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BIODYNAMIC MODELING AND ANALYSIS OF MOTOR VEHICLE ROLLOVER OCCUPANTS

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Abstract

This thesis presents a computational, three-dimensional biodynamic model of a vehicle-occupant, crash-victim during rollover accident. The model represents the human body by a series of connected bodies simulating the limbs, torso, head and neck of the human frame. The computer algorithms are developed using DYNACOMBS – a three-dimensional multibody dynamics computer simulation model. DYNACOMBS provides a dynamic analysis of arbitrary collections of bodies allowing for both translation and rotation between adjacent bodies. The relative orientation of adjacent bodies is defined by Euler parameters to avoid computational problems of singularities, which occur with other orientation angles (such as Bryan angles or Euler angles). Thus there are distinct computational advantages of the presented model over other crash-victim simulators.

The governing dynamical equations for the model are based upon Kane’s equations and their associated kinematical quantities (partial velocities, partial angular velocities, generalized speeds, and lower body arrays). Formulations based upon Kane’s equations are believed to be the most efficient and most reliable of the various methods available for studying large multibody systems. Consequently they are ideally suited for studying the dynamics of crash victims.

The developed vehicle and victim model allows for arbitrary vehicle motion. Specifically, given the vehicle rollover motion, as would occur in an accident environment, the simulation predicts the vehicle occupant movement and the forces exerted on the occupant by the seat and seat belts. The model is validated by a series of recently recorded sets of experimental data from dummies and cadavers in a variety of vehicle crash conditions.
Acknowledgements

To Dr. Huston, I offer my gratitude and respect for providing his support, care, experience and advice without which this work could not be accomplished. His management skills largely contributed in overcoming the hindrances and obstructions faced during the project.

In addition, I thank my parents for their inspiration and support during the course of this work.

Thank you all.

Yong Zhang
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1. Introduction

1.1 Background

Since the beginning of the twentieth century, vehicle accidents have been a leading cause of death and severe injury in the United States and around the world. While many steps have been taken to reduce the number of incidents and severity of the injuries, there are several factors tending to not only increase the number of accidents but also increase the severity of the injuries. These include: 1) increased number of vehicles on the highways; 2) increased speed; 3) general downsizing of the vehicles; and 4) the difficulties experienced in the design of safety systems for motor vehicles.

Federal Motor Vehicle Safety Standard (FMVSS) began to consider safety systems and occupant restraint systems in 1963. Seat belts were the first regulations that had been proposed at that time by the US government. In 1966, lapshoulder belts were set as standard equipment for all front outboard seating positions in passenger cars. Since 1967 all passenger vehicles have been equipped with manual lapshoulder belts. Since then, more than 1,000 lives have been saved annually and thousands of injuries have been lessened and mitigated. However, the use of seatbelts in passenger motor vehicles has historically been very low. It has been noticed that less than half of all motorists regularly wear safety belts. Therefore, the National Highway Traffic Safety Administration (NHTSA) has promoted passive occupant restraint systems, including airbags. The airbag technology was known for some time when NHTSA began to consider restraint systems in 1969. A patent for an airbag system was first granted in 1953, and during the 1960s
automobile manufacturers engaged in extensive research and development of airbags. However, manufacturer’s opposition to the mandatory use of passive restraints in general, and to airbags in particular, delayed the implementation of rules requiring such systems for many years. In 1986, the federal government required that automobiles satisfy the passive restraint requirement of RMVSS 208 by either including automatic safety belts or airbags in the vehicles. In 1993, the NHTSA required that driver-side airbags be provided for all automobiles.

1.2 Scope of this research

There is a pressing need not only for safety design improvements, but also for new method of analysis, which will be compatible with recently developed computational procedures in vehicle structural design. Specifically, the analysis and understanding of the kinematics and dynamics of vehicle occupants during collision is not nearly as well understood as the response of the vehicle structures. The state-of-the-art of vehicle-occupant modeling and the accompanying modeling of safety systems (e.g. seat belts, airbags, seats, steering system, and padding) lags far behind the modeling and analysis available for vehicle structural design (e.g. finite element analyses, modal analyses, CAD methods, and crash testing). This research is intended to improve the procedures available for studying the occupant movement and response in all types of vehicle collisions, but most specifically in rollover accidents.

Full scale automobile crash tests with dummies and other surrogates provide insight into occupant dynamics; however, the implementation of such tests is expensive, time-consuming, and generally impractical for comprehensive analyses, particularly for
rollover accidents. Conversely, computer simulation models are economical and practical. They provide a broader range of application. Consequently, the development of more sophisticated and explicit mathematical models for large-scale human body motions has been growing rapidly during the last decade, specifically if we consider the vast progress in computer technology. This model can be used on a PC computer rather than mainframe, which makes it easier to handle and provides flexibility for the user.

Computer simulation of occupant dynamics provides a means for predicting occupant response with or without protective systems. Furthermore, it provides an evaluation of the occupant protection systems and is thus all aid in designing the safety systems. Such evaluation may include the occupant loads, g-levels, injury indices, and other severity measures to identify the crash injury protection level needed to protect the occupant.

1.3 Objective

The objective of this research is to develop advanced modeling methods for vehicle occupants and to use these methods in studying not only the dynamics of vehicle occupants during crashes but also the response and effectiveness of vehicle safety systems with a focus on rollover accidents. The simulation model typically consists of systems of rigid bodies representing vehicle occupants.

This research focuses on the dynamics analysis of occupant during the process of rollover of motor vehicle, which is separated into two parts: the process of rollover and the process of drop. Given the data of vehicle rollover and the vehicle structure
parameters, the computer-based model predicts the occupant’s motion and forces exerted on it during the accident.

The model is intended to be both comprehensive and accurate. Shoulder and lap belts were included in the model. The modeling takes advantage of recent advances in computational mechanics including computer hardware improvement, software development, and improved numerical methods.

1.4 Solution Technique

The model employs Kane’s equations to develop the governing dynamical equations. Kane’s equations are dynamics formulation of multibody systems based upon Lagrange’s form of d’Alembert’s principle, also known as the principle of virtual power developed by Kane. Kane’s equations have the advantage, over Newton-Euler methods, in that non-working constraint forces are automatically eliminated during the analysis. Also, unlike the Lagrangian method, Kane’s method avoids the use of energy sol; hence, the tedious and lengthy differentiation problem does not arise. The formulation procedures and differential equations developed by Huston and Passerello are used to develop an efficient computer algorithm. Then, the dynamical equations can be integrated by means of a fourth order Runge-Kutta method to solve for the various variables such as the displacement, velocities and accelerations. An illustration of the proposed solution procedure is shown in figure1.1.
1.5 Overview of the Thesis

This thesis presents the concepts and the principles needed to develop and simulate the crashworthiness of a vehicle occupant. Thus, a program was developed to provide a comprehensive analysis of crash-victim event.

The input data of the computer program includes:
1. Physical parameters: these are masses, inertia dyadic, mass center positions, connection point positions and orientation angle limits for each of the bodies of the human model;

2. Cockpit geometry: this consists of a normal vector and a location point for each of the intrusion surface planes. Also, the floor position and supporting spring force constant are part of the cockpit specifications;

3. Acceleration input for the vehicle: defining the vehicle acceleration at different time intervals specifies the motion of the vehicle. Also, the motion of any body segment could be defined as input by defining the segment acceleration at different time interval depending upon the occasion need to be simulated;

4. Spring and damping coefficients: This includes the seat, restraining belts, airbags, intrusion surfaces, floor surface, orientation angle constants, and neck parameters.

5. Attach points of the restraining belts.

6. Initial conditions: This includes the initial values of the unknown variables and their derivatives. The initial time, the final time and the time-step need to be defined as well.

The output consists of four parts.

The first part consists of a copy of the input data.

The second part contains at each output time the following:

1. The values of all variables and their first derivatives;

2. The joint mass center positions in both inertia space and relative to the vehicle;

3. The absolute and relative mass center velocities and accelerations;

4. The force and moment of constraints between the adjoining bodies;
5. The restraining belts forces, airbags forces and intrusion surfaces forces.

The third part presents charts of the time history of each variable (e.g. absolute and relative displacements, velocities and accelerations…etc.). Every variable of each body is listed vs. time.

The fourth part is used to form the animation of the occupant time history, which is written using Matlab.

The balance of the thesis is divided into eight chapters. The following chapter provides a historical review of the subject. Chapter 3 provides some kinematics concepts that were useful in this research. Chapter 4 presents the governing dynamical equation of the multibody systems. Chapter 5 develops the crash victim model. Chapter 6 introduces the structure of the FORTRAN computer simulator. The rollover analysis is introduced in Chapter 7. Chapter 8 provides the summary and conclusion of this study.
2. Historical Review

2.1 Background

Computer models for crash victim simulation can be dated back at least 35 years. King and Chou (1976), Prasad (1984) and Huston (1987), describing the various characteristics of the simulators, reviewed most of the crash victim simulation models. These mathematical models can be classified into simplified simulation models, represented by one or two dimensional simulation models, and gross-motion simulation models, represented by three dimensional simulation models. A simplified simulation model was developed by McHenry in 1963. It was a 7-degrees of freedom, two-dimensional model to study the dynamics response of a vehicle occupant involved in a frontal collision event. This was followed by a two dimensional, 11-degree of freedom model developed also by McHenry and Naab in 1966. In 1967, Segal et al. made an expansion to McHenry’s model.

Glancy and Larsen (1971) developed a two-dimensional model called SIMULIA. Rods were used to simulate the human body segments using the Newtonian technique. Moreover, the masses were concentrated at the joints rather than the segment mass center. This model was modified by Twigg et al. (1974), sponsored by Boeing Computer Services (BCS), and renamed PROMETHEUS.

Sponsored by the Motor Vehicle Manufacturer’s Association, Robbins et al. (1974) developed a two-dimensional crash victim simulation model called MVMA. This model has 10 degrees of freedom and spheres were used to simulate the human body.
Concurrently, the Highway Safety Research Institute (HSRI) developed a two-dimensional model which has the same characteristics as MVMA model.

Maltha and Wismans (1980) discussed a two-dimensional model developed by the transportation research institute. This model was called MADAMO. The Lagrangian method was used for the analytical formulation. It consisted of an arbitrary number of pin-connected ellipsoids representing the human frame.

In a review, Huston (1987) reported that Robbins (1970) developed the first gross-motion simulation model (i.e. three-dimensional model). It consisted of three bodies with 12 degrees of freedom. In spite of its simplicity, the model was able to simulate forces generated by impact of the bodies with planes representing the interior of the vehicle. Two years later, Robbins expanded the model to include six bodies having 14 degrees of freedom. In 1973, Robbins modified his model to contain 6 mass segments and 17 degrees of freedom. The model contained both hinge and ball-and socket joints simulating the human joints. It had bilinear, unsymmetrical torsion springs at the joints with coupling between pitches, roll and yaw stops. There was provision for 4 seat-belt attachments to the torso of the model and forces were generated when the model strikes a cockpit intrusion surface. The motion input was via the cockpit with provision for piecewise-linear functions. The governing equations were derived using Lagrange’s equation and they were numerically integrated using a Runge-Kutta integrator. The objectives of developing this code were to formulate an efficient, user-oriented, but yet a comprehensive model. Therefore, there should be a trade-off between the complexity of the model (e.g. degrees of freedom), its capability to accurately predict physical phenomena, the economy of user and run time.
Young (1970) introduced a three-dimensional model called the TTI model. This model consisted of 12 mass segments and had 31 degrees of freedom. Spheres connected by spherical and hinge joints were used to simulate the human frame. It employed bilinear viscous damping at the joints to simulate muscle and ligament forces. It had provision for lap-shoulder belt restraints and forces were generated when the model struck a cockpit surface. The motion input was via the cockpit by specifying its linear and angular displacement as a function of time. Lagrange’s equations were used to develop the governing equations and they were integrated numerically using a Runge-Kutta technique. This code was designed primarily for automobile crashes where the automobile displacement was known as a function of time. Hence, it did not provide as much flexibility on input as some of the others. However, this allowed the code to run more efficiently. In 1974, Young modified the version and used ellipsoids to simulate the human limbs. The Lagrangian method was used also to develop the dynamic formulations.

In 1972, Bartz and associates developed a three-dimensional model called the Calspan model. It consisted of 15 human bodies and could have up to 63 degrees of freedom. Ellipsoids, connected by spherical pin joints were used to simulate the human limbs. The motion input was through the vehicle with provision for 6 piecewise linear acceleration functions. The dynamic formulation was based upon Newton-Euler methods. The model had torsional and flexural spring and viscous joint moments to simulate muscle and ligament action. It employed sliding lap and shoulder restraint belts. It had provision for contact forces when the model struck an intrusion surface of the cockpit or when segments of the model struck each other.
This model and its code were the most elaborate and complex of all the available codes to date. Indeed, probably far more effort and research had been expended in the development of this code than in any of the other previous codes. However, it was probably the most difficult and expensive code to use due to its complexity and its various options.

Huston and associates (1974) developed a three-dimensional model called the UCIN-CRASH. It had 12 body segments and 31 degrees of freedom. Ellipsoids were used to simulate the human frame. It contained hinge and ball and socket joints simulating the human joints. It had bilinear torsional and flexural viscous damping at the joints. This model included up to 10 elastic restraint belts, which could be connected arbitrarily between the model and the vehicle frame. The governing equations were base upon Kane’s equations as described by Kane and Levinson (1985). The motion input was via the cockpit with provision for 6 (3 linear and 3 angular) piecewise-linear acceleration functions.

In 1975, Laanamen developed a three-dimensional model call SOM-LA at the Dynamic Sciences Division of Ultrasystems, Inc. The model used 11 mass segments connected by hinge and ball-and-socket joints, simulating the human joints. It had 28 degrees of freedom. It employed nonlinear torsional spring and viscous dampers at the joints. It contained a finite element seat model and sliding lap and shoulder belts. The motion input was via the seat with provision for 6 piecewise linear acceleration functions, 3 linear and 3 angular. The governing equations were derived using Lagrange’s equations and they were numerically integrated using Runge-Kutta and Adams-Moulton predictor-corrector methods.
In 1981, Wisman and Maltha developed a three-dimensional model called MADAMO. It had the option of choosing any number of motion segments. The joints were simulated by spherical and hinge joints. The dynamic equations were base upon Lagrangian method.

### 2.2 Literature Summary

All of the crash victim models are summarized in Table 1 (Huston, 1987). Each of the codes needs more user-oriented documentation and better user manuals. The majority of the crash victim models developed in the last 30 years are intended to be used on the mainframe computers.

Additionally, they do not provide flexibility in the input data. The crash victim models are restricted to specific range of motion and some singularities will develop if specific angles are used.

**Table 2.1 Crash Victim Models**

<table>
<thead>
<tr>
<th>Model Name</th>
<th>Model Type</th>
<th>DOF</th>
<th>Body Segment Shape</th>
<th>Analytical Formulation</th>
<th>Developer</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAL 2D</td>
<td>2D</td>
<td>7</td>
<td>Rods</td>
<td>Lagrangian</td>
<td>McHenry Calspan</td>
</tr>
<tr>
<td>CAL 2D</td>
<td>2D</td>
<td>11</td>
<td>Spheres</td>
<td>Lagrangian</td>
<td>McHenry and Naab</td>
</tr>
<tr>
<td>CAL 2D</td>
<td>2D</td>
<td>11</td>
<td>Spheres</td>
<td>Lagrangian</td>
<td>Segal et al</td>
</tr>
<tr>
<td>Package</td>
<td>Dimension</td>
<td># of Elements</td>
<td>Geometry</td>
<td>Kinematic Model</td>
<td>Reference</td>
</tr>
<tr>
<td>-------------</td>
<td>-----------</td>
<td>---------------</td>
<td>--------------------------------</td>
<td>----------------</td>
<td>---------------------------</td>
</tr>
<tr>
<td>HSRI 2D</td>
<td>2D</td>
<td>10</td>
<td>Spheres</td>
<td>Lagrangian</td>
<td>Robbins</td>
</tr>
<tr>
<td>HSRI 3D</td>
<td>3D</td>
<td>12</td>
<td>Ellipsoids, cylinders</td>
<td>Lagrangian</td>
<td>Robbins</td>
</tr>
<tr>
<td>TTI</td>
<td>3D</td>
<td>31</td>
<td>Spheres</td>
<td>Lagrangian</td>
<td>Young</td>
</tr>
<tr>
<td>SIMULA</td>
<td>2D</td>
<td>9</td>
<td>Rods</td>
<td>Newtonian</td>
<td>Glancy and Larsen</td>
</tr>
<tr>
<td>HSRI 3D</td>
<td>3D</td>
<td>14</td>
<td>Ellipsoids, Cylinders</td>
<td>Lagrangian</td>
<td>Robbins</td>
</tr>
<tr>
<td>CAL 3D</td>
<td>3D</td>
<td>40</td>
<td>Ellipsoids</td>
<td>Newtonian</td>
<td>Bartz</td>
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<tr>
<td>MVMA 2D</td>
<td>2D</td>
<td>10</td>
<td>Spheres</td>
<td>Lagrangian</td>
<td>Robbins, MAMA</td>
</tr>
<tr>
<td>PROMETHEUS</td>
<td>2D</td>
<td>9</td>
<td>Rods</td>
<td>Lagrangian</td>
<td>Twigg, Karnes</td>
</tr>
<tr>
<td>CAL 3D</td>
<td>3D</td>
<td>N/A</td>
<td>Ellipsoids</td>
<td>Newtonian</td>
<td>Flack</td>
</tr>
<tr>
<td>UCIN</td>
<td>3D</td>
<td>31</td>
<td>Ellipsoids, cylinders, cone</td>
<td>Kane’s Equation</td>
<td>Huston</td>
</tr>
<tr>
<td>MADAMO</td>
<td>2D, 3D</td>
<td>N/A</td>
<td>Ellipsoids</td>
<td>Lagrangian</td>
<td>Maltha, Wismans</td>
</tr>
<tr>
<td>SUPER CRASH</td>
<td>3D</td>
<td>54</td>
<td>Ellipsoids, cylinders, cones</td>
<td>Kane’s Equation</td>
<td>Huston</td>
</tr>
</tbody>
</table>
In the previous models, the joints between the human segments were simulated by a hinge joint (one-rotational degree of freedom) or a ball and socket joint (three-rotation degrees of freedom). However, translational motions should be considered as well. The translational motion of a joint, such as neck stretching, will cause the soft tissues like ligaments and tendons to strain or tear off. Therefore, it is imperative to consider translational motion at the joints. As a result, some of the joints will have more than three degrees of freedom.

Of all the injuries suffered in vehicle accidents, those affecting the head/neck system are the most debilitating. Comprising 60% of the injuries, head and neck injuries, lead to partial paralysis, loss of mental function and loss of organ function. Therefore, it is believed that the head/neck joint system should be considered in this model. Among the factors during head impact that could influence the measured values of kinematics parameters are impact site, impact surface, impact mass, and motion limiting features (i.e. at the neck). All of these are considered in the model.
3. Kinematics of Multi-Body System

3.1 Introduction

A “multi-body system” is defined as a collection of linked or disjoint rigid bodies as shown in Figure 3.1. Many mechanical systems such as robots, chains, antennas, human body models, and other bio-systems can be effectively modeled as systems of flexible and rigid bodies, called multi-body systems. Adjacent bodies may be connected in many ways with three translational and three rotational degrees of freedom allowed between each pair of adjoining bodies. If some of these variables are specified then different types of joints will be formed such as prismatic, revolute, cylindrical or spherical joints.

The basic theory behind understanding the kinematics of a multi-body system is the bodies’ organization and movement relative to each other. A method developed by Huston et al. will be used. It provides a step-by-step procedure to develop the kinematics of multi-body systems.

3.2 Body connection array

Consider the multi-body system in Figure 3.1. The first body is selected arbitrarily and called B₁. Next, the bodies are numbered and labeled in ascending progression away from B₁, moving from branch to branch in the system until all the bodies are numbered (i.e. B₂, B₃, B₄, … etc).

Each body, except B₁, has an adjacent lower numbered body. Thus, a listing of the lower numbered bodies for each body is developed. Let L(K), called a “lower body
array”, be this listing where, K is the corresponding body number. Thus, L(K) defines the connection order between the bodies where $L^0(K) = K$, $L^1(K) = L(K)$, $L^2(K) = L(L(K))$, $L^3(K) = L(L(L(K)))$, …, etc. For the system shown in Figure 3.2, L(K) is

Figure 3.1 A Multi-Body System

Figure 3.2 Numbering of the Multi-Body System
\[ K = (1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8) \]
\[ L(K) = (0 \ 1 \ 2 \ 1 \ 4 \ 1 \ 6 \ 6) \]

The reference frame \( R \) is considered as \( B_0 \).

There are other arrays that are useful in developing the kinematics algorithm such as the ending body array, branching body array and intermediate body array. The ending bodies are those bodies that do not have adjacent higher numbered bodies such as \( B_3, B_5, B_7 \) and \( B_8 \). The branching bodies are those bodies at the dividing points of the system, such as \( B_1 \) and \( B_6 \). The intermediate bodies are those bodies that are neither ending nor branching.

### 3.3 Orientation angles

The orientation angles define the relative orientation of two adjacent connected bodies. The orientation of each body requires three angles say \( \alpha, \beta \) and \( \gamma \). Consider two typical adjoining bodies \( B_j \) and \( B_k \) with their axes’ system mutually aligned as shown in Figure 3.3.

![Figure 3.3 Adjacent bodies in a Multi-body system](image)
B\textsubscript{k} can be brought into a general orientation relative to B\textsubscript{j} by three successive rotations through the angles $\alpha\textsubscript{k}$, $\beta\textsubscript{k}$ and $\gamma\textsubscript{k}$ about the axes N\textsubscript{1}, N\textsubscript{2} and N\textsubscript{3}. These angles are considered the dextral angles because the positive rotation is in the right hand direction, or the dextral sense, relative to the axis. Sometime, the orientation of body B\textsubscript{k} relative to B\textsubscript{j} can be defined in terms of a single rotation about one of the coordinate axes such as the hinge joint.

### 3.4 Transformation Matrices

Consider the two adjoining bodies of a multi-body system B\textsubscript{j} and B\textsubscript{k} (Fig. 3.3). Let $n\textsubscript{m}$ and $N\textsubscript{n}$ (m, n =1, 2, 3) be two mutually perpendicular unit vectors fixed in bodies B\textsubscript{j} and B\textsubscript{k} respectively. Thus, the direction of body B\textsubscript{k} with respect to B\textsubscript{j} can be expressed in terms of $n\textsubscript{m}$ and $N\textsubscript{n}$ as follows

$$
n\textsubscript{m} = (n\textsubscript{m} \cdot N\textsubscript{1}) N\textsubscript{1} + (n\textsubscript{m} \cdot N\textsubscript{2}) N\textsubscript{2} + (n\textsubscript{m} \cdot N\textsubscript{3}) N\textsubscript{3}
$$

$$
= (n\textsubscript{m} \cdot N\textsubscript{n}) N\textsubscript{n} = SJK\textsubscript{mn} N\textsubscript{n}
$$

Where SJK is the transformation matrix from body j to body k. The components of the SJK matrix are described in the short form

$$
SJK = n\textsubscript{m} \cdot N\textsubscript{n}
$$

If we want to express the orientation of body B\textsubscript{j} with respect to body B\textsubscript{k} in terms of unit vectors then

$$
N\textsubscript{n} = SJK\textsubscript{mn}^T n\textsubscript{m}
$$

Where SJK\textsubscript{mn}^T is the transpose of SJK\textsubscript{mn}.

Thus, SJK\textsubscript{mn}^T * SJK\textsubscript{mn} = I (where I is the identity matrix).
The unit vectors can be related to each other using graphs or charts, called the configuration graphs. They are useful in developing the algorithms for determining the transformation matrices.

(a) $\alpha$ – rotation

(b) $\beta$ - rotation
For instance, the relative orientation of $B_k$ with respect to $B_j$ can be expressed in terms of the relative inclination of the $n_i$ and $N_i$, using the three relative angles $\alpha$, $\beta$, and $\gamma$. To relate the unit vectors $N_i$ and $n_i$ to each other. The following equations are developed for three subsequent rotations around the unit vectors $n_1$, $n_2$, and $n_3$ (Figs. 3.4 a, b, c).

If the unit vectors $n_1$ and $N_1$ coincide and the rotation, with $\alpha$ degrees, happens around the $N_1$ unit vector, then the following equations is developed:

\[
\begin{align*}
  n_1 &= N_1 \\
  n_2 &= C_\alpha N_2 - S_\alpha N_3 \\
  n_3 &= S_\alpha N_2 + C_\alpha N_3
\end{align*}
\]

(3.1a)

Where $C_\alpha = \cos \alpha$ and $S_\alpha = \sin \alpha$

Similarly for $\beta$ and $\gamma$ rotations, the following equations are obtained:

\[
\begin{align*}
  n_1 &= C_\beta N_1 + S_\beta N_3 \\
  n_2 &= N_2 \\
  n_3 &= -S_\beta N_1 + C_\beta N_3
\end{align*}
\]

(3.1b)
\[ n_1 = C_\gamma N_1 - S_\gamma N_2 \]
\[ n_2 = S_\gamma N_1 + C_\gamma N_2 \quad (3.1c) \]
\[ n_3 = N_3 \]

Conversely, if the \( \alpha \) rotation happens from \( N_i \) coordinates to match \( n_j \) coordinates then the relation will be

\[ N_1 = n_1 \]
\[ N_2 = C_\alpha n_2 + S_\alpha n_3 \quad (3.2a) \]
\[ N_3 = -S_\alpha n_2 + C_\alpha n_3 \]

Similarly for \( \beta \) and \( \gamma \) rotations, the following equations will develop:

\[ N_1 = C_\beta n_1 - S_\beta n_3 \]
\[ N_2 = n_2 \quad (3.2b) \]
\[ N_3 = S_\beta n_1 + C_\beta n_3 \]

\[ N_1 = C_\gamma n_1 + S_\gamma n_2 \]
\[ N_2 = -S_\gamma n_1 + C_\gamma n_2 \]
\[ N_3 = n_3 \quad (3.2c) \]

These equations can be expressed in matrix form as well. For example equation 3.1a can be written in the following format:

\[
\begin{bmatrix}
  n_1 \\
  n_2 \\
  n_3
\end{bmatrix} =
\begin{bmatrix}
  1 & 0 & 0 \\
  0 & C_\alpha & -S_\alpha \\
  0 & S_\alpha & C_\alpha
\end{bmatrix}
\begin{bmatrix}
  N_1 \\
  N_2 \\
  N_3
\end{bmatrix}
\quad (3.3)
\]

It can also be expressed in the form of configuration graphs as followed:
In this diagram there are six dots, or nodes, each representing one of the six unit vectors of Figure 6.3.3. The horizontal line then designated equality \( n_i = N_i \). The inclined line connects nodes associated with the “inside” unit vectors. Consequently, the unconnected nodes are associated with the “outside” unit vectors.

The three rotations can also be shown as in the following configuration graph:

\[
\begin{array}{ccc}
1 & N_i & n_i \\
2 \\
3 & \alpha & \beta & \gamma \\
\end{array}
\]

In the matrix forms, we have

\[
\begin{bmatrix}
n_1 \\
n_2 \\
n_3
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 \\
0 & C_\beta & -S_\beta \\
0 & S_\alpha & C_\alpha
\end{bmatrix} \begin{bmatrix}
C_\lambda & -S_\gamma & 0 \\
S_\gamma & C_\gamma & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
N_1 \\
N_2 \\
N_3
\end{bmatrix}
\]

Thus, the general transformation matrix from \( n_i \) to \( N_i \) will be

\[
\begin{bmatrix}
n_1 \\
n_2 \\
n_3
\end{bmatrix} = \begin{bmatrix}
C_\beta C_\gamma & -C_\beta S_\gamma & S_\beta \\
(C_\alpha S_\gamma + S_\alpha S_\beta C_\gamma) & (C_\alpha C_\gamma - S_\alpha S_\beta S_\gamma) & -S_\alpha C_\beta \\
(S_\alpha S_\gamma - C_\alpha S_\beta C_\gamma) & (C_\alpha S_\beta S_\gamma + S_\alpha C_\gamma) & C_\alpha C_\beta
\end{bmatrix} \begin{bmatrix}
N_1 \\
N_2 \\
N_3
\end{bmatrix}
\] (3.4)

Similarly, the general transformation matrix from \( N_i \) to \( n_i \) will be

\[
\begin{bmatrix}
N_1 \\
N_2 \\
N_3
\end{bmatrix} = \begin{bmatrix}
C_\beta C_\gamma & (C_\alpha S_\gamma + S_\alpha S_\beta C_\gamma) & (S_\alpha S_\gamma - C_\alpha S_\beta C_\gamma) \\
-C_\beta S_\gamma & (C_\alpha C_\gamma - S_\alpha S_\beta S_\gamma) & (C_\alpha S_\beta S_\gamma + S_\alpha C_\gamma) \\
S_\beta & -S_\alpha C_\beta & C_\alpha C_\beta
\end{bmatrix} \begin{bmatrix}
n_1 \\
n_2 \\
n_3
\end{bmatrix}
\] (3.5)

These expressions can be used to describe the orientation of adjacent bodies relative to each other. The specifications and variables of each body (i.e. position,
velocity, acceleration ... etc) can similarly be expressed relative to each other. To illustrate this, consider the relative angular velocity of any two adjacent bodies say B_j and B_k, of a multi-body system (Fig. 3.5). The angular velocity of B_k in the reference frame R_k is (when n_3=N_3)

\[ \overset{R_k}{\omega}^g = \dot{\gamma} N_3 = \dot{n}_3 \]  

(3.6a)

![Figure 3.5 Adjacent Bodies in a Multi-body System](image)

Similarly, the relative angular velocity between the two reference frames R_j and R_k will be

\[ \overset{R_j}{\omega}^R_k = \dot{\beta} N_2 = \dot{\beta} n_2 \]  

(3.6b)

The relative angular velocity between the reference frame R_j and the body B_j will be

\[ \overset{R_j}{\omega}^R_j = \dot{\alpha} N_1 = \dot{\alpha} n_1 \]  

(3.6c)

Then, the total transformation of the relative angular velocity between the bodies B_j and B_k will be
Now, it is better if the relative angular velocity is expressed entirely in terms of either $n_i$ or $N_i$. Thus,

$$^B_i\omega^B_i = (\alpha + \gamma S_\beta) n_1 + (\dot{\beta} C_\alpha - \ddot{\gamma} C_\beta S_\alpha) n_2 + (\dot{\gamma} S_\alpha + \ddot{\gamma} C_\beta C_\alpha) n_3$$

(3.8)

or

$$^B_i\omega^B_i = (\alpha C_\beta C_\gamma + \dot{\beta} S_\gamma) N_1 + (\dot{\gamma} C_\beta - \ddot{\gamma} C_\beta S_\gamma) N_2 + (\dot{\gamma} S_\alpha + \ddot{\gamma}) N_3$$

(3.9)

These equations can be written in the form

$$^B_i\omega^B_i = \omega_1 n_1 + \omega_2 n_2 + \omega_3 n_3$$

(3.10)

and

$$^B_i\omega^B_i = \omega_1 N_1 + \omega_2 N_2 + \omega_3 N_3$$

(3.11)

By equating each coefficient from equations 3.10 and 3.11 with equations 3.8 and 3.9 respectively and solve for the angle derivatives we get

$$\dot{\alpha} = \omega_1 - S_\beta (\omega_3 C_\alpha - \omega_2 S_\alpha) / C_\beta$$

$$\dot{\beta} = \omega_2 C_\alpha + \omega_3 S_\alpha$$

$$\dot{\gamma} = (\omega_3 C_\alpha - \omega_2 S_\alpha) / C_\beta$$

(3.12)

and

$$\dot{\alpha} = (\omega_1 C_\gamma - \omega_2 S_\gamma) / C_\beta$$

$$\dot{\beta} = \omega_1 S_\gamma + \omega_2 C_\gamma$$

$$\dot{\gamma} = \omega_3 - S_\beta (\omega_1 C_\gamma - \omega_2 S_\gamma) / C_\beta$$

(3.13)

In both equations 3.12 and 3.13 we notice that singularities might develop if $C_\beta=0$ ($\beta=90^\circ$). To avoid singularities, we can avoid using the orientation angles and instead use
employing orientation parameters or Euler parameters (Whittaker 1957; Houston 1978), as described in the next section.

### 3.5 Euler parameters

Euler parameters provide another way of describing body orientation. They consist of four parameters and can be described as follows: Consider two adjacent bodies \( B_j \) and \( B_k \) as shown in Figure 3.6.

\[
\begin{align*}
\varepsilon_{k1} &= \lambda_{k1} \sin \left( \frac{\theta_k}{2} \right) \\
\varepsilon_{k2} &= \lambda_{k2} \sin \left( \frac{\theta_k}{2} \right) \\
\varepsilon_{k3} &= \lambda_{k3} \sin \left( \frac{\theta_k}{2} \right) \\
\varepsilon_{k4} &= \cos \left( \frac{\theta_k}{2} \right)
\end{align*}
\]  

(3.14)

**Figure 3.6 Two Typical Adjoining Bodies, Rotation Axis, and Rotation angle used in Euler parameter definition**

\( B_k \) may be brought into any general orientation to \( B_j \) by means of a single rotation about an appropriate axis. If \( \lambda_k \) is a unit vector along this axis and \( \theta_k \) is the rotation angle around the same axis, the four Euler parameters are:
Where \( \lambda_{ki} \) \((i=1,2,3)\) are the projections of \( \lambda_k \) along the \( X_j, Y_j, \) and \( Z_j \) axes. These are the direction cosines of the rotation axis referred to \( X_j, Y_j, \) and \( Z_j. \) The Euler parameters, unlike the dextral angles, are dependent because there are four Euler parameters describing the directions of a body, another dependent equations is

\[
\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2 = 1 \quad (3.15)
\]

The transformation matrix \( S_{JK} \) expressed in terms of Euler parameter is

\[
S_{JK} = \begin{bmatrix}
(e_1^2 - e_2^2 - e_3^2 + e_4^2) & 2(e_1^2 e_2^2 - e_3^2 e_4^2) & 2(e_1^2 e_3^2 + e_2^2 e_4^2) \\
2(e_1^2 e_2^2 + e_3^2 e_4^2) & (-e_1^2 + e_2^2 - e_3^2 + e_4^2) & 2(e_2^2 e_3^2 - e_1^2 e_4^2) \\
2(e_1^2 e_3^2 - e_2^2 e_4^2) & 2(e_2^2 e_3^2 + e_1^2 e_4^2) & (-e_1^2 - e_2^2 + e_3^2 + e_4^2)
\end{bmatrix} \quad (3.16)
\]

### 3.6 Time derivative transformation matrices

The transformation matrix between the reference frame and a body \( B_k \) can be written as

\[
S_{OK_{ij}} = n_{oi} \times n_{kj} \quad (3.17)
\]

Where \( n_{oi} \) and \( n_{kj} \) are the unit vectors fixed in the reference frame \( R \) and body \( k \) respectively \((i,j=1,2,3)\). To develop the time derivative of transformation matrices, it is helpful to use the permutation matrix whose elements are \( e_{ijk} \) \((i,j,k = 1,2,3)\) where

\[
e_{ijk} = \begin{cases} 
1 & \text{for } i,j,k \text{ distinct in cyclic order} \\
-1 & \text{for } i,j,k \text{ distinct in anticyclic order} \\
0 & \text{for } i,j,k \text{ not distinct}
\end{cases} \quad (3.18)
\]

Thus, the derivative of \( S_{OK_{ij}} \) in reference frame \( R \) may be written as

\[
\frac{d(S_{OK_{ij}})}{dt} = n_{oi} \left( \omega^k \times n_{kj} \right) = -\omega_p e_{imp} n_{og} \cdot n_j = -e_{imp} \omega_p S_{OK_{mp}} \quad (3.19)
\]
By introducing the matrix WOK, called the dual matrix of the angular velocity vector \( \omega \), we can write

\[
WOK_{im} = -e_{imp}\omega_p = \begin{bmatrix} 0 & -\omega_3 & -\omega_2 \\ -\omega_3 & 0 & -\omega_1 \\ -\omega_2 & -\omega_1 & 0 \end{bmatrix}
\]  

(3.20)

Thus, the derivative of the transformation matrix can be written as

\[
\frac{r}{dt}d(SOK_{ij}) = -e_{imp}\omega_p SOK_{mp} = WOK_{im}SOK_{mp}
\]

(3.21)

Where \( \omega_p \) \((p=1,2,3)\) are the \( n_k \) components of the angular velocity of body B in the reference frame R, where \( n_k \) are fixed in R.

In equation 3.21, for index \( i=1 \), multiply the three equations, which correspond to \((j=1,2,3)\), by \( SOK_{31}, SOK_{32}, SOK_{33} \) respectively and then add the resulting equations. Similarly, for equations with index \( i=2 \) and \((j=1,2,3)\), by \( SOK_{11}, SOK_{12}, SOK_{13} \) respectively and then add the resulting equations. Finally, for \( i=3 \) and \((j=1,2,3)\), multiply the three equations by \( SOK_{21}, SOK_{22}, SOK_{23} \) respectively and then add the resulting equations. This lead to the following equations:

\[
\begin{align*}
\omega_2 &= SOK_{31} \frac{d(SOK_{11})}{dt} + SOK_{32} \frac{d(SOK_{12})}{dt} + SOK_{33} \frac{d(SOK_{13})}{dt} \\
\omega_3 &= SOK_{11} \frac{d(SOK_{21})}{dt} + SOK_{12} \frac{d(SOK_{22})}{dt} + SOK_{13} \frac{d(SOK_{23})}{dt} \\
\omega_1 &= SOK_{21} \frac{d(SOK_{31})}{dt} + SOK_{22} \frac{d(SOK_{32})}{dt} + SOK_{23} \frac{d(SOK_{33})}{dt}
\end{align*}
\]

(3.22)

By substituting the transformation matrix (SJK) of equation 3.16 in equation 3.22, we can obtain the relative angular velocities expressed in terms of Euler parameters as
\[
\begin{align*}
\omega_1 &= 2(\varepsilon_4 \dot{\varepsilon}_1 - \varepsilon_3 \dot{\varepsilon}_2 + \varepsilon_2 \dot{\varepsilon}_3 - \varepsilon_1 \dot{\varepsilon}_4) \\
\omega_2 &= 2(\varepsilon_3 \dot{\varepsilon}_1 + \varepsilon_4 \dot{\varepsilon}_2 - \varepsilon_1 \dot{\varepsilon}_3 - \varepsilon_2 \dot{\varepsilon}_4) \\
\omega_3 &= 2(-\varepsilon_2 \dot{\varepsilon}_1 + \varepsilon_1 \dot{\varepsilon}_2 + \varepsilon_4 \dot{\varepsilon}_3 - \varepsilon_3 \dot{\varepsilon}_4) \\
\end{align*}
\] (3.23)

These expressions are a set of three equations for the four \( \varepsilon_i \) \((i=1,2,3,4)\). Therefore, the derivatives of the four Euler parameters cannot be uniquely determined from these equations. However, if we recall that the \( \varepsilon_i \) are not independent and that they must satisfy equation 3.15, then we can obtain a unique solution. By differentiating Equation 3.15, we obtain

\[
2(\varepsilon_1 \dot{\varepsilon}_1 + \varepsilon_2 \dot{\varepsilon}_2 + \varepsilon_3 \dot{\varepsilon}_3 + \varepsilon_4 \dot{\varepsilon}_4) = 0 
\] (3.24)

By appending equation 3.18 to equation 3.17 we get a set of four equations for the Euler parameters derivatives that can be written in a matrix form as follows

\[
\begin{bmatrix}
\omega_1 \\
\omega_2 \\
\omega_3 \\
\omega_4 \\
\end{bmatrix}
= 2
\begin{bmatrix}
\varepsilon_4 & -\varepsilon_3 & \varepsilon_2 & -\varepsilon_1 \\
\varepsilon_3 & \varepsilon_4 & -\varepsilon_1 & -\varepsilon_2 \\
-\varepsilon_2 & \varepsilon_1 & \varepsilon_4 & -\varepsilon_3 \\
\varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \varepsilon_4 \\
\end{bmatrix}
\begin{bmatrix}
\dot{\varepsilon}_1 \\
\dot{\varepsilon}_2 \\
\dot{\varepsilon}_3 \\
\dot{\varepsilon}_4 \\
\end{bmatrix}
\] (3.25)

This set of four equations is linear in the Euler parameter derivative where \( \omega_4 = 0 \). By solving for the Euler parameter derivatives we obtain,

\[
\begin{align*}
\dot{\varepsilon}_1 &= \frac{1}{2}(\varepsilon_4 \omega_1 + \varepsilon_3 \omega_2 - \varepsilon_2 \omega_3) \\
\dot{\varepsilon}_2 &= \frac{1}{2}(-\varepsilon_3 \omega_1 + \varepsilon_4 \omega_2 + \varepsilon_1 \omega_3) \\
\dot{\varepsilon}_3 &= \frac{1}{2}(\varepsilon_2 \omega_1 - \varepsilon_4 \omega_2 + \varepsilon_4 \omega_3) \\
\dot{\varepsilon}_4 &= \frac{1}{2}(-\varepsilon_1 \omega_1 - \varepsilon_2 \omega_2 - \varepsilon_3 \omega_3) \\
\end{align*}
\] (3.26)

Unlike equation 3.13, there are no singularities in equation 3.26.
3.7 Generalized speeds

Considering Euler parameters with a system of N bodies, there will be 7N degrees of freedom (three translation coordinates and four rotation coordinates for each body). These coordinates can be generalized, (i.e. using one variable rather than different variables for translation coordinates and rotation coordinate). Such coordinates, called body coordinates, can be conveniently expressed as,

\[
\begin{align*}
X_i &= \begin{cases} 
\varepsilon_{km} & l = 4(k-1) + m, \quad k = 1, \ldots, N \\
S_{kn} & l = 4N + 3(k-1) + n, \quad k = 1, \ldots, N
\end{cases} \\
&\quad m = 1,2,3,4 \quad n = 1,2,3
\end{align*}
\]  

(3.27)

where \( \varepsilon_{km} \) and \( s_{kn} \) are Euler parameters and translation variable respectively.

The generalized speeds can be obtained by differentiating the body coordinates in equation (3.27) with respect to time. They can be express as

\[
\begin{align*}
Y_i &= \begin{cases} 
\sigma_{kn} & l = 3(k-1) + n, \quad k = 1, \ldots, N \\
\dot{S}_{kn} & l = 3N + 3(k-1) + n, \quad k = 1, \ldots, N
\end{cases} \\
&\quad n = 1,2,3
\end{align*}
\]  

(3.28)

where \( \sigma_{kn} \) is the relative angular velocity of body \( B_k \). Similarly, by deriving equation (3.28), the generalized accelerations can be expressed as

\[
\begin{align*}
\ddot{Y}_i &= \begin{cases} 
\ddot{\sigma}_{kn} & l = 3(k-1) + n, \quad k = 1, \ldots, N \\
\ddot{S}_{kn} & l = 3N + 3(k-1) + n, \quad k = 1, \ldots, N
\end{cases} \\
&\quad n = 1,2,3
\end{align*}
\]  

(3.29)

3.8 Angular velocities

The angular velocities of a multi-body system can be computed utilizing the lower numbered array concept. Consider the multi-body system, described in Figure3.2. The absolute angular velocities of each body can be expressed as

\[\omega_k = \sum_{p=0}^{r} \sigma_q \quad \text{where } q = L^p(k) \]  

(3.30)
where $r$ is the index such that $L'(K)$ is 1, and $\omega$ is the relative angular velocity. Thus, for example, the angular velocity of body 5 ($B_5$) in Figure 3.2 is

$$\omega_5 = \omega_5 + \omega_4 + \omega_1 \quad (3.30a)$$

The angular velocities of the bodies of a multibody system may be written in the component form as:

$$R \omega_{B_k} = R \omega_{kt} + \sum_{r=1}^{n} R \omega_{kr} \dot{q}_r \quad (3.31)$$

where the $R \omega_{kr}$ are identified by equation as

$$R \omega_{kt} = \frac{2R \omega_{B_k}}{2\dot{q}_r} \quad (3.32)$$

The vectors $\omega_t$ and $\omega_r$ are called the partial angular velocity of a body $B$ with respect to time $t$ and with respect to position $q_r$ in reference frame $R$.

Thus, the angular velocities can be expressed as

$$\omega_k = \omega_{klm} \dot{x}_{n_{om}} \quad (k = 1,...,N; l = 1,...,3N; m = 1,2,3)$$

$$= \omega_{klm} y_{l} \dot{n}_{om} \quad (l = 1,...,3N) \quad (3.33)$$

Where the $\dot{x}_l$ represent the respective orientation angles, $\omega_{klm}$ is the $n_{om}$ components of partial angular velocity vectors, and $y_l$ are the generalized speeds.

For $(3k-3) < l <= (3k)$:

$$\omega_{klm} = \begin{cases} 
SOJ_{m_1} & S_{m_2} \alpha J & S_{n_3} \beta J \\
SOJ_{m_2} & S_{m_2} \alpha J & S_{n_3} \beta J \\
SOJ_{m_3} & S_{m_3} \beta J & S_{n_3} \alpha J 
\end{cases} \quad (3.34)$$

where $J = L(K), K = k$

For $l <= (3k-3)$:

$$\omega_{klm} = \omega_{jlm}, \quad j = J \quad (3.35)$$
For \( l > 3k \):

\[
\omega_{klm} = 0 \quad \text{(3.36)}
\]

Where the first, second or third choice made in equation (3.34) depends upon whether \( l \) is an \( \alpha \), \( \beta \), or \( \gamma \) rotation.

### 3.9 Angular accelerations

The angular acceleration in reference frame \( R \) of a typical body of a system has the form of

\[
\alpha_k = \frac{d}{dt}(\dot{\omega}_{klm} \dot{x} + \omega_{klm} \dot{x}) n_{om} = (\dot{\omega}_{klm} \dot{y} + \omega_{klm} \dot{y}) n_{om} \quad \text{(3.37)}
\]

The derivative of partial angular velocity may be obtained by differentiating equation (3.34). Thus,

For \((3k-3) < l \leq (3k)\):

\[
\dot{\omega}_{klm} = \begin{cases} 
SOJ_{m1} \\
SOJ_{n\alpha}J_{n2} + SOJ_{mn} \alpha J_{n2} \\
SOJ_{mn\alpha}J_{np}J_{p3} + SOJ_{mn} \alpha J_{np}J_{p3} + SOJ_{mn} \alpha J_{np} \beta J_{p3}
\end{cases} \quad \text{(3.38)}
\]

Where \( J = L(K), K = k \),

For \( l \leq (3k-3)\):

\[
\dot{\omega}_{klm} = \dot{\omega}_{jlm}, \quad j = J
\]

For \( l > (3k)\):

\[
\dot{\omega}_{klm} = 0 \quad \text{(3.40)}
\]

Where the first, second, or third choice made in equation (3.38) depends upon whether \( l \) is an \( \alpha \), \( \beta \), and \( \gamma \) rotation.
3.10 Mass center velocities

Let $P_k$ be the mass center position vector of $B_k$ relative to a fixed point in the inertial frame $R$ (Fig. 3.7). Then, $P_k$ can be expressed as

$$P_k = \sum_{i=0}^{b} (q_i + s_i) + r_k \quad (3.41)$$

Where $v=L^i(k)$, $L^b(k)=1$

![Figure 3.7 Mass center position $P_k$ in body $B_k$ with respect to inertial reference frame $R$](image)

The mass center velocity of $B_k$ with respect to the inertial frame can be obtained by taking the time derivative of $P_k$ in equation (3.41):

$$v_k = \frac{\dot{R}}{dt} = \sum_{i=0}^{b} \left[ \omega_\nu \times (q_i + s_i) + \dot{s}_i \right] + \omega_k r_k \quad (3.42)$$

where as before $b$ is the index such that $L^b(k)$ is unity and $u$ is $L(v)$.

3.11 Mass center accelerations

The mass center acceleration of a body $B_k$ is the derivative of the mass center velocity expressed in equation 3.42; hence,
\[ a_k = \frac{r dv_k}{dt} = \sum_{i=0}^{b} \left[ a_u \times (q_v + s_v) + \omega_u \times (q_v + s_v) + \hat{s}_v + (\omega_u \times \hat{s}_v + \hat{s}_v) \right] + a_k \times r_k + \omega_k \times (\omega_k \times r_k) \] (3.43)
4. Kinetics of Multi-body Systems

4.1 Introduction

Multi-body kinetics involves the study of force and/or moment systems, applied to a multibody system. Two vectors can characterize a force system: the “resultant” and the “moment about some point”. The resultant is defined as the sum of the force vectors. The moment about any point can be defined as the sum of moments of the individual forces about that point.

4.2 Generalized forces

Suppose that numbers of forces are applied on a typical body $B_k$ of a multi-body system. These forces may be replaced by an equivalent force system consisting of a single force $F$ passing through a point $P$ of $B_k$ and a couple with torque $T$. Thus, the generalized force system $F_r$ is then defined as

$$F_r = \vec{F} \cdot \frac{\partial \vec{v}}{\partial x_r} + \vec{T} \cdot \frac{\partial \omega_k}{\partial x_r} \quad r = 1, \ldots, n \quad (4.1)$$

Where $\omega_k$ is the angular velocity of $B_k$ in R, $\vec{v}$ is the velocity of P in R, and $n$ is the number of degree of freedom.

4.3 Inertia forces

Consider a rigid body B that is composed of N particles. If B is moving in an inertia reference frame R, there will be the N inertia forces acting on B. The inertia forces can be computed from the following equation
\[ F + F^* = 0 \] (4.2)

Where \( F^* \) is the inertia force applied on a partial \( P \) of \( B \).

Since

\[ F = m^p a^p \] (4.3)

Then,

\[ F = -m^p a^p \] (4.4)

This formulation is attributed to J d’Alembert [Tuma JJ1974 Huston book] and is often referred to as “d’Alembert principle” [27]. Hence, the generalized inertia forces, analogous to equation (4.1), can be expressed as

\[ F^*_r = \ddot{F}^* \cdot \frac{\partial \ddot{v}^*}{\partial x_r} + \dddot{F}^* \cdot \frac{\partial \dddot{\hat{o}}_k}{\partial x_r} \] (4.5)

Moreover, the generalized inertia forces on body \( B_k \) can be expressed in terms of the generalized coordinate derivative \( \dot{y}_l \) as,

\[ F^*_l = \ddot{F}^*_k \cdot \frac{\partial \ddot{v}_k}{\partial \hat{y}_l} + \dddot{F}^*_k \cdot \frac{\partial \dddot{\hat{o}}_k}{\partial \hat{y}_l} \] (4.6)

Recall from equation (3.33) and (3.37) that the angular velocity \( \omega_k \) and angular acceleration \( \alpha_k \) can be expressed in terms of the generalized speeds as follows,

\[ \omega_k = \omega_{klm} y_l n_{om} \quad (l = 1, \ldots, 3N) \] (4.7)

\[ \alpha_k = - (\dot{\omega}_{klm} y_l + \omega_{klm} \dot{y}_l) n_{om} \] (4.8)

Similarly, the mass center velocities and accelerations can be expressed in terms of the generalized speed as,

\[ v_k = v_{klm} y_l n_{om} \quad (l = 1, \ldots, 3N) \] (4.9)
\[ \alpha_k = -(\dot{v}_{klm} y_i + v_{klm} \dot{y}_i)n_{pn} \quad (4.10) \]

Hence, the generalized inertia forces can be written as,

\[ F_i^* = \bar{F}_{km}^* \cdot v_{klm} + \bar{T}_{km}^* \cdot \omega_{klm} \quad (no \ sum \ on \ k) \quad (4.11) \]

The inertia force system can also be expressed in terms of the partial velocity and partial angular velocity as follows,

\[ F_k^* - m_k \alpha_k = -m_k (\dot{v}_{klm} y_i + v_{klm} \dot{y}_i)n_m \quad (4.12) \]

and

\[ T_k^* = -I_k \cdot \alpha_k - \omega_k \times (I \cdot \omega_k) \]
\[ = -I_k \cdot (\omega_{klm} \dot{y}_i + \dot{\omega}_{klm} y_i)n_m \quad (no \ sum \ on \ k) \quad (4.13) \]
\[ = -(\omega_{klm} y_i n_m) \times [I_k \cdot (\omega_{klm} y_i n_m)] \]

Where \( m_k \) is the mass center of \( B_k \).

Since \( I_k \) can be written as \( I_{kmn} n_m n_n \). Then, by substituting this expression in equation (4.13) and carrying out indicated operations, the generalized inertia moments will be

\[ T_k^* = -[I_{kmn} (\omega_{klm} \dot{y}_i + \dot{\omega}_{klm} y_i) + e_{rpm} \omega_{kmr} \omega_{kmr} I_{kmn} y_i y_p]n_m \quad (no \ sum \ on \ k) \quad (4.14) \]

By substituting equations (4.14) and (4.12) into equation (4.11), we get,

\[ F_i^* = -m_k v_{klm} (\dot{v}_{kpm} y_p + v_{kpm} \dot{y}_p) - I_{kmn} \omega_{klm} (\omega_{kpn} \dot{y}_p + \dot{\omega}_{kpn} y_p) \]
\[ - I_{kmn} \omega_{klm} e_{rpm} \omega_{kmr} \omega_{kmr} y_q y_p \quad (no \ sum \ on \ k) \quad (4.15) \]

This form can be expressed concisely, by gathering together the multipliers of the derivative of \( y_i \) such that

\[ F_i^* = -a_{lp} \dot{y}_p - h_j \quad (4.16) \]

Where \( a_{lp} \), called the generalized inertia coefficient, is
\[ \alpha_{lp} = m_k v_{klm} v_{kpm} + I_{kmn} \omega_{klm} \omega_{kpm} \quad (no \ sum \ on \ k) \]  

(4.17)

and \( h_l \), called the generalized inertia force coefficient, is

\[ h_l = m_k v_{klm} \dot{v}_{kpm} y_{p} + I_{kmn} \omega_{klm} \dot{\omega}_{kpm} y_{p} + I_{kmn} \omega_{klm} e_{nm} \omega_{lqm} \omega_{kpm} y_{q} y_{p} \quad (no \ sum \ on \ k) \]  

(4.18)

Using equations (4.2) and (4.16), the dynamical equations can be written as,

\[ a_{lp} \dot{x}_p = f_i \]  

(4.19)

Where \( f_i \) is defined as \( f_i = F_l - h_l \). Also, equation (4.19) can be expressed in terms of the generalized speeds such that,

\[ a_{lp} \dot{y}_p = f_i \]  

(4.20)

### 4.4 Translational Spring and damper forces on multi-body systems

The springs and dampers are of important interest in the dynamical analysis of multi-body systems. The forces that are exerted on the bodies could be external or internal forces. Consider two typical adjoining bodies \( B_j \) and \( B_k \) as in figure (4.1).

![Figure 4.1 Bodies B_j and B_k connected by springs and dampers.](image)

The springs and dampers, located along the line \( L \), will produce force \( F_j \) and \( F_k \) on the Bodies \( B_j \) and \( B_k \) respectively. Let \( u \) be the distance between the center of the two
bodies. The forces $F_j$ and $F_k$ can be expressed, in terms of $u$ and the unit vector $n_1$ and $n_2$, as

$$F_j = f(u, \dot{u})n \quad (4.21a)$$

and

$$F_k = -f(u, \dot{u})n = -F_j \quad (4.21b)$$

Where $f(u, \dot{u})$ is a function depending upon the physical properties of the spring and the damper. Utilizing equation (4.5), the spring and damper forces on the bodies $B_j$ and $B_k$ can be written as,

$$F_r = F_j \frac{\partial v^P_j}{\partial x_r} + F_k \frac{\partial v^P_k}{\partial x_r} \quad (4.22)$$

The velocities of $P_j$ and $P_k$ are written as,

$$v^P_j = v^P_j + \dot{u}n_1 + (\dot{u})n_2 \quad (4.23)$$

Since $u$ is a function of $x_r$, then $du/dt$ can be expressed as,

$$\dot{u} = \frac{du}{dt} = \sum_{r=1}^{n} \left( \frac{\partial u}{\partial x_r} \right) \dot{x}_r + \frac{\partial u}{\partial t} \quad (4.24)$$

Hence,

$$\frac{\partial \dot{u}}{\partial \dot{x}_r} = \frac{\partial u}{\partial x_r} \quad (4.25)$$

Similarly, for a rotation $\theta$ we can write,

$$\frac{\partial \dot{\theta}}{\partial \dot{x}_r} = \frac{\partial \theta}{\partial x_r} \quad (4.25a)$$

By differentiating equation (4.23) and using equation (4.25) we get,
\[ \frac{\partial v^R_i}{\partial x_r} = \frac{\partial v^R_j}{\partial x_r} + \frac{\partial u}{\partial x_r} n_1 + \frac{\partial}{\partial x_r} n_2 \]  

(4.26)

Thus the contribution of the forces \( F_j \) and \( F_k \) to the generalized force \( F_r \) will be,

\[
F_r = F_j \cdot \frac{\partial v^R_j}{\partial x_r} + F_k \cdot \frac{\partial v^R_k}{\partial x_r} \\
= F_j \cdot \left( \frac{\partial v^P_j}{\partial x_r} + \frac{\partial u}{\partial x_r} n_1 + \frac{\partial}{\partial x_r} n_2 \right) \\
= -f n_1 \cdot \left( \frac{\partial u}{\partial x_r} n_1 + \frac{\partial}{\partial x_r} n_2 \right) \\
= -f \frac{\partial u}{\partial x_r} 
\]

(4.27)

### 4.5 Torsional Spring and damper forces on multi-body systems

Consider two typical bodies \( B_j \) and \( B_k \), connected by a hinge where its axis of rotation is parallel to a unit vector \( \lambda \), as in Figure 4.2. To control the movement around this axis, torsional springs and torsional dampers might be applied at the hinge joint producing a moments on both bodies \( T_j \) and \( T_k \). \( T_j \) and \( T_k \) might be written in terms of \( \theta \) rotation as,

![Figure 4.2 Torsional springs and dampers at the hinge joint between Bodies \( B_j \) and \( B_k \)]

\[
T_j = g(\theta, \dot{\theta})\lambda 
\]

(4.28a)
And

$$T_k = -T_j = -g(\theta, \dot{\theta})\dot{\lambda} \quad (4.28b)$$

Where $g(\theta, \dot{\theta})$ is function of the physical properties of the torsional springs and dampers. Hence, the generalized active force $F_r$ can be expressed in terms of the moments $T_j$ and $T_k$ as,

$$F_r = T_j \cdot \frac{\partial \omega_j}{\partial x_r} + T_k \cdot \frac{\partial \omega_k}{\partial x_r} \quad (4.29)$$

Where $\omega_j$ and $\omega_k$ are the angular velocities of $B_j$ and $B_k$. From equation (3.30a), $\omega_k$ can be written as a function of $\omega_j$ as

$$\omega_k = \omega_j + \dot{\theta} \dot{\lambda} \quad (4.30)$$

By substituting equation (4.30) in equation (4.29) and using equation (4.25a), we obtain

$$F_r = -g \frac{\partial \theta}{\partial x_r} \quad (4.31)$$

### 4.6 Joint Forces and Moments

In the crash victim analysis, the human body is subjected to internal forces and moments at the joints between the bodies. Such forces and moments are generated due to friction or energy sources like muscles at the joints. Hence, these forces and moments contribute to the generalized forces in the multi-body system. Consider two typical bodies $B_j$ and $B_k$ that are connected by a joint, which allows translation and rotation (Fig 4.3).
Figure 4.3 Bodies $B_j$ and $B_k$ are connected with a joint that allows translation and rotation.

The relative translation between the bodies is represented by the variable $s_k$, which may be expressed in terms of the unit vector $n_{ij}$ as,

$$s_k = s_{ij} n_{ij} \quad (4.32)$$

Similarly, the relative orientation (rotation) is measured by the generalized coordinate derivative such as the relative angular velocity $\omega_k$. Such coordinates can be written in terms of the unit vector $n_{ij}$ as,

$$\omega_k = \omega_{ij} n_{ij} \quad (4.33)$$

The forces and moments transmitted through the joints are the forces and moments that are exerted from both bodies $B_j$ and $B_k$, which are $F_j$, $F_k$, $M_j$, $M_k$ acting on a point on either body. Let this point be on body $B_j$ that coincides with $O_k$ and let it be called $Q_k$. The forces and moments are opposite to each other such that,

$$F_j = -F_k = -f_{ij} n_{ij}$$
$$M_j = -M_k = -m_{ij} n_{ij} \quad (4.34)$$

Thus, the generalized forces $F_r$ can be expressed as,
\[
F_r = F_j \left( \frac{\partial v_{Q_j}}{\partial \dot{x}_r} \right) + M_j \left( \frac{\partial \omega_j}{\partial \dot{x}_r} \right) + F_k \left( \frac{\partial v_{Q_k}}{\partial \dot{x}_r} \right) + M_k \left( \frac{\partial \omega_k}{\partial \dot{x}_r} \right) \quad (4.35)
\]

Using the relations,

\[
v_{o_r} = v_{Q_o} + \dot{s}_k n_{ij} \quad (4.36)
\]

and,

\[
\omega_k = \omega_j + \sigma_k n_{ij} \quad (4.37)
\]

Where \( \sigma_{kj} \) is the relative angular velocity between the two bodies, the generalized active force for \( x_r \) can be obtained by substituting equations (4.36) and (4.37) into equation (4.35) as follows,

\[
F_r = F_k \left( \frac{\partial s_{ij}}{\partial \dot{x}_r} \right) n_{ij} + M_k \left( \frac{\partial \sigma_{kj}}{\partial \dot{x}_r} \right) n_{ij}
\]

\[
= f_{ij} \left( \frac{\partial s_{ij}}{\partial \dot{x}_r} \right) + m_k \left( \frac{\partial v_{ij}}{\partial \dot{x}_r} \right) \quad (4.38)
\]
5. Model Structure and Approach

5.1 Human Body Model

The modeled human body consists of 13 rigid bodies as illustrated in Figure 5.1. The body segments of the human body are connected together with spherical (ball and socket) joints, revolute (hinge) joints or spherical and one translational motion, as listed in Table 5.1.

![Figure 5.1 Segment configurations for the human body model](image-url)
Where \( B_i \) \((i=2,\ldots,14)\) represents one part of human body respectively,

\[
\begin{align*}
B_2 &: \text{Lower torso} \\
B_3 &: \text{Back} \\
B_4 &: \text{Shoulder} \\
B_5 &: \text{Left upper arm} \\
B_6 &: \text{Left lower arm} \\
B_7 &: \text{Right upper arm} \\
B_8 &: \text{Right lower arm} \\
B_9 &: \text{Neck} \\
B_{10} &: \text{Head} \\
B_{11} &: \text{Left upper leg} \\
B_{12} &: \text{Left lower leg} \\
B_{13} &: \text{Right upper leg} \\
B_{14} &: \text{Right lower leg}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Type of Joint</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_2 \sim B_3 )</td>
<td>Ball and socket (lower torso)</td>
</tr>
<tr>
<td>( B_3 \sim B_4 )</td>
<td>Ball and socket (upper torso)</td>
</tr>
<tr>
<td>( B_4 \sim B_5 )</td>
<td>Ball and socket (left shoulder)</td>
</tr>
<tr>
<td>( B_5 \sim B_6 )</td>
<td>Hinge (left elbow)</td>
</tr>
<tr>
<td>( B_4 \sim B_7 )</td>
<td>Ball and socket (right shoulder)</td>
</tr>
<tr>
<td>( B_7 \sim B_8 )</td>
<td>Hinge (right elbow)</td>
</tr>
<tr>
<td>( B_4 \sim B_9 )</td>
<td>Ball and socket + one translation (neck)</td>
</tr>
</tbody>
</table>
### Table 5.1 Types of joints in the human body

Eighty-four variables are used to describe the configuration of the model (Table 5.2). Each joint has a set of maximum six variables. The first three variables of each set describe the relative orientation of each body while the remaining variables of the same set describe the relative translation motion of the body. Variables 1~6 specify the vehicle’s frame rotation and translation motions. Variable 7~12 describe the movement of the occupant seat relative to the vehicle. Variables 13~84 describe the motions of the body segments relative to their lower numbered bodies. Specifically, the motion is described for each body relative to its adjacent lower numbered body. Since the relative motion between the seat and the vehicle is zero, the variables \( y_7, y_8, y_9, y_{10}, y_{11}, \) and \( y_{12} \), are set as known variables to provide a locked joint between the cockpit and vehicle. The variables \( y_{16}, y_{17}, y_{18}, y_{22}, y_{23}, y_{24}, y_{28}, y_{29}, y_{30}, y_{31}, y_{33}, y_{34}, y_{35}, y_{36}, y_{40}, y_{41}, y_{42}, y_{43}, y_{45}, y_{46}, y_{47}, y_{48}, y_{52}, y_{53}, y_{58}, y_{59}, y_{64}, y_{65}, y_{67}, y_{69}, y_{70}, y_{71}, y_{72}, y_{76}, y_{77}, y_{78}, y_{79}, y_{81}, y_{82}, y_{83}, y_{84} \) are set as known variables as well, to provide the hinge and ball-and-socket joints at the elbows, knees, shoulders, neck, and head.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>B_9 ~ B_{10}</td>
<td>Ball and socket + one translation (head)</td>
</tr>
<tr>
<td>B_2 ~ B_{11}</td>
<td>Ball and socket (left hip)</td>
</tr>
<tr>
<td>B_{11} ~ B_{12}</td>
<td>Hinge (left knee)</td>
</tr>
<tr>
<td>B_2 ~ B_{13}</td>
<td>Ball and socket (right hip)</td>
</tr>
<tr>
<td>B_{13} ~ B_{14}</td>
<td>Hinge (right knee)</td>
</tr>
<tr>
<td>$y_1$, $y_2$, $y_3$</td>
<td>Position of the vehicle relative to the inertial frame R</td>
</tr>
<tr>
<td>-------------------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>$y_4$, $y_5$, $y_6$</td>
<td>Orientation of the vehicle relative to the inertial frame R</td>
</tr>
<tr>
<td>$y_7$, $y_8$, $y_9$</td>
<td>Position of the reference point in $B_2$ relative to the origin of the vehicle frame</td>
</tr>
<tr>
<td>$y_{10}$, $y_{11}$, $y_{12}$</td>
<td>Orientation of $B_2$ relative to the vehicle frame</td>
</tr>
<tr>
<td>$y_{13}$, $y_{14}$, $y_{15}$</td>
<td>Position of a reference point in $B_3$ relative to an attach point in $B_2$</td>
</tr>
<tr>
<td>$y_{16}$, $y_{17}$, $y_{18}$</td>
<td>Orientation of $B_3$ relative to $B_2$</td>
</tr>
<tr>
<td>$y_{19}$, $y_{20}$, $y_{21}$</td>
<td>Position of a reference point in $B_4$ relative to an attach point in $B_3$</td>
</tr>
<tr>
<td>$y_{22}$, $y_{23}$, $y_{24}$</td>
<td>Orientation of $B_4$ relative to $B_3$</td>
</tr>
<tr>
<td>$y_{25}$, $y_{26}$, $y_{27}$</td>
<td>Position of a reference point in $B_5$ relative to an attach point in $B_4$</td>
</tr>
<tr>
<td>$y_{28}$, $y_{29}$, $y_{30}$</td>
<td>Orientation of $B_5$ relative to $B_4$</td>
</tr>
<tr>
<td>$y_{31}$, $y_{32}$, $y_{33}$</td>
<td>Position of a reference point in $B_6$ relative to an attach point in $B_5$</td>
</tr>
<tr>
<td>$y_{34}$, $y_{35}$, $y_{36}$</td>
<td>Orientation of $B_6$ relative to $B_5$</td>
</tr>
<tr>
<td>$y_{37}$, $y_{38}$, $y_{39}$</td>
<td>Position of a reference point in $B_7$ relative to an attach point in $B_6$</td>
</tr>
<tr>
<td>$y_{40}$, $y_{41}$, $y_{42}$</td>
<td>Orientation of $B_7$ relative to $B_6$</td>
</tr>
<tr>
<td>$y_{43}$, $y_{44}$, $y_{45}$</td>
<td>Position of a reference point in $B_8$ relative to an attach point in $B_7$</td>
</tr>
<tr>
<td>$y_{46}$, $y_{47}$, $y_{48}$</td>
<td>Orientation of $B_8$ relative to $B_7$</td>
</tr>
<tr>
<td>$y_{49}$, $y_{50}$, $y_{51}$</td>
<td>Position of a reference point in $B_9$ relative to an attach point in $B_8$</td>
</tr>
<tr>
<td>$y_{52}$, $y_{53}$, $y_{54}$</td>
<td>Orientation of $B_9$ relative to $B_8$</td>
</tr>
<tr>
<td>$y_{55}$, $y_{56}$, $y_{57}$</td>
<td>Position of a reference point in $B_{10}$ relative to an attach point in $B_9$</td>
</tr>
<tr>
<td>$y_{58}$, $y_{59}$, $y_{60}$</td>
<td>Orientation of $B_{10}$ relative to $B_9$</td>
</tr>
<tr>
<td>$y_{61}$, $y_{62}$, $y_{63}$</td>
<td>Position of a reference point in $B_{11}$ relative to an attach point in $B_2$</td>
</tr>
<tr>
<td>$y_{64}, y_{65}, y_{66}$</td>
<td>Orientation of $B_{11}$ relative to $B_2$</td>
</tr>
<tr>
<td>-------------------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>$y_{67}, y_{68}, y_{69}$</td>
<td>Position of a reference point in $B_{12}$ relative to an attach point in $B_{11}$</td>
</tr>
<tr>
<td>$y_{70}, y_{71}, y_{72}$</td>
<td>Orientation of $B_{12}$ relative to $B_{11}$</td>
</tr>
<tr>
<td>$y_{73}, y_{74}, y_{75}$</td>
<td>Position of a reference point in $B_{13}$ relative to an attach point in $B_2$</td>
</tr>
<tr>
<td>$y_{76}, y_{77}, y_{78}$</td>
<td>Orientation of $B_{13}$ relative to $B_2$</td>
</tr>
<tr>
<td>$y_{79}, y_{80}, y_{81}$</td>
<td>Position of a reference point in $B_{14}$ relative to an attach point in $B_{13}$</td>
</tr>
<tr>
<td>$y_{82}, y_{83}, y_{84}$</td>
<td>Orientation of $B_{14}$ relative to $B_{13}$</td>
</tr>
</tbody>
</table>

**Table 5.2** Definition of the generalized coordinates in the human body

The inertia properties for the mathematical model of the human body are specified by the mass center location of the segment and the moments of inertia and products of inertia about axes through the center of mass. The Hanavan method has been used to define the segments’ mass center locations and the inertia properties.

### 5.2 Vehicle Model

14 planes called “intrusion planes” represent the vehicle intrusion (table 5.3). The developed numerical algorithm checks whether the occupant body segments hit one or more of those intrusion planes. If the body hits or crosses an intrusion plane, a damping force is included opposite to the velocity normal to the plane. That is damping forces are employed in the direction opposite to the velocity of the body relative to the plane to simulate relative forces of the intrusion vehicle surface. Hence, the damping forces can be expressed as,

$$
F_d = C_d (V_n - V_{car})
$$

(5.1)
Where $F_d$ is the damping force produced from the intrusion plane on the body segment, $C_d$ is a positive damping coefficient, and $V_n$ is the velocity component of the body segment normal to the intrusion plane. This force is only activated in the program when there is a contact between the body segment and the intrusion plane. During the contact between the intrusion plane and the body segment, the model continues to monitor the distance of penetration. As soon as the distance goes to zero or negative, the damping force equation is deactivated.

<table>
<thead>
<tr>
<th></th>
<th>Left window</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Front window</td>
</tr>
<tr>
<td>3</td>
<td>Right window</td>
</tr>
<tr>
<td>4</td>
<td>Upper left door</td>
</tr>
<tr>
<td>5</td>
<td>Lower left door</td>
</tr>
<tr>
<td>6</td>
<td>Upper right door</td>
</tr>
<tr>
<td>7</td>
<td>Lower right door</td>
</tr>
<tr>
<td>8</td>
<td>Roof</td>
</tr>
<tr>
<td>9</td>
<td>Firewall</td>
</tr>
<tr>
<td>10</td>
<td>Top dash</td>
</tr>
<tr>
<td>11</td>
<td>Right foot</td>
</tr>
<tr>
<td>12</td>
<td>Left foot</td>
</tr>
<tr>
<td>13</td>
<td>Front dash</td>
</tr>
<tr>
<td>14</td>
<td>Bottom dash</td>
</tr>
</tbody>
</table>
The vehicle ($B_1$) is as a moving frame. To simulate the crash environment, the motion input is developed through the vehicle. It represents the vehicle acceleration or deceleration depending upon the crash type. The numerical model has the capability to accommodate up to 6 (3 linear, 3 angular) piecewise linear acceleration or deceleration values at selected time intervals creating the acceleration or deceleration profiles. This program has the capability to accept up till 25 data points to simulate an acceleration or deceleration profile (or curve).

The curves are approximated by straight lines as shown in Figure 5.2. They are the only data points needed to determine the motion characteristics of the human body aside from the initial velocity and the initial displacement. The vehicle velocity and displacement can be obtained by numerically integrating the acceleration profile with respect to time as follows,

\[
a = a_i + \frac{(a_{i+1} - a_i)}{(t_{i+1} - t_i)} (t - t_i) \quad (5.2)
\]

\[
v = v_i + a_i (t - t_i) + \frac{(a_{i+1} - a_i)}{(t_{i+1} - t_i)} \frac{(t - t_i)^2}{2} \quad (5.3)
\]

and,

\[
d = d_i + v_i (t - t_i) + a_i \frac{(t - t_i)^2}{2} + \frac{(a_{i+1} - a_i)}{(t_{i+1} - t_i)} \frac{(t - t_i)^3}{6} \quad (5.4)
\]

Where $a_i$, $v_i$, $d_i$, and $t_i$ are the acceleration, velocity, displacement and time at the beginning of the $i^{th}$ interval respectively.
5.3 Belt Model

Restraint belts are modeled as spring elements that connect body segments of the occupant model with the interior surfaces of the vehicle frame. The related data such as, the stiffness values and the location of the attachment points, are input data in the algorithm of the occupant program. Simulated by four springs, the restraint belt model used in this study is the three-point belt type. Two springs simulate the lap belt where the attachment points are between the vehicle frame and the pelvis B_2 (lower torso). The third spring connects the upper torso B_4 (chest) with a point on the vehicle frame to the left of the occupant for the driver and to the right for the front seat passenger. Finally, the fourth spring connects the middle torso B_3 and the vehicle frame. For the drive, the fourth spring attachment is to the right of the occupant at the same point as the right top belt spring attachment with the vehicle. For the passenger, the fourth spring attachment is to
the left of the occupant at the same point as the left top belt attachment. The stiffness values for these springs are all the same. Figure 5.3 and 5.4 show the geometry configuration of the restraint belt contact model.

![Figure 5.3 A three-point belt](image-url)

Figure 5.3 A three-point belt
5.5 The Seat Model

The seat is modeled by seven springs and viscous dampers. Three springs model the seat bottom and four springs model the backrest. The forces are exerted by the springs on six bodies where the point of application is on the mass center of bodies 2, 3, 4, 9, 10, and 12 (or pelvis, middle torso, upper torso, head, right and left hips). Since body 2 shares the contact with the seat bottom and the backrest, two-spring forces will be applied on the pelvis, one from the seat bottom and the other from the backrest. The dampers are used to serve as stoppers when the body segment exceeds a deflection limit. The seat deflection and spring constants are also input data in the computer program. Typical seat dimensions are shown in Figure 5.5. This sets the initial position of the driver; hence, the
occupant’s back will be tilted rearward 20° off the upright position and the hips will be tilted rearward approximately 5° off the horizontal line.

**Figure 5.5** The occupant and the seat models in initial configuration
6. The Crash Victim Simulator

6.1 Computer Model

Using the concepts described in the previous chapters, a computer program is written in FORTRAN to study the motion of the human bodies in a vehicle-occupant crash system. The program formulates and numerically integrates the governing dynamical equation using a fourth order Runge-Kutta integrator. This program is more general and comprehensive than previously developed programs such as DYNOCOMB and UCIN-CRASH. Utilizing the advances in computers, this program can interface with MATLAB for animation and graphical presentation. The program consists of fifty-eight subroutines. Also, twelve channels are used to create the input and output data files.

6.2 Input File

Using channel 5, one input data file is created. The data are submitted in the FORTRAN format. The integer data are entered in I5 format and the real data are entered in F10.9 format unless it is otherwise specified. The program can consider any kind of units whether it is SI or English units, but all the data has to be consistent.

The following paragraphs describe the order of the input data.

6.2.1 Heading and Title

To identify a particular data or particular case of study, a title is written in the first line of the input file. The title contains up to 60 characters and it will appear in the top of the output page.
6.2.2 Gravity Force

The program gives the option to include gravity force (i.e. the weight of each body segment at their mass centers). To include the gravity forces the integer number 1 must be entered, otherwise 0 is entered.

6.2.3 Vertical Direction

This option is included when the gravity forces are considered. To specify the vertical direction in the X, Y, Z directions, enter 1, 2, and 3. Hence, the vertical direction for the gravity forces is in the –Z direction, so -3 should be entered.

6.2.4 Two or Three – Dimensional Problems

If the occupant motion will take place in the x-y plane then 1 should be entered to declare a two-dimensional problem, otherwise, 0 is entered to declare a three-dimensional problem.

6.2.5 Constrained Motion Problem

If the multibody system undergoes closed loops or specified motion constraints, a non-zero number should be entered. Since, the multibody system, in this study, is an open loop system, 0 must be entered in this line to represent a non-constrained multibody system (i.e. the human body).

6.2.6 Number of Bodies

This line includes the number of body segments in the system.
6.2.7 Labels of the Mass Centers

The following series of lines include the mass center labels. Three labels per line are entered with a maximum of 20 characters. Consequently, the second and third labels will start at columns 21 and 41, respectively. The labels are entered starting with bodies 1, 2, 3… until body NB, where NB is the total number of bodies considered in the system.

6.2.8 Labels of the Reference Points

With the same format as above, labels need to be entered in a series of lines representing the reference points for each body (see section 6.2.7).

6.2.9 Body Connection Array

This line contains the bodies’ numbers that are adjacent to and have a lower body number than the corresponding body segment in the system. In this study, 14 bodies are included forasmuch as the vehicle seat as body1 (Fig 5.3). The bodies are numbered starting from the pelvis as body 2. The numbering goes in ascending progression away from B₂ through the branches of the model’s tree system. The inertial frame R is considered as body 0.

The bodies in the system are (1 2 3 4 5 6 7 8 9 10 11 12 13 14) and the corresponding lower numbered bodies are (0 1 2 3 4 5 47 4 9 2 11 2 13).

6.2.10 Gravity Constant

This line includes the acceleration of gravity. The units must be consisting with the other input data.
6.2.11 Masses of the Body Segments

The next series of lines contain the 14 masses of the 14 bodies. Five values are entered in each line.

6.2.12 Reference Point Locations

The next 13 lines contain the coordinates of the reference points. Each line contains three numbers representing the location of the reference point for each body segment. This vector is fixed and located in an adjacent and lower numbered body to a particular body.

6.2.13 Mass Center Locations

The following 14 lines contain the coordinates of the bodies’ mass centers. Each line contains three numbers representing the location of the mass center for each body segment. For a particular body, the coordinates are fixed and located in the body.

6.2.14 Inertia Matrices

The next 42 lines (3 × 14 = 42) contain the inertia matrices for the bodies. Each line contains a set of three values. Thus, each matrix requires three lines, which characterizes one body segment. The numbers will represent the moments and products of inertia of the body relative to a set of axes that pass through the bodies’ mass center.
6.2.15 Number of the angular acceleration profiles

This line contains the number of acceleration profiles to be specified for orientation angles. If 0 is entered then the next lines related to the angular acceleration profile data are omitted.

6.2.16 Angular Acceleration Profile data

The next set of lines contains the angular acceleration profile data. The first line contains the number of data points in each profile. The following line contains two numbers: the body number and the direction of the angular acceleration. The numbers 1, 2, and 3 correspond to the motion direction of the angles $\alpha$, $\beta$, and $\gamma$ respectively. The last series of lines contain the time and data values of the profile. The first of these lines contains four values: the initial time, the initial angular acceleration, the initial angular velocity, and the initial angle. The remaining lines contain the time values and the angular acceleration values.

6.2.17 Number of the linear acceleration profiles

This line contains the number of acceleration profiles to be specified for the translation variables. If 0 is entered then the next lines related to the translational acceleration profile data are omitted.

6.2.18 Linear Acceleration Profile Data

The next set of lines contains the linear acceleration profile data. Similar to the angular acceleration profiles, the first line contains the number of data points in each profile. The following line contains two numbers: the body number and the direction of
the linear acceleration. The numbers 1, 2, and 3 are specified for x, y, and z respectively. The last series of the lines contain the time and data values of the profile. The first of these lines contains four values: the initial time, the initial linear acceleration, the initial linear velocity and the initial position. The remaining lines contain the time values and the linear acceleration values.

6.2.19 Intrusion Surfaces

This option depends on the first line. If 0 is read, the following lines are omitted. If 12 is entered, then the following 12 lines are considered. In every line, six values are read. The first three values are the coordinates x, y, and z of any point in the intrusion plane relative to O, which is the vehicle origin. The last three values defined the orientation of the planes by defining the direction of the planes’ normal. The format is 6F10.9. The 12 intrusion planes are ordered as the left window, the front window, the right window, the upper left door, the lower left door, the upper right door, the lower right door, the roof, the firewall, the top dash, the front dash, and the bottom dash (see Table 5.3).

6.2.20 Floor-spring constant and distance

This line contains the spring constant value of the floor and the vertical distance from the reference point in body 2 to the floor of the vehicle when the model is in the reference position.

6.2.21 Airbags Damping Constant

This next line contains the damping constant of the airbags.
6.2.22 Seat belts

The first line contains an integer number specifying the number of belts employed. Any number from 0 to 10 can be assigned. Next a series of lines are read depending upon the number of belts entered. The belt’s characteristics are specified by a group of four lines. The first line of each group contains the number of body to which the belt is attached. The second line contains three values specifying the vector $q$ which locates the attach point of the belt in the body. The third line includes three values specifying the variable RHO which locates the attach point in the vehicle relative to the origin of the vehicle. Finally, the fourth line has one value specifying the spring constant.

6.2.23 Orientation Angle Limits

The next 12 lines contain $(12 \times 6 = 72)$ values. Each line describes the angle limits of every joint relative to its lower numbered body, starting at the joint of body 3 and body 2. The first 3 values of every line specify the maximum angle limit in the three directions $\alpha$, $\beta$, and $\gamma$. The remaining three values describe the minimum angle limits in the same direction order.

6.2.24 Neck Spring and Damping Constants

The next line contains two values: the translation spring and damping constants. These values are effective when neck stretching is considered in the model.

6.2.25 Damping Coefficients of the Joints and Joint Limits

The first line contains the values of the damping coefficient of the joints. The usual units are lb. sec / rad. The second line contains another damping coefficient of the
joints. This is usually a large number, in order to model a joint angle displacement unit. The usual units are lb. sec / rad.

6.2.26 Seat Spring Constants

The next line contains two spring constants specifying the stiffness of the seat bottom and the stiffness of the backrest. The usual units are lbs./in.

6.2.27 Seat Displacement Limits

There are two maximum deflection values: one for the seat bottom and one for the backrest. The usual units are in inches.

6.2.28 Definitions for known and unknown variables

The next 14 lines contain values of 1’s and 0’s. Each line contains six integer values indicating the variables associated with each body to be either known (1) or unknown (0). The first three values on each line refer to the orientation angles of the body and the remaining three refer to the translation variables of the body. If a value is declared known, it is assumed to be zero unless otherwise it is specified by an acceleration profile. If a value is declared unknown, the initial value must still be specified and the numerical integration process determines its value at later times.

6.2.29 Initial values of the variables and their derivatives

There are 14 sets of lines with three lines per set specifying the initial values. The first line of each set contains an integer number specifying the body number. The second line includes six values specifying the initial translation variables and their derivatives.
The third line contains six values specifying the corresponding initial angles and their derivatives.

**6.2.30 Integration parameters**

The next line contains four numbers setting the parameters of the integrator. The first and second numbers correspond to the integration starting and ending times, respectively. The third number refers to the initial integration step size and the last number specifies the upper bound on the relative integration error. If the initial step size is entered as a negative number, say $-L$, the program sets the increment to $2^{(-L)}$.

**6.2.31 Printing Increment**

This line contains one number, which correspond to the time increment for printing data. The printing amount should be greater than the integration step size.

**6.3 Output Files**

The output data units are consistent with the input data. The only exception is with the angles. The angles of the input data are in while the output data are in degrees. Twelve channels are used to form three sets of output files: the general output data file (channel 6), the variables output file (channels 1, 2, 4, 7, 8, 10, 11, and 13), and the animation output file (channel 12).

**6.3.1 general output file**

The output file contents are created through channel 6. The output data are labeled on the computer printout and thus are self-explanatory. The data comprises two parts: the
first part reprints the input data in better arrangement to double check with the numbers entered in the input data file. The second part computes the motion variables at the pre-specified time interval (see section 1.5).

6.3.2 variable output file

They are nine output files describing the variables’ behaviors between the specified starting and ending times (see section 1.5). The nine files, “positon.dat”, “angle.dat”, “belt.dat”, “velocity.dat”, “rposition.dat”, “acceler.dat”, and “introforce.dat”, are created through channels 1, 2, 4, 7, 8, 9, 10, 11, and 13, respectively such that:

- “Position.dat” describes the absolute mass center position of the body segments.
- “Angle.dat” describes the absolute angles of the body segments in degrees.
- “Belt.dat” includes the forces created on each belt, and the damping forces generated by the front and side airbags.
- “Velocity.dat” describes the absolute mass center velocities of the body segments.
- “Rposition.dat” describes the mass center velocity of the body segments relative to their lower numbered body.
- “Acceler.dat” describes the absolute mass center acceleration of the body segments.
- “Raccela.dat” describes the mass center acceleration of the body segments relative to their lower numbered body.
- “Intrforce.dat” includes the forces generated from the interior surfaces of the vehicle when contact occurs between the occupant limbs and the vehicle’s intrusion surfaces.
6.3.3 animation output file

The file that creates the animation data is “pjoint.dat”, which includes the position of the joints between bodied at specific time intervals. The file’s contents are written in FORTRAN format using channel 12. The animation itself is done using MATLAB (see section 1.5 for details).
7. Rollover Modeling and analysis

7.1 Rollover modeling

When compared to the other accident modes, rollovers have a higher risk of injuries and fatalities. While rollover accidents represent only 2.2 percent of all highway towaway crashes, they are responsible for about 19 percent of highway fatalities. In order to accurately simulate the rollover process, we make a body model and rollover-and-drop model as follows:

7.1.1 The Body Modeling

The occupant’s properties of each body segment, including mass, dimensions and inertia matrices, are summarized in Table 7.1, 7.2, 7.3 and 7.4.

<table>
<thead>
<tr>
<th>Body Segment Number (N)</th>
<th>Body Segment Name</th>
<th>Mass (slug)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Lower torso (pelvis)</td>
<td>0.916</td>
</tr>
<tr>
<td>3</td>
<td>Middle torso</td>
<td>0.916</td>
</tr>
<tr>
<td>4</td>
<td>Upper torso (chest)</td>
<td>0.75</td>
</tr>
<tr>
<td>5</td>
<td>Left upper arm</td>
<td>0.178</td>
</tr>
<tr>
<td>6</td>
<td>Left lower arm</td>
<td>0.143</td>
</tr>
<tr>
<td>7</td>
<td>Right upper arm</td>
<td>0.178</td>
</tr>
<tr>
<td>8</td>
<td>Right lower arm</td>
<td>0.143</td>
</tr>
<tr>
<td>9</td>
<td>Neck</td>
<td>0.033</td>
</tr>
<tr>
<td>10</td>
<td>Head</td>
<td>0.4</td>
</tr>
</tbody>
</table>
### Table 7.1 Mass properties of the body model

<table>
<thead>
<tr>
<th>Number (N)</th>
<th>Body Segment</th>
<th>Mass Center (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Left upper leg (left hip)</td>
<td>0.555</td>
</tr>
<tr>
<td>12</td>
<td>Left lower leg</td>
<td>0.358</td>
</tr>
<tr>
<td>13</td>
<td>Right upper leg (right hip)</td>
<td>0.555</td>
</tr>
<tr>
<td>14</td>
<td>Right lower leg</td>
<td>0.358</td>
</tr>
</tbody>
</table>

### Table 7.2 Location of the limbs’ mass center

<table>
<thead>
<tr>
<th>Number (N)</th>
<th>Body Segment</th>
<th>Mass Center (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>11.7</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>11.7</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>18.6</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td>18.6</td>
</tr>
<tr>
<td>Body segment number (N)</td>
<td>Mass Center (inches)</td>
<td></td>
</tr>
<tr>
<td>------------------------</td>
<td>----------------------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>4.46</td>
<td>0</td>
</tr>
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<td>0</td>
<td>0</td>
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<td>7</td>
<td>4.46</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
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<td>9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>9.8738</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>9.8738</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 7.3 Location of the limbs’ reference points

<table>
<thead>
<tr>
<th>Body No.</th>
<th>Inertia Matrices (slug – in²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>15.700 0.0000 0.0000 0.0000 9.5950 0.0000 0.0000 15.330</td>
</tr>
<tr>
<td>3</td>
<td>15.700 0.0000 0.0000 0.0000 9.5950 0.0000 0.0000 15.330</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>4</td>
<td>11.250</td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td>5</td>
<td>0.0082</td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td>6</td>
<td>3.0590</td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td>7</td>
<td>0.0082</td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td>8</td>
<td>3.0590</td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td>9</td>
<td>1.6430</td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td>10</td>
<td>4.0000</td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td>11</td>
<td>0.0650</td>
</tr>
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</tr>
<tr>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td>12</td>
<td>12.2316</td>
</tr>
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<td></td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td>13</td>
<td>0.0650</td>
</tr>
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<td>0.0000</td>
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<td></td>
<td>0.0000</td>
</tr>
<tr>
<td>14</td>
<td>12.2316</td>
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<td></td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Table 7.4 Inertia Matrices of the body segment of the occupant model
7.1.2 The Rollover and Dropdown Modeling

In most cases, the rollover process includes the rollover period (in the air, without contact with the ground) and the dropdown period (roof contacts with the ground and is deformed), as shown in Figure 7.1.

Fig 7.1 Rollover and Dropdown

During the rollover period (1-2-3), the car rotates along its axils $180^\circ$, with a constant angle deceleration $208 \text{ Degree/Sec}^2$. During the dropdown period the roof, with contact on the ground, has a large pulse deceleration. The deceleration profiles is shown in Figure 7.2:

Fig 7.2 Deceleration profile during the rollover and dropdown
7.2 Rollover and dropdown simulation

With the body model and rollover model, the software DYNACOMBS can simulate the vehicle and driver’s interaction during the entire process, including the rollover process (from 0.0s to 280 millisecond) and drop down process (from 280 millisecond to 320 millisecond).
Overview                          X-Direction                   Y-Direction                    Z-Direction

At Time = 0.0 second

Overview                          X-Direction                   Y-Direction                   Z-Direction

At Time = 40 milliseconds

Overview                          X-Direction                   Y-Direction                    Z-Direction

At Time = 80 milliseconds

Overview                          X-Direction                   Y-Direction                    Z-Direction

At Time = 120 milliseconds
Overview                         X-Direction                   Y-Direction                    Z-Direction

At Time = 160 milliseconds

Overview                         X-Direction                   Y-Direction                    Z-Direction

At Time = 200 milliseconds

Overview                         X-Direction                   Y-Direction                    Z-Direction

At Time = 240 milliseconds

Overview                         X-Direction                   Y-Direction                    Z-Direction

At Time = 260 milliseconds
At Time = 280 milliseconds

At Time = 290 milliseconds

At Time = 300 milliseconds

At Time = 310 milliseconds
Fig 7.3 Animation Output

At Time = 320 milliseconds

At Time = 330 milliseconds

At Time = 340 milliseconds
The animation vividly shows the reaction of drivers when rollover occurs. During this process, the driver is initially moved left along the seat, then the head contacts with the left door windows with the body twisting and limbs stretching. Combined with the following data analysis charts, we can understand the reaction with more accuracy.

![Head mass center position (x-direction)](image)

**Figure 7.4a Head mass center position (x-direction)**
Figure 7.4b Head mass center position (y-direction)

Figure 7.4c Head mass center position (z-direction)
Impact force on the head

Figure 7.4d Impact force on the head
The Figure 7.4 shows the position of head mass center on the x, y and z-direction. In the Figure 7.4b, the y-direction position of head reaches the maximum value of about 16 inches and keeps unmoved after 0.17 second. Since 0.17s, the head contacted with the left door window, and as a result, a force is generated by the short contact, as shown in Figure 7.4d. The direction of this pulse force is vertical to the plate of the window glass.

![Left arm mass center position (x-direction)](image)

**Figure 7.5a Left arm mass center position (x-direction)**
Left arm mass center position (y-direction)

Figure 7.5b Left arm mass center position (y-direction)

Left arm mass center position (z-direction)

Figure 7.5c Left arm mass center position (z-direction)
Figure 7.5d Impact force on the left arm
Similarly, Figure 7.5 shows the position of the left arm mass center. As shown in Figure 7.5b and 7.5d, the left arm contacts the left door and receives a pulse force at 0.15 second.

Figure 7.6a Chest mass center position (x-direction)
Figure 7.6 shows the position of chest mass center, which can indicate the motion of driver’s torso.
8. Summary and Conclusions

8.1 Summary

This thesis introduced a computer-oriented method to simulate occupant dynamics in vehicle rollover events. The computer simulation model provides a means of predicting occupant response to, and interaction with, protective systems. The governing equations of motion were developed using Kane’s equations. These equations are more efficient than Newton-Euler method and Lagrange method since the exact number of desired equations is automatically obtained (one for each degree of freedom). Kane’s equations are associated with other kinematical quantities such as partial velocities, partial angular velocities, partial accelerations, partial angular accelerations, and generalized speeds. The relative orientation of adjacent bodies is computed using Euler parameters to avoid computational problems of singularities.

The configuration of the crash victim model was based on the lower body array concept. The concept developed the numbering and accounting procedures for the body segments. As a result, an efficient computer algorithm was developed for the dynamic analysis of the crash-victim system. Consequently, the dynamical equations were integrated by means of fourth order Runge-Kutta method to solve for the body segments’ motion variables, such as the velocities, accelerations and displacements.

The vehicle restraint system is the seat, seat belts, and airbags. The model uses arbitrary seat geometry and intrusion surfaces to represent the vehicle interior. Moreover,
the model predicts the interactive forces between the occupant and the vehicle seat, seat belts, airbags and intrusion surfaces.

The process of rollover is so quick (0.32 second) that the program must divide the process into very small integration steps in order to get accurate simulation results. The integration increment step is 0.0005 second. With a given initial position, speed and acceleration, the integration calculation continues until the end of 0.32 s.

The computational results show that the occupant body model moves toward the left side during the rollover process, with a little displacement toward the roof. Thus, the head reaches and contacts left window glass instead of the roof. A large force (up to 4000N) is exerted on the head. Then, during the process of drop down the head will contact the roof. This force depends on the stiffness of roof, the weight of driver and other related parameters; this is not discussed in this thesis.

8.2 Discussion

The model presented in this thesis is expected to be useful for obtaining an intuitive and yet qualificative understanding of occupant behavior in rollover environment. However, there are differences between experimental tests and simulation results. These differences can be attributed to several factors:

- There are difference between the actual seat belt configuration and the model seat belt. Some parameters need to be considered to describe the behavior of a seat belt accurately such as the belt webbing characteristics, the belt spool at the retractors, deformation of the body segment in contact with the belt, and belt sliding on the surface of the segment.
The airbag forces are modeled as a function of the airbag impact relative velocity and the body segment relative velocity. When contact occurs, the airbag velocity may change due to surface deformation for both the airbag’s surface and the body segment surface.

The seat is modeled as two flat surfaces (the seat bottom and the backrest), almost perpendicular to each other. Two identical flat surfaces, parallel to each other and perpendicular to the seat bottom’s right and left edges, are added. However, in actual vehicle, the seat bottom and the backrest are curved surfaces, which help keep the occupant from sliding. The model assumes that the seat surfaces produce no friction forces between the occupant and the seat, which also may account for small differences between the model and the experimental data.

The input acceleration profile is not exactly identical to the test pulse input. Indeed, it is difficult to have a formula or data points that match the test pulse-input because it is not smooth during the simulation period.

### 8.3 Future directions

Although considerable improvement have been accomplished on the simulation model described in this thesis, more accurate results may be possible if some improvements or modifications too the current model are obtained.

The elastic properties of the head and neck joint can be enhanced better by presenting a combination of springs and dampers connected to each other in parallel or in series study the viscoelastic properties of the head and neck system.
(i.e. force displacement behavior). This also can be done for every joint of the human body.

- The springs model the simple 3-point-belt safety belt system fairly well. However, for more convoluted systems with a retractor and/or slip ring, more complicated model is needed. There is a locking mechanism accompanied with the retractor model. The retractor will lock when the vehicle acceleration exceeds a specified limit and it will remain locked after that. Therefore, the modeled belt will have at least two different stiffness constants for the periods before and after the retractor lock. This can be defined by measuring the forces on the belts and the belt outlet distance in both periods.

- The seat geometry can be improved by introducing more flat surfaces. The backrest, for instance, can be modeled by three flat surfaces. One supports most of the occupant back and the other support the sides and the shoulders of the occupant. Thus, those two surfaces will make an angle with the first (main) flat surface to simulate the curved shape of the backrest. Similarly, this can be done for the seat bottom.

- The tertiary impact (i.e. internal impact of the brain and internal organs with the skeletal system) could be presented in this model by adding other bodies to represent the brain and the internal organs and see their motion and impact with the skeletal system.
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