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A Novel Method for Accurate Evaluation of Size for Cylindrical Components

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A Novel Method for Accurate Evaluation of Size for Cylindrical Components

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ABSTRACT

The objective of this thesis is to develop a methodology to calculate the size of a cylindrical profile accurately per ANSI standards. The ANSI Y 14.5.1M – 1994 standard defines the size of a cylinder as the size of the largest ball rolling on a spine such that all points on the surface of the cylinder are external to it, or the size of the smallest ball rolling on a spine such that all points on the surface of the cylinder are internal to it.

Current methods of size evaluation reduce the complexity of the spine and model it as a straight line. A novel methodology to evaluate the control points of the spine modeled as a Bézier curve or an Open Uniform B-Spline curve of a pre-specified degree based on points collected on the surface of the cylinder has been developed in this thesis. This provides a quantitative measure of the size of the cylinder in accordance with ANSI standards. The formulations to evaluate the maximum inscribing spine and the minimum circumscribing spine are presented as multi-level optimization problems. The outer level optimization is used to identify the optimal set of control points for the spline representing the path of the rolling ball. The inner level optimization is used to find the nearest point on the curve corresponding to every point in the dataset.

The optimization formulation has been used to calculate the true size of cylinders for several published, simulated and real datasets. These results have been compared to traditional estimates for size of a cylinder, such as the maximum inscribed, minimum circumscribed and least squares cylinders [3]. The results indicate that the method presented in this research conforms better to the ANSI standards as compared to the traditional methods. Further analysis is presented to
observe the effect of sample size on the results of the algorithm. It is observed that with an increase in the sample size, the difference between the results of the presented algorithm and the traditional methods increases with the presented method providing more accurate estimates of the size of the cylinder.
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LIST OF ABBREVIATIONS

ANSI: American National Standards Institute

ASME: American Society of Mechanical Engineers

CMM: Coordinate Measuring Machine

GD&T: Geometric Dimensioning and Tolerancing

LS: Least Squares

MCC: Minimum Circumscribing Cylinder

MCT: Minimum Circumscribing Tube

MIC: Maximum Inscribing Cylinder

MIT: Maximum Inscribing Tube

PSO: Particle Swarm Optimization
1. Introduction

1.1 ASME/ANSI Definition of Size of a Cylinder

The ASME/ANSI mathematical definition of dimensioning and tolerancing principles [1] defines the tolerance zone for the size for a feature of size as the volume between two half-space boundaries formed by sweeping two balls, one with radius equal to the least-material condition limit and one with radius equal to the maximum-material condition limit, along a spine. The degree of the spine depends on the nature of the feature, as shown in figure 1. For a circle or sphere, the degree of the spine is zero, and is geometrically represented as a point (figure 1(a) and 1(b)). For cylindrical features, the spine has a degree equal to one, and is geometrically represented as a non-self-intersecting curve in space, as shown in figure 1(c). A more restrictive representation of a spine of degree one is a straight line, as shown in figure 1(d).

The ASME/ANSI mathematical definition [1] also provides guidelines for evaluating the actual value of the size using two values. The actual external size of an external feature (boss) is defined as the size of the smallest ball that can be rolled on a spine so that the entire surface of the feature is internal to the ball. Also, the actual internal size of an external feature (boss) is defined as the size of the largest ball that can be rolled on a spine so that the entire surface of the feature is external to the ball. Similarly, for an internal feature (hole), the actual internal size is defined as the size of the largest ball that can be rolled on a spine so that the entire surface of the feature is external to the ball. The actual external size of an internal feature (hole) is defined in a similar manner as the size of the smallest ball that can be rolled on a spine so that the entire surface of the feature is internal to the ball. Figure 2 illustrates these concepts for an internal
feature while figure 3 depicts the corresponding definitions for an external feature.

Figure 1: SPINES OF DEGREE ZERO AND ONE USED TO REPRESENT TOLERANCE ZONES FOR COMMONLY OBSERVED FEATURES OF SIZE
Figure 2. ASME / ANSI DEFINITION OF ACTUAL VALUE OF SIZE FOR AN INTERNAL CYLINDRICAL FEATURE

Figure 3. ASME / ANSI DEFINITION OF ACTUAL VALUE OF SIZE FOR AN EXTERNAL CYLINDRICAL FEATURE
1.2 Limitations of current methods for evaluation of size

Historically, the size of an object has been measured and evaluated using gages or instruments such as micrometers or calipers at multiple cross-sections [26]. However, neither of these methods conforms to the ANSI definition of a limit of size.

Gages can only provide a qualitative result of whether or not a size is within the prescribed limits. There is no quantitative evaluation of the actual size of the component. When micrometers or other measuring instruments are used, the reading is only a measure of the diameter across the two contact points and cannot be used as a representative of the entire component. ANSI requires that the size be within the prescribed limits at all cross-sections. It is not feasible to verify this requirement using conventional measuring instruments. Further, as explained by Suresh and Voelcker [26] and Voelcker [29], two-point measurements can be extremely misleading as shown in figure 4. All two-point measurements on the Reuleaux discs (figure 4(b) and 4(c)), and all two point measurements passing through the center on the Fourier disc (figure 4(d)) yield a constant value, and it can clearly be observed that neither of these sections is circular (figure 4(a)).

![Figure 4. SHAPES INDISTINGUISHABLE USING TWO-POINT MEASUREMENTS: (a) CIRCULAR DISC, (b) REULEAUX DISC WITH 3 SEGMENTS, (c) REULEAUX DISC WITH 7 SEGMENTS, (d) FOURIER DISC. (ADAPTED FROM VOELCKER [29])](image-url)
1.3 Motivation for the thesis

In today’s manufacturing setting, virtual manufacturing, inspection, assembly and testing has become the *mantra* to reduce design costs by minimizing prototype building at various stages. Considerable research has been carried out in the areas of machining and forming process simulation, which can be used to simulate the manufacturing conditions of various cylindrical components. Such virtually manufactured cylindrical components need to be virtually inspected and tested for their size and shape to evaluate their performance either independently or as a part of an assembly. In such a scenario, the method proposed in this research could provide a better estimate of the features of the manufactured part rather than assuming that the part has a perfect shape (straight line axis).

In recent years, there has been an increase in the use of flexible cylindrical components, especially micro scale components such as capillary tubing, etc. The method proposed in this research could provide a means to accurately estimate the size of such components which are defined in terms of the radius or diameter, but with no defined axial shape.

Present-day measuring systems such as CMMs and non-contact laser / light-based systems are extremely fast and can collect a very large number of points on the surface of a cylinder in a very short period of time. With the constant decrease in the cost of high-speed computing and increase in computing speeds, it is no longer difficult to use a large dataset to evaluate the size of a cylinder in a manner conforming to the ANSI standards.
In this research, methodologies are presented to evaluate the location and shape of the spine required to meet the ANSI criterion. The spine, which emulates the curved axis of the cylinder, is modeled either as a Bézier curve or as an Open Uniform B-Spline curve. The locations of the control points of this curve are adjusted so as to maximize the distance of the curve from any measured point on the surface of the cylinder while constraining the solid generated within the profile of the cylindrical surface. The spine is called the “Maximum Inscribing Spine” and the solid generated by sweeping the ball along the spine is termed the “Maximum Inscribing Tube” (MIT). Similar formulations can be used to compute a different spine and minimum radius of a ball such that the generated solid completely encloses the surface of the cylindrical profile. This tube is termed as the “Minimum Circumscribing Tube” (MCT), and the corresponding spine is called the “Minimum Circumscribing Spine”. The MIT represents the internal size of a boss and the external size of a hole. Correspondingly, the MCT represents the external size of a boss and the internal size of a hole.

The spines defining the MCT and the MIT have considerable flexibility depending on the nature of the curve used to represent them. In their most restrained form, the spines represent straight lines resulting in the MCT becoming the minimum circumscribing cylinder (MCC) and the MIT becoming the maximum inscribing cylinder (MIC).

A two-level optimization formulation is used to calculate the size of the MCT and MIT. The solution methodology uses the classical golden section search with parabolic interpolation method to solve the inner level optimization and particle swarm optimization (PSO) to solve the outer level optimization. The methodology and results presented in this thesis are based on the
author’s work published in Ramaswami et al. [21] and Ramaswami and Anand [22].

1.4 Outline of the thesis

Chapter 2 provides an overview of relevant literature in the area of size evaluation and the PSO algorithm. Details of the MIT and MCT formulations using Bézier and B-spline curves, their adaptation for PSO, initialization of particles and selection of PSO parameters are discussed in chapter 3. Chapter 4 illustrates the results obtained using the formulations developed in this research and compares the results with the traditional MIC and MCC formulations, along with the LS cylinder formulation. Chapter 4 also examines the effect of sample size on the results obtained from the methods proposed in this thesis, and compares them with the traditional methods. Chapter 5 summarizes the conclusions obtained from the results in chapter 4 and discusses potential avenues for improvement and additional research in this area.
2. Literature Review

2.1 Evaluation of actual size

Ever since the publication of the Y14.5.1M-1994 standards [1], researchers have attempted to provide a formal definition of the size limits and evaluate the actual size as prescribed in the standard. Srinivasan [25] discussed a procedure for evaluating the tolerance zone for any feature of size using sweeps along spines. By varying the dimension of the spine, this method can be used for spheres, cylinders or planes. The procedure involves sweeping a circular disc along the spine to produce the region. He did not present any results but only emphasized the capability of various geometric operations to represent the tolerance zones.

Suresh and Voelcker [26] described a method for evaluating the conformance of a cylindrical component to its size limits. They proposed a two-stage inspection procedure. They used the external and internal mating envelopes and the medial axis transform of the object to obtain a qualitative result on whether the part should be accepted or rejected. If this stage proved inconclusive, they proposed the use of the medial axis to define a spine with a radius function whose minimum value was greater than any other spine. A review of relevant literature indicates that this is the only paper that attempts to calculate the size limits by modeling the axis as a spine as defined in the ANSI standards [1]. However, no implementation details or results were provided in the paper.

Carr and Ferreira [3] described a sequential linear programming method for evaluating the size of a cylinder. They provided formulations for the MCC and MIC. However, the formulation
used by them also assumes perfect form of the substitute geometry (straight line axis for the cylinders) and does not conform to the ANSI definition. This results in a lower value for the MIC and a higher value for the MCC.

Nassef and ElMaraghy [14] presented a method to determine the best objective function for evaluating geometric deviations. They also selected a perfect form envelope to evaluate the size deviation. They defined the error objective function using the $L^p$-norm equation. Their methodology involves obtaining an optimal value of $p$ so as to simultaneously minimize the over-estimation error obtained by using the least squares (LS) method ($L_2$-norm) and the susceptibility to measurement errors when the minimum zone method ($L_{\infty}$-norm) is used.

Voelcker [29] discussed some of the issues associated with the indirect definition of size using Features of Size. He also emphasized on the need to solve the problem of conformance of size tolerances. He expressed the view that the problems presented by the current definition should be fixed or a new definition should be introduced.

Traband et al. [28] provided a statistical approach for evaluation of the form error and size of a cylinder. They assumed normality of the errors and created a nonlinear least-squares regression model for the cylinder. The model parameters were obtained by performing a Gauss-Newton non-linear least-squares regression. Statistical inferences on the parameters were then used to evaluate the size of the feature and check whether it was within the prescribed limits.
2.2 Particle Swarm Optimization

Proposed by Kennedy and Eberhart [9], PSO is a population based stochastic optimization technique. In PSO, the system is initialized using a population of random solutions. The optimum solution is found by coordinated movement of all the particles in the feature space over several iterations. PSO is similar to evolutionary computation techniques such as Genetic Algorithms (GA), which uses random initialization of populations and searches for optimal results by updating generations. However, the PSO algorithm is easier to implement since it does not use evolution operators such as mutation and crossover, and has fewer parameters to adjust. PSO has been used extensively to solve optimization problems in various areas such as computational metrology [10, 11, 21], supply chain management [7, 17], shop floor production planning [16, 27] antenna design [4, 8], and controller design [6]. Additional discussion of literature pertaining to initialization and selection of PSO parameters is provided in section 3.4.
3. Methodology

3.1 Mathematical Definition of Size of a Cylinder

As defined by ANSI / ASME Y14.5.1M [1], the actual external size of an external feature of size is the size of the smallest ball that can be rolled on a spine such that the entire surface of the cylindrical feature is internal to the ball. Assuming that the spine is a curve whose control points are represented by $CP$, the definition for the MCT can be mathematically represented as:

$$\text{Size}_{\text{MCT}}(CP, Q) = \text{Minimize}(\max(d(CP, \overline{Q}_j)))$$

(1)

where $d(CP, \overline{Q}_j)$ is the distance of the $j^{th}$ point of the dataset $Q$ from the curve defined by the set of control points $CP$.

If there are $L$ points in the dataset, the distance of each point in the dataset from the closest point on the curve, $d(CP, \overline{Q}_j)$, is evaluated as:

$$d(CP, \overline{Q}_j) = \text{Minimize} \left\{ \left( \frac{\left( P_x(t) - Q_{xj} \right)^2 + \left( P_y(t) - Q_{yj} \right)^2 + \left( P_z(t) - Q_{zj} \right)^2 \right)}{0.5} \right\}$$

(2)

where

$$\overline{Q}_j = \{Q_{xj}, Q_{yj}, Q_{zj}\}$$ are the coordinates of the $j^{th}$ point of the dataset

$$Q=\{\overline{Q}_1, \overline{Q}_2, ... \overline{Q}_L\}$$ is the dataset

$$P(t) = \{P_x(t), P_y(t), P_z(t)\}$$ are the coordinates of the point on the curve represented by the control points $CP$ with parameter $t$. 

Similarly, the actual internal size of an external feature of size, or the MIT can be mathematically represented as:

\[
\text{Size}_{MIT}(CP, Q) = \text{Maximize}(\min(d(CP, \overline{Q}_j)))
\]  \hspace{1cm} (3)

In this research, two different types of curves (Bézier and open-uniform B-spline) have been used to model the spine. The overall function that needs to be optimized, (equation (1) or (3)) is independent of the type of curve used to model the spine. However, the calculation of the point on the curve \( P(t) \) changes depending on the curve selected.

### 3.2 Modeling of spine using a Bézier Curve

A Bézier curve is defined by a single parameter, the degree of the curve ‘\( n \)’. The number of control points required to define this curve is \( n+1 \). Using this parameter and the Bernstein basis function, the coordinates of a point on the curve with parameter ‘\( t \)’ can be calculated as described by Rogers [23] as:

\[
P_x(t) = \sum_{i=0}^{n} CP_{ix} B_{i,n}(t) \\
P_y(t) = \sum_{i=0}^{n} CP_{iy} B_{i,n}(t) \hspace{1cm} 0 \leq t \leq 1 \\
P_z(t) = \sum_{i=0}^{n} CP_{iz} B_{i,n}(t) \\
B_{i,n}(t) = \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i}
\]  \hspace{1cm} (4)

where \( \overline{CP}_i = \{CP_{ix}, CP_{iy}, CP_{iz}\} \) are the coordinates of the \( i^{th} \) control point of the Bézier curve such that \( CP = \{\overline{CP}_0, \overline{CP}_1, \ldots, \overline{CP}_n\} \)

\( \overline{Q}_j = \{Q_{xj}, Q_{yj}, Q_{zj}\} \) are the coordinates of the \( j^{th} \) point of the dataset

\( Q = \{\overline{Q}_1, \overline{Q}_2, \ldots, \overline{Q}_L\} \) is the dataset
3.3 Modeling of spine using an Open Uniform B-spline Curve

An open uniform B-spline is defined by two parameters, the number of control points \((n+1)\) and the order of the curve, \(k\), such that \(n+1 \geq k\). Using these two parameters, the knot vector can be uniquely defined mathematically as described by Rogers [23] as:

\[
\begin{align*}
    x_i & = 0 \quad 1 \leq i \leq k \\
    x_i & = i - k \quad k + 1 \leq i \leq n + 1 \\
    x_i & = n - k + 2 \quad n + 2 \leq i \leq n + k + 1
\end{align*}
\]

where \(X = [x_1, x_2, \ldots, x_{n+k+1}]\) is the knot vector for the curve.

The values of the B-spline basis functions, as described by Rogers [23], are:

\[
N_{i,1}(t) = \begin{cases} 
1 & \text{if } x_i \leq t < x_{i+1} \\
0 & \text{otherwise} 
\end{cases} \quad 1 \leq i \leq n+1
\]

and

\[
N_{i,k}(t) = \frac{(t-x_i)N_{i,k-1}(t)}{x_{i+k-1}-x_i} + \frac{(x_{i+k}-t)N_{i+1,k-1}(t)}{x_{i+k}-x_{i+1}}
\]

Using the above relationships, the values of the basis functions for each segment of the curve can be calculated. Once the basis functions are calculated, the value of \(P(t)\) at any value of \(t\) (\(0 \leq t \leq n-k+2\)) can be evaluated as:

\[
\begin{align*}
    P_x(t) & = \sum_{i=0}^{n} CP_{x_i} * N_{i+1,k}(t) \\
    P_y(t) & = \sum_{i=0}^{n} CP_{y_i} * N_{i+1,k}(t) \\
    P_z(t) & = \sum_{i=0}^{n} CP_{z_i} * N_{i+1,k}(t)
\end{align*}
\]
\( \overline{CP}_i = \{CP_{x_i}, CP_{y_i}, CP_{z_i}\} \) are the coordinates of the \( i^{th} \) control point of the B-spline curve such that \( CP = [\overline{CP}_0, \overline{CP}_1, \ldots, \overline{CP}_n] \)

### 3.4 Solution Methodology

The sequence of steps for solving the two-level non-linear optimization formulation discussed above can be summarized as:

**Read the data points**

**Initialize the curve based on the location and orientation of the MCC / MIC axis**

**While convergence criteria for MCT / MIT not satisfied**

**For each point of the dataset**

- **Calculate the shortest distance of each point from the curve** (Inner level optimization) using equation (2)

  **End**

**If calculating MIT**

- **Select the minimum of those distances as the radius of the MIT** using equation (3)

**Else**

- **Select the maximum of those distances as the radius of the MCT** using equation (1)

  **End**

**Update the B-Spline curve control point locations** (Outer level optimization)

**End**

The functions in equations (1) and (3) are highly nonlinear and are optimized at two levels. The inner level optimization is the determination of the value of the parametric variable ‘\( t \)’ of the
curve that gives the minimum distance of a point from the axis using equation (2). The outer level optimization involves the determination of the locations of the control points of the Bézier or B-spline curve that minimize or maximize the size of the circumscribing or inscribing tube. Since the functions in equations (1) and (3) are highly non-linear and non-smooth, the use of traditional gradient-descent based optimization methods leads to sub-optimal solutions. Other non-gradient methods such as Simplex Search have also been found to yield sub-optimal results [21]. PSO has been proven effective in such situations. Also, because the particles are distributed over the function space, the chances of the optimization converging to a local optimum are reduced.

Each particle in the swarm represents the ‘m’ co-ordinates of the control points of the curve. Thus, each particle in the swarm is represented by a m-dimensional vector $X_i = (x_{i1}, x_{i2}, x_{i3}, \ldots x_{im})$, which is related to the number of control points of the curve, $n+1$, as:

$$m = 3 \times (n+1)$$

(8)

3.4.1 Swarm Size and Initialization:

The number of particles in the swarm ($N$) significantly influences the performance of the algorithm. If a small number of particles are used, the execution times are low. However, the ability of the particles to search the entire feature space is affected, causing the algorithm to converge to local optima. Increasing the swarm size leads to better coverage of the search space. However, this increases the execution time for each iteration and slows down the algorithm. In this research, a swarm size of 50 was found to provide good results in terms of a balance between consistent convergence and computational effort.
The initialization of the swarm also plays a significant role in early convergence of the swarm. Usually, the swarm is initialized using a uniform distribution over the search space [18, 19]. Other methods such as Sobol sequence generators and nonlinear simplex methods have also been proven to provide improved performance [18, 19].

The initial axis for evaluating the size is obtained by finding the location and orientation of the axis that provides the solution to the minimum zone form error problem. The minimum zone cylindricity formulation used here is based on the sequential linear programming method as described by Carr and Ferreira [3]. The axis so obtained is reconstructed using a B-spline curve with desired number of control points and order and the control points are determined. This curve is used as one of the particles in the swarm.

The next step is to generate additional particles (control point sets) around the minimum zone axis to initialize the rest of the swarm. First a set of \((N-1)\times m\) uniform random numbers between -0.5 and +0.5 is generated. These random numbers are then scaled by the minimum zone form error and translated to lie around the control points of the curve representing the minimum zone axis. Thus \(N\) random axes are obtained around the minimum zone axis of the cylinder.

3.4.2 Objective Function Evaluation:

For each particle, the minimum distance of each point in the dataset from the axis is computed. As described in equation (2), the aim is to find the value of curve parameter ‘\(t\)’ such that the distance of the point from the curve is minimized. This minimization problem is solved using
the golden section method with parabolic interpolation [2]. The limits of ‘t’ for the golden section method are set at 0 and 1 for the Bézier curve spine. For the B-spline curve spine, the limits of ‘t’ are set at 0 and \( n-k+2 \). This is the most computationally intensive section of the algorithm, as this optimization has to be performed for each point in the dataset and for each particle for each iteration.

Once the distance of each point from the curve has been computed, the function value for the particle is set as shown in equations 9 and 10. Equation 9 is used for the circumscribing tube size and equation 10 for the inscribing tube size.

\[
\text{FuncValue}_i = \min(d(CP_i, \overline{Q}_j)) \quad i = 1, 2...N, \quad j = 1, 2...L \tag{9}
\]

\[
\text{FuncValue}_i = \max(d(CP_i, \overline{Q}_j)) \quad i = 1, 2...N, \quad j = 1, 2...L \tag{10}
\]

where \( CP_i \) are the control points of the \( i \)th curve

\( \overline{Q}_j \) is a point in the dataset

\[ d(CP_i, \overline{Q}_j) = \text{minimum distance of } \overline{Q}_j \text{ from the curve defined by } CP_i \]

3.4.3 Swarm Parameter Selection:

The performance of the PSO method is affected significantly by the choice of the various parameters such as the constriction factor (\( \chi \)), the inertia weight (\( \omega \)), and the cognitive and social learning parameters (\( c_1 \text{ and } c_2 \)), which control the motion of the particles across iterations. The work of Parsopoulos and Vrahatis [20], Clerc and Kennedy [5] and Shi and Eberhart [24] extensively discusses the effects of the various parameters on the convergence properties of the
PSO. The values of the parameters selected for use in this research were based on extensive testing with different values of these parameters, and ranges obtained from prior literature.

The velocities of the particles are controlled by the constriction factor ($\chi$). Clerc and Kennedy [5] demonstrated the ability of properly defined constriction coefficients to prevent explosion and induce particles to converge at local minima. In this research, the constriction factor was set to $\chi=0.73$, providing for some damping of the velocities. The inertia weight, $w$, controls the influence of the prior particle velocities on the current velocity. Larger inertia weights cause the particles to travel to unexplored areas (exploring behavior). At smaller inertia weights, the particles perform a fine search in a localized area (exploiting behavior). The inertia weight ($w$) is initially set to 1.2 to allow adequate exploration of the function space. As the algorithm progresses, it is exponentially reduced to 0.3 over 150 iterations. This encourages local exploration capabilities once the particles are in the neighborhood of the global optimum. Once the inertia weight reaches a value of 0.3, it is held steady. The cognitive parameter $c_1$ and social parameter $c_2$ control the influence of the personal best value of the particle and the best particle in the swarm respectively on the velocity of the particle. The values of $c_1$ and $c_2$ are both set to 1.5, to allow for equal learning from the particle’s personal best value and the best value found by the entire swarm.

The positions where the best function value of the $i^{th}$ particle is recorded and the best function value recorded by any particle are stored as $P_i$ ($i=1, 2, \ldots, N$) and $P_g$ respectively. The best value is defined as the maximum value of $FuncValue_i$ for the inscribing tube and minimum value of $FuncValue_i$ for the circumscribing tube.
The velocity of the $i^{th}$ particle in the $t+1^{th}$ iteration, denoted by $V_{i}^{t+1}$ is evaluated as discussed by Parsopoulos and Vrahatis [18] as:

$$V_{i}^{t+1} = \chi \left( wV_{i}^{t} + c_{1}r_{i1}^{t}(P_{i} - X_{i}^{t}) + c_{2}r_{i2}^{t}(P_{g} - X_{i}^{t}) \right)$$  \hspace{1cm} (11)$$

where $i = 1, 2, \ldots, N$,

$r_{i1}$ and $r_{i2}$ are uniformly distributed random numbers in the range $[0, 1]$.

To prevent excessive oscillations of the particle due to very high velocities, a bound is placed on the upper limit of the velocities. The velocity limit for the particles was set to 25% of the minimum zone form error, based on trials with various fractions. If the absolute value of any velocity component increases beyond the bound, it is set to be equal to the bound with the same sign. This ensures that sudden changes do not force the particles out of the area in which they are located. When the velocity limits are set as higher fractions, the convergence of the algorithm was affected because the particles tended to overshoot the global optimum. Conversely, with very low velocity limits, the movement of the particles towards the global optimum was observed to be very slow.

Equation 12, as presented by Parsopoulos and Vrahatis [18], is used to update the position of the $i^{th}$ particle, by adding the velocity calculated in equation (11) to its current position.

$$X_{i}^{t+1} = X_{i}^{t} + V_{i}^{t+1}$$  \hspace{1cm} (12)$$

The termination criteria for the algorithm were set to 500 iterations and all the particles being within $1 \times 10^{-12}$ of the best solution.
3.5 Existing methods for Size Evaluation

Traditional measurements of size of a cylinder have been based on the diameters of the MCC and the MIC. The formulation for evaluating the MCC, as shown in figure 5 and equation 13, involves the determination of location and orientation of an axis such that the maximum distance of any point in the dataset from the axis is minimized.

![Figure 5. MINIMUM CIRCUMSCRIBING CYLINDER FORMULATION](image)

Figure 5. MINIMUM CIRCUMSCRIBING CYLINDER FORMULATION

\[
\text{Minimize}(r_{MCC}) \\
\text{s.t.} \\
\left\| \begin{array}{c} x_j - x_0 \\ y_j - y_0 \\ z_j \\ \end{array} \right\| \begin{bmatrix} \cos(\alpha) \\ \cos(\beta) \\ \cos(\gamma) \end{bmatrix} \leq r_{MCC} \\
\quad j = 1, 2, ..., L
\]

where \((x_j, y_j, z_j)\) are the coordinates of the jth point in the dataset

\((x_0, y_0, 0)\) are the coordinates of one point on the axis
\((\cos(\alpha), \cos(\beta), \cos(\gamma))\) are the direction cosines of the axis.

\(r_{\text{MCC}}\) is the radius of the MCC

Similarly, the MIC formulation shown in figure 6 and equation 14 evaluates the location and orientation of an axis such that the minimum distance of any point in the dataset from the axis is maximized.

\[
\begin{align*}
\text{Maximize}(r_{\text{MIC}}) \\
\text{s.t.} \\
\begin{bmatrix}
  x_j - x_0 \\
  y_j - y_0 \\
  z_j - z_0
\end{bmatrix} \times \begin{bmatrix}
  \cos(\alpha) \\
  \cos(\beta) \\
  \cos(\gamma)
\end{bmatrix} \geq r_{\text{MIC}} & \\
& j = 1, 2, \ldots, L
\end{align*}
\]

where \(r_{\text{MIC}}\) is the radius of the MIC
In addition to the MCC and MIC, the LS cylinder is also used as a measure of the size. To determine the LS cylinder size, the location and orientation of the LS axis is evaluated using the Least Squares Geometric Elements Matlab Toolbox [15] developed by the National Physical Laboratory. In prior research [3], the radius of the LS cylinder was used as an estimate. However, since the LS cylinder, by definition, lies in the midst of the data points, using this value directly does not appropriately quantify the size of the cylinder. To overcome this limitation, two size values derived from the LS axis are used in this research, as illustrated in figure 7. The inner size of the cylinder is calculated as the distance of the point closest to the LS axis. Similarly, the outer size of the cylinder is calculated as the distance of the point farthest from the LS axis.

![Figure 7. EVALUATION OF INNER AND OUTER SIZES BASED ON LEAST SQUARES ALGORITHM](image)

Thus, the inner and outer LS sizes can be mathematically represented as:
\[ R_{LS}^{outer} = R_{LS} + \max(d) \]
\[ R_{LS}^{inner} = R_{LS} + \min(d) \]

where \( R_{LS}^{outer} \) is the size of the outer cylinder based on the LS method

\( R_{LS}^{inner} \) is the size of the inner cylinder based on the LS method

\( R_{LS} \) is the radius of the LS cylinder

\( d \) is a vector of the radial deviations of all points in the dataset from the LS cylinder
4. Implementation and Results

4.1 Comparison of results from simulated and published datasets

The algorithms were implemented using Matlab. Both the MIT and MCT algorithms were executed for Bézier curves of degree 3 (4 control points) and 4 (5 control points); and B-spline curves of order k=4 with 6 and 8 control points. The algorithms were tested with datasets from prior literature, actual data collected using contact and non-contact inspection methods and simulated data. Cylinder datasets 2 and 3 from the paper by Carr and Ferreira [3] designated as ‘Carr 2’ and ‘Carr 3’ in tables 1 and 3 were used to compare the results. Discrete point data was also collected from four manufactured surfaces using a Brown and Sharpe Reflex 343 CMM and a Breuckmann stereoSCAN3D optical scanning system. These surfaces were produced using different manufacturing processes ranging from machining to extrusion to blow molding. The number of points sampled from these datasets varied from 122 to 25,000. These datasets are designated as ‘M1’, ‘M2’, ‘M3’, and ‘M4’ in tables 1 and 3. Four simulated datasets were also generated with 100 points each using software developed by Wang [30], where the effect of various shapes, lobing and other errors were simulated using a Chebyshev polynomial sequence. Each dataset had a different shape, amplitude of axial deviation, number of lobes and lobe amplitude. 100 points were selected randomly from the 10,000 points generated on the surface of each simulated cylinder. These datasets are designated as ‘S1’, ‘S2’, ‘S3’, and ‘S4’ in tables 2 and 4. The results obtained using the MIT and MCT formulations developed were compared with the MIC or MCC, and the LS solutions.

In this research, the locations of the particles in the PSO algorithm are initialized randomly.
Hence, each replication of the algorithm requires different number of iterations before the convergence criteria are met. To evaluate the average behavior of the algorithm, the average number of iterations required to achieve convergence was used as a measure. The average number of iterations of the PSO formulation was calculated based on 100 replications for each dataset.

Figure 8(a) illustrates the comparison between the MCT and MCC for a banana shaped cylinder. Figure 8(b) shows the results for MIT and MIC for the same dataset. It can be clearly seen from the figures that the MIT has a larger diameter than the MIC. Similarly, the MCT has a smaller diameter than the MCC. This is because the MIT and MCT conform to the overall contour of the profile by using a mathematical curve (B-Spline) to represent the axis. This provides increased flexibility, and better conformance with ANSI guidelines, as opposed to the MIC and MCC, which use a straight line axis. Since the MIT and MCT conform to the shape of the profile, as specified by ANSI, they yield more accurate estimates of the size of the profiles as compared to the MIC and MCC. Therefore, this method can more accurately predict the size of the cylinder when there is a marked deviation of the profile from the ideal cylinder.

Table 1 illustrates the results for the MIT formulation for the datasets from prior literature and the profiles obtained from the manufactured parts using a CMM. It can be observed that the MIT formulations presented in this research (either with Bézier or B-spline curves) yield larger or equal values of size as compared to the MIC formulation or the LS cylinder formulation. This is because the MIC and LS cylinder formulations assume perfect form of the profile, which is not a constraint for the MIT formulation.
Comparing the results obtained from the B-spline curves with six and eight control points, it can be observed that the results obtained using eight control points are as good as or slightly better than using six control points. A similar observation can be made while comparing the results of the 3rd and 4th degree Bézier curves. Thus, it cannot be stated comprehensively that increasing the number of control points improves the quality of the result.

A comparison of the average number of iterations indicates that the B-spline curves with eight
control points converge in higher number of iterations than the curves with six control points. However, it should be noted that the computational requirements for a curve with eight control points are not significantly higher than a curve with six control points. This increased number of iterations can be attributed to the increased dimensionality and non-linearity of the problem. However, the computational requirements for a 4th degree Bézier curve are more than a 3rd degree Bézier curve, because of the higher degree of the polynomial equation.

It should also be noted that even though the algorithm requires over 60 iterations to converge, the region of the optimal solution is reached in a few iterations. The remaining iterations are necessary for all the particles to converge to the global solution. The progress of the PSO algorithm is shown in figure 9.

The axis in iteration 0 is the B-spline representation of the MCC axis. As soon as the particles are initialized, in iteration 1, one of the other particles becomes the global best axis. As the iterations progress, the shape of the axis curve changes continuously until all the particles converge. It can be observed in figure 9 that the change in the nature of the axis after iteration 25 is minimal.

Table 2 compares the results for the MIT formulation with the MIC and LS cylinder formulations for the simulated datasets. Similar to the results in Table 1, it can be observed that the MIT yields larger (more accurate) values of the size of the profile as compared to the MIC and LS cylinder formulations.
Tables 3 and 4 illustrate the results of the MCT formulation along with the results from the MCC and LS cylinder formulations. It can be observed that the formulation presented in this research yields smaller or equal values of the radius of the profile as compared to the MCC and LS cylinder formulations.

Figure 9. PROGRESS OF THE PARTICLE SWARM OPTIMIZATION ALGORITHM
<table>
<thead>
<tr>
<th>Dataset #</th>
<th>LS Inner Size / MIC Size</th>
<th>Published Size</th>
<th>n=5, k=4 B-spline curve</th>
<th>n=7, k=4 B-spline curve</th>
<th>3rd degree Bézier curve</th>
<th>4th degree Bézier curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carr 2 *</td>
<td>59.884586</td>
<td>59.936215</td>
<td>59.938163</td>
<td>91.65</td>
<td>59.941999</td>
<td>73.81</td>
</tr>
<tr>
<td>Carr 3 *</td>
<td>49.994319</td>
<td>49.995664</td>
<td>49.995665</td>
<td>54.11</td>
<td>49.995665</td>
<td>59.33</td>
</tr>
<tr>
<td>M1</td>
<td>1.406002</td>
<td>1.406106</td>
<td>1.406166</td>
<td>47.65</td>
<td>1.406170</td>
<td>41.11</td>
</tr>
<tr>
<td>M2</td>
<td>2.407688</td>
<td>2.418480</td>
<td>2.418480</td>
<td>63.32</td>
<td>2.418480</td>
<td>60.84</td>
</tr>
<tr>
<td>M3</td>
<td>1.039679</td>
<td>1.039831</td>
<td>1.039841</td>
<td>42.41</td>
<td>1.039851</td>
<td>58.25</td>
</tr>
<tr>
<td>M4</td>
<td>0.656291</td>
<td>0.657252</td>
<td>0.657324</td>
<td>53.78</td>
<td>0.657376</td>
<td>63.16</td>
</tr>
</tbody>
</table>

* Obtained from Carr and Ferreira [3]
<table>
<thead>
<tr>
<th>Dataset #</th>
<th>LS Inner Size</th>
<th>MIC Size</th>
<th>( n=5, k=4 ) B-spline curve</th>
<th>( n=7, k=4 ) B-spline curve</th>
<th>3(^{rd}) degree Bézier curve</th>
<th>4(^{th}) degree Bézier curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>S2</td>
<td>19.650831</td>
<td>19.723271</td>
<td>19.799337</td>
<td>104.73</td>
<td>19.833579</td>
<td>98.23</td>
</tr>
<tr>
<td>S3</td>
<td>49.394313</td>
<td>49.473361</td>
<td>49.573515</td>
<td>78.73</td>
<td>49.576394</td>
<td>68.25</td>
</tr>
<tr>
<td>S4</td>
<td>98.724377</td>
<td>99.013312</td>
<td>99.154060</td>
<td>82.15</td>
<td>99.158359</td>
<td>96.10</td>
</tr>
</tbody>
</table>
TABLE 3. ACTUAL SIZE VALUES OF MINIMUM CIRCUMSCRIBING TUBE FOR DIFFERENT DEGREES OF CURVES AND EVALUATION METHODS FOR PUBLISHED DATASETS AND MEASURED CYLINDERS

<table>
<thead>
<tr>
<th>Dataset #</th>
<th>LS Outer Size</th>
<th>Published Size / MCC Size</th>
<th>n=5, k=4 B-spline curve</th>
<th>n=7, k=4 B-spline curve</th>
<th>3rd degree Bézier curve</th>
<th>4th degree Bézier curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carr 2*</td>
<td>60.09857</td>
<td>60.0578814</td>
<td>60.034085</td>
<td>92.53</td>
<td>60.033978</td>
<td>85.21</td>
</tr>
<tr>
<td>Carr 3*</td>
<td>50.004691</td>
<td>50.0041462</td>
<td>50.004146</td>
<td>42.33</td>
<td>50.004146</td>
<td>65.14</td>
</tr>
<tr>
<td>M1</td>
<td>1.407433</td>
<td>1.407206</td>
<td>1.407046</td>
<td>51.54</td>
<td>1.407043</td>
<td>49.59</td>
</tr>
<tr>
<td>M2</td>
<td>2.576050</td>
<td>2.560506</td>
<td>2.543417</td>
<td>68.23</td>
<td>2.537711</td>
<td>71.74</td>
</tr>
<tr>
<td>M3</td>
<td>1.041573</td>
<td>1.0413983</td>
<td>1.041398</td>
<td>53.72</td>
<td>1.041398</td>
<td>52.20</td>
</tr>
<tr>
<td>M4</td>
<td>0.659096</td>
<td>0.658970</td>
<td>0.658963</td>
<td>49.90</td>
<td>0.658963</td>
<td>46.45</td>
</tr>
</tbody>
</table>

* Obtained from Carr and Ferreira [3]
<table>
<thead>
<tr>
<th>Dataset #</th>
<th>LS Outer Size</th>
<th>MCC Size</th>
<th>n=5, k=4 B-spline curve</th>
<th>n=7, k=4 B-spline curve</th>
<th>3rd degree Bézier curve</th>
<th>4th degree Bézier curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>10.226134</td>
<td>10.162784</td>
<td>10.148784</td>
<td>74.58</td>
<td>10.148359</td>
<td>95.69</td>
</tr>
<tr>
<td>S2</td>
<td>20.391353</td>
<td>20.313794</td>
<td>20.306436</td>
<td>72.99</td>
<td>20.296103</td>
<td>87.92</td>
</tr>
<tr>
<td>S3</td>
<td>50.902286</td>
<td>50.622343</td>
<td>50.518268</td>
<td>86.48</td>
<td>50.481858</td>
<td>89.21</td>
</tr>
<tr>
<td>S4</td>
<td>100.842571</td>
<td>100.755936</td>
<td>100.666392</td>
<td>95.12</td>
<td>100.659334</td>
<td>78.02</td>
</tr>
</tbody>
</table>
4.2 Effect of variation of sample size

Discrete sampling of the surface has a significant effect on the result obtained from the evaluation of any GD&T parameter. In order to assess the performance of the presented method as compared to other methods with variation in sample sizes, a Monte-Carlo simulation study was carried out. A cylinder was simulated with 10,000 points on the surface. In order to know the true size of the cylinder based on ANSI definition prior to the analysis, the data points were generated by sweeping a ball of radius equal to two units along an arc spanning 40° of a circle of radius 400 units, as shown in figure 10.

Figure 10. PROFILE OF SIMULATED CYLINDER
In order to investigate the effect of sample size on the evaluation of size, random samples of sizes 16 to 4,096 in powers of 2 were drawn from the dataset. Each dataset was evaluated using the three algorithms (LS outer size, MCC, and MCT). This was repeated 100 times at each sample size. Figure 11 illustrates the distributions produced by the replications at sample sizes of 32, 256, and 2048 along with the true value and the mean of each distribution.

Figure 11. VARIATION OF SIZE RESULTS USING DIFFERENT METHODS AT VARIOUS SAMPLE SIZES

It can be observed in figure 11 that the LS method results in mean values that are farthest away from the true value at all sample sizes. The MCT method results in values which are closest to the true value of the size of the cylinder. The MCC method yields a value between the MCT and LS methods. At lower sample sizes, the variance of the MCT method is lower than the LS and
MCC methods. However, at higher sample sizes, the variances produced by all the methods converge.

Figure 12 shows the variation of the mean value of the radius for all the sample sizes. It can be observed that with an increase in the sample size, the value of size reported by all the methods move away from the true value and towards their respective asymptotic values. The deviation from the asymptotic value at any sample size is lowest for the MCT method, illustrating the consistently higher accuracy provided by the method as compared to the MCC and LS methods. The LS size provides the most conservative estimate with the MCC method providing estimates between the LS and MCT methods.

Figure 12. VARIATION IN ACCURACY WITH SAMPLE SIZE
The change in the value of the calculated size with sample size is minimum for the MCT method, indicating its accuracy at all sample sizes. The MCC and LS methods are unable to estimate the true size of the cylinder because of the definition of the axis as a straight line. The MCT method is able to provide a better estimate due to the flexibility provided by modeling the axis as a B-spline curve. However, even the MCT method is unable to provide the exact value because the B-spline curve can only approximate the true axis curve. The error introduced due to the approximation results in the deviation between the true value and the size reported by the MCT method.

Figure 13 illustrates the variation in the precision observed at different sample sizes for the different methods. The precision of any method is measured by the standard deviation of the results obtained during the repetitions at each sample size.
It can be observed that as the sample size increases, the precision levels increase, as indicated by the decreasing values of standard deviation. It can also be observed that the LS method provides the highest standard deviation at all sample sizes, indicating a lower level of precision as compared to the MCC and MCT methods. At lower sample sizes, the precision level of the MCT method is the highest, as indicated by the lowest values of standard deviation. As the sample size increases, the precision levels of all the methods converge. Similar results were observed in a comparison of the MIT method with the LS inner size and MIC methods.

From the results of the accuracy and precision in figures 12 and 13, it can be concluded that the MCT and MIT methods presented here are capable of providing a more accurate estimate of the size of a cylindrical profile as compared to the LS and MCC or MIC methods at all sample sizes. Also, since the improvement in the accuracy achieved using the MCT or MIT methods with an increase in sample size is not significant, the method can be used with lower sample sizes for obtaining more accurate and precise estimates of the size of a cylindrical profile when compared to the LS and MCC or MIC methods.
5. Conclusions

In this research, a novel method for evaluating the size of a cylinder is presented. The method can use any mathematical curve to represent the axis of the cylinders instead of straight lines, which permits the axis to conform better to the contour of the cylinder. In this research, Bézier and open uniform B-spline curves have been used to model the axis. Formulations have been developed for evaluating the maximum inscribing and minimum circumscribing tubes. The presented formulation adheres more closely to the ANSI definition of size of a cylinder and yields better results than the traditional measures such as the least squares, maximum inscribed and minimum circumscribed cylinders.

The presented method has been extensively tested with published and simulated datasets. The results of the tests clearly indicate the superiority of the presented method over other traditional methods such as MCC, MIC or LS cylinders in terms of accuracy and precision.

In this work, the axis of the tube is represented using Bézier and B-spline curves. The B-spline curve permits better local control on the nature of the curve than the Bézier curve. However, if the number of control points is increased, the dimensionality of the optimization problem and its complexity increase tremendously. An efficient formulation for evaluating the minimum distance from any datapoint to the axis is essential for the success of this research. Another direction for further research in the area could focus on adaptively changing the number of control points and order of the B-spline curve to identify the additional gains in result based on increasing the complexity of the curve.
REFERENCES


