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I, Kwame Edjah, hereby submit this original work as part of the requirements for the degree of Master of Science in Electrical Engineering.

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A PRACTICAL OBLIQUE PROJECTION METHOD FOR GPS CROSS-CORRELATION INTERFERENCE MITIGATION

Student’s name: Kwame Edjah

This work and its defense approved by:

Committee chair: H. Howard Fan, Ph.D.

Committee member: Eric T. Vinande, Ph.D.

Committee member: Xuefu Zhou, Ph.D.
A Practical Oblique Projection Method for GPS Cross-Correlation Interference Mitigation

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By

Kwame N. Edjah

BSc., Kwame Nkrumah University of Science and Technology, Ghana

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Committee Chair:
Prof. Howard Fan

Committee Members:
Dr. Frank Zhou
Dr. Eric Vinande
Abstract

Current state of the art GPS receivers employ various techniques for the acquisition of weak GPS signals as low as 21dBHz $C/N_0$ if there is no interference from other GPS satellites with strong signal levels. In reality, receivers located in places like urban canyons or under forest canopies receive weak GPS satellite signals co-existing with strong interfering GPS satellite signals. The 1023 chip Gold code sequences of the GPS C/A spreading codes limit the dynamic range of a typical GPS receiver, such that the power difference between the strongest and weakest satellite signals is limited to less than 24dB to successfully acquire the weak signal. When this condition is not satisfied, the Cross Correlation Interference (CCI) experienced during acquisition of the weak signal due to high cross correlations at the correlator output can cause difficulties in complete loss or false acquisition of the weak signal and subsequently tracking failures. The Successive Interference cancellation, Parallel Interference Cancellation and the Delayed Parallel Interference Cancellation methods have been used for CCI mitigation in GPS, but the difficulty in these methods is the continuous tracking of the amplitudes of the strong interfering satellite signals.

This thesis presents the use of a Practical Oblique projection method for a very effective removal of all strong GPS satellite signals relative to the weak signal being acquired before the acquisition process. The resulting signal from this Oblique projection method is void of any strong signal interference that produces large CCI, a threat to the acquisition process. The ensuing output of the correlator is therefore as though there were no interference from other satellite signals. The effectiveness of the Oblique Projection operator used here is compared to the Orthogonal Projection operator used in the Subtractive Orthogonal Projection method.
Acknowledgement

First, I would like to express my earnest gratitude to Prof. Howard Fan for insightful guidance throughout my thesis work and his support during my graduate work. His support has been immeasurable during my time in so far in graduate school and this wonderful experience with him will remain with me forever. I would like to submit that, my interest in Communication Systems and Digital Signal Processing was inspired by his mastery and command over this area.

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<td>Analog to Digital Convertor</td>
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<td>AGPS</td>
<td>Assisted GPS</td>
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<td>AMCS</td>
<td>Alternate Master Control Station</td>
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<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
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<tr>
<td>BOC</td>
<td>Binary Offset Carrier</td>
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<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
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<td>BP</td>
<td>Band Pass</td>
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<td>BS</td>
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<td>C/N₀</td>
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<td>C/N</td>
<td>Carrier-to-Noise Ratio</td>
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<tr>
<td>CCI</td>
<td>Cross Correlation Interference</td>
</tr>
<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
</tr>
<tr>
<td>DCO</td>
<td>Digitally Controlled Oscillator</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transforms</td>
</tr>
<tr>
<td>DoD</td>
<td>United States Department of Defense</td>
</tr>
<tr>
<td>DORIS</td>
<td>Doppler Orbitography and Radio-positioning Integrated by Satellite</td>
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<tr>
<td>DSSS</td>
<td>Direct Sequence Spread Spectrum</td>
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<tr>
<td>EGNOS</td>
<td>European Geostationary Navigation Overlay Service</td>
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<tr>
<td>FDMA</td>
<td>Frequency Division Multiple Access</td>
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<td>FFT</td>
<td>Fast Fourier Transform</td>
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<tr>
<td>GA</td>
<td>Ground Antennas</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>GAGAN</td>
<td>GPS Aided Geo Augmented Navigation</td>
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<tr>
<td>GLONASS</td>
<td>GLObal'naya NAvigatsionnaya Sputnikovaya Sistema or GLObal NAvigation Satellite System</td>
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<tr>
<td>GNSS</td>
<td>Global Navigation Satellite System</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>I</td>
<td>In-Phase Signal</td>
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<tr>
<td>IF</td>
<td>Intermediate Frequency</td>
</tr>
<tr>
<td>IIFT</td>
<td>Inverse Fast Fourier Transform</td>
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<tr>
<td>IRNSS</td>
<td>Indian Regional Navigational Satellite System</td>
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<td>JPO</td>
<td>Joint Program Office</td>
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<td>LFSR</td>
<td>Linear Feedback Shift Registers</td>
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<td>LNA</td>
<td>Low Noise Amplifier</td>
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<tr>
<td>LO</td>
<td>Local Oscillator</td>
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<td>MAI</td>
<td>Multiple Access Interference</td>
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<td>MCS</td>
<td>Master Control Station</td>
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<td>Minimum Mean Square Error</td>
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<td>Operational Control Segment</td>
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<td>PDD</td>
<td>Presidential Decision Directive</td>
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<td>PPS</td>
<td>Precision Positioning Service</td>
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<tr>
<td>PR</td>
<td>Pseudorandom</td>
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<tr>
<td>PRN</td>
<td>Pseudorandom Noise</td>
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<tr>
<td>PSP</td>
<td>Partitioned Subspace Projection</td>
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<td>Q</td>
<td>Quadrature Signal</td>
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<td>Quadrature Phase Shift Keying</td>
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<td>QZSS</td>
<td>Quasi-Zenith Satellite System</td>
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<td>RDC</td>
<td>Relative Doppler Carrier</td>
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<td>Radio Frequency</td>
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<td>Satellite Based Augmentation Systems</td>
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<td>Successive Interference Cancellation</td>
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<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
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<td>SPS</td>
<td>Standard Positioning Service</td>
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<td>SV</td>
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<tr>
<td>TOA</td>
<td>Time of Arrival</td>
</tr>
<tr>
<td>UTC</td>
<td>Coordinated Universal Time</td>
</tr>
<tr>
<td>WAAS</td>
<td>Wide Area Augmentation System</td>
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Chapter 1

Introduction

The Global Positioning System (GPS) was conceived in 1973 by the Joint Program Office (JPO) of the Space and Missile Center at El Segundo, California under the directives of the United States Department of Defense (DoD) [1], as a radio ranging system from a constellation of artificial satellites called NAVSTARs at known positions in space to receivers at unknown positions on earth with an original objective of instantaneously determining the position and velocity (navigation) of the receiver and time in a common reference system.

Initially, the primary goals of the DoD in the development of GPS were military ones so as to satisfy the navigation requirements of military forces but since United States Congress directed DoD to promote its civil use under the 1996 Presidential Decision Directive (PDD) which provided a comprehensive national policy on joint civil/military GPS management and Selective Availability turned off by 2006 [2], the applications for GPS have far exceeded those imagined by the designers when GPS was first proposed in 1973.

In all these fast growing civil applications, it is required to use the signals that are present, particularly the Legacy Coarse Acquisition (C/A) code signal, which is the dominant signal currently in use as it is designated for civil use. Unfortunately, the GPS C/A code signal was developed at a time when signal processing capabilities were significantly less than those achievable using current technology. The signal was designed as a compromise between
having good cross correlation properties and receivers achieving a fast acquisition. As such, the use of 1,023 chip length Gold-codes for the GPS C/A spreading-codes represents a compromise between the need for rapid acquisition and the cross correlation dynamic range of the spreading codes [3]. These Gold codes generally work well under normal conditions but problems are encountered in certain situations in which a receiver is required to detect GPS signals with weak signal strength. Many applications and environments present these scenarios, for example indoor localization and localization in urban canyons. These scenarios pose difficulties because some of the signals may be significantly weaker than usual thereby causing difficulties during acquisition and tracking of the weak signals. For this reason, many applications such as the construction of femtocells, which calls for the inclusion of a GPS receiver to provide timing, frequency and location information [4] may suffer. The use of the rather short C/A Gold codes represents a significant difficulty for these scenarios because of its low cross correlation dynamic range. As such, the C/A code signal structure is not ideally suited for some of the environments and applications it finds today. Techniques have therefore been developed to use the existing C/A code signal structure in dealing with the problem of weak signal acquisition and tracking.

GPS receiver sensitivity to weak signals can be improved by a technique used in high-sensitivity GPS receivers, which is to conduct coherent integration over a time period that is longer than one millisecond (one C/A code period), although in a standalone GPS receiver the coherent integration time period must not exceed the navigation data bit duration (20 ms) to avoid crossing navigation data bit boundaries. Another approach, the
“half-bits” method [5], [6], gathers consecutive 10ms correlator outputs into a collection. The collection can be viewed as two alternating sets, one of which is guaranteed to be free of bit transitions. This presents a big gain in the output of the coherent integration.

Additional gain can be obtained by a mixed coherent/non-coherent integration suggested for GPS signal acquisition by [6], [7]. In this method, there is a subsequent non-coherent integration which is to combine a set of coherent integration correlations outputs (usually longer than 1ms) that individually avoid data bit transitions, by non-coherently summing the magnitude squared (envelope) coherent integration correlations. An advantage of non-coherent integration is that the GPS signal can be integrated non-coherently for as long as desired without the concern of the navigation information bit duration or the Doppler shift offsets, since the magnitude squared operation removes the sensitivity to frequency errors [7].

Assisted GPS (AGPS) has received increased attention in its use in acquiring weak signal GPS. Applications like mobile phones containing GPS receivers primarily to permit location of the mobile handset during an emergency call [8] and construction of femtocells, where GPS receivers are used to provide timing, frequency and location information [4] employ AGPS. In the case of AGPS, the problem of weak signal acquisition is resolved through the provision of externally supplied aiding such as acquisition assistance, sensitivity assistance or externally supplied ephemeris from a reference GPS receiver. Acquisition assistance provides information on satellite visibility, Doppler frequency and code phase to reduce search time during acquisition of the signals. Sensitivity assistance provides navigation
message fragments for navigation-message data wipe-off therefore cleaning the received signal of any data bit transition. This permits very long coherent integration thereby increasing sensitivity. The direct supply of ephemeris permits navigation without the need to extract ephemeris from the transmitted navigation message, which is near impossible at extremely weak signal levels.

The techniques discussed above work well for the acquisition of weak GPS signal if they do not coexist with other strong GPS satellite signals but fail if a combination of strong and weak satellite signals is present at the receiver input which is replete in most of the scenarios in the civil use of the GPS signals. In such situations, the cross correlation interference at the correlator output caused by the presence of a single or multiple strong satellite signals prevents the acquisition and tracking of other required weaker signals.

This thesis addresses the use of an oblique projection method for the removal of strong GPS satellite signals that may co-exist with weak GPS satellite signals, so that the techniques for acquiring and tracking these weak signals could further be used effectively. An overview of the GPS follows up beginning with a short look at the more generic Global Navigation Satellite System (GNSS).

1.1 Global Navigation Satellite System (GNSS)

GNSS is a generic name used for any satellite navigation system with global coverage. The success of GPS has resulted in the invigoration of old and development of new GNSS by Russia, China and the European Union.
Russia, the formerly Soviet Union, developed the GLObal’naya NAvigatsionnaya Sputnikovaya Sistema or GLObal NAvigation Satellite System (GLONASS) beginning in 1976, and was a fully functional navigation constellation as of 1995. But since the collapse of the Soviet Union the GLONASS fell into disrepair, leading to gaps in coverage and only partial availability [9]. The Russian Federation pledged in 2000 to restore the GLONASS and it achieved full global coverage in 2011 [10]. The GLONASS signals are a mixture of Frequency Division Multiple Access (FDMA) and Code Division Multiple Access (CDMA) as multiplexing methods with Direct Sequence Spread Spectrum (DSSS) encoding and binary phase-shift keying (BPSK) modulation scheme [10].

In March 2002, the European Union (EU) and European Space Agency (ESA) agreed to develop the Galileo positioning system, originally scheduled to be operational in 2010 but now expected to be in full service in 2020 at the earliest [11]. According to ESA, Galileo will perform better in finding location within 3 feet (1 meter) compared to 16 feet (five meters) with NAVSTAR GPS. It also offers greater penetration in urban areas and the coverage will improve in northern Europe and other high latitude areas [9]. Galileo uses the CDMA multiplexing method with Binary Offset Carrier (BOC) modulation [12].

China has indicated its intent to globalize regional Navigation System called “Beidou”, which started with three satellites in orbit in 2000, now having 10 satellites in orbit offering navigation services in China and neighboring regions, by 2020 [13], [14]. The second generation of the system is known as Compass, and is expected to be a complete
GNSS consisting of 35 satellites by 2020. Compass uses a CDMA multiplexing method with BPSK and QPSK modulation schemes [15].

The GNSSs available are GPS, GLONASS, Compass and Galileo with the GPS and GLONASS currently in full use. Other satellite navigation systems with regional coverage are Beidou 1, Doppler Orbitography and Radio-positioning Integrated by Satellite (DORIS), a French precision navigation system, Indian Regional Navigational Satellite System (IRNSS) developed by the Indian Space Research Organization to be completed by 2014 [16].

Many other satellite systems called Satellite Based Augmentation Systems (SBAS) have been developed to augment the GPS systems. They include the U.S. Wide Area Augmentation System (WAAS), the Quasi-Zenith Satellite System (QZSS) covering Japan, the European Geostationary Navigation Overlay Service (EGNOS), the Japanese the Multi-functional Satellite Augmentation System (MSAS) and lastly, the Indian GPS Aided Geo Augmented Navigation (GAGAN) system.

1.2 **Global Positioning System Overview**

This section introduces the basic concept of the GPS system, how a GPS receiver makes use of the Satellite signals for Navigation and the structural segments of the GPS.
1.2.1 Basic Concept

The basic concept of the GPS system is to determine a GPS receiver’s unknown position, velocity and time using the ranging signals transmitted by NAVSTAR satellites with known positions, at equal distance from the center of the earth and in fixed orbits around the earth. Basically the position of a certain unknown point in space can be determined with knowledge of distances between this point and some other points whose positions in space are known. Fig 1.1 below illustrates this idea. The GPS uses the NAVSTAR satellites in orbit as the known positions. In Fig 1.1, if the three known points where the satellites are located are given by \( S_1 (x_1, y_1, z_1) \), \( S_2 (x_2, y_2, z_2) \) and \( S_3 (x_3, y_3, z_3) \) and the distances between the unknown points, \( U(x_u, y_u, z_u) \), and the known satellite positions can be measured as \( \rho_1 \), \( \rho_2 \) and \( \rho_3 \), then the following equations hold:

\[
\rho_1 = \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2 + (z_1 - z_u)^2} \quad \ldots \quad (1.1)
\]

\[
\rho_2 = \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2 + (z_2 - z_u)^2} \quad \ldots \quad (1.2)
\]

\[
\rho_3 = \sqrt{(x_3 - x_u)^2 + (y_3 - y_u)^2 + (z_3 - z_u)^2} \quad \ldots \quad (1.3)
\]

\( x_u, y_u \) and \( z_u \) can be solved with linearization and an iterative approach.

The satellite positions \( S_1 \), \( S_2 \) and \( S_3 \) are obtained from the navigation data transmitted from the satellites encoded in the transmitted signals from the satellites. The distances from the user or receiver (the unknown position) to the known satellite positions must be measured simultaneously at certain time instances using the satellite signals received by the receiver or user from each satellite. By measuring the travel time of the signal from the satellite to the user the distance between the user and the satellite can be estimated. These estimated distances are referred to as Pseudoranges. The use of the transmitted satellite
signal with its associated C/A code in the determination of the Pseudorange will be discussed in chapter 2.

Fig. 1.1 Use of known satellite positions ‘S₁’, ‘S₂’, and ‘S₃’ for determination of unknown 3 dimensional position ‘U’.

Since the Pseudoranges are estimates and therefore erroneous due to receiver clock errors or bias, and many other errors including satellite position error effect on range, tropospheric delay error, ionospheric delay error and receiver measurement noise error, there is a need to factor in these errors in Equations 1.1, 1.2, and 1.3. Therefore we have

\[ \rho_1 = \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2 + (z_1 - z_u)^2 + b_u} \]  
\[ \rho_2 = \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2 + (z_2 - z_u)^2 + b_u} \]  
\[ \rho_3 = \sqrt{(x_3 - x_u)^2 + (y_3 - y_u)^2 + (z_3 - z_u)^2 + b_u} \]
with $b_u$ accounting for the errors.

Hence at least four simultaneous equations are needed to solve for four unknowns, $x_1$, $x_2$, $x_3$ and $b_u$ and therefore four or more visible satellites are needed by a receiver to give a good positional solution.

1.2.1 **Structural segments of the GPS**

The GPS consists of three distinct segments illustrated in Fig. 1.2 below; the Space Segment, Control Segment and User segment.

![Fig.1.2 Structure of the GPS.](image)

a) **Space Segment**

At present the GPS space segment in its nominal constellation consists of 24 operational Satellite Vehicles (SV) each continually transmitting navigation and timing signals. The SVs are in six medium earth ecliptic orbits each with a period of 12 hours sidereal time and
with each orbit having a semi-major axis of 26,561.75 km [7]. This places the SVs at an altitude of approximately 20,181 km above the earth surface. The SVs having orbital periods of 12 hours sidereal time produces a ground track which repeats twice every day, over and over. The constellation is arranged in six evenly spaced orbit planes (A to F) each with an inclination of 55 degrees and four satellites per orbit plane.

As mentioned before, measurements on at least four SV signals must be made almost simultaneously without mutual interference to obtain a position solution. This capability is termed Multiple Access and in GPS this is achieved using CDMA where each SV has a unique spreading code for spreading its signal. If all SV signals are of equal strength, this permits measurement to be made on a signal from one SV without signals from other SVs interfering with that measurement.
Several generations or classes of satellites have been manufactured, starting with the first generation of Block I SVs used to validate the GPS concept. There have been six classes that have been deployed at present and in the past, Block I, Block II, Block IIA, Block IIR, Block IIR-M, and Block IIF SVs, each with a different hardware configuration which has evolved over time. At the time of this writing, the present SVs in orbit include 10 Block IIA SVs, 12 Block IIR SVs, 7 Block IIR-M SVs and 2 Block IIF SVs [17].

b) **Operational Control Segment**

The GPS Operational Control Segment (OCS) is responsible for monitoring the health and the status of the space segment. It consists of a ground Master Control Station (MCS), a ground backup Alternate Master Control Station (AMCS), six ground Monitoring Stations (MS), and four dedicated Ground Antennas (GA) [17]. These form a system of continuous L band tracking stations located around the world, monitoring and controlling the entire GPS constellation in the Space segment.

The ground MS receive data (orbits, altitude, location, speed) from the visible SVs, track the GPS navigation signal from these SVs, measure pseudorange and carrier phase from the satellite signals, record the broadcast navigation messages from these SVs and record local atmospheric data. The collected data are transmitted to the MCS which performs GPS command and control. With this data, the MCS generates and estimates satellite orbits information and clock corrections, generates the navigation message, monitors and maintains the GPS constellation, verifies the received navigation messages recorded by the MS and maintains synchronization between GPS and UTC time. The MCS then corrects the
orbital and clock information known as ephemeris data, and updates the satellites via the GA preventing the satellites from drifting away and ensuring that they orbit within limits. The other information data are also relayed to the GA for transmission to the satellites for broadcast to the user segment.

The locations of components of the OCS are shown in Table 1.1 [17], [19].

| Master control station          | Schriever Air Force Base (formerly Falcon AFB), Colorado Springs, CO |
| Master control station (backup) | Gaithersburg, MD                                                   |
| Monitor station                 | Schriever Air Force Base (formerly Falcon AFB), Colorado Springs, CO |
| Remote monitor station          | Cape Canaveral, FL                                                |
| Remote monitor station          | Hawaii                                                             |
| Remote monitor station          | Ascension Islands                                                 |
| Remote monitor station          | Diego Garcia                                                      |
| Remote monitor station          | Kwajalein                                                          |
| Ground antenna                  | Cape Canaveral, FL                                                |
| Ground antenna                  | Ascension Islands                                                 |
| Ground antenna                  | Diego Garcia                                                      |
| Ground antenna                  | Kwajalein                                                          |

Table 1.1 Locations of the Component of the Operational Control Segment
c) **User Segment**

The GPS user segment refers to devices used to receive the GPS signals. It is composed of hundreds of thousands of U.S. and allied military users of the secure GPS Precise Positioning Service, and tens of millions of civil, commercial and scientific users of the Standard Positioning Service. Since the user receiver does not transmit any signals, the GPS space segment can provide service to an unlimited number of users. The design, implementation and construction of these receivers require expertise from a variety of disciplines including antennas design, analogue RF electronics, digital electronics, software engineering, systems engineering, digital control systems, estimation theory and algorithm design, to name a few.

GPS receivers consist of an antenna, tuned to the frequencies transmitted by the satellites, a highly stable clock (often a crystal oscillator) and receiver-processors that measure and decode the satellite transmissions to provide positioning, velocity, and precise timing information to the user by computing the four dimensions of X, Y, Z (position) and Time. They may also include a graphical display of maps, providing location and speed information to the user. GPS receivers have parallel multi-channel/receiver circuit design and are often described by the number of channels, signifying how many satellite signals it can acquire and track simultaneously. Originally limited to four or five, this has progressively increased over the years. As of 2007, receivers typically have between 12 and 20 channels [20].
1.2.3 Applications for the Global Positioning System

Although initially intended for military use, GPS applications have diversified into many significant civilian applications like commerce, scientific uses, tracking, surveillance, and providing accurate timing. This facilitates everyday activities such as banking, mobile telephony and even the control of power grids by allowing well synchronized hand-off switching.

A common military use is the inclusion of GPS Navigation capabilities in “Commanders Digital Assistant” and lower ranks “Soldier Digital Assistant” which allows soldiers to find objectives, even in the dark or in unfamiliar territory, and to coordinate troop and supply movement [17]. Other applications include Target tracking of potential ground and air targets, missile and projectile guidance, search and rescue of downed pilots, Reconnaissance, and nuclear detonation detectors for path finding [17].

Civilian uses are numerous and cannot be exhausted in one write up especially when users create new ones almost every day. But the most common use is Navigation where Navigators provide digitally precise velocity and orientation measurements providing a graphical view in maps of the users’ position and direction. Others include Clock synchronization where the accuracy of GPS time signals (±10 ns error) is second only to the atomic clocks upon which they are based [17], Disaster relief and emergency services and Surveying where Surveyors use absolute locations to make maps and determine property boundaries.
1.3 **Multiple Access Interference (MAI) in Multiple Access Methods**

As mentioned earlier on, the GPS satellites use a Multiple Access technique, in particular the CDMA, to be able to share and transmit on the same frequency band allocations. The other most commonly used Multiple Access techniques used are the TDMA and FDMA. Associated with all multiple access techniques is an inherent interference called Multiple Access Interference (MAI) which is a type of interference caused by multiple users who are using the same frequency allocation at the same time. MAI can present a significant problem if the power level of the desired signal is significantly lower than the power level of the interfering user.

The Guard Times between user time slots in TDMA and the use of Guard Bands between user bands in FDMA help to achieve MAI free transmission from multiple users sharing the same channel resource. But in CDMA, all users share the same bandwidth resource at the same time, only separated by their unique code from a set of orthogonal pseudorandom (PR) codes with which the user employs to modulate and spread the information-bearing signal. The solution to the MAI problem in CDMA takes a very different approach.

1.4 **CDMA MAI - Cross Correlation Interference (CCI) and Existing Mitigation Methods**

In a CDMA system, the receivers use the code sequences to decode the messages transmitted by multiple users through a channel. The effects of MAI is realized at the
output of the matched filter or correlation filter (correlator) where correlation output can become very noisy due to cross correlation components within the correlator output. The MAI component in CDMA systems is therefore called Cross Correlation Interference (CCI). If these CCI components are very large, the output of the correlator becomes very noisy and less useful for decoding the messages transmitted by the desired users.

1.4.1 Overview of CDMA CCI

Fig. 1.4 General Structure of CDMA system in baseband

Fig. 1.4 above shows a block diagram of the general structure of a simple base band CDMA system. In Fig. 1.4, if \( i \) is the user index, then \( b_i \) is the binary data bit of user \( i \) and \( b_i(t) \) is its
corresponding digital waveform at the output of the digital modulator. If Bi-Phase scheme and rectangular waveform are used for digital modulation, then the binary data bits \( b_i \) will take on values of +1 or -1, and therefore \( b_i(t) \) is a rectangular waveform with amplitude +1 or -1. Also, \( s_i(t) \) is the PR waveform of user \( i \) and \( h_i \) is the channel gain between the transmitter or user \( i \) and the receiver. Lastly, \( P \) is the transmission Power, \( n(t) \) is Additive White Gaussian Noise (AWGN), \( T_b \) is the bit duration of the transmitted data bit, and \( N \) is the number of users in the system.

Assuming that all users in the system transmit synchronously, in one bit duration, the transmitted signal of user \( i \) is given as

\[
f_i(t) = \sqrt{P}b_i(t)s_i(t) = \sqrt{P}b_i(t) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (1.7)
\]

The received signal at the receiver side is given as

\[
r(t) = \sum_{i=1}^{N} \sqrt{h_i}f_i(t) + n(t) = \sum_{i=1}^{N} \sqrt{h_i}\sqrt{P}b_i(t) + n(t) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (1.8)
\]

Demodulation is done by correlation/matched filters for each transmitter or user \( i \). Let \( y_i \) be the output of the correlation filter for user \( i \), therefore

\[
y_i = \int_{T_b} r(t)s_i(t)dt = \int_{T_b} \left( \sum_{j=1}^{N} \sqrt{h_j}\sqrt{P}b_j s_j(t) + n(t) \right) s_i(t)dt
\]

\[
= \sqrt{P}\sqrt{h_i}b_i \int_{T_b} s_i(t)s_i(t)dt + \sqrt{P}\left( \sum_{j=1}^{N} \left( \sqrt{h_j}b_j \int_{T_b} s_j(t)s_i(t)dt \right) \right) + \int_{T_b} n(t)s_i(t)dt
\]
\[ y_i = \sqrt{P} \sqrt{h_i b_i \Gamma_{ii}} + \sqrt{P} \left( \sum_{j=1 \atop j \neq i}^{N} (\sqrt{h_j b_j \Gamma_{ij}}) \right) + n_i \] ... (1.9)

where

\[ \Gamma_{ij} = \text{cross correlation} = \int_{T_b} s_i(t) s_j(t) dt = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases} \] ... (1.10)

for perfectly orthogonal codes, and

\[ n_i = \int_{T_b} n(t) s_i(t) dt \] ... (1.11)

is a Gaussian random variable representing the interference caused by the AWGN.

From Equations 1.9 and 1.10, for perfectly orthogonal codes,

\[ y_i = \sqrt{P} \sqrt{h_i b_i} + n_i \] ... (1.12)

where \( \sqrt{P} \sqrt{h_i b_i} \) represents the ideal signal component of the desired user \( i \).

But due to channel distortion, the received PR codes are not perfectly orthogonal no matter how carefully designed. Therefore, the output of the Correlation filter is always as in Equation 1.9, where \( \sqrt{P} \sqrt{h_i b_i \Gamma_{ii}} \) is the signal component of the desired user \( i \),

\( \sqrt{P} \left( \sum_{j=1 \atop j \neq i}^{N} (\sqrt{h_j b_j \Gamma_{ij}}) \right) \) is the interference caused by other users to the desired user correlator output responsible for MAI. The MAI term for CDMA systems is therefore referred to as Cross Correlation Interference (CCI). At the output of each correlator responsible for each user, there always exists this CCI term which can severely impact the receiver performance if it is very high as compared to the desired signal component. The
CCI is different from other interference because it is inherent and a structured interference. The CCI term increases as the number of users increases.

From Fig. 1.4, the signal path for the transmitted signal from each user $i$ has unequal channel gain, $h_i$, due to various factors including distance. Characteristically, in a wireless transmission system, the channel gain is proportional to the inverse of the $\alpha$th power of the distance between the transmitter and the receiver where $\alpha$ is the path loss exponent [21]. This affects the received power of each user's transmitted signal in that the power of the signals received by a receiver from different users vary depending on their distance from the receiver. In a normal CDMA wireless mobile system, this phenomenon is called the Near-Far problem. Therefore, larger received power of signals from nearby interfering users can be much greater than that of the desired user far away and due to the nonzero MAI inherent in CDMA wireless systems which causes large CCI term at the output of the matched filter.

In the GPS signal, the near-far problem does not contribute much to CCI because of the vast distance between receivers and all GPS satellites so the variation in the receiver to satellite ranges is relatively small compared to its average value. Nevertheless, signals from different satellites arrive at the receiver with widely varying power level disparities [43]. This is because signal propagation from different satellites are attenuated to different power levels by a variety of conditions and objects such as cloudiness, trees, forest and buildings especially in the urban environments where most consumers live, travel and work. Also, power level disparities are caused by different forms of multipath fading
encountered by each satellite signal [43]. Therefore, the conventional assumption that all
signal power levels are the same is not realistic [43]. Meanwhile, the near-far effect is
becoming a problem with the increasing use of “pseudolites” for augmentation of the GPS
system [43].

1.4.2 Overview of General CCI Mitigation or Cancellation Methods in
Conventional CDMA Wireless Mobile Systems

In conventional CDMA systems like the wireless mobile system, where the Near-Far
problem is prevalent, it has been shown in Equation 1.9 that the CCI, \( \sqrt{\sum_{j=1}^{N_{t}} (\sqrt{h_j b_j \Gamma_{ij}})} \)
at the output of the receiver matched filter is non-zero.

One major CCI suppression method for CDMA systems with the Near-Far problem is Power
Control. With a fixed matched filter structure, and a fixed code assignment to each user, \( \Gamma_{ij} \)
in Equation 1.9 is always fixed and cannot be changed. Since \( h_j \) and \( b_j \) are independent of
the system design, the only way to suppress CCI in the matched filter receiver design is to
to control the power, \( P \) for each individual user. Equation 1.9 then becomes

\[
y_i = \sqrt{P_i} \sqrt{h_i} b_i \Gamma_{ii} + \sqrt{P_i} \sum_{j=1}^{N_{t}} \left( \sqrt{h_j b_j \Gamma_{ij}} \right) + n_i \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (1.13)
\]

Power control is implemented in the form of feedback control. For example in the CDMA
mobile network, transmission powers of the interfering users, is controlled or reduced as
much as possible while at the same time maintaining a certain QoS (quality of service) requirement for each user in the system. The Base Stations (BS) constantly communicate with each user under their cell through beacon signals received from and sent to each user in its cell. Based on its estimation, the BS then calculates the optimal transmitter power needed by each user and send power update commands back to the users through the downlink wireless channel. Upon receiving the power update commands from a BS, the mobile units update their transmitter powers to their respective optimal levels.

The Power control method does not completely remove or attempt to remove CCI. It just suppresses the CCI term \( \sqrt{P_i} \left( \sum_{j=1}^{N} (\sqrt{h_i}b_j\Gamma_{ij}) \right) \). The CCI term at the output of the matched filter is due to the sub-optimality of each matched filter in the receiver design which only depends on the user code for demodulation of data. To achieve optimality, the there is a need for an increase in the complexity of the receiver structure by designing each matched filter taking into account all user codes in the system. These types of receiver design are called Multiuser detection.

An optimum Multiuser detector design based on the maximum likelihood estimation of the transmitted bits was first discussed in [22]. The computational complexity of this method is high as the computation proved to be exponential in the number of users.

Minimum Mean Square Error (MMSE) detectors have also been designed to deal with CCI. The MMSE detectors in [23], [24], suppresses the combined effect of CCI and the ambient
noise, and minimizing the mean square error between the transmitted information bit and the output of the detector. These detectors are implemented in a way such that exact knowledge of the signature sequences of all the users is required.

Lately, CCI have been eliminated with knowledge of the interfering users by introducing training data sequences for every active user. This is called adaptive multiuser detection [25], [23].

1.5 Thesis Organization

As mentioned earlier, the GPS system is a CDMA based satellite ranging system and therefore susceptible to CCI. The GPS structure, mode of operation and requirements such as speed demands on GPS receivers make the methods mentioned above for CCI cancellation in the general CDMA mobile wireless systems impossible to apply to CCI mitigation or suppression in GPS receivers. Details of the reasons will be explained in Chapter 3. Chapter 2 takes a look at the GPS signal structure and its inherent interference susceptibility. Chapter 3 looks at GPS CCI and how different it is from the general CDMA CCI, and reviews existing GPS CCI mitigation or cancellation techniques. Chapter 4 begins with a look at subspace projections and investigates how an ideal oblique projector could be used in the complete removal of other satellite interferences before the matched filter is used in acquisition. A practical Oblique Projector is also discussed and compared to the orthogonal projection operator used in the Subtractive orthogonal projection method for CCI removal. Finally, Chapter 5 contains conclusions and suggestions for future work.
Chapter 2

GPS Signal Structure and Inherent Interference Susceptibility

2.1 GPS Signal structural Requirements as a Satellite Navigation System

With the GPS being a radio signal ranging technology, its principles and the accuracy of position and velocity determination depend strongly on the structure and nature of the signals transmitted by the satellites. The various methodologies/criteria considered in developing the signal structure include accurate position determination using radio ranging signals, exact direction and velocity determination using Doppler Effect or Doppler frequency shift of the received ranging signals, transmission of navigation data, simultaneous reception of multiple satellite signals (a Multiple Access Technique) with reduced effect of MAI, ability to withstand external interference and multipath effects, penetrative ability and lastly provision of correction for ionospheric delay.

One of the first characteristics of interest for a satellite signal transmission is a suitable carrier frequency $f_c$. First of all, the chosen frequency had to be in the range where the signal propagation is not adversely influenced by clouds, rain, snow and other weather elements. The signals had to also possess a good cloud penetrative ability. In considering ionospheric delays, it is realized that the delay is enormous for ranges below 100MHz and
above 10GHz. It is also known that, the speed of propagation for electromagnetic waves in air media deviates from the speed of light as frequency decreases, therefore \( f_c \) was chosen to be high enough. Also \( f_c \) was to be chosen to be below 2GHz to avoid the use of beam antennas for signal reception. Lastly, \( f_c \) had to be high enough for the PRN code modulation on carrier which required a high bandwidth. Based on these, frequencies located in the L-Band (1GHz to 2GHz) were chosen for different GPS applications. As of now, the GPS legacy signals are designated to the L1 (1.57542GHz) and L2 (1.2276GHz) frequencies for civilian and strictly military purposes respectively. For other purposes, the frequencies used are L3 (1.38105GHz), L4 (1.37992GHz) and L5 (1.17645GHz).

The signals at these frequencies are used to carry Navigation data to the receivers by phase modulating the Navigation message on carriers at these frequencies by BPSK. The choice of Phase Modulation, BPSK in particular, is to allow for an expansion of the bandwidth of the carrier signal leading to spread spectrum.

CDMA is used as a means to allow all the GPS satellites to use the same frequency resource allocations at the same time. Each satellite, numbered between 1 and 32, is assigned a unique PRN code sequence called a spreading code, which also serves as a means of identification for each satellite by the receivers. The PRN codes, which are continuous strings of +1 and −1 valued symbols called chips, corresponding respectively to binary values 0 and 1 generated from tapped feedback shift registers. The chips are of much shorter duration than navigation data bits and are used to phase modulate the sinusoidal signal by BPSK. Since the chip rate is much higher than the navigation data bit rate, the final
transmitted signal occupies a much higher bandwidth, a technique called Direct Sequence Spread Spectrum (DSSS). The use of these PRN codes offer resistance to intended and unintended jamming and interception. A very important use of the PRN codes in GPS is the determination of the relative time between transmission from the satellites and reception at a receiver during acquisition and tracking. This gives the receiver needed information about the distance between the receiver and each visible satellite.

The PRN codes were chosen to be a set of code sequence with strong autocorrelation and weak cross-correlation levels to make identification possible at the receiver. In this way, these PRN code sequences which are known by the receivers can therefore be used in identifying each satellite signal in the receiver’s correlation filter. Two kinds of PRN codes are used in the GPS; the Coarse Acquisition (C/A) code and a much longer Precision (P) code which may be encrypted by a highly secret Y-code to form a P(Y) code for military use providing precise position determination due to its length, jamming resistance and antispoofing capabilities.

2.2 **Legacy Signals (L1 and L2 band Signals)**

The GPS Legacy signals are transmitted by all GPS SVs up through the Block IIR class on the L1 and L2 frequencies of 1.57542GHz and 1.2276GHz respectively. The higher powered L1 signal has its in-phase component modulated by a C/A code with chip rate of approximately 1.023MHz while its quadrature component is modulated by a P or the encrypted P(Y) code with chip rate of approximately 10.23MHz. The L1 signal can therefore provide the Standard Positioning Service (SPS) for civil use and Precision
Positioning Service (PPS) for military and authorized usage. The quadrature component of L1 carrying the P(Y) code is transmitted at a power of 10.4dBW to ensure a minimum user power level of -161.5dBW at the receiver using a 3-dBi gain linearly polarized antenna. Its in-phase component carrying the C/A code is transmitted at a power of 13.4dBW for a minimum specified user power level of -158.5dBW [30] at the same kind of receiver end. The L2 signal, which is transmitted at a power of 8.2dBW for a minimum ground reception power of -164.5dBW [30] at a 3-dBi gain linearly polarized antenna, is used strictly for military and authorized usage providing the PPS and is modulated with the P or P(Y). It also provides an accurate measurement of ionospheric delays when used along with the L1 signal since the delay is related by a scale factor difference in signal’s TOA for the two carriers [30]. For the L1 single frequency users (civil users), estimation of ionospheric delay is done using modeling parameters sent to the users through the navigation data bits.

Table 2.1 shows the details of the PRN codes and the Navigation data in the GPS Legacy signals.

<table>
<thead>
<tr>
<th></th>
<th>C/A code</th>
<th>P(Y) code</th>
<th>Navigation Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Frequency(Rate)</strong></td>
<td>1.023 MHz (Mega chips/s)</td>
<td>10.23 MHz (Mega chips/s)</td>
<td>50 Hz (bps)</td>
</tr>
<tr>
<td><strong>Length per chip or bit</strong></td>
<td>977.517 ns (293 m)</td>
<td>97.7517 ns (29.31 m)</td>
<td>20 ms (5950 km)</td>
</tr>
<tr>
<td><strong>Cyclic Period</strong></td>
<td>1 ms</td>
<td>1 week</td>
<td>N/A</td>
</tr>
<tr>
<td><strong>Number of chips per cyclic period</strong></td>
<td>1023 chips</td>
<td>6.187104 × 10^{12} chips</td>
<td>N/A</td>
</tr>
<tr>
<td><strong>Code type</strong></td>
<td>Gold</td>
<td>Pseudo random</td>
<td>N/A</td>
</tr>
<tr>
<td><strong>Carrier</strong></td>
<td>L1</td>
<td>L1, L2</td>
<td>L1, L2</td>
</tr>
<tr>
<td><strong>Service/Feature</strong></td>
<td>SPS, PPS, Civil use and easy to acquire</td>
<td>PPS, jam resistance, antispoofing, available to only authorized users</td>
<td>Time, ephemeris, Almanac, ionosphere correction, satellite quality.</td>
</tr>
</tbody>
</table>

Table 2.1 *GPS Legacy signals PRN and the Navigation data.*
Fig. 2.1 shows the structure of the synthesis of the transmitted signals. All signal components are generated from the output of highly stable atomic clocks onboard each SV which generate pure sine waves at fundamental frequency, $f_o$ of 10.23MHz. Actually, to account for relativistic effects, the frequency of the SV clock is offset to 10.22999999543MHz [30] but appears as 10.23MHz to receivers on ground.

![Fig. 2.1 Structure of GPS Legacy Signal Generator [30].](image)

Table 2.2 below shows the frequencies of each satellite signal components generated from the fundamental clock frequency as can be seen in Fig. 2.1.
<table>
<thead>
<tr>
<th>Component</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 Carrier</td>
<td>(154f_0 = 1575.42 \text{ MHz})</td>
</tr>
<tr>
<td>L2 Carrier</td>
<td>(120f_0 = 1227.60 \text{ MHz})</td>
</tr>
<tr>
<td>P Code</td>
<td>(f_0 = 10.23 \text{ MHz})</td>
</tr>
<tr>
<td>C/A Code</td>
<td>(f_0/10 = 1.023 \text{ MHz})</td>
</tr>
<tr>
<td>Navigation data</td>
<td>(f_0/204600 = 50 \text{ Hz})</td>
</tr>
</tbody>
</table>

Table 2.2 Frequencies of GPS Legacy Signal Components with respect to satellite clock fundamental frequency \(f_0\).

The Legacy signals L1 and L2 transmitted by satellite \(i\) can therefore be respectively expressed as

\[
L_1^i(t) = \sqrt{2P_{PL1}^i}P_i(t)D_i(t)\sin(2\pi f_{L1}^i t) + \sqrt{2P_{CA}^i}C_i(t)D_i(t)\cos(2\pi f_{L1}^i t) ...................................(2.1)
\]

\[
L_2^i(t) = \sqrt{2P_{PL2}^i}P_i(t)D_i(t)\sin(2\pi f_{L2}^i t) .................................................. ..............................(2.2)
\]

where \(t\) is time(seconds), \(P_{CA}, P_{PL1}\) and \(P_{PL2}\) are transmission powers of the C/A and P modulated L1 signal and P modulated L2 signal respectively, \(P_i\) is the P(Y) code sequence of satellite \(i\), \(C_i\) is the C/A code sequence of satellite \(i\), \(D_i\) is the navigation data sequence of satellite \(i\) and \(f_{L1}^i\) and \(f_{L2}^i\) are the carrier frequencies of L1 and L2, respectively.

As the PPS signals in the L1 and L2 carrier are normally encrypted, civilian are mostly limited to using the SPS signals modulated with the C/A code. For this reason, only the C/A code will be described further.

### 2.3 C/A Pseudo Random Spreading Codes

The choice of the C/A code was made to satisfy the condition of permitting rapid acquisition and at the same time having good enough autocorrelation and cross correlation.
properties for a good multiple access capability. For quicker acquisition, the codes must be short but unfortunately, short codes have poor cross correlation properties and therefore signal from one satellite may interfere with the acquisition of signals from another satellite. To mitigate this problem, the Gold codes named after Robert Gold was chosen as the class of PRN code for the GPS C/A code.

2.3.1 Introduction to Gold Codes and its properties

Gold code sequences are a family of codes with each chip produced by taking the modulo-2 sum of the “nth” bit of an n-bit maximal length Linear Feedback Shift Registers (LFSR), G1 and the “nth” bit of a shifted version of another n-bit maximal length LFSR, G2, producing a code sequence of period $L = 2^n - 1$ chips. The choice of the two maximal length LFSR, G1 and G2 are such that their highest absolute crosscorrelation for their code sequences is $2^{(n+2)/2}$ [26]. Every shift or delay, $\tau$, in G2 produces a unique length- $L$ chip sequence called the Gold code sequence.

![Fig. 2.2 Structure of Gold Code Sequence Generator][27].

\[ \tau = \text{Delay/Shift} \]
\[ T_c = \text{chip rate} \]
The shifted version of G2 may be implemented in a number of different ways. In the case of the GPS C/A code, two different taps of the G2 LFSR are added and used in place of the bit of G2.

From the structure of the Gold Code generator, even though a large number of code unique sequences can be generated, the Gold codes are chosen so that over the range of chosen sequences, the autocorrelation and crosscorrelation values between the code sequences are bounded and uniform [27]. The crosscorrelation of the selected Gold code sequences have three unique values. Table 2.3 below summarizes the values and probability levels of the correlation levels.

### 2.3.2 C/A Code Generation

The GPS C/A code is generated from two 10 bits maximal length LFSR G1 and G2 with generating polynomials $f(x) = 1 + x^3 + x^{10}$ and $f(x) = 1 + x^2 + x^3 + x^6 + x^8 + x^9 + x^{10}$ respectively and with both having initial states of all ones. The structure of the C/A code generator is shown in Fig. 2.3.

At every 1023rd chip, signifying the end of one period, the content of the LFSR is reset to all ones marking the code sequence generation start over. For a different phase shift, the outputs of the two shift registers are combined in a special way; whereas G1 supplies its nth state bit, G2 supplies the modulo-2 adder of two of its state bits. The selection of the two different states in G2 solely influences the phase shift even as phase selection or shift delay determines a unique 1023 chip C/A code sequence for each satellite. Table 2.4 shows
the code phase assignments for each satellite with respect to phase selection from the register G2.

<table>
<thead>
<tr>
<th>Normalized Autocorrelation Levels</th>
<th>Normalized Peak of 1 and other bounded levels of 3-valued Crosscorrelation Levels defined below</th>
<th>Normalized 3-valued Cross Correlation Levels</th>
<th>Frequency/Probability of occurrence of Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$ = number of bits/stages in LFSR, Length/Period $L = 2^n - 1$</td>
<td>$\frac{2^{(n+1)/2} + 1}{L}$</td>
<td>$\frac{1}{L}$</td>
<td>0.25</td>
</tr>
<tr>
<td>$n$ = number of bits/stages in LFSR, Length/Period $L = 2^n - 1$</td>
<td>$\frac{2^{(n+1)/2} + 1}{L}$</td>
<td>$\frac{1}{L}$</td>
<td>0.5</td>
</tr>
<tr>
<td>$n$ = number of bits/stages in LFSR, Length/Period $L = 2^n - 1$</td>
<td>$\frac{2^{(n+1)/2} + 1}{L}$</td>
<td>$\frac{1}{L}$</td>
<td>0.25</td>
</tr>
<tr>
<td>$n$ = number of bits/stages in LFSR, Length/Period $L = 2^n - 1$</td>
<td>$\frac{2^{(n+1)/2} + 1}{L}$</td>
<td>$\frac{1}{L}$</td>
<td>0.125</td>
</tr>
<tr>
<td>$n$ = number of bits/stages in LFSR, Length/Period $L = 2^n - 1$</td>
<td>$\frac{2^{(n+1)/2} + 1}{L}$</td>
<td>$\frac{1}{L}$</td>
<td>0.75</td>
</tr>
<tr>
<td>$n$ = number of bits/stages in LFSR, Length/Period $L = 2^n - 1$</td>
<td>$\frac{2^{(n+1)/2} + 1}{L}$</td>
<td>$\frac{1}{L}$</td>
<td>0.125</td>
</tr>
</tbody>
</table>

Table 2.3 Cross Correlation Properties of Gold Codes [7], [27], [30].
Fig. 2.3 C/A Code Generator [7], [30].
<table>
<thead>
<tr>
<th>Satellite ID Number</th>
<th>GPS PRN Signal Number</th>
<th>Code Phase Selection</th>
<th>Code Delay Chips</th>
<th>First 10 Chips C/A Octal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2 ⊕ 6</td>
<td>5</td>
<td>1440</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3 ⊕ 7</td>
<td>6</td>
<td>1620</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>4 ⊕ 8</td>
<td>7</td>
<td>1710</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>5 ⊕ 9</td>
<td>8</td>
<td>1744</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>1 ⊕ 9</td>
<td>17</td>
<td>1133</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>2 ⊕ 10</td>
<td>18</td>
<td>1455</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>1 ⊕ 8</td>
<td>139</td>
<td>1131</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>2 ⊕ 9</td>
<td>140</td>
<td>1454</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>3 ⊕ 10</td>
<td>141</td>
<td>1626</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>2 ⊕ 3</td>
<td>251</td>
<td>1504</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
<td>3 ⊕ 4</td>
<td>252</td>
<td>1642</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>5 ⊕ 6</td>
<td>254</td>
<td>1750</td>
</tr>
<tr>
<td>13</td>
<td>13</td>
<td>6 ⊕ 7</td>
<td>255</td>
<td>1764</td>
</tr>
<tr>
<td>14</td>
<td>14</td>
<td>7 ⊕ 8</td>
<td>256</td>
<td>1772</td>
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<tr>
<td>15</td>
<td>15</td>
<td>8 ⊕ 9</td>
<td>257</td>
<td>1775</td>
</tr>
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<td>16</td>
<td>16</td>
<td>9 ⊕ 10</td>
<td>258</td>
<td>1776</td>
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<tr>
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<td>1 ⊕ 4</td>
<td>469</td>
<td>1156</td>
</tr>
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<td>2 ⊕ 5</td>
<td>470</td>
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<td>4 ⊕ 7</td>
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<td>474</td>
<td>1763</td>
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<td>509</td>
<td>1063</td>
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<td>4 ⊕ 6</td>
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<td>1706</td>
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<tr>
<td>25</td>
<td>25</td>
<td>5 ⊕ 7</td>
<td>513</td>
<td>1743</td>
</tr>
<tr>
<td>26</td>
<td>26</td>
<td>6 ⊕ 8</td>
<td>514</td>
<td>1761</td>
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<td>27</td>
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<td>7 ⊕ 9</td>
<td>515</td>
<td>1770</td>
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<tr>
<td>28</td>
<td>28</td>
<td>8 ⊕ 10</td>
<td>516</td>
<td>1774</td>
</tr>
<tr>
<td>29</td>
<td>29</td>
<td>1 ⊕ 6</td>
<td>859</td>
<td>1127</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>2 ⊕ 7</td>
<td>860</td>
<td>1453</td>
</tr>
<tr>
<td>31</td>
<td>31</td>
<td>3 ⊕ 8</td>
<td>861</td>
<td>1625</td>
</tr>
<tr>
<td>32</td>
<td>32</td>
<td>4 ⊕ 9</td>
<td>862</td>
<td>1712</td>
</tr>
<tr>
<td>**</td>
<td>33</td>
<td>5 ⊕ 10</td>
<td>863</td>
<td>1745</td>
</tr>
<tr>
<td>**</td>
<td>34*</td>
<td>4 ⊕ 10</td>
<td>950</td>
<td>1713</td>
</tr>
<tr>
<td>**</td>
<td>35</td>
<td>1 ⊕ 7</td>
<td>947</td>
<td>1134</td>
</tr>
<tr>
<td>**</td>
<td>36</td>
<td>2 ⊕ 8</td>
<td>948</td>
<td>1456</td>
</tr>
<tr>
<td>**</td>
<td>37*</td>
<td>4 ⊕ 10</td>
<td>950</td>
<td>1713</td>
</tr>
</tbody>
</table>

*34 and 37 have the same C/A code.

**GPS satellites do not transmit these codes; they are reserved for other uses.

Table 2.4 Satellite ID versus C/A code phase assignment [6], [30].
2.3.3 C/A Code Power Spectral Density Analysis

Fig. 2.4 shows the MATLAB simulated Power Spectrum of C/A code of satellite 9. As can be seen from the figure, with a chipping rate of 1.023MHz, the null to null bandwidth of the main lobe of the spectrum is 2.046MHz. Along this main lobe, other side lobes are transmitted in an approximately 20MHz L1 signal bandwidth to be able to carry the 20.46 MHz bandwidth of the P code.

![Power Spectrum of C/A code of satellite 9 in the 20MHz bandwidth of L1 signal. Notice that the bandwidth of the null to null main lobe is 2.046MHz](image)

2.3.4 C/A Code Correlation Properties

As discussed earlier, the autocorrelation and cross correlation properties of the spreading code plays a very important role for the transmitted signal structure, in that the selected
code must have a balance of being long enough to have a high autocorrelation peak with respect to the cross correlation “floor” levels (Table 2.3) for a high dynamic range for acquisition, and short enough for fast acquisition and tracking. The C/A is a 1023 chip length Gold Code and therefore from Table 2.3, the normalized autocorrelation levels are 1, -65/1023, -1/1023, and +63/1023 and the normalized cross correlation levels include -65/1023, -1/1023 and +63/1023 with 0.125, 0.75 and 0.125 probability of occurrence respectively.

Table 2.5 below gives the power in dB of each of the correlation levels, including the autocorrelation peak at chip shift zero.

<table>
<thead>
<tr>
<th>Correlation Levels</th>
<th>Power in dB</th>
<th>Comparison to Autocorrelation peak power</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Un-normalized</td>
<td>Normalized Level</td>
</tr>
<tr>
<td>1023</td>
<td>10 log(_{10})(1023)(^2)</td>
<td>10 log(_{10})(1)(^2)</td>
</tr>
<tr>
<td></td>
<td>= 60.1975</td>
<td>= 0</td>
</tr>
<tr>
<td>-65</td>
<td>10 log(_{10})(-65)(^2)</td>
<td>10 log(_{10})(\frac{-65}{1023})(^2)</td>
</tr>
<tr>
<td></td>
<td>= 36.2583</td>
<td>= -23.9392</td>
</tr>
<tr>
<td>-1</td>
<td>10 log(_{10})(-1)(^2)</td>
<td>10 log(_{10})(\frac{-1}{1023})(^2)</td>
</tr>
<tr>
<td></td>
<td>= 0</td>
<td>= -60.1975</td>
</tr>
<tr>
<td>+63</td>
<td>10 log(_{10})(-65)(^2)</td>
<td>10 log(_{10})(\frac{63}{1023})(^2)</td>
</tr>
<tr>
<td></td>
<td>= 35.9868</td>
<td>= -24.2107</td>
</tr>
</tbody>
</table>

**Table 2.5 Power of Correlation Levels of C/A codes**

The -65/1023 and +63/1023 side lobes can be problematic when acquiring a weak satellite signal in the presence of other strong satellite signals since they are only approximately 24dB below the autocorrelation correlation peak from Table 2.5.
Fig. 2.5 and Fig. 2.6 show the autocorrelation of the C/A code satellite 20 and cross correlation C/A codes satellite 2 and satellite 20 respectively.

Fig. 2.5a Autocorrelation C/A Code of Satellite 20.

Fig. 2.5b Close-in view of the Autocorrelation C/A Code of Satellite 20.
2.4 GPS Signal Processing and Receiver Design

Before analyzing the CCI problem within GPS receivers, it is of relevance to review the GPS receiver design and operation. There are three main signal processing stages in a GPS receiver; Acquisition, Tracking and Navigation stage. The Acquisition stage is to acquire the signal from the satellites, identify the transmitting satellites, and roughly estimate some useful parameters such as Code Phase, Carrier phase and Doppler shift. The Tracking component performs a demodulation function, splitting up the received signal into its I and Q components for code phase tracking and carrier phase and frequency tracking. Navigation data extraction is also performed at this stage. The information from these stages is passed to the Navigation stage which is for Pseudorange, velocity, and position determination. Display functions also follow in this stage. Advancement in Software
Defined Radio has made it possible for implementation of these stages in software. Before discussing these stages, a discussion on the Code Phase, Carrier phase, and Doppler Shift parameters and their relevance is in the next subsection. A look at Receiver design with software defined radio and their hardware font-end also follows.

### 2.4.1 GPS Received Signal Code Phase, Carrier Phase and Doppler Shift

#### their usefulness

Determination of the position and velocity of a receiver is the fundamental objective of the GPS navigation system. As explained in Chapter 1, the first objective in this process is to determine the Pseudorange and the relative velocity between the receiver and visible GPS satellites. The GPS signal is inherently structured so that the received signal parameters like the Code phase, Carrier phase and Doppler shift could be used in determining the Pseudorange and the relative velocities.

**a) Code Phase**

When the C/A code embedded in the GPS signal is transmitted, the GPS receiver receives the signal only after some time equivalent to the space travel time by the signal which causes a time lag in the received signal code. Every GPS receiver generates an exact replica of the transmitted C/A code. With the assumption that the GPS satellite and receiver clocks are perfectly synchronized, the transmitted code and its replica are compared through a matched filter correlation process to determine the time lag in the received code. The correlation process involves the time domain sliding of the receiver code replica over the received signal code and integration until there is a sync at which the integration value is
maximum. The amount of slide, corresponding to the time lag and defined as the Code Phase, is an estimate of the signals space travel time which is in turn used to determine Pseudorange from the transmitting satellite.

Fig. 2.7 Code Phase ($\tau$) determination

As mentioned in Chapter 1, the term Pseudorange is used because there is always a synchronization error in the clocks which are used for signal generation in the satellite and the receiver which, along with other errors, contaminate this measured range. Also, resolution of the Code Phase range measurement can introduce some errors and increase the synchronization ambiguity problem. With a C/A code chip period of 977.517ns, this corresponds to a 293m distance resolution for every slide or shift in chip during the correlation process. The P(Y) code has 10 times better resolution of 29.3m but as explained, it is reserved for military use, limiting civilian use to the C/A code.

With improvement in receiver processing speed, improvements have been made in the distance resolution during correlation at the receiver in that, the shift or slide of the codes
are performed in fractions of the chips length called Code Bins giving better resolution after correlation. In this thesis, as will be discussed later, the received signal is sampled at 5MHz, meaning for the entire 1023 chip period of the C/A code, there are 5000 samples used for Code Phase determination. This translates into a Code Bin of about one fifth of the C/A code chip length.

Innovations have also been made in the utilization of the received signal carrier phase for pseudorange determination since the signal carriers are at the GHz range, and therefore have a very high distance resolution.

b) Carrier Phase

The range from the receivers to the satellites can be obtained from the signal carrier by taking the number of complete carrier cycles plus the fractional cycle from the satellite to the receiver and by knowledge of the carrier wavelength, the range can be determined. This method has a very high resolution and using the L1 carrier at 1.57542GHz, a resolution of 19cm is achievable.

The problem with this method is that, since the carriers are pure sinusoids, all cycles are the same and the beginning of transmission cannot be marked. The receiver can only measure the last fraction of carrier cycle while the number of complete cycles remains ambiguous.
There has been ongoing research in the use of Carrier Phase and to solve the initial cycle ambiguity. More of this can be referred to [28]. For the purposes of this thesis, we will concentrate on using the code phase measurements for the pseudorange determination.

c) Doppler Shift

Because of the satellites’ orbital motion, there is always a relative motion between the GPS satellites and the receiver even when the receiver is motionless. For this reason, both carrier frequency and C/A code rate of the received signal is always offset from the signal transmitted by the satellites. This frequency is termed Doppler Shift.

Doppler Shift has an effect on the Code Phase determination and the quality of code and carrier phase tracking during the signal Acquisition and Tracking stages. Therefore information on the Doppler Shift is very useful for these stages of the receiver signal processing, as it is an important factor for determining frequency update rate especially in the tracking process [6]. Hence in addition to the Code Phase search domain, a frequency search domain is necessary during Acquisition and Tracking to account for the Doppler Shift.

In relation to the satellite transmitted signal frequency, $f_s$, the received signal frequency, $f_r$ is given in Equation 2.3 [31] as

$$f_r = f_s \left(1 - \frac{v_d}{c}\right)$$

where $c$ is the speed of light and $v_d$ is the Doppler Velocity which is the satellite velocity component toward the receiver or line-of-sight relative velocity (line-of-sight range
change rate) of satellite towards receiver. Since range reduces as a satellite approaches the receiver, this signifies a positive Doppler velocity and negative for a departing satellite. The Doppler Shift, \( f_D \), is therefore

\[
f_D = f_R - f_s = -f_s \left( \frac{V_d}{c} \right)
\]

A positive Doppler Shift therefore implies an approaching satellite and vice-versa.

With a GPS satellite having a displacement velocity of approximately \( 3874 \text{m/s} \), it has been proven that the maximum unsigned Doppler Velocity relative at a stationary receiver is \( 928.83 \text{m/s} \) [6]. Thus from Equation 2.4, the maximum Doppler Frequency of the L1 carrier \( (f_{L1} = 1575.42 \text{MHz}) \) is approximately \( \pm 4.88 \text{kHz} \). As compared to the satellite velocity, the velocity of most on-land vehicles is insignificant and therefore, relatively stationary compared to the GPS satellite velocity. For this reason, during the acquisition stage, the search range for the frequency domain is expanded by \( \pm 5 \text{kHz} \) of the carrier frequency to account for the Doppler shift. But for high speed vehicles like an aircraft with a speed that could approach and even exceed \( 928.83 \text{m/s} \), the receiver frequency search domain is expanded to \( \pm 10 \text{kHz} \) of the carrier frequency [6].

As mentioned earlier, aside from the carrier frequency, the Doppler shift slightly affects the C/A code rate also. The Doppler Shift on the C/A code rate as computed from Equation 2.4 is \( 3.17 \text{Hz} \). This causes a misalignment in the receiver replica of the C/A code and the received signal C/A code, degrading the crosscorrelation results. Though typically insignificant due to the low offset rate compared to the C/A code rate of \( 1.023 \text{MHz} \) and could be ignored, at very high receiver speed, the offset in the received signal C/A code rate
could be high, leading to a large misalignment in the receiver replica and received signal C/A code.

Even though in most cases the Doppler Effect on the C/A code is not very significant, there is always a misalignment in the receiver replica of the C/A code and the received signal C/A code presented by the Doppler Effect on the carrier which the C/A code modulates. This affects the dynamic range of the cross-correlation properties of the C/A code discussed above in section 2.3.4. The results shown in Fig. 2.5 and 2.6 were simulated for the ideal case where the Relative Doppler Carrier (RDC) frequency, which is frequency offset between the received signal and the replica signal generated by the receiver for acquisition and tracking purposes, is zero. Worse cross-correlation results are obtained when the RDC frequency is not zero and for such situations the dynamic range of the cross-correlation results at the receiver could be as low as 21.1dB [30]. There is therefore the need to update the frequency changes caused by Doppler Effect at the receiver.

Aside from the determination of the Doppler Shift for better Code and Carrier phase estimation, the Doppler shift and the frequency update rate have been used for determining the velocity of the receiver in many applications [1], [31]. The form in Equation 2.4 has widely been used for GPS receiver velocity determination. Various algorithms have also been developed to refine the application of Doppler Shift for receiver velocity determination [32], [33].
2.4.2 **Receiver Design and Signal Processing with Software Defined Radio**

There are various designs of GPS receivers from various vendors, and it is particularly difficult to define a typical receiver design. In the conventional design of the GPS receiver, the Acquisition and Tracking stages are implemented in hardware. However, modern receivers process the signal in software after the signal is digitized due to advancements in Software Defined Radio. The processing of the signal from antenna reception up to the digitization stage is still done by hardware. Fig. 2.8 shows a generic Software Defined Radio based GPS receiver design based on receiver design requirement.

**Fig. 2.8 Simplified Block Diagram of Software Defined Radio Based GPS Receivers**

a) **RF Front-end Hardware Signal Processing**

The foremost purpose of the Front-end after the antenna reception is to down-convert the received Radio Frequency (RF) GPS signal to an Intermediate Frequency (IF) which is much lower than the input frequency, and to digitize the IF signal for further processing. High carrier frequencies are essential for space and atmospheric propagation as mentioned earlier but they are not easy to amplify, filter and digitize. Aside from the fact that high frequency Amplifiers can be very expensive, the Analog to Digital Convertor (ADC) must
have a large input bandwidth to accommodate the high input frequency. Such an ADC is difficult to build for the GPS signal frequency and has fewer effective bits [6]. It is also difficult to build narrow-band filters at high frequencies which usually have relatively high insertion loss [6].

After the antenna, the GPS receiver Front-end comprises of Low Noise Amplifiers (LNA), an initial Filter for out-of-band Noise removal, a Local Oscillator (LO) for local RF signal synthesis, a Mixer, a Band Pass (BP) filter (for removal of the unwanted frequency from the mixing process, Image Noise and Frequency Harmonics), and an Analog-to-Digital Converter. Fig. 2.9 shows the details of a generic Front-end design for the GPS L1 signal receiver.

![Diagram of generic Front-end design for the GPS L1 signal receiver](image)

Fig. 2.9 *Generic Front-end design for the GPS L1 signal receiver*

Variants of the design in Fig.2.9 have been used. In some designs, the Initial Filter and LNA are housed integrated in the antenna electronics [7] and the signal after the antenna unit is the RF signal plus narrow band noise. Also, even though only the single stage down-conversion is presented here, some designs use multi-stage down-conversion to IF to reduce demands on each individual component [34].
Later simulations in this thesis are done at an IF of 21.25MHz, requiring a 1.55417GHz LO. Various designs use different values of LO frequencies and IF frequencies. A mathematical analysis of the front-end processing of the L1 signal is presented in the following equations, assuming only one satellite transmission. The analysis is also limited to the C/A code signal.

From Equation 2.1, the in-phase component of a satellite signal carrying the C/A code is

\[ L_1(t) = \sqrt{2P_{CA}} C(t)D(t)\cos(2\pi f_{L1}t) \]  

The received signal from a single satellite, \( r(t) \), assuming no multipath, is given as

\[ r(t) = \sqrt{2PC} (t - \tau)D(t - \tau)\cos[2\pi(f_{L1} + f_D)t + \phi] + n(t) \]  

where \( \tau \) is the time delay (seconds) caused by the signal time-of-flight, \( f_D \) is the Doppler shift experienced by the carrier, and \( \phi \) is the carrier phase angle (radiance) and \( n(t) \) is AWGN.

For down-conversion the LO signal at frequency \( f_{LO} \) and phase \( \phi_{LO} \) is mixed with the incoming signal. The result is as follows [7];

\[ r_{LO}(t) = r(t)\cos(2\pi f_{LO}t + \phi_{LO}) \]

\[ = [\sqrt{2PC} (t - \tau)D(t - \tau)\cos[2\pi(f_{L1} + f_D)t + \phi] + n(t)]\cos(2\pi f_{LO}t + \phi_{LO}) \]

\[ + \text{Harmonics + LO feedthrough + Image Noise} \]

\[ = A \cdot C(t - \tau)D(t - \tau)\cos[2\pi(f_{L1} - f_{LO} + f_D)t + \phi - \phi_{LO}] \]

\[ + A \cdot C(t - \tau)D(t - \tau)\cos[2\pi(f_{L1} + f_{LO} + f_D)t + \phi + \phi_{LO}] + n_{IF}(t) \]

\[ + \text{Harmonics + LO feedthrough + Image Noise} \]  

where

\[ A = \frac{\sqrt{2P}}{2} \] is the signal amplitude after down-conversion
\[ n_{IF}(t) = n(t) \cos(2\pi f_{LO}t + \phi_{LO}) \] is the down-converted noise.

The harmonic content are due to harmonic frequencies much higher than the IF generated by the non-linear nature of the mixer while the LO feed-through is a small LO frequency component that “leaks through” the mixing process [7]. Image noise is also due to noise energies at image frequencies after mixing [7]. The signal after mixing is passed through an appropriate BP filter to remove the high frequency content so that only the wanted IF signal of frequency \( f_{IF} = f_{L1} - f_{LO} \) remains. The filtering operation gives

\[ r_{IF}(t) = A \cdot C(t - \tau)D(t - \tau)\cos[2\pi(f_{IF} + f_D)t + \phi - \phi_{LO}] + n_{IF}(t) \] \hspace{1cm} (2.8)

This IF signal is then digitized by the ADC to collect real data samples. A factor considered in the choice of sampling frequency, \( f_s \), is the sampling frequency should not be a integer multiple of the C/A code rate plus its Doppler Shift [6].

There are two frequency plans that could be used in the sampling process. One method is to down-convert the incoming signal to appropriately low IF and choose a sampling frequency at least twice the highest frequency content of the IF signal taking into consideration the Doppler Shift on the incoming signal. With a C/A code bandwidth of 2.046MHz, the choice of the sampling frequency is guarded by

\[ \frac{f_s}{2} > f_{IF} + f_D + 1.023MHz \] \hspace{1cm} (2.9)

With no prior knowledge of the Doppler Shift before acquisition, the sampling frequency should be large enough to cover the range of all possible Doppler Shifts.
In the second plan, with a C/A code bandwidth of 2.046MHz at an IF, the technique of IF or BP sampling which makes use of the spectral gap by aliasing the signal into the baseband Nyquist zone (0 to $f_s/2$) is used. Loss of information in aliasing is prevented by ensuring that the bandwidth of the incoming signal, $\Delta f$, is less than half the sampling frequency [6]. The digital IF frequency, $f_o$, at the output of the ADC in this sampling method is given by

$$f_o = f_i - n\left(\frac{f_s}{2}\right), \quad f_o < \frac{f_s}{2}, \quad \Delta f < \frac{f_s}{2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2.10)$$

where $f_i$ is the input frequency and $n$ is an integer.

The second plan from Equation 2.10 is used in this thesis. With a sampling frequency of 5MHz used in this thesis, the digital frequency, $f_o$, at the output of the ADC is 1.25MHz considering an input IF of 21.25MHz after down-conversion.

The samples from the ADC can be written as

$$r_{IF}[n] = A \cdot C(nT - \tau)D(nT - \tau)\cos[2\pi T(f_{IF} + f_D)n + \phi - \phi_{LO}] + n_{IF}(nT) \quad \ldots \quad \ldots (2.11)$$

where $T$ is the sampling period. Considering a receiver with multiple satellite signals in sight, the IF received signal can be written as

$$r_{IF}[n] = n_{IF}(nT) + \sum_{i=1}^{N} A_i C_i(nT - \tau_i)D_i(nT - \tau_i)\cos[2\pi T(f_{IF} + f_{Di})n + \phi_i - \phi_{LO}] \quad \ldots (2.12)$$

where $N$ is the number of satellite signals being received and the subscript $i$ indicates that a quantity is associated with satellite $i$. Many commercial GPS receivers use only 1- or 2 bit ADCs [6].
b) GPS Acquisition Stage

The purpose of the Acquisition stage is to

- Find the satellite signals visible by the receiver
- Estimate the coarse C/A Code phase
- Estimate the coarse received signal carrier frequency due to Doppler Shift

Various forms of implementations exists but basically, during Acquisition the receiver generates replica digital C/A code and digital carrier at IF over a range of Doppler shifts for the satellites, and correlates the received signal with the generated signal. The peak correlation value is evaluated to make the decision on the visible satellites, the Code Phase and received Carrier frequency.

As explained earlier, most receivers search the frequency domain over a range of ±5 kHz or ±10 kHz. To do this, the frequency over this range is discretized by dividing it into smaller steps called Doppler Frequency Bins whose size depends on the desired correlation integration time and the desired maximum Signal-to-Noise Ratio (SNR) loss due to frequency mismatch. The commonly used Doppler Frequency Bin for Acquisition in most receivers is 500Hz. The Code Bin used in Acquisition depends on the sampling frequencies used during the digitization of the received signal.

Aside the Code Bin and Frequency Bin, the length of signal samples used in Acquisition is also important. A minimum of one full period of C/A code sequence equivalent to 1ms is
required for good correlation results to maximize the potential of the correlation properties described in Section 2.3.4. Integer multiples of the C/A Code period, $M$, could also be used in Acquisition when the incoming signal is weak. Note that $M$ should be an integer to realize the full C/A code length. Even though Correlation Integration time is increased, with $M > 1$, the SNR or correlation gain is improved and the probability of false alarm is reduced for weak signals [6], [7]. But transitions from +1 to -1 or vice-versa at Data-bit boundaries can destroy the integration results due to the subtraction operation which occurs when this happens and therefore must be avoided. Since a Data bit is 20ms long and therefore contains 20 C/A code sequences, if the beginning of the Data bit is known, then $M = 20$ could be used, otherwise $M < 20$. As mentioned in Chapter 1, the half-bit method [5], [6] and the mixed coherent/non-coherent integration method [6], [7] have been used to improve SNR and correlation gain when the signal is weak.

Correlation is done in two channels, the In-phase ($I$) and Quadrature ($Q$) components of the signal envelope during demodulation to baseband, and data from these two combined. Ideally, the signal power should be in the In-phase part since the signal is modulated onto that. But because the phase of the received carrier signal is unknown, it is necessary to investigate both channels. The results from these channels are combined to preserve the amplitude information in the correlation output.

Three commonly different Acquisition algorithms are discussed in terms of implementation and performance;
i. Serial Search in Time Domain

ii. Parallel Frequency Space Search

iii. Parallel Code Phase Search

i. **Serial Search Acquisition in Time Domain**

In this method, each possible combinations of frequency and C/A code offset is evaluated serially with correlators dedicated to acquiring a particular satellite or C/A code at a time during the process [35], [36]. For faster acquisition multiple correlators are used where each correlator uses a unique C/A code replica representing a satellite. Fig. 2.10 shows the block diagram for the Serial Search algorithm for a single correlator.

![Fig. 2.10 Block diagram for the Serial Search algorithm for a single Correlator](image-url)
The C/A Code Digitally Controlled Oscillator (DCO) generates digitized form of the code corresponding to sampling frequency used by the ADC and at a particular code phase indicated by $m$ in Fig. 2.10. After multiplication with the locally generated C/A code, the incoming signal samples are then multiplied by locally generated In-phase and Quadrature digital carriers at IF plus a Doppler

Not shown in Fig. 2.10 are the Filters to remove the High frequency content. The Doppler Shift Information is still retained, as $I$ and $Q$ components are still modulated on the Doppler frequency. The samples from each channel are separately integrated or summed over a period of an integer multiple, $M$, of the C/A code period [35] as shown in Fig. 2.10. In the figure, $L$ is the number of samples per C/A code period. If a 5MHz sampling frequency is used, there will be 5000 samples per C/A code period. The Integration results from each channel are then squared and added up. The result is compared to a predefined threshold and if exceeded, the code phase and frequency at that point are correct and passed onto the tracking stage. If this condition is not met, the correlator moves to the next code phase or Doppler shift of the carrier.

Though fairly simple to implement in software and even hardware, the serial search is very time consuming, especially for high sensitivity correlators where the resolution of frequency bins and code bins are smaller. The number of correlation operation could exponentially increase if speed is to be increased. Assuming a Code Bin of 1 chip and a Frequency Bin of 500Hz over a $\pm 10$ kHz frequency range, there are 1023 C/A code phases and 41 frequency bins $\left(\frac{20kHz}{500Hz} + 1\right)$ to search giving a combination of 41,943 (1023 * 41).
Any method that reduces the number of such combination and the search time consumed is useful. The next two methods remove the frequency and code phase serial search respectively.

**ii. Parallel Frequency Space Search**

This method parallelizes the frequency search space by utilizing a Fast Fourier Transform (FFT) method. Fig. 2.11 summarizes this method of Acquisition.

![Block Diagram of the Parallel Frequency Space Search for a single Correlator](image)

The incoming IF signal samples are multiplied by replica digital version of the C/A code phase shifted between 0 and 1022 chips indicated by $m$ in Fig. 2.11. The result is transformed into frequency domain by an FFT process. Ideally, if the replica C/A code and the local replica are aligned, a continuous wave form arises. When this happens, a distinct peak in magnitude is observed in the FFT results. The frequency location of this peak identifies the frequency of the incoming signal with the shift used in the replica C/A code identifies the Code Phase [36]. Signal components from other satellites in the incoming signal samples are minimized as a result of the cross-correlation properties of the C/A codes [36].
The frequency resolution of the FFT affects the accuracy of the determined frequency of this method. This depends on the length of FFT or number of samples used which is dependent on the sampling frequency and number of integer multiple, $M$, of the C/A code period used. From the sampling frequency, the number of samples per C/A code period, $L$, can be inferred. The frequency resolution, $\Delta f$, of the FFT is given as 

$$\Delta f = \frac{f_s}{ML}$$

Therefore with a sampling frequency of 5MHz, $L = 5000$ samples and with $M = 1$, $\Delta f$ will be 1 kHz which is only comparable to the Serial Search method with a Frequency Bin of 1kHz, and actually worse than the Serial Search when a 500Hz Frequency Bin is used.

The Code Phase and carrier frequency combination to search is considerably reduced to 1023 in this method if a Code bin of 1 chip is used. The problem with this method is that each search combination is computationally intensive due to the FFT process. The efficiency of this method depends on the speed of the FFT implementation.

iii. Parallel Code Phase Search

This method parallelizes the code phase space therefore eliminating the need to search through all possible code phases. This leaves only 41 steps or combinations of only the frequency to be searched if a $\pm 10$kHz Doppler Shift range and a 500 Hz Frequency Bin is used. Before discussing the implementation of this method, a look at circular correlation is discussed since it is the underlining principle used [6], [36].
Let the Discrete Fourier Transforms (DFT) of the finite length sequences $x[n]$ and $y[n]$ of length $N$ (but repeating with a period $N$) be given respectively as

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi nk/N}$$ ................................. (2.14)

$$Y[k] = \sum_{n=0}^{N-1} y[n]e^{-j2\pi nk/N}$$ ................................. (2.15)

The time domain cross correlation sequence between $x[n]$ and $y[n]$ is given as

$$z[m] = \sum_{n=0}^{N-1} x[n]y[n + m]$$ ................................. (2.16)

The DFT of this $N$ point cross correlation, $z[m]$ is given as

$$Z[k] = \sum_{m=0}^{N-1} \left( \sum_{n=0}^{N-1} x[n]y[n + m] \right) e^{-j2\pi mk/N}$$

$$= \sum_{n=0}^{N-1} x[n] \left( \sum_{m=0}^{N-1} y[n + m] \right) e^{j2\pi nk/N} e^{-j2\pi (m+n)k/N}$$

$$= \sum_{n=0}^{N-1} x[n]e^{j2\pi nk/N} \sum_{m=0}^{N-1} y[n + m]e^{-j2\pi (m+n)k/N}$$

$$= \left( \sum_{n=0}^{N-1} x^*[n]e^{-j2\pi nk/N} \right)^* \sum_{m=0}^{N-1} y[n + m]e^{-j2\pi (m+n)k/N}$$

$$= \left( \sum_{n=0}^{N-1} x[n]e^{-j2\pi nk/N} \right)^* \sum_{m=0}^{N-1} y[n + m]e^{-j2\pi (m+n)k/N}$$, if $x[n]$ is a real sequence

$$\therefore Z[k] = X^*[k]Y[k] \quad \overset{F^{-1}}{\longleftrightarrow} \quad x[-n]*y[n]$$ ................................. (2.17)
If the incoming signal samples can be stripped off the IF carrier and converted to baseband leaving only the received C/A code, then the Equation 2.17 which is the DFT of the correlation process can be utilized for Acquisition assuming $x[n]$ is the locally generated replica C/A code samples and $y[n]$ is the baseband version of the incoming signal C/A code samples. Since number of integer multiple, $M$, of the C/A code period used in acquisition is chosen such that the samples does not include navigation bit transition boundaries, $y[n]$ can be considered as periodic.

![Block Diagram](image)

**Fig. 2.12 Block Diagram of the Parallel Frequency Space Search for a single Correlator**

Fig 2.12 above shows the Parallel Code Phase Search process using this equation. Here, the absolute value of the Inverse Fast Fourier Transform (IFFT) represents the correlation between the input C/A code and the local replica. Notice from Fig. 2.12 that in contrast with the first two methods discussed, the C/A code is not shifted or at Code Phase of zero for all Doppler Shifts on the carrier frequency. The index at a peak in a correlation points the Code
Phase and the frequency used in local Carrier DCO is the frequency of the incoming signal samples.

The accuracy of this method depends on the number of samples used in the FFT and IFFT process which in turn depends on the sampling frequency and the number of integer multiple, $M$, of the C/A code period used in Acquisition. For a sampling frequency of 5MHz, the number of samples per C/A code period, $L$, is 5000 samples and with $M = 1$, the FFT and IFFT are of length 5000 samples for the C/A code sequence period of 1023. This is equivalent to using a Code Bin of almost one-fifth of a C/A code chip and same as results obtained for the Serial Search method. Though the number of combinations is very small compared to the other methods, the Parallel Code Phase Search process is very computationally intensive because of the FFT and IFFT processes involved.

Modern GPS receivers use multiple and parallel correlators to simultaneously implement multiple search channels on any one of the above 3 methods to reduce the search time. For the purpose and scope of this thesis, the Serial Search is implemented for its ease of implementation.

c) **MATLAB Simulation of GPS Signal and Implementation of GPS Acquisition search**

To test the Oblique Projection algorithm for CCI cancellation to be discussed in the Chapter 4, the received GPS signal is simulated in MATLAB and the Serial search method is implemented to test the performance of the interference cancellation. Also, for the topic of this thesis, it was enough to demonstrate with the implementation of acquisition which
utilizes one C/A code period, therefore as shown in Fig. 2.10 above, $M = 1$. A step by step MATLAB simulation, from the satellite transmitted GPS signal to the Serial Search Acquisition is discussed next.

i. Carrier

The incoming IF is simulated at 21.25MHz at a sampling rate of 5MHz giving a digital output IF frequency of 1.25MHz. Fig 2.13 shows the FFT of the 21.25MHz IF carrier sampled at 5MHz in frequency domain.

ii. Navigation Data

From Table 2.1, we see that the bit length of a Navigation Data bit is 20ms and the period of a C/A code sequence is 1ms. This means there are 20 full C/A code sequences used in spreading every Navigation Data bit. Implementing acquisition with $M = 1$ means only one-twentieth of a Navigation Data bit is used. Therefore there is no need simulating the Navigation Data bits. For simplicity, in this thesis only $M = 1$ will be implemented in simulations and no Navigation Data will be simulated.

![Fig. 2.13 FFT of 21.25MHz IF Carrier sampled at 5MHz](image)
iii. *Simulation of C/A code*

The generation of the C/A code is explained in Section 2.3.2. A MATLAB function with the satellite vehicle ID and the sampling frequency as arguments to the function was written to simulate a period of the C/A code. The 1ms C/A code of satellite 9, a close view of 22 chips of the code and a close view of 5MHz samples of 22 chips of the code are shown in Fig. 2.14. The sampled codes are simulated because of the sampling operation on the received signal at the ADC. The power spectrum of satellite 9’s C/A code was shown in Fig. 2.4.

![1ms period of Satellite 9 C/A Code](image)

*Fig. 2.14a 1ms period of Satellite 9 C/A Code*
Fig. 2.14b Close-up view of the first 22 chips of Satellite 9 C/A Code

Fig. 2.14c Close-up view of first 22 chips (107 samples) of Satellite 9 C/A Code sampled at 5MHz
iv. **Simulation of Satellite Transmitted**

A MATLAB function was written to simulate the original satellite transmitted GPS signal from a single satellite, based on Equations 2.11 when the AWG noise \( n_{IF} \), code phase \( \tau \), Doppler shift \( f_D \) and carrier phase \( \phi_c \) are assumed to be zero. Also, without loss of generality, the simulation of the transmitted signal is done at IF. The transmitted signal from a satellite is simulated at a transmit power of 13.4dBW which is the transmit power of the C/A code modulated quadrature component of the L1 signal, as mentioned in Section 2.2.

For the demonstration purpose, the frequency and power spectrums were simulated at a 4.8MHz IF Carrier and sampling frequency of 16MHz. Later simulations of the Acquisition stage will be done with simulated GPS signals at 21.25MHz IF Carrier and sampling frequency of 5MHz. Fig. 2.15 below shows the Frequency and Power Spectrum of the transmitted GPS signal from satellite 9 on a 4.8MHz IF Carrier sampled at 16MHz at a transmission power for the C/A coded L1 signal (13.4dBW).

v. **Simulation of Noise and Received Signal**

The received Noise Density from the receiver antenna input through the transmission lines to ADC is given by [6], [37]

\[
N_o = k T_{eq} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2.18)
\]

where

\[
k = \text{Boltzmann’s constant} = 1.38 \times 10^{-23} \text{W} / ^\circ \text{K} \cdot \text{Hz} \equiv -228.6 dBW / ^\circ \text{K} \cdot \text{Hz}
\]
\( T_{eq} \) = Equivalent Noise Temperature, which is specific to the receiver design depending on the ambient temperature, the receiver Noise Figure, the power line transmission losses and the noise temperature of the antenna [37].

Fig. 2.15a Normalized Frequency Spectrum of the 13.4dBW transmitted GPS signal of satellite 9 at IF of 4.8MHz. Notice that the bandwidth of the main lobe is approximately 2.046MHz, equal the Bandwidth of the C/A code.

Fig. 2.15b Normalized Frequency Spectrum of the 13.4dBW transmitted GPS signal of satellite 9 at IF of 4.8MHz.
At an ambient temperature of 290°K, a typical Equivalent Temperature of 219.65°K (23.42dB°K) is used, translating to a Noise Density of

\[ N_o = (1.38 \times 10^{-23} W/°K \cdot Hz)(219.65°K) = 3.0312 \times 10^{-21} W/Hz \]

\[ \equiv (-228.6 dBW/°K \cdot Hz) + 23.42 dB°K = -205.18 dBW/Hz \]

Considering the C/A code bandwidth of 2.046MHz (63.11dBHz) and a minimum expected power, \( P_c \), of -160dBW for the C/A code L1 carrier using a receiver with a 0dBi RHCP antenna, the C/A code modulated L1 signal power Spectral density is -223.11dBW/Hz (\( P_c(Watts)/2.046MHz = -160dBW - 63.11dBHz \)). This is 17.93dB below the typical Noise floor of -205.18dBW/Hz. Thus a single satellite GPS signal is not visible to a Spectrum Analyzer [37] or in simulations using the MATLAB function used here.

The Carrier-to-Noise Density \( (C/N_o) \), which is the carrier power to Noise per unit bandwidth, is unique to each satellite signal since each signal is received at a different signal power. For simulations in this work, given a received signal power of \( P_c(dBW) \),

\[ \frac{C}{N_o} = (P_c - N_o) = (P_c + 205.18)dBHz \]

Therefore for received signal strength of -160dBW, \( C/N_o \) is 45.18dBHz.

The Carrier-to-Noise Ratio \( (C/N) \), which the carrier power over the whole usable receiver bandwidth is given by

\[ \frac{C}{N} = \frac{C}{P_N} = \frac{C}{N_o B} = \frac{C}{N/o}/B \]

\[ \equiv \left[ \frac{C}{N_o} dBHz - 10 \log_{10} B \right] dB = (P_c - N_o - 10 \log_{10} B)dB \]

\[ \equiv (2.20) \]
Carrier-to-Noise Ratio is not very easy to define since there is no unique universally accepted receiver bandwidth [6] and therefore $C/N$ is unique to receiver design. For simulations using the MATLAB function written for this purpose, the bandwidth of the C/A code, 2.046MHz is used. This means for a signal of strength -160dBW, the Carrier-to-Noise Ratio is -17.93dB. From Equation 2.20, it should be noted that $C/N$ is scaled according to the received power of the signal from the satellite being acquired or tracked.

Because this thesis assesses the performance of a cross-correlation cancellation method in the acquisition of the GPS signals from multiple satellites at different strengths or powers, the received signal samples were simulated as the sum of GPS signals with different signal strengths. For simplicity sake and without loss of generality, the MATLAB function for simulating the received signal was also written with the assumption that the Noise Density is constant, keeping Noise floor constant for each satellite signal in view. Each received satellite signal was simulated at a Code Phase corresponding to the time between transmission and reception and at a Doppler Shift before combining them to obtain the total received signal. Fig. 2.16 shows the power spectrum of the received GPS signal from satellite9 at IF of 4.8MHz sampled at 16MHz, with AWGN at Noise Density of $-205.18$dBW/Hz and without noise. Notice that in the case where the received signal contains AWGN at this Noise Density, which is typical for the GPS, the signal is buried in the noise as discussed earlier.
vi. **Serial Search Acquisition**

The Serial Search was implemented for its simplicity to test the performance of the CCI removal before Acquisition and Tracking of the received GPS signal. One correlator was implemented to search for the Code Phase and Doppler shift over a range of 0 to 1022 and -10kHz to +10kHz respectively with a Doppler Bin of 500Hz making 41 different frequency steps. A simulation of the transmitted GPS satellite signal as discussed above is used as a local replica of the signal at IF plus a Frequency Shift step in the Serial Search. The incoming signal is simulated as a sum of received signals from multiple satellites simulated at different power levels, Code phases and Doppler shifts. The Acquisition is simulated for an IF of 21.25MHz and sampling frequency of 5MHz. The simulations were done for received signal strengths between -158dBW and -184dBW for different satellite signals.

![Power Spectrum](image1.png)

![Power Spectrum](image2.png)

Fig. 2.16 *Power Spectrum of a -160dBW received GPS signal from satellite9 at IF of 4.8MHz without and with AWGN at $N_o = -205.18$dBW/Hz*
This is because the strongest received L1 GPS signal has the strength of -158dBW while a typical weak signal is around -184dBW or worse due to different levels of attenuations faced by different satellite signals.

Fig. 2.17 demonstrates an Acquisition test result using MATLAB of Satellite 1 at -184dBW with the Code phase and Doppler shift pair of (372 chips, 4kHz) using the correlators responsible for acquiring the signals from Satellite 1 and Satellite 2. While the received signals for Fig. 2.17a and c are simulated in the absence of noise, the received signal in Fig. 2.17b is simulated with AWGN at Noise Density of -229.18dBW/Hz even though the noise levels of the actual received GPS signal is normally at a worse level. This Noise Density is used in order to keep the acquisition simulations of the weak GPS signals simple for the sake of demonstration and the scope of this thesis. Higher noise levels for these weak signals require Acquisition lengths with $M > 1$ and methods like the half-bit method and the mixed coherent/non-coherent integration method as mentioned in Chapter 1. As mentioned earlier, Acquisition simulations in this thesis were done with 1ms of simulated received GPS signal, i.e. $M = 1$ for simplicity. Therefore to ensure acquisition of weak signals in the simulations, the noise level cannot be as high as the actual level.

Fig. 2.17a and b show a correct peak at Code Phase and Doppler shift of 372 chips and 4kHz respectively while Fig. 2.17c shows no peak since there is no signal component from Satellite 2. The higher cross-correlation floor in Fig. 2.17b compared to that of Fig. 2.17a is because of the presence of AWGN in the simulation of the received signal used in Fig. 2.17b.
Fig. 2.17a Acquisition of Satellite1 signal simulated without Noise using the correlator for acquiring Satellite1 when only Satellite 1 is visible.
Fig. 2.17b Acquisition of Satellite1 signal simulated with Noise at $N_o = -229.18\text{dBW/Hz}$ using the correlator for acquiring Satellite1 when only Satellite 1 is visible.
Note that the correlator outputs in Fig 2.17 are under very ideal conditions i.e., only one satellite signal is present. In reality, the signal received by a receiver as explained is a combination of many signals from different satellites at different strengths. A signal being tracked by a particular correlator is considered weak relative to its interfering satellite signals if it is 24dB below any interfering satellite signal. This may adversely distort the correlator outputs acquiring and tracking the weak signal and therefore, the interfering satellite signals must be removed first before acquisition. This interference effect and its mitigation will be discussed in the next chapter.
Chapter 3

GPS Cross-Correlation Interference and Review of Existing Mitigation/Cancellation Methods

It has been stated in Chapter 2 that the GPS signal is normally received at Noise Density, $N_o$, of $-205.18\, dBW/Hz$. Therefore, a typically strong satellite signal of -160dBW strength has a Carrier-to-Noise Density, $C/N_o$, of about 45.18dBHz and a Carrier-to-Noise Ratio, $C/N$, of -17.93dB. Without considering the effect of the interference from other signals, even though such a signal is 17.93dB below the typical Noise floor, it can easily be acquired by the use of the conventional correlator at a receiver end as the signal is raised out of the noise through the receiver’s code-correlation process.

Discussions in Chapter 1 also mentioned some innovations developed to acquire GPS signals weaker than -160dBW. [45] presents the use of 10ms coherent integration time in acquiring signals with $C/N_o$ approaching 32dBHz. The paper also reports the use of combined coherent and non-coherent integration in acquiring signals with $C/N_o$ as low as 24dBHz. [5] also discusses an approach that used 4s of input data in acquiring signals with $C/N_o$ of 21dBHz representing signals as weak as $-184.18\, dBW$ if the typical $N_o$ of $-205.18\, dBW/Hz$ is assumed.

These methods for acquiring weak signals work well in the absence of strong interfering signals but problems arise when the weak signal coexist with strong interfering GPS signals as mentioned before. This Chapter discusses the GPS cross correlation interference (CCI),
beginning with simulations to take a look at the effect of CCI from interfering signals. The simulations are done in the absence of noise to assess the extent to which CCI alone can distort the output of the correlator used in acquisition.

3.1 GPS CCI

As explained in Chapter 2, the received GPS signal at the input of the correlator is a combination of signals from multiple satellite signals. Inferring from the GPS signal structure, any received satellite signal is considered weak if it is 24dB below the strongest signal amongst the visible satellite signals. This is because from Table 2.5, the dynamic range of the auto-correlation C/A spreading code is limited to about 24dB. Also, as explained in Section 2.4.1, the dynamic range could drop to as low as 21.1dB [30] due to the Doppler Effect on the C/A code chipping rate. As mentioned in Chapter 1, scenarios such as cloudiness, presence of trees, forest and buildings especially in the urban environments present varying attenuation levels for different visible satellite signals, making some of these visible signals weaker as compared to other visible signals. The presence of the strong signals presents very high CCI which affects the acquisition of the weak signals.

3.1.1 MATLAB simulations of the GPS CCI problem

The effect of CCI is demonstrated in the cross-correlation properties of multiple raw baseband C/A code summed together. Fig. 3.1 shows the auto-correlation of Satellite 1 C/A code , $g_1$, and the cross-correlation of $g_1$ with the sum of $g_1$ and multiples of other satellite C/A code, ‘$g_x$’ where $x$ is the satellite ID.
Fig. 3.1a Autocorrelation of Satellite 1 C/A Code g1

Fig. 3.1b Cross correlation of g1 with (g1+g2)
Fig. 3.1c Cross correlation of \( g_1 \) with \( (g_1+g_2+g_3+g_4) \)

Fig. 3.1d Cross correlation of \( g_1 \) with \( (g_1+5g_2+3g_3+4g_4) \)
The cross-correlation worsens as the summed number of C/A codes increases. In Fig. 3.1d, although the peak of 1023 at code phase 0 is still maintained, there is several peaking magnitude which does not make the 0 code phase peak very obvious. The cross correlation is therefore distorted. This is due to the high magnitudes of the interfering signals with respect to g1 which is to be acquired stemming from the fact that the C/A codes are not perfectly orthogonal. This property of the C/A code is apparent during the Acquisition process where correlation filters are used. As mentioned in Chapter 1, the CCI term in Equation 1.9, at the output of the correlation filter is due to the sub-optimality of each correlation filter in the receiver design which depends on a particular satellite code for acquisition and tracking. The following simulations as depicted in Fig. 3.2 are four scenarios of the effect of CCI on the Acquisition process at the output of the correlator responsible for acquiring satellite 1 signal in the presence of three other satellite interferences at different interfering strengths and at the code phase and Doppler shift pairs of (372, 4kHz), (400, 5kHz), (823,6kHz), (90, -8kHz) respectively for satellites 1 to 4.

a) **Simulation of Acquisition of Satellite 1 signal with Satellite 2, 3 and 4 signals visible at relatively strong comparable received signal strength**
   Satellite 1 strength = -160dBW  
   Satellite 3 strength = -158dBW  
   Satellite 2 strength = -161dBW  
   Satellite 4 strength = -159dBW

b) **Simulation of Acquisition of Satellite 1 signal with Satellite 2, 3 and 4 signals visible at relatively weak comparable received signal strength**
   Satellite 1 strength = -184dBW  
   Satellite 3 strength = -181dBW  
   Satellite 2 strength = -180dBW  
   Satellite 4 strength = -182dBW

c) **Simulation of Acquisition of relatively weak Satellite 1 signal in the presence of one strong interfering signal and a 20dB dynamic range**
   Satellite 1 strength = -181dBW  
   Satellite 3 strength = -181dBW  
   Satellite 2 strength = -180dBW  
   Satellite 4 strength = -161dBW

d) **Simulation of Acquisition of relatively weak Satellite 1 signal in the presence of two strong interfering signal and a 24dB dynamic range**
   Satellite 1 strength = -184dBW  
   Satellite 3 strength = -182dBW  
   Satellite 2 strength = -160dBW  
   Satellite 4 strength = -161dBW
Fig. 3.2a Acquisition of Satellite1 signal using the correlator for acquiring Satellite1 when Satellite 1, 2, 3, 4 are visible with relatively strong comparable received Signal strength
Fig. 3.2b Acquisition of Satellite1 signal using the correlator for acquiring Satellite1 when Satellite 1, 2, 3, 4 are visible with relatively weak comparable received Signal strength
Fig. 3.2(c) Acquisition of relatively weak Satellite1 signal in the presence one strong interfering satellite signal.
Fig. 3.2d Acquisition of relatively weak Satellite1 signal in the presence two strong interfering satellite signals.
Note that the received signals used in Fig. 3.2 were simulated without noise in order to view the contribution of only the CCI in the correlator output. Fig. 3.2a and b show that when the Signal strengths of the interfering satellite signals are comparable to the Signal Strength of the Satellite Signal being acquired by a particular correlator, both in the case of all signals being strong or weak, the correlator output is very good for acquisition and the acquired code phase or Doppler shifts are correct. The acquisition results worsen following the presence of one or more strong interfering satellite signals in Fig 3.2c and d. In Fig 3.2c, where satellite 4 signal is 20dB stronger than Satellite 1, even though the code phase and Doppler shift could be acquired, there are many competing peaks along with the correct peak therefore increasing the probability of false alarm. In Fig 3.2d, the presence of two strong interfering satellite signals lead to wrong acquisition of the code phase and Doppler shift for the satellite 1 signal which is made worse by the fact that satellite 2 signal is 24dB more than satellite 1 signal. The 24dB range difference cutoff between a weak signal and a strong signal strength is explained in the next section.

3.1.2 GPS CCI Time Domain Analysis

For more insight into the GPS receiver CCI, the acquisition process of the correlation filter must be analyzed in the time domain. Beginning with Equation 2.12 and ignoring the phase, $\phi_{LO}$ of the LO for down conversion to IF, the sampled received signal is given by

$$ r_{IF}[n] = n_{IF}(nT) + \sum_{i=1}^{N} A_i \cdot C_i(nT - \tau_i) \cdot D_i(nT - \tau_i) \cdot \cos[2\pi(Tf_{IF} + f_{Di})n + \phi_i] \quad \ldots \ldots (3.1) $$

Using the serial search method for analysis, from Fig. 2.10, consider the acquisition of satellite indexed $k$. With a replica C/A code, $C_k(nT)$ at a code phase of $\tau_k$ and a local
quadrature carrier DCO generating at the same IF as the incoming samples and at a Doppler shift and phase of \( f_{D_k} \) and \( \phi_k \) respectively. The local replica signal used in acquisition is given by

\[
r_{DCO}[n] = C_k(nT - \tau_k) \cdot \cos[2\pi T(f_{IF} + f_{D_k})n + \phi_k]
\]

The in-phase channel output before integration or summation is given by

\[
I_k[n] = r_{IF}[n] \cdot r_{DCO}[n]
\]

\[
= \sum_{i=1}^{N} \left\{ A_i \cdot C_i(nT - \tau_i) \cdot C_k(nT - \tau_k) \cdot D_i(nT - \tau_i) \cdot \cos[2\pi T(f_{IF} + f_{D_i})n + \phi_i] \right. \\
\cdot \cos[2\pi T(f_{IF} + f_{D_k})n + \phi_k] \left. \right\} + \eta_i[n]
\]

\[
I_k[n] = \sum_{i=1}^{N} \left\{ \frac{A_i}{2} \cdot C_i(nT - \tau_i) \cdot C_k(nT - \tau_k) \cdot D_i(nT - \tau_i) \\
\cdot \left[ \cos[2\pi T(\Delta f_{D_{i,k}})n + \Delta \phi_{i,k}] + \cos[2\pi T(2f_{IF} + f_{D_i} + f_{D_k})n + \phi_i + \phi_k] \right] \right\} \\
+ \eta_i[n]
\]

Where

\[
\eta_i[n] = n_{IF}(nT) \cdot C_k(nT - \tau_k) \cdot \cos[2\pi T(f_{IF} + f_{D_k})n + \phi_k]
\]

\[
\Delta f_{D_{i,k}} = f_{D_i} - f_{D_k}
\]

\[
\Delta \phi_{i,k} = \phi_i - \phi_k
\]

Filtering out the high frequency components and leaving the base band content,

\[
I_k[n] = \sum_{i=1}^{N} \left\{ \frac{A_i}{2} \cdot C_i(nT - \tau_i) \cdot C_k(nT - \tau_k) \cdot D_i(nT - \tau_i) \cdot \cos[2\pi T(\Delta f_{D_{i,k}})n + \Delta \phi_{i,k}] \right\} \\
+ \eta_i[n]
\]
Since \([C_k(nT - \tau_k) \cdot C_k(nT - \tau_k) = 1]\) and assuming that \(f_{D,k}\) is the correct Doppler shift, therefore \(\Delta f_{D,k} = 0\)

\[
I_k[n] = \eta_i[n] + \frac{A_k}{2} \cdot D_k(nT - \tau_k) \cdot \cos(\Delta \phi_k)
\]

\[
+ \sum_{i=1}^{N} \left( \frac{A_i}{2} \cdot C_i(nT - \tau_i) \cdot C_k(nT - \tau_k) \cdot D_i(nT - \tau_i) \cdot \cos[2\pi T(\Delta f_{D,i,k})n + \Delta \phi_{i,k}] \right)
\]...

(3.5)

The coherent integration output, \(I_k[j]\), which is the sum of the in-phase channel sample outputs, \(I_k[n]\) over a period of \((M \cdot L)\) samples, where \(M\) is the number of integer multiple of the C/A code period used in acquisition and \(L\) is the number of samples per C/A code period is given by

\[
I_k[j] = \frac{A_k}{2} \cdot \cos(\Delta \phi_k) \cdot \left( \sum_{n=j}^{j+ML-1} D_k(nT - \tau_k) \right)
\]

\[
+ \sum_{i=1}^{N} \frac{A_i}{2} \cdot \left( \sum_{n=j}^{j+ML-1} \left( C_i(nT - \tau_i) \cdot C_k(nT - \tau_k) \cdot D_i(nT - \tau_i) \cdot \cos[2\pi T(\Delta f_{D,i,k})n + \Delta \phi_{i,k}] \right) \right)
\]

\[+ \eta_i[j] \]

for \(j = 0, 1, ..., (ML - 1)\), where \(j\) is the integration dump sample number.

Since coherent integration is done for a period of samples free of data bit transition boundary, the data bit remains constant for the period of integration and can be assumed to be \(\pm 1\). Therefore

\[
\left| \sum_{n=j}^{j+ML-1} D_k(nT - \tau_k) \right| = M \cdot L
\]
The integration output is given by

\[ I_k[j] = M \cdot L \cdot a_k \cdot \cos(\Delta \phi_k) \]

\[ + \sum_{i=1}^{N} a_i \cdot \left( \sum_{n=j}^{j+ML-1} \left( C_i(nT - \tau_i) \cdot C_k(nT - \tau_k) \cdot D_i(nT - \tau_i) \cos[2\pi T (\Delta f_{D_{ik}}) n + \Delta \phi_{i,k}] \right) \right) \]

\[ + \eta_I[j] \] ...

(3.7)

where \( a_i = \frac{A_i}{2} \)

By extension, the quadrature channel integration output is

\[ Q_k[j] = M \cdot L \cdot a_k \cdot \sin(\Delta \phi_k) \]

\[ + \sum_{i=1}^{N} a_i \cdot \left( \sum_{n=j}^{j+ML-1} \left( C_i(nT - \tau_i) \cdot C_k(nT - \tau_k) \cdot D_i(nT - \tau_i) \sin[2\pi T (\Delta f_{D_{ik}}) n + \Delta \phi_{i,k}] \right) \right) \]

\[ + \eta_Q[j] \] ...

(3.8)

The CCI due to the interfering satellite signal \( i \) in the in-phase and quadrature channel at the integral output of the correlation filter for acquiring satellite signal \( k \) are given respectively as;

\[ l_{ik}^{CCI}[j] = a_i \cdot \left( \sum_{n=j}^{j+ML-1} \left( C_i(nT - \tau_i) \cdot C_k(nT - \tau_k) \cdot D_i(nT - \tau_i) \cos[2\pi T (\Delta f_{D_{ik}}) n + \Delta \phi_{i,k}] \right) \right) \] ...

(3.9)

\[ Q_{ik}^{CCI}[j] = a_i \cdot \left( \sum_{n=j}^{j+ML-1} \left( C_i(nT - \tau_i) \cdot C_k(nT - \tau_k) \cdot D_i(nT - \tau_i) \sin[2\pi T (\Delta f_{D_{ik}}) n + \Delta \phi_{i,k}] \right) \right) \] ...

(3.10)

The expected acquisition peaks for the channels are

\[ I_k^{PEAK}[j] = M \cdot L \cdot a_k \cdot \cos(\Delta \phi_k) \] ...

(3.11)

\[ Q_k^{PEAK}[j] = M \cdot L \cdot a_k \cdot \sin(\Delta \phi_k) \] ...

(3.12)
As in Table 2.5 in Chapter 2, it can be shown that for all integer values of \( M \) and number of samples \( L \), the maximum cross-correlation between two non-identical or identical but unaligned C/A codes is about 24dB below \( M \cdot L \), the peak of the autocorrelation for \( L \) samples per C/A code period and \( M \) C/A code periods. It has been explained that this value could drop to as low as 21dB due to the Doppler Effect on the C/A code chipping rate. In Equation 3.7 and 3.8, as long as \( a_i \) and \( a_k \) are of identical strengths or as long as the incoming signal are of identical strengths, the CCI is not a problem since it is always between 21 and 24dB below the acquisition peaks given in Equations 3.11 and 3.12. This is shown in Fig. 3.2(a) and (b) where all signals are almost the same strength. Nevertheless, problems arise during acquisition in situations where \( a_i \) approaches 21dB above \( a_k \). This raises the CCI terms in Equations 3.9 and 3.10 to magnitudes close to or above the peaks given in Equations 3.11 and 3.12. As seen in Fig. 3.2(c), the presence of an interfering signal 21dB gives rise to many competing peaks increasing the probability of false detection. In Fig. 3.2(d) where the strongest signal is 24dB above the satellite 1 signal being acquired, the acquired code phase and Doppler shifts are completely wrong.

3.2 Assessing Use of Conventional CDMA Cellular Network CCI Mitigation Methods for GPS CCI Mitigation

The deployment of the CDMA cellular mobile network is now wide spread. Since such a network also uses CDMA spreading code, the problem of CCI also exists in such a network. There are a number of CCI mitigation methods in CDMA cellular mobile networks, such as Power Control and the adaptive multiuser detection. It is natural to consider application of
such mitigation methods in a GPS receiver which is also based on the CDMA principle. However, major differences exist between these two kinds of CDMA systems, which account for the main differences in the solutions to the CCI problems encountered in both systems.

In a CDMA cellular mobile network, the multi users of the uplink channel in a cell are the Mobile Stations (MS) with unique spreading codes in the area of the cell, and the receiving end in each cell has one stationary Base Station (BS) which has knowledge of all the spreading codes of all the MS. Also, there is continuous uplink and downlink communication between the BS and all MS. In the GPS system, the multi users of the channel are the orbiting SVs and the receiving end has many GPS receivers as against one BS which is a reference in the former. Only one way (downlink) communication exists in the GPS system. As a result, a feedback method like the Power Control used for CCI mitigation in a cellular mobile network cannot be used. Also, the training based channel equalization methods in a cellular mobile network cannot be used either because the lack of coherence (synchronization) that is required for training to work. Besides, training based MMSE detectors in a cellular mobile network are designed to combat multipath propagation, but not for interference mitigation.

Nevertheless, some multiuser interference mitigation methods in cellular mobile networks can indeed be considered for GPS. The next section presents a discussion on the use of Successive Interference cancellation and Parallel Interference Cancellation in CCI mitigation for GPS. These methods have been used for CCI mitigation in the CDMA cellular
network. In addition, the Subtractive Orthogonal Projection Method, a subspace interference cancellation method used in both the cellular network and GPS, is also briefly discussed.

3.3 Existing GPS CCI Mitigation/Cancellation Methods

3.3.1 Successive Interference Cancellation (SIC) [44], [38], [3]

The SIC method attempts to reconstruct each interfering strong satellite signal by the use of parameters passed from a conventional detector or correlator. These reconstructed satellite signals are serially removed from the received signal prior to correlation of the weak signal. The reconstruction of the interfering signals calls for the estimation of its frequency, phase and amplitudes with the estimation of the amplitude being of particular concern. With the presence of amplitude fluctuation due to fading and multipath, the appropriate amplitude must be continuously monitored and adjusted to ensure the efficiency of this method.

The SIC method has recently been employed in [44] where signals from ground based pseudolites, which are used to aid the GPS, interfere with the GPS signals. In [44], the method was constrained to the post-processing of pre-recorded signal data using MATLAB [44]. The complexity of this method is increased due to the continuous tracking of the amplitude of the interfering signals. Estimation of the frequency and code phase of the strong interfering signals is achieved with the Conventional receiver. This could become a problem when multiple strong interferers exist, in which can they must be estimated and subtracted one-by-one, causing increase in computational complexity and latency.
3.3.2 Parallel Interference Cancellation (PIC) [43], [3]

The PIC is a variation of the SIC where there is a simultaneous reconstruction and removal of strong interfering satellite signals instead of the serial method in the SIC. This method has the advantage of reducing the delay in detecting weak satellite signals even though there is an increase in the complexity in hardware. The same challenge is encountered as the amplitude of the strong signals must be continuously tracked. In comparison with the SIC algorithms, the processing time with the PIC algorithms is greatly reduced but its hardware is considerably more complicated than that of the SIC [43]. [43] implements and compares the PIC and SIC methods.

3.3.3 Subtractive Orthogonal Projection Method [3], [39]

This is an orthogonal subspace projection approach where the received signal input is orthogonally projected onto the strong signal subspace giving a good approximation of the contribution of the strong signal to the total received signal. The estimate of the contribution of the strong interfering signals is then subtracted from the received signal leaving the weak signal for a successful detection or acquisition.

Due to the strong relevance of this method with the proposed method of this thesis, a detailed description of this method will be given in Chapter 4 where the orthogonal projection operator and the subspace projection method are discussed in more detail. As will be evident in Chapter 4, this method does not need continuous tracking of the strong signal amplitude, a major advantage over the SIC and PIC, reducing its complexity.
compared to the earlier discussed methods. [39] presents a practical implementation of the Subtractive Orthogonal Projection Method with simulated GPS signals.
Chapter 4

GPS CCI Cancellation Using Oblique Projection Method

4.1 Introduction to Subspace Projection

Interference cancellation by Subspace projection methods refer to those techniques in which the signal detector is specifically designed to reject any structured interference, by projecting the received signal onto a subspace spanned by either the desired signal or the interfering signals.

Let $h_1, h_2, \ldots, h_m \in \mathbb{C}^n$ be a set of independent column vectors forming the basis of a subspace in $\mathbb{C}^n$ defined by

$$\langle H \rangle = \text{Span}(H)$$

where

$$H = [h_1 \; h_2 \; \ldots \; h_m] \in \mathbb{C}^{n \times m}$$

In other words, the subspace $\langle H \rangle$ is spanned by the columns of $H$ and also $h_1, \ldots, h_m \in \langle H \rangle$.

A vector $v \in \mathbb{C}^n$ can be projected onto the subspace $\langle H \rangle$ to obtain a projection $\hat{v} \in \langle H \rangle$ by the use of the Projection operator or matrix $E$.

Two projection methods and operators will be discussed in the following sections; the Orthogonal and the Oblique projection.
4.1.1 Orthogonal Projections

The Orthogonal projection $\hat{v} \in \langle H \rangle$ of $v \in \mathbb{C}^n$ onto the subspace $\langle H \rangle$ is such that $(v - \hat{v})$ is perpendicular to every vector in the subspace, $w \subseteq \langle H \rangle$.

Fig. 4.1 Illustration of Orthogonal Projection onto subspace

Fig. 4.1 illustrates that for the orthogonal projection $\hat{v}$ of $v \in \mathbb{C}^n$ onto subspace $\langle H \rangle$,

$$(v - \hat{v}) \perp w, \forall w \subseteq \langle H \rangle \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (4.2)$$

The orthogonal Projection Operator or matrix, $E_H$ for the projection of every vector $v$ onto subspace $\langle H \rangle$ is given by [40]

$$E_H = H(H^\dagger H)^{-1}H^\dagger \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (4.3)$$

where $H$ is as defined in Equation 4.1 and $\dagger$ is the hermitian operator

The Orthogonal Projection Operator, $E_H$ is such that,

1. $E_H = E_H^\dagger$

2. $E_H^2 = E_H$

3. $E_H \cdot H = H$, since the columns of $H$ belong to $\langle H \rangle$
4. \( \forall v \in \mathbb{C}^n, \text{if } \hat{v} = E_H \cdot v \Rightarrow \hat{v} \in \langle H \rangle \). Therefore the subspace \( \langle H \rangle \) is the Range Space of \(E_H\).

5. \( \forall s \subseteq \langle H \rangle^\perp \), where \( \langle H \rangle^\perp \) is the subspace orthogonal to \( \langle H \rangle \): \( \langle H \rangle^\perp \perp w, \forall w \subseteq \langle H \rangle \), \( E_H \cdot s = 0 \). By this definition, any vector or subspace perpendicular to the subspace \( \langle H \rangle \) or any member of subspace \( \langle H \rangle^\perp \) is a member of the Null Space of \(E_H\). Therefore \((v - \hat{v})\) in Equation 4.2 is a member of the null space of \(E_H\).

As discussed in Chapter 3, an Orthogonal Projection Operator is used in the Subtractive Orthogonal Projection Method in estimating and removing strong interfering signals from the received signal before the correlation process during Acquisition. A more detailed discussion on this process is discussed later in this Chapter.

4.1.2 **Oblique Projection and properties**

A generalized Projection Operator can be defined such that the Range Space and Null Space are not necessarily orthogonal to each other. This form of projection is called the Oblique Projection. This means, the Range space and the Null Space must be specified in defining an Oblique Projection.

Let \( \langle H \rangle \) be a subspace spanned by the linearly independent columns of \( H \in \mathbb{C}^{n \times m} \) and \( \langle S \rangle \) be a subspace spanned by the linearly independent columns of \( S \in \mathbb{C}^{n \times t} \) where \( H \) and \( S \) are independent i.e., their columns are linearly independent of each other but may not be orthogonal to each other.
Define $E_{HS}$ as a "$n \times n$" Projection Operator or matrix onto subspace $\langle H \rangle$, meaning $\langle H \rangle$ is the Range space of $E_{HS}$, and with Null space $\langle S \rangle$. $E_{HS}$ is called an Oblique projection operator onto the subspace $\langle H \rangle$ and it is given by [41]

$$E_{HS} = H[H^\dagger(E_{S\perp})H]^{-1}H^\dagger(E_{S\perp})$$

where $E_{S\perp} = (1 - E_S)$ and as defined in Equation 4.3, $E_S$ is the Orthogonal Projection matrix onto $\langle S \rangle$, the Null space of $E_{HS}$.

The following statements then hold for $E_{HS}$.

1. $E_{HS} = (E_{HS})^2$
2. $E_{HS} \cdot H = H$, since the columns of $H$ belong to $\langle H \rangle$
3. $\forall \, v \in \mathbb{C}^n$, if $\hat{v} = E_{HS} \cdot v$, then $\hat{v} \in \langle H \rangle$
4. $\forall \, w \in \langle S \rangle$, then $E_{HS} \cdot w = 0$
5. $E_{HS} \cdot S = 0$, since the columns of $S$ belong to $\langle S \rangle$
6. The Orthogonal projection onto $\langle H \rangle$ is a special case of the Oblique projection where the Null space $\langle H \rangle^\perp$ of the Projection operator $E_H$ is Orthogonal to the Range space $\langle H \rangle$. If $\langle H \rangle$ is spanned by the linearly independent columns of $H \in \mathbb{C}^{n \times m}$ and $\langle H \rangle^\perp$ is spanned by the linearly independent columns of $H^\perp \in \mathbb{C}^{n \times t}$, then the orthogonal projection operator $E_H = E_{H \perp}$. This can be proven as follows starting from $E_{H \perp}$. From Equation 4.4

$$E_{H \perp} = H[H^\dagger(E_{(H^\perp)^\perp})H]^{-1}H^\dagger(E_{(H^\perp)^\perp})$$

since $E_{H^\perp} = 1 - E_H$, where $E_H$ is the Orthogonal projection onto $\langle H \rangle$

$$E_{(H^\perp)^\perp} = 1 - E_{H^\perp} = I - (1 - E_H) = E_H$$

$$\therefore \, E_{H \perp} = H[H^\dagger(E_H H)]^{-1}H^\dagger E_H$$

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From the properties of $E_H$ given in Section 4.1.1,

$$E_H H = H, \text{ and } E_H^\dagger = E_H$$

$$\Rightarrow H^\dagger E_H = ((E_H)^\dagger H)^\dagger = (E_H H)^\dagger = H^\dagger$$

$$\therefore E_{H H^\perp} = H [H^\dagger H]^{-1} H^\dagger$$

Therefore from Equation 4.3

$$E_H = E_{H H^\perp}$$

### 4.2 Linear Modeling of the Received GPS Signal

A GPS signal can be modeled linearly as a weighted sum of modes, where the modes are unit magnitude copies of the signal transmitted by each satellite and received at the receiver starting from the received code phase and offset by the correct Doppler frequency. Separating the satellite signal being acquired by a particular correlator from other interfering satellite signals, the received GPS signal is modeled as

$$y = H \theta + S \emptyset + v$$

$$= x + b + v \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 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$S$ is $n \times m$ matrix with each column representing the unit magnitude of the signal from the interfering satellites.

$\varnothing$ is an $m \times 1$ weight vector representing the strength of the interfering satellite signals.

$\nu \in \mathbb{C}^n$ is an $n \times 1$ AWG noise.

### 4.3 Ideal use of Oblique Projectors for CCI cancellation

#### 4.3.1 Mathematical Description

Cross Correlation becomes a problem when the satellite signal being acquired by a correlator is weak as compared to other interfering satellite signals. The oblique projection method is designed to reject any interfering satellite signal whilst still being able to observe the desired satellite signal without the need for estimating the signal weights or signal strength.

From the linear model of the received signal given in Equation 4.5 above, an oblique projection operator $E_{HS}$ is formed. The range space $\langle H \rangle$ of $E_{HS}$ is spanned by the column vector $H$, which is the unit magnitude mode of samples of the weak satellite signal being acquired, and the null space $\langle S \rangle$ of $E_{HS}$ is spanned by columns of $S$, unit magnitude modes of the samples of the strong interfering satellite signals. Therefore, $E_{HS}$ is explicitly written as

$$E_{HS} = H[H^\dagger(E_{S\perp})H]^{-1}H^\dagger(E_{S\perp})$$

A projection of the incoming signal vector $y$ onto the subspace $\langle H \rangle$ with the projection operator $E_{HS}$ is given by
\[ E_{HS} \cdot y = \tilde{y}_H = E_{HS} \cdot H\theta + E_{HS} \cdot S\Phi + E_{HS} \cdot \nu \]

\[ = H\theta + \tilde{\nu}_H \]

\[ = x + \tilde{\nu}_H \]

(4.6)

where

\[ \tilde{\nu}_H = E_{HS} \cdot \nu, \]

is a projection of the noise vector onto the subspace \( \langle H \rangle \). This effectively removes all the strong interfering satellite signals leaving only the weak satellite signal and the noise vector projected onto \( \langle H \rangle \). The weak signal can now be acquired using techniques like the non-coherent integration after long coherent integration, techniques designed for acquisition of weak GPS signals. Without removal of the strong interfering satellite signals which cause CCI, these techniques for weak signal acquisition are not effective. This is because, as shown Chapter 3, the CCI from the strong interfering signals produce peaks at the correlator output that compete with the correct peak at the Code Phase and Doppler shift being acquired increasing the probability of false detection.

### 4.3.2 Test of the Ideal Oblique Projection method for CCI cancellation using raw Baseband/Unmodulated C/A Codes (zero Doppler Shift)

In this subsection we use the ideal oblique projection method to test CCI removal in the raw baseband or unmodulated C/A code cross-correlations described in Section 3.1.1 of Chapter 3. The C/A code of satellite 1 locally generated by a receiver at zero code phase is correlated with the following cases of the sum of different C/A codes ‘gx’ at different code phases, where \( x \) is the satellite ID.
a. $g_1$

b. $(g_1 + g_2)$

c. $(g_1 + g_2 + g_3 + g_4)$

d. $(g_1 + 5g_2 + 3g_3 + 4g_4)$

In each of these cases of simulation $g_1, g_2, g_3$ and $g_4$ were generated at Code Phases 186, 465, 651 and 930 chips respectively. We first test the conventional cross-correlation method, with results shown in Fig. 4.2 below;

![Cross-Correlation of Satellite 1 C/A code and C/A codes combinations a, b, c and d](image)

**Fig. 4.2 Cross-Correlation of Satellite 1 C/A code and C/A codes combinations a, b, c and d**

The oblique projection algorithm was tested on the same signal combinations, where the C/A code $g_1$ is used as the column vector $H$ while $g_2, g_3$ and $g_4$ form the columns of $S$. The Oblique Projection Operator $E_{HS}$ is formed with range space $\langle H \rangle$ and null space $\langle S \rangle$ spanned by $H$ and $S$ respectively, as in Equation 4.4. Following Equation 4.6, the resulting projections of each case using $E_{HS}$ is the same: Leaving only $g_1$ and cancelling out the
interfering codes $g_2, g_3$ and $g_4$ regardless of the multipliers (i.e., satellite signal power levels) used in the combinations. The cross-correlation with satellite 1 C/A code after the oblique projection is shown in Fig. 4.3.

![Cross-correlation graph](image)

**Fig. 4.3** Cross-Correlation of Satellite 1 C/A code and resulting Ideal Oblique projections from combinations a, b, c and d. Results is exactly the same for each combination of C/A code.

It is noticed that Fig. 4.3 is the same as Fig. 4.2a where only $g_1$ was used and therefore no interfering C/A code was present in the cross-correlation. This confirms that all the interfering C/A codes are completely removed in the Oblique projection before cross-correlation is performed.

### 4.3.3 Demonstration of the use of the Ideal Oblique Projection for CCI Removal in Simulated Received GPS Signal

In this subsection, the effectiveness of the Ideal oblique projection algorithm is examined when both Code Phase and Doppler Shift is included in the simulation of the received signal. For simulations in the first four sets of scenarios below, the received signals are
simulated without noise to demonstrate how the effectiveness of the method in removing the CCI from the correlator output. The next four sets of received signals are simulated to contain AWGN at $N_o$ of $-229.18 dBW/Hz$ to assess the performance of the method in the presence of additive noise. As mentioned in Chapter 2, even though the noise levels of the real received GPS signal is normally at a higher level, this Noise Density is used in order to keep the acquisition simulations of the weak GPS signals simple. In both sets of simulations, the oblique projection operator, $E_{HS}$, has the range space $\langle H \rangle$ spanned by the column vector $H$ which is the unit magnitude mode of samples of the weak satellite signal being acquired. The null space $\langle S \rangle$ of $E_{HS}$ is spanned by the columns of $S$, the unit magnitude modes of the samples of the strong interfering satellite signals. Fig. 2.17a of Chapter 2 shows an example of the cross-correlation output during the Acquisition of GPS signal from Satellite 1 when there are no interfering GPS signals from other SVs. Fig. 3.2 of Chapter 3 also demonstrated the extent by which other interfering GPS Satellite signals can distort the correlation output during Acquisition. Fig. 4.4 shows the Code Phase plots and Doppler Frequency plots for the peak correlation when the conventional correlation method is used during Acquisition of Satellite 1 signal for the following scenarios. For each scenario, the following code phase and Doppler shift pairs of $(372$ chips, $4 kHz)$, $(400$ chips, $5 kHz)$, $(823$ chips, $6 kHz)$, $(90$ chips, $-8 kHz)$ were used respectively for received signals from satellites 1 to 4.

**Scenarios without Noise**

a) *Received signal from Satellite 1 only at a strength of -184dB without noise (No interfering GPS signal).*

b) *Satellite 1, 2, 3 and 4 signals visible at relatively comparable received signal strength without noise;*

<table>
<thead>
<tr>
<th>Satellite 1 strength</th>
<th>Satellite 2 strength</th>
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<tbody>
<tr>
<td>$-184 dBW$</td>
<td>$-180 dBW$</td>
</tr>
<tr>
<td>Satellite 3 strength</td>
<td>Satellite 4 strength</td>
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<tr>
<td>$-181 dBW$</td>
<td>$-182 dBW$</td>
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c) Relatively weak Satellite 1 signal in the presence of one strong interfering signal and a 20dB dynamic range with signals simulated without noise;
Satellite 1 strength = -181dBW  Satellite 2 strength = -180dBW
Satellite 3 strength = -181dBW  Satellite 4 strength = -161dBW

d) Relatively weak Satellite 1 signal in the presence of two strong interfering signals and a 24dB dynamic range with signals simulated without noise;
Satellite 1 strength = -184dBW  Satellite 2 strength = -160dBW
Satellite 3 strength = -182dBW  Satellite 4 strength = -161dBW

Scenarios with Noise

e) Received signal from Satellite 1 only strength = -184dB with AWGN at $N_o = -229.18dBW/Hz$ (No interfering GPS signal)

f) Satellite 1, 2, 3 and 4 signals visible at relatively comparable received signal strength with AWGN at $N_o = -229.18dBW/Hz$;
Satellite 1 strength = -184dBW  Satellite 2 strength = -180dBW
Satellite 3 strength = -181dBW  Satellite 4 strength = -182dBW

g) Relatively weak Satellite 1 signal in the presence of one strong interfering signal and a 20dB dynamic range with signals simulated with AWGN at $N_o = -229.18dBW/Hz$;
Satellite 1 strength = -181dBW  Satellite 2 strength = -180dBW
Satellite 3 strength = -181dBW  Satellite 4 strength = -161dBW

h) Relatively weak Satellite 1 signal in the presence of two strong interfering signals and a 24dB dynamic range with signals simulated with AWGN at $N_o = -229.18dBW/Hz$;
Satellite 1 strength = -184dBW  Satellite 2 strength = -160dBW
Satellite 3 strength = -182dBW  Satellite 4 strength = -161dBW

The Ideal Oblique projection method is now used to remove all interfering satellite signals for all the cases as described in Equation 4.6 regardless of the level or magnitude of interfering signals. Fig. 4.5a shows the correlation output for the cases b, c and d after the interferences have been removed which is the same as case a in Fig. 4.4a or the correlation output in Fig. 2.17a in Chapter 2 when there was no interfering signal and noise. The Code Phase plots and Doppler Frequency plots for the correlation for the cases f, g and h after the interferences have been removed are shown in Fig. 4.5b. Here, the cross-correlation levels are very similar to scenario e of Fig. 4.4b when there was no interfering signal. The inclusion of AWGN in the simulation of the received signal accounts for the differences.
Fig. 4.4a Code Phase plots and Doppler Frequency plots for the peak correlation during Acquisition of Satellite 1 signal for cases a, b, c and d (Signals are simulated without Noise)
Fig. 4.4b Code Phase plots and Doppler Frequency plots for the peak correlation during Acquisition of Satellite 1 signal for cases e, f, g and h (Signals are simulated with Noise at $N_o = -229.18dBW/Hz$)
Fig. 4.5a Acquisition of Satellite1 signal using the correlator for acquiring Satellite1 when only Satellite 1, 2, 3, 4 are visible after Ideal oblique projection is used to remove interfering Satellite signals for cases b, c and d.
Fig. 4.5b Acquisition of Satellite1 signal using the correlator for acquiring Satellite1 when only Satellite 1, 2, 3, 4 are visible after Ideal oblique projection is used to remove interfering Satellite signals for cases f, g and h.

4.3.4 Problems associated with using the ideal Oblique Projection Method

In the formation of matrix $H$ and $S$, the code phase and the Doppler frequency of the weak satellite signal being acquired and the strong interfering satellite signals must be estimated before a projection operator $E_{HS}$ is formed. This is very simple in the case of strong signals since they can be acquired without the need to remove the interfering satellite signals. The
Code phase and Doppler frequency information can therefore be passed on to form matrix $S$.

The problem now is when the desired weak satellite signal is buried in other strong interfering satellite signals and therefore its Code phase and Doppler frequency cannot be acquired. The formation of matrix $H$ which spans the range space of $E_{HS}$ is not feasible. Methods of inferring into the code phase and Doppler frequency of the weak signal that is being acquired must be obtained to form a good projection matrix $E_{HS}$ for practical implementation of this method.

### 4.4 A Practical Oblique projection Implementation

The requirement of knowledge of the Code Phase and Doppler Shift of the weak satellite signal to be acquired makes the oblique projection method described so far ideal but not practical. A modification to the approach that is suitable for practical application is described below.

#### 4.4.1 Mathematical Description

Following the fact that the strong interfering signals can be easily acquired, the Code phase and Doppler shift information of the strong signals can be estimated first and used to form an oblique projection operator $E_{SH_o}$ with the range space $\langle S \rangle$ spanned by the columns of $S$, the unit magnitude modes of the samples of the strong interfering satellite signals with the estimated Code phase and Doppler Shift information. The null space, $\langle H_o \rangle$, of $E_{SH_o}$ is now
spanned by column vector $H_o$ which is the unit magnitude mode of samples of the weak satellite signal at transmission. Since we cannot estimate the Code Phase and Doppler shifts of the weak signal buried in the strong interference, they are set to zero to form $E_{SH_o}$. Using the linear model in Equation 4.5, a projection of the incoming signal onto subspace $\langle S \rangle$ with projection operator $E_{SH_o}$ is given by

$$E_{SH_o} \cdot y = \tilde{y}_S = E_{SH_o} \cdot x + E_{SH_o} \cdot b + E_{SH_o} \cdot v = E_{SH_o} \cdot H\theta + E_{SH_o} \cdot S\phi + E_{SH_o} \cdot v$$

$$= \tilde{x}_S + S\phi + \tilde{v}_S$$

$$= b + \tilde{x}_S + \tilde{v}_S$$

$$\cong b + \tilde{v}_S$$

(4.7)

Since $\theta$, the strength of the weak signal, is small compared to the entries of the vector $\phi$ for the strong signal, $\tilde{x}_S$, the projection of the weak signal onto $\langle S \rangle$ is negligible even when the code phase and Doppler shift in $H_o$ do not match those of the weak satellite signal. $\tilde{y}_S$ is therefore an approximation of the sum of strong interfering signals plus $\tilde{v}_S$, the projection of the noise vector onto the subspace $\langle S \rangle$. This can be then subtracted as in Equation 4.8 from the received signal $y$ leaving only the weak signal plus a noise vector resulting from the projection and subtractive process.

$$y - \tilde{y}_S = x + b + v - (b + \tilde{v}_S)$$

$$= x + v_d$$

(4.8)

The weak signal can then be easily acquired from the signal resulting from Equation 4.8.

As demonstrated in Chapter 3, during the acquisition of a signal, the CCI from other interfering satellite signals is not a problem if the strengths of the interfering satellite signals are comparable to the strength of the signal being acquired. In a scenario where
multiple strong satellites signals and multiple weak satellite signals are being received at
the same time, each strong signal can be acquired even in the presence of the other strong
satellite signals. The Code Phase and Doppler Shift information from the acquisition of each
of the strong signals are used in constructing the columns of \( S \). The column vector \( H_0 \) is
constructed for each weak signal being acquired without the prior need for its Code Phase
and Doppler Shift Information. \( S \) and \( H_0 \) are then used in constructing \( E_{SH_0} \). Each weak
satellite signal is acquired separately since the process acquiring one weak satellite is not
disrupted by the other weak signals.

4.4.2 Tests of methodology using raw Baseband/Unmodulated C/A Codes
(zero Doppler Shift)

This method is tested with the raw C/A code in removing all other C/A codes \((5g2 + 3g3 +
4g4)\) from the sum \((g1 + 5g2 + 3g3 + 4g4)\) leaving only \( g1 \) before correlation with
satellite 1 C/A code locally generated at zero code phase to determine the correct Code
Phase of \( g1 \). Three different simulations were done with \( g2, g3 \) and \( g4 \) generated at Code
phases of 372, 558 and 744 chips respectively while the Code phase of \( g1 \) was varied at 10,
465 and 930 chips for the three simulations. The results are shown in Fig. 4.6.
Fig. 4.6a Cross-Correlation of Satellite 1 C/A code and remainder vector when $g_2$, $g_3$ and $g_4$ are removed from the sum $(g_1 + 5g_2 + 3g_3 + 4g_4)$ with $g_1$ generated at Code Phase of 10 chips.

Fig. 4.6b Cross-Correlation of Satellite 1 C/A code and remainder vector when $g_2$, $g_3$ and $g_4$ are removed from the sum $(g_1 + 5g_2 + 3g_3 + 4g_4)$ with $g_1$ generated at Code Phase of 465 chips.
The Code Phase is correctly acquired in each scenario when the Code Phases of 10, 465 and 930 chips were used. From Fig. 4.6 above it is noticed that, instead of the three distinct cross-correlation levels which is typical of Gold code sequences, very small spikes are introduced in the cross-correlation. This is due to the projection \( \tilde{x}_S \) of \( x \), the C/A code to be acquired, onto subspace \( \langle S \rangle \). This makes the cross-correlation output after the interfering C/A codes are removed by this method less perfect than the case of Fig. 4.3 when the ideal oblique projection was used to remove the interfering C/A codes. Nevertheless, the noisy cross-correlation spikes are at very small levels and do not prevent the acquisition of the correct Code Phase.
4.4.3 Tests of methodology in removing CCI in Simulated Received GPS signal at Different Code phases and Doppler shifts

The method described by Equations 4.7 and 4.8 was used in removing the CCI in the scenarios described below. For the same reasons given in the test of the Ideal Projection method, received signals of Scenarios \(a,b\) and \(c\) are simulated without noise to demonstrate how the effectiveness of this method in removing the CCI from the correlator output. The received signals of Scenarios \(d,e\) and \(f\) are simulated to contain AWGN at \(N_o\) of \(-229.18\)dBW/Hz to assess the performance of the method in the presence of additive noise.

**Scenarios without Noise**

Relatively weak Satellite 1 signal simulated at signal strength of \(-184\)dBW with Code Phase and Doppler Shift pairs varied at

a) 279 chips, -1kHz

b) 465 chips, 3kHz

c) 930 chips, 9kHz

in the presence of two strong interfering signals and a 24dB dynamic range with signals simulated without noise. The interfering signal strengths with Code Phase and Doppler Shift pairs are described below;

Satellite 2 signal (-160dBW, 400 chips, 5kHz)

Satellite 3 signal (-182dBW, 823 chips, 6kHz)

Satellite 4 signal (-161dBW, 90 chips, -8kHz)

**Scenarios with Noise**

Relatively weak Satellite 1 signal simulated at signal strength of \(-184\)dBW with Code Phase and Doppler Shift pairs varied at

d) 279 chips, -1kHz

e) 465 chips, 3kHz

f) 930 chips, 9kHz
in the presence of two strong interfering signals and a 24dB dynamic range with signals simulated with AWGN at $N_o = -229.18$dBW/Hz. The interfering signal strengths with Code Phase and Doppler Shift pairs are described below;

**Satellite 2 signal** (-160dB, 400 chips, 5kHz)

**Satellite 3 signal** (-182dB, 823 chips, 6kHz)

**Satellite 4 signal** (-161dB, 90 chips, -8kHz)

Fig. 4.7 shows the Code Phase plots and Doppler Frequency plots for the peak correlation during Acquisition of Satellite 1 signal for the scenarios a to f without CCI removal. Since the dynamic range of the strength of the received signals is 24dB used in the simulations above, none of the Code Phase and Doppler shifts were correctly acquired for cases a to f as shown in Fig. 4.7.
Fig. 4.7 Code Phase plots and Doppler Frequency plots for the peak correlation during Acquisition of Satellite 1 signal for cases a, b, c, d, e and f before CCI removal.
Fig. 4.8 below shows the output of the correlator when CCI is removed first as described in Equations 4.7 and 4.8 before Acquisition for the scenarios a to f presented above. As described in Section 4.4.1, the null space, $\langle H_o \rangle$, of $E_{SH_0}$ is spanned by column vector $H_o$ which is the unit magnitude mode of samples of the weak satellite with Code Phase and Doppler shifts set to zero.

As shown in Fig. 4.8a to f, the Code phase and Doppler shifts are correctly acquired after the interfering satellite signals are removed. Whereas Fig. 4.8 a, b and c show very low cross-correlation levels, Fig. 4.8 d, e and f have higher cross-correlation levels due to the addition of noise during the simulation of the received signals used in scenarios d, e and f. It can therefore be concluded that the oblique projection operator $E_{SH_0}$ is effective in estimating and the sum of the strong satellite signals before its removal from the received signal. The remaining weak satellite signal can then be acquired.

The next section compares the Practical Oblique projection method and the Subtractive Orthogonal Projection method discussed earlier in Section 4.1.2 by comparing the effectiveness of its Orthogonal projection operator $E_S$ to the oblique projection operator $E_{SH_0}$ in estimating the sum of the strong satellite signals.
Fig. 4.8a Acquisition of Satellite1 signal using the correlator for acquiring Satellite1 after removing interfering Satellite signals for scenario a.
Fig. 4.8b Acquisition of Satellite1 signal using the correlator for acquiring Satellite1 after removing interfering Satellite signals for scenario b.
Fig. 4.8c Acquisition of Satellite1 signal using the correlator for acquiring Satellite1 after removing interfering Satellite signals for scenario c.
Fig. 4.8d Acquisition of Satellite1 signal using the correlator for acquiring Satellite1 after removing interfering Satellite signals for scenario d.
Fig. 4.8e Acquisition of Satellite1 signal using the correlator for acquiring Satellite1 after removing interfering Satellite signals for scenario e.
Fig. 4.8f Acquisition of Satellite1 signal using the correlator for acquiring Satellite1 after removing interfering Satellite signals for scenario f.
4.5 Comparing the Effectiveness of Oblique Projection Operators $E_{SH_o}$ and Orthogonal Projection $E_S$ in estimation of interfering satellite Signals

4.5.1 The use of the Orthogonal Projection Operator in the Subtractive Orthogonal Projection Method

The Subtractive Orthogonal Projection Method was introduced in Chapter 3 as a subspace method for CCI mitigation. Here an Orthogonal projection operator $E_S$ is used in the estimation of the sum of strong signal interference. A mathematical description of the method follows.

After acquiring the strong interfering satellite signals, the Code Phase and Doppler shift information are used in constructing a matrix $S$ with its columns being the unit magnitude modes of the samples of the acquired strong interfering satellite signals. Since the columns of $S$ are linearly independent, they can be used as the basis for the strong interfering satellite signal subspace $\langle S \rangle$. Therefore from Equation 4.3 the orthogonal projection operator, $E_S$, onto the subspace $\langle S \rangle$ is defined by

$$E_S = S(S^\dagger S)^{-1}S^\dagger$$  \hspace{1cm} (4.9)

Starting from the linear model given in Equation 4.5, where vectors $x$ and $b$ are the samples of the weak signal to be acquired and the sum of strong interfering signals respectively, the orthogonal projection of the received signal onto $\langle S \rangle$ is given by

$$E_S \cdot y = \hat{y}_S = E_S \cdot x + E_S \cdot b + E_S \cdot v = E_S \cdot H\theta + E_S \cdot S\phi + E_S \cdot v$$
\[ \hat{y}_s = \hat{x}_s + S \hat{\theta} + \hat{\nu}_s = b + \hat{x}_s + \hat{\nu}_s \approx b + \hat{\nu}_s. \]  

(4.10)

\( \hat{x}_s \) and \( \hat{\nu}_s \) are the orthogonal projection of the weak signal and noise respectively onto the strong interfering signal subspace \( \langle S \rangle \). Since the magnitude of the weak signal, \( \theta \), is small compared to that of the strong interfering signals, \( \hat{y}_s \) can be used as an estimation of the sum of strong interfering signals which is subtracted from the received signal leaving only the weak signal plus a residual vector.

\[ y - \hat{y}_s = x + b + \nu - (b + \hat{\nu}_s) \]

\[ = x + \hat{\nu}_d \]  

(4.11)

Following Equation 4.11, the weak signal can then be acquired by the correlation process.

In contrast with the Practical Oblique projection Method discussed in the previous section which uses the Oblique projection operator, \( E_{SH_0} \), for estimating the sum of strong interfering signals, the Subtractive Orthogonal Projection method uses the Orthogonal projection operator, \( E_S \), for this purpose. In the process of estimating the sum of strong interfering signals, the extent to which the projection operators \( E_{SH_0} \) and \( E_S \) cancel out the weak signal being acquired is different. Both the Subtractive Orthogonal Projection Method and the Practical Oblique projection method rely on the low magnitude of the weak signal being acquired when estimating the sum of the strong interfering signals. However, the difference is in the magnitudes of \( \hat{x}_S \) (the oblique projection of the weak signal, \( x \), onto \( \langle S \rangle \)) and \( \hat{x}_S \) (the orthogonal projection of the weak signal, \( x \), onto \( \langle S \rangle \)) as seen in Equations 4.7 and 4.10. The smaller \( \hat{x}_S \) or \( \hat{x}_S \) is, the better the effectiveness of \( E_{SH_0} \) or \( E_S \) respectively. In the next section, we see by simulation that \( E_{SH_0} \) is more effective in estimating the sum of the strong interfering signals than \( E_S \) and hence \( \hat{x}_S \) is always smaller than \( \hat{x}_S \).
4.5.2 **Comparing the Oblique Projection Operators** $E_{SH_o}$ **and Orthogonal Projection** $E_S$ **in estimation of Strong Interfering Satellite Signals**

The orthogonal projection operator $E_S$ has a range space $\langle S \rangle$ spanned by the column vector $S$, the unit magnitude modes of the samples of the strong interfering satellite signals with the estimated Code phase and Doppler Shift information. From the sixth property of oblique projection matrices given in Section 4.1.2, the null space of $E_S$ is $\langle S \rangle^\perp$ which is orthogonal to the range space $\langle S \rangle$.

Considering the structure of the GPS signals, the individual satellite signals are not perfectly orthogonal to each other and therefore the subspace spanned by each satellite signal is inclined at an angle to each other [42]. Hence the weak signal subspace is inclined at an angle to the strong interference subspace $\langle S \rangle$, i.e., they are not orthogonal to each other. Therefore subtracting the sum of strong interfering satellite signals estimated by Orthogonal Projection operator $E_S$ from the received signal would also subtract a larger portion of the desired weak satellite signal than the case where the Oblique Projection operator $E_{SH_o}$ is used. For this reason, the oblique projection operator, $E_{SH_o}$, performs better in estimating and cancelling out the strong signal interference than the orthogonal projection operator $E_S$. Fig. 4.9 compares the performance of both methods for CCI mitigation as both methods are used in removing the interfering satellite signals from scenarios $a$ to $f$ as described in Section 4.4.3 before acquisition. For the same reasons as discussed before, scenarios $a$, $b$ and $c$ have received signal simulated without noise while the received signals in scenarios $d$, $e$ and $f$ are simulated to contain AWGN at $N_o$ of $-229.18$ dBW/Hz.
Oblique Projector (Scenario a)  Orthogonal Projector (Scenario a)

Oblique Projector (Scenario b)  Orthogonal Projector (Scenario b)
Fig. 4.9 Code Phase plots and Doppler Frequency plots comparing the effectiveness of oblique projection operator $E_{SH_0}$ and orthogonal projection operator $E_S$ in mitigation of CCI during Acquisition
As observed from Fig. 4.9, in all the three scenarios a to f, though the acquired Code phase and Doppler shifts are correct for both methods, the cross correlation levels are higher for the cases where the Orthogonal projection operator $E_S$ than the cases where the Oblique projection operator $E_{SH_o}$ was used. This means, the Oblique projection operator $E_{SH_o}$ performs better in estimating and cancelling the interfering signals than the Orthogonal projection operator $E_S$.

4.6 Effects of Parameter Deviation

In practice, the Acquisition and Tracking of the strong interfering signal parameters may contain errors in the estimated parameters of the true Code Phase and Doppler Shifts of the strong interfering signals. Such estimation errors are mainly due to, in addition to noise, factors such as time discretization (sampling) of the received signal code phase and the discretization into frequency bins for the range of the tested Doppler Shifts. While the former directly affects the granularity of the estimated Code Phase, the latter affects the granularity of the estimated Doppler Shift.

It has been mentioned that, the formation of the Oblique Projection Operator $E_{SH_o}$ and Orthogonal Projection Operator $E_S$ for both Subspace projection methods discussed above make use of Code Phase and Doppler Shift parameters passed down from the Acquisition and Tracking of the strong interfering satellite signal. Deviations in these parameter estimates result in errors in the estimation of the sum of strong interfering signals using the projection operators formed from these parameters.
assessed the extent to which deviations in the Doppler Shift affects the estimation of the sum of strong signal interference when the Orthogonal Projection Operator $E_S$ is used in the Subtractive Orthogonal Projection Method, by performing an orthogonal projection of a 200ms data interval, a typical data length used in weak signal acquisition. It is observed that, with even a small estimation error in Doppler Shift of 3Hz, such a long data length presents a considerable amount of error in the projection errors, when the resulting estimate of the sum of strong interfering signals is compared to the sum of strong interfering signals in the original received signal. It is also observed that, the shorter the data length, the smaller the amount of errors due to the deviation in Doppler Shift.

The following are demonstrations of the projection errors of strong interfering signals with the Oblique Projection operator $E_{SH_o}$ due to varying amount of estimation errors in Doppler Shift for a 10ms data length. The received signal composition with their strengths, Code Phase, and Doppler Shifts used for this study is given below.

Satellite 1 signal (-184dBW, 465 chips, 3kHz)
Satellite 2 signal (-160dBW, 400 chips, 5kHz)
Satellite 3 signal (-161dBW, 823 chips, 6kHz)
Satellite 4 signal (-161dBW, 90 chips, -8kHz)

The interfering signals in this case are Satellite 2, Satellite 3 and Satellite 4 signals. An Oblique Projection operator $E_{SH_o}$ is constructed with the following estimation errors in the Doppler Shift for the interfering signals;

a) 5Hz  
   b) 10Hz  
   c) 20Hz  
   d) 50Hz

  e) 55Hz  
   f) 75Hz 
   g) 85Hz  
   h) 100Hz
Fig. 4.10a shows plots of the first 100 and last 100 samples of the simulated received signal from Satellite 3 and its projections with Oblique Projection operator $E_{SH_o}$ for a 10ms data length, in order to highlight the projection errors due to each case of estimation errors for the Doppler Shift. The simulated correlator output after the sum of the strong interfering signals have been removed by the Oblique Projection operator $E_{SH_o}$ formed for each case of the Doppler Shift estimation errors are shown in Fig 4.10b. From this figure Doppler estimation errors of 85Hz and above produce wrong acquisition. Each case in Fig 4.10b was performed with simulated received signals without noise as well as with AWGN at $N_o$ of $-229.18$dBW/Hz.

The next illustrations in Fig. 4.11 show the effects of errors in the Code Phase estimation when the samples that mark the Code Phase of the acquired strong interfering signals that are used in constructing $E_{SH_o}$ are off by one or two samples. The received signal composition used here is;

*Satellite 1 signal* (-184dBW, 465 chips, 3kHz)

*Satellite 2 signal* (-160dBW, 400 chips, 5kHz)

*Satellite 3 signal* (-161dBW, 823 chips, 6kHz)

*Satellite 4 signal* (-161dBW, 90 chips, -8kHz)

where the interfering signals are Satellite 2, Satellite 3 and Satellite 4 signals. An Oblique Projection operator $E_{SH_o}$ is constructed with the following scenarios;

i) 5MHz sampled received signal with Code phase error of 1 sample for interfering signals

j) 7MHz sampled received signal with Code phase error of 1 sample for interfering signals

k) 10MHz sampled received signal with Code phase error of 1 sample for interfering signals

l) 10MHz sampled received signal with Code phase error of 2 samples for interfering signals
Fig. 4.10a Plots of the first 100 and last 100 samples received signal from Satellite3 and its projections with Oblique Projection operator $E_{Sh}$ for each case of deviation in Doppler Shift for cases a, b, c and d. Blue represents the received signal, and Red represents its Projection.
Fig. 4.10a Plots of the first 100 and last 100 samples received signal from Satellite 3 and its projections with Oblique Projection operator $E_{SHo}$ for each case of deviation in Doppler Shift for cases e, f, g and h. Blue represents the received signal, and Red represents its Projection.
**Scenarios without Noise**

a) 5Hz deviation

b) 10Hz deviation

c) 20Hz deviation

d) 50Hz deviation

**Scenarios with Noise**

a) 5Hz deviation

b) 10Hz deviation

c) 20Hz deviation

d) 50Hz deviation

Fig. 4.10b Correlator output after sum of strong interfering signals are removed by the Oblique Projection operator $E_{SH_0}$ formed with erroneous Doppler Shifts in cases a, b, c and d.
Fig. 4.10b Correlator output after sum of strong interfering signals are removed by the Oblique Projection operator $E_{S_{\text{H}_0}}$ formed with erroneous Doppler Shifts in cases e, f, g and h
Fig. 4.11a Plots of the received signal from Satellite 2 and its projections with Oblique Projection operator $E_{S_{H_o}}$ for deviation in Code Phase in cases i, j, k and l
Fig. 4.11b Correlator output after sum of strong interfering signals are removed by the Oblique Projection operator $E_{SHo}$ formed with erroneous Code Phases in cases i, j, k and l.
Fig. 4.11a shows plots of the simulated received signal from Satellite 2 and its projections with Oblique Projection operator $E_{SH_0}$ for a 10ms data length, in order to highlight the projection errors due to each case of estimation errors in the Code phase. The simulated correlator output after the sum of the strong interfering signals have been removed by the Oblique Projection operator $E_{SH_0}$ formed for each case of the Code phase estimation errors are shown in Fig 4.11b. Each case in Fig 4.11b was performed with simulated received signals without noise as well as with AWGN at $N_o$ of $-229.18$dBW/Hz. The results displayed in Fig 4.11b show that only a 1 sample deviation in the code phase of the strong interfering signals for a 10MHz sampled received signal provided the correct acquisition result for the weak satellite 1 signal. A 2 sample error in the code phase of the strong interfering signals for the same sampling frequency of 10MHz provided wrong acquisition results.

A combination of Code Phase and Doppler shift errors was tried for the same received signal composition, for the following scenarios;

m) 10MHz sampled received signal with Code phase error of 1 sample and Doppler shift error of 50Hz for each interfering signals
n) 10MHz sampled received signal with Code phase error of 1 sample and Doppler shift error of 55Hz for each interfering signals

Fig. 4.12 shows the simulated correlator output after the sum of the strong interfering signals have been removed by the Oblique Projection operator $E_{SH_0}$ formed with errors described in cases m and n. Again, the simulations were performed with simulated received signals without noise as well as with AWGN at $N_o$ of $-229.18$dBW/Hz.
Fig. 4.12 Correlator output after sum of strong interfering signals are removed by the Oblique Projection operator $E_{SH_0}$ formed with erroneous Code Phases and Doppler shifts in cases $m$ and $n$

In Fig 4.12, the error combination in case m produced the correct acquisition results for the weak signal from satellite 1. Meanwhile, with the same Code Phase error, when the Doppler Shift error for the strong interfering signals was increased to 55Hz in the case of scenario n, the acquisition result for the weak signal from satellite 1 was wrong.

From the demonstrations presented above, it can be concluded that, the Practical Oblique projection method is more tolerant towards Doppler Shift errors than Code Phase errors. It is during the acquisition stage that the impact of the Doppler shift estimation error is more
significant. From the simulations in Fig. 4.10, the Oblique Projection operator $E_{SH_o}$ performs well during the acquisition of the weak signal until the estimation error in the Doppler Shift reached 85Hz. However, since the estimation error in Doppler Shift for a steady-state tracking mode is typically within a few Hz [39], the projection errors will have negligible impact on weak signal tracking. On the other hand, the prevailing commercial GPS receivers partition the Doppler shift frequency into 500 Hz bins during acquisition. In light of the results of Fig. 4.10, such frequency bins must be made finer, to be less than 170Hz (twice 85Hz), in applying the Practical Oblique Projection method to mitigate strong interferers. Meanwhile, to deal with the problems presented by the Code Phase errors, a high sampling frequency must be used when sampling the received signal and generating receiver local replica signals.

[39] presented the Partitioned Subspace Projection (PSP), a procedure which can be used for dealing with the effects of parameter estimation error when using the Subspace Projection operators for estimation of a sum of strong signal interference. Here, the input data block is partitioned into smaller sub-blocks of length $T$. A Projection operator is formed for each sub-block using the Code Phase and Doppler Shift estimated for each sub-block in a batch based process. A projection is then obtained for the input data from each sub-block using its corresponding Projection operator. In [39], the PSP was experimented for a 200ms data interval partitioned into 10ms sub-blocks using the Orthogonal Projection Operator $E_5$. The experiment is repeated for estimation error in Doppler Shift between 0Hz to 100Hz to quantify the projection errors for strong signals of $C/N$ of -13 and -19dB when $E_5$ is used. It is observed that as the estimation error in Doppler Shift approaches 50Hz, the
projection errors for the strong signals reach a level corresponding to the input interfering signal power level which could impact the acquisition and tracking of weak signals [39]. The PSP method presented in [39] can therefore be applied in the Practical Oblique Projection method presented in this thesis.
Chapter 5

Conclusion and Recommendations

5.1 Summary

It has been noted in Chapters 1 to 3 that though there are several reported methods for Acquiring GPS signals as weak as 21dB Carrier-to-Noise Density, problems are encountered when the weak satellite signals co-exist with strong satellite signals. This presents high CCI in the output of the receiver correlator during the acquisition of the weak satellite signals, giving rise to many competing peaks and therefore increasing the probability of false detection. It was established in Chapter 3 that a particular satellite signal is considered weak if it co-exists with another satellite signal which is 24dB stronger. This could drop to as low as 21dB due to the Doppler Effect on the C/A code chipping rate. The situations where weak satellite signals co-exist with strong satellite signals arise from scenarios such as cloudiness, presence of trees, forest and buildings especially in the urban environments, which present varying attenuation levels for different visible satellite signals making some of the visible signals weaker as compared to other visible signals.

Though methods like Successive Interference cancellation, Parallel Interference Cancellation and the Delayed Parallel Interference Cancellation methods have been used for CCI mitigation in GPS, the difficulty in these methods is the continuous monitoring and adjustment of the amplitudes of the strong interfering satellite signals during their reconstruction because of amplitude fluctuation due to fading and multipath. Subspace
projection methods present a special advantage over these methods since there is no need for tracking the amplitudes of the strong interfering satellite signals.

In Chapter 4, a Practical Oblique Projection method, which is a Subspace projection method, was developed to deal with the CCI problem. The Oblique Projection operator, $E_{SH_o}$, developed for this method was compared to the Orthogonal Projection operator $E_S$ used in the Subtractive Orthogonal projection method which is an existing Subspace projection method. Simulations demonstrated in Fig. 4.9 of Chapter 4 showed that $E_{SH_o}$ is more effective than $E_S$ in cancelling the sum of the strong interfering satellite signals, therefore producing better results for the acquisition of the simulated weak signals in the presence of other strong interfering signals.

### 5.2 Suggestions for Future Research

#### 5.2.1 Methods for Dealing with the Effects of Parameter Estimation Errors

Though the presentation in [39] focuses on the Subtractive Orthogonal Projection Method, the PSP method can also be applied in dealing with the effects of parameter estimation errors when the Practical Oblique Projection operator $E_{SH_o}$ is used. In future continuation of the work presented in this thesis, the projection errors when $E_{SH_o}$ is used should be compared to cases when the Orthogonal Projection matrix $E_S$ is used in estimating a sum of strong signal interference. Also, the performance of the PSP method presented in [39] should be assessed when the Oblique Projection operator $E_{SH_o}$ is used in estimating a sum
of strong signal interference. The experiments in [39] only focused on applying the PSP method in dealing with effects presented by Doppler Shifts errors only. Future experimentation with the PSP method should assess its effectiveness in dealing with estimation errors in both Code Phase and Doppler shift parameters for both the Practical Oblique Projection operator $E_{SH_o}$ and Orthogonal Projection matrix $E_S$.

5.2.2 Effects of Projection Operators on Received GPS Signal Noise Components

From Equations 4.7 and 4.10 in Chapter 4, $\tilde{v}_S$ and $\hat{v}_S$ are the projections of the noise component, $v$, by the projection operators $E_{SH_o}$ and $E_S$ respectively onto the strong interfering signal subspace $\langle S \rangle$. $\tilde{v}_S$ and $\hat{v}_S$ contribute to the estimates of the sum of strong interfering signals by the projection operators $E_{SH_o}$ and $E_S$. Therefore the effect of these projection operators on the component, $v$, and the extent to which it affects the correlator output when these subspace projection methods are used in acquiring weak signals should be studied in future continuation of this research.

5.2.3 The Use of Real GPS signal data and Simulation of Weak Signal Acquisition Methods

Because of the lack of logistics and time, the signals used in this work were generated from simulations using MATLAB. A continuation of this work should be set up such that real GPS signal data collected under different environments can be captured with a range of conditions for the satellites under view.
Finally, weak signal acquisition methods such as the half-bit method and the mixed coherent/non-coherent integration method for $M > 1$ should be used in future work to test the effectiveness of the proposed oblique projection method under a realistic weak signal under strong CCI condition.
References


