A Thesis

entitled

Study of Hidden Corrosion on Prestressing Strands

by

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A parking garage near the north entrance of the University of Toledo built in 1976 had a prestressed spandrel beam fail. The L shaped prestressed spandrel beam exhibited a large dominant crack at the mid-span of the beam. The crack had nearly traveled to the entire depth of the beam and was few inches below the top of the beam. So it was hypothesized that this beam suffered brittle failure since there would have been micro-cracks if the failure was of ductile nature.

On visual inspection of this beam and some other beams in the garage, there were signs of corrosion of prestressing strands. Rust in the steel reinforcement was visible along with some color changes in the neighboring concrete. As these indications of corrosion were not easily visible until the concrete covering these strands fell off due to deterioration; the late diagnosis caused much damage to the prestressing strands. When the steel suffered corrosion, the steel was substituted by the rust which increased the volume of the reinforcement.
This research was aimed at understanding the cause of the brittle failure of the beam. The beam failed with two broken strands so cross sectional analysis becomes significant to know the behavior of concrete with progressive corrosion of its strands. The number of corroded strands for the beam to suffer brittle failure was determined by varying the strand number and is checked against the experimental values in Response 2000. Response2000 provided all the necessary results for the ultimate moment strength and the cracking moment. In order visualize the corrosion induced crack propagation, analysis of the beam was done in finite element software ABAQUS. It provided insight on the crack growth and the failure of the beam. Hand calculation was done to verify the results of Response2000 taking into account the load specifications of American Society of Civil Engineers/Structural Engineers Institute (ASCE/SEI) 7-10 and Precast/Prestressed Concrete Institute (PCI).

The results from Response2000 and Mathcad indicated the beam was initially designed to provide enough capacity to perform well under the service load. But over the period of time, the beam suffered corrosion due to environmental effects and loading. The flaws in the concrete made the steel strands vulnerable to corrosion attack and the degraded strands did not perform to their full capacity. According to ACI 318-11, a structure is ductile if this ratio is greater or equal to 1.2. If this ratio falls below 1.2, the beam loses its ductility and becomes brittle in nature. When the analysis was run for two broken strands as that in the parking garage beam at failure, there was shift in the failure mode from ductile to brittle suggesting the corrosion of strands has affected its ductility and strength. The progressive corrosion of strands eroded them which in reduced the strength of the concrete and eventually led to the failure of beam.
The moment capacity results from the analysis and hand calculation were in good agreement. The moment due to the estimated loading on the beam at failure was significantly less than the calculated capacity. Two hypotheses on the reason the failure load was less than the calculated capacity were examined.

Magnetic flux leakage method for corrosion inspection was carried out, in collaboration with a team led by Prof Ghorbanpoor from University of Wisconsin, Milwaukee, on the suspected beams to further support the analytical and the scan results suggested irregularity in the strands. The result obtained from the magnetic inspection was consistent with the actual strand condition in most of the beams but on the beam 8-9-4CW between column 8 and 9, the scan results did not give the real picture of the strand condition inside the beam. The magnetic inspection suggested that the strands had some general corrosion which was not the case when the beam was excavated and strands were visually inspected.
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Chapter 1

Introduction

1.1 Problem Statement

A parking garage located near the North entrance of the University of Toledo exhibited a structural failure in one of its L-shaped spandrel beams. This garage was built in 1976 and was subjected to different types of loading. Post failure inspection showed that the prestressing strands had suffered corrosion which had weakened the beam resulting in the failure.

The beam failed in the tension region as indicated by a significant through crack in mid span of the beam as seen in figure 1-1. The crack was of considerable length and had distorted the beam. There was neither micro-crack and nor early warning to suggest the ductile failure. All the signals indicated that the brittle failure of beam. On inspection of the crack and corroded tendons, the cross sectional area of some tendons was reduced to zero.

It was necessary to determine the ultimate strength of the beam and strength of the beam at failure so that proper adjustment may be done to prevent other beam from suffering such fate in the future.
Figure 1-1: Parking garage beam at failure (Hashtroodi, 2015)
1.2 Objectives

Corrosion of reinforcement is not easily visible unless it affects the surrounding concrete. By the time we know the strand is attacked by corrosion, it becomes too late. So this research is focused on determining the effects of corrosion if it goes undetected for a long period of time as well as utilizing an existing methodology to inspect the corrosion in its hidden stage.

This research is aimed at understanding the cause of the brittle failure of the parking garage beam and the role of corrosion of strands in transition of the concrete beam from ductile to brittle nature. The study covers the following:

a. Determining the initial capacity of the beam to check if it had enough ductility at the time of construction in accordance to the flexure requirement of ACI 318-11.

b. Understanding the loss in flexural strength of the beam with the removal of tensile strands.

c. Calculating the capacity of beam at failure with the two broken strands as in the real beam.

d. Verifying the results of hand calculation using Mathcad with the cross sectional analysis of beam in Response2000.

e. Conducting 3D analysis of the crack propagation in ABAQUS to estimate the cracking moment and validating it with the cracking moment generated by Response2000.
1.3 Research Approach

Corrosion is a progressive phenomenon; once it initiates on a strand, surrounding concrete and steel are at the risk as well. So in the analysis, the inability of nearby tendons to contribute in the strength of concrete is considered by removing those tendons from the beam. It enables us to simulate the real field situation and observe what really happens out there in the corrosive environment. The sectional analysis of beam was done with the help of Response2000 and the loss in ultimate flexural strength of the prestressed beam was observed for removal of each tendon gradually. Analyses are carried out for different number of tendons and tensile strength to understand the behavior of concrete once it gets weakened by corrosion. The critical number of tendons was then calculated which is required for the transition of ductile nature into brittle and the beam was analyzed for the complete brittle failure using appropriate number of tendons and choosing the parameter for the tensile strength if necessary.

Since we are dealing with the study of corrosion, it is important to understand the effect of corrosion in the concrete member. In order visualize the corrosion induced crack propagation, 3D analysis of concrete is done in finite element software ABAQUS. It provides us insight on the crack growth and the failure of the beam by providing a better view of crack in motion and the response of beam as the crack travels through it. A pre-existing crack was modeled in the beam around the mid span and loading and boundary conditions were simulated as in the field. The cracking moment for the beam was calculated based on the loading at which crack starts to propagate. Hand calculation was done to verify the results of Response2000 taking into account the load specifications of ASCE/SEI 7-10 and PCI.
Along with these virtual analyses, real field investigation was also carried out to detect the corrosion if it was hidden beneath the concrete cover and is gradually deteriorating the steel and concrete. Magnetic inspection by means of Magnetic flux leakage method was used to detect any change in the cross section of the steel due to corrosion or other factor. The magnetic flux tends to show anomalies if there is some discrepancy in the subject under investigation, whether it is change in cross section or misplaced steel bars.
Chapter 2

Threats to concrete integrity

2.1 Introduction

The reliability of any structure is primarily associated with its durability, serviceability and strength. These three parameters ensure the longevity of the structure and comfort of its occupants. During the design of a structure, it is well advised to stay within the design limits and not to take the structural components beyond their design capacity. The ultimate and serviceability limit serve as basic guidelines for the design purpose. Reinforced concrete is widely used all over the world, from land to water, as a reliable construction material but the reinforcement corrosion presents a serious threat to its structural life. Corroded rebar does not contribute much to the strength of the prestressed concrete and the ultimate limit state and serviceability limit state are affected significantly. Corrosion erodes the reinforcement cross-sectional area lowering its strength and produce rust. The corrosion product has greater volume compared to steel which results in extra tensile force; when the tensile force exceeds the tensile strength of concrete, it leads to spalling of the specimen.
Concrete strength depends on a variety of factors; the construction materials used, construction practices, and exposure conditions can potentially affect concrete degradation.

2.2 Defects and their causes

Concrete defects can be architectural (affecting the appearance of the structure) or structural (affecting the strength of the structure). Major types of defect are as explained below:

2.2.1. Crack

2.2.1. A. Crazing: They occur at the surface of the concrete in a web-like series of cracks. These type of cracks are shallow and don’t run deep into the concrete so does not threaten the structural capacity of the concrete. Typically such cracks are results of surface shrinkage.

2.2.1. B. Disintegration: Alternate cycles of freeze and thaw cause concrete surface disintegration. The in-built capillaries and pores of the concrete hold water and moisture from the atmosphere which on freezing expands. The increased volume of the capillaries disintegrates the concrete and cause the concrete surface to fall off.
2.2.1. C. Plastic Cracks: The hardened concrete shows the plastic cracks if the amount of water lost due to evaporation during curing is more than the amount of bleed water. Depending on the temperature and quantity of water used in the concrete mix, hairline cracks occur within few hours of concrete pouring. These thin hairs, though very thin in appearances, may be deceptive as it may run throughout the depth of the concrete and can be very hazardous for the safety of the building and its occupants. Moreover, the plastic cracks can act as initiator for dry shrinkage and hardened cracks.

2.2.1. D. Hardened Cracks: Drying shrinkage is the main cause of cracks in the hardened concrete. The excess water used for mixing gets evaporated while the concrete is drying thus the volume of concrete decreases. The restraints tend to restrict the contraction causing the concrete to crack. Improperly spaced joints contribute to the hardened cracks.

2.2.1. E. Scaling: Sometimes freezing and thawing may result in the cavity in the concrete surface which exposes the aggregates. Since this type of defect is associated only with the concrete surface, it does not affect the overall structural strength of the concrete.

2.2.1. F. Delamination: This defect is similar in appearance to the scaling where the surface aggregates get exposed. The underlying difference is the cause behind the defect. Scaling is due to alternate cycles of freezing and thawing whereas the delamination is related to the evaporation of water from the concrete. If the excess water during curing is not allowed to rise to surface due to premature finishing of the concrete, that excess water is trapped beneath the surface. Over the course of time as this trapped water evaporates, it leaves cavity in the concrete exposing the aggregate. Similar to the scaling, it is not of
much danger to the structural integrity except to the cantilever, as the cantilever has reinforcing bars near the top portion of the slab.

2.2.1. G. Overloading: If the structure is subjected to loading higher than the loading the structure was designed for, then it threatens the structural integrity. Overload causes cracks in the concrete and these crack patterns indicate the types of the loading stresses. The different kinds of cracks resulting from the different orientations of the loads are as explained below:

2.2.1. G. i. Flexural crack

   a) Positive Flexural Crack: Vertical cracks at the bottom and the center of the simply supported beam represents the positive flexural crack.

   b) Negative Flexural Crack: Cracks on the top of beam over the support indicates the negative flexural crack.

The splitting crack acts as a passage for the moisture to enter inside the concrete. If this moisture comes in contact with the steel reinforcement, there is a high chance of steel corrosion and as a result the concrete can not withstand much flexural stresses. Ultimately the weakened concrete specimen undergoes further flexural cracking.

2.2.1. G. ii. Shear crack: The diagonal cracks at quarter points along the beam member represent the shear crack.
2.2.2. Spalling

Spalling is the outcome of the corroded reinforcement bars and the embedded metals in the concrete. Metal corrosion causes increase in its volume by 10 fold developing internal tensile force inside the concrete. If this tensile force exceeds the capacity of concrete, there is falling off of the concrete pieces between the concrete and the corroded steel which is known as spalling.

Concrete with spall is very vulnerable to further defects. The opening created by the spall exposes the steel making it susceptible to corrosion and more corroded steel means more spall. So, even a small amount of spall acts a trigger to bigger spall. The spall on the surface may be a result of the corroded steel beneath the surface. Corrosion is contagious so there is high chance of the nearby steel being corroded as well.

If there has been considerable amount of spalling, then the reinforced concrete may not achieve the design tensile and compressive strength of the steel and concrete. Abnormal increase in the volume of the steel results in higher tensile stresses and once it reaches the tensile limit, it will fail. Beyond the tensile limit, steel merely acts a confinement to the concrete playing no part in resisting the loads acting on it.

2.2.3. Freeze thaw deterioration

Concrete contains pores and capillaries. When the concrete gets moist, the frozen water in those pores can expand by 9% and produces pressure (PCA, 2002). If the pressure in the cavity becomes more than the tensile strength of the concrete, it will increase the cavity and lead to dilation. The alternate freeze and thaw contribute to the significant expansion,
cracking and crumbling of the concrete. Intentional entrained air can act as remedy to the enlargement problem arising from freezing and thawing cycle.

### 2.2.4. Sulfate attack

Since concrete is alkaline in nature, acidic environment is deleterious for the concrete. Sulfate attack is more common in the dry areas, especially in Northern Great Plain and parts of the western United States. Sulfates are naturally present in soil and ground-water; sea water may also contain sulfate but it’s not as alarming as in groundwater. The degradation rate amplifies in the concrete subjected to wet-dry cycling. The moisture gets trapped in the pores of the wet concrete which, on drying, evaporates leaving behind the sulfates. This cycle leads to the accumulation of sulfates on the concrete surface which accelerates the deterioration process. Crystallization of sulfate salts is another threat concrete has to deal with. When supersaturated, the salt solution crystallizes resulting in change of volume and additional pressure large enough to develop crack and scaling. The hydrated compounds of the hardened concrete suffer the most from sulfate attack; the pressure due to sulfate reaction disturbs the cement paste causing it to lose strength and cohesion. The major binding constituent of hydrated cement C-S-H (calcium-silicate-hydrate) gel undergoes decalcification which in turns lowers the modulus of elasticity of concrete. As a result, even a small force is enough to expand and crack the concrete.

In case of University of Toledo’s parking garage beam, tendon corrosion is the cause for the beam failure. Over the course of time, the concrete cover deteriorated due to weathering effects and could not prevent the corrosive agents from coming in contact with the prestressing strands. As the conditions in the neighborhood of the strands became favorable
for the corrosion, the steel began to get corroded. The corroded strands lost their strength and the rust on the steel started increasing the tensile stress in the beam. Eventually the tensile stress reached the tensile limit after which the beam was no longer able to withstand the stress and cracks developed in the beam.
Chapter 3

Steel corrosion and prevention

Metals are usually thermodynamically unstable when exposed to normal atmospheric condition except gold and platinum, and steel is no exception. Steel is a treated metal which absorbs some energy on transforming from iron. It tries to release this energy and attain the natural state, the natural state being the iron oxide or rust. So the steel always has a natural tendency to rust on exposure to environment unless the steel is protected by some means. Studies have shown that slight corrosion helps to strengthen the bond between steel and concrete but the severe corrosion have very harsh impact on the bonding (Fu and Chung, 1997)

Concrete was thought to be a reliable protection against the corrosion. But in reality, it was not. It is quite ironic that the concrete itself acts a medium of corrosion reaction and it provides favorable condition for the corrosion to take place. The micro-cracks, heterogeneity and pores of concrete make the corrosion inevitable.
3.1. Initiation of corrosion in steel

As the steel is refined product of iron, energy is required to transform iron into steel. Being a thermodynamically unstable under normal atmospheric conditions, it releases energy and returns to its natural state forming iron oxide or rust known as corrosion. Corrosion is an electrochemical phenomenon constituting the flow of electrons causing gradual increase in the cross sectional area of reinforcement as shown by the diagram below:

Figure 3-1: Extractive metallurgy in reverse (Ahmad, 2006)

Corrosion is an electro-chemical reaction and it needs to meet certain criteria to initiate, such as:

a. Difference in energy level in the specimen: The energy level at different locations of reinforcement steel is not uniform. The energy varies over the length of the rebar. Since the structure is exposed to different environment at various
locations, the steel are subjected to different condition leading to the variable energy states.

b. Electrolyte: Concrete plays the role of electrolyte and offers the pathway for the movement of electrons.

c. Metallic connection: Ties, wires and other anchorage provide the connection between the steel and the concrete.

Figure 3-2: Volume expansions due to rust (Roberge, 2000)

Under normal condition, concrete serves as an excellent passive cover to the steel. It has low permeability, high electrical resistance and most importantly it is highly alkaline (pH>12.5). The alkaline surrounding of the concrete produces a passive layer of ferric oxide around the steel which protects the metal atoms from dissolving which reduces the
rate of corrosion. If there were no passive layer, the steel would corrode 1000 times faster.

(ACI 222.2R-14)

Owing to all these resistive properties of concrete, steel is protected from corrosion unless and until no external physical or chemical agents affect these properties. On exposure to the environment, the chlorides and carbonates can destroy the passive layer and make steel vulnerable to corrosion. The roles of chlorides and carbonation in corrosion are discussed below:

a) **Chloride penetration:** Chlorides present in sea water, deicing salts and accelerators act as a main catalyst for the premature corrosion of steel. Chloride can permeate even through concrete and its ions can penetrate the protective oxide layer of steel more effectively than other ions. On exceeding the permissible limit, chloride accumulated on the steel surface initiates corrosion in presence of water and oxygen. Studies show that only water-soluble chlorides promote corrosion and chlorides do not get actively involved in the rate of corrosion after initiating it.

![Figure 3-3: Chloride-induced corrosion (Hansson et al., 2006)](image)
Other than the coastal areas where natural accumulation of chlorides takes place, the chlorides result from the de-icing salts in the concrete specimens. The chlorides are not evenly distributed on the concrete specimen, some regions of the steel has higher chloride concentration whereas some areas have lower concentration. Higher concentration areas act as anode since there is excess of negatively charged chlorides here and remaining regions act as cathode. Concrete plays the role of electrolyte and steels present inside the concrete act as metallic conductor thus an electric circuit is established. When the current flows from the anode region to the cathode, iron loses electrons and becomes iron ions. These electrons upon reaching the cathode changes oxygen to hydroxyl ions in presence of moisture. Now the unstable iron ions and hydroxyl ions react to form ferrous hydroxide which gets converted to rust in the presence of oxygen and moisture. The reaction takes place as shown below:

**Corrosion of steel:**

\[
Fe \rightarrow Fe^{2+} + 2e^- \\
\frac{1}{2} O_2 + H_2O + 2e^- \rightarrow 2OH^- 
\]

**Formation of rust:**

\[
Fe^{2+} + 2OH^- \rightarrow Fe(OH)_2 \\
Fe(OH)_2 + \frac{1}{2} O_2 + H_2O \rightarrow 2Fe(OH)_3 \\
\rightarrow Fe_2O_3 \cdot H_2O
\]

When corrosion takes place, competent metal is lost and rust increases the volume of the concrete.

**b) Carbonation:** When carbon dioxide from the atmosphere comes in contact with the hydroxide of the concrete, carbonates are formed reducing the pH level of
concrete to as low as 8.5. When the alkalinity of concrete is decreased so drastically, the passive alkaline film becomes unstable and steel is in risk of corrosion. It is generally slow process depending heavily on the relative humidity of concrete. If the relative humidity falls below 25%, carbonation process gets insignificant and if it exceeds 75%, the moisture in the capillaries prevent the penetration of carbon dioxide in the concrete (ACI 201.2R-92). The concrete structure in damp areas receiving very less sunlight with inadequate concrete cover are susceptible to carbonation. Carbonation initiates the corrosion by destroying the passive layer but has no contribution in the rate of corrosion thereafter.

3.2. Types of steel corrosion in concrete:

The concrete characteristics and the exposure condition govern the different forms of corrosion in steel. Corrosion can be classified on the basis of corrosion mechanism, final damage appearances, environment that causes corrosion and other criteria. On the basis of corrosion mechanism, the corrosions are of following types:

a. Uniform corrosion:

If the two locations on the steel, acting as anode and cathode, are very close to each other, the corrosion will be uniform along the length of the steel. Smaller distance between the two opposite charged poles causes the uniform dissolution of the steel. This type of corrosion is common in carbonated concrete, this results from the reaction of harden cement paste with carbon which decreases the passivity of concrete. The uniform literally does not mean absolutely uniform distribution of corrosion rather it’s a relative concept. Uniform corrosion is more of an ideal
concept and the anode and cathode portion in the steel are not so close to each other in practical. Even if they are close to each other, their strength aren’t uniform, cathodic reaction tends to supersede the anodic reaction and vice versa (Broomfield, 1997), so the uniformity of corrosion rarely takes place in reality. The term “uniform corrosion” is not used much in publication but the fundamental research studies on corrosion are based on the concept of uniformity of corrosion.

b. **Galvanic corrosion (Bimetallic corrosion):**

This involves two dissimilar metals with a common electrolyte. Metal with low corrosion resistance acts as anode whereas other is cathode. This type of corrosion is influenced by the concrete resistivity and it usually occurs in the concrete where anode and cathode lie significant distance apart. The galvanic corrosion rate and resistivity are inversely proportional to each other and this forms the base of determining the corrosion rate of steel by means of resistivity. This Galvanic corrosion is common in large concrete structures and studies show that about 50-80% of corrosion in US bridges suffer from galvanic corrosion compared to about 5-20% in European countries (Gowers et al., 1994). The reason for such large difference is the excessive use of de-icing salt containing chloride in US. The rate of galvanic corrosion is determined by the ratio of cathodic and anodic areas, if the cathodic area is larger than the anodic area then the rate of galvanic corrosion is higher compared to the smaller ratio of cathodic to anodic area.
c. **Localized corrosion:**

This corrosion affects the steel most severely and has many forms, mainly the pitting and crevice corrosion. Localized corrosion is a special form of galvanic corrosion where the ratio of cathodic to anodic area is very high with cathodic area surrounding the quickly decreasing anodic area. Also, this type of corrosion is self-accelerated due to the fact that the decreasing anodic area lowers the pH value of the electrolyte and production of chloride ions increases the acidity. This self-catalystic feature makes this corrosion very dangerous leaving the steel reinforcement excessively vulnerable.

i. **Pitting corrosion:** When the chloride ions attack the passive film of reinforcement steel, the film breaks down and triggers rapid decrease in the area of steel. Cavity is formed at the location of attack in the steel and chloride ions get accumulated in that pit which changes the alkaline environment into acidic. This further reduces the steel in the pit and the chlorides continue to get accumulated and the cycle continues. The most severe form of localized corrosion remains unnoticed most of the time until the engineering structure fails completely. Pit refers to the hole whose surface diameter is less than or equal to the depth, pit corrosion forms similar closely spaced pits within a small area causing perforation on the metal surface. Openings of these pits are generally covered with corrosion product so they mostly remain obscure.
Figure 3-4: Typical cross sectional shapes of corrosion pits (Roberge, 2008)

ii. Crevice corrosion: It can be considered as a specific kind of pit corrosion where the pit originates physically rather chemically. Crevice or cavity or pit results due to gap in overlapping surfaces, metallic or non-metallic and other connections in the reinforced concrete. This crevice holds corrosive chloride solution forming a stagnant pool. Corrosion in crevice is due to the difference in oxygen level between two areas; crevice and external environment. The mechanism for crevice corrosion is more or less similar to that of pitting corrosion. It too comprises of reduction in steel area and acidification by chloride ion accumulation. Crevice has very low oxygen content compared to environment and
serves as anode whereas the oxygen rich environment becomes cathode; this creates a highly corrosive environment inside crevice.

Figure 3- 5: Crevice Corrosion (Roberge, 2008)

d. **External current induced corrosion:**

When the reinforced concrete is subjected to an external current, there is increase in the corrosion of steel. The higher conductivity of the steel allows almost all the current to pass through steel which results in bi-polarity. This causes the dissolution of steel and its cross sectional area is reduced. The reinforced concrete structures where electric trams and trains move are susceptible to such corrosion.
e. Stress corrosion cracking:

The stress corrosion cracking is not easily noticed but the sudden breakdown of reinforcement due to this can invite disaster. High strength steel are preferred in the prestressed concrete and stress corrosion cracking occurs mostly in the high strength steel so it is very much important to understand the effects of SSC in the prestressed concrete structure. SCC is the combination of effect of corrosion and stress in steel. There is a minimum value of stress below which stress corrosion cracking won’t occur and the crack due to SCC is perpendicular to the applied stress. Since the prestressed concrete is subjected to higher stress than the reinforced concrete, small reduction in the prestressing strand due to corrosion can lead to failure of the strand.

f. Corrosion fatigue:

The specimen with the possibility of corrosion fractures earlier if alternate loading and unloading stress is applied on it. There are several loads which produce such alternating stress, namely wind, traffic, temperature fluctuation, etc. Since these loads are ever present in our surrounding, the reinforced concrete structures are under constant threat of corrosion fatigue.
3.3. Factors that make prestressing tendons vulnerable to corrosion:

b. Voids in the vicinity of tendons,

c. Weakening of the passive alkaline layer of concrete,

d. Introduction of chloride ions in the concrete which when comes in contact with water and oxygen starts corrosion,

e. Not properly sealed joints,

f. Insufficient concrete cover,

g. Highly permeable concrete due to high water-cement ratio.

In the University of Toledo’s parking beam, the reasons for corrosion are weakening of the alkaline protective layer of concrete, improperly sealed joints, chloride-induced corrosion and high permeable concrete.

If concrete is defect-free with low permeability, it can greatly help to prevent the rate of corrosion. Properly consolidated concrete with low water-cement ratio ensures that the concrete is less permeable. There should be adequate concrete covering to prevent easy exposure of the reinforcement steel to the surrounding. The steel can be treated in order to make it corrosion resistant. The high electrical resistance of epoxy coating and reduced cathodic area helps to keep the corrosion to minimum. The construction of the prestressed concrete beam needs proper attention in order to avoid any defect that may be favorable for corrosion. The ends of prestressing strand should have covering so as to protect it from the deicing chemicals.
3.4. Remedies for steel corrosion

3.4.1. Use of corrosion inhibitors:

These are the chemicals added to the concrete mix to prevent the corrosion in the reinforcement steel. The chemicals can be used for both the conventional reinforced as well as the prestressed concrete. They not only raise the threshold of chloride ions required for the onset of corrosion but also lower the rate of corrosion avoiding much deterioration even if the corrosion starts. They resist corrosion either by forming a protective oxide layer on the reinforcing bars or by reducing the transfer of chloride ions. We should be well informed of inhibitors physical and chemical behavior since the ignorance about inhibitor can affect the structure drastically. They are meant to prolong the life of structure so they have to meet this desired goal instead of degrading the structural components. There are various factors to be considered for the use of an inhibitor which are as explained below:

First is the stability and behavior of the inhibitor. Corrosion is a long and slow process so it has to be resisted over a considerable period of time. Exposure condition and other factors should not affect the integrity of inhibitor. Its chemical characteristics should be retained for very long time.

Second is the dosage of inhibitor. It should be used proportionally, its over or under use may severely affect the specimen. Crack propagation rate can be significantly lowered with the use of inhibitor so the concrete mix should contain appropriate dose or else it can have negative effect on the crack propagation.


3.4.2. Epoxy coated steel:

Reinforcement bars are most vulnerable to the corrosion due to chloride attack. During 1970s, Epoxy Coated Reinforcement bar (ECR) came into use which can be protected from the moisture and chloride ions. It was first employed in the bridge deck in 1973 in Pennsylvania. The epoxy coat helps to insulate the bar and lower its conductivity providing high resistivity to the corrosion current.

The substrate (reinforcement bar) is cleaned and heated before the epoxy powder is sprayed over the length of the bar. Heat in the bar melts the epoxy and the epoxy gets uniformly spread on the bar. On cooling, the coating adheres to the bar and remains stable as it is thermosetting material. If due attention is given to the handling and transporting of these bars, it can serve very well; 1974 Federal Highway Administration (FHWA) report suggests that even the poor coating reduces corrosion by 11 times (Smith and Virmani, 2000).

ECR code for the substructure does not meet the high standard it sets for the superstructure. Coating gets affected when it comes in contact with salt water and also if there is not adequate concrete covering. FHWA report no FHWA-RD-96-092 states that ECR can prevent the rebar from corrosion for about 20 years. Having said all these, ECR has some limitations though. Epoxy coating protection is influenced by various factors; proper attention is required from the time of epoxy application to the embedment of the steel. Even during the service life of the ECR, they are constantly affected by the acidic nature of the concrete and other environmental factors. If the coating isn’t properly adhered
to the steel surface, the durability of the reinforcement bars come under question. ECR is not of much use if the concrete cracks and offers easy access to the corrosion agents.

3.4.3 Corrosion resistant reinforcing bars:

Considering all these limitations of ECR, corrosion-resistant reinforcing bars can be a viable alternative to the ECR. In case of these resistant bars, concrete cover, permeability of concrete, environmental factors do not offer much threat. Initial cost is definitely higher compared to the normal reinforcement bar but its overall cost of maintenance and durability makes it a profitable investment in the long run. If epoxy is applied on the corrosion resistance bars, the epoxy coating remains stable for much longer than when applied on the conventional rebar. So the combined use of such bar with epoxy coating can really strengthen the design life of the steel specimen. Multiple corrosive barriers certainly help to increase the resistance against corrosion, the initial cost for such protection are worthwhile over the service life of the structure.

3.5 Corrosion effects on reinforced structure’s service life

Corrosion degrades the quality of reinforced concrete as it weakens the steel and decreases its load carrying capacity. The reinforced concrete structure may not perform well to its service life owing to the change in the concrete parameter due to corrosion. Corrosion affects the bonding between reinforcement bar and concrete leading to the spalling and cracking of the concrete cover.
Degradation of reinforced concrete comprises of two stages (Schiessl 1987, Broomfield 1995 and Blankenvoll 1997): one is the beginning of the crack and the other is the propagation of crack. First phase involves the initiation of crack; it includes chlorine attack, carbonation whereas second phase is the spreading of crack over the concrete with time leading to the disintegration of concrete. In real time, boundary separating stage 1 and 2 is difficult to determine. (Song and Shayan, 1998)

Not much research and development have been done on the propagation of crack to deal with the service life of the structure. The propagation of crack involves many uncertainties and it is not easy to determine whether the crack rate is increasing or decreasing or constant. All these factors make the prediction of second phase very tough however Liu and Weyers (1996) have formulated an equation to predict the time of crack initiation which helps to determine the service life of reinforced concrete structure:

\[ T_{\text{corr}} = \frac{W^2}{2K} \]
Where, \( k = 9.11 \left( \frac{1}{\alpha} \right) I_{\text{corr}} \)

\( W = \) mass of the rust product

\( \alpha = \) a constant

The parking garage beam under the consideration must have undergone the stages mentioned in the above graph. It has been there for four decades so weathering and usage has definitely affected its integrity and strength. Over the course of time, the steel strands were subjected to corrosive environment due to ingress of moisture and corrosive agents through the flaws in concrete. Defects in the concrete provide easy passage to the readily available atmospheric carbon dioxide and moisture; carbon dioxide reacts with hydroxide of concrete and form carbonate resulting in the shift of alkaline nature of concrete towards the acidic. As the pH level starts to decrease, the passive alkaline layer protecting the steel strands becomes unstable and steel becomes vulnerable to corrosion. When the corrosion starts, the tensile force in the concrete increases and when this stress exceeds the tensile limit of the concrete, crack develops in the concrete. This is a slow process and is defined by the initiation time \( t_0 \) in figure 3-6.
Table 3.1: Damage caused by corrosion of steel in concrete structure (Song and Shayan, 1998)

<table>
<thead>
<tr>
<th>Event</th>
<th>Damage</th>
<th>Economic loss</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate in US</td>
<td>corrosion damage of highway bridges</td>
<td>$90-$150 billion</td>
<td>Federal Highway administration (1991)</td>
</tr>
<tr>
<td>Estimate in US</td>
<td>annual cost of repairs of bridge deck, substructures and car parks</td>
<td>$200-$450 million</td>
<td>Transportation Research Board (1991)</td>
</tr>
<tr>
<td>Estimate in UK</td>
<td>corrosion damage of motorway and trunk road bridges in England and Wales</td>
<td>GBP 616.5 million</td>
<td>Wallband (1989)</td>
</tr>
<tr>
<td>Collapse of the multi-story parking structure in Minnesota</td>
<td>Collapse</td>
<td></td>
<td>Heidersbach (1986)</td>
</tr>
<tr>
<td>Slab spalled off a bridge in New York</td>
<td>one man killed</td>
<td></td>
<td>Broomfield (1997)</td>
</tr>
</tbody>
</table>

It is always economical to identify the corrosion at early stages so that control measures can be taken immediately in order to cut the astronomical cost associated with late detection and rehabilitation. Corrosion prevention has always been a challenge for the US considering the aging steel structures and strict budget allocation for the research in the field of corrosion prevention and control. Significant loss of property has been witnessed over the last few decade due to corrosion attack as shown in Table 1 and if appropriate strategies and sufficient resources for corrosion programs are not managed well on time, the trend will continue in the future.
Chapter 4

Concrete behavior and mechanics involved

4.1. Introduction

Despite all the advances made in the field of concrete technology, concrete cracking is bound to happen. Flaws in the concrete is unavoidable even if the finest workmanship and latest techniques are used. These flaws are the origins of cracking; cracks provide easy passage for the corrosive agents to come in contact with the steel and the weak steel reinforcement results in the weak concrete structure. The steel which are meant to strengthen the concrete needs the protection of concrete cover in return to avoid corrosion. If the reinforcement does not perform to its full potential owing to corrosion, the cracks are at the risk of increasing in size and propagating since the tensile strength of steel can not resist the stress applied. All these mechanisms are inter-related and they need to be discussed in detail in order to understand the crack formation and propagation.

4.2. Fracture Mechanics

Crack formation and propagation influences the behavior of reinforced concrete and it has become very critical to understand the changes in the concrete characteristics due to the introduction of crack on it. Finite element and fracture mechanics provide us
with reliable insight on this matter; fracture mechanics deal with the basic foundation for

crack growth and finite element integrate fracture mechanics into the model and helps us
to analyze the crack properties. Finite mechanics has been recommended by RILEM
committee to be able to serve as basic approach to deal with non-linear aspects of concrete
behavior (Shah and Ouyang, 1992)

Fracture mechanics offers many solutions to the crack propagation (Hillerborg et al.,
1976), such as:

a. Stress intensity factor approach
b. Energy balance approach
c. Strip-yield model (Dugdale method)
d. Cohesive force model (Barenblatt method)

According to stress intensity factor approach, stress are highly concentrated near the
crack tip and the stress intensity factor deals with the stress near the crack tip assuming
theoretical value of stress at the tip as infinity. This approach addresses the crack
propagation without much emphasis on the crack formation phenomenon. The stress varies
with the distance from the tip as given by the formula,

\[ \sigma = \frac{k}{2\pi r} \]

\[ k_c^2 = E\cdot G_c \]

where, \( r \) is the distance from the crack tip

\( k \) is the stress coefficient factor. The crack propagates when \( k \) reaches the
critical value \( k_c \).
$G_c$ is the amount of energy absorbed on formation of a unit area of crack surface formed.

$E$ is the modulus of elasticity of concrete.

When a crack forms, energy is absorbed whereas energy is released when crack propagates. Whenever the released energy is equal or greater than $G_c$, crack starts to grow. This balance of energy is taken into account by the energy balance approach to analyze crack propagation but it does not deal with the crack formation.

Dugdale (1960) model introduces a plastic zone ahead of the crack tip and the stress at this zone is equal to the yield strength. Barenblatt (1962) further developed this concept and incorporated the changes in stress with the deformation. The analysis for this thesis is based on the fracture process zone approach which is explained on detail later.

4.3. Cohesive Crack Model (CCM)

Cohesive crack model (CCM) is a simple model based on the fracture process zone which is advantageous for the simulation of non-linear fracture behavior by utilizing tension softening curve to predict the fracture mechanism. This model is particularly useful for analysis of quasi-brittle materials like concrete. Barenblatt and Dugdale laid the foundation for the cohesive crack models assuming a fracture process zone (FPZ) at the tip of preexisting crack.

The works of Barenblatt and Dugdale were restricted only to the models containing pre-existing cracks. Hillerborg extended the basic principle of Barenblatt and Dugdale to the concrete specimen devoid of macroscopic cracks. According to Hillerborg, a cohesive crack does not necessarily need to develop just ahead of the pre-existing crack tip but can
originate anywhere in the specimen. This idea helped a great deal to understand the concrete fracture which had no major cracks. Concrete specimen do contains microcracks of the size of grains which are usually not detected by observation alone.

The popular and classical linear elastic fracture mechanics (LEFM) seems to be misleading as it correlated the toughness with the size and geometry of the specimen. Moreover, LEFM requires pre-existing crack for analyzing concrete cracking and the non-linear zone in front of crack-tip has to be insignificant which may not always be the case. Due to these reasons, LEFM was not applicable to all conditions and there was need to come up with non-linear approach to deal with the problem LEFM was incapable to handle. The result produced by the CCM is always greater than that based on LEFM because the associated inelastic zone is not negligible compared to the size of the beam in case of CCM.

Hillerborg introduced the idea of cohesive crack model (CCM) in concrete, often known as fictitious crack model, which eliminated the compulsion of pre-existing crack for the formation of cohesive zone. This idea originally developed for concrete is being widely used for other quassi-brittle materials as well with some necessary adjustments.

![Cohesive Crack Diagram](image)

**Figure 4-1: Cohesive crack as a constitutive assumption (Planas et al., 2003)**
Fracture analysis by the means of cohesive crack approach requires following three things:

1. The stress-strain behavior of concrete as defined by the classical LEFM, which means the behavior before the formation of cohesive crack.

2. The initiation criterion, which defines the orientation and necessary conditions for the crack to originate.

3. The evolution law for the cohesive crack, which associates the transferred stresses with the relative deformation in the crack tips.

In order to form crack, the stress in the concrete has to exceed the tensile strength ($f_t$). The energy required to break a unit surface area of cohesive crack is known as fracture energy ($G_f$) which is the area under the softening curve. The measure of the brittleness of the material is given by the characteristic length ($l_{ch}$),

$$l_{ch} = \frac{E* G_f}{f_t^2}$$

where $E$ is the elastic modulus of material.

The brittleness of the material is indirectly proportional to the characteristic length, which means the longer the characteristic length, the more brittle the material is.

There are various models to understand the fracture behavior of concrete. These models find their usage over a large range covering microscopic cracks to large cracks.
CCM is the intermediate approach of the crack analysis which incorporates the criteria and utilizes the advantages of discrete crack and continuum based method.

### 4.4. Tensile Softening of Concrete

When a material is subjected to tensile loading, a cohesive stress ($\sigma$) and crack opening ($w$) relation can be plotted showing a gradual decrease in stress with increasing displacement. This decrease in stress with increasing deformation is known as softening which is also known as strain softening or tensile softening. The softening curve is a property of material and is independent of geometry, loading and boundary condition. The cohesive stress decreases as the crack opening increases and when the crack opening becomes sufficiently large, the stress becomes zero. The relation between the stress and the opening can be mathematically shown as,
\[ \sigma = f(w), \; 0 \leq w \leq w_c \]

**Figure 4- 3: A cohesive constitutive law (Chen and Su, 2013)**

\( f(w) \) defines the tension softening curve

\( w \) ranges from 0 to \( w_c \).

\( w = 0 \) is the point at which the cohesive stress is maximum which is assumed to be equal to the tensile strength of the concrete (\( f_t \))

\( w_c \) is the point at which the material isn’t intact for the cohesive force to act on, so the cohesive stress becomes zero here.

This implies that with the increase in the width of the crack opening, cohesive stress becomes less than \( f_t \) and eventually a time comes when the crack separates the specimen. Softening curve is the behavior of material after it reaches the maximum strength and starts to fail. Softening indicates the gradual deterioration of material and the residual carrying
capacity keeps on decreasing as the crack propagates through the specimen’s cross-section. This also indicates the fall in load carrying capacity of material.

The elastic energy release ($\Delta W_e$) due to the crack growth should be in equilibrium with the energy required to create the new fracture area ($\Delta W_c$) which can be expressed mathematically as, $\Delta W_e \leq \Delta W_c$. From this equation, we can infer that when too large elastic energy is released, the excess available energy increases the rate of crack propagation. The size of the specimen contributes significantly to the tensile softening and the fracture energy $G_f$, if the boundary conditions and the inner region containing crack-bridging are comparable.

The tensile softening for concrete fracture has become the fundamental of fracture mechanics of concrete. The direct tensile test is used to determine the tensile softening of a material. For a quasi-brittle material like concrete and other crystalline ceramics, tensile softening or crack-bridging activities are the result of aggregate/grain interlocking and traction forces. Just after the fictitious crack-tip (after $f_i$), the interlock and traction forces does not govern the softening but is entirely a mechanical process limited by the softening zone width ($W_{sf}$).
Figure 4- 4: Limits of the softening zone (Hu, 2010)

$W_{sf}$ is restricted to the fracture activities that influence the tensile softening whereas $W_f \geq W_{sf}$ consists of isolated micro-fractures which are not associated with the softening. $W_{sf}$ range is determined by the maximum aggregate size to the stress and the stress field around the crack-tip. The tensile strength ($f_t$) governs the fracture activities in the region between $W_{sf}$ and $W_f$ as they originate near the advancing fictitious crack-tip and becomes inactive as the cracking starts.

The strength value ($f_t$) depends on the aggregate constitution and the pre-existing defects at the crack tip. If the crack has already separated more than 95% of the cross-section, there is a possibility of a long tensile softening tail for the remaining few crack areas.
4.4.1. Stages of Tension Softening

Tension softening consists of three different stages:

a. The material undergoes elastic deformation

b. The initiation of micro cracks and macro crack which degrades and weakens the material.

c. Branching and bridging of cracks which result in continuous crack.

For the analysis of bridging in crack growth, the tensile test results of Labuz et al. (1985). Figure 4-6 depicts the propagation of the crack throughout the specimen by the means of bridging. As shown in the figure, the crack initiates at the right notch and follows a curved path as it propagates towards the stronger part of the specimen. This indicates the steep part of the softening curve in figure 4-5 where crack width starts to increase with decrease in cohesive strength. When the end of the steep curve is reached, the crack has traversed to the entire cross section with overlaps at some locations. These overlaps serve as bridges for the cracks to interact with each other.
Figure 4-5: Fracture mechanisms in tension (Van Mier et al., 2008)
Figure 4-6: Macrocack growth and bridging between crack overlaps (Van Mier et al., 2008)
Bridging is also known as hand-shake crack in which the two tips of nearby cracks do not tend to form a single crack but remain as separate identities which are forced to merge under large deformations.

### 4.5. Fracture Process Zone in Concrete

The behavior of concrete, once it undergoes fracture, largely depends upon the fracture process zone (FPZ) of the concrete. FPZ is a fictitious crack zone that is assumed to develop around and ahead of the crack tip. The FPZ results in the toughening of the specimen.

#### 4.5.1. Concepts of Fracture Process Zone

FPZ is basically a non-linear zone comprising mostly micro-cracks. One concept suggests that there are several closed micro-cracks around the main crack and an opening micro-crack exists in front of these micro-cracks (Mihashi, 1987). Whereas, Wittmann (1992) thought of inner micro-crack and surrounding micro-cracks as two separate identity.
Figure 4-7: Mihashi Concept of FPZ (Otsuka and Date, 2000)

Figure 4-8: Wittman Concept of FPZ (Otsuka and Date, 2000)

WsF: Width of the inner zone with interacting microcracks
Wf: Width of the inner zone in which isolated microcracking takes place
4.5.2. Techniques to detect Fracture Process Zone

Since this is a virtual zone and cannot be identified by a naked eye, we need to take the aid of different techniques to investigate it such as ultrasonic pulse velocity or acoustic emission measurements, X-ray radiography, fluorescent spraying technique, optical observations either direct or by artificial means, electrical conductivity measurements.

FPZ being an inelastic zone at the crack tip leads to the non-linearity of the stress. The stress produced in this zone varies from that in the elastic zone. As long as this zone is very small compared to the dimensions of other cracked geometry, linear elastic fracture mechanics (LFEM) solutions hold true for the analysis. But if the yielding is the controlling factor in this zone then non-linear fracture mechanics solutions based on plasticity has to deal with the ductile fracture of the specimen.

![Stress distribution at a crack-tip and process zone](image)

Figure 4-9: Stress distribution at a crack-tip and process zone (Shah and Ouyang, 1992)
4.5.3 Factors causing Fracture Process Zone

There are several factors that contribute to the occurrence of the FPZ. High stress concentration in the crack tip causes micro-cracking at nearby faults. This micro-cracks offer resistance to the crack propagation as it endures some of the external loads which is termed as the micro-crack shielding (Shah and Ouyang, 1992).

Tough aggregates present on the path of crack propagation cause the crack to deflect towards the softer neighborhood. Sometimes aggregates act as bridge for the stress transfer between the cracks around it until the aggregate fails, this phenomenon is known as bridging. Different mechanisms of toughening are as shown in the figure below:

![Diagram](image)

Figure 4-10: Toughening mechanisms in concrete (Shah and Ouyang, 1992)
4.6. Strain softening behavior of concrete

As of now, there are two models which are used extensively to model the strain softening behavior of concrete in tension:

1. Bazant and Oh model
2. Hillerborg model

4.6.1. Bazant and Oh model

Bazant and Oh (1983) proposed a simple fictitious crack model known as “crack band theory” for the analysis of plain concrete. This theory is based on two basic assumptions: first, the width of the concrete is equal to 3 times the maximum size of aggregate and second, the concrete strains are uniform within the band. According to this theory, the value of fracture strain ($\varepsilon_o$) is given by,

$$\varepsilon_o = \frac{2 G_f}{(f_t\times b)}$$

where, $b$ is the element width

$G_f$ is the fracture energy required to form a crack.

$f_t$ is the tensile limit of concrete

4.6.2. Hillerborg model

Hillerborg et al. (1976) came up with a concept of bilinear descending curve for the tensile strain softening behavior of concrete. This concept assumes that the microcracks
are uniformly distributed over the crack band width and the fracture strain obtained by this concept is,

\[ \varepsilon_o = 18 \frac{Gf}{(5*f_t*b)} \]

These model works considerably well in the reinforced concrete member when the size of finite element mesh is small but the experimental results deviate from the analytical result if the mesh size becomes large. Both these models consider microcracks to be uniformly distributed over the finite element and if this concept is used to analyze finite element with larger mesh size, the crack is spread over larger area which is actually not the case practically as the microcrack is concentrated in a smaller cracked region of the element.

![Diagram showing typical shapes of softening branch of concrete](image)

(a) Bazant and Oh model  (b) Hillerborg model

**Figure 4-11: Typical shapes of softening branch of concrete (Kwak and Filippou, 1990)**
The conventional failure mechanism did not consider the existence of flaws and defects of the concrete which are a part of the material itself and can also form during manufacturing and handling. These flaws in the form of cracks and joints result in a redistribution of stress and strain around the discontinuous parts of the materials. So the conventional failure mechanisms based on the continuous strain and displacement field may not be reliable to assess the strength of the structure with pre-existing crack. Since this research deals with the concrete with mid-span cracking which causes discontinuity, fracture mechanics has been used to analyze the reduction in the strength of the beam.
Chapter 5

Beam description, strength and demand calculation

5.1. Beam description

The beam under consideration was an L-shaped with a span of 35’10”. Initially, it contained 28 strands; 21#1/4” strands and 7#1/2” strands. The characteristics of the beam are as listed below:

- 28 day compressive strength of concrete ($f'_c$) = 5000 psi
- Ultimate strength of prestressing strands ($f_y$) = 250 ksi
- Modulus of Elasticity of concrete ($E_c$) = 57000* $\sqrt{f'_c}$ = 4030 ksi
- Modulus of Elasticity of steel ($E_s$) = 28500 ksi
- Area of half inch tendons ($A_{h}$) = 0.144 in$^2$
- Area of quarter inch tendons ($A_{q}$) = 0.036 in$^2$
- Concrete cover = 1 inch
Figure 5-1: Cross section of beam with all strands intact

The beam suffered brittle failure with no micro cracks around the dominant crack. Upon visual inspection, it was found that its two strands had suffered severe corrosion and did not contribute to the strength of the concrete. In order to determine the capacity of the beam initially when all of its strands were healthy and functioning at full potential, flexural capacity was calculated using Mathcad considering the requirements of PCI 6th edition and ACI 318-11.

ACI 318-11 defines a structural member as ductile if the ratio of its ultimate moment capacity and cracking moment is at least 1.2. Since the beam under consideration
suffered brittle failure with 2 broken tendons, the moment capacity with 2 corroded strands was also calculated to know the capacity of the beam at the time of failure.

The beam had 3 sets of stirrups which are illustrated in figure 5-2. This is the layout specified on the drawings. However, in reality the spacing of the stirrups near the center of the beam is uniform.

Figure 5-2: Layout of stirrups in the L-shaped beam
5.2. Strength calculation

5.2.1. Initial strength with all the tendons intact

The beam’s initial capacity was calculated using Mathcad considering all the tendons as unbroken. The calculation is explained below:

\[ f'_{c} = 5000 \text{ psi} \quad f_{y} = 250 \text{ ksi} \quad \text{width of flange (b) = 10 in} \]

\[ A_{h} = 0.144 \text{ in}^{2} \quad A_{q} = 0.036 \text{ in}^{2} \quad E_{s} = 28500 \text{ ksi} \]

Gross area of concrete, \( A_{g} = 10*72+12*18 = 936 \text{ in}^{2} \)

Distance from the top extreme fiber to the steel in compressive zone, \( d' = 1.25 \text{ in} \)

Distance from the top extreme fiber to the steel in tensile zone, \( d_{t} = 82.25 \text{ in} \)

Solving height of the stress box ‘a’ by trial and error [Calculation reference: Nawy, 2003]

1. Let’s assume there is no strand in compressive zone

Area of steel in tension, \( A_{s} = 7A_{h} + 21A_{q} = 1.764 \text{ in}^{2} \)

Percentage of steel in tension, \( \rho = A_{s} / A_{g} = 0.00188 \)

Area of steel in compression, \( A'_{s} = 0 \)

Percentage of steel in compression, \( \rho' = A'_{s} / A_{g} = 0 \)

\[ A_{s} - A'_{s} = 1.764 \text{ in}^{2} \quad \rho - \rho' = 0.00188 \]

For \( f'_{c} = 5000 \), \( \beta = 0.85 - [0.05 \left( f'_{c} - 4000 \right) / 1000] = 0.8 \)
\[ f'_{s} = 87000 \left[ 1 - (0.85f'_{c},d')/(\rho - \rho')f_{y},d_{t} \right] = 77.46 \text{ ksi} \]

\[ a = (A_{s},f_{y} - A'_{s},f'_{s})/(0.85f'_{c},b) = 10.38 \text{ in} \]

Neutral axis depth, \( c = a/0.8 = 12.97 \text{ in} \)

We had already assumed that this trial had no steel in the compression zone but the value of neutral axis came out to be 12.97 in. If we look at the cross section of the beam, top strands were at the depth of 1.91 inch so this assumption was not true.

2. For second trial, let’s assume compressive zone has top 2 steels (2#1/2”)

Area of steel in tension, \( A_{s} = 5A_{h} + 21A_{q} = 1.476 \text{ in}^{2} \)

Percentage of steel in tension, \( \rho = A_{s}/A_{g} = 0.00158 \)

Area of steel in compression, \( A'_{s} = 2A_{h} = 0.288 \text{ in}^{2} \)

Percentage of steel in compression, \( \rho' = A'_{s}/A_{g} = 0.00031 \)

\( A_{s} - A'_{s} = 1.188 \text{ in}^{2} \quad \rho - \rho' = 0.00127 \)

\[ f'_{s} = 87000 \left[ 1 - (0.85f'_{c},d')/(\rho - \rho')f_{y},d_{t} \right] = 72.8 \text{ ksi} \]

\[ a = (A_{s},f_{y} - A'_{s},f'_{s})/(0.85f'_{c},b) = 8.19 \text{ in} \]

Neutral axis depth, \( c = a/0.8 = 10.24 \text{ in} \)

At depth of 10.24 in, there were 4 strands, so this assumption was invalid as well.

3. Assuming top 4 strands in compressive zone (2#1/2”+2#1/4”)

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Area of steel in tension, \( A_s = 5A_h + 19A_q = 1.404 \text{ in}^2 \)

Percentage of steel in tension, \( \rho = A_s / A_g = 0.0015 \text{ in}^2 \)

Area of steel in compression, \( A'_s = 2A_h + 2A_q = 0.36 \text{ in}^2 \)

Percentage of steel in compression, \( \rho' = A'_s / A_g = 0.00038 \text{ in}^2 \)

\[ A_s - A'_s = 1.044 \text{ in}^2 \quad \rho - \rho' = 0.00038 \]

\[ f'_s = 87000 \left[ 1 - (0.85\beta.f'_c.d')/\{(\rho - \rho')f_y.d_t\} \right] = 70.88 \text{ ksi} \]

\[ a = (A_s.f_y - A'_s.f'_s)/(0.85.f'_c.b) = 7.66 \text{ in} \]

Neutral axis depth, \( c = a/0.8 = 9.57 \text{ in} \)

At depth of 9.57 in, there are 4 strands, so this assumption is true.

Strain at the extreme steel in tensile zone, \( \varepsilon_t = 0.003(d_t - c)/c = 0.02278 > 0.005 \). So, tensile strands yielded.

From the similar triangles, strain at the top 2#1/2 strands \( \varepsilon_1 = 0.0024 \)

Stress at the top 2#1/2 strands \( \sigma_1 = E_s * \varepsilon_1 = 68.4 \text{ ksi} \)

Strain at the top 2#1/4 strands \( \varepsilon_2 = 0.00098 \)

Stress at the top 2#1/4 strands \( \sigma_2 = E_s * \varepsilon_2 = 27.93 \text{ ksi} \)

\( A_s = 1.404 \text{ in}^2 \)

Area of tendons at the bottom, 3#1/2” and 1#1/4” = 3\( A_h + A_q = 0.468 \text{ in}^2 \)

Centroid calculation
Calculating CG of tensile steel from bottom

\[
\frac{[1.25A_b+10.21*2A_h+2A_q*(14.53+21.43+28.43+35.43+42.43+50.43+57.43+64.43+71.43)]}{A_s} = 22.28
\]

CG from top, \( cg_t = 84 \text{ in} - 22.28 \text{ in} = 61.72 \text{ in} \)

**Moment calculation**

\[
A_s*f_y (cg_t - c) + \sigma_1*2A_h*8.32 + \sigma_2*2A_q*3.75 + 0.85f'_c*b*a*(c-a/2) = 1694.18 \text{ kip-ft}
\]

Moment = 1694.2 kip-ft

The beam had the initial capacity of 1694.2 kip-ft when it was constructed 40 years ago but as the beam suffered degradation due to weathering and loading, its capacity decreased over the time. When the beam suffered brittle failure, it was seen that 2 of its bottom tendons were heavily corroded making them unsuitable to contribute in the strength of beam.

The strength of beam was calculated next at the time of its failure with 2 broken tendons. The two broken tendons at bottom are removed since they have no part in the strength calculation.

**5.2.2. Strength calculation with 2 broken tendons**

The beam’s capacity at failure was calculated using Mathcad considering two tendons unbroken. The calculation is explained below:
\( f'_c = 5000 \text{ psi} \quad f_y = 250 \text{ksi} \quad \text{width of flange (b)} = 10 \text{ in} \)

\[
A_h = 0.144 \text{ in}^2 \quad A_q = 0.036 \text{ in}^2 \quad E_s = 28500 \text{ ksi}
\]

Gross area of concrete, \( A_g = 10 \times 72 + 12 \times 18 = 936 \text{ in}^2 \)

Distance from the top extreme fiber to the steel in compressive zone, \( d' = 1.25 \text{ in} \)

Distance from the top extreme fiber to the steel in tensile zone, \( d_t = 82.25 \text{ in} \)

Solving height of the stress box ‘a’ by trial and error

1. Let’s assume there is no strand in compressive zone
   
   Area of steel in tension, \( A_s = 5A_h + 21A_q = 1.476 \text{ in}^2 \)
   
   Percentage of steel in tension, \( \rho = A_s / A_g = 0.00158 \)
   
   Area of steel in compression, \( A'_s = 0 \)
   
   Percentage of steel in compression, \( \rho' = A'_s / A_g = 0 \)
   
   \( A_s - A'_s = 1.476 \text{ in}^2 \quad \rho - \rho' = 0.00158 \)
   
   For \( f'_c = 5000, \quad \beta = 0.85 - [0.05 (f'_c - 4000) / 1000] = 0.8 \)
   
   \( f'_s = 87000 \left[ 1 - (0.85\beta f'_c d') / (\rho - \rho') f_y d_t \right] = 75.6 \text{ksi} \)
   
   \( a = (A_s f_y - A'_s f'_s) / (0.85f'_c b) = 8.68 \text{ in} \)
   
   Neutral axis depth, \( c = a / 0.8 = 10.85 \text{ in} \)

We had already assumed that this trial had no steel in the compression zone but the value of neutral axis came out to be 10.85 in. If we look at the cross section of the beam, top strands were at the depth of 1.91 inch so this assumption was not true.
2. For second trial, let’s assume compressive zone has top 2 strands (2#1/2”)

Area of steel in tension, \( A_s = 3A_h + 21A_q = 1.188 \text{ in}^2 \)

Percentage of steel in tension, \( \rho = A_s / A_g = 0.00127 \)

Area of steel in compression, \( A'_s = 2A_h = 0.288 \text{ in}^2 \)

Percentage of steel in compression, \( \rho' = A'_s / A_g = 0.00031 \)

\[ A_s - A'_s = 0.9 \text{ in}^2 \]

\[ \rho - \rho' = 0.00096 \]

\[ f'_s = 87000 \left[ 1 - (0.85 \beta f'_c d')/\{(\rho - \rho') f_y d_t \} \right] = 68.3 \text{ ksi} \]

\[ a = (A_s f_y - A'_s f'_s)/(0.85 f'_c b) = 6.53 \text{ in} \]

Neutral axis depth, \( c = a/0.8 = 8.157 \text{ in} \)

At depth of 8.15 in, there were 4 strands, so this assumption was invalid as well.

3. Assuming top 4 strands in compressive zone (2#1/2” + 2#1/4”)

Area of steel in tension, \( A_s = 3A_h + 19A_q = 1.116 \text{ in}^2 \)

Percentage of steel in tension, \( \rho = A_s / A_g = 0.00119 \)

Area of steel in compression, \( A'_s = 2A_h + 2A_q = 0.36 \text{ in}^2 \)

Percentage of steel in compression, \( \rho' = A'_s / A_g = 0.00038 \text{ in}^2 \)

\[ A_s - A'_s = 0.756 \text{ in}^2 \]

\[ \rho - \rho' = 0.00081 \]

\[ f'_s = 87000 \left[ 1 - (0.85 \beta f'_c d')/\{(\rho - \rho') f_y d_t \} \right] = 64.7 \text{ ksi} \]

\[ a = (A_s f_y - A'_s f'_s)/(0.85 f'_c b) = 6.02 \text{ in} \]

Neutral axis depth, \( c = a/0.8 = 7.52 \text{ in} \)

At depth of 7.52 in, there are 4 strands, so this assumption is true.
Strain at the extreme steel in tensile zone, \( \varepsilon_t = 0.003(d_t - c)/c = 0.02981 > 0.005 \). So, tensile strands yielded.

From the similar triangles, strain at the top 2#1/2 strands \( \varepsilon_1 = 0.00224 \)

Stress at the top 2#1/2 strands \( \sigma_1 = E_s \times \varepsilon_1 = 63.84 \) ksi

Strain at the top 2#1/4 strands \( \varepsilon_2 = 0.00072 \)

Stress at the top 2#1/4 strands \( \sigma_2 = E_s \times \varepsilon_2 = 20.52 \) ksi

\( A_s = 1.116 \text{in}^2 \)

Area of tendons at the bottom, 1#1/2” and 1#1/4” = \( A_h + A_q = 0.18 \) in²

Tensile steel centroid calculation

Calculating CG of tensile steel from bottom

\[
\frac{[1.25A_b + 10.21 \times 2A_h + 2A_q \times (14.53 + 21.43 + 28.43 + 35.43 + 42.43 + 50.43 + 57.43 + 64.43 + 71.43)]}{A_s} = 27.7 \text{ in}
\]

CG from top, \( c_{g_t} = 84 \) in – 27.7 in = 56.3 in

Moment calculation

\[
A_s f_y (c_{g_t} - c) + \sigma_1 2A_h \times 6.27 + \sigma_2 2A_q \times 1.67 + 0.85f'_c b.a(c - a/2) = 1239.13 \text{ kip-ft}
\]

Moment = 1239.13 kip-ft
The moment capacity has significantly lowered compared to the initial strength of 1694.2 kip-ft as the broken two tendons at bottom did not have any role in strength in this second calculation. Corrosion definitely diminishes the strength and weakens the concrete making it vulnerable to failure.

**Cracking Moment for the beam with 2 broken tendons**

The cracking moment for the beam is obtained by using the formula:

\[ M_{\text{cracking}} = f_r \times I_g / y_t \]

Where, \( I_g \) = Moment of inertia of the cross-section about neutral axis

\[ y_t = \text{distance of neutral axis from the extreme tensile fiber} \]

\[ f_r = \text{modulus of rupture of concrete which is given by } c \sqrt{f_c} \]

\( c \) ranges from 7.5 to 12 (ACI 318-11)

For the brittle failure mode, the value of \( c \) has been raised from 7.5 to 10 in Response2000 and ABAQUS)

Location of neutral axis from the top of the beam:

\[ C_{NA} = (10 \times 72 \times 36 + 12 \times 18 \times 78)/(720+216) = 45.62 \text{ in} \]

\[ y_t = 84 - 45.62 = 38.31 \text{ in} \]

\[ f_r = 10 \sqrt{f_c} = 7.5 \sqrt{5000} = 707 \text{ psi} \]

\[ I_g = 18 \times 12^3/12 + (18 \times 12)^{(6-38.31)^2} + 10 \times 72^3/12 + (10 \times 72)^{(48-38.31)^2} = 606,700 \text{ in}^4 \]
\[
M_{\text{cracking}} = f_e * I_e / y_e = 707*606700/38.31 = 11180000 \text{ lb-in} = 931 \text{ kip-ft}
\]

### 5.3. Demand on beam at failure

All the above calculations to find the strength was based on concrete and steel characteristics. The loads on the beam at the time of failure are estimated consistent with the PCI handbook and ASCE/SEI 7-10.

For this calculation, following loads are considered:

- **DL of Double Tees** = 45 psf (maximum DL as specified in the original drawing)
- **LL on parking garage** = 40 psf (as specified in ASCE/SEI 7-10)
- **Self-weight of beam (W)** = 150 pcf

a. **For L-shaped beam**

   Cross sectional area \( A_L \) = 6.5 ft\(^2\)

   Unit load on L-shaped beam \( w_L \) = \( W \times A = 975 \text{ lb/ft} \)

b. **For Double Tees**

   Total parking garage area supported by Double Tees \( A_T \) = 58 ft * 35 ft

   DL on Double Tees = 45 psf * 58 ft = 2610 lb/ft

   LL on Double Tees = 40 psf * 58 ft = 2320 lb/ft

   LL + DL on Double Tees = 4930 lb/ft
c. For 3” topping on the Double Tees

Weight of the topping = \( \frac{3}{12} \times 58 \times 150 \text{ pcf} = 2175 \text{ lb/ft} \)

Since the Double Tees are equally distributed on 2 beams, so the unit load on one beam due to Double Tees and topping \( (w_T) = \frac{(4930+2175)}{2} = 3552.5 \text{ lb/ft} \)

Total unit load acting on beam = \( w_T + w_L = 4527.5 \text{ lb/ft} \)

Flexural moment on the beam = \( 4527.5 \times 35^2 / 8 = 693273.4 \text{ lb-ft} = 693.3 \text{ kip-ft} \)

The load on the beam is significantly less than the capacity set by the undamaged tendons. This is a mystery which could not be solved, even after numerous checks and calculations. As far as the capacity based on tendons is concerned, it has been verified by the cross sectional analysis in Response2000. The cracking moment generated by Response2000 is in good agreement with the 3D Finite Element Analysis done in ABAQUS which are discussed in chapter 6 and 7.
Chapter 6

Determining the Failure mode by RESPONSE2000

6.1. Background

There is a crucial relation between the ultimate moment and cracking moment to determine the nature of failure in flexural members. ACI 318-11 states that “The total amount of prestressed and non prestressed reinforcement in members with bonded prestressed reinforcement shall be adequate to develop a factored load at least 1.2 times the cracking load computed on the basis of the modulus of rupture $f_r$ specified in 9.5.2.3”. For the structural member to be ductile enough to show considerable deflection and prevent sudden failure, its ultimate strength should be at least greater than 1.2 times its cracking load. If the ultimate strength equals the cracking load, then the structure shows brittle failure.

Modulus of rupture in flexure is defined as a function of compressive strength and is expressed as:

$$f_r = c\sqrt{f_c}$$

where, $f_c$ is the 28-day compressive strength of concrete.
c is a constant that takes value between 7.5 and 12 if \( f_r \) is expressed in psi. [ACI 318-11]

The lower limit of the modulus of rupture is obtained by using 7.5 for \( c \) but to better simulate the actual brittle failure of the beam, \( c \) is taken as 10.

Based on the calculation with \( c \) as 7.5 and non-corroded initial condition of tendons and beam, the ratio of ultimate moment capacity and cracking moment exceeds 1.2. For the initial condition, the concrete and steel parameters were same as those in Mathcad calculation.

For the prestressing strand, an initial jacking force \( (f_{pi}) \) of 70 percent of \( f_{pu} \) is applied to each tendon. (Hashtrroodi, 2015)

- Initial jacking force, \( f_{pi} = 0.7*250=175 \) ksi

- Pre-strain of each layer, \( \varepsilon = \frac{f_{pi}}{E} = 0.00614 \)

- Ultimate flexural strength, \( \text{Mult} = 1675.3 \text{ kipft} \) [from fig 6-2]

- Cracking moment, \( \text{Mcr} = 1298.5 \text{ kipft} \) [from fig 6-3]

\[ \frac{\text{Mult}}{\text{Mcr}} = 1.299 \]

The above data results suggest that the beam was ductile enough initially as the ratio is well above 1.2 but on field inspection, it was clear that the beam suffered brittle failure. This transition in the characteristics of beam must have resulted from the loss in the strength of the beam due to the loss of prestressing strands. As the strands got corroded over the course of time, the ultimate strength of beam reduced drastically causing sudden
failure of the beam. Such adverse situation occurs when Mult lowers to the critical value of $M_{cr}$ making the ratio equal to one.

The number of tendons and the c value in the tensile strength equation were varied to account for the broken tendons due to corrosion and loss of strength over time. The objective of analyzing the beam with the help of Response software is to accurately find the number of corroded tendons at the expense of which the beam loses its ductility and experiences brittle failure.

6.2. Ductile failure of the beam

6.2.1. Prestressed beam with intact tendons (Initial conditions)

For the initial condition, all the tendons were considered corrosion free and beam was intact. The beam in the reality was an L-shaped spandrel beam but it was not available in the Response2000 module so the L-cross section was substituted by an inverted T-shaped. The mechanics involved in the T-shaped beam is very much similar to the L-shaped beam as the torsion is not significant for both the beams so the beam properties for these two beams can be used interchangeably. The factors determining the ultimate strength and cracking moment such as cross-sectional area, moment of inertia, center of gyration and other don’t differ a lot between L section and inverted T-section. So taking all these factors into account, a symmetrical inverted T-shaped beam has been used instead of non-symmetrical L-shaped beam.
The data for the initial conditions were used and above properties were obtained as output. For the initial stage, all tendons were assumed to be functioning at their full capacity and the ultimate strength and cracking moment were obtained to check the ductility of the beam.

Figure 6-1: Properties of beam at initial condition
Figure 6- 2: Ultimate flexural moment of the beam at initial condition

From the analysis result for the initial condition, the ultimate flexural strength was found to be 1675.3 kip-ft. The cracking moment was determined from the moment

Figure 6- 3: Moment curvature curve at initial condition
curvature curve. The curve remains linear until a crack occurs in the beam but as soon as the crack initiates, there is change in the slope of the curve due to change in stiffness of the member as the beam transforms from elastic to plastic. In figure 6-3, the sudden change in curve occurs for the moment of 1298.5 kip-ft. As per ACI 318-11 code, the beam is ductile if the ratio of Mult and Mcr is at least 1.2. For this initial analysis,

\[
\frac{\text{Mult}}{\text{Mcr}} = \frac{1675.3}{1298.5} = 1.299 > 1.2
\]

Since the ratio is higher than 1.2, the beam has ductility and can deform considerably before failure.

![Member Crack Diagram](image)

**Figure 6- 4: Crack diagram for initial stage**

The crack diagram indicates distributed crack around the mid-section of the beam. The diagram only shows one half of the beam. Absence of diagonal cracks near the support suggests that the beam has no shear crack.
Figure 6- 5: Load-displacement curve for beam at initial conditions

The load-displacement curve shows large deflection before the failure occurs. The distributed crack around the mid span and the large amount of deflection further validate the ductile nature of the beam.

6.2.2. Beam with one corroded (broken) tendon

Since the beam was suspected of being under corrosion attack, there was high possibility of stands being affected by corrosion. Such corroded strand does not take any part in imparting strength to the beam, so analysis is done by removing such strand from the beam until the analysis meets the requirement for complete brittle failure. One of the tendons from the bottom layer of beam was removed to simulate the real situation in the field.
As it is clear from the fig, one tendon from the bottom layer has been removed. This causes change in the ultimate flexural strength and cracking moment of beam.

**Figure 6- 6: Properties of beam with one corroded tendon**

**Figure 6- 7: Ultimate flexural moment of the beam with one broken tendon**
There is loss of one prestressing strand; this loss reduces the ultimate strength of beam as the beam gets weakens in the tensile region. Eventually, the cracking moment also lowers making the beam crack at lower load than the intact beam.

Figure 6-8: Moment-curvature curve for beam with one corroded tendon

Moment curvature curve for beam with one removed tendon has lower value than the earlier beam. The cracking moment for this beam is 1177.6 Kip-ft which is below the initial value of 1298.5 Kip-ft. The ratio Mult/Mcr = 1.26 implies that the beam still possesses ductile nature but its ratio has definitely lowered in comparison to the earlier analysis (Mult/Mcr = 1.299)

All these changes make the beam become less ductile, this fact is evident from the crack diagram and load-deflection shown below:
The crack is not widely distributed as in the previous case suggesting the lower ductility of beam. Crack distribution is an evidence of ductility of any structural member and change in crack distribution indicates the transition of nature from ductile towards brittle.

Figure 6-10: Load-displacement curve for beam with one broken tendon

The load deflection curve still supports the beam’s ductility, there is sufficient deflection before failure.
6.3. Changing the parameter to attain brittle failure

ACI suggested the modulus of rupture to be governed by the equation, \( f_t = c \sqrt{f'_c} \). The value of \( c \) varies from 7.5 to 12, the analyses run considering \( c \) as 7.5 did not simulate the real field situation as it happens to be very conservative (Hashtroodi, 2015). The analysis wasn’t able to produce brittle failure of beam by taking the lower limit of \( c \), so several trials were done for values of \( c \) between 7.5 and 12 to obtain complete brittle failure of beam. After many attempts, value of \( c \) as 10 was able to obtain the brittle failure mode with the condition of tendon similar to that in the field.

6.3.1. Considering two broken tendon and taking \( c=10 \)

![Properties of beam with 2 corroded tendon and \( c=10 \)](image)

Figure 6-11: Properties of beam with 2 corroded tendon and \( c=10 \)
The above figure shows the geometric properties of beam with 2 broken tendons and change in the modulus of rupture as the $c$ values in $f_t = c \sqrt{f'_c}$ has been changed from 7.5 to 10. The value of $f_t$ is 707 psi compared to 530 psi in the earlier analyses.

![Graph showing ultimate flexural moment](image)

**Figure 6-12**: Ultimate flexural moment of the beam with two broken tendon and $c=10$

![Graph showing moment-curvature](image)

**Figure 6-13**: Moment-curvature curve for beam with two corroded tendon and $c=10$
The ratio $\text{Mult}/\text{Mcr} = 1290.3/1217.3 = 1.05$ has drastically fallen below 1.2 which was the threshold for the ductility of the beam. The beam has significantly lost its ductility and has attained brittleness.

**Figure 6-14: Crack diagram for two broken tendons**

The crack diagram illustrates the brittle nature of this beam as there are not much cracks distribution around the mid-span. Now some trials were conducted to obtain the lowest possible ultimate flexural strength required for the brittleness of beam, for this the Mult should be equal to cracking moment.

**6.4. Response2000 result comparison**

Different analyses with the help of Response software were done for varying number of tendons and tensile strength (modulus of rupture). A table of comparison is shown with the failure mode:
Table 6.1: Results from Response2000 for varying c and tendons

<table>
<thead>
<tr>
<th></th>
<th>Initial condition (non-corroded strands, c=7.5)</th>
<th>One corroded strand, c=7.5</th>
<th>Two corroded strand, c=10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultimate Flexural Strength, Mult (Kip-ft)</td>
<td>1675.3</td>
<td>1483.6</td>
<td>1290.3</td>
</tr>
<tr>
<td>Cracking Moment, Mcr (Kip-ft)</td>
<td>1298.5</td>
<td>1177.6</td>
<td>1217.3</td>
</tr>
<tr>
<td>Mult/Mcr</td>
<td>1.29</td>
<td>1.26</td>
<td>1.06</td>
</tr>
<tr>
<td>Failure mode</td>
<td>Ductile</td>
<td>Ductile</td>
<td>Brittle</td>
</tr>
</tbody>
</table>

c varies in the equation $f_c = c\sqrt{f'c}$

It is clear from the Table 6.1 that the beam was initially designed for enough ductility. The ultimate moment is larger than the cracking moment to allow the beam to undergo enough deformation without cracking. But as the strands started to deteriorate, the flexural strength started to decrease and when two strands corroded, the beam was not ductile enough to deform under loading and it cracked.
Chapter 7

Finite element analysis using ABAQUS

7.1. Introduction

Response2000 provides all the necessary results for the ultimate moment strength and the cracking moment making it ideal software for the cross sectional analysis. Since we are dealing with the study of corrosion, it is important to understand the effect of corrosion in the concrete member. The most hazardous effect of corrosion is the weakening of strand in tension zone leading to cracking of the concrete. This crack propagates in the member over the time due to the sustained loading and corrosion progression.

In order visualize the corrosion induced crack propagation, analysis of concrete was done in ABAQUS. It provided us insight on the crack growth and the failure of the beam. Analyses of reinforced and prestressed concrete have many complications owing to its time-dependent characteristics and dissimilar behavior of its constituting materials. The complexities of the concrete behavior and safety of the structure have necessitated the use of powerful computational tool to analyze the non-linear response of reinforced concrete specimen.
Ngo and Scordelis (1967) hold the credit for the first work on the analysis of RC structures with the help of finite element method. They carried linear elastic analysis on simple beams with pre-existing crack patterns. The principal stresses in concrete, reinforcement stresses and bond stresses were analyzed in this earliest model to understand the behavior of reinforced concrete member. The analysis wasn’t directed to formulate any innovative idea about concrete but to realize the feasibility and potentiality of finite element method for the study of the reinforced concrete.

Nilson (1972) incorporated non-linear material properties for concrete and steel in the analysis. He emphasized on usage of incremental load method for non-linear analysis. Cracking was computed manually which required the analysis to stop as soon as the specimen reached its tensile strength and resuming the loading after new crack parameters were defined.

Franklin (1970) devised the automatic method of analyzing the crack with redistribution of stressed in the structure. It did not require frequent interruptions in the analysis so the system could be fully analyzed from the initial loading to failure a single continuous analysis. He extended the analysis to the special frame-type element, axial bar member, tie links in reinforced concrete frames and shear walls.

Cervenka (1970) developed constitutive relationship for concrete-steel material and used elastic stiffness matrix for the entire analysis through the uncracked, cracked and plastic stage of behavior. The ideas of progressive cracking and load-time dependent material properties were introduced by Selna (1969). The effects of heat transfer, creep and shrinkage were included in the analysis by Becker and Bresler (1974).
Joffriet and McNiece (1971) analyzed reinforced concrete slab by reducing the flexural stiffness to include the change in bending stiffness of the material. Scanlon and Murray (1974) took into account both the cracking and time-dependent effects of creep and shrinkage in slabs, assuming the cracks to propagate only parallel and perpendicular to orthogonal reinforcement. The tension stiffening effect of concrete between cracks were incorporated by Lin and Scordelis (1975). They had used layered triangular finite elements in RC shell.

The major breakthrough in the concrete crack model was made by Rashid (1968) by initiating the concept of smeared crack model. He used prestressed concrete reactor structure for his analysis which was subjected to cracking, creep and temperature effects. The smeared crack model is widely used by researchers for analyzing the non-linear cracking behavior of concrete. The limitations of the smeared crack models are regularly addressed and the model is ever improving during the last few decades.

The dependence of the analytical results of the reinforced concrete on the mesh size of the finite element and the tension stiffening was identified by the Gilbert and Warner (1978). Other researches carried out investigations to determine the accuracy of the finite element analysis based on the varying mesh size.

Tension stiffening significantly affects the under reinforced beams in bending whereas in case of over-reinforced beams and shear walls, the behavior of reinforced concrete specimen is controlled by the bond-slip of reinforcing steel. Two different approaches are used to study the bond-slip in finite element analysis of RC specimen.
The first approach is devised by Ngo and Scordelis (1967) in which the node of concrete finite element and adjacent steel element has same co-ordinate and they are connected by dimensionless bond link element. The second approach was proposed by de Groot et al (1981) who utilized the bond-zone element. He formulated a material law for the bond zone which comprises of the contact surface between concrete and steel, and the concrete in the neighborhood of rebar. This bond zone helped to establish continuous connection between the reinforcing steel and concrete.

The non-linear behavior of response of reinforced concrete can be classified into three stages: the uncracked elastic stage, the crack propagation and the plastic (yielding of concrete). The non-linearity response of concrete is majorly affected by two factors; first is the cracking of concrete in tension and second is the yielding of reinforcement or crushing of concrete in the compression. Bond-slip between the steel and adjacent concrete, aggregate interlock near the cracks and dowel action of the reinforcing steel at crack contribute to the non-linearity.

For the finite element analysis in this research, non-linear elasticity model was used for concrete as this model makes it computationally simple to solve the complexities arising due to the crack propagation. Whereas, steel was modeled by using embedded steel model for this analysis as this model had large number of elements and embedded steel model performs very well for such elements. [Kwak and Filippou, 1990]
7.2. ABAQUS modeling with boundary condition and loads

Figure 7-1: Beam with initial crack

The L-shaped spandrel beam model with a span of 35 feet 10 inches is shown in the figure with a crack in the mid-span to simulate the real field situation. It is a simply supported beam with hinge support at one end and roller support on the other. Torsional constraint was used on the upper half of the beam. The beam, crack and the tendons were modeled separately and respective material properties were assigned to them. The material properties for concrete and steel are already discussed in the Response2000 calculation.

The force applied to each half inch and quarter inch tendons are as explained below:

From the Response2000 chapter, the initial jacking force \( f_{pi} = 175 \text{ ksi} \)

Assuming 25% long term losses, the effective stress \( f_{pe} = 0.75 \times f_{pi} = 131 \text{ ksi} \)
Area of half inch tendon = 0.144 in$^2$

Area of quarter inch tendon = 0.036 in$^2$

Force applied on half inch tendon = 131 * 0.144 = 18.864 kip

Force applied on half inch tendon = 131 * 0.036 = 4.716 kip

**Figure 7-2: Beam with boundary conditions and load**

The self-weight is incorporated in the ABAQUS analysis, the additional load due to Double Tees has to be supported by the ledge of spandrel beam. Since the beam had to support the dead load and live load applied on the double Tees, analysis was conducted by subjecting ledge to a pressure force generated by the Double Tees load.

Maximum DL on the Double Tees as specified in the original drawing = 45 psf
Maximum LL of parking garage beam as recommended by ASCE/SEI 7-10 = 40 psf

The self-weight is incorporated in the ABAQUS analysis, the additional load due to Double Tees has to be supported by the ledge of spandrel beam.

Area covered by Double Tees = 58*35 = 2030 ft²

Load due to Double Tees = (45+40)*2030 = 172550 lb

Load due to 3” topping on the Double Tees = 3/12*2030*150 = 76,125 lb

Total load due to topping and Double Tees = 248,675 lb

Since the Double Tees area is supported by 2 L-beams, load on each beam = 124,338 lb

This 124,338 lb load is applied on L-beam to simulate the real field condition.

The total area of bottom part of L-beam consisting ledge = 430*8 = 3,440 in²

The pressure load on the bottom part of L-beam = 86275/(2*216) = 36 lb/in²

As obtained in the above calculation, pressure force of 36 psi is applied on the ledge to simulate the real field situation. So two sets of forces were used in the analysis, two for the tendons and one for the pressure force on ledge.

The analysis conducted for 36 psi had total of 24 increments and it was not sufficient to trigger the crack propagation. The load demand at failure obtained in chapter 5 was way too less compared to the strength of the beam so this pressure load could not
initiate the crack propagation. Several trials were done varying the pressure load to stimulate the real beam condition at failure. The final stage of crack propagation obtained with pressure load of 223 psi was similar to the real beam crack in the field.

7.3. Analysis

It is possible to follow the load and crack status in ABAQUS. During analysis, the crack started to propagate at the 27th increment of the load. The different stages of the analysis are shown in the screenshots below:

Figure 7-3: First step of analysis with pre-defined crack
In the previous two figures, there is no change in the state of crack. The top figure represents the first state of the analysis whereas the bottom figure is the screenshot of the 26th step denoted by increment 26. The crack propagation started from the 27th step and the crack tip can be seen rising above the ledge into the flange.
7.4. Results

As the analysis begins, there is continuous increment in loading but the crack propagation won’t occur till the stress is below the tensile limit of the beam. When the applied stress exceeds the tensile strength, the crack starts to propagate. At the 27th increment, the pressure force acting on the ledge is 36.53% of 223 psi force. At this stage, 81.46 psi pressure force is acting on the ledge which is the cracking load for this analysis. The cracking moment is obtained as follows:

\[ 36.53\% \text{ of } 223 \text{ psi force} = 81.46 \text{ psi} \]
Width of ledge = 8 in

Unit load acting on the ledge = 8\times 81.46 = 651.69 \text{ lb/in}

Cracking moment, \( \text{Mcr} = 651.69 \times (430)^2 / 8 = 15,062,305 \text{ lb-in} = 1255 \text{ kip-ft} \)

The cracking moment obtained from the Response2000 for 2 broken tendons is 1217.3 kip-ft. The results vary by just 3.04\% and this validates the result generated by ABAQUS. The pressure load kept on increasing until the beam failed. The crack at the final state had nearly propagated all the way to the top of the beam causing it to fail abruptly without any distributed crack around the main crack which is generally associated with brittle failure. This failure mode is very much in consistence with the failure mode in real field.

Figure 7-6: 3D view of final state of the analysis
7.5. Demand at failure versus calculated capacity

The capacity of the beam at the failure, when two of its bottom tendons were broken, along with the cracking moment obtained by hand calculation and computer analyses are as tabulated below:

Table 7.1: Comparison of capacity and cracking moment at failure obtained by various means

<table>
<thead>
<tr>
<th></th>
<th>Mathcad</th>
<th>Response2000</th>
<th>ABAQUS</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mcr (kip-ft)</td>
<td>931</td>
<td>1217</td>
<td>1255</td>
<td>1134</td>
</tr>
<tr>
<td>Mult (kip-ft)</td>
<td>1239</td>
<td>1290</td>
<td>-</td>
<td>1264</td>
</tr>
</tbody>
</table>

The demand on the beam at failure was calculated to be 693 kip-ft on chapter 5 which is smaller than both the cracking moment and the ultimate moment as seen on the table 7.1.
\[ M_{\text{demand}} = 693.3 \text{ kip-ft} \quad M_{\text{cr}} = 1134 \text{ kip-ft} \quad M_{\text{ult}} = 1264 \text{ kip-ft} \]

7.6. Discrepancy and possible causes

The discrepancy between the expected capacity and the failure load is large. The computational methods used here are widely accepted and have been used to predict both the cracking moment and ultimate capacity of new beams accurately (Morganstern, 2014). This motivates one to speculate on the cause of complete failure at such a low demand. The possible causes of the failure could be:

a. Most of the strands are corroded

b. Existing crack in the beam

For the capacity to get as low as the load demand, there has to be more number of broken strands than there was in the real beam. The number of strands that have to be removed from the beam to lower the capacity of the beam was computed using Mathcad as shown below:

7.6.1. Required number of broken tendons for the beam to fail at load demand

The beam’s initial capacity was calculated using Mathcad considering all the tendons as unbroken. The calculation is explained below:

\[ f'_c = 5000 \text{ psi} \quad f_y = 250 \text{ ksi} \quad \text{width of flange (b)} = 10 \text{ in} \]

\[ A_h = 0.144 \text{ in}^2 \quad A_q = 0.036 \text{ in}^2 \quad E_s = 28500 \text{ ksi} \]

Gross area of concrete, \[ A_g = 10*72+12*18 = 936 \text{ in}^2 \]

Distance from the top extreme fiber to the steel in compressive zone, \[ d' = 1.25 \text{ in} \]
Distance from the top extreme fiber to the steel in tensile zone, \( d_t = 82.25 \) in

Solving height of the stress box ‘a’ by trial and error (All the tendons below the 72” depth are removed)

1. Let’s assume there is no strand in compressive zone

Area of steel in tension, \( A_s = 2A_h + 20A_q = 1.008 \) in\(^2\)

Percentage of steel in tension, \( \rho = A_s / A_g = 0.00108 \)

Area of steel in compression, \( A'_s = 0 \)

Percentage of steel in compression, \( \rho' = A'_s / A_g = 0 \)

\[ A_s - A'_s = 1.008 \text{ in}^2 \quad \rho - \rho' = 0.00108 \]

For \( f'_c = 5000, \quad \beta = 0.85 - [0.05 (f'_c - 4000) / 1000] = 0.8 \)

\[ f'_s = 87000 \left[ 1 - \frac{(0.85\beta f'_c d'_t)}{(\rho - \rho') f_y d_t} \right] = 70.3 \text{ ksi} \]

\[ a = \frac{(A_s f_y - A'_s f'_s)}{(0.85 f'_c b)} = 5.93 \text{ in} \]

Neutral axis depth, \( c = a/0.8 = 7.41 \) in

We had already assumed that this trial had no steel in the compression zone but the value of neutral axis came out to be 7.41 in. If we look at the cross section of the beam, top strands were at the depth of 1.91 inch so this assumption was not true.

2. For second trial, let’s assume compressive zone has top 2 steels (2#1/2”)

Area of steel in tension, \( A_s = 20A_q = 0.72 \) in\(^2\)
Percentage of steel in tension, $\rho = \frac{A_s}{A_g} = 0.00077$

Area of steel in compression, $A'_{s} = 2A_h = 0.288$ in$^2$

Percentage of steel in compression, $\rho' = \frac{A'_{s}}{A_g} = 0.00031$

$A_{s} - A'_{s} = 0.432$ in$^2$ \hspace{1cm} $\rho - \rho' = 0.00046$

$f'_{s} = 87000 \left[ 1 - (0.85\beta.f'_{c}.d')/(\rho - \rho') f_y.d_t \right] = 48$ ksi

$a = (A_{s}.f_y - A'_{s}.f'_{s})/(0.85f'_{c}.b) = 3.91$ in in Neutral axis depth, $c = a/0.8 = 4.89$ in

There are top 2 strands at a depth of 1.91” and top 4 at 6.15”, so this assumption is correct.

Strain at the extreme steel in tensile zone, $\epsilon_t = 0.003(d_t - c)/c = 0.004749 > 0.005$. So, tensile strands yielded.

From the similar triangles, strain at the top 2#1/2 strands $\epsilon_1 = 0.0023$

Stress at the top 2#1/2 strands $\sigma_1 = E_s * \epsilon_1 = 65.5$ ksi

$A_{s} = 0.72$ in$^2$

Area of tendons at the bottom, 3#1/2” and 1#1/4” = $3A_h + A_q = 0.468$ in$^2$

**Tensile steel centroid calculation**

Calculating CG of tensile steel from bottom

$[2A_q*(14.43+21.43+28.43+35.43+42.43+50.43+57.43+64.43+71.43+78.43)]/A_{s}$

$= 46.4$

CG from top, $cg_t = 84$ in – 46.4 in = 37.6 in
Moment calculation

\[ A_s f_y (c_g - c) + \sigma_1 * 2A_h * 3.64 + \sigma_2 * 2A_q * 3.06 + 0.85f_c' * b * a * (c - a/2) = 537 \text{ kip-ft} \]

Moment = 537 kip-ft

When we assume all the tendons below 72” depth as broken then the moment capacity becomes less than the load demand but this is not the case in reality. When we excavated the beam 8-9-4CW and visually observed its strands, they were perfectly fine with no signs of corrosion on them. So the case of beam failure at load lower than the capacity due to larger number of broken tendons can be ruled out.

7.6.2. Existing crack in the beam

The second assumption for the failure of beam at significantly lower load can be the pre-existing crack in the beam. The load demand on the beam at failure was 693 kip-ft and the height of crack from the bottom of beam necessary for the cracking moment to be equal to this load demand is calculated as:

Let us assume the height of crack from the bottom of beam be \( x \). The section that includes the crack is not taken into consideration during calculation.

New depth of the ledge = 12-\( x \)

The neutral axis of the cracked beam is,

\[ Y_{\text{bot}} = \frac{[72*10*(36+12-\text{x}) + (12-\text{x})*18*(6-\text{x}/2)]/[72*10+(12-\text{x})*8]}{\text{[72*10+(12-\text{x})*8]}} \]

Gross moment of inertial of the cracked beam is,
\[ I_g = 18*(12-x)^3/12 + 10*72^3/12 + 18(12-x)\{Y_{\text{bot}} - (12-x)/2\}^2 + 72*10\{Y_{\text{bot}} - (36+12-x)\} \]

The modulus of rupture of concrete at failure is, \( f_r = 10\sqrt{5000} = 707 \text{ psi} \)

The cracking moment is given by the formula, \( M_{\text{cracking}} = f_r I_g / Y_{\text{bot}} \)

Equating the cracking moment with the load demand and solving the value of \( x \) with the help of Mathcad,

\[ f_r I_g / Y_{\text{bot}} = 639*12000 \text{ lb-in} \]

\( x = 6.9 \text{ in} \)

The calculated crack height is 6.9 inch from the bottom of the beam. Also, there was an existing crack at the failure location. Through cracks are quite possible. In fact, beam 8-9-4CW had a crack which was observed to be open and closing with temperature by Thomas Stuckey of Poggemeyer.

Considering above assumptions and calculations, it is reasonable to conclude that the beam failed due to the existing crack which affected the strength of the beam and caused the beam to fail at much lower load than expected.
Chapter 8

Magnetic flux leakage corrosion inspection

8.1. Introduction

The durability and stability of a reinforced or prestressed concrete structure depends on the integrity of the steel reinforcement. It is important to understand the state of steel reinforcement in order to determine the condition of those structures. Reinforced concrete structures are adversely affected by the corrosion of the embedded reinforcement bars; more often than not, the corrosion is the main cause of premature deterioration of concrete construction. Since the bars are covered with the concrete cover, it is difficult to locate them and determine the extent of the damage caused by the corrosion with normal visual inspection.

Corrosion cracking is of progressive nature so it is necessary to monitor the concrete cover and steel in real time to understand the extent of the degradation and performance of the structure. The maintenance and life prediction can be planned properly if the development of the corrosion is tracked on regular basis. Visual inspection yields result only when the concrete cover has degraded enough to observe the corrosion on the
tendons. Electrochemistry technique can also provide insight on the steel state, but it relies on the concrete environment rather than the actual steel on which corrosion takes place.

The inspection requires the inspection tools to be arranged on the external surfaces of structures or inserted into tubing or pipelines. Most of the corrosion inspection techniques require excavation of the concrete cover to gain access to the reinforcement, so adequate assessment of the risk should be done beforehand in order to ensure maximum safety of the inspection team and minimum degradation of the concrete structure. Usually nondestructive techniques (NDT) are advantageous compared to the destructive ones and they have proven to be fast and inexpensive as well.

Considering all these, it will be beneficial if we can physically monitor the condition of steel with the help of non-destructive technique (NDT). NDT does not demand removal of concrete cover and does not affect the concrete physically or chemically.

8.2. Different types of NDT techniques for corrosion inspection

Some of the physical tests for steel corrosion assessment which are in practice are as described below:

8.2.1. X-ray diffraction:

When a reinforced concrete specimen is subjected to the X-ray, the area containing corroded steel has less intense X-ray than the sound steel area. This change in the intensity helps to determine the extent of corrosion in the steel. Though this technique is simple to use, it is hazardous. (Lei and Zheng, 2013)
8.2.2. **Acoustic emission:**

Corrosion increases the volume of steel reinforcement by 6-10 times which results in the tensile stress in concrete (Broomfield, 1997). Whenever this tensile stress is above the concrete tensile strength, cracks are developed. When concrete starts to propagate, energy is released; part of this released energy converts into sound waves. The sensors installed in concrete specimen detect such emitted sound waves, process it and can assess the location and strength of the sound source, i.e. crack. (Kawasaki et al, 2013)

8.2.3. **Electrical resistance probe:**

This test is based on the change in the resistance of the sensor as an indicator of corrosion. Resistance of these probes (sensors) increases with decrease in cross sectional area of steel, so as the steel corrodes, it gets thinner in section causing increase in the resistance. (Lei and Zheng, 2013)

8.2.4. **Magnetic flux leakage:**

This non-destructive technique is very effective way to investigate the corrosion in steel reinforcement. The corrosion is determined by the variation in the magnetic field resulting from the reduction in the cross sectional of the steel. (Fernandes et al, 2012)

8.3. **Procedure of corrosion inspection in UT’s parking garage:**

Magnetic flux leakage method was used for corrosion inspection of prestressing strands in the University of Toledo’s east parking garage beam. This test was done in collaboration with a team from University of Wisconsin, Milwaukee led by Prof Ghorbanpoor. This reliable non-destructive technique of corrosion detection of steel...
embedded inside concrete was discovered by F. N. Kusenberger (Fernandes, 2012). This method utilizes the inducing property of magnet to magnetize the steel reinforcement in concrete members and has been demonstrated to be more effective than other non-destructive techniques on hidden corrosion. (Fernandes et al., 2012).

A constant directional flow of magnetic flux is generated in prestressing strands when an external magnetic field is applied. If the strand has flaw, some or all of the flux is leaked out of the strand. This leakage is detected by sensors and produce signals with different amplitude compared to the normal flux signal. The flaw may be change in cross sectional area of the steel due to corrosion or accidental addition or removal of steel inside the concrete. The beams considered for the magnetic test were chosen by visual inspection of the physical condition of the beam; if the surface of a beam exhibited severe crack or a beam’s orientation differed on comparison to the neighboring beams or there was some alteration on the beam’s alignment, the beam was suspected of corrosion and magnetic inspection was conducted on it.

There were some restrictions in scanning the full length of the beam. The sensor used in the inspection was housed two feet inside the robotic beam traveler frame so the scan result was possible only for the region beyond two feet from the face of the column. Also, the clear spacing on either side of the beam needed to be approximately one feet for the robotic traveler to move along the beam which made it difficult to access beam at some of the locations which did not have enough clear spacing on sides.
Figure 8-1: Magnetic flux with and without flaw [Lijan et al., 2001]

As the robotic beam traveler travelled over the length of the beam, a graph of the magnetic flux signal was plotted. The amplitude (peak) of the graph marked the presence of transverse stirrup; the distance between two regular peaks indicated the distance between the stirrups in the beam (Fernandes et al, 2012). Any irregularity in the signal implied the presence of corroded bar or wrongly placed stirrup. The concrete in the areas of abnormalities corresponding to the location in graph were later chipped off in three areas and the steel was exposed for visual observation to validate the accuracy of the magnetic inspection. Seven beams were inspected and excavation was performed in three beams. The beams which were subjected to magnetic inspection are as tabulated below:

Table 8.1: List of beams subjected to magnetic inspection

<table>
<thead>
<tr>
<th>Floor Level</th>
<th>Name of beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11-12-1A</td>
</tr>
<tr>
<td>1</td>
<td>11-12-1C</td>
</tr>
<tr>
<td>4</td>
<td>5mid-6-4CW</td>
</tr>
<tr>
<td>4</td>
<td>5-5mid-4CW</td>
</tr>
<tr>
<td>4</td>
<td>8mid-9-4B</td>
</tr>
<tr>
<td>4</td>
<td>8-8mid-4B</td>
</tr>
<tr>
<td>4</td>
<td>8-9-4CW</td>
</tr>
</tbody>
</table>
In Fig 8-2, we can see robotic beam traveler on top which is a frame mounted on wheels with magnet and Hall-effect sensors. The movement of this beam traveler can be controlled and as it travels along the length of the beam, it scans the condition of steel strands inside the beam. As this traveler scans, the graph of the flux signal is plotted on the
computer which is connected to data acquisition unit and a DC power source as shown on the bottom of Fig 8-2.

![Robotic beam traveler travelling along the beam](image)

**Figure 8-3: Robotic beam traveler travelling along the beam**

As can be seen in Fig 8-3, the tires of the robotic beam traveler gripped the exterior faces of the beam while it moved with some support from below to counter its weight and prevent from slipping down.

During our observation, we were allowed to excavate beam’s region which was 6 feet away from the faces of column. Though several beams were selected on the basis of visual inspection, only two of those beams were excavated in their existing conditions. The beam suspected of having the most heavily corroded strands (beam on level 4 between columns 8 and 9) was not excavated until a post tensioned beam was installed beneath this beam since there was danger of collapse of the beam owing to the suspected loss in strength of its strands due to corrosion as indicated by magnetic field test.
8.4. Analysis and results

A beam denoted by 11-12-1A on parking garage level 1 between column 11 and 12 was excavated to determine the accuracy of the magnetic inspection result. The areas circled in red in figure 8-4 indicates anomaly. The beam was excavated at the corresponding location of marked area in figure 8-5 and as predicted in the scan result, there was a stirrup on that region as predicted by the scan result. The graph plots are offset by 2 feet compared to the locations on real beam.

11-12-1A West Half

![Figure 8- 4: Magnetic inspection scan result of 11-12-1A West Half](image-url)
Figure 8-5: Transverse Stirrup on the level 1 beam at corresponding locations marked on the Fig 8-4

The visual observation was in agreement with the test result on level 1 beam, this prompted us to dissect another beam whose graph exhibited abnormality. The next
candidate for excavation was a 5mid-6-4CW beam between column 5mid and 6 on the level 4. On excavation, it was found that an extra piece of steel was left inside the beam due to bad workmanship as shown in figure 8-7:

**Figure 8-6:** Magnetic inspection scan result of 5mid-6-4CW West Half
Figure 8-7: Extra Steel causing deviation in the magnetic signal

This resulted in the extra amount of steel which skewed the magnetic flux plot. On study of other skewed graph, it was seen that the peaks of the graph varied in similar manner to this beam’s graph indicating that those beams were also affected by bad workmanship.

8.4.1. Inspection and excavation of beam between column 8 and 9 on level 4

Figure 8-8 suggests that the strands in the beam 8-9-4CW may have some general corrosion. Cracks were visible in the region between 18’ and 19.5’ as marked in the figure below. Since this scan results are offset by 2’, the crack region in the Fig 8-8 falls between 16’ and 17.5’.
There was an attempt made to excavate after strengthening this beam by installing a post-tensioned beam below it. The prestressed beam was marked from 18’ to 20’ from the face of column 8 which is the corresponding region of discrepancy in the graph and drilled but it did not go according to plan and cracks developed in the new post tensioned beam. The research teams was allowed to dissect the suspected beam on the level 4 between columns 8 and 9, only after these newly formed cracks on the post tensioned beam were repaired.

**Figure 8-8: Magnetic inspection scan result of 8-9-4CW West Half**
Figure 8-9: Post tensioned beam installed beneath the 8-9-4CW beam on level 4

Figure 8-10: Prestressed beam excavation in the middle of the marked region
Figure 8-11: Cracks on post tensioned beam while excavating upper pre-tensioned beam

On the second attempt, the beam was excavated in the region between 18’ and 19’-3” from the face of column 8 and a depth of 6.5” from the top to the bottom of the spandrel. But the condition of the strands was as good as fresh strand with no visible corrosion on them. The concrete on the neighborhood of the strand were chipped off and tested with phenolphthalein. The pH test confirmed that they were still alkaline in nature indicating no trace of corrosion.

Figure 8-12: Strand in healthy (non-corroded) condition
Figure 8-13: Pink color of concrete in pH test indicating its alkaline nature

8.5. Summary of the magnetic inspection

The results predicted by the magnetic inspection were true for almost all of the beams except beam 8-9-4CW. The anomaly indicated in the graph was due to incorrect orientation of stirrup or extra steel in the beam as seen on excavation of the beam. The stirrups in 8-9-4CW had not suffered any corrosion attack nor was there any extra steel in the beam. The beams which were inspected are tabulated below with their excavation results:

<table>
<thead>
<tr>
<th>Beam</th>
<th>MFL result</th>
<th>Excavation result</th>
</tr>
</thead>
<tbody>
<tr>
<td>11-12-1A</td>
<td>anomaly in the location 16 to 18 feet from the face of column 11</td>
<td>wrong orientation of stirrup</td>
</tr>
<tr>
<td>5mid-6-4CW</td>
<td>anomaly in the location 10 to 12 feet from the face of column 6</td>
<td>extra piece of steel was left in beam</td>
</tr>
<tr>
<td>8-9-4CW</td>
<td>anomaly in the location 17 to 24 feet from the face of column 8</td>
<td>strands were healthy and no extra steel</td>
</tr>
</tbody>
</table>

Table 8.2: Comparison of MFL and excavation result
Chapter 9

Conclusion

9.1. Results and Discussion

Based on findings of finite element analysis and the real field investigation, the damaging effects of corrosion in the prestressed concrete were studied. Prestressed concrete is very much vulnerable to corrosion as the corrosive agents find easy access to the strands in the prestressing strand as compared to the reinforced concrete. Not only the chemical agents but also the environmental factors and loading condition become critical for corrosion initiation and propagation.

Steel governs the tensile strength of concrete maximizing its ultimate flexural strength. If the reinforced/prestressed steel is not in healthy condition, the concrete loses its strength. Corrosion destroys the steel strand slowly and it is too late when the corrosion comes to our notice. Response2000 gave a clear picture of the behavior of prestressed concrete when the tendons become useless due to corrosion. With each broken tendon, the strength diminishes and finally a stage comes when the ultimate flexural strength falls to the cracking moment level. This is the stage when the structure is structurally weak and both the occupants and structure are at high risk.
Initially the beam had enough ductility when all of its strands were functioning at full potential but when corrosion weakens the strand; the beam started loses its ductility which is verified by the Response 2000 results as well. The corrosion started affecting other strands as well and eventually the beam failed with 2 corroded strands. For initial analysis with full healthy tendons, the ratio of ultimate moment to cracking moment is 1.29 which is well above the ductility requirement of ACI 318-11 (Mult/Mcr =1.24) but as the strands got corroded the ratio started decreasing and at the 2 broken strands, the ratio came close to being unity which means the beam has completely lost its ductility and suffered brittle failure as the real beam in parking garage.

When we analyze the results, we can see that the cracking moments obtained from Abaqus and Response2000 are consistent with each other. The capacity of beam obtained by Mathcad calculation and Response2000 don’t differ much which validates the analysis results. The comparison of initial ultimate capacity of the beam with all the tendons functioning at full potential is shown below:

<table>
<thead>
<tr>
<th>At Initial State</th>
<th>Response 2000</th>
<th>Mathcad</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment capacity</td>
<td>1675.3</td>
<td>1694.2</td>
<td>1.0</td>
</tr>
<tr>
<td>(kip-ft)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As we can see, the % difference in the capacity of the beam obtained from Response 2000 is only 1% when compared with the hand calculation (Mathcad).
The concrete strength at the brittle failure was also compared for the Mathcad and Response2000 results. At failure, the variation was slightly higher compared to the initial state. This may be due to increase in the tensile limit of the concrete in Response2000 to simulate the beam failure with two broken tendons. The tensile limit value was increased from $7.5\sqrt{f'_{c}}$ to $10\sqrt{f'_{c}}$ and this caused increase in the result of Response2000, so the variation increased.

Table 9.2: Comparison of ultimate moment capacity of beam with two tendons missing obtained from Response2000 and Mathcad

<table>
<thead>
<tr>
<th>At Failure</th>
<th>Response 2000</th>
<th>Mathcad</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment capacity (kip-ft)</td>
<td>1290.3</td>
<td>1239.13</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Crack propagation was dealt with the 3D finite element software ABAQUS as the progression of crack is very well illustrated in ABAQUS. A pre-defined crack was modeled in the mid-span of the beam as in real field. This analysis gave a clear understanding of the status of crack at each stage and the load at which crack started to propagate was considered as the cracking load for the analysis. The cracking moment obtained from ABAQUS is in good agreement with that of Response2000 as shown in the following table:

Table 9.3: Comparison of cracking moment of beam with two tendons missing obtained from Response2000 and ABAQUS

<table>
<thead>
<tr>
<th></th>
<th>ABAQUS</th>
<th>Response 2000</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cracking Moment (Kip-ft)</td>
<td>1255</td>
<td>1217.3</td>
<td>3</td>
</tr>
</tbody>
</table>
The load demand at failure is significantly lower compared to the expected capacity of beam. Numerous attempts were made to solve this discrepancy which include checking the original drawing for the load specification and studying PCI guidelines. But still the results were not anywhere near the values obtained from the tendon based capacity calculation. Larger number of broken tendons due to corrosion and possibility of pre-existing crack in the beam were assumed as the reasons for this discrepancy. On excavation of beam 8-9-4CW, no corrosion of strand was detected. Calculation of crack height based on cracking moment at failure provided by Response2000 suggested that a pre-existing crack could lower the capacity to the load at which beam failed. The 8-9-4CW had a crack which fluctuated with the change in temperature as observed by Thomas Stuckey.

Usually corrosion of reinforcement leads to abrupt failure giving no warning to fix it. Some methods of preventing corrosion were discussed which are very much essential to incorporate during manufacture of prestressed concrete. Preventive measures such as use of corrosion resistant and epoxy coated reinforcement steel can definitely lower the chances of corrosion.

Magnetic inspection was conducted to determine the corroded strands inside the beam. The result obtained from the magnetic inspection was consistent with the actual steel condition in some of the beams. But on the beam between column 8 and 9, the scan results did not agree with the strand condition inside the beam. The magnetic inspection suggested that the strands had some faults which was not the case when the beam was excavated and strands were visually inspected. No regions with heavy corrosion or clear breaks were identified on the basis of inspection results. This inspection can be beneficial in checking
the detailing of steel strands in prestressing concrete as a flaw in workmanship was detected during this inspection.

9.2. Future Studies

Corrosion detection methodology is still in its early stage and has a long way to go. The magnetic inspection equipment we used in this research was bulky and required lots of effort to use in some of the beams. If we can come up with easy-to-handle design, then the inspection will not be tiresome and time consuming.

Response2000 software does not have built-in feature for the L-shaped beam so we had to rely on the inverted T-beam for the analysis. The result would be more convincing if we can conduct the analysis on the particular beam section rather than the closely related alternative section. The geometry of specimen affects the distribution of loading and strands position.

Discrepancy in the load demand and expected capacity of the beam remained a mystery throughout this research. Assumptions were made for the reasons behind this significant variation and many computational works were required to come to conclusion. The author would like to suggest to check for the possible faults in the beam such as crack and defects in the structural member before proceeding to the analytical and computational part.
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