I, Andrew Knesnik, hereby submit this original work as part of the requirements for the degree of Master of Science in Mechanical Engineering.

It is entitled:
Feasibility Study of an Axially-Stacked, Subsonic Propeller System

Student's name: Andrew Knesnik

This work and its defense approved by:

Committee chair: Milind Jog, PhD
Committee member: Shaaban Abdallah, PhD
Committee member: J. Kim, PhD
Feasibility Study of an Axially-Stacked, Subsonic Propeller System

A thesis submitted to the Graduate School of the University of Cincinnati in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering in the College of Engineering and Applied Science by Andrew S. Knesnik

B.S. Cedarville University December 2012

Committee Chair: Milind Jog, Ph.D.
Thesis Adviser: Shaaban Abdallah, Ph.D.
ABSTRACT

With the aerospace industry pushing for an increased production and application of unmanned aerial vehicles (UAVs), humans will increasingly become exposed to noise pollution emanating from above. Fortunately, electric motors have and will continue to gain an ever-prominent role in the power plant of small, to mid-sized air vehicles, thus reducing the levels of acoustic exposure due to aircraft. However, one consequence of this electric motor emphasis is that the noise generated from an electric motor-powered UAV may become dominated by the propeller’s acoustics. In many cases, this propeller is selected off-the-shelf and only given consideration late in the design process. In an effort to reduce acoustic emission of future UAVs, this thesis explores a unique multi-stage propeller configuration, which may be applied to a vehicle post-design. It is my intention to explore the feasibility of this unique propeller configuration with aerodynamic and acoustic analysis.
This page intentionally left blank
ACKNOWLEDGEMENTS

First of all, I feel compelled to thank my Lord and Savior, Jesus Christ. His continued hand of blessing has been on my life for as long as I can remember, and it is only because of His sacrifice on the Cross that I can begin to understand the meaning of my existence.

I also feel indebted to my wife, who has proved to be incredibly patience towards me as I borrow from the ‘us time’ bank and gave it to ‘thesis time’ bank in order to complete this endeavor. Her unwavering sacrifice has provided me with the time and resources I needed to complete this thesis and her continued encouragement cannot be underestimated.
# TABLE OF CONTENTS

ABSTRACT............................................................................................................................................... i

ACKNOWLEDGEMENTS .......................................................................................................................... iii

LIST OF SYMBOLS ............................................................................................................................... vi

Chapter 1 Background Information ....................................................................................................... 1
  1.0 Introduction ...................................................................................................................................... 1
  1.1 Historic Aerodynamic Approaches to Propeller Design and Analysis ............................................. 4
  1.2 Historic Acoustic Approaches to Propeller Analysis ...................................................................... 7
  1.3 Current Approach to Propeller Analysis ...................................................................................... 8

Chapter 2 Propeller Aerodynamics ....................................................................................................... 12
  2.1 Modified blade-element/momentum theory .................................................................................. 12
  2.2 Wake analysis ............................................................................................................................. 16
  2.3 Drag polar evaluation/curve-fitting .............................................................................................. 20
  2.4 Code Validation ........................................................................................................................... 24
  2.5 Aerodynamic Approach Validation .............................................................................................. 25

Chapter 3 Propeller Acoustics ............................................................................................................... 32
  3.1 Summary of Acoustic Formulation .............................................................................................. 32
  3.2 Specifics of Acoustic Formulation .................................................................................................. 37
  3.3 Code/Acoustic Approach Validation .............................................................................................. 42

Chapter 4 Propeller Characterization ................................................................................................... 48
  4.0 Introduction ..................................................................................................................................... 48
4.1 Baseline Propeller Characterization ................................................................. 50

4.1 Stacked Propeller Characterization ................................................................. 56

4.3 Comparison of Baseline and Stacked Propellers ............................................. 63

Chapter 5 Summary of results and conclusions ................................................. 66

Works Cited ............................................................................................................. 68
# LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>Number of propeller blades</td>
</tr>
<tr>
<td>$c$</td>
<td>Speed of sound</td>
</tr>
<tr>
<td>$c_{\text{chord}}$</td>
<td>Chord of blade element</td>
</tr>
<tr>
<td>$C_d$</td>
<td>Drag coefficient</td>
</tr>
<tr>
<td>$C_l$</td>
<td>Lift coefficient</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Power coefficient, $C_p = \frac{Q}{\rho n^2 D^5}$</td>
</tr>
<tr>
<td>$C_t$</td>
<td>Thrust coefficient, $C_t = \frac{T}{\rho n^2 D^4}$</td>
</tr>
<tr>
<td>$D$</td>
<td>Tip diameter of propeller</td>
</tr>
<tr>
<td>$F$</td>
<td>Modified Prandtl tip loss correction factor</td>
</tr>
<tr>
<td>$J$</td>
<td>Propeller advance ratio, $J = \frac{V_\infty}{n D}$</td>
</tr>
<tr>
<td>$\ell$</td>
<td>Local force intensity, $\ell_i = P_{ij} \hat{n}_j$</td>
</tr>
<tr>
<td>$M$</td>
<td>Mach number</td>
</tr>
<tr>
<td>$n$</td>
<td>Rotational speed $\left( \frac{\text{rot}}{s} \right)$</td>
</tr>
<tr>
<td>$\hat{n}$</td>
<td>Unit normal vector to the blade surface</td>
</tr>
<tr>
<td>$OASPL$</td>
<td>Overall sound pressure level</td>
</tr>
<tr>
<td>$p'$</td>
<td>Acoustic pressure</td>
</tr>
<tr>
<td>$P_{ij}$</td>
<td>Compressive stress tensor</td>
</tr>
<tr>
<td>$P_{\text{ref}}$</td>
<td>Reference acoustic pressure, $P_{\text{ref}} = 20 \mu Pa$</td>
</tr>
<tr>
<td>$Q$</td>
<td>Torque</td>
</tr>
<tr>
<td>$r$</td>
<td>Radial distance from the axis of rotation</td>
</tr>
<tr>
<td>$Re$</td>
<td>Chord Reynolds number, $Re = \frac{\rho V_{\text{chord}}}{\mu}$</td>
</tr>
<tr>
<td>$RPM$</td>
<td>Rotational speed $\left( \frac{\text{rot}}{\text{min}} \right)$</td>
</tr>
<tr>
<td>$R_{\text{tip}}$</td>
<td>Tip radius of propeller</td>
</tr>
<tr>
<td>$SPL$</td>
<td>Sound pressure level, $SPL = 20 \cdot \log_{10} \left( \frac{p}{P_{\text{ref}}} \right)$</td>
</tr>
<tr>
<td>$t$</td>
<td>Observer time, taken in observer time frame</td>
</tr>
<tr>
<td>$T$</td>
<td>Thrust</td>
</tr>
<tr>
<td>$\vec{u}$</td>
<td>Externally-induced velocity vector on a blade element</td>
</tr>
<tr>
<td>$\vec{U}$</td>
<td>Intermediate velocity vector, $\vec{U} = \vec{u} + \vec{V}<em>\infty + \vec{V}</em>{\text{rot}}$</td>
</tr>
<tr>
<td>$\vec{v}$</td>
<td>Rotor-induced velocity vector on a blade element</td>
</tr>
<tr>
<td>$\vec{V}_\infty$</td>
<td>Freestream velocity vector</td>
</tr>
<tr>
<td>$\vec{V}_{\text{rot}}$</td>
<td>Rotational velocity vector</td>
</tr>
<tr>
<td>$\vec{W}$</td>
<td>Total velocity vector on a blade element</td>
</tr>
</tbody>
</table>
$\alpha$  Angle of attack on a blade element
$\beta$  Angle of a blade element, referenced to the zero-lift line
$\Gamma$  Circulation
$\eta$  Propeller efficiency, $\eta = \frac{c_t}{c_p} J$
$\lambda_w$  Wake advance ratio
$\rho$  Density of fluid
$\tau$  Retarded (emission) time, taken in source time frame
$\phi$  Angle of total velocity vector on a blade element
$\phi_i$  Angle between freestream velocity vector and total velocity vector on a blade element
$\phi_{\infty}$  Angle of freestream velocity vector on a blade element
$\Omega$  Rotational speed of propeller ($\frac{\text{rad}}{s}$)

**Subscript:**

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Vector component taken in the axial direction</td>
</tr>
<tr>
<td>$t$</td>
<td>Vector component taken in the tangential direction</td>
</tr>
<tr>
<td>$n$</td>
<td>Component taken in the direction normal to surface</td>
</tr>
<tr>
<td>$r$</td>
<td>Component taken in the radiation direction</td>
</tr>
<tr>
<td>$x, y, z$</td>
<td>Vector component taken along one direction for coordinate system defined in acoustic analysis</td>
</tr>
</tbody>
</table>
Chapter 1

Background Information

1.0 Introduction

As the title of this thesis implies, my main objective is to evaluate a unique propeller configuration in order to determine its feasibility for application on propeller-driven aircraft. Standard aircraft propellers enjoyed much attention by engineers and scientists in the earlier part of the 20th century, followed by a period of lull. In recent years, the rate at which propeller driven, unmanned aerial vehicles (UAVs) has exponentially increased, has brought on a resurgence of interest concerning propeller technology.

With the revival in propeller driven aircraft, better technologies are needed to improve the mission capability of UAVs. Typically the propeller used on a UAV is selected late in the design process as an ‘off the shelf’ selection. Ideally a propeller should be specifically designed for every UAV in production in order to optimize the entire propulsion system, both aerodynamically and acoustically. However, the difficulty of changing the culture of the aerospace community to accommodate this mindset presents me with a seemingly insurmountable hurdle.

Rather than trying to change the culture, I will attempt to explore the feasibility of a somewhat simple propeller configuration not currently in production. This propeller structure is created by displacing several off-the-shelf propellers by some axial distance on a similar shaft. If performance improvements can be realized with this setup it will serve as a simple means to improve the performance of propeller-driven aircraft with only slight modifications. A picture of this configuration can be seen in the figure below.
Figure 1.1: Two stage, axially-stacked propeller concept.

The concept of axially displacing rotors drastically expands the design field by introducing a number of new variables not present in standard, single-stage propellers. For this particular study I will confine my evaluation of this concept by assuming a similar geometry for a 2-stage configuration. The table on the following page outlines some of the propeller configurations afforded by this concept.
Table 1.1: Several of the unique design configurations available to the axially-stacked propeller concept.

<table>
<thead>
<tr>
<th>Number of stages:</th>
<th>Unique angular velocity at every stage:</th>
<th>Number of blades per stage</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Diameter of each stage:</th>
<th>Relative angular offset per stage:</th>
<th>Distance between stages:</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angular sweep:</th>
<th>Axial sweep:</th>
<th>Unique asymmetric spacing at each stage:</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
</tr>
</tbody>
</table>
1.1 Historic Aerodynamic Approaches to Propeller Design and Analysis

Prior to delving into the details of this section, I want to be up front with my intention. I am not aiming to provide a detailed historical development of every available propeller analysis technique; rather I hope to show the historical progression of the methodology I used for this endeavor.

Aircraft propeller research and analysis received heavy attention during the first half of the 20th century, although marine propellers began receiving attention more than a half a century earlier. In 1865 Rankine put forward a simple analysis method where a propeller was replaced with a disk, across which the pressure was discontinuous. This Method of analysis did not concern itself with specific blade geometry and was based on three main assertions: (1) the propeller works in an ideal fluid where frictional drag may be ignored; (2) the propeller may be substituted with an actuator disk, which is equivalent to a propeller with an infinite number of blades; and (3) the propeller can produce thrust without adding tangential swirl to the slipstream. Later in 1887 R.E. Froude removed the third assumption, which changed the name of the method from the ‘axial momentum theory’ to the Rankine-Froude momentum theory (also known as the general momentum theory). One significant outcome afforded by this technique was that it allowed for rapid computation of the efficiency ceiling for a propeller. Another famous outcome of this theory was the idea that half of the fluid’s axial acceleration within the streamtube occurs upstream of the propeller disk.

Taking quite a different approach, W. Froude in 1878 introduced what is known as the blade-element theory. In this approach the propeller is subdivided into a number of radial stations, each of which ‘sees’ the freestream velocity ($V_\infty$) and the rotational velocity ($\Omega r$) as if it were in isolation from the other blade sections. Modern theoretical approaches include an additional velocity component known as the rotor-induced velocity. The blade-element theory laid the foundation for many modern propeller analysis and design techniques.
Although the Rankine-Froude momentum theory and the blade element theories progressed for a time decoupled from each other, they were eventually coupled into what is now known as an early developmental stage of the combined blade element/momentum theory.

In 1919 Prandtl extended the idea that the same circulation emanating from a lifting surface is also the mechanism by which propellers generate thrust. One significant consequence of this was the idea that the induced propeller drag is minimized when the free vortex sheet emanating from the propeller takes on the form of a rigid helicoidal vortex sheet moving backward unreformed in the wake of the propeller.

![Figure 1.2: Helicoidal vortex sheet emanating from propeller (Wald, 2006).](image)

Prandtl discovered an approximate solution to the flow around this vortex helicoidal sheet by likening the flow around the helix structure to that around a cascade of straight lamina. One particularly useful property of the solution Prandtl found was that it was a simple, closed-form analytical equation, thus making it an easily-employed approximation method.
A number of authors sought to obtain a solution to the actual flow around the helicoidal vortex structure, although Goldstein was the first. He solved for the potential flow around this structure, although stating that the results he obtained were only applicable for small advance ratios (i.e. lightly-loaded propellers). Theodorsen removed the lightly-loaded propeller assumption by looking at the vortex structure in the far downstream wake of the propeller, after the slipstream has already contracted to its ultimate diameter. Another one of Theodorsen’s contributions was to expand the Goldstein factors by devising an ingenious experiment to measure the potential around this vortex structure using an electrical analogy.

All of the above work concerning the helicoidal structure was centered on developing methods to design optimum efficiency propellers, although the performance of a given propeller configuration can be deduced from the same work. In order to do this, certain assumptions need to be made concerning the velocity field at the blade elements of the propeller. This analysis requires the assumption that the blades may be analyzed independently; something that cannot be justified either mathematically or otherwise. However, one reason for the popularity of this method as a low-fidelity analysis tool is its satisfactory comparison with experimental data trends.
A number of other methods are available to predict the performance of propellers, from lifting line/surface methods, to vortex lattice techniques, to boundary element solvers. Indeed, computational resources have advanced to the extent that propeller can now be analyzed with full Navier-Stokes solvers, albeit at the expense of computational efficiency. However, because the intention of this thesis is to explore the design field in a way what will inform both myself and others if this propeller configuration is viable, I am not concerned with the absolute accuracy of the performance predictions, but rather with the performance trends. Because of this, a lower fidelity tool such as a blade element/momentum solver matches well with my needs.

1.2 Historic Acoustic Approaches to Propeller Analysis

Aircraft propeller noise became a research topic as far back as 1919, when airplane technology was still in its infancy. During the early stages of research, engineering focused mostly on developing empirical models for predicting rotor acoustics. Although a certain amount of progress was made in this period, it wasn’t until the advent of computers in the 1950’s that propeller acoustic prediction started to become possible for more general test cases.

Another significant introduction in the 50’s was several papers Lighthill published called “On Sound Generated Aerodynamically I&II, General Theory”. The theoretical foundation pioneered by Lighthill allowed Ffowcs Williams and Hawkings to expand on the concepts using the mathematical theory of distributions. Their benchmark paper known as “Sound Generation by Turbulence and Surfaces in Arbitrary Motion” has led the way as the most commonly used theoretical development to predict the sound of moving surfaces, a subset of which is propellers.

A number of authors continued to build on the Ffowcs Williams Hawkings (FWH) equation, one of whom was Farassat, who developed surface integral forms of the original equations to represent the
thickness, loading and quadrupole noise sources. Farassat’s contributions to the aerospace industry have allowed for rapid computations of propeller noise predictions, which form the basis for the research performed in this thesis. One significant form of the FWH equation that Farassat developed is known as Farassat’s formulation 1A. This equation represents the foundations from which I performed all my acoustic analysis.

Although this section presents a most shallow overview of the history of propeller noise prediction, a number of references do an excellent job of expanding on this topic, one of which is a NASA report titled “A Review of Propeller Noise Prediction Methodology 1919-1994” (Metzger, 1995).

1.3 Current Approach to Propeller Analysis

The path I have taken to complete this thesis has been full of fortuitous ventures, and countless failed endeavors. However, when Thomas Edison was talking about the many failures he had prior to discovering an effective light bulb design he was quoted saying “I have not failed 1,000 times. I have successfully discovered 1,000 ways to NOT make a light bulb.” I share his sentiments.

At the beginning of this thesis work I was intent on developing my own CFD code, through which I was planning on running all of the aerodynamic analysis. I started developing this code using a fractional step method, only to discover that much of the commonly-used fractional step techniques require a staggered grid. Trying to avoid this added difficulty, I decided to search for a method which was formulated specifically for a non-staggered grid, which lead me to the pressure gradient method, described by Golbraith and Abdallah (Implicit Solutions of Incompressible Navier-Stokes Equations Using the Pressure Gradient Method, 2011).

Because of the difficulty associated with generating their solution for the pressure gradient terms using first-order differentials, I started developing a technique to solve for the pressure gradients using a
second-order, Laplacian differential, which I derived. However, problem after problem arose with my specific formulation, forcing me to search for yet another numerical approach to solving the Navier-Stokes equations.

The last and final CFD technique I attempted was the Vorticity-Velocity approach. One significant advantage of this formulation was that rotating coordinates can be simply expressed as modifications to the boundary and initial conditions, rather than requiring a re-formulation. Through this approach I was able to develop and validate a 2D code to solve the full Navier-Stokes equations using general, curvilinear coordinates. However, this technique was highly dependent on the differencing scheme used for the vorticity boundary conditions, making the selection of boundary discretization more an art, rather than a science. In addition to this difficulty, ensuring a divergence-free field proved tremendously difficult; especially due to the non-staggered grid approach.

Because of the many difficulties associated with developing the full Navier-Stokes solver, I decided to search for an alternate aerodynamic approach. After much deliberation I finally selected the modified blade-element/momentum theory. Although the absolute accuracy of the predictions from this method follows a similar trend as general low fidelity analysis, the aerospace community is generally accepting of the relative performance prediction trends. Additionally, the computational efficiency of such an approach has proven unmatched by nearly any other method that has this degree of fidelity.

As far as the acoustic analysis is concerned, it was never in question as to whether I would use some form of the FWH equation to generate my solution. Most modern propeller acoustic approaches use this formulation, making it widely accepted in nearly every acoustic community as the best method for aerodynamic analysis of propellers. However, as a sub-topic to this category, I did deliberate for some time on which formulation of the FWH equations I would use.

The first choice I made regarded whether I wanted to perform my analysis in the time or the frequency domain. Historically, frequency-domain analysis of propellers was more popular in the
infancy of acoustic propeller analysis than in modern times. Because of the wealth of readily-available ‘recent’ papers on the time-domain formulations for representing the FWH equation, I decided to use this approach. Next I had to decide which time domain approach to use. Having been exposed to the work of Farassat in the past I was quite impressed with his ability to explain difficult acoustic concepts in intuitive ways. He has published quite a few papers on different time domain formulations, several of which he developed himself. I decided to use a formulation of the FWH equation known as Farassat’s 1A formulation, which I will explain in detail in the acoustic section of this thesis. One interesting thing to note about Farassat is his significant contribution to the aerospace industry through his effort in developing a computer program known as WOPWOP; a NASA software code to predict the acoustics of helicopter rotors.

Steady pressures on the blade surface are required in order to estimate the propeller’s acoustics. In order to obtain these pressures I used the program XFOIL, created by Mark Drela. This software has proven indispensable for the current effort, as it has seamlessly tied computational performance with excellent prediction capabilities.

I have coupled all of these codes together with numerous MATLAB scripts, the flow structure of which is outlined in the following chart.
Figure 1.4: Software flowchart for axially-stacked propeller analysis

In order to facilitate this exploratory study, I have decided to develop as many of my solutions as possible using software that I created. I certainly could have used existing software to complete all of the objectives, but had I done so I would have missed a great opportunity of learning. Programming each module by hand allowed me to more fully understand (and also appreciate) both the theory and the computational application of multiple modern theories, which is something I will take with me on my life-long endeavor of learning.
Chapter 2

Propeller Aerodynamics

2.1 Modified blade-element/momentum theory

As mentioned in the background section of this report, I intend to use a combined blade-element/momentum method for the aerodynamic propeller calculations. This method is detailed in the QPROP Formulation paper, although I will delve into some of the particulars here as well (Drela, 2006). The following figure shows the velocity vectors incident on a section of the propeller blade at a given radial station.

![Figure 2.1: Velocities incident on a 2D airfoil section, used for blade-element analysis](image)

In the above figure, $\vec{u}$ represents the externally-induced velocity, while $\vec{v}$ represents the rotor-induced velocity. Note that $\vec{u} = 0$ for isolated propellers, where the freestream velocity has not been modified by an upstream or a downstream influence. The velocities seen by two propellers, axially-stacked, as well as those seen by a pair of counter-rotating propellers are shown below for reference. As you can see, the main difference between the velocity vectors is the direction of the tangential, externally-induced velocity on the downstream propeller stage, relative to the tangential rotor velocity.
Table 2.1: Velocity triangles for a pair of axially-offset propellers.

<table>
<thead>
<tr>
<th></th>
<th>Upstream stage</th>
<th>Downstream stage</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stacked rotors</strong></td>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
</tr>
<tr>
<td><strong>Counter-rotating</strong></td>
<td><img src="image3" alt="Image" /></td>
<td><img src="image4" alt="Image" /></td>
</tr>
</tbody>
</table>

The tangential induced velocity ($v_t$) is related to the torque that the rotor imparts on the fluid. Helmholtz’s Theorem may be used to relate this torque, resulting in swirl, to the circulation on the rotor blades, which is described in the equation below.

$$2\pi r \bar{v}_t = \frac{1}{2} B \Gamma$$

The circumferentially-averaged tangential velocity is assumed to be related to the tangential velocity via the following empirically-derived relationship, which Drela put forward in the theory guide for QPROP (Drela, QPROP Formulation, 2006).

$$\bar{v}_t = v_t F \sqrt{1 + \left(\frac{4\lambda \omega_{tip}}{\pi Br}\right)^2}$$
In the above equation, $F$ is the modified Prandtl tip loss correction factor, while $\lambda_w$ is the wake advance ratio. The equations defining these quantities may be seen below.

$$F = \frac{2}{\pi} \cos^{-1} \left\{ \exp \left[ -\frac{B}{2} \left( 1 - \frac{r}{R_{tip}} \right) \frac{1}{\lambda_w} \right] \right\}$$  \hfill (2.3)

$$\lambda_w = \frac{r}{R_{tip}} \tan \phi$$  \hfill (2.4)

Geometrically the induced tangential velocity and the fluid velocity seen by the rotor blade are related to the freestream velocity magnitude, $U$, via the following equations:

$$v_t = U \cdot \sin \phi_l \sin \phi$$ \hfill (2.5)

$$W = U \cdot \cos \phi_l$$ \hfill (2.6)

Manipulating equations 2.3-2.6 and solving for $\Gamma$, leads to the following equation for circulation, as defined by the downstream helicoidal vortex sheet emanating from the propeller.

$$\Gamma = \frac{8r}{B} W \cdot \cos^{-1} \left\{ \exp \left[ \frac{B}{2} \left( 1 - \frac{R_{tip}}{r} \right) \cot \phi \right] \right\} \sqrt{1 + \left( \frac{4}{\pi B} \right)^2 \tan^2 \phi \cdot \tan \phi_l \sin \phi}$$ \hfill (2.7)

The above equation ties the propeller to the far-field wake, although one last equation is needed to bring closure. That relationship is found by connecting the 2-dimensional airfoil lift coefficient to the local blade circulation.

$$\Gamma = \frac{1}{2} W \cdot chord \cdot C_l$$ \hfill (2.8)

Substituting equation 2.8 into 2.7 and rearranging the results leads to the following final equation.

$$\cos^{-1} \left\{ \exp \left[ \frac{B}{2} \left( 1 - \frac{R_{tip}}{r} \right) \cot \phi \right] \right\} \sqrt{1 + \left( \frac{4}{\pi B} \right)^2 \tan^2 \phi \cdot \tan \phi_l \sin \phi - \frac{B \cdot chord}{16r} C_l} = 0$$ \hfill (2.9)

The above equation, which relates the circulation from the 2-dimensional airfoil analysis to that which springs from the blade surface (which generates a helicoidal vortex structure in the wake of the propeller) is used to solve for the induced velocity angle, $\phi_l$ at every blade element.
In order to solve for the roots of this nonlinear equation efficiently, I used a modified secant method. This is accomplished by setting the above equation equal to some variable, \( \Delta = \Delta(\phi_i) \). A small perturbation, \( \varepsilon \) is introduced in order to find the approximate derivative of the \( \Delta \) function.

\[
\Delta'(\phi_i^n) \approx \frac{\Delta(\phi_i^{n+1})-\Delta(\phi_i^n)}{\varepsilon}
\]  

(2.10)

The Newton-Raphson formula, which is historically one of the most widely used root-finding approaches, uses the tangent at a given point along a slope to iteratively converge on the root of an equation. The Newton-Raphson formula and a figure, which visually describes the Newton Raphson process, can both be seen below.

\[
x^{n+1} = x^n - \frac{f(x^n)}{f'(x^n)}
\]

(2.11)

![Newton-Raphson convergence process](image)

Figure 2.2: Newton-Raphson convergence process

The next best guess for the induced velocity angle, \( \phi_i^{n+1} \) (which will satisfy equation 2.8, forcing \( \Delta = 0 \)) is seen in the relationship below:

\[
\phi_i^{n+1} = \phi_i^n - \frac{\varepsilon \cdot \Delta(\phi_i^n)}{\Delta(\phi_i^{n+1})-\Delta(\phi_i^n)}
\]

(2.12)

See the reference “Applied Numerical Methods with MATLAB” for a more detailed description of this root finding method (Chapra, p. 150).
2.2 Wake analysis

Knowing how the flowfield develops both behind and in front a propeller stage is of utmost importance for multi-stage propeller configurations. When \( \bar{u} = 0 \), the approach outlined in the previous section applies to isolated propellers. However, for the case of multiple propeller stages, it is important to model the propeller’s influence on the fluid both upstream and downstream of each propeller, which will enter the blade element/momentum analysis through the externally-induced velocity component, \( \bar{u} \).

Consider the following streamtube, which encases the fluid far upstream to far downstream and passes through the propeller plane of a single-stage configuration.

![Figure 2.3: Velocities within and on boundaries of streamtube encapsulating the fluid passing through a 1-stage actuator disk.](image)

Following the first assumption taken by Lock, the analysis of blade elements on a multi-stage propeller will be performed assuming the interference velocity may be analyzed by considering the velocity fields from multiple propeller stages independently (Lock, 1941). However, one item still needed to perform the analysis was a means of estimating the streamtube cross-sectional area (and consequential radius) as a function of axial position. This was required so that I might develop a method to estimate the externally-induced axial velocity components at each propeller stage.

By conservation of mass through the streamtube, \( \rho_\infty V_\infty A_u = \rho_\infty(V_\infty + v_a)A_{disk} = \rho_\infty[V_\infty + f(z) \cdot v_a]A_{tube}(z) = \rho_\infty(V_\infty + 2v_a)A_d \), where subscripts “u” and “d” represent the far upstream and far downstream boundaries, respectively. Additionally, \( z \) is the axial location, treated as positive in the
downstream direction with its origin at the actuator disk. Note that $A_{disk}$ is approximately equivalent to $\pi R_{tip}^2$. It has been assumed that the far upstream and far downstream boundaries are located at an axial distance of $5 \cdot R_{tip}$ from the propeller. Although this assumption cannot be justified through any sort of mathematical means, it will allow me to come up with a closed-form analytical solution to represent the consequence of the streamtube contraction in the wake of the propeller. The function $f(z)$, which will represent the axial velocity increase from far upstream to far downstream, will be selected such that $f(z = -5R_{tip}) = 0$, $f(z = 0) = 1$, and $f(z = 5R_{tip}) = 2$. It is highly desirable to select an analytical function, which has limits at both $+\infty$ and $-\infty$. For this reason, the inverse tangent function, $f(z) = \frac{z}{\pi} \arctan\left(\frac{z}{R_{tip}}\right) + 1$ was selected. The following figure visually describes $f(z)$, which is equal to 0.0405 at $z = -5R_{tip}$, 1.0000 at $z = 0$, and 1.9595 at $z = 5R_{tip}$.

![Figure 2.4: Scale factor used to describe smooth transition from far upstream to far downstream for streamtube area calculation.](image)

In summary, the following equation will be used to evaluate the radius of the streamtube at an arbitrary axial location:

$$A(z) = \left\{ \frac{v_\infty + \bar{w}_a}{v_\infty + f(z) \bar{w}_a} \right\} A_{disk} = \left\{ \frac{v_\infty + \bar{w}_a}{v_\infty + \frac{z}{\pi} \arctan\left(\frac{z}{R_{tip}}\right) + 1} \bar{w}_a \right\} A_{disk}$$  \hspace{1cm} (2.13)
\[ R_{\text{tube}}(z) = R_{\text{tip}} \sqrt{\frac{V_{\infty} + \bar{w}_a}{V_{\infty} + \frac{2}{\pi} \arctan \left( \pi \frac{z}{R_{\text{tip}}} \right) + 1 \bar{w}_a}} \]  

(2.14)

The above equation can be expressed in fractions of tip radius using the following relationship:

\[ x_{\text{tube}}(z) = \frac{V_{\infty} + \bar{w}_a}{V_{\infty} + \frac{2}{\pi} \arctan \left( \pi \frac{z}{R_{\text{tip}}} \right) + 1 \bar{w}_a} \]  

(2.15)

It is also necessary to determine an approximation of the axial and tangential velocities present close to, but outside of the streamtube in the radial direction. As a zeroth-order approximation, I have assumed that both the averaged axial and tangential velocities outside of streamtube smoothly decrease from a value of \( \bar{w}_a \) and \( \bar{w}_t \) at the surface of the streamtube \( (R_{\text{tube}} = R_{\text{tube}}(z)) \) at a given axial location to zero at a radial location of \( 1.2 x_{\text{tube}} \). Another inverse tangent function has been selected to represent the decay of the averaged, induced velocities outside the streamtube. The equation below represents the average induced velocity component, which smoothly transitions from nearly a constant value within the streamtube to a rapid decay outside the streamtube

\[ \bar{w}(x) = g(x) \cdot \bar{w} \]  

(2.16)

\[ g(x, z) = \frac{1}{2} + \frac{1}{\pi} \arctan \left\{ -10\pi[x - x_{\text{tube}}(z)] \right\} \]  

(2.17)

The analytical expression, \( g(x) \), is approximately equal to 1 throughout the streamtube, then rapidly decays to near zero at \( x = 1.2 \). The shape of this curve appears very similar to the one described above, thus I will not show it here. However, the consequence of assuming these profiles can be seen in the figure below, which shows the induced axial velocity down the streamtube.
Figure 2.5: Induced axial velocity profiles inside the streamtube.

When performing lower fidelity analysis, aeronautical engineers have historically assumed the tangential velocity is negligible upstream of the propeller, and equal to the induced tangential velocity through the entire downstream profile. I have also utilized this assumption for my work here. To describe the induced tangential velocity decay outside the streamtube I used the same decay behavior as the induced axial velocity profile, resulting in the following tangential velocity profiles, taken at a slice of the streamtube.
2.3 Drag polar evaluation/curve-fitting

The propeller cross sections contain airfoil elements, each with unique drag and lift polars, which are dependent on both the angle of attack and Reynolds number. As an example, lift and drag polars are shown below for a NACA 4412 airfoil.

Figure 2.7: Representative lift and drag polar for a NACA 4412 at $Re = 1e5$, generated in XFOIL.
Many different approaches have been taken to estimate the lift and drag in an efficient way. As the blade element solver iterates, the angle of attack and Reynolds number seen at each radial station should converge. Often times, a database of airfoil polars is built up prior to executing the blade element/momentum solver, after which the polars are interpolated between table values. Although this approach keeps the lift/drag data bounded by the table values, it may not adequately account for the non-linear dependency on the Reynolds number.

Another approach is to establish a scaling law relationship between the lift/drag and the Reynolds number. Such was the approach of Yamauchi and Johnson when they published approximate Reynolds number scaling laws based on a literature survey (Trends of Reynolds Number Effects on Two-Dimensional Airfoil Characteristics for Helicopter Rotor Analysis, 1983). Drela of MIT has also implemented this type of Reynolds number scaling in his code QPROP/QMIL and XROTOR (Drela, XROTOR Download Page, 2003) (Drela, QPROP Download Page, 2007). Although approximate scaling laws may be established, the constants used within the scaling law relationships are highly dependent on the airfoil being analyzed, thus making these relationships somewhat crude and cumbersome for analysis of arbitrary airfoils. However, one advantage of this method is their extreme computational efficiency. All that is required of the 2-dimensional airfoil solver is to sweep each airfoil section through a range of angle of attacks at one Reynolds number, after which the value of the lift and drag coefficients at a different Reynolds number may be estimated by scaling the results either up or down.

Obviously, one accurate means of obtaining the lift/drag data is to run the 2D airfoil solver in series with the blade element solver. Apart from the accuracy, this approach also has the highly desirable feature that the analysis is generalized to the extent that nearly any rotor may be analyzed without requiring a pre-processor to obtain the airfoil polars. Because of the high degree of accuracy afforded by this approach, I decided to use this method in my aerodynamic solver. To combat the large
computation time, which would be required to run the analysis from start-to-finish with a full viscous solver, I used an inviscid panel code to predict the lift at each blade section for the initial analysis.

Analyzing the airfoil inviscidly is computationally efficient, although it is sure to over predict the lift coefficient, by implicitly assuming no frictional drag. After the angle of attack and Reynolds number at every blade section converged, I repeat the blade element/momentum analysis with the viscous airfoil solver. Typically the analysis run with the inviscid solver converges after 10-20 iterations, while the analysis with the viscous airfoil solver only requires 2-3 iterations.

XFOIL has proven itself time and time again to be an excellent means of analyzing blade sections in a viscous way; sacrificing little accuracy, but with the gain of extreme computational efficiency (compared to higher fidelity methods, such as CFD). For these reasons, I performed all viscous airfoil analysis with XFOIL.

Although XFOIL is also capable of analyzing blade sections with an inviscid panel formulation, the overhead time required for the data files to be written and read prompted me to search for an alternate inviscid solver.

An inviscid panel solver, located in the appendix of the report “Design and Analysis of Low Reynolds Number Airfoils” was utilized for this effort (Borer, 2002). Computationally the solver is efficient, and it has an added bonus that the source code was written in MATLAB, thus making it a prime candidate for this effort. In the report outlined above, Borer performs a side-by-side comparison with results he obtained from his panel code and those found by XFOIL’s inviscid solver. The following plot shows a side-by-side comparison of the pressure coefficients obtained from both the MATLAB panel solver, and XFOIL for a NACA 4412 airfoil at an angle of attack of 5°.
Notice in the above graph how closely the Borer code matches the XFOIL inviscid analysis for all but the extreme trailing edge of the airfoil. Because of the near-identical results, the Borer code was used as the inviscid airfoil solver.

The following plot shows the blade element/momentum solver converging on the angle of attacks (hence the lift coefficient). Notice that as expected, the lift coefficient converged on with the inviscid airfoil analysis was greater than the final lift coefficient, converged with the viscous airfoil analysis. The angle of attack also shows this trend.
2.4 Code Validation

Prior to delving into the details of my code validation, I want to make it clear that validating the software is much different than validating the formulation or approach used to solve the problem. This step merely involves checking to ensure the algorithms were correctly implemented, and does not address the validity of the predictions.

Because a majority of the combined blade element/momentum formulation came directly from the QPROP theory guide, I used this software to generate comparison data. Rather than using the original QPROP executables, which also run calculations for electric motor configurations, I isolated the blade element/momentum solver and converted the original FORTRAN 77 source to MATLAB source. The following plot describes the results I obtained for the APC 7x5E propeller at an advance ratio of 0.5, and a rotational speed of 5000 RPM. Note that to ensure the two codes were using the same values for 

![Figure 2.9: Convergence of angle of attack and lift coefficient.](image)
lift and drag, I used the curve-fits of a Clark Y airfoil (rather than running XFOIL in series with the blade element/momentum solver).

![Graph](image)

**Figure 2.10:** Comparison of calculated lift coefficient distribution with QPROP results for an APC7x5 propeller, assuming an advance ratio of 0.5 at an RPM of 5000 with a Clark Y airfoil across the entire span.

As is quite clear from the figure above, my program calculated a nearly identical lift coefficient distribution across the span of the propeller, thus leading me to believe I have correctly implemented the algorithm.

### 2.5 Aerodynamic Approach Validation

Despite its general acceptance within the aerospace community as a ‘good’ lower-fidelity analysis approach, I still thought it would be a good idea to test my software against experimental data. In order to do this I ran my program against test data that I found on the University of Illinois propeller database website (UIUC Propeller Database, 2012). This website contains propeller performance information, which was gathered from a number of sources.
The propeller I selected for this validation effort was an APC 9x6E, which can be seen in the figure below. One reason I chose this configuration (apart from the fact that experimental data was available) was that it is in the same general size class as the APC 7x5 propeller (the propeller I selected as the baseline configuration for this thesis). Choosing a propeller in the same size class as the 7x5 propeller is significant, due to the fact that the low-Reynolds number effects often associated with small propellers, can have a substantial effect on the performance predictions.

Figure 2.11: Front and side view of an APC 9x6E propeller, obtained from UIUC’s website (Ananda, 2012).

The following figure outlines the blade geometry of the airfoil.

Figure 2.12: Blade geometry for APC 9x6E propeller.
Experimental test data was available for this propeller across a range of advance ratios and RPM’s. I ran my analysis at 5000 RPM, over a range of advance ratios where the propeller was not wind-milling. The following plots show the results

![Figure 2.13: Comparison of predicted vs. experimental power coefficients for APC 9x6E propeller.](image)

![Figure 2.14: Comparison of predicted vs. experimental thrust coefficients for APC 9x6E propeller.](image)

Unfortunately, the predicted results prove to be a ‘mixed bag;’ at certain advance ratios, the predicted performance was nearly identical to the measured value, whereas at other run conditions the
predicted performance was nearly 20% different than the actual. Additionally, the predicted curve is not as ‘smooth’ as I would have expected the pseudo-analytical analysis to produce. However, I believe a number of the contentious results are due to incorrect propeller geometry and not of a fault of the analysis technique.

In order to run the analysis code, the airfoils along the entire blade were needed. From my personal interaction with the APC Propeller, I learned that for an APC 7x5E propeller, the blade sections linearly transition from a Clark Y airfoil at the hub, to an Eppler 63 airfoil at the tip. Because of the similarity in size between the 7x5 and 9x6 propellers, I have also assumed this airfoil transition. However, I suspect this assumption is introducing a certain amount of errors. Consider the following statements from APC Propeller’s website:

The dominant basis for the primary airfoil shape used in most APC propellers is similar to the NACA 4412 and Clark-Y airfoils, except the leading edge is somewhat lower. Also, the aft region is somewhat thicker (APC Propeller, 2012).

Cross-section geometry in and near the hub region is defined with specialized algorithms. The aerodynamic-dominant airfoil must smoothly transition into a structural-dominant shape in a manner that emphasizes strength consistent with milling machine tool constraints (APC Propeller, 2012).

Because of these factors, I believe the airfoil cross sections I have used for the analysis to be somewhat incorrect. As one ‘check’ to this assertion, I re-ran the analysis, this time using a Clark Y airfoil along the entire blade. This airfoil is extremely stable, meaning it has a large range of angle of attacks where the airfoil is not stalled. Because of this stability, XFOIL had very little difficulty executing for a vast majority of the angle of attacks seen on the blade section, thus resulting in the smooth performance curves seen below.
Another potential source of error was the blade geometry (i.e. chord and blade angle at each radial station), which was provided with the experimental test data. In their paper, “Analytical –
Experimental Comparison for Small Electric Unmanned Air Vehicle Propellers,” the authors performed a rough sensitivity analysis on the effect a small perturbation in blade angle has on the overall propeller performance (OL, Zeune, & Logan, 2008). These authors concluded that even a two degree offset in the blade angle can have a significant effect on the predicted blade performance using blade element/momentum theory. To test this result, I re-ran the analysis assuming both a $+2^\circ$ and a $-2^\circ$ blade angle offset, the results of which are shown below. Note that because of the airfoil stability, I assumed the blade sections were entirely composed of Clark Y airfoils.

![Power Coefficient Variation for APC 9x6E](image)

**Figure 2.17:** Power coefficients variation for APC 9x6E, with $2^\circ$ blade angle offsets in either direction, assuming the blade elements are entirely composed of Clark Y airfoils.
Figure 2.18: Thrust coefficients variation for APC 9x6E, with 2° blade angle offsets in either direction, assuming the blade elements are entirely composed of Clark Y airfoils.

By merely examining the above two curves, it becomes obvious to me that knowing the exact blade angle to a high degree of certainty is important when trying to predict the actual performance of a given propeller.
Chapter 3

Propeller Acoustics

3.1 Summary of Acoustic Formulation

By far, the most common acoustic formulation for predicting the noise of a rotating propeller is the Ffowcs Williams/Hawkings (FWH) equation, based on Lighthill’s acoustic analogy. An integral formulation of the FWH equation is one in which forces on the propeller’s surface are integrated, thus eliminating the need to solve the full three-dimensional flowfield, as is commonly done in Computational Fluid Dynamics (CFD) solutions, using Computational Aeroacoustics (CAA).

A mid-point quadrature, retarded-time formulation was selected for this current effort, due to its efficiency and robustness. In this formulation, the surface of the propeller is meshed into panels, at the center of which an approximate pressure magnitude is calculated. Note that the mesh must be of high enough resolution that the surface pressure is approximately linear over the panel, and the retarded time doesn’t vary significantly over the panel. If the source strengths on the panels are not linear, the midpoint value does not accurately represent the value at the vertices, hence the mesh needs further refinement.

One concept unique to acoustics is the idea of a retarded time, which can prove difficult for someone not intimately familiar with acoustic noise prediction techniques. This concept can be explained as follows: as the propeller rotates, each panel on the surface of the propeller acts as a source, from which sound waves emanate. Each element of the meshed blade surface is located at a different distance from the observer than the other elements; hence it takes a different amount time for the sound emitted by that panel to reach the observer than it might take for the sound generated by a different panel to reach the observer. In order to calculate the noise heard by the observer at a given observer...
time, $t$, one must solve for the time offset (the retarded time, $\tau$) from when the source was emitted, in order that it might reach the observer at $t$. The following equation is used to solve for the retarded time, where $t$ is time in the observer’s frame, $\tau$ is time in the source’s frame, $\vec{x}$ is the position of the observer, and $\vec{y}$ is the position of the source.

$$\tau = t - \frac{|\vec{x} - \vec{y}(\tau)|}{c}$$

(3.1)

One difficulty associated with understanding this concept is that the sound heard by an observer was generated at a different time than it is seen; hence when the sound reaches the observer, the source has already moved to a new position. The following figure describes this visually.

![Figure 3.1: Graphical description of some complexities associated with noise calculations.](image)

In order to facilitate rapid acoustic predictions, a source-time-dominant approach was taken, where the emission time is treated as primary, or dominant. The emission times are selected, from which the observer times are calculated. In essence, this approach proves more numerically efficient, because rather than figuring out the position the of a panel in the past in order that the sound generated
by it reaches the observer at the current time, the positions of the panel at every emission, or retarded time is **interpolated** to solve for their magnitude at each observer time. This approach removes the need to solve for the root of equation 3.1. The following excerpt does a phenomenal job explaining the benefits of this formulation.

A sequence of source times (i.e., the times at which the source strength is available) will lead to a sequence of *unequally* spaced observer times. This panel time history can be interpolated to provide the contributions at the desired observer times. Interpolation in time is necessary so that the contributions from all source panels can be added together at the same observer time (Brentner & Farassat, 2003, pp. 26-27).

The following plot describes an example of the relationship between the emission time and the observer time over one rotational period. This data was obtained by following a single patch located near the tip of the propeller, in order that it might emphasize the non-linearity of this relationship.

![Figure 3.2: Non-linear relationship between emission time and observer time for one patch on the surface of the propeller over one period of rotation.](image)
Farassat, whose work in rotor acoustics is renowned through the aerospace industry, detailed a number of formulations of the FWH equation, two prominent ones being Farassat’s formulation 1 and 1A. The equation defining formulation 1 is shown below for subsonic analysis with the quadrupole noise source neglected. Note that the brackets { } represent the values taken at the retarded, or emission time.

\[
4\pi p' (\vec{x}, t) = \frac{1}{c} \frac{\partial}{\partial t} \int_S \left\{ \frac{\rho \mathbf{v}_n + \ell_r}{r (1 - M_r)} \right\} dS + \int_S \left\{ \frac{\ell_r}{r^2 (1 - M_r)} \right\} dS
\]

(3.2)

The above equation was written using the short-hand notation outlined in many of Farassat’s papers. An expanded definition of these terms is shown below.

\[
\mathbf{v}_n = \vec{V} \cdot \hat{n}
\]

(3.3)

\[
\ell_r = \vec{\ell} \cdot \hat{r} \approx p \hat{n} \cdot \hat{r}
\]

(3.4)

\[
M_r = \vec{M} \cdot \hat{r}
\]

(3.5)

The compactness of the equations make this formulation somewhat inviting compared to formulation 1A, although it does have some significant drawbacks, making formulation 1A more desirable for certain applications. One particularly significant limitation of formulation 1 is the derivative, seen operating on the left-most integral of equation 3.2. This term represents a difference in the observer’s frame, thus requiring numerical differentiation for all but the most simplified scenarios. Another limitation resulting from the observer time derivative is that the observer is required to be stationary for all calculations. Ideally, it would be nice to have the derivatives all taken with respect to the emission, or source time, thus eliminating the numerical differentiation and the stationary observer constraint. Such was the motivation of Farassat when he sought to derive what is now known as formulation 1A of Farassat. Because of the limitations of formulation 1, I decided to do all my analysis with 1A, which is shown below.

35
\[ 4\pi p'_T(\vec{x}, t) = \int_{f=0} \left\{ \frac{\rho_0 (\vec{v}_n + \vec{v}_r)}{r|1-M_r|^2} \right\} dS + \int_{f=0} \left\{ \frac{\rho_0 \vec{v}_n (r \vec{M}_r + cM_r - cM^2)}{r^2|1-M_r|^3} \right\} dS \] (3.6)

\[ 4\pi p'_L(\vec{x}, t) = \frac{1}{c} \int_{f=0} \left\{ \frac{\ell_r}{r|1-M_r|^2} \right\} dS + \int_{f=0} \left\{ \frac{\ell_r - \ell_M}{r^2|1-M_r|^2} \right\} dS + \frac{1}{c} \int_{f=0} \left\{ \frac{\ell_r (r \vec{M}_r + cM_r - cM^2)}{r^2|1-M_r|^3} \right\} dS \] (3.7)

\[ p'(\vec{x}, t) = p'_T(\vec{x}, t) + p'_L(\vec{x}, t) \] (3.8)

Much of the notation I used in the above equations comes directly from the paper “Modeling Aerodynamically Generated Sound of Helicopter Rotors” (Brentner & Farassat, 2003). Note the denominator of equation (41) in the above reference has a typo, which wreaks havoc on the numerical solution. The second integral of this equation should have a \( r^2 \) term in the denominator, although it is typed as \( r \). Because of the amount of time I spent merely trying to understand what each term means, the following list of information will hopefully help someone in the future save a bit of time.

- \( t \) represents the time in the observer’s frame, which can be thought of as the time at which an observer is hearing the sound, whereas \( \tau \) represents the emission time, which is also known as the retarded time.

- The subscript \( n, r, \) and \( M \) signify a dot product with the unit normal vector, the unit radiation vector (the vector from the source to the observer), and the surface velocity vector normalized by the speed of sound, respectively.

- The dot over a variable represents the derivative of that variable with respect to \( \tau \). To give an example of this, the term \( \vec{v}_n = \frac{\partial \vec{v}}{\partial \tau} \cdot \vec{n} \) and \( \vec{v}_n = \vec{v} \cdot \frac{\partial \vec{n}}{\partial \tau} \). Once again, I would like to re-iterate that this was the notation used by Brentner.

- The brackets \( \{ \} \), represent the values taken at the emission time.

- \( \ell \) is the local force intensity (force per unit area) on the fluid, which is equivalent in this scenario to the steady pressure value on a surface panel, times the unit normal vector of that panel, neglecting the viscous shear (\( \ell = p \cdot \vec{n} \)).
• Brentner and Farassat, among others have published many papers expounding upon formulation 1A from different perspectives. I found it helpful to utilize multiple published papers to more fully understand how the equations were derived, and consequently how to apply them to the numerical algorithm.

3.2 Specifics of Acoustic Formulation

In this section I intend to explain some of the specifics of the formulations 1A used. Each sub-topic is independent of the others, thus making this section a good reference guide for someone wanting to understand specific information about the acoustic implementation.

**Kinematics of Blade Rotation**

For ease of numerical computation, the propeller is rotated about the y-axis; hence all rotation occurs in the x-z plane, as shown in the figure below.

![Coordinate system used for acoustic analysis.](image)

*Figure 3.3: Coordinate system used for acoustic analysis.*

If the real axis were coincident with the z-axis, and the imaginary axis coincident with the x-axis, the initial position within the x-z plane of the k\(^{th}\) element would be \((xz)_0 = z_0 + jx_0\). A rotation of angle \(\theta = \omega \tau\) would lead to the following value of x and z:
\[(xz)_\theta = (z_0 + jx_0)e^{i(\omega t)}\]

\[= (z_0 + jx_0)[\cos(\omega t) + j \sin(\omega t)]\]

\[= [z_0 \cos(\omega t) - x_0 \sin(\omega t)] + j[z_0 \sin(\omega t) + x_0 \cos(\omega t)]\]  \hspace{1cm} (3.9)

The position of the \(k^{th}\) element can therefore be described with the following set of equations for any emission time:

\[x = x_0 \cos(\omega t) + z_0 \sin(\omega t)\]

\[y = y_0 + V_\infty \tau\]

\[z = -x_0 \sin(\omega t) + z_0 \cos(\omega t)\]  \hspace{1cm} (3.10)

The derivative of the above equations with respect to the emission time results in the velocity components shown below.

\[\frac{\partial x}{\partial \tau} = V_x = \omega [-x_0 \sin(\omega t) + z_0 \cos(\omega t)]\]

\[\frac{\partial y}{\partial \tau} = V_y = V_\infty\]

\[\frac{\partial z}{\partial \tau} = V_z = -\omega [x_0 \cos(\omega t) + z_0 \sin(\omega t)]\]  \hspace{1cm} (3.11)

In order to verify this relationship, the velocity of a given panel may also be found by adding the axial translation in the \(y\)-direction to the rotational velocity, which results in the relationship \(\vec{V} = \vec{V}_\infty + \vec{\omega} \times \vec{r}\), where the following is true:

\[\vec{V}_\infty = V_\infty \cdot \hat{j}\]

\[\vec{\omega} = \omega \cdot \hat{j}\]

\[\vec{r} = [z_0 \sin(\omega t) + x_0 \cos(\omega t)] \cdot \hat{i} + (y_0 + V_\infty \tau) \cdot \hat{j} + [z_0 \sin(\omega t) + x_0 \cos(\omega t)] \cdot \hat{k}\]

Carrying out the cross product results in the following:

\[\vec{V} = V_\infty \cdot \hat{j} + (\omega \cdot \hat{j})\]

\[\times \left\{ [z_0 \sin(\omega t) + x_0 \cos(\omega t)] \cdot \hat{i} + (y_0 + V_\infty \tau) \cdot \hat{j} + [z_0 \sin(\omega t) + x_0 \cos(\omega t)] \cdot \hat{k} \right\}\]

\[V_x = \omega [z_0 \sin(\omega t) + x_0 \cos(\omega t)]\]
\[ V_y = V_\infty \]
\[ V_z = -\omega [z_0 \sin(\omega \tau) + x_0 \cos(\omega \tau)] \]

As expected, these results align with those originally solved for. The derivative of the equations 3.7 and 3.10 with respect to the emission time results in the acceleration components shown below:

\[
\frac{\partial^2 x}{\partial \tau^2} = \frac{\partial v_x}{\partial \tau} = -\omega^2 [x_0 \cos(\omega \tau) + z_0 \sin(\omega \tau)] \\
\frac{\partial^2 y}{\partial \tau^2} = \frac{\partial v_y}{\partial \tau} = 0 \\
\frac{\partial^2 z}{\partial \tau^2} = \frac{\partial v_z}{\partial \tau} = -\omega^2 [-x_0 \sin(\omega \tau) + z_0 \cos(\omega \tau)]
\]

(3.12)

All of these equations are necessary for applying this theory to the numerical algorithm, which is why they are presented here. Note that these equations have been simplified by the fact that I have assumed rotation is only occurring around one axis (the y-axis). For helicopter (any many other) applications, these equations need revision to account for more rotations/translations.

**Blade Surface Mesh Properties**

Out of convenience, the propeller was meshed radially along the airfoil sections, and circumferentially around the blade at the airfoil coordinates. The following figure gives a visual description of the mesh applied to a rotor surface.
After the mesh was applied to the structure, the area of each ‘patch’ and the normal vector from that area element was required. Consider the following plane, which is defined in three dimensions:

The normal vector for this plane may be found via \( \vec{N} = \vec{V}_{13} \times \vec{V}_{24} \). The area and unit normal vector of the patch may be found by computing \( A_{1234} = \frac{1}{2} |\vec{N}| \) and \( \hat{n} = \frac{\vec{N}}{|\vec{N}|} \), respectively (Chung, 2002, p. 231).

**Emission Time Dominant Formulation**

Programming a method for efficiently storing and consequently analyzing a set of acoustic data with unequally-spaced observer times proved difficult, hence I will delve into some of the detail of it.
here. The following line graph physically shows the consequence of performing the analysis with a source time-dominant algorithm.

![Graph showing emission and observer times](image)

**Figure 3.6: Visual description of the emission/observer time relationship.**

Each black dot in the figure above and to the left represents when the source contributions emitted from the $i^{th}$ panel reach the observer. Because there is a finite distance between the source and the observer, causality rules that the absolute observation time must be greater than the absolute emission time. Additionally, due to the relative motion of the propeller surface compared to the observer, the observation times when a source arrives are unequally spaced.

One significant benefit of analyzing subsonic rotors as opposed to supersonic ones is that the observation time period over one full blade rotation is constant, regardless of which panel emitted the source. As an example of this, consider the $i^{th}$ panel on the propeller surface given in the figure above. At the initial rotor orientation, this panel is a distance of $|\vec{x}_i - \vec{y}_o|$ from the source, thus the retarded time lags the emission time by $\frac{|\vec{x}_i - \vec{y}_o|}{c}$, where $\vec{x}_i$ is the absolute position vector of the $i^{th}$ panel, $\vec{y}_o$ is the absolute position vector of the observer, and $c$ is the speed of sound. So long as observer maintains a constant distance of separation from the propeller (i.e. the observer translates with the propeller), when the rotor completes one full rotation, the $i^{th}$ panel will once again be a distance of $|\vec{x}_i - \vec{y}_o|$ from the source, resulting in a retarded time lag of $T + \frac{|\vec{x}_i - \vec{y}_o|}{c}$, where $T$ is the period of one full rotation.
Therefore, the period of the source contributions for a given panel is constant in the observer frame, even though the sampling within this period is not uniform.

Knowing this information is important because in order to estimate the total acoustic signature in the observer’s frame the source contributions need to be summed at uniform observer times. To obtain the source contributions at uniform observer times for a non-uniformly spaced of data I need to interpolate the information to a uniform observer grid. Consequently, prior to interpolating I need to make sure the data I am interpolating spans an entire rotational period. In application, the ‘take-away’ from this entire discussion is the in order to properly interpolate the source data, I first analyze the propeller’s acoustics over one blade period, after which I repeat the source contributions over another period. Consider the following scenario where I am trying to interpolate the source contributions over one full propeller period, in the observers frame. Notice how the data samples need to be repeated twice in order to allow for interpolation across one period in the observer’s frame.

![Figure 3.7: Consequence of non-uniformly sampled source contributions in the observer’s frame.](image)

**3.3 Code/Acoustic Approach Validation**

Several validation methods were used to ensure correctness of the numerical implementation for the acoustic code. One of the most simple ‘checks’ was to ensure a proper acoustic attenuation with increasing observer distance from the propeller. For a dipole noise source under standard atmospheric
conditions, the acoustic wave is expected to decrease 6 dB for every doubling of distance. This number is not exact for other types of acoustic sources (i.e. those generated by a propeller), although it still provides a rough estimate of what the acoustics are ‘supposed’ to do. Spot-checks of the acoustic code revealed that the program appropriately accounted for acoustic attenuation with distance.

The next test case I ran came from a report titled “Development and Validation of a Propeller/Rotor Acoustic Prediction Program (ProRAPP)” (Taghaddosi, Gallman, & Agarwal, 1998). The propeller that I performed this analysis on was the 2-bladed Hartzell F8475D-4 propeller. In this report the authors failed to provide the blade loading used for their acoustic analysis, hence the first step for me was to execute my blade element/momentum solver. At the test condition, the propeller was rotating at 2100 RPM with a freestream inflow velocity of 51.2 m/s. According to my calculations, I predicted the overall thrust coefficient to be 0.0929, whereas their analysis showed it at 0.0930. The following table summarizes my acoustic predictions side-by-side with the experimental data presented in the report for microphones 1, 4, and 8 with a 0° inflow angle. Note that in order to calculate these values, I first digitized the waveform presented in their report, after which I was able to pull this information. As you can see from the table, the OASPL calculations are similar.

<table>
<thead>
<tr>
<th>Microphone # =</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>OASPL (dB)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculated</td>
<td>102.4</td>
<td>113.4</td>
<td>107.1</td>
</tr>
<tr>
<td>Experimental</td>
<td>102.1</td>
<td>109.9</td>
<td>107.9</td>
</tr>
</tbody>
</table>

My final validation effort was to predict the acoustics of the 2-bladed propeller I found in a Master’s thesis by Finn of the Embry-Riddle Aeronautical University. In his thesis report, he performs analysis on a number of propeller configurations, using a software developed by NASA Langley known

Of the propellers Finn analyzed for his thesis, I chose to perform my acoustic validation effort on the baseline, 2-bladed version. This propeller has a diameter of 1.9304 m and the following blade form curve.

![Blade geometry for Finn propeller.](image)

Prior to running the acoustic analysis, I first need to ensure that I was predicting a similar aerodynamic performance. I analyzed the propeller at the on-design condition. The figure below shows a side-by-side comparison of the lift coefficient predicted with my program, and that which Finn obtained from ANOPP-PAS. Clearly the two programs are predicting a similar propeller loading, which is important for the acoustic analysis.
To check the acoustic formulation of my software program I compared both near-field and far-field OASPL values. Because the aerodynamic predictions from those given in the Finn’s report are not identical to the performance calculated by my program, I do not expect the acoustic analysis to exactly match either. However, if my program is performing the aerodynamic and acoustic analysis correctly, I would at the very least expect the trends to follow a similar pattern. The near-field comparison test that I performed entailed predicting the acoustics at a distance of $5 \cdot R_{tip}$ for an angle sweep, incremented with $15^\circ$ intervals. For a pictorial description of this test see the figure below.

Figure 3.9: Predicted versus given comparison of lift coefficient across propeller span.
The following plot outlines a comparison of the acoustic directivity of the propeller noise predictions.

By observation, I can see that my program is predicting an extremely similar trend as that of the ANOPP-PAS software.
In order to assess the accuracy of my predictions with the far-field acoustics generated by the ANOPP-PAS system, I ran a fly-over pass at an altitude of 1000 ft. The following plot shows the OASPL as a function of time, where a time of zero corresponds to the propeller being directly overhead.

Figure 3.12: Far-field flyover acoustic test at an altitude of 1000 ft.

Once again, the trends of the above figure indicate that the calculated results follow a similar pattern as those predicted by the ANOPP-PAS software.
Chapter 4
Propeller Characterization

4.0 Introduction

In this chapter I have set out to characterize both the baseline and the axially-stacked propeller system, both aerodynamically and acoustically. As a test case, I selected a modified APC 7x5 propeller for all of my baseline computations. I modified the propeller geometry by assuming the airfoil sections were that of a Clark Y airfoil, rather than the linear transition from a Clark Y at the hub to an Eppler 63 at the tip. The reason for this modification was to improve the computational efficiency of XFOIL. As noted earlier in this document, the intention of this thesis is to identify the performance trends of the stacked propeller configuration, rather than to estimate the exact efficiencies, making this modification of no consequence to the conclusions obtained from the analysis.

The way in which I went about evaluating the feasibility of the stacked rotor configuration can be seen in the following procedure list.

1. Evaluate the aerodynamic and acoustic performance of the single stage, baseline propeller configuration over a large range of RPM’s and advance ratios.
2. Evaluate the aerodynamic and acoustic performance of the single stage, baseline propeller configuration with an equivalent number of blades as the stacked propeller, over a large range of RPM’s and advance ratios.
3. Evaluate the stacked rotor configuration at several RPMs, over a range of advance ratios and axial offsets and identify the maximum desirability across the spectrum of data
4. Identify the run conditions for each propeller, which results in the highest level of desirability
5. Compare performance of each propeller under conditions of highest desirability.
From my personal experience running propeller analysis and experimental tests for the Air Force Research Laboratory, I selected an RPM range of 5,000-20,000 as the run points at which to operate the axially-stacked propeller configuration. Consequently, I analyzed the standard, single-stage propellers over that same range, although with a higher RPM resolution. The reason I used such a course RPM resolution for the stacked propeller was due to computational efficiency considerations; not only did I have exponentially more potential run conditions afforded by axially-displacing the rotors, but each case took almost four times as long to execute. This increase in computational time per run was due to the fact that the two rotors needed to converge first locally their own rotor-induced velocities, then globally on the externally-induced velocities generated by the other propeller.

Because the directivity pattern of each propeller is not uniform, I ran the acoustic analysis at the following locations in several arbitrarily-selected far-field positions. Although I have not set out to design a propeller for this thesis, including the directivity pattern in the optimization of a particular propeller design is important, in order to ensure the optimizer does not converge on a blade configuration that decreases the noise at a single location at the expense of the other positions. Note that for the axially-stacked propeller analysis, the microphones were placed 10m away from the mid-point of the two propellers.

![Figure 4.1: Microphone placement for propeller acoustic characterization.](image-url)
In order to evaluate the desirability of a particular run condition for a given propeller I first selected threshold limits, both acoustically and aerodynamically, which govern the desirable propeller performance. The following example illustrates how desirability is calculated.

Let the aerodynamic efficiency of the example propeller be 75% and the acoustic signature at a given point 60 dB, while operating at a set rotational speed and freestream velocity. For the sake of this discussion, I will define the aerodynamic efficiency threshold to be 80% and the acoustic threshold to be 50 dB. I have defined desirability \( \Delta \) in the following way:

\[
\Delta = \Delta_{\text{aerodynamic}} \cdot \Delta_{\text{acoustic}},
\]

Because desirability is equal to unity when all of the threshold limits have been satisfied, \( \Delta_{\text{aerodynamic}} \) and \( \Delta_{\text{acoustic}} \) are only equal to 1.0 when the efficiency of the propeller is at least equal to the efficiency threshold, and the acoustics are less than or equal to the acoustic threshold, respectively.

The ‘local’ desirability values are seen in the equations below.

\[
\Delta_{\text{aerodynamic}} = \begin{cases} 
\frac{\eta}{\eta_{\text{threshold}}}, & \eta < \eta_{\text{threshold}} \\
1.0, & \eta \geq \eta_{\text{threshold}} 
\end{cases}
\]

\[
\Delta_{\text{acoustic}} = \begin{cases} 
\frac{\text{SPL}_{\text{threshold}}}{\text{SPL}}, & \text{SPL} > \text{SPL}_{\text{threshold}} \\
1.0, & \text{SPL} \leq \text{SPL}_{\text{threshold}} 
\end{cases}
\]

Thus for this example, \( \Delta_{\text{aerodynamic}} = \frac{0.75}{0.8} = 0.9375 \) and \( \Delta_{\text{acoustic}} = \frac{50}{60} = 0.8333 \). Based on these calculations, the overall desirability is \( \Delta = (0.9375) \cdot (0.8333) = 0.781 \).

### 4.1 Baseline Propeller Characterization

The figures on the following page describe the aerodynamic performance of the baseline 2-bladed propeller, as estimated by my aerodynamic solver.
Figure 4.2: Modified APC 7x5 efficiencies

Figure 4.3: Modified APC 7x5 thrust coefficients

Figure 4.4: Modified APC 7x5 power coefficients
One obvious and typical trend I have observed with this propeller is that the efficiency tends to improve with increasing RPM. Various attempts have been made by researchers to modify standard dimensionless quantities (such as advance ratio) in order to collapse these curves more effectively, although the theories have yet to be unified (OL, Zeune, & Logan, 2008).

Unfortunately, despite the improved propeller efficiency with increasing rotational speed, the acoustic signature is sure to degrade. In what I consider to be a benchmark paper on general noise trends, Korkan, Gregorek and Keiter show that increasing the tip Mach number has a detrimental effect acoustically on the propeller (An Acoustic Sensitivity Study of General Aviation Propellers, 1980).

The following plots describe the median OASPL value as a function of rotational speed and advance ratio.

![Figure 4.5: Mean OASPL (dB) for modified APC 7x5 propeller.](image)

The figure above depicts several interesting phenomena, which I would like to delve into. First of all, I would like to point out the very low OASPL signature for the propeller over a majority of the
contours. Purely for the sake of this discussion, I will assume a background noise level of 40 dB. Note that this value is **highly** dependent on a number of factors, which I will not delve into here. On average the human ear can only begin to discern a different in sound pressure levels that are 3 dB apart. In essence, what this would mean is that the modified APC 7x5 propeller is only starting to become audible when the propeller is rotating at approximately 12,000-14,000 RPM for an observer 10m away.

In an attempt to combine the aerodynamic information with the acoustic information, the following plot describes the desirability as a function of rotational speed and advance ratio. As desirable limits, I set the acoustic target to 40 dB, and the aerodynamic efficiency target to 70%.

![Figure 4.6: Desirability of modified APC 7x5 propeller with threshold limits of 40 dB and 70% aerodynamic efficiency.](image)

It is quite obvious from the desirability figure that the propeller is operating at its peak condition at an RPM of 12,000 and an advance ratio of 0.55.

I also analyzed a single stage, modified **4-bladed** APC 7x5 propeller. The results of this analysis are shown in the figures below.
Figure 4.7: Modified four-bladed APC7x5 efficiencies.

Figure 4.8: Modified four-bladed APC7x5 thrust coefficients.

Figure 4.9: Modified four-bladed APC7x5 power coefficients.
Figure 4.10: Mean OASPL (dB) for modified four-bladed APC 7x5 propeller.

Figure 4.11: Desirability of four-bladed, modified APC 7x5 propeller with threshold limits of 40 dB and 70% aerodynamic efficiency.
One thing to note is that the 4-bladed propeller requires an increased rotational speed versus the 2-blade baseline propeller in order to operate at its maximum desirability level, which is consequently lower. As expected, the acoustic signature of the 4-blade design is reduced, when compared with the 2-blade baseline propeller at the same RPM and advance ratio.

4.1 Stacked Propeller Characterization

I will now set out to characterize the axially-stacked propeller configuration. Note that using the same propeller for both stages will most likely have a detrimental effect on the potential performance improvements gained by adding the second stage. There are several reasons why this is so, one of which is that the second stage will introduce additional axial velocity on the upstream stage, thus causing the upstream blade sections to ‘see’ a reduced angle of attack. The consequence of this is a potentially hindered thrust capability. It is unclear what sort of affect the additional swirl and axial velocity will have on the performance of the downstream stage.

One potential source of acoustic optimization afforded by the axially-stacked propeller configuration is the ability to offset the angle of each propeller until the acoustic waves generated by each propeller interact in a destructive way. Consider the following illustration, which represents the addition of two acoustic waves that have been offset by a certain amount of phase.
Figure 4.12: Consequence of summing two acoustic waves together with a shifting phase, accomplished by offsetting each propeller’s angular positions from one stage to the next.

By shifting the two pressure signals through their entire phase range I was able to calculate the following plot, which shows the OASPL of the resulting acoustic wave as a function of the phase shift. Note that because I used a sine wave to represent the pressure signals, at a phase offset of $\pi$ the waves are exactly out of phase with each other, thus canceling out any pressure fluctuations.
Figure 4.13: OASPL resulting from the summation of two pressure signals offset by the specified amount.

Because the acoustic waves at a given location in the far-field will be nearly identical, except for a potential phase shift, there most certainly exists an optimal angular configuration in order to decrease the acoustic signature. One difficulty with optimizing in this way is the potential to increase the acoustic signature at other positions.

Unequally spacing the rotors on standard, single-stage propellers has been studied by a number of engineers. Many of these researchers concluded that although the OASPL remains the same, regardless of the blade spacing, the weighted OASPL may somewhat decrease with the shifting frequency spectral data (Lewy, 1992). However, the obvious practical limitations of only being able to unevenly space the blades on a single stage over a small range of angles does not apply to the stacked rotor configurations, hence allowing for more flexibility in optimization.

I will now show the results of the axially-stacked propeller analysis. In an effort to limit the large number of plots I generated for the axially-stacked propellers, I will only show contours for efficiency, minimum OASPL, and the desirability. Note that in order to obtain the minimum OASPL
value I shifted the phase of the pressure signals arriving at each observer until the OASPL was
minimized, after which I took a mean of the three observer OASPL values.
Figure 4.14: Efficiency of stacked propellers as a function of axial offset.
Figure 4.15: OASPL (dB) of stacked propellers as a function of axial offset.
Figure 4.16: Desirability of stacked propellers as a function of axial offset, with threshold limits of 40 dB and 70% aerodynamic efficiency.
4.3 Comparison of Baseline and Stacked Propellers

By looking at the performance of the stacked rotor across the range of operating conditions, the contours appear to show a slight sensitivity to axial offset. However, by directly computing the difference in aerodynamic efficiencies, certain operating points show nearly a 10% difference, when compared with the efficiencies obtained at the same run condition, although at a different axial offset.

By comparing the stacked propeller to the baseline 2-bladed design, it is quite obvious that the operating conditions change drastically in order to obtain the highest level of desirability. The 2-bladed baseline propeller requires an RPM of 12,000 to obtain a comparable desirability of the axially-stacked and 4-bladed propellers at 15,000 RPM.

One reason this is significant has nothing to do with the propeller itself, but rather the power plant. Typical electric motors tend to operate more efficiently at higher RPMs, thus if the stacked rotor has a similar overall performance as the 2-bladed baseline propeller, although at an increased RPM, the configuration has potential to increase the overall system efficiency.

Another item that requires attention is the tip Mach number; making the rotor transonic will likely reduce the propeller’s efficiency (a reduction not accounted for with the blade element/momentum analysis performed for this thesis) and will certainly increase the acoustic signature. For reference the following figure contains several curves showing the advance ratio at each RPM, which results in a number of tip Mach values.
Figure 4.17: Advance ratio/RPM pairs resulting in several tip Mach numbers for a 7 in diameter propeller.

Fortunately, the curve above shows that both the axially stacked propeller and the baseline propeller are operating well below a Mach number of unity.

The following table provides the numerical data for each of the three configurations (2-bladed baseline; 4-bladed baseline, and axially-stacked propellers) at the conditions identified as the highest desirability level. Note that data for the stacked propeller configuration is only shown for the axial offset of $1.0 \cdot R_{tip}$.

Table 4.1: Comparison of each propeller configuration under conditions of highest desirability.

<table>
<thead>
<tr>
<th>RPM =</th>
<th>2-Blade Baseline</th>
<th>4-Blade Baseline</th>
<th>Stacked Propeller</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advance Ratio, $J$ =</td>
<td>12,000</td>
<td>15,000</td>
<td>15,000</td>
</tr>
<tr>
<td>Efficiency, $\eta$ =</td>
<td>0.55</td>
<td>0.55</td>
<td>0.5</td>
</tr>
<tr>
<td>Thrust coefficient, $C_t$ =</td>
<td>0.0407</td>
<td>0.0569</td>
<td>0.0639</td>
</tr>
<tr>
<td>Power coefficient, $C_p$ =</td>
<td>0.0363</td>
<td>0.0559</td>
<td>0.0561</td>
</tr>
<tr>
<td>OASPL (dB) =</td>
<td>41.2</td>
<td>42.8</td>
<td>39.3</td>
</tr>
<tr>
<td>Desirability =</td>
<td>85.2%</td>
<td>81.7%</td>
<td>88.5%</td>
</tr>
</tbody>
</table>
Interestingly enough, the 4-bladed propeller was out-performed both aerodynamically and acoustically by both the 2-bladed configuration and the stacked rotors. Most certainly, the reason that the 2-bladed propeller was quieter than the 4-bladed version was due to a trade-off between acoustic signature and aerodynamic efficiency, required for the desirability calculations. However, if the desirability had included thrust as a requirement, it may have elevated the 4-blade design over the 2-blade design.

When the propellers are operating at their peak desirability conditions, the stacked propeller shows a 57% increase in the thrust coefficient compared to the 2-bladed propeller, and a 14% increase over the 4-bladed design. If the actual thrust values obtained by each propeller are compared the improvement over the 2-bladed design will become more drastic, as thrust is proportional to rotational speed squared.
Chapter 5

Summary of results and conclusions

In this thesis I explored the feasibility of an axially stacked, subsonic propeller configuration, comparing the aerodynamic and acoustic results to a single stage propeller. I performed the aerodynamic analysis using a modified blade element/momentum solver. Acoustically, I used a surface integral form of the Ffowcs Williams/Hawkings equation known as Farassat’s formulation 1A to evaluate the propeller’s performance. I coupled all of the resulting programs together into a unified MATLAB script, which allowed me to automate much of the propeller analysis.

In order to validate the aerodynamic and acoustic solvers I compared performance predictions with published data. The results of this validation effort were consistent enough to allow me to extend the analysis to the axially stacked propeller system.

The results of the axially-stacked propeller analysis have shown some of the potential acoustic advantages of this configuration, and have confirmed the possibility for optimization. One significant acoustic advantage afforded by this design is the ability to ‘tune’ the sound generated by the system as a whole by adjusting the phase of the pressure wave generated by one stage relative to the other at a particular point in space. Although the analysis showed a slight degradation in the aerodynamic performance of the axially stacked propeller configuration over the baseline propeller, it is likely that this aerodynamic performance may be improved by specifically designing each propeller for the operation conditions it will ‘see’ in the stacked rotor configuration.

When the results are compared to a single stage, 4-bladed propeller the axially-stacked design shows both aerodynamic and acoustic improvements, thus confirming the feasibility of this design.

Future research is needed to continue with this feasibility study. The first item that needs attention for future research is to specifically design the blade geometry for an axially-stacked propeller
configuration. These modifications will likely result in increasing the blade angle of the upstream stage. Additionally, the diameter of downstream stage may require a slight reduction to ensure the blade tip stays within the streamtube generated by the upstream stage. This step should be performed with a low-fidelity analysis, such as that used for this thesis.

The next effort that will be required is to perform higher fidelity aerodynamic analysis, which should include the unsteady flow oscillations. Ideally, this effort should include both CFD analysis and experimental testing. The CFD will allow for rapid visualization of the complex flow-field around the stacked rotors, while the experimental testing will validate the CFD results.
Works Cited


http://www.ae.illinois.edu/m-selig/props/propDB.html


http://web.mit.edu/drela/Public/web/xrotor/xrotor_doc.txt


68


