I, Fady Bishara, hereby submit this original work as part of the requirements for the degree of Master of Science in Mechanical Engineering.

It is entitled:
Numerical Simulation of Fully Developed Laminar Flow and Heat Transfer in Isothermal Helically Twisted Tubes with Elliptical Cross-Sections

Student's name: Fady Bishara

This work and its defense approved by:

Committee chair: Milind JogPhD
Committee member: Raj ManglikPhD
Committee member: Shaaban AbdallahPhD
Numerical Simulation of Fully Developed Laminar Flow and Heat Transfer in Isothermal Helically Twisted Tubes with Elliptical Cross-Sections

A thesis submitted to the
Graduate School
of the University of Cincinnati
in partial fulfilment of the
requirements for the degree of
Master of Science
in the Department of Mechanical Engineering
of the College of Engineering and Applied Science
by
Fady Bishara
Bachelor of Mechanical Engineering
Cleveland State University
May 2002

Committee Chair: M. A. Jog, Ph.D.
Committee Co-Chair: R. M. Manglik, Ph.D.
Abstract

Periodically fully-developed swirling laminar flows in twisted tubes with elliptical cross sections are numerically simulated. The tubes are helically twisted along the axis perpendicular to their cross-section. The helical twist geometry is described by the $180^\circ$ twist ratio and the elliptical cross-section is described by the ellipse aspect ratio. The geometries considered for this study have twist ratios with values 3, 4, 5, 6 and ellipse aspect ratios with values 0.3, 0.5, 0.7. Constant-property flow with a nominal Prandtl number of 3 (e.g. water at 60°C) and Reynolds numbers in the range $10 \leq Re \leq 1000$ was considered. The analysis quantifies the improvement in the Nusselt number as well as the increase in friction factor in order to map the effective heat transfer enhancement due to the twisted-tube-geometry-induced swirl flows. To this effect, the numerical results are compared, for a given aspect ratio, with the case of a straight elliptical tube (i.e. twist ratio tends to infinity) for which well established correlations are available. The results were also compared with those of circular tubes with twisted tape inserts for any given twist ratio. Numerical results show that the friction factor and the Nusselt number are strong functions of the twist ratio, aspect ratio and the Reynolds number. The increase in the friction factor and Nusselt number is higher for more tightly twisted tubes and for tubes with a lower ellipse aspect ratio. For Reynolds numbers below 100, the heat transfer results do not deviate significantly from the straight-tube values, but at higher values of Re, significant enhancement in heat transfer is evident for all twist ratios considered here. For example, for tubes with aspect ratio $\alpha = 0.3$ and a twist ratio $y = 3$, the enhancement of the Nusselt number relative to the straight tube Nusselt number $\frac{Nu}{Nu_{y=\infty}} = 2.7$. For the same aspect ratio, but with a twist ratio $y = 6$, the enhancement $\frac{Nu}{Nu_{y=\infty}} = 2$. On the other hand, for nearly round cross sections
(e.g. $\alpha = 0.7$) and nearly straight tubes (e.g. $y = 6$) the enhancement \( \text{Nu}/\text{Nu}_{y=\infty} \sim 1 \); i.e. there is no enhancement. The friction factor and Nusselt number results provided in this paper will give practicing engineers the necessary data to integrate twisted elliptical tubes in various heat transfer applications.
Acknowledgements

I would like to thank Dr. Milind Jog and Dr. Raj Manglik for their guidance and suggestions throughout the course of my research. This work is more refined due to their constant feedback. I would also like to thank Dr. Shaaban Abdallah for his support and patience and for asking many good questions.

As any graduate student knows, the working hours are very long. Those long hours were pleasantly broken up thanks to all of my colleagues at the Thermal-Fluids and Thermal Processing Laboratory. In particular, I would like to thank Omar Huzayyin, Gabriel Wickizer, Deepak Veettil and Advait Athavale for their camaraderie.

Last but not least, I would like to thank Bekele Badada and all of my friends in the physics department for their support and encouragement during the last phase of the writing process.
# Contents

Abstract ................................................................. ii

Acknowledgements ...................................................... iv

List of Tables .......................................................... viii

List of Figures .......................................................... ix

Nomenclature ............................................................ xii

1 Introduction ............................................................ 1

   1.1 Background .......................................................... 1

   1.2 Methods of Heat Transfer Enhancement .......................... 2

   1.3 Heat Transfer Enhancement Using Curved Walls ............... 4

   1.4 Evaluation of Enhanced Performance ............................ 5

   1.5 Scope of Research .................................................. 6

   1.6 Aim of Research ................................................... 7

2 Geometry & Mathematical Formulation .............................. 8

   2.1 Cross-Sectional Geometry ......................................... 8
2.2 Axial Geometry ........................................ 10
2.3 Mathematical Formulation ............................. 14
2.4 Boundary Conditions ................................... 15
   2.4.1 Hydrodynamic Boundary Conditions ............... 15
   2.4.2 Thermal Boundary Conditions ...................... 16
2.5 Calculation of Performance Parameters .............. 16
   2.5.1 Hydodynamic Results ............................. 16
   2.5.2 Thermal Results .................................. 17

3 Numerical Methodology .................................. 19
   3.1 Introduction ....................................... 19
   3.2 Computational Grid ................................ 19
   3.3 Convergence Criteria ............................... 20
      3.3.1 Convergence of Velocity Profile to the Fully-Developed Condition .. 22
      3.3.2 Convergence of Temperature Profile to the Fully-Developed Condition 23
   3.4 Grid Independence Study ............................ 23

4 Results and Discussion .................................. 26
   4.1 Introduction ....................................... 26
   4.2 Velocity Profile ................................... 26
   4.3 Isothermal Friction Factor .......................... 36
   4.4 Temperature Profile ................................ 46
   4.5 Nusselt Number .................................... 54
   4.6 Discussion of the Behavior of Nu for Re < 100 ....... 55
   4.7 Phenomenological Behavior of Fluid Flow Above and Below Re ≈ 100 .... 65
List of Tables

3.1  Mesh Refinement Results ......................................... 25

A.1  Numerical values of $f \cdot \text{Re}$ and $\text{Nu}$ for $y = \infty$ ................................. 78
A.2  $f \cdot \text{Re}$ numerical results ........................................ 79
A.3  $\text{Nu}$ numerical results ............................................. 80
List of Figures

1.1 Photograph of helically twisted elliptical tube ........................................... 3

2.1 Tube cross-section ......................................................................................... 9

2.2 Axial helical twist geometry .......................................................................... 11

2.3 Isometric view of tubes with the same twist ratio but different aspect ratios . 12

2.4 Isometric view of tubes with the same twist ratio but different aspect ratios . 13

3.1 Cross-sectional grid ....................................................................................... 21

4.1 Dimensionless axial velocity contours for $\alpha = 0.5$ ................................. 28

4.2 Dimensionless axial velocity contours for $\alpha = 0.5$ & $Re = 100$ ............ 29

4.3 Dimensionless axial velocity contours for $\alpha = 0.5$ & $Re = 700$ .............. 30

4.4 Dimensionless axial velocity contours for $\alpha = 0.5$ & $Re = 1000$ .......... 31

4.5 Dimensionless axial velocity contours for $y = 3$ ....................................... 32

4.6 Dimensionless axial velocity contours for $y = 3$ & $Re = 100$ ................. 33

4.7 Dimensionless axial velocity contours for $y = 3$ & $Re = 700$ ................. 34

4.8 Dimensionless axial velocity along semi-major axis for $\alpha = 0.5$ ............. 37

4.9 Dimensionless axial velocity along semi-major axis for $y = 3$ and $Re=700$ . 38

4.10 Dimensionless axial velocity along semi-major axis for $y = 3$ and $Re=100$ . 39
4.11 Streamtraces for $\alpha = 0.5$, $y = 3$ and Re = 700 with velocity contours . . . . . . . . . 40
4.12 Streamtraces for $\alpha = 0.5$, $y = 3$ and Re = 700 with velocity contours . . . . . . . . . 41
4.13 Streamtrace ribbons for $\alpha = 0.3$, $y = 3$ and Re = 700 with velocity contours . . . . . . 42
4.14 Streamtrace ribbons for $\alpha = 0.3$, $y = 3$ and Re = 700 with velocity contours . . . . . . 43
4.15 $f \cdot$ Re versus Reynolds number ................................................................. 45
4.16 Dimensionless temperature contours for $\alpha = 0.5$ ............................................... 47
4.17 Dimensionless temperature contours for $\alpha = 0.5$ & Re = 100 .......................... 48
4.18 Dimensionless temperature contours for $\alpha = 0.5$ & Re = 700 ........................... 49
4.19 Dimensionless temperature contours for $\alpha = 0.5$ & Re = 1000 .......................... 50
4.20 Dimensionless temperature contours for $y = 3$ ................................................ 51
4.21 Dimensionless temperature contours for $y = 3$ & Re = 100 ............................... 52
4.22 Dimensionless temperature contours for $y = 3$ & Re = 700 ............................... 53
4.23 Nusselt number versus Reynolds number for uniform wall temperature ... 56
4.24 Nusselt number versus Reynolds number for uniform wall temperature and
   $\alpha = 0.3$ ........................................................................................................ 57
4.25 Nusselt number versus Reynolds number for uniform wall temperature and
   $\alpha = 0.5$ ........................................................................................................ 58
4.26 Nusselt number versus Reynolds number for uniform wall temperature and
   $\alpha = 0.7$ ........................................................................................................ 59
4.27 Nusselt number versus Reynolds number for uniform wall temperature and
   $y = 3$ ................................................................................................................ 60
4.28 Dimensionless temperature contours for $\alpha = 0.3$ and Re = 10 ........................... 62
4.29 Dimensionless temperature contours for $\alpha = 0.3$ and Re = 10 ........................... 63
4.30 Dimensionless temperature contours for $\alpha = 0.7$ and Re = 10 ........................... 64
4.31 Streamtraces for $\alpha = 0.3$, $y = 3$ and $Re = 10$ (Velocity Contours Shown) . . 67
4.32 Streamtraces for $\alpha = 0.3$, $y = 3$ and $Re = 700$ (Velocity Contours Shown) . . 68
4.33 Nusselt number versus $f \cdot Re^3$ .............................................. 71
4.34 Normalized Nusselt number versus $f \cdot Re^3$ ...................................... 72
4.35 Colburn factor ($j$) versus $f \cdot Re^3$ ................................................. 74
4.36 Normalized Colburn factor ($j/j_y=\infty$) versus $f \cdot Re^3$ ...................... 75
Nomenclature

a  semi-major axis of the ellipse

A  flow cross sectional area

b  semi-minor axis of the ellipse

$C_p$  specific heat at constant pressure

$D_h$  Hydraulic diameter, $4r_h$; Eqn. 2.2

$E$  friction power expended per unit area

$E(m)$  complete elliptic integral of the second kind

$f$  fanning friction factor

$H$  $180^\circ$ twist ratio

$h$  heat transfer coefficient

$k$  thermal conductivity

$m$  elliptic integral parameter, $1 - \alpha^2$

$N$  run number
NGP  number of grid points

Nu  Nusselt number

$P$  pressure

$P_w$  flow wetted perimeter

Pr  Prandtl number

$Q$  heat

$r_h$  hydraulic radius, $A/P_w$

Re  Reynolds number

St  Stanton number

$T$  temperature

$u$  velocity vector

$u_i$  velocity vector component where $i = \{x, y, z\}$

$y$  twist ratio; Eqn. 2.3

$\alpha$  ellipse aspect ratio; Eqn. 2.1

$\mu$  dynamic viscosity coefficient

$\rho$  fluid density

$\tau$  shear stress
\( \theta \)  
normalized temperature; Eqn. 2.14

\( \varepsilon \)  
error

\( t \)  
related to temperature

\( u \)  
related to velocity

\( w \)  
quantity evaluated at the wall

\( x \)  
\( x \)-component

\( y \)  
\( y \)-component

\( z \)  
\( z \)-component

\( \langle \rangle \)  
average value
Chapter 1

Introduction

1.1 Background

Heat transfer enhancement is an active area of research because of the need to minimize the cost and physical envelope of heat exchangers. Among the many ways to achieve these goals, heat transfer enhancement by means of curved walls is widely used. This method has two advantages: increased surface area and induced secondary swirl flows to enhance mixing across the cross section of the tube [1]. The secondary swirl flows can be induced using twisted non-circular ducts or by placing a twisted tape insert in circular tubes. The latter technique has received considerable attention in the literature. Manglik and Bergles [2] provide a comprehensive survey of twisted tape inserts to induce secondary flows. They have characterized the secondary-flow induced enhancement using a swirl parameter and provided correlations for friction factor and Nusselt number. In contrast with the extensive literature covering twisted tape inserts, only a few investigations of twisted non-circular tubes are available. Masliyah and Nandakumar [3] and Kheshgi [4] investigated laminar flow
in twisted square tubes. Their results show that the $f \cdot \text{Re}$ begins to increase with Reynolds number from $\text{Re} \approx 100$. In a subsequent study, Masliyah and Nandakumar obtained heat transfer in twisted square tubes [1]. However, their results for the uniform wall temperature case does not show the expected increase in Nusselt number with an increase in Reynolds number. Todd treated this geometry analytically but as a result his treatment is restricted to low Reynolds numbers ($\text{Re} < 1$) [5].

Tube and shell heat exchangers with the twisted tube technology are being used in the process industry (see for example, http://www.kochheattransfer.com/). Despite the fact that this technology has existed for about thirty years and that it is in commercial use, its heat transfer and hydrodynamic characterization is largely missing in the scientific literature.

In the present work, we treat helically twisted tubes with elliptical cross sections numerically and characterize the friction factor and Nusselt numbers for a wide range of Reynolds numbers.

### 1.2 Methods of Heat Transfer Enhancement

Heat transfer enhancement techniques are generally divided into two main categories: active and passive enhancements[6]. Passive enhancement technologies are “installed” during the manufacturing process of the heat exchanger and require no further input during operation whereas active technologies require some mechanical or electrical power input during operation[6].

**Passive enhancement techniques include:**

- Surface treatments such as coatings or surface textures.
Figure 1.1: Photograph of helically twisted elliptical tube
- Extended surfaces such as fins.

- Swirl flow devices which generate secondary flows to enhance mixing and essentially disrupt the thermal boundary layer (e.g. twisted tape inserts).

- Coiled tubes.

- Fluid additives to (for example) change its physical properties such as the surface tension.

Active enhancement techniques, on the other hand, include:

- Surface vibration.

- Stirring of fluid by mechanical means.

- Injection or suction through a porous heat transfer surface.

1.3 Heat Transfer Enhancement Using Curved Walls

Swirl flow devices and coiled tubes, in general, belong to a broader class of passive heat transfer enhancement devices that employ curved walls to induce secondary flows in order to mix the fluid in the center of the tube with the fluid near the wall of the tube. This mixing is of course the source of heat transfer enhancement in these devices since the fluid near the wall approaches the temperature of the wall as the thermal boundary layer develops and thus lowers the temperature gradient normal to the wall which in turn lowers the heat transfer coefficient and the Nusselt number. By contrast the fluid temperature at the center of the tube (in the absence of mixing) typically differs the most from the wall temperature and thus it is desirable to move it near the wall since this would result in an increase in the temperature gradient normal to the wall and therefore increase the heat transfer coefficient. Helically
twisted tubes with elliptical cross sections fall under this class of curved wall enhancement devices. They generate the required secondary flows to produce the desired mixing.

Other examples of curved wall devices are twisted tape inserts in circular tubes. Twisted tape inserts have been studied extensively (see for example the review article by Manglik and Bergles [2]). Because of this, we have used the results from the literature as a baseline to compare the performance of twisted elliptical tubes against. In particular, we have used the twisted tape $f \cdot Re$ and Nu correlations from [2]; we used Eq. 7 in Table II for $fRe$ and Eq. 25 in Table III for Nu.

1.4 Evaluation of Enhanced Performance

There are many design options available to the engineer when it comes to heat exchanger design. One question that will undoubtedly arise is how to quantitatively evaluate the performance of a given heat exchanger design against another. To this end, there are several evaluation criteria that include but are not limited to the following[6]:

- Economic considerations such as manufacturing and operational costs.

  The reliability of the heat exchanger to minimize down time.

- Safety during operation and maintenance.

  Regarding the economic considerations, we are faced with a trade-off between enhanced heat transfer and pumping power required to operate the heat exchanger with the desired performance. For example, a more compact heat exchanger envelope may require more pumping power to achieve the same thermal performance of a larger heat exchanger.

  How, then, do we evaluate the heat exchanger? This question has received much attention
in the literature [7, 6]. Bergles [6] classifies the performance evaluation criteria under three categories:

1. **FG Criteria:** where the physical envelope of the heat exchanger is fixed and the performance enhancement comes from better surface designs (for example).

2. **FN Criteria:** where the cross-sectional envelope of the heat exchanger is fixed but the length is allowed to vary.

3. **VG Criteria:** the net tube side cross-sectional flow area is allowed to increase to accommodate a higher friction factor.

We used the FG-2a [7, 6] criterion as a measure of performance in this study (c.f. Chapter 4 Sections 4.5 & 4.8).

### 1.5 Scope of Research

We considered laminar, periodically fully developed single phase flow of a Newtonian fluid with the constant wall temperature (isothermal) boundary condition. Tubes with ellipse aspect ratios $\alpha = 0.3, 0.5$ and $0.7$ and twist ratio values $y = 3, 4.5$ and $6$ are considered for a fluid with a Prandtl number of 3 (e.g. water at 60°C).

The Nusselt number and isothermal friction factor were calculated for Reynolds numbers in the range $10 \leq \text{Re} \leq 1000$ for the set of geometrical parameters discussed above. Suggested extensions to the present study are discussed in the last chapter.
1.6 Aim of Research

The aim of the research presented in this thesis is to provide a characterization of twisted tubes with elliptical cross sections. Namely, the benefits (heat transfer enhancement) and drawbacks (increased pressure drop) of this technology. This characterization will provide the practicing engineer and heat exchanger designer with the data that they require to successfully incorporate twisted elliptical tubes into their designs.
Chapter 2

Geometry & Mathematical Formulation

2.1 Cross-Sectional Geometry

This study considers helically twisted tubes with elliptical cross sections. The cross sections are described by the aspect ratio parameter $\alpha$ given by

$$\alpha = \frac{2b}{2a}$$  \hspace{1cm} (2.1)

where $a$ and $b$ are the semi-major and semi-minor axes of the ellipse respectively as shown in Fig. 2.1(a).

In the limit $\alpha \rightarrow 1$, the ellipse reduces to a circle with a diameter equal to the hydraulic diameter. This limit, in combination with the limit $y \rightarrow \infty$, is useful for comparison between results obtained for twisted elliptical tubes and known results for straight circular and elliptical tubes.
(a) Elliptical cross-sectional geometry

(b) Cross sectional geometry with different aspect ratios

Figure 2.1: Tube cross-section
The hydraulic diameter is often used as the relevant length scale in describing fluid flow and heat transfer in ducts with non-circular cross-sections. For the duct cross-section shown in Fig. 2.1, the hydraulic diameter is given by

\[ D_h = \frac{4A}{P_w} = \frac{4\pi ab}{4aE(m)} = \frac{\pi b}{E(m)} \quad (2.2) \]

where \( E(m) \) is the complete elliptic integral of the second kind with parameter \( m = 1 - \alpha^2 \).

Figure 2.1(b) shows three ellipses with the aspect ratios that are considered in this study. All ellipses have the same hydraulic diameter. In addition, a circle (\( \alpha = 1 \)) with the same diameter is shown for comparison.

### 2.2 Axial Geometry

The cross section twists helically along the axis of the tube. The direction of the twist is clockwise in the positive z-direction as shown in Fig. 2.2. The amount of twist is described by the 180° twist ratio \( y \) defined as

\[ y = \frac{H}{D_h} \quad (2.3) \]

where \( H \) is the length of one half of a full period as shown in Fig. 2.2. The parameter \( y \) is, thus, dimensionless and measures the half period of the twist (180°) in units of number of hydraulic diameters.

It is evident from Eq. 2.3 that a smaller twist ratio leads to a shorter 180° twist length \( H \). This behavior can be seen in Fig. 2.3 which shows tubes with the same aspect ratio (\( \alpha = 0.5 \)) and hydraulic diameter but with different twist ratios. Each tube is one period long (360° twist). Indeed, the figure shows that a smaller twist ratio \( y \) results in a more tightly twisted tube. Figure 2.4 shows the same tubes as in Fig. 2.3 with shading.
Figure 2.2: Axial helical twist geometry
Figure 2.3: Isometric view of tubes with the same twist ratio but different aspect ratios
Figure 2.4: Isometric view of tubes with the same twist ratio but different aspect ratios

\[ \alpha = 0.5, \ y = 6 \]

\[ \alpha = 0.5, \ y = 4.5 \]

\[ \alpha = 0.5, \ y = 3 \]
2.3 Mathematical Formulation

Tube and shell heat exchangers are widely used in the process industries (e.g. chemical, petro-chemical, food, and pharmaceutical) [8, 2, 9]. Hence, on the tube side, the working fluids tend to be highly viscous and as a result the fluid flows generally fall in the laminar regime. Further, the heating on the shell side (for example) is typically done using steam or turbulent fluids. This leads to a uniform wall temperature (UWT) thermal boundary condition.

For the above-mentioned reasons, the following assumptions were made:

1. Incompressible flow.

2. Constant-property flow. Specifically, under this assumption, the fluid viscosity and thermal conductivity were taken as independent of temperature.

3. Laminar flow.

4. Steady state, hydrodynamically and thermally fully developed flow.

5. Peripherally and axially uniform wall temperature (Dirichlet) boundary condition.

6. The body forces due to gravity are ignored.

Again, these conditions prevail for most viscous Newtonian liquid flows (Pr ≥ 1) in long ducts of small hydraulic diameters \((L/D_h \gg 1)\) [10].

The incompressible Navier-Stokes equations for a constant property flow without body forces or viscous dissipation can be found in many references (e.g. Tannehill, Anderson and Pletcher [11]). The general Navier-Stokes equations were specialized by incorporating the assumptions listed above and are written below in vector form.
Continuity equation:
\[ \nabla \cdot \mathbf{u} = 0 \quad (2.4) \]

Momentum equation:
\[ \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \mu \nabla^2 \mathbf{u} \quad (2.5) \]

Energy equation:
\[ \rho \mathbf{u} \cdot \nabla T = k \nabla^2 T \quad (2.6) \]

For helically twisted tubes, the applicable Navier-Stokes momentum equation (Eq. 2.5) is non-linear due to the presence of the convective acceleration term \( \rho \mathbf{u} \cdot \nabla \mathbf{u} \) and, thus, cannot be readily solved analytically. In the absence of twist, the equation can be reduced to one dimension in the weighted radius variable but in the presence of twist this technique no longer works [5]. For this study, the three dimensional Navier-Stokes equations were solved numerically using finite volume discretization.

### 2.4 Boundary Conditions

#### 2.4.1 Hydrodynamic Boundary Conditions

a) Wall: no slip boundary condition (i.e. \( u_x = u_y = u_z = 0 \)).

b) Inlet: initially specified as a uniform velocity profile with a value equal to \( \langle u_z \rangle \) to give the desired Reynolds number. But, as velocity profile develops the dimensionless profile at the outlet is repeatedly applied at the inlet until the convergence criterion is met (c.f. Section 3.3.1).

c) Outlet: a constant pressure condition is applied uniformly at the outlet cross-section.
2.4.2 Thermal Boundary Conditions

a) Wall: a constant reference temperature is applied to the entire wall of the tube and is normalized such that it has a value of unity.

b) Inlet: initially, a uniform temperature that is an arbitrary fraction of the wall temperature $T_w$ is applied to the entire inlet cross-section (e.g. $T = \langle T \rangle = 0.5 T_w$). But, as the temperature profile developed, the dimensionless temperature profile at the outlet is applied at the inlet as a boundary condition for the subsequent run until the convergence criterion is satisfied (c.f. Section 3.3.2).

c) Outlet: a backflow temperature was specified but did not enter into the simulation since there was no backflow.

2.5 Calculation of Performance Parameters

2.5.1 Hydodynamic Results

To evaluate the hydraulic performance of a heat exchanger, the designer’s main concern is to compute the power needed to deliver the desired mass flow rate of fluid through the heat exchanger. This means that the designer needs to know the pressure drop across the heat exchanger or, equivalently, the pressure drop per unit length of tube (e.g. for tube side of a tube and shell heat exchanger). The pressure drop is of course not dimensionless but it is related to the friction factor follows

$$\frac{dP}{dz} = \frac{-2\mu f}{D^2_h} \sim f$$  \hspace{1cm} (2.7)
To calculate the friction factor from the results of the numerical simulation, it is more convenient to write it in terms of the average wall shear stress as

$$f = \frac{2\langle \tau_w \rangle}{\rho \langle u_z \rangle^2}$$

(2.8)

To report the results, however, we used the more natural parameter $f \cdot \text{Re}$ in lieu of $f$. It is more natural because the product $f \cdot \text{Re}$ is constant for fully developed laminar flows in straight tubes ($y = \infty$) with arbitrary cross-sections.

The Reynolds number is defined in the usual way

$$\text{Re} = \frac{\rho \langle u_z \rangle D_h}{\mu}$$

(2.9)

where

$$\langle u_z \rangle = \frac{1}{A} \int_A u_z dA$$

(2.10)

### 2.5.2 Thermal Results

Typically, the heat transfer coefficient $h$ is used to quantify the heat transfer at the boundary for convective flows. But $h$ is not a dimensionless quantity so we used the Nusselt number which is related to $h$. The Nusselt number is defined in the usual way

$$\text{Nu} = \frac{h D_h}{k} \sim h$$

(2.11)

It was calculated from the results of the numerical simulation using the log mean temperature difference. The heat transfer coefficient is given by
\[ h = \frac{\dot{m} C_p (\langle T_o \rangle - \langle T_i \rangle)}{A \Delta T_{lm}} \]  \hspace{1cm} (2.12) 

where

\[ \Delta T_{lm} = \frac{(T_w - \langle T_i \rangle) - (T_w - \langle T_o \rangle)}{\ln \left( \frac{T_w - \langle T_i \rangle}{T_w - \langle T_o \rangle} \right)} \]  \hspace{1cm} (2.13) 

For the constant wall temperature boundary condition, the temperature profile is considered fully developed when the dimensionless temperature profile becomes constant. That is, when the dimensionless temperature profile at the inlet approaches that at the outlet. The dimensionless temperature is given by [12]

\[ \theta = \frac{\langle T_w \rangle - T}{\langle T_w \rangle - \langle T \rangle} \]  \hspace{1cm} (2.14) 

In general the average wall temperature is calculated as follows

\[ \langle T_w \rangle = \frac{1}{A_w} \int_{A_w} T_w \, dA_w \]  \hspace{1cm} (2.15) 

but in our case it is constant and was applied as a boundary condition at the tube wall. The bulk mean temperature is given by

\[ \langle T \rangle = \frac{1}{A \langle u_z \rangle} \int_A u_z \, T \, dA \]  \hspace{1cm} (2.16)
Chapter 3

Numerical Methodology

3.1 Introduction

For incompressible and constant property flow, the momentum equations (Eq. 2.5) are de-coupled from the energy equation (Eq. 2.6). For this reason, we solved the momentum equations first and then used the resulting velocity field to solve the energy equation. The discretized three-dimensional equations were solved in three dimensional cartesian coordinates. Therefore, each combination of geometrical parameters (i.e. twist ratio $y$ and aspect ratio $\alpha$) required a separate computational grid.

3.2 Computational Grid

The cross-sectional grid (in the $x$-$y$ plane) is divided into two regions: a core region and a mantle region. As Figure 3.1 shows, the mantle region is close to the wall of the tube and extends a small distance into the domain. In this region, the grid is orthogonal and was created using elliptic cylinder coordinates. The spacing of the cells in the direction normal to
the wall grows as the distance from the wall grows. This serves two purposes: 1) it provides good grid resolution close to the wall where it is required for the calculation of velocity and temperature gradients; 2) it reduces the total number of cells in each domain and provides a gradual transition to the core region.

The mesh in the core region of the cross-sectional grid can no longer follow the elliptic cylindrical coordinates because of the singularity that is present at each focus of the ellipse. Instead, we used a mapped grid that was smoothed using an elliptic solver for structured grids. Grid smoothing was not used for the mantle region in order to preserve the orthogonality of the grid.

In the axial (z) direction, we maintained a separation between successive grid points equal to 0.05 × $D_h$. By definition, the 180° twist pitch is given by: $H = y \times D_h$. Therefore, the domain length in the z-direction is equal to $2 \times H = 2 \times y \times D_h$ and thus the number of grid points in the z direction is given by $NGP_z = 2 \times y \times D_h/(0.05 \times D_h) = 40 \times y$. For example, for a twist ratio $y = 6$, there are 240 grid points in the z direction and for $y = 3$ there are 120.

### 3.3 Convergence Criteria

The numerical solution was obtained by solving the discretized equations iteratively until the numerical convergence criteria were met. Since the computational domain consisted of one period only, numerical convergence neither implied nor guaranteed that the velocity and temperature profiles were fully developed. This meant that we had to have two distinct sets of convergence criteria. The first set ensured that, for each numerical run, the hydrodynamic solution satisfied the momentum and continuity equations and that the temperature solution
Figure 3.1: Cross-sectional grid
satisfied the energy equation. The second set of criteria ensured that the velocity and temperature profiles were fully-developed.

Once the flow solution converged, the exit velocity profile from the converged run was taken as the input profile for the next run. In this way, each run simulated the flow in a subsequent period. Thus, the process was repeated until a fully developed flow condition was reached.

A similar process was used for the temperature profile. However, a properly normalized temperature must be used in this case. This process is described in detail in Appendix B.

### 3.3.1 Convergence of Velocity Profile to the Fully-Developed Condition

For internal developing laminar flows in any given geometry, the friction factor $f$ asymptotically approaches the constant value that it takes when the velocity profile becomes fully developed. Thus by ensuring that the change in the friction factor $f$ or the product $f \cdot \text{Re}$ is within a certain error limit, we ensure that the velocity profile is fully developed. We considered the flow to be fully developed when the percentage error in $(f \cdot \text{Re})$ between successive runs was $\varepsilon_u \leq 0.5\%$. The percentage error is given by

$$
\varepsilon_u = \frac{(f \cdot \text{Re})_N - (f \cdot \text{Re})_{N-1}}{(f \cdot \text{Re})_N} \times 100
$$

(3.1)

where the subscript $N$ is any given run number.

We found that this method provided a better check than comparing the numerical values of the velocity at each grid point at a given location (e.g. inlet or outlet) from successive runs until the difference everywhere is within a certain error limit.
3.3.2 Convergence of Temperature Profile to the Fully-Developed Condition

In the case of the temperature profile, the Nusselt number asymptotically approaches a constant value as the temperature profile becomes fully developed. Again, we considered the temperature profile fully developed when the percentage error in the Nusselt number between successive runs was $\varepsilon_t \leq 0.5\%$. The percentage error in this case is given by

$$\varepsilon_t = \frac{(Nu)_N - (Nu)_{N-1}}{(Nu)_N} \times 100$$  \hspace{1cm} (3.2)

where the subscript N is any given run number.

This proved to be a better measure of convergence than comparing the normalized temperatures at each grid point in a given cross section (e.g. inlet or outlet) between successive runs. Please note that since we are solving the energy equation with a Dirichlet boundary condition, the bulk mean temperature of the fluid will asymptotically approach the specified temperature at the wall. In this case we cannot determine whether or not the temperature profile is fully developed by comparing the temperatures at each grid point in a chosen cross section (e.g. inlet or outlet) between successive runs. However, we can normalize the temperature such that the the normalized temperature (Eq. 2.14 becomes constant when the temperature profile becomes fully developed.

3.4 Grid Independence Study

To guarantee that the results were independent of the computational grid, we compared the results obtained from grids with different spacings between the grid points. We changed the
spacing in both the cross-sectional plane and the $z$ (or axial) direction. We then compared the error in $f \cdot \text{Re}$ and $\text{Nu}$ between the results of the simulations of two successive mesh refinements. Let us call this error $\varepsilon_{\text{mesh}}^a$ where

$$
\varepsilon_{\text{mesh}}^a = \frac{(f \cdot \text{Re})_N - (f \cdot \text{Re})_{N-1}}{(f \cdot \text{Re})_N} \times 100
$$

(3.3)

this is similar to Eq. 3.1. We also compared the results obtained from all the meshes to the results obtained from the final mesh. Let us call this error $\varepsilon_{\text{mesh}}^b$ where

$$
\varepsilon_{\text{mesh}}^b = \frac{(f \cdot \text{Re})_N - (f \cdot \text{Re})_{\text{final}}}{(f \cdot \text{Re})_N} \times 100
$$

(3.4)

where the subscript $N$ refers any mesh while the subscript final refers to the final mesh. The mesh was considered adequate when the results of the numerical simulation became grid independent after successive refinement. This condition was implemented by comparing the error in the $f \cdot \text{Re}$ results between any given mesh and the baseline mesh. The results were considered grid independent when the error was $\varepsilon_{\text{mesh}}^{a,b} \leq 0.5\%$.

Initially, grid independence was checked for a grid with a twist ratio $y = 6$, an aspect ratio $\alpha = 0.7$ and for $\text{Re} = 200$; these results are listed in Table 3.1. The spacing between grid points along the circumference of the tube $\Delta s$ and in the $z$ direction are non-dimensionalized by dividing them by the hydraulic diameter $D_h$.

Subsequently, we conducted a smaller study for the case with $y = 3$, $\alpha = 0.3$ and $\text{Re} = 1000$ which has the most severe gradients using two meshes. The result of the subsequent study gave an error $\varepsilon_{\text{mesh}} \sim 0\%$ and thus justifies our claim that the results for the whole range of geometrical parameter combinations considered in this study are indeed grid independent.
<table>
<thead>
<tr>
<th>Mesh No.</th>
<th>Number of Cells in Cross-Section</th>
<th>Number of Grid Points</th>
<th>Circumferential</th>
<th>Axially</th>
<th>$\Delta S/D_h$</th>
<th>$\Delta z/D_h$</th>
<th>$f \cdot \text{Re}$</th>
<th>Nu</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1360</td>
<td>40</td>
<td>28</td>
<td>0.082</td>
<td>0.429</td>
<td>0.0</td>
<td>-0.25%</td>
<td>1.45%</td>
</tr>
<tr>
<td>2</td>
<td>960</td>
<td>64</td>
<td>98</td>
<td>0.082</td>
<td>0.316</td>
<td>0.0</td>
<td>-0.75%</td>
<td>1.21%</td>
</tr>
<tr>
<td>3</td>
<td>340</td>
<td>40</td>
<td>48</td>
<td>0.082</td>
<td>0.250</td>
<td>0.0</td>
<td>-0.13%</td>
<td>1.21%</td>
</tr>
<tr>
<td>4</td>
<td>340</td>
<td>40</td>
<td>73</td>
<td>0.082</td>
<td>0.164</td>
<td>0.0</td>
<td>-0.06%</td>
<td>1.02%</td>
</tr>
<tr>
<td>5</td>
<td>340</td>
<td>40</td>
<td>28</td>
<td>0.082</td>
<td>0.429</td>
<td>0.0</td>
<td>-0.06%</td>
<td>1.02%</td>
</tr>
<tr>
<td>6</td>
<td>340</td>
<td>40</td>
<td>48</td>
<td>0.082</td>
<td>0.250</td>
<td>0.0</td>
<td>0.01%</td>
<td>0.28%</td>
</tr>
<tr>
<td>7</td>
<td>340</td>
<td>40</td>
<td>73</td>
<td>0.082</td>
<td>0.164</td>
<td>0.0</td>
<td>-0.08%</td>
<td>0.21%</td>
</tr>
<tr>
<td>8</td>
<td>340</td>
<td>40</td>
<td>28</td>
<td>0.082</td>
<td>0.429</td>
<td>0.0</td>
<td>0.00%</td>
<td>0.20%</td>
</tr>
<tr>
<td>9</td>
<td>340</td>
<td>40</td>
<td>48</td>
<td>0.082</td>
<td>0.250</td>
<td>0.0</td>
<td>-0.24%</td>
<td>-0.03%</td>
</tr>
<tr>
<td>final</td>
<td>2700</td>
<td>120</td>
<td>240</td>
<td>0.027</td>
<td>0.050</td>
<td>0.0</td>
<td>0.03%</td>
<td>-0.46%</td>
</tr>
</tbody>
</table>

Table 3.1: Mesh Refinement Results
Chapter 4

Results and Discussion

4.1 Introduction

In this chapter, we present the results of the numerical simulation subject to the assumptions and constraints described in the previous chapters. For the hydrodynamic results we will discuss the velocity profile and isothermal friction factor and for the heat transfer results we will discuss the resulting temperature profile and Nusselt number. The results cover three twist ratios $y = \{3, 4.5, 6\}$, three aspect ratios $\alpha = \{0.3, 0.5, 0.7\}$ and Reynolds numbers in the range $10 \leq Re \leq 1000$.

4.2 Velocity Profile

The axial velocity profile in helically twisted elliptical tubes deviates from that of a straight elliptical tube as a function of twist ratio, aspect ratio and Reynolds number. For a given aspect ratio $\alpha$, the straight tube solution can be taken as the limit $y \to \infty$ or for a given twist ratio $y$, as the limit $Re \to 0$. The former case gives the actual straight tube solution
but for our purposes the latter case allows us to compare our solution for every twist ratio that we considered to that of a straight tube as a check. Furthermore, the combined limit $\alpha \to 1$ and $y \to \infty$ approaches the straight circular tube solution.

The deviation of the velocity profile from the straight tube profile (which is a paraboloid) can be understood as the result of the induced mixing of flow within a given cross section. In a straight tube where the axis of the tube is oriented along $z$ axis, the $x$ and $y$ velocities are zero. This is not the case in twisted tubes, the $x$ and $y$ velocities are not zero and they cause in-plane mixing which, of course, is the desired effect. This mixing affects the velocity gradients near the tube wall and is the cause of the increase in the friction factor.

To see the effects of the axial twist, let us examine a specific aspect ratio and look at the variation in the axial velocity profile as a function of Reynolds number and twist ratio. Figure 4.1 shows the axial velocity profile for various values of the twist ratio $y$ and Reynolds numbers for an aspect ratio $\alpha = 0.5$ where the Reynolds number increases from left to right and the twist ratio decreases from top to bottom. Consequently, the deviation from the straight tube axial velocity profile solution increases from the top left corner ($y = 6$, $Re = 100$) to the bottom right corner ($y = 3$, $Re = 1000$).

Figures 4.2, 4.3 and 4.4 show the same axial velocity profiles as in Figure 4.1; they correspond to Reynolds numbers of 100, 700 and 1000 respectively. These figures give a more detailed view of the dimensionless axial velocity profile.

Further, the aspect ratio itself affects the axial velocity profile. Since an aspect ratio of unity corresponds to a circle, we would expect that our solution will tend to that of a straight circular tube as $\alpha \to 1$ and $y \to \infty$. Given the range of parameters that we are considering in this study, the closest combination is $\alpha = 0.7$ and $y = 6$. This combination is shown at the top left corner of Fig. 4.5 along with the axial velocity profile for different
Figure 4.1: Dimensionless axial velocity contours for $\alpha = 0.5$
Figure 4.2: Dimensionless axial velocity contours for $\alpha = 0.5$ & Re = 100
Figure 4.3: Dimensionless axial velocity contours for $\alpha = 0.5$ & Re = 700
Figure 4.4: Dimensionless axial velocity contours for $\alpha = 0.5$ & $Re = 1000$
Figure 4.5: Dimensionless axial velocity contours for $y = 3$
$\alpha = 0.7$

$\alpha = 0.5$

$\alpha = 0.3$

Figure 4.6: Dimensionless axial velocity contours for $y = 3$ & Re = 100
Figure 4.7: Dimensionless axial velocity contours for $y = 3$ & Re = 700
aspect ratios and Reynolds numbers but all with a twist ratio $y = 3$. In this case, the Reynolds number increases from left to right and the aspect ratio $\alpha$ decreases from top to bottom. Consequently, the deviation between the axial velocity profiles of helically twisted elliptical tubes and straight circular tubes increases from the top left corner to the bottom right corner.

Figures 4.6 and 4.7 contain the same information as in Figure 4.5 but allow for a more detailed study of the dimensionless velocity profile simply because they are magnified.

To further illustrate the change in the axial velocity profile as a function of the Reynolds number and twist ratio, let us take a look at the axial velocity profile along the semi-major axis. This is, effectively, a “slice” through the cross-sections of the previous figures. Figure 4.8 shows the dimensionless velocity profile for $\alpha = 0.5, y = \{3, 4.5, 6\}$ and $Re = \{100, 1000\}$ plotted along the semi-major axis. It is evident that at low Reynolds numbers (in this case for $Re = 100$) the axial dimensionless velocity $u_z/\langle u_z \rangle$ rises monotonically from a value of zero at the tube wall to a maximum value at the center of the tube. Whereas, for high Reynolds numbers (in this case $Re = 1000$) the dimensionless velocity exhibits a local minimum between its zero value at the tube wall and its maximum value at the tube center. This is a manifestation of the enhanced mixing that occurs at high Reynolds numbers ($Re > 200$) thanks to the curved walls of the tube.

Figure 4.9 shows this effect for a fixed value of the twist ratio $y = 3$. For $\alpha = 0.7$, the dimensionless velocity rises monotonically from its zero value at the wall to its maximum value at the tube center. On the other hand, for $\alpha = 0.3$, the dimensionless velocity has a distinct local minimum.

Finally, Figure 4.10 shows that the mixing effect is rather muted at low Reynolds numbers (in this case $Re = 100$). While the dimensionless velocity for the curve with $\alpha = 0.3$ is slightly
deformed in comparison with a parabola, it certainly does not have a local minimum between the tube wall and the tube center.

Figures 4.11 and 4.12 each show an isometric view of a full period of the twisted elliptical tube for \( y = 3, \alpha = 0.5 \) and \( \text{Re} = 700 \). Figures 4.13 and 4.14 each show an isometric view of a full period of the twisted elliptical tube for \( y = 3, \alpha = 0.3 \) and \( \text{Re} = 700 \). Velocity contours are shown at six cross-sectional location in each figure along the tube and velocity streamtrace ribbons connect those sections and show (to the extent possible) two features of the flow: firstly, the streamtrace ribbons that are (mostly) outside of the core region near the center of the tube sample a different location in each cross section as the fluid flows downstream. This feature is more pronounced in the case where \( \alpha = 0.3, y = 3 \) and \( \text{Re} = 700 \) as we would expect and is a manifestation of the enhanced mixing that increases as \( y \to 0, \alpha \to 0 \) and \( \text{Re} \to \infty \). Secondly, each streamtrace ribbon twists as the fluid flows downstream relative to the rotating coordinate system at each cross-section. Again, this is a manifestation of the enhanced fluid mixing.

### 4.3 Isothermal Friction Factor

In the previous section, we discussed how the axial velocity profiles for helically twisted elliptical tubes deviate from the axial velocity profile of a straight elliptical tube with the same aspect ratio. In this section, we will quantify these deviations by discussing how the isothermal friction factor changes as a function of the Reynolds number and the geometrical parameters \( y \) and \( \alpha \).

The friction factor is given by
Figure 4.8: Dimensionless axial velocity along semi-major axis for $\alpha = 0.5$
Figure 4.9: Dimensionless axial velocity along semi-major axis for $y = 3$ and $Re=700$
Figure 4.10: Dimensionless axial velocity along semi-major axis for $y = 3$ and Re= 100
Figure 4.11: Streamtraces for $\alpha = 0.5$, $y = 3$ and $Re = 700$ with velocity contours
Figure 4.12: Streamtraces for $\alpha = 0.5$, $y = 3$ and $Re = 700$ with velocity contours
Figure 4.13: Streamtrace ribbons for $\alpha = 0.3$, $y = 3$ and $Re = 700$ with velocity contours.
Figure 4.14: Streamtrace ribbons for $\alpha = 0.3$, $y = 3$ and $Re = 700$ with velocity contours.
\[ f = \frac{2\tau_w}{\rho(u_z)^2} \]  

(4.1)

Alternatively, it could be written as

\[ f = \frac{-dp/dzD^2}{2\mu} \]  

(4.2)

For a straight tube of arbitrary cross section, the friction factor \( f \) is inversely proportional to the Reynolds number but the product \( f \cdot \text{Re} \) is constant for laminar flow with which we are dealing exclusively in this study). We normalize \( f \cdot \text{Re} \) for any twist ratio by dividing it by the value of \( f \cdot \text{Re} \) for the straight tube \( (y = \infty) \) with the same aspect ratio. For a straight tube, the value of \( f \cdot \text{Re} \) is given by

\[ f \cdot \text{Re} = 2 \left( 1 + \alpha^2 \right) \left( \frac{\pi}{E(m)} \right)^2 \]  

(4.3)

(see [14], Section 3.6.1). The normalized product \( f \cdot \text{Re} \), then, provides a quantitative measure for the deviation between the twisted tube and the straight tube with the same aspect ratio and at the same Reynolds number.

These variations are shown in Fig. 4.15. For low Reynolds numbers, the normalized product \( f \cdot \text{Re} \to 1 + \delta \) where \( \delta \) is a small positive offset that depends on the aspect ratio and the twist ratio; \( 0 < \delta = \delta(\alpha, y) \ll 1 \). This offset is due to the increased length of the helical streamlines in helically-twisted tube flows at very low Reynolds numbers in comparison with the straight axial-streamlines that arise in straight tube flows at any Reynolds number provided that the flow is laminar. As the Reynolds number increases, the normalized product \( f \cdot \text{Re} \) monotonically increases. Furthermore, the increase is greater for smaller twist ratios and smaller aspect ratios as might be expected from the qualitative results presented in the
Reynolds Number

\[ f \cdot \frac{Re}{Re_\infty} = \infty \]

\[ \alpha = 0.3, \ y = 3 \]
\[ \alpha = 0.3, \ y = 6 \]
\[ \alpha = 0.5, \ y = 3 \]
\[ \alpha = 0.5, \ y = 6 \]
\[ \alpha = 0.7, \ y = 3 \]
\[ \alpha = 0.7, \ y = 6 \]

Twisted Tape Correlation \( y = 3 \) \[2\]
Twisted Tape Correlation \( y = 6 \) \[2\]

Figure 4.15: \( f \cdot Re \) versus Reynolds number
last section.

Figure 4.15 also shows the normalized $f \cdot \text{Re}$ curves for circular tubes with twisted tape inserts (for the twist ratios of interest) to provide a baseline for comparison. These curves are plotted from the twisted tape correlations given by Manglik and Bergles [2].

4.4 Temperature Profile

For the parameters considered in this study, the Peclet number ($\text{Pe} = \text{Re} \cdot \text{Pr}$) is much greater than unity. Consequently, convective heat transfer always dominates over diffusive heat transfer. This is evident in that the variation in the temperature profiles with Reynolds number and the geometrical parameters follows the same trend as the variation in the velocity profile. Figures 4.16 and 4.20 are setup using the same Reynolds numbers and geometrical parameters as the velocity profiles figures (Figs. 4.1 and 4.5 respectively).

Figure 4.16 shows the variation of the temperature profile as a function of Reynolds number and twist ratio for an aspect ratio $\alpha = 0.5$. The top left profile ($y = 6, \text{Re} = 100$) approaches the solution for a straight tube with the same aspect ratio. The deviation from the straight tube profile increases as the Reynolds number increases (moving from left to right in the figure) and as the twist ratio decreases (moving from top to bottom in the figure). The bottom right profile ($y = 3, \text{Re} = 1000$) shows the most deviation from the straight tube profile.

Figures 4.17, 4.18 and 4.19 show the same temperature profiles as in Figure 4.16; they correspond to Reynolds numbers of 100, 700 and 1000 respectively. These figures give a more detailed view of the dimensionless temperature profile.

This deviation is a direct result of the in-plane velocity mixing as discussed in Sec. 4.2
Figure 4.16: Dimensionless temperature contours for $\alpha = 0.5$. 

$\Re = 1000$

$\Re = 700$

$\Re = 100$

$y = 6$

$y = 4.5$

$y = 3$
Figure 4.17: Dimensionless temperature contours for $\alpha = 0.5$ & $Re = 100$
Figure 4.18: Dimensionless temperature contours for $\alpha = 0.5$ & $Re = 700$
Figure 4.19: Dimensionless temperature contours for $\alpha = 0.5$ & $Re = 1000$
Re=700
Re=100

α = 0.7
α = 0.5
α = 0.3

Figure 4.20: Dimensionless temperature contours for $y = 3$
$\alpha = 0.7$

$\alpha = 0.5$

$\alpha = 0.3$

Figure 4.21: Dimensionless temperature contours for $y = 3$ & Re = 100
$\alpha = 0.7$

$\alpha = 0.5$

$\alpha = 0.3$

Figure 4.22: Dimensionless temperature contours for $y = 3$ & Re = 700
since convection effects dominate diffusive effects (there is non-trivial behavior for Re < 100; this is discussed later). The mixing is of course responsible for increasing the temperature gradients near the tube wall and, consequently, the increase in Nusselt number.

As in the case of the velocity profile, the aspect ratio, too, affects the temperature profile. Figure 4.20 shows how the temperature profile changes as a function of the twist and Reynolds number for a fixed twist ratio \( y = 3 \). The top left profile (\( \alpha = 0.7, \text{Re} = 100 \)) approaches the straight circular tube solution while the bottom right profile (\( \alpha = 0.3, \text{Re} = 1000 \)) deviates from it the most.

Figures 4.21 and 4.22 contain the same information as in Figure 4.20 but allow for a more detailed study of the dimensionless temperature profile simply because they are magnified.

### 4.5 Nusselt Number

The qualitative changes in the temperature profile presented in the previous section show that the “mixing” increases as the Reynolds number increases and the twist and aspect ratios decrease. The enhanced mixing results in higher temperature gradients near the wall of the tube (cf. Figs. 4.20 and 4.20). This is of course leads to a higher Nusselt number. Again, we will work with a normalized Nusselt number as we did for \( f \cdot \text{Re} \). To normalize, we divide the Nusselt number by the Nusselt number for a straight tube (\( y = \infty \)) with the same aspect ratio. For a straight tube with aspect ratio \( \alpha \), the value of Nu is given by

\[
\text{Nu}_T = 0.3536 \left( 1 + 0.9864 \alpha - 0.7189 \alpha^2 + 3.3364 \alpha^3 - 3.0307 \alpha^4 + 1.0130 \alpha^5 \right) \left( \frac{\pi}{E(m)} \right)^2
\]

(see [14], Table 3.49). Figure 4.23 shows the variation of the normalized Nusselt number as
a function of Reynolds number for the entire range of geometrical parameters \( y \) and \( \alpha \). The Nusselt number behaves as expected for \( Re \geq 100 \), i.e. it increases with increasing Reynolds number and decreasing twist and aspect ratios. For \( Re < 100 \), however, the Nusselt number decreases below that of a straight tube with the same aspect ratio. Furthermore, the decrease is higher for lower aspect ratios as is evident from Fig. 4.23.

Figure 4.23 also shows the normalized Nusselt number curves for circular tubes with twisted tape inserts (for the twist ratios of interest) to provide a baseline for comparison. These curves are plotted from the twisted tape correlations given by Manglik and Bergles [2]. For \( \alpha = 0.3 \) and \( y = 3 \), the twisted elliptical tube provides (approximately) a three-fold improvement over a straight elliptical tube or a straight circular tube.

Figures 4.24, 4.25 & 4.26 show the \( Nu/Nu_{y=\infty} \) curves for \( \alpha = \{0.3, 0.5, 0.7\} \) respectively. In addition to the twist ratios shown in Figure 4.23, an additional curve for \( y = 4.5 \) is also shown for each \( \alpha \).

Finally, Figure 4.27 shows the behavior of \( Nu/Nu_{y=\infty} \) as a function of the aspect ratio \( \alpha \) for three value of the Reynolds number. As in Figure 4.23, the value of \( Nu/Nu_{y=\infty} \) for \( \alpha = 0.3 \) at low Reynolds numbers (\( Re < 200 \)) is less than unity. This behavior is explained in the next section. For higher Reynolds numbers, however, the value of \( Nu/Nu_{y=\infty} \) is higher than unity as expected and desired.

### 4.6 Discussion of the Behavior of Nu for Re < 100

For Reynolds numbers \( Re < 100 \), the normalized Nusselt number is less than unity for all combinations of geometrical parameters of twisted elliptical tubes. This behavior is most prominent in the case where \( \alpha = 0.3 \) and as the Reynolds number tends to zero,
Reynolds Number

\[ \frac{N_u}{N_{u_{y=\infty}}} = \alpha \]

\( \alpha = 0.3, \ y = 3 \)

\( \alpha = 0.3, \ y = 6 \)

\( \alpha = 0.5, \ y = 3 \)

\( \alpha = 0.5, \ y = 6 \)

\( \alpha = 0.7, \ y = 3 \)

\( \alpha = 0.7, \ y = 6 \)

Figure 4.23: Nusselt number versus Reynolds number for uniform wall temperature
Reynolds Number

\[ \frac{\text{Nu}}{\text{Nu}_y} = \infty \]

\[ y = 3 \]

\[ y = 4.5 \]

\[ y = 6 \]

Figure 4.24: Nusselt number versus Reynolds number for uniform wall temperature and \( \alpha = 0.3 \)
Figure 4.25: Nusselt number versus Reynolds number for uniform wall temperature and $\alpha = 0.5$.
Figure 4.26: Nusselt number versus Reynolds number for uniform wall temperature and $\alpha = 0.7$. 

[2] Twisted Tape Correlation $y = 3$

[2] Twisted Tape Correlation $y = 6$
Figure 4.27: Nusselt number versus Reynolds number for uniform wall temperature and $y = 3$
for definiteness, let us consider the case with Re = 10. Figure 4.28 shows the normalized temperature profile for twist ratios $y = \infty$, $y = 6$ and $y = 3$ in sub-figures (a), (b) and (c) respectively. The dashed vertical lines provide a visual guide to compare the first three contour lines in all sub-figures. The figure clearly shows that as the twist ratio $y$ decreases (tighter twist), the approximately isothermal area of the region between the wall and the first contour line grows in size.

This can be understood in the following way, as the twist ratio decreases, the flow path becomes essentially restricted away from the core region of the tube which is bounded by the inscribed circle inside the ellipse. This circle sweeps a cylinder in the direction of the flow. Thus, the flow path is completely unobstructed inside this cylinder. Outside of the cylinder, however, the flow path is continuously confronted with the tube walls which force the streamlines to turn and follow a helical path instead of a straight one. For low Reynolds numbers (Re < 100) there is virtually no mixing between the streamlines that go through the center and the ones that follow the helical path along the tube wall (this is discussed in more detail in the next section). Consequently, the fluid elements that follow the paths near the wall well outside of the core region have a longer residence in one period than those that follow paths near the wall but close to the core region.

Evidently, this reduces the value of the average wall temperature gradient which, in turn, reduces the value of the average Nusselt number. The peripheral (local) Nusselt number is shown in Figure 4.29 for two twist ratios: $y = \infty$ and $y = 3.0$ to illustrate this behavior. Indeed, The figure supports this conclusion since the Nusselt number curve for $y = 3$ is almost entirely lower than the curve for $y = \infty$. From this, we can see that the average Nusselt number for the case with $y = 3$ must be lower than the Nusselt number for the straight tube case ($y = \infty$).
Figure 4.28: Dimensionless temperature contours for $\alpha = 0.3$ and Re = 10.
Figure 4.29: Dimensionless temperature contours for $\alpha = 0.3$ and $Re = 10$
Figure 4.30: Dimensionless temperature contours for $\alpha = 0.7$ and $Re = 10$
The behavior where the Nusselt number for a case with $y < \infty$ is lower than the Nusselt number for a case with the same tube cross section but with $y = \infty$ is more pronounced as $\alpha \rightarrow 0$ as can be seen in Fig. 4.23. We have already shown that the first three normalized temperature contour lines for $\alpha = 0.3$ become more widely separated as $y \rightarrow 0$ (cf. Fig. 4.29). We would expect, then, the separation between the first few normalized temperature contour lines to remain relatively constant with the decrease in $y$ for a case where $\alpha \rightarrow 1$. This is indeed the case and is demonstrated in Fig. 4.30 which shows the normalized temperature contours for aspect $\alpha = 0.7$ and with twist ratios $y = \infty, 6, 3$ in sub-figures (a), (b) and (c) respectively. The vertical dashed lines are again shown to facilitate the visual comparison. The expected behavior is realized since the separation between the temperature contour lines near the wall is approximately constant.

The qualitative argument of the previous paragraphs is quantified by the value $\frac{\text{Nu}}{\text{Nu}_{y=\infty}}$ and corroborates the observed behavior. As an example, for $\{\alpha = 0.3, y = 3, \text{Re} = 10\}$, $\frac{\text{Nu}}{\text{Nu}_{y=\infty}} = 0.7988$ whereas for $\{\alpha = 0.3, y = 3, \text{Re} = 10\}$, $\frac{\text{Nu}}{\text{Nu}_{y=\infty}} = 0.9453$. On the other hand, for $\{\alpha = 0.7, y = 3, \text{Re} = 10\}$, $\frac{\text{Nu}}{\text{Nu}_{y=\infty}} = 0.9785$ and for $\{\alpha = 0.7, y = 3, \text{Re} = 10\}$, $\frac{\text{Nu}}{\text{Nu}_{y=\infty}} = 0.9891$ (cf. Table A.3).

4.7 Phenomenological Behavior of Fluid Flow Above and Below $\text{Re} \approx 100$

Let us consider the paths that fluid elements follow in two cases: the first with $\text{Re} = 10$ and the second with $\text{Re} = 700$. The paths in these two cases are shown in Figures 4.31 and 4.32 respectively. Both figures show an isometric view of the twisted tube with $\alpha = 0.3$ and $y = 3$. 

65
Further, each shows five cross-sections that are labeled 1 thru 5 (encircled numbers next to the cross-sections) and some paths of fluid elements that start in different locations in section 1. The idea is to follow each path to see where it crosses the four sections downstream of section 1 to give a qualitative idea of how much “mixing” occurs in both cases.

We begin by examining Fig. 4.31 with Re = 10. There are two labeled paths: path ‘a’ starts on the major axis near the tube wall and path ‘b’ starts below the major axis midway (along a radius) between the centerline of the tube and the wall. Following path ‘a’ downstream, we see that it crosses sections 2, 3, 4 and 5 near the wall. Similarly, path ‘b’ starts midway between the centerline and the tube wall in section 1 and crosses sections 2, 3, 4 and 5 approximately midway between the centerline and the tube wall. We note, however, that the paths end at a rotated location with respect to the start locations; the rotations in both cases are not equal. Notwithstanding, both paths remained at the same radial location from the centerline as they travelled through the entire period.

Turning to Fig. 4.32, we again have two labeled paths: path ‘a’ starts on the major axis close to the tube wall and path ‘b’ starts on the major axis midway between the centerline and the tube wall along a radius. Path ‘a’ crosses section 2, 3 and 4 close the tube wall but crosses section 5 much closer to the centerline. On the other hand, path ‘b’ progressively gets closer to the tube wall as it travels downstream and ends very close to the tube wall in section 5. Following the unlabeled paths gives the same trend; namely, the paths don’t maintain the same distance from the tube wall as they travel downstream.

We can see from Figs. 4.31 and 4.32 that, phenomenologically, the fluid flow exhibits different characters in the regimes Re < 100 and Re > 100. For Re < 100, there is virtually no mixing in the cross section. The fluid elements that start near the wall maintain the same distance from the wall as they travel downstream. In addition, the fluid elements that
Figure 4.31: Streamtraces for $\alpha = 0.3$, $y = 3$ and $Re = 10$ (Velocity Contours Shown)
Figure 4.32: Streamtraces for $\alpha = 0.3$, $y = 3$ and $Re = 700$ (Velocity Contours Shown)
start near the wall close to the major axis of the ellipse travel a longer distance in one period than the elements that start near the wall on the minor axis. Consequently, a portion of the circumference of the tube in the neighborhood of the major axis does not contribute significantly to the heat transfer between the fluid and the tube wall. This can be seen in Fig. 4.29, the portions of the circumference near locations ‘b’ and ‘d’ have the lowest Nusselt number along the circumference.

On the other hand, for Re > 100, the fluid elements “wander” in the cross section as they travel downstream. This results in greater mixing (qualitatively) and give a higher average Nusselt number (quantitatively). These observations support the results shown in Fig. 4.23 and discussed in previous sections.

### 4.8 Comparison of Heat Transfer and Friction Power

Finally, we consider the combined thermal and hydrodynamic performance of twisted elliptical tubes to quantify the performance gain by heat transfer enhancement versus the extra power expended in overcoming the additional pressure drop as discussed by Kays and London [13]. They write the heat transfer coefficient as

\[
h = \frac{C_p\mu}{Pr^{2/3}r_h} \left( St \cdot Pr^{2/3} \right) Re
\]  

(4.5)

and the friction power expended per unit area (with units of W/m²) as

\[
E = \frac{1}{2} \rho^3 \left( \frac{1}{4r_h} \right)^3 f \cdot Re^3
\]  

(4.6)

One disatvantage of these parameters is that they are dimensional. It is much more useful (and more general) to use dimensionless parameters.
Since, the heat transfer coefficient \( h \) is proportional to the Nusselt number with a constant of proportionality \( k/D_h \), it is clear that the Nusselt number is the appropriate dimensionless parameter to replace \( h \) with. Similarly, the friction power expended per unit area is proportional to \( f \cdot \text{Re}^3 \) with a constant prefactor and so, \( f \cdot \text{Re}^3 \) is the appropriate dimensionless parameter to use. In summary, we can write

\[
h \propto \text{Nu} \quad \text{and} \quad E \propto f \cdot \text{Re}^3
\]  

(4.7)

Figure 4.33 shows the Nusselt number as a function of the friction factor times the Reynolds number cubed for the best combinations of geometrical parameters \( \{\alpha = 0.3, y = 3, 4.5\} \) and \( \{\alpha = 0.5, y = 3\} \). Figure 4.34 is similar to Figure 4.33 except the Nusselt number is “normalized” by the Nusselt number of a straight tube \( (y = \infty) \) with the same aspect ratio. The limit \( \{y \to \infty, \alpha \to 0\} \) (i.e. straight circular duct) is shown and provides a lower bound on the combined hydrodynamic and thermal performance. The values of Nusselt number and friction factor for the straight circular tube are calculated from the correlations given in the Handbook of Single-Phase Convective Heat Transfer [14]. Further, two curves for straight circular tubes with twisted tape inserts are shown and provide an upper bound on the combined performance. The Nusselt number and friction factor for the twisted tape case were plotted from the correlation given by Manglik and Bergles [2].

It is worth noting that Fig. 4.34 quantifies the increased heat load with respect to a straight tube (with the same cross-sectional geometry). The measure \( \text{Nu}/\text{Nu}_{y=\infty} \) is equal to \( Q_{\text{twisted}}/Q_{\text{straight}} \) and implements the FG-2a criterion for the evaluation of the thermal performance of a compact heat exchanger as discussed by Yerra, Manglik and Jog [7].

For moderate Prandtl numbers, \( 0.5 \leq \text{Pr} \leq 10 \) [6], it is also useful to look at the variation
$\alpha = 0.3$, $y = 3$

$\alpha = 0.3$, $y = 4.5$

$\alpha = 0.5$, $y = 3$

$\alpha = 0.5$, $y = 2$

Figure 4.33: Nusselt number versus $f \cdot Re^3$
$\alpha = 0.3, \ y = 3$

$\alpha = 0.5, \ y = 3$

Straight Tube

$\alpha = 0.3, \ y = 4.5$

Twisted Tape Correlation $y = 3$

Twisted Tape Correlation $y = 6$

Figure 4.34: Normalized Nusselt number versus $f \cdot Re^3$
of the Colburn factor $j$ as a function of $f \cdot \text{Re}^3$. This is shown in Figure 4.35 where the Colburn factor is given by

$$j = \text{St} \frac{\text{Pr}^{2/3}}{\text{Nu}} \left( \frac{\text{Re} \text{Pr}^{1/3}}{}\right)$$  \hspace{1cm} (4.8)

On the other hand, in Figure 4.36 where $j$ for given aspect and twist ratios is normalized by $j$ for a straight tube with the same aspect ratio, the variation in $j/j_{y=\infty}$ is identical to that of $\text{Nu}/\text{Nu}_{y=\infty}$ since the dependence on Re and Pr is the same in the numerator and the denominator and so $j/j_{y=\infty} = \text{Nu}/\text{Nu}_{y=\infty}$.
Figure 4.35: Colburn factor ($j$) versus $f \cdot Re^3$
Figure 4.36: Normalized Colburn factor \((j/j_{y=\infty})\) versus \(f \cdot Re^3\)
Chapter 5

Conclusions

Numerical solutions of periodically fully developed flows in helically twisted tubes of elliptical cross sections are presented. Twist ratios of $y = \{3, 4.5, 6\}$ were considered for a range of Reynolds numbers from $10 \leq Re \leq 1000$ and a Prandtl number of 3. The friction factor and heat transfer results along with normalized velocity and temperature distributions were presented. The friction factor and Nusselt number are strongly dependent on the twist ratio and the Reynolds number in twisted elliptical tubes. The friction factor and the Nusselt number increase with a decrease in twist ratio (tighter twist) and increase in Reynolds number due to the enhanced mixing caused by the induced secondary swirl flows. For $\alpha = 0.3$, $y = 3$ and $Re = 1000$, Nu increased by nearly three times compared to the corresponding straight tube value. Comparison to straight circular tubes with twisted tape inserts shows that twisted elliptical tubes provide comparable heat transfer enhancement at a comparable friction factor increase albeit not as effectively.
5.1 Suggestions for Future Work

This study characterizes the hydrodynamic and thermal behavior of twisted tubes with elliptical cross sections for laminar Newtonian flows with a Prandtl number of 3 for the constant wall temperature boundary condition. Extending this characterization to Prandtl numbers spanning a few orders of magnitude would provide a more complete data set (e.g. $0.7 < \text{Pr} < 1000$). Further, investigating the constant wall heat flux case for the same range of geometrical parameters would cover even more heat transfer applications. Obtaining correlations for $f \cdot \text{Re}$ and $\text{Nu}$ would be the ultimate goal of these extensions but they require much more data including more twist and aspect ratios.
Appendix A

Results of Numerical Simulation

Table A.1 lists the numerical values of $fRe$ and Nu for $y = \infty$ and the aspect ratios used in this study for reference. These results are calculated from Equations 4.4 & 4.3 (see [14], Chapter 3, Table 3.49).

The results of the numerical simulations are tabulated in Tables A.2 & A.3.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$f \cdot Re$</th>
<th>Nu</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>17.90</td>
<td>3.771</td>
</tr>
<tr>
<td>0.5</td>
<td>16.82</td>
<td>3.742</td>
</tr>
<tr>
<td>0.7</td>
<td>16.24</td>
<td>3.711</td>
</tr>
<tr>
<td>1</td>
<td>16</td>
<td>3.658</td>
</tr>
</tbody>
</table>

Table A.1: Numerical values of $f \cdot Re$ and Nu for $y = \infty$
\[ \alpha = 0.3 \]

\[ \alpha = 0.5 \]

\[ \alpha = 0.7 \]

<table>
<thead>
<tr>
<th>Re</th>
<th>( y = 3 )</th>
<th>( y = 4.5 )</th>
<th>( y = 6 )</th>
<th>( y = 3 )</th>
<th>( y = 4.5 )</th>
<th>( y = 6 )</th>
<th>( y = 3 )</th>
<th>( y = 4.5 )</th>
<th>( y = 6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.0419</td>
<td>1.0256</td>
<td>1.0159</td>
<td>1.0241</td>
<td>1.0143</td>
<td>1.0094</td>
<td>1.0079</td>
<td>1.0048</td>
<td>1.0031</td>
</tr>
<tr>
<td>20</td>
<td>1.0519</td>
<td>1.0285</td>
<td>1.0171</td>
<td>1.0280</td>
<td>1.0154</td>
<td>1.0099</td>
<td>1.0092</td>
<td>1.0051</td>
<td>1.0032</td>
</tr>
<tr>
<td>30</td>
<td>1.0654</td>
<td>1.0324</td>
<td>1.0186</td>
<td>1.0333</td>
<td>1.0169</td>
<td>1.0105</td>
<td>1.0110</td>
<td>1.0057</td>
<td>1.0034</td>
</tr>
<tr>
<td>50</td>
<td>1.1025</td>
<td>1.0440</td>
<td>1.0229</td>
<td>1.0478</td>
<td>1.0218</td>
<td>1.0123</td>
<td>1.0156</td>
<td>1.0074</td>
<td>1.0041</td>
</tr>
<tr>
<td>100</td>
<td>1.2229</td>
<td>1.1006</td>
<td>1.0476</td>
<td>1.0925</td>
<td>1.0446</td>
<td>1.0233</td>
<td>1.0270</td>
<td>1.0139</td>
<td>1.0077</td>
</tr>
<tr>
<td>200</td>
<td>1.4558</td>
<td>1.2592</td>
<td>1.1521</td>
<td>1.1801</td>
<td>1.1046</td>
<td>1.0636</td>
<td>1.0457</td>
<td>1.0264</td>
<td>1.0170</td>
</tr>
<tr>
<td>500</td>
<td>2.0265</td>
<td>1.6598</td>
<td>1.4680</td>
<td>1.3903</td>
<td>1.2730</td>
<td>1.2015</td>
<td>1.0919</td>
<td>1.0578</td>
<td>1.0410</td>
</tr>
<tr>
<td>700</td>
<td>2.3662</td>
<td>1.8889</td>
<td>1.6407</td>
<td>1.5013</td>
<td>1.3626</td>
<td>1.2758</td>
<td>1.1206</td>
<td>1.0770</td>
<td>1.0556</td>
</tr>
<tr>
<td>1000</td>
<td>2.8362</td>
<td>2.1978</td>
<td>1.8726</td>
<td>1.6523</td>
<td>1.4706</td>
<td>1.3698</td>
<td>1.1642</td>
<td>1.1055</td>
<td>1.0769</td>
</tr>
</tbody>
</table>

Table A.2: \( f \cdot \text{Re} \) numerical results
<table>
<thead>
<tr>
<th>Re</th>
<th>$\alpha = 0.3$</th>
<th>$\alpha = 0.5$</th>
<th>$\alpha = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$y = 3$</td>
<td>$y = 4.5$</td>
<td>$y = 6$</td>
</tr>
<tr>
<td>10</td>
<td>0.7988</td>
<td>0.9026</td>
<td>0.9453</td>
</tr>
<tr>
<td>20</td>
<td>0.8154</td>
<td>0.9130</td>
<td>0.9544</td>
</tr>
<tr>
<td>30</td>
<td>0.8286</td>
<td>0.9266</td>
<td>0.9660</td>
</tr>
<tr>
<td>50</td>
<td>0.8547</td>
<td>0.9513</td>
<td>0.9878</td>
</tr>
<tr>
<td>60</td>
<td>0.8723</td>
<td>0.9638</td>
<td>0.9980</td>
</tr>
<tr>
<td>80</td>
<td>0.9212</td>
<td>0.9954</td>
<td>1.0201</td>
</tr>
<tr>
<td>100</td>
<td>0.9790</td>
<td>1.0399</td>
<td>1.0477</td>
</tr>
<tr>
<td>120</td>
<td>1.0285</td>
<td>1.0909</td>
<td>1.0823</td>
</tr>
<tr>
<td>150</td>
<td>1.0905</td>
<td>1.1623</td>
<td>1.1474</td>
</tr>
<tr>
<td>200</td>
<td>1.1736</td>
<td>1.2456</td>
<td>1.2486</td>
</tr>
<tr>
<td>250</td>
<td>1.2459</td>
<td>1.3043</td>
<td>1.3156</td>
</tr>
<tr>
<td>300</td>
<td>1.3209</td>
<td>1.3560</td>
<td>1.3632</td>
</tr>
<tr>
<td>400</td>
<td>1.4839</td>
<td>1.4565</td>
<td>1.4394</td>
</tr>
<tr>
<td>500</td>
<td>1.6749</td>
<td>1.5706</td>
<td>1.5142</td>
</tr>
<tr>
<td>700</td>
<td>2.0676</td>
<td>1.8492</td>
<td>1.6924</td>
</tr>
<tr>
<td>1000</td>
<td>2.6754</td>
<td>2.3394</td>
<td>2.0467</td>
</tr>
</tbody>
</table>

Table A.3: Nu numerical results
Appendix B

Scaling the Temperature Profile

The energy equation was solved using the Dirichlet boundary condition. As a result, the bulk mean temperature asymptotically approached the wall temperature. For the cases with low Reynolds numbers, this resulted in a small temperature difference ($< 1K$) between the wall temperature and the bulk mean temperature at the outlet (end of the period) before the temperature profile became fully-developed (in some cases). This, of course, is not desirable because the outlet temperature profile is used as the inlet profile for the next run. In that case, the bulk mean temperature in the entire domain of the subsequent run would have been very close to the wall temperature which is problematic for a dimensional solver.

To solve this problem, we rescaled the outlet temperature profile before using it as the inlet profile. However, the temperature at each grid point cannot not be simply rescaled. The temperatures had to rescaled under a contraint to preserve the “developed-ness” of the profile. Recall that a fully-developed temperature profile with the Dirichlet boundary condition means that the normalized temperature (cf. Eqn. 2.14) becomes constant. Therefore, the scaling constraint is
\[ \theta_i = \theta_o \]  

(B.1)

where \( \theta_i \) is the rescaled temperature.

Implementing this constraint gives the following equation the temperature at each grid point in the profile

\[
T_i = \frac{T_o \cdot \langle T_i \rangle - T_o \cdot T_w - T_w \cdot \langle T_i \rangle + T_w \cdot \langle T_o \rangle}{\langle T_o \rangle - T_w}
\]  

(B.2)

where the subscript \( o \) refers to the outlet profile from a given run and the subscript \( i \) refers the to rescaled profile to be used as the inlet profile for the subsequent run.

The above procedure was implemented using the Matlab .m file listed below.

```matlab
clear;
rho = 983.2;
mu = 46.68e-5;
Dh = 0.02;
Tw = 373;
Pr = 3;
Revision = 0;

% List of twist ratios for a given run number
% --------------------------------------------------------------
% This option could be set to one twist ratio
% only. It is useful for scaling multiple profiles
% at once.
% --------------------------------------------------------------
TR = [3.0,4.5,6.0];

% List of Reynolds numbers for a given run number
% --------------------------------------------------------------
% This option could be set to one Reynolds number
% only. It is useful for scaling multiple profiles
% at once.
% --------------------------------------------------------------
```
\[ Re = [10, 20, 30, 50, 100, 200, 500, 700, 1000]; \]

\% Run number to scale
\[ R0 = 21; \]

dateStr = '2009-08-30';
dateStrTrn = '2009-08-09';
dateStrCas = '2009-08-24';

\%
cNs1 = 'Elliptical_Pipe_Twist_1P_Y=';
cNs2 = '_ba=0.3_Re=';
pNs1 = 'Elliptical_Pipe.ba=0.3_Re=';

var = ['(x ',' '); ...
     '(y ',' '); ...
     '(z ',' '); ...
     '(temperature',' '); '];

\% Runs
for i = 1:length(TR)
    TRStr = sprintf('%0.1f',TR(i));
    for j = 1:length(Re)
        wm = (Re(j)*mu)/(rho*Dh);
        ReStr = sprintf('%04d',Re(j));
        runs = ceil((0.05*Re(j)*Pr)/(TR(i)*2));
        RI = sprintf('%02d',R0);
        RN = sprintf('%02d',R0);

        \% Build profile name strings for input and output profiles
        profNameI = strcat(dateStr,cNs1,TRStr,cNs2,ReStr,'-R',RI,'-T','.prof');
        profNameO = strcat(dateStr,cNs1,TRStr,cNs2,ReStr,'-R',RN,'s-T','.prof');

        \% Variable starts and ends in Fluent profile file
        nPts = dlmread(profNameI,'',[0 2 0 2]);2
        Xs = 2;
        Xe = 2 + nPts - 1;
Ys = Xe + 3;
Ye = Ys + nPts - 1;
Zs = Ye + 3;
Ze = Zs + nPts - 1;
Ws = Ze + 3 + 2*(nPts - 1 + 3);
We = Ze + 3*(nPts - 1 + 3);
Ts = We + 3;
Te = Ts + nPts - 1;
As = Te + 3;
Ae = As + nPts - 1;

% Read input profile from Fluent profile file

X = dlmread(profNameI,'',[Xs 0 Xe 0]);
Y = dlmread(profNameI,'',[Ys 0 Ye 0]);
Z = dlmread(profNameI,'',[Zs 0 Ze 0]);
To = dlmread(profNameI,'',[Ts 0 Te 0]);
W = dlmread(profNameI,'',[Ws 0 We 0]);
A = dlmread(profNameI,'',[As 0 Ae 0]);
s0 = sprintf('%d',nPts);

Wavg = sum(W.*A)./sum(A);
Tmo = sum(To .* W .* A)./(sum(A)*Wavg);

% Specify new inlet bulk mean temperature
Tmi = 303;

% Rescale the temperature at each node
Ti = (To.*Tmi - Tw.*Tmi - Tw.*To + Tw.*Tmo) ./ (Tmo - Tw);

minTi = min(Ti);

% Check that no negative temperature exist in the new profile
while ((min(Ti) < 0 ) && (Tmi < 370))
    display('Negative temperature found in inlet profile!');
    display('delta revised');
    Tmi = Tmi+1;
    Ti = (To.*Tmi - Tw.*Tmi - Tw.*To + Tw.*Tmo) ./ (Tmo - Tw);
end;

data = cat(2,X,Y,Z,Ti);
% Write output profile

fid = fopen(profNameO,'w');
fprintf(fid,'%s %s%s','((pressure-outlet-6 point',s0,')');

for k = 1:4
    fprintf(fid,'
%s',strtrim(var(2*k-1,:)));
    fprintf(fid,'
%1.9f',data(:,k));
    fprintf(fid,'
%s',strtrim(var(2*k,:)));
end;
fclose(fid);

end;
end;

end;
Bibliography


