I, Allakh Kulkarni, hereby submit this original work as part of the requirements for the degree of Master of Science in Civil Engineering.

It is entitled:
An Application of Strut-and-Tie Model to Deep Beams

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An Application of Strut-and-Tie Models to Deep Beams

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Abstract

Strut-and-tie modeling (STM) is an experimentally proven technique to analyze and design D-regions. STM is easy to model if the truss configuration is available. The flow of forces and stresses within the beam can be visualized with STM, and an appropriate truss can be assembled to represent the stress pattern. The required reinforcement to resist the tension at different locations can be detailed from the forces in the truss members.

The study presented herein analyzes and designs deep beams subjected to point loads and uniformly distributed loads. Beams with high-strength as well as normal-strength concrete were modeled in this study. The clauses from ACI 318 (2008) are followed throughout this research. Strut-and-tie technique is an iterative process. MATLAB programs were written to perform the iterative calculations. The output from the MATLAB programs includes the longitudinal reinforcement as well as shear reinforcement required to resist the applied loads on the beam.

The cases considered herein included simply supported beams subjected to single or two point loads. The loads were placed symmetrically as well as asymmetrically. The output from the MATLAB programs was verified with the software CAST. The results from MATLAB were found to be in agreement with CAST output. A comparative study between two different models proposed in literature was performed and the results were included to justify the selection of a particular model in this research. An attempt was made in this research to generate an optimum design. The design was subjected to a large number of iterations. These iterations generated an optimum truss height and, hence, the most efficient design for the given beam properties. The strut-and-tie model considered in this research requires design of shear reinforcement as well as reinforcement to resist the transverse tensile force in the bottle shaped struts. The required
reinforcement to resist the two actions (shear and transverse tension) was detailed such that excessive use of bars was avoided.

The detailing of the longitudinal reinforcement was performed in a manner that would ensure ease of field installation. Preference was given to straight developed bars and smaller diameter bars. Similarly, large diameter bars were avoided as shear reinforcement. Adequate space was ensured within the bars from the same layer, with adjacent layers and shear reinforcement to facilitate concreting of the section.
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List of symbols and abbreviations

$A_{cs}$ - Area of cross section of struts

$A_h$ - Area of reinforcement required to resist the horizontal component of the transverse tensile force in a bottle shaped strut

$A_{hp}$ - Area of the reinforcement provided to resist the horizontal component of the transverse force in a bottle shaped strut

$A_{nz}$ - Area of cross section at face of the nodes

$A_s$ - Area of reinforcement required to resist shear

$A_{sl}$ - Reinforcement area for a bottle shaped strut

$A_{st}$ - Area of the longitudinal reinforcement

$A_{vp}$ - Area of the reinforcement provided to resist the vertical component of the transverse tensile force in a bottle shaped strut

$F_h$ - Horizontal component of the transverse tensile force in a bottle shaped strut

$F_n$ - Nominal strength of nodes

$F_{ns}$ - Nominal strength of struts

$F_{nt}$ - Nominal strength of a tie

$F_{rp}$ - Resistance provided by reinforcement towards the horizontal component of the transverse tensile force in a bottle shaped strut

$F_{strut}$ - Force at the end nodes of a bottle shaped strut

$F_{tie}$ - Ultimate tensile force in a tie

$F_{tie}$ - Transverse tensile force in a bottle shaped strut

$F_u$ - Ultimate load in a strut, tie, or node

$F_v$ - Vertical component of the transverse tensile force in a bottle shaped strut

$R$ - Minimum span to depth ratio
\( V_n \) - Nominal shear strength of a cross section

\( V_u \) - Ultimate shear strength of a cross section

\( b_w \) - Beam width

\( b_s \) - Space available to place bars in a single layer after deducting cover and stirrup diameter from the beam width

\( d_b \) - Diameter of reinforcing bars

\( d \) - Distance between the top most compression fiber to the centroid of the longitudinal reinforcement

\( f_{cn} \) - Allowable stress in the nodes

\( f_{cs} \) - Allowable stress in struts

\( f_y \) - yield strength of reinforcement

\( f_c' \) - Allowable stress in the struts

\( l_{available} \) - Available length for development of reinforcement at supports

\( l_d \) - Required development length for straight bars

\( l_{dth} \) - Required development length for 90-degree hooked bars

\( n_b \) - Maximum number of bars in a single layer

\( s_b \) - Width of base plate at load location

\( s_t \) - Spacing of reinforcement for a bottle shaped strut

\( s_p \) - Shear span of the beam

\( s_v \) - Spacing of vertical reinforcement in the shear span

\( sw \) - Support width

\( w_{tie} \) - Width of bottom longitudinal tie

\( x_1 \) and \( x_2 \) - Half the distance between center of the support and the load
\( z' \) - Available width to anchor struts at a node

\( \beta_n \) - Factor to account for the effect of the anchorage of ties on the effective compressive strength of a nodal zone

\( \beta_s \) - Factor to account for the effect of cracking and confining reinforcement on the effective compressive strength of the concrete in a strut

\( \theta \) - Angle of inclination of the bottle shaped struts with respect to the longitudinal tie

\( \Phi \) - Strength reduction factor
Chapter 1

1.1 Introduction

ACI-318 (2008) defines a deep beam as the one that has a clear span less than or equal to four times the member depth or a region with a concentrated load within twice the member depth from the face of the support. Deep beams are non-flexural members and have a higher shear capacity as compared to flexural members. The higher shear capacity is due to the force transfer mechanism that characterizes a deep beam. The strut and tie truss carries the load from the point of application to the support.

Disturbed regions, also called D-regions, are an inherent part of a deep beam. The strut-and-tie model (STM) is a tool to analyze and design the D-regions, and is better suited for everyday design as compared to finite element modeling. The strut-and-tie models explain the behavior of the stresses acting within a deep beam through the action of compression and tension members that are joined at the nodes (Schlaich et. al. 1987). These models present a clear picture of the stresses within the member. This research aims at the application of the strut-and-tie models to some particular cases of deep beams. The outcome of the research is a detailed procedure to analyze and design selected cases of deep beams based on the procedure outlined in Appendix –A of ACI-318 (2008), herein referred to as the Code.

Strut-and-tie model is not a cook book approach and requires some judgment on the part of the designer. Moreover, it requires a few iterations to reach a configuration of member sizes (struts, ties, and nodes) and a reinforcement layout that would satisfy the Code requirements. MATLAB programs can be written to perform the iterative calculations. Design tables can be generated using these MATLAB programs as per the requirement of a designer. Different cases of loadings, beam dimensions, and concrete strengths may be considered in development of
MATLAB programs that would serve design aids. Loading types that are considered in this research include point loads and uniformly distributed loads. The design concrete strength is also considered to be a variable. An application of normal-strength concrete as well as high-strength concrete is included. Hence, designers would have the freedom of choosing concrete strength as per site requirements or design needs. Beam span and width may be varied based on the design requirement.

The output of the MATLAB programs was verified for a limited number of cases with the aid of a computer program, CAST (Computer Aided Strut and Tie), developed by Tjhin and Kuchma (2002).

1.2 Background of STM

1.2.1 Early applications of truss models to beams

ASCE-ACI Committee 445 on Shear and Torsion (1998) provides a history of the development of truss models in concrete beams. A truss approach was first suggested by Ritter in 1899 to analyze the shear stresses in a beam. Morsch extended the truss problem to explain torsion in beams. The primary application of this procedure was to study the action of shear forces on beams. Ritter suggested that after formation of cracks in the concrete, a parallel chord truss is formed with diagonal compression members. The truss analogy was also applied to the study of combined shear and torsion in a reinforced concrete beam by Lampert and Thrulimann (Brown and Bayrak 2008).

1.2.2 Brief background on the research work for D regions and STM

A beam can be divided into two regions, the B-region and the D-region. As per the Code, any portion of the beam where the application of flexure theory is applicable is defined as the B-region. The B – regions exhibit a linear variation of strain through the depth. A D-region
represents discontinuities in the beam. The discontinuity could be a result of a change in geometry or presence of a concentrated load. Discontinuity leads to non-linear stress distribution and the Bernoulli’s hypothesis – ‘plane sections remain plain after bending’ does not apply. As per St. Venant’s principle, the stress distribution remains non-linear up to a distance equal to the depth of the member from the discontinuity.

The stress in an uncracked section can be calculated using section properties like cross-sectional area, and moment of inertia. Once the section is cracked, that is, the tensile stresses in concrete exceed the tensile capacity, truss model or its variations need to be applied (Schlaich et. al. 1987). It should be noted that the truss models do not account for shear strength in concrete due to dowel action, aggregate interlocking, and friction directly (Committee 445 report – 1998).

Marti (1985) and Marti (1999) applied truss models to study the relationships between the loads, reactions, and the internal forces within the concrete and reinforcement. Struts, ties, and nodal zones were used to develop the truss. The pin connections in a truss were idealized to nodes in the strut-and-tie model.

In the analysis of deep beams, the load is transferred directly to the supports from the point of application through the action of compressive struts, and ties. The struts are subjected to axial compressive loads while the ties take up the tensile forces. The D-regions are modeled as truss with compressive struts and ties that behave like compression members and tension members, respectively. The points of intersection of the truss members, that is, struts and ties, are defined as nodes (Marti 1985).

1.3 Basics of STM

This section describes the basic components of a STM and the guiding principles for generating a model that represents the actual behavior in a structure.
Ramirez and Yun (1996) outline the procedure to design a reinforced beams in the following three steps:

1. Selection of member dimensions and material properties based on practical or architectural constraints.

2. Calculation of stresses in concrete and reinforcement detailing is included in the second step. The assumptions made in the first step may be revised based on the results of this step.

3. Estimating the deflection at service loads and deformations completes the third step.

A strut-and-tie model is used in the second step to analyze and design a deep beam.

Assumptions outlined by Rogowsky (1986) are:

1. The STM truss should be in equilibrium.

2. Concrete resists only compression.

3. Reinforcement that is provided in the beam should resist all the tensile forces.

4. The failure of a deep beam is when the concrete from a strut fails in compression or a certain number of reinforcing bars yield in tension.

1.3.1 Struts

Struts are primarily compression members. The Code identifies a strut as a region of parallel or fan shaped compression field.
**Types of struts**

MacGregor (2002) and ACI-318 (2008) define the following types of struts.

1. **Prismatic strut**

   A prismatic strut can be idealized as strut with parallel boundaries. It may also have a tapered shaped depending on the nodes that it connects. Prismatic struts are normally observed in beams in regions where the width of the strut is restricted by the neutral axis. This type of strut represents the compressive stress block in beams. (Brown et al 2006).

   Prismatic struts occur in regions of constant bending moments in a beam (Brown and Bayrak 2008).

2. **Bottle shaped strut**

   A bottle shaped strut is formed when there is space available for the strut to expand laterally. Tension forces are generated in the mid-section of the bottle shaped strut on account of lateral expansion of the strut.

   The failure of a prismatic strut would be crushing of concrete. The nominal strength of struts should be high enough to allow the yielding of the ties prior to crushing of concrete in the strut. Yielding of the ties in a bottle shaped strut would give a warning of the impending failure. (Febres et al 2006). The Code provisions applicable to the design of struts are discussed in Chapter 2.

1.3.2 **Ties**

Ties are tension members. The capacity of a tie depends on the yield strength of the reinforcement that reinforces the tie. Yielding of reinforcement or loss of anchorage indicates the failure of ties. The behavior of ties is discussed in Chapter 2.
1.3.3. Nodes

Nodes are points of intersection of struts and/or ties. As explained by MacGregor (2002), nodes are the actual intersection points while a nodal zone is the area around the nodal zone within which the transfer of forces is assumed to take place. Nodal zones in a STM are complex members and the theoretical definition of a node is an approximation (Brown and Bayrak 2008). The theory of STM proposes a simple idealization of a complex stress problem. A nodal zone should have sufficient strength to allow safe transfer of forces.

Two types of nodal zones are defined in literature and included in the Code, namely, the hydrostatic nodal zone and the extended nodal zone. Hydrostatic nodal zones assume the stresses to be equal on all in-plane sides. The faces of the hydrostatic nodal zones are perpendicular to the axes of the struts and ties framing in to the nodal zone. Extended nodal zone is defined by the outlines of the compression members. These outlines also include the prism of concrete that is a part of the tie (MacGregor 2002). To satisfy the equilibrium condition, at least three forces should act at a nodal zone.

Three types of nodes are generally observed depending upon the number of struts or ties framing into a node. These are C-C-C (three struts framing at a node), C-T-T (one strut and two ties) and C-C-T (two struts and one tie). Failure of nodal zones such as crushing of concrete around the node or loss of anchorage zone for the tie reinforcement also implies failure of the model.

1.3.4 Description of strut-and-tie models

STM technique is applied to the analysis of the D-region for reinforced concrete deep beams. STM reduces a complex and nonlinear strain distribution to a simple truss problem that can be solved using basic equations of equilibrium. The aim of STM is to assemble a truss that resembles the flow of stresses in a deep beam. It is extremely important to get the flow of forces
correct. Typically, a few iterations are required before an appropriate truss can be assembled.

MacGregor (2002) has outlined the important aspects that need to be considered before arriving at a suitable truss model. These issues include:

1. The layout of struts and ties.
2. Strength of struts and nodal zones.
3. Anchorage and detailing requirements for reinforcement.

Applications of cracking pattern, if available, indicates the location of struts and ties that are formed upon loading of a deep beam up to failure. Crack patterns can be used to build a hypothetical truss, analysis of which would give the stresses within the D-region. Some guidelines mentioned in Appendix A of the Code are as follows:

1. The external factored loads and the reactions at the supports must be in static equilibrium.
2. The nominal strength of struts, ties, and nodal zones should be greater than the forces acting on them, as calculated from the analysis of the truss.
3. Struts cannot overlap each other.
4. The minimum angle between a strut and tie joined at a node is 25 degrees

1.4 Objective of the research

The research presented in this report has two main objectives. First, robust strut-and-tie models (STM) for nearly all cases encountered in practice are discussed. Various beam dimensions, loading, and concrete strengths were considered. Using the models and the associated discussions, practitioners would be able to use STM for their design with confidence. The second objective is to develop MATLAB programs to facilitate the use of STM for the practical cases
presented herein. Using these programs, tabulated design charts can be generated. These charts can be used in lieu of running the MATLAB program.

1.5 Cases considered in this research

The combinations of loads, beam dimensions, concrete strength, and load placement summarized in Table 1.1 were undertaken. The basic principles of strut-and-tie modeling remain the same for each case. The difference lies in the model used. A detailed description of the Code provision and their application in the research are provided in Chapter 2.
Table 1.1 Description of the various cases considered in this research

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Details of the case</th>
<th>Location of load on the beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Beam subjected to two equal point loads.</td>
<td>Loads placed symmetrically at a distance equal to beam depth from the face of the support</td>
</tr>
<tr>
<td></td>
<td>Concrete strength less than or equal to 6000 psi</td>
<td></td>
</tr>
</tbody>
</table>
Table 1.1 (cont’d) Description of the various cases considered in this research

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Details of the case</th>
<th>Location of load on the beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>Beam subjected to two unequal point loads.</td>
<td>Load placed at some random points on the beam within some</td>
</tr>
<tr>
<td></td>
<td>Concrete strength more than 6000 psi</td>
<td>constraints.</td>
</tr>
<tr>
<td>9</td>
<td>Beam subjected to a single point load.</td>
<td>Load placed at any point on the beam within some constraints.</td>
</tr>
<tr>
<td></td>
<td>Concrete strength less than or equal to 6000 psi</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Beam subjected to a single point load.</td>
<td>Load placed at any point on the beam within some constraints.</td>
</tr>
<tr>
<td></td>
<td>Concrete strength more than 6000 psi</td>
<td></td>
</tr>
</tbody>
</table>

1.6 Brief description of the design tables

As mentioned in Section 1.1, the aim of this research is to develop MATLAB programs that analyze and design deep beams for the cases mentioned in Table 1.1. These MATLAB programs could be used to prepare design tables. The design of a deep beam would require the following details.

1. Flexural reinforcement – The tables include the total number of bars and number of layers of reinforcement required. The tables also include the location of the center of gravity for the main longitudinal reinforcement. The reinforcement layout should be such that the location of the center of gravity of all the reinforcement taken together should match the centerline of the tie from STM. Various sketches included in Section 2.6.1 shed some light on the Code requirements for the placement of bars.
2. Shear reinforcement – The shear reinforcement is provided from the face of the support up to the load for point loads. In case of uniformly distributed loads, the reinforcement is to be provided for the entire span of the beam.

3. Reinforcement for the bottle shaped strut – Vertical and horizontal reinforcing bars are to be provided on both the vertical faces of the beam to resist the transverse tension along the length of the bottle shaped strut. The Code handles deep beams with concrete strengths up to 6000 psi in a manner different from the deep beams with concrete strength more than 6000 psi.

The input parameters that are required for design are:

1. dimensions of the beam
2. material properties (concrete strength and reinforcement yield stress), and
3. loads on the beam.
Chapter 2

Case 1 – Beam subjected to two equal point loads (Normal strength concrete).

2.1 Description of the beam and geometry

The beam selected for design in this case is simply supported and has a single span. The geometry of the beam is as sketched in Fig. 2.1. The loads on the beam are equal and symmetrically placed.

![Geometry of the beam and loads (elevation and side view)](image)

Fig.2.1 Geometry of the beam and loads (elevation and side view)

The conditions that are necessary for any beam to be qualified as a deep beam are:

1. The loads are at a distance equal to the depth of the beam, from the face of the support.

2. The span of the beam is less than four times the depth of the beam.

Thus, the conditions that are necessary for any beam to be qualified as a deep beam are fulfilled by the beam geometry from Fig. 2.1. The complete span of the beam can be considered to be a D-region as per St. Venant’s principle. Hence, the application of strut-and-tie model to analyze and design the beam is justified.
The loads on the beam (Load 1 and Load 2) are inclusive of the self-weight of the beam. The total self-weight is equally divided and added to each point load. Although the self-weight of the beam acts over the complete span and is in the form of a uniformly distributed load, it can be modeled as point loads without much loss of accuracy. This simplification is required in the modeling as a combined model for point loads and uniformly distributed loads makes the truss and other calculations complicated, and the advantages from such a model are offset by the complications involved. This argument is applied for all the models discussed in this research. A series of steps are described in this section to detail the procedure adopted for modeling the deep beam (Fig. 2.1) using strut-and-tie model.

2.2 Step by step approach to model, analyze, and design a deep beam with STM

The strut-and-tie model for the case under consideration is shown in Fig. 2.2. This model is appropriate for concrete strengths up to 6000 psi.

![Fig. 2.2 Strut-and-tie model for Case 1 (Alcocer and Uribe – 2002)](image)

**Step 1 – Selection of an appropriate strut-and-tie model**

Selection of a reasonable truss model is the first and most crucial step in the design of deep beams with the strut-and-tie model. An appropriate truss model, as defined by Rogowsky (1986),
is the one that depicts the stress pattern at failure and also yields an appropriate reinforcement that would allow a balanced or tension failure. The strut-and-tie model used in this research is selected from the work published by Alcocer and Uribe (2002). The model is shown in Fig. 2.2. Another approach to model the same beam is proposed by Kuchma and Tjhin (2002). A comparison of designs done with the two approaches is included in Section 2.12.

Members S1 through S7 are described as struts while members T1 through T5 are modeled as ties (refer to Fig. 2.2). The point of connection between the ties and struts are called as nodes.

**Step 2 – Calculation for the forces in the struts and ties**

This step is straight forward once STM and the height of the truss are selected. The forces are calculated by applying the principles of basic statics. The equations used to calculate the forces in the individual members are listed in Table 2.1 (refer to Fig. 2.2).

<table>
<thead>
<tr>
<th>Strut/Tie</th>
<th>Equations</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1 and S4</td>
<td>Reaction 1$^*$ [ \frac{\sin \theta}{\sin \theta} ]</td>
<td>Strut</td>
</tr>
<tr>
<td>T3 and T5</td>
<td>S1 \cos \theta</td>
<td>Tie</td>
</tr>
<tr>
<td>S7 and S5</td>
<td>S1 \cos \theta</td>
<td>Strut</td>
</tr>
<tr>
<td>T1 and T2</td>
<td>S1 \sin \theta</td>
<td>Tie</td>
</tr>
<tr>
<td>S2 and S3</td>
<td>Load 1 [ \frac{\sin \theta}{\sin \theta} ]</td>
<td>Strut</td>
</tr>
<tr>
<td>S6</td>
<td>S7 + S2 \cos \theta</td>
<td>Strut</td>
</tr>
<tr>
<td>T4</td>
<td>T3 + S2 \cos \theta</td>
<td>Tie</td>
</tr>
</tbody>
</table>

* Reaction 1 is equal to Load 1 for this case.
The forces in the truss members are a function of the angle $\theta$ between the inclined struts (S1, S2, S3 and S4) and the longitudinal tie (T3, T4 and T5) at the bottom of the beam. The angle ($\theta$) depends on the height of the truss and the distance $x_1$. Height of the truss is defined as the distance between the centerlines of the prismatic strut S6 and tie T4. The distance $x_1$ is constant (location of the load is fixed). Therefore, the depth of the truss is the only variable that controls the forces in the individual struts and ties.

An arbitrarily selected truss height is incapable of generating a compatible and efficient STM. A large value for the center to center distance between top strut (S6) and bottom longitudinal tie (T4) might lead to an incompatible truss model that cannot be accommodated within the physical dimensions of the beam. On the other side, a very small truss height might lead to large forces within the members that would make the beam design inefficient. An optimum truss height can be reached after successive iterations. Starting from a value close to the beam depth, the truss height is reduced in successive iterations. This process continues until a compatible truss configuration is reached. The reduction in truss heights for the successive iterations is small to ensure the most optimum design of the beam.

For the first iteration, the truss height is taken as 99% of the total beam depth minus the cover. The cover for the longitudinal reinforcement is a variable and is decided based on the provisions of the Code. The cover is deducted from the overall depth of the beam prior to calculating the truss height. The details of truss height and cover are shown in Fig.2.3.
Step 3 – Strut and nodal capacity calculation

2.3.1 Basic design equations for struts and nodes

The design of struts, ties, and nodes is governed by the following equation (Clause A.2.6 of the Code).

\[ \Phi F_n \geq F_u \]  \hspace{1cm} (2.1)

\( F_u \) – Ultimate load in the strut or at the face of the node or the tie

\( F_n \) – Nominal strength of the strut or at the face of the node or of the reinforcement in the tie

\( \Phi \) – Strength reduction factor. The Code specifies a value of 0.75 for design of struts, ties, and nodal zone.

2.3.2 Nominal capacity of struts and nodes

Clause A.3 of Appendix A from the Code governs the acceptable effective compressive stress in a strut. The Code introduces a factor \( \beta_s \) to account for reduction in strength as a result of the
cracking along the length of the strut caused by lateral expansion. The compressive forces at the nodes result in lateral expansion along the span of the strut.

The factor $\beta_s$ also accounts for the effect of confinement for concrete provided by the horizontal and vertical reinforcement within the strut length. The Code allows a lower reduction in strength for beams with reinforcement that controls the cracking in a strut. The values of $\beta_s$ for different types of struts are tabulated in Table 2.2.

**Table 2.2 Values of $\beta_s$ for struts (Section A.3 of Appendix A from the Code)**

<table>
<thead>
<tr>
<th>Type of Strut</th>
<th>$\beta_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Struts with a uniform cross section (Prismatic struts).</td>
<td>1.0</td>
</tr>
<tr>
<td>Struts with width at mid span greater than the width at the nodes (Bottle shaped struts).</td>
<td>0.75 (with confining reinforcement)</td>
</tr>
<tr>
<td>Struts with width at mid span greater than the width at the nodes (Bottle shaped struts).</td>
<td>0.6 (without confining reinforcement)</td>
</tr>
</tbody>
</table>

The strut-and-tie models in this research are applicable to normal-weight concrete. Light-weight concrete is not considered in this report. Similarly, unreinforced bottle shaped struts are not considered.

Analogous to the struts, the Code mentions a factor $\beta_n$ for nodes. These factors are included in Table 2.3.
Table 2.3 Values of $\beta_n$ for nodes (Section A.5 of Appendix A from the Code)

<table>
<thead>
<tr>
<th>Type of Nodes</th>
<th>$\beta_n$</th>
<th>Relevant nodes in Fig. 2.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-C-C node</td>
<td>1.0</td>
<td>N6 and N7</td>
</tr>
<tr>
<td>C-C-T node</td>
<td>0.8</td>
<td>N1, N4, N5 and N8</td>
</tr>
<tr>
<td>C-T-T node</td>
<td>0.6</td>
<td>N2 and N3</td>
</tr>
</tbody>
</table>

2.3.3 Calculations for the required widths of struts and nodal zones

The equations and theory discussed in this section are based on the provisions of the relevant clauses of Appendix A of the Code.

The nominal strength of a strut, mentioned in Eq. 2.1, is calculated as

$$F_{ns} = f_{cs} A_{cs} \quad (2.2)$$

$A_{cs}$ – Cross sectional area at one end of the strut given by,

$$A_{cs} = b_w w_s \quad (2.3)$$

$b_w$ – Width of the beam

$w_s$ – Width of the strut

Clause RA.3.1 of the Code allows the use of entire beam width in strength calculations for ties, nodes and struts for two-dimensional members.

The effective compressive strength ($f_{cs}$) of concrete used in Eq. 2.2 is given by,

$$f_{cs} = 0.85 \beta_s f'_c \quad (2.4)$$

$f'_c$ - Cylinder compressive strength of concrete

$\beta_s$ - Refer to Table 2.2

Similarly, the nominal strength for node is calculated as
\[ F_{nn} = f_{cn} A_{nz} \]  \hspace{1cm} (2.5)

Where \( A_{nz} \) is the area of cross section at the face of node given by

\[ A_{nz} = b_w w_s \]  \hspace{1cm} (2.6)

The effective compressive strength \( (f_{cn}) \) of concrete in the nodal zone used in Eq. 2.5 is given by

\[ f_{cn} = 0.85 \beta_n f'_c \]  \hspace{1cm} (2.7)

Refer to Table 2.3 for the value of \( \beta_n \). The capacity of a tie is controlled by the reinforcement while the width of a tie is a function of the capacity of the nodes that anchor the tie. The calculation for the capacity of a tie is summarized in Step 6.

**Step 4 – Calculation for the required dimensions of struts and nodal zones**

The demand from a strut, node, or a tie is linked to the capacity in Eq. 2.1. The required dimensions of struts and nodal zones are computed by equating the capacities from Step 3. The ultimate force in a strut or tie is known from Step 2. The areas \( A_{nz} \) and \( A_{cs} \) are unknowns in the calculation for the dimensions of the struts, ties, and nodal zones. The goal of this step is to compute the value of \( w_s \) for a strut or at the face of a node that would keep the resistance more than the demand. A number of important points need to be taken into account prior to calculating the required dimensions. These points are summarized in the next few paragraphs.

For the calculation of width for struts S1 and S2 either Eq. 2.2 or Eq. 2.5 would govern depending upon the value of \( \beta_s \) or \( \beta_n \). Strut S2 (Fig. 2.2) is modeled as bottle shaped. The value of \( \beta_s \) is 0.75 (from Table. 2.2) the strut. The node N2 is a C-T-T node since two ties frame into this node. Hence, the value of \( \beta_n \) would be 0.6 (from Table. 2.3). Thus, the strength of node N2 govern while calculating the required width a strut. The details are shown in Fig. 2.4.
The opposite is true for strut S1. The value of $\beta_s$ for the strut is 0.75 as it is a bottle shaped strut, but $\beta_n$ is 0.8 for the C-C-T nodes (nodes N1 and N5). Hence, the strength of the strut would govern the required width of the strut. The width so calculated would also satisfy the strength requirement at nodes. The details for strut S1 are shown in Fig. 2.5.
The width of tie T4 is governed by the required width at node N2 and N3 (Fig. 2.2). The longitudinal reinforcement is same throughout the length of the beam and, hence, the widths of ties T3 and T5 (Fig. 2.2) are same as T4 (refer to Fig. 2.6).

Fig. 2.6 Detailing of ties T3 and T4

From the discussion in the preceding paragraphs it is clear that while deciding the β factor for a strut, the strengths at the nodes at the ends as well as member strength (for struts) need to be compared and the minimum values of β are chosen for design. The final values of β used for the case under consideration are summarized in Table 2.4.
Table 2.4 Governing $\beta$ values for the strut-and-tie model

<table>
<thead>
<tr>
<th>Strut/Tie</th>
<th>Governing element</th>
<th>$\beta$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1 and S4</td>
<td>Strut strength</td>
<td>0.75</td>
</tr>
<tr>
<td>S2 and S3</td>
<td>Node strength (Node N2 and N3)</td>
<td>0.6</td>
</tr>
<tr>
<td>S5 and S6</td>
<td>Node strength (Node N5 and N8)</td>
<td>0.8</td>
</tr>
<tr>
<td>S7</td>
<td>Strut strength equals node strength</td>
<td>1.0</td>
</tr>
<tr>
<td>T1 and T2</td>
<td>Node strength (Node N2 and N3)</td>
<td>0.6</td>
</tr>
<tr>
<td>T3 and T5</td>
<td>Node strength (Node N2 and N3)</td>
<td>0.6</td>
</tr>
<tr>
<td>T4</td>
<td>Node strength (Node N2 and N3)</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Step 5 – Assembling of the truss

All the required dimensions for the ties and struts are available at this point. The truss can now be assembled. The truss with all the member dimensions (struts and ties) should fit within the external boundaries of the beam. Once this condition is met, the required reinforcement in the ties and the minimum reinforcement in the bottle shaped struts are calculated in Step 6 and Step 7, respectively. If the dimensions fail to fit within the outer boundaries of the beam, the truss height is reduced and steps 1 through 5 are repeated. The first check is based on the truss configuration shown in Fig. 2.3. The total height of the truss (inclusive of the truss member widths) plus the cover should be equal to the beam depth.

If the truss configuration is acceptable, additional checks are performed in this step. These checks include the shear capacity of the proposed model, and the angle between the longitudinal tie (T3) and the inclined strut (S1) shown in Fig. 2.2.
The bottle shaped struts need to be accommodated at the nodes. The nodes that are to be checked are N1 and N6 (Fig. 2.2). These nodes are more crucial than nodes N2 and N5 because their configuration is controlled by the tie width, and the width of the support or the base plate (for loads). Hence, the next check is to confirm the availability of sufficient width at nodes to fit the bottle shaped struts. The available width cannot be less than the actual required for the bottle shaped strut from strength point.

The checks mentioned in the preceding paragraphs are now discussed in detail.

2.5.1 Shear capacity of the section

The shear capacity of the cross section is calculated with the following equation from the Code. (Clause 10.7.4 of the Code)

\[ V_n = 10 \sqrt{f'_c} \cdot b_w \cdot d \]  (2.8)

d – The distance between the top most compression fiber and the centroid of longitudinal reinforcement.

The shear capacity is checked at the supports or at loads. It is confirmed at each trial height of the truss. If the shear capacity of the section is less than the actual shear force at any section, the height of the truss cannot be reduced any further. No further iterations are performed beyond this point. The beam configuration has to be modified if a member has to be detailed to carry the selected load.

2.5.2 Angle \( \theta \) between the longitudinal tie and the inclined strut

The Code specifies that the angle between the longitudinal reinforcement and the inclined strut should not be less than 25 degrees. This angle is checked for each height of the truss. If the angle falls below 25 degrees, the load cannot be carried by the beam within the provisions of the Code.
The beam geometry or the concrete strength has to be changed to design a STM for the given load.

2.5.3 Space available to accommodate the bottle shaped struts

The bottle shaped struts require some finite width at the nodes. This width is needed to accommodate the bottle shaped strut at the node. The required width is calculated in Step 4. The actual available width is a function of the inclination of the bottle shaped strut and the dimensions of the other truss members framing in to the nodes. The available width at the face of the nodes cannot be less than the required widths of the bottle shaped strut.

The framing of the bottle shaped struts into nodes N1 and N6 are crucial in the design of the deep beam modeled with the STM shown in Fig. 2.2. The geometry of the nodes N1 and N6 are shown in Fig. 2.7 and Fig. 2.8 respectively.

The support width (node N1) and the base plate width (at node N6) affect the node geometries. Hence, this check is performed only at these two nodes. Nodes N4 and N7 (refer to Fig. 2.2) are similar to nodes N1 and N6 on account of symmetry of the model.

![Figure 2.7](image)  
*Fig 2.7 Geometry of node N1 (Adopted from Appendix A, Fig. R.A. 1.5, of the Code)*
The width required for the strut from strength point of view is calculated in Section 2.3.3. Figure 2.7 gives the details needed to calculate the actual available space to accommodate the bottle shaped strut. The available width at the face of the node (N1) that anchors strut S1 is calculated as

\[ z' = wt \cos \theta + sw \sin \theta \]  \hspace{1cm} (2.9)

![Fig. 2.8 Geometry of node N5](image)

Calculations similar to node N1 are performed for node N6. The strut S2 should have a width less than or equal to the available space at the face of node N6. The equation is summarized below.

\[ z' = ws \cos \theta + sb \sin \theta \]  \hspace{1cm} (2.10)

The value of required strut widths \((z)\), calculated in Step 4, should be less than or equal to the available widths \((z')\) calculated from Eq. 2.9 and Eq. 2.10. If the available width is less than the required width, the beam geometry or concrete strength has to be changed to design a strut-and-tie model for the given load.
The checks performed in this section are required to confirm the acceptability of the strut-and-tie model. After satisfying the checks described earlier, a truss that fits within the physical boundaries of the beam is established.

2.5.4 Check for crushing of concrete at supports and at under the load

The concrete at supports and under the loads should have enough capacity to resist the applied loads. The capacity can be checked by calculating the strengths of the struts formed at the concerned nodes (N1 and N6) in a manner similar to that discussed in Section 2.3.3. Node N6 is a C-C-C node and, hence, a suitable strength reduction factor is to be selected from Table 2.3. Node N1 is also to be checked in the same fashion. The required widths of the struts connecting the load or the support to these nodes should be less than or equal to the actual width available of base plate at load or the support.

Steps one through five are illustrated through the flow chart shown in Fig. 2.9. The remaining steps involve the calculation of longitudinal reinforcement and detailing of the bottle shaped strut.
Fig. 2.9 Flow chart to explain the iterative process to generate a compatible truss model
Step 6 – Calculation for the longitudinal reinforcement

An acceptable truss configuration is obtained from the previous 5 steps. At this stage, the demands in the all the members are known. In the steps that follow, the reinforcement required to resist the tension in the bottom longitudinal tie (T4 in Fig. 2.2) is calculated. The procedure for detailing the required reinforcement is based on the relevant Code requirements.

2.6.1 Calculation of the total area of reinforcement

Clause A.4 from Appendix A of the Code specifies the following equation to calculate the nominal capacity in the longitudinal reinforcement.

\[ F_{nt} = f_y A_{st} \]  \hspace{1cm} (2.11)

\( F_{nt} \) – Nominal capacity of the tie

\( f_y \) – Yield strength of the reinforcement

\( A_{st} \) – Area of reinforcing steel

The nominal strength provided by the reinforcement should be greater than the demand, refer to Eq. 2.1. Demand in the tie is the force in the tie T4 while Eq. 2.11 gives the resistance. Equation 2.1 and Eq. 2.11 are combined to calculate the total reinforcement area required to resist the tension in the tie.

\[ A_{st} = \frac{F_{tie}}{0.75 f_y} \]  \hspace{1cm} (2.12)

\( A_{st} \) – Area of reinforcement required

\( F_{tie} \) – Force in the bottom tie, from Step 2

\( f_y \) – Yield strength of reinforcement
2.6.2 Detailing requirements for the reinforcement

The total area reinforcement required is calculated in Section 2.6.1. The next task is to detail the reinforcement. Detailing of reinforcement includes the selection of an appropriate bar diameter, number of bars per layer, number of layers, and development of the bars at the support. A check is also to be made at this point to confirm whether the required reinforcement can be accommodated within the width calculated for tie T4 in Step 4 based on the strength of nodes N2 and N3.

The longitudinal reinforcement in tie T4 is analogous to the flexural reinforcement for a slender beam. The detailing clauses from the Code that are applicable to slender beams also govern deep beams and are adhered to in this research. Thus, the provisions of the Code related to spacing of reinforcement, and development of bars needs to be conformed along with the requirements of STM. The relevant clauses and requirements are presented in conjunction with the detailing of reinforcement are discussed in the following sections.

2.6.3 Limits on the reinforcing bar size for longitudinal reinforcement

In this research, the minimum bar used as longitudinal reinforcement is #6 while the maximum bar size is #9. Bar sizes larger than #9 are avoided as large diameter bars require longer development lengths. For the majority of the cases, the bars need to be hooked at the support in order to develop the full tension capacity in accordance with Clause A.4.3.4 of the Code. Large diameter bars pose more problems during bending, and, hence, an attempt is made to use small diameter bars. Multiple layers of small diameter bars are preferred to single layer of large diameter bars, provided that there is space available to place multiple layers.
2.6.4 Development of reinforcing bars in tension

The total available space for development of reinforcing bar has the highest impact on detailing of reinforcement. The bar forming the longitudinal tie has to be embedded in the support at a distance equal to or more than its development length. The total space available for development dictates the maximum bar diameter that can be used as reinforcement for the longitudinal tie. The relevant clauses from the Code are followed to establish the total available space for bar development and the length required.

Reinforcing bars can be developed as straight or as hooked bars. Hooked bars require less development length as compared to straight bars. The space available for bar development is calculated in Section 2.6.5. The available space is then compared to the space required (for straight and hooked bars) for development. The maximum bar diameter bars that can be used as reinforcement is decided based on this comparison.

2.6.5 Space available for development of reinforcement

Figure 2.10, which is adopted from the Fig. R.A. 1.5 of the Code defines the available development length at the support. Equation 2.13 is derived based on this figure.

The available development length is calculated as

\[ l_{\text{avail}} = sw - \text{side cover} + 0.5 \frac{w_{\text{tie}}}{\tan(\theta)} \]  \hspace{1cm} (2.13)

\( sw \) – Width of the support

\( w_{\text{tie}} \) – Width of longitudinal tie (Tie T4 from Fig. 2.2)

\( \theta \) – Inclination of the bottle shaped strut with respect to the longitudinal tie

The last term in Eq. 2.13 (indicated by variable ‘x’ in Fig. 2.10) is based on the theory of extended nodal zone. The Code specifies a critical section up to which all the bars have to be fully developed. Side cover in this research is considered to be 2.5 inches (minimum) for hooked
as well as straight bars. The side cover of 2.5 inches is to be provided as it allows for a reduction in the required development length required (Clause 12.5.3 of the Code).

Reinforcing bars placed in multiple layers and developed as straight bars have equal available length for each layer. Equation 2.13 can be applied without any modifications for straight developed reinforcement as bars in all the layers have the same available length for development.

When hooked bars are to be developed in multiple layers, the bars in the top most layer have the least length to develop their full tension capacities. Hence, the check for development length is controlled by the available length of the top most layers. This point is illustrated in the sketches shown in Fig. 2.11 to 2.13. MacGregor and Wight (2005) have suggested developing checks for longitudinal reinforcement similar to those described previously.

**Fig. 2.10** Critical section for development of reinforcement (Adopted from Appendix A, Fig.R.A.1.5, of the Code)
Fig. 2.11 Development length available for single layer reinforcement configuration

Fig. 2.12 Development length available for two layers reinforcement configuration
2.6.6 Choice of straight or hooked reinforcing bars

The available development length (from Section 2.6.5) sets a limit on the maximum bar size that can be used as reinforcement and whether the bars can be developed straight or as a hooked bar. Preference is always given to smaller size bars that can be developed as straight bars. Hooked bars, especially larger diameter bars, may pose problem while bending in the required profile. Hence, the space required to develop straight bars is calculated first and compared to the actual available space from STM. The bar sizes that can be developed in the given space can then be considered for further detailing.

The available development length will determine the bar size that can be used as reinforcement to carry the load in the tie. All other parameters remaining constant, the development length varies directly with the diameter of the bar. Thus, the result from Eq. 2.13 will control the maximum bar diameter that can be developed in the available space. If bars cannot be developed in a straight manner within the given length, the alternative of hooked bars will be utilized.
2.6.7 Calculation for the development length of reinforcement

The development lengths of straight bars are calculated based on Clause 12.2 of the Code.

For #6 and smaller bars,

\[ l_d = \left( \frac{f_y \psi_t \psi_e}{25 \sqrt{f_c'}} \right) d_b \] (2.14)

For #7 and larger bars,

\[ l_d = \left( \frac{f_y \psi_t \psi_e}{20 \sqrt{f_c'}} \right) d_b \] (2.15)

\( d_b \) – Bar diameter.

\( \psi_t \) and \( \psi_e \) = 1.0 for all the cases considered herein (Clause 12.2.4 of the Code). Concrete strength, \( f_c' \) is in psi for Eq. 2.14 and Eq. 2.15.

The equation used to calculate the development length for hooked bars is given by Clause 12.5.2 of the Code.

\[ l_{dh} = \left( \frac{0.02 \psi_e f_y}{\sqrt{f_c'}} \right) d_b \] (2.16)

\( \psi_e \) is equal to 1.0 and strength of concrete \( f_c' \) is in psi. The Code allows the required development length be reduced by 30% if the provided cover normal to the plane of the hook satisfies the Code prescribed minimum values (Clause 12.5.3 of the Code). Hence, the value obtained from Eq. 2.16 is multiplied by 0.7 for the cases considered in this research.

2.6.8 Limitation on the maximum bar diameter

The available length for developing bars and requirement of development length are calculated in the previous two sections. The maximum bar size that can be used is now known and the
detailing part is to be completed with these allowable bar dimensions. As mentioned earlier, straight developed bars are preferred to hooked bars, and smaller diameter bars are favored to larger diameter bars.

2.6.9 Spacing limits for reinforcement

Clause 7.6 of the Code specifies the minimum clear spacing between parallel longitudinal bars. The minimum distance between two parallel bars is the larger of 1 inch and bar diameter. The minimum spacing between two layers of reinforcement is 1 inch (Clause 7.6.2 of the Code). These requirements are followed herein. The next two sections discuss the application of these provisions as a part of the detailing of the longitudinal reinforcement.

2.6.10 Calculation for the maximum number of bars in a single layer

The minimum spacing between bars in a single layer is described in the previous section. A sketch is included here to make the requirements more graphic. Figure 2.14 shows the layout of bars in multiple layers. The side and bottom covers shown are the minimum values prescribed by the Code. These covers may change depending on the site specific requirements or Code provisions for minimum cover in case of concrete exposed to corrosive environment.

Fig. 2.14 Minimum distances between reinforcing bars (cross section)
Thus, the maximum number of bars that can be accommodated in a single layer is calculated as

\[ d_b n_b + (n_b - 1) s = b_s \]  (2.17)

The diameter of the bar is \( d_b \). The minimum spacing between two parallel bars is the smaller of 1 inch and bar diameter \( (d_b) \), denoted by \( s \). The number of bars that can be placed in a layer is \( n_b \). The width of the beam is \( b_w \). Number 5 bars are assumed to be used as stirrups for this calculation. The stirrups with smaller diameter bars can be used depending on the actual requirements. In case with smaller diameter stirrups, the calculations presented in this section would still remain valid. Bar sizes larger than #5 is not envisaged as they are not practical for shear reinforcement. From Fig. 2.14, the space available to place the bars, after deduction of cover and stirrup diameter from the total beam width, is \( b_s \). The maximum number of bars that can be placed \( (n_b) \) is now calculated by substituting the known values in Eq. 2.17.

### 2.6.11 Calculation for the maximum number of layers of reinforcement

The minimum space between two layers is limited to 1 inch (refer to Fig. 2.14). The total width of the tie is available from the calculations in Section 2.2. The available width sets a limit on the maximum number of layers of bars that can be provided in a tie.

The total space required to accommodate \( n \) number of layers within the tie width is given by Eq. 2.18

\[ (n - 1) + n d_b = \text{width required} \leq \text{tie width} \]  (2.18)

Where \( n \) is the number of layers.

### 2.6.12 Final layout of reinforcement

The calculations performed in the previous sections are now compiled to develop a reinforcement layout that fulfills all the requirements of the Code and STM. The maximum bar size that can be used is based on development length requirements. The method used for bar
developing the bars, i.e., straight or hooked is now known. The maximum number of bars per layer as well as the maximum number of layers is now calculated.

The minimum bar size (#6) is adopted in the first iteration. The total number of bars required is the ratio of the total required area to the area of each bar. The number of layers required is the ratio of the total number of bars required to the maximum number of bars in each layer. The number of layers required is now checked against the maximum number of layers that can be accommodated (calculated from Eq. 2.17). If #6 bars do not fit the criterion summarized in the earlier section, #7 bars are to be checked based on the same criteria checked for #6 bars. The condition here is that #7 bars can be developed within the given space. This process continues until an acceptable configuration is reached and complete tension capacity of the bars can be developed within the available space. It should be noted that the center of gravity of reinforcement layout has to match the centroid of the longitudinal tie as shown in Fig. 2.14.

**Step 7 – Calculation of the confining reinforcement for bottle shaped struts**

The value of $\beta_s$ (bottle shaped strut strength reduction factor) is assigned as 0.75 in Step 4. The strength and performance of a bottle shaped strut is enhanced with the introduction of reinforcement within the strut length. The reinforcement prevents brittle failure of the strut and also allows the strut to carry some load even after the splitting cracks are formed (Brown et al 2006).

The transverse tension present along the length of a bottle shaped strut causes cracking parallel to the length of strut. These cracks reduce the compression capacity of the strut. The transverse tension can be resisted by reinforcement provided in a direction normal to the strut axis. The ideal placement of the reinforcement would be perpendicular to the cracked strut. However, the cracks have a variable angle and hence vertical and horizontal bars are recommended in the
Code. The component of the reinforcement capacity in the direction normal to the cracks is used in the calculation for the actual resistance of the bars (refer to Fig. 2.15).

The Code specifies a simplified method for calculating the minimum reinforcement required to resist the transverse tension in a bottle shaped strut. This simplified procedure bypasses the actual modeling of a bottle shaped strut for concrete grades up to 6000 psi.

The minimum reinforcement ratio is given by Eq. 2.19, which is the same as Eq. A.4 from Appendix A of the Code. In this equation, the horizontal and vertical bars are considered separately, and their individual contributions are added together.

\[
\sum \frac{A_{si}}{b_w s_i} \sin(\theta_i) \geq 0.003
\]  

(2.19)

The variables in Eq.2.19 are

\(A_{si}\) – Area of reinforcement

\(s_i\) – Spacing for vertical or horizontal bars

\(b_w\) – Width of the beam

\(\theta_i\) – Angle between the bar and the axis of the strut

Equation 2.19 can be satisfied by vertical steel, horizontal steel, or a combination of both. In this research, equal amounts of steel are provided in the horizontal and vertical directions. That is, the reinforcement in each direction is taken as at least 0.0015. The horizontal and vertical reinforcement ratios are computed from Eq. 2.20 and Eq. 2.21.

\[
\text{Horizontal component} = \frac{A_h}{b_w s_h} \sin(\theta) \geq 0.0015
\]  

(2.20)

\[
\text{Vertical component} = \frac{A_v}{b_w s_v} \cos(\theta) \geq 0.0015
\]  

(2.21)
The reinforcement in both the directions resists the splitting force that is assumed to be equally divided amongst them.

2.7.1 **Limit on the bar sizes for bottle shaped strut reinforcement**

The minimum bar size used as reinforcement for the bottle shaped strut is #3, and the maximum bar size is limited to #5. The reinforcement for a bottle shaped strut is detailed in a manner similar to the shear reinforcement in a slender beam. The maximum bar size is limited to #5 in order to facilitate bending and installation.

2.7.2 **Spacing limits for horizontal and vertical bars**

As per Clauses 11.7.4 and 11.7.5 of the Code, the maximum spacing permissible for the bars in both horizontal and vertical direction is smaller 12 inches and d/5. The value of d is the distance between the top most compression fiber and the centroid of the flexural reinforcement. The minimum spacing is limited to 4 inches to facilitate concrete placement.
2.7.3 Calculation of the horizontal reinforcement

Equation 2.20 gives the required minimum ratio (0.0015) for horizontal reinforcement. There are two variables involved in Eq. 2.20. The first variable is the bar area and the second is the spacing. Selection of an appropriate bar size at an optimum spacing is an iterative process. The aim of the iterations is to make sure that the ratio of reinforcement provided does not exceed the minimum required ratio (0.0015) by a large amount. This iterative process is explained next.

For the first iteration, the spacing is set to the limit imposed by the provisions of Clause 11.7.4. The reinforcement is set to the area of #3 bars with two legs. The values of the inclination of the strut with respect to the bottom longitudinal tie and the width of the beam are known quantities. The ratio calculated by substituting the reinforcement area and the spacing in Eq. 2.20 is checked against the minimum required. If the ratio is more than 0.0015, the reinforcement pattern (#3 bars – two legs at maximum spacing) is acceptable. If the ratio is less than the Code requirements, the spacing between the bars is reduced. The spacing is reduced in two-inch increments. It should be noted that the spacing between the bars is restricted to even numbers.

The calculations similar to the first iteration are performed to check the adequacy of reinforcement. If required, the spacing is further reduced keeping the area of reinforcement constant. As mentioned in Section 2.7.2, the minimum spacing between the bars is limited to 4 inches. If the ratio from Eq. 2.20 fails to meet the Code requirements at the minimum spacing (4 inches) and #3 bar area, the steel area is increased for the next iteration. The spacing is again set to the maximum permissible (Section 2.7.2). Thus, for the next iteration, #4 bars – two legs at the maximum allowable spacing are checked by Eq. 2.20. This procedure is repeated until a successful combination of area and spacing is achieved. This iterative procedure is illustrated with a flow chart shown in Fig. 2.16.
Input data – Beam width and inclination of the bottle shaped strut with respect to the bottom longitudinal tie.

Calculate the maximum spacing as per Code requirements.

Reinforcement = #3 bar, two legs at the maximum spacing

Calculate reinforcement ratio with Eq. 2.20

Is reinforcement spacing less than 4 inches?

Yes

Adopt a larger reinforcement area. Set reinforcement spacing to the maximum permissible

No

Is reinforcement ratio more than 0.0015?

Yes

Reinforcement pattern admissible as per Code provisions

No

Reduce reinforcement spacing by 2 inches

Fig. 2.16 Flow chart for selection of horizontal reinforcement in a bottle shaped strut
2.7.4 Calculation of the vertical reinforcement for the bottle shaped strut-and-tie T1

The calculations performed while finalizing a reinforcement layout in the vertical direction is slightly different from the approach taken for the horizontal reinforcement. The vertical reinforcement for the bottle shaped strut is to be combined with the tension reinforcement required for tie T1 and T2 (refer to Fig. 2.2). Hence, a more refined method is needed to detail the vertical reinforcement for the bottle shaped strut.

The iterations performed while deciding the pattern (area and spacing) of horizontal reinforcement were based on the area provided by an actual bar. For example, the first iteration considered #3 bars placed at the maximum permissible spacing. Whereas, the technique adopted for the vertical bar considers just the area that may not be available in the form of a bar. Once the required area at a spacing that satisfies Eq. 2.21 is obtained, the area required for the vertical ties (T1 and T2) can be added to the area for the bottle shaped strut. The spacing of the combined reinforcement is used in the calculations.

In the following sections, the procedure to detail the vertical reinforcement for the bottle shaped strut is discussed first. Subsequently the combination of the vertical tie reinforcement and bottle shaped strut reinforcement is presented.

2.7.5 Vertical reinforcement for bottle shaped strut

The first step in the iterations for deciding the vertical reinforcement layout is the area. The area is selected to be 0.1 square inches for the first iteration. The value of area selected is based on parametric studies performed prior to confirming the final algorithm. The area and spacing of reinforcement for a bottle shaped strut in beams with concrete strength is independent of the tensile force transverse to the strut (for concrete strengths below 6000 psi). The area and spacing depend on the beam width and the angle of inclination of the strut. It was observed that the actual
area required as per Code provisions is small for many cases. The value of 0.1 square inch is based on these observations.

Thus, for the first iteration, the area assumed to satisfy the Code requirement is 0.1 inch square at the maximum spacing calculated in Section 2.7.2. The reinforcement configuration is checked against the requirements of Eq. 2.21. If this check fails, the spacing of the reinforcement is reduced by two inches. As in the case of horizontal reinforcement, the vertical reinforcement is evenly placed. The minimum permissible spacing of reinforcement is limited to four inches. If the conditions from Eq. 2.21 are not met at a spacing of four inches, the area of reinforcement is increased by 0.01 square inches. The spacing is again set to the maximum permissible value (refer to Section 2.7.2). These iterations are continued until an acceptable reinforcement layout is obtained.

2.7.6  Reinforcement for the vertical tie T1

The reinforcement required to resist the tension in the vertical ties is calculated in a manner similar to the method used for the bottom longitudinal ties (Ties T3, T4 and T5 – Fig 2.2). The required reinforcement area is calculated by using Eq. 2.12. The reinforcement is distributed evenly over the shear span of the beam. The spacing requirements from Chapter 11 of the Code are applicable for the tie reinforcement.

2.7.7   Combination of the vertical reinforcement for bottle shaped strut and tie T1

The strut-and-tie model from Fig. 2.2 has two bottle shaped struts in each shear span. The spacing of the bottle shaped strut reinforcement is calculated in Section 2.7.5. As mentioned in Section 2.7.6, the total reinforcement for ties T1 and T2 is evenly distributed within the shear span. The total tie reinforcement is divided in two equal halves and added to the vertical reinforcement required for each bottle shaped strut, as discussed in the following.
If the total area for shear reinforcement is $a_v$ and the shear span is $s_p$, the number of bars and the area required at any reinforcement location can be calculated as

$$N = \frac{s_p}{s_v}$$ \hspace{1cm} (2.22)

$N$ – Total number of bars required

$s_v$ – Spacing required for bottle shaped reinforcement

$$a_b = \frac{0.5 \ a_v}{N}$$ \hspace{1cm} (2.23)

$a_b$ – Area required at each bar location

The total vertical reinforcement provided in each shear span is equal to the sum of $a_b$ and the output of the iterations performed in Section 2.7.5. This procedure is explained in flow chart shown in Fig. 2.17.

As reported by Ramirez et al. (2002), the strains in the vertical reinforcement depend on the location of the reinforcement and the corresponding crack pattern. Hence, the spacing of the vertical reinforcement calculated in Section 2.7.5 is kept unchanged when combining it with the vertical reinforcement for the ties.
Fig. 2.17 Flow chart for detailing of vertical reinforcement - bottle shaped strut and vertical ties

(T1 and T2)
2.8 Illustrative example for Case 1

2.8.1 Strut-and-tie model

This section includes a fully solved example for the deep beam discussed in Section 2.2 (refer to Fig. 2.2). The input parameters for this example are included in Table 2.5.

![Figure 2.18 Load and beam details](image-url)
Table 2.5 Geometry of the model, load, and material properties

<table>
<thead>
<tr>
<th>Input parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth of the beam</td>
<td>48 inches</td>
</tr>
<tr>
<td>Width of the beam</td>
<td>16 inches</td>
</tr>
<tr>
<td>Support width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Distance of load from the center of the support</td>
<td>56 inches</td>
</tr>
<tr>
<td>Distance between the loads</td>
<td>48 inches</td>
</tr>
<tr>
<td>Load 1</td>
<td>200 kips</td>
</tr>
<tr>
<td>Load 2</td>
<td>200 kips</td>
</tr>
<tr>
<td>Concrete strength</td>
<td>4000 psi</td>
</tr>
<tr>
<td>Yield strength of reinforcement (Longitudinal)</td>
<td>60000 psi</td>
</tr>
<tr>
<td>Yield strength of reinforcement (Transverse)</td>
<td>60000 psi</td>
</tr>
</tbody>
</table>

Step 1 – Check for the shear capacity of the section

The allowable shear load on the beam, \( V_u \) is given by Eq. 2.8

Substituting the values from Table 2.5,

\[
V_u = \Phi(10\sqrt{f_c} b_w d) = 0.75 \times 10 \sqrt{4000} \times 16 \times 0.99 \times 48 = 360.65 \text{ kips}
\]

At this step, the location of the centroid of the longitudinal fiber is not known. For the first iteration, the value of \( d \) is taken as 0.99 times the depth of the beam. For the subsequent iterations, the factor will decrease as the height of the truss reduces. The maximum shear, \( V_u \), is 200 kips (at the support) and it is less than the maximum permissible as per the Code provisions.
Step 2 – Height of the truss for the first iteration

For the current iteration, the depth of the truss is taken as 0.95 times the overall depth of the beam. Thus, the depth of the truss will be 0.95 x 46.5 = 44.175 inches.

Step 3 – Calculate $\theta_1$ and $\theta_2$ (angle between the inclined struts and longitudinal tie)

The depth of the truss is obtained from Step 2, and the geometry is known (see Table 2.5). The calculation for $\theta_1$ and $\theta_2$ is straightforward given as,

$$\theta_1 = \arctan\left(\frac{44.175}{28}\right) = 57.61 \text{ degrees}.$$  

Similarly, $\theta_2$ would be 57.61 degrees.

The limiting value of $\theta_1$ and $\theta_2$ is 25 degrees in accordance with the requirement of the Code.

Step 4 – Forces in members of the truss

The vertical reaction at node N1, i.e., the support reaction, is 200 kips. The forces in the tie T3 and strut S1 are calculated as follows.

$$S_1 = \frac{V}{\sin(\theta_1)} = \frac{200}{\sin(57.61)} = 236.84 \text{ kips}$$

Where, V is the support reaction.

$$T_3 = S_1 \cos(\theta_1) = 234.84 \times \cos(57.61) = 125.8 \text{ kips}$$

Similarly, the forces in the truss members are calculated using the procedure for solving a determinate truss. The forces are tabulated in Table 2.6.
Table 2.6 Forces in the members of the truss

<table>
<thead>
<tr>
<th>Member</th>
<th>Force</th>
<th>Nature of force</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>236.84 kips</td>
<td>Compressive</td>
</tr>
<tr>
<td>S2</td>
<td>236.84 kips</td>
<td>Compressive</td>
</tr>
<tr>
<td>S3</td>
<td>236.84 kips</td>
<td>Compressive</td>
</tr>
<tr>
<td>S4</td>
<td>236.84 kips</td>
<td>Compressive</td>
</tr>
<tr>
<td>S5</td>
<td>125.8 kips</td>
<td>Compressive</td>
</tr>
<tr>
<td>S6</td>
<td>252.67 kips</td>
<td>Compressive</td>
</tr>
<tr>
<td>S7</td>
<td>125.84 kips</td>
<td>Compressive</td>
</tr>
<tr>
<td>T1</td>
<td>200 kips</td>
<td>Tensile</td>
</tr>
<tr>
<td>T2</td>
<td>200 kips</td>
<td>Tensile</td>
</tr>
<tr>
<td>T3</td>
<td>125.84 kips</td>
<td>Tensile</td>
</tr>
<tr>
<td>T4</td>
<td>252.67 kips</td>
<td>Tensile</td>
</tr>
<tr>
<td>T5</td>
<td>125.8 kips</td>
<td>Tensile</td>
</tr>
</tbody>
</table>

Step 5 – Capacities of struts and nodes

The struts are divided into two groups. The prismatic struts and bottle shaped struts. The theory applied to calculate the strut width based on the strength reduction factors for struts is described in Step 3 and 4 of Section 2.2. The resulting values are summarized in Table 2.7.
### Table 2.7 Allowable compressive stresses in struts

<table>
<thead>
<tr>
<th>Strut</th>
<th>Type</th>
<th>β factor</th>
<th>Allowable compressive strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>Bottle shaped strut</td>
<td>0.75</td>
<td>2550 psi</td>
</tr>
<tr>
<td>S2</td>
<td>Bottle shaped strut</td>
<td>0.75</td>
<td>2550 psi</td>
</tr>
<tr>
<td>S3</td>
<td>Bottle shaped strut</td>
<td>0.75</td>
<td>2550 psi</td>
</tr>
<tr>
<td>S4</td>
<td>Bottle shaped strut</td>
<td>0.75</td>
<td>2550 psi</td>
</tr>
<tr>
<td>S5</td>
<td>Prismatic strut</td>
<td>1.0</td>
<td>3400 psi</td>
</tr>
<tr>
<td>S6</td>
<td>Prismatic strut</td>
<td>1.0</td>
<td>3400 psi</td>
</tr>
<tr>
<td>S7</td>
<td>Prismatic strut</td>
<td>1.0</td>
<td>3400 psi</td>
</tr>
</tbody>
</table>

As described in Step 3 and 4 of Section 2.2, the nodes are divided into three groups depending on the number of struts and ties meeting at a node. The nodes encountered in this example are C-C-C, C-C-T and C-T-T nodes. The capacities are shown in Table 2.8.
Table 2.8 Allowable stresses at face of nodes

<table>
<thead>
<tr>
<th>Node</th>
<th>Type</th>
<th>β factor</th>
<th>Allowable stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>C-C-T</td>
<td>0.8</td>
<td>2720 psi</td>
</tr>
<tr>
<td>N2</td>
<td>C-T-T</td>
<td>0.6</td>
<td>2040 psi</td>
</tr>
<tr>
<td>N3</td>
<td>C-T-T</td>
<td>0.6</td>
<td>2040 psi</td>
</tr>
<tr>
<td>N4</td>
<td>C-C-T</td>
<td>0.8</td>
<td>2720 psi</td>
</tr>
<tr>
<td>N5</td>
<td>C-C-T</td>
<td>0.8</td>
<td>2720 psi</td>
</tr>
<tr>
<td>N6</td>
<td>C-C-C</td>
<td>1.0</td>
<td>3400 psi</td>
</tr>
<tr>
<td>N7</td>
<td>C-C-C</td>
<td>1.0</td>
<td>3400 psi</td>
</tr>
<tr>
<td>N8</td>
<td>C-C-T</td>
<td>0.8</td>
<td>2720 psi</td>
</tr>
</tbody>
</table>

Step 5 – Required widths of struts and ties

In Step 4, the allowable stresses in struts and the faces of nodal zones are calculated. These stresses control the required widths of struts and ties. The area of concrete available in a strut and at the face of a node would determine the maximum factored nominal force that can be sustained. This force has to be equal to or greater than the ultimate force (calculated in Step 3) in the strut or at the face of the node. The governing widths for struts based on strut strength requirement are calculated first. The required strut widths are summarized in Table 2.9. The calculated strut and tie widths from strength consideration at the face of nodes are included in Table 2.10. Note that only one half of the beam needs to be modeled due to symmetry.
### Table 2.9 Required strut widths from strut strength perspective

<table>
<thead>
<tr>
<th>Strut</th>
<th>Type</th>
<th>Allowable compressive strength</th>
<th>Force in strut</th>
<th>Width required</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>Bottle shaped strut</td>
<td>2550 psi</td>
<td>236.84 kips</td>
<td>7.74 inches</td>
</tr>
<tr>
<td>S2</td>
<td>Bottle shaped strut</td>
<td>2550 psi</td>
<td>236.84 kips</td>
<td>7.74 inches</td>
</tr>
<tr>
<td>S3</td>
<td>Bottle shaped strut</td>
<td>2550 psi</td>
<td>236.84 kips</td>
<td>7.74 inches</td>
</tr>
<tr>
<td>S4</td>
<td>Bottle shaped strut</td>
<td>2550 psi</td>
<td>236.84 kips</td>
<td>7.74 inches</td>
</tr>
<tr>
<td>S5</td>
<td>Prismatic strut</td>
<td>3400 psi</td>
<td>125.8 kips</td>
<td>3.08 inches</td>
</tr>
<tr>
<td>S6</td>
<td>Prismatic strut</td>
<td>3400 psi</td>
<td>252.67 kips</td>
<td>6.19 inches</td>
</tr>
<tr>
<td>S7</td>
<td>Prismatic strut</td>
<td>3400 psi</td>
<td>125.8 kips</td>
<td>3.08 inches</td>
</tr>
</tbody>
</table>

* See Table 2.2 for the values of $\beta_s$
Table 2.10 Required strut and tie widths from node strength perspective

<table>
<thead>
<tr>
<th>Node</th>
<th>Strut/Tie Type</th>
<th>Type</th>
<th>Allowable compressive strength in node</th>
<th>Force in strut/tie</th>
<th>Width required</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1 (C-C-T node)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S1</td>
<td>Bottle shaped strut</td>
<td>2720 psi</td>
<td>236.84 kips</td>
<td>7.26 inches</td>
<td></td>
</tr>
<tr>
<td>T3⁺</td>
<td>Tie</td>
<td>2720 psi</td>
<td>125.80 kips</td>
<td>10.32 inches</td>
<td></td>
</tr>
<tr>
<td>SS1*</td>
<td>Prismatic strut</td>
<td>2720 psi</td>
<td>200.00 kips</td>
<td>6.13 inches</td>
<td></td>
</tr>
<tr>
<td>N2 (C-T-T node)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S2</td>
<td>Bottle shaped strut</td>
<td>2040 psi</td>
<td>236.84 kips</td>
<td>9.67 inches</td>
<td></td>
</tr>
<tr>
<td>T3⁺</td>
<td>Tie</td>
<td>2040 psi</td>
<td>125.80 kips</td>
<td>10.32 inches</td>
<td></td>
</tr>
<tr>
<td>T4</td>
<td>Tie</td>
<td>2040 psi</td>
<td>252.67 kips</td>
<td>10.32 inches</td>
<td></td>
</tr>
<tr>
<td>T1</td>
<td>Tie</td>
<td>2040 psi</td>
<td>200.00 kips</td>
<td>8.17 inches</td>
<td></td>
</tr>
<tr>
<td>N5 (C-C-T node)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S1</td>
<td>Bottle shaped strut</td>
<td>2720 psi</td>
<td>236.84 kips</td>
<td>7.26 inches</td>
<td></td>
</tr>
<tr>
<td>S7</td>
<td>Prismatic strut</td>
<td>2720 psi</td>
<td>125.80 kips</td>
<td>3.85 inches</td>
<td></td>
</tr>
<tr>
<td>T1</td>
<td>Tie</td>
<td>2720 psi</td>
<td>200.00 kips</td>
<td>6.13 inches</td>
<td></td>
</tr>
<tr>
<td>N6 (C-C-C node)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S7</td>
<td>Prismatic strut</td>
<td>3400 psi</td>
<td>125.80 kips</td>
<td>3.08 inches</td>
<td></td>
</tr>
<tr>
<td>S2</td>
<td>Bottle shaped strut</td>
<td>3400 psi</td>
<td>236.84 kips</td>
<td>5.80 inches</td>
<td></td>
</tr>
<tr>
<td>S6</td>
<td>Prismatic strut</td>
<td>3400 psi</td>
<td>252.67 kips</td>
<td>6.20 inches</td>
<td></td>
</tr>
</tbody>
</table>

Strut SS1 is the strut at support of the beam

⁺ Reinforcement from tie T4 is continued up to the support. Therefore the width of tie T3 is equal to the width of tie T4.

** See Table 2.3 for the values of $\beta_n$

The maximum width required to satisfy the nodal strength of the strut’s strength would control the required width of strut. As an example, consider strut S2, which is a bottle shaped strut with C-T-T node at one end. The width required to satisfy the strut’s strength is 7.74 inches (refer to Table 2.9). At node N2 (C-T-T node), the required width at the face of this node is 9.67 inches,
whereas at node N6 (C-C-C node) the width required is 5.80 inches (refer to Table 2.10). The maximum width would be used in the model. That is, the strength at node N2 will govern. Table 2.11 summarizes the governing widths of various ties and struts.

**Table 2.11** Governing widths of struts and ties

<table>
<thead>
<tr>
<th>Member</th>
<th>Force</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>236.84 kips</td>
<td>7.74 inches</td>
</tr>
<tr>
<td>S2</td>
<td>236.84 kips</td>
<td>9.67 inches</td>
</tr>
<tr>
<td>S3</td>
<td>236.84 kips</td>
<td>9.67 inches</td>
</tr>
<tr>
<td>S4</td>
<td>236.84 kips</td>
<td>7.74 inches</td>
</tr>
<tr>
<td>S5</td>
<td>125.80 kips</td>
<td>3.85 inches</td>
</tr>
<tr>
<td>S6</td>
<td>245.60 kips</td>
<td>6.20 inches</td>
</tr>
<tr>
<td>S7</td>
<td>122.80 kips</td>
<td>3.85 inches</td>
</tr>
<tr>
<td>T1</td>
<td>200 kips</td>
<td>8.17 inches</td>
</tr>
<tr>
<td>T2</td>
<td>200 kips</td>
<td>8.17 inches</td>
</tr>
<tr>
<td>T3</td>
<td>122.80 kips</td>
<td>10.32 inches</td>
</tr>
<tr>
<td>T4</td>
<td>245.60 kips</td>
<td>10.32 inches</td>
</tr>
<tr>
<td>T5</td>
<td>122.80 kips</td>
<td>10.32 inches</td>
</tr>
</tbody>
</table>

**Step 6 - Verification of the geometry for the strut-and-tie model**

**First check**

The total height of the truss (Fig. 2.4) is equal to the sum of the distance between the centerlines of the bottom tie – T4 and top strut – S6 plus half of the width of the bottom tie and half of the
width of the top strut. This total height cannot be more than the beam depth; otherwise, the truss cannot be fitted inside the physical dimensions of the beam.

For the first iteration, the truss height is available from the calculations performed in Step 2, and the strut/tie dimensions are those determined in Step 5. For this iteration: The height of truss + half of bottom tie + half of top strut = 44.175 + 0.5 (6.20 + 10.32) = 52.435 inches. The limit on the truss height is beam depth – cover = 48 – 1.5 = 46.5 inches.

Thus, the truss cannot be accommodated within the beam depth, and the height of the truss has to be reduced for the next iteration. This reduction increases the forces, and hence, the width of the individual struts and ties.

For the next iteration, the depth of the truss is taken as

(0.95 - 0.005) times (depth of beam – cover) = 0.945 (48 – 1.5) = 43.9425 inches.

For subsequent iterations, the height is reduced by small values to ensure the accuracy of the results and efficiency of design. Successive iterations are performed until this check is satisfied. The final solution to the problem is obtained and taken as the best possible truss height. The final truss “height factor” is 0.7840, which results in the overall height of the truss is 0.7840 (48-1.5) = 36.456 inches. Using this truss height, the final member forces and dimensions are computed. These values are summarized in Table 2.12.
Table 2.12 Final forces and member sizes after iterations

<table>
<thead>
<tr>
<th>Member</th>
<th>Force</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>252.18 kips</td>
<td>8.24 inches</td>
</tr>
<tr>
<td>S2</td>
<td>252.18 kips</td>
<td>10.30 inches</td>
</tr>
<tr>
<td>S3</td>
<td>252.18 kips</td>
<td>10.30 inches</td>
</tr>
<tr>
<td>S4</td>
<td>252.18 kips</td>
<td>8.24 inches</td>
</tr>
<tr>
<td>S5</td>
<td>153.61 kips</td>
<td>4.71 inches</td>
</tr>
<tr>
<td>S6</td>
<td>307.27 kips</td>
<td>7.53 inches</td>
</tr>
<tr>
<td>S7</td>
<td>153.61 kips</td>
<td>4.71 inches</td>
</tr>
<tr>
<td>T1</td>
<td>200 kips</td>
<td>7.53 inches</td>
</tr>
<tr>
<td>T2</td>
<td>200 kips</td>
<td>4.71 inches</td>
</tr>
<tr>
<td>T3</td>
<td>153.61 kips</td>
<td>12.55 inches</td>
</tr>
<tr>
<td>T4</td>
<td>307.27 kips</td>
<td>12.55 inches</td>
</tr>
<tr>
<td>T5</td>
<td>153.61 kips</td>
<td>12.55 inches</td>
</tr>
</tbody>
</table>

The check for compatibility of the truss is repeated. The height of truss + 0.5 (bottom tie + top strut) width = 36.456 + 0.5 (7.53 + 12.55) = 46.496 inches. The limit on the truss height is the beam depth – cover = 48 – 1.5 = 46.5 inches. Thus, the first check is satisfied.

**Second Check**

The shear capacity of the section is

\[ V_u = \Phi(10\sqrt{f'_c} b_w d) = 0.75 \times 10 \times \sqrt{4000} \times 16 \times (36.456 + 0.5 \times 7.53) = 305.26 \text{ kips} \]

The shear capacity is greater than the actual shear force in the shear span (200 kips). Hence, this check is satisfied.
Third check

The inclination angle of the bottle shaped struts with respect to the longitudinal tie has to be greater than 25 degrees. This angle, $\theta$, is calculated as

$$\theta = \arctan\left(\frac{\text{height of truss}}{x_1}\right) = \arctan\left(\frac{36.456}{28}\right) = 52.47 \text{ degrees.}$$

The configuration of the truss is acceptable as the angle is greater than 25 degrees.

Fourth check

The required width of bottle shaped struts $S_1$ and $S_2$ must be less than or equal to the actual available width at the faces of the nodes that support them. This check is performed for nodes $N_1$ and $N_6$.

For strut $S_1$ at node $N_1$ –

Width required from strength consideration = 8.24 inches (see Table 2.12).

The width of tie $T_3$ (wt) = 12.55 inches (see Table 2.12).

Support width (sw) = 16 inches.

The actual available space ($z'$) is calculated from Eq. 2.10, i.e.,

$$z' = \text{wt} \cos \theta + \text{sw} \sin \theta = 12.55 \cos (52.47) + 16 \sin (52.47) = 20.33 \text{ inches}$$

The space available at the face of the node is greater than the actual space required (8.24 inches, refer to Table 2.12) for the strut.

For strut $S_2$ at node $N_6$ –

Width required from strength consideration = 10.30 inches (see Table 2.12).

The width of strut $S_6$ (ws) = 7.53 inches (see Table 2.12).

Base plate width (sb) = 16 inches.

The actual available space ($z'$) is calculated from Eq. 2.10,

$$z' = \text{ws} \cos \theta + \text{sb} \sin \theta = 7.53 \cos (52.47) + 16 \sin (52.47) = 17.28 \text{ inches}$$
The space available at the face of the node is greater than the actual space required (10.30 inches, refer to Table 2.12) for the strut.

2.8.2 Longitudinal reinforcement

The member forces and dimensions computed in Section 2.8 form the input parameters for calculating the longitudinal reinforcement. These parameters are summarized in Table 2.13.

**Table 2.13 Input parameters required for design of longitudinal reinforcement**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete strength</td>
<td>$f'_c$</td>
<td>4000 psi</td>
</tr>
<tr>
<td>Yield strength of reinforcing bar</td>
<td>$f_y$</td>
<td>60000 psi</td>
</tr>
<tr>
<td>Width of the bottom tie available</td>
<td>$w_t$</td>
<td>12.55 inches</td>
</tr>
<tr>
<td>Width of the inclined strut at support</td>
<td>$w_s$</td>
<td>16 inches</td>
</tr>
<tr>
<td>Support width</td>
<td>$s_w$</td>
<td>16 kips</td>
</tr>
<tr>
<td>Angle of inclination of strut with respect to bottom tension tie</td>
<td>$\theta$</td>
<td>52.54 degrees</td>
</tr>
<tr>
<td>Force in the tie</td>
<td>$F_{tie}$</td>
<td>307.3 kips</td>
</tr>
<tr>
<td>Beam width</td>
<td>$b_w$</td>
<td>16 inches</td>
</tr>
<tr>
<td>Cover to stirrups</td>
<td>cover</td>
<td>2 inches</td>
</tr>
<tr>
<td>Diameter of stirrups</td>
<td>$d_s$</td>
<td>0.625 inches</td>
</tr>
</tbody>
</table>

The area of reinforcement required is

$$A_{st} = \frac{F_{tie}}{0.75 f_y} = \frac{500}{0.75 \times 60} = 6.83 \text{ square inches}$$

The available length to develop a straight bar is

$$s_{\text{avail}} = s_w - \text{cover} + \frac{w_t}{2 \tan(\theta)} = 16 - 2.5 + \frac{12.55}{2 \tan(52.45)} = 18.31 \text{ inches}$$

The space available to develop a hooked bar in single layer is
The space available to develop a hooked bar in two layers is

\[ s_{\text{avail}} = s_w - \text{cover} + \frac{w_t}{2 \tan(\theta)} = 16 - 2.5 + \frac{12.55}{2 \tan(52.45)} = 18.31 \text{ inches} \]

The space available to develop a hooked bars in three layers is

\[ s_{\text{avail}} = s_w - \text{cover} - 2 + \frac{w_t}{2 \tan(\theta)} = 14 - 2.5 + \frac{12.55}{2 \tan(52.45)} = 16.31 \text{ inches} \]

The available length is reduced by 2 and 4 inches in the previous two calculations due to the reasons discussed in Section 2.6.5. The topmost reinforcement layer would have the least space available on account of the hook. The bar size assumed in these sketches and above equation is #8. A minimum spacing of 1 inch is assumed between the hooked bars. Thus, adding the bar diameter and the spacing within the hooks leads to the reduction indicated in the above equation.

For #6 bar and smaller

\[ l_d = \left( \frac{f_y \psi_t \psi_e}{25 \sqrt{f'_c}} \right) d_b = \left( \frac{60000 \times 1 \times 1}{25 \sqrt{4000}} \right) \times 0.75 = 28.46 \text{ inches.} \]

For #7 bar and larger

\[ l_d = \left( \frac{f_y \psi_t \psi_e}{20 \sqrt{f'_c}} \right) d_b = \left( \frac{60000 \times 1 \times 1}{20 \sqrt{4000}} \right) \times 0.875 = 41.50 \text{ inches} \]

The required development lengths are summarized in Table 2.14.
Table 2.14 Development lengths required (straight development)

<table>
<thead>
<tr>
<th>Bar designation</th>
<th>Diameter</th>
<th>Development length required</th>
</tr>
</thead>
<tbody>
<tr>
<td>#6</td>
<td>0.75 inches</td>
<td>28.46 inches</td>
</tr>
<tr>
<td>#7</td>
<td>0.875 inches</td>
<td>41.50 inches</td>
</tr>
<tr>
<td>#8</td>
<td>1.00 inch</td>
<td>47.43 inches</td>
</tr>
<tr>
<td>#9</td>
<td>1.28 inches</td>
<td>60.715 inches</td>
</tr>
</tbody>
</table>

The development length required for all cases shown in Table 2.14 is greater than the available space calculated in Eq. 2.13. Hence, the use of hooked bars needs to be examined.

The development length for #6 hooked bars is

\[
l_{dh} = \left( \frac{0.02 \psi_e f_y}{\sqrt{f'_c}} \right) d_b = \left( \frac{0.02 \times 1 \times 60000}{\sqrt{4000}} \right) \times 0.75 = 14.23 \text{ inches}
\]

Similar calculations are performed for larger-diameter bars. The development length can be reduced by 30 percent as per Code provisions. The required development lengths are summarized in Table 2.15.
**Table 2.15 Development lengths required (hooked bars)**

<table>
<thead>
<tr>
<th>Reinforcement designation</th>
<th>Diameter</th>
<th>Development length required</th>
<th>Reduced development length</th>
</tr>
</thead>
<tbody>
<tr>
<td>#6</td>
<td>0.75 inches</td>
<td>14.23 inches</td>
<td>9.96 inches</td>
</tr>
<tr>
<td>#7</td>
<td>0.875 inches</td>
<td>16.6 inches</td>
<td>11.62 inches</td>
</tr>
<tr>
<td>#8</td>
<td>1.00 inch</td>
<td>18.98 inches</td>
<td>13.28 inches</td>
</tr>
<tr>
<td>#9</td>
<td>1.28 inches</td>
<td>24.29 inches</td>
<td>17.00 inches</td>
</tr>
</tbody>
</table>

The bar sizes from #6 to #9 can be used as the main longitudinal reinforcement if placed in a single layer (Table 2.15). The development length available is sufficient to develop the reinforcement in a 90-degree hooked configuration. Larger-diameter bars cannot be used in multiple layers due to lack of development length.

Preference is given to #6 as the longitudinal reinforcement.

The total required number of #6 bars \(n_6\) is

\[
n_6 = \frac{A_s}{A_6} = \frac{6.83}{0.44} = 15.52 \approx 16
\]

The total number of bars calculated is to be accommodated within the area of the bottom tie.

From Section 2.4.2, the maximum number of #6 bars in a single layer is (Eq. 2.15),

\[
0.75 n_b + (n_b - 1) = (16 - 1.5 \times 2 - 0.625 \times 2) = 11.75 \text{ inches}
\]

Solving for the unknown yields \(n_b = 7.28 \approx 7\) bars per layer.

Thus, to accommodate 16 bars, 3 layers of reinforcement would be required. The available development length for 3 layers of reinforcement is 14.31 inches from the calculations shown.
The required development length for #6 bars is 9.96 inches (Table 2.15). The bars can, therefore, be developed within the space available.

It is necessary to check if 3 layers can be accommodated in the given tie width. From Eq. 2.18, the space required to accommodate 3 layers is

\[(3 - 1) + 3 \times 0.75 = 4.25\text{ inches} \leq 12.55\text{ inches}\]

Thus, sixteen #6 bars placed in 3 layers would be sufficient to resist the force of the tension tie.

The location of the C.G. for the reinforcement layout, from the bottom of the beam is \((0.5 \times 12.55 + 1.5) = 7.775\) inches.

The layout of reinforcing bars is crucial as it affects the behavior of the longitudinal tie. As per the requirement of the Code, the C.G. of the reinforcement layout should be the same as the centerline of the tie. The reinforcement pattern for the example is drawn in Fig. 2.19 and Fig. 2.20.

![Fig. 2.19 Suggested reinforcement layout for the solved example (front elevation)](image)
Fig. 2.20 Suggested reinforcement layout for the solved example (section)

The following points need to be considered while deciding the reinforcement layout:

1. The reinforcement is to be placed within the tie width calculated.
2. The Code requirements for minimum distance between bars within the same layer and distance between layers are to be satisfied.
3. The C.G. of the reinforcement layout and center line of the tie needs to be the same.
4. For cases where larger bars are needed and sufficient development length is not available, mechanical anchors may be provided.

2.8.3 Reinforcement for bottle shaped strut and vertical tie

The design of reinforcement for a bottle shaped strut in concrete strengths below 6000 psi revolves around the provisions of Eq. 2.19. The bars are to be provided in vertical and horizontal directions and on both the faces of the beam. The relevant data from the previous sections are provided in Table 2.16. These values are needed to calculate the reinforcement for the bottle shaped struts.
Table 2.16 Input parameters in the design of reinforcement for bottle shaped strut

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Abbreviation</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of inclination of strut with respect to bottom tension tie</td>
<td>$\theta$</td>
<td></td>
<td>52.54 degrees</td>
</tr>
<tr>
<td>Beam width</td>
<td>$b_w$</td>
<td></td>
<td>16 inches</td>
</tr>
<tr>
<td>Yield strength of reinforcement</td>
<td>$f_y$</td>
<td></td>
<td>60000 psi</td>
</tr>
<tr>
<td>Height of the truss</td>
<td>$d_1$</td>
<td></td>
<td>36.45 inches</td>
</tr>
<tr>
<td>Width of the top prismatic strut</td>
<td>-</td>
<td></td>
<td>7.53 inches</td>
</tr>
</tbody>
</table>

Calculation of horizontal reinforcement for bottle shaped strut

Iteration 1 – Calculation for the maximum permissible spacing as per Code requirements

The maximum spacing equals the smaller of 12 inches or 0.2 times ($d_1 + 0.5 \times$ top strut width. Width $d_1 =$ the truss height). The quantity in the parenthesis is the shear depth of the beam. Thus, the shear depth is equal to 0.2 x (36.45 +0.5 x 7.53), that is, 8.043 inches. Hence, the maximum permissible width is set at 8 inches. Number 3 bars are selected for the first iteration and the bars are placed 8 inches apart. It should be noted that two legs of reinforcement are provided at every bar location. Using Eq. 2.20

$$
\text{Reinforcement ratio} = \frac{2 \times 0.11}{16 \times 8} \sin (52.54) = 0.00136
$$

The horizontal reinforcement contribution towards Eq. 2.19 is less than 0.0015. Hence, the spacing of reinforcement is reduced for the next iteration.

Iteration 2 – #3 bars with spacing equal to 6 inches for the next iteration. The resulting reinforcement ratio is 0.001819, which is greater than the required ratio. Hence, the reinforcement configuration in the form of #3 bars placed at 6 inches is acceptable.
Calculation of vertical reinforcement for bottle shaped strut

Iteration 1 – Calculation of maximum permissible spacing as per Code requirements

The maximum permissible spacing for the vertical reinforcement remains the same as the one adopted for horizontal bars, that is, 8 inches.

The area of reinforcement selected for the first iteration is 0.1 square inches. This value was selected based on the results from a number of parametric studies. This area is independent of the number of legs at this stage. The total number of legs and bar size will be selected after the reinforcement for the vertical ties (T1 and T2) is computed and combined with the strut reinforcement. From Eq. 2.21,

\[
\text{Reinforcement ratio} = \frac{0.1}{16 \times 8} \cos (52.54) = 0.000475
\]

The vertical reinforcement contribution towards Eq. 2.19 is less than 0.0015. Hence, the spacing of reinforcement is reduced for the next iteration.

Iteration 2 – The spacing is reduced to 6 inches and the reinforcement ratio (Eq. 2.21) is 0.000633. The spacing is reduced to 4 inches for the next iteration. It is observed that the required reinforcement ratio is not obtained in the next iteration. Hence, the area of reinforcement is increased by 0.01 square inches and the spacing is set to the maximum permissible (8 inches) for the subsequent iteration. The procedure remains similar to the one used for the earlier iterations. After a series of calculations and iterations, it is observed that the following reinforcement ratio fulfills the requirements set in Eq. 2.21.

Reinforcement area = 0.17 square inches

Spacing = 4 inches

\[
\text{Reinforcement ratio} = \frac{0.17}{16 \times 4} \cos (52.54) = 0.001615
\]
The requirement of Eq. 2.21 is satisfied for this particular reinforcement configuration and, hence, the selected reinforcement area and spacing are acceptable.

**Calculation of reinforcement for vertical ties T1 and T2**

The force in each vertical tie is equal to the reaction at the support.

Tensile force in the tie T1 = 200 kips

The total area of reinforcement \( (a_v) \) required to resist the tension
\[
\frac{200 \text{ kips}}{0.75 \times 60} = 4.44 \text{ square inches.}
\]

The shear span is equal to 48 inches. The total number of bars that are provided for bottle shaped reinforcement equals,

\[
N = \frac{48 \text{ in.}}{4 \text{ in.}} = 12
\]

Area required at each bar location = \( a_b = \frac{4.44}{12} = 0.37 \) square inches.

**Combination of the vertical reinforcement for the bottle shaped strut and the vertical ties T1 and T2**

From the calculations performed earlier, the total area of reinforcement at each bar location for the bottle shaped strut is 0.17 square inches. The required reinforcement for the vertical ties is 0.37 square inches. The areas required for the struts and ties are added to obtain the total reinforcement area required, that is, 0.54 square inches. The spacing is limited to 4 inches as it was required from the bottle shaped strut.

A suitable bar size with either two or four legs is selected next to provide 0.54 square inches of reinforcement. The required iterations are as follows.
**Iteration 1** - #3 bars with two legs. The total area available is equal to 0.22 square inches. This iteration fails to provide the required area.

**Iteration 2** - #4 bars with two legs. The total area available is equal to 0.40 square inches. This iteration fails to provide required area.

**Iteration 3** - #3 bars with four legs. The total area available is equal to 0.44 square inches. This iteration fails to provide required area.

**Iteration 4** - #5 bars with two legs. The total area available is equal to 0.62 square inches. This iteration is successful.

Hence, #5 bars (two legs at each reinforcement location) spaced at 4 inches apart would provide the sufficient reinforcement to resist the splitting cracks in the bottle shaped strut and the tension in the vertical ties T1 and T2.

The final reinforcement layout is shown in Fig. 2.21

![Final reinforcement layout for the bottle shaped strut and vertical ties](image)

**Fig. 2.21** Final reinforcement layout for the bottle shaped strut and vertical ties
2.9 MATLAB program to analyze and design deep beam

The design of deep beams with strut-and-tie models is an iterative procedure. Hand calculations can be used to perform these iterations, but manual computations may not always offer an economical design. MATLAB programs are developed to make the overall beam design more efficient and less time consuming. Such programs give the designer the liberty to use different concrete strengths as well as beam dimensions. The aim of a MATLAB program is to generate a set of longitudinal and shear reinforcement for a given design parameters, which include the beam dimensions, material properties, and loads. The MATLAB programs are included in Appendix C.

2.10 Verification of the output from MATLAB program with the aid of CAST

CAST is a program developed by Kuchma and Tjhin (2002). The design of a deep beam with CAST is based on the requirement of the Code. This program is used to verify the output generated by the MATLAB program developed as a part of the reported research. These checks were performed for a few sample deep beams. As shown in Appendix A, the output of MATLAB program is found to be in agreement with the result of CAST. Thus, it can be concluded that the results from the MATLAB program are in accord with the design guidelines. The details of the comparison are included in the Appendix B.

2.11 Sample table for longitudinal reinforcement

Table 2.17 is a sample output for longitudinal reinforcement generated by the MATLAB program. The beam dimensions and material properties are similar to the data included in Table 2.5. The minimum load for which the MATLAB program designs the beam is 50 kips and the load step is of 10 kips. The maximum load for which the beam can be designed is controlled by the checks described in Step 5 of Section 2.2. These conditions have to be met before the
longitudinal reinforcement can be designed. The sample table includes design reinforcement for various trial beam widths. Similar tables can be generated using the MATLAB program. The design parameters like beam span, depth, and concrete strength can be changed as per design requirements.
Table 2.17a Sample design table for longitudinal reinforcement

<table>
<thead>
<tr>
<th>load values (kips)</th>
<th>Location of CG (inches)</th>
<th>N</th>
<th>Longitudinal Reinforcement</th>
<th>#</th>
<th>Location of CG (inches)</th>
<th>N</th>
<th>Longitudinal Reinforcement</th>
<th>#</th>
<th>Location of CG (inches)</th>
<th>N</th>
<th>Longitudinal Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>6</td>
<td>1.28</td>
<td>4</td>
<td>one layer hooked</td>
<td>6</td>
<td>1.14</td>
<td>4</td>
<td>one layer hooked</td>
<td>6</td>
<td>1.0</td>
<td>4</td>
</tr>
<tr>
<td>60</td>
<td>6</td>
<td>1.56</td>
<td>4</td>
<td>one layer hooked</td>
<td>6</td>
<td>1.38</td>
<td>4</td>
<td>one layer hooked</td>
<td>6</td>
<td>1.2</td>
<td>4</td>
</tr>
<tr>
<td>70</td>
<td>6</td>
<td>1.84</td>
<td>5</td>
<td>one layer hooked</td>
<td>6</td>
<td>1.62</td>
<td>5</td>
<td>one layer hooked</td>
<td>6</td>
<td>1.4</td>
<td>5</td>
</tr>
<tr>
<td>80</td>
<td>6</td>
<td>2.13</td>
<td>6</td>
<td>one layer hooked</td>
<td>6</td>
<td>1.87</td>
<td>6</td>
<td>one layer hooked</td>
<td>6</td>
<td>1.6</td>
<td>6</td>
</tr>
<tr>
<td>90</td>
<td>6</td>
<td>2.42</td>
<td>6</td>
<td>one layer hooked</td>
<td>6</td>
<td>2.13</td>
<td>6</td>
<td>one layer hooked</td>
<td>6</td>
<td>1.8</td>
<td>6</td>
</tr>
<tr>
<td>100</td>
<td>6</td>
<td>2.71</td>
<td>7</td>
<td>two layers hooked</td>
<td>6</td>
<td>2.38</td>
<td>7</td>
<td>one layer hooked</td>
<td>6</td>
<td>2.1</td>
<td>7</td>
</tr>
<tr>
<td>110</td>
<td>6</td>
<td>3.02</td>
<td>8</td>
<td>two layers hooked</td>
<td>6</td>
<td>2.65</td>
<td>8</td>
<td>two layers hooked</td>
<td>6</td>
<td>2.3</td>
<td>8</td>
</tr>
<tr>
<td>120</td>
<td>6</td>
<td>3.34</td>
<td>9</td>
<td>two layers hooked</td>
<td>6</td>
<td>2.92</td>
<td>9</td>
<td>two layers hooked</td>
<td>6</td>
<td>2.5</td>
<td>9</td>
</tr>
<tr>
<td>130</td>
<td>6</td>
<td>3.66</td>
<td>10</td>
<td>two layers hooked</td>
<td>6</td>
<td>3.19</td>
<td>9</td>
<td>two layers hooked</td>
<td>6</td>
<td>2.7</td>
<td>9</td>
</tr>
<tr>
<td>140</td>
<td>6</td>
<td>3.99</td>
<td>10</td>
<td>two layers hooked</td>
<td>6</td>
<td>3.48</td>
<td>10</td>
<td>two layers hooked</td>
<td>6</td>
<td>3.0</td>
<td>10</td>
</tr>
<tr>
<td>150</td>
<td>6</td>
<td>4.34</td>
<td>11</td>
<td>two layers hooked</td>
<td>6</td>
<td>3.77</td>
<td>11</td>
<td>two layers hooked</td>
<td>6</td>
<td>3.2</td>
<td>11</td>
</tr>
</tbody>
</table>
Table 2.17a (cont’d) Sample design table for longitudinal reinforcement

<table>
<thead>
<tr>
<th>load values (kips)</th>
<th>#</th>
<th>width of the beam (inches)</th>
<th>width of the beam (inches)</th>
<th>width of the beam (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>16</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Location of CG (inches)</td>
<td>Location of CG (inches)</td>
<td>Location of CG (inches)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Longitudinal Reinforcement</td>
<td>Longitudinal Reinforcement</td>
<td>Longitudinal Reinforcement</td>
</tr>
<tr>
<td>160</td>
<td>6</td>
<td>4.7</td>
<td>12</td>
<td>two layers hooked</td>
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<tr>
<td>170</td>
<td>6</td>
<td>5.07</td>
<td>13</td>
<td>two layers hooked</td>
</tr>
<tr>
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<td>6</td>
<td>5.45</td>
<td>14</td>
<td>three layers hooked</td>
</tr>
<tr>
<td>190</td>
<td>6</td>
<td>5.85</td>
<td>15</td>
<td>three layers hooked</td>
</tr>
<tr>
<td>200</td>
<td>6</td>
<td>6.28</td>
<td>16</td>
<td>three layers hooked</td>
</tr>
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<td>210</td>
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</tr>
<tr>
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<td>18</td>
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<td>240</td>
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<td>8.24</td>
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<td>three layers hooked</td>
</tr>
<tr>
<td>250</td>
<td>7</td>
<td>8.84</td>
<td>17</td>
<td>three layers hooked</td>
</tr>
<tr>
<td>260</td>
<td>8</td>
<td>9.51</td>
<td>14</td>
<td>three layers hooked</td>
</tr>
</tbody>
</table>
### Table 2.17a (cont’d) Sample design table for longitudinal reinforcement

<table>
<thead>
<tr>
<th>load values (kips)</th>
<th>Location of CG (inches)</th>
<th>N</th>
<th>Longitudinal Reinforcement</th>
<th>#</th>
<th>Location of CG (inches)</th>
<th>N</th>
<th>Longitudinal Reinforcement</th>
<th>#</th>
<th>Location of CG (inches)</th>
<th>N</th>
<th>Longitudinal Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>270</td>
<td>8</td>
<td>10.28</td>
<td>15</td>
<td>three layers hooked</td>
<td>6</td>
<td>8.24</td>
<td>23</td>
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<td>6</td>
<td>6.6</td>
<td>22</td>
</tr>
<tr>
<td>280</td>
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<td></td>
<td></td>
<td></td>
<td>7</td>
<td>8.77</td>
<td>18</td>
<td>three layers hooked</td>
<td>6</td>
<td>7.0</td>
<td>23</td>
</tr>
<tr>
<td>290</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7</td>
<td>9.35</td>
<td>20</td>
<td>three layers hooked</td>
<td>6</td>
<td>7.3</td>
<td>25</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>8</td>
<td>10</td>
<td>16</td>
<td>three layers hooked</td>
<td>6</td>
<td>7.7</td>
<td>26</td>
</tr>
<tr>
<td>310</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7</td>
<td>8.1</td>
<td>20</td>
<td>three layers hooked</td>
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<td></td>
<td></td>
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<td>8.5</td>
<td>21</td>
<td>three layers hooked</td>
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</tr>
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<td>7</td>
<td>9.0</td>
<td>23</td>
<td>three layers hooked</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>340</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8</td>
<td>9.5</td>
<td>18</td>
<td>three layers hooked</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

C.G. is the location of the centroid of the bars measured from the beam soffit

# denotes the bar size

N denotes the total number of bars
Table 2.17b Sample design table for horizontal reinforcement for bottle shaped strut

<table>
<thead>
<tr>
<th>load values (kips)</th>
<th>Horizontal Reinforcement (#)</th>
<th>Number of legs</th>
<th>Spacing (inches)</th>
<th>Horizontal Reinforcement (#)</th>
<th>Number of legs</th>
<th>Spacing (inches)</th>
<th>Horizontal Reinforcement (#)</th>
<th>Number of legs</th>
<th>Spacing (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>4</td>
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<td>4</td>
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<td>3</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>4</td>
</tr>
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<td>110</td>
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<td>2</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>120</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>3</td>
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<td>4</td>
</tr>
<tr>
<td>130</td>
<td>3</td>
<td>2</td>
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<td>4</td>
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<td>4</td>
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Table 2.17b (Cont’d) Sample design table for horizontal reinforcement for bottle shaped strut

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Table 2.17b (Cont’d) Sample design table for horizontal reinforcement for bottle shaped strut

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Table 2.17c Sample design table for vertical reinforcement for bottle shaped strut and the tie T1

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Table 2.17c (cont’d) Sample design table for vertical reinforcement for bottle shaped strut and the tie T1

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Table 2.17c (cont’d) Sample design table for vertical reinforcement for bottle shaped strut and the tie T1

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2.12 **Comparison of the two approaches of modeling the deep beam from Case 1**

Two different strut-and-tie models may be used to analyze and design a deep beam subjected to two equal and symmetrically placed loads can be modeled with two different approaches. The first approach (Model 1) was proposed by Alcocer and Uribe (2002), refer to Fig. 2.22. The second approach (Model 2) was proposed by Kuchma and Tjhin (2002), see Fig. 2.23. The basic load transfer mechanism remains the same for both the models, i.e., the struts and ties act together to transfer the loads to the supports. The two models are compared in terms of the design shear strengths.

![Diagram of deep beam models](image)

**Fig. 2.22** Model 1 – Proposed by Alcocer and Uribe (2002)
2.12.1 Shear capacity calculation for the cross section

The Code provides the following equation to calculate the maximum ultimate shear at any cross section,

\[ V_u = \Phi (10\sqrt{f'_c} \cdot b_w \cdot d) \]  
(2.24)

\( f'_c \) – Strength of concrete

\( b_w \) – Width of the beam

\( d \) – Distance between the outermost compression fiber and the center of gravity of reinforcement

A model that offers a higher shear capacity is preferred.

2.12.2 Case study example

A beam with the following properties was selected.

Span = 144 inches

Depth = 48 inches

\( f'_c = 4000 \) psi

Two equal loads placed symmetrically
Based on Model 1, the beam can resist 260 kips, while the maximum load carrying capacity from Model 2 is 200 kips. This difference is explained with reference to Fig. 2.24. In this figure, the required and the available widths of diagonal struts (z and z', in Fig. 2.9, respectively) as from the two models are compared. At point A, the strut width (Strut S1 from Fig. 2.22) required by Model 2 exceeds the available width. As a result the total available shear capacity (10 \( \Phi \sqrt{f'_c} b_w d \)) is not realized. On the other hand, the capacity from Model 1 is limited by the total available shear capacity.

It should be noted that the total available shear capacities are different between the two models.

**Fig. 2.24** Comparison of the width available to the width required for the bottle shaped struts at the load
2.12.3 Comparison of the member forces and capacities for similar truss heights

As evident from Table 2.18, the two models result in different forces in the prismatic struts (S6 for Model 1, in Fig. 2.21, and S2 for Model 2, in Fig. 2.22) and the ties (T4 for Model 1, in Fig. 2.21, and T1 for Model 2.22, in Fig. 2.22). To satisfy the available strengths the width of strut S6 and the width of tie T4 are larger than their counterparts. As a result the truss depth (d) for Model 1 is smaller than that for Model 2. The shear capacities are directly proportional to this depth.

Hence, Model 1 results in lower shear capacity than Model 2.

Table 2.18 Comparison of Model 1 and Model 2

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<th>Abbreviation</th>
<th>Value</th>
<th>Equality/Inequality</th>
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<td>d1 &lt; d2</td>
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<td>Allowable stress in inclined strut</td>
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<td>Fixed</td>
<td>Equal</td>
</tr>
<tr>
<td>Angle theta</td>
<td>θ₁</td>
<td>Arctangent (d₁/x)</td>
<td>θ₁ &gt; θ₂</td>
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<tr>
<td>Force in inclined strut</td>
<td>F₁</td>
<td>F₁ = R/ sin(θ₁)</td>
<td>F₁ &lt; F₂</td>
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<tr>
<td>Width of inclined strut</td>
<td>ws₁</td>
<td>w₁ = F₁ / fs₁</td>
<td>w₁ &lt; w₂</td>
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<td>Force in the bottom tie</td>
<td>T₁</td>
<td>T₁ = 2 F₁ cos(θ₁)</td>
<td>T₁ &gt; T₂</td>
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<td>Quantity under consideration</td>
<td>Abbreviation</td>
<td>Value</td>
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<td>fn₂ (node N1)</td>
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<td>S₂</td>
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**2.12.4 Advantages and Disadvantages of Model 1 and Model 2**

The numbers of variables (struts and ties, and nodes) in Model 2 are fewer as compared to Model 1. As a result, Model 2 is more convenient for hand calculations as successive iterations can be
performed more easily. However, this model predicts a lower load carrying capacity than Model 1. On the other hand, Model 1 is more complex than Model 1, and is less appropriate for hand calculations in comparison to Model 2. Model 1, though, indicates a larger load carrying capacity than Model 2.

The research work conducted by Bayrak et al (2008) suggests that Model 2 represents load transfer mechanism in beams with no or relatively low shear reinforcement as it does not correctly capture the contribution of shear reinforcement. Model 1 incorporates the contribution of shear reinforcement more effectively.

In view of the effectiveness of Model 1 to capture the contribution of shear reinforcement, and the larger load-carrying capacity computed by this model, the reported research is based on Model 1. As discussed in the following chapters, variations of this model are employed for cases with unsymmetrical loads, concrete strengths larger than 6000 psi, a single load, and uniform loads.
Chapter 3

Case 2 – Beam subjected to two point loads placed symmetrically (High-strength concrete).

3.1 Description of the beam and geometry

The beam selected for design is simply supported and has a single span. The geometry of the beam is as shown in Fig. 3.1. The loads on the beam are equal and symmetrically placed. The strut-and-tie model is sketched Fig. 3.3 and it is applicable to beams with concrete strengths more than 6000 psi.

Fig. 3.1 Geometry of the beam and loads

The beam geometry is similar to Case 1 discussed in Section 2.1. The only difference is in the strengths of concrete. The higher strength of concrete requires a more refined approach to design the reinforcement to resist the transverse tension in the bottle shaped struts. The STM selected herein is different from the one used in Chapter 2 in terms of how to model a bottle shaped strut. The procedure to develop a strut-and-tie model and design the deep beam is similar to the method adopted for Case 1. The Code imposed limits on the maximum stress in struts, ties, and
at the face of nodes remains unchanged. The governing equation (Eq. 2.1) from Case 1, used to calculate the required widths of struts, ties, and nodal zones, remains unchanged for Case 2. The only difference is the modeling of struts S1 through S4 for Case 1 (Fig. 2.2). This chapter describes the methodology used in the modeling and detailing of the bottle shaped strut suggested for deep beams with concrete strength more than 6000 psi.

### 3.2 Modeling of the bottle shaped strut

A bottle shaped strut has a transverse tensile force present along its length. It accounts for the tendency of the strut to expand laterally as there is space available for such an expansion. The tensile force causes cracking along the length of the strut. The crack is assumed to be formed along the center line of the strut. The cracking reduces the strength of the strut and requires reinforcement perpendicular to the crack. The reinforcement is required to maintain equilibrium in the bottle shaped strut. Absence of this equilibrium leads to excessive cracking in the strut (Bayrak and Brown 2006). It is not always feasible to provide reinforcement perpendicular to the centerline of the strut. The Code prescribes reinforcement along the vertical and horizontal directions along the strut length.
The Code deals with struts that have concrete strengths below 6000 psi differently than the struts with concrete strengths more than 6000 psi. The Code allows the bottle shaped strut to be modeled as a uniform width strut provided that an appropriate strength reduction factor ($\beta = 0.75$ or 0.6) is applied to the strength of the strut. This assumption or simplification is permissible only for concrete strengths up to 6000 psi. While dealing with concrete strengths above 6000 psi, the Code expects the designer to actually model the bottle shaped strut based on Fig. 3.2 in lieu of using Eq. 2.19, which is applicable to bottle shaped struts with concrete compressive strength below 6000 psi. It is important to note that cracks are formed in struts for all concrete strengths. The high-strength concrete beams are treated differently because they are expected to carry higher loads which lead to higher transverse tensile forces along the strut length. The Code requirements from Eq. 2.19 are insufficient to resist the higher transverse tensile forces generated. Hence, for concrete strengths above 6000 psi, bottle shaped struts have to be directly...
modeled. In the following, a procedure is presented to model an actual bottle shaped strut that accounts for the dispersion of the compressive stresses and the tensile stresses perpendicular to the length of the strut.

3.3 Calculation for the forces within the bottle shaped strut

The strut-and-tie model shown in Fig.3.3 is selected in the reported research. The procedure to solve the truss and decide the member forces and dimensions is similar to the discussion from the Section 2.2. The steps in Section 2.2 for modeling the top prismatic struts and the bottom longitudinal tie are also applicable to Case 3. The optimization of the truss height, similar to what done for Case 2, is required for Case 3. The compatibility conditions and the other checks performed on the truss model remain unchanged. The procedure adopted to configure the longitudinal reinforcement for Case 3 is exactly similar to the one used for Case 2. The only difference in the two models (Fig. 2.2 and Fig. 3.3) is analysis and design of the bottle shaped struts.

![Fig. 3.3 Strut-and-tie model for Case 2](image)
The compression forces present at the end nodes of the bottle shaped struts are calculated from static analysis. The forces within the members of the global strut are shown in Fig. 3.4 and are summarized in Table 3.1. The equations for calculating the forces in the various STM elements as proposed by Bayrak and Brown (2006) are

\[ C = \frac{F_{\text{strut}}}{2 \cos(\Phi)} \]  
\[ F_{\text{tie}} = C \sin(\Phi) \]  

<table>
<thead>
<tr>
<th>Member</th>
<th>Force in Members</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>( \frac{F_{\text{strut}}}{2 \cos(\Phi)} )</td>
</tr>
<tr>
<td>S2</td>
<td>0.5 ( F_{\text{strut}} )</td>
</tr>
<tr>
<td>S3</td>
<td>( \frac{F_{\text{strut}}}{2 \cos(\Phi)} )</td>
</tr>
<tr>
<td>T1</td>
<td>( C \sin(\Phi) )</td>
</tr>
<tr>
<td>T2</td>
<td>( C \sin(\Phi) )</td>
</tr>
</tbody>
</table>

Table 3.1 Forces in individual members of a bottle shaped strut
The individual struts are modeled as prismatic struts. The angle $\Phi$ is calculated as arctangent of $(1/2)$ and equals 26.55 degrees. The dispersion ratio of $(1/2)$ is recommended by the Code.

### 3.4 Detailing of the reinforcement for the bottle shaped strut

The bottle shaped has a splitting force present along the length. Reinforcement in the vertical as well as horizontal direction has to be provided to resist this force. Vertical reinforcement from the bottle shaped strut is to be combined with the reinforcement required for the ties T1 and T2. This combination is achieved in a manner similar to the procedure discussed in Chapter 2.

#### 3.4.1 Horizontal reinforcement

Detailing of the horizontal reinforcement for the bottle shaped strut involves selection of an appropriate bar size placed at some spacing that would resist the force in the horizontal direction force calculated in Section 3.3. The horizontal component of the splitting force in the tie is

$$F_h = 2 \ F_{\text{tie}} \sin (\theta) \quad (3.3)$$

$F_{\text{tie}}$ — Force in ties T1 and T2 from Fig. 3.4
θ – Inclination of the bottle shaped strut with respect to the longitudinal tie (refer to Fig. 3.4)

The factor 2 appears in Eq. 3.3 on account of the presence of two ties in each bottle shaped strut.

The required reinforcement area ($A_h$)

$$A_h = \frac{F_h}{0.75 \, f_y} \quad (3.4)$$

$f_y$ – Yield strength of reinforcement

The resistance provided by reinforcement is equal to

$$F_{rp} = 0.75 \, f_y \, A_{hp} \quad (3.5)$$

$A_{hp}$ - Area of reinforcement provided to resist the transverse tensile force in the horizontal reinforcement

Resistance from Eq. 3.5 has to be greater than or equal to the actual splitting force in the tie. That is

$$F_{rp} \geq 2 \, F_{tie} \sin (\theta) \quad (3.6)$$

The total area is to be distributed over the height ‘H’ as shown in Fig. 3.5. The spacing of reinforcement and a suitable bar size is selected through an iterative process in order to achieve an efficient and economical reinforcement layout.
In the first iteration, #3 bars are selected as reinforcement, and the spacing is set to the maximum permissible. The maximum permissible spacing is calculated in a manner similar to the procedure discussed in Section 2.7.2.

The number of bars that can be placed within this space is determined next. The total area of bars are equal to the product of the area of #3 bar, number of legs of reinforcement, and the total number of bars that can be accommodated. If the area so provided is more than the required area calculated from Eq. 3.4, the reinforcement pattern is sufficient to resist the splitting force component in the horizontal direction. If the resistance provided by this configuration is less than the actual force from Eq. 3.5, the spacing between the #3 bars is reduced. The spacing may be reduced to 4 inches. If the transverse tensile force along the strut length is still greater than the...
resisting force provided by #3 bars at 4 inch spacing, #4 bars have to be used. The spacing is set to the maximum permissible as it was in case of #3 bars. The procedure discussed in this section is explained in the flow chart shown in Fig. 3.6. The ratio calculated from Eq. 2.19 has to be checked for the provided reinforcement area and spacing and it should be more than 0.003 for Case 3 (Bayrak and Brown - 2006).
Fig. 3.6 Flow chart for the calculation of the horizontal reinforcement
3.4.2 Vertical reinforcement

The calculations performed to determine the vertical reinforcement for the bottle shaped strut are similar to the computations for horizontal reinforcement. The vertical component of transverse tension along the strut length is

\[ F_v = 2 \ F_{\text{tie}} \cos (\theta) \]  

The required reinforcement can be computed by Eq. 3.4 by substituting \( F_h \) with \( F_v \).

The reinforcement so calculated is required for one bottle shaped strut. Two bottle shaped struts are to be modeled in each shear span. Hence, the total reinforcement is to be multiplied by 2 because the forces in the two bottle shaped struts are equal in direction and value. The spacing of the reinforcement and the bar size are not selected yet as the reinforcement calculated in this section is to be added to the reinforcement required for the vertical ties T1 and T2 calculated in the next section.

3.4.3 Reinforcement for the tie T1

The procedure to detail the required reinforcement for ties T1 and T2 is similar to the approach used in Section 2.7.6. The reinforcement area is calculated based on the tensile force and the yield strength of reinforcement.

3.4.4 Combination of the vertical reinforcement for the bottle shaped strut and the tie T1

The reinforcement (vertical reinforcement) for the bottle shaped struts and the required reinforcement for the vertical tie are added to calculate the total reinforcement that is to be provided in the shear span. The calculated reinforcement is spread over the total shear span at spacing not more than the maximum permissible. The minimum spacing is limited to 4 inches. Two or four legs of reinforcement may be used depending on the total requirement. The
minimum bar size is #3 and the maximum bar size is set as #5. The procedure to detail the reinforcement is iterative, and an attempt is made to economize the configuration.

3.5 Space available to accommodate the bottle shaped struts

The forces from the individual struts forming the bottle shaped strut are combined to produce an equivalent load. This approach is consistent with Clause A.3.2 of the Code. From the geometry of the bottle shaped strut, the angle between the individual struts (Φ in Fig. 3.4) is always equal to 26.55 degrees. The geometry of the system is shown in Fig. 3.7. Note that angle (θ) shown in this figure is a function of the geometry of the STM.

Fig. 3.7 Details of end node for the bottle shaped strut at the support

The equivalent load is used to establish whether the node dimensions are adequate. At node N1, the available width to accommodate a strut required for this equivalent load is established based
on the procedure discussed in Section 2.5.3. This equivalent load is also used to check the adequacy of node N6 similar to the method presented in Section 2.5.3.

3.6 Illustrative example for Case 2

Input parameters for the example

A sketch of the beam is shown in Fig. 3.8. The required input quantities are summarized in Table 3.2. These values are based on the results from steps 1 through 6 described in Section 2.2. The additional computation required is the actual modeling of the bottle shaped strut.

![Beam geometry for the solved example in Section 3.6](image)

**Fig. 3.8** Beam geometry for the solved example in Section 3.6
Table 3.2 Basic input parameters for design of bottle shaped struts

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete strength</td>
<td>$f'_c$</td>
<td>8000 psi</td>
</tr>
<tr>
<td>Yield strength of reinforcement</td>
<td>$f_y$</td>
<td>60000 psi</td>
</tr>
<tr>
<td>Inclined length of strut*</td>
<td>$l_s$</td>
<td>52.01 inches</td>
</tr>
<tr>
<td>Beam width</td>
<td>$b_w$</td>
<td>16 inches</td>
</tr>
<tr>
<td>Force at the end nodes of the strut</td>
<td>$F_{strut}$</td>
<td>237.34 kips</td>
</tr>
<tr>
<td>Angle of inclination of strut</td>
<td>$\theta$</td>
<td>57.4 degrees</td>
</tr>
<tr>
<td>Top prismatic strut width</td>
<td>$w_s$</td>
<td>3.13 inches</td>
</tr>
<tr>
<td>Bottom longitudinal tie width</td>
<td>$w_t$</td>
<td>5.22 inches</td>
</tr>
</tbody>
</table>

* The distance between the end nodes is considered to be the length of the strut. It is not the distance between the faces of the nodes.

The angle of spread of the bottle shaped strut is arctangent of 0.5 as mentioned earlier.

The forces in the members of the strut are (refer to Fig. 3.4)

$$C = \frac{F_{strut}}{2 \cos(\Phi)} = \frac{237.34 \text{ kips}}{2 \cos (26.565)} = 132.66 \text{ kips}$$

The force in the tension tie, for which the reinforcement needs to be designed, is

$$F_{tie} = C \sin(\Phi) = 132.66 \sin(26.526) = 59.24 \text{ kips}$$

The total splitting force in the strut is twice $F_{tie}$, i.e., 118.84 kips.

**Horizontal reinforcement for the bottle shaped strut**

Using Eq. 3.3, the component of the splitting force in the horizontal direction is

$$F_h = 2 \times 59.24 \sin (57.4) = 99.82 \text{ kip.}$$

The reinforcement required to resist this force is computed by Eq. 3.4.

$$\text{Area of reinforcement required} = A_h = \frac{99.82}{0.75 \times 60} = 2.22 \text{ square inches.}$$
Effective depth = 48 – 1.5 – 0.5 x 5.22 = 43.9 inches

The maximum permissible spacing for the horizontal reinforcement is smaller of 12 inches or 0.2 times the effective depth (0.2 x 43.9 = 8.79 inches, rounded to 8 inches).

**Iteration 1** - #3 bars at 8-inch spacing.

The space available to provide the required horizontal bars is calculated from the details shown in Fig. 3.4, i.e.,

\[ H = 48 - 1.5 - 3.13 - 5.22 = 38.15 \text{ inches}. \]

Five #3 bars can be placed at 8 inches on the center, which fit within the available space. The total area of reinforcement is equal to

\[ A_{hp} = 2 \times 0.11 \times 5 = 1.1 \text{ square inches} \]

The resistance provided by this reinforcement is equal to

\[ F_{rp} = 0.75 \times 60 \times 1.1 = 49.5 \text{ kips} \]

The resistance is less than the actual force (99.82 kips). Hence, the spacing has to be reduced for the next iteration.

**Iteration 2** - #3 bars at 6-inch spacing.

Seven #3 bars can be placed at 6 inches on the center, which fit within the available space. The total area of reinforcement is equal to

\[ A_{hp} = 2 \times 0.11 \times 7 = 1.54 \text{ square inches} \]

The resistance provided by this reinforcement is equal to

\[ F_{rp} = 0.75 \times 60 \times 1.54 = 69.3 \text{ kips} \]

The resistance is less than the actual force (99.82 kips). Hence, the spacing has to be reduced for the next iteration.
**Iteration 3** - #3 bars at 4 inch spacing.

Ten #3 bars can be placed at 4 inches on center, which fit within the available space. The total area of reinforcement is equal to

\[ A_{hp} = 2 \times 0.11 \times 10 = 2.2 \text{ square inches} \]

The resistance provided by this reinforcement is equal to

\[ F_{rp} = 0.75 \times 60 \times 2.2 = 99 \text{ kips} \]

The resistance is slightly less than the actual force (99.82 kips). The spacing for the last iteration was equal to 4 inches. It cannot be reduced further. Hence, #4 bars are selected for the next set of iterations. The spacing between the bars is set to 8 inches (the maximum permissible as per the Code).

**Iteration 4** - #4 bars at 8-inch spacing.

Five #4 bars can be placed at 8 inches on center, which fit within the available space. The total area of reinforcement is equal to

\[ A_{hp} = 2 \times 0.2 \times 5 = 2.0 \text{ square inches} \]

The resistance provided by this reinforcement is equal to

\[ F_{rp} = 0.75 \times 60 \times 2.0 = 90 \text{ kips} \]

The resistance is less than the actual force (99.82 kips). Hence, the spacing has to be reduced for the next iteration.

**Iteration 5** - #4 bars at 6-inch spacing.

Seven #4 bars can be placed at 6 inches on center, which fit within the available space. The total area of reinforcement is equal to

\[ A_{hp} = 2 \times 0.2 \times 7 = 2.8 \text{ square inches} \]

The resistance provided by this reinforcement is equal to
\[ F_{rp} = 0.75 \times 60 \times 2.8 = 128 \text{ kips} \]

The resistance is greater than the actual force (99.82 kips). Hence, this configuration is acceptable as per Code requirements. It is also the best possible solution within the constraints of bar diameters available. This selection can be treated as an economical option.

**Vertical reinforcement for the bottle shaped strut**

Component of the splitting force in the vertical direction is

\[ F_v = 2 \times 59.24 \cos (57.4) = 63.83 \text{ kips}. \]

The reinforcement required to resist this force is

Area of reinforcement required \( A_{vp} = \frac{63.83}{0.75 \times 60} = 1.42 \text{ square inches}. \)

This area represents the reinforcement for one bottle shaped strut. There are two such struts in each shear span. Hence, the required area is to be multiplied by 2.

The spacing of this reinforcement is not calculated at this step. The area required for the vertical ties T1 and T2 (see Fig. 3.2), is added to the vertical reinforcement needed to resist the transverse tension in the bottle shaped strut. The total area will be used to establish the required spacing.

**Reinforcement for the vertical ties T1 and T2**

The total force in the vertical ties is equal to the load in each shear span, i.e., 200 kips.

The reinforcement required to resist this force is equal to

\[ A_s = \frac{200}{0.75 \times 60} = 4.44 \text{ square inches}. \]

**Combination of the reinforcement for bottle shaped struts and vertical ties**

The total vertical reinforcement to be provided within the shear span is the summation of the areas required for the bottle shaped struts and vertical ties.

Combined requirement \( = 2 \times 1.42 + 4.44 = 7.82 \text{ square inches}. \)
The combined reinforcement is distributed at some spacing. The necessary spacing is determined through an iterative procedure. The iteration steps are similar to those for establishing the horizontal reinforcement of bottle shaped strut.

The shear span is equal to 48 inches and the maximum spacing is equal to 8 inches (from Section 3.5.1).

**Iteration 1** - #3 bars at 8-inch spacing.

Seven #3 bars can be placed at 8 inches on center, which fit within the shear span. The total area of reinforcement (two legs at each bar location) is equal to

\[ A_{vp} = 2 \times 0.11 \times 7 = 1.54 \text{ square inches} \]

The area provided is less than the required area. Hence, the spacing is reduced in the next iteration.

The intermediate iteration for #3 bars at 6 inch on center is not performed here because the earlier calculations suggest that it will not be sufficient.

**Iteration 2** - #3 bars at 4-inch spacing.

Thirteen #3 bars can be placed at 4 inches on center, which fit within the shear span. The total area of reinforcement (two legs at each bar location) is equal to

\[ A_{vp} = 2 \times 0.11 \times 13 = 2.86 \text{ square inches} \]

The area provided is less than the required area. Hence, the next bar size is used for the next iteration.

**Iteration 3** - #4 bars at 8-inch spacing. The spacing is set to the maximum permissible as bar diameter is increased.

Seven #4 bars can be placed at 8 inches on center, which fit within the shear span. The total area of reinforcement (two legs at each bar location) is equal to
The area provided is less than the required area. Hence, the spacing is reduced in the next iteration.

**Iteration 4** - #4 bars at 4-inch spacing.

Thirteen #4 bars can be placed at 4 inches on center, which fit within the shear span. The total area of reinforcement (two legs at each bar location) is equal to

\[ A_{vp} = 2 \times 0.2 \times 13 = 5.2 \text{ square inches} \]

The area provided is less than the required area. Hence, additional iterations are necessary.

For the next iteration, adequacy of four legs of #3 bars is checked. Following the trend established in the previous iterations, #5 bars (two legs) should have been the next choice for further iteration. The reason for not using #5 bars is that the area of two legs of #5 bars is 0.62 square inches whereas four legs of #3 bars provide 0.44 square inches. If #3 bars are found to be sufficient, the reinforcement configuration would be more economical as compared to the one obtained by using #5 bars.

**Iteration 5** - #3 bars (four legs) at 8 inches on center. The spacing is set to the maximum permissible value as the area can be increased by using a larger number of bars.

Seven #3 bars can be placed at spacing of 8 inches on center, which fit within the shear span.

The total area of reinforcement (two legs at each bar location) is equal to

\[ A_{vp} = 4 \times 0.11 \times 7 = 3.08 \text{ square inches} \]

The area provided is less than the required area. Hence, the spacing is reduced in the next iteration.
**Iteration 6** - #3 bars (four legs) at 4-inch spacing.

Thirteen #3 bars can be placed at 4 inches on center within the shear span. The total area of reinforcement (two legs at each bar location) is equal to

$$A_{vp} = 4 \times 0.11 \times 13 = 5.72 \text{ square inches}$$

The area provided is less than the required area. The bar spacing of 4 inches cannot be reduced further. Hence #5, bars (two legs) are checked for sufficiency in the next iteration.

**Iteration 7** - #5 bars at 8-inch spacing. The spacing is set to the maximum permissible as the bar diameter is increased.

Seven #5 bars can be placed at 8 inches on center, which fit within the shear span. The total area of reinforcement (two legs at each bar location) is equal to

$$A_{vp} = 2 \times 0.31 \times 7 = 4.34 \text{ square inches}$$

The area provided is less than the required area. Hence, the spacing is reduced in the next iteration.

**Iteration 8** - #5 bars at 6-inch spacing.

Nine #5 bars can be placed at 6 inches on center, which fit within the shear span. The total area of reinforcement (two legs at each bar location) is equal to

$$A_{vp} = 2 \times 0.31 \times 9 = 5.58 \text{ square inches}$$

The area provided is less than the required area. Hence, the spacing is reduced in the next iteration.

**Iteration 9** - #5 bars at 4-inch spacing.

Thirteen #5 bars can be placed at 4 inches on center, which fit within the shear span. The total area of reinforcement (two legs at each bar location) is equal to

$$A_{vp} = 2 \times 0.31 \times 13 = 8.06 \text{ square inches}$$
The area provided is more than the area required (7.82 square inches). Hence, this configuration is acceptable. The reinforcement layout that includes bars for the bottle shaped strut (vertical and horizontal) and the vertical ties is shown in Fig. 3.9.

![Fig. 3.9 Final reinforcement layout for the bottle shaped strut](image)

3.7 MATLAB program

The procedure to calculate the reinforcement for a bottle shaped strut is repetitive and is the same for all cases and beam configurations. A MATLAB code was developed to perform the iterative calculations. The input parameters for this program are similar to the data summarized in Table 3.2.

MATLAB codes were prepared for all the cases that are examined in this research. The codes solve the STM truss to determine an acceptable configuration of struts, ties and nodal zones. A separate MATLAB program was developed to design the longitudinal and bottle shaped strut reinforcement for a given set of load and beam geometry. The results obtained using the method
proposed in this chapter was found to be in agreement with the results from the software CAST
developed by Kuchma and Tjhin (2002), refer to Appendix A.
Chapter 4

Case 3 – Beam subjected to two unequal point loads placed symmetrically (Normal-strength concrete).

4.1 Geometry of the beam and the strut-and-tie model

The cases discussed in the previous two chapters were symmetrically loaded. Symmetrical loading yields a symmetrical bending moment diagram. The case considered in this chapter is that of a deep beam subjected to unequal symmetrically placed loads. The location of the loads and the geometry of the beam are similar to the previous cases. The loads are placed a distance equal to the depth of the beam from the face of the support.

Fig. 4.1 Geometry of beam and loads for Case 3 and Case 4

The strut-and-tie model for the case under consideration remains similar to the models discussed in the preceding chapters. The truss configuration changes according to the bending moment diagram. The strut-and-tie model follows the pattern of bending moment diagram (Mac Gregor – 2005). The selected model is shown in Fig. 4.2. Zhang et al (2009) have proposed a direct modeling method based on Mohr’s interactive failure criterion. The procedure explained in this
research work is quite involved, and was not found suitable for the scope of the reported research. The approach proposed by MacGregor (2005) is more intuitive. The results obtained using the method proposed in this chapter was found to be in agreement with the results from the software CAST developed by Kuchma and Tjhin (2002), refer to Appendix A.

![Fig. 4.2 Strut-and-tie model for Case 3](image)

The heights \(d_1\) and \(d_2\) shown in the strut-and-tie model are proportional to the moments \(M_1\) and \(M_2\) shown in the bending moment diagram (refer to Fig. 4.3). Hence,

\[
\frac{d_1}{d_2} = \frac{M_1}{M_2}
\]  

(4.1)

![Fig. 4.3 Bending moment diagram for the beam](image)
The procedure to solve the STM is similar to the one described in Chapter 2. The difference lies in the orientation of the top prismatic strut. The height of the truss at the location of the heavier load governs. The height of the truss at the location of the smaller load is multiplied by the ratio in Eq.4.1. For a new iteration, distance $d_1$ is reduced; distance $d_2$ is obtained from Eq. 4.1.

4.2 Calculation for the forces in the struts and ties

Figure 4.4 is a detail of the shear span for Load 1 (refer Fig. 4.1). The demands in each member of the truss can be computed using basic statics.

![Fig. 4.4 Member details of the shear span for Load 1](image)

The equations used for these calculations are provided in Table 4.1.
Table 4.1 Demands in the truss members from shear span for Load 1

<table>
<thead>
<tr>
<th>Member</th>
<th>Demand</th>
<th>Type of member</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>$\frac{\text{Reaction 1}^*}{\sin (\theta)}$</td>
<td>Bottle shaped strut</td>
</tr>
<tr>
<td>S2</td>
<td>$\frac{\text{Reaction 1}}{\sin (\theta)}$</td>
<td>Bottle shaped strut</td>
</tr>
<tr>
<td>S7</td>
<td>S1 cos (\theta)</td>
<td>Prismatic strut</td>
</tr>
<tr>
<td>T1</td>
<td>Reaction 1</td>
<td>Vertical tie</td>
</tr>
<tr>
<td>T3</td>
<td>S1 cos (\theta)</td>
<td>Longitudinal tie</td>
</tr>
<tr>
<td>T4</td>
<td>T3 + S2 cos (\theta)</td>
<td>Longitudinal tie</td>
</tr>
</tbody>
</table>

*Reaction at the support nearer to Load 1

Similar computations can be performed for the other shear span. The details of this shear span are shown in Fig. 4.5. The angle $\theta$ shown in Fig. 4.4 is different from the angle $\theta$ in Fig. 4.5.

Fig. 4.5 Member details of the shear span for Load 2
4.3 Design of the struts, ties and nodal zones

The design of struts, ties, and nodal zones is straightforward once the member forces are known. The steps followed in Section 2.2 can be used to design the beam. Care should be taken in the design of bottle shaped struts. Struts in the two shear spans have different forces and need to be addressed individually. The main longitudinal reinforcement has to be checked for available development length at each support. The design procedure is shown graphically in the flow chart shown in Fig. 4.6.
Fig. 4.6 Flow chart to model a deep beam for Case 5
Case 4 – Beam subjected to two unequal point loads placed symmetrically (High-strength concrete).

4.4 Geometry of the beam and the strut-and-tie model

This case is a combination of the approaches used while addressing Case 3 and Case 2. The mechanics of the truss is similar to the one for Case 3. Similar to Case 2, the struts S1, S2, S3 and S4 are modeled as bottle shaped struts. The sketch is included in Fig. 4.7.

![Fig. 4.7 Strut-and-tie model for Case 4](image)

For a new iteration, the height of the truss at the location of the heavier load (Load 1) is established first. The height at the location of the other load (Load 2) is calculated using Eq. 4.1. The bottle shaped struts (BS1, BS2, BS3 and BS4) are modeled using an approach described in Section 3.2. The modeling of the other truss members is similar to the procedure adopted in Section 2.2.

4.5 Modeling of the bottle shaped strut

The strut-and-tie model discussed in Chapter 2 (Case 2) is for a beam that is symmetrically loaded. The forces within the struts (in both the shear spans) are equal. On account of equal
forces, the required reinforcement is the same for all the bottle shaped struts. For the particular case under consideration, the forces within the two bottle shaped struts are different, and, hence, the bottle shaped struts in each shear span has to be designed separately. The horizontal reinforcement from the strut with higher loads (and hence more reinforcement) may be used for the other shear span for the ease of construction.

4.6 Development for bottom longitudinal tie reinforcement

The force in the tie T4 (Fig. 4.2) governs the total quantity of the longitudinal reinforcement required. Reinforcement in the bottom longitudinal tie is not curtailed within the beam span, and hence equal development lengths are needed at each support. Depending on the widths of the bottle shaped strut formed at each support, the available development strength would vary. A check has to be performed at both the supports to ensure for development of the bars. Note that a check at any one support was sufficient in the case of symmetrically loaded beams.
Chapter 5

Case 5 – Beam subjected to uniformly distributed loads (Normal strength concrete).

5.1 Description of the beam and the strut-and-tie model

The beam is simply supported and has a single span. The geometry of the beam is shown in Fig. 5.1. The uniformly distributed load is placed within the clear span of the beam and along the center line of the beam in the plan view.

![Fig. 5.1 Geometry of the beam and loads](image)

The strut-and-tie model shown in Fig. 5.2 is applicable to concrete strength less than or equal to 6000 psi. In the reported research, total load on the beam is modeled as a two single point loads placed at the quarter spans (with the span taken as the center to center distance between the supports).
This technique of modeling a uniformly distributed load as two single point loads was adopted based on the work published by Muttoni et al (1997). A deep beam supports a uniformly distributed load through the action of a compression fan. The fan is a collection of a large number of struts that carry an axial load to the support. This axial load in a strut from the fan region corresponds to a point load. The point load is the product of a finite span and the load per unit length. To model the actual fan action, a large number of struts have to be modeled. In the most general form, any distributed load could be broken into a collection of point loads placed at one inch on center. Each of these point loads would have to be connected to the support through inclined struts. Such a model would give rise to a large number of struts and would make the computations cumbersome. Since the design procedure using STM is iterative, the same set of calculations has to be performed for each trial. Hence, the calculations will be more time consuming and prone to error. To overcome these difficulties, the application of point load is adopted in this research. The system of two point loads was validated through the studies described in the next section.
5.2 Model 1 – Beam with the given uniformly load modeled as equivalent point loads placed inch apart

The beam is subjected to a uniformly distributed load with the units of kips per inch. The uniformly distributed load is converted to an equivalent system of point loads. The point loads are placed an inch apart. The strut-and-tie model so generated is presented in Fig. 5.3. The crucial outputs from this model are the forces in the tension tie and the prismatic strut.

![Diagram of uniformly distributed load and modeled point loads](image)

**Fig 5.3 STM for Model 1**

The truss height is arbitrarily selected, and a detailed design of struts or ties is not performed at this stage. The purpose of this exercise is to study the effect on member forces using point loads in lieu of the distributed load. The procedure to calculate the forces in each truss member is similar to computations performed for Case 1.
5.3 Model 2 – Beam with given uniformly distributed load modeled as equivalent point loads placed at a distance more than an inch

The point loads are combined together to form larger point loads placed at greater distances. The distance between the point loads should be fractions of the span. A representative STM is sketched in Fig. 5.4.

![STM for Model 2](image)

**Fig 5.4** STM for Model 2

The STM is remodeled with the new configuration. The truss height from the earlier model is kept unchanged and the forces in each member are calculated.

5.4 Model 3 – Beam with given uniformly distributed load modeled as equivalent point loads placed at quarter points

As done in the second model, the point loads for half of the span are combined as a concentrated load placed at the quarter point. The resulting strut-and-tie model shown in Fig. 5.2 is generated. The truss height is kept same as the earlier two models. The point load for Model 3 is equal to the reaction at the supports.
5.5 Synthesis of the analysis results of the three models

1. **Top strut** – The maximum force in the top prismatic strut at the mid span remains the same for each model. Hence, the width (and hence the strength) of the strut that would be required to resist the compressive force equal for all the three models.

2. **Inclined bottle shaped struts** – The forces in the bottle shaped struts vary for each model. Although there is a difference in the forces, this difference has little or no bearing on the design and detailing. As per the Code provisions, the reinforcement that resists the transverse tension in these struts is independent of the force in the struts for concrete strengths below 6000 psi.

3. **Longitudinal tie** – The force in the bottom longitudinal tie remains constant for the three models. Hence, the width of the tie is the same for the models. The equal force; also, results in a similar longitudinal reinforcement for the three models.

4. **Truss height** – The forces along the centerline of the bottom tie and top prismatic strut are equal for each of the three models. The widths for the top prismatic strut and the longitudinal tie are identical for each model. The truss height is a function of the widths of the top prismatic strut and the bottom longitudinal tie (provided the clear cover is constant). Hence, it can be concluded that the truss height for each model remains the same. This statement is true if the loads are symmetrically placed, and the load locations are in terms of the span.

5.6 Comparison of the bending moment diagrams for the model proposed and the actual beam

The loads on the selected model are different from the original uniformly distributed load. The beam is subjected to a uniformly distributed load whereas; the model is subjected to two point
loads. It is important to check the key characteristics of the original and modified loading patterns. For this purpose, the bending moments as well as the shear force diagrams are compared for a beam with a clear span equal to 120 inches and a uniformly distributed load of 2 kips per inch. As shown in Fig. 5.5, although the bending moment diagram for the strut-and-tie model is not parabolic like the original beam, the maximum values remain unchanged. Moreover, the bending moment curve for the original beam is within the area covered by the bending moment curve for the model. The proposed model is, hence, conservative.

**Fig. 5.5** Comparison of bending moment diagrams from each model
5.8 Comparison of the shear force diagrams for the model proposed and the actual beam

Similar to bending moment diagram, the shear force diagram of the model needs to be checked against that for uniformly distributed load. As shown in Fig. 5.6, the model represents the shear conservatively for each of the first quarter spans. For the region between the quarter spans and the mid span of the beam, the model predicts zero shear force. However, the shear force diagram for uniformly distributed load is not zero within this region. The maximum shear force at the quarter spans is equal to half of the numerical value at the support. Design of shear reinforcement with the results from the STM would lead to insufficient design in the region between the quarter spans.

Fig. 5.6 Comparison of shear force diagrams from each model
5.9 **Shear reinforcement in the mid-section of the beam**

As explained in the previous section, the model proposed falls short of representing the actual shear forces between the quarter spans. This shortcoming is overcome by providing shear reinforcement in the mid-section of the beam even though the STM indicates zero shear force. The quantity of this reinforcement is taken as half of the amount used in the shear span; i.e., between the support and concentrated load.

5.10 **Analysis and design of the deep beam with STM**

The analysis and design of deep beam supporting a uniformly distributed load is similar to the methodology adopted in Chapter 2. The steps described in Chapter 2 are applicable to the case under consideration. The only change is in the location of the load on the beam. The point load at each quarter span represents the load acting on that half of the beam. The span is taken as the center to center dimensions between supports. This load is equal to the reaction at the support.

**Case 6 – Beam subjected to uniformly distributed loads (High-strength concrete).**

The strut-and-tie model to analyze and design a deep beam subjected to uniformly distributed loads is shown in Fig. 5.7. The design procedure is similar to the method adopted in Chapter 2 and Chapter 3. The bottle shaped struts are to be actually modeled because the concrete strength is more than 6000 psi.
A deep beam subjected to uniformly distributed loads and with concrete strength more than 6000 psi can be designed by modeling it as per the discussion in the Section 3.2. The discussion from Case 5 related to modeling of the uniformly distributed loads with point loads remains valid and hence is not repeated.
Chapter 6

Case 7 – Beam subjected to two equal/unequal loads placed at symmetrical/unsymmetrical locations (Normal-strength concrete).

6.1 Description of the beam and the strut-and-tie model

The strut-and-tie models discussed in the preceding chapters had some limitations in terms of the load locations and/or relation between the two loads. The assumptions made while detailing the models in earlier section are as follows.

Section 2.1

1. The loads are equal.
2. The spacing between the loads is equal to the beam depth.
3. The spacing between the load and support is identical as the depth of the beam.
4. The span to depth ratio of the beam is 3.

Section 3.1

1. The spacing between the loads is equal to the beam depth.
2. The spacing between the load and support is identical as the depth of the beam.
3. The span to depth ratio of the beam is 3.

This chapter describes the application of a single strut-and-tie model to design deep beams subjected to randomly placed point loads. Hence, the assumptions and limitations from the earlier sections are eliminated. The Strut-and-tie model used in this chapter is a combination of those employed in the preceding chapters.
The geometry of the deep beam under consideration in this chapter is shown in Fig. 6.1. The loads shown in this sketch may or may not be equal. The shear spans shown in the figure may be the same or different. Certain conditions are imposed on the minimum and maximum shear spans. These constraints are a result of the limitations imposed on the applicability of strut-and-tie models. These limitations on the shear spans are discussed in Section 6.2.

**Fig. 6.1 Geometry of the beam for Case 7 and Case 8**

The truss model remains similar to those used in the earlier section. The model is depicted in Fig. 6.2. The purpose of keeping the truss consistent is to maintain a uniform design basis for similar cases. The STM in Fig. 6.2 is applicable to cases where Load 1 is greater than Load 2. The inclination of the prismatic strut S6 changes depending on the relation between the two loads. A combination of the techniques used in earlier chapters is applied in this chapter.
6.2 Limits on the load position

Two factors are considered while determining the limits on the load placement. These are:

1. The minimum distance between the load and the support (Fig. 6.3)
2. The minimum distance between the loads (Fig. 6.4)

6.2.1 Minimum distance between the load and the support

The minimum distance between the load and the center line of the support is governed by the angle between the inclined strut and longitudinal tie for model adopted. MacGregor and Wight (2005) propose the maximum angle between the inclined strut (strut S1 and S4 – Fig. 6.2) and the longitudinal tie (tie T3 and T5) as 65 degrees.

The inclined lines in Fig. 6.3 are the bottle shaped struts and the truss height equals the beam depth. The distance between the load and the center of the support is calculated next.

\[
\text{distance } x_1 \text{ and } x_2 = \frac{\text{depth of beam (D)}}{\tan (65)} = \frac{\text{depth of beam (D)}}{2.147} = 0.466 \times \text{beam depth (D)}.
\]

The distances \( x_1 \) and \( x_2 \) in Fig. 6.2 must be equal to or greater than 0.466 times the beam depth. The truss height would always be less than the beam depth; hence, the angle of inclination would
always be less than 65 degrees. Therefore, a load is placed at distance equal to $2 \times 0.466$ times the beam depth from the support or further away would always maintain the angle between the longitudinal tie and inclined strut less than 65 degrees.

![Diagram](image)

**Fig.6.3** Minimum distance of the load from the support

For the first iteration, the height of the truss is assumed to be 99 percent of the depth of the beam. The load is at the limiting distance ($0.932$ times beam depth) from center line of the support. Thus, the angle is calculated as

$$\tan (\theta) = \frac{\text{height of the truss}}{x1} = \frac{0.99 \times \text{depth of beam}}{0.466 \times \text{depth of beam}} = 2.124.$$ 

Thus, $\theta$ equals 64.77 degrees (less than 65). As a result, the condition that $\theta$ not to exceed 65 degrees is always satisfied.
6.2.2 Minimum distance between the loads

The loads on the deep beam were represented as single line in all the models discussed earlier. The loads would be from columns, which have physical dimensions that have to be taken into account while deciding the space between the loads. The minimum dimension of a column specified in the Code is 10 inches. Thus, the spacing between the two loads is the sum of the distance between the face of the columns and the physical dimension of the column. The two columns are assumed to be 30 inches apart due to physical considerations. Hence, the center-to-center distance between loads is equal to 40 inches as shown in Fig. 6.4.

![Diagram showing minimum distance between the two columns that load a deep beam](image)

Fig.6.4 Minimum distance between the two columns that load a deep beam

6.2.3 Span to depth ratio

The Strut-and-tie model shown in Fig. 6.2 can be used for deep beams for span to depth ratios ranging from a minimum value up to 4. The maximum limit of 4 is set by the Code. The minimum possible value of span to depth ratio is controlled by the limitations imposed on the
load placement as discussed in Sections 6.2.1 and 6.2.2. Using the variables listed in Table 6.1, the minimum span to depth ratio was established.

**Table 6.1 Variables for calculating the minimum span to depth ratio**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Span of the beam</td>
<td>L</td>
</tr>
<tr>
<td>Depth of the beam</td>
<td>D</td>
</tr>
<tr>
<td>Minimum span to depth ratio</td>
<td>R</td>
</tr>
</tbody>
</table>

To establish the minimum span to depth ratio, the load has to be placed such that its location satisfies both the minimum distance from the support (i.e., 0.932 D as shown in Fig. 6.3) and the minimum distance from the center line (i.e., 20 inches as shown in Fig. 6.4). This condition is shown in Fig. 6.5.

![Fig. 6.5 Minimum span to depth ratio](image)

With respect to Fig. 6.5, the following equation is derived.
0.5 L − 0.932 D − 20 = 0 \hspace{1cm} (6.1)

The value 0.932 D comes from Fig. 6.3 (the minimum distance between load and the support).

From the definition of span to depth ratio, \( D = \frac{L}{R} \)

Substituting the value of D in Eq. 6.1 and simplifying

\[
R = \frac{0.932 L}{(0.5L - 20)} \hspace{1cm} (6.2)
\]

The value of R from Eq. 6.2 is the minimum span to depth ratio for which the model is applicable.

6.3 Analysis and design methodology

The STM is shown in Fig. 6.2. The techniques described in the previous chapters remain unchanged once the model is selected. However, the locations of the loads and the relationships between the loads have to be also considered. These two steps are summarized in this section. MATLAB codes are used to perform the iterative design calculations. The MATLAB code written for this section is applicable to equal or unequal loads. The loads can be placed at any location on the beam, within the constraints of the minimum distance between the loads (Fig. 6.4) and the minimum distance between the loads and supports (Fig. 6.3). The additional steps incorporated to the earlier MATLAB codes are discussed in the following.

**Step 1: Fix the location of the loads**

The limiting load locations are described in sections 6.2.1 and 6.2.3. The program sets the location of the two loads at a distance equal to the minimum permissible space between the load and the support (see Fig. 6.3). The distance x1 and x2 (Fig. 6.2) are the variables that control the load placement. The different combination of these distances is elaborated in the flow chart shown below.
Set $x_1 = \text{minimum permissible from Fig. 6.3}$

Set $x_2 = \text{minimum permissible from Fig. 6.3}$

Assemble the truss and iterate the STM to get an acceptable truss configuration for all possible ratios between loads

Design individual STM components as per relevant clauses of the Code for each load case

Increase $x_2$ by a user defined value

Is $x_2$ greater than the maximum permissible

Yes

Increase $x_1$ by a pre-determined value

No

Is $x_1$ greater than the maximum permissible given in Fig. 6.4?

No

Yes

All possible combinations of $x_1$ and $x_2$ have been checked by the program

Fig 6.6 Flow chart to decide the load location


Step 2: Ratio of loads for each set of load spacing

The program offers a flexibility to have different or identical values for Load 1 and Load 2 shown in Fig. 6.1. The relationship between the two loads can be specified and varied as per the requirement of the user. Load 1 (Fig. 6.1) is considered to be a variable load and the program calculates the value for Load 2 as per the relation specified by the user. The minimum value taken by the program for Load 1 is 50 kips although it can be a variable. The maximum value is controlled by the maximum shear force that the model can resist. The program solves the truss and generates an output for each load combination. The flowchart shown in Fig. 6.7 presents an overview of the MATLAB code developed for the particular case discussed in this section. The minimum angle of inclination of the inclined strut to the longitudinal tie is 25 degrees as specified in the Code. The program checks this condition for an iteration. The height of truss is set at 99 percent of the beam depth for each new set of load or load location. The height of the truss is reduced by 0.5 percent for every cycle of iteration. The process of iteration is similar to that presented in Section 2.2 or Section 4.2. The output of a successful iteration is then subjected to further analysis that includes the calculation for longitudinal reinforcement, reinforcement for bottle shaped struts, and shear reinforcement calculations. The methodology adopted for these calculations is analogous to that discussed in Section 2.2.
Fig. 6.7 Flow chart to describe the working of the MATLAB code
Case 8 – Beam subjected to two equal/unequal loads placed at symmetrical/unsymmetrical locations (High-strength concrete).

6.4 Geometry of the beam and the strut-and-tie model

The beam geometry is the same as that shown in Fig. 6.1. The load locations may or may not be symmetrical, and the loads may or may not be equal. The STM shown in Fig. 6.8 is applicable for the case where Load 1 is greater than Load 2. As mentioned earlier, the inclination of the prismatic strut S2 would vary depending on the location of loads and the ratio between them.

![Fig. 6.8 Strut-and-tie model for Case 8](image)

6.5 Limitations on the location of loads and load combinations

Section 6.2 describes the limitations imposed on the location of the loads. These limitations are applicable to the case under consideration. The STM can be assembled, analyzed, and designed after the load locations are established according to Section 6.3. Similar to Section 6.3, the ratio between the loads can be any specified value.

6.6 Analysis of the truss and design of individual members

The procedure followed to analyze and design the beam is a combination of the approaches adopted for the cases discussed in Chapter 3 and 4. The design of the bottle shaped struts is
similar to the one discussed in Chapter 3. The struts in the two shear spans may have different configurations depending on the nature of loading. Separate calculations have to be performed if the forces in the bottle shaped struts are not equal.

The analysis of the overall truss is the same as that discussed in Chapter 4. The inclination of strut S2 (with respect to horizontal) would depend on the ratio of the bending moments at each load location.

The main longitudinal reinforcement is calculated in a manner similar to that discussed in Section 2.6.1. This reinforcement should resist the total tensile force present in tie T4. The development of the main longitudinal reinforcement has to be checked at both the supports.
Chapter 7

Case 9 – Beam subjected to a single point load (Normal-strength concrete).

7.1 Description of the beam geometry

The models discussed in the previous chapters were for beams subjected to two point loads. The remaining two cases of this research focus on deep beams supporting a single point load. The geometry of the beam is shown in Fig. 7.1.

![Fig. 7.1 Geometer of the beam for Case 9 and Case 10](image)

The Code has restrictions on the maximum distance of load from the support and a limiting span to depth ratio for which STM can be used for analysis and design of deep beams. These limitations and their implications on the use of the Code provisions in this research are discussed in the sections that follow. The actual modeling of the beam is discussed after presenting the limits on cases that can be designed based on STM.
7.2 Influence of span to depth ratio, location of load and Code provisions

The Code defines a deep beam as the one with a clear span less than or equal to four times the beam depth, or a span with the load placed within a distance that is less than or equal to two times the beam depth. These clauses highlight the importance of the ratio of clear span to beam depth on the applicability of STM. The following cases are considered to illustrate the influence of span to depth ratio.

7.2.1 Span to depth ratio more than four

The sketch shown in Fig. 7.2 is for a beam with span to depth ratio more than 4. The nature of loading and support condition lead to the formation of four D – Regions within the beam clear span. The length of each D – region is equal to the depth of the beam (St. Venant’s principal).

![Diagram of beam for depth to span ratio more than 4](image)

**Fig. 7.2** Details of the beam for depth to span ratio more than 4

The D – regions 1 and 4 (Refer Fig. 7.2) are formed due to the supports while the D – regions 2 and 3 are created on account of the presence of the point load. The geometry of the beam, shown in Fig. 7.2, leads to two B – Regions within the span of the beam. For some cases, depending on the geometry and the load location, a single B – region may be formed. Hence, there would be at
least one B – region within the span. The Code defines a beam with B – Regions along the span as a ‘slender beams’. The shear strength of such beams is governed by clauses from Chapter 11 of the Code. Cases like the once shown in Fig. 7.2 are not included in this research because such cases do not require the use of STM. The strut-and-tie model discussed in this chapter is applicable to beams with a clear span less than or equal to four times the beam depth.

7.2.2 Span to depth ratio equal to four

This case is a limiting case for maximum span to depth ratio clause of the Code. This case can be further divided into two different cases based on location of the load. Case 2a is a beam with a single point load at the center of the beam. Case 2b is a beam with a single point load placed away from the center of the beam.

Case 2a – The distance between the face of the support and the load is exactly equal to two times the member depth when the load is placed exactly at the centerline of the clear span. The two conditions that define a deep beam (the maximum clear span equals four times the member depth, and the maximum distance between the face of the support and the load equals two times the beam depth) are fulfilled in this case. Hence, the beam can be defined as a deep beam. The D – Regions are formed within the complete span of the beam (refer to Fig. 7.3). The D – regions 1 and 4 are formed due to the supports while D – regions 2 and 3 are created due to the point load. The Code provisions from Appendix A can be applied to this case, and the beam can be designed with STM approach.
Case 2b – In this case, the load is placed away from the center line of the beam (refer to Fig. 7.4). The D – regions 1 and 4 are formed at the supports while D – regions 2 and 3 are attributable to the point load. The location of the load causes the D – regions from the support and the load to overlap in once span, and a B – region is created in the other shear span. Hence, the beam is defined as a slender beam according to the Code provisions and can be designed as per the requirements of Chapter 11 of the Code. STM is not needed for this case, and it is not considered in this research.
7.2.3 Span to depth ratio less than four

For Case 2a, the location of the load was limited to the centerline of the beam. In the case of beams with span to depth ratio less than 4, the load may be placed away from the centerline provided that there is no B – Region within the beam span. If the load is to be placed at a desired location, the span to depth ratio would have to be such that a B – region a not formed.

A strut-and-tie model can be used if the load is placed not more than twice the beam depth from the farthest support (refer to Fig. 7.5). In an equation form, the limitation on the load location can be written as

\[ L - X = 2D \]  
(7.1)

From the definition of span to depth ratio (R)

\[ D = \frac{L}{R} \]  
(7.2)

Combining Eq. 7.1 and Eq. 7.2

\[ L - X = \frac{2L}{R} \]  
(7.3)
Equation 7.3 can be used to calculate the minimum span to depth ratio required for a given load location, or it can be used to calculate the maximum distance between the load and the closer support.

If the load is placed at a distance more than twice the depth from a support, the beam configuration will be similar to that shown in Fig. 7.6. The beam is a slender beam on account of the presence of a B – region within the span.
Fig. 7.6 Details of the beam for depth to span ratio less than 4 (load at a distance more than twice depth from the farther support)

7.3 Verification of Eq. 7.3

Equation 7.3 is verified by solving two cases. Eq. 7.3 should yield the minimum span to depth ratio (R) for a given span (L) and load location (X). There should be no D-regions formed within the beam span if the computed ratio (R), and hence, the beam depth (D) was to be adopted for design.

Case 1 for verification – Calculate the minimum span to depth ratio for a load placed away from center line of the beam (refer to the case shown Fig. 7.7).

Clear span of the beam (L) = 160 inches

Location of the load from the face of the support closest to the load = 60 inches

From Eq. 7.3,

\[
160 - 60 = \frac{2 \times 160}{R}
\]

The minimum value of R = 3.2.
Hence, the depth of the beam has to at least be 50 inches if it is to be designed by using strut-and-tie model. If the beam depth is reduced as a part of design optimization, a B-region would be formed within the span, and STM would not be required for design. On the other hand, if the beam depth is increased, the procedure discussed in Section 7.4 would be applicable.

**Case 2 for verification** – Calculate the minimum span to depth ratio for a load at the center line of the beam

A load is placed at the centerline of the beam for this case. As discussed in section 7.2.1, the minimum depth required should be one fourth of the clear span.

Clear span of the beam = 160 inches (similar to Case 1 of Section 7.3)

Location of the load from the face of the support = 80 inches

From Eq. 7.3,
\[160 - 80 = \frac{2 \times 160}{R}\]

The maximum value of \( R = 4 \). Hence, the depth of the beam has to at least be 40 inches if it is to be designed by using strut-and-tie model.

### 7.4 Strut-and-tie model for Case 9

The STM for the beam with single point load (concrete strength less than 6000 psi) is shown in Fig. 7.8. The model is based on the work published by Schlaich et al (1987).

**Fig. 7.8 Strut-and-tie model for Case 9**

As discussed in Section 6.2, the minimum distance between a load and the center of the support has to be 0.932 times depth of the beam. As a result, the span for which STM (refer to Fig. 7.8) is applicable cannot be less than \( 2 \times 0.932 \) times the depth of the beam.

### 7.4.1 Forces in the individual members of the Strut and Tie Model

The forces in the individual strut and ties of the truss are calculated using basic statics. Table 7.1 summarizes the equations necessary for computing the forces in various members, and if applicable, the type of strut.
### Table 7.1 Forces in the individual struts and ties

<table>
<thead>
<tr>
<th>Member</th>
<th>Force</th>
<th>Type of strut</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>(\frac{\text{Reaction 1}}{\sin(\theta_1)})</td>
<td>Bottle shaped strut</td>
</tr>
<tr>
<td>S2</td>
<td>(\frac{\text{Reaction 1}}{\sin(\theta_1)})</td>
<td>Bottle shaped strut</td>
</tr>
<tr>
<td>S3</td>
<td>(\frac{\text{Reaction 2}}{\sin(\theta_2)})</td>
<td>Bottle shaped strut</td>
</tr>
<tr>
<td>S4</td>
<td>(\frac{\text{Reaction 2}}{\sin(\theta_2)})</td>
<td>Bottle shaped strut</td>
</tr>
<tr>
<td>S5</td>
<td>(S_1 \cos(\theta_1))</td>
<td>Prismatic strut</td>
</tr>
<tr>
<td>S6</td>
<td>(S_4 \cos(\theta_2))</td>
<td>Prismatic strut</td>
</tr>
<tr>
<td>T1</td>
<td>Reaction 1</td>
<td>-</td>
</tr>
<tr>
<td>T2</td>
<td>Reaction 2</td>
<td>-</td>
</tr>
<tr>
<td>T3</td>
<td>(S_1 \cos(\theta_1))</td>
<td>-</td>
</tr>
<tr>
<td>T4</td>
<td>(S_1 \cos(\theta_1) + T_3)</td>
<td>-</td>
</tr>
<tr>
<td>T5</td>
<td>(S_4 \cos(\theta_2))</td>
<td>-</td>
</tr>
</tbody>
</table>

#### 7.4.2 Calculation for the capacities and dimensions of struts, nodes, and ties

The design of struts, ties, and nodal zones is similar to the discussion presented in Section 2.2. The allowable stresses in struts and the faces of the nodes remain unchanged. The calculations for the required strut widths and the areas at the faces of a node are the same as those performed in Section 2.2. The strength reduction factors from Tables 2.2 (for struts) and 2.3 (for nodes) are also applicable in this case.
7.4.3 Design and detailing of the deep beam for Case 9

The selection of an optimum truss height is an iterative procedure as discussed in Section 2.2. The height reduction factor that was used in Chapter 2 remains valid. The check for shear and the inclination of the bottle shaped strut is conducted in conjunction with confirming that the truss fits within the physical dimensions of the beam. The longitudinal reinforcement is computed based on the methods discussed previously in Chapter 2. Considering the unsymmetrical nature of loading, the development of bars has to be checked at both the supports. The reinforcement that is provided for the longitudinal tie should be enough to resist the tensile force in T4 (refer to Fig. 7.8).

The procedure adopted in the design for the required reinforcement for the bottle shaped strut for this chapter is the same as that utilized in Chapter 2 (for concrete strengths less than or equal to 6000 psi). Bottle shaped struts from one shear span may have different requirements as compared to the struts in the other shear span if the beam is asymmetrically loaded. The vertical tie (T1 or T2) is designed to carry a load that is equivalent to the reaction at the support closer to the load point. The required reinforcement for this tie is combined with the vertical reinforcement from the bottle shaped struts within that shear span. The resulting reinforcement is distributed over each shear span, as discussed in Chapter 2.

7.5 Detailing of node N6

Node N6 is a C-C-C. Five struts meet at this node. Struts S5 and S6 are prismatic struts that can be described as the compression block of the beam. Struts S2 and S3 are bottle shaped. A prismatic strut connects node N6 to the load (see Fig. 7.8). The detailing of N6 is crucial because the bottle shaped struts S2 and S3 are to be anchored at this node. The geometry of the node is shown in Fig. 7.9.
The applied load is divided into two parts. Load 1 and Load 2 represent the reactions, or the shear, within each shear span. Widths $x_1$ and $x_2$ are a function of the loads and are calculated as follows.

\[
x_1 = \frac{\text{Load 1}}{\text{allowable axial stress at node N6}}
\]

Similarly,

\[
x_2 = \frac{\text{Load 2}}{\text{allowable axial stress at node N6}}
\]

It should be noted that the sum of the two widths ($x_1$ and $x_2$) cannot be greater than the actual width of the bearing plate or the column that transfers the load.

![Diagram](image)

**Fig. 7.9** Details of node N6 for Case 9

The node N6 is divided as shown in Fig. 7.9, which is considered to be a hydrostatic node for the purpose of this analysis. Although hydrostatic nodes are considered to be obsolete, they were selected because an appropriate extended nodal zone could not be easily established. The loads
acting on the node are assumed to act perpendicular to the face of the node. Using the theory of hydrostatic nodes,

Available width to accommodate strut S2 at node N6

\[ \text{ws}_2 = \sqrt{\text{ws}_5^2 + x_1^2} \] (7.6)

Similar calculations can be performed for the bottle shaped strut S3, i.e.,

Available width to accommodate strut S3 at node N6

\[ \text{ws}_3 = \sqrt{\text{ws}_6^2 + x_2^2} \] (7.7)

The widths required for the prismatic strut, S5 and S6 (i.e., \( \text{ws}_5 \) and \( \text{ws}_6 \), respectively) are obtained from the analysis of the truss shown in Fig. 7.8.

If the available widths from Eq. 7.6 and Eq. 7.7 (for the bottle shaped struts S2 and S3, respectively) are greater than the actual required widths (obtained from the analysis of the truss shown in Fig. 7.8), the struts can be accommodated at these nodes. Otherwise, the truss configuration (truss) has to be altered to increase the width of the prismatic struts S5 and S6, provided the shear capacity of the section is not exceeded and the angle of inclination of the bottle shaped struts is more than 25 degrees, else the beam configuration (beam dimensions or concrete strength) have to be modified to design the beam for the applied load.
Case 10 – Beam subjected to single point load (High-strength concrete).

7.6 Description of the beam geometry and the strut-and-tie model

![Strut-and-tie Model for Case 10](image)

**Fig. 7.10** Strut-and-tie Model for Case 10

The beam dimensions and the limitations imposed on span to depth ratio are similar to those discussed for Case 9. The difference between the two models is the concrete strength that is more than 6000 psi. Hence, bottle shaped struts have to be modeled. Each bottle shaped strut can be modeled according to the procedure described in Chapter 3. The struts in each shear span may have different configurations depending on the location of the load. In such cases, separate analyses have to be performed to compute the required reinforcement. The anchoring of the bottle shaped struts at their end nodes (nodes N1 through N7) is required, as discussed in Chapter 3.

The truss analysis and the subsequent iterations are performed based on the theory detailed in Chapter 2. The development length of the longitudinal reinforcement is to be checked at each support if the load is placed unsymmetrically.
Summary and Conclusion

Analysis and design of deep beams is complex. Non-linear finite element analysis techniques may be used for this purpose. However, such analyses are not appropriate for routine designs and require in-depth analytical skills. Strut-and-tie modeling (STM) offers a powerful alternative for analysis and design of deep beams. STM allows the designer to model a deep beam for a variety of cases. The modeling procedure is easy, provided that an appropriate truss model is available. Although iterations for the truss geometry are required, STM still has the ability to yield good results. The overall flow of forces within the beam is presented by STM and can be verified and improved upon with FEA.

Strut-and-tie models for some general cases of loading for simply supported beams were presented in this research. MATLAB programs were written to facilitate the iterative design process of STM. The programs would enable the user to establish a preliminary design that may be further improved by FEA. The output of the MATLAB programs was found to be in agreement with the results from the software called CAST.

The MATLAB programs provide the user with the required longitudinal reinforcement as well as the shear reinforcement. The nature of reinforcement development (90-degree hook or straight developed bars) and the number of layers are included in the output. Similarly, the bar size and spacing for shear reinforcement is calculated by the programs. The applicable lauses from ACI-318 - 08 were incorporated in the programs.

Normal-strength as well as high-strength concrete were considered. For beams with high-strength concrete, bottle shaped struts were modeled as per the Code requirements, analyzed, and corresponding reinforcement was designed and detailed. For normal-strength concrete beams, the detailing for bottle shaped struts was done based on simplified Code requirements.
The required input parameters for the MATLAB programs are the beam dimensions and material properties. One or more combinations of beam dimensions and material properties can be specified in the input file. This feature of the program would allow the user to make a comparative study with varying input parameters and help in choosing an optimum design.

The programs are written in manner that could be used to generate design tables. For any given input parameters (beam dimensions and material properties), the program can generate an output (longitudinal and shear reinforcement) for a range of loads. The minimum load for such a design table can be selected by the user, while the maximum load is controlled by design clauses from ACI 318-08. The design tables could be used as a quick reference.

**Recommended Future Research**

ACI 318-08 Code does not provide any clear guidelines that could be used as a reference for generating an appropriate STM for a given problem. The available literature has some suggestions for the selection of a model that resembles the load path and stress flow. The lack of unambiguous models for some common cases adds to the difficulty in using STM. The selection of the model depends on the designer’s judgment and experience, which may lead to error. These difficulties could be overcome if the Code includes truss models for some commonly used cases. The load path represented in the truss models included herein can be verified with FEA. The load paths, and hence the truss models can be further optimized for some particular cases with topology optimization programs and neural networks.
References


Appendix A

Comparison of CAST results with MATLAB output

Case 1 – Beam subjected to two equal point loads (Normal-strength concrete).

Example 1

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam dimensions</td>
<td></td>
</tr>
<tr>
<td>Clear Span</td>
<td>144 inches</td>
</tr>
<tr>
<td>Depth</td>
<td>48 inches</td>
</tr>
<tr>
<td>Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Width of Base Plate at Load</td>
<td>16 inches</td>
</tr>
<tr>
<td>Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td>Concrete strength</td>
<td>4000 psi</td>
</tr>
<tr>
<td>Yield strength of Reinforcement</td>
<td>60000 psi</td>
</tr>
<tr>
<td>Load</td>
<td>Two Equal Point Loads</td>
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</tbody>
</table>
### Table A.2 Comparison of output

<table>
<thead>
<tr>
<th>Parameters for comparison</th>
<th>CAST output</th>
<th>MATLAB output</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strut and tie member dimensions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottle shaped strut S1</td>
<td>7.223 inches</td>
<td>7.238 inches</td>
</tr>
<tr>
<td>Bottle shaped strut S2</td>
<td>9.029 inches</td>
<td>9.054 inches</td>
</tr>
<tr>
<td>Prismatic S7</td>
<td>4.024 inches</td>
<td>4.05 inches</td>
</tr>
<tr>
<td>Prismatic S6</td>
<td>6.439 inches</td>
<td>6.476 inches</td>
</tr>
<tr>
<td>Tie T3</td>
<td>10.733 inches</td>
<td>11 inches</td>
</tr>
<tr>
<td><strong>Longitudinal Reinforcement</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>2 layers</td>
<td>2 layers</td>
</tr>
<tr>
<td>Number of bars</td>
<td>12</td>
<td>12</td>
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<tr>
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<td>Number 7</td>
</tr>
<tr>
<td>Type of development</td>
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<td>90-degree hook</td>
</tr>
</tbody>
</table>
**Fig. A.1** Typical output of CAST for Case 1 - Example 1

**Example 2**

**Table A.3** Input parameters for comparison

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Beam dimensions</strong></td>
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<tr>
<td>Clear Span</td>
<td>144 inches</td>
</tr>
<tr>
<td>Depth</td>
<td>48 inches</td>
</tr>
<tr>
<td>Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Width of Base Plate at Load</td>
<td>16 inches</td>
</tr>
<tr>
<td>Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td><strong>Concrete strength</strong></td>
<td>4000 psi</td>
</tr>
<tr>
<td><strong>Yield strength of Reinforcement</strong></td>
<td>60000 psi</td>
</tr>
<tr>
<td><strong>Load</strong></td>
<td>Two Equal Point Loads 300 kips (each)</td>
</tr>
</tbody>
</table>
Table A.4 Comparison of output

<table>
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<tr>
<th>Parameters for comparison</th>
<th>CAST output</th>
<th>MATLAB output</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strut and tie member dimensions</strong></td>
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<td></td>
</tr>
<tr>
<td>Bottle shaped strut S1</td>
<td>11.91 inches</td>
<td>11.838 inches</td>
</tr>
<tr>
<td>Bottle shaped strut S2</td>
<td>15 inches</td>
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</tr>
<tr>
<td>Prismatic S7</td>
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<td>7.511 inches</td>
</tr>
<tr>
<td>Prismatic S6</td>
<td>12.241 inches</td>
<td>12.017 inches</td>
</tr>
<tr>
<td>Tie T3</td>
<td>20.7 inches</td>
<td>20.031 inches</td>
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</tbody>
</table>

**Longitudinal Reinforcement**

<table>
<thead>
<tr>
<th></th>
<th>CAST output</th>
<th>MATLAB output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of layers</td>
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<td>Number 7</td>
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<tr>
<td>Type of development</td>
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</table>
Case 2 – Beam subjected to two point loads placed symmetrically (High-strength concrete).

Example 1

Table A.5 Input parameters for comparison

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<th>Input Parameters</th>
<th>Value</th>
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<tr>
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</tr>
<tr>
<td>Clear Span</td>
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</tr>
<tr>
<td>Depth</td>
<td>48 inches</td>
</tr>
<tr>
<td>Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Width of Base Plate at Load</td>
<td>16 inches</td>
</tr>
<tr>
<td>Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td>Concrete strength</td>
<td>8000 psi</td>
</tr>
<tr>
<td>Yield strength of Reinforcement</td>
<td>60000 psi</td>
</tr>
<tr>
<td>Load</td>
<td>Two Equal Point Loads, 430 kips (each)</td>
</tr>
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</table>
Table A.6 Comparison of output

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<th>MATLAB output</th>
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<td><strong>Strut and tie member dimensions</strong></td>
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<tr>
<td>Prismatic S6</td>
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<td></td>
</tr>
<tr>
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<td>2 layers</td>
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<tr>
<td>Number of bars</td>
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<td>12</td>
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<tr>
<td>Bar denomination</td>
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<td>Number 7</td>
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</tr>
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</table>
Example 2

**Table A.7 Input parameters for comparison**

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<td>Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Width of Base Plate at Load</td>
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</tr>
<tr>
<td>Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td>Concrete strength</td>
<td>4000 psi</td>
</tr>
<tr>
<td>Yield strength of Reinforcement</td>
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<td>Load</td>
<td>Two Equal Point Loads</td>
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</table>
Table A.8 Comparison of output

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<th>MATLAB output</th>
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<td></td>
</tr>
<tr>
<td>Prismatic S7</td>
<td>7.7 inches</td>
<td>7.511 inches</td>
</tr>
<tr>
<td>Prismatic S6</td>
<td>12.241 inches</td>
<td>12.017 inches</td>
</tr>
<tr>
<td>Tie T3</td>
<td>20.7 inches</td>
<td>20.031 inches</td>
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<td><strong>Longitudinal Reinforcement</strong></td>
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<td>3 layers</td>
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<td>Number 8</td>
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</table>
Case 3 – Beam subjected to two unequal point loads placed symmetrically (Normal-strength concrete).

Example 1

**Table A.9** Input parameters for comparison

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<tr>
<td>Clear Span</td>
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</tr>
<tr>
<td>Depth</td>
<td>48 inches</td>
</tr>
<tr>
<td>Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Width of Base Plate at Load</td>
<td>16 inches</td>
</tr>
<tr>
<td>Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td>Concrete strength</td>
<td>4000 psi</td>
</tr>
<tr>
<td>Yield strength of Reinforcement</td>
<td>60000 psi</td>
</tr>
<tr>
<td>Load</td>
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<tr>
<td>Point Load (left shear span)</td>
<td>200 kips</td>
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<td>Point Load (right shear span)</td>
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### Table A.10 Comparison of output

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<td>Prismatic S6</td>
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<tr>
<td>Tie T3</td>
<td>8.5 inches</td>
<td>8.457 inches</td>
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<td>5.867 inches</td>
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<td>Bottle shaped strut S2</td>
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<td>6.47 inches</td>
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<td><strong>Longitudinal Reinforcement</strong></td>
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<tr>
<td>Number of layers</td>
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<td>2 layers</td>
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<tr>
<td>Number of bars</td>
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<td>12</td>
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<tr>
<td>Bar denomination</td>
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<td>Number 6</td>
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</table>

Example 2

Table A.11. Input parameters for comparison

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<th><strong>Value</strong></th>
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<td>Clear Span</td>
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</tr>
<tr>
<td>Depth</td>
<td>48 inches</td>
</tr>
<tr>
<td>Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Width of Base Plate at Load</td>
<td>16 inches</td>
</tr>
<tr>
<td>Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td>Concrete strength</td>
<td>4000 psi</td>
</tr>
<tr>
<td>Yield strength of Reinforcement</td>
<td>60000 psi</td>
</tr>
<tr>
<td>Load</td>
<td></td>
</tr>
<tr>
<td>Point Load (left shear span)</td>
<td>300 kips</td>
</tr>
<tr>
<td>Point Load (right shear span)</td>
<td>150 kips</td>
</tr>
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</table>
Table A.12 Comparison of output

<table>
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<th>MATLAB output</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strut and tie member dimensions</strong></td>
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<td></td>
</tr>
<tr>
<td>Prismatic S7</td>
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<td>5.41 inches</td>
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<tr>
<td>Prismatic S6</td>
<td>8.84 inches</td>
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<td>Tie T3</td>
<td>14.5 inches</td>
<td>14.43 inches</td>
</tr>
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<td>9.26 inches</td>
<td>9.22 inches</td>
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<tr>
<td>Bottle shaped strut S2</td>
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<td>11.52 inches</td>
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<tr>
<td>Bottle shaped strut S3</td>
<td>10.36 inches</td>
<td>10.30 inches</td>
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<tr>
<td>Bottle shaped strut S4</td>
<td>8.29 inches</td>
<td>8.24 inches</td>
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<tr>
<td><strong>Longitudinal Reinforcement</strong></td>
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<td></td>
</tr>
<tr>
<td>Number of layers</td>
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<td>3 layers</td>
</tr>
<tr>
<td>Number of bars</td>
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<td>Number 6</td>
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<tr>
<td>Type of development</td>
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<td>90-degree hook</td>
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</table>
Case 4 – Beam subjected to two unequal point loads placed symmetrically (High-strength concrete).

Example 1

Table A.13 Input parameters for comparison

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<th>Value</th>
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<td>Clear Span</td>
<td>144 inches</td>
</tr>
<tr>
<td>Depth</td>
<td>48 inches</td>
</tr>
<tr>
<td>Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Width of Base Plate at Load</td>
<td>16 inches</td>
</tr>
<tr>
<td>Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td>Concrete strength</td>
<td>8000 psi</td>
</tr>
<tr>
<td>Yield strength of Reinforcement</td>
<td>60000 psi</td>
</tr>
<tr>
<td>Load</td>
<td></td>
</tr>
<tr>
<td>Point Load (left shear span)</td>
<td>300 kips</td>
</tr>
<tr>
<td>Point Load (right shear span)</td>
<td>200 kips</td>
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</table>
Table A.14 Comparison of output

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<th>MATLAB output</th>
</tr>
</thead>
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<tr>
<td><strong>Strut and tie member dimensions</strong></td>
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<td></td>
</tr>
<tr>
<td>Prismatic S7</td>
<td>2.5 inches</td>
<td>2.55 inches</td>
</tr>
<tr>
<td>Prismatic S6</td>
<td>3.95 inches</td>
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</tr>
<tr>
<td>Tie T3</td>
<td>6.8 inches</td>
<td>6.81 inches</td>
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<tr>
<td><strong>Longitudinal Reinforcement</strong></td>
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<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>3 layers</td>
<td>3 layers</td>
</tr>
<tr>
<td>Number of bars</td>
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<td>Number 6</td>
</tr>
<tr>
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</table>
Fig. A.2 Typical output of CAST for Case 4 - Example 1

Example 2

Table A.15 Input parameters for comparison

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<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Beam dimensions</strong></td>
<td></td>
</tr>
<tr>
<td>Clear Span</td>
<td>144 inches</td>
</tr>
<tr>
<td>Depth</td>
<td>48 inches</td>
</tr>
<tr>
<td>Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Width of Base Plate at Load</td>
<td>16 inches</td>
</tr>
<tr>
<td>Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td><strong>Concrete strength</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4000 psi</td>
</tr>
<tr>
<td><strong>Yield strength of Reinforcement</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>60000 psi</td>
</tr>
<tr>
<td><strong>Load</strong></td>
<td></td>
</tr>
<tr>
<td>Point Load (left shear span)</td>
<td>300 kips</td>
</tr>
<tr>
<td>Point Load (right shear span)</td>
<td>150 kips</td>
</tr>
</tbody>
</table>
### Table A.16 Comparison of output

<table>
<thead>
<tr>
<th>Parameters for comparison</th>
<th>CAST output</th>
<th>MATLAB output</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strut and tie member dimensions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prismatic S7</td>
<td>3.65 inches</td>
<td>3.58 inches</td>
</tr>
<tr>
<td>Prismatic S6</td>
<td>5.77 inches</td>
<td>5.74 inches</td>
</tr>
<tr>
<td>Tie T3</td>
<td>10 inches</td>
<td>10.04 inches</td>
</tr>
<tr>
<td><strong>Longitudinal Reinforcement</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>3 layers</td>
<td>3 layers</td>
</tr>
<tr>
<td>Number of bars</td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>Bar denomination</td>
<td>Number 7</td>
<td>Number 7</td>
</tr>
<tr>
<td>Type of development</td>
<td>Not available</td>
<td>90-degree hook</td>
</tr>
</tbody>
</table>
Case 5 – Beam subjected to uniform loads (Normal-strength concrete).

Example 1

Table A.17 Input parameters for comparison

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam dimensions</td>
<td></td>
</tr>
<tr>
<td>Clear Span</td>
<td>144 inches</td>
</tr>
<tr>
<td>Depth</td>
<td>48 inches</td>
</tr>
<tr>
<td>Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Width of Base Plate at Load</td>
<td>16 inches</td>
</tr>
<tr>
<td>Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td>Concrete strength</td>
<td>4000 psi</td>
</tr>
<tr>
<td>Yield strength of Reinforcement</td>
<td>60000 psi</td>
</tr>
<tr>
<td>Load</td>
<td>Uniform load</td>
</tr>
</tbody>
</table>
### Table A.18 Comparison of output

<table>
<thead>
<tr>
<th>Parameters for comparison</th>
<th>CAST output</th>
<th>MATLAB output</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strut and tie member dimensions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottle shaped strut S1</td>
<td>7.223 inches</td>
<td>7.238 inches</td>
</tr>
<tr>
<td>Prismatic S7</td>
<td>5.1 inches</td>
<td>5.08 inches</td>
</tr>
<tr>
<td>Prismatic S6</td>
<td>8.16 inches</td>
<td>8.14 inches</td>
</tr>
<tr>
<td>Tie T3</td>
<td>13.75 inches</td>
<td>13.567 inches</td>
</tr>
<tr>
<td><strong>Longitudinal Reinforcement</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>2 layers</td>
<td>2 layers</td>
</tr>
<tr>
<td>Number of bars</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>Bar denomination</td>
<td>Number 7</td>
<td>Number 7</td>
</tr>
<tr>
<td>Type of development</td>
<td>Not available</td>
<td>90-degree hook</td>
</tr>
</tbody>
</table>
**Example 2**

**Table A.19** Input parameters for comparison

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Beam dimensions</strong></td>
<td></td>
</tr>
<tr>
<td>Clear Span</td>
<td>144 inches</td>
</tr>
<tr>
<td>Depth</td>
<td>48 inches</td>
</tr>
<tr>
<td>Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Width of Base Plate at Load</td>
<td>16 inches</td>
</tr>
<tr>
<td>Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td><strong>Concrete strength</strong></td>
<td>4000 psi</td>
</tr>
<tr>
<td><strong>Yield strength of Reinforcement</strong></td>
<td>60000 psi</td>
</tr>
<tr>
<td><strong>Load</strong></td>
<td>Uniform load</td>
</tr>
<tr>
<td></td>
<td>2.80 kips/inch</td>
</tr>
<tr>
<td>Parameters for comparison</td>
<td>CAST output</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>-------------</td>
</tr>
<tr>
<td><strong>Strut and tie member dimensions</strong></td>
<td></td>
</tr>
<tr>
<td>Bottle shaped strut S1</td>
<td>4.89 inches</td>
</tr>
<tr>
<td>Prismatic S7</td>
<td>3.05 inches</td>
</tr>
<tr>
<td>Prismatic S6</td>
<td>6.63 inches</td>
</tr>
<tr>
<td>Tie T3</td>
<td>8.1 inches</td>
</tr>
<tr>
<td><strong>Longitudinal Reinforcement</strong></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>2 layers</td>
</tr>
<tr>
<td>Number of bars</td>
<td>12</td>
</tr>
<tr>
<td>Bar denomination</td>
<td>Number 6</td>
</tr>
<tr>
<td>Type of development</td>
<td>Not available</td>
</tr>
</tbody>
</table>
Case 6 – Beam subjected to uniform loads (High-strength concrete).

Example 1

Table A.21 Input parameters for comparison

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam dimensions</td>
<td></td>
</tr>
<tr>
<td>Clear Span</td>
<td>144 inches</td>
</tr>
<tr>
<td>Depth</td>
<td>48 inches</td>
</tr>
<tr>
<td>Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Width of Base Plate at Load</td>
<td>16 inches</td>
</tr>
<tr>
<td>Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td>Concrete strength</td>
<td>8000 psi</td>
</tr>
<tr>
<td>Yield strength of Reinforcement</td>
<td>60000 psi</td>
</tr>
<tr>
<td>Load</td>
<td>Uniform load</td>
</tr>
<tr>
<td></td>
<td>4.15 kips/inch</td>
</tr>
<tr>
<td>Parameters for comparison</td>
<td>CAST output</td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>-------------</td>
</tr>
<tr>
<td><strong>Strut and tie member dimensions</strong></td>
<td></td>
</tr>
<tr>
<td>Prismatic S7</td>
<td>3.05 inches</td>
</tr>
<tr>
<td>Prismatic S6</td>
<td>6.63 inches</td>
</tr>
<tr>
<td>Tie T3</td>
<td>8.1 inches</td>
</tr>
<tr>
<td><strong>Longitudinal Reinforcement</strong></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>2 layers</td>
</tr>
<tr>
<td>Number of bars</td>
<td>12</td>
</tr>
<tr>
<td>Bar denomination</td>
<td>Number 7</td>
</tr>
<tr>
<td>Type of development</td>
<td>Not available</td>
</tr>
</tbody>
</table>
Example 2

Table A.23 Input parameters for comparison

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam dimensions</td>
<td></td>
</tr>
<tr>
<td>Clear Span</td>
<td>144 inches</td>
</tr>
<tr>
<td>Depth</td>
<td>48 inches</td>
</tr>
<tr>
<td>Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Width of Base Plate at Load</td>
<td>16 inches</td>
</tr>
<tr>
<td>Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td>Concrete strength</td>
<td>8000 psi</td>
</tr>
<tr>
<td>Yield strength of Reinforcement</td>
<td>60000 psi</td>
</tr>
<tr>
<td>Load</td>
<td>Uniform load</td>
</tr>
</tbody>
</table>
Table A.24 Comparison of output

<table>
<thead>
<tr>
<th>Parameters for comparison</th>
<th>CAST output</th>
<th>MATLAB output</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strut and tie member dimensions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prismatic S7</td>
<td>3.55 inches</td>
<td>3.35 inches</td>
</tr>
<tr>
<td>Prismatic S6</td>
<td>5.51 inches</td>
<td>5.51 inches</td>
</tr>
<tr>
<td>Tie T3</td>
<td>9.22 inches</td>
<td>9.19 inches</td>
</tr>
<tr>
<td><strong>Longitudinal Reinforcement</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>3 layers</td>
<td>3 layers</td>
</tr>
<tr>
<td>Number of bars</td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>Bar denomination</td>
<td>Number 7</td>
<td>Number 7</td>
</tr>
<tr>
<td>Type of development</td>
<td>Not available</td>
<td>90-degree hook</td>
</tr>
</tbody>
</table>
Case 7 – Beam subjected to two equal/unequal loads placed at symmetrical/unsymmetrical locations (Normal-strength concrete).

Example 1

**Table A.25** Input parameters for comparison

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Beam dimensions</strong></td>
<td></td>
</tr>
<tr>
<td>Clear Span</td>
<td>144 inches</td>
</tr>
<tr>
<td>Depth</td>
<td>48 inches</td>
</tr>
<tr>
<td>Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Width of Base Plate at Load</td>
<td>16 inches</td>
</tr>
<tr>
<td>Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td><strong>Concrete strength</strong></td>
<td>4000 psi</td>
</tr>
<tr>
<td><strong>Yield strength of Reinforcement</strong></td>
<td>60000 psi</td>
</tr>
<tr>
<td><strong>Load</strong></td>
<td></td>
</tr>
<tr>
<td>Load 1</td>
<td>300 kips</td>
</tr>
<tr>
<td>Load 2</td>
<td>200 kips</td>
</tr>
<tr>
<td><strong>Load Location</strong></td>
<td></td>
</tr>
<tr>
<td>Center of support to Load 1 (twice x1, refer to Fig. 6.2)</td>
<td>60 inches</td>
</tr>
<tr>
<td>Center of support to Load 2 (twice x2, refer to Fig. 6.2)</td>
<td>60 inches</td>
</tr>
<tr>
<td>Parameters for comparison</td>
<td>CAST output</td>
</tr>
<tr>
<td>---------------------------------</td>
<td>-------------</td>
</tr>
<tr>
<td><strong>Strut and tie member dimensions</strong></td>
<td></td>
</tr>
<tr>
<td>Bottle shaped strut S1</td>
<td>10.01 inches</td>
</tr>
<tr>
<td>Bottle shaped strut S2</td>
<td>13.00 inches</td>
</tr>
<tr>
<td>Bottle shaped strut S3</td>
<td>12.34 inches</td>
</tr>
<tr>
<td>Bottle shaped strut S4</td>
<td>9.88 inches</td>
</tr>
<tr>
<td>Prismatic S6</td>
<td>10.64 inches</td>
</tr>
<tr>
<td>Prismatic S7</td>
<td>6.64 inches</td>
</tr>
<tr>
<td>Tie T3</td>
<td>17.7 inches</td>
</tr>
<tr>
<td><strong>Longitudinal Reinforcement</strong></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>3 layers</td>
</tr>
<tr>
<td>Number of bars</td>
<td>15</td>
</tr>
<tr>
<td>Bar denomination</td>
<td>Number 8</td>
</tr>
<tr>
<td>Type of development</td>
<td>Not available</td>
</tr>
</tbody>
</table>
### Example 2

**Table A.27 Input parameters for comparison**

<table>
<thead>
<tr>
<th><strong>Input Parameters</strong></th>
<th><strong>Value</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam dimensions Clear Span</td>
<td>144 inches</td>
</tr>
<tr>
<td>Beam dimensions Depth</td>
<td>50 inches</td>
</tr>
<tr>
<td>Beam dimensions Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Beam dimensions Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Beam dimensions Width of Base Plate at Load</td>
<td>16 inches</td>
</tr>
<tr>
<td>Beam dimensions Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td>Concrete strength</td>
<td>4000 psi</td>
</tr>
<tr>
<td>Yield strength of Reinforcement</td>
<td>60000 psi</td>
</tr>
<tr>
<td>Load Load 1</td>
<td>250 kips</td>
</tr>
<tr>
<td>Load Load 2</td>
<td>200 kips</td>
</tr>
<tr>
<td>Load Location Center of support to Load 1</td>
<td>60 inches</td>
</tr>
<tr>
<td>Load Location (twice x1, refer to Fig. 6.2)</td>
<td></td>
</tr>
<tr>
<td>Load Location Center of support to Load 2</td>
<td>60 inches</td>
</tr>
<tr>
<td>Load Location (twice x2, refer to Fig. 6.2)</td>
<td></td>
</tr>
<tr>
<td>Parameters for comparison</td>
<td>CAST output</td>
</tr>
<tr>
<td>---------------------------</td>
<td>-------------</td>
</tr>
<tr>
<td><strong>Strut and tie member dimensions</strong></td>
<td></td>
</tr>
<tr>
<td>Bottle shaped strut S1</td>
<td>8.59 inches</td>
</tr>
<tr>
<td>Bottle shaped strut S2</td>
<td>10.74 inches</td>
</tr>
<tr>
<td>Bottle shaped strut S3</td>
<td>10.38 inches</td>
</tr>
<tr>
<td>Bottle shaped strut S4</td>
<td>8.31 inches</td>
</tr>
<tr>
<td>Prismatic S6</td>
<td>8.04 inches</td>
</tr>
<tr>
<td>Prismatic S7</td>
<td>5.05 inches</td>
</tr>
<tr>
<td>Tie T3</td>
<td>13.39 inches</td>
</tr>
<tr>
<td><strong>Longitudinal Reinforcement</strong></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>2 layers</td>
</tr>
<tr>
<td>Number of bars</td>
<td>14</td>
</tr>
<tr>
<td>Bar denomination</td>
<td>Number 8</td>
</tr>
<tr>
<td>Type of development</td>
<td>Not available</td>
</tr>
</tbody>
</table>
Case 8 – Beam subjected to two equal/unequal loads placed at symmetrical/unsymmetrical locations (High-strength concrete).

Example 1

Table A.29 Input parameters for comparison

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Beam dimensions</strong></td>
<td></td>
</tr>
<tr>
<td>Clear Span</td>
<td>144 inches</td>
</tr>
<tr>
<td>Depth</td>
<td>48 inches</td>
</tr>
<tr>
<td>Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Width of Base Plate at Load</td>
<td>16 inches</td>
</tr>
<tr>
<td>Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td><strong>Concrete strength</strong></td>
<td>8000 psi</td>
</tr>
<tr>
<td><strong>Yield strength of Reinforcement</strong></td>
<td>60000 psi</td>
</tr>
<tr>
<td><strong>Load</strong></td>
<td></td>
</tr>
<tr>
<td>Load 1</td>
<td>250 kips</td>
</tr>
<tr>
<td>Load 2</td>
<td>200 kips</td>
</tr>
<tr>
<td><strong>Load Location</strong></td>
<td></td>
</tr>
<tr>
<td>Center of support to Load 1 (twice x1, refer to Fig. 6.2)</td>
<td>60 inches</td>
</tr>
<tr>
<td>Center of support to Load 2 (twice x2, refer to Fig. 6.2)</td>
<td>60 inches</td>
</tr>
</tbody>
</table>
**Table A.30** Comparison of output

<table>
<thead>
<tr>
<th>Parameters for comparison</th>
<th>CAST output</th>
<th>MATLAB output</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strut and tie member dimensions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prismatic S7</td>
<td>2.16 inches</td>
<td>2.15 inches</td>
</tr>
<tr>
<td>Prismatic S6</td>
<td>3.47 inches</td>
<td>3.44 inches</td>
</tr>
<tr>
<td>Tie T3</td>
<td>5.8 inches</td>
<td>5.73 inches</td>
</tr>
<tr>
<td><strong>Longitudinal Reinforcement</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>2 layers</td>
<td>2 layers</td>
</tr>
<tr>
<td>Number of bars</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>Bar denomination</td>
<td>Number 6</td>
<td>Number 6</td>
</tr>
<tr>
<td>Type of development</td>
<td>Not available</td>
<td>90-degree hook</td>
</tr>
</tbody>
</table>
**Example 2**

**Table A.31 Input parameters for comparison**

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam dimensions</td>
<td></td>
</tr>
<tr>
<td>Clear Span</td>
<td>144 inches</td>
</tr>
<tr>
<td>Depth</td>
<td>48 inches</td>
</tr>
<tr>
<td>Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Width of Base Plate at Load</td>
<td>16 inches</td>
</tr>
<tr>
<td>Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td>Concrete strength</td>
<td>8000 psi</td>
</tr>
<tr>
<td>Yield strength of Reinforcement</td>
<td>60000 psi</td>
</tr>
<tr>
<td>Load</td>
<td></td>
</tr>
<tr>
<td>Load 1</td>
<td>250 kips</td>
</tr>
<tr>
<td>Load 2</td>
<td>300 kips</td>
</tr>
<tr>
<td>Load Location</td>
<td></td>
</tr>
<tr>
<td>Center of support to Load 1 (twice x1, refer to Fig. 6.2)</td>
<td>48 inches</td>
</tr>
<tr>
<td>Center of support to Load 2 (twice x2, refer to Fig. 6.2)</td>
<td>60 inches</td>
</tr>
</tbody>
</table>
Table A.32 Comparison of output

<table>
<thead>
<tr>
<th>Parameters for comparison</th>
<th>CAST output</th>
<th>MATLAB output</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strut and tie member dimensions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prismatic S7</td>
<td>2.84 inches</td>
<td>2.83 inches</td>
</tr>
<tr>
<td>Prismatic S6</td>
<td>4.58 inches</td>
<td>4.56 inches</td>
</tr>
<tr>
<td>Tie T3</td>
<td>7.6 inches</td>
<td>7.54 inches</td>
</tr>
<tr>
<td><strong>Longitudinal Reinforcement</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>3 layers</td>
<td>3 layers</td>
</tr>
<tr>
<td>Number of bars</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>Bar denomination</td>
<td>Number 6</td>
<td>Number 6</td>
</tr>
<tr>
<td>Type of development</td>
<td>Not available</td>
<td>90-degree hook</td>
</tr>
</tbody>
</table>
Case 9 – Beam subjected to single point load (Normal-strength concrete).

Example 1

Table A.33 Input parameters for comparison

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam dimensions</td>
<td></td>
</tr>
<tr>
<td>Clear Span</td>
<td>100 inches</td>
</tr>
<tr>
<td>Depth</td>
<td>50 inches</td>
</tr>
<tr>
<td>Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Width of Base Plate at Load</td>
<td>16 inches</td>
</tr>
<tr>
<td>Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td>Concrete strength</td>
<td>4000 psi</td>
</tr>
<tr>
<td>Yield strength of Reinforcement</td>
<td>60000 psi</td>
</tr>
<tr>
<td>Load</td>
<td></td>
</tr>
<tr>
<td>Load 1</td>
<td>300 kips</td>
</tr>
<tr>
<td>Load Location</td>
<td></td>
</tr>
<tr>
<td>Center of support to Load (Span 1, refer to Fig. 7.8)</td>
<td>48 inches</td>
</tr>
<tr>
<td>Center of support to Load (Span 2 ,refer to Fig. 7.8)</td>
<td>68 inches</td>
</tr>
</tbody>
</table>
Table A.34 Comparison of output

<table>
<thead>
<tr>
<th>Parameters for comparison</th>
<th>CAST output</th>
<th>MATLAB output</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strut and tie member dimensions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottle shaped strut S1</td>
<td>5.82 inches</td>
<td>5.83 inches</td>
</tr>
<tr>
<td>Bottle shaped strut S2</td>
<td>7.28 inches</td>
<td>7.29 inches</td>
</tr>
<tr>
<td>Bottle shaped strut S3</td>
<td>5.71 inches</td>
<td>5.71 inches</td>
</tr>
<tr>
<td>Bottle shaped strut S4</td>
<td>4.57 inches</td>
<td>4.57 inches</td>
</tr>
<tr>
<td>Prismatic S5</td>
<td>2.63 inches</td>
<td>2.63 inches</td>
</tr>
<tr>
<td>Tie T3</td>
<td>7.05 inches</td>
<td>7.02 inches</td>
</tr>
<tr>
<td><strong>Longitudinal Reinforcement</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>2 layers</td>
<td>2 layers</td>
</tr>
<tr>
<td>Number of bars</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Bar denomination</td>
<td>Number 6</td>
<td>Number 6</td>
</tr>
<tr>
<td>Type of development</td>
<td>Not available</td>
<td>90-degree hook</td>
</tr>
</tbody>
</table>
**Example 2**

**Table A.35 Input parameters for comparison**

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Beam dimensions</strong></td>
<td></td>
</tr>
<tr>
<td>Clear Span</td>
<td>100 inches</td>
</tr>
<tr>
<td>Depth</td>
<td>50 inches</td>
</tr>
<tr>
<td>Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Width of Base Plate at Load</td>
<td>16 inches</td>
</tr>
<tr>
<td>Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td><strong>Concrete strength</strong></td>
<td>4000 psi</td>
</tr>
<tr>
<td><strong>Yield strength of Reinforcement</strong></td>
<td>60000 psi</td>
</tr>
<tr>
<td><strong>Load</strong></td>
<td></td>
</tr>
<tr>
<td>Load 1</td>
<td>380 kips</td>
</tr>
<tr>
<td><strong>Load Location</strong></td>
<td></td>
</tr>
<tr>
<td>Center of support to Load</td>
<td>48 inches</td>
</tr>
<tr>
<td>(Span 1, refer to Fig. 7.8)</td>
<td></td>
</tr>
<tr>
<td>Center of support to Load</td>
<td>68 inches</td>
</tr>
<tr>
<td>(Span 2, refer to Fig. 7.8)</td>
<td></td>
</tr>
</tbody>
</table>
Table A.36 Comparison of output

<table>
<thead>
<tr>
<th>Parameters for comparison</th>
<th>CAST output</th>
<th>MATLAB output</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strut and tie member dimensions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottle shaped strut S1</td>
<td>7.45 inches</td>
<td>7.44 inches</td>
</tr>
<tr>
<td>Bottle shaped strut S2</td>
<td>9.31 inches</td>
<td>9.30 inches</td>
</tr>
<tr>
<td>Bottle shaped strut S3</td>
<td>7.34 inches</td>
<td>7.33 inches</td>
</tr>
<tr>
<td>Bottle shaped strut S4</td>
<td>5.87 inches</td>
<td>5.86 inches</td>
</tr>
<tr>
<td>Prismatic S5</td>
<td>3.5 inches</td>
<td>3.45 inches</td>
</tr>
<tr>
<td>Tie T3</td>
<td>9.4 inches</td>
<td>9.20 inches</td>
</tr>
<tr>
<td><strong>Longitudinal Reinforcement</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>2 layers</td>
<td>2 layers</td>
</tr>
<tr>
<td>Number of bars</td>
<td>10</td>
<td>14</td>
</tr>
<tr>
<td>Bar denomination</td>
<td>Number 7</td>
<td>Number 6</td>
</tr>
<tr>
<td>Type of development</td>
<td>Not available</td>
<td>90-degree hook</td>
</tr>
</tbody>
</table>
Case 10 – Beam subjected to single point load (High-strength concrete).

Example 1

Table A.37 Input parameters for comparison

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam dimensions Clear Span</td>
<td>100 inches</td>
</tr>
<tr>
<td>Beam dimensions Depth</td>
<td>50 inches</td>
</tr>
<tr>
<td>Beam dimensions Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Beam dimensions Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Beam dimensions Width of Base Plate at Load</td>
<td>16 inches</td>
</tr>
<tr>
<td>Beam dimensions Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td>Concrete strength</td>
<td>8000 psi</td>
</tr>
<tr>
<td>Yield strength of Reinforcement</td>
<td>60000 psi</td>
</tr>
<tr>
<td>Load Load 1</td>
<td>400 kips</td>
</tr>
<tr>
<td>Load Location Center of support to Load (Span 1, refer to Fig. 7.8)</td>
<td>48 inches</td>
</tr>
<tr>
<td>Load Location Center of support to Load (Span 2, refer to Fig. 7.8)</td>
<td>68 inches</td>
</tr>
<tr>
<td>Parameters for comparison</td>
<td>CAST output</td>
</tr>
<tr>
<td>---------------------------</td>
<td>-------------</td>
</tr>
<tr>
<td><strong>Strut and tie member dimensions</strong></td>
<td></td>
</tr>
<tr>
<td>Prismatic S5</td>
<td>1.7 inches</td>
</tr>
<tr>
<td>Tie T3</td>
<td>4.5 inches</td>
</tr>
<tr>
<td><strong>Longitudinal Reinforcement</strong></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>2 layers</td>
</tr>
<tr>
<td>Number of bars</td>
<td>10</td>
</tr>
<tr>
<td>Bar denomination</td>
<td>Number 7</td>
</tr>
<tr>
<td>Type of development</td>
<td>Not available</td>
</tr>
</tbody>
</table>
### Example 2

**Table A.39 Input parameters for comparison**

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Beam dimensions</strong></td>
<td></td>
</tr>
<tr>
<td>Clear Span</td>
<td>100 inches</td>
</tr>
<tr>
<td>Depth</td>
<td>50 inches</td>
</tr>
<tr>
<td>Width</td>
<td>18 inches</td>
</tr>
<tr>
<td>Support Width</td>
<td>16 inches</td>
</tr>
<tr>
<td>Width of Base Plate at Load</td>
<td>16 inches</td>
</tr>
<tr>
<td>Cover</td>
<td>1.5 inches</td>
</tr>
<tr>
<td><strong>Concrete strength</strong></td>
<td>8000 psi</td>
</tr>
<tr>
<td><strong>Yield strength of Reinforcement</strong></td>
<td>60000 psi</td>
</tr>
<tr>
<td><strong>Load</strong></td>
<td></td>
</tr>
<tr>
<td>Load 1</td>
<td>380 kips</td>
</tr>
<tr>
<td><strong>Load Location</strong></td>
<td></td>
</tr>
<tr>
<td>Center of support to Load (Span 1, refer to Fig. 7.8)</td>
<td>48 inches</td>
</tr>
<tr>
<td>Center of support to Load (Span 2, refer to Fig. 7.8)</td>
<td>68 inches</td>
</tr>
<tr>
<td>Parameters for comparison</td>
<td>CAST output</td>
</tr>
<tr>
<td>---------------------------</td>
<td>-------------</td>
</tr>
<tr>
<td><strong>Strut and tie member dimensions</strong></td>
<td></td>
</tr>
<tr>
<td>Prismatic S5</td>
<td>2.40 inches</td>
</tr>
<tr>
<td>Tie T3</td>
<td>6.40 inches</td>
</tr>
<tr>
<td><strong>Longitudinal Reinforcement</strong></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>3 layers</td>
</tr>
<tr>
<td>Number of bars</td>
<td>18</td>
</tr>
<tr>
<td>Bar denomination</td>
<td>Number 6</td>
</tr>
<tr>
<td>Type of development</td>
<td>Not available</td>
</tr>
</tbody>
</table>
Appendix B

B.1 User manual for the MATLAB program

1. Creating the input file

The input file for all the MATLAB programs is a text file. Each program has a unique file with different set of parameters. The user must refer to the part of the program that reads the data from the input file prior to creating it. The order of the data included in the input file must be similar to the order in which MATLAB reads from the text file. Sample input files for each case discussed previously are included herein.

2. Reading the output file

The output from the MATLAB programs is stored in the output file. Each program has a unique output file with different set of parameters. Separate output files are generated that include details of longitudinal reinforcement and the shear reinforcement.

3. Actions required from the user

The user is required to create an input text file as detailed earlier. Each case includes a main MATLAB files that generates the STM data, detail longitudinal reinforcement, and detail the shear and bottle shaped strut reinforcement. The MATLAB files are to be run in the following order

1. Main MATLAB program
2. MATLAB program to detail the longitudinal reinforcement
3. MATLAB program to detail the shear and bottle shaped strut reinforcement.
B.2 Details of input files

1. Input text file for Case 1 and Case 2

The data for a beam configuration is mentioned in a single line and is separated by spaces. The beam dimensions and material properties are entered in the input file in the same sequence as mentioned in the MATLAB program. If multiple beam configurations are to be entered, they should be typed on different lines. Figure B.1 is a screen shot of a typical input file for a single beam configuration. The numbers from the input file are in the following sequence.

1. The number ‘4’ represents the concrete strength in ksi.
2. The number ‘48’ represents the total depth of the beam in inches.
3. The number ‘144’ represents the clear span of the beam in inches.
4. The number ‘16’ represents the beam width in inches.
5. The number ’16’ represents the support width in inches.
6. The number ‘16’ represents the width of base plate or width of the column that transfer the load to the beam.
7. The number ‘1.5’ is the clear cover to the longitudinal reinforcement in inches.

![Input file](image)

**Fig. B.1** Input file for main MATLAB program (single beam configuration- Case 1 and Case 2)
The screen shot from Fig. B.2 is an input file where more than one beam configurations are to be analyzed with the program. A variable ‘n’ is declared in initial few lines of every MATLAB program (main file). The value of ‘n’, that represents the number of cases included in the input text file, should be same as the number of cases included in the input file. The user has to modify the value of ‘n’ depending on the number of cases to be analyzed. For example, the value of ‘n’ for the sample input file shown in Fig. B.2 is 3. Similar multiple beam configurations can be checked for all the other cases.

![Input file for main MATLAB program (multiple beam dimensions - Case 1 and Case 2)](image)

**Fig. B.2** Input file for main MATLAB program (multiple beam dimensions - Case 1 and Case 2)
2. Input text file for Case 3 and Case 4

![Input file for main MATLAB program](image)

**Fig. B.3** Input file for main MATLAB program (Case 3 and Case 4)

1. The number ‘4’ represents the concrete strength in ksi.
2. The number ‘144’ represents the clear span of the beam in inches.
3. The number ‘16’ represents the beam width in inches.
4. The number ‘48’ total depth of the beam in inches.
5. The number ’16’ represents the support width in inches.
6. The number ‘16’ represents the width of base plate or width of the column that transfer the load to the beam (units - inches).
7. The number ‘1.5’ is the clear cover to the longitudinal reinforcement in inches.
8. The number ‘100’ represents the minimum value of Load 1 (refer to Fig. 4.2) on the beam in kips.
9. The number ‘150’ represents the minimum value of Load 2 (refer to Fig. 4.2) on the beam in kips.
10. The number ‘10’ represents the increment in the loads for successive iterations in kips.
3. **Input text file for Case 5 and Case 6**

![Input file for main MATLAB program (Case 5 and Case 6)](image)

**Fig. B.4** Input file for main MATLAB program (Case 5 and Case 6)

1. The number ‘4’ represents the concrete strength in ksi.
2. The number ‘48’ represents the total depth of the beam in inches.
3. The number ‘144’ represents the clear span of the beam in inches.
4. The number ‘16’ represents the beam width in inches.
5. The number ’16’ represents the support width in inches.
6. The number ‘1.5’ is the clear cover to the longitudinal reinforcement in inches.

The minimum value for the uniform load is set to 0.05 kips/inch for each beam configuration. The maximum value is controlled by the shear capacity of the model.
4. **Input text file for Case 7 and Case 8**

![Input file for main MATLAB program](image)

**Fig. B.5** Input file for main MATLAB program (Case 7 and Case 8)

1. The number ‘4’ represents the concrete strength in ksi.
2. The number ‘144’ represents the clear span of the beam in inches.
3. The number ‘16’ represents the beam width in inches.
4. The number ‘16’ represents the support width in inches.
5. The number ‘16’ represents the width of base plate or width of the column that transfer the load to the beam.
6. The number ‘1.5’ is the clear cover to the longitudinal reinforcement in inches.
7. The number ‘3’ represents the span to depth ratio.
8. The number ‘100’ represents the minimum value of Load1 (refer to Fig. 6.2) on the beam in kips.
9. The number ‘150’ represents the minimum value of Load2 (refer to Fig. 6.2) on the beam in kips.
10. The number ‘4’ represents in inches, the distance between the original load location and the new load location after a successful iteration.
5. **Input text file for Case 9 and Case 10**

![Input file for main MATLAB program (Case 7 and Case 8)](image)

**Fig. B.6** Input file for main MATLAB program (Case 7 and Case 8)

1. The number ‘4’ represents the concrete strength in ksi.
2. The number ‘40’ represents the total depth of the beam in inches.
3. The number ‘100’ represents the clear span of the beam in inches.
4. The number ‘40’ represents the location of the load from the left support in inches.
5. The number ‘16’ represents the beam width in inches.
6. The number ‘16’ represents the support width in inches.
7. The number ‘16’ represents the width of base plate or width of the column that transfer the load to the beam.
8. The number ‘1.5’ is the clear cover to the longitudinal reinforcement in inches.

The minimum point load on the beam is set to 50 kips for each beam configuration.

**B.3 Output file MATLAB programs with details the longitudinal reinforcement**

Figure B.6 is a screen shot of the output file of the program with details of longitudinal reinforcement. The output for longitudinal reinforcement is the same for all the cases considered in this research. Each row from the output file is the detailing data for a particular set of point load on the beam.
Fig. B.7 Typical output file for the MATLAB program that details the longitudinal reinforcement

The values from the output data file are in the following sequence (numbers from a single row).

1. The number ‘6’ represents the denomination of the bar (#6 bar).

2. The number ‘11’ represents the number of layers as well as the type of bar development at support. If the entry in the second column is a single number (1, 2, or 3), it means the bars are developed straight and placed in a number as layers mentioned by the second digit. If the entry in the second column has two digits (11, 21, or 31), it means that the bars are developed as 90-degree hooks at support in a number of layers given by the first digit.

3. The number ‘4’ represents the total number of bars required to resist the tensile force in the bottom longitudinal tie.

4. The number ‘0.63’ represents the distance of C.G. of the longitudinal tie from the bottom of the beam minus the cover. To calculate the location of CG from the beam soffit, cover has to be added to the value from column 4.

5. The number ‘16’ represents the beam width (inches).
6. The number ‘50’ represents the load acting on the beam for which the reinforcement has been detailed.

B.4 Output file MATLAB program with details the horizontal reinforcement for the bottle shaped strut

Figure B.7 is a screen shot of the output file of the program with details of horizontal reinforcement for a bottle shaped strut (normal-strength and high-strength concrete). Each row from the output file is the detailing data for a particular set of point load on the beam.

![Output file screenshot](image)

**Fig. B.8** Typical output file for the MATLAB program that details the horizontal reinforcement for the bottle shaped strut

The values from the output data file are in the following sequence (numbers from a single row).

1. The number ‘50’ represents the load acting on the beam for which the reinforcement has been detailed.
2. The number ‘3’ represents the bar denomination (#3 bar).
3. The number ‘2’ represents the total number of legs of reinforcement required.
4. The number ‘10’ represents the spacing of the horizontal reinforcement (inches).
B.5 Output file MATLAB program with details the horizontal reinforcement for the bottle shaped strut

Figure B.8 is a screen shot of the output file of the program with details of vertical reinforcement for a bottle shaped strut and the shear reinforcement (normal-strength and high-strength concrete) for a shear span. Each row from the output file is the detailing data for a particular set of point load on the beam.

![Image](vertical-shear-Notepad.png)

**Fig. B.9** Typical output file for the MATLAB program that details the vertical reinforcement and the shear reinforcement for a shear span

The values from the output text file are in the following sequence (numbers from a single row).

1. The number ‘50’ represents the load acting on the beam for which the reinforcement has been detailed.
2. The number ‘3’ represents the bar denomination (#3 bar).
3. The number ‘2’ represents the total number of legs of reinforcement required.
4. The number ‘4’ represents the spacing of the horizontal reinforcement (inches).
Appendix C

MATLAB Programs

%MATLAB program for Case 1 – Chapter 2. Main program that creates a STM.

clear all
clc

%’q’ is a counter that keeps track of the number of successful iterations
q=0;

%the input data for the program is stored in a text file.
fid=fopen('input.txt');

%the value of ‘n’ indicates the number of beam configurations entered in the input file. Refer to %user
guidelines for details of ‘n’.
n=1;

for i=1:n

    %MATLAB reads the data from the input file
    z=textscan(fid,'%f %f %f %f %f %f %f',1);

    %’fc’ is the concrete strength (units - ksi).
    fc=z{1};

    %’d’ is the total depth of the beam (units - inches).
    d=z{2};

    %’l’ is the clear span of the beam (units - inches).
    l=z{3};

    %’b’ is the width of the beam (units - inches).
    b=z{4};

    %’sw’ is the support width (units – inches).
    sw=z{5};
'sb' is the width of the base plate or column that transfers the load to the deep beam (units – inches).

\[ sb = z_6 \]

% clear cover provided to the longitudinal reinforcement (units – inches)

\[ cover = z_7 \]

% load for the first iteration

\[ load = 40 \]

% trial ultimate shear strength of the cross section.

\[ shear = 0.75 \times 10 \times ((fc \times 1000)^{0.5}) \times b \times 0.99 \times d / 1000 \]

while load < shear

\[ load = load + 10 \]

% y controls the truss height. ‘y’ is set to 0.99 for first iteration.

\[ y = 0.99 \]

\[ m = 0 \]

while m != 1

% height of the truss for first iteration

\[ d_1 = y \times d \]

% x1 is the distance between nodes N1 and N2 (refer Fig. 2.2)

\[ x_1 = 0.5 \times d + 0.25 \times sw \]

% x2 is the horizontal distance between nodes N2 and N6 (refer Fig. 2.2)

\[ x_2 = x_1 \]

% theta is the inclination of the bottle shaped strut (S1) with respect to horizontal

\[ \theta_1 = \text{atan}(d_1 / x_1) \times (180 \times 7 / 22) \]

if (\( \theta_1 < 25 \)) \( || \) (\( \theta_1 > 65 \))

break

end
%f3 represents bottle shaped strut S1 from Fig. 2.2.

\[ f_3 = \text{load} / \sin(\theta_1 \times 22 / (7 \times 180)) \]

%f6 represents tie T3 from Fig. 2.2.

\[ f_6 = f_3 \times \cos(\theta_1 \times 22 / (7 \times 180)) \]

%f1 represents prismatic strut S7 from Fig. 2.2.

\[ f_1 = f_3 \times \cos(\theta_1 \times 22 / (7 \times 180)) \]

%f5 represents tie T1 from Fig. 2.2.

\[ f_5 = f_3 \times \sin(\theta_1 \times 22 / (7 \times 180)) \]

%theta2 is the inclination of the bottle shaped strut S2 with respect to horizontal

\[ \theta_2 = \theta_1 \]

%f4 represents bottle shaped strut S2 from Fig. 2.2.

\[ f_4 = f_5 / \sin(\theta_2 \times 22 / (7 \times 180)) \]

%f7 represents the tie T4 from Fig. 2.2.

\[ f_7 = f_4 \times \cos(\theta_2 \times 22 / (7 \times 180)) + f_6 \]

%f2 represents prismatic strut S6 from Fig. 2.2.

\[ f_2 = f_1 + f_4 \times \cos(\theta_1 \times 22 / (7 \times 180)) \]

%calculation for nodal capacities

%node N1: CCT node, beta = 0.8

%reference ACI 318(2008), Section A.5.2 - page 393

\[ f_{cu1} = 0.75 \times 0.85 \times 0.8 \times f_c \]

%node N5: CCT node, beta = 0.8

\[ f_{cu2} = 0.75 \times 0.85 \times 0.8 \times f_c \]

%node N2: CTT node, beta = 0.6

\[ f_{cu3} = 0.75 \times 0.85 \times 0.6 \times f_c \]

%node N6: CCC node, beta = 1.0

\[ f_{cu4} = 0.75 \times 0.85 \times 1.0 \times f_c \]
calculation for strut capacities

strut S1 is a bottle shaped strut, beta = 0.75
reference ACI 318(2008), Section A.3.2 - page 388

fc3 = 0.75*0.85*0.75*fc;

strut S7 is a prismatic strut, beta = 1.0

fc1 = 0.75*0.85*1*fc;

strut S6 is a rectangular strut, beta = 1.0

fc2 = 0.75*0.85*1*fc;

strut S2 is a bottle shaped strut, beta = 0.75

fc4 = 0.75*0.85*0.75*fc;

'p' keeps a track of the anchoring of the bottle shaped struts at support and at load locations

p = 0;

width required for tie#6

the node capacity at node N2 governs the width required for this tie

w6reqd = f7/(fcu3*b);

calculation of the width available for strut#3 at node 1

w3avail = sw*sin(theta1*22/(7*180))+w6reqd*cos(theta1*22/(7*180));

calculation for the requirement of width for strut S1 from strength

considerations. The strut width is governed by the bottle shaped strut

beta factor.

w3reqd = f3/(fc3*b);

check width required to the width available to anchor strut S1 at support.

if w3avail > w3reqd
  p = p + 1;
end

Width required for strut S6 is governed by the capacity of node N6.
\[
\text{w}_2\text{reqd} = \frac{f_2}{(b \cdot \text{fcu}_4)};
\]

% calculation for the requirement of width for strut S7 from strength

% considerations
\[
\text{w}_1\text{reqd} = \frac{f_1}{(b \cdot \text{fcu}_1)};
\]

% calculation for tie width for tie T4
\[
\text{w}_7\text{reqd} = \frac{f_7}{(b \cdot \text{fcu}_3)};
\]

% calculations for available width to anchor strut width at node N6 for strut S2
\[
\text{w}_4\text{avail} = s_b \cdot \sin(\theta_2 \cdot \frac{22}{(7 \cdot 180)}) + \text{w}_2\text{reqd} \cdot \cos(\theta_2 \cdot \frac{22}{(7 \cdot 180)});
\]

% calculation for the actual width required for strut S2. The capacity of node N2 governs this width
\[
\text{w}_4\text{reqd} = \frac{f_4}{(b \cdot \text{fcu}_3)};
\]

% check width required to the width available to anchor strut S2 at load.

if \( \text{w}_4\text{avail} > \text{w}_4\text{reqd} \)
\[
\text{p} = \text{p} + 1;
\]

end

% ultimate shear capacity of the cross section. Refer to Section 2.5.1 for details.
\[
\text{shear} = \left(0.75 \cdot 10 \cdot (\text{fc} \cdot 1000)^{(0.5)} \cdot b \cdot (d_1 + 0.5 \cdot \text{w}_2\text{reqd})\right) / 1000;
\]

% check whether the calculated dimensions of the truss fit in the physical dimensions of the beam
\[
\text{dmodel} = d_1 + \text{w}_7\text{reqd} \cdot 0.5 + \text{w}_2\text{reqd} \cdot 0.5;
\]

% if the dmodel is greater than depth of beam, the value of height of
% truss is reduced for next iteration. If the bottle shaped struts cannot be anchored
% at the nodes, the truss height is to reduce for further iterations. If shear % capacity is less than the actual shear force, beam dimensions have to be % modified.

if \( \text{dmodel} \leq (d - \text{cover}) \) \&\& \( \text{p} = 2 \) \&\& \( \text{shear} \geq \text{load} \)

% MATLAB prints the output in a text files
% data required for longitudinal reinforcement
fid1=fopen('reininput.txt','a');
fprintf(fid1,'%.2f %.2f %.2f %.2f %.2f %.2f %d %.2f\n',fc,f7,theta1,theta2,w7reqd,b,load,sw);
fclose(fid1);
%data required to calculate the shear reinforcement + bottle shaped strut %reinforcement
fid2=fopen('shear+strut.txt','a');
fprintf(fid2,'%d %.2f %.2f %d %d %d %.2f %.1f %d %d %d \r\n',load,theta1,(d1+0.5*w2reqd),b,l/3);
fclose(fid2);
q=q+1;
m=1;
else
%the reduction in ‘y’ is dependent on the accuracy required.
y=y-0.0001;
end
end
end
close(fid);
fid=fopen('qh.txt','a');
fprintf(fid,'%.2f %.2f \n',q);
fclose(fid);

%MATLAB program for Case 2 – Chapter 3. Main program that creates a STM.
clear all
%‘q’ is a counter that keeps track of the number of successful iterations.
q=0;
fid=fopen('input.txt');

%the data read from the input file in stored in column matrices for the
%each variable
%the value of n depends on the number of cases in the input file
n=1;
for i=1:n
    z=textscan(fid,'%f%f%f%f%f%f',1);
%the data read from the input file is stored in column matrices for the
%respective variables
%concrete compressive strength (units - ksi)
fc=z{1};
%total depth of the beam (units - inches)
d=z{2};
%distance between the loads (units - inches)
l=z{3};
%width of the beam (units - inches)
b=z{4};
%support width (units - inches)
sw=z{5};
%width of the base plate at load point (units - inches)
spb=z{6};
%clear cover provided to reinforcement (units - inches)
cover=z{7};
%factor to control truss height
y=0.99;
%height of the truss (center to center between the top prismatic strut
% and the bottom longitudinal tie

\[ d_1 = y \times (d - \text{cover}); \]

% ultimate shear capacity of the section

\[ \text{shear} = \frac{(0.75 \times 10 \times (f_c \times 1000)^{0.5}) \times b \times d_1}{1000}; \]

% load for the first iteration

\[ \text{load} = 40; \]

while \( \text{load} < \text{shear} \)

\[ \text{shear} = \frac{(0.75 \times 10 \times (f_c \times 1000)^{0.5}) \times b \times d_1}{1000}; \]

\[ \text{load} = \text{load} + 10; \]

\[ m = 0; \]

while \( m \neq 1 \)

% height of the truss

\[ d_1 = y \times (d - \text{cover}); \]

% \( x_1 \) is the distance between the nodes N1 and N3 (refer Fig. 3.2).

\[ x_1 = 0.5 \times d + 0.25 \times \text{sw}; \]

% \( x_2 \) is the horizontal distance between nodes N3 and N4 (refer Fig. 3.2).

\[ x_2 = x_1; \]

% \( \theta_1 \) is the inclination of the bottle shaped strut BS1 and BS2 with respect to horizontal (refer Fig. 3.2)

\[ \text{theta}_1 = \tan(d_1/x_1) \times (180 \times 7/22); \]

if \((\text{theta}_1 < 25) \|| (\text{theta}_1 > 65)\)

\[ \text{break} \]

end

% \( f_3 \) is modeled as a bottle shaped strut

\[ f_3 = \text{load} / \sin(\text{theta}_1 \times 22 / (7 \times 180)); \]

% division of forces in the bottle shaped strut
\[
\begin{align*}
\text{phi}1 &= \text{atan}(1/2); \\
\% \text{fb1 represents S1, and S3 from Fig. 3.3} \\
\text{fb1} &= \frac{f3}{(2 \cos(\text{phi1}))}; \\
\% \text{fb2 represents S2 from Fig. 3.3} \\
\text{fb2} &= \frac{f3}{(2)}; \\
\% \text{bt1 represents T1 from Fig. 3.3} \\
\text{bt1} &= \frac{f3}{(4)}; \\
\% \text{f6 represents tie T3 from Fig.3.2.} \\
\text{f6} &= \frac{f3 \cos(\theta1 \times 22/(7 \times 180))}{\sin(\text{theta2} \times 22/(7 \times 180))}; \\
\% \text{f1 represents strut S1 from Fig.3.2.} \\
\text{f1} &= \frac{f3 \cos(\theta1 \times 22/(7 \times 180))}{\sin(\text{theta2} \times 22/(7 \times 180))}; \\
\% \text{f5 represents tie T1 from Fig. 3.2.} \\
\text{f5} &= \frac{f3 \sin(\theta1 \times 22/(7 \times 180))}{\sin(\text{theta2} \times 22/(7 \times 180))}; \\
\% \text{theta2 is the inclination of the bottle shaped strut BS3 and} \\
\% \text{BS4 with respect to horizontal (refer Fig 3.2)} \\
\text{theta2} &= \text{theta1}; \\
\% \text{BS4 is modeled as a bottle shaped strut} \\
\text{f4} &= \frac{f5}{\sin(\text{theta2} \times 22/(7 \times 180))}; \\
\% \text{division of forces in the bottle shaped strut} \\
\text{phi2} &= \text{atan}(1/2); \\
\text{fb3} &= \frac{f4}{(2 \cos(\text{phi2}))}; \\
\text{fb4} &= \frac{f4}{(2)}; \\
\text{bt2} &= \frac{f4}{(4)}; \\
\% \text{end of bottle shaped strut} \\
\text{f7} &= \frac{f4 \cos(\theta1 \times 22/(7 \times 180))}{\sin(\text{theta2} \times 22/(7 \times 180))} + \text{f6}; \\
\text{f2} &= \text{f1} + \frac{f4 \cos(\theta1 \times 22/(7 \times 180))}{\sin(\text{theta2} \times 22/(7 \times 180))};
\end{align*}
\]
%nodal capacities

%node N1 is a CCT node. beta = 0.8
fcu1=0.75*0.85*0.8*fc;

%node N2 is a CCT node. beta = 0.8
fcu2=0.75*0.85*0.8*fc;

%node N3 is a CTT node. beta = 0.6
fcu3=0.75*0.85*0.6*fc;

%node N4 is a CCC node. beta = 1.0
fcu4=0.75*0.85*1.0*fc;

%strut capacities

%strut S1 is a prismatic strut
fc1=0.75*0.85*1*fc;

%strut S2 is a prismatic strut
fc2=0.75*0.85*1*fc;

%strut BS1 is a bottle shaped strut
fc3=0.75*0.85*0.75*fc;

%strut BS2 is a bottle shaped strut
fc4=0.75*0.85*0.8*fc;

%calculations for nodal capacities

%node N1

%p’ keeps a track of the anchoring of the bottle shaped struts at support and at %load locations
p=0;

%width required for tie T6 (refer Fig. 3.2).

w6reqd=f7/(fcu3*b);

%calculation of the width available for strut BS1 at node N1
%(refer Fig. 3.2).
w3avail = \( w_3 \cdot \sin(\theta_1 \cdot \frac{22}{7} \cdot 180) + w_{6 \text{reqd}} \cdot \cos(\theta_1 \cdot \frac{22}{7} \cdot 180) \);  

% calculation for the requirement of width for strut BS1 from strength considerations  

w3reqd = \( \frac{f_3}{f_c3 \cdot b} \);  

% check width required to the width available at node N1  

if w3avail > w3reqd  
    \( p = p + 1 \);  
end  

% node N4  

% width required for prismatic strut - S2 (refer Fig. 3.2).  

w2reqd = \( \frac{f_2}{b \cdot f_{c2}} \);  

% width available at node N4 to anchor the bottle shaped strut BS2  

w4avail = \( w_{2 \text{reqd}} \cdot \sin(\theta_2 \cdot \frac{22}{7} \cdot 180) + s_b \cdot \cos(\theta_1 \cdot \frac{22}{7} \cdot 180) \);  

% actual width required for the bottle shaped strut BS2  

w4reqd = \( \frac{f_4}{b \cdot f_{c4}} \);  

if w4avail > w4reqd  
    \( p = p + 1 \);  
end  

% check whether the given dimensions fit in the beam  

dmodel = d1 + w_{6 \text{reqd}} \cdot 0.5 + w_{2 \text{reqd}} \cdot 0.5 + \text{cover};  

% if the dmodel is greater than depth of beam, the value of height of truss is reduced for next iteration  

% check whether the conditions required for a successful STM are met.  

if dmodel > d || \text{shear} <= \text{load} || p < 2  
    % if the conditions are not met, truss height is reduced and
% the earlier calculations are performed once again with the
% new truss configuration

y=y-0.001;

else

m=1;

% calculation for the inclined length of the bottle shaped strut

ls=(d1^2+x1^2)^0.5;

fid1=fopen('reinfinput.txt','a');

fprintf(fid1,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f
r\n',fc,f7,theta1,theta2,w6reqd,b,load,sw);

fid2=fopen('strut1plustie.txt','a');

fprintf(fid2,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f
r\n',load,f5,theta1,b,l/3,bt1,ls,d1+0.5*w6reqd);

fclose(fid2);

q=q+1;

close(fid1);

end

end

end

close(fid);

fid=fopen('qh.txt','a');

fprintf(fid,'%2f\n',q);

fclose(fid);

% MATLAB program for Case 3 – Chapter 4. Main program that creates a STM.
clear all
clc

%input.txt is the input file
fid=fopen('input.txt');

%counter - load
q=0;

%number of input beam configuration in the input file
n=1;

for i=1:n

%the data read from the input file is stored in column matrices for the
%respective variables
z=textscan(fid,'%f %f %f %f %f %f %f %f %f %f %f %f',1);

%concrete strength. (units - ksi)
fz=z{1};

%clear span of the beam (units - inches)
span=z{2};

%beam width (units - inches)
b=z{3};

%Beam depth (units - inches)
depth=z{4};

%support width (units - inches)
sw=z{5};

%width of the base plate at load (units - inches)
sb=z{6};

%Cover to the longitudinal reinforcement (units - inches)
cover=z{7};
%Load 1 (units - kips)
load1=z[8];

%Load 2 (units - kips)
load2=z[9];

%Increment in load
loadincrement=z[10];

%factor to control the truss height
y=0.99;

%height of the truss (center to center between the top prismatic strut and %bottom longitudinal tie)
d1=y*(depth-cover);

%ultimate shear capacity of the section
shear=(0.75*10*((fc*1000)^(0.5)*b*d1)/1000);

%Reaction at left support - refer Fig. 4.2
Va=(load1*(2*depth+0.5*sw)+load2*(depth+0.5*sw))/(3*depth+sw);

%Reaction at right support - refer Fig. 4.2
Vb=load1+load2-Va;

while Va<shear && Vb<shear
  m=0;
  y=.99;
  x1=0.5*(depth+0.5*sw);
  x2=x1;
  %Reaction at left support - refer Fig. 4.2
  Va=(load1*(2*depth+0.5*sw)+load2*(depth+0.5*sw))/(3*depth+sw);
  %Reaction at right support - refer Fig. 4.2
  Vb=load1+load2-Va;

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%ratio of the bending moments at the two load points - refer
%Fig. 4.3 and Eq. 4.1

ratio = Vb/Va;

while m~ = 1
    if ratio > 1
        d2 = y*(depth-cover);
        d1 = d2*ratio;
    else
        d1 = y*(depth-cover);
        d2 = d1*ratio;
    end

%inclination of the strut S1 and S2 - refer Fig. 4.2
theta1 = atan(d1/x1);

%inclination of struts S3 and S4 with respect the
%longitudinal tie - refer Fig. 4.2
theta2 = atan(d2/x2);

%distance between the two loads
l = depth;

%theta3 is the angle between the inclined sturt (S3)
%and horizontal - refer Fig. 4.2
if d1 > d2
    theta3 = atan((d1-d2)/l);
else
    theta3 = atan((d2-d1)/l);
end

%calculation of forces in the truss members
%Strut S1 is a bottle shaped strut (refer Fig. 4.2)

\[ s_1 = \frac{V_a}{\sin(\theta_1)} \]

%Tie T3 (refer Fig. 4.2)

\[ t_3 = s_1 \cos(\theta_1) \]

%Strut S4 is a bottle shaped strut (refer Fig. 4.2)

\[ s_4 = \frac{V_b}{\sin(\theta_2)} \]

%Tie T5 (refer Fig. 4.2)

\[ t_5 = s_4 \cos(\theta_2) \]

%Strut S5 is a prismatic strut (refer Fig. 4.2)

\[ s_5 = s_1 \cos(\theta_1) \]

%Tie T1 and T2 (refer Fig. 4.2)

\[ t_1 = s_1 \sin(\theta_1) \]
\[ t_2 = s_4 \sin(\theta_2) \]

%Strut S2 is a bottle shaped strut (refer Fig. 4.2)

\[ s_2 = \frac{t_1}{\sin(\theta_1)} \]

%Strut S3 is a bottle shaped strut (refer Fig. 4.2)

\[ s_3 = \frac{t_2}{\sin(\theta_2)} \]

%Strut S7 is prismatic strut (refer Fig. 4.2)

\[ s_7 = s_4 \cos(\theta_2) \]

%Strut S6 is a prismatic strut (refer Fig. 4.2)

\[ s_6 = (s_5 + s_2 \cos(\theta_1))/\cos(\theta_3) \]
\[ t_4 = t_3 + s_2 \cos(\theta_1) \]

%Nodal capacities and strut capacities

%node N1:CCT node, beta = 0.8

%reference ACI 318(2008) section A.5.2 - page 393

\[ f_{cu1} = 0.75 \times 0.85 \times 0.8 \times f_c; \]
percent node N5: CCT node, beta = 0.8
fcu2 = 0.75 * 0.85 * 0.8 * fc;

percent node N2: CTT node, beta = 0.6
fcu3 = 0.75 * 0.85 * 0.6 * fc;

percent node N6: CCC node, beta = 1.0
fcu4 = 0.75 * 0.85 * 1.0 * fc;

percent node N7: CCC node, beta = 1.0
fcu5 = 0.75 * 0.85 * 1.0 * fc;

percent node N8: CCT node, beta = 0.8
fcu6 = 0.75 * 0.85 * 0.8 * fc;

percent node N3: CTT node, beta = 0.6
fcu7 = 0.75 * 0.85 * 0.6 * fc;

percent node N4: CCT node, beta = 0.8
fcu8 = 0.75 * 0.85 * 0.8 * fc;

% strut capacities

% strut S1 is a bottle shaped strut, beta = 0.75

% reference ACI 318(2008) section A.3.2 - page 388
fc1 = 0.75 * 0.85 * 0.75 * fc;

% strut S2 is a bottle shaped strut, beta = 0.75
fc2 = 0.75 * 0.85 * 0.75 * fc;

% strut S3 is a bottle shaped strut, beta = 0.75
fc3 = 0.75 * 0.85 * 0.75 * fc;

% strut S4 is a bottle shaped strut, beta = 0.75
fc4 = 0.75 * 0.85 * 0.75 * fc;

% strut S5 is a prismatic strut, beta = 1.0
fc5 = 0.75 * 0.85 * 1.0 * fc;
%strut S6 is a prismatic strut, beta = 1.0
fc6=0.75*0.85*1.0*fc;

%strut S7 is a prismatic strut, beta = 1.0
fc7=0.75*0.85*1.0*fc;

%calculations for strut and tie widths
%calculation for tie width T3
t3reqd=t4/(fcu3*b);

%calculation for strut width S1
p=0;

%required strength for Strut S1
s1reqd=s1/(fc1*b);

%available space to anchor the strut at support - refer Fig. 2.7
s1avail=sw*sin(theta1)+t3reqd*cos(theta1);

%if the available width is greater than the required
%width, the check is complete
if s1reqd <= s1avail
    p=p+1;
end

%calculation for tie width T5. the width of the tie is
%calculated to resist the force from Tie T4. Refer to
%Section 2.2 for details.
t5reqd=t4/(fcu3*b);

%required strut width for Strut S4
s4reqd=s4/(fc4*b);

%available width to anchor the strut at support - refer Fig. 2.7
s4avail=sw*sin(theta2)+t5reqd*cos(theta2);
if \( s_{4\text{reqd}} \leq s_{4\text{avail}} \)

\[
p = p + 1;
\]

end

\( s_{6\text{reqd}} = s_6 / (b \cdot f_{c6}) \);

\( s_{5\text{reqd}} = s_5 / (b \cdot f_{cu2}) \);

% available width to anchor the strut at load point -
% refer Fig. 2.8

\( s_{2\text{avail}} = s_b \cdot \sin(\theta_1) + s_{6\text{reqd}} \cdot \cos(\theta_1) \);

% required strut width for Strut S2

\( s_{2\text{reqd}} = s_2 / (b \cdot f_{cu3}) \);

if \( s_{2\text{avail}} > s_{2\text{reqd}} \)

\[
p = p + 1;
\]

end

\( s_{7\text{reqd}} = s_7 / (b \cdot f_{cu6}) \);

% available width to anchor the strut at load point -
% refer Fig. 2.8

\( s_{3\text{avail}} = s_b \cdot \sin(\theta_2) + s_{6\text{reqd}} \cdot \cos(\theta_2) \);

% required strut width for Strut S2

\( s_{3\text{reqd}} = s_3 / (b \cdot f_{cu7}) \);

if \( s_{3\text{avail}} > s_{3\text{reqd}} \)

\[
p = p + 1;
\]

end

\( s_{4\text{reqd}} = s_4 / (f_c \cdot b) \);

\( s_{4\text{avail}} = s_w \cdot \sin(\theta_2) + t_{5\text{reqd}} \cdot \cos(\theta_2) \);

\( t_{4\text{reqd}} = t_4 / (b \cdot f_{cu7}) \);

% check whether the given dimensions fit in the beam
dmodel=d1+t4*reqd*0.5+(s6*reqd*0.5/cos(theta3));

%either d1 or d2 would govern the model depending on
%the ratio between the two point loads.
if d2>d1
    dmodel=d2+t4*reqd*0.5+(s6*reqd*0.5/cos(theta3));
end

%if the dmodel is greater than depth of beam, the value of height of
%truss is reduced for next iteration
shear=(0.75*10*((fc*1000)^(0.5))*b*(d1+0.5*s6))/1000;
if d2>d1
    shear=(0.75*10*((fc*1000)^(0.5))*b*(d2+0.5*s6))/1000;
end
if shear < Va || shear < Vb
    m=1;
    break
end
if dmodel<=(depth-cover) && p==4
    fid1=fopen('reinfinput.txt','a');
    fid2=fopen('shear+strut1.txt','a');
    fid3=fopen('shear+strut2.txt','a');
    fprintf(fid1,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f
\r\n',fc,t4,theta1*(180*7/22),theta2*(180*7/22),t4*reqd,b,load1,sw);
    fprintf(fid2,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f
\r\n',load1,Va,theta1*(180*7/22),d1+0.5*s6,load1,span/3);
    fprintf(fid3,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f
\r\n',load1,Vb,theta2*(180*7/22),d2+0.5*s6,load1,span/3);
q=q+1;
m=1;
else
y=y-0.001;
end
end

%Reaction at left support - refer Fig. 4.2
Va=(load1*(2*depth+0.5*sw)+load2*(depth+0.5*sw))/(3*depth+sw);

%Reaction at right support - refer Fig. 4.2
Vb=load1+load2-Va;
ratio=Vb/Va;
if ratio >1
    d2=y*(depth-cover);
    d1=d2*ratio;
else
    d1=y*(depth-cover);
    d2=d1*ratio;
end

shear=(0.75*10*((fc*1000)^(0.5))*b*(d1+0.5*s6reqd))/1000;
if d2>d1
    shear=(0.75*10*((fc*1000)^(0.5))*b*(d2+0.5*s6reqd))/1000;
end
load1=load1+loadincrement;
load2=load2+loadincrement;
end
fid4=fopen('qh.txt','a');
fprintf(fid4,'%d\n',q);
fclose(fid4);
fclose(fid1);
fclose(fid2);
fclose(fid3);

%MATLAB program for Case 4 – Chapter 4. Main program that creates a STM.
clear all
clc

%input.txt is the input file
fid=fopen('input.txt');
%counter - load
q=0;

%number of input beam configuration in the input file
n=1;
for i=1:n
    %the data read from the input file is stored in column matrices for the
    %respective variables
    z=textscan(fid,'%f %f %f %f %f %f %f %f %f %f %f',1);
    %concrete strength. (units - ksi)
    fc=z{1};
    %clear span of the beam (units - inches)
    span=z{2};
    %beam width (units - inches)
    b=z{3};
    %Beam depth (units - inches)
depth=z{4};
% support width (units - inches)
sw=z{5};
% width of the base plate at load (units - inches)
sb=z{6};
% Cover to the longitudinal reinforcement (units - inches)
cover=z{7};
% Load 1 (units - kips)
load1=z{8};
% Load 2 (units - kips)
load2=z{9};
% Increment in load
loadincrement=z{10};
% Factor to control the truss height
y=0.99;
% Height of the truss (center to center between the top prismatic strut and
% bottom longitudinal tie)
d1=y*(depth-cover);
% Ultimate shear capacity of the section
shear=(0.75*10*((fc*1000)^0.5)*b*d1)/1000;
% Reaction at left support - refer Fig. 6.1
Va=(load1*(2*depth+0.5*sw)+load2*(depth+0.5*sw))/(3*depth+sw);
% Reaction at right support - refer Fig. 6.1
Vb=load1+load2-Va;
while Va<shear && Vb<shear
  m=0;
x1=0.5*(depth+0.5*sw);

x2=x1;

%Reaction at left support - refer Fig. 6.1
Va=(load1*(2*depth+0.5*sw)+load2*(depth+0.5*sw))/(3*depth+sw);

%Reaction at right support - refer Fig. 6.1
Vb=load1+load2-Va;

y=0.99;

while m~=-1

%ratio of the bending moments at the two load points - refer
%Fig. 4.3 and Eq. 4.1
ratio=Vb/Va;

if ratio >1

d2=y*(depth-cover);
d1=ratio*d2;
else

d1=y*(depth-cover);
d2=ratio*d1;
end

%inclination of the strut S1 and S2 - refer Fig. 6.2
theta1=atan(d1/x1);

%inclination of struts S3 and S4 with respect the
%longitudinal tie - refer Fig. 6.2
theta2=atan(d2/x2);

%distance between the two loads
l=depth;

%theta3 is the angle between the inclined sturt (S3)
% and horizontal - refer Fig. 6.2

if d1>d2
    theta3=atan((d1-d2)/l);
else
    theta3=atan((d2-d1)/l);
end

% calculation of forces in the truss members

%Bottle shaped strut – BS1 (refer Fig. 4.7)
s1=Va/sin(theta1);

% division of forces in the bottle shaped strut
phi1=atan(1/2);

% fb1 represents S1, and S3 from Fig. 3.3
fb1=s1/(2*cos(phi1));

% fb2 represents S2 from Fig. 3.3
fb2=s1/2;

% bt1 represents T1 from Fig. 3.3
bt1=s1/4;

% Tie T3

t3=s1*cos(theta1);

%Bottle shaped strut – BS4 (refer Fig. 4.7)
s4=Vb/sin(theta2);

% division of forces in the bottle shaped strut
phi2=atan(1/2);

% fb3 represents S1, and S3 from Fig. 3.3
fb3=s4/(2*cos(phi2));

% fb2 represents S2 from Fig. 3.3
fb4=s4/2;
%bt1 represents T1 from Fig. 3.3
bt2=s1/4;
%Tie T5
t5=s4*cos(theta2);
%Prismatic strut S1
s5=s1*cos(theta1);
%Tie T1
t1=s1*sin(theta1);
%Tie T2
t2=s4*sin(theta2);
%Bottle shaped strut – BS2 (refer Fig. 4.7) – distribution of forces similar to BS1
s2=t1/sin(theta1);
%Bottle shaped strut – BS3 (refer Fig. 4.7) – distribution of forces similar to BS1
s3=t2/sin(theta2);
%Prismatic strut S3
s7=s4*cos(theta2);
%Prismatic strut S2
s6=(s5+s2*cos(theta1))/cos(theta3);
t4=t3+s2*cos(theta1);
%nodal capacities and strut capacities
%node 1:CCT node, beta = 0.8
%reference ACI 318(2008) section A.5.2 - page 393
fcu1=0.75*0.85*0.8*fc;
%node 2:CCT node, beta = 0.8
fcu2=0.75*0.85*0.8*fc;
%node 3: CTT node, beta = 0.6
fcu3 = 0.75 * 0.85 * 0.6 * fc;

%node 4: CCC node, beta = 1.0
fcu4 = 0.75 * 0.85 * 1.0 * fc;

%node 5: CCC node, beta = 1.0
fcu5 = 0.75 * 0.85 * 1.0 * fc;

%node 6: CCT node, beta = 0.8
fcu6 = 0.75 * 0.85 * 0.8 * fc;

%node 7: CTT node, beta = 0.6
fcu7 = 0.75 * 0.85 * 0.6 * fc;

%node 8: CCT node, beta = 0.8
fcu8 = 0.75 * 0.85 * 0.8 * fc;

%strut capacities

%strut 1 is a bottle shaped strut, beta = 0.75
%reference ACI 318(2008) section A.3.2 - page 388
fc1 = 0.75 * 0.85 * 0.75 * fc;

%strut 2 is a bottle shaped strut, beta = 0.75
fc2 = 0.75 * 0.85 * 0.75 * fc;

%strut 3 is a bottle shaped strut, beta = 0.75
fc3 = 0.75 * 0.85 * 0.75 * fc;

%strut 4 is a bottle shaped strut, beta = 0.75
fc4 = 0.75 * 0.85 * 0.75 * fc;

%strut 5 is a prismatic strut, beta = 1.0
fc5 = 0.75 * 0.85 * 1.0 * fc;

%strut 6 is a prismatic strut, beta = 1.0
fc6 = 0.75 * 0.85 * 1.0 * fc;
%strut 7 is a prismatic strut, beta = 1.0

fc7=0.75*0.85*1.0*fc;

%calculations for strut and tie widths
%calculation for tie width t3

t3reqd=t4/(fcu3*b);

%calculation for strut width s1

p=0;

%required strength for Strut S1

s1reqd=s1/(fc1*b);

%available space to anchor the strut at support - refer Fig. 2.7

s1avail=sw*sin(theta1)+t3reqd*cos(theta1);

%if the available width is greater than the required
%width, the check is complete

if s1reqd <= s1avail
    p=p+1;
end

%calculation for tie width t5. the width of the tie is
%calculated to resist the force from Tie T4. Refer to
%Section 2.2 for details.

t5reqd=t4/(fcu3*b);

%required strut width for Strut S4

s4reqd=s4/(fc4*b);

%available width to anchor the strut at support - refer Fig. 2.7

s4avail=sw*sin(theta2)+t5reqd*cos(theta2);

if s4reqd <= s4avail
    p=p+1;
end
s5reqd=s5/(b*fcu2);
% available width to anchor the strut at load point -
% refer Fig. 2.8
s6reqd=s6/(b*fc6);
s2avail=sb*sin(theta1)+s6reqd*cos(theta1);
% required strut width for Strut S2
s2reqd=s2/(b*fc2);
if s2avail>s2reqd
    p=p+1;
end

s7reqd=s7/(b*fcu6);
% available width to anchor the strut at load point -
% refer Fig. 2.8
s3avail=sb*sin(theta2)+s6reqd*cos(theta2);
% required strut width for Strut S2
s3reqd=s3/(b*fc3);
if s3avail>s3reqd
    p=p+1;
end

s4reqd=s4/(fc4*b);
s4avail=sw*sin(theta2)+t5reqd*cos(theta2);
if s4avail>s4reqd
    p=p+1;
end

t4reqd=t4/(b*fcu7);
% check whether the given dimensions fit in the beam
dmodel=d1+t4reqd*0.5+s6reqd*0.5;
% either d1 or d2 would govern the model depending on
% the ratio between the two point loads.
if d2>d1
    dmodel=d2+t4reqd*0.5+s6reqd*0.5;
end
% if the dmodel is greater than depth of beam, the value of height of
% truss is reduced for next iteration
shear=(0.75*10*((fc*1000)^(0.5))*b*(d1+0.5*s6reqd))/1000;
if d2>d1
    shear=(0.75*10*((fc*1000)^(0.5))*b*(d2+0.5*s6reqd))/1000;
end
if shear < Va || shear < Vb
    m=1;
    break
end
if dmodel>(depth-cover) || p<4
    y=y-0.0001;
else
    m=1;
    fid1=fopen('reinfinput.txt','a');
    fprintf(fid1,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f
\r\n',fc,t4,theta1*(180*7/22),theta2*(180*7/22),t4reqd,b,load1,sw);
    q=q+1;
    fclose(fid1);
end
%calculation for the inclined length of the bottle shaped strut BS1
ls1=(d1^2+x1^2)^0.5;

%calculation for the inclined length of the bottle shaped strut BS4
ls2=(d1^2+x1^2)^0.5;

fid2=fopen('strut1plustie1.txt','a');
fprintf(fid2,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f
\r\n',load1,Va,theta1*(180*7/22),b,span/3,bt1,ls1,d1+0.5*s6reqd);
fclose(fid2);

fid3=fopen('strut1plustie2.txt','a');
fprintf(fid2,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f
\r\n',load1,Vb,theta2*(180*7/22),b,span/3,bt2,ls2,d2+0.5*s6reqd);
fclose(fid2);

load1=load1+loadincrement;
load2=load2+loadincrement;

end

end

y=0.99;
m=1;

%Reaction at left support - refer Fig. 6.1
Va=(load1*(2*depth+0.5*sw)+load2*(depth+0.5*sw))/(3*depth+sw);

%Reaction at right support - refer Fig. 6.1
Vb=load1+load2-Va;
shear=(0.75*10*((fc*1000)^(0.5))*b*(d1+0.5*s6reqd))/1000;
if d2>d1
    shear=(0.75*10*((fc*1000)^(0.5))*b*(d2+0.5*s6reqd))/1000;
end
% MATLAB program for Case 5 – Chapter 5. Main program that creates a STM.

clear all
clc

% the input data for the program is stored in a text file.
fid=fopen('input.txt');

% this is a counter that keeps track of the number of successful iterations
q=0;

% the value of ‘n’ indicates the number of beam configurations entered in the input file
n=1;

for i=1:n
    % MATLAB reads the data from the input file
    z=textscan(fid,'%.2f %.2f %.2f %.2f %.2f %.2f',1);
    % ‘fc’ is the concrete strength (units - ksi)
    fc=z{1};
    % ‘d’ is the total depth of the beam (units - inches)
    d=z{2};
    % ‘l’ is the clear span of the beam (units - inches)
    l=z{3};
    % ‘b’ is the width of the beam (units - inches)
    b=z{4};
    % ‘sw’ is the support width (units – inches)
sw=z{5};
%clear cover provided to the longitudinal reinforcement (units – inches)

cover=z{6};
%uniformly distributed load (units - kips/inch). The uniformly distributed load increases post successful iteration.

udl=0.05;
%y controls the truss height. ‘y’ is set to 0.99 for first iteration.

y=0.99;
%initial trial height of the truss
d1=y*(d-cover);

%shear capacity of the beam
shear=0.75*10*((fc*1000)^{(1/2)})*b*y*d/1000;
%Reaction at supports (units – kips). The reaction is the applied load (Load 1) at the quarter spans (refer to Fig. 5.2).

R=udl*l*0.5;
while shear>R

    udl=udl+0.05;

    m=0;

    R=udl*l*0.5;

    while m~=1

        d1=y*(d-cover);

        %x1 is the distance between nodes N1 and N2 (refer Fig. 5.2)
        x1=(0.25*l+0.5*sw)*0.5;

        %x2 is the horizontal distance between nodes N2 and N6 (refer Fig. 5.2)
        x2=x1;

        %theta is the inclination of the bottle shaped strut with respect to horizontal
\[
\theta_1 = \arctan\left(\frac{d_1}{x_1}\right) \times \left(\frac{180 \times 7}{22}\right);
\]

if (\theta_1 < 25) || (\theta_1 > 65)
    break
end

% calculation for the individual member forces

%f3 represents the bottle shaped strut S1 from Fig. 5.2

\[ f_3 = \frac{R}{\sin(\theta_1 \times \frac{22}{7 \times 180})}; \]

%f6 represents tie T3 from Fig. 5.2.

\[ f_6 = f_3 \times \cos(\theta_1 \times \frac{22}{7 \times 180}); \]

%f1 represents prismatic strut S7 from Fig. 5.2.

\[ f_1 = f_3 \times \cos(\theta_1 \times \frac{22}{7 \times 180}); \]

%f5 represents tie T1 from Fig. 5.2.

\[ f_5 = f_3 \times \sin(\theta_1 \times \frac{22}{7 \times 180}); \]

%theta2 is the inclination of the bottle shaped strut S2 with respect to horizontal

\[ \theta_2 = \theta_1; \]

%f4 represents bottle shaped strut S2 from Fig. 5.2.

\[ f_4 = \frac{f_5}{\sin(\theta_2 \times \frac{22}{7 \times 180})}; \]

%f7 represents tie T4 from Fig. 5.2.

\[ f_7 = f_4 \times \cos(\theta_2 \times \frac{3.142}{180}) + f_6; \]

%f2 represents prismatic strut S6 from Fig. 5.2.

\[ f_2 = f_1 + f_4 \times \cos(\theta_1 \times \frac{3.142}{180}); \]

% nodal capacities

% node 1: CCT node, beta = 0.8

% reference ACI 318(2008) section A.5.2 - page 393

\[ f_{cu1} = 0.75 \times 0.85 \times 0.8 \times f_c; \]

% node 2: CCT node, beta = 0.8
fcu2=0.75*0.85*0.8*fc;

%node 3: CTT node, beta = 0.6
fcu3=0.75*0.85*0.6*fc;

%node 4: CCC node, beta = 1.0
fcu4=0.75*0.85*1.0*fc;

%strut capacities
%strut 3 is a bottle shaped strut, beta = 0.75

%reference ACI 318(2008) section A.3.2 - page 388
fc3=0.75*0.85*0.75*fc;

%strut 1 is a rectangular strut, beta = 1.0
fc1=0.75*0.85*1*fc;

%strut 2 is a rectangular strut, beta = 1.0
fc2=0.75*0.85*1*fc;

%strut 4 is a bottle shaped strut, beta = 0.75
fc4=0.75*0.85*0.75*fc;

%calculations for nodal capacities
%node 1
p=0;

%the node capacity at node 3 governs the width required for this tie
%calculation for tie width. tie 7
w7reqd=f7/(b*fcu3);

%width required for strut#6 (prismatic strut)
w6reqd=f6/(fcu2*b);

%calculation of the width available for strut#3 at node 1
w3avail=sw*sin(theta1*22/(7*180))+w7reqd*cos(theta1*22/(7*180));

%calculation for the requirement of width for strut#3 from strength
considerations. The strut width is governed by the bottle shaped strut beta factor.

\[ w_{3\text{reqd}} = \frac{f_3}{(f_{c3} \cdot b)}; \]

% check width required to the width available
if \( w_{3\text{avail}} > w_{3\text{reqd}} \)
    \[ p = p + 1; \]
end

% Width required for strut 2 is governed by the node capacity of node 2.
\[ w_{2\text{reqd}} = \frac{f_2}{(b \cdot f_{cu4})}; \]

% calculation for the requirement of width for strut#1 from strength considerations
\[ w_{1\text{reqd}} = \frac{f_1}{(b \cdot f_{cu1})}; \]

% calculation of width for tie#5 from strength requirement of strut#3
\[ w_{5\text{avail}} = \sqrt{(w_{3\text{avail}})^2 - (w_{1\text{reqd}})^2}; \]

% ultimate shear capacity
\[ \text{shear} = (0.75 \times 10 \times ((f_c \times 1000) \times (0.5)) \times b \times (d_1 + 0.5 \times w_{2\text{reqd}}))/1000; \]

% check whether the given dimensions fit in the beam
\[ \text{dmodel} = d_1 + w_{7\text{reqd}} \times 0.5 + w_{2\text{reqd}} \times 0.5 + \text{cover}; \]

% if the dmodel is greater than depth of beam, the value of height of truss is reduced for next iteration
if \( \text{dmodel} > d \ || \ p < 1 \ || \ \text{shear} < R \)
    % reduce the truss height for the next iteration
    \[ y = y - 0.001; \]
else
    % MATLAB prints the output in a text file
%data required for longitudinal reinforcement

fid1=fopen('reininput.txt','a');
fprintf(fid1,'%.2f %.2f %.2f %.2f %.2f %d %.2f %d
',fc,f7,theta1,theta2,w7reqd,b,udl,sw);
q=q+1;
 fid2=fopen('shear+strut.txt','a');
 fprintf(fid2,'%.2f %.2f %.2f %.2f %d %d
',udl,R,theta1,(d1+0.5*w2reqd),b,l/4);
 fclose(fid2);
 m=1;
end
end
end
fclose(fid);
clear fid
fid=fopen('qh.txt','a');
 fprintf(fid,'%d
',q);

%MATLAB program for Case 6 – Chapter 5. Main program that creates a STM.
clear all
clc
fid=fopen('input.txt');

%’q’ is a counter that keeps track of the number of successful iterations
q=0;
n=1;
for i=1:n
%the input data for the program is stored in a text file.
z=textscan(fid,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f',1);

%concrete strength (units - ksi)
fc=z{1};

%total depth of the beam (units - inches)
d=z{2};

%total length of the beam, face to face between supports (units - inches)
l=z{3};

%width of the beam (units - inches)
b=z{4};

%support width
sw=z{5};

%width of base plate
sb=z{6};

%clear cover provided
cover=z{7};

%intitial uniformy distributed load - kip/inch
udl=0.05;

%y controls the truss height. ‘y’ is set to 0.99 for first iteration.
y=0.99;

%intitial height of the truss
d1=y*(d-cover);

%shear capacity of the beam
shear=0.75*10*((fc*1000)^(1/2))*b*0.99*d/1000;

%reaction = load at quarter span of the beam - refer Fig. 5.7
\[ R = udl \cdot l \cdot 0.5; \]

while shear > \( R \)

\[ udl = udl + 0.05; \]

\[ m = 0; \]

\[ R = udl \cdot l \cdot 0.5; \]

while \( m = 1 \)

\[ d1 = y \cdot (d - \text{cover}); \]

\[ x1 = (0.25 \cdot l + 0.5 \cdot sw) \cdot 0.5; \]

\[ x2 = x1; \]

\[ \theta_1 = \tan(d1/x1) \cdot (180 \cdot 7/22); \]

if \( \theta_1 < 25 \) \( \text{or} \) \( \theta_1 > 65 \)

break

end

% calculation for the individual member forces

%Bottle shaped strut BS1 from Fig. 5.7

\[ f3 = R / \sin(\theta_1 \cdot 22/ (7 \cdot 180)); \]

% division of forces in the bottle shaped strut

\[ \phi_1 = \tan(1/2); \]

% \( fb1 \) represents \( S_1 \), and \( S_3 \) from Fig. 3.3

\[ fb1 = f3/2 \cdot \cos(\phi_1); \]

% \( fb2 \) represents \( S_2 \) from Fig. 3.3

\[ fb2 = f3/2; \]

% \( bt1 \) represents \( T_1 \) from Fig. 3.3

\[ bt1 = f3/4; \]

% Tie \( T_3 \) from Fig. 5.7

\[ f6 = f3 \cdot \cos(\theta_1 \cdot 22/ (7 \cdot 180)); \]
%Prismatic strut S7 from Fig. 5.7
\[ f_1 = f_3 \cos(\theta_1 \times 22/(7 \times 180)) ; \]

%Tie T1 from Fig. 5.7
\[ f_5 = f_3 \sin(\theta_1 \times 22/(7 \times 180)) ; \]
\[ \theta_2 = \theta_1 ; \]

%Bottle shaped strut BS3 from Fig. 5.7
\[ f_4 = f_5 / \sin(\theta_2 \times 3.142/180) ; \]

%tie T4 from Fig. 5.7
\[ f_7 = f_4 \cos(\theta_2 \times 3.142/180) + f_6 ; \]

%Prismatic strut S6 from Fig. 5.7
\[ f_2 = f_1 + f_4 \cos(\theta_1 \times 3.142/180) ; \]

%nodal capacities

%node N1:CCT node, beta = 0.8
%reference ACI 318(2008) section A.5.2 - page 393
\[ f_{cu1} = 0.75 \times 0.85 \times 0.8 \times f_c ; \]

%node N5:CCT node, beta = 0.8
\[ f_{cu2} = 0.75 \times 0.85 \times 0.8 \times f_c ; \]

%node N2:CTT node, beta = 0.6
\[ f_{cu3} = 0.75 \times 0.85 \times 0.6 \times f_c ; \]

%node N6:CCC node, beta = 1.0
\[ f_{cu4} = 0.75 \times 0.85 \times 1.0 \times f_c ; \]

%strut capacities

%strut BS1 is a bottle shaped strut, beta = 0.75
%reference ACI 318(2008) section A.3.2 - page 388
\[ f_{c3} = 0.75 \times 0.85 \times 0.75 \times f_c ; \]

%strut S7 is a rectangular strut, beta = 1.0
fc1 = 0.75 * 0.85 * 1 * fc;

% strut BS2 is a bottle shaped strut, beta = 0.75

fc4 = 0.75 * 0.85 * 0.75 * fc;

% calculations for nodal capacities

% node N1

p = 0;

% width required for tie T3

% the node capacity at node N2 governs the width required for this tie

w6reqd = f7 / (fcu3 * b);

% calculation of the width available for strut BS1 at node N1

w3avail = sw * sin(theta1 * 22 / (7 * 180)) + w6reqd * cos(theta1 * 22 / (7 * 180));

% calculation for the requirement of width for strut BS1 from strength

% considerations. The strut width is governed by the bottle shaped strut

% beta factor.

w3reqd = f3 / (fc3 * b);

% check width required to the width available

if w3avail > w3reqd
    p = p + 1;
end

% Width required for strut S6 is governed by the node capacity of

% node N2.

w2reqd = f2 / (b * fcu4);

% calculation for the requirement of width for strut S7 from strength

% considerations

w1reqd = f1 / (b * fcu1);

% node N2
% calculation for tie width. tie T4
w7reqd=f7/(b*fcu3);

% calculations for strut BS2 at load
w4avail=lb*sin(theta2*22/(7*180))+w2reqd*cos(theta1*22/(7*180));
% calculation for the requirement of width for strut BS1 from strength
% considerations. The strut width is governed by the bottle shaped strut
% beta factor.

w4reqd=f4/(b*fc3);

if w4avail>w4reqd
    p=p+1;
end

% ultimate shear capacity
shear=(0.75*10*((fc*1000)^(0.5))*b*(d1+0.5*w2reqd))/1000;
% check whether the given dimensions fit in the beam

m=1;
% calculation for the inclined length of the bottle shaped strut
ls=(d1^2+x1^2)^0.5;

fid1=fopen('reinfinput.txt','a');
fprintf(fid1,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f \n',fc,f7,theta1,theta2,w6reqd,b,udl,sw);
```

 fid2=fopen('strut1plustie.txt','a');
 fprintf(fid2,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f
\r\n',udl,R,theta1,b,l/4,bt1,ls,d1+0.5*w6reqd);
 fclose(fid2);
 q=q+1;
 fclose(fid1);
 end
 end
 end

close(fid);
 clear fid
 fid=fopen('qh.txt','a');
 fprintf(fid,'%d
\r\n',q);

%MATLAB program for Case 7 – Chapter 6. Main program that creates a STM.

clear all
clc

%input.txt is the input file
fid=fopen('input.txt');

%counter - load
q=0;

%number of input beam configuration in the input file
n=1;

for i=1:n
    z=textscan(fid,'%f%f%f%f%f%f%f%f%f',1);
```
%concrete strength. (units - ksi)
fe=z{1};

%clear span of the beam (units - inches)
span=z{2};

%beam width (units - inches)
b=z{3};

%support width (units - inches)
sw=z{4};

%width of the base plate at load (units - inches)
sb=z{5};

%Cover to the longitudinal reinforcement (units - inches)
cover=z{6};

%Span to depth ratio
ratio=z{7};

%Load 1 (units - kips)
load1=z{8};

%Load 2 (units - kips)
load2=z{9};

%increment in distance
distanceincrement=z{10};

%maximum case for deep beam
depth=span/ratio;

%this variable governs the load spacing
theta=65*22/(180*7);

%this is the first location of the load
dist=2*(floor(depth/tan(theta))+1);
while dist <=(0.5*(span+sw)-0.5*2.5*12-5)
    x1=dist/2;
    y=0.99;
    d1=y*(depth-cover);
    dist1=2*(floor(depth/tan(theta))+1);
while dist1<=(0.5*(span+sw)-0.5*2.5*12-5)
    shear=0.75*10*(fc*1000)^0.5*d1*b/1000;
    x2=dist1/2;
    m=0;
    y=0.99;
    d1=y*(depth-cover);
    Va=(load1*(span+sw-dist)+load2*dist1)/(span+sw);
    Vb=load1+load2-Va;
    ratio1=Vb*dist1/(Va*dist);
    theta1=atan(d1/x1);
    while m/=1 && theta1 > (25*22/(180*7))
        ratio1=Vb*dist1/(Va*dist);
        d1=y*(depth-cover);
        d2=ratio1*d1;
        theta1=atan(d1/x1);
while m/=1 && theta1 > (25*22/(180*7))
    ratio1=Vb*dist1/(Va*dist);
    d1=y*(depth-cover);
    d2=ratio1*d1;
    %this part of the program calculates the forces in the truss members
    theta1=atan(d1/x1);
    %this is the distance between the two loads
    l=span+sw-(dist+dist1);
    theta2=atan(d2/x2);
    theta3=atan((d1-d2)/l);
    %calculation of forces in the truss members
%s1 is a bottle shaped strut

s1=Va/sin(theta1);
s4=Vb/sin(theta2);
t5=s4*cos(theta2);
s5=s1*cos(theta1);
t1=s1*sin(theta1);
t2=s4*sin(theta2);
s2=t1/sin(theta1);

%s3 is a bottle shaped strut

s3=t2/sin(theta2);
t3=s1*cos(theta1);
s7=s4*cos(theta2);
s6=(s5+s2*cos(theta1))/cos(theta3);
t4=t3+s2*cos(theta1);
t41=t5+s3*cos(theta2);
if (t4-t41)>0.1
    disp('error 1')
end

%check whether the calculations are correct

if abs(s3*sin(theta2)-s6*sin(theta3))- load2 > 0.1
    disp('error 2')
end

%nodal capacities and strut capacities

%node 1:CCT node, beta = 0.8

%reference ACI 318(2008) section A.5.2 - page 393

fcu1=0.75*0.85*0.8*fc;
%node 2: CCT node, beta = 0.8
fcu2 = 0.75 * 0.85 * 0.8 * fc;

%node 3: CTT node, beta = 0.6
fcu3 = 0.75 * 0.85 * 0.6 * fc;

%node 4: CCC node, beta = 1.0
fcu4 = 0.75 * 0.85 * 1.0 * fc;

%node 5: CCC node, beta = 1.0
fcu5 = 0.75 * 0.85 * 1.0 * fc;

%node 6: CCT node, beta = 0.8
fcu6 = 0.75 * 0.85 * 0.8 * fc;

%node 7: CTT node, beta = 0.6
fcu7 = 0.75 * 0.85 * 0.6 * fc;

%node 8: CCT node, beta = 0.8
fcu8 = 0.75 * 0.85 * 0.8 * fc;

%strut capacities
%strut 1 is a bottle shaped strut, beta = 0.75
reference ACI 318(2008) section A.3.2 - page 388
fc1 = 0.75 * 0.85 * 0.75 * fc;

%strut 2 is a bottle shaped strut, beta = 0.75
fc2 = 0.75 * 0.85 * 0.75 * fc;

%strut 3 is a bottle shaped strut, beta = 0.75
fc3 = 0.75 * 0.85 * 0.75 * fc;

%strut 4 is a bottle shaped strut, beta = 0.75
fc4 = 0.75 * 0.85 * 0.75 * fc;

%strut 5 is a prismatic strut, beta = 1.0
fc5 = 0.75 * 0.85 * 1.0 * fc;
% strut 6 is a prismatic strut, beta = 1.0
fc6=0.75*0.85*1.0*fc;

% strut 7 is a prismatic strut, beta = 1.0
fc7=0.75*0.85*1.0*fc;

% calculations for strut and tie widths
% calculation for tie width t3
 t3reqd=t4/(fcu3*b);
% calculation for strut width s1
 p=0;
 s1reqd=s1/(fc1*b);
 s1avail=sw*sin(theta1)+t3reqd*cos(theta1);
 if s1reqd <= s1avail
   p=p+1;
 end

% calculation for tie width t5
 t5reqd=t4/(fcu1*b);
% calculation for strut width 4
 s4reqd=s4/(fc4*b);
 s4avail=sw*sin(theta2)+t5reqd*cos(theta2);
 if s4reqd <= s4avail
   p=p+1;
 end

 s5reqd=s5/(b*fcu2);
 s6reqd=s6/(b*fc6);
 s2avail=sb*sin(theta1)+s6reqd*cos(theta1);
 s2reqd=s2/(b*fcu3);
if s2avail>s2reqd
    p=p+1;
end

s7reqd=s7/(b*fcu6);
s3avail=sb*sin(theta2)+s6reqd*cos(theta2);
s3reqd=s3/(b*fcu7);
if s3avail>s3reqd
    p=p+1;
end
t4reqd=t4/(b*fcu7);

%check whether the given dimensions fit in the beam

dmodel=d1+t4reqd*0.5+s6reqd*(0.5/cos(theta3));
if d2>d1
    dmodel=d2+t4reqd*0.5+s6reqd*0.5/cos(theta3);
end

%if the dmodel is greater than depth of beam, the value of height of
%truss is reduced for next iteration

Va=(load1*(span+sw-dist)+load2*dist1)/(span+sw);
Vb=load1+load2-Va;
shear=(0.75*10*((fc*1000)^(0.5))*b*(d1+0.5*s6reqd))/1000;
if d2>d1
    shear=(0.75*10*((fc*1000)^(0.5))*b*(d2+0.5*s6reqd))/1000;
end

if dmodel>depth-cover|| p<4
    y=y-0.0001;
else
fid1=fopen('reininput.txt','a');
fprintf(fid1,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f
%2f\n',fc,t4,theta1*(180*7/22),theta2*(180*7/22),t4reqd,b,load1,sw,dist,dist1);
q=q+1;
fclose(fid1);
ls1=(d1^2+(0.5*dist)^2)^0.5;
fid2=fopen('shear+strut1.txt','a');
fprintf(fid2,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f
%2f\n',load1,Va,theta1*(180*7/22),d1+0.5*s6reqd,b,dist,dist1);
fclose(fid2);
ls2=(d2^2+(0.5*dist1)^2)^0.5;
fid3=fopen('shear+strut2.txt','a');
fprintf(fid3,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f
%2f\n',load1,Vb,theta2*(180*7/22),d2+0.5*s6reqd,b,dist1,dist);
fclose(fid3);
m=1;
end
end
dist1=dist1+distanceincrement;
end
dist=dist+distanceincrement;
end
end
fid=fopen('qh.txt','a');
fprintf(fid,'%d
%2f\n',q);
fclose(fid);
%MATLAB program for Case 8 – Chapter 6. Main program that creates a STM.

clear all
clc

%input.txt is the input file
fid=fopen('input.txt');

%counter - load
q=0;

%number of input beam configuration in the input file
n=1;

for i=1:n
    z=textscan(fid,'%f %f %f %f %f %f %f %f %f %f',1);
    %concrete strength. (units - ksi)
    fc=z{1};
    %clear span of the beam (units - inches)
    span=z{2};
    %beam width (units - inches)
    b=z{3};
    %support width (units - inches)
    sw=z{4};
    %width of the base plate at load (units - inches)
    sb=z{5};
    %Cover to the longitudinal reinforcement (units - inches)
    cover=z{6};
    %Span to depth ratio
    ratio=z{7};
%Load 1 (units - kips)
load1=z{8};

%Load 2 (units - kips)
load2=z{9};

%increment in distance
distanceincrement=z{10};

%maximum case for deep beam
depth=span/ratio;

%this variable governs the load spacing
theta=65*22/(180*7);

%this is the first location of the load
dist=2*(floor(depth/tan(theta))+1);
while dist <=(0.5*(span+sw)-0.5*2.5*12-5) 
x1=dist/2;
y=0.99;
d1=y*(depth-cover);
dist1=2*(floor(depth/tan(theta))+1);
while dist1<=(0.5*(span+sw)-0.5*2.5*12-5) 
shear=0.75*10*(fc*1000)^0.5*d1*b/1000;
x2=dist1/2;
m=0;
y=0.99;
d1=y*(depth-cover);
Va=(load1*(span+sw-dist)+load2*dist1)/(span+sw);
Vb=load1+load2-Va;
ratio1=Vb*dist1/(Va*dist);
\[ \theta_1 = \arctan\left(\frac{d_1}{x_1}\right) \]

while \( m = 1 \&\& \theta_1 > \frac{(25*22)}{(180*7)} \)

\[ \text{ratio1} = \frac{V_b \cdot \text{dist1}}{(V_a \cdot \text{dist})} \]

if \( \text{ratio1} > 1 \)
\[ d_2 = y \cdot (\text{depth} - \text{cover}) \]
\[ d_1 = d_2 / \text{ratio1} \]
else
\[ d_1 = y \cdot (\text{depth} - \text{cover}) \]
\[ d_2 = \text{ratio1} \cdot d_1 \]
end

\% this part of the program calculates the forces in the truss members

\[ \theta_1 = \arctan\left(\frac{d_1}{x_1}\right) \]

\% this is the distance between the two loads
\[ l = \text{span} + \text{sw} - (\text{dist} + \text{dist1}) \]
\[ \theta_2 = \arctan\left(\frac{d_2}{x_2}\right) \]

if \( d_1 > d_2 \)
\[ \theta_3 = \arctan\left(\frac{d_1 - d_2}{l}\right) \]
else
\[ \theta_3 = \arctan\left(\frac{d_2 - d_1}{l}\right) \]
end

\% calculation of forces in the truss members

\% s1 is a bottle shaped strut
\[ s_1 = \frac{V_a}{\sin(\theta_1)} \]
\[ \phi_1 = \arctan(1/2) \]
\[ f_{b1} = s_1 / (2 \cdot \cos(\phi_1)) \]
\[ b_{t1} = s_1 / 4 \]
s4=Vb/sin(theta2);

t5=s4*cos(theta2);

s5=s1*cos(theta1);

t1=s1*sin(theta1);

t2=s4*sin(theta2);

s2=t1/sin(theta1);

%s3 is a bottle shaped strut

s3=t2/sin(theta2);

phi2=atan(1/2);

fb2=s3/(2*cos(phi2));

bt2=s3/4;

t3=s1*cos(theta1);

s7=s4*cos(theta2);

s6=(s5+s2*cos(theta1))/cos(theta3);

t4=t3+s2*cos(theta1);

t41=t5+s3*cos(theta2);

if (t4-t41)>0.1
    disp('error 1')
end

%check whether the calculations are correct

if abs(s3*sin(theta2)-s6*sin(theta3))- load2 > 0.1
    disp('error 2')
end

%nodal capacities and strut capacities

%node 1:CCT node, beta = 0.8

%reference ACI 318(2008) section A.5.2 - page 393
fcu1=0.75*0.85*0.8*fc;
%node 2: CCT node, beta = 0.8
fcu2=0.75*0.85*0.8*fc;
%node 3: CTT node, beta = 0.6
fcu3=0.75*0.85*0.6*fc;
%node 4: CCC node, beta = 1.0
fcu4=0.75*0.85*1.0*fc;
%node 5: CCC node, beta = 1.0
fcu5=0.75*0.85*1.0*fc;
%node 6: CCT node, beta = 0.8
fcu6=0.75*0.85*0.8*fc;
%node 7: CTT node, beta = 0.6
fcu7=0.75*0.85*0.6*fc;
%node 8: CCT node, beta = 0.8
fcu8=0.75*0.85*0.8*fc;

%strut capacities
%strut 1 is a bottle shaped strut, beta = 0.75
%reference ACI 318(2008) section A.3.2 - page 388
fc1=0.75*0.85*0.75*fc;
%strut 2 is a bottle shaped strut, beta = 0.75
fc2=0.75*0.85*0.75*fc;
%strut 3 is a bottle shaped strut, beta = 0.75
fc3=0.75*0.85*0.75*fc;
%strut 4 is a bottle shaped strut, beta = 0.75
fc4=0.75*0.85*0.75*fc;
%strut 5 is a prismatic strut, beta = 1.0
\begin{align*}
\text{fc5} &= 0.75 \times 0.85 \times 1.0 \times \text{fc}; \\
\% \text{ strut 6 is a prismatic strut, } \beta = 1.0 \\
\text{fc6} &= 0.75 \times 0.85 \times 1.0 \times \text{fc}; \\
\% \text{ strut 7 is a prismatic strut, } \beta = 1.0 \\
\text{fc7} &= 0.75 \times 0.85 \times 1.0 \times \text{fc}; \\
\% \text{ calculations for strut and tie widths} \\
\% \text{ calculation for tie width } t_3 \\
t_3\text{reqd} &= t_4 / (\text{fcu3} \times b); \\
\% \text{ calculation for strut width } s_1 \\
p &= 0; \\
s_1\text{reqd} &= s_1 / (\text{fc1} \times b); \\
s_1\text{avail} &= s_1 \times \sin(\theta_1) + t_3\text{reqd} \times \cos(\theta_1); \\
\text{if } s_1\text{reqd} \leq s_1\text{avail} \\
& \quad p = p + 1; \\
\% \text{ calculation for tie width } t_5 \\
t_5\text{reqd} &= t_4 / (\text{fcu1} \times b); \\
\% \text{ calculation for strut width } s_4 \\
s_4\text{reqd} &= s_4 / (\text{fc4} \times b); \\
s_4\text{avail} &= s_1 \times \sin(\theta_2) + t_5\text{reqd} \times \cos(\theta_2); \\
\text{if } s_4\text{reqd} \leq s_4\text{avail} \\
& \quad p = p + 1; \\
\text{s5reqd} &= s_5 / (b \times \text{fcu2}); \\
\text{s6reqd} &= s_6 / (b \times \text{fc6}); \\
\text{s2avail} &= s_1 \times \sin(\theta_1) + s_6\text{reqd} \times \cos(\theta_1); \\
\end{align*}
s2reqd=s2/(b*fcu3);
if s2avail>s2reqd
    p=p+1;
end

s7reqd=s7/(b*fcu6);
s3avail=sw*sin(theta2)+s6reqd*cos(theta2);
s3reqd=s3/(b*fcu7);
if s3avail>s3reqd
    p=p+1;
end

s4reqd=s4/(fc4*b);
s4avail=sw*sin(theta2)+t5reqd*cos(theta2);
t4reqd=t4/(b*fcu7);

%check whether the given dimensions fit in the beam
dmodel=d1+t4reqd*0.5+s6reqd*(0.5/cos(theta3));
if d2>d1
    dmodel=d2+t4reqd*0.5+s6reqd*(0.5/cos(theta3));
end

%if the dmodel is greater than depth of beam, the value of height of %truss is reduced for next iteration
Va=(load1*(span+sw-dist)+load2*dist1)/(span+sw);
Vb=load1+load2-Va;
shear=(0.75*10*((fc*1000)^(0.5))*b*(d1+0.5*s6reqd))/1000;
if d2>d1
    shear=(0.75*10*((fc*1000)^(0.5))*b*(d2+0.5*s6reqd))/1000;
if dmodel>(depth-cover)|| p<4
  
y=y-0.0001;
else
  fid1=fopen('reinfinput.txt','a');
  fprintf(fid1,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f 
\r\n',fc,t4,theta1*(180*7/22),theta2*(180*7/22),t4reqd,b,load1,dist,dist1,sw);
  q=q+1;
  fclose(fid1);
  ls1=(d1^2+(0.5*dist)^2)^0.5;
  fid2=fopen('strut1.txt','a');
  fprintf(fid2,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f 
\r\n',load1,Va,theta1*(180*7/22),b,dist,bt1,ls1,d1+0.5*s6reqd);
  fclose(fid2);
  ls2=(d2^2+(0.5*dist1)^2)^0.5;
  fid3=fopen('strut2.txt','a');
  fprintf(fid3,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f 
\r\n',load1,Vb,theta2*(180*7/22),b,dist1,bt2,ls2,d1+0.5*s6reqd);
  fclose(fid3);
  m=1;
end
end

dist1=dist1+distanceincrement;
end

dist=dist+distanceincrement;
end
fid=fopen('qh.txt','a');
fprintf(fid,'%d\n',q);
fclose(fid);

%MATLAB program for Case 9 – Chapter 7. Main program that creates a STM.
clear all
clc
q=0;
fid=fopen('input.txt');
%the data read from the input file i stored in column matrices for the
%each variable
%the value of n depends on the number of cases in the input file
n=1;
for i=1:n
    z=textscan(fid,'%f %f %f %f %f %f %f %f',1);
    %the data read from the input file is stored in column matrices for the
    %respective variables
    %concrete compressive strength (units - ksi)
    fc=z{1};
    %total depth of the beam (units - inches)
    d=z{2};
    %clear span of the beam (units - inches)
    l=z{3};
    %distance between the face of the left support and load
    span1=z{4};
%width of the beam (units - inches)
b=z{5};

%support width (units - inches)
sw=z{6};

%width of the base plate at load point (units - inches)
sb=z{7};

%clear cover provided to reinforcement (units - inches)
cover=z{8};

%Single point load on the beam
load=40;

span2=l-span1;

%Reaction at left support (refer Fig. 7.8).
Va=load*(span2+0.5*sw)/(l+sw);

%Reaction at right support (refer Fig. 7.8).
Vb=load-Va;

%factor to control truss height
y=0.99;

%height of the truss (center to center between the top prismatic strut
%and the bottom longitudinal tie)
d1=y*(d-cover);
theta1=atan(d1/(0.5*(span1+0.5*sw)));
theta2=atan(d1/(0.5*(span2+0.5*sw)));

m=0;
shear=(0.75*10*(fc*1000)^0.5*b*d1)/1000;
k=0;
while shear > Va && shear>Vb && k<1
m=0;

while m~1 && theta1>(25*22/(180*7)) && theta2>(25*22/(180*7))

d1=y*(d-cover);

%angle of inclination of the bottle shaped struts -S1 and S2 with respect to the
%bottom longitudinal tie (refer Table 7.1)
theta1=atan(d1/(0.5*(span1+0.5*sw)));

%angle of inclination of the bottle shaped struts -S3 and S4 with respect to the
%bottom longitudinal tie (refer Table 7.1)
theta2=atan(d1/(0.5*(span2+0.5*sw)));

%check for shear capacity of the section
shear=(0.75*10*(fc*1000)^0.5*b*d1)/1000;

if shear<Va || shear<Vb
    m=1;
    break
end

%S1 is a bottle shaped strut (refer Fig. 7.8)
bs1=Va/sin(theta1);

%S2 is a bottle shaped strut
bs2=Va/sin(theta1);

%Force in tie T3 (refer Fig. 7.8)
t3=bs1*cos(theta1);

%Force in tie T1 (refer Fig.7.8)
t1=Va;

%Strut S5 is a prismatic strut (refer Fig. 7.8)
s5=bs1*cos(theta1);

%BS4 is a bottle shaped strut (refer Fig. 7.8)
bs4 = Vb / sin(theta2);
%S3 is a bottle shaped strut
bs3 = Vb / sin(theta2);
%Strut S6 is a prismatic strut
s6 = bs4 * cos(theta2);
%Force in tie T5
t5 = bs4 * cos(theta2);
%force in tie T4
t4 = t3 + bs2 * cos(theta1);
%Computation for allowable stresses in struts and nodes
%Strut S1 is a bottle shaped strut
fc1 = 0.75 * 0.85 * 0.75 * fc;
%Strut S2 is a bottle shaped strut
fc2 = 0.75 * 0.85 * 0.6 * fc;
%Strut S3 is a bottle shaped strut
fc3 = 0.75 * 0.85 * 0.6 * fc;
%Strut S4 is a bottle shaped strut
fc4 = 0.75 * 0.85 * 0.75 * fc;
%strut S5 - prismatic strut. Node N7 is a CCT node. Beta of the node
%governs
fc5 = 0.75 * 0.85 * 0.8 * fc;
%Sturt S6 - prismatic strut. Node N5 is a CCT node. Beta of the node
%governs
fc6 = 0.75 * 0.85 * 0.8 * fc;
%Tie T3, T4 and T5 are anchored at nodes N2 and N3 which are CTT nodes. Beta of the node
%governs
ft4=0.75*0.85*0.6*fc;

% Computation for the widths of the individual struts and ties.

% Ties T3, T4 and T5 have uniform width.
wt4=t4/(ft4*b);

% Width of prismatic strut S5
ws5=s5/(fc5*b);

% Width of prismatic strut S6
ws6=s6/(fc6*b);

% Check for the anchorage of bottle shaped strut S1 at Support
% width required for the strut - strength consideration
ws1reqd=bs1/(fc1*b);

% Calculation of the width available for strut S1 at node N1
% (refer Fig. 7.9).
ws1avail=sw*sin(theta1)+wt4*cos(theta1);
P=0;
if ws1reqd<=ws1avail
p=p+1;
end

% Check for the anchorage of bottle shaped strut S4 at load
% width required for the strut - strength consideration
ws4reqd=bs4/(fc4*b);

% Calculation of the width available for strut S4 at node N4
% (refer Fig. 7.9).
ws4avail=sw*sin(theta2)+wt4*cos(theta2);
if ws4reqd<=ws4avail
p=p+1;
%check for the anchorage of bottle shaped strut S2 at load
%width required for the strut - strength consideration
ws2reqd=bs2/(fc2*b);
%calculation of the width available for strut S4 at node N4
%(refer Fig. 7.9).
x1=Va*sb/load;
x2=Vb*sb/load;
ws2avail=(x1*x1+ws5*ws5)^0.5;
if ws2reqd<= ws2avail
    p=p+1;
end

%check for the anchorage of bottle shaped strut S3 at load
%width required for the strut - strength consideration
ws3reqd=bs3/(fc3*b);
%calculation of the width available for strut S4 at node N4
%(refer Fig. 7.9).
ws3avail=(x2*x2+ws6*ws6)^0.5;
if ws3reqd<= ws3avail
    p=p+1;
end
%total height of the truss computed from STM calculations
dmodel=0.5*wt4+0.5*ws5+d1+cover;
shear=(0.75*10*(fc*1000)^0.5*b*(d1+0.5*ws5))/1000;
%check for the bottle shaped struts can be anchored at the supports
%and at the load location
k=0;

%Check if theta1 and theta2 are greater than 25 degrees
if theta1<0.4365 || theta2<0.4365
    k=1;
    m=1;
    break
end

%check for the shear capacity and compatibility of the
%truss with the physical dimensions of the beam
if p==4 && shear>=Va && shear>=Vb && dmodel<=d
    fid1=fopen('reinfinput.txt','a');
    fid2=fopen('shear+strut1.txt','a');
    fid3=fopen('shear+strut2.txt','a');
    fprintf(fid1,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f
\r\n',fc,t4,theta1*(180*7/22),theta2*(180*7/22),wt4,b,load,sw);
    fprintf(fid2,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f
\r\n',load,Va,theta1,d1+0.5*ws5,b,span1);
    fprintf(fid3,'%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f
\r\n',load,Vb,theta2,d1+0.5*ws5,b,span2);
    fclose(fid1);
    fclose(fid2);
    fclose(fid3);
    m=1;
    q=q+1;
else
    y=y-0.0001;
    if dmodel<=d && p<4
        m=1;
    end
end
k=1;
break;
end
end
end
load=load+10;

%Reaction at left support (refer Fig. 7.9)
Va=load*(span2+0.5*sw)/(l+sw);

%Reaction at right support (refer Fig. 7.9).
Vb=load-Va;
shear=(0.75*10*(fc*1000)^0.5*b*(d1+0.5*ws5))/1000;
end
end
fid=fopen('qh.txt','a');
fprintf(fid,'%d
',q);
close(fid);

%MATLAB program for Case 10 – Chapter 7. Main program that creates a STM.
clear all
clc
q=0;
fid=fopen('input.txt');
%the data read from the input file i stored in column matrices for the
%each variable
%the value of n depends on the number of cases in the input file
n=1;
for i=1:n
    z=textscan(fid,'%f %f %f %f %f %f %f %f',1);
    \%the data read from the input file is stored in column matrices for the
    \%respective variables
    \%concrete compressive strength (units - ksi)
    fc=z{1};
    \%total depth of the beam (units - inches)
    d=z{2};
    \%clear span of the beam (units - inches)
    l=z{3};
    \%distance between the face of the left support and load
    span1=z{4};
    \%width of the beam (units - inches)
    b=z{5};
    \%support width (units - inches)
    sw=z{6};
    \%width of the base plate at load point (units - inches)
    sb=z{7};
    \%clear cover provided to reinforcement (units - inches)
    cover=z{8};
    \%Single point load on the beam
    load=40;
    span2=l-span1;
    \%Reaction at left support (refer Fig. 7.9).
    Va=load*(span2+0.5*sw)/(l+sw);
    \%Reaction at right support (refer Fig. 7.9).
Vb=load-Va;

% factor to control truss height
y=0.99;

% height of the truss (center to center between the top prismatic strut
% and the bottom longitudinal tie)
d1=y*(d-cover);
m=0;
shear=(0.75*10*(fc*1000)^0.5*b*d1)/1000;
k=0;
while shear > Va && shear> Vb && k~=1
    m=0;
    while m~=1
        d1=y*(d-cover);
        % angle of inclination of the bottle shaped struts -BS1 and BS2 with respect to the
        % bottom longitudinal tie (refer Fig. 7.10)
        theta1=atan(d1/(0.5*(span1+0.5*sw)));
        % angle of inclination of the bottle shaped struts -BS3 and BS4 with respect to the
        % bottom longitudinal tie (refer Fig. 7.10)
        theta2=atan(d1/(0.5*(span2+0.5*sw)));
        % check for shear capacity of the section
        shear=(0.75*10*(fc*1000)^0.5*b*d1)/1000;
        if shear<Va || shear<Vb
            m=1;
            break
        end
    end
    % BS1 is a bottle shaped strut (refer Fig. 7.10)
bs1 = \frac{V_a}{\sin(\theta_1)};
\%\text{division of forces in the bottle shaped strut}

\phi_1 = \arctan(1/2);
\%\text{fb1 represents S1, and S3 from Fig. 3.3}

fb1 = \frac{bs1}{2 \cos(\phi_1)};
\%\text{fb2 represents S2 from Fig. 3.3}

fb2 = \frac{bs1}{2};
\%\text{bt1 represents T1 from Fig. 3.3}

bt1 = \frac{bs1}{4};
\%\text{BS2 is a bottle shaped strut}

bs2 = \frac{V_a}{\sin(\theta_1)};
\%\text{division of forces in the bottle shaped strut}

\phi_2 = \arctan(1/2);
\%\text{fb3 represents S1, and S3 from Fig. 3.3}

fb3 = \frac{bs2}{2 \cos(\phi_2)};
\%\text{fb4 represents S2 from Fig. 3.3}

fb4 = \frac{bs2}{2};
\%\text{bt2 represents T1 from Fig. 3.3}

bt2 = \frac{bs2}{4};
\%\text{Force in tie T3 (refer Fig. 7.10)}

t3 = bs1 \cos(\theta_1);
\%\text{Force in tie T1 (refer Fig. 7.10)}

t1 = V_a;

\%\text{Strut S5 is a prismatic strut (refer Fig. 7.10)}

s5 = bs1 \cos(\theta_1);
\%\text{BS4 is a bottle shaped strut (refer Fig. 7.10)}
bs4 = Vb / sin(theta2);
% division of forces in the bottle shaped strut
phi3 = atan(1/2);
% fb5 represents S1, and S3 from Fig. 3.3
fb5 = bs4 / (2*cos(phi3));
% fb6 represents S2 from Fig. 3.3
fb6 = bs4 / 2;
% bt3 represents T1 from Fig. 3.3
bt3 = bs4 / 4;
% BS3 is a bottle shaped strut
bs3 = Vb / sin(theta2);
% division of forces in the bottle shaped strut
phi4 = atan(1/2);
% fb7 represents S1, and S3 from Fig. 3.3
fb7 = bs3 / (2*cos(phi3));
% fb8 represents S2 from Fig. 3.3
fb8 = bs3 / 2;
% bt4 represents T1 from Fig. 3.3
bt4 = bs3 / 4;
% Strut S6 is a prismatic strut
s6 = bs4 * cos(theta2);
% Force in tie T5
t5 = bs4 * cos(theta2);
% force in tie T4
t4 = t3 + bs2 * cos(theta1);
% Computation for allowable stresses in struts and nodes
Strut BS1 is a bottle shaped strut
\[ f_{c1} = 0.75 \times 0.85 \times 0.75 \times f_c; \]

Strut BS2 is a bottle shaped strut
\[ f_{c2} = 0.75 \times 0.85 \times 0.75 \times f_c; \]

Strut BS3 is a bottle shaped strut
\[ f_{c3} = 0.75 \times 0.85 \times 0.75 \times f_c; \]

Strut BS4 is a bottle shaped strut
\[ f_{c4} = 0.75 \times 0.85 \times 0.75 \times f_c; \]

Strut S5 - prismatic strut. Node N7 is a CCT node. Beta of the node governs
\[ f_{c5} = 0.75 \times 0.85 \times 0.8 \times f_c; \]

Strut S6 - prismatic strut. Node N5 is a CCT node. Beta of the node governs
\[ f_{c6} = 0.75 \times 0.85 \times 0.8 \times f_c; \]

Tie T3, T4 and T5 are anchored at nodes N2 and N3 which are CTT nodes. Beta of the node governs
\[ f_{t4} = 0.75 \times 0.85 \times 0.6 \times f_c; \]

Computation for the widths of the individual struts and ties.
Ties T3, T4 and T5 have uniform width.
\[ w_{t4} = t_4 / (f_{t4} \times b); \]

Width of prismatic strut S5
\[ w_{s5} = s_5 / (f_{c5} \times b); \]

Width of prismatic strut S6
\[ w_{s6} = s_6 / (f_{c6} \times b); \]

Check for the anchorage of bottle shaped strut BS1 at Support
Width required for the strut - strength consideration
ws1reqd = bs1/(fc1*b);

%calculation of the width available for strut BS1 at node N1
%(refer Fig. 7.10).
ws1avail = sw*sin(theta1) + wt4*cos(theta1);
p = 0;
if ws1reqd <= ws1avail
    p = p + 1;
end

%check for the anchorage of bottle shaped strut BS4 at load
%width required for the strut - strength consideration
ws4reqd = bs4/(fc4*b);
%calculation of the width available for strut BS4 at node N4
%(refer Fig. 7.10).
ws4avail = sw*sin(theta2) + wt4*cos(theta2);
if ws4reqd <= ws4avail
    p = p + 1;
end

%check for the anchorage of bottle shaped strut BS2 at load
%width required for the strut - strength consideration
ws2reqd = bs2/(fc2*b);
%calculation of the width available for strut BS4 at node N4
%(refer Fig. 7.10).
x1 = Va*sb/load;
x2 = Vb*sb/load;
ws2avail = (x1*x1 + ws5*ws5)^0.5;
if ws2reqd <= ws2avail
\begin{verbatim}
p=p+1;
end

%check for the anchorage of bottle shaped strut BS3 at load
%width required for the strut - strength consideration
ws3reqd=bs3/(fc3*b);
%calculation of the width available for strut BS4 at node N4
%(refer Fig. 7.10).
ws3avail=(x2*x2+ws6*ws6)^0.5;
if ws3reqd<= ws3avail
    p=p+1;
end

%total height of the truss computed from STM calculations
dmodel=0.5*wt4+0.5*ws5+d1+cover;
shear=(0.75*10*(fc*1000)^0.5*b*(d1+0.5*ws5))/1000;
%check for the bottle shaped struts can be anchored at the supports
%and at the load location
%check for the shear capacity and compatibility of the
%truss with the physical dimensions of the beam
if p==4 && shear>=Va && shear>=Vb && dmodel<=d
    fid1=fopen('reinfinput.txt','a');
    fid2=fopen('strut1.txt','a');
    fid3=fopen('strut2.txt','a');
    %Check if theta1 and theta2 are greater than 25 degress
    if theta1 >=(25*22/(180*7)) && theta2 >=0.0442
        sl1=(d1*d1+((0.5*(span1+0.5*sw))*(0.5*(span1+0.5*sw)))/0.5;
        sl2=(d1*d1+((0.5*(span2+0.5*sw))*(0.5*(span2+0.5*sw)))/0.5;
\end{verbatim}
fprintf(fid1, '%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f
\r\n', fc, t4, theta1 * (180 * 7 / 22), theta2 * (180 * 7 / 22), wt4, b, load, sw);
fprintf(fid2, '%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f
\r\n', load, Va, theta1 * (180 * 7 / 22), b, span1, bt1, sl1, d1 + 0.5 * ws5);
fprintf(fid3, '%.2f %.2f %.2f %.2f %.2f %.2f %.2f %.2f
\r\n', load, Vb, theta2 * (180 * 7 / 22), b, span2, bt4, sl2, d1 + 0.5 * ws5);
fclose(fid1);
fclose(fid2);
fclose(fid3);
m = 1;
q = q + 1;
end
else
y = y - 0.001;
if p < 4 && dmodel <= d
k = 1;
m = 1;
break
end
end
end
load = load + 10;

% Reaction at left support (refer Fig. 7.9)
Va = load * (span2 + 0.5 * sw) / (l + sw);
% Reaction at right support (refer Fig. 7.9).
Vb = load - Va;
shear=(0.75*10*(fc*1000)^0.5*b*(d1+0.5*ws5))/1000;

end
end

fid4=fopen('qh.txt','a');
fprintf(fid4,'%d
',q);
close(fid4);

%MATLAB program for detailing the longitudinal reinforcement.
clear all

%area of reinforcement required
fid2=fopen('qh.txt');
h=textscan(fid2,'%d',1);
n=h{1};
close(fid2);

fid=fopen('reinfinput.txt');
for i=1:n
    c=textscan(fid,'%f %f %f %f %f %f %d %f',1);
    %concrete strength (ksi)
    fc=c{1};
    %force in the longitudinal tie
    f7=c{2};
    %angle of inclination of strut in the left shear span
    theta1=c{3};
    %angle of inclination of the strut in the right shear span
    theta2=c{4};
    %width of the bottom longitudinal tie (inches)
w7reqd=c{5};
%width of the beam (inches)
b=c{6};
%load on the beam (kips)
load=c{7};
fy=60;
%support width (inches)
sw=c{8};
ast=f7/(0.75*fy);
%length available to develop reinforcement
stravl1=sw-2+(0.5*w7reqd/(tan(theta1*22/(180*7))));
stravl2=sw-2+(0.5*w7reqd/(tan(theta2*22/(180*7))));
%check development length at both the supports
if stravl1>stravl2
    stravl=stravl2;
else
    stravl=stravl1;
end
%diameter of each bar
d6=0.75;
d7=0.875;
d8=1;
d9=1.28;
%area of each bar
a6=0.44;
a7=0.6;
a8 = 0.79;
a9 = 1;

% number of bars of each size required
n6 = floor(ast/a6) + 1;
n7 = floor(ast/a7) + 1;
n8 = floor(ast/a8) + 1;
n9 = floor(ast/a9) + 1;

% straight development length for each bar
ld6 = fy * 1000 * d6 / (25 * (1000 * fc)^0.5);
ld7 = fy * 1000 * d7 / (20 * (1000 * fc)^0.5);
ld8 = fy * 1000 * d8 / (20 * (1000 * fc)^0.5);
ld9 = fy * 1000 * d9 / (20 * (1000 * fc)^0.5);

% hooked development length for each bar - assume 2.5in cover min. hence
% factor 0.7 appears in the equation. refer clause 12.5.3.(a)
ldh6 = 0.7 * 0.02 * fy * 1000 * d6 / ((1000 * fc)^0.5);
ldh7 = 0.7 * 0.02 * fy * 1000 * d7 / ((1000 * fc)^0.5);
ldh8 = 0.7 * 0.02 * fy * 1000 * d8 / ((1000 * fc)^0.5);
ldh9 = 0.7 * 0.02 * fy * 1000 * d9 / ((1000 * fc)^0.5);

% maximum number of bars per layer. assume no 5 bar as the stirrup and 2in
% cover for the side. minimum 1in space between each bar
spaceavailable = b - 2 * 2 - 2 * (5/8);

nf6 = round((spaceavailable + 1) / (d6 + 1));
max6 = nf6 * 3;

% rem(nf6, 2);
nf7 = round((spaceavailable + 1) / (d7 + 1));
max7 = nf7 * 3;
\%r7 = \text{rem}(nf7,2);

nf8 = \text{round}((\text{spaceavailable}+1)/(d8+1));

max8 = nf8*3;

\%r8 = \text{rem}(nf8,2);

nf9 = \text{round}((\text{spaceavailable}+1)/(d9+1));

max9 = nf9*3;

\%r9 = \text{rem}(nf9,2);

\%maximum number of layers that can be fit into the available width of the
\%tie. 1.5in bottom cover and 1in space between two layers.

\%3 layers of reinforcement

w36 = 2+2*d6;

if w36 < w7\text{reqd}
    n36 = nf6*3;
else
    n36 = 0;
end

w37 = 2+2*d7;

if w37 < w7\text{reqd}
    n37 = nf7*3;
else
    n37 = 0;
end

w38 = 2+2*d8;

if w38 < w7\text{reqd}
    n38 = nf8*3;
else

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n38 = 0;
end
w39 = 2 + 2 * d9;
if w39 < w7reqd
    n39 = nf9 * 3;
else
    n39 = 0;
end
% 2 layers of reinforcement
w26 = 1 + 2 * d6;
if w26 < w7reqd
    n26 = nf6 * 2;
else
    n26 = 0;
end
w27 = 1 + 2 * d7;
if w27 < w7reqd
    n27 = nf7 * 2;
else
    n27 = 0;
end
w28 = 1 + 2 * d8;
if w28 < w7reqd
    n28 = nf8 * 2;
else
    n28 = 0;
end

w29=1+2*d9;

if w29<w7reqd
    n29=nf9*2;
else
    n29=0;
end

%single layer of reinforcement
w16=1.5+0.5*d6;
w17=1.5+0.5*d7;
w18=1.5+0.5*d8;
w19=1.5+0.5*d9;

%start optimisation
m=0;
while m~= 1
    %check one layer of #6 bar. straight developed
    if n6<nf6 && stravl> ld6
        m=1;
        ans1=6;
        ans2=1;
        ans3=n6;
        ans4=nf6;
        break
    end
    %check one layer of #7 bar. straight developed
    if n7<nf7 && stravl> ld7
        m=1;
        ans1=7;
        ans2=1;
        ans3=n7;
        ans4=nf7;
        break
    end
end

%start optimisation
m=1;
ans1=7;
ans2=1;
ans3=n7;
ans4=nf7;
break
end

%check one layer of #8 bar. straight developed
if n8<nf8 && stravl> ld8
    m=1;
    ans1=8;
    ans2=1;
    ans3=n8;
    ans4=nf8;
    break
end

%check one layer of #9 bar. straight developed
if n9<nf9 && stravl> ld9
    m=1;
    ans1=9;
    ans2=1;
    ans3=n9;
    ans4=nf9;
    break
end

%check two layer of #6 bar. straight developed
if n6<n26 && stravl> ld6
    m=1;
    ans1=6;
    ans2=2;
    ans3=n6;
    ans4=nf6;
    break
end
%check two layer of #7 bar. straight developed
if n7<n27 && stravl> ld7
    m=1;
    ans1=7;
    ans2=2;
    ans3=n7;
    ans4=nf7;
    break
end
%check two layer of #8 bar. straight developed
if n8<n28 && stravl> ld8
    m=1;
    ans1=8;
    ans2=2;
    ans3=n8;
    ans4=nf8;
    break
end
%check two layer of #9 bar. straight developed

if n9<n29 && stravl> ld9
    m=1;
    ans1=9;
    ans2=2;
    ans3=n9;
    ans4=nf9;
    break
end

%check three layer of #6 bar. straight developed

if n6<n36 && stravl> ld6
    m=1;
    ans1=6;
    ans2=3;
    ans3=n6;
    ans4=nf6;
    break
end

%check three layer of #7 bar. straight developed

if n7<n37 && stravl> ld7
    m=1;
    ans1=7;
    ans2=3;
    ans3=n7;
    ans4=nf7;
    break
end

%check three layer of #8 bar. straight developed
if n8<n38 && stravl> ld8
    m=1;
    ans1=8;
    ans2=3;
    ans3=n8;
    ans4=nf8;
    break
end

%check three layer of #9 bar. straight developed
if n9<n39 && stravl> ld9
    m=1;
    ans1=9;
    ans2=3;
    ans3=n9;
    ans4=nf9;
    break
end

%check one layer of #6 bar. 90 degree hooked
if n6<nf6 && stravl> ldh6
    m=1;
    ans1=6;
    ans2=11;
    ans3=n6;
    ans4=nf6;
break
end

%check two layer of #6 bar. 90 degree hooked
if n6<n26 && stravl> (ldh6+1+d6)
    m=1;
    ans1=6;
    ans2=21;
    ans3=n6;
    ans4=nf6;
    break
end

%check three layer of #6 bar. 90 degree hooked
if n6<n36 && stravl> (ldh6+2+2*d6) && (n36-1)<=max6
    m=1;
    ans1=6;
    ans2=31;
    ans3=n6;
    ans4=nf6;
    break
end

%check one layer of #7 bar. 90 degree hooked
if n7<nf7 && stravl> ldh7
    m=1;
    ans1=7;
    ans2=11;
    ans3=n7;
ans4=nf7;
break
end
%check two layer of #7 bar. 90 degree hooked
if n7<n27 && stravl> (ldh7+1+d7)
    m=1;
    ans1=7;
    ans2=21;
    ans3=n7;
    ans4=nf7;
    break
end
%check three layer of #7 bar. 90 degree hooked
if n7<n37 && stravl> (ldh7+2+2*d7) && (n37-1)<=max7
    m=1;
    ans1=7;
    ans2=31;
    ans3=n7;
    ans4=nf7;
    break
end
%check one layer of #8 bar. 90 degree hooked
if n8<nf8 && stravl> ldh8
    m=1;
    ans1=8;
    ans2=11;
ans3=n8;
ans4=nf8;
break
end
%check two layer of #8 bar. 90 degree hooked
if n8<n28 && stravl> (ldh8+1+d8)
  m=1;
  ans1=8;
  ans2=21;
  ans3=n8;
  ans4=nf8;
  break
end

%check three layer of #8 bar. 90 degree hooked
if n8<n38 && stravl> (ldh8+2+d8) && (n38-1)<=max8
  m=1;
  ans1=8;
  ans2=31;
  ans3=n8;
  ans4=nf8;
  break
end

%check one layer of #9 bar. 90 degree hooked
if n9<nf9 && stravl> ldh9
  m=1;
  ans1=9;
ans2=11;
ans3=n9;
ans4=nf9;
break
end

%check two layer of #9 bar. 90 degree hooked
if n9<n29 && stravl> (ldh9+1+d9)
    m=1;
    ans1=9;
    ans2=21;
    ans3=n9;
    ans4=nf9;
    break
end

%check three layer of #9 bar. 90 degree hooked
if n9<n39 && stravl> (ldh9+2+2*d9) && (n39-1)<=max9
    m=1;
    ans1=9;
    ans2=31;
    ans3=n9;
    ans4=nf9;
    break
else
    ans1=0;
    ans2=0;
    ans3=0;
ans4=0;
m=1;
end
end
fid1=fopen('reinfoutput.txt','a');
format short
ans5=w7reqd/2;
fprintf(fid1,'%d %d %d %d %.2f %d %d
',ans1,ans2,ans3,ans4,ans5,b,load);
end
fclose(fid1);
fclose(fid);

%MATLAB program for bottle shaped strut + tie reinforcement (Normal strength concrete).
clear all
clc
%calculation of reinforcement for the bottle shaped strut
%area of reinforcing rebar
area=0.11;
fid4=fopen('qh.txt');
z=textscan(fid4,'%.2f',1);
n=z{1};
fclose(fid4);
fclose(fid);
for i=1:n
    area=0.11;
y=textscan(fid,%2f %2f %2f %2f %2f,1);
end
% Load is required for the calculation of the shear reinforcement
load=y{1};

% angle of inclination for the bottle shaped strut
theta1=y{2};

% shear depth of the beam
d1=y{3};

% width of the beam
b=y{4};

% shear span
shearspan =y{5};

% design of vertical rebar
m=0;

% two number of legs for the vertical and horizontal reinforcement
p=1;
s1=d1/5;
if s1>12
  sv=12;
else
  sv=floor(s1)+1;
  maxsv=sv;
  % the spacing of the vertical rebar should be an even number
  if rem(sv,2) ~=0
    sv=sv-1;
  
  maxsv=sv;
  end
end
end
while m==1

% maximum allowable spacing as per the code requirements

sum=(p*area*sin(theta1*22/(180*7)))/(b*sv);

if sum >= 0.0015
    m=1;
    break
else
    sv=sv-2;
end

if sv<4
    area=area+0.01;
    sv=maxsv;
end

if sv<4 && area>0.31
    m=1;
    disp('error - vertical reinforcement')
end
end

% two number of legs for the vertical and horizontal reinforcement

areah=0.11;
p=1;
s1=d1/5;

if s1>12
    sh=12;
else
    sh=floor(s1)+1;
maxsh=sh;

% the spacing of the horizontal rebar should be an even number
if rem(sh,2) ~= 0
    sh=sh-1;
    maxsh=sh;
end
end
m=0;
while m~=1
    % maximum allowable spacing as per the code requirements
    sum=(p*areah*cos(theta1*22/(180*7)))/(b*sh);
    if sum >= 0.0015
        m=1;
        break
    else
        sh=sh-2;
        if load == 200
            disp(sh)
            disp(sum)
        end
    end
    if sh<4
        areah=areah+0.001;
        sh=maxsh;
    end
    if sh<4 && areah>0.31

m = 1;

disp('error - horizontal reinforcement')

end

end

if areah <= 2 * 0.11
    ans1 = 3;
    ans2 = 2;
end

if areah <= 2 * 0.2 && areah > 2 * 0.11
    ans1 = 4;
    ans2 = 2;
end

if areah <= 4 * 0.11 && areah > 2 * 0.2
    ans1 = 3;
    ans2 = 4;
end

if areah <= 2 * 0.31 && areah > 4 * 0.11
    ans1 = 5;
    ans2 = 2;
end

if areah <= 4 * 0.2 && areah > 2 * 0.31
    ans1 = 4;
    ans2 = 4;
end

if areah <= 4 * 0.31 && areah > 4 * 0.2
    ans1 = 5;

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ans2=4;

end

fid2=fopen('horizontalreinf.txt','a');
fprintf(fid2,'%d %d %d %d \
', load,ans1,ans2,sh);
%calculation for the shear reinforcement required to resist the tension in
%the vertical tie
%shear reinforcement required
as=load/(0.75*60);
%total area for shear + bottle shaped strut
atot=as+2*area;
%spacing for the combined reinforcement should be similar to the spacing
%adopted for bottle shaped strut
a32=0.11*2;
a34=0.11*4;
a42=0.2*2;
a44=0.2*4;
a52=0.31*2;
a54=0.31*4;
% #3 rebar, 2 legs
num=atot/a32;
sv=shearspan/num;
if sv>maxsv
    sv=maxsv;
end
svf=floor(sv)+1;
s=0;
if svf >= 5 && s==0
    ans1=3;
    ans2=2;
    s=1;
    if rem(floor(svf),2)==0
        ans3=svf;
    else
        ans3=svf-1;
    end
end
% #3 rebar, 4 legs
num=atot/a34;
sv=shearspan/num;
if sv>maxsv
    sv=maxsv;
end
svf=floor(sv)+1;
if svf >= 5 && s==0
    ans1=3;
    ans2=4;
    s=1;
    if rem(floor(svf),2)==0
        ans3=svf;
    else
        ans3=svf-1;
    end
end
end

% #4 rebar, 2 legs

num=atot/a42;

sv=shearspan/num;

if sv>maxsv
    sv=maxsv;
end

svf=floor(sv)+1;

if svf >= 5 && s==0
    ans1=4;
    ans2=2;
    s=1;
    if rem(floor(sv),2)==0
        ans3=svf;
    else
        ans3=svf-1;
    end
end

end

% #4 rebar, 4 legs

num=atot/a44;

sv=shearspan/num;

if sv>maxsv
    sv=maxsv;
end

svf=floor(sv)+1;
if svf >= 5 && s==0
    ans1=4;
    ans2=4;
    s=1;
    if rem(floor(svf),2)==0
        ans3=svf;
    else
        ans3=svf-1;
    end
end

% #5 rebar, 2 legs
num=atot/a52;
sv=shearspan/num;
if sv>maxsv
    sv=maxsv;
end
svf=floor(sv)+1;

if svf >= 5 && s==0
    ans1=5;
    ans2=2;
    s=1;
    if rem(floor(svf),2)==0
        ans3=svf;
    else
        ans3=svf-1;
    end
% #5 rebar, 4 legs
num=a54/atot;
sv=shearspan/num;
if sv>maxsv
  sv=maxsv;
end
svf=floor(sv)+1;
if svf >= 5 && s==0
  ans1=5;
  ans2=4;
  s=1;
  if rem(floor(svf),2)==0
    ans3=svf;
  else
    ans3=svf-1;
  end
end
fid4=fopen('vertical+shear.txt','a');
fprintf(fid4,'%d %d %d %d ',load,ans1,ans2,ans3);
end

%MATLAB program for bottle shaped strut + tie reinforcement (High strength concrete).
clear all
close all
cle
fid1=fopen('qh.txt');

z=textscan(fid1,'%.2f',1);

n=z{1};
fclose(fid1);

fid=fopen('strut1plustie.txt');

for i=1:n
    y=textscan(fid,'%f %f %f %f %f %f %f %f',1);
    pointload=y{1};
    %Reaction at left support - required to calculate the shear
    %reinforcement in the shear span
    load=y{2};
    %inclination of the strut
    theta1=y{3};
    %width of the beam
    b1=y{4};
    shearspan=y{5};
    %tension force in the tie
    t=y{6};
    %length of the strut
    l=y{7};
    %distance between the top compression fiber and the C.G. of
    %longitudinal reinforcement
    d1=y{8};
    %total tensile force in the bottle shaped strut = twice the force in
    %each tie
    f=2*t;

fclose(fid);

%yield strength of the reinforcement
fy=60;
s1=d1/5;
if s1>=12
    sh=12;
    sv=12;
    maxsh=sh;
    maxsv=sv;
else
    sh=floor(s1)+1;
    sv=floor(s1)+1;
    maxsh=sh;
    maxsv=sv;
    if rem(sh,2)~=0
        sh=sh-1;
        maxsh=sh;
    end
    if rem(sv,2)~=0
        sv=sv-1;
        maxsv=sv;
    end
end

%splitting force in the horizontal direction
fh=f*sin(theta1*22/(180*7));

%splitting force in the vertical direction
fv=f*cos(theta1*22/(180*7));
% calculation for the tensile force resisting reinforcement in the
% horizontal direction

areah = 0.11;

while areah <= 0.31 * 4;
    while sh > = 4;
        ph = areah / (b1 * sh);
        while ph * sin(theta1 * 22 / (180 * 7)) < 0.0015
            if sh <= 4
                areah = areah + 0.01;
                sh = maxsh;
            else
                sh = sh - 2;
            end
            ph = areah / (b1 * sh);
        end
        rh = 0.75 * ph * fy * b1 * l;
    end
    if rh >= fh
        if areah <= 2 * 0.11
            ans1 = 3;
            ans2 = 2;
        end
        if areah <= 2 * 0.2 && areah > 2 * 0.11
            ans1 = 4;
            ans2 = 2;
        end
        if areah <= 4 * 0.11 && areah > 2 * 0.2

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ans1=3;
ans2=4;
end

if areah <= 2*0.31 && areah > 4*0.11
    ans1=5;
    ans2=2;
end

if areah <= 4*0.2 && areah > 2*0.31
    ans1=4;
    ans2=4;
end

if areah <= 4*0.31 && areah > 4*0.2
    ans1=5;
    ans2=4;
end

fid2=fopen('horizontalreinf1.txt','a');
fprintf(fid2,'%.2f %d %d %d
', pointload,ans1,ans2,sh);
fclose(fid2);
break

else
    sh=sh-2;
end
end

if rh>fh
    break
end
sh=maxsh;
areah=areah+0.01;
end
%calculation for the tensile force resisting reinforcement in the
%vertical direction
areav=fv/(0.75*fy);
%the ratio of vertical reinforcement should be more than 0.0015
sv=areav*cos(theta1*(22/(180*7)))/(0.0015*b1);
while areav*cos(theta1*(22/(180*7)))/(sv*b1)<0.0015
    areav=areav+0.01;
    sv=areav*cos(theta1*(22/(180*7)))/(0.0015*b1);
if sv<4 || sv> maxsv
    areav=areav+0.01;
    sv=maxsv;
end
end
%combination of the vertical reinforcement and the shear reinforcement
%shear in the beam equals the load on the beam
%total area of reinforcement required
av=load/(0.75*fy);
%additional area for shear reinforcement per bar
atot=av+2*areav;
a32=0.11*2;
a34=0.11*4;
a42=0.2*2;
a44=0.2*4;
\[ a_{52} = 0.31 \times 2; \]
\[ a_{54} = 0.31 \times 4; \]

% #3 rebar, 2 legs

\[ \text{num} = \text{atot} / a_{32}; \]
\[ \text{sv1} = \text{shearspan} / \text{num}; \]

if \( \text{sv1} > \text{sv} \)

\[ \text{sv1} = \text{sv}; \]

end

if \( \text{sv1} > \text{maxsv} \)

\[ \text{sv1} = \text{maxsv}; \]

else

\[ \text{sv1} = \text{shearspan} / \text{num}; \]

end

\[ \text{svf} = \text{floor}(\text{sv1}) + 1; \]
\[ s = 0; \]

if \( \text{svf} \geq 5 \) \&\& \( s = 0 \)

\[ \text{ans1} = 3; \]
\[ \text{ans2} = 2; \]
\[ s = 1; \]

if rem(floor(svf), 2) == 0

\[ \text{ans3} = \text{svf}; \]

else

\[ \text{ans3} = \text{svf} - 1; \]

end

end

% #4 rebar, 2 legs
num=atot/a42;  
sv1=shearspan/num;  
if sv1>sv  
    sv1=sv;  
end  
if sv1>maxsv  
    sv1=maxsv;  
else  
    sv1=shearspan/num;  
end  
svf=floor(sv1)+1;  
if svf >= 5 && s==0  
    ans1=4;  
    ans2=2;  
    s=1;  
    if rem(floor(svf),2)==0  
        ans3=svf;  
    else  
        ans3=svf-1;  
    end  
end  

% #3 rebar, 4 legs  
num=atot/a34;  
sv1=shearspan/num;  
if sv1>sv  
    sv1=sv;  
end
end

if sv1>maxsv
    sv1=maxsv;
else
    sv1=shearspan/num;
end

svf=floor(sv1)+1;
if svf >= 5 && s==0
    ans1=3;
    ans2=4;
    s=1;
    if rem(floor(svf),2)==0
        ans3=svf;
    else
        ans3=svf-1;
    end
end

end

% #4 rebar, 4 legs
num=atot/a44;
sv1=shearspan/num;
if sv1>sv
    sv1=sv;
end

if sv1>maxsv
    sv1=maxsv;
else
sv1 = shearspan/num;
end
svf = floor(sv1) + 1;
if svf >= 5 && s == 0
    ans1 = 4;
    ans2 = 4;
    s = 1;
    if rem(floor(svf), 2) == 0
        ans3 = svf;
    else
        ans3 = svf - 1;
    end
end
% #5 rebar, 2 legs
num = atot/a52;
sv1 = shearspan/num;
if sv1 > sv
    sv1 = sv;
end
if sv1 > maxsv
    sv1 = maxsv;
else
    sv1 = shearspan/num;
end
svf = floor(sv1) + 1;
if svf >= 5 && s == 0
ans1=5;
ans2=2;
s=1;
if rem(floor(svf),2)==0
    ans3=svf;
else
    ans3=svf-1;
end
end

% #5 rebar, 4 legs
num=atot/a54;
sv1=shearspan/num;
if sv1>sv
    sv1=sv;
end
if sv1>maxsv
    sv1=maxsv;
else
    sv1=shearspan/num;
end
svf=floor(sv1)+1;
if svf >= 5 && s==0
    ans1=5;
    ans2=4;
    s=1;
    if rem(floor(svf),2)==0
ans3=svf;
else
    ans3=svf-1;
end

fid4=fopen('shearbot1.txt','a');
fprintf(fid4,'%.2f %d %d %d \
',pointload,ans1,ans2,ans3);
end