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TRANSFORMATIONAL NETWORKS AND VOICE LEADINGS
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Abstract

This thesis examines the use of transformation approaches for Anton Webern’s the First Cantata Op.29, No.1. The thesis is divided primarily into two parts: the summary of a theory of transformation and analyses of Op.29, No.1, which is studied in a transformation attitude.

After presenting the introduction, Chapter 1 summarizes a theory of transformation based on David Lewin’s analyses about Webern’s works, and it researches three different approaches of voice leading, especially focusing on Joseph Straus’s transformational attitude. Before getting to the analyses, Chapter 2 presents the background of Webern’s Cantatas No.1, Op.29 as a basic for understanding this piece. The main contents of Chapter 2 are Webern’s thought about Op.29 and the reason why he collected three different poems as one work. The highlight of this study, Chapter 3 contains the two kinds of analyses of the first movement of Webern’s Op.29. The first analysis is focused on transformations of Hexachords and is presented by constructing networks based on those transformative. The second analysis is examined in terms of voice leadings in relation to “Uniformity, Balance, and Smoothness” of Straus’s concepts.
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Introduction

The concept of transformation has been applied to a musical space and distance or measurement since the 1980s.\(^1\) The new theoretical paradigm was developed by David Lewin and formalized by theorists such as David Lewin, Henry Klumpenhouwer, Philip Lambert, Edward Gollin, Richard Cohn.

The transformational attitude asks, “If I would like to change Gestalt A to Gestalt B, what sorts of acceptable transformations in my space will do the best job?” That kind of question tries to find a function among objects and to organize those objects by functions. After all, it helps us to discover a generalizing power of certain transformations.

This transformational attitude is not only applied to the relationship of pitches but also to voice leading. Simply speaking, to examine what kind of relationship exists between two pitch-class sets in succession is the core of the transformational approach in a study of voices. Hence, it will acquire the mapping of pitch-classes induced by transposition or inversion.

This study is to apply to a theory of transformation in the first movement of Webern’s Cantata No. 1, Op. 29, which is a serial piece. Chapter 1 presents a theory of transformation based on Lewin’s *Generalized Musical Intervals and Transformations* and introduces approaches of voice leadings, especially focusing on the transformational attitude. In order to provide a better understanding of his first cantata, there is a characteristic of symmetry, which is one of Webern’s core concepts. Chapter 2 explains the background of Webern’s thoughts about Op. 29 and the reason why he collected three different poems as one work. The highlight of this study,

Chapter 3, contains the two analyses of the first movement of Webern’s Op. 29. The first analysis focuses on the transformations of hexachords. The second analysis examines voice leadings with respect to the transformational attitude.

The goal of this study is to summarize a theory of transformation and to experiment with the theory in Webern’s music. Through experimentation, new understandings of the piece will be acquired: the relationships and networks of hexachords and voice leadings shown in this piece.
I. Theoretical Issues

A. David Lewin’s transformational network models of Webern’s pieces

David Lewin’s *Generalized Musical Intervals and Transformations* develops structures he calls “transformational networks” which are node- and -arrow systems.¹ On the basis of the concept of transformation, he tries to analyze various pieces. Among his analyses, let us review the analyses of Webern’s pieces.

Before starting the review of transformational network models of Webern’s pieces, let us think about “transformation.” The dictionary meaning of transformation is to change the form of something. Like the definition, a transformational attitude seeks a way to change from object A to object B. Example 1 shows how the melodic motive of the “falling minor ninth” is developed in Schoenberg’s piano piece Op.19, No.6, from the intervallic structure of the opening chord in the right hand (Ex.1).² The opening chord (a) consists of three pitch intervals: -5, -9, and -14 semitones. Example 1 (b) displays the same relationship of intervals, shown in repeating the falling minor ninth motives. Thus, observing the process of transformation and seeking acceptable transformations (T₅, T₉, and Tₑ₄) in pc space is the core in analyzing pieces in terms of transformational attitude.

² Ibid. 159-60.
Example 1) Lewin’s example for Schoenberg Op.19, No.6

\[
\begin{align*}
\text{a) } & \quad \text{b)} \\
-5 & \quad -9 \\
-14
\end{align*}
\]

In Webern’s *Five Pieces for string quartet*, Op.5, No.4, Lewin explains interesting serial operations.\(^3\) The operation TLAST transposes a series by its last interval, and the operation TFIRST transposes a series by its first interval. TFIRST -1 transposes a series by the complement of its first interval. Example 2 displays three series of pitch classes, arranged in a network that involves TFIRST-1 and TLAST. The first interval of \(<\text{C- E- F#- B – C# - Bb}>\) is four semitones. Thus, TFIRST -1 means transposing by 8 semitones. As a result, \(<\text{A flat-C- D- G- A- E flat- F sharp}>\) is the T8 of \(<\text{C- E - F#- B – C# - Bb}>\). Also, when we apply the operation TLAST to \(<\text{F- A- B- E- F# - C- Eb}>\), \(<\text{Ab- C- D- G- A- Eb - F#}>\) becomes the T3 of \(<\text{F- A- B – E- F# - C- Eb}>\), because the last interval is three semitones. However, why does Lewin use the operation TFIRST-1 and the operation TLAST instead of T8 and T3? The reason is that the transpositional labels would not reveal the balancing centrality of the form beginning on Ab, the form that ends the piece.

Example 2) The operation TFIRST-1 and the operation TLAST

\[
\begin{align*}
\text{C- E- F#- B- C #- G- Bb} \\
\text{TFIRST -1} \downarrow \\
\text{Ab- C - D- G- A - Eb - F#} \\
\text{TLAST} \uparrow \\
\text{F- A- B- F# - C - Eb}
\end{align*}
\]

\(^3\) Ibid., 188.
Lewin’s article “Transformational Techniques in Atonal and Other Music Theories” introduces the opening of Anton Webern’s *Five Pieces for String Quartet*, Op. 5, No. 2. Example 3 shows the motive in the Viola, set X = \{G, B, C\#\}, and several of its forms. The accompaniment chord of measure 2 produces a form of X, which is T₈ of X. The first chord in the accompaniment does not have any form of X. However, according to Lewin’s analysis⁴, the first chord projects strongly the structuring force of pitch-class inversion about D. Under this inversion the pitch class D mirrors itself, the dyad \{A, F\} mirrors the \{G, B\} of the viola motive, and the Ab mirrors itself. He calls the inversion “I”, which signifies inversion about D or about Ab. In m.1 the dyad \{G, B\} becomes extended to X= \{G, B, C\#\}. The pairing of Eb with C#, as an I – related partnership about D, establishes \{A, F, Eb\}= I (X) (Ex.4).

Example 3) Anton Webern’s *Five Pieces for String Quartet*, Op. 5, No. 2, mm.1-4

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⁴ David Lewin, “Transformational Techniques in Atonal and Other Music Theories,” *Perspectives of New Music* Fall-Winter (1982), 312.
Example 4) Lewin’s example showing Motive X and I (X)

The four note sonority \{G, Eb, A, F\} in m.2 consists of two X-forms. They are T8(X), represented by the three lowest notes of the sonority, and I(X), represented by the three highest notes of the sonority. In addition, the four note sonority \{G, Eb, A, F\} establishes a new inversional symmetry which he denotes by “J”, that is inversion about F#.

Inversion J continues into the viola melody \{F#, B, G, F, C#\} over mm. 2-3. The pentachord \{F#, B, G, F, C#\} is its own J-inversion. Within this melodic fragment two x-forms exist; these are the inverted form K(X) = \{B, G, F\} and the transposed form T6(X) = \{G, F, C#\}. K is an inversion that transforms the pitch classes G and B each into the other. Both K(X) and T6(X) leave invariant two members of X; K(X) and X share the pitch classes G and B, and T6(X) and X share the pitch classes G and C#. Also, when the viola moves to C# in m.3, a new form of X is formed as T2(X) = \{Eb, A, C#\}.

Finally, the transformational journey of X ends as (X) comes back in m.3.

Example 5 presents the various forms of X in a convenient format and Example 6 shows a transformational network of X-forms.
Example 5) Lewin’s example showing various X-forms

Furthermore, Lewin introduces a non-intervallic transformation network in connection with serial transformation shown in Webern’s Piano Variations Op. 27.  

Webern is fond of using RICH as row transformations in his serial works. RI-chaining operation RICH is that retrograde-inverted form of a pitch-class set, preserving the last two elements. Thus if \( p = B-D-E-F \), then RICH(p) is E-F-G-Bb. RICH(RICH(p)) is G-Bb-C-Db. The RICH transform of RICH(p), being a retrograde-inverted form of a retrograde-inverted form of p, must always be

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some transposed form of p. In the example just described, the interval of transposition is 8: G-Bb
-C- Db is the 8 transpose of B-D-E-F. The prime row of his Op.27 is Eb - B - Bb - D - C#- C -F#
- E- G- F- A- G#, RICH (P) is A- G#- C- Bb- C#- B- F- E- Eb- G- F#- D. Example 7 presents the
row transformation in mm.19-37 of the first movement.

Example 7) Lewin’s example showing Webern’s Piano Variations Op. 27, I mm.19-37

Summarizing the method of constructing transformational networks based on Lewin’s
analyses, I believe that it is important first to identify what my object is, and seek the common
factor shown in a certain piece. The common factor may be a hexachord which has a certain pc
set or a pentachord, or even a twelve-tone row. A piece may comprise many hexachords or
pentachords, or twelve-tone rows, which have the same character. Keep in mind that objects
have the same quality; for example, if I decide on H={C, C#, D, D#, E, F} as an object, other
objects that have any forms of H are all set class (012345). The second is to examine the
relationships among objects. The objects are related by inversion or transposition. In the case of
inversion, one must distinguish diverse inversions. There are twelve kinds of inversions
according to axes: inversion about C or F#, inversion about C and C# or about F# or G, inversion
about C# or G, etc. In order to prevent confusing various inversions, we could use the names of
inversions. Sometimes, one can discover that a certain inversion has a specific result; for example, a certain operation maps the object into the unique form of the pentachord which inverts the given form and leaves invariant the four-note chromatic tetrachordal subset as Lewin has shown in an analysis of Stockhausen’s *Klavierstück III*. When \( P = \{C, C\#, D, Eb, G\} \), \( I(P) = \{Ab, C, C\#, D, Eb\} \). In this case, one can focus not only on the inversion about \( C\# \) and \( D \) but also on the character of four-note chromatic tetrachordal subset. The final step is to construct transformational networks based on objects and functions that one has already found. There are some methods of constructing networks: a “blow-by-blow” network, a network for a spatial structure, and a “narrating” network based on the spatial structure\(^6\). A blow-by-blow network has a chronological and narrative structure, so the network progresses according to the flow of the piece. The two other networks reflect a more spatial sense.

Lewin’s analyses display how the musical objects transform and how the transformed objects become the basis for a network. The transformational sense allows us to notice the movement and mapping between pc sets.

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\(^6\) The good examples of three kinds of networks are presented in Lewin’s “Making and Using a Pc set network for Stockhausen’s *Klavierstück III*,” *Musical Form and Transformation* (New Haven: Yale University Press, 1993).
B. Approaches to Post-Tonal Voice Leading

Before presenting various voice leadings in post-tonal music, let’s think about the definition of voice leading. Robert Morris and Brian Alegant say in their articles that Voice-leading denotes the progression of pitches to form horizontal voices or lines. Also, John Roeder defines voice leading as follows:

When one simultaneity succeeds another of the same size, we hear a voice as the succession of pitches in the same registral order position, and we consider the voice leading to be the intervals spanning those pitch successions. In the pc-series model of simultaneities, we present the voice leading between two simultaneities by a list of the ordered pitch-class intervals connecting the order corresponding members of the pc series.

Example 8 represents the voice-leading by the list of the ordered pitch-class intervals according to Roeder’s definition. The ordered pc interval from the lowest member of the first chord to the lowest member of the second chord is 1, the ordered pc interval from the second-lowest member of the first chord to the second-lowest member of the second chord is 3, and so forth. Thus, the voice leading is <1, 3, 1, 11, 7>.

Example 8) Voice leading (Schoenberg’s Six Short Piano Pieces Op. 19, No. 4, m. 6)

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There are three different approaches to post-tonal voice leading. According to Joseph Straus, “approaches to voice leading have generally had one of three distinct theoretical orientations.” The first approach is to adapt or to extend models of tonal voice leading. Scholars such as Felix Salzer, Roy Travis, James Baker have attempted to apply Schenkerian prolongation to post-tonal music. Straus regards this approach as “prolongational.” The second approach is to describe the linear organization of atonal music in terms of pitch lines linked through common registers, timbres, metrical placements, etc. Straus views this approach as “associational.” This voice leading model is based on “the pitch-class set theory of Allen Forte, Robert Morris, and John Rahn.” The third approach to “linear pitch continuity is found in the various transformational models of musical structure devised by David Lewin, Henry Klumpenhouwer, and others.” “A simple but effective approach to voice leading as transformation is to examine the relations of the intervals between the pitch class set in chords and their influence on the intervals between different chords.”


13 Robert Morris, 176.
1. Prolongational approach

Despite the fact that Schenker believed that organic structural coherence is lacking in music after Brahms, more recent theorists have sought to extend his principles of structural levels and prolongation to the analysis of post-tonal music. The trials have not always been successful. Even though some theorists, such as Joseph N. Straus, have suggested problems with this method, the prolongational approach can lead to interesting analyses.\(^{14}\)

James M. Baker’s analysis of Schoenberg’s *Six Little Piano Pieces*, Op.19, no.1 (1911) is very interesting.\(^{15}\) There is no fundamental line and fundamental bass, which are the basis of a Schenkerian analysis, for this piece reflects Schoenberg’s famous aim “to base our thought, not on the seven tones of the major scale, rather, on the twelve of the chromatic scale.”\(^{16}\) However, the graph shows various prolongational techniques of Schenker (Example 9).

In order to adopt Schenkerian techniques for atonal music, one may begin by taking note of any conspicuous repetitions of pitches or sets of pitches. The reason is that a repetition helps to ensure continuity and reinforces our perception of the antecedent and consequent relationship. For instance, in Op.19, No.1, the final trichord of the first phrase (m.1), the G-C-F\(^\#\), is restated in the second phrase (m.2).

\(^{14}\) Joseph Straus explains the problems of the prolongational approach in two ways in his article “The Problem of Prolongation in Post-Tonal Music(1987).” First, there is the problem of harmonic support. If one does not know which harmonies are consonant and dissonant, one cannot decide which notes are structural and which are embellishing. In atonal music, to distinguish between consonance and dissonance is not systematical. Therefore, deciding whether a note is structural or embellishing becomes arbitrary. Second, there is that of embellishment. In tonal music, there are three types of embellishments: passing tones, neighboring tones, and arpeggations. Conversely, atonal music makes it impossible to identify the embellishments with any confidence.


Example 9 presents some specific characters by a prolongational approach. First, the repetition of B-A [11-9] stands out prominently. The B-A [11-9] in mm.1-2 is repeated in m.3. The time span of the B-A [11-9] in m.3 is shorter than that in mm.1-2. Also, the vertical A-B [9, 11] endorses the B-A [11-9]. The second is the succession of ic3. The order of succession is the A-C(m.1), the Bb-Db(m.2), the B-D(m.2-3), the C-Eb, the B-D, and the C-Eb (m.3). The first three ic3s move forward the C-Eb by the chromatic progression. The B-D in m.2 is prolonged to that in m.3.

Example 9) The voice leading by prolongational approach
(Excerpt from Baker’s analysis)
2. Associational approach

Associational analyses generally find linear projections of harmonies or harmony-types on the musical surface. Musical tones are related with any contextual means, including register, timbre, metrical placement, dynamics, and articulation. These notes may form the linear structures.

As Straus mentioned before, the important thing is that the entities being discussed in associational analyses are not voices but lines. A line is a series of notes that share some distinctive musical characters such as they are all played in the lowest register or the highest register, or they are all played forte or piano. Conversely, a voice is an aspect of counterpoint and polyphony that recognizes each part as an individual line, not merely as an element of the resultant harmony. Voices result from the workings of the system such as the resolution of the seventh chord in tonal music. In short, lines depend on the contextual means, and voices are operational.

The right hand in Schoenberg’s *Three Piano Pieces* Op.11, no.1, mm.34-35 consists of the same trichord-type (Example 7). [D-flat, E, F], [B-flat, C-sharp, D], [A, C, D-flat], [C-flat, D, E-flat], and [G, B-flat, B] are all members of set-class 3-3 (014). Also, according to the register, the horizontal lines in measure 34 are set-class 3-3 (014): [F, D, D-flat], [D-flat, B-flat, A] and [E, C-sharp, C]. Thus, each registral line presents the intervallic content of chords.

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18 Ibid.

Example 10) The associational approach (Schoenberg Op. 11, No.1, mm.34-35)

Scriabin’s *Prelude* Op. 74, No. 4 is an interesting example. The first chord is a member of set-class 4-17(0347). Example 8 shows a large-scale statement of the same set class (0347), [C, D-sharp, E, G] in mm.1-3. The voice leading is related with register and dynamics. Dynamics start at C and D-sharp, and E and G also have a crescendo. The pitches go to the climax for the first phrase.

Example 11) Scriabin, *Prelude* Op. 74, No.4, mm.1-3

Webern’s *Drei Lieder*, Op.25, no.1 consists of many pc sets 3-3 (014). All triple sixteen notes are pc set 3-3, and many trichordal segmentations present pc set 3-3 in the voice. Now, when one studies this piece in terms of voice leading, one can find a large-scale statement of pc
set 3-3 according to register in the voice of mm.1-5. The first note G, the lowest note B, and the highest note G-sharp produce pc set 014 (Example 12).

Example 12) Webern, *Drei Lieder*, Op. 25, No.1, mm. 1-5
3. Transformational approach

While the associational approach is related with lines, the transformational approach is a study of voices, which is simply the mapping of pitch-classes induced by transposition or inversion. The core of this approach is a research of relationships between pitch-class sets in chords. Roeder explains the benefits of describing voice leading as a generalized interval. It allows us to understand voice leading in ways that are structurally analogous to the pitch operations of transposition and inversion. Another benefit is that we can hear voice-leading-like gestures in successions of musical structures that are not simply simultaneities.

However, the notations of voice leading in the transformational approach are different according to scholars such as Roeder, Straus, and Klumpenhouwer. Roeder notates voice leading like example 1, <1, 3, 1, 11, 7>. The order of intervals in the bracket is from the lowest voice to the highest voice. Some voice leading has characteristic properties according to his explanation. The first example is the voice leading in which every voice contains the same interval. The second characteristic type of voice leading is that all intervals consist of 11, 0, or 1. This type is shown in atonal repertories, such as Webern and Schoenberg. The third distinctive type is complementary contrary motion, in which each interval is complemented, mod 12. In example 10, interval 1 of the second lowest voice is the complement of interval 11, the highest voice. Interval 1 of the third lowest voice is the complement of interval 11, the second highest voice. Example 6 also has the character of the second type. All intervals are 11, 0 or 1.

21 John Roeder, 46.
22 Ibid, 42.
Example 13) Webern, *Fünf Lieder*, Op. 4, No. 1, m. 3

Roeder asserts that we can proceed to define a transposition-like operation on pc series with voice leading defined as a generalized interval connecting element series. In Example 14, the voice leading connecting these two series, \(<F, Db, A, E>\) and \(<E, C, Ab, F>\), is the element-series interval \(<11, 11, 11, 1>\). The next voice leading between \(<E#, C#, A, G#>\) and \(<E, C, G#, A>\) is also \(<11, 11, 11, 1>\). Thus, we can regard the second pc series to be a transposition.

Example 14) Transposition of voice leading \(<11, 11, 11, 1>\), Scriabin, Op. 74, No. 4, mm.14-15

In the case of inversion, Roeder suggests an interesting example, Schoenberg’s *Five Pieces for Orchestra* Op. 16, No. 4, mm.2-3.\(^{23}\) The diagram shows that every pair of successive

\(^{23}\) John Roeder, 47-9.
simultaneities is measured by the same element-series interval, \(<11, 11, 11, 1, 1, 1\>\).

Interestingly, inversion relations are shown in two ways. When one regards the middle two chords as the axes, the outer pairs of notes establish inversionsal relations. For example, the highest voice is related by inversion \(I_{F#}^{G}\), and the lowest voice is related by \(I_{E}^{F}\). Therefore, the last chord is the pc series inversion about the two middle chords of the first pc series. Another inversion relation can be found between each voice. The two middle voices are related by \(I_{C}^{C}\). On the other hand, the outer voices are related by \(I_{B}^{C}\). This means that the highest voices and the lowest voices are paired and they are related by \(I_{B}^{C}\). Also, the relationship of the second-highest voices and the second-lowest voices is the same.

Example 15) Roeder’s example showing inversionsal relation, Schoenberg’s *Five Pieces for Orchestra*, Op.16, No.4, mm. 2-3

\[
\begin{align*}
D\# & --1----E --1---- F-----1---- F# --1---- G --1---- G# --1---- A --1---- Bb \\
B & --1----C --1----C# --1---- D --1---- D# --1---- E --1---- F --1---- F# \\
G & --1----G# --1---- A --1---- Bb --1---- B --1---- C --1---- C# --1---- D \\
F & --1---- E --1---- Eb --1---- D --1---- Db --1---- C --1---- B --1---- Bb \\
C & --1---- B --1---- Bb --1---- A --1---- Ab --1---- G --1---- F# --1---- F \\
G# & --1---- G --1---- Gb --1---- F --1---- E --1---- Eb --1---- D --1---- Db
\end{align*}
\]
Even though Roeder’s method explains how individual voice-leading intervals connect to the interval structure of simultaneities, it does not adapt to more general textures. A valuable solution to this problem was suggested by Klumpenhouwer. He presents a “Generalized Model of Voice leading” as a permutation of registral order positions in his dissertation.

Klumpenhouwer explains the voice leading paradigm as follows:

The voice leading paradigm is applied in the contexts of various chord models. One is the pitch-class collection, an idea that involves a family of T- or I-related chords. Another chord model is developed and expanded from Hauptmann’s discussion of major and minor triad. Yet another model interprets chords as transformational networks, whose nodes reference pitch-classes related under various transposition and inversion operations.24

After all, he treats voice leading essentially as a single reordering gesture, one that is conceivable in the context of other models of chords besides the one just described.25

Whereas Klumpenhouwer is primarily concerned with the registral permutations that result from the motion of the voices, Straus focuses instead on the integrity of the voices themselves, over potentially large musical spans, and their interaction with contextual lines.26 Straus’s study of voice leading is very practical for a novice to apply to his research. Thus, I will review briefly the method.

When two chords are related by transposition or inversion, each note in the first chord maps onto a corresponding note in the second. Thus, we can create a network of linear connections between two chords such as in example 16.


25 John Roeder, 43.

Example 16) Voice leading from Chord A to Chord B by transposition or inversion

\[
\begin{align*}
A_1 \rightarrow B_1 \\
A_2 \rightarrow B_2 \\
A_3 \rightarrow B_3 \\
A \rightarrow B \\
\text{Tn} \quad \text{In}
\end{align*}
\]

Example 17) The chords by transposition: Webern’s *Five Pieces for String Quartets*  
Op. 5, No. 2
Example 17 shows the voice leading between two chords by transposition. The first chord [3, 7, 9] and the second chord [7, e, 1] are set class 3-8 (026). The first chord is transposed by four semitones. The pitch class mappings may or may not be coincident with register or instrumentation. In the case of Example 17, there is not coincident with register and instrumentation. The pitch A of violin 2 goes to C sharp, the lowest note of the cello.

Inversion works the same way. It maps notes in one harmony onto corresponding notes in the next, and thus creates transformational voices. Example 18 traces the voice leading between chords by inversion. The first trichord [1, 2, 5] in the violin and [4, 7, 8] in the piano are members of 3-3 (014) in Schoenberg’s *Violin Phantasy*, m. 85. The two chords are related by $I_{v}$.

Example 18) Schoenberg’s *Violin Phantasy*, m. 85
Sometimes inversionally symmetrical harmonies may be understood as related by either transposition or inversion. In Example 19, [0, 4, 8] and [5, 9, 1] have six possible operations. At the transpositional interpretation, T₁₁, T₇ and T₃ are possible. Also, T₁₁, T₅₁ and T₉₁ are established. Among these six operations, we can select the most appropriate transformation. In Schoenberg’s *Violin Phantasy*, I prefer T₉₁ because each pair of trichord is in the relationship with T₉₁ (Example 20).

Example 19) Transpositionally and inversionally symmetrical set:
Schoenberg’s *Violin Phantasy*, m. 87

---

27 Some set classes contain sets that can map entirely onto themselves under inversion. Such set classes are called *inversionally symmetrical*. The intervals reading from left to right will be the same as the intervals reading from right to left in inversionally symmetrical sets.
Example 20) Schoenberg’s *Violin Phantasy*, mm.85-88

<table>
<thead>
<tr>
<th>mm. 85</th>
<th>86</th>
<th>87</th>
<th>88</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vln. [1,2,5] (014) [5,9,1] (048) [9, e,3 ] (026) [9, e, 3] (026) [0,4,8] (048) [6,7,t] (014) [0,4,8] (048)</td>
<td>T9I</td>
<td>T9I</td>
<td>T9I</td>
</tr>
<tr>
<td>Pf. [4,7,8] (014) [6,t,0] (026) [6,t,0] (026) [5,9,1] (048) [e,2,3] (014) [6,7,t] (014)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

However, all chords are not related with each other by transposition or inversion. Thus, Straus contrived two criteria, *uniformity* and *balance*, based on transposition and inversion, which lie at the core of most of our theories of atonal harmony.\(^{28}\) The definitions of two criteria are following:

Uniformity refers to the extent to which the voices move by the same intervallic distance. The more uniform the voice leading, the more closely it approximates traditional transposition. Balance refers to the extent to which the voices flip around the same axis of inversion, that is, the extent to which they move by the same index number. The more balanced the voice leading, the more closely it approximated traditional inversion. Uniformity and balance thus represent generalization of transposition and inversion for the purpose of describing voice leading.\(^{29}\)

Example 21 arranges the six voice leadings according to uniformity, measured by the extent to which the voices move by the same or the closest interval. Two pitch-class sets of the same size are related by neither transposition nor inversion. The first voice leading approximates a transposition at T1. The soprano and the bass move by T1, and the alto move by T4. This is an instance of “near-transposition.” Two of the three voices move by the same interval. The alto has *offset* by three semitones from a transposition at T1. Offset is used as the principal means for

\(^{28}\) See Straus 2003, 305-52.

\(^{29}\) Joseph Straus (2003), 314.
measuring degree of uniformity. Straus explains that if voice leading has a higher offset, it is considered less uniform. Because the third voice leading has offset (4), they are treated less uniform than the first.

Example 21) Voice leading uniformity in the progression from \{C, E, G\} to \{E, F, G#\}

![Example 21 diagram]

\[\begin{align*}
\text{G}^i & \rightarrow \text{F} \\
\text{C}^4 & \rightarrow \text{E} \\
\text{E}^1 & \rightarrow \text{G#} \\
\text{T}_1 & \quad \text{(3)} \\
\text{G}^i & \rightarrow \text{F} \\
\text{C}^5 & \rightarrow \text{E} \\
\text{E}^0 & \rightarrow \text{G#} \\
\text{T}_1 & \quad \text{(5)} \\
\text{G}^i & \rightarrow \text{F} \\
\text{C}^8 & \rightarrow \text{E} \\
\text{E}^0 & \rightarrow \text{G#} \\
\text{T}_1 & \quad \text{(4)} \\
\text{G}^9 & \rightarrow \text{F} \\
\text{C}^5 & \rightarrow \text{E} \\
\text{E}^8 & \rightarrow \text{G#} \\
\text{T}_5 & \quad \text{(5)} \\
\text{G}^i & \rightarrow \text{F} \\
\text{C}^4 & \rightarrow \text{E} \\
\text{E}^4 & \rightarrow \text{G#} \\
\text{T}_4 & \quad \text{(6)} \\
\text{G}^9 & \rightarrow \text{F} \\
\text{C}^8 & \rightarrow \text{E} \\
\text{E}^1 & \rightarrow \text{G#} \\
\text{T}_8 & \quad \text{(8)}
\end{align*}\]

However, there are two other measures of uniformity: consistency and span. Like example 14, if voices move by the same interval, a condition is called consistency. If all of the voices move consistently by the same ordered pitch-class interval, the voice leading is entirely uniform. Another way of measuring voice leading uniformity involves the extent to which the voice-leading intervals diverge. The smaller the range of difference is among the voice leading intervals, the more uniform the voice leading.

The G, the C, and the E move by the different pitch-class intervals, 10, 8, and 0. Among three pc intervals, pc interval 10 plays a role as a convergence point. Thus, offset can be calculated by adding the distance of 10 and 8 and the distance of 10 and 0. Therefore, offset is 4 (2 + 2).
Example 22 shows the same progression of Example 21 but reorders the six voice leadings according to their balance. The superscripted numbers represent the index of inversion. The first voice leading is the most balanced because one of its voices describes index 9 and the axes of the remaining voices are offset by one semitone and offset by two semitones. This means that the first has the fewest offsets. In the second voice leading and the third voice leading, two of their voices describe the same index, 0 and 8. However, the axes of the remaining voice are offset by four semitones. As a result, the second and the third are less balanced than the first. Noticeably, just as offset numbers work for voice-leading uniformity, they work for voice-leading balance.

Example 22) Voice leading balance in the progression from \{C, E, G\} to \{E, F, G#\}
Straus suggests one additional factor to consider in voice leading: *smoothness*. Straus defines smoothness as emphasizing minimal total displacement of the voices:

Voice-leading smoothness is measured by the *total displacement*, the sum of the intervals traversed by each note from its origin in X to its destination in Y. The intervals in question are *unordered pitch-class intervals (interval classes)*. . . . the one that has the lower total displacement is the smoother. Unlike transposition and inversion, smoothness is a quality, not an operation.\(^{32}\)

Example 23 reconstructs the previous example in terms of voice-leading smoothness. Seek displacement of each voice and sum the displacements. The voice leading which has the lowest total displacement is the smoothest one. The total displacements of the first, the second and third voice leading are all six. Thus, the first three voice leadings are smoother than the others.

\(^{32}\) Joseph Straus (2003), 320-22.
In sum, the core of transformational attitude is a study of the relationship of pitch-classes induced by transposition or inversion. Even though there are differences of method according to scholars, the basic ideas are the same. Based on these studies of voice leading, I will examine Webern’s *Cantata No. 1*, Op. 29, No. 1.
C. Symmetry

An outstanding characteristic of Webern’s music is its condensation into structures based on a high degree of symmetry. The first cantata is also a good example of symmetry. Thus, before analyzing the piece, it is crucial to know the concept of symmetry. Even though the concept of symmetry can be applied to a wide range of phenomena, this section focus on a brief introduction to the concept of symmetry and to the principal symmetrical operations applicable to music.

There are three kinds of symmetry according to Robert Morgan’s article: spatial symmetry, pitch-class symmetry, and time symmetry. His article introduces and summarizes the basic concepts of symmetry in the abstract. Borrowing his terms of symmetry, I would like to develop the ideas with musical examples. However, I do not treat time symmetry because it is not necessary in my analysis.

In pitch-class symmetry, symmetrical pitch-class collections are those that remain invariant in pitch-class space under a group of transformations. Pitch-class sets are unordered collections occupying position in the pitch-class space, which is a cyclical pitch space. Thus, they are rotational. The rotational character stems from cyclicity: each pitch class returns to its initial position after traversing the other eleven equidistant positions by chromatic scale or circle of fifths.

A pitch-class set with transpositional symmetry is one that divides pitch-class space into equidistant smaller segments. The simplest instances are 6-35 (02458t) (the whole-tone scale), 4-35 (01358t) (the major third and minor sixth scale), and 3-4 (015t) (the tritone scale).

---


34 Ibid, 7.
28 (0369) (the diminished triad), and 3-12 (048) (the augmented triad). These sets are capable of mapping entirely onto themselves under transposition. Pc set 4-9 (0167) has intervals of 1, 5, and 1. All map onto themselves at T0, T6. ([0,1, 6, 7] -> T6 -> [6, 7, 0,1] ) It thus has a degree of transpositional symmetry of 2.

Whereas transpositional symmetry is derived from equal octave division, inversional symmetry is derived from corresponding placement of the reflection line that divides the pitch-class space into two mirroring halves. Sets that are inversionally symmetrical can be written so that the intervals reading from left to right will be the same as the intervals reading from right to left. Pc set 4-9 (0167) also has the character of inversional symmetry. The intervallic palindrome of 1, 5, 1 is apparent, and the set maps onto itself under T7 I and T7 I (Example 24).

Example 24) Inversional symmetry of pc set (0167)

Next, spatial symmetry helps to grasp the structure of tones in pitch-space. Webern’s music, especially, presents spatial symmetry in many cases. When one arrays the tones in pitch-space, one can notice that distance between notes has a certain rule in the center of a certain note.

Webern’s *Five Pieces for Orchestra* Op.10, No.4 consists of three sections: m.1, mm.2-4, and mm.5-6. Look at the second section. Example 18 is an analysis applied to spatial symmetry. This diagram is the observation of the distance between notes in pitch space. The interval of A₄ and D₅ is 5, and that of G#₄ and Eb₄ is the same. The distance between A₄ of the clarinet and Bb₅ of the solo viola is 13, and that between G#₄ of the trombone and G₃ of the same instrument is also 13. The rest notes shown in the second section are related to the notes of the third section. The interval of A₄ of the clarinet and B₄ of the trumpet is 2, and that of G#₄ of the trombone and F#₄ of the harp is the same. In addition, the distance between A₄ of the clarinet and F₅ of the trumpet is 8, and that between G#₄ of the trombone and C₄ of the clarinet is also 8. As a result, one can notice that A₄ of the clarinet and G#₄ of the trombone play the role of the center. In the pitch space, the notes have a symmetrical structure based on the center tones, A₄ and G#₄.

Example 25) Spatial symmetry, Webern’s *Five Pieces for Orchestra*, Op. 10, No. 4, mm. 2-5
IV.

Webern op.10 No.4

**Fließend, äußerst zart**

\[ \text{Tempo} \quad \text{vi} \quad \text{tempo} \quad \text{vi} \quad \text{Tempo} \quad \text{vi} \quad \text{Tempo} \]

**Col.**

\[ \text{ppp} \quad \text{pp} \]

**Haf.**

\[ \text{pp} \quad \text{pp} \]

**Kl. T.**

\[ \text{PPP} \]

**Solo-Gs. in Epf.**

\[ \text{PPP} \]

**Solo-Dr. in Dpf.**

\[ \text{PP} \]
If one tries a different symmetrical reading of the same passage, one can acquire a different axis. In terms of pitch-class symmetry, D, Eb, F, G, G#, and B of the middle section can make the intervallic palindrome. The intervals of B, D, Eb, F, G, G#, and B are 3, 1, 2, 2, 1, and 3. As a result, it establishes inversive symmetry around F or B, while the symmetry in pitch space is around G-sharp and A₄ in the middle section of Webern, Op. 10, No. 4. Therefore, we can seek different answers according to whether symmetry occurs in pitch-class space or in pitch space.
II. Background of Webern’s *Cantata No. 1*, Op. 29

Webern’s *Cantata No. 1*, Op. 29 is written for soprano solo, mixed chorus and small orchestra and is a three-movement work. The first and third movements are choral, and the second movement is a soprano solo with orchestra. The three poems from which Webern selected excerpts as texts for his *Cantata No. 1* were from different collections of Hildegard Jone’s poetry.

Letters, which Webern sent to Hildegard Jone and Humplik Jone, present not only the process of the composition but also the work itself and Webern’s attitude to poetry. Based on the letters, knowing the background helps to understand of the composition.

The first movement is quite pictorial in the sense of a pastoral scene. After a twelve-measure introductory instrumental section, chordal phrases are interspersed with loud, polyphonic sections and end with a low *sforzando* note on muted trombone, followed by a fortissimo stroke on the timpani along with a cymbal crash. The effect is like the approach and arrival of a storm. All of the choral phrases of the first movement are chordal. They are almost entirely homorhythmic, although they are not in a strict sense, homophonic. In comparing the choral sections and the instrumental episodes, the first movement is based on the contrast between ‘firm’ choral sections and ‘loose’ instrumental episodes.

The other choral movement of Op. 29, No. 3, was called by Webern a four-part double fugue. He also used the terms “schezo” and “variations” as he described it to Hildegard Jone;
“but” he said, it is “still a fugue.”\textsuperscript{2} These explanations of the last movement were often shown in his letters to Jone and Willi Reich. The choral section of this movement is mostly polyphonic and involves generally the same type of imitative relationships such as his explanations.

He not only selected three poems from different collections but also did not use the whole text that he thought of originally; the musical form demanded it differently. The texts for the three movements of Cantata No.1 are:

I. ‘Blitz und Donner’ from the unpublished cycle \textit{Der Mohnkopf}.

Zündender Lichtblitz des Lebens schlug ein aus der Wolke des Wortes, Donner der Herzschlag folgt nach, bis er in Frieden verebbt.

II. ‘Kleiner Flügel Ahornsamen’ from Farbenlehre.

Kleiner Flügel Ahornsamen schwebst im Winde! Musst doch in der Erde Dunkel sinken. Aber du wirst auferstehn dem Tage, all den Düften und der Frühlingszeit; wirst aus Wurzeln in das Helle steigen, bald im Himmel auch verwurzelt sein. Wieder wirst aus dir du Kleine Flügel senden, die in sich schon tragen deine ganze schweigend Leben sagende Gestalt.

III. A verse from the unpublished poem ‘Verwandlung der Chariten.’

Tönen die seligen Saiten Apolls, wer nennt sie Chariten? Spielt er durch den wachsenden Abend, wer denket Apollon? Sind doch im Klange die früheren Namen alle verklungen; sind doch im Aorte die schwächeren Worte lange gestorben; und auch die blasseren Blider zum Siegel des Specktrums geschmolzen. Chairs, die Gabe des Höchsten:

die Anmut der gnade erglänzet!
Schenkt sic him Dunkel dem werdenden Herzen
als Tau der Vollendung.

Why did he feel that the poems had close connections in spite of the fact that the poems
came from three different collections? In a letter to Jone dated 2 December 1939, he wrote:

. . . Aren’t the “little wings” (‘Kleiner Flügel’) and “Lightning and thunder” (‘Blitz und
Donner’) answering the questions posed in the “Chariten” verses, dear Hildegard? Aren’t they saying what is implied by the latter, by the “sound,” the “word,” the “seal of the
spectrum”?³

Webern thought that the “sound,” the “word,” and the “seal of the spectrum” are elements
which serve to bind the third movement to the first and second movements. Thus, he combined
three poems as a work.

³ Anton Webern, Letters to Hildegard Jone and Josef Humplik, edited by Josef Polnauer, trans. Cornelius Cardew,
III. The Analyses of Webern’s *Cantata No.1*, Op. 29, No. 1

Before starting the analyses in terms of a transformational attitude, I will identify the general features, especially about the series of this piece. Because all three movements of Op. 29 employ the same series, it will be necessary to grasp the series itself.

One controversial issue is what the prime form is. The first movement of Webern’s Op. 29 has been analyzed by George Rochberg¹, Jonathan Kramer², Graham H. Phipps³ and Robin Hartwell⁴. Also, there are some books that treat an introduction of Anton Webern’s works⁵. However, the prime form, which each author is discussing, is different. Mainly, these articles can be divided into two opinions. The first opinion is that the prime form is the first soprano melody of the choir from m.14: A- F- Ab- G- B- Bb - Db - C- E- D#- F# - D. It is supported by Rochberg, Kolneder, Hartwell, and Kramer. Second, according to Phipp’s article, the prime form of the series is defined as that which begins in m.1 of the first movement in the trumpet: G- B- G#- A- F- F# - Eb - E - C- Db - Bb - D. The series that starts from the trumpet has the highest register in m.1. In addition, Friedrich Wildgans introduces prime form D# - B- D- C# - F- E- G- F# - Bb - A- C- Ab.

The different views of the prime form might result for two reasons. The first opinion is which section is the core of the piece. The scholars who have the first opinion consider that the movement has a ternary structure consisting of instrumental prelude and postlude that frame the exposition, a contrasting middle choral section. Kolneder expresses that the first movement is based on the contrast between “firm” choral sections and “loose” instrumental episodes. No matter how the authors think of the form, the basic view is that the choral section is both the highlight and the core of the movement as a theme. For that reason they select the first soprano melody as the prime form, despite the fact that the row appears in the middle of the piece.

Phipps follows the ordinary method to figure out a prime form step by step from measure 1. Although he also agrees that the first movement has a ternary structure, the form falls into ‘A’, ‘B’, and ‘A’” section instead of using the term of prelude, exposition, and postlude in his essay. This means that he has no intention to emphasize the choral section, ‘B’. In his viewpoint, section ‘A’ and section ‘A’” do not function as just an introduction or a codetta, and they themselves are characteristic sections. Accordingly, he considers that the prime form starts from the first measure.

Although most scholars who study Webern’s Op. 29 consider the tone row of the choral section as the prime form, I agree with Phipp’s opinion. It is true that the choral section is the core because Webern called this piece “Cantata.” However, each section is equally composed in the structural aspect. Furthermore, the ratio of instrumental sections is extensive. Therefore, in my paper, I will use G- B- G#- A - F- F# - Eb - E- C- Db - Bb - D as a prime form of the series, but I call the prime form not P0 but P7 because the pitch class of G is 7.
The second assumption is that the prime form might be decided by which row strands are more essential for this piece. Interestingly, Webern’s row strands have cycles. The selection of the row form depends on a certain rule; the prime forms always go to 3 intervals of transposition, and the inversion forms go to 9 intervals of transposition. The subsequent form of Pi is Pi+3, and the subsequent form of Ij is Ij+9. When a row form is P2, the next form is P5. If a row form is I8, the subsequent form is I5. Consequently, after four successive rows, we return to the original forms (3+3+3+3=0, and 9+9+9+9=0).

However, Webern did not continue his row strands for a full cycle in section ‘A’. He just used a half cycle. On the other hand, section ‘B’ has a full cycle of new material and section ‘A’ is made up of the same material with section ‘B’ but a half cycle. Therefore, the first row of section ‘B’ plays an important role as the start of the full cycle. Accordingly, most scholars except Phipps selected the first row of the soprano melody, I9 as a prime form.

Next, it will be necessary to examine row properties. The row in Opus 29 includes identifying marks of Webern’s rows. The first feature is a derived series. A derived series is one in which the discrete segmental trichords or tetrachords are all members of the same set class. In this piece the discrete trichords of the row are all pc set 3-3 (014). Second, when one form moves to another form, he uses an invariant in all of Opus 29. Thus the last two notes of one form become the first two notes of another (Ex.26).

Example 26) invariance

\[
P7: \begin{array}{ccccccccccc}
7 & e & 8 & 9 & 5 & 6 & 3 & 4 & 0 & 1 & t & 2 \\
\end{array}
\]

\[
P_t: \begin{array}{ccccccccccc}
t & 2 & e & 0 & 8 & 9 & 6 & 7 & 3 & 4 & 1 & 5 \\
\end{array}
\]

---

6 Any musical quality or relationship preserved when the series is transformed is called an invariant.
In addition to these normal features of Webern’s row, the row is limited to three interval-class adjacencies: IC 1, IC 3, and IC 4. This means that saturated trichords are made up of only 3-2 (013) and 3-3 (014) (Ex. 27).

Example 27) Saturated trichords of the row in Opus 29

Also, the row has only 24 forms, since R=I or P=RI. One can observe the result through a matrix (Ex.25).

Example 28) A matrix

\[
\begin{array}{cccccccccc}
1 & 7 & e & 8 & 9 & 5 & 6 & 3 & 4 & 0 & 1 & t & 2 \\
7 & G & B & G\# & A & F & F\# & Eb & E & C & Db & Bb & D & 7 \leftarrow R \\
3 & Eb & G & E & F & Db & D & B & C & Ab & A & F\# & Bb & 3 \\
6 & Gb & Bb & G & Ab & E & F & D & Eb & B & C & A & Db & 6 \\
5 & F & A & F\# & G & Eb & E & C\# & D & Bb & B & Ab & C & 5 \\
9 & A & C\# & Bb & B & G & Ab & F & F\# & D & Eb & C & E & 9 \\
8 & Ab & C & A & Bb & F\# & G & E & F & C\# & D & B & Eb & 8 \\
e & B & Eb & C & C\# & A & Bb & G & Ab & E & F & D & F\# & e \\
t & Bb & D & B & C & Ab & A & F\# & G & Eb & E & C\# & F & t \\
2 & D & F\# & Eb & F & C & Db & Bb & B & G & Ab & F & A & 2 \\
1 & Db & F & D & Eb & B & C & A & Bb & F\# & G & E & Ab & 1 \\
4 & E & Ab & F & F\# & D & Eb & C & Db & A & Bb & G & B & 4 \\
0 & C & E & Db & D & Bb & B & Ab & A & F & F\# & Eb & G & 0 \\
RI & 7 & e & 8 & 9 & 5 & 6 & 3 & 4 & 0 & 1 & t & 2 \\
\end{array}
\]
A. Hexachordal Transformation Networks in the first movement of
Webern’s *Cantata No.1*

The ‘A’ section (mm.1-13) of the first movement may be considered as a mirror canon at the tritone in which each canonic element is itself a mirror canon at the major seventh. Webern uses prime forms on P7 and P2 with I8 and I1. P7 and I8 are a pair as a mirror canon A, and I1 and P2 are a couple as a mirror canon B (Ex. 26) Pitch class pairing by canon A and canon B produces the same result. P7 - I8 and I1- P2 have the same inversion center: G-G sharp or C sharp-D. In other words, pitch class pairing in canon A is preserved in pc pairing of canon B; pc pairing of canon A is G-G#, B-E, G#-G, A-Gb, F-Bb, F- F#, Eb-C, E-B, C-D#, Db-D, Bb-F, etc., and pc pairing of canon B is C#-D, A-F#, C-D#, B-E, Eb-C, D-C#, F-Bb, etc. (Ex.30).

Example 29) section ‘A’

```
0 1 2 3 4 5 6 7 8 9 t e
0 1 2 3 4 5 6 7 8 9 t e
a b c b' c' b' d c a d' c' b c b c' a b d a b'
```

| Mirror canon A |   G   |   B   |   G#  |   A   |   F   |   F#  |   Eb  |   E   |   C   |   Db  |   Bb  |   D   |   B   |   C   |   Ab  |   A   |   F#  |   G   |   Eb  |   E   |   C#  |   F   |
|----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| P7             |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       | 130   |
| Mirror canon B |   C#  |   A   |   C   |   B   |   Eb  |   D   |   E   |   F   |   G#  |   G   |   Bb  |   Gb  |   A   |   Ab  |   C   |   B   |   D   |   C#  |   F   |   E   |   G   |   Eb  |
| I1             |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       | 130   |
| P2             |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       | 130   |

---

7 Graham H. Phipps, 130.
Example 30) Inversion center of pitch class pairing

Example 29 also shows that only four different vertical sonorities are found in section ‘A’. They are pc set 4-9 (0167), 4-23 (0257), 4-20 (0158) and single perfect fourth with doublings of both pitch class (05). 4-23 (0257), 4-20 (0158) and (05) appear at two transpositional levels: the original one and a transposition to the tritone. On the other hand, pc set 4-9 (0167) has only one level: [C#, D, G, Ab]. Because 4-9 (0167) has the property of transpositional symmetry, which is capable of mapping entirely onto them under transposition, the tetrachord at T6 is the same at T0.

Kramer summarizes these four simultaneities of mm.1-13 as shown in example 31-1 and suggests that four pc collections are interrelated. Four vertical sonorities are formed by combining six dyads. Dyad C- Eb is common to [Bb, C, Eb, F] and [G, Ab, C, Eb], and dyad F#-A is common to [E, F#, A, B] and [C#, D, F#, A]. Dyad G-Ab is mutual to [G, Ab, C, Eb] and [G, Ab, C#, D], and dyad C#-D is mutual to [C#, D, F#, A] and [G, Ab, C#, D]. Besides, dyad F-Bb and dyad B-E are common to [Bb, C, Eb, F] and [E, F#, A, B] and establish themselves as (05).

---

8 In order to understand transpositional levels of four vertical sonorities, I borrowed Phipps’s notation in example 4. The vertical sonority a is pc set 4-16, b is 4-23, c is 4-20 and d is perfect fourth. Also, b’, c’ and d’ refer to a transposition to the tritone.

9 Jonathan Kramer, 162-65.
(Example 31-1) Kramer’s example showing basic simultaneities of mm.1-13

\[
\begin{array}{cccc}
4-20 & (0158) & 4-23 & (0257) & 4-9 & (0167) & 2-5 & (05)
\end{array}
\]

(Example 31-2) Kramer’s example showing relationship between basic simultaneities of mm.1-13

When the respective four set forms are aligned vertically in the section `B’\(^{10}\) (mm.14- 35) and section `A’ (mm. 36- 47), there are only three different chord types: pc set 4-1 (0123), 4-23 (0257), and 4-10 (0235). Each set is labeled as x, y, and z, and their tritone transpositions are x', y', and z' respectively (Ex. 32) There is a mutual tetrachord, 4-23 (0257), between section `A’ and section `B’. The chord b of section `A’ is identical with the chord y' of section `B’, and the chord b' is same with the chord y. The last chord of section `A’ is the chord b', which has a function to connect with section `B’ as a common chord.

\(^{10}\) In regard to this question why Webern adjusts the canonic relationship at m.14, Phipps suggests that the organization of this choral section is based on a cantus firmus principle reflecting Webern’s interest in early music. The tenor voice in this new canonic structure begins with G (Ex.7), the original transpositional level of the treble voice of the first canonic structure in section `A’; the first row form of section `A’ is P7, and the tenor cantus firmus is I7.
Example 32) Vertical alignment in the section ‘B’ and ‘A’

Section ‘B’ (Section ‘A’ – Only half cycle)

| x  | y  | x  | z  | y' | z  | z' | y' | x' | y' | z' | x  | z  | x' | z' | y' | x' | y' |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |

<table>
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<tr>
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<th>G</th>
<th>B</th>
<th>B</th>
<th>C#</th>
<th>C</th>
<th>E</th>
<th>Eb</th>
<th>F#</th>
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<td>E</td>
<td>F</td>
<td>C#</td>
<td>D</td>
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<td>A</td>
<td>G#</td>
<td>B</td>
<td>Bb</td>
<td>D</td>
<td>C#</td>
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<td>Bb</td>
<td>G</td>
<td>G#</td>
<td>E</td>
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<td>D</td>
<td>F#</td>
<td>D</td>
<td>F#</td>
<td>P2</td>
</tr>
<tr>
<td></td>
<td>P8</td>
<td>A</td>
<td>F</td>
<td>D</td>
<td>Eb</td>
<td>B</td>
<td>C</td>
<td>A</td>
<td>C#</td>
<td>Bb</td>
<td>G</td>
</tr>
</tbody>
</table>

| x' | z' | y  | z' | x' | z' | z' | x' | z' | y' | x' | y' | z' | x' | z' | y' | x' | y' |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |

<table>
<thead>
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<th>E</th>
<th>G</th>
<th>F#</th>
<th>Bb</th>
<th>A</th>
<th>C</th>
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<td>G</td>
<td>Ab</td>
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<td>D</td>
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<td>B</td>
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<td>Eb</td>
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<td>Ab</td>
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<td>Bb</td>
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<td>Ab</td>
<td>C</td>
<td>B</td>
<td>D</td>
<td>C#</td>
<td>F</td>
<td>E</td>
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<tr>
<td></td>
<td>C#</td>
<td>D</td>
<td>Bb</td>
<td>G#</td>
<td>A</td>
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<td>F#</td>
<td>Eb</td>
<td>P</td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>E</td>
<td>F</td>
<td>C#</td>
<td>D</td>
<td>B</td>
<td>C</td>
<td>G#</td>
<td>A</td>
</tr>
</tbody>
</table>
Kramer proves again that the six dyads used in section ‘B’ and section ‘A’ have the same result as the six dyads in example 33. Three basic tetrachords and their tritone transpositions share dyads; for example, dyad G-Ab is common to [F#, G, Ab, A] and [F, G, Ab, Bb], and dyad F-Bb is mutual to [Bb, C, Eb, F] and [F, G, Ab, Bb] (Ex. 33).

Example 33) Kramer’s example showing relationship between vertical tetrachords in mm.14-47

We have examined the general features shown in the first movement of Op. 29 based on theorists’ research. Now, let us look into this piece in terms of a transformational attitude. While most researchers studied vertical tetrachords of Op. 29 such as seen in previous pages, I would like to examine the movement at a horizontally hexachordal level. As I mentioned before, because Webern selected the row forms according to the certain rule, Pi+3 and Ij+9, there is no specific feature in the case of a serial transformation. Thus it would be appropriate to construct the networks at a hexachordal level.
Each tone row consists of two hexachords: H1 and H2. In the case of the prime form, the first hexachord of each row is called “H1.” The second hexachord is called “H2.” Conversely, in the case of inversion form, the first hexachord is called “H2.” The second hexachord is called “H1.” The reason that the hexachord order of inversion form is reverse is to keep their feature as H1 itself and H2 itself. Both H1 and H2 are pc set 6-2 (012346), and they are related by an inversion. In detail, while all H1 hexachords have a type of 6-2 (012346) as a normal form, all H2 hexachords have a type of (023456) as a normal form.

The row name applies to the name of hexachord; for example, two hexachords of P7 are called “P7H1” and “P7H2,” and those of I8 are “I8H2” and “I8H1.” After the row is partitioned by two hexachords, each hexachord had better be inscribed as a normal form in order to compare other hexachords easily instead of leaving the tone order of the hexachord as it stands. Thus, I shall use an operation “n.f.” to display the normal form of a hexachord. P7H1 <7, e, 8, 9, 5, 6>\(^{11}\) presents the pitch set of the first hexachord of P7, and n.f. P7H1 {5, 6, 7, 8, 9, e}\(^{12}\) displays the normal form of <7, e, 8, 9, 5, 6>.

On the basis of example 29, section ‘A’ is inscribed by using H1 and H2 in example 34. The arrow \(\rightarrow\) describes the function between two hexachords in the same tone row, and the arrow \(\longrightarrow\) illustrates it between two hexachords from different tone rows. As a transformation, only inversion is used not only in \(\rightarrow\) but also in \(\longrightarrow\) because of the continuous alternation of H1 and H2. Notice that the cycle of tone row is applied at the hexachord level: Pi+3 and Ij+9. In addition, Pt and I5 establish combinatoriality. Pt consists of n.f. PtH1 {8, 9, t, e, 0, 2} and n.f. PtH2 {1, 3, 4, 5, 6, 7}, and I5 is made up of n.f. I5H2 {1, 3, 4, 5, 6, 7} and n.f. I5H1 {8, 9, t, e, 0, 2, x, y, z> means the ordered set comprising the three things x, y, and z.

\(\text{\{x, y, z\}}\) means the unordered set.
Thus, $\text{PtH1}$ and $\text{I5H2}$ make an aggregate, and $\text{PtH2}$ and $\text{I5H1}$ do as well. Combinatoriality happens more frequently in section ‘B’ and ‘A’.

Example 34) The basic transformations of section ‘A’ seen on the hexachord level

\[
\begin{align*}
\text{n.f. P7H1} & \rightarrow \text{n.f. P7H2} & \rightarrow & \text{n.f. PtH1} & \rightarrow & \text{n.f. PtH2} \\
\{5, 6, 7, 8, 9, e\} & \rightarrow \{t, 0, 1, 2, 3, 4\} & \rightarrow & \{8, 9, t, e, 0, 2\} & \rightarrow & \{1, 3, 4, 5, 6, 7\} \\
T_9 & \rightarrow & T_0 & \rightarrow & T_3 & \rightarrow \\
\text{n.f. I8H2} & \rightarrow \text{n.f. I8H1} & \rightarrow & \text{n.f. I5H2} & \rightarrow & \text{n.f. I5H1} \\
\{1, 3, 4, 5, 6, 7\} & \rightarrow \{e, 0, 1, 2, 3, 5\} & \rightarrow & \{1, 3, 4, 5, 6, 7\} & \rightarrow & \{8, 9, t, e, 0, 2\} \\
T_9 & \rightarrow & T_6 & \rightarrow & T_3 & \rightarrow \\
\text{n.f. I1H2} & \rightarrow \text{n.f. I1H1} & \rightarrow & \text{n.f. ItH2} & \rightarrow & \text{n.f. ItH1} \\
\{9, e, 0, 1, 2, 3\} & \rightarrow \{4, 5, 6, 7, 8, t\} & \rightarrow & \{6, 8, 9, t, e, 0\} & \rightarrow & \{1, 2, 3, 4, 5, 7\} \\
T_7 & \rightarrow & T_4 & \rightarrow & T_1 & \rightarrow \\
\text{n.f. P2H1} & \rightarrow \text{n.f. P2H2} & \rightarrow & \text{n.f. P5H1} & \rightarrow & \text{n.f. P5H2} \\
\{0, 1, 2, 3, 4, 6\} & \rightarrow \{5, 7, 8, 9, t, e\} & \rightarrow & \{3, 4, 5, 6, 7, 9\} & \rightarrow & \{8, t, e, 0, 1, 2\} \\
T_c & \rightarrow & T_2 & \rightarrow & T_5 & \rightarrow \\
\end{align*}
\]

While the basic transformations in Example 34 present the chronological progress of the section ‘A’ at the hexachord level, Example 35 shows the relationships between hexachords and between adjacent parts. Each box stands for each row cycle. The order of boxes follows the order of the vertical sonorities from top to bottom. The horizontal arrows within boxes indicate transformations between two hexachords from the same row form. The vertical arrows within boxes display transformations between two hexachords from a different row. The arrows between boxes exhibit relationship between adjacent parts.
Example 35) A transformational network of section ‘A’

![Diagram of transformational network]

\[ \text{n.f. } P7H1 \{5, 6, 7, 8, 9, e\} \leftrightarrow \text{n.f. } P7H2 \{t, 0, 1, 2, 3, 4\} \]

\[ \Downarrow T0I \]

\[ \text{n.f. } PtH2 \{1, 3, 4, 5, 6, 7\} \leftrightarrow \text{n.f. } PtH1 \{8, 9, t, e, 0, 2\} \]

\[ \Downarrow T3I \]

\[ 0+6=6 \]

\[ 9+3=12 \]

\[ \text{n.f. } I8H2 \{1, 3, 4, 5, 6, 7\} \leftrightarrow \text{n.f. } I8H1 \{e, 0, 1, 2, 3, 5\} \]

\[ \Downarrow T0I \]

\[ \text{n.f. } I5H1 \{8, 9, t, e, 0, 2\} \leftrightarrow \text{n.f. } I5H2 \{1, 3, 4, 5, 6, 7\} \]

\[ \Downarrow T5 \]

\[ 4+2=6 \]

\[ 7+5=12 \]

\[ 1+e=12 \]

\[ \text{n.f. } P2H1 \{0, 1, 2, 3, 4, 6\} \leftrightarrow \text{n.f. } P2H2 \{5, 6, 7, 8, t, 0\} \]

\[ \Downarrow T2I \]

\[ \text{n.f. } P5H2 \{8, t, e, 0, 1, 2\} \leftrightarrow \text{n.f. } P5H1 \{3, 4, 5, 6, 7, 9\} \]

\[ \Downarrow T5I \]
Interestingly, there are certain regulations in the network of Example 35. Two boxes of the top make a pair and two boxes of the bottom form a couple. The sum of inversion-transposition numbers of vertical arrows is 6 between the boxes of the top (0+6= 6). In the case of the horizontal arrows at the two boxes of the top, an arrow (←→) between P7H1 and P7H2 of the first box and the other arrow (←→) between I5H1 and I5H2 of the second box (outer arrows) make a pair. The sum of two arrows is 12 (9+3= 12). Also, ←→ between PtH1 and PtH2 of the first box and ←→ between I8H2 and I8H1 make a pair, and the sum of inversion-transposition numbers is 12 (3+9=12). Two boxes of the bottom also have the same result although the numbers are different. The sum of vertical arrows is 6 (4+2= 6). In the case of horizontal arrows, the sum of outer arrows is 12 (7+5=12) and the sum of the inner arrows is also 12 (1+11= 12). Finally, transformations between boxes work as the order of T3I, T5, and T3I.

Section ‘B’ can also be described on the hexachord level on the basis of example 32 (Ex.36). Section ‘A’ is just half of section ‘B’, so I will explain only section ‘B’. While section ‘A’ has only the half cycle, section ‘B’ is made up of the full cycle (3+3+3+3=0 or 9+9+9+9=0). Therefore, transformations between hexachords in section ‘B’ are expanded double. In the soprano, the first half cycle consists of T4I, T8I and T5I and the second half does the same. T2I plays a role for the connection of two half cycles.

The soprano and the alto make a pair and the tenor and bass do as well. Operations of solid arrows are the same at the pair. When the first operation of the soprano is T4I, that of the alto is also T4I. However, transformations of dotted arrows have different results at the pair. The order of dotted transformations is T8I-T2I-T8I in the soprano, while the alto has the order of T2I-T8I-T8I-T2I. The regulations of dotted arrows and solid arrows are equally applied to the tenor and the bass.
Example 36) Basic transformations of section ‘B’ seen on the hexachord level

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<tr>
<th>Sop.</th>
<th>n.f. I9H2</th>
<th>n.f. I9H1</th>
<th>n.f. I6H2</th>
<th>n.f. I6H1</th>
<th>n.f. I3H2</th>
<th>n.f.I3H1</th>
<th>n.f. I0H2</th>
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<td>{6,7,8,9,t,0}</td>
<td>{8,t,e,0,1,2}</td>
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</tr>
<tr>
<td></td>
<td>T3I</td>
<td>T7I</td>
<td>T5I</td>
<td>T2I</td>
<td>T3I</td>
<td>T8I</td>
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<td>T8I</td>
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<table>
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<th>n.f. I4H2</th>
<th>n.f. I4H1</th>
<th>n.f. I1H2</th>
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<td>T4I</td>
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The arrows with a solid line indicate transformations between hexachords within the same row forms and dotted arrows show them between two hexachords from different row forms such as Example 30.
The second and the fourth row forms make combinatoriality. The second row form of the soprano, I6, consists of n.f. I6H2{2,4,5,6,7,8} and I6H1{9,t,e,0,1,3}, and that of the alto, Pe, is made up of n.f. PeH1{9,t,e,0,1,3} and n.f. PeH2{2,4,5,6,7,8}. Thus I6H2 and IeH1 make an aggregate, and I6H1 and PeH2 do the same. The second row form of the tenor, I4, and the second of the bass, P9, form combinatoriality. In addition, the fourth row forms do the same: I0 and P5, and It and P3. Notice that combinatoriality used in this piece follows the formula, Pn, 1n+7.

Example 37 presents relationships between hexachords in each part and between parts. Remember that horizontal arrows within boxes denote transformations between hexachords from the same row form, and vertical arrows indicate those between hexachords from the different row form. The soprano and the alto have the same transformations in horizontal arrows, and the tenor and the bass also have the same. In terms of relationships of parts, the tenor is 10-transpose of the soprano, and the bass is also 10-transpose of the alto. Thus, the arrows of the soprano and the tenor within the boxes make 12-combination of inversion. In the case of the horizontal arrows, TcI as an operation between I9H2 and I9H1 in the soprano makes a pair with TcI as an operation of I4H2 and I4H1 in the tenor. In the result, the sum of transformations is 12 (11+1=12). Also, TcI which is the transformation of I6H2 and I6H1 in the soprano makes a couple with TcI which is that of I7H2 and I7H1 in the tenor. After all, the sum is 12 (5+7=12). In the case of the vertical arrows, isomorphic arrows\(^{14}\) form 12-combinations of transformations. TcI as an operation between I9H1 and I6H2 in the soprano is isomorphic with TcI as an operation between I7H1 and I4H2 in the tenor. The sum of transformations is also 12 (8+4=12). These kinds of rules are also applied to the relationship of the alto and the bass.

\(^{14}\) When two arrows are located in an isomorphic place of the boxes, I call the arrows isomorphic.
Example 37) A transformation networks of section ‘B’
All inversion operations within the boxes follow these rules. The horizontal arrow has an operation of an odd number, and the vertical arrow has that of an even number. In addition, the operations between adjacent boxes are that the alto is T₃I of the soprano, the tenor is T₃I of the alto, and the bass is T₃I of the tenor. All operations between adjacent boxes are also odd.

Let us now examine the relationships between adjacent hexachords regardless of parts. As we have studied above, all adjacent hexachords are related by inversion. Six sorts of inversions are found in the first movement of Webern’s *Cantata No. 1*, Op. 29. First, an operation “J” maps it onto the unique form of the hexachord that inverts the given form and leaves invariant the five-note chromatic pentachord subset. When a hexachord (H) is {9, e, 0, 1, 2, 3}, J(H)= {e, 0, 1, 2, 3, 5}. Second, an operation “K” is that a hexachord preserves five notes, and only the second or the second to the last note is different from the original hexachord. When H= {5, 6, 7, 8, 9, e}, K(H)= {5, 7, 8, 9, t, e}. If H= {1, 3, 4, 5, 6, 7}, K(H)= {1, 2, 3, 4, 5, 7}. The third operation is “L.” “L” maps it onto the hexachord which inverts the given form and preserves the first chromatic dyad or the last chromatic dyad. When H= {t, 0, 1, 2, 3, 4}, L(H)= {3, 4, 5, 6, 7, 9}. If H= {t, e, 0, 1, 2, 4}, L(H)= {5, 7, 8, 9, t, e}. Fourth, “LWTDYAD” operation is similar to “L” operation. “LWTDYAD” maps it onto hexachord which inverts the given form and preserves not the first or the last chromatic dyad but the first or the last whole tone dyad. Thus, L of LWTDYAD signifies the relationship with “L” operation and WTDYAD of that means the common tones that consist of whole tone dyad. Fifth, an operation “M” leaves invariant the first or last four-note chromatic tetrachord when a hexachord is inverted. When H= {4, 5, 6, 7, 8, t}, M(H)= {1, 3, 4, 5, 6, 7}. If H= {5, 7, 8, 9, t, e}, M(H)= {8, 9, t, e, 0, 2}. Last, “N” signifies inversion about E- and -F, or about Bb- and -B; N- arrow means that N(H) is the N-inversion of H.
Example 38) The various inversions between hexachords in section ‘A’

The first hexachord group

The second

The third

The fourth

n.f. P7H1
{5, 6, 7, 8, 9, e}

n.f. P7H2
{t, 0, 1, 2, 3, 4}

n.f. PtH1
{8, 9, t, e, 0, 2}

n.f. PtH2
{1, 3, 4, 5, 6, 7}

n.f. I8H2
{1, 3, 4, 5, 6, 7}

n.f. I8H1
{e, 0, 1, 2, 3, 5}

n.f. I5H2
{1, 3, 4, 5, 6, 7}

n.f. I5H1
{8, 9, t, e, 0, 2}

n.f. I1H2
{9, e, 0, 1, 2, 3}

n.f. I1H1
{4, 5, 6, 7, 8, t}

n.f. ItH2
{6, 8, 9, t, e, 0}

n.f. ItH1
{1, 2, 3, 4, 5, 7}

n.f. P2H1
{0, 1, 2, 3, 4, 6}

n.f. P2H2
{5, 7, 8, 9, t, e}

n.f. P5H1
{3, 4, 5, 6, 7, 9}

n.f. P5H2
{8, t, e, 0, 1, 2}
Example 38 shows various inversions used in section ‘A’. There are two patterns: “J/K complex” and “L/M complex.” Between the first hexachord group and the second, and between the third hexachord group and the fourth, only K- arrows and J-arrows are shown. Thus let us call it the “J/K complex.” Both operation J and operation K share five common tones when they are inverted (Ex.39).

Meanwhile, L- arrow and M- arrow exist between the second hexachord group and the third; let us call it the “L/M complex.” In the L/M complex, interval class 1 is served as a character of common tones. Operation L has a chromatic dyad (01) as common tones, and operation M has a chromatic tetrachord (0123) (Ex.40). While hexachords of the J/K complex are originated from four row forms, eight hexachords of the L/M complex are derived from eight different row forms. This means that the L/M complex plays a role of the connection among various row forms. L-arrows in the section ‘A’ are served as a transition to link section ‘A’ and section ‘B’ (Ex. 41). All the fourth hexachord group of section ‘A’ and the first group of section ‘B’ are linked by L operation and LWTDYAD operation.

Example 39) The number of common tones in J/K complex

\[
\begin{align*}
J & \quad n.f. \ P2H1 \quad \left\{0, 1, 2, 3, 4, 6\right\} \quad \text{n.f.} \ P7H2 \quad \left\{t, 0, 1, 2, 3, 4\right\} \\
& \quad \text{Common tones} \\
K & \quad n.f. \ P7H1 \quad \left\{5, 6, 7, 8, 9, e\right\} \quad \text{n.f.} \ P2H2 \quad \left\{5, 7, 8, 9, t, e\right\}
\end{align*}
\]
Example 40) The common character of L/ M complex

\[ \begin{align*}
\text{L} & \\
n.f. P7H2 & \rightarrow n.f. P5H1 \\
\{t, 0, 1, 2, 3, 4\} & \rightarrow \{3, 4, 5, 6, 7, 9\} \\
\text{interval class } (01) & \\
\text{M} & \\
n.f. I1H1 & \rightarrow n.f. I5H2 \\
\{4, 5, 6, 7, 8, t\} & \rightarrow \{1, 3, 4, 5, 6, 7\} \\
& (0123)
\end{align*} \]

Example 41) The transition by L- arrows and LWTDYAD arrows

The last hexachords of Section ‘A’ Section ‘B’

\[ \begin{align*}
\text{n.f. PtH2} & \quad \text{L} & \quad \text{n.f. I9H2} \\
\{1, 3, 4, 5, 6, 7\} & \rightarrow & \{5, 7, 8, 9, t, 2\} \\
\text{n.f.I5H1} & \quad \text{L} & \quad \text{n.f. P8H1} \\
\{8, 9, t, e, 0, 2\} & \rightarrow & \{6, 7, 8, 9, t, 0\} \\
\text{n.f.ItH1} & \quad \text{LWTDYAD} & \quad \text{n.f. I7H2} \\
\{1, 2, 3, 4, 5, 7\} & \rightarrow & \{3, 5, 6, 7, 8, 9\} \\
\text{n.f.P5H2} & \quad \text{LWTDYAD} & \quad \text{n.f. P6H1} \\
\{8, t, e, 0, 1, 2\} & \rightarrow & \{4, 5, 6, 7, 8, t\}
\end{align*} \]

In section ‘B’, relationships between adjacent hexachords regardless of parts consist of operations L, LWTDYAD, N, and K. Example 42 presents inversion-operation shown in section ‘B’. Operation L, LWTDYAD, and K are used again in section ‘B’ and operation N is a new concept. Operation N is inversion about E and F or about Bb and B. The inversion N appears at the first and the third of the row cycle.
Example 42) The various inversions between hexachords in section ‘B’
Mapping in Section ‘B’ is classified by three areas depending on sorts of pairs of inversions and combinatoriality. First, L-arrow works with N-arrow, so it forms L/N complex. It appears at the first rows and the third rows of the cycle. Second, K-arrow works with LWTDYAD arrow: K/LWTDYAD complex. Its role is a connection between rows: between the first rows and the second rows, between the second and third, and between the third and fourth. The last areas are the second and the fourth rows which form combinatoriality. Thus, the pitch classes of the third hexachord group are the same as those of the fourth. The content of the seventh is also the same as that of the eighth (Ex 43).

(Example 43) Combinatoriality of the second and fourth rows in section ‘B’

<table>
<thead>
<tr>
<th>the second row</th>
<th>the fourth row</th>
</tr>
</thead>
<tbody>
<tr>
<td>I6H2</td>
<td>I6H1</td>
</tr>
<tr>
<td>PeH1</td>
<td>PeH2</td>
</tr>
<tr>
<td>=</td>
<td>=</td>
</tr>
<tr>
<td>I4H2</td>
<td>I4H1</td>
</tr>
<tr>
<td>P9H1</td>
<td>P9H2</td>
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<tr>
<td>=</td>
<td>=</td>
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</tbody>
</table>

The various inversions seen above are summarized in Example 44 that shows all possibilities of these inversions between hexachords. The horizontal stands for all H1s in normal form, and the vertical stands for all H2s in normal form. n.f. I3H1\{6, 7, 8, 9, \(t\), 0\} is equal to n.f. P8H1\{6, 7, 8, 9, \(t\), 0\}, and n.f. I9H2\{5, 7, 8, 9, \(t\), \(e\)\} is equal to n.f. P2H2\{5, 7, 8, 9, \(t\), \(e\)\}. Accordingly, I mark all hexachords as only normal forms (\{6, 7, 8, 9, \(t\), 0\}) rather than the names of hexachords (I3H1) because of hexachord invariance.
Example 44) The matrix of inversions

<table>
<thead>
<tr>
<th></th>
<th>012346</th>
<th>123457</th>
<th>234568</th>
<th>345679</th>
<th>45678t</th>
<th>56789e</th>
<th>6789t0</th>
<th>789te1</th>
<th>89te02</th>
<th>9te013</th>
<th>te0124</th>
<th>e01235</th>
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</thead>
<tbody>
<tr>
<td>H1</td>
<td>K</td>
<td>J</td>
<td>M / N</td>
<td>L</td>
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<td>H2</td>
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<tr>
<td>46789t</td>
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<td>K</td>
<td>J</td>
<td>M</td>
<td>L</td>
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<td>J</td>
<td>M / N</td>
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<td>L / N</td>
<td>LWTYAD</td>
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</tbody>
</table>
Inversions J, K, L, LWTDYAD, and M are based on common tones, while inversion N is based on axis or index number. Thus, inversions J, K, L, LWTDYAD, and M are displayed from the top of the left side to the bottom of the right side in the matrix, and inversion N is presented from the bottom of the left to the top of the right. As a result, inversion N is overlapped by other inversions M and L twice.

Rather than trying to make transformations denote phenomenological presences, we can regard them as ways of structuring an abstract space of hexachords. Example 45 presents a network that reflects a more spatial sense of hexachords in Section ‘A.’ The network starts at the bottom of the left. Inside any part, all the horizontal arrows denote transposition relations, and all the vertical arrows and all the diagonal arrows have inversion relations. The two of the left networks are isomorphic in terms of transposition relation, and the two of the right networks are also isomorphic in the transposition relations. On the other hand, in terms of inversion relation, only the two of the bottom networks are isomorphic in the vertical and diagonal arrows. Inside any part, every hexachord projects each other.

Also, all four parts are related each other by inversion or transposition. The left networks are T₃I of the right networks. P₅H₁ at the top of the left network is T₃I of I₅H₂ at the top of the right network. P₂H₁ at the top of the left network is T₃I of I₁H₂ at the top of the right network. When a square-network shifts from one to another, a network maps another into. I call this SHIFT operation. On the other hand, the bottom network is T₅ of the top network in the case of the left side, and the top network is T₅ of the bottom network in the case of the right side. P₇H₁ at the bottom network is T₅ of P₂H₁ at the top network in the left side. I₅H₂ at the top network is T₅ of I₅H₂ at the bottom in the right side. Accordingly, when the bottom networks flip, the result of transposition comes at the top network. It is called FLIP operation.
Example 45) A network that reflects a more spatial sense of hexachords in Section ‘A’
Example 46 is a network of section ‘B’ in the same sense with Example 45. The size of the network in Section ‘B’ becomes the double of section ‘A.’ While there is only one square-network in each voice of section ‘A,’ each voice has two square-networks in each voice of Section ‘B.’ The two square-networks are connected by a single arrow (→ or ←), which shows progression within the same voice part. All the single arrows denote T6-relations because a square-network presents a half cycle of Webern’s row strand (3+3+3+3). A pair of square-networks is isomorphic in each voice part; they have the same operations in the vertical, horizontal, and diagonal arrows. For example, the all the horizontal arrows have T9, the vertical arrows denote T5I and T3I, and all the diagonal arrows show T8I-relations at two square-networks of the soprano (the bottom of the left). After all, the two square-networks are bonded by a bracket as a couple.

All four parts are related each other by inversion or transposition. The soprano is T5I of the alto. The tenor is T3I of the bass. The relationship of the soprano and the alto is formed when the soprano shifts to the location of the alto. The relationship of the tenor and the bass is also applied to SHIFT operation. On the other hand, the soprano is T3I of the bass and the alto is also T3I of the tenor. However, it is established when the soprano flips, the result of T3I-relation appear at the bass. The relationship of the alto and the tenor is also applied to FLIP operation. In addition, the tenor is T3 of the soprano, and the bass is also T3 of the alto. When a network of the soprano flips, the result of transposition comes at a network of the tenor. The relation of the alto and the bass is the same as well.
Example 46) A network that reflects a more spatial sense of hexachords in section ‘B’
We have explored Webern’s Op. 29, No. 1 in terms of a transformational attitude. I have focused on two aims to analyze the piece. The first aim was to examine interaction among hexachords. For example, “what kind of operation exists between hexachords?”, “what is the regulation of the operation?” and so on. The second was to look into hexachords in the spatial sense. We regarded them as not only phenomenological ways but also existence in the structural space.

Finally, let us compare to approaches of atonal pieces and approaches of serial twelve-tone pieces. Lewin analyzed Webern’s atonal pieces except Piano Variations Op. 27 as I introduced in Chapter one, and I have analyzed a serial piece. There are some differences between atonal pieces and twelve tone pieces in using a transformational technique. First of all, the difference is whether components of the object are doubling or not. In the case of atonal pieces, when one segment the object such as pentachord or tetrachord, the objects are saturated with each other in many cases. On the other hand, serial pieces usually have the discrete segmentations. For instance, Lewin selects twelve-tone segmentation in Serial Transformation Networks in Dallapiccola’s Simbolo,¹⁵ and I have chosen hexachord segmentation in the analysis of Webern’s Op. 29, No. 1. Both apply to discrete segmentation except invariance of the tone rows themselves. The second difference is whether all notes of a piece are included as objects or not. In the case of atonal pieces, all notes are selected as objects. When we construct the network which consists of 4-3 (014), some notes are excluded. While on the other, because a serial piece is made up of the same material of the twelve tone, we can form networks that include all notes of the piece.

If I were a performer with little knowledge of Webern’s *Cantata No. 1*, I might feel comfortable to use other theorists’ analyses that I introduced before. The performer is apt to notice features in a tangible way; the row properties, vertical sonorities as a choral work, rhythmic structure, etc. However, my network analysis based on two purposes might help one understand the music at the deeper level. The structure seen through interaction of hexachords reminds me of Webern’s outstanding feature, condensation.
B. Transformational Voice Leadings in Webern’s First Cantata, No.1

Following the analysis through transformation networks shown in Webern Op. 29, No. 1, this section study focuses on a voice leading based on a transformational attitude that arises from harmonic successions motivated by transposition or inversion. As shown in Chapter One, there are various viewpoints to analyze voice leading in a transformational attitude: Klumpenhouwer, Straus, Roeder, Lewin, etc. Especially, this study adopts Straus’s concepts of “Uniformity”, “Balance,” and “Smoothness.”

Before starting an analysis, it is useful to think about voice leading in a basic way for a generalization. If cardinality of pitch-class sets X and Y are the same as n, there are n! (n factorial) voice leadings. For example, if X and Y are dyads, two voice leadings are possible (x1 – y1, x2 - y2 and x1- y2, x2 – y1). If X and Y are tetrachords, twenty-four voice leadings are possible (4!= 4*3*2*1). If we examine voice leadings of the first two tetrachords \{A, Ab, G, F#\} and \{F, C, Eb, Bb\} in section ‘B’, the result is like Example 47. There are the twenty-four voice leadings according to their relative uniformity (transposition), and there are also the twenty four voice leadings according to the balance (inversion). In other words, forty-eight possibilities exist in the progression from \{A, Ab, G, F#\} and \{F, C, Eb, Bb\}. 
(Example 47) Voice leading between \{A, Ab, G, F#\} and \{F, C, Eb, Bb\}

<table>
<thead>
<tr>
<th>Example</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>T8 or T9 (10)</td>
<td><img src="image1" alt="Diagram" /></td>
</tr>
<tr>
<td>T8 (10)</td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td>T4 or T8 (10)</td>
<td><img src="image3" alt="Diagram" /></td>
</tr>
<tr>
<td>T3 or T9 (12)</td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
<tr>
<td>T5 or T9 (8)</td>
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<tr>
<td>T6 (8)</td>
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<tr>
<td>T4 (8)</td>
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<tr>
<td>T6 (10)</td>
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<td>T9 (8)</td>
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<tr>
<td>T8 or T9 (8)</td>
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<tr>
<td>T1 or T1 (10)</td>
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<tr>
<td>T7 or T1 (10)</td>
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<tr>
<td>T7, Tc, or T1 (12)</td>
<td><img src="image13" alt="Diagram" /></td>
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</tbody>
</table>
Whenever one looks into a voice leading from set X to set Y, one cannot research every possibility. The cardinalities of the set are higher, and the possibilities are more increased. The number of voice leadings between two pentachords is 120. The number of voice leadings by two criteria, which are uniformity and balance, is 240. Therefore, we need a convenient way of measuring the distance between two set classes.

The best method is to use the concept of an “optimal offset,” which is a best fit between two set classes. In Example 47, various offsets of (6) to (14) exist among forty eight pairs of voice leadings. Among those, an offset of (6) is an optimal offset because it is the most transposition-like or the most inversion-like. An optimal offset is related with not only uniform or balance but also smoothness. The optimal offset is the minimum displacement value associated with the smoothest way of moving between two sets. At the transposition level of Example 47, there are two offsets of (6). Both voice leadings have the total displacement of (18). At the inversion level, there is one offset of (6). The total displacement is (10). Thus, the voice leading is the most suitable among forty-eight pairs. Accordingly, if one seeks the most uniform or balanced voice leading using an optimal offset, one does not need to consider all possibilities of voice leading.

Straus presents a spatial map of an optimal offset. Because my research is on voice leadings between tetrachords, I introduce his map for only tetrachords (Ex.48). Lines link tetrachord classes by an optimal offset of (1). For instance, 4-10 (0235) is offset by (1) in relation to 4-2 (0124), 4-11 (0135), and 4-12 (0236). Each move on the map thus represents an offset of

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17 The lowest total displacement is the smoothest.

18 Straus, 336-9.
(1), and the moves are cumulative.\textsuperscript{19} In addition, 4-10 has offset by (4) in relation to 4-23 because there are four moves in the map.

Example 48) Straus’s example showing optimal offsets for tetrachords as the map of a voice leading space

\textsuperscript{19} Ibid, 336.
Based on these basic concepts, let’s examine the voice leadings of section ‘B’. As seen before, section ‘B’ consists of only three different tetrachords and their 6-transpose: x-(0123), y-(0257), and z-(0235). Therefore, there are only three kinds of offsets according to adjacent chords: x and y, z and y, and x and z. x (0123) or x' has offset by (6) in relation to y (0257) or y'. z' (0235) or z has offset by (4) in relation to y (0257). x (0123) or x' has offset by (2) to z (0235) or z'.

Using these optimal offsets, I construct voice leadings of section ‘B’ (ex.49). Also, Example 50 presents the relative restricted portion of the tetrachordal map. The relationship of x (A, Ab, G, F#) and y (F, C, Eb, Bb) has already been explained in example 47. Among forty-eight possibilities, I9 is the most balanced and the smoothest because it has the optimal offset by (6)²¹, and the total displacement is the smallest. The progression of y to x is identical to that of x to y. After all, tetrachords x, y, and x can be summarized as <x>.

Example 50) The tetrachordal map of section ‘B’

²⁰ x', y', and z' mean the 6-transpose of x, y, z.

²¹ The two voices describe index 9 (they flip around a shared axis). The axis of the G to the F is offset by three semitones from I9, and the axis of the Ab to the Bb is also offset by three semitones from I9. Therefore, x has offset by six semitones in relation to y.
Example 49) Voice leadings in section ‘B’

The First Rows of the Full Cycle

\[
\begin{array}{cccccccccccc}
  x & y & x & z & y' & z & z' & y & z' & x' & y' & x' \\
 I9 & A^9 & F^9 & Ab^9 & G^9 & B^9 & Bb^6 & C#^e & C^1 & E^9 & Eb^9 & F#^9 & D \\
 P8 & Ab^6 & C^9 & A^1 & Bb^1 & F#^1 & G^6 & E^1 & F^e & C#^0 & D^6 & B^0 & Eb \\
 I7 & G^9 & Eb^9 & Gb^9 & F^e & A^9 & G#^6 & B^5 & Bb^1 & D^0 & C#^0 & E^6 & C \\
 P6 & F#^9 & Bb^6 & G^9 & G#^1 & E^1 & F^6 & D^1 & Eb^e & B^1 & C^9 & A^9 & C# \\
\end{array}
\]
The second rows

\[
\begin{array}{cccccccccccc}
& y' & x' & y & z' & x & z & z' & x & z' & y & x' & y' \\
I6 & F# & D & E & E & G & Bb & A & C# & C & Eb & B & B \\
P# & B & E & C & C# & A & Bb & F & F & F & D & F# & F \\
I4 & E & C & Eb & D & G & Ab & F# & G & F & Bb & C# & A \\
P9 & A & C# & Bb & B & F & E & T1 or Te & T0 & T0 & T6 & I3 & I9 \\
\end{array}
\]

\[
\begin{array}{cccccccccccc}
& I9 & I3 & T_e or T_1 & T_6 & T_0 & T_0 & T_0 & T_6 & T_1 or T_e & I3 & I9 \\
(6) & (6) & (4) & (2) & (2) & (0) & (2) & (2) & (4) & (6) & (6) \\
\end{array}
\]

F# I
B e
E 1
A e

T_1 or T_e
(4)

G i
Bb 1
F e
Ab 1

T_1 or T_e
(4)

B
F#
A
E

F# B
B F#
A E

T_0
(0)
The third rows

\[
\begin{array}{cccccccccccc}
  x' & y'' & x' & z' & y & z' & z & y' & z' & x & y & x \\
I3 & Eb^9 & B^0 & D^0 & C#^e & F^e & E^6 & G^e & F#^i & Bb^5 & A^9 & C^9 & Ab \\
P2 & D^6 & F#^9 & Eb^1 & E^1 & C^i & C#^e & Bb^1 & B^6 & G^0 & Ab^6 & F^0 & A \\
I1 & C#^0 & A^9 & C^e & B^6 & Eb^e & D^6 & F^e & E^1 & Ab^0 & G^0 & Bb^6 & F# \\
P0 & C^9 & E^6 & C#^6 & D^1 & Bb^1 & B^6 & G#^1 & A^e & F^1 & F#^9 & Eb^5 & G \\
\end{array}
\]

I9 | I9 | T0 | T1 or Te | Te or T1 | T6 | T1 | Te | T0 | I9 | I9 
---|---|---|---------|---------|---|---|---|---|---|--- 
(6) | (6) | (2) | (4) | (4) | (0) | (4) | (4) | (2) | (6) | (6) 

The fourth rows

\[
\begin{array}{cccccccccccc}
  y & x & y' & z & x' & z' & z' & x' & z & y' & x & y \\
I0 & C^9 & Ab^0 & B^5 & Bb^5 & D^0 & C# & E^e & D#^7 & G^9 & F#^3 & A^9 & F \\
P5 & F^0 & A^3 & F#^6 & G^6 & Eb^1 & E & C#^0 & D^6 & Bb^1 & B^6 & Ab^6 & C \\
It & Bb^6 & F#^3 & A^e & Ab^6 & C^e & B & D^9 & C#^6 & F^1 & E^0 & G^0 & Eb \\
P3 & Eb^9 & G^6 & E^3 & F^7 & C#^0 & D & B^1 & C^5 & G#^1 & A^3 & F#^9 & Bb \\
\end{array}
\]

I9 | I3 | Te | T6 | T0 | T0 | T0 | T6 | T1 | I3 | I9 
---|---|---|---|---|---|---|---|---|---|--- 
(6) | (6) | (4) | (2) | (2) | (0) | (2) | (2) | (4) | (6) | (6) 

→ Section ‘A’
The next tetrachord group is z, y' and z in the first rows of the full cycle. In the progression of z (G, Bb, F, G#) to y' (B, F#, A, E), three voices move by T₁ (Bb - B, F - F#, G# - A), and the remaining voice has offset by four semitones from a transposition at T₁. Thus, z and y' are related by a near-transposition at T₁. Meanwhile, z and y' can be related by a near-transposition at Tₑ. We could say that three voices move by Tₑ (Bb - A, F - E, G - F#), and the remaining voice (G#-B) moves by T₃. Consequently, z is also offset by four semitones from a transposition at Tₑ. If z and y' are related by uniformity at T₁, y' and z are related at Tₑ. Conversely, when z and y' are connected by uniformity at Tₑ, y' and z are connected at T₁.

On the other hand, when we consider “smoothness” we can acquire a different result. z and y' are related by another uniformity at T₁ or Tₑ. Two voices move by Tₑ (G – F#, G# - A) and the remaining voices move by T₁ (Bb – B, F- F#). Note that T₁ or Tₑ by moving of two voices is different from when three voices move by Tₑ or T₁. T₁ or e by moving of two voices has the displacement of (4) because the unordered pc interval of each voice is 1. T₁ or Tₑ by moving three voices has the displacement of (6) because the unordered pc interval of three voices is 1 and it of the remaining voice is 3. Accordingly, voice leading which has the displacement of (4) is the smoother (Example 51). Thus, the smoother voice leading is the more suitable transformation.

In addition, tetrachords z, y' and z can be reduced as <z> like the previous <x> group.
Example 51) Comparison of two kinds of T₁ or Te

<table>
<thead>
<tr>
<th></th>
<th>z</th>
<th>y'</th>
<th>z</th>
<th>y'</th>
<th>z</th>
<th>y'</th>
</tr>
</thead>
<tbody>
<tr>
<td>G⁹</td>
<td>B</td>
<td>G⁹</td>
<td>B</td>
<td>G⁹</td>
<td>B</td>
<td>G⁹</td>
</tr>
<tr>
<td>Bb</td>
<td>F#</td>
<td>Bb</td>
<td>F#</td>
<td>Bb</td>
<td>F#</td>
<td>Bb</td>
</tr>
<tr>
<td>F ¹</td>
<td>A</td>
<td>F ¹</td>
<td>A</td>
<td>F ¹</td>
<td>A</td>
<td>F ¹</td>
</tr>
<tr>
<td>G# ¹</td>
<td>E</td>
<td>G# ¹</td>
<td>E</td>
<td>G# ¹</td>
<td>E</td>
<td>G# ¹</td>
</tr>
</tbody>
</table>

T₁ or Te

Offset: (4) or (4) or (4)
Displacement: (6) or (6) or (4)

The third tetrachord group in the first rows is 6-transpose of <z> group: z', y, and z'. It would be called <z'> group. This group is just transposed, and the function is the same with <z> group. Also, the fourth tetrachord group, <x'>, in the first rows is 6-transpose of <x> group: x', y', x'.

Let’s examine voice leadings between four groups. As seen before, the first rows of the full cycle are made up of <x>, <z>, <z'> and <x'> group. <x> and <z>²² are related by a near-transposition at T₀. The Ab and the G move by T₀ and the A and the F# move by T₁ and Te, so T₀ has an optimal offset of (2). <z> and <z'> are related by crisp-T₆²³. <z'> and <x'> have the same transformation with <x> and <z>; the fuzzy-T₀. Consequently, the two fuzzy-T₀ and the crisp-T₆ combine to created a single crisp-T₆ that spans the progression from the first chord x to the last chord x'.

---

²² Also, it could be said not only <x> and <z> but also “x and z” because two tetrachords are actually adjacent.

²³ Straus says that Tn or In that has an offset of (0) is called “crisp”- Tn or In. If Tn or In has some offset, it is called “fuzzy”- Tn or In.
Let me explain symmetry in the first rows. Each small group, such as \(<x>\) and \(<z>\), has the symmetrical form; for example, \(<x>\) consists of \(x, y,\) and \(x\). In terms of the bigger view, the first rows are also made up of mirror form: \(<x>, <z>, <z'>\) and \(<x'>\). For \(<x>\) is equal to \(<x'>\) and \(<z>\) is identical to \(<z'>\). As a result, voice leadings seen in the transformational attitude are symmetrical. The order of the voice leadings is \(I_9\ (6), I_9\ (6), T_1\ or\ e\ (4), T_1\ or\ e\ (4), T_6\ (0), T_1\ or\ e\ (4), T_1\ or\ e\ (4), T_0\ (2), I_9\ (6), I_9\ (6).\) \(T_6\ (0)\) operates on the axis of the symmetry. This structure is a self-retrograde.

The second rows of the full cycle have a different structure from the first. While the first rows can be divided by four groups, \(<x>, <z>, <z'>\) and \(<x'>\), the second rows exist as each chord itself because it forms a mirror structure in the center of two \(z\) tetrachords (Ex. 52). Thus, voice leadings also form exact symmetry: \(I_9\ (6), I_3\ (6), T_1\ or\ e\ (4), T_6\ (2), T_0\ (2), T_0\ (0), T_0\ (2), T_6\ (2), T_1\ or\ e\ (4), I_3\ (6), I_9\ (6).\) \(T_0\ (0)\) is the axis of the symmetry.

Example 52) The symmetry of the second rows

\[
\begin{array}{cccccccccc}
y' & x' & y & z' & x & (z & z) & x & z' & y & x' & y'
\end{array}
\]

The second rows pursue a strategy of gradually decreasing levels of offset and gradually increasing, as shown in Example 49. The first two moves are by fuzzy-In and the levels of offset are decreased by the order of \((6) - (6) - (4) - (2)\) until the center of a mirror structure, chord \(z\).

---

24 Symmetry is one of the important factors in Webern’s musical thought. Rochberg has expressed in his article “Webern’s Search for Harmonic Identity” that symmetry forms a syntactical basis in Webern’s music. As symmetry implies regulation of proportion, balance and unity of pattern and design, it was a wonderful tool for his music. For symmetry makes possible compact, densely packed, self-enclosed organic structure permitting of no waste space or energy. For that reason, a study of symmetry cannot be omitted in Webern’s music.
The center of the progression involves the smallest amount of offset, (0); thereafter, the levels of offset are increased by the order of (2) – (4) – (6) – (6).

The third rows of the full cycle are a retrograde of the first rows. While the first rows consist of $x$, $y$, $x$, $z$, $y'$, $z'$, $y$, $z'$, $x'$, $y'$, $x'$, the third rows have the order of $x'$, $y'$, $x'$, $z'$, $y$, $z'$, $z$, $y'$, $z$, $x$, $y$, $x$. Also, we could say that the third rows are 6-transpose of the first rows. Consequently, the content of the voice leadings in the third rows is identical to those in the first rows. Like the preceding, the fourth rows are also 6-transpose of the second rows.

Voice leadings of Example 49 shown in a transformational attitude can be examined in a different way. In the first rows of the full cycle of example 49, the middle-ground\textsuperscript{25} manifests the common linear character. All horizontal lines have pc set 4-7 (0145). In the second rows of the full cycle, the middle-ground presents all horizontal lines having pc set 3-4 (015). However, there is a problem to interpret 4-7 (0145) or 3-4 (015) lines. If I pick any notes by row order, all linear voice leading have the same results because this cantata is a serial piece; it might be all 4-25 (0268) or other pc sets depending on the selection of notes. Therefore, I believe it is not the appropriate method to analyze voice leadings.

Voice leadings seen in terms of a transformational attitude help to observe hidden fascinations of Webern’s Op.29, No.1. Using transposition and inversion, the traditional sources in the post-tonal music, the latent characters of the linear organization have been discovered. Although I have not examined voice leadings of registral pitches because Webern’s Op.29 is a twelve-tone composition, I also believe the study through pitch intervals produces a hearable, theoretically secure voice leading.

\textsuperscript{25} I just borrowed a Schenkerian term “the middle-ground” in order to express the middle stage, which shows the main structures and delete delicate notes.
Conclusion

There are the various methods to analyze twelve-tone pieces: examining combinatoriality, common tones, isomorphic partition, and sets. Among those, I chose a theory of transformation to analyze the first movement of Webern’s *Cantata No.1*, Op. 29. I emphasized both networks and transformations after David Lewin, and voice leading after Joseph Straus.

In my research of the relationship of pitches, the basic unit is a hexachord. I first examined various inversions between adjacent hexachords, and I furthermore looked all hexachords. As a result, all hexachords that appear in the piece are related with each other by transposition or inversion. Finally, I have constructed the transformational networks in relations to the process of the piece.

In my research of voice leading, I adopted Straus’s concepts, “Uniformity, Balance, and Smoothness.” I studied that what sort of relationship exists between two pitch-class sets in chord in terms of voice leading. As a result, voice leadings, which have the mapping of pitch-classes induced by transposition or inversion, present certain regulations in relation to offsets and symmetry.

Musical transformation is applied as a method to analyze the twelve-tone pieces in terms of not only pitch structure but also voice leading. The two analyses using transformation help us to discover the new viewpoints about the first movement of Webern’s *Cantata No.1*, Op. 29.
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**On Voice Leading**


**On Webern Study**


