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It is entitled Analysis and Modeling of Space-Time Organization of Remotely Sensed Soil Moisture

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ANALYSIS AND MODELING OF SPACE-TIME ORGANIZATION OF REMOTELY SENSED SOIL MOISTURE

A dissertation submitted to the

Division of Research and Advanced Studies of the University of Cincinnati

in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in the Department of Civil and Environmental Engineering of the College of Engineering, University of Cincinnati

2001

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The characterization and modeling of the spatial variability of soil moisture is an important problem for various hydrological, ecological, and atmospheric processes. Without the knowledge of soil moisture distribution, linkages between soil moisture observations and hydrological models at different scales cannot be characterized reliably. Due to the labor- and capital-intensive nature of the ground-based measurement of soil moisture and the calibration difficulties of remote sensing data, various efforts have been devoted to explore the feasibility of obtaining soil moisture distribution from easily measured environmental factors. Among the environmental factors controlling soil moisture distribution, the soil properties, topography and mean soil moisture have drawn significant attention because their influence is easily measured from observational and modeling studies. However, it is hard to identify the combined effects of these three environmental factors unless a large amount of observations are available. Inferred relationships between soil moisture and other environmental factors based on limited observation are often site-specific, and apparently contradictory findings have appeared in the literature. Therefore, a compact representation of interdependencies among soil moisture distribution, mean soil moisture, soil properties and topography is necessary.
This study attempts to provide such a compact representation using two complimentary approaches.

In the first approach, we develop a stochastic framework to evaluate the influence of spatial variability in topography and soil physical properties, and mean soil moisture on the spatial distribution of soil moisture. The problem of water flow in an unsaturated zone is expressed as a partial differential equation that depends on three stochastic properties: the heterogeneity of soil properties, the variability of topography and the change of mean soil moisture. A perturbation method is applied to solve the soil moisture problem. The resulting model provides closed form analytical solutions for (a) the variance of soil moisture distribution, and (b) the covariance between soil moisture distribution and soil properties, and (c) the covariance between soil moisture distribution and topography as a function of soil heterogeneity, topography and soil moisture. The key findings from the stochastic analysis are summarized below. First, topography appears to have dominant control on soil moisture distribution when the area is dominated by coarse-texture soil or by mixed soil with small correlation scale for topography (i.e., small $\lambda_c$). In such cases the soil moisture variability increases as the soil becomes wet. Second, soil properties is likely to have dominant control on soil moisture distribution for fine-texture soil or for mixed soil with large $\lambda_c$. In such cases, the soil moisture variability decreases as the soil becomes wet. Finally, both topography and soil properties appear to have similar control for medium-texture soil with moderate value of $\lambda_c$. In such cases, the soil moisture variability initially decreases and then increases as the
soil changes from dry to wet. Comparisons between above findings and a number of field observations show qualitative agreement for various environmental conditions.

The application of the proposed stochastic framework requires the statistical information of different soil characteristics. Since the measurement of soil properties is time consuming and expensive, it is desirable to develop simplified methods to characterize soil media properties over large areas. In the second approach, we explore the recent developments in Artificial Neural Network (ANN) to develop nonparametric space-time relationships between soil moisture and readily available remotely sensed surface variables. We have used remotely sensed brightness temperature data in a single drying cycle from Washita ’92 Experiment and two different ANN architectures (Feed-Forward Neural Network (FFNN), Self Organizing Map (SOM)) to classify soil types into three categories. The results show that FFNN yield better classification accuracy (about 80%) than SOM (about 70% accuracy). However, SOM has an advantage because it requires very little information regarding soil properties during the training phase while FFNN require more information regarding observed soil textural data. Our attempt to classify soil types into more than three categories resulted in about 50% accuracy when a FFNN was used and even lesser accuracy when a SOM was used. We also note that about 90% of the classification errors made by a FFNN for more than three soil categories can be accounted from soil types that are similar to each other (e.g., silty loam been predicted as loam). Such systematic error indicates that there is room for improvement in classifying soil types into more than three groups. We further explore the use of Artificial Neural Networks (ANN) models and brightness temperature from the
Southern Great Plains in the United States to classify soil into more than three groups. We have shown that the remotely sensed brightness temperature \textit{in a single drying cycle} might contain sufficient features to classify soil types into three categories. If we desire to classify soil texture into more than three groups, however, the single-drying-cycle brightness temperature data appear to be inadequate. To classify soil into more than three groups and to explore the limits of classification accuracy, this study suggests the use of \textit{multiple-drying-cycle} brightness temperature data. We have performed several experiments with FFNN models and the results suggest that the maximum achievable classification accuracy through the use of multiple-drying-cycle brightness temperature is about 80%. It appears that the requirement of rapidly changing decision boundary, in the case of space-time evolution of brightness temperature over large areas, will restrict the FFNN model to yield better accuracy. Motivated by these observations, we have used a simple prototype-based classifier, known as 1-NN model, and achieved 86% classification accuracy for six textural groups. A comparison of error regions predicted by both models suggests that, for the given input representation, maximum achievable accuracy for classification into six soil texture types is about 94%.

Results from this dissertation are expected to be useful to characterize and model soil moisture variability over large areas where little or no site-specific information of soil variability is available.
This research was supported by a National Aeronautics and Space Administration (NASA) Earth System Science Fellowship, grants from the National Science Foundation (NSF) and the United States Department of Agriculture (USDA). These research supports are gratefully acknowledged.

The work was performed by Dyi-Huey Chang and presented here as fulfillment of the thesis requirement for the degree of Doctor of Philosophy in Environmental Engineering at the University of Cincinnati. It is with great gratitude and appreciation that the author acknowledges the continuous and invaluable guidance and support provided by his advisor – Professor Shafiqul Islam throughout the course of his studies at the University of Cincinnati.

Appreciation is extended to the members of this dissertation committee, Professors Steven G. Buchberger, James G. Uber in the Department of Civil and Environmental Engineering, and Professor Ravi Kothari in the Department of Electrical & Computer Engineering & Computer Science at the University of Cincinnati, for their assistance towards the improvement of this thesis. Especially, Professor Ravi Kothari’s guidance and suggestions on various parts of the study have significantly improved the scope and depth of the author’s work.
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<th>Definition</th>
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<tr>
<td>1-NN</td>
<td>One Nearest Neighbor Classifier</td>
</tr>
<tr>
<td>ANN(s)</td>
<td>Artificial Neural Network(s)</td>
</tr>
<tr>
<td>CC</td>
<td>Correlation Coefficient</td>
</tr>
<tr>
<td>CV</td>
<td>Coefficient of Variation</td>
</tr>
<tr>
<td>DD</td>
<td>Degree of Difference</td>
</tr>
<tr>
<td>DEM</td>
<td>Digital Elevation Model</td>
</tr>
<tr>
<td>DI</td>
<td>Disparity Index</td>
</tr>
<tr>
<td>ESTAR</td>
<td>Electronically Scanned Thinned Array Radiometer</td>
</tr>
<tr>
<td>FFNN</td>
<td>Feed-Forward Neural Network</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
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</tbody>
</table>
**PPSPP** Percentage of Patterns with Similar Posterior Probability

**REO** Ratio of Estimated to Observed

**RSC** Ratio of Sand to Clay

**SGP** Southern Great Plains

**SOM** Self Organizing Map

**USDA** United States Department of Agriculture

**USGS** United States Geological Survey

**UTM** Universal Transverse Mercator
LIST OF SYMBOLS

\( \bar{\chi}_{uv} \) Scalar, defined by Equation (2.17). The variables \( u \) and \( v \) represent \( K_s, A, B \) or \( z \).

\( \bar{V}_{uv} \) Scalar, defined by Equation (2.17). The variables \( u \) and \( v \) represent \( K_s, A, B \) or \( z \).

\( \bar{u} \) Domain average of variable \( u \). The variables \( u \) represents \( K_s, A, B \) or \( z, K \) or \( \psi \)

\( u' \) Perturbation term of variable \( u \). The variables \( u \) represents \( K_s, A, B \) or \( z, K \) or \( \psi \)

\( \eta \) Normalized soil moisture, defined as \( \eta = (\theta - \theta_d)/(\theta_s - \theta_d) \).

\( \theta \) Volumetric soil moisture.

\( \psi \) Hydraulic suction of water.
$\beta$  Vertical hydraulic gradient near land surface.

$\xi$  In chapter 2 and 3, $\xi$ is a scalar defined by Equation (2.10). In Chapter 4 and 5, $\xi$ represents input prototype of ANN models.

$\zeta$  Scalar, defined by Equation (2.10).

$\lambda$  Slope parameter of activation function. See Equation (5.2).

$\omega$  In chapter 2 and 3, $\omega$ is a scalar defined by Equation (2.10). In Chapter 4 and 5, $\omega$ represents interconnection weights of ANN models.

$\beta$  Mathematical group, defined by Equation (2.8)

$\Theta_{\text{CAP}}$  Soil moisture at field capacity.

$\Theta_{\text{WP}}$  Soil moisture at permanent wilting point.

$\theta$  Residual value of volumetric soil moisture.

$\Theta$ or $\theta_{\text{sat}}$  Saturated value of volumetric soil moisture.

$\sigma$  Standard deviation of variable $u$. The variables $u$ represents $K_s$, $A$, $B$ or $z$. 
\( \lambda_u \) Correlation scale of variable \( u \). The variables \( u \) represents \( K_s, A, B \) or \( z \).

\( \sigma^2_u \) Variance of variable \( u \). The variables \( u \) represents \( K_s, A, B \) or \( z \).

\( \sigma_{uv} \) Covariance between variable \( u \) and \( v \). The variables \( u \) represents \( K_s, A, B \) or \( z \).

\( A \) A dimensionless soil parameter, which is defined as the product of \( B \) and \( C_s \).

\( B \) Thickness of capillary fringe.

\( C \) Specific capacity of soil moisture.

\( C_s \) Specific capacity of normalized soil moisture, \( C_s = C / (\theta_s - \theta_r) \).

\( D \) Depth of root zone soil.

\( f_c \) A mathematical term defined by Equation (3.3), representing the impact from correlation between topography and soil properties to \( \sigma_{uv} \).
$f_v$ A mathematical term defined by Equation (3.3), representing the impact from variability of topography to $\sigma F$.

$g_{\lambda u}$ Dimensionless parameters, defined by Equation (2.20). The variables $u$ represents $K_s, A, B$ or $z$.

$h$ The number of hidden neurons in a FFNN model.

$K$ Unsaturated hydraulic conductivity.

$K_s$ or $K_{sat}$ Saturated hydraulic conductivity.

$k_x, k_y$ Wave numbers in $x$ and $y$ directions.

$P$ The number of days in of multi-temporal brightness temperature data.

$R$ Effective rainfall defined as the portion of rainfall infiltrating into soil.

$R_0$ The portion of effective rainfall infiltrating to a unit depth of soil ($R_0 = R/D$).

$s$ Evaporation flux, parameterized as $s = \beta K$. 
$S_{uv}$ Spectral density function between $u$ and $v$. The variables $u$ and $v$ represent $K_s, A, B$ or $z$.

$T_B$ Brightness temperature.

$V_{uv}$ Scalar, defined by Equation (2.7). The variables $u$ and $v$ represent $K_s, A, B$ or $z$.

$W_0, W_u$ Scalar, defined by Equation (2.8). The variables $u$ represents $K_s, A, B$ or $z$.

$X_{uv}$ Scalar, defined by Equation (2.7). The variables $u$ and $v$ represent $K_s, A, B$ or $z$.

$y_i^l$ The output of the $i^{th}$ neuron in the output layer of FFNN model for pixel $l$.

$z$ Elevation (positive upward).

$Z(k_x, k_y)$ Fourier-Stieltjes spectral amplitudes

$z_j^l$ The output of the $j^{th}$ hidden neuron of FFNN model for pixel $l$. 
CHAPTER 1

Introduction
1.1. **Motivation and Objective**

Surface soil moisture is an important variable for a range of hydrological processes. It controls surface runoff and infiltration process, serves as a diluting media for contamination transport, and works as a buffer zone for land-atmosphere interactions. The highly heterogeneous nature of soil moisture variability and the non-linearity between hydrological processes and soil moisture distribution, although difficult to tackle from observational and modeling viewpoint, are accepted as reality [Entekhabi and Rodriguez-Iturbe, 1994]. Without the knowledge of soil moisture distribution, possible linkages between soil moisture observations and hydrological models at different scales cannot be characterized accurately. There is a growing consensus that a unified approach is necessary to monitor, characterize, and model distribution of soil moisture over a range of scale.

In order to accurately characterize the soil moisture distribution, observations should be obtained on a high-resolution basis. In general, the measurement of soil moisture data can be categorized as direct and indirect methods. Direct measurement, namely, is based on ground-based monitoring at discrete data points. Collection of high-resolution soil moisture data through direct measurement is labor-intensive, time-consuming, and thus impractical. On the other hand, indirect measurement refers to obtaining soil moisture from other easily measured complementary data, such as remotely sensed brightness temperature, topography and soil properties. Currently, passive microwave remote sensing is arguably the most feasible way to measure soil brightness temperature and derive soil moisture over large areas. However, calibration of
such technique is still a difficult problem that requires the knowledge of the relationship between soil moisture and environmental factors (e.g., topography, soil properties, radiation, precipitation, and vegetation) [Famiglietti et al., 1999]. One of the objectives of this study is to derive a unified expression that can link soil moisture distribution with other environmental factors.

Among the environmental factors controlling soil moisture distribution, the soil properties, topography and mean soil moisture have drawn significant attention because their influence is easily measured from observational and modeling viewpoint. Topography introduces gradient of hydraulic head that forces the gravity flow. Soil properties affect the distribution of soil moisture through their differential capacity of in holding, conducting and storing water. Mean soil moisture reflects the wet or dry soil condition over large scale. As the soil dries down, the effects of topography decrease due to the lowering of average hydraulic conductivity [Yeh and Eltahir, 1998]. A number of previous studies have explored the influence of one of these three factors (soil properties, topography and mean soil moisture) on soil moisture distribution [Hills and Reynolds, 1969; Reynolds, 1970; Henninger et al., 1976; Bell et al., 1980; Hawley et al., 1982; Hawley et al., 1983; Ladson and Moore, 1992; Charpentier and Groffman, 1992; Niemann and Edgell, 1993; Robinson and Dean, 1993; Western et al., 1999; Famiglietti et al., 1999, among others]. It is extremely difficult to infer the relative importance of each of these factors [Famiglietti et al., 1998]. There are, however, few studies that attempt to investigate the influence of multiple factors on the soil moisture distribution [i.e., Reynolds, 1970; Nyberg, 1996; Crave and Gascuel-Odoux, 1997; Famiglietti et al.,
A considerable amount of apparent contradiction exists in the literature regarding the relative influence of multiple factors [for detailed review, see Chapter 2, Section 2.1].

Stochastic analysis has the potential to investigate the combined influence of environmental factors on soil moisture distribution [Gelhar, 1993]. A series of papers by Mantoglou and Gelhar [1987a, 1987b, 1987c] presented a stochastic methodology to derive a large-scale model of transient unsaturated flow in spatially variable soil formations. Their resulting model relates the variability of capillary pressure with the variability and covariance of various soil parameters. Yeh and Eltahir [1998] formulated the stochastic problem of water flow in unsaturated zones, in which topography is the main forcing term for the movement and distribution of soil moisture. In reality, both soil physical properties and topography will control spatial variations of soil moisture over large areas. To our knowledge, there is no stochastic approach attempt to incorporate the effects of mean soil moisture, spatially variable topography, and soil heterogeneity on soil moisture distribution. In this study, a stochastic framework will be developed to investigate the mutual dependency among soil moisture distribution and above three environmental factors.

The application of the proposed stochastic framework requires the information of topographic and soil distribution. Topographic data can be obtained from digital elevation model (DEM) by U.S Geological Survey (USGS) at a resolution as fine as 30 m. However, the estimation of soil properties over large area is a challenging task. Field
measurement of soil properties is laborious, time-consuming and expensive. The soil survey maps available from national or local agencies (e.g. U.S. Department of Agriculture (USDA)) are derived from a small number of soil samples interpolated onto maps. For example, the soils maps for the Williams River catchment, which are provided at a resolution of 1km, are derived from 46 field collected soil samples over the 1300 km² of the catchment. While interpolation shortens the time and labor necessary to collect data, given the heterogeneity of soil properties, such an interpolation makes the resulting texture database highly uncertain at finer resolution. Therefore, development of a method that can render reliable estimation of soil texture at finer resolution would be of great value.

The remotely sensed, especially by electronically scanned thinned array radiometer (ESTAR), brightness temperature images has been shown to have the potential to detect the soil heterogeneity over large areas [Hollenbeck et. al, 1996]. The relationship between brightness temperature and soil properties is complicated and non-linear because the state of soil moisture is influenced by a variety of environmental factors (e.g. precipitation, vegetation, etc.). A number of previous studies have explored the feasibility of estimating soil texture through brightness temperature and soil data from limited number of sampled points [i.e., Camillo et al., 1984; Hollenbeck et. al, 1996; Mattikalli et al., 1995]. Many of these methodologies, however, are either based on land-atmosphere model that requires several input data or based on regression methods that provide non-unique solutions [for detailed review, see Chapter 4, Section 4.1]. One possibility to link the non-linear relationship between brightness temperature and soil
properties is through Artificial Neural Networks (ANN). ANN, although at its early stages of hydrologic applications, are rapidly becoming an attractive tool to characterize, model, and predict complex multi-source remotely sensed hydrologic data.

To address and consider the issues reviewed above, this study will attempt to (i) understand the combined influence of soil properties, topography, and mean soil moisture on the distribution of soil moisture through a stochastic analysis; and (ii) provide a simplified methodology for estimating soil properties over large area through remotely sensed data and artificial neural networks. Specific objectives for this dissertation are to:

- Develop a stochastic framework for relating the spatial variation of soil moisture to mean soil moisture as well as the variation of topography and soil properties.
- Evaluate the combined influence of topography, soil properties and mean soil moisture on the distribution of soil moisture, and identify the main factors that control soil moisture variability.
- Construct two ANN architectures (Feed-Forward Neural Network (FFNN), Self Organizing Map (SOM)) based on the physical relationship among brightness temperature, soil moisture, and soil properties. Use these ANN models to classify soil into three soil textural groups (coarse, medium, and fine).
- Refine the ANN architectures to classify soil into more than three soil textural groups. Estimate the limits of accuracy that can be achieved for the estimation of soil properties based on remote sensing and ANN.
This dissertation hopes to deliver a compact methodology to evaluate the combined effects of various environmental factors on soil moisture distribution and provide a simplified methodology for obtaining soil textural data over large areas. Although the proposed methodology only focuses on the combined effects of topography, soil properties, and mean soil moisture, it can be extended to evaluate the effects other environmental factors such as precipitation. Expected results have the potential to characterize soil moisture distribution without heavily relying on ground-based measurements.

1.2. Organization of the Dissertation

Following five chapters provide detailed literature review, formulate specific questions, outline approaches, and present key findings that address above objectives. The first two objectives are addressed in Chapter 2 and 3 and the last two objectives are tackled in Chapter 4, and 5. Summary, conclusion, and future works are presented in Chapter 6.

Chapter 2 starts with a more detailed review of the previous attempts attempting to use different environmental factors to characterize soil moisture distribution and outlines the issues that will be addressed by the proposed stochastic frameworks. The proposed stochastic frameworks may be summarized as: (i) Formulate the equation of unsaturated flow based on mass balance theory and widely accepted physical principles, (ii) Decompose the continuum equation into large-scale (spatial mean) and small-scale (perturbation) equations, and (iii) Use spectral theory to convert the small-scale equation into statistical expressions that relate the variation of unsaturated hydraulic properties
(i.e., unsaturated hydraulic conductivity, capillary pressure, and soil moisture) with the variation of environmental factors. The product of this chapter is a theoretical framework, with tractable analytical solutions that relates the spatial distribution of soil moisture to mean soil moisture as well as the statistical properties of soil physical properties and topography.

The resulting model of Chapter 2 is evaluated in Chapter 3. We first discuss the physical and mathematical meaning of relevant parameters of the stochastic model. The typical values of relevant parameters are obtained or estimated from the literature. Various simulations are performed based on reasonable values of relevant parameters. The combined effect of topography, soil properties, and mean soil moisture on the distribution of soil moisture is evaluated. The major findings of stochastic analysis are validated with several observation-based approaches from the literature.

In Chapter 4 and 5, we explore the feasibility of estimating soil physical and hydraulic properties using only remotely sensed data with little or no information about soil texture. In Chapter 4, we provide a detailed review of different approaches to estimate soil properties using remote sensing. The physical linkage among soil moisture, remotely sensed brightness temperature, and soil properties are discussed. Apparent linkage of these three attributes are illustrated using remotely sensed data from Washita 92’ Experiment. A general description and theoretical background of ANN are also addressed. In this chapter, we first construct two Artificial Neural Network (i.e., SOM and FFNN) models based on the physical linkages among space-time distribution of
brightness temperature, soil moisture and soil media properties. Using a sequence of remotely data from Washita 92' experiment, we will show that it is possible to classify soil texture into three categories based on multi-temporal brightness temperature and soil moisture data.

In Chapter 5, we focus on the limit of classification accuracy by using brightness temperature data and ANN to classify soil types into more than three categories. This chapter takes a closer look at the performance of the FFNN. In particular, we focus on the ways of minimizing the error made by the FFNN in categorizing the soil texture into more than three categories. In part, the questions that arise include the number of wetting and drying cycles (amount of time) for which a pixel must be observed to categorize it as well the topology and other associated parameters of the FFNN. Our results motivate the use of a prototype-based classifier as being more appropriate for predicting the soil texture based on the brightness temperature. We will show that using a simple prototype based classifier (1-Nearest Neighbor or 1-NN in short) can obtain significantly better classification accuracy. Finally, the limit of classification accuracy is discussed by comparing and contrasting the inaccurately predicted regions by both FFNN and 1-NN models.
CHAPTER 2

Effects of Topography, Soil Properties and Mean Soil Moisture on the Spatial Distribution of Soil moisture: A Stochastic Analysis
2.1. Introduction

For each kilogram of water on Earth, only one milligram is stored as soil moisture. Yet this miniscule amount of water exerts significant control over various hydrological, ecological and meteorological processes ranging from boundary layer dynamics to the global water cycle. There is a growing consensus that a unified approach is necessary to monitor, characterize, and model distribution of soil moisture over a range of scale.

Analysis of several field-measured soil moisture data [Hills and Reynolds, 1969; Reynolds, 1970; Henninger et al., 1976; Bell et al., 1980; Hawley et al., 1982; Hawley et al., 1983; Ladson and Moore, 1992; Charpentier and Groffman, 1992; Niemann and Edgell, 1993; Robinson and Dean, 1993; Famiglietti et al., 1998; Famiglietti et al., 1999, Western et al., 1999; among others] and remote sensing data [Rodriguez-Iturbe et al., 1995; Hu et al., 1997; among others] has been carried out to characterize and model the spatial variation of soil moisture. Such analyses provide fundamental knowledge about the dependencies between soil moisture distribution and environmental factors. Several recent studies suggested that topography, soil physical properties, vegetation, and climate are the key environmental factors that control soil moisture variations over large scales [e.g., Yeh and Eltahir, 1998; Famiglietti et al., 1998].

A considerable amount of apparent contradiction, however, appears in the literature about the influence of these factors. With respect to topography, several studies [e.g. Hawley et al., 1983; Famiglietti et al., 1998] noted the influence of topography on soil moisture variability. Charpentier and Groffman [1992], Niemann and Edgell [1993]
and Ladson and Moore [1992] found no obvious relationship between topography and soil moisture. Hawley et al., however, [1983] reported that the variation of soil types has minimum impact on the soil moisture variation. Rodriguez-Iturbe et al [1995], on the other hand, suggested that the spatial organization of soil moisture is a consequence of that of soil properties. Chang and Islam [2000] illustrated the capability of inferring the spatial map of soil properties through remotely sensed brightness temperature maps, in which soil moisture is strongly related with brightness temperature [Jackson et al., 1995]. Several investigations have suggested that the variance of soil moisture increases with increasing mean soil moisture [Hills and Reynolds, 1969; Reynolds, 1970; Henninger et al., 1976; Bell et al., 1980; Hawley et al., 1982; Robinson and Dean, 1993 and Famiglietti et al., 1998]. Charpentier and Groffman [1992], however, found no systematic relationship between the variance of soil moisture and mean soil moisture. Famiglietti et al. [1999] reported that variance of soil moisture decreases with increasing mean soil moisture.

In order to perform a reliable statistical analysis that can decipher dependencies between soil moisture and other environmental factors, one needs a large number of samples under different environmental conditions. Due to the time and economic constraints related to field measurement and remote sensing data, inferred relationships between soil moisture and other environmental factors are often site-specific. It is possible that the reported controversies or apparent inconsistencies in the interpretation of soil moisture data come from certain site-specific parameters, for example the differences in mean soil-textural type and the correlation scale of topography, among others.
To overcome the problem of limited information about the spatial soil variability and topography, a stochastic framework has been suggested. This approach assumes that spatial variability of an attribute (e.g. soil physical properties or topography) is a realization of a random field. The large-scale model structure is then derived by averaging local governing flow equation over the ensemble of realization of the underlying soil properties or topography random field. Mantoglou and Gelhar [1987] presented a stochastic methodology to derive a large-scale model of transient unsaturated flow in spatially variable soil formations. Yeh and Eltahir [1998] formulated the stochastic problem of water flow in unsaturated zone such that topography becomes the forcing term for the movement and distribution of soil moisture. In reality both soil physical properties and topography will control spatial variations of soil moisture over large areas. It is conceivable under certain situations topographic control will dictate the distribution of soil moisture while in some other cases soil physical properties will be the key factor that controls variations of soil moisture.

A primary objective of this study is to develop a general stochastic methodology for relating the spatial variation of soil moisture to mean soil moisture as well as the variation of topography and soil properties. The stochastic methodology explicitly considers the effects of spatial variability of soil physical properties and topography. A product of this chapter will be a theoretical framework, with tractable analytical solutions, that relates the spatial distribution of soil moisture to statistical properties of soil physical properties and topography. Clearly, the problem is quite complex and several
assumptions will be needed to arrive at analytically tractable results. Nevertheless, a
general stochastic framework would allow us to better understand the spatial organization
of soil moisture with explicit considerations for heterogeneity in soil physical properties
and topography.

We will examine the effects of topography, soil properties and mean soil moisture
on the soil moisture distribution at the root zone of shallow unsaturated soil. The
interaction between root zone and groundwater table as well as the effects from other
environmental factors, such as vegetation, will not be addressed in this study. This
chapter is organized as the follows. In Section 2.2, a general stochastic framework for
characterizing the soil moisture distribution is outlined. In Section 2.3, the general
problem considered in this study is formulated and the perturbation method is applied to
solve the problem. In Section 2.4, a preliminary analysis based on one-dimensional
analytical solution of the problem is provided. The discussion and conclusions are
included in the last section.

2.2 Methodology

The general methodology to characterize soil moisture distribution is summarized
schematically in Figure 2.1. The problem is formulated as a soil-water continuum
equation including source and sink terms. The input is statistical information about
topography, soil type, and climatic variables (precipitation and evaporation). The water
source is from effective precipitation while evaporation--formulated as functions of
precipitation and mean soil moisture-- serves as the sink of mean soil moisture. The
perturbation method is applied to solve the soil moisture dynamics. The resulting model
formulates the spatial distribution of soil moisture as a function of topography, soil properties, and mean soil moisture. We will compare and contrast the modeled results with previous studies based on analysis and interpretation of field measurements.

**Figure 2.1. Methodology of Proposed Stochastic Framework**

**Stochastic Analysis**
1. Formulate the soil-water continuum equation relating moisture flow with topography, soil properties, precipitation and evaporation.
2. Decompose the continuum equation into large-scale (spatial mean) and small-scale (perturbation) equations.
3. A spectral theory is applied to convert the small-scale equation into statistical expressions that relate the distribution of soil wetness with that of topography, soil properties and mean soil moisture.

**Observed Data Analysis**
Observed relationship between the distribution of soil moisture and that of topography, soil properties and information about mean soil moisture.

**Validation**
Compare the proposed statistical expressions with observed data analysis.

**Resulting Model**
Characterization of the spatial distribution of soil wetness as a function of topography, soil properties and mean soil moisture.
In particular, we will focus on the effects of topography, soil properties and mean soil moisture on the spatial distribution of soil moisture. We are not trying to render a comprehensive study that includes all related effects from these three environmental factors. Rather, this study focuses on the following aspects. For soil properties, we are interested in the soil parameters needed to characterize the soil-water characteristic curve (i.e., the $\psi-K-\theta$ relationship). Other properties such as soil color and the existence of macro-porosity and organic matters are not discussed in this study. For topography, we focus on the effects of relative elevation. Topographic indices, such as aspects, specific contributing areas and curvature, are beyond the scope of this study. Furthermore, we are interested in the following spatial scales: (1) horizontal scale smaller than 10 km so that the heterogeneous precipitation are assumed to have minor impacts at such scales, and (2) vertical spatial scale (soil depth) of upper 1 m so that we assume that the vertical moisture below 1 m is much smaller than that in upper depth. In this study the soil moisture content is represented by normalized soil moisture ($\eta$), defined as $\eta = (\theta - \theta_r) / (\theta_s - \theta_r)$ where $s$ and $r$ represent, the saturated and residual values of volumetric soil moisture ($\theta$), respectively.

2.3. Stochastic Analysis of Steady-State Unsaturated Flow

In this section, we develop a stochastic framework for characterizing the spatial distribution of soil moisture. The proposed approach assumes that the spatial variables (soil properties, unsaturated soil hydraulic properties and elevation) are realizations of a two-dimensional, cross-correlated, second-order stationary random field. Similar
assumption has also been made by other stochastic analyses [i.e., Mantoglou and Gelhar, 1987; Yeh and Eltahir, 1998].

We begin with a Richardson-type equation derived by Yeh and Eltahir [1998]. They have combined the continuum equation with Darcy’s Law to develop the equation of moisture flux in shallow soil in a two-dimensional domain (see Appendix 2.A):

\[
D \frac{\partial \Theta}{\partial t} = D \frac{\partial}{\partial x_i} \left[ K \frac{\partial (-\psi + z)}{\partial x_i} \right] - s + R = 0
\]  

(2.1)

where the direction \(x_i\) (\(i = 1, 2\)) represent the horizontal coordinates, \(D\) is the depth of root zone with the assumption that the soil below such depth is completely impermeable, \(\Theta\) is volumetric soil moisture, \(K\) is unsaturated hydraulic conductivity, \(\psi\) is hydraulic suction of water, \(z\) is elevation (positive upward), \(s\) is evaporation flux, parameterized as \(s = \beta K\) by assuming that evaporation is proportional to unsaturated hydraulic conductivity where \(\beta\) is equivalent to the vertical hydraulic gradient near land surface, and \(R\) is effective rainfall defined as the portion of rainfall infiltrating into soil. The typical ranges of above relevant parameters will be discussed in Section 5. In the following, a steady state case is considered. This assumption is reasonable when we consider a long time scale. Similar to Yeh and Eltahir [1998] we also assume that the parameters \(D\), \(\beta\) and \(R\) are homogeneous in space. A key difference between Yeh and Eltahir [1998] and this study is the inclusion of effects of soil heterogeneity.
Three unsaturated soil hydraulic parameters \((K, \theta, \psi)\) of equation (2.1) are correlated with each other through the so-called soil hydraulic functions (i.e. \(K-\theta-\psi\) relationship). A significant knowledge base exists, in the soil physics and groundwater literature that focuses on the development and refinement of methods to describe such \(K-\theta-\psi\) relationship. Most of them, however, are expressed in a highly nonlinear fashion and consequently analytical result is extremely difficult to obtain when such relationships are used with other modeling approaches. As an alternative, a linear approximation of \(\psi\theta\) relationship [Mantoglou and Gelhar, 1987],

\[
\theta = \theta_s - C\psi 
\]  

(2.2)

and a quasilinear approximation of \(\psi K\) relationship [Gardner, 1958],

\[
\psi = -B \ln(K / K_s) 
\]  

(2.3)

have been applied [Yeh et al., 1985; Mantoglou and Gelhar, 1987; Yeh and Eltahir, 1998], where \(C\) is the specific capacity of soil moisture, \(B\) is the thickness of capillary fringe and \(K_s\) is the saturated hydraulic conductivity. Such approximations of soil hydraulic functions provide a dramatic simplification for various modeling applications. Although the gains in simplification are achieved at the cost of accuracy, the reality is that even the most complicated soil hydraulic functions cannot precisely address the behavior of soil-moisture flow because so much is unknown and uncertain in the unsaturated porous media.

By converting the volumetric soil moisture into normalized soil moisture, equation (2.3) becomes:

\[
\eta = 1 - C_s\psi 
\]  

(2.4)
where \( C_s = C / (\Theta_s - \Theta_r) \) represents the specific capacity of normalized soil moisture. By combining Equation (2.3) and (2.4) to eliminate \( \psi \) one can derive the \( K-\eta \) relationship as below:

\[
\eta = 1 + A \ln \left( \frac{K}{K_s} \right)
\]  
(2.5)

where \( A \), a dimensionless soil parameter, is defined as the product of \( B \) and \( C_s \). Equation (2.5) shows a power-law relationship between normalized soil moisture and hydraulic conductivity. Similar power-law functions have also been reported by Gardner et al. [1970] and Campbell [1974]. Note that equations (2.3–2.5) involve three soil parameters \( (A, B, \text{ and } K_s) \), while the parameter \( C_s \) can be termed as \( C_s = A/B \). Usually, these parameters are considered independent of unsaturated soil hydraulic properties \( (\psi, K \text{ and } \Theta) \) and assumed to vary only as a function of soil type. Mantoglou and Gelhar [1987] argue that such constant assumption is valid for an intermediate range of \( \psi \) values provided local hysteresis is relatively small. In this study, the linear-type expressions of \( K-\psi \) relationship (equation 2.3) and \( K-\eta \) relationship (equation 2.5) will be applied to equation (2.1) in order to represent the soil-water flow equation with a single-dependent variable \( \eta \).

Following Mantoglou and Gelhar [1987], a stochastic analysis is applied to the steady-state unsaturated flow equation (equation 2.1), where the soil hydraulic functions are described by equation (2.3–2.5). The proposed stochastic analysis involves a two-step approach: (i) derivation of the large-scale and perturbation model (see Appendix 2.B), and (ii) solution of the perturbation equations using spectral techniques (see Appendix 2.C). Consequently, we derive the following statistical relationship that characterizes the
spatial distribution of normalized soil moisture as a function of topography, soil properties and mean soil moisture:

\[
\sigma^2 = V_{K_xK_y} \sigma^2_{K_x} + V_{BB} \sigma^2_B + V_{zz} \sigma^2_z + V_{AA} \sigma^2_A \\
+ 2(V_{AB} \sigma_A \sigma_B + V_{AK_x} \sigma_A \sigma_{K_x} + V_{A_z} \sigma_A \sigma_z) \\
+ V_{BK_x} \sigma_B \sigma_{K_x} + V_{Bz} \sigma_B \sigma_z + V_{K_xz} \sigma_{K_x} \sigma_z 
\]

(2.6a)

\[
\sigma_{\eta A} = X_{AA} \sigma^2_A + X_{BA} \sigma_A \sigma_B + X_{K_xA} \sigma_A \sigma_{K_x} + X_{A_z} \sigma_A \sigma_z 
\]

(2.6b)

\[
\sigma_{\eta B} = X_{AB} \sigma_A \sigma_B + X_{BB} \sigma^2_B + X_{K_xB} \sigma_B \sigma_{K_x} + X_{zB} \sigma_B \sigma_z 
\]

(2.6c)

\[
\sigma_{\eta K_x} = X_{AK_x} \sigma_A \sigma_{K_x} + X_{BK_x} \sigma_B \sigma_{K_x} + X_{K_xK_x} \sigma^2_{K_x} + X_{zK_x} \sigma_{K_x} \sigma_z 
\]

(2.6d)

\[
\sigma_{\eta z} = X_{Az} \sigma_A \sigma_z + X_{Bz} \sigma_B \sigma_z + X_{K_xz} \sigma_{K_x} \sigma_z + X_{zz} \sigma^2_z 
\]

(2.6e)

To simplify the notation, here we use the variables \( u \) and \( v \) to represent \( K_x, A, B \) or \( z \). In equation (2.6), \( \sigma_u^2 \) and \( \sigma_v \) represent the variance and standard deviation of variable \( u \), respectively, \( \sigma_{uv} \) represents the covariance between variable \( u \) and \( v \), and the scalars \( V_{uv} \) and \( X_{uv} \) are defined as:

\[
V_{uv} = V_{uv} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(k_x + k_y)\tau} W_u W_v \frac{S_{uv}(k_x, k_y)}{\sigma_u \sigma_v} dk_x dk_y \bigg|_{\tau=0}
\]

(2.7)

\[
X_{uv} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(k_x + k_y)\tau} W_u \frac{S_{uv}(k_x, k_y)}{\sigma_u \sigma_v} dk_x dk_y \bigg|_{\tau=0}
\]

where \( x, y \) are spatial coordinates, \( k_x, k_y \) are wave numbers in \( x \) and \( y \) directions, \( i = \sqrt{-1} \), \( S_{uv} \) is the spectral density function between \( u \) and \( v \), and \( W_u \) is a function of large-scale soil and topographic properties, wave numbers, effective rainfall \( (R) \), root zone soil depth \( (D) \) and vertical hydraulic gradient \( (\beta) \):
\[ W_{K_s} = \frac{A}{K_s}(W_0 - 1), W_B = \left(1 - \frac{A}{B}\right)W_0, W_z = -\frac{A}{B}W_0, W_A = \left(1 - \frac{A}{B}\right), \]

\[ W_0 = \frac{(k_x^2 + k_y^2)}{(k_x^2 + k_y^2) + \beta_i}, \quad \beta_i = \frac{\beta}{BD} = \frac{R_0}{K_s e^{-\frac{1-\pi}{\alpha}}}, \quad R_0 = \frac{R}{D} \quad (2.8) \]

2.4. Preliminary Analysis

A general stochastic theory for characterizing the spatial distribution of soil moisture was provided in the previous section. The resulting model is a function of the variance of soil properties and the variance of topography as well as the covariance between topography and soil properties. Evaluation of the resulting model (equation 2.6) requires knowledge of the covariance and cross-covariance functions of soil properties \((A, B, K_s)\) and topography \((\xi)\). In this section, we provide some analytical results of the proposed stochastic model (equation 2.6) based on the frequently used exponential covariance functions [Gelhar, 1993] in one-dimensional horizontal field. A comparison between our analytical results and those of Yeh and Eltahir [1998] will also be discussed.

2.4.1 Evaluation of the Soil Moisture Variance and its Covariance with Topography and Soil Properties

Previous studies have used exponential covariance functions for the topographic [Yeh and Eltahir, 1998] and soil distribution [Przewłocki, 2000]. Here, we consider the one-dimensional unsaturated flow with explicit considerations for heterogeneous topography and heterogeneous soil properties. The covariance-spectrum pair for the exponential covariance function is expressed as:
\[
\frac{R_u(\tau)}{\sigma_u^2} = \exp\left(-\frac{\tau}{\lambda_u}\right)
\]

\[
\frac{S_u(k)}{\sigma_u^2} = \frac{\lambda_u}{\pi(1 + \lambda_u^2 k^2)}
\]

(2.9)

where the variable \( u \) represents \( K_s, A, B \) or \( z \); \( \tau \) is the lag, \( \lambda \) is the correlation scale and \( k \) is the wave number in the horizontal flow direction.

The assignments of cross-covariance functions of soil properties and topography need attention. Since the work of Jenny [1941], it has been a well-known fact in soil science that topography is an important factor in influencing soil distribution. There are virtually no systematic field studies that quantify cross covariance properties of topography and soil properties. To develop some insights regarding such relationships between topography and soil properties, here we examine the relationship among \( A, B, K_s \) and \( z \) for the following three limited cases: (1) \( A, B, K_s \) and \( z \) being uncorrelated, (2) the \( A, B \) and \( K_s \) are perfectly correlated but the soil properties are uncorrelated with \( z \) and (3) \( A, B, K_s \) and \( z \) are perfectly correlated. Above three limited cases are selected in order to evaluate the relationship between \( \sigma_{A}^2 \) and \( \sigma_{\eta_i}^2 \) (\( i = A, B, K_s \) or \( z \)). Let \( \xi^2 \), \( \zeta^2 \) and \( \omega^2 \) be the ratios of the variance of \( B, K_s \) and \( z \) to the variance of \( A \), i.e.,

\[
\xi^2 = \sigma_{\kappa}^2 / \sigma_{A}^2
\]

\[
\zeta^2 = \sigma_{B}^2 / \sigma_{A}^2
\]

(2.10)

\[
\omega^2 = \sigma_{z}^2 / \sigma_{A}^2
\]

There cross-spectral density functions of \( (B, K_s \text{ and } z) \) are then related to that of \( A \) by:
Case 1: A, B, Ks, z uncorrelated

\[ S_{KsKs} = \xi^2 S_{AA} \]
\[ S_{BB} = \xi^2 S_{AA} \]
\[ S_{AKs} = 0 \]
\[ S_{AB} = 0 \]
\[ S_{BKs} = 0 \]
\[ S_{zA} = 0 \]
\[ S_{zKs} = 0 \]
\[ S_{zB} = 0 \]

Case 2: A, B, Ks perfectly correlated but uncorrelated with z

\[ S_{KsKs} = \xi^2 S_{AA} \]
\[ S_{BB} = \xi^2 S_{AA} \]
\[ S_{AKs} = \xi S_{AA} \]
\[ S_{AB} = -\xi S_{AA} \]
\[ S_{BKs} = -\xi S_{AA} \]
\[ S_{zA} = 0 \]
\[ S_{zKs} = 0 \]
\[ S_{zB} = 0 \]
Case 3: $A$, $B$, $K_s$ and $z$ perfectly correlated

\[
\begin{align*}
S_{KAK_s} &= \zeta^2 S_{AA} \\
S_{BB} &= \zeta^2 S_{AA} \\
S_{zz} &= \omega^2 S_{AA} \\
S_{AK_s} &= \zeta S_{AA} \\
S_{AB} &= -\zeta S_{AA} \\
S_{BK_s} &= -\zeta^2 S_{AA} \\
S_{cA} &= s \omega S_{AA} \\
S_{cK_s} &= s \omega \zeta S_{AA} \\
S_{zB} &= -s \omega \zeta S_{AA}
\end{align*}
\]

where $s$ is a switch with a value of either $+1$ or $-1$. The switch $s = +1$ represents that the study area contain more coarse-texture soil in higher elevation than that in lower elevation and $s = -1$ represents the opposite case. Such a switch is necessary to explicitly acknowledge the nature of correlation between soil physical properties and topography. For instance, if higher elevation has coarser-texture soil and lower elevation has finer-texture soil than the correlation between saturated hydraulic conductivity and topography is positive. On the other hand, if higher elevation has finer-texture soil and lower elevation has coarser-texture soil than the correlation between saturated hydraulic conductivity and topography will be negative. It is a commonly accepted in soil science that finer-texture soil often appears in low-lying places and better-drained soils are located in higher lying areas [Jenny, 1941]. Therefore, we will use $s = +1$ for subsequent calculation. Also note that finer-texture soils generally have smaller $\bar{A}$ and $\bar{K}_s$ values.
but larger $\bar{B}$ values; and coarser-texture soils have larger ($\bar{A}, \bar{K}_s$) values but smaller ($\bar{B}$) (See Appendix 2.D for the typical values of $A$, $B$, and $K_s$ for various soil types). Consequently, the negative signs appear in $S_{BK_s}$ and $S_{AB}$.

Applying equations (2.11), (2.12) and (2.13) to equation (2.6) yields the following results:

Case 1: $A, B, K_s, z$ uncorrelated

\[
\sigma^2_\eta = \left(\xi^2 \tilde{V}_{k,k} + \zeta^2 \tilde{V}_{BB} + \tilde{V}_{AA}\right)\sigma^2_A + V_{zz} \sigma^2_z \tag{2.14a} \\
\sigma_{\eta A} = \tilde{X}_A \sigma^2_A \tag{2.14b} \\
\sigma_{\eta B} = \zeta^2 \tilde{X}_B \sigma^2_A \tag{2.14c} \\
\sigma_{\eta K_s} = \xi^2 \tilde{X}_{K_s} \sigma^2_A \tag{2.14d} \\
\sigma_{\eta z} = X_{zz} \sigma^2_z \tag{2.14e}
\]

Case 2: $A, B, K_s$ perfectly correlated but uncorrelated with $z$

\[
\sigma^2_\eta = \left(\xi^2 \tilde{V}_{k,k} + \zeta^2 \tilde{V}_{BB} + \tilde{V}_{AA} - 2\xi\xi \tilde{V}_{k,B} + 2\xi\tilde{V}_{k,A} - 2\xi\tilde{V}_{BA}\right)\sigma^2_A + V_{zz} \sigma^2_z \tag{2.15a} \\
\sigma_{\eta A} = \left(\tilde{X}_A + \xi \tilde{X}_{K_s} - \zeta \tilde{X}_B\right)\sigma^2_A \tag{2.15b} \\
\sigma_{\eta B} = -\zeta \left(\tilde{X}_A + \xi \tilde{X}_{K_s} - \zeta \tilde{X}_B\right)\sigma^2_A \tag{2.15c} \\
\sigma_{\eta K_s} = \xi \left(\tilde{X}_A + \xi \tilde{X}_{K_s} - \zeta \tilde{X}_B\right)\sigma^2_A \tag{2.15d} \\
\sigma_{\eta z} = X_{zz} \sigma^2_z \tag{2.15e} \]
Case 3: A, B, \( K_s \) and \( z \) perfectly correlated

\[
\sigma_\eta^2 = (\xi^2 V_{K_s K_s} + \xi^2 V_{BB} + \omega^2 V_{zz} + V_{AA} - 2\xi \tilde{V}_{K_s B} + 2\xi \tilde{V}_{K_s A} - 2\xi \tilde{V}_{B A} \sigma_{BA} + 2\alpha \tilde{V}_{K_s z} - 2\alpha \tilde{V}_{B z} + 2\alpha \tilde{V}_{z A}) \sigma_A^2
\]  
(2.16a)

\[
\sigma_{\eta A} = (\tilde{X}_A + \xi \tilde{X}_{K_s} - \xi \tilde{X}_B + \omega \tilde{X}_z) \sigma_A^2
\]  
(2.16b)

\[
\sigma_{\eta B} = -\xi (\tilde{X}_A + \xi \tilde{X}_{K_s} - \xi \tilde{X}_B + \omega \tilde{X}_z) \sigma_A^2
\]  
(2.16c)

\[
\sigma_{\eta K_s} = \xi (\tilde{X}_A + \xi \tilde{X}_{K_s} - \xi \tilde{X}_B + \omega \tilde{X}_z) \sigma_A^2
\]  
(2.16d)

\[
\sigma_{\eta z} = \omega (\tilde{X}_A + \xi \tilde{X}_{K_s} - \xi \tilde{X}_B + \omega \tilde{X}_z) \sigma_A^2
\]  
(2.16e)

In equation (2.14-2.16), the scalars \( \tilde{V}_{uv} \), \( \tilde{X}_u \), \( V_{zz} \) and \( X_{zz} \) are defined as:

\[
\tilde{V}_{uv} = \tilde{V}_{vw} = \iint_{-\infty \to \infty} e^{i(k_x + k_y)\tau} W_u W_v \frac{S_{AA}(k_x, k_y)}{\sigma_A^2} dk_x dk_y \bigg|_{\tau=0}
\]

\[
\tilde{X}_u = \iint_{-\infty \to \infty} e^{i(k_x + k_y)\tau} W_u \frac{S_{AA}(k_x, k_y)}{\sigma_A^2} dk_x dk_y \bigg|_{\tau=0}
\]  
(2.17)

\[
V_{zz} = \iint_{-\infty \to \infty} e^{i(k_x + k_y)\tau} W_z^2 \frac{S_{zz}(k_x, k_y)}{\sigma_z^2} dk_x dk_y \bigg|_{\tau=0}
\]

\[
X_{zz} = \iint_{-\infty \to \infty} e^{i(k_x + k_y)\tau} W_z^2 \frac{S_{zz}(k_x, k_y)}{\sigma_z^2} dk_x dk_y \bigg|_{\tau=0}
\]

where the variables \( u \) and \( v \) represent \( K_s \), \( A \), \( B \) or \( z \).

Several important features regarding the relationship between soil properties and topography may be identified from equations 2.14-2.16. First, if \( A \), \( B \), \( K_s \) and \( z \) are uncorrelated (Case 1), variability in soil moisture is composed of individual variability of soil properties and topography (i.e., equation 2.14a). In other words, variability of soil
moisture can be viewed as a sum of four random variables \((A, B, K_s, \text{and } z)\). In such cases, covariance between soil moisture and the attributes (i.e., \(A, B, K_s, \text{or } z\)) will be a function of those attributes only. For instance, covariance between soil moisture and topography (equation 2.14e) is only a function of variability in topography. If soil properties are correlated among themselves but uncorrelated with topography (Case 2), variability in soil moisture is composed of individual variability of soil properties and topography as well as the covariance among soil properties (i.e., equation 2.15a). Similarly, covariance between soil moisture and individual soil property \((A, B \text{ or } K_s)\) will be the results in Case 1 plus the terms with respect to the correlation among soil properties. However, for the covariance between soil moisture and topography, Case 1 and Case 2 yield the same result (i.e., equation 2.14e and 2.15e). In other words, if topography is uncorrelated with soil properties, the correlation among soil properties has no influence on the relationship between topography and soil moisture. In the third case \((A, B, K_s \text{ and } z\) are perfectly correlated), variability in soil moisture is the results in Case 2 plus the terms with respect to the covariance between soil properties and topography. In such cases, the covariance between soil moisture and other attributes \((A, B, K_s \text{ or } z)\) show the following relationship:

\[
\frac{\sigma_{nA}}{\sigma_n \sigma_A} = -\frac{\sigma_{nB}}{\sigma_n \sigma_B} = \frac{\sigma_{nK_s}}{\sigma_n \sigma_{K_s}} = \frac{\sigma_{nz}}{\sigma_n \sigma_z} \tag{2.18}
\]

In other words, in Case 3 the correlation coefficients between soil moisture and any attributes \((A, B, K_s \text{ or } z)\) are the same.
2.4.2 One-Dimensional Analysis

By substituting the covariance functions of soil parameter ($S_{AA}$) and topography ($S_{zz}$) in equation (2.17) with an exponential type covariance model (equation 2.9) and after integration, we obtain the analytical results of the scalars $\tilde{V}_{vv}$, $\tilde{X}_{u}$, $V_{zz}$ and $X_{zz}$ as:

\[
\tilde{V}_{AA} = \left(1 - \frac{\eta}{A}\right)^2
\]

\[
\tilde{V}_{BB} = \left(1 - \frac{\eta}{B}\right)^2 \left[ \frac{g_{\lambda,AA}^{y^2} - 3g_{\lambda,AA}^{y^2} + 2}{2(g_{\lambda,AA} - 1)^2} \right]
\]

\[
\tilde{V}_{K,K} = \left(\frac{A}{K_s}\right)^2 \left[ 1 - \frac{2}{g_{\lambda,AA}^{y^2} + 1} \frac{g_{\lambda,AA}^{y^2} - 3g_{\lambda,AA}^{y^2} + 2}{2(g_{\lambda,AA} - 1)^2} \right]
\]

\[
\tilde{V}_{zz} = \left(\frac{A}{B}\right)^2 \left[ \frac{g_{\lambda,AA}^{y^2} - 3g_{\lambda,AA}^{y^2} + 2}{2(g_{\lambda,AA} - 1)^2} \right]
\]

\[
\tilde{V}_{K,A} = \tilde{V}_{A,K} = \left(\frac{A}{K_s}\right)^2 \left(1 - \frac{1}{g_{\lambda,AA}^{y^2} + 1}\right)
\]

\[
\tilde{V}_{BA} = \tilde{V}_{AB} = -\left(\frac{1 - \eta}{AB}\right)^2 \left(\frac{1}{g_{\lambda,AA}^{y^2} + 1}\right)
\]

\[
\tilde{V}_{zA} = \tilde{V}_{Az} = \left(\frac{1 - \eta}{B}\right) \left(\frac{1}{g_{\lambda,AA}^{y^2} + 1}\right)
\]

\[
\tilde{V}_{K,B} = \tilde{V}_{kB} = \frac{A(1 - \eta)}{BK_s} \left(\frac{2}{g_{\lambda,AA}^{y^2} - 3g_{\lambda,AA}^{y^2} + 2} - \frac{1}{g_{\lambda,AA}^{y^2} + 1}\right)
\]

\[
\tilde{V}_{K,z} = \tilde{V}_{zK} = -\frac{A^2}{BK_s} \left[ \frac{g_{\lambda,AA}^{y^2} - 3g_{\lambda,AA}^{y^2} + 2}{2(g_{\lambda,AA} - 1)^2} - \frac{1}{g_{\lambda,AA}^{y^2} + 1} \right]
\]

\[
\tilde{V}_{Bz} = \tilde{V}_{zB} = -\frac{A(1 - \eta)}{B^2} \left[ \frac{g_{\lambda,AA}^{y^2} - 3g_{\lambda,AA}^{y^2} + 2}{2(g_{\lambda,AA} - 1)^2} \right]
\]
\[ \tilde{X}_A = -\left( \frac{1 - \eta}{A} \right) \]

\[ \tilde{X}_B = \left( \frac{1 - \eta}{B} \right) \left( \frac{1}{g_{\lambda,\alpha}} + 1 \right) \]

\[ \tilde{X}_K = -\left( \frac{A}{K} \right) \left[ 1 - \frac{1}{g_{\lambda,\alpha}} + 1 \right] \]

\[ \tilde{X}_z = -\left( \frac{A}{B} \right) \left( \frac{1}{g_{\lambda,\alpha}} + 1 \right) \]

\[ V_{zz} = \left( \frac{A}{B} \right)^2 \left[ \frac{g_{\lambda,\alpha} - 3g_{\lambda,\alpha} + 2}{2(g_{\lambda,\alpha} - 1)^2} \right] \]

\[ X_{zz} = -\left( \frac{A}{B} \right) \left( \frac{1}{g_{\lambda,\alpha}^2 + 1} \right) \]

where \( g_{\lambda,\alpha} \) and \( g_{\lambda,\alpha} \) are dimensionless parameters defined as:

\[ g_{\lambda,i} = \lambda \beta_i, i = A \text{ or } z \quad (2.20) \]

A computer code (not shown) in Mathematica was written and executed for the required calculation. In the following section, we will apply and discuss the proposed stochastic model based on the preliminary analysis described in this section. The results of these three limited cases will be used to address the three main issues defined in the introduction of this chapter.

Beside the three limited cases discussed in Section 4.1, there are two particular cases of interest: (I) homogeneous soil properties and (II) homogeneous topography. The results of those two cases can be easily obtained by setting \( \sigma_{\lambda}^2 = 0 \), \( A = \bar{A} \), \( B = \bar{B} \) and
\( K_s = \bar{K}_s \) for Case I and \( \sigma_{z}^2 = 0 \) for Case II. The assumption of homogeneous soil properties has been made by Yeh and Eltahir [1998] in developing their stochastic model. Their two main results for characterizing the variation of soil moisture and the covariance between soil moisture and topography, respectively, are as follows:

\[
\sigma_{\theta}^2 = C^2 \left[ \frac{g_{\lambda,z}^2 - 3g_{\lambda,z}^2 + 2}{2(g_{\lambda,z} - 1)^2} \right] \sigma_{z}^2 \tag{2.21a}
\]

\[
\sigma_{\theta z} = -C \left( \frac{1}{g_{\lambda,z} + 1} \right) \sigma_{z}^2 \tag{2.21b}
\]

where \( C \) is the specific capacity of soil moisture, \( \theta \) is the volumetric soil moisture and \( g_{\lambda,z} \) is the same as equation (2.20). Here we provide a brief comparison between our proposed stochastic model in the case of homogeneous soil properties with the stochastic model developed by Yeh and Eltahir [1998]. In the case of homogeneous soil properties, the three limited cases are simplified into one single result:

\[
\sigma_{\eta}^2 = \left( \frac{A}{B} \right)^2 \left[ \frac{g_{\lambda,z}^2 - 3g_{\lambda,z}^2 + 2}{2(g_{\lambda,z} - 1)^2} \right] \sigma_{z}^2 \tag{2.22a}
\]

\[
\sigma_{\eta z} = -\left( \frac{A}{B} \right) \left( \frac{1}{g_{\lambda,z} + 1} \right) \sigma_{z}^2 \tag{2.22b}
\]

We note here that the results in equation (2.22) are exactly the same compared to those of Yeh and Eltahir [1998] (equation 2.21) given the following relationship, as described in Section 3:

\[
\sigma_{\eta}^2 = \frac{\sigma_{\theta}^2}{(\theta_s - \theta_r)^2}
\]

\[
\sigma_{\eta z} = \frac{\sigma_{\theta z}}{\theta_s - \theta_r}
\]
\[
\frac{A}{B} = C_s
\]

(2.23)

\[
C_s = \frac{C}{\Theta_s - \Theta_i}
\]

In addition to the aforementioned three limited cases and two particular cases, there are two extreme cases related to mean soil moisture: Case (a) extremely wet \((\overline{\eta} = 1)\) and Case (b) extremely dry \((\overline{\eta} = 0)\). Case (a) is valid in absence of evaporation (i.e., \(\beta = 0\)); and Case (b) occurs when there is no precipitation (i.e., \(R = 0\)). In these two extreme situations, the three limited cases are simplified into one single result:

\[
\sigma_{\eta}^2 = \sigma_{\eta_h} = \sigma_{\eta_\theta} = \sigma_{\eta_K} = \sigma_{\eta_z} = 0
\]

(2.24)

Equation (2.24) shows mathematical consistency when mean soil moisture is at its extreme value.

### 2.5 Summary and Discussion

A stochastic framework is proposed in this study for characterizing the steady-state soil moisture distribution in a heterogeneous-soil and -topography field under the influence of precipitation and evaporation. The problem of water flow in an unsaturated zone is expressed as a partial differential equation that depends on three stochastic variables: the heterogeneity of soil properties, the variability of topography and the change of mean soil moisture. This is perhaps the first attempt to include the effects of topography and soil properties on soil moisture distribution. A perturbation method and spectral technique are applied to solve the soil moisture dynamics. The resulting model provides closed form analytical solutions for (a) the variance of soil moisture distribution
(\(\sigma_n^2\)), and (b) the covariance between soil moisture distribution and soil properties (\(\sigma_{\text{A}}, \sigma_{\text{B}}, \sigma_{\text{K}_s}\)), and (c) the covariance between soil moisture distribution and topography (\(\sigma_{\text{T}}\)) as a function of soil heterogeneity, topography and mean soil moisture.

The proposed approach assumes that the spatial variables (soil properties, unsaturated soil hydraulic properties and elevation) are realizations of a two-dimensional, cross-correlated, second-order stationary random field. Evaluation of the influences of these spatial variables on the soil moisture distribution requires knowledge of the cross-covariance functions of these variables. There are very little field observations that can quantify cross-covariance functions of topography and soil properties. To develop an insight regarding the interdependencies of these variables and their influence on variability of soil moisture, we focus on three limiting cases in this study: (i) soil properties (\(A, B, K_s\)) and topography (\(z\)) are uncorrelated; (ii) the soil properties are correlated among themselves but uncorrelated with topography; and (iii) soil properties and topography are perfectly correlated.

Several important features regarding the relationship between soil properties and topography may be identified from these three limiting cases. First, if soil properties and topography are uncorrelated, variability of soil moisture can be viewed as a sum of the individual variability of soil properties and topography. In such cases, covariance between soil moisture and the attributes (i.e., \(A, B, K_s\) or \(z\)) will be a function of those attributes only. If soil properties are correlated among themselves but uncorrelated with topography, variability in soil moisture is composed of individual variability of soil
properties and topography as well as the covariance among soil properties. In the third case, when $A$, $B$, $K_s$ and $z$ are perfectly correlated, cross correlation between soil moisture and soil physical properties or soil moisture and topography are equivalent.
Appendix 2.A. Formulation of the Problem

The Richardson-type equation derived by Yeh and Eltahir [1998] assumes that the vertical moisture flux below a root zone depth ($D \approx 1$ m) is much smaller than horizontal moisture flux. Soil moisture in such layer is assumed to be homogeneous and the vertical flux only occurs in the near surface due to evaporation and precipitation.

Figure 2.A.1. A free-body diagram of the soil-water flow in a root zone depth ($D$), where $q$ is the flow velocity in $x_1$, $x_2$ and $x_3$ directions
Conservation of mass of soil moisture moving in a rigid soil matrix (as shown in Figure 2.A.1) leads to:

\[-(\partial x_1 \partial x_2 D) \frac{\partial \rho \Theta}{\partial t} = (\partial x_2 D) \partial \rho q_{x_1} + (\partial x_1 D) \partial \rho q_{x_2} + (\partial x_1 \partial x_2) (\rho q_{x_3,\text{out}} - \rho q_{x_3,\text{in}})\]  \hspace{1cm} (2.A.1)

where \( q \) is the flow velocity in \( x_1, x_2 \) and \( x_3 \) directions and \( \rho \) is the density of water.

Equation (2.A.1) can be simplified as the so-called continuum equation by assuming constant density:

\[-\frac{\partial \Theta}{\partial t} = \frac{\partial q_{x_1}}{\partial x_1} + \frac{\partial q_{x_2}}{\partial x_2} + \frac{q_{x_3,\text{out}} - q_{x_3,\text{in}}}{D}\]  \hspace{1cm} (2.A.2)

The flow velocities in the horizontal directions (i.e., in \( x_1 \) and \( x_2 \) directions) follow Darcy Law:

\[q_{x_1} = -K \frac{\partial (\psi + z)}{\partial x_1}\]  \hspace{1cm} (2.A.3a)

\[q_{x_2} = -K \frac{\partial (\psi + z)}{\partial x_2}\]  \hspace{1cm} (2.A.3b)

where \( q \) is the flow velocity (L/T), \( K \) is unsaturated hydraulic conductivity, \( \psi \) is hydraulic suction of water, and \( z \) is elevation (positive upward). On the other hand, the flow velocity in the vertical direction (i.e., the \( x_3 \) direction) is parameterized as:

\[q_{x_3,\text{in}} = 0\]  \hspace{1cm} (2.A.3c)

\[q_{x_3,\text{out}} = s - R\]  \hspace{1cm} (2.A.3d)
where $s$ is evaporation flux (positive upward) and $R$ is effective rainfall (positive upward).

Combination of the continuum equation (2.A.2) with the equations of flow velocity (2.A.3) leads to equation (2.1):

$$D \frac{\partial \Theta}{\partial t} = D \frac{\partial}{\partial x_i} \left[ K \frac{\partial (-\psi + z)}{\partial x_i} \right] - s + R = 0 \quad (2.1)$$
Appendix 2.B. Derivation of the Large-scale and Perturbation Models

The general form of the unsaturated flow model (equation 2.1) at steady state can be divided into the large-scale and small-scale components. The large-scale model represents the “mean” water balance at large scales. On the other hand, the small-scale perturbation model relates the distribution of soil moisture with heterogeneous topography and soil properties. The first step in the derivation of the large-scale and perturbation models is to decompose the variables $K$, $\psi$, and $z$ into their spatial mean ($K, \bar{\psi}, \bar{z}$) and perturbation terms ($K', \psi', z'$), so that equation (2.1) becomes:

$$\frac{\partial}{\partial x_i} \left[ K' \frac{\partial (-\psi' + z')}{\partial x_i} + K' \frac{\partial (-\bar{\psi} + \bar{z})}{\partial x_i} + K' \frac{\partial (-\bar{\psi} + \bar{z})}{\partial x_i} \right] - \frac{\beta}{D} (K' + K') + \frac{R}{D} = 0$$

(2.B1)

The large-scale model is obtained by averaging the local equation (2.B1) over the entire domain, and it becomes:

$$\frac{\partial}{\partial x_i} \left[ \bar{K} \frac{\partial (-\bar{\psi} + \bar{z})}{\partial x_i} \right] - \frac{\beta}{D} \bar{K} + \frac{R}{D} = -\mathbb{E} \left[ \frac{\partial}{\partial x_i} \left( K' \frac{\partial (-\psi' + z')}{\partial x_i} \right) \right]$$

(2.B2)

If the spatial-dependant variables are assumed as the realization of second-order stationary random field and the expected value of higher-order perturbation terms in the right-hand side of equation (2.B2) is relatively small to other large-scale terms, equation (2.B2) can be further simplified following Yeh and Eltahir [1998]:

$$\beta = \frac{R}{\bar{K}}$$

(2.B3)
Equation (2.B3) implies that the mean soil moisture, which is related to mean unsaturated hydraulic conductivity $\bar{K}$, is the net result of effective rainfall ($R$) and vertical hydraulic gradient ($\beta$). Such relationship is reasonable based on the theory of mass conservation which indicates that the residual moisture is primarily controlled by the input (i.e., effective rainfall) and output (i.e., evaporation) terms.

The perturbation model is obtained by subtracting the large-scale equation (2.B2) from (2.B1):

$$
\frac{\partial}{\partial x_i} \left[ \frac{-\psi' + z'}{K} \right] - \frac{\beta}{D} K' = 0
$$

Equation (2.A4) is derived under the assumption that the higher-order perturbation terms can be approximated by their expected values. Similar assumption is also made by other stochastic analyses [i.e., Mantoglou and Gelhar, 1987; Yeh and Eltahir, 1998].

The derivation of perturbation equation is not complete without considering the effects of heterogeneous soil properties, which are introduced into equation (2.B4) through the $K-\psi$ relationship (equation 2.3) and $K-\eta$ relationship (equation 2.5). By decomposing the variables $\psi$, $K$ and $K_s$ in equation (2.3) into their spatial mean and perturbation term, it becomes:

$$
\psi + \psi' = -B' \left[ \ln \left( 1 + \frac{K'}{K} \right) - \ln \left( 1 + \frac{K_s'}{K_s} \right) + \ln \left( \frac{\bar{K}}{\bar{K}_s} \right) \right] - B' \left[ \ln \left( 1 + \frac{K'}{K} \right) - \ln \left( 1 + \frac{K_s'}{K_s} \right) + \ln \left( \frac{\bar{K}}{\bar{K}_s} \right) \right]
$$

(2.B5)
Assuming that $K'/\bar{K} << 1$, by using Maclaurin series ($\ln(1+\xi) = \xi - \xi^2/2 + \xi^3/3 - ... = \xi$ for $|\xi|<<1$), equation (2.B5) is approximated as:

$$\Psi + \psi' = -B\left[K' - \frac{K'}{K} + \ln\left(\frac{K}{K_s}\right)\right] - B'\left[K' - \frac{K'}{K} + \ln\left(\frac{K}{K_s}\right)\right]$$  \hspace{1cm} (2.B6)

By taking the expected value of equation (2.B6), we obtain the following large-scale $\Psi - \bar{K}$ relationship:

$$\Psi = -B\ln\left(\frac{\bar{K}}{K_s}\right) - E\left[B'\left(\frac{K'}{K} - \frac{K'}{K_s}\right)\right]$$  \hspace{1cm} (2.B7)

Using the same process to derive equation (2.B4), the perturbation expression of hydraulic suction is derived as:

$$\psi' = -B\left(\frac{K'}{K} - \frac{K'}{K_s}\right) - B'\ln\left(\frac{\bar{K}}{K_s}\right)$$  \hspace{1cm} (2.B8)

By substituting the perturbation expression of $K$-$\psi$ relationship (equation 2.B8) into equation (2.B4), we have the following differential equation:

$$\frac{\partial}{\partial x_i} \left\{ \bar{K} \frac{\partial}{\partial x_i} \left[B\left(\frac{K'}{K} - \frac{K'}{K_s}\right) + B'\ln\left(\frac{\bar{K}}{K_s}\right) + z'\right] \right\} - \frac{\beta}{D} K' = 0$$  \hspace{1cm} (2.A9)

Similar to the derivation of equation (2.B4), we assume that all spatially-related variables follow the second-order stationary process, and thus equation (2.B9) becomes:

$$-\frac{1}{K_s} \frac{\partial^2 K_s'}{\partial x_i^2} + \frac{1}{K} \frac{\partial^2 K'}{\partial x_i^2} + \frac{1}{B} \ln\left(\frac{\bar{K}}{K_s}\right) \frac{\partial^2 B'}{\partial x_i^2} + \frac{1}{B} \frac{\partial^2 z'}{\partial x_i^2} - \frac{\beta}{K B D} K' = 0$$  \hspace{1cm} (2.B10)
Equation (2.B10) is the perturbation equation of unsaturated hydraulic conductivity, which can be rewritten as the perturbation equation of soil moisture through the $K - \eta$ relationship (equation 2.5). By decomposing the variables $A$, $K$ and $K_s$ in equation (2.5) into their spatial mean and perturbation term, and using similar process for deriving $\psi - \bar{K}$ and $\psi' - K'$ relationship as in equations (2.B7) and (2.B8), respectively, the following $\eta - \bar{K}$ and $\eta' - K'$ relationships are derived as:

$$\eta = 1 + A \ln \left( \frac{K}{K_s} \right) + E \left[ A' \left( \frac{K' - K'_s}{K} \right) \right]$$

$$\eta' = A' \left( \frac{K' - K'_s}{K} \right) + A' \ln \left( \frac{K}{K_s} \right)$$

Equation (2.B11) can be further simplified by assuming that the expected value of higher-order perturbation is much smaller than other large-scale terms. In such case, the mean relative hydraulic conductivity can be estimated as:

$$\ln \left( \frac{K}{K_s} \right) = - \frac{1 - \eta}{A}$$

The final form of the perturbation model is represented by two simultaneous equations (2.B14a) and (2.B14b), through substituting the large-scale equation (2.B3) and (2.B13) into equations (2.B10) and through substituting the large-scale equation (2.B13) into equations (2.B12), respectively:

$$- \frac{1}{K_s} \frac{\partial^2 K'_s}{\partial x_i^2} + 1 \frac{\partial^2 K'}{\partial x_i^2} - \frac{1}{B} \left( 1 - \frac{\eta}{A} \right) \frac{\partial^2 B'}{\partial x_i^2} + 1 \frac{\partial^2 z'}{\partial x_i^2} - \frac{\beta}{K} K' = 0$$

$$\eta' = A' \left( \frac{K' - K'_s}{K} \right) - A' \left( \frac{1 - \eta}{A} \right)$$
\[ \beta_i = \frac{\beta}{BD} = \frac{R_0}{K_e} \left( \frac{1}{B} \right) \]

The parameter \( R_0 \) is defined as the portion of effective rainfall infiltrating to a unit depth of soil \( (R_0 = \frac{R}{D}) \). Based on previous derived equation (2.B3), the effective rainfall \( (R) \) is equivalent to large-scale evaporation \( (\beta K) \).
Appendix 2.C. Solving the Perturbation Equations using Spectral Techniques

The spectral techniques are applied to solve above simultaneous perturbation equations. Following Gelhar [1993], the two-dimensional stationary random fields $K'$, $\eta'$, $A'$, $B'$, $K'_s$, and $z'$ can be expressed in their wave number domain as below:

\[
K' = \int_{-\infty}^{\infty} e^{i(k_x x + k_y y)} dZ_K(k_x, k_y)
\]

\[
\eta' = \int_{-\infty}^{\infty} e^{i(k_x x + k_y y)} dZ_\eta(k_x, k_y)
\]

\[
A' = \int_{-\infty}^{\infty} e^{i(k_x x + k_y y)} dZ_A(k_x, k_y)
\]

\[
B' = \int_{-\infty}^{\infty} e^{i(k_x x + k_y y)} dZ_B(k_x, k_y)
\]

\[
K'_s = \int_{-\infty}^{\infty} e^{i(k_x x + k_y y)} dZ_{K'_s}(k_x, k_y)
\]

\[
z' = \int_{-\infty}^{\infty} e^{i(k_x x + k_y y)} dZ_z(k_x, k_y)
\]

where $x, y$ are spatial coordinates, $k_x, k_y$ are wave numbers in $x$ and $y$ directions, $i = \sqrt{-1}$ and $Z(k_x, k_y)$ is the Fourier-Stieltjes spectral amplitudes. By substituting the spectral relationships (2.C1) into equation (2.B14a), we can derive the Fourier-Stieltjes spectral amplitude of unsaturated hydraulic conductivity as:

\[
dZ_K = \frac{K}{K_s} W_0 dZ_{K_s} + \frac{K}{B} \left( \frac{1 - \eta}{A} \right) W_0 dZ_B - \frac{K}{B} W_0 dZ_z
\]

(2.C2)

where

\[
W_0 = \frac{(k_x^2 + k_y^2)}{(k_x^2 + k_y^2) + \beta_i}
\]

(2.C3)
Similarly, by substituting the wave number relationship into equation (2.B14b), we can derive the Fourier-Stieltjes spectral amplitude of soil moisture as:

$$dZ_\eta = A \left( \frac{1}{K} dZ_K - \frac{1}{K_s} dZ_{K_s} \right) \left( \frac{1 - \eta}{A} \right) dZ_A$$  \hspace{1cm} (2.C4)

By substituting equation (2.C2) into equation (2.C4), we can relate \(dZ_\eta\) with \(dZ_A\), \(dZ_B\), \(dZ_{Ks}\), and \(dZ_z\) as the following:

$$dZ_\eta = W_{K_s} dZ_{K_s} + W_B dZ_B + W_z dZ_z + W_A dZ_A$$  \hspace{1cm} (2.C5)

where

$$W_{K_s} = \frac{A}{K_s} (W_0 - 1), \hspace{0.5cm} W_B = \left( \frac{1 - \eta}{B} \right) W_0, \hspace{0.5cm} W_z = - \frac{A}{B} W_0, \hspace{0.5cm} W_A = - \left( \frac{1 - \eta}{A} \right)$$  \hspace{1cm} (2.C6)

Equation (2.BC5) relates the normalized soil moisture (\(\eta\)), soil properties (\(A, B, K_s\)) and topography (\(z\)) in Fourier-Stieltjes Domain.

An inverse transform is applied to convert equation (2.C5) from wave-number domain into the real domain. Following Gelhar [1993], the variation of soil moisture, \(\sigma_\eta^2\), can be derived through the inverse Fourier-Stieltjes transformation of the spectral density function of soil moisture (\(S_\eta\)) as:

$$\sigma_\eta^2 = \int \int e^{ik_x k_y} S_\eta (k_x, k_y) dk_x dk_y \bigg|_{k=0} \hspace{1cm} (2.C7a)$$

where \(S_\eta dk_x dk_y = E \left[ dZ_\eta \cdot dZ_\eta^* \right] \) and the prescript “*” indicates the conjugative form.

Similarly, the covariance between soil moisture and variable \(u\) (where \(u = \text{parameters } A, B, K_s \text{ or } z\)), \(\sigma_{\eta u}\), can be derived through the following:
\[ \sigma_{\eta u} = \int e^{i(k_x + k_y)\tau} S_{\eta u}(k_x, k_y) \, dk_x \, dk_y \bigg|_{\tau=0} \tag{2.7b} \]

where \( S_{\eta u} \, dk_x \, dk_y = E\left[dZ_\eta \cdot dZ_u^*\right] \). Using equation (2.5), the spectral density function of soil moisture is derived as:

\[
S_{\eta \eta} = W_{K_x}^2 S_{K_x K_x} + W_{B}^2 S_{B B} + W_{z}^2 S_{zz} + W_{A}^2 S_{AA} \\
+ 2\left( W_{K_x} W_{B} S_{K_x B} + W_{K_x} W_{z} S_{K_x z} + W_{K_x} W_{A} S_{K_x A} + W_{B} W_{z} S_{B z} + W_{B} W_{A} S_{B A} + W_{z} W_{A} S_{z A} \right) 
\tag{2.8a}
\]

Similarly, the cross-spectral density function of soil moisture and the variable \( u \) (where \( u = \text{parameters } A, B, K, \text{ or } z \)) is derived as:

\[
S_{\eta \eta A} = W_{A} S_{AA} + W_{B} S_{BA} + W_{K_x} S_{K_x A} + W_{z} S_{z A} \tag{2.8b}
\]

\[
S_{\eta \eta B} = W_{A} S_{BA} + W_{B} S_{BB} + W_{K_x} S_{K_x B} + W_{z} S_{z B} \tag{2.8c}
\]

\[
S_{\eta \eta K_x} = W_{A} S_{K_x A} + W_{B} S_{K_x B} + W_{K_x} S_{KK_x} + W_{z} S_{K_x z} \tag{2.8d}
\]

\[
S_{\eta \eta z} = W_{A} S_{z A} + W_{B} S_{z B} + W_{K_x} S_{K_x z} + W_{z} S_{zz} \tag{2.8e}
\]

By substituting equation (2.8a) into equation (2.7a) and substituting equations (2.8b-e) into equation (2.7b), equation (2.6) is thus derived.
Appendix 2.D. Estimation of Soil Physical Properties ($A$, $B$, and $K_s$)

To make the proposed stochastic framework applicable in field observations, it is essential to have the knowledge about the typical values of related soil parameters ($A$, $B$, and $K_s$) for different soil types. Unfortunately, although values of saturated hydraulic conductivity ($K_s$) can be found in the literature (e.g., Le et al. [1976]; Carsel and Parrish [1988]), the information about the parameters $A$ and $B$ is very limited. The objective of this appendix is to estimate the parameters $A$ and $B$ from widely used van Genuchten’s [1980] soil hydraulic functions.

To do so, we rewrite equation (2.3) and (2.5) as linear forms so that the soil deficit $\phi$ defined as $\phi = 1 - \eta$, is the independent variable:

\[ \psi = \frac{1}{C_s} \phi \]  \hspace{1cm} (2.D1) \[
\ln(K_r) = -\frac{1}{A} \phi \]  \hspace{1cm} (2.D2) \[ \text{where } K_r = K/K_s, \text{ is the relative hydraulic conductivity. Parameters } A, B \text{ and } C_s \text{ are estimated by using other more accurate and widely used } K-\eta-\psi \text{ expressions to approximate the above linear relationship. Here we use van Genuchten’s } [1980] \text{ soil hydraulic functions. By rewriting van Genuchten’s } K-\eta-\psi \text{ expressions in a form such that } \phi \text{ becomes the independent variable, and the following expression is obtained:} \]

\[ \psi = a^{-1} \left[ (1-\phi)^{\frac{1}{m}} - 1 \right]^{\frac{1}{n}} \]  \hspace{1cm} (2.D3)
\[
\ln(K_s) = \ln \left\{ \frac{1}{2} \left[ 1 - \left( 1 - \varphi \right)^{1/n} \right]^m \right\}
\] (2.D4)

where parameters \( n \) and \( a \) are constants primarily depending on soil types and \( m = 1 - 1/n \). Values of \( n \) and \( a \) for twelve different soil textures are as in Carsel and Parrish [1988]. A comparison between equation (2.D1) and equation (2.D3) shows that the linear fit of the \( \psi \varphi \) relationship using equation (2.D3) can be applied to estimate parameter \( C_s \), where the slope of the approximated linear line is equivalent to \( 1/C_s \). Similarly, a comparison between equation (2.D2) and equation (2.D4) show that the linear fit of the \( \ln(K/K_s) \)-\( \varphi \) relationship using equation (2.D4) can be applied to estimate parameter \( A \), where the slope of the approximated linear fit is equivalent to \( -1/A \). Finally parameter \( B \) is derived as \( B = A/C_s \). Table 2.1 lists the estimated values of parameters \( A \), \( B \) and \( C_s \) and their valid range of \( \eta \) and \( \psi \) as well as observed \( K_s \) values from Carsel and Parrish (1988). We note that these four parameters show monotonic relationships with soil types. For example, the \( K_s \) and \( C_s \) values monotonically decreases from coarse-texture soil to fine-texture soil. On the other hand, the \( B \) values monotonically increases from coarse-texture soil to fine-texture soil.
Table 2.D.1, Relevant parameters for the linear-type hydraulic function \cite{Mantoglou and Gelhar, 1987; Gardner, 1958}

<table>
<thead>
<tr>
<th>Texture</th>
<th>$A$</th>
<th>$B$ (m)</th>
<th>$C_s$ (m$^{-1}$)</th>
<th>$K_s$ (m/hr)</th>
<th>Valid ranges of $B$ and $C_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\psi$ (m)</td>
</tr>
<tr>
<td>Sand</td>
<td>0.141</td>
<td>0.021</td>
<td>6.625</td>
<td>0.2970</td>
<td>0.03–0.12</td>
</tr>
<tr>
<td>Loamy Sand</td>
<td>0.127</td>
<td>0.027</td>
<td>4.763</td>
<td>0.1459</td>
<td>0.04–0.14</td>
</tr>
<tr>
<td>Sandy Loam</td>
<td>0.109</td>
<td>0.058</td>
<td>1.878</td>
<td>0.0442</td>
<td>0.04–0.31</td>
</tr>
<tr>
<td>Sandy Clay Loam</td>
<td>0.079</td>
<td>0.085</td>
<td>0.929</td>
<td>0.0131</td>
<td>0.07–0.39</td>
</tr>
<tr>
<td>Loam</td>
<td>0.086</td>
<td>0.124</td>
<td>0.697</td>
<td>0.0104</td>
<td>0.12–0.55</td>
</tr>
<tr>
<td>Silt Loam</td>
<td>0.072</td>
<td>0.290</td>
<td>0.249</td>
<td>0.0045</td>
<td>0.18–1.39</td>
</tr>
<tr>
<td>Silt</td>
<td>0.068</td>
<td>0.402</td>
<td>0.169</td>
<td>0.0025</td>
<td>0.21–1.99</td>
</tr>
<tr>
<td>Clay Loam</td>
<td>0.061</td>
<td>0.416</td>
<td>0.146</td>
<td>0.0026</td>
<td>0.12–2.18</td>
</tr>
<tr>
<td>Sandy Clay</td>
<td>0.050</td>
<td>0.297</td>
<td>0.167</td>
<td>0.0015</td>
<td>0.20–1.39</td>
</tr>
<tr>
<td>Silty Clay Loam</td>
<td>0.050</td>
<td>0.801</td>
<td>0.062</td>
<td>0.0007</td>
<td>0.53–3.76</td>
</tr>
<tr>
<td>Agricultural Clay</td>
<td>0.024</td>
<td>2.440</td>
<td>0.010</td>
<td>0.0020</td>
<td>2.18–12.51</td>
</tr>
<tr>
<td>Silty Clay</td>
<td>0.024</td>
<td>3.904</td>
<td>0.006</td>
<td>0.0002</td>
<td>3.49–20.02</td>
</tr>
</tbody>
</table>
CHAPTER 3

Validation and Major Findings from the Stochastic Analysis
3.1. Introduction

The previous chapter develops a stochastic framework for characterizing the steady-state soil moisture distribution in a heterogeneous-soil and -topography field under the influence of precipitation and evaporation. The resulting model formulates the spatial distribution of soil moisture with relatively few parameters. Those parameters describe the statistical properties of topography, and soil properties, as well as the information regarding mean soil moisture, precipitation and soil depth.

The goal of this chapter is to evaluate the proposed stochastic framework with reasonable values of related parameters. The major findings from this evaluation will allow us to better understand the spatial organization of soil moisture with explicit considerations for heterogeneity in soil physical properties and topography. In particular, such evaluation would help us to explore, for example, under what conditions: (i) the relationship between soil moisture and topography will be enhanced or reduced? (ii) the relationship between soil moisture and soil properties will be enhanced or reduced? and (iii) the variation of soil moisture will decrease (or increase) with increasing mean soil moisture?

This chapter is organized as the follows. In Section 3.2, physical and mathematical features of related parameters are discussed. From Section 3.3 to Section 3.5, we examine the effects of topography, soil properties and mean soil moisture on the soil moisture distribution at the root zone of shallow unsaturated soil. In Section 3.6, a
qualitative comparison between simulation results and observations are provided. The summary and discussion are included in the last section.

3.2. General Discussion of Relevant Parameters in Proposed Stochastic Framework

The relevant parameters associated with the steady-state horizontal distribution of soil moisture can be categorized into four groups: (1) the correlation scales of soil properties (\( \lambda_d \)) and relative elevation (\( \lambda_e \)), (2) large-scale soil properties (\( A, B \) and \( K\)), (3) large-scale water source and mean soil moisture (\( R_0, \bar{R} \)), and (4) variability of relative elevation (\( \sigma_z^2 \)) and soil properties (\( \sigma^2_A, \sigma^2_B \) and \( \sigma^2_K \)). The physical meaning and the typical range of these parameters are discussed in this subsection.

First the correlation scales, \( \lambda_d \) and \( \lambda_e \), are related to the geologic process and undulation of soil and topographic fields. Previous study [Yeh and Eltahir, 1998] has shown that smooth undulating fields of relative elevation (large \( \lambda_e \)) tend to result in smaller variability of soil moisture. Similarity, large correlation scale of soil properties will have smaller variability in moisture distribution. It is possible that in a study area with large \( \lambda_e \) the soil moisture distribution will be controlled by \( \lambda_d \), provided \( \lambda_d \ll \lambda_e \). To our knowledge, the comparative effects of \( \lambda_d \) and \( \lambda_e \) on soil moisture distribution have not drawn much attention. The relative scale of \( \lambda_d \) and \( \lambda_e \), however, is likely to have significant impact on resulting soil moisture distribution. A typical value of \( \lambda_d \) is about one meter [Mantoglou and Gelhar, 1987] and the typical range of \( \lambda_e \) is \( 10^2 \) –\( 10^4 \) meter [Yeh and Eltahir, 1998]. In the following three subsections, different values of both
correlation scales will be used to evaluate the comparative effects of $\lambda_d$ and $\lambda_c$ on spatial distribution of soil moisture.

Second parametric groups include the large-scale soil properties, namely $\bar{A}, \bar{B}, \bar{K}_s$. In general, finer-texture soils have smaller $\bar{A}$ and $\bar{K}_s$ values but larger $\bar{B}$ values; and coarser-texture soils have larger $\bar{A}$ and $\bar{K}_s$ values but smaller $\bar{B}$ (See Appendix C in Section 2.6). Philip [1969] suggested that if the thickness of capillary fringe ($\bar{B}$) were large (i.e., finer-texture soil) the gravity flow introduced by elevation gradient would be less obvious. Thus, in an area mainly composed of finer-texture soils, the heterogeneous soil properties may have more control then topography on the distribution of soil moisture. Therefore, large-scale distribution of soil properties must be carefully considered in examining the soil moisture distribution. We will evaluate the effects of large-scale soil properties by comparing the distribution of soil moisture based on four soil types (1) sandy loam, (2) silty loam (3) clay loam and (4) mixed soil. The mixed soil is assumed to contain even portion of sandy loam, silty loam and clay loam. The soil properties of these four soils are summarized in Table 3.1, in which the properties ($\bar{A}$ and $\bar{B}$) were estimated (See Appendix C in Section 2.6) based on the soil database complied by Carsel et al. [1988]. The soil properties of mixed soil are assumed to be the average values of the soil properties of sandy loam, silty loam and clay loam.
### Table 3.1, The soil properties of sandy loam, silt loam, clay loam and mixed soil

<table>
<thead>
<tr>
<th></th>
<th>Sandy loam</th>
<th>Silt loam</th>
<th>Clay loam</th>
<th>Mixed soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$ (dimensionless)</td>
<td>0.179</td>
<td>0.072</td>
<td>0.061</td>
<td>0.104</td>
</tr>
<tr>
<td>$B$ (m)</td>
<td>0.058</td>
<td>0.290</td>
<td>0.416</td>
<td>0.255</td>
</tr>
<tr>
<td>$K_s$ (m/hr)</td>
<td>0.0442</td>
<td>0.0045</td>
<td>0.0026</td>
<td>0.0171</td>
</tr>
</tbody>
</table>

The third parametric group includes the mean soil moisture ($\overline{\Pi}$) and effective rainfall per depth ($R_0$). In equation (2.20), we have derived the expression of vertical hydraulic gradient ($\partial$) as functions of $\overline{\Pi}$ and $R_0$ and mean soil properties ($A$, $B$, $K_s$). This expression contains the information of large-scale water balance. A large vertical gradient creates a large vertical moisture sink and $\overline{\Pi}$ approaches small limiting values. In such cases, variability of soil moisture is primary controlled by variability in soil properties. In this study, typical ranges for evaporation ($0.5$~$1$ m yr$^{-1}$) and root zone depth ($D = 1$ m) were used following Yeh and Eltahir [1998]. From these values we can estimate the range of effective rainfall per depth, where effective rainfall is taken to be $50\%$ of evaporation and $R_0$ is a ratio of effective rainfall and root-zone depth.
Parameters in the last group ($\sigma_A^2, \sigma_B^2, \sigma_{K_s}^2$ and $\sigma_z^2$) represent variations of heterogeneous soil properties and topography. Here we express the variability in terms of coefficient of variation (CV):

\[
\begin{align*}
\sigma_A^2 &= \bar{A}^2 CV_A^2 \\
\sigma_B^2 &= \bar{B}^2 CV_B^2 \\
\sigma_{K_s}^2 &= \bar{K}_s^2 CV_{K_s}^2 \\
\sigma_z^2 &= \bar{z}^2 CV_z^2
\end{align*}
\] (3.1)

Note that CV is a dimensionless parameter. To make a fair comparison between the contributions from topography and soil properties, in the following experiment we will assume that the coefficients of variation are equivalent for topography and soil properties, i.e., $CV_A = CV_B = CV_{K_s} = CV_z = cv$. The mean soil properties ($\bar{A}, \bar{B}$ and $\bar{K}_s$) are as in Table 3.1 for four different soil types and the mean relative elevation is assumed to be $\bar{z} = 30$ m. In such cases, equation (2.10) is approximated as:

\[
\begin{align*}
\xi^2 &= \left(\frac{\bar{K}_s}{\bar{A}}\right)^2 \\
\zeta^2 &= \left(\frac{\bar{B}}{\bar{A}}\right)^2 \\
\omega^2 &= \left(\frac{\bar{z}}{\bar{A}}\right)^2
\end{align*}
\] (3.2)

3.3. Relationship between Soil Moisture Distribution and Topography

Here we evaluate the covariance between soil moisture and topography, $\sigma_{s\cdot z}$, based on the preliminary analysis (equations 2.14e, 2.15e and 2.16e) and the typical values of relevant parameters discussed above. We first look at the situation when soil
properties are entirely uncorrelated with topography (Case 1 and Case 2). Note that the expression of \( \sigma_F \) in Case 1 is the same as that in Case 2 (see Subsection 2.4.1 for discussion). The \( \sigma_F \) in Case 1 and Case 2 is shown in Figure 3.1 for sandy loam, silty loam, clay loam and mixed soil. For each soil type, the normalized covariance between topography and soil moisture, \( \frac{\sigma_{\varphi}}{c v^2} \), is plotted as a function of mean soil moisture \( \bar{\varphi} \) for two different correlation scales \( \lambda_c = 10^2 \text{ m and } 10^4 \text{ m} \). Several general features are identified in Figures 3.1. First, soil moisture is negatively correlated with relative elevation for different soil types. The negative covariance between soil moisture and topography has also been identified in a number of previous studies [Krumback, 1959; Henninger et al., 1976; Hawley et al., 1983; Robinson and Dean, 1993; Nyberg, 1996]. Second, \( |\sigma_F| \) is smaller for finer-texture soil and larger for coarser-texture soil, where “\(|\cdot|\)” denotes the absolute value. A weaker covariance between soil moisture and topography for finer-texture soil results from smaller pore and large surface area, which yields more resistance on the topographically induced moisture flow. On the other hand, coarser-texture soil with larger pore size and smaller surface area appears to enhance the topographically induced moisture flow. Third, \( |\sigma_F| \) is smaller if the topographic field is gently rolling (i.e., large \( \lambda_c \)) and larger if the topographic field is highly undulated (i.e., small \( \lambda_c \)). Usually, soil moisture tends to accumulate in the valley area and dissipate at the ridge area. Larger \( \lambda_c \) tends to smooth the appearance of valley and ridge and consequently reduce the soil moisture variability. Finally, \( |\sigma_F| \) decreases for dry soil condition and increases for wet soil condition. This feature stems from direct impact of conductivity on the topographically induced moisture flow. In dry soil, low unsaturated
hydraulic conductivity prohibits the topographically induced moisture flow. On the other hand, high conductivity (wet soil) reinforces such redistribution.

**Figure 3.1.** The normalized covariance between topography and soil moisture ($\sigma_{hc} / cv^2$) for Case 1 and Case 2 as a function of mean soil moisture ($E[h] \})$ and two different correlation scales of topography [$l_z = 10^2$ m (solid line) and $\varphi = 10^4$ m (dash line)]. Figure 3.1a refers to sandy loam. Figure 3.1b refers to silt loam. Figure 3.1c refers to clay loam. Figure 3.1d refers to mixed soil.
The $\sigma_{TF}$ in Case 3 (topography and soil properties perfectly correlated) are shown in Figure 3.2 by assigning equivalent correlation scales for topography and soil properties ($\lambda_d = \lambda_c = 10^2$ m and $\lambda_d = \lambda_c = 10^4$ m). It can be seen in Figure 3.2 that the appearance of negative covariance and the effect of mean soil properties (i.e., smaller $|\sigma_T|$ for finer-texture soil) as well as the effect of correlation scale of topography (i.e., smaller $|\sigma_T|$ for larger $\lambda_c$) are similar to that in Figure 3.1. However, the impacts from $\tilde{\eta}$ (i.e., increasing $|\sigma_T|$ for wetter soil moisture) are altered by the soil-heterogeneity effects on $\sigma_{TF}$. To inspect the effects of soil properties on $\sigma_{TF}$, equation (2.16e) is decomposed into two terms:

$$\sigma_{\eta_c} = f_c + f_v$$

$$f_c = \alpha \left( \tilde{X}_A + \xi \tilde{- X}_B - \xi \tilde{X}_B \right) \sigma_{AA}^2$$

$$f_v = \omega^2 \tilde{X}_A \sigma_{AA}^2$$  \hfill (3.3)

where $f_c$ represents the impact from correlation between topography and soil properties, and $f_v$ represents the impact from variability of topography. Figure 3.3 plots the $f_c/\sigma_{AA}^2$ and $f_v/\sigma_{AA}^2$ for four different soil types as in Figure 3.2. A comparison between Figure 3.2 and Figure 3.3 shows that in coarser-texture soil (e.g., sandy loam) $f_v$ has more dominant control on $\sigma_{TF}$. In other words, for coarser-texture soils $\sigma_{TF}$ in Case 3 is similar to that in Case 1 and 2. Also for areas with fine-texture soil (i.e., clay loam) or with a large $\lambda_c$, $f_c$ will have more dominant control on $\sigma_{TF}$. In such cases, $|\sigma_T|$ increases in dry soil condition and decreases in wet soil condition. In a mixed soil and small $\lambda_c$, $f_v$ has more dominant control on $\sigma_{TF}$ in wet soil condition and $f_c$ has more dominant control for
dry soil condition. In such cases, the negative covariance between soil moisture and topography is initially reduced and then enhanced as the soil changes from dry to wet.

**Figure 3.2.** Similar to Figure 3.1 but for Case 3. Note that solid line refers to $\lambda_D = \lambda_E = 10^2$ m and dash line refers to $\lambda_D = \lambda_E = 10^4$ m.
Figure 3.3. The $f_c/cv^2$ and $f_v/cv^2$ for four different soil types as in Figure 3.2, where $f_c$ represents the impact from correlation between topography and soil properties, and $f_v$ represents the impact from variability of topography. The relationship between Figure 3.2 and Figure 3.3 is “Figure 3.2$x$ = Figure 3.3$x[1]$ + Figure 3.3$x[2]$”, where the figure indices “$x$” represents “a” for sandy loam, “b” for silt loam, “c” for clay loam and “d” for mixed soil.
3.4. Relationship between Soil Moisture Distribution and Soil Properties

The covariances between soil moisture and soil properties \((A, B\) and \(K_s\)) are represented respectively by \((\sigma_Y, \sigma_F\) and \(\sigma_{nK_s}\)). For illustrative purposes, we only discuss the results for the covariance between soil moisture and saturated hydraulic conductivity \((\sigma_{nK_s})\). The results of Case 1 and 2 are shown in Figures 3.4 and 3.5, respectively, for four soil types described before. For each soil type, the normalized covariance between soil moisture and saturated hydraulic conductivity \((\sigma_{nK_s}/cv^2)\) is plotted as a function of mean soil moisture \((\bar{n})\). In Figure 3.4 and 3.5, two different correlation scales of soil properties \((\rho_A = 0.1\) m and \(10\) m) are investigated. Because the evaporation rate is higher in soils with higher conductivity and slower in smaller conductivity, soil moisture and saturated hydraulic conductivity show negative covariance. A general trend appears in Case 1 and 2 is that the negative covariance between saturated hydraulic conductivity and soil moisture is enhanced as the soil becomes dry. In other words, the heterogeneous soil properties have more pronounced influence on the distribution of soil moisture for dry soil condition. We note here that \(|\sigma_{nK_s}|\) in Case 1 comes from the variance of \(K_s\) only, whereas \(|\sigma_{nK_s}|\) in Case 2 is the net results from the variability of three soil properties \((A, B\) and \(K_s\)). Consequently, a comparison between Figure 3.4 and Figure 3.5 shows that \(|\sigma_{nK_s}|\) in Case 2 is about one order of magnitude larger than those in Case 1. We also note in Case 2 that the correlation scale of soil properties \((\rho_A)\) has a relatively minor impact on \(\sigma_{nK_s}\). As discussed in Subsection 2.4.1, the following relationship holds if soil properties and topography are perfectly correlated (Case 3):
\[
\frac{\sigma_{\eta K_s}}{\sigma_{\eta}} \frac{\sigma_{\eta}}{\sigma_{K_s}} = \frac{\sigma_{\eta z}}{\sigma_{\eta}} \frac{\sigma_{\eta}}{\sigma_{z}}
\] (3.4)

Therefore, the result of \(\sigma_{\eta K_s}\) in Case 3 is similar to that of \(\sigma_T\) in Case 3.

**Figure 3.4.** The normalized covariance between saturated conductivity and soil moisture \((\sigma_{\eta K_s} / c v^2)\) for Case 1 as a function of mean soil moisture \((\bar{\eta})\) and two different correlation scales of soil property \(A\) \([\lambda_a = 10^{-1} \text{ m} \text{ (solid line)} \text{ and } \lambda_a = 10^2 \text{ m} \text{ (dash line)}]\). Figure 3.4a refers to sandy loam. Figure 3.4b refers to silt loam. Figure 3.4c refers to clay loam. Figure 3.4d refers to mixed soil.
Figure 3.5. Similar to Figure 3.4 but for Case 2.
Figure 3.6. Similar to Figure 3.4 but for Case 3.
3.5. The Impact of Mean Soil Moisture on the Soil Moisture Variation

In previous two subsections, we have shown that the mean soil moisture has different influences on the topographically induced or soil-heterogeneity induced moisture distribution. Specifically, in the case when soil properties and topography are entirely uncorrelated (Case 1 and 2), $|\sigma_\eta| \text{ decreases for dry soil condition and increases in wet soil condition, on the other hand, } |\sigma_{\eta K}| \text{ increases for dry soil condition and decreases in wet soil condition. If soil properties and topography are perfectly correlated (Case 3), both } |\sigma_\eta| \text{ and } |\sigma_{\eta K}| \text{ show similar response with mean soil moisture. Because both } |\sigma_\eta| \text{ and } |\sigma_{\eta K}| \text{ in Case 3 is the net results of topographically induced and soil-heterogeneity induced moisture distribution, the response of } |\sigma_\eta| \text{ and } |\sigma_{\eta K}| \text{ with mean soil moisture will depend on whether topography or soil properties have more dominant control on soil moisture distribution. In coarser-texture soil with small } \lambda_c, \text{ topography tends to have more dominant control on soil moisture distribution. In such situations, both } |\sigma_\eta| \text{ and } |\sigma_{\eta K}| \text{ in Case 3 decreases for dry soil condition and increases in wet soil condition, which is similar to the response of } |\sigma_\eta| \text{ with mean soil moisture in Case 1 and 2. In fine-texture soil or with large } \lambda_c, \text{ soil properties tends to have more dominant control on soil moisture distribution. In such situations, both } |\sigma_\eta| \text{ and } |\sigma_{\eta K}| \text{ in Case 3 increases for dry soil condition and decreases in wet soil condition, which is similar to the response of } |\sigma_{\eta K}| \text{ with mean soil moisture in Case 1 and 2. In a mixed soil, topography have more dominant control in wet soil condition and soil properties have more dominant control on soil moisture distribution for dry soil condition. Therefore, for a mixed soil in Case 3,}
both $|\mathbf{\sigma}_T|$ and $|\mathbf{\sigma}_{\eta,s}|$ will decreases initially and then increase as the soil changes from dry to wet.

Similar to $|\mathbf{\sigma}_T|$ and $|\mathbf{\sigma}_{\eta,s}|$ in Case 3, the variance of soil moisture ($\sigma^2_\eta$) is the net results of topographically induced and the soil-heterogeneity induced moisture distribution. An important implication from the above discussion is that the impact of mean soil moisture on $\sigma^2_\eta$ will also depend on whether topography or soil properties have more dominant control. Based on above discussion, we summarizes that (1) topography will have dominant control on soil moisture distribution when the area is dominated by coarse-texture soil or by mixed soil with small $\lambda_e$; (2) soil properties will have dominant control on soil moisture distribution for fine-texture soil or by mixed soil with large $\lambda_e$; and (3) both topography and soil properties will have comparable influence for medium-texture soil with moderate value of $\lambda_e$.

Figures 3.7-3.9 show the variances of soil moisture as a function of different soil types for three different cases. For each soil type the normalized soil moisture variability ($\sigma^2_\eta/cv^2$) is plotted as a function of mean soil moisture ($\eta$). In Case 1 and 2, two different correlation scales of topography $\varrho_c = 10^2$ m and $10^4$ m) as well as two different correlation scales of soil property $A$ ($\lambda_a = 10^{-1}$ m and 10 m) are evaluated. In Case 3, equivalent correlation scales are assigned for topography and soil property $A$ ($\lambda_a = \lambda_c = 10^2$ m and $\lambda_a = \lambda_c = 10^4$ m). A common and important feature is identified in Figures 3.7-3.9. If topography has more dominant control on soil moisture distribution, the soil
moisture variability increases as the soil becomes wet. On the other hand, if soil properties have more dominant control on soil moisture distribution, the soil moisture variability decreases as the soil becomes wet. In cases when topography and soil properties have similar control on soil moisture distribution, the soil moisture variability initially decreases and then increases as the soil changes from dry to wet. For instance, if the area is dominated by fine-texture soil such as clay loam (Figures 3.7c[1], 3.7c[2], 3.8c[1], 3.9c[2], 3.9c[1], 3.9c[2]) soil properties tend to have more dominant control on soil moisture distribution and the soil moisture variability decreases as the soil becomes wet. We also note that the correlation between soil properties will reinforce the impact of soil properties on the soil moisture distribution. Take the sandy loam with $\lambda_e = 10^4$ m for example (Figure 3.7a[2] for Case 1 and Figure 3.8a[2] for Case 2). If soil properties are entirely uncorrelated (Case 1), topography has more dominant control on soil moisture distribution and thus soil moisture variability increases as the soil becomes wet. However, if soil properties are entirely uncorrelated (Case 1), the impact of soil properties on the soil moisture distribution becomes more significant and thus the soil moisture variability initially decreases and then increases as the soil changes from dry to wet.
Figure 3.7.1. The normalized variance of soil moisture ($\sigma^2_\eta / cv^2$) for Case 1 as a function of mean soil moisture ($\bar{\eta}$) and different correlation scales of $K_s$ and $z$. This Figure is for small correlation scale of topography ($\gamma_k = 10^2$ m). The solid line refers to $\gamma_k = 10^{-1}$ m and the dash line refers to $\gamma_k = 10^2$ m. Figure (a) refers to sandy loam; (b) refers to silt loam; (c) refers to clay loam; and (d) refers to mixed soil.
Figure 3.7.2. Similar to Figure 3.7.1, but for large correlation scale of topography ($\lambda_c=10^4$ m).
Figure 3.8.1. Similar to Figure 3.7.1, but for Case 2.
Figure 3.8.2. Similar to Figure 3.7.2, but for Case 2.
Figure 3.9.1. Similar to Figure 3.7.1, but for Case 3. Note that $\lambda_\delta = \lambda_\varepsilon = 10^2$ m.
Figure 3.9.2. Similar to Figure 3.7.2, but for Case 3. Note that $\lambda_6 = 10^4$ m.
3.6. Comparison with Previous Studies

To evaluate the robustness of the proposed stochastic theory in modeling the soil moisture distribution, it is essential to compare the results to relevant field observations. Unfortunately, only few and often incomplete observational data exist. It is therefore extremely difficult to provide a quantitative comparison. Here we discuss a number of field observations that is in qualitative agreement with the results of this study. We will focus on the effects of mean soil properties and the correlation scale of topography on the relative importance of topographically induced moisture flow and soil-heterogeneity induced moisture flow as well as the impact of mean soil moisture on the soil moisture distribution.

Qiu et al. [2001] discussed the soil moisture variation in relation to topography and land-use in a hillslope catchment in the Loess Plateau (China). The attitude ranges from 1000 m to 1350 m in a 3.5 km$^2$ area. A highly undulated topographic field is expected with such elevation difference. In such cases, topography is likely to control the soil moisture distribution. However, they have reported the land-use and soils have more pronounced control on soil moisture distribution than topography. We note here that the catchment is mostly composed of fine-texture soil (fine silt and silt). Consequently, results of Qiu et al. [2001] are consistent with the suggestions in Subsection 3.4 in that finer-texture soil tends to reduce the influence of topographically induced moisture distribution.
Western et al. [1999] investigated the relationship between soil moisture distribution and several topographic indices in Tarrawarra catchment, which is dominated by finer-texture soil. They found weak correlation in most topographic indices (i.e., aspect, specific contribution area), with the exception of tangent curvature. According to the observation by Famiglietti et al. [1998], aspect and specific contribution area have much higher correlation with relative elevation than that with curvature. In other words, the results of Western et al. [1999] suggest that the finer-texture soil might result in weak correlation between soil moisture and elevation. These findings are also consistent with our results.

Crave and Gascuel-Odoux [1997] analyzed the influence of elevation difference on soil moisture distribution within a subcatchment of the Coët-Dan catchment (Brittany, France). The topographic field is observed as highly undulated (small \( \lambda_c \)). The average saturated hydraulic conductivity in their study area is \( 3.01 \times 10^{-5} \) m/s in the hillslope domain and \( 2.15 \times 10^{-6} \) m/s in the valley. Data suggest that the correlation between topography and soil properties exist in this area and the soil type ranges from coarse to medium-coarse. One of their results shows (in their Figure 4c and 4d) that the relationship between topography and soil moisture is enhanced in wet condition and reduced in dry condition. Such relationship is similar to the solid line in Figure 3.2a (soil properties and topography are perfectly correlated and the study area is dominated by coarse-texture soil with small \( \lambda_c \)).
*Famiglietti et al.* [1999] investigated the soil moisture variability within remote sensing footprints during Southern Great Plains 1997 (SGP97). They have found, in sites LW13 and LW21, no correlation between surface soil moisture and downslope decrease in elevation. In addition, the variances of soil moisture in both sites are found to decrease as the soil become wet. We note here that the soil types in both sites are dominated by medium-fine soil (i.e., LW13 is dominated by loam and LW21 is dominated by silt loam). In addition, the topographic fields are gently rolling in LW13 and flat in LW21 (i.e., large $\lambda$). This result agrees with our finding in that a smooth topography field and the domination of fine-texture soil will lead to weak relationship between elevation and soil moisture. In such cases, the variance of soil moisture will decrease as the soil become wet.

The field observations discussed above shows qualitative agreement with the findings from proposed stochastic theory with respect to the effects of mean soil properties and the correlation scale of topography on the relative importance of topographically induced moisture flow and soil-heterogeneity induced moisture flow. We note that inference from the proposed stochastic theory will not work as well if precipitation is highly heterogeneous in spatial and temporal scale. For instance, *Famiglietti et al.* [1998] investigated the soil moisture variability along a hillslope transect at Rattlesnake Hill (Texas). The topographic field is gently rolling (i.e., large $\lambda$) and the soil field is evenly composed of clay, silt and sand. A strong correlation between topography and soil properties appears in the study area. Therefore the soil and topographic conditions in this area is similar to that in the solid line of Figure 3.2d. The
solid line of Figure 3.2d shows that the negative correlation between soil moisture and topography is minimized as the soil becomes wet. This result partially agrees with the observation of Famiglietti et al. [1998]. They found that the negative correlation is minimized as the soil changes from dry to medium wet. However, as soil changes from medium wet to extreme wet, they found that the correlation between topography and soil moisture alters from negative to positive correlation. Probably, such positive correlation results from highly spatial-temporal variation in precipitation during their experiment.

3.7. Summary and Discussion

In this Chapter, we apply the proposed stochastic model to evaluate the influence of topography, soil physical properties, and mean soil moisture on the variation of soil moisture distribution. To do this, we have investigated the role of correlation scales for different attributes, large-scale properties, and variance of soil properties and topography on the variability of soil moisture distribution. To keep the analytical approach tractable, we have examined three limiting cases in detail. Clearly, observed data will fall somewhere in between these limiting cases and the extension of these analytical results to actual field situations should be done with caution.

Our results suggest that the relative importance of topographic or soil-properties effects are one of the key factors control the appearance of either positive or negative correlation between mean soil moisture and the variability of soil moisture. The mean soil types and the correlation scale of topography determine the contribution of topographic effects. The topographic effect becomes stronger if the study area is dominated by coarser-texture soil or by undulated topographic field (i.e., smaller $\lambda_c$) and weaker in
other cases. On the other hand, the effect of soil heterogeneity on soil moisture
distribution does not change with mean soil types and the correlation scale of soil
properties. Because the topographic effect changes with mean soil types and \( \lambda \),
while the effects of soil heterogeneity is relatively constant, the importance of these two effects
becomes distinctive with different mean soil types and \( \lambda \). The following items summarize
the effects of topography, soil types, and mean soil moisture on the soil moisture
distribution. First, topography appears to have dominant control on soil moisture
distribution when the area is dominated by coarse-texture soil or by mixed soil with small
correlation scale for topography (i.e., small \( \lambda \)). In such cases the soil moisture variability
increases as the soil becomes wet. Second, soil properties is likely to have dominant
control on soil moisture distribution for fine-texture soil or for mixed soil with large \( \lambda \).
In such cases, the soil moisture variability decreases in wet soil condition. Finally, both
topography and soil properties appear to have similar control for medium-texture soil
with moderate value of \( \lambda \). In such cases, the soil moisture variability initially decreases
and then increases as the soil changes from dry to wet.

Soil moisture variability is influenced by a number of environmental factors,
including topography, soil properties, vegetation and climate. Our proposed analytical
approach provides a systematic framework to evaluate the importance of different
interacting variables, for example coupled effects of topography and saturated hydraulic
conductivity, on the variation of soil moisture distribution. Future work will address the
corresponding transient problem, the issues of spatially variable climatic forcing and
vegetation on the soil moisture distribution.
CHAPTER 4

Estimation of Soil Physical and Hydraulic Properties using Remote Sensing and Artificial Neural Network
4.1. Introduction

An accurate estimation of spatially variable soil media properties is necessary to develop reliable models of flow and transport throughout the soil-plant-atmosphere continuum, for efficient management of resources, and for maintenance of environmental quality. A significant knowledge base exists, in the soil physics and groundwater literature, that focuses on development and refinement of methods to estimate soil physical and hydraulic properties [e.g., van Genuchten and Nielsen 1985; Feddes et al., 1988; Jury et al., 1991; Ahuja et al., 1993]. Most of these studies, however, have used laboratory scale or in-situ measurements to describe and predict soil properties.

Measurements of soil physical and hydraulic properties are time consuming and expensive. In addition, a large number of measurements are necessary to quantify their space-time variability. Reliable measurement of these properties is confounded by the extreme spatial heterogeneity and inherent nonlinearity of soil characteristics. Therefore, it is desirable to develop simplified methods to characterize soil media properties over large areas. Our goal here is to obtain soil media properties over large remote areas where conventional in-situ techniques may not be readily available or will be too expensive and time consuming.
Currently, passive microwave remote sensing is arguably the most feasible way to measure soil brightness temperature and derive soil moisture over large areas. Camillo et al., [1984] were perhaps the first to estimate soil hydraulic parameters using passive microwave measurements (e.g., brightness temperature) and atmospheric forcing data (e.g., air temperature, wind speed, humidity, net radiation). They have used a parameter estimation technique within an energy and moisture balance model to estimate soil hydraulic parameters. Recently, Hollenbeck et al., [1996] have shown that temporal sequence of remotely sensed data may be used to qualitatively detect soil hydraulic heterogeneity. Mattikalli et al., [1995] suggested that changes in soil moisture could be used as an indicator of soil texture and average saturated hydraulic conductivity. In addition to remote sensing observations, they have used in-situ measurements and a one dimensional model that requires atmospheric forcing data. Mattikalli et al. [1998] developed regression relationships for the ratio of sand to clay (RSC) and effective saturated hydraulic conductivity in terms of temporal changes in brightness temperature ($T_B$) and soil water content change. Extensive information about RSC or soil texture is needed to develop such regression relationships.
A key objective of this study is to examine the feasibility of estimating soil physical and hydraulic properties using only remotely sensed data with little or no information about soil texture. Our approach of identifying a distributed map of soil physical and hydraulic properties is based on recent developments in Artificial Neural Network (ANN) and the principles of self-organization. This methodology would allow us to estimate soil physical properties based on the physical linkage between soil hydraulic properties and soil moisture during the drydown phase. Using a sequence of remotely sensed data from the Washita ’92 experiment, we will examine the feasibility and robustness of estimating soil media properties. From the Washita ’92 data, we have spatially distributed time series of near surface brightness temperature and soil moisture. Since the soil moisture data in Washita ’92 experiment is derived from brightness temperature [Jackson et al., 1995], we first explore the possibility inferring soil physical properties from a sequence of remotely sensed brightness temperature. We also compare and contrast results of using remotely sensed soil moisture data to infer soil physical properties.
4.2. Relationships Between Remotely Sensed Surface Features and Soil Physical Properties

The brightness temperature and soil moisture data used in this study have been collected by National Aeronautics and Space Administration (NASA), the US Department of Agriculture (USDA), several other government agencies and Universities from the Little Washita watershed located in southwest Oklahoma in the Great Plains region of the United States during the so-called Washita’92 experiment [Jackson et al. 1995 and references therein]. Eight days of soil moisture images were collected from June 10 to June 18 (except June 15) in 1992. For a complete description of the Little Washita watershed, we refer to Allen and Naney (1991). The soil moisture images in the form of volumetric soil moisture were derived from ESTAR sensed microwave brightness temperature images. These data are stored in 228 by 93 pixels covering an area of 45.6 km by 18.6 km (848 sq. km) with a pixel resolution of 200 m by 200 m. In this study, we use only the 228 by 85 pixels instead of 228 by 93 pixels since most of the soil moisture data are missing for the horizontal coordinate greater than eighty-five in the eight-day period.
Figure 4.1, Remotely sensed brightness temperature in degree K (a: June 10, b: June 12, c: June 14, d: June 16, e: June 18) and derived soil moisture in percent (f: June 10, g: June 12, h: June 14, i: June 16, j: June 18) for the Washita '92 experiment. Figure 1K refers to soil texture (1: Sand, 2: Loamy Fine Sand, 3: Fine Sandy Loam, 4: Loam, 5: Silt Loam, 6: Silty Clay Loam / Clay Loam, 7: Pits, Quarries, Urban, 8: Gypsum, 9: Water).
Figure 4.1 shows a sequence of remotely sensed brightness temperature, $T_B(x,t)$, and corresponding near surface soil water content, $\theta(x,t)$, from the Washita ’92 research experiment carried out during June 10-18 1992. Here, “x” is the position vector and “t” is the time. This sequence of images reflects both the spatial and temporal variations. For example, for June 10, $T_B(x,t)$ ranges between 190$^0$K (eastern and western part) and 250$^0$K (middle region) over the watershed while for June 18 this range in $T_B(x,t)$ has changed to 230$^0$K (eastern and western part) and 270$^0$K (middle region). On the other hand, on June 10, $\theta(x,t)$ ranged between 0.35 (eastern and western part) and 0.15 (middle region) while for June 18, $\theta(x,t)$ range changed to 0.20 (eastern and western part) and 0.05 (middle region).

Figure 4.1k shows the spatial variation of soil texture for the Little Washita watershed. There appears to be a significant degree of spatial coherence between brightness temperature and soil texture as well as soil moisture and soil texture. This suggests that spatial structure of $T_B$ and its temporal variations may be used to identify soil media properties. Mattikalli et al. [1998] have used regression relationships between $T_B$ and RSC to identify saturated hydraulic conductivity. In addition to requiring extensive soil textural information, Mattikalli et al., [1998] regression relationships show
orders of magnitude change between one day and eight-day change in $T_B$ relationships.

For example, for one day the change relationship between $T_B$ and RSC is 

$$(1.3025 \times 10^{41}) T_B^{-45.81}$$

while for eight-day change the relationship is

$$(8.0543 \times 10^{20}) T_B^{-13.46}.$$ 

There are over 20 order of magnitude change between one day and eight day change relationships. Such a large change in exponent of $T_B$ is likely to introduce a significant degree of uncertainty in the estimation of soil properties for a small error in $T_B$.

Figures 4.2a and 4.2b show temporal pattern of average brightness temperature and near surface soil water content as a function of different soil types. There was no rainfall in the watershed during this experiment and the daily average hydrometeorological forcing (e.g., solar radiation, evaporation) was approximately constant [Mattikalli et al., 1998]. Thus, the observed space-time change of brightness temperature and soil water content may be attributed to the redistribution of soil water. Physical characteristics of the redistribution phase are primarily controlled by the soil media properties [Brutsaert and Chen 1995; Salvucci 1997].
Figure 4.2, Temporal variation of site average brightness temperature (Fig. 4.2a) and volumetric soil moisture content (Fig. 4.2b) as an indicator of different soil textural class for the Washita '92 experiment (day 1 refers to June 10 while day 18 is June 18, June 15 data is missing).
If the changes in soil moisture with time (Figure 4.2b) is assumed to be represented by a gravity drained rectangle model (Jury et al., 1991) and a $K-\psi$ parameterization of Clapp and Hornberger [1978] then one can derive a relationship between surface soil moisture and time, for a given profile depth $L$, as

$$\Theta(t) = \left[\frac{\Theta_{sat}^{2b+4}L}{(2b+3)K_{sat}}\right] \psi^{-1/(2b+3)}$$

(4.1)

where $\Theta_{at}$, $K_{sat}$, and $b$ are saturated moisture content, saturated hydraulic conductivity and a texture parameter for the Clapp and Hornberger [1978] parameterization. Although in Equation (4.1) for simplicity, $\Theta(t)$ is modeled as a function of time only we recognize that in reality soil moisture is a function of both space and time i.e., $\Theta(x,t)$. Equation (4.1) has four parameters ($\Theta_{at}$, $K_{sat}$, $b$, and $L$); in theory, it is possible to identify soil parameters using inverse modeling. In practice, however, such an inverse problem is ill-posed and the solutions are non-unique and unstable for small errors in the observed data [Yeh, 1986; Morshed and Kaluarachchi 1998]. In addition, $\Theta(x,t)$ is not a directly observable quantity by remote sensing. It is derived from remotely sensed images of $T_B(x,t)$ by using several other surface parameters including soil texture [Jackson et al.,
Thus, it is necessary to examine the feasibility of identifying soil media properties from $T_B(x,t)$ alone.

A possibility is to use complex pattern recognition algorithms which can utilize space-time evolution of $T_B(x,t)$ (Figure 4.1 and 4.2a) to identify soil types. Rule-based pattern recognition methods work well for patterns with known features and, more importantly, well-defined shapes [Ranjithan et al., 1993]. Unfortunately, remotely sensed images of $T_B(x,t)$ do not exhibit such characteristics. As discussed earlier, the general pattern of changes in $T_B(x,t)$ during a drydown phase are related to soil physical and hydraulic properties. The details of the changes in $T_B(x,t)$ are, however, likely to be affected by other environmental factors (e.g., meteorological forcing, topography, etc.). A regression relationship developed for one scenario may not accurately fit another scenario of environmental conditions even for the same location. Therefore, to improve the accuracy and reliability of identifying soil media properties from remote sensing, we seek an alternative model structure that would enable us to extract information from multiple input features (e.g., a sequence of $T_B(x,t)$ images during a drydown phase) that are related to soil media properties, and perform robust classification and mapping in a multidimensional input-output space.
Our objective is to develop a tool that satisfies above requirements through the use of methods that synthesize recent developments in ANN with the physical linkages among the space-time pattern of $T_B(x,t)$, soil moisture depletion, and soil media properties. In this chapter, we will develop and examine the feasibility of two types of ANN models to predict soil media properties from remotely sensed data. ANN model structure is well-suited for solving problems that involve symbolically inexpressible knowledge about a mapping between one space (e.g., brightness temperature) of distributed pattern and another (e.g., soil type). We begin with a brief review of ANN models relevant to this study.

4.3. A Brief Overview of ANN Models

Artificial neural networks are model free estimators in that they do not rely on an assumed form of the underlying data. Rather, based on some observed data they attempt to obtain an approximation of the underlying system that generated the observed data. In general, one can look at computation in neural networks from the perspective of estimating an unknown function based on these observations. This general framework of learning from examples makes it possible to use neural networks in situations where exact cause-and-effect relationships are not known. This does not, however, imply that
neural networks by themselves are heuristic techniques. Far from it, neural networks have a rich theory underlying them and are based on sound mathematical principles. Criticisms of the technique primarily arise from people who do not fully explore this underlying theory and attempt to use neural networks as off-the-shelf black boxes.

Underlying a neural network are three main ingredients. (i) The model of a single neuron that is assumed, (ii) How these simple neurons are interconnected to form a network, and (iii) How the strengths of the interconnections between the neurons are determined. Most models of neural networks arise by varying these constituents, so as to ensure that the resulting network has properties which are useful and which can be guaranteed.

A neuron accepts an input $\xi$ (e.g., brightness temperature at a given pixel $l$) and produces an output $y^l$. Each component of $\xi$ is carried to the neuron through adjustable weights. The two common forms of neurons are the so-called linear basis function neurons and the radial basis function neurons. The more popular forms of neural networks are based on a network of either of these two models of neurons. As an example, linear basis function neurons, compute their output as,
\[ z_j^l = f(s_j^l) = f \left( \sum_{k=1}^{p} w_{jk} x_k^l \right) \]  

(4.2)

where, \( w_{jk} \) is the strength of the interconnection from input \( k \) (in a network this could be the output of another neuron) to neuron \( j \), and \( f(.) \) is the activation function. Thus the neuron computes a weighted sum \( (s_k^l) \) of its input and passes it through the activation function to produce an output. In most models a numerical value called bias is added to the net input. \( f(.) \) itself can be linear (this leads to linear networks which have limited computational ability), or may be non-linear. When non-linear, \( f(.) \) is usually taken to be the sigmoid i.e. \( f(x) = \frac{1}{1+e^{-x}} \), though, in general, a large class of functions exist from which \( f(.) \) can be chosen.

In particular, when the neurons are arranged in a layered fashion, such that output of neurons in a layer serve as the inputs to the neurons in the second layer one gets a feed-forward neural network. Connectivity is uni-directional from neurons in one layer to neurons in the next layer. Such feed-forward neural networks can serve as (static) function approximators or as classifiers. One of the most popular learning schemes for multi-layered network is the back propagation algorithm [Rumelhart and McClelland 1986]. Alternatively, it is also possible to interconnect the neurons such that each neuron is connected to every other neuron and itself i.e. a neuron influences other neurons and is
also influenced by itself. These networks also called recurrent networks allow (temporal) function approximation such as that required in time-series prediction. Finally, it is also possible to interconnect the neurons such that they compete in which case one obtains interesting properties of self-organization.

In a Feed-Forward Neural Network (FFNN), the neurons are generally grouped into three layers: input layer, hidden layer, and output layer. Signals flow always from input layer to the output layer through the hidden layer via unidirectional connections. In FFNN, neurons are connected from one layer to the next but not within the same layer. It has been shown that a FFNN can approximate any continuous function to an arbitrary precision [e.g., Cybenko 1989; Hecht-Nielson 1990]. Several studies, however, have reported difficulties in training the FFNN due to parameter insensitivity, parameter independence, and local optima [e.g., Hecht-Nielson 1990; Haykin 1994]. Also, the architecture of a traditional FFNN is static and it cannot adequately capture the space-time evolution of a physical system.

A particularly powerful and flexible ANN to identify input-output relationships from a distributed data set is the Self-Organizing Map (SOM) proposed by Kohonen
In SOM, input-hidden layer identifies similar patterns and groups them into clusters. Compared to traditional pattern recognition approaches, SOM offers two distinct advantages: (i) they allow for a non-parametric estimation of the underlying field, and (ii) they provide a representation which fully preserves the topological order in the underlying field. A main difference between SOM and classical pattern recognition techniques is that it utilizes the parallel architecture of a network, provides a graphical organization of pattern relationships, and provides an asymptotic estimate of the underlying probability density function. If the input patterns have an uneven probability density, then the organization of neurons in the competitive layer will reflect the probability density of the input pattern after adaptation is completed. We have successfully utilized SOM to provide a compact spatial characterization of remotely sensed soil moisture [Kothari and Islam, 1999].

As discussed in Section 4.2, there is a close relationship between space-time evolution of $T_B(x,t)$ and soil types. We will explore the use of such a relationship to identify soil types from a sequence of remotely sensed images of $T_B(x,t)$ and Self Organizing Maps (See Subsection 4.1). We will also examine whether use of space-time evolution of $T_B(x,t)$ in conjunction with same soil media properties can improve the
identification of soil physical properties using a feed-forward neural network (see Subsection 4.2)

4.4. Methodology

4.4.1. Identification of Soil Media Properties Using Remotely Sensed Brightness Temperature and SOM

Space-time variability of soil brightness temperature is a consequence of complex interactions among input (e.g., rainfall, net radiation), output (e.g., surface fluxes, infiltration, runoff), and physical properties (e.g., soil hydraulic properties, topography) of a soil surface. For example, for bare soil on a flat terrain, temporal pattern of soil moisture (and hence brightness temperature) responds to atmospheric forcing and soil properties. To filter the confounding effects of input, output, and physical properties related to $T_B(x,t)$ variability, we will focus on certain aspects of this variability. During a drydown sequence (i.e., no precipitation), spatial variability and temporal gradient of $T_B(x,t)$ will be primarily controlled by soil physical and hydraulic properties. During this sequence, we expect a monotonic initial wetness-drydown relationship of surface soil moisture. Information about spatial variability of initial $T_B(x,0)$ and temporal gradient of $T_B(x,t)$ at the pixel scale can then be used to infer soil physical and hydraulic properties.
For example, after a rainfall sequence, during the drydown period a medium (e.g., loam) soil will have higher initial soil wetness (and hence a lower brightness temperature) and a faster depletion rate compared to a coarse (e.g., sand) soil (Figures 4.2a and 4.2b). An advantage of using changes in $T_B(x,t)$ is that it can filter out confounding effects of roughness and vegetation; in addition, over short period of time changes in $T_B(x,t)$ are primarily controlled by changes in soil moisture [Hollenbeck et al., 1996].

Each time a drainage basin goes through a drying cycle, it is expected that the same spatial pattern would evolve each time [Engman 1997]. The rate of evolution may vary based on hydrometeorological forcing but the relative patterns are expected to be similar. We plan to identify such patterns from remotely sensed $T_B(x,t)$. To explore a possible relationship between $T_B(x,t)$ and soil types, we will focus on the redistribution stage when the assumption of gravity drainage of soil water is appropriate. Physical characteristics of this stage are very similar to those of the stage-two evaporation [Brutsaert and Chen 1995; Salvucci 1997]. We will identify the onset of this stage from sharp change in remotely sensed surface albedo [Salvucci 1997] or ratio of air temperature and radiometric surface temperature [Shouse et al., 1982].
We assume that we have a series of $T_b(x,t)$ with $M$ by $N$ pixels over $P$ days to examine the feasibility and robustness of our approach. The pixel resolution and the size of $M$ and $N$ will depend on the sensor and its flight path while $P$ will depend on the length and frequency of data collection. Input pattern for our proposed SOM network will be this series of brightness temperature for a given pixel. The function of SOM is to detect and classify patterns in the input data, without any reference to the output data. This lack of reference to output data in SOM is particularly attractive for us because it allows us to classify soil into different types with only $T_b(x,t)$ as an input. The number of neurons, $Q$, in the hidden layer (also called competitive layer within the context of a SOM shown in Figure 4.3) correspond to desired number of classes to be identified from the input data.

The connections between the input layer and the competitive layer are initialized to have random weights. When an input vector (say a temporal sequence of brightness temperature at a location) is presented as an input, the first step in the operation of a SOM is to compute a matching value for each unit in the competitive layer. This value measures the extent to which the weights of each processing unit match the corresponding values of the input pattern. The processing unit that has closest match to
the input is identified as a winning unit. Interconnection weights of the winning unit and its nearest neighbors are updated to go further closer to the input pattern. Next, another input pattern is chosen from the data set and the process is repeated until the interconnection weights between the input data and the processing units stop changing.

\[ \xi^I = [T_B(x,1), T_B(x,2), \cdots, T_B(x,P)]^T \]

**Figure 4.3, A schematic of self-organizing map (SOM)**
We will use the SOM network to cluster a sequence of spatially distributed
$T_B(x,t)$ data into several groups. Given a time series of brightness temperature for a given	pixel, $\{\xi^l\}$, defined as $(\xi^l = [T_B(x,1), T_B(x,2), \ldots, T_B(x, P)])^T$, we expect to minimize

$$J = \min_i \| \xi^l - w_i \|$$  \hspace{1cm} (4.3)

Note, here the position vector “$x$” for a given pixel is denoted by a scalar pixel
identifier “$l$” for notational simplicity. Here, $w_i$ is a vector of length $P$ referring to the
weight for neuron $i$. In the second step SOM updates the parameters of the best matching
neuron $c$, and all other neurons within a certain topological neighborhood, $N_c$, of the best
matching neuron. That is,

$$w^i(l + 1) = w^i(l)\alpha(l, d)[\xi^l - w^i(l)]_i \in N_c$$ \hspace{1cm} (4.4a)

$$w^i(l + 1) = w^i(l) \text{; otherwise}$$ \hspace{1cm} (4.4b)

where, $N_c$ denotes those neurons which are less than the correlation scale of the best
matching neuron. The term $\alpha(\cdot)$ is a real valued scalar, which is a decreasing function of
both spatial location and distance “$d$” from the best matching neuron. To prevent the
same neuron from frequently winning, a “conscience” term is added to penalize the
frequently winning neuron. Once the update in (4a and 4b) is completed, next input pattern from pixel $k+1$ is given as input and this process is continued till all the pixel information is used as input and the interconnection weights between the input and neurons reach a stable state.

Upon convergence, the SOM detects and classifies the temporal sequence of brightness temperature in the input layer into $Q$ groups through the weights in the competitive layer. Each group represents a typical dry-down curve for certain type of soil texture. Training is terminated when the location of each textural class becomes stable. Since there are a limited number of neurons in the competitive layer, the competitive layer becomes a filter that associates each input vector with one of the $Q$ textural classes. As a consequence of the training strategy used, the competitive neurons “self organize” such that neighboring neurons are also functionally similar.

We randomly choose a certain percentage of multi-temporal $T_B(x,t)$ pixels as the input layer. The input data are connected with $Q$ neurons in the competitive layer. As a result, the weights on these $Q$ neurons represent $Q$ typical curves of $Q$ different soil textures. The problem now is to detect what soil texture classes these curves belong to.
During a dry-down phase, the values of soil moisture should fall into the range of field capacity \( (\theta_{CAP}) \) and permanent wilting point \( (\theta_{PWP}) \) for a given soil texture. These two values are distinguishable for different soil textural classes. Based on the similarities of Figures 4.2a and 4.2b we assume that the brightness temperature corresponding to wilting point and field capacity will also be distinguishable.

We use the length of overlap between the “curves” of these \( Q \) neurons and the range of \( \theta_{PWP} \) and \( \theta_{CAP} \) for \( Q \) different soil texture to determine the attributes of each neurons—the longest overlap decide the soil texture. To predict the soil texture for a given point, we first find the neurons that are close to the input vector through the minimum distance approach. These neurons are then related to soil textural classes. One particular feature in the above framework we emphasize is the “unsupervised” nature of the ANN i.e., a classification is achieved based on the inherent physical and hydraulic properties of soil media without any explicit knowledge about the soil media. In other words, we determine the soil texture map directly from the changes in soil brightness temperature maps.
4.4.1.1 Use of brightness temperature to identify soil types

We randomly choose a percentage (e.g., 5 % and 10 %) of multi-temporal brightness temperature data from the 228 by 85 pixels as the training data. Initially, we classify soils into three (i.e., $Q = 3$ corresponding to coarse, medium and fine soil) textural groups as a function of ratio of sand to percent clay. A comparison of the estimated soil texture map with the observed soil texture map is performed to evaluate the efficacy of this approach.

Figure 4.4a and 4.4b show spatially distributed texture classification for the Little Washita watershed using 5 % and 10 % randomly chosen $T_B(x,t)$. A comparison of Figure 4.4a and 4.4b with observed soil texture classes (Figure 4.4c) show a very close spatial similarity. In these Figures, “others” represents the non-soil points (i.e., water, gypsum, pits, etc.) that are not included in our analysis. Table 4.1 shows some statistics of the performance of SOM model. We use the correlation coefficient ($CC$) and disparity index ($DI$) to compare and evaluate the performance of the proposed ANN models. Eq. (4.5) and Eq. (4.6) shows the mathematical expression for these two statistics.

$$CC = \frac{1}{N} \sum_{i=1}^{N} \frac{(x_i - \bar{x})(y_i - \bar{y})}{\sigma_x \sigma_y}$$

$$DI = \frac{1}{N} \sum_{i=1}^{N} \|B(x_i - y_i)\|$$

101
\[
B(\bullet) = \begin{cases} 
1, & \bullet > 0 \\
0, & \bullet = 0 \\
-1, & \bullet < 0 
\end{cases}
\]

where \(x_i\) is the observed soil textural index (e.g., 1 for coarse, 2 for medium and 3 for fine); \(y_i\) is the soil textural index inferred from ANN models, \(\bar{x}\) and \(\bar{y}\) are the domain average of \(x_i\) and \(y_i\) respectively, \(\sigma_x\) and \(\sigma_y\) are the standard deviation of \(x_i\) and \(y_i\) respectively.

Comparison of Figure 4.4a and 4.4b as well as results from Table 4.1 show that sensitivity of using a greater percentage (5% vs. 10%) of multi-temporal brightness temperature is minimal. In other words, 5% of randomly chosen \(T_B(x,t)\) training data may be sufficient to classify soil into three different classes.
Figure 4.4, Comparison of SOM estimated (Fig. 4.4a, 4.4b) and observed (Fig. 4.4c) soil texture field from Washita '92. Soil, which is classified as three (coarse, medium, and fine) textural groups. In Fig. 4.4a and 4.4b, 5% and 10% of brightness temperature data is applied as training data, respectively. In all Figures, “others” represent the non-soil points (i.e., water, gypsum, pits, etc.) that are not included in our analysis.
Figure 4.5, Similar to Figure 4.4, but soil moisture data is applied as training data
4.4.1.2 Use of soil moisture to identify soil types

Now we will repeat the experiment described in 4.1.1 by using remotely sensed soil moisture data, instead of brightness temperature, and SOM to classify soil into different textural types. We note here that \( q(x,t) \) is not a directly observed quantity by remote sensing. It is derived from remotely sensed \( T_B(x,t) \) by using several other surface parameters [Jackson et al., 1995].

Figure 4.5a and 4.5b show spatially distributed texture classification for the Little Washita watershed using 5% and 10% randomly chosen \( q(x,t) \). A comparison of Figure 4.4a and 4.4b with observed soil texture classes (Figure 4.4c) show a very close spatial similarity. Again, sensitivity of using a greater percentage (5% vs. 10%) of multi-temporal soil moisture is minimal (see Table 4.1 for statistics). Table 4.1 also shows that the use of soil moisture as an input produces slightly better results compared to those using brightness temperature. This is somewhat expected because the physical linkages between soil moisture and soil texture is much stronger than those between brightness temperature and soil texture. In addition, remotely sensed soil moisture data were derived
from brightness temperature using a model [Jackson, 1995] with input factors including soil texture.

Table 4.1, The performance of SOM model in predicting three textural classes, using brightness temperature as training data

<table>
<thead>
<tr>
<th>Training data</th>
<th>Brightness temperature</th>
<th>Soil moisture</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>Correlation coefficient</td>
<td>0.58</td>
<td>Fig. 4.4a</td>
</tr>
<tr>
<td></td>
<td>Disparity index</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>10%</td>
<td>Correlation coefficient</td>
<td>0.58</td>
<td>Fig. 4.4b</td>
</tr>
<tr>
<td></td>
<td>Disparity index</td>
<td>0.29</td>
<td></td>
</tr>
</tbody>
</table>

4.4.2. Use of a Multi-layered Feed-Forward Neural Network to Improve Soil Classification Accuracy

The SOM model provides a methodology to classify soil into different textural groups using an “unsupervised” ANN. This methodology of identifying soil media properties would be invaluable for many regions where no detailed soil information are available. We must note, however, that classification in an unsupervised ANN is guided by the changes in $T_B(x,t)$. If changes in $T_B(x,t)$ are affected by other environmental
factors (e.g., variability in atmospheric forcing, topography, etc.) than a classification based on unsupervised ANN would involve uncertainty. We plan to use a Feed-Forward Neural Network (FFNN) to improve our classification accuracy.

The Feed-Forward Neural Network consists of three layers of neurons: an input layer, a hidden layer, and an output layer (Figure 4.6). The input neuron receives and delivers the signal without changing it. The output neuron weights and sums the coming signals, and then the net result is passed through a linear activation function. The hidden neurons are similar to the output neurons except that a binary sigmoid is used as the activation function. For a certain pixel \( k \), each neuron in the hidden layer receives input \((\xi^i = [T_B(x,1), T_B(x,2), ..., T_B(x,P)]^T)\). If the number of neurons in the input layer and hidden layer is \( P \) and \( I \) and \( \xi_m^k \) represent the \( m^{th} \) component of \( \xi^k \), then the output of the \( i^{th} \) neuron in the hidden layer is:

\[
z_j^i = f_1\left(\sum_{k=1}^{n} w_{jk} \xi^i_k\right)
\]

where \( f_1(x) = \frac{1}{1 + e^{-x}} \)

The result of the output layers is:

\[
y_j^i = f_2\left(\sum_{j=1}^{h} (w_{ji}z_j^i)\right)
\]
where \( f_2(x) = \text{Round}(x) \)

where \( w_k \) representing the weight in the hidden layer, and \( w_j \) representing the weight in the output layer. As a result, an integer number is generated from the output \( y^l \) for the pixel \( l \), which represents the soil texture class.

\[
\xi^l = [T_B(x,1), T_B(x,2), \ldots, T_B(x,P)]^T
\]

**Figure 4.6**, A schematic of a Feed Forward Neural Network (FFNN)
To obtain the weights in the inter-connection, we randomly choose a certain percentage (e.g., 5%) of multi-temporal $T_B(x,t)$ pixels to train the FFNN. For this set of pixels, we assume that we have soil textural information. This computed output signal is compared with the desired output (i.e. textural class in our case) and an optimization rule is used to update the connection weights. In our approach, we use the optimization rule by Levenberg-Marquardt \cite{Levenberg,1944;Marquardt,1963} since it converges faster than other training rule. For a detailed discussion of Levenberg-Marquardt method we refer to Bishop \cite{1996}). This procedure is continued until all the training patterns are presented and connections weights are optimized. Then, the remainder of the brightness temperature patterns will be presented and texture classification for a given pixel will be estimated by the network.

4.4.2.1 Use of brightness temperature to identify soil types

Initially, we classify soils into three (i.e., $Q = 3$ corresponding to coarse, medium and fine soil) textural groups as a function of ratio of sand to percent clay using FFNN architecture. Based on the results of this preliminary classification scheme, further classification into six (i.e., $Q = 6$) soil classes will be considered later (Subsection 4.2.3). Figure 4.7a and 4.7b show spatially distributed texture classification for the Little Washita watershed using 5 % and 10 % randomly chosen $T_B(x,t)$. Using brightness
temperature as the input, the results using FFNN architecture (Fig. 4.7a and 4.7b) shows better classification accuracy than SOM (Fig. 4.4a and 4.4b). These comparative performances are also shown in statistical parameters (Table 4.1 for SOM and Table 4.2 for FFNN). For example, the disparity index for FFNN is as low as 19% compared to 28% for SOM.

**Table 4.2, The performance of FFNN model in predicting three textural classes, using brightness temperature as training data.**

<table>
<thead>
<tr>
<th>Training data</th>
<th>Brightness temperature</th>
<th>Figure</th>
<th>Soil moisture</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>Correlation coefficient</td>
<td>0.70</td>
<td>Fig. 4.7a</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>Percent difference</td>
<td>0.19</td>
<td></td>
<td>0.17</td>
</tr>
<tr>
<td>10%</td>
<td>Correlation coefficient</td>
<td>0.71</td>
<td>Fig. 4.7b</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>Percent difference</td>
<td>0.19</td>
<td></td>
<td>0.15</td>
</tr>
</tbody>
</table>
To explore the reason why the performance of FFNN model is better than that of SOM model in the case of using brightness temperature as the input, we now examine the basic structure of SOM model. The SOM model self-organizes the “feature” of training data and classifies the training data into several groups based on the feature. In the case of using brightness temperature as training data, the “feature” is the “physical relationship” between brightness temperature decaying curve and the soil texture. In particular, SOM infers that different soil textures have different decaying curve. When other factors (e.g., vegetation types, vegetation water content, soil surface roughness, etc.) are considered, different soil textures are likely to have similar brightness temperature decaying curve. On the other hand, the FFNN model uses space-time evolution of brightness temperature in conjunction with some soil media properties (e.g., 5% or 10 % pixels with soil textural information) as training data. The interconnection weights in the FFNN network are adjusted through the given examples of soil textural information. This adjustment appears to compensate for the lack of information in using brightness temperature alone.

4.4.2.2 Use of soil moisture to identify soil types

Now we will use remotely sensed soil moisture data and FFNN to classify soil into different textural types. Figure 4.8a and 4.8b show spatially distributed texture
classification for the Little Washita watershed using 5% and 10% randomly chosen $\mathcal{O}(x,t)$. Comparison of Figure 4.8 and Figure 4.7 suggests that FFNN architecture using soil moisture as the input does not have significant improvement compared to those using brightness temperature as input. This is partly because soil moisture is not a directly observed quantity; soil moisture is derived from brightness temperature by using other surface information including soil texture.
Figure 4.7, Comparison of FFNN estimated (Fig. 4.7a, 4.7b) and observed (Fig. 4.7c) soil texture field from Washita '92. Soil, which is classified as three (coarse, medium, and fine) textural groups. In Fig. 4.7a and 4.7b, 5% and 10% of brightness temperature data is used as training data, respectively. In all Figures, “others” represent the non-soil points (i.e., water, gypsum, pits, etc.) which are not included in our analysis.
Figure 4.8, Similar to Figure 4.7, but soil moisture data is used as training data.
Figure 4.9, Comparison of the FFNN model estimated (Fig 4.9a, 4.9b) and observed (Fig 4.9c) soil texture field from Washita '92. Soil, which is classified as six ((1) sand, (2) loamy fine sand, (3) fine sandy loam, (4) loam, (5) silt loam, and (6) silty clay loam / clay loam.) textural groups.
**Figure 4.10**, The degree of difference between FFNN estimated and observed soil texture.
4.4.2.3 Identification of Six Textural Classes Using FFNN

To explore the capability of FFNN model further, we use this model to estimate soil texture in six classes: (1) sand, (2) loamy fine sand, (3) fine sandy loam, (4) loam, (5) silt loam, and (6) silty clay loam / clay loam. The output of FFNN is an integer corresponds to a given textural class. Figure 4.9 compares the FFNN estimated (Fig 4.9a and 4.9b) and observed (Fig 4.9c) soil texture field from Washita ’92. Soil, where, in Figure 4.9a, the model is trained with 5% of brightness temperature while in Figure 4.9b it is trained with 5% of soil moisture. The correlation coefficients between model estimated and observed soil texture is as high as 0.65 for training with brightness temperature and 0.70 for training with soil moisture. Figure 4.10 shows the degree of difference between model estimated and observed soil texture. The degree of difference (DD) is defined as:

\[
DD = |x_i - y_i|
\]  

where \(x_i\) is the observed soil textural index (1 for sand, 2 for loamy fine sand, 3 for fine sandy loam, 4 for loam, 5 for silt loam, and 6 for silty clay loam / clay loam); \(y_i\) is the
soil textural index inferred from FFNN models. A zero value of $DD$ represents that the
model estimated results and observed data perfectly match each other. It is interesting to
note that over 45% of the pixels are identified with a $DD$ value of zero. A $DD$ value of
one implies that the model estimated and the observed soil textures are in the nearest
classes (two different textural classes with the difference of indexes equal to one). The
worst case is a $DD$ value of five, that is, for a certain pixel the model estimated texture is
sand but the observed texture is silty clay loam / clay loam, or vice versa. Our results
suggest that less than 0.01 % of grid point is estimated with a $DD$ value of five.
According to the Washita92 data, there are 432 pixels in the total of 228* 85 pixels that
are Silty Clay Loam / Clay Loam. Our results shows that there are only 2 pixels that are
wrongly predicted for sand to Silty Clay Loam / Clay Loam or Silty Clay Loam / Clay
Loam to sand. In other words, there is only 0.5% chance to yield the worst case ($DD=5$)
using the FFNN model.

4.5. Concluding Remarks

We have used multi-temporal remotely sensed maps during a drydown phase to
physically relate brightness temperature and soil moisture to soil physical properties
within the framework of a set of Artificial Neural Network Models. Two ANN models
are developed and tested to classify soil texture using remotely sensed brightness temperature or remotely sensed soil moisture without any land surface information. There is a close relationship between space-time evolution of remotely sensed brightness temperature and remotely sensed soil moisture and soil types and such a relationship can be used to identify soil types from a sequence of remotely sensed images of remotely sensed brightness temperature and remotely sensed soil moisture and SOM. The FFNN Model use space-time evolution of remotely sensed brightness temperature and remotely sensed soil moisture in conjunction with some soil media properties (e.g., 5% pixels with soil textural information) to improve classification accuracy. A classification based on FFNN can yield better classification accuracy in such situations. However, if no soil textural data are available, the unsupervised SOM model would provide reasonable classification accuracy.

As multi-temporal maps of near surface brightness temperature maps are used for the estimation of soil texture map, this map essentially corresponds to near surface textural properties. Thus, this study cannot provide explicit information regarding the profile soil moisture or texture. In many cases, however, there is a strong relationship between near surface soil moisture and root zone soil moisture. In such cases, profile soil
properties can be inferred from near surface [e.g., Calvet et al., 1998]. There are indications that a sequence of surface soil moisture data can be used to predict saturated hydraulic conductivity [e.g., Ahuja et al., 1993; Mattikalli et al., 1998]. We also note that with significant heterogeneity in topography, there can be appreciable redistribution of soil moisture such that the valley bottoms would evaporate at nearly potential rate whereas evaporation would be very low elsewhere. In such cases, an unsupervised classification based on SOM is likely to produce large errors. However, a classification based on FFNN is expected to yield better classification accuracy in such situations.

The proposed methodology is based on the assumption that the dry-down curves of brightness temperature and soil moisture at different locations with the similar soil texture would exhibit similar behavior. This assumption may not always be appropriate if the dry-down curves for a certain location is influenced, for example, by other physical attributes (e.g., lateral flow, topography, etc.) of neighboring grid points. For such cases, additional input variables from neighboring pixels will be needed to improve the classification accuracy.
CHAPTER 5

Estimation of Soil Physical Properties using Remotely Sensed Brightness Temperature Over the Southern Great Plains
5.1. Introduction

Soil physical and hydraulic properties are the key parameters for a variety of surface and atmosphere processes, including the soil-plant-atmosphere interaction, drainage, erosion, and solute transport. Currently, soil survey maps from local or national agencies are the major sources for the information of spatially variable soil properties. The incompatibility of soil survey maps, usually at the resolution of kilometers, with other high-resolution terrain data has been reported [Band and Moore, 1995; Zhu, 1997]. This inadequacy largely results from the algorithm to derive soil survey maps. In general, soil survey maps are interpolated from a small number of sample points of soil texture. While interpolation shortens the time and labor necessary to collect data, given the heterogeneity of soil properties, such an interpolation makes the resulting texture database highly uncertain at finer resolution. Therefore, development of a method that can render reliable estimation of soil texture at finer resolution would be of great value.

To overcome this problem of limited information of soil properties from discrete sampled points, some easily measured complementary data has been used. Based on the relationships between complementary data and soil properties, soil properties in the non-sampled pixels can be estimated. For example, Zhu [2000] has demonstrated the
feasibility of using topographic indices to estimate soil distribution in a watershed located in Lubrecht Experimental Forest. The use of topographic indices to estimate soil properties, however, may not be suitable for flat or gently rolling topographic field due to the different spatial frequency in topography and soil properties. Remotely sensed, especially by electronically scanned thinned array radiometer (ESTAR), brightness temperature images were shown to have the potential to detect soil heterogeneity [Hollenbeck et al., 1996]. The remotely sensed brightness temperature responds to the water content of the surface soil layer. Consequently, space-time evolution of brightness temperature has the potential to infer soil moisture and soil physical properties over large areas [Hollenbeck et al., 1996; Rodriguez-Iturbe et al., 1995].

The relationship between brightness temperature and soil properties is complicated and non-linear because the state of soil moisture is influenced by a variety of environmental factors (e.g. precipitation, vegetation, etc.). A number of previous studies have explored the feasibility of estimating soil texture through brightness temperature and soil data from limited number of sampled points [Camillo et al., 1984; Hollenbeck et al., 1996; Mattikalli et al., 1998]. Many of these methodologies, however, are either based on land-atmosphere model that requires several input data or based on regression
methods that provide non-unique solutions (for a review, see Chang and Islam, 2000)).

One possibility to link the non-linear relationship between brightness temperature and soil properties is through Artificial Neural Networks (ANN). ANN, although at its early stages of hydrologic applications, are rapidly becoming an attractive tool to characterize, model, and predict complex multi-source remotely sensed hydrologic data.

In Chapter 4, we have demonstrated the feasibility of classifying soil types into three categories (coarse, medium and fine textures) using ESTAR sensed brightness temperature from Washita ‘92 Experiment (Jackson et al., 1995) and ANN models (i.e., Self-Organizing Map (SOM) and Feed-Forward Neural Network (FFNN)). Our results suggest that more than 70% classification accuracy can be obtained from either SOM or FFNN networks. Yet such a three-category classification may not be sufficient for a variety of soil related applications due to the high variability of soil properties within each class. Our attempt to classify soil types into more than three categories resulted in about 50% accuracy when a FFNN was used and even lesser accuracy when a SOM was used. SOM is an unsupervised ANN model and has the advantage that the observed data used for training the network need not be labeled with the true soil type. In one sense, SOM can be considered to perform clustering of the observed data. When some labeled
data is available, then the clusters can be associated with a soil type. Nonetheless, the performance of unsupervised classifiers consistently underperforms that of a supervised classifier. Theoretically, a well-trained FFNN model is capable of making precise decision boundaries between arbitrary classes defined by the example input-out relationship. This capability is demonstrated in the fact that about 90% of the classification errors made by a FFNN for more than three soil categories can be accounted from soil types that are similar to each other (e.g., silty loam been predicted as loam). The confusion that the FFNN displays in soil types that are similar is not surprising because it is quite difficult to distinguish some soil types apart, based on the input data used in Chapter 4, from their physical properties (e.g., saturated hydraulic conductivity, porosity).

This study takes a closer look at the performance of the FFNN model and in particular at ways of minimizing the error made by the FFNN model in categorizing the soil into different texture types. In part, the questions that arise include the number of wetting and drying cycles (amount of time) for which a pixel must be observed to categorize it as well the topology and other associated parameters of the FFNN. Our investigation yield results that motivate the use of a prototype based classifier as being
more appropriate for predicting the soil texture based on the brightness temperature. Indeed, using a simple prototype based classifier (1-Nearest Neighbor or 1-NN in short), we obtain significantly better performance. We discuss the reason for this later in this chapter.

5.2. Background

The brightness temperature and soil data used in this study have been collected by National Aeronautics and Space Administration (NASA), the US Department of Agriculture (USDA), and several other government agencies and universities during the so-called Southern Great Plains 1997 Hydrology Experiment (SGP ’97). During the SGP ’97 experiment, observations were made in central Oklahoma through various microwave sensors, including ground-based, aircraft, and space-borne sensors. Among them, the L-band imagery from airborne Electronically Scanned Thinned Array Radiometer (ESTAR) provides high spatial and temporal resolution of brightness temperature ($T_B$) data. The soil moisture images in the form of volumetric soil moisture were then derived from brightness temperature. Sixteen days of brightness temperature and soil moisture data were collected during the period from June 18 through July 16, 1997. These data are stored in 206 by 621 pixels covering a 10,000 km$^2$ area with a pixel resolution of 800 m by 800 m. For a complete description of SGP ’97, we refer to [Jackson et al., 1999]. In
this study, we will apply only 15 days of ESTAR sensed brightness temperature data. The data from July 11 are excluded because of much fewer data point for that day. In addition, since the flight line measuring $T_B$ images changes slightly each day, there are only 18,699 pixels with complete 15-day $T_B$ data. Therefore, this study focuses on a smaller 160 by 400 pixel area that encloses a polygon containing those 18,699 pixels. The Universal Transverse Mercator (UTM) coordinate of the test area is $<543600$ E, $4117000$ N> for the upper left corner and $<671600$ E, $3797000$ N> for the lower right corner. Note that this test area covers the domain of Washita '92 Experiment (UTM coordinates: $<563844$ E, $3872666$ N> for the upper left corner; $<609444$ E, $3854066$ N> for the lower right corner), which was used in Chapter 4.

Figure 5.1 shows a sequence of remotely sensed brightness temperature, $T_B(x,t)$, and corresponding near surface soil water content, $\theta(x,t)$, within the polygon. Here, “x” is the position vector and “t” is the time. It appears that there is a strong spatial and temporal coherence between $T_B$ and $\theta$. For example, for July 1, $T_B(x)$ is low in the northern parts (about $180^\circ$K) and high in the middle and south regions (about 260 K). On the other hand, $\theta(x)$ is wetter in the northern part (about 0.45) and drier in the middle and south regions (about 0.1). Regions with higher brightness temperature tend to have drier
soil moisture. Figure 5.1 also shows the spatial variation of soil texture. Soil texture is classified into twelve categories based on United States Department of Agriculture (USDA) systems, although only six soil classes existed in the study area. In Figure 5.1, there appears to be a significant degree of spatial coherence between brightness temperature and soil texture as well as soil moisture and soil texture. For example, the sand region has much higher brightness temperature (about 270 K) and lower soil moisture (about 0.5) in most $T_B(x,t)$ and $\theta(x,t)$ maps, respectively. In brief, Figure 5.1 suggests that spatial structure of $T_B$ and its temporal variations may be used to estimate soil texture.
Figure 5.1, Remotely sensed brightness temperature (in degree K) and derived volumetric soil moisture (in percent) for 15 days between June 18 and July 16 from the Southern Great Plains 1997 Experiment. The last figure refers to soil texture.
In Chapter 4 we have used ANN and multi-temporal sequence of brightness temperature from Washita ’92 Experiment to estimate the distribution of soil types. These data are measured after a period of heavy rainfall and no precipitation was observed during the experiment. In other words, the multi-temporal sequence of brightness temperature reflects the soil moisture data in a single drying cycle. If the precipitation is homogeneous (such as the case in Washita ’92 Experiment), the state of soil moisture in similar soil types (e.g., Sand \approx Sandy Loam; Silty Loam \approx Loam) would have similar initial values and also decreases at similar evaporation rate. Consequently, a sequence of brightness temperature from a single drying cycle after a homogeneous precipitation event may not contain sufficient features to distinguish the soil types with acceptable accuracy. That is the reason why we obtain comparative low accuracy (i.e. 45\%) in Chapter 4 when we intend to classify soil types into more than three categories.

As an illustration, we analyze the sequence of brightness temperature and soil moisture data for two soil types with similar soil physical and hydraulic properties—Silty Loam and Loam— in the Washita catchment during the SGP ’97 Experiment. These two soil types are considered as the same soil group (i.e., medium soil) in the three-category classification. As shown in Figure 5.2, there are four drying cycles (initiated on the 1st, 129
5\textsuperscript{th}, 8\textsuperscript{th}, and 12\textsuperscript{th} days) during the SGP '97 Experiment. Note that for the first drying cycle, beginning with June 18, brightness temperature and soil moisture for these two soil types are distinctly different and continue to be different for the entire drying cycle for the following three days. On the other hand, during the last drying cycle, brightness temperature and soil moisture curves for these two soil types begin with similar initial values and tend to overlap in the following days. If we use the first drying cycle to train the ANN model, the model might be able to distinguish between these two soil types. However, if we use the last drying cycle to train the ANN model, the model would have difficulty distinguishing these two soil types. To reduce such uncertainties during the training phase and to provide multiple scenarios of drying sequences, for this study, we use four drying cycles to train the ANN model. This would allow us to compare and contrast our classification accuracy to those of Chapter 4 where only one drying cycle was used. We begin with a Feed Forward Neural Network.
Figure 5.2, Temporal variation of site average brightness temperature (a) and volumetric soil moisture content (b) as an indicator of different soil textural class in the Washita catchment (day 1 to day 15 refer to the following days in 1997: June 18, 19, 20, 25, 26, 27, 29, 30, July 1, 2, 3, 12, 13, 14, 16)
Figure 5.3, A schematic diagram of a Feed Forward Neural Network (FFNN)
5.3. Feed forward neural network

5.3.1. Identification of Soil Media Properties Using Remotely sensed Brightness Temperature and Feed-Forward Neural Network (FFNN)

A Feed-Forward Neural Network consists of an input layer, one or more hidden layers, and an output layer of neurons. Each neuron in a layer is connected to all the neurons in the next layer as shown in Figure 5.3. Let there be \( n \) inputs, a single hidden layer with \( h \) neurons and an output layer with \( m \) neurons. Let the weights between the \( k^{th} \) input and the \( j^{th} \) neuron in the hidden layer be denoted by \( w_{jk} \) and the weight between the \( j^{th} \) neuron in the hidden layer and the \( i^{th} \) neuron in the output layer be denoted by \( w_{ij} \).

When an input \( \xi^I = [\xi_1, \xi_2, \ldots, \xi_k, \ldots, \xi_p] \) is applied, the \( j^{th} \) hidden neuron produces as output,

\[
z_j^I = f_j^I \left( \sum_{k=1}^{p} w_{jk} \xi_k^I \right)
\]  

(5.1)

where, \( f_j^I(\cdot) \) is a non-linear function (often called the activation function). The sigmoidal activation function defined as,

\[
f_j^I(x) = \frac{1}{1 + e^{-\lambda x}}
\]  

(5.2)

where \( \lambda \) controls the steepness of the sigmoid, which is often called the slope parameter. Neuron \( i \) in the output layer produces,
\[ y'_j = f_2 \left( \sum_{j=1}^{h} (w_{ij} z'_j) \right) \]  

(5.3)

where, \( f_2(\cdot) \) as before is an activation function. Often a linear activation function is used in the output layer such that,

\[ f_2(x) = x \]  

(5.4)

From one perspective, each output can be viewed as being produced by the superposition of \( h \) (the number of hidden layer neurons) sigmoids similar to say, when, a signal is approximated by the superposition of sine and cosine functions as in Fourier series. However, the basis functions in a FFNN are not orthogonal and are adapted on the basis of a given training data set. As a result, complex approximations can be produced by a FFNN and it can be shown that they can approximate continuous functions defined on a compact set \( \mathbb{R}^n \) [Cybenko, 1989].

For each pixel \( x \), we formed the input as \((\xi'_x = [T_b(x,1), T_b(x,2), \ldots, T_b(x, P)]^T)\). Note that the brightness temperature over \( P \) observation intervals corresponds to the number of inputs of the FFNN. We used a 1-of-\( m \) encoding for the output, i.e., if there are \( m \) soil types then the FFNN has \( m \) outputs. If a particular pixel belongs to the \( q^{th} \) soil texture type, then the \( q^{th} \) output is 1 and all other outputs are 0. In this study, the test area
in SGP '97 contains six soil types (Sand, Sandy loam, Silt Loam, Loam, Silty Clay Loam, Clay loam). The soil type is then represented by a vector of six components as shown in Table 5.1.

<table>
<thead>
<tr>
<th>Soil Texture Class</th>
<th>Sand</th>
<th>Sandy loam</th>
<th>Silt Loam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Vector</td>
<td>[1, 0, 0, 0, 0, 0]</td>
<td>[0, 1, 0, 0, 0, 0]</td>
<td>[0, 0, 1, 0, 0, 0]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Soil Texture Class</th>
<th>Loam</th>
<th>Silty Clay Loam</th>
<th>Clay loam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Vector</td>
<td>[0, 0, 0, 1, 0, 0]</td>
<td>[0, 0, 0, 0, 1, 0]</td>
<td>[0, 0, 0, 0, 0, 1]</td>
</tr>
</tbody>
</table>

We chose 15% of multi-temporal brightness temperature data from the 18,699 pixels of multi-temporal pixels to train the FFNN. For this set of pixels, we assume that we have soil textural information. The output of the network when each of these inputs are presented is compared with the desired output (i.e. textural class in our case) and an optimization rule is used to update the connection weights such that the network output agrees with the desired output. In our approach, we use the optimization rule by Levenberg-Marquardt [Levenberg, 1944; Marquardt, 1963] since it converges faster than other training rule. For a detailed discussion of Levenberg-Marquardt method we refer to [Bishop, 1996]. This procedure is continued until all the training patterns are presented.
and connections weights are optimized. Then, the remainder of the brightness
temperature patterns will be presented and texture classification for a given pixel will be
estimated by the network.

Once trained, when an input is presented to the network the response may
typically be for example, [0.1, 0.2, 0.9, 0.3, 0, 0]. In such cases, the maximum component
(i.e., 0.9) is considered as ‘one’ and other components are considered as ‘zero’. Thus the
output is treated as [0, 0, 1, 0, 0, 0] which represents Silt Loam.

We have noted that in the 15% of sampled pixels, the number of pixels for each
soil texture is not uniformly distributed. For example, the sample size of Silt Loam is
much larger than other soil texture classes due to the abundance of Silt Loam over the
domain. The approximate ratios of sample size in Silt Loam to that of other soil texture
class are about 10:1, 4:1, 3:1, 40:1, 30:1 for Sand, Sandy Loam, Loam, Silty Clay Loam,
and Clay Loam, respectively. In such cases, the FFNN may not be trained adequately for
the soil texture classes with relatively smaller sample sizes. To overcome this skewed
distribution of soil texture, a resample process is suggested. Taking the Clay Loam for
example, where ratio of sample size in Silt Loam to that of Clay Loam is about 1:30, by
adding a small perturbation in the brightness temperature data for Clay Loam and repeating such process 30 times, the resample process will generate a data set of Clay Loam with sample size equal to that of Silty Loam. In this example, we take the standard deviation of daily brightness temperature data in Clay Loam multiplying by 0.1 as the perturbation. A similar resampling process is also applied to other texture types.

5.3.2. The comparison between the FFNN-predicted results based on brightness temperature in single and multiple drying cycles

In this subsection, we compare the FFNN-predicted results based on brightness temperature in single and multiple drying cycles. Initially, we classify soil types into three (coarse, medium, and fine soil texture) categories (based on the portions of sand, silt and clay) using a FFNN model with the number of hidden neurons $h = 15$ and slope parameter $\lambda = 1$. Figure 5.4a, 5.4b and 5.4c show the FFNN-predicted results based on a three-category classification. For Figure 5.4a and 5.4b, the inputs of FFNN model are the 4-day brightness temperature data, which contains a single drying cycle. For Figure 5.4c, the inputs are 15-day brightness temperature data, which contains four individual drying cycles. A comparison between Figure 5.4a, 5.4b, 5.4c and the observed textural classes (Figure 5.4c) shows that, for a three-category classification, the predicted results based on a single-drying-cycle and multiple-drying-cycle of brightness temperature data are
spatially consistent with observations. We also note that the use of single-drying-cycle brightness temperature data generates 81% (for Figure 5.4a) and 80% (for Figure 5.4b) accuracy and the use of multiple-drying-cycle data produces 83% accuracy. Figure 5.4e, 5.4f and 5.4g show the FFNN-predicted results based on the six-category classification. Our attempt to classify soil into six categories using brightness temperature data in a single drying cycle only provides about 61% (for Figure 5.4e) and 59% (for Figure 5.4f) accuracy. On the other hand, if the multiple-drying-cycle brightness temperature is used (Figure 5.4g), the performance of FFNN-predicted results shows significant improvement (about 73% accuracy). In brief, Figure 5.4 suggests that the brightness temperature data in a single drying cycle may be sufficient to classify soil types into three categories. However, if we desire to classify soil types into more than three groups, the use of brightness temperature data in multiple drying cycles can yield better classification accuracy.
Figure 5.4, Comparison of FFNN-predicted and observed soil-texture map. Figure (a), (b), (c), (d) are based on 3-category classification. (a) FFNN is trained with brightness temperature data for a single drying cycle (June 18, 19, 20, 25), (b) FFNN is trained with brightness temperature data for another single drying cycle (July 12, 13, 14, 16), (c) FFNN is trained with brightness temperature data for multiple (15-day) drying cycles, and (d) is observed soil texture data. Figure (e), (f), (g), (h) are similar to (a), (b), (c), (d), respectively, but are based on 6-category classification.
5.3.3. Robustness of FFNN model

The previous subsection demonstrates that the use of multiple-drying-cycle brightness temperature data can improve the classification accuracy compared to single-drying-cycle data. With the use of multiple-drying-cycle brightness temperature data and a FFNN model with the number of hidden neurons $h = 15$ and slope parameter $\lambda = 1$, about 73% accuracy can be obtained. While a FFNN can form arbitrarily complex decision boundaries when the number of hidden layer neurons is increased, this is not always feasible in practice. In part, this is due to the increased training time that results from an increased network size, increased requirements for the amount of data required to estimate the large(r) number of weights and from the requirement that a smaller network has less chances of over-fitting and consequently producing better generalization. However, in the present situation it is evident that similar soil types produce a brightness signature that is at times only slightly different. In the input space then, different classes (soil texture type) are present as one moves a small distance. Consequently, we varied the slope of the activation function. Our motivation being that the superposition of steeper activation functions can allow for more rapid change of the
decision boundary. We thus performed three more experiments with $h = 20$ and $\lambda = 1$, $h = 20$ and $\lambda = 3$, $h = 20$ and $\lambda = 10$.

**Figure 5.5**, Comparison of observed and FFNN-predicted soil-texture map. (a) 15 neurons, $\lambda = 1$, (b) 20 neurons, $\lambda = 1$, (c) 20 neurons, $\lambda = 3$, (d) 20 neurons, $\lambda = 10$, (e) observed soil texture data.
Figure 5.5a, 5.5b, 5.5c and 5.5d show spatially distributed texture classification for the test area in SGP '97 Experiment for these different values. Figure 5.5e shows the observed soil texture classes. A visual comparison between the FFNN-predicted results and observation shows spatial agreement for all of these four experiments. We use a conditional probability, termed as *Ratio of Estimated to Observed (REO)*, to compare and evaluate the performance of the FFNN models:

\[
REO(B|A) = \frac{\text{Number of Texture } A \text{ being predicted as Texture } B}{\text{Total number of Texture } A} \times 100\%
\]  

(5.5)

Table 5.2 shows a detailed comparison on the performance of these four experiments using *REO* as defined above. In these experiments, the training set accuracy is about two to five percent higher than the global accuracy. Although we did not test the FFNN with larger number of neurons (i.e., \(h > 20\)), as mentioned above, too large network size will increase the chances of over-fitting and results in the loss of the generation property. In addition, increasing the slope parameter \(\lambda\) results in initial improvement of FFNN model and deterioration in performance with a further increase of \(\lambda\). Too large \(\lambda\) value might introduce too strong undulation in the decision and consequently lose the ability to generalize. Our experiment suggests that the best FFNN classification accuracy appears in Experiment (3) (i.e., 20 hidden neurons and \(\lambda = 3\)), in which about 80% accuracy can be obtained.
Table 5.2, Performance of FFNN model measured by *Ratio of Estimated to Observed (REO)*

<table>
<thead>
<tr>
<th>Observed/ Predicted</th>
<th>FFNN 15neurons</th>
<th>FFNN 20neurons</th>
<th>FFNN 20neurons</th>
<th>FFNN 20neurons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda=1$</td>
<td>$\lambda=1$</td>
<td>$\lambda=3$</td>
<td>$\lambda=10$</td>
</tr>
<tr>
<td>Global Accuracy</td>
<td>(Observed=Predicted)</td>
<td>73</td>
<td>76</td>
<td>80</td>
</tr>
<tr>
<td>Sand (1,027 pixels)</td>
<td>Sand</td>
<td>56</td>
<td>60</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td>Sandy Loam</td>
<td>24</td>
<td>16</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>Silt Loam</td>
<td>17</td>
<td>19</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Loam</td>
<td>2</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Silty Clay Loam</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Clay Loam</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Sandy Loam (2,693 pixels)</td>
<td>Sand</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Sandy Loam</td>
<td>61</td>
<td>63</td>
<td>66</td>
</tr>
<tr>
<td></td>
<td>Silt Loam</td>
<td>15</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>Loam</td>
<td>19</td>
<td>17</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Silty Clay Loam</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Clay Loam</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Silt Loam (11,128 pixels)</td>
<td>Sand</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Sandy Loam</td>
<td>6</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Silt Loam</td>
<td>77</td>
<td>88</td>
<td>88</td>
</tr>
<tr>
<td></td>
<td>Loam</td>
<td>11</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Silty Clay Loam</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Clay Loam</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Loam (3,451 pixels)</td>
<td>Sand</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Sandy Loam</td>
<td>12</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Silt Loam</td>
<td>10</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>Loam</td>
<td>76</td>
<td>76</td>
<td>78</td>
</tr>
<tr>
<td></td>
<td>Silty Clay Loam</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Clay Loam</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Silty Clay Loam (275 pixels)</td>
<td>Sand</td>
<td>1</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Sandy Loam</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Silt Loam</td>
<td>61</td>
<td>49</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>Loam</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Silty Clay Loam</td>
<td>38</td>
<td>37</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>Clay Loam</td>
<td>0</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>Clay Loam (392 pixels)</td>
<td>Sand</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Sandy Loam</td>
<td>7</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Silt Loam</td>
<td>33</td>
<td>44</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>Loam</td>
<td>21</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Silty Clay Loam</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Clay Loam</td>
<td>35</td>
<td>45</td>
<td>56</td>
</tr>
</tbody>
</table>
We also note that the brightness temperature data is influenced by various environmental factors including climatologic factors, vegetation, radiation, and topography. It is conceivable that similar patterns of brightness temperature may result from soils with different texture because of confounding effects from other environmental factors.

Due to the existence of such confusing patterns, the use of brightness temperature to estimate soil types have an upper limit of classification accuracy. Here we use *Percentage of Patterns with Similar Posterior Probability (PPSPP)*, defined below, to illustrate the effects of confusing patterns on the performance of FFNN model. It is well known that the output of FFNN estimates the posterior probability [Bishop, 1996]. As mentioned earlier, the output of FFNN is a vector containing real-number components. If we present a brightness temperature sequence to FFNN and obtain an output as \([0, 0, 0.9, 0.6, 0, 0]\), based on Table 5.1 this pixel is estimated as Silty Loam because the third component is the largest one. This output is interpreted by FFNN as Silt Loam with a probability of 0.6 (i.e., \(0.6 = 0.9/(0.9+0.6)\)). Since we assign only one class label to each pixel, we choose Silty Loam even though there exists a finite probability (0.4 in this case) that this pixel may belong to other soil classes (Loam in this case). To capture the
proportion of patterns where such ambiguities could occur, we use $PPSPP$ defined as follows:

$$PPSPP = P[|\text{max}_1-\text{max}_2| \leq y] \quad (5.6)$$

where $\text{max}_1$ and $\text{max}_2$ are the largest and second largest components in the output vector, respectively; and $y$ is an arbitrary number between zero and one. If the value of $y$ is close to zero (i.e., $\text{max}_1$ is close to $\text{max}_2$) and $PPSPP$ is approximately zero, it suggests that all brightness temperature patterns are distinct for different soil types and no confusing patterns exist. On the other hand, if the value of $y$ is close to zero and $PPSPP$ is close to one, it suggests a given brightness temperature pattern is confused with at least one soil type. Table 5.3 shows the $PPSPP$ for Experiments (1) through (4). It appears that the classification accuracies for four experiments in Table 5.3 are approximately “1 - $P(|\text{max}_1-\text{max}_2| \leq 0.2)$”. Specifically, the classification accuracies of Experiments (1) through (4) are about 73%, 76%, 80%, and 68%, respectively; while the $PPSPP$ with $y = 0.2$ are about 30%, 27%, 15%, and 32%, respectively. In other words, if the output vector of FFNN model shows that the absolute value of the difference between the largest and the second largest components is smaller than 0.2, such output vector is possibly been predicted as inaccurate soil texture.
In brief, above experiments suggest the use of FFNN and brightness temperature to classify soil types into more than three categories can achieve a maximum accuracy of about 80%. While there may exist a particular network configuration that can provide yet higher accuracy, obtaining such a network through trial and error is difficult. However, our conclusion is that similar soil types have overlapping signatures and the decision boundary required to obtain proper categorization is non-smooth. This suggests that a prototype based classifier, such as the One Nearest Neighbor (1-NN) classifier, might in fact be more suited to the task of predicting the soil texture from brightness temperature.

5.4. One-Nearest Neighbor (1-NN) Classifier

Given a set of prototypes (training data set), a 1-NN classifier assigns a class label to a test input that is the same as that of the nearest pattern in the training data. The 1-NN is thus a non-parametric classifier that has excellent practical performance [Mitiche and
Aggarwal, 1996]. It has been shown [Cover and Hart, 1967] that 1-NN has asymptotic error rate that is at most twice the Bayes error rate, independent of the distance metric used. The $k$-Nearest Neighbors Rule is an extension of the 1-NN, where a test input is assigned to the class with more elements in the set of $k$ prototypes nearest to the test input.

We applied the 1-NN Rule temperature to classify ESTAR sensed brightness temperature into several groups identified a specific types of soil texture. The reason we use 1-NN instead of $k$-NN is that the distribution of soil types in the land surface is often non-uniform. For example, if a study area is dominated by finer-textural soil such as clay or silty clay, than the use of $k$-NN rule tend to assign other soil types as finer-texture soil.
**Table 5.4**, Performance of 1-NN model measured by Ratio of Estimated to Observed ($REO$)

<table>
<thead>
<tr>
<th>Observed</th>
<th>Predicted</th>
<th>$REO$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Global Accuracy</strong> (18,966 pixels)</td>
<td>(Observed=Predicted)</td>
<td>86</td>
</tr>
<tr>
<td>Sand (1,027 pixels)</td>
<td>Sand</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>Sandy Loam</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Silt Loam</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>Loam</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Silty Clay Loam</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Clay Loam</td>
<td>0</td>
</tr>
<tr>
<td>Sandy Loam (2,693 pixels)</td>
<td>Sand</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Sandy Loam</td>
<td>77</td>
</tr>
<tr>
<td></td>
<td>Silt Loam</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>Loam</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Silty Clay Loam</td>
<td>0</td>
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<tr>
<td></td>
<td>Clay Loam</td>
<td>0</td>
</tr>
<tr>
<td>Silt Loam (11,128 pixels)</td>
<td>Sand</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Sandy Loam</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td><strong>Silt Loam</strong></td>
<td>91</td>
</tr>
<tr>
<td></td>
<td>Loam</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Silty Clay Loam</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Clay Loam</td>
<td>1</td>
</tr>
<tr>
<td>Loam (3,451 pixels)</td>
<td>Sand</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Sandy Loam</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Silt Loam</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td><strong>Loam</strong></td>
<td>83</td>
</tr>
<tr>
<td></td>
<td>Silty Clay Loam</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Clay Loam</td>
<td>0</td>
</tr>
<tr>
<td>Silty Clay Loam (275 pixels)</td>
<td>Sand</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Sandy Loam</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Silt Loam</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>Loam</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td><strong>Silty Clay Loam</strong></td>
<td>57</td>
</tr>
<tr>
<td></td>
<td>Clay Loam</td>
<td>0</td>
</tr>
<tr>
<td>Clay Loam (392 pixels)</td>
<td>Sand</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Sandy Loam</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Silt Loam</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>Loam</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Silty Clay Loam</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td><strong>Clay Loam</strong></td>
<td>72</td>
</tr>
</tbody>
</table>
Figure 5.6, Comparison of 1-NN-predicted (a) and observed (b) soil-texture map.
**Figure 5.7**, The regions inaccurately predicted by 1-NN and FFNN models.

- **Sample region**
- **Wrongly predicted by 1-NN**
- **Wrongly predicted by FFNN, 20 neurons, \( \lambda = 3 \)**
- **Wrongly predicted by both methods**
We randomly choose a certain percentage (e.g., 15%) of multi-temporal $T_b(x,t)$ pixel as the labeled prototypes in the 1-NN classifier by assuming that their respective soil types are known. Our objective is to predict the soil types in the test area in SGP ’97 with 1-NN and the labeled patterns. Figure 5.6a shows spatially distributed texture classification for the SGP ’97 Experiment with 15% randomly chosen $T_b(x,t)$. A comparison of Figure 5.6a with observed soil texture classes (Figure 5.6b) shows a very close spatial similarity. Table 5.4 shows detailed performance of 1-NN model based on REO. It can be seen in Table 5.4 that the global accuracy is about 86% and the specific classifier accuracy of each soil texture class is in general high. Comparison between Table 5.2 and Table 5.4 shows that the classification accuracy based on 1-NN model is in general slightly better than that from FFNN model. Figure 5.7 shows regions inaccurately predicted by 1-NN and FFNN models (Experiment 3). It can be seen in Figure 5.7 that the error regions generated by both models are in spatial consistence. The red regions are the pixels inaccurately predicted by FFNN but can be accurately predicted by 1-NN model. This suggests that the brightness temperature patterns in those regions are highly variable that cannot be defined by boundary-based classifier such as FFNN. The yellow regions are the pixels inaccurately predicted by 1-NN but can be accurately predicted by FFNN model. Finally, the black regions, which are inaccurately predicted by both
models, suggest that the brightness temperature patterns in these regions might be pure noise that cannot be used to estimate soil texture. Our 1-NN classifier has an approximate classification error of 14% (see Table 5.4) while we note that black pixel in Figure 5.7 comprise of about 6% of the total pixels. Thus, it seems plausible that the optimum error rate that can be achieved with the given input representation is about 6%~7%.

5.5. Conclusion Remarks

Accurate characterization of spatially variable soil properties is necessary for various applications, such as the soil-plant-atmosphere interaction, drainage, erosion, and solute transport. These properties are often derived from soil hydraulic models that require the information of various soil physical and hydraulic properties. The measurement of soil physical and hydraulic properties over large area, however, is tedious and expensive. Thus, the development of a simplified method that can render reliable estimation of soil texture at fine resolution would be helpful. In Chapter 4, we have demonstrated the feasibility of using remotely sensed brightness temperature and ANN to classify soil types into three categories (coarse, medium, and fine soil texture). This chapter is an extension of previous chapter; here, we focus on the limit of classification accuracy by using brightness temperature data and ANN to classify soil types into more than three groups.
Based on the previously used FFNN model, we have shown that the ESTAR sensed brightness temperature *in a single drying cycle* might contain sufficient features to classify soil types into three categories. If we desire to classify soil texture into more than three groups, however, the single-drying-cycle brightness temperature data appear to be inadequate. Such inadequacy comes from the overlapping patterns among different soil texture in single-drying-cycle brightness temperature. Especially, when the precipitation and other environmental factors are highly heterogeneous, the chance of such overlapping patterns may be significant. To reduce such possibility and to increase the predicted accuracy, this study suggests the use of *multiple-drying-cycle* brightness temperature data to classify soil types into more than three categories.

To evaluate the robustness of FFNN model, we performed several experiments with different numbers of hidden neurons and different values of slope parameter in activation function. A comparison of these experiments suggests that the maximum achievable accuracy through the use of FFNN model and multiple-drying-cycle brightness temperature is about 80. We also note that the influence of overlapping patterns, can be reduced, but cannot be totally removed by the use of multiple-drying-
cycle brightness temperature. Such overlapping signatures and the decision boundary required to obtain proper categorization is non-smooth. While the proposed FFNN architecture can only provide smooth decision boundary, it is clear that the requirement of rapidly changing decision boundary will restrict the FFNN model to yield better accuracy. This suggests that a prototype-based classifier might be more suited to the task of predicting the soil texture from brightness temperature.

Motivated by these observations, we used a simple prototype-based classifier, known as 1-NN model, and multiple-drying-cycle brightness temperature to classify soil type into more than three categories. Our results show that the use of 1-NN model outperforms the FFNN model. About 86% accuracy can be obtained. A comparison between the error regions predicted by 1-NN model and FFNN shows that there are common regions, which cover about 6% of total pixels, which are inaccurately predicted by both model. We argue that such 6% error rate is the minimum error that can be achieved with the given input representation. Conversely, the maximum achievable accuracy for classification into the six soil texture types considered in this study based on 15 days of brightness temperature is about 94%. Designing a classifier to achieve such accuracy remains to be an open challenge and it reserves the room for future study.
CHAPTER 6

SUMMARY OF FINDINGS AND FUTURE WORK
6.1 Major findings

The goal for this dissertation was to develop a better understanding of the spatial organization of soil moisture fields. To achieve this goal, we proposed a stochastic framework that relates the spatial distribution of soil moisture with that of major environmental factors (i.e., soil properties, topography and mean soil moisture). The application of the proposed stochastic framework requires the information of topographic and soil distribution. Motivated by this requirement, we have also provided a simplified methodology for estimating soil properties over large areas through remote sensing and artificial neural networks. Four specific objectives were identified and addressed. In this chapter, each of these objectives is reiterated followed by the major findings. More detailed conclusions have been given at the end of chapters 2 through 5.

Develop a stochastic framework for relating the spatial variation of soil moisture to mean soil moisture as well as the variation of topography and soil properties.

A stochastic framework describing the spatial organization of soil moisture field is proposed based on the problem of water flow in an unsaturated zone, the soil hydraulic functions from Mantoglou and Gelhar (1987) and Gardner (1958), perturbation method, and spectral techniques.

The resulting model from stochastic framework provides closed form analytical solutions for (a) the variance of soil moisture distribution ($\sigma_h^2$), and (b) the covariance between soil moisture distribution and soil properties ($\sigma_{Ah}$, $\sigma_{Bh}$, $\sigma_{Kh}$).
and (c) the covariance between soil moisture distribution and topography ($\sigma_T$) as a function of soil heterogeneity, topography and mean soil moisture.

The resulting models show that the spatial distribution of soil moisture can be expressed the function of the following parametric groups: (a) the correlation scales of soil properties ($\lambda_a$) and relative elevation ($\lambda_z$), (b) large-scale soil properties ($\overline{A}$, $\overline{B}$ and $K_s$), (c) large-scale water source and mean soil moisture ($R_0$, $\overline{H}$), and (d) variability of relative elevation ($\sigma_z^2$) and soil properties ($\sigma_A^2$, $\sigma_B^2$ and $\sigma_{K_s}^2$). Therefore, the resulting model can provide a test bed for evaluating the combined influence of major environmental factors on the spatial distribution of soil moisture, without heavily relying on observed data.

Evaluate the combined influence of topography, soil properties and mean soil moisture on the distribution of soil moisture, and identify the main factors that control soil moisture variability.

We first evaluate three limiting cases regarding the interdependencies among soil properties and topography: (i) soil properties ($A$, $B$, $K_s$) and topography ($z$) are uncorrelated; (ii) the soil properties are correlated among themselves but uncorrelated with topography; and (iii) soil properties and topography are perfectly correlated. Several important features may be identified from these three limiting cases:

- In Case (i), covariance between soil moisture and the attributes (i.e., $A$, $B$, $K_s$ or $z$) will be a function of those attributes only.
In Case (ii), variability in soil moisture is composed of individual variability of soil properties and topography as well as the covariance among soil properties.

In Case (iii), cross correlation between soil moisture and soil physical properties or soil moisture and topography are equivalent.

Second, we have investigated the role of correlation scales for different attributes, large-scale properties, and variance of soil properties and topography on the variability of soil moisture distribution. Our results suggest that:

- Topography appears to have dominant control on soil moisture distribution when the area is dominated by coarse-texture soil or by mixed soil with small correlation scale for topography (i.e., small $\lambda_c$). In such cases the soil moisture variability increases as the soil becomes wet.

- Soil properties is likely to have dominant control on soil moisture distribution for fine-texture soil or for mixed soil with large $\lambda_c$. In such cases, the soil moisture variability decreases as the soil becomes wet.

- Both topography and soil properties appear to have similar control for medium-texture soil with moderate value of $\lambda_c$. In such cases, the soil moisture variability initially decreases and then increases as the soil changes from dry to wet.

Comparisons between above results and a number of field observations show qualitative agreement under various environmental conditions.
Construct two ANN architectures (Feed-Forward Neural Network (FFNN), Self Organizing Map (SOM)) based on the physical relationship among brightness temperature, soil moisture, and soil properties. Use these ANN models to classify soil into three soil textural groups (coarse, medium, and fine).

We have used remotely sensed brightness temperature data in a single drying cycle from Washita '92 Experiment and two different ANN architectures (FFNN and SOM) to classify soil types into three categories. The results show that FFNN yield better classification accuracy about (80%) than SOM (about 70% accuracy). However, SOM has an advantage because it requires little or no information regarding soil properties in the training phase while FFNN requires more information regarding observed soil textural data.

Our attempt to classify soil types into more than three categories resulted in about 50% accuracy when a FFNN was used and even lesser accuracy when a SOM was used. We also note that about 90% of the classification errors made by a FFNN for more than three soil categories can be accounted from soil types that are similar to each other (e.g., silty loam been predicted as loam). Such systematic error indicates that there are rooms for improvement in classifying soil into more than three groups.
Refine the ANN architectures to classify soil into more than three soil textural groups. Estimate the limits of accuracy that can be achieved for the estimation of soil properties based on remote sensing and ANN.

Based on the previously used FFNN model, we have shown that the ESTAR sensed brightness temperature in a single drying cycle might contain sufficient features to classify soil types into three categories. If we desire to classify soil texture into more than three groups, however, the single-drying-cycle brightness temperature data appear to be inadequate. To classify soil into more than three groups and to explore the limits of classification accuracy, we suggest the use of multiple-drying-cycle brightness temperature data.

We have performed several experiments with FFNN models and the results suggest that the maximum achievable classification accuracy through the use of multiple-drying-cycle brightness temperature is about 80%. It appears that the requirement of rapidly changing decision boundary, in the case of space-time evolution of brightness temperature over large areas, will restrict the FFNN model to yield better accuracy. Motivated by these observations, we have used a simple prototype-based classifier, known as 1-NN model, and achieved 86% classification accuracy for six textural groups.

A comparison of error regions predicted by both models suggests that, for the given input representation, maximum achievable accuracy for classification into six soil texture types is about 94%.
6.2 Future work

The accurate characterization of the spatial distribution of soil moisture and the development of simplified method for estimating soil physical properties over large area will have significant practical values for a variety of surface and atmosphere processes, including the soil-plant-atmosphere interaction, drainage, erosion, and solute transport. Results from this dissertation hope to have an impact by providing several approaches to characterize and model soil moisture variations. Several research topics are suggested for further investigation.

- Validate the proposed stochastic framework for different field conditions and space-time resolutions.
- Extend the scope of stochastic framework by including the influences of heterogeneous precipitation, vegetation, and soil depth.
- Develop a transient solution to the stochastic formulation.
- Validate the proposed ANN architecture for a variety of remote sensing platforms with different spatial resolutions and different lengths of multiple drying cycles.
- Design a classifier to achieve the theoretical maximum classification accuracy.
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