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A Blind Space-Time Decorrelating RAKE Receiver in a DS-CDMA system in Multipath Channels

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A Blind Space-Time Decorrelating RAKE Receiver in a DS-CDMA System in Multipath Channels

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Abstract

This paper addresses the problem of blind multiple access interference (MAI) and inter-symbol interference (ISI) mitigation in direct sequence code division multipath access (DS/CDMA) systems. Studies show that multiuser detection can be performed without the knowledge of users’ channel parameters in a frequency-selective fading channel, and these approaches are of high computational complexity. In this thesis, a space-time decorrelating RAKE receiver is developed in the AWGN channel. This method is blind because it only needs to know the desired user’s signature sequence and timing information. A time-varying multipath channel that has the desired temporal-spatial correlation properties is simulated, and is used to evaluate the performance of the receiver. We also develop an adaptive algorithm for the proposed receiver. Simulations show that the proposed receiver is near-far resistant, able to eliminate MAI and ISI effectively, and has high performance with low complexity.
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Chapter 1 Introduction

1.1 Research Background

Wireless Communications for mobile telephone and data transmission is undergoing very rapid development. Many of the emerging wireless systems will incorporate considerable signal-processing intelligence in order to provide advanced services such as multimedia transmission. In order to make optimal use of available bandwidth and to provide maximal flexibility, many wireless systems operate as multiple access systems in which channel bandwidth is shared by many users on a random-access basis. As demand for wireless communications continues to grow, third-generation cellular communications systems are being standardized to provide better voice and data services, and direct-sequence Code-division multiple access (DS-CDMA) techniques are being used as its basis. In North America, the second generation DS-CDMA standard, IS-95, is the basis for a third-generation system (CDMA2000). In Japan and Europe, a third-generation wideband CDMA (WCDMA) system is also being developed [1], [2]. An effort is being made to merge these systems into a common, globe third generation CDMA standard.

While CDMA possesses many intrinsic advantages over the earlier access techniques such as time-division multiple access (TDMA) and frequency-division multiple access (FDMA) [1], the capacity of current practical CDMA systems is limited by the multiple access interference (MAI) caused by code non-orthogonality due to diverse phenomena such as asynchronous transmission, multi-path propagation, or limited bandwidth. Moreover, the presence of inter-symbol interference (ISI) due to the time-dispersive nature of wireless channels is often neglected in low rate CDMA systems, but it becomes a major problem in wideband CDMA systems. In addition, a major technological hurdle
of CDMA systems is the near-far problem: the bit-error-rate (BER) of conventional receiver (a matched filter for the user of interest) is so sensitive to differences between the received energies of the desired user and interfering users that reliable demodulation is impossible unless stringent power control is exercised. The optimum multiuser receiver for asynchronous multi-access Gaussian channels shows that the near-far problem is overcome by a more sophisticated receiver that accounts for the presence of other interferers in the channel.

This receiver (optimum receiver) was shown [4] to attain essentially single-user performance assuming that the receiver knows the following:

1) The signature waveform of the desired user;
2) The signature waveform of the interfering users;
3) The timing (bit-epoch and carrier phase) of the desired user;
4) The timing (bit-epoch and carrier phase) of each of the interfering users;
5) The received amplitudes of the interfering users (relative to that of the desired user).

Different techniques have been proposed to suppress MAI as well as ISI using linear filtering. The conventional receiver only requires 1) and 3), but it is severely limited by the near-far problem, and even in the presence of perfect power control, its BER is orders of magnitude far from optimal. The optimum detector for the asynchronous multiple-access Gaussian channel [4] shows near-far resistance and significant performance improvement over that of the conventional receivers by jointly detecting all of the users’ signals. However, the optimum receiver needs to know 1), 2), 3), 4), and 5), and the computational complexity of the optimum receiver grows exponentially with the number
of active users. Decorrelating receivers [5] are sufficient in order to achieve optimum resistance against the near-far problem, and, at the expense of slight increase over the minimum BER, the decorrelating receiver avoids the exponential complexity in the number of active users of the optimum multiuser detector. But decorrelating receivers require 1), 2), 3), and 4). Conventional linear minimum mean square error (MMSE) receivers [5], [6] offer superior performance with linear complexity. However, they need to know 1), 2), 3), 4), and 5). The adaptive MMSE detectors in [6] and [7] substitute the need to know 2), 4), and 5) by the need to have training data sequences for every active user. The typical operation of those adaptive multiuser detectors requires each transmitter to send a training sequence at start-up which the receiver uses for initial adaptation. After the training phase ends, adaptation during actual data transmission occurs in decision-directed mode. However, any time there is a drastic change in the interference environment, decision-directed adaptation becomes unreliable, and the data transmission of the desired user must be suspended and a fresh training sequence is required. Thus, the reliance on training sequence is cumbersome in most CDMA systems, where one of the most important advantages is the ability to have completely asynchronous and uncoordinated transmissions that switch on and off autonomously. So that implies the need for blind adaptive receivers, and many papers have talked about it [10]. However, most proposed blind estimation algorithms involve computationally intensive operations such as Singular Value Decomposition (SVD) and therefore may be prohibitive in practice. Thus, several receivers based on the linear constrained minimum variance (LCMV) have been proposed to decrease the computational complexity. The authors in [10] and [11] introduce blind adaptive receivers which minimize the mean output energy
(MOE) with the knowledge of the desired user’s signature sequences and its timing, and they also show that the canonical MOE detector is, in fact, the MMSE detector. The authors in [26] and [35] introduce constrained MOE receivers which are used in a single channel environment. Based on a LCMV criterion similar to that used in [26] and [35], the authors in [28] develop a decorrelating receiver in a multipath environment for a synchronous CDMA system by using a particular constraint. All of these receivers are developed for a single antenna at the receiver.

Another approach to interference suppression in wireless systems is through space-time processing using an antenna array at the receiver. This approach applies various diversity techniques to improve the detected data quality so as to increase the system capacity. Among these techniques, the spatial and temporal combiner such as the beamformer and the RAKE receiver has been proposed and its superior performance has already been verified in many papers [8], [9]. To emphasize the specified user’s signal, the temporal combiner, such as the RAKE receiver, performs a coherent combining of the temporally spread multipath components whereas the beamformer combines the spatially spread components.

The goal of this thesis is to investigate and obtain a blind receiver, which does not require training sequences and requires knowledge of only 1) and 3), that is, the same knowledge as the conventional receiver, while this receiver is near-far resistant. In the meantime, this receiver also applies space-time processing techniques to improve its performance in a frequency-selective, asynchronous channel.
1.2 Organization of Thesis

The rest of the thesis is structured as follows. Chapter 2 begins with a brief introduction to radio propagation because this is important to signal detection. Then we introduce various CDMA receiver structures: time-only, space-only, and space-time processing, and compare their performances in detection. Chapter 3 introduces a time-varying multipath vector channel simulator that will be used to analyze the performance of the space-time receiver. The simulator generates channel coefficients that simulate a time-varying multipath channel and has the desired temporal and spatial correlations. Chapter 4 models and proposes the space-time decorrelating receiver. Performances based on computer simulation are given. In Chapter 5, an LMS-based adaptive algorithm is developed. Chapter 6 provides conclusion and future work.
Chapter 2  Spread Spectrum and CDMA Signal Detection

This chapter will present some of the characteristics of spread spectrum systems. There are several ways to spread the signal spectrum, each with its own particular set of advantages [5]. The common measure is called Processing Gain that reflects the degree of spectral spreading. We will focus on direct-sequence spread spectrum systems. First, we will review the propagation environment in which the radio signals are passed. By doing so, we will understand the problems that various receivers have tried to solve. Then, we will briefly discuss several signal detection methods: time-only, space-only, and space-time, and compare their performance by their BER behavior.

2.1 Radio Propagation Environment

Radio signals generally propagate according to three mechanisms: reflection, diffraction, and scattering. When signals arrive at the receiver, the original signal arrives from several different paths, called multipath, as Figure 2.1 shows. Every path has its own delay, attenuation, and direct of arrival (DOA). This section introduces the radio channel and its effects.
2.1.1 Path Loss

It is well known that the radio signals propagate in the free space in the form of an electromagnetic wave, and the intensity decays with the square of the radio path length. Thus the received power is [12]

\[ P_r = P_t \left( \frac{\lambda}{4\pi d} \right)^2 g_t g_r \]  

(2.1)

where \( P_t \) and \( P_r \) are transmitted and received signal power, respectively, \( \lambda \) is wavelength, \( g_t \) and \( g_r \) are the gains of transmitter antenna and receiver antenna, respectively, and \( d \) is the distance between transmitter and receiver. However, free space propagation does not apply in a mobile radio environment and the propagation path loss depends not only on the distance and wavelength, but also on the antenna heights of the mobile stations (MS) and base stations (BS), and the local terrain characteristics such as buildings and hills. The site specific nature of radio propagation makes the theoretical prediction of path loss
difficult and there are no easy solutions. In practice, we can approximate the received signal power as

$$P_r = P_t \left( \frac{h_t h_r}{d^2} \right)^2 g_t g_r$$

(2.2)

where $h_t$ and $h_r$ stand for the effective heights of transmitter antenna and receiver antenna, respectively. The effective path loss follows an inverse fourth power law (exponent equal to four) that result in a loss of 40dB/decade [12].

### 2.1.2 Fading

In addition to path loss which causes signal attenuation, received signal power is also attenuated or varied because of the environmental disturbance, called fading, which is represented by [31]

$$\gamma(t) = \gamma_s \gamma_r$$

(2.3)

where $\gamma_s$ is called slow fading and represents long-term time variations of the received signal, whereas $\gamma_r$ is called fast fading, short-term fading or multipath fading. The slow fading $\gamma_s$ is the envelope of the signal $\gamma(t)$.

### 2.1.3 Slow Fading (Shadowing)

Slow fading is usually caused by the shadowing effect of mountains or buildings, and is affected by antenna height, transmission frequencies and environments. From [12], slow fading can be described by a log-normal distribution. Its probability density function (pdf) is
where $x$ is a random variable, representing signal level, and $\mu$ and $\sigma$ are the mean and standard deviation, respectively, all in dB.

### 2.1.4 Fast Fading

In a fast fading channel, the channel impulse response changes rapidly within the symbol duration. Suppose that a scattered signal has random amplitude with angle of arrival uniformly distributed in $[0, 2\pi]$. Then the received signal envelope has a Rayleigh distribution with pdf [12]

$$
p(y) = \begin{cases} 
\frac{1}{\sigma^2} \exp\left(-\frac{y^2}{2\sigma^2}\right) & y \geq 0 \\
0 & y < 0
\end{cases} 
$$

If there exists line of sight (LOS) between transmitter and receiver, then the distribution is Ricean with pdf [13]

$$
p(y) = \begin{cases} 
\frac{1}{\sigma^2} \exp\left(-\frac{y^2 + s^2}{2\sigma^2}\right) J_0\left(\frac{ys}{\sigma^2}\right) & y \geq 0 \\
0 & y < 0
\end{cases} 
$$

where $s^2$ is the average power, $J_p$ is the modified $p$th-order Bessel function of the first kind.

### 2.1.5 Doppler Spread: Time-Selective Fading

When a channel is time-variant it is referred to as possessing time-selective fading. The mobile station introduces a Doppler, or frequency, shift into the received signal. Doppler spreading results in the signal bandwidth being stretched so that the received signal’s...
bandwidth is different (greater or less) from that of the transmitted signal. Taking the Doppler power spectrum to be the Fourier transform of the time autocorrelation of the channel impulse response, the Doppler spread is defined as the support of the Doppler power spectrum [13].

In a simple case that assumes scatters uniformly distributed in angle \([0, 2\pi]\) around the mobile station, the baseband power spectrum density (psd) of the vertical electrical field has the following form [13]

\[
S(f) = \frac{3\sigma}{2\pi f_m} \left[ 1 - \left( \frac{f - f_c}{f_m} \right)^2 \right]^{-1/2} \quad f_c - f_m < f < f_c + f_m
\]  

(2.7)

where \(f_m = \frac{\nu}{\lambda_c}\) is the maximum Doppler shift, \(\lambda_c\) is the wavelength of the carrier, and \(\sigma\) is the total average received power from all multipath components. We can estimate at what transmitted signal duration distortion becomes noticeable by referring to the channel’s coherence time. The channel’s coherence time is approximately inversely proportional to the maximum Doppler shift experienced by the signal [13].

### 2.1.6 Delay Spread: Frequency-Selective Fading

A mobile radio channel causes both delay and Doppler spreading. Delay spreading results in two effects: time dispersion and frequency-selective fading. Time dispersion is a result of the signals taking different times to cross the channel by traveling different propagation paths. Frequency-selective fading occurs because the electrical length of each propagation path can expressed as a function of frequency. A measure of the transmission bandwidth at which distortion becomes appreciable is often based on the channel’s coherence bandwidth. The coherence bandwidth indicates the frequency
separation at which the attenuation of the amplitudes of two frequency components becomes decorrelated such that the envelope correlation coefficient reaches a designated value. That is, when the transmission bandwidth is greater than the coherence bandwidth, the channel is called frequency-selective fading; when the transmission bandwidth is less than coherence bandwidth, the channel is called non-frequency-selective, or flat fading. Coherence bandwidth is approximately inversely proportional to the delay spread [13].

2.1.7 Angle Spread: Space-Selective Fading

In the presence of multipath, the signal transmitted by each user arrives at the receiver not from one direction, but from a continuum. The angle spread gives the range of angle values for which significant energy is received. At the receiver, angle spread refers to the spread of angles of arrival of the multipaths at the antenna array; likewise, at the transmitter, angle spread refers to the spread of departure angles of the multipaths. Angle spread causes space-selective fading, which means that signal magnitude depends on the spatial location of the antenna. Coherence distance represents the maximum spatial separation for which the channel responses at two antennas remain strongly correlated. Coherent distance is used to characterize the space-selective fading. The larger the angle spread, the shorter the coherence distance.

2.2 Space-Time Processing

In this section, we discuss the characteristic of temporal, spatial processing techniques, and then discuss temporal-spatial processing. Temporal processing corresponds to
equalizers that use a weighted sum of signal samples and spatial processing corresponds to simple beamforming that uses a weighted sum of antenna outputs.

### 2.2.1 Temporal Processing

In a frequency selective channel, there are multiple replicas (that are resolvable in time) of the transmitted signal at the receiver, traversing different multipath. These multiple copies can be combined to improve the signal to noise ratio (SNR) at the receiver. Since the signals are coming from different paths, they encounter independent fading. This means that if one of the paths is undergoing a deep fade, it is very unlikely that the signals from the other paths are also encountering fading. As a result the receiver still has a good chance to attain acceptable fidelity. In a CDMA system, the receiver can employ multiple correlators to separate the multiple copies of the signal and mitigate fading. This receiver, commonly known as a Rake receiver [14], is shown in Figure 2.2.

![Figure 2.2 Temporal (time-only) processing.](image-url)
The structure shown consists of a bank of $L$ RAKE fingers, each correlating to a different delay of the received signal. The finger outputs are then combined to form a decision statistic. The structure is equivalent to more practical forms in which the received signal is first filtered with a chip pulse shaping matched filter and despreading is performed using received chip samples and the spreading code sequence. Temporal processing by the RAKE receiver lets the CDMA system exploit multipath diversity and make it inherently resistant to fading.

If the channel is described as a tapped-delay-line (TDL) model, temporal processing can be implemented by an equalizer, as shown in Figure 2.3. The desired signal $s(t)$ is transmitted through the channel, and received at the receiver antenna as $x(t)$. The received signal $x(t)$ is continuous, and sampled to obtain the discrete signal $x(k)$ which is then filtered by a linear filter to get $y(k)$. The equalizer is used to eliminate inter-symbol interference (ISI) and multi-access interference (MAI) [16].

![Figure 2.3 Equalizer.](image-url)
Assuming there are \( Q \) users and the channel is noise-free, we obtain the received discrete signal as

\[
x(k) = \sum_{q=1}^{Q} h_q^H s_q(k)
\]  

where \( h_q = [h_{q1}, h_{q2}, \ldots, h_{qL}] \) is the discrete channel gain coefficients corresponding to the multipath components and \( L \) is the number of resolvable paths. Here, we model the channel as a time-invariant finite impulse response (FIR) channel. \( s_q = [s_q(k), s_q(k-1), \ldots, s_q(k-L+1)]^T \) is the data sequence, and the dimension of the equalizer’s weight vector is \( M \times I \), so the output of the equalizer is given by

\[
y(k) = w^H x_k
\]

where \( x_k = [x(k), x(k-1), \ldots, x(k-M+1)]^T \) is given by

\[
x_k = \sum_{q=1}^{Q} H_q \bar{s}_q(k),
\]

\( H_q \) is a Toeplitz matrix with dimension \( M \times (M+L-1) \) given by

\[
H_q = \begin{bmatrix}
h_{q1} & h_{q2} & \cdots & h_{qL} & 0 & \cdots & 0 \\
0 & h_{q1} & h_{q2} & \cdots & h_{qL} & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \cdots & 0 & h_{q1} & h_{q2} & \cdots & h_{qL}
\end{bmatrix},
\]

and \( \bar{s}_q(k) = [s_q(k), s_q(k-1), \ldots, s_q(k-L-M+2)]^T \) is a vector with dimension \( (M+L-1) \times 1 \). We rewrite the output of the equalizer in (2.9) as

\[
y(k) = w^H H \bar{S}(k)
\]
where $H = [H_1, H_2, ..., H_Q]$ with dimension being $M \times Q(M+L-1)$ and

$\bar{S}(k) = [\bar{s}_1^T(k), \bar{s}_2^T(k), ..., \bar{s}_Q^T(k)]$ is a vector with dimension $Q(M+L-1) \times 1$.

If we want to recover the signal $s_1(k)$ from the received signal $x(k)$ where $s_1(k)$ is the desired signal and cancel interference, the following zero-forcing (ZF) requirement must be satisfied in order to eliminate MAI and ISI:

$$w^H H = [1, 0, \ldots, 0].$$

Equation (2.13) requires that $H$ must be of full-column rank [15], but it is clear that $H$ has the dimension of $M \times Q(L+M-1)$, and usually $M < Q(L+M-1)$. This implies that ISI and MAI cannot be eliminated simultaneously if only temporal-processing is used. One solution to this problem is to oversample the received signal by a factor $P$ so that $H$ has the dimension of $MP \times Q(L+M-1)$ where $MP > Q(L+M-1)$. By oversampling, $H$ can reach full-column rank, but the disadvantages are less efficiency of the system and noise enhancement.

### 2.2.2 Spatial Processing

The adaptive antenna array can achieve spatial diversity and mitigate multipath fading. This is in addition to the interference cancellation attained from steering beams towards the desired user and/or steering nulls in the direction of interferers. The signal envelopes observed across the elements of an antenna array should have low cross-correlation in order to achieve diversity gain. As a result, if the signal at one of the elements is going through a deep fade, it is highly unlikely that the signals at the other elements are encountering similar fades at the same time. So there is nearly always good signal
reception on one of the antenna elements. Therefore combining the signals from various elements will increase the SNR and the fidelity of the received signal.

Spatial processing can be implemented by a beamformer, as shown in Figure 2.4.

Figure 2.4 Beamformer.

Assuming there are $Q$ active users, and the channel has no multipath effect, the received signal is given by

$$
x(k) = \sum_{q=1}^{Q} h_q^H s_q(k)
$$

(2.14)

where $h_q$ is a $J \times 1$ vector that represents space-only channel in the absence of delay spread, and therefore $s_q(k)$ is a scalar. $J$ is the number of antenna array elements. Let $s_1(k)$ be the desired signal. The output of the beamformer is given by

$$
y(k) = w^H x(k)
$$

(2.15)

where $w$ is the $J \times 1$ weight vector. The above equation can be rewritten in matrix form

$$
y(k) = w^H H s(k)
$$

(2.16)
where $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, ..., \mathbf{h}_Q]_{(J \times Q)}$ and $\mathbf{s}(k) = [s_1(k), s_2(k), ..., s_Q(k)]^\top_{(Q \times 1)}$.

The ZF condition to retrieve the first user and cancel all MAI will then be

$$\mathbf{w}^\top \mathbf{H} = [1, 0, \cdots, 0].$$  \hspace{1cm} (2.17)

The above equation requires $\mathbf{H}$ must be a full-column rank matrix. This requirement implies $J \geq Q$, i.e., the number of antenna array elements must be greater or equal to the number of active users. Furthermore, considering multipath effects, the requirement becomes

$$J \geq \sum_{q=1}^{Q} L_q$$ \hspace{1cm} (2.18)

where $L_q$ is the number of multipaths for each user. When $L_q$ and $Q$ are large numbers, the number of antenna array elements is required to be large, which is not practical. But unlike time-only processing, space-only processing can be quite effective against MAI.

The space-only and time-only configurations discussed above use a ZF structure and did not balance the effect of noise. As stated in [5], the single user matched filter receiver is optimized to fight the background white noise exclusively, whereas the conventional decorrelating detector eliminated the multiuser interference disregarding the background noise, and in the contrast, the MMSE (minimum mean-square error) receiver can be seen as a compromise solution that takes into account the relative importance of each interfering user and the background noise. Moreover, the relative performance of the two structures will depend heavily on the channel parameters [17].

### 2.2.3 Space-time processing

A Beamformer-RAKE cascades a beamformer with RAKE reception to process the signal both in the spatial and the temporal domains. For each finger of the temporal
RAKE processor, there is a beamformer to improve the fidelity of the signal of that particular branch. At the front end of the receiver is an antenna array. The signals from the array are fed into a set of spatial combiners that perform beamforming for different multipath and each weight vector accentuates the signal from a particular multipath component of the desired user. A temporal combiner follows the spatial combiner where the contribution from different multipath (from their corresponding spatial combiner) is combined to exploit the multipath diversity.

The structure is shown in Figure 2.5. The received signal at each antenna array element is fed into an equalizer. The weight matrix acts partially as a beamformer.

![Figure 2.5 Space-time receiver.](image)

We use the system model described in Section 2.2.1. We assume $Q$ is the number of active users, $M$ is the number of taps in the equalizer, and $J$ is the number of antenna array elements. The antenna array response should be taken into account in the channel model. Without loss of generality and to simplify the analysis, we assume the direction of
arrival (DOA) for all multipaths of a user be the same, \( \theta_q \), that is, the DOA at the antenna array for the \( q \)th user, or at least they all fall into the same beam. The array response is \( \mathbf{a}_\theta \) which is a \( J \times 1 \) vector and is also known as a steering vector. We get

\[
\mathbf{a}_\theta \mathbf{h}_q^T = [\mathbf{a}_\theta^1, \mathbf{a}_\theta^2, \ldots, \mathbf{a}_\theta^q] = \begin{bmatrix} \mathbf{a}_\theta h_{q1} & \mathbf{a}_\theta h_{q2} & \cdots & \mathbf{a}_\theta h_{qL} \end{bmatrix}.
\] (2.19)

Substituting the above into (2.11), we get a modified channel matrix

\[
\mathbf{H}_q = \begin{bmatrix}
\mathbf{a}_\theta h_{q1} & \mathbf{a}_\theta h_{q2} & \cdots & \mathbf{a}_\theta h_{qL} & 0 & \cdots & 0 & 0 \\
0 & \mathbf{a}_\theta h_{q1} & \mathbf{a}_\theta h_{q2} & \cdots & \mathbf{a}_\theta h_{qL} & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & \mathbf{a}_\theta h_{q1} & \mathbf{a}_\theta h_{q2} & \cdots & \mathbf{a}_\theta h_{qL}
\end{bmatrix}.
\] (2.20)

The dimension of the space-time channel matrix for the \( q \)th user is \( (JM) \times (L+M-1) \). The received signal at the antenna array is given by

\[
\mathbf{x}(k) = \begin{bmatrix}
x_{11} \\
x_{21} \\
\vdots \\
x_{J1} \\
x_{12} \\
\vdots \\
x_{J2} \\
x_{1M} \\
\vdots \\
x_{JM}
\end{bmatrix} = \mathbf{H}_1 \begin{bmatrix}
S_{1,k} \\
S_{1,k-1} \\
\vdots \\
S_{1,k-L-M+1}
\end{bmatrix}_{(L+M-1) \times 1} + \sum_{q=2}^{Q} \mathbf{H}_q \mathbf{s}_q + \mathbf{n},
\] (2.21)

where \( s \) is the transmitted information, and we assume the noise \( \mathbf{n} \) is spatially and temporally white and Gaussian. From above, \( \mathbf{H} \) has dimension \( (JM) \times (L+M-1) \), where \( JM \geq (L+M-1) \), so \( \mathbf{H} \) is full column rank and satisfies the ZF condition, and therefore, cancels both ISI and MAI. In the case of noise, we no longer consider the ZF condition, but instead discuss how to optimize the solution by using spatial-temporal techniques.
A popular optimality criterion is ST-MMSE (space-time minimum mean-square error). In ST-MMSE, we obtain an estimate of the transmitted signal as a space-time weighted sum of the received signal and seek to minimize the mean square error between the estimate and the true signal at any time instant. The receiver structure is shown in Figure 2.5.

In a space-time filter, the weight $\mathbf{W}$ has the following form

$$\mathbf{W}(k) = \begin{bmatrix}
w_{11}(k) & \cdots & w_{1M}(k) \\
\vdots & \ddots & \vdots \\
w_{J1}(k) & \cdots & w_{JM}(k)
\end{bmatrix}. \quad (2.22)$$

We define the operator $\text{vec}(.)$ as

$$\text{vec}([\mathbf{v}_1 \ \cdots \ \mathbf{v}_M]) = \begin{bmatrix}
\mathbf{v}_1 \\
\vdots \\
\mathbf{v}_M
\end{bmatrix}. \quad (2.22)$$

Therefore,

$$\mathbf{w}(k) = \text{vec}^{\top} \mathbf{W}(k) \quad (JMx1). \quad (2.23)$$

Thus, we obtain a convenient formulation for the space-time filter output:

$$y(k) = \mathbf{w}(k)^{\top} \mathbf{x}(k). \quad (2.24)$$

The ST-MMSE filter chooses the space-time filter weights to achieve the MMSE (minimum mean square error), i.e.,

$$\arg \min_{\mathbf{w}} E \left\| \mathbf{w}^{\top} \mathbf{x}(k) - s(k) \right\|^2. \quad (2.25)$$

The solution to this LS (least squares) problem is given by [16]

$$\mathbf{w} = \mathbf{R}_{xx}^{-1} \mathbf{z} \quad (2.26)$$
where $\mathbf{R}_{xx} = \mathbf{E}[\mathbf{x}x^H]$ is the auto-correlation matrix of the received signal at the receiver, or the input signal to the filter. The vector $\mathbf{z} = \mathbf{E}[\mathbf{x}_s^r]$ is the cross-correlation vector between the received signal and the desired signal [16].

ST-MMSE combines the strengths of time-only and space-only processing, and trades MAI and ISI reduction against noise enhancement. It will cancel the MAI primarily in the spatial dimension, and ISI in the temporal dimension.

### 2.3 Simulation Examples

In this section, we compare the performance difference between the temporal processing and space-time processing by computer simulation.

First we consider temporal processing. Assume the number of users $Q=3$, the number of multipaths $L=4$, the number of the equalizer taps $M=4$, and the signal-to-noise ratio $\text{SNR}=10\text{dB}$. The received signal is given by

$$
\begin{bmatrix}
    x_k \\
    x_{k-1} \\
    x_{k-2} \\
    x_{k-3}
\end{bmatrix} =
\begin{bmatrix}
    h_{11} & h_{12} & h_{13} & h_{14} & 0 & 0 & 0 \\
    0 & h_{12} & h_{13} & h_{14} & 0 & 0 & 0 \\
    0 & 0 & h_{11} & h_{12} & h_{13} & h_{14} & 0 \\
    0 & 0 & 0 & h_{11} & h_{12} & h_{13} & h_{14}
\end{bmatrix}
\begin{bmatrix}
    s_{1,k} \\
    s_{1,k-1} \\
    \vdots \\
    s_{1,k-6}
\end{bmatrix} + \mathbf{H}_2 \mathbf{s}_2 + \mathbf{H}_3 \mathbf{s}_3 + \mathbf{n}.
$$

(2.27)

The equalizer's output is given by

$$
y(k) = \mathbf{w}^H \begin{bmatrix}
    \mathbf{H}_1 \\
    \mathbf{H}_2 \\
    \mathbf{H}_3
\end{bmatrix} \begin{bmatrix}
    \mathbf{s}_1 \\
    \mathbf{s}_2 \\
    \mathbf{s}_3
\end{bmatrix}.
$$

(2.28)

In the ideal case, the following should be satisfied:

$$
\mathbf{w}^H \begin{bmatrix}
    \mathbf{H}_1 \\
    \mathbf{H}_2 \\
    \mathbf{H}_3
\end{bmatrix} = [1 \ 0 \ \cdots \ 0]_{3 \times (L+M-1)}.
$$

(2.29)

The simulation result is given below. The result is a 21 x 1 column vector.

$$
\mathbf{s}s = \mathbf{w}^H \begin{bmatrix}
    \mathbf{H}_1 \\
    \mathbf{H}_2 \\
    \mathbf{H}_3
\end{bmatrix}
$$
ss = [0.1059 + 0.0026i  0.0373 + 0.0673i  0.0452 - 0.0730i -0.0082 + 0.0557i -0.0222 + 0.0134i  0.0151 - 0.0172i  0.0129 - 0.0033i  0.0710 + 0.0129i  0.0836 + 0.0008i 0.0163 + 0.0774i  0.0224 - 0.1125i -0.0404 + 0.0626i  0.0264 + 0.0144i  0.0103 - 0.0250i  0.1660 + 0.0008i  0.0697 + 0.0191i -0.0374 + 0.0263i -0.0030 - 0.0178i 0.0058 + 0.0146i  0.0169 - 0.0040i  0.0048 - 0.0086i]T.

From the result above, we can see the first element is much larger than others. The ratio of 2-norms of the first element and the rest is shown below

0.1060 : 0.3041

This result is far from the ideal case (1:0), and it proves that the time-only processing cannot provide a satisfactory performance.

The Figure 2.6 shows the QPSK constellation of the desired user. The modulated signal is presented by a+jb. Figure 2.7 shows the received signal at the equalizer input. Due to the channel effects, multipath effects, interference, and noise, the received signals are corrupted and no longer hold QPSK characteristics. The receiver must process the received signal to eliminate the MAI, ISI and noise in order to recover the transmitted signal.

Figure 2.8 shows the signal constellation at the equalizer output. The signals have similar amplitudes to the original transmitted signal, but cannot be effectively recovered. The performance of time-only processing is poor.

Next, we consider space-time processing. A uniform linear antenna array with five elements and half-wavelength spacing is used. The DOAs are [0°  -60°  50°], and 0° is the DOA of the desired signal.

The simulation result is given below. The result is a 21 x 1 column vector:
The ratio of 2-norms of the first element and the rest is

\[ \frac{0.9378}{0.0848} \approx 0.9378 : 0.0848. \]

The result is very close to (1:0). Figure 2.9 shows the signal constellation after space-time processing. It shows the recovered signals have similar amplitudes with the original transmitted signals, and are distributed around each center of the original QPSK modulated signals. The received signals can be effectively recovered.

The beam pattern for the above space-time processing is illustrated in Figure 2.10. As we can clearly observe, the antenna gain reaches the largest magnitude at the DOA of the desired user, i.e., \[0^\circ\], and has a null at the directions of the interferers, i.e., \([-60^\circ, 50^\circ]\]. Therefore, MAI can be effectively eliminated in the space-time processing.

In this chapter, we presented a description of a wireless channel and discussed different physical effects such as path loss, fading, delay spread, angle spread, and Doppler spread that make the received signal much weaker than the transmitted signal. We also discussed three processing techniques that are used at the receiver: time-only, space-only and space-time processing. As said, time-only and space-only processing techniques have fundamental drawbacks that make the receiver very difficult to eliminate MAI and ISI simultaneously. In the contrast, space-time processing have overcome the drawbacks and
have significant advantages over the other two processing techniques, and the receiver’s performance have improved dramatically.

Figure 2.6 QPSK modulated signal constellation.

Figure 2.7 Received signal constellation at the equalizer input.
Figure 2.8 Signal constellation after time-only processing.

Figure 2.9 Signal constellation after space-time processing.
Figure 2.10 Beam pattern for the space-time processing.
Chapter 3 Time-varying Multipath Vector Channel

Simulator

In a wireless channel, the change of surrounding structures and the movement of mobile users make the channel time-varying. For the purpose of analyzing the performance of communication systems, there is a need for effectively simulating the radio channel. With the employment of an antenna array, the signals are received at the receiver by multiple antennas, and therefore, the channel model becomes a vector as opposed to a scalar where only a single antenna is employed. In this chapter, we introduce a time-varying multipath (fast fading) vector channel simulator that can be used to evaluate the performance of antenna array receiver.

3.1 Time-varying Multipath Vector Channel

In [18], a multipath channel simulator was introduced which is a multi-channel generalization of the scalar channel presented in [32]. Taking into account an antenna array, an expansion is made to simulate the new channels in this chapter.

We consider the transmission from a mobile station to a base station. The mobile is surrounded by reflecting structures without line of sight between the mobile and base station and causes multipath effects. The mobile station’s movement also causes Doppler shift. The faster the mobile station moves, the faster the channel response changes. In order to present the channel model, the following parameters are defined:

- \( J \) number of antenna array elements
- \( f_c \) carrier frequency (\( \omega_c = 2 \pi f_c \))
• $B$ the transmitted signal bandwidth

• $\lambda_c$ carrier wavelength

• $v$ speed of the mobile

• $\theta$ direction of arrival of the received signal; a plane wave is assumed

• $d$ the distance between mobile station and base station

• $\tau = d / c$, the propagation delay where $c$ is the velocity of light. $\tau_{\text{min}}$ and $\tau_{\text{max}}$ are the minimum and maximum propagation delay, respectively.

• $D$ array spacing

• $L$ number of multipaths

The antenna array used here is a Uniform Linear Array (ULA), i.e., the spacing between the elements of a linear array is equal, as illustrated in Figure 3.1

![Figure 3.1 Uniform Linear Array with three elements.](image)

We assume:
1. The signals originate far away from the array and the plane wave associated with the signal advances through a non-dispersive medium that only introduces a propagation delay. Under such circumstances, the signal at any other element can be represented by a time-delayed version of the signal at the first element.

2. \( D \ll c/B \), i.e., the bandwidth of the impinging signal is much smaller than the reciprocal of the propagation time across the array. This is commonly known as narrowband assumption. This assumption makes it possible to represent the propagation delay within the elements of the array by phase shifts in the signal.

With the assumptions mentioned above, the received signal at the antenna array is given by

\[
x(t) = \sum_{m=0}^{J-1} s(t)a_m(\theta)
\]

where \( s(t) \) is the information and \( a_m(\theta) \) is the array response.

The above equation can be written in a vector form as

\[
x(t) = s(t)a(\theta)
\]

where each element of \( x(t) \) contains the received signal at the corresponding array element and

\[
a(\theta) = \begin{bmatrix}
1, & e^{-j\omega \rho_1(\theta)}, & \cdots, & e^{-j\omega \rho_{J-1}(\theta)}
\end{bmatrix}^T
\]

\[
= \begin{bmatrix}
1, & e^{-j2\pi \frac{D}{\lambda} \sin(\theta)}, & \cdots, & e^{-j2\pi \frac{D}{\lambda} \sin((J-1)\sin(\theta))}
\end{bmatrix}^T
\]

where \( \rho \) is the propagation delay relative to the first array element.
The vector \( \mathbf{a}(\theta) \) is known as the array response vector or the steering vector of a ULA, which is a function of DOA under such circumstances. It can be shown that the baseband equivalent of \( J \times 1 \) vector channel impulse response is given by [22]

\[
\mathbf{g}(t; \tau) = \sum_{i=1}^{L} \delta(\tau - \tau_i) \mathbf{h}_i(t)
\]

(3.4)

where \( t \) is the observation time, \( \tau_i \) is the propagation delay of the \( i \)th path, \( L \) is the number of time differentiable paths which are composed of a large number of time non-differential subpaths with the similar DOA. The DOA of each time differential path is the mean of angles of all subpaths. The spread of angles of subpaths is defined as angle spread. Similarly, \( \tau_i \) is the mean of the delays of the subpaths. The larger the \( \tau_i \), the less correlation between those paths.

The parameter \( \mathbf{h}_i \) is the complex path vector for the \( i \)th path and is given by

\[
\mathbf{h}_i = [h_{i0}, h_{i1}, \ldots, h_{i(M-1)}]^T
\]

(3.5)

where the elements of \( \mathbf{h}_i \) are the channel coefficients which are functions of time and frequency. If we assume all propagation paths lie in the same plane as the array (elevation angle \( \psi = 0 \)), the \( i \)th complex path vector is given by

\[
\mathbf{h}_i(t) = \int \int \alpha(\theta, \tau) \mathbf{a}(\theta) e^{j(\omega_d t - \omega_i \tau)} d\theta d\tau
\]

(3.6)

where \( \tau_i \) ranges in \( [\tau_{\min} + i/B, \tau_{\min} + (i+1)/B] \), \( \theta_i \) ranges \( [\bar{\theta} - \Delta/2, \bar{\theta} + \Delta/2] \), \( \bar{\theta} \) is the mean angle of arrival, \( \Delta \) is the angle spread, \( \alpha(\theta, \tau) \) is the amplitude intensity function for each path, and \( \mathbf{a}(\theta) \) is the array response. The Doppler shift and path delay result in phase variation, \( e^{j(\omega_d t - \omega_i \tau)} \), where Doppler shift is
\[ f_d = f_c \frac{v}{c} \cos(\psi) \]  

(3.7)

with \( \omega_d = 2\pi f_d \) and we will assume \( \psi = 0 \).

Usually, all time differentiable paths are composed of a large number of non-differentiable subpaths. By the central limit theorem, we find the channel coefficients to be well approximated by complex Gaussian variables [16]. Since Gaussian variables are entirely characterized by their first and second order statistics, we can simulate the channel simulator coefficients by generating Gaussian variables that have some predefined appropriate first and second order statistics. Next, we introduce the first and second characteristics of the channel coefficients.

### 3.2 Second Order Characterization

Since the mobile is surrounded by local reflecting structures, we have many indirect transmission subpaths each of which exhibits different properties. With such a scenario, we assume:

1. Subpaths corresponding to different delays or angles of arrival have uncorrelated amplitudes, i.e.,

\[ E[\alpha(\theta, \tau)\alpha^*(\theta', \tau')] = \delta(\theta - \theta', \tau - \tau') f(\theta, \tau) \]  

(3.8)

where \( f(\theta, \tau) \) is the joint pdf of \( \theta \) and \( \tau \), and * the complex conjugate. This is also referred to as the wide sense stationary uncorrelated scattering (WSSUS) assumption [20] [36].

2. For a given subpath, the delay, angles of arrival are mutually independent, i.e.,
\[ f(\theta, \tau) = f_a(\theta)f_b(\tau) \] (3.9)

where \( f_a(\theta) \) and \( f_b(\tau) \) are the power density function of angles of arrival, \( \theta \), and transmission delay \( \tau \), respectively.

3. The power density function of the transmission delay \( \tau \) is given by [20]

\[ f_b(\tau) = \frac{1}{\tau} e^{-\frac{\tau}{\tau}} \] (3.10)

where \( \bar{\tau} \) is the mean delay. Integrating \( f_b(\tau) \) with respect to the delay domain for the \( i \)th path, we get

\[ F(i) = \int_{t_{\text{min}}}^{t_{\text{max}}-i+1/B} f_b(\tau)d\tau \]

\[ = [1 - e^{-1/(\bar{\tau})}] e^{-i/(\bar{\tau})} \] (3.11)

where \( F(i) \) is simply the power fraction associated with the \( i \)th path.

Because of the random phase associated with each individual time indifferentable subpath in a given time differentiable path, the channel coefficients are well modeled by a zero mean complex variable [19].

Under the above assumptions, the cross-correlation matrix of the complex path vectors is given by [22]

\[ R_y(t_1, t_2) = E[h_i(t_1)h_j^H(t_2)] \]

\[ = \delta_{ij}J_0(\omega_0 \Delta t)F(i)R_{a,j} \] (3.12)

where \( i, j \) represent the \( i \)th and \( j \)th path, \( \delta_{ij} = 0 \) when \( i \neq j \) which implies the time differentiable paths are independent from each other, \( \Delta t = |t_1 - t_2| \) is the time lag, and \( J_0(\cdot) \) is the Bessel function of the first kind and of order 0. \( R_{a,j} \) is the spatial correlation matrix for the \( i \)th path, and is given by
\[
R_{a,i} = \int_{\theta_i} f_a(\theta) a(\theta) a^H(\theta) d\theta \\
(3.13)
\]

If we assume the power density function with respect to the angles of arrival, \( f_a(\theta) \), is uniform in \( \bar{\theta} \pm \Delta / 2 \), (3.13) can be written as

\[
R_{a,i} = \frac{1}{\Delta} \int_{\bar{\theta} - \Delta/2}^{\bar{\theta} + \Delta/2} a(\theta) a^H(\theta) d\theta . \\
(3.14)
\]

Thus, the smaller the angle spread, the higher the spatial correlation between the signals at the antenna elements. The spatial correlation is also a function of antenna element spacing \( D \). The larger \( D \), the lower the spatial correlation. In macrocellular mobile communications, since the base is usually far from the mobile and the local reflectors surrounding it, the angle spread induced by the local reflectors is, therefore, often relatively small [17], and the spatial correlation can remain relatively high even if the antenna elements are spaced by many \( \lambda_c \).

### 3.3 Vector Channel Simulator

The vector channel simulator structure is shown in Figure 3.2. It is single-input multiple-output (SIMO) discrete-time FIR system with time-varying coefficients, based on a TDL (tapped delay line) model with taps evenly spaced one sample apart. The simulator has \( J \) branches corresponding to each antenna array element. Each branch has \( L \) weights with each of which represents a resolvable path coefficient. The sampling interval is \( T_c \) (i.e., \( 1/B \)). The power of the noise signal is chosen according to the required SNR. The input to the simulator is a baseband transmitted signal, \( s(t) \), and it is multiplied by the corresponding weight, \( h_{ij} \), which is the channel coefficient corresponding to the \( i \)th path.
of the $j$th antenna element. AWGN is added at the output and the received signal at the $j$th antenna element is $y_j(t)$. Finally, we simulate the signals received at the antenna traveling through the time-varying multipath channel. Next, we describe how to generate the channel coefficients.

![Diagram of a time-varying multipath vector channel simulator.](image)

**Figure 3.2 Time-varying multipath vector channel simulator.**

### 3.4 Complex Path Vector Generator

We have known that the time differentiable paths are independent from each other and the channel coefficients can be generated independently and approximated by Gaussian variables. The channel coefficients have zero mean and second order characteristics
discussed above, including spatial-temporal correlation properties. Therefore, we can apply some kind of space-time correlation shaping transformation to uncorrelated Gaussian white noise sequences in order to obtain time-varying path vectors that exhibit the appropriate spatial-temporal correlation properties.

The time-varying channel is mostly due to mobile station motion. The extent of how the channel varies in time is measured by the Doppler shift. As long as the complex path vectors are obtained at a rate higher than twice the Doppler frequency, their temporal correlation structure is preserved.

Figure 3.3 shows the structure of the $i$th path vector generator.

Figure 1.3 The complex path vector generator for the $i$th path
We take the sampling interval of $T$ where $\frac{1}{T} = 3f_d$, then feed the samples into the time-correlation transformer and space-correlation transformer, and finally the simulated complex path vectors are interpolated in time to the desired sampling rate, $1/T_c = B$.

Equation (3.12) can be rewritten in the following way for the case $i=j$:

$$\mathbf{R}_{ii}(\Delta t) = J_0(\omega_d \Delta t) F(i) \mathbf{R}_{a,i}$$

(3.15)

where $F(i)$ is independent of $\Delta t$, and therefore, we define

$$C_i = F(i) \mathbf{R}_{a,i}.$$  

(3.16)

This suggests that in order to obtain the appropriate space-time correlation characteristics for a given path, one can use a time transformation followed by a spatial transformation. The time index in Figure 3.3 refers to the sampling interval $T$ of the complex path vectors prior to the interpolator. The noise generator produces zero-mean complex Gaussian vectors:

$$\mathbf{n}(m) = [n_{i0}(m) \quad n_{i1}(m) \quad \ldots \quad n_{i(M-1)}(m)]^T.$$  

(3.17)

The time-correlation shaping filter denoted by $H(z)$ is designed so that the temporal correlation of its output, $y_{ij}(m) = H(z)n_{ij}(m)$, is approximately equal to temporal component in (3.15), i.e., $J_0(\omega_d \Delta t)$. The space-correlation transformation takes care of the spatial component of the correlation, i.e., $C_i$. This transformation is applied to the vector $y_i(m)$. Finally the simulated complex path vectors are interpolated in time to the desired sampling rate.
3.4.1 Time-correlation Shaping Filter

A time correlation shaping filter for obtaining the desired temporal correlation is widely used. The desired power spectral density function is the Fourier transform of the Bessel function in (3.15), and is given by [18]

\[
s(\omega) = \begin{cases} 
\frac{2}{\sqrt{\omega^2_d - \omega^2}} & \text{if } |\omega| \leq \omega_d \\
0 & \text{otherwise}
\end{cases}
\]  \hspace{1cm} (3.18)

For the time correlation shaping filter, we must have \( |H(e^{j\omega})|^2 = s(\omega) \). Since \( s(\omega) \) has singularities at \( \omega = \pm \omega_d \), the design of this filter is unrealizable. In practice, the singularities at \( \omega = \pm \omega_d \) are replaced by sharp peak. If FIR is selected to implement this filter, the order of the filter must be high to obtain the sharp frequency response. To reduce the computational complexity, a low order IIR filter is used.

We design an analog filter that satisfies \( |H(e^{j\omega})|^2 = s(\omega) \), and then apply AD conversion.

The sampling rate is

\[
\frac{1}{T} = \frac{3}{2} \times 2f_d = 3f_d
\]  \hspace{1cm} (3.19)

i.e., the sampling rate is 3/2 times the Nyquist frequency. Finally, we get a 4th-order IIR filter [21]

\[
H(z) = \frac{0.717 + 1.705z^{-1} + 20251z^{-2} + 1.513z^{-3} + 0.536z^{-4}}{1 + 1.743z^{-1} + 2.334z^{-2} + 1.343z^{-3} + 0.596z^{-4}}. \hspace{1cm} (3.20)
\]

We will simulate the result later by comparing the correlation of the channel coefficients after the filter with Bessel function.
3.4.2 Spatial Transformation

The spatial transformation used to obtain the complex path vector \( h_i \) takes the form of a simple matrix operation to \( y_i \):

\[
h_i = M_i y_i
\]  
(3.21)

The matrix \( M_i \) must satisfy \( M_i M_i^H = C_i \), i.e., the correlation matrix of the spatial transformation matrix must be equal to \( C_i \).

The development of \( M_i \) is based on Karhunen-Loeve expansion for vectorial random process. Since the \( C_i \) is a correlation matrix of a discrete time random process, it is non-negative definite Hermitian matrix [16]. A Hermitian matrix is diagonalizable, so we can write

\[
C_i = Q_i \Lambda_i Q_i^H
\]  
(3.22)

where \( Q_i \) is the matrix whose columns are orthonormalized eigenvectors of \( C_i \), denoted by \( q_{ij} \) \( (j = 0, \ldots, J-1) \) and \( \Lambda_i \) is the diagonal matrix whose entries correspond eigenvalues, \( \lambda_{ij} \). From \( q_{ij}^H q_{ij} = 1 \), we get \( q_{ij}^H = q_{ij}^{-1} \). The fact that \( C_i \) is nonnegative definite implies that \( q_{ij}^H C_i q_{ij} \geq 0 \). By definition \( C_i q_{ij} = \lambda_{ij} q_{ij} \), so we have \( q_{ij}^H C_i q_{ij} = \lambda_{ij} ||q_{ij}||^2 = \lambda_{ij} \geq 0 \), so the eigenvalues of \( C_i \) are real and nonnegative. We can therefore get

\[
C_i = Q_i \Lambda_i^{1/2} \Lambda_i^{-1/2} Q_i
\]

\[
= [Q_i \Lambda_i^{1/2}] [\Lambda_i^{-1/2} Q_i]
\]

\[
= M_i M_i^H
\]

where \( M_i = Q_i \Lambda_i^{1/2} \) is the desired spatial transformation matrix.

Premultiplication of the spatially uncorrelated signal vector \( y_i \) by \( M_i = Q_i \Lambda_i^{1/2} \) gives the complex path vector such that
\[ E[h_i(m)h_i^H(k)] = Q_i \Lambda_i^{1/2} E[y_i(m)y_i^H(k)] \Lambda_i^{1/2} Q_i \]

\[ \approx [Q_i \Lambda_i^{1/2}] J_0(\omega_d T(k - m)) [\Lambda_i^{1/2} Q_i] \]

\[ \approx J_0(\omega_d T(k - m)) Q_i \Lambda_i Q_i^H \]

\[ \approx J_0(\omega_d T(k - m)) \mathbf{C}_i \]

which is the desired result.

Figure 3.4 illustrates the space-correlation transformation for the \( i \)th path. Each element of \( y_i \) is first scaled by the square root of the eigenvalue. The resultant vector is then multiplied by the eigenvector matrix \( Q_i \). The output of the path vector generator has the appropriate time-space correlation properties.

3.4.3 Interpolator

As mentioned before, the sampling rate to sample the Gaussian variable generator is \( 1/T = 3f_d \), and the samples are applied to the time-space correlation transformation. The frequency associated with the path vector generator is much smaller than the one associated with channel filtering \( 1/T_c = B \). We therefore need to obtain the high rate
channel coefficients required for channel filtering via interpolation. When a signal with a sampling frequency much higher than the Nyquist frequency has to be interpolated, simple methods such as linear or cubic interpolation can be used without compromising the precision [33]. Since $1/T_c = B$ is much larger than $1/T$, it is possible to decompose the interpolation process to reduce the computational requirements, as illustrated in Figure 3.5. The system is composed of a linear interpolator followed by a cubic interpolator. The linear interpolator increases by a factor $I \geq 30$ the sampling rate to become much higher than the Nyquist frequency, and then a cubic interpolator is used to further increase the sampling frequency to its desired value of $1/T_c$.

Figure 3.5 Interpolator.

Figure 3.6 illustrates the linear interpolation: Figure 3.6A represents the original signals, Figure 3.6B the signals after adding $(I-1)$ zeros between samples (zero-padding), and then the signals are fed into a low-pass filter with a radial cutoff frequency $\pi / I$. Figure 3.6C represents the signals after linear interpolation. Linear interpolation is computationally complex, but has more precision. When linear interpolation is completed, a cubic interpolator is used.
Figure 3.7 illustrates the cubic spline interpolation, between the sample interval $[T_{i-1}, T_i]$, where the signal is represented by a third-order polynomial:

$$p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3. \tag{3.25}$$

Therefore, samples can be obtained by sampling the above function at the required sample time instant so that the rate is increased.
3.5 Simulation Examples

We simulate and examine the channel realization via the path vector generator. Consider a DS/CDMA system with carrier frequency being $f_c = 1\text{GHz}$, transmitted signal bandwidth is $B = 1.2288\text{MHz}$ ($T_c = 1/B$), the number of antenna array elements is $M=3$, the mean delay is $\bar{\tau} = 2/B = 2T_c$, the speed of the mobile is $v = 30\text{m/s}$, the Doppler frequency is $f_d = f_c \cdot \frac{V_c}{c} = 100\text{Hz}$, and therefore, the sampling interval before the interpolator is $T = \frac{1}{3f_d} = 4096T_c$. The linear interpolator decreases the interval to $128T_c$ ($J=32$), and then the cubic interpolation reduces it from $128T_c$ to $T_c$.

By passing a white Gaussian noise sequence through the IIR filter $H(z)$ developed previously, we get filtered outputs whose temporal correlation is approximately equal to the temporal component in (3.15), as illustrated in Figure 3.8 which plots the desired and obtained temporal correlation. The smaller the time lag, the more they are correlated.
After passing through the time correlation shaping filter, the simulated path vector needs to be spatially transformed to get an appropriate spatial correlation. First of all, assume there are three time differentiable paths, $L = 3$, whose DOAs are $[-60^\circ, 0^\circ, 50^\circ]$, respectively. The angle spreads are $[10^\circ, 2^\circ, 5^\circ]$, respectively.

Figure 3.9 illustrates the plot of the magnitude of the channel coefficients at the first antenna element. Since there is high correlation of the channel coefficients between antenna elements, the channel coefficients are very similar for different antennas, and the plots of the channel coefficients versus time are nearly the same. The time interval of the plot along x-axis is $1/B$, corresponding to a chip in DS-CDMA. As illustrated, the channel coefficients are rapidly time-varying.
The interpolator is used to interpolate the channel coefficients to increase the final sample rate to the transmitted signal bandwidth $B$. Figure 3.10 illustrates the plot of the channel coefficient magnitudes vs. time for the first path of the first antenna element before and after the interpolator. As illustrated, the curve is not smooth before the interpolator, and has sharp corners. After the interpolator, the curve is very smooth, and very similar to that of a typical wireless channel [20].
In this chapter, a new multipath channel simulator is introduced. The simulation results agree with the discussion above, and later, we will use this time-varying multipath vector channel simulator to generate the desired channel coefficients to analyze the receiver’s performance.

Figure 3.10 Channel coefficient magnitudes before and after interpolation for Path 1 of the first antenna element.
Chapter 4 Space-Time Decorrelating RAKE Receiver for a DS/CDMA System

A direct-sequence code division multiple access (DS/CDMA) system suffers from multiple access interference (MAI), inter-symbol interference (ISI) and the near-far problem that degrade its link quality. Space-time processing techniques have been proven to be effective in combating MAI, ISI, and near-far resistant, and therefore provide significant performance improvements over time-only or space-only processing. The use of antennas in CDMA is expected to improve system capacity, quality, and coverage, which make space-time processing important for CDMA. In this chapter, we will develop a space-time receiver based on LCMV (linear constrained minimum variance) criterion.

4.1 Introduction

In CDMA, the users operate in the same frequency and time channel by using spread spectrum techniques. In DS/CDMA, each user has a unique spreading code that operates at chip rate $N$ times greater than the information data rate. The DS/CDMA link therefore needs a large bandwidth channel that can be shared by multiple users. The user code can be designed to be orthogonal. If orthogonal codes are used and there is no multipath, code orthogonality can be maintained; thus users do not interfere with each other and signal detection is noise limited. On the other hand, if quasi-orthogonal or orthogonal with multipath codes are used, the users interfere with each other and the detection becomes interference limited, i.e., MAI limited, and various multiuser detection techniques have been introduced to combat MAI among which space-time processing is one of the most
promising while multiuser detection techniques has been introduced to combat the near-far effect.

There are several types of receiving structures in which space-time processing is used [24],[25].

![Diagram of space-time receivers](image)

Figure 2.1 Structures of space-time receivers.
Figure 4.1(a) and (b) show two types of receiving structures consisting of multiuser detection together with a RAKE combiner. Among several schemes of sub-optimal multiuser detection, the decorrelating detector has been developed due its capability of perfect cancellation, and resistance to the near-far problem [23]. However, the decorrelating receiver demands high computational complexity, especially in the multipath environment [34]. To reduce the computational burden, the RAKE combiner is placed prior to multipath detection so that the multipath components corresponding to each user are integrated in constructive fashion, as illustrated in Figure 4.1(a). Figure 4.1(c) illustrates the structure of a space-time receiver in which a beamformer is involved, and therefore, the multipath components are collected and combined over the spatial as well as over the temporal domain.

The above structures require de-spreading against the received signals before space-time processing. The received signals include MAI and ISI, and therefore this degrades the performance of the receiver that de-spread the signals before processing. The receiver will perform better if the ISI and MAI are eliminated before de-spreading, or doing so at the same time as de-spreading.

As stated before, in [10-11, 26, 28, 35], a receiver based on LCMV criterion was developed into which we will apply the space-time structure in Figure 4.1(c) based on the above consideration.

4.2 System Model

Let us consider a baseband DS/CDMA mobile radio network with \( K \) simultaneous users. A binary data sequence of each user is multiplied by a unique signature waveform to
produce a transmitted baseband signal. The normalized signature waveform for the $k$th user is given by

$$a_k(t) = \sum_{n=0}^{N-1} c_k(n)\varphi(t-nT_c)$$  \hfill (4.1)$$

where $a_k(t)$ is the signature waveform for the $k$th user, $N = T_b/T_c$ is the processing gain, $c_k(n)$ is the $n$th element of a signature sequence, $T_c$ is the chip duration, and $\varphi(t)$ is a normalized chip waveform.

So, the transmitted baseband signal of $k$th user is given by

$$s_k(t) = A_k \sum_{m=0}^{M-1} b_k(m)a_k(t - mT_b)$$

$$= \sum_{m=0}^{M-1} b_k(m) \sum_{n=0}^{N-1} c_k(n)\varphi(t - mT_b - nT_c)$$

where $M$ is the number of data symbols per user per frame, $b_k(m) \in \{+1,-1\}$ is the $m$th data bit for $k$th user, $T_b$ is the bit duration, $A_k$ is the amplitude of the $k$th user. It is assumed that $a_k(t)$ has a duration of $[0, T_b]$ and unit energy. It is also assumed that each user transmits independent equiprobable symbols, the symbols from different users are independent, and the spreading waveform has normalized energy, i.e., $\int_0^{T_b} |a_k(t)|^2 = 1$.

Assume that an antenna array of $J$ elements is employed at the receiver and that each transmitter has a single antenna. The baseband impulse response of the channel from the $k$th user to the $j$th antenna at the receiver is
\[ c_{j,k}(t) = \sum_{l=1}^{L} c_{j,k,l}(t) \delta(t - \tau_{k,l}) \] 
\[ = \sum_{l=1}^{L} \alpha_{k,l} \vartheta(\theta_{j,k,l}) \delta(t - \tau_{k,l}) \] 

where \( c_{j,k,l}(t) = \alpha_{k,l} \vartheta(\theta_{j,k,l}) \), \( L \) is the number of resolvable paths, \( \alpha_{k,l}(t) \) and \( \tau_{k,l} \) are the complex gain and delay of the \( l \)th path of the \( k \)th user, respectively, \( \vartheta(\theta_{j,k,l}) \) is the array response corresponding to the \( l \)th path of the \( k \)th user at the \( j \)th antenna, and \( \theta_{j,k,l} \) is the direction of arrival (DOA) from the \( k \)th user along the \( l \)th finger to the \( j \)th antenna. So the baseband impulse between the \( k \)th user and receiver’s antenna array is given by 
\[ c_k(t) = \sum_{j=1}^{L} c_{j,k}(t) \delta(t - \tau_{k,j}) \] 
\[ = \sum_{l=1}^{L} \alpha_{k,l} \vartheta(\theta_{j,k,l}) \delta(t - iT_b - \tau_{k,l}) \] 

where \( c_{k,l} = \alpha_{k,l}(t) \vartheta(\theta_{j,k,l}) \), \( \vartheta(\theta_{j,k,l}) = [\vartheta(\theta_{1,k,l}), \ldots, \vartheta(\theta_{J,k,l})]^T \). 

The received baseband signal at the \( j \)th antenna is given by 
\[ r_j(t) = \sum_{i=0}^{M-1} \sum_{k=1}^{K} s_k(t) \otimes c_{j,k}(t) + \sigma_n_j(t) \] 
\[ = \sum_{i=0}^{M-1} \sum_{k=1}^{K} A_k b_k(i) \sum_{l=1}^{L} c_{j,k,l}(t) s_k(t - iT_b - \tau_{k,l}) + \sigma_n_j(t) \] 
\[ j = 1,2,\ldots,J \] 

where \( n_j(t) \) is independent zero-mean complex white Gaussian noise with power of \( \sigma^2 \), and \( \otimes \) denotes convolution. 

The total received signal at the receiver is given by
\[ \mathbf{r}(t) = \sum_{i=0}^{M-1} \sum_{k=1}^{K} s_k(t) \otimes \mathbf{c}_{k,i}(t) + \mathbf{n}(t) \]  

(4.6)

where \( \mathbf{r}(t) = [r_1(t), \cdots, r_J(t)]^T \) and \( \mathbf{n}(t) = [n_1(t), \cdots, n_J(t)]^T \). We assume that the channel is designed as the tapped-delay-line (TDL) model, and each tap in the TDL model is delayed by one chip duration \( (T_c) \), and the channel is constant during \( M \)-bit intervals.

The discrete-time \( c_{j,k,i}(t) \) will be generated by the vector channel simulator introduced in Chapter 3. Without loss of generality, let the first user be the user of interest. We also assume that the receiver knows this user’s spreading waveform \( a_1(t) \) and its multipath delays \( \tau_{1,1}, \ldots, \tau_{1,L} \), and that the propagation delay for the first path arrives at each element of the antenna array at the same time, likewise for the second path, etc.

The received signal at the \( j \)th antenna is first passed through a chip-matched filter (MF), and then sampled at the chip rate to collect \( (L+N) \) samples. The output sample of the chip matched-filter for the \( n \)th chip of the \( m \)th symbol for the \( l \)th finger of \( j \)th antenna is given by

\[
\int_{mT_b+nT_c}^{mT_b+(n+1)T_c} r_{j,n}^m(t)^{-t-mT_b-nT_c} \varphi(t) dt.
\]

(4.7)

\( (L+N) \) output samples make a received signal vector

\[
\mathbf{r}_j(m) = [r_{j,0}(m), r_{j,1}(m), \ldots, r_{j,L+N-1}(m)]^T
\]

(4.8)

and those samples are chosen to capture the desired user’s signal from all paths.

The received signal vector for the \( j \)th antenna is given by

\[
\mathbf{r}_j(m) = A_i b_i(m) \sum_{l=1}^{L} c_{j,i,l}(m) a_{1,l} + \sum_{l=1}^{L} I_{j,1,l} + \sum_{k=2}^{K} \sum_{l=1}^{L} I_{j,k,l} + \mathbf{n}_j(m)
\]

(4.9)
where \( \mathbf{a}_{1,l} = [a_{1,l,0}, a_{1,l,1}, \ldots, a_{1,l,L+N-1}]^T \) is the signature sequence vector for the first user along the \( l \)th path, and it is the discrete version of the delayed signature waveform of the first user with the \( n \)th element given by

\[
a_{1,l,n} = \int_{\tau_{1,1} + nT_c}^{\tau_{1,1} + (n+1)T_c} a_{1,l}(t - \tau_{1,1}) \varphi(t - \tau_{1,1} - nT_c) \, dt.
\] (4.10)

\( \mathbf{I}_{j,l}(m) \) is inter-symbol interference (ISI), \( \mathbf{I}_{j,k}(m), \ k=2,3,\ldots,K \), is multiple access interference (MAI) from the \( k \)th user, and \( \mathbf{n}_j(m) \) is a noise vector with \( L+N \) independent zero-mean complex Gaussian random variables with unit variance.

We assume the propagation delay for the first path of the \( k \)th user is \( d_{j,k}T_c \), where \( d_{j,k} \) is a positive integer, and the relative time delay \( \tau_{j,k,l} \) lies in the interval \([0,T_b]\) for the convenience of performance evaluation. Then, ISI and MAI terms are given by

\[
\mathbf{I}_{j,l}(m) = A_1 c_{j,1,l} b_l(m-1) \mathbf{a}_{1,l} + A_1 c_{j,1,l} b_l(m+1) \mathbf{a}_{1,l} \quad (4.11)
\]

\[
\mathbf{I}_{j,k,l}(m) = A_k c_{j,k,l} b_k(m-1) \mathbf{a}_{k,l} + A_k c_{j,k,l} b_k(m) \mathbf{a}_{k,l} + A_k c_{j,k,l} b_k(m+1) \mathbf{a}_{k,l},
\]

respectively.

We can rewrite the above results in matrix form. We use the synchronous-equivalent model described in [10]. We have assumed each relative delay is within the interval \([0,T_b]\) for all paths and users. All paths corresponding to one bit from a user are assumed to be within a \( 2T_b \) interval, and therefore three consecutive bits are required for each user [11].

Let \( \mathbf{b}_k(m) = \begin{bmatrix} b_k(m-1) \\ b_k(m) \\ b_k(m+1) \end{bmatrix} \), and \( \mathbf{b}(m) = \begin{bmatrix} b_1(m) \\ \vdots \\ b_K(m) \end{bmatrix} \), and
The received signal after the chip-MF at the \( j \)th antenna is given by

\[ r_j(m) = a C_j A b(m). \quad (4.12) \]

Let \( H_j = a C_j \), so \( H_j \) is \((L+N)\times K\), and let \( H = \begin{bmatrix} H_1 \\
\vdots \\
H_J \end{bmatrix}_{J(L+N)\times K}. \)

The matrix form of the received signal in the absence of noise is given by

\[ r(m) = H A b(m) \quad (4.13) \]

where \( r(m) = [r_1(m)^T, \ldots, r_J(m)^T]^T \), \( C_j \) represents the spatial array channel impulse response in multipath fading and \( a \) represents the temporal channel impulse response with spreading code and path delay.

### 4.3 Space-Time Decorrelating Detector

The objective of this thesis is to develop a low complexity, with high performance receiver that can recover information bits from received signal. If the channel response and signature sequences for all users are known to the receiver, then linear MMSE reception can be performed. The linear decorrelating detector can also completely
eliminate the interference at the expense of enhancing the noise. Both of these detectors involve inversion of a large matrix which has high computational complexity. In addition, sometimes the receiver knows only its own signature sequence and its own channel response especially on the downlink. The proposed Space-Time Decorrelating RAKE Receiver only needs to have the knowledge of the desired user’s signature sequence and timing information while the conventional RAKE receiver needs information on the multipath channel gain and array response of the desired user.
Figure 4.2 Receiver structure.

The proposed Space-Time Decorrelating RAKE Receiver has two stages. Figure 4.2(a) gives the overall structure of this receiver. The first stage has a set of matrix $W_j$. The idea is as follows: at each antenna element $j$, for each path $l$, a linear filter $w_{j,l}$ is applied to the received signal $r_j(m)$ to extract the desired user’s signal from the $l$th path and to suppress the signals from other paths and the interfering signals. The second stage is a linear combiner that enhances the output SINR. Figure 4.2(b) is a straightforward extension to the antenna array of the receiver proposed in [28]. In Figure 4.2(b), the first step is an adaptive de-spread of the received signals along each path at each antenna element. The second is a beamformer that combines the outputs of the $l$th path from all $J$ antenna
elements. The third step is a linear combiner that combines the outputs from all paths, which actually acts as a RAKE receiver. Because of the decorrelating nature of this receiver, the MAI and ISI are effectively eliminated in the outputs of the first step, and therefore, we integrate the second and third step into one step that leads to the structure illustrated in Figure 4.2(a).

Without loss of generality, let the first user be the one of interest. Let the multipath filter bank at the \( j \)th antenna element be \( \mathbf{W}_j = [w_{j,1}, w_{j,2}, \ldots, w_{j,L}] \), \( j = 1, \ldots, J \), and let \( \mathbf{S}_1 = [\mathbf{a}_{i,1}, \ldots, \mathbf{a}_{i,L}] \). The \( \mathbf{w}_j \) are chosen according to the linear constrained minimum variance (LCMV) criterion:

\[
\begin{align*}
\mathbf{W}_j &= \arg \min_{\mathbf{w}_j} \quad E\{\|\mathbf{W}_j^H \mathbf{r}_j(i)\|^2\} \\
&= \arg \min_{\mathbf{w}} \quad tr(\mathbf{W}_j^H \mathbf{R}_j \mathbf{W}) \quad \text{subject to} \quad \mathbf{w}_j^H \mathbf{S}_1 = \mathbf{I}_L 
\end{align*}
\]

where \((\cdot)^H\) denotes Hermitian transposition, \(tr(\cdot)\) denotes the matrix trace operator, and \( \mathbf{R}_j = E\{\mathbf{r}_j(m)\mathbf{r}_j(m)^H\} \) is the autocorrelation matrix of \( \mathbf{r}_j \). The constraints ensure that the desired user’s signal from the \( l \)th path at the \( j \)th antenna is held constant at the filter output, while the desired signal from other paths are eliminated, and signals can be perfectly recovered in the absence of noise.

The closed-form optimum matrix to the above constrained problem can be obtained via Lagrange multipliers [26], [27]

\[
\mathbf{W}_j = \mathbf{R}_j^{-1} \mathbf{S}_1 (\mathbf{S}_1^T \mathbf{R}_j^{-1} \mathbf{S}_1)^{-1} .
\]

Substituting (4.15) into (4.9), we get the output of the \( l \)th finger at the \( j \)th antenna

\[
\mathbf{w}_{j,l}^H \mathbf{r}_j(m) = A_l B_1(m) c_{j,l,d} + \sigma \mathbf{n}_{j,l}(m)
\]
where \( l = 1, 2, \ldots, L \) and \( j = 1, 2, \ldots, J \).

Clearly, the linear filter \( w_{j,l} \) can pick up the desired signal from the \( l \)th finger and eliminate the signals from other paths (MAI and ISI). By doing so, the receiver performs as a decorrelating receiver, and therefore, has the property of near-far resistance of a decorrelating receiver.

The outputs of all linear multipath filters are stacked to form a \( L J \times 1 \) vector

\[
y(m) = [(W_i^H r_i(m))^T, \ldots, (W_j^H r_j(m))^T]^T
\]

From the above two equations, we get

\[
y(m) = A_iB_i(m)h_i + \sigma n(m)
\]

where \( h_k = [c_{i,k,1}, \ldots, c_{i,k,L}, c_{j,k,1}, \ldots, c_{j,k,L}, \ldots, c_{j,k,L}]^T \),

\[
n(m) = [(W_i^H n_i(m))^T, \ldots, (W_j^H n_j(m))^T]^T \sim \mathcal{N}(0,T),
\]

where \( T = \text{diag}(W_i^H W_i \ldots W_j^H W_j) \).

The second stage is a linear combiner. The combining vector \( \mathbf{g} \in \mathbb{C}^{LJ} \) is applied to \( y(m) \) to coherently combine the output from all fingers to further enhance the SINR and give the decision statistic for the \( m \)th symbol of the first user according to

\[
z(m) = \mathbf{g}^H y(m).
\]

Denote \( T^{-(1/2)} = \text{diag}[(W_i^H W_i)^{-1/2} \ldots (W_j^H W_j)^{-1/2}] \) as the inverse of the Hermitian square-root of \( T \). We can get

\[
\tilde{y}(m) = T^{-(1/2)} y(m)
\]

\[
= A_iB_i(m)(T^{-(1/2)} h_i) + \sigma(T^{-(1/2)} n(m))
\]

\[
= A_iB_i \tilde{h}_i + \sigma \tilde{n}(m)
\]
where $\bar{\mathbf{n}}(m) \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_L)$. It then follows that the optimal combining weight vector $\mathbf{g}$ for detecting $b_1(m)$ is given by

$$\mathbf{g} = \bar{\mathbf{h}}_1. \quad (4.21)$$

From this, the autocorrelation matrix of $\bar{\mathbf{y}}(m)$ is given by

$$E[\bar{\mathbf{y}}(m)\bar{\mathbf{y}}(m)^H] = A_1^2 \bar{\mathbf{h}}_1 \bar{\mathbf{h}}_1^H + \sigma^2 \mathbf{I}_L. \quad (4.22)$$

Therefore, the combining weight vector $\bar{\mathbf{h}}_1$ is given by the principal eigenvector of the autocorrelation matrix of the signal $\bar{\mathbf{y}}(m)$. In principal, construction of the optimum combining vector requires the channel parameters. Regardless, because of ISI and MAI cancellation in the first stage, the total signal power in $\mathbf{y}(m)$ is much higher than that of the noise.

To see this, we rewrite the received signal vector at the $j$th antenna as

$$\mathbf{r}_j(m) = A_1 b_1(m) \sum_{l=1}^L c_{j,l,1}(m) \mathbf{a}_{1,l} + \sum_{l=1}^L \mathbf{I}_{j,l,1} + \sum_{k=2}^K \sum_{l=1}^L \mathbf{I}_{j,k,l} + \mathbf{c}_n(m) \quad (4.23)$$

$$= A_1 b_1(m) \sum_{l=1}^L c_{j,l,1}(m) \mathbf{a}_{1,l} + \mathbf{u}(m)$$

where $\mathbf{u}(m)$ denotes interference and noise. So, the outputs of the $l$th path at the $j$th antenna after the first stage is

$$\mathbf{y}_{j,l}(m) = \mathbf{W}_{j,l}^H \mathbf{r}_j(m) = A_1 b_1(m) \sum_{i=1}^L c_{j,l,i}(m) \mathbf{W}_{j,l}^H \mathbf{a}_{i,l} + \mathbf{W}_{j,l}^H \mathbf{u}(m) \quad (4.24)$$

$$= A_1 b_1(m) c_{j,l,1} + \mathbf{W}_{j,l}^H \mathbf{u}(m)$$

where $\mathbf{c}_{j,l} = [c_{j,l,1} \ldots c_{j,l,L}]^T$. 

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The signal-to-interference-and-noise-ratio (SINR) of the \(l^{th}\) path at the output of the \(j^{th}\) antenna is given by

\[
\text{SINR}_l = \frac{A_i^2 E[c_{j,l}^* c_{j,l}^*]}{E[W_j^H uu^H W_j]}
\]

\[
= \frac{A_i^2}{W_j^H R_u W_j}
\]

Since the output signal power is fixed at \(\|c_{j,l}\|^2\), maximizing SINR is equivalent to minimizing the output power of \(y_j(m)\). That is what the proposed receiver does.

To obtain the theoretical BER expression, we re-write (4.19)

\[
z(m) = g^H y(m) = \overline{h}_1^H y(m) = A_i B_i \overline{h}_1^H h_i + \sigma \overline{h}_1^H n(m)
\]

The noise variance is

\[
E[(\sigma^2 h_1^H n(m)(\sigma^2 h_1^H n(m))^T] = E[\sigma h_1^H T^{-\frac{1}{2}} n(m) n^T (m) T^{-\frac{1}{2}} h_1^* \sigma]
\]

\[
= \sigma^2 h_1^H T^{-\frac{1}{2}} T^{-\frac{1}{2}} h_1^*
\]

\[
= \sigma^2 h_1^H h_1^*
\]

Consequently, the first user’s theoretical BER is

\[
p_i(\sigma) = Q\left(\frac{A_i h_1^H h_1^*}{\sigma \sqrt{h_1^H h_1^*}}\right)
\]

We adopt a simple combining strategy by choosing the combining vector \(g\) as principal eigenvector of the autocorrelation matrix of \(y(m)\), which can be obtained blindly using standard decomposition techniques.
The combining step is critical in mobile communications. Due to channel variations, performance of the single-finger receiver is susceptible to shifts of the dominant multipath component and thus lacks the robustness for practical applications. The proposed receiver constructively combines signals from all $L \times J$ fingers, and therefore has strong resistance against fading and timing ambiguity. So the output SINR is improved, and thus the capacity of the wireless system increases. It is also worth noting that one of the major drawbacks of the conventional decorrelating detector is the contribution to noise enhancement that occurs from multiplying the inverse of the autocorrelation matrix of the effective signature waveform. To withstand this, the space-time decorrelating detector is introduced [34].

4.4 Simulation Results

The performance of the receiver is investigated under the multipath fading channel from the aspect of BER behavior. The simulation uses eight users, that is $K=8$, with a spreading gain $N=31$. We assume a multipath environment with $L=3$ paths per user and a receiver antenna array with $J=3$. The antenna array is linearly employed with spacing being on the order of $\lambda/2$ (where $\lambda$ is the carried wavelength). Each resolvable multipath component arrives as a planar wavefront. Let the direction of arrival (DOA) of $k$th user’s signal along the $l$th path with respect to the antenna array be $\theta_{k,l}$. Then, the array response is given by

$$\tilde{\rho}(\theta_{p,k,l}) = \exp(j(p-1)\pi \sin(\theta_{k,l}))$$

where $p$ stands for the $p$th antenna element.
In the simulation, the directions of arrival $\theta_{k,l}$ are uniformly distributed over $(-\pi/2, \pi/2)$, and other parameters are set the same with those in Chapter 3. We use length 31 Gold sequences as spreading sequences. Multipath delays are randomly generated within one symbol interval. All users have equal power, i.e., $A_1 = \ldots = A_K$.

The channel coefficients are generated by the vector channel simulator introduced in Chapter 3. The received signal powers are not equal due to the unequal strength of the multipath gain for each user. The total strength of each user’s multipath channel is measured by the norm of the channel complex gain vector. We assume the first user is the desired user, and we assign the weakest channel to the first user so the near-far situation exists.

Figure 4.3 shows the BERs of conventional RAKE receiver and the new receiver. It shows the new receiver achieves performance improvement over the conventional RAKE receiver. The nature of decorrelation eliminates the ISI and MAI from interference and collects the desired signal along each path of each antenna element, while the conventional RAKE receiver is not able to effectively eliminate the ISI and MAI.

Figure 4.4 illustrates that the new receiver is near-far resistant as we expect. We fix the signal strength for the desired user (the first user) and adjust the power level of all the other interfering users. SNR is set to be 5dB. The result of a conventional RAKE receiver is also plotted. As we can see, the conventional receiver suffers from near-far problem.

When the channel has different response caused by Doppler shift, the receiver’s performance varies accordingly. The faster the mobile moves, the larger the Doppler frequency. A larger Doppler frequency causes the channel change faster, and makes the receiver more difficult to track the channel coefficients, and therefore the receiver
performs worse, as illustrated in Figure 4.5. In this simulation, the mobile’s speeds are $15m/s \ (f_d = 50Hz)$ and $30m/s \ (f_d = 100Hz)$.

Figure 4.3 BER performance.
Figure 4.4 Near-far resistance.

Figure 4.5 BER performances at $f_d = 50Hz$ and $f_d = 100Hz$. 
In this chapter, we introduced a space-time decorrelating RAKE receiver that has many advantages over the conventional RAKE receiver. The proposed receiver is near-far resistant, and able to effectively eliminate MAI and ISI with the knowledge of the desired user’s signature sequences and its timing. The simulation results agree with the discussions. And what is more, the receiver is easily adaptively implemented, as we will discuss in the next chapter, to make it more attractive compared with MMSE receiver that needs to know more information.
Chapter 5 Adaptive Implementation

In the receiver presented in the last chapter, both the weight matrix in the first stage and the coherent combining vector in the second stage need to be estimated from the received signals. When utilized in a non-stationary environment, it is necessary to implement to the receiver adaptively in order to track the time-varying channels. On the other hand, while the closed-form result is a solution to the constrained optimization problem, it is computationally complex in the sense that a data correlation matrix must be estimated regularly and then inverted to arrive at the solution. A simplified, iterative technique for computing the solution needs to be investigated.

5.1 LMS-based Algorithm

We adopt the projection approach in [26] to find the LMS solution to the constrained optimization problem because of its good performance and ease of implementation. The weight matrix is chosen to minimize the mean square value of the outputs using constrained optimization as discuss in previous chapter

\[
W_j = \arg \min_{W_j} \mathbb{E}\{\|W_j^H r_j(i)\|^2\} \\
= \arg \min_{W} \text{tr}(W^H R_j W) \quad \text{subject to } W_j^H S_1 = \mathbf{I}_L. \tag{5.1}
\]

The closed-form optimal solution to the problem is given by

\[
W_j = R_j^{-1} S_1 (S_1^T R_j^{-1} S_1)^{-1}. \tag{5.2}
\]

The optimal receiver output power is then given by
\[ \xi_{\text{min}} = \sum_{j=1}^{L} (S_j^T R_j^{-1} S_j)^{-1}. \] (5.3)

A simplified, iterative technique for computing the solution can be found by decomposing the solution matrix into two orthogonal components via a projection matrix associated with the constrained equation, i.e., the projection approach is adopted here to find the LMS solution to the constrained optimization problem [13], [26].

Define the following projection matrix \( P_s \) that projects a vector onto the column space of \( S_1 \):

\[ P_s = S_1 (S_1^H S_1)^{-1} S_1^H. \] (5.4)

Decompose the optimal linear filter bank \( W_j \) into two orthogonal components as follows

\[ W_j = W_j^s - M^H W_j^a. \] (5.5)

where

\[ W_j^s = P_s W_j = S_1 (S_1^H S_1)^{-1}. \] (5.6)

\( W_j^s \) is the projection of the columns of \( W_j \) onto the column space of \( S_1 \), and is independent of data and represents the non-adaptive portion of the weight matrix \( W_j \). The matrix \( M \) is \( N \times (L+N) \) and satisfies \( MS_1 = 0_{(N \times L)} \), that is, \( M \)'s row space spans the null space of \( S_1 \). \( W_j^a \) is the adaptive portion and a \( N \times (L+N) \) weight matrix. By forcing the weight vector components to be orthogonal, signal cancellation (resulting in zero output power) is avoided. Moreover, since the motivation behind the weight vector decomposition is to derive an iterative solution, the adaptive weight vector should be found using some iterative update scheme.
The structure is shown in Fig 5.1. The upper branch represents the constrained non-adaptive portion of the solution. The lower branch first removes the signal of interest by blocking a particular code then adapting the weight to remove remaining interference.

\[
W_s^{(i)} \rightarrow W_a^{(i)} \rightarrow y_f(i) \rightarrow r_f(i)
\]

Figure 5.1 LMS algorithm structure.

The constraint equation is satisfied by the nonadaptive weighting vector in the upper branch, leaving the lower branch weights free to adapt in order to reduce the interference. The blocking matrix is used to prevent signal cancellation from occurring as well as to ensure orthogonality between the weight vector components:

\[
0 = MS_t (S_t^H S_t)^{-1} = M W_s^{(i)} = MS_t (S_t^H S_t)^{-1} = 0 .
\]  

There are many possible choices for blocking matrix \( M \). In order to reflect explicitly the loss of degrees of freedom associated with the constraints, \( M \) is chosen to have dimension \( N \times (L+N) \). \( M \) can be obtained during simulation by using MATLAB function NULL so that \( MS_1 = 0_{(N,L)} \).

By using the orthogonal decomposition of the weight matrix, the constrained optimization is converted into unconstrained optimization
\[
W_j^a = \arg \min_{w^a \in C^{(\text{NLL})}} E\{\| (W_j^a - \mathbf{M}^H W_j^a)^H r_j(i) \|^2 \}
= \arg \min_{w^a \in C^{(\text{NLL})}} (W_j^a - \mathbf{M}^H W_j^a)^H \mathbf{R}_j (W_j^a - \mathbf{M}^H W_j^a).
\]

The LMS algorithm for adapting the weights \( W_j^a \) is given by

\[
W_j^a (i+1) = W_j^a (i) - \frac{\mu}{2} g(W_j^a (i))
\]

where the stochastic gradient \( g(W_j^a (i)) \) is given by

\[
g(W_j^a (i)) = \frac{d}{d W_j^a (i)} \| [W_j^a - \mathbf{M}^H W_j^a (i)]^H r_j(i) \|^2
= \frac{d}{d W_j^a (i)} \text{tr}([W_j^a - \mathbf{M}^H W_j^a (i)]^H r_j(i)r_j(i)^H [W_j^a - \mathbf{M}^H W_j^a (i)])
= -2 \mathbf{M} r_j(i) \{ [W_j^a - \mathbf{M}^H W_j^a (i)]^H r_j(i) \}^H.
\]

Thus, we get the LMS algorithm for updating \( W_j^a (i) \)

\[
y_j(i) = (W_j^a - \mathbf{M}^H W_j^a (i))^H r_j(i)
W_j^a (i+1) = W_j^a (i) + \mu \mathbf{M} r_j(i) y_j(i)^H.
\]

The choice of step size \( \mu \) represents a compromise between rate of convergence and steady-state excess error.

\( y_j(i) \) is the output of the multipath filter at the \( j \)th antenna element. The output at all antenna elements are concatenated to obtain

\[
y(i) = [y_1(i)^T \ldots y_j(i)^T]^T.
\]

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Note that since the adaptive branch is trying to minimize the overall output power instead of mean square error (MSE), no training signal for the LMS adaptation is required. Both standard and fast adaptive eigen-decomposition techniques can be employed to estimate the principal vector of $\mathbf{R}_y$ for linear combining in the second stage [29]. We adopt here a simple algorithm to track the largest eigenvalue $\lambda$ and the corresponding eigenvector $\mathbf{g}$ of the autocorrelation matrix of $\mathbf{y}(i)$ [27]

$$z(i) = \mathbf{g}(i-1)^H \mathbf{y}(i)$$
$$\lambda(i) = \beta \lambda(i-1) + |z(i)|^2$$
$$\mathbf{g}(i) = \mathbf{g}(i-1) + [\mathbf{y}(i) - \mathbf{g}(i-1)z(i)]z^*(i)/\lambda(i)$$

where $0 < \beta < 1$ is a forgetting factor, $z(i)$ the decision statistic. In simulation, the initial values $\mathbf{g}(0)$ and $d(0)$ are set to be $[1\ 0\ ...\ 0]$ and 1, respectively.

### 5.2 Simulation Results

The simulation parameters are set to be the same with those in the previous chapter. Figure 5.2 illustrates the performance of the LMS-based adaptive blind algorithm discussed earlier. The step size of the first 500 iterations is set to be $\mu = 0.01$, for the next 250 iterations $\mu = 0.005$, and finally the step size is set to be $\mu = 0.001$ for the rest iterations. The convergence behavior of the multipath filter in shown in Figure 5.2, where total mean output energy $\|\mathbf{y}(i)\|^2$ is plotted vs. the numbers of iterations. Here we compute the short-term average of the output energy. The averaging interval is set to be 100. The theoretical minimum mean output energy that is given by equation (5.3) is also plotted in the same figure as the dashed line. As illustrated, after 1000 iterations, the LMS algorithm reaches the steady-state and the gap between simulation and theoretical value is very small.
Figure 5.3 illustrates the steady-state bit error rate (BER) performance. Error is averaged over 100 independent runs after initial 1500 iterations.

Figure 5.4 illustrates the performance of adaptive algorithm for eigenvector estimation that tracks the largest eigenvalue and corresponding eigenvector. A larger forgetting factor ($\beta$) will lead to a smaller deviation from the theoretical eigenvector. When $\beta = 1$, the adaptive algorithm will reach the true eigenvector. When $\beta < 1$, it will lead to a nonzero steady state value of deviation. However, when $\beta$ is set to be 1, after 100 iterations, the deviation from true value is less than 0.5%, as Figure 5.4 illustrated.
Figure 5.3 Steady-state BER performance.

Figure 5.4 Performance of adaptive algorithm for eigenvector estimation.
In this chapter, we introduced a LMS-based adaptive implementation of the presented space-time RAKE receiver. This algorithm was noted to have advantages over the approach in [10], and is well known to possess robustness properties with respect to finite precision effects [26]. The simulation demonstrated the algorithm’s performance.
Chapter 6 Conclusions and Future Work

In wireless communication systems, in order to achieve high speed and reliable transmission and reception with efficiency on a band limited channel, space-time receiver has the advantages of temporal and spatial processing that combines the equalizer and beamformer, and therefore has the capability of eliminating MAI and ISI that limit the performance of DS/CDMA systems. The performance of a space-time receiver that employs an antenna array has been improved significantly over that of a receiver with single antenna.

When mobile stations move around, or the surrounding reflectors change, the channel becomes a time-varying multipath channel. A complete channel must have the temporal and spatial characteristics. The spatial characteristic varies with DOAs changing that refers to angle spread. A higher angle spread causes a larger change in spatial characteristic. Doppler shift cases channel time-varying due to the mobile’s motion, and causes carrier frequency shift. A time-varying vector channel simulator generates channel coefficients that simulates a time-varying multipath channel and has the desired temporal, spatial correlation.

In some scenarios, it is difficult for the receiver to know channel parameters and all users’ signature sequences due to a large number of co-channel users and fast varying nature of the channel. A constrained optimization is used to develop a space-time receiver that only needs to know the desired user’s signature sequence and timing. This receiver is blind, relatively simple and decorrelating in nature, and can be adaptively implemented. It significantly outperforms the conventional RAKE receiver.
The LMS-based adaptive algorithm is developed to track the time-varying channel. This structure is different from and has advantage over the typical LMS algorithm in [10] that is susceptible to signal cancellation resulting from the accumulation of errors due to finite precision arithmetic. On the contrary, the structure discussed here is known to possess robustness properties with respect to finite precision effects [30]. Simulation results demonstrate the algorithm’s performance.

The simulation results have shown that a better performance can be achieved with the space-time receiver of relatively low complexity in a time-varying multipath channel. However, with the system’s processing gain increasing, the computational complexity becomes high. So do the numbers of antenna array elements and multipath. Further study is needed to decrease the computational complex in such a situation. Furthermore, it is better if the requirement of the desired user’s timing information is removed. Studies have been done to deal with this issue, but with a high computational burden.


X. Bernstein and A. M. Haimovic, “Space-time processing for increased


Jan. 1997


