM) (Bažant and Kazemi, 1990). The location of the kink point was assumed to be at 25% of the tensile strength of the specimen.

The same simulation concept found in Roesler et al. (2007a) was extended to idealize the fracture process in simply supported beams with functionally graded concrete materials (Roesler et al., 2007b). A specimen made partially of plain concrete and partially of fiber reinforced concrete was tested so as to obtain fracture properties. The numerical simulation employed a trilinear softening curve that was constructed by accounting for the bridging effect of the fiber reinforced concrete on the fracture process zone on the bilinear curve used by Roesler et al. (2007a). The parameters that define the trilinear curve were obtained from experiments conducted during the study (ASTM D790, 2010). A UEL created for ABAQUS® implementing the trilinear curve was used to idealize the cohesive zone.
The corresponding case involving a three-dimensional (3D) PCC pavement slab-on-grade was first considered by Ioannides and Peng (2004), using once again *JOINTC* elements from the *ABAQUS*® library. A more versatile 3D UEL implementing a cohesive zone idealization was subsequently formulated by Song (2006), and was applied to a cylindrical asphalt concrete specimen.

Such efforts received a boost with the release in early 2005 of *ABAQUS*® version 6.5, which for the first time included “a family of cohesive elements for modeling deformation and damage in finite-thickness adhesive layers between bonded parts. Cohesive elements are typically connected to underlying elements with surface-based *TIE CONSTRAINTS*, so the mesh used for the cohesive layer can be independent of the mesh used for the bonded components” (*ABAQUS*, 2004). Gaedicke and Roesler (2009) reported using these cohesive elements in their study of crack propagation in concrete beams and slabs.

The investigation presented in this paper is a continuation of the step-by-step development and application of fracture mechanics tools using the FCM in pavement engineering, initiated at the University of Cincinnati in the late 1990s. The main objective is to implement the built-in cohesive elements that have recently been added to the *ABAQUS*® library, and to compare the performance of these elements to that reported in earlier investigations. It is hoped that in this manner, a contribution will be made to the ongoing effort for more mechanistic-empirical pavement design procedures that will utilize fracture mechanics concepts, thereby replacing the purely statistical/empirical transfer functions and Miner’s hypothesis, which are currently in use. Validating a FE simulation of crack propagation in a simply supported beam is considered to be
a necessary precursor to a more comprehensive analysis of slabs-on-grade, required for *in situ* pavement systems.

### 2.2 Methodology

The present study focuses on the post-cracking response of simply supported PCC beams, using commercial general purpose FE program *ABAQUS®/STANDARD*, version 6.7-1. The geometric and material properties of the beams considered in this paper are shown in Table 2.1. To begin with, however, a linear elastic analysis using 2D and 3D elements is described, and the results are compared with available closed-form solutions. This initial step is considered essential in ensuring the robustness of the proposed FE idealization. Upon the successful conclusion of the linear elastic analysis, the simulation of crack propagation can be carried out, by implementing the built-in cohesive elements of *ABAQUS®*, in accordance with the FCM for fracture. In all analyses, elements *CPS4* and *C3D27* are used for 2D and 3D discretizations of the intact material, respectively, while the cohesive fracture zone is simulated with *COH2D4* (2D) and *COH3D8* (3D) cohesive elements. Program runs reported herein capture the effects of numerical analysis technique, mesh fineness, cohesive zone width, and type of softening curve. The resulting simulation is finally used to reproduce numerical and experimental studies conducted by other independent researchers, thereby adding to its credibility.

### 2.3 Analysis of Linear Elastic Response

This section reports numerical testing to ascertain the robustness of the proposed FE idealization through simulation of the linear elastic response of the simply supported beam designated as Beam A in Table 2.1. This beam was originally used by Sengupta (1998) and was later adopted
by Ioannides et al. (2005), whose results are, therefore, available for comparison purposes, along with published closed-form solutions. Both concentrated and uniformly distributed loads are considered.

Beam A was assumed to be simply supported on rollers, requiring degree of freedom 2 (vertical displacement) to be fixed at the two support nodes. Because of symmetry, half of the beam was meshed uniformly with $64 \times 48$ CPS4, and $5 \times 40 \times 30$ C3D27 elements, for the 2D and 3D discretizations, respectively. All elements used were nearly square, which thereby eliminated any significant aspect ratio effects. Once the mesh for the right-hand-side half of the beam was defined, a mirror image was created along the plane of symmetry, and surface-based TIE CONSTRAINTS were used to connect surfaces on either side of the symmetry plane.

The simulations first considered a concentrated load of 33.7 kips, applied at the midspan. For the 2D simulation, the load was applied on the top node at the midspan. To avoid localized effects in the 3D idealization, a small loaded area of size 0.4 by 1.5 in. (two elements along the symmetry line) was defined at the center of the beam, on which a pressure of 56.17 ksi was applied. The same beam was also subjected to a uniformly distributed load of magnitude 10 kips/in. This load was applied as a pressure of magnitude 6.667 ksi (10 kips/in. divided by 1.5 in.) over the top surface elements.

Simulation results were compared with closed-form solutions. For a beam subjected to a concentrated load ($P$), the maximum midspan deflection ($v_{\text{max}}$) is given by Timoshenko and Goodier (1970) as:
\[
\nu_{\text{max}} = \frac{PL^3}{48EI} + \frac{PL}{4c} \left( \frac{3}{4G} - \frac{3}{10E} - \frac{3\mu}{4E} \right) - 0.21 \frac{P}{Eb} 
\]

(2.1)

where \( c \) is half of the beam thickness, \( h; b, L, I \) are beam’s width, length and moment of inertia; 
\( \mu \) is the Poisson’s ratio, \( E \) is the Young’s modulus, and \( G \) is the shear modulus of the material, which can be computed from:

\[
G = \frac{E}{2(1 + \mu)} 
\]

(2.2)

The results are shown in Table 2.2, where it is observed that the FE discretizations show a discrepancy from the theoretical solution of 2.3% and 0.3%, for 2D and 3D, respectively. For the beam subjected to uniformly distributed load \((q_o)\), results were compared with values computed according to the following theories, as summarized by Shames and Dym (1985):

Euler-Bernoulli’s Theory:

\[
v(x) = \frac{q_o L^4}{24EI} \left[ \left( \frac{x}{L} \right)^4 - \frac{3}{2} \left( \frac{x}{L} \right)^2 + \frac{5}{16} \right] 
\]

(2.3)

 Theory of Elasticity:

\[
v(x) = \frac{q_o L^4}{24EI} \left[ \left( \frac{x}{L} \right)^4 - \frac{3}{2} \left( \frac{x}{L} \right)^2 + \frac{5}{16} + \left( \frac{h}{L} \right)^2 \left( \frac{12}{5} + \frac{3\mu}{2} \right) \left( \frac{1}{4} - \frac{x}{L} \right)^2 \right] 
\]

(2.4)

Timoshenko Beam Theory:
Note that $x$ and $v$ in the above equations represent any point of interest along the span of the beam and the corresponding deflections, respectively. It was found that the FE idealization exhibited near perfect agreement with the theory of elasticity accounting for Poisson’s ratio effect. Moreover, the FE solutions provided excellent approximations (98.86% and 98.66% for 2D and 3D, respectively) to the theoretical solution for a beam with shear deformation (Timoshenko theory), as shown in Fig. 2.1. The Euler beam theory, on the other hand, gave lower results (77%) as compared to other theories as well as the FE results. It is, therefore, concluded that the FE discretization implemented in this study is robust as far as the linear elastic aspects of the analysis are concerned, and that it may be considered to be a good candidate for investigating the more demanding fracture mechanics issue of crack propagation in simply supported beams.

### 2.4 Simulation of Crack Propagation Using Built-in Cohesive Elements

This section outlines the FE formulation incorporating cohesive elements recently introduced in *ABAQUS®*, and their implementation in accordance with the FCM for investigating the post-fracture response of simply supported concrete beams. The sensitivity of the FE solution to a variety of aspects of the numerical procedure employed is also examined.
2.4.1 Idealization of Cohesive Zone

Among the three classes of cohesive elements that are available in ABAQUS®, those implementing the so-called “traction-separation” formulation are the most suitable for use in crack propagation studies that use the FCM. Accordingly, the load-displacement responses can be subdivided into three stages: pre-crack; initiation of crack; and post-peak (or softening) behavior.

a. Pre-Crack Behavior

During the pre-crack stage, the material is considered to experience a very small but finite separation, and the cohesive element response is governed by the following elastic strain-displacement relation (ABAQUS, 2007):

\[
\begin{bmatrix}
    \varepsilon_n \\
    \varepsilon_s \\
    \varepsilon_t
\end{bmatrix} = \frac{1}{T_0} \begin{bmatrix}
    w_n \\
    w_s \\
    w_t
\end{bmatrix}
\]

(2.6)

where: \( \varepsilon \) and \( w \) are the nominal strain and the elastic separation vectors, respectively, in the normal (n) and two shear directions (s and t); and \( T_0 \) is initial width of the cohesive zone. The elastic traction stress components can then be computed from Eq. (2.7):

\[
\begin{bmatrix}
    t_n \\
    t_s \\
    t_t
\end{bmatrix} = \begin{bmatrix}
    K_{nn} & K_{ns} & K_{nt} \\
    K_{sn} & K_{ss} & K_{st} \\
    symm & K_{tn} & K_{tt}
\end{bmatrix} \begin{bmatrix}
    \varepsilon_n \\
    \varepsilon_s \\
    \varepsilon_t
\end{bmatrix}
\]

(2.7)

where \( K \) is a nominal stiffness (also referred to as penalty stiffness), illustrated in Fig. 2.2, and \( t \)
is the nominal stress, in the normal and two shear directions, respectively. If the shear and normal components are uncoupled, Eq. (2.7) will reduce to:

\[
\begin{bmatrix}
    t_n \\
    t_s \\
    t_t
\end{bmatrix} =
\begin{bmatrix}
    K_{nn} & 0 & 0 \\
    0 & K_{ss} & 0 \\
    0 & 0 & K_{tt}
\end{bmatrix}
\begin{bmatrix}
    \varepsilon_n \\
    \varepsilon_s \\
    \varepsilon_t
\end{bmatrix}
\]

(2.8)

For a 2D idealization, only the first two rows and columns of Eq. (2.7) are used. Selection of the initial width of the cohesive zone and of the penalty stiffness, \( K \), relies largely on prior experience with the software, yet it can influence the solution convergence significantly. *ABAQUS* (2007) recommends computing the penalty stiffness from: \( K_{nn} = E/T_0 \), where \( E \) is the Young’s modulus of the intact (or uncracked) material. Similarly, it may be assumed that \( K_{ss} = K_{tt} = G/T_0 \), where \( G \) is the corresponding shear modulus of the intact material.

b. Initiation of Crack and Post-Peak Behavior

Crack initiation refers to the beginning of the degradation of the material. In PCC crack propagation studies, it is often assumed that the crack initiates when the stress reaches the tensile strength, \( f'_t \), of the material (Liu, 1994; Ioannides and Sengupta, 2003; Roesler et al., 2007a).

Once the crack initiates, material damage evolves according to a predefined softening law. Park et al. (2008) provided a comprehensive list of softening options proposed for concrete. At first, Hillerborg et al. (1976) used a linear softening curve. Nowadays, it is more common to employ a bilinear curve characterized by three points, as shown in Fig. 2.2: (a) the crack initiation point, defined by traction stress \( f'_t \) and separation \( w_{cr} \); (b) the kink point, at which the separation-
traction stress pair is \((w_k, \psi f')\); and (c) the critical (or maximum) separation, \(w_f\), for which the traction stress is zero.

The crack evolution for any kind of softening is facilitated by using a dimensionless damage variable defined as \((ABAQUS, 2007; Gaedicke and Roesler, 2009)\):

\[
D = 1 - \frac{t'}{t_i} \quad \text{for} \quad w_{cr} \leq w \leq w_f
\]  

(2.9)

where: \(t'_s\) is the traction stress for separation \(w\), along the softening curve; \(t'_i\) is the traction stress that would have corresponded to \(w\) had the pre-crack stiffness endured, as explained in Fig. 2.2.

The traction stress for a given separation before the kink point can easily be derived as:

\[
t'_s = f_i \left[1 + (\psi - 1) \left(\frac{w - w_{cr}}{w'_k - w_{cr}}\right)\right] \quad \text{for} \quad w_{cr} \leq w \leq w_k
\]  

(2.10)

Similarly, for the second softening segment, \(t'_s\) can be obtained as:

\[
t'_s = \psi f_i \left(\frac{w_f - w}{w_f - w'_k}\right) \quad \text{for} \quad w_k \leq w \leq w_f
\]  

(2.11)

The elastic traction stress that would have been obtained had the material not become damaged, \(t'_i\), can easily be obtained from:

\[
t'_i = K_{nn} w
\]  

(2.12)

Substituting Eqs. (2.12) and (2.10) into Eq. (2.9) and simplifying, one can obtain:
A similar approach can be followed to obtain an expression for $D$ pertaining to the remaining segment $w_k \leq w \leq w_f$. ABAQUS® requires $D$ to be specified as a function of the effective separation, $w_e$, defined as the separation with respect to the crack initiation point, $w_{cr}$.

The above equations are useful for a bilinear softening curve, whose kink point location is known. Instead of assuming the location of the kink point, Roesler et al. (2007a) used the initial fracture energy, $G_f$, a parameter defined as the area under the first two limbs of softening curve shown in Fig. 2.2. This is used to determine the separation at which the second limb of the softening curve intersects the separation line, $w_i$. From Fig. 2.2, therefore, $w_i$ can be found as:

$$w_i = \frac{2G_f}{f_t} \quad (2.14)$$

The traction $t'_s$, is now easily determined from:

$$t'_s = f'_t \left( \frac{w_i - w}{w_i - w_{cr}} \right) \text{ for } w_{cr} \leq w \leq w_k \quad (2.15)$$

The equation for the traction stresses for separation values between $w_k$ and $w_f$ remains the same as Eq. (2.11). The kink point separation, $w_k$, can be calculated using Eq. (2.14) for $t'_s = \psi f'_t$ as follows:

$$w_k = w_i - \psi (w_i - w_{cr}) \quad (2.16)$$
The separation at the zero traction stress, $w_f$, depends on the difference between the total and initial fracture energy, and is given as (Roesler et al., 2007a):

$$w_f = \frac{2}{\psi f_t} \left[ G_F - (1 - \psi)G_f \right]$$

(2.17)

*ABAQUS*® incorporates only linear and exponential softening curves. The bilinear or any other kind of curve, however, may be specified by the user in tabular form, as illustrated below.

c. Numerical Example

Typical calculation procedures that are used to generate the necessary input values of material properties for cohesive elements in accordance with a bilinear curve are demonstrated below for Beam A and Beam C.

i. Beam A

The material properties for this beam are shown in Table 2.1 as: $E = 4000$ ksi; $f_t^* = 0.46327$ ksi. The bilinear curve parameters are (Ioannides et al., 2006): $\psi f_t^* = 0.15442$ ksi (thus $\psi = 1/3$), $w_k = 0.000745$ in., $w_f = 0.0033525$ in. The initial width of the cohesive zone is assumed to be $T_0 = 0.001$ in.

The initial stiffness is: $K_{nn} = E/T_0 = 4 \times 10^6$ ksi/in. The critical separation, $w_{cr}$, can be computed from Eq. (2.12) as $w_{cr} = f_t^* / K_{nn} = 1.158 \times 10^{-7}$ in. Considering a point on the bilinear softening curve at $w = 0.0005$ in., for example, Eq. (2.10) yields:
\[ t'_s = 0.46327 \left[ 1 + \left( \frac{1}{3} - 1 \right) \left( \frac{0.0005 - 1.158(10)^{-7}}{0.000745 - 1.158(10)^{-7}} \right) \right] = 0.256006 ksi \]  

(2.18a)

The elastic traction stress that would have been obtained had there been no damage is: \( t'_i = K_{nn} w \) = 4 × 10^6 × 0.0005 = 2000 ksi. The damage variable \( D \) is now computed by using Eq. (2.9) as:

\[ D = 1 - \frac{0.256006}{2000} = 0.9998719969 \]  

(2.18b)

The effective displacement, \( w_e \), which will be defined along with \( D \), is: \( w_e = w - w_{cr} = 0.0005 - 1.158 \times 10^{-7} = 4.998842 \times 10^{-4} \) in. Next, the damage variable and the effective displacement are specified in the input file as \((D, w_e)\). At \( w = 0.0005 \) in., it is \((0.9998719969, 0.0004998842)\). A similar approach can be used for Beam B, which has the same softening curve and material properties, except for \( E \), as that of Beam A.

ii. Beam C

The material properties of this beam are also shown in Table 2.1 as: \( E = 32000 \) MPa; \( f'_i = 4.15 \) MPa; \( G_f = 56.57 \) N/m; \( G_F = 167 \) N/m. \( T_0 \) is assumed to be 1 mm; the stress ratio at the kink point, \( \psi \), is taken to be 0.25 (Roesler et al., 2007a). Using Eq. (2.14) and (2.17) one can obtain:

\[ w_i = \frac{2 \times 56.57(10)^{-3}}{4.15} = 0.02726265 mm \]

\[ w_f = \frac{2}{0.25 \times 4.15} [167 - (1 - 0.25) \times 56.57] \times 10^{-3} = 0.24014 mm \]  

(2.18c)

The penalty stiffness is: \( K_{nn} = E/T_0 = 32000 \) MPa/mm. The critical separation is: \( w_{cr} = f'_i/K_{nn} = 1.296875 \times 10^{-4} \) mm. The kink point separation, \( w_k \) can be obtained from Eq. (2.16) as: \( w_k = 0.02726265 - 0.25(0.02726265 - 1.296875 \times 10^{-4}) = 0.0204794 \) mm. Considering a separation, \( w \), of
0.01 mm (which is less than \( w_k \)), for example, Eq.(2.15) gives:

\[
\begin{align*}
t'_{s} &= 4.15 \left( \frac{0.02726265 - 0.01}{0.0272626500 - 0.0001296875} \right) = 2.64033 \text{ MPa} \\
t'_{i} &= 32000 \times 0.01 = 320 \text{ MPa} \\
D &= 1 - \frac{2.64033}{320} = 0.991748966
\end{align*}
\]

The effective displacement is: \( w_e = w - w_{cr} = 0.0098703125 \) mm. The softening behavior is provided in the form of a table in the input file; for example, \((D, w_e)\) is \((0.991748966, 0.0098703125)\) at \( w = 0.01 \) mm. It is noted that in this section, SI units have been retained per Roesler et al. (2007a) as an illustration of such use in this field.

### 2.4.2 Sensitivity Study for Proposed Discretization

Previous investigators have examined in detail the effect of several variables influencing numerical solutions analogous to that proposed in the present paper. Thus, Song et al. (2006) studied the effect of total fracture energy, \( G_F \) (defined as the area under the bilinear curve shown in Fig. 2.2), of tensile strength, \( f'_t \), and of cohesive zone width, \( T_0 \), on the fracture of asphalt concrete beams. For his part, Park (2005) examined the sensitivity of the solution to the initial fracture energy, \( G_f \), as well as to the location of the kink point, for PCC specimens. The latter was also investigated by Gaedicke and Roesler (2009), who employed built-in cohesive elements.

In this section, the sensitivity of the proposed FE discretization to the analysis technique, mesh size, notch depth, cohesive zone width and type of softening curve is investigated. The beams are idealized using 3D elements, \( C3D27 \) for the intact material and \( COH3D8 \) for the cohesive zone, respectively.
### a. Effect of Analysis Technique

For the purposes of this investigation, there are currently two analysis options in *ABAQUS®*: the general (or default) procedure, which uses a Newton-Raphson method; and the modified Riks approach (*ABAQUS*, 2007). The latter is particularly suited for potentially unstable problems that occasionally exhibit negative stiffness values, or present convergence difficulties, e.g., buckling and snap-back of the load-displacement curve. Since cohesive elements are inserted in materials experiencing softening resulting from progressive damage, their application may also be fraught with such numerical problems. To overcome such challenges when using the Newton-Raphson approach, “viscous regularization” (*ABAQUS*, 2007) may be applied, but a preferable alternative in certain cases is to employ the modified Riks procedure. For example, Song et al. (2005) encountered divergence problems when using the Newton-Raphson method, whereas the modified Riks approach produced convergence. For their part, Yang and Proverbs (2004) studied the efficacy of various solution strategies for fracture, and concluded that arc-length solvers (such as that used in the modified Riks method) would capture the softening behavior in snap-back type of load-displacement relations.

To study the effect of the two analysis options currently available in *ABAQUS®*, Beam A and B in Table 2.1 are considered in an unnotched configuration. The cohesive zone width is set to 0.001 in. The penalty stiffnesses, $K_{nn}$, $K_{ss}$, $K_{tt}$, are computed as noted earlier. Linear softening is used for simplicity.

For the Newton-Raphson method (Crisfield, 1991), the loading parameters that need to be specified are: initial time increment ($ITI$), time period of the step ($TPS$), minimum time increment ($MnTI$) and maximum time increment ($MxTI$). These are set to $6 \times 10^{-3}$, 1.0, $1 \times 10^{-9}$,
$3\times10^{-2}$, respectively, on the basis of previous work by Ioannides et al. (2005). The maximum number of time increments is set to 200.

The modified Riks approach (Riks, 1979) requires four parameters to be defined: the initial increment in the arc-length along the static equilibrium path, $\Delta l_{in}$; the total arc-length scale factor, $l_{period}$; and the minimum and maximum arc-length increments, $l_{min}$, and $l_{max}$, respectively. A convenient way to assign values to these parameters is to retain those specified above: $6\times10^{-3}$, 1.0, $1\times10^{-9}$, $3\times10^{-2}$, respectively. Moreover, the terminal increment is similarly set to 200.

The results obtained in this manner are plotted in Fig. 2.3 and Fig. 2.4 for Beams A and B, respectively, as pairs of load versus crack mouth opening displacement ($P$-$CMOD$) and load versus load line displacement ($P$-$LLD$) curves. It is clear in Fig. 2.3 that for Beam A, the trends captured by the two methods are significantly different from one another; those according to the modified Riks approach are considered to be more realistic because they reflect the snap-back behavior expected in the softening stage. For Beam B, however, both techniques give identical results when the $P$-$LLD$ curve is examined, and only a minor difference is observed in the in the two $P$-$CMOD$ curves, as shown in Fig. 2.4. Evidently, the difference between the results of the two techniques is more pronounced in deeper beams (lower span-to-thickness ratio), and in such cases resorting to the modified Riks procedure may be advantageous when built-in cohesive elements are employed.

b. Effect of Mesh Fineness

A set of runs involving three types of mesh were considered for Beams A and B in Table 2.1; these discretizations involved multiple mesh configurations for the intact region of the beams,
and a constant mesh for the cohesive zone. Linear softening was used for simplicity. The coarsest mesh consisted of $8 \times 6 \times 1$ and $24 \times 6 \times 2$ $C3D27$ elements in the length, depth and width directions, whereas the finest mesh had $32 \times 24 \times 4$ and $60 \times 15 \times 8$ $C3D27$ elements; the median mesh had $16 \times 12 \times 2$ and $36 \times 9 \times 4$ $C3D27$ subdivisions for Beams A and B, respectively. The cohesive zone mesh was maintained as 10 times finer than the median mesh in the depth and width directions, and had one element in the length direction. The results are shown in Fig. 2.5 and Fig. 2.6 for Beams A and B, respectively. It is observed that the effect of mesh fineness is surprisingly insignificant for the meshes adopted, especially regarding the $CMOD$ response. In as much as there are differences between the coarsest and the finest meshes for the $LLD$ response, it is observed that a finer mesh generally results in a slightly lower load before the peak, and a slightly higher load after the peak, but such differences do not exceed 5%.

c. **Effect of Notch Depth**

The effect of notch depth, $a$, on the $P-CMOD$ response was studied by using the $60 \times 15 \times 8$ element discretization of Beam B. The cohesive zone mesh used previously was retained here. It is observed from Fig. 2.7 that as the notch depth increases, the $CMOD$ value increases but the peak load decreases. For the 6.67% notch depth-to-beam thickness ratio, for example, the peak load is 12.5% lower than for the unnotched beam, whereas the $CMOD$ value is 28.9% higher.

d. **Effect of Cohesive Zone Width**

To investigate the influence of the cohesive zone width, three levels were considered: $T_0 = 1.0$, 0.01, and 0.001 in. Three-dimensional FE discretization of Beam B with linear softening was carried out. Figure 2.8 shows that as the cohesive zone width increased, the elastic deformation
of the cohesive zone increased, increasingly contributing to the elastic strain energy until damage began. Once damage started, the responses were not sensitive to the cohesive zone width. An increase in cohesive zone width decreased the peak load that could be supported by the beam. The 0.01 and 0.001 in. width, however, yielded almost the same result, and this led to the decision to use 0.001 in. in all subsequent sections.

e. Effect of Softening Curve

In the cases considered above, linear softening was used for its simplicity. A comparison between linear and bilinear curves employed in conjunction with Beam B indicated that linear softening would over-predict the peak load by about 11%, as shown in Fig. 2.9. After the peak, however, the bilinear curve eventually gave a higher load than the linear one, and there was a crossover on the curves. The areas under $P$-$CMOD$ curves for both softening assumptions were approximately the same, reflecting the equality of the areas under the two softening curves.

On the basis of the preceding investigations into the sensitivity of the proposed fracture formulation to the various discretization parameters examined, it can be concluded that careful attention should be given to the selection of the type of solver, of the width of the cohesive zone and of the type of softening curve in idealizing crack propagation in simply supported beams when cohesive elements are employed. Equipped with enhanced understanding of the effect of each parameter involved in cohesive zone FE analysis, one may proceed with the application of the approach to reproducing numerical and experimental results obtained by other researchers, as described in the following sections.
2.5 Comparison with Previous Numerical Studies

In this section, comparison is made of simulation results obtained from this study with those from numerical studies conducted by other independent researchers. The purpose is to validate the proposed FE procedure using corroborating evidence confirming its credibility.

2.5.1 GTSTRUDL/CRACKIT by Sengupta (1998)

In the earliest University of Cincinnati effort to simulate crack propagation in simply supported concrete beams, Sengupta (1998) developed a combination approach that employed a commercial software FE package, GTSTRUDL (GTSTRUDL, 1993), for the elastic response and the associated flexibility matrix, in tandem with a specially coded FORTRAN computer program named CRACKIT for the ensuing fracture behavior, in accordance with a bilinear softening law and the FCM. To illustrate his approach, Sengupta (1998) reproduced the response of a beam that had first been studied by Liu (1994), and discretized it with 4-node plane stress elements of size 0.2×0.5 in. Liu’s beam is Beam B in Table 2.1, and is considered in the present study with a 3D idealization using C3D27 elements of size 0.2×0.2×0.2 in. Results obtained using the 2D GTSTRUDL/CRACKIT combination approach by Sengupta (1998) and the 3D COH3D8 procedure employed for the cohesive fracture zone in this study are shown in Fig. 2.10. The P-LLD curves appear to agree better than the P-CMOD curves. The difference between the P-LLD curves can be explained by the difference in the intact region elements used (2D 4-node element versus 3D 27-node element), whereas the P-CMOD discrepancy is mainly due to the implicit assumption in CRACKIT that the notch remains undeformed until the crack begins to propagate.
2.5.2  *ABAQUS*® - JOINTC by Ioannides et al. (2005)

In order to simulate crack propagation in concrete beams using *ABAQUS*®, Ioannides et al. (2005) used *CPS4* elements for the intact region and a nonlinear spring element, *JOINTC*, for the fracture zone, prescribing the same bilinear FCM curve as Sengupta (1998). Beam A in Table 2.1 was considered and discretized with a coarse FE mesh consisting of elements of size 1 x 1 in. The notch depth-to-beam thickness ratio was 1/3. Identical mesh pattern and element type were used in the present 2D study, in which each intact zone element was a *CPS4* and the cohesive zone was discretized by using *COH2D4* instead of *JOINTC* elements.

The results are shown in Fig. 2.11. The cohesive fracture simulation gave higher load for a given *CMOD* as compared to the *JOINTC* fracture idealization in the post-peak stage. In general, however, it can be said that the two approaches are in good agreement for the particular mesh considered. Nonetheless, the wavy curves in Fig. 2.11 suggest some convergence difficulties, which may easily be overcome when the mesh is refined. To complicate matters, however, mesh refinement is also found to increase the discrepancy between *JOINTC* and the cohesive elements, as shown in Table 2.3. Further research is required to identify the cause of this phenomenon; the preliminary postulate is that it is related to differences in the assumptions of the two elements regarding pre-crack behavior.

2.5.3  *ABAQUS*® - 2D UEL by Roesler et al. (2007a)

Roesler et al. (2007a) employed a UEL for the cohesive zone, and a 4-node plane stress element for the intact material, in a mesh that was significantly finer near the cohesive zone than further away. The geometry and material properties of their beam are shown in Table 2.1, where it is
designated as Beam C. The following parameters were also adopted: $T_0 = 0.04$ in.; $E = 4.6$ Mpsi; $G_f = 0.323$ lb/in.; $G_F = 0.954$ lb/in.; $\psi = 0.25$.

To reproduce the results of the 2D analysis presented by Roesler et al. (2007a), Beam C was meshed uniformly in this study with $0.2 \times 0.2$ in. elements in both directions for the intact material, whereas the mesh of the cohesive zone was made 5 times as fine. The penalty stiffnesses for the given $E$ and $T_0$ values were computed as $K_{nn} = 118$ Mpsi/in. and $K_{ss} = 51$ Mpsi/in.

The $P$-$CMOD$ curve obtained in the present study along with that presented by Roesler et al. (2007a) is shown in Fig. 2.12. As it can be seen, good agreement is obtained between the two numerical simulations. The small difference in the elastic region may be due to the respective approaches followed in establishing the penalty stiffnesses.

2.5.4 *ABAQUS*® - 2D COH2D4 by Gaedicke and Roesler (2009)

Gaedicke and Roesler (2009) were the first to use the built-in 2D *ABAQUS*® cohesive element *COH2D4* to idealize the fracture process of simply supported beam for a variety of kink point locations. Their mesh pertained to Beam C and was similar to that used by Roesler et al. (2007a). Their $P$-$CMOD$ curve for $\psi = 0.25$ is plotted in Fig. 2.13, along with the corresponding results from the present study. The peak load predictions differ by about 7%; Gaedicke and Roesler (2009) had reported that their discretization “under-predicted the peak load by 12% and 7% with respect to the average and minimum experimental peak load, respectively.” This may again be attributed to penalty stiffness differences.
From the comparisons with previously reported results, it may be concluded that the proposed use of built-in *ABAQUS*® cohesive elements is effective in simulating PCC fracture in simply supported beams. Comparison with experimental measurements reported by other researchers appears in the following section.

### 2.6 Comparisons with Experimental Results

In this section, FE simulations conducted by means of the proposed procedure that implements cohesive elements in *ABAQUS*® are compared with experimental measurements reported by various independent researchers. The simulations employ 3D discretizations with bilinear softening curves in all cases.

#### 2.6.1 Experimental Results by Liu (1994)

Liu (1994) tested notched beam specimens under center-point loading. The pertinent geometry and average material properties reported are shown in Table 2.1, under the Beam B designation. A comparison of the $P$-$CMOD$ and $P$-$LLD$ curves is shown in Fig. 2.14. Good agreement is observed between the numerical solution in the present study and Liu’s experimental results, especially for the $P$-$CMOD$ curve. The small difference in the elastic portion of the $P$-$LLD$ was explained by Liu (1994) as “the result of support settlement, which can cause the measured load-point deflection larger than the actual one. As a result, the predicted curves are stiffer in comparison with the measured ones. A more sophisticated testing set-up is needed to overcome this problem.”
2.6.2 Experimental Results by Roesler et al. (2007a)

Beam C of Table 2.1 is considered here. Good agreement is obtained between the experimental results reported by Roesler et al. (2007a) and the FE simulation conducted in the present study, as shown in Fig. 2.15. In the elastic range, the present idealization gave a smaller load for any given CMOD as compared to the experimental results. This can be attributed to the use of low penalty stiffness. The numerical procedure reproduced the peak load very well. The post-peak behavior is accurately reproduced up to CMOD of 0.0063 in. If one keeps in mind the variability in the experimental results from replicate specimens reported by Roesler et al. (2007a), it can be concluded that the numerical simulation has reasonably captured the fracture process.

The results shown in Fig. 2.14 and 2.15 indicate the potential use of cohesive elements available in ABAQUS® to simulate crack propagation in concrete on the basis of the FCM. The findings from this study affirm the potential of the proposed numerical procedure when this is extended to PCC slabs-on-grade in the near future.

2.7 Conclusions

This study focused on the use of 2D and 3D cohesive elements that have recently become available in the commercial FE package ABAQUS® in studying crack propagation in simply supported concrete beams. The input parameters required for traction-separation cohesive elements include the tensile strength, the fracture energy, and the softening curve type. These elements were inserted in the anticipated cohesive fracture zone, and their top and bottom faces were tied to the beam elements on either side of the crack plane. Analyses conducted examined the effect of solution technique, mesh size, and width of cohesive zone.
In a comparison of the proposed approach with other numerical studies, good agreement was found with Sengupta’s GTSTRUDL/CRACKIT combination. The small discrepancy observed can be ascribed to assumptions implicit in CRACKIT. The proposed FE formulation also gave good agreement with the results from the UEL created by Park (2005). Results from the present study were also compared with experimental data reported by different researchers and good agreement was again found.

The main advantages of cohesive elements over other numerical simulations presented can be summarized as: (a) capability to be applied in 3D FE analysis; (b) feasibility of implementing different kind of softening curve assumptions; and (c) possibility of accommodating a variety of different failure criteria. Cohesive elements, however, are computationally demanding and tax an analyst’s discretization skills. For example, exploring material damage introduces nonlinearity to the system, which may result in convergence problems especially if the Newton-Raphson solution algorithm is used. To avoid this problem, the modified Riks method (or the Newton-Raphson method with viscous regularization) can be used.

From the findings in this study, it is anticipated that cohesive elements implementing traction-separation will be used in realistic problems involving PCC pavement slabs resting on layered foundations. It is hoped that the use of fracture mechanics concepts will eventually lead to the definition of more reliable and realistic failure criteria, which will address the current weaknesses of statistical/empirical transfer functions commonly employed in pavement design guides.
2.8 References


Table 2.1 Geometry and Material Properties of Beams Studied

<table>
<thead>
<tr>
<th>Beam</th>
<th>Thickness</th>
<th>Width</th>
<th>Length</th>
<th>Span</th>
<th>Young's Modulus</th>
<th>Tensile Strength</th>
<th>Fracture Energy</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$h$ (in.)</td>
<td>$b$ (in.)</td>
<td>$L$ (in.)</td>
<td>$S$ (in.)</td>
<td>$E$ (ksi)</td>
<td>$f'$ (ksi)</td>
<td>$G_F$ (lb/in.)</td>
</tr>
<tr>
<td>A (Sengupta, 1998)</td>
<td>6</td>
<td>1.5</td>
<td>16</td>
<td>16</td>
<td>4000</td>
<td>0.463</td>
<td>0.431</td>
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<tr>
<td>B (Liu, 1994)</td>
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<td>1</td>
<td>12</td>
<td>12</td>
<td>5405</td>
<td>0.463</td>
<td>0.431</td>
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<tr>
<td>C (Roesler et al., 2007a)</td>
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<td>43.3</td>
<td>39.4</td>
<td>4641</td>
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<td>0.954</td>
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Table 2.2 Deflection at Midspan of Beam A for Different Types of Elements

<table>
<thead>
<tr>
<th>Element Type</th>
<th>No. of Elements</th>
<th>$v_{max} \times 10^{-2}$ (in.)</th>
<th>Percentage (%)</th>
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<tr>
<td>Theory</td>
<td>NA</td>
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<td>$B22$</td>
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<td>3.673</td>
<td>104.1</td>
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<tr>
<td>$CPS4$</td>
<td>64×48</td>
<td>3.609</td>
<td>102.3</td>
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<tr>
<td>$C3D27$</td>
<td>5×40×30</td>
<td>3.538</td>
<td>100.3</td>
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Table 2.3 Comparison of *JOINTC* and *COH2D4* for Different Mesh Sizes (Beam A)

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Response</th>
<th>JOINTC (a)</th>
<th>COH2D4 (b)</th>
<th>Col. b ÷ Col. a (%)</th>
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<tr>
<td>8×6</td>
<td>$P_{\text{max}}$ (kips)</td>
<td>0.5658</td>
<td>0.5656</td>
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<td>CMOD/2 (in.)</td>
<td>0.0007</td>
<td>0.0006</td>
<td>92.12</td>
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<td></td>
<td>LLD (in.)</td>
<td>0.0016</td>
<td>0.0016</td>
<td>100.00</td>
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<td>16×12</td>
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<td>CMOD/2 (in.)</td>
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<td>LLD (in.)</td>
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<td>LLD (in.)</td>
<td>0.0018</td>
<td>0.0024</td>
<td>134.87</td>
</tr>
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</table>

Note: Applied vertical displacement = 0.03 in., $a/h = 33.33\%$, bilinear softening curve, $P_{\text{max}} =$ maximum load resisted by the beam
Figure 2.1 Comparison of Theoretical and Finite Element Results (Beam A)

Note: FE = finite element, TB = Timoshenko beam, TE = theory of elasticity, EB = Euler’s beam
Figure 2.2 Bilinear Softening Curve
Figure 2.3 Effect of Analysis technique (Beam A)
Figure 2.4 Effect of Analysis technique (Beam B)

Note: NR = Newton-Raphson
Figure 2.5 Effect of Mesh Fineness (Beam A)
Figure 2.6 Effect of Mesh Fineness (Beam B)
Figure 2.7 Effect of Notch Depth (Beam B)

Note: $a$ is notch depth, and $h$ is beam thickness
Figure 2.8 Effect of Cohesive Zone Width (Beam B)

Note: $T_0$ is cohesive zone width
Figure 2.9 Effect of Softening Curve (Beam B)
Figure 2.10 Comparison of 2D CRACKIT with 3D Cohesive Element (Beam B)
Figure 2.11 Comparison between JOINTC and Cohesive Element (Beam A)
Figure 2.12 Comparison with Numerical Results by Roesler et al. (2007a) (Beam C)
Figure 2.13 Comparison with Numerical Results by Gaedicke and Roesler (2009) (Beam C)
Figure 2.14 Comparison with Experimental Results by Liu (1994) (Beam B)
Figure 2.15 Comparison with Experimental Results by Roesler et al. (2007) (Beam C)
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Chapter 3  Numerical Analysis of Fracture Process in Pavement Slabs²

Abstract

This paper presents a numerical analysis of the fracture behavior of pavement slabs, using special-purpose cohesive elements. Hilleborg’s fictitious crack model is employed in sensitivity studies exploring the effect of a number of parameters on edge loading responses. Moreover, the case of interior loading is investigated, anticipating a future thermal stress analysis. Results are compared with previous experimental as well as numerical investigations conducted by other independent researchers. It is shown that cohesive elements are suitable for studying crack propagation as required in pavement engineering. It is envisaged that the approach presented in this study can be extended to more realistic in situ pavement systems, thereby addressing the limitations of current mechanistic-empirical pavement design procedures.

² A shorter version of this chapter has appeared in Canadian Journal of Civil Engineering, Vol. 39, No. 5, 2012, pp. 506-514.
3.1 Introduction

Portland cement concrete (PCC) slabs-on-grade are subjected to traffic loads and environmental stressors, mainly temperature and moisture, resulting in the formation of discrete cracks (Westergaard, 1927). Analytical as well as numerical idealizations of the slab behavior up to its elastic limit under those actions have been well documented (Huang and Wang, 1973). Additionally, considerable research has been devoted to post-elastic characteristics, following crack initiation (Ang et al., 1963; Ramsamooj, 1993; Ioannides and Peng, 2004). Naturally, the performance of the system is greatly affected by the processes of cracks formation and propagation. The present study is intended as yet another small contribution in this evolving field of research, which aspires to eventually replace the purely statistical/empirical transfer functions and Miner’s cumulative linear fatigue hypothesis (Miner, 1945), which are currently in use in mechanistic-empirical pavement design guides (AASHTO, 2008). More specifically, two problems frequently encountered in dealing with post-crack slab behavior are addressed in this paper. The first is that the complexity of the phenomenon defies closed-form theoretical treatment, especially for quasi-brittle materials like concrete, while the second is that its numerical simulation poses enormous computational challenges of its own.

For this study, post-crack analysis of slab-on-grade pavements is accomplished using a commercial finite element package ABAQUS®️, in which a nonlinear fracture mechanics (NLFM) application of the fictitious crack model (FCM), first proposed by Hillerborg et al. (1976), is implemented. A crucial aspect of the simulations presented is the use of cohesive elements for capturing deformation and damage, leading to a detailed discussion of how related computational issues may be resolved.
3.2 Literature Review

The majority of analytical approaches developed to date for crack simulation have been limited to linear elastic fracture mechanics (LEFM) concepts (Ramsamooj, 1993, 1994; Shah and Ouyang, 1994; Roesler and Khazanovich, 1997; Gotlif et al., 2006). Such approaches neglect the quasi-brittle nature of concrete and the creation of a large fracture process zone (FPZ) (Kaplan, 1961; Shah and McGarry, 1971). The relatively fewer attempts to employ concepts of NLFM to characterize concrete fracture have been summarized by Shah et al. (1995). These aim primarily at capturing the physics of the problem, while simultaneously remaining rather simplistic, lest their numerical implementation become prohibitively demanding. The FCM, first proposed by Hillerborg et al. (1976), is a case in point, since it has been found to simulate well the cracking process, while lending itself well for numerical algorithms. The FCM takes into account the interdependence of the traction pressure transferred at the crack tip and the corresponding material delamination occurring in the FPZ. Energy expended in creating new crack surfaces is neglected, since it is assumed to be very small compared to that needed to cause crack propagation.

The FCM has been implemented in finite element (FE) investigations of the crack process, beginning with two-dimensional problems (Hillerborg et al., 1976; Bažant and Oh, 1983; Liu, 1994). In order to extend the approach to increasingly more complex practical situations, a step-by-step approach has been pursued by researchers at the University of Cincinnati since the late 1990s, following an extensive review of fracture mechanics applications in pavement engineering (Ioannides, 1997a, 1997b). At first, a standalone computer program was coded and applied to simply supported beams (Ioannides and Sengupta, 2003). Subsequently, appropriate
elements in the commercially available FE program ABAQUS® Version 5.6 were sought (Ioannides et al., 2005). At the time, nonlinear spring element, JOINTC, was found to be suitable for this purpose, and this was adopted for concrete beams and slabs-on-grade subjected to mechanical loads (Ioannides and Peng, 2004; Ioannides et al., 2006). Similarly, investigators at the University of Illinois, created user elements (UEL) relying on FCM, and implemented them locally in commercial software to simulate crack propagation in concrete beam specimens (Park, 2005; Roesler et al., 2007), in asphalt specimens (Song, 2006; Song et al., 2006a, 2006b), and in functionally graded concrete materials (Evangelista et al., 2009; Park et al., 2010). Although these elements were two-dimensional, responses obtained were compared with experimental measurements and adequately good agreement was reported. Additional theoretical progress is to be expected following the recent formulation of a more general cohesive zone concept using potential energy fields (Park et al., 2009).

Numerical implementation of the FCM has been boosted significantly by the 2005 release of ABAQUS® Version 6.5, which for the first time included “a family of cohesive elements for modeling deformation and damage in finite-thickness adhesive layers between bonded parts.” Both aforementioned University research groups quickly embarked on related but independent simulations of crack propagation in beams and slabs-on-grade subjected to mechanical loads, with some very encouraging results (Gaedicke and Roesler, 2009; Aure and Ioannides, 2010). The present paper is the most recent product of this continuing effort, which may eventually enable engineers to develop rational failure criteria that can replace the statistical/empirical algorithms in current pavement design practice (AASHTO, 2008).
3.3 Finite Element Discretization

The present study utilizes general purpose FE program ABAQUS/STANDARD® version 6.9-2 for the analysis of pavement slabs-on-grade, whose geometry plus material properties are shown in Table 3.1. Unless and otherwise stated, the slab designated as SL1 in Table 3.1 is considered in most sections; this slab was previously analyzed by Ioannides et al. (2006), using JOINTC elements for the FPZ idealization.

To minimize resources expended, bulk material and cohesive FPZ are discretized independently: a coarse mesh of C3D27 elements is used for the intact material, pursuant to a conclusion from a linear elastic analysis of the slab to be presented in the next section, and a relatively fine mesh of 0.001×0.12×0.12 COH3D8 elements for the FPZ idealization, as indicated in Fig. 3.1. The two mesh sections are connected using surface-based TIE CONSTRAINTS, which enforce identical boundary conditions on nodes that lie on the interface.

Cohesive elements, COH3D8, are inserted along the anticipated fracture plane, in accordance with a discrete crack approach. This is deemed a reasonable representation for pavement slabs loaded either at the edge or at the interior, since the fracture plane is anticipated in the direction of the maximum stress. Moreover, Mode I or opening mode is assumed, since the contribution of the shear stresses is expected to be negligible.

Two Winkler-type subgrade idealizations are considered: SPRING1 elements that can only support compression; and FOUNDATION elements that can resist both tension and compression. Moreover, two loading scenarios are considered during the fracture analysis of the slabs: a unit displacement applied at two nodes on either side of the symmetry line at \((x, y, z) = (3, 6, 6 \text{ in.})\)
from the edge, as shown in Fig. 3.1, and a unit displacement applied over the nodes that lie within a 12 by 12 in. loading area, so that a rigid plate type of loading can be realized.

The responses monitored are as follows: load line displacement (LLD), i.e., vertical displacement at the center of the loaded area to the right of the symmetry (also, fracture) plane; maximum bending stress at the bottom of the loaded area ($\sigma_{\text{max}}$); crack mouth opening displacement (CMOD), i.e., the horizontal displacement at the bottom of the FPZ directly under the applied load; and total applied load ($P$), i.e., the sum of reaction forces at all loaded nodes.

### 3.4 Linear Elastic Analysis

In order to select an appropriate element for the discretization of its intact (uncracked) mass, a linear elastic analysis of slab SL1 is first conducted. Three discretization trials employing C3D27, C3D20R and C3D8R elements, which are described as 27-node full integration, 20-node reduced integration and 8-node reduced integration solid elements, respectively, are considered. The mesh is uniform in all three cases with $40 \times 20 \times 3$ elements in the length, width and depth directions, respectively, resulting in an element size of $6 \times 6 \times 2$ in. The subgrade is simulated by using the tension capable linear elastic FOUNDATION option available in ABAQUS®.

Pressure is incrementally applied at the slab edge over an area of $12 \times 12$ in., The maximum bending stress ($\sigma_{\text{max}}$) and load line displacement (LLD) values attained at the specified maximum pressure of 100 psi are given in Table 3.2, along with the analytical results obtained from Westergaard’s edge load solution (Westergaard, 1948). It is clearly observed that linear solid elements, C3D8R, result in a maximum stress in the slab that is only about 65% of that predicted by Westergaard and reproduced more precisely by the higher order elements.
To improve the performance of the $C3D8R$ elements, the alternating mesh pattern shown in Fig. 3.2 is considered; this is similar to the meshes used by Gaedicke and Roesler (2009). At the symmetry line of the slab, the element size is $0.1875 \times 0.1875 \times 0.1875$ in. whereas at the far ends of the slab, the element size is $12 \times 12 \times 3$ in. The total number of $C3D8R$ elements employed is 174,240, resulting in a total of 202,718 nodes. This is about 8 and 15 times the number of nodes generated with uniformly meshed $C3D27$ and $C3D20R$ elements, respectively. The maximum bending stress and deflection values for the alternating $C3D8R$ simulation are also shown in Table 3.2. The maximum stress now approaches 95% of the Westergaard value, but the cost of such an improvement in terms of computational resources expended is deemed prohibitive for general application. Therefore, simpler and coarser meshes of higher order elements are recommended, in order to ensure savings in computational resources without compromising the accuracy of the responses. Additionally, $C3D8R$ elements exhibit spurious modes in the vicinity of the loaded area, particularly when a concentrated load is applied. Therefore, unless otherwise stated, the full integration $C3D27$ element with a mesh size of $6 \times 6 \times 2$ in. is adopted in all subsequent sections in discretizing the intact material.

3.5 Sensitivity Studies

3.5.1 Effect of Concrete Softening Curve

The FCM assumes that the traction stress is purely a material property, independent of specimen geometry and size. The softening curve that relates the traction stress to the opening displacement is defined in terms of the total fracture energy, $G_F$, the concrete tensile strength, $f'_t$, and the shape of the curve. Several researchers have argued that the shape of the softening curve affects structural response significantly, particularly local failure behavior (Petersson, 1981;
Gustafsson, 1985; Roelfstra and Wittmann, 1986; Alvaredo and Torrent, 1987). Consequently, numerous different curve shapes have been proposed (Shah et al., 1995; Park et al., 2008).

Hillerborg et al. (1976) started with a simple linear softening curve, as shown in Fig. 3.3. Since the total fracture energy is equal to the area under the curve, the final (zero traction) displacement, \( w_f \), is:

\[
 w_f = 2 \frac{G_f}{f_t} 
\]  

Subsequently, Petersson (1981) and Gustafsson (1985) employed a bilinear softening curve characterized by an additional kink point, as shown in Fig. 3.3. If the coordinates of the kink point, \((w_k, \psi f_t)\), are set at \( (0.8, \frac{1}{3} f_t) \), the final displacement will be:

\[
 w_f = 3.6 \frac{G_f}{f_t} 
\]  

The location of the kink point, leading to Eq. (3.2), is contingent on two empirical assertions, represented by the coefficients 0.8 and 1/3, above, and has, therefore, been subject to debate. Bažant (2002) essentially eliminated the first of these through the introduction of initial fracture energy, \( G_f \), a specimen size dependent parameter, whose determination had been outlined by Bažant and Kazemi (1990). With regard to the bilinear softening curve, \( G_f \) is the area under the first branch, thereby defining the (extrapolated) horizontal intercept of this branch as:

\[
 w_1 = \frac{2G_f}{f_t} 
\]
Bažant (2002) retained the second coefficient, $\psi$, as an empirical assertion, but estimated that its value ranged from about 1/6 to 1/3. Accordingly, the final displacement can be written as:

$$w_f = 2 \psi f_i \left[ G_F - (1 - \psi) G_f \right]$$  \hspace{1cm} (3.4)

Eager to determine $\psi$ experimentally, Park et al. (2008) suggested using yet another parameter obtained from testing, this time as described by Jenq and Shah (1985), namely the elastic critical crack tip opening displacement ($CTOD_c$). Assuming that this is equal to the displacement at the kink point, the stress ratio, $\psi$, can then be established as:

$$\psi = 1 - \frac{CTOD_c f'_i}{2G_f}$$  \hspace{1cm} (3.5)

Additional softening curves have been proposed by various investigators, including exponential and power forms. For example, an exponential traction stress versus displacement is used by Gaedicke and Roesler (2009):

$$t'_s = f'_s \left[ 1 - \left( \frac{1 - \exp^{-\alpha w_c w_f}}{1 - \exp^{-\alpha}} \right) \right]$$  \hspace{1cm} (3.6)

In this expression, $w_{cp}$ is the cohesive zone separation at the crack initiation point. The softening parameter $\alpha$ may be determined by equating the integrand of Eq. (3.6) to the total fracture energy. In this study, numerical computing package MATLAB (Moler, 2004) was used for this purpose. Similarly, a power law softening shape is used by Song (2006):
Integration can be used again to compute the exponent parameter $\alpha$, for a given value of $w_f$.

In *ABAQUS*®, the softening curves are sub-divided into two regions for the purpose of integration: the elastic or pre-crack response and the damage or post-crack behavior. The pre-crack response is characterized by:

\[
\begin{bmatrix}
\varepsilon_n \\
\varepsilon_s \\
\varepsilon_t
\end{bmatrix} = \frac{1}{T_0} \begin{bmatrix} w_n \\ w_s \\ w_t \end{bmatrix}
\]  

(3.8)

where: $\varepsilon$ is a nominal strain and $w$ is the elastic separation vectors, in the normal and two shear directions, respectively; and $T_0$ is initial width of the cohesive zone. The elastic traction stress components can then be computed from:

\[
\begin{bmatrix} t_n \\ t_s \\ t_t \end{bmatrix} = K \begin{bmatrix} \varepsilon_n \\ \varepsilon_s \\ \varepsilon_t \end{bmatrix}
\]  

(3.9)

where $K$ is a nominal stiffness (also referred to as penalty stiffness) and $t$ is the nominal traction stress vector, in the normal and two shear directions, respectively. If uncoupled traction is assumed (as done in the present study), the off-diagonal terms in Eq. (3.9) are zero. The values of the penalty stiffness can be approximated by using the elastic moduli, $E$ and $G$ of the intact material, and the so-called characteristic width of the cohesive zone, $T_0$, i.e., $K_{nn} = E/T_0$, and $K_{ss} = K_{tt} = G/T_0$ (Daudeville et al., 1995).
Following the onset of the crack, and for as long as the strength of the cohesive zone exceeds that of the intact material, damage evolves according to the stiffness degradation variable, $D$, which is defined as:

$$D = 1 - \frac{{t_s'}}{{t_i'}} \quad \text{for} \quad w_{cr} \leq w \leq w_f$$

(3.10)

where: $t_s'$ is the traction stress for separation $w$, along the softening curve; $t_i'$ is the traction stress that would have corresponded to $w$ had the pre-crack stiffness endured (ABAQUS, 2009).

In the present study, the repercussions of softening curve selection are examined using as test slab the one designated as SL1 in Table 3.1. Four softening curves are considered, namely, linear, bilinear, exponential and power. All correspond to the same total fracture energy given in Table 3.1, as shown Fig. 3.3. For bilinear softening, the location of the kink point is situated per Petersson (1981) and Gustafsson (1985). Setting the final displacement for the exponential and power softening curves to the same value as obtained from Eq. (3.2) results in parameter $\alpha$, determined by integration of Eqs. (3.6) and (3.7), as 3.06 and 2.60, respectively. Results of the FE runs with each of the four softening curves are shown in Fig. 3.4. Linear softening is observed to give a slightly higher peak load than the rest. The difference between the linear and the bilinear curves is less than 2%, which is even lower than that reported for simply supported beams (Aure and Ioannides, 2010). In fact, all differences are negligible from a practical point of view. Recalling that previous studies (Roesler et al, 2007; Gaedicke and Roesler, 2009; Aure and Ioannides, 2010) indicated that bilinear softening predicts results that are in good agreement with experimental measurements, this relatively simple representation was adopted for the
present study. Convergence concerns (Gaedicke and Roesler, 2009) are addressed through viscous regularization, below. The analysis is carried out using both Newton-Raphson and modified Riks, which are to be discussed in a subsequent section below.

### 3.5.2 Effect of Viscous Regularization

A major challenge posed by materials exhibiting progressive damage and stiffness degradation is achieving a convergent solution, since this is sensitive to the type of softening curve, mesh size, penalty stiffness and type of solver used (Chaboche et al., 2001; Camanho and Dávila, 2002; Gao and Bower, 2004). The use of linear softening, moderately low penalty stiffness, and arc-length solvers, along with a very fine mesh may be expected to result in convergent solutions, but refining the mesh in large structural systems, like concrete pavements, can be computationally demanding. To overcome this problem, several researchers have resorted to viscous regularization. The idea was first proposed by Needleman (1988) for simulating plastic behavior, and was subsequently widely accepted (Chaboche et al., 2001; Gao and Bower, 2004; Maimi et al., 2007; Lapczyk and Hurtado, 2007; Hamitouche et al., 2008). By adjusting the value of a viscous term introduced in the constitutive equations of the degrading material, the rate of viscous energy dissipation is controlled, numerical instability is avoided, and the solution is regularized.

In order to address convergence problems with cohesive elements, ABAQUS® incorporates this concept in the form of a viscous stiffness degradation parameter, \( D_v \), defined by:

\[
\frac{d}{dt} \left( D_v \right) = \frac{1}{\mathcal{M}} \left( D - D_v \right)
\]  \hspace{1cm} (3.11)
where \( m \) is “the viscosity parameter representing the relaxation time of the viscous system,” \( D \) is “the degradation variable evaluated in the inviscid backbone model”, and \( t \) denotes time (\textit{ABAQUS}, 2009). The traction stress is then computed using \( D_v \) as:

\[
 t_s' = (1 - D_v) t_i'
\]  

(3.12)

The viscous system approaches the inviscid ideal as \((t/m)\) tends to infinity, making the viscous energy dissipated negligible. Viscous energy during unstable crack growth depends on the specimen geometry and material properties (Gao and Bower, 2004).

Accordingly, a sensitivity study was conducted in order to select the appropriate size of \( m \) for the problem size considered in the present study. Ideally, \( m \) should be as small as possible since it represents an artificial departure from the actual material constitutive relationship, but this requirement is tempered by the fact that larger \( m \) values are computationally less demanding.

Setting \( m \) to zero, results in abrupt solution termination due to lack of convergence. Different values of \( m \) ranging from \( 10^{-9} \) to \( 10^{-3} \) were, therefore, considered, and the results are presented in Fig. 3.5. Four stages are discernible in the load-displacement curve: linear, nonlinear, softening, rebound. The linear stage corresponds to the elastic behavior of the system, prior to crack initiation. Departure from linearity heralds the onset of cracking, but its location is sensitive to the value of \( m \) used. The second stage is nonlinear, suggesting that stable cracks are forming but the maximum load capacity of the slab is not yet attained. In the third stage, softening occurs evincing the formation of unstable cracks as the material continues to deteriorate. Finally, the fourth stage is reached, during which the subgrade springs assume the responsibility of sustaining the load. Figure 3.5 also indicates that the results for \( m = 10^{-6} \) and \( m = 10^{-9} \) are almost
identical, i.e., $m$ values lower than $10^{-6}$ do not affect the response; therefore, $m = 10^{-6}$ was adopted for the present study. Beyond this value, the solution becomes very sensitive to $m$ and diverges from the inviscid (actual) behavior, presumably because in its search for equilibrium, the solver employs grossly fictitious traction levels from Eq. (3.12) that exceed the actual material tensile strength, and increase the viscous energy dissipated.

The effect of viscous parameter is more pronounced when Newton-Raphson solution method is used compared to modified Riks method. For $m$ values greater than $10^{-6}$, the modified Riks method predicts a lower peak load than the Newton-Raphson (e.g., 32 versus 69 kips for $m = 10^{-3}$). For $m$ values lower than $10^{-6}$, however, the two solvers predicted identical peak loads, as shown in Fig. 3.5. The differences between the two solvers will be discussed in a subsequent section below.

3.5.3 Effect of Subgrade Idealization

The Winkler subgrade in concrete pavement systems is generally idealized using spring elements that can support both tension and compression or compression alone. The former approach is commonly preferred for its numerical simplicity in spite of the fact that it hardly represents in situ concrete pavement behavior. This section investigates the effect of both simulation approaches on the post-crack load-displacement responses. The tension and compression supporting subgrade is idealized using the built-in elastic FOUNDATION, providing the subgrade reaction as an input, while SPRING1 elements are employed in discretizing a subgrade that supports compression alone. The same modulus of subgrade reaction value is used in both cases by converting it to equivalent forces on the spring element depending on its contributing area. To compare the two simulations, a FE run of the SL1 slab is made and the load-
displacement curves are shown in Fig. 3.6. It can be clearly observed from the FOUNDATION simulation result that although there is a slight change in the slope of the curve evincing the initiation of cracking in the slab, there is no softening region, i.e., there is no indication of a complete slab failure. The absence of softening can be explained by the fact that the FOUNDATION elements pull down any slab elements attempting to lift up along the unloaded edge of the slab. For the SPRING1 discretization, on the other hand, the crack initiation point and the peak load point are clearly exhibited. It should be noted that the SPRING1 simulation requires larger amounts of computational resources since the solver utilizes smaller time increments to capture the solution path in the softening region. Generally, it can be concluded that the load-displacement response as well as the amount of computational resources expended are very much influenced by the subgrade idealization, and, therefore, the analyst must carefully weigh both options against prevailing in situ conditions before deciding to adopt either of the idealization schemes.

3.5.4 Effect of Cohesive Zone Width

The characteristic width of the cohesive zone, $T_0$, is used to determine the penalty stiffness components in the normal and shear directions for the cohesive element, as discussed above. In beam fracture analysis, a cohesive width of 0.001 in. was found to be a reasonable value to give a convergent solution (Aure and Ioannides, 2010).

The same approach may be extended to slabs. Four different $T_0$-values were tried: 0.001, 0.01, 0.1, and 1.0 in. Judging from the load-displacement curves shown in Fig. 3.7, changes in the cohesive zone width do not change the response substantially, although an increase in $T_0$ widens the crack mouth opening displacement slightly. This is due to the fact that increasing cohesive
width decreases the penalty stiffness and consequently increases the horizontal elastic displacement of the cohesive zone. As for beams, the 0.001 in. width is found to be computationally attractive without compromising the stability and accuracy of the solution. Therefore, it was adopted in this study.

3.5.5 Effect of Cohesive Zone Mesh

Three cohesive zone meshes, with aspect ratio of 3, 2, and 1, respectively, are considered. These give rise to COH3D8 element sizes of 0.75×0.25 in., 0.40×0.20 in., and 0.12×0.12 in, along the width and depth directions, respectively. For these cases, 1×160×24, 1×300×30, and 1×1000×50 cohesive elements, respectively, correspond to the 40×20×3 slab elements. Refining the cohesive zone mesh is found to decrease the number of increments needed to complete the applied displacement. Nonetheless, the load-displacement curves shown in Fig. 3.8 for all three cases are observed to be essentially identical, and, therefore, the least demanding 0.12×0.12 in. cohesive zone mesh is adopted in this study.

3.5.6 Effect of Solution Technique

In materials that exhibit snap-back type of load-displacement curves, it has been reported that the Newton-Raphson method fails to capture the softening region (Riks, 1979; Crisfield, 1991; Verhoosel et al., 2009). To remedy this situation, Yang and Proverbs (2004) conducted a study of several numerical methods and recommended the use of arc-length solvers, like the modified Riks, for problems involving material softening. In implementing the modified Riks algorithm, ABAQUS (2009) suggests that it can also be used for problems involving snap-back and snap-through load-displacement history. This suggestion was verified by Aure and Ioannides (2010),
who demonstrated the failure of Newton-Raphson to capture the softening region appropriately during the fracture of simply supported beams. The usefulness of modified Riks was especially noticeable for beams with smaller span-to-thickness ratios.

A similar comparison of the two methods is made in the present study for slab-on-grade fracture. Responses are compared in Fig. 3.4, where it is clearly seen that the modified Riks method captures the post-peak softening region, whereas the Newton-Raphson shows a vertical drop in the load at constant displacement. Despite this numerical disadvantage, however, careful adjustment of the modified Riks arc-length increment is required if the reloading curve is to be reproduced, as well. This requires increased effort, which accrues increased computational demand compared to Newton-Raphson. The inaccuracy observed by using the latter may in fact be tolerable in certain cases, owing to its numerical simplicity and shorter computational time, especially if only the peak load supported by the slab is of primary interest. In this study, Newton-Raphson was, therefore, retained only in the study of viscous regularization and of interior loading, for reasons explained elsewhere herein.

3.5.7 Effect of Loading Mode

In the preceding sections, the displacement is applied at the edge over two nodes located at \((x, y, z) = (3, 6, 6)\) in., per Gaedicke and Roesler (2009). This approach was compared to the case of a unit displacement applied at all nodes that lie within a 12 by 12 in. area, per Ioannides et al. (2006). The two scenarios correspond roughly to the application of concentrated and rigid plate loads, respectively. It is found that the cohesive elements start to be damaged earlier under the concentrated load scenario, resulting in a significantly lower peak load: 18 compared to 43 kips, respectively, as shown in Fig. 3.9. This is probably attributable to stress redistribution occurring
when a rigid plate load is applied, toward the corner nodes of the loaded area and away from the assumed fracture plane. The actual loading condition on \textit{in situ} slabs lies between these two extreme scenarios, probably being closer to the concentrated load case. The latter is, therefore, retained in this study for edge loading; for interior loading the rigid plate load is deemed more appropriate, as discussed in the pertinent section, below.

### 3.5.8 Effect of Tensile Strength and Fracture Energy

The softening curves depicted in Fig. 3.3 make it apparent that the two main material parameters influencing the fracture process are tensile strength, $f'_t$, and total fracture energy. To quantify the influence of these parameters on the load versus displacement response of the slab, one of them may be kept constant while changing the other. To illustrate this more simply, the linear softening curve and the Newton-Raphson solution method may be employed, as detailed next.

Let the tensile strength increase in 10\% increments over the value indicated in Fig. 3.3, to $1.1 f'_t$, $1.2 f'_t$, $1.3 f'_t$, and $1.4 f'_t$. The load versus displacement responses obtained are shown in Fig. 3.10: whereas the load at which the crack initiates increases slightly, the peak load and the post-peak softening behavior remain unaffected. In all cases, the material exhibits brittle behavior, with localized softening immediately following crack initiation. It is evident that although the tensile strength has a limited influence along the small softening segment of the load-displacement curve, it does not seem to play a major post-cracking role. Consequently, the peak load and the corresponding crack mouth opening and vertical displacements remain almost the same for all cases considered.
Similarly, five different values of total fracture energy, i.e., 0.9 \( G_F \), 1.1 \( G_F \), 1.2 \( G_F \), 1.3 \( G_F \) and 1.4 \( G_F \) may be considered, keeping the tensile strength constant. This simulation results in Fig. 3.11, which indicates that increasing the fracture energy gives rise to higher peak loads. The dimensionless ratios \( \beta = \frac{P}{P_{1GF}} \) of the peak load, \( \zeta = \frac{LLD}{LLD_{1GF}} \) of the vertical displacement at the peak load, and \( \eta = \frac{CMOD}{CMOD_{1GF}} \) of the crack mouth opening displacement at the peak load for each case considered, to the corresponding peak load, \( LLD \) and \( CMOD \) for the baseline fracture energy (1.0 \( G_F \)), are plotted against a dimensionless parameter called the brittleness number, \( B \), defined by Bache and Vinding (1990) as:

\[
B = \frac{f_t^2 h}{E G_F} = \frac{h}{l_{ch}} \tag{3.13}
\]

where \( l_{ch} \) is the characteristic length of the material, first introduced by Hillerborg (1985), \( E \) is Young’s modulus, and \( h \) is the slab thickness. The definition in Eq. (3.13) suggests that as the fracture energy decreases, \( B \) increases, i.e., the material becomes more brittle. Figure 3.12 confirms \( \beta \) is inversely proportional to \( B \). It may be postulated that a unique relationship exists that would allow one to determine the peak load for a particular value of \( B \), given the corresponding peak load for a different brittleness number, thereby providing a linkage between laboratory testing and field applications. A similar trend is observed in the plot of \( B \) versus \( \zeta \) and \( \eta \) corresponding to the peak load. Such observations are very encouraging in the quest for mechanistic post elastic failure criteria in pavement engineering, and underscore the desirability for treating fracture energy as equally important as the tensile strength.
3.6 Crack Propagation under Interior Loading

For PCC pavement slabs subjected to mechanical loads alone, edge loading causes maximum bending stress. If both thermal and mechanical loads act together, however, critical stress may sometimes occur at the interior of the slab. Anticipating a future study of thermal fracture, cohesive elements are used in this section to idealize interior loading cracks and a comparison is made with edge loading. The crack is assumed to propagate in two orthogonal directions from the center of the slab, per Meda et al. (2004). Therefore, cohesive elements are inserted centrally along the x- and y-axes, and are tied to the intact slab elements. The cohesive element properties are identical in both directions. In order to ensure a direct comparison between interior and edge loading, the rigid plate load scenario is used for both, in addition to the concentrated load (point displacement) case.

As expected, Fig. 3.13 shows that the maximum load supported by the slab is higher under interior than under edge loading: 72 versus 43 kips for the rigid plate load; 46 versus 18 kips for the concentrated loads. The trace in the vicinity of the peak load on the load-displacement curve indicates that the maximum stress is attained not at the center but at the corner of the loaded area, closest to the fracture plane. The first peak shows the progression of damage of the cohesive zone toward the center, whereas the second peak shows the complete failure of the cohesive zone as the crack proceeds along the prescribed fracture plane. It is also found that at any given applied load, the crack opens up more along the width (shorter dimension) than along the length.

Crack growth patterns along the bottom of the FPZ in the length and width directions at load levels shown in Fig. 3.13 are plotted in Fig. 3.14. It is observed that at load level point 3, nodes directly under the loaded area have exceeded the final (zero traction) displacement, i.e., the
displacement at which traction stress is zero, in the length direction, while the crack remains stable in the width direction. At load level point 7, the displacement at the bottom of the slab in both width and length directions has exceeded the zero traction value, indicating that the load is now supported by the subgrade.

### 3.7 Crack Propagation in a Slab Supported by Base Layer

In general, concrete pavement structures incorporate a second man-made layer, commonly called the base layer, on which the concrete slab is poured and compacted, in order to enhance constructability. This section investigates the effect of the stiffness of the base layer on the post-crack response of the slab by considering a base layer of the same plan dimensions as SL1, and 8 in. in thickness. To simplify the analysis, it is grossly assumed that the base layer remains intact, behaves elastically and remains in full contact with the subgrade during the loading period. Moreover, any influence of the base layer on the traction-separation relationship of the concrete slab is neglected. An untreated aggregate base (UAB) and a cement treated base (CTB) with Young’s modulus values of 25 and 1500 ksi, respectively, are assumed. The Poisson ratios of the two materials are taken to be 0.35 and 0.2, respectively.

The slab mesh is kept identical to the previous cases, i.e., $40 \times 20 \times 3$ C3D27 elements, whereas the base is discretized with relatively coarser mesh size, $20 \times 10 \times 2$ C3D27 elements. The interface between the two layers is idealized using contact interactions, specifically INTERACTIONS, HARD CONTACT, in which unlimited normal compressive pressure is transmitted from the slab to the base layer. The two layers are assumed to be unbonded, i.e, no frictional stresses arise at the interface. This is realized by assuming a zero coefficient of friction between slab and base.
Since the subgrade is assumed to remain in full contact with the base layer during the loading period, the tension supporting elastic *FOUNDATION* is employed to simulate the subgrade.

The analysis results for the two base types are presented in Fig. 3.15. As may be anticipated, the presence of the UAB increased the peak load only very slightly (by about 5%), as compared to the more prominent contribution by the CTB, which quadrupled this load. It can also be seen from the CTB curve that the difference between the load at which the crack initiates, which is made manifest by the change in slope of the curves, and the peak load attained is much larger when the CTB replaces the UAB (35 compared to 4 kips, respectively). This indicates the development of stable cracks in the system incorporating a CTB. The results reconfirm the fact that the presence of a stiff base enhances the strength and fracture resistance of the slab, provided drainage issues that may be associated with the use of a CTB are addressed.

### 3.8 Comparison with Other Numerical and Experimental Studies

This section compares the FE idealization employed in the present study with numerical analyses provided by Ioannides et al. (2006) and by Gaedicke and Roesler (2009), as well as the experimental results reported by Gaedicke and Roesler (2009).

#### 3.8.1 Comparison with Ioannides et al. (2006)

Ioannides et al. (2006) used nonlinear spring element *JOINTC*, available in *ABAQUS*® to simulate the fracture process in concrete pavement slabs implementing the FCM. The geometry and material properties of the slab was identical to SL1 shown in Table 3.1. The slab was discretized with a relatively coarser mesh of $40 \times 10 \times 3$ *C3D27R* element as compared to the $40 \times 20 \times 3$ *C3D27* element outlined above. An initial notch depth of one-tenth and one-third of the
slab width and thickness, respectively, was provided to initiate the crack. Displacement control was used with a maximum displacement of 1 in. over an area of 12×12 in., thereby simulating a rigid plate loading scenario. The subgrade was idealized using *FOUNDATION*, which supports tension as well as compression as described above.

In the present study, an identical FE simulation is adopted except that cohesive elements are used instead of *JOINTC* elements for the idealization of the FPZ, and the *C3D27* element is employed instead of the *C3D27R* for the idealization of the intact material. The load-displacement results, shown as *P-CMOD* to be consistent with Ioannides et al. (2006), are presented in Fig. 3.16. It is observed that the simulation with the cohesive elements gives a lower load for a given *CMOD*, which is a similar observation to that made in an earlier study for simply supported beams (Aure and Ioannides, 2010). The main reason for these discrepancies may be the development of high tensile stresses at the nodes of the elements shared by *JOINTC* and *C3D27* elements along the fracture plane. Ioannides et al. (2006) speculated that the occurrences of such tensile stresses that are greater than the material failure stress are related to the “stress gradient criteria of strength for quasi-brittle materials” (Kharlab and Minin, 1989). In the cohesive fracture reproduction, on the other hand, the stresses along the fracture plane do not exceed the tensile strength, since the latter is defined as the failure criterion. In either case, post-peak softening is not exhibited on the load-displacement curves on account of the tension supporting foundation, as explained earlier.

### 3.8.2 Comparison with Gaedicke and Roesler (2009)

Gaedicke and Roesler (2009) also used built-in traction-separation cohesive elements to simulate cracking in PCC slabs-on-grade in *ABAQUS®*. Two slab geometries had been considered: 2.48
in. and 5.9 in. thick, by 78.74 × 78.74 in. in plan. For the present comparison, the first of these is reproduced, assuming a one-third notch. The geometric and material properties of this slab are described in Table 3.1, under slab label SL2.

A total of 283,904 elements had been employed out of which 21,504 were COH3D8 and the remaining were C3D8R, the latter being used for the intact material. In view of the computational intensity of this discretization, a coarser but uniform mesh is employed in its reproduction during the present study, using a higher order element, C3D27, as delineated in Table 3.1. A total of 12,160 elements out of which 2,560 were COH3D8 and 9,600 were C3D27 were employed. For simplicity, linear softening is retained herein, along with a penalty stiffness computed from $E/T_0$, where $E$ is the slab Young’s modulus and $T_0$ is the cohesive zone width, set to 0.039 in. The foundation is simulated using tensionless SPRING1 elements, and horizontal springs with a stiffness of $1/10^{th}$ of that of the vertical springs are used to provide lateral restraint, both per Gaedicke and Roesler (2009).

The simulation result is shown in Fig. 3.17. It is observed that the load-displacement curves from the two studies are almost identical up to 4 kips. Beyond that point, cracking is initiated, and the two simulations diverge until the peak load, which is also almost the same. The post-peak unloading, during which the cohesive elements become completely damaged, occurs at a much lower load in the present study. This is probably related to the much coarser mesh used in the present study.

The corresponding experimental result reported by Gaedicke and Roesler (2009) is also plotted along with the two numerical simulations in Fig. 3.17. Good agreement is obtained up to the
peak load. The post-peak behavior shows differences primarily due to the linear elastic
idealization of the subgrade. The experimental result shows a pronounced post-peak vertical drop
in the load, mainly due to local plastic yielding of the \textit{in situ} soil. Considering the overriding
practical importance of the peak load, rather than of the softening region, the results obtained
from the simulation approach followed in the present study are deemed to be adequately accurate
as well as computationally efficient.

3.9 Conclusions

The use of cohesive elements in the simulation of PCC pavement slab fracture has been
investigated by studying the main parameters that affect pre- and post-peak responses.
Discretization with cohesive elements is computationally demanding as it involves material
softening, which influences the stability of the solution algorithm. Consequently, appropriate
solution control strategies (such as viscous regularization) need to be employed.

The effects of softening curve, cohesive zone width and mesh design, analysis technique, loading
mode, tensile strength and fracture energy has been investigated. It is found that the type of
softening curve, cohesive zone width and mesh design do not influence the response
significantly. They do, however, play a significant role in the convergence of the solution. The
influence of loading placement (at the interior or at the edge of the slab) has been examined, on
account of its importance for \textit{in situ} pavements under simultaneous thermal and mechanical
loads. It is found that the fracture process is more sensitive to the fracture energy than to the
tensile strength.
It can be concluded from this study that the application of cohesive elements in fracture analysis of PCC pavement slabs is promising, as demonstrated by the comparison with experimental results. It is anticipated that the approach can be extended to pavements subjected to thermal stresses and incorporating load transfer. This effort will contribute to the ongoing development of rational failure criteria that can substitute the statistical/empirical algorithms along with Miner’s hypothesis used in current mechanistic-empirical pavement design procedures.

3.10 References


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Ioannides, A. M. (1997b). "Pavement fatigue concepts: a historical review." Proc., Sixth International Purdue Conference on Concrete Pavement Design and Materials for High Performance, Purdue University, Indianapolis, IN, 147-159.


Table 3.1 Geometry, Material Properties and Discretization of Slabs Considered

<table>
<thead>
<tr>
<th>Slab Geometry</th>
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<tr>
<td>Slab</td>
<td>SL1</td>
<td>SL2</td>
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<tr>
<td>Length (in.)</td>
<td>240</td>
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</tr>
<tr>
<td>Width (in.)</td>
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<td>78.74</td>
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<td>Thickness (in.)</td>
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<td>Tensile Strength (ksi)</td>
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<td>Fracture Energy (lb/in.)</td>
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<th>Subgrade Characteristics</th>
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<td>2×2×0.4</td>
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<tr>
<td>Cohesive Zone Element Size (in.)</td>
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<td>Concrete Softening Curve</td>
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<td>Linear</td>
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<td>Riks or Newton-Raphson</td>
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<tr>
<td>Subgrade Idealization</td>
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<td>SPRING1</td>
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</table>

Notes: 1 lb = 4.444 N; 1 in. = 25.4 mm; 1 ksi = 6.89 MPa
Table 3.2 Comparison of Different Solid Elements Available in *ABAQUS*

<table>
<thead>
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<th>Element</th>
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<td>743.56</td>
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<td>768.37</td>
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Figure 3.1 Finite Element Discretization of Slab (SL1)

Figure 3.2 Finite Element Mesh Pattern Using C3D8R Elements (SL1)
Figure 3.3 Different Types of Concrete Softening Curves

\[ f'_t = 0.46327 \text{ ksi} \]

\[ \alpha = 2.60 \]

\[ \alpha = 3.06 \]

\[ (w_k, \psi f'_t) = (0.000745 \text{ in.}, 0.1544233 \text{ ksi}) \]

\[ w_f = 0.0033525 \text{ in.} \]

\[ w_f = 0.0018618 \text{ in.} \]

\[ w_j = 0.0018618 \text{ in.} \]

\[ w_j = 0.0033525 \text{ in.} \]
Figure 3.4 Effect of Concrete Softening Curve and Analysis Technique

Note: NR = Newton-Raphson
Figure 3.5 Effect of Viscous Regularization on Slab Response
Figure 3.6 Effect of Subgrade Idealization on Slab Response
Figure 3.7 Effect of Cohesive Zone Width
Figure 3.8 Effect of Cohesive Zone Mesh
Figure 3.9 Effect of Loading Mode
Figure 3.10 Effect of Tensile Strength

Note: Baseline $f'_t = 0.463$ ksi
Figure 3.11 Effect of Fracture Energy

Note: Baseline $G_F = 0.431$ lb/in.
Figure 3.12 Variation of Normalized Peak Load, $LLD$ and $CMOD$ with Brittleness Number

\[ \beta = -0.766B + 1.5735 \]

\[ R^2 = 0.9929 \]
Figure 3.13 Slab Responses under Different Loading Scenarios Applied at Interior and Edge
Figure 3.14 Crack Growth Along Slab Length and Width at Points Shown in Fig. 3.13
Figure 3.15 Effect of Base Stiffness on Slab Response
Figure 3.16 Comparison with Numerical Results by Ioannides et al. (2006)
Figure 3.17 Comparison with Numerical and Experimental Results by Gaedicke and Roesler (2009)
Chapter 4  Curling Effects on Concrete Pavement Fracture

Abstract

This paper discusses finite element analysis of the behavior of pavement slabs-on-grade undergoing cracking due to temperature and wheel loads. Traction-separation cohesive elements are used to simulate the fracture process according to the fictitious crack model. To verify the proposed discretization, pre-crack analysis of the slab subjected to curling alone, as well as to curling combined with a wheel load, is carried out first, and the results are compared with previous finite element and analytical solutions available in the literature. The three-dimensional analysis presented herein produces lower curling stresses compared to earlier two-dimensional calculations, but excellent agreement is confirmed with previous three-dimensional simulations. The investigation is subsequently extended to the post-cracking stage, for which the effects of linear and nonlinear temperature distributions, notch depth, slab size, slab self-weight, and concrete age are investigated. All these factors are found to influence cracking due to curling, except for slab self-weight, which is observed to be less significant for this loading stage than before cracking. Findings presented elucidate concrete pavement fracture under wheel load and curling. The procedures adopted may be extended to more complex in situ pavement systems incorporating load transfer, thereby addressing the limitations in current pavement design procedures that exclusively rely on statistical algorithms for the prediction of pavement distresses.
4.1 Introduction

The first analytical attempt to address the problem of concrete pavement curling was due to Westergaard (1927), who provided the well-known equations for the critical responses at the slab edge and interior. In the derivation of these equations, Westergaard employed grossly restrictive assumptions, including infinite slab self-weight, linear temperature distribution through the slab thickness, and applicability of the principle of superposition when combining temperature-related and load-induced stresses (Ioannides et al., 1999). Recognizing the importance of thermal stresses, subsequent studies have endeavored to eliminate Westergaard’s assumptions, particularly following the development of computer programs implementing two-dimensional (2D) finite element (FE) analysis (Huang and Wang, 1974; Tia et al., 1987; Korovesis, 1990; Khazanovich, 1994). Application of such curling investigations, however, has pertained solely to the pre-crack aspects of slab behavior, in view of the limitations of 2D analysis.

More recently, three-dimensional (3D) FE analysis has attracted the attention of pavement engineers, who have used either commercially available packages, such as ABAQUS® (Kuo, 1994; Masad et al., 1996; Pane et al., 1998), ANSYS (Chen et al., 2002; Mahboub et al., 2004; Dere et al., 2006; Siddique et al., 2006), and LS-DYNA (Shoukry et al., 2007), or stand-alone 3D FE codes developed specifically for pavements, such as EverFE (Davids et al., 1998) and Pave3D (Nishizawa et al., 2001; Shimomura et al., 2008). The most promising application of 3D FE analysis is probably in the realm of post-fracture pavement behavior, and this has already prompted a number of independent, yet complementary investigations of pavement slabs subjected to wheel load acting without curling (Barros and Figueiras, 2001; Meda et al., 2004; Ioannides et al., 2006; Gaedicke and Roesler, 2009; Aure and Ioannides, 2012). Such efforts,
however, have yet to be extended to the consideration of thermal curling, either by itself or in combination with the wheel load. The only research effort in this direction has been that of Channakeshava et al. (1993), who considered a dowel-jointed concrete pavement under wheel load and linear temperature gradient, and who employed a smeared crack approach and a plastic material constitutive relation. The latter two features are not pursued in this thesis, and this precludes further consideration of this pioneering publication.

The main objective of this paper is to investigate the post-cracking behavior of a concrete pavement slab subjected to both thermal curling and static wheel load. The concepts of nonlinear fracture mechanics (NLFM) are employed, as incorporated in traction-separation cohesive elements, recently implemented in a general purpose FE package, ABAQUS® (ABAQUS, 2009). The presentation below is organized as follows. The proposed FE discretization is validated first by application to the elastic (pre-crack) stage of the pavement slab under curling alone, as well as curling combined with wheel load. The simulation is subsequently extended to the corresponding post-cracking behavior.

4.2 Finite Element Discretization

The present study utilizes general purpose FE program ABAQUS/STANDARD® version 6.9-2 for the analysis of pavement slabs-on-grade subjected to wheel load and thermal curling. Unless and otherwise stated, the slab considered is 240 in. long, 120 in. wide and 6 in. thick. Typical concrete material properties are: Young’s modulus, $E$, of 4,000 ksi; Poisson’s ratio, $\mu$, of 0.2; concrete unit weight, $\gamma$, of 150 pcf; and coefficient of thermal expansion, $\alpha$, of $5\times10^{-6}$/°F. The modulus of subgrade reaction, $k$, was taken as 200 pci; this slab was previously analyzed by the authors (Aure and Ioannides, 2012) considering wheel load alone.
The fracture plane is anticipated perpendicular to the length direction of the slab, in accordance with a discrete crack approach. This is deemed a reasonable representation for pavement slabs loaded either at the edge or at the interior, since the fracture plane is anticipated in the direction of the maximum stress. Moreover, Mode I or opening mode is assumed, since the contribution of the shear stresses is expected to be negligible. Further discussions on the FE discretization of the entire pavement system is given in the following sections.

4.2.1 General Considerations

Among available options for discretizing a slab-on-grade system, previous researchers had used either the 20-node quadratic element (Pane et al., 1998; Chen et al., 2002; Mahboub et al., 2004; Siddique et al., 2006) or the reduced integration 27-node element (Kuo, 1994; Hammons, 1997; Ioannides et al., 2006). The main disadvantage of the 20-node element is the lack of mid-face nodes, which are helpful in specifying the temperature variation through the slab thickness. On the other hand, the reduced integration 27-node element is often found to produce spurious deformations at unrestrained nodes, particularly when the slab loses contact with the subgrade due to curling (ABAQUS, 2009). Consequently, the 27-node full integration element, available in ABAQUS® under the name C3D27, is adopted for the analyses presented below. A uniform mesh pattern is used for the pavement slab, with 6×6×2 in. elements (length×width×thickness, respectively).

The subgrade is idealized using SPRING1 elements supporting only compression. The force-displacement relationship for each element of this type is hand-calculated with reference to the tributary areas surrounding its node, and is provided in the input file as a table. For numerical stability, lateral restraint is provided using additional SPRING1 elements, whose horizontal
stiffness is set to $1/10^\text{th}$ of the value of the vertical springs, per Gaedicke and Roesler (2009). This may be justified as simulating friction between the slab and the subgrade.

Temperature variation through the slab thickness is idealized by assigning nodal values. This is accomplished using the keyword TEMPERATURE. It is assumed that the temperature remains identical at all nodes lying on the same horizontal plane of the slab. When needed, a wheel load is applied as a point displacement at two nodes on either side of the fracture plane at $(x, y, z) = (3, 6, 6)$ in. from the symmetry line at the edge, as explained further below.

4.2.2 Fracture Process Zone (FPZ)

To minimize resources expended, the fracture process zone (FPZ) and the bulk uncracked material are discretized independently. The FPZ is idealized using traction-separation cohesive elements ($COH3D8$) of finer mesh size of $0.12 \times 0.12 \times 0.001$ in. This is connected to the much coarser mesh of $C3D27$ elements representing the intact mass using surface-based TIE CONSTRAINTS, which enforce identical boundary conditions at the nodes that lie on the interface.

The cohesive elements are inserted along the anticipated fracture plane of the slab, consistent with a discrete crack approach; this is deemed to be a reasonable assumption for pavement slabs loaded at the edge, and it is justified by the commonly observed in situ crack patterns. Moreover, Mode I fracture is assumed, since the contribution of shear stresses is expected to be negligible. The fracture process can be distinguished into pre- and post-crack initiation phases, depending on the stress level developing in the FPZ, as detailed in two earlier publications pertaining to cracking in a simply supported beam, as well as an isolated slab-on-grade under static load (Aure...
and Ioannides, 2010; Aure and Ioannides, 2012). The penalty stiffness and viscous regularization techniques used in those earlier studies are retained here, as well. Moreover, linear and bilinear concrete softening is retained for the curling-only and combined curling-with-wheel load cases, respectively, chiefly on the grounds of expediency. A complete discussion of the softening options available is provided by Aure and Ioannides (2012).

The responses monitored are as follows: longitudinal horizontal stress through the slab thickness ($\sigma_{xx}$); load line displacement ($LLD$), i.e., the vertical displacement at the center of the loaded area to the right of the fracture plane; crack mouth opening displacement ($CMOD$), i.e., the horizontal displacement at the top or bottom of the FPZ; and the total applied load ($P$), i.e., the sum of the conjugate “reaction” forces at the two loaded nodes, $RF3$.

4.3 Pre-Crack Analysis of Slab under Thermal Curling Alone

As a prelude to the study of fracture behavior, pre-crack elastic analysis of the slab subjected to a temperature distribution alone is carried out first, and the results are compared with available analytical, experimental and FE results found in the literature. This initial step is considered essential for assessing the robustness of the proposed FE discretization of the intact slab mass and the subgrade.

4.3.1 Comparison with 3D FE Analysis

In the aforementioned 3D FE study by Pane et al. (1998), a 30-ft long, 24-ft wide, and 9-in. thick slab was subjected to a temperature distribution described by a cubic polynomial, per Thompson et al. (1987) and Mohamed and Hansen (1996). The temperature profile at 3 pm is re-examined in the present study, retaining the following cubic variation through the slab thickness:
\[ T(z) = 1.4762 - 0.5291z + 0.2685z^2 - 0.0514z^3 \]  

(4.1)

Here, \( z = 0 \) denotes the mid-depth of the slab and \( z = h/2 \) its bottom, where \( h \) is the thickness of the slab. Typical material properties are retained from Pane et al. (1998): Young’s modulus, \( E \), of 4,000 ksi; Poisson’s ratio, \( \mu \), of 0.15; concrete unit weight, \( \gamma \), of 150 pcf; and coefficient of thermal expansion, \( \alpha \), of \( 5 \times 10^{-6}/^\circ F \). The modulus of subgrade reaction, \( k \), was taken as 100 pci.

The variation of the longitudinal stress \( \sigma_{xx} \) (positive if tensile) through the thickness of the slab occurring at the slab center is shown in Fig. 4.1. It can be verified that the 3D FE results obtained in this study agree well with those reported by Pane et al. (1998), despite differences in the type and number of elements used.

As an additional comparison, the same slab is analyzed in this study using 3D FE software EverFE v. 2.24 (Davids et al., 1998). The latter can only accommodate a trilinear temperature distribution, and this is easily extracted from Eq. (4.1) by dividing the slab thickness into three equal segments. Surprisingly, the simulation results plotted in Fig. 4.1 disclose that EverFE overestimates the stress at the bottom of the slab (\( z = +4.5 \) in.) by about 15%. This may be due to the linearization of the temperature profile, differences in element types used as well as numerical solution strategies implemented in the two softwares.

The 3D FE simulation scheme adopted in this study is also applied in reproducing experimentally measured curling stresses reported by Teller and Sutherland (1935) on a concrete pavement slab 20-ft long, by 10-ft wide, and 6-in. thick. Temperature differential values (\( \Delta T \)) along with corresponding curling stresses had been provided (measured between April 18 and May 19, 1934). Kuo (1994) adopted a linear temperature distribution in his own 3D FE
reproduction of the measurements, and assumed Young’s modulus of 5000 ksi, Poisson’s ratio of 0.15, concrete unit weight of 0.083 lb/in.$^3$, and modulus of subgrade reaction of 200 pci. Table 4.1 presents a three-way comparison of the center bending stresses from these two previous investigations, as well as from the present study. It is observed that the 3D FE idealizations agree with one another, yet overestimate the measured curling stresses, typically by about 10%. This discrepancy could be the result of the relatively coarse meshes adopted, as well as the fact that “the observations extended over a considerable period of time… [and] the temperature differentials [and therefore the corresponding stresses] were typical of the highest average values”, per Teller and Sutherland (1935). Nonlinearity in the in situ temperature distribution, as well as the existence of any so-called built-in upward curling arising during the construction process could also contribute to the observed differences (Eisenmann and Leykauf, 1990).

4.3.2 Comparison with 2D FE Analysis and Westergaard (1927)

Choubane and Tia (1992) monitored pavement temperatures in six slab-on-grade pavements using thermocouples embedded at the center of one of the slabs at 1.0, 2.5, 4.5, 6.5, and 8.0 in. below its top surface, and reported that the variation of the temperature profile was nonlinear, for which the following simple quadratic form was suggested:

$$T = A + Bz + Cz^2$$  \hspace{1cm} (4.2)

where: $A$, $B$, and $C$ are coefficients that can be determined from the measured data; $T$ is the temperature in degrees Fahrenheit; $z$ is the slab depth, with $z = 0$ at the top and $z = h$ at the bottom; and $h$ is the slab thickness. Note that Choubane and Tia (1992) adopted the quadratic assumption merely for the sake of expediency, even though several researchers caution that the
actual temperature variation may require a higher order polynomial (Teller and Sutherland, 1935; Thompson et al., 1987).

Each slab was 20-ft long, 12-ft wide and 9-in. thick. Choubane and Tia (1992) conducted a 2D FE analysis of one of the slabs using the program FEACONS-IV and compared the results with the theoretical solution by Westergaard (19927), which they misguidedly attributed to Bradbury (1938). The following material properties were assumed: Young’s modulus of 4,500 ksi; Poisson’s ratio of 0.2; coefficient of thermal expansion of $6 \times 10^{-6}$/°F; and unit weight of 150 pcf. The modulus of subgrade reaction was taken to be 300 pci.

The same slab was re-examined using the 3D FE simulation of the present study, retaining the nonlinear temperature profiles reported for the month of June. The corresponding coefficients for Eq. (4.2) are shown in Table 4.2.

Figures 4.2 and 4.3 show the temperature variation, in °F, and the computed longitudinal bending stress, $\sigma_{xx}$, respectively, at the slab center through the slab thickness for the quadratic distribution considered in this study between 9 am and 8 pm (called daytime). Similarly, Fig. 4.4 and 4.5 show the temperature profile and the corresponding longitudinal bending stress, $\sigma_{xx}$, respectively, for the nighttime (10 pm to 6 am). As expected, the stress profiles reflect the quadratic shape of the temperature distribution; any kinks observed in Fig. 4.3 and 4.5 are attributable to the relatively coarse mesh employed. Almost without exception, the critical bending stress occurs at the slab’s extreme fibers, where it may trigger either bottom-up or top-down cracking. Nonetheless, at 9:00 am, maximum tension occurs at about 3 in. from the bottom, which suggests that thermal cracks may in fact initiate within the slab mass. The overall maximum tensile stress is observed at 3:00 pm, which may be the time of traffic congestion, as
well. Such an unfavorable combination of traffic and thermal loads will certainly aggravate the formation of pavement cracks.

The bending stresses at the top ($\sigma_t$) and at the bottom ($\sigma_b$) of the slab computed by 3D nonlinear curling analysis are compared with those from an additional 3D run of the corresponding linear distribution using the temperature differential ($\Delta T$) between the top and bottom of the slab given in Table 4.2. Results are shown in Table 4.3. It is observed that the linear profile overestimates the bottom-fiber stresses and underestimates the top-fiber stresses, except at 3:00 pm and 8:00 pm. The off-trend characteristic exhibited at these two times can be explained by inspecting more closely the temperature profiles shown in Fig. 4.2 and 4.4. During the daytime, it can be seen that the temperature profile is convex to the right, except at 3:00 pm, when it exhibits an opposite curvature. The latter gives rise to lower temperatures according to the linear profile as compared to the nonlinear one. At 8:00 pm, the temperature is almost identical at the top and bottom. Consequently, tensile stresses of almost the same magnitude are obtained at the top and at the bottom for the nonlinear temperature profile. At some other points, however, compressive stresses are observed. This observation has significant consequences for the slab post-crack response, which will be shown later to be influenced by the overall stress distribution through the thickness of the slab.

The 3D FE results obtained in this study for the linear distribution are also compared in Table 4.4 to those reported by Choubane and Tia (1992), from their 2D FE analyses. For the sake of completeness, the results from the analytical solution by Westergaard (1927) are also presented. It is observed that the present 3D FE idealization generally gives results that are lower than those reported from the 2D analysis, discrepancies ranging between 2 to 25%. A similar trend is also
observed when comparison is made with the analytical result. These discrepancies can be partly explained by the relatively coarse 3D mesh size adopted in this study. Another cause may be the theoretical differences between 2D plate theory and 3D continuum FE formulations, as inferred from the closeness of the 2D FE result to that of Westergaard. It is recalled that Westergaard’s “special theory” is an effort to recapture some of the lost strain energy in the plate-on-dense liquid idealization (Ioannides et al., 1985)

For the nonlinear temperature distribution, the stresses obtained at the top and bottom using the present 3D FE analysis are compared in Table 4.5 to those reported by Choubane and Tia (1992) as resulting from a combination of 2D FE analysis and their proposed analytical solution for nonlinear curling. As for the linear distribution, the 3D FE output records smaller bending stresses than the 2D FE result (in the range of 4 to 40%), except at 8:00 pm, presumably for the same reasons explained above.

4.4 Pre-Crack Analysis of Nonlinear Curling-and-Wheel Load

A pre-crack 3D FE discretization similar to that described in the previous section is also employed here for the case of a nonlinear temperature distribution combined with a wheel load. The 3D FE analysis of the slab under both actions is achieved by splitting the solution process into two steps, namely temperature and wheel loading, per Mahboub et al. (2004). In the temperature step, the responses due to curling and self-weight are computed, whereas in the second step the effect of the wheel load on the already temperature-stressed slab is assessed. A general nonlinear Newton-Raphson solution technique is employed (Aure and Ioannides, 2012) to capture the geometric nonlinearities caused by curling and the tensionless subgrade.
The proposed *ABAQUS*® 3D FE simulation is first applied to a 12-ft wide, 15-ft long, and 9-in. thick concrete pavement slab resting on a 200 pci subgrade, previously investigated by Khazanovich (1994) using the 2D FE code *ILSL2*. Typical material properties are retained: Young’s modulus of 4,000 ksi; Poisson’s ratio of 0.15; and coefficient of thermal expansion of $5 \times 10^{-6}/\degree F$. The slab is loaded by a wheel load of $P = 10,000$ lbs, at a pressure $p = 100$ psi applied at its corner in addition to its self-weight of 150 pcf. The temperature distribution reported by Richardson and Armaghani (1987) in the month of August at Gainesville, FL, is adopted. A quadratic temperature profile per Eq. (4.2), with coefficients for the daily cycle given in Table 4.6, is assumed.

Simulation runs are made for the temperature profiles given in Table 4.6, in which the principal bending stresses at the top and bottom of the slab are recorded for two scenarios. The first set of results is termed the combination approach and corresponds to the proposed FE solution, in which the application of the temperature distribution is followed by the wheel load applied on the deformed shape of the slab. The second scenario is termed the superposition approach, according to which individual responses from each of the two actions are simply summed up. For this purpose, the 3D FE run of the slab under the wheel load alone is conducted and the maximum principal bending stress at the top of the slab is found to be about 0.245 ksi, and occurs at 28.69 in. from the corner of the slab. Table 4.6 indicates that the two approaches generally yield similar results, except at noon and 4 pm, when the top is decidedly warmer than the bottom. At these times, superposition leads to gross overestimates of the maximum bending stress: 175% and 155% of the corresponding combination stresses, respectively. The discrepancy is attributed to the difference in boundary conditions in the two scenarios, exacerbated by the
application of a relatively light load on a relatively stiff subgrade. The adequacy of superposition, therefore, cannot be taken for granted, confirming previous observations in the published literature (Ioannides and Salsilli-Mura, 1989). The overall largest diurnal maximum principal bending stress is observed at midnight, presumably because the critical stress due to both corner loading and nighttime temperature distribution occurs at the top.

Results from the proposed *ABAQUS*® simulation are compared in Fig. 4.6 to those reported by Khazanovich (1994) on the basis of 2D FE results using program *ILSL2*, and to those from the 3D FE program *EverFE* v. 2.24 executed during this study. Similar trends are exhibited by all three software. It is observed, however, that *EverFE* tracks the *ABAQUS*® results more closely than *ILSL2*, especially at the slab top. Comparing stress values at any specified time may point to discrepancies sometimes exceeding 50% (see Table 4.7), and may lead to misleading conclusions as to the repercussions of using each particular analytical tool. The sensitivity of the results to mesh refinement in each case should be carefully assessed in the context of the respective theory employed, i.e., continuum or plate.

The discrepancies between 2D and 3D FE results certainly warrant further exploration, preferably employing carefully measured field responses. For the purposes of this study, however, the 3D FE formulation proposed is judged to be consistent with reliable state-of-the-art efforts, and is, therefore, suitable for the investigations presented in the remainder of this paper.

### 4.5 Fracture Response of Pavement Slab Subjected to Thermal Curling Alone

Cracking in concrete pavement slabs may occur under thermal curling alone especially at an early age, when the slab exhibits lower material strength. Using 2D FE methods, Jenq et al.
(1993) studied the effect of temperature on the formation of cracks in concrete pavement slabs at the age of 16 hours, 24 hours, and 36 hours. They investigated the effect of notch depth, $a$, ranging from 0 to 6 in.; and slab thickness, $h$, ranging from 6 to 12 in. The notch is formed at the top of the slab, just as saw-cutting is conducted in the field; consequently, a nighttime temperature differential is considered, which causes maximum curling stresses at the top, and encourages a crack beneath the notch to propagate to the slab bottom. They reported that the notch depth and saw-cut timing influenced significantly the critical temperature resisted by the slab. For a 6-in. thick unnotched slab of age 16 hours, for example, a nighttime (cooler top) temperature differential of about 30°F would cause random thermal cracks, and, therefore, a cut should be made earlier. The 2D FPZ idealization adopted by Jenq et al. (1993) entailed node-by-node crack progression. The applied temperature resisted may, therefore, have been overestimated, whereas the corresponding crack mouth opening displacement may have been underestimated.

In this study, the material properties provided by Jenq et al. (1993) as shown in Table 4.8, are retained in an investigation of the fracture behavior of a concrete pavement slab, 240-in. long, 120-in. wide, and 6-in. thick, under thermal curling alone. The subgrade reaction and the concrete unit weight are assumed to be 200 pci and 150 pcf, respectively. A linear concrete softening curve is used in this section for simplicity (Aure and Ioannides, 2012). Consequently, the FPZ damage initiates when the tensile strength is attained; thereupon the cohesive stress that resists cracking decreases linearly until all the fracture energy is consumed. In order to capture accurately the post-peak characteristics of the crack mouth opening versus applied temperature differential and profile responses, the modified Riks solver is employed (Aure and Ioannides,
The effects of notch depth, concrete age, self-weight, nonlinearity of temperature profile, and slab size on the fracture process are discussed below.

### 4.5.1 Effect of Notch Depth

Joints are cut in concrete pavement slabs in order to avoid random cracks that may result from thermal stresses at an early age. The depth and width of the cut and its timing are very important in controlling the extent of the cracks (Jenq et al., 1993; Soares, 1997). In this study, the effect of notch depth, $a$, is investigated by adopting the material properties of concrete at the age of 16 hours given in Table 4.8. Four different notch depths of 0, 1/2, 2, 3 in., are considered, corresponding (for the 6-in. slab adopted) to notch depth-to-slab thickness ratios ($a/h$) of 0, 1/12, 1/3, and 1/2, respectively. Cracks are assumed to propagate from top to bottom as a nighttime (cooler top) linear temperature differential is applied, toward a specified maximum temperature difference of 54°F.

Figure 4.7 shows the variation of applied temperature differential ($\Delta T$) versus $CMOD$. As might be expected, as the notch depth increases, the critical temperature differential supported by the slab decreases. Stable cracks are formed until the critical temperature differential is attained (e.g., 33°F for the unnotched case); this is followed by softening, signaling the complete damage of the cohesive elements. Once the cohesive elements along the assumed fracture plane are damaged completely, the slab is divided into two essentially independent segments and the location of the maximum thermal stress moves toward their respective centers. If the temperature differential exceeds the values indicated in Fig. 4.7, unstable cracks will form and the slab would fail at the age of 16 hours.
The results from this study are also compared with those from Jenq et al. (1993) in Fig. 4.8, in which a similar trend is exhibited. The discrepancies between the two responses may be attributed to differences in the respective simulations, including in the idealization of the FPZ, the linear versus quadratic temperature profiles employed, the inclusion of a base layer and the adoption of an elastic foundation by Jenq et al. (1993), as well as mesh fineness. The effect of the temperature profile is pursued further in subsequent sections.

### 4.5.2 Effect of Concrete Age

The effect of concrete age on the fracture process is examined at 16 hours, 24 hours, and 36 hours, through increases in the concrete Young’s modulus, tensile strength and fracture energy, as measured in the laboratory by Jenq et al. (1993). A constant notch depth of 0.5-in. at the top of the slab is retained here along with the nighttime (cooler top) linear temperature differential specified above. Results in Fig. 4.9 indicate that, as expected, the critical temperature differential resisted by the slab increases significantly with age, reflecting concrete maturation. Concomitantly, the corresponding CMOD decreases slightly with age. It is also noted that in the post-peak or softening portion of the response curves, a much more pronounced reduction in the temperature differential resisted is experienced as concrete maturity increases. This is attributable to the increased fracture energy with age (see Table 4.8), which results in slower damage in the FPZ, exhibited as a smaller (fracture energy-to-temperature differential loss) ratio.

The effect of age on the critical linear temperature differential resisted by the slab may be examined using dimensionless parameters $\alpha \Delta T$, and $B$, in which: $\alpha$ is the coefficient of thermal expansion, a concrete material characteristic assumed to remain constant over time; $\Delta T$ is the
critical temperature differential between the bottom and top of the slab; and $B$ is the brittleness number, defined by Bache and Vinding (1990) as:

$$B = \frac{f'_t h}{E G_F} = \frac{h}{l_{ch}}$$

(4.3)

where $l_{ch}$ is a characteristic length of the material; $f'_t$ is the tensile strength; $E$ is Young’s modulus; $G_F$ is the fracture energy; and $h$ is the slab thickness. The result is plotted in Fig. 4.10, where it is observed that as the brittleness number increases, the maximum linear temperature differential resisted by the slab increases almost linearly. The corresponding results from Jenq et al. (1993) are also plotted in Fig. 4.10. Differences between the two simulations outlined above notwithstanding, a similar trend are observed in their results, inviting further exploration of dimensional analysis for interpretation of post-crack responses in pavement slabs.

### 4.5.3 Effect of Slab Self-Weight

Concrete self-weight can exacerbate curling stresses and the formation of discrete cracks. For the sake of comparison, consider the same 6-in. concrete slab as in previous sections, with four different assumed unit weights of 37.5, 75, 150, and 300 pcf, at 16 hours of age. The slab has a 0.5-in. top notch, and is subjected to the nighttime (cooler top) linear temperature differential specified above. As the slab curls upward, the portion of the slab that loses contact with the subgrade is acted upon by its self-weight, in a cantilever action with the support in the FPZ, contributing to curling stresses. Consequently, lower curling stresses develop in slabs of a smaller self-weight, and such lighter slabs fail at a higher critical temperature differential, as observed in the results presented in Fig. 4.11. Nonetheless, this effect is not significant for typical concrete unit weight values (e.g., between 50% and 200% of the commonly assumed self-
weight of 150 pcf). Similar observations are anticipated for a daytime (warmer top) temperature differential, as well, since the latter has an analogous effect, except for causing bottom-up cracking. For their part, Jenq et al. (1993) did not study the effect of concrete self-weight.

4.5.4 Effect of Nonlinearity of Temperature Distribution

In the pre-fracture analysis of curling stresses presented above, it was found that the bending stress distribution through the slab thickness depends on the profile of the temperature distribution. In some instances, it was observed that a linear temperature variation results in higher top or bottom bending stresses than the corresponding nonlinear distributions considered. To study this effect in the aftermath of crack initiation, the 6-in. pavement slab at 16 hours of age with 0.5-in. notch at the top is considered again. Three nighttime (cooler top) temperature profiles are investigated: a linear; a quadratic with downward curvature; and a quadratic with upward curvature, as shown in Fig. 4.12 (inset). Note that in all three cases, the applied temperature differential between the bottom and top remains at 54°F. The responses shown in Fig. 4.12 reflect the pattern displayed by the temperature distributions, inasmuch as the linear one lies in-between its two quadratic counterparts. The results also indicate the sensitivity of slab response to the entire temperature profile and not merely to the temperature differential, retained as constant in these analyses. Moreover, recalling the results in Figs. 4.2 through 4.5, it is asserted that pavement slab response here reflects the entire distribution of curling stresses, and not merely the extreme-fiber values. Slab failure results from the dissipation of fracture energy through the entire slab thickness. Such dissipation depends on the overall prevailing stress state and not merely on the extreme-fiber stresses. The presumed conservative nature of the commonly used linear temperature distribution in concrete pavement analysis that only considers
extreme-fiber stresses is, therefore, called into question. This is especially true at an early age, when temperature variation is the sole cause of cracking; the degree to which the same observation also applies once the concrete hardens and the pavement is opened to traffic will be taken up again in Section 4.6, below.

4.5.5 Effect of Slab Size

In his pioneering paper on curling, Westergaard (1927) demonstrated that pre-crack curling stress increases almost linearly as slab size increases, until a dimensionless slab size, $L/l$ or $W/l$, of about 8 is reached, beyond which stresses remain nearly constant. Here, $L$ denotes the slab length, $W$ the slab width and $l$ the radius of relative stiffness of the slab-subgrade system. In this study, the effect of slab size on the post-crack curling response is investigated by considering once again the 16-hour old 6-in. slab, with a 0.5-in. top notch and nighttime (cooler top) linear temperature variation scenario. The following sizes are considered: $L = 120, 180, 240, 360, and 480$ in., keeping $W = 120$ in.; $W = 120, 150, 180, and 240$ in., keeping $L = 240$ in.

The dimensionless critical temperature differential, $\alpha \Delta T$, for each case is plotted in Fig. 4.13, as a function of the dimensionless length and width, $L/l$ and $W/l$. It is observed that as slab length increases, the critical temperature differential drops drastically, until $L/l$ reaches 10. Beyond $L/l = 10$, however, the response remains almost unaffected, with only a slight increase of the critical temperature differential after about $L/l = 13$, hearkening back to Westergaard’s own observations. On the other hand, for the particular cases considered in the present study, increasing the slab width leads to critical temperature differentials that decrease slightly in a linear manner. Note that the FPZ is assumed to extend along the shorter (width) dimension of the slab.
For their part, Jenq et al. (1993) simulated the same slab supported on 9-in. base of modulus 50 ksi resting on elastic solid foundation of modulus 30 ksi, underlain by a rigid base. Their FE analysis employed plane strain conditions, i.e., their implicit slab width is infinite. They retained slab length of 24 ft and considered thicknesses of 6, 9, and 12 in., reporting the attained $\Delta T$ value in each case. The complete data were reported in Liaw (1992), and are also plotted in Fig. 4.13 (assuming $k = 500$ pci), in which they appear to confirm the trends obtained in this study.

From the preceding investigations of slab fracture due to curling alone, the following conclusions can be reached:

a. Thermal cracking is significantly affected by the age of the concrete, which dictates the time and size of joint sawing. The dimensionless brittleness number, $B$, can be used to characterize the effect of concrete age on the fracture process;

b. Surprisingly, the slab self-weight is observed to be unimportant with regard to the fracture process under curling alone, except possibly for very light-weight materials;

c. The complete temperature profile through the slab thickness exerts a significant influence on the fracture process under curling alone, and evinces the importance of the entire stress distribution rather than of merely the extreme-fiber stresses. This result calls for the accurate determination of pavement temperature profiles;

d. The fracture process due to curling is particularly sensitive to slab sizes below $L/l$ of 10, at least for the cases considered in the present study.

### 4.6 Fracture Response of Slab Subjected to Wheel Load and Thermal Curling

In this section, the post-crack behavior of a pavement slab subjected to both thermal curling and a wheel load is investigated for the first time in the published literature. Geometry and material
properties for the slab are selected to be consistent with earlier analyses conducted by Ioannides et al. (2006) and Aure and Ioannides (2012); these are provided in Table 4.9. The FE discretizations for both the fracture process zone and the bulk (intact) material remain identical with those in Section 4.3, above.

When examining post-crack behavior, a wide range of options is available with regard to the combined curl-plus-load condition. Two extreme situations are easily envisaged: (a) a constant thermal gradient is applied in combination with an increasing applied wheel load; (b) a constant applied load exists in combination with an increasing thermal gradient. The actual field situation is probably somewhere between these two extremes, with the proportion of stress contributed by each of the two factors being in a state of constant flux. Only the two extreme combinations among the host of possibilities will be pursued below.

4.6.1 Slab under Constant Thermal Gradient and Increasing Wheel Load

In this situation, the two-step pre-crack FE procedure adopted by Mahboub et al. (2004) is retained. In the first step, the constant temperature distribution is applied along with the slab self-weight, whereas in the second step, the increasing wheel load is additionally applied. In the discretization proposed herein, measured nonlinear temperature profiles are considered. The temperature distributions reported by Thompson et al. (1987) and by Choubane and Tia (1992) cannot be used, since in both those cases the slabs were thicker. Consequently, the measured temperature profiles for a 6-in. slab reported by Teller and Sutherland (1935) were adopted. Diurnal and monthly variations are available. In this study, the profiles for July 12 and 13, 1932 are selected, and these are discretized using a cubic polynomial, so that temperature values may be assigned at each node in the FE mesh. The equation of the cubic polynomial is of the form:
\[ T = Az^3 + Bz^2 + Cz + D \]  \hspace{1cm} (4.4)

where \( z = 6 \) in. at the bottom of the 6-in. thick slab and \( z = 0 \) at its top. Coefficients, \( A, B, C, \) and \( D \) of the polynomial, are given in Table 4.10. For comparison purposes, temperature differentials corresponding to each hour are also listed. The wheel load is applied as an increasing point displacement placed symmetrically at \((x, y, z) = (3, 6, 6)\) in., per Fig. 4.14. Post-crack responses are tracked, as noted in Section 4.2.2, above. These load-plus-curl results are compared to the corresponding load-only outputs from Chapter 3 (Aure and Ioannides, 2012) in order to assess the influence of introducing thermal regime considerations. More specifically, the sensitivity of post-fracture response to the diurnal cycle, to the nonlinearity of the temperature profile, and to the geometric and material properties of the system is investigated in the following sections.

a. Effect of Daytime and Nighttime Variations on Fracture

The FE simulation is performed for the constant temperature profiles described in Table 4.10 combined with an increasing applied load. Selected results are presented in Fig. 4.15, along with previously published data by Aure and Ioannides (2012) for the no-curling (i.e., wheel load only) case. During the daytime, at 9:30 am, the peak load resisted by the slab is approximately 50% of the value obtained under wheel load alone (8 kips versus 18 kips). This is explained by the fact that a significant portion of the bending strength of the slab is consumed by thermal stresses, prior to the application of the wheel load. The reverse is observed at 7:00 pm, when the peak sustainable load reaches more than 200% of the curl-free condition (50 kips versus 18 kips).

During the daytime the slab curls down and acts as a plate supported solely along its boundaries, since only its periphery remains in contact with the subgrade. Unstable cracks form, followed by a pronounced softening effect. During the nighttime, on the other hand, as the slab curls up, a
very substantial portion of its bottom surface remains supported by the subgrade. This condition, combined with curling stresses that counteract the loading stresses, result in stable crack formation, followed by only a relatively mild capacity reduction during softening. It is interesting to note in Fig. 4.15 that during the night, the bulk of the wheel load is applied after crack initiation (manifested in an abrupt slope change at load level 1), whereas during the day, the peak load is reached shortly after the onset of cracking, at load level 1. This also evinces the greater stability of fracture during the nighttime as compared to the daytime.

Figures 4.16 and 4.17 show the variation of crack mouth opening displacement across the bottom of the width of the slab along the assumed fracture plane. The curves correspond to specific load levels of interest as already indicated in Fig. 4.15, for the temperature profiles reported at 9:30 am and 7:00 pm. Also plotted is the failure displacement, $w_f$, i.e., the CMOD at which the traction stress reaches zero. At 9:30 am, Fig. 4.16 shows that about 40% of the slab (45 to 55 in.) width has experienced bottom cracking at load levels 1 and 2. The failure displacement has exceeded over more than half the cracked portion (35 to 45 in.). By load level 3, the slab has completely failed, since the cracks along the entire width of the slab exceeded the failure displacement; evidently, any additional load beyond load level 3 is carried by the subgrade.

For the 7:00 pm temperature profile, Fig. 4.17 indicates that the failure displacement is not exceeded at all at load level 1, suggesting that the observed cracks are stable and cohesive elements are still offering support. As the load increases, unstable cracks begin to form. At load levels 2 and 3, for example, more than 55% and 90%, respectively, of the width of the slab exhibits crack mouth opening displacements greater than the failure displacement. By load levels 4 and 5, the entire slab width has experienced crack mouth openings greater than the failure
displacement, indicating that cohesive elements no longer offer resistance to the applied load. Comparing crack growth at the two peak load levels (load level 2 at 9:30 am and load level 3 at 7:00 pm), it is apparent that crack mouth opening displacement is much larger at 7:00 pm than at 9:30 am. For example, at 30 in. from the origin, the CMOD is 0.035 in at 7:00 pm (Fig. 4.17, load level 3) compared to 0.006 in. at 9:30 am (Fig. 4.16, load level 2). Therefore, it may be concluded that although the slab exhibits a substantially higher strength during the nighttime, its concomitant larger crack mouth openings may lead to other unforeseen problems, such as pumping. It is envisaged that this type of analysis will become increasingly meaningful as the profession advances toward design procedures that rely on distress determination from mechanistic analysis rather than from statistical algorithms.

b. Comparison of Linear and Nonlinear Temperature Distributions

Analysis of a slab subjected to thermal curling alone presented earlier in this Chapter has confirmed fracture responses are sensitive to the shape of the temperature profile through the slab thickness. It has also been observed that these responses depend on the entire distribution and not merely on the extreme-fiber values. This effect is re-examined in this section by considering the linear and cubic temperature profiles (per Table 4.10) at 9:30 am and 7:00 pm, in combination with an increasing wheel load.

The load-displacement responses from the pertinent FE runs are plotted in Fig. 4.18. For both cases considered, results show that the peak loads when linear temperature distribution is assumed are faintly greater than the corresponding values when cubic temperature profile is adopted. At 9:30 am, for example, the peak load corresponding to the linear temperature distribution is 15% greater than that of the cubic profile (8.8 kips versus 7.7 kips). This may
appear at first to be an unexpected result: the FE output indicates that prior to the load application, the curling-only extreme-fiber bending stress due to the linear temperature profile is higher than for the cubic distribution, by 13%. The latter observation is also consonant with some earlier conclusions (Section 4.5.4) that a linear distribution may lead to higher extreme-fiber stresses, which may facilitate crack initiation. The peak load, however, depends not only on those stresses, but also on the overall stress gradient through the slab thickness. This phenomenon was first noted by Kharlab and Minin (1989), who proposed using a “stress gradient criterion of strength” rather than relying on the maximum stress alone. The FE results obtained in this study confirm the necessity of accurate temperature profile determination, as well as the need for a 3D FE analysis if fracture under curling is to be tracked.

c. Effect of Slab Length and Width

As discussed earlier in Section 4.5.5 for cracking due to curling alone, the critical temperature differential resisted by the slab is not sensitive to its length for $L/l$ value when this is greater than 10. In this section, it is desired to examine whether the same conclusion holds true when the slab is subjected to thermal curling and wheel load. To investigate this, a constant daytime (warmer top) linear temperature differential of $24^\circ$F is applied followed by an increasing wheel load. The slab sizes used earlier are retained.

The peak load, $P_{\text{max}}$, i.e., the load beyond which softening occurs, may be rendered dimensionless and plotted against the dimensionless slab length, $L$, or slab width, $W$; this is accomplished by retaining the ratios $(L/l)$ and $(W/l)$, per Westergaard (1927), and introducing $(P_{\text{max}} / f'fh^2)$, in accord with Bache and Vinding (1990). The trend revealed in Fig. 4.19 is similar to that from the case of cracking due to curling only: slab size beyond $(L/l)$ of 10 does not
affect the peak load supported by the slab. The width effect is similar, albeit much less pronounced, as might be expected. A more comprehensive dimensionless representation of phenomena like this would entail dimensionless parameters for the geometry of the applied load (tire print radius, $a$) and for the applied temperature distribution, e.g., $(a/l)$ and $(\alpha \Delta T)$, per Ioannides and Salsilli-Murua (1989), as well as for the slab’s self-weight, $D_\gamma$. The latter was first suggested in the former Soviet Union by Korenev and Chernigovskaya (1962), and subsequently applied in the United States by Lee and Darter (1994); its influence is discussed further below.

d. Effect of Slab Self-Weight

In order to investigate the effect of the self-weight of the slab, $\gamma$, the typical value of 150 pcf assumed in the previous sections is incremented through multiplication by a factor of 0.5, 2, 5, and 10, specified for the 240- by 120-in. slab. A constant daytime (warmer top) linear temperature differential of 24°F is applied, followed by an increasing wheel load, as done for the size effect study above. Results pertaining to the load-displacement behavior, shown in Fig. 4.20, indicate that a change in the slab self-weight does not significantly affect the peak load resisted by the slab. The vertical load line displacement at the peak load, $\Delta$, however, increases with an increase in self-weight. This displacement may be rendered dimensionless using the standard $(\Delta kl^2/P)$ form (Ioannides et al., 1985), and plotted against the dimensionless self-weight, $D_\gamma$, which is defined as:

$$D_\gamma = \frac{\gamma h^2}{kl^2}$$

(4.5)

where $k$ is the modulus of subgrade reaction, $l$ is the radius of relative stiffness; $P$ is $P_{\text{max}}$, $\Delta$ is the LLD corresponding to $P_{\text{max}}$, and $h$ is the slab thickness, respectively. The result shown in Fig.
4.21 confirms that the lighter the slab, the smaller the curling-plus-load deflection at the peak load, presumably because the edge load location on the lighter slab lifts up more. As its self-weight increases, more of the slab remains in contact with the subgrade and a more gradual fracture process takes place. The softening segment of the load-displacement curves in Fig. 4.20 is seen to decrease.

e. Effect of Tensile Strength and Total Fracture Energy

During fracture analysis of a pavement slab subjected to wheel load alone (Aure and Ioannides, 2012), it was found that tensile strength influences the load at which cracking begins (manifested in an abrupt slope change in the P-LLD curve), whereas fracture energy determines the peak load at which the slab ruptures completely along the assumed fracture plane (evinced by softening). A similar investigation is replicated in this section using the same 120 by 240-in. slab subjected first to a constant linear daytime (warmer top) temperature differential of 24°F, followed by an increasing wheel load. The tensile strength, $f'_t$, baseline value of 0.463 ksi (per Table 4.9) is modified to 0.9$f'_t$ and 1.4$f'_t$, keeping the total fracture energy constant. Similarly, in examining the effect of total fracture energy ($G_F$), the baseline value of 0.431 lb/in. (see Table 4.9) is incremented to 0.9$G_F$, 1.2$G_F$ and 1.4$G_F$, keeping the tensile strength constant. The load-displacement response for each case is shown in Fig. 4.22 for the tensile strength and Fig. 4.23 for the fracture energy factorials. Also shown in both figures are some of the corresponding results for a slab under wheel load alone, presented in Chapter 3 (Aure and Ioannides, 2012). Figure 4.22 shows that as tensile strength increases, the peak load also rises modestly, but the change in the softening segment is barely significant, since the fracture energy is assumed to be constant. It is observed that the presence of curling causes the peak load to be reached
immediately after crack initiation. On the contrary, when only wheel load is applied, there is a localized softening after crack initiation, followed by a stable crack growth until peak load is attained. This is caused by the pre-existing widespread curling stresses along the entire assumed fracture plane when wheel load is applied, compared to the localized stresses that first develop when the wheel load is applied by itself, and that continue to increase gradually along the fracture plane.

Figure 4.23 shows that increasing total fracture energy results not only in a higher peak load, but also in a diminished post-peak loss due to softening. Moreover, displacement experienced during the softening stage also increases, and the softening segment becomes less precipitous, i.e., the material exhibits increased ductility, indicating the development of a stable crack. In general, it can be concluded that daytime curling combined with wheel load reduces the extent of stable crack growth in the slab, a phenomenon that can be compensated by increasing total fracture energy, for example, by using fibers.

4.6.2 Slab under Constant Wheel Load and Increasing Thermal Gradient

In this loading combination situation, a constant magnitude of wheel load is applied in the first step, followed by an increasing daytime differential temperature toward a maximum value $\Delta T = 96^\circ F$. To investigate the effect of the magnitude of the applied load ($P$), FE runs for $P$ values ranging from 0.001 kips to 7 kips are made. The differential temperature ($\Delta T$) versus load line displacement ($LLD$) response for selected cases is shown in Fig. 4.24. As may be anticipated, when the applied load increases, the differential temperature needed to cause the slab failure decreases, since some of the strength of the material is consumed by the applied load prior to the application of temperature. When 4 kips and 7 kips wheel loads are applied, for example, the
peak temperature differentials (i.e., the values corresponding to slab failure) are only about 22°F and 12°F, respectively.

Examining the deformed shape of the slab under the 7-kip load alone, it is found that crack starts propagating before temperature application begins. Therefore, as temperature differential increases gradually, so does the load line displacement, evincing stable crack growth. In contrast, under the 4-kip or 1-kip loads, crack initiation occurs at a specific temperature differential, and is reflected in the flat segment of the response in Fig. 4.24.

It is also clearly seen from Fig. 4.24 that as the applied load decreases, \( LLD \) increases in the negative direction (upward): the slab curls down significantly with a small load (i.e., the midpoint of the edge where the load is applied moves upward with respect to the subgrade). This is clearly evident, for example, for the curve with \( P = 0.1 \) kips. For \( P \) values less than about 0.3 kips, Fig. 4.24 shows that there is no softening at all (i.e., no flat segment) in the responses, indicating no cracking. Therefore, the presence of a very small cut or notch is necessary to initiate crack growth under daytime curling alone or curling combined with very small wheel load.

To find out the notch size at which crack initiation under curling alone occurs, five different notch depths of 0.001, 0.1, 0.5, 1 and 2 in. are introduced at the bottom of the slab, corresponding to notch depth-to-slab thickness (\( a/h \)) ratios of 1/6000, 1/60, 1/12, 1/6 and 1/3. Finite element runs are made for the slab subjected to daytime curling alone. The variation of the peak differential temperature with notch size is plotted in Fig. 4.25. As expected, a peak temperature differential is observed even for the smallest notch depth of 0.001 in. (\( a/h = 1/6000 \)).
As notch depth increases, the peak differential temperature decreases almost linearly. Therefore, for daytime temperature to initiate bottom-up cracking, a very small notch should exist at the bottom of the slab. In contrast, as noted in Section 4.5.1 (see Fig. 4.7), under nighttime temperature alone (warmer bottom), top-down cracking may initiate without any notch in the slab. This may be related to the different effect slab self-weight has on the respective curled boundary conditions, but this issue requires further exploration.

To compare the two extreme loading scenarios presented in this study, i.e., scenario (a) constant daytime temperature differential combined with increasing wheel load, and scenario (b) constant wheel load followed by increasing daytime temperature differential, the variation of the peak temperature differential versus the applied load is plotted in Fig. 4.26. Note that responses for scenario (a) are obtained from Section 4.6.1(a) corresponding to the peak loads obtained at constant daytime temperature differentials for the time periods given in Table 4.10 (see also Fig. 4.18). The curve for scenario (b) is extracted from Fig. 4.24. A similar trend is observed in both cases: an increase in the magnitude of one of the stressors decreases the magnitude of the other stressor required to cause slab failure. Under pre-crack elastic analysis, the two scenarios may be expected to yield the same curve. Yet, other things being equal, Westergaard’s curling analysis (which assumed infinite slab weight) appears to conform better with scenario (b), which is a more conservative approach than scenario (a). Scenario (a) is preferable to scenario (b), since it allows a higher load a given temperature and a higher temperature at a given load.

Stresses at the bottom of the slab caused by a wheel load and daytime temperature are of the same sense (both tensile), but the corresponding deformations are in the opposite direction, changing the boundary condition of the slab as one of the stressors is applied compared to what
prevailed under the other. Moreover, as mentioned above, cracking under wheel load first initiates locally beneath the load and then gradually propagates, following the assumed failure path. Under thermal curling, on the other hand, cracking may begin almost at the same time at all points along the assumed fracture plane, since temperature (therefore, curling stress) is assumed to be constant for all points on the same plane. Wheel load alone results in a relatively gradual failure of the slab, whereas the curling only causes a sudden collapse of the slab once the failure strength is attained. Depending on the magnitude of the stressor applied first, the responses under combined analysis can, therefore, exhibit stable or unstable cracking behaviors. The large flat softening segment exhibited in Fig. 4.24 (especially at the lower applied loads), for example, replicates the sudden failure of the slab under curling only. As the load increases, on the other hand, for example at $P = 7$ kips, stable crack growth is observed without almost any softening, reminiscent of load-only behavior. Observations in Fig. 4.24 and Fig. 4.26 evidently challenge the validity of linear superposition if it were to be applied for post-crack responses due to individual stressors.

4.7 Summary and Conclusions

The post-crack behavior of a concrete pavement slab subjected to two main actions, temperature and wheel load, has been investigated, preceded by a pre-crack (elastic) analysis. The proposed 3D FE simulation gave thermal stresses smaller than those from 2D FE approaches and analytical results. Nonetheless, excellent agreement is observed with other 3D FE results available in the literature. Post-crack behavior of an early age concrete slab-on-grade subjected to thermal curling alone has also been investigated. The age of concrete is the dominant factor affecting material properties, and determines the timing and depth of joint sawing. It has also
been observed that slab self-weight is not significant with regard to the critical temperature differential resisted by the slab.

The influence of a fixed diurnal temperature distribution through the slab thickness acting in combination with increasing wheel load has been examined. It has been observed that daytime conditions significantly reduce the peak load capacity of the slab. During the nighttime, cracks are stable, mainly due to the counteraction of curling stresses with the loading stresses, as well as the full contact between the slab and the subgrade. In both cracking due to curling alone and fixed curling combined with increased wheel loading, responses are observed to depend not only on the extreme-fiber stresses that may initiate the crack, but also on the stress distribution through the slab thickness, reflecting the temperature profile.

From the parametric studies conducted, it can be concluded that the influence of slab size on the fracture process is negligible for $L/l$ ratios greater than 10. Moreover, the peak load is insensitive to the self-weight of the slab. The deflection at the peak load increases as the slab self-weight increases. An increase in the fracture energy is observed to increase the post-peak load and to result in stable (more ductile) crack formation, whereas an increase in the tensile strength affects the load at which cracks initiate and results in brittle failure.

The loading scenario involving a fixed wheel load followed by an increasing daytime temperature has also been considered. It is observed that the stability of the crack and the magnitude of the peak temperature differential largely depend on the magnitude of the wheel load. A relatively high wheel load, encourages stable crack growth with almost no softening, beginning at a lower temperature differential. In contrast, a rather small wheel load, allows the
temperature differential to increase significantly before crack initiation, but the peak is followed by sudden slab failure with pronounced softening.

It can be concluded that the application of cohesive elements to the fracture analysis of concrete pavement slabs is promising. It is anticipated that the approach can be extended to the idealization of fracture in \textit{in situ} jointed concrete pavements. This step-by-step effort can contribute to the ongoing development of rational failure criteria for pavement structures. Consequently, the exclusively statistical algorithms that are used to predict pavement distresses on the basis of Miner’s hypothesis (Miner, 1945) in current pavement design procedures may eventually be eliminated.

4.8 References


Soares, J. B. (1997). "Concrete characterization through fracture mechanics and selected pavement applications," Ph.D. dissertation, Texas A and M University, College Station, TX.


Table 4.1 Comparison with Experimental and Numerical Results

<table>
<thead>
<tr>
<th>$\Delta T$ (°F)</th>
<th>$\sigma_{xx}$ (ksi) Teller and Sutherland (1935)</th>
<th>$\sigma_{xx}$ (ksi) Kuo (1994)</th>
<th>$\sigma_{xx}$ (ksi) Present study</th>
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<td>0.187</td>
<td>0.187</td>
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<tr>
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</tr>
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</tr>
<tr>
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<td>0.251</td>
</tr>
<tr>
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<td>0.264</td>
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<tr>
<td>21</td>
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</table>
Table 4.2 Coefficients $A$, $B$, and $C$ for Daily Cycle in June (Choubane and Tia, 1992)

<table>
<thead>
<tr>
<th>Time</th>
<th>$A$ (°F)</th>
<th>$B$ (°F/in.)</th>
<th>$C$ (°F/in.$^2$)</th>
<th>$\Delta T$ (°F)</th>
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Table 4.3 Comparison of 3D Linear and Nonlinear Curling

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<td>$\sigma_b$</td>
<td>$\sigma_t$</td>
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<td>0.140</td>
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<td>0.154</td>
<td>-0.251</td>
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<td>0.266</td>
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</tr>
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</tr>
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Note: $\sigma_t$ is the top bending stress in ksi; $\sigma_b$ is the bottom bending stress in ksi
Table 4.4 Comparisons for Linear Curling

<table>
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<th>Time</th>
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<th>2D FE</th>
<th></th>
<th>3D FE</th>
<th>Percentage</th>
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<tr>
<td></td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
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<td>(f)</td>
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<td>0.266</td>
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<td>0.086</td>
<td>-0.086</td>
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<td>-0.085</td>
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</table>

Note: $\sigma_t$ is the top bending stress in ksi; $\sigma_b$ is the bottom bending stress in ksi.

Analytical: After Westergaard (1927)

2D FE: Choubane and Tia (1992) using 2D FE program *FEACONS IV*

3D FE: Present study employing *ABAQUS®*
Table 4.5 Comparison with 2D FE Results for Quadratic Temperature Profile

<table>
<thead>
<tr>
<th>Time</th>
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<th>3D FE (This Study)</th>
<th>Percentage</th>
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<tr>
<td></td>
<td>( \sigma_t ) ( \sigma_b )</td>
<td>( \sigma_t ) ( \sigma_b )</td>
<td>c/a d/b</td>
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<td>0.151 -0.074</td>
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<td>0.157 -0.094</td>
<td>NA NA</td>
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Note: \( \sigma_t \) is the top bending stress in ksi; \( \sigma_b \) is the bottom bending stress in ksi; NA: not available
### Table 4.6 Comparison of 3D FE Combination and Superposition Analysis

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<th>Coefficients</th>
<th>Combination</th>
<th>Superposition</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
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<td>$A$ (°F)</td>
<td>$B$ (°F/in.)</td>
<td>$C$ (°F/in.$^2$)</td>
<td>(a) $\sigma_t$ (ksi)</td>
</tr>
<tr>
<td>Midnight</td>
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<tr>
<td>4:00 am</td>
<td>90</td>
<td>1.0000</td>
<td>-0.0741</td>
<td>0.2784</td>
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<tr>
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<td>1.0000</td>
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<td>0.2645</td>
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<tr>
<td>Noon</td>
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<td>0.1589</td>
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<tr>
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Note: $\sigma_t$ is the top principal bending stress in ksi; $\sigma_b$ is the bottom principal bending stress in ksi

### Table 4.7 Comparison Among Different FE Programs

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<th>Hour</th>
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<th>ABAQUS®</th>
<th>EverFE</th>
<th>Percentage</th>
</tr>
</thead>
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<tr>
<td></td>
<td>(a) $\sigma_t$</td>
<td>(b) $\sigma_b$</td>
<td>(c) $\sigma_t$</td>
<td>(d) $\sigma_b$</td>
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<tr>
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Note: $\sigma_t$ is the top principal bending stress in ksi; $\sigma_b$ is the bottom principal bending stress in ksi
Table 4.8 Material Properties for Concrete at Different Ages (Jenq et al., 1993)

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<tr>
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<th>72 hrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ (ksi)</td>
<td>1209</td>
<td>2130</td>
<td>3051</td>
</tr>
<tr>
<td>$G_F$ (kips/in.)</td>
<td>$5.80 \times 10^{-5}$</td>
<td>$17.2 \times 10^{-5}$</td>
<td>$29.80 \times 10^{-5}$</td>
</tr>
<tr>
<td>$f'_t$ (ksi)</td>
<td>0.084</td>
<td>0.209</td>
<td>0.348</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>$\alpha$ ($\times 10^{-6}/^\circ F$)</td>
<td>5.562</td>
<td>5.562</td>
<td>5.562</td>
</tr>
<tr>
<td>Slab Geometry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------------------------</td>
<td>-------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length (in.)</td>
<td>240</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Width (in.)</td>
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<td></td>
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<tr>
<td>Thickness (in.)</td>
<td>6</td>
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<table>
<thead>
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<th>Material Properties</th>
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<tr>
<td>Young's Modulus (ksi)</td>
<td>4000</td>
</tr>
<tr>
<td>Coefficient of Thermal Expansion (in./in./°F)</td>
<td>$5 \times 10^{-6}$</td>
</tr>
<tr>
<td>Tensile Strength (ksi)</td>
<td>0.463</td>
</tr>
<tr>
<td>Fracture Energy (lb/in.)</td>
<td>0.431</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>0.15</td>
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</table>

<table>
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<tr>
<th>Subgrade Characteristics</th>
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<tbody>
<tr>
<td>Modulus of Subgrade Reaction (psi/in.)</td>
<td>200</td>
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<th>Discretization Characteristics</th>
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<tr>
<td>Slab Element Size (in.)</td>
<td>$6 \times 6 \times 2$</td>
</tr>
<tr>
<td>Cohesive Zone Element Size (in.)</td>
<td>$0.12 \times 0.12$</td>
</tr>
<tr>
<td>Concrete Softening Curve</td>
<td>Bilinear</td>
</tr>
<tr>
<td>Solver Type</td>
<td>Riks or Newton-Raphson</td>
</tr>
<tr>
<td>Subgrade Idealization</td>
<td>$SPRING1$</td>
</tr>
</tbody>
</table>

Note: 1 lb = 4.444 N; 1 in. = 25.4 mm; 1 ksi = 6.89 MPa
Table 4.10 Coefficients of Cubic Polynomial Used to Estimate Temperature Distribution

<table>
<thead>
<tr>
<th>Time</th>
<th>Temp. Differential</th>
<th>Cubic: $T(z) = Az^3 + Bz^2 + Cz + D$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta T$ (°F)</td>
<td>$A$ (°F/in.$^3$)</td>
</tr>
<tr>
<td>07:00 am</td>
<td>-3.4806</td>
<td>-0.0306</td>
</tr>
<tr>
<td>04:00 am</td>
<td>-4.8438</td>
<td>-0.0389</td>
</tr>
<tr>
<td>01:00 am</td>
<td>-3.7638</td>
<td>-0.0333</td>
</tr>
<tr>
<td>10:00 am</td>
<td>-4.1064</td>
<td>-0.0083</td>
</tr>
<tr>
<td>07:00 pm</td>
<td>-3.9780</td>
<td>0.0167</td>
</tr>
<tr>
<td>09:30 am</td>
<td>21.8274</td>
<td>0.3333</td>
</tr>
<tr>
<td>11:30 am</td>
<td>18.6882</td>
<td>0.1389</td>
</tr>
<tr>
<td>04:00 am</td>
<td>16.6674</td>
<td>0.1111</td>
</tr>
<tr>
<td>02:00 pm</td>
<td>14.9652</td>
<td>0.0556</td>
</tr>
</tbody>
</table>

Source of data: Teller and Sutherland (1935).
Figure 4.1 Comparison Among Different 3D FE Simulations (Tension is Positive)
Figure 4.2 Variation of Daytime Temperature (9 am - 8 pm)
Figure 4.3 Variation of Longitudinal Bending Stress Caused by Daytime Temperature

(3D Nonlinear Curling)
Figure 4.4 Variation of Nighttime Temperature (10 pm - 6 am)
Figure 4.5 Variation of Longitudinal Bending Stress Caused by Nighttime Temperature

(3D Nonlinear Curling)
Figure 4.6 Comparison Among Three FE Programs
Figure 4.7 Effect of Notch Depth on Cracking Due to Curling
Figure 4.8 Comparison with Jenq et al. (1993)

(16 hrs age concrete)
Figure 4.9 Effect of Concrete Age on Fracture Due to Curling

(Notch size of 0.5 in.)
Figure 4.10 Variation of Maximum Dimensionless Temperature Differential with Britteness Number

(16-72 hrs, notch depth of 0.5 in.)
Figure 4.11 Effect of Concrete Unit Weight on Fracture Due to Curling

Note: $\gamma = 150$ pcf
Figure 4.12 Effect of Temperature Profile on Fracture Due to Curling

Case 1: $T(z) = 1.5z^2$
Case 2: $T(z) = -1.5z^2 + 18z$
Case 3: $T(z) = 9z$
Figure 4.13 Effect of Slab Size on Fracture Due to Curling
Figure 4.14 Finite Element Idealization of Slab with Properties Shown in Table 4.9
Figure 4.15 Load-Plus-Curling Response Curves under Diurnal Thermal Distributions

Note: Points 1, 2, 3, 4 and 5 represent load levels.
Figure 4.16 Crack Growth along Slab Width for Load Levels Shown in Fig. 4.15
Figure 4.17 Crack Growth along Slab Width for Load Levels Shown in Fig. 4.15

(7:00 pm)
Figure 4.18 Comparison of Linear and Cubic Temperature Profiles
Figure 4.19 Effect of Slab Length and Width
Figure 4.20 Effect of Slab Self-Weight

Note: $\gamma = 150$ pcf
Figure 4.21 Variation of Dimensionless Peak Load versus Dimensionless Self-Weight
Figure 4.22 Effect of Tensile Strength

Note: $f'_t = 0.463$ ksi
Figure 4.23 Effect of Total Fracture Energy

Note: $G_F = 0.431$ lb/in.
Figure 4.24 Fracture Responses of Slab under Constant Wheel Load and Increasing Daytime Temperature Differential
Figure 4.25 Effect of Notch Depth on Peak Linear Temperature Differential (Curling Only)
Figure 4.26 Comparison of Two Loading Scenarios
Chapter 5  Crack Propagation in Pavement Slabs with Aggregate Interlock

Abstract

This paper discusses finite element analysis of crack propagation in pavement slabs with aggregate interlock joints, using the finite element package ABAQUS® 6.9-2. The fracture process is idealized using cohesive elements, per the fictitious crack model. The joint mechanism is simulated in accordance with linear and nonlinear approaches available in the literature. The proposed discretization is first verified by comparing the pre-crack (elastic) responses with analytical solutions and with experimental results published by independent researchers. Parametric studies are conducted concerning the effects of joint stiffness, joint opening and aggregate size. It is observed that when linear aggregate interlock is employed, load transfer efficiency does not change significantly even after slabs undergo cracking. Under nonlinear aggregate interlock discretization, on the other hand, load transfer efficiency increases substantially and attains its maximum value at the peak loads resisted by the slabs and decreases continuously thereafter. A daytime temperature profile is observed to reduce both the peak load supported by the slab system and the load transfer efficiency of the joint, while a nighttime temperature distribution results in modest increases in these metrics. It is concluded that the proposed approach can be used as a basis for further exploration of crack propagation in concrete pavement systems. The step-by-step methodology implemented in this study may contribute to the ongoing development of rational failure criteria that can replace the statistical/empirical algorithms currently used in pavement design procedures.
5.1 Introduction

In jointed concrete pavement slabs, load transfer may be accomplished by two primary mechanisms: dowel bars and aggregate interlock. Dowel bars are often placed across a joint to guarantee the longevity of the mechanism, which relies almost exclusively on each bar’s shear resistance. Aggregate interlock, on the other hand, results from shear forces developing at the rough interfaces constituting the joint due to the mechanical interaction of aggregate particles, which may degrade with time. Critical responses in jointed concrete pavements are very sensitive to the load transfer efficiency of the joint, i.e., its ability to transfer load applied on one slab to the other. Therefore, it is essential to incorporate realistic joint characteristics in a fracture analysis of such pavement systems.

Published literature indicates that the mechanics of aggregate interlock is a complex phenomenon that depends on several parameters, including aggregate size and distribution, concrete compressive strength, friction between the aggregate and the cement paste, crack (or joint) opening, and interface sliding (Bažant and Gambarova, 1980; Walraven, 1981). Normal and shear stresses developing along a cracked concrete interface have been observed to be nonlinear functions of the corresponding displacements. A few researchers have accounted for such nonlinearities for concrete pavements (Davids and Mahoney, 1999; Wattar, 2001), but most have adopted a simplified approach by assuming a linear pure-shear interlocking mechanism. The first use of the linear pure-shear assumption was made by Skarlatos (1950), who derived an analytical relationship between responses in the loaded and unloaded slabs, respectively, assuming that shear springs were distributed over the length of the pavement slab joint. This approach is essentially identical to that subsequently implemented in two-dimensional (2D) finite
element (FE) programs, such as KENSLABS and ILLI-SLAB, commonly used in pavement engineering (Huang and Wang, 1973; Tabatabaie and Barenberg, 1978). Results from ILLI-SLAB for different slab geometries, load sizes and joint spring stiffness values were interpreted using dimensional analysis by Ioannides and Korovesis (1990a).

A major limitation of the pure-shear approach is that it requires specifying a joint stiffness value, which is not easy to determine with reference to the joint’s physical characteristics. Some investigators have resorted to analysis of laboratory data conducted on aggregate interlock jointed concrete pavements (Colley and Humphrey, 1967; Brink et al., 2005; Jensen and Hansen, 2006; Maitra et al., 2010), their conclusions, however, are only applicable to the particular set of conditions they each considered and fail to address all the mechanistic phenomena observed. Consequently, joint stiffness is commonly backcalculated from field observations (Ioannides and Korovesis, 1990b).

The present study investigates the effect of both linear and nonlinear aggregate interlock on post-crack responses of a typical concrete pavement under edge loading. The fracture process is tracked using the fictitious crack model (FCM) first suggested for concrete by Hillerborg et al. (1976), and employs cohesive elements, also implemented in earlier phases of this research effort (Aure and Ioannides, 2010, 2012). A fixed fracture path is assumed along the center of both the loaded and unloaded slabs, where principal stresses are expected, in accordance with a discrete crack approach.

This study is organized as follows. In the first section, a review of aggregate interlock mechanics is presented and a suitable nonlinear approach is identified. In the second section, pre-fracture
analysis of the slabs is conducted. Linear and nonlinear aggregate interlock idealizations are verified by comparing with analytical, numerical and experimental results available in the literature. In the third section, crack propagation in jointed slabs subjected to wheel loading is examined. A similar approach is implemented in the fourth section for the case of a constant temperature distribution, which is considered in addition to the increasing wheel load. The effects of daytime and nighttime temperature variations on the post-crack responses of the slabs are also investigated.

5.2 Mechanics of Aggregate Interlock: A Review

This section outlines a general formulation for load transfer due to aggregate interlock. A cracked concrete surface contains irregularly-shaped, protruding aggregate particles. When their surfaces slide with respect to one another, the aggregate particles interlock, thereby transferring shear and normal stresses. These stresses depend on the size and distribution of the aggregate particles, friction between the cement paste and the aggregate, compressive strength of concrete, and the size of crack opening. According to Bažant and Gambarova (1980), the normal and shear stresses at a cracked concrete interface, in a two-dimensional plane, are functions of the normal and shear displacements of the interface, as follows:

\[ \sigma_n = f_n(\delta_n, \delta_t) \]  \hspace{1cm} (5.1a)

\[ \sigma_t = f_t(\delta_n, \delta_t) \]  \hspace{1cm} (5.1b)

where: \( \sigma_n \) is the normal stress; \( \sigma_t \) is the shear (or tangential) stress; \( \delta_n \) and \( \delta_t \) are the normal and shear displacements, respectively, and \( f_n \) and \( f_t \) are pertinent (generally, nonlinear) functions to be determined.
Differentiation of Eq. (5.1) results in:

\[
\begin{bmatrix}
    d\sigma_n \\
    d\sigma_t
\end{bmatrix} = \begin{bmatrix}
    \frac{\partial f_n}{\partial \delta_n} & \frac{\partial f_n}{\partial \delta_t} \\
    \frac{\partial f_t}{\partial \delta_n} & \frac{\partial f_t}{\partial \delta_t}
\end{bmatrix} \begin{bmatrix}
    d\delta_n \\
    d\delta_t
\end{bmatrix} = \begin{bmatrix}
    K_{nn} & K_{nt} \\
    K_{tn} & K_{tt}
\end{bmatrix} \begin{bmatrix}
    d\delta_n \\
    d\delta_t
\end{bmatrix}
\]

(5.2)

in which: \( K_{nn} = \frac{\partial f_n}{\partial \delta_n}, K_{nt} = \frac{\partial f_n}{\partial \delta_t}, K_{tn} = \frac{\partial f_t}{\partial \delta_n}, K_{tt} = \frac{\partial f_t}{\partial \delta_t} \)

are crack stiffness coefficients that can be determined once the functions \( f_n \) and \( f_t \) are established. If the variation of functions \( f_n \) and \( f_t \) is nonlinear with \( \delta_n \) and \( \delta_t \), the crack stiffness coefficients may be sensitive to stress level, and therefore, are subject to change as the load is applied. This behavior is identified as nonlinear aggregate interlock mechanism. On the other hand, if crack stiffness coefficients remain constant as the load is applied, linear aggregate interlock behavior results. When the two off-diagonal terms of the crack stiffness matrix are identical (\( K_{nn} = K_{nt} \)), the joint exhibits symmetrical behavior, otherwise it is termed as unsymmetrical. Setting the two off-diagonal terms of the crack stiffness matrix to zero results in uncoupled behavior, whereas if all the off-diagonal coefficients are non-zero, the stiffness matrix defines coupled aggregate interlock.

Based on these definitions, therefore, three feasible combinations of aggregate interlock behavior can be envisaged for each of the linear and nonlinear cases: linear-coupled-symmetrical, linear-coupled-unsymmetrical and linear-uncoupled-symmetrical (pure-shear); nonlinear-coupled-symmetrical, nonlinear-coupled-unsymmetrical and nonlinear-uncoupled-symmetrical (pure-shear). In the present study, only linear-uncoupled-symmetrical and nonlinear-coupled-unsymmetrical cases are considered.
According to Bažant and Gambarova (1980), interface stresses $\sigma_n$ and $\sigma_t$ must conform to the following conditions: normal stress, $\sigma_n$, is always less than or equal to zero (i.e., always compressive), whereas normal displacement, $\delta_n$, is always greater than zero (i.e., crack only opens); stress-displacement relations are continuous and smooth, ensuring that the differentiation of functions $f_n$ and $f_t$ is feasible; normal stress, $\sigma_n$, is zero at zero shear displacement, $\delta_t$, irrespective of the value of the normal displacement, $\delta_n$, since there is no contact between the two crack surfaces; at constant shear displacement, $\delta_t$, both the normal, $\sigma_n$, and shear, $\sigma_t$, stresses decrease as crack opening increases; at constant crack opening, both stresses increase as shear displacement increases.

Following Bažant and Gambarova (1980), the majority of aggregate interlock mechanics studies have been devoted to the determination of the functions $f_{n,t}(\delta_n, \delta_t)$ using either experimental (Reinhardt and Walraven, 1982; Divakar et al., 1987) or theoretical (also called micro-mechanical) approaches (Walraven, 1981; Divakar and Fafitis, 1992; Walraven, 1994). Notable among these is the contribution by Walraven (1981), who formulated a theoretical constitutive equation assuming concrete to be a two-phase material, and implemented “a statistical analysis of the crack [or interface] structure and the associated contact areas between the crack faces [in terms] of the displacements.” The stresses he computed from the most probable projected contact areas of the aggregate particles are as follows:

\[
\sigma_n = \sigma_{pu} \left( A_x - \mu A_y \right) \quad (5.3a)
\]

\[
\sigma_t = \sigma_{pu} \left( A_y + \mu A_x \right) \quad (5.3b)
\]
where $\mu$ is the coefficient of friction between the aggregate and the cement paste, usually taken as 0.4; $A_x$ is the most probable projected contact area of a unit crack area in the $x$-direction; and $A_y$ is the most probable projected contact area of a unit crack area in the $y$-direction. The aggregate-paste matrix yielding strength ($\sigma_{pu}$) depends on the uniaxial compressive strength of concrete $f_c'$ and is given as:

$$\sigma_{pu} = 56.7 f_c^{0.56} \text{ psi}$$

(5.4)

Walraven (1981) provides formulae for determining the contact areas obtained using statistical analysis of the size distribution of the aggregate particles and of the resulting deformation modes when the aggregate bears against the cement paste. The areas are correlated to the maximum aggregate size, crack opening, sliding displacement and aggregate particle size distribution. Aggregate particle size distribution is described by the so-called Fuller curve (Fuller and Thompson, 1907), given by:

$$p = \sqrt[6]{\frac{D}{D_{max}}}$$

(5.5)

where $D_{max}$ is the maximum aggregate particle size; $D$ denotes a given particle size, and $p$ gives the percent of the aggregate that is finer than $D$. Walraven’s complete solutions for the contact areas are given in Table 5.1.

After examining other micro-mechanics approaches, Davids (1998) implemented Walraven’s constitutive relations in three-dimensional (3D) nonlinear FE program *EverFE*. He deemed the approach to be relatively simple, suitable for numerical implementation, validated by other independent researchers and of reasonable accuracy when estimating responses. Wattar (2001)
validated Walraven’s equations with experimental measurements, verifying their applicability to the idealization of concrete pavement aggregate interlock joints. Walraven’s constitutive relations were further extended to cyclic loads, experienced by concrete pavements (Walraven, 1994). This general (nonlinear) approach is, therefore, adopted in the present study for the idealization of the nonlinear aggregate interlock mechanism in the broadest conceivable way. Normal and shear stress variations, generated by numerical integration of Walraven’s constitutive equations using numerical computing package MATLAB (Moler, 2004), are shown in Fig. 5.1. These curves correspond to the typical properties of concrete for pavements given in Table 5.2, and different crack openings ($\delta_n$); they are used in the post-crack analysis of the pavement slab in Section 5.4, below.

5.3 Pre-Crack Aggregate Interlock Load Transfer Analysis

5.3.1 Finite Element Discretization

Each of the two slabs, whose geometric and material properties are given in Table 5.2, is discretized using a quadratic continuum element, $C3D27$, with a uniform mesh size of $6\times6\times2$ in. in the width, length, and depth directions, respectively. The subgrade is simulated using $SPRING1$ elements that can only support compression. For computational stability and reflecting the friction between slab and subgrade, horizontal restraint is provided by assigning the $SPRING1$ elements with horizontal stiffness at $1/10^{th}$ the value of their vertical stiffness, per Gaedicke and Roesler (2009). Furthermore, the lateral thrust exerted by neighboring slab panels is simulated by restraining the two slab sides opposite to the loaded edge in the direction perpendicular to the joint plane.
There are three candidate elements for linear aggregate interlock simulation, namely, joint elements (JOINTC), traction-separation cohesive elements (COH3D8), and PROJECTION CARTESIAN type connector elements (CONN3D2). The nonlinear aggregate interlock mechanism can only be simulated using CONN3D2. The application of each of these element types in the discretization of both categories of aggregate interlock mechanism is described in detail in the following sections.

a. Joint Elements for Linear Aggregate Interlock Idealization

Analysis of load transfer mechanism by aggregate interlock in concrete pavements dates back to the late 1940s, culminating in the work of Skarlatos (1950), in which the behavior of the joint was exclusively characterized as a pure-shear load transfer mechanism, i.e., all the crack stiffness coefficients except $K_\alpha$ were assumed to be zero. This approach was subsequently implemented in 2D FE programs, KENSLABS (Huang and Wang, 1973) and ILLI-SLAB (Tabatabaie and Barenberg, 1978). Ioannides and Korovesis (1990a) employed dimensional analysis in interpreting numerical results for different slab geometries and load sizes obtained from ILLI-SLAB so as to extract the necessary joint stiffness terms. In this manner, they obtained a relationship between a dimensionless joint stiffness parameter ($\text{AGG}/kl$) and the load transfer efficiency with respect to deflection ($\text{LTE}_d$) and stress ($\text{LTE}_\sigma$). Here, $k$ is the subgrade modulus and $l$ is the radius of relative stiffness of the slab-subgrade system. Ioannides and Hammons (1996) demonstrated that the joint stiffness parameter $\text{AGG}$, used in ILLI-SLAB to define aggregate interlock factor, corresponds to Skarlatos’s symbol $q$.

Skarlatos’s aggregate interlock idealization was adopted in a 3D FE load transfer analysis by Hammons (1997), who employed a general flexible joint element in ABAQUS® called JOINTC.
This is similar to conventional spring elements, but it also has the capability to simulate damping that may be necessary in cyclic loading. The element can be ascribed linear or nonlinear properties in three orthogonal directions, and, it can be used to idealize either linear-uncoupled-symmetrical or nonlinear-uncoupled-symmetrical aggregate interlock behavior. In the present study, therefore, this element is preferred over other spring elements available in ABAQUS® (e.g., SPRING2 and SPRINGA), in anticipation of future research involving dynamic loading.

The use of Skarlatos’s pure-shear aggregate interlock idealization in 3D FE simulation as employed by Hammons (1997) results in the interpenetration of the bulk elements lying along the joint, as the two crack surfaces slide with respect to each other. To circumvent this problem, normal stiffness ($K_{nn}$) is provided in the present study along with the shear stiffness ($K_{tt}$). By first assuming a particular value of $K_{tt}$, a trial-and-error approach is used to obtain $K_{nn}$; accordingly, the normal stiffness is set to 2.3 times the shear stiffness:

$$K_{nn} = 2.3K_{tt} \tag{5.6}$$

This relationship may be justified by noting that the ratio of the Young’s modulus ($E$) to the shear modulus ($G$) is $2 \ (1 + \mu)$; for a concrete with Poisson’s ratio, $\mu$, of 0.15, this ratio yields 2.3.

Following Hammons (1997), each JOINTC element is inserted between two nodes symmetrically located on either side of a joint plane. For every assumed value of the dimensionless joint stiffness $AGG/kl$, the corresponding JOINTC shear stiffnesses, ($K_{tt} [\text{FL}^{-1}]$), of elements located at the corner of the joint plane are determined per Hammons (1997), as follows:
where $AGG$ [FL$^{-2}$] is the joint stiffness per unit length, $\lambda$ [L] is the joint length, $N_R$ is the number of nodal rows, and $N_C$ is the number of nodal columns. The primary dimensions are abbreviated here as [L] for length and [F] for force. Since the mesh adopted is uniform, the stiffnesses at the edge and interior are two and four times those at the corner, respectively. Equation (5.6) is used to obtain the corresponding normal stiffness, $K_{nn}$. As noted earlier, the mesh was 6×6×2 in., in the width, length and depth directions, respectively; from these, the number of nodal rows and nodal columns can be determined. The JOINTC shear stiffnesses for elements located at the corner of the joint corresponding to the assumed $AGG/kl$ values are computed using Eq. (5.7) and presented in Table 5.3.

b. **Cohesive Elements for Linear Aggregate Interlock Idealization**

Traction-separation cohesive elements (COH3D8), commonly employed for fracture analysis, are also potential candidates for the discretization of linear aggregate interlock load transfer mechanism. In such usage, a cohesive element performs the function of an interface element with a linear constitutive relation. Its linear-coupled-symmetrical characteristics are similar to Eq. (5.2), which when extended to 3D becomes (*ABAQUS*, 2009):

\[
\begin{bmatrix}
\sigma_n \\
\sigma_s \\
\sigma_t
\end{bmatrix} = \frac{1}{T_0} \begin{bmatrix}
K_{nn} & K_{ns} & K_{nt} \\
K_{sn} & K_{ss} & K_{st} \\
Symm & K_{tt}
\end{bmatrix} \begin{bmatrix}
\delta_n \\
\delta_s \\
\delta_t
\end{bmatrix}
\]  

(5.8)
where: \( n, s, \) and \( t \) represent the normal and two shear directions, \( K \) is the stiffness along these directions, \( T_0 \) is the initial size of the cohesive element. The latter is assumed to be 0.001 in. as suggested by a previous study (Aure and Ioannides, 2010).

Setting the off-diagonal terms to zero, the pure-shear aggregate interlock behavior becomes:

\[
\begin{bmatrix}
\sigma_n \\
\sigma_s \\
\sigma_t
\end{bmatrix} = \frac{1}{T_0} \begin{bmatrix}
K_{nn} & 0 & 0 \\
0 & K_{ss} & 0 \\
0 & 0 & K_{tt}
\end{bmatrix} \begin{bmatrix}
\delta_n \\
\delta_s \\
\delta_t
\end{bmatrix}
\]

(5.9)

The two shear stiffness coefficients, \( K_{ss} \) and \( K_{tt} \), can be related to the joint stiffness per unit length, \( AGG \), by using the following simple relation:

\[
K_{ss} h = K_{tt} h = AGG
\]

(5.10)

where \( h \) is the thickness of the slab. The factor of 2.3 suggested earlier may be used to relate the normal and shear stiffnesses, as for \( JOINTC \) elements:

\[
K_{nn} = 2.3 K_{ss} = 2.3 K_{tt}
\]

(5.11)

A large failure stress and negligible fracture energy is assumed for a \( COH3D8 \) element, so that it remains elastic at all times. The top and bottom surfaces of the element are tied to the slab elements using surface-based \( TIE CONSTRAINTS \). This approach was found to work well in earlier studies (Aure and Ioannides, 2010), and is especially useful in discretizing the joint with a relatively finer mesh than the rest of the slab. It should be noted that linear-coupled-unsymmetrical behavior cannot be accommodated by cohesive elements; the same is also true for any kind of nonlinear behavior.

c. Connector Elements for General Aggregate Interlock Idealization
The main limitation of the JOINTC and COH3D8 elements is their inability to discretize the more general nonlinear aggregate interlock load transfer mechanism represented by Eq. (5.2). Consequently, another element type was sought in this study, and 3D connector element (CONN3D2) was found to be the best candidates. This element can define a relative displacement and rotation between two nodes in three orthogonal directions. ABAQUS® provides varieties of connector types to accommodate all possible relative displacements and rotations between the connected nodes, such as simple translational and rotational displacements. Among a library of connector types, PROJECTION CARTESIAN was deemed to be appropriate for idealizing both linear and nonlinear aggregate interlock behavior, provided adequate techniques are devised to establish the required material properties, represented by the complete set of matrix coefficients in Eq. (5.2).

Like JOINTC, a connector element is defined between two symmetrically located nodes along the joint plane. When a connector element is used to discretize linear aggregate interlock behavior, its stiffness coefficients ($K_{nn}$ and $K_{nt}$) are identical to those of a JOINTC element. Nonlinear aggregate interlock behavior, on the other hand, requires the definition of the entire force-displacement rule to be followed by ABAQUS® during the analysis.

In order to generate the force-displacement data required as inputs to the program from Walraven’s constitutive equations, the following steps are followed: (a) Assume suitable ranges for the normal and shear displacements. From Fig. 5.1, it is apparent that the interface stresses are not influenced by normal and shear displacement values greater than 0.1 in. Therefore, the upper limit in both cases may be set to 0.1 in. The lower limit can be zero for shear but needs to be a small finite value for normal displacement, as indicated above (e.g. 0.0001 in.). To generate
a smooth curve, thereby avoiding any convergence problems during the FE analysis to follow, each range should be subdivided into a sufficient number of intervals. The value of 100 was found to ensure a convergent FE solution for both displacements. (b) Select an appropriate numerical computing package to perform the integration for the most probable projected contact areas, $A_x$ and $A_y$, from Walraven’s equations given in Table 5.1. The required variables are the maximum aggregate size ($D_{max}$) and the aggregate volume per unit volume of concrete ($p_k$). The latter is usually assumed to be 0.75 (Walraven, 1981). In this study MATLAB was selected for computing the integrands “to within an error of $10^{-12}$ using recursive adaptive Lobatto quadrature” (Moler, 2004). (c) Use Eqs. (5.3) and (5.4) to obtain the interface normal and shear stresses ($\sigma_n$ and $\sigma_t$). (d) Compute the corresponding normal and shear forces by multiplying these stresses by the contributing areas defining the connector element, i.e., the areas of those elements located at the corner, edge, and interior of the joint plane, as done for the linear aggregate interlock simulation. These forces are then provided as inputs to ABAQUS® manually in tabular form, as functions of the displacements.

Examining the nonlinear aggregate interlock constitutive equations provided by Walraven, it is observed that the crack stiffness coefficients given in Eq. (5.2) are unsymmetrical (i.e., $K_{nt} \neq K_{tn}$). Consequently, unsymmetrical matrix storage is adopted in this study for nonlinear aggregate interlock load transfer simulation; this is accomplished by using the keyword UNSYMM (ABAQUS, 2009).

5.3.2 Validation of Linear Aggregate Interlock Discretization
The concrete pavement system described in Table 5.2 is used in this section to verify the proposed linear aggregate interlock FE discretization. A pressure of 0.5 ksi is applied over a $2\epsilon \times 2\epsilon$ (12 by 12 in.) area, symmetrically located about the midpoint of the joint on the edge of the loaded slab side. Here $2\epsilon$ represents the side of the square area. The modulus of subgrade reaction, $l$, is determined using the pertinent properties given in Table 5.2 to be 24.63 in. The equivalent radius of the applied load, $a$, is 6.77 in. Therefore, $a/l$ and $\epsilon/l$ becomes 0.275 and 0.244, respectively. Finite element runs are made for the eight assumed ($AGG/kl$) values given in Table 5.3, using each of the three linear aggregate interlock discretization options outlined above, involving $JOINTC$, $COH3D8$ and $CONN3D2$ elements. The following results are monitored: load transfer efficiency with respect to deflection ($LTE_\delta$), calculated as the ratio of the deflection at the center of the edge of the unloaded slab to the deflection of the loaded slab at that point; transferred load efficiency ($TLE$), which is equal to the ratio of the sum of the vertical reaction forces in the subgrade springs under the unloaded slab to the total applied load; and load transfer efficiency with respect to bending stress ($LTE_\sigma$), determined as the ratio of the bending stress at the bottom of the unloaded slab at a point corresponding to the center of the edge of the slab divided by the bottom fiber bending stress at that point on the loaded side of the joint.

The results are presented in Figs. 5.2 and 5.3, in which it is observed that the three linear aggregate interlock idealizations exhibit excellent agreement with one another for all the responses monitored. The $CONN3D2$ and $JOINTC$ elements give exactly identical results with each other, affirming the use of either of these elements in idealizing linear aggregate interlock. The responses resulting from $COH3D8$ elements are observed to be slightly smaller than those from the other two options. This is attributable to the surficial load transferring mechanism.
pertaining to the \textit{COH3D8}, in contrast to the nodal load transfer prevailing for the other two. For $LTE_\delta$ and $TLE$, almost identical results are obtained by the present study and those from Ioannides et al. (1996) and Ioannides and Korovesis (1990a), respectively. For the $LTE_\sigma$, on the other hand, as joint stiffness increases, the present study points are lower than that by Ioannides et al. (1996). This may be attributed to mesh fineness, plate versus continuum formulation and point versus surficial load transfer mechanism. All three FE options in Fig. 5.3 result in data points located slightly above the curve for the 2D FE results established by Ioannides and Hammons (1996). The discrepancies may be caused by a combination of the theoretical approach (continuum versus plate theory) as well as the difference in joint discretization.

It should also be noted that unlike \textit{JOINTC} and \textit{CONN3D2} elements, meshing of traction-separation \textit{COH3D8} elements can be refined as needed without any changes to the mesh of the rest of the slab. Moreover, these elements might be useful in the future for simulation of linear aggregate interlock with a coupled-symmetrical crack stiffness matrix, a capability not shared by \textit{JOINTC}. Therefore, in the subsequent post-crack analysis of the slabs, cohesive and connector (\textit{COH3D8} and \textit{CONN3D2}) elements are used to idealize linear and nonlinear aggregate interlock mechanisms, respectively. This constitutes a departure from the conventional approach employed for linear aggregate interlock idealization to date, which has favored the use of spring-type (\textit{JOINTC}) elements, per Hammons (1997).

A second observation is warranted at this point with regard to the method of load application. It has been found that the uniform pressure approach described above gives rise to convergence problems when retained in investigating post-fracture behavior. Consequently in what follows, it is explained that a point load is applied symmetrically from the assumed fracture plane, instead.
5.3.3 Validation of Nonlinear Aggregate Interlock Discretization

The accuracy of the proposed nonlinear aggregate interlock simulation is verified by reproducing the test data reported by Colley and Humphrey (1967). They examined the load transfer efficiency of an aggregate interlock joint between two slab panels, for different concrete slab thicknesses, joint openings, and base layers under static and cyclic loads. Davids and Mahoney (1999) also reproduced this experimental study using EverFE, and their results are available for comparison, as well. According to Colley and Humphrey (1967), the so-called “equivalent subgrade reaction of the [sand-gravel] base and silt-clay soil” was taken as 145 pci. The size of the joint opening was controlled by steel rods, anchored in the vertical faces of the two slabs parallel to the joint line. The upward movement of the far ends of the two slabs during the loading process was constrained by beams spanning the slab width. A static load of 9 kips was applied over a 16-in. diameter steel bearing plate, placed at 1 in. from the joint, at mid-width on the loaded slab.

The FE simulation conducted in the present study considers the two 7-in. thick, 48-in. wide and 108-in. long slab panels, resting on SPRING1 elements representing modulus of subgrade reaction, $k$, of 145 pci (see Fig. 5.4). The concrete Young’s modulus is taken to be 4495 ksi, with Poisson’s ratio of 0.2, per Davids and Mahoney (1999). Parameters needed for the nonlinear aggregate interlock constitutive relations in Table 5.1 are selected per Walraven (1981) and Davids and Mahoney (1999), as follows: maximum aggregate size of 1.5 in.; cement paste-aggregate particle coefficient of friction of 0.4; concrete compressive strength of 5.5 ksi; and aggregate volume per unit volume of concrete ($p_{\text{v}}$) of 0.75. The required input forces for the CONN3D2 elements are evaluated using Table 5.1, Eqs. (5.3) and (5.4). MATLAB is employed to
compute the most probable projected contact areas given in Table 5.1, using the steps given earlier. The interface normal and shear stresses are then computed from Eqs. (5.3) and (5.4). These stresses are multiplied by the contributing areas around the nodes that define the connector element to obtain the corresponding normal and shear forces. Finally, the forces are provided as input to ABAQUS® in a tabular form, depending on assumed interface normal and shear displacements, as already discussed.

The two slabs are discretized using a uniform mesh with $3 \times 3 \times 1.75$ in. elements, in the length, width and thickness directions, respectively. The lateral supports provided by the anchored steel rods during the experiment by Colley and Humphrey (1967) is simulated by restraining the nodal displacements in both horizontal directions along the slab width at mid-depth. Moreover, the top nodes along the far end edges are also restrained against vertical movement, so as to idealize the effect of the transverse beams used in those locations. For mesh expediency, the 9-kip static load is applied over a 12 by 12-in. area, resulting in a pressure of 0.0625 ksi.

Colley and Humphrey (1967) presented their results in terms of joint effectiveness ($Eff$), first defined by Teller and Sutherland (1935) as:

$$Eff(\%) = \frac{2\delta_U}{\delta_L + \delta_U} \times 100 = \frac{2LTE_\delta}{1 + LTE_\delta} \quad (5.12)$$

where: $\delta_L$ is the vertical deflection of the loaded slab; $\delta_U$ is the corresponding deflection for the unloaded slab; and $LTE_\delta$ is the load transfer efficiency with respect to deflection, given as:

$$LTE_\delta(\%) = \frac{\delta_U}{\delta_L} \times 100 \quad (5.13)$$
The variation of $LTE_\delta$ with respect to the assumed values of the initial joint opening, $\omega_i$, obtained from the present FE study is plotted in Fig. 5.5, along with the experimental data from Colley and Humphrey (1967) and the EverFE results obtained by Davids and Mahoney (1999), respectively. The three studies generally exhibit good agreement, especially at smaller joint openings. As the joint opening increases, EverFE estimates $LTE_\delta$ values between those from the experiment and from the present study. This is probably attributable to the fact that Davids and Mahoney (1999) accounted for a 1-in. groove at the top and bottom of the joint during the experiment, by inserting zero stiffness interface elements. To avoid the additional complexity it would entail, this effect was not considered in the present study. In general, the disagreements noted are insignificant for practical purposes. Therefore, the present nonlinear aggregate interlock joint discretization can be deemed suitable for the post-crack analysis of jointed pavement slabs in subsequent sections.

5.4 Post-Crack Analysis of Slabs with Aggregate Interlock under Wheel Load Alone

The geometric and material properties of the two identical slabs considered in Section 5.3.2 are retained here, per Table 5.2 and Fig. 5.6. The fracture process zone (FPZ) is idealized using the FCM per Hillerborg et al. (1976), realized by traction-separation cohesive elements, as in a previous phase of this research (Aure and Ioannides, 2012). Cohesive elements are inserted along the pre-defined fracture planes, where principal tensile stresses are expected, as shown in Fig. 5.6.

A bilinear concrete softening rule proposed by Petersson (1981) and used by Gustafsson (1985) is employed. The pertinent fracture parameters defining the softening rule are given in Table 5.2.
These values are identical to those used in previous single-slab fracture analysis by Ioannides et al. (2006) and by Aure and Ioannides (2012).

The load is applied as a unit displacement at each of two points, 3 in. away from the face of the joint and 3 in. on either side of the anticipated fracture plane. To avoid convergence problems that commonly arise during stiffness degradation of cohesive elements, viscous regularization is employed with a viscosity, $\mu$, of $1 \times 10^{-6}$. This value had been obtained from a sensitivity study conducted for single-slab fracture analysis (Aure and Ioannides, 2012). A general Newton-Raphson solution method is used, since it is less computationally demanding.

In addition to $LTE_\delta$ and $TLE$ defined earlier, the following responses are monitored: load line displacement ($LLD$), i.e., the vertical displacement at either point of the applied load; the total applied load ($P$), i.e., the sum of the conjugate “reaction” forces at the two loaded nodes, $RF3$; and, load transfer efficiency with respect to crack mouth opening displacement ($LTE_\delta$), calculated as the ratio of the crack mouth opening displacement ($CMOD$) at the bottom edge of the unloaded slab to that at the corresponding point on the loaded side of the slab joint. Analysis results for both linear and nonlinear aggregate interlock simulations are presented in the following sections.

5.4.1 Linear Aggregate Interlock

Finite element runs are made for the $AGG/kl$ values assumed in Table 5.3, and the load-displacement responses for some of the joint stiffnesses are shown in Fig. 5.7. Note that $COH3D8$ elements are used to discretize the joint. It is observed that at smaller joint stiffness values, e.g., $AGG/kl = 0.01$, the curves exhibit only one softening segment. This indicates that
the loaded slab has completely failed along the assumed fracture plane, while the unloaded slab has remained intact. As the joint stiffness increases, a larger portion of the applied load begins to be transferred and cracks begin to develop in the unloaded slab, as well. Consequently, two softening regions are exhibited in the load-displacement curves for intermediate joint stiffness values (e.g., $AGG/kl = 0.1$ and $1.0$), indicating the failure of both slabs: the first peak load indicates the failure of the loaded slab, whereas the second evinces the succumbing of the unloaded slab. For $AGG/kl = 10.0$ and $100.0$, only one softening event is again observed, signifying that the two slabs are acting monolithically; the peak load represents the maximum load that the two slabs can support together, before transverse cracking develops simultaneously across both. In general, as the joint stiffness increases, the pavement system becomes more competent, thereby causing the first peak load to increase continuously. The second peak load, on the other hand, may develop at a relatively high applied load, but upon further joint stiffening, its value quickly diminishes and it eventually coincides with the first peak load.

The effect of the dimensionless joint stiffness on several response parameters computed at the first peak load is shown in Fig. 5.8. These parameters are: load transfer efficiency with respect to deflection, $LTE_{δ}$, and with respect to crack mouth opening displacement, $LTE_{C}$; and transferred load efficiency, $TLE$. Also shown are the elastic (pre-crack) curves reproduced from the FE runs in Section 5.3.2. It is observed that after cracking, $LTE_{δ}$ remains almost identical to the corresponding elastic curve, particularly for those cases exhibiting one softening segment. This might be anticipated since joint characteristics are not altered by cracking, beyond any slab size effects. The post-crack $TLE$, on the other hand, is slightly higher than the corresponding pre-crack response particularly at higher joint stiffnesses, which may be due to differences in the
mode of load application in the two cases. These observations have the potential for some interesting practical implications: knowledge of pre-crack joint responses, per Skarlatos (1950), may hold clues for predicting aspects of post-cracking behavior, assuming that the dimensionless joint stiffness does not depend on the magnitude of the applied load. The $LTE_O$ is observed to be lower than the $LTE_δ$ for all joint stiffness values, but the two curves approach each other as joint stiffness increases.

The variation of $LTE_δ$ with the total applied load for two typical joint stiffness values is further examined in Fig. 5.9. It is observed that for $AGG/kl = 1$, $LTE_δ$ remains constant (0.46) until the total applied load equals 13.6 kips, which from Fig. 5.7 corresponds to the point at which the load-displacement curve begins to change in slope. This point evinces the beginning of cracking in the loaded slab. The $LTE_δ$ then slightly decreases until the total applied load becomes 41.2 kips ($LTE_δ = 0.39$), which corresponds to the first peak load (failure of loaded slab). After this load level, it increases until the total load equals 54.3 kips, which is the second peak load (failure of unloaded slab) on the load-displacement curve shown in Fig. 5.7. A similar behavior is exhibited for $AGG/kl = 100$, except that the offset in $LTE_δ$ becomes very small for the latter between the beginning and end of cracking in the slabs.

5.4.2 Nonlinear Aggregate Interlock

In this section, the proposed nonlinear aggregate interlock joint discretization using $CONN3D2$ elements is employed to investigate post-crack responses of concrete pavement slabs. The geometry of the two slabs is shown in Fig. 5.6, and the pertinent material properties as well as the FE discretization employed are given in Table 5.2. The parameters required for Walraven’s
constitutive relations are also given in Table 5.2. The effects of initial joint opening and maximum aggregate sizes on the responses are considered in subsequent sections.

a. Effect of Initial Joint Opening

Shrinkage and seasonal temperature variations may cause opening or closing of joint openings in situ concrete pavement slabs. Application of mechanical loads may cause additional opening changes, which may even be non-uniform over the joint depth, resulting in concomitant changes in joint stiffness. Such fluctuations influence the transfer of deflections and of stresses to the unloaded slab. The linear elastic aggregate interlock simulation presented previously cannot accommodate this aspect of behavior, since that idealization simply assumes a constant joint stiffness irrespective of the change in joint characteristics. The proposed nonlinear aggregate idealization, on the other hand, accounts for the initial joint width, as well as for its subsequent variation. Accordingly, the shear and normal forces occurring at the vertical faces of the joint are allowed to vary in response to all possible normal and shear displacements.

To investigate the effect of initial joint opening ($\omega_i$), FE runs considering slabs with $\omega_i$ values ranging from 0.02 in. to 0.09 in. are conducted, and the load-displacement responses for some of the joint openings are presented in Fig. 5.10. It is observed that as initial joint opening increases, the slopes of the curves gradually decrease, denoting a reduction in the stiffness of the system. This is due to the fact that in larger openings fewer aggregate particles can participate in interlocking. As had been the case for linear aggregate interlock, two softening segments are observed on these curves. The first (and lower) peak in the applied load indicates the failure of the loaded slab, whereas the second (and higher) applied load peak denotes the failure of the
unloaded slab. With increasing joint opening, the second peak load becomes even higher, and occurs much later than the first one, signaling that the decreased load transfer efficiency causes the unloaded slab to fail at a higher level of the total applied load. This ensures the longevity of the unloaded slab, whose diminished participation in resisting the applied load occurs at the expense of the loaded slab. Also noteworthy in Fig. 5.10 is the nonlinearity of the curves, especially discernible in the portions near the origin. Nonlinearity during the initial stages of loading has never been noted before, and may be caused by the nonlinearity of the aggregate interlock and/or cracking in the loaded slab.

Figure 5.11 presents plots of post-crack $TLE$, $LTE_\delta$, and $LTE_\delta$ versus initial joint opening, determined at the first peak load values. It is observed that all responses decrease linearly as initial joint opening increases, although the trend exhibited by the $TLE$ is much fainter, evincing a smaller sensitivity to initial joint opening. Variations in the transferred load are muted by the inertia of the pavement system, and reflect more global phenomena than localized cracking, thereby thwarting uncontrollable behaviors.

The variation of $LTE_\delta$ with respect to the applied load is plotted in Fig. 5.12 for initial joint openings of 0.02 in. and 0.09 in. The results indicate that as the load is applied, $LTE_\delta$ initially increases up to a certain maximum value at the first peak load (representing the failure of the loaded slab), and then gradually decreases as additional load is applied. This phenomenon is caused by Walraven’s constitutive relations, which result in high interface stresses up to a certain maximum value when the shear displacement increases and the normal (joint opening) displacement decreases, as shown in Fig. 5.1. At early stages of load application, joint opening is small and, therefore, aggregate particles interlocking capacity increases with applied load.
thereby enhancing joint stiffness. As the joint opening and shear displacements continue to increase, aggregate particles are no longer in contact, and therefore, the interface stresses decrease, as depicted in Fig. 5.1. Consequently, beyond the second peak load (at which the unloaded slab fails), the joint’s load transfer capability declines as the total applied load increases. The effect of the size of initial joint opening on load transfer efficiency is also clearly observed in Fig. 5.11 by comparing curves corresponding to the 0.02 in. and 0.09 in. openings.

b. Effect of Aggregate Size

One of the influential parameters in Walraven’s aggregate interlock constitutive equations is the maximum diameter ($D_{\text{max}}$) of the aggregate particles. Walraven (1981) conducted a sensitivity study to investigate the effect of this parameter on interface stresses. He observed that the interface shear stress is more influenced than the normal stress; the larger the maximum aggregate size, the stiffer the shear stress versus shear displacement curve, especially at large initial crack openings. This observation is important in aggregate interlock joints of concrete pavements that rely on the interface shear stresses as load transfer mechanism.

The effect of maximum aggregate size on post-crack responses of pavement slabs is examined in the present study by considering aggregate sizes of 0.65 in. and 1.3 in., while keeping other parameters in Walraven’s constitutive relations unchanged (see Table 5.2). It may be argued that the change in aggregate size may influence other material properties (e.g., compressive strength), but this is considered beyond the scope of the proposed analysis. The simulation is conducted for the two initial joint openings of 0.02 in. and 0.09 in., and the load-displacement responses are presented in Fig. 5.13. It is observed that aggregate size matters little when the initial joint
opening is small. On the other hand, as initial joint opening increases, more interlocking can be expected as $D_{\text{max}}$ increases, and therefore, larger aggregate particles result in stiffer joint behavior, manifested in the reduced second peak load at which the unloaded slab failed. A dimensionless parameter, $\omega_i/D_{\text{max}}$, in conjunction with the peak load may be a useful parameter to represent the effect of aggregate size on the slab response. It is concluded, therefore, that nonlinear aggregate interlock simulation in tandem with fracture analysis is essential in assessing the effect of joint opening and aggregate size.

5.5 Pre- and Post-Crack Analysis of Slabs with Aggregate Interlock under Constant Temperature Distribution and Increasing Wheel Load

Diurnal variations of temperature cause stresses in concrete pavement slabs, especially when combined with wheel loads, thereby reducing the system’s longevity. During the daytime, the top surface of the slab is warmer than the bottom and the slab curls downward; the bottom of the slab experiences tensile stresses, as well as widening of the joint opening. During the nighttime, the opposite phenomenon occurs: the slab curls up, and tensile stresses arise at the top of the slab, where joint opening broadens.

The effect of curling stresses on the pre- and post-crack responses of a single pavement slab has been investigated in Chapter 4 of this study. It was observed that daytime temperature variation through the slab thickness results in unstable crack formation and reduces significantly the peak load resisted by the slab. In contrast, a nighttime temperature gradient spawns stable cracks and a higher peak load. It was also observed that the fracture process depends not merely on the extreme-fiber curling stresses, but on the entire stress distribution through the thickness, re-emphasizing the importance of temperature profile estimation in concrete pavement structures.
In the present study, the effect of curling stresses on the pre- and post-crack response of the two slabs interconnected by aggregate interlock is investigated. The geometry of the setup is shown in Fig. 5.6, and the pertinent properties are given in Table 5.2. Constant temperature profiles measured by Teller and Sutherland (1935) at 7:00 pm and 9:30 am of June 12-13, 1932 are considered; these are typical nighttime and daytime variations through a 6 in. slab. A cubic polynomial was found in Chapter 4 to fit adequately these temperature variations; the following best fit equations are used to assign temperature values at nodes through the slab thickness:

\[
T(z) = -0.333z^3 + 3.446z^2 - 5.042z + 78.036 \quad \text{at 9:30 am} \quad R^2 = 0.9595 \quad (5.12a)
\]

\[
T(z) = -0.017z^3 - 0.060z^2 + 0.295z + 90.995 \quad \text{at 7:00 pm} \quad R^2 = 0.9969 \quad (5.12b)
\]

Here \( z \) is measured upward from the bottom of the slab. It is assumed that the temperature remains the same at all nodes lying on the same horizontal plane.

As described in Chapter 4, the analysis is carried out in two steps. In the first instance, the constant temperature distributions given in Eq. (5.12) are applied; the application of the increasing wheel load follows thereafter. The latter is represented by a unit displacement as described in earlier sections. Both linear and nonlinear aggregate interlock idealizations are considered. Note that the second loading scenario, in which a constant wheel load is applied first followed by an increasing thermal curling, is not considered in this chapter.

### 5.5.1 Linear Aggregate Interlock

The resulting load-displacement curves for the specified nonlinear daytime temperature profile (resulting in temperature differential, \( \Delta T = 22^\circ \text{F} \)) at three selected joint stiffness values are shown in Fig. 5.14. The peak loads in this case are significantly lower than those for no-curling.
presented earlier in Fig. 5.7, which had ranged between 30 and 60 kips. This is attributed to the curling stresses, which consume a substantial portion of the slab’s load-carrying capacity, even prior to the application of the wheel load. At lower joint stiffness values, only one softening kink is observed, indicating the failure of the loaded slab alone; this was also observed earlier in the case of no curling. As the joint stiffness increases, the unloaded slab starts to share the applied load and two softening regions may begin to be manifest. Finally, at very large joint stiffness values, for example, $AGG/kl=100$, only one softening is observed once more, evincing that the two slabs act monolithically. The translation of the load-displacement curves to the right with decreasing joint stiffness is worth noting. This reflects the curl-only deflections at the onset of loading, which are determined by the degree of monolithic or independent action of the two slabs, depending on the joint stiffness.

A similar approach has been followed for the analysis of the slab under nonlinear nighttime temperature distribution (resulting in $\Delta T = 4^\circ F$) and wheel loading. The corresponding load-displacement curves obtained are shown in Fig. 5.15, in which the peak loads are significantly higher than those for no-curling (see Fig. 5.7). Comparing the peak loads from these curves with those in Fig. 5.14, the first peak nighttime load (at which softening begins) is found to be much larger than for daytime temperatures for each joint stiffness value considered. At $AGG/kl = 100$, for example, this load is six times higher than during the daytime (96 kips compared to 12 kips). This change is attributable to compressive curling stresses arising during the nighttime at the bottom of the slab, counterbalancing the tensile stresses due to the wheel load.

As a result of temperature variation through the thickness of the slab, the two joint faces, which are initially vertical and parallel to one another (resembling the letter H), rotate with respect to
one another and assume inclined orientations, resembling letters V or Λ. Such movement consists of displacements normal to the joint surfaces, which contribute to a change in the joint stiffness. This is the essence of the coupling phenomenon between normal and shear interface stresses, which the linear aggregate interlock idealization cannot simply accommodate. This limitation is addressed by nonlinear aggregate interlock, as discussed below.

5.5.2 Nonlinear Aggregate Interlock

For the case of nonlinear aggregate interlock, load-displacement results for daytime temperature are shown in Fig. 5.16 for three selected initial joint opening values. Two softening regions are observed once again, an effect captured by the cascading failure of the loaded and unloaded slabs, respectively. When compared to the responses from the cases without curling presented in Fig. 5.10, the daytime peak loads are significantly lower (8 to 15 kips compared to 70 to 95 kips), as had been the case for linear aggregate interlock, as well.

During the daytime, the joint widens at the bottom and progressively closes at the top of the slab, assuming a configuration resembling the letter Λ. When the wheel load is applied, the joint opening continues to broaden at the bottom and remains closed at the top, thereby causing a decrease in load transfer efficiency. This results in a reduction of the pavement system competence, manifested in the reduced initial slope of the load-displacement curves as initial joint opening increases.

Comparison of Fig. 5.17 with Fig. 5.10 indicates that peak loads increase very slightly during the nighttime. A nighttime temperature variation results in progressive joint closing at the bottom of the slab and widening at the top, in a configuration resembling the letter V. This may be
expected to increase joint efficiency, an effect that can be particularly significant at large nighttime temperature differentials. This is verified by conducting additional nonlinear aggregate interlock runs involving an initial joint opening of 0.07 in., positive (i.e., daytime) and negative (i.e., nighttime) linear temperature differentials of 6, 12, and 18°F. Variations of transferred load efficiency at the first peak load with respect to the temperature differentials are presented in Fig. 5.18. It is observed that during the nighttime, load transfer efficiency remains unaffected by an increase in the temperature differential, while during the daytime it is inversely proportional to it. Therefore, daytime temperature is deemed critical, not only because of the additional curling stresses induced at the bottom of the slab, but also due to the loss of joint efficiency, which hastens the failure of the loaded slab.

5.6 Summary and Conclusions

Simulations using different techniques for idealizing aggregate interlock load transfer in jointed concrete pavement slabs are presented. Both pre- and post-crack analyses are conducted. Previously published numerical and experimental studies are available only for pre-crack conditions, and they are used to validate the proposed formulation. Post-crack responses are used to investigate crack growth in both the loaded and the unloaded slabs, for linear as well as nonlinear aggregate interlock idealizations. For linear aggregate interlock, it is found that at smaller joint stiffness values, there is only one peak load, indicating the failure of only the loaded slab. As the joint stiffness increases, however, softening begins to be manifest at two points on the load-displacement curve, evincing the successive failure of both the loaded and unloaded slabs. At a very large joint stiffness, the two slabs act as one unit, and only one softening region is observed once again, at a relatively large peak load. The post-crack load transfer efficiencies
with respect to vertical deflections and the applied load are observed to be similar to those reported in the literature for uncracked slabs. In general, the load transfer efficiency with respect to crack mouth opening displacement ($LTE_\omega$) is found to be lower than load transfer efficiency with respect to deflection ($LTE_\delta$).

Unlike its linear precursor, nonlinear aggregate interlock idealization takes into account the change in the joint opening during the loading process, and the concomitant change in the load transfer efficiency of the joint. It is observed that the load transfer efficiency with nonlinear aggregate interlock decreases linearly with increasing initial joint opening. It is also found that $LTE_\delta$ continuously increases until the peak load is attained and decreases thereafter.

The effects of both nighttime and daytime temperature are also investigated for the linear as well as the nonlinear aggregate interlock idealizations. Daytime temperature variations result in a lower peak load, and in decreased joint efficiency on account of a widened joint opening at the bottom of the slab. Nighttime temperature curling, on the other hand, has the opposite effect and may prevent a decrease of joint efficiency, if it does not actually enhance joint behavior.

When viewed in conjunction with earlier studies (Aure and Ioannides, 2010; Aure and Ioannides, 2012), this study affirms the conclusion that application of cohesive elements to the fracture analysis of concrete pavement slabs is promising. The approach will be extended to more complex in situ pavement systems that involve dowel-jointed slabs under combined wheel and thermal loadings in the next Chapter. Experimental studies that can verify such numerical approaches need to be conducted both in the field and in the laboratory, thereby contributing to the ongoing development of rational failure criteria that can replace the statistical/empirical
algorithms along with Miner’s (Miner, 1945) hypothesis that are currently relied upon in mechanistic-empirical pavement design procedures.

5.7 References


Table 5.1 Walraven’s Equations for Contact Areas per Unit Crack Area

Case A: \( \delta_i < \delta_n \)

\[
\bar{A}_y = \int_0^{D_{\max}} \frac{4p_k}{\pi} \times F \left( \frac{D}{D_{\max}} \right) \times G_1(\delta_n, \delta_i, D) \, dD
\]

\[
\bar{A}_x = \int_0^{D_{\max}} \frac{4p_k}{\pi} \times F \left( \frac{D}{D_{\max}} \right) \times G_2(\delta_n, \delta_i, D) \, dD
\]

Case B: \( \delta_i > \delta_n \)

\[
\bar{A}_y = \int_0^{D_{\max}} \frac{4p_k}{\pi} \times F \left( \frac{D}{D_{\max}} \right) \times G_3(\delta_n, \delta_i, D) \, dD + \int_0^{D_{\max}} \frac{4p_k}{\pi} \times F \left( \frac{D}{D_{\max}} \right) \times G_4(\delta_n, \delta_i, D) \, dD
\]

\[
\bar{A}_x = \int_0^{D_{\max}} \frac{4p_k}{\pi} \times F \left( \frac{D}{D_{\max}} \right) \times G_5(\delta_n, \delta_i, D) \, dD + \int_0^{D_{\max}} \frac{4p_k}{\pi} \times F \left( \frac{D}{D_{\max}} \right) \times G_6(\delta_n, \delta_i, D) \, dD
\]

with

\[
G_1(\delta_n, \delta_i, D) = D^{-3} \left[ \sqrt{D^2 - \left( \delta_n^2 + \delta_i^2 \right)} \times \frac{\delta_i}{\sqrt{\delta_n^2 + \delta_i^2}} \times u_{\max} - \delta_n \times u_{\max} - u_{\max}^2 \right]
\]

\[
G_2(\delta_n, \delta_i, D) = D^{-3} \left[ \delta_i - \sqrt{D^2 - \left( \delta_n^2 + \delta_i^2 \right)} \times \frac{\delta_n}{\sqrt{\delta_n^2 + \delta_i^2}} \times u_{\max} + \left( u_{\max} + \delta_i \right) \times \frac{1}{4} \left( D^2 - \left( \delta_n + u_{\max} \right)^2 \right) \right. \\
- \delta_n \times \frac{1}{4} \left( D^2 - \delta_n^2 \right) + \frac{1}{4} D^2 \arcsin \left( \frac{\delta_n + u_{\max}}{2 D} \right) - \frac{1}{4} D^2 \arcsin \left( \frac{2 \delta_n}{D} \right) \right]
\]
Table 5.1 (Continued)

\[ G_3(\delta_n, \delta_i, D) = D^{-3}\left(\frac{1}{2}D - \delta_n^2\right)^2 \]

\[ G_4(\delta_n, \delta_i, D) = D^{-3}\left(\frac{\pi}{8}D^2 - \delta_n^2\sqrt{\frac{1}{4}D^2 - \delta_n^2} - \frac{1}{4}D^2 \arcsin\left(\frac{2\delta_n}{D}\right)\right) \]

\[ F\left(\frac{D}{D_{\text{max}}}\right) = 0.532\left(\frac{D}{D_{\text{max}}}\right)^{0.5} - 0.212\left(\frac{D}{D_{\text{max}}}\right)^4 - 0.072\left(\frac{D}{D_{\text{max}}}\right)^6 - 0.036\left(\frac{D}{D_{\text{max}}}\right)^8 - 0.025\left(\frac{D}{D_{\text{max}}}\right)^{10} \]

\[ u_{\text{max}} = \frac{\frac{1}{2}\delta_n(\delta_n^2 + \delta_i^2) + \frac{1}{2}\sqrt{\left(\frac{\delta_n^2}{\delta_i^2}\right)^2 - \left(\frac{\delta_n^2}{\delta_i^2}\right)^2 - 4\delta_i^2\left(\frac{D}{2}\right)^2}}{\delta_n^2 + \delta_i^2} \]

\[ p_k = \text{aggregate volume/unit volume of concrete} \]
Table 5.2 Geometry, Material Properties and Discretization of Two Slabs Considered

<table>
<thead>
<tr>
<th>Geometry of Each Slab</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (in.)</td>
<td>240</td>
</tr>
<tr>
<td>Width (in.)</td>
<td>120</td>
</tr>
<tr>
<td>Thickness (in.)</td>
<td>6</td>
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<table>
<thead>
<tr>
<th>Material Properties</th>
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<tbody>
<tr>
<td>Young's Modulus (ksi)</td>
<td>4000</td>
</tr>
<tr>
<td>Thermal Expansion Coefficient (in./in./°F)</td>
<td>$5 \times 10^{-6}$</td>
</tr>
<tr>
<td>Tensile Strength (ksi)</td>
<td>0.463</td>
</tr>
<tr>
<td>Fracture Energy (lb/in.)</td>
<td>0.431</td>
</tr>
<tr>
<td>Compressive Strength (ksi)</td>
<td>5</td>
</tr>
<tr>
<td>Unit Weight (pcf)</td>
<td>150</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>0.15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Walraven’s Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Aggregate size (in.)</td>
<td>0.65</td>
</tr>
<tr>
<td>Coefficient of Friction</td>
<td>0.4</td>
</tr>
<tr>
<td>Aggregate Volume per Unit Volume of Concrete</td>
<td>0.75</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subgrade Characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of Subgrade Reaction (pis/in.)</td>
<td>200</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Discretization Characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Slab Element Size (in.)</td>
<td>$6 \times 6 \times 2$</td>
</tr>
<tr>
<td>Cohesive Zone Element Size (in.)</td>
<td>$0.12 \times 0.12$</td>
</tr>
<tr>
<td>Softening</td>
<td>Bilinear</td>
</tr>
<tr>
<td>Solver</td>
<td>Newton-Raphson</td>
</tr>
<tr>
<td>Subgrade Idealization</td>
<td>$SPRING1$</td>
</tr>
</tbody>
</table>

Notes: 1 lb = 4.444 N; 1 in. = 25.4 mm; 1 ksi = 6.89 Mpa
Table 5.3 Joint Stiffness Characteristics Used for Linear Aggregate Interlock

<table>
<thead>
<tr>
<th>Assumed $\frac{AGG}{kl}$ values</th>
<th>$AGG$ (ksi)</th>
<th>$JOINTC$ or $CONN3D2$ $K_{nn}$ (kips/in.) By Eq. (5.7)</th>
<th>$COH3D8$ $K_{nn}$ (kips/in.) By Eq. (5.10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>0.00493</td>
<td>0.00062</td>
<td>0.00082</td>
</tr>
<tr>
<td>0.010</td>
<td>0.04930</td>
<td>0.00616</td>
<td>0.00821</td>
</tr>
<tr>
<td>0.100</td>
<td>0.49300</td>
<td>0.06159</td>
<td>0.08212</td>
</tr>
<tr>
<td>1.000</td>
<td>4.93000</td>
<td>0.61587</td>
<td>0.82116</td>
</tr>
<tr>
<td>10.000</td>
<td>49.3000</td>
<td>6.15870</td>
<td>8.21160</td>
</tr>
<tr>
<td>100.000</td>
<td>493.000</td>
<td>61.5870</td>
<td>82.1160</td>
</tr>
<tr>
<td>1000.000</td>
<td>4930.00</td>
<td>615.870</td>
<td>821.160</td>
</tr>
<tr>
<td>10000.000</td>
<td>49300.00</td>
<td>6158.70</td>
<td>8211.60</td>
</tr>
</tbody>
</table>

Note: For $JOINTC$ or $CONN3D2$: $K_{ss} = 0$ and $K_{nn}$ is determined from Eq. (5.6);

Edge stiffness = $2 \times$ corner stiffness; Interior stiffness = $4 \times$ corner stiffness;

For $COH3D8$: $K_{ss} = K_{nn}$ and $K_{nn}$ is determined from Eq. (5.11);

$$l = \frac{Eh^3}{12(1 - \mu^2)k}$$

is radius of relative stiffness and $k$ is modulus of subgrade reaction.
Figure 5.1 Typical Normal and Shear Stress Variations at Different Crack Openings
Figure 5.2 Comparison Among Three Linear Aggregate Interlock Idealizations and Previous Studies

Note: $a/l = 0.275$
Figure 5.3 Verification of Relationship between Load Transfer Efficiencies

(Linear Aggregate Interlock)

Note: $a/l = 0.275$
Figure 5.4 Top View, Boundary Conditions and Applied Load

(Colley and Humphrey, 1967)
Figure 5.5 Comparison with Experimental and FE Results by Other Researchers

(Nonlinear Aggregate Interlock)
Figure 5.6 Planar View, Location of Applied Load and Assumed Fracture Planes
Figure 5.7 Effect of Joint Stiffness on Slab System Response

(Linear Aggregate Interlock)
Figure 5.8 Pre-Crack and Post-Crack Load Transfer Responses

(Linear Aggregate Interlock)
Figure 5.9 Variation of Load Transfer Efficiency with Total Applied Load

(Linear Aggregate Interlock)
Figure 5.10 Effect of Initial Joint Opening on Load-Displacement Response

(Nonlinear Aggregate Interlock, $D_{\text{max}} = 0.065$ in.)
Figure 5.11 Effect of Initial Joint Opening on Load Transfer at Peak Load

(Nonlinear Aggregate Interlock, $D_{\text{max}} = 0.065$ in.)
Figure 5.12 Variation of Load Transfer Efficiency with Total Applied Load

(Nonlinear Aggregate Interlock, $D_{\text{max}} = 0.065\text{in.}$)
Figure 5.13 Effect of Maximum Aggregate Size (Nonlinear Aggregate Interlock)
Figure 5.14 Effect of Daytime Temperature (Linear Aggregate Interlock)
Figure 5.15 Effect of Nighttime Temperature (Linear Aggregate Interlock)
Figure 5.16 Effect of Daytime Temperature

(Nonlinear Aggregate Interlock, $D_{max} = 0.065$ in.)
Figure 5.17 Effect of Nighttime Temperature

(Nonlinear Aggregate Interlock, $D_{max} = 0.065$ in.)
Figure 5.18 Effect of Temperature on Joint Efficiency

(Nonlinear Aggregate Interlock, $D_{\text{max}} = 0.065$ in., $\omega_i = 0.07$ in.)
Chapter 6  Numerical Analysis of Cracking in Doweled Concrete Pavements

Abstract

This paper presents numerical analyses of crack propagation in concrete pavement slabs with doweled joints using the finite element package ABAQUS® 6.9-2. Traction-separation cohesive elements are employed in simulating the fracture process according to the fictitious crack model. Both wheel load and thermal curling are considered. The effects of joint width, dowel-concrete interaction idealization, and dowel slip on slab responses are investigated. It is observed that joint width does not affect the maximum load supported, but it reduces the load transfer efficiency of the pavement system. Dowel slip is found to decrease the peak load that is resisted by the slab system. A daytime temperature variation significantly reduces the peak load, whereas a nighttime temperature profile increases it. This step-by-step effort may contribute to the ongoing development of rational failure criteria that can substitute the statistical/empirical algorithms used in current pavement design procedures.
6.1 Introduction

Load transfer in jointed concrete pavements is accomplished by two primary mechanisms: aggregate interlock and dowel bars. Aggregate interlock relies only on shear forces developing on rough vertical joint interfaces due to mechanical interlock between aggregate particles. Numerical idealization of aggregate interlock joints and its post-crack implications on pavement system responses have been discussed extensively in Chapter 5.

Dowel bars are often placed across a joint to complement the aggregate interlock load transfer mechanism, and to ensure its permanence. Friberg (1940) idealized dowel bars as beam elements encased in an elastic medium, per an earlier suggestion by Timoshenko and Lessels (1925). This approach has been incorporated in the two-dimensional (2D) finite element (FE) code ILLI-SLAB, by considering both the bending and shear resistance of dowel bars (Tabatabaie and Barenberg, 1978). Huang and Chou (1978) argued that the bending contribution of the dowel bar over the very short span of the joint opening could be neglected, leaving shear resistance as the sole load transfer mechanism of doweled joints. Korovesis (1990) adapted ILLI-SLAB to permit activation of dowel bending, shear and torsional degrees of freedom. Ioannides and Korovesis (1992) then used dimensional analysis to interpret results obtained from ILLI-SLAB and derived an independent joint stiffness parameter \( D/skl \) corresponding to the dimensionless aggregate interlock joint stiffness \( AGG/kl \). Here, \( D \) [FL\(^{-1}\)] is composite joint shear stiffness, \( AGG \) [FL\(^{-2}\)] is aggregate interlock joint stiffness per unit length, \( k \) [FL\(^{-3}\)] is the subgrade modulus, \( l \) [L] is the radius of relative stiffness of the slab-subgrade system, and \( s \) [L] is dowel bar spacing. The primary dimensions are abbreviated here as [L] for length and [F] for force.
A slightly different formulation for the interaction of the dowel beam element with the slab was first presented by Nishizawa et al. (1989), who divided the dowel bar into three segments: two bending components embedded in the slab and one shear-and-bending component across the joint. This approach was further refined by Guo et al. (1995), who implemented it in an update of 2D FE code JSLAB (Tayabji and Colley, 1984). Subsequently, an analogous, if more elaborate, concept was also employed in three-dimensional (3D) FE analysis (Davids and Mahoney, 1999; Davids, 2000; Kim and Hjelmstad, 2003).

In the present study, 3D numerical pre- and post-crack analysis of a dowel-jointed pavement system is conducted, using the general purpose finite element package ABAQUS® 6.9-2. The slab system is subjected to both wheel and temperature loading. The fracture process of the slabs is tracked using traction-separation cohesive elements inserted along the anticipated crack path in both slabs. The effects on slab responses of dowel-concrete interaction idealization, joint opening and dowel looseness are investigated.

6.2 Finite Element Idealization of Doweled Pavement System

6.2.1 Discretization of Slab and Subgrade

The geometry, material properties and FE discretization characteristics employed in the present study are shown in Table 6.1. Properties, including tensile strength, \( f' \), and fracture energy, \( G_F \), are selected to be consistent with previous studies (Ioannides and Peng, 2004; Ioannides et al., 2006; Aure and Ioannides, 2012). The FE mesh shown in Fig. 6.1 is generated using software application ABAQUS®/CAE 6.9-2, a pre- and post-processing package. Reduced integration 20-node quadratic elements, \( C3D20R \), are employed for the bulk of the concrete slab material, since
these are the highest order continuum elements supported in ABAQUS®/CAE. The mesh is considerably finer along the assumed fracture plane and the joint than elsewhere. Tetrahedron elements, C3D10, are used in the transition zones to the coarser mesh regions, as shown in Fig. 6.1, since these are the highest order tetrahedron elements compatible with C3D20R.

The adoption of this graded mesh increases the amount of labor required in defining the stiffness of individual spring elements for the supporting Winkler subgrade, which must not resist any tension. To overcome this difficulty, membrane elements, M3D8R, of negligible Young’s modulus (1 ksi) and zero thickness are defined beneath each slab, onto which FOUNDATION elements that can support tension and compression are attached; the stiffness of the latter is manually provided in the input file. This approach was first proposed by Kuo (1994) to idealize slab-subgrade and slab-base interfaces. The membrane elements are discretized with the same mesh size as the slabs; their horizontal interaction with the slabs is defined as FRICTION, ROUGH (i.e., infinite coefficient of friction), whereas in the normal direction only compressive forces are resisted. Consequently, the slab and the membrane can separate when a tensile force acts between them.

The wheel load is applied as a unit displacement at nodes located at \((x, y, z) = (114, 3, 6)\) in. and \((x, y, z) = (114, -3, 6)\) in., as shown in Fig. 6.2. This is intended to represent a 12 by 12 in. area at mid-edge of the loaded slab. It is anticipated that the maximum stress will develop at the slab bottom, along the joint, half-way between the points of displacement application. The two vertical planes perpendicular to the joint are, therefore, assumed to be the fracture planes. The results monitored are: the total applied load \((P)\), i.e., the sum of the conjugate “reaction” forces at
the two loaded nodes, $RF_2$; and the vertical displacement at the nodes where the displacement is applied.

### 6.2.2 Discretization of Dowel Bars

The diameter, spacing, length, and material properties of the dowel bars employed in the present study are shown in Table 6.1. These values are selected to represent prevailing practical guidelines (Huang, 2004). The $C3D20R$ element used for the intact slab material is also employed in discretizing the dowel bars, thereby eliminating any loss of precision incurred when dissimilar elements are connected. Two approaches are investigated when idealizing dowel-concrete interaction. In the first approach, the dowel bars are placed in holes “drilled” in the concrete slab using ABAQUS®/CAE instruction $cut extrude$, and $SURFACE INTERACTION$ options are used to describe their relative movement. If no relative slip is allowed, the $TIE CONSTRAINT$ option may be used, instead.

The second approach involves embedding the dowel bars in the slab without pre-drilling holes. This method is often used to idealize reinforcement bars in concrete structures. In ABAQUS®, it is achieved by the use of the $EMBEDDED ELEMENT$ key word. The main advantage of this approach over the former is that it simplifies the discretization of the slab near the dowels, which has repercussions on convergence and computational resources required. The nodes of the embedded element are constrained to the nodes on the host (concrete mass) element. The program will “search for the geometric relationships between nodes on the embedded elements and the host elements. If a node on an embedded element lies within a host element, the degrees of freedom at the node will be eliminated by constraining them to the interpolated values of the degrees of freedom of the host element” ABAQUS (2009). The main drawbacks of this approach
are that it does not simulate the in situ dowel-concrete interaction, which involves sliding on one side of the joint, and that it does not account for any gaps present between the bar and the concrete slab. The effects of such differences between the two approaches on the post-crack response of the pavement system are examined in sections that follow.

As a baseline, linear elastic material properties are assumed for the dowel bars. It might be argued, however, that the load transfer system could fail by yielding even before the slab fails, i.e., before the peak load is attained. Therefore, yielding properties of the dowel bars are also specified in Table 6.1. The resulting slab response is compared to the baseline observations later in the study.

6.2.3 Discretization of the Fracture Process

Three-dimensional traction-separation cohesive elements, COH3D8, are used to discretize a very narrow fracture process zone (FPZ), whose width is set to 0.001 in. as suggested by previous experiences (Aure and Ioannides, 2010; Aure and Ioannides, 2012). The FPZ mesh involves a uniform pattern of element size 0.2 by 0.2 in., which is much finer than the slab mesh size. The cohesive elements are inserted along the assumed fracture plane, and surface-based TIE CONSTRAINTS are used to connect them to the intact slab surfaces. This technique has been used by the authors in earlier work (Aure and Ioannides, 2010; Aure and Ioannides, 2012) and found to be efficient. A bilinear concrete softening curve is employed per Petersson (1981) and Gustafsson (1985); the coordinates of the kink point are set at \( \left( 0.8 \frac{G_f^{c}}{f_t^{c}}, \frac{1}{3} f_t^{c} \right) \), as in a previous study (Aure and Ioannides, 2012). The pertinent fracture parameters are given in Table 6.1. Since this is a problem involving stiffness degradation, viscous regularization is employed with the
general (Newton-Raphson) solver option in order to ensure a convergent solution; the so-called viscosity is set to $1 \times 10^{-6}$ (ABAQUS, 2009).

6.3 Validation of Finite Element Discretization: Pre-Crack Analysis

The pre-crack phase, during which all materials are linear and elastic, affords the opportunity to verify the FE discretization, i.e., the adequacy of the mesh size, element types, and idealizations of dowel-concrete interaction and subgrade. For this purpose, the experimental study conducted by Hammons (1997) is reproduced. Among the six small-scale tests reported, LSM-2 is considered. The system consists of two slabs, each of which is $36 \times 48 \times 2$ in., connected by 0.25 in. diameter dowel bars that are 15.5 in. long and are spaced at 4 in. center-to-center. The joint opening is 0.0625 in. The moduli, $E$, for the slab and for the dowel bar are taken as 4 Mpsi and 29 Mpsi, respectively; the corresponding values of Poisson’s ratio are 0.18 and 0.3 (per Hammons, 1997). Additional details about the experimental setup are provided by Hammons (1997). The rubber pad foundation used in the experiment was assigned a subgrade modulus, $k$, of 0.33 ksi/in., backcalculated using a FE analysis by Hammons (1997).

The FE discretization of the test slab-dowel system is shown in Fig. 6.3. It is similar to the one discussed in Section 6.2, above, which will be employed in the next section. The boundary conditions are selected to represent the set-up in the experiment: translations in all directions at nodes along the two outer 48-in. sides, as well as planar translations at nodes along the two outer 72-in. sides, are restrained. The load is applied as a pressure of 145 psi at a corner by the joint over a square area of 16 in.$^2$. The two dowel-concrete interaction idealizations discussed above are implemented. In the first case, the dowel bars are assumed to be encased in the slabs (without a hole), i.e., dowels experience no relative movement with respect to the slabs. Consequently, the
**EMBEDDED ELEMENT** approach is employed to discretize dowel-concrete interaction. In the second case, which is closer to reality, holes are created in the unloaded slab and then dowel bars are placed in therein. It is noteworthy that Hammons (1997) had applied grease to the dowel bars and encased them in drinking straws on the unloaded side of the joint, a common practice in order to ensure debonding during load transfer. While reproducing Hammons experimental results, Davids and Mahoney (1999) estimated the thickness of the drinking straw to be 3.15 mills (0.08 mm). In the present study, this effect is accounted by providing a clearance of 3.15 mills between the dowel bars and the slab holes. The effect of grease is idealized, by providing a coefficient of friction of 0.01.

Figure 6.4 shows the resulting deflection profile at a distance of 7 in. from the edge closest to the loaded area; this is the location at which the displacement transducers had been placed during the experiment (Hammons, 1997). The same experiment had been previously reproduced by Davids and Mahoney (1999) in verifying the FE program *EverFE*, and their result is also shown. Good agreement is observed between the present study and Davids and Mahoney (1999) for both cases, no gap (embedded) and with gap. As compared to the experiment, it is observed that the **EMBEDDED ELEMENT** approach results in a very stiff joint and results in a perfect load transfer efficiency. The second approach, in which the gap between the dowel bars and the slab holes is accounted, gives almost perfect deflection profile as compared to the experiment especially around the joint. Relatively larger differences between the present study and the experiment in the displacements as one moves away from the joint may be attributed to the effect of the boundary condition of the slab, which is idealized in this study as perfectly rigid around the peripheries of the slab. Obviously, perfect rigidity may not be attained in the experiment.
Other reasons may include numerical discretization errors, such as mesh fineness and contact interaction between the slab and dowel bar. It worth noting that the idealization of dowel-concrete interaction when a gap exists between the slab holes and the dowel bars is computationally demanding, especially when such an approach is augmented with cracking in the slabs.

From the results shown in this section, the numerical discretization adopted is deemed reliable and applicable to the investigation of the more complex phase involving fracture of the slabs subjected to both wheel load and thermal curling, as discussed in the following sections.

### 6.4 Crack Propagation in Slabs with Doweled Joints

Post-crack responses of the doweled pavement system described earlier are studied in this section. Recall that geometry, material properties and FE discretization are shown in Table 6.1 and Figs. 6.1 and 6.2; slabs are connected by dowel bars that are nominally 14 in. long and of diameter 0.75 in., spaced at 12 in. center-to-center, inserted in holes “drilled” into the concrete slab. A parametric study is carried out to investigate the effect of joint opening, dowel embedment, dowel-concrete interaction (dowel looseness), and yielding of the dowel bar, as well as thermal curling.

#### 6.4.1 Effect of Initial Joint Opening

The significance of joint opening, \( \omega_i \), on the load-displacement response of the two-slab system during the fracture process is investigated by considering three initial joint openings: 0.2 in. (baseline), 0.4 in. and 0.8 in. The length of the dowel bars is adjusted slightly depending on the joint width, so that 6.9 in. remain encased in the slab on each side of the joint; for the baseline
opening of 0.2. in., the length of the dowels is, therefore, 14 in. The dowel bar surfaces are connected to the corresponding holes ‘drilled’ in the slab using the TIE CONSTRAINT option; this eliminates dowel slip, but it is selected because of its implementation simplicity.

The load-displacement responses obtained in this manner are plotted in Fig. 6.5. The curves exhibit only one softening event, evincing that the two slabs failed almost at the same time. The more competent dowel system evidently results in monolithic action of the two slabs. It is observed that an increase in joint opening decreases slightly the stiffness of the pavement system, discernible as a decrease in the slope of the load-displacement curves. The reduction in slope at about half the peak load evinces the onset of cracking in the loaded slab. The value of the peak load resisted by the slabs remains largely unaffected by changes in joint opening. It appears, therefore, that while an increase in initial joint opening may increase slightly the displacement at which the peak load occurs, its influence on the peak load supported by the slabs is rather insignificant. This observation is confirmed by Fig. 6.6, which shows the variation of the load transfer efficiency with respect to deflection ($LTE_\delta$) as the initial joint opening increases. It is evident that as the joint opening increases, load transfer efficiency decreases almost linearly.

6.4.2 Effect of Dowel–Concrete Interaction

The interaction between the concrete mass and the dowel bar is one of the major parameters influencing load transfer efficiency in a doweled pavement system. Consequently, there have been several attempts by different researchers to discretize this interaction realistically in 3D FE analysis. In general, such efforts fall into two main categories: those employing spring elements and those relying on contact elements.
An early attempt employing the former approach is described by Channakeshava et al. (1993). The same methodology was subsequently implemented by Davids (2000) during the development of 3D FE program *EverFE*. Accordingly, spring elements were used to represent the relative movements of the dowel bar with respect to the concrete mass in which it is encased, in the three orthogonal directions. Such movements are characterized using three distinct stiffnesses, but since the normal and transversal components are physically indistinguishable, only two spring stiffnesses need be specified. The first is the conventional modulus of dowel support, $K \ [FL^{-3}]$ (Timoshenko and Lessels, 1925). The second stiffness, corresponding to the slip of the dowel bar along its longitudinal direction, was dubbed the “dowel-slab restraint modulus”. It has the same dimensions as $K$, but it is typically assumed to have only a fraction of its magnitude. A similar approach was adopted by subsequent investigators (Bhattacharya, 2000; Dere et al., 2006). The main limitations of this idealization technique are: (a) There are no sound experimental or theoretical methods to determine the modulus of dowel support, $K$ (Ioannides and Korovesis, 1992), let alone its ratio to the dowel-slab restraint modulus; (b) This approach ignores dowel looseness, and is incapable of simulating possible dowel-concrete separation; (c) A gap alternative provided by Davids (2000), ignores geometric nonlinearities arising at the dowel-concrete interface during the continual alternation between contact and separation. Moreover, this alternative necessitates the elimination of dowel sliding, effectively requiring full-bond between the dowel and the concrete upon gap closing.

The second approach employs contact elements, already implemented in FE programs selected for simulation of dowel-concrete interaction in concrete pavement systems (William and Shoukry, 2001; Riad et al., 2009; Maitra et al., 2009). Coulomb’s friction law is adopted to
idealize dowel slip, according to a specified coefficient of friction. A certain clearance is provided between the dowel and the concrete to represent the gap; upon closure, the specified modulus of dowel support is activated. Determination of these parameters is still fraught with difficulties.

In the present study, the second approach is adopted, because it is deemed to be more comprehensive and realistic, even though for numerical expediency, it is assumed that there is no gap between the dowel bar and the slab. The bearing (or normal) resistance is idealized using the **HARD CONTACT** option under **SURFACE INTERACTION**, which obviates the need to specify \( K \) since the transfer of normal pressure between the two surfaces is controlled by their respective stiffnesses and the FE mesh. Dowel slip is idealized using the Coulomb **FRICITION** option.

As noted in Section 6.2.2 above, fully bonded dowel bars can be simulated alternatively using the **EMBEDDED ELEMENT** discretization, which eliminates the presence of holes in the concrete mass and thereby simplifies the numerical discretization of the dowel-concrete region.

Four cases are considered involving dowels inserted in pre-drilled holes. In the first instance, no relative movement is allowed between the dowel and slab (i.e., the two are assumed to be fully bonded), whereas for the remaining two cases sliding between the dowel and slab is allowed, and the coefficient of friction is set at 0.01 or 0.2. To compare with the fully bonded case, the **EMBEDDED ELEMENT** discretization is considered as a fourth case. Only the loaded side segment of the dowel bar is allowed to slip. In all the cases, the initial joint opening is kept at 0.2 in.
The load-displacement responses obtained are shown in Fig. 6.7. It is observed that there is a significant reduction in the peak load (by about 20%) as the dowel slips. It is interesting to see that there is also a noticeable decrease in stiffness of the system with slip, occurring after the onset of cracking at about half the peak load. The load transfer efficiency with respect to deflection ($LTE_\delta$) is also computed. It is found that for fully bonded, coefficient of friction of 0.2 and 0.01, $LTE_\delta$ is 0.705, 0.700, and 0.696, respectively at an applied load of 35 kips. This indicates a slight reduction of load transfer efficiency of the joint with increasing slip.

The load-displacement response corresponding to the *EMBEDDED ELEMENT* discretization option is also shown in Fig. 6.7. The results indicate that this approach gives a slightly lower peak load value as compared to the no slip case. The stiffness of the system, however, remains the same in these two cases. It can be concluded that for dowel bars fully bonded to the concrete slab, the *EMBEDDED ELEMENT* approach is not only attractive for its numerical simplicity but also compares well with the more elaborate no-slip alternative.

6.4.3 Effect of Dowel Bar Yielding

As the load continues to increase past crack initiation, dowel bars may yield and precipitate load transfer failure, even before ultimate structural collapse of the concrete. To investigate whether this situation may happen for the joint configuration specified earlier (Table 6.1), dowel bars are assigned the ABAQUS® nonlinear material constitutive relation of isotropic hardening suitable for steel. Two yield stress levels of 36 ksi and 55 ksi are considered; these correspond to ultimate stresses of 50 and 70 ksi, respectively. In both instances, ultimate plastic strain is assumed to be 10% of the elastic strain (Salmon et al., 2009).
The simulation result is presented as a load-displacement curve in Fig. 6.8. It is observed that dowel bar yielding occurs much later than the peak load. Yielding is manifested in the slope decrease after the end of softening, a phenomenon not observed in the linear, no-yield case (no-slip in Fig. 6.7). It might be expected that premature load transfer failure due to dowel yielding may occur only if dowels are of inadequate diameter, are too widely spaced, or are made of low grade material.

### 6.4.4 Effect of Temperature Differential

The effect of temperature on the fracture of a single concrete pavement slab has been studied in Chapter 4 of the present research. Two typical daytime and nighttime temperature cycles have been considered, as originally recorded by Teller and Sutherland (1935) at 7:00 pm and 9:30 am on July 12 and 13, 1932, respectively. The pavement system consisted of a 6-in. thick concrete slab-on-grade, resting on silty clay subgrade. That case study is retained in this Chapter, which also considers load transfer provided by steel dowels, as described in Table 6.1, above. Per Chapter 4, cubic polynomials are used to assign temperature values, \( T(y) \), through the slab thickness, as follows:

\[
T(y) = -0.333y^3 + 3.446y^2 - 5.042y + 78.036 \quad \text{at 9:30 am} \quad R^2 = 0.9595
\]

\[
T(y) = -0.017y^3 - 0.060y^2 + 0.295y + 90.995 \quad \text{at 7:00 pm} \quad R^2 = 0.9969
\]

in which \( y \) is the distance measured up from the bottom of the slab. It is assumed that the temperature is constant at all points lying on the same horizontal plane.

The load is applied in two steps: first, temperature effects are considered alone, and subsequently the wheel load is applied incrementally as a unit displacement, per Section 6.2.1, above. The
deformed shape of the cracked slab for the daytime temperature differentials obtained in this manner is shown in Fig. 6.9; the corresponding nighttime (curled-up) depiction is very similar. In this enlarged presentation, the transition elements described in Section 6.2.1 are clearly visible. Figure 6.10 shows both daytime and nighttime load-displacement responses, along with the corresponding response for the case without curling (no-slip case in Fig. 6.7). As might be expected, the nighttime peak load (~95 kips) is significantly higher than the no-curling value (~45 kips); the latter is itself more than double the corresponding daytime capacity (~18 kips). These results reinforce the conventional understanding that load-plus-curling stresses are cumulative during the daytime, both being tensile at the bottom of the slab, whereas they tend to offset each other during the nighttime, as curling tension migrates to the top.

6.5 Summary and Conclusions

The effects of joint characteristics on the load carrying capacity and fracture process of a doweled pavement system have been studied. An initial pre-crack analysis provided verification for the robustness of the proposed FE idealization, which was confirmed by comparisons with previous laboratory and numerical results. Fracture analysis of the slabs has been carried out to investigate the maximum load the system can resist before failure. It is found that an increase in initial joint opening decreases slightly the system stiffness (manifest in a slope reduction of the load-displacement curve), but does not change significantly the peak load resisted. A major parameter that influences slab response is identified to be the interaction between the dowel bar and the slab, i.e., dowel slip; the peak load decreases by about 20% for the range of coefficient of friction considered in this study (no slip, 0.01 and 0.2). If the dowel is fully bonded to the slab, the computationally efficient EMBEDDED element approach for dowel placement is found to be
no less precise than the more demanding options. The influence of daytime and nighttime temperature distributions on the peak load capacity of dowel jointed slabs is also investigated. It is observed that daytime temperatures result in approximately half the peak load resisted during the nighttime, due to their combination with load effects.

It can be concluded from this study that the application of cohesive elements in a fracture analysis of doweled concrete pavement slabs is promising and may be extended to concrete pavements subjected to repeated loading. This effort may contribute to the ongoing development of rational failure criteria that can substitute the statistical/empirical algorithms employed along with Miner’s hypothesis (Miner, 1945) in current mechanistic-empirical pavement design procedures.

6.6 References


### Table 6.1 Geometry, Material Properties and Discretization of Pavement System

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<tr>
<th>Slab Geometric Properties</th>
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<tr>
<td>Length (in.)</td>
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<td>Width (in.)</td>
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<td>Thickness (in.)</td>
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<tr>
<td>Thermal Expansion Coefficient (in./in./°F)</td>
<td>$5 \times 10^{-6}$</td>
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<th>Discretization Characteristics</th>
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<td>Cohesive Zone Element Size (in.)</td>
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<tr>
<td>Softening Curve</td>
<td>Bilinear</td>
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<tr>
<td>Solver</td>
<td>Newton-Raphson</td>
</tr>
<tr>
<td>Subgrade Idealization</td>
<td>FOUNDATION</td>
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</table>
Total number of nodes: 302464
Total number of elements: 85656
37000 elements of type C3D20R
388 elements of type C3D10
40960 elements of type COH3D8

Figure 6.1 Finite Element Mesh
Figure 6.2 Slab Geometry and Loading Layout

- Expected fracture plane
- Location of applied unit displacement
- 20-0.75 in. dia. dowels @ 12 in.

C.L.

240 in.

120 in.

x

z
Dowel bar mesh

Slab mesh

$p = 145$ psi

48 in. 72 in.

2 in.

Figure 6.3 Finite Element Mesh for Slab LSM-2 Tested by Hammons (1997)
Figure 6.4 Comparison with Other Experimental and Numerical Studies
Figure 6.5 Effect of Initial Joint Opening (D/skl ~ 30)
Figure 6.6 Effect of Initial Joint Opening on Load Transfer Response
Figure 6.7 Effect of Dowel Slip and Embedment ($\omega_i = 0.2$ in.)
Figure 6.8 Effect of Dowel Bar Yielding
Figure 6.9 Cracked Concrete Pavement Slab under Daytime Temperature and Wheel Load

\( (\omega_i = 0.2 \text{ in.}) \)
Figure 6.10 Effect of Temperature
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Chapter 7  Summary, Conclusions and Recommendations for Future Work

7.1  Research Summary

The main objective of this research has been to implement nonlinear fracture mechanics concepts, particularly the fictitious crack model (FCM), in the study of crack propagation in concrete pavement slabs, using cohesive elements that have been recently incorporated in the general purpose finite element (FE) package ABAQUS®. This has been accomplished through a step-by-step procedure that investigates successively simply supported beams, individual slabs-on-grade, as well as concrete pavement systems equipped with a load transfer mechanism and subjected to load-plus-curling.

The approach was first verified by reproducing experimental and numerical data for simply supported beam specimens reported by previous independent researchers. Next, the sensitivity of post-crack responses was studied, as several pertinent parameters involved in the fracture process were varied, including the width of the cohesive zone, mesh fineness of the bulk (intact) and cohesive zone, concrete softening curve and solution techniques. Results were found to be more sensitive to solution technique and concrete softening curve than to mesh fineness and width of the cohesive zone.

A similar technique was then implemented for an individual slab-on-grade under a wheel load. The main challenge during this phase was obtaining an efficient simulation, i.e., one that is economical in its requirements for computing resources, while at the same time providing a convergent solution without compromising the accuracy of the results. The challenge may be largely attributed to the increase in the dimensions of the structural system (i.e., slab versus
beam), as well as the presence of a supporting subgrade. This problem was addressed by designing an optimal mesh for both the cohesive zone and intact region, and by implementing a numerical technique termed as viscous regularization, specifically developed to address convergence issues caused by the damage of the cohesive zone. A sensitivity study was first conducted to obtain the best value of the viscosity term. Simulation results in this phase have been also verified by reproducing experimental and numerical data reported by other researchers.

Next, the effects of softening curve, cohesive zone width and mesh, analysis technique, loading mode, tensile strength and fracture energy were investigated for the single slab-on-grade. It was found that the type of softening curve and cohesive zone width and mesh do not influence post-peak responses significantly, especially as compared to beams. They do, however, play a significant role in the convergence of the solution. It was found that the fracture process is more affected by the fracture energy than by the tensile strength. Newton-Raphson and modified Riks solution methods have been compared, and it has been observed that the load-displacement responses obtained by the latter exhibit more pronounced post-peak softening compared to the former. Despite capturing accurately the post-peak responses, the modified Riks method was found to be computationally more demanding and relatively inefficient in large problem sizes. The influence of loading placement (at the interior or at the edge of the slab) was also examined, on account of its importance for in situ pavements under both thermal and mechanical loads. In this case, two softening regions were observed on the load-displacement curves, evincing collapse of the slab first parallel to its width and subsequently parallel to its length.

The effect of curling with and without wheel load on the fracture behavior of a single slab-on-grade was studied in the third phase of the present research. To begin with, the accuracy of the
numerical discretization was verified by conducting a pre-crack analysis of the slab under curling alone and comparing responses obtained with analytical, as well as experimental, data available in the literature. Subsequently, the post-cracking behavior of the concrete slab under thermal curling alone was examined. The effects of parameters such as concrete age, notch depth, slab size, temperature profile through slab thickness, and slab’s self-weight were investigated. It has been observed that thermal cracks are affected by concrete age, which, therefore, dictates the timing and the size of joint sawing. Cracking due to thermal curling was observed to be insensitive to slab sizes greater than 10 times the radius of relative stiffness of the slab-foundation system, an observation similar to Westergaard’s (Westergaard, 1927). Under curling alone, the effect of slab’s self-weight on pavement slab cracking was found to be negligible. The temperature profile through slab thickness was found to exert a significant influence on the fracture process, and this evinces the importance of considering the entire temperature (and stress) distribution, rather than relying merely on the maximum top and bottom temperature differentials. This result calls for the accurate determination of pavement temperature profiles.

Fracture analysis of a single slab under both wheel load and thermal curling has been also carried out by considering two loading scenarios: (a) constant temperature followed by increasing wheel load; and (b) constant wheel load followed by increasing temperature. In the first loading case, the effects of daytime and nighttime temperature profile have been investigated. It has been observed that daytime curling forms unstable cracks and significantly reduces the slab’s resistance to wheel load. Nighttime temperature, on the other hand, results in stable cracks and increases resistance of the slab, but it results in a relatively large crack width at the bottom of the
slab. The effect of temperature profile through the slab thickness has once again been established as the major parameter affecting the fracture process.

The loading scenario involving a fixed wheel load followed by an increasing daytime temperature has also been considered. It is observed that the stability of the crack and the magnitude of the peak temperature differential largely depend on the magnitude of the wheel load. A relatively high wheel load, encourages stable crack growth with almost no softening, beginning at a lower temperature differential. In contrast, a rather small wheel load, allows the temperature differential to increase significantly before crack initiation, but the peak is followed by a sudden slab failure with pronounced softening.

The fourth phase of the present research has focused on cracking in concrete pavement slabs with aggregate interlock load transfer. Both linear and nonlinear aggregate interlock idealizations have been considered. Post-crack responses were used in the study of crack growth in both the loaded and the unloaded slabs, for linear as well as nonlinear aggregate interlock. For linear aggregate interlock, it was found that at smaller joint stiffness values, there was only one peak load, indicating the failure of the loaded slab alone. As the joint stiffness increased, however, softening began to be manifest at two points on the load-displacement curve, evincing the successive failure of both slabs. At very large joint stiffness values, the two slabs act monolithically and only one softening region was noted once again, manifested at a relatively large peak load. The load transfer efficiency with respect to vertical deflection and the transferred load efficiency were observed to be similar to the corresponding values for uncracked slabs.
Unlike linear aggregate interlock, the nonlinear aggregate interlock idealization takes into account the change in joint opening occurring during the loading process, as well as the concomitant alteration in the load transfer efficiency of the joint. It was observed that the load transfer efficiency with nonlinear aggregate interlock decreases linearly with an increase in initial joint opening.

The effects of both nighttime and daytime temperature distributions were also investigated for linear as well as nonlinear aggregate interlock idealizations. Daytime temperature variations result in a lower peak load, and in decreased joint efficiency, on account of a widened joint opening at the bottom of the slab. Nighttime temperature curling, on the other hand, has the opposite effect and may prevent a decrease in joint efficiency, if it does not actually enhance it.

The fifth and final phase of the present research was devoted to cracking in dowelled concrete pavements. From the post-cracking analysis of pavement slabs, it has been found that an increase in initial joint opening decreases the system stiffness, but it does not change the peak load resisted. A major parameter that influences slab response was identified to be the interaction between the dowel bar and the slab, i.e., dowel looseness. If the dowel is fully bonded to the slab, the *EMBEDDED* element approach for dowel placement has proven to be computationally efficient. The influence of daytime and nighttime temperature distributions on the peak load capacity of dowel jointed slabs was also investigated. It was observed that a daytime temperature variation resulted in approximately a 50% reduction of the peak load resisted during the nighttime.
7.2 Concluding Remarks

The present study demonstrates that the application of cohesive elements to fracture analysis of plain concrete pavements holds significant potential as a means of exploring facets of performance that have been dealt with only in a phenomenological way until now. It may even be anticipated that with the advent of new techniques incorporating different fracture modes and the emergence of the extrinsic cohesive element formulation from computational mechanics (Park et al., 2009; Paulino et al., 2010), the development of rational failure criteria may be forthcoming, so that reliance on the statistical/empirical algorithms used in current pavement design guides may be drastically reduced, if not eliminated altogether. The main conclusions of the study presented herein can be classified into two main categories, associated with numerical simulation and structural system aspects, respectively. The former group includes those factors that influence the accuracy of the results and the convergence of the solution, whereas the latter class incorporates the effects of material characterization as well as of structural behavior of the pavement slab, as summarized below.

a. Element choice

Quadratic elements (for example, C3D27) are generally found to produce better results than linear elements (for example, C3D8R), even when comparing meshes with the same total number of nodes. This is particularly important in fracture analysis of structural systems requiring significant computational resources.

b. Subgrade characterization

The subgrade idealization exerts a significant influence on the convergence of the solution. The preferred manner of simulating the subgrade in the present study involves using spring-type
elements, namely $SPRING1$. Each such spring is assigned properties reflecting that it supports only compression; slab and subgrade separation can thereby be realized. On the other hand, when the subgrade is represented using the $FOUNDATION$ option which enforces permanent contact between the slab and the soil, no softening in the load-displacement curves can be observed, which may explain the relatively rapid convergence of the solution in these cases (i.e., fewer time increments are needed). When a tensionless subgrade is employed, softening is clearly exhibited on the load-displacement curve, but the solution requires a large number of increments to convergence.

c. **Inter-element interaction**

This refers to the approaches adopted in simulating the interactions between independent elements: for example, between cohesive elements and intact material (solid elements); between dowel bars and the concrete slab; and between the slab and the underlying elements. Numerical techniques were selected for this purpose so as to yield significant savings in computational resources. This objective was tempered by concerns about undesirable side effects on the convergence of the solution and the accuracy of the results. Surface-based $TIE CONSTRAINTS$, $HARD CONTACT INTERACTIONS$, and $EMBEDDED ELEMENT$ formulations were the interaction idealization approaches employed in this study. Surface-based $TIE CONSTRAINTS$ are used to ensure independently discretized cohesive elements remain connected to the intact material. This technique saves a significant amount of computational memory since the intact material is discretized using a relatively coarser mesh. The $HARD CONTACT INTERACTIONS$ approach plays a major role in accelerating solution convergence, and it is mainly used to simulate sliding friction between two individual components, such as between the dowel bar and the slab, or between a base layer and the slab.
d. Solution technique

The load-displacement curves obtained during the post-peak responses of the simply supported beam and of the pavement slab-on-grade most often exhibit snap-back instabilities. Moreover, when the slab completely fails and the subgrade springs pick up the load, the load-displacement curve exhibits a sudden change in slope making solution convergence very difficult. A Newton-Raphson solution method fails to capture such behavior, since it employs a positive gradient in the search for an equilibrium point in the solution space. A preferable solution approach is available using arc-length type of solvers, as, for example, in the modified Riks method. The main drawback of the latter, however, is that it requires smaller load increments in order to ensure that the snap-back path of the load-displacement curve is successfully traversed; consequently, the method consumes large amounts of computational resources. On the other hand, the Newton-Raphson method is preferable over the Riks method if saving computational resources is an overriding concern, especially when only the peak load needs to be determined.

e. Loading approach

In the present study, displacement and loading controls have been used, and the former was found to be most appropriate in capturing post-peak responses of the slab. Loading control used in conjunction with the Newton-Raphson solution approach fails to converge after the peak load is attained. If loading controls as well as post-peak response are desired, the modified Riks can be used, with the load applied over a point. When the load is applied as a pressure, the solution fails to converge after the peak load even with the modified Riks method.

A uniform displacement was also stipulated over a certain area, as is the case when a rigid plate is applied on the slab. This, however, resulted in stress concentrations at the corners of the
loaded area, creating convergence issues caused by localized failure. It was concluded, therefore, that a single displacement at the centroid of the loaded area is the most suitable method of load application for practical representation of the wheel load; at the same time the amount of computer memory consumed was reduced.

f. Viscous regularization

To address the convergence challenge posed by an increase in the size of the structural system, the present study adopted viscous regularization. This technique allows the solver to iterate using values within a small but finite extent beyond the pre-defined concrete softening curve without compromising the accuracy of the solution. The slab responses were observed to be very sensitive to the viscosity value used. Therefore, a sensitivity study was conducted as the structural size, element type, and mesh size employed changed. The viscous regularization technique permits simulation of large and complex structural systems at a relatively low computational resource expense.

g. Fracture parameters

Parameters considered include fracture energy, tensile strength and the concrete softening curve. From the present study, it was concluded that fracture energy is the major parameter influencing post-crack structural response. Tensile strength, on the other hand, was observed to have a minimal effect on slab response, except for slabs subjected to temperature curling and wheel load. As the tensile strength increases, load-displacement responses for slabs under wheel load and temperature were observed to exhibit relatively pronounced softening, i.e., reduction in the applied load from its peak level before reloading occurs, when compared to the corresponding load-displacement responses for slabs under wheel load alone. Since fracture energy and tensile
strength are interrelated, it is necessary to consider their interaction, and the dimensionless parameter called brittleness number, \( B \), was selected for this purpose. The type of concrete softening curve provided as input, e.g., linear, bilinear, etc., was found to have a minimal effect on slab responses. It had, however, an effect on the convergence of the solution and the amount of computational resources expended. In general, smoother softening curves were found to be computationally more efficient.

h. Slab size

The effect of slab width and length on the post-crack response of the slab was found to be similar to what has been observed in previous, more conventional (pre-crack) research after Westergaard (1927). Slabs with dimensionless length \( (L/l) \) or width \( (W/l) \) greater than 10 have a negligible effect on load-displacement responses. Due to the lack of experimental information concerning the sensitivity of fracture energy and tensile strength to the slab thickness, as well as concerning temperature variations, this conclusion was reached from analyses using only one slab thickness (6 in.). Consequently, its generality has not been established. Given that fracture parameters exhibit sensitivity to size, it is conceivable that Westergaard’s linear elastic observations may not hold true for much thicker slabs. A more comprehensive future study would account for size effects on material properties.

i. Curling effect

The temperature distribution along the thickness of the concrete slab was observed to be one of the major factors influencing post-crack slab response. The peak load resisted by the slab was most significantly altered not by the temperature differential between the top and bottom of the slab, which is conventionally considered in pre-crack curling analysis, but by the temperature
profile through the slab thickness. This is precisely what necessitates the use of three-dimensional finite element methods for post-crack analysis of concrete pavements.

Unstable cracks form when stresses due to the wheel load are of the same sense (compressive or tensile) as those from thermal curling. For a slab loaded at its edge while subjected to temperatures that decrease from top to bottom (daytime temperature profile), the load-displacement response exhibits pronounced softening upon attaining the peak load. This is attributed to the loss of contact between a large portion of the slab’s underside and the subgrade. On the other hand, the same slab under temperatures increasing from top to bottom (nighttime temperature profile) will remain in contact with the subgrade over a much larger area. This explains the relatively smaller softening observed in the corresponding curves, as well as the more stable crack growth. On the other hand, wheel loads cause nighttime cracks at the bottom of the slab that were found to be wider than those developing during the daytime, and this may lead to other undesirable effects, such as pumping. Temperature distribution was also observed to alter joint opening, and thereby its load transfer efficiency.

j. **Joint effect**

The effect of aggregate interlock and dowelled joints on concrete slab response was examined. The commonly adopted pure-shear aggregate interlock idealization oversimplifies the mechanics, but it is used since it has huge computational advantage. To simulate joint behavior and its effect on post-crack slab response more realistically, it is necessary to adopt nonlinear aggregate interlock mechanics, so that the effects of concrete material properties, aggregate size and distribution, as well as the change in joint width during the loading process, may be accommodated. The effect of such joint characteristics on concrete pavement post-crack
response under static load and temperature curling was analyzed herein, possibly for the first
time. It was found that joint load transfer efficiency decreases linearly with increasing joint
width. The load-displacement curves exhibited one or two peak loads depending on the stiffness
of the joint. Dimensional analysis was employed for the limited sets of slab properties
considered in an effort to generalize observations made regarding the interaction among
individual design features affecting slab response.

7.3 Recommendations for Future Research

Following the development of new computational technologies, application of nonlinear fracture
mechanics to concrete structures has recently gained increased attention (Park et al., 2009). A
comparable implementation of such mechanistic approaches to pavement systems is still in its
infancy (Ioannides et al., 2006; Gaedicke and Roesler, 2009; Aure and Ioannides, 2010). In
contrast, the newly released (AASHTO, 2008) mechanistic-empirical pavement design guide
(MEPDG) employs computer programs but only to determine pavement responses, relying on
statistical algorithms for the prediction of pavement performance. Despite the considerable
mechanistic component of MEPDG, its empirical aspects are still thoroughly invested in the
statistical paradigm the new approach has inherited from its predecessors following the AASHO
Road Test (1958-1960), half a century ago. To overcome the lingering and significant
limitations, it is imperative that pavement analysis incorporate post-crack phenomena using self-adaptive mesh generation strategies and crack branching techniques (Park et al., 2012). The
present study has endeavored in this direction through the assessment of the importance of
parameters pertaining to fracture analysis and post-crack behavior of concrete pavements. In
order to elucidate the complex interactions among these parameters, leading to the formulation of
more mechanistic pavement analysis and design procedures, the following are submitted as potential areas for future investigations.

a. Fracture analysis of concrete pavements under mixed mode failure

The effects of shear stresses may be significant in thick concrete pavements, and in pavements subjected to corner loads. Under such circumstances, the concrete may crack due to the combined action of bending and shear, producing what is termed as mixed-mode failure.

b. Effects of cracking in non-surficial man-made layers

In general, pavement structures are constructed in such a way that the strength of man-made layers decreases from top to bottom. Consequently, damage or cracking may occur in the base or subbase before failure of the concrete slab, or simultaneously with it. Therefore, it may be necessary to simulate the damage processes in non-surficial layers as well, in order to capture the global response and behavior of the system.

c. Experimental verification of analytical studies into jointed concrete pavement cracking

Field and laboratory tests may be used to validate numerical studies, such as those conducted in the present research on jointed concrete pavements. Early laboratory tests on jointed concrete pavements had focused on pre-crack behavior (Colley and Humphrey, 1967; Hammons, 1997). Consequently, the effects of joint characteristics on the fracture process of the slabs, and conversely, the effect of slab cracking on load transfer efficiency have not been documented. Such tests can also be used to identify variables that are not apparent in numerical analysis.

d. Fracture analysis of concrete pavements under fatigue loading

Pavement structures are subjected to cyclic environmental and traffic loads. Consequently, failure in such structures is due to repeated action of such loads, but capturing this phenomenon
in a computer simulation poses significant numerical challenges. The quasi-brittle nature of
concrete and the difference between its compressive and tensile behaviors compound the
complexities commonly encountered in computational fracture analysis. The study of fatigue
cracking using cohesive zone simulation is barely at its earliest stage at this time (Park, 2009),
and much additional research is required before it can be applied to pavement systems.

e. Application of extrinsic numerical simulation in concrete pavements

In the present study, a fracture plane has been assumed along a fixed direction (usually termed as
discrete or intrinsic approach), along which cohesive elements are inserted. This assumption
neglects crack branching, a very common observation in concrete structures. Recent
developments in adaptive cohesive fracture simulation await application to concrete pavements
(Paulino et al., 2010; Park et al., 2012).

f. Application of dimensional analysis

Dimensional analysis can be invaluable in interpreting numerical data obtained from multiple
finite element runs for a factorial of geometric and material inputs. Its suitability in formulating
broad guidelines to be implemented by practicing engineers is particularly attractive, since
conclusions from theoretical developments must always be of consequence to design
applications.

g. Specimen size effect on fracture parameters

In contemporary practice, material properties for concrete pavement design are usually derived
from tests on small, simply supported beam specimens in a laboratory. It is almost universally
admitted that this approach may seriously misrepresent the strength characteristics of in situ
pavement slabs. Experimental and numerical studies are sorely needed in order to elucidate the
repercussions of the so-called size effect on concrete fracture parameters, as well as on the field responses of concrete pavement systems.

h. Effects of shrinkage and creep

Early age concrete shrinkage may result in premature cracking shortly after a pour, especially in longer jointed concrete pavements. The constitutive material relations adopted in the present study do not account for the effects of shrinkage and creep. Further research is necessary to account for the effects of such phenomena on the fracture process.

7.4 References


