I, SREENIDHI KRISHNAMOORTHY, hereby submit this work as part of the requirements for the degree of:

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Experimental Testing and Mathematical Modeling of a Thermoelectric Based Hydronic Cooling and Heating Device with Transient Charging of Sensible Thermal Energy Storage Water Tank

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Experimental Testing and Mathematical Modeling of a Thermoelectric Based Hydronic Cooling and Heating Device with Transient Charging of Sensible Thermal Energy Storage Water Tank

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Abstract

Sensible charging of cold and hot water thermal energy storage tanks has been studied experimentally and theoretically using a heat exchanger equipped with multiple thermoelectric (TE) modules. The primary objective was to design a simple, but effective, modular Peltier heat pump system component to provide chilled or hot water for domestic use at the appliance level; and when arranged in multiple unit combinations, a system that can potentially satisfy small home cooling and heating requirements. Moreover, when the TEs are directly energized using solar PV panels, the system provides a renewable, pollution free and off-the-grid solution to supplement home energy needs.

The present work focuses on the (1) design, (2) testing and (3) theoretical modeling of a thermoelectric heat exchanger component that consists of two water channels machined from two aluminum plates with an array of three or five thermoelectric modules placed in between to transiently cool and/or heat the water in the thermal energy storage tank. The water passing over either the cold or hot side of the TE modules is recirculated to charge the cold or hot thermal storage tank, respectively. The temperatures in the prototype Peltier heat exchanger test component and thermal energy water storage tank were measured during both cold tank charging and hot tank charging operation. In addition, a mathematical model was developed and numerically solved to predict the charging of cold and hot water tanks using thermoelectric modules heat exchanger device. Equations are developed for the heat pumped by the TE module as a function of the temperature difference across it for the appropriate values of the heat sink temperatures. These equations, along with those for the three lumps, are then finite differenced with a stable time step, so that a smooth variation of temperatures could be obtained. The temperature history of the tank water, thus obtained as a function of time using a three-lumped parameter model is compared to the experimental data. The thermal efficiencies of TE heat pump cooling/heating system are reported. The effects of TE power input, number of TE units and rate of fluid flow are studied.
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Nomenclature

\( A \) = area
\( b \) = y-intercept of the performance curve
\( c_p \) = specific heat
\( \text{COP} \) = Coefficient of performance
\( E \) = rate of energy transfer
\( \bar{h}A \) = average convective heat transfer loss coefficient
\( I \) = current drawn by the TE module
\( k \) = thermal conductivity
\( L \) = length
\( m \) = slope of the performance curve
\( m \) = mass flow rate
\( n \) = number of modules used
\( P \) = total power input
\( \rho \) = density (of water)
\( q \) = heat pumped
\( t \) = time
\( T \) = temperature
\( \Delta T \) = temperature difference
\( \Delta t \) = time step
\( T_c \) = cold plate temperature
\( T_h \) = hot plate temperature
\( v \) = volume
\( V \) = voltage applied to the TE module
\( \nu \) = volumetric flow rate

Subscripts:

\( \text{air} \) = ambient air
\( \text{airgap} \) = stagnant between successive TE modules
\( \text{alum} \) = aluminum plate
\( \text{b} \) = bulk
\( \text{bath} \) = constant temperature bath
\( \text{bot} \) = bottom
\( \text{c} \) = cold
\( \text{cc} \) = cold channel
\( \text{h} \) = hot
\( \text{hc} \) = hot channel
\( \text{h-c} \) = difference between hot and cold plates
\( \text{i} \) = in to the system
\( \text{ins} \) = insulation material
\( \text{lm}_{\text{cw}} \) = log mean temperature difference between the cold water and plate
\( \text{lm}_{\text{hw}} \) = log mean temperature difference between the hot water and plate
max = maximum
o = out of the system
p = plate
s = surface
screws = screws
top = top
T = tank
w = water
Chapter 1

Introduction

Achieving thermal comfort in different climatic conditions throughout the year is a basic need of human body. The fundamental principles for achieving this have remained essentially the same since the 1950’s in the form of mechanical vapor compression cooling and fossil fuel heating. Typical air conditioning systems suffer from the basic drawback of the use of environmentally harmful refrigerants such as R-11, R-22, R-134 ammonia and many more. The exposure of these refrigerants to the atmosphere contributes to the ozone depletion and global warming that is being addressed prominently these days. Moreover, conventional refrigeration systems use heavy hermetic compressors which fail after so many years of service due to its numerous moving parts and the technology involved. With regards to winter heating season, natural gas, propane and fuel oil have been the primary resources used to accomplish the task. These fuels, and indirectly coal, used in electrical power generating stations (which power resistance heating furnaces) are also polluting releasing greenhouse gases, and moreover, they offer limited supply as non-renewable resources and tend to be an increasingly expensive fuel for the homeowner.

Thus, the need is to develop an environmental friendly system which is different from the above conventional systems and which can mitigate the inherent problems described above. In this context, the design of a hydronic cooling/heating system with solar-powered thermoelectric modules has been proposed, which would help to control the residential environment year round. The use of thermoelectric units is a promising technology which, due to recent increase in energy fuel costs and the advancement in available materials is gaining momentum as a potential candidate in supplementing and in replacing conventional cooling and heating systems. This technology is already being used to a large extent in various electronic industries to help cool the integrated circuits of computers and electronic equipment in small enclosures. But thermoelectrics, as yet, have not been used on the larger scale in commercial equipment in home HVAC systems to take care of the cooling and heating demands for achieving thermal comfort.
1.1 Problem Definition

In the context of the above needs, this thesis specifically addresses the use of thermoelectrics in a hydronic system for residential year-round climate control. The integration of small thermoelectrics in the building application area to take care of the demand of cooling and heating needs, without switching to different fuels and/or system hardware at different point of time throughout the year is the main motto of the present work. Initially, the basic system component which could provide chilled or hot water for domestic use at the appliance level is studied in the laboratory. Then the system is run on the non-conventional but renewable energy source, specifically solar energy via PV panels to power the thermoelectrics. Ultimately the system then would be enlarged in multiple unit combinations to handle larger loads of the size needed for residential year-round climate control. Before all this, however, first the performance and reliability of the basic system component was to be determined by experimentation and theoretical analysis.

1.2 Solution Methodology

The approach to the above stated problem consisted of two components. First, the basic heat exchanger component consisting of TE modules for providing chilled or hot water for domestic use at the appliance level has been designed and tested in the laboratory (1). Tests were conducted in both steady state and transient modes, and several different combinations of these tests conducted were selected from: 1) whether the aim was to cool or heat the water in the storage tank; 2) vary the number of TE modules used, - either 3 or 5; 3) vary the flow rate through the test section; and 4) vary the total power supplied to the TEs, i.e. most tests were performed nominally at 300W for the majority of the runs except for two charging experiments set at 180W (low power cases). Complementing the data collection in those tests, a mathematical model has been developed that would theoretically predict the tank temperature rise and fall with time. The model has been developed and presented in this work. In addition, the procedure also involved not only determining and plotting the temperature history of the aluminum plate lumps.
Chapter 2

Literature review

2.1 Introduction

The review of prior work is divided into two groups: thermoelectric devices and sensible tank charging. The use of thermoelectric modules has been studied in depth from the point of view of its principle of operation, applications in building heating and cooling, power generation, electronic cooling and for miscellaneous applications. Previously published articles in International conferences are also briefly discussed and reviewed. As regards sensible tank charging, approaches towards both sensible heating and cooling in different forms have been studied and finally, studies concerning the economic analysis of such systems has been presented.

2.2 Thermoelectric Devices

A thermoelectric module is a small solid state device that can operate as a heat pump. The TE device works on the Peltier Effect, discovered by Jean Peltier of France in 1834 [2]. A TE module consists of several semiconductors doped to form P and N junctions that are connected electrically in series and thermally in parallel. When current passes through the junctions of such different conductors, it results in a temperature change. The heat transferred is a linear function of the current passed through the junction causing cooling on the side where energy is removed and heating on the opposite side.

For practical applications we need the semiconductors to be a good conductor of electricity but a poor conductor of heat. Such a material is Bismuth Telluride. The semiconductors, sandwiched between two ceramic plates, are powered by a DC source.

To date, due to their low heat pumping capacity, TEs so far have mainly been used for small thermal load applications such as cooling electronic components [3-5] and portable electric food coolers/warmers (such as deluxe car cup holders and chilled cream dispenser to name but two) [6,7]. Fortunately, commercially available thermoelectric
modules are designed so that multiple TE modules can be used side-by-side or they can be stacked up one on top of the other to increase the heat pumped and the temperature difference, respectively. To produce temperature difference of less than 80˚C, it is sufficient to use TE modules mounted side by side. Ninety per cent of all practical applications for thermoelectric technology fall within a very narrow range of temperatures, 20˚C to 50˚C.

With regards to proposed applications of TEs incorporated into building HVAC powered with PVs, recently Xu et al. [8] analyzed TEs in an Active Building Envelope (ABE) component. Their design proposes to use solar energy with the help of PV panels to power the TE modules to augment heating and cooling of the building enclosure, specifically focusing on an ABE window-system unit with the TE modules mounted within the window frame structure. An external heat sink attached to the TE dissipated (or absorbed) heat by natural convection to (from) the surrounding air and passed it to an aluminum water filled tube (thermal mass) that then slowly absorbed or dissipated heat toward the building’s interior. Two types of modules were analyzed with electrical connections in single stand alone position or multiple series and parallel combination. The COP of the TE modules was experimentally determined based on the data collected in the prototype setup by running them under different voltage steps, powered by a bench top DC source on different time basis, to simulate the practical condition of changing PV output power.

Another study based on the utilization of solar energy by using PV panels was done by Melero et al. [9] which utilize the thermoelectric cooling principle for air-conditioning of a room. They used an iterative approach to optimize the required design of the PV panels and the number of TE modules needed for the given operating conditions and the system was compared with the vapor compression economy.

A hybrid domestic hot water system was developed [10] using the waste heat rejected from an array of 30 TEs mounted in a duct used for air-conditioning in conjunction with solar energy from a solar thermal collector/storage tank. The system produced 120 liters of 50˚C hot water within 2 hours and the cold side of the TE heat exchanger provided
space air cooling of the room. Test were conducted under two different water flow and air flow rates. The highest effective COP of the combined hybrid system was 3.12.

Some unique and interesting applications of solar powered TE units on the individual level worth mentioning include the development of a headgear [11], i.e. baseball cap, to provide personal cooling while in sunny conditions that utilize a small flexible PV strip fixed on the cap to power a TE element set at the front of the headgear to cool the forehead; refrigerator needed for food and medical drugs [12] powered directly from the PV panel while storing the excessive power in the batteries to provide backup for no sun condition; refrigeration in outdoor conditions [13] based on the solar irradiance available at different time of the day with arrangements for day and night time power supply mode to minimize weight and maintain low cost operation.

Lastly, for completeness, it should be mentioned that TEs are often used in reverse, i.e. for electric power generation, given two fixed thermal reservoirs at different temperatures, but are not discussed here any further, but however, TE systems of this types, based on a variety of different waste heat sources can be found described in references [14-21] for the interested reader.

2.3 Sensible Tank Charging

Sensible heat storage of thermal energy is perhaps, conceptually, the simplest form of storing thermal energy. This kind of thermal storage is possible when the medium of energy transfer remains in the same state/phase throughout the process and is associated with an increase in temperature. The energy source for sensible TES is obtainable from several paths, however the most common renewable source is the energy obtained from the sun. The energy thus collected from the sun, can be stored and saved for future use. Different approaches to sensible heat storage are possible and can be classified with respect to the (1) medium use to energy transfer and (2) the medium of the storage itself. Rock bed storage, tank storage, thermocline energy storage and aquifers are the common classifications of sensible TES based on the storage medium. The fluids used are primarily water and air, although pressurized fluids like steam etc. may also be used. Heat storage in all these cases occur by increasing the temperature of the heat transfer
medium and the unit storage capacity is defined by the product of the heat capacitance and the temperature change. A broad classification of sensible TES systems has been presented in Table 1 of Dincer et al. [22] and also by Yunku et al. [23] in his book “Solar Energy Utilization”.

A prior study by Kazmierczak et al. [1] described a novel technology that could utilize the energy collected from solar PV panels but converted to sensible chilled/heated water tanks using TE modules. More precisely, it explored an air-conditioning technique that could use this solar electric energy to power an array of thermoelectric modules and store the energy in the form of sensible cold/hot water.

Such a water tank thermal storage system is very common and applicable for large-scale central air conditioning systems in today’s era of energy sustainability. Hot and chilled water produced by solar power can be stored in water tanks and circulated around a small home or a building to produce heating/cooling. Studies to this effect were first carried out on a comprehensive scale by Packer and Glicksman [24], although the concept of sensible TES was known well before that time. This work also included a computer simulation performed using New York City weather data. It was shown that sensible TES had technically good potential for home applications but was economically unfeasible due to the high initial cost in those times.

That same year, Wildin et al. (1979) collected data on the operation of a heat pump system with TES installed in a small office building in Albuquerque, New Mexico. Upon analyzing the performance, high COP values (4.02 to 4.43) were achieved in that case, particularly for the heating mode. Fast forwarding 20 years, another interesting study was carried out by Lee and Jones (1999), which focused on storing the compressor heat of rejection produced by a heat pump in storage tanks as hot water and use it for building heating. The experimental work conducted in this regard involved charging and discharging of a DHW tank from 95 °F to 128 °F and 95 °F to 85 °F respectively in a time span of three to four hours.
Similar to sensible heat storage, TES for building cooling systems has also been gaining prominence. “During the past two decades, TES technology, especially cold storage, has matured and is now accepted by many as a proven energy-conservation technology.”[27]. A notable work worth mentioning is Bahnfleth and Joyce’s [28] study on the use a chilled water storage system, adopted to improve the efficiency of energy usage in a university district cooling system. Likewise, Evans, W.S [29] demonstrated an improvement in the air-conditioning system efficiency of a campus building by means of ice-storage cooling. The two main components involved in TES cooling concepts, namely (1) the chiller (to cool the fluid) and (2) the distribution system (to transport the cold fluid) have been studied by several authors. Saito,A. [30] presents a comprehensive review of these works that discusses more than 130 articles.

Interesting literature carried out from prior studies on the performance of sensible storage water tanks were also reviewed. Ghaddar, N.K. [31] compared the energy storage efficiencies of a fully stratified water tank and a fully mixed water tank. Furbo and Berg [32] performed experiments on a water thermal storage tank with mantle heat exchanger for use in solar domestic hot water systems, and Baur et. al [33] simulated these experiments by analytically solving a system of differential equations. Other interesting papers in relation to the present work include the papers written by Bejan, A. [34], Han et. al [35], Krane, R.J. [36], Rosen, M.A. [37], Rosen, M.A and Hooper, F.C. [38] etc.

Lastly, as a starting point for the final aim of the present work, a literature survey was carried out on the studies involving sensible TES from an economic analysis point of view. The economic viability of the present sensible storage system has not been discussed here, however, prior studies [Badar, M.A and Zubair, S.M. [39]; Domanski and Fellah [40]] have been cited as references for the interested reader. While the analysis performed in the first literature was studied for the charging process only, the latter one was extended to the discharging cycle as well.

It should be noted here that the significance of this work not only involves the fact that energy sustainability is achieved by the use of sensible TES, but also with equal importance, the use of a thermoelectric based heat pump.
2.4 Other Reviews

In addition to the above mentioned studies, the reader may want to investigate other previously published review articles on the applications of TE modules. Ample literature on the design, development and applications of thermoelectric modules, including but not limited to HVAC applications, can be found in annually held TE conferences (1976-present). From these published articles, the literatures on the use of TEs for electronic cooling (Chu and Simons (1999), Simons and Chu (2000), Palacios et al. (2001)) and for interesting applications like the development of portable food coolers /warmers (Riffat and Ma, (2003)) have been specifically reviewed for those select topics. However, in the context of using TEs for building heating and cooling, the study on thermoelectric cooling principle for room air conditioning by Melero et al. (2003), analysis of TEs in an active building envelope component by Xu et al. (2007), and the development of a hybrid domestic hot water system by Khedari et al. (2001) are worth mentioning and was found very relevant to the present work.
Chapter 3

Experimental Set up

3.1 Test Section

The test section is shown in Figures 1-2. It consists of two aluminum plates which rectangular channels were machined for water to flow. Each plate is 25.36” long (Fig. 2a) and is covered by transparent Plexiglas lid (Fig. 2b) that allows viewing the flow of water inside the channels. The cover is fixed to the top of the plates with the help of machine screws. A rubber gasket separates the cover and aluminum plate to ensure the prevention of water leakage. In between the two aluminum plates, an array of three or five thermoelectric modules (Fig. 3) are placed which act as heat pumps and when energized create the temperature difference between the plates. To minimize the heat conduction from the cold plate to the hot plate, styrofoam is inserted in the gap between them and around the TE modules. The ends of the channels are fitted with 3/8” NPT pipes to facilitate the entry and exit of water through the hoses connected to them from the test loop circuit. Five thermocouples, fixed on one of the side walls of each aluminum plate with the help of aluminum tape and high thermal conductivity paste, are used for the measurement of wall plate temperature positioned lengthwise along the plate and located at the center of each TE. The test section is covered all around with a 1” thick foam thermal insulation material (k=0.04W/mK) to prevent any heat loss (or heat gain) to the channel to (or from) the ambient air. The entire unit is mounted on wooden blocks placed at the work area.

3.1.1 Design methodology of the test section

As earlier discussed, the design of the heat exchanger component involves the incorporation of thermoelectric modules; the other major issues to be taken care of included proper flow of water at both sides in the channel, the inlet and outlet of water stream, attachment of the system to other components required for experiments. The length of the plate was estimated based on the power requirement of the system. Each
thermoelectric module has a maximum power supply limit of 100 W. Hence in our experiments, the test section was designed with the intention to hold a maximum of 10

Figure 1: Assembled drawings of TE-HX test section shown in (a) planar and (b) isometric views.
Figure 2: Dimensions of test section water channel shown by (i) top view (ii) side view (iii) end view with Section A-A in (iv) exploded and (v) assembled views with top Plexiglas cover.
Figure 3: Photographs of TE-HX test section shown in (a) exploded and (b) assembled views.
thermoelectric modules (1000W power). The width and spacing between adjacent thermoelectric modules are known; hence the total length of the plate was calculated. Further, the size of the thermoelectric module determined the width of the system too as the water stream was to be made of the same dimension as of the module to equalize the heat transfer area. The thicknesses of the plates were calculated based on the minimum thickness required for the water stream to achieve a 2 GPM flow rate in turbulent conditions. The aluminum plates of nearest available size were procured and machined according to the estimated dimensions. The cover plate was another medium to regulate the gap thickness for water stream. Initially, a gap thickness of 0.25” was set and left enough material in cover plate to further decrease the gap thickness by removing the extra material, if needed, at later stage of experiments.

As per thermoelectric mounting technology, the proper means of fixing the modules to have them contact properly with the plates and without damaging them is of utmost importance. It is advised to have screws at the sides in the center of the modules to apply even pressure from both sides. As the number of modules was determined, the position to fix them was marked and it helped in determining the place for screwing the through holes on the plates.

Once the plate dimensions and the various holes’ location were calculated, the designing of the cover plate was the next task to be completed. A transparent epoxy glass cover plate was used for the experiment. All the screws were to be passed through the cover plate so their respective location in Aluminum plates was to be replicated on the cover plate to match and fix it perfectly.

The two channels were identical and replica of each other in design to achieve symmetry. Once everything was set for one side of the system i.e. one channel, the same was applied on the other channel too.

3.1.2 TE modules

Tellurex Z-Max® thermoelectric cooling modules (Fig. 4a) measuring 54mm x 54mm x 3.2mm were used in the experiments. Each module’s maximum current is 8.1 Amps and provides a maximum heat pumping capacity of $Q_{c, \text{max}} = 139$ Watts at the maximum
current for an operating surface $\Delta T_{s,h-c} = T_h - T_c = 0$. The performance curves for the module have been supplied by the manufacturer [7] as shown in Fig. 4b for different heat sink temperatures. These curves show the parameters that affect the performance of the module. The key parameters that determine the heat pumping capacity are the hot side temperature ($T_h$) and the resultant temperature differential across the hot side and the cold side of the module ($\Delta T_{s,h-c}$) which depend on the design of the heat sinks. As the temperature differential increases, the cooling ability, $Q_c$, and the coefficient of performance of the modules drops rapidly. Thus, it is important to note that the heat pumping capability of the thermoelectric module is significantly influenced by the combined thermal resistances of the heat sinks, which should be kept as small as possible to keep $Q_c$ large. While conducting our different set of tests, the cold side to the hot side temperatures are expected to have different values of $\Delta T$, which results in different values of system COP to be shown later in the results section.

Fig. 4(a) The thermoelectric module

3.1.3 Machining

As the thermoelectric modules were to be sandwiched in between the two rectangular Aluminum plates, the flatness and the smoothness of the surface was of prime importance as, while mounting, the unevenness could have led to the damage of thermoelectric modules due to uneven pressure at different surface of the modules.
The inside surface of the plate was to be milled to the width equal to the thermoelectric module width. A diverging section after the inlet and a converging section before the outlet were provided to give space to water to have the regularity of the flow once it comes after the 3/8” NPT pipe inlet and enters the same at the outlet.

### 3.2 The Testing Loop

In addition to the aluminum plate water channel TE-HX test section, the apparatus consists of several other pieces of hardware needed to operate the testing loop (Fig. 5). The water circuit is configured either as a closed loop (recirculated) or as open loops (once through) systems depending on whether the system is desired to be operated in the
transient tank charging mode or in the single pass testing mode configuration. Tap water is used as the working fluid for both testing modes. Figure 5a is the schematic diagram of

Fig 5 (a): Schematic diagram of test flow circuits tank charging experiments

the assembly used for the transient tank charging tests. In this case, the tank is initially filled with 5 gallons of water and sealed, so that no water escapes. The tank is also covered on all 4 sides, top and bottom with thermal insulation. Water from the exit of the top channel returns back to the closed tank. The reservoir tank in the top flow circuit is connected to a progressive cavity pump, which pumps the water from the reservoir through the aluminum channel at a certain desired flow rate. The water flow rate was controlled by the pump rpm. A flow versus pump rpm curve was generated by collecting a certain volume of water during the corresponding elapsed period of time. A five gallon container was used to measure the volume of the accumulated water. The measurement is initiated and continued until the water level in the container reaches a pre-set value. The elapsed time is then recorded. Water flowing across the bottom plate is pumped through the channel with the help of a self-pumping water bath. The water bath maintains the water at a pre-set temperature in addition to pumping the water through the channel at a given flow rate. The flow rate through the constant temperature heat sink/source side of the testing loop was set at 1.8 GPM. This
volume flow rate was measured using a rotameter which was check against a large graduated cylinder. The water exiting the bottom channel returns back to the constant temperature bath to form a full cycle. Thermocouples were used to measure the bulk water temperatures at the inlet and outlet of both flow channels inserted into the PVC pipes located upstream and downstream of the channels. The thermocouples were cemented into the top of the pipe to avoid water leakage. The hoses on the entire top loop as well as the bottom loop were covered by 1” thick thermal insulation. Finally, a thermocouple probe was also inserted midheight into the well-mixed tank in order to measure the tank water temperature.

The transient tank charging tests were conducted in the cooling tank mode to produce cold water as well as the hot tank charging mode to produce hot water. In the cold tank charging tests, the initial tank temperature and the constant temperature water bath were set nearly the same as the ambient air temperature in the laboratory, i.e. +/- 0.5 °C, although it should be noted that in actuality the heat sink ground water temperature may vary throughout the summer months.

Switching operation from the cooling to heating tank charging modes (and vice-versa) was easily accomplished by flipping the polarities of the switch to which the TEs are connected. When the operating mode is switched from the cooling mode to the heating mode the goal is now to provide hot water on a typical winter day. This requires that the tank temperature, which is now to be heated, starts at the ambient air temperature, but whereas however, the heat source from which it draws energy, may be either lower or higher in temperature as the initial tank temperature depending on the available heat source used, such as the building water supply line (being lower), or preferably, the heat source being grey water collected from the shower drains (and thus being higher). For the present study the constant temperature water bath were set nearly the same, +/- 0.5 °C, as the initial tank temperature and the ambient air temperature in the laboratory for the duration of the hot tank charging tests reported in the results section.

Figure 5b is the schematic for the assembly of the single pass tests whereas water from the supply tap directly passes through the channels and exits right to drain without recirculation. The flow rate on both sides were set at 2 gpm. The top channel flow rate was controlled again by the pump speed but an overflow drain pipe was attached to the
reservoir to escape the overflow of the incoming water from the tap installed to maintain constant pump head. The bottom plate water flow rate was controlled by a valve

Fig 5(b): Schematic diagram of test flow circuits for single pass experiments.

positioned in line with the tap water supply line. Water exiting both outlets of the channels passed directly to the drain through a combination of pipe-hose routing.

3.3 Additional hardware

The additional equipments which are necessary for the operation of the circuit are discussed in further detail below, and Table 1 indicates values of key system parameters.

3.3.1 Progressive Cavity Pump

The SP300 series progressive cavity pump used in the recirculation loop is a compact positive displacement pump. It is quite smooth in operation without much slip as compared to centrifugal pumps. Other than the compression of the fluid, the pulsing is almost minimal. The characteristic curves of the pump shows its almost linear behavior in performance and due to this, the flow can be assumed steady. Although it can be used for
high viscosity fluids, here, the pump was used to circulate the water into the top channel at required flow rate. The volumetric flow rate output from the pump can be regulated by changing the pump rpm. The versatility to use many liquids, long lasting pump life, easy maintenance and metering capability makes it more attractive and easy to use. Further specifications for the pump are as following:

Make: Robbins and Myers  
Model: Moyno 33104  
Capacity: 0.1 – 3.0 gpm at 50 psi  
Motor: ¾ hp A/C Duty Master, 1800 rpm  
Motor Make: Reliance Electric

<table>
<thead>
<tr>
<th>Voltage AC</th>
<th>Normal Current</th>
<th>Inrush Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>110-120</td>
<td>3.0</td>
<td>30</td>
</tr>
<tr>
<td>220-240</td>
<td>1.5</td>
<td>15</td>
</tr>
</tbody>
</table>

The motor is 3 phase and can be driven at 60 Hz. It is controlled by a programmable variable frequency drive type SP500, Model ISU21001 from Reliance Electric. The SP500 controller operates on a 3 phase (single phase) AC power source at the appropriate rated input voltage. When power is supplied to the controller's input, the conventional inverter bridge transfers energy from the AC input line to the AC motor. The AC line voltage is rectified through the input diode module which in turn generates the constant DC bus voltage.

The digital panel allows controlling feeds easily and can be set at the exact RPM required to operate the pump.

### 3.3.2 Power Supply

A DC programmable power supply of Hewlett Packard make was used. The specifications are as follows:

Model No: 6032 A  
Voltage Range: 0-60 V  
Current Range: 0-50 Amps
Maximum Power: 1000 W

This power unit could operate in either of two conditions a) constant current (CC) and b) constant voltage (CV). For powering thermoelectric modules, this power supply was used and was set for constant current condition as the characteristics curves from the manufacturer are available in constant current conditions only.

Output voltage and current are continuously indicated on individual meters. LED indicators assist in keeping the track of the state of the unit. Easy setting from front panel, over voltage protection and foldback protection are some other basic feature of the unit that makes the usage safe. This is a reliable unit and gives steady readings.

3.3.3 Constant temperature water bath

When the system was being tested in transient mode, the water inlet temperature would continuously alter at the inlet of the bottom loop. However, it was desired to maintain the inlet water temperature constant in the bottom loop for higher resolution and understanding of experimental results. This was achieved by using a water bath (chiller) in connection with the pump. The outlet of the chiller was connected with the water reservoir placed on the top of the progressive pump unit. The device is basically a refrigerated circulator.

The specifications of the device are listed below:

Make: Neslab RTE 111

Capacity: 500 W (Heat Removal) at 20°C

Volumetric capacity: 4 gpm

The water in the water tank (top loop) and the chiller (bottom loop) were hence both in closed loops and was continuously circulated through the two aluminum plates. The constant temperature bath gives the flexibility of maintaining the water temperature within the accuracy of ±0.1°C.
3.3.4 Water Circuit Piping

As the working fluid throughout the system is normal tap water, the system is not that critical in terms of equipment material. In most of the system setup, rubber tubing and PVC piping is used to supply water. In the starting, the tap water reaches the tank at one side and to the volumetric flow meter at other side with the help of two separate hose pipes. The outlet of the progressive pump is connected with the $\frac{1}{2}$ " PVC piping which enables the flow to be directed in the desired directions with the help of easily available joints. Just before the inlet of water channel, the PVC piping is joined to a $\frac{3}{8}$ " NPT thread pipe of steel. Apart from the progressive pump outlet section, this is the only metallic part in the piping circuit. The other threaded potion of the metal goes in the aluminum plate. The same joints are applied at the outlet of the water channel plate. A $\frac{1}{2}$ “rubber tube is used to connect volumetric flow meter to the PVC piping of bottom water channel.

The water tank, seated at the top of pump, is of 5 gallons capacity and the material of the same is Translucent Polyethylene. The model no of the ball valve before the pump inlet is Whitney SS-45S8 $\frac{1}{2}$, which is kept fully open in normal mode of operation. Both the tank and piping are well insulated using Armaflex insulating material ($k=0.04$ W/m K).

3.4 Instrumentation

The instrumentation and the sensors used for measuring data consisted of flow meters, thermocouples, pressure gauges, multi-meter for voltage and current measurements. These are described below.

3.4.1 Flow meter

The measurement of volumetric flow rate at each side was of prime importance as the system was being analyzed for symmetry in terms of flow rate. At top side of water channel, the progressive cavity pump was attached in the line. The pump’s characteristics vary linearly and with the graphs, the pump was set at the particular rpm equivalent of a specific volumetric flow rate. Thus, at top side of channel, the flow rate was governed by pump rpm panel.
On the other side, the flow measurement was done with the help of a flow rotameter. The tap water was directly supplied to the flow meter to measure and control the rate. The specifications are:

Make: Omega

Flowmeter Cat. No. Size: 5

Serial No.: E9780 – E9879

Float type: Stainless Steel

The range of the meter is from 0.104 l/min (0.0275 gpm) to 4.83 l/min (1.28 gpm) at a scale of 0-100 calibrated divisions. The float type could be changed from Stainless Steel to Glass if the requirement is to measure low flow rates more precisely. (Ranging from 0.0224 l/min to 2.07 l/min). The reading was taken at the center of the ball (float).

To measure a flow rate above the range of the meter, bucket measurement method was used by measuring flow in a particular time and adjusting it accordingly to attain the desired flow rate.

3.4.2 Thermocouples

Type-T copper-constantan thermocouples were used for measuring the temperature at various locations in the set-up. The locations for thermocouples were at the inlet and outlet of both the channels for measuring bulk water temperatures and on the surface of the Aluminum plate for measuring the surface temperature of water at different locations along the plate. For bulk temperature, grooves were made in PVC pipe and thermocouple was inserted deep enough up to mid of the stream and glued with high thermal conductivity paste to the pipe tightly. At surface of the plate, thermocouples were stuck with the help of Aluminum tape. Five thermocouples on each plate were used to measure the temperature along the plate at the distances of 2.25 inch, 6.75 inch, 11.25 inch, 15.75 inch and 20.25 inch respectively. The location of the thermocouples was equidistant from both the inlets of the channel.
The thermocouples were made with a Tigtech 116SRL thermocouple welder from 30 gauge wire. The T-type thermocouple can measure temperatures by providing an accuracy of 0.5°C. A total of fifteen thermocouples were used including the one used for measuring the ambient temperature.

### 3.4.3 Pressure Gauges

In the experiment, the pressure drop calculation and the impact of pressure loss in the system was not of high significance and very little care was taken to consider that into account. Although, to see if there is really a high pressure drop when the water flows through the channel, two pressure gauges were added in the line, one at the inlet and another at the outlet of the channel.

The pressure gauges were Omega PGS-25L Bourdon tube pressure gauges with 1% full scale accuracy. The range of both was 0-30 psi. The gauges were attached to PVC pipe with the help of a T-joint at that section.

### 3.4.4 Handheld multi-meter

The current, voltage and resistance in the electrical wiring and heater was measured and calculated with the help of a handheld multi-meter. The general description of Multi-meter is as following:

Model: HP E2378A

Operational Temperature: 0-40°C

Max common mode voltage: ±1000V DC or 750V rms (1000V peak)

Power supply: IEC LR03 (AAA) 1.5x2
Chapter 4

Experimental Tests and Data Reduction

A total of ten experiments (Table 2) were conducted with this experimental set up which consisted of six transient charging tests and four steady-state single pass tests. During the experiments, the bulk water inlet and outlet temperatures on both sides, the plate surface temperatures at the different locations, the tank temperature and the current and voltage were recorded at regular intervals of time. The intervals were every 15 minutes at the beginning but increased to 30 minutes as time progressed.

Table 2a is the summary sheet of the cold tank charging tests. This table reports the final total drop in tank temperature as well as the difference in the plate temperatures located at the very center of the test section at large time. When steady state was reached the tank temperature and the water channel bulk inlet temperatures remained constant in the charging loop. Similar tests were conducted by recirculation of the hot side (switching voltage polarities on the TE module) to heat the hot tank (hot tank charging experiments) with the key results summarized in Table 2b.

Since the generation of chilled water and hot water was a major focus of this research, the rate of cooling / heating of the system was calculated on the basis of the water flow rate and the temperature difference between bulk outlet (return water to the tank) and inlet (entering the heat exchanger) as follows:

\[ Q_c = \rho \cdot v \cdot c_p \cdot (T_{bc,o} - T_{bc,i}) \]  
\[ Q_h = \rho \cdot v \cdot c_p \cdot (T_{bh,o} - T_{bh,i}) \]

The coefficient of performance for the refrigeration and the heat pump modes are then calculated as follows:

\[ \text{COP}_{\text{cold}} = \frac{Q_c}{P} \]  
\[ \text{COP}_{\text{hot}} = \frac{Q_h}{P} \]
where the corresponding total power input supplied to thermoelectric modules was determined as the product of the voltage times current

\[ P = V I \quad (5) \]

For the transient charging tests, the COPs listed in Table 2a and Table 2b are based on the very last set of measured data for which the system had achieved steady state. The first law energy balance shows the range within which all the measurements agree with each other as the very last column of Table 2.

The summary of results for the single pass tests are tabulated in Table 2c. Among the once through steady state tests, two were conducted at 2 GPM and the other two at 1 GPM. For each flow rate, tests were conducted with either 3 thermoelectric modules or 5 thermoelectric modules. The purpose of conducting the tests with different number of TE modules was to compare the effect of system performance with more localized heating/cooling due to fewer TEs but at same total power and at different flow rates. Also the once through tests allowed one to analyze the system’s COP for smaller values of plate temperature difference \( \Delta T_s = T_h - T_c \).

The experimental uncertainties of the collected data were performed following the procedure of Kline and McClintock [41] and Moffat [42]. The sources of error in the system are the flow rate measurement, the thermocouple, and the digital instruments used for the thermocouple readings and in the HP power supply. The thermo-physical properties are estimated to have a bias limit of 0.5% [43-44] and their precision limits are zero with the property value being fixed. A bias limit of 0.5°C for the thermocouples was the major source of uncertainty. With a bias limit of 0.5 second for the stop watch used in the flow rate measurement check, 0.05°C for the thermocouple instrument readings, 0.36% and 0.025% for the current and the voltage, respectively, and taking into account the corresponding precision limits, the total uncertainty in COP has been calculated and is 23.61% for the cold tank charging experiments and 8.95% for the hot tank tests. Arguably, the former has a higher uncertainty due to the smaller change in the bulk temperature differences. A detailed uncertainty analysis has been presented in the appendix and also tabulated in Table A1.
Chapter 5

MATHEMATICAL MODEL

5.1 Governing equations

Here a theoretical model to predict the temperature history of the charging process of the well-mixed, sensible storage tank is developed. A transient three-lumped system has been used to model the water temperature inside the tank and the TE-HX system. The three thermal masses consist of the TES water storage tank and the two aluminum plates, respectively. Each plate temperatures vary with time, with the temperature difference across them affecting the TE cooling performance which in turn, the temperature drop/rise history of the tank water. In the present model, it is assumed that the tank is well-mixed, and the entire top and bottom plates are isothermal, but at different temperatures. Based on these assumptions, the first law energy balance equations for each of the three lumps are formulated, i.e.

\[ \rho V_c \frac{dT}{dt} = E_i - E_o \quad [45] \]  

(6)

Expanding the heat gain and heat loss terms shown schematically in Fig. 6 for each of the three thermal masses for the cold tank charging tests [Runs 1-3], the governing equations then become:

\[ \left( \rho V_c \right)_{w} \frac{dT_T}{dt} = \left( \bar{h} A \right)_{w} \left( T_{air} - T_T \right) - \left( \bar{h} A \right)_{cw} \left( \Delta T_{lm\_cw} \right) \]  

(7)

\[ \left( \rho V_c \right)_{alum} \frac{dT_{alum,top}}{dt} = \left( \bar{h} A \right)_{cc} \left( \Delta T_{lm\_cw} \right) + \left( \bar{h} A \right)_{p} \left( T_{air} - T_{alum,top} \right) - nq_c + q_{screws} + q_{airgap} \]  

(8)

\[ \left( \rho V_c \right)_{alum} \frac{dT_{alum,bot}}{dt} = nq_c + P - \left( \bar{h} A \right)_{hc} \left( \Delta T_{lm\_hw} \right) - \left( \bar{h} A \right)_{p} \left( T_{alum,bot} - T_{air} \right) - q_{screws} - q_{airgap} \]  

(9)

As shown by the above equations, that in addition to the convective heat transfer through the two plates \( (q_{cc}, q_{hc}) \) and the TE modules \( (q_c) \), additional paths of heat gain / heat loss were included for greater accuracy in the model. These consisted of (1) heat gain or heat loss by the ambient air to the tank or plates; (2) the thermal bypass from the hot to the
Figure 6: First law energy balance for the tank, top and bottom aluminum plates for tank chilling runs.

Note: $T_{\text{alum, top}} < T_{\text{tank}} < T_{\text{air}}$ and $T_{\text{alum, bot}} > T_{\text{air}}$

cold plate which passes through the machine screws used to clamp the plates and TE devices together; and (3) the heat leakage through the stagnant air gap between the plates. Explicit expression (in terms of overall $\overline{U}$ values) for heat transfer from the ambient to the tank ($q_a$) and heat gain/loss to the plates ($q_p$) have already been inserted into equations (7)-(9). Explicit expressions for the latter two thermal paths (pure conduction) are given by the following:
\[ q_{screws} = \pm 24 \frac{T_{alum,bot} - T_{alum,top}}{L_{screws}} \frac{1}{k_{screws} A_{screws}} \]  

\[ q_{airgap} = \pm \frac{T_{alum,bot} - T_{alum,top}}{L_{airgap}} \frac{1}{k_{airgap} A_{airgap}} \]

Note that \( q_c \) is obtained from the performance curves supplied by the manufacturer [www.tellurex.com] and is a linear function of the temperature difference between the two plates expressed as follows:

\[ q_c = \pm [m (T_{alum,top} - T_{alum,bottom}) + b] \]  

The values of \( m \) and \( b \), in turn, are dependent on the current and the heat sink temperature through each module. Hence the exact equation for \( q_c \) is obtained by first digitizing the performance curves shown in Fig. (4b) and then extrapolating them for the right current and the right heat sink temperature. The tabulated values for the coefficients are presented in Table 3 for all of the numerical simulations to be presented.

Note that for the cold tank charging experiments i.e. Runs [1-3], the system is designed to remove energy from the tank and to chilled water. Hence the water in the tank is at a colder temperature than the ambient, and the tank will gain thermal energy from the outside air. At the same time, the energy is extracted out of the tank by the flow of water leaving the tank through convection heat transfer between the water and the top aluminum plate, \( q_{cc} \) (see energy balance on the TES tank in Figure 6 and Eq. (7)). The model for the top aluminum plate is developed (Eq. (8)), taking into consideration that the array of thermoelectric modules removes heat from the plate on one side (\( q_c \)) while the plate absorbs heat from the tank water (\( q_{cc} \)), the ambient air (\( q_p \)) and heat from screws and air gap. In contrast, energy is added to the bottom aluminum plate (Eq. (9)) via the heat pumped from the top plate plus the energy used to power the thermoelectric device (\( nq_c + P \)). The bottom heated plate loses energy to the ambient air and to the cold
plate through the screws and the air gap, but primarily via forced internal convection \(q_{hc}\) through the constant temperature water bath, maintained at a fixed temperature prior to the start of each test. The flow through both aluminum channels, being the case of internal flow, the ideal temperature difference for convective heat transfer between the water and aluminum plates is defined in terms of the log mean temperature difference, and the convection coefficient is defined for the average value over the entire channel length as described by Incropera and Dewitt, (2007).

The schematic diagram depicting the first law energy balance for the hot tank charging tests (Runs 4-6) has not been shown for brevity, but is very similar to Figure 6, with the main difference being that the direction of heat flow through the TEs is reversed. Equations (7)-(12) for these tests remain essentially the same with the exceptions that Eq. (10)-(12) will be used with the negative sign; as the heat loss through the screws and the air gap between the plates now transfer energy through conduction to the cold bottom aluminum plate. It should be noted here that the \(m\) & \(b\) coefficients calculated for Runs 4-6 in Eq. (12) is only an approximation as far as the hot tank charging tests are concerned, since the heat sink temperature is continuously changing. However, the model could be rendered more accurate by regenerating the performance curves with the linear coefficients being functions of heat sink temperatures as follows:

\[
m = m(T_h), \quad b = b(T_h).
\]

### 5.2 Governing equations simplification and convection parameters:

The log mean temperature difference is difficult to calculate in the three lump model formulation used for this analysis since it would then involve two additional unknown temperatures being the bulk outlet temperatures of the channels. Therefore, the following simplifying assumptions were made in order to make the formulation solvable.

\[
\Delta T_{lb_{cw}} \approx T_r - T_{alum, top} = T_{b,i} - T_s
\]

(8)

\[
\Delta T_{lb_{bw}} \approx T_{alum, hot} - T_{bath} = -(T_{b,i} - T_s)
\]

(9)
This approximation is valid since the fluid temperature rise, $\Delta T_b = (T_{b,\beta} - T_{b,\alpha})$, through the channel is fairly small relative to the surface and bulk temperature $|T_s - T_b|$ difference. The model requires the convective thermal resistance input parameters for the external natural convection with air, and forced internal convection in the cold and hot water channels. These values have been determined from the equations (15) – (20) given in Table B1 of the Appendix. In all the numerical simulations, the external $(\overline{UA})$ values used were 5.65 W/mK for the insulated tank and 0.37 W/mK for the insulated aluminum plates; 70.04 W/mK and 64.35 W/mK are the internal $(\overline{hA})$ values for the top aluminum plate water channel and the bottom loop water channel, respectively.

5.3 Solution Technique

The governing equations (7), (8), (9) coupled with the supplementary equations (10) - (14) were solved numerically with the finite-difference method. Both implicit and explicit numerical solvers were considered, however, the explicit Euler method has been used in the present model due to its ease of implementation in spite of its time step limitation. The stability requirement for this model was determined to be $\Delta t \leq 29.65$ seconds, but to obtain accurate results, a time step of 1 second was chosen for all numerical simulations. The basic form of the Euler’s method is described in any standard numerical textbook. This method was coded using the commercial MATLAB software and was used to calculate all three temperatures as a function of time for our given system. The necessary initial conditions for each thermal mass, specifically $T_r(0)$, $T_c(0)$ and $T_h(0)$ for each of Runs 1-6, were obtained from the experimental data.
Chapter 6

Results and Discussion

6.1 Charging tests experimental results

Figure 7 (runs 1-3) shows the chilled water sensible tank charging experiments. The measured tank temperature history reveals that the system cools rapidly at first with the drop in tank temperature exponentially decreasing with time to some final constant steady state value. Fig. 7a, displaying the plots of tank temperature versus time for all three cold cycle tests, shows that the water cools down faster and to a lower temperature for the case of higher power supply (300 W vs. 180W). Moreover, from the same graph, it is shown that at fixed power, that the test with five thermoelectric modules achieves a lower final temperature by 2.1°C (Table 2a) than the test conducted using three modules. The temperature of the bulk outlet on the hot stream side of the test section remains relatively constant, since the temperature of the water entering the channel is maintained constant by the thermal bath and the heat pumped by the module is fairly the same. The time it takes to achieve steady state is similar for all cooling tests.

Fig 7a: Cold tank and hot side bulk outlet temperatures versus time
Figure 7b reports the surface temperature variation along the plate length and bulk temperatures at steady state for run 1 (3 TEs at 300 W). The measured bulk temperature values at the inlet and outlet locations are plotted as well as the linearly interpolated values in between at the three different thermocouple locations. At steady state, the bulk temperature rise for the hot side is 0.8°C and 0.2°C decrease for the cold side. The hot plate’s surface temperature increases at a steeper rate at the entrance of the channel than it does further along the plate.

![Steady state temperatures for 3 TEs test 300 W](image)

Fig 7b: Steady state axial hot and cold side bulk water and plate surface temperatures

Similar transient behavior is observed for the case of the hot tank charging experiments, runs 4-6 (Table 2b), except that the increase in power causes a greater change in tank temperature (Fig. 8) than for the cold tank charging experiments. Specifically, an increase in power supply by about 67% causes the temperature of water to rise by about 25% more. Observing the graph from the point of view of the number of TE modules used, the system with five TEs performs better than with three TE modules in this case as well. With respect to the spatial variation of temperatures with distance, Fig. 8b was plotted showing the lengthwise steady state plate and bulk temperatures. Only run 4 (3 TE
modules at 300 W power) has been plotted in Fig. 8b and it depicts relatively the same behavior as shown in Fig. 7b.

Figure 8: Results of hot tank charging tests. (a) Hot tank and cold side bulk outlet temperatures versus time and (b) steady state axial hot and cold side bulk water and plate surface temperatures.
6.2 Effect of number of TE modules

In order to study the effect of the number of TE units on the performance of the system in more detail, the results of the once through steady state tests (Runs 7-10 of Table 2c) were analyzed in addition to those in the charging modes. These once through tests provide a closer look at the system performance with constant inlet water temperature as opposed to time-dependent bulk inlet temperature. The bulk and surface temperatures at different locations along the test section for once-through tests were recorded and are plotted in Fig. 9 for the case of same power input and at 2 GPM flow rate (Runs 7-8) but for different number of TEs. The bulk temperature distributions of the water flowing through the test section channels follow similar pattern irrespective of whether 3 TEs or 5 TEs were used, increasing or decreasing depending on what side (hot or cold) of the TE module the water channel flow traverses. Closer inspection reveals that at each location, the difference between the bulk temperature values of the two tests data sets lie within 0.5°C [Table (2c)]. However we can note with sufficient resolution that the test with 5 TEs results in colder bulk temperatures than the 3 TE test on the cooling channel. While comparing the plate temperatures, we note that the hot plate is colder while using 5 TEs. The temperatures on the cold plate are also lower at the same 3 TE thermocouple locations. This leads us to conclude that more effective performance can be achieved when 5 TEs are used. Analyzing in terms of the heat conduction, using 5 TE modules causes less localization of the heat flux distribution of the TEs on the aluminum plates as compared to using only three TEs, and this in turn, result in more uniform plate temperature which would allow each individual TE module to operate better (smaller localized ∆T across each module). Finally note, by comparing the surface temperatures at the end of the cold plate, observe a clearly lower value of the cold plate temperature when using 5 TE modules during testing (even with the 4th TE module apparently not working properly). In general, for these single-pass test runs the difference noted when using five versus three TE modules is somewhat subtle but is still very measurable.

6.3 Effect of Flow Rate

A larger change in the resulting temperature performance of the device is observed when
the flow rate is changed (while keeping the same number of TE modules). Figure 10 shows the surface and bulk temperatures (bulk being interpolated at all locations along

![Surface and bulk temperatures vs location at 2 GPM](image)

Figure 9: Plots of hot and cold side plate surface and bulk water temperatures versus distance for single pass tests for different number of TEs at two GPM flow rate.

the plates) for two different flow rates when 3 TE modules were used (runs 8 and 10, Table 2c). Note the significant difference in the values of $T_s - T_b$ (vertical line with arrowheads in Fig. 10) on comparing the result at 1 GPM versus 2 GPM. The higher water flow rate through the channel results in higher rate of convective heat transfer and lower heat sink resistance and therefore results in overall better system performance. Vice-versa, at 1 GPM, the coefficient of convection is less than what it would be at 2 GPM, hence the corresponding $T_s - T_b$ is higher. When the flow rate is decreased, the slopes of the bulk temperature lines in the figure also change as expected. The magnitude of $T_s - T_b$ are not the same for the top and bottom plates due to the differences in the values of $Q_s$. The value of $Q_b$ is approximately 1.6 times the power input and that of $Q_c$ is
about 0.6 times, hence the value of $T_c - T_h$ at a given flow rate is higher for the hot stream side than for the cold stream. Finally note that the surface temperature profile has a higher slope at the entrance of the channel, which reduces as the water travels further downstream. This is because hydrodynamically developing flow at the entrance of the channel causes the convection coefficient to be significantly higher than farther down the plate. The analysis of the entrance effect in the evaluation of the local convection coefficient and the quantification of the Nusselt number is currently being studied in detail and will be presented in a future study. Nonetheless, we can safely conclude from the data presented here that the effect of the velocity has a larger impact on the system performance than the effect of the number of TE modules. The effect of flow rate was also studied for the steady state once through tests for the same power with 5 TE modules (comparison of

![Figure 10: Plots of hot and cold side plate surface and bulk water temperatures versus distance for single pass test with 3 TEs at two different flow rates.](image)

- $1/h_{hot}$
- $1/h_{cold}$

Location (cm)
runs 7 and 9). Similar behavior was noticed for this case as was observed for the system with 3 TEs, but for brevity, these plots have not been presented.

6.4 Coefficient of Performance

Next we analyze the results in terms of the coefficient of performance (COP), as defined by Equations (3-5). These values are listed in Table 2 and are shown plotted in Fig. 11. For the present system, the variation in COP has been examined with respect to the type of test conducted (charging/once through), the number of TE modules used, the temperature difference across the plates and also compared with respect to standard vapor compression and vapor absorption cycles.

6.4.1 COP versus number of TE modules

The COP is graphed versus the number of TEs in Fig. 10a for all the 300W tests with 2 GPM flow rate. Note that the COP for heating is always greater than one. This shows that the TE modules in heating mode works better than a simple resistance heater which has a COP = 1 (reference line). The COP for cooling mode is always less than one and vary between 0.21 – 0.88 (Table 2). Observe that the channel with 5 TEs produce higher COPs; 5 TEs always work better than 3 TEs in all of the operating conditions (heating, cooling and once-through) tested. It is speculated that further increase in system COP may be achieved by using more than 5 TE modules per channel but a large increase is not expected. Ranking in descending order from highest to smallest value, the present results show that once-through tests have the highest COP followed by the cold tank charging tests. The hot recirculation tests produce the lowest coefficient of performance. The highest COP of cold tank charging achieved was COP\textsubscript{c} = 0.54 and the highest COP of the system for hot tank charging was COP\textsubscript{h} = 1.38 although the COPs for once through (heating or cooling) were larger.

6.4.2 COP versus temperature difference across the plates

The variation in COP and performance of the system was examined further by plotting the COP versus the temperature difference of the aluminum plates, i.e. ΔT across the TE modules. The results for the 300 W runs with three TEs installed are plotted in Fig. 11b
and show why the hot tank charging tests produced the lowest value of COP. The tank water temperature rose up to 67.2 °C during the hot side recirculation tests with 3 TEs, thus the temperature differential across the TE modules was notably higher. In contrast, the TE modules in all the other tests operated over a narrower range of temperatures with the single pass configuration having the smallest. It is well known that a linear relationship exists between $Q_c$ and $\Delta T_{s,h-c}$ as can be seen from the characteristic curves provided by the manufacturer [Fig.(4b)] and as confirmed by Palacios and Sanz-Bobi [47]. The higher temperature differential causes a smaller amount of heat to be pumped by the module towards the hot side, causing the module performance to deteriorate. This is also consistent with the observation that towards the end of the hot charging experiments, the current drawn by the modules for the applied fixed voltage decreased as time progressed. The cold side recirculation tests had a lower $\Delta T_{s,h-c}$ across the module; hence the COP values were higher. The once through tests, with the smallest $\Delta T_{s,h-c}$ produced the highest COP among all the sets of experiments.

Figure 11(a): COP for different runs - comparison between 3 and 5 TEs
6.4.3 Comparison of TE system to standard cycles

Finally, for completeness the COP of the present system has been compared to a standard vapor compression and vapor absorption refrigerators. The data for the vapor cycles have been obtained from Bansal and Martin [48] who carried out a recent comparative study. Note that our results confirm those in [48], that the present TE system (COP = 0.54) is ahead of the vapor absorption system (COP=0.47) but runs a distant second to the vapor compression cycle (COP = 2.59) when evaluated using system COP as the measure. Despite this fact, a TE based air-conditioning unit offers several advantages and offers tremendous potential for HVAC applications over conventional AC/heat pump systems powered by typical home electricity or by the non-traditional PV panels. When compared to the non-standard PV electrical source, one should note that the TEs based system can run on the PV panels directly and thus can completely eliminate the use of the costly inverters. On the thermal load end, the TE system can be used for much more precise temperature control and load matching, in fact an infinite number of load adjustments.
steps are possible as opposed to just single or two-stage load control in available commercial air-conditioning systems, because the current directly controls the cooling ability in the TE systems. And finally from an environmental point of view, the absence of CFCs and inherent noise reduction justify the use of TE air conditioners in an increasing number of “green” applications.

6.5 Mathematical model for cold tank charging transient tests

The temperature history of the water in the sensible storage tank and the two aluminum plates were simulated for the entire experimental duration for each of the six runs. Figures 12-14 show a comparison between the experimental measurements and the model predictions for the three thermal masses (tank, cold plate and hot plate) for Runs 1-3 respectively. It can be seen that the mathematical model predicts

![Figure 12: Mathematical model to experimental data comparison for Run 1 for tank water, top aluminum plate and bottom aluminum plate temperature history](image-url)
Figure 13: Mathematical model to experimental data comparison for Run 2 for tank water, top aluminum plate and bottom aluminum plate temperature history

the exponential decrease in the tank temperature history curve as observed during the experiments. For the cold tank charging test with a total of 180 W power and the use of 3 TE modules, the three lumped model way under-predicts the final temperature of the tank water. However, on simulating the test with a higher power (300 W) for the same 3 TE modules, the model more closely matches the experimental results. Note that for Run 2 (5 TEs at 300 W) involving higher power and more TE modules, the experimentally obtained steady state temperatures of the chilled water as well as that predicted by the model are the lowest among all the three tests.
Figure 14: Mathematical model to experimental data comparison for Run 3 for tank water, top aluminum plate and bottom aluminum plate temperature history

The temperatures of cold plate show similar behavior as that of the water in the tank, except is lower in value by approximately fixed amounts. The predicted temperatures in the three cold tank charging tests for the cold plate is always lower than the temperatures measured during the experiments. Hence there is a marked variation in the experimental and theoretical curves for the tank water temperature history curve and the cold plate temperature history curve. This indicates that the external convection heat loss coefficient for the tank through natural convection of ambient air was higher for the experimental conditions than assumed in the simulation. However, as regards the internal channel flows, the $\overline{hA}$ values used in the simulation were calculated using standard correlations and found to be within realistically acceptable limits (Table B1, Appendix). In increasing order of accuracy, the temperatures predicted by the model for the cold plate can be arranged as Run 2, 3 and 1; with the Run 2 reporting the lowest temperature of 3.64°C. In other words, the use of a higher number of same TE modules for the same power input
predicts a lower steady state temperature for the plate. The results obtained from the simulation are confirmed to be true from the experimental tests in this regard, and the reasons for the particular behavior have been outlined in Kazmierczak et.al (2008). The mathematical model for Runs 1-3 is also consistent with the observation that the heat sink temperature remains more or less the same (27.91°C, 26.84°C, and 27.53°C respectively) throughout the experimental duration. This is an expected behavior since the bulk inlet temperature of water being pumped out of the bath is constant.

6.6 Comparison of the TE performance: Model Vs Manufacturer

In an attempt to determine how closely the module behavior has been estimated in the simulation, the variation of $q_c$ per TE module with $\Delta T$ generated by our mathematical model is plotted (Fig.15) for the charging tests. For the cooling tests with the heat sink temperature at approximately 26°C, the total current through each TE module was averaged to be 5.66 A, 4.22 A, 4.4 A in this order. Note that the model shows a linear variation in the $Q_c - \Delta T$ plot. The value of $Q_c$ is much higher at the start of the tests; however, after the charging process starts, $Q_c$ sharply decreases in the first few minutes of the experimental duration due to increasing $\Delta T$ between the module faces. This behavior is true for all Runs 1-3, along with the heat pumped per TE module being larger in magnitude for higher amps. The above-mentioned observations play a major role in confirming that the low theoretical temperature history for Run 2 over 1 is not only the consequence of the choice of external $\overrightarrow{n}A$ values in the model, but a result of the predicted values of $Q_c$. It should be noted here that the tellurex performance curves may not exactly match the heat pumping capacity of the TE module observed during the tests. The reasons for these have already been studied and presented by Palacios, R. [47]. Moreover, the manufacturer’s graphs are based on the temperatures on the module faces and not on the heat sink temperatures. The two temperatures differ due to the contact resistance drop between the module and the heat sink. The values of ‘m’ and ‘b’ obtained through extrapolation of these graphs are not true in reality because the modules show quite an important dependency with the temperature. However, the simulation in the present work with respect to the TE equations could be safely assumed to be within acceptable limits of error.
Another overall comment for the mathematical simulation in Runs 1-3 is that the model over-estimates the closed system behavior of the experimental set up; i.e. the tank and the cold plate are predicted to have lower temperatures, whereas a slightly warmer temperature is calculated for the hot plate lump in each of the three runs. This led to analyzing the effect of further heat loss mechanisms in detail such as the heat loss through the stainless steel screws. The results have been reported in conjunction with the effect of screw bypass for the hot tank charging tests later in the section.

Figure 15: $Q_c$ vs $\Delta T$ as predicted by the mathematical model.
6.7 Mathematical model for hot tank charging transient tests

The transient behavior observed for the hot tank charging tests (Runs 4-6) are graphed in Fig. 16-18. From the results obtained, we note that when 300 W of power is supplied to the HX system, the tank water temperature as well as the heat plate temperature rises by about 45°C in a time span of over 10 hours experimentally. The model predicts nearly the same behavior. The use of 3 TEs at 300W power is predicted to have a steady state temperature of 63.6°C for the tank water, and the use of higher number of TE modules (5) at the same power is observed to further increase the steady state temperature of the sensible storage water tank by another 4.1°C. On the contrary, the use of 3 TE modules at 180 W instead of 300 W power results in lower steady state final tank temperature (by 11.5 °C); a behavior that is also observed experimentally by comparing Run 5 and Run 6 of Kazmierczak et. al [1].

![Figure 16: Mathematical model to experimental data comparison for Run 4 for tank water, top aluminum plate and bottom aluminum plate temperature history](image-url)
6.8 Time scale comparison

Examining the surface temperature of the hot plate shows that this temperature is always higher than the tank water temperature both theoretically and experimentally. Concurrently, it is also observed that the theoretical prediction for the hot aluminum plate temperature is close to the measured values. As far as the bottom circuit is concerned, the aluminum plate is initially cooled by extraction of energy by the TE modules but the bulk inlet temperature of the water is maintained constant by the isothermal bath. Each aluminum plate, treated as a single mass, has a system time constant (defined by $\frac{\rho V c_p}{h A}$) that is about 40 times shorter than that of the tank lump, i.e. $\frac{(\rho V c_p)_w}{(\rho V c_p)_al} = 38.76$ with $\overline{(h A)}$ for the tank and plates being the same. This explains the reason why the aluminum plate reaches steady state in about 15 minutes, whereas the tank water which takes 10 hours to do so.
6.9 Effect of heat bypass

The thermal bypass from the hot to cold aluminum plates through the two additional modes, namely (1) stainless steel screws and (2) air gap was studied to determine its impact on system performances. The magnitude of their effect was studied in detail for both cold tank charging tests and the hot tank charging mode, by numerically solving the equations (7)-(9), first without the air gap, and then without both screws and the air gap. Figure 19(a) and 19(b) compare simulation cases of 3 TEa running at 180 W power with and without these additional heat loss paths for both heating and cooling to the experimental data. Here we see that the absence of air gap slightly improves the charging performance of the tank. For the cooling test, a lower final steady state temperature is predicted than what was simulated in the original run. As far as the heating is concerned, again note that a fine improvement in the system performance could result if the stagnant air gap could be insulated. The value of the heat bypass through the air gap has been
calculated to be approximately 5 – 10% of the total power input for various tests conducted.

In contrast, note the extremely degraded performance that the uses of metal screws cause during the experimental tests. Without the use of screws and by perfect insulation in the air gap, the model shows that the 5 gallon water reservoir could be cooled to about 10°C lower and heated by about 14°C higher with the same 180W power supply, a tremendous improvement, in fact even better performance than the system with screws at 300 W. Hence a tremendous amount of electrical power can be saved if the thermal bypass between the plates can be eliminated.

At this juncture, it is worth to note that the system greatly simplifies from a mathematical point of view as well upon absence of these additional heat bypass modes. Moreover, an analytical solution could be obtained for the TES tank temperature history if the thermal storage in the two water channel aluminum plates are also neglected. A discussion of this analytical solution and its comparison to the above mentioned numerical model without heat bypass has been presented herewith.
6.10 Code validation

The simulation results of the developed three-lumped parameter computer codes are compared with analytical solutions of a simpler model. This model consists of a single lump in the form of the thermal storage tank (i.e. the heat storage terms in the plates neglected) and the heat bypasses through the screws and the air/gap have not been considered. For such a case, the governing equations (7) – (9) reduce to the following, with the equation for $q_c$ remaining unchanged.

\[
\begin{align*}
(\bar{U} A)_w (T_{\text{air}} - T_T) - (hA)_{cc} (T_T - T_{\text{alum,top}}) &= (\rho V c_p)_w \frac{dT_T}{dt} \\
(hA)_{cc} (T_T - T_{\text{alum,top}}) + (U A)_p (T_{\text{air}} - T_{\text{alum,top}}) - nq_c &= 0 \\
nq_c + P - (hA)_{hc} (T_{\text{alum,hot}} - T_{\text{bath}}) - (U A)_p (T_{\text{alum,hot}} - T_{\text{air}}) &= 0
\end{align*}
\]
The temperature history of the tank \( T_T(t) \), can then be easily obtained by solving the first order linear differential equation, yielding

\[
T_T(t) = \frac{y}{x} + (T_T(0) - \frac{y}{x}) e^{-(xt)}
\]  

(16)

where for Runs 1-3,

\[
x = \frac{1}{(\rho V c_p)_w} \left[ (\hat{h}A)_{cc} + (\hat{h}A)_w - \frac{(mn + (\hat{h}A)_{hc} + (\hat{h}A)_p) (\hat{h}A)_{cc}^2}{(mn + (\hat{h}A)_{hc} + (\hat{h}A)_p)^3} \right]
\]

(17)

\[
y = \frac{1}{(\rho V c_p)_w} \left[ (\hat{h}A)_w T_{air} + \frac{(mn + (\hat{h}A)_{hc} + (\hat{h}A)_p) (\hat{h}A)_w T_{air} - bn}{(mn + (\hat{h}A)_{hc} + (\hat{h}A)_p)} + \frac{mn (bn + P + (\hat{h}A)_{hc} T_{bath} + (\hat{h}A)_p T_{air})}{(mn + (\hat{h}A)_{hc} + (\hat{h}A)_p)^{1/2}} \right]
\]

(18)

As noted in our model, we see here that \( T_T(t) \) is an exponential function of time. Note also that the temperatures of the two aluminum plates, are clearly linear functions of the tank temperature \( T_T(t) \) (as can be understood from equations (14) and (15)). These could also be explicitly expressed as functions of time and are expected to exhibit nearly exponential behaviors (Figure 20a and b). From an energy balance point of view the heat pumping rate \( q_c \), and hence the energy input \( q_h \) to the hot plate (for chilled water runs) decrease with time. This causes \( T_h \) and \( T_c \) to reduce exponentially, although the drop in \( T_h \) is with an extremely gentle slope.

To conclude, the temperatures of the three thermal masses obtained from this analytical solution have been compared to our numerical model, but without heat bypass. This comparison carried out for one set of heating and cooling cases yielded very tight (close) matching of all temperature histories; in fact each lump temperature history obtained from the analytical solution nearly superposed on that predicted by the numerical model without the heat bypass. This means that the heat storage in the two plate lumps is negligible and do not cause any delay in the response time towards steady state. Nevertheless on closer inspection, a very small shift of the analytical solution curves towards the left can be noted which is an indication of the heat storage in the plates.
Fig 20: Comparison of analytical solution of the simplified system to the 3 lump numerical model without additional heat bypass modes: (a) 3 TE cooling test at 180W. (b) 3 TE heating test at 180 W.
6.11 Additional simulation

Fig 21a: Additional runs to predict temperature histories when run with 10 TEs at 1000 W power for the cooling test.

Here now, the successfully validated model is now used to predict the temperature distribution of the TE-HX system for a larger application. It is desired that this system, powered by PV panels in a solar house, would supply 10 TEs with a total power of 1000 W. For such a system, the estimated current per TE module is same as Run 1 for the cooling test simulation and Run 4 for the heating test simulation yielding same corresponding ‘m’ and ‘b’ values as shown in Table 3. With \( q_c \) calculated using these numbers, and using the same initial conditions as in Run 1 for cooling and Run 4 for heating, the model predicts that the temperature history of the tank water and plates would be as shown in Fig. 21(a) and 21(b), respectively. The top figure shows, at this higher power, that the chilled water would nearly reach the freezing point at steady state. The water leaving the heat sink, due to much larger heat dissipation, would be preheated to above 40°C as shown, and could be cooled by the water fountain. At the same time,
the heating test shows a high probability of the water being past its boiling temperature. Observe that in both the tests, there is a significant $\Delta T$ between the plates.

![Graph showing temperature over time](image)

**Fig 21b:** Additional runs to predict temperature histories when run with 10 TEs at 1000 W power for heating test.
Chapter 7

Conclusions

7.1 Broad-Based Findings

A thermoelectric heat exchanger with an array of three and five TE heat pump modules has been studied experimentally in the laboratory and the data collected shows the performance characteristics of the basic system component operating under three different modes. The results highlight the potential use of thermoelectric modules in hydronic heating and cooling systems for home use whereas the TEs can be powered directly using solar energy, a plentiful source of renewable energy that is nonpolluting and that does not contribute to global warming. The major results of the laboratory tests are briefly summarized below.

1. The experimental set up was successful in sensibly charging a small cold and hot water tank for two given power inputs and flow rate within a reasonable amount of time.
2. The system is found to have slightly higher values of COP when 5 TEs are used than for 3 TEs due, presumably, to less localized heating/cooling.
3. The flow rate strongly effects the system performance with higher flow rate reducing the heat sink resistance and the temperature difference across the TE module and allowing them to operate more efficiently (i.e. higher COP).
4. As the power required for this unit to operate is fairly low, the objective of using PV panels to supply power to the TEs can be conveniently and realistically accomplished.
5. The experimental validation of the sensible tank charging process model is successfully accomplished using a three-lumped parameter model using the presented data set.
7.2 Future Direction

In addition to running the given system more realistically (i.e. outside the lab environment and directly on solar PV panels) as depicted in Figure 22a, and developing a transient charging mathematical model, ongoing work in progress is to develop a comprehensive developing internal convective heat transfer model in the TE-HX channel component and also the 3-D heat conduction in the plates. Note that only a single test section utilizing a small number of TE modules was studied in the lab experiments (and with each TE not powered up to their maximum capacity). Future plans are to greatly scale-up the system performance by: 1) running each TE at their higher (and optimum) current; 2) increasing the number of TEs in the test section from 5 modules to 10 modules and; 3) to increase the number of test section units from 1 to 4 arranged in a series-parallel combination so as to increase the total size of the system capacity to 1 ton and more in order to support small residential home cooling or heating needs. For example, to achieve 3.5 KW (1 Ton) cooling, would require a total number of about 40 TEs modules, i.e. 40 units x 90 W/unit = 3600 W ~ 1 Ton (where 90 Watts/unit is for a 20-30°C temperature difference across the hot and cold surfaces as shown in Figure 4b.) As depicted in Figure 22b, the 40 TE modules, all working together, would be divided into 4 test sections, with each test section having 10 TEs each. The four test sections would be arranged in two parallel paths, with each parallel path having two test sections in series, thus doubling the system flow rate and temperature rise as compared to the performance of one test section alone. In addition to this, the cold (or hot) water storage volume would increase from the small 5 gallons plastic laboratory tank to a commercially purchased 139 gallon well/insulated water tank resulting in an additional 1.75 ton-hr (2.21x10^4 kJ) of cold thermal energy storage capacity when fully charged assuming an allowable 10°C working ΔT temperature change (see Fig. 22b tables for greater amount of thermal energy storage capacity for larger sized tanks).

A simple sketch of the proposed home cooling/heating system using the TE system with multiple TE-HX units is depicted in Figure 23 (note only one TE-HX unit is drawn for simplicity). As shown, the TE-HX system is powered by DC current from the solar panels to produce chilled building water in the summer cooling mode operation (Figure
**Figure 22:** TE – HX systems: (a) Laboratory experiments operating directly on solar power; (b) Scale-up equivalent 1 ton air-conditioning system with sensible TES chilled water tank (heat sink loop not shown).

<table>
<thead>
<tr>
<th>Gallons</th>
<th>Ton-hr</th>
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<td>15.07</td>
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</table>

ΔT_{allowable} = 80-60 = 20°C

ΔT_{allowable} = 5-15 = 10°C
Figure 23: Sketch of proposed home cooling/heating system using thermoelectrics with building supply/drain water as possible heat sink/source: (a) Summer cooling mode; (b) Winter heating mode.
23a) and the heat sink would be the domestic building water supply line (supplying both normal cold and hot tap water), and when building water supply is not required, then the hot side heat sink will be cooled via an open evaporative type cooling pond (shown as the water fountain) in the bypass leg install around the TE-HX unit, to ensure continuous TE cooling operation with nearly constant heat sink water temperature. For the winter heating (Figure 23b) operation, on the other hand, instead of using the building supply water, ideally the heat source would come from the grey water discharge stream, since it would be at a higher temperature and would make the TE system run more efficiently by minimizing the temperature differential across the TE modules, not to mention the overall total energy gain due to the grey water heat recovery. Continuous hot water heating is accomplished by the TE-HX unit, without using any building discharge flow whatsoever, by using a similar closed recirculation loop installed around the TE-HX unit but utilizing a thermal solar panel (acting as a heat source instead of the fountain heat dump) as the “bottoming” thermal cycle to maintain nearly constant heat source temperature. Of course a fairly smart controller and valve assemble hardware must be installed and programmed to implement this bypass flow control strategy for the summer cooling and winter heating modes to provide non-interrupted TE-HX operation during varying domestic water needs.
References


### Tables

#### (a) Hose, tank and plate Information

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$L_{hose}$</td>
<td>3.65</td>
<td>m</td>
</tr>
<tr>
<td>$D_{hose}$</td>
<td>0.01905</td>
<td>m</td>
</tr>
<tr>
<td>$A_{hose}$</td>
<td>0.2189</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$H_{tank}$</td>
<td>1.04</td>
<td>ft</td>
</tr>
<tr>
<td>$W_{tank}$</td>
<td>1</td>
<td>ft</td>
</tr>
<tr>
<td>$D_{tank}$</td>
<td>1</td>
<td>ft</td>
</tr>
<tr>
<td>$V_{tank}$</td>
<td>1.042</td>
<td>ft$^3$</td>
</tr>
<tr>
<td>$A_{tank}$</td>
<td>6.167</td>
<td>ft$^2$</td>
</tr>
<tr>
<td>$A_{tank}$</td>
<td>0.5729</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$V_{alum}$</td>
<td>$8.571 \times 10^{-4}$</td>
<td>m$^3$</td>
</tr>
</tbody>
</table>

#### (b) Bypass screw information

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
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<tbody>
<tr>
<td>$L$</td>
<td>0.0032</td>
<td>m</td>
</tr>
<tr>
<td>$k$</td>
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<td>W/mK</td>
</tr>
<tr>
<td>$A_c$</td>
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<td>m$^2$</td>
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#### (c) Insulation information

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
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<td>$k_{ins}$</td>
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<tr>
<td>$L_{ins}$</td>
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<td>cm</td>
</tr>
<tr>
<td>$A_p$</td>
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<td>m$^2$</td>
</tr>
</tbody>
</table>

Table 1: Physical dimensions and thermophysical properties of test loop
### a) Cold Tank Charging Tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Number of TEs</th>
<th>Flow Rate [GPM]</th>
<th>Avg. Power In [W]</th>
<th>Tank [°C]</th>
<th>SS, T&lt;sub&gt;b,i&lt;/sub&gt; [°C]</th>
<th>Center ∆T&lt;sub&gt;s&lt;/sub&gt; (T&lt;sub&gt;h&lt;/sub&gt;-T&lt;sub&gt;c&lt;/sub&gt;)</th>
<th>COP at SS</th>
<th>First Law Error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Initial</td>
<td>Final ΔT</td>
<td>Cold</td>
<td>Hot</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1.8</td>
<td>297.5</td>
<td>23.1 12.5 10.6</td>
<td>12.9 23.5</td>
<td>18</td>
<td>0.36 1.28</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>2</td>
<td>1.8</td>
<td>297.0</td>
<td>23.8 11.1 12.7</td>
<td>11.7 23.6</td>
<td>17.9</td>
<td>0.54 1.61</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1.8</td>
<td>178.0</td>
<td>24.5 15.0 9.5</td>
<td>15.3 24.5</td>
<td>14.5</td>
<td>0.30 1.35</td>
</tr>
</tbody>
</table>

### b) Hot Tank Charging Tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Number of TEs</th>
<th>Flow Rate [GPM]</th>
<th>Avg. Power In [W]</th>
<th>Tank [°C]</th>
<th>SS, T&lt;sub&gt;b,i&lt;/sub&gt; [°C]</th>
<th>Center ∆T&lt;sub&gt;s&lt;/sub&gt; (T&lt;sub&gt;h&lt;/sub&gt;-T&lt;sub&gt;c&lt;/sub&gt;)</th>
<th>COP at SS</th>
<th>First Law Error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Initial</td>
<td>Final ΔT</td>
<td>Cold</td>
<td>Hot</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1.8</td>
<td>251.5</td>
<td>24.5 67.5 43.0</td>
<td>24.6 67.2</td>
<td>42.4</td>
<td>0.21 1.16</td>
</tr>
<tr>
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<td>5</td>
<td>2</td>
<td>1.8</td>
<td>256.4</td>
<td>24.2 69.3 45.1</td>
<td>24.0 69.1</td>
<td>44.2</td>
<td>0.41 1.38</td>
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<tr>
<td>6</td>
<td>3</td>
<td>2</td>
<td>1.8</td>
<td>154.7</td>
<td>21.9 53.8 31.9</td>
<td>22.3 53.8</td>
<td>33.7</td>
<td>0.33 1.48</td>
</tr>
</tbody>
</table>

### c) Single Pass Test

<table>
<thead>
<tr>
<th>Test</th>
<th>Number of TEs</th>
<th>Flow Rate [GPM]</th>
<th>Avg. Power In [W]</th>
<th>Cold [°C]</th>
<th>Hot [°C]</th>
<th>Center ∆T&lt;sub&gt;s&lt;/sub&gt; (T&lt;sub&gt;h&lt;/sub&gt;-T&lt;sub&gt;c&lt;/sub&gt;)</th>
<th>COP at SS</th>
<th>First Law Error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>T&lt;sub&gt;b,i&lt;/sub&gt;</td>
<td>T&lt;sub&gt;b,o&lt;/sub&gt; ΔT</td>
<td>T&lt;sub&gt;b,i&lt;/sub&gt;</td>
<td>T&lt;sub&gt;b,o&lt;/sub&gt; ΔT</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>300.8</td>
<td>8.6 8.1 0.5</td>
<td>8.7 9.8 1.2</td>
<td>12.4</td>
<td>0.88 1.92</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>300.8</td>
<td>8.9 8.5 0.4</td>
<td>8.8 9.8 1</td>
<td>12.1</td>
<td>0.70 1.75</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>300.2</td>
<td>9.3 8.4 0.9</td>
<td>9.3 11.1 1.8</td>
<td>17.7</td>
<td>0.79 1.67</td>
</tr>
<tr>
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<td>3</td>
<td>1</td>
<td>1</td>
<td>301.0</td>
<td>9.4 8.7 0.7</td>
<td>9.3 11 1.7</td>
<td>16.3</td>
<td>0.61 1.49</td>
</tr>
</tbody>
</table>

*First law error % = \[ \frac{Q_k - (Q_f + P)}{Q_f + P} \times 100 \]*

Table 2 - Summary of experiments conducted for different operating conditions and select results.
Table 3: Slopes and y-intercepts of the interpolated performance curves for Runs 1 through 6 and the additional simulation for heating and cooling cases

<table>
<thead>
<tr>
<th>Run</th>
<th>n</th>
<th>P (Watts)</th>
<th>Current per TE (Amps)</th>
<th>T_h (°C)</th>
<th>m (W/K)</th>
<th>b (Watts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>295.8</td>
<td>5.66</td>
<td>27.3</td>
<td>-1.56011</td>
<td>104.8659</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>295.6</td>
<td>4.22</td>
<td>26.0</td>
<td>-1.40926</td>
<td>91.2660</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>176.6</td>
<td>4.40</td>
<td>25.9</td>
<td>-1.4263</td>
<td>92.7523</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>302.5</td>
<td>4.70</td>
<td>68.8</td>
<td>-1.6792</td>
<td>128.9616</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>311.97</td>
<td>3.53</td>
<td>69.7</td>
<td>-1.6607</td>
<td>121.3044</td>
</tr>
<tr>
<td>6</td>
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<td>179.66</td>
<td>3.80</td>
<td>55.27</td>
<td>-1.5709</td>
<td>111.5243</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>1000</td>
<td>5.66</td>
<td>27.3</td>
<td>-1.56011</td>
<td>104.8659</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>1000</td>
<td>4.70</td>
<td>68.8</td>
<td>-1.6792</td>
<td>128.9616</td>
</tr>
</tbody>
</table>

Table 3: Slopes and y-intercepts of the interpolated performance curves for Runs 1 through 6 and the additional simulation for heating and cooling cases
APPENDIX A

Uncertainty Analysis

As most of the engineering studies are established with single sample experiments, the theory has been developed [Kline and McClintock, 1953], advanced and presented in elaborate way to evaluate the uncertainty associated with these kinds of experimentation.

Uncertainty is attributed to the possible error that the obtained experimental results might have. The error sources are divided into two categories a) Bias or fixed error b) Precision or random error [Coleman and Steele, 1989]. The uncertainty is determined by the residual error after correction. As described by Moffat, 1988, the sources of error can be different based on the true value assumptions. Also, the results published should be within 95% of confidence level [Kim et al., 1993]. As suggested, the root square method is used to evaluate the overall uncertainty for bias and precision errors.

The uncertainty analysis on the Coefficient of Performance is presented as following. The value of bias limit is considered as per calibration or manufacturer’s description and the precision limit is determined as per half the value of the least count of the instrument (Table A1).

Bias and Precision limits for different sources of error:

The total uncertainty of a measured value can be evaluated as following:

\[
U = (B_U^2 + P_U^2)^{1/2}
\]

If the relationship between the experimental results, \( r \), and measured quantities, \( a_i \), is of the form \( r = f(a_1, a_2, a_3, a_4 \ldots \ldots, a_n) \)

For single measurement experiments, the propagation of uncertainty to the calculated results can be determined by the following [Kline and McClintock, 1953]:

\[
(\delta r)^2 = \sum_{i=1}^{n} \left[ \frac{\partial f}{\partial a_i} \delta a_i \right]^2
\]

Where, \( \delta r \) represents the uncertainty of \( r \).
Table A1: Bias and precision errors in different measurements

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Measured Property</th>
<th>Instrument</th>
<th>Bias Error (B)</th>
<th>Precision Error (P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Time for flow rate (t)</td>
<td>Watch</td>
<td>0.5 sec/min</td>
<td>0.5 sec/min</td>
</tr>
<tr>
<td>2</td>
<td>Temperature (T)</td>
<td>Thermocouple Probe</td>
<td>0.5°C</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>Temperature (T)</td>
<td>Thermocouples</td>
<td>0.05°C</td>
<td>0.05°C</td>
</tr>
<tr>
<td>4</td>
<td>Current (I)</td>
<td>HP Power Supply</td>
<td>0.36%</td>
<td>0.01 A</td>
</tr>
<tr>
<td>5</td>
<td>Voltage (V)</td>
<td>HP Power Supply</td>
<td>0.025%</td>
<td>0.01 V</td>
</tr>
<tr>
<td>6</td>
<td>Specific heat (C_p)</td>
<td></td>
<td>0.5%</td>
<td>0</td>
</tr>
</tbody>
</table>

Uncertainty calculation for COP:

Bias Uncertainty:

\[
\left( \frac{F_{\text{COP}}}{\text{COP}} \right) = 1.036 \times 10^{-2} = 1.036\% 
\]

Precision Uncertainty:

The change in the bulk temperature is in the range of 0.3°C at cold side and 0.8°C at hot side.

\[
\left( \frac{F_{\text{COP}}}{\text{COP}} \right) = 0.99\% \quad \text{[for} (T_{b,c} - T_{b,h}) = 0.8] \\
= 23.59\% \quad \text{[for} (T_{b,c} - T_{b,h}) = 0.3]
\]

Total Uncertainty in COP

\[
\left( \frac{U_{\text{COP}}}{\text{COP}} \right)^2 = \left( \frac{F_{\text{COP}}}{\text{COP}} \right)^2 + \left( \frac{F_{\text{COP}}}{\text{COP}} \right)^2
\]

\[
\left( \frac{U_{\text{COP}}}{\text{COP}} \right) = 0.95\% \quad \text{[for} (T_{b,c} - T_{b,h}) = 0.8] \\
= 23.61\% \quad \text{[for} (T_{b,c} - T_{b,h}) = 0.3]
\]
APPENDIX B

Determination of internal convective resistance and overall UA values

Convective heat loss has been considered to be the primary mechanism of heat transfer in the three lumped model discussed in this paper. The values of convective resistances involved in the model were calculated from basic principles for both cold tank charging and the hot tank charging runs. Each of these runs consisted of internal and external flow parameters. The values of the internal flow convection resistances for the channels were estimated using an ‘h’ value obtained from well-known correlations. For this purpose, first the Reynolds number was determined to discern whether the flow is laminar or turbulent as shown below. Following that, the Prandtl number was calculated. The Reynolds number was found using a hydraulic diameter of .01143 meters.

\[ \text{Re}_D = \frac{m \cdot D_h}{\mu \cdot A_c} = \frac{0.126 \ kg/s \cdot 0.01143m}{1.12E-3 \ N \cdot s/m^2 \cdot 3.629E-4 \ m^2} = 3543.33 \]  

\[ \text{Pr} = \frac{C_p \cdot \mu}{k} = \frac{4180 \ J/kg \cdot K \cdot 1.12E-3 \ N \cdot s/m^2}{0.62 \ W/m \cdot K} = 7.55 \]  

This Reynolds Number indicates that the flow in the channel is transitioning into the turbulent regime. With this known, the Dittus-Boelter equation was used to approximate the Nusselt number.

\[ Nu_D = 0.023 \text{Re}_D^0.8 \cdot \text{Pr}^{3} = 0.023 \cdot 3543.33^{0.8} \cdot 7.55^{0.3} = 29.15 \]  

Finally, the Nusselt number was used to determine the convection coefficient of the water flowing through the channel.

\[ h = \frac{Nu_D \cdot k}{D_h} = \frac{29.15 \times 0.62 W/m \cdot K}{0.01143m} = 1582 W/m^2 \cdot K \]  

Similarly, this value was also determined for the bottom channel and the value of the bath side channel convection coefficient was obtained as 1453 W/m²-K. These ‘h’ values, and the calculated value of .04429 square meters for the internal surface area were used to
determine the \((\bar{h}A)_c\) and \((\bar{h}A)_h\) values reported in Table 4 and used in the numerical solutions.

For external natural convection air flow, a heat transfer coefficient of 20 W/m\(^2\)-K was assumed. Since the tank and aluminum plates were insulated on the outside by applying a layer of foam, the overall \((\bar{U}A)\) values are needed. The plate overall thermal resistance consisted of two resistances in parallel, namely the heat conduction through the insulation and the natural convection through air, as shown below.

\[
(\bar{U}A)_p = \frac{1}{L_{\text{ins}}} + \frac{1}{k_{\text{ins}}A_p hA_p}
\]

(23)

Finally, the tank resistance also consisted of these two resistances in parallel with the difference that the tank surface area was different from that of the plate.

\[
(\bar{U}A)_w = \frac{1}{L_{\text{ins}}} + \frac{1}{k_{\text{ins}}A_w hA_w}
\]

(24)

The \((\bar{U}A)\) values and the h values for Runs 1-3 are listed in Table [B1]. For Runs 4-6, the table remains the same, except that \((\bar{U}A)_c\) and \((\bar{U}A)_h\) is interchanged.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Numerical value (W/K)</th>
<th>Corresponding ‘h’ (W/m(^2)K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\bar{U}A)_w)</td>
<td>5.65</td>
<td>20</td>
</tr>
<tr>
<td>((\bar{h}A)_c)</td>
<td>70.07</td>
<td>1582</td>
</tr>
<tr>
<td>((\bar{h}A)_h)</td>
<td>64.35</td>
<td>1453</td>
</tr>
<tr>
<td>((\bar{U}A)_p)</td>
<td>0.37</td>
<td>20</td>
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</tbody>
</table>