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Three Essays in Finance

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Three Essays in Finance

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Abstract

This dissertation consists of three loosely related essays. In Essay I, I study the relationship between firm specific risk and return. In Essay II, I study the managerial and investor short-termism. And in Essay III, I study investors heterogeneous preference for skewness and its effect on the idiosyncratic volatility puzzle.

Essay I: A spurious positive relation between EGARCH estimates of expected month $t$ idiosyncratic volatility and month $t$ stock returns arises when the month $t$ return is included in estimation of model parameters. We illustrate via simulations that this look-ahead bias is problematic for empirically observed degrees of stock return skewness and typical monthly return time series lengths. Moreover, the empirical idiosyncratic risk-return relation becomes negligible when expected month $t$ idiosyncratic volatility is estimated using returns only up to month $t-1$.

Essay II: The paper considers a model in which (1) managers allocate effort to both short- and long-term projects, and (2) there is feedback between the managerial incentive contract and the number of speculators collecting information on each type of project. More weight placed on near-term price results in more speculation based on information about the short-term project, which induces further increases in the weight placed on near-term price. This feedback effect can result in short-term speculation crowding out the collection of long-term information, which in turn results in the withdrawal of incentives aimed at inducing effort in more profitable long-term projects. The paper shows that the equilibrium that obtains depends upon adjustment costs and initial conditions and is, in general, not efficient. Such outcomes are consistent with concerns about managerial and investor short-termism recently expressed by policy makers and market participants (e.g., the Aspen Institute). The paper considers the efficacy of various corporate and public policy remedies.

Essay III: Consistent with models that incorporate investors heterogeneous preference for skewness, I show that (1) high skewness stocks are primarily held by investors with the
strongest affinity for lottery-like payoff, (2) the negative skewness-return relation is the strongest for those stocks primarily held by agents with the strongest affinity for lottery-like payoff, (3) the idiosyncratic volatility-return relation is the strongest for those stocks held by agents with the strongest affinity for lottery-like payoff, and (4) investors heterogeneous preference for skewness help explain the idiosyncratic volatility puzzle. Taken together, the results provide evidence for the importance of investors heterogeneous preference for skewness in asset pricing and its implication on the idiosyncratic volatility puzzle.
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Essay I:
On the Relation between EGARCH Idiosyncratic Volatility and Expected Stock Returns §

ABSTRACT

A spurious positive relation between EGARCH estimates of expected month $t$ idiosyncratic volatility and month $t$ stock returns arises when the month $t$ return is included in estimation of model parameters. We illustrate via simulations that this look-ahead bias is problematic for empirically observed degrees of stock return skewness and typical monthly return time series lengths. Moreover, the empirical idiosyncratic risk-return relation becomes negligible when expected month $t$ idiosyncratic volatility is estimated using returns only up to month $t - 1$.

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I. Introduction

A positive tradeoff between systematic risk and return is the cornerstone of standard rational expectations asset pricing models with a representative agent. However, financial economists have long recognized that idiosyncratic risk is potentially an important determinant of expected stock returns because the portfolio held by a typical U.S. household can only loosely be characterized as diversified (see, e.g., Blume and Friend (1975) and Goetzmann and Kumar (2008)). In particular, many authors, e.g., Levy (1978), Merton (1987), and Malkiel and Xu (2002), argue that these investors with their poorly diversified portfolios require extra compensation for holding stocks that expose them to greater idiosyncratic volatility. The empirical evidence for this prediction, however, has been elusive. In fact, Ang, Hodrick, Xing, and Zhang (2006, 2009; hereafter AHXZ) report that the cross-sectional relation between lagged realized idiosyncratic risk and returns is negative. In contrast, Fu (2009) has uncovered a strong positive relation between conditional idiosyncratic volatility estimated using EGARCH models and expected stock returns. Fu’s (2009) results are theoretically appealing as well. Consistent with Merton’s (1987) conjecture, Fu (2009) documents a positive relation between market capitalization and conditional stock returns when controlling for EGARCH idiosyncratic volatility in cross-sectional regressions. Because of its potentially significant contribution to the idiosyncratic risk literature, the EGARCH approach has been widely adopted in related empirical studies (e.g., Spiegel and Wang (2005) and Hwang, Liu, Rhee, and Zhang (2010)).

We must alert researchers, however, that they need to be especially careful when implementing the EGARCH idiosyncratic volatility methodology. A common estimation strategy accidentally introduces a look-ahead bias into recursive volatility forecasts by including the month $t$ return in the estimation of EGARCH parameters that are used to construct the expected month $t$ idiosyncratic volatility. We show analytically that the in-sample EGARCH idiosyncratic volatility can have a strong dependence on the contemporaneous stock return in relatively small samples event though the expected month
$t$ idiosyncratic volatility depends on volatility data only through $t - 1$. In particular, when we include the month $t$ return in the estimation of EGARCH model parameters, the month $t$ EGARCH idiosyncratic volatility has an upward bias when the month $t$ return is large in magnitude.\footnote{This result is quite intuitive. We estimate EGARCH models using the maximum likelihood method. An extreme return in month $t$ leads to a particularly low likelihood for this observation. To improve the likelihood of all observations, the month $t$ conditional volatility should increase.} This bias correlates positively with the month $t$ return if the latter is positive, and the correlation is negative if the latter is negative. Significantly, the cross-section of stock returns is positively skewed (e.g., Duffee (1995)), i.e., there are more stocks with extreme positive returns than stocks with extreme negative returns. Thus, the positive intertemporal correlation between the bias in in-sample EGARCH idiosyncratic volatility and one-period-ahead stock returns dominates in stock return data. As a result, the look-ahead bias may generate a spurious predictability of cross-sectional stock returns.

Of course, just because it exists, that does not necessarily imply that the look-ahead bias is so large that it affects statistical inference. This is especially true considering that the ‘look ahead’ in this case consists of a single monthly return. Therefore, we conduct Monte Carlo simulations to evaluate the impact of the look-ahead bias. The simulation results show that at skewness levels similar to, or even smaller than, those exhibited by monthly CRSP data the look-ahead bias is significant. In addition, the bias is monotonically increasing in skewness. Moreover, despite the fact that the simulations show that the bias is monotonically decreasing in the length of the return series used in estimation, the bias is still significant for return series equivalent to the entire length of the monthly CRSP stock return series.

Having established analytically and via simulations the existence of a potentially significant look-ahead bias in in-sample EGARCH idiosyncratic volatility estimates, we turn to the empirical relation between idiosyncratic risk and the cross-section of stock returns. In one of the best known papers in this literature, Fu (2009) reports three major findings. First, idiosyncratic risk and returns are positively related in the cross-section. Second, after controlling for idiosyncratic risk, there is a positive size effect. Third, he argues that
EGARCH idiosyncratic volatility is a better measure of conditional idiosyncratic volatility than the lagged realized idiosyncratic volatility used in AHXZ (2006). We will show that these results are driven by the look-ahead bias introduced by incorporating the month $t$ return into the estimate of the month $t$ EGARCH idiosyncratic volatility.

We should point out that although we will show that the positive EGARCH idiosyncratic risk-return relation reflects a look-ahead bias, this does not mean that our paper represents an unqualified confirmation of AHXZ (2006). It may well be that the true relationship between idiosyncratic risk and the cross-section of stock returns is positive, as Fu (2009) maintains (or even zero as implied by traditional asset pricing models); but, this particular evidence is unreliable due to the look-ahead bias. Therefore, it is premature to conclude that there is strong evidence of a positive relation between idiosyncratic volatility and returns.

To ensure that our results are directly comparable to those reported in existing studies, we use Fu’s (2009) monthly estimates of EGARCH idiosyncratic volatility obtained through his website. As a baseline we replicate the result that idiosyncratic risk is positively related to returns. We then show that this is due to the look-ahead bias in two ways. First, we document a strong positive cross-sectional relation between our proxies for the look-ahead bias (e.g., unexpected changes in in-sample EGARCH idiosyncratic volatility) and expected returns. Moreover, including unexpected changes in in-sample EGARCH idiosyncratic volatility in cross-sectional regressions substantially attenuates the explanatory power of the level of in-sample EGARCH idiosyncratic volatility. This result is especially strong when we use log returns instead of simple returns as the dependent variable because log returns have a smaller skewness than do simple returns and, thus, should reduce the magnitude of the look-ahead bias.

---

2Bali and Cakici (2008) argue that the AHXZ (2006) result is sensitive to different weighting schemes and the estimation of idiosyncratic risk with daily versus monthly return data. Huang, Liu, Rhee, and Zhang (2010) suggest that it relates to the shorthorizon return reversal anomaly. Bali, Cakici, and Whitelaw (2011) find that the negative effect of idiosyncratic volatility is driven by its close relation with the maximum daily return in a month, proxying for demand for lotterylike stocks. Jiang, Xu, and Yao (2009) hypothesize that firms with high price volatility tend to be opaque in their earnings disclosures. Han and Lesmond (2011) argue that microstructure noise factors have substantial effects on the realized variance measure.
The second way that we show that the positive EGARCH idiosyncratic risk and return relation is due to the look-ahead bias is more direct. We simply replace the in-sample EGARCH idiosyncratic risk estimates with our own truly out-of-sample forecast of EGARCH idiosyncratic volatility. In particular, to obtain month $t$ conditional idiosyncratic volatility, we estimate EGARCH model parameters using stock return data up to month $t - 1$. We find that while out-of-sample EGARCH idiosyncratic volatility has strong predictive power for one-month-ahead realized idiosyncratic volatility, it does not forecast cross-sectional stock returns.$^3$ That is, the positive relation between idiosyncratic risk and returns goes away when we estimate the risk without the look-ahead bias.

In the CRSP data, small capitalization stocks tend to have higher expected returns than large capitalization stocks. Fu (2009), however, shows that this size effect becomes significantly positive after controlling for EGARCH idiosyncratic volatility in cross-sectional regressions. He highlights this finding as direct support for Merton (1987). As with the idiosyncratic risk and return result, we initially illustrate the nature of the bias in in-sample estimates with his own data. We first replicate the result and then we show that the positive effect of market capitalization on expected returns disappears when we control for the look-ahead bias in cross-sectional regressions.$^4$ When we control for out-of-sample idiosyncratic risk we find the traditional size effect. Namely, size is significantly, negatively related to expected returns. Thus, as with the positive idiosyncratic risk and return relationship, we find that when we employ out-of-sample idiosyncratic risk estimates the reported results

$^3$Bali, Scherbina, and Tang (2010) have independently verified our main finding that out-of-sample EGARCH idiosyncratic volatility estimates have negligible predictive power for the cross-section of stock returns. After the first draft of this paper was circulated, Fink, Fink, and He (2012) confirmed the weak relation between out-of-sample EGARCH idiosyncratic volatility and the cross-section of stock returns.

$^4$In Fu’s (2009) data, the difference between months $t$ and $t - 1$ EGARCH idiosyncratic volatilities is a proxy for the look-head bias. Specifically, when including both variables in the cross-sectional regression, we show that the former correlates positively with month $t$ stock returns, while the relation is negative for the latter. There is a strong negative correlation of month $t - 1$ EGARCH idiosyncratic volatility with month $t - 1$ market capitalization. Therefore, a positive relation between month $t$ stock returns is found in conjunction with month $t$ EGARCH idiosyncratic volatility because the former serves as an instrumental variable for month $t - 1$ EGARCH idiosyncratic volatility. As expected, the positive size effect goes away when we control for the (unexpected) change in EGARCH idiosyncratic volatility as a proxy for the look-ahead bias.
AHXZ (2009) show that lagged realized idiosyncratic volatility has strong explanatory power for one-month-ahead realized idiosyncratic volatility. Fu (2009) suggests that his findings differ qualitatively from AHXZ (2006) because EGARCH idiosyncratic volatility is a better measure of conditional idiosyncratic volatility than is lagged realized idiosyncratic volatility, for example, as used in AHXZ (2006). We corroborate the AHXZ (2009) finding by showing that the explanatory power of lagged realized idiosyncratic volatility remains statistically significant after controlling for in-sample EGARCH idiosyncratic volatility. In addition, it remains statistically significant after controlling for our out-of-sample EGARCH idiosyncratic volatility estimates. Therefore, lagged realized idiosyncratic volatility provides important information about one-month-ahead realized idiosyncratic volatility beyond EGARCH idiosyncratic volatility. These results cast doubt on the argument that the difference between Fu (2009) and AHXZ’s (2006) findings reflects mainly the fact that EGARCH idiosyncratic volatility is a better measure of conditional idiosyncratic volatility than is lagged realized idiosyncratic volatility.

The remainder of the paper proceeds as follows. In Section II, we discuss the look-ahead bias introduced by including the month $t$ return in the estimation of month $t$ EGARCH model parameters. We illustrate the significance of the bias on inference by conducting Monte Carlo simulations. In Section III, we show that the positive cross-sectional relation between EGARCH idiosyncratic volatility and expected stock returns reflects mainly this look-ahead bias. We then show that truly out-of-sample EGARCH idiosyncratic volatility has negligible explanatory power for the cross-section of stock returns. We also revisit Merton’s (1987) conjecture that after controlling for idiosyncratic volatility the size effect is positive. In Section IV we conduct robustness tests. In Section V, we discuss the look-ahead bias in EGARCH idiosyncratic volatility estimated using the full sample. In Section VI, we offer some concluding remarks.
II. Look-ahead Bias in In-Sample EGARCH Idiosyncratic Volatility

A. EGARCH Models

Many authors estimate idiosyncratic risk using the Fama and French (1996) three factors as proxies for systematic risk,

\[ R_{i,t} - r_{f,t} = \alpha_i + \beta_i (R_{m,t} - r_{f,t}) + s_i SMB_t + h_t HML_t + \varepsilon_{i,t} \]  

(1)

where \( R_{i,t} \) is stock \( i \)'s return, \( r_{f,t} \) is the risk-free rate, \( R_{m,t} - r_{f,t}, SMB_t \) and \( HML_t \) are the excess market return, the size premium, and the value premium, respectively, as in the Fama and French (1996) three-factor model. The idiosyncratic return, \( \varepsilon_{i,t} \), is assumed to have a serially independent normal distribution

\[ \varepsilon_{i,t} \sim N(0, \sigma_{i,t}^2) \]  

(2)

and its conditional variance, \( \sigma_{i,t}^2 \), follows an EGARCH process

\[ \ln \sigma_{i,t}^2 = a_i + \sum_{l=1}^{p} b_i \ln \sigma_{i,t-l}^2 + \sum_{k=1}^{q} c_{i,k} \left\{ \theta \left( \frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}} \right) + \gamma \left[ \frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}} \right] - (2/\pi)^{1/2} \right\} \]  

(3)

Under these assumptions, the log likelihood of the month \( t \) return, \( R_{i,t} \), is

\[ L(R_{i,t}) = -\frac{1}{2} \log(2\pi) - \frac{1}{2} \log(\sigma_{i,t}^2) - \frac{\varepsilon_{i,t}^2}{2\sigma_{i,t}^2} \]  

(4)

Researchers commonly use the maximum likelihood (or quasi-maximum likelihood if the error term in equation (1) has a nonnormal distribution) method to estimate EGARCH model parameters. That is, the parameter values in equations (1)-(3) are selected to maximize the sum of the log likelihood of stock returns in a given sample period.
B. The Problem with In-Sample Estimates of EGARCH Idiosyncratic Volatility

Many authors rely on the maximum likelihood method to estimate EGARCH model parameters. They refer to the resulting recursive EGARCH idiosyncratic volatility estimates as out-of-sample forecasts. Because equation (3) shows that the month $t$ EGARCH idiosyncratic volatility, $\sigma_{i,t}^2$, depends on its own lags and lagged idiosyncratic returns, it is plausible that these estimates do in fact provide an out-of-sample forecast of the month $t$ EGARCH idiosyncratic volatility. However, it is important to note that, if specified inappropriately, this estimation strategy can actually result in an in-sample estimate of EGARCH idiosyncratic volatility. Specifically, if we set the sample period to be from month 1 to month $t$, we will include the month $t$ stock return in the calculation of the sum of the log likelihood

$$\sum_{\tau=1}^{t} L(R_{i,\tau}) = -\frac{t}{2} \log(2\pi) - \frac{1}{2} \sum_{\tau=1}^{t} \log(\sigma_{i,\tau}^2) - \sum_{\tau=1}^{t} \frac{\varepsilon_{i,t}^2}{2\sigma_{i,\tau}^2}$$

(5)

Typically, the next step would be to estimate the EGARCH model by choosing values of the parameters in equations (1)-(3) to maximize the sum of the log likelihood of returns over the period from month 1 to month $t$ in equation (5). The problem with this approach is that, via equation (5), the parameter estimates depend (to an asymptotically vanishing degree) on the month $t$ return. In particular, the conditional month $t$ EGARCH idiosyncratic volatility, $E(IVOL_t)$, has a look-ahead bias because it depends on EGARCH model parameters that are estimated using the month $t$ return

$$E(IVOL_t) = \exp \left[ \ln \sigma_{i,t}^2 \right] = \exp \left[ a_{i,t} + \sum_{l=1}^{p} b_{i,l} \ln \sigma_{i,t-l}^2 + \sum_{k=1}^{q} c_{i,k} \left\{ \theta \left( \frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}} \right) + \gamma \left[ \frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}} \right] - \left( \frac{2}{\pi} \right)^{1/2} \right\} \right]$$

(6)

In equation (6), we use the subscript $t$ on the EGARCH model parameter estimates, $a_{i,t}, b_{i,l,t}$...
and \(c_{i,k,t}\) to highlight their dependence on the month \(t\) return. Because of its inclusion of the information from month \(t\), the conditional idiosyncratic volatility in equation (6), \(E(IVOL_t)\), is actually an in-sample EGARCH idiosyncratic volatility estimate. To obtain one-month-ahead forecasts of EGARCH idiosyncratic volatility that are truly out-of-sample, we must restrict equation (5) to include only the returns up to month \(t-1\) in the calculation of the sum of the log likelihood,

\[
\sum_{\tau=1}^{t-1} L(R_{i,\tau}) = -\frac{t-1}{2} \log(2\pi) - \frac{1}{2} \sum_{\tau=1}^{t-1} \log(\sigma_{i,\tau}^2) - \sum_{\tau=1}^{t-1} \frac{\varepsilon_{i,\tau}^2}{2\sigma_{i,\tau}^2} \tag{7}
\]

Obviously equation (7) is obtained from equation (5) by excluding the month \(t\) return. Then, as is standard, we can estimate an EGARCH model by searching for values of the parameters in equations (1)-(3) that maximize the sum of the log likelihood in equation (7). We then substitute these parameter estimates into equation (3) to obtain the out-of-sample forecast of month \(t\) EGARCH idiosyncratic volatility, \(E(IVOL_{O_t})\),

\[
E(IVOL_{O_t}) = \exp\left[\ln \sigma_{i,t}^2\right] = \exp\left[a_{i,t-1} + \sum_{l=1}^{p} b_{i,l,t-1} \ln \sigma_{i,t-1}^2 + \sum_{k=1}^{q} c_{i,k,t-1} \left\{ \theta \left( \frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}} \right) + \gamma \left[ \left( \frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}} \right)^2 - \left( \frac{2}{\pi} \right)^{1/2} \right] \right\}\right] \tag{8}
\]

In equation (8), we use the subscript \(t-1\) on the parameter estimates, \(a_{i,t-1}, b_{i,l,t-1}\) and \(c_{i,k,t-1}\) and to emphasize the fact that we obtain them using information available at month \(t-1\).

The month \(t\) in-sample EGARCH idiosyncratic volatility from equation (6) has a look-ahead bias because it depends on the month \(t\) return. This relation is quite intuitive. Suppose that there is an extreme return in month \(t\) due to an extreme month \(t\) idiosyncratic return, \(\varepsilon_{i,t}\). As a result, equation (4) shows that, ceteris paribus, the log likelihood of the month \(t\) return is likely to be particularly low. One way to improve the in-sample fit, as illustrated in equation (4), is to raise the month \(t\) idiosyncratic volatility, \(\sigma_{i,t}^2\), by, for example, increasing...
the constant term, $a_{i,t}$, in conditional volatility of equation (6). That is, the look-ahead bias in the in-sample month $t$ EGARCH idiosyncratic volatility correlates positively with the magnitude of the month $t$ return.

The magnitude of the look-ahead bias depends on the length of the return series used. For example, if we have a large number of stock return observations, the estimates of the EGARCH model parameters in equations (1)-(3) converge asymptotically to their population values. Intuitively, when we have millions of return observations, an extreme return in month $t$ should have a negligible effect on the sum of the log likelihood in equation (5); therefore, the EGARCH parameter estimates do not have to change much to accommodate this extreme observation. In this case, the look-ahead bias in in-sample EGARCH idiosyncratic volatility converges asymptotically to zero. On the other hand, if the number of observations is relatively small, the look-ahead bias can have a substantial impact on the in-sample estimate of EGARCH idiosyncratic volatility. What qualifies as ‘relatively small’ is an empirical issue that we will investigate in Monte Carlo simulations in sub-section II.D and in an examination of Fu’s (2009) major findings in section III.

At this point, the reader may be persuaded that the distortion from including month $t$ returns imparts noise to our volatility estimates but still question whether there is any bias. To see that there often will be, note that when the stock return, $R_{i,t}^+$, is positive, it correlates positively with the bias of its in-sample EGARCH idiosyncratic volatility, $EIVOL_{i,t}^B$

$$R_{i,t}^+ = \alpha EIVOL_{i,t}^B \tag{9}$$

where $\alpha$ is a positive parameter. Likewise, when the stock return, $R_{j,t}^-$, is negative, it correlates negatively with the bias of its in-sample EGARCH idiosyncratic volatility.

---

5 Consider equation (4). When conditional idiosyncratic volatility increases, the second term on the right-hand-side (RHS) implies that the log likelihood decreases with $\log(\sigma)$ while the third RHS term implies that the log likelihood increases with $\sigma^2$. The more extreme the return, the more the latter dominates the former; hence, the log likelihood increases with conditional idiosyncratic volatility.
If cross-sectional stock returns are symmetrically distributed, then the positive relation in equation (9) and the negative relation in equation (10) should approximately cancel each other out in the cross-sectional regression and our idiosyncratic volatility estimates will be noisy but unbiased. However, untabulated results show that cross-sectional stock returns are not symmetrically distributed; but, rather have a strong positive realized skewness in most months, indicating that there are substantially more stocks with extreme positive returns than stocks with extreme negative returns (see also Duffee (1995)). Thus, for the empirical distribution, the positive relation in equation (9) will dominate the negative relation in equation (10) in cross-sectional regressions. That is, there will be a look-ahead bias; a positive cross-sectional relation between in-sample EGARCH idiosyncratic volatility and expected stock returns.

In sub-section II.D, we illustrate the dependence of the EGARCH parameter estimates in equation (6) on the month $t$ return via Monte Carlo simulations. They verify the existence of a look-ahead bias that is due to a dependence of the month $t$ in-sample EGARCH idiosyncratic volatility on the month $t$ return. As the foregoing discussion suggests, the bias is increasing in the skewness of the cross-section of returns and is decreasing in the length of the return series.

\[ R_{j,t}^- = -\alpha EIVOL_{j,t}^B. \] (10)

---

$^6$The fact that the EGARCH methodology could not truly be making \textit{ex ante} predictions is illustrated by the results of a preliminary experiment of forecasting market returns. Specifically, when we aggregate equations (9) and (10) across all stocks, the equal-weighted market return should correlate positively with the average look-ahead bias if stock returns are positively skewed. As conjectured, we find that changes in average monthly EGARCH idiosyncratic volatility, a proxy for the look-ahead bias, forecast one-month-ahead market returns. A simple switching strategy between a market index and a risk-free Treasury bond based on the information content of EGARCH idiosyncratic volatility generates a Sharp ratio twice as high as buying-and-holding the market index.
C. Estimating Out-of-Sample EGARCH Idiosyncratic Volatility

We use SAS to construct the one-month-ahead out-of-sample forecast of month \( t \) EGARCH idiosyncratic volatility. That is, we set the sample over the period from month 1 to month \( t - 1 \) when estimating EGARCH model parameters. We then substitute the parameter estimates into equation (8) to calculate the month \( t \) EGARCH idiosyncratic volatility.\(^7\) As in Spiegel and Wang (2005), we require at least sixty monthly return observations to estimate EGARCH models. In contrast, other authors, e.g., Fu (2009), utilizes only thirty monthly return observations. We adopt Spiegel and Wang’s (2005) specification because many authors, e.g., Scruggs (1998) and Lundblad (2007), emphasize the need for a large number of observations to obtain precise parameter estimates of GARCH-type nonlinear models. We do not impose a larger minimum because we want our estimates to be comparable to Fu (2009). To further alleviate concerns about the small sample bias, we use an expanding sample starting from July 1926 in the recursive estimations.\(^8\) As in Fu (2009), we consider nine EGARCH specifications, i.e., EGARCH(p,q), where \( 1 \leq p \leq 3 \) and \( 1 \leq q \leq 3 \), and choose the one that converges with the lowest Akaike Information Criterion.

The minimum requirement of sixty monthly return observations is at best a partial solution to the small sample problem. For example, the EGARCH(3,3) model has over ten parameters, and we would not expect to obtain a sensible estimation of these parameters using only sixty observations. Due to the small sample sizes involved, the EGARCH estimates in these studies can be quite sensitive to tuning parameters such as the initial parameter values, the number of iterations, and the convergence criteria. These technical issues highlight the potentially serious problems associated with using EGARCH idiosyncratic

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\(^7\)In an alternative approach, we set the sample over the period from month 1 to month \( t \) and arbitrarily set the month \( t \) return to be a missing observation. This effectively tricks SAS into computing an in-sample month \( t \) EGARCH idiosyncratic volatility that does not depend on information from month \( t \). That is, because we have intentionally set the month \( t \) return to be a missing observation, the EGARCH parameter estimates in equation (6) depend on returns up to month \( t-1 \) and thus have no look-ahead bias. We have confirmed that the EGARCH idiosyncratic volatilities obtained from these two approaches are identical. David Manzler deserves special thanks for suggesting this alternative approach to us.

\(^8\)While CRSP monthly stock return data begin in January 1926, the Fama and French three factor data, which we use as a proxy for systematic risk, are available from July 1926.
volatility estimates. The potential sensitivity of our results raises the possibility that we are accidentally generating the look-ahead bias that we detect below because we are using different convergence criteria, etc. We address this concern in three ways. First, we rely on Fu’s (2009) estimates to illustrate the look-ahead bias. Second, when generating our own estimates we select the tuning parameters in such a way that our in-sample EGARCH results closely match those reported in Fu (2009). We then use the same tuning parameters to estimate out-of-sample EGARCH idiosyncratic volatility. In this way, we ensure that the different results obtained from in-sample and out-of-sample EGARCH idiosyncratic volatility estimates reflect only the look-ahead bias. Third, in Section IV, we estimate out-of-sample EGARCH idiosyncratic volatility using a two-year rolling window of daily return data with a minimum of 252 daily returns. The EGARCH estimation is less sensitive to tuning parameters for daily return data because they allow for substantially more observations. Again, we find that the EGARCH idiosyncratic volatility has negligible predictive power for the cross-section of stock returns.

D. Look-ahead Bias: Monte Carlo Simulations

We have clearly demonstrated that incorporating the month t return into the EGARCH estimates induces a bias. On the other hand, we are talking about one observation. It is not at all obvious that one observation could have such a statistically meaningful impact on our inferences to account for the positive EGARCH idiosyncratic risk and return relation reported in the previous studies. In this sub-section we examine this very point. We employ Monte Carlo simulation to gauge the effect of the look-ahead bias on statistical inference about the relation between EGARCH idiosyncratic volatility and future stock returns. For simplicity, we generate simulated monthly return data using the EGARCH(1,1) specification. We set the unconditional volatility to 0.06, which is smaller than the median stock return volatility of 0.14 for CRSP common stocks with at least 60 monthly return observations. This conservative calibration ensures that our results are not driven mainly by extreme
returns. We set the conditional mean return to zero. Therefore, the simulated data can be interpreted as idiosyncratic returns and by construction, there is no relation between EGARCH idiosyncratic volatility and future stock returns. We generate the i.i.d. error term of the EGARCH model using Ramberg and Schmeiser’s (1974) Generalized Lambda Distribution (GLD) algorithm.\textsuperscript{9} Specifically, we set the kurtosis of the simulated error term to 3.2, the median kurtosis of CRSP stock returns. In the benchmark case, we set the skewness to 1.1, the median skewness of CRSP common stocks. In simulated data, each stock has 260 return observations, which is slightly more than the median of 230 monthly stock returns for CRSP common stocks. For illustration, we investigate cross-sectional implications using 120 stocks; more stocks should not affect our results in any qualitative manner. Overall, the benchmark case is a reasonably good proxy for the actual data.

As we noted in sub-section II.B, the look-ahead bias increases with skewness. Therefore, we will first attempt to get a feel for the degree of cross-sectional skewness required to affect our inferences by considering different parameterizations of the skewness. Table 1 reports the Fama and MacBeth (1973) estimation results of regressing stock returns on conditional EGARCH volatility obtained using simulated data. For comparison, we consider three specifications. First, under the column “Out of Sample”, we estimate the time \( t \) EGARCH volatility recursively using the information available up to time \( t - 1 \). Second, under the column “In Sample”, we estimate the time \( t \) EGARCH volatility recursively using the information available up to time \( t \). Last, under the column “Full Sample”, we estimate the EGARCH volatility using the full 260 month sample. In the first two cases, we require a minimum of 60 observations for EGARCH model estimations. Because the simulated data were generated with the EGARCH(1,1) model, we use the same specification to estimate the conditional volatility of the simulated data. We set the maximum iterations to 1,000 and adopt the default SAS convergence tolerance criterion of 0.001. In the cross-sectional regression, we include only stocks that converged in the EGARCH estimations.

\textsuperscript{9}We also used Fleishman’s (1978) power transformation and found similar results.
Panel D of Table 1 is the benchmark case, in which we set the skewness equal to 1.1. The in-sample EGARCH idiosyncratic volatility correlates positively with future stock returns, and the relation is statistically significant at the 1% level.\textsuperscript{10} Note that the presence of a relatively high degree of skewness can facilitate the spurious correlation between expected idiosyncratic volatility and returns. For example, the relation is significantly positive when the skewness is 0.8 (Panel C). For comparison the mean (median) cross-sectional skewness of CRSP common stock returns is 0.8 (1.1). Similarly, we find a positive and significant relation between the full-sample EGARCH idiosyncratic volatility and future stock returns. Moreover, as conjectured, Table 1 shows that the look-ahead bias increases monotonically with the skewness for both the in-sample and full-sample estimates. The out-of-sample results stand in sharp contrast. There is a negligible relation between out-of-sample EGARCH idiosyncratic volatility and future stock returns even for skewness as high as 1.6 (Panel E). These results indicate that, in fact, it is possible for the look-ahead bias to fully account for the positive EGARCH idiosyncratic risk and return relation reported in the literature.\textsuperscript{11}

It is a common practice to estimate specifications of the EGARCH model and choose the one that fits the data best according to the Akaike Information Criterion. This specification-selection approach generates an even stronger look-ahead bias than the approach we adopted in Table 1 by using a fixed EGARCH(1,1) specification. Intuitively, when there is an extreme return at time $t$, the specification that produces the largest time $t$ conditional volatility is most likely to be selected as the best model because it will generally have the highest likelihood. In light of this possibility, we re-run the cross-sectional regression using

\textsuperscript{10}Interestingly, the coefficient on the EGARCH idiosyncratic volatility in the simulation is 0.118, which is comparable in magnitude to 0.138 obtained using Fu’s (2009) data, as reported below in row 1 of Table 4.  
\textsuperscript{11}For example, Fu (2009) reports a skewness of 2.35 for his data of pooled CRSP common stocks with at least 30 monthly return observations.
nine specifications (mimicking Fu’s (2009) approach) and report the results in Table 2. As expected, the look-ahead bias in Table 2 is noticeably larger than its counterpart in Table 1. For example, in the benchmark case (Panel D), the coefficient on the in-sample EGARCH idiosyncratic volatility is 0.153, compared with 0.118 reported in Table 1. Moreover, a significant positive relation is present at even lower degrees of skewness; a positive relation is present for both the in-sample and full-sample estimates when skewness is 0.4. This is only half (about 1/3rd) the mean (median) cross-sectional skewness of monthly CRSP stock returns.

[Insert Table 3 here]

Lastly, in Table 3, we investigate the effect of the sample size on the look-ahead bias. We consider only the case of the full-sample EGARCH estimation because computation becomes forbiddingly intensive for the recursive in-sample EGARCH estimation when the sample size is large. We set the skewness equal to 0.8 and choose the best model from the nine EGARCH specifications. As conjectured, the look-ahead bias decreases monotonically with the length of the return series; however, it remains statistically significant even when the sample size grows to 5,000 observations. Considering that the entire history of CRSP is roughly 1000 months, this indicates that in practice the monthly return series is never long enough to eliminate the look-ahead bias. For example, in Table 3, when T=1000, the coefficient on the full-sample idiosyncratic volatility is 0.132 with a t-statistic of 6.556. For shorter series, the bias is substantially larger.

Note that, because the look-ahead bias is likely to decrease with the sample size, as we confirm in Table 3, the look-ahead bias of the full-sample EGARCH estimation should be weaker than that of the in-sample EGARCH estimation. In practice, however, we have reason to suspect that full-sample EGARCH idiosyncratic volatility estimates may be prone to even larger measured look-ahead biases than in-sample estimates. For a given length return series, say 260 months, the in-sample estimate requires an initial estimation period, say 60 months;
therefore there are only 200 months available for the cross-sectional regressions. The full-sample estimates for the same 260 month return series has 60 (or 30%) more months in the cross-sectional regression. As indicated by the simulation results reported in Table 3, the look-ahead bias should be decreasing with the length of the return series. Therefore, we would expect less look-ahead bias in the full-sample. On the other hand, in-sample EGARCH idiosyncratic volatility is estimated over far fewer months and is, hence, noisier. Thus, when we conduct the second stage regression, there will be a more serious error-in-variables problem for the in-sample estimates than for the full-sample estimates. Therefore, the second stage regression coefficients will be more downward biased (the ‘attenuation effect’) for the in-sample estimates than for the full-sample estimates. It is not clear which one of these effects should dominate; but, in the Monte Carlo simulations reported in Tables 1 and 2 we found the spuriously (yet, frequently, significantly) correlated $E[IVOL]$ coefficients were consistently greater for the full-sample estimates than for the in-sample estimates. So, it appears that the attenuation effect may offset the shorter return series available for the cross-sectional regressions when using in-sample EGARCH estimates.

III. How Important is Look-ahead Bias in Practice?

In Section II we showed analytically and via Monte Carlo simulation that it is possible for EGARCH estimates of idiosyncratic volatility to contain a significant look-ahead bias. In this section we will show that an example of this in practice is Fu’s (2009) study of the relation between idiosyncratic risk and the cross-section of stock returns. He reports three major results. First, he finds a positive cross-sectional relationship between his measure of expected idiosyncratic volatility and returns. Second, he finds a positive relation between firm size and returns (as predicted in Merton (1987)). Third, he suggests that his EGARCH measure of expected idiosyncratic volatility provides a superior forecast of future realized idiosyncratic volatility than the lagged values of realized idiosyncratic volatility advocated in AHXZ (2006).
We will first use Fu’s (2009) own estimates of idiosyncratic risk to replicate the positive idiosyncratic risk-return relation he reports. We will then show that this result is attenuated or even eliminated when we control for proxies for the look-ahead bias. More directly, we will then show that there is no relation between EGARCH idiosyncratic volatility and returns when we eliminate the look-ahead bias from the idiosyncratic risk estimates. Additionally, we will show that the positive size effect disappears when we control for the influence of the look-ahead bias. Finally, we will show that the in-sample EGARCH measure of idiosyncratic volatility is superior to lagged realized volatility is also due to the look-ahead bias.

A. In-Sample EGARCH Idiosyncratic Volatility and the Cross-Section of Stock Returns

We begin with monthly estimates of stock-level EGARCH idiosyncratic volatility over the July 1963 to December 2007 period obtained from Fangjian Fu at Singapore Management University. We denote this measure $E(IVOL)$ and will frequently refer to it as an in-sample estimate for the reasons outlined in Section II. In row 1 of Table 4, we replicate Fu’s (2009) main finding of a strong positive cross-sectional relation between and expected stock returns in the univariate Fama and MacBeth (1973) cross-sectional regression. The point estimate is 0.138 and the adjusted $R^2$ is 3%, compared with 0.11 and 3%, respectively, as reported in Fu’s (2009) Table 5.

Fu (2009) emphasizes that EGARCH idiosyncratic volatility is a good measure of conditional idiosyncratic volatility because it is quite persistent. This finding suggests that $E(IVOL_{t-1})$ should have explanatory power similar to that of $E(IVOL_t)$ for the cross-section of stock returns. This conjecture has also been proposed in a similar context by AHXZ (2006), who show that two-month lagged realized idiosyncratic volatility has explanatory power for realized idiosyncratic volatility, albeit with an adjusted $R^2$ about 2/3rds that of $E(IVOL_t)$.

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12 In row 3 of Table 7 below, we confirm this point by showing that one-month lagged EGARCH idiosyncratic volatility, $E(IVOL_{t-1})$, has strong predictive power for realized idiosyncratic volatility, albeit with an adjusted $R^2$ about 2/3rds that of $E(IVOL_t)$. 

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power for the cross-section of stock returns qualitatively similar to that of one-month lagged realized idiosyncratic volatility. Contrary to this conjecture, Table 4 reports that $E(IVOL_{t-1})$ does not correlate with expected stock returns in the univariate cross-sectional regression, with a $t$-statistic close to zero (row 2). More surprisingly, when we include both $E(IVOL_t)$ and $E(IVOL_{t-1})$ as the explanatory variables in the cross-sectional regression, the effect on expected returns remains significantly positive for the former, while it becomes negative and highly significant for the latter (row 3). Because of the strong correlation between $E(IVOL_t)$ and $E(IVOL_{t-1})$, it is tempting to believe that this result reflects a multicollinearity problem. This interpretation, however, does not account for the fact that the $t$-statistics in row 3 are substantially larger in magnitudes than are their univariate counterparts, as reported in rows 1 and 2, respectively.

The result in row 3 of Table 4 may reflect the fact that (unexpected) changes in EGARCH idiosyncratic volatility (which are shocks and, thus, a proxy for the look-ahead bias) have a strong positive correlation with one-month-ahead stock returns. To account for this possibility, we measure unexpected changes in EGARCH idiosyncratic volatility in two ways. First, we consider the difference between $E(IVOL_t)$ and $E(IVOL_{t-1})$, which we dub $\Delta E(IVOL_t)$. Row 4 of Table 4 confirms our conjecture—the first difference, $\Delta E(IVOL_t)$, has a significantly positive correlation with one-month-ahead stock returns even when we control for $E(IVOL_t)$. Because EGARCH idiosyncratic volatility is quite persistent, we also control for the difference between $E(IVOL_t)$ and $E(IVOL_{t-2})$, which we dub $\Delta_2 E(IVOL_t)$. Row 5 of Table 4 reports that $\Delta_2 E(IVOL_t)$ has strong incremental explanatory power as well. In contrast, the explanatory power of $E(IVOL_t)$ attenuates substantially after we control for its changes. For example, in row 5 of Table 4, the parameter estimate and the $t$-statistic of $E(IVOL_t)$ are 0.065 and 2.641, respectively, which are substantially smaller than are their univariate counterparts, as reported in row 1 of Table 4.

As a second proxy for the look-ahead bias, for each stock, we regress its EGARCH idiosyncratic volatility on the two lags, and use the residual from the time-series regression,
\textit{UE(IVOL_t)}, as a measure of unexpected changes in \textit{E(IVOL_t)}. In row 6 of Table 4 we see that \textit{UE(IVOL_t)} has a strong positive correlation with one-month-ahead stock returns in the cross-sectional regression even when we control for \textit{E(IVOL_t)}. Again, the explanatory power of \textit{E(IVOL_t)} attenuates substantially, as compared with the univariate regression results reported in row 1 of Table 4.

The analysis of Section II, particularly the Monte Carlo simulations, clearly demonstrated that the look-ahead bias in in-sample EGARCH idiosyncratic volatility increases with the skewness of returns. An interesting way to see this in Fu’s (2009) data is to use log returns instead of simple returns as the dependent variable in cross-sectional regressions. Naturally, using log returns will alter the distribution of returns, making it more ‘Normal’ and, thus, reduce the skewness in the data. Moreover, Asparouhova, Bessembinder, and Kalcheva (2011) show that, unlike simple returns, log returns are not subject to biases resulting from microstructure noises. These estimation results are reported in Panel B of Table 4. As expected, when using log returns, we find that the relation between \textit{E(IVOL_t)} and one-month-ahead stock returns becomes statistically insignificant in the univariate regression at conventional significance levels (row 7). When we control for unexpected changes in \textit{E(IVOL_t)} in the cross-sectional regression, the relation becomes even negative, and statistically significant in some cases, as shown in rows 10 to 12. Note that in both Panels A and B, unexpected changes in \textit{E(IVOL_t)} are always significantly positively correlated with expected stock returns. This result reflects the same underlying phenomenon; returns are positively correlated with \textit{E(IVOL_t)} estimates because the estimates are contaminated by a look-ahead bias.\footnote{As a robustness check, Fu (2009) also uses log returns in cross-sectional regressions but only tabulates the results for multivariate regressions that include both \textit{E(IVOL_t)} and market capitalization as independent variables. As we explain in sub-section III.C, this specification strengthens the look-ahead bias and generates a positive size effect.}

\textbf{B. Out-of-Sample EGARCH Idiosyncratic Volatility and the Cross-Section of Stock Returns}

[Insert Table 5 here]
We have shown that Fu’s (2009) $E(IVOL_t)$ estimates are not positively associated with expected returns when we control for a variety of measures that proxy for the look-ahead bias. We now turn to the most direct demonstration that Fu’s (2009) result is due to the look-ahead bias. Table 5 reports on the relation between out-of-sample EGARCH idiosyncratic volatility and expected stock returns. In Panel A, we report the cross-sectional regression results for the July 1963 to December 2006 period, mirroring the sample period in Fu (2009). We also consider his three empirical specifications: univariate regression (row 1); controlling for market capitalization and the book-to-market equity ratio (row 2); and, also controlling for past returns, the turnover, and the coefficient of variation of the turnover (row 3). As in Fu (2009), we use the log transformations of firm characteristics except for past stock returns. We find that out-of-sample EGARCH idiosyncratic volatility, $E(IVOL,O_t)$, has a positive correlation with expected stock returns in all three specifications; however, the correlation is always statistically insignificant at conventional significance levels. Simply put, the positive relation between idiosyncratic risk and returns goes away when we estimate the risk without the look-ahead bias.

As a robustness check, we also consider two different samples, both beginning in September 1931. Because book equity data are unavailable for this early period, we cannot control for the book-to-market equity ratio in cross-sectional regressions for these two samples. Panel B of Table 5 reports the results for the early sample spanning the September 1931 to June 1963 period. We again find a positive, albeit insignificant, relation between $E(IVOL,O_t)$ and expected stock returns. Second, Panel C reports the results for the full sample spanning the September 1931 to December 2009 period. Row 7 shows that the relation is significantly positive at the 5% level in the univariate regression. It, however, becomes statistically insignificant when we control for market capitalization (row 8). Because small stocks have higher expected returns than do big stocks partly because the former are less liquid (Amihud and Mendelson (1980)), our findings are consistent with those reported by Spiegel and Wang (2005), who document a strong positive relation between EGARCH
idiosyncratic volatility and various measures of illiquidity. Row 9 shows that $E(I VOL, O_t)$ remains statistically insignificant when we control for other commonly used determinants of expected stock returns.

C. The Positive Size Effect Revisited

In the CRSP data, there is a pervasive negative relation between market capitalization and expected stock returns. That is, small stocks tend to have higher expected returns than do big stocks. Fu (2009), however, shows that the size effect becomes significantly positive after controlling for EGARCH idiosyncratic volatility in cross-sectional regressions. He highlights this finding as direct support for a novel prediction of Merton’s (1987) model—the relation between market capitalization and expected stock returns should be positive when we control for the effect of conditional idiosyncratic volatility on expected stock returns.

In Table 5 we control for out-of-sample idiosyncratic risk and find that size (column 1) is significantly, negatively related to returns in the 1963-2006 period that Fu (2009) analyzes. Furthermore, this holds as well in the 1931-1963 and 1931-2009 periods. Thus, as with the positive idiosyncratic risk and return relationship, we find that when we employ out-of-sample idiosyncratic risk estimates the results disappear.

[Insert Table 6 here]

To further illustrate the nature of the bias in in-sample estimates we again use $E(I VOL, t)$. In Table 6, we first confirm that there is a positive size effect after controlling for $E(I VOL, t)$. Following Fu (2009), we consider two specifications. First, in row 1, we include market capitalization, the book-to-market equity ratio, and $E(I VOL, t)$ as the explanatory variables. Second, in row 3, we add the stock return over the past six months, the turnover, and the coefficient of variation of the turnover to the cross-sectional regression. For both specifications, we replicate the finding of a significantly positive relation between market capitalization and expected stock returns when controlling for $E(I VOL, t)$. 
As we did in analyzing the relation between idiosyncratic risk and return in Table 4, we re-run the regressions while controlling for the look-ahead-bias. To this end, we include both $\Delta_1 E(IVOL_t)$ and $\Delta_2 E(IVOL_t)$ as proxies for the look-ahead bias, and find that the positive effect of market capitalization on expected stock returns disappears for both specifications (rows 2 and 4).\footnote{We find qualitatively similar results using $UE(IVOL_t)$ as a proxy for the look-ahead bias; for brevity, we do not report these results here but they are available upon request.} The results reported in row 5 show that controlling for market beta does not qualitatively change our results. In Panel B, we show that the results are qualitatively similar when using log returns as the dependent variable. Our results are quite intuitive. In row 3 of Table 4, we show that while $E(IVOL_t)$ correlates positively with future stock returns, the relation is negative for $E(IVOL_{t-1})$. Because of their strong negative correlation, market capitalization serves as an instrumental variable for $E(IVOL_{t-1})$ when deployed in conjunction with $E(IVOL_t)$. Therefore, its predictive power disappears when we control for the look-ahead bias.

D. Forecasting One-Month-ahead Realized Idiosyncratic Volatility

Fu (2009) suggests that his findings differ qualitatively from those in earlier studies, e.g., AHXZ (2006), because EGARCH idiosyncratic volatility is a better measure of conditional idiosyncratic volatility than is lagged realized idiosyncratic volatility, for example, as used in AHXZ (2006). We investigate this conjecture in Table 7. As in AHXZ (2009), we use the Fama and MacBeth (1973) cross-sectional regression method to investigate the relation between EGARCH idiosyncratic volatility and one-month-ahead realized idiosyncratic volatility.\footnote{Realized volatility is a proxy for a latent variable—the ‘true’ one-month-ahead volatility—and is estimated with measurement error that is sensitive to methodology, estimation frequency, and estimation window, etc. Therefore, as a robustness check, we employ future options-implied volatility rather than future realized idiosyncratic volatility as the benchmark. Options-implied volatility is the markets estimate of future volatility. Moreover, it should contain relatively little measurement error because it is available mainly for large optionable stocks. We find qualitatively similar results (untabulated) using the future options-implied volatility as the benchmark.} This approach is appropriate because our purpose is to understand the cross-sectional relation between conditional idiosyncratic volatility and stock returns.
As illustrated in Section II, in-sample EGARCH idiosyncratic volatility tends to be high when the one-month-ahead return takes an extreme value, which in turn implies a high realized idiosyncratic volatility. Therefore, the look-ahead bias tends to strengthen the positive relation between and one-month-ahead realized idiosyncratic volatility. To illustrate this point, we report univariate cross-sectional regression results using both in-sample estimate, $E(IVOL_t)$, and out-of-sample estimate, $E(IVOL_{O_t})$, in rows 1 and 2, respectively, of Table 7. While both variables have a strong positive correlation with one-month-ahead realized idiosyncratic volatility, the adjusted $R^2$ is substantially larger for the former. With this caveat in mind, we discuss below the relative predictive power of EGARCH idiosyncratic volatility versus lagged realized idiosyncratic volatility for one-month-ahead realized idiosyncratic volatility.

In Table 7, we show that, consistent with the results reported in AHXZ (2009), lagged realized idiosyncratic volatility, $IVOL_{t-1}$, has significant predictive power for one-month-ahead realized idiosyncratic volatility (row 4). Noticeably, the adjusted $R^2$ is 45%, which is substantially higher than the adjusted $R^2$ of 28% for $E(IVOL_t)$, as reported in row 1. Moreover, row 5 shows that when we include both variables in the cross-sectional regression, lagged realized idiosyncratic volatility remains highly significant, and the adjusted $R^2$ increases only moderately from 45% in the univariate regression (row 4) to 50% in the multivariate regression. We find qualitatively similar results when controlling for other firm characteristics, including market capitalization, ME, the book-to-market equity ratio, BE/ME, the return over the past six months, RET(-2,-7), the turnover, TURN, and the coefficient of variation of the turnover, CVTURN, in the cross-sectional regression (row 10). Therefore, lagged realized idiosyncratic volatility provides important information about one-month-ahead realized idiosyncratic volatility beyond EGARCH idiosyncratic volatility. These results cast doubt on the argument that the difference between Fu (2009) and AHXZs (2006) findings reflects mainly the fact that EGARCH idiosyncratic volatility is a better
IV. Additional Robustness Tests

A. Daily Data

For monthly data, we have to use a relatively small number of return observations to estimate EGARCH idiosyncratic volatility. To address the concern that the EGARCH estimation can be quite sensitive to tuning parameters due to the small sample sizes involved, we estimate the EGARCH idiosyncratic volatility using a two-year rolling window of daily returns with a minimum of 252 observations over the period July 1964 to December 2009. Specifically, we estimate the nine EGARCH specifications using a two-year rolling window of daily stock returns through the last business day of month $t$, and use the specification with the lowest Akaike Information Criterion to make an out-of-sample idiosyncratic volatility forecast for (1) the next day or (2) the next $d$-days, where $d$ is the number of trading days in month $t + 1$. For the first measure, we multiply the daily idiosyncratic volatility estimate by $\sqrt{22}$ to obtain the expected idiosyncratic volatility of month $t + 1$, $E(I VOL_D1)$. For the second measure, we aggregate $d$ conditional daily volatility estimates to get a monthly measure, $E(I VOL_D2)$. While both alternative EGARCH idiosyncratic volatilities have highly significant predictive power for realized idiosyncratic volatility or options-implied volatility (untabulated), in Table 8, we again find that neither measure forecasts the cross-section of stock returns in either univariate or multivariate regressions. These results cast further doubt on the existing evidence of a positive relation between EGARCH idiosyncratic volatility and future stock returns.

\footnote{We thank an anonymous referee for suggesting this alternative measure of EGARCH idiosyncratic volatility.}
B. Size, Liquidity, and Price Screens

Bali, Cakici, Yan, and Zhang (2005) and Bali and Cakici (2008) show that AHXZs (2006) finding of a negative relation between realized idiosyncratic volatility and future stock returns is sensitive to a screen for size, price, and illiquidity. As a robustness check, following Bali and Cakici (2008), we exclude (1) the smallest decile stocks by NYSE breakpoints, (2) the most illiquid decile stocks, and (3) stocks with a price below $10. After screening for size, price, and illiquidity, we sort stocks into two portfolios by market capitalization. Interestingly, we find that in-sample EGARCH idiosyncratic volatility measure forecasts returns only for small stocks but has negligible predictive power for large stocks. This result is not too surprising in light of the Monte Carlo simulation results reported in sub-section II.D above. Small stocks tend to have larger skewness than do large stocks and, thus, are more susceptible to the look-ahead bias in in-sample EGARCH idiosyncratic volatility measure. In contrast, the out-of-sample EGARCH idiosyncratic volatility estimated using either monthly or daily return data always has negligible predictive power for both small and large stocks. For brevity, we do not tabulate these results but they are available on request.

C. Illiquidity and Idiosyncratic Skewness

We have shown via simulations that the lookahead bias in in-sample EGARCH idiosyncratic volatility increases monotonically with skewness. Consistent with this prediction, in Panel B of Table 6, we show that the predictive power of in-sample EGARCH idiosyncratic volatility attenuates substantially when we use log returns. Recent studies, e.g., Boyer, Mitton, and Vorkink (2010), document a strong negative relation between idiosyncratic skewness and future stock returns. Moreover, Bali and Cakici (2008) provide strong evidence for the interaction of illiquidity and idiosyncratic volatility and the effect of this interaction on future stock returns. As a robustness check, we include both idiosyncratic skewness and the Amihud (2002) illiquidity measures as additional control variables and redo the empirical analyses reported in Tables 5 to 7. We find qualitatively similar results (untabulated).
D. Instrumental Variables

In Table 5, we reported that out-of-sample EGARCH idiosyncratic volatility has a positive, albeit weak, correlation with expected stock returns. On the other hand, we reported in Table 4 that the correlation with expected returns is significantly positive for in-sample EGARCH idiosyncratic volatility. Some may argue that the latter is a better measure of conditional idiosyncratic volatility than is the former because we need a large number of return observations to obtain precise estimates of EGARCH model parameters. Although investors cannot exploit its correlation with expected stock returns for their portfolio choices, in-sample EGARCH idiosyncratic volatility is nevertheless useful because it provides a powerful test of economic theories such as Mertons (1987) under-diversification hypothesis. We tested this idea formally using several instrumental variables specifications (available upon request). None of the instrumental variable specifications generated a significantly positive relation between idiosyncratic volatility and expected stock returns.

V. EGARCH Idiosyncratic Volatility Estimated Using the Full Sample

While Fu (2009) estimates EGARCH idiosyncratic volatility recursively, he indicates in his footnote 10 that he finds the same results using the full period data to estimate EGARCH model parameters. Because full-sample EGARCH estimation is computationally less intensive than is recursive EGARCH estimation, many authors, e.g., Brockman and Schutte (2007) and Peterson and Smedema (2011), have subsequently relied upon only the full-sample EGARCH estimates in their studies. The simulation results reported in sub-section II.D suggest that full-sample estimates are subject to a similar look-ahead bias. Therefore, it is not surprising that (in results that, for brevity, are not reported here) we find that the full-sample EGARCH estimates also have the look-ahead bias we documented for Fu’s (2009) recursively estimated EGARCH estimates. As we noted in sub-section II.D, full-sample estimates may actually yield even larger cross-sectional coefficients in the second stage regression than recursive in-sample estimates.
VI. Conclusion

We contribute to the empirical literature on the relation between expected idiosyncratic volatility and the cross-section of stock returns by reconsidering findings that EGARCH idiosyncratic volatility is positively related to returns in the cross-section. We show both analytically and empirically that the positive idiosyncratic risk-return relation is driven by a look-ahead bias accidentally introduced by standard methods of estimating month \( t \) EGARCH idiosyncratic volatility. We show that when month \( t \) EGARCH idiosyncratic volatility is forecasted using returns only up through month \( t - 1 \), there is no significant cross-sectional relation between EGARCH idiosyncratic volatility and returns. EGARCH estimates can be quite sensitive to the tuning parameters in small samples. To allay fears that this may account for our conclusions, we document that our results continue to hold when we estimate EGARCH volatility from large samples of daily returns.

More generally, we demonstrate that, somewhat counter intuitively, the look-ahead bias introduced by incorporating one extra monthly return is so large that it affects statistical inference. To aid our intuition, we conduct Monte Carlo simulations to evaluate the impact of the look-ahead bias. The simulation results show that at skewness levels similar to, or even smaller than, those exhibited by monthly CRSP data the look-ahead bias is significant. In addition, the bias is monotonically increasing in skewness. Therefore, the bias will be more pronounced in samples that exhibit greater return skewness (e.g., notably, small stocks). Moreover, despite the fact that the simulations show that the bias is monotonically decreasing in the length of the return series used in estimation, the bias is still significant for return series equivalent to the entire length of the monthly CRSP stock return series.
References


Table I. The Impact of Skewness on the Look-ahead Bias: Monte Carlo Simulation Using EGARCH (1,1) Estimation

<table>
<thead>
<tr>
<th>Panel</th>
<th>Skewness</th>
<th>Intercept</th>
<th>In Sample</th>
<th>Full Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Out of Sample</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Intercept</td>
<td>E(IVOL)</td>
<td>Adj. $R^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.000</td>
<td>-0.005</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-0.099)</td>
<td>(-0.134)</td>
<td>(-0.148)</td>
</tr>
<tr>
<td>Panel A</td>
<td>0.0</td>
<td>0.001</td>
<td>-0.021</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-0.670)</td>
<td>(0.022)</td>
<td>(2.362)</td>
</tr>
<tr>
<td>Panel B</td>
<td>0.4</td>
<td>-0.001</td>
<td>0.076*</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-0.005)</td>
<td>(0.069*)</td>
<td>(-2.133)</td>
</tr>
<tr>
<td>Panel C</td>
<td>0.8</td>
<td>0.002</td>
<td>-0.039</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.217)</td>
<td>(-0.076)</td>
<td>(-2.603)</td>
</tr>
<tr>
<td>Panel D</td>
<td>1.1</td>
<td>0</td>
<td>0.043</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-0.012)</td>
<td>(3.774)</td>
<td>(-10.722)</td>
</tr>
<tr>
<td>Panel E</td>
<td>1.6</td>
<td>-0.002</td>
<td>0.135*</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-1.464)</td>
<td>(-4.331)</td>
<td>(-14.869)</td>
</tr>
</tbody>
</table>

Notes: The table reports the OLS results of the cross-sectional regression of returns on EGARCH estimated idiosyncratic volatility in simulated data. For each of the iterations, 120 artificial stock returns are simulated for 260 months. The unconditional volatility is 0.06 and the conditional mean is set equal to 0. For each of these 120 stock return series, we run out-of-sample, in-sample, and full-sample EGARCH (1,1) to estimate idiosyncratic volatility which we denote by E(IVOL). The default SAS convergence criterion of 0.001 is used, and we set the maximum iterations to 1,000. We require 60 months to start the expanding window volatility estimation for the out-of-sample and in-sample estimates. The two samples differ only in the last return. For each panel, we generate the i.i.d. error term of the EGARCH model using the Generalized Lambda Distribution algorithm of Ramberg and Schmeiser (1974) with kurtosis of 3.2 and the stated skewness. Newey-West corrected t-statistics are reported in parentheses. Asterisks * or ** indicate significance at the 5% and 1% levels, respectively.
Table II. The Impact of Skewness on the Look-ahead Bias: Monte Carlo Simulation Using Nine EGARCH Combinations

<table>
<thead>
<tr>
<th></th>
<th>Out of Sample</th>
<th>In Sample</th>
<th>Full Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.000</td>
<td>-0.000</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>(-0.152)</td>
<td>(-0.344)</td>
<td>-1.695</td>
</tr>
<tr>
<td>E(IVOL)</td>
<td>0.005</td>
<td>0.008</td>
<td>-0.036</td>
</tr>
<tr>
<td></td>
<td>-0.272</td>
<td>-0.495</td>
<td>(-1.356)</td>
</tr>
<tr>
<td>Adj. ( R^2 )</td>
<td>0.005</td>
<td>0.006</td>
<td>0.009</td>
</tr>
</tbody>
</table>

| **Panel B**   |              |           |             |
| Intercept     | 0.001        | -0.004*   | -0.008**    |
|               | -0.877       | (-2.593)  | (-5.684)    |
| E(IVOL)       | -0.019       | 0.078*    | 0.138**     |
|               | (-0.734)     | -2.568    | -5.574      |
| Adj. \( R^2 \) | 0.002        | 0.005     | 0.006       |

| **Panel C**   |              |           |             |
| Intercept     | -0.001       | -0.007**  | -0.025**    |
|               | (-0.476)     | (-6.653)  | (-18.642)   |
| E(IVOL)       | 0.003        | 0.117**   | 0.435**     |
|               | -0.138       | -6.693    | -16.679     |
| Adj. \( R^2 \) | 0           | 0.003     | 0.02        |

| **Panel D**   |              |           |             |
| Intercept     | 0.001        | -0.009**  | -0.034**    |
|               | -0.623       | (-7.008)  | (-28.440)   |
| E(IVOL)       | -0.018       | 0.153**   | 0.612**     |
|               | (-0.871)     | -6.611    | -26.195     |
| Adj. \( R^2 \) | 0.001        | 0.007     | 0.037       |

| **Panel E**   |              |           |             |
| Intercept     | -0.001       | -0.009**  | -0.036**    |
|               | (-1.621)     | (-6.312)  | (-25.895)   |
| E(IVOL)       | 0.025        | 0.186**   | 0.689**     |
|               | -1.496       | -6.44     | -24.401     |
| Adj. \( R^2 \) | -0.001       | 0.010     | 0.050       |

Notes: The table reports the OLS results of the cross-sectional regression of returns on EGARCH estimated idiosyncratic volatility in simulated data. For each of the iterations, 120 artificial stock returns are simulated for 260 months. The unconditional volatility is 0.06 and the conditional mean is set equal to 0. For each of these return series, we run out-of-sample, in-sample, and full-sample EGARCH \((p,q)\) for 1 \(\leq\) \(p\) or \(q\) \(\leq\) 3 to estimate its idiosyncratic volatility which we denote by \(E(IVOL)\). The default SAS convergence criterion of 0.001 is used for each of the nine EGARCH combinations, and we set the maximum iterations to 1,000. We select the one that converges with the lowest AIC. We require 60 months to start the expanding window volatility estimation for the out-of-sample and in-sample estimates. The two samples differ only in the last return. For each panel, we generate the i.i.d. error term of the EGARCH model using the Generalized Lambda Distribution algorithm of Ramberg and Schmeiser (1974) with kurtosis of 3.2 and the stated skewness. Newey-West corrected t-statistics are reported in parentheses. Asterisks * or ** indicate significance at the 5% and 1% levels, respectively.
Table III. The Impact of the Length of the Estimation Period on the Look-Ahead Bias: Monte Carlo Simulation Using Nine EGARCH Combinations over the Full Sample with SKEW = 0.8

<table>
<thead>
<tr>
<th></th>
<th>T = 100</th>
<th>T = 200</th>
<th>T = 300</th>
<th>T = 400</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-0.032**</td>
<td>-0.027**</td>
<td>-0.023**</td>
<td>-0.018**</td>
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<tr>
<td></td>
<td>(-17.860)</td>
<td>(-13.489)</td>
<td>(-11.898)</td>
<td>(-9.436)</td>
</tr>
<tr>
<td>E(IVOL)</td>
<td>0.599**</td>
<td>0.482**</td>
<td>0.399**</td>
<td>0.321**</td>
</tr>
<tr>
<td></td>
<td>-17.191</td>
<td>-12.805</td>
<td>-11.327</td>
<td>-8.953</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.053</td>
<td>0.022</td>
<td>0.014</td>
<td>0.011</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>T = 500</th>
<th>T = 600</th>
<th>T = 700</th>
<th>T = 800</th>
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<tbody>
<tr>
<td>Intercept</td>
<td>-0.016**</td>
<td>-0.012**</td>
<td>-0.010**</td>
<td>-0.009**</td>
</tr>
<tr>
<td></td>
<td>(-9.381)</td>
<td>(-7.801)</td>
<td>(-7.522)</td>
<td>(-7.994)</td>
</tr>
<tr>
<td>E(IVOL)</td>
<td>0.274**</td>
<td>0.202**</td>
<td>0.176**</td>
<td>0.157**</td>
</tr>
<tr>
<td></td>
<td>-8.873</td>
<td>-7.383</td>
<td>-7.052</td>
<td>-7.441</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.008</td>
<td>0.006</td>
<td>0.005</td>
<td>0.004</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>T = 900</th>
<th>T = 1000</th>
<th>T = 2000</th>
<th>T = 5000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-0.008**</td>
<td>-0.008**</td>
<td>-0.005**</td>
<td>-0.002**</td>
</tr>
<tr>
<td></td>
<td>(-6.788)</td>
<td>(-6.842)</td>
<td>(-6.250)</td>
<td>(-3.208)</td>
</tr>
<tr>
<td>E(IVOL)</td>
<td>0.135**</td>
<td>0.132**</td>
<td>0.083**</td>
<td>0.045**</td>
</tr>
<tr>
<td></td>
<td>-6.444</td>
<td>-6.556</td>
<td>-6.119</td>
<td>-3.337</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.004</td>
<td>0.004</td>
<td>0.003</td>
<td>0.006</td>
</tr>
</tbody>
</table>

Notes: The table reports the OLS results of the cross-sectional regression of returns on EGARCH estimated volatility in simulated data. The unconditional volatility is 0.06 and the conditional mean is set equal to 0. For each of the iterations, 120 artificial stock returns are simulated over a sample of T months. For each of these 120 stock return series, we run full-sample EGARCH (p,q) for $1 \leq p \leq 3$ and $1 \leq q \leq 3$ to estimate its idiosyncratic volatility which we denote by E(IVOL). The default SAS convergence criterion of 0.001 is used for each of the nine EGARCH combinations, and we set the maximum iterations to 1,000. We select the one that converges with the lowest AIC. For each simulation, we generate the i.i.d. error term of the EGARCH model using the Generalized Lambda Distribution algorithm of Ramberg and Schmeiser (1974) with skewness of 0.8 and kurtosis of 3.2. Newey-West corrected t-statistics are reported in parentheses. Asterisks * or ** indicate significance at the 5% and 1% levels, respectively.
### Table IV. In-Sample EGARCH Idiosyncratic Volatility and Expected Stock Returns

<table>
<thead>
<tr>
<th></th>
<th>$E(IVOL_t)$</th>
<th>$E(IVOL_{t-1})$</th>
<th>$\Delta_1E(IVOL_t)$</th>
<th>$\Delta_2E(IVOL_t)$</th>
<th>$UE(IVOL_t)$</th>
<th>Adj. $R^2$</th>
</tr>
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<tbody>
<tr>
<td><strong>Panel A Simple Returns</strong></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.138**</td>
<td>0.000</td>
<td>0.030</td>
<td></td>
<td></td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>(6.607)</td>
<td>(0.019)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.211**</td>
<td>-0.125**</td>
<td>0.037</td>
<td></td>
<td></td>
<td>0.037</td>
</tr>
<tr>
<td></td>
<td>(11.746)</td>
<td>(-13.040)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.086**</td>
<td>0.125**</td>
<td></td>
<td></td>
<td>0.228**</td>
<td>0.039</td>
</tr>
<tr>
<td></td>
<td>(3.685)</td>
<td>(13.040)</td>
<td></td>
<td></td>
<td>(11.471)</td>
<td></td>
</tr>
<tr>
<td><strong>Panel B Log Returns</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.019</td>
<td>-0.070**</td>
<td></td>
<td></td>
<td></td>
<td>0.026</td>
</tr>
<tr>
<td></td>
<td>(0.983)</td>
<td>(-4.068)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.094**</td>
<td>-0.129**</td>
<td></td>
<td></td>
<td></td>
<td>0.033</td>
</tr>
<tr>
<td></td>
<td>(5.881)</td>
<td>(-13.258)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-0.035</td>
<td>0.129**</td>
<td></td>
<td></td>
<td></td>
<td>0.033</td>
</tr>
<tr>
<td></td>
<td>(-1.553)</td>
<td>(13.258)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>10</td>
<td>-0.057*</td>
<td>0.100**</td>
<td>0.083**</td>
<td></td>
<td></td>
<td>0.037</td>
</tr>
<tr>
<td></td>
<td>(-2.365)</td>
<td>(12.376)</td>
<td>(10.580)</td>
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<td></td>
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<tr>
<td>11</td>
<td>-0.050*</td>
<td>0.238**</td>
<td></td>
<td></td>
<td></td>
<td>0.035</td>
</tr>
<tr>
<td></td>
<td>(-2.066)</td>
<td>(12.005)</td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The table reports Fama and MacBeth (1973) cross-sectional regressions of forecasting one-month-ahead stock returns. $E(IVOL_t)$ is EGARCH idiosyncratic volatility that we obtain from Fangjian Fu at Singapore Management University. $E(IVOL_{t-1})$ is one-month lag of $E(IVOL_t)$. $\Delta_1E(IVOL_t)$ is the difference between $E(IVOL_t)$ and $E(IVOL_{t-1})$. $\Delta_2E(IVOL_t)$ is the difference between $E(IVOL_t)$ and its two-month lag, $E(IVOL_{t-2})$. $UE(IVOL_t)$ is the residual from the time-series regression of $E(IVOL_t)$ on a constant and its one-month and two-month lags. We report Newey-West corrected t-statistics in parentheses. The data span the July 1963 to December 2006 period. Asterisks * or ** indicate significance at the 5% and 1% levels, respectively.
<table>
<thead>
<tr>
<th>LN(ME)</th>
<th>LN(BE/ME)</th>
<th>RET(-2,-7)</th>
<th>LN(TURN)</th>
<th>Ln(CVTURN)</th>
<th>$E(I V O L_{O_t})$</th>
<th>Adj. $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<tr>
<td><strong>Panel A July 1963 to December 2006</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>1</td>
<td>0.015</td>
<td>0.014</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(0.995)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-0.089*</td>
<td>0.211**</td>
<td>0.006</td>
<td>0.033</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(-2.296)</td>
<td>(3.714)</td>
<td>(0.475)</td>
<td></td>
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<td></td>
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<tr>
<td>3</td>
<td>-0.145**</td>
<td>0.171**</td>
<td>0.702**</td>
<td>-0.059</td>
<td>-0.453**</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(-3.693)</td>
<td>(3.224)</td>
<td>(3.987)</td>
<td>(-0.794)</td>
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<td>(0.460)</td>
</tr>
<tr>
<td><strong>Panel B September 1931 to June 1963</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.028</td>
<td>0.015</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(1.523)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>5</td>
<td>-0.260**</td>
<td>0.002</td>
<td>0.035</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(-3.021)</td>
<td>(0.130)</td>
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</tr>
<tr>
<td>6</td>
<td>-0.309**</td>
<td>0.745</td>
<td>-0.121</td>
<td>-0.300**</td>
<td></td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>(-3.912)</td>
<td>(1.760)</td>
<td>(-1.844)</td>
<td>(-2.614)</td>
<td></td>
<td>(0.373)</td>
</tr>
<tr>
<td><strong>Panel C September 1931 to December 2009</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.024*</td>
<td>0.015</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.185)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-0.208**</td>
<td>0.002</td>
<td>0.030</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-5.010)</td>
<td>(0.247)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-0.264**</td>
<td>0.700**</td>
<td>-0.089</td>
<td>-0.384**</td>
<td>0.004</td>
<td>0.056</td>
</tr>
<tr>
<td></td>
<td>(-6.727)</td>
<td>(-3.307)</td>
<td>(-1.808)</td>
<td>(-6.259)</td>
<td></td>
<td>(-0.703)</td>
</tr>
</tbody>
</table>

Notes: The table reports Fama and MacBeth (1973) cross-sectional regressions of forecasting one-month-ahead stock returns. Ln(ME) is log market capitalization. Ln(BE/ME) is log book-to-market equity ratio. RET(-2,-7) is the return over the previous 7th to 2nd months. Ln(TURN) is log turnover. Ln(CVTURN) is log coefficient of variation of the turnover. $E(I V O L_{O_t})$ is out-of-sample EGARCH idiosyncratic volatility. We report Newey-West corrected t-statistics in parentheses. Asterisks * or ** indicate significance at the 5% and 1% levels, respectively.
Table VI. Size, In-Sample EGARCH Idiosyncratic Volatility, and the Cross-Section of Stock Returns

<table>
<thead>
<tr>
<th>Beta</th>
<th>LN(ME)</th>
<th>LN(BE/ME)</th>
<th>RET(-2,-7)</th>
<th>LN(TURN)</th>
<th>LN(CVTURN)</th>
<th>E(IVOL_t)</th>
<th>Δ1E(IVOL_t)</th>
<th>Δ2E(IVOL_t)</th>
<th>Adj. R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.204**</td>
<td>0.444**</td>
<td></td>
<td></td>
<td></td>
<td>0.164**</td>
<td>(5.504)</td>
<td>(8.379)</td>
<td>0.045</td>
</tr>
<tr>
<td>2</td>
<td>0.049</td>
<td>0.308**</td>
<td></td>
<td></td>
<td></td>
<td>0.070**</td>
<td>(1.437)</td>
<td>(6.191)</td>
<td>0.053</td>
</tr>
<tr>
<td>3</td>
<td>0.127**</td>
<td>0.392**</td>
<td>0.910**</td>
<td>-0.360**</td>
<td>-0.730**</td>
<td>0.184**</td>
<td>(3.453)</td>
<td>(8.151)</td>
<td>0.065</td>
</tr>
<tr>
<td>4</td>
<td>-0.003</td>
<td>0.281**</td>
<td>0.891**</td>
<td>-0.218**</td>
<td>-0.579**</td>
<td>0.088**</td>
<td>(-0.096)</td>
<td>(6.011)</td>
<td>0.070</td>
</tr>
<tr>
<td>5</td>
<td>-0.087</td>
<td>-0.011</td>
<td>0.275**</td>
<td>0.907**</td>
<td>-0.208**</td>
<td>-0.585**</td>
<td>(-0.534)</td>
<td>(6.102)</td>
<td>0.076</td>
</tr>
</tbody>
</table>

Panel A Simple Returns

| 6    | 0.223** | 0.476**   |            |          |            | 0.049**   | (5.976)     | (8.685)     | 0.043  |
| 7    | 0.065   | 0.412**   |            |          |            | -0.047**  | (1.923)     | (8.502)     | -2.983 |
| 8    | 0.146** | 0.412**   | 1.044**    | -0.406** | -0.698**   | 0.074**   | (3.930)     | (8.502)     | 0.051  |
| 9    | 0.017   | 0.301**   | 1.025**    | -0.264** | -0.547**   | -0.023    | (0.499)     | (6.446)     | 0.063  |
| 10   | -0.112  | 0.007     | 0.295**    | 1.037**  | -0.248**   | -0.551**  | (-0.673)    | (6.549)     | 0.073  |

Panel B Log Returns

Notes: The table reports Fama and MacBeth (1973) cross-sectional regressions of forecasting one-month-ahead stock returns. Beta is the loading on the market risk. Ln(ME) is log market capitalization. Ln(BE/ME) is log book-to-market equity ratio. RET(-2,-7) is the return over the previous 7th to 2nd months. Ln(Turn) is log turnover. Ln(CVTURN) is log coefficient of variation of the turnover. $E(IVOL_t)$ is EGARCH idiosyncratic volatility that we obtain from Fangjian Fu at Singapore Management University. $Δ1E(IVOL_t)$ is the difference between $E(IVOL_t)$ and its one-month lag, $E(IVOL_{t-1})$. $Δ2E(IVOL_t)$ is the difference between $E(IVOL_t)$ and its two-month lag, $E(IVOL_{t-2})$. We report Newey-West corrected t-statistics in parentheses. Asterisks * or ** indicate significance at the 5% and 1% levels, respectively. The data span the July 1963 to December 2006 period.
### Table VII. The Cross-Section of Expected Idiosyncratic Volatility

<table>
<thead>
<tr>
<th>LN(ME)</th>
<th>LN(BE/ME)</th>
<th>RET(-2,-7)</th>
<th>LN(TURN)</th>
<th>LN(CVTURN)</th>
<th>E(IVOL&lt;sub&gt;t&lt;/sub&gt;)</th>
<th>E(IVOL&lt;sub&gt;t−1&lt;/sub&gt;)</th>
<th>E(IVOL&lt;sub&gt;t−1&lt;/sub&gt;)</th>
<th>IVOL&lt;sub&gt;t−1&lt;/sub&gt;</th>
<th>Adj. R&lt;sup&gt;2&lt;/sup&gt;</th>
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<tr>
<td>1</td>
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<td></td>
<td></td>
<td></td>
<td>0.631**</td>
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<td></td>
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<td>0.277</td>
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<td>(45.683)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.507**</td>
<td></td>
<td></td>
<td></td>
<td>0.163</td>
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<td>3</td>
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<td>0.579**</td>
<td></td>
<td></td>
<td></td>
<td>0.234</td>
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<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
<td>(41.139)</td>
<td></td>
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<td>4</td>
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<td></td>
<td></td>
<td>0.172**</td>
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<td>(46.470)</td>
<td></td>
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<td></td>
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<tr>
<td>5</td>
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<td></td>
<td></td>
<td></td>
<td>0.678**</td>
<td></td>
<td></td>
<td></td>
<td>0.482</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(85.633)</td>
<td></td>
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<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
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<td>0.294**</td>
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<td></td>
<td>0.502</td>
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<td></td>
<td></td>
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<td></td>
<td>(57.239)</td>
<td></td>
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<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.445**</td>
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<td>0.322**</td>
<td></td>
<td></td>
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<td>0.480</td>
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<td>(39.679)</td>
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<td>9</td>
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<td></td>
<td></td>
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<td>0.211**</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
<td>(33.346)</td>
<td></td>
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<tr>
<td>10</td>
<td>-0.823**</td>
<td>-0.343**</td>
<td>-1.241**</td>
<td>0.376**</td>
<td>0.214**</td>
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<td>0.482**</td>
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<tr>
<td></td>
<td>(-20.357)</td>
<td>(-11.912)</td>
<td>(-9.745)</td>
<td>(10.452)</td>
<td>(4.005)</td>
<td></td>
<td></td>
<td></td>
<td>(42.507)</td>
</tr>
<tr>
<td>11</td>
<td>-0.702**</td>
<td>-0.216**</td>
<td>-1.259**</td>
<td>0.212**</td>
<td>0.041</td>
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<tr>
<td></td>
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<td>(-8.604)</td>
<td>(-10.627)</td>
<td>(6.341)</td>
<td>(0.849)</td>
<td></td>
<td></td>
<td></td>
<td>(14.096)</td>
</tr>
</tbody>
</table>

Notes: The table reports the Fama and MacBeth (1973) cross-sectional regressions of forecasting one-month-ahead realized idiosyncratic volatility. LN(ME) is log market capitalization. LN(BE/ME) is log book-to-market equity ratio. RET(-2,-7) is the return over the previous 7th to 2nd months. LN(TURN) is log turnover. LN(CVTURN) is log coefficient of variation of the turnover. E(IVOL<sub>t</sub>) is EGARCH idiosyncratic volatility that we obtain from Fangjian Fu at Singapore Management University. E(IVOL<sub>t−1</sub>) is one-month lag of E(IVOL<sub>t</sub>). E(IVOL<sub>t−1</sub>) is out-of-sample EGARCH idiosyncratic volatility. IVOL<sub>t−1</sub> is one-month lagged realized idiosyncratic volatility. We report Newey-West corrected t-statistics in parentheses. Asterisks * or ** indicate significance at the 5% and 1% levels, respectively. The data span the July 1963 to December 2006 period.
<table>
<thead>
<tr>
<th></th>
<th>LN(ME)</th>
<th>LN(BE/ME)</th>
<th>RET(-2,-7)</th>
<th>LN(TURN)</th>
<th>LN(CVTURN)</th>
</tr>
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<tr>
<td>1</td>
<td>0.017</td>
<td>0.037</td>
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<td>0.027</td>
</tr>
<tr>
<td>2</td>
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<td>-0.300</td>
<td>-0.080</td>
<td>0.019</td>
</tr>
<tr>
<td>3</td>
<td>-0.001</td>
<td>0.011</td>
<td>0.055</td>
<td>0.027</td>
<td>0.027</td>
</tr>
<tr>
<td>4</td>
<td>-0.800</td>
<td>0.040</td>
<td>0.057</td>
<td>0.027</td>
<td>0.027</td>
</tr>
<tr>
<td>5</td>
<td>0.005**</td>
<td>0.005**</td>
<td>0.005**</td>
<td>0.005**</td>
<td>0.005**</td>
</tr>
<tr>
<td>6</td>
<td>0.019</td>
<td>0.011</td>
<td>0.055</td>
<td>0.027</td>
<td>0.027</td>
</tr>
<tr>
<td>7</td>
<td>0.014</td>
<td>0.027</td>
<td>0.055</td>
<td>0.027</td>
<td>0.027</td>
</tr>
<tr>
<td>8</td>
<td>-0.001</td>
<td>0.011</td>
<td>0.055</td>
<td>0.027</td>
<td>0.027</td>
</tr>
<tr>
<td>9</td>
<td>-0.800</td>
<td>0.040</td>
<td>0.057</td>
<td>0.027</td>
<td>0.027</td>
</tr>
<tr>
<td>10</td>
<td>0.005**</td>
<td>0.005**</td>
<td>0.005**</td>
<td>0.005**</td>
<td>0.005**</td>
</tr>
<tr>
<td>11</td>
<td>0.019</td>
<td>0.011</td>
<td>0.055</td>
<td>0.027</td>
<td>0.027</td>
</tr>
<tr>
<td>12</td>
<td>-0.001</td>
<td>0.011</td>
<td>0.055</td>
<td>0.027</td>
<td>0.027</td>
</tr>
</tbody>
</table>

Notes: The table reports Fama and MacBeth (1973) cross-sectional regressions of forecasting one-month-ahead stock returns. LN(ME) is log market capitalization. LN(BE/ME) is log book-to-market equity ratio. LN(TURN) is log turnover. LN(CVTURN) is log coefficient of variation of the turnover. E(IVOL,D1) and E(IVOL,D2) are out-of-sample EGARCH idiosyncratic volatilities estimated using daily return data. We report Newey-West corrected t-statistics in parentheses. Asterisks * or ** indicate significance at the 5% and 1% levels, respectively.
Essay II:
The Inevitable Tension between Long-term and Short-term Managerial and Investor Incentives

ABSTRACT
The paper considers a model in which (1) managers allocate effort to both short- and long-term projects, and (2) there is feedback between the managerial incentive contract and the number of speculators collecting information on each type of project. More weight placed on near-term price results in more speculation based on information about the short-term project, which induces further increases in the weight placed on near-term price. This feedback effect can result in short-term speculation crowding out the collection of long-term information, which in turn results in the withdrawal of incentives aimed at inducing effort in more profitable long-term projects. The paper shows that the equilibrium that obtains depends upon adjustment costs and initial conditions and is, in general, not efficient. Such outcomes are consistent with concerns about managerial and investor short-termism recently expressed by policy makers and market participants (e.g., the Aspen Institute). The paper considers the efficacy of various corporate and public policy remedies.

§This essay is coauthored with Steve Slezak. We have benefited from discussions with Yong Kim, Chen Xue, Alexander Borisov, Brian Kluger, Michael Ferguson, Hui Guo, Brian Hatch, Doina Chichernia, David Manzler, Shaun Bond and seminar participants at Baruch College, Bentley University, Kansas State University, Miami University, University of Cincinnati, and University of Toledo.
I. Introduction

Among the many forces that contributed to the recent financial crisis, many policy makers, market participants, and market observers blame managerial short-termism and the prevalence of managerial incentives that focus too much on near-term stock price rather than long-term value; presumably, by focusing on their firm’s current stock price, managers neglect risks that threaten long-term solvency and stability.¹ This argument clearly views near-term price as not adequately reflecting long-run value; if it did, then management would take actions to mitigate long-run vulnerabilities in an effort to boost near-term price (i.e., by caring about the near-term stock price, managers care about long-term value). As such, many of the discussions about the financial crisis also blame investor short-termism for not making current prices reflect long-term risks.² The argument is that, although equity prices are supposed to impound available information about the long-term viability of the firm, since the majority of investment decisions (mostly made by professional portfolio managers

¹For example, in an October 2009 article in the Atlantic, former Senior Vice President for Law and Public Affairs at General Electric and current senior fellow at Harvard’s Kennedy School of Government and Harvard Law School’s Program on Corporate Governance Ben W. Heineman Jr. stated: “As we now know all too well, the credit crisis and the global recession stemmed, in important part, from stark failures of boards of directors and operating business leadership in important financial institutions: the witchs brew of leverage, poor risk management, creation of toxic products, lack of liquidity all made more poisonous by compensation systems which rewarded short-term revenues/profits without regard to risk.” In remarks to the National Press Club in July of 2011, former Chairman of the Federal Deposit Insurance Corporation (FDIC) Sheila Bair concurred: “The compensation of loan officers, portfolio managers and bank CEOs was typically based on current-year loan volume, earnings or stock price, with little regard for the risks that were building up in the system” (italics added).

²For example, a committee of the Aspen Institute, consisting of 30 current or retired CEOs, legal experts, and government officials (including such notable members as Berkshire Hathaway CEO Warren E. Buffett, Vanguard founder John C. Bogle, and TIAA-CREF President and CEO Roger W. Ferguson, Jr.), signed a statement decrying short-termism, proclaiming that “...fund managers with a primary focus on short-term trading gains have little reason to care about long-term corporate performance or externalities, and so are unlikely to exercise a positive role in promoting corporate policies, including appropriate proxy voting and corporate governance policies, that are beneficial and sustainable in the long-term.” This position is also echoed by UCLA Law Professor Lynn Stout (also a member of the Aspen Institute committee) in a Wall Street Journal blog: “The pursuit of short-term trading profits also has a corrosive effect on the wider economy because it distracts corporate managers who must respond to short-term investors demands from the important business of planning for and investing in the future. ...[C]orporations build railways, specialized manufacturing facilities, trusted international brand names, mass-produced software, new drugs and medical devices. Yet, it can be difficult or impossible for corporate directors and executives to focus their attention on such projects when they are constantly being called upon instead to meet quarterly earnings targets and to raise tomorrows stock price.”
who are gauged by their short-term performance) are driven by short-term gain, very little information about the long-term performance of the firm is collected, resulting in near-term stock prices that contain little information about the long run. Thus, when incentive contracts are written on near-term equity values, managers have little incentive to take actions that improve long-run value.

These arguments have led many to suggest or impose that incentive contracts place more weight on longer-term returns/value. For example, in what amounts to a reallocation of contractually specified weight from short- to long-term performance metrics, the FDIC recently adopted a rule that “allows the agency to claw back two years’ worth of compensation from senior executives and managers responsible for the collapse of a systemic, non-back financial firm...” (Bair (2011)). Also, New York Times banking reporter Eric Dash noted “[o]ne idea is for ‘clawbacks’ to be strengthened to include bad performance, not just wrongdoing or fraud.” Further, he suggests “...it might be worthwhile to consider withholding a bigger portion of trader’s bonus over a more extended period of time, perhaps several years or more, in an escrow account. That way, it could be adjusted up or down based on the trader’s actual results....” In addition, others have suggested investors be “encouraged” to take a longer-term view too. For example, the Aspen Institute committee (see footnote 2) suggests three policies: (1) “Revise capital gains tax provisions or implement an excise tax in ways that are designed to discourage excessive share trading and encourage long-term share ownership....” (2) “Remove limitations on capital loss deductibility for very long-term holdings....” (3) “In exchange for enhancing shareholder participation rights, consider adopting minimum holding periods or time-based vesting....”

Although statements/beliefs such as those identified above appear to be fairly common following the financial crisis, these beliefs raise a whole raft of questions - many of which don’t have obvious answers in the academic literature. One question is whether (and why) there is an excessive incentive to collect information on short-term prospects - especially if, as is presumed in many of the statements above, long-term prospects are more important to
long-run value. Presumably, any investor that can uncover a lack of long-term viability would seem to be able to profit in the short run by publicizing such inadequacies. Is there some sense in which short-term information “crowds out” more important long-term information?\(^3\) In addition, it is also unclear how putting greater weight on far-term value would affect the incentives to collect information on short- versus long-term projects. Indeed, prices are supposed to transport future values to the present; anything that improves long-term value (and the manager’s compensation based on long-term value) should also improve near-term value (and the manager’s compensation based on near-term value). Moreover, implicit in recommendations for centralized regulation (e.g., as in the FDIC’s clawback provisions and the Aspen Institute Committee’s suggested revisions to capital gains taxation) is the belief that there are externalities that private contracting cannot solve; what is the nature of these externalities (especially in terms of contracting)?

In this paper, we develop a simple model to investigate the issues raised above. Specifically, the paper develops an agency model in which a manager allocates (personally costly) effort to both a short- and a long-term project, both of which positively affect the long-run value of the firm. In order to induce effort, the manager is compensated according to an incentive contract that specifies pay contingent on both near- and far-term stock prices. Hereafter, we refer to the sensitivities of the manager’s compensation to the near-term price and far-term price as, respectively, the near-term weight and the far-term weight. The paper also considers the incentives of speculators to collect and trade on information on each of the projects. Depending upon speculators’ ability to profitably trade on such information, the amount of each type of information collected in the first place and the amount of that information that is reflected in the near-term stock price may vary by project type. In order to determine whether the types of distortions described in the quotes above are possible, we examine how the initial, near-term, and long-term values of the firm depends upon the

\(^3\) This question was explicitly asked in Heineman (2009): “Indeed, important questions have been raised about the role institutional investors played in causing the melt-down by pressuring financial service entities to take undue risk for short-term profits. Did the short-term investors crowd out long term investors in influencing corporations and, if so, is this likely to be the future pattern?” (italics added.)
incentive contract, the characteristics of the equity market, and the market for information.

The model generates four main results. First, when the far-term value of the firm contains greater uncertainty independent of the manager's current effort, it is optimal to place compensation weights on both near-term and far-term equity values. The far-term firm value fully reflects the impact of the manager's short-term and long-term effort choices, while the near-term price only partially reflects speculators' (potentially noisy) information about the manager's effort choices. As a result, far-term weight is more effective than near-term weight at inducing the manager to exert effort on both the short- and long-term projects. But, since the far-term value depends upon more things beyond the manager's control at the time the manager exerts effort, weight placed on the far-term value generates more risk to the manager. Since the manager must be compensated for this risk via a higher average wage, more weight placed on far-term value is more expensive to shareholders. Thus, shareholders opt to place weight on both the near-term and the far-term firm value in order to reduce compensation costs.

Second, the nature of the equity and information markets affects the relative ability of the near-term price to induce effort on each type of project – independently of the productivity of each type of effort on long-run outcomes. In particular, when the number of speculators collecting short-term information is higher than the number collecting long-term information, then the near-term price will reflect relatively more of the short-term information, making the near-term price more effective (and less costly) at inducing effort on the short-term project. As a result, if there are more speculators collecting information on the short-term project, the market will “encourage” the short-term project relative to the long-term project.

The third result is that more weight placed on the near-term price increases the expected trading profits of the speculators collecting and trading on information about the short-term project. Further, as the number of speculators collecting information on the short-term project increases with the increase in expected profits, the near-term price reflects more short-term information, which in turn makes shareholders want to increase the weight on
the near-term price. This happens because when more weight is placed on the near-term price the manager exerts relatively more effort on the short-term project and, as a result, a larger component of the firm’s future value is determined by the short-term project. This then leads to an increase in the incentive for speculators to collect information on the short-term project. And, as more speculators collect (and compete) on information about the short-term project, incentives based on the near-term stock price become cheaper (as the prices have less noise related to liquidity trading), which encourages shareholders to place more weight on the near-term stock price. As a consequence, there exist equilibria in which more weight is placed on the near-term price and the manager exerts more effort on the short-term project, even when managerial effort is more productive if exerted on the long-term project.

The fourth result is that trade based on information on the short-term project crowds out the collection of information about the long-term project. This is true because the future value of the firm depends more on the manager’s effort and productivity in the short-term project, which is less correlated with information on the long-term project collected by long-term informed traders. Thus, the market sustains fewer informed traders collecting information on the long-term project.

In addition to the theoretical literature that shows that there can be a divergence between the private incentive to collect information and its social value (e.g., Hirshleifer (1971), Marshall (1974) and Hakansson, Kulkel, and Olson (1982), the literature has developed models that specifically imply a trade-off between short- and long-term projects. For example, the signal jamming models in Narayanan (1985) and Stein (1989) show that, even though no one is fooled, managers may attempt to manipulate shareholders’ beliefs about long-run value by boosting short-term earnings via excessive borrowing from the future. Peng and Roell (2012) develop a model for optimal executive compensation where managers are prone to manipulate and their propensity to manipulate is random. The resulting equilibrium contract weights (written on near-term stock price and far-term value) depend on the degree of manipulation uncertainty: when manipulation uncertainty is high, more weight is placed
on near-term stock price. In addition, Bolton, Scheinkman, and Xiong (2009) provide a model in which the market, by assumption, generates a near-term price that provides a poor signal of long-term value. In their model, the equilibrium incentive contract places weight on both near-term price and long-term value. As a result, the managers make sub-optimal decisions relative to situations in which the market price is a better signal of long-term value. These models rely on the existence of uncertainty about the information content of market prices with respect to long-term firm value. These elements are not present in the model we develop herein.

A closely related paper is Paul (1992), which also considers the efficacy of equity-based contracts when the manager must allocate effort across multiple projects. A key difference between our model and Paul (1992) is that in Paul (1992) the productivity of the manager is known by the market and the parameters of the compensation contract do not affect the speculators' incentive to collect information. Furthermore, in Paul (1992), the allocation of information across projects is taken as given. In our model, informed traders endogenously choose the type of information to obtain given the manner in which the contract affects the incentive to collect specific type of information.

The remainder of the paper is organized as follows. Section I describes the model. Section II characterizes the equilibrium in the securities market taking the contract as given. This section derives (1) the optimal amounts of effort to exert on the short- and long-term projects given the contract and the price function, (2) the optimal trades of speculator as a function of the information they collect on short- and/or long-term projects and the parameters of the price function, (3) the unconditional expected speculative profits as a function of the number of speculator collecting each type of signal and the parameter of the managerial incentive contract, and (4) the equilibrium price function. Section III discusses the setting of the parameters of the incentive contract and the overall equilibrium. This section also shows the nature of the contracting inefficiency. Section IV considers potential remedies. Section V concludes.
II. The Model

A. Managerial Effort and Long-run Value

The principal-agent model runs over three periods ($t = 0, 1, \text{ and } 2$) as depicted in the timeline in Table I. At $t = 2$, the terminal value of the firm $v$ is distributed to claimants (which include the shareholders and the manager). The terminal value of the firm is simply the sum of the values from a short-term project ($v_S$) and a long-term project ($v_L$), both of which are initiated at $t = 0$:

$$v = v_S + v_L \quad (1)$$

The values of the projects (realized at $t = 2$) depend upon the amount of effort (denoted $e_S$ and $e_L$) the manager (i.e., the agent) exerts on each of these projects at $t = 0$. Specifically, the realized value of the short-term project is

$$v_S = \gamma_S e_S + \eta_S, \quad (2)$$

where $\gamma_S$ is the marginal productivity of the manager’s short-term effort $e_S$ and $\eta_S$ is a short-run random component beyond the manager’s control, where $\eta_S \sim N(0, \sigma_{\eta_S}^2)$. The marginal productivity of effort with respect to the short-term project $\gamma_S$ is, with respect to the markets information set, random, with $\gamma_S^2 \sim N(\bar{\gamma}_S^2, \sigma_{\gamma_S}^2)$.

However, the realized value of $\gamma_S$ (or $\gamma_S^2$) is known by the manager at the time the manager exerts effort on the short-term project.

The realized value of the long-term project is

$$v_L = \gamma_L e_L + \eta_L + \eta_T, \quad (3)$$

As will be obvious later, we assume that the square of the marginal productivity (rather than the level of the marginal productivity itself) is normally distributed in order to obtain closed-form solutions for various endogenous variables.
where $\gamma_L$ is the marginal productivity of long-term effort $e_L$, $\eta_L$ is a short-run random component with respect to the long-term project that is beyond the manager’s control and is uncorrelated with $\eta_S$ (i.e., $E(\eta_S \eta_L) = 0$), and $\eta_T$ is a long-run random component that affects the outcome of the long-term project that is also beyond the manager’s control and is independent of both $\eta_S$ and $\eta_L$.\footnote{For example, a long-term project may be to develop new markets in developing countries while a short-term project might entail investing in a new technology that reduces production costs. While the impact of the cost saving technology will become apparent in the near term, whether the new market turns out to be lucrative or not depends upon both the actions and efforts of the management but also on many factors beyond the managers control such as political reform that might liberalize or restrict foreign investment.} Both $\eta_L$ and $\eta_T$ are normally distributed; $\eta_L \sim N(0, \sigma_{\eta_L}^2)$ and $\eta_T \sim N(0, \sigma_{\eta_T}^2)$. As with the short-term project, the marginal productivity of effort with respect to the long-term project $\gamma_L$ is, with respect to the market’s information set, random, with $\gamma_L^2 \sim N(\bar{\gamma}_L^2, \sigma_{\gamma_L}^2)$. Again, the realized value of $\gamma_L$ (or $\gamma_L^2$) is known by the manager at the time the manager exerts effort on the long-term project.

One of the main differences between the short- and long-term projects is that, since the long-term project is realized farther in the future, its realization depends upon more things beyond the manager’s control at $t = 0$ (when effort is exerted). That is, the far-term payoff to the long-term projects includes $\eta_T$ (with $\sigma_{\eta_T}^2 > 0$) while no such random component exists for the long-term payoff of the short-term project. As is discussed below, the timing of the realization and availability of information on each of these components is such that $\sigma_{\eta_T}^2 > 0$ is not equivalent to $\sigma_{\eta_L}^2 > \sigma_{\eta_S}^2$ with $\sigma_{\eta_T}^2 = 0$. In addition, in keeping with the presumption that long-term projects lead to greater long-run value than short-term projects, we assume that the expected managerial productivity of effort is greater when exerted on the long-term rather than the short-term project: $\bar{\gamma}_L^2 > \bar{\gamma}_S^2$.$^6$

\textit{B. Speculators’ Information on Projects}

An important part of the contracting environment is the amount of information about the manager’s effort exerted on each type of project that will be reflected in the near-term price

$^6$All of the expressions derived herein are general and do not rely on a particular ranking of the various parameters. We merely maintain certain ranking assumptions in order to simplify the exposition and relate the analysis to the claims found in the business press.
and the terminal value. The incentive to collect and trade on such information depends upon the informational efficiency of the market for the firm’s shares, which we assume is open at $t = 1$. Investors (or speculators) in this market are risk neutral and value the stock at the expected value of the terminal value conditional on any information that is available at $t = 1$. The information that is available at $t = 1$ includes both a costly noisy signal $\theta_S$ of the value of the short-term project and a costly noisy signal $\theta_L$ of the short-term component of the value of the long-term project $\gamma_L e_L + \eta_L$. Specifically, the signals are related to project values as follows:

$$\theta_S = \gamma_S e_S + \eta_S + \varepsilon_S,$$

$$\theta_L = \gamma_L e_L + \eta_L + \varepsilon_L$$

where $\varepsilon_i \sim N(0, \sigma_{\varepsilon_i}^2)$ and $E[\varepsilon_i \eta_j] = E[\varepsilon_i \eta_T] = 0$ for $i, j \in S, L$. Since the signal $\theta_S$ is based on information about short-run outcomes that may have already come to fruition while $\theta_L$ is with respect to outcomes that are farther in the future, then it is likely that the long-term signal is noisier (i.e., $\sigma_{\varepsilon_L}^2 > \sigma_{\varepsilon_S}^2$) and/or more costly (i.e., $C_L \geq C_S$, where $C_i$ denotes the cost of obtaining an $i$-type signal, for $i \in S, L$). We will refer to any agent that obtains $\theta_S$ as a short-term informed investor; any agent that obtains $\theta_L$ is a long-term informed investor.

By obtaining signals $\theta_S$ and/or $\theta_L$, traders obtain information on $\eta_S$ and/or $\eta_L$; it is assumed that signals that provide information on $\eta_T$ are not available (for any cost) at $t = 0$ or $t = 1$. Thus, $\sigma_{\eta_T}^2 > 0$ but $\sigma_{\eta_L}^2 = \sigma_{\eta_S}^2$ is not equivalent to $\sigma_{\eta_L}^2 > \sigma_{\eta_S}^2$ with $\sigma_{\eta_T}^2 = 0$.

\footnote{If the investors were risk averse, then the price would be the expected terminal value minus a risk premium. If the market participants know the parameters of the model, then the risk premium will be a non-random constant with respect to investors’ information sets. As a result, for a given amount of information collected, the information content of the price in a risk-averse economy will be the same as in a risk-neutral economy. However, since in a risk-averse economy the amount of information collected may depend upon the risk aversion of investors, the amount of information collected may be different in a risk-averse versus a risk-neutral economy.}
C. Market Prices

Initially (i.e., at $t=0$), the price of the stock is the unconditional expected terminal value: $p_0 = E[v']$, where $v'$ denotes the terminal value net of the manager’s compensation. The stock price at $t=1$ is this unconditional expected terminal value plus any deviation created by the information content of investors trading at $t=1$. That is, the price $p$ in the secondary market at $t=1$ is the expected terminal value conditional on the net order flow given informed and liquidity trades:

$$p = E[v' | \Psi] = p_0 + \lambda \Psi$$

where $\Psi \equiv \sum_{m=1}^{N_{IS}} X_{Sm} + \sum_{m=1}^{N_{IL}} X_{Lm} + Z$ is the net order flow at $t=1$, $Z$ denotes the trades of liquidity traders (with $Z \sim N(0, \sigma_Z^2)$), $X_{Sm}$ is the demand of the $m$th short-term informed investor (of which there are $N_{IS}$), and $X_{Lm}$ is the demand of the $m$th long-term informed investor (of which there are $N_{IL}$). With competitive market makers who make zero expected profits, $\lambda = \frac{\text{cov}(\Psi, v')}{\text{var}(\Psi)}$.

To the extent that liquidity traders may have some discretion as to which securities to trade to satisfy their liquidity needs, the variance of liquidity trades $\sigma_Z^2$ for a particular firm may depend upon cross-sectional variation in both endogenous and exogenous characteristics of firms traded in the market. For now, however, we take the value of $\sigma_Z^2$ as a parameter and derive results conditional upon its value. After these conditional results are obtained, the forces that determine how liquidity trades will be distributed (and their impact on the over-all equilibrium) will be discussed.

D. Incentive Compensation and the Manager’s Objective Function

At $t = 1$, the manager is (partially) compensated based on a short-term measure of performance. Specifically, the compensation at $t = 1$ is $\omega_pp$, where “performance” is
measured by the near-term price $p$ and $\omega_p$ is the short-term pay-for-performance sensitivity. The manager will also receive compensation in the long-term (at $t = 2$) based on the long-term value of the firm realized at $t = 2$. In particular, at $t = 2$, the manager receives compensation $\omega_v v$, where $v$ is the realized far-term value of the firm gross of managerial compensation and $\omega_v$ is the long-run pay-for-performance sensitivity. At $t = 0$, the manager is paid a fixed wage $\omega_0$. Thus, the manager’s compensation contract is characterized as an ordered triple $\omega = (\omega_0, \omega_p, \omega_v)$. To simply focus on the incentives and the risks associated with these incentives, we assume that the manager has no time preference; that is, the objective function of the manager is based on the simple sum of his compensation payments:

$$ w = \omega_0 + \omega_p p + \omega_v v \quad (7) $$

That is, even though the payments $\omega_0$, $\omega_p p$, and $\omega_v v$ are made at different times, they all receive equal weight; thus, the timing of the compensation payments is unimportant. Under this assumption, there is no benefit to paying the manager early; in actual markets, one reason to base the manager’s pay on near-term market price and/or performance is that the manager faces liquidity constraints and capital market imperfections that prevent him/her from being able to borrow against expected future performance in order to consume during earlier periods. This motivation for placing weight on the near-term value does not exist under the assumptions made herein. Thus, if there is an incentive to put weight on near term price here, then there will be even stronger incentives (based on liquidity) in actual markets. The issue here is simply whether there is a justification for placing weight to near-term price based on incentives independent of any liquidity justification.

Although the manager has no time preference, s/he is risk averse and has disutility to both short- and long-run effort. Specifically, the manager has a negative exponential utility
with a constant absolute risk averse (CARA) risk preference given by

$$u(w, e_S, e_L) = -\exp \left[ -\gamma \left[ w - \sum_{i \in (S,L)} \psi(e_i) \right] \right],$$  \hspace{1cm} (8)

where $\gamma$ is the manager’s coefficient of constant absolute risk aversion (i.e., $\gamma = -\frac{w''}{w} > 0$), and the disutility function $\psi(e_i)$ is given by

$$\psi(e_i) = \frac{\delta_i e_i^2}{2}$$  \hspace{1cm} (9)

for $i \in S, L$. Given effort levels $e_S$ and $e_L$ and compensation $w$, the manager’s expected utility satisfies

$$E[u(w, e_S, e_L)] \propto E[w] - \frac{\gamma}{2} \text{var}(w) - \frac{\delta_S e_S^2}{2} - \frac{\delta_L e_L^2}{2}.$$  \hspace{1cm} (10)

The manager is willing to accept any contract that produces an expected utility greater than or equal to his reservation utility, which, without a loss of generality, we assume is zero. Since we allow for the manager’s marginal productivity with respect to each type of project to differ (i.e., $\gamma_S$ need not equal $\gamma_L$), there is no need to have the disutility of effort exerted differ by the type of project; thus, we can have $\delta_i = \delta$ for $i \in S, L$.

We further assume that no one in the model has time preference; thus, the net interest rate is zero.

**E. The Shareholders’ Problem**

Prior to the manager exerting effort at $t = 0$, the board of directors, acting on behalf of long-term shareholders (i.e., shareholders that will hold the equity in the firm until $t = 2$),
picks the contract \( \omega = (\omega_0, \omega_p, \omega_v) \) to solve the following problem:

\[
\max_{\omega_0, \omega_p, \omega_v} \ E[v(\hat{e}_S, \hat{e}_L)] - E[w(\hat{e}_S, \hat{e}_L)]
\]  

subject to (PC) \( E[w|\hat{e}_S, \hat{e}_L] - \frac{\gamma}{2} \text{var}(w|\hat{e}_S, \hat{e}_L) - \frac{\delta_S}{2} (\hat{e}_S)^2 - \frac{\delta_L}{2} (\hat{e}_L)^2 = 0 \)

and (IC) \( \hat{e}_S, \hat{e}_L \in \arg \max E[w|e_S, e_L] - \frac{\gamma}{2} \text{var}(w|e_S, e_L) - \frac{\delta_S}{2} (e_S)^2 - \frac{\delta_L}{2} (e_L)^2 \)

That is, the board picks the contract to maximize the expectation of the terminal value minus the manager’s wage (i.e., the firm’s expected residual value) taking into consideration how the manager’s short-term and long-term efforts depend upon the parameters of the contract (via the incentive compatibility constraint IC), such that, at the optimal levels of effort, the manager achieves at least his reservation utility (as in the participation constraint PC).

F. Discussion

There are a couple of features of the above environment to note. First, since the terminal value of the firm is simply the sum of the realized values of both the short-term and long-term projects, both projects contribute equally to the long-term value of the firm. Thus, the long-term project is no better or more important than the short-term project. Second, short-term value does not lead to short-term results that the market may confuse with long-term value. Thus, in this model there is no manipulation, whereby manager takes actions to manipulate observable results in an attempt to fool shareholder into believing that long-term value (which is reflected in near-term stock price) is higher than it actually is. (That is, there is no signal jamming as in Narayanan (1985) and Stein (1989).) However, to the extent that the manager’s effort may, on average, be more productive if exerted on the long-term project (i.e., \( \gamma_L > \gamma_S \)), then a contract that induces the manager to exert more effort on the long-term project will produce more value. Thus, the issue is not whether the manager manipulates short-term performance, but whether the contracting environment is such that the equilibrium contract induces more managerial effort exerted on long-term projects when
\( \gamma_L > \gamma_S \).

Third, although the manager’s marginal productivity of effort on terminal value is \( \gamma_i \) (for \( i \in S, L \)), the marginal impact of the manager’s efforts on the manager’s total compensation depend upon the marginal impact of each type of effort on both the market price at \( t = 1 \) and the terminal value. The marginal impact of effort on the market price at \( t = 1 \) depends upon the equilibrium price function then, which in turn depends upon the aggressiveness with which informed traders trade on their private information about the manager’s effort with respect to a particular project. If more informed traders collect information on the short-term project, for example, then the competition among these traders will result in a price that reflects more of this type of information than regarding the long-term project. In addition, if there is more noise in investors’ signals of the long-term project’s value than for the short-term project (i.e., \( \sigma^2_{\varepsilon_S} > \sigma^2_{\varepsilon_S} \)), then investors trades will be less informative about the long-term project (holding constant the amount of intrinsic uncertainty – \( \sigma^2_{\eta_S} = \sigma^2_{\eta_L} \)).

III. Equilibrium in the Securities Market (Taking the Contract as Given)

A. The Benchmark Case: First-Best Effort (No Agency Problem)

Before the equilibrium in the securities market is characterized, we first specify the first-best effort levels and value as a benchmark for which to compare the equilibrium levels of effort and value.

LEMMA 1: The first-best level of effort and the resulting expected terminal value (net of the cost of compensation for the disutility of effort) is as follows:

\[
\begin{align*}
\epsilon_{FB}^S &= \frac{\gamma_S}{\delta_S} \\
\epsilon_{FB}^L &= \frac{\gamma_L}{\delta_L}
\end{align*}
\]
and the resulting residual value is

\[ E(v) - E(w) = \frac{\gamma_S^2}{2\delta_S} + \frac{\gamma_L^2}{2\delta_L} \]  

(14)

Proof: See the appendix.

Lemma 1 is intuitive: the manager’s short-term and long-term efforts should be higher, the higher the marginal productivity of effort \( \gamma_i \), and the lower the marginal disutility of each type of effort \( \delta_i \) (for \( i \in S, L \)). The unconditional expectation of the expected residual value is simply \( E[E(v) - E(w)] = \frac{\gamma_S^2}{2\delta_S} + \frac{\gamma_L^2}{2\delta_L} \).

B. The Price Function

In order to determine the manager’s effort as a function of the contract terms offered, we first derive the time \( t = 1 \) secondary market price function. The following lemma specifies an expression of the price as a function of the net-order flow at \( t = 1 \) and the contract terms – given that the manager’s compensation depends on that price (and the expected terminal value).

LEMMA 2: The price function at \( t = 1 \), given by equation (6) \( \text{i.e., } p = p_0 + \lambda \Psi \), is as follows:

\[
    p_0 = \left( \frac{1 - \omega_0}{1 + \omega_p} \right) \left[ E[v] - \frac{\omega_0}{1 - \omega_v} \right],
\]

\[
    \lambda = \left( \frac{1 + \omega_v}{1 + \omega_p} \right) \left( \frac{\text{cov}(v, \Psi)}{\text{var}(\Psi)} \right)
\]

where \( E[v] \) depends upon the manager’s equilibrium levels of effort (specified below).

Proof: See the appendix.

For a fixed expected gross terminal value \( E[v] \) and a fixed wage \( \omega_0 \), the price is decreasing in both \( \omega_v \) and \( \omega_p \). This is true since the higher either \( \omega_v \) or \( \omega_p \), the higher is the manager’s
pay (which reduces the residual value retained by the shareholders). However, the values of $\omega_v$ and $\omega_p$ determine the manager’s incentive to exert effort, which increases $E[v]$. In addition, they also affect the amount of risk in the manager’s compensation, which affects the fixed wage $\omega_0$ that must be paid to induce the manager to take the job in the first place.

In order to determine the variance of the net order flow $\Psi$ and the correlation between the net-order-flow and the gross terminal value $v$, the optimal trades of the informed investors must be determined. We conjecture that the trades of investors who are informed about either the short- or long-term project will be linear in the unexpected component of their signal (i.e., either $\theta_s - E(\theta_s)$ or $\theta_L - E(\theta_L)$). If the trades of informed investors are linear in their signals, then the net order flow $\Psi$ will also be linear in these signals (and the trades of the liquidity traders). Thus, we assume that $\Psi = \pi_S[\theta_S - E(\theta_S)] + \pi_L[\theta_L - E(\theta_L)] + Z$, where $\pi_S$ and $\pi_L$ are, as yet, undetermined coefficients that quantify the sensitivities of the collective trades of the investors informed about the short-term and long-term project, respectively. Given this, the price function can be written as

$$p = p_0 + \lambda \left( \pi_S[\theta_S - E(\theta_S)] + \pi_L[\theta_L - E(\theta_L)] + Z \right)$$

where $\lambda_S \equiv \lambda \pi_S$ and $\lambda_L \equiv \lambda \pi_L$. Thus, the sensitivity of the price to a given signal $\theta_i$ depends upon both the sensitivity of the collective informed investors’ trades to that signal ($\pi_i$) and the resulting correlation between all of the net order flow $\Psi$ and the terminal value

$$\lambda = \left( \frac{1+\omega_p}{1+\omega_p} \frac{\text{cov}(v,\Psi)}{\text{var}(\Psi)} \right).$$

If $\theta_i$ is completely uninformative, informed investors’ trades will not be sensitive to that signal, resulting in $\pi_i = 0$ and $\lambda_i = 0$ (even though $\lambda \neq 0$). If, relative to the completely uninformative–signal case, the signal is more informative and the collective trades of the informed investors are more sensitive to that signal, then the price is more sensitive to that signal. Of course, the sensitivity of the collective trades of the informed investors depends upon the number of investors who obtain that signal and how
sensitive the optimal demand of a typical informed investor is to that signal.

C. Optimal Effort Conditional on the Contract and Prices

Before we can completely specify the price function, first we must determine the optimal effort the manager exerts on each of the two types of projects given his compensation so that the unconditional expected gross terminal value \( E[v] \) can be specified. For this we take the form of the price function from Lemma 2 as given. Later, to the extent that the parameters of the price function depend upon the managerial effort levels and the contract offered by the shareholders, we will have to solve for reduced-form fixed point for the price function that is internally consistent. The following lemma, providing the first step in this process, specifies the optimal effort levels given arbitrary parameters for both the manager’s compensation contract and the price function.

**Lemma 3:** For a given contract \( \omega = (\omega_0, \omega_p, \omega_v) \) and the parameters \((\lambda_S, \lambda_L, \lambda)\) of the \( t = 1 \) price function as in equation (15), the optimal effort levels (for \( i = S \) and \( L \)) are:

\[
\hat{e}_i(\omega) = \frac{\gamma_i}{\delta_i} W_i. \tag{16}
\]

where \( W_i \equiv \omega_p \lambda_i + \omega_v \).

**Proof:** See the appendix.

Trivially, the higher is the marginal productivity of type-\( i \) effort (i.e., \( \gamma_i \)), the greater the manager’s incentive to exert type-\( i \) effort. This is true since any amount of type-\( i \) effort affects both the terminal value \( v \) and (via the signals of the value of projects) the intermediate price \( p \); for positive contract weights placed on either value (i.e., \( W_i > 0 \)), the more productive that effort (i.e., the greater \( \gamma_i \)), the greater is the manager’s incentive to exert that effort. Also, since \( \delta_i \) is the manager’s marginal cost of effort spent on project \( i \), the lower \( \delta_i \), the greater the incentive to exert effort on project \( i \).
The variable $W_i$ specifies the extent to which the manager’s effort on project $i$ affects his total compensation: (1) The manager’s effort on project $i$ increases the informed investors’ signal on project $i$, which (a) raises those informed investors’ trades by $\pi_i$ and (b) raises the near-term price by $\lambda_i \equiv \lambda \pi_i$; given the contract weight $\omega_p$ on the near-term price, the manager’s compensation thus rises by $\omega_p \lambda_i$. (2) The manager’s effort on project $i$ also increases the expected gross terminal value directly; given the contract weight $\omega_v$ on the gross terminal value, the manager’s compensation rises by $\omega_v$. Thus, the manager’s efforts on both the short-term and the long-term projects are increasing in the contract’s weights $\omega_p$ and $\omega_v$. But, for a fixed set of contract weights $(\omega_p, \omega_v)$, the manager will exert relatively more effort on the project that has a larger $\lambda_i$, which depends upon the number of investors obtaining the type-$i$ signal and the aggressiveness with which they collectively trade on that signal. The greater the relative sensitivity of the collective trades to the type-$i$ signal, the greater the effort on the type-$i$ project.

Weight placed on the terminal value is relatively more effective at inducing effort than weight placed on intermediate price (i.e., $\frac{\partial \hat{e}_i}{\partial \omega_p} < \frac{\partial \hat{e}_i}{\partial \omega_v}$) because the terminal value exactly reflects the manager’s effort whereas the intermediate price only reflects his effort multiplied by the regression coefficient $\lambda_i < 1$. Thus, since the terminal value is more sensitive to a given amount of type-$i$ effort than the intermediate price, the manager will exert more type-$i$ effort per-unit of weight placed on the terminal value than on the intermediate price.

Finally, if $\omega_p = 0$ and $\omega_v > 0$, then the equilibrium levels of effort will be proportional to the first-best levels (specified in Lemma 1); the agency problem uniformly reduces effort, but does not reduce the effort of one project relatively more than the other. However, if $\omega_p > 0$, then there will exist variation in the relative reduction of one type of project compared to the other due to differences in the extent to which the market reflects information on the two types of projects. The size of this distortion depends upon the variation in $\lambda_i$ and the amount of weight $\omega_p$ placed on the near-term price. Thus, incentive compensation based on near-term price is potentially distortive.
D. Optimal Speculative Trades

Notice that the information structure assumed above is such that signals provide information on both the managers effort (i.e., the component $\gamma_i e_i$ in the signal $\theta_i = \gamma_i e_i + \eta_i + \varepsilon_i$) and the component of the outcome that is beyond the manager’s control (i.e., $\eta_i$). Given the noise $\varepsilon_i$ in the signal, informed traders optimally extract the information content of the signal with respect to both $\gamma_i e_i$ and $\eta_i$, both of which have relevance with respect to future value. However, with respect to the manager’s information set, both $\varepsilon_i$ and $\eta_i$ represent risk to the manager. While the optimal extraction of the information content from the signal by informed traders minimizes (but cannot eliminate) the effect of the noise, the informed traders want to use the information in the signal on $\eta_i$. Thus, there is a benefit and a cost associated with the number of informed traders in a particular type of signal being high: the price reflects less of the noise (which makes contracts written on the near-term price more efficient and, thus, less costly) but it also reflects more information about $\eta_i$ (which makes contracts written on the near-term price riskier to the manager and, thus, more costly).

Given optimal levels of effort for the short- and long-term projects as a function of the price function parameters and the incentive contract, the expectation of the terminal residual value conditional on the net order flow can be calculated under the assumption that optimal informed trades are linear in the signals. Given these conditional expectations, it can then be verified that the optimal demands of informed traders are indeed linear in their signals. This then allows us to characterize the equilibrium price function and the resulting incentives they provide. Lemma 4 below specifies the optimal trades of informed traders given a price function of the form specified in equation (15); Lemma 5 (in the next sub-section) specifies the equilibrium price function given these optimal demands.

**LEMMA 4:** The optimal trade $X_i^*$ of an individual trader who observes $\theta_i$ (for $i \in S, L$) is
as follows:

\[ X^*_i = \frac{\frac{W_i}{\delta_i} E[\gamma_i^2 - \bar{\gamma}_i^2 | \theta_i]}{\lambda(N_i^I + 1)} + E[\eta_i | \theta_i] \]  

(17)

where

\[ E[\gamma_i^2 - \bar{\gamma}_i^2 | \theta_i] = \mu_i \left( \theta_i - E[\theta_i] \right) = \mu_i \left( \frac{W_i}{\delta_i} (\gamma_i^2 - \bar{\gamma}_i^2) + \eta_i + \varepsilon_i \right) \]  

(18)

\[ E[\eta_i | \theta_i] = \mu_{\eta_i} \left( \theta_i - E[\theta_i] \right) = \mu_{\eta_i} \left( \frac{W_i}{\delta_i} (\gamma_i^2 - \bar{\gamma}_i^2) + \eta_i + \varepsilon_i \right) \]  

(19)

\[ \mu_i \equiv \frac{\frac{W_i}{\delta_i}}{\frac{1}{\sigma^2_{\gamma_i}}} \frac{2 \sigma^2_{\gamma_i} + \sigma^2_{\eta_i} + \sigma^2_{\varepsilon_i}}{2 \sigma^2_{\gamma_i} + \sigma^2_{\eta_i} + \sigma^2_{\varepsilon_i}} \]  

(20)

\[ \mu_{\eta_i} \equiv \frac{\frac{W_i}{\delta_i}}{\frac{1}{\sigma^2_{\eta_i}}} \frac{2 \sigma^2_{\eta_i} + \sigma^2_{\gamma_i} + \sigma^2_{\varepsilon_i}}{2 \sigma^2_{\gamma_i} + \sigma^2_{\eta_i} + \sigma^2_{\varepsilon_i}} \]  

(21)

*Proof*: See the appendix.

Given these demands, the collective trades of the informed traders imply that the price varies with the signals according to

\[ \lambda(N_i^I X^*_S + N_i^L X^*_L) = \lambda_S (\theta_S - E[\theta_S]) + \lambda_L (\theta_L - E[\theta_L]), \]

where \( \lambda_S = \left( \frac{N_i^I}{N_S^I} \right) \left( \frac{1-\omega_v}{1+\omega_p} \right) \left( \frac{W_S}{\delta_S} \mu_S + \mu_{\eta S} \right) < 1 \) and \( \lambda_L = \left( \frac{N_i^I}{N_L^I} \right) \left( \frac{1-\omega_v}{1+\omega_p} \right) \left( \frac{W_L}{\delta_L} \mu_L + \mu_{\eta L} \right) < 1 \).

Thus, the relative sensitivity of the price to each of the two types of signals (i.e., \( \lambda_S \) relative to \( \lambda_L \)) depends on (1) the number of each type of informed traders (i.e., the size of \( \frac{N_i^I}{N_S^I+1} \) relative to \( \frac{N_i^L}{N_L^I+1} \)) and (2) the size of \( \frac{W_S}{\delta_S} \mu_S + \mu_{\eta S} \) relative to \( \frac{W_L}{\delta_L} \mu_L + \mu_{\eta L} \). The following corollary specifies which type of signal is reflected most in the price.

**COROLLARY 1**: If the short- and long-term projects are the same (in terms of \( \delta_i, N_i^I, \sigma^2_{\gamma_i}, \bar{\gamma}_i^2, \) and \( \sigma^2_{\eta_i} \)) except that the long-term signal is noisier than the short-term signal (i.e., \( \sigma^2_{\varepsilon_L} > \sigma^2_{\varepsilon_S} \)), then for a given set of contract weights \( (\omega_p, \omega_v) \), the price at \( t = 1 \) will reflect more of the signal on the short-term project than the signal on the long-term project.

*Proof*: See the discussion below.
The values of $\mu_i$ and $\mu_{\eta_i}$ quantify the sensitivity of the expectations of $\gamma_{2i}^2$ and $\eta_i$, respectively, to the signal $\theta_i$. The higher the quality of the signal on project $i$, the less noise in the signal (i.e., $\sigma_{\varepsilon_i}^2$ is smaller) and the greater $\mu_i$ and $\mu_{\eta_i}$. Importantly, the sensitivity of informed expectation is also increasing in the contract weights $(\omega_p, \omega_v)$ through $W_i \equiv \omega_p \lambda_i + \omega_v$. Thus, informed expectations depend on the contract and, ceteris paribus, will be relatively more sensitive to the short- (long-) term signal if $\lambda_S > (\leq) \lambda_L$. That is, a larger $\lambda_i$ reinforces itself. Thus, if $\sigma_{\varepsilon_L}^2 > \sigma_{\varepsilon_S}^2$, we have $\mu_S > \mu_L$ and $\mu_{\eta_S} > \mu_{\eta_L}$. Then, $\lambda_S$ will be large (due to the direct effect), which will further increase $\lambda_S$, resulting in the price reflecting a large amount of the short-term project signal. When that happens, the manager will have a more intense incentive to exert effort on the short-term project (relative to the long-term project) since $\lambda_S > \lambda_L$.

Of course, since the profits to be gained and the costs to be incurred vary across the types of signals, $N^I_S$ need not equal $N^I_L$. For example, if, with $N^I_S = N^I_L$ and $\sigma_{\varepsilon_L}^2 > \sigma_{\varepsilon_S}^2$, the price reflects more of the short-term signal, then the profits to be gained from collecting and trading on the short-term signal will be less than those associated with the signal on the long-term project. As a result, unless the cost of the signal on the short-term project is less than that for the long-term project, then either speculators collecting short-term information will have to exit or speculators collecting long-term information will enter, resulting in $N^I_S < N^I_L$. This then results in the price reflecting less short-term information and more long-term information which could improve the efficacy of the price at inducing effort on the long-term project.

Lemma 5 in the next section specifies the equilibrium price function for given numbers of short-term and long-term informed traders. Given that price function, the expected trade profits for each type of trader can be derived. As will be shown, the aggressiveness with which each type of informed trader trades on their information depends upon the contract weights. This implies that the contract weights will have an influence on the informativeness of the price with respect to each type of signal, which will affect the cost of inducing effort.
via the near-term price versus the long-term value.

E. Equilibrium Price Function

Given the demands specified in Lemma 3, the price function at \( t = 1 \) is provided next.

**LEMMA 5**: The price function at \( t = 1 \) is

\[
p = p_0 + \lambda_S \gamma_S (e_S - \bar{e}_S) + \lambda_L \gamma_L (e_L - \bar{e}_L) + \hat{\eta}_S + \hat{\epsilon}_S + \hat{\eta}_L + \hat{\epsilon}_L + z
\]

where

\[
p_0 = \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left[ \bar{v}_S + \bar{v}_L \right] - \frac{\omega_0}{1 + \omega_p}
\]

with, for \( i \in S, L \)

\[
\bar{e}_i = \frac{\gamma_i}{\delta_i} W_i, \quad \bar{v}_i = \frac{\gamma_i^2}{\delta_i} W_i, \quad \hat{\eta}_i = \lambda_i \eta_i, \quad \hat{\epsilon}_i = \lambda_i \epsilon_i, \quad z \equiv \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \Gamma Z
\]

\[
\Gamma = \frac{1}{\sigma_Z} \left[ \sum_{i \in (S, L)} \left\{ \left( \frac{W_i}{\lambda_i} \right)^2 \sigma_{\gamma_i}^2 + \sigma_{\eta_i}^2 \right\} \left( \frac{N_i^I M_i}{N_i^I + 1} \right) \left( 1 - \frac{N_i^I M_i}{N_i^I + 1} \right) \right]^{1/2}
\]

\[
M_i \equiv \frac{W_i}{\delta_i} \mu_i + \mu_{\eta_i}
\]

and, \( \lambda_i \) is the fixed point value that satisfies the following expression (given that \( \mu_i \) and \( \mu_{\eta_i} \) from Lemma 4 depend upon \( \lambda_i \) via \( W_i \) defined in Lemma 3):

\[
\lambda_i = \frac{\left( \frac{N_i^I}{N_i^I + 1} \right) \left( \frac{\omega_{\eta_i}}{\delta_i} \mu_i + \mu_{\eta_i} \right)}{\left( \frac{1 + \omega_p}{1 - \omega_v} \right) - \left( \frac{N_i^I}{N_i^I + 1} \right) \left( \frac{\omega_{\eta_i}}{\delta_i} \mu_i \right)}
\]

**Proof**: See the appendix.

The equilibrium level of \( \lambda_i \) depends upon the number of informed speculators trading on information in project \( i \) according to the following corollary.
COROLLARY 2: The value of $\lambda_i$, the sensitivity of the price to speculators expectations of the manager’s effort in project $i$, increases with the number of speculators receiving information about project $i$.

Thus, if there are more speculators obtaining information on the short-term project, then the price will reflect more of their expectation of the manager’s short-term effort. As a result, if there are more speculators obtaining short-term information on the productivity of the short-term project, since the near-term price is more sensitive to that information, the near-term price induces the manager to exert (relatively) more effort on the short-term project. As is discussed in Section F below, the question is whether the manager’s greater focus on the short-term project induces speculators to also focus more on the short-term project (i.e., inducing more speculators to collect information on the short-term project). If so, then the increase in the number of speculators collecting short-term information reinforces the incentive of the manager to exert relatively more effort on the short-term project.

Before we examine how the expected profits of speculators depend upon the contract and the quality of information available, we first need to relate the initial price of the firm (i.e., the expected residual value) to the contract and the price function parameters. This will allow us to see the trade-off shareholders face when setting the contract weights, given the potential effects of the contract weights on the price process. Given the form of the price function (as in equation (22)), the expected residual value can be written as a function of the parameters of the price function and the incentive contract. Since the participation constraint will bind, $\omega_0$ will be a function of $\omega_p$ and $\omega_v$ (and the exogenous parameters), and the optimal contract will simply be a function of $\omega_p$ and $\omega_v$ (with $\omega_0$ set as a function of $\omega_p$ and $\omega_v$). Specifically, we have the following result:

LEMMA 6: For $i = S, L$, and for given values of $\omega_p$ and $\omega_v$, the fixed wage that satisfies the

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The participation constraint is
\[
\omega^* = (1 + \omega_p) \left\{ \frac{\gamma}{2} \left[ \sum_{i \in \{S,L\}} \left( W^2_i \sigma^2_{\eta_i} + (\omega_p \lambda_i)^2 \sigma^2_{\varepsilon_i} \right) + \omega^2_p \sigma^2_z + \omega^2_v \sigma^2_{\eta_T} \right] \right\} + \frac{1}{2} \sum_{i \in \{S,L\}} \left( \frac{\tilde{\gamma}^2_i}{\delta_i} W^2_i \right) - \omega_p \left( \frac{1 + \omega_p}{1 - \omega_v} \right) \sum_{i \in \{S,L\}} \frac{\tilde{\gamma}^2_i}{\delta_i} W_i - \omega_v \sum_{i \in \{S,L\}} \frac{\tilde{\gamma}^2_i}{\delta_i} W_i \right\} \]  

The expected residual value \( E[RV] \) is given by

\[
E[RV] = E[v] - E[w] = \frac{1}{2} \left[ \sum_{i \in \{S,L\}} \frac{\tilde{\gamma}^2_i}{\delta_i} \left( \omega_p \lambda_i + \omega_v \right) \right] - \frac{\gamma}{2} \left\{ \sum_{i \in \{S,L\}} \left[ (\omega_p \lambda_i + \omega_v)^2 \sigma^2_{\eta_i} + (\omega_p \lambda_i)^2 \sigma^2_{\varepsilon_i} \right] + \omega^2_p \sigma^2_z + \omega^2_v \sigma^2_{\eta_T} \right\} \]

where \( \lambda_i \) characterizes the price function in the securities market (see Lemma 5).

The expected residual value consists of fractions of each component of the first-best expected residual value minus the risk premium that must be paid to the manager to compensate for the risk in the incentive compensation. Although the expected gross future value is increasing in both \( \omega_p \) and \( \omega_v \), the risk premium is also increasing in these contract parameters. A given increase in \( \omega_p \) increases the expected terminal value \( E[v] \) by \( \frac{\tilde{\gamma}^2_S}{\sigma^2_S} \lambda_S + \frac{\tilde{\gamma}^2_L}{\sigma^2_L} \lambda_L \). Thus, an increase in the sensitivity of the manager’s compensation to the near-term price increases the expected terminal value via both the short-term and long-term project. However, since the near-term price is affected by liquidity shocks, the impact is moderated by \( \lambda_S \) and \( \lambda_L \). As a result, a same-sized increase in \( \omega_v \) increases the expected terminal value by more (since \( \frac{\tilde{\gamma}^2_S}{\sigma^2_S} + \frac{\tilde{\gamma}^2_L}{\sigma^2_L} > \frac{\tilde{\gamma}^2_S}{\sigma^2_S} + \frac{\tilde{\gamma}^2_L}{\sigma^2_L} \) ).
As is standard in a Kyle-type model, for given numbers of informed investors of each type, the variance of the price is not a function of the variance of liquidity trades; that is

$$\sigma_z^2 = \left(1 - \omega_v\right)^2 \Gamma^2 \sigma_Z^2 = \left(1 - \omega_p\right)^2 \left[ \sum_{i \in \{S,L\}} \left\{ \left[ \frac{W_i}{\delta_i} \right]^2 \sigma_{\gamma_i}^2 + \sigma_{\eta_i}^2 \right\} \left( \frac{N_i^I M_i}{N_i^I + 1} \right) \left(1 - \frac{N_i^I M_i}{N_i^I + 1}\right) \right]$$

and \(\lambda_i\) is not a function of \(\sigma_z^2\). This is true because the informed traders alter the aggressiveness with which they trade on their information in response to changes in the amount of noise created by liquidity traders. As a result, for fixed numbers of informed investors, the component of the manager’s compensation based on the near-term price is not risky due to the noise created by liquidity shocks.

However, two sources of variability in the near-term price create risk in the manager’s compensation. First, the price variance is a function of the noise in the informed traders’ signals \(\varepsilon_i\); greater noise in the signal generates less volatility in the price since traders trade less aggressively on lower quality (i.e., more noisy) signals. Second, the number of speculators choosing to get information on each type of project affects the information content of their collective trades, which affects the extent to which the price reflects the signals of the manager’s effort in each project. The greater the number of speculators collecting information on a particular project, the more intense is the competition and the more their collective trades reflect the signal (and its noise). On net, if there are more speculators informed about the short-term project than the long-term project, then the variance of expected short-term effort conditional on the price is smaller than the conditional variance of long-term effort, making it relatively cheaper for a contract written solely on the near-term price to induce short-term effort. Thus, if the market sustains a higher number of informed traders in a particular type of project, then that type of project will be “encouraged” in the sense that it is relatively more efficient, given the cost of inducing effort, to induce effort in that project. Again, a critical variable in determining if the short- or long-term project is excessively encouraged by the market is the equilibrium number of short- or long-term
informed speculators, which is taken up in the next section.

When $\sigma^2_{\eta_T} = 0$, the optimal contract will solely consist of weight placed on the terminal value. That is, for any value of $\lambda_i > 0$, the expected residual value is maximized at $\omega_p = 0$. When $\sigma^2_{\eta_T} > 0$, solely placing weight on the terminal value will be expensive since the manager is forced to bear risk not present in the near-term price. Consequently, it is optimal to place some weight on $p$. In that event, the composition of the investors in terms of the type of signals they collect affects the efficacy of the price.

Also note that for a given set of contract weights, the expected residual value is quadratic and concave in the values of $\lambda_S$ and $\lambda_L$. Thus, for a given set of contract weights, there exists a maximal expected residual value. Figure 1 depicts how the expected residual value of the firm depends upon the price parameters $\lambda_S$ and $\lambda_L$ for a given contract. Figure 1 shows the level sets for various values of the expected residual value given variation in $\lambda_S$ and $\lambda_L$ when the short-term signal is less noisy than the long-term signal. Note that, given Corollary 2, such variation in $\lambda_S$ and $\lambda_L$ may be due to variation in the number of speculators obtaining short- and long-term signals. The arrows in the figure show the direction of increasing value. Since both $\lambda_S$ and $\lambda_L$ are less than 1, the level sets are depicted only under this restriction. Also, when the short-term signal is less noisy than the long-term signal, the maximal expected residual value is at the dot, where $\lambda_S > \lambda_L$. In that case, Lemma 3 implies that the manager exerts more effort on the short-term project than on the long-term project.

F. Equilibrium in the Information Market and Expected Trade Profits

The problem considered here is different from the problem in which there is a short-term price (which depends upon the liquidity trading in the short-term market) that reflects information about the short-term project and a long-term price (which depends upon the liquidity trading in the long-term market) that reflects information about the long-term project. In that case, the difference in the expected profits of the short-term and long-
term informed traders will depend upon the difference of the variance of liquidity trading in
the short-term and long-term markets. Specifically, those markets with a higher amount of
liquidity trading variance will sustain a higher number of informed traders, resulting in that
market reflecting more information about the effort for that market’s project; then, that
project will be the cheapest to generate incentives for.

But, in our model, and in actual markets in which shares in multi-project firms are traded,
separate markets for each project do not exist. Rather, there is a single set of liquidity trades
for a single firm that is comprised of both short-term and long-term projects. Thus, this
single set of liquidity trades is responsible for generating profitable trade opportunities for
informed traders who collect information on either (or both) the short-term or long-term
project. The issue, which is taken up next, is to identify the features in the near-term price
process that creates differential incentives to collect the different types of information.

Whether or not an investor obtains a specific type of signal for a specific firm depends
upon the expected profit and the cost associated with obtaining that signal. The equilibrium
number of investors who obtain the short-term signal (denoted $N_{IS}^*$) and the equilibrium
number of investors who obtain signals of the long-term project (denoted $N_{IL}^*$) must be such
that

$$E[\Pi_{IS} | N_{IS}^*] > C_S > E[\Pi_{IS} | N_{IS}^* + 1]$$

(25)

and

$$E[\Pi_{IL} | N_{IL}^*] > C_L > E[\Pi_{IL} | N_{IL}^* + 1]$$

(26)

That is, the equilibrium number of speculators of each type must be such that the last
speculator of either type to enter can cover the cost of obtaining their signal (i.e., no exist)
but that any additional speculator that enters will make insufficient trade profits to cover
the cost of the signal (no entry).
Given the price function from Lemma 5, expected trade profits are given by the following lemma.

**LEMMA 7:** The expected profit of an informed trader who observes the signal $\theta_i$ is

$$E[\Pi_i | \theta_i] = E[X_m^I (v' - p)] = (1 - \omega_v) \left\{ \left( \frac{W_i}{\delta_i} \right) \mu_i + \mu_n \right\} \left( \frac{W_i}{\delta_i} \Delta_i + \mu_i + \eta_i + \varepsilon_i \right) \right\} \equiv E\left[ X_{1m}^I \left( v' - p \right) \right]$$

where $\Delta_i \equiv \gamma_i^2 - \bar{\gamma}_i^2$, for $i = S$ or $L$. Integrating over the possible values of the signal $\theta_i$, the unconditional expected profit for a trader who will obtain a type-i signal ($i = S$ or $L$) is

$$E[\Pi_i] = (1 - \omega_v) \left( \frac{W_i}{\delta_i} \right)^2 \sigma^2_{\gamma_i} + \sigma^2_{\eta_i} + \sigma^2_{\varepsilon_i} \left\{ \sum_{i \in (S, L)} \left[ \left( \frac{W_i}{\delta_i} \right)^2 \sigma^2_{\gamma_i} + \sigma^2_{\eta_i} \right] \left( \frac{N_i^I M_i}{N_i^I + 1} \right) \left( 1 - \frac{N_i^I M_i}{N_i^I + 1} \right) \right\}^{1/2} \frac{1}{\Gamma(N_i^I + 1)^2} \Gamma(N_i^I + 1)^2$$

$$= \frac{(1 - \omega_v) M_i^2 \sigma^2_Z}{(N_i^I + 1)^2} \left\{ \sum_{i \in (S, L)} \left[ \left( \frac{W_i}{\delta_i} \right)^2 \sigma^2_{\gamma_i} + \sigma^2_{\eta_i} \right] \left( \frac{N_i^I M_i}{N_i^I + 1} \right) \left( 1 - \frac{N_i^I M_i}{N_i^I + 1} \right) \right\}^{1/2} \cdot M_i \sigma_Z$$

$$= \frac{(1 - \omega_v) M_i \sigma^2_Z \sqrt{2 \sqrt{\frac{G_i}{\bar{H}_i}}}}{(N_i^I + 1)^2}$$

where $G_i \equiv \left( \frac{W_i}{\delta_i} \right)^2 \sigma^2_{\gamma_i} + \sigma^2_{\eta_i}$, $\bar{H}_i \equiv \frac{H_{-i} + \left( \frac{G_{-i}}{2} \right) H_{-i}}{2}$, and $H_i \equiv \left( \frac{N_i^I M_i}{N_i^I + 1} \right) \left( 1 - \frac{N_i^I M_i}{N_i^I + 1} \right)$

**Proof:** See the appendix.

As can be verified by equation (28), the expected profit of either type of informed trader is increasing in the variance of liquidity trades $\sigma^2_Z$. If the variance of liquidity trades increases, then the number of informed traders in each type of signal will increase until the increased profits generated by the greater amount of liquidity trading is competed away and the expected profit from obtaining information on project $i$ drops to the signal cost $C_i$. Furthermore, as in the Kyle (1985) model, as the number of informed in a particular
signal increases, the market price reflects more of the information content of the signal these informed traders obtain. Thus, as $\sigma_Z^2$ increases, the number of informed in both the short- and the long-term projects increases, making the price more informative with respect to both short- and long-term effort, which improves the efficacy of contracts written on the near-term price at inducing the manager to exert effort. This result is similar to that in Holmstrom and Tirole (1993), except that here we have two projects (and the price creates the incentive for the manager to exert effort on both projects rather than just one).

The following set of corollaries show how the expected profits depend upon the characteristics of the signals and the contract.

COROLLARY 3: If $N^I_S = N^I_L = N$ and the short- and long-term projects are the same (i.e., $\sigma_{\gamma_S}^2 = \sigma_{\gamma_L}^2$, $\delta_S = \delta_L$, $\sigma_{\varepsilon_S}^2 = \sigma_{\varepsilon_L}^2$) except that the $i$-th project has more uncertainty beyond the manager’s control (i.e., $\sigma_{\eta_i}^2 > \sigma_{\eta_{-i}}^2$), then $\lambda_i > \lambda_{-i}$, $G_i > G_{-i}$, $M_i > M_{-i}$, $H_i > H_{-i}$, $\bar{H}_i < \bar{H}_{-i}$, $E[\Pi_i] > E[\Pi_{-i}]$, $\frac{\partial E[\Pi_i]}{\partial \omega_v} < \frac{\partial E[\Pi_{-i}]}{\partial \omega_v}$ and $\frac{\partial E[\Pi_i]}{\partial \omega_p} < 0$, and $\frac{\partial E[\Pi_i]}{\partial \omega_v} < \frac{\partial E[\Pi_{-i}]}{\partial \omega_p}$ and $\frac{\partial E[\Pi_i]}{\partial \omega_p} < 0$.

Since $E[\Pi_i] > E[\Pi_{-i}]$ when $N^I_S = N^I_L = N$, then it must be the case that $N^I_i > N^I_{-i}$ if the information market is in equilibrium. As a result, the price will reflect more project $i$ information, inducing the manager to exert relatively more effort on project $i$ than on $-i$.

For example, if there is more uncertainty beyond the manager’s control in the long-term project, ceteris paribus, the information market will induce more effort in the long-term project. But, the corollary also indicates that if the Board increases either $\omega_v$ or $\omega_p$ (perhaps in an attempt to induce the manager to exert more effort on one or both projects), then there will be a relative drop in the expected profits associated with trading on the signal in the project with the greater uncertainty beyond the manager’s control. This will result in an exit of speculators obtaining that signal, resulting in the near-term price being a less precise indicator of the effort exerted on that project and, as a result, less effective at inducing effort on that project. Thus, an increase in $\omega_v$ or $\omega_p$ reduces the relative distortion in the incentive to exert effort on the project with the greater uncertainty beyond the manager’s control.

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The above corollary assumes that the amount of noise in the short- and long-term project signals is the same. The following corollary considers when signal noise varies by project type.

COROLLARY 4: If $N_L^I = N_S^I = N$ and the short- and long-term projects are the same (i.e., $\sigma_{I_S}^2 = \sigma_{I_L}^2 = \sigma_I^2, \delta_S = \delta, \sigma_{I_S}^2 = \sigma_{I_L}^2 = \sigma_I^2$) except that the $i$-th signal is noisier (i.e., $\sigma_{\varepsilon_i}^2 > \sigma_{\varepsilon_{-i}}^2$), then $\lambda_{-i} < \lambda_i, G_{-i} < G_i, M_{-i} < M_i, H_{-i} < H_i, \bar{H}_{-i} > \bar{H}_i, E[\Pi_{-i}] < E[\Pi_i], \frac{\partial E[\Pi_{-i}]}{\partial \omega_v} > \frac{\partial E[\Pi_i]}{\partial \omega_v}$

For example, if there is more noise in the signal of the long-term project, then an increase in $\omega_v$ will make the expected profit associated with trading on the long-term signal increase. As a result, an increase in $\omega_v$ results in more long-term informed and fewer short-term informed. Thus, the statement that there will be less of a bias toward short-term projects if more weight is placed on far-term price is correct. But, increasing weight on the far-term price may result in the manager’s compensation being riskier (due to $\eta_T$). Thus, although there will be more long-run informed speculators making the intermediate price more informative with respect to the long-term project, that increased information content comes from placing relatively more weight on the far-term value rather than on the near-term price.

COROLLARY 5: The unconditional expected profit $E[\Pi_i]$ of speculators receiving a type-$i$ signal is decreasing in the number of speculators ($N_I^i$) receiving that signal and increasing in the number of speculators ($N_{-i}^I$) receiving the other type of signal.

Note that $\frac{\partial H_i}{\partial N_I^i} < 0$ for $N_I^i > 1$. Thus, $\frac{\partial H_i}{\partial N_I^i} < 0$ and $\frac{\partial \bar{H}_i}{\partial N_{-i}^i} < 0$. Then, since $E[\Pi_i]$ is decreasing in $\bar{H}_i$, we have $\frac{\partial E[\Pi_i]}{\partial N_{-i}^i} > 0$, $\frac{\partial E[\Pi_i]}{\partial N_I^i} < 0$ since the effect of an increase in $N_I^i$ on the $(N_I^i + 1)^2$ term in the denominator of the expected profit is larger than the effect of an increase in $N_I^i$ on the drop in $\bar{H}_i$ in the denominator of the ratio under the radical in the expected profit expression.

COROLLARY 6: For any given contract and liquidity trade variance, there are multiple equilibria in the information market. In particular, there are multiple pairs of $N_L^I$ and $N_S^I$...
that satisfies the conditions for equilibrium in the information market. The set of equilibrium $N^I_L$ and $N^I_S$ pairs is such that there is an inverse relationship between the equilibrium number of speculators obtaining one type of signal and the number obtaining the other type of signal. That is, there is a crowding out in the sense that if in one equilibrium there are more speculators getting one type of signal than in another equilibrium, then the number of speculators getting the other type of signal is less than in the other equilibrium. So, if the market is currently in an equilibrium in which there are many speculators obtaining the signals on the short-term project, then there will be relatively few speculators collecting information on the long-term project.

Consider a combination $(N^I_S, N^I_L)$ such that $E[\Pi_S|N^I_S, N^I_L, (\omega_p, \omega_v)] > C_S > E[\Pi_S|N^I_S + 1, N^I_L, (\omega_p, \omega_v)]$ and $E[\Pi_L|N^I_S, N^I_L, (\omega_p, \omega_v)] > C_L > E[\Pi_L|N^I_S, N^I_L + 1, (\omega_p, \omega_v)]$. Holding fixed $N^I_L$, if the value of $N^I_S$ is increased (to $N'^I_S > N^I_S$), then $E[\Pi_S|N'^I_S, N^I_L, (\omega_p, \omega_v)]$ drops. If this drop is large enough such that $E[\Pi_S|N'^I_S, N^I_L, (\omega_p, \omega_v)] < C_S$, then the equilibrium with $N^I_L$ no longer exits. However, by the lemma above, $E[\Pi_S|N^I_S, N^I_L, (\omega_p, \omega_v)]$ is decreasing in $N^I_L$; thus, for $N'^I_S$, it must be the case that the number of long-term informed speculators must drop in order to raise $E[\Pi_S|N'^I_S, N^I_L, (\omega_p, \omega_v)]$ to be above $C_S$. Thus, there are multiple equilibrium combinations of $N^I_S$ and $N^I_L$, with an inverse relationship between the numbers of speculator informed about each type of project. This, combined with Corollary 2, implies that there is an inverse relationship between $\lambda_S$ and $\lambda_L$.

Let the set of possible combinations of $N^I_S$ and $N^I_L$ such that equations (25) and (26) hold for a given contract pair $(\omega_p, \omega_v)$ be denoted $\mathcal{N}(\omega_p, \omega_v)$. Let the $i$th particular combination be denoted $N^i(\omega_p, \omega_v) \in \mathcal{N}(\omega_p, \omega_v)$. Associated with this particular combination is an expected residual value (or initial price for the firm) $ERV(N^i(\omega_p, \omega_v))$. Also let $\Pi^{\max}(\omega_p, \omega_v) = \max_i \left(ERV(N^i(\omega_p, \omega_v))\right)$. Also let $N^{\max}(\omega_p, \omega_v)$ denote the pair of values of $N^i(\omega_p, \omega_v)$ associated with the maximal expected residual value. For any choice of contract $(\omega_p, \omega_v)$, there are multiple equilibrium, only one of which maximizes the expected residual value of the firm. To see this, consider Figure 2. In that figure, the downward sloping dashed
lines indicate combinations of $\lambda_S$ and $\lambda_L$ (associated with the inversely related values of $N_{IS}^I$ and $N_{IL}^I$) that are associated with a particular contract. For each particular contract, each point on the line indicating the combinations of $\lambda_S$ and $\lambda_L$ resulting from equilibrium in the information market is associated with a different level set. The point where the highest level set is tangent to the $\lambda_S$ and $\lambda_L$ combination line is where the expected residual value is maximized; all other points on the $\lambda_S$ and $\lambda_L$ combination line are associated with lower expected residual values.

Corollary 3 and Corollary 6 together imply that if the short- and long-term projects are the same except that $\sigma_{\eta L}^2 \geq \sigma_{\eta S}^2$ and $C_S \leq C_L$, then, for all of the combinations of $N_{IS}^I$ and $N_{IL}^I$ consistent with equilibrium in the information market, it is always true that $N_{IS}^I \geq N_{IL}^I$. Thus, if there is more uncertainty beyond the manager’s control in the long-term project, then the market will “encourage” effort in the short-term project because the market will reflect more information on the short-term project. Similarly, Corollary 4 and Corollary 6 imply that if the only difference between the short- and long-term projects is $\sigma_{\varepsilon_L}^2 > \sigma_{\varepsilon_S}^2$ and $C_S \leq C_L$, then the information market equilibrium is such that $N_{IS}^I \geq N_{IL}^I$. Again, in this case, the market encourages the short-term project over the long-term project. It must be stressed, however, that in both of these cases, the market encourages the short-term project over the long-term project independent of the expected productivity of the manager’s effort in each of these projects. This, however, does not necessarily imply that the outcomes are inefficient; if the short-term signals are less noisy, then, given the cost of inducing effort, it may make sense to encourage short-term effort. The question is whether the market encourages short-term effort too much. The next section considers the optimality of equilibrium.

IV. The Equilibrium Incentive Contract

Lemma 7 and Corollary 6 above imply that if the number of informed speculators can adjust quickly, then the firm cannot, by picking the compensation contract alone, maximize the expected residual value of the firm. If the firm picks a particular contract, there will be
multiple pairs \((N^S, N^L)\) such that the expected trade profits just cover the information collection costs. For example, consider Figure 3 below. For a given set of primitive parameters (e.g., \(\gamma, \sigma^2, \epsilon_i\), etc) denoted \(\Omega \in \Omega\) (where \(\Omega\) is the set of all possible parameters), the line labeled \(E(\Pi|\omega_p, \omega_v), \Omega_l = 0\) consists of \((N^S, N^L)\) pairs such that equations (25) and (26) hold when the contract is \((\omega_p, \omega_v)\). (Although these pairs are depicted by a continuous line, since the number of speculators is discrete, the actual relationship is a set of discrete points on that line.)

All along (the set of discrete points on) \(E(\Pi|\omega_p, \omega_v), \Omega_l = 0\), the expected residual value of the firm varies, along with market liquidity and, as a result, the information content of the market price with respect to effort exerted on the two types of projects. Thus, if the number of informed speculators is not fixed, then any choice of contract \((\omega_p, \omega_v)\) merely determines a set of possible expected residual values for the firm (depending upon the specific \((N^S, N^L)\) pair that obtains). If there is a change in the underlying environment (say from \(\Omega = \Omega_0\) to \(\Omega = \Omega_1\)), there will be a shift in the set of \((N^S, N^L)\) pairs that satisfy equations (25) and (26) from \(E(\Pi|\omega_p, \omega_v), \Omega_0 = 0\) to \(E(\Pi|\omega_p, \omega_v), \Omega_1 = 0\) . (In the case depicted, the change in the underlying environment causes expected profits to increase (for any given value of \((N^S, N^L)\)).

Given the change in the underlying environment, the change in the endogenous variables that occurs depends upon which variables can adjust more quickly. For example, consider what happens if, in the short run, (1) the contract is fixed (that is, the contracts are legal documents that take time to renegotiate) and (2) the total number of speculators who are trained to interpret and trade on information is fixed. While the total number of speculators is fixed at \(N^{Tot}_0 = N^S_0 + N^L_0\), it is likely that some long-term speculators may be able to quickly become short-term speculators (and vice versa). The line labeled \(N^{Tot}_0\) denotes all of the combinations of \(N^S\) and \(N^L\) such that \(N^{Tot}_0 = N^S + N^L\). Then, given the change in underlying environment from \(\Omega_0\) to \(\Omega_1\), the short-run outcome will be where \(E(\Pi|\omega_p, \omega_v), \Omega_1 = 0\) crosses \(N^{Tot}_0\). In Figure 3, since the \(E(\Pi|\omega_p, \omega_v), \Omega_l\) lines are steeper than the \(N^{Tot}_0\) line,
the change in the environment that increases profits results in at least a short-run increase
in the number of long-term speculators and a drop in the number of short-term speculators.
But, note that, as in Figure 4, if the \( E(\Pi|(\omega_p, \omega_v), \Omega_l) \) lines are flatter than the \( N_0^{Tot} \) line,
then such an environmental change creates a short-run increase in short-term speculators at
the cost of a drop in long-term speculators.

Given the environmental change, and the resulting movement along \( N_0^{Tot} \), what happens
to the incentive to change other variables that may be adjusted in the long run? For example,
at the new point on \( N_0^{Tot} \), the initial contract (denoted \( (\omega_{p0}, \omega_{v0}) \) ) may be sub-optimal; there
may be an incremental gain in terms of expected residual value to renegotiating the contract
to say \( (\omega_{p1}, \omega_{v1}) \). Such a change will produce another shift in the \( E(\Pi|(\omega_p, \omega_v), \Omega_l) \) line
(since the expected profits from each type of signal depend upon the parameters of the
compensation contract). In fact, the new line may not even cross \( N_0^{Tot} \) at \( (N_0^S, N_0^L) \). Thus,
if, in the short-run, there is a fixity in \( N^{Tot} \) but flexibility in the mixture \( (N^S, N^L) \), then the
shareholders can pick the contract \( (\omega_p, \omega_v) \) that maximizes expected residual value as long as
\( (N^S, N^L) \) is on \( N_0^{Tot} \). That is, for every pair \( (N^S, N^L) \) on \( N_0^{Tot} \), the shareholders can find the
contract \( (\omega_p, \omega_v) \) that produces a \( E(\Pi|(\omega_p, \omega_v), \Omega_l) \) line that crosses \( N_0^{Tot} \) at that \( (N^S, N^L) \)
point. Associated with that contract will be an expected residual value. The shareholders
should find the set of contracts for each \( (N^S, N^L) \) pair on \( N_0^{Tot} \) and pick the single contract
associated with the specific point on \( N_0^{Tot} \) that has the highest expected residual firm value.

Figure 5 illustrates the determination of the equilibrium contract. Let the current
environment be described by \( \Omega_1 \). Consider a particular point on \( N_0^{Tot} : (N_0^S, N_0^L) \). There
are multiple contracts that produce \( E(\Pi|(\omega_p, \omega_v), \Omega_1) = 0 \) lines that go through the point
\( (N_0^S, N_0^L) \). Let this set of contracts be denoted \( \omega(N_0^S, N_0^L) \) and let a particular member of
this set be denoted \( \omega_k(N_0^S, N_0^L) \in \omega(N_0^S, N_0^L) \). Each contract \( \omega_k(N_0^S, N_0^L) \in \omega(N_0^S, N_0^L) \)
produces a different expected residual value. Let the contract \( (\omega_{p0}, \omega_{v0}) \in \omega(N_0^S, N_0^L) \)
be the contract that is associated with the highest expected residual value among the
expected residual values that obtain given this set of contracts. In the figure, the maximal
expected residual value for \((N^S_0, N^L_0)\) is labeled \(E(RV)_0\). Similarly, there exists a contract \((\omega_{p1}, \omega_{v1}) \in \omega(N^S_1, N^L_1)\) that produces a maximal expected residual value (subject to \((N^S_1, N^L_1)\)), labeled in the figure as \(E(RV)_1\). \(E(RV)_2\) is similarly derived. Let, for example, \(E(RV)_0 > E(RV)_1 > E(RV)_2\). Also let \(E(RV)_0\) be the largest of any expected residual value obtainable for all \((N^S, N^L) \in N^T_0\). If the shareholders know that there is a fixity in \(N^T_0\) but not in its composition, then the shareholders will be able to achieve \(E(RV)_0\) by picking the contract \((\omega_{p0}, \omega_{v0})\). That is, the shareholders pick the contract that maximizes expected residual value subject to producing \((N^S_0, N^L_0) \in N^T_0\). (Note this is true only if this contract is unique – that is, if this contract is not also the optimal contract for another combination \((N^S, N^L) \in N^T_0\). If this is not true, then there is an indeterminacy.)

On the other hand, consider what happens if there is less fixity in the total number of speculators than in the contract. Once again consider the change in environment to \(\Omega_1\). For concreteness consider the example in Figure 6, in which the change in environment immediately causes a drop in long-term speculators and an increase in short-term speculators. Before the shareholders can change the contract (which, by assumption, takes time), the expected profits under the old contract are positive and will create the incentive for more speculators or either type to enter. In this case, any of the combinations of \((N^S, N^L)\) on \(E(\Pi|\omega_{p0}, \omega_{v0}, \Omega_1) = 0\) will be feasible in the short-run. However, since there may be start-up or adjustment costs to entering, it is likely that the \((N^S, N^L)\) pairs that obtain will not be extreme relative to the initial pair \((N^S_0, N^L_0)\). For example, as indicated in Figure 6, let the new pair be \((N^S_1, N^L_1)\). Under this pair, however, \((\omega_{p0}, \omega_{v0})\) is likely not the optimal contract. Thus, given \((N^S_1, N^L_1)\), the shareholders will adjust the contract to be optimal, say to \((\omega_{p1}, \omega_{v1})\). But notice that there were many combinations of \((N^S, N^L)\) that are possible, with other combinations generating different optimal contracts and potentially lower expected residual values. That is, the shareholders are at the mercy of the entry decision of speculators, which are not determined by the contract choice of the shareholders. Thus, the shareholders essentially act as if they must take the number of speculators are
given. And if the shareholders have some influence, they have limited control. Thus, the concerns expressed by the *Aspen Institute* are understandable in the context of this model.

V. Corporate and Public Policy Remedies

In this section we examine the efficacy or justifications for a variety of corporate and/or public policies that can produce a Pareto improvement in the equilibrium. To keep the discussion that follows concrete, we focus on the situation in which there are only two differences between the short- and the long-term projects: (1) the noise in the long-term signal is greater than the noise in the short-term signal and (2) the expected productivity of the manager’s effort exerted on the long-term project is greater than that for the short-term project. In this situation, the market encourages the manager to exert more effort on the short-term project even though effort exerted on the long-term project is more productive.

One set of potential remedies is to subsidize the collection of information on the long-term project. Under the conditions specified above, the set of \((N^S, N^L)\) pairs consistent with equilibrium in the information market consist only of pair for which \(N^S > N^L\) since \(C^S = C^L\). In this case, *ceteris paribus*, it may be possible to affect the numbers of speculators of each type to increase expected residual value. There are a couple of possible ways to achieve this possibility. First, the firm could subsidize information collection on the long-term project. It could do this by providing greater access to management concerned with the long-term project relative to managers concerned with the short-term project. If this greater access merely serves to lower the cost of collecting information, this policy is equivalent (from the information collection side) to subsidizing information collection in the long-term project. That is, let \(S_i\) denote the subsidy for the type-\(i\) project, \(i \in \{S, L\}\). Then, by setting \(S_L > 0\) and \(S_S = 0\), the effective cost of collecting information becomes \(C^S > C^L - S_L\). In the case of greater access, the cost to the firm associated with lowering the cost of information collection for the long-term project by \(S_L\) may cost less than \(S_L\); in the direct subsidy case, the firm’s cost will exactly equal \(S_L\). Thus, the firm will opt for a policy (greater access or
direct subsidy) with the lowest cost.

The impact of the above (implicit/indirect or explicit/direct) subsidy is depicted in Figure 7 below. Without the subsidy, the set of equilibria is at the downward sloping dotted line labeled $E(\Pi|\omega_p, \omega_v), \Omega_0, T_S = T_L = S_S = S_L = 0) = 0$ going through point A. With the subsidy, the zero-expected profit combination line shifts right to the line labeled $E(\Pi|\omega_p, \omega_v), \Omega_0, T_S = T_L = S_S = S_L > 0) = 0$, going through point B. If the equilibrium was a point A prior to the offer of the subsidy, then the equilibrium might move to point B. Whether the firm wants to do this or not depends upon whether the increase in expected residual value from A to B is larger than the total cost of the subsidy $N_i L S_L$. In the case shown, such a movement increases expected residual value (the dotted arrows indicate the direction of increasing level curves). Note that from a public policy perspective, if there is any increase in the expected residual value (even if $\Delta ERV < N_i L S_L$), the subsidy satisfies the Pareto Criterion since it is merely a transfer (from shareholders to speculators). Thus, under certain sets of parameters, there is a role for a centralized government.

As an alternative to a subsidy, a tax on short-term speculation will also allow for equilibria with $N_L > N_S$. Let $T_i$ denote the per-signal tax associated with signals in project $i$. If $T_S > 0$ and $T_L = 0$, then the expected zero profit combination line shifts down (as depicted in Figure 7 to the dotted line that goes through point C labeled $E(\Pi|\omega_p, \omega_v), \Omega_0, T_S > 0, T_L = S_S = S_L = 0) = 0$). In this case, the number of short-term speculators fall (from A to C), which reduces the amount of effort exerted on the short-term project induced by the near-term price. Although, without the tax, the amount the near-term price reflects the short-term project is too large relative to the long-term project, a drop in the extent to which the near-term price reflects the manager’s effort on the short-term project does reduce the efficacy of the near-term price at inducing short-term effort. Thus, although such a tax may reduce the relative distortion on short-term versus long-term effort, it nonetheless reduces total effort. As a result, the tax is justified only if the drop in value due to the reduction in total effort is more than offset by a gain created by reducing the relative bias created by the market...
plus the value of any tax revenue (redistributed in a manner independent of the collection of information). In the case depicted in Figure 7, this movement generates an increase in expected residual value. However, Figure 8 shows how a tax on short-term speculation may cause a drop in expected residual value.

Depending upon the circumstances, it may be easier to implement a tax versus a subsidy. For example, in order for the firm to implement an effective subsidy, it will need to tie the subsidy to the collection of long-term signals. Yet, it may be hard to differentiate between types of information. Alternatively, if the gains from trading on information concerning short-term projects will be realized over shorter intervals of time relative to those over which profits form long-term signals are made, then a tax on short-term capital gains and/or a forgiveness of tax on long-term capital gains will achieve the downward shift in the zero expected profit combination line as depicted in Figure 7.

One question is what would be the effect of a policy in which the firm publically discloses more information about one type of project relative to the other. An issue is whether that would potentially lead to an incentive to manipulate such statements (which might make the disclosures – and the resulting near-term price – less informative with respect to a particular type of project). Also, in order for the information to be credible, it must be collected and held privately so that the collection costs can be recouped via trading. If more information is publicly disclosed, then there may actually be less private information reflected in price.

VI. Conclusion

The paper develops a model in which the efficacy of incentive contracts and the information content of prices are jointly determined. Given the interaction between these variables, there are, for every possible contract, multiple combinations of numbers of short- and long-term speculators, with the expected residual value of the firm varying across these combinations. The paper shows that in some equilibria the market excessively encourages managers to exert effort on inferior short-term projects. The effectiveness of regulation designed to moderate
this encouragement will depend upon adjustment costs outside the model.
References


A. Appendix

Proof of Lemma 1: First-Best Results
When effort is contractible, the problem is as follows:

$$\max E[v] - E[w]$$

subject to $E[u] = \bar{u}$, where $\bar{u}$ is the agents reservation utility (which, without a loss of generality, we set to $\bar{u} = 0$). In the first best, the principal pays the agent his disutility level and the principal bears all the risk (the var($w$) = 0). This implies that the wage is simply

$$E(w) = \frac{\delta_S}{2} e_S^2 + \frac{\delta_L}{2} e_L^2$$

The principals problem reduces to

$$\max \gamma_S e_S + \gamma_L e_L - \frac{\delta_S}{2} e_S^2 - \frac{\delta_L}{2} e_L^2$$

The first order conditions with respect to $e_S$ and $e_L$ imply

$$e_S^* = \frac{\gamma_S}{\delta_S} \quad e_L^* = \frac{\gamma_L}{\delta_L}$$

The resulting residual value is thus

$$E[v] - E[w] = \frac{\gamma_S}{2\delta_S} + \frac{\gamma_L}{2\delta_L}$$

QED.

Proof of Lemma 2: The Price function at $t = 1$
The price is simply the conditional expectation of the residual value, which is the terminal value minus the wage payments made to the manager. The realized residual value is simply

$$v' = v - w = v - [\omega_0 + \omega_p p + \omega_i v] = (1 - \omega_i) v - \omega_0 - \omega_p p$$  \hfill (A.1)

The unconditional expectation is thus

$$E[v'] = (1 - \omega_i) E[v] - \omega_0 - \omega_p p_0$$

where $p_0 \equiv E[p]$. The price at $t = 1$ is simply

$$p = p_0 + \lambda \Psi$$  \hfill (A.2)

where $\lambda \Psi = E[v' - E[v']|\Psi]$. Since $p_0 \equiv E[p] = E[v'] = (1 - \omega_i) E[v] - \omega_0 - \omega_p p_0$ we have

$$p_0 = \frac{1}{1 + \omega_p} \left[ (1 - \omega_i) E[v] - \omega_0 \right]$$  \hfill (A.3)

Note that the difference between the realized terminal residual value and its unconditional expectations (i.e., $v' - E[v']$) is

$$v' - E[v'] = v' - p_0 = (1 - \omega_i) v - \omega_0 - \omega_p p_0 - \omega_p \lambda \Psi - p_0$$

$$= (1 - \omega_i) v - \omega_0 - (1 + \omega_p) \left[ \frac{1}{1 + \omega_p} \left[ (1 - \omega_i) E[v] - \omega_0 \right] \right] - \omega_p \lambda \Psi$$

$$= (1 - \omega_i) [v - E[v]] - \omega_p \lambda \Psi$$

Thus

$$\lambda \Psi = E[v' - E[v']|\Psi]$$

$$= E \left[ (1 - \omega_i) [v - E[v]] - \omega_p \lambda \Psi | \Psi \right]$$

$$= E \left[ (1 - \omega_i) [v - E[v]] \right] - \omega_p \lambda \Psi$$

$$= \left[ \frac{1 - \omega_i}{1 - \omega_p} \right] E \left[ v - E[v] \right]$$  \hfill (A.4)
Since $E[E[v]|\Psi] = E[v]$, the price function is simply

$$p = \frac{1}{1 + \omega_p} \left[ \left( 1 - \omega_v \right) E[v] - \omega_0 \right] + \left( \frac{1 - \omega_v}{1 + \omega_p} \right) E[v - E[v]|\Psi]$$

$$= \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left[ E[v] - \frac{\omega_0}{1 - \omega_v} \right] + \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \Gamma \Psi$$

(\text{A.5})

where $\Gamma \Psi = E[v - E[v]|\Psi] = \frac{\text{Cov}(v, \Psi)}{\text{var}(\Psi)} \Psi - E[\Psi]$. (i.e. $\Gamma = \frac{\text{Cov}(v, \Psi)}{\text{var}(\Psi)}$). Thus $p_0 = \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left[ E[v] - \frac{\omega_0}{1 - \omega_v} \right]$, and $\lambda \Psi \equiv \Gamma \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \Psi$ (i.e. $\lambda \equiv \Gamma \left( \frac{1 - \omega_v}{1 + \omega_p} \right)$) QED.

**Proof of Lemma 3: Optimal Short- and Long-term Effort as a Function of the Contract**

We conjecture that the trades of the informed agents are linear in their signals. Specifically, we conjecture that $\Psi$ is linear in the unexpected components of the signals (e.g., $\theta S \equiv E[\theta S] = \gamma S (\varepsilon S - \varepsilon S) + \eta S + \varepsilon S$):

$$\Psi = \pi S (\theta S - E[\theta S]) + \pi L (\theta L - E[\theta L]) + Z$$

Since the price is linear in the net order flow, the price is also linear in the signals. Specifically,

$$p = \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left[ E[v] - \frac{\omega_0}{1 - \omega_v} + \Gamma \Psi \right]$$

$$= \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left[ E[v] - \frac{\omega_0}{1 - \omega_v} + \Gamma \left( \pi S (\theta S - E[\theta S]) + \pi L (\theta L - E[\theta L]) + Z \right) \right]$$

$$= \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left[ \sum_{i=S,L} \left( \gamma_i e_i^* \right) - \omega_0 \right] + \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \Gamma \left\{ \sum_{i=S,L} \left[ \pi_i (\gamma_i (e_i - e_i^* + \eta_i + \varepsilon_i)) \right] + Z \right\}$$

Conditional on the managers information, the price is distributed with the following mean and variance:

$$E[p|\varepsilon S, \varepsilon L] = \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left( \sum_{i=S,L} \left( \gamma_i e_i^* \right) + \sum_{i=S,L} \Gamma \pi_i (\gamma_i (e_i - e_i^*)) \right) - \frac{\omega_0}{1 - \omega_v}$$

$$\text{Var}[p|\varepsilon S, \varepsilon L] = \left( \frac{1 - \omega_v}{1 + \omega_p} \right)^2 \left( \pi S \Gamma^2 (\sigma_{\varepsilon S}^2 + \sigma_{\varepsilon S}^2) + (\pi L \Gamma)^2 (\sigma_{\varepsilon L}^2 + \sigma_{\varepsilon L}^2) + \sigma_Z^2 \right)$$

The incentive compatibility constraint is as follows:

$$E[u] = E[w] - \frac{\gamma}{2} \text{var}(w) - \sum_{i=S,L} \frac{\delta_i}{2} e_i^2$$

where

$$E[w] = \omega_0 + \omega_p E[p|\varepsilon S, \varepsilon L] + \omega_v E[v]$$

$$= \omega_0 + \omega_p \left( \sum_{i=S,L} \left( \gamma_i e_i^* \right) + \sum_{i=S,L} \Gamma \pi_i (\gamma_i (e_i - e_i^*)) \right) - \frac{\omega_0}{1 - \omega_v} + \omega_v \sum_{i=S,L} \gamma_i e_i$$

and

$$\text{Var}[w|\varepsilon S, \varepsilon L] = \omega_p^2 \text{var}(p|\varepsilon S, \varepsilon L) + \omega_v^2 \text{var}(v|\varepsilon S, \varepsilon L) + 2 \omega_v \omega_p \text{cov}(v, p|\varepsilon S, \varepsilon L)$$

$$= \omega_p^2 \left( \frac{1 - \omega_v}{1 + \omega_p} \right)^2 \left( \pi S \Gamma^2 (\sigma_{\varepsilon S}^2 + \sigma_{\varepsilon S}^2) + (\pi L \Gamma)^2 (\sigma_{\varepsilon L}^2 + \sigma_{\varepsilon L}^2) + \sigma_Z^2 \right)$$

$$+ \omega^2 \sum_{i=S,L} \sigma_i^2 + 2 \omega_v \omega_p \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left( \pi S \Gamma \sigma_{\varepsilon S}^2 + \pi L \Gamma \sigma_{\varepsilon L}^2 \right)$$

$$= (W_S)^2 \sigma_{\varepsilon S}^2 + (\omega_p \lambda S)^2 \sigma_{\varepsilon S}^2 + (W_L)^2 \sigma_{\varepsilon L}^2 + (\omega_p \lambda L)^2 \sigma_{\varepsilon L}^2 + \sigma_Z^2 + (\omega_v)^2 \sigma_{\eta T}^2$$

Thus
\[ E[u] = E[w] - \frac{1}{2} \nu(w) - \sum_{i=S,L} \delta_i e_i^2 \]
\[ = \omega_0 + \omega_p \left\{ \left( 1 - \omega_v \right) \left( \sum_{i=S,L} (\gamma_i e_i^*) \right) + \sum_{i=S,L} \Gamma \pi_i (e_i - e_i^*) \right\} - \frac{\omega_0}{1 - \omega_v} + \omega_p \sum_{i=S,L} \gamma_i e_i \]
\[ - \frac{\gamma}{2} \nu(w) - \sum_{i=S,L} \delta_i e_i^2 \]

The FOC, \( \frac{\partial E[u]}{\partial e_i^*} = 0 \), is
\[ \omega_p \gamma_i \lambda_i + \omega_v \gamma_i - \delta_i e_i^* = 0 \]
where \( \lambda_i \equiv \lambda_i \). Thus, the optimal effort level (for \( i = S, L \)) are:
\[ e_i^* = \frac{\gamma_i}{\delta_i} \left[ \omega_p \lambda_i + \omega_v \right] = \frac{\gamma_i}{\delta_i} W_i \]
where \( W_i \equiv \omega_p \lambda_i + \omega_v = \omega_p \pi_i \lambda + \omega_v = \omega_p \left( 1 + \frac{\omega_v}{1 + \omega_p} \right) \pi_i \Gamma + \omega_v \). QED.

**Proof of Lemma 4: The Optimal Trades of an Individual Trader**

The optimization problem faced by a trader who has a signal of the short-term project is
\[ \max X_m^S (v' - p) | \theta_S \]

The objective function can be rewritten as
\[ E \left[ X_m^S \left( (1 - \omega_v) v - \omega_0 - \omega_p p - p \right) | \theta_S \right] \]
\[ = E \left[ X_m^S \left( (1 - \omega_v) v - \omega_0 - (1 + \omega_p) (p_0 + \lambda \Psi) \right) | \theta_S \right] \]
\[ = E \left[ X_m^S \left( (1 - \omega_v) v - \omega_0 - (1 + \omega_p) \psi - (1 + \omega_p) \lambda \Psi \right) | \theta_S \right] \]
\[ = E \left[ X_m^S \left( (1 - \omega_v) v - \omega_0 - \left[ (1 - \omega_v) E[v] - \omega_0 \right] - (1 + \omega_p) \lambda \Psi \right) | \theta_S \right] \]
\[ = E \left[ X_m^S \left( (1 - \omega_v) \left( v - E[v] \right) - (1 + \omega_p) \lambda \Psi \right) | \theta_S \right] \]
\[ = E \left[ X_m^S \left( (1 - \omega_v) \left( v - E[v] \right) - (1 + \omega_p) \Gamma \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \Psi \right) | \theta_S \right] \]
\[ = (1 - \omega_v) E \left[ X_m^S \left( v - E[v] - \Gamma \Psi \right) | \theta_S \right] \]

Thus, the informed traders’ problem is equivalent to
\[ \max X_m^S \left[ v - (E[v] + \Gamma \Psi) \right] | \theta_S \]

Given the optimal effort functions, the problem is
\[ \max X_m^S \left[ \left( 1 \frac{\gamma^2}{\delta_S} WS + \eta_S + \left( 1 \frac{\gamma^2}{\delta_L} WL + \eta_L + \eta_T \right) - (E[v] + \Gamma \Psi) \right) | \theta_S \right] \]

The objective function can be rewritten as
\[
E[\Pi_*|\theta_S] = E\left[X^S_m\left(\frac{\gamma^2}{\delta_S}W_S + \eta_S + \frac{\gamma^2}{\delta_L}W_L + \eta_L + \eta_T\right) - (E[v] + \Gamma \Psi)\right]|\theta_S
\]
\[
= X^S_m\left[\frac{W_S}{\delta_S}E[\gamma^2_S - \gamma^2_S|\theta_S] + E[\eta_S|\theta_S] + \frac{W_L}{\delta_L}E[\gamma^2_L - \gamma^2_L|\theta_S] + E[\eta_L|\theta_S] + E[\eta_T|\theta_S]\right] - \Gamma E[\Psi|\theta_S]
\]
\[
= X^S_m\left[\frac{W_S}{\delta_S}E[\gamma^2_S - \gamma^2_S|\theta_S] + E[\eta_S|\theta_S]\right] - \Gamma \left(X^S_m + (N^S_S - 1)E[X^S_S|\theta_S]\right)
\]

where \(X^S_m\) denotes the demands of the other investors who have the signal for the short-term project. The first order condition is

\[
\frac{\partial E[\Pi_*|\theta_S]}{\partial X^S_m} = \left[\frac{W_S}{\delta_S}E[\gamma^2_S - \gamma^2_S|\theta_S] + E[\eta_S|\theta_S]\right] - \Gamma \left(X^S_m + (N^S_S - 1)E[X^S_S|\theta_S]\right) - X^S_m \Gamma = 0
\]

which implies

\[
X^S_m = \frac{1}{2\Gamma} \left[\frac{W_S}{\delta_S}E[\gamma^2_S - \gamma^2_S|\theta_S] + E[\eta_S|\theta_S]\right] - \Gamma (N^S_S - 1)E[X^S_S|\theta_S]
\]

Since, in equilibrium, all investors that have the signal of the short-term project will have the same demand, the symmetric equilibrium demands are the value \(X^S_S\) such that \(X^S_S = E[X^S_S|\theta_S] = X^S_S\). This value is

\[
X^S_S = \frac{W_S}{\delta_S}E[\gamma^2_S - \gamma^2_S|\theta_S] + E[\eta_S|\theta_S]
\]

\[
(N^S_S - 1)\Gamma
\]

Similarly, the optimal demand of investors who receive the signal of the long-term project is

\[
X^L_* = \frac{W_L}{\delta_L}E[\gamma^2_L - \gamma^2_L|\theta_L] + E[\eta_L|\theta_L]
\]

\[
(N^L_L - 1)\Gamma
\]

We next need expressions for the expectations.

\[
E[\gamma^2_S - \gamma^2_S|\theta_S] = E[\gamma^2_S|\theta_S] = \gamma_S^2 \epsilon_S + \eta_S + \epsilon_S - \gamma_S^2
\]

\[
E[\gamma^2_S|\theta_S] = \frac{W_S}{\delta_S} \gamma_S^2 + \eta_S + \epsilon_S - \gamma_S^2
\]

\[
= \gamma_S^2 + \frac{\text{cov}(\gamma_S, W_S \gamma_S + \eta_S + \epsilon_S)}{\text{var}(\gamma_S, W_S \gamma_S + \eta_S + \epsilon_S)} \left(\frac{W_S}{\delta_S} \gamma_S^2 + \eta_S + \epsilon_S - \frac{W_S}{\delta_S} \gamma_S^2\right) - \gamma_S^2
\]

\[
= \frac{\text{cov}(\gamma_S, W_S \gamma_S + \eta_S + \epsilon_S)}{\text{var}(\gamma_S, W_S \gamma_S + \eta_S + \epsilon_S)} \left(\frac{W_S}{\delta_S} \Delta_S + \eta_S + \epsilon_S\right)
\]

\[
= \left(\frac{W_S}{\delta_S}\right)^2 \frac{\sigma^2_S + \sigma^2_S + \sigma^2_S}{\sigma^2_S + \sigma^2_S + \sigma^2_S} \left(\frac{W_S}{\delta_S} \Delta_S + \eta_S + \epsilon_S\right)
\]

\[
= \mu_S \left(\frac{W_S}{\delta_S} \Delta_S + \eta_S + \epsilon_S\right)
\]

where \(\Delta_S \equiv \gamma^2_S - \gamma_S^2\) and \(\mu_S = \left(\frac{W_S}{\delta_S}\right)^2 \frac{\sigma^2_S}{\sigma^2_S + \sigma^2_S + \sigma^2_S}\). Similarly,

\[
E[\gamma^2_L - \gamma^2_L|\theta_L] = \mu_L \left(\frac{W_L}{\delta_L} \Delta_L + \eta_L + \epsilon_L\right)
\]

where \(\Delta_L \equiv \gamma^2_L - \gamma^2_L\) and \(\mu_L = \left(\frac{W_L}{\delta_L}\right)^2 \frac{\sigma^2_L}{\sigma^2_L + \sigma^2_L + \sigma^2_L}\). We also have

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\[ E[\eta S|\theta S = \frac{W_S}{\delta S} \gamma_S^2 + \eta_S + \epsilon_S] = \frac{\text{cov}\left(\eta_S, \frac{W_S}{\delta S} \gamma_S^2 + \eta_S + \epsilon_S\right)}{\text{var}\left(\frac{W_S}{\delta S} \gamma_S^2 + \eta_S + \epsilon_S\right)} \left(\frac{W_S}{\delta S} \gamma_S^2 + \eta_S + \epsilon_S - \frac{W_S}{\delta S} \gamma_S^2\right) \]
\[ = \frac{\sigma^2_{\eta_S}}{(\frac{W_S}{\delta S})^2 \sigma^2_{S} + \sigma^2_{\eta_S} + \sigma^2_{\epsilon_S}} \left(\frac{W_S}{\delta S} \Delta S + \eta_S + \epsilon_S\right) \]
\[ = \mu_{\eta_S} \left(\frac{W_S}{\delta S} \Delta S + \eta_S + \epsilon_S\right) \]

where \( \mu_{\eta_S} = \frac{\sigma^2_{\eta_S}}{(\frac{W_S}{\delta S})^2 \sigma^2_{S} + \sigma^2_{\eta_S} + \sigma^2_{\epsilon_S}} \). Similarly, with \( \mu_{\eta_L} = \frac{\sigma^2_{\eta_L}}{(\frac{W_L}{\delta L})^2 \sigma^2_{S} + \sigma^2_{\eta_L} + \sigma^2_{\epsilon_L}} \),

\[ E[\eta L|\theta_L = \frac{W_L}{\delta L} \gamma_L^2 + \eta_L + \epsilon_L] = \mu_{\eta_L} \left(\frac{W_L}{\delta L} \Delta L + \eta_L + \epsilon_L\right) \]

Let \( \bar{\eta}_S = \left(\frac{\gamma_S}{\gamma_S}\right)^2 W_S \) and \( \bar{\eta}_L = \left(\frac{\gamma_L}{\gamma_L}\right)^2 W_L \). Then,

\[ X^S \equiv \frac{W_S}{\delta S} \mu_S + \mu_{\eta_S} \left(\frac{W_S}{\delta S} \Delta S + \eta_S + \epsilon_S\right) \]
\[ = \frac{(\frac{W_S}{\delta S})^2 \mu_S + \mu_{\eta_S} \left(\gamma_S (\epsilon_S - \bar{\epsilon}_S) + \eta_S + \epsilon_S\right)}{(N^S + 1) \Gamma} \]

and

\[ X^L \equiv \frac{W_L}{\delta L} \mu_L + \mu_{\eta_L} \left(\frac{W_L}{\delta L} \Delta L + \eta_L + \epsilon_L\right) \]
\[ = \frac{(\frac{W_L}{\delta L})^2 \mu_L + \mu_{\eta_L} \left(\gamma_L (\epsilon_L - \bar{\epsilon}_L) + \eta_L + \epsilon_L\right)}{(N^L + 1) \Gamma} \]

QED.

Proof of Lemma 5: The Equilibrium Price Function

Define \( M_S \equiv \frac{W_S}{\delta S} \mu_S + \mu_{\eta_S} \) and \( M_L \equiv \frac{W_L}{\delta L} \mu_L + \mu_{\eta_L} \). In equilibrium, we have

\[ \Gamma = \frac{\text{Cov}(\Psi, v)}{\text{Var}(\Psi)} \]

where

\[ \text{Cov}(\Psi, v) = \text{Cov}\left(\gamma_S \epsilon_S + \gamma_S \epsilon_L + \eta_S + \gamma_T, \frac{N^S_S M_S \left(\frac{W_S}{\delta S} \Delta S + \eta_S + \epsilon_S\right)}{(N^S_S + 1) \Gamma} + \frac{N^L L M_L \left(\frac{W_L}{\delta L} \Delta L + \eta_L + \epsilon_L\right)}{(N^L_L + 1) \Gamma} + Z\right) \]
\[ \quad = \text{Cov}\left(\frac{W_S}{\delta S} (\gamma_S^2 + \Delta S) + \eta_S + \frac{W_L}{\delta L} (\gamma_L^2 + \Delta L) + \eta_L + \eta_T, \frac{N^S_S M_S \left(\frac{W_S}{\delta S} \Delta S + \eta_S + \epsilon_S\right)}{(N^S_S + 1) \Gamma} \right) \]
\[ + \frac{N^L L M_L \left(\frac{W_L}{\delta L} \Delta L + \eta_L + \epsilon_L\right)}{(N^L_L + 1) \Gamma} + Z\right) \]
\[ = \left(\frac{N^S_S M_S}{(N^S_S + 1) \Gamma}\right) \left(\frac{W_S}{\delta S} \gamma_S^2 + \sigma^2_{\eta_S}\right) + \left(\frac{N^L L M_L}{(N^L_L + 1) \Gamma}\right) \left(\frac{W_L}{\delta L} \gamma_L^2 + \sigma^2_{\eta_L}\right) \]

and
\[
\text{Var}(\Psi) = \left( \frac{N^j_i M^j_S}{(N^j_S + 1) \Gamma} \right)^2 \left( \frac{W^j_S}{\delta^j_S} \right)^2 \sigma^2_{\gamma^j_S} + \sigma^2_{\eta^j_S} \right) + \left( \frac{W^j_L}{\delta^j_L} \right)^2 \sigma^2_{\gamma^j_L} + \sigma^2_{\eta^j_L} + \sigma^2_Z
\]
since \( \gamma^j_S = \frac{W^j_S}{\pi^j_S} (\gamma^j_S + \Delta^j_S) \) and \( \gamma^j_L = \frac{W^j_L}{\pi^j_L} (\gamma^j_L + \Delta^j_L) \). We can solve for \( \Gamma \) in the following equation

\[
\Gamma = \frac{1}{\sigma^2_Z} \left( \left( \frac{W^j_S}{\delta^j_S} \right)^2 \sigma^2_{\gamma^j_S} + \sigma^2_{\eta^j_S} \right) ^2 \left( \frac{W^j_S}{\delta^j_S} \right)^2 \left( 1 - \frac{N^j_i M^j_S}{N^j_S + 1} \right) + \left( \frac{W^j_L}{\delta^j_L} \right)^2 \sigma^2_{\gamma^j_L} + \sigma^2_{\eta^j_L} \left( 1 - \frac{N^j_i M^j_L}{N^j_L + 1} \right)^2 \right]
\]
which implies

Recall that \( p = p_0 + \lambda \Psi \). Thus,

\[
p = p_0 + \left( \frac{1 - \omega^i_S}{1 + \omega^i_S} \right) \Gamma_N \left( \frac{W^j_S}{\pi^j_S \sigma^j_S \mu^j_S + \mu^j_S} \right) \left( \gamma^j_S(e^j_S - \bar{e}^j_S) + \eta^j_S + \bar{e}^j_S \right) \left( N^j_S / (N^j_S + 1) \right)
\]
\[
+ \frac{W^j_L}{\pi^j_L \sigma^j_L \mu^j_L + \mu^j_L} \left( \gamma^j_L(e^j_L - \bar{e}^j_L) + \eta^j_L + \bar{e}^j_L \right) + Z
\]
Since \( p = p_0 + \lambda \Psi = p_0 + \left( \frac{1 - \omega^i_S}{1 + \omega^i_S} \right) \Gamma_N \left( \pi^j_S(\theta^j_S - E[\theta^j_S]) + \pi^j_L(\theta^j_L - E[\theta^j_L]) + Z \right) \) and \( \theta^j_L - E[\theta^j_L] = \gamma^j_L(e^j_L - \bar{e}^j_L) + \eta^j_L + \bar{e}^j_L \), we have

\[
\pi^j_S = \frac{1}{\lambda} \left( \frac{1 - \omega^i_S}{1 + \omega^i_S} \right) \left( N^j_i / (N^j_S + 1) \right) \left( W^j_S / \delta^j_S \sigma^j_S \mu^j_S + \mu^j_S \right)
\]
\[
\pi^j_L = \frac{1}{\lambda} \left( \frac{1 - \omega^i_S}{1 + \omega^i_S} \right) \left( N^j_i / (N^j_S + 1) \right) \left( W^j_S / \delta^j_S \sigma^j_S \mu^j_S + \mu^j_S \right)
\]
Recall that \( W_i = \omega^j_S \pi^j_i + \omega^j_S \). Thus, the coefficients in the price function \( \pi^j_S \) and \( \pi^j_L \) are given by the following (for \( i = S, L \)):

\[
\pi^j_i = \frac{1}{\lambda} \left( \frac{1 - \omega^i_S}{1 + \omega^i_S} \right) \left( N^j_i / (N^j_L + 1) \right) \left( \omega^j_S \pi^j_i \lambda + \omega^j_S \delta^j_i \mu^j_i + \mu^j_i \right)
\]
\[
\Rightarrow \pi^j_i = \frac{1}{\lambda} \left( \frac{1 - \omega^i_S}{1 + \omega^i_S} \right) \left( N^j_i / (N^j_L + 1) \right) \left( \omega^j_S \mu^j_S \delta^j_i \right)
\]
\[
\Rightarrow \pi^j_i = \frac{1}{\lambda} \cdot \frac{1}{1 - \left( \frac{1 + \omega^i_S}{1 + \omega^i_S} \right) \left( N^j_i / (N^j_L + 1) \right) \left( \omega^j_S \mu^j_S \delta^j_i \right)}
\]
\[
\Rightarrow \lambda^j_i = \frac{\pi^j_i}{\lambda} \left( N^j_i / (N^j_L + 1) \right) \left( \frac{\omega^j_S \mu^j_S \delta^j_i}{1 - \left( \frac{1 + \omega^i_S}{1 + \omega^i_S} \right) \left( N^j_i / (N^j_L + 1) \right) \left( \omega^j_S \mu^j_S \delta^j_i \right)} \right)
\]
Thus, the price function can be written as

\[
p = p_0 + \lambda S \gamma^j_S(e^j_S - \bar{e}^j_S) + \lambda L \gamma^j_L(e^j_L - \bar{e}^j_L) + \hat{\eta}^j_S + \hat{\epsilon}^j_S + \hat{\eta}^j_L + \hat{\epsilon}^j_L + z
\]
where \( \hat{\eta}^j_S \equiv \lambda^j_i \eta^j_S \) and \( \hat{\epsilon}^j_i \equiv \lambda^j_i \epsilon^j_i \), and \( z = \left( \frac{1 - \omega^i_S}{1 + \omega^i_S} \right) \Gamma Z \). QED.

**Proof of Lemma 6: Expected Residual Value**
Recall that the price function is

\[
p = p_0 + \lambda S \gamma^j_S(e^j_S - \bar{e}^j_S) + \lambda L \gamma^j_L(e^j_L - \bar{e}^j_L) + \hat{\eta}^j_S + \hat{\epsilon}^j_S + \hat{\eta}^j_L + \hat{\epsilon}^j_L + z
\]
Note that the unconditional expectation of $p$ is $E[p] = p_0$. The expected utility of the agent is
\[
E[u] = \omega_0 + \omega_p \left[ \sum_{i=S,L} \lambda_i \gamma_i (e - \bar{e}_i) \right] + \omega_v E \left[ \sum_{i=S,L} \gamma_i e_i \right] - \left( \frac{\gamma}{2} \var(w) + E \left[ \sum_{i=S,L} \delta_i w_i^2 \right] \right)
\]
where
\[
\var(w) = \var(\omega_p + \omega_v p + \omega_v v) = \var \left( \omega_p \left[ \sum_{i=S,L} \lambda_i \gamma_i (e - \bar{e}_i) \right] + \omega_v \left[ \sum_{i=S,L} \gamma_i e_i \right] \right) + \omega_v \left[ \gamma \left( \sum_{i=S,L} \lambda_i \gamma_i (e_L - \bar{e}_L) + \lambda_S \eta S + \lambda_L \eta L + \lambda_L \xi_L + \xi L \right) \right]
\]
Substituting for $e_i = \bar{e}_i = \frac{\epsilon_i}{\xi_i} W_i = \frac{\xi_i}{\xi_i} [\omega_p \lambda_i + \omega_v]$ and taking expectations yields
\[
E[u] = \omega_0 + \omega_p p_0 + \omega_v \sum_{i=S,L} \frac{\xi_i^2}{\delta_i} W_i - \frac{\gamma}{2} \left( W_S^2 \sigma_{\epsilon S}^2 + (\omega_p \lambda_S)^2 \sigma_{\epsilon S}^2 + (W_L^2) \sigma_{\epsilon L}^2 + (\omega_p \lambda_L)^2 \sigma_{\epsilon L}^2 + (\omega_v)^2 \sigma_{\epsilon L}^2 \right)
\]
Substituting $p_0 = \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left[ \frac{\xi_S^2}{\delta_S} W_S + \frac{\xi_L^2}{\delta_L} W_L \right] - \frac{\gamma}{1 + \omega_p}$ (from Lemma 5) and set to zero to solve for $\omega_0$:
\[
0 = E[u]
\]
\[
= \omega_0 + \omega_p \left[ \frac{1 - \omega_v}{1 + \omega_p} \right] \left[ \frac{\xi_S^2}{\delta_S} W_S + \frac{\xi_L^2}{\delta_L} W_L \right] - \frac{\omega_v}{1 + \omega_p} \sum_{i=S,L} \frac{\xi_i^2}{\delta_i} W_i
\]
\[
- \frac{\gamma}{2} \left( W_S^2 \sigma_{\epsilon S}^2 + (\omega_p \lambda_S)^2 \sigma_{\epsilon S}^2 + (W_L)^2 \sigma_{\epsilon L}^2 + (\omega_p \lambda_L)^2 \sigma_{\epsilon L}^2 + (\omega_v)^2 \sigma_{\epsilon L}^2 \right) - \frac{1}{2} \sum_{i=S,L} \frac{\xi_i^2}{\delta_i} (W_i)^2
\]
\[
\Rightarrow \omega_0 \left[ 1 - \frac{\omega_0}{1 + \omega_p} \right] = \frac{\gamma}{2} \left( W_S^2 \sigma_{\epsilon S}^2 + (\omega_p \lambda_S)^2 \sigma_{\epsilon S}^2 + (W_L)^2 \sigma_{\epsilon L}^2 + (\omega_p \lambda_L)^2 \sigma_{\epsilon L}^2 + (\omega_v)^2 \sigma_{\epsilon L}^2 \right)
\]
\[
+ \frac{1}{2} \sum_{i=S,L} \frac{\xi_i^2}{\delta_i} (W_i)^2 - \omega_p \left[ \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left( \frac{\xi_S^2}{\delta_S} W_S + \frac{\xi_L^2}{\delta_L} W_L \right) \right] - \omega_v \sum_{i=S,L} \frac{\xi_i^2}{\delta_i} W_i
\]
\[
\Rightarrow \omega_0^* = \left( 1 + \omega_p \right) \left\{ \frac{\gamma}{2} \left( W_S^2 \sigma_{\epsilon S}^2 + (\omega_p \lambda_S)^2 \sigma_{\epsilon S}^2 + (W_L)^2 \sigma_{\epsilon L}^2 + (\omega_p \lambda_L)^2 \sigma_{\epsilon L}^2 + (\omega_v)^2 \sigma_{\epsilon L}^2 \right) \right.
\]
\[
+ \frac{1}{2} \sum_{i=S,L} \frac{\xi_i^2}{\delta_i} (W_i)^2 - \omega_p \left[ \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left( \frac{\xi_S^2}{\delta_S} W_S + \frac{\xi_L^2}{\delta_L} W_L \right) \right] - \omega_v \sum_{i=S,L} \frac{\xi_i^2}{\delta_i} W_i \right\}
\]
Moving to the principals problem, we have
\[
\max_{\omega_p, \omega_v} E[v] - E[u]
\]
We have
\[
E[v] = \frac{\xi_S^2}{\delta_S} W_S + \frac{\xi_L^2}{\delta_L} W_L
\]
and

\[
E[w] = \omega_0^* + \omega_p\alpha_0 + \omega_v\left(\frac{\gamma_2^2}{\delta_S} W_S + \frac{\gamma_2^2}{\delta_L} W_L\right)
\]

\[
= \omega_0^* + \omega_p\left(\frac{1 - \omega_v}{1 + \omega_p} + \omega_v\right)\left(\frac{\gamma_2^2}{\delta_S} W_S + \frac{\gamma_2^2}{\delta_L} W_L\right)
\]

\[
= \omega_0^* \frac{1}{1 + \omega_p} + \omega_p\left(\frac{1 - \omega_v}{1 + \omega_p} + \omega_v\right)\left(\frac{\gamma_2^2}{\delta_S} W_S + \frac{\gamma_2^2}{\delta_L} W_L\right)
\]

\[
= \left\{\frac{\gamma}{2}\left[(W_S)^2\sigma^2_{\eta_S} + (\omega_p\lambda_S)^2\sigma^2_S + (W_L)^2\sigma^2_{\eta_L} + (\omega_p\lambda_L)^2\sigma^2_L + (\omega_v)^2\sigma^2_T\right]
\right.
\]

\[
+ \frac{1}{2} \sum_{i=S,L} \frac{\gamma^2}{\delta_i} (W_i)^2 - \omega_p\left[\frac{1 - \omega_v}{1 + \omega_p}\left(\frac{\gamma^2}{\delta_S} W_S + \frac{\gamma^2}{\delta_L} W_L\right)\right] - \omega_v \sum_{i=S,L} \frac{\gamma^2}{\delta_i} W_i
\]

\[
+ \omega_p\left(\frac{1 - \omega_v}{1 + \omega_p} + \omega_v\right)\left(\frac{\gamma_2^2}{\delta_S} W_S + \frac{\gamma_2^2}{\delta_L} W_L\right)
\]

\[
= \frac{1}{2} \left\{\frac{\gamma}{2}\left[(W_S)^2\sigma^2_{\eta_S} + (\omega_p\lambda_S)^2\sigma^2_S + (W_L)^2\sigma^2_{\eta_L} + (\omega_p\lambda_L)^2\sigma^2_L + (\omega_v)^2\sigma^2_T\right]
\right.
\]

\[
+ \frac{1}{2} \sum_{i=S,L} \frac{\gamma^2}{\delta_i} (W_i)^2 - \omega_p\left[\frac{1 - \omega_v}{1 + \omega_p}\left(\frac{\gamma^2}{\delta_S} W_S + \frac{\gamma^2}{\delta_L} W_L\right)\right] - \omega_v \sum_{i=S,L} \frac{\gamma^2}{\delta_i} W_i
\]

Thus, the objective function for the shareholders is

\[
E[v] - E[w] = \frac{1}{2} \left(\frac{\gamma^2}{\delta_S} W_S + \frac{\gamma^2}{\delta_L} W_L\right) - \frac{\gamma}{2}\left[(W_S)^2\sigma^2_{\eta_S} + (\omega_p\lambda_S)^2\sigma^2_S + (W_L)^2\sigma^2_{\eta_L} + (\omega_p\lambda_L)^2\sigma^2_L + (\omega_v)^2\sigma^2_T\right]
\]

\[
+ \frac{1}{2} \sum_{i=S,L} \frac{\gamma^2}{\delta_i} (W_i)^2 - \omega_p\left[\frac{1 - \omega_v}{1 + \omega_p}\left(\frac{\gamma^2}{\delta_S} W_S + \frac{\gamma^2}{\delta_L} W_L\right)\right] - \omega_v \sum_{i=S,L} \frac{\gamma^2}{\delta_i} W_i
\]

\[
= \frac{1}{2} \left(\frac{\gamma^2}{\delta_S} (\omega_p\lambda_S + \omega_v) + \frac{\gamma_2^2}{\delta_L} (\omega_p\lambda_L + \omega_v)\right)
\]

\[
- \frac{\gamma}{2}\left[(W_S)^2\sigma^2_{\eta_S} + (\omega_p\lambda_S)^2\sigma^2_S + (W_L)^2\sigma^2_{\eta_L} + (\omega_p\lambda_L)^2\sigma^2_L + (\omega_v)^2\sigma^2_T\right]
\]

\[
\text{where}
\]

\[
\lambda_i = \left(\frac{N^f_{\xi}}{N^f_{\xi} + 1}\right) \left(\frac{\gamma_2^2}{\delta_S} \mu_i + \mu_s\right)
\]

QED.

**Proof of Lemma 7: Expected Trade Profits**

The expected profit of an investor receiving a signal of the short term project is defined as \(E[X^S_m((v' - p)|\theta_S)]\). From page 84, we have,

\[
E\left[X^S_m((v' - p)|\theta_S)\right] = (1 - \omega_v)E\left[X^S_m((v - E(v)) - \Gamma\Psi)|\theta_S\right]
\]

From page 85,

\[
= (1 - \omega_v)X^S_m\left[\frac{W_S}{\delta_S}E[\gamma^2 - \gamma^2_3|\theta_S] + E\{\eta_S|\theta_S\}\right] - \Gamma\left(X^S_m + (N^f_S - 1)E[X^S_m|\theta_S]\right)
\]

where \(X^S_m\) denotes the trades of the other investors with a type-S signal. Let \(\alpha \equiv E[\gamma^2 - \gamma^2_3|\theta_S]\) and \(\beta \equiv E[\eta_S|\theta_S]\) and rewrite as

\[
E\left[X^S_m((v' - p)|\theta_S\right] = (1 - \omega_v)X^S_m\left[\frac{W_S}{\delta_S}\alpha + \beta - \Gamma X^S_m N^f_S\right]
\]

From page 85, we have the optimal demand of an investor with signal type-S equal to

\[
X^{S*}_m = \frac{W_S}{\Gamma(N^f_S + 1)}\alpha + \beta - \Gamma X^S_m N^f_S
\]

This implies
\[
E\left[ X_m^{S}(v' - p) | \theta_S \right] = (1 - \omega_v) \left( \frac{W_S}{\delta_S} \alpha + \beta \right)^2 \frac{1}{\Gamma(N_S^I + 1)}
\]

Now substitute \( \alpha \) and \( \beta \) from previous results: \( \alpha = \mu_S \left( \frac{W_S}{\delta_S} \Delta_S + \eta_S + \varepsilon_S \right) \), and \( \beta = \mu_{\eta_S} \left( \frac{W_S}{\delta_S} \Delta_S + \eta_S + \varepsilon_S \right) \). Thus,

\[
E\left[ X_m^{S}(v' - p) | \theta_S \right] = (1 - \omega_v) \left( \frac{W_S}{\delta_S} \mu_S \left( \frac{W_S}{\delta_S} \Delta_S + \eta_S + \varepsilon_S \right) + \mu_{\eta_S} \left( \frac{W_S}{\delta_S} \Delta_S + \eta_S + \varepsilon_S \right) \right)^2 \frac{1}{\Gamma(N_S^I + 1)^2}
\]

Similarly, the expected profit of an investor receiving a signal of the long term project is:

\[
E\left[ X_L^{L}(v' - P) | \theta_L \right] = (1 - \omega_v) \left( \frac{W_L}{\delta_L} \mu_L \left( \frac{W_L}{\delta_L} \Delta_L + \eta_L + \varepsilon_L \right) + \mu_{\eta_L} \left( \frac{W_L}{\delta_L} \Delta_L + \eta_L + \varepsilon_L \right) \right)^2 \frac{1}{\Gamma(N_L^I + 1)^2}
\]

To calculate the unconditional expected profit, we use the identity \( E[X|Y] = E[X] \). Taking the expectation of the results above, \( E[\Pi_S | \theta_S] = E[\Pi_S] \) where \( \Pi_S \) is the profit from receiving a short signal.

\[
E \left\{ (1 - \omega_v) \left( \frac{W_S}{\delta_S} \mu_S \left( \frac{W_S}{\delta_S} \Delta_S + \eta_S + \varepsilon_S \right) + \mu_{\eta_S} \left( \frac{W_S}{\delta_S} \Delta_S + \eta_S + \varepsilon_S \right) \right)^2 \frac{1}{\Gamma(N_S^I + 1)^2} \right\}
\]

\[
= (1 - \omega_v) \left( \frac{W_S}{\delta_S} \mu_S \left( \frac{W_S}{\delta_S} \Delta_S + \eta_S + \varepsilon_S \right) + \mu_{\eta_S} \left( \frac{W_S}{\delta_S} \Delta_S + \eta_S + \varepsilon_S \right) \right)^2 \frac{1}{\Gamma(N_S^I + 1)^2}
\]

Recall that \( \Delta_S = \gamma_S^2 - \gamma_S^2 \) and \( \Delta_S, \eta_S, \varepsilon_S \) are all uncorrelated. In the expectation, after multiplying the variables in parenthesis by itself, the only thing that survives the expectation is the square terms. That is

\[
E \left[ \left( \frac{W_S}{\delta_S} \Delta_S + \eta_S + \varepsilon_S \right)^2 \right] = \left( \frac{W_S}{\delta_S} \right)^2 E[\Delta_S]^2 + E[(\eta_S)^2] + E[(\varepsilon_S)^2]
\]

And we know that \( \text{Var}[X] = E[X^2] - E[X]^2 \) since each as a mean zero,

\[
\left( \frac{W_S}{\delta_S} \right)^2 E[\Delta_S]^2 + E[(\eta_S)^2] + E[(\varepsilon_S)^2] = \left( \frac{W_S}{\delta_S} \right)^2 \sigma_{\Delta_S}^2 + \sigma_{\eta_S}^2 + \sigma_{\varepsilon_S}^2
\]

Substitute this back into A.6, yielding

\[
E[\Pi_S] = \frac{(1 - \omega_v) \left( \frac{W_S}{\delta_S} \mu_S + \mu_{\eta_S} \right)^2}{\Gamma(N_S^I + 1)^2} \left[ \left( \frac{W_S}{\delta_S} \right)^2 \sigma_{\Delta_S}^2 + \sigma_{\eta_S}^2 + \sigma_{\varepsilon_S}^2 \right]
\]

Similarly,

\[
E[\Pi_L] = \frac{(1 - \omega_v) \left( \frac{W_L}{\delta_L} \mu_L + \mu_{\eta_L} \right)^2}{\Gamma(N_L^I + 1)^2} \left[ \left( \frac{W_L}{\delta_L} \right)^2 \sigma_{\Delta_L}^2 + \sigma_{\eta_L}^2 + \sigma_{\varepsilon_L}^2 \right]
\]

QED.
Table I. Time Line of the Model

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• The board of directors offers the manager a contract ( (\omega_0, \omega_p, \omega_v) ).</td>
<td>• Investors (or speculators) observe costly noisy signals ( \theta_S(\theta_L) ) of the value of short-term (long-term) project.</td>
<td>• The terminal value of the firm ( v ) is realized.</td>
</tr>
<tr>
<td></td>
<td>• The short-term ( v_S ), and the long-term ( v_L ) projects are initiated by the manager.</td>
<td>• The market for the firm’s shares opens for trade; and the resulting market price is ( p ).</td>
<td>• The manager is (partially) compensated an amount of ( \omega_v v ).</td>
</tr>
<tr>
<td></td>
<td>• The initial price of the stock ( p_0 ) (unconditional expectation of the net terminal value) is determined by the investors.</td>
<td>• The manager is (partially) compensated an amount of ( \omega_p p ).</td>
<td>• The manager is (partially) compensated a fixed wage of ( \omega_0 ).</td>
</tr>
</tbody>
</table>
Figure 1. Level Sets for Expected Residual Value given Combinations of $\lambda_S$ and $\lambda_L$. 
Figure 2. Market Fails to Achieve the Maximal Expected Residual Value
Figure 3. Effect of Changing Conditions (Case 1)
Figure 4. Effect of Changing Conditions (Case 2)

\[ E(\Pi|\omega_{p0}, \omega_{v0}, \Omega_0) = 0 \]

\[ E(\Pi|\omega_{p0}, \omega_{v0}, \Omega_1) = 0 \]
Figure 5. Fixity in Total Number of Speculator and the Achievement of Maximal Expected Residual Value

\[ E(RV) = E(RV)_2 \]
\[ E(\Pi|\omega_{p1}, \omega_{v1}, \Omega_1) = 0 \]
\[ E(\Pi|\omega_{p2}, \omega_{v2}, \Omega_1) = 0 \]
\[ E(\Pi|\omega_{p0}, \omega_{v0}, \Omega_1) = 0 \]
Figure 6. Less Fixity in the Total Number of Speculators than the Contract
Figure 7. Either a Short-term Tax or a Long-Term Subsidy Increases Expected Residual Value

\[ E(\Pi|(\omega_p, \omega_v), \Omega_0, T_S = T_L = S_S = S_L = 0) = 0 \]

\[ E(\Pi|(\omega_p, \omega_v), \Omega_0, T_S = T_L = S_S = 0, S_L > 0) = 0 \]

\[ E(\Pi|(\omega_p, \omega_v), \Omega_0, T_S > 0, T_L = S_S = S_L = 0) = 0 \]
Figure 8. A Tax on Short-Term Speculation Lower Expected Residual Value

\[ E(\Pi|\omega_{p0}, \omega_{v0}, \Omega_0, T_S = T_L = S_S = S_L = 0) = 0 \]

\[ E(\Pi|\omega_{p0}, \omega_{v0}, \Omega_0, T_S = T_L = S_S = 0, S_L > 0) = 0 \]

\[ E(\Pi|\omega_{p0}, \omega_{v0}, \Omega_0, T_S > 0, T_L = S_S = S_L = 0) = 0 \]
Essay III:
Heterogenous Preference for Skewness, Idiosyncratic Volatility, and Expected Stock Returns

ABSTRACT

Consistent with models that incorporate investors heterogeneous preference for skewness, I show that (1) high skewness stocks are primarily held by investors with the strongest affinity for lottery-like payoff, (2) the negative skewness-return relation is the strongest for those stocks primarily held by agents with the strongest affinity for lottery-like payoff, (3) the idiosyncratic volatility-return relation is the strongest for those stocks held by agents with the strongest affinity for lottery-like payoff, and (4) investors heterogeneous preference for skewness help explain the idiosyncratic volatility puzzle. Taken together, the results provide evidence for the importance of investors heterogeneous preference for skewness in asset pricing and its implication on the idiosyncratic volatility puzzle.

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I. Introduction

Classical asset pricing models such as Capital Asset Pricing Model (CAPM) drive equilibrium results by assuming *identical* representative agents. However, recent theoretical asset pricing models started relaxing this assumption. One such a model that gained momentum recently is investors *heterogenous* preference for skewness. In this article, I establish the empirical relevance of this assumption for equilibrium asset prices and its implication on the low returns associated with high idiosyncratic volatility stocks.

Models that incorporate investors *heterogenous* preference for skewness typically extend the classical mean-variance optimization problem to a mean-variance-skewness optimization problem and allow the magnitude of investors preference for skewness to vary with the type of investors. Often times two-agent models are used to capture this variation in skewness preference where the first agent is assumed to have little or no preference and the optimization problem for this agent is reduced to mean-variance, and the second agent is assumed to have some degree of preference for skewness and she is faced with a mean-variance-skewness optimization problem. The two agents are then to maximize their expected utility subject to their respective budget constraint. The resulting equilibrium price will be the one that clears the optimal market demand for the risky assets. Prominent examples of models that incorporate skewness to the traditional mean-variance optimization problem include Mitton and Vorkink (2007), Coiken and Tamarkin (1981), and Kraus and Litzenberger (1976).

The need for heterogeneous investor models arose because typical U.S. households hold underdiversified portfolios (see, e.g. Blume and Friend (1975) and Goetzmann and Kumar (2008)) and researchers hypothesize that perhaps one reason why investors hold underdiversified portfolios is to capture the low probability of high returns associated with lottery-like payoff stocks. There are other plausible reasons why some investors held underdiversified portfolios including costs associated with information acquisition e.g. Van Nieuwerburgh and Veldkamp (2010).

In this paper, I provide evidence for the importance of investors heterogenous preference
for skewness in asset pricing and its implication on the idiosyncratic volatility puzzle. First, I show that highly skewed stocks are held primarily by agents with the strongest affinity for lottery-like payoff. Second, I show that the negative skewness-return relation is the strongest for those stocks held by agents with the strongest affinity or lottery-like payoff. Third, the negative idiosyncratic volatility-return relation is the strongest for those stocks held by agents with the strongest affinity or lottery-like payoff. Fourth, investors heterogenous preference for skewness help explain the idiosyncratic volatility puzzle. Taken together, these results suggest that heterogenous investor models help explain the low returns associated with highly skewed and highly volatile stocks.

Kumar (2009) show that the socioeconomic characteristics of state lottery players (e.g. the poor, young, relatively less educated, and those who live in urban areas) can be mapped to investments in stocks. Using proprietary data from discount brokerage house, Mitton and Vorkink (2007), Kumar (2009) and others show that underdiversified investors seek lottery-like payoffs and hold highly skewed stocks. I use institutional ownership ratio from a readily available Thomson Reuters 13F data to capture investors preference for lottery-like payoffs. I show that highly skewed stocks are held primarily by retail investors who seek lottery-like payoff. Portfolio level analysis shows that stocks in the lowest institutional ownership ratio (IOR) quintile have an average skewness of 0.28 whereas stocks in the highest institutional ownership ratio (IOR) quintile have an average skewness of 0.19. The skewness difference between the highest and the lowes IOR quintile portfolios is -0.09 and it is statistically significant with a $t$-statistics of -6.37.

In the mean-variance-skewness optimization setting, investors will hold underdiversified portfolios (relative to the mean-variance optimization) to maximize their utility via skewness preference. Empirically, this will imply that the negative skewness-return relation will be the strongest for those investors with the highest preference for skewness. Portfolio-level analysis shows the relation between skewness and expected stock returns is decreasing in the institutional ownership ratio (IOR) of the firm. An investment strategy that goes long in
the highest skewness stocks and short in the lowest skewness stocks produces a statistically significant average return of -6.12% per annum in the lowest IOR quintile, and a statistically not significant average return of -0.12% per annum in the highest IOR quintile.

A related result is that the negative idiosyncratic volatility-return relation is the strongest for those stocks held by agents with the strongest affinity or lottery-like payoff because these agents are more underdiversified. Portfolio level analysis shows the relation between idiosyncratic volatility and expected stock returns is decreasing in the institutional ownership ratio (IOR) of the firm. An investment strategy that goes long in the highest idiosyncratic volatility stocks and short in the lowest idiosyncratic volatility stocks produces a statistically significant average return of -14.04% per annum in the lowest IOR quintile, and a statistically not significant average return of -3.6% per annum in the highest IOR quintile. Taken together, these results suggest that the low returns associated with high idiosyncratic volatility stocks might be consistent with heterogenous preference for skewness models. We test this directly and find some support to it.

My paper adds to the growing literature that shows a negative relationship between skewness and stock returns.\footnote{There are a some papers that show a positive relation between skewness and stock returns. See, for example, Rehman and Vilkov (2010), Bali and Hovakimian (2009), Cremers and Weinbaum (2010), Xing, Zhang, Zhao (2010).} For example, Kraus and Litzenberger (1976) and Harvey and Saddique (2000) develop a model that incorporates higher moments in the maximization of investors’ utility function and show a negative relation between skewness and expected stock returns. There is ample empirical support to this prediction as well. For example, Boyer, Mitton and Vorkink (2010) document a negative correlation between expected idiosyncratic skewness and stock returns. Similarly, Bali, Cakici, Whitelaw (2011) show that stocks with recent extreme positive returns, which can be thought of as a crude measure of skewness, have low future returns. Conrad, Dittmar, Ghysels (2009) show a negative relation between risk-neutral skewness and future stock returns. Chang, Christoffersen, and Jacobs (2013) show that stocks with high exposure to innovations in risk-neutral market skewness have low...
II. Data

The data I use in this paper is from the Center for Research Security Prices (CRSP), Compustat, and Thomson Reuters 13F data via the Wharton Research Data Services (WRDS). The main variables are constructed for all common stocks (i.e. share code (SHRCD) = 10 and 11) that are traded in the New York Stock Exchange (NYSE), American Stock Exchange (AMEX) and National Association of Securities Dealers Automated Quotations (NASDAQ) (i.e. exchange code (EXCHCD) = 1, 2, or 3) as follows.

1) Idiosyncratic Volatility: I follow the standard Ang, Hodrick, Xing, and Zhang (2006) methodology to construct monthly idiosyncratic volatility. Every month I regress the daily excess return of each stock ($R_{i,t} - r_{f,t}$) on daily Carhart-four factors $MKTRF_t$, $HML_t$, $SMB_t$, and $UMD_t$ – the daily excess market return, the value premium, the size premium, and momentum factors respectively. i.e.

$$R_{i,t} - r_{f,t} = \beta_0 + \beta_1 * MKTRF_t + \beta_2 * HML_t + \beta_3 * SMB_t + \beta_4 * UMD_t + \varepsilon_{i,t} \quad (1)$$

and the monthly idiosyncratic volatility is the standard deviation of the residuals from this daily regression. i.e.

$$\sigma_i = \sqrt{\text{var}(\varepsilon_{i,t})} \quad (2)$$

I require at least fifteen trading days within a month to be included in my sample.

2) Total Realized Skewness: Every month I calculate the sample skewness from daily data

$$total\_skew_i = \text{skew}(R_{i,t} - r_{f,t}) \quad (3)$$

and I require at least fifteen trading days within a month to be included in the sample.

3) Institutional Ownership ratio: Institutional ownership ratio is from the Thomson Reuters 13F data. It is the ratio of the institutional ownership level and total shares
III. Empirical Results

In this section, we investigate whether (1) investors have a heterogeneous preference for stocks with lottery-like payoffs, and (2) if this heterogeneous preference help explain the low returns associated with high idiosyncratic volatility stocks. The conjecture is potentially consistent with the model presented in Mitton and Vorkink (2007). Mitton and Vorkink (2007) argue that investors have a heterogeneous preference for skewness and, in equilibrium, some investors may remain underdiversified in order to increase the skewness in their portfolio holdings. We proxy investors heterogeneity in their preference for lottery-like payoffs with institutional ownership ratio. The extant evidence suggests that retail investors and state lottery players have similar characteristics – in that they gamble for the small probabilities of winning big (see, for example, Kumar (2009), Statman (2002), Barberis and Huang (2008), among many others). The percentage of institutional ownership captures the investors affinity for lottery-like payoff for that stocks.

A. Investor Heterogeneity, Lottery-like Payoffs and Expected Stock Returns

In Table I, we investigate whether investors have a heterogenous preference for stocks with lottery-like payoffs. We sort stocks based on the previous month institutional ownership ratio and report the average skewness, the average idiosyncratic volatility, and the average institutional ownership ratio for each quintile portfolio. The table show a monotonic relation between average skewness and institutional ownership ratio. The stocks in the portfolio with the lowest institutional ownership ratio (or, equivalently, the portfolio with the highest retail investors) have an average skewness of 0.28. And the stocks in the highest institutional ownership ratio (or, equivalently, the portfolio with the lowest retail investors) have an average skewness of 0.19. The average skewness difference between the two extreme quintile portfolios is -0.09 and it is statistically significant with a $t$-statistics of -6.37. This result
shows that retail investors have a preference for high skewness stocks. If, as is standard in the literature, realized skewness proxies the asymmetry in the payoff distribution, this result shows that investors have a heterogeneous preference for stocks with lottery-like payoffs. Additionally, the table shows that stocks with low institutional ownership ratio have high idiosyncratic volatility. Stocks in the lowest quintile institutional ownership ratio have an average idiosyncratic volatility of 16% and and stocks in the high institutional ownership ratio quintile have an average idiosyncratic volatility of 8% and the idiosyncratic volatility difference between the highest and lowest institutional ownership quintile is -8% with a highly significant t-statistics of -18.63. Last, there is enough variation in the institutional ownership ratio within our data set. For a typical stock, only 4% of the shares are held by institutions in the lowest quintile 71% of the shares are held by institutions in the highest quintile. Taken together, this table shows that there is a monotonic negative relation between the percentage of shares owned by institutions and the skewness of a typical stock. Highly skewed firms are held primarily by retail investors where as low skewed stocks are held primarily by institutional investors. The result confirms the hypothesis that investors have heterogeneous preference for lottery-like payoff stocks. Additionally, the idiosyncratic volatility of a firm is negatively correlated with the percentage of institutional holdings of the firms shares.

In Table II, we do a mirror image of Table I in that we sort stocks into quintile portfolios by the previous month realized skewness and we report the portfolio characteristics in Panel A and the portfolio returns in Panel B. We note a couple of interesting statistical results here. First, the level of average idiosyncratic volatility increases as we move from the low to high skewness quintile although this increase is modest. However, the average idiosyncratic volatility difference between the high and low skewness quintile is statistically significant. Similarly, the the average institutional ownership ratio shows a slight decrease as we move from the highest to the lowest skewness quintile portfolio. Last, the average skewness varies modestly from 0.21 in the lowest quintile to 0.28 in the highest quintile. This confirms the previously reported results that, on average, stocks returns are positively skewed. Panel
B report the value weighted return, the CAPM alpha, the Fama-French (1996) three factor alpha, and the Carhart four alpha for each quintile portfolio sorted by the previous month skewness. The table shows that the average return decreases as you move from the lowest skewness portfolio to the highest skewness portfolio. The average return in lowest skewness is 1.20% per month and the average return in the highest skewness portfolio is 0.90% per month. The difference in the average return between the highest and the lowest skewness portfolio is -0.30% and it is statistically significant with a t-statistics of just -2.79. The CAPM, Fama-French (1996) three factor and Carhart four factor risk adjusted average returns are also significant at conventional levels. Taken together, Tables I and II suggests that stocks primarily held by retail traders are highly skewed and a double sort long-short strategy ought provide more information about the investors preference for lottery-like payoff stocks.

In Table III, we double sort stocks: first we form quintile portfolios by the previous month institutional ownership ratio, and then, within each institutional ownership ratio quintile, we form quintile portfolios by the previous month skewness and report the value weighted raw returns (in Panel A), the CAPM alpha (in Panel B), and the Fama and French (1996) three factor alpha (in Panel C). The table shows that as you move from low institutional ownership ratio (Low IOR) quintile to high institutional ownership ratio (High IOR) quintile in Panel A, the absolute return difference between high skewness and low skewness portfolios decreases monotonically. For example, for the lowest institutional ownership quintile (look at the row labeled ‘Low IOR’ in Panel A), the average return for the lowest skewness quintile portfolio is 0.93% per month with a t-statistics of 2.85 whereas the highest skewness portfolio has an average return of 0.42% per month with a t-statistics of 1.03. The difference in returns between these two portfolios is -0.51% per month with a t-statistics of -2.53. Similarly, the difference in returns between the high skewness and low skewness for the next four IOR portfolios is -0.46%, -0.31%, -0.29% and -0.01% per month respectively. This clearly shows that after accounting for the type of investors holding the stocks, there is a variation in returns between stocks with high and low realized skewness. Specifically, retail investors
have a preference for high skew stocks and they pay a higher premium for it. However, institutional investors don’t seem to show a preference for skewed stocks.

The inference we draw from Panel A of Table III can also be made from Panels B and C. Panel B shows the abnormal return after controlling for the market risk factor. For example, the first row shows that within the low institutional ownership portfolios, the abnormal return difference between the high skewness and low skewness stocks is -0.63% with a t-statistics of -3.30. However, the last row of Panel B shows that, within the highest institutional ownership quintile portfolio, the abnormal return difference between the high skewness and low skewness portfolio is -0.03% with a t-statistics of -0.39. This shows that there is alpha associated in the skewness of stocks in the lowest institutional ownership quintile portfolio whereas there is no alpha for the highest institutional ownership portfolio. The pattern is also what we expected. If you move down (in Panel B and C) from the lowest institutional ownership portfolio to the highest institutional ownership portfolio, the absolute difference in abnormal returns between the highest and lowest skewness portfolios decreases. In sum, the table shows that there is investor heterogeneity in the preference for skewness and retail traders have a preference whereas institutional traders do not have a preference.

We further investigate investors heterogeneous preference for lottery-like across market conditions. Kumar (2009) argues that the demand for state lottery-like stocks increases during economic contraction than expansion. We use the Chicago Fed’s National Economic Activities Index three month moving average series to distinguish between economic expansion and contraction. We use the average of the Chicago Fed’s index to separate the expansion and contraction. Panel A of Table V shows sequential double sort during economic contraction and Panel B shows during economic expansion. The table clearly show that that retail investors pay more premium for high skewness stocks during economic contraction than economic expansion. The value weighted return difference between high skewness stocks and low skewness stocks for low institutional ownership ratio portfolios
during economic contraction (the first row) is -0.81% and it is statistically significant with a t-statistics of -2.88, whereas the difference during economic expansion is -0.19% and it is not statistically significant with a t-statistics of -0.72. The pattern in the high minus low column for Panel A is similar to Table III in that as you move down from retail investors to institutional investors, the difference in returns gets closer to zero. Over all, the present table (Table V) shows an indirect evidence of an increase in the demand for lottery-like stocks during economic contraction than expansion.

B. Investor Heterogeneity, Idiosyncratic Volatility and Expected Stock Returns

Having documented investors heterogeneous preference for lottery-like stocks, next we show that this heterogeneous preference helps explain the low returns associated with high idiosyncratic volatility stocks – the so-called idiosyncratic volatility puzzle.

We start by confirming the previous findings that low returns are associated with high idiosyncratic volatility stocks. In Table VI, we form decile portfolios using the previous month idiosyncratic volatility and we report the associated value weighted returns, the CAPM alpha, the Fama and French (1996) three factor alpha, and the Carhart four factor alpha. The last column labeled ‘H - L’ shows the raw and risk-adjusted return difference between the highest and lowest idiosyncratic volatility portfolio. The table confirms the previous finding that low returns are associated with high idiosyncratic volatility stocks. For example, the value weighted raw return difference between the highest and the lowest idiosyncratic volatility stocks is -0.68% per month and it is statistically significant with a t-statistics of -2.21. Similarly, the Fama and French (1996) three factor risk-adjusted return difference is -1.06% per month which is highly significant with a t-statistics of -5.83.

After having confirmed the low returns associated with high idiosyncratic volatility we move on to investigate whether this result is consistent to the equilibrium underdiversification hypothesis posited by Mitton and Vorkink (2007). To reiterate, the main idea here is that some investors may have a preference for lottery-like payoff stocks and they may remain
underdiversified to reap the benefits of having a higher skewness portfolio in equilibrium. We show that this equilibrium underdiversification helps explain the low returns associated with high idiosyncratic volatility stocks. In this subsection, we show that the so-called idiosyncratic volatility puzzle is concentrated in the portfolio of stocks with the lowest institutional ownership ratio and the highest skewness – proxies we use for investors heterogeneous preference for lottery-like payoffs and the lottery-like payoff characteristics of the stock respectively. And in the next subsection, we will show that after controlling for institutional ownership and skewness, low returns are no longer associated with high idiosyncratic volatility on average.

In Table VII, we sort stocks into quintile portfolios first by the previous month institutional ownership ratio and then by the previous month idiosyncratic volatility. In Panel A, we report the value weighted portfolio returns from this sequential sort. It shows that once you control for the institutional ownership ratio of stocks, the low returns associated with high idiosyncratic volatility is present only in the bottom two quintile portfolios of institutional ownership ratio. In the lowest institutional ownership ratio quintile (labeled 'Low IOR’), the low idiosyncratic volatility stocks have a value weighted return of 1.09% per month and the high idiosyncratic volatility stocks have a value weighted return of -0.08% per month. The difference in the value weighted returns between the highest and the lowest idiosyncratic volatility (labeled ‘H-L’) within the ‘low IOR’ quintile portfolio is -1.17% per month and it is statistically significant with a t-statistics of -3.08. As you move to the next highest institutional ownership quintile portfolio, the return difference between the high and low idiosyncratic volatility stocks decreases (in absolute value). The second institutional ownership ratio quintile portfolio has a return difference between high and low idiosyncratic volatility of -1.04% per month and it is statistically significant with a t-statistics of -2.54. The return difference between the high and low idiosyncratic volatility portfolios for the next three institutional ownership ratio quintile are -0.62%, -0.47%, and -0.30% per month which are not statistically significant at conventional levels with a t-statistics of -1.51, -1.42,
and -1.16 respectively. Over all, Panel A shows that the low returns associated with high idiosyncratic volatility stocks are concentrated at low institutional ownership ratio portfolios, and the return difference between high and low idiosyncratic volatility decreases monotonically (in absolute value) as we move from low to high institutional ownership ratio portfolios, and the difference is not significant starting the third quintile portfolio. The result herein shows that the so-called idiosyncratic volatility puzzle is concentrated in stocks that are primarily held by retail investors which is consistent with the hypothesis that some investors pay a higher price for stocks with lottery-like payoff characteristics.

In Panel B and Panel C of Table VII, we report the average idiosyncratic volatility and the average skewness associated with the 25 portfolios sequential sorted by the previous month institutional ownership ratio and the previous month idiosyncratic volatility respectively. In doing so, we try to understand the stock characteristics for the finding that the low returns associated with high idiosyncratic volatility stocks are more pronounced stocks with the highest retail traders. We note two results from Panel B. First, the stocks held primarily by retail traders have high idiosyncratic volatility on average. This is so because the average idiosyncratic volatility decreases as you move from the lowest to the highest institutional ownership ratio. Second, there is more variation in idiosyncratic volatility in the lowest institutional ownership portfolio than in the highest institutional ownership portfolio quintile. In other words, the difference between the highest and lowest idiosyncratic volatility (labeled ‘H-L’) decreases as you move from the portfolio with low institutional ownership ratio to high institutional ownership ratio. The lowest institutional ownership ratio quintile portfolio has an average idiosyncratic volatility difference of twice as much as that of the highest institutional ownership ratio (0.18 vs. 0.07). The two results taken together imply that idiosyncratic volatility and institutional ownership ratio have a monotonic and convex relation – high idiosyncratic volatility of the stock attract retail traders, and perhaps, the presence of retail traders increases the idiosyncratic volatility of the stock.

Similarly, Panel C of Table VII shows the variation in the average skewness in the
25 portfolios sequential sorted by the previous month institutional ownership ratio and idiosyncratic volatility respectively. The average skewness difference in the low institutional ownership ratio portfolio is much higher than that of high institutional ownership ratio portfolio (0.21 vs. 0.06). Additionally, this difference is monotonic as you move from the lowest to the highest institutional ownership ratio portfolio – 0.21, 0.15, 0.13, 0.10, and 0.06. Taken together, Panel B and C show that stocks held primarily by retail traders have high idiosyncratic volatility and high skewness and therefore have low returns. This is consistent to the story that retail traders pay a higher price for stocks with lottery-like payoff features.

We argued earlier in Table V that the demand for lottery-like payoff stocks increases during economic contraction and decreases during economic expansion. In Table VIII, we show that the low returns associated with high idiosyncratic volatility is present during economic contraction only. Moreover, within the economic contraction period, the low returns associated with high idiosyncratic volatility is present only in the lowest two institutional ownership ratio quintile (see Panel A). Whereas, Panel B shows that during economic expansion periods, the average return difference between high and low idiosyncratic volatility is is not statistically significant even in the lowest institutional ownership ratio quintile. This table gives indirect evidence that investors heterogenous preference for lottery-like payoff stocks helps explain the idiosyncratic volatility puzzle.

**C. Lottery-like Payoffs, Idiosyncratic Volatility and Expected Stock Returns**

Thus far we have documented that investors have a preference for skewness (a proxy for lottery-like payoffs), and this preference is not uniform in that some investors have a stronger affinity for skewness than others. We showed this via double sort, first by intuitional ownership ratio and then by skewness. This sorts showed that retail investors pay on average a higher price for stocks with lottery-like payoffs, whereas institutional investors on average do not. We also showed that the idiosyncratic volatility is concentrated on stocks with high lottery-like payoffs. A double sort, first by institutional ownership ratio and
then by idiosyncratic volatility, shows that that retail investors (not institutional investors) pay a higher price for stocks with high idiosyncratic volatility. The so-called idiosyncratic volatility puzzle is concentrated in the lowest two institutional ownership ratio quintile. Taken together, the results indicate that the idiosyncratic volatility puzzle might in fact be consistent with investors preference for lottery-like payoff. However, the reverse might also hold - i.e. investors pay a higher price for stocks with high volatility and just it happened to be that these stocks have low institutional ownership ratio and high skewness. We investigate these two hypotheses next.

In Table IX, we sort stocks into quintile portfolios first by last month skewness and then by last month idiosyncratic volatility. If it is true that retail investors pay a higher price for stocks with lottery like payoffs, then we expect that the idiosyncratic volatility effect should be concentrated in the high skew quintile. Ideally, if the idiosyncratic volatility puzzle is just investors preference for skewness, then the puzzle should disappear after we control for skewness. The table shows that the low returns associated with high idiosyncratic volatility stocks becomes more pronounced as we move to the highest skewness portfolio quintile. The return difference between the highest and lowest idiosyncratic volatility quintile for each skewness quintile portfolio is -0.14%, -0.46%, -0.63%, -0.96% and -1.42% per month respectively, of which the last two skewness quintile are statistically significant at 5%. This shows that, even after controlling for skewness, there is enough return dispersion when we sort stocks by idiosyncratic volatility for the highest two skewness quintile.

However, Table X shows that this result is only present in the lowest institutional ownership ratio portfolio. We show this via a triple sort. We first sort the universe of stocks into quintile portfolio by institutional ownership ratio and then within each institutional ownership ratio, we first sort by skewness and then by idiosyncratic volatility. We report the lowest and the highest institutional ownership ratio quintile. The results are qualitatively similar in the middle three institutional ownership ratio quintile portfolio. Pane A shows that for the lowest institutional ownership ratio quintile, the idiosyncratic volatility puzzle is
concentrated in the highest two skewness quintile portfolios. The return difference between the high and low idiosyncratic volatility portfolios (for the highest two skewness portfolios) are -1.29% and -1.74% and they are statistically highly significant. The lowest three skewness quintile portfolios do not show enough return dispersion between the high and low idiosyncratic volatility portfolios. Panel B shows results for the highest invitational ownership ratio portfolio. In this panel, we sort stocks first by skewness and then by idiosyncratic volatility and the results show that there is no return dispersion between the high and low idiosyncratic volatility for each skewness quintile. Taken together, Panels A and B show that the so-called idiosyncratic volatility puzzle is concentrated on stocks with high skewness that are held primarily by retail investors. Now, it is unclear whether this puzzle is truly a puzzle or whether high skewness stocks that are held primarily by retail traders have high idiosyncratic volatility, and in equilibrium, these stocks (at the corner of a triple sort) have no return dispersion. The results thus far suggest that investors don’t pay a higher price for stocks that have a higher idiosyncratic volatility but rather they do so to capture some other stock features such as the lottery-like payoff feature of the stocks. This is evident from the present table, Table X, as the low returns associated with high idiosyncratic volatility stocks is only present in the highly skewed stocks that are held primarily by retail traders.

IV. Conclusion

In this paper, we argue that the so-called idiosyncratic volatility puzzle is consistent with investors heterogenous preference for lottery-like payoff stocks. Since retail traders have innate preference for lottery-like payoff, and institutional investors have little to no preference, we used institutional ownership ratio to capture investors preference for lottery-like payoff and showed that this variable is positively correlated with lottery-like payoff stock characteristics. Stocks held primarily by individual investors have high skewness than stocks held by institutions. Further, we showed that there is a positive correlation between
institutional ownership ratio and the idiosyncratic volatility of a stock. Retail investor hold high idiosyncratic volatility stocks than institutional investors. Further, we showed that high idiosyncratic volatility stocks have high skewness as well.

After documenting the correlation among institutional ownership ratio, lottery-like payoff and the idiosyncratic volatility characteristics of a stock, we investigated the corresponding returns via double sort. First, we showed in Table III that within each institutional ownership ratio quintile portfolio, high skewness stocks are associated with low returns and low skewness stocks are associated with high returns. Moreover, the relation between skewness and returns is the strongest for low institutional ownership ratio portfolio. As you move from low to high institutional ownership ratio quintile, a high minus low strategy on skewness becomes less and less significant (economically and statistically) suggesting that retail investors pay a higher price for stocks higher skewness. Second, we showed in Table VII that the so-called idiosyncratic volatility puzzle is concentrated on stocks that are held primarily by retail investors. When we sort stocks first by institutional ownership ratio and then idiosyncratic volatility, the return difference between high and low idiosyncratic volatility stocks is only significant in the lowest two institutional ownership ratio quintile. This shows that retail investors pay a higher price for stocks with high idiosyncratic volatility.

Third, in Table IX a double sort, first by skewness and then by idiosyncratic volatility shows that low returns are associated with high idiosyncratic volatility stocks in the highest two skewness quintile portfolios. Taken together, these results show that low returns are associated with stocks with high idiosyncratic volatility, low institutional ownership ratio, and high skewness.
References


### Table I. Single Sort by Institutional Ownership Ratio (IOR)

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Notes: The table reports quintile portfolio characteristics sorted by Institutional Ownership Ratio (IOR) over the January 1980 to December 2012 period for skewness (SKEW), idiosyncratic volatility (IVOL), and institutional ownership ratio (IOR). SKEW is the realized skewness of monthly returns from daily data. IVOL is the standard deviation of the residuals from a four-factor model using a daily data within a month. IOR is the percentage of the shares of the firm owned by institutions. The column ‘H-L’ is the average portfolio characteristics (i.e. SKEW, IVOL and IOR) difference the high IOR and low IOR quintile. Newy-West t-statistics are reported in parenthesis.
Table II. Single Sort by Skewness

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Notes: The table reports quintile portfolios sorted by skewness (SKEW) over the January 1980 to December 2012 period. Panel A shows the average portfolio characteristics for skewness (SKEW), idiosyncratic volatility (IVOL), and institutional ownership ratio (IOR). SKEW is the realized skewness of monthly returns from daily data. IVOL is the standard deviation of the residuals from a four-factor model using a daily data within a month. IOR is the percentage of the shares of the firm owned by institutions. Panel B reports value weighted returns for the quintile portfolio. We also report value weighted returns adjusted for risk factors - the CAPM alpha adjusts for the market risk factor, the FF3 alpha adjusts for the Fama and French (1996) three factor, and the Carhart4 alpha adjusts for the momentum risk factor (e.g. Jegadeesh and Titman (1993)) in addition to the Fama and French three risk factors. Newy-West t-statistics are reported in parenthesis.
Table III. Double Sort - First by Institutional Ownership Ratio (IOR), then by Skewness

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Panel B: CAPM Alpha

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Panel C: FF3 Alpha

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<td></td>
<td>SKEW 2</td>
<td>SKEW 3</td>
</tr>
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<td>(0.88)</td>
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Notes: The table reports quintile portfolios sorted by skewness (SKEW) over the January 1980 to December 2012 period. Panel A shows the average portfolio characteristics for skewness (SKEW), idiosyncratic volatility (IVOL), and institutional ownership ratio (IOR). SKEW is the realized skewness of monthly returns from daily data. IVOL is the standard deviation of the residuals from a four-factor model using a daily data within a month. IOR is the percentage of the shares of the firm owned by institutions. Panel B reports value weighted returns for the quintile portfolio. We also report value weighted returns adjusted for risk factors - the CAPM alpha adjusts for the market risk factor, the FF3 alpha adjusts for the Fama and French (1996) three factor, and the Carhart4 alpha adjusts for the momentum risk factor (e.g. Jegadeesh and Titman (1993)) in addition to the Fama and French three risk factors. Newy-West t-statistics are reported in parenthesis.
Table IV. Double Sort - First by Institutional Ownership Ratio (IOR), then by SKEWNESS

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<th>High IOR</th>
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<th>4</th>
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<td>0.21</td>
<td>0.23</td>
<td>0.25</td>
<td>0.04***</td>
<td></td>
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<td></td>
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<tr>
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<td>0.17</td>
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<td>0.19</td>
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<td>0.04***</td>
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</tbody>
</table>

Note: The table reports the average skewness of a five by five sequential sort. I sort stocks into quintile portfolios first by institutional ownership ratio (IOR) and then with each IOR quintile, I sort stocks into quintile portfolio by skewness (SKEW). The column labeled ‘H - L’ reports the skewness difference between stocks in the highest and lowest skewness quintile within each IOR quintile. The Newy-West $t$-statistics are reported in the parenthesis. The data is from January 1980 up to December 2012.
Table V. Double Sort - First by Institutional Ownership Ratio (IOR), then by SKEWNESS

<table>
<thead>
<tr>
<th></th>
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<th>4</th>
<th>High IOR</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<tr>
<td>SKEW</td>
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<td>0.78</td>
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<td></td>
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<tr>
<td></td>
<td>(1.77)</td>
<td>(2.03)</td>
<td>(1.39)</td>
<td>(1.07)</td>
<td>(0.11)</td>
<td>(-2.88)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.10</td>
<td>1.14</td>
<td>0.93</td>
<td>0.76</td>
<td>0.56</td>
<td>-0.55**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.09)</td>
<td>(2.18)</td>
<td>(1.79)</td>
<td>(1.36)</td>
<td>(0.91)</td>
<td>(-2.49)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.30</td>
<td>1.10</td>
<td>1.03</td>
<td>1.04</td>
<td>0.96</td>
<td>-0.34*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.49)</td>
<td>(2.21)</td>
<td>(1.94)</td>
<td>(1.75)</td>
<td>(1.69)</td>
<td>(-1.94)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.13</td>
<td>1.03</td>
<td>0.93</td>
<td>1.08</td>
<td>0.71</td>
<td>-0.43***</td>
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<tr>
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<td>(2.24)</td>
<td>(2.05)</td>
<td>(1.89)</td>
<td>(2.14)</td>
<td>(1.36)</td>
<td>(-3.50)</td>
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<tr>
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<td>0.90</td>
<td>0.88</td>
<td>0.84</td>
<td>0.79</td>
<td>0.88</td>
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</tr>
<tr>
<td></td>
<td>(1.81)</td>
<td>(1.74)</td>
<td>(1.65)</td>
<td>(1.55)</td>
<td>(1.77)</td>
<td>(-0.18)</td>
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</tr>
</tbody>
</table>

The table reports the value weighted returns of a two by five by five sequential sort. First I sort stocks into two by market wide economic conditions into contraction period (Panel A) and expansionary period (Panel B). I use mean of the three month moving average Chicago Fed’s National Economic Activities Index to distinguish between expansionary and contractions period. Then, within the two economic activities, I sort stocks into quintile portfolios first by institutional ownership ratio (IOR) and then within each IOR quintile portfolio, I sort stocks into quintile portfolios by skewness (SKEW). The column labeled ‘H - L’ reports the return difference between stocks in the highest and lowest skewness quintile within each IOR quintile. The Newy-West t-statistics are reported in the parenthesis. The data is from January 1980 up to December 2012.
Table VI. Single Sort on Idiosyncratic Volatility

<table>
<thead>
<tr>
<th>Low IVOL</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>High IVOL</th>
<th>H - L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Return</td>
<td>1.05</td>
<td>1.18</td>
<td>1.23</td>
<td>1.31</td>
<td>1.35</td>
<td>1.34</td>
<td>1.26</td>
<td>1.19</td>
<td>0.90</td>
<td>0.37</td>
</tr>
<tr>
<td>(6.20)</td>
<td>(6.09)</td>
<td>(5.76)</td>
<td>(5.67)</td>
<td>(5.50)</td>
<td>(4.93)</td>
<td>(4.21)</td>
<td>(3.64)</td>
<td>(2.56)</td>
<td>(0.96)</td>
<td>(-2.21)</td>
</tr>
<tr>
<td>CAPM Alpha</td>
<td>0.3</td>
<td>0.34</td>
<td>0.36</td>
<td>0.41</td>
<td>0.45</td>
<td>0.34</td>
<td>0.27</td>
<td>0.14</td>
<td>-0.12</td>
<td>-0.65</td>
</tr>
<tr>
<td>(2.97)</td>
<td>(3.52)</td>
<td>(3.50)</td>
<td>(3.89)</td>
<td>(3.84)</td>
<td>(2.66)</td>
<td>(1.82)</td>
<td>(0.80)</td>
<td>(-0.60)</td>
<td>(-2.55)</td>
<td>(-3.52)</td>
</tr>
<tr>
<td>FF3 Alpha</td>
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<td>0.14</td>
<td>0.14</td>
<td>0.18</td>
<td>0.22</td>
<td>0.13</td>
<td>0.05</td>
<td>-0.10</td>
<td>-0.38</td>
<td>-0.96</td>
</tr>
<tr>
<td>(1.35)</td>
<td>(2.17)</td>
<td>(2.33)</td>
<td>(3.12)</td>
<td>(4.11)</td>
<td>(2.53)</td>
<td>(0.77)</td>
<td>(-1.41)</td>
<td>(-4.12)</td>
<td>(-6.73)</td>
<td>(-5.83)</td>
</tr>
<tr>
<td>Carhart4 Alpha</td>
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<td>0.16</td>
<td>0.19</td>
<td>0.25</td>
<td>0.30</td>
<td>0.24</td>
<td>0.20</td>
<td>0.09</td>
<td>-0.15</td>
<td>-0.70</td>
</tr>
<tr>
<td>(1.44)</td>
<td>(2.80)</td>
<td>(3.41)</td>
<td>(4.37)</td>
<td>(5.65)</td>
<td>(4.31)</td>
<td>(3.04)</td>
<td>(1.19)</td>
<td>(-1.42)</td>
<td>(-4.47)</td>
<td>(-4.30)</td>
</tr>
</tbody>
</table>

The table reports the average returns, CAPM alpha, Fama and French (1996) three factor alpha and Carhart four factor alpha of decile portfolios formed by sorting stocks by idiosyncratic volatility (IVOL). The column labeled ‘H - L’ reports the respective differences between stocks in the highest and lowest IVOL decile portfolios. The Newy-West $t$-statistics are reported in the parenthesis. The data is from January 1980 up to December 2012.
Table VII. Double Sort - First by Institutional Ownership Ratio (IOR), then by Idiosyncratic Volatility (IVOL)

<table>
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<tr>
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<th>H - L</th>
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</tr>
<tr>
<td>IVOL</td>
<td></td>
<td></td>
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</tr>
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</tr>
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<td>Low IOR</td>
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<td>1.15</td>
<td>0.80</td>
<td>-0.08</td>
<td>-1.17***</td>
</tr>
<tr>
<td></td>
<td>(4.21)</td>
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</tr>
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<td>1.45</td>
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<td>-0.62</td>
</tr>
<tr>
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<td>(5.17)</td>
<td>(4.32)</td>
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</tr>
<tr>
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<td>(4.79)</td>
<td>(4.62)</td>
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<td>(-1.42)</td>
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<tr>
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<td>0.11</td>
<td>0.15</td>
<td>0.09***</td>
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<td>0.08</td>
<td>0.09</td>
<td>0.12</td>
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<td>Panel C: Average SKEW</td>
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<td>0.33</td>
<td>0.37</td>
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<td>0.24</td>
<td>0.28</td>
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<td>0.23</td>
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</table>

The table reports average returns (Panel A), average idiosyncratic volatility (Panel B), and average skewness (Panel C) from a five by five sequential sort. I form quintile portfolios first by institutional ownership ratio (IOR) and then within each IOR quintile, I sort stocks into quintile portfolio by idiosyncratic volatility (IVOL). The column labeled ‘H - L’ reports the respective differences between stocks in the highest and lowest IVOL quintile within each IOR quintile. The Newy-West t-statistics are reported in the parenthesis. The data is from January 1980 up to December 2012.
Table VIII. Double Sort - First by Institutional Ownership Ratio (IOR), then by Idiosyncratic Volatility (IVOL)

<table>
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<tr>
<th>Low IVOL</th>
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<th>4</th>
<th>High IVOL</th>
<th>H - L</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: During Economic Contraction</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low IOR</td>
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<td>0.65</td>
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<td>(2.22)</td>
<td>(1.73)</td>
<td>(1.00)</td>
<td>(-0.77)</td>
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<tr>
<td>2</td>
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<td>1.17</td>
<td>1.21</td>
<td>1.05</td>
<td>-0.27</td>
</tr>
<tr>
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<td>(1.57)</td>
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<td><strong>Panel B: During Economic Expansion</strong></td>
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<tr>
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<td>(5.20)</td>
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<td>1.64</td>
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<td>(5.79)</td>
<td>(5.64)</td>
<td>(4.34)</td>
<td>(2.44)</td>
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<td>(5.07)</td>
<td>(5.58)</td>
<td>(4.62)</td>
<td>(3.31)</td>
</tr>
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</table>

The table reports the value weighted returns of a two by five by five sequential sort. First I sort stocks into two by market wide economic conditions into contraction period (Panel A) and expansionary period (Panel B). I use mean of the three month moving average Chicago Fed's National Economic Activities Index to distinguish between expansionary and contractions period. Then, within the two economic activities, I sort stocks into quintile portfolios first by institutional ownership ratio (IOR) and then within each IOR quintile portfolio, I sort stocks into quintile portfolios by idiosyncratic volatility (IVOL). The column labeled ‘H - L’ reports the return difference between stocks in the highest and lowest idiosyncratic volatility quintile within each IOR quintile. The Newy-West t-statistics are reported in the parenthesis. The data is from January 1980 up to December 2012.
Table IX. Double Sort - First by SKEWNESS, then by Idiosyncratic Volatility (IVOL)

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<th></th>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.64)</td>
<td>(4.82)</td>
<td>(4.29)</td>
<td>(2.81)</td>
<td>(1.29)</td>
<td>(-1.82)</td>
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</tr>
<tr>
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<td>1.29</td>
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<td>0.29</td>
<td>-0.96**</td>
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<td>-1.42***</td>
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<td>(1.82)</td>
<td>(-0.31)</td>
<td>(-3.56)</td>
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</table>

The table reports average returns from a five by five sequential sort. I form quintile portfolios first by skewness and then within each skewness quintile, I sort stocks into quintile portfolio by idiosyncratic volatility (IVOL). The column labeled ‘H - L’ reports the return differences between stocks in the highest and lowest IVOL quintile within each IOR quintile. The Newy-West t-statistics are reported in the parenthesis. The data is from January 1980 up to December 2012.
Table X. Triple Independent Sort - by Institutional Ownership Ratio (IOR), Skewness (SKEW), and Idiosyncratic Volatility

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<th>4</th>
<th>High</th>
<th></th>
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<td>(3.05)</td>
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<td>(2.63)</td>
<td>(1.81)</td>
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<td>1.34</td>
<td>1.43</td>
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<td>(4.30)</td>
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<td>(1.71)</td>
<td>(-1.09)</td>
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<td>1.24</td>
<td>1.52</td>
<td>1.31</td>
<td>0.56</td>
<td>-0.47</td>
</tr>
<tr>
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<td>(3.84)</td>
<td>(3.87)</td>
<td>(4.07)</td>
<td>(2.86)</td>
<td>(1.18)</td>
<td>(-1.20)</td>
</tr>
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<td>1.33</td>
<td>1.04</td>
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<td>-1.29***</td>
</tr>
<tr>
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<td>(5.24)</td>
<td>(3.50)</td>
<td>(2.37)</td>
<td>(0.38)</td>
<td>(-3.14)</td>
</tr>
<tr>
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</tr>
<tr>
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<td>1.28</td>
<td>1.04</td>
<td>0.84</td>
<td>0.71</td>
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<td>(2.88)</td>
<td>(1.70)</td>
<td>(-1.03)</td>
</tr>
</tbody>
</table>

The table reports average returns from a five by five by five sequential sort. I form quintile portfolios first by institutional ownership ratio (IOR) and then within each IOR quintile, I sort stocks into quintile portfolio by skewness and then within each skewness quintile portfolio, I sort stocks into quintile portfolios by idiosyncratic volatility (IVOL). Panel A reports the average returns in the lowest quintile institutional ownership ratio (IOR) and Panel B reports the average returns in the highest institutional ownership ratio (IOR). The middle three institutional ownership ratio (IOR) quintile are not shown here. The column labeled ‘H - L’ reports the respective differences between stocks in the highest and lowest IVOL quintile within each IOR quintile. The Newy-West t-statistics are reported in the parenthesis. The data is from January 1980 up to December 2012.