UNIVERSITY OF CINCINNATI

Date: 3/6/2008

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hereby submit this work as part of the requirements for the degree of:
Master of Science

in:
Aerospace Engineering

It is entitled:
The Design and Implementation of an Acoustic Flow Resistance Apparatus for Manufacturing Process Control.

This work and its defense approved by:

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The Design and Implementation of an Acoustic Flow Resistance Apparatus for Manufacturing Process Control

A dissertation submitted to the Graduate School of the University of Cincinnati in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

In the Department of Aerospace Engineering of the College of Engineering

By

Michael Perrino

Committee Chair: Dr. Asif Syed
Abstract:

This thesis presents the theory and application of flow resistance measurements in the development of acoustic liners. The principal focus of the thesis is on the design and development of a flow resistance measurement apparatus that is used in a factory environment to assist in the development and manufacturing process control of honeycomb, embedded with Mesh-Cap septa, used in double layer acoustic liners. The operating environment as well as stringent accuracy and reliability requirements require an apparatus that is ergonomic, user friendly, and robust.

The theoretical basis of the flow resistance measurement and its application to acoustic liners is described in detail. The apparatus hardware and software requirements are studied to assure the final design meets or exceeds all the design criteria. The statistics used in the measurement process and in process control theory are also described in detail.
Acknowledgements

I would like to thank Dr. Asif Syed for giving me the opportunity to work on this project and the guidance he has provided over the years. I would also like to thank the following: Earl Ayle and Hexcel Corporation for providing funding and assistance in constructing the apparatus in Casa Grande, Dr. Ephraim Gutmark and Dr. Jandro Abot for being members of my advisory committee, Curtis Fox for his expertise, and Fumitaka Ichihashi for writing the operation manual for the improvements he has made to the apparatus.
Nomenclature

Z  -  The impedance
f  -  Frequency
p  -  Acoustic pressure
u  -  Particle velocity
c  -  Speed of sound
ρ  -  Fluid density
R  -  Acoustic resistance
X  -  Acoustic reactance
i  -  Irrational component of a complex number
α  -  Absorption coefficient
Z_c  -  Impedance of the cavity
Z_{fs}  -  Impedance of the liner face sheet
f_{cut-on}  -  Cut-on frequency
D  -  Widest section of the honeycomb cavity
R  -  Complex coefficient of acoustic wave equation
L  -  Complex coefficient of acoustic wave equation
k  -  Acoustic wave number
R_f  -  Resistance of the porous sheet
m  -  Coefficient of mass reactance of the porous sheet
t  -  Thickness of the porous sheet
d  -  Hole diameter of the porous sheet
σ  -  Porosity of the porous sheet
<table>
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<td>A</td>
<td>Intercept</td>
</tr>
<tr>
<td>B</td>
<td>Slope</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Absolute coefficient of viscosity of air</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Discharge coefficient</td>
</tr>
<tr>
<td>$u_{rms}$</td>
<td>Root mean squared value of particle velocity</td>
</tr>
<tr>
<td>$p_{rms}$</td>
<td>Root mean squared value of pressure</td>
</tr>
<tr>
<td>$R_{rms}$</td>
<td>Root mean squared value of resistance</td>
</tr>
<tr>
<td>$R(105)$</td>
<td>Resistance at 105cm/s</td>
</tr>
<tr>
<td>NLF</td>
<td>Non-linearity factor</td>
</tr>
<tr>
<td>$d_t$</td>
<td>Thread diameter</td>
</tr>
<tr>
<td>$s_{se}$</td>
<td>Effective porosity</td>
</tr>
<tr>
<td>S</td>
<td>Spacing between the threads in the warp direction</td>
</tr>
<tr>
<td>$\Delta P_s$</td>
<td>Pressure drop across porous sheet</td>
</tr>
<tr>
<td>$U$</td>
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<tr>
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<td>Sample area</td>
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<td>$p_0$</td>
<td>Reference pressure</td>
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<td>$T_0$</td>
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<tr>
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<td>$T_f$</td>
<td>Static temperature just upstream of flow meter</td>
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<tr>
<td>$p_s$</td>
<td>Static pressure just upstream of porous sheet</td>
</tr>
<tr>
<td>$T_{amb}$</td>
<td>Ambient temperature</td>
</tr>
<tr>
<td>$\Delta P_f$</td>
<td>Pressure drop across flow meter</td>
</tr>
</tbody>
</table>
\(B_f\) - Calibration constant of flow meter

\(C_f\) - Calibration constant of flow meter

\(\bar{R}(105)\) - Average resistance of standard sample for a flow velocity of 105cm/s

\(mR_i\) - Fluctuation in the standard sample resistance

\(\overline{mR}\) - Average fluctuation in the standard sample measurement

UNPL - Upper natural process limit

LNPL - Lower natural process limit

URL - Upper range limit

\(\bar{\rho}\) - Average static pressure

\(p_i\) - Discrete frequency components of the static pressure

\(p_r\) - Random perturbations in pressure

\(u\{Z\}\) - Uncertainty of function “Z”

M - Mach number
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1. Introduction

1. 1. Fan Noise Suppression

1. 1. 1. Noise regulation

Noise generation by commercial engines during takeoff and landing is of great concern as the population density around airports and the number of commercial aircraft traffic increase. The two main sources of noise are:

   a. Jet noise created by sharp pressure gradients aft of the engine due to turbulent mixing of the high speed jet with static ambient air. Jet noise is generated outside the engine nacelle.

   b. Fan noise generated by fan blades due to unsteady lift forces on the fan blades and on the outlet guide vanes. Fan noise is generated within the engine nacelle.

Acoustic liners are the primary method for reducing fan noise. *Figure 1* shows the sources of engine noise for low and high bypass ratio engines.
The progression towards higher bypass ratios to improve engine efficiency has resulted in the fan noise becoming a larger part of the overall engine noise, especially at the Approach power. Acoustic liners are placed on the walls of the inlet and the fan-exhaust ducts to attenuate the noise generated by the fan. Figure 2 shows the regions where acoustic liners are located on a GE CF6-80C2 Propulsion System.

Figure 1: Sources of Engine Noise for Low and High Bypass Ratio Engines
Figure 2: Locations of Acoustic Liners on a GE CF6-80C2 Propulsion System
1.1.2. Acoustic Liners

The acoustically absorptive liners used in the nacelle of aircraft engines are based on the Helmholtz resonator principle. They are of a light-weight sandwich construction, consisting of honeycomb layers and thin porous sheet materials. The two basic types of acoustic liners used to suppress fan noise are single layer liners and double layer liners. Single layer liners provide great sound attenuation over a relatively short frequency range while double layer liners have a much wider sound absorption frequency range. Figure 3 is a schematic diagram of a single layer liner, Figure 4 is a diagram of a double layer liner, and Figure 5 is an image of a double layer liner.

![Diagram of a Single-layer (1-DoF) Acoustic Liner](image)

*Figure 3: Diagram of a Single-layer (1-DoF) Acoustic Liner*
Figure 4: Diagram of a Double-layer (2-DoF) Acoustic Liner

Figure 5: Image of a Double-layer (2-DoF) Acoustic Liner
1.2. Basis for Flow Resistance Measurement

It has been well established that the acoustic resistance of porous sheet materials is correlated with their flow resistance characteristics \[^{1, 2, 3}\]. Therefore, it is normal to specify porous sheet materials in terms of their flow resistance parameters after all manufacturing processes are completed. Thus an apparatus to measure the flow resistance is essential to developing the manufacturing processes for acoustic liners.
1.2.1. Necessity for a Flow Resistance Apparatus

The University of Cincinnati has a custom built flow resistance apparatus for research in acoustic liners. There is also a need for a flow resistance apparatus at the Hexcel Corporation’s honeycomb manufacturing plant in Casa Grande, Arizona for the purpose of manufacturing process control for acoustic liners. The design and development of this apparatus is the principal part of this research thesis. The software and instrumentation as well as the measurement specific hardware are based on the UC flow resistance apparatus but the specific needs of Hexcel are also taken into account.

This flow resistance test stand measures the flow resistance parameters of porous materials used in the construction of acoustic liners of aircraft engines. This apparatus should provide the capability for quality control in the manufacture of Hexcel honeycomb with embedded Mesh-Cap™ septua \(^\text{[4]}\). Figure 6 is an image of honeycomb with embedded Mesh-Cap™ septua. These honeycomb panels, based on the new Acousti-Cap™ technology, will be used in the fabrication of double layer acoustic liners.

![Image of Honeycomb with Embedded Mesh-Cap™ Septua](image)

*Figure 6: Image of Honeycomb with Embedded Mesh-Cap™ Septua*
1. 2. 2. Apparatus Requirements

The flow resistance test stand must meet the testing requirements of Hexcel Corporation and must be capable of reliable and accurate operation in a factory environment by personnel who have little knowledge of the technical details of the measurement methodology. To assure reliability, the apparatus must be capable of detecting any deviation in or loss of accuracy in the measurement. If a loss of measurement accuracy occurs, the measurement apparatus should be capable of diagnosing the cause of the deviation so that corrective actions may be taken.
1.3. Thesis Overview

This thesis describes the design of Hexcel Corporation’s flow resistance test stand beginning with the basic theory of single and double layer acoustic liners and how acoustic resistance measurement fit into the design process of these types of liners. The concept of the discharge coefficient of the perforated face sheet of an acoustic liner and its correlation with the flow resistance and the porosity is examined. The measurement hardware considerations are specified and the design is described in detail. The software process maps are presented and each step of the measurement is described in detail. The theory of statistical process control maps and how they are quantified to alert the user, if the measurements are out of control, is described.
2. Theoretical Notes

2.1. Acoustic Liner Theory

2.1.1. Single Layer Acoustic Liners

The most basic type of acoustic liner is the single layer or single-degree-of-freedom (1-DoF) acoustic liner shown in Figure 3. This type of acoustic liner consists of a porous face sheet and a back sheet that is impervious to sound. They are separated by a honeycomb core of specified depth to provide the required tuning characteristics. Figure 7 is a model of the single layer acoustic liner with acoustic waves.

![Figure 7: Single Layer Acoustic Liner Model](image)

The physical effect this type of acoustic liner has on sound is described in terms of the acoustic impedance. The acoustic impedance is a measure of the acoustic liner’s impact on the sound waves. It is a complex value represented as $Z(f) = \frac{-p(f)}{\rho c u(f)}$ (1)
where “Z” is the impedance, “p” is the acoustic pressure, “u” is the particle velocity and “f” is
the frequency of the acoustic waves. The impedance is made non-dimensional by dividing by the
speed of sound “c” and the fluid density “ρ”. The impedance is a complex value because the
pressure and the acoustic particle velocity are functions of frequency and they are generally out
of phase. The normalized impedance can be written in terms of its real and imaginary
components

\[ Z(f) = R(f) + i \cdot X(f) \]  

(2)

where “R” is defined the normalized acoustic resistance, “X” is defined as the normalized
acoustic reactance of the liner, and “i” is the irrational component of a complex number (i² = -1).

Impedance of an acoustic liner is used to calculate the absorption coefficient of the acoustic liner.
The absorption coefficient is defined as the ratio between the wave energy absorbed by the liner
and the incident wave energy. The absorption coefficient can be written in terms of the liner
resistance and reactance

\[ \alpha = \frac{4R(f)}{(1 + R(f))^2 + X(f)^2} \]  

(3)

where “α” is the absorption coefficient.

Using this model, the impedance on the right side of the porous face sheet in this figure can be
written as the summation of the impedance of the cavity between the liner face sheet and the
impervious back sheet and the change in impedance caused by the liner face sheet \[\text{[2]}\]

\[ Z(f) = Z_{fs}(f) + Z_{C}(f) \]  

(4)
where “Z_c” is the impedance of the cavity and “Z_{fs}” is the change in the impedance across the porous sheet.

Figure 8 is a 1-Dimensional model of the cavity with plane wave propagation. The plane wave assumption is valid if the width of the cavity is much smaller than its depth, h. Moreover, the width of the liner cavity must be small compared to the acoustic wavelength. The frequency at which an acoustic mode begins to propagate is called the cut-on frequency. The cut-on frequency for the first non-planar mode can be calculated

\[ f_{\text{cut-on}} = \frac{c}{2D} \]  

where “D” is the width of the widest part of the cavity. The cells of the honeycomb with embedded Mesh-Cap\textsuperscript{TM} septua are 1cm wide and the cut-on frequency for the first non-planar mode through the cells at 70ºF is approximately 17000Hz. The frequencies of interest generally do not exceed 6000Hz; therefore, the plane wave assumption is valid.

There are instances when the liner cavity width is larger than the cavity depth. When this occurs, the sound pressure distribution across the entrance of the acoustic liner must be specified in order to solve a multi-modal sound propagation model. There is currently no way to accurately measure sound pressure level throughout the engine duct; therefore, the sound pressure level throughout the fan duct is assumed to be constant. When the sound pressure level across the entrance of a single acoustic liner is constant, the plane wave is the only mode propagating through the cavity and the plane wave assumption is valid.
The pressure and particle velocity at any location in the cavity is given by the 1-dimensional wave propagation equations \[^5\]

\[
p(x, f) = R e^{-ikx} + Le^{ikx}
\]

(6)

&

\[
- \rho cu(x, f) = R e^{-ikx} - Le^{ikx}
\]

(7)

where “R” and “L” are complex coefficients and “k” is the acoustic wave number given by

\[
k = \frac{2 \pi f}{c}
\]

(8)

The particle velocity is 0 at the impervious back sheet (x=0) for all frequencies. Applying this boundary condition to Equation 7 yields

\[
R = L
\]

(9)

for any location in the cavity. Substituting Equations 6, 7, and 9 into Equation 1 yields
The impedance of the cavity is

\[ Z_c(x, f) = -\frac{e^{-ikx} + e^{ikx}}{e^{-ikx} - e^{ikx}} = -\text{Cot}(kx) \quad (10) \]

The acoustic impedance of the cavity can be calculated if the geometry of the cavity and the fluid properties are known. The acoustic impedance of the liner face sheet is more complex to calculate \[^2\]

\[ Z_{fs} = R_{fs} + imk \quad (12) \]

where “\( R_{fs} \)” is the normalized resistance of the face sheet and “\( m \)” is the coefficient of mass reactance of the face sheet. The mass reactance of a perforated sheet is \[^2\]

\[ m = \frac{t + \varepsilon d}{\sigma} \quad (13) \]

where “\( t \)” is the thickness of the perforated sheet, “\( d \)” is the hole diameter, “\( \sigma \)” is the porosity of the perforated sheet, and “\( \varepsilon \)” can be approximated as \[^2\]

\[ \varepsilon = 0.85 - 0.595\sqrt{\sigma} \quad (14) \]

Figure 9 is a general diagram of a perforated sheet.
The porosity (open area ratio) of a perforated sheet is

\[ \sigma = \frac{\pi r^2}{b s} \quad (15) \]

The resistance of the porous sheet is a function of its geometrical details, the fluid particle velocity traveling through it, the grazing flow Mach number, and the boundary layer thickness associated with the grazing flow [6]. Figure 10 is a diagram of air particles moving through a porous sheet due to incident sound waves and no grazing flow.
The resistance of the porous sheet with no grazing flow can be linearly approximated

\[ R_{fs}(u) = A + Bu \]  

(17)

or written as

\[ \frac{\Delta p}{\rho c} = Au + Bu^2 \]  

(18)

where “\(\Delta p\)” is the pressure drop across the sample at a given time, “\(u\)” is the particle velocity at a given time, “\(A\)” is the linear viscous resistance component and is a measure of the frictional losses caused by the air traveling through the holes. “\(B\)” is the nonlinear turbulent jet component.
These approximations are adequate for incompressible flow at low Reynolds numbers. Figure 11 shows the resistance vs. particle velocity.

![Figure 11: Resistance vs. Particle Velocity](image)

The particle velocity can be either positive or negative but the resistance term is always positive. The linear approximation is shown in blue and the resistance for a given sound wave is shown in red. “A” and “B” are constants for a particular porous sheet. The intercept, “A,” for a perforated sheet can be calculated \[^2\]

\[
A = \frac{32 \mu t}{\rho c \sigma C_D d^2} \quad (19)
\]

where “\(\mu\)” is the absolute coefficient of viscosity of air and “\(C_D\)” is the orifice discharge coefficient.

The slope, “B,” for a perforated sheet can be written as \[^2\]

\[
B = \frac{1}{2 c C_D^2} \left( \frac{1 - \sigma^2}{\sigma^2} \right) \quad (20)
\]
The particle velocity through the porous sheet fluctuates over time but the root mean squared value of overall acoustic particle velocity is constant with respect to time for given incident sound waves and liner geometry. The root mean square (rms) value of the overall acoustic particle velocity depends on the incident sound pressure level (SPL) and the impedance of the porous sheet \[^2\]

\[ u_{\text{rms}} = \sqrt{\sum u_i^2} \]  

(21)

where “\(u_{\text{rms}}\)” is the root mean square velocity of the \(i\)th frequency band summed over all frequency bands. The root mean square velocity of the \(i\)th frequency band is determined from the impedance relationship \[^2\]

\[ u_i = \frac{(p_{\text{rms}})_i}{\rho c Z_i} \]  

(22)

Where “\((p_{\text{rms}})_i\)” is the root mean squared value of acoustic pressure at the \(i\)th frequency band and “\(Z_i\)” is the liner impedance at the \(i\)th frequency band. \(Figure 12\) is the \(R_{\text{rms}}\) vs. \(u_{\text{rms}}\).
Figure 12: $R_{\text{rms}}$ vs. $u_{\text{rms}}$ for a Particular Porous Sheet and Incident SPL

The $R_{\text{rms}}$ at a particular impedance and incident SPL is indicated by the red dot and the blue line is the $R_{\text{rms}}$ at several SPLs given by

$$R_{\text{rms}} = A + B u_{\text{rms}}$$

$u_{\text{rms}}$ is independent of frequency since it is the rms value of acoustic particle velocity integrated over all frequencies. $R_{\text{rms}}$ has a very small dependency on frequency; however, the DC flow component is orders of magnitude larger. Note that in Equation 23 the intercept is dimensionless and the slope has the dimension of $cm^{-1}$. 
2. 1. 2. Acoustic Liner Design Parameters

The actual SPLs inside an engine are hard to measure with accuracy without drilling holes into the engine walls for acoustic pressure transducers. Therefore, estimates are made based on limited amount of measured data. These estimated values of sound pressure level are only rough approximations of the actual values experienced in the engine ducts. For the porous sheet materials used as face sheets or septa, the linear viscous resistance component and the nonlinear turbulent jet component are used to compute the resistance at 105cm/s (\( R(105) \) cgs Rayl units) and the Non-Linearity Factor (NLF). These are the main parameters used in specifying the acoustic liner design. \( R(105) \) in cgs Rayl units is given by \cite{2}

\[
R(105) = \rho c (A + B105) \quad \text{(24)}
\]

The NLF(200:20) is given by \cite{2}

\[
NLF(200 : 20) = \frac{A + B200}{A + B20} \quad \text{(25)}
\]

The NLF(200:20) ranges between 1 and 10 and is the ratio of the resistance of a porous sheet at a high particle velocity (200cm/s) to the resistance at a low particle velocity (20cm/s). The NLF is a measure of how linear the relationship between the pressure drop across the porous sheet and the particle velocity through it is. The NLF is small when the nonlinear turbulent jet component is small compared to the linear viscous resistance component of the resistance equation. Figure 13 shows the resistance curves of perforated sheets and acoustic mesh sheets.
Figure 13: Resistance Curves of Various Porous Sheets

The data for the mesh sheets show that their nonlinear turbulent jet components are of the same order of magnitude as for the perforated sheets; however, their linear viscous resistance components are substantially larger. This results in low NLFs for the mesh sheets. A NLF of 1 is desirable for engine design purposes due to the fact that the resistance would no longer be a function of particle velocity and the liner could be designed without knowing accurately the SPL (dB) in the fan duct. This can only be achieved if the passages through the liner are designed to completely eliminate the rotational losses. This is not practical. Therefore, the resistance is always a function of particle velocity. The so-called linear materials, such as the fine mesh sheets of Figure 13, are preferred because of low values of NLF. Table 1 is a list of the NLFs for the perforated sheets and Table 2 is a list of the NLFs for the mesh sheets shown in Figure 13.
### Table 1: List of NLFs for Various Perforated Sheets

<table>
<thead>
<tr>
<th>Hole Diameter (cm)</th>
<th>$\sigma$ (%)</th>
<th>NLF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>38%</td>
<td>9.97</td>
</tr>
<tr>
<td>0.32</td>
<td>6%</td>
<td>9.90</td>
</tr>
<tr>
<td>0.1</td>
<td>13%</td>
<td>7.55</td>
</tr>
<tr>
<td>0.32</td>
<td>11%</td>
<td>2.17</td>
</tr>
</tbody>
</table>

### Table 2: List of NLFs for Various Mesh Sheets

<table>
<thead>
<tr>
<th>Mesh Sheets</th>
<th>NLF</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 Rayls</td>
<td>1.39</td>
</tr>
<tr>
<td>80 Rayls</td>
<td>1.40</td>
</tr>
<tr>
<td>100 Rayls</td>
<td>1.45</td>
</tr>
</tbody>
</table>
2.1.3. Model of the Acoustic Mesh Material

Flow resistance measurements are useful for characterizing the flow resistance of porous sheets when analytical models are inaccurate or unavailable \cite{2}. There are currently no analytical models available for the Acousti-Cap\textsuperscript{TM} technology; however, Rice has created an analytical model for metallic wire mesh. The non-dimensional linear resistance of mesh sheets has been obtained \cite{7}

$$ A_s = \frac{C \mu (1 - \sqrt{\sigma_{se}})^2}{\rho c d_t} \quad (26) $$

Where “\(d_t\)” is the mesh thread diameter, “\(C\)” is an empirical constant approximately equal to 272.6 when the thread diameter is given in centimeters, and “\(\sigma_{se}\)” is the effective porosity of the mesh sheet. The effective porosity of the mesh sheet is defined as \cite{7}

$$ \sigma_{se} = \sigma_s C_{Ds} \quad (27) $$

Where “\(\sigma_s\)” is the mesh sheet porosity and “\(C_{Ds}\)” is the discharge coefficient of the mesh sheet. The non-dimensional non-linear resistance of mesh sheets has also been obtained \cite{7}

$$ B_s = \frac{1}{2c} \left( \frac{1}{\sigma_{se}^2} - 1 \right) \left( 1 - \sigma_{se} \right) \quad (28) $$

Rice’s resistance model was studied to determine the effect of the thread diameter and the effective porosity of the mesh on the values of R(105) and NLF(200:20). Figure 14 R(105) vs. the effective porosity and Figure 15 is the NLF(200:20) vs. the effective porosity for thread diameters ranging from 0.001cm to 0.005cm.
Figure 14: $R(105)$ vs. Effective Porosity of a Mesh Sheets

Figure 15: Non-Linearity Factor vs. Effective Porosity of Acoustic Mesh Sheets
The plots show that decreasing the effective porosity increases the R(105) and NLF(200:20) while decreasing the thread diameter increases the R(105) and decreased the NLF(200:20).

This analytical model was compared with the R(105) and the NLF(200:20) of the acoustic mesh material used for Hexcel’s Acousti-Cap™. The acoustic mesh sheets used for Hexcel’s Acousti-Cap™ were designed to have high resistances (between 50 Rayls and 100 Rayls) and very low non-linearity factors. This is achieved by using extremely small diameter threads and producing sheets with very low porosities [7].

Images of the mesh material were taken under a microscope to determine the porosity and thread diameter. Figure 16 is an image of the 80 Rayl mesh material under a microscope.

Figure 16: Image of 80 Rayl Mesh Material under a Microscope
This image shows that the spacing of the threads in the warp direction is approximately 60% the width of the threads and there appears to be no spacing between the threads in the weft direction.

*Figure 17* is an image of the threads in the warp direction and *Figure 18* is an image of the threads in the weft direction.

*Figure 17: Image of the Threads in the Warp Direction*

*Figure 18: Image of the Threads in the Weft Direction*
Rice provides an equation to calculate the porosity of wire mesh sheets. Using his method, the porosities of the acoustic mesh sheets are 0. This is because there is no spacing between the threads in the weft direction. Rice’s equation doesn’t account for the gap perpendicular to the air flow. This gap is negligible when the spacing between the wires is sufficiently large but must be taken into account when the spacing is small. Figure 19 is a diagram of the area where the threads cross. The diagram shows is cross section of the mesh sheet parallel to the threads in the warp direction and perpendicular to the threads in the weft direction.

![Figure 19: Diagram of the Open Area where the Threads Cross](image)

The porosity of the mesh sheets was calculated by approximating the area of the gap and dividing it by the total surface area of one side of the mesh. This leads to an approximate mesh sheet porosity of

\[
\sigma_m = \frac{S d_t^2}{4d_t + \frac{4S}{\cos\left(\tan^{-1}\left(\frac{2d_t}{S}\right)\right)}}
\]  \hspace{1cm} (29)
where “S” is the spacing between the threads in the warp direction. This approximation assumes
the shape of the hole where the threads cross is triangular. The threads in the warp direction
appear to be the same thickness as the threads in the weft direction and the thickness of the mesh
sheet is approximately 3 times the thickness of a single thread. The thread diameter of the mesh
sheets is approximately 0.001cm. The porosities of the 50 Rayl, 80 Rayl and 100 Rayl are
roughly 5%, 3.5% and 2.5% respectively. The measured non-linearity factor of the mesh
materials is 1.4. These values are close to the results predicted by Rice’s model. The differences
can be attributed to the inaccuracies in the approximation of the thread diameter and the mesh
sheet porosity.
2.1.4. Double Layer Acoustic Liners

A double layer or two-degrees-of freedom (2-DoF) acoustic liner works along the same principles as a 1-DoF acoustic liner. This type of acoustic liner consists of two porous face sheets and a back sheet that is impervious to sound. They are separated by two honeycomb cavities to provide the required impedance characteristics. The acoustic impedance of a 2-DoF acoustic liner can be calculated in a similar fashion as for a 1-DoF acoustic liner \cite{5}. \textit{Figure 20} is a 1D acoustic model of a 2-DoF acoustic liner.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{double_layer_acoustic_liner}
\caption{1D Acoustic Model of a Double-layer (2-DoF) Acoustic Liner}
\end{figure}

The method of calculating the impedance of a 2-DoF acoustic liner is described in \textit{Appendix A}. 
2. 2. Flow Resistance Measurement

2. 2. 1. Approximate Acoustic Resistance

The purpose of the flow resistance measurement is to measure the acoustic resistance of porous sheets used in acoustic liners. In Figure 10, the incident acoustic waves are normal to the acoustic liner. The fluid particles traveling through the porous face sheet are assumed to be moving normal to the face sheet, and as a result, the particle velocity is only a function of the fluid particles axial location relative to the face sheet as well as time. This is similar to air particles traveling through the porous face sheet with a constant velocity. Figure 21 is a diagram of a particle of air traveling in one direction through an acoustic liner.

![Diagram of a Particle Traveling Through an Acoustic Liner](image)

*Figure 21: Diagram of a Particle Traveling Through an Acoustic Liner*

In this analogy, the constant velocity “U” through the porous face sheet is equivalent to the $u_{\text{rms}}$ of Equation 23 and the average face sheet resistance “Rs” is equivalent to $R_{\text{rms}}$ of Equation 23. The acoustic resistance of the face sheet can be calculated by measuring the pressure drop across the sample and the mean flow velocity through the sample $^{[1, 2]}$
\[ Rs = \frac{\Delta Ps}{U} \]  

(30)

Where “\(\Delta Ps\)” is the pressure drop across the porous sheet (dynes/cq.cm.) and “U” is the corresponding mean flow velocity (cm/sec) through the porous sheet. The total resistance of the 1-DoF acoustic liner can then be easily measured by causing air flow through the face sheet, measuring the pressure drop across it and the corresponding average particle velocity through it. Note that the resistance defined by Equation 30 has dimensions of \(\rho c\); where \(\rho\) is the density of air (grams/cubic centimeter) and \(c\) is the speed of sound (cm/sec). Thus, the flow resistance defined by Equation 30 is generally specified in cgs Rayl units. Therefore, the normalized resistance can be obtained by the by dividing the flow resistance by \(\rho c\) (≈ 41.3 cgs Rayl) at sea level pressure and 70ºF.
2.2.2. Discharge Coefficient Measurement

Flow resistance measurements are also used to determine the “effective porosity” of perforated face sheets of acoustic liners. These types of measurements can be used to ensure that a manufactured acoustic liner meets design specifications. A previous correlation between the porosity ($\sigma$) and discharge coefficient ($C_d$) of punched aluminum perforated face sheets was established by Syed and Perrino\textsuperscript{[3]}

$$C_d = 1 - 0.00813 \sqrt{\frac{1 - \sigma^2}{\sigma^2}}$$ \hspace{1cm} (31)

It can be shown that for incompressible flow through a perforated sheet, the measured slope of the flow resistance chart can be expressed in terms of the porosity and the discharge coefficient as shown\textsuperscript{[3]}

$$B = \frac{\rho}{2 * C_d^2} \left( \frac{1 - \sigma^2}{\sigma^2} \right)$$ \hspace{1cm} (32)

where “B” is the slope of the resistance as a function of mean flow velocity measured using a flow resistance measurement apparatus. The face sheet porosity can be accurately measured using pin gauges and calipers. The discharge coefficient can be easily calculated

$$C_d = \sqrt{\frac{\rho}{2 * B} \left( \frac{1 - \sigma^2}{\sigma^2} \right)}$$ \hspace{1cm} (33)

The derivation of the relationship between the discharge coefficient of a perforated sheet, its porosity, and the measured flow resistance, using the incompressible fluid assumption, is shown in Appendix B.
2. 2. 3. Measurement Setup

The two main quantities being measured to calculate the resistance of a porous sheet are the average particle velocity through the test sample and the static pressure drop across it. *Figure 22* is a schematic diagram of the DC flow resistance measurement apparatus in the suction mode of operation.

![Diagram of Flow Resistance Measurement Setup in Suction Mode](image)

*Figure 22: Diagram of Flow Resistance Measurement Setup in Suction Mode*

The measurement can also be setup with the direction of the airflow reversed. The suction mode setup is used at Hexcel Corporation because of its simplicity. The average flow velocity through the porous face sheet sample is calculated by measuring the volumetric flow rate at standard pressure and temperature through the sample and the sample area $^{[1]}$.

\[ U = \frac{Q_s}{A_s} \]  

(34)
where “Qs” is the standard volumetric flow rate through the sample and “As” is the sample area. The standard volumetric flow rate is measured downstream of the sample using a flow meter and is assumed to be equal to the standard volumetric flow rate through the sample area. This assumption is true if there is no leakage in the pipe work of the apparatus. The difference in static pressure across the flow meter is necessary to calculate the volumetric flow rate. The static pressure just upstream of the flow meter is used to correct to the volumetric flow rate at the actual temperature and pressure. The static pressure drop across the sample is measured using a differential pressure transducer. The ambient conditions are also obtained to correct the measurement to the standard conditions of pressure and temperature (70°F, 14.7psi).
2. 2. 4. Correcting to Standard Conditions

The average velocity can be corrected to standard conditions using the methodology described by Motsinger et al\textsuperscript{[1, 7]}

\[ U_c = \frac{Q}{A} \left( \frac{P_f}{P_o} \right) \left( \frac{T_o}{T_f} \right)^{2.5} \] (35)

Where “Uc” is the corrected average flow velocity, “Pf” & “Tf” are the static pressure and temperature just upstream of the flow meter and “P0” & “To” are values of the reference pressure and temperature. The corresponding measured resistance can also be corrected to standard conditions\textsuperscript{[1, 7]}

\[ R_c = \Delta P_s \times \frac{A}{Q} \left( \frac{P_s}{P_f} \right) \] (36)

where “Ps” is the static pressure upstream of the sample.
2. 2. 5. Measurement Assumptions

Some assumptions are made to simplify the computations of the flow resistance parameters. The first assumption is that the fluid temperature throughout the measurement section of the apparatus is constant and equal to the ambient temperature

\[ T_f = T_s = T_{amb} \]  \hspace{1cm} (37)

Were “Tf” is the temperature just upstream of the flow meter and “Ts” is the temperature just upstream of the sample. This is a good assumption as long as there is no heat generation in the apparatus and the Mach number is sufficiently low. This limitation will be discussed in the future improvements section of the thesis.

The third assumption is that the fluid flow through the apparatus is steady state. This assumption requires that the fluid has sufficient time to reach a quasi steady state before measurements are taken. Also, this assumption requires that the flow is quasi-stationary at the measurement locations and there are a sufficient number of data points to resolve random pressure fluctuations. These assumptions are validated in the data uncertainty analysis section.
3. Flow Resistance Apparatus Design

3.1. Hardware Design

Hexcel Corporation was responsible for the design and construction of the sample holder as well as the test stand. There was oversight from the University of Cincinnati to assure that the design does not adversely affect the accuracy of the measurements. The sample holder is opened and closed pneumatically using an electric switch. The pneumatic head also aligns the hole in the test head with the sample area below. Hexcel Corporation also designed an optional attachment that holds the sample in place. This attachment is used if multiple measurements need to be taken at the same location on the sample.

The size of the sample holder test area needed to be large enough so that the area of the sample tested is representative of the sample properties as a whole. The sample holder also has an upper limit on its size. The sample area must be small enough so that the volumetric flow rate through the sample holder does not exceed the limitations of the flow meter and the power of the vacuum pump. A diameter of 0.127m (5in) was chosen for the sample area. The sample area is approximately 0.013m² (19.5 in²).

To ensure the accuracy of the measurement, the sample holder must provide an air tight seal around the sample. Therefore, rubber gaskets were placed above and below the sample. The inner diameter of these gaskets is the same as the inner diameter of the sample holder. This is to ensure that the samples are always held in place and the sample area always lines up correctly.
The plenum chamber is attached to the sample holder by 8 bolts. A flange gasket is used to ensure that there is no air leakage. The pipe work starting with the plenum chamber under the sample holder was designed and fabricated at the University of Cincinnati with input from Hexcel Corporation.

*Figure 23* is a diagram of the plenum chamber. The plenum chamber is a flanged pipe nipple with a blind flange as a bottom. A porous sheet is used in the plenum chamber to condition the flow. This sheet is necessary to ensure the velocity profile in the plenum chamber is nearly uniform. Another hole was also drilled in flanged pipe nipple and a pipe fitting was welded on. This is where a pressure transducer is connected to measure the static pressure inside the plenum chamber. A hole was drilled through the center of the pipe flange and a 2 inch diameter pipe nipple was welded on. A pipe elbow is attached to the pipe nipple.

*Figure 23: Diagram of Plenum Chamber*
The next section of piping is composed of two sections of 2 inch diameter pipes with a flow meter separating them. The pipe upstream of the flow meter is required to be at least 10 diameters long (20 inches) to allow the flow to become fully developed. The pipe downstream of the flow meter is required to be at least 5 diameters long (10 inches). To allow for the flow meter to be replaced easily, the pipes are held in place at either end with flanges. Gaskets were placed between the flanges to make sure there were no air leaks. A fine wire mesh was placed between the set of flanges upstream of the flow to condition the flow after the sharp turn after the plenum chamber. The wire mesh also provides a last line of defense to stop debris from entering the flow meter and affecting its calibration.

Piping starting at the sample holder and ending after the flow meter must be air tight to ensure accurate measurements. If there is an air leak in this section of the apparatus, then the assumption that the mass flow rate through the sample is the same as the mass flow rate through the flow meter would be invalid.

The air downstream of the flow meter continues through the piping that takes the air through a wall and into another room. This room is where the control valve and vacuum pump operate. The vacuum pump and the control valve are placed in a different room because their operation produces high intensity noise that would require the operator and others within close proximity to the apparatus to wear hearing protection. They are placed in a sound absorbing enclosure to reduce the intensity of the noise.
The vacuum pump and the control valve were connected in parallel to the pipe work using a T-section. *Figure 24* indicates the air flow through the T-section. The vacuum operates at a constant speed and the volumetric air flow rate through the apparatus is controlled by opening and closing the control valve.

![Diagram of Air Flow through T-section](image)

*Figure 24: Diagram of Air Flow through T-section*

A NuTone vacuum was chosen as the vacuum pump because it is an off the shelf vacuum pump capable of the volumetric flow rate of 150 CFM (4320 in$^3$/s). This will allow for an average particle velocity up to 5.5 m/s (~216 in/s). The actual particle velocity will never reach this maximum particle velocity because there will be resistances in the flow such as the flow meter, porous sheet in the plenum chamber, as well as the sample being measured.

The control valve is a 2 inch ball valve with an electrical actuator installed on it. The electronic actuator was installed so that the computer is capable of controlling the ball valve. The ball valve
has the potential to seriously injure someone by crushing their hand so an elbow pipe was attached to the side of the valve open to the atmosphere. The ball valve is attached to the T-section using a quick detachment because the actuator has the potential of failing and is located in a small space. If the actuator fails the ball valve can be easily removed and sent out to be repaired.
3. 2. Software and Instrumentation

The need for the system to operate reliably and autonomously increased the complexity of the software. The greater complexity is attributed to the incorporation of hardware control algorithms to perform all the necessary tasks to run the system and measure the data efficiently as well as system checks and statistical analysis used to assure the system produces accurate results. Two separate programs were written for the apparatus. The first is a calibration check program that measures the resistance of a standard sample to see if the apparatus is operating accurately from day to day. The second program is the actual measurement program that accurately measures the resistance of various samples.

The underlying code used by both programs to measure the acoustic resistance remains the same. The program is identical from the time the operator pushes the run button till after the sample resistance is measured and the slope and intercept of the data is calculated. Each step of this process and the specific considerations of each step are described in detail.
3.2.1. Resistance Measurement Program

*Figure 25* is a process map for the resistance measurement program. The first step in the software is to measure the local atmospheric conditions that will be used to normalize the measured data to standard conditions as well as calculating absolute pressures throughout the apparatus. The atmospheric pressure and temperature are measured using a Fisher Scientific digital barometer that is interfaced with the computer through one of the four comports. The barometric gauge has an accuracy of ±1°C and ±5 mbar. The gauge is located away from the sample holder to prevent the airflow through the sample from affecting the results. It is assumed that the ambient conditions at the barometric gauge are the same as the ambient conditions directly upstream the test sample.

*Figure 25: Process Map for the Resistance Measurement Program*
The vacuum pump is controlled using a voltage output of either 0 volts or 1 volt. The signal is transferred through a National Instruments BNC-2110 data acquisition board to a solid-state relay. The data acquisition board is connected to one of the PNC ports of the computer. The vacuum pump is connected to the solid-state relay, which is connected to an AC power source. The solid-state relay acts as a computer controlled on/off switch.

A control valve is used to vary the flow speed through the sample. The control valve consists of a ball valve and an actuator. It is controlled by the software using a voltage output between 0 volts and 10 volts. The signal is transferred through the same National Instruments BNC-2110 data acquisition board as the solid-state relay.

The signal is sent to a valve actuator, which moves to the position specified by the output voltage. The static pressure drop across the sample is measured using a differential pressure transducer. One static pressure port senses the static pressure in the plenum chamber just downstream of the sample and the other static pressure port senses the ambient pressure.

The next step in the program is to take the measurements at a particular flow rate. Three differential pressure measurements are taken using Mensor 6115 pressure transducers. The first measures the pressure drop across the sample. The second transducer measures the pressure upstream of the flow meter relative to the ambient pressure in the lab, and the third is used to measure the static pressure drop across the flow meter (Meriam 50MW20-2 laminar flow element). The Mensor 6115 pressure transducers were chosen for two reasons. First, they are capable of measuring small pressure differences with an accuracy of 0.01%FS. Second, the
pressure transducers have built-in analog to digital converters and connect to the computer using RS-232 cables. This allows the signal to be accurately transmitted over 150 feet of cable.

The first pressure transducer measures the pressure drop across the sample. One static pressure port is connected to the static pressure tap in the plenum chamber and the other is open to the atmosphere. The pressure tap location is shown in Figure 16. The pressure transducer is calibrated to read ±0.5psi.

The static pressure upstream of the flow meter is measured using a second differential pressure transducer. This configuration is shown in Figure 26. The other pressure port will measure the ambient pressure. This pressure transducer is calibrated to read ±2.0psi.

Figure 26: Pressure Tap Configuration for Meriam Flow Meter
The mass flow rate is measured using a Meriam 50MW20-2 laminar flow element and a third Mensor 6115 differential pressure transducer. The Meriam 50MW20-2 laminar flow element was chosen because it is extremely accurate over the velocity range of interest with an accuracy of ±0.86% of the reading. The static pressure drop across the flow meter is measured using a differential pressure transducer with the first pressure port measuring the static pressure directly before the flow meter and the other measuring the static pressure directly after the flow meter. The volumetric flow rate through the flow meter at standard pressure and temperature is given by [7]

\[ Q_f = B_f \Delta P_f + C_f \Delta P_f^2 \]  \hspace{1cm} (38)

where “B_f” and “C_f” are calibration constants for the flow meter and “ΔP_f” is the measured static pressure drop across the flow meter.

The pressures are measured in series. 40 pressure measurements are taken over a period of 6 seconds from each transducer and these measurements are averaged. The statistical methods used for determining the number of measurements are described in the data analysis and uncertainty section.

After the pressure measurements at a particular flow velocity have been taken, the control valve closes slightly by outputting a larger voltage from the computer to the control valve. The position of the control valve is compared with the desired position. The program waits until the position of the control valve matches the desired position. Adjusting the control valve takes time and the control valve must be stationary when the pressure measurements are taken or else the
assumption that the flow is quasi steady will be invalidated. Closing the control valve slightly decreases the volumetric flow rate through the control valve and increases the volumetric flow rate through the sample. There is a two second wait between the time the control valve finishes adjusting and the time when measurements are taken to assure that the flow through the apparatus has reached steady state conditions.

A feedback loop is not used to control the particle velocities measured for two reasons. First, the resistance is linear as a function of particle velocity in the velocity measurement range and the slope and intercept of this function can be accurately calculated using only measuring the resistance at two or more flow velocities that are sufficiently different. Second, a feedback loop drastically increases the amount of time to run the program with the current hardware. The drastic increase in measurement time is due to the lag between the time when the signal to the control valve is sent and the time when the control valve finishes adjusting to the new position.

The flow velocity and corrected resistance is calculated after each measurement. The linear region for the resistance as a function of velocity is between 50cm/s and 200cm/s. If a data point lies below 50cm/s, then the data is rejected and the apparatus continues to the next data point. If a data point lies above 200cm/s, then the program exits the measurement loop.

Once the measurements have been taken, a least squares fit is used to estimate the slope and the intercept of the data. *Figure 27* shows an example of the measured flow resistance data and *Figure 28* is an example of the flow resistance measurement data file.
This linear fit is only applicable in the regime where the flow is incompressible. This is a good assumption for Mach numbers less than 0.2 and past experience has shown that this threshold will not be reached for acoustic testing. This assumption puts a limitation on the maximum particle velocity through the sample and the minimum porosity of the sample. There are correction factors that can be implemented if the Mach number is increased beyond 0.2 \(^1\). These correction factors are derived in Appendix C.
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<td>13.45604</td>
<td>0.16003</td>
<td>0.176836</td>
<td>26.106291</td>
<td>87.0457</td>
<td>131.758157</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>13.343303</td>
<td>0.195555</td>
<td>0.210243</td>
<td>30.746059</td>
<td>101.66089</td>
<td>137.776094</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>13.158339</td>
<td>0.24739</td>
<td>0.281307</td>
<td>37.655451</td>
<td>122.777502</td>
<td>147.210997</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>12.843757</td>
<td>0.319558</td>
<td>0.383999</td>
<td>47.786214</td>
<td>152.081183</td>
<td>162.067592</td>
<td></td>
</tr>
</tbody>
</table>

*Figure 28: Example of Data File*
3. 2. 2. Calibration Check Program

3. 2. 2. 1. Calibration Check Method

The calibration check program is used to determine if the apparatus is measuring correctly. The calibration check program operates similarly to the resistance measurement program in that it measures the resistance of a sample. *Figure 29* is a process map for the calibration check program.

![Figure 29: Process Map for the Calibration Check Program](image)

The Calibration check program works by measuring the resistance of a standard sample. The standard sample is a custom made aluminum plate with holes drilled in a circular pattern. The resistance of the sample and the standard deviation of the measured resistance are known. The program operates by measuring $R(105)$ value of this sample. Process behavior charts are used to determine if the system is working within the operating limits by comparing the most recent
resistance measurements of the standard sample with its established process limits. The user is notified if a calibration check measurement passes or fails the process control criteria. The R(105) value and the time the check was performed are saved in two different files on the computer. The first file stores the resistance measurements of the calibration checks that pass so that they can be used in future calibration checks. The second file saves the R(105) values of all calibration checks. The second file can be used to determine when the calibration began to go out of control.
3.2.2.2. Process Behavior Charts

The calibration check program uses both “individual value” and “moving range” process behavior charts. The moving range process control chart is used to determine if the random variation of the signal is out of control and the individual value process control chart is used to determine if there is a bias error.

The individual value process control chart works by comparing the most recent measurements of the standard sample’s resistance with the average and moving range of the past measurements of the same standard sample’s resistance. The average standard sample resistance is calculated \[ \bar{R}(105) = \frac{1}{N} \sum_{i=1}^{N} R(105)_i \]  

where \( R(105) \) is the resistance of the standard sample at 105 cm/s.

The measurements are plotted in the order that they are taken and they are compared with the average standard sample resistance. An example of an individual value process control chart is shown in Figure 30.
The moving range process control chart works by comparing the most recent fluctuation in the resistance measurement of the standard sample with the past fluctuations in the resistance measurements of the standard sample’s resistance. The fluctuation in the standard sample resistance is calculated \([8]\)

\[
mR_i = |R(105)_i - R(105)_{i-1}|	ag{40}
\]

The average fluctuation in the standard sample measurement is calculated \([8]\)

\[
mR = \frac{\sum_{i=2}^{N} |R(105)_i - R(105)_{i-1}|}{N}	ag{41}
\]

The fluctuations in the standard sample resistance are plotted in the order that they are taken along with the average fluctuation. The moving range process control chart for the above individual value process control chart is shown in Figure 31.

The average standard sample’s R(105) value and the average moving range are used to calculate the upper natural process limit (UNPL) and lower natural process limit (LNPL) \([8]\)
\[ UNPL = \bar{R}(105) + 2.66 \times \bar{mR} \quad (42) \]
\[ LNPL = \bar{R}(105) - 2.66 \times \bar{mR} \quad (43) \]

The process limits are used to provide bounds for a system that is under control. The individual value process control chart is divided into four different regions that represent varying degrees of deviation from the average standard sample resistance value. The boundaries of the regions A, B, C, and D are defined as \[^8\]

\[
\text{Region A: } \frac{2}{3} \times 2.66 \times \bar{mR} \leq A \leq \frac{2}{3} \times 2.66 \times \bar{mR} \\
\text{Region B: } B_l < A \leq \frac{2}{3} \times 2.66 \times \bar{mR} \quad \& \quad \frac{2}{3} \times 2.66 \times \bar{mR} < A < \text{Region C} \quad \& \quad B \leq \text{Region C} \leq \frac{2}{3} \times 2.66 \times \bar{mR} \\
\text{Region D: } D_l < C_l \quad \& \quad C_h < D_h
\]

*Figure 32* is the individual value process control chart with the different regions indicated in different colors.
The regions are used as a quantitative way to determine if the measurement is under control. If the measurement is under control, most measurements will occur in region A, some measurements will occur in region B, and a few points will occur in region C. The points will also be randomly distributed above and below the average standard sample resistance. There are five indicators that the program uses to determine if the flow resistance apparatus is out of control [8].

1. One point occurs in region D.
2. Two of three successive points occur in region C.
3. Four of five successive points occur in regions B or C
4. Seven successive points occur above the average standard sample resistance
5. Seven successive points occur below the average standard sample resistance

The program will indicate that the apparatus is in control 98% of the time for the specified upper and lower natural process limits if the apparatus measurements follow a normal distribution about the mean value.
The moving range process control chart is quantified similarly. The natural range limit (URL) can be calculated using the average moving range and is a method to determine if the measurement deviations are out of control. The upper range limit is defined as \[^{[8]}\]

\[ URL = 3.27 \times \overline{mR} \]  

(44)

*Figure 33* is an example of a moving range process control chart with the upper range limit. The process is considered out of control if any one of the moving range values is more than the upper range limit. The program will indicate that the apparatus is out of control 2% of the time for the specified upper range limit if the apparatus measurements follow a normal distribution about the mean value.

*Figure 33: Moving Range Process Control Chart with the Upper Range Limit*
3. 2. 2. 3. Hexcel’s Process Control Charts

Figures 34 and 35 are the calibration check measurement process control charts taken between November 2006 and October 2007. [11]

**Figure 34: Calibration Check Individual Value Process Control Chart**

**Figure 35: Calibration Check Moving Range Process Control Chart**
These figure shows that the apparatus has been operating accurately on a day to day basis since November 2006. *Figure 36* is the distribution of the calibration check data where the average is 34.26 cgs Rayls and the standard deviation is 0.36 cgs Rayls.

Figure 36: Distribution of Calibration Check Data

This figure shows that the measurement has a distribution that is close to a Gaussian distribution.
3. 3. Data Uncertainty Analysis

The flow resistance measurements will be used for process control. This requires the measurements to be repeatable and reproducible. The two types of errors that have to be resolved for these requirements to be fulfilled are random errors and bias errors. The effect of random errors can be mitigated using statistical methods. Bias errors can be resolved by making sure the measurement assumptions are valid and the transducers are calibrated. Testing a standard sample and comparing the results with measurements taken by several other flow resistance apparatuses provides a check of the reproducibility of the measurement. This check was performed by Hexcel Corporation.
3. 3. 1. Repeatability

The major concerns for deciding on the number of averages that are taken are

- the random static pressure perturbations,
- the frequency component of the pressure, and
- the computer computation time.

Static pressure can be written as[9]

\[ p(t) = \bar{P} + P_r + P_r \]

Where “\( \bar{P} \)” is the average static pressure, “\( P_r \)” are discrete frequency components of the static pressure, and “\( P_r \)” are random variations in the static pressure.

Random errors can be caused by random fluctuations in the signal being measured or random noise can enter a signal anywhere between the transducer to the data acquisition board. Mensor 6115 pressure transducers are used because they convert the analog signal to a digital signal within the transducer to reduce the effects of noise between the transducer and the computer. Random errors cause the measured data to be normally distributed around an average measured value. As the sample size increases, the average of the measurements approaches the actual mean of the value being measured[9].

A minimum of 40 samples should be taken at a sampling rate of 10Hz so that the mean of the sample set is an approximation of the actual value[10]. As the number of data points increases,
the approximation becomes more accurate and the time needed to run the test increases. Table 3 is a chart of the error as a function of the number of averages. The error is based on 90% confidence interval of the measurement as well as a bias error. The bias error is calculated assuming the resistance measured using 80 averages has a negligible bias error. This is a good assumption because the average calculated resistance using 40 averages was approximately equal to the average calculated resistance using 80 averages.

<table>
<thead>
<tr>
<th># Avgs</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.51</td>
</tr>
<tr>
<td>10</td>
<td>1.15</td>
</tr>
<tr>
<td>20</td>
<td>0.96</td>
</tr>
<tr>
<td>40</td>
<td>0.64</td>
</tr>
<tr>
<td>80</td>
<td>0.48</td>
</tr>
</tbody>
</table>

*Table 3: Approximate Error of the Measurement as a Function of the Number of Averages*

The number of data points measured at each flow velocity should be a balance between accuracy and measurement time. Figure 37 is a plot of the measurement time as a function of the number of data points averaged at each flow velocity. The number of flow velocities measured was held constant at 17. There was no discernable difference between the standard deviation of the measurement taken with 40 data points and 80 data points; however, there was a significant difference in the amount of time it took to measure the data. Therefore, the number of pressure measurements averaged is set at 40.
Figure 37: Measurement Time as a Function of the Number of Data Points Averaged

The number of flow velocities between the maximum flow velocity and the minimum flow velocity was also examined. The acoustic resistance values of three face sheet samples with different resistance values were measured using 3 to 17 distinct flow velocities. The error is based on 90% confidence interval of the measurement as well as a bias error. The bias error is calculated assuming the resistance measured using 17 flow velocities has no bias error. This is a good assumption because the average calculated resistance using 9 flow velocities was approximately equal to the average calculated resistance using 17 flow velocities. Table 4 is a comparison of the total error of the measurement and the number of flow velocities.

<table>
<thead>
<tr>
<th># Pts</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>0.40</td>
</tr>
<tr>
<td>9</td>
<td>1.19</td>
</tr>
<tr>
<td>5</td>
<td>2.23</td>
</tr>
<tr>
<td>3</td>
<td>2.85</td>
</tr>
</tbody>
</table>

Table 4: Approximate Error of the Measurement as a Function of the Number of Different Flow Velocities Measured between 50 cm/s and 200 cm/s
Figure 38 is a plot of the measurement time as a function of the number of flow velocities measured. The number of averages used at each point was held constant at 40.

Figure 38: Measurement Time as a Function of the Flow Velocities Measured

There was no discernable difference between the resistance measurements taken using 9 flow velocities and 17 flow velocities; however, there was a significant difference in the amount of time it took to measure the data. Therefore, the number of flow velocities is set at 9.
3.3.2. Accuracy

Each one of the transducers has an accuracy associated with it. The accuracy of the final measurement can be calculated using the accuracy of the individual measurements and their effect on the final measurement. The computer round off error is added to the specified transducer accuracy. The round off error can be decreased at the cost of increasing the computation time. The final accuracy of the measurement can then be calculated using the general uncertainty equation. The general uncertainty equation is defined as

\[
\left( \frac{\partial Z}{\partial A} \right)^2 u_A^2 + \left( \frac{\partial Z}{\partial B} \right)^2 u_B^2 + \ldots = u_Z^2
\]

(46)

where \( Z = f(A, B, \ldots) \). The measurement accuracies are calculated in Appendix D. The resulting accuracy of the particle velocity is ±2.95 cm/s, and the accuracy of the resistance measurement is ±0.345 c.g.s. Rayls.
4. Conclusions

4. 1. Hexcel Flow Resistance Apparatus Operation

Hexcel Corporation’s flow resistance apparatus is an integral part of the development of acoustic liners. The apparatus is easy to operate and is robust. A user’s manual written by Fumitaka Ichihashi describes how to operate the apparatus and what to do in case of a malfunction[12].

4. 2. Implemented Improvements

There have been several minor additions made to the apparatus since it was first put into operation. These improvements include a new sample holder capable of holding large acoustic liner panels. Also, the program was modified to save multiple measurements of a sample to the same data file.

4. 3. Future Improvements

The Hexcel flow resistance apparatus has one minor drawback. The direction of the air through the apparatus causes debris to accumulate in the test section. The main source of debris is epoxy dust that has not fully cured on the honeycomb samples. When the sample is placed in the sample holder, the sample bends slightly causing the epoxy to flake off and get sucked into the apparatus. This dust accumulation occurs on the screen upstream of the flow meter and causes a malfunction in the apparatus when the blockage becomes too large. The temporary solution for this problem is to clean the screen upstream of the flow meter every two weeks. A long term solution, currently being tested at the University of Cincinnati, requires a change in the direction of the airflow through the apparatus. This change requires a slight modification to how the data
is normalized to standard conditions as well as the measurement of the air temperature throughout the apparatus.
References:

5. Syed, A.A., *The Impedance Modeling in a MDOF Cavity with Variable Area of Cross Section*, Unpublished (see the Appendix A)
The cavity of interest is illustrated above. The acoustic field in this cavity is one-dimensional. The following parameters are known:

- Let $A_1$, $A_2$, and $A_3$ be the areas of cross section of the three tubular segments
- $l_1$, $l_2$, and $l_3$ are the lengths of the three segments
- $\Delta Z_1$ and $\Delta Z_2$ are step changes in impedance at the interfaces between segments 1 & 2 and between segments 2 & 3
- The Impedance $Z_0(f)$ at $x = 0$

We need to compute the Impedance, $Z_3(f)$ from known data.

The acoustic pressure, $p(f)$, and velocity, $u(f)$ are given by
\[ p(x) = A \exp(-i k x) + B \exp(i k x) \quad \cdots \quad (1) \]
\[ \rho c u(x) = A \exp(-i k x) - B \exp(i k x) \quad \cdots \quad (2) \]
\[ \rho \text{ is the density of air} \quad ; \quad c \text{ is the speed of sound} \quad k = \frac{2 \pi f}{c} \]
\[ f \text{ is the acoustic frequency} \]

\( A \) and \( B \) are complex coefficients to be determined from boundary conditions.

The acoustic Impedance, \( Z(f) \), is defined as follows

\[ Z(x) = \frac{-p(x)}{\rho c u(x)} = \frac{-\{A \exp(-i k x) + B \exp(i k x)\}}{\{A \exp(-i k x) - B \exp(i k x)\}} \quad (3) \]

**Segment #1**

First, consider the acoustic field in segment #1. From equation (3), at \( x = 0 \), it follows that

\[ Z_0 = \frac{-(A_1 + B_1)}{(A_1 - B_1)} \]

\[ B_1 = A_1 \left( \frac{Z_0 + 1}{Z_0 - 1} \right) = \phi_0 A_1 \]

therefore the pressure \( p_1 \) and the velocity \( u_1 \) in the region #1, are given by

\[ p_1(x) = A_1 \{\exp(-i k x) + \phi_0 \exp(i k x)\} \]
\[ \rho c u_1(x) = A_1 \{\exp(-i k x) - \phi_0 \exp(i k x)\} \]

Let \( SPL_0 \) be the sound pressure level (dB) at \( x = 0 \) then

\[ A_1 = \left\{ 200 \times 10^{\left( \frac{SPL_0}{20} - 6 \right)} \right\} \frac{1}{(1 + \phi_0)} \quad \cdots \quad (4) \]

\[ Z(l_1) = \frac{-\{\exp(-i k l_1) + \phi_0 \exp(i k l_1)\}}{\{\exp(-i k l_1) - \phi_0 \exp(i k l_1)\}} \quad \cdots \quad (5) \]
If, \( Z_0 \) approaches infinity (hard wall at \( x=0 \)), then \( \phi_0 = 1 \), and \( Z(l_1)_1 = -i \cot(k l_1) \).

Therefore at \( x = l_1 \), the values of the acoustic pressure and the acoustic velocity are given by

\[
p_1(l_1) = 2 A_1 \cos(k l_1) \quad \ldots \quad (6)
\]

\[
\rho c u_1(l_1) = -i 2 A_1 \sin(k l_1) \quad \ldots \quad (7)
\]

\[
|u_1(l_1)| = \left| \frac{2 A_1 \sin(k l_1)}{\rho c} \right| \quad \ldots \quad (8)
\]

the rms magnitude of acoustic velocity

Note: The rms acoustic velocity will be used to compute the resistance and mass reactance of the porous septum placed at location \( x = l_1 \).

At the interface between the segments #1 and #2, continuity of mass requires that

\[
a_1 u_1 = a_2 u_2 \quad , \quad \text{therefore} \quad u_2 = u_1 \left( \frac{a_1}{a_2} \right) \quad , \quad \text{and}
\]

\[
Z(l_1)_2 = Z(l_1)_1 \left( \frac{a_2}{a_1} \right)
\]

\[
Z_1 = (Z(l_1)_1 + \Delta Z_1) \left( \frac{a_2}{a_1} \right) \quad \ldots \quad (9)
\]

\[
\Delta Z_1 = R_1 + i m_1 k
\]

\( m_1 \) - is the coefficient of mass reactance of the porous septum, \( R_1 \) is its normalized resistance. These values are to be determined from separate impedance tests or from design methods.
Segment #2

In segment 2, the pressure, velocity and impedance, at any location $x$, can be determined as follows

$$B_2 = \phi_1 A_2$$

$$\phi_1 = \left( \frac{Z_1 + 1}{Z_1 - 1} \right) \quad \ldots \quad (10)$$

$$p_2(x) = A_2 \left\{ \exp(-ik\Delta x) + \phi_1 \exp(ik\Delta x) \right\} \quad \ldots \quad (11)$$

$$\rho c u_2(x) = A_2 \left\{ \exp(-ik\Delta x) - \phi_1 \exp(ik\Delta x) \right\} \quad \ldots \quad (12)$$

where $\Delta x = x - l_i$

$$A_2 = \left( \frac{-i 2 A_1 \sin(kl_1)}{1 - \phi_1} \right) \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad \ldots \quad (13)$$

$$Z(x) = \left( \frac{-p(x)}{\rho c u(x)} \right)_2 = \frac{\left\{ \exp(-i k \Delta x) + \phi_1 \exp(i k \Delta x) \right\}}{\left\{ \exp(-i k \Delta x) - \phi_1 \exp(i k \Delta x) \right\}} \quad \ldots \quad (14)$$

From (11) and (12), the impedance at the end of segment 2 (just inside the segment) is given by

$$Z(l_2)_2 \left( a_2 \right) = \frac{\left\{ \exp(-i k l_2) + \phi_1 \exp(i k l_2) \right\}}{\left\{ \exp(-i k l_2) - \phi_1 \exp(i k l_2) \right\}} \quad \ldots \quad (15)$$

$$Z_2 = Z(l_2)_2 \left( \begin{pmatrix} a_3 \\ a_21 \end{pmatrix} \right) + \Delta Z_2 \quad \ldots \quad (16)$$

Segment #3

In segment 3, the pressure, velocity and impedance, at any location $x$, are determined as follows
\[ B_3 = \phi_2 A_3 \]
\[ \phi_2 = \left( \frac{Z_2 + 1}{Z_2 - 1} \right) \quad \ldots \quad (17) \]
\[ p_3(x) = A_3 \{ \exp(-i k \Delta x) + \phi_2 \exp(i k \Delta x) \} \quad \ldots \quad (18) \]
\[ \rho c u_3(x) = A_3 \{ \exp(-i k \Delta x) - \phi_2 \exp(i k \Delta x) \} \quad \ldots \quad (19) \]
\[ Z(x) = \left( \frac{-p(x)}{\rho c u(x)} \right)_3 = \frac{-\{\exp(-i k \Delta x) + \phi_2 \exp(i k \Delta x)\}}{\{\exp(-i k \Delta x) - \phi_2 \exp(i k \Delta x)\}} \quad \ldots \quad (20) \]

where \( \Delta x = x - (l_1 + l_2) \)

\[ A_3 = A_2 \{ \exp(-ikl_2) - \phi_1 \exp(ikl_2) \} \frac{a_3}{a_2} \frac{1}{(1 - \phi_2)} \quad \ldots \quad (21) \]

The acoustic impedance at the open end

\[ Z_3 = Z(l_3)_3 = \frac{-\{\exp(-i k l_3) + \phi_2 \exp(i k l_3)\}}{\{\exp(-i k l_3) - \phi_2 \exp(i k l_3)\}} \quad \ldots \quad (22) \]

In the example I computed, \( \Delta Z_1 \) in equation (5) is zero. \( \Delta Z_2 \) in equation (9) is given by

\[ \Delta Z_2 = \left( \frac{A + B u}{41.3} \right) + i m k \]

\( A \) and \( B \) are the Flow Resistance data I used and \( u \) is the acoustic velocity (cm/s). I used \( u = 30 \).

\( m \) - is the coefficient of mass reactance for the woven material of the caps bonded inside the honeycomb cells. I chose to use \( m = 0 \) as a first approximation. Please use the same for your code.

Please note that the products, \( kl_{l_2} \), in the above technical notes are non-dimensional. It is necessary to use consistent units of length for computing these values.
Given conditions are:
- The SPL dB at the open end of the honeycomb cell
- Particle velocity at the closed end (back sheet) is zero. $\Phi_0 = 1$
- The Flow Resistance (cgs Rayl units) Intercept (A) and Slope (B)
- The geometry of the cell and the septum ($l_1, l_2, l_3, a_2/a_1, a_3/a_2$)

Step 1: Conduct following calculations

- Assume SPL0 at back sheet
  \[
  SPL_0 = 200 \times 10^{\left(\frac{SPL0}{20} - 6\right)}
  \]
- Compute $A_1$ from:
  \[
  A_1 = \frac{200 \times 10^{\left(\frac{SPL0}{20} - 6\right)}}{(1 + \phi_0)}
  \]
- Compute particle velocity, $u_1$ as follows
  \[
  \left|u_1(l_1)\right| = \frac{2 A_1 \sin(k l_1)}{\rho c}
  \]
- Compute Impedance at $x = l_1$ as follows
  \[
  Z(l_1)_1 = -i \cot(k l_1)
  \]
- Compute the Resistance of the septum at $x = l_1$ as follows
  \[
  R_1 = \frac{\text{Intercept} + \text{slope} \left|u_1(l_1)\right|}{\rho c}
  \]
- Compute mass reactance from given correlation or data: $m_k$
  \[
  \Delta Z_1 = R_1 + i m_k
  \]

Step 2: Conduct following calculations

- Compute Impedance $Z_1$ at start of segment #2 as follows
  \[
  Z_1 = \left\{Z(l_1)_1 + (R_1 + m_k)\right\} \left(\frac{a_2}{a_1}\right)
  \]
• Compute $\phi_1$ as follows

$$\phi_1 = \left( \frac{Z_1 + 1}{Z_1 - 1} \right)$$

• Compute the Coefficient, $A_2$, as follows

$$A_2 = \left( \frac{-i 2A_1 \sin(kl_1)}{1 - \phi_1} \right) \left( \frac{a_1}{a_2} \right)$$

• Compute the velocity at end of segment #2, as follows

$$\rho \ c \ u_2(l_2)_2 = A_2 \left\{ \exp(-ik \ l_2) - \phi_1 \ \exp(i k \ l_2) \right\}$$

$$|u_2(l_2)_2| = \frac{|A_2 \ \{ \exp(-i k \ l_2) - \phi_1 \ \exp(i k \ l_2) \}|}{\rho \ c}$$

• Compute Impedance at the end of segment #2

$$Z(l_2)_2 = \frac{- \{ \exp(-i k \ l_2) + \phi_1 \ \exp(i k \ l_2) \}}{\{ \exp(-i k \ l_2) - \phi_1 \ \exp(i k \ l_2) \}}$$

• Compute the impedance of the septum between segments #2 and #3

$$\text{Intercept + slope} \ |u_2(l_2)| \left( \frac{a_2}{a_3} \right)
\ R_2 = \frac{\rho \ c}{\rho \ c}$$

• Compute the mass reactance $m_2 \ k$

• Compute Impedance $Z_2$ at start of segment #3 as follows

$$Z_2 = Z(l_2)_2 \left( \frac{a_3}{a_2} \right) + (R_2 + m_2 \ k)$$
Step 3: Conduct following calculations

- Compute the complex coefficient, $A_3$, as follows

$$
\phi_2 = \left( \frac{Z_2 + 1}{Z_2 - 1} \right)
$$

$$
A_3 = A_2 \left\{ \exp(-i k l_2) - \phi_1 \exp(i k l_2) \right\} \frac{a_3}{a_2} \frac{1}{(1 - \phi_2)}
$$

- Compute Impedance, $Z_3$, at the open end

$$
Z_3 = Z(l_3) = \frac{-\{\exp(-i k l_3) + \phi_2 \exp(i k l_3)\}}{\{\exp(-i k l_3) - \phi_2 \exp(i k l_3)\}}
$$

- Compute SPL$_3$ at the open end as follows

$$
|p_3(l_3)| = |A_3 \left\{ \exp(-i k l_3) + \phi_2 \exp(i k l_3) \right\}|
$$

$$
SPL_3 = 20 \log_{10} \left( \frac{|p_3(l_3)|}{0.0002} \right)
$$
Appendix B

*The Relationship between Flow Resistance, Porosity & Discharge Coefficient*

The flow resistance measurements can be used to experimentally calculate the discharge coefficient of a sample if the sample porosity is known. Experiments are also being performed to correlate the porosity and discharge coefficients of perforated sheets [3].

![Diagram of flow through perforated sheet](image)

The porosity of the perforated sheet is defined as

\[ \sigma = \frac{A_2}{A_1} \]  

(B1)

Where “\( \sigma \)” is the porosity of the perforated sheet, “\( A_1 \)” is the flow area upstream of the perforated sheet at location 1. This area is equal to the total sample area. “\( A_2 \)” is the total area of the perforated holes.

Equation of continuity

\[ Q_1 = Q_2 = Q_{\text{ideal}} \]  

(B2)
where “Q₁” is the volumetric flow rate through the sample area “A₁”, “Q₂” is the volumetric flow rate through the perforated holes “A₂”, and “Q_{ideal}” is the ideal volumetric flow rate.

The continuity equation can be written as

\[ \rho_1 \cdot U_1 \cdot A_1 = \rho_2 \cdot U_2 \cdot A_2 \]  \hspace{1cm} (B3)

Where “\(\rho_1\)” and “\(\rho_2\)” are the fluid densities at locations 1 and 2, and “\(U_1\)” and “\(U_2\)” are the average flow velocities at locations 1 and 2.

Bernoulli’s Equation

\[ p_1 + \frac{\rho_1 \cdot U_1^2}{2} = p_2 + \frac{\rho_2 \cdot U_2^2}{2} \]  \hspace{1cm} (B4)

where “\(p_1\)” and “\(p_2\)” are the static pressures at locations 1 and 2. The Mach numbers of the flow for acoustics purposes are much less than 0.2. The fluid is assumed to be incompressible.

\[ \rho_1 = \rho_2 \]  \hspace{1cm} (B5)

Bernoulli’s equation with the incompressible assumption

\[ p_1 + \frac{\rho \cdot U_1^2}{2} = p_2 + \frac{\rho \cdot U_2^2}{2} \]  \hspace{1cm} (B6)

This equation can be written as

\[ \Delta p = p_1 - p_2 = \frac{\rho \cdot U_2^2}{2} - \frac{\rho \cdot U_1^2}{2} = \frac{\rho}{2} (U_2^2 - U_1^2) \]  \hspace{1cm} (B7)

The equation of continuity with the assumption of incompressibility

\[ U_1 \cdot A_1 = U_2 \cdot A_2 \]  \hspace{1cm} (B8)
The average flow velocity at location 1 can be written in terms of the average flow velocity at location 2 and the porosity of the perforated sheet.

\[ U_1 = \sigma * U_2 \]  

(B9)

Substituting this into the incompressible Bernoulli’s equation

\[ \Delta p = \frac{\rho}{2} (U_2^2 - (\sigma * U_2)^2) \]  

(B10)

This leads to

\[ \Delta p = \frac{\rho}{2} U_2^2 (1 - \sigma^2) \]  

(B11)

And finally

\[ \Delta p = \frac{\rho * Q_{ideal}^2}{2 * A_2^2} (1 - \sigma^2) \]  

(B12)

where

\[ U_2^2 = \frac{Q_{ideal}^2}{A_2^2} \]  

(B13)

The actual volumetric flow rate through the perforated sheet is not equal to the ideal volumetric flow rate but less than the ideal volumetric flow rate due to losses.

\[ Q = C_D * Q_{ideal} \]  

(B14)

Where “Q” is the actual volumetric flow rate and “C_D” is the discharge coefficient. The discharge coefficient can be written in terms of the static pressure drop and the sample geometry.

First, the ideal volumetric flow rate is written as the ratio of the actual volumetric flow rate and the discharge coefficient.
Then the ideal volumetric flow rate is substituted into

\[ \Delta p = \frac{\rho * Q^2}{2 * A^2 * C_D^2} (1 - \sigma^2) \]  

(B16)

Then the equation is solved for the discharge coefficient

\[ C_D = \frac{Q}{A_2} \sqrt{\frac{\rho}{2 * \Delta p}} (1 - \sigma^2) \]  

(B17)

The equation can be written in terms of \( A_1 \) by multiplying by unity

\[ C_D = \frac{Q}{A_1} \frac{A_1}{A_2} \sqrt{\frac{\rho}{2 * \Delta p}} (1 - \sigma^2) \]  

(B18)

And finally by substituting porosity into the equation

\[ C_D = \frac{Q}{A_1} \sqrt{\frac{\rho}{2 * \Delta p}} \frac{(1 - \sigma^2)}{\sigma^2} \]  

(B19)

The equation can be written in terms of \( \Delta p/u \)

\[ \frac{\Delta p}{u_1} = \frac{\rho}{2 C_D^2} \left( \frac{1 - \sigma^2}{\sigma^2} \right) u_1 \]  

(B20)

A correlation between the discharge coefficient and sample porosity has been established for a particular set of metallic perforated samples \(^3\). If this holds, then the sample resistance can be written as only a function the sample porosity.
Appendix C

Compressibility Effects on the Flow Resistance Measurement

The theory used for the acoustic flow resistance measurement assumes that the fluid is incompressible. This is a very good assumption for the acoustic liner designs with low Mach number flows; however, there are other applications where compressibility effects can alter the results. It is important to know how to take the fluid compressibility into account. This is performed using the isentropic relations to determine how the density change effects the resistance measurements. The methodology for determining the compressibility effects on the flow resistance measurement has been established \[2\].

The flow resistance measurements taken at very low Mach numbers were the fluid is incompressible and the resistance vs. flow velocity is linear. The data can be extrapolated to higher Mach numbers were the fluid is compressible by multiplying the resistance and flow velocities by correction factors that are functions of the flow Mach number and the fluid properties.
The isentropic flow equation relating the ratio of the total and static temperatures to the flow Mach number and the specific heat ratio

\[
\frac{T_0}{T} = \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^\frac{1}{\gamma - 1}
\]  

(C1)

Where “\(T_0\)” and “\(T\)” are the total and static temperatures respectively, \(M\) is the flow Mach number, and \(\gamma\) is the specific heat ratio. The isentropic flow equation relating the ratio of the total and static pressures to the ratio of the total and static temperatures

\[
\frac{P_0}{P} = \left( \frac{T_0}{T} \right)^{\frac{\gamma}{\gamma - 1}}
\]  

(C2)

where “\(P_0\)” and “\(P\)” are the total and static pressures respectively. The equation of state is used to relate the static pressure to the density of air and the static pressure.

\[
P = \rho \cdot R_{	ext{air}} \cdot T
\]  

(C3)
where “ρ” is the density of air and “R\text{air}” is the gas constant for air. The continuity

\[ \rho_s \ast U_s \ast A_s = \rho_c \ast U_c \ast A_c \quad \text{(C4)} \]

where “ρ_c” is the density of air in the constricted area of the sample, “U_c” is the average particle velocity in the constricted area of the sample, “A_c” is the area of the constriction “ρ_s” is the density of air upstream of the sample, “U_s” is the average particle velocity upstream of the sample, and “A_s” is the area upstream of the constriction. The state equation is substituted into both sides of the continuity equation

\[ \frac{\rho_s}{R_{\text{atm}} \ast T_s} \ast U_s \ast A_s = \frac{\rho_c}{R_{\text{atm}} \ast T_c} \ast U_c \ast A_c \quad \text{(C5)} \]

The average particle velocity in the constricted area can be written in terms of the average particle velocity upstream of the sample and the sample porosity.

\[ U_c = \frac{U_s}{\epsilon} \ast Cor \quad \text{(C6)} \]

where “Cor” is the compressibility correction factor for the average particle velocities.

\[ Cor = \frac{P_s}{P_c} \ast \frac{T_s}{T_c} \quad \text{(C7)} \]

This is the correction factor that can be applied to the resistance calculation.

\[ R_{\text{compressible}} = R_{\text{incompressible}} \ast Cor \quad \text{(C8)} \]

This correction factor can be written in terms of the average particle Mach number before the sample and the average particle Mach number in the constricted area. The ratio of the total static temperatures can be written in terms of the Mach numbers.
The ratio of the total static pressures can be written in terms of the ratios of the static temperatures and the specific heat ratio. These ratios can then be written in terms of Mach numbers and the specific heat ratio.

\[ \frac{T_s}{T_0} = \frac{T_s}{T_0} \left( \frac{1 + \frac{\gamma - 1}{2} M_c^2}{1 + \frac{\gamma - 1}{2} M_s^2} \right) \]  
(C9)

These equations are substituted into the compressibility correction factor.

\[ \frac{P_s}{P_0} = \frac{P_s}{P_0} \left( \frac{T_s}{T_0} \right)^{\frac{\gamma}{\gamma - 1}} \left( \frac{1 + \frac{\gamma - 1}{2} M_c^2}{1 + \frac{\gamma - 1}{2} M_s^2} \right)^{\frac{\gamma}{\gamma - 1}} \]  
(C10)

The equation is simplified to.

\[ Cor = \frac{\left(1 + \frac{\gamma - 1}{2} M_c^2\right)^{\frac{\gamma}{\gamma - 1}} \left(1 + \frac{\gamma - 1}{2} M_s^2\right)}{\left(1 + \frac{\gamma - 1}{2} M_c^2\right)^{\frac{\gamma}{\gamma - 1}} \left(1 + \frac{\gamma - 1}{2} M_s^2\right)^{\frac{\gamma}{\gamma - 1}}} \]  
(C11)

For: \(1 > M_c >> M_s\)

\[ Cor = \left(1 + \frac{\gamma - 1}{2} M_c^2\right)^{\frac{1}{\gamma - 1}} \]  
(C13)
A compressibility correction also needs to be applied to the calculation of the particle velocity.  Were

\[ U_{\text{compressible}} = U_{\text{incompressible}} \times \text{Cor}_u \]  \hspace{1cm} (C14)

The particle velocity is calculated assuming an incompressible flow dynamic head. The correction factor applied to the velocity is given as the ratio of the incompressible flow dynamic head over the compressible flow dynamic head.

\[ \text{Cor}_u = \frac{q_{\text{inc}}}{q_{\text{comp}}} \]  \hspace{1cm} (C15)

The incompressible flow dynamic head in the constricted area

\[ q_{\text{inc}} = \frac{\rho_c U_c^2}{2} \]  \hspace{1cm} (C16)

Substituting in the state equation

\[ q_{\text{inc}} = \frac{P_e U_c^2}{2 R_{\text{air}} T_c} \]  \hspace{1cm} (C17)

The incompressible flow dynamic head is then written in terms of the Mach number

\[ q_{\text{inc}} = \frac{\gamma}{2} P_e M_c^2 \]  \hspace{1cm} (C18)

The definition of the compressible flow dynamic head

\[ q_{\text{comp}} = P_0 - P_e = P_e \left( \frac{P_0}{P_e} - 1 \right) \]  \hspace{1cm} (C19)

Substituting the isentropic relationship
\[ q_{\text{comp}} = P_c ((1 + \frac{\gamma - 1}{2} M_c^2)^\frac{\gamma}{\gamma - 1} - 1) \quad \text{(C20)} \]

The compressible and incompressible dynamic heads are substituted into the flow velocity correction factor.

\[ Cor_u = \frac{\gamma * M_c^2}{2 \left(1 + \frac{\gamma - 1}{2} M_c^2 \right)^\frac{\gamma}{\gamma - 1} - 1} \quad \text{(C21)} \]

The relationship between the compressible and resistance and flow velocity

\[ R_{\text{compressible}} = A + B * U_{\text{compressible}} \quad \text{(C22)} \]

where the values of A and B are the slope and intercept obtained from the flow resistance measurement.
Appendix D

Measurement Uncertainty Calculation

The manufacturer specified measurement accuracy and the truncation error of the differential pressure measurement across the acoustic liner sample.

\[ u\{\Delta P_s\} = \pm 0.0001 \times 0.5\ psi + 0.0001\ psi = \pm 0.0006\ psi \]  \hspace{1cm} (D1)

The manufacturer specified measurement accuracy and the truncation error of the differential pressure measurement across the flow meter.

\[ u\{\Delta P_f\} = \pm 0.0001 \times 1\ psi + 0.0001\ psi = \pm 0.00011\ psi \]  \hspace{1cm} (D2)

The manufacturer specified measurement accuracy and the truncation error of the differential pressure measurement between the ambient pressure and the pressure in the apparatus just upstream of the flow meter and atmospheric pressure.

\[ u\{\Delta P_{uf}\} = \pm 0.0001 \times 2\ psi + 0.0001\ psi = \pm 0.00021\ psi \]  \hspace{1cm} (D3)

The manufacturer specified measurement accuracy and the truncation error of the ambient pressure measured by the barometric gauge.

\[ u\{P_s\} = \pm 0.073\ psi + 0.015\ psi = \pm 0.088\ psi \]  \hspace{1cm} (D4)

The manufacturer specified measurement accuracy and the truncation error of the ambient temperature measured by the barometric gauge.

\[ u\{T_s\} = \pm 1.8R + 1R = \pm 2.8R \]  \hspace{1cm} (D5)
The general uncertainty equation applied to the pressure upstream of the flow meter and the resulting measurement accuracy

\[ u\{P_f\} = \sqrt{u(\Delta P_{uf})^2 + u\{P_s\}^2} \]

\[ u\{P_f\} = \sqrt{(0.00021 \text{ psi})^2 + \{0.088 \text{ psi}\}^2} = \pm 0.088 \text{ psi} \] \hspace{1cm} (D6)

The general uncertainty equation applied to the flow rate through flow meter and the resulting measurement accuracy corresponding to the maximum pressure drop

\[ u\{Q\} = (B * + C * \Delta P_f) * u\{\Delta P_f\} \]

\[ u\{Q\} = (5.51885 + 0.0440690 \times 13.88 \text{ in} H_2O) \times 0.003045 \text{ in} H_2O = \pm 0.03543 \text{ CFM} \]

\[ u\{Q\} = \pm 0.00001672 \frac{m^3}{s} \] \hspace{1cm} (D7)

The general uncertainty equation applied to the calculation of the corrected particle velocity and the resulting measurement accuracy

\[ u\{Uc\} = \left( \frac{P_f T_0^{2.5}}{P_0 T_f^{2.5}} \right)^2 \times u\{Q\}^2 + \left( \frac{Q P_f T_0^2}{P_0 T_f^2} \right)^2 \times u\{A\}^2 + \left( \frac{Q T_0^{2.5}}{P_0 T_f^{2.5}} \right)^2 \times u\{P_f\}^2 + \left( 2.5 \frac{Q P_f T_0^{2.5}}{P_0 T_f^{2.5}} \right)^2 \times u\{T_f\}^2 \]

\[ u\{Uc\} = \pm 2.95 \text{ cm/s} \] \hspace{1cm} (D8)

The general uncertainty equation applied to the calculation of the corrected resistance and the resulting measurement accuracy

\[ u\{R_e\} = \left( \frac{A P_f}{Q P_f} \right)^2 \times u\{\Delta P_e\}^2 + \left( \frac{\Delta P_e P_f}{Q P_f} \right)^2 \times u\{A\}^2 + \left( \frac{\Delta P_e A P_f}{Q^2 P_f} \right)^2 \times u\{Q\}^2 + \left( \frac{\Delta P_e A}{P_f Q} \right)^2 \times u\{P_f\}^2 + \left( \frac{\Delta P_e A P_f}{Q^2 P_f^2} \right)^2 \times u\{P_f\}^2 \]

\[ u\{R_e\} = \pm 0.345 \text{ c.g.s. Rayls} \] \hspace{1cm} (D9)