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Genetic Fuzzy Controller for a Gas Turbine Fuel System

A thesis submitted to the Graduate Faculty of the University of Cincinnati in partial fulfillment of the requirements for the degree of Master of Science in the School of Aerospace Systems of the College of Engineering and Applied Science

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by

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Committee Chair: Dr. Kelly Cohen
Abstract

In this study, a fuel system controller for a gas turbine engine was examined. Controller design in this application is challenging due to nonlinearities in the closed loop system, as well as uncertainties associated with hardware components from part variation or degradation. Current closed loop design methodologies are discussed, as are the limitations or challenges facing these systems. Details on fuzzy logic control and its benefits in this type of application are explored. Information on genetic algorithms is presented, along with a study on how this optimization approach can be utilized to enhance the fuzzy logic controller process. A fuzzy logic controller structure was developed for providing closed loop fuel control in the gas turbine application, using a genetic algorithm to tune the system to provide an accurate and fast response to changing input demands. With a genetic fuzzy controller in place, closed loop analysis was performed, along with a stochastic robustness analysis to assess controller performance in an uncertain environment. Results show that the genetic fuzzy system performed well in this application, resulting in a system with fast rise and settling times to stepping inputs, while also minimizing overshoot and steady state error. Robustness characteristics of the fuzzy controller were also demonstrated, as the stochastic robustness analysis yielded acceptable performance in each simulation of the closed loop system with uncertainties included.
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Chapter 1

Introduction

With the dawn and advancement of the industrial age, means for controlling mechanical systems and processes have been sought. From early electronic circuits to modern digital controls, systems have been developed to provide automation, improve accuracy, increase efficiency, or expand capability. This trend is expected to continue, as new technological challenges are undertaken.

Control systems cover a broad spectrum of applications, as well a broad spectrum of involvement in a given system. Some control systems function merely in watchdog roles, monitoring critical parameters in a system and taking mitigating action in the event an unsatisfactory condition is detected. In other systems, the control system is absolutely critical to the performance and capabilities of the system. In fact, many modern tactical aircraft are designed to be aerodynamically unstable, and when augmented with a well-designed control system, provide enhanced flying qualities and increased operational capabilities to give an edge in their designed missions.

In an age driven by technology, where systems are constantly being pushed to meet new challenging requirements, advancements in control system design are needed to keep up. We want our mechanical systems to be faster, more accurate, and more robust. We want our systems to be intelligent, to be able to analyze current conditions and objectives, and to make decisions
on appropriate behavior, instead of just applying generic global behavior. To this end, intelligent control techniques are gaining traction and increased focus.

Fuzzy logic control is one such intelligent control technique, and will be a pillar of this study. This nonlinear control design technique provides significant benefits in terms of flexibility and optimization space. When coupled with the ability to capture expert or heuristic knowledge, and the ability to tune behavior in local envelopes of the operating space, fuzzy logic can be an indispensable control design tool in many applications. Fuzzy logic control also possesses inherent robustness properties due to having knowledge-based properties, making them good candidates for stochastic systems.

As a result of the flexibility of fuzzy logic controllers, one challenge facing control designers is the tuning process. Fuzzy logic controllers can have a variety of handles to impact performance, from the fuzzy input and output sets to the governing rule base. Genetic algorithms, a branch of evolutionary algorithms, will be utilized in this study to provide an autonomous guided search of the design space to develop a more optimized solution against the design requirements. This approach uses a search scheme based on evolutionary principles, a ‘survival of the fittest’ type technique, including genetic recombination and variation.

In addition to ensuring a controller design meets performance requirements and objectives, a designer will also want to make sure the controller will be robust enough to meet those requirements in all operating conditions. A controller may perform admirably for a given mechanical system design, but will it uphold that performance when part-to-part variation due to
manufacturing capabilities is taken into account? Will it stand up to all operating conditions and system degradation? A study in controller robustness is needed to answer these questions, and in this study, a stochastic robustness analysis will be performed to ensure the controller maintains adequate performance in an uncertain environment.
Chapter 2

Background Information and Literature Survey

In this chapter, background information on current closed-loop control system design approaches and methodologies will be presented, as well as some of the limitations on those systems. Information on intelligent control techniques, *Fuzzy Logic Control* in particular, will also be presented, including why these type systems are good candidates in the face of the limitations on traditional design methods. Details on *Genetic Algorithms* and their use in fuzzy systems will also be covered. Lastly, information will be presented on *Stochastic Robustness Analysis*, a useful technique for characterizing controller performance robustness.
2.1 Current Design Methods

Through its history, control system design has produced a wide variety of approaches and techniques that can be used in various systems. In this section, a few of the more common and broadly used schemes will be discussed. Basic background information on the approaches will be covered, along with their benefits and constraints.

One of the simplest controller forms is an on-off controller, commonly referred to as a ‘bang-bang’ controller. This type of controller has two states it can switch between, such as a heating element switching on or off, or a valve switching between two positions. Implementation is fairly straightforward, switching states whenever certain criteria are met. This type controller has its roots in optimal control theory, and as such represents a minimal-time solution, making it very effective in many design situations [21].

Even with its simple structure, this type controller has many applications. Many heating and air conditioning systems run on a bang-bang controller, targeting a temperature band around a given set point. For a residential heating system, the controller will turn on whenever the temperature dips below the set point (perhaps with some hysteresis), adding heat to the system until the set point is reached. The furnace then switches off, leaving the temperature of the system to float until it drops below the trip point again. An air conditioning system works similarly, in the opposite direction. Bang-bang controllers are more than adequate in these type scenarios, aided by the relatively large time constants of the system under control (internal temperature).
In the temperature control system scenarios considered here, the active element has only two states, on or off, making a bang-bang controller suitable. Once the furnace is triggered on, it provides heat at a designed rate, which it cannot vary during operation. Whether the temperature is just slightly below the set point or significantly below, the output will have the same response.

This illustrates a few of the major limitations of a bang-bang controller, in its differentiation and granularity. This type of controller may lead to decreased efficiency in some areas, as it only has two actions paths to take. Using the temperature control example again, a more efficient controller might apply heat at varying rates, depending on the current temperature relative to the set point. This could be accomplished through cascading bang-bang controllers, but this adds complexity to an otherwise simple system, while a change in controller structure may be better suited for the task.

Another area where bang-bang controllers struggle is robustness properties in systems with uncertainty. A bang-bang controller may lead to an oscillatory response with uncertainties in the system, leading to stability concerns [31]. Prakash and Nair have studied a means of handling uncertainty using intelligent control techniques, demonstrating feasibility in power flow control with their fuzzy bang-bang approach [15].

Another class of controllers that is quite prevalent in industrial applications is a PID controller, an acronym for a controller with ‘proportional’, ‘integral’, and ‘derivative’ components. The performance of this type of feedback controller can be tuned using three different handles to influence behavior, offering significantly enhanced capabilities over the simple bang-bang

6
controller. Each of the different components of this type of controller can be tuned individually with a different overall impact on system behavior, as discussed below.

The proportional component takes controller action proportional to the current error of the system, or the difference between the target point of the system and the current feedback. The error is multiplied by the proportional gain to influence the output on a varying scale, having a larger impact for larger errors. Larger proportional gains can lead to faster rise times (the time it takes for the output to move from 10% to 90% of a step in the input demand), but can also lead to overshoot of the target value, which can be an important concern in some applications. Generally, steady-state errors decrease when incorporating increasing proportional gains, however, the risk of instability is also a concern, as with too high a gain the controller can overreact to an input error, leading to increasingly larger errors.

The integral component of the controller integrates over the error in the system, and issues an output response proportional to this integrated error. This component will take larger actions for larger errors similar to the proportional component, but also takes larger actions for errors that last for some duration. A controller with an integral component strives to remove steady-state error completely, as any error present in the system will show up in the integral component, which will take effect to drive out the error. Integral terms also have tendencies to increase overshoot, and can also increase the setting time of a system, or the time it takes for a system to stay within a certain percentage of the target point, typically within 2% or 5%. Integral terms can also degrade stability capabilities, so this must be checked carefully throughout the operating envelope of the controller.
The final term, the derivative component, acts on the derivative of the current error. This piece can be interpreted as a predictor of future error, and when combined with the past-capturing properties of the integral term and the proportional term acting on the current error, provides a controller with influence from all three time regions, a reason for the broad appeal of PID controllers [2]. Incorporation of derivative control can help reduce overshoot and settling time to stepping input demands, and can be used to balance the negative effects of the proportional and integral terms in these areas. Derivative control can also improve stability properties of the closed loop system.

It is not necessary to use all three components in every instance; the individual components can be combined in any permutation to suit the needs of the control application at hand. Sometimes only a simple proportional (P) controller is needed, or perhaps adding in an integral component to help with steady state accuracy (PI controller). This freedom in design, along with their simple structure and vast experience base, enable PID controllers to have a broad appeal.

Many controllers of this class can provide reasonable control in terms of stability with minimal tuning, however many applications may have additional requirements in terms of timing, overshoot, or error that may require additional tuning. Even though this type controller typically has up to three parameters that influence behavior, tuning can become cumbersome to achieve optimum operating points. Manual tuning is possible, but it is often best to start with a set of heuristics to serve as guidelines, such as shown in Table 2.1 suggested by Ang and others [1]. This table shows the impact of increasing an individual gain while holding the others constant.
Table 2.1: Closed Loop Effects of Tuning Individual PID Gains

<table>
<thead>
<tr>
<th>Gain</th>
<th>Rise Time</th>
<th>Overshoot</th>
<th>Settling Time</th>
<th>Steady-State Error</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>Decrease</td>
<td>Increase</td>
<td>Small Increase</td>
<td>Decrease</td>
<td>Degrade</td>
</tr>
<tr>
<td>$K_i$</td>
<td>Small Decrease</td>
<td>Increase</td>
<td>Increase</td>
<td>Large Decrease</td>
<td>Degrade</td>
</tr>
<tr>
<td>$K_d$</td>
<td>Small Decrease</td>
<td>Decrease</td>
<td>Decrease</td>
<td>Minor Change</td>
<td>Improve</td>
</tr>
</tbody>
</table>

There are various other tuning methods that have been developed for PID controllers, such as the Ziegler-Nichols Method, which is another heuristic approach that can provide acceptable closed loop response in many instances, but may result in higher overshoots. PID tuning software packages are also prevalent throughout the industry, allowing for more optimal controller design with less manual analysis.

In an offline-tuning environment, the performance of the PID controller can be heavily dependent on the accuracy of the model of the closed loop system. Differences from the actual closed loop system due to un-modeled effects or noise introduction can degrade expected performance in the target system.

Optimization of PID controllers in a system with nonlinearities can also be difficult. At their heart, PID controllers are linear controllers, which provide inadequate design space for optimization in a nonlinear regime. As Bonissone and others point out, the output region of a PI controller can be mapped to a single plane, which leads to difficulties for finding true optimization points for a given control system [4]. Some of these optimization difficulties can be met in part using techniques such as cascading multiple PID controllers together, but this adds complexity, which can introduce additional tuning difficulties. Gain scheduling is another
option, which modifies the PID gains depending on certain conditions, which incorporates techniques similar to intelligent control, discussed later in this study.

Robustness is a concern in any controller design application, and this is no different for PID controllers. This type of controller can have a difficult time dealing with uncertainty in the closed loop system. Small shifts in characteristic parameters of the closed loop system can shift the controller to a lower performance point, or could erode stability margins. Intelligent control techniques, such as fuzzy logic controllers, have some inherent improvements in robustness characteristics, and may be a better fit in stochastic systems. Work has been done in the area of fuzzy-PID controllers, providing the benefits of the PID structure with the added robustness and flexibility that results from the nonlinear fuzzy structure [28].

There are a number of other closed-loop control schemes besides bang-bang and PID controllers, although the design of these systems is often more involved than these simple forms, and they are not as widely used. One such approach is Optimal Control, which, as the name would suggest, is a technique that tries to operate at the most optimized conditions, such as a minimum time or minimum fuel burn case. Paiewonsky provides some background into optimal control theory, as well as a variety of examples illustrating application [14].

Robust control is another control design approach, one that tries to ensure controller stability in the face of uncertainty in the closed loop system. This approach assumes a bounded distribution for system disturbances, and is designed to maintain adequate control over any disturbance within the range. H-infinity loop-shaping is the most common robust control technique, and is
being used more frequently in a variety of systems. You, Chen, and He implement H-infinity control for vibration control in flexible structures, an area that is increasing in popularity [32].

This section is not exhaustive in its discussion of various control schemes; there are additional control system design approaches not highlighted here that may be well suited for a given application. A designer with in-depth knowledge of the system and the control objectives would need to consider which approach is best suited for a given application.
2.2 Fuzzy Logic Controller

Fuzzy sets were introduced by Lotfi Zadeh [33] as a means of handling imprecision in a classification process. Membership to a fuzzy set is defined by membership functions, continuous functions with values between 0 (no membership) and 1 (full membership). Zadeh suggested these sets for use in problems where precision may not be possible or necessary, and the desired outcome could still be attained.

In addition to the definitions associated with fuzzy sets and membership functions, Zadeh also extended the definitions of operations associated with set theory into fuzzy set theory, such as union and intersection. The union of two fuzzy sets was defined as the max of the two sets, while intersection was defined as the min. With these definitions, the behavior for crisp or singleton sets is preserved, yielding the same result as union or intersection in the classical sense.

Membership functions are a mathematical representation for the degree to which something belongs to a fuzzy set. Say you wanted to classify a given temperature ([0°C 100°C] °C) using the categories Cold, Warm, and Hot. If using crisp sets, this would be difficult to do. Where would you draw the line between a Warm temperature and a Hot temperature? The difference between any adjacent categories would only be 1 °C, barely discernable to most human sense observations, yet the temperatures would be placed in two distinct categories. Fuzzy logic can help deal with this type scenario, by defining some membership to both adjacent categories.
Using the example discussed above, Figure 2.1 illustrates a sample set of membership functions for the three different temperature categories. To illustrate how membership functions operate, the membership towards the different categories will be determined for a single temperature, 70 °C. Moving to this temperature on the horizontal axis, the corresponding membership to each category can be found by determining the membership function value at the temperature of interest. As illustrated in Figure 2.2, this temperature has 0 membership in the category of *Cold*, belongs to *Warm* to degree 0.75, and shows a 0.25 membership to the category of *Hot*. This shows the temperature of interest has some membership to both the *Warm* and *Hot* categories, but still shows the temperature is more associated with a *Warm* temperature. The membership function values can be used in some sort of weighting scheme for determining the output of a
system based on this input temperature, or also can be used in a Fuzzy Logic Control system, which will be demonstrated shortly.

![Temperature Membership Functions](image)

**Figure 2.2: Membership Function Evaluation for Set Temperature**

Membership function categories and their values are up to the designer, and could be adjusted based on system needs. In the example above, trapezoidal membership functions were used (truncated for limits), but membership functions can take a number of different forms. Isosceles triangles are fairly common, but it is possible to use non-isosceles triangles as well. Gaussian distributions and exponential distributions are also available for use to the designer following scaling, as well as any custom function with values over the interval from [0 1].

Mamdani expanded on Zadeh’s use of fuzzy sets to develop a control system based on fuzzy logic principles [10]. A *Fuzzy Logic Control* system can operate in a closed loop environment...
similar to other control techniques. The system takes inputs from the system, makes decisions on the desired response based on classification of those inputs, and determines appropriate outputs. The result is a fairly simple nonlinear control system that can be tuned for desired performance.

The basic structure of a fuzzy logic controller is illustrated below in Figure 2.3. This figure illustrates the four main components of this type of controller: The Fuzzifier, Rule Base, Defuzzifier, and the Inference Engine. These components work in tandem to provide closed loop control for a given system of interest. Details on each of these components and their roles are discussed below.

![Figure 2.3: Fuzzy Logic Controller Structure](image-url)
Fuzzifier:

The purpose of the fuzzifier is to transform the crisp inputs into the fuzzy logic controller into fuzzy inputs. The inputs to this type of system can range from sensed quantities, estimated quantities, or even system demands for a desired state. The fuzzifier uses series of membership functions to categorize the crisp inputs into their respective fuzzy set representations. The fuzzifier determines which fuzzy sets are ‘active’ (non-zero membership) based on each of the crisp inputs, and also to what degree. The membership function example using the temperature scale could be viewed as a sample fuzzifier. The crisp input coming in would be the temperature, measured using a sensor or estimated from a mathematical model. As shown, the output would be the membership function values associated with each fuzzy set.

Fuzzifiers must provide *coverage* of the input space; every potential input to the fuzzy system must result in membership to one or more of the possible fuzzy sets. This helps ensure *completeness* for the fuzzy logic control system, to be discussed in more detail in the section on rule bases. *Computation* is also an important consideration when designing a fuzzifier. For each fuzzy set that becomes active based on a given input, the mathematics of the inference engine become more involved as a result, especially when considering controllers with multiple inputs, with different fuzzy sets associated with each. Therefore, it is standard practice to ensure that only two fuzzy sets are active at a time for each input, leading to the guideline that only adjacent fuzzy sets should be allowed to overlap in their membership function distributions.
Rule Base:

The rule base of a fuzzy logic controller can be thought of as the ‘brains’ of the control system, or its means of determining what actions to take based on a given set of inputs. A rule base is made up of series of IF-THEN rules corresponding to the fuzzy inputs and leading to the fuzzy outputs. The rules can be developed using knowledge from experts or operators in the field, as well as historical experience. This allows for control system behavior to be developed without requiring expertise in control system, or can also serve as a means for adapting a controller based on system behavior. Cordon and others suggest methods for tuning rule bases to achieve desired performance, but this approach is beyond the scope of this study [7].

As a simple example, a sample rule base will be developed for an automobile’s cruise control system. After all, most drivers have a basic understanding of a cruise control system, and would be able to develop basic rules for governing behavior. This example will use the difference (delta) between the desired set speed and the current speed as the input, classified into fuzzy sets Low, Even, and High by a fuzzifier. Using this information, a sample rule could be “IF speed delta is Low, THEN Accelerate”. Here, the term Accelerate corresponds to a fuzzy output set, which would be defined in the defuzzifier. With little thought, most drivers would be able to develop the set of rules illustrated in the table below.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>Accelerate</td>
<td>IF speed delta is Low, THEN Accelerate</td>
</tr>
<tr>
<td>Even</td>
<td>Maintain</td>
<td>IF speed delta is Even, THEN Maintain</td>
</tr>
<tr>
<td>High</td>
<td>Decelerate</td>
<td>IF speed delta is High, THEN Decelerate</td>
</tr>
</tbody>
</table>

Table 2.1: Simple Automobile Cruise Control Rule Base
This rule base is fairly crude, and may not deliver adequate performance in all driving conditions. Enhancements to this rule base may involve adding more fuzzy sets to the input and output systems, increasing granularity. For example, modified categories like Very Low or Slightly High could be added as additional fuzzy sets to the fuzzifier, and the output could be expanded to include additional classes for Accelerate and Decelerate, perhaps to provide varying degrees of these actions. Another option would be to include an additional input to the fuzzifier, perhaps a rate term in addition to the speed difference. Examples of rules using these expanded fuzzy sets could include statements such as “IF speed delta is Slightly Low AND speed is Increasing, THEN Maintain” or “IF speed delta is Slightly Low AND speed is Decreasing, THEN Accelerate”. This could provide a means of differentiating different driving cases, leading to different controller behavior dependent on a given scenario. While this example is fairly basic and not complete, it should provide insight into the rule-making process and how working knowledge of a system could be captured without the need for expertise in control system design.
Defuzzifier:

The defuzzifier fulfills the inverse role of the fuzzifier; it takes the information from the fuzzy output sets and transforms them into crisp outputs that can be used by the control system. This output could be a number of different things, from controller loop gains, to modeled parameters, to the system demands themselves. The typical defuzzifier consists of membership functions similar to a fuzzifier, but the system works in the opposite direction. Instead of working from the input on the horizontal axis to determine the fuzzy set memberships, the active fuzzy sets are used to find a crisp output on the horizontal axis. The active fuzzy sets in the defuzzifier are determined by the rule consequents in the inference engine, and some mathematical approach is used to combine these sets into a single output value. A Centroid method is one of the more common and simplistic approaches, and will be employed in this study and demonstrated below. Numerous other methods have been proposed as alternatives to try to accommodate some of the potential pitfalls or special cases that can arise [11].
Inference Engine:

The inference engine is the component that ties all the other components of the fuzzy logic controller into a complete system. The engine takes the fuzzy inputs from the fuzzifier, checks them against the rules in the rule base, and determines which fuzzy output sets are activated based on exercising the rules. The degree of membership for each of the fuzzy input sets is used to determine the ‘degree of activation’ of the corresponding output sets. Using the temperature classification example illustrated above again, a temperature of 70 °C had membership of 0.75 to fuzzy set *Warm* and 0.25 to *Hot*. Therefore, the output fuzzy set(s) tied to the input set *Warm* through the rule base would be activated to degree of 0.75, and those outputs tied to the set *Hot* would be activated to degree 0.25. To better illustrate this, consider the temperature classification example as the fuzzifier for a water heater fuzzy logic controller application. The goal of the heater is to maintain temperatures above 70 °C, temperatures predominant in the *Hot* category. The output fuzzy sets associated with this application may be actions like *Maintain*, *Low Heat*, and *High Heat*, activated through the simple rule base below.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cold</td>
<td>High Heat</td>
<td>IF input temperature is Low, THEN apply High Heat</td>
</tr>
<tr>
<td>Warm</td>
<td>Low Heat</td>
<td>IF input temperature is Warm, THEN apply Low Heat</td>
</tr>
<tr>
<td>Hot</td>
<td>Maintain</td>
<td>IF input temperature is Hot, THEN Maintain</td>
</tr>
</tbody>
</table>

Table 2.2: Simple Water Heater Control Rule Base

The figure below illustrates how the output sets are activated based on the membership values on the input sets. These output sets are normalized on an interval of 0 to 1, but could be scaled to
any appropriate level. The output sets are triggered into action to the same degree of the membership of the inputs. The resulting area on this figure would represent the combination of the outputs that are active for this set of inputs, and the crisp output would be determined based on this information.

![Heater Output Membership Functions](image)

**Figure 2.4: Output Fuzzy Sets for Water Heater Example**

This example used a simple case with single-antecedent rules. It is quite common for rule antecedents to be combined, such as in forms “IF $x_1$ is $A$ AND $x_2$ is $B$, THEN $y_1$ is $C$” or “IF $x_1$ is $A$ OR $x_2$ is $B$, THEN $y_2$ is $D$”. In these cases, the membership function values of the inputs are combined using fuzzy definitions of these operations. The AND operator (intersection) would result in the minimum of the two input membership values, or the product of the two if this alternative definition for fuzzy intersection is used. The OR operator (union) would result in
maximum of the two input membership values being used, or any one of a number of alternative definitions [11]. Of course, these operators could be expanded to whatever number of rule antecedents are used for a given rule base system.

As with any new design approach, it will have benefits and drawbacks compared to other design methods. Fuzzy logic controllers allow for a simple implementation of a nonlinear control system design. This approach is also a candidate for embedded systems because of its relative ease for computation. The structure of this type of controller also provides for compartmentalization, leading to the possibility of the controller to be tuned for specific behavior in smaller, local operating regions, instead of having to tune or check the performance over the entire operating envelope following controller adjustments [27]. Fuzzy logic controllers are also able to perform admirably in the face of uncertainty in a given control application, making them good candidates for systems that need to meet higher performance objectives in environments affected by uncertainties.

Fuzzy logic controllers have been explored and implemented for a number of different applications. Kelly, Weller, and Ben Asher develop a controller for a linear second-order system proposed as a benchmark problem by the American Control Conference [6]. This problem consisted of a system with plant uncertainty, and had a non-colocated sensor and actuator pair, leading to a nontrivial design problem. They cite a fuzzy logic controller as a good candidate for this application due to its ease of implementation, and its inherent robustness characteristics. Also mentioned is the ability of the controller to emulate the minimum-time solution derived from optimal control theory using heuristics in its rule base. The controller in this paper was
developed with a Luenberger observer in the loop; the current study will have access to necessary control states. Following controller development, the paper also examined the robustness characteristics of the control system through Stochastic Robustness Analysis, examined further in this study.

Nelson and Lakany have studied use of fuzzy logic in the fuel control system for industrial gas turbines [12]. In their paper, they discuss how traditional control system design methods may encounter difficulties in systems that can degrade over time, especially in areas of emissions control. They suggest that fuzzy logic controllers may be a good candidate for this type of environment, lending to their nonlinear properties and robustness in cases where the plant may not be well understood (as in cases of degradation). They develop a fuzzy logic controller to compare performance against an existing PI controller. Through various operational scenarios, performance of the two controllers was comparable, although the PI controller typically had better response and settling times. In some scenarios, however, the fuzzy logic controller did show better performance in terms of exhaust temperature overshoot, which is a critical performance measure in industrial gas turbine operation.

A key comment Nelson and Lakany mention in their paper is the difficulty to tune the fuzzy logic controller to a more optimum design point. They implemented a controller with two inputs, Error and Error Rate ($E$ and $\dot{E}$), using five fuzzy sets for each. The output, an incremental change to fuel demand, also had five sets, although singleton sets were used. The system used 25 rules in the base for the inference engine. The result is a large number of parameters that can be tuned to affect performance, although some simplifications were made.
By assuming a consistent shape for the membership functions, the five fuzzy sets for each input can be defined by two parameters, or assuming symmetry about the zero axis, can be reduced to one. However, fuzzy sets with such a simple structure may be constraining in terms of optimization. The authors proposed an alternative fuzzy set definition, using symmetry about the zero point in the input space, but using non-isosceles triangular membership functions where each set is allowed to be a different size. This results in a more flexible fuzzy set definition that can be better tuned for performance than a rigid structure with identically shaped membership functions, but still only needs two defining parameters. As illustrated in the figure below, x1 and x2 can entirely define the membership functions for the five fuzzy sets.

Figure 2.5: Non-Isosceles Fuzzy Membership Functions
Of course, this approach can be expanded for inputs with more fuzzy sets, adding an additional tuning parameter for each pair of additional sets. The symmetry constraint is common in fuzzy logic controller designs, as it is often desired to have consistent system response when moving in different directions. This approach may still provide some constraints on a truly optimal solution, but it does add flexibility to the design space and ensures that total membership across fuzzy sets always sums to one, which can be an important factor for controller consistency.

A fuzzy logic controller for a fuel system test bench is developed in an article by Zilouchian and others [35]. This test bench is designed to allow for proper simulation testing of a jet engine fuel delivery system, by controlling the combustor pressure that would be experienced in a given operating condition. The fuel system test bench is currently controlled with a traditional PI controller, but performance shortfalls are noted. This paper looks to fuzzy logic as a solution due to its nonlinear characteristics and the ability for tuning in small operating envelopes (compartmentalization). The authors also make note of the difficulties in tuning fuzzy logic controllers due to the number of design options. In this paper, they chose to use a consistent structure of seven triangular membership functions with equal size normalized over the range [-1, 1] for both the inputs (Error and Change in Error, E and ΔE) and output (Valve Positions), and chose to optimize using a scaling parameter up and downstream of the fuzzy sets. This restricts flexibility, but provides a simple structure for a first cut at controller design. The authors went on to develop separate fuzzy logic controllers for ‘coarse’ and ‘fine’ control, utilizing the compartmentalization capabilities of fuzzy logic schemes.
As discussed above, one of the major drawbacks to a fuzzy logic controller cited in various sources is the tuning process. The membership function types and the distributions themselves can present a great number of design options for the input/output fuzzy systems, which can be paralyzing for a designer trying to optimize a given control system. The rule base provides another set of inputs that can be used to alter the performance of the loops. Use of genetic algorithms can help in this area, performing a guided search towards an optimal design and taking some of the burden off of the control system designer. Details of genetic algorithms will be discussed in the next section.
2.3 Genetic Algorithm

Genetic Algorithms are a class of optimization approaches based on evolutionary principles. First introduced by John Holland in 1975, this type of algorithm searches for an optimized solution based on the premise of “survival of the fittest” [8]. The approach emulates Charles Darwin’s research on natural selection, that those members of a population best suited for their environment are more likely to survive and pass on their genetic traits. In a genetic algorithm, members of a population are represented by numeric strings (dubbed *chromosomes*), and evaluated for fitness for a certain function. Similar to natural selection, those that receive higher fitness ratings are more likely to see their chromosomes passed on to subsequent generations of the population, and through an iterative process, this algorithm will converge to a better performing design. This algorithm also incorporates the ideas of *genetic variation*, which will be discussed further in this chapter.

One of the first steps in developing a genetic algorithm is to determine how members of the population will be represented. Binary strings are the most common and often simplest representation, but it is also possible to represent members of the population with non-binary, or real-value coded strings. Fixed-length strings are more common, but Cordon, and others discuss details of variable-length strings and their benefits and application [7]. For this study, fixed-length binary-coded strings will be employed in the optimization algorithm for their inherent simplicity.
One problem that can arise when using a binary-coded string in the traditional base-two sense is related to the large hamming distances that are realized for successive integers. Integer values 7 and 8 for example have binary representations of 0111 and 1000 respectively, illustrating a difference in every character in the string for a relatively small difference in numeric value. The iterative genetic algorithm process may have a difficult time converging to a true optimized point using basic genetic variations (discussed below) if it would have to cross a large hamming distance en route. Cordon and others propose the use of a \textit{gray code}, in which only hamming distances of 1 separate successive integers [7]; however, single bit value changes may still have a large impact on the represented numeric value. This study will utilize multiple runs of the genetic algorithm, each with a different random initial population to try to mitigate this shortcoming.

Once the representation format for the members of the population has been determined, the initial population needs to be generated. The number of members in the population is left up to the designer. Larger populations will provide more opportunities to search the design space and provide population variation, but will also take longer to determine performance and the makeup of successive populations. The initial set can be generated in a number of different ways, ranging from purely random generation for each member string, to sparse sets that try to cover the range of the design space, or even initial predetermined candidate sets that correspond to known performance levels for which improvement is desired. Hybridization of these methods can also be utilized to generate an initial population.
With the initial population in place, the next step is to develop a means of assessing the performance of each member for a given function. A common example could be a genetic algorithm that tries to minimize the time for a given process. For this type case, the fitness function could simply be the inverse of the calculated time, yielding higher fitness ratings corresponding to members with the lowest process times. In this study into closed loop control applications, the fitness function will be based on common controller performance metrics such as response time, settling time, and control efforts.

As future populations are often developed using probabilities of selection based on the fitness ratings, it may be important to ensure that the fitness values are all positive. Another common pitfall in the optimization process is for a member to stand out from other members in terms of vastly superior fitness rating, which can lead to premature convergence if it occurs in early generations depending on the selection schemes used. Also, in later generations when most members begin to illustrate improved performance, it may make it difficult for any particular member to stand out, slowing the process of convergence to an optimized solution. As a means for mitigating these scenarios, Cordon and others provide suggestions for a number of fitness function scaling methods [7].

With the performance for a current population known, this information can be used to determine the candidate makeup of the next generation. Some selection schemes place direct copies of the top performers of a population into the subsequent population, known as elitist selection, ensuring that best performer of the next generation is always at least as good as the one before it. This approach also ensures that the candidate pool contains the genetic strings of the best known
solutions are always available for future generations, immune to being removed due purely to random occurrence. Similarly, discarding the poorest performers is another option, although these candidates may still have some desirable qualities that could become part of future solutions, and would tend to be removed from future generations anyway as part of the genetic algorithm process.

Most selection schemes choose candidates from the existing population using a probability measure determined from their fitness. The simplest of these measures is to divide the fitness of a given candidate by the sum of fitness for all candidates, although other probability methodologies have been suggested [7]. Using the simple approach, the candidates can then be thought of as being arranged on a roulette wheel, filling area proportional to their probability level. The wheel is then spun to select a candidate as many times as needed to fill out the full population.

This roulette wheel approach has some inherent flaws, one being the potential for large discrepancy between the expected and actual number of selections for a given candidate. Alternative methods for selection have been proposed, one of which being stochastic universal sampling [7]. In this sampling method, the roulette wheel is only spun once, while pointers spaced equi-circumferentially around the wheel determine the candidates needed to fill the population. There are numerous other schemes that can be employed in the selection process, each trading off simplicity for the likelihood of a better population sample.
With the candidates for the next generation of a population selected from the previous, the genetic variation process begins. Three different processes will be used to introduce genetic variation into the existing candidates: crossovers, mutations, and inversions.

Crossovers involve swapping genetic material between two chromosomes. In this way, optimal solutions are sought in the area of current knowledge in the design space. The simplest of these is the one-point crossover, where a number less than the length of the chromosome strings is chosen at random, and the genetic strings to the right of that number are swapped between two parent chromosomes. This is illustrated in Figure 2.6 below for two sample binary strings swapping the rightmost two bits.

![Figure 2.6: Illustration of One-Point Crossover](image)

The two-point crossover is also common, and helps alleviate the bias that the latter part of the chromosome strings are more likely to be swapped. In this process, two random numbers are chosen, and the string of characters between the two numbers is swapped in the parent chromosomes, as illustrated in Figure 2.6. Of course, this process could be extrapolated for higher order crossovers, up to the limiting case of the uniform crossover, where the child
chromosome is developed one element at a time, choosing each character independently from the
two parent chromosomes.

Another process to introduce new genetic combinations is mutation. In this process, each
individual character in a genetic string is independently subjected to the potential to change
value. In binary genetic strings, the outcome is simple; 1’s turn to 0’s and 0’s turn to 1’s. Where
as crossovers tend to search for new solutions near the current ones, mutations have the
possibility to ‘unlock’ new regions of the design space that haven’t been explored. This aids in
avoiding pre-mature convergence to local extremes, helping to find the overall optimal solution
in the available envelope.

Figure 2.7: Illustration of Two-Point Crossover

Figure 2.8: Illustration of Binary Mutation
The final means of introducing genetic variation considered in this study is inversion. Similar to mutation, this process is another means of exploring the design space. In this process, the binary chromosome string of the least fit candidate(s) are ‘inverted’, such that all 0’s become 1’s and vice-versa. While this type of operation does not have as strong a tie to genetic processes, it may be a helpful tool in developing new ‘genetic material’. The premise is that the least fit candidates are far from an optimal solution, and that by jumping to a new point in the design space, a more optimal solution can be found. This approach may also explore new regions that may not be optimal overall, but may have some desirable qualities that could become intertwined with existing solutions through the crossover process.

![Figure 2.9: Illustration of Genetic Inversion](image)

Figure 2.9 illustrates the entire process of generating a new ten-member population for the next iteration of a genetic algorithm once the fitness of the current population has been determined. This figure is intended for illustrative purposes, and is not representative of any data generated for this study. Part a) illustrates the chromosomes from the current population, sorted from highest fitness to lowest. Demonstrating an elitist scheme, the top two performers are placed directly into the next population. Also placed into the next population is the binary compliment of the worst performer, or an inversion of all the bits within the chromosome. These three chromosomes will be tested in the next population as-is, they are exempt from genetic variation introduced by crossovers and mutations. The base chromosomes for remaining members of the
population are selected with some randomness from the current members, typically weighted in some fashion using the current populations’ fitness ratings. Part b) shows the resulting chromosomes from the selection process.

Following selection, the population (minus elites and inverts) is subjected to crossovers. A random number is generated for each chromosome, and compared to the probability number selected for crossovers. For this example, both one- and two-point crossovers were employed. Part c) of Figure 2.10 shows the results of the crossover phase. The first column (CO) indicates which type of crossover resulted (one-point, two-point, or none), the second column (PC) indicated the partner chromosome with which genetic material was swapped, and the third (Bits) shows which bits bound the crossover.

Part d) of the figure illustrates the mutation phase. A random number is generated for each bit of each member of the population, and compared to the selected probability for mutations. Each bit that meets the criteria is inverted, as illustrated in the first column of the figure. This phase completes the process for generating the next population, which is illustrated in Part e) of Figure 2.10.
Figure 2.10: Illustration of Population Selection and Genetic Variation Process
Genetic algorithms or other types of evolutionary-based techniques have been used as an optimization tool in a number of different applications. Cordon and others illustrate a simple example for path optimization under gravitational influence [7]. The genetic algorithm tries to define the path for minimum time travel between two points. The path is discretized using a number of intermediate points, with each point represented by a six-character binary string using a simple base 2 encoding. Using an initial population of 20 individuals developed randomly, successive populations are determined using an elitist scheme and using only one-point crossovers and mutations as genetic operators. The time to travel between each point is determined and the total summed, using the inverse of this path time as the fitness function. The results were determined from a number of different initial populations, but the minimum paths found were always relatively close to each other, illustrating the robustness of the process.

Nyongesa examines optimization of fuzzy-neural systems using genetic algorithms and other evolutionary strategies [13]. The author describes the benefits of fuzzy logic controllers in areas such as nonlinear behavior and the ability to capture expert or system knowledge. By incorporating neural network capabilities, which are renowned for their knowledge acquisition and adaptive learning capabilities, the paper discusses how fuzzy systems can be further enhanced for robustness and performance in uncertain environments, but points out that optimization of such a system is still cumbersome. Citing genetic and evolutionary algorithms, the author discusses how these techniques can be incorporated to develop fuzzy-neural systems optimized for a given application.
Genetic algorithms have been used to help mitigate the effort associated with fuzzy logic system optimization, collectively known as genetic fuzzy systems. In their paper on diesel engine pressure modeling, Radziszewski and Kekez describe the use of genetic fuzzy systems to develop an accurate model of the complex pressure profiles experienced in the diesel engine cycle [16]. Fuzzy logic was fitting for the application due to its ability to represent nonlinear systems, and also for robustness properties as several different fuel sources were used in the engine modeling. Use of genetic algorithms allows for an easier method for tuning the various fuzzy system parameters (membership functions and rule base) to generate an accurate model of the pressure cycle that was robust enough across several different fuel types. As a fitness function, the authors used a measure of accuracy to experimental data, along with a factor associated with the number of rules in the resulting rule base, such that systems with fewer rules may be rewarded for simplicity. The results consisted of several different genetic fuzzy system models that met desired accuracies over specified ranges.
2.4 Stochastic Robustness Analysis

Controller robustness is a concern in any design application, but this is particularly true in stochastic systems, or systems characterized by some level of uncertainty. The designer will want to ensure the controller performs acceptably throughout the operating envelope, taking into account operating conditions and variation within the system itself. Controller systems designed for a single nominal operating point may have inadequate performance in another operating condition, or may become unstable when hardware part-to-part variation is taken into account. Analysis that considers these factors must be employed as part of the design process to develop a robust system capable of operating in a range of conditions in the face of uncertainty.

Classically, gain and phase margins have been used as a robustness measurement tool. However, in a stochastic system, margins typically deemed adequate can’t necessarily guarantee stability. Uncertainty in the plant or measurement system could shift the characteristics of the system, such that a controller that is stable on one version of the system may not be stable on another. Even if stable, key performance characteristics may be lost, leaving a system unable to meet its requirements. If information about the variation is known or can be estimated, it would be possible to develop a controller to satisfy performance requirements for the ‘worst-case’ system. However, as Stengel and others point out, this analysis can be prohibitively difficult or could lead to an over-conservative controller design [25].

The stochastic robustness analysis proposed and demonstrated by Stengel and others [17-19, 24-26, 29-30] uses Monte Carlo to simulate a number of different systems. The various sources of
uncertainty within the system are represented with distributions, typically uniform or Gaussian, but others are possible if enough information about the variation is known to use a more fitting distribution. For each Monte Carlo run, random numbers are generated using the distribution information to develop a different sample of the system. Controller performance is evaluated for each case, using a straightforward approach that can give a good indication of performance across a fleet or operating range.

To evaluate controller performance as part of the stochastic robustness analysis, classical performance measures can be used. Stability is often of greatest concern, but measures like rise time, settling time, or control effort can also be used in conjunction. As Stengel discusses, it is best to mold these performance measures into sets of pass/fail criteria, such as stable/unstable, or settling times that are acceptable/unacceptable [24]. By using this approach, the resulting performance from all of the runs could be congregated using a binomial distribution, and all the tools associated with this type of distribution can be used in the analysis.

The process for stochastic robustness analysis is illustrated in Figure 2.10. Prior to running the robustness analysis, a design for the controller needs to be determined. Oftentimes, design of the controller is an iterative process, trying to balance controller design parameters against performance objectives. Once the controller is in place, the robustness analysis can begin.

The first step in the stochastic analysis is to generate a value for each of the identified sources of uncertainty using predetermined distributions (Gaussian, uniform, etc) based on generated data or knowledge of the system. The collection of uncertain parameters in conjunction with other
system information would make up one sample of the system, such as a member of a fleet or a
certain operating condition. With the system characteristics fully defined, simulation or mission
analysis can be performed as a means of determining performance. Following simulation,
stability or other performance measures can be assessed.

These steps make up one iteration of the analysis, typically referred to as a Monte Carlo analysis.
This process is then repeated as many times as the designer would like, with the benefit of
increased confidence coming with increased simulations. Once the system and uncertainty
distributions are set up, the only cost to additional iterations is computational time, which is
becoming less of an issue with increases in processing capabilities.

Once all of the Monte Carlo runs have been completed, the performance of the controller with
respect to the various selected measures for each run can be combined to develop an overall
sense of robustness across the simulated conditions. If performance against each measure was
assessed in a binary sense (pass/fail), then the binomial distribution can be used to aggregate
controller performance across all the runs. Based on these results, the control designer can then
determine whether performance was adequate across the sampling, or if additional adjustments
are needed. This process can also highlight regions of the design space that may be more critical
in terms of meeting requirements, where additional information about the limiting conditions
may lead to a better overall design.
Figure 2.11: Illustration of Stochastic Robustness Analysis Process
Chapter 3

Hardware Model Description and Formulation

In this study, a closed loop controller will be developed for a gas turbine fuel system. Turbine engines often have multiple variable geometries, each with their own set of controlling hardware, but rarely do these systems compare to the design challenges of the fuel system. Fuel control is directly tied to gas turbine performance and operability, and is often associated with engine constraints as well.

The hardware models for the gas turbine fuel system that will be used in this study are physics-based. The models are initially developed from first principles, and later fine tuned once testing data becomes available. Details of each of the components of the closed loop system are modeled, including nonlinear characteristics. As with any manufactured system, some degree of uncertainty is expected due to part-to-part variation and operating condition variation. These uncertainties are included in the model for controller performance and robustness analysis.

The actuation system consists of an electro-hydraulic servo valve (EHSV), a single actuator, and a throttling valve (also known as a Fuel Metering Unit, FMU). The EHSV takes an electrical current signal from the controller and provides corresponding fuel flow to the actuator. The actuator slides based on the flows and adjusts the fuel port areas as a function of stroke. The throttling valve works to maintain a constant fuel pressure across the actuator fuel ports, which yields a predictable fuel flow for a given port area. This fuel flow leads to a series of manifolds
and eventually to the combustor’s fuel nozzles. The actuator position is fed back to the controller through a linear variable displacement transducer, or LVDT. The diagram below shows a simple schematic of the closed loop system.

**Figure 3.1: Diagram of Closed Loop Fuel Control System**

In the EHSV model, a null current is subtracted from the input current from the controller, and then passes through a hysteresis model to account for current direction changes. This current then passes through the first of two first-order lags to account for current rise time. The lagged current value is input into an interpolating lookup table to determine the corresponding fuel flow, and to which side of the actuator it is flowing. This lookup is linear over large regions, but does have some nonlinear effects near the zero crossing as well as the end points. The calculated flow is corrected for pressure and density conditions as compared to the rated conditions. The resulting flow passes through a second first-order lag, the output of which is the flow to the actuator.

There are a number of uncertainties associated with the EHSV model. Part-to-part variation can lead to slight differences in null current level as well as the magnitude of the hysteresis term, leading to instances of uncertainty. The time constants on the two first-order lags are also
modeled with uncertainty to account for fleet variation. A gain modifier on the flow output of the EHSV is also included, centered on unity.

The actuator model is fairly simple and straightforward; the flow provided by the EHSV is divided by the actuator bore area, and integrated to determine the actuator stroke position. The actuator stroke can be correlated to a fuel port area, and the resulting flow is related to the pressure across this area. Uncertainty in the actuator is tied to bore and cylinder area variations combined with frictional differences, but the levels of these are very small in comparison to some of the other uncertainties in the loop, so they will not be considered in this study.

The throttling valve introduces additional nonlinearities and uncertainties, but these characteristics are all downstream of the closed loop system. The actuator position is the feedback used by the control, the fuel flow output of the system is not measured. The high level closed loop control can adjust fuel flow demands by using other sensed quantities in the gas turbine, so the dynamics of the throttling valve will not be considered for this study on the inner loop controller.

The sensing system model consists of a second-order lag to represent the LVDT. Uncertainty is included in this model as well, variation taken into account in both the natural frequency and damping terms. The output of the LVDT is a pair of voltage signals corresponding to the position of the actuator. It is customary to model noise on these signals, but this factor was left out of the model for this study.
Usually in this type of system, the controller hardware itself can introduce some slight dynamics to the closed loop system. However, in many cases such as this instance, the contribution is small compared to the other components in the system. As a result, the dynamics from the control hardware will be neglected in this study.

Also in this study, normalization will be applied to the actuator position and the fuel demands and resulting flows. This gives the results a more universal appeal without getting caught up in details of the specific application.

The current controller for this fuel system is a Proportional-Integral (PI) controller. As discussed in Chapter 2.1, the proportional component is implemented to try to give fast response times to changing demands, and the integral component is in place to try to eliminate steady state error. Derivative control elements can be difficult to design for this type of system. As mentioned above, the voltage feedback signals are susceptible to noise, which present a challenge for mathematical derivatives. It is possible to filter out some of the noise, but it’s not a trivial task, and can reduce the effectiveness of the derivative component. For the current system, control can be achieved without this component.

The current controller doesn’t use fixed gains for proportional and integral components. Instead, the controller employs a technique known as gain scheduling. With this approach, the constant gains are replaced with arrays, selecting different gains based on current conditions. This essentially transforms the output space of the PI controller from a single plane to a series of planes, enabling for better optimization for the control surface. The tuning for this approach can
be a challenge, having to determine the appropriate gains for each of the different operating conditions.

The gain scheduling technique shares some similarities with fuzzy logic controllers in the fact that it changes controller outputs based on input conditions, mirroring the decision-making behavior of intelligent controls. A fuzzy logic controller is inherently nonlinear, providing the optimization space similar to the gain scheduling technique, but perhaps in a more straightforward manner. One of the goals of this study is to try to evaluate a unique tuning approach for fuzzy logic controllers to try to take some of the burden off of the designer.
Chapter 4

Genetic Fuzzy System Approach

A fuzzy logic controller will be utilized to provide closed loop control of the gas turbine fuel system in this study. As mentioned, a genetic algorithm will be used to aid in tuning the membership functions of the controller, forming a member of the generic class of systems known as genetic fuzzy systems.

The fuzzy logic controller will be based on the structure proposed by Zilouchian, et al, in their closed loop controller for a gas turbine combustor pressure simulation system [35]. The fuzzy logic controller used in that study consisted of two inputs and one output. The inputs consisted of the Error, the difference between the desired set point and the current point, and the Delta-Error, or the difference between the current error and the previous. The output of the system was the control action, tied to positions for pressurizing and depressurizing valves. Here the output will be associated with the electrical current supplied to the EHSV of the actuation system.

Each of the inputs and output will be normalized on a [-1 1] scale, with scalar multipliers up and downstream of the fuzzy controller to reach the desired range. For the inputs, the error will be normalized by the range of the actuator, and the delta-error will follow suit. On the output, the control action will be scaled by the maximum torque motor current that can be output, covering the full output range. This scaling allows for simplification within the fuzzy membership functions while still providing coverage throughout the operating envelope. Also, by using
normalized membership functions, the fuzzy controller has a more universal appeal to transfer for use in other applications.

In the study, Zilouchian, et al, used seven membership functions for both the inputs and the output [35]. This study will also use seven membership functions for each parameter, although their shapes will be set a little differently. The linguistic variables that will be used are shown below, along with Figure 4.1 showing membership functions for identically shaped isosceles triangles. This distribution will be used for comparative purposes.

NL: Negative Large
NM: Negative Medium
NS: Negative Small
ZE: Zero
PS: Positive Small
PM: Positive Medium
PL: Positive Large
The membership functions themselves will be triangles, although they will not necessarily be isosceles triangles as in the study by Zilochian, et al. The triangles will be non-isosceles, with the exception of the zero fuzzy set (ZE) as the fuzzy sets will be symmetric about the zero point of the axis. Using a similar approach as Nelson and Lakany demonstrate, with a few assumptions, the membership functions can be specified by a reduced number of parameters [12]. By assuming symmetry, and taking advantage of the normalization on the inputs and output, the entire set of seven membership functions can be fully specified with just two parameters. Figure 4.2 demonstrates how two parameters, x1 and x2, can fully specify the seven non-isosceles membership functions. In this figure, x1 had a value of 0.55 and x2 had a value of 0.82.
With this structure shown in the figure, a maximum of two fuzzy sets will be active at any time for each of the inputs, which reduces complexity of the input-to-output relationship as well as computational requirements. This method will also ensure that the sum of membership across all sets is unity, a measure of consistency.

With the membership functions in place, a rule base is needed. The set proposed by Zilochian, et al. will be used here [35]. The rule base consists of 49 IF-THEN rules, forming a complete set for the two inputs with seven fuzzy sets each. The rule base is illustrated in the table below. As
an example, the first rule would read, “IF ERROR is NL AND DELTA-ERROR is NL, THEN CURRENT is NL”.

<table>
<thead>
<tr>
<th>Error</th>
<th>Delta-Error</th>
<th>NL</th>
<th>NM</th>
<th>NS</th>
<th>ZE</th>
<th>PS</th>
<th>PM</th>
<th>PL</th>
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<td>NL</td>
<td>NL</td>
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<td>NS</td>
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<td>NL</td>
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<tr>
<td>PS</td>
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<td>PL</td>
<td>PL</td>
</tr>
</tbody>
</table>

Table 4.1: Fuzzy Logic Controller Rule Base for Seven Membership Functions

Using the seven membership functions for the fuzzy output sets, the last piece of the fuzzy controller needed is the defuzzification method. With the selected membership functions and rule base, the fuzzy sets of the output will always be adjacent. As a result, the Centroid defuzzification method can be used, as this fuzzy logic controller will not encounter some of the shortcomings that prevent this simple method from being utilized in other contexts.

With the details of the fuzzy logic controller in place, the representation of these parameters in a form the genetic algorithm can act upon is needed. As mentioned previously, both of the inputs and the output will each have two parameters defining their membership functions, leading to a total of six parameters to be used for tuning. Simple binary encoding will be used, and due to the reduced number of parameters to be represented, more bits can be used to provide more granularity for optimization. Eight bits will be used for each parameter, leaving each
chromosome to be encoded with 48 bits total. The first two sets of eight bits will be used for input one (ERROR), the second set for input two (DELTA-ERROR), and the last pair will be used for the output (CURRENT). The two parameters for each of the input/output will be determined using the relation below.

\[
y = y_{\text{min}} + \frac{y_{\text{max}} - y_{\text{min}}}{2^n - 1} \cdot \sum_{i=0}^{n-1} s_i \cdot 2^i
\]  

[4.1]

Taking advantage of the normalization of the inputs and output, the determining parameters will be bounded between [0 1], giving the minimum and maximum limits. Each set of eight bits will be passed through this formula to determine the governing parameters for the fuzzy sets. An example is illustrated below.

Sample Chromosome:

01100011 00010010 01111100 01100011 11101011 11100111

First Parameter String:

0 1 1 0 0 0 1 1
First Parameter Determination:

\[ y = \frac{1}{2^8 - 1} \cdot \sum_{i=0}^{7} s_i \cdot 2^i \]

\[ y = \frac{1}{2^8 - 1} \cdot (1 \cdot 2^0 + 1 \cdot 2^1 + 0 \cdot 2^2 + 0 \cdot 2^3 + 0 \cdot 2^4 + 1 \cdot 2^5 + 1 \cdot 2^6 + 0 \cdot 2^7) \] \[ \text{[4.2]} \]

\[ y = \frac{99}{255} = 0.3882 \]

Parameter Table:

<table>
<thead>
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<th>String</th>
<th>Value</th>
</tr>
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<tbody>
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<td>01100011</td>
<td>0.3882</td>
</tr>
<tr>
<td>2</td>
<td>00010010</td>
<td>0.0706</td>
</tr>
<tr>
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<td>0.4863</td>
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</tbody>
</table>

Table 4.2: Sample Fuzzy Membership Function Characteristic Parameters
After the fuzzy sets have been developed, the closed loop system was simulated to determine the closed loop response. This process was then repeated for each member of the population to assess performance.

In order for the genetic algorithm to optimize, it must have a mathematical measure to assess the performance of each member of the population. In control system design, performance is often measured against several different criteria, each important in some aspect. The designer tries to meet various requirements, often trading performance against one criterion for another, trying to
develop an overall balanced design. This system isn’t any different, requiring a design that tries to maximize performance against several different criteria.

In this study, the performance of the closed loop system to a step input will be assessed using an aggregate of the classical measures of rise time (Trise), settling time (Tset - 2%), percent overshoot (%OS), and percent steady state error (%SSE). It is necessary to combine these measures into a simple, comprehensive form on which the genetic algorithm can optimize. For this study they will be summed, but first they need to be non-dimensionalized in some manner and placed on the same order of magnitude. For this task, each of the classical measures will be divided by the respective performance measure of a fuzzy logic controller with identically shaped isosceles triangles, resulting in a percent comparison to this baseline configuration. The sum will then be utilized by the genetic algorithm for optimization.

It is common practice to use these parameters in the design process as measures of controller performance, but often times some measures are more important overall than others. In this study, in addition to normalization, these parameters will have weightings applied within the fitness function to try to reflect priorities in the design with respect to these parameters.

In this design, it was determined that steady state error was the most important of these measures, as accurate fuel control is vital to gas turbine performance. Without accurate fuel control, the engine may lose efficiency or violate operating constraints, negatively impacting
range or payload of aircraft, or power output by generation-based turbines. The steady state error term in the fitness function was given a weighting of 4. Settling time was also deemed more important than the remaining two factors, but not quite to the level of steady state error. When a given operating condition is demanded, the operator will want the turbine engine to attain that condition rapidly. An extreme example that illustrates this importance could be seen in a combat aircraft, relying on fast and accurate throttle control providing a combat advantage. A weighting of 2 will be applied to the normalized settling time term. Rise time is an important measure for getting the engine most of the way to the desired point, but typically to a lesser degree than accuracy and settling time. A rapid response is typically desired, but fast rise times can lead to conflict in terms of other performance characteristics, as they can lead to higher overshoots and longer settling times. By incorporating all of these measures into the fitness function for the genetic algorithm, a design that is balanced with respect to these criteria can be sought.

This priority system discussed above will be reflected within the fitness function. By defining the fitness function as the sum of weighted performance measures, the genetic algorithm can work on improving system performance in a manner consistent with priorities. Using this approach, performance improvements in one area will be held in check with performance in the other categories, making sure the system won’t completely sacrifice performance in one area for improvement in others. The weighting scheme on the normalized performance components is illustrated in the fitness function relation below. The second relation is optional, depending on how the genetic algorithm is setup (to minimize or maximize fitness function values).
The population for the genetic algorithm will consist of 10 members, each represented with a 48-bit binary string. This number was chosen to give opportunities for optimization but try to limit the computation of each iteration. The genetic algorithm will run through 200 iterations for each run. With any genetic algorithm, premature convergence has the potential to hinder some optimization if an early candidate performs extremely well compared to the other candidates. To try to alleviate this shortcoming, this study will perform multiple runs of the genetic algorithm, and the best solution overall will be chosen. The initial population for each trial will be developed at random, each bit having equal opportunity of being a 0 or 1. With each initial population being chosen randomly, multiple runs will also help explore the design space.

Following closed loop simulation of each member, the fitness of each is determined. Then, the candidates are ranked according to their fitness. Using an elitist scheme, the top 2 performers are kept for the next population. Also, the worst performing member undergoes inversion, flipping every bit character in the string, and is placed into the next population. The remaining members are determined through a selection scheme and have opportunities for genetic variation to be introduced.

\[
f = \left( 4 \times \frac{\%SSE}{\%SSE_{bl}} + 2 \times \frac{Tset}{Tset_{bl}} + \frac{Trise}{Trise_{bl}} + \frac{\%OS}{\%OS_{bl}} \right) / Wsum
\]

\[
Wsum = 4 + 2 + 1 + 1 = 8
\]

\[
F = 1 / f
\]
The remaining members of the next population are selected at random from the current population, with a probability proportional to their fitness. The fitness of each member is divided by the fitness sum across all members; yielding the percent chance that member is selected for the next population. This can be thought of as the roulette wheel approach, discussed in section 2.3.

After the members of the population have been determined, these members undergo genetic variation, crossovers and mutations. This scheme will include both one and two-point crossovers. A design parameter of the genetic algorithm is the rates for these crossovers. The overall chance of a member undergoing a crossover is set at 0.8, of which 2/3 will be one-point crossovers and two-point crossovers will be performed on the residual. Using an elitist scheme enables a high crossover rate, as the best performing candidates are already preserved. The goal of crossovers is to try to recombine pieces of existing members to form new members, which may be better suited for the controller.

Following crossovers, the population (excluding elites and inverts) is subjected to mutation. Again, the mutation rate is a design parameter for the algorithm developer, and in this case was chosen as 0.02. This means that each bit in each chromosome string has a 2% chance of being inverted. Multiple mutations can occur within the same string, or some may be left without any alteration. Mutations are a way of trying to develop new or unexplored genetic material, which may move closer to an optimized solution.
Once the successive population has been selected and has had opportunities for genetic variation to be introduced, it is ready for the next iteration of the genetic algorithm. The members of the new population go through the same process, having their fitness determined from closed loop controller performance, determining the best and worst performers, and determining the makeup of the next population. This process is repeated for each iteration, and over time, a more optimized solution than the original population should be produced.
Chapter 5

Controller Selection and Closed Loop Analysis

With the binary representation, population selection, and genetic variation rates in place, the genetic algorithm is ready to be run. For this study, the algorithm was run five different times, each with a different random initial population, and the best performing candidate and its corresponding fitness rating was saved from each run. In the end, the member of the population with the best overall fitness rating was selected for the final controller design.

A comparison of the best performer from each of the runs is shown below in Table 5.1. This table shows the chromosome string from each candidate, the results for the performance measures, and the corresponding fitness value.

<table>
<thead>
<tr>
<th>Run</th>
<th>Chromosome</th>
<th>Trise</th>
<th>Tset</th>
<th>%OS</th>
<th>%SSE</th>
<th>Fitness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100101100000001000010000111000001000111111111</td>
<td>0.250</td>
<td>0.363</td>
<td>0.0002</td>
<td>0.0003</td>
<td>13.01</td>
</tr>
<tr>
<td>2</td>
<td>0011101000010110001100000000011000101110101</td>
<td>0.263</td>
<td>0.363</td>
<td>0.0092</td>
<td>0.0092</td>
<td>12.07</td>
</tr>
<tr>
<td>3</td>
<td>001110100100100100000100000100011000111010</td>
<td>0.288</td>
<td>0.413</td>
<td>0.0045</td>
<td>0.0045</td>
<td>11.11</td>
</tr>
<tr>
<td>4</td>
<td>01101100010001000001100010110000101111010</td>
<td>0.288</td>
<td>0.413</td>
<td>0.0014</td>
<td>0.0008</td>
<td>11.35</td>
</tr>
<tr>
<td>5</td>
<td>001110100100100010000100000100011000111010</td>
<td>0.288</td>
<td>0.413</td>
<td>0.0045</td>
<td>0.0045</td>
<td>11.11</td>
</tr>
</tbody>
</table>

Table 5.1: Genetic Algorithm Performance Comparison

In general, the performance of each of the runs is pretty comparable. In fact, runs 3 and 5 produced the exact same solution for the best performing candidate. The solution found from the first run had the best overall performance, with a fitness rating of 13.01. Even though the other solutions did not quite result in the same level of performance, they all still demonstrated
significant improvement over the symmetric case, whose performance parameters are shown below. The values listed in Table 5.2 served as the baseline case for normalizing the fitness function components. By definition, the fitness for this case is unity.

<table>
<thead>
<tr>
<th>Trise</th>
<th>Tset</th>
<th>%OS</th>
<th>%SSE</th>
<th>Fitness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.21</td>
<td>1.78</td>
<td>1.193</td>
<td>1.193</td>
<td>1.00</td>
</tr>
</tbody>
</table>

*Table 5.2: Symmetric Fuzzy System Performance Chart*

The figures below show illustrate the performance of the genetic algorithm over the iterations of the first run, which resulted in the best performing controller to be used for future evaluation. In the first figure, the fitness rating of the best performer is shown over iteration count. Since an elitist scheme was used, the fitness will gradually increase from the initial population, as new members created through genetic variation perform better than their predecessors. In the second figure, Figure 5.2, the controller performance parameters are shown against the iterations of the genetic algorithm.
Figure 5.1: Fitness Function Value History of Selected Fuzzy Controller

Figure 5.2: Performance Measure History of Selected Fuzzy Controller
Using an elitist scheme ensures the best performer is kept from one iteration to the next, however since overall performance is based on a weighted combination of four individual performance parameters, it is possible to see some increases in these figures in the general downward trend.

Details of the final controller design are shown below, starting with the binary representation found through the genetic algorithm, showing a table of the binary translation, and lastly a plot of the fuzzy sets.

Final Design Chromosome:

**10010110 00000010 00010000 00011000 00100011 1111110**

Final Design Parameter Table:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>String</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10010110</td>
<td>0.5882</td>
</tr>
<tr>
<td>2</td>
<td>00000010</td>
<td>0.0078</td>
</tr>
<tr>
<td>3</td>
<td>00010000</td>
<td>0.0627</td>
</tr>
<tr>
<td>4</td>
<td>00011000</td>
<td>0.0941</td>
</tr>
<tr>
<td>5</td>
<td>00100011</td>
<td>0.1373</td>
</tr>
<tr>
<td>6</td>
<td>11111110</td>
<td>0.9961</td>
</tr>
</tbody>
</table>

Table 5.3: Selected Fuzzy Membership Function Characteristic Parameters
Final Design Fuzzy Sets:

Figure 5.3: Selected Fuzzy Membership Functions
Some conclusions can be drawn from the resulting distributions of the fuzzy sets. On the input side (Error and Delta-Error), the membership functions are aggregated near the zero point. This is likely driven by the high weighting on the steady state error term in the fitness function. By increasing the granularity in this area, the system can better respond to drive out any error resulting in the low steady state errors seen from simulating the system. This structure for the inputs also serves to reduce overshoot, as the system can tone down the output demands as the system gets close to the target value.

On the output side (TM Current), it is interesting to note how the large classes (NL, PL) have been pushed way out towards the normalized limits. Looking back at Table 4.1, these categories come in to play in the upper left and lower right corners, where the inputs tend to be in the Medium and Large categories. This will result in a large controller output, leading to a fast response of the system when in a large error or delta error region. In the closed loop system, this results in lower rise times, aggressively driving the system to smaller errors (and deltas).

Figure 5.3 and the accompanying discussion demonstrate the compartmentalization property of fuzzy control systems. Different regions in the input space will elicit different behaviors resulting from the output space, allowing specific regions to be tuned to achieve desired performance. This specialization offered by the simple nonlinear controller demonstrates a key advantage over a traditional PI controller, whose control response could be mapped to a single plane.
Another way of visualizing the resulting fuzzy membership functions is through a surface plot, shown below in Figure 5.4. This plot combines inputs, the output, and the fuzzy controller rule base into a single figure that can help illustrate key characteristics of the controller. This figure reinforces visually the behavior that large outputs will result from larger categories of the inputs when they share signs, and the finer granularity in the output as the inputs approach zero.

Figure 5.4: Surface Plot of Selected Fuzzy Controller
With the membership functions for the fuzzy sets delivered by the genetic algorithm, the closed loop system can be simulated for any mission profile or fleet sample. One of the obvious cases is a simple step response, the case for which the controller was tuned to. Figure 5.5 demonstrates the response to both a positive and negative step, to check for controller consistency as expected as a result of the symmetry in the rule base and about the zero axis in the membership functions.

![Figure 5.5: Fuzzy Logic Controller Step Responses](image)

To determine the benefits of the genetic algorithm, the tuned non-isosceles fuzzy controller can be compared to the symmetric case using the same rule base. This is illustrated in Figure 5.6, and is a visual representation of the data illustrated in Tables 5.1 and 5.2.
While a step input was used to tune the controller as well as for initial simulation results, this type of input may not be the only type seen by the controller in deployment. It would be important to check the response characteristics of the controller for any input that may be experience in operation. The controller response to a ramping input and a sinusoidal input are illustrated below in Figures 5.7 and 5.8. In these figures, fuel flow tracking is plotted along with the actuator position tracking to illustrate some of the nonlinear behavior. Note that while the
controller was not tuned for these cases, the controller still performs very well to both inputs, showing good tracking behavior to both sets of input types.

![Fuel Flow Tracking](image1)

![Actuator Position Tracking](image2)

Figure 5.7: Fuzzy Logic Controller Ramp Response
The types of inputs shown above in Figures 5.7 and 5.8 could also be used for tuning the control with a genetic algorithm, if it was more fitting to the controller application or design requirements. If ramp tracking was more critical to the design, or if certain control bandwidth requirements were mandated by the application, than the optimization process could be turned towards such. This information would be tied to the fitness determination, and the genetic algorithm could work to optimize to these conditions. The fitness function could also combine
performance against various criteria or requirements to drive towards a balanced design in a competing space.
Chapter 6

Controller Robustness Analysis

With the controller design selected in Chapter 5, the next step is to gauge the performance robustness of the closed loop system. This study will use the stochastic robustness analysis demonstrated by Stengel and others in many applications characterized by some level of uncertainty. As discussed in Chapter 2.4, this analysis involves a Monte Carlo simulation of the closed loop system, considering variation within the components or across operating conditions.

This study will consider variation within the modeled hardware components, representing part-to-part variation within a fleet or perhaps component degradation over time. Component characteristics with uncertainty considered in this study are summarized in Table 6.1 below.

<table>
<thead>
<tr>
<th>Electro-Hydraulic Servo Valve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null Current</td>
</tr>
<tr>
<td>Hysteresis</td>
</tr>
<tr>
<td>Current Time Constant</td>
</tr>
<tr>
<td>Flow Time Constant</td>
</tr>
<tr>
<td>Flow Gain</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Linear Variable Displacement Transducer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Frequency</td>
</tr>
<tr>
<td>Damping</td>
</tr>
</tbody>
</table>

Table 6.1: Summary of Stochastic Parameters in Closed Loop System
The uncertainty associated with each of the parameters listed in Table 6.1 will be represented by a Gaussian distribution, with mean values and standard deviations representative of variation within each component.

For this study, a total of 1000 closed loop simulations will be run, each using a different set or variation parameters determined randomly from the Gaussian distributions for each uncertainty term. The system will be simulated with a step input in fuel demand, as in the case used to tune the controller. Each simulation will be gauged for performance against parameters discussed above, mainly the rise and settling times, overshoot, and steady state error. The fitness function will also be generated for each case as means to compare performance to the design case and gauge performance across all the Monte Carlo runs.

In some cases, it may make sense for the robustness analysis to be performed on a collection of simulation cases, such as a complete mission analysis or cases known to be challenging in terms of performance. The selection is up to the designer, and at a minimum should be able to assess controller performance against the design requirements.

In order to assess controller robustness, criteria needs to be developed for determining if performance is adequate. These criteria will be selected to provide a binary determination, or pass/fail criteria. For the purposes of this study, performance will be deemed acceptable for rise and settling time requirements if the times are less than double those of the design case, or less than 0.50 and 0.725 seconds respectively. In terms of overshoot, a limit of 0.5% will be used; anything more and performance will be considered inadequate. For the steady state error limit, a
value of 0.1% will be used. The performance of each run will be compared to these limits, and the robustness of the controller will be determined based on the aggregate results across all simulations.

The results of the stochastic robustness analysis are illustrated below, beginning with Figure 6.1. This figure illustrates the fitness function values calculated based on the results of each of the individual simulations. The results generally hover in the 9-12 range with a few outliers. The extreme values (8.06, 13.8) are also highlighted in the figure.

![Fitness Function Value](image)

**Figure 6.1: Fitness Function Values for Each Monte Carlo Simulation**

Another means of looking at this data is using a histogram, which basically counts the number of occurrences within a set of ranges. This is illustrated in Figure 6.2 below.
Figure 6.2 helps illustrate the importance of performing a robustness analysis once an iteration of the controller design is in place. The fitness function value of the selected controller was just over 13 for the design case, but as shown above, a vast majority of the simulation cases are below this value. This demonstrates how uncertainty in the loop can lead to a decrease in expected performance, and why a stochastic robustness study is important to show how the controller will perform across a fleet or set of operating conditions.
Figure 6.3, below, compares performance of the cases with the minimum and maximum fitness function values to the base case from the original design. The figure includes both a plot of the full response and a plot zoomed in at the top of the step to capture some of the details of the compared cases. As shown, the differences are very slight, with the biggest differences seen in overshoot and steady state error. This figure demonstrates the robustness properties of the fuzzy logic controller, that even in a nonlinear environment with uncertainties, the controller can maintain a high level of performance.
Figure 6.3: Step Input Results Comparison of Minimum, Maximum, and Base Cases
The figures above took a look at the fitness ratings from each of the simulations and some comparisons between the extreme cases. What is really important in a stochastic robustness analysis is to look at the performance measures and how well they stack up to the limits. Figure 6.4 shows histogram plots of the four main performance parameters across all the runs of the Monte Carlo simulation.

![Histograms of Performance Parameter Values](image)

**Figure 6.4: Histograms of Performance Parameter Values**

As the figure above shows, each of the simulations demonstrates adequate performance against the criteria outlined previously. Gas turbine rotor dynamics are typically on the order of a second in terms of response time, so even though some of the simulations show a slight decrease
in performance with respect to the design case, the controller still should respond fast enough to maintain adequate control.

It is important to note that uncertainties in a few terms of the control loop were neglected for this study. However, these terms are known to be small in comparison to the uncertainties that were considered, and are not expected to affect the results to a large degree. In addition, as demonstrated below in Table 6.2, the performance parameters have margins ranging from 30-50% to the selected limits, leaving some flexibility for these effects.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Base Case</th>
<th>Average Case</th>
<th>Min Case</th>
<th>Max Case</th>
<th>Limit Case</th>
<th>Margin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fitness</td>
<td>13.0</td>
<td>10.1</td>
<td>8.06</td>
<td>13.8</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Rise Time (sec)</td>
<td>0.250</td>
<td>0.247</td>
<td>0.225</td>
<td>0.300</td>
<td>0.500</td>
<td>40%</td>
</tr>
<tr>
<td>Settling Time (sec)</td>
<td>0.363</td>
<td>0.363</td>
<td>0.325</td>
<td>0.438</td>
<td>0.726</td>
<td>40%</td>
</tr>
<tr>
<td>Overshoot (%)</td>
<td>0.0002</td>
<td>0.060</td>
<td>0.000</td>
<td>0.248</td>
<td>0.500</td>
<td>50%</td>
</tr>
<tr>
<td>Steady State Error (%)</td>
<td>0.0003</td>
<td>0.0397</td>
<td>0.0001</td>
<td>0.0703</td>
<td>0.1000</td>
<td>30%</td>
</tr>
</tbody>
</table>

Table 6.2: Stochastic Robustness Analysis Performance Summary

Table 6.2 underscores the importance of performing a robustness analysis on a controller design. Looking at the first row, the average fitness across the runs was just over 10, a significant drop when compared to the design case of 13. As mentioned previously, this is a strong indicator of how uncertainty can impact performance. Looking at the two rows for the timing measures, the mean case did not deviate significantly from the design case. In fact, the rise time average was actually lower than base case, albeit only by a small degree. Each of these parameters has 40% margin to the selected design limit.
Whereas the performance measures for time did not shift dramatically from the base design case, the percent overshoot and percent steady state error measures did experience a significant change. The averages for these measures was found to be 0.06% and 0.04%, respectively, which is a relatively large change considering the design case had basically eliminated these measures. This demonstrates that the overshoot and steady state error are more sensitive to the uncertainties associated with the closed loop system, and again highlights the need for robustness analysis. The steady state error reached as high as 0.07% in the Monte Carlo simulations, leaving just 30% margin to the selected design limit. As this criteria was stated to be the most important factor to the closed loop control design, it is something to keep an eye on to evaluate carefully in future studies.

If the control did not perform acceptably in all cases, the designer has a few options. Controller redesign is one option, depending on the severity of the violations found in the robustness analysis. The designer could examine the cases in which performance was deemed unacceptable, and try to capture this information into the tuning process. After redesign, the robustness analysis would be performed again, making sure no new issues were introduced in the process, as illustrated in Figure 2.11.

It is also possible to tie the stochastic robustness analysis process into the design process. Instead of having the genetic algorithm focus on a specific design case, the fitness could be based on average or minimum performance results from a robustness analysis. In this manner, the stochastic robustness analysis becomes part of the design, rather than just a check at the end to gauge performance.
Another option would be to use information from the specific performance failure cases to adjust tolerances for the system. If the stochastic analysis represented fleet variation, than an inspection process could be put in place to prevent hardware exhibiting those qualities from being deployed in the field if the component was on the fringe of the Gaussian curve. The designer could also choose to live with the performance exception, depending on the conditions and how critical to the overall control system that simulated case represents.

As demonstrated, stochastic robustness analysis is a critical tool when evaluating controller design in an uncertain environment. Considering manufacturing processes, component tolerances, and operating conditions, one would be hard pressed to find a system without some level of uncertainty. This type analysis is critical to ensuring a designer develops a robust control system, one that will be able to meet its design objectives in foreseeable environments.
Chapter 7

Conclusions and Future Research

In this study, a fuzzy logic controller was designed for a gas turbine fuel system. The design process began with the basic structure used by Zilouchian and others in their study for a fuel system simulation bench [35]. This study contributed the input/output definition, as well as the rule base. Their approach utilized identically shaped isosceles triangles for the fuzzy set definitions, which can be constraining when trying to tune the controller for improved performance. Nelson and Lakany evaluated the use of non-isosceles triangular membership functions for increased tuning flexibility, defined using an approach that reduces the number of tunable parameters to a more manageable level [12]. These two studies formed the backbone of the fuzzy logic controller utilized in this study, while a genetic algorithm was utilized in a unique tuning approach.

The genetic algorithm was used with simple binary encoding for each of the parameters governing the fuzzy sets for the inputs and output of the controller. Multiple runs of the genetic algorithm, each with a unique random initial population, were employed to try to workaround the potential for premature convergence. As Table 5.1 illustrates, each of the genetic algorithm runs converged to a similar solution, speaking to the robustness of the algorithm itself. As the simulation results indicate, each of the solutions demonstrated significant performance improvements over the symmetric isosceles fuzzy set case. This stands to show that even with the simplifications put in place to reduce the number of tuning parameters, enough flexibility
was maintained to allow for performance improvements. Looking at Table 7.1 below, the rise time and settling time for the genetic-algorithm-tuned case are roughly 20% of what they were in the baseline case. The operator will experience marked improvement in the response of the gas turbine, in both the time it takes to get close to the operating point and the time it takes to settle there. Looking at the percent overshoot and steady state error, the results show an even greater improvement. Both the overshoot and error were practically eliminated from the closed loop system. The improvements in each of these performance measures resulted in a fitness function of 13 as compared to the baseline case of 1.

<table>
<thead>
<tr>
<th></th>
<th>Trise</th>
<th>Tset</th>
<th>%OS</th>
<th>%SSE</th>
<th>Fitness</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA Tuned Non-Isosceles</td>
<td>0.250</td>
<td>0.363</td>
<td>0.0002</td>
<td>0.0003</td>
<td>13.01</td>
</tr>
<tr>
<td>Identical Isosceles (Baseline)</td>
<td>1.21</td>
<td>1.78</td>
<td>1.19</td>
<td>1.19</td>
<td>1.00</td>
</tr>
<tr>
<td>Percentage of Baseline</td>
<td>21%</td>
<td>20%</td>
<td>0.02%</td>
<td>0.03%</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.1: Fuzzy Logic Controller Comparison

It is important to note that these improvements were merely due to a change in the fuzzy membership function definitions, the inputs, outputs, and rule base remained the same between the two cases. This illustrates just how important a role the membership functions play in overall controller performance, and how powerful a design handle they can be. In this study, isosceles triangular membership functions were switched to non-isosceles functions. It may be possible to find an even more optimal solution by employing a different membership function structure, such as a Gaussian distribution or some custom function.
The performance of the fuzzy logic controller was improved in several measures based on the fitness function developed in Chapter 4. The fitness function was normalized, weighted, and inverted to yield a relation that could be maximized by the genetic algorithm, where successive populations could be selected proportional to fitness. With the flexibility of genetic algorithms, there are a number of different approaches that can be used. Instead of inverting the fitness function, the algorithm could be set up to minimize the performance function if a different population selection scheme was used. Another alternative would be to multiply the fitness function by $-1$ instead of inverting, and setup the algorithm to maximize $-f$. Either of these approaches would change the appearance of the fitness history shown in Figure 5.1, where the fitness jumps up in large segments due to incremental changes in the performance characteristics because of the inversion.

A few other design decisions were made regarding the genetic algorithm that could also influence the results. Utilizing multiple runs for this study was designed to avoid the risk of premature convergence. There are a few other techniques that could be explored to try to accomplish this as well, such as using a different representation of the design parameters. The population size for the runs was also set to 10, which is on the smaller side compared to some genetic algorithm runs. This was done to try to balance computational iterations but still give a chance to optimize. Of course, the population could be expanded to a much larger base, and perhaps multiple runs wouldn’t be needed. The algorithm could have also been set up to run for a longer number of iterations, but this study tried to strike a balance between computational time and incrementally small improvements to the closed loop system.
Inversion was a variation process introduced into the genetic algorithm, with mixed results. In several cases, the inverted chromosome ended up performing somewhere in the middle of the population in the subsequent generation, but only in one instance did the inverted chromosome end up being a top performer in the next generation. This one case was early in the iteration process, and didn’t end up being the final chromosome selected, but may have accelerated the process to converging to the solution by introducing a string of good genetic material. One trend to note was that this process was less effective in later iterations as the population collectively improved overall, and inversions generally led to poorer performance in successive iterations.

With a controller design in place, a stochastic robustness analysis was performed using several simulations of the closed loop system to determine how the controller would behave in a range of environments resulting from uncertainty in the closed loop system. This type of analysis can be more useful than classical measures such as gain and phase margins, which can be misrepresentative in stochastic environments. This analysis only used a single test case for illustrative purposes, but could be expanded for a more complete mission analysis. The results of the analysis here showed the controller performed adequately in each simulation of the Monte Carlo analysis, with little degradation when variation was considered. This illustrates the inherent robustness properties of fuzzy logic controllers, and their abilities to operate in varying environments.

The fuzzy logic controller was shown to be a viable option for the given environment, and the genetic algorithm proved to be very beneficial in the tuning process. The algorithm did take a bit of computational time considering each iteration of each run, however once a streamlined
process is setup, computational time can be much more efficient than a manual tuning process.

While the tuning case and fitness functions were pretty straightforward here, additional cases or complexities could be molded into the process using the same tool set outlined here. This study demonstrated the viability of genetic fuzzy systems for hardware control, a partnership that may be used in future control system design projects in the push for more intelligent systems.
References


