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OPTIMIZATION AND CONSTRUCTION OF PASSIVE SHIM COILS FOR HUMAN BRAIN AT HIGH-FIELD MRI

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Abstract

When scanning the human brain with high-field MRI, local susceptibility variations can lead to field inhomogeneities that cause artifacts such as image distortion and signal loss. Active shimming improves homogeneity by applying corrective fields generated from shim coils, but requires calculation of optimal current for each shim coil. In chapter two, we discuss the computation of the calibration table to generate optimal active shim field and improve the field homogeneity within the human brain and a phantom. A modified 3D gradient-echo pulse sequence was implemented to obtain $B_0$ field maps of the phantom at 4T. The magnetic field distributions of the first- and second-order shim coils were determined by assuming that the spherical harmonic coefficients of each shim coil follow the linear relationship with the settings of the DAC (Digital to Analog Converter, which controls the voltage across the different shim coils). The calibration table was determined for the optimization of all first-order and second-order shim currents. By updating the first-order and second-order active shim system using the derived calibration table, a significantly improved field distribution within the phantom was obtained.

Local susceptibility variations within the human brain can lead to field inhomogeneities that degrade image quality. Improvements have been demonstrated using high-order shims at fields greater than 3T. However, due to space limitations within the magnet bore, most MRI systems provide, at most, second-order active shims. In chapter three, we present a local third-order passive shim design for improvement of field homogeneity in the brain. A 3D gradient-echo pulse sequence was used to obtain brain $B_0$ field maps of four subjects at 4T. The $B_0$ field
maps for each subject were then decomposed into third-order spherical harmonic coefficients and averaged. Optimized positions for placement of shim elements on a cylindrical shim tube that fits over the RF coil were then computed to reduce the third-order harmonics over the entire human brain. When combined with first- and second-order active shimming, the passive shim tube significantly improved $B_0$ homogeneity within the brain.

The orbitofrontal cortex (OFC) area plays a critical role in human brain functions. However, the great mismatch in magnetic susceptibility between paranasal sinuses and the surrounding tissues of the nasal cavity leads to local susceptibility-induced field variations in the OFC that lead to imaging artifacts such as image distortion and signal loss. In chapter four, we discuss the use of magnetic material to generate the modified dipole passive shim field and improve the field homogeneity within a group of subjects’ brains, particularly in the OFC. A 3D gradient-echo pulse sequence was used to obtain $B_0$ field maps. The half cylindrical geometry (i.e. the basic passive shim geometry) for mounting shim elements was determined. The positions of the shim elements for each harmonic component were computed with adaptation for passive shim design, which is to be placed below the chin. After the selection of appropriate positions and the corresponding dipole field amplitudes, the required susceptibilities and dimensions were determined. This study has demonstrated that the accurate placement of the appropriate diamagnetic and ferromagnetic material on the surface of the half-cylinder could generate the desired local dipole passive shim field, which causes the $B_0$ inhomogeneity to reduce significantly over the sinus region within the human brain, especially in the OFC.

In conclusion, a shim system that combines the first- and second-order active shimming, the third-order passive shimming, along with a local dipole neck-passive semi-cylinder shimming, can significantly improve $B_0$ homogeneity within the brain at 4T.
Dedication

To my Father, my Mother, my Wife, my brother and my sisters
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Chapter 1

Introduction

1.1. Overview of magnetic resonance imaging

In 1973, Lauterbur [1-2] published the first 2D image of two water-filled capillaries by means of nuclear magnetic resonance (NMR) zeugmatography. His technique was based on applying several linear gradients to the capillaries with 45° incremental intervals in a series of experiments. From a sufficient number of back-projections onto different directions, the 2D images of two capillaries were obtained. Lauterbur termed it a zeugmatogram. In 1975, Ernst and his co-workers [3] introduced a new and more efficient technique for obtaining 2D or 3D images of a macroscopic sample by applying three orthogonal (x, y and z) linear gradients to the sample after the excitation of a radiofrequency (RF) pulse. The free induction decay (FID) was recorded during a “readout” gradient and the 2D or 3D images were reconstructed with 2D or 3D Fourier transformation, respectively. In 1977, Mansfield et al [4, 5] introduced a technique which allowed a cross-sectional picture to be obtained from a 3D volume object. The first in vivo cross-section image was produced by switching the linear magnetic field gradient during the NMR signal collections. Mansfield’s novel technique is now widely used for medical applications. Both P. Mansfield and P.C Lauterbur were awarded the Nobel Prize in Medicine in 2003 for their work of the development of MRI.
Magnetic resonance imaging (MRI) is a popular medical imaging modality primarily used in biomedical research and routine clinical examinations to visualize interior structures non-invasively and to study the function of the human body. MRI provides detailed images of the body in any plane. Compared with traditional imaging modalities such as CT (computed tomography), MRI provides a better contrast for soft tissues, making it particularly useful in neurological, musculoskeletal, cardiovascular, and oncological (cancer) imaging. Magnetic resonance spectroscopy (MRS) is another powerful tool to measure the alteration of in vivo metabolism, which may be linked to certain diseases [6-8].

In general, MRI uses an intense, static magnetic field ($B_0$), a radio frequency field ($B_1$) perpendicular to $B_0$, and three independently orthogonal gradient fields to create an image of an object [6, 9]. When the object (ensemble of protons) is placed in an applied magnetic field ($B_0$) the protons tend to align with $B_0$ and precess with Larmor frequency ($\omega_0$), which is dictated by the fundamental properties of nuclear spin physics and is given by the Larmor Equation:

$$\omega_0 = \gamma B_0.$$  \[1.1\]

Here $\gamma$ is the scaling factor between the Larmor frequency and magnetic field and is known as the gyromagnetic ratio. The gyromagnetic ratio is constant and every nucleus with a non-zero spin has its unique Larmor frequencies for a given magnetic field strength. The collective magnetic property of spins is a vector corresponding to the bulk magnetization ($M$), which is normally aligned parallel to the $B_0$ field. In order to generate an NMR signal, the net magnetization is tipped toward the transverse plain that is perpendicular to $B_0$. To accomplish this, the spins are exposed to an alternating $B_1$ field which is tuned to the Larmor frequency and
applied in the transverse plane. The spins absorb energy from the applied RF field and are tipped away from the longitudinal axis to the transverse plane. After the transmission energy turns off, the transverse magnetization couples with the RF coil and induces an NMR signal.

MRI is a unique medical imaging method in terms of the relationship between the detected signal and the final image. The MR images are achieved based on the fact that the spin precesses at a specific frequency that is associated with the magnetic field it experiences [10-11]. The goal of MRI is to find the intensity of the NMR signal for an array of pixels corresponding to different locations through the object. However, the NMR signal obtained by the RF coil cannot be collimated to restrict to a specific location. In fact, the NMR signal originates from the entire object rather than a single location in a very homogeneous magnetic field environment. In order to provide the spatial encoding, it is necessary to introduce magnetic field gradients in the very homogeneous $B_0$. Specifically, three orthogonal gradient fields $G_x$, $G_y$ and $G_z$ are oriented in the $x$, $y$, and $z$ direction, respectively (Fig 1.1). Depending on their functions, these are called slice-selection gradient, frequency-encoding (or readout) gradient, and phase-encoding gradient [12]. The magnitudes of the magnetic field gradients are proportional to the position from the isocenter of the magnet as shown in Fig 1.1. The main magnetic field, $B_0$, varies linearly after introducing these field gradients and, as a result, spins in each of the regions experience a unique magnetic field which allows for the identification of their positions. This is because the spin in each location experiencing a unique magnetic field gives a specific Larmor frequency ($\omega_L$), as shown below [11-12].

$$\omega_L = \omega_0 + \gamma \vec{G} \cdot \vec{r}$$  \[1.2\]
Figure 1.1 3D variations of the magnetic field gradients. (A) $G_X$, (B) $G_Y$ and (C) $G_Z$. 
Here $\vec{G}, r$, and $t$ are the gradient strength, radial distance, and time, respectively. The MRI signal obtained from a plane defined as a ‘slice’ is excited to visualize 2D images of an object, a process first presented by Garroway et al in 1974 [13]. By introducing an RF pulse containing a finite range of frequencies, along with the linear magnetic field gradient, during the period that the RF pulse applied, a “slice” of signal from a volume can be selected. The application of field gradient causes a linear variation of the field strength along the slice selection direction, consequently varying the resonance frequency linearly. An RF pulse with an excitation bandwidth frequency of $\omega = (\omega_0 \pm 1/2 \Delta \omega)$ excites only those spins with frequency corresponding to the location of $z = (z_0 \pm 1/2 \Delta z)$ (see Figure 1.2). By controlling the bandwidth and the frequency of the excitation pulse, the thickness and location of the slice can be controlled. [14].

Ignoring the relaxation effects, the MRI signal observed by the receiver at the position $r$ is proportional to the effective spin density $\rho(\vec{r})$, and based on the space- and time- dependent precession frequency $\omega_L(r,t)$ from time $t = 0$ to $t = t$

$$S(r,t) \propto \rho(r) \cdot e^{\int_0^t \omega_L(r,t')dt'}$$ \hspace{1cm} [1.3]

The signal from a 3D volume sample is

$$S(r,t) \propto \iiint \rho(r) \cdot e^{\int_0^t \omega_L(r,t')dt'} \, dx \cdot dy \cdot dz$$ \hspace{1cm} [1.4]
Figure 1.2 Slice selection. Excitation of spins in a slice of thickness $\Delta z$ corresponding to frequency range of $G_z$ represents the slice select gradient.
The time-dependent precessional angular frequencies for spins depend on the applied magnetic field; and \( \omega_t (r, t) \) can be determined by the magnetic field experienced by protons. Therefore, the signal equation can be expressed as

\[
S(r, t) \propto \iiint \rho(r) \cdot e^{-i(\omega_n + \frac{1}{\gamma} \int_0^r G(r') dr')} \, dx \cdot dy \cdot dz
\]

[1.5]

Here, time-dependent gradient can be expressed as

\[
\bar{G}(t') = G_x(t') \hat{x} + G_y(t') \hat{y} + G_z(t') \hat{z}.
\]

[1.6]

At the rotating frame with frequency of \( \omega_0 \), Eq. [1.5] can be simplified as

\[
S(r, t) \propto \iiint \rho(r) \cdot e^{-iG \cdot r} \, dx \cdot dy \cdot dz
\]

[1.7]

Thus, \( S(r, t) \) is the 3D Fourier transformation of effective spin density, \( \rho(r) \). The goal of MRI is to generate images by acquiring a set of signals so that the information about effective spatial spin density of the object can be obtained by applying inverse Fourier transformation to the acquired signal \( S(r, t) \)[15-17].

Spatial information of the NMR signal can be encoded with the frequency-encoding gradient \( \bar{G}(t') = G_x \hat{i} \) in one direction when the signal is measured from \( t' = 0 \) to \( t' = t \).
Here \( k_x = \gamma \cdot G_x \cdot t \) is the spatial frequency. In the presence of \( G(t') \), the resonance frequency of the spins is linearly related to their spatial location; a unique precessional frequency is assigned to spins at a different spatial location along the gradient direction. Therefore, the time domain signal, \( S(r, t) \), consists of a range of frequencies, each corresponding to the spins’ precession at a different location. The inverse Fourier transformation of the received signal, \( S(r, t) \), discloses the frequency content and consequently the spatial information of the effective spin density [15-16].

In addition to applying frequency encoding, the phase-encoding gradient \( G(t') = G_y \hat{y} \), which is orthogonal to the frequency-encoding direction, is also utilized for spatial localization. During the application of phase-encoding gradient, \( G_y \), the phase of the MR signals alters linearly with spatial location. After introduction of phase-encoding \( G_y \), and then the readout \( G_x \) gradients, the acquired spatial information of the MR signals during the period of read gradient is

\[
S(r, t) \propto \iiint \rho(r) \cdot e^{-i\phi_y - i\gamma x G_y \cdot t} \, dx dy dz.
\]  

[1.9]

where \( \phi_y = \gamma \cdot y \cdot G_y \cdot t \) is the phase of the MR signals due to the phase encoding gradient prior to data acquisition. The inverse Fourier transformation of Eq. [1.9] discloses information about the distribution of the effective spin density in a 2D plane [16, 18].

The fundamental principle of MRI is that the signals are acquired in the time domain, also known as \( k \)-space. The \( k \)-space is a representation of the spatial frequency information in
two or three dimensions of an object. In practice, the $k$-space data are directly manipulated by the strength or amplitude of the gradient as well as the duration of the gradient. By changing the gradient strength and duration, the data points are sampled uniformly in the $k$-space. Every data point in a $k$-space matrix contains a portion of the information for the complete image. During the slice excitation, only the $G_Z$ gradient is applied and the frequency of the RF pulse is adjusted according to the slice of interest. After completion of the slice excitation, the signal is acquired in the presence of a constant $G_x$ gradient and is sampled along the $k_x$-direction. This step proceeds multiple times by altering phase-encoding gradient strength until all $k$-space data points are collected. By applying 2D Fourier transformation to a single slice data, a 2D image can be reconstructed.

The center of $k$-space data represents the signal from the low spatial frequencies, whereas the peripheral represents the high spatial frequencies. If $k$-space is only partially filled, the image may not be a complete representation of the object. Spatial frequency is a measure of how rapidly the image intensity varies over space. If signals with low spatial frequencies are eliminated, then overall image intensity, contrast, and general features will be lost. Additionally, if signals with high spatial frequencies are ignored, then the information regarding edges and sharpness of the image is lost [17, 19-21].

A MRI scanner is composed of a main magnet, gradient coils, and RF coils, which generate $B_0$ and $B_1$ fields, as well as shim coils that optimize the magnetic field homogeneity (see Figure 1. 3). The superconducting electromagnet is the principal component of an MRI scanner and consists of a large coil made by niobium-titanium (Nb$_3$Ti) alloy wire with a critical temperature of 10 K. In order to maintain the temperature below the critical temperature, the superconducting coil of Nb$_3$Ti is immersed in liquid helium with a boiling temperature of 4.2 K.
Figure 1.3 MRI scanner diagram. Illustration of the (A) cross sectional and (B) longitudinal sectional view of an MRI scanner.
By passing current through the main superconducting coil, an intense magnetic field is generated with almost no resistance and heat generated [22]. Shim coils are placed either inside the cryostat (called “superconducting shims”) or inside the “bore” of the magnet (called “resistive shims”) to improve or “shim” the homogeneity of the static magnetic field, $B_0$. Gradient coils, which are used for spatial localization, are placed inside the magnet bore. Gradient strength and slew rate are two main properties for gradient coils. The variation of field strength over a distance is called the gradient strength. The rate of driving a gradient coil from zero to its maximum amplitude, either positive or negative, is called the slew rate. The units of gradient strength and slew rate are mT/m (or Gauss/cm) and T/m/s, respectively. The RF coil is placed as close as possible to the object to be imaged. An oscillating RF magnetic field, which is applied through an RF coil, has to be introduced to the subject for “resonance” to occur. The RF coil is used to excite the protons within the tissue at their resonance frequency, and is also used to detect the NMR signal. Some RF coils can be used as both transmitter and receiver; others can function only as transmitter or receiver [12, 17, 23].

1.2. Magnetic field inhomogeneity

In MRI, an intense, static magnetic field, $B_0(z)$, a radio frequency field, $B_1$, perpendicular to $B_0$, and three independently orthogonal gradient fields are utilized to create image information [24-26]. The acquisition of excellent MR imaging and spectroscopy strongly relies on the availability of a strong, highly homogeneous, static magnetic field. However, to have a uniform magnetic field is unlikely because many aspects can cause inhomogeneities within the field, including constraints on generating uniform current density and fabrication imperfections in the
main magnet. In addition, the physical geometry of the magnet coils may distort the magnetic field due to the electromagnetic forces acting on superconducting windings of the main magnet [27-28]. The field inhomogeneity due to manufacturing limits and local environment is typically up to a few hundred parts per million (ppm).

The introduction of a sample in the magnet further perturbs the magnetic field locally. The induced magnitude and spatial distribution of the inhomogeneity is dependent on the geometry and magnetic susceptibility of that sample and the strength of the $B_0$ field. Disturbance of the field homogeneity is greatest in structures where there is a mismatched susceptibility boundary, such as between air cavities ($\chi \approx 0.3 \text{ ppm}$) and biological tissue ($\chi \approx -9.2 \text{ ppm}$) [29], which is aligned perpendicularly to the $B_0$ field [26]. Additionally, the field inhomogeneity could also be caused by variation of the macroscopic susceptibility distributions attributed to different subjects and subject’s motion (e.g. due to respiration).

Inhomogeneous magnetic field distribution of \textit{in vivo} experiments can be grouped into microscopic, macroscopic and mesoscopic contributions based on the relative scale of the inhomogeneity compared with an imaging voxel [30]. The field inhomogeneity on a spatial scale comparable to atomic and molecular size; i.e.; smaller than an imaging voxel size, is defined as microscopic contribution, whereas magnetic field variation on a spatial larger than imaging voxel is known as macroscopic contribution. Further, mesoscopic scale refers to magnetic field variation over distance that is smaller than the voxel size but bigger than the atomic and molecular size. Uncorrected microscopic and mesoscopic fields cause irreversible signal loss characterized by longitudinal (T1) and transverse (T2) relaxation and BOLD (Blood-Oxygen-Level-Dependent) contrast in functional MRI [31]. Conversely, the macroscopic field
inhomogeneity could cause signal loss and image distortions, so that the information regarding physiological and anatomical data is affected.

1.2.1. Effects of magnetic field inhomogeneity

The quality of MR images mainly depends on field homogeneity. The magnetic susceptibility influences heavily on the $k$-space data acquisition. The existence of any uncorrected field during the frequency encoding alters the precessional frequency of spins. This gives misregistration of spin’s spatial position, which is known as the image distortion, as the spatial localization in MRI relies on the one-to-one relationship between the spin’s position and its frequency. Additionally, if the field inhomogeneity presents, the slice thickness is either compressed or enlarged depending on either $\Delta B_0 > 0$ or $\Delta B_0 < 0$, and at the same time, the position of the slice is spatially shifted. This leads to changes in the imaging intensity across the slice [12, 31-34].

Using high-resolution echo-planar imaging (EPI) that facilitates the detection of the blood-oxygen-level dependence (BOLD) signal change in blood vessels with 50 – 100 $\mu$m in diameter, high-field MRI scanners are recommended [35-37]. However, one of the challenges of the high-field MRI is field inhomogeneity: that magnetic susceptibility increases with field strength. The local inhomogeneous static magnetic field distributions causing imaging pixel shifts are the most difficulty with functional MRI (fMRI) studies because the EPI is much more sensitive to susceptibility artefacts than other sequences [39-40].

Magnetic resonance spectroscopy (MRS) is a powerful technique to quantify in vivo metabolite concentration non-invasively. MRS is widely used to evaluate disorders related to the central nervous system, such as bipolar disorder, epilepsy, Alzheimer’s disease, and others [41-
1.3. Shimming

As described above, it is important to reduce these undesirable spatial variations in the magnetic field by applying correcting fields, a procedure known as shimming. Shimming methods can be divided into two categories: active and passive shimming.

1.3.1. Active shimming

An active shim set comprises a collection of electromagnetic coils that are energized with suitable current levels. The required field homogeneity over a selected region of interest is achieved by passing proper currents through appropriately distributed shim coils; this is known as active shimming [42,44-49]. The shim coils are designed to produce the spherical harmonic fields in most MR systems. Based on spherical harmonics, we named the first orders for $X$, $Y$, and $Z$ coils and the second orders for $XY$, $X^2Y^2$, $Z^2$, $XZ$, and $ZY$ coils [42,44-47]. Shim coils are generally placed inside the magnet bore (see Fig 1.3). The gradient amplifiers used for
imaging spatial encoding are also used to generate the first-order shim fields. However, second-
and higher-order shim field required additional shim amplifiers, one for each coil.

Many studies suggest that providing higher-order shims can improve field homogeneity
significantly for in vivo MRI and MRS quantification. However, in most human and animal MRI
systems, the existing electromagnetic shim coils are limited to the second order (or the third at
most in some cases) due to the restriction of space within the magnet bore. In addition, the
current shim power supplies in human and animal MRI systems are restricted by the maximum
current of 2-4A and 4-20A [42], respectively, which are less than adequate.

The manual alteration of the shim currents in each shim coil is the most primitive
technique to enhance field homogeneity in NMR procedure. However, this technique is
inefficient for in vivo studies on human or animal due to time constraints [42, 50-51]. Hence,
active shimming on human or animal imaging is achieved in automatic fashion; that is,
determining the required shim currents in each shim coil by measuring inhomogeneous field
maps of the object.

A number of automated shim techniques have been developed and are categorized into
(1) magnetic-field-map-based shimming, and (2) projection-based shimming [52]. For the
former, generally two or three-dimensional field maps are acquired using dual- or multi-echo
gradient-echo sequences and projected onto the spherical harmonic model to determine the
requisite shim currents for each shim coil. For projection-based shimming, the magnetic field
distributions are mapped based on MR signal phase alternations along a certain number of
column projections, which are used to determine the optimal shim currents. Current projection-
based automatic active shim methods such as FASTMAP (Fast Automatic Shimming Technique
for Mapping Along the Projection) are effective for finding optimal currents and work well at
high-field strengths, but are restricted to shimming cubical regions of interest. Also, shim power with the commercial power supplies is sometimes limited at certain anatomical regions [47].

A most recent advance for automated shim technique is Dynamic Shim Updating (DSU), which is designed for optimizing shimming for multiple locations at the same time. The field distribution over each slice (or sub-volume) is acquired to determine the required shim currents corresponding to each slice (or sub-volume) during the data acquisition to enhance field homogeneity slice-by-slice or by sub volume [53-54] during one single scan.

1.3.2. Passive shimming

Unlike active shimming, passive shimming is the placement of diamagnetic or para/ferromagnetic materials with known magnetic susceptibility (see Table 1.1), proper size, and position inside the magnet bore to improve the local field homogeneity.

The major goal of this work is to develop a local passive shim system to optimize the shim field for human brain MRS/MRI scans at high-field scanners (≥3T). A number of experiments have been carried out using this approach for improving the $B_0$ field homogeneity in the human and animal brain.

Cusack et al. [55] and Wilson et al. [56-59] reduced the field inhomogeneity and resultant artifacts of brain images using a diamagnetic passive shim piece (highly oriented pyrolytic graphite: HOPG) placed in the subject’s mouth. However, in our opinion, it is probably best to place the shim elements externally to reduce discomfort.

The optimization of field homogeneity in the visual cortex of a monkey brain was introduced by Juchem et al. [43] using a combined active and passive shimming method.
<table>
<thead>
<tr>
<th>Material</th>
<th>Density (g/cm³)</th>
<th>Susceptibility (10⁻⁶ c.g.s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graphite</td>
<td>2.26</td>
<td>-595</td>
</tr>
<tr>
<td>Carbon</td>
<td>2.26</td>
<td>-204</td>
</tr>
<tr>
<td>Bismuth</td>
<td>9.75</td>
<td>-164</td>
</tr>
<tr>
<td>Antimony</td>
<td>6.69</td>
<td>-67</td>
</tr>
<tr>
<td>Indium</td>
<td>7.31</td>
<td>-51</td>
</tr>
<tr>
<td>Thallium</td>
<td>11.85</td>
<td>-37</td>
</tr>
<tr>
<td>Gold</td>
<td>19.32</td>
<td>-34</td>
</tr>
<tr>
<td>Tin</td>
<td>5.75</td>
<td>-23</td>
</tr>
<tr>
<td>Carbon</td>
<td>3.51</td>
<td>-21.8</td>
</tr>
<tr>
<td>Alumina</td>
<td>3.97</td>
<td>-18.1</td>
</tr>
<tr>
<td>Silica</td>
<td>2.64</td>
<td>-16.3</td>
</tr>
<tr>
<td>Lead</td>
<td>11.35</td>
<td>-15.8</td>
</tr>
<tr>
<td>Zinc</td>
<td>7.13</td>
<td>-15.7</td>
</tr>
<tr>
<td>Pyrex Glass</td>
<td></td>
<td>-13.88</td>
</tr>
<tr>
<td>Copper</td>
<td>8.92</td>
<td>-9.63</td>
</tr>
<tr>
<td>Water</td>
<td>0.933</td>
<td>-9.05</td>
</tr>
<tr>
<td>Human Tissue</td>
<td>1.00-1.05</td>
<td>-11.0 to -7.0</td>
</tr>
<tr>
<td>Whole Blood</td>
<td>1.057</td>
<td>-7.9</td>
</tr>
<tr>
<td>Air</td>
<td>0.00129</td>
<td>0.36</td>
</tr>
<tr>
<td>Tin</td>
<td>7.31</td>
<td>2.4</td>
</tr>
<tr>
<td>Rubidium</td>
<td>1.532</td>
<td>3.8</td>
</tr>
<tr>
<td>Cesium</td>
<td>1.873</td>
<td>5.2</td>
</tr>
<tr>
<td>Sodium</td>
<td>0.971</td>
<td>8.5</td>
</tr>
<tr>
<td>Magnesium</td>
<td>1.74</td>
<td>11.7</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2.70</td>
<td>20.7</td>
</tr>
<tr>
<td>Calcium</td>
<td>1.55</td>
<td>21.7</td>
</tr>
<tr>
<td>Tungsten</td>
<td>19.3</td>
<td>77.2</td>
</tr>
<tr>
<td>Titanium</td>
<td>4.54</td>
<td>182</td>
</tr>
<tr>
<td>Niobium</td>
<td>8.57</td>
<td>237</td>
</tr>
<tr>
<td>Platinum</td>
<td>21.45</td>
<td>279</td>
</tr>
<tr>
<td>Chromium</td>
<td>7.19</td>
<td>320</td>
</tr>
<tr>
<td>Vanadium</td>
<td>6.11</td>
<td>384</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>8.0</td>
<td>3520-6700</td>
</tr>
<tr>
<td>Nickel</td>
<td>8.9</td>
<td>600</td>
</tr>
<tr>
<td>Iron</td>
<td>7.84</td>
<td>200000</td>
</tr>
<tr>
<td>Supermalloy</td>
<td>8.77</td>
<td>1000000</td>
</tr>
</tbody>
</table>

Table 1.1 Magnetic susceptibilities of diamagnetic, paramagnetic and ferromagnetic materials [9].
Shim pieces were placed on a cylindrical surface placed inside the magnet bore, surrounding the subject’s head and RF coil. However, they only implemented a passive system with a second-order shim, which is usually available in most of MRI scanners.

Hsu et al. [60] improved the local field homogeneity within the frontal lobe by placing current carrying electromagnetic shim coils within subjects’ mouths. However, heat dissipation due to the current flowing over the coils may cause the intensity of generated shim field to change. In addition, placing a shim setup inside the mouth is not comfortable, particularly for elderly patients and children.

Koch et al. [61] introduced a sample-specific diamagnetic (bismuth, $\chi = -164$ ppm and zirconium, $\chi = +92$ ppm) passive shim technique to improve the field homogeneity within mouse brain at high fields. However, for human studies, extension of the shim system may require a large number of shim inserts, which may not be practical.

Juchem et al. [62] improved the field homogeneity in the human prefrontal cortex using localized shim fields provided by a limited number of electrical coils. The coils were fixed on the surface of the facial mask that was positioned above the subject’s mouth/chin and inside the RF coil. However, this method may affect the existing active shimming over the subject’s brain. In addition, current through the electric coils may not be uniform due to the resistance of the coils and heat dissipation; that may cause difficulties in producing the required strength for the shim field. Furthermore, placing the shim setup inside the head coil reduces the available room (or patient space) and may cause discomfort for subjects.

In this work, we propose to develop a local high-order (i.e. third order and higher) passive shim method, which will provide improved MRI quality and spectral resolution in MRS.
1.3.3. Specific aims

Our specific aims are as follows.

**Aim 1: To recognize the limitations of the existing active shim set.**

High magnetic field strength (>3T) leads to better spectral dispersion in MR spectroscopy; and the benefit of simplified spectra at high field can only be appreciated if an optimal shimming is achieved. However, some conditions may require increased shim power, which may not always be available, to correct significant field non-uniformity. In most MRI scanners, only the first-order and second-order shim coils are available. Higher-order shims such as the third-order shim, however, are limited due to space limitation in most of MRI scanners. Therefore, the first aim is to determine the capability and limitation of the existing active shim set (see Chapter 2).

**Aim 2: To construct a desired passive shim coil.**

The magnetic susceptibility variations lead to image artifacts and signal losses. Therefore, due to insufficient of the existing active shim power, local passive shimming is used to reduce the field deviations over a particular region of interest at high field MRI. Hence the second aim is to design and construct a desired third order shim set. This work involves with the determination of the correct position, dimensions, and magnetic susceptibility of the shim pieces to be used to generate the desired magnetic field on the surface of a cylinder (see Chapter 3). This shim set will first be test on a phantom and then be test on human subjects.
Aim 3: To evaluate the passive shim system on human brain MRI and MRS.

Passive shim fields can optimize the field variation due to the third order perturbations over the entire brain. Therefore, the third aim is to implement a third order passive shim coil along with the existing active shim set to improve the local magnetic field over the entire brain (see Chapter 4).

1.4. References


Chapter 2

Magnetic field mapping using gradient-echo pulse sequence

2.1. Introduction

One of the major tasks of this work is to develop an automatic shim technique to optimize the magnetic field at high-field scanners (≥3T). A number of electronic shim techniques have been carried out for improving $B_0$ field homogeneity and shimming techniques on a desired volume of interest using localization techniques have been demonstrated [1-5,13-19]. A novel active shim technique was developed and tested on both phantom and human subjects by Holtz et al [3]. The volume of interest (VOI) is determined by using a surface coil that produces signals from the selected VOI. The signal integral over the free induction decay (FID) was used for field optimization; and the shim technique has to be performed iteratively [3]. Hence, this shim technique is time-consuming and is undesirable for in vivo application. Additionally, conventional localized active shimming techniques have been somewhat constrained to the alteration of the X, Y and Z shim coils by observing the FID or spectral peak amplitudes as shim settings varied. The reason for using linear shim coils alone is because the alteration of a second- or higher-order shim will couple with the linear terms when the VOI origin is away from the isocenter of the shim coil set, [4, 5]. FASTMAP (fast automatic shimming technique by mapping
along projections) [13] is an effective technique for finding optimal currents for higher-order shims and works well at high-field. Its alteration of optimal shim currents in first- and second-order shim coils is achieved by mapping the magnetic field along six column projections. However, FASTMAP technique is restricted to shim a specific (i.e. selected) region of interest [12].

In this work, a spherical phantom was used to calibrate the active shim set. The magnetic field distributions of the first- and second-order shims were acquired by stepping the shim current or DAC settings (i.e. Digital to Analog Converter that controls the voltage across the different shim coils). The first- and second-order spherical harmonic amplitudes of the magnetic field distribution were obtained and the calibration table (i.e. harmonic coefficient constants vs. coils) was established.

2.2. Theory

A collection of electromagnetic coils (the shim set) is energized with suitable current level to correct the inhomogeneous magnetic field in space by producing an additional magnetic field. Each shim coil generates one particular magnetic field distribution that corresponds to a specific spherical harmonic function [14]. The first- and second-order spherical harmonic functions are shown in Table 2.1. Since the spherical harmonics are orthogonal, the alteration of currents in each shim coil should be independent [20]. The numerical analysis of inhomogeneous magnetic fields, derivation of the calibration table, and evaluation of appropriate shim currents for the first- and second-order shim coils are given below.
Table 2.1 The short-hand notation, amplitude, spherical, and Cartesian descriptions of the first- and second-order spatial-dependent spherical harmonic functions.

<table>
<thead>
<tr>
<th>n</th>
<th>m</th>
<th>Shorthand notation</th>
<th>Coefficients $(\alpha_{nm})$</th>
<th>Spatial dependence function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>$X$</td>
<td>$\alpha_{11}$</td>
<td>$r \sin \theta \cos \phi$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>$Z$</td>
<td>$\alpha_{10}$</td>
<td>$r \cos \theta$</td>
</tr>
<tr>
<td>1</td>
<td>−1</td>
<td>$Y$</td>
<td>$\alpha_{1-1}$</td>
<td>$r \sin \theta \sin \phi$</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>$X2−Y2$</td>
<td>$\alpha_{22}$</td>
<td>$r^2 \sin^2 \theta \cos 2\phi$</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>$ZX$</td>
<td>$\alpha_{21}$</td>
<td>$r^2 \sin \theta \cos \theta \cos \phi$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>$Z2C$</td>
<td>$\alpha_{20}$</td>
<td>$r^2(3\cos^2 \theta − 1)/2$</td>
</tr>
<tr>
<td>2</td>
<td>−1</td>
<td>$ZY$</td>
<td>$\alpha_{2-1}$</td>
<td>$r^2 \sin \theta \cos \theta \sin \phi$</td>
</tr>
<tr>
<td>2</td>
<td>−2</td>
<td>$XY$</td>
<td>$\alpha_{2-2}$</td>
<td>$r^2 \sin \theta \cos \theta \sin \phi$</td>
</tr>
</tbody>
</table>
2.2.1. Static magnetic field analysis

In a region of interest, the current density is zero ($\mathbf{J} = 0$); and the static magnetic field $\mathbf{B}_z$ is described by Laplace’s equation.

$$\nabla^2 \mathbf{B}_z = 0$$  \[2.1\]

The solution of the static magnetic field $\mathbf{B}_z$ within the region of interest (i.e. within the phantom and human brain in this case) can be expressed as spherical harmonic series [16-18].

The static magnetic field within any object can be expressed as sum of spherical harmonic functions [14, 20, 21].

$$\mathbf{B}_z (r, \theta, \phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} A_{n,m} r^n \cdot P_{n,m} (\cos \theta) \cdot e^{im\phi}$$  \[2.2\]

Here, $r, \theta$ and $\phi$ are the radial positions, polar angle, and azimuthal angle, respectively; $n$ and $m$ are integers obeying the conditions $n \geq m \geq 0$; $n$ is the order and $m$ is the degree of the spherical harmonics. Additionally, $A_{n,m}$ are the coefficients of harmonic components and consistent with the boundary condition, while $P_{n,m} (\cos \theta)$ is the Ferrer’s associated Legendre polynomial [22].

Eq. (2.2) can be expressed in terms of Cartesian coordinate as Eq. (2.3), and here, $x, y$ and $z$ are Cartesian coordinates describing a 3D space.
\[ B_z(x, y, z) = c + \alpha_{11}x + \alpha_{10}z + \alpha_{1-1}y + \alpha_{22}(x^2 - y^2) + \alpha_{21}2x + \alpha_{20}(z^2 - 1/2(x^2 + y^2)) + \alpha_{2-1}zy + \alpha_{2-2}xy + \alpha_{33}(x^3 - (3y^2)x) + \alpha_{32}(x^2 - y^2)z + \alpha_{31}(4z^2x - (x^3 + y^2)x) + \alpha_{30}(1/2(2z^3 - 3x^2z - 3y^2z) + \alpha_{3-1}(4z^2x - (x^3 + y^2)x) + \alpha_{3-2}2xyz + \alpha_{3-3}(3x^2y - y^3) + \ldots \]  

where \( c \) and \( \alpha \) represent the 0\(^{th}\) and higher order coefficients of spherical harmonic components. The matrix representation of the magnetic field for Eq. (2.3) is given in Eq. (2.4).  

\[ B_z(x, y, z) = \sum_{n=0}^{\infty} \sum_{m=0}^{n} F_{n,m}(x_i, y_j, z_k) \cdot \alpha_{nm} \]  

where \( \alpha_{nm} \) are the spherical harmonic coefficients and \( F_{n,m}(x_i, y_j, z_k) \) is the function of spatial dependent spherical harmonics in the Cartesian coordinate. The symbols of \( x_i, y_j, \) and \( z_k \) as well as the subscripts of \( i, j, \) and \( k \) represent the orientation and the matrix position of the Cartesian coordinate, respectively.  

2.2.2. Magnetic field mapping  

The static magnetic field map can be evaluated from comparing two phase images acquired by using the gradient-echo pulse sequence with different echo times. The distribution of field inhomogeneity, \( \Delta B_0(x, y, z) \), is related to the phase evolution, \( \Delta \phi(x, y, z) \), in each voxel location at a given different echo time (\( \Delta TE \)), which can be described as the following equation:[6]
\[ \Delta B_0(x, y, z) = \Delta \phi(x, y, z) / (\gamma \cdot \Delta T E). \] \[ \text{[2.5]} \]

Here, \( \gamma \) is the gyromagnetic ratio in units of radian/s/T and depends on the particle or nucleus.

For proton, the gyromagnetic ratio is \( \gamma(H) = 2.675 \times 10^8 \text{ rad} \cdot s^{-1} \cdot T^{-1} \). Since the phase can only have magnitude of \(-2\pi < \phi < 2\pi\), phase unwrapping is performed as necessary.

The distribution of precessional frequency, \( f(x, y, z) \), is related to the inhomogeneous field distribution in each voxel location and can be given as

\[ f(x, y, z) = \frac{\gamma \cdot \Delta B_0(x, y, z)}{2\pi}. \] \[ \text{[2.6]} \]

Hence, substituting Eq. (2.6) with Eq. (2.3), it leads to Eq. (2.7).

\[ f(x, y, z) = \left( c' + \eta_{11}x + \eta_{10}z + \eta_{1-1}y + \cdots \right. \]
\[ \left. + \eta_{22}(x^2 - y^2) + \eta_{21}zx + \eta_{20}(z^2 - 1/2(x^2 + y^2)) + \eta_{2-1}zy + \eta_{2-2}xy + \cdots \right) \] \[ \text{[2.7]} \]

Here, \( f(x, y, z) \) are the measured phase shift in frequencies (Hz), \( c' \) and \( \eta \) represent the 0th and higher order coefficients of spherical harmonic components. The matrix representation of Eq. (2.7) is written as

\[ f(x, y, z) = \sum_{n=0}^{\infty} \sum_{m=0}^{n} F_{n,m}(x, y, z) \cdot \eta_{nm}. \] \[ \text{[2.8]} \]
where $\eta_{nm}$ are the spherical harmonic coefficients.

### 2.2.3. Determination of calibration table

Phantom study was used to calibrate shim current settings by systematically varying the DAC settings. Optimized spherical harmonic coefficients of the first- and second-order shim coil \((i.e., X, Y, Z, X2 - Y2, XZ, Z2C, YZ and XY\) coil) can be obtained using least-squares method as

$$
\eta_{nm,g,i} = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left( F_{n,m}(x_i,y_j,z_k) \cdot f(x,y,z) \right) \frac{1}{DAC_{i,g}}.
$$

Here $\eta_{nm,g,i}$ are the spherical harmonic coefficients of the $g^{th}$ shim coil at $i^{th}$ DAC step. It is assumed that the spherical harmonic coefficients of each shim coil linearly vary with the DAC steps which can be written as follows.

$$
\eta_{nm,i} = C_{nm,g} \cdot DAC_{i,g}.
$$

Here $C_{nm,g}$ is defined as the calibration constant for each spherical harmonic. Hence, the $C_{nm,g}$ can be determined as

$$
C_{nm,g} = \left( DAC_{i,g} \cdot DAC_{i,g} \right)^{-1} \cdot DAC_{i,g} \cdot \eta_{nm,g,i}.
$$

Updated shim values are obtained from...
\[ DAC'_g = DAC_g + C_{nm,g} \cdot \eta_{nm,g} \]  

[2.12]

where \( DAC_g \) and \( DAC'_g \) are the present and updated shim settings, respectively.

### 2.3. Methods

#### 2.3.1. Experimental set-up and data acquisition

All experimental shim procedures were carried out on a 4T whole-body Varian INOVA (Palo Alto, CA) MRI scanner, which was equipped with the following resistive shim coils: \( X, Z, Y \) \((n = 1, m = 1, 0, -1 \text{ where } n \text{ is the order and } m \text{ is the degree})\) and second-order \((X^2 - Y^2, ZX, Z^2, ZY, XY \text{ (} n = 2, m = -2, -1, 0, 1, 2))\). A TEM volume head coil was used for RF transmission and reception. A modified 3D gradient-echo pulse sequence as shown in Fig. 2.1 was implemented to obtain images for the calculation of \( B_0 \) field maps. A 3D magnetic field map is produced by comparing two phase images acquired with two different echo times \((\text{TE1} = 5.25 \text{ ms and TE2} = 7 \text{ ms})\) (Eq. (2.5)). Magnetic field gradient pulses can produce eddy-currents in nearby conducting structures, which may affect the accuracy of the filed map determination. The artifacts of this influence can be reduced by fixing the relative timing of the gradient pulses immediately preceding excitation pulses or acquisition windows during \( \delta_1 \) and \( \delta_2 \). All acquisitions used a 256x256x256mm field of view, 128x64x64 acquisition matrix, 10° pulse flip angle, a repetition time (TR) of 16ms, and echo times of 5.25ms and 7ms.
Figure 2.1 Pulse sequence of the modified 3D gradient-echo used for collecting magnetic field maps.
Phase-difference reconstruction of the two different echo times is used to extract the $B_0$ magnetic field. 3D phase unwrapping was performed as necessary.

2.3.2. Evaluation of the Calibration table for active shimming on a phantom

The magnetic field distributions of 8 shim coils were determined by stepping each shim current (DAC steps) setting from -15000 to 15000 by an increment of 5000 each step. The corresponding field maps of the phantom were determined using Eq. 2.5. Frequency distributions of all shim coils at each DAC step were projected onto a spherical harmonic model (Eq. (2.9)) to obtain the first- and second-order spherical harmonic coefficients. The spherical harmonic calibration constant of each shim coil was determined by calculating the slope from a plot of spherical harmonic coefficients vs. DAC values. Furthermore, the corresponding linear correlations were determined.

2.3.3. Shimming on a spherical phantom at 4T MRI

As described in the section 2.3.1, a 3D gradient-echo pulse sequence with modifications to reduce eddy currents was used to obtain $B_0$ field maps of the phantom. A 3D phase image is produced by comparing phase maps between two images acquired at different echo times and phase-difference images were computed with 3D phase unwrapping as necessary. A 3D magnetic field map is generated by using the 3D phase images following frequency distortion correction along read-out direction. The field distribution of the phantom was measured before shimming, and the optimal 1$^{st}$ and 2$^{nd}$ order shim currents were determined using Eq. (2.12). Then the field distribution was measured by using the updated the 1$^{st}$ and 2$^{nd}$ shim described above. The experimental results were compared to the theoretical values.
2.3.4. Comparison of FASTMAP and simulated $B_0$ field map shimming for a human brain MRI at high-field

The field maps of the human brain were measured using FASTMAP. The optimal shim currents for the first- and second-order coils were determined using Eq. (2.12). The measured and simulated field maps were compared for the human brains. Furthermore, the full width at half maximum (FWHM) of the Gaussian fitted frequency distributions were compared as well.

2.4. Results

2.4.1. Calibration table for the first- and second-order active shimming coils

The magnetic field maps of the spherical phantom in the axial, coronal and sagittal plane for the $X$, $Z$, $Y$, $X2-Y2$, $ZX$, $Z2C$, $ZY$ and $XY$ shim coil with stepping DAC steps from -15000 to 15000 at steps of 5000 are illustrated in Fig 2.2 (i)-(vi), respectively. Then the spherical harmonic coefficients, slopes (i.e. the calibration constants), and the linear correlation coefficients were calculated for each coil using Eq. (2.9) and are presented in Table 2.2 (i)-(viii). The slopes and linear correlation coefficients of spherical harmonic coefficients are computed based on a linear model assumption. Here, the slopes are the spherical harmonic calibration constants. Additionally, the plot of the spherical harmonic coefficients as a function of DAC values for all coils are shown in Fig. 2.3 (i)-(viii). It is clearly seen in Fig 2.3 (i) and Table 2.2 (i) that the performance of the X gradient coil has a strong linear correlation ($R^2 = 1$) with the X amplifier, while other coils have little or weak correlations ($< 0.8$) with the X amplifier. Similar to the $x$
all other harmonic components (i.e. \( z, y, x^2 - y^2, xz, z^2 - (x^2 + y^2)/2, zy \) and \( xy \)) showed a strong self-correlation (see Fig 2.3 (ii - viii) and Table 2.2 (ii – viii)).

Table 2.3 and 2.4 display the table of spherical harmonic calibration constants and its corresponding linear correlation coefficients for the first- and second-order coils. The direct term of harmonic calibration coefficient constant of both \( x \) and \( y \) is \(-0.0023 \text{ Hz.cm}^{-1}.\text{DAC}^{-1}\) whereas the \( z \) component is \(0.0023 \text{ Hz.cm}^{-1}.\text{DAC}^{-1}\), while the calibration constants (direct and cross terms) of the rest of the spherical harmonic components are in the range of \(10^{-7} \text{ Hz.cm}^{-1}.\text{DAC}^{-1}\) and \(10^{-8} \text{ Hz.cm}^{-2}.\text{DAC}^{-1}\) that are much smaller than the direct term.

Furthermore, the direct term of the second-order spherical harmonic components of \( x^2 - y^2, xz, z^2 - (x^2 + y^2)/2, zy \) and \( xy \) have the calibration constants of \(-0.00021, -0.00018, -0.00045, 0.00018 \text{ and } 0.00021 \text{ Hz.cm}^{-2}.\text{DAC}^{-1}\), respectively. Again, the cross terms are much weaker and are in the range of \(10^{-5}- 10^{-6} \text{ Hz.cm}^{-1}.\text{DAC}^{-1}\) and \(10^{-7}- 10^{-8} \text{ Hz.cm}^{-2}.\text{DAC}^{-1}\) for the first and second orders, respectively. However, it is also clearly observable in Table 2.4 that the second-order coils showed some strong cross-correlation with the first-order coils and in some case other second-order coils. The ones with stronger cross-correlations (i.e. \( R^2 \geq 0.9 \)) are highlighted in light green (Table 2.4). For example, the \( xy \) coil shows a strong correlation with all first-order coils and some second-order coils (i.e. \( x^2-y^2, xz, \) and \( zy \)) as well as the main coil (i.e. \( c \), the zero order term). The corresponding calibration constants are \(-0.00001, -0.0002, 0.00002 \text{ (Hz.cm}^{-1}.\text{DAC}^{-1}\)), \(0.000003, 0.000005, 0.0000009 \text{ (Hz.cm}^{-2}.\text{DAC}^{-1}\)), and \(-0.0002 \text{ (Hz.DAC}^{-1}\)) for \( x, y, z, x^2-y^2, xz, zy, \) and \( c \), respectively (highlighted in light blue in Table 2.3). Their corresponding linear correlation coefficients are highlighted in light green in Table 2.4. The strong correlation implies changes in magnetic field are significantly associated with changes in DAC values.
Figure 2.2(i) Phantom study used to calibrate shim current settings. The magnetic field distributions of X shim coil were determined by setting the shim current values (i.e. DAC) with (A) -15000, (B) -10000, (C) -5000, (D) 0, (E) 5000, (F) 10000, and (G) 15000.

Figure 2.2(ii) Phantom study used to calibrate shim current settings. The magnetic field distributions of Y shim coil were determined by setting the shim current values (i.e. DAC) with (A) -15000, (B) -10000, (C) -5000, (D) 0, (E) 5000, (F) 10000, and (G) 15000.
Figure 2.2(iii) Phantom study used to calibrate shim current settings. The magnetic field distributions of Z shim coil were determined by setting the shim current values (i.e. DAC) with (A) -15000, (B) -10000, (C) -5000, (D) 0, (E) 5000, (F) 10000, and (G) 15000.

Figure 2.2(iv) Phantom study used to calibrate shim current settings. The magnetic field distributions of X2 – Y2 shim coil were determined by setting the shim current values (i.e. DAC) with (A) -15000, (B) -10000, (C) -5000, (D) 0, (E) 5000, (F) 10000, and (G) 15000.
Figure 2.2(v) Phantom study used to calibrate shim current settings. The magnetic field distributions of XZ shim coil were determined by setting the shim current values (i.e. DAC) with (A) -15000, (B) -10000, (C) -5000, (D) 0, (E) 5000, (F) 10000, and (G) 15000.

Figure 2.2(vi) Phantom study used to calibrate shim current settings. The magnetic field distributions of Z2C shim coil were determined by setting the shim current values (i.e. DAC) with (A) -15000, (B) -10000, (C) -5000, (D) 0, (E) 5000, (F) 10000, and (G) 15000.
Figure 2.2(vii) Phantom study used to calibrate shim current settings. The magnetic field distributions of ZY shim coil were determined by setting the shim current values (i.e. DAC) with (A) -15000, (B) -10000, (C) -5000, (D) 0, (E) 5000, (F) 10000, and (G) 15000.

Figure 2.2(viii) Phantom study used to calibrate shim current settings. The magnetic field distributions of XY shim coil were determined by setting the shim current values (i.e. DAC) with (A) -15000, (B) -10000, (C) -5000, (D) 0, (E) 5000, (F) 10000, and (G) 15000.
Table 2.2(i) The first- and second-order spherical harmonic coefficients ($Hz.cm^{-1}$ and $Hz.cm^{-2}$) at a given DAC value for X shim coil along with its corresponding spherical harmonic calibration constants ($Hz.cm^{-1}.DAC^{-1}$ and $Hz.cm^{-2}.DAC^{-1}$) and the linear correlation coefficients.

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Table 2.2(ii) The first- and second-order spherical harmonic coefficients ($Hz.cm^{-1}$ and $Hz.cm^{-2}$) at a given DAC value for Y shim coil along with its corresponding spherical harmonic calibration constants ($Hz.cm^{-1}.DAC^{-1}$ and $Hz.cm^{-2}.DAC^{-1}$) and the linear correlation coefficients.

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Table 2.2(iii) The first- and second-order spherical harmonic coefficients (*Hz.cm⁻¹* and *Hz.cm⁻²*) at a given DAC value for Z shim coil along with its corresponding spherical harmonic calibration constants (*Hz.cm⁻¹.DAC⁻¹* and *Hz.cm⁻².DAC⁻¹*) and the linear correlation coefficients.

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Table 2.2(iv) The first- and second-order spherical harmonic coefficients (*Hz.cm⁻¹* and *Hz.cm⁻²*) at a given DAC value for X2 – Y2 shim coil along with its corresponding spherical harmonic calibration constants (*Hz.cm⁻¹.DAC⁻¹* and *Hz.cm⁻².DAC⁻¹*) and the linear correlation coefficients.

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<td>-8.6E-02</td>
<td>-6.1E-02</td>
<td>3.1E-06</td>
<td>9.6E-01</td>
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<tr>
<td>A₂₂ – X2 - Y2</td>
<td>3.2E+00</td>
<td>2.1E+00</td>
<td>1.1E+00</td>
<td>2.9E+00</td>
<td>-1.1E+00</td>
<td>-2.1E+00</td>
<td>-3.2E+00</td>
<td>-2.1E+00</td>
<td>1.0E+00</td>
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<tr>
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<td>ZX</td>
<td>8.6E-02</td>
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<td>-6.3E-06</td>
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<td>Z₂C</td>
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<td>7.1E-01</td>
</tr>
<tr>
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<td>ZY</td>
<td>-4.2E-02</td>
<td>-2.5E-02</td>
<td>-9.6E-03</td>
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<td>3.2E-02</td>
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<td>6.9E-02</td>
<td>3.7E-06</td>
<td>9.9E-01</td>
</tr>
<tr>
<td>A₂₁</td>
<td>XY</td>
<td>-4.3E-02</td>
<td>-3.0E-02</td>
<td>-1.5E-02</td>
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<td>-2.6E-01</td>
<td>1.4E+00</td>
<td>3.4E-04</td>
<td>9.9E-01</td>
</tr>
<tr>
<td>A₂₁</td>
<td>C</td>
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<td>-7.0E+00</td>
<td>-5.6E+00</td>
<td>-4.5E+00</td>
<td>-2.1E+00</td>
<td>-2.6E+00</td>
<td>1.4E+00</td>
<td>3.4E-04</td>
<td>9.9E-01</td>
</tr>
</tbody>
</table>
Table 2.2(v) The first- and second-order spherical harmonic coefficients ($Hz.cm^{-1}$ and $Hz.cm^{-2}$) at a given DAC value for ZX shim coil along with its corresponding spherical harmonic calibration constants ($Hz.cm^{-1}.DAC^{-1}$ and $Hz.cm^{-2}.DAC^{-1}$) and the linear correlation coefficients.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Notation</th>
<th>-15000</th>
<th>-10000</th>
<th>-5000</th>
<th>0</th>
<th>5000</th>
<th>10000</th>
<th>15000</th>
<th>Slope</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{31}$</td>
<td>X</td>
<td>-2.8E-01</td>
<td>-2.7E-01</td>
<td>-2.1E-01</td>
<td>-1.1E-01</td>
<td>-3.0E-02</td>
<td>4.0E-02</td>
<td>5.9E-02</td>
<td>1.3E-05</td>
<td>9.7E-01</td>
</tr>
<tr>
<td>$A_{30}$</td>
<td>Z</td>
<td>-1.9E-01</td>
<td>-3.2E-01</td>
<td>-4.7E-01</td>
<td>-6.1E-01</td>
<td>-7.2E-01</td>
<td>-8.0E-01</td>
<td>-8.4E-01</td>
<td>-2.2E-05</td>
<td>9.7E-01</td>
</tr>
<tr>
<td>$A_{31}$</td>
<td>Y</td>
<td>-4.2E-01</td>
<td>-3.3E-01</td>
<td>-2.2E-01</td>
<td>-1.1E-01</td>
<td>6.0E-03</td>
<td>1.2E-01</td>
<td>2.1E-01</td>
<td>2.2E-05</td>
<td>1.0E+00</td>
</tr>
<tr>
<td>$A_{22}$</td>
<td>X2 - Y2</td>
<td>1.8E-02</td>
<td>1.1E-02</td>
<td>3.1E-03</td>
<td>2.9E-03</td>
<td>5.2E-03</td>
<td>1.0E-02</td>
<td>1.0E-02</td>
<td>-1.6E-07</td>
<td>1.0E-01</td>
</tr>
<tr>
<td>$A_{33}$</td>
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<td>-2.6E+00</td>
<td>-1.8E-04</td>
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<td>4.4E-02</td>
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<td>-6.4E-07</td>
<td>9.6E-01</td>
</tr>
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<td>ZY</td>
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<td>1.9E-02</td>
<td>1.6E-02</td>
<td>1.7E-02</td>
<td>1.6E-02</td>
<td>1.5E-02</td>
<td>1.4E-02</td>
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<td>$A_{22}$</td>
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<td>1.8E-03</td>
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<td>2.2E-03</td>
<td>2.9E-03</td>
<td>-3.4E-04</td>
<td>-2.3E-03</td>
<td>-1.4E-03</td>
<td>-1.5E-07</td>
<td>6.5E-01</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>-7.1E+00</td>
<td>-6.3E+00</td>
<td>-5.4E+00</td>
<td>-4.5E+00</td>
<td>-3.2E+00</td>
<td>-2.0E+00</td>
<td>-6.1E-01</td>
<td>2.2E-04</td>
<td>9.9E-01</td>
</tr>
</tbody>
</table>

Table 2.2(vi) The first- and second-order spherical harmonic coefficients ($Hz.cm^{-1}$ and $Hz.cm^{-2}$) at a given DAC value for Z2C shim coil along with its corresponding spherical harmonic calibration constants ($Hz.cm^{-1}.DAC^{-1}$ and $Hz.cm^{-2}.DAC^{-1}$) and the linear correlation coefficients.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Notation</th>
<th>-15000</th>
<th>-10000</th>
<th>-5000</th>
<th>0</th>
<th>5000</th>
<th>10000</th>
<th>15000</th>
<th>Slope</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{31}$</td>
<td>X</td>
<td>-4.8E-01</td>
<td>3.8E-01</td>
<td>-2.5E-01</td>
<td>-7.5E-02</td>
<td>-4.6E-03</td>
<td>1.6E-01</td>
<td>2.9E-01</td>
<td>2.6E-05</td>
<td>9.9E-01</td>
</tr>
<tr>
<td>$A_{30}$</td>
<td>Z</td>
<td>3.4E+00</td>
<td>3.1E-01</td>
<td>-2.7E-01</td>
<td>-5.0E-01</td>
<td>1.1E-01</td>
<td>1.2E+00</td>
<td>2.7E+00</td>
<td>4.6E-05</td>
<td>4.9E-01</td>
</tr>
<tr>
<td>$A_{31}$</td>
<td>Y</td>
<td>2.6E-01</td>
<td>3.2E-01</td>
<td>6.7E-02</td>
<td>-3.1E-02</td>
<td>3.4E-02</td>
<td>-1.2E-01</td>
<td>-1.8E-01</td>
<td>-1.2E-05</td>
<td>9.9E-01</td>
</tr>
<tr>
<td>$A_{22}$</td>
<td>X2 - Y2</td>
<td>-1.2E-02</td>
<td>-1.5E-02</td>
<td>-1.4E-02</td>
<td>-1.0E-02</td>
<td>-1.8E-02</td>
<td>-2.4E-02</td>
<td>-2.4E-02</td>
<td>-3.9E-07</td>
<td>6.3E-01</td>
</tr>
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<td>ZX</td>
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<td>-1.1E-02</td>
<td>-5.1E-03</td>
<td>-2.1E-03</td>
<td>2.6E-03</td>
<td>3.5E-03</td>
<td>8.4E-07</td>
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<td>9.0E-01</td>
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<tr>
<td>$A_{30}$</td>
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<td>4.5E+00</td>
<td>2.2E+00</td>
<td>6.5E-02</td>
<td>2.2E+00</td>
<td>4.4E+00</td>
<td>-6.7E+00</td>
<td>-4.3E-04</td>
<td>1.0E+00</td>
</tr>
<tr>
<td>$A_{31}$</td>
<td>ZY</td>
<td>-3.1E-02</td>
<td>-2.0E-02</td>
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<td>-1.0E-02</td>
<td>-2.0E-02</td>
<td>-7.9E-03</td>
<td>-1.4E-03</td>
<td>-8.3E-07</td>
<td>8.7E-01</td>
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<tr>
<td>$A_{22}$</td>
<td>XY</td>
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<td>-5.7E-03</td>
<td>-4.2E-03</td>
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<td>3.5E-04</td>
<td>3.5E-07</td>
<td>8.6E-01</td>
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<tr>
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Table 2.2(vii) The first- and second-order spherical harmonic coefficients (Hz.cm\(^{-1}\) and Hz.cm\(^{-2}\)) at a given DAC value for ZY shim coil along with its corresponding spherical harmonic calibration constants (Hz.cm\(^{-1}\).DAC\(^{-1}\) and Hz.cm\(^{-2}\).DAC\(^{-1}\)) and the linear correlation coefficients.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Notation</th>
<th>(-15000)</th>
<th>(-10000)</th>
<th>(-5000)</th>
<th>0</th>
<th>5000</th>
<th>10000</th>
<th>15000</th>
<th>Slope</th>
<th>R(^2)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>X</td>
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<td>-3.4E-01</td>
<td>-2.4E-01</td>
<td>-1.3E-01</td>
<td>-1.6E-02</td>
<td>6.1E-02</td>
<td>1.9E-01</td>
<td>2.1E-05</td>
<td>1.0E+00</td>
</tr>
<tr>
<td>(A_{12})</td>
<td>Z</td>
<td>-4.1E-01</td>
<td>-3.8E-01</td>
<td>-5.5E-01</td>
<td>-5.9E-01</td>
<td>-6.3E-01</td>
<td>-6.4E-01</td>
<td>-6.0E-01</td>
<td>-7.0E-06</td>
<td>8.0E-01</td>
</tr>
<tr>
<td>(A_{13})</td>
<td>Y</td>
<td>1.8E-01</td>
<td>1.1E-01</td>
<td>2.9E-03</td>
<td>-1.2E-01</td>
<td>-2.4E-01</td>
<td>3.1E-01</td>
<td>3.1E-01</td>
<td>3.7E-01</td>
<td>2.0E-05</td>
</tr>
<tr>
<td>(A_{22})</td>
<td>X2 - Y2</td>
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<td>-8.4E-03</td>
<td>-3.1E-03</td>
<td>1.2E-03</td>
<td>-3.7E-03</td>
<td>-7.6E-03</td>
<td>-7.4E-03</td>
<td>1.7E-07</td>
<td>1.3E-01</td>
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<td>ZY</td>
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<td>-1.0E-02</td>
<td>-8.8E-03</td>
<td>1.0E-02</td>
<td>-1.0E-02</td>
<td>-7.6E-03</td>
<td>-7.4E-03</td>
<td>2.4E-07</td>
<td>6.3E-01</td>
</tr>
<tr>
<td>(A_{32})</td>
<td>ZC</td>
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<td>5.0E-02</td>
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<td>4.4E-02</td>
<td>4.4E-02</td>
<td>3.9E-02</td>
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<td>-5.7E-07</td>
<td>9.8E-01</td>
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<td>-1.8E+00</td>
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<td>2.6E+00</td>
<td>1.8E-04</td>
<td>1.0E+00</td>
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</table>

Table 2.2(viii) The first- and second-order spherical harmonic coefficients (Hz.cm\(^{-1}\) and Hz.cm\(^{-2}\)) at a given DAC value for XY shim coil along with its corresponding spherical harmonic calibration constants (Hz.cm\(^{-1}\).DAC\(^{-1}\) and Hz.cm\(^{-2}\).DAC\(^{-1}\)) and the linear correlation coefficients.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Notation</th>
<th>(-15000)</th>
<th>(-10000)</th>
<th>(-5000)</th>
<th>0</th>
<th>5000</th>
<th>10000</th>
<th>15000</th>
<th>Slope</th>
<th>R(^2)</th>
</tr>
</thead>
<tbody>
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<td>(A_{11})</td>
<td>X</td>
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<td>2.5E-02</td>
<td>-2.7E-02</td>
<td>-1.3E-01</td>
<td>-1.9E-01</td>
<td>-2.4E-01</td>
<td>-3.2E-01</td>
<td>-1.4E-05</td>
<td>9.9E-01</td>
</tr>
<tr>
<td>(A_{12})</td>
<td>Z</td>
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<td>1.5E+00</td>
<td>5.2E-01</td>
<td>-5.7E-01</td>
<td>-1.7E+00</td>
<td>-2.7E+00</td>
<td>-3.7E+00</td>
<td>-2.1E-04</td>
<td>1.0E+00</td>
</tr>
<tr>
<td>(A_{13})</td>
<td>Y</td>
<td>-5.4E-01</td>
<td>-4.1E-01</td>
<td>-3.2E-01</td>
<td>-1.1E-01</td>
<td>4.4E-02</td>
<td>1.6E-01</td>
<td>3.3E-01</td>
<td>2.9E-05</td>
<td>9.9E-01</td>
</tr>
<tr>
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<td>X2 - Y2</td>
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<td>-3.5E-02</td>
<td>-1.9E-02</td>
<td>7.1E-04</td>
<td>2.2E-02</td>
<td>3.9E-02</td>
<td>5.7E-02</td>
<td>3.8E-06</td>
<td>1.0E+00</td>
</tr>
<tr>
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<td>ZY</td>
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<td>-6.5E-02</td>
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<td>4.0E-02</td>
<td>6.8E-02</td>
<td>5.4E-06</td>
<td>1.0E+00</td>
</tr>
<tr>
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<td>2.9E-02</td>
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<td>1.1E-01</td>
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<tr>
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<td>8.4E-03</td>
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<td>1.6E-02</td>
<td>1.8E-02</td>
<td>2.3E-02</td>
<td>9.7E-07</td>
<td>9.2E-01</td>
</tr>
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<td>2.1E-04</td>
<td>1.0E+00</td>
</tr>
<tr>
<td>(A_{32})</td>
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<td>-3.5E-01</td>
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<td>-4.8E+00</td>
<td>-5.8E+00</td>
<td>-7.1E+00</td>
<td>-2.7E-04</td>
<td>9.9E-01</td>
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</tbody>
</table>
Figure 2.3(i) The spherical harmonic coefficients as the function of DAC values for X shim coil.

Figure 2.3(ii) The spherical harmonic coefficients as the function of DAC values for Y shim coil.
Figure 2.3(iii) The spherical harmonic coefficients as the function of DAC values for Z shim coil.

Figure 2.3(iv) The spherical harmonic coefficients as the function of DAC values for X2 – Y2 shim coil.
Figure 2.3(v) The spherical harmonic coefficients as the function of DAC values for XZ shim coil.

Figure 2.3(vi) The spherical harmonic coefficients as the function of DAC values for Z2C shim coil.
Figure 2.3(vii) The spherical harmonic coefficients as the function of DAC values for ZY shim coil.

Figure 2.3(viii) The spherical harmonic coefficients as the function of DAC values for XY shim coil.
Table 2.3 The calibration table of spherical harmonic coefficients for the first- and second-order coils. The light orange highlight indicates a strong direct correlation and light blue highlight indicates a strong cross correlation.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Notation</th>
<th>X1_DAC</th>
<th>Z1_DAC</th>
<th>Y1_DAC</th>
<th>X2Y2_DAC</th>
<th>XZ_DAC</th>
<th>Z2C_DAC</th>
<th>ZY_DAC</th>
<th>XY_DAC</th>
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</thead>
<tbody>
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<td>X</td>
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<td>-8.0E-07</td>
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<td>-1.4E-05</td>
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<td>2.3E-03</td>
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<td>-2.1E-04</td>
</tr>
<tr>
<td>A_{11}</td>
<td>Y</td>
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<td>2.2E-05</td>
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<td>-2.0E-05</td>
<td>2.9E-05</td>
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Table 2.4 The linear correlation coefficient table for the first- and second-order coils. The light yellow highlight indicates a strong direct correlation and light green highlight indicates a strong cross correlation.

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<th>Coefficient</th>
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<th>Z1_DAC</th>
<th>Y1_DAC</th>
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2.4.2. Application of the active shimming on a spherical phantom at high field

Figure 2.4 illustrates the measured field distribution within the phantom (A) before and (B) after active shimming. First, the field distribution without active shimming was measured to determine the requisite DAC values to optimize the field homogeneity over the entire phantom. Spherical region of interest (ROI) was used for the field measurement (Fig 2.4 (A)). The required DAC values were obtained using the calibration constants in Table 2.3. After updating the first- and second-order shim current, the field distribution of the phantom was obtained (see Fig 2.4(B)). Again the spherical ROI was used for the active shimming process. According to the comparison between measured field distribution before and after active shimming, the improvement of the field homogeneity over the entire phantom is clearly illustrated (see Fig 2.4(A) and (B)). Figure 2.5 shows the histograms of magnetic field distribution over the entire phantom (A) before and (B) after the active shimming. The full width at the half maximum (FWHM) of the phantom field distribution following the active shimming procedure is reduced approximately by 94.8%, as shown in Fig 2.5(B). Hence, the measured field maps and the corresponding histograms shown in Fig 2.4 and Fig 2.5 clearly reveal that generated active shim fields improve the field homogeneity within the entire phantom.

2.4.3. Comparison of FASTMAP and simulated $B_0$ field map shim for a human brain MRI at high field

The comparison of magnetic field map before and after FASTMAP shimming (i.e. the first and second orders) are shown at the center slice of axial, coronal and sagittal planes for a subject as displayed in Fig. 2.6.
Figure 2.4 Comparison of the measured magnetic field homogeneity within the phantom (A) before and (B) after active shimming. The calibration table used to achieve the optimization of the field homogeneity.

Figure 2.5 The left and right histograms of magnetic field variation of the phantom corresponding to Figures 2.4 (A) and (B), respectively. A narrower histogram indicates a more uniform magnetic field over the entire phantom.
Figure 2.6 illustrates (A) the measured magnetic field distribution following the FASTMAP shimming, (B) the simulated residual magnetic field distribution that is corrected by the simulated corrected fields (i.e. the optimized field by adding the calculated first- and second-order shim corrections), and (C) the optimal simulated residual magnetic field distribution following the optimal first-, second- and third-order magnetic field corrections. The corresponding FWHMs of the measured and simulated magnetic field distributions are presented in Fig 2.7. The smaller the FWHM indicates the more uniform magnetic field over the entire brain. The FWHM of the measured field distribution is always greater when FASTMAP shims are introduced as compared to that with simulations. The main drawback of FASTMAP is its restrictions on the selection of the corrected region of interest. As demonstrated in Figure 2.6(A), the magnetic field distribution inside the brains is not entirely uniform after FASTMAP shimming. After introduction of the FASTMAP shimming, the FWHM of the measured field distribution is about 127.1 Hz for the subject. The actual magnetic field, as one expects, is a combination of the first-, second-, and higher-order spherical harmonic field components. By removing more unnecessary contributions, the magnetic field reaches more homogeneity. Even with the optimal simulated residual magnetic field distribution (see in Fig 2.6(B)) after the first- and second-order field corrections, the region near the nasal sinus and prefrontal regions remains difficult to shim due to large susceptibility variation in that area. Simulations in Figure 2.6 (C) imply that the remaining field inhomogeneity may result from uncorrected higher order shims (i.e. third order and above). Therefore, in order to further improve shimming for these regions, high-order shims must be considered.
Figure 2.6 Comparison of magnetic field map of one typical subject’s brain both before and after FASTMAP shimming. (A) Measured magnetic field distribution after the active (FASTMAP) shimming. The simulated residual field maps after introduction of (B) both the optimal first- and second-order shims and (C) the optimal first-, second- and third-order shims.

Figure 2.7 The left, middle, and right histograms of magnetic field variation of the brain corresponding to Figures 2.6 (A), (B), and (C), respectively. A narrower histogram indicates a more uniform magnetic field over the entire brain.
2.5. Discussion and Conclusion

The active shimming can be used to improve the field homogeneity over the entire phantom and human brain at high-field MRI scanners. Magnetic fields related to spherical harmonic functions are generated by passing current through the corresponding shim coils. It is well known that the spherical harmonics are orthogonal so that ideally, magnetic fields created by shim coils are independent. According to data displayed in Table 2.4, the first-order shim coils are perfectly uncoupled to other coils. However, the second-order coils seem to be coupled with some coils. Table 2.4 clearly illustrates that all coils have a strong direct-correlation (i.e. diagonal terms), which is expected (i.e. linear correlation coefficients are ~1.0). The cross-correlations (i.e. off-diagonal terms) of the first-order coils with other coils are relatively small (less than 0.5), which implies that the first-order coils are generally uncoupled to others. However, some cross-correlation coefficients of the second-order coils are greater than 0.9, which implies that some second-order coils do couple with other coils. Therefore, when correcting currents for those coils, one needs to adjust the coupled coils accordingly to avoid the coupling contributions. For example, adjustments of DAC settings of the first-order coils that are coupled by a second-order coil are performed as follows

\[
DAC_{1,m_1}^{\text{Corrected}} = DAC_{1,m_1} - \frac{C_{2,m_2}}{C_{1,m_1}} \cdot DAC_{2,m_2}.
\]  

[2.13]

Here, \(DAC_{1,m_1}\) is the updated shim setting of the 1\(^{st}\) order, \(m_1\)\(^{th}\) degree shim coils and \(DAC_{1,m_2}\) is the updated shim setting of the 2\(^{nd}\) order, \(m_2\)\(^{th}\) degree shim coil. Both \(C_{1,m_1}\) and \(C_{2,m_2}\) are the 1\(^{st}\)
order, \( m_1 \)\textsuperscript{th} degree and 2\textsuperscript{nd} order, \( m_2 \)\textsuperscript{th} degree calibration coefficients of the corresponding shim coils, respectively.

In conclusion, our data show (Figs. 2.4 (A) and (B)) that generated active shim field significantly improves the field homogeneity over the entire phantom. Similarly, the simulated data (Figs 2.6) demonstrate that optimal shim currents derived from the field map can provide better overall shimming for the human brain. However, high-order shims are required.

2.6. References

Chapter 3

Construction of an optimized local third-order passive shim insert for human brain imaging at 4T MRI

3.1. Introduction

The optimal MRI quality relies on a homogeneous magnetic field. However, local susceptibility variations within human brain can lead to field inhomogeneity that causes artifacts such as image distortion and signal drop-out, which become worse with increasing magnetic field strength. In Chapter 2, the calibration table was evaluated for global first- and second-order active shimming. Many evidences showed that high order shims (i.e., > 2nd order) are required for optimal MRI at field greater than 3T [1-6]. However, due to limited space, many MRI systems provide only up to second-order active shims. In this chapter, we present a technique to improve \( B_0 \) field homogeneity by combining a generic third-order local passive shim device with active first- and second-order shimming. The potential for utilizing an averaged field map for the brain to construct a third-order passive shim system is demonstrated and validated using theoretical calculations and experimental measures. The averaged field map is created by decomposing individual field maps into spherical harmonic components and averaging the harmonic coefficients across subjects. The requisite magnetic susceptibility, saturation magnetization and dimensions for shim elements are then determined and verify the accuracy of the technique using
simulations. Additionally, correct position for the shim pieces on a cylindrical surface is essential. We also introduce a method for optimizing placement of ferro-shim elements on a cylindrical surface to generate spherical harmonic fields.

3.2. Theory

Local susceptibility variations can result in field inhomogeneities [7]. Improved magnetic field homogeneity over the entire human brain volume has been attempted [1, 7-9]. The numerical analysis of the inhomogeneous magnetic field as well as the design and construction of the third-order passive shim coil are given briefly below for reducing the field variation.

3.2.1. Numerical analysis of shared magnetic field variations across subjects

It has been shown in Chapter 2 that the inhomogeneous magnetic field distribution within the region of interest (i.e. within the human brain in this case) is given by the matrix representation (Eq. (2.4)). The matrix $F_{n,m}(x_i, y_j, z_k)$ can be partitioned into spherical harmonics that are used for active, $A$, or passive, $P$, components’ shimming-dependent terms and can be expressed in Eq. (3.1).

$$F_{n,m}(x_i, y_j, z_k) = [A, P]$$  \[3.1\]

where $A$ represents the sub-matrix consisting of the first- and second-order spherical harmonics that are compensated for by active shimming; and $P$ represents the third-order spherical harmonics that will be corrected by a passive shim insert. Given an individual subject’s field
map, Eq. (2.4) in Chapter 2 can be solved to determine the optimal coefficients required to correct for the measured inhomogeneity. To combine multiple subjects using the same passive shim insert the individual $F_{n,m}(x_i, y_j, z_k)$ matrices can be assembled into $g(x_i, y_j, z_k)_{n'}$ matrix in Eq. (3.2) when the $n'$ is number of subjects in the study.

$$g(x_i, y_j, z_k)_{n'} = \begin{bmatrix} A_1 & 0 & 0 & \cdots & 0 & P_1 \\ 0 & A_2 & 0 & \cdots & 0 & P_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & A_{n'} & P_{n'} \end{bmatrix}$$

[3.2]

where $A_1, A_2 \ldots A_{n'}$ represent the first- and second-order spherical harmonics that are to be corrected by active shimming for each subject, and $P_1, P_2 \ldots P_{n'}$ represent the effect of common third-order passive shim on each subject. An optimized third-order passive shim coefficients $\beta_{nm}$ for the sub-population can be obtained using least-squares as

$$\beta_{nm} = [g(x_i, y_j, z_k)_{n'}^T \cdot g(x_i, y_j, z_k)_{n'}]^{-1} \cdot g(x_i, y_j, z_k)_{n'}^T \cdot \Delta f_z(x_i, y_j, z_k)_{n'}$$

[3.3]

where $\Delta f_z(x_i, y_j, z_k)_{n'}$ is the measured frequency shift due to field inhomogeneity at position $x_i, y_j$ and $z_k$ of the $n^{th}$ subject, and the -1 and T superscripts represent matrix inversion and transposition, respectively. The corresponding symmetric 95% confidence interval for $\beta_{nm}$ is given by

$$\beta_{nm} \pm t_{(N-r,0.025)} \cdot \sqrt{[g(x_i, y_j, z_k)_{n'}^T \cdot g(x_i, y_j, z_k)_{n'}]^{-1} \cdot \hat{\sigma}^2}$$

[3.4]
where \( t_{(N-r, 0.025)} \) is the critical t value for \( N-r \) degrees of freedom and \( \hat{\sigma}^2 \) is the estimated residual variance given by [10]

\[
\hat{\sigma}^2 = \left( \frac{\sum \Delta f_z(x_i, y_j, z_k) \Delta f_z(x_i, y_j, z_k) - \beta^T g(x_i, y_j, z_k) \Delta f_z(x_i, y_j, z_k)}{N-r} \right) [3.5]
\]

where \( N \) and \( r \) are the total number of voxels and the total number of parameters used in the fit, respectively.

### 3.2.2. Determination of the positions of the shim elements at the surface of the cylinder

Figure 3.1 shows that a magnetic induction \( (dH_z) \) at the point \( P(r, \theta, \phi) \) is produced by a ferromagnetic shim element with an elementary dipole of volume \( dV \) and susceptibility \( \chi \) placed on a cylindrical surface at point \( Q(s, \gamma, \phi) \) in a main magnetic field \( B_0(z) \) [5, 11]. The strength of the induced magnetization is given by

\[
dH_z = \frac{\chi \cdot B_0 \cdot dV}{4\pi \cdot s^3} \cdot \mathcal{E}_m \cdot \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left( \frac{r}{s} \right)^n \left( \frac{(n-m+2)!}{(n+m)!} \right) P_{n,m}(\cos \theta)P_{n+2,m}(\cos \gamma)\cos(m(\phi-\varphi)) [3.6]
\]

where \( s, \gamma, \) and \( \phi \) are the radial distance, the polar, and azimuthal angles, respectively, of the shim piece on the cylinder; and \( r, \theta, \) and \( \phi \) are the radial distance, the polar and azimuthal angles of the target shim location.
Figure 3.1 A schematic plot to illustrate the coordinates and orientation of a ferromagnetic shim element placed at point $Q$ on a cylindrical surface for the generation of desired passive shim field.
Additionally, $n$ and $m$ are the order and degree of the spherical harmonics and $\varepsilon_m$ and $P_{n+2, m}$ ($\cos \gamma$) are the Neumann factor ($\varepsilon_m = 1$ if $m = 0$ otherwise $\varepsilon_m = 2$) and associated Legendre polynomials, respectively.

It is assumed that the fields produced by fourth- and higher-order multi-poles are negligible and that the shim elements are small. Eq. (3.6) can be further simplified into the following polar and azimuthal angle dependent magnetic field component, i.e. Eqs. (3.7) and (3.8), respectively. By careful selection of azimuthal ($\gamma_p$) and polar ($\varphi_q$) angles independently, a passive shim coil can be designed with desired spherical harmonics by using Eqs. (3.7) and (3.8).

$$dH_z \propto \sum_p \frac{\sin^{(n+3)}(\gamma_p) \cdot P_{(n+2,m)} \cos(\gamma_p)}{R^{(n+3)}} \quad \text{(when } \varphi \text{ is fixed)} \quad [3.7]$$

$$dH_z \propto \int_{-\Delta \varphi}^{\Delta \varphi} d\varphi_q \cdot \cos(m(\phi - \varphi_q)) \quad \text{(when } \gamma \text{ is fixed)} \quad [3.8]$$

where $p$ and $q$ represent the location of shim element in the azimuthal and polar angles, respectively. These equations imply that the angles $\gamma_p$ and $\varphi_q$ can be optimized independently. Furthermore, a passive shim coil with a desired order and degree of spherical harmonics can be designed as a “constellation” of small shim elements on a cylinder with radius $R$ (See Fig. 3.1.).
3.2.3. Selection of magnetic susceptibility and dimensions of the shim elements

Following selection of shim element locations using Eqs. (3.7) and (3.8), one can then determine the susceptibility and shim element dimensions required to produce a magnetic field with spherical harmonic amplitudes ($\beta_{nm}$) from Eq. (3.9) [7].

$$\chi = -4\pi \cdot \beta_{n,m} \cdot \int \sum_{p,q} \sum_{mn} \left( \frac{(n-m+2)! P_{n+2,m} \cdot \cos(\gamma_p) \cdot \cos(m\varphi_q)}{(n+m)! s_{n+3}^p} \right) dV_{pq}$$  \hspace{1cm} [3.9]

where $dV_{pq}$ is the shim element unit volume.

3.3. Methods

3.3.1. Data acquisition and field mapping

Four healthy subjects (ages between 20 and 40; 2 females and 2 males) were recruited and consented to participate in this study. As described in Chapter 2 (2.3.1), a 3D gradient-echo pulse sequence with modifications to reduce eddy currents and the same approaches were used to obtain $B_0$ field maps. AFNI's 3dSkullStrip program was used to segment the brain from magnitude reconstructed images. A $B_0$ field map for each individual brain was then calculated from the corresponding phase-difference images; and these were projected onto the first-, second- and third-order spherical harmonic functions (Eqs. (2.3) – (2.4)) to obtain the desired
amplitudes along with the corresponding confidences for construction of the average brain field map.

### 3.3.2. Design of the passive shim system

A cylindrical geometry (OD = 36.5 cm; ID = 35.5 cm) was selected for the passive shim system in order to generate the designed fields and fit around the RF coil [12]. The optimized amplitudes for the third-order spherical harmonic passive shim components along with the confidences were obtained (Eqs. (3.2) – (3.4)) for the correction of field inhomogeneity. The positions of arc- and rod-shaped shim elements in the azimuthal and polar angles for each harmonic component were computed (Eqs. (3.6) – (3.8)) for a cylindrical design [13, 7]. After positions were selected, the necessary susceptibility and shim elements dimensions were determined according to Eq. (3.9).

Based on the computed magnetic susceptibility and saturation magnetization, a set of mu-metal (Ad-Vance Magnetics, Inc.; Ni(77%)-Fe(16%)-Cu(5%)-Cr(2%) by mass composition) was selected for construction of the third-order passive shim system. The maximum relative permeability, the coercive force and the saturation polarization of the mu-metal alloy are 20000, 4 Am^{-1}, and 0.75 T, respectively. The density of the mu-metal is 8.75 g/cc [14].

### 3.3.3. Construction of the passive shim system

The masses and dimensions of each rod and arc shim element were calculated. The passive shim elements were firmly fixed to the computed positions on the surface of a paper at the corresponding angles of 0°, 90°, 180°, 270°, and 360° (21.6 cm in length); the paper then was inserted around the inner surface of the plastic shim tube. The shim tube was placed around the
RF coil and carefully positioned so that the 0° and 90° markings were closely aligned with the X and Y axes of the main magnet [15].

3.3.4. Measurement of third-order passive shim fields on a phantom

To validate the theory and shim design, a series of measurements on a spherical phantom were performed. The shim tube was placed to surround the phantom and the RF coil. Following the first- and second-order active shimming using FASTMAP, the 3D gradient-echo pulse sequence was utilized to acquire imaging data, which are used determine $B_0$ field maps of the phantom first with and then without the passive shim tube by comparing the phase difference between two images acquired at two echo times as described above. The magnetic field distribution within the phantom that was induced by passive shim for each degree of the third-order spherical harmonics was measured separately and independently. Finally, measurements for the fully-assembled system were obtained.

3.3.5. Combined passive and active shimming in vivo

To verify the theory, the same four healthy volunteers recruited for this study were scanned. The field distributions within the brain when using only active shimming (first and second orders using FASTMAP) and with combined active and passive shimming (i.e. third-order) were measured as before. The brains were masked and the FWHM of the field distributions over the entire human brain were compared with and without passive shimming.
3.4. Results

3.4.1. Average brain field map for third-order passive shimming

For each subject, the magnetic field distribution following the first- and second-order active shimming was projected into the first-, second- and third-order spherical harmonic function according to Eq. (2.3) to obtain the third-order spherical harmonic amplitudes of each individual. The magnitudes of the third-order spherical harmonic coefficients of each subject are presented in Fig 3.2. It shows that the inter-subject variations of the third-order coefficients are quite small except for the coefficients of $m = 0$ and $\pm 2$.

Figure 3.2 Plot of the variation of amplitudes of the third-order spherical harmonic coefficients of each subject.
The magnetic field distributions of all subjects were used to obtain the averaged subject-dependent third-order spherical harmonic amplitudes using Eqs. (3.2) and (3.3). The magnitudes of the third-order spherical harmonic coefficients computed using all possible combinations of three subjects (i.e. the leave-one-out approach) (Fig. 3.3a-d), all four subjects together (Fig 3.3e), and the mean of the third-order coefficients (Fig 3.3f), are presented. The results clearly illustrate that the values of the third-order coefficients obtained by the various approaches are quite consistent. This suggests that using the averaged subject-dependence third-order spherical harmonics may be feasible to correct magnetic field inhomogeneity for a large segment of study populations. Furthermore, while some harmonics (e.g. \( m = -1 \) and 2) contribute significantly to field inhomogeneity, others contribute relatively weakly (e.g. \( m =1 \) and 3). The higher the amplitudes are, the greater the contribution to field inhomogeneity.

Figure 3.4 displays (a) the measured brain \( B_0 \) map of subject 4 at center slices of axial, coronal, and sagittal planes with an initial active shim as well as simulated residual \( B_0 \) maps after (b) optimal 1st and 2nd order active shimming, (c) combines optimal active shimming with averaged third-order passive shimming that was computed using only the other three subjects’ data (i.e. the leave-one-out method), (d) combined the optimal active and the averaged third-order passive shimming that was computed using all four subjects, and (e) combined the optimal active and the passive shimming that was obtained from this individual. Their corresponding FWHMs of the magnetic field variation of the entire brain volume for all subjects are presented in Fig. 3.5. A smaller FWHM indicates the more uniform magnetic field over the entire brain. According to the comparison between uncorrected and simulated \( B_0 \) maps in Fig 3.4, an improvement in field homogeneity throughout the entire brain is demonstrated in simulation field maps.
Figure 3.3 Plot of the variation of amplitudes of the third-order spherical harmonic coefficients calculated based on three subjects at a time (i.e. the leave-one-out approach), (a) 2nd, 3rd and 4th, (b) 1st, 3rd and 4th, (c) 1st, 2nd and 4th, (d) 1st, 2nd and 3rd, (e) all four subjects together and (f) the mean of the third-order coefficients of all four subjects.
Figure 3.4 The $B_0$ map for axial, coronal and sagittal slices of subject 4. (a) The measured $B_0$ map with active shimming, the simulated residual field map after introduction of (b) the optimal active shimming, (c) both the optimal active and the averaged third-order passive shimming (obtained by the leave-one-out approach), (d) both the optimal active and the averaged third-order passive shimming (all four subjects together), and (e) both the optimal active shimming and the individual (i.e. the subject 4) third-order passive shimming.
Furthermore, the improvement of field homogeneity is seen particularly within the frontal lobe after the introduction of the third-order passive shim device when combined with optimal active shimming (see Figs 3.4c-e) is significantly better than when using optimal active shimming without the passive shim device (Fig 3.4b). These simulations demonstrate that a third-order passive shim system has the potential to suppress field inhomogeneity in the human brain. Furthermore, the FWHM of the field distribution after initial active shimming is always greater than that of the simulated field maps for all other conditions (Fig 3.5). After applying optimal active shimming, the FWHM of the residual field maps are reduced by approximately 8.7%, 66.1%, 7.5% and 20.2% for subject S1, S2, S3 and S4, respectively. Furthermore, the introduction of the averaged third-order passive shim field along with the optimal active shimming further reduced the FWHM for S1, S2, S3 and S4 by about 21.3%, 37.8%, 17.7%, and 25.2%, respectively. However, the FWHM of field distributions for images in Figs 3.4(c), (d) and (e) were comparable as shown in Fig 3.5. Even though S2 and S4 show greater FWHM reductions after introduction of active shimming, the improvement in the field homogeneity within the frontal lobe can be appreciated only after the introduction of both active shimming and third-order passive shimming.

These simulations demonstrate that an averaged $B_0$ map measured from a subset of subjects can be used to design a third-order passive shim system for suppressing the $B_0$ inhomogeneity within the human brain for a group of subjects.

3.4.2. Design of a cylindrical passive shim insert for human brain imaging at high field

Accurate positions of the shim pieces at the surface of the cylindrical surface to generate the desired third-order spherical harmonic passive shim field functions were computed using
Eqs. (3.7) and (3.8) [11]. With the positions selected, the required susceptibility $\chi$ and shim element dimensions were determined from Eq. (3.9) for generation of the correct amplitude of the third-order spherical harmonic passive shim fields. The brain $B_0$ map of subject 3 at a center slice as decomposed into third-order spherical harmonics of $m = 2$, $m = -1$, and $m = -3$ are presented in Fig. 3.6(a). Fig. 3.6(b) shows computed positions for shim elements on the surface of the cylindrical shim tube that produce each harmonic component. Assuming the shim pieces in a magnetic field $B_0$, the passive shim field can be calculated correspondingly to each spherical harmonic shown in the Fig 3.6(a).

![Figure 3.5 Comparison of the FWHM](Figure_3.5.png)

**Figure 3.5 Comparison of the FWHM** of (a) magnetic field following the active shimming as well as the simulated residual magnetic field (b) after optimized by active shimming only, (c) after optimized by active shimming along with the third-order average passive shimming that is computed by selecting arbitrarily three subjects at a time, (d) after optimized by active shimming along with the third-order average passive shimming that is computed by considering all four subjects together, and (e) after optimized by active and third-order passive shimming.
Figure 3.6 Validated designs of third-order cylindrical passive shim insert. (a) Axial slices of the average $B_0$ field of the subject S3’s brain decomposed into $n = 3, m = 2$; $n = 3, m = -1$ and $n = 3, m = -3$ spherical harmonic fields. (b) 3D configuration of shim inserts on the surface of the cylinder for the labelled corresponding fields. (c). Simulated magnetic field after optimized by the proposed cylindrical placement of shim elements.
The optimized simulated residual magnetic field after introduction of the cylindrical passive shim insert is almost negligible as shown in Fig 3.6(c). These results demonstrate that this approach can be used to correctly estimate the $\chi$ values and shim element dimensions needed to reduce the $B_0$ inhomogeneity.

3.4.3. Validation of experimental and theoretical third-order spherical harmonic fields on a phantom

Two pairs of shim pieces were used to generate the $Z(X^2 - Y^2)$ (i.e., $n = 3, m = 2$) passive shim field. Each shim piece had length, width, and thickness of 3.2 cm, 0.6 cm, and 0.03 cm, respectively, and were mounted in the computed positions on the surface of the shim tube (left panel of Fig.3.6(b)). Three arcs of passive shim inserts with 60.5 cm in length and 0.02 cm thickness were mounted on the shim tube to generate the $YZ2$ (i.e., $n = 3, m = -1$) passive shim field (middle panel of Fig.3.6(b)). However, the width of the shim inserts at the middle (0.2 cm) and the top and the bottom (0.7 cm) of the shim tube are not the same. For the $Y3$ (i.e., $n = 3, m = -3$) passive shim field (right panel of Fig.3.6(b)), three rod-shaped passive shim inserts with length, thickness, and width of 21.3 cm, 0.02 cm, and 0.3 cm, respectively, were mounted on the surface of the shim tube such that all shim inserts were separated by 120°. Figure 3.7 shows the results of experimental measurement of the magnetic fields generated by the $Z(X^2 - Y^2)$ (Fig 3.7(a)), $YZ2$ (Fig 3.7(c)), and $Y3$ (Fig 3.7(e)) shim tube configurations and the corresponding theoretical field maps (Figs 3.7(b), 3.7(d), and 3.7(f), respectively) within a spherical phantom in axial, coronal, and sagittal planes. The experimental and theoretical field distributions demonstrate that experimentally measured third-order spherical harmonic field distributions agree well with the theoretical predictions of the third-order spherical harmonic field functions.
Figure 3.7 Comparison between experimental and theoretical field maps of the third-order coils with $n = 3$, $m = 2$ (a), (b); $n = 3$, $m = -1$ (c), (d); and $n = 3$, $m = -3$ (e), (f). Unit of the field maps is in Hz.
The magnitudes of the third-order spherical harmonic coefficients of the field distribution within the phantom both with and without the shim tube are presented in Fig 3.8. It is clearly visible that, with the presence of each specific passive shim system, the corresponding magnitude of spherical harmonic coefficient is significantly higher than that of the other spherical harmonic coefficients. For example, the coefficient for $A_{32}$ is larger than the others when the $Z(X^2 – Y^2)$ passive shim inserts is present (Fig. 3.8(a)). This is also true for the $YZ_2$ (Fig. 3.8(b)) and $Y_3$ (Fig. 3.8(c)) shim inserts. These plots demonstrate that the proposed passive shim system produces the desired third-order spherical harmonic fields. These data also suggest that unwanted coupling among different degrees of the third order harmonics is relatively weak.

### 3.4.4. Passive and active shimming for human brain at 4T

In order to select the most significant third-order spherical harmonic coefficients for brain imaging at 4T, we conducted a one-sample t-test for all in vivo data. The p-values of the third-order harmonic coefficients are $YZ_2 = 0.01$, $Y_3 = 0.01$, $Z(X^2 – Y^2) = 0.02$, $Z_3 = 0.09$, $XYZ = 0.64$, $XZ_2 = 0.74$ and $X_3 = 0.97$ (Table 3.1). According to the computed p-values, the third-order spherical harmonic coefficients were ordered according to significance, i.e. $YZ_2 > Y_3 > Z(X^2 – Y^2) > Z_3 > XYZ > XZ_2 > X_3$. From this order, the FWHM of the magnetic field distribution of all four subjects’ brains were computed and compared with the results of removing the least significant component one at a time, which is shown in Fig. 3.9.

These results confirm that only the first three components - $YZ_2$, $Y_3$, and $Z(X^2 – Y^2)$ – contribute significantly to the perturbation of the magnetic field in this sample. The contribution of other components to field inhomogeneity is negligible.
Figure 3.8 Plots of the comparison of the amplitudes of the third-order spherical harmonic coefficients with (i) and without (ii) introduction of (a) \( n = 3, m = 2 \); (b) \( n = 3, m = -1 \); and (c) \( n = 3, m = -3 \) passive shim systems.
Table 3.1 The third-order spherical harmonic coefficients of subjects S1, S2, S3, and S4 and their corresponding p-values

<table>
<thead>
<tr>
<th>Harmonic Coefficient</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>YZ2</td>
<td>-0.08</td>
<td>-0.10</td>
<td>-0.09</td>
<td>-0.04</td>
<td>0.01</td>
</tr>
<tr>
<td>Y3</td>
<td>0.05</td>
<td>0.07</td>
<td>0.05</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>Z(X2 - Y2)</td>
<td>0.25</td>
<td>0.13</td>
<td>0.24</td>
<td>0.08</td>
<td>0.02</td>
</tr>
<tr>
<td>Z3</td>
<td>0.08</td>
<td>0.00</td>
<td>0.10</td>
<td>0.04</td>
<td>0.09</td>
</tr>
<tr>
<td>X3</td>
<td>-0.01</td>
<td>0.01</td>
<td>0.03</td>
<td>-0.01</td>
<td>0.64</td>
</tr>
<tr>
<td>XZ2</td>
<td>0.01</td>
<td>0.02</td>
<td>-0.02</td>
<td>0.00</td>
<td>0.74</td>
</tr>
<tr>
<td>XYZ</td>
<td>0.07</td>
<td>-0.01</td>
<td>-0.10</td>
<td>0.03</td>
<td>0.97</td>
</tr>
</tbody>
</table>
Figure 3.9 Plot of the variation of FWHM of the simulated residual magnetic field distribution after optimized accumulatively, sequentially by shimming of $n = 3, m = -1$ (YZ2); $n = 3, m = -3, -1$ (YZ2,Y3); $n = 3, m = -3,-1, 2$ (YZ2,Y3, Z(X2–Y2)); $n = 3, m = -3, -1, 0, 2$ (YZ2,Y3, Z(X2–Y2), Z3); $n = 3, m = -3, -2, -1, 0, 1, 2$ (YZ2,Y3, Z(X2–Y2), Z3, X3); $n = 3, m = -3, -2, -1, 0, 1, 2, 3$ (YZ2,Y3, Z(X2–Y2), Z3, X3, XZ2), and $n = 3, m = -3, -2, -1, 0, 1, 2, 3$ (YZ2,Y3, Z(X2–Y2), Z3, X3, XZ2,XYZ) components of each subject.
Therefore, for this work, only the YZ2, Y3, and \( Z (X2 – Y2) \) third-order spherical harmonic components were used for construction of the final passive shim device.

The magnetic field homogeneity within the human brain \textit{in vivo} after introduction of the third-order, \( Z(X2 – Y2) \), YZ2 and Y3 passive shim field device when combined with active shimming is presented in Fig. 3.10. The measured magnetic field maps within the subject’s brain following (1\textsuperscript{st} and 2\textsuperscript{nd} order) active shimming only and following combined active and passive shimming are shown in Figs. 3.10(a) and 3.10(b), respectively. The results show that active shimming with 1\textsuperscript{st} and 2\textsuperscript{nd} order generally optimizes \( B_0 \) field within the brain with the exception of the region of the prefrontal lobe (Fig.3.10(a)). Using both active shimming and the third-order passive shim insert, the field homogeneity over the entire brain, particularly in the regions of prefrontal lobe and the area of the hippocampus, was improved significantly, which is noticeable in Fig.3.10(b). Additionally, it is evident that \( B_0 \) homogeneity within other regions such as the parietal and occipital lobes was slightly improved as well. The FWHM of the frequency distributions both with and without third-order passive shimming were compared over the entire brain of the subject as shown in Fig. 3.10(c). The FWHM was reduced from 135.2 Hz to 107.8 Hz after addition of the third-order passive shimming. The magnitudes of the third-order spherical harmonic coefficients on the subject’s brain field distribution following active shimming only (green line) and following combined active and passive shimming (red line) also are presented in Fig.3.11. The amplitudes of the third-order spherical harmonic coefficient are significantly reduced after the introduction of the passive shimming device. Specifically, the amplitudes of the \( Z(X2 – Y2) \), YZ2, and Y3 were reduced from 0.233 (Hz/cm\(^3\)), 0.072 (Hz/cm\(^3\)), and 0.082 (Hz/cm\(^3\)) to 0.012 (Hz/cm\(^3\)), 0.029 (Hz/cm\(^3\)), and 0.021 (Hz/cm\(^3\)), respectively.
Figure 3.10 The field map of the entire brain of the 4th subject using active shimming alone and both active and third-order passive shimming was compared. (A) The field distributions with active shimming only (a) and with both active and passive shimming (b) are presented. The $B_0$ field in-homogeneity remained in some regions of the brain after active shimming, but it was reduced by the addition of passive shimming. (B) The FWHM of the Gaussian fitted frequency distribution was reduced from (green) 135.2 to (red) 107.8 Hz. (The units of the field maps are in Hz.)
Figure 3.11 Plots of the comparison of amplitudes for the third-order spherical harmonic coefficients with (blue) only active shimming and with (violet) both active and third-order passive shimming.
Hence, the third-order passive shim field improves field homogeneity within the subject’s brain, particularly around the prefrontal lobe.

### 3.5. Discussion and Conclusion

It is common to use both active and global passive shimming to improve the field homogeneity over a volume of interest. Field inhomogeneity due to magnetic susceptibility variations increases with main field strength. Often, it is insufficient to use first- and second-order active shimming to adequately correct for magnetic susceptibility variations present within \textit{in vivo} samples. Even with optimal first- and second-order active shimming, improvements in field homogeneity in some regions of interest (i.e. human brain in our case) is frequently inadequate due to the presence of uncorrected high-order shim components.

In this work, we have demonstrated through simulations and experiments that third-order local passive shimming significantly improves field homogeneity in the brain over a group of subjects studied at high field. However, results of the simulations in Fig 3.4 indicate that inhomogeneous field distributions remain within the sinus region, despite significant improvements in field homogeneity over other regions of the brain. Therefore, one can assume that the field inhomogeneity within the sinus region is due to the contribution of higher-order spherical harmonic fields, (>third-order) which are possibly not corrected by third-order shimming. Fig. 3.12 shows the measured field distribution within the volunteer’s brain following (a) the first- and second-order shimming, as well as simulated magnetic field distribution
optimized by the addition of (b) third-order correction, (c) third- and fourth-order correction, and (d) third-, fourth-, and fifth-order shimming.

Even if field homogeneity were improved within the subject’s brain, the field inhomogeneity is not significantly reduced near the sinus region, which is clearly visible toward the bottom of the coronal and sagittal images.

Hence, the simulations in Fig. 3.12 verify that the correction of the third or higher orders is not fully effective in the sinus region. Indeed, the general solutions of the Laplace’s equation for the magnetic field is \[ \bar{B}_z(r, \theta, \phi) = \sum_{n=0}^{\infty} \sum_{m=0}^{n} (C_{nm} r^n + \frac{D_{nm}}{p_{n+1}^2}) P_{n,m} (\cos \theta) e^{im\phi}, \]
where \( C_{nm} \) and \( D_{nm} \) are the constants that are analogous to \( A_{nm} \) and \( B_{nm} \) in Eq. (2.2).

**Figure 3.12** Comparison between measured and simulated field maps within a volunteer’s brain. Axial, coronal and sagittal slices of the magnetic field distribution following (a) the active shimming, and the simulated residual field distributions after optimization by (b) the first-,
second- and third-order shimming, (c) first-, second-, third- and fourth-order shimming, and (d) first-, second-, third-, fourth- and fifth-order shimming.

We suspect that the field inhomogeneity near the sinus region is mainly due to external unperturbed field components \[ \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left( \frac{D_{nm}}{r^{n+1}} \right) P_{n,m}(\cos \theta) e^{im\phi}, \]
which are difficult to compensate for using this method. The presence of the air filled sinus cavity in the main magnetic field gives rise to the external magnetic field nearby the sinus region.

The simulated field maps in Fig.3.6(c) can be used to validate designs of cylindrical passive shim inserts. Some field inhomogeneities may not be entirely suppressed due to the presence of higher order contributions. These manifest in the Z(X2−Y2), YZ2, and Y3 \( B_0 \) maps (Fig.3.6(c)), which retain some residual inhomogeneity. Additionally, the amplitude of the spherical harmonics depends on the estimated \( \chi \) values, dimensions, and positions of the shim inserts on the surface of the cylinder. Incorrect estimation of susceptibility \( \chi \) and positions of the shim inserts on the shim tube can yield unwanted fields. Precise placement of the shim tube around the subject is essential to minimize the generation of other unwanted spherical harmonic field components which may jeopardize field homogeneity. The results of Fig 3.8 indicate that the magnitudes of each desired third-order coefficient (i.e. \( m = 2, m = -1, \) and \( m = -3 \)) with the shim tube are clearly higher than those of other coefficients and those without the shim tube.

This implies that the placement of the shim tube surrounding the subject and RF coil relative to the gradient system is fairly accurate. However, it can be noted in Fig 3.8 that (a) the coefficients of (3, 3), (3, 1), (3, 0), and (3, -2) with \( n = 3, m = 2 \) shim tube, (b) the coefficients of (3, 1) and (3, 0) with \( n = 3, m = -1 \) shim tube, and (c) the coefficients of (3, 3), (3, 1), (3, 0), and (3, -1) with \( n = 3, m = -3 \) shim tube are slightly higher than the coefficients without the shim
tube. The cause of these unwanted fields is slight imperfection in alignment of the shim tube surrounding the RF coil. It was found that even errors in the azimuthal angle as small as 1° can result in unwanted spherical harmonic field components. It is, therefore, essential to ensure accurate alignment of the shim tube during the experiment in order to generate only the desired spherical harmonic field components.

Passive shimming is not flexible. The generated passive shim field cannot easily be changed. Therefore, it sometimes causes overcompensation of the field homogeneity in some regions within the brain (see, for example, in Fig 3.10(b)).

The method proposed in this work used the combination of first- and second-order active shimming and third-order passive shimming to improve field homogeneity over the entire brain; and is expected to be of great benefit for studies of the prefrontal lobe (MRI/MRS) at high field. These results suggest that the third order magnetic field component due to inter-subject variation in the same MRI system may be small, and this can be exploited to design and construct an optimal local passive shim coil.

In conclusion, this study has demonstrated that the accurate placement of appropriate ferromagnetic material on the surface of a cylinder can generate the desired third-order passive shim field to reduce $B_0$ inhomogeneity significantly over the entire human brain, especially in the prefrontal lobe. Both Eqs. (3.3) and (3.9) allow one to determine the required magnetic susceptibility for the optimization of field in using a spherical harmonic model. Additionally, Eqs. (3.2) and (3.3) allow estimation of an average brain field map of any order and degree desired for passive shimming. The average brain field map can be used to construct a passive shim system to improve field homogeneity over the human brain for an entire group of subjects.
3.6. References


Chapter 4

Construction and optimization of novel passive shim system for human orbitofrontal cortex at 4T MRI

4.1. Introduction

The orbitofrontal cortex (OFC) region plays a critical role in human brain functions such as cognitive processing and decision-making [1]. However, the great variation in magnetic susceptibility between the ethmoid, frontal, and sphenoid sinuses and nasal cavity (see Fig 4.1) is attributed to imaging artefacts in the OFC. In Chapter 3, the local third-order passive shim insert was unable to fully reduce the field inhomogeneity in the region of the OFC. Localized passive shimming, as demonstrated, reduces field deviations within the OFC [2-8]. It is advantageous to position shim elements away from the subject to reduce discomfort. In this work, we propose the use of a passive shim element positioned beneath the subject’s chin to generate a locally-targeted passive shim field for the OFC region.

Based on multiple subjects’ inhomogeneous brain field maps, the average amplitudes of spherical harmonics for the local dipole passive shimming are derived. The potential of utilizing the local dipole passive shim system to enhance the field homogeneity within the OFC is verified using the simulations and measures of its corresponding FWHMs.
Figure 4.1 Sagittal view of the head anatomy. Nasal cavity, sphenoid sinus, ethmoid sinus, maxillary sinus and frontal sinus are mainly filled with air and cause field in-homogeneity, which affects imaging in the orbitofrontal cortex. This figure is adapted from reference [1].
With the strength of the average amplitude and positions selected, the requisite magnetic susceptibilities and dimensions are determined.

4.2. Theory

The great variation in magnetic susceptibility between the brain tissue, paranasal sinuses, and nasal cavity leads to significant local susceptibility-induced field variations in the orbitofrontal cortex (OFC). Improvements of magnetic field homogeneity over the human brain, particularly in the regions of inferior frontal cortex (IFC) and inferior temporal cortex (ITC), have been attempted [2-7]. The numerical analysis of the inhomogeneous magnetic field as well as the design and construction of the localized neck-passive shim system are given below.

4.2.1. Design of the neck passive shim system

Fig.4.2 shows that spherical shim elements, which can be diamagnetic and/or para/ferromagnetic material, with radius \( a \) and susceptibility \( \chi_{l(z)} \) placed on a half-cylindrical surface at the predefined position \((x_{0l}, y_{0l}, z_{0l})\) of a passive shim system in a main magnetic field give rise to a magnetic induction outside the sphere at the position \((x, y, z)\) as shown in Eqn. (4.1) [7-12].

\[
\Delta B_z(x, y, z) = \sum_l a_l^3 \chi_{l(z)} B_0 \frac{(2(z-z_{0l})^2-(x-x_{0l})^2-(y-y_{0l})^2)}{3 \left( (z-z_{0l})^2 + (x-x_{0l})^2 + (y-y_{0l})^2 \right)^{\frac{3}{2}}} \hat{k} \tag{4.1}
\]
Here, $x$, $y$, and $z$ are Cartesian coordinates describing 3D space and $l$ represents the number of shim pieces. The simplified version of Eq. (4.1) is given below [7].

$$\Delta \mathbf{B}_l(x,y,z) = \sum_l G(x_i,y_j,z_k) \cdot \xi_l$$  \[4.2\]

Here, $G(x_i,y_j,z_k)$ and $\xi_l$ are the spatial-dependent components and the amplitude of the induced passive shim field, respectively. Further, as described in Chapter 2 (2.2.1), the symbols of $x_i$, $y_j$, $z_k$ and the subscript of $i$, $j$, $k$ represent the orientation and the matrix position of the Cartesian coordinate, respectively. The amplitude $\xi_l$ depends on the magnetic susceptibility and the dimension of each individual shim insert. The least-squares optimal $\xi_{n,l}$ for the individual subjects are given by Eq. (4.3) below ($n'$ is the number of subjects in the group).

$$\xi_{n',l} = \left[ G(x_i,y_j,z_k)_{n'}^T \cdot G(x_i,y_j,z_k)_{n'} \right]^{-1} \cdot G(x_i,y_j,z_k)_{n'}^T \cdot \Delta f_Z(x_i,y_j,z_k)_{n'}$$  \[4.3\]

where $\Delta f_Z(x_i,y_j,z_k)_{n'}$ is the measured frequency shift due to the field inhomogeneity at voxel position $x_i$, $y_j$, and $z_k$ of the $n^{th}$ subject, and the -1 and T superscripts represent matrix inversion and transposition, respectively.

The average amplitude of each spherical harmonic component $\xi_{av,l}$ is given by Eq. (4.4) below.

$$\xi_{av,l} = \frac{1}{n'} \sum_{S=1}^{n'} \xi_{S,l}$$  \[4.4\]
Figure 4.2 Diamagnetic and/or para/ferromagnetic spherical shim elements placed at point $(x_{0l}, y_{0l}, z_{0l})$ on a half-cylindrical surface that induces the dipole passive shim field at position $(x, y, z)$. 
4.3. Methods

4.3.1. Data acquisition and field mapping

A 3D gradient-echo pulse sequence (Fig.2.4.4) was used in this study. The magnetic field maps of each subject’s brain were projected onto a spherical harmonic model (Eq.4.1) to obtain the desired amplitudes of the induced passive shim field for the subject. Then the amplitudes were averaged (Eq.4.4) from all subjects for the final design.

4.3.2. Passive shim system design

Semi-cylindrical geometry (OD = 20.5 cm; ID = 20.0 cm and L = 4 cm) was chosen for the basic geometry of the passive shim system. The positions of the shim elements were evaluated with the adaptations for passive shim design. With the positions and average $\xi_{av,l}$ selected, the required susceptibilities $\chi_{l(\pm)}$, dimensions and masses were determined.

4.3.3. Magnetic properties of the passive shim insert

Based on the computed magnetic susceptibilities, a ferromagnetic material of mu-metal (see Chapter 3 (3.3.2)) and a diamagnetic material of bismuth (Chemical Store, Main Avenue, NJ; Bi) were utilized to construct the localized dipole passive shim system. The density of the bismuth is 9.78 g/cc.
4.3.4. Construction of localized dipole passive shim system

The combined ferro- and dia-magnetic passive shim inserts were firmly fixed to the predefined positions on the surface of a paper (4.0 cm in length and 50.0 cm in width) at positions of 0, ±5, ±10, ±15, ±20 in cm along X and 10.0, 10.5, 11.0, 11.5, 12.0, 12.5, 13.0 and 13.5 in cm along Z. The paper then was wrapped around the outer surface of the plastic semi-cylinder (i.e. the dipole passive shim system) such that the position of X = 0 was aligned with the vertical rod of the passive shim system. The positions were computed with respect to the magnet’s iso-center. The best mounting position of the semi-cylindrical passive shim system was determined based on the YZ plane (sagittal plane). The passive shim system then was mounted to the RF coil such that the YZ plane aligned along the movable vertical and horizontal rods of the dipole passive shim system.

4.3.5. Both local dipole passive and active shimming in vivo

The field distributions in the human brain with only active shimming (using FASTMAP) and with both active and local dipole passive shimming were measured using a modified 3D gradient-echo pulse sequence (see section (3.3.1)) on the 4T scanner. The field variation of the orbitofrontal cortex region was compared between the field maps using active shimming only and the combination of the local dipole passive and active shimming.

4.3.6. Combined local dipole, third-order passive and global active shimming in vivo

The field distributions in the human brain were measured using two different approaches: (a) local dipole and third-order passive shimming and (b) local dipole, third-order passive and global
active shimming. In global active shimming, the entire brain was selected as the region of interest (ROI) during the online shimming. AFNI's 3dSkullStrip program was used to segment the brain region of interest. The field map of the subject’s brain, particularly within the orbitofrontal cortex region, at 4T was compared among the two methods described above. Furthermore, the FWHM of the corresponding frequency distributions, fitted with a Gaussian function, were also compared.

4.4. Results

4.4.1. Local dipole passive shim model for human OFC at 4T.

The enhancement of the magnetic field homogeneity within the human brain, particularly in the OFC, after the introduction of the localized dipole magnetic passive shim field along with the active shimming was presented in Fig. 4.3. Figure 4.3 displays (A) the measured magnetic field maps of the brain following the active shimming, (B) a simulated dipole magnetic field induced by a passive shim element, as well as the simulated residual $B_0$ maps after optimizing by (C) the dipole passive shim that is placed in the subject’s mouth, and (D) the active and dipole passive shim combined. According to the comparison between measured field map (Fig 4.3A) and simulated $B_0$ map optimized by both active and localized dipole passive shimming (Fig 4.3D), the significant improvement of the field homogeneity of the brain, particularly with in the OFC, is demonstrated.
Figure 4.3 The measured and simulated $B_0$ map of a human brain at 4T. (A) The measured $B_0$ map for the axial, coronal and sagittal slices of the 2nd subject’s brain with first- and second-order active shim. (B) A dipole magnetic induced by a magnetic passive shim element. (C) Simulated residual $B_0$ map optimized by a local dipole passive shim element inside the subject’s. (D) Simulated residual $B_0$ map optimized by the first- and second-order active shim as well as the localized dipole passive shim element inside the subject’s mouth.
As shown in Fig 4.3B, the dipole field is relative small. Thus, the shim piece has to be placed in close proximity to the sinus region. However, placing the shim piece within subject’s mouth is uncomfortable for the subject. Therefore, we proposed to use a dipole passive shim element that is positioned beneath the subject’s chin, which induces a locally-targeted dipole field to compensate for the field inhomogeneity within the OFC. Figure 4.4 illustrates the passive shim system comprising of diamagnetic (green) and para-/ferro-magnetic (red) shim pieces for inducing the effective dipole passive shim field.

The magnetic field distribution following active shimming (i.e. the first and second order) of each subject was projected into the spherical harmonic function (Eq. (4.3)) to obtain the spherical harmonic amplitudes, $\xi_{r,l}$, of each individual.

**Figure 4.4** Subject-friendly the passive shim system to be placed below the chin with predefined positions at a half-cylindrical geometry support. The color green and red represent diamagnetic and para-/ferro-magnetic materials, respectively.
Table 4.1 The spherical harmonic coefficients of subject S1, S2, S3, and S4, as well as the average coefficients of all subjects. The unit of the spherical harmonic coefficients is Hz.cm$^3$.

<table>
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<th>Coefficient Number</th>
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<th>S3</th>
<th>S4</th>
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Figure 4.5 Amplitudes of the spherical harmonic coefficients for the first, second, third and fourth subjects computed by Eq. (4.3) as well as the average amplitudes of all subjects.
For the final shim set design, the average amplitudes were computed using Eq. (4.4). The magnitudes and the variation of the spherical harmonic coefficients of each subject, as well as the average amplitudes of all subjects, are presented in Table 4.1 and Fig. 4.5. The most challenging conditions would be those harmonics whose values fluctuate between positive and negative field. To generate a magnetic field with positive amplitudes requires a para- or ferromagnetic material, while a negative filed requires a diamagnetic element.

To estimate the best mounting positions of the dipole passive shim geometry, Fig. 4.6 displays the FWHM of the brain field distribution as a function of the shim set in the Y and Z coordinates on the sagittal plane. The blue regions (i.e. small field variation) in the plot indicate more successful shimming, whereas red regions indicate poor shimming. As Fig 4.6 illustrates, the best mounting position along the Y and Z axis is about 5 - 6.5 cm and 11 – 13.5 cm, respectively.

Figure 4.7 displays (A) the measured brain $B_0$ map of the four subjects at the axial, coronal and sagittal planes with an initial active shim as well as the simulated $B_0$ maps optimized by (B) the first- and second-order active shimming, (C) combined the active and the averaged local dipole passive shimming, and (D) combined the active, the averaged local dipole passive, and the averaged third-order passive shimming. Their corresponding FWHMs of the magnetic field variation of the entire brain volume for all subjects are presented in Fig. 4.8. According to the comparison between uncorrected (Fig 4.7(A) for all four subjects) and simulated $B_0$ maps (Figs 4.7 (B), (C), and (D) for all four subjects), the improvement of the field homogeneity within the entire brain is clearly demonstrated. Furthermore, the improvement of the field homogeneity, particularly within the OFC and the frontal regions, in the simulated field maps that are optimized by all three shimming methods (see (Figs 4.7 (D) for all four subjects) is
significantly greater than that by active shimming only ((Figs 4.7 (B) for all four subjects). These simulation results demonstrate that the constructed local dipole passive shim system and third-order passive shim system can be used to suppress the field inhomogeneity in the human brain, including the OFC and the frontal lobe regions. Furthermore, the FWHM of the field distributions after the initial active shimming is always greater than that of the simulated field maps that include at least one passive shim (Figs 4.8). After the introduction of the optimal active shimming, the FWHMs of the simulated field maps (Figs 4.8 (B)) are reduced approximately by 11.1%, 33.3%, 18.6%, and 13.5% for the first, second, third, and fourth subjects, respectively. Furthermore, the introduction of the averaged local dipole, third-order passive shim, and the optimal active shimming (Figs 4.8 (D)), further reduced the FWHMs for subjects one, two, three, and four by 27.1%, 51.3%, 56.2% and 39.9%, respectively.

**Figure 4.6** Determination of best mounting positions of the half-cylindrical geometry (i.e. the dipole passive shim geometry) position in the sagittal plane. Blue indicates a more effective shimming. Plot of the FWHM of the field distribution of the (A) 1st, (B) 2nd, (C) 3rd and (D) 4th subject’s brain based on the y and z position of the half-cylindrical passive shim system.
Figure 4.7 Comparison of measured and predicted magnetic field map of four subjects' brains. (A) $B_0$ map of the 1st, 2nd, 3rd and 4th subject’s brain following an initial active shimming. The predicted $B_0$ maps of the 1st, 2nd, 3rd and 4th subjects optimized by (B) the first- and second-order active shimming, (C) the active and the local dipole passive shimming, and (D) the active, local third-order passive, and local dipole passive shimming.
Figure 4.8 Histograms of magnetic field distribution within the brains of all four subjects corresponding to Figures 4.7 (A), (B), (C), and (D), respectively.
The simulation field maps (Fig 4.7) and corresponding histograms (Fig 4.8) for all four subjects show that this technique has potential to improve field homogeneity, particularly within OFC, for a group of subjects.

4.4.2. The local dipole passive shim field on phantom at 4T.

Diamagnetic (Bi) and ferromagnetic (mu-metal) passive shim pieces were used to generate the local dipole field. Seven Bi shim pieces with volume 0.2 cm$^3$ (3 pieces), 0.5 cm$^3$ (2 pieces), 1.0 cm$^3$ (1 piece), and 2.0 cm$^3$ (1 piece) as well as ten mu-metal pieces with volume 0.00002 cm$^3$ (2 pieces), 0.0001 cm$^3$ (2 pieces), 0.0003 cm$^3$ (2 pieces), 0.003 cm$^3$ (3 pieces), and 0.007 cm$^3$ (1 piece) were placed in the predefined positions on the surface of the half-cylindrical (Fig. 4.4). The experimental generation of the localized dipole magnetic field within the phantom in the axial, coronal and sagittal planes are illustrated in Fig. 4.9. The dipole passive shim field maps within the phantom, particularly in the axial and sagittal planes, reveal that the passive shim system generate the desired dipole field.

![Field Maps](image)

Figure 4.9 The generation of the localized dipole passive shim field within the phantom. Axial, coronal and sagittal slices of the magnetic field distribution within the phantom in the presence of local dipole passive shim system.
4.4.3. The local dipole passive and active shimming on human brain at 4T.

The enhancement of the magnetic field homogeneity within the human brain after introduction of the averaged local dipole passive shim field component along with the active shimming was presented in Fig. 4.10. The measured magnetic field distributions within the brain after (A) active shimming only and (B) combined active and local dipole passive shimming are shown in Fig. 4.10.

![Figure 4.10](image)

**Figure 4.10** The simply active shimming and with both active and local dipole passive shimming for field homogenization of the 3rd subject’s orbitofrontal cortex region at 4T were compared. The $B_0$ field with (A) active shimming only and (B) both active and local dipole passive shimming are presented. The $B_0$ field inhomogeneity over the OFC region remained after simply active shimming but was reduced after the introduction of a local dipole passive shimming.
It is clearly shown in Fig. 4.10(A) that, even after active shimming, there remains inhomogeneity in some areas of the brain, including the frontal lobe, OFC, parietal lobe, temporal lobe, the upper part of the cerebellum, and the sinus regions. Nevertheless, after adding the localized dipole passive shimming, the field homogeneity is improved over several brain regions, including the regions closer to the ethmoid and sphenoid sinuses and parietal lobe, which is clearly noticeable in the field map of Fig 4.10(B). Additionally, a slight improvement in the \( B_0 \) homogeneity within the frontal lobe is also noticeable in comparing the sagittal and axial field maps in Fig 4.10(A) and (B).

4.4.4. Application of the averaged local dipole, the third-order passive and global active shimming on human brain at 4T

Figure 4.11 displays (A) the measured brain \( B_0 \) map of subject 3 for the axial, coronal and sagittal planes with the averaged local dipole and averaged third-order passive shimming, as well as the predicted \( B_0 \) maps after optimized by (B) the global active shimming, and (C) combined optimal global active and averaged third-order passive shimming. Their corresponding FWHMs of the magnetic field variation are presented in Fig. 4.12. According to the results (Fig. 4.11 (A)) vs. Figs. 4.11 (B) and (C)), the improvement of the field homogeneity using the dipole shim set is clearly demonstrated. Further, the FWHMs of the predicted field maps (see Fig 4.12 (B) and (C)) are reduced by 58.1% and 59.3% respectively. These simulated figures and corresponding FWHMs demonstrate that the optimal global active shimming along with the averaged third-order and local dipole passive shimming can be used to optimize the field inhomogeneity in human brain, particularly within the OFC.
Figure 4.11 Comparison of measured and predicted magnetic field map of the 3rd subject’s brain. (A) The measured $B_0$ map of the 3rd subject’s brain following the local dipole and third-order passive shimming, (B) the predicted $B_0$ maps optimized by the active shimming, and (C) the predicted $B_0$ maps optimized by the active and local third-order passive shimming.

Figure 4.12 The left, middle and right histograms of magnetic field distribution within the brain corresponding to Figures 4.11 (A), (B), and (C), respectively.
Further (A) the measured brain $B_0$ map of subject 3 for the axial, coronal and sagittal planes with the global active, averaged local dipole and averaged third-order passive shimming, and the predicted $B_0$ maps after optimization by (B) the global active shimming, and (C) combined optimal global active and averaged third-order passive shimming are shown in Fig 4.13. According to the comparison between measured (Fig. 4.13 (A)) and simulated $B_0$ maps (Fig. 4.13 (B) and (C)), the improvement of the field homogeneity within the entire brain is not significant. However, a slight improvement of the magnetic field homogeneity is noticeable in predicted field maps (see Fig 4.13 (B) and (C)) within the frontal lobe and parietal lobe of the axial and sagittal images and in the cerebral hemispheres of the coronal images. The FWHMs of the magnetic field variations in Fig 4.13 are presented in Fig. 4.14. The FWHM of 101.1 Hz, Fig 4.14 (A) was reduced to 93.4 Hz, Fig 4.14 (B), or by 7.6% for the subject’s frequency distribution from the measured to the predicted with optimal active shimming only. In addition, the FWHM of 101.1 Hz for the measured field distribution over the entire brain was reduced to 89.1 Hz, which is about 11.8% for the predicted field map following combined optimal active and the third-order passive shimming (see Fig 4.14 (C)). However, the FWHM of Figs. 4.14 (B) and (C) were comparable. Although Fig 4.14 (B) and (C) shows the reducton of the FWHM after introduction of the (1) optimal global active shimming only and (2) combined optimal global active and the third-order passive shimming, the improvement of the field homogeneity within the sinus and orbitofrontal cortex can be appreciated only after the introduction of local dipole passive shimming. Therefore, the measured field maps in Fig 4.13 and the corresponding plot of the FWHM in Fig 4.14 clearly reveal that generated local dipole and the third-order passive shim field suppress the field in-homogeneity and improves the field homogeneity within the subject’s brain, particularly within OFC and sinus regions.
Figure 4.13 Comparison of measured and predicted magnetic field map of the 3rd subject’s brain. (A) The measured $B_0$ map of the 3rd subject’s brain following the local dipole, third-order passive and active shimming, (B), the predicted $B_0$ maps following simulated active shimming only, and (C) the predicted $B_0$ maps following simulated active and local third-order passive shimming.

Figure 4.14 The left, middle, and right histograms of magnetic field distribution within the brain corresponding to figures 4.13 (A), (B) and (C) above, respectively.
4.5. Discussion and Conclusion

Based on stimulations, even if first-and second-order active shim power is sufficient, the improvement of the field homogeneity in some regions within the human brain remains inadequate. The method of using the combined active, averaged local dipole, and averaged third-order passive shimming, may minimize this limitation. Our result illustrates that the third-order passive shim system along with the existing active shimming can improve the field homogeneity in some brain areas, but not all. For example, the area near the sinus remains difficult. Nevertheless, the addition of a local dipole passive shim can overcome that difficulty, as demonstrated by our stimulated and experimental results. The results from the simulated field maps (Fig 4.3) demonstrate that field homogeneity can be improved over the sinus region after the introduction of the local dipole passive shim piece in the subject’s mouth.

The induced localized dipole passive shim field is proportional to $1/r^3$. Therefore, the distance between passive shim inserts and the target shimming area needs to be short in order to be effective. The proposed half-cylindrical passive shim system is placed below the chin, which is relative remote to the OFC region. Due to the relatively long distance between the OFC and shim inserts, the required amount and magnitude of the magnetic susceptibility of passive shim inserts are considerably large in our design. In addition, in order to generate a strong dipole field, it is essential to use shim material with highly negative magnetic susceptibility that is not usually available in nature, or to use a large quantity of shim elements that may be limited by the dimension of the design.

The dipole passive shim system is to be placed below the chin and slightly above the neck with defined positions at a half-cylindrical geometry support. Sometimes, the shim system may not be able to be placed at the best mounting position for some subjects due to the variation
of neck sizes and the limitation of the shim system’s geometry. Hence, this approach may not be able to be used with all subjects.

In general, the local dipole passive shim field is possible for use in any localized region within the brain. In conclusion, this study has shown that the accurate placement of appropriate diamagnetic and ferromagnetic materials on the surface of the half-cylinder could generate the desired local dipole to significantly reduce $B_0$ inhomogeneity in the orbitofrontal cortex. This approach has proved to be operator- and subject-friendly as compared to other invasive methods. Both Eqs (4.3) and (4.4) can be used to determine the magnetic susceptibility and dimension required for field homogeneity optimisation. Incorrect estimating the magnetic susceptibility or its dimension may potentially jeopardize the outcome.

4.6. References


Chapter 5

Discussion and summary

5.1. Discussion, Summary, and Implication

The optimal image quality in MRI and better spectral dispersion in MR spectroscopy rely on a homogeneous magnetic field. However, local susceptibility variation within the human body leads to field inhomogeneity that causes artifacts such as image distortions and signal drop-out. Furthermore, this problem becomes worse with increasing field strength. Given the trend of increasing MRS and MRI studies performed at high-field strength ($\geq 3$ Tesla), an effective shimming system for high fields is needed.

Current automatic active shim techniques such as FASTMAP (Fast Automatic Shimming Technique for Mapping Along the Projection) are effective techniques for finding optimal currents that work well at high field strength but are restricted by several limitations [1]. For example, perfect magnetic field homogeneity can only be approached with low-order spherical harmonic shims and the shim power is insufficient for certain anatomical regions. In this work, we proposed to develop a combined active and passive approach to address shimming issues at high-field strength. Our goal is to achieve optimal magnetic field homogeneity with this approach, and eventually leading to improved MRI resolution and better spectral quality in MRS.
The higher fields (>3T) lead to better spectral dispersion in MR spectroscopy but the benefit of simplified spectra at high field can be exploited only with an excellent homogeneous magnetic field. To achieve perfect field homogeneity by shimming requires stronger shim powers, which may not be possible in some cases. For example, regions with great magnetic-susceptibilities mismatches, such as tissue-air transitions (e.g., in the vicinity of the ear canals or the cranial bone) are prone to image artifacts and signal-loss artifacts and hence may require shim currents that exceed the capacity of shimming amplifiers. In addition, most MRI scanners currently are equipped only with the first- and second-order active shim coils. The higher-order shim such as the third-order term is unavailable on most of the human MRI scanners due to the space limitation and cost. In addition to the active shimming, local passive shimming is also effective to reduce the field inhomogeneities over a particular region of interest at high-field MRI. It has been reported that a desired spherical harmonic magnetic field component can be generated with proper shim materials [2]. The passive shimming approach is also effective in reducing the field variation caused by the third- order perturbations in the brain $B_0$ field. The implementation of the passive shimming approach can be a cost effective alternative for the expensive third-order active shim coil on high-field MRI scanners. The present work identifies several brain regions (e.g. the OFC), which are difficult to shim with the existing first-and second -order shim coils on the 4 Tesla MRI whole-body scanner. We address those issues with the combined method of active and passive shimming.

It is known the MRI and MRS data in the orbitofrontal cortex (OFC) region have suffered the image distortion and signal loss due to susceptibility-induced field variations, which are caused by the great mismatch in magnetic susceptibility between the paranasal sinuses and the surrounding tissues. The simulation and experimental results showed that the correction with
the third or higher orders is not fully effective in the sinus region. To overcome this problem, we also developed the local dipole passive shim system to produce the desired dipole magnetic fields over the sinus region. We demonstrate that, combined with the third-order active shim; this local dipole passive shim system can minimize this problem.

In automatic shim technique using a gradient-echo pulse sequence experiment, a modified 3-D multiple-echo gradient-echo pulse sequence was used to acquire phase maps for computing the field distributions over the subjects (a phantom and the human brain). The calibration table was derived to be used in the optimization of the field homogeneity. First, the first- and second-order shim settings were updated and field distribution over the phantom was measured. The FWHM of the phantom field distribution after using the calibration table was reduced approximately by 95% (see Figure 2.4 and 2.5).

In our initial attempt to improve the field homogeneity over the human brain with the third-order passive shim, we created $B_0$ field maps with a 3D gradient-echo pulse sequence in four subjects’ brains at 4 Tesla. The $B_0$ field maps for each subject were then decomposed into the third-order spherical harmonic coefficients and averaged. Optimized positions for placement of shim elements on a cylindrical shim tube that fits over the RF coil were then computed to reduce the third-order harmonics over the entire human brain. The combined active-passive shim methodology demonstrated that the accurate placement of appropriate ferromagnetic material on the surface of a cylinder is effective to generate the desired third-order fields to improve the field homogeneity significantly over the entire human brain, particularly in the regions of prefrontal lobe and the hippocampus. Additionally, $B_0$ homogeneity within other regions such as the parietal and occipital lobes was slightly improved as well. Furthermore, the FWHMs of the frequency distributions were reduced significantly after addition of the third-order passive shim.
However, this initial approach using the combined method of active and passive shimming did not generate sufficient field homogeneity in the OFC. This is because the presence of the air-filled sinus cavity in the main magnetic field gives rise to an additional externally magnetic field near the sinus region (i.e. the frontal lobe). Additionally, the third-order passive shim field generated by our models was not adjustable; therefore, it can cause overcompensation of the field in some cases. We also demonstrated that the third-order magnetic field component does not significantly contribute to the inter-subject variation in the same MRI system, and this can be exploited to design and construct an optimal local passive shim coil. The proposed method used the combination of active and third-order passive shimming, and is expected to have great benefits for studies of the prefrontal lobe (MRI/MRS) at high field.

As noted above, the introduction of the third-order passive shim along with the active shim was not immensely effective in improving field homogeneity in the OFC. In a separate study, we developed and optimized a local dipole passive shim technique to optimize the field inhomogeneity over the OFC region. The required susceptibility and dimensions of the shim inserts were determined and we fixed them at the predefined positions on the subject-friendly, half-cylindrical passive shim system, which was placed below the chin. The selection of magnetic properties and mass of the shim inserts depend on the inhomogeneous field intensity of the target regions. As noted, the intensity of field inhomogeneity within the sinus region varies from subject to subject, depending on the volume of the air-filled sinus cavity. Therefore, materials with higher negative susceptibility may be required when there is a significantly higher inhomogeneous field in a subject’s sinus region. It is hard to find material with highly negative magnetic susceptibility in nature. Thus, it may require a large number of shim elements for this condition; although that number may be limited by the dimension of the design. This is one of
the challenges for this type of approach. Furthermore, the selection of brain ROI to precede the global active shimming is another challenge. During the selection of ROI, it is impracticable to exclude regions such as mouth, tongue, and others; this could cause failure in achieving the optimization over the entire brain. Inaccurate estimation of the magnetic susceptibility or dimension of shim inserts may potentially generate unwanted fields that jeopardize the field homogeneity.

5.2. Future Studies

In this work, we demonstrate that improving field homogeneity over the entire human brain is possible by using a combined method of active, third-order shims and dipole passive shims. This work also provides several important directions for future studies.

In working with our subjects, we noticed that the combined method of active and passive shimming technique sometimes over-compensated field homogeneity in certain brain regions, including the prefrontal lobe, hippocampus, parietal lobe, and occipital lobes. These over-compensated results were found mostly in subjects who were not included in the original volunteer group when the model was built. This suggests the overcompensation may result from an incomplete model. For future studies, we need to increase the number of subjects for building an effective higher order passive shim system. Additionally, specific parameters such as subject’s age, head size, head shape, and dental fillings also should be taken into consideration.

The third-order passive shim device was designed by placing the ferromagnetic shim inserts with pre-calculated mass at predefined positions around the surface of the cylindrical surface. The generated passive shim field, thus, was not flexible, and was limited by the fixed
number of ferromagnetic shim inserts and their position. To overcome this limitation, the fixed-mass of shim inserts can be replaced with an adjustable shim tube which allows the user to adjust the mass of the shim insert and consequently generate the desired field in real time.

Even though this work focused on the human brain, the methodology we developed is also suitable for other regions of the human body, such as breasts and knees [3]. It is well known that severe field inhomogeneity is commonly encountered in MRI/MRS of breasts and knees [3]. Moreover, the anatomical locations of breast, knee, or hand are distant from the isocenter of MRI magnet and have made the shimming even more challenging. The passive shim technique can be a very effective solution to address this issue. Combined with existing active shim coil, the introduction of a passive shim device would generate optimal homogenous field in those difficult-shim regions and ultimately improve MRI/MRS studies.

Moreover, the localized dipole passive shim technique alone can be used to improve the field homogeneity over a particularly localized region within the human brain (i.e. the temporal lobe), by placing appropriate passive shim inserts with accurate mass where needed on the subject’s head.

5.3. References

Appendix

Determination of the dielectric constant for passive RF shimming at high field

Abstract

Optimal image quality for Magnetic Resonance Imaging (MRI) at high fields requires a homogeneous RF ($B_1$) field; however, dielectric properties of the human brain cause $B_1$ field inhomogeneities and signal loss in the periphery of the head. These result from constructive and destructive RF interactions of complex wave behavior, which become worse with increasing Larmor frequency. Placement of a shim object with high dielectric constant adjacent to the body has been proposed as a method for reducing $B_1$ inhomogeneity by altering wave propagation within the volume of interest [1], [2], [3], [4] and [6]. Selecting the appropriate permittivity and the quantity of material for the shim object is essential. While previous work has determined the dielectric properties of the shim object empirically, this work introduces an improved theoretical framework for determining the requisite dielectric constant of the passive shim material directly and verifies the accuracy using the simulation field maps.

Keywords: RF field inhomogeneity, dielectric constant, high-field MRI
1.1. Introduction

High-field MR imaging systems are increasingly utilized for \textit{in vivo} human studies due to enhanced signal-to-noise ratio. The MRI at high field also faces new technical challenges such as RF field inhomogeneity [3, 5, 6]. Optimal image quality at high field requires a homogeneous RF field; however, local dielectric property variations within the human brain lead to sample-induced $B_1$ field inhomogeneities which become worse with increasing main magnetic field strength [1, 3, 4, 5 and 7]. Additionally, the geometry and relative position of the human head within the RF coil also contribute to RF inhomogeneity. RF field inhomogeneity arises from constructive and destructive RF interactions and wave behavior [1]. At high field, the effective wavelength of the RF field is comparable to, or less than, the size of the human brain.

It is, therefore, important to reduce these undesirable spatial variations in the RF magnetic field. RF field homogeneity within the brain can be improved by placing shim objects made of high dielectric material around the head. With appropriate dielectric constant of the RF passive shim material, the placement of dielectric material around the head generates displacement current within the shim material and consequently produces a separate RF field close to the shim material [1, 4 and 5]. A variety of experiments have demonstrated improved $B_1$ field homogeneity within the brain through the use of high dielectric materials such as water and calcium titanate pads surrounding the head [1, 4]. In these studies, the appropriate dielectric constant of the RF passive shim material was selected through trial-and-error and experimental measurement of relative dielectric constant as a function of frequency [1].

In this work, we introduce a method to compute the dielectric constant of the shim object required to increase the axial propagation constant of the B1 field and thereby improve RF field homogeneity within the human brain.

1.1. Theory

The head is approximated by a cylinder of radius $r_1$. The head lies inside the birdcage coil and RF shield with radii $r_2$ and $r_3$, respectively. The head, coil and shield are assumed to be coaxial. Let regions 1, 2 and 3 define the spaces of brain, brain-to-coil and coil-to-shield (see Fig. 1). The relative permittivity and conductivity of brain is $\varepsilon_r$ and $\sigma$ respectively and both the brain-to-coil and coil-to-shield spaces are filled with air with permittivity $\varepsilon_0$. 


It is assumed that the $B_1$ electromagnetic field propagates in the z direction [7]. The general solutions of the propagating electromagnetic waves satisfy the scalar Helmholtz wave equation and the propagation constant can be determined from the wave equation solutions [5, 7 and 8]. The propagation constant in the brain is found to be [7, 8]

$$k^2 = \omega^2 \mu \varepsilon - j \omega \mu \sigma. \quad [1]$$

Similarly the propagation constant in air is given by

$$k_0^2 = \omega^2 \mu \varepsilon_0. \quad [2]$$

The general solution of the propagating electromagnetic wave from the vector Helmholtz equation is a function of unmodified Bessel functions. Each region has a unique electromagnetic wave solution with unknown amplitude coefficients. At each boundary, (i.e. $r = r_1$, $r = r_2$ and $r = r_3$) the propagating RF field satisfies different boundary conditions. The boundary conditions select the amplitude coefficients of the general wave solution [3, 7 and 8]. After applying the boundary conditions to wave solutions in each region, the characteristic equation can be determined. The characteristic equation is simplified by substituting the first terms in series expansion of the Bessel function [9]. The simplified characteristic equation is

$$f(k_{p0}, k_p, k_0, k, k_z) = (r_1^2.k_z^2.k_{p^2}.(k^2-k_0^2), (r_2^2 - r_f^2), \alpha_i) + (k_p^2.k_{p^0.2}(2.k^2.k_{p^0.2}(r_2^2 - r_f^2) + k_{p^2}.k_{p^0.}.(r_2^2 + r_f^2)) \cdot (r_1^2.k_z^2.k_{p^2}.\alpha' - k_{p^0.2}.\alpha_i) = 0. \quad [3]$$

where, $k_z$ and $k_p$ are the axial and radial propagation constants which together characterize the RF field amplitude distribution within the head, and $\alpha$ are $\alpha'$ are

$$\alpha = (-2./ (r_3^2.k_{p^0}.\pi))(r_1./ (r_3.\pi) + r_2./ (r_1.\pi)), \quad [4]$$

$$\alpha' = (-2./ (\pi^2r_1.r_2.r_3^2.k_{p^0}))(r_1 + \pi. r_3^2.k_{p^0}). \quad [5]$$

The radial propagation constant inside the brain $k_p$ depends on the dielectric properties of the brain [3, 6]
\[ k_\rho^2 = k^2 - k_z^2. \quad [6] \]

Likewise, the radial propagation constant in air \( k_{\rho 0} \) depends on the dielectric properties within the brain-to-coil region [3, 6].

\[ k_{\rho 0}^2 = k_0^2 - k_z^2. \quad [7] \]

The brain-to-coil space is filled with air. Hence, both the axial and free space propagation constants are similar, which indicates that the axial propagation constant is relatively small. In contrast, the radial propagation constant, \( k_\rho \) is relatively large at high frequencies which results in small wave lengths and large amplitude variation of the \( B_1 \) magnetic field. Large variations of RF field amplitudes result in \( B_1 \) field inhomogeneities within the target object i.e., subject’s brain. The RF field homogeneity can be improved if the radial wavelength of \( B_1 \) field can be increased to match the dimensions of the subject’s head or greater. Radial wave length, \( \lambda_\rho = 2\pi / k_\rho \), can be increased by minimizing the radial propagation constant. According to the Eq. [6], this can be achieved either by decreasing the propagation constant within the subject’s brain, which is impractical, or by increasing the axial propagation constant.

The axial propagation constant is therefore essential to improving field homogeneity with the increasing main magnetic field. Loading high-dielectric material into the brain-to-coil air region significantly influences the propagation characteristics of the head coil, which is demonstrated by changes in axial propagation constant, \( k_z \). The characteristic equation (Eq. [3]) allows the requisite effective dielectric constant for the loading substance to be determined.

Loading with high-dielectric material into the brain-to-coil air region significantly influences the propagation characteristics of the head coil, which is demonstrated by changes in \( k_z \). Combining Eqs. (2), (3), (4) and (5), the \( f(k_{\rho 0}, k_{\rho}, k_0, k, k_z) \) can be expressed as a function of axial and radial propagation constants. The modified characteristic equation can be factored to express the \( k_z \) as a function of the \( k_{\rho} \). Based on this relationship between radial and axial propagation constants, it is possible to maximize \( k_z \) and minimize \( k_\rho \). Additionally, the \( f(k_{\rho 0}, k_{\rho}, k_0, k, k_z) \) can also be expressed as a function of \( k_z \) and effective dielectric constant (\( \varepsilon \)) in the body-to-coil region, i.e. \( g(k_z, \varepsilon) \). The range of required effective \( \varepsilon \) and corresponding \( k_z \) can then be
determined when the modified characteristic function \( g(k_z, \varepsilon) \) approaches zero. Once the maximal \( k_z \) has been determined, the requisite effective \( \varepsilon \) for \( B_1 \) passive shimming can be determined.

1.1. Results and Discussion

Brain, RF coil and RF shield diameters of 20, 28 and 34 cm, respectively, were assumed throughout this calculation and the dielectric constant of the brain tissue is considered \( \varepsilon_b=58 \) [6]. Fig. 2 shows the relationship between axial and radial propagation constants at 4T and 7T \( B_0 \) field strengths. The global minimum \( k_\rho \) of 0.34 and 0.24 cm\(^{-1} \) are obtained for maximums \( k_z \) of 1.82 and 2.93 cm\(^{-1} \) at 4T and 7T respectively. Additionally, at the first maximums of \( k_z \), the \( \lambda_\rho s \) are about 18.5 and 26.2 cm which are comparable to or greater than the size of the brain. However, variation of \( k_z \) and \( \lambda_z \) influences the RF field variation along the z-direction. Due to the increasing \( k_z \) and decreasing \( \lambda_z \) with main field strength, RF field inhomogeneity is enhanced at high field. The relationship of the modified characteristic function and \( k_z \) and \( \varepsilon \) at 4T and 7T field strengths are shown in Fig. 3 (A) and (B) respectively. The best solutions of both \( k_z \) and \( \varepsilon \) are expected as the modified function, \( g(k_z, \varepsilon) \) reaches zero. Therefore, according to Fig. 3(A) and (B) the appropriate magnitude range of the effective \( \varepsilon \) of 92 - 94 and 108 - 112 are determined corresponding to \( k_z \) of 1.82 and 2.93 cm\(^{-1} \) as the \( g(k_z, \varepsilon) \) reaches zero. The bag containing the suspension of 40% v/v calcium titanate (CaTiO\(_3\)) in de-ionized water with effective \( \varepsilon \) of 110 and the bag containing water with \( \varepsilon \) of 80.0 were placed on either sides of the subject’s head and determined that greater signal from the area closer to the CaTiO\(_3\) bag at 7T by Haines et al., 2010 [1]. The selected effective \( \varepsilon \) of 110 for the 40% v/v CaTiO3 suspension by Haines et al., is validated the estimated range of the effective \( \varepsilon \) of 108 – 112 for the RF shim material at 7T using the theoretical model introduced in this article.

A suspension of an appropriate substance in de-ionized or pure water can be prepared by mixing two different materials with different volume fractions to achieve the required effective dielectric constant. Based on Lichtenecker’s logarithmic mixture law [10], the required volume fraction of the substance, \( \alpha \) can be found from

\[
\alpha = \frac{\log (\varepsilon_{\text{eff}}) - \log (\varepsilon_{\text{de-ionized water}})}{\log (\varepsilon_{\text{substance}}) - \log (\varepsilon_{\text{de-ionized water}})} \tag{8}
\]
where $\varepsilon_{\text{eff}}$ is the desired effective dielectric constant. $\varepsilon_{\text{de-ionized water}}$ and $\varepsilon_{\text{substance}}$ are the dielectric constants of de-ionized (or pure) water and the substance respectively. Hence, based on the Eq. [8], the required volume fraction of CaTiO$_3$ is about 22.5% to attain the effective dielectric constant of 92 - 94 of the suspension for the RF shimming at 4T. Further, Fig. 4 shows the measured dielectric constant with variation of volume fraction of CaTiO$_3$ in de-ionized water by Haines et al., 2010 and simulated results of the variation of dielectric constant with the volume fraction of CaTiO$_3$ in de-ionized water based on the Eq. [8]. According to Fig. 4, the measured dielectric constants by Haines et al., are comparable to the simulated results of the dielectric constants. Therefore, it is further validated that the estimated required volume fraction of CaTiO$_3$ of 22.5% is accurate for RF shimming at 4T.

Due to the introduction of high dielectric material, the RF field distribution is changed according to Maxwell’s Laws [1, 5, 10]

$$\nabla \times \mathbf{H} = J_c + J_d = \sigma \mathbf{E} + i\varepsilon_r \varepsilon_o \omega \mathbf{E}. \quad [9]$$

where $H$, $J_c$ and $J_d$, are the magnetic field, conductive and displacement current and $\sigma$, $\varepsilon_r$, $\varepsilon_o$ are the conductivity, relative and vacuum permittivity, respectively. The introduction of external dielectric materials produce conductive and displacement currents. The displacement current generates a separate RF field which leads to enhanced RF field homogeneity, whereas conductive current generates eddy current which causes $B_1$ field inhomogeneity. Therefore it is important to prepare the suspension with high dielectric constant but low or zero conductivity for the RF passive shimming.

Figure 5 shows simulated RF field distribution over an axial image of the brain at 4T (a) without any suspension and with a suspension of (b) $\varepsilon = 48$ (c) $\varepsilon = 94$ placed around the head. Each field map has been normalized to the field amplitude at the center of the FOV. It is assumed that the entire brain has a uniform dielectric constant of $\varepsilon_b = 58$. It is clearly visible in Fig. 5 that the ripples of the RF field distribution within the brain reduce as the dielectric constant within the body-to-coil region increases. Based on the Eq. [10], the $B_1$ field distribution within the subject’s brain can be determined [7, 9].

$$B_r \propto (1/r) \sum_{n = 0}^{\infty} ((-1)^n / n! (n + 1)!) (k_\rho r / 2)^{l+2n}. \quad [10]$$
where $r$ is the radial distance and $n$ is an integer. The proportional constant is related to the dielectric constant of the loading material within the brain-to-coil region. Furthermore, the axial RF field profiles for the subject’s brain based on Eq. [10] are plotted in Figure 6. The amplitudes of curves for different dielectric materials are normalized at the center point. Again, the plots verify that the ripples of the curves reduced as the dielectric constant increased. This implies that RF field homogeneity enhances as the dielectric constant increases. Inappropriate selection of suspension with inaccurate dielectric constant will not achieve the optimal RF field homogeneity. For example, in Figures 5(B) and 6, the RF field homogeneity is not improved over the entire brain even though the selected dielectric constant, $\varepsilon$ is about 48, which is greater than that of air. Therefore, it is essential to choose the accurate dielectric constant in order to achieve the optimal RF field homogeneity.

1.1. Conclusion

A technique for determining the dielectric properties of a passive shim element for improving RF field homogeneity by loading the appropriate dielectric material in the body-to-coil region has been shown. The simulation results shown in Fig. 5 and 6 indicate that this method has potential to improve RF field homogeneity within the subject’s brain. Due to the introduction of a dielectric material with high dielectric constant, the axial propagation constant is modified. This allows one to change the radial propagation appropriately. Accurate estimation of effective dielectric constant potentially diminishes RF field in-homogeneity over the subject’s brain. Eqs (3), (4) and (5) allow one to determine the effective dielectric constant of the mixture necessary to improve uniformity of the B1 field. Incorrect estimation of the dielectric properties of the suspension may potentially diminish signal intensity over the entire brain due to the presence of $B_1$ field in-homogeneity.
References


Figure 1 A long cylindrical model used to determine requisite dielectric properties of the passive shim material for $B_1$ passive shimming of the human brain.

Figure 2 Plot of the relationship between axial and radial propagation constants at 4T and 7T main magnetic field strength. The minimum radial propagation constant is chosen at the maximum axial propagation constant.
Figure 3 The distribution of the modified characteristic equation of $g(k_2, \varepsilon)$ according to variation of both axial propagation constant and dielectric constant at (A) 4T and (B) 7T. The appropriate dielectric constant is selected to ensure that $g(k_2, \varepsilon)$ is zero.

Figure 4 Plot of the measured by Haines et al., 2010 and simulated dielectric constant vs. the volume fraction of CaTiO$_3$ in de-ionized water.
Figure 5  Simulated RF field map for axial slices of a subject’s brain after filling the body-to-coil region with (A) air, (B) an arbitrary chosen dielectric material with $\varepsilon=48$, and (C) a dielectric material with $\varepsilon=94$ which is obtained based on the Eq. [3].

Figure 6  Radial field profiles for different dielectric materials filling the body-to-coil region at 4T. The amplitudes of curves are normalized at the center point.
Design of a cylindrical passive shim insert for human brain imaging at high field
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Introduction: Local susceptibility-induced field variations can lead to inhomogeneities that cause artifacts such as image distortion and signal loss. Therefore, in addition to active shimming, localized passive shimming has been used to reduce field deviations over desired regions of interest for high field MRI [1, 2]. For passive shimming, it is advantageous to position shim elements away from the subject and reduce discomfort. After computing positions for the shim elements using method introduced by Romeo and Hoult [4], determining the correct magnetic susceptibility and dimensions of the shim pieces at the surface of the selected geometries is essential for generation of the correct amplitude of the spherical harmonic passive shim fields. In this work, we introduce a method to evaluate the magnetic susceptibility and the dimension of the shim elements and verify the accuracy using the simulated field maps.

Theory: After selection of shim element locations, one can evaluate the susceptibility \( \chi \) and shim element dimensions required to produce a spherical harmonic field of order \( n \) and degree \( m \) with amplitude \( A_{n,m} \) based on Eq (1), which is adapted from Holt’s works [3-4].

\[
\chi = -4A_{n,m}\pi \left( e_{n}B_{0}\int_{R}^{r} \int_{\rho}^{z} \int_{\theta}^{\phi} \frac{(n-m+2)!}{(n+m)!} A_{n+m} \left( \cos\gamma \cos(m\phi) \right) d\rho d\theta d\phi \right)
\]

Here \( \rho, \gamma \) and \( s_{j} \) are the azimuthal, polar and radial positions of the shim elements and \( \omega_{m} \) is the Neumann factor (\( \omega_{m} = 1 \), if \( m = 0 \); otherwise, \( \omega_{m} = 2 \)). Assuming shim elements are very small compared to the dimension of the shim coil, the thickness, width and height of the shim elements can be estimated from the integration limits.

The \( A_{n,m} \) is given in Eq (2).

\[
A_{n,m} = \int f(x_{i,j,k}) \Delta H_{z}(x_{i,j,k})
\]

where the columns of \( f(x_{i,j,k}) \) are the \( n^{th} \) order and \( m^{th} \) degree spherical harmonics evaluated at \( x_{i,j,k} \) and \( \Delta H_{z}(x_{i,j,k}) \) is the measured inhomogeneous magnetic induction at position \( x_{i,j,k} \). The denotes -1, \( T \) represent, inverse, and transpose, respectively.

Methods: A 3D gradient-echo pulse sequence with modifications to reduce eddy currents was used to calculate a \( B_{0} \) field map of a volunteer’s brain by comparing the phase of two images acquired at two echo times (TE=5.25 or 7ms, TR=16ms, 10 spring, 256x256x256mm FOV, 128x64x64 matrix) on a 4T whole-body Varian INOVA. Phase-difference reconstruction was used to extract the flip, 256x256x256mm FOV, 128x64x64 matrix) on a 4T whole-body Varian INOVA.

Results: Assuming the shim pieces in a magnetic field \( B_{0} \), the passive shim field can be calculated correspondingly to each spherical harmonics shown in the top panel of Fig 2. These figures demonstrate that using both Eqs. (1) and (2) allow us to estimate the \( \chi \) of shim element along with the dimensions to generate the amplitude of the designed magnetic fields based on spherical harmonic in any specific orders and degrees.

Discussion: The amplitude of the spherical harmonics depends on the estimated \( \chi \) and incorrect estimation gives the unwanted fields. Hence, the \( \chi \) inversely proportional to the dimensions Eq. (1), the small pieces of shim elements are allowed to estimate the \( \chi \) which satisfies an approximation of the selected position. Inhomogeneous spherical harmonic fields may not be able to correct perfectly due to the presence of higher order fields. This manifest \( H_{z} \) remain some residual unwanted filed. The proposed method allows one to determine the magnetic susceptibility to uniform an uncorrected magnetic field in any spherical harmonic fields. Reference: [1] C. Juchem et al., JMR 2006; 183:278-289; [2] J. L. Wilson et al., MRM 2002;48:906-914; [3] Hoult and Lee, Rev.Sci.Instrum.1985; 56:131-135; [4] Romeo and Hoult, MRM, 1984, 1, 44-65
A novel localized passive shim technique for optimizing magnetic field of the human orbitofrontal cortex at high field

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Introduction: The orbitofrontal cortex (OFC) area plays a critical role in human brain functions. However, the great distinction in magnetic susceptibility between paranasal sinuses and the surrounding tissues of the nasal cavity leads to local susceptibility-induced field variations in the OFC that cause artifacts such as image distortion and signal loss. In addition to active and global passive shimming, localized passive shimming has been used to reduce field deviations over the OFC region for high field MRI.1 For passive shimming, it is advantageous to position shim elements away from the subject and reduce discomfort. 2 In this work, we propose the use of magnetic material to generate the passive shim field and improve the field homogeneity within a group of subjects’ brains, particularly in the OFC. The simulated residual field maps of each individual subject’s brain show significant improvement in the field homogeneity and verify the accuracy of using our novel passive shim technique.

Theory: Spherical shim elements, which can be diamagnetic and/or para/ferromagnetic material, with radius a and susceptibility \( x_{oi}(\pm) \) placed at the predefined position \((x_{oi}, y_{oi}, z_{oi})\) of a passive shim system in a main magnetic field gives rise to a magnetic induction out side the sphere at the position \((x, y, z)\) as shown in Eq. (1).

\[
\Delta B_{OCT}(x_{oi}, y_{oi}, z_{oi}) = \sum_{i} \frac{A_i}{3} x_{ijkl} B_0 \left[ \frac{(2(z-z_{oi})- (x-x_{oi})^2 - (y-y_{oi})^2)}{(z-z_{oi})^2 + (x-x_{oi})^2 + (y-y_{oi})^2} \right] \hat{k} \tag{1}
\]

Here, \( x, y \) and \( z \) are Cartesian coordinates describing 3D space and \( i \) represents the number of shim pieces. The simplified version of the Eq. (1) is given below.

\[
\Delta B_{OCT}(x_{oi}) = \sum_{i} \beta_i f(x_{oi}) \cdot \hat{k} \tag{2}
\]

Here, \( \beta_{3ijk} \) and \( \beta_i \) are spatial dependent components and amplitude of the generated passive shim field, respectively. The amplitude, \( \beta_i \), depends on the magnetic susceptibility and dimensions of each individual shim inserts. The least-squares optimal \( \beta_i \) is given by

\[
\beta_i = [ f(x_{oi})^\gamma f(x_{oi})^\gamma f(x_{oi})^\gamma \Delta H_z(x_{oi}) ] \tag{3}
\]

Here, \( \Delta H_z(x_{oi}) \) is the measured inhomogeneous magnetic induction at position \( x_{oi} \) (The -1, T superscripts represent matrix inversion and transposition, respectively). The average amplitude is given by Eq. (4) below (\( n \) is the number of subjects in the group).

\[
\beta_{av,i} = \frac{1}{n} \sum_{i} \beta_{3ijk} \tag{4}
\]

Methods: A 3D gradient-echo pulse sequence with modifications to reduce eddy currents was used to calculate \( B_0 \) field map of four volunteers’ brains by comparing the phase of two images acquired at two echo times (TE=5.25 and 7ms, TR=16ms, 10° flip, 256x256x256mm FOV, 128x64x64 matrix) on a 4T whole-body Varian INOVA system. Phase-difference reconstruction was used to generate the \( B_0 \) map with 3D phase unwrapping as necessary. The best mounting positions of the half cylindrical geometry (i.e. the basic passive shim geometry) were determined. The positions of the shim elements for each field component were evaluated with the adaptations for passive shim design. The measured, uncorrected field distribution of each subject was then projected onto Eq. (3) to obtain the desired amplitudes \( \beta_i \) of each individual subject. Then the average amplitudes were computed using Eq.4 for correction of the inhomogeneity. With the positions and average \( \beta_{av,i} \) selected, the required susceptibilities \( x_{oi}(\pm) \) and dimensions were determined.

Results: Fig 1(A) shows the passive shim system with diamagnetic (green) and para/ferromagnetic (red) shim pieces for generating the average dipole passive shim field. Fig. 1(B) shows the field map following first- and second order active shimming. The simulated residual field map after removal of the average dipole passive shim and optimal active shim field is shown in Fig. 1(C). These figures demonstrate that the presented technique can be used to improve the field homogeneity considerably over the entire brain particularly in the OFC.

Discussion: Even if the field homogeneity is improved within the subject’s brain after introduction of the 3rd order shim system, the field inhomogeneity is not significantly reduced within the OFC (data not shown). The simulation results show that this method effectively improves the field homogeneity, particularly in the OFC for a group of subjects (n=4). The generated passive shim field may sometimes overly compensate the field in some regions within the brain of some individuals. However, the results showed that the system worked generally well for all subjects if optimal positions and shim materials were determined. Reference; [1] Osterbauer et al., NeuroImage 29, 245-53 (2006); [2] M. Jayatilake et al.,Proc. Intl. Soc. Mag. Reson. Med. 17 (2009); [3] F.Schenck et al., Med. Phys. 815 – 850 (1996); [4] J.L. Wilson et al., MRM 2002; 48:906-914.
Construction and optimization of local 3rd order passive shim system for human brain imaging at 4T MRI

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Introduction: The optimal MRI quality relies on a homogeneous magnetic field. However, local susceptibility variations within human brain can lead to field inhomogeneity that causes artifacts such as image distortion and signal drop-out, which become worse with increasing magnetic field strength. Many evidences showed that high order shims (i.e. > 2nd order) are required for optimal MRI at field greater than 3T [1, 2]. However, due to limited space, many MRI systems provide only up to second order active shims. In this work, we introduce a third order local passive shimming along with the active 1st and 2nd order shimming to improve field homogeneity within the human brain for a group of subjects.

Theory: By careful selection of azimuthal, γ and polar φ angles independently, a passive shim coil can be designed with desired 3rd order spherical harmonics by using Equations (1) and (2) [3, 4]. Here, i and j represent the location of shim element in γ and φ angles; n and m are the order and the degree of spherical harmonics and R is the radius of the cylinder (see Fig 1).

\[
dH_i \propto \int_{-\pi/2}^{\pi/2} \cos(\phi) \cos(\phi') d\phi \cos(m(\phi' - \phi))
\]

(1)

\[
dH_i \propto \sum_{n, m} \sin(\gamma_i) \cdot P_{n+2m}(\gamma_j) \cos(\gamma_j)/R^{n+3}
\]

(2)

Following the selection of shim element locations, the susceptibility and the dimensions of shim pieces can be evaluated to produce a magnetic field with an optimal 3rd order passive shim coefficients (β_m,n) for the sub population using Eq. (3) [4]

\[
\chi = -4\pi \cdot \beta_m,n / \langle \epsilon_m \rangle \cdot B_i \sum_{n, m} \left( (n-m+2)! P_{n+2m}(\cos(\gamma_j)) \cdot \cos(m\phi_j) / (n+m)! \right) s_j^{n+1} dy_j
\]

(3)

Here, \( s_j \) is the radial positions of the shim elements and \( \epsilon_m \) is the Neumann factor (\( \epsilon_m = 1 \), if \( m = 0 \); otherwise, \( \epsilon_m = 2 \)). Assuming shim elements are very small compared to the dimension of the shim coil, the thickness, width and height of the shim elements can be estimated from the integration limits. The \( \beta_m,n \) is given in Eq (4).

\[
\beta_m,n = \left \{ \begin{array}{l}
g(x_{i,j,k}) \cdot g(y_{i,j,k}) \cdot T \\
g(x_{i,j,k}) \cdot T
\end{array} \right.
\]

(4)

where the columns of \( g(x_{i,j,k}) \) are subject dependent 1st, 2nd and the averaged 3rd order spherical harmonics evaluated at \( x_{i,j,k} \) of the \( n \)th subject and \( \Delta F_j \) is the measured inhomogeneous magnetic induction at position \( x_{i,j,k} \). The denotes -1 and T represent inverse and transpose, respectively.

Methods: A 3D gradient-echo pulse sequence was modified to reduce sensitivity to eddy currents and used to obtain field maps of 4 subjects’ brains at a 4T Varian INOVA system. Measured field maps were used to evaluate the subject dependent 3rd order spherical harmonic coefficients of the passive shim (Eq. 4). The optimized positions (Eq.1, 2), the required susceptibility and dimensions (Eq.3) of shim elements were evaluated on a cylindrical surface to generate the desired magnetic field that can optimize the field variation over the entire human brain. A ferromagnetic material of Ni (77%)-Fe (16%)-Cu (5%) -Cr (2%) was used to construct the 3rd order passive shim system. Then the constructed 3rd order passive shim tube (36.5 cm OD and 35.5 cm in length) (Fig 2A) was mounted surrounding the RF coil. The adjustable 1st and 2nd order active shimming was applied following introduction of the desired 3rd order passive shim fields.

Results: Fig 2(A) shows the cylindrical passive shim system with ferromagnetic shim pieces for generating the Z(X2 – Y2) (green), YZ2 (blue), and Y3 (red) 3rd order passive shim field. Fig 2(B) shows the measured field maps within the subject’s brain (a) following active shimming only and (b) following both active and 3rd order, Z(X2 – Y2); YZ2 and Y3 passive shimming. The whole brain field histogram line width (Fig 2C) 135.2 Hz (blue) was reduced to 107.8 Hz (red) when using both active and the 3rd order passive shimming as comparing to using the active shimming only.

Discussion: This study shows that the introduction of both active and the 3rd order passive shimming the field homogeneity over the entire brain particularly in the regions of prefrontal lobe and the hippocampus regions is improved significantly, which is clearly noticeable in the field map shown in Fig. 2B(b). Additionally, it is also noticeable that the slight improvement in the \( B_i \) homogeneity within the top of the parietal and occipital lobe as revealed by the comparison of the sagittal field maps of Fig 2B(a) and (b). This technique will greatly benefit studies of the prefrontal lobe (MRI/MRS) at high field. The average brain field map can be used to construct any higher order passive shim system to improve the field homogeneity over the human brain for a group of subjects. This work demonstrated that a 3rd order local passive shimming is effective and easy to construct.

Theoretical determination of the dielectric constant for passive RF shimming at high field
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Introduction: Optimal image quality for MRI at high fields requires a homogeneous RF ($B_1$) field among others; however, dielectric properties of the human body result in severe $B_1$ field inhomogeneity. These are resulted from constructive and destructive RF interactions of complex wave behaviour, which become worse with increasing Larmor frequency. Placement of a shim object with high-dielectric constant adjacent to the body has been proposed as a method for reducing $B_1$ inhomogeneity by altering wave propagation within the volume of interest [1, 2, 3, and 4]. Selecting the appropriate permittivity and the quantity of material for the correction of $B_1$ field is essential. While previous work determined primarily empirically the dielectric properties of the shim object, this work introduces a theoretical framework for calculating the requisite dielectric constant of the passive shim material and verifies the accuracy using simulated field maps.

Theory and Methods: The human head is approximated by a cylinder with radius $r_1$ of 20 cm. The head lies inside a TEM coil with inner and outer radii $r_2$ and $r_3$ of 28 and 34 cm, respectively. The head and coil are coaxial (Fig. 1). The relative permittivity and conductivity of brain is $\varepsilon_r = 58$ and $\sigma = 0.35$ S/m, respectively; and both the brain-to-coil and inner-to-outer coil spaces are filled with air with permittivity $\varepsilon_0$ [2]. It is assumed that $B_1$ field propagates along the $B_0$ direction. The general solution of the propagating electromagnetic wave has a unique solution at each region with unknown amplitudes. After applying the boundary conditions to wave solutions in each region, the characteristic equation can be determined [5]. The characteristic equation can further be simplified by substituting the first terms in series expansion of the Bessel function. The simplified characteristic equation is

$$f(k_{00}, k_x, k_y, k_z) = r_1^2 - k_x^2 - k_y^2 (k^2 - k_z^2) (r_2^2 - r_3^2) \alpha + k_x^2 k_y^2 (2k^2 - k_z^2) + k_x^2 k_z^2 (r_2^2 - r_3^2) + k_y^2 k_z^2 (r_2^2 + r_3^2) = 0 \quad (1)$$

Here $k$, $k_0$, $k_x$, and $k_y$ are the brain, air, axial and radial propagation constants respectively. The $k_x$ and $k_y$ characterize the RF field amplitude distribution within the brain, where $\alpha$ and $\alpha'$ are given below.

$$\alpha = -2 \left( \frac{r_1^2 + \pi \cdot r_2}{r_2} \cdot r_1 \cdot r_2 \cdot k_{00} \right) \quad (2); \quad \alpha' = -2 \left( \frac{r_1 + \pi \cdot k_{00} \cdot r_2}{r_2} \cdot r_1 \cdot r_2 \cdot k_{00} \right) \quad (3)$$

The values of $k_x$ and $k_{00}$ depend on the dielectric properties within the brain and brain-to-coil region.

$$k_0^2 = k_1^2 - k_z^2 \quad (4); \quad k_{00}^2 = k_0^2 - k_z^2 \quad (5)$$

Loading with high dielectric material into the brain-to-coil air region significantly influence on the propagation characteristics of the head coil which is demonstrated by changes in $k_z$. Combining Eqs. (2), (3), (4) and (5), the $f(k_{00}, k_x, k_0, k_z)$ can be expressed as a function of $k_x$ and $k_0$. The modified characteristic equation can be factored to express the $k_x$ as a function of the $k_0$. Based on this relationship between radial and axial propagation constants, it is possible to maximize $k_x$ and minimize $k_0$. Additionally, the $f(k_{00}, k_x, k_0, k_z)$ can also be expressed as a function of $k_x$ and effective dielectric constant ($\varepsilon$) in the body-to-coil region, i.e. $g(k_x, \varepsilon)$. The range of required effective $\varepsilon$ and corresponding $k_x$ can then be determined when the modified characteristic function $g(k_x, \varepsilon)$ approaches zero.

Results: Based on the relationship between $k_0$ and $k_x$ (Fig. 2A) the minimum $k_0$ of 0.34 cm$^{-2}$ is obtained at the maximum $k_x$ of 1.82 cm$^{-2}$ for a 4T magnet. Additionally, at the first maximum of $k_x$, the radial wave length of $\lambda_x = 2\pi / k_x$ is about 18.5 cm which is comparable to the size of the subject’s brain. According to Fig. 2B the appropriate magnitude range of the effective $\varepsilon$ of 92 - 94 is determined corresponding to $k_x$ of 1.82 cm$^{-2}$ as $g(k_x, \varepsilon)$ reaches zero. Fig. 3 shows the simulated $B_1$ field map after filling the body-to-coil region with (A) air and (B) a dielectric material with $\varepsilon$ of 94 which is obtained according to Eq. (1). The $B_1$ field map is normalized to the field amplitude at the center of the FOV with the assumption that the entire brain has a uniform dielectric constant. The results clearly show that the ripples of the $B_1$ field distribution within the brain are significantly reduced as the $\varepsilon$ within the body-to-coil region increased.

Discussion: We have demonstrated with simulation data, as shown in Fig. 3, that this method provides an effective solution for selecting an appropriate material to improve $B_1$ field homogeneity within the human brain at 4T. This approach should be applicable to all fields. Experimental verification of the method is currently being conducted in vivo.