A Thesis

entitled

Maximizing Channel Capacity based on Antenna and MIMO Channel Characteristics and its Application to Multimedia Data Transmission

by

Andrew Pottkotter

Submitted to the Graduate Faculty as partial fulfillment of the requirements for the Master of Science Degree in Electrical Engineering

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An abstract of
Maximizing Channel Capacity based on Antenna and MIMO Channel Characteristics and its Application to Multimedia Data Transmission

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Communication transmission between electronic devices is evolving at an ever faster pace. There are now more electronic handheld devices that we communicate with on a daily basis. The allotted bandwidth and speed for these devices are limited by hardware, software, handshaking capabilities between each electronic application. The demand for information at high data rates without the loss of reliability has evolved antenna technology and digital signal processing into more complex systems utilizing multiple processors and multiple antennas. This paper discusses the various techniques used to increase data speed, enhance channel capacity, and reliability of application specific devices with respect to the Multiple-Input-to-Multiple-Output (MIMO) based methods. MIMO based applications can improve the data speed, channel capacity, and reliability of the system with maximum limitations based on hardware, coding schemes, and handshaking abilities between devices.
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LIST OF ABBREVIATIONS

AS ............................................. azimuth spread
AWGN ............................................. additive white Gaussian noise
BER ............................................. bit error rate
BPSK ............................................. binary phase shift keying
BS ............................................. base station
CDF ............................................. cumulative distribution function
CIR ............................................. channel impulse response
dB ............................................. decibel
FDMA ............................................. frequency division multiple access
FIR ............................................. finite impulse response
FWGN ............................................. filtered white Gaussian noise
IBI ............................................. inter-block interference
ICI ............................................. inter-carrier interference
ISI ............................................. inter-symbol interference
LOS ............................................. line of sight
MIMO ............................................. multiple-input, multiple-output
MIS ............................................. multiple-input, single-output
MSE ............................................. mean square error
NLOS ............................................. non line of sight
NMSE ............................................. normalized mean square error
OFDM ............................................. orthogonal frequency division multiplexing
OFDMA .............................................orthogonal frequency division multiple access
PAS .............................................power azimuth spread
PDP .............................................power delay profile
QoS .............................................quality of service
QPSK .............................................quadrature phase shift keying
SIR .............................................signal-to-interference ratio
SIMO .............................................single-input-multiple-output
SISO .............................................single-input single-output
SNR .............................................signal-to-noise ratio
LIST OF SYMBOLS

I: Identity matrix
N_r: Number of receive antennas
N_t: Number of transmit antennas
rand: Random number uniformly distributed in [0, 1]
T_s: Symbol duration
σ^2: Noise covariance
λ: carrier wavelength
C: Correlation matrix or Channel Capacity
σ_τ: RMS delay
P_n: power of the nth path
σSF: lognormal shadow fading
M: number of subpaths per-path.
θ_n,m,AoD: AoD for the mth subpath of the nth path.
θ_n,m,AoA: AoA for the mth subpath of the nth path.
GBS θ_n,m,AoD: BS antenna gain of each array element.
GMS θ_n,m,AoA: MS antenna gain of each array element
K: wave number 2π / λ
λ: carrier wavelength
ds: distance in meters from BS antenna element s from the reference antenna s = 1, d_1 = 0.
du is the distance in meters from MS antenna element u from the reference (u = 1) antenna. For the reference antenna u = 1, d1 = 0.

Φ_{n,m} phase of the mth subpath of the nth path.

|V| magnitude of the MS velocity vector

θ_v angle of the MS velocity vector

d antenna spacing distance

β phase

E E-plane of Electric Field

H H-plane of Magnetic Field

D Directivity

G Gain

ε_r dielectric constant

W width of patch antenna

h height of patch antenna

L_e effective length of patch antenna

L # of Taps

AF Array Factor

ψ maximum directivity

f_m Doppler frequency

σ^{sf} shadowing fading
Preface

This Thesis outlines the principles of MIMO technology and how it can influence multimedia data communication. To describe how the increasing demand for information can be met with MIMO technology, the following paper will discuss the principles of MIMO technology and its dependent counterparts such as propagation path and physical antenna characteristics.
Chapter 1

Introduction

1.1 General Introduction

MIMO (Multiple-input and multiple-outputs) channel “is the use of multiple antennas at both the transmitter and receiver to improve communication performance. MIMO has evolved to refer to as “a method for multiplying the capacity of a radio link by exploiting multipath propagation.” MIMO functions are categorized into three focuses, precoding or beamforming, spatial multiplexing, and diversity coding. Each category is dependent on the antenna and its channel characteristics. MIMO Channel characteristics can be model by the three categorizing listed above. Beamforming offers an increase in receive power and directivity. Spatial Multiplexing offers multiple data rates and spatially orthogonal channels. Diversity offers mitigate fading and space-time coding. The directivity or diversity at the receiver and transmit ends of a MIMO link determines the extent at which the channel supports. Each channel is affected by the wireless propagation and its fading path.

Antenna is defined “as a means for radiating or receiving radio waves.” Antenna parameters such as directivity, gain, radiation intensity, beam pattern, and efficiency can enhance or limit beamforming, spatial multiplexing, and diversity.
Through multimedia devices, there is an increasing need for transmitting data over multiple devices at the same time to multiple users. For example, in a typical home a family may be using multiple media devices, such as phones, tablets, and laptops. Accessing the information through Single-Input-Single-Output devices can limited bandwidth, resulting in longer downloads, timeout errors, and unable to hold communication connection. The need for information through multimedia devices at a seamlessly fast pace requires the advancement of technology. MIMO methods can be used with other MIMO systems and can communicate with non-based MIMO systems existing today.

1.2 Motivation for the Thesis Work

Thesis work was motivated by designing Single-Input-Single-Output devices (SISO) for 2.4GHz Bluetooth-to-Bluetooth applications. The Bluetooth device was designed using PCB board. Recent devices such as the I-PAD, Wi-Fi routers (Wi-MAX), and mobile devices (LTE) are moving towards MIMO systems for a reliable and faster way of handling and transferring data between devices. To keep up with antenna design and MIMO systems, I have found an interest in expanding my knowledge and background with antenna systems and to study MIMO based system to increase channel capacity, and reliability of data of communication between devices.
1.3 Objectives of the Thesis Work

To design a theoretical MIMO antenna channel that uses microstrip type of antenna to enhance the reliability and increase channel capacity of antenna system. The system is to be carried out using accessible MIMO models while implementing them to multimedia applications. Microstrip type antennas have become more popular in the last decade due to its low cost and small package design.

1.4 Outline of the Thesis

The thesis outlines how to create microstrip antenna to help decide if the antenna meets requirements and needs of the design application. After designing for single antenna, uniform linear array (ULA) system of antennas will be implemented resulting in Beamwidth, Antenna Gain, and Radiation Patterns. The following Antenna parameters will be incorporated into SISO and MIMO channel models. The following channel models will give an insight into the benefits of incorporating MIMO channel over SISO channel. As well as benefits of the MIMO channel, the complexity of the channel increases as well. This paper discusses full implementation from antenna design to channel characteristics.
Chapter 2

Antenna Modeling and Simulations

2.1 Introduction

An Antenna is defined “as a means for radiating or receiving radio waves.” The listed antennas in Table 2.1 are the most common types of antennas used in practice today.

<table>
<thead>
<tr>
<th>Antenna Type</th>
<th>Description</th>
<th>Application Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wire</td>
<td>Straight wire dipole, loop, and helix</td>
<td>automobiles, buildings, ships, aircraft</td>
</tr>
<tr>
<td>Aperture</td>
<td>Pyramidal horn, Conical Horn, Rectangular waveguide</td>
<td>aircraft or spacecraft</td>
</tr>
<tr>
<td>Microstrip</td>
<td>Consist of metallic patch can vary in configurations</td>
<td>spacecraft, satellites, missiles, cars, mobile devices</td>
</tr>
<tr>
<td>Array antennas</td>
<td>An arrangement of radiating elements that add up to give maximum in a particular direction or directions, minimum to others</td>
<td>yagi-Uda array, aperture array, microstrip patch array, slotted-waveguide array</td>
</tr>
<tr>
<td>Reflector</td>
<td>Parabolic reflector with front feed, parabolic reflector with Cassegrain feed, Corner reflector</td>
<td>Used to communicate over great distances</td>
</tr>
<tr>
<td>Lens</td>
<td>Used to collimate incident divergent energy to prevent it from spreading in undesired directions. It transforms forms of divergent energy into plane waves</td>
<td></td>
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Each type of antenna has its advantages and disadvantages with respect to the application of use. For instance the microstrip antenna is easy to fabricate, has low cross-polarization radiation, low profile, mechanically robust, versatile resonant frequency, polarization pattern and impedance but may be limited in frequency bandwidth. Reflector antennas are larger than the other antennas with diameters up to 305 meters, but can transmit...
signals over very large distances such as outer space exploration. Wire antennas can vary in size and can be easily adjusted to specific range of frequency and bandwidth and are found in in automobiles, buildings, ships, aircraft, spacecraft, and so on. Aperture antennas are for higher frequencies with unique characteristics. Array antennas are combinations of single implemented antennas that can give maximum directivity and range to the antenna system. [2]

2.2 Antenna Characteristics

Designing MIMO system involves exploiting the antenna characteristics in relation to the application it is meant for. When designing the application system, the type of antenna used will ultimately determine the limitations of the system. The following characteristics in Table 2.2 are affected by the type of antenna chosen when designing the system.

Table 2.2: Characteristic Parameters of Antenna Design

<table>
<thead>
<tr>
<th>Antenna Characteristics</th>
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<td>Radiation Power Density</td>
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<tr>
<td>Radiation Intensity</td>
</tr>
<tr>
<td>Beam Pattern</td>
</tr>
<tr>
<td>Directivity</td>
</tr>
<tr>
<td>Bandwidth</td>
</tr>
<tr>
<td>Gain</td>
</tr>
<tr>
<td>Power handling capability</td>
</tr>
<tr>
<td>Manufacturability</td>
</tr>
<tr>
<td>Cost</td>
</tr>
</tbody>
</table>

An antenna radiation pattern is defined as “a mathematical function or a graphical representation of the radiation properties of the antenna as a function of space coordinates. The radiation pattern is determined in the far-field region and is represented
as a function of the directional coordinates. Radiation Properties include power flux density, radiation intensity, field strength, directivity, phase or polarization. An antenna radiation pattern can be described and represented by three individual patterns. Field Pattern represents a plot of the magnitude of the electric or magnetic field as a function of the angular space and is depicted in Figure 2-1 for a 10-Element Linear Array with 0.25 $\lambda$ spacing between elements.

![Figure 2-1: Electric Field Pattern for 10-Element Linear Array with 0.25 $\lambda$ spacing between elements](image)

Power Pattern typically represents a plot of the square of the magnitude of the electric or magnetic field as a function of the angular space. Power pattern (in linear scale) in Figure 2-2 represents the magnitude of the electric or magnetic field in decibels, as a function of the angular space for 10-element linear array with 0.25 $\lambda$ spacing between elements and phase of $\beta = -0.6\pi$. [2]

![Figure 2-2: Represents a Power Pattern plot of the square of the magnitude of the electric or magnetic field as a function of the angular space.](image)
The radiation pattern can also be expressed as power patterns in terms of decibel. Figure 2-3 represents the magnitude of the electric or magnetic field, in decibels, for a 10-element linear array of isotropic sources, with spacing of $d = 0.25\lambda$ between elements and phase of $\beta = -0.6\pi$. [2]

Figure 2.3: A plot of the magnitude of the electric or magnetic field as a function of the angular space in decibels.

The electric and magnetic fields of the radiation pattern can be modeled 3-D can be combined with the 2-dimensionsonal radiation patterns in each E and H plane. Figure 2-4 represents the 3-D model of the magnitude of the electric or magnetic radiation field pattern (in dB) as a function of angular space for 10-element linear array of isotropic sources, with spacing of $d = 0.25\lambda$ between elements and phase of $\beta = -0.6\pi$. 
Figure 2-4: A 3-dimensional normalized field pattern for 10-Element Linear Array with 0.25 λ spacing between elements and phase of $\beta = -0.6\pi$.

Figure 2-4 show the main beam and directivity of the 10-element linear array of isotropic sources. From this model you can see the side lobes as well as the main lobes of the radiation pattern. As will in the +z direction there is small emitted lobes in the –z direction. The side lobes can be modified and adjusted by changing the distance spacing between adjacent antennas and the phase. The unintentional radial pattern in the –z direction can eliminated by adding ground plane underneath the antenna. For Microstrip antennas, it is common for ground plane to exist. [2]

2.2.1 Field Regions

The space surrounding the antenna is divided into three regions of the radiating field. They are the reactive near-field, radiating near-field (Frensel) region, and Far-field (Fraunhofer) region. The reactive near-field region is the portion of the radiating field closest to the antenna where the reactive field predominates. The radiating near-field region is between the Reactive near-field and far-field regions where the radiation fields predominate and where the angular field distribution is dependent upon the distance from the antenna. If the antenna has a maximum dimension that is not large compared to the
wavelength, this region may not exist. The Far-field region is the furthest radiating field of an antenna where the angular field distribution is independent of the distance from the antenna.

2.2.2 Radiation Power Density

The radiation power density describes the electromagnetic waves that are used to transfer information through a wireless medium. [2] The average power density associated with the electromagnetic fields of an antenna in its far-field region is given by

\[ W_{av} = \frac{1}{2} Re[ExH^*] \]  \hspace{1cm} (2-1)

The average radiated power radiated by the antenna is given by

\[ P_{rad} = P_{av} = \iint_s W_{av} \cdot ds = \iint_s Re[ExH^*] \cdot ds \]  \hspace{1cm} (2-2)

The power density which is uniformly distributed over the surface of a sphere of radius \( r \) is given by

\[ W_0 = \frac{P_{rad}}{4\pi r^2} \left( W/m^2 \right) \]  \hspace{1cm} (2-3)

2.2.3 Radiation Intensity

Radiation intensity in a particular direction is defined as “the power radiated from an antenna per unit solid angle.” [2] The radiation intensity is a far-field parameter and is given by
\[ U = r^2 W_{rad} = B_0 F(\theta, \phi) \cong \frac{r^2}{2\eta} \times \left[ |E_\theta(r, \theta, \phi)^2| + |E_\phi(r, \theta, \phi)^2| \right] \]  

(2-4)

2.2.4 Beam Pattern

Beam Pattern is defined as the angular separation between two identical points on opposite side of the pattern maximum. The most common beamwidth used is the Half-Power Beamwidth defined by IEEE as “In a plane containing the direction of the maximum of a beam, the angel between the two directions in which the radiation intensity is one-half value of the beam. The First-Null Beamwidth (FNBW) the angular separation between the first nulls of the patterns. As the beamwidth decreases the side lobe increases and vice versa. The beamwidth is used to describe the resolution capabilities of the antenna to distinguish between two adjacent radiating sources. The resolution capability of an antenna to distinguish between two sources is equal to half the first null beamwidth (FNBW/2), which is usually used to approximate the half power beamwidth (HPBW). [2]

2.2.5 Directivity

Directivity of an antenna is defined as “the ration of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions.” [2]. For an isotropic source, the radiation pattern is equally in all directions. Therefore the maximum directivity will always be greater than unity, and it is a relative “figure of merit” which gives an indication of the directional properties of the antenna as compared with those of an isotropic source. The directivity of a non-isotropic source is equal to the
ratio of its radiation intensity in a given direction over that of an isotropic source and is given by

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}} = \frac{4\pi}{\Omega_A}$$ (2-5)

The beam solid angle of an antenna is used to compute directivity. It is defined as “the solid angle through which all the power of the antenna would flow if its radiation intensity is constant for all angles within $$\Omega_A$$,” and is given by

$$\Omega_A = \int_0^{2\pi} \int_0^\pi F_n(\theta, \phi) \sin \theta d\theta d\phi$$ (2-6)

Where $$F_n(\theta, \phi)$$ normalizes the radiation intensity.

$$F_n(\theta, \phi) = \frac{F(\theta, \phi)}{|F(\theta, \phi)|_{max}}$$ (2-7)

If the direction of the radiation intensity is not specified, then the maximum directivity is given by

$$D_{max} = D_0 = \frac{U_{max}}{U_0} = \frac{4\pi U_{max}}{P_{rad}} = D_\theta + D_\phi$$ (2-8)

For antennas with orthogonal polarization components the partial directivity’s are given for an antenna in a given direction by equations (2-9) and (2-10)

$$D_\theta = \frac{4\pi U_\theta}{P_{rad}} = \frac{4\pi U_\theta}{(P_{rad})_\theta + (P_{rad})_\phi}$$ (2-9)

$$D_\phi = \frac{4\pi U_\phi}{P_{rad}} = \frac{4\pi U_\phi}{(P_{rad})_\theta + (P_{rad})_\phi}$$ (2-10)

2.2.6 Bandwidth

Bandwidth of an antenna is defined as “the range of frequencies within which the performance of the antenna, with respect to some characteristic, and conforms to a specified standard.” [2] Associated with power bandwidth are gain, side lobe level,
beamwidth, polarization, and beam direction while input impedance and radiation
efficiency are related to impedance bandwidth.

2.2.7 Gain

Gain is closely related to directivity, but it takes into account the efficiency of the
antenna as well as its directional capabilities. Gain is defined as “the ratio of intensity, in
a given direction, to the radiation intensity that would be obtained if the power accepted
by the antenna were radiated isotopically.” The radiation intensity corresponding to the
isotopically radiated power is equal to the power accepted by the antenna divided by $4\pi$.
The Gain of the antenna in the $U(\theta, \phi)$ direction is given by

$$G = \frac{4\pi U(\theta, \phi)}{P_{in}} = e_{cd} \left[ \frac{4\pi U(\theta, \phi)}{P_{rad}} \right] = e_{cd} D(\theta, \phi)$$

(2-11)

where

$$P_{rad} = e_{cd} P_{in}$$

(2-12)

2.2.8 Efficiency

Losses due to an antenna are contributed to reflections because of the mismatch
between the transmission line in the antenna, and the $I^2R$ losses (conduction and
dielectric). The antenna radiation efficiency used in equation (2-11) and (2-12) is found
by

$$e_{cd} = \frac{R_r}{R_r + R_L}$$

(2-13)

2.3 Antenna Design for Microstrip Antenna
Microstrip Antennas are becoming more commonly used due to its smaller size, weight, cost, performance, and ease of installations. Microstrip antennas advantages are its low profile, simple and inexpensive to manufacture, and mechanically robust when mounted on rigid surfaces compared to other conformable planer and nonplanar antenna designs. Disadvantages of microstrip antennas are their low efficiency performance, spurious feed radiation and very narrow frequency bandwidth. Efficiency can be enhanced by increasing the height of the substrate. Although this can lead to degradation of the antenna pattern and polarization by reducing power from the radiating beam which can lead to standing waves within the substrate. Surface waves can be eliminated by using cavities. Stacking can lead to an increase bandwidth.

![Figure 2-5: Physical outline of Microstrip patch antenna [2]](image)

Microstrip antennas consist of a very thin metallic strip placed on a small fraction of a wavelength above the ground plane as shown in Figure 2-5. The patch is design so its pattern maximum is normal to the patch (broadside radiator). This in accomplished by properly choosing the mode (field configuration) of excitation beneath the patch. The patch and ground plane are separated by a dielectric sheet known as the substrate. Common dielectric constants of substrates are in-between $2.2 \leq \epsilon_r \leq 12$. Rectangular antennas with thicker substrates with lower dielectric constants provides better efficiency,
larger bandwidth, loosely bound field for radiation into space, but it is at the expense of larger element size. Typical Length of Antenna Patches is between $\frac{\lambda_0}{3} \leq L \leq \frac{\lambda_0}{2}$.

Typical height of substrate is between $0.003\lambda_0 \leq h \leq 0.05\lambda_0$. Microstrip dipoles are attractive because they inherently possess a large bandwidth and occupy less space, which makes them attractive for arrays. Arrays can contribute to scanning capabilities and achieve greater directivities. There are four main methods that are used to feed microstrip antennas are as follows: microstrip line, coaxial probe, aperture coupling, and proximity coupling. The analysis methods for modeling microstrip antennas consist of the transmission-line, cavity, and full wave models. The transmission-line and cavity models will be discussed in sections 2.3.1 and 2.3.2 respectively [2].

2.3.1 Transmission-Line Model

The transmission line model is the easiest of all to model but is the least accurate of the models. The transmission-line model represents the microstrip antenna by two slots, separated by a low-impedance $Z_c$ transmission line of length $L$. As a result of the fields at the edges of the path along finite lengths and widths, fringing occurs. Fringing is a function of the ratio of the length of the patch $L$ to the height $h$ of the substrate ($L/h$) and the dielectric constant $\varepsilon_r$ of the substrate. It can affect the overall resonating frequency. Fringing ultimately makes the microstrip line look wider electrically compared to its physical dimensions. Some of the waves will travel in air and the others through the substrate. The dielectric constant of the substrate will be represented by $\varepsilon_r$ and the effective dielectric constant is defined by $\varepsilon_{reff}$ will be represented by the following.
\[ \varepsilon_{eff} = \frac{\varepsilon_{r+1}}{2} + \frac{\varepsilon_{r-1}}{2} \* \left[ 1 + 12 \frac{h}{W} \right]^{-1/2} \]  

(2.14)

To reduce fringing it is common that the W/h > > 1 and \( \varepsilon_r >> 1 \). This implies that the electric field lines are concentrated in the substrate rather than air. The Effective dielectric constant, \( \varepsilon_{eff} \), increases as the dialectic constant increases, assuming width and height of the substrate are constant. At low frequencies the effective dielectric constant is essentially constant. As the frequencies increase, the effective dielectric constant approaches the values of the dielectric constant of the substrate [3]. The normalized extension length \( \Delta L \) due to fringing is given by

\[ \frac{\Delta L}{h} = 0.412 * \frac{(\varepsilon_{eff}+0.3)(\frac{W}{h}+0.264)}{(\varepsilon_{eff}-0.258)(\frac{W}{h}+0.8)} \]  

(2.15)

The Effective Path Length is given by

\[ L_{eff} = L + 2\Delta L \]  

(2.16)

In our design process we are going to develop an antenna for multiple frequency applications and comparing their overall features. The following is an example for government use only applications used by the Department of Defense (DoD). I have chosen a Band of operations between 15.7 - 17.3 GHz. The applications that are used by these given frequencies are Command Control Link, Navaids, Multi-mode Airborne Radar, Fire-control, Navigation and Mapping, Terrain Following/Avodiance, RDT &E, and PGM. The Rogers RT/duroid 5880 high frequency laminate substrate will be used in designing the microstrip antenna. This substrate is commonly used in commercial airline broadband antennas, microstrip and stripline circuits, millimeter wave applications, military radar systems, missile guidance systems, and point to point digital radio antennas. The benefits of this antenna include low electrical loss, low moisture
absorption, stable dielectric constants over frequency, and low outgassing for space applications. [3]

2.3.1.1 Design Rectangular Patch Antenna using Transmission Line Model

\[ \epsilon_r = 2.2 \] Substrate material

\[ h = 0.1588 \text{cm}(0.0625 \text{ in.}) \] Height of 4 Layers Printed Circuit Board

\[ f = 16 \text{GHz} \] Expressed in Hz

Determine Width (W) by the following formula,

\[ W = \frac{1}{2f_r/\sqrt{\epsilon_0 \epsilon_r}} \sqrt{\frac{2}{\epsilon_r+1}} = \frac{v_0}{2f_r} \sqrt{\frac{2}{\epsilon_r+1}} \quad (2.17) \]

Using Eq. 2.19 the width is given by

\[ W = \frac{3 \times 10^{10} \text{cm}}{2 \times 16 \times 10^9 \text{Hz}} \sqrt{\frac{2}{\epsilon_r+1}} = \frac{2}{2f_r} \sqrt{\frac{2}{\epsilon_r+1}} = 0.9375 \times \sqrt{\frac{2}{2.2+1}} \]

\[ = 0.9375 \times \sqrt{0.625} = 0.741158 \text{ cm} \]

Determine Effective dielectric constant using Eq. 2.16

\[ \epsilon_{ref} = \frac{2.2+1}{2} + \frac{2.2-1}{2} \times \left[ 1 + 12 \times \frac{0.1588}{0.714458} \right]^{-\frac{1}{2}} = 1.917504371 \]

The extend incremental length of the patch ∆L using Eq. 2.17

\[ \Delta L = 0.1588 \times 0.412 \times \frac{(1.9175 + 0.3)(4.6672 + 0.264)}{(1.9175 - 0.258)(4.6672 + 0.8)} = 0.078853 \text{ cm} \]

Actual Length L of the patch is calculated as

\[ L = \frac{\lambda}{2} - 2\Delta L = \frac{30}{2 \times 16 \times \sqrt{1.9175}} - 2 \times 0.078853 = 0.519315 \text{ cm} \]

Effective Length using Eq. 2.18 is calculated as

\[ L_e = L + 2\Delta L = 0.519315 + 2 \times 0.078853 = 0.677022 \text{cm} = 0.266544 \text{ inches} \]
For dominant $TM_{010}$ mode, the resonant frequency of the microstrip antenna is a function of its length and is given by assuming no fringing.

$$ (fr)_{010} = \frac{1}{2L\sqrt{\varepsilon_r} \sqrt{\mu_0 \varepsilon_0}} = \frac{v_0}{2L\sqrt{\varepsilon_r}} $$  \hspace{1cm} (2.18)

To take into account of fringing affects, the modified equation below is modified by

$$ (frc)_{010} = \frac{1}{2L_{eff} \sqrt{\varepsilon_{reff}} \sqrt{\mu_0 \varepsilon_0}} = \frac{v_0}{2L_{eff} \sqrt{\varepsilon_{reff}}} = \frac{1}{2(L+2\Delta L)\sqrt{\varepsilon_{reff}} \sqrt{\mu_0 \varepsilon_0}} $$  \hspace{1cm} (2.20)

$$ = q \frac{1}{2L\sqrt{\varepsilon_r} \sqrt{\mu_0 \varepsilon_0}} = q \frac{v_0}{2L\sqrt{\varepsilon_r}} $$  \hspace{1cm} (2.19)

Where, $\frac{fr}{fr} = \frac{(frc)_{010}}{(fr)_{010}}$, which is referred to as the fringe factor (length reduction factor). As the substrate height increases, fringing will increase. This will lead to larger separations between the radiating edges and lower resonant frequencies. Without taking into account of fringing effects the calculated resonant frequency of the rectangular antenna design is $(fr)_{010} = 19.47$ GHz. When taking fringing affects into consideration the resonant frequency is calculated to be $(frc)_{010} = 16.00$ GHz. From the calculated resonant frequency results, it is important to take fringing into consideration for tuning the antenna system properly. From above, using the transmission line model, the following fringing affects have been taken into consideration for designing the single element system to be 16 GHz. Table 2.2 depicts several high frequency laminate substrates that can be used in antenna design. The frequency and height of the substrate were held constant while the dielectric constant of the substrate changes. From table 2.3 as the dielectric constant of the substrate material increases, the overall size of rectangular patch antenna decreases.
Table 2.3: Microstrip Transmission Line Model for High Frequency Laminates

<table>
<thead>
<tr>
<th>Substrate Material</th>
<th>RT/duroid 5880 (PTFE Random Glass Fiber)</th>
<th>TM66 (Hydrocarbon Ceramic)</th>
<th>R03010 (PTFE Ceramic)</th>
<th>Silicone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Er (dielectric constant)</td>
<td>2.2</td>
<td>6.3</td>
<td>11.2</td>
<td>11.7</td>
</tr>
<tr>
<td>Frequency (GHz)</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>Height (cm)</td>
<td>0.1588</td>
<td>0.1588</td>
<td>0.1588</td>
<td>0.1588</td>
</tr>
<tr>
<td>Width (cm)</td>
<td>0.7411</td>
<td>0.4907</td>
<td>0.3796</td>
<td>0.3720</td>
</tr>
<tr>
<td>Ereff (effective dielectric constant) (cm)</td>
<td>1.9175</td>
<td>4.8490</td>
<td>8.1785</td>
<td>8.5122</td>
</tr>
<tr>
<td>L (extended length) (cm)</td>
<td>0.7885</td>
<td>0.0633</td>
<td>0.0582</td>
<td>0.0579</td>
</tr>
<tr>
<td>L (Length of Patch) (cm)</td>
<td>0.5193</td>
<td>0.2990</td>
<td>0.2113</td>
<td>0.2055</td>
</tr>
<tr>
<td>Leff (Effective Length) (cm)</td>
<td>0.6770</td>
<td>0.4257</td>
<td>0.3278</td>
<td>0.3213</td>
</tr>
<tr>
<td>resonant frequency (fr) (GHz)</td>
<td>19.47</td>
<td>19.97</td>
<td>21.21</td>
<td>21.34</td>
</tr>
<tr>
<td>resonant frequency (fr) (GHz)</td>
<td>16.00</td>
<td>16.00</td>
<td>16.00</td>
<td>16.00</td>
</tr>
</tbody>
</table>

The following equations describe how to determine mutual conductance, feed resistance, feed insert point for rectangular patch antenna. Each patch antenna comes with a radiating slot which is represented as a parallel equivalent admittance Y (with conductance G and susceptance B) and is modeled in Figure 2-6 for the two radiating slots.

![Figure 2.6: Transmission Line Model Circuit Diagram for two Radiating Slots.](image)

The equivalent conductance $G_1$ and susceptance $B_1$ for a rectangular microstrip patch can be calculated by Eq. 2.20 and Eq. 2.21.

$$G_1 = \frac{W}{120\lambda_0} \left[ 1 - \frac{1}{24} (k_0 h)^2 \right], \frac{h}{\lambda_0} < \frac{1}{10} \quad (2.20)$$

$$B_1 = \frac{W}{120\lambda_0} \left[ 1 - 0.636\ln(k_0 h) \right], \frac{h}{\lambda_0} < \frac{1}{10} \quad (2.21)$$

The following recessed microstrip rectangular path has a resonating input resistance related to the total admittance at slot #1 by transformation of equation of transmission lines of the admittance of slot #2 from the output terminals to the input terminals. The
two slots should ideally be separated by \(\lambda/2\). But due to fringing the overall separation of the two slots is slightly less than \(\lambda/2\). At \(\lambda/2\), the Gains of slot #1 and slot #2 are equal to each other and the susceptance \(B\) are opposite but equal in magnitude. Therefor the total resonant input admittance is real which means the resonant input impedance is real, and is given by

\[
Z_{in} = \frac{1}{Y_{in}} = Y_1 + \bar{Y}_2 = R_{in} = \frac{1}{2G_1}
\]  

(2.22)

This resonant input resistance does not take into account of the mutual effects between the slots. The modified formula is given by

\[
R_{in} = \frac{1}{\frac{1}{2(G_1 \pm G_{12})}}, \quad \text{for modes with odd symmetry}
\]

\[
= \frac{1}{\frac{1}{2(G_1 \pm G_{12})}}, \quad \text{for modes with even symmetry}
\]

(2.23)

Mutual Conductance between slot #1 and slot #2 is defined in terms of the far-zone field as

\[
G_{12} = \frac{1}{|V_0|} Re \int_s E_1 \times H_2^* \cdot ds
\]  

(2.24)

\(E_1\) – Electric field radiated by slot #1,

\(H_2\) – magnetic field radiate by slot #2,

\(V_0\) – Voltage across the slot and it is integrated over a sphere of large radius.

It is found that the input resistance is not strongly dependent upon the substrate height \(h\). For very small values of \(k_0h<<1\), the input resistance is not dependent on \(h\). From the above equations the resonant input resistance can be decreased by increasing the width \(W\) of the patch as long as the ratio of \(W/L < 2\). If the ratio is above 2, the aperture efficiency begins to decline.

The resonant input resistance is calculated by equation 2.27. The resonant input resistance can be changed by using an inset feed, recessed a distance \(y_0\) from slot #1.
This technique can be used effectively to match the patch antenna using a microstrip-line feed whose characteristic impedance is given by

\[
Z_c = \left\{ \frac{60}{\epsilon_{\text{reff}}} \ln \left( \frac{8h}{W_0} \right) \left[ \frac{W_0}{4h} \right] \right\} \left( \frac{\epsilon_{\text{reff}}}{\frac{W_0}{h} + 1.393 + 0.667 \ln \left( \frac{W_0}{h} + 1.444 \right)} \right) \left( \frac{\epsilon_{\text{reff}}}{\frac{W_0}{h} + 1} \right)
\]

\(W_0\) is the width of the microstrip line. Therefore the resistance for given inset feed point distance is given by the following for no coupling and coupling

\[
\begin{align*}
\text{No coupling} & \quad R_{\text{in}}(y = y_0) = R_{\text{in}}(y = y_0) \cos^2 \left( \frac{\pi}{L} y_0 \right) = \frac{1}{2G_1} \cos^2 \left( \frac{\pi}{L} y_0 \right) \\
\text{Coupling} & \quad R_{\text{in}}(y = y_0) = R_{\text{in}}(y = y_0) \cos^2 \left( \frac{\pi}{L} y_0 \right) = \frac{1}{2(G_1 + G_{12})} \cos^2 \left( \frac{\pi}{L} y_0 \right)
\end{align*}
\]

For the following similar materials used in Table 2.3 and example, the input impedance, the inset feed point distance \(y_0\) will be calculated using similar substrate materials at 16 GHz frequency.

| Table 2.4: Microstrip Transmission Line Model for High Laminate Frequencies |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| Substrate Material          | RT/duroid 5880 (PTFE Random Glass Fiber) | TM66 (Hydrocarbon Ceramic) | R03010 (PTFE Ceramic) | Silicon |
| Er (dielectric constant)    | 2.2                         | 6.3                        | 11.2                       | 11.7    |
| Frequency (GHz)             | 16                          | 16                        | 16                         | 16      |
| Height (cm)                 | 0.1588                      | 0.1588                     | 0.1588                     | 0.1588  |
| Width (cm)                  | 0.7411                      | 0.4907                     | 0.3796                     | 0.3720  |
| Ereff (effective dielectric constant) (cm) | 1.9175                      | 4.8490                     | 8.1785                     | 8.5122  |
| \(\Delta L\) (extended length) (cm) | 0.7885                      | 0.0633                     | 0.0582                     | 0.0579  |
| L (Length of Patch) (cm)    | 0.5193                      | 0.2990                     | 0.2113                     | 0.2055  |
| Leff (Effective Length) (cm) | 0.6770                      | 0.4257                     | 0.3278                     | 0.3213  |
| resonant frequency (fr)010 (GHz) | 19.47                      | 19.97                      | 21.21                      | 21.34   |
| resonant frequency (frf010) (GHz) | 16.00                      | 16.00                      | 16.00                      | 16.00   |
| \(\lambda_0\) (wavelength)(cm) | 3                          | 3.0000                     | 3.0000                     | 3.0000  |
| G1 (Conductance of Slot #1) | 0.003278872                 | 0.0022                     | 0.0165                     | 0.0016  |
| G12(Mutual conductance)     | 0.00074278                  | 0.0059                     | 0.0004                     |         |
| Rin (Input Resistance to Inset Field) | 288                        | 657.0000                   | 1098.0000                  | 1143.0000 |
| Desired impedance          | 50                          | 50.0000                    | 50.0000                    | 50.0000  |
| \(W_0\) (microstrip line characteristic impedance)(cm) | 0.475                      | 0.2030                     | 0.1100                     | 0.1050  |
| \(y_0\) (inset feed point distance) | 0.167821111                 | 0.095450936                | 0.06732684                 | 0.065459387 |
The inset feed point distance decreases as dielectric constant increases. The input resistance to inset field increases as dielectric constant increases. The width of the microstrip line characteristic impedance decrease as the dielectric constant increases.

2.3.2 Cavity Model

The Cavity Model represents microstrip antennas as a dielectric-loaded cavity. The normalized fields within the dielectric substrate between the patch and the ground plane can be found more accurately by treating that region as a cavity bounded by electric conductors and by magnetic walls along the perimeter of the patch. The cavities will be represented by field Configuration modes, $TM^x$. To determine the dominant mode with the lowest frequencies, placing the resonant frequencies in ascending order determines the order of the modes of operation. For all microstrip antennas, $h<<L, H<<W$.

Mode $L > W > h$

$$ (fr)_0^{10} = \frac{1}{2L\sqrt{\epsilon_r}\sqrt{\mu_0\epsilon_0}} \quad (2.27a) $$

$$ (fr)_0^{10} = \frac{1}{2L_{eff}\sqrt{\epsilon_{reff}}\sqrt{\mu_0\epsilon_0}} \quad (2.27b) $$

Mode $L > W > L/2 > h$

$$ (fr)_0^{01} = \frac{1}{2W\sqrt{\epsilon_r}\sqrt{\mu_0\epsilon_0}} \quad (2.28) $$

Mode $W > L > h$

$$ (fr)_0^{02} = \frac{1}{L\sqrt{\epsilon_r}\sqrt{\mu_0\epsilon_0}} \quad (2.29) $$

For each radiating slot in the far-zone electric fields, the current densities can be written in terms of

$$ E^t_{\phi} = E_{\phi}(single \ slot) \times AF \quad (2.30) $$
Array factor (AF) describes the two elements of the same magnitude and phase, separated by a distance $L_e$ (effective length) along the $y$ direction.

\[
(AF)_y = 2\cos \left( \frac{k_0 L_e}{2} \sin \theta \sin \phi \right) \tag{2.31}
\]

For microstrip antenna, the $x$-$y$ plane ($\theta = 90^\circ$, $0^\circ \leq \phi \leq 90^\circ$ and $270^\circ \leq \phi \leq 360^\circ$) is the principle E-Plane. For example using RT/duroid 5880 (PTFE Random glass Fiber with dielectric constant of 2.2, the following in Figure 2-6 represent the E-Plane and H-Plane of the designed microstrip antenna with $E_r = 2.2$, $h = 0.1588$ cm, $L = 0.906$ cm and $L_e = 1.068$ cm.

![E- and H-plane Patterns of Rectangular Microstrip Antenna](image)

**Figure 2-6:** E-Plane and H-Plane of 16 GHz microstrip antenna

The fields generated by the non-radiating slots of each effective Length $L_e$ and height $h$ are found using similar procedure of the radiating slots except they are facing the $+z$ axis. The two non-radiating slots form an array of two elements, of the same magnitude but of opposite phase, separate along $z$ axis by a distance $W$. The following fields in the $H$-plane are zero because the fields radiate by each quarter cycle of each slot are cancelled by the fields radiated by the other quarter. It is also the same for the E-Plane, in that the
fields radiated by each slot are cancelled by the fields radiated by the other. This is why they are referred to as non-radiating slots.

Directivity as stated before is given by

\[ D_{\text{max}} = D_0 = \frac{U_{\text{max}}}{U_0} = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = D_\theta + D_\phi \]  

(2.32)

The following is to derive the directivity for single slot and the condition where \((k_0 h) \ll 1\).

1. \(U_{\text{max}}\) (Maximum radiation) and \(P_{\text{rad}}\) (radiated power) can be written as

\[ U_{\text{max}} = \frac{|V_0|}{2\eta_0 \pi^2} \left(\frac{\pi W}{\lambda_0}\right)^2 \]  

(2.33)

\[ P_{\text{rad}} = \frac{|V_0|^2}{2\eta_0 \pi^2} \int_0^\pi \left[ \sin \left(\frac{k_0 W \cos \theta}{2} \right) \right]^2 \sin^3 \theta \, d\theta \]  

(2.34)

The directivity of a single slot can be expressed as

\[ D_0 = \left(\frac{2\pi W}{\lambda_0}\right)^2 \frac{1}{I_1} \]  

(2.35)

Where

\[ I_1 = \int_0^\pi \left[ \sin \left(\frac{k_0 W \cos \theta}{2} \right) \right]^2 \sin^3 \theta \, d\theta = \left[ -2 + \cos(X) + XS_i(X) + \frac{\sin(X)}{X} \right] ; X = k_0 W \]  

(2.36)

For two slots and condition that \((k_0 h) \ll 1\), the directivity can be written as

\[ D_2 = \left(\frac{2\pi W}{\lambda_0}\right)^2 \frac{1}{I_2} = \frac{2}{15\text{Grad}} \left(\frac{W}{\lambda_0}\right)^2 \]  

(2.37)

Where Grad is the radiation conductance and is given by

\[ I_2 = \int_0^\pi \int_0^\pi \left[ \sin \left(\frac{k_0 W \cos \theta}{2} \right) \right]^2 \sin^3 \theta \cos^2 \left(\frac{k_0 L_e}{2} \sin \theta \sin \phi\right) \, d\theta \, d\phi \]  

(2.38)

The broadside directivity \(D_2\) for the two radiating slots separated by dominant \(TM_{010}\) mode field (antisymmetric voltage distribution) can be written as
\[ D_2 = D_0 D_{AF} = D_0 \frac{2}{1 + g_{12}} \quad (2.39) \]

\[ D_{AF} = \frac{2}{1 + g_{12}}; g_{12} \approx 2 \text{ for } g_{12} \ll 1 \quad (2.40) \]

Using the electric field of the radiating slot for very small heights \((k_0 h) \ll 1\), the maximum radiation intensity and radiated power can be simplified for single slot if the following conditions are met for directivity for 1 slot

\[ D_0 = \begin{cases} 
3.3 \text{(dimensionless)} = 5.2 \text{dB}; & W \ll \lambda_0 \\
4 \frac{W}{\lambda_0}; & W \gg \lambda_0 
\end{cases} \quad (2.41) \]

and directivity for 2 slots

\[ D_0 = \begin{cases} 
6.6 \text{(dimensionless)} = 8.2 \text{dB}; & W \ll \lambda_0 \\
8 \frac{W}{\lambda_0}; & W \gg \lambda_0 
\end{cases} \quad (2.42) \]

The directivity can also be approximated by Kraus’s and Tai & Periera’s in terms of the E-plane and H-plane beamwidths given by

\[ \Theta_E = 2 \cos^{-1} \left( \frac{7.03 \lambda_0^2}{4(3L_e + h^2) \pi^2} \right) \quad (2-43) \]

\[ \Theta_H = 2 \cos^{-1} \left( \frac{1}{2 + k_0 + W} \right) \quad (2-44) \]

Using the same high frequency laminates and same height of the substrate, using the Cavity Model the following directivities have been calculated.
### Table 2.5: Directivity of Antenna using Cavity Model

<table>
<thead>
<tr>
<th>Substrate Material</th>
<th>RT/duroid 5880 (PTFE Random Glass Fiber)</th>
<th>TM66 (Hydrocarbon Ceramic)</th>
<th>R03010 (PTFE Ceramic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Er (dielectric constant)</td>
<td>2.2</td>
<td>6.3</td>
<td>11.2</td>
</tr>
<tr>
<td>Frequency (GHz)</td>
<td>16</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>Height (cm)</td>
<td>0.1588</td>
<td>0.1588</td>
<td>0.1588</td>
</tr>
<tr>
<td>Width (cm)</td>
<td>0.7411</td>
<td>0.4907</td>
<td>0.3796</td>
</tr>
<tr>
<td>Ereff (effective dielectric constant) (cm)</td>
<td>1.9175</td>
<td>4.8490</td>
<td>8.1785</td>
</tr>
<tr>
<td>ΔL (extended length) (cm)</td>
<td>0.7885</td>
<td>0.0633</td>
<td>0.0582</td>
</tr>
<tr>
<td>L (Length of Patch) (cm)</td>
<td>0.5193</td>
<td>0.2990</td>
<td>0.2113</td>
</tr>
<tr>
<td>Leff (Effective Length) (cm)</td>
<td>0.6770</td>
<td>0.4257</td>
<td>0.3278</td>
</tr>
<tr>
<td>resonant frequency (fr)010 (GHz)</td>
<td>19.47</td>
<td>19.97</td>
<td>21.21</td>
</tr>
<tr>
<td>resonant frequency (frc)010 (GHz)</td>
<td>16.00</td>
<td>16.00</td>
<td>16.00</td>
</tr>
<tr>
<td>λ0 (wavelength)(cm)</td>
<td>3</td>
<td>3.0000</td>
<td>3.0000</td>
</tr>
<tr>
<td>G1 (Conductance of Slot #1)</td>
<td>0.00157243</td>
<td>0.0022</td>
<td>0.0165</td>
</tr>
<tr>
<td>G12 (Mutual conductance)</td>
<td>0.0074278</td>
<td>0.0059</td>
<td>0.0004</td>
</tr>
<tr>
<td>Rin (Input Resistance to Inset Field) - odd symmetry</td>
<td>215.963</td>
<td>380.0300</td>
<td>593.0149</td>
</tr>
<tr>
<td>Rin (Input Resistance to Inset Field) - even symmetry</td>
<td>602.6601</td>
<td>3561.0000</td>
<td>11468.0000</td>
</tr>
<tr>
<td>Desired impedance</td>
<td>50</td>
<td>50.0000</td>
<td>50.0000</td>
</tr>
<tr>
<td>W0 (microstrip line characteristic impedance)(cm)</td>
<td>0.475</td>
<td>0.2030</td>
<td>0.1100</td>
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<tr>
<td>y0 (inset feed point distance) - odd symmetry</td>
<td>0.1767</td>
<td>0.1142</td>
<td>0.0858</td>
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<tr>
<td>y0 (inset feed point distance) - even symmetry</td>
<td>0.2114</td>
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<td>E -Plane HPBW</td>
<td>5.3837</td>
<td>3.7537</td>
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<tr>
<td>H-Plane HPBW</td>
<td>7.3108</td>
<td>5.7446</td>
<td>5.3054</td>
</tr>
</tbody>
</table>

#### 2.3.3 Summary of Rectangular Patch Antenna Design

The overall height of the substrate was held constant throughout this process and was designed with 50 Ohm matching impedance microstrip line. As the dielectric constant of the substrate material increased, the overall width and length of the patch decreased, the overall input resistance to the inset feed point increased, inset feed point distance decreased, and the directivity of the rectangular patch decreased, the HPBW increased in the E and H planes.

Typically there are radiation (space wave) losses, conduction (ohmic) losses, dielectric losses, and surface wave losses listed respectively in order. All of these factors contribute to an overall quality factor represented as $Q_t$. 

25
\[ \frac{1}{Q_t} = \frac{1}{Q_{rad}} + \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_{sw}} \]  

(2.45)

For very thin substrates, the losses due to surface waves are very small and can be neglected. For thicker substrates they must be taken into account. By using cavities, the losses for using thicker substrates can be eliminated. Where \( h \ll \lambda_0 \), the following quality factors can be expressed by

\[ Q_c = h \sqrt{\pi f \mu \sigma} \]  

(2.46)

\[ Q_{cd} = \frac{1}{\tan \delta} \]  

(2.47)

\[ Q_{rad} = \frac{2 \omega \varepsilon_r}{h G_t / l} K \]  

(2.48)

where, \( \tan \delta \) is the loss tangent of the substrate material, \( \sigma \) is the conductivity of the conductors associated with the patch and ground plane, \( G_t / l \) is the total conductance per unit length of the radiating aperture = \( G_{rad} / W \). 

K = \( L / 4 \) when operating in the dominant \( TM_{010} \).

The \( Q_{rad} \) is inversely proportional to the height of the substrate, and for very thin substrates is usually the dominant factor [5]. The fractional bandwidth of the antenna is inversely proportional to the quality factor ( \( Q_t \) ) and is given by.

\[ \frac{\Delta f}{f_0} = \frac{1}{Q_t} \]  

(2.49)

The following does not take into account the impedance matching at the input terminals of the antenna. A more meaningful definition of the fractional bandwidth is over a band of frequencies where the VSWR at the input terminals is equal to or less than a desired maximum value. This requires the VSWR is unity at the design frequency. A modified form takes into account the impedance matching [4].

\[ \frac{\Delta f}{f_0} = \sqrt{\frac{\text{VSWR} - 1}{Q_t \sqrt{\text{VSWR}}}} \]  

(2.50)
The Bandwidth is approximately $= \frac{1}{\sqrt{\varepsilon_r}}$. This means that the bandwidth is inversely proportional to the square root of the dielectric constant of the substrate. The bandwidth increases as the height of the substrate increases. The radiation efficiency of an antenna is defined as the power radiated over the input power. It is expressed in terms as the following

$$e_{cdsw} = \frac{1/Q_{rad}}{1/Q_t} = \frac{Q_t}{Q_r}$$

(2.51)

Antenna arrays can be made up of single microstrip antenna elements. They can synthesize a required pattern that cannot be achieved with a single element. They can be used to scan the beam of an antenna system, increase the directivity, and perform various other functions which would be difficult with any one single element. The next section will discuss smart antenna design which will lead into MIMO Antenna Design.
Chapter 3

Smart Antenna Design

3.1 Introduction to Smart Antennas

Smart Antenna Benefits sole purpose is to increase channel capacity. Wireless technology has been increasing due to the improved infrastructure, services, and lower costs. Spatial Processing is the idea of “adaptive or smart antenna systems.” Adaptive antennas date back to World War II with the conventional Bartlett Beam-former. Today they are more common due to advancements of technology in low-cost Digital Signal Processors (DSP), application Specific Integrated Circuits (ASICs), software-based signal processing algorithms. Smart antennas concentrate on the principle of adaptive direction-of-arrival (DOA) and signal of interest (SOI). The need for advancements in adaptive antennas is due to the need for larger coverage area, higher transmission quality. As the demand for wireless services increased to maintain the capacity the Federal Communications Commission (FCC) created a hexagon Cell structure (Figure 3-1) to accommodate the increasing number of users in cellular radio design.
Each hexagon represents a small geographical area named cell with maximum radius R. In the center of each cell, a base station exists with omnidirectional antennas with a given band of frequencies. Adjacent hexagon cells have different band of frequencies which reduces multiple bands. This allows for multiple non adjacent cells to have the same frequencies without interference with one another. Cells with the same shaded pattern use the same frequency spectrum. In the first cellular radio system, the base station used an omnidirectional antenna. Only a small percentage of the total energy reached the desired user, the remaining energy would be radiated in an undesired direction. As the number of users increased, the interference increased, resulting in reduced capacity.

The first solution to the Cellular Radio interference was to take each cell and subdivide it into microcells of same shape. This idea is known as “Cell Splitting” Each microcell consisted of its own base station and corresponding reduction in antenna height and transmitter power. Each Cell split improves Capacity by decreasing the cell radius R and keeping the D/R ratio unchanged. D is the distance between Cells. The disadvantage of cell splitting are costs incurred from the installation of new base stations, and the increase in number of handoffs (the process of transferring communication from one
base station to another when the mobile unit travels from one cell to another), and a higher processing load per subscriber. Frequency Reuse is “The design process of selecting and allocating the same bands of frequencies to different cells of cellular base stations within a system.”

The demand for wireless services grew higher, and the number of frequencies assigned to each cell became insufficient to support the required number of subscribers. A cellular design technique, known as “Cell Sectoring,” replaces the base station’s single omnidirectional antenna with several antennas. Each cell was sectorized into three sectors of 120° as shown in Figure 3-2.

Figure 3-2: Sectorized base-station antenna

Each sector, improved capacity while keeping the cell radius unchanged and reducing the D/R ration [2]. Improvement in capacity is achieved by reducing the number of cells in a cluster and increasing the frequency reuse. In order for Sectoring to work properly, it is necessary to reduce the relative interference without decreasing the transmitting power. The co-channel interference was reduced in cellular systems since only two neighboring cells interfere instead of six (from 120° sectoring). With the improvement of signal to noise interference (S/I) ratio and capacity, the number of antennas at a base station had to
increase, and a decrease in trunking efficiency. Trunking efficiency is “a measure of number of users that can be offered service with at particular configuration of fixed number of frequencies.

As the demand for wireless services continued to grow, sectoring did not provide solution for capacity problem, and smart antennas system were designed. Smart antenna systems can be referred to as a “as system that can dynamically sectorized a cell.” Smart antenna system is composed of multiple beams. Smart antennas focus their radiation pattern or beams toward the desired user while rejecting unwanted interferences. This allows for greater coverage area for each base station. The higher rejection interference results in lower bit error rate (BER) and therefore can improve capacity. These systems are classified as Switched Beam or Adaptive Array.

Switched Beam antenna systems form multiple beams with heightened sensitivity in a particular direction. The antenna system detects the signal strength of the mobile unit as it moves throughout a cell. The system will than choose the appropriate redefined beam pattern, and continually switches the beams as necessary to provide the maximum gain according to the location of the user. The downside to predefined switched beams is that the user may not be in the center of the main beam. If an interferer is at the center of the main beam, its signal may be enhanced more than the desired user causing unwanted interference.

Adaptive array systems represent the most advanced smart antenna system approaches. It provides more degrees of freedom by adapting in real time to radiation pattern to the RF signal environment. They can provide maximum gain by directing the main beam at the desired user while suppressing the antenna pattern in the direction of
the interferers (SNOI). In low level interference the, both types of smart antennas provide significant gains over the conventional sectored systems. For high level interference, the capability for adaptive array systems to rejected unwanted SNOI can prove more coverage than conventional or switched-beam.

In advancements of smart antenna systems, Spatial Division Multiple Access (SDMA), its spatial-processing enables it to locate many users by creating different beam for each user. This allows for more than one user to be allocated to the same physical communication channel in the same cell simultaneously, with only an angle of separation. This requires the base station to have N parallel beamformers, operating independently. Each beamforming algorithm determines its own DOA and time delay of each user system. This increases frequency reuse, improves interference suppression decreases infrastructure cost, and improves capacity.

Capacity in smart antenna systems is not only based on co-channel interference. It is also affected by the environment. For an isotropic radiated signal, the environment causes multipath fading. At the receiver, direct signals or multipath signals (delayed signals due to reflections in the environment) are communicated with. The ability for the receiver to exploit or reject the reflected signals can increase channel capacity. Smart antennas have the ability to extract information form the direct path of SOI as well as extracting information from the reflected version of the SOI while rejecting all interferers or SNOI. The ability of the antenna to manage multiple signal paths improve link quality. The signals differ in amplitude and phase. The delay phases of the multiple signal paths can combine destructively over a narrow bandwidth, leading to fading of the received signal level. This results in reduced signal strength. Examples of this
destructive fading are the Rayleigh, Ray-Based, Rician Fading. During phase cancellation, a call cannot be maintained for a long period of time, and it is dropped. This phenomenon of destructive interference from the effect of multipath signals is known as delay spread. When inter-symbol interference (ISI) occurs, the bit error rate (BER) rises, resulting in degradation of quality of signal. Co-channel interference is a signal propagation issue when a user signal interferes with a cell having the same set of frequencies [2].

Smart antenna systems can increase channel capacity, increase the useful received signal level, lower interference level, increases signal to noise ratio, increases range by optimizing more directional than omnidirectional and sectorized antennas, increases security by making it more difficult to intercept information, and spatial detection can be used to locate humans in emergencies or for any other location-specific service. The drawbacks of smart antennas system includes as follows: transceivers are more complex than traditional base station transceivers, antennas need separate transceiver chains for each array antenna element, real time calibration is needed, and beamforming is computationally intensive and requires powerful processors.

Smart Antenna systems can use radiators such as dipoles, monopoles, loops, apertures, horns, reflectors, and micro strips. Since technology is desirable to be cheap, lightweight, small package size, microstrip antennas will be concentrated mainly on design in the particular military frequencies of interest.

Array Design along with smart antenna system increases complexity of system and has many benefits. The two common types of array geometries are the rectangular and circular patches. Each array possesses the ability to provide the necessary bandwidth,
scanning capabilities, beamwidth, and sidelobe level. A Linear array is two dimensional and a planar array has the added ability to scan in 3-D space. The Planar array can scan the main beam in any direction of θ (elevation) and φ (azimuth). A linear array is important to analyze and demonstrate the basic principles of array theory.

There are 5 controls that can be used to shape the overall pattern of the antenna.

1. The geometrical configuration of the overall array (linear, circular, rectangular, spherical),
2. The relative displacement between the elements,
3. The excitation amplitude of the individual elements,
4. The excitation phase of the individual elements,
5. The relative pattern of the individual elements.

### 3.2 N-Element Array

An array with identical elements all of identical magnitude and each with a progressive phase is referred to as a uniform array. The array factor can be obtained by considering the elements to be point sources. If the elements are not isotropic sources, the total field can be formed by multiplying the array factor of the isotropic sources by the field of a single element. For arrays of identical elements, the array factor is given by

\[
AF = \sum_{n=1}^{N} e^{j(n-1)\psi}; \quad \text{where } \psi = kdcos\theta + \beta
\]  

### 3.3 Linear Arrays

Figure 3-3 represents the geometry of a two-element array positioned along the z-axis. Antenna systems may use multiple uniform linear array (ULA), especially in mobile communication. Figure 3-2 would be made up of three ULA with each sector of 120° and is used in models such as Yagi-Uda array model.
This uniform linear array will be used to help represent the Broadside, Ordinary Endfire, Hanson-Woodyard, and Scanning Arrays [2].

### 3.3.1 Uniform Broadside Array

Broadside Uniform Arrays is desirable to have the maximum radiation of an array directed normal to the axis of the array or $\theta_0 = 90^\circ$. The first maximum of the array factor occurs when

$$\psi = k d \cos \theta + \beta$$

(3-2)

In order to have the first maximum directed toward $\theta_0 = 90^\circ$, broadside, the following conditions in Eq. 3-3 must be met.
\[ \psi = k d \cos \theta + \beta = 0, \quad \text{when } \theta_0 = 90^\circ \text{ and } \beta = 0. \] (3-3)

The Broadside array requires that the elements have the same phase excitation in addition to the same amplitude excitation [2].

### 3.3.2 Uniform End-fire Array

If the goal of the radiation is desirable to direct it along the axis of the array, in other words towards \( \theta_0 = 0^\circ \) and \( \theta_0 = 180^\circ \). The End fire radiation is accomplished when \( \beta = -kd, \text{ for } \theta_0 = 0 \) or \( \beta = kd, \text{ for } \theta_0 = 180 \). If the element spacing is \( \lambda/2 \), maxima exist in both directions at \( \theta_0 = 0 \) and \( \theta_0 = 180 \). If \( d = n\lambda \), with \( n \) being an integer \( n=1, 2, 3, \ldots \), the end fire will also have maxima in the broadside directions. To have only one end-fire maximum and to avoid any gating lobes, the maximum spacing between the elements should be less than \( d < \lambda/2 \). By controlling the progressive phase difference between the elements, the maximum radiation can be pointed in any desired direction to from a scanning array. This is the basic principle of electronic scanning phased array operation. This process of monitoring and controlling the phase shift can be done electronically by the use of a ferrite or diode phase shifters. The End-fire array shows similar directivities in any direction when using scanning arrays by setting the phase \( \beta = -kd \cos \theta_0 \) [2].

### 3.3.3 Uniform Hansen-Woodyard End-Fire Array

The Hansen-Woodyard End-Fire Array allows the enhancement of the directivity of an end-fire array without destroying the other characteristics of the radiation pattern. This is done by setting the phase to
\[
\beta = - \left( kd + \frac{2.94}{N} \right) \approx - \left( kd + \frac{\pi}{N} \right) \rightarrow \text{for maximum at } \theta_0 = 0^\circ \quad (3-4)
\]

\[
\beta = + \left( kd + \frac{2.94}{N} \right) \approx + \left( kd + \frac{\pi}{N} \right) \rightarrow \text{for maximum at } \theta_0 = 180^\circ \quad (3-5)
\]

These conditions lead to larger directivity than the conditions given by the Ordinary End Fire Array. These conditions do not necessarily yield the maximum possible directivity at \( \theta_0 = 0 \) or \( \theta_0 = 180^\circ \). These other conditions must apply to yield the maximum directivity. For maximum directivity along \( \theta_0 = 0 \)

\[
|\psi| = |kd\cos\theta + \beta|_{\theta=0^\circ} = \frac{\pi}{N} \quad \text{and} \quad |\psi| = |kd\cos\theta + \beta|_{\theta=180^\circ} = \pi
\]

For maximum directivity along \( \theta_0 = 180^\circ \),

\[
|\psi| = |kd\cos\theta + \beta|_{\theta=180^\circ} = \frac{\pi}{N} \quad \text{and} \quad |\psi| = |kd\cos\theta + \beta|_{\theta=0^\circ} = \pi
\]

This will require that the spacing distance between elements be

\[
d = \left( \frac{N-1}{N} \right) \frac{\lambda}{4} \approx \frac{\lambda}{4}, \quad \text{for very large number of elements} \quad (3-8)
\]

The Hansen-Woodyard condition can improve directivity provided that the spacing between the elements is approximately \( \lambda/4 \). It has been found that the Hansen-Woodyard end-fire array is approximately 1.805 times (or 2.54 dB) greater than the directivity of an ordinary end-fire array. The trade off in increasing directivity of the pattern was at the increase of the side lobe level.

The following Tables below describes the nulls, maxima, half-power points, minor lobe maxima, FNBW, HPBW, FSLBW for the describe Uniform and Non-uniform arrays.

The following Table describes the Directivities for the Broadside, End-fire, and Hansen-Woodyard Arrays [2].
3.3.4 Non Uniform Binomial Arrays

The array factor for N-elements along Z-Axis separated by a distance d is given by

\[
AF = \sum_{n=1}^{N} a_n e^{j(n-1)\psi}; \quad \text{where } \psi = k\text{d} \cos \theta + \beta
\]  

(3-9)

where \(a_n\) is the amplitude excitation of coefficients and \(\gamma\) is the angle between the axis of the array and the radial vector from the origin to the observation point. \(\gamma\) can be obtained from the dot product of a unit vector along the axis of the array with a unit vector directed toward the observation point. To simplify things the Dot Products per X, Y, Z axis is shown below. The Z axis is not affected by variations in \(\phi\) (x-y plane).

\[
\begin{align*}
\gamma &= \cos \theta; \quad \text{Z axis} \\
\gamma &= \cos^{-1}(\sin \theta \cos \phi); \quad \text{X axis} \\
\gamma &= \cos^{-1}(\sin \theta \cos \phi); \quad \text{Y axis}
\end{align*}
\]  

(3-10)

To illustrate the above, let's design a two-half-wavelength dipole (\(l=\lambda/2\)) and are positioned along the x-axis and are separated by a distance d. The lengths of the dipoles are parallel to the z-axis. The total field of the array is solved by assuming uniform amplitude excitation and a progressive phase difference of \(\beta\). The total field of the array is given by

\[
E_{\theta total} = E_{\theta} \cdot (AF)_n
\]  

(3-11)

where, \(E_{\theta} = j\eta \frac{I_0 e^{-jkr}}{2\pi r} \left[ \frac{\cos(\frac{\pi}{2} \cos \theta)}{\sin \theta} \right] \)  

(3-12)

and \( (AF)_n = \frac{\sin(kd \sin \theta \cos \phi + B)}{2 \sin \left[ \frac{B}{2} \right]} \)  

(3-13)
Figure 3-4 shows Binomial array directivities for 10-element array with antennas spacing $d = 0.25\lambda$. The binomial arrays do not exhibit any minor lobes provided the spacing between the elements is equal to or less than one-half of a wavelength. Broadside, End-Fire, Hansen-Woodyard End-Fire array, and scanning arrays would have all exhibited side lobes [2].

3.3.5 Non-uniform Dolph-Tschebyscheff

Broadside arrays with uniform spacing but non-uniform amplitude distribution will be considered. The methods to solve this will be using binomial, Dolph-Tschebyscheff or Chebyshev. Array Factor for even and odd number of isotropic elements where they are positioned symmetrically along the $z$-axis separated by a distance $d$, is given by

$$
(AF)_{2M}^{(even)} = \sum_{n=1}^{M} a_n \cos[(2n - 1) u]
$$

$$
(AF)_{2M+1}^{(odd)} = \sum_{n=1}^{M+1} a_n \cos[2(n - 1) u]
$$

$$
M = \text{elements place on each side of the origin}
$$

$$
u = \frac{\pi d}{\lambda} \cos \theta
$$

(3-14)

The excitation coefficients $a_n$ are found by using the Binomial expansion method.
\[(1 + x)^{m-1} = 1 + (m - 1)x + \frac{(m-1)(m-2)}{2!}x^2 + \frac{(m-1)(m-2)(m-3)}{3!}x^3 \]  

(3-15)

Closed form expressions have not been developed for directivity and half-power beamwidth for binomial arrays. Since the spacing is \(\lambda/2\) with no side lobes, the approximate closed form expression for binomial arrays are given by

\[
HPBW\left(d = \frac{\lambda}{2}\right) \cong \frac{1.06}{\sqrt{n-1}} = \frac{1.06}{\sqrt{2L/\lambda}} = \frac{0.75}{\sqrt{L/\lambda}}
\]  

(3-16)

\[
D_0 = \frac{2}{\int_0^{\pi/2} \left[ \frac{\cos \left( \frac{\pi}{2} \cos \theta \right) }{\sin \theta} \right] (N-1) \sin \theta \, d\theta}
\]  

(3-17)

\[
D_0 = \frac{(2N-2)(2N-4)\ldots2}{(2N-3)(2N-5)\ldots1}
\]  

(3-18)

\[
D_0 \cong 1.77\sqrt{N} = 1.77\sqrt{1 + 2L/\lambda}
\]  

(3-19)

The uniform amplitude array yields the smallest half-power beamwidth, than Doph-tschebyscheff, and then binomial arrays. In contrast, the binomial arrays usually possess the smallest side lobes followed in the order of Doph-tschebyscheff, and then uniform arrays. The Binomial arrays with element spacing equal or less than \(\lambda/2\) have no side lobes. The designer must compromise between the side lobe level and beamwidth. The Dolph-Tschebyscheff Array is a compromise between uniform and binomial arrays. Its excitation coefficients are related to the Tschebyscheff polynomials. The Dolph-Tschebyscheff array with no side lobes reduces to the binomial design. The Tschebyscheff polynomials is given by

\[
T_m(z) = 2zT_{m-1}(z) - T_{m-2}(z)
\]

Conditions \(-1 \leq z \leq 1\)

\[
T_m(z) = \cos[m\cos^{-1}(z)]; -1 \leq z \leq 1
\]

\[
T_m(z) = \cosh[m\cosh^{-1}(z)]; z < 1, z > 1
\]  

(3-20)
When designing the Dolph-Tschebyscheff array, first specify the side lobe level (in dB) and the number of elements N. Then transform the side lobe level from decibel to a voltage ratio using

\[ R_0(\text{Voltage Ratio}) = [R_0(\text{VR})] = 10^{R_0(\text{dB})/20} \]  

(3-21)

Than calculate the order of the Tschebyscheff polynomials. The order of the polynomial should be one less than the total number of elements of the array.

\[ P = \text{number of elements} - 1 \]  

(3-22)

Determine \( Z_0 = Z \) such that \( T_m(z_0) = R_0(\text{voltage ratio}) \) or by

\[ z_0 = \cosh \left[ \frac{1}{P} \cosh^{-1}[R_0(\text{VR})] \right] \]  

(3-23)

Calculate the excitation coefficients using

\[
\begin{align*}
    a_n &= \begin{cases} 
    \sum_{q=n}^{M} (-1)^{M-q} (z_0)^{2q-1} \frac{(q+M-2)(2M-1)}{(q-n)!(q+n-1)!(M-q)!} & \text{for even } 2M \text{ elements} \\
    \sum_{q=n}^{M+1} (-1)^{M-q+1} (z_0)^{2(q-1)} \frac{(q+M-2)(2M)}{\epsilon_n(q-n)!(q+n-2)!(M-q+1)!} & \text{for odd } 2M + 1
    \end{cases} \\
    \quad n = 1, 2, \ldots, M
\end{align*}
\]  

(3-24)

Determine the beam broadening factor using

\[ f = 1 + 0.636 \left( \frac{2}{R_0} \cosh \left[ \sqrt{(\cosh^{-1}[R_0])^2 - \pi^2} \right] \right)^2 \]  

(3-25)

Than find the half-power beamwidth of the Tschebysceff array by multiplying the half-power beamwidth of the uniform array by the beam broadening factor.

\[ HPBW = f(\text{broadning factor}) \times HPBW \text{ of Uniform Array} \]  

(3-26)

The maximum spacing between the elements should not exceed

\[ d_{\text{max}} \leq \frac{\lambda}{\pi} \cos^{-1} \left( \frac{-1}{z_0} \right) \]  

(3-27)
as a requirements not to introduce a minor lobe with a level exceeding the others. 

Than the directivity of the array can be determined by

\[ D_0 = \frac{2R_0^2}{1+(R_0^2-1)f} \frac{\lambda}{\theta+d} \]  

(3-28)

The directivity of a dolph-Tschebyscheff array, with a given side lobe level, increases as the array size or number of element increase. For any given length of an array, the directivity may not necessarily increase as the side lobe level decreases [2].

### 3.4 Planar arrays

Planar arrays in practice are not easily measured. They are typically reconstructed using 2-dimensional linear arrays. For Planar arrays, the array factor is very important in constructing 3-dimensional arrays. Planar arrays are more versatile and can provide more symmetrical patterns with lower side lobes. They can scan the main beam of the antenna toward any point in space. Applications of planar arrays include tracking radar, search radar, remote sensing, communications, and many others.

The normalized form for the array factor of a planar array is given by

\[ AF_n(\theta, \phi) = \left\{ \frac{1}{M} \sin \left( \frac{M}{2} \psi_x \right) \right\} \left\{ \frac{1}{N} \sin \left( \frac{N}{2} \psi_y \right) \right\} \]

\[ \psi_x = k d_x \sin \theta \cos \phi + \beta_x \]

\[ \psi_y = k d_y \sin \theta \cos \phi + \beta_y \]

When spacing between elements greater than \( \lambda/2 \), multiple maxima of equal magnitude can be formed. The principle maximum is referred to as the major lobe, and the remaining lobes are referred to as grating lobes. A grating lobe is defined as “a lobe”, other than the main lobe produced by an array antenna when the inter element spacing is sufficiently large to permit the in-phase addition of radiated field in more than one
direction. In planar arrays, to avoid grating lobes in the x and y directions, the spacing between elements should be that \( d_x \leq \frac{\lambda}{2} \) and \( d_y \leq \frac{\lambda}{2} \). In typical applications (such as rectangular arrays) it is beneficial to have only one main beam that its maxima are directed towards the same direction. In order for this to happen the progressive phase shift between the elements in the x and y directions must be equal to

\[
B_x = -kd_x \sin \theta_0 \cos \phi_0 \\
B_y = -kd_y \sin \theta_0 \cos \phi_0
\]  

(3-30)

The grating lobes can be located by

\[
\phi = \tan^{-1} \left[ \frac{\sin \theta_0 \sin \phi_0 \pm n \lambda/d_y}{\sin \theta_0 \cos \phi_0 \pm m \lambda/d_x} \right] \\
\theta = \sin^{-1} \left[ \frac{\sin \theta_0 \sin \phi_0 \pm n \lambda/d_y}{\cos \phi} \right]
\]

(3-31)

A grating lobe will occur when both forms of the above are satisfied simultaneously.

For large array, with its maximum near broadside, the elevation plane half-power beamwidth is given by

\[
\Theta_h = \frac{1}{\cos^2 \theta_0 \left[ \Theta_{x0}^2 \cos^2 \phi_0 + \Theta_{y0}^2 \sin^2 \phi_0 \right]}
\]

(3-32)

Where \( \Theta_{x0} \) represents the half-power beamwidth of a broadside linear array of M elements and \( \Theta_{y0} \) represents the half-power beamwidth of a broadside linear array of N elements. These values can be obtained by using linear array for uniform distribution and for Tschebyscheff distribution by multiplying each uniform distribution value by the beam broadening factor. This can be done for other distributions if the beam broadening factors are available. For square array the beamwidth is given by

\[
\Theta_h = \Theta_{x0} \csc \theta_0 = \Theta_{y0} \csc \theta_0
\]

(3-33)

The half-power beamwidth \( \psi_h \) in the plane that is perpendicular to the \( \phi = \phi_0 \) elevation is given by
\[ \psi_h = \frac{1}{\sqrt{\Theta_x \sin^2 \phi_0 + \Theta_y \cos^2 \phi_0}} \]  

(3-34)

For a square array the half-power beamwidth reduces to

\[ \psi_h = \Theta_{x0} = \Theta_{y0} \]  

(3-35)

For a planar array it is useful to define a beam solid angle \( \Omega_A = \Theta_h \psi_h \). The directivity for array factor is complex and can be reduced its computing by using directivity of a linear array. For large planar arrays which are nearly broadside can be reduced to

\[ D_0 = \pi \cos \theta_0 D_x D_y \]  

(3-36)

Each of the directivities can be formed by using the appropriate beam broadening factor \( f \). For most practical amplitude distributions, the directivity is related to the beam solid angle of the same array which is given by

\[ D_0 \approx \frac{\pi^2}{\Omega_A (\text{rads}^2)} = \frac{32,400}{\Omega_A (\text{degrees}^2)} \]  

(3-37)

It has been shown that controlling the excitation phase can significantly alter the radiation pattern of an array. It has also been shown that the excitation amplitude between the elements can be used to control the beamwidth and side lobe level. The side lobes level can be controlled by tapering the distribution across the array. Smoother the taper from the center of the array toward the edges, lower the side lobe level and the larger the half-power beamwidth. For a binomial distribution, it would result in lower side lobes but larger half-power beamwidth and likewise for a uniform distribution has smaller half-power beamwidth and larger side lobe levels. It is desired to achieve low side lobe level and small half-power beamwidth, which the Dolph-Tschebyscheff design uses. Other good examples that achieve the desired affects is the Taylor line Source (Tschebyscheff-
Error) and Taylor Line-Source (one-Parameter). Typical designs incorporate the uniform array, due to large number of elements [2].

3.4.1 Antenna Beamforming

For planar arrays it is important to process the information and directivity of array by understanding the direction of arrival (DOA) of all impinging signals, and the appropriate weights to ideally steer the maximum radiation of the antenna pattern toward the SOI and to place nulls toward the SNOI. Between signal processing and introduction of planar arrays, smart antenna designs have are being developed [2].

3.4.2 Antenna Direction of Arrival (DoA)

The direction of arrival (DoA) of the incoming waves are based on time delays of the incoming signals. When an incoming wave, carrying a baseband signal s(t) impinges at an angle (θ, ϕ), on the antenna array, it produces time delays relative to the other antenna elements. The time delay for M x N planar array is given by

\[ \tau_{mn} = \frac{\Delta r}{v_0}, \text{where } \Delta r = d_{mn}\cos(\psi) \]  \hspace{1cm} (3-38)

\[ d_{mn} = \sqrt{m^2d_x^2 + n^2d_y^2} \]  \hspace{1cm} (3-39)

where, \( \cos(\psi) = \frac{\hat{a}_r \cdot \hat{a}_\rho}{|\hat{a}_r||\hat{a}_\rho|} \), \( \hat{a}_r \) and \( \hat{a}_\rho \) unit vectors along the direction of the incoming signals s(t). The time delay of the (m, n) element with respect to the element at the origin (0,0) is given by

\[ \tau_{mn} = \frac{md_x\sin\theta\cos\phi + nd_y\sin\theta\sin\phi}{v_0} \]  \hspace{1cm} (3-40)
DOA techniques used to analyze the data are based in 4 different areas; conventional methods, subspace-based methods, maximum likelihood methods, and integrated methods. The number of array elements affects the beamwidth of a radiation pattern. The more elements are used in an array, the narrower the main beam. Furthermore, the narrower beamwidth will resolve more accurately the SOIs and SNOIs. The smaller beamwidth and lower side lobes lead to lower co-channel interference [2].
Chapter 4

Introduction to MIMO System Design

Multiple-Input-Multiple-Output (MIMO) Antenna Design will allow for several platforms of devices to communicate with one other without giving up societies fast pace changing needs in the ways we communicate. Over the past couple of decades, the way we communicate between people has really advanced. In the 90s, most of the communication was done through email, fax, or over the phone. With the advancements in communication multiple people across the globe can connect to each other at any given moment using devices (Cellphones, PDA’s, Gaming Industry, and internet). The advancement of communication systems as they change and how they evolve with other devices can potentially be accomplished by incorporating MIMO Antenna Design. The ideal MIMO system consists of the following:

1. Forms several concurrent overlapping beams in any azimuth direction,
2. Responds to signals with any polarization, provides isolation between outputs,
3. Provides outputs with low cross correlation, and has high efficiency.

Currently today, there are many existing devices and standards that are incorporating MIMO technology and how it will continue to grow in the market based on our needs.
Current applications and standards for existing MIMO systems are incorporated in MIMO-OFDMA IEEE802.11n, MIMO-OFDM IEEE 802.11n, and ITI-T & G.9963 home networking standards. Future applications such as in standards 3GPP and 3GPP2, High Speed Packet Access plus (HSPA+), Long Term Evolution (LTE), and Cellular networks that include IST-MASCOT are currently being developed and are gradually going to be seen in the market.

There are many things surround MIMO Technology that can limit its full potential. In this paper we will discuss limiting factors and potential ways to improve the design theoretically using Matlab Simulink software tools. MIMO design is impacted by the following environmental and hardware limitations. One of the biggest design challenges is the propagation channel path through the environmental. The environment can be unknown as well as changing if object is in motion. It makes it difficult than to communicate across multiple antennas systems and applications due to unknowns in hardware and equipment. Each individual antenna within system of antennas must be unique but yet identifiable. Software challenges include how to handle the channel state information (CSI) when it is unknown at the transmitter or receiver. For devices to know the CSI at any given moment in time can be very difficult, but can be predicted. Hardware design challenges include physical antenna limitations, handshaking capabilities between devices, unknown hardware on receiving devices, and unknown coding schemes of the receiving devices.
4.1 MIMO Propagation and Channel Modeling

The MIMO channel is part of the communication system that cannot be engineered or known at all times. By studying the propagation channels, than are we able to exploit opportunities and extend the limitations of the way we communicate wirelessly. MIMO offers three main benefits: beamforming Gain, Spatial Diversity, and Spatial Multiplexing. Figure 4-1 illustrates the signal path that will be use in MIMO system.

![Figure 4.1: Multi Signal Path Environment](image)

Here the transmitted signals between the radio tower and mobile client have different scenarios paths of propagation for the signal to reach the mobile user as well have the same signal received from multiple paths.

4.1.1 Beamforming

In beamforming, transmit and receive antenna patterns can be focused into a specific angular direction by the appropriate choice of complex baseband antenna.
weights. The more correlated the antenna signals, the better for beamforming. Under certain propagation paths such as Line of Sight (LOS), the Receiver Rx and Transmitter Tx gains may add up, leading to an upper limit of \( m \cdot n \) for the beamforming gain of a MIMO system (where \( m \) is number of Transmitter Antennas and \( n \) is the number of Receivers). Precoding is a form of multi-stream beamforming. All the spatial processing occurs at the transmitter. Single beamforming is when the signal is emitted from the transmit antenna with the appropriate phase and gain weighting such that the signal power is maximized at the receiver input. The benefits of beamforming are to: increase received signal gain by making signals emitted from different antennas add up constructively and to reduce the multipath fading effect. (In line-of-sight propagation, beamforming results in a directional pattern. With multiple antennas, the transmit beamforming cannot simultaneously maximize the signal level at all of the receive antennas. Precoding beamforming requires knowledge of channel state information (CSI) at the transmitter and receiver [1].

### 4.1.2 Spatial Diversity

Spatial diversity offers multiple directions in space with an increase in transmission reliability of the link between transmitter and receiver. Spatial Diversity is on the order of \( m \cdot n \) limit. Spatial Diversity is reduced by spatial correlation. Diversity Coding is used when there is no channel knowledge at the transmitter. A single stream is transmitted, but the signal is coded using techniques called space-time coding. The signal is emitted from each of the transmit antennas with full or near orthogonal coding. Diversity coding exploits the independent fading the multiple antenna links to enhance
signal diversity. Diversity coding can be combined with spatial multiplexing when some channel knowledge is available [1].

4.1.3 Spatial Multiplexing

Spatial Multiplexing involves both space and time. MIMO systems are constantly moving, and MIMO channels allow for parallel data streams by transmitting and receiving on orthogonal spatial channels. The number of channels for spatial multiplexing gain limits depends on the min (m, n). Beamforming and Diversity/multiplexing is dictated by channel properties. For a directive channel, beamforming would be more beneficial, where a nondirective channel allows for diversity and multiplexing. The tradeoff between diversity and multiplexing is also dictated by the channel. High data rates can be achieved by multiplexing, and high reliability is achieved by diversity. Diversity can be observed at the uncorrelated link end. If both are uncorrelated at the link, both are possible. Spatial Multiplexing consists of multiple antennas in a specified configuration. In spatial multiplexing a high-rate signal is split into multiple lower-rate streams and each stream is transmitted from a different transmit antenna in the same frequency channel. When the signal arrives at the receiver antenna array with sufficiently different spatial signatures and the receiver has accurate Channel State Information (CSI), it can separate each stream into parallel channels.

Spatial Multiplexing results in as following. The channel capacity increases, but has higher signal to noise rations (SNR). The number of spatial streams is limited by the lesser of the number of antennas at the transmitter or receiver (min[M,N]). Spatial multiplexing can be used without CSI at the transmitter and can be combined with
precoding if CSI is available. Spatial multiplexing allows for simultaneous transmission
to multiple receivers, known as space-division multiple access or multi-user MIMO
requires CSI at transmitter), and the scheduling of receivers with different spatial
signatures which allows for good separability.

Diversity and spatial multiplexing results in the following for traditional antenna
diversity duplicate copies of a single information stream and is sent in order to increase
the reliability of detection. In spatial multiplexing, different information streams are sent
over separate spatial channels to increase throughput and spectral efficiency. The MIMO
system can achieve a mixture of these benefits, trading them against each other. As
spatial multiplexing gain increases, the diversity gain decreases and vice versa [1].

4.2 Types of Antenna Configurations

1. SISO – Single-input-single-output (Transmitter and Receiver has only 1 antenna)
2. SIMO – Single-input-Multiple-output (Case where transmitter has only 1 antenna)
3. MISO – Multiple-input-Single-Output (Case where receiver has only 1 antenna)
4. MIMO – Multiple-Input-Multiple-Output
4.2.1 SISO Antenna Type Configuration

SISO antenna type configurations offer point-to-point transmission. The multipath propagation creates fading and signal loss. By using diversity techniques, the degradation can be restored but the channel is no better than an unobstructed single path. There are many models for SISO channels, and some are included and represented in Chapter 5.

4.2.2 SIMO Antenna Type Configuration

SIMO Antenna type configuration involves more than one receiver antenna to only one transmit antenna. For SIMO systems, system is limited by m x n antennas.
4.2.3 MISO Antenna Type Configuration

MISO antenna type configuration involves more than one transmit antenna and only one receiver antenna. Again, the system is limited by $m \times n$ antennas. MISO and SIMO models are capable of easily be represented compared to SISO system since the CSI at the transmitter or receiver is known.

4.2.4 MIMO Antenna Type Configuration

MIMO Antenna Type configurations involve multiple transmit and multiple receiver information. MIMO antenna design offers enhanced data rate with no increase in bandwidth. The transmitting array and receiving array must undergo a form of scattering for multi-path wave propagation. For a 2x2 MIMO system, both arrays must be capable of resolving the two paths. Each path carries different data streams and will increase the throughput without increasing bandwidth. The transmit vector for MIMO system is projected onto the channel matrix $H(w)$, the number of independent data streams that can be supports is limited by the rank of channel matrix $H(w)$. The properties of $H(w)$ channel matrix determines the potential performance for a MIMO system. Figure 4.3 shows MIMO channel matrix $H(w)$ [14].
4.2.4.1 Channel Matrix

The channel matrix $H(w)$ combines the effects of both transmitting and receiving antennas. As discussed in Chapter 3, Antenna Parameters such as the array size, configuration, element pattern, element coupling, antenna impedance matching and multipath propagation characteristics affect antenna performance for the desired application [14].

4.2.4.2 Correlation Matrix

A MIMO system must be capable of making the best use of the signals available in any configuration of the system and the environment it is in. Antenna system design and processing algorithms must take account for the environmental changes. Between each adjacent antenna separated by distance $d$, there exists a correlation between each antenna and the antenna elements surrounding it. For MIMO system, low correlation between antenna outputs is necessary but not sufficient for good MIMO performance. Low correlation is achieved when each antenna provides a unique weight to each individual
multipath component. The weight can be due to antenna location (spatial diversity, angle diversity, or polarization diversity). Low correlation generally occurs for a large set of multipath components with large angular spread. MIMO needs a rich scattering environment to achieve low signal-to-noise-ratio (SNR). A low SNR will increase channel capacity. The radiation pattern involved with MIMO system in different regions of space can be met by different methods involving switching between individual directional antennas, and switching between multiple beams formed from a single multi-element array. With a rich multipath, the correlation between signals from even closely spaced antennas is very small, but for very small spacing’s the outputs of two antennas will be influenced by mutual coupling. There are three types of correlation involved with antenna systems and are as follows:

1. Temporal Correlation which is model well in SISO Channels
2. Spatial Correlation which is modeled with MISO or SIMO Channels.
3. MIMO is correlation between each channel between each transmitting and receiving antenna.

SISO, MISO, SIMO can determine the correlation at either the transmitting or receiving antennas. MIMO correlation is more difficult because TX and RX must be synchronized either by cables or feedback network, which in practical use does not exist.

**4.2.4.3 Channel Knowledge**

In a mobile environment, there is no knowledge of the channel. When you have m x n channels, everything will have changed and we will have no useful result. In a portable application the rate of change could allow effective channel sounding. In a rich
multipath environment, a MIMO system with M transmitting and M receiving antennas provides \( M^2 \) transmissions channels and has potential throughput up to M times that of a single channel occupying the same bandwidth. Every property of a MIMO system depends on the statistical properties of the environment. The channel randomness is broken up into six types as follows:

Type 1: Random changes that result when any of the antennas used in a MIMO link moves continuously in an otherwise physically static environment.

Type 2: Random changes that result when Interacting Objects in the critical region of MIMO link move, but all antennas remain static.

Type 3: Random changes that result when both Type I and Type II activity occurs.

Type 4: Random differences among instantaneous physical link transfer functions between physically static antenna elements.

Type 5: Random Changes that occur when either the Tx array or the Rx array or both are moved in steps to different locations within a local area throughout which shadowing by obstructions remains constant.

Type 6: Random changes that occur when either the Tx array or Rx array or both are moved in steps beyond the boundaries of a local area with stationary characteristics.

4.3 MIMO modulation Schemes

MIMO Technology can be carried out in various ways. Spatial multiplexing is typically combined with Orthogonal-Frequency-Division-Multiplexing (OFDM) or with Orthogonal Frequency Division Multiple Access (OFDMA) modulation. The use of
OFDM and OFDMA is capable of handling multi-path channel efficiently. Antenna Diversity and Space-Time coding Techniques are often used for modeling 2x2 MIMO systems. They include Receive Diversity, Transmit Diversity, Space-Time Coding (STC), Space-Time Block Code (STBC), and Space-Time Trellis Code. These coding schemes are used to help maximize capacity based on unknown environment.
Chapter 5

SISO Channel Modeling

5.1 Basic Wireless Channel: Propagation and Fading

SISO Channel Modeling is part of the building blocks that will also be used in implementing path for the MIMO fading channel setup used in the I-METRA and SCM Modeling of MIMO channel. Figure 5-1 describes the classification of various fading channels that are used in SISO and MIMO channel models.

Figure 5.1: Classification of fading channels.
5.1.1 Free Space Propagation Model

The free space propagation model is used for predicting the received signal strength in the line of sight (LOS) environment. For Large Scale Fading the General Path Loss Model for a non-isotropic antennas with transmit gain of $G_t$ and receive gain of $G_r$, L is system loss factor, the received power at distance $d$, $P_r(d)$ is given by

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L} \quad (5-1)$$

The free space path loss without any system loss ($L = 1$) is given by

$$PL_f(d) (dB) = 10 \log \left( \frac{P_t}{P_r} \right) = -10 \log \left( \frac{G_t G_r \lambda^2}{(4\pi)^2 d^2} \right) = 20 \log \left( \frac{4\pi d}{\lambda} \right) \quad (5-2)$$

The following Figure 5-2 below represents the Free Path Loss for given range of frequencies assuming $G_t = G_r = 1$.

![Figure 5-2: General Free Space Path Loss Model](image.png)

From the graphs we can see that the path loss increase as distance between transmitter and receiver increase. Since the free space path loss model is exponential in behavior it
can be modeled with the path loss exponent \( n \) with path loss at distance \( d \), where \( d_0 \) is the reference distance at which or closer to the path loss inherits the characteristics of free space loss is given by

\[
PL(d) = PL(f(d_0)) + 10n\log\left(\frac{d}{d_0}\right)
\]

**5.1.2 Log Normal Path Loss Model**

Table 5.1 describes the path loss exponent for various environments for log-distance path loss model.

<table>
<thead>
<tr>
<th>Environment</th>
<th>Path Loss Exponent (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free Space</td>
<td>2</td>
</tr>
<tr>
<td>Urban area cellular radio</td>
<td>2.7-3.5</td>
</tr>
<tr>
<td>Shadowed urban cellular radio</td>
<td>3-5</td>
</tr>
<tr>
<td>In building line of sight</td>
<td>1.6-1.8</td>
</tr>
<tr>
<td>Obstructed in building</td>
<td>4-6</td>
</tr>
<tr>
<td>Obstructed in factories</td>
<td>2-3</td>
</tr>
</tbody>
</table>

The log distance path loss model at a distance \( d \) is given by

\[
PL_{LD}(d) (dB) = PL_f(d_0) + 10n\log\left(\frac{d}{d_0}\right)
\]  

(5-3)

The Log-Normal Path loss model (shadowing model) allows the transmitter and the same distance \( d \) to have a different path loss, which varies with the random showing effect \( X_\sigma \). \( X_\sigma \) is define as a Gaussian random variable with a zero mean and a standard deviation of \( \sigma (dB) \). The Log-normal shadowing path loss model is given by

\[
PL_{LD}(d) (dB) = PL_f(d_0) + 10n\log\left(\frac{d}{d_0}\right) + X_\sigma
\]

(5-4)

For development with military communications of our microstrip rectangular antenna for 16 GHz signal for Free Path-Loss Model, Log-Distance Path Loss Model, and Log-normal Path-Loss Model is plotted for various gains, path loss exponent, and random paths [1].
It is clear that from Free-Path Loss Model that the Path Loss increases as distance increase of the traveled signal. The path loss for the Log-Distance Path Loss model increase with path loss exponent $n$, even if the distance between the transmitter and receiver is equal to each other. This implies that the path loss surrounding environments vary with location. The Log-Normal path loss model illustrates the random effect of shadowing that is imposed on the deterministic nature of the log-distance path loss model.

### 5.1.3 Okumura/Hata Model

The Okumura/Hata Model has been obtained through extensive experiments to compute the antenna height and coverage for mobile communications systems. This model mainly covers the typical mobile communications system characteristics with a frequency band of 500-1500MHz, cell radius of 1-100km, and an antenna height of 30m to 100km. The path loss at a distance $d$ for the Okumura Model is given by

$$PL_{\text{okumura}}(d)[dB] = PL_F + A_{mu}(f,d) - G_{Rx} - G_{Tx} + G_{Area}$$

(5-5)
The more popular and widely used Hata Model for urban areas is given as

\[
PL_{Hata,U}(d)[dB] = 69.55 + 26.16\log_{10}(fc) - 13.82\log_{10}(h_{TX}) - C_{RX} + \left\{44.9 - 6.55\log_{10}(h_{TX})\right\}\log_{10}(d)
\]  

(5-6)

\(C_{RX}\) is the correlation coefficient of the receive antenna, which depends on the size of coverage and for small and large coverage is given as

**Small to Medium Coverage**

\[C_{RX} = 0.8 + \{1.1 \log_{10}(fc) - 0.7\}h_{RX} - 1.56\log_{10}(fc)\]

**Large Size Coverage Distance**

\[
\begin{align*}
C_{RX} &= 8.29\left\{\log_{10}(1.54h_{RX})\right\}^{2} - 1.1 & \text{if } 150 \text{ MHz} \leq f_{c} \leq 200 \text{ MHz} \\
C_{RX} &= 3.2\left\{\log_{10}(11.75h_{RX})\right\}^{2} - 4.9 & \text{if } 200 \text{ MHz} \leq f_{c} \leq 1500 \text{ MHz}
\end{align*}
\]

(5-7)

The following Hata models have been developed for suburban and open area respectively by

\[
PL_{Hata,SU}(d)[dB] = PL_{Hata,U}(d) - 2\left\{\log_{10}\left(\frac{f_{c}}{28}\right)\right\}^{2} - 5.4
\]  

(5-8)

\[
PL_{Hata,O}(d)[dB] = PL_{Hata,U}(d) - 4.78\left\{\log_{10}(f_{c})\right\}^{2} + 18.33\log_{10}(f_{c}) - 40.97
\]  

(5-9)

Figure 5-4 represents the path loss graphs for the Urban, Sub-urban, and Open area models.

![Figure 5-4: Hata Path Loss Model (Urban, Suburban, Open area)](image-url)
Figure 5-3 shows that urban area has the most significant path loss compared to suburban and open area.

5.1.4 IEEE 802.16d Model

The IEEE 802.16d uses the log normal shadowing path loss model. The model for IEEE 802.16d environment is based on macro-cell suburban with environment conditions for hilly terrain with moderate-to-heavy- tree densities, for intermediate path loss condition, and for flat terrain with light tree densities.

Table 5.2: Parameters for IEEE 802.16.16d type A, B, and C models.

<table>
<thead>
<tr>
<th>TYPE</th>
<th>Description</th>
<th>Parameter Type A</th>
<th>Parameter Type B</th>
<th>Parameter Type C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Macro-cell suburban, ART to BRT for hilly terrain with moderate to heavy tree densities</td>
<td>4.6</td>
<td>4</td>
<td>3.6</td>
</tr>
<tr>
<td>B</td>
<td>Macro-cell suburban, ART to BRT for intermediate path loss condition</td>
<td>0.0075</td>
<td>0.0065</td>
<td>0.005</td>
</tr>
<tr>
<td>C</td>
<td>Macro-cell suburban, ART to BRT for flat terrain with light tree densities</td>
<td>12.6</td>
<td>17.1</td>
<td>20</td>
</tr>
</tbody>
</table>

The IEEE 802.16d model is given by

\[
PL_{802.16}(d)[dB] = PL_F(d_0) + 10\gamma log_{10}\left(\frac{d}{d_0}\right) + C_f + C_{RX} \quad \text{for } d > d_0
\]

(5-10)

where, \( d_0 = 100m; \gamma = a - bh_{Tx} + \frac{c}{h_{Tx}} \); \( h_{Tx} \) is height of antenna

\( C_f \) is the correlation coefficient for the carrier frequency and is given by

\[
C_f = 6log_{10}\left(\frac{f_c}{2000}\right)
\]

(5-11)

\( C_{RX} \) is the correlation coefficient for the receive antenna and is given by
\[ C_{RX} = \begin{cases} -10.8 \log_{10} \left( \frac{h_{RX}}{2} \right) & \text{for Type A and B} \\ -20 \log_{10} \left( \frac{h_{RX}}{2} \right) & \text{for Type C} \end{cases} \] (5-12)

Or

\[ C_{RX} = \begin{cases} -10.8 \log_{10} \left( \frac{h_{RX}}{3} \right) & \text{for } h_{RX} \leq 3 \text{m} \\ -20 \log_{10} \left( \frac{h_{RX}}{3} \right) & \text{for } h_{RX} > 3 \text{m} \end{cases} \] (5-13)

The Path Loss has discontinuity and can be seen in part A of Figure 5-5. At Distance greater than \( d_0 \) (100m), the path loss is less at 101m which doesn’t comply with increase in loss with an increase in distance. This implies that the existing model needs to be modified for distances greater than \( d_0 \). The new reference distance is given as

\[ d'_0 = d_0 \left( \frac{C_f + C_{RX}}{10^\gamma} \right) \] (5-14)

The new IEEE 802.16 path loss model using shadowing affect is given by

\[ PL_{MB02.16}(d)[dB] = \begin{cases} 20 \log_{10} \left( \frac{4\pi d}{\lambda} \right) & \text{for } d \leq d'_0 \\ 20 \log_{10} \left( \frac{4\pi d_0}{\lambda} \right) + 10 \log_{10} \left( \frac{d}{d_0} \right) + C_f + C_{RX} & \text{for } d > d'_0 \end{cases} \] (5-15)

The new reference distance \( d'_0 \) across the range of distances removes the discontinuity and provides better estimation.
The IEE802.16d path loss model for different heights provide discontinuity and when Eq. 5.10 is equated into Eq. 5.2, the determined path loss no longer shows discontinuity for different variations in height of the antennas [1].

5.2 Indoor Models

Small scale fading is referred to as fading in short. Fading is the rapid variation of the received signal level in the short term as the user terminal moves a short distance. With multiple inputs arrive approximately at the same time of the receiver, constructive and deconstructive interference occurs with different phases. Small scale fading is attributed to multi-path propagation, mobile speed, speed of the surrounding objects, and transmission bandwidth of the signal parameters of Small-Scale Fading include the Power Delay Profile (PDP) or Power Spectrum Profile (PDP). If the scattering components are much stronger than most of the components, the fading process no longer follows the Rayleigh distribution, but instead follows the Rician fading. Rician fading
follows The Line of Sight (LOS) probability density function of the signal received. The non-Line of Sight (NLOS) follows the Rayleigh distribution.

Figure 5-6 displays the Rayleigh and Rician probability density function (PDF). As you can see as the Rician K factor decreases to K=-40 dB, the Rician fading approaches Rayleigh fading.

There has been a lot of study done with indoor and outdoor channel models. To show the building blocks of SISO, SIMO, MISO channels, the following section describes SISO channel for indoor and outdoor environments [7]. The indoor channel model represents small coverage areas inside buildings, such as offices and shopping malls. Under these conditions the power azimuth spectrum (PAS) can be approximated to be uniform. The scattering components will be received from all directions with the same power. The channel is represented more as a static channel due to low mobility, but occasionally must be represented by the power delay profile (PDP) to represent the channel delays and average power as channel changes. If the channel was truly static, the
environment in which a channel condition does not change for the duration of data transmission at the given time and location. For indoor channel models, general models include the 2-ray model and exponential model.

5.2.1 2-ray Model vs. Exponential Model

The 2-ray model depicts two rays with the first ray being direct path (LOS) with time delay of zero. The second ray is based on a reflection in the environment and is received at a later time with the same magnitude. The maximum excess delay is given by \( \tau_m = \tau_1 \). The mean excess delay is given by \( \bar{\tau} = \frac{\tau_1}{2} = \sigma_\tau \), and for the ray model is also equal to the RMS delay. This model may be inaccurate do to the fact that the second ray or reflected ray is usually much less in amplitude than the first ray. The 2-ray model Power delay profile is depicted in Figure 5-7. In the exponential model, the average channel power decreases exponentially with the channel delay and is given by \( P(\tau) = \frac{1}{\tau_d} e^{-\tau/\tau_d} \). In the exponential model, the mean excess delay and RMS delay spread are equal to each other as it is in the 2-ray model. The maximum excess delay is different and is given by \( \tau_m = -\tau_d \ln A \). “A” is a ratio of non-negligible path power to the first path power and is given by \( A = \frac{P(\tau_m)}{P(0)} = \exp(-\frac{\tau_m}{\tau_d}) \). The exponential model can be represented by a discrete-time model with a sampling period of \( \tau_S \), time index of the last path \( p_{max} = \frac{\tau_m}{\tau_S} \) and is given by

\[
P(p) = \frac{1}{\sigma_\tau} e^{-p \tau_S/\sigma_\tau}, p = 0,1, \ldots, p_{max}
\]  

(5-16)

The total power of the power delay profile (PDP) is given by

\[
P_{total} = \sum_{p=0}^{p_{max}} P(p) = \frac{1}{\sigma_\tau} \cdot \frac{1-e^{-(p_{max}+1)\tau_S/\sigma_\tau}}{1-e^{-\tau_S/\sigma_\tau}}
\]  

(5-17)
And the normalized total power of the above equation is given by

\[ P(p) = P(0)e^{-\frac{p\tau_s}{\sigma_t}}, p = 0, 1, ..., p_{max} \]  \hspace{1cm} (5-18)

Where \( P(0) \) is the first path power, \( P(0) = 1/(P_{total} \cdot \sigma_t) \). Figure 5-7 illustrates the PDP for exponential model with 10,000 channel realizations with RMS delay spread of 30ns and sampling period of 10ns.

The ray model depicts the same channel power for each tap delay, whereas the exponential depicts a more realistic model in what the channel power is likely to decrease at later delay time [1].

5.2.2 IEEE 802.11 Channel Model

Applications that incorporate Exponential model includes IEEE 802.11 Channel Model, Saleh-Valenzuela (S-V) Channel Model, UWB Channel Model. The IEEE 802.11 Channel model is based on 2.4 GHz indoor channel that represents an impulse response represented by the output of finite impulse response filter with each channel tap is modeled by independent complex Gaussian random variable with its average power that follows the exponential PDP, while taking the time index of each channel tap by the
integer multiples of sampling periods [10]. The maximum number of paths is given by RMS delay spread which is

\[ p_{max} = \left[ 10 \cdot \frac{\sigma_\tau}{t_s} \right] \]  
(5-19)

The impulse response assuming the power of the pth channel tap has the mean of 0 and variance of \( \sigma_p^2/2 \) is given by

\[ h_p = Z_1 + jZ_2, \quad P = 0,1, ..., p_{max} \]  
(5-20)

Where \( Z_1 \) and \( Z_2 \) are statistically independent and identical Gaussian random variables each with \( \mathcal{N}(0, \frac{\sigma_p^2}{2}) \). The IEEE 802.11 Channel Model maximum excess delay is computed by a path of the least non-negligible power level with it being fixed to 10 times the RMS delay spread. The power of each channel tap is given as

\[ \sigma_p^2 = \sigma_0^2 e^{-p\tau_s/\sigma_\tau} \]  
(5-21)

\( \sigma_0^2 \) is the power of the first tap, which is used to determine the average received power equal to one. The first tap is given by

\[ \sigma_0^2 = \frac{1 - e^{-t_s/\sigma_\tau}}{1 - e^{-\left(p_{max}+1\right)t_s/\sigma_\tau}} \]  
(5-22)

Figure 5-8: IEEE802.11 channel model
The IEE802.11 Channel Model in Figure 5-8 is based on 10,000 channel realizations, $\sigma_t = 25$ ns and $t_s = 50$ ns. Due to the RMS delay spread being relatively small, the power variation in the frequency domain is with at most 15dB, which implies that the frequency selectivity is not that significant [1].

### 5.2.3 Saleh-Valenzuela (S-V) Channel Model

The Saleh-Valenzuela (S-V) channel model is based on indoor channel with multipath-delayed components that can be modeled as a Poisson process. After measuring indoor channel measurements, they have created a new channel model (S-V) that uses multiple clusters, each with multiple rays, in the delay profile. The arrival time of the first ray in the mth cluster, denoted by $\tau_m$, is modeled by a Poisson process with an average arrival rate of $\Lambda$ while the arrival times of rays in each cluster is modeled by a Poisson process with an average arrival rate of $\lambda$. The channel model shows that a distribution of inter-cluster arrival times and a distribution of inter-ray arrival times are given by the following exponential distributions.

![Figure 5-9: Saleh-Valenzuela channel model.](image)
The Saleh-Valenzuela channel model as shown in Figure 5-9 depicts the transmitted signal arriving at the receiver over a cluster distribution arrival time. Each received cluster has an average delay time, and the magnitude of the received cluster signals decreases with each delay cluster. The S-V model followed the log-normal fading path for each cluster of received signals [1].

5.2.4 UWB Channel Model

The UWB Channel model follows a log-normal multipath fading vs the Rayleigh distribution. It incorporates the same clustering as in the S-V channel model. Its model is depicted in Figure 5-10.

Figure 5-10: UWB Channel Model

Figure 5-10 represents the indoor channel model with Rayleigh fading subjected to long normal fading as well as the S-V channel cluster channel modeling.
5.3 Outdoor Channel Models

5.3.1 FWGN Model

The Clarke/Gans Model has been developed under the assumption that scattering components around a mobile station are uniformly distributed with an equal power for each component. This model incorporates various types of Doppler spectrum by a filtered white Gaussian noise (FWGN) model. The Clarke/Gans model is implemented in both frequency and time depending on Doppler filter used. The Clarke/Gans model constructs a complex channel gain using IFFT filter block. Figure 5-11 shows frequency and time characteristics for Doppler frequency of $f_m=100$Hz at sampling period of 50us.

![Figure 5-11: Clark/Gans Model (FWGN Model)](image)

The Clark/Gans model has average power distribution making for average received power in the time domain. Due to Clark/Gans Model’s computation complexity, the I-METRA model incorporates a modified Frequency-Domain FWGN Model. This model allows for generating the fading signal with a given fading duration without taking the
maximum Doppler frequency into account. It advantages for simulation since the time-domain signal can be obtained by interpolation with the maximum Doppler frequency $f_m$. 

![Figure 5-12: Modified Frequency-Domain FWGN Model](image)

The modified frequency domain FWGN model in Figure 5-12 depicts 2.4 GHz signal with 1024 channel realizations with oversampling of eight times the sampling frequency. As the Doppler frequency increases for the individual paths, so did the variations in magnitude in time and frequency [1].

### 5.3.2 Jakes Model

Jakes Model is constructed from a Rayleigh fading channel to a given Doppler spectrum and is generated by complex sinusoids. The complex output of Jakes model is represented as

$$h(t) = \frac{E_0}{\sqrt{2N_0+1}} \{ h_I(t) + jh_q(t) \}$$  \hspace{1cm} (5-23)

and the frequency Doppler shift is expressed as

$$w_n = w_d \cos \theta = 2\pi f_m \cos \left( \frac{2\pi n}{N} \right)$$  \hspace{1cm} (5-24)
Figure 5-13 depicts Jakes model for Doppler frequency of 926 Hz at sampling rate of 1 us. The Jakes model applies that the radio waves propagate horizontally at the receiver. The angles of arrival of the radio waves are uniformly distributed over \([-\pi, \pi]\). The radio waves applied here suggest that the antenna is omnidirectional [1].

5.3.3 Ray-Based Channel Model

Ray based channel model is frequently used in a MIMO channel since it can take a spatiotemporal correlation into account. In this section we will represent it as a SISO channel. The ray based model is given by a sum of arriving plane waves. It can model plane wave’s incoming from an arbitrary direction around the mobile terminal, which can deal with the scattering environment. The power azimuth spectrum (PAS) is not uniform. The Doppler spectrum is not given in a U-shape, but in various forms, depending on the scattering environments. The following \( h_{u,s,n}(t) \) denote a channel response of the nth path (cluster) between the sth transmit antenna and uth receive antenna which is given by
\[
\begin{align*}
\mathcal{h}_{u,s,n}(t) = \\
\sqrt{\frac{P_n}{M}} \sum_{m=1}^{M} \left( \sqrt{G_{BS}(\theta_{n,m,AoD})} \exp\left(j[kd_s \sin(\theta_{n,m,AoD}) + \phi_{n,m}] \right) \times \sqrt{G_{BS}(\theta_{n,m,AoD})} \exp\left(jk d_u \sin(\theta_{n,m,AoD}) \right) \exp(jk \|v\| \cos(\theta_{n,m,AoA} - \theta_v) t) \right)
\end{align*}
\]

(5-25)

The parameters for Ray based model for SISO channel are given as

- \(P_n\) is the power of the \(n\)th path.
- \(\sigma_{SF}\) is the lognormal shadow fading applied as a bulk parameter to the \(n\) paths for a given drop.
- \(M\) is the number of subpaths per-path.
- \(\theta_{n,m,AoD}\) is the the AoD for the \(m\)th subpath of the \(n\)th path.
- \(\theta_{n,m,AoA}\) is the the AoA for the \(m\)th subpath of the \(n\)th path.
- \(G_{BS} \theta_{n,m,AoD}\) is the BS antenna gain of each array element.
- \(G_{MS} \theta_{n,m,AoA}\) is the MS antenna gain of each array element.
- \(K\) is the wave number \(2\pi / \lambda\) where \(\lambda\) is the carrier wavelength in meters.
- \(d_s\) is the distance in meters from BS antenna element \(s\) from the reference \((s = 1)\) antenna. For the reference antenna \(s = 1\), \(d_1 = 0\).
- \(d_u\) is the distance in meters from MS antenna element \(u\) from the reference \((u = 1)\) antenna. For the reference antenna \(u = 1\), \(d_1 = 0\).
- \(\phi_{n,m}\) is the phase of the \(m\)th subpath of the \(n\)th path.
- \(|V|\) is the magnitude of the MS velocity vector.
- \(\theta_v\) is the angle of the MS velocity vector.

Ignoring all parameters associated with the spatial correlation and ignore the effect of the log-normal shadowing \((\sigma_{SF} = 1)\), the following impulse response for SISO channel is given by

\[
\begin{align*}
\mathcal{h}_n(t) = \sqrt{\frac{P_n}{M}} \sum_{m=1}^{M} \left( \exp(\mathcal{f}[\phi_{n,m}] \right) \times \exp(jk \|v\| \cos(\theta_{n,m,AoA} - \theta_v) t) \right) 
\end{align*}
\]

(5-26)

In the ray-based model, any channel with a given PAS can be modeled by allocating the angle and power to each subray in accordance with the PAS. The uniform power subray method and Laplacian methods are ways of determining PAS of the angle and power allocation to each subray. The uniform power sub-ray method allocates the same power
to each subray while arranging the subray angels in a non-uniform manner. Uniform

Power Subray method under the given PAS is

\[
\int_{\theta_1}^{\theta_2} P(\theta, \sigma) d\theta = \int_{\theta_1}^{\theta_2} \frac{1}{\sqrt{2}\sigma} e^{\frac{-\sqrt{2}|\theta|}{\sigma}} d\theta = \frac{1}{2} \left( e^{\frac{-\sqrt{2}|\theta_2|}{\sigma}} - e^{\frac{-\sqrt{2}|\theta_1|}{\sigma}} \right)
\]

\begin{equation}
= \frac{1}{a(M+1)} \	ext{ for } M \text{ is odd} \quad a = 2 \text{ for } M \text{ is even} 
\end{equation} (5-27)

The offset angle with the same power is given by

\[
\theta_{m+1}[deg] = -\frac{\alpha}{\sqrt{2}} \left[ \ln \left( e^{\frac{-\sqrt{2}\theta_{m}}{\sigma}} - \frac{2}{a(M+1)} \right) \right], \quad m = 0,1, ..., \left[ \frac{M}{2} \right] - 1, \quad \theta_{0} = 0^\circ
\]

\begin{equation}
(5-28)
\end{equation}

The Laplacian Method uses the power of the sub-rays follows the Laplacian PAS with their offset angles asymmetrically centered around the average angle of arrival (AoA).

The first M reference offset angels are generated by allocating them uniformly with an equal distance of \( \delta = \frac{2\alpha}{M} \) over the range of \( \phi = [-\alpha, \alpha] \) centered around the average AoA. Once the reference offset angels are generated, the actual offset angels are determined by adding an arbitrary random number selected over [-0.5, 0.5] to the reference offset angles. The offset angle allocation in the sampled Laplacian method is given by

\[
\theta_m = -\alpha + m \cdot \delta + \phi \text{ for } m = 0,1, ..., M - 1
\]

\begin{equation}
(5-29)
\end{equation}

Figure 5-14 depicts the ray based model for frequency of 900 MHz, as speed of 120kmh with AS = 2 and angle-of-departure (AoD) = 50 at the BS and AS = 35 and AoA =67.5 with DoT = 22.5 at the MS.
The Ray-based fading channel depicted in Figure 5-14 shows the path game for single transmit to receiver antenna for random Rayleigh fading channel [1].

5.3.4 Frequency-Selective Fading Channel Model

Tapped Delay Line (TDL) Model is used for implementing the multi-path channel. The TDL model is implemented as a FIR filter with the following output

\[ y(n) = \sum_{d=0}^{N_D-1} h_d(n)x(n-d) \]  \hspace{1cm} (5-30)

The new tap delay is given by

\[ \tau'_d = floor\left(\frac{t_d}{t_s} + 0.5\right) \cdot t_s \]  \hspace{1cm} (5-31)

The relative distance of new tap delay is given by

\[ t_r = \frac{t_d}{t_s} - t_l \]  \hspace{1cm} (5-32)

The new tap adjustment by interpolation from channel delay \( t_l \) is given by
\[ h'_t(n) = \tilde{h}_t(n) + \sqrt{1 - \tau_r h_{ra}(n)} \]  

(5-33)

And the temporary complex channel coefficient for the new tap delay of \( t_i + 1 \) is given as

\[ \tilde{h}_t(n) = \sqrt{\tau_r} h_{ra}(n) \]  

(5-34)

The following tapped delay line method will be implemented in I-METRA model as discussed in chapter 6 [1].

### 5.3.5 SUI Channel Model

The SUI Channel Model IEEE802.16d is classified into three different terrain types depending on tree density and path-loss condition. Table 5-3 depicts the terrain type.

The Doppler power spectrum (PSD) for SUI channel in truncated form is given as

\[ S(f) = \begin{cases} 
1 - 1.72f_0^2 + 0.785f_0^4 & \text{for } f_0 \leq 1 \\
0 & \text{for } f_0 > 1 
\end{cases} \]  

(5-35)

Where, \( f_0 = f / f_m \).

<table>
<thead>
<tr>
<th>Terrain Type</th>
<th>SUI Channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>SUI-5, SUI-6</td>
</tr>
<tr>
<td>B</td>
<td>SUI-3, SUI-4</td>
</tr>
<tr>
<td>C</td>
<td>SUI-1, SUI-2</td>
</tr>
</tbody>
</table>
Terrain type A is for hilly terrain with moderate to heavy tree densities. The SUI channel in Figure 5-15 shows the PDP profile for the three taps of SUI-6 channel. The parameters for the PDP with time delays are used. The SUI fading channel is generated using the FWGN model. Than interpolation or resampling process for frequency selective fading channel using the tapped delay line is used to produce the wireless transmission. For each path the ideal and simulated PSD for each path is shown. Ideal is with PDP of 1 for each path, but in reality sees a decrease in PSD for each path [1].
Chapter 6

MIMO Channel Models

6.1 Statistical MIMO Model

The statistical model for a MIMO system will be represented by a vector of the received signals in the uniform linear array of M elements can be expressed as

\[ y(t) = \sum_{i=1}^{I} \alpha_i c(\phi_i) x(t - \tau_i) + n(t) \]  

(6-1)

Where \( \alpha_i = \text{channel gain} \), \( \tau_i = \text{delay time} \), \( \phi_i = \text{angle of arrival} \), \( x(t) = \text{transmitted signal} \). The array steering vector is defined as

\[ c(\phi) = [c_1(\phi), c_2(\phi), ..., c_M(\phi)]^T \]  

(6-2)

where,

\[ c_M(\phi) = f_m(\phi) e^{-j2\pi(m-1)(\frac{\lambda}{2})\sin\phi}, \quad m = 1, 2, ..., M \]  

(6-3)

where, \( f_m \) is the complex field pattern for the mth array element and \( \lambda \) is a carrier wavelength. The received signal is expressed as

\[ y(t) = \int \int c(\phi) h(\phi, \tau) x(t - \tau) d\tau d\phi + n(t) \]  

(6-4)

Where \( h(\phi, \tau) \) represents the channel as a function of the Azimuth-Delay Spread. The instantaneous power azimuth delay spectrum (PADS) is given as

\[ P_{\text{inst}}(\phi, \tau) = \sum_{i=1}^{I} |\alpha_i|^2 \delta(\phi - \phi_i, \tau - \tau_i) \]  

(6-5)
The average PADS is defined as an expected value of Equation (6-5), such that

\[ P(\phi, \tau) = E\{P_{\text{inst}}(\phi, \tau)\} \quad (6-6) \]

The PAS (Power Azimuth Spectrum or Power Angular Spectrum) is obtained by the integral of PADS over delay.

\[ P_A(\phi) = \int P(\phi, \tau) d\tau \quad (6-7) \]

The AS (Azimuth Spread or Angular Spread) is defined by the central moment of PAS, and is given by

\[ \sigma_A = \sqrt{\int (\phi - \phi_0)^2 P_A(\phi) d\phi} \quad (6-8) \]

To find the Power Delay Spectrum, the integral over PADS over AoA and is given as

\[ P_D(\tau) = \int P(\phi, \tau) d\phi \quad (6-9) \]

The DS (Delay spread) defined as the central moment of PDS is given as

\[ \sigma_D = \sqrt{\int (\tau - \tau_0)^2 P_D(\tau) d\tau} \quad (6-10) \]

and \( \tau_0 \) is an average delay spread. The joint PDF of AoA and delay is given by equations (6-11) and (6-12).

\[ f_A(\phi) = \int f(\phi, \tau) d\tau \quad (6-11) \]

\[ f_D(\tau) = \int f(\phi, \tau) d\phi \quad (6-12) \]

Spatial Correlation for MIMO channel determines how the received signals are related to each other [1]. The receiving signals with mean AoA, the difference in their traveled distance is given by

\[ \tau_0 = (d/c)\sin \phi_0 \quad (6-13) \]

The channel impulse response between adjacent antennas is defined as

\[ h_a(\phi) = \alpha e^{j\beta} \sqrt{P(\phi)} \quad (6-14a) \]
\[ h_b(\phi) = \alpha e^{i(\beta + \frac{2\pi d \sin(\phi)}{\lambda})} \sqrt{P(\phi)} \]  

(6-14b)

For normalization of PAS that is \( \int_{-\pi}^{\pi} P(\phi) d\phi = 1 \), and normalized distance \( D = 2\pi d / \lambda \)

The general spatial correlation functions can be written as

\[ R_{xx}(D, \phi_0) = \int_{-\pi}^{\pi} \cos(D\sin\phi) \cdot P(\phi - \phi_0) \, d\phi \]  

(6-15)

\[ R_{xy}(D, \phi_0) = \int_{-\pi}^{\pi} \sin(D\sin\phi) \cdot P(\phi - \phi_0) \, d\phi \]  

(6-16)

### 6.2 PAS Model

The Power Azimuth Spectrum or Power Angular Spectrum (PAS) helps determine the correlation between antenna elements. Table 6.1 below describes what PAS model to choose for type of environment for the corresponding base station and mobile station.

<table>
<thead>
<tr>
<th></th>
<th>BS</th>
<th>MS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outdoor</td>
<td>Macrocell</td>
<td>Uniform</td>
</tr>
<tr>
<td></td>
<td>Truncated Laplacian</td>
<td></td>
</tr>
<tr>
<td></td>
<td>nth power of a cosine function</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Microcell</td>
<td>Truncated Gaussian</td>
</tr>
<tr>
<td></td>
<td>Picocell</td>
<td>Almost Uniform</td>
</tr>
<tr>
<td>Indoor</td>
<td></td>
<td>Uniform</td>
</tr>
</tbody>
</table>

Table 6-1: PAS model for different environments
6.2.1 ‘n’th Power of a Cosine Function PAS Model

The nth power of a Cosine Function PAS Model can be represented by the nth power of a cosine function and is given by

\[ P(\phi) = \frac{Q}{\pi} \cos^n(\phi), \quad -\frac{\pi}{2} + \phi_0 \leq \phi \leq \frac{\pi}{2} + \phi_0 \]  

(6-17)

Value \( n \) is an even integer related to antenna beamwidth, \( Q \) is factor to normalize PAS into 1, and \( \phi_0 = \text{mean AoA} \). The spatial correlation coefficients for the nth power of a cosine function PAS Model are given by

\[ R_{xx}(D, \phi_0) = \int_{-\pi/2}^{\pi/2} \cos(D\sin \phi) \cdot \frac{Q}{\pi} \cos^n(\phi - \phi_0) d\phi \]  

(6-18)

\[ R_{xy}(D, \phi_0) = \int_{-\pi/2}^{\pi/2} \sin(D\sin \phi) \cdot \frac{Q}{\pi} \cos^n(\phi - \phi_0) d\phi \]  

(6-19)

The nth power of a cosine function is shown below in Figure 6-1. As \( n \) increases the width of PAS decreases.

Figure 6-1: nth Power of a cosine function PAS
6.2.2 Uniform PAS Model

Uniform PAS model is for modeling a rich-scattering environment such as indoor environment. It represents a uniform power distribution of the specified range angle.

The power distribution of the uniform PAS model is given by

\[ P(\phi) = Q \cdot 1, \quad -\Delta \phi + \phi_0 \leq \phi \leq \Delta \phi + \phi_0 \]  

(6-20)

The specified range of angles are limited by the angular spread and is given by

\[ \Delta \phi = \sqrt{3} \sigma_A \]  

(6-21)

Q is the normalization factor to set PAS to 1 and is found by

\[ Q = 1/(2\Delta \phi) \]  

(6-22)

The spatial correlation coefficients for Uniform PAS model are found by

\[ R_{xx}(D, \phi_0) = J_0(D) + 4Q \sum_{m=1}^{\infty} J_{2m}(D) \cos(2m\phi_0) \sin(2m \cdot \Delta \phi) / 2m \]  

(6-23)

\[ R_{xy}(D) = 4Q \sum_{m=1}^{\infty} \frac{J_{2m+1}(D, \phi_0) \sin((2m+1)\phi_0) \sin((2m+1)\Delta \phi)}{(2m-1)} \]  

(6-24)

Where, \(J(D)\) is the first-kind \(m\)'th order Bessel function. As the angular spread \(\sigma_A\) increases the \(R_{xy}\) phase differences approach 0. Figure 6-2 displays the Uniform PAS model with uniform power distribution and the resulting spatial correlation coefficients for given spacing between antenna elements for the angular spread values of 10, 20 and 30. With Uniform power distribution the imaginary spatial correlation coefficients are 0.
6.2.3 Truncated Gaussian PAS Model

Truncated Gaussian PAS Model is used for macro-cell and microcell environments. Its power distribution is given as

\[ P(\phi) = \frac{Q}{\sqrt{2\pi}\sigma} \int_{-\Delta\phi}^{\Delta\phi} e^{-\frac{\phi^2}{2\sigma^2}} d\phi = 1 \]  

(6-25)

To normalize Q the \(\Delta\phi\) is usually set = \(\pi\). The normalization factor Q is found by

\[ Q = \frac{1}{\text{erf}\left(\frac{\Delta\phi}{\sqrt{2}\sigma}\right)} \]  

(6-26)

\[
R_{xx}(D, \phi_0) = J_0(D) + Q \sum_{m=1}^{\infty} J_{2m}(D)e^{-2\sigma^2m^2} \cos(2m\phi_0) \cdot \text{Re}\left\{ \text{erf}\left(\frac{\Delta\phi}{\sigma\sqrt{2}}\right) - \frac{\Delta\phi}{\sigma\sqrt{2}} - jm\sigma\sqrt{2}\right\} 
\]  

(6-27)
\[ R_{xx}(D, \phi_0) = Q \sum_{m=1}^{\infty} J_{2m+1}(D) e^{-2\sigma^2 m^2} \sin((2m+1)\phi_0) \cdot \text{Re} \left[ \text{erf} \left( \frac{\Delta \phi}{\sigma \sqrt{2}} - j\sigma \sqrt{2}(m + 1/2) \right) \right] - \text{erf} \left( -\frac{\Delta \phi}{\sigma \sqrt{2}} - j\sigma \sqrt{2}(m + 1/2) \right] \] (6-28)

Figures 6-3 represent the Truncated Gaussian PAS for AoA = 0 degrees for given AS of 10, 20, 30 degrees.

As AS increases, the power distribution becomes broad over large angle while the spatial correlation coefficient at the same antenna spacing decreases. As the AS for Gaussian PAS is greater than 30 degrees, the standard deviation \( \sigma \) and AS significantly differ.
6.2.4 Truncated Laplacian PAS model

The truncated Laplacian PAS model has a broader range of linearity than the Gaussian PAS model and therefore will be used in simulation results if AS is above 30 degrees. The power distribution of the Truncated Laplacian PAS model is given by

\[ P(\phi) = \frac{Q}{\sqrt{2}\sigma} e^{-\frac{|\phi-\phi_0|}{\sigma}}, \quad -\Delta\phi + \phi_0 \leq \phi \leq \Delta\phi + \phi_0 \] (6-29)

To normalize \( Q \) the \( \Delta\phi \) is usually set = \( \pi \). The normalization factor \( Q \) is found by

\[ Q = \frac{1}{1-e^{-\left(\frac{\sqrt{2}\Delta\phi}{\sigma}\right)}} = 1 \] (6-30)

The spatial correlation coefficients for Truncated Laplacian PAS are given by

\[ R_{xx}(D, \phi_0) = J_0(D) + 4Q \sum_{m=1}^{\infty} J_{2m}(D) \cos(2m\phi_0) \cdot \frac{\sqrt{2}+e^{-\left(\frac{\sqrt{2}\Delta\phi}{\sigma}\right)}}{\sqrt{2}\sigma \left[ \left(\frac{\sqrt{2}}{\sigma}\right)^2 + (2m)^2 \right]} \] (6-31)

\[ R_{xy}(D, \phi_0) = J_0(D) + 4Q \sum_{m=1}^{\infty} J_{2m+1}(D) \sin(2m\phi_0) \cdot \frac{\sqrt{2}+e^{-\left(\frac{\sqrt{2}\Delta\phi}{\sigma}\right)}}{\sqrt{2}\sigma \left[ \left(\frac{\sqrt{2}}{\sigma}\right)^2 + (2m+1)^2 \right]} \] (6-32)

The spatial correlation coefficients for Truncated Laplacian PAS model are found by using Eq. 6-31 and 6-32.
The spatial correlation coefficients for the truncated Laplacian model in Figure 6-4 increase as the AS decreases and as the normalized distance between antennas decrease. For low spatial correlation, the maximum distance between antennas should be achieved and higher the AS. With an increase an AS, the signal has overall broader range of degrees and that the power density is distributed evenly in the overall transmitted signal. For very low AS, the PDP has very narrowband distribution.

6.3 Doppler Spectrum

The Doppler spectrum is fading process that is independently generated by any fading channel model for the SISO channel. Various types of Doppler Spectrum includes
flat, classical, Laplacian Doppler and are used in the I-METRA model. More Doppler Spectrum models include Bell, Rounded, Jakes, Gaussian, and Bigaussian Doppler models.

For Line of Sight in the I-Metra model, the Rician fading channel can be implemented as well in the MIMO channel. The Rician fading channel for the first path H1 is given by

\[ H_1 = \sqrt{K} \sqrt{P_1 H_{LOS}} + \sqrt{P_1 H_{Rayleigh}} \]  \hspace{1cm} (6-33)

\( P_1 \) is the average power of the first path and \( K \) is the power ratio of the LOS to Rayleigh components. \( f_d = \left( \frac{v}{\lambda} \right) \cos(\alpha) \), where \( \alpha \) is the angle between the Direction of Movement (DoM) and LOS component. \( d_{Rx} \) and \( d_{Tx} \) is the antennas spacing in the receiver and transmitter. \( AoA_{Rx} \) and \( AoD_{Tx} \) represent the angel of arrival at the receiver and the angle of departure at the transmitter.

\[ H_{LOS}(t) = e^{j2\pi f_d t} \begin{bmatrix} 1 \\ e^{j2\pi \frac{d_{Rx}}{\lambda} \sin(AoA_{Rx})} \\ \vdots \\ e^{j2\pi \frac{d_{Rx}(M-1)}{\lambda} \sin(AoA_{Rx})} \\ e^{j2\pi \frac{d_{Tx}}{\lambda} \sin(AoD_{Tx})} \\ \vdots \\ e^{j2\pi \frac{d_{Tx}(N-1)}{\lambda} \sin(AoD_{Tx})} \end{bmatrix} \]  \hspace{1cm} (6-34)

### 6.4 Steering Matrix

The antenna radio pattern incurs a phase difference of \( d \sin \varphi \) between two adjacent antenna elements. The beamforming system deals with the phase difference between antenna elements. These phase differences from the antenna radio pattern create a mean DoA when all the scatters are located near the MS and the impinging field at the BS are subject to a delay of \( \tau = (d/c)\sin \varphi \). The received signal is given by

\[ y(t) = W(\varphi_{BS}) \int H(\tau) s(t - \tau) d\tau \]  \hspace{1cm} (6-35)
Where the steering diagonal matrix for the given mean DoA of $\varphi$ is given by

$$W(\varphi) = \begin{bmatrix}
W_1(\varphi) & 0 & \ldots & 0 \\
0 & W_2(\varphi) & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & W_M(\varphi)
\end{bmatrix}$$

(6-36)

$W_m(\varphi)$ represents the average phase shift relative to the first antenna element for the mean azimuth DoA of the impinging field equal to $\varphi$. The steering matrix for a uniform linear antenna array with element spacing $d$, is given as

$$W_m(\varphi) = f_m(\varphi)e^{-j2\pi(m-1)(\frac{d}{\lambda})\sin\varphi}$$

(6-37)

$f_m(\varphi)$ is the complex radiation pattern of the mth antenna element. If correlation between adjacent antennas are statistically independent, random variation in phase between the two antennas are expected. If this is the case, than the mean DoA and steering diagonal matrix is not applicable.

### 6.5 I-Metra MIMO Channel Model

I-METRA (Intelligent Multi-element Transmit and Receive Antennas) MIMO Channel model is implemented by using statistical characteristics including spatial correlation for PAS. The statistical model of correlated MIMO fading channel consists of M antennas at the base station and N antennas at the mobile station. The narrowband MIMO channel $H$ is statistically expressed with M x N matrix as

$$H = \Theta_R^{1/2} A_{iid} \Theta_T^{1/2}$$

(6-38)

$\Theta_R$ and $\Theta_T$ are the correlation matrices for the receive antennas and transmit antennas. $A_{iid}$ represents the independent and identically distributed fading channel [20]. Equation (6-28) represents the correlation matrices as being independent from one another.
assuming that the antenna spacing in the transmitter and receiver is smaller than the
distance between the transmitter and receiver. The MIMO channel can be modeled using
tapped delay line (TDL).

\[
H(\tau) = \sum_{l=1}^{L} A_l \delta(\tau - \tau_l)
\]  

(6-39)

\(A_l\) is the complex channel gain matrix for the \(l\)th path and delay \(\tau_l\). \(A_l\) consists of
channel coefficient between each BS antenna and nth MS antenna for the \(l\)th path. The
complex channel gain can be represented by Rayleigh, rank-1 Rician and Rician channel
with an arbitrary phase.

\[
A_l = \begin{bmatrix}
\alpha_{11}^{(l)} & \alpha_{12}^{(l)} & \ldots & \alpha_{1N}^{(l)} \\
\alpha_{21}^{(l)} & \alpha_{22}^{(l)} & \ldots & \alpha_{2N}^{(l)} \\
\vdots & \vdots & \ddots & \vdots \\
\alpha_{M1}^{(l)} & \alpha_{M2}^{(l)} & \ldots & \alpha_{MN}^{(l)}
\end{bmatrix}
\]  

(6-40)

The signals between the MS and BS can be expressed by the following

\[
y(t) = \int H(\tau)s(t - \tau)d\tau
\]  

(6-41)

where the received signals at the BS are given as \(y(t) = [y_1(t), y_2(t), \ldots, y_M(t)]^T\) and
the transmitted signals at the MS are given as \(x(t) = [x_1(t), x_2(t), \ldots, x_N(t)]^T\).
The correlation coefficient of a channel gain for two different MS antennas, \(n_1\) and \(n_2\)
can be expresses as

\[
\rho_{n_1n_2}^{MS} = \langle |\alpha_{mn_1}^{(l)}|^2, |\alpha_{mn_2}^{(l)}|^2 \rangle, \quad m = 1, 2, \ldots, M
\]  

(6-42)

As long as the antenna spacing at BS is relatively small when Tx and Rx are sufficiently
apart and the MS does not depend on the Tx antenna. When the MS antennas are
separated by more then \(\lambda/2\), which means Eq. 6-32 is approximately equal to zero, for
\(n_1 \neq n_2\). [15] The spatial correlation matrix for the MS is given by
As similar as the MS, the BS can also be expressed as independent of MS antennas and the correlation coefficient of a channel gain for two different BS antennas, m1 and m2 can be expressed as

\[ \rho_{m1m2}^{BS} = \left( |\alpha_{m1n1}^{(l)}|, |\alpha_{m2n2}^{(l)}| \right)^2, \quad n1 \neq n2, m1 \neq m2 \] (6-44)

Similar to \( R_{MS} \), the spatial correlation matrix for the BS is given as

\[ R_{BS} = \begin{bmatrix} \rho_{11}^{(BS)} & \rho_{12}^{(BS)} & \cdots & \rho_{1M}^{(BS)} \\ \rho_{21}^{(BS)} & \rho_{22}^{(BS)} & \cdots & \rho_{2M}^{(BS)} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{M1}^{(BS)} & \rho_{M2}^{(BS)} & \cdots & \rho_{MM}^{(BS)} \end{bmatrix} \] (6-45)

\[ \rho_{n1m1}^{n2m2} = \left( |\alpha_{m1n1}^{(l)}|, |\alpha_{m2n2}^{(l)}| \right)^2 \] (6-46)

There is no theoretical solution to the equation (6-36) but can be approximated as

\[ \rho_{n1m1}^{n2m2} \approx \rho_{n1n2}^{MS} \rho_{m1m2}^{BS} \] (6-47)

The complex channel gain matrix for the lth path can be represent by

\[ \bar{A}_l = \sqrt{P_l} C a_l \] (6-48)

Where \( P_l \) is the average power of the ‘1’th path, C is the correlation shaping matrix and \( a_l \) is the correlated MN x 1 channel vector with correlate MIMO fading channel coefficients [19]. The correlation shaping matrix C is generated from the spatial correlation matrix in equations 6-33 and 6-35.

\[ R = \{ R_{BS} \otimes R_{MS}, R_{MS} \otimes R_{BS} \} \] (6-49)
Root power correlation matrix $\Gamma$ is found using the Kronocker product. The root power correlation matrix with ‘field’ type uses the square root decomposition for real matrices.

$$\Gamma = \begin{cases} \sqrt{R}, & \text{for field type} \\ R, & \text{for complex type} \end{cases} \quad (6-50)$$

The matrix with ‘complex’ type uses the Cholesky decomposition for complex matrices.

$$\Gamma = CC^T \quad (6-51)$$

6.6 I-METRA Block Diagram

The I-METRA MIMO Channel Model is based on the stochastic MIMO channel model which generates a correlated MIMO fading channel using the spatial correlation derived for Uniform Linear Array (ULA) subject to single or multiple clusters with the Uniform, Truncated Gaussian, or Truncated Laplacian PAS models. The I-METRA model or COSSAP can be modeled by the block diagram in Figure 6-5 [8]. The MIMO channel uses tapped delay line. It takes into account for the delay and power profiles. It takes the uncorrelated fading channel generated and multiplies it by a spatial correlation mapping matrix to generate a correlated fading channel. The given PDP characteristics are implement by passing the correlated fading signal through a FIR filter that is design to satisfy the given average power and delay specification for each path.
6.7 SCM Channel Model

The SCM Channel model is proposed by a joint work of Ad Hoc Group (AHG) in 3GPP and 3GPP2, which specifies the parameters for spatial channel model. The SCM channel model is a ray-based channel model, which superposes sub-ray components on the basis of PDP, PAS, and antenna array structure. The SCM benefits are as follows:

Directly models the statistical characteristics of MIMO channel, Maintains the statistical characteristics in the time, space, and frequency domains. It is simple to implement, flexible in changing the various types of PDP and PAS, supports both LOS and NLOS channels, and its effective rank of channel matrix $H$ depending on the number of sub-rays in each path, $M$. [13]

The PAS of a path arriving at the BS follows a Laplacian distribution. For any AoD $\bar{\theta}$ and RMS angular spread $\sigma$, the per-path BS Pas at any angle $\theta$ is given by $\sigma$
\[ P(\theta, \sigma, \tilde{\theta}) = N_0 e^{-\frac{\sqrt{2}|\theta - \tilde{\theta}|}{\sigma}} G(\theta) \]  

(6-52)

The BS antenna gain \( G(\theta) \) and normalization factor \( N_0 \) is given by

\[ \frac{1}{N_0} = \int_{-\pi+\theta}^{\pi+\theta} e^{-\frac{\sqrt{2}|\theta - \tilde{\theta}|}{\sigma}} G(\theta) d\theta, -\pi + \tilde{\theta} \leq \theta \leq \pi + \tilde{\theta} \]  

(6-53)

For a 3-sector or 6-sector antenna at the BS, the antenna gain must be taken into account for computing PAS. The antenna gain for BS is given as

\[ A(\theta) = -\min \left[ 12 \left( \frac{\theta}{\theta_{3dB}} \right)^2, Am \right] \text{[dB]}, \text{for } -180 \leq \theta \leq 180 \]  

(6-54)

The PAS arriving at the MS for incoming path that arrives at AoA \( \tilde{\theta} \) with RMS delay spread, for Laplacian PAS of sub-ray arriving at an angle \( \theta \) is given by

\[ P(\theta, \sigma, \tilde{\theta}) = N_0 e^{-\frac{\sqrt{2}|\theta - \tilde{\theta}|}{\sigma}} G(\theta), -\pi + \tilde{\theta} \leq \theta \leq \pi + \tilde{\theta} \]  

(6-55)

The MS antenna gain \( G(\theta) \) and normalization factor \( N_0 \) is given by

\[ \frac{1}{N_0} = \int_{-\pi+\theta}^{\pi+\theta} e^{-\frac{\sqrt{2}|\theta - \tilde{\theta}|}{\sigma}} d\theta = \sqrt{2}\sigma \left( 1 - e^{-\sqrt{2}\pi/\sigma} \right) \]  

(6-56)

The uniform PAS of the sub-ray arriving at angle \( \theta \) for MS is given by

\[ P(\theta, \sigma, \tilde{\theta}) = N_0 \ast 1, -\sqrt{3}\sigma + \tilde{\theta} \leq \theta \leq \sqrt{3}\sigma + \tilde{\theta} \]  

(6-57)

Figure 6-6 displays the overall PAS with antenna gain for antenna with Laplacian PAS distribution for AoA = 22.5° and AS = 35°. The antenna gain for beamwidth \( \theta_{3dB} = 70^\circ \) and maximum attenuation of \( A_m = 20 \text{ dB} \). The total PAS is the product of PAS \times \text{Antenna Gain}.  

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Figure 6-6 shows the truncated Laplacian PAS for transmitted signal with AoA of 35° and AS of 35°. The antenna gain of the system is also displayed for given range of -180° to 180°. The overall PAS of the system taking the gain of the antenna into account is the product of the truncated Laplacian PAS x Antenna gain of the system [1]. The antenna gain of the system depends on the array structure and capabilities of the antenna hardware.

6.7.1 SCM Link-Level Channel Parameters

The SCM block diagram is shown in Figure 6-7. Case I, II, III, and IV involve selecting the corresponding PDP profile that follows pedestrian A, vehicular A, pedestrian B, and single path respectfully. Each PDP profile comes with Relative Path power and delay time for each path. The SCM involves selecting the speed of the mobile user. Each profile also has set of guidelines for the base station and mobile station involving the PAS, AoA, AoD, and DoT. Once the parameters are defined, the model will produce the corresponding offsets, gains, and phases of each respected path. The last section involves combining the generated values to produce the channel coefficient for each individual path.
6.7.2 SCM Link-Level Channel Modeling

The Ray based SCM angle parameters in Figure 6-8 are defined as

\[
\begin{align*}
\Omega_{\text{BS}} & \quad \text{BS antenna array orientation, defined as the difference between the broadside of the BS array and the absolute North (N) reference direction.} \\
\theta_{\text{BS}} & \quad \text{LOS AoD direction between the BS and MS, with respect to the broadside of the BS array.} \\
\delta_{n,\text{AoD}} & \quad \text{AoD for the nth (n = 1 \ldots N) path with respect to the LOS AoD } \theta_0 .
\end{align*}
\]
$\Delta n,m,\text{AoD}$ .... Offset for the mth ($m = 1 \ldots M$) subpath of the nth path with respect to $\delta n,\text{AoD}$.

$\theta n,m,\text{AoD}$ .... Absolute AoD for the mth ($m = 1 \ldots M$) subpath of the nth path at the BS with respect to the BS broadside.

$\Omega_{\text{MS}}$ .......... MS antenna array orientation, defined as the difference between the broadside of the MS array and the absolute North reference direction.

$\theta_{\text{MS}}$ ............ Angle between the BS-MS LOS and the MS broadside.

$v$ .................... MS velocity vector.

$\theta v$ ................... Angle of the velocity vector with respect to the MS broadside: $\theta v = \arg(v)$.

The General Ray-based SCM model high level language describing the channel is given by

$$h_{u,s,n}(t) = \sqrt{\text{nth Path Power}} \sum_{m=1}^{M} \left\{ \left( \text{BS PAS} \right) \left( \text{Phase due to BS Array} \right) \ast \left( \text{MS PAS} \right) \left( \text{Phase due to MS Array} \right) \right\}$$

(5-58)

The channel for Uniform Power Sub Ray Model for MIMO channel is given by

$$h_{u,s,n}(t)$$

$$= \sqrt{\frac{P_n \sigma_{\text{SF}}}{M}} \sum_{m=1}^{M} \left( \sqrt{G_{\text{BS}}(\theta_{n,m,\text{AoD}})} \exp(j[kd_s \sin(\theta_{n,m,\text{AoD}}) + \phi_{n,m}]) \times \sqrt{G_{\text{MS}}(\theta_{n,m,\text{AoD}}) \exp(jkd_u \sin(\theta_{n,m,\text{AoA}})) \exp(jk\|v\| \cos(\theta_{n,m,\text{AoA}} - \theta v) t)} \right)$$

(5-59)

The Ray-based SCM model for Laplacian model is given by

$$h_{u,s,n}(t)$$

$$= \sqrt{P_n} \sum_{m=1}^{M} \left( \sqrt{P_{\text{BS}}(\theta_{n,m,\text{AoD}})G_{\text{BS}}(\theta_{n,m,\text{AoD}})} \exp(j[kd_s \sin(\theta_{n,m,\text{AoD}}) + \phi_{n,m}]) \times \sqrt{P_{\text{MS}}(\theta_{n,m,\text{AoD}})G_{\text{BS}}(\theta_{n,m,\text{AoD}}) \exp(jkd_u \sin(\theta_{n,m,\text{AoA}})) \exp(jk\|v\| \cos(\theta_{n,m,\text{AoA}} - \theta v) t)} \right)$$
The Ray-based SCM model for Rician channel model for LOS is given by [1]

\[ h_{\text{LOS},u,s,n}(t) = \sqrt{\frac{K}{K+1}} h_{u,s,n}(t) \]

\[ + \sqrt{\frac{K}{K+1}} \sum_{m=1}^{M} \left( \sqrt{G_{\text{BS}}(\theta_{\text{BS}})} \exp(j[kd_s \sin(\theta_{\text{BS}})]) \times \sqrt{G_{\text{MS}}(\theta_{\text{MS}})} \exp(jk|\nu| \cos(\theta_{\text{MS}} - \theta_{\nu}) t) + \phi_{\text{LOS}} \right) \]

\[ \rho_{\text{SCM}}(d) = \frac{1}{M} \sum_{m=1}^{M} e^{\frac{j2\pi d \sin \theta_{n,m,\text{AoA}}}{\lambda}} \]

where \( \theta_{n,m,\text{AoA}} \) is the AoA for the mth sub-ray with mean \( \bar{\theta} \). The corresponding sub-ray is given as

\[ \theta_{n,m,\text{AoA}} = \bar{\theta} + \Delta_{n,m,\text{AoA}} \]

Figure 6-9 describes the signal model for uniform linear antenna for Ray-based SCM channel.
The temporal correlation derived from the uniform-power sub-ray is given as

\[ \rho_{SCM}^{te}(\tau) = E\{h_{s,u,n}(t) \cdot h_{s,u,n}^*(t)\} = \frac{1}{M} \sum_{m=1}^{M} e^{\frac{j2\pi v||\cos(\theta_{n,m,\text{AoA}}-\theta_v)}{\lambda}} \]  (6-65)

As the mean AoA increases for given AS, the correlation magnitude increases for given normalized antenna spacing distance \(d/\lambda\). For when AS increases, the lower the correlation magnitude [11]. For lower spatial correlation coefficients using the SCM Ray-based model, it is wise to design with higher AS, and larger the normalized antenna spacing distance \(d/\lambda\).
Chapter 7

Channel Capacity

Channel capacity for multiple antenna system with $N_t$ transmit and $N_r$ receive antennas can be increased by the factor of $\min (N_t, N_r)$, without using additional transmit power or spectral bandwidth. For any given wireless channel with high channel capacity, techniques used to achieve high-speed data rate transmission are done through diversity techniques and spatial-multiplexing [12]. Diversity techniques involve converting Rayleigh fading wireless channel into more stable AWGN-like channels. Spatial multiplexing involve the independent data streams simultaneously transmitted by the multiple transmit antennas, which achieves higher transmission speed.

MIMO system capacity involves Matrix theory using singular value decomposition (SVD) given as

$$H = U\Sigma V^H$$  \hspace{1cm} (7-1)

Where $U \in \mathbb{C}^{N_R \times N_T}$ and $V \in \mathbb{C}^{N_T \times N_T}$ are unitary matrices, and $\Sigma \in \mathbb{C}^{N_R \times N_R}$ is rectangular matrix whose diagonal elements are no-negative real numbers and whose off-diagonal elements are zero. The received signal of the system is given by

$$y = \sqrt{\frac{E_x}{N_t}} Hx + z$$  \hspace{1cm} (7-2)
Where $z$ is the noise vector. The autocorrelation of transmitted signal vector is defined as

$$R_{xx} = E\{xx^H\}$$

(7-3)

When the transmission power for each transmit antenna is assumed to be 1, $\text{Tr}(R_{xx}) = N_t$.

The channel capacity is the maximum mutual information that can be achieved by varying the probability density function of the transmit signal vector. The capacity of a deterministic channel is defined as

$$C = \max I(x; y) \text{ bits/channel use}$$

(7-4)

When CSI is known to the transmitter side for channel capacity, the deterministic MIMO channel is expressed as

$$C = T_r\max (R_{xx}) = N_T\log_2 \left( I_{NR} + \frac{E_x}{N_TN_0}HR_{xx}H^H \right) \text{ bps/Hz}$$

(7-5)

When CSI is not available at the transmitter side, the total power is equally allocated to all transmit antennas, and that is the autocorrelation function of the transmit signal vector $x$ is given as

$$R_{xx} = I_{NT}$$

(7-6)

and the channel capacity is given as

$$C = \log_2 \left( I_{NR} + \frac{E_x}{N_TN_0}HH^H \right) \text{ bps/Hz}$$

(7-7)

The channel capacity of SIMO system is given by

$$C_{SIMO} = \log_2 (1 + \frac{E_x}{N_0}N_R)$$

(7-8)

The channel capacity of MISO system is given by

$$C_{MISO} = \log_2 (1 + \frac{E_x}{N_0}N_T)$$

(7-9)
From these equations, the capacity of MISO channel is the same as that of the SISO channel. Its benefits may be incorporated using space-time coding technique which improves the transmission reliability. [16] The capacity of SIMO channel increases logarithmically as the number of antennas increases, but since availability if CSI at the transmitter side does not improve the channel capacity at all. The channel capacity of random MIMO channels is given as

$$C = \log_2 \left( I_{N_R} + \frac{E_x}{N_T N_0} H R x x^H H \right) \text{bps/Hz} \quad (7-10)$$

and is known as ergodic channel capacity. The cumulative distribution function (CDF) of the capacity for the random MIMO channel when CSI is not available at the transmitter side [9]. Figure 7-1 shows the CDFs of the random 2x2 and 4x4 MIMO channel capacities when SNR = 10dB, and outage channel capacity $\epsilon = 0.01$

![Ergodic Channel Capacity of MIMO Channel](image)

**Figure 7-1: CDF of MIMO Channel**

It shows that by increasing the number of transmit and receive antennas, the overall rate increases, but the Capacity CDF is still the same. Figure 7-2 shows the ergodic capacity
of MIMO channel for various numbers of transmitters and receivers when CSI is not available at the transmitter.

![Diagram](image-url)

**Figure 7-2: Ergodic Channel Capacity when Transmitter is Unknown**

The ergodic channel capacity increases for a given SNR as number of transmitters and receive antennas increase especially when the SNR increases in magnitude. [17]

The ergodic MIMO capacity for correlated channel is compared with uncorrelated Rayleigh channel. The correlated matrix is based off of spatially correlated matrices used in simulation results for I-METRA model found in Chapter 7. You can tell that the correlated matrix for 2x2 MIMO system is nearly uncorrelated. As the SNR increases, the channel capacity decreases for 2x2 MIMO system.
Figure 7-3: Ergodic MIMO Capacity

Figure 7-3 shows the channel capacity for uncorrelated 2x2 Rayleigh fading channel. Using the simulated generated channel for 2x2 in the I-METRA model detailed in Section 8.2, the following correlated Rayleigh fading channel shows that the capacity decreases with a correlated fading channel especially as the SNR increases in magnitude. Using SVD composition Figure 7-4 represents general Capacity capabilities for SISO, 2x2 and 4x4 MIMO antenna systems for uncorrelated Rayleigh fading channel [7].
The Channel Capacity greatly improves for an application system with an increase in number of antennas for MIMO system.[18] For high SNR, the MIMO channel can greatly enhance the channel capacity, while at low SNR, the gain in capacity may have little or no affect.

7.1 Antenna Diversity vs Space Diversity

Antenna Diversity techniques are used to mitigate degradation in the error performance due to unstable wireless fading channels. Space-diversity is accomplished sufficiently when antennas are separated by more than 10λ which implements independent wireless channels. This allows for polarization diversity, time diversity, frequency diversity, and angle diversity. For receiver diversity, increasing the number of
receiving antennas improves performance. The maximal ratio combining (MRC) technique is used. As you can see the BER increases with SNR and performance improves by increasing number of receiving antennas.

Figure 7-5: BPSK MRC performance for Rayleigh fading channels

Figure 7-5 represents Binary Phased Key Modulation (BPSK) for SISO channel model and as the number of receiver antennas increases. The MRC performance matches the received signals from each antenna and uses maximum ratio combining (MRC) technique to weight each received antenna signal and combine them for maximum channel capacity based on SNR.
Chapter 8

Simulation Results

8.1 Uniform Linear Arrays

The uniform linear arrays were plotted using MATLAB program. The following sub-sections will depict the describe Broadside, End-fire, Hansan-Woodyard, and Scanning arrays as describe in Chapter 3.

8.1.1 Broadside Uniform Linear Arrays (ULA)

The Broadside array is designed to have maximum directivity along the 90 degrees axis as depicted in Figure 3-3 by setting $\psi = kdcos\theta + \beta = 0$, when $\theta_0 = 90^\circ$ and $\beta = 0$. 
Figure 8-1: Two element ULA Broadside Array (Linear Uniform Directivity, Array Factor, Polar Plot of Directivity, and 3-D plot of directivity)

Figure 8.1 depicts the linear uniform broadside array for two element ULA with $\lambda=0.5$ spacing. The 2D-uniform linear directivity model displays only the $[0,180^\circ]$, while the polar directivity model displays directivity from $[0,360^\circ]$. The 3-Dimensional plot shows that for 2-element array, the HPBW is relatively large, and this antenna could be considered as close to omnidirectional antenna in the X-Y plane.
Figure 8.1: 4-Element ULA and 10-Element ULA for antenna spacing of 0.5λ

From figure 8.2, as the number of elements in the ULA increase, the directivity increases in magnitude, but the HPBW decreases. Therefore there is a trade of between directivity and HPBW.
Table 8.1 shows the following generated directivities and HPBW for the following 2, 4, and 10 ULA for varies antenna spacing. It looks like the maximum directivity is typically found with antenna spacing of 0.5λ and without having more than one $\theta_{\text{max}}$. The $\theta_{\text{max}}$ majority of designed ULA’s typically follow a 0.5 λ for antenna design systems for its one maximum directivity in a particular direction.
8.1.2 Ordinary END-FIRE Array

The Ordinary End-Fire array is designed to have maximum directivity along the 90 degrees axis as depicted in Figure 3-3 by setting $\psi = kd\cos\theta + \beta = 0$, when $\theta_0 = 0^\circ$ or $180^\circ$ and $\beta = -kd$ or $kd$ respectively.

Figure 8-3: Two element ULA End-Fire Array (Linear Uniform Directivity, Array Factor, Polar Plot of Directivity, and 3-D plot of directivity)

Figure 8.3 depicts the linear uniform end-fire array for two element ULA with $\lambda=0.5$ spacing. Compared to the broadside array, the maximum directivity in the $\theta_{max}$, occurs at $0^\circ$ or $180^\circ$. 
From figure 8.4, as the number of elements in the ULA increase, the directivity increase in magnitude, but the HPBW decreases. Therefore there is a trade off between directivity and HPBW.
Table 8.2: End-Fire ULA for 2, 4, and 10 Elements

<table>
<thead>
<tr>
<th>Antenna Spacing 'd'</th>
<th>0.25λ</th>
<th>0.5λ</th>
<th>0.75λ</th>
<th>1.0λ</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2-Element ULA</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Directivity (dB)</td>
<td>3.0103</td>
<td>3.0103</td>
<td>3.0103</td>
<td>3.103</td>
</tr>
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<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>HPBW #1 [degree]</td>
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<td>0</td>
</tr>
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<td>28.955</td>
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</tr>
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<td>90</td>
<td></td>
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<td>HPBW #3 [degree]</td>
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<table>
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<th>0.5λ</th>
<th>0.75λ</th>
<th>1.0λ</th>
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</thead>
<tbody>
<tr>
<td><strong>4-Element ULA</strong></td>
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</tr>
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<td>Directivity (dB)</td>
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<td>6.0206</td>
<td>6.0206</td>
<td>6.0206</td>
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<tr>
<td>Directivity dimensionless</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>HPBW #1 [degree]</td>
<td>114.004</td>
<td>78.877</td>
<td>63.9666</td>
<td>55.212</td>
</tr>
<tr>
<td>θ Max #1 [degree]</td>
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<td>0</td>
<td>18.5618</td>
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<td>θ Max #2 [degree]</td>
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</tr>
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<table>
<thead>
<tr>
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<th>0.5λ</th>
<th>0.75λ</th>
<th>1.0λ</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>10-Element ULA</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Directivity (dB)</td>
<td>10</td>
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<td>10</td>
<td>10</td>
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<tr>
<td>Directivity dimensionless</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>HPBW #1 [degree]</td>
<td>64.4185</td>
<td>48.7049</td>
<td>34.6655</td>
<td>34.308</td>
</tr>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>HPBW #2 [degree]</td>
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<td>7.216</td>
<td>5.0997</td>
<td></td>
</tr>
<tr>
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<td>109.5</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>HPBW #3 [degree]</td>
<td></td>
<td>34.308</td>
<td></td>
<td></td>
</tr>
<tr>
<td>θ Max #3 [degree]</td>
<td></td>
<td>180</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8.2 shows the following generated directivities and HPBW for the following 2, 4, and 10 ULA for varies antenna spacing. The maximum directivity is the same regardless of antenna spacing d. For when antenna spacing is 0.5λ, the end-fire radiation exists simultaneously in both \( \theta = 0^\circ \) and \( \theta = 180^\circ \) directions. For End-Fire arrays with antenna spacing \( d = (1\lambda, 2\lambda, \ldots) \) there exist four maximum, 2 in broadside and 2 in End-Fire array. To have only one end-fire maximum and to avoid any grating lobes, the maximum spacing between the elements should be less than \( d_{max} < \lambda/2 \).
8.1.3 HANSEN-WOODYARD END-FIRE ARRAY

The Hansen-Woodyard End-Fire array is designed to enhance the maximum directivity along the $\theta_0 = 0^\circ$ or $180^\circ$ degrees without destroying any other characteristics. This array is accomplished by shifting the phase between elements. Using the same setting as before $\psi = kdcos\theta + \beta = 1$, when $\theta_0 = 0^\circ$ or $180^\circ$ and $\beta = -\left( kd + \frac{\pi}{N} \right)$ or $\beta = \left( kd + \frac{\pi}{N} \right)$ respectively.

Figure 8-5: Two element Hansen-Woodyard End-Fire ULA (Linear Uniform Directivity, Array Factor, Polar Plot of Directivity, and 3-D plot of directivity)

Figure 8.5 depicts the linear uniform end-fire array for two element ULA with $\lambda=0.25$ spacing. The maximum directivity in the $\theta_{max}$, occurs at $0^\circ$ in Figure 8.5.
Figure 8-6: 4-Element and 10-Element End-Fire ULA for antenna spacing of $0.25\lambda$.

From figure 8.6, as the number of elements in the ULA increase, the directivity increase in magnitude, but the HPBW again decreases. Therefore there is a trade of between directivity and HPBW for Hansen-Woodyard End-Fire array.
Table 8.3: Hansen-Woodyard End-Fire ULA for 2, 4, and 10 Elements.

<table>
<thead>
<tr>
<th>Hansen-Woodyard End-Fire Uniform Linear Array</th>
<th>Antenna Spacing 'd'</th>
<th>0.25λ</th>
<th>0.5λ</th>
<th>0.75λ</th>
<th>1.0λ</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-Element ULA</td>
<td>0.25λ</td>
<td>4.3964</td>
<td>3.0103</td>
<td>4.0462</td>
<td>3.103</td>
</tr>
<tr>
<td>Directivity (dB)</td>
<td>2.752</td>
<td>2</td>
<td>2.5387</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>HPBW #1 [degree]</td>
<td>96.3782</td>
<td>89.8788</td>
<td>38.9425</td>
<td>30</td>
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<tr>
<td>θ Max #1 [degree]</td>
<td>0</td>
<td>120</td>
<td>90</td>
<td>75.5</td>
<td></td>
</tr>
<tr>
<td>HPBW #2 [degree]</td>
<td>96.3797</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>HPBW #3 [degree]</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>θ Max #3 [degree]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-Element ULA</td>
<td>0.25λ</td>
<td>8.3478</td>
<td>6</td>
<td>7.2705</td>
<td>6.0206</td>
</tr>
<tr>
<td>Directivity (dB)</td>
<td>6.8356</td>
<td>4</td>
<td>5.334</td>
<td>4</td>
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</tr>
<tr>
<td>HPBW #1 [degree]</td>
<td>62.6061</td>
<td>46.3893</td>
<td>17.7182</td>
<td>13.1797</td>
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</tr>
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<td>θ Max #1 [degree]</td>
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<td>138.6</td>
<td>99.6</td>
<td>82.8</td>
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</tr>
<tr>
<td>HPBW #2 [degree]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>θ Max #2 [degree]</td>
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<td></td>
</tr>
<tr>
<td>HPBW #3 [degree]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>θ Max #3 [degree]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-Element ULA</td>
<td>0.25λ</td>
<td>12.5017</td>
<td>10</td>
<td>11.2885</td>
<td>10</td>
</tr>
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<td>Directivity (dB)</td>
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<td>10</td>
<td>13.4539</td>
<td>10</td>
<td></td>
</tr>
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<td>HPBW #1 [degree]</td>
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<td>27.2878</td>
<td>7.0584</td>
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</tr>
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<td>θ Max #1 [degree]</td>
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<td>105.5</td>
<td>87.1</td>
<td></td>
</tr>
<tr>
<td>HPBW #2 [degree]</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>θ Max #2 [degree]</td>
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<td></td>
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</tr>
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<td>HPBW #3 [degree]</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>θ Max #3 [degree]</td>
<td></td>
<td></td>
<td></td>
<td>161.8</td>
<td></td>
</tr>
</tbody>
</table>

Table 8.3 shows the following generated directivities and HPBW for the following 2, 4, and 10 ULA for varies antenna spacing. The maximum directivity is given when antenna spacing d=λ/4. The Hansen-Woodyard End-Fire array can only yield improvement in directivity provided that the spacing between antennas is d=λ/4.

### 8.1.4 SCANNING ARRAY

The Scanning array is designed to enhance the maximum directivity in any direction for $0^\circ \leq \theta_0 \leq 180^\circ$. The maximum radiation in any direction is accomplished
by shifting the phase between elements for given angle. Using the same equation as before $\psi = kd\cos\theta + \beta = 1$, when $\theta = \theta_0$ and $\beta = -kd\cos\theta_0$.

Figure 8-7: Two Element Scanning ULA for $\theta_0 = 30^\circ$ (Linear Uniform Directivity, Array Factor, Polar Plot of Directivity, and 3-D plot of directivity)

Figure 8.7 depicts the uniform scanning array for two element ULA with $\lambda=0.5$ spacing. The maximum directivity in the $\theta_{\text{max}}$, occurs at $30^\circ$ in Figure 8.7.
Figure 8-8: 4-Element and 10-Element Scanning array for antenna spacing of \(d=0.5\lambda\) and \(\theta_0 = 30^\circ\).

From figure 8.8, as the number of elements in the ULA increase, the directivity increase in magnitude, but the HPBW again decreases. The Scanning ULA follows the End-Fire values for when \(\theta_0 = 0^\circ\) or \(180^\circ\). For all other \(\theta_0\) follows the same magnitude of directivity as End-Fire ULA, but differ in the HPBW.

8.2 2x2 Uniform Linear Array I_METRA MIMO Model

The following will discuss the Rayleigh fading for a 2x2 Uniform Linear Array I_METRA MIMO Model. Table 8.4 defines the parameter inputs for the I-METRA MIMO Model that we will be using. The parameters used in are based off of I-METRA channel parameters in 3GPP.
Table 8.4: I-METRA Input Parameters

<table>
<thead>
<tr>
<th>Fading Path</th>
<th>Rayleigh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>2.5 GHz</td>
</tr>
<tr>
<td>Power Delay Profile</td>
<td>ITU-R Vehicular A</td>
</tr>
<tr>
<td>Doppler Spectrum</td>
<td>Classical</td>
</tr>
<tr>
<td>Speed (km/h)</td>
<td>120</td>
</tr>
<tr>
<td>Transmitter Antenna Spacing</td>
<td>.5λ</td>
</tr>
<tr>
<td>Transmitter PAS</td>
<td>Truncated</td>
</tr>
<tr>
<td>Transmitter Azimuth Spread</td>
<td>10°</td>
</tr>
<tr>
<td>Transmitter AoA</td>
<td>20°</td>
</tr>
<tr>
<td>Transmitter DoM</td>
<td>0°</td>
</tr>
<tr>
<td>Receiver Antenna Spacing</td>
<td>.5λ</td>
</tr>
<tr>
<td>Receiver PAS</td>
<td>Truncated</td>
</tr>
<tr>
<td>Receiver Azimuth Spread</td>
<td>35°</td>
</tr>
<tr>
<td>Receiver AoA</td>
<td>-67.5°</td>
</tr>
<tr>
<td>Receiver DoT</td>
<td>-22.5°</td>
</tr>
</tbody>
</table>

The power delay profile being used is from the ITU-R model table parameters for Pedestrian A. The values can be found below in Table 8-5.

Table 8.5: ITU-R Model Parameters

<table>
<thead>
<tr>
<th>TAB</th>
<th>Pedestrian A</th>
<th>Pedestrian B</th>
<th>Vehicular A</th>
<th>Vehicular B</th>
<th>Doppler Spectrum</th>
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<td>1</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>110</td>
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<td>3</td>
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<td>-4.9</td>
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</tr>
<tr>
<td>4</td>
<td>410</td>
<td>-22.8</td>
<td>1200</td>
<td>-8.0</td>
<td>1090</td>
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<td>5</td>
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<td>-15.0</td>
<td>17100</td>
</tr>
<tr>
<td>6</td>
<td>3700</td>
<td>-23.9</td>
<td>2510</td>
<td>-20.0</td>
<td>20000</td>
</tr>
</tbody>
</table>

Figure 8-9 displays the PAS distribution for Transmitting/Base Station Antenna for AS = 10° and AoA = 20°.
Figure 8-9: Laplacian PAS Transmitter

Figure 8-10 displays the Laplacian PAS distribution for the receiving/mobile station antenna for AS of 35° and AoA of 67.5°.

Figure 8-10: Laplacian PAS of Receiver

The following spatial correlation matrices generated for the following 2x2 MIMO antennas space \( \lambda \) apart from each other are given by

\[
R_{BS}(Transmitter) = \begin{bmatrix} 1 & 0.428989579371720 \\ 0.428989579371720 & 1 \end{bmatrix}
\]
Figure 8-11 describes the fading for the given paths with Doppler shifts of $f_m = 100\text{Hz}$, $50\text{Hz}$, and $10\text{Hz}$. The Modified FWGN fading path uses the classical model of Doppler shift and Rayleigh fading distribution.

Using the same spatial correlation matrixes, other PDP profiles can be used in the I-METRA model. For example, the SUI model for Terrain type A ‘SUI-6’ will be modeled. Figure 8-12 displays the PDP profile for SUI-6 type channel.
Figure 8-12: PDP of SUI-6 Fading Channel

SUI-6 uses 3 TAP modeling system. Figure 8-13 models each transmitting to receiver antenna path for PDP of each tap above in Figure 8-12.

Figure 8-13: 2x2 MIMO SUI Fading Channel for each path

Figure 8-13 shows that for each received path antenna, the relative channel power for each tap approximately has roughly the same channel gains. For choice of environments,
under which the channel path is chosen, the I-METRA model allows for tapped-delay
line that follows set of PDP and time delays. These environmental parameters can be
applied to tapped-delay line along with Doppler shift to formulate the MIMO channel for
\(N_T\) transmitters and \(N_R\) receivers.

8.2 SCM Link Channel Model Results

The SCM channel link model parameters are similar to I-METRA model used for
3GPP systems. Using the same PDP profile as in I-METRA results above for ITU-R
model for Vehicular A. The difference in parameters between the two models is the PAS
with a Laplacian distribution with RMS angle spread of 5 degrees versus 10 degrees in
the I-METRA. For 2.5 GHZ frequency and for six channel taps, the following SCM
model for each transmitter to receiver path follows a Rayleigh distribution and is
displayed in Figure 8-14.
Figure 8-14: 2x2 MIMO Rayleigh Ray-based SCM Fading Channel

For Ray-based SCM model with LOS present will be using the Rician Distribution.

Using the same PDP and time delays except with LOS component, the following fading channel for Ray-based SCM for each transmitter to receiver path has been generated and is shown in Figure 8-15.

Figure 8-15: 2x2 MIMO Rician Ray-based SCM fading Channel
The MIMO Rician Ray-base SCM fading shows that for the first path, the random generated channel gain has potential to have increase in gain. In most MIMO systems, the LOS component of the transmitted signal tends not to exist. The LOS component tends to occur when the base station is elevated above the ground where the mobile user can get LOS signal.
Chapter 9

Conclusions

Antenna design for multimedia devices begin with the desired frequency, the application of interest, and the size requirements of the application. For microstrip antennas, lower the frequency larger the patch antenna will become. For lower frequencies in the kilo-hertz range microstrip antennas are not very useful due to the size of the antenna and the required spatial correlation between the antennas. To maximize the gain of a single antenna, Table 2.3 indicated that by selecting material with lower dielectric constant, the directivity gain can increase, but may reduce the HPBW range of the antenna. We can alter the gain or directivity of the antenna by using multiple antennas such as the ULA and changing the phase of the antenna to have desired output directivities and directions. As you increase the number of antennas in the system, the gain of the system increases, but results in smaller beamwidth.

The MIMO channel model is more complex than the SISO channel models. The SISO channel are the building blocks for each transmitted path of the MIMO channel. The MIMO channel creates a correlation between adjacent antennas and must be taken into account to include the mutual affects. Different radiation patterns and environments can result in various AS and AOA. As the AS increases, the overall power distribution becomes broad over the large angle. When the AS decreases, the power
distribution becomes narrow and the the spatial correlation tends to increase. In order to maintain low correlation for small AS, the distance between adjacent antennas have to increase. This results in larger antenna system.

The I-METRA model which is used in 3GPP networks, allows for statistical MIMO channel to be generated. Its unique function is to incorporate the steering vector of the radiation properties of the antenna. The radiation pattern can be continuously changing to maximize the performance of the MIMO channel system. The channel matrix $H(w)$ can be used to generate the channel capacity of the system. From Chapter 7, we found that by increasing the number of transmitters and receivers, the performance and capacity of the MIMO channel can increase. For diversity coding, the maximum capacity may not be greater with MIMO system, but with SIMO or MISO. It can be match or greater depending on the coding techniques used such as Trellis Coding, STC, or OSTC to maximize the performance of the system.

SCM ray-based systems was easier to implement then the I-METRA model. The SCM directly modeled the statistical behaviour of the MIMO channel. It maintains the statistical charactersitic in time, space, and frequency domains. It was flexible in changing the various types of PDP and PAS. It supported both LOS and NLOS channels. Its overall effect rank of channel matrix $H$ depend on the number of sub-rays in each path. Channel capacity and diversity gain decrease as the correlation between the antenna elements increases, and therefore antenna spacing must be set large enough to reduce the correlation. The direction of travel for Ray-based SCM system also affected the overall correlation. When direction of Travel was Broadside direction the correlation magnitude
was very large. When direction of Travel was 90° from Broadside, the correlation magnitude was very low.

These are the proposed recommended steps of designing an antenna system. For the application of use, decide the desired frequency of communication between devices. If it has to work with existing devices, the range of frequencies may be limited by the device. Wire antennas are capable of being adjusted whereas fixed antenna structures such as microstrip antennas are limited in resonant frequency and the physical properties of the antenna may limit frequency range. For instance, if we decided that we wanted to be able to communicate with existing wireless devices that operated at 2.4GHz, than a single antenna element should be designed using the appropriate software models (Ex. Microstrip - Transmission Line, Cavity Model). The user would design for the appropriate radiation pattern, gains, efficiencies, polarization, beamwidth, and bandwidth. The next step is to decide for the application, are other devices capable of communicating with multiple antennas. You would have to factor in coding schemes, hardware, algorithms to be able to decode multiple antenna systems. At the point you have decide that MIMO system is a practical and feasible option for the application. It is important to decide the correct antenna spacing. Are you limited by size of application? If so, is the spatial correlation between adjacent antennas acceptable limits. This can be predicted by the I-METRA and SCM Ray-based models as described. Each model you can characterize the fading channel with respect to its indoor/outdoor environments. How many antennas should be used? This could be determined by running channel capacity simulations for the application. SISO, SIMO, MISO, and MIMO should all be modeled to see where the tradeoffs between increasing channel capacity or wasting resources due
to limitations of the other device you are trying to communicate with. Other factors include, beamforming or precoding capabilities, spatial multiplexing, and spatial diversity. By simulations we are capable of estimating, maximum capacity, data rates speeds, BER vs SNR for reliability, and maximum transmitting and receiving distances. Will the final product meet the application necessities and will it be technologically advanced enough for future systems to come?

To increase MIMO capacity, spatial diversity, and precoding even further, planar arrays systems as describe in the smart antenna systems are ways to maximize beam gains, SOI, elimination SNOI, 3-dimension scanning capabilities, and increase the overall performance of the system by utilizing time, space, and frequency. As applications such as in multimedia device, Tablets, PDAs, gaming systems, PCs/Laptops have to communicate between multiple users at once, the complexity of the application and software challenges arise again to utilize time, space, and frequency capabilities of the antenna system or multiple antennas systems used together. For single user MIMO to MIMO systems, the means of increasing capacity can be supported by spatial multiplexing while providing spatial diversity gain. As Multi-MIMO systems are developed and used, each system must be divided into maximum number users without degrading the quality of systems. MIMO systems can help increase channel capacity, signal reliability, and security, but its limitations are factored upon antenna hardware and software processing capabilities.
References


Appendix

A. Antenna Design Structure

The antenna design for microstrip antennas was using code from “Antenna Theory Analysis and Design written by Constantine A. Balanis. The code consisted of model for designs for the following. Chapter 2 in the CD gives outlines for solving for directivity as well as creating 2-D and 3-D models for antenna. Chapter 6 gives code to calculate directivities and HPBW for the Linear Arrays of Broadside, End-Fire, Hansen-Woodyard, Scanning, Binomial, and Chevby.

Source Code Details Outlined in Appendix A.

A-1 Directivity.m

A-2 Arrays.m

A-3 Microstrip.m

A-1 Directivity.m

%**********************************************************
%*********************************
% DIRECTIVITY.m
%**********************************************************
% This is a MATLAB based program that calculates the:
% I. radiated power
% II. maximum directivity (dimensionless and in dB)
% of any antenna. The maximum directivity is calculated
% using the trailing edge method in increments of 1 degree in theta and 1 degree in phi.
% Input parameters:
%  1. TL, TU: Lower and upper limits in theta (in degrees)
%  2. PL, PU: Lower and upper limits in phi (in degrees)
%  3. U(theta,phi): The radiation intensity function (in file 'U.m')
%
% **NOTE: The radiation intensity function F must be provided for a given
% antenna and should be inserted into the subroutine U.
%
% **EXAMPLE: If the antenna is radiating only in the upper hemisphere,
% the input parameters are TL = 0, TU = 90, PL = 0, PU = 360
%
--------------------------------------------------------------------------------------------------------------------------
% Converted from fortran to MATLAB 3/2002 by Kelly O'Dell
% Modified by Marios Gkatzianas
% --------------------------------------------------------------------------------------------------------------------------

close all;
clear all;
format long;

fprintf('!!!WARNING: Make sure you define the radiation intensity in the file U.m !!! \n');
s=which('U.m');
if isempty(s),
    disp(['File U.m not found in current directory. Make sure that the radiation intensity is' ...
         ' defined in that file!!']);
end;

device=3;

while ((device~=1)&(device~=2)),
    fprintf('Output device option \n\tOption (1): Screen \n\tOption (2): File \n');
    device=input('Output device=');
    if device==2,
filename=input('Input the desired output filename: 
%s');
elseif device~=1,
    fprintf('
Outputting device number should be either 1 or 2
');
end;
end;

TL=[]; TU=[]; PL=[]; PU=[];

while isempty(TL),
    TL = input('
The lower bound of theta in degrees = '); 
end;

while isempty(TU),
    TU = input('The upper bound of theta in degrees = '); 
end;

while isempty(PL),
    PL = input('The lower bound of phi in degrees = '); 
end;

while isempty(PU),
    PU = input('The upper bound of phi in degrees = '); 
end;

theta=(TL:TU)*pi/180; % convert into radians
phi=(PL:PU)*pi/180;

dth=theta(2)-theta(1);
dph=phi(2)-phi(1);

[THETA,PHI]=meshgrid(theta,phi)
x=U(THETA,PHI);

Prad=sum(sum(x.*sin(THETA)*dth*dph));
D=4*pi*max(max(x))/Prad;
D_db=10*log10(D);

if (device==2),
    fid=fopen(filename,'wt');
else
    fid=2; % standard output
end;

fprintf(fid,'
Input parameters:
-----------------');
fprintf(fid,'\nThe lower bound of theta in degrees = %3.0f',TL);
fprintf(fid,'\nThe upper bound of theta in degrees = %3.0f',TU);
fprintf(fid,'\nThe lower bound of phi in degrees = %3.0f',PL);
fprintf(fid,'\nThe upper bound of phi in degrees = %3.0f',PU);
fprintf(fid,'\nOutput parameters:
------------------
Radiated power (watts) = %6.4f',Prad);
fprintf(fid,'\nDirectivity (dimensionless) = %6.4f',D);
fprintf(fid,'\nDirectivity (dB) t= %6.4f',D_db);
fprintf(fid,'\n');

if(device == 2)
    fclose(fid);
end

function y = U(THETA, PHI)
% **Example**
% y = sin(THETA).* (sin(PHI)).^2;
y = sin(THETA).* (sin(PHI)).^2;

A-2 Arrays.m

% ******************************************************************
% ARRAYS
% This is a MATLAB based program that computes the
% radiation characteristics of:
% I. LINEAR ARRAYS (UNIFORM & BROADSIDE NONUNIFORM)
% II. PLANAR ARRAY (BROADSIDE UNIFORM)
% III. CIRCULAR ARRAY (BROADSIDE UNIFORM)
% THE UNIFORM AND BROADSIDE NONUNIFORM LINEAR ARRAYS HAVE N
% ELEMENTS
% PLACED EQUIDISTANTLY ALONG THE Z-AXIS.
% BROADSIDE PLANAR UNIFORM ARRAY HAS M x N ELEMENTS PLACED
% EQUIDISTANTLY ALONG THE X AND Y AXES
% OPTION I. LINEAR ARRAYS
% OPTION A. UNIFORM
% ** CHOICES: ARRAY TYPE
% 1. BROADSIDE (MAXIMUM ALONG THETA = 0 DEGREES)
2. ORDINARY END-FIRE
   (a). MAXIMUM ALONG THETA = 0 DEGREES
   (b). MAXIMUM ALONG THETA = 180 DEGREES
3. HANSEN WOODYARD END-FIRE
   (a). MAXIMUM ALONG THETA = 0 DEGREES
   (b). MAXIMUM ALONG THETA = 180 DEGREES
4. SCANNING (MAXIMUM ALONG THETA = THETA_MAX)

** INPUT PARAMETERS:

1. NUMBER OF ELEMENTS
2. SPACING BETWEEN THE ELEMENTS (IN WAVELENGTHS)
3. DIRECTION OF ARRAY MAXIMUM (THETA_MAX IN DEGREES)

** PROGRAM OUTPUT:

1. NORMALIZED ARRAY FACTOR
2. DIRECTIVITY (DIMENSIONLESS & IN dB) USING NUMERICAL INTEGRATION OF THE ARRAY FACTOR
3. HALF-POWER BEAMWIDTH (IN DEGREES) USING AN ITERATIVE METHOD (FOR ALL MAXIMA IN THE PATTERN)

OPTION B. NONUNIFORM BROADSIDE
** CHOICES: ARRAY TYPE

1. BINOMIAL
2. DOLPH-TSCHEBYSCHEFF

** BINOMIAL ARRAY INPUT PARAMETERS:

1. NUMBER OF ELEMENTS
2. SPACING BETWEEN THE ELEMENTS (IN WAVELENGTHS)

** DOLPH-TSCHEBYSCHEFF INPUT PARAMETERS:

1. NUMBER OF ELEMENTS
2. SPACING BETWEEN THE ELEMENTS (IN WAVELENGTHS)
3. SIDE LOBE LEVEL (IN POSITIVE dB; i.e., 30 dB)

** PROGRAM OUTPUT:

1. NORMALIZED EXCITATION COEFFICIENTS (An)
2. NORMALIZED ARRAY FACTOR
3. DIRECTIVITY (IN dB) USING NUMERICAL INTEGRATION OF THE ARRAY FACTOR
4. HALF-POWER BEAMWIDTH (IN DEGREES) USING AN ITERATIVE METHOD (FOR ALL MAXIMA THAT OCCUR IN THE PATTERN)

OPTION II. PLANAR ARRAY

** ARRAY INPUT PARAMETERS:

1. NUMBER OF ARRAY ELEMENTS IN X-DIRECTION
2. SPACING BETWEEN THE ELEMENTS IN X-DIRECTION (IN WAVELENGTHS)
3. NUMBER OF ARRAY ELEMENTS IN Y-DIRECTION
4. SPACING BETWEEN THE ELEMENTS IN Y-DIRECTION (IN
WAVELENGTHS)
5. MAXIMUM BEAM DIRECTION - ANGLE THETA (IN DEGREES)
6. MAXIMUM BEAM DIRECTION - ANGLE PHI (IN DEGREES)
7. THE PHI ANGLE (IN DEGREES) AT WHICH THE 2-D ANTENNA
    PATTERN NEEDS TO BE EVALUATED (PHIEVAL IN DEG.)

NOTE: ONLY THE ELEVATION ANTENNA PATTERN IS EVALUATED. THIS
----- PATTERN RANGES FROM THETA=0 DEG. TO THETA=180 DEG. WHEREAS
PHI REMAINS CONSTANT AT PHIEVAL. IF THE PATTERN NEEDS TO
BE EVALUATED IN THE BACKSIDE REGION OF THE 2-D ARRAY,
THEN THE PROGRAM NEEDS TO BE RE-RUN FOR A NEW PHIEVAL
WHICH MUST BE EQUAL TO THE PREVIOUS VALUE PLUS 180 DEG.

** PROGRAM OUTPUT:
1. NORMALIZED ARRAY FACTOR EVALUATED AT A GIVEN ANGLE
   (PHIEVAL)
2. UNIFORM PROGRESSIVE PHASE SHIFT IN X AND Y DIRECTIONS
   (IN DEGREES)
3. DIRECTIVITY (IN dB) USING NUMERICAL INTEGRATION OF THE
   ARRAY FACTOR
4. HALF-POWER BEAMWIDTH (IN DEGREES) FOR ALL MAXIMA THAT
   OCCUR IN THE ELEVATION PLANE OF THE 2-D ANTENNA PATTERN

OPTION III. CIRCULAR ARRAY
** CHOICES: EQUATION TYPE
1. EXPONENTIAL FUNCTION FORM
2. BESSEL FUNCTION FORM

ALL THE INPUT PARAMETERS ARE IN TERMS OF THE WAVELENGTH.
*****************************************************
Written by: Seunghwan Yoon, Arizona State University, 08/12/2002
Revised by: Seunghwan Yoon, Arizona State University, 12/03/2002
*****************************************************

function[]=ARRAYS;
close all;
clc;
option_a=0;
while((option_a~=1)&(option_a~=2)),
disp(strvcat('
OUTPUT DEVICE OPTION FOR THE OUTPUT PARAMETERS','
  OPTION (1):SCREEN','OPTION (2):OUTPUT FILE'));
  option_a=input('OUTPUT DEVICE =');
end
if option_a==2, %OUTPUT FILE
  filename=input('INPUT THE DESIRED OUTPUT FILENAME <in single quotes>
    = ','
    'a');
  fid=fopen(filename,'wt');
end
disc=181;
MM=disc;
NN=disc;
theta_low=0;
theta_up=180;
phi_low=0;
phi_up=360;

% CHOICE OF LINEAR OR PLANAR ARRAY OR CIRCULAR ARRAY
option_b=0;
while ((option_b~=1)&(option_b~=2)&(option_b~=3)),
disp(strvcat('LINEAR OR PLANAR ARRAY','OPTION (1):LINEAR ARRAY','OPTION (2):PLANAR ARRAY','OPTION (3):CIRCULAR ARRAY'));
option_b=input('OPTION NUMBER =');
end;

if option_b==1, 
% LINEAR ARRAY
% CHOICE OF UNIFORM OR NONUNIFORM
option_c=0;
while ((option_c~=1)&(option_c~=2)),
disp(strvcat('UNIFORM OR NONUNIFORM ARRAY','OPTION (1):UNIFORM ARRAY','OPTION (2):NONUNIFORM ARRAY'));
option_c=input('OPTION NUMBER =');
end;
M=1800;
%MM=180;
%NN=180;
k=2*pi;
theta=linspace(0,pi,M+1);
theta3=linspace(theta_low*pi/180,theta_up*pi/180,MM+1);
phi3=linspace(phi_low*pi/180,phi_up*pi/180,NN+1);
[THETA,PHI]=meshgrid(theta3,phi3);
dtheta=pi/M;

if option_c==1, 
% UNIFORM ARRAY
% CHOICE OF ARRAY TYPE
% (BROADSIDE,ORDINARY END-FIRE,HANSEN-WOODYARD END-FIRE,SCANNING)
option_d=0;
while ((option_d~=1)&(option_d~=2)&(option_d~=3)&(option_d~=4)),
option_d=input('OPTION NUMBER =');
end;

if option_d==1, 
% BROADSIDE
Nelem=0;
while (Nelem<1),
Nelem=floor(input('NUMBER OF ELEMENTS ='));
end
d=input('SPACING d BETWEEN THE ELEMENTS (IN WAVELENGTHS) =');
beta=0;
psi=k.*d.*cos(theta)+beta;
psi3=k.*d.*cos(THETA)+beta;
AF = \text{sinc}((Nelem \cdot \psi / 2) / \pi) / \text{sinc}(\psi / 2 / \pi); \quad \% \text{I used sinc function.}
AF3 = \text{sinc}((Nelem \cdot \psi3 / 2) / \pi) / \text{sinc}(\psi3 / 2 / \pi); \quad \% 
\text{sinc}(x) = \sin(\pi x) / (\pi x). \quad \text{not, } \sin(x) / x.

\textbf{elseif} \quad \text{option}_d = 2, \quad \% \text{ORDINARY END-FIRE}
\text{Nelem} = 0;
\text{while} \quad (\text{Nelem} < 1),
\text{Nelem} = \text{floor}(\text{input('NUMBER OF ELEMENTS =')});
\text{end}
d = \text{input('SPACING d BETWEEN THE ELEMENTS (IN WAVELENGTHS) =')};
\text{thmax} = 90;
\text{while} ((\text{thmax} = = 0) \& (\text{thmax} = = 180)),
\quad \text{thmax} = \text{input('ANGLE WHERE MAXIMUM OCCURS (THETA = 0 OR 180 DEG.) =')};
\text{end}
\text{if} \quad \text{abs}(\text{thmax}) < \varepsilon, \quad \text{beta} = -k \cdot d;
\text{elseif} \quad \text{abs}(\text{thmax} - 180) < \varepsilon, \quad \text{beta} = k \cdot d;
\text{end}
\psi = k \cdot d \cdot \cos(\theta) + \text{beta};
\psi3 = k \cdot d \cdot \cos(\Theta) + \text{beta};
AF = \text{sinc}((\text{Nelem} \cdot \psi / 2) / \pi) / \text{sinc}(\psi / 2 / \pi);
AF3 = \text{sinc}((\text{Nelem} \cdot \psi3 / 2) / \pi) / \text{sinc}(\psi3 / 2 / \pi);

\textbf{elseif} \quad \text{option}_d = 3, \quad \% \text{HANSEN-WOODYARD END-FIRE}
\text{Nelem} = 0;
\text{while} \quad (\text{Nelem} < 1),
\text{Nelem} = \text{floor}(\text{input('NUMBER OF ELEMENTS =')});
\text{end}
d = \text{input('SPACING d BETWEEN THE ELEMENTS (IN WAVELENGTHS) =')};
\text{thmax} = 90;
\text{while} ((\text{thmax} = = 0) \& (\text{thmax} = = 180)),
\quad \text{thmax} = \text{input('ANGLE WHERE MAXIMUM OCCURS (THETA = 0 OR 180 DEG.) =')};
\text{end}
\text{if} \quad \text{abs}(\text{thmax}) < \varepsilon, \quad \text{beta} = -k \cdot d + \pi / (\text{Nelem} - 0.00001); \quad \% \text{what is the value (0.00001)?}
\text{elseif} \quad \text{abs}(\text{thmax} - 180) < \varepsilon, \quad \text{beta} = k \cdot d + \pi / \text{Nelem};
\text{end}
\psi = k \cdot d \cdot \cos(\theta) + \text{beta};
\psi3 = k \cdot d \cdot \cos(\Theta) + \text{beta};
AF = \text{sinc}((\text{Nelem} \cdot \psi / 2) / \pi) / \text{sinc}(\psi / 2 / \pi);
AF3 = \text{sinc}((\text{Nelem} \cdot \psi3 / 2) / \pi) / \text{sinc}(\psi3 / 2 / \pi);

\textbf{elseif} \quad \text{option}_d = 4, \quad \% \text{SCANNING}
\text{Nelem} = 0;
\text{while} \quad (\text{Nelem} < 1),
\text{Nelem} = \text{floor}(\text{input('NUMBER OF ELEMENTS =')});
\text{end}
d = \text{input('SPACING d BETWEEN THE ELEMENTS (IN WAVELENGTHS) =')};
\text{thmax} = \text{input('ANGLE WHERE MAXIMUM OCCURS (THETA = 0 OR 180 DEG.) ='});
\beta = \text{k} \cdot \text{d} \cdot \cos(\text{thmax} \cdot \pi / 180);
\psi = \text{k} \cdot \text{d} \cdot \cos(\theta) + \beta;
\psi3 = \text{k} \cdot \text{d} \cdot \cos(\Theta) + \beta;
AF = \text{sinc}((\text{Nelem} \cdot \psi / 2) / \pi) / \text{sinc}(\psi / 2 / \pi);
AF3 = \text{sinc}((\text{Nelem} \cdot \psi3 / 2) / \pi) / \text{sinc}(\psi3 / 2 / \pi);
elseif option_c==2, % NONUNIFORM ARRAY
% CHOICE OF ARRAY TYPE
% (BINOMIAL,DOLPH-TSCHEBYSCHEFF)

option_e=0;
while ((option_e~=1)&&(option_e~=2)),
    disp(strvcat('ARRAY NAMES','OPTION (1):BINOMIAL','OPTION (2):DOLPH
    TSCHEBYSCHEFF'));
    option_e=input('OPTION NUMBER =');
end;

if option_e==1, % BINOMIAL
    Nelem=0;
    while (Nelem<1),
        Nelem=floor(input('NUMBER OF ELEMENTS ='));
    end
    d=input('SPACING d BETWEEN THE ELEMENTS (IN WAVELENGTHS) =');
    beta=0;
    for i=1:M+1
        [AF,Ncoef,Coef]=bin(theta,Nelem,d);
    end
    for i=1:MM+1
        [AF3,Ncoef3,Coef3]=bin(THETA,Nelem,d);
    end
end

elseif option_e==2, % DOLPH-TSCHEBYSCHEFF
    Nelem=0;
    while (Nelem<1),
        Nelem=floor(input('NUMBER OF ELEMENTS ='));
    end
    d=input('SPACING d BETWEEN THE ELEMENTS (IN WAVELENGTHS) =');
    beta=0;
    RdB=input('SIDE LOBE LEVEL (IN dB) =');
    for i=1:M+1;
        [AF,Ncoef,Coef]=tscheby(theta,Nelem,d,RdB);
    end
    for i=1:MM+1;
        [AF3,Ncoef3,Coef3]=tscheby(THETA,Nelem,d,RdB);
    end
end

Coef=Coef(1:Ncoef);
Ncoef=Coef(1:Ncoef)/Coef(Ncoef);
end

U=(abs(AF)./max(abs(AF))).^2;
Prad=2*pi*sum(U.*sin(theta).*dtheta);
D=4*pi*U/Prad;
DdB=10.*log10(D+eps);
Do=max(D);
DodB=max(DdB);
if Nelem==1|max(AF)<2*min(AF)
    hp=0;
    thmax=0;
else
    [hp,thmax]=hpbw(U,M);
end
else if option_b==2, % PLANAR ARRAY
\%M=180;
\%N=180;
k=2*pi;
dtheta=pi/MM;
dphi=2*pi/NN;
Mx=0;
while (Mx<2),
Mx=floor(input('NUMBER OF ELEMENTS IN THE X-DIRECTION ='));
end
Ny=0;
while (Ny<2),
Ny=floor(input('NUMBER OF ELEMENTS IN THE Y-DIRECTION ='));
end

dx=input('SPACING dx BETWEEN THE ELEMENTS (IN WAVELENGTHS) =')-1e-10;
dy=input('SPACING dy BETWEEN THE ELEMENTS (IN WAVELENGTHS) =')-1e-10;
thmax2=input('MAXIMUM BEAM DIRECTION - ANGLE THETA (IN DEGREES - INTEGER #) =');
phimax2=input('MAXIMUM BEAM DIRECTION - ANGLE PHI (IN DEGREES - INTEGER #) =');
phieval=input('THE PATTERN IS EVALUATED AT AN ANGLE PHI (IN DEGREES - INTEGER #) =');
dtor=pi/180;
betax=-k*dx*sin(dtor*thmax2)*cos(dtor*phimax2);
betay=-k*dy*sin(dtor*thmax2)*sin(dtor*phimax2);

theta=linspace(theta_low*pi/180,theta_up*pi/180,MM+1);
phi=linspace(phi_low*pi/180,phi_up*pi/180,NN+1);

[THETA,PHI]=meshgrid(theta,phi);
AF3=af10(THETA,PHI,Mx,Ny,dx,dy,betax,betay);

Prad=sum(sum(abs(AF3).^2.*sin(THETA)*dtheta*dphi));
D1=4*pi*abs(AF3).^2/Prad;
D1dB=10.*log10(D1);
Do=4*pi*max(max(abs(AF3).^2))/Prad;
DodB=10.*log10(Do);

theta=linspace(0,pi,10*MM+1);
phi=phieval*dtor;
AF=af10(theta,phi,Mx,Ny,dx,dy,betax,betay);
D=4*pi*abs(AF).^2/Prad;
DdB=10.*log10(D);
U=(abs(AF)./max(abs(AF))).^2;

phi180=phieval*pi;
AF180=af10(theta,phi180,Mx,Ny,dx,dy,betax,betay);
D180=4*pi*abs(AF180).^2/Prad;
D180dB=10.*log10(D180);
U180=(abs(AF180)./max(abs(AF180))).^2;
elseif option_b==3, % CIRCULAR ARRAY
    option_f=0;
while ((option_f~=1)&(option_f~=2)),
    disp(strvcat('FUNCTIONS', 'OPTION (1): EXPONENTIAL FUNCTION FORM', 'OPTION (2): BESSEL FUNCTION FORM'));
    option_f=input('OPTION NUMBER =');
end;

%M=180;
%N=180;
x=2*pi;
dtheta=pi/MM;
dphi=2*pi/NN;

Nelem=0;
while (Nelem<2),
    Nelem=floor(input('NUMBER OF ELEMENTS ='));
end
rad=input('RADIUS (IN WAVELENGTHS) =');
if option_f==2, % BESSEL
    m=-1;
    while (m<0),
        m=floor(input('Residuals[>2]='));
    end
end

theta0=input('MAXIMUM BEAM DIRECTION - ANGLE THETA (IN DEGREES - INTEGER #)=');
phi0=input('MAXIMUM BEAM DIRECTION - ANGLE PHI (IN DEGREES - INTEGER #)=');
phieval=input('THE PATTERN IS EVALUATED AT AN ANGLE PHI (IN DEGREES - INTEGER #)=');

if option_f==1, % EXPONENTIAL
    dtor=pi/180;
    theta=linspace(theta_low*pi/180,theta_up*pi/180,MM+1);
    phi=linspace(phi_low*pi/180,phi_up*pi/180,NN+1);
    [THETA,PHI]=meshgrid(theta,phi);
    AF3=afc(THETA,PHI,theta0,phi0,Nelem,rad);
    Prad=sum(sum(abs(AF3).^2.*sin(THETA)*dtheta*dphi));
    D1=4*pi*abs(AF3).^2/Prad;
    D1dB=10.*log10(D1);
    Do=4*pi*max(max(abs(AF3).^2))/Prad;
    Dodb=10.*log10(Do);

    theta=linspace(0,pi,10*MM+1);
    phi=phieval*dtor;
    AF=afc(theta,phi,theta0,phi0,Nelem,rad);
    D=4*pi*abs(AF).^2/Prad;
    DdB=10.*log10(D);
    U=(abs(AF)./max(abs(AF))).^2;
AFdB = 10 * log10(U);

phi180 = phi.eval * dtor - pi;
AF180 = afc(theta, phi180, theta0, phi0, Nelem, rad);
D180 = 4 * pi * abs(AF180).^2 / Prad;
D180dB = 10 * log10(D180);
U180 = (abs(AF180) ./ max(abs(AF180))).^2;

end

if option_f == 2, % BESSEL
  dtor = pi / 180;
  theta = linspace(theta_low * pi / 180, theta_up * pi / 180, MM + 1);
  phi = linspace(phi_low * pi / 180, phi_up * pi / 180, NN + 1);
  [THETA, PHI] = meshgrid(theta, phi);
  AF3 = afc3(THETA, PHI, theta0, phi0, Nelem, rad, m);
  Prad = sum(sum(abs(AF3).^2 .* sin(THETA) .* dtheta .* dphi));
  D1 = 4 * pi * abs(AF3).^2 / Prad;
  D1dB = 10 * log10(D1);
  Do = 4 * pi * max(max(abs(AF3).^2)) / Prad;
  DodB = 10 * log10(Do);

  AF30 = afc3(THETA, PHI, theta0, phi0, Nelem, rad, 0);
  Prad0 = sum(sum(abs(AF30).^2 .* sin(THETA) .* dtheta .* dphi));
  D10 = 4 * pi * abs(AF30).^2 / Prad0;
  D10dB = 10 * log10(D10);
  Do0 = 4 * pi * max(max(abs(AF30).^2)) / Prad0;
  DodB0 = 10 * log10(Do0);

  theta = linspace(0, pi, 10 * MM + 1);
  phi = phi.eval * dtor;
  AF = afc3(theta, phi, theta0, phi0, Nelem, rad, m);
  D = 4 * pi * abs(AF).^2 / Prad;
  DdB = 10 * log10(D);
  U = (abs(AF) ./ max(abs(AF))).^2;
  AFdB = 10 * log10(U);

  AF0 = afc3(theta, phi, theta0, phi0, Nelem, rad, 0);
  D0 = 4 * pi * abs(AF0).^2 / Prad0;
  DdB0 = 10 * log10(D0);
  U0 = (abs(AF0) ./ max(abs(AF0))).^2;
  AFdB0 = 10 * log10(U0);

  phi180 = phi.eval * dtor - pi;
  AF180 = afc3(theta, phi180, theta0, phi0, Nelem, rad, m);
  D180 = 4 * pi * abs(AF180).^2 / Prad;
  D180dB = 10 * log10(D180);
  U180 = (abs(AF180) ./ max(abs(AF180))).^2;

end
end

if option_a == 2
  diary(filename);
end
scale=0;
while ((scale~1)&(scale~2)),
disp(strvcat('DIMENSIONLESS OR dB SCALE IN 3D DIRECTIVITY PLOT','OPTION (1):DIMENSIONLESS SCALE','OPTION (2):dB SCALE'));
scale=input('OPTION NUMBER =');
end;

% Let's go output!!!
disp(strvcat('********************************************************'));
disp(strvcat('PROGRAM OUTPUT'));
disp(strvcat('********************************************************'));
disp(strvcat('INPUT SPECIFICATION'));
disp(strvcat('--------------------------------------------------------'));
if option_b==1
  if option_c==1&option_d==1
    disp(strvcat('UNIFORM BROADSIDE ARRAY'));
  end
  if option_c==1&option_d==2
    disp(strvcat('UNIFORM ORDINARY END-FIRE ARRAY'));
  end
  if option_c==1&option_d==3
    disp(strvcat('UNIFORM HANSEN-WOODYARD END-FIRE ARRAY'));
  end
  if option_c==1&option_d==4
    disp(strvcat('UNIFORM SCANNING ARRAY'));
  end
  if option_c==2&option_e==1
    disp(strvcat('NONUNIFORM BINOMIAL ARRAY'));
  end
  if option_c==2&option_e==2
    disp(strvcat('NONUNIFORM DOLPH-TSCHEBYSCHEFF ARRAY'));
  end
  disp(['NUMBER OF ARRAY ELEMENTS = ',num2str(Nelem)]);
  disp(['SPACING BETWEEN THE ELEMENTS (IN WAVELENGTHS) = ',num2str(d)]);
  if option_c==1&option_d~1
    disp(['MAXIMUM NEEDS TO OCCUR AT = ',num2str(thmax)]);
  end
  if option_c==2&option_e==2
    disp(['SIDE LOBE LEVEL (IN dB) = ',num2str(RdB)]);
  end
  disp('OUTPUT CHARACTERISTICS OF THE ARRAY');
  disp('--------------------------------------------------------');
  disp(['DIRECTIVITY = ',num2str(DodB),' dB']);
  disp(['DIRECTIVITY = ',num2str(Do),' dimensionless']);
  disp(['NUMBER OF MAXIMA BETWEEN 0 AND 180 DEGREES = ',num2str(No_maxima)]);
  for i=1:No_maxima;
    disp(['HPBW FOR MAXIMUM # =',num2str(i),' ',num2str(hp(i)),' degrees THMAX = ',num2str(thmax(i)),' degrees']);
  end
if option_b==1&option_c==2
    if 2*round(Nelem/2) ~= Nelem
        Coef(1)=2*Coef(1);
    end
    disp('TOTAL EXCITATION COEFFICIENTS FOR THE ARRAY DESIGN');
    disp(Coef);
    disp('NORMALIZED TOTAL EXCITATION COEFFICIENTS (RELATIVE TO EDGE)');
    Ncoef=Coef/Coef(round(Nelem/2));
    disp(Ncoef);
    disp('NORMALIZED TOTAL EXCITATION COEFFICIENTS (RELATIVE TO CENTER)');
    Ncoef=Ncoef./Ncoef(1);
    disp(Ncoef);
end
end

if option_b==2
    disp(strvcat('UNIFORM PLANAR ARRAY'));
    disp([strvcat('NUMBER OF ELEMENTS IN X-DIRECTION = ',num2str(Mx))]);
    disp([strvcat('SPACING BETWEEN THE ELEMENTS IN X-DIRECTION (IN WAVELENGTHS) = ',num2str(dx))]);
    disp([strvcat('NUMBER OF ELEMENTS IN Y-DIRECTION = ',num2str(Ny))]);
    disp([strvcat('SPACING BETWEEN THE ELEMENTS IN Y-DIRECTION (IN WAVELENGTHS) = ',num2str(dy))]);
    disp([strvcat('MAXIMUM BEAM DIRECTION - THETA (IN DEGREES) = ',num2str(thmax2))]);
    disp([strvcat('MAXIMUM BEAM DIRECTION - PHI (IN DEGREES) = ',num2str(phimax2))]);
    disp([strvcat('THE 2D ANTENNA PATTERN IS EVALUATED AT AN ANGLE PHI (IN DEGREES) = ',num2str(phieval))]);
    disp(strvcat('OUTPUT CHARACTERISTICS OF THE ARRAY'));
    disp(strvcat('PROGRESSIVE PHASE SHIFT IN X-DIRECTION = ',num2str(betax/dtor),' degrees'));
    disp(strvcat('PROGRESSIVE PHASE SHIFT IN Y-DIRECTION = ',num2str(betay/dtor),' degrees'));
    disp([strvcat('DIRECTIVITY BASED ONLY ON THE FIELDS ABOVE THE XY-PLANE')]);
    disp([strvcat('DIRECTIVITY = ',num2str(DodB+10*log10(2))),' dB']);
    disp([strvcat('DIRECTIVITY = ',num2str(Do*2),' dimensionless')]);
    if max(AF3)<2*min(AF3)
        hp=0;
        thmax=0;
    else
        [hp,thmax]=hpbw(U,10*MM);
    end
    No_maxima=length(thmax);
    disp([strvcat('DIRECTIVITY BASED ON THE FIELDS ABOVE AND BELOW THE XY-PLANE')]);
    disp([strvcat('DIRECTIVITY = ',num2str(DodB),' dB')]);
    disp([strvcat('DIRECTIVITY = ',num2str(Do),' dimensionless')]);
    disp([strvcat('EVALUATION PLANE: NUMBER OF MAXIMA BETWEEN 0 AND 180 DEGREES = ',num2str(No_maxima))]);
    for i=1:No_maxima;
        disp([strvcat('HPBW FOR MAXIMUM #',num2str(i),', ',num2str(hp(i)),' degrees THMAX = ',num2str(thmax(i)),' degrees')]);
    end
end

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if option_b==3
    disp(strvcat('UNIFORM CIRCULAR ARRAY'));
    disp(['NUMBER OF ELEMENTS = ',num2str(Nelem)]);
    disp(['RADIUS (IN WAVELENGTHS) = ',num2str(rad)]);
    disp(['MAXIMUM BEAM DIRECTION - THETA (IN DEGREES) = '
         ,num2str(theta0)]);
    disp(['MAXIMUM BEAM DIRECTION - PHI (IN DEGREES) = ',num2str(phi0)]);
    disp(['THE 2D ANTENNA PATTERN IS EVALUATED AT AN ANGLE PHI (IN DEGREES) = '
         ,num2str(phieval)]);
    disp(strvcat('OUTPUT CHARACTERISTICS OF THE ARRAY'));
    disp(strvcat('--------------------------------------------------------'));
    disp(['DIRECTIVITY BASED ONLY ON THE FIELDS ABOVE THE XY-PLANE'])
    disp(['DIRECTIVITY = ',num2str(DodB+10*log10(2)),' dB'])
    disp(['DIRECTIVITY = ',num2str(Do*2),' dimensionless'])
    if max(AF3)<2*min(AF3)
        hp=0;
        thmax=0;
    else
        [hp,theta0]=hpbw(U,10*MM);
    end
    No_maxima=length(theta0);
    disp(['DIRECTIVITY BASED ON THE FIELDS ABOVE AND BELOW THE XY-PLANE'])
    disp(['DIRECTIVITY = ',num2str(DodB),' dB'])
    disp(['DIRECTIVITY = ',num2str(Do),' dimensionless'])
    disp(['EVALUATION PLANE: NUMBER OF MAXIMA BETWEEN 0 AND 180 DEGREES = '
         ,num2str(No_maxima)]);
    for i=1:No_maxima;
        disp(['HPBW FOR MAXIMUM #',num2str(i),' ',num2str(hp(i)),' degrees THMAX = ',num2str(theta0(i)),' degrees'])
    end
end

*** NOTE:'

*** *** THE NORMALIZED ARRAY FACTOR (in dB) IS STORED IN

*** *** AN OUTPUT FILE CALLED ............ ArrFac.dat

*** *** ===============================================

*** *** *** ***

AFdB=10.*log10(U);
if option_b==1,
    for i=1:901
        thetarec(i)=theta(i*2-1);
        AFdBrec(i)=AFdB(i*2-1);
    end
    for i=902:1801
        thetarec(i)=2*pi-theta(3601-i*2+3);
        AFdBrec(i)=AFdB(3601-i*2+3);
    end
elseif option_b==2|option_b==3,
    for i=1:MM+1
        thetarec(i)=theta(i*10-9);
        AFdBrec(i)=AFdB(i*10-9);
end

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end
for i=MM+2:2*MM+1
    thetarec(i)=2*pi-thetarec(2*MM+2-i);
    AFdBrec(i)=AFdBrec(2*MM+2-i);
end

fidaf=fopen('ArrFac.dat','wt');
fprintf(fidaf,'%7.3f          %9.5f
',[thetarec.*180/pi; AFdBrec]);
fclose(fidaf);

% PLOT THE GRAPHS
% ARRAY FACTOR
clf;
plot(theta*180/pi,AFdB,'m','linewidth',2);
xlabel(['\theta' (degrees)]),ylabel('ARRAY FACTOR (dB)')
grid on;
axis([0 180 max(min(AFDdB)-1,-60) 1])
t1=text(1,1,['\text{HPBW} = ',num2str(max(hp)),' (degrees)'])
set(t1,'units','normalized','position',[1 1.05],'horizontalalign','right');

if option_b==1&option_c==1&option_d==1
    s1=title('UNIFORM BROADSIDE','Fontsize',15);
    set(gca,'units','normalized');
    set(s1,'position',[0 1],'horizontalalign','left');
end
if option_b==1&option_c==1&option_d==2
    s2=title('UNIFORM ORINARY END-FIRE','Fontsize',15);
    set(gca,'units','normalized');
    set(s2,'position',[0 1],'horizontalalign','left');
end
if option_b==1&option_c==1&option_d==3
    s3=title('UNIFORM HANSEN-WOODYARD END-FIRE','Fontsize',15);
    set(gca,'units','normalized');
    set(s3,'position',[0 1],'horizontalalign','left');
end
if option_b==1&option_c==1&option_d==4
    s4=title('UNIFORM SCANNING','Fontsize',15);
    set(gca,'units','normalized');
    set(s4,'position',[0 1],'horizontalalign','left');
end
if option_b==1&option_c==2&option_e==1
    s5=title('NONUNIFORM BINOMIAL','Fontsize',15);
    set(gca,'units','normalized');
    set(s5,'position',[0 1],'horizontalalign','left');
end
if option_b==1&option_c==2&option_e==2
    s6=title('NONUNIFORM DOLPH-TSCHEBYSHEFF','Fontsize',15);
    set(gca,'units','normalized');
    set(s6,'position',[0 1],'horizontalalign','left');
end
if option_b==2
    s7=title('UNIFORM PLANAR','Fontsize',15);
if option_b==3&option_f==1
    s8=title('UNIFORM CIRCULAR(EXPONENTIAL)', 'Fontsize',15);
    set(gca, 'units', 'normalized');
    set(s8, 'position', [0 1], 'horizontalalign', 'left');
end

if option_b==3&option_f==2
    hold on;
    plot(theta*180/pi,AFdB0,'b--','linewidth',2);
    legend('principal+residuals','principal')
    s9=title('UNIFORM CIRCULAR(BESSEL)', 'Fontsize',15);
    set(gca, 'units', 'normalized');
    set(s9, 'position', [0 1], 'horizontalalign', 'left');
end

figure;

diff=Do-min(D);
subplot(2,1,1)
plot(theta*180/pi,D,'r','linewidth',2);
xlabel(['\theta', ' (degrees)']),ylabel('DIRECTIVITY(dimensionless)')
grid on;
axis([0 180 floor(min(D)-0.1*diff-.1) ceil(Do+0.1*diff+.1)]);
t2=text(1,1,'D_0 = ',num2str(Do), '(dimensionless)');
set(t2, 'units', 'normalized', 'position', [1 1.05], 'horizontalalign', 'right');

if option_b==1&option_c==1&option_d==1
    s1=title('UNIFORM BROADSIDE', 'Fontsize',15);
    set(gca, 'units', 'normalized');
    set(s1, 'units', 'normalized', 'position', [0 1], 'horizontalalign', 'left');
end

if option_b==1&option_c==1&option_d==2
    s2=title('UNIFORM ORDINARY END-FIRE', 'Fontsize',15);
    set(gca, 'units', 'normalized');
    set(s2, 'units', 'normalized', 'position', [0 1], 'horizontalalign', 'left');
end

if option_b==1&option_c==1&option_d==3
    s3=title('UNIFORM HANSEN-WOODYARD END-FIRE', 'Fontsize',15);
    set(gca, 'units', 'normalized');
    set(s3, 'units', 'normalized', 'position', [0 1], 'horizontalalign', 'left');
end

if option_b==1&option_c==1&option_d==4
    s4=title('UNIFORM SCANNING', 'Fontsize',15);
    set(gca, 'units', 'normalized');
    set(s4, 'units', 'normalized', 'position', [0 1], 'horizontalalign', 'left');
end

if option_b==1&option_c==2&option_e==1
    s5=title('NONUNIFORM BINOMIAL', 'Fontsize',15);
    set(gca, 'units', 'normalized');
end
set(s5,'units','normalized','position',[0 1],'horizontalalign','left');
end
if option_b==1&option_c==2&option_e==2
    s6=title('NONUNIFORM DOLPH-TSCHEBYSCHEFF','Fontsize',15);
    set(gca,'units','normalized');
    set(s6,'units','normalized','position',[0 1],'horizontalalign','left');
end
if option_b==2
    s7=title('UNIFORM PLANAR','Fontsize',15);
    set(gca,'units','normalized');
    set(s7,'units','normalized','position',[0 1],'horizontalalign','left');
end
if option_b==3&option_f==1
    s8=title('UNIFORM CIRCULAR(EXPONENTIAL)','Fontsize',15);
    set(gca,'units','normalized');
    set(s8,'units','normalized','position',[0 1],'horizontalalign','left');
end
if option_b==3&option_f==2
    %         hold on;
    %         plot(theta*180/pi,D0,'b--','linewidth',2);
    %         legend('principal+residuals','principal')
    s9=title('UNIFORM CIRCULAR(BESSEL)','Fontsize',15);
    set(gca,'units','normalized');
    set(s9,'units','normalized','position',[0 1],'horizontalalign','left');
end
diffdB=DodB-min(DDB);
subplot(2,1,2)
plot(theta*180/pi,DDB,'b','linewidth',2);
%if option_b==3&option_f==2
%    hold on;
%    plot(theta*180/pi,D0,'r--','linewidth',2);
%    legend('principal+residuals','principal')
end
t3=text(1,1,['D_0 = ',num2str(DodB),'(dB)]);
set(t3,'units','normalized','position',[1 1.05],'horizontalalign','right');
xlabel(['\theta', '(degrees)']),ylabel('DIRECTIVITY(dB)')
grid on;
axis([0 180 max(-50,10*floor(min(DDB)/10)) 10*ceil(DodB/10)]);
if option_b==1&option_c==2
    for i=1:Nelem;
        Ncoef(i)=1;
    end
end
if option_b==1;
    figure;
    x=(1-Nelem)*d/2:d:(Nelem-1)*d/2;
    y=(1-Nelem)/2:1:(Nelem-1)/2;
    [AX,H1,H2]=plotyy(x,Ncoef(ceil(abs(y)+0.1)),x,beta.*y.*180./pi);
end
set(get(AX(1),'Ylabel','AMPLITUDE','color','r');
set(get(AX(2),'Ylabel','PHASE (degrees)','color','b');
set(AX(1),'ycolor','r');
set(AX(2),'ycolor','b');
set(H1,'Linestyle','-','color','r','linewidth',2,'marker','s');
set(H2,'Linestyle':'','color','b','linewidth',2,'marker','o');
xlabel(['ARRAY LENGTH', ' (\lambda)']);
grid on;
l2=legend('PHASE',2);
[hle,l3]=legend('AMPLITUDE',1); set(hle,'color',[1 1 1]);
if option_b==1&option_c==1&option_d==1
  s1=title('UNIFORM BROADSIDE','Fontsize',15);
  set(gca,'units','normalized');
  set(s1,'units','normalized','position',[0 1],'
  horizontalalign','left');
end
if option_b==1&option_c==1&option_d==2
  s2=title('UNIFORM ORDINARY END-FIRE','Fontsize',15);
  set(gca,'units','normalized');
  set(s2,'units','normalized','position',[0 1],'
  horizontalalign','left');
end
if option_b==1&option_c==1&option_d==3
  s3=title('UNIFORM HANSEN-WOODYARD END-FIRE','Fontsize',15);
  set(gca,'units','normalized');
  set(s3,'units','normalized','position',[0 1],'
  horizontalalign','left');
end
if option_b==1&option_c==1&option_d==4
  s4=title('UNIFORM SCANNING','Fontsize',15);
  set(gca,'units','normalized');
  set(s4,'units','normalized','position',[0 1],'
  horizontalalign','left');
end
if option_b==2&option_c==1&option_e==1
  s5=title('NONUNIFORM BINOMIAL','Fontsize',15);
  set(gca,'units','normalized');
  set(s5,'units','normalized','position',[0 1],'
  horizontalalign','left');
end
if option_b==2&option_c==1&option_e==2
  s6=title('NONUNIFORM DOLPH-TSCHEBYSCHEFF','Fontsize',15);
  set(gca,'units','normalized');
  set(s6,'units','normalized','position',[0 1],'
  horizontalalign','left');
end
end
if option_b==2
  for i=1:Mx*Ny;
    Ncoef(i)=1;
end
end

figure;
x=(1-Mx)*dx/2:dx:(Mx-1)*dx/2;
y=(1-Mx)/2:1:(Mx-1)/2;
subplot(2,1,1)
[AX,H1,H2]=plotyy(x,Ncoef(ceil(abs(y)+0.1)),x,betax.*y.*180./pi);
set(get(AX(1), 'Ylabel'), 'String', 'AMPLITUDE(X)', 'color', 'r');
set(get(AX(2), 'Ylabel'), 'String', 'PHASE (degrees)', 'color', 'b');
set(AX(1), 'ycolor', 'r');
set(AX(2), 'ycolor', 'b');
set(H1, 'Linestyle', ' ', 'color', 'r', 'linewidth', 2, 'marker', 's');
set(H2, 'Linestyle', ' ', 'color', 'b', 'linewidth', 2, 'marker', 'o');
xlabel(['ARRAY LENGTH ' ' (\lambda)']);
%axis([(1-Mx)*dx/2 (Mx-1)*dx/2 0 max(Ncoef)+0.1]);
grid on;
l2=legend('PHASE', 2);
[hle,l3]=legend('AMPLITUDE', 1); set(hle, 'color', [1 1 1]);
s7=title('UNIFORM PLANAR', 'Fontsize', 15);
set(gca, 'units', 'normalized');
set(s7, 'units', 'normalized', 'position', [0 1], 'horizontalalign', 'left');
x1=(1-Ny)*dy/2:dy:(Ny-1)*dy/2;
y1=(1-Ny)/2:1:(Ny-1)/2;
subplot(2,1,2)
[AX,H1,H2]=plotyy(x1,Ncoef(ceil(abs(y1)+0.1)),x1,betay.*y1.*180./pi);
set(get(AX(1), 'Ylabel'), 'String', 'AMPLITUDE(Y)', 'color', 'r');
set(get(AX(2), 'Ylabel'), 'String', 'PHASE (degrees)', 'color', 'b');
set(AX(1), 'ycolor', 'r');
set(AX(2), 'ycolor', 'b');
set(H1, 'Linestyle', ' ', 'color', 'r', 'linewidth', 2, 'marker', 's');
set(H2, 'Linestyle', ' ', 'color', 'b', 'linewidth', 2, 'marker', 'o');
xlabel(['ARRAY LENGTH ' ' (\lambda)']);
%axis([(1-Ny)*dy/2 (Ny-1)*dy/2 0 max(Ncoef)+0.1]);
grid on;
l2=legend('PHASE', 2);
[hle,l3]=legend('AMPLITUDE', 1); set(hle, 'color', [1 1 1]);
end
figure;
if option_b==1,
polar_dB(theta*180/pi,DdB,max(-60,6*floor(min(DdB)/6)),10*ceil(DdB/10),12,'-')
hold on
polar_dB(-theta*180/pi,DdB,max(-60,6*floor(min(DdB)/6)),10*ceil(DdB/10),12,'-')
title('Polar plot of Directivity (0< \phi <360 degrees)', 'Fontsize', 15)
else
polar_dB(theta*180/pi,DdB,max(-60,6*floor(min(DdB)/6)),10*ceil(max(DdB)/10),12,'-')
hold on
polar_dB(-theta*180/pi,D180dB,max(-60,6*floor(min(DdB)/6)),10*ceil(max(DdB)/10),12,'-')
end

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title(['Polar plot of Directivity (\phi = ',num2str(phieval),',
degrees)'],'Fontsize',15)
end

figure;
if option_b==1,
polar_dB(theta*180/pi,DdB-max(DdB),max(-60,6*floor(min(DdB)/6)),0,12,'-')
hold on
polar_dB(-theta*180/pi,DdB-max(DdB),max(-60,6*floor(min(DdB)/6)),0,12,'-')
title('Polar plot of Relative Directivity (0 < \phi < 360
degrees)','Fontsize',15)
else
    polar_dB(theta*180/pi,DdB-max(DdB),max(-60,6*floor(min(DdB)/6)),0,12,'-')
    hold on
    polar_dB(-theta*180/pi,DdB-max(DdB),max(-60,6*floor(min(DdB)/6)),0,12,'-')
    title(["Polar plot of Relative Directivity (\phi = ',num2str(phieval),'
degrees)"],'Fontsize',15)
end

%Spherical Plot3D
D3=4*pi*abs(AF3).^2/Prad;
D3dB=10.*log10(D3);
D3dB=D3dB-min(min(D3dB));

%figure;
%[x,y,z]=sph2cart(PHI,pi/2-THETA,D3dB);
%surf(x,y,z);
%title('Directivity(Spherical Plot3D)','Fontsize',15);

if scale==1
    DD=D3;
elseif scale==2
    DD=D3dB;
end
disc=size(DD,1);
spherical_plot(DD,THETA,PHI,disc);
if scale==1
    ss=title('3D Spherical plot of Directivity (Dimensionless)','Fontsize',15);
elseif scale==2
    ss=title('3D Spherical plot of Directivity (dB)','Fontsize',15);
end
%title('3D Spherical plot of Directivity','Fontsize',15)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Subroutines
% HPBCALC
function [hp,thmax]=hpbw(U,M)
tol=0.001;
imax=0;
j=0;
for i=1:M+1;
    if abs(U(i)-1)<tol & floor((j+2)/2)==imax+1,
        imax=imax+1;
        thmax(imax)=(i-1)/10;
    end
    if i>1 & abs(U(i)-1)<tol & U(i)>U(i-1) & j~=0,
        thmax(imax)=(i-1)/10;
    end
    if i>1,
        y(1)=U(i)-0.5;
        y(2)=U(i-1)-0.5;
        x(1)=(i-1)/10;
        x(2)=(i-2)/10;
        sign=y(1)*y(2);
        if sign<0,
            j=j+1;
            root(j)=x(2)-y(2)*((x(2)-x(1))/(y(2)-y(1)));
            if j>=2 & y(2)>y(1),
                hp(imax)=root(j)-root(j-1);
            elseif j==1 & y(2)>y(1),
                hp(imax)=2.*root(j);
            end
        end
    end
    if thmax(imax)>root(j),
        hp(imax)=2.*(180-root(j));
    end
end
end
end

% BINOMIAL
function [AF,Ncoef,Coef]=bin(theta,Nelem,d)
    if 2*floor(Nelem/2)==Nelem,Ncoef=Nelem/2;
    else Ncoef=(Nelem+1)/2;
    end
    for i=1:Ncoef;
        Coef(i)=1;
        for j=1:Ncoef-i;
            Coef(i)=Coef(i).*(Nelem-j)./(j);
        end
    end
    if 2*floor(Nelem/2)==Nelem,Coef(1)=Coef(1)/2;
end
    u=pi*d*cos(theta);
    if 2*floor(Nelem/2)==Nelem,AF=0;
        for i=1:Ncoef;
            AF=AF+Coef(i).*cos((2.*i-1).*u);
        end
    elseif AF=0;
        for i=1:Ncoef;
            AF=AF+Coef(i).*cos(2.*(i-1).*u);
        end
end

% TSCEBY(THETA,NELEM,D,RDB)
function [AF,Ncoef,Coef]=tscheby(theta,Nelem,d,RdB);
Ro=10^{(RdB/20)};
P=Nelem-1;
Zo=0.5*(\sqrt{(Ro^2-1)}^P+(Ro-\sqrt{(Ro^2-1)})^P);
if 2*floor(Nelem/2)==Nelem,
   Ncoef=Nelem/2;
   M=Ncoef;
for i=1:M;
   Coef(i)=0;
   for j=i:M;
      Coef(i)=Coef(i)+(-1)^{(M-j)}*Zo^{(2*(j-1))}*fact(j+M-2)*((2*M-1))/(fact(j-i)*fact(j+i-1)*fact(M-j));
   end
end
elseif 2*floor((Nelem+1)/2)==Nelem+1,
   Ncoef=(Nelem+1)/2;
   M=Ncoef-1;
for i=1:M+1;
   Coef(i)=0;
   for j=i:M+1;
      if i==1,EN=2;
      else EN=1;
      end
      Coef(i)=Coef(i)+(-1)^{(M-j+1)}*Zo^{(2*(j-1))}*fact(j+M-2)*((2*M-1))/(EN*fact(j-i)*fact(j+i-2)*fact(M-j+1));
   end
end
end
u=\pi*d*cos(theta);
if 2*floor(Nelem/2)==Nelem,
   AF=0;
   for i=1:Ncoef;
      AF=AF+Coef(i)*cos((2*i-1)*u);
   end
elseif 2*floor((Nelem+1)/2)==Nelem+1,
   AF=0;
   for i=1:Ncoef;
      AF=AF+Coef(i)*cos(2*(i-1)*u);
   end
end

% FACT(IARG)
function [f7]=fact(iarg)
f7=1;
for j=1:iarg;
   f7=j*f7;
end

% PLANAR(THETA,PHI,MX,NY,DY,BETAX,BETAY)
function [f10]=af10(theta,phi,Mx,Ny,dx,dy,betax,betay)
k=2*pi;
psix=k.*dx.*sin(theta).*cos(phi)+betax;
psiy=k.*dy.*sin(theta).*sin(phi)+betay;
AFx=sinc((Mx.*psix./2)./pi)./sinc((psix./2)./pi);
AFy=sinc((Ny.*psiy./2)./pi)./sinc((psiy./2)./pi);
f10=AFx.*AFy;
% CIRCULAR(THETA, PHI, THETA0, PHI0, NELEM, RAD)
function [fc] = afc(theta, phi, theta0, phi0, Nelem, rad)
k = 2*pi;
dtor = pi/180;
%n = linspace(1, Nelem, Nelem);
% AF = sum(exp(i.*(k.*rad.*sin(theta).*cos(phi-phin(n)) + alpha(n))));
AF = 0;
for n = 1:Nelem
    phin(n) = 2*pi*n/Nelem;
    alpha(n) = -k.*rad.*sin(dtor.*theta0).*cos(dtor.*phi0-phin(n));
    AF = AF + exp(i.*(k.*rad.*sin(theta).*cos(phi-phin(n)) + alpha(n)));
end
AFabs = abs(AF);
fc = AFabs;

% bessel with residuals
function [fc3] = afc3(theta, phi, theta0, phi0, Nelem, rad, m)
if theta0 == 0 | phi0 == 0,
    theta0 = theta0 + 0.000001;
    phi0 = phi0 - 0.000001;
end
k = 2*pi;
dtor = pi/180;
rho0 = rad.*sqrt((sin(theta).*cos(phi) - sin(theta0).*cos(phi0)).^2 + (sin(theta).*sin(phi) - sin(theta0).*sin(phi0)).^2);
zeta = atan((sin(theta).*sin(phi) - sin(theta0).*sin(phi0))./(sin(theta).*cos(phi) - sin(theta0).*cos(phi0)));
AFpri = besselj(0, k.*rho0);
AFres = 0;
for n = 1:m
    AFres = AFres + (i^(n.*Nelem)) * besselj(n.*Nelem, k.*rho0) .* cos(n.*Nelem.*zeta);
end
AFres = 2*AFres;
AF = AFpri + AFres;
AFabs = abs(AF);
fc3 = AFabs;

% spherical plot
function spherical_plot(r, THETA, PHI, disc)
% theta = linspace(theta_low, theta_up, disc);
% phi = linspace(phi_low, phi_up, disc);

% [THETA, PHI] = meshgrid(theta, phi);

% spherical to rectangular conversion
x = abs(r).*sin(THETA).*cos(PHI);
y = abs(r).*sin(THETA).*sin(PHI);
z = abs(r).*cos(THETA);
% do the plot
figure; surf(x,y,z); view(135,20);
C = [.8 .8 .8]; colormap(C); axis off equal;

% Draw x, y, and z axes
set(line([1e-8:max(max(x))+3],[1e-8;1e-8],[1e-8;1e-8]),'Color','r');
set(line([1e-8;1e-8],[1e-8:max(max(y))+3],[1e-8;1e-8]),'Color','r');
set(line([1e-8;1e-8],[1e-8;1e-8],[1e-8:max(max(z))+3]),'Color','r');

% Label x, y, and z axes
text(max(max(x))+4,0,0,'x','FontSize',14,'FontName','Times','FontAngle','italic','Color','r');
text(0,max(max(y))+4,0,'y','FontSize',14,'FontName','Times','FontAngle','italic','Color','r');
text(0,0,max(max(z))+4,'z','FontSize',14,'FontName','Times','FontAngle','italic','Color','r');

% Fill surface using patches
patch_1 = zeros(3,disc+1); patch_2 = zeros(3,disc+1);
patch_1(1,1:disc) = x(1,:); patch_2(1,1:disc) = x(disc,:);
patch_1(2,1:disc) = y(1,:); patch_2(2,1:disc) = y(disc,:);
patch_1(3,1:disc) = z(1,:); patch_2(3,1:disc) = z(disc,:);
patch(patch_1(1,:),patch_1(2,:),patch_1(3,:),C);
patch(patch_2(1,:),patch_2(2,:),patch_2(3,:),C);

%---------------------------------------------------------------------
% polar_dB(theta,rho,rmin,rmax,rticks,line_style)
% POLAR_DB is a MATLAB function that plots 2-D patterns in
% polar coordinates where:
% 0 <= THETA (in degrees) <= 360
% -infinity < RHO (in dB) < +infinity
% Input Parameters Description
% -------------------------------
% - theta (in degrees) must be a row vector from 0 to 360 degrees
% - rho (in dB) must be a row vector
% - rmin (in dB) sets the minimum limit of the plot (e.g., -60 dB)
% - rmax (in dB) sets the maximum limit of the plot (e.g., 0 dB)
% - rticks is the # of radial ticks (or circles) desired. (e.g., 4)
% - linestyle is solid (e.g., '-' or dashed (e.g., '--')

% Credits:
% S. Bellofiore
% S. Georgakopoulos
% A. C. Polycarpou
% C. Wangsvick
% C. Bishop

% Tabulate your data accordingly, and call polar_dB to provide the
% 2-D polar plot
% Note: This function is different from the polar.m (provided by
function hpol = polar_dB(theta,rho,rmin,rmax,rticks,line_style)

% Convert degrees into radians
theta = theta * pi/180;

% Font size, font style and line width parameters
font_size  = 16;
font_name  = 'Times';
line_width = 1.5;

if nargin < 5
  error('Requires 5 or 6 input arguments.');
elseif nargin == 5
  if isstr(rho)
    line_style = rho;
    rho = theta;
    [mr,nr] = size(rho);
    if mr == 1
      theta = 1:nr;
    else
      th = (1:mr)';
      theta = th(:,ones(1,nr));
    end
  else
    line_style = 'auto';
  end
elseif nargin == 1
  line_style = 'auto';
  rho = theta;
  [mr,nr] = size(rho);
  if mr == 1
    theta = 1:nr;
  else
    th = (1:mr)';
    theta = th(:,ones(1,nr));
  end
end
if isstr(theta) | isstr(rho)
  error('Input arguments must be numeric.');
end
if any(size(theta) ~= size(rho))
  error('THETA and RHO must be the same size.');
end

% get hold state
ca = newplot;
next = lower(get(ca,'NextPlot'));
hold_state = ishold;

% get x-axis text color so grid is in same color
tc = get(ca,'xcolor');
% Hold on to current Text defaults, reset them to the % Axes' font attributes so tick marks use them.
fAngle  = get(cax, 'DefaultTextFontAngle');
fName   = get(cax, 'DefaultTextFontName');
fSize   = get(cax, 'DefaultTextFontSize');
fWeight = get(cax, 'DefaultTextFontWeight');
set(cax, 'DefaultTextFontAngle', get(cax, 'FontAngle'), ...
    'DefaultTextFontName', font_name, ...
    'DefaultTextFontSize', font_size, ...
    'DefaultTextFontWeight', get(cax, 'FontWeight') )

% only do grids if hold is off
if ~hold_state

% make a radial grid
hold on;
% v returns the axis limits
% changed the following line to let the y limits become negative
hhh=plot([0 max(theta(:))],[min(rho(:)) max(rho(:))]);
v = [get(cax,'xlim') get(cax,'ylim')];
ticks = length(get(cax,'ytick'));
delete(hhh);

% check radial limits (rticks)
if rticks > 5  % see if we can reduce the number
    if rem(rticks,2) == 0
        rticks = rticks/2;
    elseif rem(rticks,3) == 0
        rticks = rticks/3;
    end
end

% define a circle
th = 0:pi/50:2*pi;
xunit = cos(th);
yunit = sin(th);
% now really force points on x/y axes to lie on them exactly
inds = [1:(length(th)-1)/4:length(th)];
xunits(inds(2:2:4)) = zeros(2,1);
yunits(inds(1:2:5)) = zeros(3,1);

rinc = (rmax-rmin)/rticks;

% label r
% change the following line so that the unit circle is not multiplied % by a negative number. Ditto for the text locations.
for i=(rmin+rinc):rinc:rmax
    is = i - rmin;
    plot(xunit*is,yunit*is,'-','color',tc,'linewidth',0.5);
    text(0,is+rinc/20,["' num2str(i)"],'verticalalignment','bottom');
end
% plot spokes

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th = (1:6)*2*pi/12;
cst = cos(th); snt = sin(th);
cs = [-cst; cst];
sn = [-snt; snt];
plot((rmax-rmin)*cs,(rmax-rmin)*sn,'-', 'color',tc,'linewidth',0.5);

% plot the ticks
george=(rmax-rmin)/30; % Length of the ticks
th2 = (0:36)*2*pi/72;
cst2 = cos(th2); snt2 = sin(th2);
cs2 = [(rmax-rmin-george)*cst2; (rmax-rmin)*cst2];
sn2 = [(rmax-rmin-george)*snt2; (rmax-rmin)*snt2];
plot(cs2,sn2,'-', 'color',tc,'linewidth',0.15);
plot(-cs2,-sn2,'-', 'color',tc,'linewidth',0.15);

% annotate spokes in degrees
% Changed the next line to make the spokes long enough
rt = 1.1*(rmax-rmin);
for i = 1:max(size(th))
    text(rt*cst(i),rt*snt(i),int2str(abs(i*30-90)));
    if i == max(size(th))
        loc = int2str(90);
    elseif i*30+90<=180
        loc = int2str(i*30+90);
    else
        loc = int2str(180-(i*30+90-180));
    end
    text(-rt*cst(i),-rt*snt(i),loc);
end

% set view to 2-D
view(0,90);

% set axis limits
% Changed the next line to scale things properly
axis((rmax-rmin)*[-1 1 -1.1 1.1]);
end

% Reset defaults.
set(cax, 'DefaultTextFontAngle', fAngle , ... 
     'DefaultTextFontName', font_name, ... 
     'DefaultTextFontSize', fSize, ... 
     'DefaultTextFontWeight', fWeight );

% transform data to Cartesian coordinates.
% changed the next line so negative rho are not plotted on the other side
for i = 1:length(rho)
    if rho(i) > rmin
        if theta(i)*180/pi >=0 & theta(i)*180/pi <=90
            xx(i) = (rho(i)-rmin)*cos(pi/2-theta(i));
            yy(i) = (rho(i)-rmin)*sin(pi/2-theta(i));
        end
    end
end
elseif theta(i)*180/pi >=90
    xx(i) = (rho(i)-rmin)*cos(-theta(i)+pi/2);    
    yy(i) = (rho(i)-rmin)*sin(-theta(i)+pi/2);
elseif theta(i)*180/pi < 0
    xx(i) = (rho(i)-rmin)*cos(abs(theta(i))+pi/2);  
    yy(i) = (rho(i)-rmin)*sin(abs(theta(i))+pi/2);
else
    xx(i) = 0;
    yy(i) = 0;
end

% plot data on top of grid
if strcmp(line_style,'auto')
    q = plot(xx,yy);
else
    q = plot(xx,yy,line_style);
end
if nargout > 0
    hpol = q;
end
if ~hold_state
    axis('equal');axis('off');
end

% reset hold state
if ~hold_state, set(cax,'NextPlot',next); end

A-3  Microstrip.m

%*******************************************************************************%
% MICROSTRIP
%*******************************************************************************%
% THIS PROGRAM IS A MATLAB PROGRAM THAT DESIGNS AND THEN COMPUTES THE
% ANTENNA RADIATION CHARACTERISTICS OF:
% % I.  RECTANGULAR
%    II.  CIRCULAR
% MICROSTRIP PATCH ANTENNAS BASED ON THE CAVITY MODEL AND DOMINANT
% MODE OPERATION FOR EACH.  THAT IS:
% % A.  TM(010) MODE FOR THE RECTANGULAR PATCH
%    B.  TM(011) MODE FOR THE CIRCULAR PATCH
% ** INPUT PARAMETERS
% 1.  FREQ  = RESONANT FREQUENCY (in GHz)
% 2.  EPSR  = DIELECTRIC CONSTANT OF THE SUBSTRATE
% 3.  HEIGHT = HEIGHT OF THE SUBSTRATE (in cm)
% 4.  Y0    = POSITION OF THE RECESSED FEED POINT (in cm)
%        RELATIVE TO LEADING RADIATING EDGE OF RECTANGULAR
%        PATCH.  NOT NECESSARY FOR CIRCULAR PATCH. 

**OUTPUT PARAMETERS**

A. RECTANGULAR PATCH:

1. PHYSICAL WIDTH OF THE PATCH \( W \) (in cm)
2. EFFECTIVE LENGTH OF PATCH \( L_e \) (in cm)
3. PHYSICAL LENGTH OF PATCH \( L \) (in cm)
4. NORMALIZED E-PLANE AMPLITUDE PATTERN (in dB)
5. NORMALIZED H-PLANE AMPLITUDE PATTERN (in dB)
6. E-PLANE HALF-POWER BEAMWIDTH (in degrees)
7. H-PLANE HALF-POWER BEAMWIDTH (in degrees)
8. DIRECTIVITY (dimensionless and in dB)
   a. AT LEADING RADIATING EDGE \( (y = 0) \)
   b. AT RECESSED FEED POINT FROM LEADING RADIATING EDGE \( (y = y_0) \)

B. CIRCULAR PATCH:

1. PHYSICAL RADIUS OF THE PATCH \( a \) (in cm)
2. EFFECTIVE RADIUS OF THE PATCH \( a_e \) (in cm)
3. NORMALIZED E-PLANE AMPLITUDE (in dB)
4. NORMALIZED H-PLANE AMPLITUDE (in dB)
5. E-PLANE HALF-POWER BEAMWIDTH (in degrees)
6. H-PLANE HALF-POWER BEAMWIDTH (in degrees)
7. DIRECTIVITY (dimensionless and in dB)

******************************************************************************
Programmed by : Sung-Woo Lee, Arizona State University
Modified by   : Zhiyong Huang, Arizona State University
Nov. 23, 2004
******************************************************************************

function []=MICROSTP;
clear all;
close all;
warning off;

option=[];
while isempty(option)|(option~=1&option~=2),
    option=input(['SELECT OUTPUT METHOD
n' 'OPTION (1): SCREEN
n' 'OPTION (2): OUTPUT FILE
n' 'SELECT OPTION: ']);
end;

filename=[];
if option==2,
    while isempty(filename),
        filename=input('INPUT THE DESIRED OUTPUT FILENAME <in single quotes> = ', 's');
    end;
end;
addpath(pwd);
if exist(filename,'file')&isa(filename,'char'),
delete(filename);
end;
rmpath(pwd);

patchm=[];
while isempty(patchm)|(patchm~=1&(patchm~=2)),
    patchm=input(['PATCH GEOMETRY OPTION
', 'OPTION (1) : RECTANGULAR PATCH
', 'OPTION (2) : CIRCULAR PATCH
', 'SELECT OPTION NUMBER: ']);
end;

if (patchm==1),  % Rectangular
    rect(option,filename);
else  % Circular
    circ(option,filename);
end;

warning on;

%%%%%%%%%%%%%%%%%%%
function rect=rect(option_a,filename);
%%%%%%%%%%%%%%%%%%%

% Input Parameters (freq, epsr, height, Yo)
freq=[];
while isempty(freq),
    freq=input('INPUT THE RESONANT FREQUENCY (in GHz) = ');
end;

er=[];
while isempty(er),
    er=input('INPUT THE DIELECTRIC CONSTANT OF THE SUBSTRATE = ');
end;

h=[];
while isempty(h),
    h=input('INPUT THE HEIGHT OF THE SUBSTRATE (in cm) = ');
end;

option1=[];
while isempty(option1)|(option1~=1&option1~=2),
    option1=input(['OPTIONS
', 'OPTION (1): FIND INPUT IMPEDANCE Zin AT FEED-POINT Yo
', 'OPTION (2): DETERMINE Yo FOR A GIVEN DESIRED Zin
', 'SELECT OPTION NUMBER: ']);
end;

if option1==1
    Yo=[];
    while isempty(Yo),
        Yo=input(['INPUT THE POSITION OF THE RECESSED FEED POINT ']}); ...
'RELATIVE TO THE LEADING RADIATING EDGE
OF THE
RECTANGULAR PATCH (in cm) = ']);

end
else
    Zin=[];
    while isempty(Zin),
        Zin=input(['INPUT THE DESIRED INPUT IMPEDANCE Zin (in ohms) = ']);
    end
end

% Compute W, ereff, Leff, L (in cm)
W=30.0/(2.0*freq)*sqrt(2.0/(er+1.0));
ereff=(er+1.0)/2.0+(er-1)/(2.0*sqrt(1.0+12.0*h/W));
dl=0.412*h*((ereff+0.3)*(W/h+0.264))/((ereff-0.258)*(W/h+0.8));
lambda_o=30.0/freq;
lambda=30.0/(freq*sqrt(ereff));
Leff=30.0/(2.0*freq*sqrt(ereff));
L=Leff-2.0*dl;
ko=2.0*pi/lambda_o;
Emax=sinc(h*ko/2.0/pi);

% Normalized radiated field
%         E-PLANE RADIATION PATTERN
%         E-PLANE RADIATION PATTERN
% H-PLANE RADIATION PATTERN
% NOT E: THIS PATTERN IS ROTATED CCW BY 90 DEGREES
phi=0:360; phir=phi.*pi./180; [Ethval,Eth]=E_th(phir,h,ko,Leff,Emax);

th=0:180; thr=th.*pi/180.0;    [Ephval,Eph1]=E_ph(thr,h,ko,W,Emax);
Eph(1:91)=Eph1(91:181); Eph(91:270)=Eph1(181); Eph(271:361)=Eph1(1:91);

% Output files
fid_e=fopen('Epl-Micr_m.dat','wt');
fid_h=fopen('Hpl-Micr_m.dat','wt');
fprintf(fid_e,'# E-PLANE RADIATION PATTERN
');
fprintf(fid_e,'# -------------------------
');
fprintf(fid_h,'# H-PLAN RADIATION PATTERN
');
fprintf(fid_h,'# NOTE: THIS PATTERN IS ROTATED CCW BY 90 DEGREES
');
fprintf(fid_h,'# -------------------------
');

Epl=[phi;Eth];
fprintf(fid_e,' %7.4f\t%7.4f
',Epl);
close(fid_e);
Hpl=[[0:90^270:360];[Eph(1:91) Eph(271:361)]];
fprintf(fid_h,' %7.4f\t%7.4f
',Hpl);
close(fid_h);

% Plots of Radiation Patterns
% Figure 1
% ******
Etheta=[Eth(271:361),Eth(2:91)];
xs=[0 20 40 60 80 90 100 120 140 160 180];
xs1=[270 290 310 330 350 0 10 30 50 70 90];
hli1=plot(Etheta,'b-');
set(gca,'Xtick',xs);
set(gca,'Xticklabel',xsl);
set(gca,'position',[0.13 0.11 0.775 0.8]);
h1(gca); h2=copyobj(h1,gcf);
xlim([0 180]); ylim([-60 0]);
set(h1,'xcolor',[0 0 1]); set(hli1,'erasemode','xor'); hx=xlabel('\phi (degrees)',12);

axes(h2); hli2=plot(Eph1,'r:'); axis([0 180 -60 0]);
set(h2,'xaxislocation','top','xcolor',[1 0 0]);
legend([hli1 hli2],{'E_{\phi} (E-plane)','E_{\phi} (H-plane)'},4);
xlabel('\theta (degrees)',12);
set([hli1 hli2], 'linewidth',2); set(hx,'erasemode','xor');
ylabel('Radiation patterns (in dB)',12);

figure(2);

hp1=semipolar_micror(phi,Eth,-60,0,4, '-', 'b'); hold on;
hp2=semipolar_micror(phi*pi/180,Eph,-60,0,4,':','r');
title('E- and H-plane Patterns of Rectangular Microstrip Antenna', 'fontsize',12);
hle=legend([hp1 hp2],{'E_{\phi} (E-plane)','E_{\phi} (H-plane)'},0);

% E-plane HPBW and H-plane HPBW
% ******************************
an=phi(Eth>3);
an(an>90)=[];
EHPBW=2*abs(max(an));
HHPBW=2*abs(90-min(th(Eph1>3))); 

% Directivity
[D,DdB]=dir_rect(W,h,Leff,L,ko);

% Input Impedance at Y=0 and Y=Yo
[G1,G12]=sintegr(W,L,ko);
Rin0P=(2.*(G1+G12))^(-1);
Rin0M=(2.*(G1-G12))^(-1);
if option1==1
    RinYoP=Rin0P*cos(pi*Yo/L)^2;
    RinYoM=Rin0M*cos(pi*Yo/L)^2;
else
    YP=acos(sqrt(Zin/Rin0P))*L/pi;
    YM=acos(sqrt(Zin/Rin0M))*L/pi;
end

% Display (rectangular)
clc;
if(option_a==2)
diary(filename);
end
disp(strvcat('INPUT PARAMETERS','================'));
disp(sprintf('
RESONANT FREQUENCY (in GHz) = %4.4f',freq));
disp(sprintf('DIELECTRIC CONSTANT OF THE SUBSTRATE = %4.4f',er));
disp(sprintf('HEIGHT OF THE SUBSTRATE (in cm) = %4.4f',h));
if option1==1
disp(sprintf('POSITION OF THE RECESSED FEED POINT (in cm) = %4.4f',Yo));
else
fprintf('DESIRED RESONANT INPUT IMPEDANCE (in ohms) = %4.4f
',Zin);
end
disp(strvcat('OUTPUT PARAMETERS','================='));
disp(sprintf('
PHYSICAL WIDTH OF PATCH (in cm) = %4.4f',W));
disp(sprintf('EFFECTIVE LENGTH OF PATCH (in cm) = %4.4f',Leff));
disp(sprintf('PHYSICAL LENGTH OF PATCH (in cm) = %4.4f',L));
disp(sprintf('E-PLANE HPBW (in degrees) = %4.4f',EHPBW));
disp(sprintf('H-PLANE HPBW (in degrees) = %4.4f',HHPBW));
disp(sprintf('DIRECTIVITY OF RECTANGULAR PATCH (dimensionless) = %4.4f',D));
disp(sprintf('DIRECTIVITY OF RECTANGULAR PATCH (in dB) = %4.4f',dB));
disp(sprintf('G1 (Using (14-12)) = %4.8f', G1));
disp(sprintf('G12 (Using (14-18a)) = %4.8f', G12));
disp(sprintf('RESONANT INPUT RESISTANCE AT LEADING RADIATING EDGE (y=0) Rin0P (Using + sign in (14-17)) = %4.4f ohms',Rin0P));
disp(sprintf('RESONANT INPUT RESISTANCE AT LEADING RADIATING EDGE (y=0) Rin0M (Using - sign in (14-17)) = %4.4f ohms',Rin0M));
if option1==1
fprintf('RESONANT INPUT RESISTANCE AT RECESSED FEED POINT (y=%4.4f cm) RinYoP (Using + sign in (14-17)) = %4.4f ohms
',Yo, RinYoP);
fprintf('RESONANT INPUT RESISTANCE AT RECESSED FEED POINT (y=%4.4f cm) RinYoM (Using - sign in (14-17)) = %4.4f ohms
',Yo, RinYoM);
else
fprintf('FOR DESIRED IMPEDANCE %4.4f ohms, THE FEED POINT POSITION YoP (Using + sign in (14-17)) = %4.4f cm\n\',Zin, YP);
fprintf('FOR DESIRED IMPEDANCE %4.4f ohms, THE FEED POINT POSITION YoM (Using - sign in (14-17)) = %4.4f cm\n\',Zin, YM);
end
disp(strvcat('*** NOTE:',...'
  THE E-PLANE AMPLITUDE PATTERN IS STORED IN Epl-Micr_m.dat',...'
  THE H-PLANE AMPLITUDE PATTERN IS STORED IN Hpl-Micr_m.dat',...'
  ==============================================================='));
diary off;
% Subfunctions
% ************
function [Ethval,Eth]=E_th(phir,h,ko,Leff,Emax)
ARG=cos(phir).*h.*ko./2;
Ethval=(sinc(ARG./pi).*cos(sin(phir).*ko*Leff./2))./Emax;
Eth = 20*log10(abs(Ethval));
Eth(phir>pi/2 & phir<3*pi/2) = -60;
Eth(Eth < -60) = -60;

function [Ephval, Eph1] = E_ph(thr, h, ko, W, Emax)
ARG1 = sin(thr).*h.*ko./2;
ARG2 = cos(thr).*W.*ko./2;
Ephval = sin(thr).*sinc(ARG1./pi).*sinc(ARG2./pi)./Emax;
Eph1 = 20.0*log10(abs(Ephval));
Eph1(Eph1 < -60) = -60;

function [D, DdB] = dir_rect(W, h, Leff, L, ko)
th = 0:180; phi = [0:90 270:360];
[t, p] = meshgrid(th.*pi/180, phi.*pi/180);
X = ko*h/2*sin(t).*cos(p);
Z = ko*W/2*cos(t);
Et = sin(t).*sinc(X/pi).*sinc(Z/pi).*cos(ko*Leff/2*sin(t).*sin(p));
U = Et.^2;
dt = (th(2) - th(1))*pi/180;
dp = (phi(2) - phi(1))*pi/180;
Prad = sum(sum(U.*sin(t)))*dt*dp;
D = 4.*pi.*max(max(U))./Prad;
DdB = 10.*log10(D);

function [G1, G12] = sintegr(W, L, ko)
th = 0:1:180; t = th.*pi/180;
ARG = cos(t).*(ko*W/2);
res1 = sum(sinc(ARG./pi).^2.*sin(t).^2.*sin(t).*((pi/180)*(ko*W/2).^2));
res12 = sum(sinc(ARG./pi).^2.*sin(t).^2.*besselj(0, sin(t).*(ko*L)).*sin(t ).*((pi/180)*(ko*W/2).^2));
G1 = res1./(120*pi^2); G12 = res12./(120*pi^2);

% Input Parameters (freq, epsr, height)
freq = []; while isempty(freq),
    freq = input('INPUT THE RESONANT FREQUENCY (in GHz) = ');
end;

er = []; while isempty(er),
    er = input('INPUT THE DIELECTRIC CONSTANT OF THE SUBSTRATE = ');
end;

h = []; while isempty(h),
    h = input('INPUT THE HEIGHT OF THE SUBSTRATE (in cm) = ');
end;

con = input('PLEASE INPUT THE CONDUCTIVITY (DEFAULT VALUE IS 10^7):');
if isempty(con)
    con = 10^7;
lt=input('PLEASE INPUT THE LOST TANGENT (DEFAULT VALUE OF DOMINANT MODE TM110 IS 0.0018):');
if isempty(lt)
    lt=0.0018;
end

%input of the rho0 or zin
option1=[];
while isempty(option1)||(option1~=1&option1~=2),
    option1=input(['OPTIONS
', '   OPTION (1): FIND INPUT IMPEDANCE Zin AT FEED-POINT RHOo
', '   OPTION (2): DETERMINE RHOo FOR A GIVEN DESIRED Zin
', 'SELE1CT OPTION NUMBER: ']);
end;

if option1==1
    RHOo=[];
    while isempty(RHOo),
        RHOo=input(['INPUT THE POSITION OF THE RECESSED FEED POINT '...
', 'RELATIVE TO THE CENTER OF THE CIRCULAR PATCH (in cm) = ']);
    end
else
    Zin=[];
    while isempty(Zin),
        Zin=input(['INPUT THE DESIRED INPUT IMPEDANCE Zin (in ohms) = ']);
    end
end

% Compute the Physical Radius a (in cm) and Effective Radius ae (in cm)
lambda_o=30.0/freq;
ko=2.0*pi/lambda_o;
F=8.791/(freq*sqrt(er));
a=F/sqrt(1+2*h/(pi*er*a)*(log(pi*a/(2*h))+1.7726));
ae=a*sqrt(1+2*h/(pi*er*a)*(log(pi*a/(2*h))+1.7726));

% Normalized radiated field
% E-plane and H-plane patterns : 0 < th < 90
th=0:90; thr=th.*pi./180;
x=sin(thr).*ko.*ae;
J0=besselj(0,x);
J2=besselj(2,x);
Eth1=J0-J2;
Eph1=(J0+J2).*cos(thr);

Eth2=20.*log10(Eth1./max(Eth1));
Eph2=20.*log10(Eph1./max(Eph1));
Eth2(Eth2<=-60)=-60;
Eph2(Eph2<=-60)=-60;
E\text{th}(1:91) = \text{Eth2}(1:91); \ E\text{th}(91:270) = \text{Eth2}(91); \ E\text{th}(271:361) = \text{Eth2}(91:1:1); 
E\text{ph}(1:91) = \text{Eph2}(1:91); \ E\text{ph}(91:270) = \text{Eph2}(91); \ E\text{ph}(271:361) = \text{Eph2}(91:1:1);

% Output files
fid\_e=fopen('Epl-Micr\_m.dat','wt');
 fid\_h=fopen('Hpl-Micr\_m.dat','wt');
 fprintf(fid\_e,'# E-PLANE RADIATION PATTERN\n');
 fprintf(fid\_e,'# -------------------------\n#');
 fprintf(fid\_h,'# H-PLANE RADIATION PATTERN\n');
 fprintf(fid\_h,'# -------------------------\n#');

Epl=[(0:90 270:360);[Eth(1:91) Eth(271:361)]];
 fprintf(fid\_e, ' %7.4f t%7.4f\n',Epl);
 fclose(fid\_e);
 Hpl=[(0:90 270:360);[Eph(1:91) Eph(271:361)]];
 fprintf(fid\_h, ' %7.4f t%7.4f\n',Hpl);
 fclose(fid\_h);

% Plots of Radiation Patterns
 phi=0:360;

% Figure 1
%********
 hli1=plot(-90:90,[fliprl(Eth2) Eth2(2:end)],'b-');
 set(gca,'position',[0.13 0.11 0.775 0.8]);
 h1=gca; h2=copyobj(h1,gcf); axis([-90 90 -60 0]);
 set(h1,'xcolor',[0 0 1]); set(hli1,'erasemode','xor');
 hx=xlabel('\theta (degrees)','fontsize',12);

 axes(h2); hli2=plot(-90:90,[fliprl(Eph2) Eph2(2:end)],'r:'); axis([-90 90 -60 0]);
 set(h2,'xaxislocation','top','xcolor',[1 0 0]);
 set([hli1 hli2],'linewidth',2);
 legend([hli1 hli2],{'E_{\theta} (E-plane)' , 'E_{\phi} (H-plane)'},4);
 xlabel('\theta (degrees)','fontsize',12);

% Figure 2
%********
 figure(2);
 thr=(-90:90)*pi/180;
 hpl=semipolar_microc(thr,[fliprl(Eth2) Eth2(2:end)],-60,0,4,'-','b');
 hold on;
 hp2=semipolar_microc(thr,[fliprl(Eph2) Eph2(2:end)],-60,0,4,'-','r');
 hle=legend([hpl hp2],{'E_{\theta} (E-plane)' , 'E_{\phi} (H-plane)'},0);
 title('E- and H-plane Patterns of Circular Microstrip Antenna','fontsize',[12]);

% E-plane and H-plane HPBW
 an=th(Eth2>3);
 bn=th(Eph2>3);

 EHPBW=2*abs(max(an));
 HHPBW=2*abs(max(bn));
%resonant input resistance

t=[0:0.001:pi/2];
x=ko*ae*sin(t);
j0=besselj(0,x);
j2=besselj(2,x);
j0p=j0-j2;
j02p=j0+j2;
grad=(ko*ae)^2/480*sum((j0p.^2+(cos(t)).^2.*j02.^2).sin(t).*0.001);
emo=1;
m=1;
mu0=4*pi*10^(-7); 
k=ko*sqrt(er);
gc=emo*pi*(pi*mu0*freq*10^9)^(-3/2)*((k*ae)^2-m^2)/(4*(h/100)^2*sqrt(con));
gd=emo*lt*((k*ae)^2-m^2)/(4*mu0*h/100*freq*10^9);

gt=grad+gc+gd;
Rin0=1/gt;

if option1==1
    Rin=Rin0*besselj(1,k*RHOo)^2/besselj(1,k*ae)^2;
else
    temp1=Zin/Rin0*besselj(1,k*ae)^2;
    minrho=0;
    tempk=1;
    while tempk>0.00001
        nk=0;
        rhox=linspace(minrho,maxrho,100);
        temp=besselj(1,k.*rhox).^2;
        for kk=1:99
            if temp(kk)-temp1<=0
                if temp(kk+1)-temp1>0
                    nk=nk+1;
                    minrho=rhox(kk);
                    maxrho=rhox(kk+1);
                end
            else
                if temp(kk+1)-temp1<=0
                    nk=nk+1;
                    maxrho=rhox(kk);
                    minrho=rhox(kk+1);
                end
            end
        end
        if nk>1
            display("*****Warning, there are more than one solutions for RHOo and this program only provides you one exact solution!*****/n");
        end
        [tempk,kk]=min(abs(temp-temp1));
        RHOo=rhox(kk);
    end
end

% Directivity
[D,DdB]=dir_cir(a,ae,ko);

% Display (circular)
clc;
if (option_a==2),
    diary(filename);
end

disp(strvcat('INPUT PARAMETERS','================'));
disp(sprintf('
RESONANT FREQUENCY (in GHz) = %4.4f',freq));
disp(sprintf('DIELECTRIC CONSTANT OF THE SUBSTRATE = %4.4f',er));
disp(sprintf('HEIGHT OF THE SUBSTRATE (in cm) = %4.4f
',h));
disp(strvcat('OUTPUT PARAMETERS','================='));
disp(sprintf('
PHYSICAL RADIUS OF THE PATCH (in cm) = %4.4f',a));
disp(sprintf('EFFECTIVE RADIUS OF THE PATCH (in cm) = %4.4f
',ae));
disp(sprintf('E-PLANE HPBW (in degrees) = %4.4f
',EHPBW));
disp(sprintf('H-PLANE HPBW (in degrees) = %4.4f
',HHPBW));
disp(sprintf('DIRECTIVITY OF CIRCULAR PATCH (dimensionless) = %4.4f',D));
disp(sprintf('DIRECTIVITY OF CIRCULAR PATCH (in dB) = %4.4f
',DdB));
fprintf('*** TM110 MODE ***
');
fprintf('RESONANT INPUT RESISTANCE AT RHO=ae : Rin0= %4.4f
',Rin0);
if option1==1
    fprintf('RESONANT INPUT RESISTANCE AT RECESSED FEED POINT (RHO=%4.4f cm) RIN= %4.4f
',RHOo, Rin);
else
    fprintf('FOR DESIRED IMPENDANCE %4.4f ohms, THE FEED POINT POSITION
RHOo=%4.4f cm\n',Zin, RHOo);
end

disp(strvcat('*** NOTE:',...
    '    THE E-PLANE AMPLITUDE PATTERN IS STORED IN Epl-Micr_m.dat',...
    '    THE H-PLANE AMPLITUDE PATTERN IS STORED IN Hpl-Micr_m.dat',...
    '    ========================================================='));
diary off;

% Subfunction
function [D,DdB]=dir_cir(a,ae,ko)
th=0:90; phi=0:360;
[t,p]=meshgrid(th.*pi/180,phi.*pi/180);
x=sin(t).*ko.*ae;
J0=besselj(0,x); J2=besselj(2,x);
J02P=J0-J2; J02=J0+J2;
Ucirc=(J02P.*cos(p)).^2 + (J02.*cos(t).*sin(p)).^2;
Umax=max(max(Ucirc));
Ua=Ucirc.*sin(t).*(pi./180).^2;
Prad=sum(sum(Ua));
D=4.*pi.*Umax./Prad;
DdB=10.*log10(D);
B. SISO Channel Models

The book “MIMO-OFDM Wireless Communication with MATLAB” in References [1] provided examples of code for the following SISO channel models:

B-1 Log-normal-shadowing pg. 7 (textbook)
B-2 Free Space pg. 7 (textbook)
B-3 HATA pg. 8 (textbook)
B-4 IEEE802.16d pg. 10 (textbook)
B-5 Rayleigh fading pg. 24 (textbook)
B-6 Rician fading pg. 24 (textbook)
B-7 2-Ray fading pg. 26 (textbook)
B-8 Exponential pg. 26 (textbook)
B-9 IEEE 802.11 pg. 28 (textbook)
B-10 Saleh-Valenzuela pg. 32 (textbook)
B-11 UWB pg. 35 (textbook)
B-12 Clarke/Gans pg. 41 (textbook)
B-13 FWGN/Modified FWGN pg. 46 (textbook)
B-14 Jakes pg. 53 (textbook)
B-15 Ray-based pg. 54 (textbook)
B-16 SUI pg. 68 (textbook)

B-1 Log-normal-shadowing

function PL=PL_logdist_or_norm(fc,d,d0,n,sigma)
%Log-distance or Log-normal shadowing path loss model
%Inputs:   fc: Carrier frequency [HZ]
%          d: Distance between base station and mobile station [m]
%          d0: Reference distance [m]
%          n: Path loss exponent
%          sigma: Variance [dB]

lamda = 3e8/fc
\[
PL = -20 \times \log_{10} \left( \frac{\lambda}{(4\pi d_0)} \right) + 10 \times n \log(d/d_0) \\
\text{Eq. (1.4)}
\]

\[
\text{if}\ n\text{argin} > 4,\ PL = PL + \sigma \times \text{randn(size(d))};\ \text{end}\ \text{Eq. (1.5)}
\]

### B-2 Free Space

```matlab
function PL=PL_free(fc,d,Gt,Gr)

% Free Space Path Loss Model
% Inputs: fc : Carrier frequency
% d : Distance between base station and mobile station [m]
% Gt/Gr : Transmitter / Receiver Gain
% Outputs: PL : Path Loss [dB]

\[
\lambda = \frac{3 \times 10^8}{fc};
\]

\[
tmp = \frac{\lambda}{(4\pi d)};
\]

\[
\text{if}\ n\text{argin} > 2,\ tmp = tmp \times \sqrt{Gt};\ \text{end}
\]

\[
\text{if}\ n\text{argin} > 3,\ tmp = tmp \times \sqrt{Gr};\ \text{end}
\]

\[
PL = -20 \times \log_{10}(tmp)\ \text{Eq. (1.2)/(1.3)}
\]

### B-3 HATA

```matlab
function PL=PL_Hata(fc,d,htx,hrx,Etype)

% Inputs: fc : Carrier Frequency [Hz]
% d : Distance between base station and mobile station [m]
% htx : Height of transmitter [m]
% hrx : Height of Receiver [m]
% Etype : Environment type ('urban', 'suburban', 'open')
% Output: PL : path loss [dB]

if nargin<5, Etype = 'urban'; end

fc = fc/(1e6);

if fc>=150&&fc<=200, C_Rx = 8.29*(log10(1.54*hrx))^2-1.1; elseif fc>=200&&fc<1500, C_Rx = 3.2*(log10(11.75*hrx))^2-4.97; else C_Rx = 0.8+(1.1*log10(fc)-0.7)*hrx-1.56*log(fc); end

PL= 69.55 + 26.16*log10(fc) - 13.82*log10(htx) - C_Rx...
+(44.9-6.55*log10(htx))*log10(d/1000); %Eq. (1.7)

EType=upper(Etype);

if EType(1)== 'S', PL = PL-2*(log10(fc/28))^2-5.4; %Eq. (1.10)
elseif EType(1) == 'O'
    PL=PL+(18.33-4.78*log10(fc))*log10(fc)-40.97; %Eq. (1.11)
end
```

### B-4 IEEE802.16d

```matlab
function PL=PL_IEEE80216d(fc,d,type,htx,hrx,corr_fact,mod)

%IEEE 802.16d model
% Inputs
% fc : Carrier frequency
% d : Distance between base and terminal
% type : selects 'A', 'B', or 'C'
```

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% htx     :Height of Transmitter
% hrx     :Height of receiver
% corre_fact :if shadowing exists, set to "ATnT" or 'Okumur'.
% Otherwise, 'NO'
% mod     :set to ;mod; to obtain modified IEEE 802.16d model

%Output
% PL: path loss[dB]
Mod = 'UNMOD';
if nargin>6, Mod=upper(mod);
end
if nargin==6&&corr_fact(1)=='m', Mod='MOD';corr_fact='NO';
else if nargin<6, corr_fact='NO'
  if nargin==5&hrx(1)=='m',Mod='MOD';hrx=2;
  elseif nargin<5, hrx=2;
    if nargin==4&htx(1)=='m',Mod='MOD';htx=30;
      elseif nargin <4, htx=30;
        if nargin==3&type(1)=='m',Mod='MOD';type='A'
          elseif nargin<3, type='A';
        end
      end
    end
  end
end
end
d0 = 100;
Type = upper(type);
if Type~='A'&Type~='B'&Type~='C'
disp('Error:The selected type is not supported');return;
end
switch upper (corr_fact)
  case 'ATNT',  PLf=6*log10(fc/2e9);
    PLh=-10.8*log10(hrx/2);
  case 'OKUMURA', PLf=6*log10(fc/2e9);
    if hrx<=3, PLh=-10*log10(hrx/3);
    else PLh =-20*log10(hrx/3);
  case 'NO',   PLf = 0; PLh = 0;
end
if Type =='A', a=4.6; b=0.0075; c=12.6;  % Eq(1.3)
  else a=3.6; b=0.005; c=20;
end
lamda=3e8/fc;
gamma=a-b*htx+c/htx;
d0_pr=d0;
if Mod(1)=='M'
d0_pr=d0*10^(-((PLf+PLh)/(10*gamma)));
end
A=20*log10(4*pi*d0_pr/lamda)+PLf+PLh;
for k=1:length(d)
  if d(k)>d0_pr, PL(k)=A+10*gamma*log10(d(k)/d0);
    else PL(k)=20*log10(4*pi*d(k)/lamda);
  end
B-5 Rayleigh fading

```matlab
function H = Ray_model(L)
% Rayleigh channel model
% Input: L = Number of channel realizations
% Output: H = Channel vector
H = (randn(1,L)+ j*randn(1,L))/sqrt(2);
```

B-6 Rician fading

```matlab
function H = Ric_model(K_dB,L)
% Rician channel model
% Input: K_dB = K Factor [dB]
% Output: H = Channel vector
K = 10^(K_dB/10);
H = sqrt(K/(K+1)) + sqrt(1/(K+1))*Ray_model(L);
```

B-7 2-Ray fading & Exponential

```matlab
%plot_2ray_exp_model
clear, clf
scale=2.4e-9; %nano
Ts=10*scale; % Sampling Time
t_rms=30*scale % RMS delay spread
num_ch=10000; % # of channel
%2-Raymodel
pow_2=[0.5 0.5];
delay_2 = [0 t_rms*2]/scale;
H_2 = [Ray_model(num_ch); Ray_model(num_ch)].'*diag(sqrt(pow_2));
avg_pow_h_2=mean(H_2.*conj(H_2));
subplot(221)
stem(delay_2,pow_2,'ko'), hold on, stem(delay_2,avg_pow_h_2,'k.');
xlabel('Delay[ns]'),ylabel('Channel Power[linear]');
title('2-ray Model');
legend('Ideal','Simulation'); axis([-10 140 0 0.7]);
%Exponential Model
pow_e=exp_PDP(t_rms,Ts);
delay_e=[0:length(pow_e)-1]*Ts/scale;
for i=1:length(pow_e)
    H_e(:,i)=Ray_model(num_ch).'*sqrt(pow_e(i));
end
avg_pow_h_e = mean(H_e.*conj(H_e));
subplot(111)
stem(delay_e,pow_e,'ko'), hold on, stem(delay_e,avg_pow_h_e,'k.');
xlabel('Delay[ns]'),ylabel('Channel Power[linear]');
title('Exponential Model');axis([-10 140 0 0.7])
legend('Ideal','Simulation')
```

```matlab
function [PDP,lmax]=exp_PDP(tau_d,Ts,A_dB,norm_flag)
% Exponential PDP generator (Power Delay Profile)
% inputs:
% tau_d : rms delay spread[sec]
% Ts : Sampling time[sec]
```
% A_dB : smallest noticeable power [dB]
% normm_flag : normalize total power to unit
% Output:
% PDP : PDP vector
if nargin<4,
   norm_flag=1;
end  % normalization
if nargin<3,
   A_dB=-20;
end  % 20 dB below
sigma_tau=tau_d;
A=10^(A_dB/10);
lmax=ceil(-tau_d*log(A)/Ts);
% Eq. (2.2) excess delay
% Compute normalization factor for power normalization
if norm_flag
   p0=(1-exp(-Ts/sigma_tau))/(1-exp(-(lmax+1)*Ts/sigma_tau));  % Eq. (2.4)
else
   p0 = 1/sigma_tau;
end  % Exponential PDP
l=0:1:lmax;
PDP = p0*exp(-l*Ts/sigma_tau);  % Eq. (2.5)
lmax=lmax+1;

B-8 IEEE 802.11

% plot-IEEE80211-model.m
clear, clf
scale=2.4e-9;  % nano
Ts=50*scale;  % sampling time
t_rms=25*scale;  % RMS delay spread
num_ch=10000;  % Number of channels
N=128;  % FFT size
PDP=IEEE802_11_model(t_rms, Ts);
for k=1:length(PDP)
   h(:,k)=Ray_model(num_ch).*sqrt(PDP(k));
   avg_pow_h(k)=mean(h(:,k).*conj(h(:,k)));
end
H=fft(h(1,:),N);
subplot (221)
stem((0:length(PDP)-1),PDP,'ko'), hold on,
stem([0:length(PDP)-1],avg_pow_h,'k.');
xlabel('Channel tap index p');
ylabel('Average Channel Power [linear]');
title ('IEEE 802.11 Model, \sigma_\tau=25 ns, T_s=50 ns');
legend ('Ideal','simulation'); axis ([1 7 0 1]);
subplot (222)
plot ([N/2+1:N/2]/N/Ts*1e6, 10*log10(H.*conj(H)),'k-');
xlabel ('Frequency [MHz]'), ylabel('Channel power [dB]');
title ('Frequency response, \sigma_\tau=25 ns, T_s=50 ns')

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function PDP=IEEE802_11_model(sigma_t,Ts)

%Input:
%   Sigma_t     :RMS delay spread
%   TS          :Sampling time
%Output
%   PDP         :Power delay profile
lmax=ceil(10*sigma_t/Ts); % Eq. (2.6)
sigma02=(1-exp(-Ts/sigma_t))/(1-exp(-1*(lmax+1)*Ts/sigma_t)); %Eq. (2.9)
l=0:lmax;
PDP=sigma02*exp(-l*Ts/sigma_t); %Eq.(2.8)

B-9 Saleh-Valenzuela

%plot_SV_model_ct.m
clear, clf
Lam=0.0233;
lambda=2.5;
Gam=7.4;
gamma=4.3;
N=1000;
power_nom=1;
std_shdw=3;
t1=0:300;
t2=0:0.01:5;
p_cluster=Lam*exp(-Lam*t1); % ideal exponential pdf
h_cluster=exprnd(1/Lam,1,N); % of random numbers generated
[n_cluster, x_cluster] = hist(h_cluster,25); %obtain distribution
subplot (221), plot(t1, p_cluster, 'k'), hold on,
plot(x_cluster, n_cluster*p_cluster(1)/n_cluster(1), 'k:'); % plotting
legend ('Ideal','Simulation')
title ('Distribution of Cluster Arrival Time, \n\Lambda =',num2str(Lam))
xlabel ('T_m-T_{m-1}[ns]'), ylabel ('p(T_m|T_{m-1})

p_ray=lambda*exp(-lambda*t2) %ideal exponential pdf
h_ray=exprnd (1/lambda, 1, 1000); %of random numbers generated
[n_ray, x_ray]=hist(h_ray,25); % obtain distribution
subplot (222), plot(t2, p_ray, 'k:'), hold on,
plot (x_ray, n_ray*p_ray(1) /n_ray(1), 'k:');
legend ('Ideal','Simulation')
title ('Distribution of Ray Arrival Time, \lambda =',num2str(lambda))
xlabel ('\tau_{r,m}-\tau_{(r-1),m}[ns]'),
ylabel ('p(\tau_{r,m}|\tau_{(r-1),m})
[h, t, to, np]=SV_model_ct(Lam,lambda,Gam,gamma,N,power_nom,std_shdw);
subplot(223), stem (t(1:np(1),1),abs(h(1:np(1),1)), 'ko');
title ('Generated Channel Impulse Response')
xlabel ('Delay [ns]'), ylabel ('Magnitude')
X=10.^(std_shdw*randn(1,N)./20);[temp,x]=hist(20*log10(X),25);
subplot(224), plot(x, temp, 'k-'), axis ([-10 10 0 120])
title ('Log-normal Distribution, \sigma_X=', num2str(std_shdw),'dB')
xlabel('20*log10(X)[dB]'), ylabel('occasion')

function [h,t0,np]=sv_model_ct(Lam,lambda,Gam,gamma,num_ch,b002,sdi,nLos)
%s-v channel model

% Inputs
% Lam  : cluster arrival rate in GHz (ave # of clusters nsec)
% lam  : Ray arrival rate in GHz (ave # of rays per nsec)
% Gam  : cluster decay factor (time constant, nsec)
% gam  : Ray decay factor (time constant, nsec)
% num ch : Number of random realizations to generate
% b002 : Power of first ray first cluster
% sdi  : Standard deviation of log-normal shadowing of entire impulse response in dB
% nlos : Flag to specify generation of NLOs channels

% Outputs
% h      : a matrix with num_ch columns, each column having a random realization of channel model (impulse response)
% t      : Time instances (in nsec) of the paths whose signed amplitudes are stored in h
% t0     : Arrival time the first cluster for each realization Bumber of paths for each realization.

if nargin<8, nLos=0; end  % LOS environment
if nargin<7, sdi=0; end    % dB
if nargin<6, b002=1; end   % Power of 1st ray of 1st cluster

h_len=1000;
for k=1:num_ch % Loop over number of channels
    tmp_h=zeros(h_len,1);
    tmp_t=zeros(h_len,1);
    if nLos, Tc=exprnd (1/Lam);
    else Tc=0;  % first cluster arrival occurs at time 0
    end
    t0(k)=Tc; path_ix=0;
    while (Tc<10*Gam)
        Tr=0;
        while (Tr<10*gam)
            t_val=Tc+Tr;  % time of arrival of this ray
            bk12=b002*exp(-Tc/Gam)*exp(-Tr/gam)  % ray power, Eq. 12.14)
            r=sqrt(randn^2 + randn^2)*sqrt(bk12/2);
            h_val=exp(j*2*pi*rand)*r;  % Uniform Phase
            path_ix=path_ix+1;  % Row index of this ray
            tmp_h(path_ix)=h_val; tmp_t(path_ix)=t_val;
            Tr=Tr + exprnd (1 /Lam);
        end
        Tc=Tc + exprnd(1/lam);  % Cluster arrival time based on Eq. 12.10)
    end
    np(k)=path_ix;  % Number of rays paths for this realization
end

[sort_tmp_t, sort_ix]=sort(tmp_t(1:np(k)));  % in ascending order

for k=1:num_ch % Loop over number of channels
    tmp_h=zeros(h_len,1);
    tmp_t=zeros(h_len,1);
    if nLos, Tc=exprnd (1/Lam);
    else Tc=0;  % first cluster arrival occurs at time 0
    end
    t0(k)=Tc; path_ix=0;
    while (Tc<10*Gam)
        Tr=0;
        while (Tr<10*gam)
            t_val=Tc+Tr;  % time of arrival of this ray
            bk12=b002*exp(-Tc/Gam)*exp(-Tr/gam)  % ray power, Eq. 12.14)
            r=sqrt(randn^2 + randn^2)*sqrt(bk12/2);
            h_val=exp(j*2*pi*rand)*r;  % Uniform Phase
            path_ix=path_ix+1;  % Row index of this ray
            tmp_h(path_ix)=h_val; tmp_t(path_ix)=t_val;
            Tr=Tr + exprnd (1 /Lam);  % Ray arrival time based on Eq. (2.11)
        end
        Tc=Tc + exprnd(1/lam);  % Cluster arrival time based on Eq. 12.10)
    end
    np(k)=path_ix;  % Number of rays paths for this realization
end

% Log-normal shadowing on this realization
fac=10^(sdi*randn(20))/sqrt(h(1:np(k),k)'*h(1:np(k),k));
h(1:np(k),k)=h(1:np(k),k)*fac;  % Eq. (2.15)
\%plot\_UWB\_channel.m

%clear, clf
Ts = 0.167;
num\_ch=100;randn\('state' ,12);
rand\('state',12);
cm=1;
[Lam, lam, Gam, gam, nlos, sdi, sdc, sdr]=UWB\_parameters(cm);
[h\_ct, t\_ct, t0, np]=...
    UWB\_model\_ct(Lam, lam, Gam, gam, num\_ch, nlos, sdi, sdc, sdr);
[hN, N]=convert\_UWB\_ct(h\_ct, t\_ct, np, num\_ch, Ts);
h=resample(hN,1,N);
h=h\*N;
channel\_energy=sum(abs(h).^2);
\t\h\_len=size(h,1);
\t\t=t[0:(\h\_len-1)\]*Ts;
\t\for k=1: num\_ch
\t\t sq\_h=abs(h(:,k)).^2/channel\_energy(k);
\t\t t\_norm= t-t0(k);
\t\t excess\_delay(k)=t\_norm*sq\_h;
\t\t rms\_delay(k)=sqrt((t\_norm-excess\_delay(k)).^2*sq\_h);
\t\t temp_h=abs(h(:,k));
\t\t threshold\_dB=-10;
\t\t temp\_thresh=10^(threshold\_dB/20)*max(temp_h);
\t\t num\_sig\_paths(k)=sum(temp\_h>temp\_thresh);
\t\t temp\_sort=sort(temp\_h.^2);
\t\t cum\_energy=cumsum(temp\_sort(end:-1:1));
\t\t x=0.85;
\t\t index\_e=min(find(cum\_energy >= x*cum\_energy(end)));
\t\t num\_sig\_e\_paths(k)= index\_e;
\endfor

energy\_mean=mean(10*log10(channel\_energy));
energy\_std\_dev=std(10*log10(channel\_energy));
mean\_excess\_delay=mean(excess\_delay);
mean\_rms\_delay=mean(rms\_delay);
mean\_sig\_paths=mean(num\_sig\_paths);
mean\_sig\_e\_paths=mean(num\_sig\_e\_paths);
\t\temp\_average\_power= sum(h'.\*h')/num\_ch;
\t\t temp\_average\_power= temp\_average\_power/max(temp\_average\_power);
\t\t average\_decay\_profile\_dB= 10*log10(temp\_average\_power);
\t\t fprintf(1,'Model Parameters\n''Lam=%.4f, lam=%.4f,
Gam=%.4f,gam=%.4f\n NLOS flag = %d, std\_shdw=%.4f, td\_ln\_1=%.4f,
td\_ln\_2=%.4f\n''
),...
\t\t Lam, lam, Gam,gam, nlos, sdi, sdc, sdr);
\t\t fprintf(1,'Model characteristics\n'');
\t\t fprintf(1,'Mean delays : excess(tau\_m)=%.1fns,
RMS(tau\_rms)=%.1f\n'',
\t\t mean\_excess\_delay, mean\_rms\_delay);
\t\t fprintf(1,'#paths:\#NP 10dB=%.1f, NP 85=%.1f\n'',
\t\t mean\_sig\_paths, mean\_sig\_e\_paths);
\t\t fprintf(1,'Channel energy:mean=%.1fdB, std\_deviation =%.1fdB\n'',
\t\t energy\_mean, energy\_std\_dev);
\t subplot (421), plot (t,h)
\t title('Impulse response realizations'), xlabel('Time[ns]')
\t subplot (422), plot([1:num\_ch],excess\_delay,'b-','...
title ('Excess delay [n]') subplot(423), title('RMS delay [ns]'), xlabel ('Channel number') plot([1: num_ch], rms_delay, 'b-', [1 num_ch], mean_rms_delay*[1 1], 'r-'); subplot (424), plot ((1:num_ch), num_sig_paths, 'b-', [1 num_ch], mean_sig_paths*[1 1], 'r-'); grid on, title ('Number of significant paths within 10 dB of peak') subplot (425), plot(t, average_decay_profile_dB); grid on title ('Average Power Decay Profile'), axis ([0 t(end) -60 0]) subplot (426), title('Channel Energy'); figh = plot([1:num_ch], 10*log10(channel_energy), 'b-',... [1 num_ch], energy_mean*[1 1], 'g-',... [1 num_ch], energy_mean+energy_stddev*[1 1], 'r-',... [1 num_ch], energy_mean-energy_stddev*[1 1], 'r:') legend (figh, 'Per-channel energy', 'Mean', '\pm Std. deviation',0)

%UWB channel model Program 28 "UWB parameters" to set the parameters function [Lam, lam, Gam, gam, nlos, sdi, sdc, sdr]=UWB_parameters(cm)

%Table 2.1: tmp = 4.8 /sqrt(2);
Tb2_1=[0.0233 2.5 7.1 4.3 0 3 tmp tmp; 0.4 0.5 5.5 6.7 1 3 tmp tmp; 0.0667 2.1 14.0 7.9 1 3 tmp tmp; 0.0667 2.1 24 12 1 3 tmp tmp];
Lam=Tb2_1(cm, 1);
lam=Tb2_1(cm, 2);
Gam=Tb2_1(cm, 3);
gam=Tb2_1(cm, 4);
nlos= Tb2_1(cm, 5);
sdi= Tb2_1(cm, 6);
sdc= Tb2_1(cm, 7);
sdr= Tb2_1(cm, 8);

%Program 2.10 "UWB model ct" for a continuous_time UwB channel model [25]
function [h, t, t0, np]=UWB_model_ct(Lam,lam,Gam,gam,num_ch,nlos,sdc,sdr,sdi)
std_L=1/sqrt(2*Lam);
std_lam=1/sqrt(2*lam);
mu_const=(sdc^2+sdr^2)*log(10)/20;
h_len=1000;
for k=1:num_ch
 tmp_h= zeros(h_len,1);
 tmp_t=zeros(h_len,1);
 if nlos
 Tc=(std_L*randn)^2+(std_L*randn)^2;
 else
 Tc=0;
 end
 t0(k)=Tc;
 path_ix=0;
 while (Tc<10*Gam)
 Tr=0; ln_xi=sdc*randn;
 while (Tr<10*gam)
 t_val = Tc+Tr;
 mu=(-10*Tc/Gam-10*Tr/gam)/log(10)-mu_const; %Eq. 12.19

end

ln_beta= mu + sdr*randn;
pk= 2*round(rand)-1;
h_val=pk*10^((ln_xi+ln_beta)/20)
path ix=path ix +1;
tmp_h(path ix)=h_val;
tmp t(path ix)=t_val;
Tr= Tr+ (std_lam *randn)^2+(std_lam*randn)^2;
end
Tc=Tc+(std_L*randn)^2+(std_L*randn)^2;
end
np(k)= path ix;
[sort_tmp_t, sort ix]=sort(tmp t(1:np(k)));
t(1:np(k),k)=sort_tmp_t;
h(1:np(k),k)=tmp h(sort ix(1:np(k)));
fac = 10^(sdi*randn/20)/sqrt(h(1:np(k),k)'*h(1:np(k),k));
h(1:np(k),k)=h(1:np(k),k)*fac;
end

%Program 2.9 "convert UWBct" to convert a continuous-time channel into a discrete-time one
function [hN, N]= convert_UWB_ct(h_ct,t,np,num_ch,Ts)
min Nfs=100;
N=2^nextpow2(max(1,ceil(min Nfs*Ts)));
Nfs= N/Ts;
t_max=max (t(:));
h_len=1+floor(t_max*Nfs);hN=zeros(h_len,num_ch);
for k=1: num_ch
np_k=np(k);
t_Nfs= 1+floor(t(1:np_k,k)*Nfs);
for n=1:np_k
hN(t_Nfs(n),k)= hN(t_Nfs(n),k)+ h_ct(n,k);
end
end

B-11 Clarke/Gans

%MATLAB Programs: FWGN Channel Model Program 2.11 "plot FWGN.m" to plot an FWGN model plot FWGN clear, clf
fm=100;
scale=1e-6; % Maximum Doppler frequency and mu
ts_mu=50;
ts=ts_mu*scale;
fs=1/ts; %Sampling time/frequency
Nd=1e6; %Number of samples
%obtain the complex fading channel
[h, Nfft, Nifft, doppler coeff]= FWGN_model (fm, fs, Nd);
subplot (211), plot ([1:Nfft]*ts, 10*log10(abs(h)));
str=sprintf('Clarke /Gan Model, f_m=%d[Hz], T_s=%d[us]', fm,ts_mu);
title(str),axis ([0 0.5 -30 5])
subplot(223), hist(abs(h),50),subplot(224),hist(angle(h),50)
subplot(211), plot ([1:Nd]*ts, 10*log10(abs(h)))
str=sprintf('Clarke/Gan Model, f_m=%d[Hz], T_s=%d[us]', fm, ts_mu);
title(str), axis ([0 0.5 -30 51])
subplot (223), hist(abs (h), 50), subplot (224), hist(angle(h),50)
%Program 2.12 FWGN model
function [h, Nfft, Nifft, doppler_coeff]=FWGN_model( fm, fs, N)
%FWGN (Clarke/Gan) Model
%Input: fm=Maximum Doppler frequency
%       fs= Sampling frequency,
%       N= Number of samples
%Output:
%h=Complex fading channel
Nfft=2^max(3, nextpow2 (2*fm/fs*N));  %Nfft=2^n
Nifft=ceil(Nfft*fs/ (2 * fm));
%Generate the independent complex Gaussian random process
GI=randn(1, Nfft);  GQ= randn (1, Nfft);
%Take FFT of real signal in order to make hermitian symmetric
CGI= fft(GI); CGQ=fft(GQ);
%NFFT sample Doppler spectrum generation
doppler_coeff=Doppler_spectrum(fm,Nfft);
%Do the filtering of the Gaussian random variables here
f_CGI=CGI.*sqrt(doppler_coeff);
f_CGQ=CGQ.*sqrt(doppler_coeff);
%Adjust sample size to take IFFT by (Nifft-Nfft) sample zero-padding
Filtered_CGI=[f_CGI(1:Nfft/2) zeros(1, Nifft-Nfft) f_CGI(Nfft/2+1 :Nfft)];
Filtered_CGQ=[f_CGQ(1:Nfft/2) zeros(1, Nifft-Nfft) f_CGQ(Nfft/2+1 :Nfft)];
hI=ifft(Filtered_CGI);
hQ=ifft(Filtered_CGQ);
%Take the magnitude squared of the I and Q components and add them
rayEnvelope=sqrt(abs(hI).^2 + abs(hQ).^2);
%Compute the root mean squared value and normalize the envelope
rayRMS=sqrt(mean(rayEnvelope(1:N).*rayEnvelope(1:N)));
h=complex(real(hI(1:N)), real(hQ(1:N)))/rayRMS;

%Program 2.13 Doppler spectrum
function y=Doppler_spectrum(fd,Nfft)
%fd = Maximum Doppler frequency
%Nfft = Number of frequency domain points
df= 2*fd/Nfft;  %frequency spacing
%DC component first
f(1)=0;  y(1)=1.5/(pi*fd);
%The other components for one side of the spectrum
for i=2:Nfft/2,
    f(i)=(i-1)*df;  % frequency indices for polynomial fitting
    y(i)=1.5/(pi*df* sqrt(1-(f(i)/fd).^2));
end
%Nyquist frequency applied polynomial fitting using last 3 samples
nFitPoints=3;  kk=[Nfft/2-nFitPoints:Nfft/2];
polyFreq=polyfit(f(kk), y(kk), nFitPoints);
y((Nfft/2)+1)= polyval(polyFreq, f(Nfft/2)+df);
%MATLABO Programs: Modified Frequency-Domam FWGN CHANNEL Model
%Program 2.14 "plot modified FWGN.m" to plot modified FWGN channel models
%plot modified FWGN.m
clear, clf
Nfading=1024; %IFFT size for Npath x Nfading fading matrix
Nfosf=8; % Fading over sampling factor
Npath=2; %Number of paths
N=10000;
FadingType='class';
fm = [100 10]; % Doppler frequency
[FadingMatrix,tf]=FWGN_ff(Npath, fm, Nfading, Nfosf,FadingType);
subplot(211),
plot((1:Nfading)*tf,10*log10(abs(FadingMatrix(1,:))),'k:'),
hold on, plot((1:Nfading)*tf,10*log10(abs(FadingMatrix(2,:))),'k-')
title(['Modified FWGN in Frequency Domain, Nfading=',num2str(Nfading),',Nfosf=',num2str(Nfosf)]);
xlabel('time[s]'),ylabel('Magnitude [dB]'),
legend ('Path 1, f_m=100Hz', 'Path 2, f_m=10Hz'), axis([0 0.5 -20 10])

[FadingMatrix, tf]=FWGN_tf(Npath, fm, N,Nfading,Nfosf, FadingType);
set(gcf,'CurrentAxes',gca),
subplot(212), plot([1:N]*tf, 10*log10(abs(FadingMatrix(1,:))),'k:'),
hold on, plot([1:N]*tf, 10*log10(abs(FadingMatrix(2,:))),'k-'),
title([' Modified FWGN in Time Domain, Nfading=',num2str(Nfading),',Nfosf=',num2str(Nfosf),', T_s=',num2str(tf),'s']);
xlabel('time[s]'), ylabel('Magnitude [dB]'), legend ('Path 1, f_m=100Hz', 'Path 2, f_m=10Hz'), axis([0 0.5 -20 10])

%Program 2.15 FWGN_ff" for a modified frequency-domain FWGN channel model
function [FadTime, tf]=FWGN_ff(Np, fm_Hz,Nfading,Nfosf,FadingType,varargin)
%Fadng generation based on FIGN method in the frequency domain
%FadTime= FWGN_ff(Np, fm_Hz,Nfading, Nfosf, Fading Type, sigma, phi)
%Inputs:
%NP :number of multipaths
%fm_Hz :a vector of max. Doppler frequency of each path [Hz]
%nfading :Doppler filter size (IFFT size)
%Nfosf :oversampling factor of Doppler bandwith
%FadingType :Doppler type, 'laplacian'/'class'/'flat'
%sigma :angle spread of UE in case of 'laplacian' Doppler type
%phi :DoM-AoA in case of 'laplacian' Doppler type
%output:
%FadTime :Np x Nfading, fading time matrix
fmax_Hz=max(fm_Hz);
%Doppler frequency spacing respect to maximal Doppler frequency
dfmax=2*Nfosf*fmax_Hz/Nfading;
%obtain a function corresponding to Doppler spectrum of Fading(TYPE)
ftype_DOP=Doppler_PSD_function(FadingType);
err_msg='The difference between max/min Doppler frequencies is too large.\n increase the IFFT size';
if isscalar(fm_Hz), fm_Hz=fm_Hz*ones(1, Np); end
if strcmp(lower(FadingType(1:3)),'Lap')
% Laplacian constrained PAS
for i=1:Np
    Nd=floor(fm_Hz(i)/dfmax)-1;  %Nd=fm_Hz/dfmax=Nfading/(2*Nfosf)
    if Nd<1, error (err_msg); end
    tmp= ftnt_PSD([-Nd:Nd]/Nd, varargin{1}(i), varargin{2}(i));
    tmpz= zeros(1,Nfading-2*Nd+1);
    FadFreq(i,:)= [tmp(Nd+1:end-1) tmpz tmp(2:Nd)];
end
else %symmetric Doppler spectrum
    for i=1:Np
        Nd=floor(fm_Hz(i)/dfmax)-1;
        if Nd<1, error (err_msg); end
        tmp=ftn_PSD([0:Nd]/Nd); tmpz= zeros(1, Nfading-2*Nd+3);
        FadFreq(i,:)= [tmp(1:Nd) tmpz fliplr(tmp(2:Nd))];
    end
end
%Add a random phase to the Doppler spectrum
FadFreq=sqrt(FadFreq).*exp(2*pi*j*rand(Np, Nfading));
FadTime=ifft(FadFreq, Nfading, 2);
FadTime=FadTime./sqrt(mean(abs(FadTime).^2,2)*ones(1,size(FadTime,2)));
%Normalize to 1
tf=1/(2*fmax_Hz*Nfosf);  %fading sample time = 1/(Doppler BW*Nfosf)

%Program 2.16 "Doppler PSD_function" for Doppler function
function ftn=Doppler_PSD_function(type)
% Doppler spectrum function for type =
% 'flat'     :S(f)=1, |f0(=f/fm)|
% 'class'    :S(f)=A/(sqrt(1-f0.^2)), |f0|<1 (A: a real number)
% 'laplacian':
%    S(f)=1./sqrt(1+1e-9-f0.^2).*exp(-sqrt(2)/sigma*abs(acos(f0)-phi))
%      +exp(-sqrt(2)/sigma*abs(acos(f0)-phi))
%     with sigma(angle spread of UE) and phi(=DoM-AoA)
%     in the case of laplacian' Doppler type
% 'sui'     :S(f)=0.785*fo.^4-1.72*fo.^2+1, |f0|<1
% '3gpprice' :S(f)=0.41./(2*pi*fm*sqrt(1+1e-9*(f./fm).^2))+
%     0.91*delta_ftn(f, 0.7*fm),|f|<fm
% 'dr', S(f)=inline('1./sqrt(2*pi*Dsp/2)*exp(-(f-Dsh).^2)/Dsp
%       f','Dsp','Dsh');
%f0: Normalized Doppler frequency defined as f0=f/fm
%fm: Maximum Doppler frequency
switch lower(type (1:2))
    case 'fl', ftn=inline('ones(1,length(f0))');
    case 'cl', ftn=inline('1./sqrt(1+1e-9-f0.^2)');
    case 'la', ftn=inline('(exp(-sqrt(2)/sigma*abs(acos(f0)-phi)))+exp(-sqrt(2)/sigma*abs(acos(f0)-phi))))./sqrt(1+1e-9-f0.^2)';
    case 'sui', ftn=inline('0.785*fo.^4-1.72*fo.^2+1.');
    case '3gp', ftn=inline('0.41./(2*pi*fm*sqrt(1+1e-9-(f./fm).^2))+0.91*delta_ftn(f, 0.7*fm)', 'f','fm');
    case 'dr', ftn=inline('1./sqrt(2*pi*Dsp/2)*exp(-(f-Dsh).^2)/Dsp', 'f', 'Dsp', 'Dsh');
    otherwise, error('Unknown Doppler type in Doppler_Psp_function()');
end

%Program 2.18 gen filter for Doppler filter coefficients
function filt=gen_filter(fm_Hz, fmax_Hz, Nfading, Nfosf, type, varargin)
%FIR filter weights generation
%Inputs:
% Nfading: Doppler filter size, i.e., IFFT size
% Outputs:
% filt : Filter coefficients
% Doppler BW = 2*fm*Nfosf ==> 2*fmax_Hz*Nfosf
% dfmax = 2*Nfosf*fmax_Hz/Nfading; % Doppler frequency spacing
% respect to maximal Doppler frequency
Nd = floor(fm_Hz/dfmax);
if Nd<1, error ('The difference between max/min Doppler frequencies is too large.\nincrease the IFFT size?');
end
ftn_PSD=Doppler_PSD_function(type);  % Corresponding Doppler function
switch lower(type (1:2))
  case '3g'
    PSD=ftn_PSD([-Nd:Nd],Nd);
    filt=[PSD(Nd+1:end-1) zeros(1,Nfading-2*Nd+1) PSD(2:Nd)];
  case 'la'
    PSD=ftn_PSD([-Nd:Nd]/Nd,varargin{:});
    filt=[PSD(Nd+1:end-1) zeros(1,Nfading-2*Nd+1) PSD(2:Nd)];
  otherwise
    PSD=ftn_PSD([0:Nd]/Nd);
    filt=[PSD(1:end-1) zeros(1, Nfading-2*Nd+3) PSD(end-1:-1:2)];
% constructs a symmetric Doppler spectrum
end
filt=real(ifftshift(ifft(sqrt(filt))));
filt=filt/sqrt(sum(filt.^2));

function [FadMtx, tf]= FWGN_tf(Np,fm_Hz,N,M,Nfosf,type,varargin)
% fading generation using FWGN with filtering in the time domain
% Inputs:
% NP        : Number of multipaths
% fm_Hz     : A vector of maximum Doppler frequency of each path [Hz]
% N         : Number of independent random realizations
% M         : Length of Doppler filter, i.e., size of IFFT
% Ntosf     : Fading oversampling factor
% type      : Doppler spectrum type
%            'flat'=flat, 'class'=classical, 'sui'=spectrum of SUI channel
%            '3gpprice'=rice spectrum of 3GPP
% Outputs:
% FadMtx   Np x N fading matrix
% tf        fading sample time = 1/(Max. Doppler BW*Nfosf)
if iscalar(fm_Hz), fm_Hz=ones(1,Np); end
fmax=max(fm_Hz);
path_wgn= sqrt(1/2)*complex(randn(Np,N),randn(Np,N));
for p=1:Np
  filt=gen_filter(fm_Hz(p), fmax,M,Nfosf,type,varargin{:});
  path(p,:)=fftfilt(filt,[path_wgn(p,:) zeros(1,M)]); % filtering WGN
end
FadMtx=path(:,N/2+1: end-N/2);
if 1/(2*fmax*Nfosf); % fading sample time = 1/(Max. Doppler BW*Nfosf)
FadMtx=FadMtx./sqrt(mean(abs(FadMtx).^2,2)*ones(1, size (FadMtx ,2)));

B-13  Jakes

%MATLAB Programs: Jakes Channel Model Program 2.19
"plot Jakes model.m"
%to plot a jakes channel model
%plot_Jakes_model.m

clear all, close all

%Parameters
fd=926; Ts=1e-6; %Doppler frequency and Sampling time
M=2^12; t=[0:M-1]*Ts; f=[-M/2:M/2-1]/(M*Ts*fd);
Ns=50000;  t_state=0;

%channel generation
[h, t_state]=Jakes_Flat(fd, Ts, Ns, t_state,1,0);
subplot(311), plot([1:Ns]*Ts,10*log10(abs(h)));
title([' Jakes Model, f_d=' num2str(fd), 'Hz, T_s=' num2str(Ts), 's']);
axis ([0 0.05 -20 10]), xlabel ('time[s]'), ylabel ('Magnitude[dB]')

subplot (323), hist(abs (h), 50);
title ([ 'Jakes Model, f_d=' num2str(fd), 'Hz, T_s_'], num2str(Ts), 's']);
xlabel ('Magnitude'), ylabel ('Occasions')
subplot (324), hist (angle (h), 50);
title ([ 'Jakes Model, f_d=' num2str(fd), 'Hz, T_s=' num2str(Ts), 's']);
xlabel ('Phase [rad]'), ylabel ('Occasions')

%Autocorrelation of channel
temp= zeros (2, Ns);
for i=1:Ns
    j=i:Ns;
    temp1(1:2,j-i+1)=temp(1:2,j-i+1)+[h(i)'*h(j); ones(1,Ns-i+1)];
end
for k=1:M
    Simulated_corr(k)=real(temp(1, k))/temp(2,k);
end
Classical_corr=besselj(0, 2*pi*fd*t);

Fourier Transform of autocorrelation
Classical_Y=fftshift(fft(Classical_corr));
Simulated_Y=fftshift(fft(Simulated_corr));
subplot(325), plot(t, abs(Classical_corr), 'b:', t, abs(Simulated_corr), 'r:');
title([ 'Autocorrelation, f_d=' num2str(fd), 'Hz']);
grid on, xlabel('delay\tau[s]'), ylabel('Correlation')
legend ('Classical', 'Simulation')
subplot (326), plot(f, Classical_Y, 'b:', f, abs(Simulated_Y), 'r:');
title ([ 'Doppler Spectrum, f_d=', num2str(fd), 'Hz']);
axis([-1 10 0 600]), xlabel('f/f_d'), ylabel('Magnitude')
legend ('Classical', 'Simulation')

%Program 2.20 "Jakes Flat" for fading signal with Jakes model
function [h, tf]=Jakes_Flat(fd, Ts, Ns, t0, E0, phi_N)

if nargin<6, phi_N=0; end
if nargin<5,  E0=1;  end
if nargin<4,  t0=0;  end
N0=8;  %As suggested by Jakes
N=4*N0+2;  %an accurate approximation
wd=2*pi*fd;  %Maximum Doppler frequency [rad]
t=t0+[0:Ns-1]*Ts;  %Time Vector
tf=t(end)+Ts;  %Final time
coswt=[sqrt(2)*cos(wd*t); 2*cos(wd*cos(2*pi /N*[1:N0]')*t)];  % Eq. (2.26)
h=E0/sqrt(2*N0+1)*exp(j*[phi_N pi /(N0+1)*[1:N0]])*coswt;  % Eq. (2.23)

B-14  Ray-based

%MATLAB" Programs: Ray-Based Channel with Uniform Power subray Method
%Program 2.21 "plot_ray_fading.m" to plot array-based channel model
%plot_ray_fading.m
clear, clf
fc=2.5e8;
fs=5e4;
speed_kmh=120;
Ts=1/fs;
v_ms=speed_kmh/3.6;
w_l_m=3e8/fc;
%Channel parameters setting: scM case 2
PDP_dB=[0. -1. -9. -10. -15. 20];
t_ns=[0 310 710 1090 1730 2510];
BS_theta_LOS_deg=0;
MS_theta_LOS_deg=0;
BS_AS_deg=2;  % Laplacian PAS
BS_AoD_deg=50*ones(size(PDP_dB));
 MS_AO_deg=35;  %for Laplacian PAS
DoT_deg=22.5;
MS_AoA_deg=35;
%generates the phase of a subray
[BS_theta_deg, MS_theta_deg, BS_PHI_rad]=gen_phase(BS_theta_LOS_deg, ...
  BS_AS_deg, BS_AoD_deg, MS_theta_LOS_deg, MS_AO_deg, MS_AoA_deg);
PDP=dB2W(PDP_dB);
%generates the coefficients
t=[0:1e5-1]*Ts;
h=ray_fading(20, PDP, BS_PHI_rad, MS_theta_deg, v_ms, DoT_deg, w_l_m, t);
plot (t, 10*log10(abs (h (1,:))))
title([['Ray Channel Model, f_c=', num2str(fc),',Hz,
T_s=',num2str(Ts),'s']])
xlabel('time[s]'), ylabel('Magnitude[dB]')

function theta=equalpower_subray(AS_deg)
%obtain angle spacing for equal power Laplacian PAS in scM Text equal
%Input:
% AS_deg: angle spread with valid values of 2,5 (for BS), 35 for (MS)
%Output:
%theta offset angle with M=20 as listed in Table 2.2
if AS_deg==2
    theta=[0.0894 0.2826 0.4984 1.0257 1.3594 1.7688 2.2961 3.0389 4.3101];
elseif AS_deg==5
    theta=[0.2236 0.7064 1.2461 1.8578 2.5642 3.3986 4.4220 5.7403 7.5974 10.7753];
elseif AS_deg==35
    theta=[1.5649 4.9447 8.7224 13.0045 17.9492 23.7899 30.9538 40.1824 53.1816 75.4274];
else
    error('Not Ssupport AS');
end

%Program 2.23 "assign_offset" to allocate the offset angel for each subray
function theta_AoA_deg=assign_offset(AoA_deg, AS_deg)
    %Assigns AoA/AoD offset to mean AoA/AoD
    % Inputs: AoA_deg = mean AoA/AoD, AS is a= angle spread
    % Output: theta_AoA_deg = AoA_deg+offset_deg
    offset = equalpower_subray(AS_deg);
    theta_AoA_deg=zeros(length(AoA_deg),length(offset));
    for n=1: length (AoA_deg)
        for m=1:length(offset),
            theta_AoA_deg(n,[2*m-1:2*m])=AoA_deg(n)+[offset(n)-offset(m)];
        end
    end
end

%Program 2.24 "gen phase" to generate the phase for each subray
function [BS_theta_deg, MS_theta_deg, BS_PHI_rad]=
gen_Phase(BS_theta_LOS_deg, BS_AS_deg, BS_AoD_deg, MS_theta_LOS_deg, MS_AS_deg, MS_AoA_deg, M)
    %Generates phase at BS and Ms
    %Inputs
    %BS_theta_LOS_deg   : AoD of LOS path in degree at BS
    %BS_AS_deg          : As of Bs in degree
    %BS_AoD_deg         : AOD of BS in degree
    %MS_theta_LOS_deg   : AoA of LOS path in degree at MS
    %MS_AS_deg          : As of Ms in degree
    %MS_AoA_deg         : AOA of MS in degree
    %M                  : # of subrays
    %Outputs:
    % BS_theta_deg     : (Npath x M) DoA per Path in degree at BS
    % MS_theat_deg     : (Npath x M) DoA per path in degree at MS
    % BS_PHI_deg       : (Npath x M) random phase in degree at BS
    if nargin==6, M=20; end
    BS_PHI_rad=2*pi*rand (length (BS_AoD_deg),M); %uniform phase
    BS_theta_deg= assign_offset(BS_theta_LOS_deg+BS_AoD_deg, BS_AS_deg);
    MS_theta_deg= assign_offset(MS_theta_LOS_deg+MS_AoA_deg, MS_AS_deg);
    %random pairing
    index=randperm(M); MS1=size(MS_theta_deg, 1);
    for n=1:MS1,
    MS_theta_deg(n,:)= MS_theta_deg(n,index);
    end
end

%Program 2.25 "ray_fading" to generate the fading for each subray
function h=rayfading(M, PDP, BS_PHI_rad, MS_theta_deg, V_ms, theta_v_deg, lam, t)
% Inputs:
% M : Number of subrays
% PDP : 1 x Npath Power at delay
% Bs_theta_deg : (Npath x M) DoA per th in Bs degree at
% Bs_PHI_rad : (Npath x M) random phase in degree at Bs
% MS_theta_deg : (Npath x M) DoA per path in degree at Ms
% v_ms : Velocity in m/s
% theta_deg : DoT of mobile in degree
% lam : Wavelength in meter
% t : Current time
% Output
% h : length(PDP) x length(t) channel coefficient matrix

MS_theta_rad = deg2rad(MS_theta_deg);
theta_v_rad = deg2rad(theta_v_deg);

% To generate channel coefficients using Eq. (2.32)
for n=1:length(PDP)
    What1 = exp(-j*BS_PHI_rad(n,:))' * ones(size(t));
    What2 = exp(-j*2*pi/lam*v_ms*cos(MS_theta_rad(n,:)') - theta_v_rad)*t;
    tmp = exp(-j*BS_PHI_rad(n,:))' * ones(size(t)).*exp(-j*2*pi/lam*v_ms*cos(MS_theta_rad(n,:)') - theta_v_rad)*t;
    h(n,:) = sqrt(PDP(n)/M) * sum(tmph);
end

function y=dB2w(dB)
y=10.^(0.1*dB);

B-15 SUI

% Program 2.27 .plot sur channel m" to plot an suI channel model
% plot_SUI_channel.m
clear, clf
ch_no=6;
fc=2.5e9;
fs_Hz=1e7;
Nfading=1024; % size of Doppler filter
N=10000;
Nfosf=4;
[Delay_us, Power_dB, K_factor, Doppler_shift_Hz, Ant_corr, Fnorm_dB] = SUI_parameters(ch_no);
[FadTime, tf] = SUI_fading(Power_dB, K_factor, Doppler_shift_Hz, Fnorm_dB, N, Nfading, Nfosf);
c_table = ['b', 'r', 'm', 'k'];
subplot (311)
stem(Delay_us, 10.^(Power_dB/10)), axis([-1 21 0 1.11])
grid on, xlabel('Delay time [ms]'), ylabel('Channel gain');
title(['PDP of Channel No.', num2str(ch_no)]);
subplot (312)
for k=1:length(Power_dB)
    plot((0: length(FadTime(k,:))-1)*tf,20*log10(abs(FadTime(k,:))), c_table(k)); hold on
end
grid on, xlabel('Time[s]'), ylabel('Channel Power [dB]');
title(['Channel No.', num2str(ch_no)]);
axis([0 60 -50 10])
legend(['Path 1', 'Path 2', 'Path 3'])
idx_nonz=find(Doppler_shift_Hz);
FadFreq=ones(length(Doppler_shift_Hz),Nfading);
for k=1: length(idx_nonz)
    max_dsp=2*Nfosf*max(Doppler_shift_Hz);
    dfmax= max_dsp/Nfading;
    %Doppler frequency spacing respect to maximal Doppler frequency
    Nd=floor(Doppler_shift_Hz(k) /dfmax) -1;
    f0=[-Nd+1:Nd] / (Nd);
    % Frequency vector
    f=f0.*Doppler_shift_Hz(k);
    tmp=0.785*f0.^4 -1.72*f0.^2+ 1.0 ;
    %Eq. (2.41)
    %Hpsd=pwelch(FadTime(idx_nonz(k),:),max_dsp);
    %'Fs',max_dsp,'SpectrumType','twosided'); %works
    %Hpsd=pwelch(FadTime(idx_nonz(k),:),max_dsp,'twosided','dsp');
    %Hpsd=pwelch(FadTime(idx_nonz(k),:)); % Works
    %Hpsd=pwelch(FadTime(idx_nonz(k),:),64,[],64),max_dsp,'twosided','psd');
    %Great
    h = spectrum.welch;
    %Default segLen = 64, ovlpPct = 50, Sampling Flag 'Symmetric),
    window
    %is Hamming window
    hPSD = psd(h,FadTime(idx_nonz(k),:),'Fs',max_dsp,'Spectru
    %mType','twosided');
    %Great
    %segLen = 64
    %ovlpPct=50;
    %win = hamming(segLen,'symmetric');
    %Noverlap = ceil((ovlpPct/100)*segLen);
    %[Hpsd,F]=pwelch(FadTime(idx_nonz(k),:),win,Noverlap,[],max_dsp,'twosided');
    %pxx = pwelch(x,segmentLength,[],nfft);
    % nrom_f=Hpsd.Frequencies-mean(Hpsd.Frequencies);
    %PSD_d=fftshift(hpsd.Data);
    %Do not remove incase of replacing syntax with pwelch function
    %segLen = 64
    %ovlpPct=50;
    %win = hamming(segLen,'symmetric');
    %Noverlap = ceil((ovlpPct/100)*segLen);
    %[Hpsd,F]=pwelch(FadTime(idx_nonz(k),:),win,Noverlap,[],max_dsp,'twosided');
    %pxx = pwelch(x,segmentLength,[],nfft);
    % nrom_f=Hpsd.Frequencies-mean(Hpsd.Frequencies);
    %PSD_d=fftshift(hpsd.Data);
    subplot(3, 3, 6+ k), plot(nrom_f, PSD_d,
    xlabel('Frequency [Hz]'),
    title ('h_\text{'},num2str(idx_nonz(k)),')
end

%Program 2.28 SUI parameters" to set the sui channel model parameters
function [Delay_us,Power_dB,K,Doppler_shift_Hz,
Ant_corr,Fnorm_dB]=SUI_parameters(ch_no)
%SUI Channel Parameters from Table 2.8
%Inputs
% ch_no :channel scenario number
%Outputs:
% Delay_us :tap Delay[us]
function [FadMtx, tf]=SUI_fading(Power_dB, K_factor, Doppler_shift_Hz, Fnorm_dB,N,M,Nfosf)
% SUI fading generation using FWGN with filtering in frequency domain

% Inputs:
% Power_dB : power in each tap in dB
% K_factor : Rician K-factor in linear scale
% Doppler_shift_Hz : a vector containing frequency of each path in Hz
% Fnorm_dB : gain normalization factor in dB
% N : # independent random realizations
% M : length of Doppler filter, i.e., size of IFFT
% Nfosf : fading oversampling factor

% Outputs:
% FadMtx : length (Power_dB) x N fading matrix
% tf : fading sample time=1/(Max. Doppler BW*Nfosf)

Power=10.^(Power_dB/10); % calculate linear power
s2= Power./(K_factor+1); % calculate variance
s=sqrt (s2);

m2=Power.*(K_factor./(K_factor+1)); % calculate constant power
m=sqrt(m2); % Calculate constant part

I=length(Power); % # of tabs
fmax=max(Doppler_shift_Hz); % Sample spacing

if iscalar(Doppler_shift_Hz)
    Doppler_shift_Hz=Doppler_shift_Hz*ones(1,L);
end

path_wgn=sqrt(1/2)*complex(randn(L,N),randn(L,N)); % Rayleigh fading
for p=1:L % filt is filter coefficients
    filt=gen_filter(Doppler_shift_Hz(p), fmax, M, Nfosf, 'sui');
    path(p,:)=fftfilt(filt,[path_wgn(p,:),zeros(1,M)]); % filtering WGN
end

FadMtx=path(:,M/2+1:end-M/2);
for i=1:L, FadMtx(i,:)=FadMtx(i,:)*s(i) +m(i)*ones(1, N); end
FadMtx= FadMtx*10^(Fnorm_dB/20);
C  MIMO Channel Modeling

The MIMO channel Model was developed using equations based off I-METRA and SCM Ray-based modeling from the book “MIMO-OFDM Wireless Communication with MATLAB” as in References [1] This book however did not provide code to simulate the I-METRA model. It gave reference to generating channel coefficients for the I-METRA model assuming that the correlation matrixes have been generated. To generate the corresponding spatial correlation coefficients, the spatial correlation coefficients for each transmit and receive antenna were generated by creating PAS models for Uniform and truncated Laplacian. Once the PAS models were created, different scenarios were chosen, and the PDP profile and delay times were appropriately applied to the model. The spatial correlation matrixes were than generated. Than the correlation channel was generated and subject to Rayleigh or Rician distribution. The PDP and time delays were appropriately applied into the model and the MIMO channel model was generated. If antenna radiation pattern was generated for the uniform linear arrays, it could be applied to add directionality and Gain of the antenna system. Than random input signals were passed through the MIMO channel and the channel itself has been plotted in time as well as the expected channel capacity of the system.

The SCM-Ray based was based off single Ray-based model except for applying the spatial correlation matrices. It was modified to simulate each individual path between each transmit and receive antenna.

C-1  I METRA Model

clear, clf

% Inputs
%  NT: Number of Transmitters
%  NR: Number of Receivers
function hh=channel_coeff(NT,NR,N,Rtx,Rrx,type)

NT = 2;
NR = 2;
L = 1; %Number of Paths L
Rtx = [.5 , 0; 0 , .5];
Rrx = [.5 , 0; 0 , .5];
% Implement MIMO Channel Model based on I-Metra Model
hMod = comm.PSKModulator;
Channels = 100; %Channel Length
L = Channels/NT;
modData = step(hMod, randi([0 hMod.ModulationOrder-1],100,1)); %1e5
Channel_Input = reshape(modData,[NT,1,Channels/NT])

%Make all antennas Transmit same signal at same time
Channel_Input = ones(2,1,50);

% Split modulated data into two spatial streams
%channelInput = reshape(modData, [2, 5e4]).'; %5e4

% Step 1:

frequency = 2.4E9; %Given Hz
lambda = 3.00*10^8 / frequency; % wavelength (cm) =speed of light m/s /
carrier frequency
spacing_tx = .5;
spacing_rx = .5;
    spacing_txd = spacing_tx*lambda; % spacing is based on percentage
    spacing_rxd = spacing_rx*lambda; % spacing is based on percentage
    of lamda of receiver antenna
    of lamda of receiver antenna
n = 2;
%%Enter input parameters for correleation Matrixes
Tx_PAS_type = 'Truncated_Laplacian';
    % PAS Types
    %Truncated_Laplacian
    %nth_power_of_a_cosine_function
    %Truncated_Gaussian
    %Uniform
 Rx_PAS_type = 'Truncated_Laplacian';
    % PAS Types
    %Truncated_Laplacian
    %nth_power_of_a_cosine_function
    %Truncated_Gaussian
    %Uniform
Tx_AS = 10; %Aszmith Spread [degrees]
Tx_AoA = 20; %Angle of Arrival [degrees]
Rx_AS = 35; %Aszmith Spread [degrees]
Rx_AoA = 67.5; %Angle of Arrival [degrees]
type = 'Complex'; %'Complex' or 'Field'
%Select Fading Channel 'Rayleigh' or 'Rician' or 'Doppler' 
fading = 'Rayleigh'; %Fading Channel
K_dB = 6; %[dB]
K = 10^(K_dB/10);
DoM = 22.5; %[degrees] direction of movement
%tau = 30; %ns  RMS Delay spread
d = spacing_txd;
tau = (d/(3*10^8))*sind(Tx_AoA)*1E9; %delay spread
velocity = 120; %choices 3km, 30km, or 120km

%Select Power Delay Profile 
environment = 'Indoor'; %'Indoor' or 'Outdoor'
doppler_type = 'SUI'; %'Classical' or 'SUI' or 'None'
%Generate an exponential PDP Profile for indoor using exponential
if strcmp(environment, 'Indoor')
scale = 1e-9; %nano
Ts = .1*scale; %Sampling time
%t_rms = 30*scale; %30ns RMS delay spread
%num_ch = 1000;
tau_PDP = tau*scale;
[pow_e, npaths] = exp_PDP(tau_PDP, Ts);
x = [0:length(pow_e)-1]*Ts/scale;
i=1:1:npaths
stem(x, pow_e(1,i))
xlabel('Delay[ns]')
ylabel('Channel Power [Linear]')
title('Exponential Model')
legend(['Simulation for delay =', num2str(tau), 'ns'])
end
%Generate PDP profile for outdoor model. 
if strcmp(environment, 'Outdoor')
end
end

%Solve for BS (Tx) Spatial Correlation Matrix
N_antennas = NT;
R_BS = Master_PAS_Selection_Model(N_antennas, frequency, spacing_tx, 
Tx_AoA, Tx_AS, Tx_PAS_type, n)
%Solve for MS (Rx) Spatial Correlation Matrix
N_antennas = NR
R_MS = Master_PAS_Selection_Model(N_antennas, frequency, spacing_rx, 
Rx_AoA, Rx_AS, Rx_PAS_type);
%Functions only take positive numbers
R_BS = abs(R_BS);
R_MS = abs(R_MS);

%Calculate MIMO Channel Coefficients based on R_BS and R_MS
%Output is vector format (Nt*Nr,N)
for N=1:npaths
%N = length of channel matrix
%[hh,H_LOS] =
channel_coeff(Nt, Nr, npaths, R_BS, R_MS, type, fading, K_dB, lambda, DoM, spacing_rxd, spacing_txd, Rx_AoA, Tx_AoA, tau, N);

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if strcmp(doppler_type, 'SUI')
  ch_no=6;
  fs_Hz=1e7;
  Nfading=1024; % size of Doppler filter
  NP=10000;
  Nfosf=4;
  [Delay_us, Power_db, K_factor, Doppler_shift_Hz, Ant_corr, Fnorm_dB]=SUI_parameters(ch_no);
  [FadTime1,FadTime2,FadTime3, tf]= SUI_fading2(Power_db, K_factor, Doppler_shift_Hz,Fnorm_dB, NP, Nfading, Nfosf,C);
  c_table= ['b', 'r', 'm', 'k'];
  subplot (111)
  stem(Delay_us, 10.^(Power_db/ 10)), axis([-1 21 0 1.11])
  grid on, xlabel ('Delay time [ms]'), ylable('Channel gain');
  title (['PDP of Channel No.', num2str(ch_no)]);
  %subplot (312)
  path = ['Tx_1 to Rx_1'; 'Tx_1 to Rx_2'; 'Tx_2 to Rx_1'; 'Tx_2 to Rx_2']
  for k = 1:4
    path1 = path(k,:);
    subplot(5,1,k+1)
    subplot(111)
    plot ((0: length(FadTime1(k,:))-1)*tf,20*log10(abs(FadTime1(k,:))), c_table(1)); hold on
    plot ((0: length(FadTime2(k,:))-1)*tf,20*log10(abs(FadTime2(k,:))), c_table(2)); hold on
    plot ((0: length(FadTime3(k,:))-1)*tf,20*log10(abs(FadTime3(k,:))), c_table(3)); hold on
  grid on, xlabel('Time [s]'), ylabel('Channel Power [dB]');
  title (['MIMO Fading Channel for Path ', num2str(path1)]), axis([0 60 -50 10])
  legend('Tap 1', 'Tap 2', 'Tap 3')
end

idx_nonz=find(Doppler_shift_Hz);
FadFreq=ones(length(Doppler_shift_Hz),Nfading);
for k=1:length(idx_nonz)
  max_dsp=2*Nfosf*max(Doppler_shift_Hz);
  dfmax= max_dsp/Nfading;
  %Doppler frequency spacing respect to maximal Doppler frequency
  Nd=floor(Doppler_shift_Hz(k)/dfmax) -1;
  f0=[-Nd+1:Nd]/(Nd); % Frequency vector
  f=f0.*Doppler_shift_Hz(k);
  tmp=0.785*f0.^4 -1.72*f0.^2+ 1.0 ; %Eq. (2.41)
  %Hpsd=pwelch(FadTime(idx_nonz(k),:),max_dsp);
  %'Fs',max_dsp,'SpectrumType','twosided'); %works
  %Hpsd=pwelch(FadTime(idx_nonz(k),:),max_dsp,'twosided','dsp');
  %Hpsd=pwelch(FadTime(idx_nonz(k),:)); % Works
  %Hpsd=pwelch(FadTime(idx_nonz(k),:),N,Nfosf,max_dsp,'SpectrumType','twosided');
% Hpsd = dspdata.psd(spectrum.welch, FadTime(idx_nonz(k),:), 'Fs', max_dsp, 'SpectrumType', 'twosided');
% Great
Hpsd = pwelch(FadTime(idx_nonz(k),:), [64, [], 64], max_dsp, 'twosided', 'psd');
% Great
h = spectrum.welch;
% Default segLen = 64, ovlpPct = 50, Sampling Flag 'Symmetric'),
window
% is Hamming window
hPSD = psd(h, FadTime1(idx_nonz(k),:), 'Fs', max_dsp, 'SpectrumType', 'twosided');
Pxx = hPSD.Data;
F = hPSD.Frequencies;
nrom_f = hPSD.Frequencies - mean(hPSD.Frequencies);
PSD_d = fftshift(hPSD.Data);

% Do not remove incase of replacing syntax with pwelch function
% segLen = 64
% ovlpPct=50;
% win = hamming(segLen, 'symmetric');
Noverlap = ceil((ovlpPct/100)*segLen);

[Hpsd, F] = pwelch(FadTime(idx_nonz(k),:), win, Noverlap, [], max_dsp, 'twosided');
pxx = pwelch(x, segmentLength, [], nfft);

% nrom_f = Hpsd.Frequencies - mean(Hpsd.Frequencies);
% PSD_d = fftshift(hpsd.Data);
subplot(3, 3, 6 + k), plot(nrom_f, PSD_d, 'b'), f, tmp, 'r'
xlabel('Frequency [Hz]'), axis([-1 1 0 1.1 * max([PSD_d tmp])])
title(['h_', num2str(idx_nonz(k)), 'path']);
end
end

% Other Doppler Spectrum Flat Classical and Laplacian
if strcmp(doppler_type, 'Classical')
    fs_Hz = 1e7;
    Nfading = 1024; % size of Doppler filter
    NP = 10000;
    Nfosf = 8;
    FadingType = 'class';
    Npath = NR * NT; % N
    fm = [100 50 10];
    % Npath = 2;
    % fm = [100 10];
    [FadingMatrix1, FadingMatrix2, FadingMatrix3, tf1, tf2, tf3] =
    FWGN_tf2(Npath, fm, NP, Nfading, Nfosf, FadingType, C);
    path = ['Tx_1 to Rx_1'; 'Tx_1 to Rx_2'; 'Tx_2 to Rx_1'; 'Tx_2 to
    Rx_2'];
    for k = 1:Npath
        subplot(4, 1, k),
        plot([1:NP]*tf1, 10*log10(abs(FadingMatrix1(k, :))), 'k:'), hold on
        plot([1:NP]*tf2, 10*log10(abs(FadingMatrix2(k, :))), 'g'), hold on
        plot([1:NP]*tf3, 10*log10(abs(FadingMatrix3(k, :))), 'r'), hold on
        % plot([1:N]*tf3, 10*log10(abs(FadingMatrix3(1, :))), 'k:'), hold on
    end
end
%plot([1:N]*tf, 10*log10(abs(FadingMatrix(2,:))),'k-')
title([' Modified FWGN in Time Domain, Nfading=',
num2str(Nfading),', Nfosf=',num2str(Nfosf),', T_s=',num2str(tf),'s']);
xlabel('time[s]'),
ylabel('Magnitude [dB]')
legend(['Path',num2str(path(k,:))],'f_m=',num2str(fm(1,1)),'Hz'],
'Path',num2str(path(k,:)),'f_m=',num2str(fm(1,2)),'Hz'],
'Path',num2str(path(k,:)),'f_m=',num2str(fm(1,3)),'Hz']), axis([0 0.5 -20 10])
end
end

if strcmp(doppler_type,'None')
npaths = 6;
pow_e = [0 -0.9 -4.9 -8.0 -7.8 -23.9];
pow_e = 10.^(pow_e/10);
delay = [0 310 710 1090 1730 2510]*1e-9;
% Raleigh Fading Characteristics
if strcmp(fading,'Rayleigh')
h = sqrt(1/2)*(randn(NT*NR, npaths) + 1i*randn(NT*NR,npaths));
H_LOS = zeros(NT,NR,npaths);
%H = (randn (1,L)+ j*randn(1,L))/sqrt(2);
%sqrt(2)/2 = sqrt(1/2)
% Apply correlation to channel matrix
hh = zeros(NR,NT,npaths);
for i = 1:npaths,
tmp=C*h(:,i);
    hh(:,:,i)=reshape(tmp,NR,NT);
end
end

if strcmp(fading,'Rician')
h = sqrt(1/2)*(randn(NT*NR, npaths) + 1i*randn(NT*NR, npaths));
%Rayleigh Portion
hh = zeros(NR,NT,npaths);
for i = 1:npaths,
tmp=C*h(:,i);
    hh(:,:,i)=reshape(tmp,NR,NT);
end
end
% Rician Portion
K = 10^(K_dB/10);
alpha = 30;  %DOM - LOS;
fd = (velocity/3.6)/lambda*cosd(alpha);
%for i = 1:NT
Factor = exp(1i*2*pi()*fd*(i-1)*tau*(r-1)*10^-9);
H_Part1 = zeros(NR,1);
H_Part2 = zeros(NT,1);
H_Part1(1,1) = 1;
H_Part2(1,1) = 1;
%spacing_txd/lambda
for r = 2:NR
    H_Part1(r,1) = exp(1i*2*pi()*dRx/lambda*(r-1)*sind(AoA_Rx));
end
for r = 2:NT
    H_Part2(r,1) = exp(1i*2*pi()*dTx/lambda*(r-1)*sind(AoD_Tx));
end
H_LOS = Factor * H_Part1 * H_Part2;

H_path = zeros(NT, NR, npaths);
What1 = sqrt(K) * sqrt(pow_e(1,1)) * H_LOS(:, :, 1);
What2 = sqrt(pow_e(1,1)) * hh(:, :, 1);
What3 = What1 + What2;
H_path(:, :, 1) = What3;
What1 = sqrt(K) * sqrt(pow_e(1,1)) * H_LOS(:, :, 1) + sqrt(pow_e(1,1)) * h(:, :, 1);
for i = 2:npaths
H_path(:, :, i) = sqrt(pow_e(1,i)) * hh(:, :, i);
end
HW = 0;
for i = 1:npaths
HW = HW + H_path(:, :, i)
end
DoA = Tx_AoA;
i = 1;
spacing = [0:d:NT*d]
%At DoA 20 degrees for antenna
steering_fm = 10^(-21.35/10);
%steering_fm = 10^(0/10);
for k = 1:NT
wm(k,k)=steering_fm*exp(-1i*2*pi()*(k-1)*(spacing(1,k)/lambda)*sind(DoA));
end
Nfading=1024; % size of Doppler filter
NP=10000;
Nfosf=8;
FadingType='class';
Npath = NR * NT; % N
%fms = 120/3.6/lambda;
%Npath = 2;
fm=[100 50 10];
[FadingMatrix1,FadingMatrix2,FadingMatrix3, tf1,tf2,tf3] =
FWGN_tf3(Npath,fm,NP,Nfading,Nfosf,FadingType,C);
path = ['Tx_1 to Rx_1'; 'Tx_1 to Rx_2'; 'Tx_2 to Rx_1'; 'Tx_2 to Rx_2'];
for k = 1:Npath
subplot(4,1,k),
plot([1:NP]*tf1, 10*log10(abs(FadingMatrix1(k,:))),'k:'), hold on
plot([1:NP]*tf2, 10*log10(abs(FadingMatrix2(k,:))),'g'), hold on
plot([1:NP]*tf3, 10*log10(abs(FadingMatrix3(k,:))),'r'), hold on
%plot([1:N]*tf3, 10*log10(abs(FadingMatrix3(1,:))),'k:'), hold on
%plot([1:N]*tf, 10*log10(abs(FadingMatrix(2,:))),'k-')
title([' Modified FWGN in Time Domain, Nfading=',num2str(Nfading),', Nfosf=',num2str(Nfosf),', T_s=',num2str(tf),'s']);
xlabel('time[s]'),
ylabel('Magnitude [dB]')
legend(['Path',num2str(path(k,:))],'
f_m=',num2str(fm(1,1)),'Hz'],['Path',num2str(path(k,:))],'
f_m=',num2str(fm(1,2)),'Hz'],['Path',num2str(path(k,:))],'
f_m=',num2str(fm(1,3)),'Hz'], axis([0 0.5 -20 10])
C-2 SCM Ray-based Model

%AndyRAY_BASED_MIMO_Model
%Generate Parameters
%Subray model: Equal Power or sampled Laplacian
%Determines AoA of each subray and power
%Generates random phases
%Randomly pairing subray
%Computes antenna Gain

%Generates Channel Coefficients

clear, clf
fc = 2.5e9;
s = 5e4;
speed_kmh=120;
Ts = 1/fs';
v_ms = speed_kmh/3.6;
w = 3e8/fc;
Nt = 2;
Nr = 2;
spacing_tx = .5*wx_m % percentage of lambda
spacing_rx = .5*wl_m % percentage of lambda
%Channel Parameters
SCM_Case = '2'; %Case = 1, 2, or 3
if strcmp(SCM_Case,'2')
PDP_dB=[0. -1. -9. -10. -15. -20.];
t_ns=[0 310 710 1090 1730 2510];
%BS Parameters
BS_theta_LOS_deg=0;
BS_AS_deg=35; % Laplacian PAS
BS_AoD_deg=50*ones(size(PDP_dB));
BS_PAS = 'Truncated'
%MS Parameters
MS_theta_LOS_deg=0;
MS_AS_deg=35; % for Laplacian PAS
MS_AoA_deg=67.5*ones(size(PDP_dB));
DoT_deg=22.5;
MS_PAS = 'Truncated'
end;

for MS = 1:Nr
for BS = 1:Nt
%generates the phase of a subray
[BS_theta_deg, MS_theta_deg,
BS_PHI_rad]=gen_phase(BS_theta_LOS_deg,...
BS_AS_deg, BS_AoD_deg, MS_theta_LOS_deg, MS_AS_deg, MS_AoA_deg);
PDP=dB2W(PDP_dB);
%generates the Gain coefficients of BS and MS
[GBS, GMS, PAS_BS, PAS_MS, GBS_LOS, GMS_LOS] = SCM_Link_Model2(BS_theta_deg, MS_theta_deg, BS_PHI_rad, BS_PAS, MS_PAS, BS_theta_LOS_deg, BS_AS_deg, BS_AoD_deg, MS_theta_LOS_deg, MS_AS_deg, MS_AoA_deg);

% generates the coefficients

Channel = 'Rayleigh';

h = ray_fading2(20, PDP, BS_PHI_rad, BS_theta_deg, MS_theta_deg, v_ms, DoT_deg, wl_m, t, GBS, GMS, PAS_BS, PAS_MS, spacing_tx, spacing_rx, MS, BS, Channel, BS_theta_LOS_deg, MS_theta_LOS_deg, GBS_LOS, GMS_LOS);

c_table = ['b'; 'r'; 'm'; 'k'; 'y'; 'g']

for z = 1 : size(h)
    plot (t, 10*log10(abs(h(z,:))), c_table(z)), hold on
    title([strcmpio Ray Channel Model, f_c=', num2str(fc), 'Hz,
    T_s=', num2str(Ts), 's'])
    xlabel('time[s]'), ylabel('Magnitude[dB]')
    legend('Path1', 'Path2', 'Path3', 'Path4', 'Path5', 'Path6')
end
end

D Channel Capacity

Capacity was built in MATLAB using code examples generated for uncorrelated Rayleigh channel. Instead, the I-METRA channel spatial correlation matrices were implemented in place of the random Rayleigh channel. The channel capacity models were based on

D-1 Ergodic Capacity CDF

D-2 Ergodic Capacity Correlation

D-3 Ergodic Capacity vs SNR

D-5 SISO vs. MIMO Channel Capacity

D-1 Ergodic Capacity CDF

%Ergodic_Capacity_CDF.m

SNR_dB=10;
SNR_linear = 10.^(SNR_dB/10.);
N_iter = 50000;
sq2 = sqrt(0.5);
grps = ['b'; 'b-'];
for Icase = 1:2
    if Icase==1,
nT=2;
nR =2;  %2x2
else
    nT =4;
nR=4;  %4x4
end
n = min(nT, nR);
I = eye(n);
for iter=1:N_iter
    H=sq2*(rand(nR,nT) + 1i*randn(nR,nT));
    C(iter)=log2(real(det(I+SNR_linear/nT*H'*H)));
end
[PDF,Rate]=hist(C,50);
PDF = PDF/N_iter;
for i = 1:50
    CDF(Icase,i) = sum(PDF([1:i]));
end
plot(Rate,CDF(Icase,:),grps(Icase,:)); hold on
end
xlabel('Rate[bps/Hz]');
ylabel('CDF');
axis([1 18 0 1]); grid on;
set(gca,'fontsize',10);
title('Ergodic Channel Capacity of MIMO Channel');
legend('{$N_T}=2$','$N_R}=2$','{$N_T}=4$','$N_R}=4$');

D-2 Ergodic Capacity Correlation
%Ergodic Capacity Correlation
%Capacity reduction due to correlation of MIMO channels (Fig. 9.8)
clear all, close all;
SNR_db=[0:50];
SNR_linear = 10.^(SNR_db/10);
N_iter =1000;
N_SNR= length(SNR_db);
N_T = 2;
N_R = 2;
n=min(nT,nR);
I = eye(n);
sq2 = sqrt(0.5);  %4x
Rr = [1 -0.03042; -0.03042 1];
Rt = [1 0.464025; 0.464025 1];
C_44_iid = zeros(1,N_SNR);
C_44_corr=zeros(1,N_SNR);
for iter=1:N_iter
    H_iid = sq2*(rand(nR,nT)+1i*rand(nR,nT));
    H_corr = Rr^(1/2)*H_iid*Rt^(1/2);
    tmp1 = H_iid'*H_iid/nT;
    tmp2 = H_corr'*H_corr/nT;
    for i = 1:N_SNR  % Eq. 9.48
        C_44_iid(i) = C_44_iid(i)+log2(det(I+SNR_linear(i)*tmp1));
        C_44_corr(i) = C_44_corr(i)+log2(det(I+SNR_linear(i)*tmp2));
    end
end
C_44_iid = real(C_44_iid)/N_iter;
C_44_corr=real(C_44_corr)/N_iter;
D-3 Ergodic Channel Capacity vs SNR

```matlab
%Erogodic_Capacity_vs.SNR.m
clear all, close all
SNR_dB=[0:5:20];
SNR_linear =10.^(SNR_dB/10);
N_iter=1000;
sq2 = sqrt(0.5);
for Icase = 1:5
    if Icase ==1,
        nT =1;
        nR = 1; 1x1
    elseif Icase==2
        nT =1;
        nR =2; 1x1
    elseif Icase==3
        nT =2;
        nR =1; 1x1
    elseif Icase==4
        nT =2;
        nR =2; 1x1
    else
        nT=4;
        nR =4; 4x4
    end
    n=min(nT,nR);
    I = eye(n);
    C(Icase,:)=zeros(1,length(SNR_dB));
    for iter=1:N_iter
        H=sq2*(randn(nR,nT)+1i*randn(nR,nT));
        if nR>=nT,
            HH=H'*H;
        else
            HH=H*H';
        end
        for i = 1:length(SNR_dB) % Random channel generation
            C(Icase,i)=C(Icase,i)+log2(real(det(I+SNR_linear(i)/nT*HH)));
        end
    end
end
C=C/N_iter;
plot(SNR_dB,C(1,:),SNR_dB,C(2,:),SNR_dB,C(3,:),SNR_dB,C(4,:));
hold on, plot(SNR_dB, C(5,:),SNR_dB,C(6,:),SNR_dB,C(7,:));
xlabel('SNR[db]');
ylabel('bps/Hz');
title('Ergodic MIMO Channel Capacity when CSI is not available at the
trasmitter')
legend({\it N_T}=1,{\it N_R}=1',{\it N_T}=1,{\it N_R}=2',{\it N_T}=2,{\it N_R}=1',{\it N_T}=2,{\it N_R}=2',{\it N_T}=4,{\it N_R}=4');
```

D-4 SISO vs MIMO Channel Capacity
% This program will give the comparison between two schemes: SISO and MIMO, in which, with SISO scheme, the capacity will follow the equation of Shannon. It shows that the capacity receiving from MIMO scheme will be larger than in SISO

% Input:
%   +) Signal Noise Rate: SNR
%   +) The number of receiving antennas Nr and transmitting antennas Nt
%   +) The channel co-efficient will follow Rayleigh distribution in which the dimension of H depends on the number of receiving and transmitting antennas
%   +) Bandwidth of channel: B = 1

clc;
clear all;
SNR_DB = [0:0.01:50];
SNR = 10.^(SNR_DB/10);

% The capacity of SISO model
C_SISO = log2(1+SNR);
figure
plot(SNR_DB,C_SISO,'-','LineWidth',1.5,'Color','g');
hold on

% The capacity of MIMO model
Nr = 2;
Nt = 2;
N = min(Nr,Nt);
H = zeros(Nr,Nt);
for k=1:Nr
    for l=1:Nt
        H(k,l) = randn(1) + j*randn(1);
    end
end

[S V D] = svd(H*H');
[C_MIMO] = 0;
% When CSI is not available at the transmitter side
for k=1:N
    lamda(k) = V(k,k);
    C_MIMO = C_MIMO + log2(1+SNR*lamda(k)/Nt);
end
plot(SNR_DB,C_MIMO,'LineWeight',1.5,'Color','r');
legend('SISO','MIMO');
xlabel('SNR');
ylabel('Capacity');

% The capacity of MIMO model
Nr = 3;
Nt = 3;

N = min(Nr,Nt);
H = zeros(Nr,Nt);
for k=1:Nr
    for l=1:Nt
        H(k,l) = randn(1) + j* randn(1);
    end
end

[S V D] = svd(H*H');

C_MIMO = 0;
% When CSI is not available at the transmitter side
for k=1:N
    lamba(k) = V(k,k);
    C_MIMO = C_MIMO + log2(1+SNR*lamba(k)/Nt);
end

plot(SNR_DB,C_MIMO, 'LineWidth', 1.5 , 'Color', 'y');
legend('SISO','MIMO');
xlabel('SNR');
ylabel('Capacity');

% The capacity of MIMO model
Nr = 4;
Nt = 4;

N = min(Nr,Nt);
H = zeros(Nr,Nt);
for k=1:Nr
    for l=1:Nt
        H(k,l) = randn(1) + j* randn(1);
    end
end

[S V D] = svd(H*H');

C_MIMO = 0;
% When CSI is not available at the transmitter side
for k=1:N
    lamba(k) = V(k,k);
    C_MIMO = C_MIMO + log2(1+SNR*lamba(k)/Nt);
end

plot(SNR_DB,C_MIMO, 'LineWidth', 1.5);
legend('SISO','2x2 MIMO','3x3 MIMO', '4x4 MIMO');
xlabel('SNR');
ylabel('Capacity');