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Process capability analysis for tolerance assignment in discrete part manufacturing

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Abstract

Tolerance control is critical in assuring product quality in discrete part manufacturing. This work deals with an important aspect of tolerance control, namely, the assignment of production tolerances and determination of mean working dimensions. The guiding principle is that the tolerance is used to account for the inevitable errors in manufacturing, which are collectively reflected in process capability. Therefore, a methodology based on process capability analysis is proposed to determine process tolerances for pre-finish machining operations. The mean working dimensions are determined by examining the inter-relationship between various tolerances to calculate the minimum material removal required for each machining operation. The methodology is based on the use of existing production equipment where historical data is available. An analytical method of estimating process capability based on tolerance normalization is proposed and the results are validated using Monte Carlo Simulation. Algorithms are developed and their use illustrated using several simplified actual parts. This work is a step towards developing a scientific method of solving the tolerance assignment problem based on rigorous analysis of manufacturing errors. This will not only eliminate the trial and error process usually involved in a tolerance allocation process but also result in the realistic tolerances possible to achieve using existing equipment.
Acknowledgements

First and foremost, I wish to thank my advisor Dr. Samuel H. Huang whose technical guidance and support has made this work possible. Right from the selection of topic to the very end, he has been very supportive of the new ideas. Only because of his valuable tips and suggestions, I have been able to put my thoughts together and present this work in a professional manner. I am genuinely thankful to Dr. Sam Anand and Dr. Bruce Shultes for agreeing to serve on my thesis defense committee. Just the opportunity of presenting my ideas to them has encouraged me to carry out this work in a more diligent manner. I am thankful to all my colleagues who helped me accomplish this task but I am especially indebted to two of my colleagues, Ranganath Kothamasu and Rami Musa, who spent a lot of time in discussing the new ideas with me. Last but not the least, I am grateful to my parents who loved and supported me through all the good and bad phases of my life. They may not have even the slightest idea about this work, but without their love, encouragement and support it would not have been possible at all. To them, I dedicate this thesis.
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1. Introduction

Tolerance as defined by ASME Y14.5M-1994 is “the total amount a specific dimension is permitted to vary”. The tolerance is the difference between the maximum and minimum limits. Achieving the specified tolerances at a minimum cost is the underlying objective of the manufacturing industry. US manufacturing industry is $60 billion industry and every year millions of dollars are spent in scrapped material, labor hours required for reworking and increased inspection cost due to high failure rate. The main reason for this is that the manufacturing processes and methods employed are not able to meet the tolerances specifications for the product in hand. Manufacturing industry has grown rapidly over past few decades to invent better processes, more capable machines and highly accurate measuring instruments to ease out some of the major problems in manufacturing. On the other hand, very few people have questioned the rationale behind the way tolerances are determined in first place. A fractional effort in this direction can not only avoid expensive material, machining time, labor hours etc. going waste down the line but can also reduce the manufacturing cost in future.

1.1 Tolerance assignment

The key steps involved in finalizing the tolerance specifications for a newly designed and manufactured product are outlined in figure 1-1. The first step is to clearly understand and define the customer needs and expectations. The design engineer then converts the customer requirements into product specifications by defining the overall product dimensions and performance criterion. The overall product specifications are then broken down into the specifications (dimensions, tolerances etc.) of each discrete part going into
that product. These specifications are communicated to the manufacturing engineer in the form of blue prints. Hereafter, it’s the responsibility of the manufacturing engineer to produce the parts as per the blue print specifications. Most of the industries have a structured design and development method in place to ensure that the product specifications are developed to suit the customer requirements. Since most of the real world parts can’t be produced in a single setup, manufacturing engineer needs to develop a setup plan to machine the part from the raw material stage to the finished stage. The working dimensions and tolerances maintained in each setup have to be carefully chosen so as not to violate the overall blue print specification. Most of the time, this particular aspect of tolerancing i.e. choosing the correct working dimensions and tolerances for each machining cut, is not given due importance. The process design and development method normally doesn’t define any structured methodology to solve this problem. The
problem is commonly referred to as “tolerance assignment” problem but includes
tolerance allocation and working dimension calculation.

1.2 Problem statement and objective

The objective of this work is to develop a scientific method of determining the working
dimensions and tolerances for a given setup plan to meet the blue print specifications of
the machined part. Consider, for example, the part shown on figure 1-2 and the setup plan
developed to machine the part in figure 1-3.

Figure 1-2: Machined part

Figure 1-3: Setup plan
The tolerance assignment problem is concerned with finding the working dimensions $A_i$, $B_i$, $C_i$ and $D_i$ and the corresponding tolerances $a_i$, $b_i$, $c_i$ and $d_i$ for every setup $i$ ($i = 1, 2, \ldots, 4$). Following are the most important guiding factors for a solution to the tolerance assignment problem:

a. Working dimension and tolerance for each machining cut should be determined in such a way that the enough machining stock is ensured for the subsequent cuts.

b. The allocated tolerances should be possible to achieve using existing equipment.

c. The allocated tolerances when stacked up should not violate blueprint specification.

The review of current literature and industrial practices reveals that traditional tolerance charting is still the most commonly used method to solve tolerance assignment problem. Many researchers have tried to automate this process but tolerance chart still forms the basis for finding a solution. Though the tolerance charting method has its own advantages but it has several shortcomings. It doesn’t ensure that the workpiece has enough machining stock at each stage for it to be machined as per assigned tolerances. Moreover there is no guarantee that the available production equipment will be able to achieve the allocated tolerances. Therefore it can be said that any tolerance assignment method has to take into account the capability of current production equipment.

1.3 Understanding process capability

In the most simplistic way, process capability can be defined as a measure of inherent variability in a process as compared to the specification requirements of the product.
The process capability for a product characteristic following a normal distribution with mean $\mu$ and variance $s^2$ is mathematically expressed as a ratio of the tolerance specification and six times standard deviation i.e.

$$\text{Process capability, } C_p = \frac{T}{6\sigma}$$

Tolerance $T$ is the difference between upper specification limit (USL) and the lower specification limit (LSL) and $\sigma$ is the standard deviation of the characteristic being evaluated.

As shown in figure 1-4, if the USL and LSL coincide with the $\pm3\sigma$ limits of a normally distributed process, $C_p = 1$ and 99.73% of the parts being produced are likely to fall within the tolerance limits. In other words, the process is likely to produce 0.27% non-confirming parts. This suggests that there are two possible ways of improving the capability or yield (fraction of confirming parts produced) of the process:

a. To bring down the process variance $s^2$ for a given tolerance specification limit $T$. 

![Figure 1-4: Process capability](image-url)
b. To have the maximum possible tolerance specification limit $T$ for a given process variance $s^2$.

The above discussion clearly suggests a relationship between process capability (or process variance) and the tolerance allocated for the process. The common industry practice is to aim for the process capability of 1.33. In that case, the tolerance specification limits have to be determined in a way such that they coincide with the ±4$s$ limits. For the sake of simplicity, ±3$s$ limits have been used in this work for the purpose of tolerance allocation.
2. Literature review

Section 2.1 presents the basic tolerance charting method. Various steps involved in constructing a tolerance chart are outlined. Section 2.2 presents the work done by various researchers in the area of dimensioning and tolerance allocation. Section 2.3 compares all the major dimensioning and tolerancing methods proposed in the past. Section 2.4 justifies the need of a new method for tolerance allocation.

2.1 Tolerance charting method

![Tolerance Chart Example](Image)

Figure 2-1 Tolerance chart example [2]
Used for decades [1, 2] tolerance chart still remains the most powerful tool for tolerance allocation (or to be more accurate, for checking the allocated tolerances). A typical tolerance chart looks like as shown in figure 2-1. The main idea behind tolerance chart is to identify the dimensional chains for the final blue print dimensions and then allocate the individual tolerances so that the blue print tolerance is not violated even in the worst case scenario. The steps involved in constructing a tolerance chart can be summarized as follows (refer [1] for a detailed procedure on constructing a tolerance chart):

1. List all the final (blue print) dimensions and tolerances.
2. Find out the dimensional chains involved in these final dimensions.
3. Allocate tolerances to all the individual links of these chains so as to achieve the tolerances associated with blue print dimensions, even in worst case.
4. Allocate tolerances to all other remaining cuts based on process knowledge.
5. From these allocated tolerances, find out the variation in stock removal for each machining cut.
6. Add a safety stock to get the mean stock removal for each cut.
7. Based on stock removal for each cut, work backwards from the finishing cuts, to find the working dimension for each of the pre-finishing cuts.

Looking into these seven steps it can be said that constructing a tolerance chart manually is quite a laborious task. For a complex part, this manual procedure could not only be inefficient but also error prone. For this very reason, number of researchers in past several years have tried to automate the process of tolerance charting.
2.2 Dimensioning and tolerancing schemes

In this section we present a review of the key operational dimensioning and tolerancing schemes proposed in the past. In the next section we will evaluate these schemes against the three criterion mentioned in the introduction.

Xiaoqing and Davies [3] proposed a matrix tree chain method for automatically tracing the dimensional chains instead of doing manually as in tolerance chart. Mean working dimensions were calculated similar to tolerance chart. Initial tolerance to each working dimension was assigned based on experience or industrial standards and iterative adjustments were then made until satisfactory results are obtained. Whybrew et al. [4] proposed a rooted tree method based on graph theoretic approach to represent dimensional chains. It was assumed that stock removal and tolerance for each step is already known. It was just a method to check the allocated tolerances and mean working dimensions using an algorithm instead of doing it manually using tolerance chart. Mittal et al. [5] proposed a method of representing machining sequence graphically. An algorithm was then used for identifying the machining cuts which contribute to the stack up of tolerances for every B/P dimension and stock removal. A LP model was then used to allocate tolerances to each individual machining cut. Ngoi and Teck [6] proposed a path tracing method to trace the process links and then represent them using linear equations. These linear equations were then solved to obtain working dimensions using gauss elimination technique. Tolerances were allocated based on a LP with sum of weighted tolerances as an objective function and blue print tolerances and process capabilities as constraints. Ji [7, 9] proposed a method of allocating tolerances based on a LP model with an objective function of maximizing the sum of assigned tolerances. The
stock removal, blue print tolerances and economic tolerance for each machining operation were used as constraints. Ji, Ke and Ahluwalia [8] proposed another tree theoretic representation of tolerance chart to identify dimensional chains. A mathematical model of linear equations based on dimensional chains was then used to obtain mean working dimensions. Tolerances were then allocated based on a LP model proposed in [7]. Tang et al. [10] proposed yet another LP model for the optimal allocation of process tolerances and stock removals. Minimization of production cost was used as objective function with process tolerances and stock removal as decision variables. The constraints for the LP model were derived from the dimensional chains associated with tolerance chart. Ji [11] proposed a backward derivation approach for determining working dimensions based on stock at removal each stage. Though it was a simple and realistic method of determining working dimensions, its major drawback was the assumption that the stock removal at each stage is known. Dong and Hu [12] used non-linear optimization to determine the optimal production tolerances with least production cost. He and Gibson [13] attempted to incorporate the relationship between geometrical and dimensional tolerances, ignored by previous researchers, in formulating the tolerance allocation problem. The objective function used was the minimization of scrap cost and the constraints used were the design requirements, geometrical tolerance requirement and the machining allowances. Ngoi and Tang [14] presented a comprehensive review of the methodologies used by various researchers for dimensioning and tolerancing in process planning. Most of the methods either model the tolerancing problem with a linear or non linear cost minimization objective function or use tolerance chart as the basic tool for solving the problem. Wei and Lee [15] tried to improve upon Ngoi’s method by taking
into account the capability of the process producing the workpiece. They proposed a way to estimate the reliability of the entire process and hence the need to review the manufacturing processes if the failure rate is found to be unacceptable. The basis of finding out the working dimensions was still the tolerance chart. This was just an attempt to automate the process of tolerance charting.

### 2.3 Comparison of current dimensioning and tolerancing schemes

The literature review of tolerance allocation methods reveals that most of the methods formulate the problem as an optimization problem but are based on the principles of tolerance charting. Methods differ from each other in the choice of objective function and constraints. Most of the papers consider the cost minimization or tolerance maximization as an objective function. Design tolerance requirements, machining allowance etc are normally considered as constraints. To summarize, we can say that most of the methods try to automate the tolerance charting procedure by the method of linear or non-linear programming. Table 2-1 compares all the schemes described above against the three important characteristics of any dimensioning and tolerancing algorithm i.e. method of determining machining stock, stack up analysis and tolerance assignment. Looking into table 2-1, it can be concluded that most of the algorithms proposed in the past carry out a stack up analysis using an automated procedure based on tolerance charting. Also the machining stock and the process tolerances are still being determined based on process knowledge and experience.
2.4 Need for a new method

As discussed in section 2.2 and 2.3, current techniques for tolerance control i.e. methods to determine mean working dimensions and production tolerances used in the industry are heuristic in nature. They typically evolve from the experience of individual machinists, lack scientific rigor and involve trial and error. This is a hindrance in achieving fully automated computer aided design and manufacturing. Many researchers have tried to automate this process using linear and/or non-linear programming but these approaches suffer from various drawbacks as outlined in previous section. The other problem with most of the traditional methods is that they are static in nature in the sense that the allocated working dimensions and tolerances remains same through out the life cycle of
the part. This is necessarily not the case always as any improvement or deterioration in
the existing manufacturing process will often demand a fresh look at the allocated
tolerances. In nutshell, there is a need of a new dimensioning and tolerancing method
which:

a. provides a logical way of determining working dimensions and tolerances instead of
   relying in experience and heuristics.
b. can allocate realistic tolerances possible to achieve using existing equipment.
c. provides a method of re-defining the allocated dimensions and tolerances based on
   any improvement or deterioration in the capability of manufacturing equipment.

The remaining sections of this work will try to address these points.
3. Proposed Methodology

Figure 3-1 outlines the architecture of a comprehensive tolerance assignment and inspection planning system proposed jointly by several researchers working on different aspects of the problem in ICAMS lab at University of Cincinnati.

![Figure 3-1: Proposed system architecture](image)

The setup plan developed to machine the given part serves as a basic input. The setup plan contains the information on the sequence of machining operations, datums and the
machined surfaces for every setup. The algorithm takes two routes depending on the availability of process capability data for the selected machining processes. If the capability based method is chosen, the process capability extrapolation is performed to estimate the capability for the given part based on the capability data for the similar parts available in process capability database. Working dimensions for each machining cut are then determined based on the stock removal required for each operation. Process tolerances are then assigned based on desired sigma level. Monte Carlo simulation is then used for Tolerance Analysis to verify if the allocated tolerances stack up to violate the blue print requirements. In the absence of capability data, simulation based method is used which uses raw material specification and distributions for locating and machining errors are basic inputs. The workpiece if represented in a 3-dimensional space using the random points generated based on the flatness error in all the raw part surfaces. The types and ranges for locating and machining errors to be included in the simulation are then selected based on the available equipment and their distributions obtained from database. The part is then virtually machined by transforming the sample points in 3-D space based on the errors involved in each machining operation. Computational metrology is then used to perform a virtual inspection of the machined part. All sets of error distributions which meet the final blue print requirements are termed as feasible solutions. Amongst the various feasible solutions obtained, a solution is selected based on the availability and cost associated with various manufacturing equipment. The selected solution is then used to estimate the process capability based on the variance obtained in multiple simulation runs. The estimated process capability is then used for determining stock removal and working dimensions for each operation, similar to the capability based method. The
process tolerances are assigned based on variance obtained. Assigned process tolerances and the capability of the chosen processes serve as basic input for developing an inspection plan. This work is focused on the use of capability based method for calculating working dimensions and assigning process tolerances. Some ideas on process capability extrapolation and inspection planning are also proposed. Refer [24] for details on tolerance stackup analysis and [25] for setup plan evaluation.

3.1 Determining working dimensions

The blue print of a part contains the information on the final features size and tolerances. As in most of the cases, it takes more than one machining operation/setup to machine the part from raw material to blue print dimensions and tolerances. The manufacturing engineer needs to know the dimensions and tolerances to be achieved in each of these machining operations. These dimensions are called as working dimensions or mean working dimensions to be more precise. The working dimensions can be easily determined if the machining stock required for each machining operation can be calculated.

3.1.1 Machining Stock Calculation

Determining minimum stock required for each operation is critical to ensure that part can be machined as per process plan. If at any stage of machining, it is found that the stock is not sufficient to carry out the machining, part can not be machined as per process plan and blue print dimensions and tolerances can not be ensured. The capability of machining
processes and tolerance requirements on the part can be used to determine the minimum machining stock required at each stage.

Consider the machining of a cylindrical part with a turning operation followed by a grinding operation. The objective here is to determine the machining stock required for grinding. Figure 3-2(a) shows the cross section of the shaft after turning which is a pre-finishing operation. The circularity of the part at this stage is assumed to be 0.1mm.

![Figure 3-2: Machining Stock Calculation](image)

Let’s assume that the only tolerance requirement on the shaft after the grinding operation in the circularity of 0.005mm. Fig 3-2(b) shows that for achieving a perfect circularity after grinding operation, a minimum stock removal of 0.1mm is required. For a part with the final circularity of 0.005mm, the minimum stock of $0.1 - 0.005 = 0.095$ is required. We can be little conservative here in determining minimum stock requirement and can conclude that for the grinding operation, machining stock of 0.1mm is required. Circularity of the part after the turning operation can be known from the standard deviation (of circularity) derived from the process capability.

### 3.1.2 Tolerance inter-relationship

The case described in previous section was a simple theoretical case where it was assumed that the circularity is the only requirement on the finished part. In the most
likely case of more than one type of tolerance requirement on the finished part, interrelationship between various tolerances needs to be examined for calculating the required material removal. The cylindrical shaft described above might have a run-out tolerance requirement apart from circularity. If the capability of turning operation to maintain run out on the shaft is worse than the capability for circularity, we might need more stock for grinding than as determined by circularity tolerance alone. Following general rules have been derived for determining machining stock requirements in the case of multiple tolerance requirements on the finished part:

1. Any two tolerances can be compared only if they control the same feature and have the same datum(s).

2. For the purpose of calculating machining stock, any two tolerances are dependent if the improvement in one also improves another.

3. For the tolerances which are dependent, required stock removal will be the maximum of the stock removal required for individual tolerances.

4. For the tolerances which are independent, required stock removal is the sum of the stock removal required for individual tolerances.

5. For a surface feature having more than two tolerance requirements, minimum machining stock can be calculated using the following notations:

\[
N \quad \text{number of different sets of dependent tolerance specifications}
\]

\[
M_i \quad \text{number of tolerance specifications in set } i; \quad i = 1, 2, \ldots, N
\]
$S_i$   machining stock required for set $i$

$S_{ij}$   machining stock required for
tolerance $j$ in set $i$

Minimum machining stock = $\sum_{i=1}^{N} S_i$

Where $S_i = \max (S_{ij});$  $j = 1,2,\ldots, M_i$

Relationship of the various tolerances can be determined from the matrix in Table 3-1 where ‘0’ represents tolerances independent of each other and ‘1’ represents tolerances dependent on each other. $S_{ij}$ can be determined from Table 3-2 where $s$ is the process standard deviation.

![Table 3-1: Tolerance relationship matrix](image-url)
3.1.3 Algorithm for determining working dimensions

The purpose of setup planning is to arrange manufacturing features into an appropriate sequence of groups in order to assure best product quality. The setup plan contains information only about the best combination of datum and machined surfaces for every setup. The total number of setups and manufacturing process to be used for each setup is not known at this point. Manufacturing process and the total number of setups needs to be finalized before determining working dimensions and production tolerance assignment.

Process dispersion is a good measure of the accuracy of a manufacturing process. An approximate finishing process can be selected by comparing the design tolerance of the given part with the process dispersion of the given set of machines. For example, if the design tolerance of a given part is ± 0.01 and none of the turning machines have

<table>
<thead>
<tr>
<th>Tolerance Type</th>
<th>Minimum stock removal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straightness</td>
<td>2(6s)</td>
</tr>
<tr>
<td>Flatness</td>
<td>6s</td>
</tr>
<tr>
<td>Circularity</td>
<td>2(6s)</td>
</tr>
<tr>
<td>Cylindericity</td>
<td>2(6s)</td>
</tr>
<tr>
<td>Parallelism</td>
<td>2(6s)</td>
</tr>
<tr>
<td>Perpendicularity</td>
<td>2(6s)</td>
</tr>
<tr>
<td>Angularity</td>
<td>2(6s)</td>
</tr>
<tr>
<td>Profile of a line</td>
<td>6s</td>
</tr>
<tr>
<td>Profile of a surface</td>
<td>6s</td>
</tr>
<tr>
<td>Circular runout</td>
<td>2(6s)</td>
</tr>
<tr>
<td>Total runout</td>
<td>2(6s)</td>
</tr>
<tr>
<td>Position</td>
<td>2(6s)</td>
</tr>
<tr>
<td>Centricity</td>
<td>2(6s)</td>
</tr>
<tr>
<td>Symmetry</td>
<td>2(6s)</td>
</tr>
<tr>
<td>Dimensional</td>
<td>2(6s)</td>
</tr>
</tbody>
</table>

Table 3-2: Minimum stock removal for different types of tolerances
process dispersion (±3s) less than 0.01, we can conclude that a machining operation with process dispersion less than 0.01 (finish turning, grinding etc.) needs to be performed after turning operation.

Figure 3-3: Algorithm for determining working dimensions

Once an approximate process is selected, number of setups required is also known. After this, working dimensions and production tolerances for each machining cut needs to be
determined. The main reason for leaving the machining stock in the pre-finishing operation is to have enough material for finishing operation which results in a closer dimensional and geometrical control of the features. This idea can be used for determining the working dimensions for all the setups as outlined in Figure 3-3. Machining stock required for an operation can be calculated based on required precision and the process dispersion of pre-finishing operation.

Determination of working dimensions for rotational parts can be formulated mathematically using the following notations:

\[ N \] \hspace{1cm} \text{number of setups}

\[ M_i \] \hspace{1cm} \text{number of machining cuts in setup } i \hspace{1cm} i = 1, 2, \ldots N

\[ c_{ij} \] \hspace{1cm} \text{machining cut } j \text{ in the setup } i \hspace{1cm} j = 1, 2, \ldots, M_i

\[ f_i \] \hspace{1cm} \text{primary datum surface used for setup } i

\[ f_{ij} \] \hspace{1cm} \text{surface machined by cut } j \text{ in setup } i

\[ d_{ij} \] \hspace{1cm} \text{dimension between } f_i \text{ and } f_{ij} \hspace{1cm} \text{(Note that } d_{ij} = d_{ji})

\[ w_i(d_{ij}) \] \hspace{1cm} \text{working dimension between } f_i \text{ and } f_{ij} \text{ in setup } i

\[ \Psi(f_{ij}) \] \hspace{1cm} \text{a constant, } \Psi(f_{ij}) = 1 \text{ if } f_{ij} \text{ is an external feature; otherwise, } \Psi(f_{ij}) = -1

\[ \Gamma_i(d_{ij}) \] \hspace{1cm} \text{a constant, } \Gamma_i(d_{ij}) = 1 \text{ if } d_{ij} \text{ is altered in setup } i; \text{ otherwise } \Gamma_i(d_{ij}) = 0

\[ \Phi_i(d_{ij}) \] \hspace{1cm} \text{nominal dimension between } f_i \text{ and } f_{ij} \text{ in setup } i

\[ S(d_{ij}, a) \] \hspace{1cm} \text{stock removal affecting } d_{ij} \text{ in setup } a
The algorithm is as follows:

Assign values to all $\Psi(f_{ij})$

Assign values to all $\Gamma_i(d_{ij})$

for $i = N$ to 1 step -1

for $j = 1$ to $M_i$

if $i = N$

$w_i(d_{ij}) = \Phi_i(d_{ij})$

else

$a = \min\{t \mid \Gamma_t(d_{ij}) \neq 0\}, \forall t > i$

$w_i(d_{ij}) = \Phi_i(d_{ij}) + \Psi(f_{ij}) \cdot S(d_{ij}, a)$

end

end

For the purpose of calculating the minimum machining stock requirements in setup $a$, $S(d_{ij}, a)$, interrelationship between various tolerances needs to be examined.

3.2 Tolerance allocation

Once the working dimensions for every setup is determined, production tolerance can be assigned to each working dimension based on process dispersion ($\pm 3s$). Tolerance assignment based on process dispersion is a realistic one as there is no point in assigning the tolerances which cannot be achieved with the available machines.
3.3 Process capability extrapolation

The discussion in the previous sections clearly suggests that the availability of process capability data (to know process dispersion) is a prerequisite for using the suggested algorithm. In an ideal case, this information can be obtained from the machine records but this may not be the case always. Consider a machined surface having the following specifications: surface roughness - 5µm, flatness - 0.015mm, dimension requirement in relation to bottom surface - 20±0.1mm and parallelism in relation to bottom surface - 0.020mm. Then it is easy to see that at least four capability indices are required for this process, namely, roughness, flatness, dimension tolerance and parallelism. It is reasonable to assume that these indices for roughness, dimension tolerance and flatness can be obtained from some database in the industry because the capability of maintaining the roughness, dimension and flatness of machined surface is an inherent characteristic of the milling process. But this is not the case with parallelism since the resultant parallelism is not only determined by the milling operation itself, the positioning and clamping conditions of the machined part in machine tool is a major contributor as well. Actually all datum-required geometric tolerance share this common attribute; furthermore, once the position error comes into the scope, the size of machined surface and datum surface need to be taken into consideration. Our general process capability analysis for tolerance analysis will not be complete if we could not address process capability for maintaining the datum-required geometric tolerance. The concept of tolerance normalization proposed by Liu and Huang [16] is used to solve this problem. A preliminary method of estimating the process capability based on available data is
presented in Appendix. This is a complex problem and needs further research before developing a rigorous extrapolation method.
4. Case Studies

This section presents three different case studies to illustrate the proposed algorithm of determining working dimensions and allocating tolerances. Stepped shaft A represents a relatively simple case with the machining datums same as design datums. Stepped shaft B represents a case where the tolerance stackup problem occurs because of machining datums being different from design datums. In the end, the proposed algorithm is extended to Delphi spindle, a simplified actual part manufactured at Delphi Automotive Systems.

4.1 Stepped shaft A

Stepped shaft shown in Figure 4-1 is used as an example to illustrate the algorithm for determining working dimensions and allocating process tolerances. Initial setup plan to machine the part is shown in figure 4-2.

Figure 4-1: Stepped shaft A
The machines available to machine the part to the required dimensions are shown in figure 4-3. There are two grinding machines (G1 and G2) and two turning machines (T1 and T2). Their capability values in terms of process dispersion ($6s$) for various types of tolerances are shown in the table.

<table>
<thead>
<tr>
<th>S. No</th>
<th>Characteristic</th>
<th>$6s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Linear dimensions</td>
<td>0.100</td>
</tr>
<tr>
<td>2.</td>
<td>Parallelism</td>
<td>0.120</td>
</tr>
<tr>
<td>3.</td>
<td>Flatness</td>
<td>0.120</td>
</tr>
</tbody>
</table>

Turning Machine (T1)

<table>
<thead>
<tr>
<th>S. No</th>
<th>Characteristic</th>
<th>$6s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Linear dimensions</td>
<td>0.030</td>
</tr>
<tr>
<td>2.</td>
<td>Parallelism</td>
<td>0.035</td>
</tr>
<tr>
<td>3.</td>
<td>Flatness</td>
<td>0.020</td>
</tr>
</tbody>
</table>

Grinding Machine (G1)

<table>
<thead>
<tr>
<th>S. No</th>
<th>Characteristic</th>
<th>$6s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Linear dimensions</td>
<td>0.070</td>
</tr>
<tr>
<td>2.</td>
<td>Parallelism</td>
<td>0.080</td>
</tr>
<tr>
<td>3.</td>
<td>Flatness</td>
<td>0.085</td>
</tr>
</tbody>
</table>

Turning Machine (T2)

<table>
<thead>
<tr>
<th>S. No</th>
<th>Characteristic</th>
<th>$6s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Linear dimensions</td>
<td>0.001</td>
</tr>
<tr>
<td>2.</td>
<td>Parallelism</td>
<td>0.002</td>
</tr>
<tr>
<td>3.</td>
<td>Flatness</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Grinding Machine (G2)

Process selection needs to be done before beginning with tolerance assignment. Since the finished tolerance requirement of the given part is of the order of 0.001mm, none of the available turning
machines is capable of reaching this tolerance requirement. Hence the need of grinding operation after turning is necessitated. Now the setup plan needs to be extended to include grinding operations. For simplicity purpose, same sequence of operations as in original setup plan is repeated here for grinding. Hence the Setup Plan is modified to include grinding operations as shown in figure 4-4.
Once the machine selection is over and the setup plan is finalized, working dimensions for all the intermediate cuts can be calculated according to the proposed algorithm as follows:

\[
\Psi(1) = 1 \quad \Psi(2) = 1 \quad \Psi(3) = 1 \quad \Psi(4) = 1 \quad \Psi(5) = 1
\]

\[
\Gamma_1(d_{15}) = 1, \quad \Gamma_1(d_{25}) = 0, \quad \Gamma_1(d_{35}) = 0,
\]

\[
\Gamma_1(d_{45}) = 0
\]

\[
\Gamma_2(d_{15}) = 1, \quad \Gamma_2(d_{25}) = 1, \quad \Gamma_2(d_{35}) = 1,
\]

\[
\Gamma_2(d_{45}) = 1
\]

\[
\Gamma_3(d_{15}) = 1, \quad \Gamma_3(d_{25}) = 0, \quad \Gamma_3(d_{35}) = 0,
\]

\[
\Gamma_3(d_{45}) = 0
\]

\[
\Gamma_4(d_{15}) = 1, \quad \Gamma_4(d_{25}) = 1, \quad \Gamma_4(d_{35}) = 1,
\]

\[
\Gamma_4(d_{45}) = 1
\]
\( i = 4 \) (setup 4)
\( f_i = 5 \)

\( j = 1 \) (first cut)
\( f_y = 4 \)
\( w_d(d_{54}) = \Phi_d(d_{54}) = 1.00 \)

\( j = 2 \) (second cut)
\( f_y = 3 \)
\( w_d(d_{53}) = \Phi_d(d_{53}) = 2.00 \)

\( j = 3 \) (third cut)
\( f_y = 2 \)
\( w_d(d_{52}) = \Phi_d(d_{52}) = 3.00 \)

\( j = 4 \) (fourth cut)
\( f_y = 1 \)
\( w_d(d_{51}) = \Phi_d(d_{51}) = 4.00 \)

\( i = 3 \) (setup 3)
\( f_i = 1 \)

\( j = 1 \) (first cut)
\( f_y = 5 \)
\( a = \min\{4\} = 4 \)
\[ \Psi(5) = 1 \]
\[ w_3(d_{15}) = \Phi_3(d_{15}) + (1) \Phi_3(d_{15}) + (1)S(d_{15}, 4) \]
\[ = 4.00 + (0.035 - 0.002) \]
\[ = 4.04 \]

\( i = 2 \) (setup 2)
\[ f_i = 5 \]

\( j = 1 \) (first cut)
\[ f_j = 4 \]
\[ a = \min \{4\} = 4 \]
\[ \Psi(4) = 1 \]
\[ w_2(d_{54}) = \Phi_2(d_{54}) + (1)S(d_{54}, 4) \]
\[ = (1.00 + 0.033) + (0.080 - 0.002) \]
\[ = 1.11 \]

\( j = 2 \) (second cut)
\[ f_j = 3 \]
\[ a = \min \{4\} = 4 \]
\[ \Psi(3) = 1 \]
\[ w_2(d_{53}) = \Phi_2(d_{53}) + (1)S(d_{53}, 4) \]
\[ = (2.00 + 0.033) + (0.080 - 0.002) \]
\[ = 2.11 \]

\( j = 3 \) (third cut)
\[ f_\gamma = 2 \]
\[ a = \min\{4\} = 4 \]
\[ \Psi(2) = 1 \]
\[ w_2(d_{57}) = \Phi_2(d_{57}) + (1) \cdot S(d_{57}, 4) \]
\[ = (2.00 + 0.033) + (0.080 - 0.002) \]
\[ = 3.11 \]

\( j = 4 \) (fourth cut)
\[ f_\gamma = 1 \]
\[ a = \min\{3, 4\} = 3 \]
\[ \Psi(1) = 1 \]
\[ w_2(d_{51}) = \Phi_2(d_{51}) + (1) \cdot S(d_{51}, 3) \]
\[ = 4.04 + (0.085 - 0.02) \]
\[ = 4.1 \]

\( i = 1 \) (setup 1)
\[ f_i = 1 \]

\( j = 1 \) (first cut)
\[ f_\gamma = 5 \]
\[ a = \min\{2, 3, 4\} = 2 \]
\[ \Psi(5) = 1 \]
\[ w_1(d_{15}) = \Phi_1(d_{15}) + (1) \cdot S(d_{15}, 2) \]
\[ = 4.1 + (0.12 - 0.02) \]
\[ = 4.2 \]
Once the working dimensions are determined, tolerances for all the individual cuts can be assigned based on ±3σ limits. The complete setup plan for stepped shaft A along with working dimensions and assigned tolerances for all the machining cuts is shown in figure 4-5.
4.2 Stepped shaft B

The stepped shaft A used to illustrate the calculation of working dimensions and tolerance assignment had the machining datums same as design datums. This however may not be the case all the time and machining datums might differ from the design datums. The example given here illustrates that the same methodology for determining working dimensions and tolerance assignment can still be used in such cases.
Consider the stepped shaft shown in figure 4-6. The setup plan for machining this part is shown next in figure 4-7. Note that setup datums differ from design datums in this case.

As the tolerance requirements are as low as 0.006, it can be concluded that none of the turning machines will be able to meet the design tolerance requirements. Hence the need to grinding operation after turning operation is necessitated. The setup plan shown in figure 4-7 is modified to include grinding operations also as shown in figure 4-8.
As explained earlier, the first step is the selection of machines to be used. Since the machining datums for the example part are different from the design datums, there might be tolerance stackup between successive cuts. In case of tolerance stackup, tolerance schematics for the various dimensions might be useful to decide on the minimum accuracy required for the machines. Figure 4-9 shows the tolerance schematics for all the three design dimensions. As it can be seen, the minimum accuracy required for the finishing process (grinding in this case) is 0.004. Hence the grinding machines selected should have a minimum 6σ value of 0.004.
The machines selected for machining the part are shown in figure 4-10 along with their 6s values. Once the machine selection is over, working dimensions can be determined using the methodology proposed earlier and tolerances can be calculated based on 6s values.

<table>
<thead>
<tr>
<th>S. No</th>
<th>Characteristic</th>
<th>6s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Linear dimensions</td>
<td>0.100</td>
</tr>
<tr>
<td>2.</td>
<td>Parallelism</td>
<td>0.120</td>
</tr>
<tr>
<td>3.</td>
<td>Flatness</td>
<td>0.120</td>
</tr>
</tbody>
</table>

Turning Machine (T1)

<table>
<thead>
<tr>
<th>S. No</th>
<th>Characteristic</th>
<th>6s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Linear dimensions</td>
<td>0.070</td>
</tr>
<tr>
<td>2.</td>
<td>Parallelism</td>
<td>0.080</td>
</tr>
<tr>
<td>3.</td>
<td>Flatness</td>
<td>0.085</td>
</tr>
</tbody>
</table>

Turning Machine (T2)

<table>
<thead>
<tr>
<th>S. No</th>
<th>Characteristic</th>
<th>6s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Linear dimensions</td>
<td>0.004</td>
</tr>
<tr>
<td>2.</td>
<td>Parallelism</td>
<td>0.035</td>
</tr>
<tr>
<td>3.</td>
<td>Flatness</td>
<td>0.020</td>
</tr>
</tbody>
</table>

Grinding Machine (G1)

<table>
<thead>
<tr>
<th>S. No</th>
<th>Characteristic</th>
<th>6s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Linear dimensions</td>
<td>0.001</td>
</tr>
<tr>
<td>2.</td>
<td>Parallelism</td>
<td>0.002</td>
</tr>
<tr>
<td>3.</td>
<td>Flatness</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Grinding Machine (G2)

Figure 4-10: Available machines for stepped shaft B

Working dimensions can be determined as explained next.

\[
\Psi(1) = 1 \quad \Psi(2) = 1 \quad \Psi(3) = 1 \quad \Psi(4) = 1
\]

\[
\Gamma_1(d_{32}) = 1, \quad \Gamma_1(d_{31}) = 1, \quad \Gamma_1(d_{24}) = 1
\]

\[
\Gamma_2(d_{32}) = 1, \quad \Gamma_2(d_{31}) = 1, \quad \Gamma_2(d_{24}) = 1
\]

\[
\Gamma_3(d_{32}) = 1, \quad \Gamma_3(d_{31}) = 1, \quad \Gamma_3(d_{24}) = 1
\]

\[
\Gamma_4(d_{32}) = 1, \quad \Gamma_4(d_{31}) = 1, \quad \Gamma_4(d_{24}) = 1
\]
\[ i = 4 \text{ (setup 4)} \]
\[ f_i = 3 \]

\[ j = 1 \text{ (first cut)} \]
\[ f_{ij} = 1 \]
\[ w_{d}(d_{ij}) = \Phi_{d}(d_{ij}) = 10.00 \]

\[ j = 2 \text{ (second cut)} \]
\[ f_{ij} = 2 \]
\[ w_{d}(d_{ij}) = \Phi_{d}(d_{ij}) = 4.00 \]

\[ i = 3 \text{ (setup 3)} \]
\[ f_i = 2 \]

\[ j = 1 \text{ (first cut)} \]
\[ f_{ij} = 4 \]
\[ a = \min \{4\} = 4 \]
\[ \Psi(4) = 1 \]
\[ w_{d}(d_{ij}) = \Phi_{d}(d_{ij}) + (1) \cdot S(d_{ij}, 4) \]
\[ = 10.00 + (0.035 - 0.002) \]
\[ = 10.033 \]

\[ j = 2 \text{ (second cut)} \]
\( f_y = 3 \)
\( a = \min\{4\} = 4 \)
\( \Psi(3) = 1 \)

\[ w_3(d_{32}) = \Phi_3(d_{32}) + (1) \cdot S(d_{32}, 4) \]
\[ = 4.00 + (0.035 - 0.002) \]
\[ = 4.033 \]

\( i = 2 \) (setup 2)
\( f_i = 3 \)

\( j = 1 \) (first cut)
\( f_y = 1 \)
\( a = \min\{3, 4\} = 3 \)
\( \Psi(2) = 1 \)

\[ w_2(d_{31}) = \Phi_2(d_{31}) + (1) \cdot S(d_{31}, 3) \]
\[ = 10.033 + (0.1 - 0.004) \]
\[ = 10.129 \]

\( j = 2 \) (second cut)
\( f_y = 2 \)
\( a = \min\{3, 4\} = 3 \)
\( \Psi(2) = 1 \)

\[ w_2(d_{32}) = \Phi_2(d_{32}) + (1) \cdot S(d_{32}, 3) \]
\[ = 4.033 + (0.1 - 0.004) \]
\[ = 4.129 \]
\[ i = 1 \text{ (setup 1)} \]
\[ f_i = 2 \]

\[ j = 1 \text{ (first cut)} \]
\[ f_y = 4 \]
\[ a = \min\{2, 3, 4\} = 2 \]
\[ \Psi(4) = 1 \]
\[ w_j(d_{24}) = \Phi_j(d_{24}) + (1) \cdot S(d_{24}, 2) \]
\[ = 10.129 + (0.12 - 0.08) \]
\[ = 10.169 \]

\[ j = 2 \text{ (second cut)} \]
\[ f_y = 3 \]
\[ a = \min\{2, 3, 4\} = 2 \]
\[ \Psi(3) = 1 \]
\[ w_j(d_{23}) = \Phi_j(d_{23}) + (1) \cdot S(d_{23}, 2) \]
\[ = 4.129 + (0.12 - 0.08) \]
\[ = 4.169 \]

Once the working dimensions are determined, tolerances for all the individual cuts can be assigned based on ±3s limits. The complete setup plan for stepped shaft B along with working dimensions and assigned tolerances for all the machining cuts is shown in figure 5-11.
Setup 1 (Turning)

Setup 2 (Turning)

Setup 3 (Grinding)

Setup 4 (Grinding)

Figure 4-11: Final setup plan for stepped shaft B
4.3 Delphi spindle

Figure 4-12 shows Delphi spindle, a simplified actual part manufactured by Delphi Automotive Systems.

Figure 4-12: Delphi spindle

Figure 4-13 shows an initial setup plan developed to machine the part based on the following criterion derived by examining the geometrical and tolerance relationships in the blueprint:

1. Surfaces $S_1$-$S_5$ can easily be classified in group 1 and surfaces $S_7$-$S_{13}$ in another group 2 based on their geometrical relationship.

2. Surface $S_6$ can be assigned to any group as it doesn’t have any tolerance relationship with any other surface. It is assigned to group 1 to balance the number of surfaces machined in each setup.
3. Since $S_{11}$ uses $S_5$ as a datum, $S_5$ should be machined before $S_{11}$. Since $S_5$ belongs to group 1, group 1 is assigned to setup 1.

![Initial setup plan for Delphi spindle](image)

Figure 4-13: Initial setup plan for Delphi spindle

Figure 4-14 shows the capability values of the rough and finish turning machines available to machine the part to the required dimensions and tolerances. It can be easily inferred that the tolerance requirements of the finished part can not be met only with rough turning operation as ‘6s’ for rough turning is larger then the finished part tolerances. Hence a finish turning operation is necessitated. The modified setup plan along with working dimensions and tolerances calculated using proposed algorithm is shown in figure 4-15.
<table>
<thead>
<tr>
<th>S. No</th>
<th>Characteristic</th>
<th>6s</th>
<th>± 3s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Linear dimensions</td>
<td>0.3</td>
<td>± 0.15</td>
</tr>
<tr>
<td>2.</td>
<td>Diameter</td>
<td>0.5</td>
<td>±0.25</td>
</tr>
<tr>
<td>3.</td>
<td>Flatness</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>4.</td>
<td>Parallelism</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>5.</td>
<td>Cylindricity</td>
<td>0.15</td>
<td>-</td>
</tr>
<tr>
<td>6.</td>
<td>Run-out</td>
<td>0.1</td>
<td>-</td>
</tr>
</tbody>
</table>

Rough Turning

<table>
<thead>
<tr>
<th>S. No</th>
<th>Characteristic</th>
<th>6s</th>
<th>± 3s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Linear dimensions</td>
<td>0.1</td>
<td>± 0.05</td>
</tr>
<tr>
<td>2.</td>
<td>Diameter</td>
<td>0.1</td>
<td>±0.05</td>
</tr>
<tr>
<td>3.</td>
<td>Flatness</td>
<td>0.05</td>
<td>-</td>
</tr>
<tr>
<td>4.</td>
<td>Parallelism</td>
<td>0.06</td>
<td>-</td>
</tr>
<tr>
<td>5.</td>
<td>Cylindricity</td>
<td>0.03</td>
<td>-</td>
</tr>
<tr>
<td>6.</td>
<td>Run-out</td>
<td>0.06</td>
<td>-</td>
</tr>
</tbody>
</table>

Finish Turning

Figure 4-14: Available machines for Delphi spindle
Figure 4-15: Final setup plan for Delphi spindle

Setup 2 (Rough Turning)

Setup 3 (Finish Turning)

Setup 4 (Finish Turning)
5. Inspection Planning

After the working dimensions and tolerances are assigned, an inspection plan needs to be developed to check the parts being produced and make the adjustment to the process if required to ensure that no nonconforming parts are being produced. The purpose of any inspection plan is to catch the shift in the process before any defective parts are produced. Since the tolerances are allocated based on process capability, the statistical methods can be used to determine the sample size and sample frequency required for the inspection plan.

If the process dispersion remains at the same level as was used for determining the process tolerances, the process is likely to operate within the influence of only chance variations. But in a most likely scenario, as the time passes, the process is bound to be affected by one or more assignable causes. An assignable cause will have process operating at a mean or standard deviation different from what was used for determining the control limits or process tolerances. In such a situation, an inspection plan to catch and correct the assignable cause is required. The goal of an inspection plan is to catch and eliminate the non-confirming parts produced by the process keeping the inspection cost minimum at the same time. Assuming the process to be normally distributed, the expected percentage of non-conformance depends on the following:

1. Process mean $\mu$
2. Standard deviation of the process $s$
3. Upper Specification Limit (USL) and Lower Specification Limit (LSL)

The process operation at target mean with USL and LSL coinciding with the $\pm3s$ limits, the process is expected to produce only 0.26% non-confirming parts. In a continuous
production environment, a process may shift from its target mean because of various reasons. With the shift in process mean, the same process is expected to produce higher percentage of non-confirming parts. The idea is illustrated in figure 6-1.

Consider a process operating at target mean \( \mu_1 = 1.5 \) with standard deviation \( s = 0.01 \). With USL and LSL of the product coinciding with the \( \pm 3s \) limits of 1.47 and 1.53, the process is expected to produce 0.26\% non-confirming parts. With the shift in process by \( \Delta \mu = \mu_2 - \mu_1 = 0.01 \), new process mean becomes \( \mu_2 = 1.49 \) with the same standard deviation of \( s = 0.01 \). With the USL and LSL of the product being same, now the process is expected to produce 2.28\% of non-confirming parts. It can be inferred from the above description that the inspection plan for a given process should incorporate the analysis of process shift.

The other reason for a process to produce more than expected non-confirming parts, even with mean remaining at same level is when the process standard deviation increases. Any sampling effort design for a production process should be able to detect the shift in the process. Consider a production process with a true mean \( \mu \) and standard deviation \( s \).
As discussed by Montgomery [23] while determining the sample size $n$ for an inspection plan, we should keep in mind the size of shift $d$ we are trying to detect and the degree of uncertainty $a$. The objective is to have error in estimated mean $\bar{x}$ less than $d$ with a $1-a$ uncertainty i.e.

$$P\{\bar{x} - \mu \leq \delta\} \geq 1 - \alpha \quad --------------- (1)$$

From the confidence interval computation, the following relationship holds:

$$P\left\{\bar{x} - \mu \leq z_{\alpha/2} \frac{\sigma}{\sqrt{n}}\right\} = 1 - \alpha \quad --------------- (2)$$

For both the equations (1) and (2) to be true, following equation must hold with a sample size of $n$:

$$\delta = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Thus the sample size $n$ to detect a shift $d$ in the process mean with a confidence of 100(1- $a$)% should be

$$n = \left(\frac{z_{\alpha/2}}{\delta}\right)^2 \sigma^2$$

Once the sample size has been determined, the concept of average run length (ARL) or average time to signal (ATS) outlined by Montgomery can be used to determine the sampling frequency. ARL is the average number of samples that must be checked before an out of control condition is indicated. For uncorrelated samples, ARL can be calculated as, $\text{ARL}=1/p$; where $p$ is the probability of any points exceeding the control limits (or the specifications limits). If the tolerances are allocated to coincide with ±3$s$ limits, for a normally distributed process, $p=0.0027$ and $\text{ARL}=1/0.0027=370$ i.e. if the process remains in control, an out of control signal will be generated every 370 samples. If the
samples are taken at regular intervals of time that are \textit{h} hours apart, then \textit{ATS}=\textit{ARL}.\textit{h}.

Consider a shift in process resulting in a probability of point falling outside the limits of 0.50, \textit{ARL}=1/0.5=2 i.e. two samples will be required to detect the shift in the process and if each sample is taken every one hour, average time to signal the out of control process will be 2 hrs.
6. Conclusion and Future Research

Computer-aided design (CAD) and computer-aided manufacturing (CAM) systems are now standard engineering tools used in manufacturing companies. Although these tools allow generation of cutting tool path and visualization of machining, they cannot assure the quality of finished parts due to the lack of tolerance control capability. In this work, a methodology to address production tolerance assignment problem is proposed. A tolerance allocation model for machining discrete parts in multiple setups has been developed based on process capability analysis. It is appropriate for a plant where existing production equipment is used and historical process capability data is available. A process capability extrapolation technique based on tolerance normalization approach is presented to estimate capability in the absence of exact capability data.

The proposed method has several advantages over other traditional methods of tolerance allocation. The proposed method combines the problem of determining working dimensions and tolerances instead of solving them independently as done in the past. This will help generate more realistic setup plans, as not only the allocated tolerances but also the stock required for a machining operation is governed by capability of the process and the desired tolerances in the subsequent operations. Since the tolerances are allocated using process capability, the desired defect \textit{ppm} level can be achieved using existing production equipment. Moreover the proposed method can be used continuously to estimate the best possible working dimensions and tolerances based on any improvements or deterioration in capability of the process over time.

In future, this work could be extended to develop a method of selecting the production equipment (machines, tool, fixtures etc) based on the capability and blue print
tolerance requirements. The accuracies of all the available equipment can be specified in terms of appropriate probability distribution. These distributions can then be fed into the simulation model to generate a multi-dimensional response surface. The response surface will signify the tolerances capable to be achieved with various combinations of equipment errors. All the points on the response surface with achievable tolerance less then or equal to the blue print tolerance signify a possible solution. Various solutions can then be evaluated and the most optimum one selected.
References


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Appendix

Tolerance normalization

A normalized tolerance is an angle representing the maximum permissible rotation error when locating a component. It is calculated based on rigorous analysis of manufacturing errors involved in component setups. Following table shows how to calculate normalized tolerance for different datum-required geometric tolerance. The tolerance normalization concept can be applied to develop a methodology for modeling datum-required geometric tolerance process capability. Assuming that process capability has certain stability, i.e. the ability to setup and machine a part and hence the workpiece locating errors, tool errors and the machine errors remain same, then the expected parallelism tolerance for a given part can be estimated if the parallelism value achieved in a similar part of other dimensions is known by applying the concept of tolerance normalization.

<table>
<thead>
<tr>
<th>Tolerance Type</th>
<th>Toleranced Feature</th>
<th>Normalized Tolerance</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallelism</td>
<td>surface</td>
<td>$\sin^{-1}\left(\frac{\delta}{L}\right)$</td>
<td>$L$: length of the largest line segment whose two end points are on the surface boundary</td>
</tr>
<tr>
<td>Angularity</td>
<td>axis</td>
<td>$\tan^{-1}\left(\frac{\delta}{L}\right)$</td>
<td>$L$: length of the axis</td>
</tr>
<tr>
<td>Perpendicularity</td>
<td>all</td>
<td>$\tan^{-1}\left(\frac{\delta}{L}\right)$</td>
<td>$L$: height of the surface or length of the axis</td>
</tr>
<tr>
<td>Position</td>
<td>all</td>
<td>$\tan^{-1}\left(\frac{\delta}{L}\right)$</td>
<td>$L$: length of the axis or center line, or height of the center plane</td>
</tr>
<tr>
<td>Concentricity</td>
<td>all</td>
<td>$\tan^{-1}\left(\frac{\delta}{L}\right)$</td>
<td>$L$: length of the axis or center line, or height of the center plane</td>
</tr>
<tr>
<td>Symmetry</td>
<td>all</td>
<td>$\tan^{-1}\left(\frac{\delta}{L}\right)$</td>
<td>$L$: length of the axis or center line, or height of the center plane</td>
</tr>
<tr>
<td>Circular Runout</td>
<td>axis</td>
<td>$\sin^{-1}\left(\frac{\delta}{2L}\right)$</td>
<td>$L$: length of the axis</td>
</tr>
<tr>
<td>Total Runout</td>
<td>surface</td>
<td>$\sin^{-1}\left(\frac{\delta}{2R}\right)$</td>
<td>$R$: radius of the circular surface</td>
</tr>
</tbody>
</table>

Tolerance normalization quick reference table [16]
The process capability extrapolation approach is illustrated on both the prismatic and rotational parts. Parallelism and perpendicularity are selected as two example tolerances to illustrate the approach on prismatic workpiece. Total run out is selected for application of process capability extrapolation on cylindrical parts. As it can be seen from tolerance normalization table, these three tolerances are most representative of all the other remaining geometric tolerances. Specific applications for other tolerances can be developed based on the applications presented.

**Analytical Method**

**Parallelism**

Let the parallelism tolerance achieved in workpiece 1 with representative length $L_{r_1}$ be $\delta_1$. Then the deviating angle $\varphi_{//1}$ in setting the workpiece can be written as (Liu and Huang, 2000)

$$\varphi_{//1} = \sin^{-1}\left(\frac{\delta_1}{L_{r_1}}\right) \quad (1)$$

As discussed before, the deviating angle in setting up any similar workpiece will remain same. Thus for the workpiece 2 having the representative length $L_{r_2}$ (different from workpiece 1) and deviating angle $\varphi_{//2}$, expected parallelism tolerance can be calculated using the following equation:

$$\varphi_{//2} = \sin^{-1}\left(\frac{\delta_2}{L_{r_2}}\right) \quad (2)$$

Since $\varphi_{//2} = \varphi_{//1}$ from equation (1) and (2), we have

$$\sin^{-1}\left(\frac{\delta_1}{L_{r_1}}\right) = \sin^{-1}\left(\frac{\delta_2}{L_{r_2}}\right) \quad \text{Or}$$
\[ \delta_2 = a \cdot \delta_1 \quad \text{where} \ a \ \text{is a constant} \quad \text{(3)} \]

**Perpendicularity**

Consider the prismatic block in with a perpendicularity requirement between two surface \( S_0 \) and \( S_1 \). To facilitate the analysis, it is assumed that the surface \( S_1 \) is machined using a horizontal machining center with surface \( S_0 \) as a primary datum. As per tolerance normalization table, the maximum deviating angle in setting the work piece to achieve the required perpendicularity tolerance \( \delta \) can be written as:

\[ \phi_\perp = \tan^{-1} \left( \frac{\delta}{L} \right) \quad \text{------------------- (4)} \]

Using the same logical approach as in parallelism it can be shown that with the same errors in locating a workpiece, the perpendicularity tolerance achieved in machining a workpiece of given length is a function of length itself. In other words, perpendicularity \( \delta_1 \) achieved in a workpiece of length \( L_1 \) is directly proportional to perpendicularity \( \delta_2 \) achieved in workpiece with length \( L_2 \) i.e.

\[ \delta_2 = a \cdot \delta_1 \quad \text{where} \ a \ \text{is a constant} \quad \text{(5)} \]

**Circular run-out**

Previous two sections presented an analytic approach for extrapolating the parallelity and perpendicularity tolerances for a prismatic part. This section extends the same analytic approach to rotational parts. Run-out tolerance is used as an example to illustrate the approach. It is assumed that cylindrical part is held between centers and diameter is machined using a single point cutting tool on a turning center. As per tolerance
normalization table, the maximum deviating angle in setting the work piece to achieve the required total run-out $\delta$ can be written as:

$$\varphi = \sin^{-1}\left(\frac{\delta}{2L}\right) \quad \text{(6)}$$

Using the same logical approach as in earlier section it can be shown that with the same errors in locating a workpiece, the total run-out achieved in machining a workpiece of given length is a function of length itself. In other words, run-out $\delta_1$ achieved in a workpiece of length $L_1$ is directly proportional to run-out $\delta_2$ achieved in workpiece with length $L_2$ i.e.

$$\delta_2 = a \cdot \delta_1 \quad \text{where } a \text{ is a constant} \quad \text{(7)}$$

Equation (1), (4) and (7) were derived to account for only workpiece locating error in a given setup. The effect of other errors in machining a part, namely, tool wear, motion errors, vibration, thermal errors etc. can be accounted for if $\delta_1$ (achievable parallelism tolerance for workpiece 1) is derived from the process capability value for parallelism.

**Monte Carlo Simulation**

Monte Carlo Simulation is a stochastic technique to solve mathematical problems. It creates multiple scenarios of a problem by selecting random values of the involved input variables from a set of predefined probability distributions. Each set of random variables and the corresponding solution represents one scenario of the problem. This process when repeated several times provides probability distributions of the output variables. Monte Carlo Simulation is normally used when the given problem can’t be solved mathematically, is too complex to solve mathematically or involves lot of randomness.
difficult to predict. In past several years, MC Simulation has been increasingly used to solve various manufacturing problem. Huang and Zhang [17] used MC Simulation to evaluate the setup planning algorithm developed to generate setup plans based on tolerance analysis. Lin et al. [18] used statistical simulation method to optimize the tolerance allocation in an assembly with minimum cost. MC Simulation was used to simulate the dimension variance of each part and dimensional chain. An iterative method was then used to select the tolerances on individual components or sub-assemblies capable of meeting the overall tolerance on full assembly. Gerth and Hancock [19] did a similar work to improve the product specification by predicting the overall variation in the product due to the variations in the assembled parts. The statistical distributions of the individual parts were estimated from the available data like SPC, receiving inspection records, supplier control charts etc. Variation Simulation Analysis, a MC Simulation package was then used to predict assembly variations due to tolerance stacks. Shan, Roth and Wilson [20] used MC Simulation for 3D statistical tolerance analysis to test the probability of successful assemblies as per engineering specifications. Several other researchers [21, 22] used MC Simulation to generate experimental data to develop an objective function for mathematically solving the tolerancing problems. Even though the MC Simulation in the past has been mainly applied to predict assembly variations, the same concept can also be used to predict the variations resulting from a series of machining process if the variations from various error sources can be statistically modeled. The big factor in favor of MC Simulation is its randomness which is very typical of the errors present in a manufacturing setting. In this work, MC Simulation has been mainly used to validate the methodology of process capability extrapolation.
Validation using MC Simulation

Monte Carlo Simulation is used to verify equation 3, 5 and 7 i.e. the parallelism, perpendicularity and run-out tolerance achieved in a given workpiece is proportional to the parallelism, perpendicularity and run-out achieved in a similar workpiece of different representative length. Such a relationship, if established, could be effectively used for the purpose of process capability extrapolation. The objective is to try and find out a relationship between the parallelism, perpendicularity and run-out tolerances achieved in two parts when machined using the same equipment.

Parallelism

The two types of error sources considered are: Y-axis angular deviation and Z-axis linear error. The Y-axis angular deviation achieved while locating and machining the Part A is derived from process capability based on tolerance normalization and is assumed to be known. Following general procedure is used to virtually locate, machine and inspect the parts in order to obtain the parallelism tolerance achieved under various conditions:

1. The part is located using the bottom face as a primary datum. The bottom face is represented by generating a set of sample points from a specified distribution depending on the flatness error in the raw part.
2. The top of the part is then represented using another set of sample points randomly generated based on part height distribution and the distribution of flatness error on the top face.
3. The sample points representing the top and bottom face are then transformed to account for errors in locating and clamping the part in the fixture.

4. A fresh set of sample points is then generated to represent the machining of top face. The sample is generated based on stock removal and the machining error distribution in Z-axis.

5. The sample points representing top and bottom face are then transformed back by the angle considered in step 3, to represent the actual workpiece geometry.

6. The parallelism achieved between top and bottom face is then calculated using the computational metrology techniques.

It can be easily shown that the length of the part is the only factor influencing the parallelism tolerance if the Y-axis rotational error is the only source of locating error. Hence the simulation experiment is performed on the parts of various lengths for a given Y-axis rotational error. The experiment is repeated for a range of Y-axis rotational errors. The results from the simulation experiment are summarized in the following table. To verify the fact that there is indeed a linear relationship between the parallelism values achieved for different part lengths at a given rotational angle, a simple linear regression model of the form $Y = \beta_0 + \beta_1 X$ is fitted and the results evaluated. Various statistical tests are then performed to make inferences on the coefficients and check the validity of the linear regression models.
The table below presents a summary of the regression results:

<table>
<thead>
<tr>
<th>Rotation angle (θ)</th>
<th>Regression Coefficients</th>
<th>R - Square</th>
<th>t-test (p value)</th>
<th>F -test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>β₀</td>
<td>β₁</td>
<td>H₀: β₁ = 0</td>
<td>H₁: β₁ ≠ 0</td>
</tr>
<tr>
<td>0.05°</td>
<td>0.300692</td>
<td>0.000498</td>
<td>0.986022</td>
<td>1.17E-09</td>
</tr>
<tr>
<td>0.1°</td>
<td>0.290049</td>
<td>0.001198</td>
<td>0.995009</td>
<td>1.14E-11</td>
</tr>
<tr>
<td>0.2°</td>
<td>0.272685</td>
<td>0.002828</td>
<td>0.997970</td>
<td>1.98E-13</td>
</tr>
<tr>
<td>0.3°</td>
<td>0.260306</td>
<td>0.004499</td>
<td>0.997684</td>
<td>3.58E-13</td>
</tr>
<tr>
<td>0.4°</td>
<td>0.248843</td>
<td>0.006238</td>
<td>0.999407</td>
<td>7.74E-16</td>
</tr>
<tr>
<td>0.5°</td>
<td>0.247594</td>
<td>0.007876</td>
<td>0.999020</td>
<td>7.74E-15</td>
</tr>
</tbody>
</table>

F*=MSR/MSE, F=F(1-α; 1, n-2)

Relationship between parallelism tolerance and length

Looking into the R-Square values, t-test and F-test results it can be easily concluded that there is indeed a linear relationship between the parallelism tolerance and the length of
the workpiece at a given rotational error. In addition to the above statistical inferences, the key assumptions of a linear regression model were verified by examining various plots. There were no reasons found to doubt these assumptions. The plots for one of the regression models (?=0.05) are presented below as an example.

Thus the linear equation presented in section 4.2 is verified using MC Simulation. Such a linear relationship can be used as a powerful tool to predict the capability/achievable tolerance on a workpiece given the tolerance achieved on a similar workpiece(s) using
the same equipment. Parallelism tolerance was used as an example for illustration but the same approach can be used for any type to tolerance. Moreover only one type of error source was considered to keep the illustration simple but any type and number of error sources can me modeled and incorporated in the simulation program.

**Perpendicularity**

A similar approach as in case of parallelism is used to validate equation (5) developed for perpendicularity tolerance. It is easy to see that the two errors affecting the perpendicularity between surface $S_0$ and $S_f$ are Z-axis rotational and X-axis rotational. Z-axis rotational error is considered the primary source of error because of $L > H$. For the same magnitude of rotational error, error contribution due to Z-axis rotation will supercede the X-axis contribution. Therefore, to keep the simulation model simple X-axis rotational error is ignored. The tool repeatability error of 0.08mm is also incorporated in the model. The simulations results for various lengths of example part are summarized in the following table.

<table>
<thead>
<tr>
<th>Length (mm)</th>
<th>$\theta = 0.05^\circ$</th>
<th>$\theta = 0.1^\circ$</th>
<th>$\theta = 0.2^\circ$</th>
<th>$\theta = 0.3^\circ$</th>
<th>$\theta = 0.4^\circ$</th>
<th>$\theta = 0.5^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.1705</td>
<td>0.1856</td>
<td>0.2185</td>
<td>0.2522</td>
<td>0.2863</td>
<td>0.3211</td>
</tr>
<tr>
<td>30</td>
<td>0.1790</td>
<td>0.2028</td>
<td>0.2528</td>
<td>0.3047</td>
<td>0.3569</td>
<td>0.4099</td>
</tr>
<tr>
<td>40</td>
<td>0.1870</td>
<td>0.2200</td>
<td>0.2882</td>
<td>0.3558</td>
<td>0.4252</td>
<td>0.4947</td>
</tr>
<tr>
<td>50</td>
<td>0.1962</td>
<td>0.2387</td>
<td>0.3226</td>
<td>0.4095</td>
<td>0.4963</td>
<td>0.5835</td>
</tr>
<tr>
<td>60</td>
<td>0.2409</td>
<td>0.2553</td>
<td>0.3577</td>
<td>0.4616</td>
<td>0.5663</td>
<td>0.6701</td>
</tr>
<tr>
<td>70</td>
<td>0.2140</td>
<td>0.2734</td>
<td>0.3921</td>
<td>0.5143</td>
<td>0.6357</td>
<td>0.7573</td>
</tr>
<tr>
<td>80</td>
<td>0.2240</td>
<td>0.2905</td>
<td>0.4270</td>
<td>0.5662</td>
<td>0.7067</td>
<td>0.8446</td>
</tr>
<tr>
<td>90</td>
<td>0.2310</td>
<td>0.3079</td>
<td>0.4620</td>
<td>0.6189</td>
<td>0.7762</td>
<td>0.9331</td>
</tr>
</tbody>
</table>
As done in previous section, a regression model between length and perpendicularity is developed for each given rotational error based on data in table above. The regression results are summarized below.

Looking into the R-Square values, t-test and F-test results it can be easily concluded that there is indeed a linear relationship between the perpendicularity tolerance and the length of the workpiece at a given rotational error. In addition to the above statistical inferences, the key assumptions of a linear regression model were verified by examining various plots. There were no reasons found to doubt these assumptions.

### 4.3.1.3 Circular run-out

For the given method of machining, total circular run out on the cylindrical shaft is dependent on raw material variation, length L, offset between centers along Z axis.
leading to Y rotational errors, off set between centers in Y axis leading to Z rotational errors and tool repeatability errors. For simplification purpose it is assumed that off set between centers in Y-axis is close to zero, hence Z axis rotational errors are ignored. All other sources of errors are in built in the simulation program. The total run out achieved for various lengths of shaft is as given in table below.

<table>
<thead>
<tr>
<th>Length (mm)</th>
<th>Total run-out</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>θ =0.05°</td>
</tr>
<tr>
<td>20</td>
<td>0.067</td>
</tr>
<tr>
<td>30</td>
<td>0.0653</td>
</tr>
<tr>
<td>40</td>
<td>0.075</td>
</tr>
<tr>
<td>50</td>
<td>0.0752</td>
</tr>
<tr>
<td>60</td>
<td>0.0858</td>
</tr>
<tr>
<td>70</td>
<td>0.08</td>
</tr>
<tr>
<td>80</td>
<td>1.0129</td>
</tr>
<tr>
<td>90</td>
<td>0.0979</td>
</tr>
<tr>
<td>100</td>
<td>0.0844</td>
</tr>
<tr>
<td>110</td>
<td>0.0758</td>
</tr>
<tr>
<td>120</td>
<td>0.0821</td>
</tr>
</tbody>
</table>

Raw material diameter: 22 ± 0.2mm  Machined diameter: 22 ± 0.005mm

Run-out tolerance achieved in simulation experiment

As done in previous section, a regression model between length and total circular run-out is developed for each given rotational error based on data in table above. The regression results are summarized in the following table.
<table>
<thead>
<tr>
<th>Rotation angle (θ)</th>
<th>Regression Coefficients</th>
<th>R - Square</th>
<th>t-test (p value)</th>
<th>F-test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>β₀</td>
<td>β₁</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05°</td>
<td>0.065164</td>
<td>0.000216</td>
<td>0.347826</td>
<td>0.0561</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.7999</td>
</tr>
<tr>
<td>0.1°</td>
<td>0.06301</td>
<td>0.000407</td>
<td>0.729614</td>
<td>0.000816</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>24.2858</td>
</tr>
<tr>
<td>0.2°</td>
<td>0.050094</td>
<td>0.001239</td>
<td>0.948274</td>
<td>4.3E-7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>164.9937</td>
</tr>
<tr>
<td>0.3°</td>
<td>0.029589</td>
<td>0.002278</td>
<td>0.96839</td>
<td>4.658E-8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>275.719</td>
</tr>
<tr>
<td>0.4°</td>
<td>0.030897</td>
<td>0.003151</td>
<td>0.984421</td>
<td>1.91E-9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>568.7127</td>
</tr>
<tr>
<td>0.5°</td>
<td>0.037596</td>
<td>0.003878</td>
<td>0.989839</td>
<td>2.79 E-10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>876.7797</td>
</tr>
</tbody>
</table>

Table 4-7: Relationship between total run-out and length

Looking into the R-Square values, t-test and F-test results it can be easily concluded that there is indeed a linear relationship between the perpendicularity tolerance and the length of the workpiece at a given rotational error except for very low rotational error. This can be attributed to the fact that the variation in the shaft diameter in such a case is more then the variation caused by the rotational error. In addition to the above statistical inferences, the key assumptions of a linear regression model were verified by examining various plots. There were no reasons found to doubt these assumptions.