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STUDY OF FRICTION EFFECTS ON SYSTEM DYNAMICS USING LOW-ORDER LUMPED-PARAMETER MODELS

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**ABSTRACT**

Friction is one of the oldest problems that has been studied intensively for several hundred years by many researchers. The underlying reason for this is the complex nature of friction and its strong influence on the behavior of various physical systems. Vast amount of research work can be found in literature that primarily aims at understanding the underlying mechanism of these kinds of system behavior. This thesis work is a step in the same direction. It investigates the effect of variation of system parameters on the stick-slip response of the following three models that are widely used for modeling various physical systems with dry sliding friction:

- Self-excited friction oscillator without external excitation
- Friction oscillator with external excitation (Den Hartog’s model)
- Friction interface modeling using bilinear hysteresis elements

Simple, low order, lumped, macroslip mathematical models used in these studies do not accurately predict the actual system behavior. However these studies give insight into the similarities and differences between the three models and aid designers and researchers to select appropriate model for analyzing the behavior of actual physical system with dry friction.

A new approach based on continuous microslip model for analyzing the response of frictionally damped turbine blades is also discussed in this work with emphasis on the need to integrate the model based on this more accurate approach with existing industrial computational tools for the system response predictions.
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- $c$  System viscous damping coefficient
- $k$  System stiffness
- $k_d$  Shear stiffness of the asperities/ Friction damper spring stiffness
- $m$  Main mass of the system
- $x(t)$  Absolute displacement$^1$ of the main mass
- $x_o$  Characteristic length
- $\dot{x}$  Velocity of the main mass
- $\ddot{x}$  Acceleration of the main mass
- $\hat{x}$  Normalized absolute displacement$^1$ of the main mass
- $\dot{x}'$  Normalized velocity of the main mass
- $\ddot{x}''$  Normalized acceleration of the main mass
- $x_d(t)$  Displacement$^2$ of the friction damper element
- $\hat{x}_d$  Normalized absolute displacement$^2$ of the friction damper element
- $\dot{x}_d$  Velocity of friction damper element
- $t$  Time
- $E_{\text{damper}}$  Energy dissipated by the friction damper
- $E_{\text{friction}}$  Energy dissipated by the friction interface
- $E_{\text{viscous}}$  Energy dissipated by the viscous damper
- $F(t)$  External excitation force$^3$ applied on the system in the tangential direction
- $F_d$  Friction damper force
- $F_f$  Friction force
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<td>$F_0$</td>
<td>Amplitude of the external harmonic force$^3$</td>
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<td>$F_{d_{\text{max}}}$</td>
<td>Maximum friction damper force</td>
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<td>$F_{f_{\text{max}}}$</td>
<td>Maximum friction force</td>
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<td>$N(t)$</td>
<td>Normal load applied on the main mass</td>
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<td>$\bar{S}$</td>
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<td>$V_{\text{ref}}$</td>
<td>Reference velocity</td>
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<td>$V_{\text{rel}}$</td>
<td>Relative velocity</td>
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<td>$\gamma$</td>
<td>Force ratio</td>
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<td>$\delta$</td>
<td>Effective system damping parameter</td>
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<td>$\delta_{\text{st}}$</td>
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<td>$\zeta$</td>
<td>Damping ratio</td>
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<td>$\zeta_{\text{eq}}$</td>
<td>Equivalent damping ratio</td>
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<td>$\Phi$</td>
<td>Phase difference between harmonic forcing function and the main mass displacement</td>
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\( \Omega \)  
Frequency ratio

* Stick-slip response

• Pure-slip response

1. Displacement of main mass is taken from the unstrained position of the main system spring.

2. Displacement of friction damper element is taken from the original position of friction damper when both the springs i.e. the main system spring and the friction damper element spring, are completely unstrained.

3. External excitation force is applied in the direction tangent to the motion of the main mass.
CHAPTER 1

INTRODUCTION

1.1 Motivation

Friction occurs whenever there is relative motion between two surfaces in contact. Occurrence of friction in countless number of physical systems and its effect on their behavior has been the primary reason for its study for the past several hundred years. However, it was not until the later half of twentieth century that the friction-related research received great impetus. The main reason behind this development, which is also the principal reason for the importance of this work, is the recognition of the strong influence of friction on the system dynamics that is also closely related to the economic effects. The economic losses in the USA resulting from lack of friction and wear related design knowledge were estimated at 5% of the gross national product [11].

The presence of friction in power generating systems results in energy losses and in many machines the heat release due to friction can cause undesirable distortions. Also, it reduces the life of machinery due to the continuous wear of contacting parts during its operation. Friction related problems of industry are not only confined to the losses due to friction and wear, but also include friction-induced vibrations. Some of the very common problems of these kinds occurring in real physical systems are: stick-slip vibrations, chaotic response, brake squeal, machine tool chatter, motion disturbances caused by friction at mechanical joints and railway corrugation. Ibrahim [15] has presented an excellent review of the vast amount of literature available for each of these friction related problems. From the foregoing discussion it might appear that friction is always disadvantageous, but this is not true. In many engineering systems, friction is provided to achieve desired functioning and to
enhance the performance of the system. For example, brakes, clutches, driving wheels of cars, trains, etc all operate because of the existence of friction. The application of friction can widely vary from its usage in a simple nut-bolt arrangement to its usage as a damping mechanism for the blades of a turbomachinery.

The extensive theoretical and experimental friction related research across varied engineering disciplines led to the emergence of various friction models that are used to predict the system behavior. However, due to the wide variety of applications of friction, it is not simple to choose a particular friction model to predict the system response accurately. Moreover, developing an accurate friction model is also very difficult because of the complex nature of friction. Friction depends on several factors like the material and the geometry of surfaces in contact, surface roughness, sliding speed, temperature, normal load, etc [18, 21]. Besides its dependence on variety of parameters, it is highly nonlinear microscale phenomenon, random with respect to time and hard to investigate experimentally. Inspite of all the difficulties, till today the research continues to appropriately model friction and incorporate it in the system dynamic model that will promote better understanding of the various friction mechanisms and provide valuable guidelines for the designers to achieve satisfactory performance of the system.

1.1.1 Friction interface modeling

In general, two approaches are used to model the friction interface, namely:

- Low order, lumped parameter approach (Macroslip)
- High order, continuum approach (Microslip or Partial slip)
Consider a body on a rigid support as shown in Figure 1.1(a), with dry friction present at the contact interface. It is subjected to a uniform pressure $p$ per unit length and a force $F$ in the tangential direction.

![Figure 1.1](image-url)

(a) Body on rigid support

(b) Macroslip

(c) Microslip

Figure 1.1: Macro- and micro-slip
According to the Coulomb’s friction law, the body will start moving when the force $F$ exceeds the frictional force that is given by $\mu pL$ where $\mu$ is the coefficient of friction and $L$ is the length of the body. This is illustrated in Figure 1.1(b) where the gross slip is marked by $s$. On the other hand, there will be no motion if $F$ is less than the frictional force. The body does not deform under the application of the force i.e. in this model, it is assumed that the body is rigid and all the points at the interface experience identical motion. This is equivalent to considering a model that consists of a single point contact. This assumption is the basis of lumped parameter approach which is also known as the macroslip approach since the body is either considered to be completely slipping or is completely stuck. However, in practice the tangential force will produce strain within the body and the slip will not occur simultaneously throughout the contact area as depicted in Figure 1.1(c). The body deforms elastically resulting in partial slip, also known as microslip, marked by $\Delta s$ in Figure 1.1(c). Thus, the regions of interfacial slip are small compared to the size of contact area and hence this approach is also known as microslip approach. The slip zone gradually increases depending on the magnitude of the tangential force relative to the frictional resistance and if the force $F$ is large enough, the total slip (gross slip) of the body occurs. In this approach, both system and the interface are discretized to account for elastic deformation of the system and the microslip that occurs at the contact interface. This approach captures the mechanics of the friction interface more accurately, because of which it predicts the system response more precisely as compared to the macroslip approach. Menq et al. [19, 20] analyzed microslip in friction dampers used in aerospace turbine blades and presented a new model based on this approach and compared the theoretical predictions of system response with the experimental results. Berger et al. [4] formulated a new model based on microslip approach for the
accurate determination of system dynamic response under a variety of contact conditions. Berger and Begley [3] showed that the blade root stress in turbomachinery could be predicted more accurately by considering the microslip effects between the blade and the damper. Though, the microslip approach is more accurate, the simplified assumption of single point contact in macroslip approach, which results in higher computational efficiency, makes it more popular in the study of complicated friction related problems.

Both the macro- and micro-slip approaches have their own advantages and the choice to use a particular approach for system modeling depends on what we are interested in and the severity of contact conditions. For example, experiments have shown that partial slip occurs at high normal loads [19]. So if the system response is to be predicted accurately the microslip approach should be used. However, to study only the qualitative behavior of the system response, macroslip approach can be used to model the friction interface. Moreover, if the normal loads are less, macroslip approach also predicts the system response with high accuracy.

1.1.2 Objective of this study

Many researchers have modeled variety of friction related problems using various friction models, following different approaches for modeling the friction interface. The primary objective of these researches is to understand the close inter-relationship that exists between friction and system dynamics and also to determine the appropriate system model (dynamic + friction model) for a particular problem that can predict the response of the system, which agrees with the experimental results. This thesis work is also an effort to attain the same objective.
This work investigates the qualitative behavior of three different discrete system models in the presence of friction. Each model represents a class of friction related problems. The effect of dry sliding friction on the system dynamics is explored using simulations and the effect of various system parameters on its steady state response is studied. It is very important to note that throughout this study, we are interested only in the steady state response of the system i.e. the response of the system as time tends to infinity is considered. In order to attain steady state, the simulations have been done for sufficiently longer time durations.

The mathematical models used in this study are rather simple and they are usually not suitable for simulating the response of the actual physical system. But this study will help designers and researchers to gain insight into the interaction of system dynamics and friction and identify major system parameters that affect the system response. These models are discussed in section 1.4. Since a good understanding of friction is needed to analyze these systems, theories of friction are presented in section 1.2. Presence of friction in physical systems results in intermittent stick-slip motion, which is studied for each model in depth. The stick-slip phenomenon is briefly described in this chapter in section 1.3.

1.2 Theories of friction

1.2.1 Earliest theories

The first scientific approach to friction was postulated by Leonardo da Vinci (1452-1519). He recognized that the friction force is proportional to the applied normal load and independent of the nominal area of contact. However, it was almost two hundred years later that Amonton (1699) postulated these laws, independently of Leonardo da Vinci’s observations, and is usually credited with their discovery. Later, in 1780’s Coulomb studied...
the influence of materials of surfaces in contact, surface area, normal load and the duration of contact, on friction. He recognized the concept of limiting static friction: that forces applied to a static body will not cause the body to slide unless they exceeded this limit, which is greater than the coefficient of kinetic friction. These, and the observation that frictional force is nearly independent of the sliding speed, are generally known as Coulomb’s friction laws.

Though it was known for a long time that condition of surfaces in contact affects friction, but a formal theory was given by Bowden and Tabor in 1939 according to which interaction of asperities, which may exist on micro- or nano-scale, gives rise to the frictional resistance. Based on asperity interaction theory, friction can be considered to have three components:

- Adhesive
- Roughness
- Plowing

An excellent description of this can be found in the review article by Oden and Martins [21] and is discussed here in brief. The adhesive component arises due to the adhesive forces between the two surfaces in contact. The shear strength of the adhesive bond determines the force of friction. Therefore the friction force is proportional to the real area of contact. The relative motion between the two surfaces occurs only when external forces are sufficient to break these adhesive bonds. The second component based on the roughness theory originates due to the interlocking of asperities, which offers resistance to motion. When the relative motion occurs the asperities move past each other as shown in Figure 1.2. The third component of friction results from the digging in of a harder asperity into and through an asperity on the softer opposing surface as shown in Figure 1.3. Hence this is known as the
plowing component. It is not necessary that all the three components will contribute equally to the force of friction. Depending on the parameters like surface roughness, material properties etc., one or more of the three components may be negligible. Expression for coefficient of friction, considering all three components is given by:

\[
\mu = \frac{S_y}{H} \left( \frac{\tan \theta}{\text{roughness}} + \frac{\tan \alpha}{\text{plowing}} \right)
\]

where

\(S_y\) and \(H\) are the yield strengths and hardness of the weaker material respectively. \(\theta\) is the roughness angle and \(\alpha\) is the conical asperity angle.

Figure 1.2: Roughness component of friction

Figure 1.3: Plowing component of friction
A detailed account of these theories and several other developments from the pre-historic times that led to the emergence of the science of tribology can be found in the text by Dowson [10]. Recently Feeny et al. [11] presented an interesting historical review containing 304 references on dry friction. The text by Rabinowicz [23] is also a good reference for understanding the basic theories of friction.

1.2.2 Contemporary theories and friction models

The earliest theories of friction took into account the effect of surface geometry, real area of contact, adhesion of the contacting bodies and the material hardness. However, many researchers have shown experimentally that friction not only depends on the factors mentioned above, but is also influenced by many more parameters like relative velocity, kinematic state, deformation of surface asperities, local contact temperature, lubrication, history and duration of contact and system dynamics, etc. A detailed account of some of the theories that describe the effect of each parameter on friction is presented in the reviews by Oden and Martins [18, 21], Ibrahim [14] and Berger [1]. Also, large number of references for parameter dependence of friction are given in the historical review of dry friction by Fenny et al. [11].

Tolstoi [28] was the first to observe microscale vibrations in the normal direction accompanying frictional sliding. He concluded that the difference between static and kinetic coefficients of friction is because of these normal vibrations. The observation and ideas presented by Tolstoi led Oden and Martins [21] to a new approach in the analysis of dynamic friction. They considered a relative simple constitutive model of the interface, with a power law normal response and the coefficient of friction independent of velocity. This model was combined with an analysis of motion to give an apparent kinetic coefficient of friction. In the
numerical studies also they obtained a good qualitative modeling of general experimental observations. They also introduced the idea of angular contact degree of freedom. As the friction force acts only on one side of the component, a net twisting moment acts on the body and an angular deflection results. This angular-normal coupling has been investigated and found to play a significant role in contact dynamics.

Canudas de Wit et al. [7] presented a bristle model called “LuGre” model for controls applications. This model captures a variety of behaviors observed in experiments, from velocity and acceleration dependence of sliding friction to hysteresis effects and the pre-slip displacement. This friction force is given by:

\[
F_r = \left( \sigma_o z + \sigma_1 \frac{dz}{dt} + \sigma_2 V_{rel} \right) F_n
\]  

(1.2)

where \( \sigma_o \) is the characteristic bristle stiffness, \( \sigma_1 \) is the damping parameter, \( \sigma_2 \) is the viscous damping coefficient, \( z \) is the average bristle deflection, \( V_{rel} \) is the relative velocity and \( F_n \) is the contact normal force. Though, this model can predict the system response accurately when coupled with an appropriate dynamic model, but determination of six parameters makes it very complex because of which it is used only in limited applications. Rice and Ruina [26] put forth a new rate- and state- dependent friction law that was later used by Gu et al. [13] and Rice [25] for very large scale geomechanics simulations.

The literature cited above focuses on the unforced problems, however enormous amount of work has been done to predict the forced response of systems in the presence of friction. The friction in these problems often acts as a damping mechanism to control the system response and enhance the performance of the system. Ferri [12] has presented a review of literature related to the modeling of friction as a damping mechanism for wide variety of applications. One of the main applications where friction is used as a passive
1. Introduction

damping mechanism is in the control of the forced response of the blades of turbomachinery. Berger [2] predicted the blade response by modeling friction using bilinear hysteresis elements that were applied to modeling elasto-plastic material behavior by Iwan [16]. These models are based on the macroslip approach, which has been discussed briefly in the previous section. However, experimental observations of occurrence of partial slipping at the contact interface led to the development of friction models that captures this complex microslip behavior. This approach is again discussed in brief in the previous section. Menq et al. [19, 20] developed a new microslip model to accurately predict the response of turbine blades.

The foregoing discussion on friction theories and models examines a very small percentage of the vast amount of literature available on friction. The primary objective of this discussion is to present the research efforts related to friction across varied disciplines and to realize the fact that it is very difficult to develop a unique model for friction that can be used along with the dynamic model to predict the system response accurately. Further discussion on friction related research is given in several review articles. Oden and Martins [18, 21] have critically reviewed a large body of experimental and theoretical literature on friction and interpreted as a basis for models of dynamic friction phenomena. Ibrahim [14, 15] in his two parts review article, has presented a thorough literature survey related to the parameter dependence of friction and mechanics of contact. He also has described some important nonlinear dynamic problems like friction-induced vibrations, squeal, chatter and chaos along with their modeling techniques. Berger [1] has reviewed literature on friction across various engineering disciplines that gives an insight into the available friction modeling tools and techniques.
Inspite of the fact that friction depends on so many different parameters, some of which are listed above, only the velocity dependence of friction has been given the most consideration in the literature related to the problem of vibrations in systems with friction. In this study also, only the dependence of friction on relative velocity of surfaces in contact is considered. Moreover, only the sliding motion of surfaces without any lubrication is considered.

1.2.3 Velocity dependence of friction

The coefficient of kinetic friction depends on the relative velocity of sliding. Rabinowicz [23] observed negative slope of friction-velocity characteristic indicating that kinetic friction decreases with increase in velocity. Thereafter, various models have been developed by researchers to study the effect of this behavior on system dynamics. In this study the rate- and state- dependent friction model developed by Rice and Ruina [26] is used. According to this model the friction-velocity dependence can be represented mathematically by the following expression:

\[ \mu = \mu_o + \mu_1 e^{-\alpha V_{\text{rel}}} \]  

(1.3)

where \( \mu_o, \mu_1 \) and \( \alpha \) are constants determined by experiments. \( V_{\text{rel}} \) is the relative sliding velocity. At zero sliding velocity the friction is multivalued. Its value changes with time. The maximum value of friction at zero velocity is called the coefficient of static friction and is higher than the kinetic coefficient of friction. In this study the maximum value of coefficient of kinetic friction is assumed to be equal to the coefficient of static friction.

It is observed that the relative sliding motion of two bodies in contact without lubrication is not continuous i.e. they exhibit intermittent stick-slip motion. In this type of
motion, it is observed that the accelerating and decelerating branches of friction-velocity characteristic are distinct and their slope and separation not only depends on the material properties of the surfaces in contact, but also on the dynamics of the system [21]. As a result, the system response predicted by considering friction model represented by equation (1.3) will obviously deviate from the experimental results. However, using a friction model that captures this behavior and incorporating it in the system model will further add to the complexities of modeling the problems that exhibits so complicated behavior. Moreover, in this study we are more concerned in exploring the effect of friction on system dynamics rather than studying the reverse effect. So in this study the velocity-friction characteristic is assumed to be reversible i.e. it is assumed that only one branch of friction-velocity curve exists which is given by equation (1.3).

1.3 Stick-slip oscillations

Non-uniform sliding of two parts in contact is generally observed in the presence of dry friction which is known as stick-slip motion. There are many examples of this behavior which can both be observed in everyday life such as squealing of brakes, creaking of hinges, jerkiness which may occur when large mass is driven at slow speed along slideways and also in very complicated mechanical systems like turbomachinery components. Earliest research of stick-slip oscillations was done by Den Hartog [8] who developed an exact analytical solution for a harmonically excited single degree of freedom system undergoing stick-slip motion. Blok [5] linearized the friction velocity curve to predict a critical damping value to suppress stick-slip oscillations. Derjaguin et al. [9] developed an expression for the critical speed at which stick-slip oscillations can occur between sliding surfaces. Brockley et al. [6] derived a critical velocity expression for the suppression of stick-slip vibrations.
The stick-slip response of the system plays a crucial role in providing energy dissipation because of which friction is used as a damping mechanism in various physical systems to control the structural response. However, it is only during slipping, that the friction interface dissipates energy. So it is very important to accurately model the contact interface to predict the energy dissipation precisely, and hence the response of the system. As discussed in the previous sections, microslip approach accurately captures the mechanics of friction interface by considering its partial slipping. Menq et al. [19, 20] have made significant contributions to the development of models based on this approach, which are used to determine the exact response of the system. Till today, many researchers are focusing on developing dynamic and friction models that can account for this complicated phenomena so that the response of the system can be predicted accurately.

1.4 Preview of models

This thesis work primarily investigates the influence of friction on the dynamics of three different system models, each of which represents a class of friction related problems. These three models shown in Figure 1.4 are introduced in this section. The friction related problems represented by these models can broadly be categorized as:

- Self-excited problems
- Forced response problems

The self excited problems like machine tool chatter, brake squeal, aeroplane wing flutter etc. can be represented using model shown in Figure 1.4 (a). In this model an oscillator (m-c-k system) comes into contact with an energy source and dry friction present at the contact interface controls the energy flow into the vibrating system. The vibration amplitude increases if the energy flow into the system is greater than the dissipation within it. If the
dissipation is greater than the energy flow in, then vibration amplitude decreases. Periodic motion occurs when the energy flow in is equal to the dissipation. In these problems, the friction force that governs the response of the system, itself depends on the motion of the mass.

Figure 1.4: Reduced-order lumped-prameter models

(a) Model for self excited problem

(b) Den Hartog’s model

(c) Bilinear hysteresis model with massless damper

Figure 1.4: Reduced-order lumped-prameter models
In the forced response problems, the motion of the mass is governed by an external excitation force. The first model studied in this category is shown in Figure 1.4 (b) and is commonly referred in the literature as Den Hartog’s model. This model can be used to predict the energy dissipation at the interface of machinery components subject to some external excitation. Another model shown in Figure 1.4 (c) uses the bilinear hysteresis element to model the elastic deformation of asperities at the friction interface. This model is widely used in the forced response predictions of blades of turbomachinery. All these models are discussed in detail in the subsequent chapters with a review of the literature available pertaining to each of the three modeling techniques.

These reduced-order lumped-parameter models are simple and computationally efficient, because of which many researchers have used these to model various physical systems to investigate their dynamic response in the presence of friction.

1.5 Organization of thesis

This document is organized in five chapters. This chapter serves as an introduction to the thesis work. The next three chapters focus on the influence of friction on the steady state response of physical systems represented by three different models discussed in the previous section. In each of these three chapters the numerical results of simulations are presented in the form of figures that gives an insight into the qualitative behavior of the system response. The quantities of engineering interest like energy dissipation and percent sticking are also calculated and presented in the course of discussion.

Chapter two deals with the problem of self-excited vibrations. In chapter three, stick-slip vibrations arising in dynamic systems in the presence of external excitation are discussed in detail. Chapter four introduces a different methodology of modeling friction interface
1. Introduction

considering the elastic deformation of asperities of the surfaces in contact. Bilinear hysteresis element based on this methodology is used to study the forced response of the system in this chapter. These elements can be coupled with any system dynamic model to predict its response, but this chapter focuses on the prediction of forced response of blades of turbomachinery.

The similarities and dissimilarities between three modeling techniques are presented in chapter five which serve as the guidelines for the selection of an appropriate model to predict the response of physical system. At the end of this chapter a new approach based on continuous microslip model is discussed briefly and certain areas of future work are recommended.
CHAPTER 2
SELF-EXCITED VIBRATIONS

2.1 Introduction

When there occurs relative sliding motion between two surfaces, self-excited vibrations can often be observed. The force that sustains the motion is governed by some part of the motion itself and is attributed to the friction forces between the surfaces. The presence of these types of vibrations can be highly detrimental to the mechanical systems. For example, in machining processes the friction between the machine tool slide and its way results in vibrations that affect the quality of the product significantly. The systems that exhibit this type of response can be modeled using reduced-order lumped-parameter model shown in Figure 1.4 (a) in the previous chapter.

Two general regimes of self-excited vibrations can be identified (shown later in the phase planes of Figure 2.3), namely:

- Stick-slip or quasi-harmonic regime, and
- Pure slip regime

Stick-slip regime (Figure 2.3(a)) is characterized by sawtooth displacement-time evolution as illustrated later in Figure 2.4, which has clearly defined regions of stick and slip. This regime is also called quasi-harmonic regime, as the displacement during slip is nearly sinusoidal, whereas during stick it is linear. There is intermittent sticking and slipping motion between the two surfaces in contact. The motion is governed by static friction force in the stick region and velocity dependent kinetic friction force in the region of slip. In the pure slip regime, the motion throughout is maintained in the slip phase and the response of the system decays (Figure 2.3(b)) with time.
2. Self-Excited Vibrations

2.2 Early research results

Many researchers have studied self-excited vibrations following both experimental and theoretical approaches. Blok [5] linearized the friction-velocity curve and obtained analytical solutions. His work suggested that a sufficient increase in damping could eliminate the stick-slip phenomenon. He concluded that the essential condition for the occurrence of stick-slip motion is a decrease in the frictional force with increasing slipping speeds. Derjaguin, et al. [9] presented an extensive treatment of stick-slip vibrations assuming linear friction velocity relation and derived an expression for the critical speed at which stick-slip can occur between sliding surfaces. Brockley, et al. [6] analyzed the stick-slip motion using phase plane method and derived an expression for the critical velocity that limits the incidence of vibrations. This critical velocity depends on damping, normal load, stiffness and friction characteristics of the surfaces in contact. Oden and Martins [21] described some fundamental experimental observations on stick-slip motion and most common interpretations for the phenomenon. They concluded that the theory of stick-slip motion based on the assumption of simplified velocity dependent friction will always deviate from the experimental results due to the complex nature of friction. Berger [1] presented steady sliding stability calculations and stability maps for single degree of freedom structural model with velocity dependent friction and time dependent normal force. He also examined stick-slip motion from quantitative standpoint. Only a small amount of literature has been cited in this discussion although the body of literature of self-excited vibrations is very large. The interested reader may consult the review article by Oden and Martins [21] and Ibrahim [14, 15] for discussion and details of earlier research on self excited vibrations.
2.3 Present study

2.3.1 Single degree of freedom model

The single degree of freedom (SDOF) model considered for this study is shown in Figure 2.1. It consists of a slider of mass $m$ with linear stiffness and viscous damping elements and loaded with normal force $N(t)$ acting on it to impress it against a lower surface moving with constant velocity $V_{\text{ref}}$. The absolute displacement $x(t)$ of the mass is measured from the unstrained position of the stiffness element.

![Figure 2.1: SDOF model for the study of self-excited vibrations](image)

2.3.2 Objective

The objective of this study is to understand the effect of variation of system parameters ($m$, $c$, $k$, $N$, $V_{\text{ref}}$) on the response of the system shown in Figure 2.1. It is quite complex to study this effect in the parameter space of five variables and moreover it is difficult to physically interpret the effect of variation of each individual parameter. To reduce the number of parameters and to provide physical context to the model and the equations, normalization approach is followed as discussed in section 2.3.3. The resulting normalized non-linear equations of motion are solved with high accuracy using direct time integration described in...
section 2.3.4. In this study friction is considered to be dependent on velocity and is assumed to be independent of the time of stationary contact. Also, this study is confined to the systems where normal force \( N \) is constant i.e. \( N \) is independent of time. Moreover, the normal vibrations of the system during sliding are not considered, which implies that there is no loss of contact.

### 2.3.3 Governing equations

The nonlinear equation of motion for the SDOF system is given by:

\[
m\ddot{x} + c\dot{x} + kx = F_f
\]

(2.1)

The frictional force \( F_f \) is represented by the following mathematical model that is discontinuous with respect to the relative slip velocity \( (V_{rel}) \):

\[
F_f = \begin{cases} 
\pm \mu N & : \text{slip (} V_{rel} \neq 0) \\
\leq \mu_{\text{max}} N & : \text{stick (} V_{rel} = 0) 
\end{cases}
\]

(2.2)

where \( \mu \) is the kinetic friction coefficient that decreases exponentially with relative velocity and is expressed as:

\[
\mu = \mu_o + \mu_i \ e^{-d |V_{rel}|}
\]

(2.3)

where \( \mu_o, \mu_i \) and \( d \) are constants. If \( V_{rel} = 0 \),

\[
\mu = \mu_{\text{max}} = \mu_o + \mu_i
\]

(2.4)

The constant \( \mu_o \) is the lower bound of the kinetic friction coefficient i.e. \( \mu = \mu_o \) at very large relative velocities and \( \mu_{\text{max}} \) is the upper bound i.e. \( \mu = \mu_{\text{max}} \) as relative velocity tends to become equal to zero. The constant \( d \) can be regarded as the decay rate of the kinetic coefficient with the relative slipping velocity. The actual values of \( \mu_o, \mu_i \) and \( d \) depend on the conditions of the friction surfaces in contact and can be determined only by experiments.
This velocity dependent friction model is a specific form of rate- and state-dependent friction described by Rice and Ruina [26]. In this study the values of $\mu_o$, $\mu_i$ and $d$ are considered to be 0.10, 0.02 and 0.50 respectively. During sticking ($V_{rel} = 0$) the friction force is multi-valued and its magnitude and sign are determined by force equilibrium considerations. Velocity dependence of friction force is shown in Figure 2.2.

Before integrating, the equation of motion is normalized using the following scaling parameters:

$$\hat{x} = \frac{x}{x_o}$$  \hspace{1cm} (2.5)

$$\tau = \omega_n t$$ \hspace{1cm} (2.6)

$$\omega_n = \sqrt{\frac{k}{m}}$$ \hspace{1cm} (2.7)

where $x_o$ is a characteristic length and $\omega_n$ is the natural frequency of the system. This scaling scheme results in the following slipping equation of motion:

$$\dddot{x} + \zeta \dot{x} + x = \pm \mu \hat{N}$$ \hspace{1cm} (2.8)

where

$$(...) = \frac{d (...)}{d\tau}$$ \hspace{1cm} (2.9)

$$\hat{N} = \frac{N}{m\omega_n^2 x_o}$$ \hspace{1cm} (2.10)

$$\zeta = \frac{c}{\sqrt{km}}$$ \hspace{1cm} (2.11)

This normalization approach reduces the mass and stiffness terms to unity, so the natural frequency of this system is equal to one. The relative velocity between the two surfaces in contact is given by:
During sticking ($V_{rel} = 0$), velocity of the system mass equals the reference velocity. The sticking equations of motion can be written as:

\[ \dot{x}' = \dot{V}_{ref} \]  
\[ \ddot{x}'' = 0 \]

and the sticking friction is calculated as:

\[ \mu = \frac{\zeta \dot{x}' + \dot{x}}{\dot{N}} \]
This scheme of normalization reduces the parameter space from five variables (m, c, k, N, \( V_{\text{ref}} \)) to three variables defined by equations (2.10), (2.11) and (2.13).

The transition from one kinematic state to other in stick-slip oscillation is clearly described in Table 2.1. The transition from stick to slip occurs when the sum of all other forces acting on the system mass exceeds the friction force acting in the opposite direction. When the velocity of the system mass becomes zero and the resultant of all the other forces is less than the friction force then the transition from slip to stick takes place.

<table>
<thead>
<tr>
<th>Transition criteria</th>
<th>Slip to Stick</th>
<th>Stick to Slip</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{\text{ref}} = 0 )</td>
<td>( \zeta V_{\text{ref}} + \dot{x} &lt; \mu_{\text{max}} \hat{N} )</td>
<td>( \zeta \dot{V}<em>{\text{ref}} + \ddot{x} \geq \mu</em>{\text{max}} \hat{N} )</td>
</tr>
</tbody>
</table>

2.3.4 Direct time integration

The second-order ordinary differential equation of motion formulated in the previous section is integrated numerically using C++ code based on an efficient algorithm developed by Berger et al [4]. The integration is done using fifth order Runge Kutta method and is started with the following initial conditions:

\[
\begin{align*}
\text{At } \tau = 0, \\
\dot{x} = \dot{x}' = 0; \text{kinematic state } &= \text{slip}
\end{align*}
\]

After each time step the state vector (state vector stores the information of the kinematic state at each time step) is inspected on a possible change of kinematic state within this timestep (e.g. stick-to-slip transition). If there is a change in state in a given timestep then the integration process is stopped and an iteration procedure, based on golden section search method is started, to find the transition time step with predefined tolerance limits of time and velocity. After the transition point is determined the integration process is restarted for the
new ordinary differential equation (equation of motion during sticking or sliding phase) with initial conditions as the ones determined at the point of transition.

2.3.5 Previous experimental observations

In this section some of the results of later analyses are anticipated to give a qualitative appreciation of the problem.

Effect of reference velocity

Many experimenters have observed that oscillations are usually worst at low driving speed. If reference velocity is high then the oscillation decays [6, 23]. If the velocity is low, the oscillations become more severe and sawtoothed. The lowest reference velocity for which the oscillation decays is often called the critical speed.

Oscillation Amplitude

In the stick-slip oscillations the displacement-time curves are not symmetrical about the unstrained spring element position as shown in Figure 2.3. The amplitude of stick-slip oscillations depends on the normal force and the stiffness of the system [23]. It also depends on the ratio of the driving velocity and the natural frequency of the undamped system [6, 23].

Oscillation frequency

The stick-slip oscillation frequency in general, is similar to, but not the same as damped natural frequency of the system. For low viscous damping it is nearly equal to the natural frequency of the system, which in this case is equal to one.

2.3.6 Results

In this section results of numerical integration of the equations of motion are presented in the form of plots showing the effect of various system parameters on the steady state response. The stick-slip response of the system is indicated by asterisk mark in the plots and pure slip
response \( \dot{x}' = 0 \) is marked by a dot. Figure 2.3 illustrates the stick-slip and pure slip regimes that characterizes the system response, in the form of phase plane plots. Figure 2.4 depicts the typical stick-slip response of the system. Various terms related to the system response, used in this discussion are shown in Figure 2.5.

**Description of the motion**

Before the analysis, the main features of the stick-slip motion are discussed, starting from a condition where the mass is at rest, the spring is unstrained and the driving surface commences to move with constant velocity \( V_{\text{ref}} \). The mass moves in the direction of motion of the belt due to the friction at the interface of the two surfaces in contact. The frictional force also increases with the increase in its velocity due to the negative slope of the friction-velocity curve. The mass continues to accelerate until the restoring force of the spring and viscous damping force reaches a value equal to the maximum friction force. At this point the mass sticks \( (V_{\text{rel}} = 0) \) until eventually the spring force and viscous force exceeds the maximum friction force. The mass decelerates and subsequently its velocity falls to zero after which there is a reversal in the direction of its motion. The mass moves in a direction opposite to the direction of motion of the driving surface until its velocity again becomes zero and the system is then in the same state as at the beginning of the first slip phase. This cycle is repeated till the driving surface is in motion.

From the energy standpoint, during sticking the energy of the system increases and is stored in the form of potential energy in the spring by virtue of its stretching. During slipping the energy of the system is dissipated through the viscous damper and slipping friction interface. Therefore the stick-slip limit cycle is a closed trajectory on the phase plane, which balances energy input during sticking portion and energy dissipation during slipping portion.
2. Self-Excited Vibrations

Figure 2.3: Regimes of self-excited vibrations

Figure 2.4: Evolution of system response with time
2. Self-Excited Vibrations

**System Stability**

The stability of the SDOF system considered for this study can be determined analytically by linearizing the equation of motion around the steady sliding equilibrium position \((\dot{x} = 0)\).

The linearized equation of motion for this system is:

\[
\ddot{x}^* + \zeta \dot{x}^* + x = (\mu^* - \bar{S}\dot{x}^*)N
\]

where the friction-velocity curve slope through the steady equilibrium position is shown in Figure 2.6 and is given by:

\[
\bar{S} = \left. \frac{\partial \mu}{\partial V_{rel}} \right|_{V_{rel} = \dot{V}_{ref}} \tag{2.19}
\]

and

\[
\mu^* = \mu \bigg|_{V_{rel} = \dot{V}_{ref}} \tag{2.20}
\]
Figure 2.6: Effect of reference velocity on the friction-velocity curve slope and coefficient of friction at the steady sliding equilibrium position

Equation (2.18) can be re-written as the following homogeneous equation:

\[ \ddot{y}^* + \delta \dot{y}^* + \dot{y} = 0 \]  

(2.21)

where

\[ \dot{y} = \dot{x} - \mu^* \dot{N} \]  

(2.22)

and \( \delta \) is the effective system damping parameter and is given by,

\[ \delta = \zeta + S \dot{N} \]  

(2.23)

A linear system is dynamically stable if, when disturbed from its equilibrium position it comes back there after the transients die out. In case any disturbance causes the amplitude to build with time, the system is said to be dynamically unstable. Effectively, the system becomes unstable when negative damping appears in its differential equation of motion. A
more general definition of stability is that the roots of the characteristic equation of the system should either be negative real numbers or complex numbers with negative real parts. Therefore, the system is stable only when $\delta \geq 0$

$$\Rightarrow \quad \overline{S} \leq \frac{\zeta}{N} \quad \text{(Stable Response)} \quad (2.24)$$

and the system is unstable if $\delta < 0$.

$$\Rightarrow \quad \overline{S} > \frac{\zeta}{N} \quad \text{(Unstable Response)} \quad (2.25)$$

It is clear from equations (2.24) and (2.25) that the stability of the system depends on the slope of the friction velocity curve. A sufficiently high negative slope of this curve destabilizes the system resulting in stick-slip oscillations. This can be interpreted from energy standpoint also. When mass is stationary, the friction coefficient between mass and the driving surface is $\mu^*$. When mass is moving in the direction of the motion of the driving surface, the relative velocity decreases and the coefficient of kinetic friction increases. On the other hand when the mass moves in the opposite direction, the relative velocity increases and therefore the friction coefficient decreases. Since the friction force on the mass is always in the direction of motion of the driving surface, the helping friction force when mass moves in the direction of driving surface, is always greater than the opposing friction force when mass moves in the opposite direction. This means a certain net energy is put into the system in each cycle resulting in amplitude growth. Figure 2.7 shows the stability boundary in the parameter space based on the linearized equation of motion (equation (2.21)). The system stability is also investigated numerically and the results are shown in Figure 2.8 and Figure 2.9. All these three plots indicate that the system becomes stable with the increase in driving velocity and with the increase in viscous damping. Arrow marked ‘a’ in Figure 2.7 indicates...
that at high reference velocities the system response is stable even at very low viscous damping. However, at low reference velocities the system exhibits stick-slip oscillations at higher normal loads for large values of viscous damping present in the system as indicated by arrow marked ‘b’ in the Figure 2.7. The increase in $\dot{V}_{\text{ref}}$ shifts the steady sliding equilibrium position on the friction-velocity curve towards the right, thereby reducing the slope, as shown in Figure 2.6. This decrease in slope or increase in viscous damping increases the effective system damping, thus stabilizing the system. Also, arrow marked ‘b’ in the figure depicts that at high normal loads the system exhibits stick-slip response

Figure 2.7: Stability boundary in the parameter space
2. Self-Excited Vibrations

The effect of viscous damping on the system response is clearly shown in Figure 2.10. The plot indicates that for moderately small damping, stick-slip oscillations are sustained. However, with the increase in viscous damping the stick-slip oscillations can no longer be sustained and the trajectory tends towards the equilibrium position point in the phase plane. At this equilibrium position the mass is stationary ($\dot{x} = 0, \ddot{x} = 0$).
2. Self-Excited Vibrations

Figure 2.10: Shift of stick-slip regime to steady sliding regime with increase in viscous damping ($\dot{N} = 2, \dot{V}_{\text{ref}} = 0.10$)

**Steady sliding equilibrium position and offset of stick-slip oscillations**

The equilibrium position for the steady sliding response can be easily determined using equation (2.8) and is given by:

$$\dot{x} = \mu^* \dot{N}$$  \hspace{1cm} (2.26)

where $\mu^*$ is the coefficient of kinetic friction at $\dot{x}' = 0$. As discussed in section 2.3.5, there is an offset in the displacement-time curves from the unstrained spring position as shown in Figure 2.2. This offset is given by:

$$\text{Offset} = \frac{\text{Max. displacement} + \text{Min. displacement}}{2}$$  \hspace{1cm} (2.27)

The displacement of mass is maximum or minimum when its velocity is zero. Using this condition equation (2.8) can be written as,
Equation (2.28) can be re-written as a homogeneous equation in terms of parameter $\hat{y}$ given by equation (2.22).

$$\hat{y}'' + \hat{y} = 0$$  \hspace{1cm} (2.29)

Equation (2.29) has a standard harmonic solution given by,

$$\hat{y} = A \sin(\omega_n t + \phi)$$  \hspace{1cm} (2.30)

$$\Rightarrow \hat{x} = A \sin(\omega_n t + \phi) + \mu \hat{N}$$  \hspace{1cm} (2.31)

where $A$ is the amplitude of oscillations and $\phi$ is the phase angle.

It is clear from equation (2.31) that the maximum and minimum values of $\hat{x}$ occur at maxima and minima of sine function.

$$\therefore \hat{x}_{\text{max}} = A + \mu \hat{N}$$  \hspace{1cm} (2.32)

$$\hat{x}_{\text{min}} = -A + \mu \hat{N}$$  \hspace{1cm} (2.33)

The offset for stick-slip oscillations is determined using equations (2.27), (2.32) and (2.33). It can be easily calculated by simple substitution and the final expression for the offset is the same as equation (2.26). This result indicates that steady equilibrium position and offset of stick-slip oscillations depends on the friction force relative to the spring force in the system. This can be explained in physical context also. In both steady sliding and stick-slip motion, the spring stretches to attain force balance at the steady sliding state or the stick state. The amount of stretching is determined by frictional force and the stiffness of the system. Higher the frictional force compared to the stiffness, higher will be the offset. The friction force depends on the coefficient of friction and the normal force. If any of these two parameters increases the offset will increase. So at higher normal loads relative to the system
stiffness, the offset is more. Moreover, for low driving velocities the coefficient of friction is more as shown in Figure 2.6, again resulting in higher offset. This increase in offset of stick-slip response and increasing shift of equilibrium position for steady sliding response is shown in Figure 2.11. All the curves are linear and the slope of each curve is determined by equation (2.26). An interesting thing to note is that the offset and equilibrium position is independent of the damping present in the system.

Figure 2.11: Variation of steady sliding equilibrium position and offset of stick-slip oscillations with $\hat{N}$ and $\hat{V}_{\text{ref}}$. 
Amplitude of stick-slip oscillations

It is observed from Figure 2.12 that the amplitude of stick slip oscillations is significantly affected by the increase in driving velocity. The variation in normal load does not have a significant effect on the amplitude.

While increasing driving velocity decreases the offset slightly, the increase in approximate diameter of the trajectory in phase plane is of the same order as the increase in driving velocity. This is clearly shown in Figure 2.13 that depicts the increase in amplitude with increasing reference velocity.

Percentage sticking

The system stabilizes as the effective damping of the system increases. For low viscous damping, stick-slip vibrations are observed at low driving velocities.

Figure 2.12: Variation of stick-slip oscillations amplitude with \( \hat{N} \) and \( \hat{V}_{\text{ref}} \)
Figure 2.13: Effect of reference velocity on the offset and amplitude of stick-slip oscillations

The increase in driving velocity increases effective system damping (equation (2.23) and Figure 2.6), thereby reducing the amount of stick. At very high velocities the stick-slip phenomena is not observed, even for low damping. Also, for high normal loads the frictional resistance increases and additional potential energy is stored in the spring to overcome the friction force. The energy is added to the system only during sticking so at high normal loads the amount of sticking increases. This variation of amount of stick/cycle is shown in Figure 2.14. Percentage stick per cycle is defined as:

\[
\text{Percentage Stick/cycle} = \frac{\text{Total time of stick in one cycle}}{\text{Total cycle time}} \quad (2.34)
\]
2. Self-Excited Vibrations

The stick-slip oscillation frequency is similar to the damped natural frequency of the system. The expression for the oscillation frequency of the linearized system (equation (2.21)) is

\[ \omega = \omega_n \sqrt{1 - \delta^2} = \sqrt{1 - \delta^2} \quad (2.35) \]

where \( \delta \) is given by equation (2.23). In the real system damping is low, so the frequency of stick-slip oscillations is approximately equal to its natural frequency. The value of \( \delta \) for stick slip oscillations is negative so with the increase in viscous damping, \( \delta \) becomes more positive and its absolute value decreases. Also, with the increase in driving velocity the slope of friction-velocity curve reduces (Figure 2.6), again decreasing absolute value of \( \delta \). As the absolute value of \( \delta \) decreases or in other words the effective damping of the system increases,
the frequency of stick-slip oscillations approaches the natural frequency of the system as shown in Figure 2.15.

![Figure 2.15: Frequency of stick-slip response as a function of reference velocity and viscous damping.](image)

2.4 Concluding remarks

In this study of the self-excited vibrations various aspects of frictional sliding phenomena and interface behavior is presented. A non-dimensional form of the governing system of equations is derived and the governing non-dimensional parameters are identified. Linear stability analysis of the steady sliding equilibrium positions is presented and the regions of dynamic instability are then studied, assuming a simple velocity-dependent friction model.
The negative slope of friction-velocity curve is established to be the reason for the occurrence of stick-slip oscillations in a sliding system.

It is observed that the behavior of the system is not only influenced by the nature of the surfaces in contact, but also, by the dynamic characteristics (m, c, k) of the system. The amplitude of the stick-slip oscillations is found to increase with the increase in $\dot{V}_{\text{ref}}$ and $\dot{N}$. It is also observed that the frequency of stick-slip motion approaches its undamped natural frequency with the increase in the speed of driving surface.
CHAPTER 3

STICK-SLIP VIBRATIONS OF A SYSTEM WITH EXTERNAL EXCITATION

3.1 Introduction

This chapter is a continuation of the previous chapter where various aspects of self-excited vibrations resulting from velocity dependent friction characteristic are presented. An important characteristic of self-excited vibrations is that the fluctuating force that sustains the motion is controlled by some part of the motion itself. However, in many engineering systems, the sliding components may be subjected to external dynamic disturbances that may induce the stick-slip motion. In this chapter we are concerned with the dynamics of such stick-slip vibrations that occur in the presence of an external dynamic force, independent of the motion of the system. Usually these vibrations are detrimental to the system in terms of reduction of performance and service life and may sometimes endanger the equipment. But in some rotating machinery applications this stick-slip behavior may be desirable to provide a dissipation mechanism [22].

3.2 Past researches

Earliest research on stick-slip vibration with external excitation was done by Den Hartog [8]. In his work he obtained exact solution for symmetric steady state stick-slip motion of a single degree of freedom system in the presence of both viscous and Coulomb (i.e. dry friction) damping, with at most 2 stops per cycle. Pratt and Williams [22] investigated the stick-slip motion of a two-degree of freedom system by means of combined analytical and numerical procedure. In their work they obtained steady state solution for stick-slip motion with multiple lockups per cycle, which is usually observed at low excitation frequencies. Shaw [27] considered different static and kinetic coefficients of friction and found the exact
solution for the stick-slip motion for the problem considered earlier by Den Hartog [8]. He also used bifurcation theory to analyze the stability of the single degree of freedom system. It was shown that for positive viscous damping the non-sticking steady state solutions of the same period as the forcing function are nearly always stable, but for negative viscous damping such motions can become unstable. He also showed that the symmetric motions with two stops per cycle could be unstable. In all these researches, friction was considered to be independent of velocity. Ko, et al. [17] considered the velocity dependence of friction and studied the dynamics of friction-induced vibrations, both theoretically and experimentally. They showed that the application of transverse dynamic force of proper frequency and forcing amplitude could reduce the amplitude of friction-induced vibrations. Berger [1] and Ibrahim [15] have presented excellent reviews of the vast amount of literature available for the extensive work done by many researchers concerning the forced vibrations of systems with friction.

3.3 Present study

3.3.1 Single degree of freedom model

The simple mass-damper-spring single degree of freedom (SDOF) model considered for this study is illustrated in Figure 3.1. This model is similar to the one used for the study of self-excited problem. In this case the lower surface at the contact interface is considered to be stationary. Apart from a force \( N(t) \) applied in the normal direction, a force \( F(t) \) is applied to the sliding mass \( m \) in the transverse direction.
3. Stick-Slip Vibrations of a System with External Excitation

3.3.2 Objective

The objective of this study is to investigate the effect of various system parameters (m, c, k, N(t), F(t)) on the steady state response of the SDOF system shown in Figure 3.1. In addition, effect of system parameters on various quantities of engineering interest like percentage stick, energy dissipation, frequency shift and phase angle is also studied. A simple SDOF model with linear stiffness and damping elements is considered for this study, and it provides insight into the stick-slip motion arising in real physical systems.

The following assumptions have been made for the analysis of the problem:

- The coefficient of friction is considered to be velocity dependent.
- Normal force N(t) is assumed to be constant (i.e. independent of time).
- The force in the transverse direction is considered to be harmonic (i.e. of the from \( F_0 \sin(\omega t) \)).

3.3.3 Governing equations

The equation of motion for the SDOF system is nonlinear due to presence of friction nonlinearity and can be written as:
3. Stick-Slip Vibrations of a System with External Excitation

\[ m\ddot{x} + c\dot{x} + kx = F_0 \sin \omega t \pm F_f \]  \hspace{1cm} (3.1)

where

- \( F_0 \) is the amplitude of harmonic forcing function.
- \( \omega \) is the excitation frequency.
- \( F_f \) is the force of friction that is defined by the velocity-dependent discontinuous friction model discussed in the previous chapter (equations (2.2), (2.3) and (2.4)).

The first step before integrating the equation of motion using numerical tool, is to normalize the system so as to reduce the parameter space of variables in which the behavior of the system is studied. Equation (3.1) is normalized using the following scaling scheme:

\[ x_o = \frac{F_0}{k} \] \hspace{1cm} (3.2)
\[ \omega_n = \sqrt{\frac{k}{m}} \] \hspace{1cm} (3.3)
\[ \tau = \omega_n t \] \hspace{1cm} (3.4)

\[ \Rightarrow (...) = \frac{d (...)}{dt} = \frac{d (...)}{d\tau} = \omega_n \frac{d (...)}{d\tau} = \omega_n (...)' \] \hspace{1cm} (3.5)

where

- \( x_o \) is a characteristic length that may be defined as zero frequency deflection of the spring mass system under a steady force \( F_0 \) in the absence of friction.
- \( \omega_n \) is the natural frequency of the system.

Applying this scheme of scaling, equation (3.1) can be re-written as:

\[ \ddot{x}'' + 2\zeta \dot{x} + \dot{x} = \sin \Omega \tau \pm \frac{\mu}{\mu_{\text{max}}} \gamma \] \hspace{1cm} (3.6)

where
3. Stick-Slip Vibrations of a System with External Excitation

\[ \zeta = \frac{c}{2m\omega_n} \]  
(3.7)

\[ \Omega = \frac{\omega}{\omega_n} \]  
(3.8)

\[ \gamma = \frac{\mu_{\text{max}} N}{F_o} \]  
(3.9)

Equation (3.6) is the slipping equation of motion where \( \mu \) is determined from the exponential friction-velocity relation (equation (2.3)) and \( \mu_{\text{max}} \) is given by equation (2.4).

During stick, relative velocity of the system is zero and is given by equation (2.12) where

\[ \dot{V}_{ref} = 0 \]  
(3.10)

So the equations of motion during stick are:

\[ \dot{x} = 0 \]  
(3.11)

\[ \ddot{x} = 0 \]  
(3.12)

Using these conditions, equation (3.6) can be written as:

\[ \ddot{x} = \sin \Omega \tau \pm \frac{\mu}{\mu_{\text{max}}} \gamma \]  
(3.13)

An interesting thing to note is that the slipping equation (equation (3.6)) is a nonlinear second order differential equation, whereas equation (3.13) that describes sticking is an algebraic equation. The static friction force does not have a predetermined value and is determined by the force balance of equation (3.13). For solutions of stick-slip motion both equations (3.6) and (3.13) have to be considered. It is quite complicated to arrive at the solutions of these equations analytically. Therefore the solution for this kind of problem can be obtained by following numerical approach as discussed in section 2.3.4 of the previous chapter. The
criterion for transition from one kinematic state to other is described in Table 3.1 and is similar to the self-excited case (Table 2.1) as discussed in the previous chapter.

Table 3.1

<table>
<thead>
<tr>
<th>Transition Criteria</th>
<th>Equation</th>
</tr>
</thead>
</table>
| Slip to Stick         | \( \dot{x} = 0 \)  
                        | \( \dot{x} + \sin\Omega \tau < \frac{\mu}{\mu_{\text{max}}} \gamma \) |
| Stick to Slip         | \( \dot{x} + \sin\Omega \tau \geq \frac{\mu}{\mu_{\text{max}}} \gamma \) |

3.3.4 Results

The presence of dry friction at the contact interface of the system shown in Figure 3.1 yields a nonlinear response. This section explores this nonlinear response in the parameter space of three normalized parameters given by equations (3.7), (3.8) and (3.9). Later in the section the results of simulations are presented in the form of figures that give a better understanding and appreciation of the problem. In the discussion, the results are only presented for excitation frequencies greater than 0.3 because response of the system below this frequency is quite complicated as shown in Figure 3.2 and also pointed by Shaw [27] and Pratt and Williams [22].

In the figures, the stick-slip response of the system is indicated by asterisk and pure slip response is marked by a dot.

Description of motion

The system subjected to external excitation can either exhibit pure slip response or can have intermittent sticking and slipping at the contact interface. It is also possible that the mass remains stationary and this occurs only when the maximum frictional force is greater than or equal to the amplitude of the forcing function i.e. only when force ratio \( \gamma \geq 1 \). So for this
discussion only force ratios less than one are considered. The regime of motion (pure slip or stick-slip) for force ratios less than one is governed by the three normalized parameters: the damping ratio (equation (3.7)), the frequency ratio (equation (3.8)) and the force ratio (equation (3.9)).

In the friction-induced vibrations with external excitation the frequency of motion of the system at steady state is the same as the excitation frequency irrespective of the type of regime or the system parameters (m, c, k, N). This is unlike the case of self-excited vibrations where the frequency of oscillations depends on the system parameters (equation (2.35)). The motion in the pure slip regime is nearly sinusoidal as shown in Figure 3.3. In the stick-slip regime there is no motion in the stick-phase while during slipping the motion is harmonic as illustrated in Figure 3.4.

Figure 3.2: Multiple stops per cycle at low excitation frequency
The physics of stick-slip motion is discussed here by considering the steady state of the system and assuming that the mass starts slipping with positive velocity at the end of stick state. The velocity of the mass increases till it reaches the equilibrium position and then it decelerates, thereby increasing the frictional forces due to the decrease in the relative velocity. At one extreme the velocity becomes zero and the sum of all the other forces acting on the mass is unable to overcome the static frictional force. So the mass remains stationary till the spring force and the externally applied force exceeds the static frictional resistance. At this instant the mass accelerates back towards its equilibrium position. After it crosses the equilibrium position it decelerates and again sticks at the other extreme and this motion repeats on.

Figure 3.3: Pure-slip regime
3. Stick-Slip Vibrations of a System with External Excitation

Figure 3.4: Stick-slip regime

Effect of force ratio and damping ratio on the system response

The amplitude of oscillations of the system mass can be computed analytically for both non-sticking and stick-slip motions by using the complex results presented by Den Hartog [8]. However, a simpler approach can also be used provided the friction forces in the system are low compared to the amplitude of the forcing function. The nonlinear equation of motion (equation (3.6)) can be linearized by defining an equivalent viscous damper model such that the energy dissipated remains the same in both the cases. Then the equivalent damping ratio is:

\[
\zeta_{eq} = \zeta + \frac{4}{2\pi\hat{X}_o} \frac{\mu_{\text{max}}}{\Omega}\gamma
\]  

(3.14)

where \( \hat{X}_o \) is the amplitude of vibrations at steady state.
3. Stick-Slip Vibrations of a System with External Excitation

Now the nonlinear system reduces to a linear system and the equation of motion can be written as:

\[ \ddot{x}'' + 2\zeta_{eq}\dot{x} + x = \sin \Omega \tau \]  \hspace{1cm} (3.15)

The amplitude of this linear equation is given by the following expression:

\[ \hat{X}_o = \frac{1}{\sqrt{(1 - \Omega^2)^2 + (2\zeta_{eq}\Omega)^2}} \]  \hspace{1cm} (3.16)

This normalized amplitude is actually the magnification factor of the system that is defined as the ratio of the actual amplitude of vibrations at steady state to the deflection that would occur in the spring-mass system without friction, at zero frequency under a steady force \( F_0 \). By rearranging the terms, equation (3.16) can be re-written as a quadratic equation:

\[ P\hat{X}_o^2 + Q\hat{X}_o + R = 0 \]  \hspace{1cm} (3.17)

where

\[ P = (1 - \Omega^2)^2 + (2\zeta\Omega)^2 \]  \hspace{1cm} (3.18)

\[ Q = 2 \cdot (2\zeta) \cdot \left( \frac{4 \left( \frac{\mu}{\mu_{max}} \right) \gamma}{\pi \Omega} \right) \]  \hspace{1cm} (3.19)

\[ R = \left( \frac{4 \left( \frac{\mu}{\mu_{max}} \right) \gamma}{\pi} \right)^2 - 1 \]  \hspace{1cm} (3.20)

The solution of equation (3.17) gives the amplitude of vibration of the system with both viscous and Coulomb damping. A constant value of 0.105 for the coefficient of friction is assumed i.e. between its maximum and minimum value \((0.10 \leq \mu \leq 0.12)\), to calculate the
equivalent viscous damping and linearize the system. To avoid complex roots the following condition should be satisfied:

\[ Q^2 - 4PR \geq 0 \]  \hspace{1cm} (3.21)

This condition limits the range of \( \gamma \) for which this model can be implemented. The equations to linearize the model have been formulated based on the methodology given in Rao’s text [24] and have been modified according to the normalization approach followed in this discussion. Broken lines in Figure 3.5(a) shows the result of this linearization approach. Also this figure shows the results of simulations for various parameter values. The amplitude in the simulations is computed at the steady state as shown in Figure 3.6. In Figure 3.5(a) the curves for linear model deviate from the curves obtained from simulations for higher force ratios, indicating that as the force of friction increases the linearized model cannot predict the response of the system accurately. This is the reason why this model is used only when the friction forces are low. Also in the plot, the linear model curves are shown up till certain values of \( \gamma \) because beyond this value the roots of the quadratic equation (3.17) become complex as the condition given by equation (3.21) is not satisfied. The figure also shows that for higher force ratios and for more viscous damping, the response of the system decreases since the overall damping of the system increases with the increase in these two parameters. Also, with the increase in friction forces relative to the external force, the amount of stick increases as shown in Figure 3.5(b).

The percentage stick is the percentage of the total cycle time for which the system is in the state of stick as shown in Figure 3.6. It is a convenient measure of how close the system is to pure stick regime. It vanishes for system without friction and increases monotonically with \( \gamma \); \( \gamma \) equal to one implies the system is in the state of pure stick. Figure
3.5(b) also shows that if the system exhibits stick-slip response then the amount of stick increases with the increase in viscous damping.

![Figure 3.5: Effect of force ratio and damping ratio on the amplitude of vibrations and the amount of stick](image)

**Effect of frequency ratio on the system response**

Frequency ratio plays an important role in the dynamics of the system as shown in the frequency response curves (Figure 3.7) and the curves showing the variation of phase difference with frequency ratio (which is presented later in Figure 3.10). These figures have been plotted from the results of the simulation. These curves reveal a lot of interesting and useful information regarding the response of the system to harmonic excitation in the presence of friction.
3. Stick-Slip Vibrations of a System with External Excitation

Figure 3.6: General terms for the system response

Figure 3.7 depicts that if a system exhibits stick-slip response at low frequencies, then its response regime changes to pure slip with the increase in excitation frequency and with a further increase in frequency it again exhibits stick-slip motion. This implies that the amount of stick decreases with the increase in frequency ratio, up to a certain value, beyond which it again increases as shown in Figure 3.8. It can also be seen from Figure 3.7 that as the value of force ratio increases, the lowest frequency at which stick-slip is observed also increases. Also the value of force ratio at which stick-slip is observed reduces with the increase in damping. For very high force ratios, pure slip does not persist irrespective of the excitation frequency.

The stick-slip boundary of different plots in Figure 3.7 is shown by broken magenta line. These have been plotted using the results of Den Hartog [8] which are given below:
3. Stick-Slip Vibrations of a System with External Excitation

\[ X_o \geq \frac{S_i(I - G)}{q\sqrt{H^2 + [S_iI + (1 - S_i)G]^2}} \quad (3.22) \]

where

\[ S_i = 1 \quad (3.23) \]

\[ I = \frac{2H\zeta}{\Omega} + \frac{(1 + G)}{\Omega^2} \quad (3.24) \]

\[ H = \frac{1}{\Omega\sqrt{1 - \zeta^2}} \cdot \frac{\sin\left(\frac{\pi\sqrt{1 - \zeta^2}}{\Omega}\right)}{\cosh\left(\frac{\pi\zeta}{\Omega}\right) + \cos\left(\frac{\pi\sqrt{1 - \zeta^2}}{\Omega}\right)} \quad (3.25) \]

\[ G = \frac{\sinh\left(\frac{\pi\zeta}{\Omega}\right) - \sqrt{\frac{\zeta}{1 - \zeta^2}} \cdot \sin\left(\frac{\pi\sqrt{1 - \zeta^2}}{\Omega}\right)}{\cos\left(\frac{\pi\zeta}{\Omega}\right) + \cos\left(\frac{\pi\sqrt{1 - \zeta^2}}{\Omega}\right)} \quad (3.26) \]

Note that the expressions (3.24), (3.25) and (3.26) have been modified to consider the effect of different normalization used in this study from the one used by Hartog [8]. Figure 3.7 clearly indicates that Hartog’s result does not predict exactly the actual stick-slip boundary because in his study he considered friction to be constant i.e. independent of the relative sliding velocity.

It is observed from the frequency response curves that the response of the system at any frequency is lower for higher value of viscous damping. As a result the curves are flattened for higher amount of viscous damping. A very interesting thing to note from these curves is the shift of the peaks with the increase in \( \gamma \) and this shift is known as frequency shift. For any given system the frictional characteristic and external force is almost the same
throughout the operating cycle. So this shift in frequency is primarily because of the increase in normal load $N$. If the friction force is not present in the system i.e. $\gamma$ is equal to zero then the frequency ratios at which peaks will be observed is given by:

$$\Omega = \sqrt{1 - \zeta^2}$$  \hspace{1cm} (3.27)

So the peaks will shift towards the left of $\Omega=1$ with the increase in system damping. For the maximum damping ratio considered for this study i.e. $\zeta=0.25$ the maximum frequency shift in the absence of friction will be approximately 6.5%. But for the same damping ratio a frequency shift of approximately 70% is observed in the presence of friction force at very high force ratios as shown in Figure 3.9. This figure also shows that frequency shift increases further with the increase in damping ratio.

Figure 3.10 presents another set of interesting curves that show the effect of frequency ratio on the phase difference between the displacement and the excitation force, for varying force ratios and for different amounts of viscous damping present in the system. The phase difference is calculated from the simulations as depicted in Figure 3.6. The phase angle always lies between $0^\circ$ and $180^\circ$. It will be $0^\circ$ at zero excitation frequency and $180^\circ$ for very high values of frequency ratios. In the absence of frictional forces i.e. for $\gamma=0$, the phase angle is always $90^\circ$ at $\Omega=1$, independent of the amount of viscous damping present in the system. However, with the increase in frictional forces due to the increase in normal load, the phase angle of $90^\circ$ no longer occurs at $\Omega=1$. It shifts towards the left of $\Omega=1$ i.e. it occurs at lower excitation frequencies for higher force ratios as shown in Figure 3.10. This shift in the phase angle with increasing force ratio is also the reason for frequency shift. When phase angle is $90^\circ$ the harmonic force does the maximum amount of work resulting in maximum response of the system. Since the phase shifts with higher force ratios, so does the response,
3. Stick-Slip Vibrations of a System with External Excitation

Figure 3.7: Effect of variation of frequency ratio on the system response.
3. Stick-Slip Vibrations of a System with External Excitation

Figure 3.8: Variation of the amount of stick per cycle with frequency ratio

\[ \Omega \]

\[ \Omega \]

\[ \Omega \]
resulting in occurrence of resonance at frequencies lower than the natural frequency of the system. This effect is more dominant in heavily damped systems.

The variation in phase angle with force ratio is shown in Figure 3.11. For frequency ratios less than or equal to one, the phase angle increases with increase in force ratio whereas for excitation frequencies greater than the natural frequency of the system, the phase angle decreases with increase in force ratio. This effect amplifies with the increase in system damping. As expected, the phase angle is $90^\circ$ for very low force ratios at $\Omega=1$.

This variation in phase can also be determined analytically by simply extending the linearization approach discussed previously. The phase angle for the linear system (equation (3.15)) is given by the following expression:

$$\text{Phase angle, } \varphi = \tan^{-1}\left(\frac{2\zeta \Omega}{1-\Omega^2}\right)$$

(3.28)
3. Stick-Slip Vibrations of a System with External Excitation

Figure 3.10: Effect of frequency ratio on the phase angle
where $\zeta_{eq}$ is obtained from equation (3.14) after the amplitude is calculated by solving equation (3.17). The phase angle predicted by this model is shown by broken lines in Figure 3.11. The figure clearly shows that the linear model accurately predicts the phase angle for low force ratios. However, as the force of friction increases thereby increasing force ratio, the solution obtained from the linear approach diverges from the simulation results that are more reliable and accurate. Moreover for $\Omega=1$, the phase angle predicted by linear model is 90°, independent of the system damping and force ratio. So the linearization approach is suitable to predict phase angle at only low force ratios.

![Figure 3.11: Phase shift as a function of force ratio](image)

**Energy dissipation**

The amount of energy dissipated by the frictional interface and the viscous damper depends on the system parameters. For this discussion, energy dissipated is calculated for one cycle at
the steady state. Energy dissipated by friction interface $E_{\text{friction}}$ and energy dissipated by the viscous damper $E_{\text{viscous}}$ is calculated using the following expressions:

$$
E_{\text{friction}} = \int_{t_a}^{t_f} \left( \frac{2\pi}{\Omega} \right) F_x \cdot \dot{x} \, d\tau
$$

$$
E_{\text{viscous}} = \int_{t_a}^{t_f} \left( \frac{2\pi}{\Omega} \right) \zeta \dot{x} \cdot \dot{x} \, d\tau
$$

The results of numerical integration are plotted in Figures 3.12 and 3.13. As can be seen from Figure 3.12, the energy dissipated at the friction interface vanishes at both extremes i.e. at $\gamma = 0$ and $\gamma = 1$. The figure shows that the maximum energy dissipated by friction interface occurs at some intermediate value of force ratio. At lower force ratios, although the response of the system is more but due to insufficient damping at the friction interface, the amount of energy dissipated by friction is low. At very high values of force ratio the response of the system is low thereby reducing the energy dissipated. Moreover for frequency ratios where stick-slip motion is observed, for high force ratios the amount of stick increases resulting in low energy dissipation. So there is an optimal value of $\gamma$ at which the energy dissipated by friction is maximum. This optimal value depends on the frequency ratio and also on the amount of viscous damping present in the system. For moderately low viscous damping, the maximum energy dissipated by friction occurs at excitation frequency equal to the natural frequency of the system, even for high force ratios. But for very high force ratios the maximum amount of energy is dissipated at frequency ratios less than one. This is due to the frequency shift at high force ratios. Since the frequency shift effect is more predominant at higher amount of viscous damping, the amount of energy dissipated by friction interface at high values of $\zeta$ is maximum at frequency ratios less than one. In the figure the broken line
corresponds to the frequency ratio of one. The important thing to note about this line is that it gets suppressed by the low frequency ratio lines at high viscous damping for higher force ratios. For very low force ratios the energy dissipated is still maximum at frequency ratios in the vicinity of $\Omega=1$.

![Graphs showing energy dissipated by friction interface]

*Figure 3.12: Variation of energy dissipated by the friction interface with the variation in force ratio, for different frequency ratios and viscous damping*

The ratio of energy dissipated by the friction interface to the total amount of energy dissipated by the system is another important parameter that plays critical role in controlling the response of the system. This ratio is zero when $\gamma=0$ since there is no frictional resistance present in the system. Figure 3.13 shows that with the increase in $\gamma$ this ratio increases. Moreover the figure clearly depicts that even for moderately low values of $\gamma$, the friction interface dissipates more than half of the energy dissipated by the viscous damper at low as well as at very high frequency ratios. However with the increase in excitation frequency this
3. Stick-Slip Vibrations of a System with External Excitation

does the amount of energy dissipated by the viscous damper increases due to the increase in the response of the system at higher frequencies. This trend reverses with the increase in frequency ratios beyond $\Omega=1$ since the response of the system again decreases with further increase in excitation frequency. If a given system is tuned such that its frequency ratio is kept low, then the amount of energy dissipated by the friction interface will increase. This might give rise to stick-slip oscillations but the response of the system will be less. So in some systems it might be desirable to have stick-slip motion for more energy dissipation.

Figure 3.13: Ratio of energy dissipated by the friction interface to the total energy dissipated by the system, as a function of force ratio and frequency ratio

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3.4 Concluding remarks

This study of friction-induced vibrations provides insight into the implications of presence of friction on the system dynamics. It is observed that an external excitation force, independent of the motion gives rise to stick-slip motion in the systems with friction. With properly chosen frequency and force ratios, this external excitation can eliminate the occurrence of this intermittent motion and control the amplitude of oscillations. Increasing the system damping can also reduce the system response. A shift in the resonant frequency is observed at high normal loads that is attributed to the shift in the phase angle and is more dominant in heavily damped systems. This affects the optimal value of force ratio at which friction interface dissipates the maximum energy.
4. Bilinear Hysteresis Modeling of Friction Interface

CHAPTER 4

BILINEAR HYSTERESIS MODELING OF FRICTION INTERFACE

4.1 Introduction

The presence of dry friction in physical systems might give rise to intermittent stick-slip motion as seen in the previous two chapters. These stick-slip oscillations can cause excessive wear of components, surface damage, fatigue failure and noise as presented in the review article by Ibrahim [16]. It is because of these problems that arise in everyday life, the friction is always viewed to have deteriorating effects on the performance of various systems. However, the fact is that dry friction can also enhance system performance due to its damping properties. For example, in systems like gas turbines where the operating temperatures are very high, friction damping is provided to maximize energy dissipation so that the dynamic response of the blades is reduced, henceforth increasing their fatigue life. In this chapter theoretical model of friction damper that is used to control vibration of turbine blades is presented and its effects on the response of the blades are discussed.

The blades of a jet engine turbine have the following two types of construction:

- Free standing blades
- Grouped blades

In the free standing blade design each blade stands alone and a blade-to-ground friction damper is used to control its vibration as shown schematically in Figure 4.1. In grouped blade design, the blades are grouped together using shroud or tie-wire that, besides stiffening the blades, also provide dry friction damping. The shroud is attached somewhere along the airfoil or at the tip of the blade and the centrifugal load on the blade during operation untwists the twisted blade, causing shrouds on adjacent blades to come in contact providing...
blade-to-blade damping mechanism as shown schematically in Figure 4.2. In grouped blade design with tie wire, a hole is drilled through each blade airfoil and a wire is drawn through the holes. This wire provides damping by interfacial slip between the wire and the blade as shown in Figure 4.3.

Figure 4.1: Single blade with blade-to-ground damper

Figure 4.2: Grouped-blade design with integral shroud
Both blade-to-ground and blade-to-blade dampers can dissipate vibration energy only when relative motion occurs between the friction interfaces. Hence, the modeling and analysis of the relative motion between the friction interfaces is the key problem in designing a friction damper.

There are two theoretical approaches to model dry friction interfaces: the macroslip and the microslip approach. In the macroslip approach, the entire friction interface is assumed to be either slipping or sticking. In the microslip approach, detailed analysis of the friction interface is carried out taking into account the partial slipping of the interface that predicts the system response more accurately at high normal loads [19, 20]. Several researchers [1, 3, 4, 19, 20] have used a microslip approach to predict the system response that is discussed in further detail in the next chapter. This approach gives more accurate results, yet the macroslip approach is more commonly used because of its simple assumption that requires less computational efforts. In this chapter, the dynamic behavior of blades is discussed following macroslip approach. The modeling of the friction interface is discussed first and then the response of the system is studied considering this friction interface model.
4.2 Dynamic modeling of friction interface

It is important to model the motion of friction interface first, before we can apply the friction damper to study its effect on the response of the system. Consider a rigid body of mass \( m \) subjected to force \( N \) in the normal direction and a force \( F \) in the tangential direction as shown in Figure 4.4. If force \( F \) is sufficiently large then the rigid body will begin to move. The process from state of rest to the state of motion can be viewed microscopically as shown in Figure 4.5, in order to understand the interface behavior in response to the tangential force. Figure 4.5 schematically represents the microscopic view of the contact interface of a small region ‘A’ indicated in Figure 4.4.

![Figure 4.4: Rigid body on a friction surface](image)

(a)

![Asperities in tight contact under the effect of normal force](image)
When the tangential force $F_t$ is applied the locked asperities begin to deform and the rigid body begins micro-motion. When the tangential force becomes large enough the asperities are completely deformed and are no longer locked together. At this instant the body begins to slip on the surface. From the mathematical standpoint, the behavior of friction interface can be represented by the model shown in Figure 4.6. The mass $m$ represents the rigid body and the block represents the macroslip element that considers the effect of asperities interaction. The spring $k_d$, between the mass $m$ and the block correlates to the shear stiffness of the asperities. When the force is applied in the tangential and the normal direction, the asperities are locked together and the mass $m$ is subjected to the spring force $k_d(x-x_d)$. When a sufficiently large displacement $x$ is reached, the spring force makes the spring yield completely and slipping occurs. While slipping, the mass is subjected to dry friction force, $\mu N sgn(x_d)$. So $x_d$ can be regarded as the motion of the friction damper.

Figure 4.5: Microscopic view of the contact interface at ‘A’
This model is based on the models proposed by Iwan [16] to describe the elasto-plastic material behavior. His models consist of \( N \) macroslip elements arrayed as shown in Figure 4.7 that allow for partial slipping of the interface since each contact point sticks and slips independently of each other.
4. Bilinear Hysteresis Modeling of Friction Interface

Iwan suggested that parallel-series formulation is more suitable for dynamic system modeling since this formulation always leads to equations of motion which have displacements as the independent variables. This model with multiple damping elements predicts the system response more accurately, but is not very commonly used because of the computational complexities involved. Moreover if we consider N elements then the calibration of the model parameters i.e. the stiffness and force at which each element slips, against experimental data may be difficult and non-unique. So in this study we will model the friction interface with only single damping element which will obviously limit the quantitative accuracy of the results. However the qualitative behavior of the system is similar to the experimental data.

4.3 Bilinear hysteresis model with massless damper

A typical single blade structure with blade-to-ground damper is shown in Figure 4.1. As discussed in the previous section only single damping element is considered to study the response of the blade. Moreover the friction damper is considered massless since the violent vibrations of blade appears only when it is subjected to an excitation force with a frequency near one of the natural frequencies of the blade. Assuming mass of the damper to be zero, makes its natural frequency to be far beyond the operating range of the system. This reduces
the system to a single degree of freedom model shown in Figure 4.8. Also, the number of parameters needed to describe the friction damper reduces to only two, namely:

- Damper stiffness, and
- The force at which it slips.

This model appropriately captures the dynamics of the friction interface. The sticking of the damper represents the locked state of the asperities and its slip state is representative of the relative motion of the complete interface. So only the damper exhibits the stick-slip motion. The blade parameters in the model are represented by its mass \( m \), stiffness \( k \) and equivalent viscous damping coefficient \( c \). \( N(t) \) and \( k_d \) represent the normal preload of the friction damper and the stiffness of the damper in the direction of vibration. \( F(t) \) is the external excitation force that can be expressed as a harmonic function. The coefficient of friction at the friction interface is represented by \( \mu \). The response of blade is indicated by \( x(t) \) and the elastic deformation of the asperities is accounted for, by the motion of damper which is denoted by \( x_d(t) \).

![Figure 4.8: Bilinear hysteresis model with massless damper](image)

Figure 4.8: Bilinear hysteresis model with massless damper
4.3.1 Equations of motion

The equation of motion of the SDOF model shown in Figure 4.8 can be written as:

\[ m\ddot{x} + c\dot{x} + kx = F_0\sin\omega t - F_d \]  

where \( F_0 \) is the amplitude and \( \omega \) is the excitation frequency of the external harmonic force.

\[
\text{Damper force, } F_d = \begin{cases} 
F_t & \text{(Slipping)} \\
= k_d(x - x_d) & \text{(Sticking)}
\end{cases}
\]  

\[
\text{Friction force, } F_f = \begin{cases} 
\mu N & \text{(if } \dot{x} < 0) \\
= -\mu N & \text{(if } \dot{x} > 0) 
\end{cases}
\]

The normal load \( N \) and the coefficient of friction \( \mu \) are assumed to be constants for this study. The first step towards the study of this system is to reduce the number of variables through non-dimensional grouping by using appropriate scaling parameters. The scaling parameters used to obtain non-dimensional equation of motion are:

\[
\tau = \omega t
\]

\[
\Rightarrow (\ldots) = \omega (\ldots)'
\]

Static deflection, \( \delta_{st} = \frac{F_0}{k} \)

\[
\hat{x} = \frac{x}{\delta_{st}}
\]

Natural frequency of the system without damper, \( \omega_n = \sqrt{\frac{k}{m}} \)
4. Bilinear Hysteresis Modeling of Friction Interface

After applying this scaling scheme, equation (4.1) can be written as,

\[ \ddot{x}^\ast + 2 \zeta \dot{x}^\ast + \Omega^2 \dot{x} = \Omega^2 \sin \tau - \Omega^2 \frac{F_d}{F_0} \]  

(4.8)

where

Damping ratio, \( \zeta = \frac{c}{2m\omega} \)  

(4.9)

Frequency ratio, \( \Omega = \frac{\omega_n}{\omega} \)  

(4.10)

**Slipping equation of motion**

The equation of motion for the damper can be written as

\[ m_d \ddot{x}_d = F_d - \text{Sgn}(\dot{x}_d) \mu N \]  

(4.11)

Since damper is massless, so to maintain force balance,

\[ F_d = \text{Sgn}(\dot{x}_d) \mu N \]  

(4.12)

The inertia of the damper being zero implies that during slipping the whole system will oscillate as one rigid body. Physically this means that in the slip state the spring maintains its state of tension or compression depending on its state at the end of sticking. As a result, the system mass is subjected to the total frictional force acting on the damper.

The normalized equation of motion (equation (4.8)) can be re-written using equation (4.12) as:

\[ \ddot{x}^\ast + 2 \zeta \dot{x}^\ast + \Omega^2 \dot{x} = \Omega^2 \sin \tau - \text{Sgn}(\dot{x}^\ast)\Omega^2 \gamma \]  

(4.13)

where force ratio, \( \gamma = \frac{\mu N}{F_0} \)  

(4.14)

**Sticking equation of motion**

During sticking, force exerted by the damper is given by:
4. Bilinear Hysteresis Modeling of Friction Interface

\[ F_d = k_d (x - x_d) < \mu N \]  \hspace{1cm} (4.15)

Using this, equation (4.8) can be written as

\[ \ddot{x}'' + 2 \zeta \dot{x}' + \Omega^2 (1 + \alpha) \dot{x} = \Omega^2 \sin \tau + \Omega^2 \alpha \dot{x}_d \]  \hspace{1cm} (4.16)

where,

\[ \alpha \text{ is the stiffness ratio given by } \frac{k_d}{k} \]  \hspace{1cm} (4.17)

If the damper force during slipping (equation (4.12)) and sticking (equation (4.15)) is plotted against the displacement of the system mass, a closed parallelogram shape is obtained as shown in Figure 4.9. This indicates that the series spring/damper arrangement considered in this model behaves like a spring with hysteretic stiffness. Due to the straight line segments that make up the force-displacement relation, it is often termed as a bilinear hysteresis element. The slope of the stick branch is the measure of the stiffness of the damper spring.

![Figure 4.9: Damper force–displacement relation. Branches 1-2-3-4 correspond to slip-stick-slip-stick state of the damper.](image-url)
4. Bilinear Hysteresis Modeling of Friction Interface

**Transition Criterion**

1. Slip to stick transition is determined by the change in the direction of velocity.
2. Stick to slip transition occurs when the damper force during stick exceeds the maximum frictional force acting on the damper i.e. whenever \( k_d(x-x_d) \) exceeds \( \mu N \), there is a change in state from stick to slip.

4.4 Results

The nonlinear equations of motion (equations (4.13) and (4.16)) are solved using the direct time integration method as discussed in Chapter 2 (Section 2.3.4). In this section the numerical results of this integration are presented in the form of plots that give insight into qualitative behavior of the system where the friction interface is modeled using a single macroslip element. Figure 4.10 (a) shows the response of the system subjected to external harmonic force. The damper force-displacement curve for the system is shown in Figure 4.10 (b). In this plot the slope of the branch that represents stick region is equal to the damper stiffness. This is clearly shown in Figure 4.11 (a) and (b). As the damper stiffness increases, thereby increasing the stiffness ratio, the displacement of the system mass during stick reduces. For very high values of stiffness ratios, almost instantaneous transition from one slip branch to another takes place implying that there is negligible amount of sticking. However, with the increase in normal load, the sticking displacement increases as indicated in Figure 4.11 (c) and (d). For very high values of normal load i.e. for high force ratios, the damper continues in the state of pure stick.
4. Bilinear Hysteresis Modeling of Friction Interface

Figure 4.10: (a) Response of the system (b) Damper force-displacement curve

Figure 4.11: Effect of stiffness ratio and force ratio on the system response. Plots (a) and (b) show the effect of stiffness ratio $\alpha$. Plots (c) and (d) show the effect of force ratio $\gamma$
The resonant response of the system is shown in Figure 4.12 which indicates that with the increase in force ratio, the response of the system first decreases, then after a particular value it again starts increasing. This optimal value of $\gamma$ can be used to minimize the response of the system. The reason for this behavior can be well explained from the energy standpoint. Below this optimal value, the friction force does not provide sufficient damping so the energy dissipation is less as shown in Figure 4.13. Above this value, the amount of sticking is very high as shown in Figure 4.14, again resulting in less dissipation of energy by the friction interface, thus increasing the response. Moreover, the response can further be reduced by increasing the stiffness ratio that results in more amount of slipping of the damper thereby increasing the energy dissipation at the friction interface. The horizontal lines in Figure 4.12 indicate the regions of pure stick. Pure stick will occur only when the friction forces are sufficiently high such that the maximum damper force is not able to overcome the frictional resistance at the interface. Analytically this can be determined by assuming that the system is in the state of pure stick. The maximum damper force will then be given by:

$$F_{d,\text{max}} = \Omega^2 \alpha \hat{x}_{\text{stick}}$$  \hspace{1cm} (4.18)

where $\hat{x}_{\text{stick}}$ is the maximum amplitude of vibration at the time of stick and is given by the following expression derived from the sticking equation of motion (equation (4.16)):

$$\hat{x}_{\text{stick}} = \frac{1}{(1 + \alpha)} \sqrt{\left(1 - \frac{1}{\Omega^2 (1 + \alpha)}\right)^2 + \left(\frac{2\zeta}{\Omega^2 \sqrt{1 + \alpha}}\right)^2}$$  \hspace{1cm} (4.19)

The maximum frictional force at the frictional interface is given by:

$$F_{f,\text{max}} = \Omega^2 \gamma$$  \hspace{1cm} (4.20)
The damper will be in the state of pure stick if,

\[ F_{f_{\text{max}}} \geq F_{d_{\text{max}}} \]  \hspace{1cm} (4.21)

For the regions indicated by horizontal lines in Figure 4.12, equation (4.21) is satisfied for the given set of parameters. The resonant responses \( \dot{X}_{od} \) in the figure are normalized by the resonant response of the system in the absence of friction damper, which is given by:

\[ \dot{X}_o = \frac{1}{2\zeta} \]  \hspace{1cm} (4.22)

In Figure 4.13 the energy dissipated by the friction damper is calculated over a period of one forcing cycle using the following expression:

\[ E_{\text{damper}} = \int F_d \cdot \dot{x}' \, d\tau \]  \hspace{1cm} (4.23)

When the damper is stuck, there will not be any dissipation of energy. In the figure the energy dissipated by damper is normalized by the energy dissipated by a viscous damper in the absence of friction damper which is given by:

\[ E_{\text{viscous}} = \pi \cdot 2\zeta \cdot \dot{X}_o^2 \]  \hspace{1cm} (4.24)

It can be seen from the figure that the friction damper can dissipate approximately 35% of the energy dissipated by the viscous damper for particular parameter values, which again corresponds to the optimal value of the force ratio. The damper is designed so as to maximize this ratio for the operating range of parameters.
4. Bilinear Hysteresis Modeling of Friction Interface

Figure 4.12: Resonant response variation with force and stiffness ratios

Figure 4.13: Energy dissipation as a function of force and stiffness ratio
Figure 4.14: Variation of amount of stick per cycle with force ratio for different stiffness ratios

Energy dissipated by the friction damper also depends on the frequency ratio and this effect is shown in Figure 4.15. The maximum energy dissipation for any frequency ratio occurs at an optimal value of force ratio. At low force ratios maximum energy is dissipated when the frequency of excitation is equal to the natural frequency of the system. At high force ratios the frequency at which maximum energy is dissipated by the damper is more than the natural frequency of the system i.e. $1/\Omega > 1$. This is because of the frequency shift that occurs at high force ratios as is discussed later in the section. As the excitation frequency increases, the width of the energy dissipation curve increases. At very high very frequencies no regions of pure stick are observed within the parameter range of this study which indicates
that at these frequencies, even for very high force ratios the energy is dissipated by the damper.

Figure 4.15: Effect of frequency ratio on the energy dissipated by the friction damper

**Frequency Shift**

It is observed that with the increase in force ratio, the maximum response of the system does not occur at its natural frequency. Instead, the maximum amplitude of vibrations is observed at a frequency higher than the natural frequency as indicated by the shifting of peaks in Figure 4.16. This frequency is termed as the resonant frequency. An interesting thing to note from this figure is that the peak response first decreases up to a force ratio of 0.4 and then again increases. This is the same behavior as was observed in Figure 4.12.
4. Bilinear Hysteresis Modeling of Friction Interface

Figure 4.16: Response of the system in frequency domain

Figure 4.17: Frequency shift with increase in force ratio. * indicates pure-stick response. ● indicates stick-slip response
Figure 4.16 depicts that the shift in the resonant frequency from the natural frequency increases with the increase in stiffness ratio. For high stiffness ratios, a frequency shift of approximately 20% is observed at high force ratios.

### 4.5 Concluding remarks

A friction damper is a powerful device to control the vibrations of any structure. To predict the response of the structure, the complex friction interface behavior is modeled using a massless bilinear hysteresis element. This friction interface model is different from the discontinuous models used in the previous chapters as it accounts for the deformation that can occur prior to the interfacial slip. The response of the system is studied based on the macroslip approach that gives a good insight into the qualitative behavior of the system. An optimal value of force ratio can be determined from this study to minimize the resonant response of the system.
CHAPTER 5

CONCLUSIONS AND FUTURE WORK

5.1 Comparisons and conclusions

In this study we have seen that friction plays a key role in a wide variety of physical systems. Vast amount of research has been done across varied engineering disciplines to model the sliding frictional behavior between two bodies in contact and these efforts are still continuing. Past researches have shown that friction depends on several parameters, which makes it difficult to develop a unique friction model that represent all kinds of behavior. Friction is a highly non-linear phenomenon and random with respect to time, that further worsens the task of modeling it exactly, even for a particular system based on available experimental data.

It has always been a challenge for designers of sliding system to develop a model that can predict the system response accurately. The system model consists of a friction interface model and a dynamic model. The accuracy of prediction of system response depends on the choice of both dynamic and friction model. Past researches have put forth many choices for the designers and researchers to select an appropriate friction and system dynamic model. Diversity of friction-related problems and strong coupling between system dynamics and friction, requires designers and researchers to have a good understanding of the applications and limitations of the available modeling tools and techniques, so that they can utilize the best available options for friction and system dynamic modeling or if necessary and feasible, develop a new modeling technique.

This study presents three different modeling techniques namely:

- Self-excited friction oscillator without external excitation
5. Conclusions and Future Work

- Friction oscillator with external excitation (Den Hartog’s model)
- Modeling of friction using bilinear hysteresis elements

The effect of system parameters (mass, damping, stiffness, normal load, external excitation, reference velocity) on its response is investigated in detail. Each of the models considered in this work represents a class of friction-related problems. The common characteristic of all these three problems is that the system generally exhibits stick-slip oscillations depending on the system parameters. The negative friction-velocity characteristic is responsible for these oscillations in the self-excited problems. As the name itself suggests, in the self-excited case there is no external force applied on the system to sustain the stick-slip oscillations. It is a force that is governed by some part of the motion itself, which sustains the oscillations. In the other two models, an external tangential force is applied that supplies energy for the motion of the system. The multivalued characteristic of friction at zero relative velocity gives rise to stick-slip oscillations in the case of forced response problems.

The viscous damping reduces the system response because of the additional energy dissipated by the viscous damper. This is observed in all the three models. In self-excited problem increase in viscous damping tends to make the slope of the friction-velocity curve positive resulting in pure sliding response, thus stabilizing the system. In the forced response problem increase in damping increases the amount of sticking. As a result, the amount of energy dissipated by the friction damping element in the forced response problems reduces. This implies that the friction damping is effective only up to a certain amount of viscous damping present in the system. The normal load on the system increases the amount of friction force, thereby increasing the efficiency of the friction element. However, after a certain value of normal load, the increase in the amount of stick reduces the efficiency of the
5. Conclusions and Future Work

friction element to dissipate energy. Thus, it is very important to limit the value of normal load and the viscous damping for the efficient working of the friction element to control the system response.

In the two techniques used to predict the forced response of the system, the primary difference is in the modeling of the friction interface. The bilinear hysteresis model takes into account the elastic deformation of the asperities at the contact interface. Also, in this approach the main system mass never experiences sticking. It is only the massless damper that either sticks or slips. In the Den Hartog’s model, the main mass itself sticks and slips. Because of the more realistic modeling of the friction interface using bilinear elements, this approach is able to predict the system response more precisely. Both these models show an interesting behavior in the resonant frequency shifts at high normal loads. In the Den Hartog’s model the resonance occurs at an excitation frequency less than the natural frequency of the system at higher normal loads. However, for the bilinear hysteresis model the trend is reverse i.e. with the increase in normal loads the resonance occurs at frequencies higher than the natural frequency of the system.

The rich dynamic behavior observed in a variety of systems in the presence of dry friction present at the contact interface, is discussed in this thesis work using the low-order lumped-parameter models. These models have the limitation of predicting the system response accurately due to the occurrence of partial slip at the friction interface depending on the severity of contact conditions, which cannot be accounted by these models based on the macroslip approach. Despite their limitations, these models are invariable used to study the qualitative behavior of physical systems, which helps in understanding the mechanism of friction and the coupling between system dynamics and friction.
Along with the use of low order models, research efforts still continue for developing a new model that can predict the system response accurately.

5.2 A new microslip modeling approach

The main focus of this thesis work has been to investigate the response of the system, considering dry friction to be present at the contact interface. The system and the friction interface are modeled based on the macroslip approach because of its computational efficiency and simple analytical formulations. However, the low-order lumped-parameter models based on this approach, which are considered for this study, cannot capture the highly localized interface behavior known as microslip that is observed experimentally. The accuracy of the predictions of the response by these models depend on how precisely the model is calibrated, for which extensive experimental data is required. Moreover, the macroslip models consider the normal force to be spatially constant which is generally not the case in real physical systems.

A new formulation based on microslip approach was presented by Berger et al. [4] which overcomes the limitations of the models based on macroslip approach as are mentioned above. The main features of this formulation are:

- Variation of normal force with time and in space is taken into account.
- Normal motion of the sliding system accompanied by frictional sliding, as observed for the first time by Tolstoi [28], is included in the formulation that allows for analyzing all contact situations, including loss of contact.
- Sliding system is considered as a continuous elastic system. As a result model parameters (stiffness, damping) are defined in terms of material properties such as Young’s modulus, material density etc.
5. Conclusions and Future Work

- The formulation precisely captures full range of system response i.e. it is applicable for pure slip, pure stick and partial slip situations.
- Finite element method is used to obtain discrete equations of motion.
- Friction force at each node of the contact interface not only depends on the friction coefficient of that node but also on the friction coefficients of the neighboring nodes.
- In partial slip situation, the motion of the slip contact node is unknown whereas the friction coefficient is known from its velocity dependent characteristic. For sticking contact node the motion is known but the friction coefficient is unknown. This results in complex equations of motion for the system. These stick-slip equations can be represented as a mixed differential-algebraic equation (MDAE), which can be solved directly for the unknown coefficients of friction.

This robust formulation provides an efficient and accurate numerical formulation to account for partial slip situation that exists in actual physical situations. The main advantages of this formulation over the formulation based on the macroslip approach are:

- Since the model parameters are derived from the continuum approach, the parameter calibration is not required.
- Spatial and temporal variations in normal force are considered that have significant effect on the local friction force calculations, which affects the system response.
- High spatial resolution of the friction interface is obtained which can precisely capture the mechanics of the interface.
5. Conclusions and Future Work

- Interface energy dissipation can be accurately calculated, thus the response of the system can be predicted precisely.

5.3 Future research

In turbomachinery, friction dampers are employed to attenuate the resonant vibration of blades so as to increase their fatigue life. The prediction of resonant amplitude and frequency relies on how accurately the friction interface between the blade and damper is modeled. Berger and Begley [3] showed that the accurate microslip modeling of the blade-damper interface is critical to predict blade root stress and hence life of the blade and is also crucial to capture the coupled interface-structural dynamics problem. However, the existing tools available with the industry are not accurate enough to capture this complex interface behavior, which ultimately affects the reliability of the jet engines. There is a strong need to develop a numerical design tool for the exact analysis of mechanics of friction interfaces.

Berger and Begley [3] demonstrated by that the microslip approach discussed in the previous section predicts the blade response precisely. A new model based on this approach needs to be developed that can analyze various damper configurations like blade-to-ground, blade-to-blade, wedge damper etc. Moreover, the new model will have to be integrated with the existing design tools available in the industry to provide an efficient and user-friendly design tool for the optimal design of friction dampers and to predict the forced response of turbine blades accurately.
REFERENCES


