I, Michael A Kelly, hereby submit this original work as part of the requirements for the degree of Doctor of Philosophy in Theory.

It is entitled:
A Theory of Spatial Acquisition in Twelve-Tone Serial Music

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University of Cincinnati
A Theory of Spatial Acquisition in Twelve-Tone Serial Music

Ph.D. Dissertation

submitted to the
University of Cincinnati College-Conservatory of Music
in partial fulfillment of the requirements
for the degree of

Ph.D. in Music Theory

by
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Abstract

This study introduces the concept of spatial acquisition and demonstrates its applicability to the analysis of twelve-tone music. This concept was inspired by Krzysztof Penderecki’s distinctly spatial approach to twelve-tone composition in his *Passion According to St. Luke*. In the most basic terms, the theory of spatial acquisition is based on an understanding of the cycle of twelve pitch classes as contiguous units rather than discrete points. Utilizing this theory, one can track the gradual acquisition of pitch-class space by a twelve-tone row as each of its member pitch classes appears in succession, noting the patterns that the pitch classes exhibit in the process in terms of directionality, the creation and filling in of gaps, and the like.

The first part of this study is an explanation of spatial acquisition theory, while the second part comprises analyses covering portions of seven varied twelve-tone works. The result of these analyses is a deeper understanding of each twelve-tone row’s composition and how each row’s spatial characteristics are manifested on the musical surface.
## Contents

### Part One

**Theory**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Established twelve-tone theories <em>vis-à-vis</em> the present study</td>
<td>5</td>
</tr>
<tr>
<td>The concept of spatial acquisition</td>
<td>13</td>
</tr>
<tr>
<td>Row phases and spatial row functions</td>
<td>28</td>
</tr>
<tr>
<td>Spatial row segments</td>
<td>38</td>
</tr>
<tr>
<td>Spatial acquisition and musical forces</td>
<td>45</td>
</tr>
<tr>
<td>Unidirectional row segments</td>
<td>48</td>
</tr>
<tr>
<td>Gaps</td>
<td>52</td>
</tr>
<tr>
<td>Row classes and spatial row profiles</td>
<td>59</td>
</tr>
<tr>
<td>Phase-by-phase spatial data</td>
<td>66</td>
</tr>
<tr>
<td>The analytical process</td>
<td>74</td>
</tr>
</tbody>
</table>

### Part Two

**Analysis**

<table>
<thead>
<tr>
<th>Composer/composition</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schoenberg, <em>Suite</em> for Piano, Op. 25</td>
<td>86</td>
</tr>
<tr>
<td>Berg, Violin Concerto</td>
<td>98</td>
</tr>
<tr>
<td>Dallapiccola, <em>Quaderno musicale di Annalibera</em></td>
<td>119</td>
</tr>
<tr>
<td>Babbitt, <em>Composition for Twelve Instruments</em></td>
<td>130</td>
</tr>
<tr>
<td>Penderecki, <em>The Passion According to St. Luke</em></td>
<td>140</td>
</tr>
<tr>
<td>J. S. Bach, <em>Das wohltemperierte Klavier</em>, Book 1, Fugue in B Minor</td>
<td>157</td>
</tr>
<tr>
<td>Appendices</td>
<td>167</td>
</tr>
<tr>
<td>Bibliography</td>
<td>178</td>
</tr>
</tbody>
</table>
Part One

Theory

_Established twelve-tone theories vis-à-vis the present study_

This study germinated in the fall of 2004, when I took on the task of analyzing the hymn _O crux ave_ from Penderecki’s _Passion According to St. Luke_. This hymn is the first of twenty-four movements constituting a work that, in terms of its length, thematic material, texture, and dramaturgy, is truly monumental in scope. The role that _O crux ave_ plays within the _Passion’s_ overall form seems to be introductory, as it presents, in a clearly expository fashion, two distinct and complete twelve-tone rows. Strangely, however, these are the only full statements of these rows that the entire work contains. Except for a few fragmentary examples, the rest of the _Passion’s_ movements do not exhibit transposed, inverted, retrograde, rotated, or otherwise altered forms of these rows, but instead contain a wealth of diverse musical material, including microtonal clusters; aleatoric elements; and dense counterpoint, the simultaneities within which often fill the chromatic aggregate.

The isolation of the _Passion’s_ twelve-tone rows amidst a sea of pitch material that is organized in diverse ways generates doubt as to whether the _Passion_ is actually a serial work at all. Over time, as my studies came to regard not just the first movement but the work as a whole, I found that the _Passion_ is indeed in effect a serial composition, the structure of which comprises not the interaction of various transpositions, inversions, retrogressions, and partitions of one or more twelve-tone rows, but rather consists of a voluminous catalog of gestures, each of which in some way evokes the intervallic patterns and contours that exist within one of the work’s twelve-
tone rows. This distinctly differentiates this work’s twelve-tone rows from the vast majority of
their counterparts in other serial works, although one essential commonality exists between Pen-
derecki’s compositional method and “classic” techniques of twelve-tone organization: The order
in which the twelve pitch classes are organized within a row is fundamental to the structure of
the work built upon that row, regardless of how often—or how seldom—any part of the row is
actually presented as such.

George Perle has stated that pitch-class ordering is a “necessary consequence” of twelve-
tone composition; doing so allows the twelve pitch classes to function as a “unitary structure
whose elements are not functionally differentiated.”1 Indeed, a major factor in the eventual ele-
vation of the twelve-tone method developed by Arnold Schoenberg over those formulated by his
contemporaries Josef Hauer and Herbert Eimert was the establishment of a standard order for the
twelve pitch classes, allowing a twelve-tone row to serve as a unifying resource for the musical
work built upon it.2 Consequently, it is actually the series of intervals between the pitch classes
within a twelve-tone row that defines the row itself, rather than the pitch classes themselves,
since it is these intervals that remain invariant regardless of how the row is transposed, inverted,
retrograded, or parsed. In Penderecki’s practice, as described above, this sense of integration
through the correspondence of every pitch collection within a musical work to the intervallic se-
ries that defines that work’s fundamental twelve-tone row is stretched almost—but not quite—to
the breaking point.

Rather than projecting an overt sense of intervallic integration, the Passion’s twelve-tone


rows primarily exhibit a sense of gradual motion, proceeding through both pitch and pitch-class space incrementally and following simple and evident spatial patterns while doing so, creating a clear sense of progress along definite paths. For example, as illustrated in Figure 1.1 below—which depicts pitch rather than pitch-class space for the sake of graphic simplicity—the first eight pitch classes of the Passion’s second row form four discrete chromatic dyads that exhibit an orderly pattern of expansion in alternating directions from PCs 4 and 5. The remaining four pitch classes form the historically significant BACH motto, which is not only an ostensible homage to J. S. Bach as a fellow setter of passion texts, but also comprises two more chromatic dyads. In completing the chromatic aggregate, these dyads alter the directional pattern that the initial eight pitch classes of the row exhibit. Reinforcing a sense of progress through a finite span is the fact that Penderecki presents this row with maximal compactness in pitch space: within a range of one octave and with the smallest possible interval from each pitch to the next.

In order to discuss the relationship between established twelve-tone theoretical approaches and Penderecki’s spatial approach to twelve-tone composition, and thereby to provide a basis for the theory outlined in this study, a brief overview of the most basic and universally employed elements of the established body of twelve-tone theories is in order. For the purposes of this overview, I have broadly categorized these elements into three areas: pitch-class and pitch-class-set invariance, set-class derivation, and intervallic symmetry.

When twelve-tone rows are analyzed, the term “invariance” usually refers to situations in which individual pitch classes or pitch-class sets occupy the same or related order positions within two or more row forms that are related by one of the canonical operations—transposition, inversion, or retrogression. Inversion is the operation that, by its nature, produces patterns of invariance among different forms of a twelve-tone row reliably, as demonstrated in the early work
The existence of pitch-class sets that are invariant in terms of their intervallic structure within a single row form, on the other hand, falls into the category of set-class derivation.

Milton Babbitt characterized the derivation of segments within a twelve-tone row from pitch-class collections that “are distinguished by pitch or intervallic content alone [rather than] by ordering” as combinational, whereas the derivation of “order-corresponding” segments within a row from a single set class can be called combinatorial. The more pitch classes within a row there are that belong to such derived subsets, the stronger the row’s combinatorial property is. True combinatoriality, whether hexachordal, tetrachordal, trichordal, dyadic, or unitary, allows for a systematic integration of all twelve pitch classes in the melodic and harmonic dimensions.

If invariance under retrogression or retrogression and inversion exists within the ordered interval series that defines a particular twelve-tone row, that row is symmetrical, with the result that it has only half as many unique forms as asymmetrical rows do. Utilizing symmetrical rows is an effective way to limit the material from which a twelve-tone work is drawn, and thereby to compose economically.

The concepts sketched in the last few paragraphs underlie many of the processes at work in a multitude of twelve-tone rows and compositions. When applied to Penderecki’s aforementioned row, however, the information that results does not shed much light on the spatial patterns within it that are nevertheless evident on the musical surface. Invariance and combinatoriality

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among row forms does not apply to the music based on that row, in which multiple row forms do
not appear simultaneously, although the row is derived from the all-combinatorial hexachordal
set class (012345). On the other hand, an approach based on set-class derivation shows that the
row is derived from the chromatic dyad, and that the set classes to which the row’s discrete tetra-
chords belong exhibit a symmetrical formation: (0123), (0167), (0123). These findings reflect
the fact that chromatic dyads are the building blocks within Penderecki’s row, and that those dy-
ads are farther-flung in the middle portion of the row’s presentation than at either end, but they
do not address the sense of movement conveyed by the row.

The ordered interval series for Penderecki’s row is \(<e, 2, 1, 8, e, 6, 1, 2, e, 3, e>\). There is
some information to be found here as well, as the alternation of elevens and ones among the odd-
numbered intervals in the beginning and middle of this series gives a clue as to the alternating
directions in which the chromatic dyads within the row move through pitch-class space. All in
all, what is needed to address the row’s spatial properties comprehensively is a more diachronic
approach, one through which a narrative of motion through pitch-class space can be constructed.
A key element in such an approach is a method of segmenting twelve-tone rows that is based on
the pitch classes’ positions in pitch-class space relative to the row’s other pitch classes rather
than on order positions. To apply this approach to Penderecki’s row, the initial segment
\(<43562178>\) can be parsed into the spatially determined segments 4321 and 5678. The pitch
classes within each of these segments retain their order positions relative to each other from the
original row segment, and each of those pitch classes is exactly one unordered semitone distant
from at least one other pitch class within the segment, isolating the patterns of incremental pro-
gress through pitch-class space noted above. The relationship between spatial segments such as
these and discrete row segments is essential to understanding a row’s spatial properties.
It is worth noting at this point that it is possible to segment twelve-tone rows based on their pitch classes’ positions along not only the chromatic cycle, but also the circle of fifths. Such an approach would not be optimal where Penderecki’s music is concerned, given his frequent and pointed use of semitones in pitch as well as pitch-class space. For twelve-tone rows in which perfect-fifth or interval-class-5 relationships are abundant, however, it could be useful. In this study, I utilize the chromatic cycle exclusively to represent pitch-class space, and I have two primary reasons for doing so. The first is to maintain correspondence with the conceptual underpinnings of not only the established theories pertaining to twelve-tone music, but also traditional music theory. For example, in standard practice in both theoretical worlds, we consider the unordered interval between pitch classes C and Db as 1 or one semitone, not 5 or five fifths. The other reason I prioritize the chromatic cycle is that, for all of the traditional concepts abandoned—or transcended—by twelve-tone composers, octave equivalence has always remained. Two pitches separated by any number of octaves are still considered a “doubling,” just as they have been for centuries, so that the sense of proximity exhibited by pitches that are separated by a semitone is extended as well to pitches that are separated by major sevenths, minor ninths, major fourteenths, and so on. The effect of this extended sense of proximity is that interval class 1 makes more sense as a spatial “building block” than interval class 5 does not only within the span of an octave but across the entire spectrum of pitch.

The division of a twelve-tone row into spatially-oriented segments that each comprise a set of pitch classes that may or may not be consecutive within the row recalls certain methods of partitioning particularly constructed rows that have been demonstrated by Babbitt. Through one of these methods, for example, given two different partitionings of the same row, the ordered pitch-class sets obtained by one partitioning will map onto the ordered pitch-class sets obtained
by the other partitioning through transposition or inversion. An important difference between the type of partitioning described above and the type of partitioning prescribed by spatial acquisition theory is that the former is applicable specifically to twelve-tone rows that possess certain types of intervallic structure, whereas the latter is designed to uncover the patterns of spatial acquisition that exist in every twelve-tone row, as the analyses in the second part of this study will demonstrate.

As this study is a spatial investigation of post-tonal music, it has been influenced to some degree by Jonathan Bernard’s landmark work *The Music of Edgard Varese*. In his work, Bernard states that he uses as his theoretical field the entire “equal-tempered system,” which he defines as “a neutral calibration of that portion of the frequency spectrum (seven octaves plus) available to conventional instruments, [which] provides a uniform measure of absolute interval size, of distance between upper and lower boundaries ...” The key difference between spatial analytical investigations such as Bernard’s and this one is that the field of pitch classes used in this study, although also uniformly measured and neutrally calibrated, differ from the field of pitches in that they are greatly limited by their number—twelve—within their cyclical span. The spectrum of pitches, on the other hand, is unaffected by octave equivalence, and therefore bounded only by the limits of audibility.

Babbitt wrote that, whereas octave equivalence in tonal music “serves to define classes of


equivalent function, … in the twelve-tone system [it] serves to define classes of equivalent order position.”

This study is concerned with the correlation between those order positions within a given twelve-tone row and the roles that individual pitch classes play in the gradual acquisition of pitch-class space by the row. In the following section of this study, I will present the foundational concepts and basic elements of the theory that I have developed to elucidate that correlation.

The concept of spatial acquisition

In 1973, Penderecki made a significant statement regarding his conceptual differences with the composers of the Darmstadt school: “… we never had a real point of contact … you could say that the difference between Darmstadt and me was that they were interested in dots, whereas I was interested in lines.” This simple opposition of pointillistic versus linear composition crystallizes what is unique about Penderecki’s approach to twelve-tone composition and how that approach was the conceptual basis for the theory on which this study is based.

In order to differentiate the pointillistic and the linear concepts of pitch-class space adequately, we must define each one clearly. To this end I present the schemata in Figure 1.2. The pointillistic representation, which is the schema that underlies classic twelve-tone composition and analysis, casts the twelve pitch classes as equidistant points along a circle, similar to the


marks that represent hours on a clock. A composer who creates a twelve-tone row through the use of this conceptual model must, as always, fulfill the most basic requirement of dodecaphonic compositional practice that all twelve pitch classes are utilized, and in doing so in effect completes a sort of pitch-class checklist, relating the various pitch classes’ order positions and the distances between them in the process.

The schema in Figure 1.2 that represents the linear concept of pitch-class space differs from the pointillistic model mainly in terms of one important characteristic: contiguity. In the linear model, pitch classes are seen as substantial units that exist in more than one dimension and, importantly, directly border each other, as expressed by the hash marks along the circle.

Figure 1.2: Pointillistic and linear models of pitch-class space

The idea that pitch classes exist as segmental units of the chromatic cycle rather than as points along it has a converse, which is derived from the fact that we can conceive of the spatial unit along that cycle in which any pitch class resides as being either “empty,” before that pitch
class is presented within the presentation of a twelve-tone row, or “full,” after it is presented. This has led me to develop the concept of what I call *pitch-class places* (PCPs), which are those twelve spatial units situated around the chromatic cycle, whether empty or full. The use of this concept allows us to consider the twelve pitch-class places to be gradually filled up as the pitch classes that make up a twelve-tone row are presented, then reset to an empty status every time that process is completed. In light of these spatial metaphors of fullness and emptiness, it is helpful to extrapolate the linear model of pitch-class space from a one-dimensional representation to a two-dimensional one, as shown in Figure 1.3.

Figure 1.3
One of the most basic mandates that underlies the practice of twelve-tone serial composition is that, within any composition, the chromatic aggregate must be completed frequently and systematically; as noted above, this is accomplished through the organization of musical material around a particular ordering of the twelve pitch classes. From a spatial perspective, this means that the directional patterns by which a work’s underlying twelve-tone row fills the twelve pitch-class places are of fundamental importance to the work in general. A primary concern for the theorist is to find a way to categorize both the spatial characteristics that twelve-tone rows display and the rows themselves according to those patterns.

For each of the 479,001,600—or (12!)—possible permutations of the twelve pitch classes, there are twenty-three other permutations that are related to it by transposition and/or inversion. When considered as twelve-tone rows, the permutations that make up any of these groups of twenty-four are identical in terms of their spatial characteristics: Whether or not they begin at the same point along the pitch-class cycle or unfold in the same direction, transpositionally and/or inversionally related permutations of the twelve pitch classes follow the same patterns of spatial acquisition, which are identified with the series of ordered pitch-class intervals that define those permutations. If we group permutations of the twelve pitch classes according to these spatial patterns, 19,958,400—or (12!)/24—unique sequences would result, which I term spatial row forms. The following is a general overview of the concepts underlying the elements of spatial acquisition theory, a theory that is designed to define and describe categories of spatial row forms for the purposes of analysis.

The building blocks of twelve-tone rows according to spatial acquisition theory are spatial row segments (SRSs). These differ from standard row segments in that, whereas the pitch classes that constitute standard row segments are, by definition, contiguous in terms of order
numbers, the pitch classes that constitute SRSs are contiguous in terms of pitch-class space. For example, a row that begins with the PCs <016278> could be parsed according to standard analytical practice into the discrete trichordal segments 016 and 278; these segments are called “discrete” because 1) they include all six given pitch classes of this partial row, 2) the pitch classes within each of these segments are in order relative to the row, and 3) the order numbers of the pitch classes within each of these segments are contiguous. The analytically salient aspect of this partition is that each of these trichordal segments are members of the set class (016) and therefore equivalent in terms of interval content.

In terms of spatial acquisition theory, the row segment <016278> would be parsed into SRSs 012 and 678. These segments, like the discrete ones mentioned above, also include all of the row’s six initial pitch classes, and the pitch classes within each of these segments are also in order relative to the row segment to which they belong, but the order numbers of these segments’ pitch classes are not contiguous. Instead, the pitch classes within each SRS are contiguous in terms of their positions in pitch-class space. This interpretation considers each SRS to be in the gradual process of acquiring pitch-class space, the point of origination for the former SRS being PC 0 and the point of origination for the latter SRS being PC 6. Since a twelve-tone row’s SRSs generally do not form row segments that are discrete in terms of order numbers, the interaction between standard and spatial partitions of a given row tends to be complex and enriching to the process of analysis, as will be demonstrated in the second part of this study. In order to reveal the SRS content of a twelve-tone row fully, we must discern the role that each pitch class within the row plays in the unfolding of that row’s SRSs.

Once the initial pitch class of a twelve-tone row has been presented—and, in terms of spatial acquisition theory, has established the row’s initial SRS—the pitch class that follows can
perform one of two functions from a spatial standpoint: It can establish a separate SRS or it can expand the one established by the initial pitch class. An example of the former would be a row that begins with the PCs 0 and 5; an example of the latter would be a row that begins with the PCs 0 and 1. In cases in which the space, or gap, between two established SRSs is minimal, as is, for example, the space between PCs 2 and 4 or 3 and 5, a third function is possible for a newly presented pitch class: It can fill that gap, thereby conjoining two previously established SRSs.

In more abstract terms, the three spatial row functions that I have just described can be defined as follows: A pitch class within a twelve-tone row that, when presented, has no adjacent neighbors among the row’s previously presented pitch classes serves as the establishment of an SRS. A pitch class that, when presented, has one adjacent neighbor among the row’s previously presented pitch classes functions as an expansion of an existing SRS. A pitch class that, when presented, has two adjacent neighbors—one on either side—among the row’s previously presented pitch classes is a conjunction of SRSs. When necessary in the course of this study, I abbreviate the names of these functions as E, X, and C. These three functional roles that a row’s pitch classes can play in the process of acquiring pitch-class space impact all of that row’s spatial characteristics and serve to delineate the structure of that row’s SRSs, as I will explain below.

This is an appropriate moment to mention the applicability to spatial acquisition theory of protocol pairs, a group-theoretical concept utilized by David Lewin in his Generalized Musical Intervals and Transformations. As applied to musical analysis, Lewin defines protocol pairs as “an ordered pair … of distinct … chromatic pitch classes,” displayed in the format \((x,y)\). Applying protocol pairs to the study of twelve-tone rows, I use \(x\) and \(y\) to represent any two pitch

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classes within the $P_0$ version of a particular row, with the stipulation that $x$ occurs before $y$ within the row. Thus, from the field of 132 possible pairs of non-equivalent whole numbers from zero to eleven, there are sixty-six that define any particular twelve-tone row; taken together, these protocol pairs fix the order position of each pitch class relative to every other pitch class within the row. Figure 1.4 provides an example of the protocol pairs that define the hypothetical row 0982341e576t.

Figure 1.4: The protocol pairs that define the hypothetical row 0982341e576t

\[
\begin{align*}
(0,9) & \quad (9,8) & \quad (8,2) & \quad (2,3) & \quad (3,4) & \quad (4,1) & \quad (1,e) & \quad (e,5) & \quad (5,7) & \quad (7,6) & \quad (6,t) \\
(0,8) & \quad (9,2) & \quad (8,3) & \quad (2,4) & \quad (3,1) & \quad (4,e) & \quad (1,5) & \quad (e,7) & \quad (5,6) & \quad (7,t) \\
(0,2) & \quad (9,3) & \quad (8,4) & \quad (2,1) & \quad (3,e) & \quad (4,5) & \quad (1,7) & \quad (e,6) & \quad (5,t) \\
(0,3) & \quad (9,4) & \quad (8,1) & \quad (2,e) & \quad (3,5) & \quad (4,7) & \quad (1,6) & \quad (e,t) \\
(0,4) & \quad (9,1) & \quad (8,e) & \quad (2,5) & \quad (3,7) & \quad (4,6) & \quad (1,t) \\
(0,1) & \quad (9,e) & \quad (8,5) & \quad (2,7) & \quad (3,6) & \quad (4,t) \\
(0,e) & \quad (9,5) & \quad (8,7) & \quad (2,6) & \quad (3,t) \\
(0,5) & \quad (9,7) & \quad (8,6) & \quad (2,t) \\
(0,7) & \quad (9,6) & \quad (8,t) \\
(0,6) & \quad (9,t) \\
(0,t)
\end{align*}
\]

The protocol pairs that define any twelve-tone row display the following characteristics:

1) Among all of the pairs involved, each pitch class occurs eleven times; 2) given that $n = a$ pitch class’s mod-12 order number within the row, the number of times that that pitch class occurs as $x$ can be determined by the formula 11-$n$; and 3) given the same value for $n$, the number of times a pitch class occurs as $y$ is equal to $n$. If, within such a group of protocol pairs as is displayed above, we isolate those whose members are maximally proximal from a spatial standpoint, that smaller group of protocol pairs can be used to determine the SRS structure and the layout of spatial functions within the row.
Within a collection of protocol pairs that defines a particular twelve-tone row, if we isolate those pairs in which \( x \) and \( y \) are neighboring pitch classes—or, in mathematical terms, in which \(|x-y|=1\)—the number of those pairs would be twelve, and they would reveal the number and content of SRSs as well as the role that each pitch class plays in terms of spatial functions within that row. Figure 1.5 displays the SRS-related protocol pairs for the same hypothetical row used for Figure 1.4, 0982341e576t, as well as the number of times each pitch class occurs as \( x \) and as \( y \) among those pairs. An explanation of the significance of these latter data follows.

![Figure 1.5: The protocol pairs that define the characteristics of the hypothetical row 0982341e576t in terms of spatial functions and SRSs](image_url)

<table>
<thead>
<tr>
<th>Pitch class</th>
<th>Occurs as ( x )</th>
<th>Occurs as ( y )</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>twice</td>
<td>never</td>
</tr>
<tr>
<td>9</td>
<td>twice</td>
<td>never</td>
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<tr>
<td>8</td>
<td>once</td>
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<tr>
<td>6</td>
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<td>twice</td>
</tr>
<tr>
<td>t</td>
<td>never</td>
<td>twice</td>
</tr>
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The protocol pairs that apply to spatial functions and SRS characteristics within a twelve-tone row display the following characteristics, which parallel, but are more restrictive than, those displayed by the protocol pairs that define twelve-tone rows in general: 1) Among all of the pairs involved, each pitch class occurs twice, and 2) each pitch class occurs either twice as \( x \), twice as \( y \), or once as \( x \) and once as \( y \). The distribution of a pitch class’s occurrences among the
$x$ and $y$ positions corresponds to that pitch class’s spatial function: Any pitch class that occurs as $x$ twice functions as an establishment, any pitch class that occurs as $y$ twice functions as a conjunction, and any pitch class that occurs once as $x$ and once as $y$ functions as an expansion.

We can see that the use of protocol pairs to explore a twelve-tone row’s characteristics in terms of spatial acquisition brings to the fore a more synchronic way to conceive of spatial functions than to compare the positions of newly presented pitch classes to the positions of already presented pitch classes during the presentation of a twelve-tone row, as follows: Within any row, the order number of a pitch class that serves as an establishment is lower than that of either of its neighbors in pitch-class space, the order number of a pitch class that serves as a conjunction is higher than that of either of its neighbors in pitch-class space, and the order number of a pitch class that serves as an expansion is between those of its neighbors in pitch-class space.

Having defined spatial row segments and the spatial row functions that their constituent pitch classes play, I will present the third category of row characteristics within the theory of spatial acquisition, which I have termed unidirectional row segments. In actuality, these are multiple-pitch-class subsets of SRSs that display two specific characteristics as they unfold within a twelve-tone row’s presentation that cause them to have special significance: They unfold both unidirectionally and their member pitch classes are contiguous in terms of order numbers within the row to which they belong. The marked sense of directionality that they thereby display in comparison to all other types of row segments has motivated me to provide them with their own separate category among spatial row features that are valuable in the process of row analysis.

So far, I have described features that relate only to the patterns that a twelve-tone row’s pitch classes form as they gradually acquire pitch-class space. To complete the conceptual pic-
ture of spatial acquisition, we also need to consider the converse of this process: the gradual dis-
appearance of non-acquired pitch-class space as the twelve pitch-classes gradually fill in the
twelve pitch-class places over the course of the row’s presentation. As any row unfolds, one or
more gaps appear, shrink, and eventually are filled in. The number of gaps that exist over the
course of a row’s presentation depends on the spatial properties of that row. Since rows exist in
cyclical pitch-class space, we could assert that at least one gap exists at every stage of a row’s
unfolding; even after only one pitch class has been presented, the other eleven pitch-class places
could be considered a large gap waiting to be filled.

The gaps that are particularly important for analytical purposes are those that have an im-
pact on spatially-oriented events that follow their appearance based on their size. I have termed
such gaps critical gaps. There are two types of critical gaps: 1) a dyadic gap, which contains
two pitch-class places and whose appearance makes it impossible for a new SRS to be estab-
lished within it, and 2) a monadic gap, which contains only one pitch-class place and within
which only a conjunction between the two SRSs that border it can occur.

Now that I have described the elements of spatial acquisition theory, it is necessary to
address an issue that has important implications regarding the analytical effectiveness of this
theory. The theoretical apparatus that I present in this study is designed to be applied to a totally
ordered rendition of the twelve-tone aggregate. However, it is rare in any twelve-tone work for
aggregates, as they unfold on the musical surface, to be presented consistently in strict order as
derived from the work’s underlying row and without two or more pitch classes occurring simul-
taneously. With this fact in mind, it becomes apparent that spatial acquisition theory can be
meaningfully applied to twelve-tone serial music in two distinct senses, the first more abstract
and the second more concrete: It can thoroughly identify the spatial properties of a work’s
twelve-tone row and apply those findings to manifestations of all or part of that row within the music itself, and it can identify the spatial properties of aggregates in general as they occur over the course of the work. The former type of application is straightforward; for the latter, the following problems present themselves: 1) Any simultaneities that occur within the presentation of a particular aggregate can obscure the roles that particular pitch classes within them play in terms of spatial functions, the formation of unidirectional row segments, or the creation and closure of gaps, and 2) the multiplicity of orderings of the twelve pitch classes that are not consistently related to a work’s underlying row or to each other corresponds to a multiplicity of unrelated spatial patterns that can be cumbersome to the analytical process. The following is an exploration of the interaction between established and spatial approaches to the analysis of aggregates in twelve-tone serial music.

Analyzing a twelve-tone serial work in terms of aggregate completion engages a body of established twelve-tone theory that deals with the generally partially or differently ordered nature of aggregates on the musical surface as compared to the totally—and singularly—ordered nature of twelve-tone rows. As implied above, aggregates can appear in a partially or differently ordered form for two non-exclusive reasons: 1) the presentation of at least two of the aggregate’s pitch classes simultaneously and 2) the presentation of at least two of the aggregate’s pitch classes in different order positions than they would occupy in the work’s definitive twelve-tone row or a transformation thereof. These order manipulations come about through either the compositional alteration of a particular row form, the presentation of two or more row forms in counterpoint, or both.

Brian Alegant has documented that, in some twelve-tone compositions, row forms in “strict” order are intentionally presented in opposition to row forms that are in “loose” order, the
strictly ordered row forms being associated with each other in terms of contour or other musical characteristics as are the loosely ordered forms.\textsuperscript{11} When analyzing an aggregate in the loose category, it is common to find at least some correspondence between its order and that suggested by the fundamental row for the work in question. The ordering of many loosely ordered aggregates can be defined at the pitch-class-set but not the pitch-class level; Robert Morris defines such aggregates, or subsets thereof, as partially ordered sets of pitch classes, or posets, partially based on Daniel Starr’s investigation into the properties of partially ordered aggregates, or POAs.\textsuperscript{12} An example of a hexachordal poset is $\langle\{0,1,2\}\{3,4,5\}\rangle$, in which the PCs 0, 1, and 2 must occur before 3, 4, and 5 do, but any ordering is possible beyond that restriction, including the obscuration of order through the use of simultaneities. Partially ordered aggregates can also occur “two-dimensionally” as free arrays, such as the simultaneous presentation of the aforementioned poset $\langle\{0,1,2\}\{3,4,5\}\rangle$ and the poset $\langle\{6,7,8\}\{9,te\}\rangle$; this sort of aggregate rendition may produce simultaneities as well as an ordering of the twelve pitch classes that is still further removed from an ordering that the underlying twelve-tone row would suggest.

We can see that the use of posets allows us to correlate a work’s twelve-tone row and any aggregates whose orderings are only partially derived from it, whether or not those aggregates include simultaneities. Although this is useful from any perspective, it is necessary for the purposes of spatial acquisition theory to maintain a consistent distinction between aggregates that include

\begin{itemize}
\item \textsuperscript{11} Brian Alegant, \textit{The Twelve-Tone Music of Luigi Dallapiccola}, (Rochester, New York: University of Rochester Press, 2010), 19.
\item \textsuperscript{12} Daniel Starr, “Derivation and Polyphony,” \textit{Perspectives of New Music} 23 (1984): 185.
\end{itemize}
simultaneities on the one hand and lynes\textsuperscript{13}—by which I mean aggregates that do not include simultaneities—whose orderings differ from orderings that are generated by the underlying row on the other. Since the spatial properties described in this study relate to the unfolding of the twelve-tone aggregate at the pitch-class level, an aggregate that differs from a row form only in that it contains simultaneities can be partially related to a twelve-tone row, since its order is obscured rather than in contradiction to that of the row. For lynes that are differently ordered relative to the row, however, that contradiction is in force, so that the lyne’s spatial data will differ distinctly from that of the row. To a large extent, lynes that are “differently ordered” as described above must be considered without regard for the row from a spatial standpoint, whereas the spatial properties of aggregates that include simultaneities can be correlated to those of the row, albeit limitedly.

A twelve-tone row is, in Bruce Samet’s words, an “inventory of pitch-class collections.”\textsuperscript{14} Partitioning schemes are generally motivated either by a desire to group the pitch classes within a row in a certain predetermined configuration, to highlight intervallic patterns within the row, or to reflect musical events that suggest a particular segmentation. Andrew Mead’s studies of the music of Milton Babbitt contains numerous examples of all three types;\textsuperscript{15} this reflects the fact that Babbitt sometimes composed utilizing arrays of different partitioning schemes on transformations of a single row simultaneously and used musical features to correlate the member pitch classes of a particular partition. All three types of partitioning can all be useful in analyses in

\textsuperscript{13} I have appropriated the term “lyne,” using it to refer to both abstract and literal ordered aggregates, from: Michael Kassler, “Toward a Theory that Is the Twelve-Note-Class System,” \textit{Perspectives of New Music} 5 (1967), 14.


volving spatial acquisition theory, especially if they either 1) determine a previously undetermined totally ordered aggregate or 2) determine a collection of simultaneities that correspond to partitions of the work’s fundamental twelve-tone row. In turn, the spatial properties of a row’s constituent pitch classes can determine numerous partitioning schemes of their own.

Michael Cherlin has described a distinction between partitionings of aggregates—both those aggregates that include simultaneities and lynes—that are “naturally induced” and those that are abstract.\(^\text{16}\) Whereas abstract partitionings are imposed on the aggregate without regard to its inherent characteristics as suggested by its ordering, so that the aggregate can be said to be “indifferent to its partitions,” naturally induced partitions are generated by the aggregate’s own properties, whatever degree of ordering the aggregate exhibits. Partitionings that are determined through the use of spatial acquisition theory fall into the naturally induced category, since those characteristics are generated by the arrangement in pitch-class space of the individual pitch classes of the series in question as they are presented in a particular order. Furthermore, since a particular role in terms of spatial functions and membership in a particular SRS can be identified for every pitch class within any twelve-tone row, we can see that every row generates such naturally-induced partitions, as does every lyne that may or may not be a row form \textit{per se}. As described above, the same cannot be said for all partially ordered aggregates that include simultaneities; some provide enough information for such partitionings and others do not, as demonstrated in the following examples.

Given an aggregate that begins with a dyadic simultaneity followed by a trichordal simultaneity that, together, form a member of set class (02469), we can arrange the member pitch classes of

that set along the schema of pitch-class space represented in Figure 1.3 and see that none of them is a neighbor to any of the others. Therefore, according to the definitions of spatial functions presented above, all five of those pitch classes serve as establishments. This fact would prove salient for partitioning purposes if, for example, none of the rest of the pitch classes in that aggregate served as establishments, putting the functional role played by the first five into relief. On the other hand, given an aggregate that begins with two simultaneities that, together, form a member of set class (01234), the situation is much more vague: A totally ordered version of such a set could contain one, two, or three establishments; zero, two, or four expansions; and zero, one, or two conjunctions. Without total ordering, there is no way to discern the specific roles that the set’s member pitch classes play in terms of spatial functions, and no way to discern patterns involving unidirectional row segments or critical gaps as well. In other words, there are no solid spatial criteria in this case for the purposes of partitioning.

For certain partially ordered aggregates that are more conducive to spatial analysis, spatial acquisition theory can provide unique partitioning schemes. The following example, based on the hypothetical aggregate shown in Figure 1.6, demonstrates such a case.

Figure 1.6: A hypothetical aggregate (verticals indicate simultaneities)

```
  8  6  5  e  t  4
  0  2  3  9  7  1
```

This aggregate does not display any obvious characteristics that would lead us to a particular partitioning unless we consider it from the perspective of spatial acquisition theory. The first tetrachord presented within this aggregate is a member of set class (0268), none of the
member pitch classes of which is a neighbor to any of the others in pitch-class space, and therefore all function as establishments. The next tetrachord is also a member of set class (0268); each of its member pitch classes is a neighbor to one of the initial four pitch classes of the aggregate in pitch-class space but none of its member pitch classes fills a gap, meaning that these four pitch classes all function as expansions. All four member pitch classes of the final tetrachord, a member of set class (0369), fill previously existing gaps and therefore serve as conjunctions. Thus, these three tetrachords are neatly defined by the spatial roles that their member pitch classes play, facilitating a partitioning of this aggregate that is realizable only through the use of spatial acquisition theory.

The material presented so far in this study falls into four broad categories: approaching twelve-tone rows using existing theoretical techniques, approaching twelve-tone rows using spatial acquisition theory, approaching aggregates using existing theoretical techniques, and approaching aggregates using spatial acquisition theory. Salient interactions among these four areas of inquiry provide the most useful information for analytical purposes, as shown in the second part of this study. However, because of the aforementioned necessity to consider pitch classes diachronically in order to utilize the concept of spatial acquisition fully, the rest of the theoretical part of this study will prioritize totally ordered aggregates, to which I will continue to refer generally as twelve-tone rows. What follows immediately is an explication of the properties inherent in the four categories of spatial row features.

*Row phases and spatial row functions*

In order to perform a spatial analysis of a twelve-tone row with exactitude, we must be able to define the points, whether in actual time or conceptual, at which spatially significant
events in the unfolding of the chromatic aggregate occur. For this purpose, I use the term *phase* to denote the literal or abstract moment when a particular pitch class of a row is presented. For example, in a twelve-tone row’s first phase, its initial pitch class appears; in its seventh phase, its seventh pitch class appears amidst the six that have already been presented. With this concept in place, I will now explore the possible spatial functions that a row’s pitch classes may exhibit in each of a given row’s phases.

Unique to the first, eleventh, and twelfth phases of any twelve-tone row is the fact that only one spatial arrangement is possible for each: a single pitch class for the first, the set class (0123456789t) for the eleventh and the total aggregate for the twelfth. Another way of expressing this fact would be to state that, for any row, the segment has been presented by the time the phase in question occurs can, in those cases, only belong to one particular set class. Correspondingly, the only possible function that the initial pitch class of a row can perform is that of SRS establishment, and the only possible function that the twelfth pitch class of a row can perform, since the eleventh phase can leave nothing other than one monadic gap for it to fill, is that of SRS conjunction. The second and eleventh row phases offer limited possibilities in terms of spatial functions as well. Since conjunction can only happen when two pitch classes with a monadic gap between them are already present, this function would not be possible in phase 2 of any row, in which only one pitch class has been previously presented; in this phase, only establishment or expansion could occur. Conversely in the eleventh phase, in which one of only two remaining pitch-class places are filled, establishment would not be possible, since it can only be achieved within a gap of at least three pitch-class places; in this phase, only expansion or conjunction could occur. The third through tenth phases of twelve-tone rows can accommodate all three spa-
tial-function possibilities. Figure 1.7 shows the distribution of possible spatial functions throughout all twelve phases of an abstract row.

Figure 1.7: The distribution of possible spatial functions among row phases

<table>
<thead>
<tr>
<th>Phase</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Function</td>
<td>E</td>
<td>E,X</td>
<td>E,X,C</td>
<td>E,X,C</td>
<td>E,X,C</td>
<td>E,X,C</td>
<td>E,X,C</td>
<td>E,X,C</td>
<td>E,X,C</td>
<td>X,C</td>
<td>C</td>
<td></td>
</tr>
</tbody>
</table>

The ways in which the three spatial functions interact with each other impose further limitations on their distribution among the twelve row phases. Establishments and conjunctions in particular share a special relationship because of the fact, due to the cyclical nature of pitch-class space, that the completion of a twelve-tone row is the same as the completion of the chromatic aggregate. The relationship is as follows: For every SRS that is established, it must eventually be conjoined either with another SRS or with itself; therefore, there is a 1:1 correspondence between the number of establishments and the number of conjunctions within any twelve-tone row. In turn, the number of expansions that a row contains is determined by the number of establishments and conjunctions within it, as follows: If \( n \) = the number of establishments within a particular row, then \( n \) = the number of conjunctions within the row as well, and, since the row contains a total of twelve pitch classes, \( 12 - 2n \) = the number of expansions within the row.

Given the relationships between the cardinalities of establishments, expansions, and conjunctions within a twelve-tone row as described above, we can easily determine a limited number of possible combinations of values for those cardinalities. I have already shown that the first pitch class of a row can only be an establishment; therefore, the minimum number of establishments for any row is one. Since the number of a row’s conjunctions matches the number of its establishments, the minimum number of conjunctions for any row is also one, and the maximum
number of establishments as well as conjunctions is six, since their sum cannot be greater than twelve. As for the number of expansions within any row, we now know that the “n” in the “12-2n” formula introduced above, representing the number of a row’s establishments or conjunctions, can equal any whole number from one to six. Assigning these values to “n” determines that the number of expansions within a row can be any even number from zero to ten. Therefore, there are only six fixed sets of cardinalities that can apply to the establishment(s), expansions, and conjunction(s) of a twelve-tone row, as shown in Figure 1.8.

Figure 1.8: The six possible sets of spatial-function cardinalities

<table>
<thead>
<tr>
<th></th>
<th>Establishments</th>
<th>Expansions</th>
<th>Conjunctions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
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<td>6</td>
<td>3</td>
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<tr>
<td>5</td>
<td>5</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>0</td>
<td>6</td>
</tr>
</tbody>
</table>

As illustrations of how these six sets of spatial-function cardinalities can manifest themselves within actual twelve-tone rows, I will present hypothetical rows in which the distribution of spatial functions among the row’s pitch classes corresponds to each set of cardinalities shown in Figure 1.8. In the interest of clarity, the spatial functions of these rows’ constituent pitch classes are represented graphically in Figure 1.9, and each row’s SRS content is described below. 1) One establishment, ten expansions, and one conjunction would exist in a row with only one SRS, such as 0et987654321. 2) Two establishments, eight expansions, and two conjunctions would exist in the row 061728394t5e, the SRSs of which are 012345e and 6789t5e, with PCs 0 and 6 being the establishments and PCs 5 and e the conjunctions. 3) Three establishments, six expand-
sions, and three conjunctions would exist in the row 01245689t37e, the SRSs of which are
0123e, 45637, and 89t7e, with PCs 0, 4, and 8 being the establishments and PCs 3, 7, and e the
conjunctions. 4) Four establishments, expansions, and conjunctions would exist in the row
0369147t258e, the SRSs of which are 012e, 3425, 6758, and 9t8e, with PCs 0, 3, 6, and 9 being
the establishments and PCs 2, 5, 8, and e being the conjunctions. 5) Five establishments, two
expansions, and five conjunctions would exist in the row 013468t2579e, the SRSs of which are
012e, 3425, 657, 879, and t9e, with PCs 0, 3, 6, 8, and t being the establishments and PCs 2, 5, 7,
9, and e being the conjunctions. Finally, 6) Six establishments, no expansions, and six conjunc-
tions would exist in the row 02468t13579e, the SRSs of which are 01e, 213, 435, 657, 879, and
t9e, with the initial six pitch classes comprising the establishments and the final six comprising
the conjunctions.
Figure 1.9: Representations of hypothetical rows that demonstrate the possible combinations of spatial-function cardinalities

1. Row 0et987654321

2. Row 061728394t5e

3. Row 01245689t37e

4. Row 0369147t258e

5. Row 013468t2579e

6. Row 02468t13579e

The formats of each of the hypothetical rows presented above with regard to spatial functions are fairly straightforward in the sense that, for each row, the total number of the row’s es-
establishments occur before any of its conjunctions do and the layout of spatial functions overall is relatively orderly; however, for many twelve-tone rows this is not the case. To circumscribe the full range of possible distributions for the spatial functions of a row’s pitch classes, I will present its limitations. 1) From phase 1 through phase 11, the number of existing establishments within a row must be at least one greater than the number of existing conjunctions, since equal numbers of establishments and conjunctions signify aggregate completion. 2) Since the initial pitch class of a row must be an establishment, the final pitch class must be a conjunction, and their numbers must not be equal until phase 12, the sequence of establishments in a multi-SRS row must begin with two establishments and, therefore, end with two conjunctions. If any more establishment/conjunction pairs exist within the row, they may be distributed in any fashion, as long as limitation 1 is observed. As for expansions, the limitation on their distribution is simple: They may be interspersed among a row’s establishments and conjunctions in any fashion within the phases in which expansions can exist, namely phases 2 through 11.

In addition to describing the possible combinations in which the three spatial functions can appear among the pitch classes that compose a twelve-tone row, analyzing the positions pitch classes that perform those functions can occupy in pitch-class space is useful as well. Since each SRS acquires pitch-class space in two directions, beginning with a single establishment and expanding as far as its two conjunctions, it is evident that a representation of a row’s establishments and conjunctions in pitch-class space will always reveal an alternation of Es and Cs around the chromatic cycle. Since expansions may or may not occur within any particular SRS, these alternating Es and Cs can be adjacent to one another or widely spaced depending on the structure of the row in question.
It is possible for the distributional format of a row’s establishments and conjunctions—or the format of merely its establishments or its conjunctions—to exhibit analytical significance. For example, if a musical work built upon the row 0e321586794t frequently featured members of the (0369) set class as prominent sonorities, a correspondence between the row and set class (0369) would not be evident by virtue of a non-spatial analysis of the row. It would only come to light if the row’s conjunctions were plotted graphically. For this purpose, Figure 1.9 contains what I have termed an E/C map, which displays a row’s establishments as well as its conjunctions. This row’s conjunctions are the PCs 1, 4, 7, and t, which, taken together, form a member of set class (0369). In turn, if this work prominently featured members of set class (0358), it would be the distribution of the row’s expansions, which are the PCs 0, 3, 5, and 8, that would be analytically significant.

Figure 1.9: An E/C map for the row 0e321586794t

Now that I have fully defined the extent to which the establishments and conjunctions of a twelve-tone row can be arranged along the chromatic cycle, I will explore the analytically salient qualities that various types of spatial-function sequences can display, focusing on establish-
ments and conjunctions in particular. Since the sequence of spatial functions exhibited by any multi-SRS row begins with two establishments and two conjunctions and the number of already presented establishments must be greater than the number of already presented conjunctions until the row’s twelfth phase, we can say that, on the whole, establishments generally occur before conjunctions do. However, there are plenty of cases in which a row’s conjunctions and establishments are interspersed within the middle phases of the row’s presentation. To address this sort of situation, I use the term *reversal* to refer to a point within a row’s presentation at which a conjunction occurs after an establishment does, thereby “reversing” whatever trend of SRS establishments existed up to that point. Of course, every row contains at least one such case; therefore, a row’s number of reversals is more salient the greater it is. The maximum number of reversals that a spatial-function sequence can contain is five, which would occur in the sequence that defines six SRSs and whose establishments and conjunctions alternate as much as possible: EECECECECECC. In fact, the maximum number of reversals that could occur during the presentation of a multi-SRS row that contains $n$ SRSs would be $n-1$, since each but the last of a multi-SRS row’s conjunctions could follow one or more expansions as in the preceding example.

Another example of multiple reversals within a twelve-tone row’s sequence of spatial functions would exist within the sequence EXECEECCECXC. This sequence contains three reversals, at phases 4, 7, and 10. By isolating the sequence’s reversals in this way, we can note the number of consecutive establishments and conjunctions—putting expansions aside for the moment—that occur over the course of the sequence. In this example, there are, in order, two establishments, one conjunction, two establishments, two conjunctions, one establishment, and two conjunctions. Each of these single or multiple establishments or conjunctions in succession relative to each other constitutes what I have termed an *establishment group* or a *conjunction group*
within the row’s spatial-function sequence. We can see that, in the case of this sequence, these
groups form a symmetrical pattern in terms of the numbers of “Es” or “Cs” within them: 2, 1, 2, 
2, 1, 2. Depending on the relationships between the establishments and conjunctions that make 
up these groups and the SRSs that they define, patterns such as these can provide another useful 
layer of information for the purposes of spatial row analysis.

At this point, it is appropriate to discuss the effect of the canonical twelve-tone opera-
tions—transposition, inversion, retrograde, and combinations thereof—on a row’s sequence of 
spatial functions. In twelve-tone music, the compositional exploitation of these operations is 
ubiquitous, making the definition of a relationship between the elements of spatial acquisition 
theory and these operations essential for analytical purposes. The effects of transposition and 
inversion on a row’s sequence of spatial functions are easy to describe, since, in actuality, they 
do not exist; the pitch classes of any two row forms acquire pitch-class space according to exact-
ly the same pattern regardless of both the pitch-class level at which they begin and the direc-
tion(s) in which they unfold.

The retrograde operation is the only one of the canonical twelve-tone operations that can 
actually change a row’s sequence of spatial functions. This occurs as follows: The pitch classes 
that serve as establishments exchange functional roles with those that are conjunctions, with the 
expansions remaining as expansions. For this reason, only sequences of spatial row functions 
that are symmetrical would remain invariant under all of the canonical twelve-tone operations. 
The fact that some rows that are intervallically asymmetrical exhibit symmetrical spatial row 
function sequences and vice-versa enriches the analytical process in many situations, some of 
which can be found in the analytical portion of this study.
Spatial row segments

The unfolding of spatial row segments, as defined by the three spatial row functions—establishment, expansion, and conjunction—within a given twelve-tone row corresponds to one of two narratives. For a row containing more than one SRS, the narrative proceeds as follows: The SRS is established, may or may not be expanded by a number of pitch classes, is conjoined with another SRS, may or may not be expanded some more, then is conjoined with a third SRS. For single-SRS rows, the narrative is simpler: The SRS is established, may or may not be expanded by a number of pitch classes, then is conjoined with itself. An example of a single-SRS row would be the simplistic hypothetical row 0123456789te, the final pitch class of which would close the gap between the pitch class that preceded it—t—and the row’s initial pitch class—0—to complete the total aggregate.

SRSs are the spatial building blocks of any twelve-tone row, and SRS narratives, as described above, serve to explain the spatial patterns inherent in the structure of any twelve-tone row. Certain characteristics among the many that SRSs display prove salient for the purpose of analysis. These characteristics can be divided into three categories: cardinality, contour, and continuousness. The first two of these are self-explanatory; the third refers to the degree to which the pitch classes that make up an SRS are contiguous in terms of order numbers within the row to which they belong as well as contiguous from a spatial standpoint. An exploration of these three categories of SRS characteristics follows.

As the defining elements of each SRS, the spatial functions of a twelve-tone row’s constituent pitch classes are the keys to SRS cardinality as well. The fact that each conjunction is shared by two SRSs causes the SRSs to overlap, their combined cardinalities therefore exceeding the number of pitch classes in the row, except in the rare rows that feature only one all-
encompassing SRS. Single-SRS rows need not be considered in this discussion of cardinality in any case, since the cardinality of such a row’s SRS is always, of course, twelve. The functions of the pitch classes that build each SRS in a multi-SRS row can be generalized as follows: Since each pitch class of any multi-SRS row that serves as a conjunction can be claimed by two SRSs, the combined cardinalities of the row’s SRSs would outnumber the pitch classes in the row by the number of conjunctions in the row. In other words, if \( c \) = the number of a multi-SRS row’s conjunctions, the combined cardinalities of that row’s SRSs would be \( 12 + c \).

As for the cardinalities of individual SRSs within multi-SRS rows, that fact that each SRS must comprise at least an establishment and two conjunctions means that the minimum possible cardinality for an SRS is three. The maximum possible cardinality for an SRS depends on the number of SRSs in its row. An SRS is maximal in size if all of the other SRSs in its row are minimal in size—i.e. comprising three pitch classes each—in which case the number of pitch classes within a maximally large SRS would be the difference between the total number of pitch classes in the row—twelve—and the sum of 1) one fewer than the number of establishments within the row and 2) two fewer than the number of conjunctions within the row. The reason for this is that, within a row that contained one maximally-sized SRS and a number of minimally-sized SRSs that, by virtue of being minimally-sized, contained establishments and conjunctions but no expansions, only one of that row’s establishments would belong to the maximally-sized SRS, while only the two conjunctions that formed the ends of the maximally-sized SRS would belong to it. Taking these constraints into consideration leads to the establishment of a simple formula: If \( e \) = the number of a multi-SRS row’s establishments—which, of course, is the same as the number of its conjunctions—the maximum possible cardinality for any abstract SRS with-
in that row would be $12 - ((e - 1) + (e - 2))$, or $12 - (e - 1 + e - 2)$, or $12 - e + 1 - e + 2$, or, simply, $15 - 2e$.

The formula just presented makes it clear that the possible disparity between the cardinalities of the SRSs within any particular twelve-tone row is a function of the number of SRS that the row contains. For example, in a row with only two SRSs, those SRSs may have numbers of pitch classes as divergent as three and eleven. However, in a row with six SRSs, the only possible cardinality for each of those SRSs is three. Of analytical interest, then, are two implications of this finding: 1) The larger the number of SRSs within a twelve-tone row, the more homogeneous the cardinalities of those SRSs will be, and, conversely, 2) creating a row with a small number of SRSs affords the composer more freedom as to whether or not those SRSs are to be balanced in terms of cardinality. Both of these situations exist within the musical works addressed in the analytical part of this study.

In terms of contour, all SRSs share a few distinct features: They radiate from a single point of origination—the establishment—and, after going through some number of expansions, end with two conjunctions that are on opposite “sides” relative to the establishment in pitch-class space. The pitch classes of an SRS create no gaps as they unfold, so that the space taken up by an SRS expands with each new pitch class that is added to it. In order to explore the effects of these features on SRS contour in general, I will utilize Robert Morris’ model of contour analysis with regard to pitch, in which the members of an ordered set of pitches of cardinality $n$ are represented by integers from 0 to $(n - 1)$ according to the pitches’ relative levels.17 For example, the melody C-D-F-E-G, the range of which is from C4 to G4, would be represented as <01324>.

Whereas the member pitch classes of an SRS of cardinality \( n \) can be represented by integers from 0 to \((n - 1)\) as well, with the integers—like the numerical names of the twelve pitch classes—increasing in value as one moves “clockwise” around the chromatic cycle, the radiational nature of SRS contours allows us to predict that only the two conjunctions that form the boundaries of an SRS could be represented as 0 and \((n - 1)\) in this schema. Of the remaining “internal” members of such an SRS contour representation, any one could represent the establishment and the others would represent expansions. In addition to this correspondence that SRS contours have to the spatial functions of SRSs’ constituent pitch classes, these contours shed light on the patterns of directional changes that occur as SRSs unfold.

Each SRS contour must include at least one directional change. This is due to the fact that every SRS begins with an establishment, which is followed, perhaps after intervening expansions, by a conjunction, which in turn is followed, perhaps after more intervening expansions, by another conjunction in the opposite direction. This satisfies the requirement that, in spatial terms, the two conjunctions must lie at the outer ends of the SRS, with the establishment being somewhere between them. The maximal number of directional changes that an SRS contour may display is two fewer than the cardinality of the SRS itself, since, after the initial two pitch classes of an SRS have been presented, each of the remaining pitch classes can change the direction in which the SRS is unfolding in turn.

The following are examples of SRSs that display the least and greatest possible numbers of directional changes. In order to demonstrate the directional changes within each SRS, I will present each SRS’s contour adjacency series (CAS), as defined in the analytical works of Mi-
Michael Friedmann. The hypothetical SRS 012et9, the CAS for which is \(<+, +, -, -, ->,\) unfolds with the minimum of one directional change, occurring between PCs 2 and e. The hypothetical SRS 0e1t29, on the other hand, generates a CAS of \(<-, +, -, +, ->,\) and, therefore, changes direction four times. As established above, four is the maximum number of directional changes that could occur throughout the unfolding of an SRS with a pitch-class cardinality of six such as this one. Differences in directional patterns such as these prove at times to be salient data for analytical purposes.

Continuousness, the third and final category of SRS features, is, of course, not a feature of any particular SRS but the way in which the members of multiple SRSs are distributed within a twelve-tone row. In order for an SRS to be continuous, its member pitch classes must all be contiguous with regard to the row’s order numbers; in other words, a continuous SRS forms an unbroken segment of the row to which it belongs. This type of SRS is rare in actual musical literature. A continuous SRS can possibly exist only in these three cases: 1) as the sole SRS of a single-SRS row, 2) as the second SRS to be established within a row with only two SRSs, or 3) as an SRS that is established within the pitch-class space between two previously established SRSs. In cases two and three, of course, the establishment of an SRS that is to be continuous must be followed immediately by each of that SRS’s own expansions and conjunctions.

The following are hypothetical examples of cases two and three: The row 0e12678954t3 contains the SRSs 0e12t3 and 678954t3. The establishment pitch class of the first SRS is 0, that of the second SRS is 6, and they share the conjunctions t and 3. Since the establishment of the second SRS is followed by that SRS’s five expansions and two conjunctions and no pitch classes

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that belong exclusively to the first SRS, the second SRS is continuous, an example of case two above. The first SRS in a row with two SRSs such as this could not be continuous, since the establishment of the second SRS would have to occur between the establishment of the first SRS and either of the first SRS’s conjunctions. An example of case three would be the row 016t7843259e, the four SRSs of which are 012e, 67859, 4325, and t9e. Of these four, only 4325 exists as a continuous SRS; it fills in the space between SRSs 012e and 67859, sharing its first conjunction—2—with SRS 012e and its second—5—with SRS 67859.

As opposed to those SRSs that are continuous within the twelve-tone rows to which they belong, other SRSs are presented with interruptions—intervening pitch classes belonging to other SRSs—that, for any particular SRS, can number from zero to one fewer than the cardinality of the SRS in question. This variability of the number of interruptions possible for an SRS facilitates potential disparities between how frequently one SRS is interrupted as opposed to other SRSs within the same row. For example, the hypothetical row 07169t8e2534 would contain three SRSs, 01e234, 76854, and 9t8e. The unfolding of SRS 01e234 is broken up by three interruptions, the unfolding of SRS 76854 contains four, the most interruptions possible for an SRS of cardinality five, and, contrastingly, the unfolding of SRS 9t8e contains no interruptions; it is continuous. The unbroken nature of SRS 9t8e’s unfolding is marked within the unfolding of the row to which it belongs, a fact that would have analytical implications if a musical work built on this row emphasized iterations of this particular SRS.

Further enriching the issue of SRS continuousness is the fact that, in multiple-SRS rows, patterns may exist in terms of the sequence in which pitch classes from various SRSs are presented. An example of such a pattern exists within the hypothetical row 0415e328697t, the SRSs of which are 01e2t, 453267, and 897t. For the purposes of this example, I will label these SRSs
$a$, $b$, and $c$, respectively. Figure 1.10 displays the pattern in which these three SRSs unfold as the row is presented, allowing for the fact that each of the conjunctions, PCs 2, 7, and $t$, are by definition members of two SRSs.

Figure 1.10: Pattern of SRS unfoldings within the row 0415e328697t

PCs:

| 0 | 4 | 1 | 5 | e | 3 | 2 | 8 | 6 | 9 | 7 | t |

SRS membership:

\[
\begin{array}{cccccccccc}
a & b & a & b & a & b & a/b & c & b & c & b/c & a/c \\
\end{array}
\]

Over the course of this row’s first seven phases, members of SRS $a$ alternate with members of SRS $b$ until those SRSs are conjoined; then, members of SRS $b$ alternate with members of SRS $c$ until the row’s final conjunctions occur. With this being the case, we can identify SRS $a$ more directly with, roughly speaking, the first half of the row, while identifying SRS $b$ with the row’s second half, with SRS $b$ serving as a sort of partner to both SRSs $a$ and $b$ throughout the row’s presentation. Within a hypothetical musical work based on this row, it would be analytically salient if, for instance, the member pitch classes of SRSs $a$ and $c$ were placed in a noticeably different register than the member pitch classes of SRS $b$, with their conjunctions placed midway between the two registers. Such an arrangement would highlight the spatial contiguousness within each of the three SRSs while downplaying the lack of order-position contiguousness within each of them.

Whereas the cardinalities, contours, and continuity of SRSs can vary greatly, it is the SRS subsets that are maximally smooth and uninterrupted—URSs—that exhibit the properties of spatial acquisition more prominently than do any others on the musical surface. Similarly, the crea-
tion and closure of critical gaps emphasize the sense of closure that comes from the gradual filling of pitch-class space that is at the heart of the concept of spatial acquisition. Before discussing the properties of URSs and gaps *per se*, I will address the musical forces that they make manifest in the works in which they appear.

Spatial acquisition and musical forces

One of the most distinctive features of the concept of spatial acquisition is its dynamic quality. The process by which a twelve-tone row’s pitch classes acquire pitch-class space clearly implies the senses of both motion and purposeful action that Steve Larson has identified as the primary spatial metaphors, grounded in bodily experience, that apply to music in general.19 On this conceptual foundation, Larson has identified three distinct forces that influence patterns of pitch-class and/or pitch motion in tonal music: inertia, magnetism, and gravity.20 Although the pitch-class dynamics at work in twelve-tone serial music differ greatly from those at work in tonal music, two of those forces are important to the concept of spatial acquisition as well, as explained below.

Inertia is defined in two ways that pertain to tonal music: 1) as a pattern of musical motion that tends to continue in the same direction, and 2) the tendency of a stepwise collection of pitches unfolding in a single direction to keep unfolding in that direction.21 Both of these

21. ibid.
tendencies translate to the concept of spatial acquisition. The second definition pertains directly
to the phenomenon of URSs, and the first can be applied to various spatial patterns observable in
twelve-tone rows, for example: The initial row segment <0t1928> exhibits the spatial-function
sequence EEXXXX and contains the SRSs 012 and t98. The fact that each of these SRSs unfold unidirectionally—in opposite directions—and their constituent pitch classes are alternated strict-
ly within this segment causes the segment to establish a clear spatial and intervallic pattern,
which implies that the next pitch classes to be presented within this row will be 3, 7, 4, 6, and 5.
If the pitch classes that follow the given segment within this row were to discontinue this pattern,
it would be a noteworthy event specifically because of the disruption of the inertia of musical
motion that the pattern had generated.

In Larson’s model, magnetism refers to the tendency of an unstable pitch to move to a
more stable pitch that is maximally proximal.22 The issues of both proximity and stability make
it so that this force applies to twelve-tone music much differently than it does to tonal music. As
far as proximity is concerned, the chromatic aggregate does not offer differentiations in size be
 tween pitch classes that are considered to be adjacent, unlike the whole and half steps that exist
among adjacent pitch classes in the tonal world. The classification of some pitch classes as sta-
ble and others as unstable by definition is similarly foreign within the undifferentiated chromatic
cycle, obtaining in tonal music because of the segregation of pitch classes into chord tones and
non-chord tones based on a normative triadic harmonic texture. For this reason, a discernment
of magnetism in tonal music is more dependent upon previous experience with tonal music than

22. ibid.
is the discernment of either of the other musical forces.23

Since twelve-tone serial music assumes no *a priori* harmonic norms, stability and instability exist only contextually within it. The most heightened sense of magnetism in this music occurs between the appearance of a gap between maximally proximal pitch classes—a monadic gap—and that gap’s closure. A dyadic gap, because of the lack of room for a new SRS within it, would display a lesser but also notable degree of magnetism. The presence of magnetism in a twelve-tone serial work is not guaranteed as it is in tonal music, in which the magnetism between the leading tone and the tonic scale degrees always exists, waiting to be exploited, but is completely dependent upon the spatial structure of the row upon which that work is based.

Gravity, the tendency of unstable pitches to descend to more stable ones in tonal music,24 is the one musical force that cannot apply in pitch-class space. It is based exclusively in pitch space, not pitch-class space, since pitch-class space has no low extreme that could exert a gravitational pull.

In Larson’s model, a musical gesture or phrase is said to end when its pitches “gives in” to one or more of the aforementioned forces in a distinct way.25 In some cases, a melody might give in to one or more forces while overcoming one or more others. For example, a melodic line in a major-key tonal work that ascends from fifth to the sixth to the seventh to the first scale degree is “giving in,” as it approaches the tonic, to the inertia created by its initial upward motion and the magnetism between the leading tone and tonic scale degrees. These forces overcome the

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contradictory force, gravity, which favors downward motion. Since, in twelve-tone serial music, all gaps that appear are eventually filled, the force of magnetism is always satisfied. Inertia, however, can certainly be thwarted in this music in favor of magnetism, in the sense that a spatial pattern within a twelve-tone row’s presentation can be discontinued in order for a monadic gap to be filled. If the fulfillment of a spatial pattern causes a monadic gap to be filled, then both forces would be satisfied, for example: A row that concludes with the segment \(<t6789>\) would exhibit inertia as the pitch classes 6, 7, 8, and 9 are presented as well as magnetism as PC 8 is presented. Both of these forces would be satisfied by PC 9’s completion of the chromatic aggregate. It is the presence of these forces that make URSs and critical gaps particularly important among the spatial features of any twelve-tone row.

*Unidirectional row segments*

As discussed earlier, a row segment is contiguous with regard to its member pitch classes’ order numbers, whereas an SRS is contiguous with regard to its member pitch classes’ positions in pitch-class space. A portion of a twelve-tone row that satisfies both of these criteria is a continuous SRS. A continuous segment of an SRS that unfolds in only one direction—or, to put it another way, has a completely smooth contour—is a URS. Because SRS contours always contain at least one change of direction, a URS can only exist as a part of one—and only one—larger SRS. The unmitigated inertia that URSs display makes them particularly important for analytical purposes; the following is a description of their properties.

Similar to the three functions that a row’s pitch classes perform in the unfolding of SRSs in general, each of the pitch classes that constitute a URS performs one of three functions that are unique to the unfolding of URSs: *initiation, extension, and termination*. Initiation is the function
of the initial pitch class of a URS, termination is the function of a URS’s final pitch class, and extensions are those pitch classes in between.

The three aforementioned URS functions interact with the three spatial row functions—establishment, expansion, and termination—as follows: 1) Only pitch classes that serve as SRS establishments or expansions can also serve as URS initiations. 2) Only pitch classes that serve as SRS expansions can also serve as URS extensions. 3) Only pitch classes that serve as SRS expansions or conjunctions can also serve as URS terminations. Therefore, if we replace the names of SRS and URS functions, in order, with the words “beginning,” “middle,” and “end,” we can see that a URS can begin either at the beginning or in the middle of an SRS and can end either in the middle or at the end of an SRS. The middle of a URS will always be contained within the middle of the SRS to which it belongs. For this reason, when abbreviating the names of the URS functions, I use the same letter to represent extensions—X—as I do to represent SRS expansions, since pitch classes that serve as URS extensions always also serve as SRS expansions. To represent URS initiations and terminations, I use the letters I and T respectively. Figure 1.11 represents the interactions between spatial row functions and URS functions graphically.

Figure 1.11: Interactions between spatial row functions and URS functions

<table>
<thead>
<tr>
<th>Spatial Row Functions:</th>
<th>Establishment (E)</th>
<th>Expansion (X)</th>
<th>Conjunction (C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>URS Functions:</td>
<td>Initiation (I)</td>
<td>Extension (X)</td>
<td>Termination (T)</td>
</tr>
</tbody>
</table>
One key difference between SRSs and URSs is that, whereas two SRSs that are adjacent in pitch-class space share a pitch class that serves as a conjunction for each, URSs do not share pitch classes that serve as terminations. For example, in the case of a hypothetical row that begins with the segment 0143, PCs 0 and 1 would represent one SRS and PCs 4 and 3 would represent another. In addition, since both PCs 0 and 1 and PCs 4 and 3 are adjacent both in pitch-class space and with regard to order numbers, each of these budding SRSs would also be a URS. If we were to add PC 2, this PC would serve as a conjunction between the two SRSs, but would not be the termination for both URSs. With PC2 in place, the URSs would now be 01 and 432; their respective terminations would be PCs 1 and 2. The overall initiation/termination sequence for row segment 01432 would be I, T, I, X, T. If we were to witness the rest of this hypothetical row’s unfolding, it might turn out that PCs 0 and 1 as well as PCs 4, 3, and 2 belong to larger SRSs. This is not the case, however, regarding URSs; whereas SRSs are only “finished” once two conjunctions have been presented, the initiation, any extensions, and the termination of a URS are always contiguous in terms of order numbers within the row to which they belong.

In addition to the fact that URSs do not overlap, URSs differ from SRSs in that, for the most part, their analytically valuable characteristics do not fall into the same categories—cardinality, contour, and continuity—that the analytically valuable characteristics of SRSs do. Of those three, contour and continuity are fixed where URSs are concerned, since URSs are by definition unidirectional and completely continuous. The cardinalities of URSs are, of course, variable, and therefore useful for analytical purposes. The fact that all URSs are unidirectional and continuous causes direction itself to be the other analytically useful characteristic that URSs display. A URS’s direction can be either clockwise or counterclockwise, and the interaction of
adjacent URSs in terms of the directions in which they unfold leads to more significant analytical information.

Adjacent URSs that unfold in the same direction can exhibit either a smooth or jagged contour, depending on the direction in which the URSs themselves are presented. For example, the hypothetical row segment <017923> contains the URSs 01 and 23. If we separate these URSs from the rest of the row segment, retaining the order of the URS’s constituent pitch classes within the row segment, the result is the unidirectional segment 0123. In this case, two URSs form the beginning of a single SRS. On the other hand, the row segment <107932> reverses the direction in which each of these URSs unfolds, so that if we separate these URSs from the rest of this row segment, the result is the segment 1032. This segment displays a jagged contour, since the direction in which the pitch classes within the URSs unfold differs from the direction in which the URSs themselves unfold. Here, each of the two URSs forms part of a different SRS; these SRSs are conjoined by the final pitch class of the second URS.

Adjacent URSs that unfold in different directions can display inertia of musical motion on more than one level, for example: The hypothetical initial row segment <0et1239845> contains four URSs, the first and third of which unfold counterclockwise and the second and fourth of which unfold clockwise. All four of these URSs belong to a single SRS, which is established by PC 0. The alternating directions of these URSs away from PC 0 lend this row segment the quality of a wedge-shaped compound melody, expressing clockwise and counterclockwise momentum simultaneously. This spatial profile mirrors that of the second twelve-tone row from Penderecki’s *Passion According to St. Luke*, which I cited at the beginning of this study as providing motivation for the development of the concept of spatial acquisition itself. This moti-
vation was precisely due to the striking directional inertia described above, made possible exclusively by the presence of multiple URSs.

As noted previously, gaps between SRSs—or the gap that stretches from one end of an SRS to the other in a row that contains only one SRS—can comprise from one to eleven pitch-class places. In this section, I will explore the ways in which gaps appear, reach critical status, and are closed over the course of a twelve-tone row’s presentation, as well as the importance of those events to the process of analyzing the spatial characteristics of twelve-tone rows in general.

Usually, I characterize each phase of a twelve-tone row’s unfolding as a snapshot displaying the presentation of one of the row’s pitch classes. From the standpoint of gaps, however, each phase represents a reduction by one pitch-class place of the amount of pitch-class space that has not yet been acquired by the row. These reductions can take three forms: a newly presented pitch class can either shrink, divide, or close a previously existing gap. The spatial function that a pitch class performs determines which role it plays. If a newly-presented pitch class functions as an expansion, then, from the perspective of gaps, it fills a pitch-class place that had been at the edge of an existing gap, and therefore shrinks it. On the other hand, if a newly-presented pitch class functions as an establishment, it fills a pitch-class place that had been within, and not at the edge of, a previously existing gap, and therefore divides it. A pitch class that functions as a conjunction serves simply to close an existing monadic gap.

A demonstration of the two forms of gap reduction—as opposed to gap closure—is as follows: If, in phase 5, PC 8 were added to the row segment <01t9>, it would shrink the existing gap between PCs 1 and 9 from seven pitch-class places to six. On the other hand, if PC 5 were
added to the same row segment, it would divide the existing gap between PCs 1 and 9 into two three-pitch-class-place gaps.

The relationship between establishments, conjunctions, and gaps mirrors the relationship between establishments, conjunctions, and SRS cardinality. Just as a pitch class that serves as an establishment creates a new SRS, it also creates a new gap by dividing a gap that previously existed. In turn, just as a pitch class that serves as a conjunction fuses two SRSs together, thereby reducing the number of existing SRSs at a particular point within the unfolding of a twelve-tone row by one, the same conjunction closes a monadic gap, reducing the number of a partially unfolded row’s existing gaps by one. Therefore, we can see that every twelve-tone row has the same number not only of establishments, conjunctions, and SRSs, but also of gaps. Based on previous findings within this study regarding establishments, conjunctions, and SRSs, this means that there can be from one to six gaps that are created over the course of a twelve-tone row’s unfolding. Like the number of a twelve-tone row’s existing SRSs, the number of a twelve-tone row’s existing gaps is not affected by expansions, since each expansion shrinks, rather than divides, an existing gap.

Whereas the spatial narrative of any twelve-tone row regarding SRSs involves the establishment of from one to six SRSs, the possible expansion of some or all of them, and the inevitable conjunctions between each of them, the gap-oriented narrative is as follows: A single eleven-pitch-class-place gap is created, which may gradually be divided into two to six smaller gaps; whatever the number of gaps, some or all of them may be shrunk, and all will eventually be closed. As the various gaps among a twelve-tone row’s SRSs are divided and/or shrunk, they eventually reach a critical status. It is possible for several critical gaps to exist simultaneously during a portion of a row’s presentation, and it is also possible, even in the cases of rows that
contain a large number of SRSs—and, therefore, a large number of gaps—for no more than one critical gap to exist at any particular phase.

At this point, I will address the limitations on the number of critical gaps that can exist over the course of a twelve-tone row’s presentation as well as the range of row phases in which they can exist, beginning with dyadic gaps. As many as four dyadic gaps may appear over the course of a twelve-tone row’s presentation—as would occur by the fourth phase within the presentation of a hypothetical row that begins with the segment \(<0369>\)—it is also possible for a row to contain no dyad gaps in any of its phases. This is due to the fact that it is possible, whenever a three-pitch-class-place gap is present, for a newly-presented pitch class to divide this gap into two monadic gaps rather than shrinking it to a dyadic gap. An example of this scenario would occur in the presentation of a row that begins with the segment \(<042>\). Here, the trichord-sized gap created in phase 2 is divided into two monadic gaps in phase 3.

As for monadic gaps, as many as six can appear during the presentation of a twelve-tone row, as would occur by the sixth phase within the presentation of a hypothetical row that begins with the segment \(<02468t>\). As opposed to dyad gaps, which in certain spatial row forms do not present themselves at all, there is a minimum number of monadic gaps that must appear during the presentation of any twelve-tone row. In fact, this minimum number is equal to the total number of the row’s conjunctions—and, therefore, equal to the number of the row’s establishments, SRSs, and gaps in general as well—since, as established earlier, a conjunction’s role where gaps are concerned is to close them, and the only type of gap that can be closed by a single pitch class is a monadic gap.

The role that a newly presented pitch class plays as either “shrinker” or “divider” of an existing gap can have significant consequences. For example, if, in phase 6, PC 5 were added to
the row segment <01678>, it would shrink the gap that existed between PCs 1 and 6 from four pitch-class places to three. If PC 3 were added to the same row segment instead, dividing the existing gap between PCs 1 and 6 into two smaller gaps, the situation would be distinctly different. As was the case with the addition of PC 5, the remaining non-acquired pitch-class places between PCs 1 and 6 would, of course, number three. However, whereas the single three-pitch-class-place gap created by the addition of PC 5 would not be critical for the purposes of analysis, the monadic and dyadic gaps created by the addition of PC 3 definitely would be.

When a newly presented pitch class shrinks an existing non-critical gap, a dyadic gap can only result if that existing gap is three pitch-class places in size. In turn, in order for a monadic gap to result from the shrinkage of an existing gap, that existing gap would have to be a dyad gap. When a newly-presented pitch class divides an existing gap, the situation is more complex. If, at any time during the unfolding of a twelve-tone row, a gap from seven to eleven pitch-class places in size were to exist, a newly-presented pitch class that appears within and not at the edge of that gap—i.e. a newly-presented pitch class that functions as an establishment, not an expansion—would divide the existing gap into either two smaller but non-critical gaps or one critical gap—either dyadic or monadic—and one non-critical gap. For example, given a hypothetical row that begins with the segment <01t9>, if the pitch class presented in phase 5 were PC 5, the seven-pitch-class-place gap that existed in phase 4 would be divided into two non-critical three-pitch-class-place gaps. If the newly-presented pitch class in phase 5 were PC 6 or PC 4, the result would be a non-critical four-pitch-class-place gap and a dyad gap. Finally, if the newly-presented pitch class in phase 5 were PC 7 or PC 3, the result would be a non-critical five-pitch-class-place gap and a monadic gap.
In the above example, we can see that, for the five pitch classes—3, 4, 5, 6, and 7—that can divide the gap between PCs 1 and 9, the results that the presentation of each of those pitch classes would bring about are arrayed in a symmetrical fashion in pitch-class space: The presentation of PC 5 results in two non-critical gaps, the presentation of PC 6 or 4 results in a non-critical gap and a dyadic gap, and the presentation of PC 7 or 3 results in a non-critical gap and a monadic gap. Figure 1.12 displays this symmetrical pattern of gap-division results for gaps that are seven pitch-class places or larger in size.

Figure 1.12: The division of gaps from seven to eleven pitch-class places in size

<table>
<thead>
<tr>
<th>Existing gap size</th>
<th>Number of pitch classes each of which could divide the existing gap into:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>two non-critical gaps</td>
</tr>
<tr>
<td></td>
<td>and one dyadic gap</td>
</tr>
<tr>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
</tr>
</tbody>
</table>

Gaps that are smaller than seven pitch-class places in size are distinct from larger gaps where gap division is concerned because dividing a gap that comprises six pitch-class places or fewer—as opposed to dividing a larger gap—will always produce at least one critical gap. Furthermore, dividing gaps that are three or four pitch-class places in size will exclusively produce critical gaps. The possible results that the division of gaps of these sizes can bring about are as follows: In the case of an existing gap that is six pitch-class places in size, a newly presented pitch class could divide it into either a non-critical gap of three pitch-class places and a dyadic gap or a non-critical gap and a monadic gap. Five-pitch-class-place gaps also present two divi-
sion-related possibilities: A newly presented pitch class could either divide the existing gap into a three-pitch-class-place—and therefore non-critical—gap and a monadic gap, or it could divide the existing gap into two dyad gaps. The division of a four-pitch-class-place gaps results exclusively in the creation of one dyadic gap and one monadic gap, whereas a newly-presented pitch class could only divide a three-pitch-class-place gap one way: into two monadic gaps. This completes the range of gap sizes within which division can occur; smaller gaps than these can only be shrunk or closed. Figure 1.13 lists the distribution of possible gap-division results for existing gaps that are from three to six pitch-class places in size.

Figure 1.13: The division of gaps from three to six pitch-class places in size

<table>
<thead>
<tr>
<th>Existing gap size</th>
<th>Number of pitch classes each of which could divide the existing gap into (see key below):</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Key:
A: one non-critical gap and one dyadic gap
B: one non-critical gap and one monadic gap
C: two dyadic gaps
D: one dyadic gap and one monadic gap
E: two monadic gaps

At a certain point in the unfolding of any twelve-tone row, its last non-critical gap disappears. From this point on, it is impossible for a new SRS to be established; the row’s remaining pitch classes can only function as either expansions or conjunctions. This being the case, the phase in which a newly-presented pitch class shrinks or divides the last non-critical gap within the row to which it belongs, an event that I term critical-gap saturation, is important for analytical purposes. Critical gap saturation can occur as early in a row’s unfolding as phase 4; this
would occur only in the case of a row that begins with a member of set class (0369), the only tet-
trachordal set class that includes no gaps larger than two pitch-class places among its member
PCs. The latest point at which critical gap saturation could occur is phase 10; this is due to the
fact that only in phases 10, 11, and 12 of a twelve-tone row’s presentation are there too few non-
acquired pitch-class places remaining for a non-critical gap—i.e. a gap of at least three pitch-
class places—to exist.

Another analytically important moment in the unfolding of a twelve-tone row is the phase
in which monadic-gap saturation, or the elimination of all but monadic gaps, occurs. From this
point on, new SRSs cannot be established and existing SRSs cannot be expanded; the row’s re-
maining pitch classes can only serve as conjunctions. The conditions in which monadic-gap sat-
uration can occur as early or as late as possible are similar to the conditions in which critical-gap
saturation can occur as early or as late as possible. The earliest point at which monadic-gap satu-
ration can occur is phase 6, which is only possible in the case of a row that begins with a member
of set class (02468t), the only hexachordal set class that includes no gaps larger than one pitch-
class place among its members. The latest point at which monadic-gap saturation can occur is
phase 11, since a gap of more than one pitch-class place can exist as late as phase 10 of a twelve-
tone row’s presentation.

In light of the preceding findings, phases 4 through 11 emerge as a window within the
unfolding of a twelve-tone row within which some type of gap saturation can occur, and phases 6
through 10 define the window within which both critical- and monadic-gap saturation can occur.
Both types of gap saturation tend to occur toward the later phases of each of these windows, un-
less a twelve-tone row opens either with a tetrachordal set that is derived from the set class
or with a hexachordal set that is derived from the set class (02468t), thereby displaying either type of gap saturation at the earliest possible phase.

We have already seen that, for the purposes of analysis, the most salient events in the course of a twelve-tone row’s acquisition of pitch-class space have to do primarily with the structure of that row’s SRSs and the roles that individual pitch classes play in their unfolding. The study of gaps provides us with conversely related information that is just as salient, such as:

1) critical-gap saturation—the point at which it is no longer possible for establishments to occur—
2) monadic-gap saturation—the point at which it is no longer for expansions to occur as well—and
3) how long a monadic gap—along with the magnetism it exerts—exists in terms both of row phases and of musical time, delaying the conjunction of two SRSs. The conceptual balance between SRSs and gaps is of great value as it deepens our understanding of the dynamics at work in the process of twelve-tone spatial acquisition in general.

\[ \text{Row classes and spatial row profiles} \]

As mentioned earlier, there are 19,958,400 unique spatial row forms. I have found that the most logical way to classify them is according to the most basic spatial attributes of the pitch classes that constitute a twelve-tone row: their spatial functions.

Using the letter \( E \) to represent an establishment, the letter \( X \) to represent an expansion, and the letter \( C \) to represent a conjunction, I can use a series of twelve letters to display the sequence of spatial functions for any twelve-tone row. I call such a series a spatial function profile, and although it is a useful and concise format, there are too many possible permutations of Es, Xs, and Cs that could be found in such a profile—2188 to be exact—for it to be viable as a basis for row classification.
A better choice is what I informally termed an establishment/conjunction series earlier in this study. As this type of series displays only the sequence of a row’s establishments and conjunctions, it can comprise from two to twelve letters. There are only sixty-five possible unique sequences of this type that could apply to a twelve-tone row, making them an optimal basis for the spatial classification of twelve-tone rows; to refer to such a sequence, I use the term E/C profile.

Appendix B contains a list of row classes, each of which is defined by a particular E/C profile. Each row class is designated by three numerals in the format (x-y-z); I will explain the significance of these three numerals presently. The x refers to the SRS cardinality of the profile in question; therefore, its value is limited to the numbers 1 through 6. Because of this limited number of possible values, SRS cardinality is a useful value for organizing definitive spatial function profiles into manageable groups, although the number of definitive spatial function profiles in each of these groups is far from equal: Whereas, within the spatial function profile of any twelve-tone row, an establishment must occur first and, in the cases of multiple-SRS rows, two establishments must occur before the first conjunction does, there is only one definitive spatial function profile that applies to a single-SRS row—EC—and only one that applies to a row that contains two SRSs as well—EECC. On the other hand, the possible permutations of six establishments and six conjunctions that could occur during a twelve-tone row’s presentation cause there to be forty-two definitive spatial function profiles that apply to a row that contains six SRSs.

The ys in the aforementioned row-class format signify the number of establishment groups—and, by definition, the number of conjunction groups and reversals as well—for each definitive spatial function profile. This value allows us to observe, especially in a profile of a
row that contains a high number of SRSs, how intricate the narrative of SRS establishments and conjunctions within it is; additionally, it serves as a viable basis for determining subgroups of definitive spatial function profiles within the basic groups determined by SRS cardinality. With the use of these subgroups, the number of definitive spatial function profiles that belong to any particular subgroup is never greater than twenty. The zs merely number the definitive spatial function profiles within each subgroup according to a consistent permutational pattern, so that each one has a particular row-class name.

Figure 1.14 displays the number of row classes that belong to each subgroup—as determined by SRS and establishment-group cardinality—of definitive spatial function profiles. We can see that the subgroups that contain the greatest numbers of row classes are those, within any particular category of SRS cardinality, whose establishment-group cardinality is not extremely low or high.

Figure 1.14: Number of row classes within each subgroup of definitive spatial function profiles

<table>
<thead>
<tr>
<th>SRS Cardinality</th>
<th>Establishment-group cardinality</th>
<th>Number of row classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>
It is important to note the interaction between E/C profiles and spatial function profiles, i.e. between those profiles that list only establishments and conjunctions and those that list expansions as well. For any E/C profile—and, in turn, for the row class that profile represents—there may be one, forty-five, or 210 spatial function profiles that could be built upon it. The numbers forty-five and 210 are derived from the number of permutations in which a multi-SRS row’s establishments and conjunctions, from its second establishment to its second-last conjunction, may be arrayed among the row’s second through eleventh phases. The row’s first establishment and its last conjunction are not considered, since they must always occupy, respectively, the row’s initial and final phases. Figure 1.15 displays the total possible number of combinations of establishments and conjunctions that can exist within a row’s spatial function profile, broken up according to the six possible SRS cardinalities.

Figure 1.15: The number of possible combinations of establishments and conjunctions within a spatial function profile

<table>
<thead>
<tr>
<th>SRS cardinality</th>
<th>Number of possible combinations of establishments and conjunctions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>45</td>
</tr>
<tr>
<td>3</td>
<td>210</td>
</tr>
<tr>
<td>4</td>
<td>210</td>
</tr>
<tr>
<td>5</td>
<td>45</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>

If we multiply the numbers of these combinations by the number of row classes that exist within each of the six categories of SRS cardinality, we can determine the number of spatial function profiles that exist within each of those categories. These numbers are displayed in Figure 1.16.
The data in Figure 1.16 make two things clear: 1) There are significantly more spatial function profiles that define four SRSs than define any other number of SRSs, and 2) all but eighty-eight of the 2188 unique spatial function profiles define three, four, or five SRSs. This does not necessarily mean that twelve-tone rows with one, two, or six SRSs are commensurately rare, however, since a multitude of unique rows can share any particular spatial function profile.

At this point, I will prepare the ground for the complete analysis of both twelve-tone rows and the musical works that are built upon them by demonstrating how data within all four categories of a twelve-tone row’s spatial characteristics—the spatial functions of its constituent pitch classes, its spatial row segments, its unidirectional row segments, and its gaps—can be coordinated into a single analytical format, which I term a spatial row profile. This profile takes the form of a table, the horizontal axis of which represents the various phases of a row’s unfolding and the vertical axis of which represents the four aforementioned categories of spatial row characteristics, organized into a spatial function profile, an SRS profile, a URS profile, and a gap profile. Figure 1.17 displays the spatial row profile for the row from Schoenberg’s Phantasie for Violin as an example. An explanation of the abbreviations and symbols used in such profiles follows.
Figure 1.17: A sample spatial row profile for row 0e317962t584

The row (row class (4-1-1)):
0 e 3 1 7 9 6 2 t 5 8 4

Spatial function profile:
E X E E X E E C C X C C

SRS profile:
1 1cc 2 1c 3 4 3cc 1c2cc 4c1cc 3cc 3c4cc 2c3cc

URS profile:
I T

Gap profile:
1d2 1m2 3m4m1 2d3 2m3

The notations within the spatial function profile require no explanation, as I have already used the abbreviations E, X, and C to represent SRS establishment, expansion, and conjunction, the three spatial functions that the constituent pitch classes of a twelve-tone row can perform. The signifiers within the SRS profile below it identify the SRSs to which each of the row’s pitch classes belongs and provide information related to SRS contour as well. Since notations within an SRS profile vary depending on whether they pertain to establishments, expansions, or conjunctions, I will provide separate explanations of them for each of those situations.

Below each E, I notate a numeral representing the SRS that is being established. For the first E within a given profile I notate a “1,” for the second I notate a “2,” and so on. These numerals, in effect, become the names for the SRSs within the twelve-tone row being analyzed, such as “SRS 1” or “SRS 2.”

Below each X, I notate the numeral that represents the SRS to which the X belongs accompanied by a one- to two-letter suffix that represents the direction in which the expansion in question leads away from the SRS’s establishment. This suffix is either c, representing “clock-
wise,” or cc, representing “counterclockwise.” For example, the above row begins with the segment <0e31> and its SRS profile begins with the corresponding sequence 1, 1cc, 2, 1c. These sequences make it clear that PC e expands the SRS established by PC 0 in a counterclockwise direction and that PC 1 expands the same SRS in a clockwise direction. This format makes it easy to identify the contour-based patterns that SRSs exhibit as they unfold.

Below each C, I make a double entry, since each of a twelve-tone row’s conjunctions belongs to two SRSs. Both parts of this double entry follow the same form as the notation under each X, for example: The first conjunction within the above row is PC 2, which is accompanied in the row’s SRS profile by the entry 1c2cc. This entry indicates that, as PC 2 conjoins SRSs 1 and 2, it also expands SRS 1 in a clockwise direction and SRS 2 in a counterclockwise direction.

In the URS profile, I use the letter I, X, or T to label a pitch class that performs one of the three URS functions, initiation, extension, and termination. Since a URS is by definition a subset of only one SRS and unfolds unidirectionally—so that its direction is made clear by the pitch classes within the row that constitute it—no elaboration of the URS functional signifiers is necessary. As opposed to the signifiers within the spatial function and SRS profiles, which always number twelve, there are not necessarily as many URS signifiers as there are pitch classes in the twelve-tone row to which it pertains; in fact, it is possible for no URS profile to exist within a spatial row profile at all, since twelve-tone rows do not automatically contain any number of URSs. Gap profiles also may contain fewer than twelve signifiers, but must contain at least as many as the row contains conjunctions, whose number parallels that of monadic gaps that exist within the row. The following is an explanation of gap-profile symbology.

Signifiers within gap profiles only exist in those phases in which non-critical gaps are shrunk or divided to the size of dyadic and/or monadic gaps, including cases in which a newly
presented pitch class shrinks a dyadic gap to monadic-gap size. The maximum number of gap-related notations that a twelve-tone row’s spatial structure can warrant is seven; an example of a twelve-tone row in which this number of notations would occur is 0369147258e. The presentations of the second, third, and fourth pitch classes of this hypothetical row each establish a dyadic gap, and the presentations of the row’s fifth, sixth, seventh, and eighth pitch classes each shrink one of those dyadic gaps so that it becomes a monadic gap.

The form of gap-related notations within spatial row formats is as follows: Either the letter \textit{d}, representing “dyadic,” or the letter \textit{m}, representing “monadic,” is framed by numerals representing the two SRSs that surround the gap, in clockwise order. For example, PC 3 in the above row is accompanied in the row’s gap profile by the entry 1d2. This signifies that the occurrence of PC 3 causes a dyadic gap to be present between SRSs 1 and 2. When a pitch class divides a previously existing gap into two critical gaps, the corresponding entry in the gap profile reflects this fact. For example, PC 9 in the above row, as it establishes SRS 4, also divides a three-PCP gap between SRSs 3 and 1 into two monadic gaps; therefore, the gap-profile entry that corresponds to PC 9 is 3m4m1.

I will save a demonstration of how a spatial row profile works in the process of analysis until the upcoming discussion of phase-by-phase spatial data is complete. Even without such a demonstration, it is clear that the schematic coordination of data that is facilitated by the use of a spatial row profile yields a great amount of analytical information for any twelve-tone row.

\textit{Phase-by-phase spatial data}

At each phase of a twelve-tone row’s presentation, the pitch classes that have been presented form a pitch-class set the cardinality of which is equal to the number of the phase itself,
with each of these pitch-class sets being a member of a particular set class. For the purpose of analyzing the sequence of twelve set classes formed in this way by any twelve-tone row over the course of its presentation, Appendix C provides a list of set-class types for the purposes of spatial acquisition theory. This list is based on both the standard list of set-class types, as defined by their prime forms, and various data that are important from a spatial perspective. I represent set classes in this list not with numerical names *per se*, as I represented row classes earlier, but with the prime forms of the set classes themselves.

The list of set classes that I have compiled differs with the familiar list that is commonly used to examine the intervallic structure and content of pitch-class sets in two general ways, the first of which is that the cardinalities of the set classes included within it extend from one to twelve. This is due to the fact that, although in the world of pitch-class set analysis the monad, dyads, decachords, the undecachord, and the dodecachord are not of particular value, mainly because of the limited number of their intervallic forms, their inclusion is useful to this study because of the comparison of their spatial properties to those exhibited by set classes of other cardinalities that this inclusion makes possible.

The second and most obvious main way in which the list of set classes in Appendix C differs with the standard list is in terms of the data that accompany each set class. As opposed to the familiar indications of interval content and levels of symmetry that the standard list provides, the data in the list given here relate specifically to the SRS, URS, and gap content of twelve-tone rows at various phases of their presentation. The following is an explanation of these data and their importance to the spatial analysis of twelve-tone music.

The left-hand vector accompanying each set class, in the format \(<x,y>\), displays the minimum—\(x\)—and maximum—\(y\)—number of partial or complete SRSs that can possibly exist with-
in an initial row segment whose constituent pitch classes form a member of the set class in ques-
tion. The minimum number is most easily calculated by noting the number of gaps, in pitch-
class space, within the prime form that represents the set class, for example: The prime form
(012457) contains three gaps: between PCs 2 and 4, 5 and 7, and 7 and 0. Since, over the course
of a twelve-tone row’s presentation, every gap that appears will eventually be filled by a con-
junction, and conjunctions form the borders between SRSs, there must be at least three SRSs
within an initial row segment that forms a member of this set class. For example: Given a
twelve-tone row whose first six pitch classes form a member of set class (012457), at least three
SRSs would have to be established by phase 6 of its presentation, as would be the case if the ini-
tial segment of the row were indeed <012457>, the spatial function profile of which would be
EXXEXE.

It would not be correct to assume, however, that there could only be three SRSs within an
initial row segment that forms a member of set class (012457). In order to determine the maxi-
mum number of SRSs that could exist within an initial row segment that forms a member of a
particular set class, it is necessary to examine what I have termed the contiguous-pitch-class sub-
sets (CPCSs) that exist within members of that set class. These are subsets of a pitch-class set
whose member pitch classes are contiguous in pitch-class space. For example, the prime form
(01346) contains two CPCSs, comprising the dyads [0,1] and [3,4]. The way in which CPCSs
affect the number of SRSs that an initial row segment that forms a member of a particular set
class can contain is explained below, using the set class (023457).

Two possible initial row segments that one could build from a member of set class
(023457) are <023457> and <025437>. The former, with a spatial function profile of EEXXXE,
contains three partial SRSs, whereas the latter, with a spatial function profile of EEEXCE, con-
tains four. This difference is facilitated by the presence of the CPCS [2,3,4,5] within the prime form of that set class; a more linear ordering of the pitch classes that constitute such a CPCS results in fewer SRSs being present in the resulting initial row segment and a less linear ordering results in more SRSs being present. We can determine the difference between the minimum and maximum numbers of SRSs that an initial row segment that forms a member of a particular set class can contain by using the following formula: for each CPCS that the prime form of the set class in question contains, given that \( n \) = the number of pitch classes that make up the CPCS, \((n-2)/2\), rounded up to the nearest whole number if necessary. This applies to the above examples as follows: Since the prime form of set class (023457) contains three gaps, the minimum number of SRSs that an initial row segment that forms a member of that set class can contain is three; however, the presence of a four-pitch-class CPCS within that prime form makes it so that the maximum number of SRSs that that initial row segment can contain is one greater than the minimum, or four.

The centrally-located vector that accompanies each set class displays the URS-related possibilities for that set class. The three numerals within each of these central vectors, in the format \(<x,y,z>\), represent the number of CPCSs that exist within any member of the set class that it corresponds to—\(x\), the maximum possible number of URSs that can exist within an initial row segment that forms a member of that set class—\(y\), and the number of pitch classes that comprise the CPCSs that exist within any member of that set class—\(z\). I have used these particular data and put them in this order to illustrate, for each set class, the minimum and maximum numbers of URSs that would result—and how many pitch classes they would contain—if one were to construct an initial row segment from a member of that set class using the maximum possible number of pitch classes to construct URSs. In such an endeavor, \(x\) would represent the minimum
possible number of URSs that would exist within the segment, \( y \) would represent the maximum possible number of URSs that would exist within the segment, and \( z \) would represent the number of pitch classes that these URSs would comprise, whether there were \( x \) URSs, \( y \) URSs, or some number in between. The following example illustrates how this works: The URS vector for the set class represented by the prime form \( (01235) \) is \( <1,2,4> \). If one uses all four of the pitch classes within this prime form’s CPCs, \([0,1,2,3]\) for URS construction, one can either construct one URS, as in the initial row segment \( <01235> \), or two, as in the initial row segment \( <23015> \). On the other hand, it is perfectly possible to construct a row segment from a member of this set class that contains no URSs at all; such a segment is \( <03152> \).

The right-hand vector that accompanies each set class addresses gap content. The numerals within this vector, in the format \( <x,y> \), represent the number of dyadic—\( x \)—and monadic—\( y \)—gaps that exist within the prime form of the set class to which it corresponds. If gap saturation exists within a prime form, it is noted with a \( c \) following the vector to denote critical-gap saturation or an \( m \) to denote monadic-gap saturation.

At each phase of a twelve-tone row’s presentation, the pitch class that occurs during that phase and the pitch classes that have already occurred form a member of a particular set class. When the twelve set classes formed in this way are taken together as a series, the result is what I have termed a *phase profile*, which is designed to be utilized in conjunction with the list of set classes in Appendix C. The names of the pitch classes within the row that have occurred prior to a particular phase along with the name of the pitch class that occurs within it are underlined and in boldface within the spatial diagram that corresponds to that phase. The row segment that has been presented by the time that phase occurs and the set class that it forms are presented below.
the spatial diagram. Figure 1.18 displays a sample phase profile, once again using the row from Schoenberg’s *Phantasie*.

Figure 1.18: A phase profile for row 0e317962t584

![Diagram of phase profile](image-url)
The phase representations within a phase profile that are the most salient for analytical purposes are those that display set classes that possess unusual spatial characteristics, and therefore can represent turning points in the row’s spatial narrative as well as the degree of spatial
usualness or unusualness that a row displays at any particular phase. For example, utilizing the
list of set classes in Appendix C and the accompanying data vectors, we can see that a row
whose first five pitch classes form a member of set class (01356) could only contain three SRSs
by phase 5, whereas if its next two pitch classes cause it to form a member of set class (013568t)
by phase 7, only five SRSs could exist within it at that point. Therefore, any row constructed in
that way could be said to have a spatial row profile that preordains the appearance of establish-
ments in phases 6 and 7, not to mention conjunctions in its last five phases, which are needed to
close the five SRSs that were established through phase 7. This finding provides a way to relate
rows that may be unique intervallically but form members of the same set classes at certain phas-
es.

An example of a row for which its phase profile would illuminate its URS content would
be a row whose first eight pitch classes form a member of the set class (0124568t), such as
<0452186t>. It might seem unremarkable that this segment contains two URSs, but the fact that
two is the maximum number of URSs that this set class could possibly contain would likely be
an important facet of a spatial analysis of that row.

The narrative of a twelve-tone row’s critical-gap appearances, closures, and saturation is
often revealed by the row’s phase profile more than the narrative of any of its other spatial fea-
tures is. For instance, given a row whose first seven pitch classes form a member of set class
(0123569) and whose first ten pitch classes form a member of set class (0123456789), the row’s
phase profile would make it evident that, while the row reaches critical-gap saturation at phase 7,
a statistically normal time to do so, it does not reach monadic-gap saturation until phase 11, the
latest possible phase at which that could occur. The unique lateness and quickness with which
the row’s final gap is shrunk to monadic size and filled would be a marked feature among
twelve-tone rows in general. Findings such as these, when combined with those facilitated by a spatial function profile, form a collection of information that makes the full analysis of a twelve-tone row and the music based upon it possible.

The analytical process

Now that I have presented all of the elements of the theory of spatial acquisition, I will give a demonstration of their use in the process of musical analysis. In order to show the amount and variety of information pertaining to spatial acquisition theory that a twelve-tone composition can potentially contain, I have composed two musical examples, the pitch-class organization of which is based on an interaction between established twelve-tone theoretical concepts and spatial acquisition theory. As an introduction to the analytical use of spatial acquisition theory, the present section will address only those two musical examples; in the second part of this study that follows, I will present analyses of excerpts from a variety of actual and well-known twelve-tone musical works.

In the present section, as in the analyses within the second part of this study, I will follow the following format: I will provide an analysis of the work’s twelve-tone row according to established twelve-tone theoretical principles, as generally described at the beginning of this study, followed by an analysis of the row according to the principles of spatial acquisition, noting the ways in which the two analyses complement each other. Following the row-based analysis, I will explore the musical material itself, including the disposition of aggregates and pitch-class sets that may or may not function as transformations of either the row or any of the row’s subsets, again comparing and contrasting findings that are based on established theory with those that are based on spatial acquisition theory.
The hypothetical musical work from which the examples in this analytical presentation come is a string quartet based on the twelve-tone row 0278163t549e. Perhaps the most obvious characteristic of this row is that it is derived from the set class (027). In turn, its discrete hexachords belong to the set class (012678), which is self-complementary at two levels both transpositionally and inversionally as well as being symmetrical at two levels both transpositionally and inversionally. These characteristics make it possible for this row to exhibit two levels each of prime, inversional, retrograde, and retrograde-inversional hexachordal combinatoriality, not to mention combinatoriality at the trichordal level.

The ordered interval series for this row is <2, 5, 1, 5, 9, 7, 7, e, 5, 2>. This series is not symmetrical overall; however, there are symmetrical segments within it: The first two and last two intervals in the series—<2, 5> and <5, 2>—are retrogrades of each other, while intervals 3 through 5 and intervals 7 through 9—<1, 5, 5> and <7, 7, e>—have a retrograde-inversional relationship. Therefore, the row segments to which these segments of the row’s interval series correspond will have the following relationships: the row segments <027> and <49e> are retrograde inversions of each other, while the row segments <7816> and <3t54> are retrogrades of each other. The pitch classes 7 and 4 belong to both sets of row segments; in order to determine a set of symmetrical row segments that is balanced in terms of size, we could choose to consider PCs 7 and 4 to belong only to the retrograde-inversionally-related segments. By virtue of this segmentation, the row’s discrete trichords are bound by symmetrical relationships in addition to their membership in a particular set class, as shown in Figure 1.19.
Figure 1.19: Trichordal symmetry within row 0278163t549e

```
The row:   (0   2   7)   (8   1   6)   (3   t   5)   (4   9   e)
Ordered interval series:     2   5    1    5    5    9    7   7    e    5   2
```

Grouping these “inner” and “outer” trichordal sets with each other reveals an additional set-class association: The outer trichords combine to form the noncontiguous row segment 02749e and the inner trichords combine to form the segment 8163t5, each of which belongs to the set class (024579), the diatonic hexachord. The differences in interval content between this set class and the set class to which this row’s discrete hexachords belong, (012678), is striking, as, on the one hand, set class (024579) constitutes an unbroken segment of the circle of fifths, containing only one member of interval class 1 and no tritones, and, on the other, set class (012678) contains four members of interval class 1 and three tritones. The difference in interval content between these two set classes allows for chromaticism and diatonicism to be starkly juxtaposed on the musical surface. The spatial structure of this row reinforces these segmentations; Figures 1.20 and 1.21 display its spatial row and phase profiles.
Figure 1.20: The spatial row profile for row 0278163t549e

The row (row class (4-2-1)):

```
0 2 7 8 1 6 3 t 5 4 9 e
```

Spatial function profile:
```
E E E X C X X E X C C C
```

SRS profile:
```
1 2 3 3c 1c2cc 3ce 2c 4 3cc 2c3cc 3c4cc 4c1cc
```

URS profile:
```
I T
```

Gap profile:
```
1m2
```

```
2d3 3m4m1 2m3
```
Figure 1.21: The phase profile for row 0278163t549e

Phase 1
Row segment: 0
Set class: (0)

Phase 2
Row segment: 02
Set class: (02)

Phase 3
Row segment: 027
Set class: (027)

Phase 4
Row segment: 0278
Set class: (0157)

Phase 5
Row segment: 02781
Set class: (01267)

Phase 6
Row segment: 027816
Set class: (012678)
It is important to note that, although this row itself is not symmetrical in terms of its ordered interval series, it is in terms of spatial functions, as its spatial function profile reveals.
Therefore, the spatial function profile for this row will remain invariant no matter what row forms are utilized over the course of the composition. It should be noted that, since any row derived from the set class (027) automatically begins with three establishments and end with three conjunctions, it is the spatial structure of the middle phases of this row that determines the symmetry of this row’s spatial function profile.

If we isolate the pitch classes within this row that serve as establishments, expansions, or conjunctions, we find that they are subsets of the diatonic collection. Additionally, the pitch classes within this row that constitute the row’s two URSs—7, 8, 5, and 4—mark the boundaries between the aforementioned “outer” and “inner” interval-class-5-saturated hexachords, reinforcing an emphasis on set class (024579) rather than set class (012678).

Turning our attention to this row’s SRS makeup, as shown in Figure 1.22, we can see that SRS 3 has a markedly large cardinality of six, in comparison to the cardinality of SRSs 1 and 4—three—and the cardinality of SRS 2—four. In addition, SRS 3 also contains both of the row’s URSs. An investigation of the contours of each of these four SRSs reveals that they are balanced in terms of the directions in which they unfold—two initially unfolding clockwise and one initially unfolding counterclockwise—and that SRS 3 displays two changes of direction, further distinguishing it from the other SRSs, each of which display one change of direction.

Figure 1.22 : The content and contour of the SRSs within row 0278163t549e

<table>
<thead>
<tr>
<th>SRS</th>
<th>SRS content</th>
<th>SRS contour</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRS 1</td>
<td>01e</td>
<td>&lt;120&gt;</td>
</tr>
<tr>
<td>SRS 2</td>
<td>2134</td>
<td>&lt;1023&gt;</td>
</tr>
<tr>
<td>SRS 3</td>
<td>786549</td>
<td>&lt;342105&gt;</td>
</tr>
<tr>
<td>SRS 4</td>
<td>t9e</td>
<td>&lt;102&gt;</td>
</tr>
</tbody>
</table>
Notable about this row’s gap profile is how quickly, after the row’s initial critical gap appears in phase 2 and is closed in phase 5, three other critical gaps appear and are closed. Whereas prior to phase 7 the row contains no critical gaps at all, by phase 9 there are three monadic gaps. Phases 10 through 12 are spent closing them. It could be said that the markedly distinct roles that phases 7 through 9 and 10 through 12 play in terms of critical gap appearance and closure further reinforce the parsing of this row into discrete trichords. A musical situation whose analysis would necessitate this gap-related insight into the spatial structure of this row would be one in which the suddenness and quickness with which these gaps appear and are closed is manipulated, such as a case in which the seventh, eighth, and ninth pitch classes of the row are presented repeatedly but the presentation of the row’s final three pitch classes is delayed.

The main insight provided by this row’s phase profile regards the set class—(012678)—to which its discrete hexachords belong. It is one of only four hexachordal set classes that contain no critical gaps. By virtue of this fact, a row that contains four SRSs, as this one does, that is built from this set class will have a dearth of gaps in its middle phases; its gaps will open and be closed only in either its early phases, its late phases, or both, as is the case here.

The musical examples from the hypothetical string quartet that follow demonstrate some ways in which the characteristics of row 0278163t549e, as revealed by both established and spatial theoretical principles, could shed light on music that is based upon it. Figure 1.23 pertains to the subsets of the diatonic collection into which the row easily parses.
The registral and rhythmic profiles of this example reveal a distinction between the brief, melodious statements of the first violin and the more static material of the other three voices. This distinction is borne out in an analysis of the pitch classes used in this example as they relate to the work’s twelve-tone row: While the lower three voices combine to make one statement of \( P_0 \), the first violin presents a rotated version of the row by itself, beginning with the row’s fourth pitch class and ending with its third. This rotation, divided by rests into two statements of six pitch classes each, clearly emphasizes the row’s diatonically oriented “inner” and “outer” hexachords. As for the lower three parts, the cello’s initial three and the second violin’s final three pitch classes outline the outer hexachords, whereas the inner hexachords are represented by the material that is positioned between those statements.
The spatial functions performed by the pitch classes sounded by the lower three instruments are clearly distinguished. Simply put, the cello presents all of the establishments—PCs 0, 2, 7, and t—the second violin presents all of the expansions—PCs 8, 6, 3, and 5—and the viola presents all of the conjunctions—PCs 1, 4, 9, and e. Among these pitch classes, the ones that sound simultaneously—[7, 8, 1] and [4, 5, t]—represent the only segments of the row in which the three spatial functions occur in succession, occurring in the order E, X, C in each case.

This excerpt demonstrates a way in which an overt emphasis on a particular hexachoral segmentation of the row, based mainly on the derivation of those hexachords from a particular trichordal set class, can be combined with a segmentation based primarily on spatial functions. Figure 1.24 displays a musical excerpt that addresses the issues mentioned in the above discussion of this row’s gap profile, and reflects this row’s URS content as well.

Figure 1.24: Musical example no. 2
In this excerpt, the first violin opens with the first nine pitch classes of the $P_6$ version of the work’s row, while the cello part begins with the first nine pitch classes of $R_0$. Meanwhile, the inner voices sustain the pitch classes with which the outer voices began, 6 and e. From the standpoint of established theory, this juxtaposition of row versions displays retrograde hexachordal and trichordal combinatoriality, guaranteeing that, although the outer voices are homorhythmic, they will not sound members of the same pitch class at the same time. According to spatial acquisition theory, because of the symmetry inherent in the layout of this row’s spatial functions, each establishment within one of the outer voices sounds at the same time as a conjunction in the other and vice-versa. The fact that the outer voices’ presentations of their row versions end at phase 9, leaving out the row’s final three conjunctions, becomes important in light of what follows.

The outer-voice sixteenth notes in m. 3 comprise all six conjunctions within the two row forms used in this example. Their appearance, following repeated soundings of the pitch classes that neighbor them in m. 2, close the gaps delineated by the pitch classes of m. 2 in dramatic fashion, as the pitch classes of m. 2 serve to heighten the magnetism that the row’s monadic gaps exhibit. An analysis based on established theoretical principles might explain the pitch classes present in m. 2 as a superset of the $(027)$ set class and a subset of the $(024579)$ set class: the pentatonic set class $(02479)$; however, the gap closure scenario described above and represented in Figure 1.25 would seem to be a more salient motivation for those particular pitch classes to be involved.
Figure 1.25: The pitch classes sounding in m. 2 versus those sounding in m. 3

Measure 2
(sounding pitch classes are underlined, the other pitch classes of the row are not)

\[
\begin{array}{cccccccc}
6 & 8 & 1 & 2 & 7 & 0 & 9 & 4 & e \\
\end{array}
\]

Measure 3

\[
\begin{array}{cccccccc}
t & 9 & 4 & 5 & t & 3 & 6 & 1 & 8 \\
\end{array}
\]

t, closing the gap between 9 and e
3, closing the gap between 2 and 4
5, closing the gap between 4 and 6
7, closing the gap between 6 and 8
2, closing the gap between 1 and 3
0, closing the gap between e and 1

One additional detail within this excerpt highlights the URSs that exist within each row form used in this example: The jagged pitch contour of the outer voices highlights the extremely high or low pitch classes that they present; in the first violin part these pitch classes are 1, 2, e, and t, and in the cello part they are 4, 5, 8, and 7. These eight pitch classes are indeed those that constitute URSs within the two row forms in this example. It is worth noting that in the R₀ version of this row, just as in the P₀ version, both URSs belong to the same SRS, which is the largest one within the row.

It is clear from this analytical demonstration that a spatial investigation of a twelve-tone row and the music based upon it can provide insight that is complementary to that provided by established twelve-tone theory. The questions at this point, then, are: 1) Do composers of actual twelve-tone music—as opposed to the hypothetical excerpts addressed in this section—utilize the spatial properties of twelve-tone rows and aggregates that are described in this study to any meaningful extent, either consciously or unconsciously, and 2) can the analytical process detailed and demonstrated above reveal information embedded within that music that is analytically useful? The second part of this study is intended as a first step in answering these questions.
Schoenberg, Suite for Piano, Op. 25

Although Arnold Schoenberg wrote a number of works from 1909 on that, by his account, were free “from the shackles of tonality,” the Suite for Piano, Op. 25, composed from 1921-23, was the first in which he used a twelve-tone serial compositional methodology. The result was that he was not reliant on “intuition or a text,” as he had largely been in the composition of his earlier atonal works, to determine the Suite’s form.¹ In this work, as in all subsequent twelve-tone serial works, the sequence of pitch-class intervals contained within the twelve-tone row remains invariant regardless of transposition, inversion, and retrogression and functions as a fundamental formal determinant.

Although Schoenberg takes many “liberties with the expectations of the Baroque dance forms”² in this work by the use of syncopated rhythms and frequently large melodic leaps, not to mention the use of a twelve-tone row to begin with, several features of the Suite hearken back to traditional compositional practice. The most evident of these are 1) the fact that each movement begins with an iteration of the P₄ version of Schoenberg’s row, and 2) the presence of a double drone in the Musette, which, instead of outlining a perfect fifth as a traditional dance drone

². Glenn Watkins, Soundings, 331.
would, outlines a tritone\textsuperscript{3}. In addition, the only levels of row transposition that Schoenberg used in the \textit{Suite} are 4 and 10. As the transpositional levels of these row forms differ by a member of interval class 6, the tritone is reinforced as a substitute for the perfect fifth or fourth that defines the relationship between the tonic and dominant scale degrees in a tonal work.

The twelve-tone row at the heart of this work, in $P_0$ form, is 01392e4t7856. The prominence of the tritone within the \textit{Suite} supports Monte Keene Pishny-Floyd’s assertion that tritone pairs are fundamental to the structure of this work’s twelve-tone row. In Pishny-Floyd’s analysis, he demonstrates that Schoenberg’s row contains two pairs of consecutive pitch classes that are separated by a tritone and, more saliently, that the opening and closing pitch classes of the row are separated by the same interval.\textsuperscript{4} Therefore, the opening pitch class of $P_0$ matches the closing pitch class of $P_6$ and vice-versa, as shown in figure 2.1. The same holds true, of course, for any two forms of this row that are transpositionally related by interval class 6.

Figure 2.1: Tritone relationships within and between $P_0$ and $P_6$

\begin{center}
\begin{tabular}{cccccccc}
$P_0$: & 0 & 1 & 3 & 9 & 2 & e & 4 & t & 7 & 8 & 5 & 6 \\
$P_6$: & 6 & 7 & 9 & 3 & 8 & 5 & t & 4 & 1 & 2 & e & 0 \\
\end{tabular}
\end{center}

\textsuperscript{3} Glenn Watkins, \textit{Soundings}, 329.

The two-year span in which Schoenberg composed the *Suite for Piano* is noteworthy, especially regarding the *Prelude*, which, along with the work’s *Intermezzo*, were written in 1921, while Schoenberg’s twelve-tone method of composition was still taking shape. The *Prelude* and *Intermezzo* seem to be not quite as “twelve-tone” as the rest of the work’s movements; the row in the *Prelude*, for example, often seems to be broken up into three separately malleable groups of four ordered pitch classes each.\(^5\) On the other hand, however, this sort of segmentation corresponds to what Schoenberg wrote about twelve-tone composition twenty years later: that, while he had “always … strictly observed” the “succession of tones according to their order in the set …, the set [can be] divided into groups … of six, … four, … or three tones, [in order] to provide a regularity in the distribution of the tones.”\(^6\) This tetrachordal segmentation of the *Suite’s* row proves to be related to the row’s spatial structure as well, as discussed below.

From the standpoint of established twelve-tone theory, Schoenberg’s row is derived from the set class (012346), the structure of which, like the structure of many of Schoenberg’s later twelve-tone rows, accommodates inversional combinatoriality, although that possibility is not realized here. An investigation of the *Suite’s* row in terms of symmetry suggests an uneven segmentation, in addition to the tetrachordal one described above. The segment of the row from order numbers 3 through 8, \(<392e4t>\), is symmetrical; the sequence of pitch-class intervals within this segment is 6, 5, 3, 5, 6. The row segments that precede and follow this one, \(<01>\) and \(<7856>\), are not only also symmetrical, but share the same axis of symmetry—6-7/0-1—with


segment <392e4t>. This two-, six-, and four-pitch-class segmentation is also strongly suggested by the row’s spatial data, as shown below.

The row segment <7856>, which belongs to both the tetrachordal and the uneven segmentations of the row described above, is significant as a retrograde of the BACH motto,7 which is also quoted in the movement Simbolo from Dallapiccola’s Quaderno musicale di Annalibera8 as well as the second twelve-tone row of Penderecki’s Passion According to St, Luke, as already discussed. The presence of this figure in multiple and varied twelve-tone works is evidence of the influence on composers of serial music that Bach, who Schoenberg wrote could be called “the first composer with twelve tones,” had.9 In addition to its historical import, the BACH motto also contributes to the distinguishing spatial features of Schoenberg’s row, as explicated beginning with the presentation of the row’s spatial function and phase profiles in figures 2.2 and 2.3.


Figure 2.2: The spatial row profile for row 01392e4t7856

The row (row class (4-2-2)):

\[
\begin{array}{cccccccccccc}
0 & 1 & 3 & 9 & 2 & e & 4 & t & 7 & 8 & 5 & 6 \\
\end{array}
\]

Spatial function profile:

\[
\begin{array}{cccccccccccc}
E & X & E & E & C & X & X & C & E & C & X & C \\
\end{array}
\]

SRS profile:

\[
\begin{array}{cccccccccccc}
1 & 1c & 2 & 3 & 1c2cc & 1ce & 2c & 3c1cc & 4 & 4c3cc & 2c & 2c4cc \\
\end{array}
\]

URS profile:

\[
\begin{array}{cccccccccccc}
I & T & I & T & I & T & I & T \\
\end{array}
\]

Gap profile:

\[
\begin{array}{cccccccccccc}
1m2 & 3d1 & 3m1 & 2d4m3 & 2m4 \\
\end{array}
\]
Figure 2.3: The phase profile for row 01392e4t7856

Phase 1
Row segment: 0
Set class: (0)

Phase 2
Row segment: 01
Set class: (01)

Phase 3
Row segment: 013
Set class: (013)

Phase 4
Row segment: 0139
Set class: (0236)

Phase 5
Row segment: 01392
Set class: (01236)

Phase 6
Row segment: 01392e
Set class: (012346)
As figure 2.2 demonstrates, this row belongs to row class (4-3), since its initial three establishments are followed by two conjunctions, which in turn are followed by the final estab-
lishment and the two final conjunctions. This is the only row covered by this study in which consecutive conjunctions occur before the row’s final establishment has been presented. The result is that, although by the fourth phase of this row three SRSs have already been established, by the eighth phase the number of gaps is down to one. This causes phase 8 of this row to possess a sense of internal closure, reinforcing the idea of dividing this row into segments of—in order—two, six, and four pitch classes. Further investigation into the second of these segments, namely <392e4t>, reveals that SRSs 2 and 3, which are established by the initial two pitch classes of this segment, are each conjoined to SRS 1 by PCs 2 and t, the latter of which closes this segment. The boundaries of this segment within the row as a whole are thereby defined by the establishment and conjunction of SRSs.

The three URSs contained within this row reinforce not the row’s SRS structure but rather the aforementioned segments <01>, <392e4t>, and <7856>, the last of these being the BACH motto. This is due to the fact that every pitch class of segments <01> and <7856> belongs to a URS and no pitch class within segment <392e4t> does. Further uniting segments <01> and <7856> as a combined counterpart to segment <392e4t> is the fact that all three of the URSs that those segments comprise unfold clockwise.

The pattern of gaps within this row further differentiates segments <01>, <392e4t>, and <7856>, but also serves to provide connections between the first and second and the second and third of those segments. Both critical gaps that are created—and closed—within the second segment border on SRS 1, which of course had been established in the first segment. On the other hand, both critical gaps that are created—and closed—within the third segment border on SRS 4, which exists entirely within the third segment. Among the four monadic gaps within this row, there is even a pattern among the lengths of the gaps’ existences that reinforces the importance of
gap creation and closure within segments $<392e4t>$ and $<7856>$. Both monadic gaps that appear in segment $<392e4t>$ are closed two phases after their creation, still within that segment, and the closure of the segment’s first monadic gap is followed immediately by the creation of the second. Both monadic gaps that appear in segment $<7856>$ are closed only one phase after creation, but, once again, the closure of the segment’s first monadic gap is followed immediately by the creation of the other. This means that, in every phase within segments $<392e4t>$ and $<7856>$, a monadic gap either exists or is being closed.

It is important to note that the two different segmentations of Schoenberg’s row that I have discussed are not conflicting, but complementary. Although not mentioned so far, a tetra-chordal partitioning of the row obtains from a spatial standpoint as well as from a traditional standpoint, since such a partitioning separates the row’s initial establishment and conjunction groups, and corresponds to the uneven segmentation in its treatment of the row’s final tetra-chord—the BACH motto—as a distinct unit. However, most of the row’s spatial data, as detailed above, favor the uneven segmentation.

The set classes that Schoenberg’s row forms at phases 8 and 10 of its presentation—(01234567) and (0123456789)—are notable for their compactness: (01234567) is the only octachordal set class that does not contain any critical gaps, and (0123456789) is the only decachordal set class that does not display monadic-gap saturation. These facts highlight the markedness of the fact that the final four pitch classes of this row, which form the BACH motto, are contiguous in pitch-class space.

Figure 2.4 contains the first five measures of the Suite’s prelude, in which we find tetra-chordal subsets belonging to row forms $P_{10}$, $P_{4}$, $I_{10}$, and $R_{10}$. The repeated Bbs, As, and Gs of mm. 3, 4, and 5 serve to bridge the last three of those row forms, as shown in figure 2.5 below.
Figure 2.4: The first five measures
Michael Friedmann has observed that Schoenberg often “simulates a sequential transition in tonal music by the repeated use of contour and durational units.”\textsuperscript{10} Accordingly at the beginning of the movement, tetrachords 1 and 2 of row forms $P_{4}$ and $P_{10}$ are presented with either the contour adjacency series $<+, -, +>$ or its inversion, so that the tritones that conclude those tetrachords are highlighted. Interestingly, if we apply the uneven segmentation of Schoenberg’s row that most of the row’s spatial data suggest to the $P_{4}$ version that opens this movement, its first eight pitch classes are parsed into an opening dyad and a segment that features tritones at its beginning and its end. The most obvious tritone relationship among different row forms that appear in this excerpt is transpositional, existing between the row forms that open the Prelude, $P_{4}$ and $P_{10}$. The prominent repetition of the G-Db dyad between these two row forms in mm. 1 and 2 makes use of the pitch-class invariance that results from this relationship.

The pitch-class content of the Prelude’s first two measures completes the chromatic ag-
aggregate, with six unique pitch classes occupying the first measure and the other six occupying the second. If we consider this aggregate to be an ordered series, it conforms somewhat to the 2-6-4 segmentation mentioned earlier, with the opening dyad in the left hand sounding alone to inaugurate the movement and invariant dyads comprising pitch classes 1 and 7 continually present as the next six pitch classes, 1 and 7 included, are presented. This composite series is derived from the set class (013679), within which it is only possible for four SRSs to exist, and which is critical-gap saturated. Therefore, as opposed to this work’s prototypical row, this series contains no spatial-function interruptions, with all of its establishments and all of its conjunctions occurring neatly in separate measures.

As the *Prelude* enters m. 4, its simultaneous pitch-class content comes only from one row form, so that aggregate completion happens more quickly than at the beginning of the movement. The tetrachordal segments of the I$_{10}$ version of the row that generally inhabit m. 4, when taken together, form a series that is derived from the set class (012367). The spatial function profile that this series exhibits is, like that of the previous aggregate-completion series, devoid of reversals, and is nearly symmetrical as well: EEEXCEXXCCC. In this musical section, PCs 1 and 7 are again important, as they form the row’s final two conjunctions. Dyadic-gap saturation in this series occurs later than in the main row, but monadic-gap saturation happens late in both of them, occurring in this aggregate-completion series only with the final simultaneity of m. 4.

The intervallic properties of the pitch-class content within m. 5 have a striking similarity to those of m. 4, although the organization of that content is quite different on the musical surface. The row form utilized is R$_{10}$ instead of I$_8$, but, as with the series from the previous measure, the set class from which this aggregate-completion series is derived is (012367). The spatial function profile of this series is a sort of middleground between the spatial function profiles of
the work’s main row and the series from m. 4: EEXEXCEXXCCC, which contains an interior reversal as does the main row although that reversal is brought about by only one conjunction rather than two. We can observe that the pitch-class series by which aggregate completion occurs in this section of the Suite reflect the spatial patterns inherent in the work’s main row, and in some ways complete the aggregate in a more orderly fashion than the row itself does.

_Berg, Violin Concerto_

This work, the last that Berg completed, is infused with traditional elements, such as folk song quotations and the chorale _Es ist genug_. In fact, the PCs 4, 6, 8, and t with which the row ends match the distinctive and recurring four-note ascending whole-tone motive within _Es ist genug_, as attested by Berg himself in a letter to Arnold Schoenberg at the time of the concerto’s composition.\(^\text{11}\) Berg’s row in its entirety, in \(P_0\) form, is \(037e2591468t\). In traditional terms, the initial nine pitch classes of this row comprise a chromatic chain of triads while the final four pitch classes form a segment, as mentioned above, of a whole-tone scale. Figure 2.6 reflects Craig Ayrey’s representation of the first eight pitch classes of Berg’s row as outlining the tonic and dominant triads in two minor keys whose tonics are separated by a whole step.\(^\text{12}\)

\(^{11}\) Juliane Brand, Christopher Hailey, Donald Harris, eds., _The Berg-Schoenberg Correspondence_ (New York: W. W. Norton & Co., Inc., 1987), 466.

Symmetry plays a large role in the organization of the *Violin Concerto*. Three of this work’s four movements display some type of arch form, and the row itself abounds with symmetry, as follows: The sequence of pitch-class intervals within this row is 3, 4, 4, 3, 4, 4, 3, 2, 2, 2. This sequence can be divided into three segments—3, 4, 4, 3; 3, 4, 4, 3; and 2, 2, 2—that are symmetrical, and the first and second of these segments can be combined into a symmetrical eight-interval segment: 3, 4, 4, 3, 4, 4, 3. Therefore, the following row segments, each of which contains one of these symmetrical interval-sequence segments, are symmetrical: <037e2>, <25914>, <037e25914>, and <468t>. The symmetrical relationship among the first three of these row segments is as follows: The axis of symmetry for segment <037e2> is 1/7, whereas the axis for segment <25914> is 3/9; the midpoint between these axes is 2/8, which is the axis for segment <037e25914>, and, by definition, the axis for the row segment comprising the remaining three pitch classes, <68t>.

An additional feature of Berg’s row that figures prominently in the *Violin Concerto* itself is that its structure can also be explained in terms of interval cycles, the use of which was a distinct part of Berg’s compositional process. If we isolate the first, third, fifth, seventh, and ninth


classes of Berg’s row—PCs 0, 7, 2, 9, and 4—we see that the ordered interval series for this partition is 7, 7, 7, 7, so that those pitch classes constitute a 7-cycle within the row. In turn, the second, fourth, sixth, and eighth pitch classes of this row—PCs 3, e, 5, and 1—form a 2-cycle, as do the row’s final four pitch classes.\textsuperscript{15} As is the case with the row’s first nine pitch classes overall, symmetry exists between the 7-cycle and 2-cycle partitions of those pitch classes: The axis of symmetry for the row segment \textless07924\textgreater and the row segment \textless3e51\textgreater is 2/8, the same axis that bisects the row segments that comprise the row’s first nine and the final three pitch classes. It is the various intervallic patterns within Berg’s row that I have described rather than segmentations of the row based on axes of symmetry that affect the row’s spatial features, as can be seen in the discussion that follows, beginning with displays of the row’s spatial function and phase profiles in figures 2.7 and 2.8.

Figure 2.7: The spatial row profile for row 037e2591468t

The row (row class (5-1-1)):

```
0 3 7 e 2 5 9 1 4 6 8 t
```

Spatial function profile:

```
E E E X X E E C C C C C
```

SRS profile:

```
1 2 3 1cc 2cc 4 5 1c2cc 2c4cc 4c3cc 3c5cc 5c1cc
```

URS profile:

(N/A)

Gap profile:

```
1d2 1m2 2m4m3 3m5m1
```
Figure 2.8: The phase profile for row 037e2591468t

Phase 1
Row segment: 0
Set class: (0)

Phase 2
Row segment: 03
Set class: (03)

Phase 3
Row segment: 037
Set class: (037)

Phase 4
Row segment: 037e
Set class: (0148)

Phase 5
Row segment: 037e2
Set class: (01348)

Phase 6
Row segment: 037e25
Set class: (013468)
Figure 2.8 cont’d.

Phase 7
Row segment: 037e259
Set class: (013468t)

Phase 8
Row segment: 037e2591
Set class: (0123468t)

Phase 9
Row segment: 037e25914
Set class: (01234568t)

Phase 10
Row segment: 037e259146
Set class: (012345678t)

Phase 11
Row segment: 037e2591468
Set class: (0123456789t)

Phase 12
Row segment: 037e2591468t
Set class: (0123456789te)

The most distinguishing spatial feature of this row is the fact that, as a member of row class (5-1-1), it contains five SRSs, more than any other row addressed in this study besides that
of Webern’s *Symphonie* Opus 21, and, unlike Webern’s row, this row’s establishments are presented without any reversals; i.e. all of this row’s establishments occur before any of its conjunctions do. The only two pitch classes that do not function either as establishments or conjunctions—6 and 9—are presented in two consecutive phases—4 and 5.

Because of the fact that this row is constructed as a series of ascending thirds and major seconds in pitch-class space, a cyclical pattern of presentation emerges when we consider the row in terms of its constituent pitch classes’ spatial functions. Since the ordered interval series that defines row 037e2591468t only contains intervals that are smaller than 5, the row can be seen as completing three clockwise trips around the chromatic cycle: The first involves pitch classes 0, 3, 7, and e, which function as the row’s first three establishments and the first expansion, the second involves pitch classes 2, 5, and 9, which function as the row’s final expansion and final two establishments, and the third involves pitch classes 1, 4, 6, 8, and t, which function as all five of the row’s conjunctions.

As is usually the case in rows containing a large number of SRSs, the five SRSs within row 037e2591468t are relatively uniform in terms of cardinality, each of them containing either three or four pitch classes. Because the acquisition of pitch-class space by this row is decidedly non-incremental, there is a high number of interruptions; in fact, there are no examples of SRS continuousness within this row until phase 9. Because the SRSs within this row are numerous, small, and generally not continuous, they do not play a large role for analytical purposes.

The Violin Concerto’s row is the only one covered by this study that does not contain any URSs, due mainly to the row’s high number of SRSs. In turn, the row’s high number of SRSs, like the discontinuousness of those SRSs, is due to the row’s aforementioned triadic and whole-tone basis.
Whereas the placement of the first four pitch classes of Berg’s row is notable in that only one critical gap exists by the row’s fourth phase, the fifth, sixth, and seventh phases of this row are notable in that they contain the creation of all five of the row’s monadic gaps. At phase 6, the row forms a member of set class (013468), which, like the majority of hexachordal set classes, does not display critical-gap saturation. However, the row in its seventh phase forms a member of set class (013468t), one of only three septachordal set classes to display not only critical- but monadic-gap saturation. This finding demonstrates how distinctive the shift is between phases six and seven of this row’s presentation in terms of its gap profile. It should also be noted that, from this row’s seventh phase on, the minimum number of SRSs possible for the set class formed by the row is as great as possible among set classes of the same cardinality.

In the opening excerpt of the *Violin Concerto* reproduced in figure 2.9, Berg’s row does not actually appear in its prototypical form. The discussion that follows will compare the characteristics of the content in this excerpt with those of the row already addressed.
The pitch-class content of the concerto’s opening three measures, despite not featuring a statement of Berg’s row per se, has a spatial-function structure that strongly resembles that of the row: The composite series formed by all of the pitch classes in this excerpt is <t5072{69}413e>, which, like the main row, exhibits five SRSs and no reversals. This fact is distinctive when we consider that this series is not constructed in the same way that the main row is; whereas the main row is based on a triadic chain, this series begins with a five-pitch-class segment of the circle of fifths.
The twelfth unique pitch class of this work—8—actually does not occur until m. 6. The spatial function profile exhibited by this incomplete series is either EEEECXXCCC or EEEEEEXCXCCC, the first of which is a near-mirror image of the main row’s spatial function profile in that its two expansions occur between the first and second conjunctions rather than, as with the main row, between the third-last and second-last establishments. This finding provides another spatial correlation between a work’s main row and an aggregate-completion series that exists within that work.

The second excerpt from the Violin Concerto that I will address is the first aggregate completion effected by the violin at the opening of the concerto’s second part. This Hauptstimme and its accompaniment are reproduced in figure 2.10.
This excerpt features a presentation of the I₆ version of Berg’s row, but only after the pitch classes 7, 2, and t—a spelling of a G-minor triad—are sounded. The emphasis of G here mirrors the use of a row form starting on G to begin the work. The spatial properties of the row itself have already been presented; of interest here is the relationship between the row and the series that results when we count the violin’s opening three pitch classes as well.

The series comprising the violin’s first three pitch classes as well as those that follow is $<7t63e419508>$. The most striking difference between this series and the main row is its ar-
rangement of spatial functions. As opposed to the five establishments and five conjunctions within the main row, this aggregate-completion series contains only three of each, which occur as close to the beginning and the end of the series as possible, with expansions occurring in the series’ interior six phases. Additionally, as was the case with the aggregate-completion series that opens the *Violin Concerto* as well as the two aggregate-completion series within Schoenberg’s *Suite* cited above, this series contains no reversals.

Although, by the time the PC 9 is sounded, three establishments have occurred in the aggregate-completion series and five have occurred in the main row, it is that pitch class in both series that brings about monadic-gap saturation, just as PC 8 serves as the final pitch class for both of them. We should also note the fact that, in the phase prior to the one in which PC 9 occurs—in the aggregate-completion series as well as the I₆ row form—critical-gap saturation is not even present. These correlations arise from the tonally related fact that, because the three pitch classes that precede the statement of the main row in this section spell a G-minor triad, it is those same pitch classes that are redundant when this entire passage is examined in terms of aggregate completion. Therefore, it is the G-minor triad that serves as a tonal touchstone for this work from which both the differences and correspondences between these two series arise.

*Webern, Symphonie, Op. 21, Second Movement*, Thema

In this work, Webern shows an inclination to reach back not only to forms that are associated with traditional symphonies, such as sonata and variation forms, but to still earlier formal constructs as well, such as the use of rounded binary formal units and canon.¹⁶ This work also

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displays Webern’s characteristic concentration of pitch-class material and “dependen[ce] on inversion and retrograde” operations to accomplish that concentration.  

The main features of this row as assessed by established theoretical tools fall into the categories of symmetry and set-class derivation. In terms of symmetry, the series of adjacent ordered intervals for this row is 3, e, e, 4, e, 6, 1, 8, 1, 1, 9; the fact that this series displays retrograde-inversional symmetry means that the row itself displays retrograde symmetry.

In terms of set-class derivation, the row’s discrete trichords belong, respectively, to set classes (013), (014), (014), and (013), forming an additional symmetrical pattern. The row’s discrete hexachords are derived from set class (012345), so that the row is maximally combinatorial at the hexachordal level. Allen Forte has shown the importance of the octatonic set class, (0134679t), and its subsets, especially those that are “linear,”—i.e. those that form an unbroken segment of the octatonic scale—within Webern’s atonal—or pre-twelve-tone—compositions. Two of these octatonic subsets, (013) and (014), turn out, as described above, to be definitive for this serial work as well. Figures 2.11 and 2.12 provide a spatial row profile and a phase profile for Webern’s row for the purposes of the spatial analysis that follows.


The row (row class (5-4-1)):
\[
\begin{array}{cccccccccccc}
0 & 3 & 2 & 1 & 5 & 4 & t & e & 7 & 8 & 9 & 6 \\
\end{array}
\]

Spatial function profile:
\[
\begin{array}{cccccccccccc}
E & E & X & C & E & C & E & C & E & X & C & C \\
\end{array}
\]

SRS profile:
\[
\begin{array}{cccccccccccc}
1 & 2 & 2cc & 1c2cc & 3 & 2c3cc & 4 & 4c1cc & 5 & 5c & 5c4cc & 3c5cc \\
\end{array}
\]

URS profile:
\[
\begin{array}{cccccccccccc}
I & X & T & I & T & I & T & I & X & T \\
\end{array}
\]

Gap profile:
\[
\begin{array}{cccccccccccc}
1d2 & 1m2 & 2m3 & 4m1 & 3m5d4 & 5m4 \\
\end{array}
\]
Figure 2.12: The phase profile for row 032154te7896

Phase 1
Row segment: 0
Set class: (0)

Phase 2
Row segment: 03
Set class: (03)

Phase 3
Row segment: 032
Set class: (013)

Phase 4
Row segment: 0321
Set class: (0123)

Phase 5
Row segment: 03215
Set class: (01235)

Phase 6
Row segment: 032154
Set class: (012345)
The spatial function profile of this row is unique among those of the other twelve-tone rows addressed by this study in that, as a member of row class (5-4-1), it contains four reversals;
no other rows addressed by this study contain more than two. The reversals within this row distinctly differentiate it from the row upon which Berg’s *Violin Concerto* is based, although both have the same number of SRSs. Because of the presence of these reversals, the maximum number of gaps that exist at any point within the presentation of this row is only two, the minimum possible number for any multi-SRS row and a number made all the more striking by the disparity between it and the high number of SRSs—five—that the row contains. As mentioned above, this row displays retrograde symmetry; therefore, the pattern of spatial functions among the pitch classes it comprises is symmetrical as well, with the first of the row’s two expansions occurring immediately after the row’s first two establishments and the second expansion occurring immediately before the row’s final two conjunctions.

As was the case with the SRSs within the twelve-tone row belonging to Berg’s *Violin Concerto*, the SRSs within Webern’s row are maximally uniform in terms of pitch-class cardinality. They also display solidarity in terms of pitch-class contour: The three SRSs that contain three pitch classes have either the contour <+, -> or the contour <-, +>, which are the only two contours possible for SRSs of that size. As for the two SRSs that contain four pitch classes, SRS 2 has the contour <-, -, +> and SRS 5 has its inverse, <+, +, ->, so that the contour of each of this row’s five SRSs contains only one change of direction, which is brought about by its final pitch class.

The orderliness of SRS contour in this row is matched by the orderliness of the pattern by which SRSs are established and conjoined within this row. Every establishment of an SRS within this row except for the first is followed in the next phase by member of the same SRS. The SRSs of this row are also highly continuous: Except for SRS 1, which is interrupted twice, the
SRSs of this row are interrupted only once or not at all over the course of this row’s presentation, a remarkable fact considering how numerous this row’s SRSs are.

This row’s URS profile, like its spatial function profile, its ordered interval series, and its pattern of trichordal derivation, is symmetrical. Ten of this row’s pitch classes are part of URSs, more than in any other rows covered by this study except those composed by Penderecki. Notable about the relationship between URS functions and SRS functions within this row is the fact that each of this row’s URS initiations corresponds to an SRS establishment and each of this row’s URS terminations corresponds to an SRS conjunction. A result of these correspondences is that each of this row’s URSs comprises all but the last pitch classes of each of this row’s SRSs except for SRS 1.

The unidirectional and uninterrupted progress of most of this row’s SRSs from establishment to conjunction causes the gap profile of this row also to be rather orderly—and somewhat symmetrical—as well. The conjunctions of four out of the five neighboring pairs of SRSs in this row occur exactly one phase after the monadic gap between them appears. Because of the fact that the final four pitch classes within Webern’s row are spatially contiguous, critical-gap and monadic-gap saturation do not occur until the phases—9 and 10—that immediately precede the row’s final two phases, which contain the row’s final two conjunctions.

The lateness of critical-gap and monadic-gap saturation in this row is especially marked considering the set classes formed by the row as it is presented. The majority of septachordal set classes—and, therefore, the majority of twelve-tone rows at phase 7—display at least some sort of gap saturation, as do all but a mere five octachordal set classes. In fact, the set class formed by Webern’s row at phase 8, (01234567), is the only octachordal set class that contains no critical gaps at all, corresponding to the rows addressed by this study that were composed by
Schoenberg and Penderecki; however, there is a clear reason that Schoenberg’s and one of Penderecki’s rows exhibit such late critical-gap saturation: the presence of the BACH motto, which does not play a part in this work.

All in all, this row’s symmetrical spatial function profile, uniformity of SRS cardinality and contour, and symmetrical URS profile as well as the consistent proximity—in terms of row phases—between the appearances of the row’s monadic gaps and the closure of those gaps bear witness to the symmetry and orderliness generally associated with the structure of Webern’s twelve-tone rows, as do the row’s intervallic symmetry and symmetrical derivation from two set classes as described above. The clarity of the musical texture in the opening section, or Thema, of the Symphonie’s second movement allows those qualities to be clearly evident. Figure 2.13 provides a representation of that section.
The symmetrical quality of the rhythm and contour of this excerpt is evident. Here, Webern exploits the combinatorial possibilities of his row by presenting its $I_5$ and $I_{11}$ form simultaneously, which allow for trichordal as well as hexachordal combinatoriality. In fact, the trichords themselves form a symmetrical pattern that can be represented as follows: The trichord sequence A, B, C, D is played by the clarinet and the trichord sequence D, C, B, A is played by the horns and harp. This arrangement makes it so that the aggregate is completed by the two row forms halfway through this section, as is discussed below.
It is of interest that, for both the *Thema* voiced by the clarinet and the row statement sounded simultaneously by the rest of the instruments, the markedness of the four notes in the center of each row form that are uniquely not quarter notes is matched by the markedness of their spatial function: They represent the two dyadic URSs within the row, in each of which—uniquely—an establishment and conjunction occur successively.

The repeated notes with each row-form statement deserve attention as well. In the clarinet statement, the first and last three pitch classes in the row form are repeated, emphasizing the row’s initial establishments and final conjunctions but leaving out its first conjunction and last establishment. In the other row statement, the first and last four pitch classes in the row statement are repeated, including rather than excluding the row’s first conjunction and last establishment. In the later portion of each of those row statements, this emphasis also applies to, in the clarinet part, the point at which the row reaches monadic-gap saturation, and, in the other parts, the point at which it reaches dyadic-gap saturation.

A consideration of the row statements combined is also in order. Aggregate completion occurs with the appearance of PCs 9 and 3, the central pitch classes in the central measure of this variation, and produces the series <53{21}8076t4{93}>. As was the case when aggregate-completion series were compared with fundamental twelve-tone rows in the two works discussed earlier, this aggregate series contains fewer reversals than the actual row does. Actually, this is an understatement; the opposition between the aggregate-completion series and the work’s main row in terms of reversals is somewhat remarkable. The spatial function profile for the aggregate-completion series is either EEXXEXXCCXXCC or EEECEXXCXXCC. As opposed to the incremental, URS-laden nature of the main row itself, in which gaps generally are closed as soon as they are opened, the nature of the aggregate-completion series is generally to use the first half
of the row to create gaps and the second half to close them, providing a subtle sense of balance that corresponds with the numerous manifestations of symmetry in this musical section.

_Dallapiccola, Quaderno musicale di Annalibera_

When Luigi Dallapiccola heard Schoenberg’s *Variations* Op. 31 for the first time, he noticed “something [he] had never been taught at the Conservatoire: that one of the most marked differences between classical music [in] … sonata form … and music based on a note series … [is that] … in classical music the theme is nearly always subjected to melodic transformation while its rhythm remains unaltered, [whereas] in music based on a note series the task of transformation is concerned with the arrangement of the notes, independent of rhythmic considerations.” In the *Quaderno*, Dallapiccola demonstrates a great deal of ingenuity in arranging pitch classes; this ingenuity is identifiable by established as well as spatial approaches to twelve-tone analysis.

The twelve-tone row upon which this work is based, in $P_0$ form, is $0158t43792e6$. It is not derived from any trichordal or tetrachordal set class. Instead, set-class invariance manifests itself within this row most saliently in its discrete hexachords and the pentachords formed by its first through fifth and eighth through twelfth pitch classes. Specifically, the hexachords belong to set class (014579) and the pentachords to set class (01358). Taken together, the pentachords form a member of set class (0123456789), leaving a chromatic dyad in the center of the row as their complement.

The juxtaposition of set classes (01358) and (014579) shed light on the juxtaposition of

diatonic and non-diatonic content within the *Quaderno*. Set class (01358) is a subset of the diatonic collection; a member of this set class is formed when one combines the pitch classes that make up the tonic and dominant triads in a major key. Pivotedly, each of the two pitch classes at the center of Dallapiccola’s row brings about a significant intervallic change when it joins with five of the row’s outer ten pitch classes to form a member of set class (014579), any member of which includes as subsets three members of set class (014) and one member of set class (048), which are two of the three trichordal set classes that are not subsets of the diatonic collection. This presence of diatonic pentachords within a chromatic twelve-tone framework corresponds to the juxtaposition of diatonic and chromatic elements on the musical surface.

This row does not display intervallic symmetry, but there is near symmetry in its spatial function and gap profiles, as shown in Figures 2.14 and 2.15. These features complement the symmetrical pattern of set-class derivation described above, as explained below.
Figure 2.14: The spatial row profile for row 0158t43792e6

The row (row class (4-1-1)):

\[
\begin{array}{ccccccccccc}
0 & 1 & 5 & 8 & t & 4 & 3 & 7 & 9 & 2 & e & 6 \\
\end{array}
\]

Spatial function profile:

\[
\begin{array}{cccccccc}
E & X & E & E & E & X & X & X & C & C & C & C \\
\end{array}
\]

SRS profile:

\[
\begin{array}{cccccccccccc}
1 & 1c & 2 & 3 & 4 & 2cc & 2cc & 3cc & 3c4cc & 1c2cc & 4c1cc & 2c3cc \\
\end{array}
\]

URS profile:

\[
\begin{array}{cccc}
I & T & I & T \\
\end{array}
\]

Gap profile:

\[
\begin{array}{cccccccc}
2d3 & 3m4m1 & 1d2 & 1m2 & 2m3 \\
\end{array}
\]
Figure 2.15: Phase profile for row 0158t43792e6

Phase 1
Row segment: 0
Set class: (0)

Phase 2
Row segment: 01
Set class: (01)

Phase 3
Row segment: 015
Set class: (015)

Phase 4
Row segment: 0158
Set class: (0158)

Phase 5
Row segment: 0158t
Set class: (01358)

Phase 6
Row segment: 0158t4
Set class: (014579)
Figure 2.15 cont’d.

Phase 7
Row segment: 0158t43
Set class: (0124579)

Phase 8
Row segment: 0158t437
Set class: (0134578t)

Phase 9
Row segment: 0158t4379
Set class: (01235679t)

Phase 10
Row segment: 0158t43792
Set class: (012345789t)

Phase 11
Row segment: 0158t43792e
Set class: (0123456789t)

Phase 12
Row segment: 0158t43792e6
Set class: (0123456789te)
In terms of this row’s spatial functions, all of its establishments occur from phase 1 to phase 5 and all of its conjunctions occur from phase 9 to phase 12. A single expansion between the first two establishments keep this row’s spatial function profile from being symmetrical.

Among the row’s SRSs, SRS 2 in particular has three marked features: 1) Its cardinality—five pitch classes—is greatest among the SRSs of this row, 2) its first four pitch classes form a unidirectional and counterclockwise contour, and 3) its first four pitch classes also occupy a symmetrical arrangement of row phases—3, 6, 7, and 10—not to mention the fact that the SRSs that interrupt its unfolding in phases 4, 5, 8, and 9 are members only of SRSs 3 and 4. The arrangement of this row’s URSs provides an emphasis on the initial two pitch classes that belong to SRS 1, and emphasizes the central dyad that separates the two members of set class (01358) within the row as well.

The gap profile for Dallapiccola’s row is compact and orderly: No critical gaps appear in the row’s first three phases, in each of phases 4 through 8 occurs the establishment of critical gaps, and each of phases 9 through 12 serves to fill a monadic gap. It is notable that both of the pitch classes that form the URS of phases 6 and 7 shrink the gap between SRSs 1 and 2, first to dyadic and then to monadic size.

The phase profile of this row reveals that both critical-gap saturation—occurring in phase 6—and monadic-gap saturation—occurring in phase 8—happen earlier than is typically the case. This profile also shows that, although this row only contains two URSs, two is the maximum number of URSs possible by phase 7 of this row’s presentation based on its intervallic structure, so that the URSs’ presence and placement become more distinctive. In light of these facts, it makes sense that all but one of this row’s expansions is part of a URS.
In the beginning of this work’s opening movement, *Simbolo*, Dallapiccola clearly and asymmetrically parses his row, through the arrangement of vertical sonorities, into segments of two, two, two, three, and three pitch classes, as follows and as shown in Figure 2.16: The initial dyadic oscillation in the left hand isolates PCs t and e, while the successive sonorities in the right hand present the remaining four segments through the beginning of the 7/8 measure; this opening section is a manifestation of the $P_{10}$ version of this work’s row.

Figure 2.16: The opening measures of *Simbolo*

The set classes to which these segments belong are (01), (03), (06), (026), and (037); note that because Dallapiccola partitioned the row so that its central URS is divided, the only chromatic interval within any of these sets is between the pitch classes that constitute the opening dyad. In addition, Dallapiccola exploits the more diatonic content of his row’s outer pentachords by voicing the two trichordal sets that complete the row statement triadically, so that they sound
like a deceptive cadential gesture from an altered G-dominant-7 chord to an A-minor triad. This progression turns out to be a unifying element in this movement overall.

The use of both of rows and aggregates drawn from them relatively equally reflects the compositional practice of Schoenberg more than that of Webern, which reflects the greater influence Schoenberg had on Dallapiccola as a twelve-tone composer. If we consider the right-hand sets at the opening of this movement contrapuntally, creating a new set of aggregates, the BACH motive is immediately evident as the top voice. Its relationship to the spatial aspects of Dallapiccola’s row is of interest: SRS 2, the row’s largest SRS and whose first four pitch classes unfold counterclockwise, begins as the top voice, sounding the first two pitch classes of the BACH motive, and, with the advent of the row statement’s closing triads, becomes the middle voice. Meanwhile, SRS 3, which was established along with SRS 2 in the opening right-hand dyad, becomes the upper voice of the triads, completing the BACH motive with PC 4, which is the conjunction between SRSs 2 and 3. Thus SRSs 2 and 3 effect closure in this section of music through both gap closure and motivic completion.

In the phase of Dallapiccola’s career in which he composed the Quaderno, he frequently layered row forms that had undergone different canonical operations directly, at times utilizing P, I, R, and RI row forms as a quartet, as in the final movement—the Quartina—of this work. A more simple layering is present in the fourth movement of the Quaderno, entitled Linee, which is more texturally transparent than any of the work’s other movements, comprising only two


lines throughout that state P and I versions of his row. Specifically, the right hand presents P₉ and the left hand presents I₁₀, with each voice making one full statement of the row, as shown in Figure 2.17.
Figure 2.17: Linee
In the right hand, the first eight pitch classes of $P_9$ are parsed into oscillating dyads, while the remaining four form a rhapsodic melody, including several pitch-class repetitions that I will address below. The left hand begins with a rhythmically measured statement of $I_{10}$’s initial five pitch classes; the fifth pitch class is subsequently repeated, beginning a pattern of oscillating dyads that continue to the end of the movement. There is one notable pair of invariant dyads among these row forms, owing to the row forms’ inversional relationship, that occur in phases 9 and 10 and involve PCs 6 and e. These pitch classes are also prominent within this movement by virtue of the fact that, in mm. 6 and 7, the appearance of PCs 6 and e complete the chromatic aggregate between the two voices overall; shortly following this event, the repetition of PCs 6 and e in the right hand in m. 7 immediately precedes the oscillation between the same two pitch classes in the left hand during m. 8, so that the invariant dyad pair between this movement’s two row forms is made manifest on the musical surface.

The segmentation of this movement’s two row forms on its musical surface reflects the spatial structure of the row in a significant way: In the right hand, the oscillating dyads comprise the row form’s establishments and expansions while the melody that follows exclusively contains all of its conjunctions. In turn, the opening pitch classes in the left hand contain all of its row form’s establishments, along with one expansion; the left-hand pitch classes in mm. 6 and 7 contain the row form’s remaining expansions; and the left hand’s remaining pitch classes, beginning with the PCs 6 and e, exclusively contain all of the row form’s conjunctions. With this being the case, we can see that the sustained pitch in m. 5 as well as the repeated pitches in both hands in m. 7 occur when one of the two parts has reached monadic gap saturation, and serve to heighten tension before the pitch classes that conclude the movement fill the gaps that remain.
When this movement’s two row forms are taken together, there is a distinct correlation between the spatial function profile of the combined series that results with that of the work’s fundamental row. Aggregate completion occurs with the right hand’s presentation of PC e at the beginning of m. 7, which also completes the first of the invariant dyads comprising PCs 6 and e that were mentioned above; to that point, both hands together produce a series whose spatial function profile is EXXEEXXXXXCCC, which parallels the main row’s spatial function profile in that it contains no reversals and the establishments—rather than the conjunctions—are separated by expansions on exactly one occasion. In containing no reversals, it also parallels the spatial function profiles of all of the aggregate-completion series that this study has addressed so far.

Also of note is the fact that all of this composite series’ establishments and conjunctions occur in the right hand; each of the first four pitch classes of the left hand’s opening melody therefore appears with one and only one of its neighbors having already been presented. Whether or not it was intentional on his part, the choices that Dallapiccola made in composing this movement make effective use of the spatial properties inherent in the structure of his twelve-tone row.

_Babbitt, Composition for Twelve Instruments_

In the 1950s, Miilton Babbitt criticized some composers of serial music for attempting to achieve “total organization” in music by applying “dissimilar, essentially unrelated criteria to each of the [musical] components,” so that “mere simultaneity is termed ‘polyphony.’”22 By his own account, Babbitt looked to Schoenberghian combinatoriality and Webernian set-class deriv-
tion as origins for his compositional method, \(^{23}\) bringing both of these features of twelve-tone music to new levels of intricacy, as demonstrated below.

The \(P_0\) version of the row upon which this work is based is 0149583\(\text{t}2\)e67. This row is derived from two trichordal set classes, (014) and (015), as well as the hexachordal set class (014589). A prime feature of this row is its combinatoriality at several levels. Since the hexachordal set class from which this row is derived is transpositionally symmetrical, inversionally symmetrical, and self-complementary, the row—as well as any other row derived from this set class—is all-combinatorial at the hexachordal level. It is of note that the initial hexachord of any of this row’s prograde forms contains two trichordal sets derived from set class (014), while its second hexachord contains two trichordal sets derived from set class (015), an asymmetrical arrangement that prevents the row itself from displaying intervallic symmetry overall. Because of this characteristic, trichordal combinatoriality in this row occurs only in conjunction with hexachordal combinatoriality. Furthermore, by utilizing transpositional levels whose values combine to form subsets of set class (014589), Babbitt is able to achieve dyadic and unitary combinatoriality. All aforementioned levels of combinatoriality are essential to the structure of this work.

The spatial structure of Babbitt’s row further demonstrates the high degree of organization with which he composed it. Figures 2.18 and 2.19 display its spatial row and phase profiles.

\(^{23}\) ibid.
Figure 2.18: The spatial row profile for row 0149583t2e67

The row (row class (3-1-1)):

0 1 4 9 5 8 3 t 2 e 6 7

Spatial function profile:

E X E E X X X X C C X C

SRS profile:

1 1c 2 3 2c 3ce 2cc 3c 1c2ce 3c1cc 2c 2c3cc

URS profile:

I T

Gap profile:

1d2 3d1 2d3 1m2 3m1 2m3
Figure 2.19: The phase profile for row 014958t2e67

Phase 1
Row segment: 0
Set class: (0)

Phase 2
Row segment: 01
Set class: (01)

Phase 3
Row segment: 014
Set class: (014)

Phase 4
Row segment: 0149
Set class: (0347)

Phase 5
Row segment: 01495
Set class: (01458)

Phase 6
Row segment: 014958
Set class: (014589)
As the spatial function profile for row 0149583t2e67 reveals, the row is spatially symmetrical, although not intervalically symmetrical, as discussed above. It belongs to row class (3-
1-1); its establishments and conjunctions are contained compactly within the first four and the last four of its phases. As this row contains only three establishments, i.e. only three pitch classes that are necessarily spatially separate from any pitch classes that preceded them within the row’s presentation, it is notable that this row contains only two minimally sized URSs, occurring—symmetrically—at the first two and last two of its phases.

SRS continuity—or the lack thereof—within this row is one of its most distinguishing spatial features. SRS 1, containing four pitch classes, is interrupted once, while SRS 2, containing six pitch classes, is interrupted four times and SRS 3, containing five pitch classes, is interrupted a maximal four times. These interruptions display a simple pattern concerning SRSs 2 and 3: Following this row’s first two phases, in which SRS 1 is established and expanded once, the pitch classes presented in its remaining phases alternate between members of SRSs 2 and SRS 3.

Because of the orderliness with which this row’s SRSs unfold, there is a marked gradualness with which its gaps appear and are closed as well: Single dyadic gaps are created in the third, fourth, and fifth phases of the row’s presentation, so that by phase six critical-gap saturation has occurred, and in the row’s seventh, eighth, and eleventh phases those gaps are shrunk to monadic size, effecting monadic-gap saturation. Additionally, the order of SRSs between which dyadic gaps appear is the same as the order of SRSs between which monadic gaps subsequently appear: 1 and 2, 3 and 1, 2 and 3.

The phase profile of Babbitt’s row reveals special qualities about the hexachord from which the row is derived beyond its all-combinatorial status. It is a particularly specific determinant of SRS cardinality: A row built from this set class can only contain three SRSs by phase six, and, since this set class displays critical-gap saturation—which only eighteen hexachordal
set classes do—no more SRSs can be established in the row’s remaining phases. The early appearance of critical-gap saturation within this row is offset by the fact that, since the row ends with a URS, monadic-gap saturation only occurs in phase eleven, as late as it possibly can. It is also worth mentioning that Babbitt maintains intervallic symmetry within this row on a regular basis, as the row forms inversionally symmetrical set classes in phases 1, 2, 4, 6, 8, 10, 11, and 12. The high degree of orderliness among all of this row’s spatial characteristics is perhaps a result of the fact that this row is not present on the musical surface of this work, but instead functions as a prototype for the numerous and varied ordered sets that Babbitt utilizes, and is designed to facilitate combinatorial arrangements of sets of monadic, dyadic, trichordal, and hexachordal cardinalities as discussed above.\(^{24}\)

*Composition for Twelve Instruments* opens with two twelve-tone aggregates, each of which are articulated by all twelve instruments sounding one note each.\(^{25}\) Figure 2.20 shows the pitch classes they play and the order in which they occur.


\(^{25}\) David Hush, “Asynordinate Twelve-Tone Structures,” 178.
Figure 2.20: The opening measures
Considering the pitch-class content of this section overall, the first series sounded is \(<78e23t905461>\) and the second is \(<7{03e}48t6{92}1>\). From the standpoint of the instrumental parts themselves, these two aggregates are arranged so that the two notes that each player plays belong to set class (01). Since the pitch classes that constitute these dyads fill the chromatic aggregate twice over and the arrangement of these dyads begins on PC 0, the ensemble can be divided into four equal groups, two of which sound the pitch classes that make up the row’s first hexachordal segment, \(<014958>\), and two of which sound the row’s second hexachordal segment, \(<3t2e67>\).26 This is significant from a spatial standpoint, since each instrumental part in this excerpt thus articulates part of the unfolding of one of the SRSs within this work’s main row. The arrangement of the instrumental parts in this section also highlights the row’s two URSs, as the parts whose dyads include the first and last pitches sounded in this section—harp and celesta—and the parts whose dyads sound within the shortest span of time—clarinet and bassoon—are the ones to which PCs 0, 1, 6, and 7, which form the row’s URSs, are assigned.

The horizontally realized series cited above form an interesting progression. The initial series, \(<78e23t905461>\), is derived not from the set class (014589) but from the also symmetrical but spatially uneven set class (024579), the diatonic hexachord. The spatial function profile for the initial series is EXEEXCXEXCC, the first half of which matches the spatial function profile of the work’s main row. Its second half differs however, including an interior reversal and an establishment in phase 9. The lateness of this establishment causes this series to exhibit late critical-gap saturation, unlike the main row.

The second horizontally realized series in this section resembles the work’s main row much more than the first does; a primary reason for this is that the second series, like the main

row and unlike the first series, is derived from the set class (014589). Furthermore, if we interpret the order of this series’ simultaneously sounding pitch classes accordingly, the spatial function profile of this series also matches that of the work’s prototypical row, with the added feature that the simultaneities near the beginning and the end of the series cause all three establishments and all three conjunctions to occur successively, without the intervention of expansions. Critical-gap saturation in this series occurs in phase 6, so that yet another association between the second series and the main row—and a differentiation between them and the first series—is established.

One aspect of the URS profile displayed by these two series as a whole parallels the URS profile of the main row: Just as the main row begins and ends with URSs, the first series begins with one and the second series ends with one, although several exist in the interiors of those series. Despite this link, however, the overall progression implied by the spatial aspects of this work’s twelve-tone series, as articulated by a simple chromatic dyad played by each instrument, is from difference to similarity with regard to the work’s prototypical row.

*Penderecki, The Passion According to St. Luke*

As mentioned in the opening pages of this study, Penderecki’s *Passion According to St. Luke* is built upon not one but two twelve-tone rows. The first row that is presented within the *Passion*, to which I will refer as row 1, is, in $P_0$ form, 01432567t98e. The second, to which I will refer as row 2, is, in $P_0$ form, 0e12t9346587. As mentioned previously, I have found that, in investigating the properties of these rows, the use of spatial acquisition theory produces more meaningful analytical results by far than does any other approach. As also mentioned previously, Penderecki’s *Passion* contains hardly any presentations of actual row forms or discrete row
segments. For these reasons, the present section of this study differs somewhat from the study’s other sections; specifically, it does not include a comparison or synthesis of spatially-based information and information revealed through established twelve-tone theoretical methods and it addresses manifestations of the work’s twelve-tone rows on the musical surface only briefly. Instead, the main features of this section are full spatial investigations of each row, as these rows are exemplars in terms of spatial acquisition, and investigations of two salient features of the Passion that are not statements of row forms or segments thereof per se.

Quotations help to determine the structure of each of the Passion’s rows: The manifestation of set class (0134) by the opening pitches of the Polish hymn Swiety boze is found at the outset of row 1,\(^{27}\) and the BACH motto comprises the final pitch classes of row 2. As noted at the beginning of this study, Penderecki’s presentation of these rows with maximal compactness in pitch space highlights how fundamental spatial considerations are to their structure. These rows’ spatial properties are evident in Figure 2.21, which reproduces the spatial representation of row 2 given at the beginning of this study and supplements it with a similar representation of row 1.

The vivid spatial qualities of these rows are evocative, especially in the context of the large-scale, dramatic, and religious work to which they belong. Lydia Rappoport-Gelfand has created referential labels for both rows based on their spatial features that—while speculative—are compelling, demonstrating the semiotic power contained in the contours of these series: The undulating but generally ascending shape of the first row represents Jesus climbing the Mount of Olives, while the wedge-like pattern of the second row, crowned by the BACH motto, depicts the image of the cross itself.28

From the standpoint of spatial acquisition theory, the most notable feature that both of Penderecki’s rows exhibit is the pervasiveness of URSs within them. Between the two rows, all but one of their pitch classes belong to a URS that is either two or three pitch classes in size; this is the feature that causes these rows to exhibit an inertia of movement throughout their presenta-

tion that is rare in twelve-tone serial literature. I will address row 1 first, the spatial characteristics of which are displayed in Figures 2.22 and 2.23.
Figure 2.22: The spatial row profile for row 1, 01432567t98e

The row (row class (3-2-1)):

0 1 4 3 2 5 6 7 t 9 8 e

Spatial function profile:

E X E X C X X X E X C C

SRS profile:

1 1c 2 2cc 1c2cc 2c 2c 2c 3 3cc 2c3cc 3c1cc

URS profile:

I T I X T I X T I X T

Gap profile:

1d2 1m2 2d3m1 2m3
Figure 2.23: The phase profile for row 1, 01432567t98e

Phase 1
Row segment: 0
Set class: (0)

Phase 2
Row segment: 01
Set class: (01)

Phase 3
Row segment: 014
Set class: (014)

Phase 4
Row segment: 0143
Set class: (0134)

Phase 5
Row segment: 01432
Set class: (01234)

Phase 6
Row segment: 014325
Set class: (012345)
The directional alternation within row 1 causes its interior establishments and conjunctions to occur in short order. In turn, the linear nature of this row causes the rate of SRS inter-
ruption to be remarkably low, with SRS 1 being interrupted twice and the other two SRSs only once each.

Although it does not cover every phase of row 1’s presentation, the pattern formed by this row’s URS functions from phases 3 to 11 is so clear and simple as to imply that a rotation of this row may have been the prototype for it: If row 1 is rotated so that its final pitch class serves as its first pitch class, all of the row’s pitch classes would be members of three-pitch-class URSs that unfold in alternating directions.

The gap profile for row 1 corresponds to the URSs whose pitch classes serve as establishment/conjunction pairs; only within those URSs do critical gaps appear. The URSs within this row that unfold counterclockwise thus can be exclusively associated with the opening and closing of critical gaps.

The incremental nature of this row’s spatial form is perhaps most evident through an examination of its phase profile, which shows that only four of the set classes formed by this row at each of its phases are not chromatic. At phase seven, the set class formed is unusual in that it contains no critical gaps. However, this situation is reversed in phase 8, at which point critical-gap saturation occurs, to be followed by monadic-gap saturation in phase 9. This quick change highlights the initiation of the row’s final counterclockwise URS in phase 8.

The features of row 2 from the standpoint of established twelve-tone theory were laid out in the first section of part 1 of this study; despite the fact that row 2 is derived from dyadic and hexachordal set classes, as well as from tetrachordal set classes to some extent, it is its spatial features that are the most notable, as exhibited by its spatial function and phase profiles in Figures 2.24 and 2.25.
Figure 2.24: The spatial row profile for row 2, 0e12t9346587

The row (row class (2-1-1)):

0  e  l  2  t  9  3  4  6  5  8  7

Spatial function profile:

E  X  X  X  X  X  X  X  E  C  X  C

SRS profile:

1  lcc  lcc  lcc  lcc  lcc  lcc  lcc  lcc  lcc  lcc  lcc  2  lcc  lcc  lcc  1cc  2cc  lc2cc  1cc  2cc

URS profile:

I  T  I  T  I  T  I  T  I  T  I  T

Gap profile:

1m2d1  2m1
Figure 2.25: The phase profile for row 2, 0e12t9346587

Phase 1

Row segment: 0
Set class: (0)

Phase 2

Row segment: 0e
Set class: (01)

Phase 3

Row segment: 0e1
Set class: (012)

Phase 4

Row segment: 0e12
Set class: (0123)

Phase 5

Row segment: 0e12t
Set class: (01234)

Phase 6

Row segment: 0e12t9
Set class: (012345)
Figure 2.25 cont’d.

Row 2 stands out among all of the twelve-tone rows covered by this study both because it contains a mere two SRSs and because the establishments of those SRSs are fully eight phases.
apart. Interestingly, while the BACH motive that concludes the row contains one establishment and both of the row’s conjunctions, the row’s first eight phases contain only the establishment and multiple expansions of SRS 1, setting the BACH motive apart from the incrementally unfolding gesture that comes before.

The contour of the first eight pitch classes within row 2 creates a completely regular pattern of counterclockwise-unfolding and clockwise-unfolding dyads. This pattern is broken by the BACH motive, which not only establishes a second SRS but is made up of two dyads that unfold in the same direction. The delay of the row’s second establishment until phase 9 causes a maximal disparity in size between SRSs 1 and 2, which comprise eleven and three pitch classes respectively. In fact, the incipit pitch class of the BACH motive is the only one within this row that does not belong to SRS 1.

An additional feature of row 2 is the inclusion of all of its pitch classes in URSs, all of which are dyadic. This regularity increases the sense of inertia created by the row’s first four URSs, making its disruption by the BACH motive all the more marked.

The BACH motive’s distinction from the rest of this row’s content is apparent once again by virtue of the row’s gap profile, as the only appearances of critical gaps—as well as the only conjunctions—occur in the row’s final four phases. This fact is reflected in the row’s phase profile, which shows that the lateness of critical-gap saturation in row 2 is even more extreme—occurring in phase 8—than it is in row 1. It is notable that, given a chromatic set class with a high cardinality such as (01234567), this row contains the minimum possible number of SRSs and the maximum possible number of URSs by its eighth phase.

The inertia exhibited by row 1’s URSs is made evident at the opening of the Passion, as each instrument presents the row within the span of one octave. After this presentation—and the
chromatic cluster that follows it—row 2 is presented by the choirs and organ. The use of differing sections of the choirs to present the row’s first through eighth, ninth and tenth, and eleventh and twelfth pitch classes highlights the divisions—already illuminated by the row’s SRS structure, the row’s gap profile, and the breakage of the row’s URS-based inertia—between the two dyads that constitute the BACH motive and all that comes before. Also highlighting a division between row 2’s eighth and ninth phases is the organ’s presentation of a retrograded BACH motive while the chorus is singing the row’s first eight pitch classes. Figure 2.26 displays the presentation of these two rows.
Figure 2.26: The presentation of row 1 and row 2 at the opening of the *St. Luke Passion*
A marked feature of the *Passion* in addition to its twelve-tone rows that occurs several times over the course of the work is a choral cadence on the word *Domine*. Presented at key dramatic points within the *Passion*, this cadential figure has been characterized as an extremely condensed Bach-style chorale, functioning as “a pause in the … narration, [compelling] … the listener … to meditate on the Passion story.”29 In terms of the pitch-class progression that it comprises, this recurring cadence is a chromaticized and bifurcated version of the *clausula vera* that was the standard cadential figure in pre-tonal polyphonic compositions. Penderecki’s *clausula* differs from the classic model in two ways: 1) Instead of utilizing two fundamental—and perhaps triadically harmonized—voices that expand from a major sixth to an octave, Penderecki employs four voices—two sets of two—to resolve to two separate octaves that are separated by a minor third; and 2) each set of voices resolves not from a major sixth to an octave, as in the classic model, but from an augmented sixth to an octave. Penderecki’s *clausula* is represented in Figure 2.27.

Taking all four voices together, the pitch classes that constitute the first sonority of Penderecki’s *clausula* forms a member of set class (0235), while the pitch classes that constitute the second sonority belong to the dyadic set class (03). Combined, the two sonorities form a member of set class (012345); thus, Penderecki uses this cadential figure to highlight the resolution effected by the closure of gaps in pitch-class space. The set class (0235) is special in a particular way: It is the only tetrachordal set class that is inversionally symmetrical, fits within one half of the chromatic aggregate, and contains two monadic gaps. The dyad that follows it in the *clausula* both completes half of the aggregate and is comparatively much simpler in harmonic and textural terms, so that the figure truly functions as a harmonic cadence despite its non-tonal status.
Before departing this work, its final sonority is well worth considering. The sonority in question, unlike those that constitute the *clausula* described above, harkens quite clearly to the tonal idiom. It is a gigantic E-major triad sounded by the full orchestra and chorus, which, considering the highly chromatic—and often microtonal and sound-mass—content of the *Passion* in general, is a marked simultaneity to say the least. In order to demonstrate the connection that exists between this chord and the spatial structure of row 2, Figure 2.28 reproduces the representation of that row in pitch space—as it is presented in the opening movement of the *Passion*—that was displayed in Figure 2.21, except that letters rather than numerals represent pitch classes, and the pitch classes E, G#, and B are highlighted.

Figure 2.28: The E-major triad within row 2

```
G#  G  F#
    F
  E  D#
  D  C#
C   B
Bb  A
```

Considering that, as mentioned earlier, row 2 is a sort of compound melody, one part comprising dyadic URSs the member pitch classes of which unfold clockwise in pitch-class space and the other comprising dyadic URSs the member pitch classes of which unfold counterclockwise in pitch-class space, it becomes clear that the pitch classes E, G#, and B play important syntactical roles within the row. Respectively, E is the row’s initial pitch class, G# is the
final pitch class within the clockwise-unfolding group, and B is the final pitch class within the
counterclockwise-unfolding group, as well as being the final pitch class of the row. It is notable
that B is the final pitch class of the counterclockwise-unfolding group but not the farthest pitch
class within that group from the group’s initial pitch class, E; the farthest pitch class from E
within that group is instead A. This is due to the fact that B and A are members of the BACH
motive within row 2, the two dyads within which unfold clockwise while their member pitch
classes unfold counterclockwise. This demonstrates Penderecki’s compositional ingenuity: He
utilizes the BACH motive not only as an extramusical reference within row 2 but also as a device
by which to achieve a final and dramatic connection between serial syntax and triadic structure,
made possible by the row’s spatial properties.

*J. S. Bach, Das wohltemperierte Klavier, Book 1, Fugue in B Minor*

I have chosen to conclude the series of analytical investigations that constitute the second
part of this study, which is designed to address twelve-tone serial music, with the consideration
of a work that, admittedly, is not serial at all. It is, however, twelve-tone in that its subject com-
pletes the aggregate with a good deal of efficiency. With this being the case, the portion of this
investigation that addresses this fugue in terms of established theoretical techniques will engage
both twelve-tone and tonal theory. I intend for this final analytical section to demonstrate the
applicability of spatial acquisition theory to any music, not just that of twentieth-century serialist
composers, that features the relatively equal use of all twelve pitch classes. Figure 2.29 presents
the subject of Bach’s fugue.
From a tonal standpoint, the subject of this fugue is one that begins on the tonic scale degree and ends on the dominant. After the subject spells out the tonic triad with its first three pitch classes, what follow are seven successive chromatic dyads. If we interpret the first pitch classes in each of these dyads to resolve to the second, since all but one of them are descending, a logical progression emerges, leading to the spelling of the minor dominant triad and a cadence on the dominant scale degree that completes the subject.

The second pitch classes within each dyad are, in order, F#, A#, D#, B, E#, C#, and another C#, which begins the spelling of the F#-minor triad mentioned above. This sequence effectively forms a compound melody, both parts of which—F#, D#, E# and A#, B, C#—prepare the ground for the arrival on F# that follows.

Dieter Zahn has shown that pitch-class-set theory may be applied effectively to the subject of this fugue in that the chromatic dyads that populate the subject’s interior portion form an ascending sequence of pitch-class sets that belong to the set class (0145). 30 It is the systematic

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nature of this pattern that facilitates a fairly orderly process of aggregate completion in the sub-
ject.

In order to approach this subject from a twelve-tone perspective, it is necessary to deal
with the pitch-class redundancy that is inherent in even the most chromatic tonal music, includ-
ing the trill that involves the subject’s final two pitch classes. Figure 2.30 presents the pitch
classes that make up the subject as well as the twelve-tone row that they define.

Figure 2.30: The subject and the twelve-tone row derived from it

Subject:
6 2 e 7 6 e t 4 3 0 e 6 5 2 1 0 1 9 6 9-8 6
(trill)

Row:
6 2 e 7 t 4 3 0 5 1 9 8

One might expect that a row derived from a work that is not explicitly serial would not
exhibit many salient characteristics as defined by twelve-tone theory. This is generally the case,
although Monte Keene Pishny-Floyd has pointed out the fact that the row’s discrete hexachords
are derived from the same set class, (014579).31 There are no further patterns of derivation from
set classes of smaller sizes, and the row does not display intervallic symmetry.

From a tonal perspective, it is no surprise that all but two of the pitch classes that occur
more than once within the subject are members of either the tonic or the minor dominant triad in
the key of B minor, since those two triads represent, harmonically speaking, the subject’s points
of origination and destination. It is notable, moreover, that only two of the first ten of the sub-

ject’s pitches are redundant at the pitch-class level. In order to investigate this row from a spatial perspective, Figure 2.31 and 2.32 provide the spatial row and phase profiles for Bach’s row.

Figure 2.31: The spatial row profile for row 08514t96e732

The row (row class (4-1-1)):

```
0 8 5 1 4 t 9 6 e 7 3 2
```

Spatial function profile:

```
E E E X X E C X C C X C
```

SRS profile:

```
1 2 3 1c 3cc 4 2c4cc 3c 4c1cc 3c2cc 3cc 1c3cc
```

URS profile:

```
I T
```

Gap profile:

```
3d2 1d3 2m4m1 3m2 1m3
```
Figure 2.32: The phase profile for row 08514t96e732
The spatial function profile for this row is nearly symmetrical. The row’s first hexachord contains its four establishments, while its second hexachord contains its four conjunctions. The
first, third, fifth, and—later—fourth scale degrees in B minor function as the establishments, whereas the first three conjunctions occur, in order, between scale degrees 3 and 4, 4 and 5, and 1 and 3. The final conjunction—located between scale degrees 5 and 1, where the space between establishments is greatest—occurs following an expansion of SRS 3 in phase 11, forming a URS with it; this is somewhat marked, since the fact that the final three conjunctions of this row do not occur successively is solely what keeps this row’s spatial function profile from being symmetrical. Also notable is the fact that the row’s final establishment and its first conjunction occur in consecutive phases—6 and 7, the central phases within the row—and that they form one of the row’s two URSs. Heightening the distinction of this dyad is the fact that, in the P₆ version of the row that underlies the fugal subject, the dyad’s second pitch class is D#, the first pitch class to appear within the subject that is not functional in the key of B minor.

The row’s URS profile has a salient relationship with the metrical profile of the fugal subject, as do its gap and phase profiles as well. If we parse the row so that its pitch classes are grouped according to which of three measures they belong to on the musical surface—not counting the measure that contains only the final note of the subject, which is redundant in pitch-class terms—we arrive at segments of five, five, and two pitch classes. In this segmentation, the second measure is inaugurated by the row’s first URS, which is also the row’s only presentation of an establishment and a conjunction in succession. The second measure ends with the row’s second and third conjunctions, so that the only pitch classes that lie within the third measure of the subject but were not previously presented within the subject form the row’s second URS and its final conjunction.

As for the correspondence between gaps and the measure-based segmentation of this row, we can note that, at the end of the subject’s initial measure, only two dyadic gaps exist, whereas,
in the subject’s second measure, three monadic gaps appear and are closed. This leaves one dyadic gap in place at the opening of the subject’s third measure, which the row’s final two pitch classes close.

A further reinforcement to a segmentation that separates the final two pitch classes of this row from the rest is revealed through a consideration of the row’s phase profile, specifically phase 10 thereof. At this phase, the extant pitch classes of the row form a member of a symmetrical set class, namely (0123456789). This is distinctive given that, aside from phases 1, 2, 11, and 12, in which—as in phase 10—the only pitch-class sets that a twelve-tone row’s extant pitch classes can form are symmetrical, this is the only phase of this row besides its fourth phase in which a symmetrical pitch-class set is formed. This achievement of intervallic symmetry after a hiatus of six phases, along with the gap-related data mentioned above as well as the segmentation of the subject by its barlines, encourages a sense of separation between the A-G# URS of the subject’s third measure and what came before, marking those pitch classes’ descent to the tonal arrival on F#. Despite this separation, however, the A and G# belong to the SRS established by the B that serves as the tonic of the fugue’s subject, a fact that could be said to connect the initial pitch class of the subject and the cadential figure with which it ends.

At this point, we will move beyond a consideration of only the presentation of the subject of this fugue and consider what follows. The answer to the subject begins in the left hand in m. 4, in counterpoint with the fugue’s first countersubject in the right hand. Figure 2.33 displays the initial presentations of this fugue’s answer and first countersubject.
If redundant pitch classes are skipped, the answer comprises an incomplete row, which, in what we might call P₁₁ form—as found in the fugue itself—is <e9624385t71>. For its part, the countersubject also comprises a series that has only eleven pitch classes, found here in what we might call P₆ form: <65891e042t7>. In terms of the hexachordal set classes to which their initial six pitch classes belong, the answer displays more spatial affinity to the subject than does the countersubject, in that their gap structure is identical: The set class associated with the subject, (014579), and the set class associated with the answer, (012479), each contain two dyadic gaps and two monadic gaps and display critical-gap saturation. On the other hand, the hexachord with which the countersubject begins is a member of set class (013468), which does not display critical-gap saturation yet contains three monadic gaps. This similarity between the spatial structure of the opening pitch classes within the subject and the answer seems to be due to the more triadic nature of both the subject’s and the answer’s initial gestures. This is borne out by the URS content of the answer and the countersubject, the former containing only one URS and the latter containing three.
When considered together, the answer and countersubject form the aggregate-completion series <6e59812043t7>. As has been the case fairly consistently throughout this study, this series contains no reversals, mirroring the row associated with the fugal subject by virtue of that fact as well as by the fact that it contains four SRSs. At the same time, it relates to the row associated with the countersubject in that its initial hexachord is a member of the set class (013468) and that it contains three URSs, reflecting the chromatic nature of the passage overall more than the pitch-class content of any single statement within it.

In the preceding exploration of this fugue, as well as in the analytical investigations that came before it, the many complementary and supplementary relationships between data discovered through the methods of established twelve-tone theory and those discovered through the theory of spatial acquisition are evidence of the basic viability of a spatial approach to twelve-tone analysis. The one general trend among twelve-tone compositions that this study seems to have uncovered is a tendency, found to some degree in all of the works addressed here, for the spatial function profile of an aggregate-completion series not to exhibit reversals, regardless of the number of reversals that might exist in the twelve-tone row at the heart of the work in question. Further analytical investigations will determine the reliability of this finding; I believe that it may be a result of the desire among twelve-tone composers to distribute pitch classes around the chromatic cycle with frequency on the musical surface—again regardless of the spatial structure of the row used to compose that music—in order to maintain a consistently integrated pitch-class texture. In the interest of addressing that issue and many others, I look forward to the wider and deeper application of the theory of spatial acquisition to twelve-tone and other musical literature in the future.
Appendix A

Glossary of terms unique to spatial acquisition theory

Conjunction: One of the three spatial row functions that a pitch class within a twelve-tone row can perform, along with establishment and expansion. A pitch class that serves as a conjunction can be defined both in that it is shared by two SRSs and in that it occurs within a twelve-tone row after the two pitch classes that neighbor it within the chromatic cycle have already occurred.

Conjunction group: A single conjunction or multiple consecutive conjunctions within a twelve-tone row’s E/C profile. For example, the E/C profile EECEECCECC contains three conjunction groups.

Contiguous pitch-class subset (CPCS): A subset of the abstract set of pitch classes that defines a particular set class whose constituent pitch classes are consecutive within the chromatic cycle. For example, the set class defined as (012467) contains two CPCSs, (012) and (67).

Continuousness: The degree to which the constituent pitch classes of an SRS have consecutive order positions within a twelve-tone row.

Critical gap: A gap of one or two pitch-class places that exists during one or more phases of a twelve-tone row’s presentation, termed critical because, once such a gap has appeared, a pitch class that occurs within it can perform a limited number of spatial functions (see dyadic gap and monadic gap).

Critical-gap saturation: A situation within the presentation of a twelve-tone row in which all of the gaps between pitch classes that have occurred are critical. Once arrived at, this situation lasts until the row’s final phase.

Dyadic gap: A gap of two pitch-class places that exists during one or more phases of a twelve-tone row’s presentation. It is considered a type of critical gap because, once it has appeared, a pitch class that occurs within it can function only as an expansion or a conjunction.

E/C map: A schematic representation of all of a twelve-tone row’s establishments and conjunctions on the chromatic cycle.

E/C profile: A serial representation, using Es and Cs, of a twelve-tone row’s establishments and conjunctions. Unique E/C profiles define row classes.

Establishment: One of the three spatial row functions that a pitch class within a twelve-tone row can perform, along with expansion and conjunction. A pitch class that serves as an establishment can be defined both in that it is the first of a particular SRS’s constituent pitch
classes to occur and in that it occurs before the two pitch classes that neighbor it within the chromatic cycle do.

Establishment group: A single establishment or multiple consecutive establishments within a twelve-tone row’s E/C profile. For example, the E/C profile EECEECCECC contains three establishment groups.

Expansion: One of the three spatial row functions that a pitch class within a twelve-tone row can perform, along with establishment and conjunction. A pitch class that serves as an expansion can be defined in that 1) it is neither the first of a particular SRS’s constituent pitch classes to occur nor is it shared by two SRSs, and 2) it occurs before one of the two pitch classes that neighbor it within the chromatic cycle does.

Extension: One of the three URS functions that a pitch class within a URS can perform, along with initiation and termination. Within the presentation of the twelve-tone row to which a particular URS belongs, A pitch class that serves as an extension can be defined as neither the first nor the last of the URS’s constituent pitch classes to occur.

Gap profile: A serial representation of the appearances of monadic and dyadic gaps among the phases of a twelve-tone row’s presentation.

Initiation: One of the three URS functions that a pitch class within a URS can perform, along with extension and termination. Within the presentation of the twelve-tone row to which a particular URS belongs, A pitch class that serves as an initiation can be defined as the first of the URS’s constituent pitch classes to occur.

Interruption: A situation in which two pitch classes that occupy consecutive order positions within a twelve-tone row belong to two different SRSs.

Monadic gap: A gap of one pitch-class place that exists during one or more phases of a twelve-tone row’s presentation. It is considered a type of critical gap because, once it has appeared, a pitch class that occurs within it can function only as a conjunction.

Monadic-gap saturation: A situation within the presentation of a twelve-tone row in which all of the gaps between pitch classes that have occurred are monadic. Once arrived at, this situation lasts until the row’s final phase.

Phase: A unit of abstract or real time in which one of a twelve-tone row’s constituent pitch classes is presented.

Phase profile: A representation of the twelve phases of a twelve-tone row’s presentation in the form of twelve graphic representations of the chromatic cycle. For any particular phase, the pitch class that occurs within that phase as well as any that have already occurred are plotted on the cycle corresponding to that phase. This allows the reader to view the structures of the set classes formed by the row’s extant pitch classes at any particular phase.
Pitch-class place (PCP): Utilizing the concept that the twelve pitch classes are contiguous units rather than discrete points, the chromatic cycle can be divided into twelve pitch-class places, in each of which a pitch class can be said to reside. As each pitch class of a twelve-tone row is presented, it fills a previously empty pitch-class place.

Reversal: A situation in which a conjunction follows an establishment in a twelve-tone row’s E/C profile. An E/C profile will contain from one to six reversals.

Row class: A grouping of twelve-tone rows by E/C profiles. As there are sixty-five unique E/C profiles, a twelve-tone row will belong to one of sixty-five row classes.

SRS profile: A serial representation of the SRSs—each of which is identified by a numeral—to which a twelve-tone row’s pitch classes belong. For example, an SRS profile that begins with the numerals 1, 2, and 1 indicates that the row’s first and third pitch classes belong to SRS 1 whereas its second pitch class belongs to SRS 2. Entries that correspond to SRS expansions and conjunctions also include contour-related indications.

Spatial row function: One of the three functions that a pitch class within a twelve-tone row can perform regarding the SRS to which it belongs: establishment, expansion, or conjunction.

Spatial function profile: A serial representation, using Es, Xs, and Cs, of a twelve-tone row’s establishments, expansions, and conjunctions.

Spatial row form: A permutation of the twelve pitch classes that is unique in terms of the pattern of spatial acquisition that it exhibits. The family of spatial row forms is equivalent to the family of permutations of the twelve pitch classes that are not related by transposition or inversion.

Spatial row profile: A comprehensive schema that coordinates all of the spatial data for a twelve-tone row. This profile includes a representation of the row itself accompanied by the row’s spatial function profile, SRS profile, URS profile, and gap profile.

Spatial row segment (SRS): A segment of a twelve-tone row that is determined according to the concept of incremental pitch-space acquisition that is central to this study, as follows: Over the course of a twelve-tone row’s presentation, 1) a pitch class that occurs before either of the two pitch classes that neighbor it within the chromatic cycle do is the initial pitch class, or establishment, of a particular SRS, and 2) subsequently, any pitch classes that occur that neighbor a member pitch class of this SRS within the chromatic cycle belong to this SRS as well. For example, the row <0615e724t839> contains two SRSs, <01e2t39> and <6574839>. Note that SRSs overlap with each other; a pitch class that two SRSs share is termed a conjunction. Pitch classes that belong to only one SRS and do not function as establishments are termed expansions. A row may comprise from one to six SRSs.
Termination: One of the three *URS functions* that a pitch class within a *URS* can perform, along with *initiation* and *extension*. Within the presentation of the twelve-tone row to which a particular *URS* belongs, a pitch class that serves as a termination can be defined as the last of the *URS*’s constituent pitch classes to occur.

**URS function:** One of the three functions that a pitch class within a *URS* can perform: *initiation*, *extension*, or *termination*.

**URS profile:** A serial representation of the pitch classes that serve as the *initiations*, *extensions*, and *terminations* of *URSs*—represented by *Is*, *Xs*, and *Ts*—among the phases of a twelve-tone row’s presentation.

**Unidirectional row segment (URS):** A row segment comprising pitch classes that are consecutive within the chromatic cycle and that occupy consecutive order positions within the twelve-tone row to which they belong. For example, the row segment *<35674t>* contains the URS *<567>*. A URS is always a subset of a particular *SRS*. 
# Appendix B

## A list of row classes

Key to row class names:
(SRS cardinality-establishment group cardinality-row class signifier)

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Appendix C

A list of pitch-class-set types for the purposes of spatial acquisition theory

Key to data vectors:

SRS (left-hand) vector: <minimum SRSs, maximum SRSs>
URS (central) vector: <CPCSs, maximum URSs, CPCS membership>
Gap (right-hand) vector: <dyadic gaps, monadic gaps> indication of critical-gap (c) or monadic-gap (m) saturation, if present

The dodecachord:

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The monad:

(0) <1,1> <0,0,0> <0,0>

The undecachord:

(0123456789t) <1,6> <1,5,11> <0,1>m

Dyads:

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