A Thesis

entitled

Hume, Skepticism, and the Search for Foundations

by

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Submitted to the Graduate Faculty as partial fulfillment of the requirements for the

Master of Arts Degree in

Philosophy

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May 2014
In this paper I present an account of how epistemology should be pursued. I challenge epistemological projects which focus exclusively on how our fundamental beliefs about the world – specifically our beliefs about inductive and mathematical knowledge – can be foundationally justified. To their detriment, these projects often ignore the naturalistic question of why we have these beliefs in the first place. Chapter one begins with an account of David Hume’s doctrine of knowledge, which is read most often as bifurcating knowledge into two epistemological classes. I refer to this bifurcation as the Received View of Hume’s epistemology. On this interpretation, knowledge divides exhaustively into relations of ideas, on the one hand, and matters of fact on the other. Chapter One concludes that attempts to justify either epistemological class unduly overemphasizes the importance of this distinction and risks undermining Hume’s actual epistemological goals. I argue that Hume sought a naturalistic explanation of how humans acquire (inductive and mathematical) beliefs as opposed to an explanation that restricts epistemology to a skeptical project of demonstrating why our beliefs are ultimately unjustifiable. Skepticism plays an important role in Hume’s
epistemology, but this skepticism is less important than his more positive naturalistic project to explain how and why we have different kinds of beliefs. I argue that this latter point has significance beyond interpretive studies of Hume as it has normative implications for the study of knowledge in general: epistemologists should not only establish why our beliefs about the world are justified, but also provide a naturalistic explanation of the etiology of our beliefs. This latter project is often ignored – yet restricting epistemology to the foundational search for justifications cannot succeed on its own, for we can only articulate how our beliefs might be justified by expanding epistemology to include an account of how we acquire our beliefs in the first place.

To make this broader point, I focus on movements within the philosophy of science and the philosophy of mathematics which attempt to establish an epistemic foundation to justify our knowledge claims. In Chapter Two, I analyze attempts within the philosophy of science to provide a solution to Hume’s problem of induction via some sort of foundational a priori premise or axiom. In Chapter Three, I analyze the logicist and neo-logicist projects within the philosophy of mathematics to provide a foundation for mathematical knowledge – or at least arithmetic – using only basic logical principles. Both of these chapters discuss how these epistemological projects focus exclusively on securing foundations for inductive or mathematical knowledge. Interestingly, both are unsuccessful in achieving their respective justificatory goals. I argue that the ultimate reason for this failure, in both cases, is their exclusive focus on foundations. Thus, in light of these difficulties, Chapter Four concludes by suggesting that epistemology is better served by expanding its project to include a more Humean, naturalistic, and scientific understanding of both inductive and mathematical beliefs in
lieu of projects focused *exclusively* on the epistemic justification of these beliefs. These two projects are “two sides of the same coin,” so to speak. If we want to know what justifies our beliefs we have to know how we come to have our beliefs, and vice versa.
For Hilary (and Bravo, of course).
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Chapter 1

The Epistemology of David Hume: An Overview

This chapter begins by considering the Received View’s understanding of Hume’s distinction between relations of ideas and matters of fact as found in both the *Treatise*¹ and *Enquiry*. Then, in section 1.2, I review some of the authors who have endorsed the Received View. In section 1.3, I briefly consider Non-Traditional interpretations of Hume’s epistemology, which read him as either maintaining a singular epistemological class of knowable propositions or maintaining a bifurcation different from that of the Received View. While the claim that Hume divides knowledge into two distinct classes is supported via textual evidence, I argue in section 1.4 that the attempt to justify either epistemological class unduly overemphasizes the importance of this bifurcation. This narrow focus risks undermining Hume’s actual epistemological project of articulating a naturalistic explanation of how humans come to have justifiable beliefs as opposed to an abstract dismissal of those very beliefs as inherently unjustifiable. Skepticism plays an

¹ References to Hume’s *Treatise* are to *A Treatise of Human Nature*, ed. David Fate Norton and Mary J. Norton (New York: Oxford University Press, 2000). All in-text references to the *Treatise* will hereafter be cited as “T” followed by Book, part, section, and paragraph numbers as necessary.
important part in Hume’s philosophy, but this skepticism is less important than his more positive naturalistic project to explain how and why we have beliefs in the first place.

1.1 The Received View

Perhaps the passage referenced most often by those who advocate the Received View is found in Section IV of the *Enquiry*:

All the objects of human reason or enquiry may naturally be divided into two kinds, namely, *relations of ideas* and *matters of fact*. Of the first kind are the sciences of geometry, algebra, and arithmetic, and, in short, every affirmation which is either intuitively or demonstratively certain… Propositions of this kind are discoverable by the mere operation of thought, without dependence on what is anywhere existent in the universe. Though there never were a circle or triangle in nature, the truths demonstrated by Euclid would forever retain their certainty and evidence. Matters of fact, which are the second objects of human reason, are not ascertained in the same manner; nor is our evidence of their truth, however great, of a like nature with the foregoing. The contrary of every matter of fact is still possible, because it can never imply a contradiction and is conceived by the mind with the same facility and distinctness, as if ever so conformable to reality.²

This distinction between relations of ideas and matters of fact divides knowledge into two classes of knowable propositions. This epistemological division has come to be known as “Hume’s Fork.”³ On the one hand there are relations of ideas, which are either intuitively or demonstratively *certain* and do not assert the existence of any non-abstract entities

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³ It is important to note the different accounts of what constitutes knowledge between the *Treatise* and *Enquiry*. In the former work, Hume seems to restrict knowable propositions to relations of ideas, whereas in the latter both relations of ideas and matters of fact constitute knowledge. In short, “knowledge” seems to be used in a narrower sense in the *Treatise*. Despite this difference between the two texts, it does not seem problematic for the purposes of this paper. I read the popular interpretation of Hume as arguing that both sides of the “Fork” are constitutive of knowledge, but relations of ideas represent “a more perfect species of knowledge.” Hume, *Enquiry*, p. 393.
(e.g., physical objects, minds, physical or mental states, etc). As Hume explains, they do not depend “on what is anywhere existent in the universe.” At their core, relations of ideas are the propositions of algebra, arithmetic, and geometry; they are certain because to deny any such proposition would be to assert a contradiction. Hume contrasts these “certain” propositions with matters of fact, which are neither intuitively nor demonstratively certain and do assert the existence of non-abstract entities. They are “merely probable” propositions, because, as the above passage indicates, the contrary of any matter of fact is still conceivable and therefore possible to take place in the future.

In both the Treatise and Enquiry Hume focuses on matters of fact and their role in causal inferences, while he tends to offer only a perfunctory analysis of relations of ideas. Before examining his more extended treatment of matters of fact, however, let us first clarify what exactly Hume means when he suggests that relations of ideas are known


5 Hume, Enquiry, p. 337.

6 These three types of propositions are what Hume explicitly mentions in the Enquiry, although it’s likely that other types of propositions fall into this class as well, such as basic logical principles. Some authors even include synthetic a priori propositions in this class. See, for example, ibid., p. 54; Dorothy Coleman, “Is Mathematics for Hume Synthetic A Priori?” Southwestern Journal of Philosophy 10 (Summer 1979), pp. 113-126. Furthermore, it should be noted that in the Enquiry Hume changes his original position on geometry as found in the Treatise. In the latter work, he denies geometry’s status as an “exact science.” That is, he denies it represents either intuitive or demonstrative knowledge. See, for example, T 1.3.1.4.

7 For a more detailed discussion of Hume’s use of conceivability and possibility, see below (e.g., pp. 10-12). Also see Barry Stroud, Hume (London: Routledge, 1977), pp. 46-49. Note that there are historical objections and alternative explanations for what is actually meant by these terms. See, for example, Tamar Szabó Gendler and John Hawthorne, eds., Conceivability and Possibility (Oxford: Oxford University Press, 2002).
either intuitively or demonstratively, as well as his claim that they do not assert the existence of any non-abstract entities.

In the *Treatise*, Hume puts forth seven philosophical relations.⁸ Four of these relations (i.e., *resemblance*, *proportion in quantity or number*, *degrees in any quality*, and *contrariety*) provide us with the basis for *certain* knowledge, whereas the other three (i.e., *identity*, *relations of time and place*, and *causation*) provide us with the grounds for *probable* belief. Of the first four relations, *resemblance*, *degrees in any quality*, and *contrariety* provide us with certain knowledge through intuition. Here Hume borrows extensively from Locke’s account of intuition, in that he believes these relations of ideas are known because they are “self-evident.”⁹ This is how, for example, we perceive that two is not three, or that white is not black. On the other hand, the relation *proportion in*
quantity or number generally requires demonstration in order to provide us with certain knowledge.\textsuperscript{10} Hume again borrows from Locke: demonstrative certainty is knowledge that is inferentially deduced from the self-evident propositions of intuition. For example, in order to ascertain the equality of two complex numerical ideas, other ancillary ideas must be utilized in order to demonstrate the equality of those two original ideas.\textsuperscript{11} Both intuition and demonstration, then, are classed in terms of certain knowledge; but the former is more immediately certain than the latter.

The second criterion for relations of ideas is that they are discoverable “without dependence on what is anywhere existent in the universe.”\textsuperscript{12} Again, this is a fundamental difference that holds between relations of ideas and matters of fact. In the case of the latter, existence is indeed asserted by any such proposition.\textsuperscript{13} Propositions involving relations of ideas, on the other hand, do not require that the entities involved actually exist. As Hume explains, “though there never were a circle or triangle in nature, the truths, demonstrated by Euclid, would forever retain their certainty and evidence.”\textsuperscript{14} His point here is that the truths of Euclid, for example, are true regardless of whether or not any non-abstract entities exist (e.g., non-abstract triangles, circles, etc). Consequently,

\textsuperscript{10} I say generally because there are some numerical ideas, like the fact that two is not equal to three, that are intuited. But our more complicated numerical ideas (e.g., $49 \times 356 = 17,444$) cannot be intuited and instead require demonstration.

\textsuperscript{11} As Locke explains: “Those intervening ideas, which serve to show the agreement of any two others, are called proofs; and where the agreement and disagreement is by this means plainly and clearly perceived, it is called demonstration; it being shown to the understanding, and the mind made to see that it is so.” John Locke, \textit{An Essay Concerning Human Understanding}, ed. Alexander Campbell Fraser (New York: Dover Publications, 1959), p. 179. Emphasis in original.

\textsuperscript{12} Hume, \textit{Enquiry}, p. 337.

\textsuperscript{13} This holds true for all but a particular class of propositions expressing matters of fact. See page six and seven below.

\textsuperscript{14} Ibid.
propositions about relations of ideas do not assert the existence of non-abstract entities, for if they did their truth would indeed depend on the existence of non-abstract entities and thus would not be true regardless of whether or not any such non-abstract entity exists.

Propositions expressing matters of fact, on the other hand, do assert the existence of non-abstract entities. Hume says “That the sun will not rise tomorrow is no less intelligible a proposition and implies no more contradiction than the affirmation that it will rise,”15 because the real existence of the physical (non-abstract) entity in question, namely, the sun, means that both propositions are conceivable. Simply put, the truth of the proposition that the sun will rise tomorrow depends upon the real existence of the sun. Such a proposition cannot be true unless the sun exists, just as the denial of that proposition cannot be true unless the sun does not in fact exist.

However, as prefaced in the footnote above, there is a particular class of propositions expressing matters of fact that represents an exception to this rule. Hume points out that there is a kind of matter of fact proposition that does not assert, but rather implies, the existence of non-abstract entities. This class encompasses any proposition used to make an inductive inference about the unobserved from the observed. Such propositions can generally be construed in terms of causal laws (e.g., the “causal maxim” that every event has a cause, the idea that the future will continue to resemble the past, etc.). Such propositions on their own do not assert the existence of anything, but when

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15 Ibid. Emphasis in original.
conjoined with other propositions (e.g., the observation of lightning) *imply* the existence of a yet unobserved phenomenon (e.g., the expectation of thunder).\(^\text{16}\)

Matters of fact, then, either assert or imply the existence of non-abstract entities. This fundamentally distinguishes them from relations of ideas, which need not assert the existence of non-abstract entities. Furthermore, because matters of fact do assert the existence of non-abstract entities, they cannot be either intuitively or demonstratively certain. For to assert or imply existence demands that the entity in question actually exist in order for the proposition to be true. But any such assertion or implication, it turns out, could conceivably be otherwise. As such, according to Hume we can never be certain – either intuitively or demonstratively – in our reasoning concerning any matter of fact, for it is always possible for any such inference to be conceived otherwise. That is, it does not imply contradiction to conceive of such a proposition in terms of its opposite truth-value.\(^\text{17}\)

Thus, intuition and demonstration are reserved for the class of propositions expressing relations of ideas (i.e., those propositions that do not depend on, or assert, the existence of non-abstract entities in order to be true), whereas propositions expressing matters of fact can only be “known” in terms of probability and the relation of cause and effect. Again, Hume’s central focus in both the *Treatise* and *Enquiry* concerns matters of fact and how it is we are justified in both inferring and believing in these (merely) probable assertions. Let us then further examine this branch of knowledge concerning

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\(^{16}\) I am indebted to Dicker (1998) for his succinct clarification of this exception to the rule concerning matters of fact.

\(^{17}\) For example, Bravo the dog may have four legs, but we can imagine a world in which he has, say, three legs. In contrast, a triangle must have three sides; we cannot imagine a triangle with greater or fewer sides without ceasing to think about triangles altogether.
matters of fact – or what Hume calls *moral reasoning* – before moving on into more contemporary accounts of Hume’s Fork.

In T 1.3.2 and onward Hume concerns himself to show how it is we make inferences beyond present experiences. Of the three “natural” philosophical relations restricted to matters of fact, it is only the relation of causation that allows us to make inferences about unobserved matters of fact, for this relation leads us to connect the observed with the unobserved and generate broader (causal) conclusions about the world beyond what we presently experience. As Hume explains:

> Here then it appears, that of those three relations, which depend not upon the mere ideas, the only one, that can be trac’d beyond our senses, and informs us of existences and objects, which we do not see or feel, is *causation*.

After establishing the efficacy of causation over the other two relations, Hume then proceeds to discuss our *idea of causation*. For if our inferences beyond present experience are to be justified then it is incumbent on us to also provide a justification for founding such inferences upon causation. As he explains:

> Let us therefore cast our eye on any two objects, which we call cause and effect, and turn them on all sides, in order to find that impression, which produces an idea of such prodigious consequence.

So our idea of causation (and how it is that we have the idea of causation) must be accounted for in order for our inferences beyond present experience to be justified.

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18 As already noted, problems arise with Hume’s classification of the philosophical relations. Specifically in this section, one is left somewhat confused as to Hume’s classification of “relations of time and place,” as they seem capable of being placed in either class of the philosophical relations. As Smith explains, “These relations of time and place thus fall midway between Hume’s two classes.” Smith, *The Philosophy of David Hume*, p. 355.

19 T 1.3.2.3. For Hume’s dismissal of the other two relations, namely, relations of time and place and identity, see T 1.3.2.2.

20 T 1.3.2.3
Accordingly, then, in order to understand the origin of our idea of causation we must discover the impression (or impressions) from which it is derived.\textsuperscript{21}

In answering this question, Hume begins by identifying what he takes to be the three essential features of any causally connected objects: their contiguity, succession, and necessary connection. The first two features are discoverable or recognizable in any instance of causation: causes are found to always be \textit{contiguous} and \textit{prior to} their corresponding effects. However, these relations alone are insufficient to indicate causation: “An object may be contiguous and prior to another, without being consider’d as its cause.”\textsuperscript{22} Accordingly causation must also involve the \textit{necessary connection} of causes and effects. Hume views necessity as the most important feature of causation and the one that most urgently needs to be understood if we’re to understand causation, for the observation of contiguous and temporally successive objects or events is not enough to reveal why we take those objects or events as causally connected. Necessity therefore must be included. But even with these three features of causation in mind, Hume finds that the idea of causation is not derived from an impression of causation \textit{qua} causation. For, as explained above, contiguity and succession are insufficient, whereas necessary connection is not empirically observable as a known relation holding between causally connected objects. In short, one does not find any instance in which an impression of necessary connection is derived from observation. Moreover, although one might believe

\textsuperscript{21} For as Hume explains, “‘Tis impossible to reason justly, without understanding perfectly the idea concerning which we reason; and ‘tis impossible perfectly to understand any idea, without tracing it up to its origin, and examining that primary impression, from which it arises. The examination of the impression bestows a clearness on the idea; and the examination of the idea bestows a like clearness on all our reasoning” (T 1.3.2.4).

\textsuperscript{22} T 1.3.2.10
through observation that causally connected objects are contiguous and successive – features which are observable as relations holding between objects – these features are insufficient in themselves when it comes to accounting for the necessary connection between causes and their effects. In sum, contiguity and succession cannot explain the necessary connection between causes and their effects, nor can this necessary connection be otherwise observed empirically. Thus, the impression or impressions from which our idea of causation is derived is yet to be found.

Nevertheless, Hume does not let “the despair of success” force him to abandon his fundamental thesis that all of our ideas are derived from corresponding impressions. More must certainly be said about the nature of necessary connection “which enters into our idea of cause and effect,” but in light of the above difficulties surrounding its discovery qua impression Hume instead seeks recourse in the inference whereby we associate causes with effects. As Smith summarizes:

As we cannot find any impression, nor consequently any idea, of what can be meant by the term ‘causation,’ [Hume] invites us to turn aside and examine the inference which is based upon it. Perhaps examination of the inference will give us the clue we are seeking, and so guide us to the impression which we have failed to find by the more direct method of approach.

Hume goes on to identify two questions whose answering (he “hopes”) will reveal the nature of our idea of this necessary connection. The first of these is why it is we label a cause as always being necessary to the production of an effect – that is, why it is that anything that begins to exist must have a cause for its existence. In answering this question, Hume seeks to reject traditional rationalist thinkers who thought that this

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23 T 1.3.2.13
25 T 1.3.3.1
“causal maxim” could be shown to be either intuitively or demonstratively certain. For Hume shows that conclusive deductive proof of this kind is not possible. It cannot be intuitively certain because it does not represent a proposition whose truth is “self-evident” and based solely upon the terms involved. Likewise, it cannot be demonstratively certain because the ideas of cause and effect are distinguishable, therefore making it impossible to provide a demonstrative proof that every event resembling an effect must have a cause. In other words, it’s possible to conceive of some matter of fact in terms of totally contrary, even seemingly impossible, causal circumstances. Therefore, one cannot assert a contradiction in denying that any matter of fact must necessarily be attended with particular causes or effects. Simply put, the conceivability of the claim implies its possibility, in the sense that what we can conceive does not imply any contradiction.

Having thus established that neither intuition nor demonstration can prove the necessary connection of causes with effects, Hume concludes that the only method that can account for such an idea is observation and experience. The next inquiry then leads us into the second of the two aforementioned questions, namely, “How experience gives rise to such a principle [of necessary connection].” Hume believes that this can be answered by contesting a similar question: “Why we conclude [from experience] that

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26 On the other hand, it does assert a contradiction to deny the truth of any demonstrative proof in general.
27 Stroud explains: “And since it is impossible to demonstrate the necessity of anything except by reasoning from mere ideas, the necessity of a cause for everything that begins to exist can never be demonstrated. So the traditional causal maxim is not demonstratively certain.” Stroud, *Hume*, p. 47.
28 T 1.3.4.9
such particular causes must necessarily have such particular effects, and why we form an
inference from one to another.”

As explained above, direct observation of our present experience shows us that causation is characterized by the contiguity and succession of two particular objects. But again, these features alone are insufficient for our forming the idea of necessary connection of causes with effects, or more precisely, of allowing us to infer the existence of an absent object or quality from that of one present to us. Additionally, reflective observation of both our present and past experiences reveals that causation is also characterized by the constant conjunction of causes and effects, whereby in all similar instances we observe (or expect) two corresponding objects to be constantly associated with one another. It is thus the reflective observance of this constant union, in addition to the direct observation of the succession and contiguity of any two causally connected objects, through which experience habituates us to associate particular causes with particular effects.

Unfortunately, however, this constant union does not shed light on how it is we experientially derive the idea of the necessary connection of objects, but instead only reveals the inferential process by which we link particular causes with particular effects. In order to determine the derivation of our idea of necessary connection, we must examine the nature of how we infer an absent quality or object from a present one (i.e.,

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29 Ibid.
30 T 1.3.6.3. This is why, as Smith explains, Hume identifies causation as a philosophical relation: “he is treating it as descriptive not of single instances of causal connexion, but of the type or kind to which the instances belong, and as obtained in and through comparison of them. Experience, when thus discursively reflected upon, shows that the instances fall into types or kinds, and that for each type or kind the relations of contiguity and priority are as a matter of fact constant.” Smith, The Philosophy of David Hume, pp. 371-72. My emphasis.
examine how it is we generate the idea of an absent phenomenon from a present impression. According to Hume, such an inference is made with our recounting from past experience the constant conjunction of two objects and then applying this knowledge to present and future cases. The task then is to determine whether this process is governed by reason (i.e., by intuition or demonstration) or by a different method (i.e., what Hume calls “a certain association and relation of perceptions”).

If it is reason that constitutes how we make such inferences, then it follows that, if all such inferences are to be justified, we would need to assert as a fundamental (enthymematic) premise, namely, the maxim that the future will continue to resemble the past. This premise, if true, would enable us to logically justify all of our causal inferences. Often referred to as the Uniformity Principle, this maxim itself needs to be justified if we’re to establish reason as the solution to how we infer absent causes from present effects, or present causes from absent effects. As it turns out, if we use our same account of demonstrative reasoning given above, it’s not the case that the Uniformity Principle is demonstratively certain, as it’s not a contradiction to imagine the world behaving in ways different from what we’d expect based on our past experiences. Reasoning, or rational analysis, therefore, cannot explain how it is we infer absent phenomena from those that are present to us (i.e., it cannot account for our idea of the necessary connection of causes with effects).

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31 T 1.3.6.4
32 Similarly, intuition cannot justify it, as the principle does not accord with Hume’s account of intuitive certainty given above.
Thus, as it appears that the Uniformity Principle cannot be justified through deductive reasoning, Hume is led to seek justification instead in probability.\textsuperscript{33} That is, his next task is to ascertain whether or not probability – or the mind’s inferring from some present impression the probable occurrence of a connected impression that has always attended it in our past experiences – can justify the Uniformity Principle. Unfortunately, this method fails as well, as our probable inferences derive themselves from our causal ones, and so cannot, without being circular, be used both to justify our causal inferences (i.e., justify the Uniformity Principle) while simultaneously being a product of that very same justification. That is, probable inferences depend on our experiences with causal phenomena and therefore cannot be used to justify causal phenomena without begging the question.\textsuperscript{34}

Hume’s conclusion is that intuition, demonstration, and probability all fail to justify the Uniformity Principle, thereby dismissing any chance of reason being the solution to where our idea of the necessary connection of causes with effects arises. Smith elegantly outlines the consequences of Hume’s analysis:

In the final outcome, so-called causal inference is found not to be inference at all: the apprehension of matters of fact and existence is not in

\textsuperscript{33} Having just established that deductive reasoning cannot justify the Uniformity Principle, Hume now seeks to establish whether or not inductive reasoning can do so.\textsuperscript{34} Essentially, it begs the question to use past experiences to justify the Uniformity Principle, as it is this very principle on which all past experiences and their likelihood of following the same patterns in future cases are based on. Wesley Salmon nicely summarizes: “We cannot justify any kind of ampliative inference. If it could be justified deductively it would not be ampliative. It cannot be justified nondemonstratively because that would be viciously circular. It seems, then, that there is no way in which we can extend our knowledge to the unobserved.” Wesley Salmon, \textit{The Foundations of Scientific Inference} (Pittsburgh: Pittsburgh Press, 1966), p. 11. This modernized version of Hume’s problem of induction is further developed in Chapter Two.
idea any more than in sense-perception and memory obtainable in an intellectualist or rationalist manner by way of inference.\textsuperscript{35}

Hume instead simply argues that connecting particular causes with particular effects is the result of our past experience. That is, past experience leads\textsuperscript{36} us to associate like causes with like effects and therefore generate the idea that these etiological relationships should be necessary ones through custom and habit. Thus, there is no logical reason for believing that the future will resemble the past. However, that does not mean we shouldn’t believe it. In fact we have a natural propensity to believe it, for the tribunal of past experience demands that we live our lives in accordance with this principle, despite the fact we have no logical or rational reason to support it. Similarly, we cannot logically assert that any causal idea of ours should continue to manifest itself in future cases bearing the appropriate conditions. So to sum up, although we continue to believe in the universality of causes and the uniformity of nature, we cannot logically prove either. Again, Hume does not deny that we have these beliefs. But we cannot logically know them to be the case, for neither experience nor reason is capable of justifying them.

Hume provides us with an answer for our idea of necessary connection in 1.3.14 of the Treatise, where he explains that we are led to this idea by a “determination of the mind.” As he explains:

For after a frequent repetition, I find, that upon the appearance of one of the objects, the mind is determin’d by custom to consider its usual attendant, and to consider it in a stronger light upon account of its relation to the first object. ‘Tis this impression, then, or determination, which affords me the idea of necessity.\textsuperscript{37}

\textsuperscript{35} Smith, The Philosophy of David Hume, p. 372. Emphasis in original.
\textsuperscript{36} Or to borrow from Smith (1941), past experience “causally conditions” us.
\textsuperscript{37} T 1.3.14.1
Thus, there is no quality within causally connected objects themselves that affords us with the idea of their necessary connection. Similarly, in our experiences of causally related objects we have no experience of a causal connection. Instead, all that we perceive is their contiguity, succession and constant conjunction, which subsequently lead us to expect similar causal relationships in future cases. As Hume explains later on:

"Upon the whole, necessity is something, that exists in the mind, not in objects; nor is it possible for us ever to form the most distant idea of it, consider’d as a quality in bodies. Either we have no idea of necessity, or necessity is nothing but that determination of the thought to pass from causes to effects and from effects to causes, according to their experience’d union." 38

The idea of necessity, then, is simply an impression of an expectation that the mind naturally imposes on our experiences. We are naturally inclined to project this causal order on the world.

Hume’s overall point is that reason alone is incapable of supplying a justification for causal inferences. Indeed, in Hume’s final analysis our so-called causal inferences are found not to be inferences made by reason at all. Causation is instead found to be a natural feeling of the “imagination” that attends our experiences of causally connected objects. Any attempt via rational inquiry to logically justify our causal beliefs is doomed to fail. But while this fact might seem unfortunate, we must keep in mind that nature has imposed upon us the inclination to make and believe in our causal inferences regardless of what rational inquiry might skeptically conclude about them. Thus, any epistemological project attempting to justify our beliefs will also need to account for this naturalistic influence, for to not do so will be to shackle the project’s fate to the icy slopes of skepticism.

38 T 1.3.14.22
1.2 Modern Exponents of the Received View

The above section represents a close textual exegesis of the Received View’s interpretation of Hume’s doctrine of knowledge, the bifurcation of knowledge types and the consequences therein as expressed by Hume. I begin this section with the Received View’s origination in Kant’s analysis of Hume as found in his *Critique of Pure Reason*\(^{39}\) and *Prolegomena to Any Future Metaphysics*. His analysis of Hume is characterized by the following quote in the *Prolegomena*:

Hume, when he felt the call, worthy of a philosopher, to cast his eye over the whole field of pure knowledge *a priori*, in which the human understanding presumes to such large possessions, negligently cut off from it a whole, indeed its most considerable, province, namely pure mathematics. He imagined that the nature and so to speak the constitution of this province rested on quite different principles, namely, on the principle of contradiction alone; and although he did not make as formal and universal a classification of propositions as I do here, or use the same names, it was exactly as if he had said: pure mathematics merely contains analytic propositions, but metaphysics contains synthetic propositions *a priori*.\(^{40}\)

Again, like the expository work above, Kant reads Hume as dividing knowledge between (analytic) relations of ideas and (synthetic) matters of fact.\(^{41}\) His main inspiration for this

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\(^{39}\) References to Kant’s *Critique of Pure Reason* are to Immanuel Kant, *Critique of Pure Reason*, trans. Norman Kemp Smith (New York: St. Martin’s Press, 1965). All in-text references to the *Critique of Pure Reason* will hereafter be cited as “CPR” followed by page numbers in the original first (A) and second (B) editions.


\(^{41}\) This move, however, is mistaken according to Kant. He believes Hume would have been more consistent had he taken mathematics as a posteriori knowledge. Specifically, classifying mathematics as a posteriori would have allowed him to remain consistent with his theory of ideas and the dependence on ideas to their corresponding precedent impressions. And if he had done this, according to Kant, he would have recognized that under such a conception our mathematical knowledge, as synthetic, is just as prone to skepticism as our knowledge of causation. As Kant explains, “But he would then never have been able to ground his metaphysical propositions on mere experience, because otherwise he would also have submitted the axioms of pure mathematics to experience;
interpretation is the passage quoted above from the *Enquiry*,\(^{42}\) and he is quite convinced that it offers direct support for an epistemological dichotomy within Hume’s work that is primarily focused on skeptically denying our ability to rationally justify our beliefs concerning matters of fact.

Kant importantly adds new terminology to the original distinction he reads Hume as making, specifically the a priori versus a posteriori *epistemological* classification of propositions and the analytic versus synthetic *semantic* classification of propositions. He furthers Hume’s distinction by adding that propositions expressing relations of ideas are analytic and a priori, whereas matters of fact are synthetic and a posteriori. In other words, those statements that are “knowable just by thinking” encompass the class of relations of ideas, whereas those statements that are “not knowable without experience” encompass the class of matters of fact. Thus, according to this modernized version of Hume’s Fork, all knowable propositions are classified as either analytic a priori or synthetic a posteriori.

This modernized version of Hume’s doctrine of knowledge represents the standard contemporary account, what I have heretofore referred to as the Received View. It reads Hume as dividing knowledge into the synthetic statements of empirical matters of fact and the analytic statements of mathematics and geometry. Examples of this interpretation of Hume are ubiquitous, but for the sake of clarity I will presently overview and he had too much insight to do this. The good company into which metaphysics would then have been introduced would have saved it from the danger of vile maltreatment, for the blows intended for metaphysics would certainly have also fallen on mathematics, which was not and could not be his intention; and so the sagacious man would have been drawn into considerations which must have been similar to those that now occupy us, but which would have gained immeasurably from his inimitably fine style.” Ibid., p. 22.

\(^{42}\) See pp. 2-3.
some standard versions before proceeding into a discussion of what I call Non-Traditional interpretations of Hume’s doctrine of knowledge.

As an empiricist, Hume is challenged to explain the apparent necessary nature of mathematical truth.\textsuperscript{43} Indeed, this necessity is generally seen as more easily explained by rationalist accounts of knowledge, whereas empiricism traditionally faces difficulties in this regard.\textsuperscript{44} So the Received View faces this challenge via the appeal to analyticity. For example, as Alexander Rosenberg explains in regard to Hume:

Mathematical statements can be established by considering the relations of ideas that the terms of these statements name. If these ideas give the meanings of the terms, then Hume’s claim is that mathematical statements are true in virtue of the relations between the meanings of the terms.\textsuperscript{45}

Like Kant, Rosenberg reads Hume as treating mathematical statements as analytic truths. Treating mathematical knowledge as knowledge of terms and the relations that hold between them allows Hume to account for the apparent necessity of mathematical truths. Mathematical facts are necessarily true because they do not depend on any empirical fact. On the other hand, empirical facts are contrasted with these analytic truths because they are contingent. It’s conceivable that such claims could, in fact, not be the case. The denial of any mathematical truth, however, is self-contradictory and therefore inconceivable. So empirical knowledge is not analytic. These claims are incapable of being falsified with


\textsuperscript{44} Of course, rationalism faces its own problems, especially regarding how our innate mathematical concepts are applicable to the (fundamentally different) objects of perception we study in science. The rationalist is challenged to account for what seems to come down to an “interaction problem” between physical and mental entities.

\textsuperscript{45} Ibid., p. 81.
the Law of Contradiction. They thus represent synthetic statements in this interpretation because their denials are capable of being conceived.

Similarly, Stewart Shapiro (2002) reads Hume as referring to the truths of arithmetic, algebra and geometry as non-empirical, analytic propositions. Again, this characterization of mathematics allows Hume to account for the apparent necessity of mathematics while still maintaining an overall empirical account of knowledge in general. A.J. Ayer holds this view as well – in fact he even adopts it into his own philosophy. As he explains in the preface to *Language, Truth and Logic*:

Like Hume, I divide all genuine propositions into two classes: those which, in his terminology, concern “relations of ideas,” and those which concern “matters of fact.” The former class comprises the a priori propositions of logic and pure mathematics, and these I allow to be necessary and certain only because they are analytic… Propositions concerning empirical matters of fact, on the other hand, I hold to be hypotheses, which can be probable but never certain.\(^46\)

Again, the overwhelming source of this interpretation comes from the above passage in Hume’s *Enquiry*\(^47\) and Kant’s analysis of Hume’s philosophy in general. Through this interpretation these authors and others\(^48\) read Hume as explicitly dividing epistemology into two branches of knowledge, which furthermore allows Hume to account for the apparent difference between necessary mathematical propositions and contingent empirical ones.

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\(^{47}\) See pp. 2-3.

1.3 Non-Traditional Interpretations

This section reviews what I call Non-Traditional interpretations of Hume’s epistemology. I call them Non-Traditional in that they either read Hume as maintaining a single epistemological class of knowable propositions or read Hume as maintaining two epistemological classes of knowable propositions but different from that of the Received View. My point in this review is to show why Hume might be read differently from that of the Received View, but that regardless of one’s interpretation it is misguided to read him as strictly focused on the justificatory status of the propositions within either epistemological class.

Dorothy Coleman (1979) contends that Hume classifies mathematics as synthetic a priori knowledge. To advance this position she begins with an interesting conflict within Hume’s philosophy. On the one hand we have the “Copy Principle,” the claim Hume makes that all of our ideas are derived from previously experienced impressions. Mathematical ideas, then, would appear to be just as empirically based as my idea of the desk in front of me. Hence Kant’s claim that Hume would have been more consistent had he viewed mathematics as a posteriori knowledge. However, this directly conflicts with Hume’s other claim that mathematics involves relations of ideas. Again, he claims that mathematics is a priori knowledge “discoverable by the mere operation of thought, without dependence on what is anywhere existent in the universe.”49 Despite these difficulties Coleman believes that mathematics can still be derived from experience and yet consistently be considered a priori. Mathematical ideas might indeed have to be experienced in precedent impressions, but once we have these ideas we become

cognizant of their a priori and necessary nature. We need not appeal to experience in order to verify them. Mathematical knowledge can thus be a priori as a product of inquiry even if the process of inquiry is empirical. It can be a priori for Hume without being inconsistent with his empiricism.\(^{50}\)

Coleman’s next move is to assess Hume’s statements about mathematics in terms of analyticity. She cites three often used criteria for analytic statements: 1) statements whose denials result in contradiction, 2) statements asserting only formal relations between concepts and requiring no further empirical verifications in order to determine their truth or falsity, and 3) statements lacking empirical content. She concludes that Hume’s statements regarding mathematics are inconsistent with all of these general criteria. Instead, then, a synthetic a priori interpretation of Hume’s mathematics most adequately represents his position.\(^{51}\) She therefore objects to the Received View’s interpretation of Hume that reads him as classifying mathematics in terms of analytic a priori knowledge.

Similar to the position of Coleman is that of Mark Steiner (1987). Like Coleman, he rejects the Received View of Hume as classifying all a priori knowledge as analytic. For Steiner, the above often-quoted passage in the Enquiry where Hume first distinguishes between relations of ideas and matters of fact does not “lend any credence


\(^{51}\) Importantly, however, Coleman is not using “synthetic a priori” in the way Kant does. Rather, as she explains: “The sense in which [Hume] would maintain that there are synthetic a priori judgments must be understood only as indicating a necessary relation between empirical concepts which cannot be determined by the formal relations between these concepts, or by an analysis of these concepts” (Ibid., p. 124).
to Kant’s assertion that Hume’s view is that mathematical propositions are analytic.”

Instead, according to Steiner, all Hume is saying is that such propositions are a priori. Thus, like Kant, Hume classifies mathematics as synthetic a priori knowledge. The only difference between their philosophies of mathematics is in their characterization of mathematical necessity and its relation to the mathematical a priori. For Kant, mathematical knowledge is knowledge that holds a priori for all possible experiences. Hume, on the other hand, is not concerned with whether mathematics will hold for possible experiences. For him, the a priori and necessary nature of mathematics only applies to actual experiences. It need not transcend into possible ones.

Coleman and Steiner still read Hume as bifurcating knowledge, but they differ from the Received View in that they believe Hume sees mathematics as synthetic a priori knowledge as opposed to analytic. Very different from these authors is Kevin Meeker (2007), who focuses on Part Four of the Treatise. He argues that the passage “all knowledge degenerates into probability,” taken from T 1.4.1, provides sufficient evidence for interpreting Hume as a global skeptic. Accordingly, Meeker reads Hume as claiming that we are incapable of certain knowledge of anything, even relations of ideas. As he explains, “if Hume defines knowledge as a type of certainty arising from a comparison of ideas, and there is no such certainty, then there is no knowledge whatsoever.” In other words, Meeker reads Hume as believing that knowledge requires certainty. Certainty is confined to relations of ideas and not to probable matters of fact. However, it turns out

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53 Ibid., pp. 407-408.
that even relations of ideas are capable of being doubted. For example, if something like mathematical proof represented absolute certain knowledge, then there would be no need for us to check on the correctness of such a proof. But mathematicians have to evaluate their proofs for errors, so such knowledge can’t be constitutive of absolute certainty. Instead, such knowledge is merely probable – though much more probable than matters of fact. Meeker then advances the position that in light of this reduction of all knowledge into probability, Hume can be read as anticipating the collapse of the analytic/synthetic distinction made by Quine. Just as Quine sees no distinction between analytic and synthetic statements, Hume similarly sees no real distinction between our knowledge of relations of ideas and our knowledge of matters of fact. For Hume, there are varying levels of probability between the two types of knowledge, but in the end neither resembles absolute certain knowledge.

Meeker’s reading of Hume as endorsing an epistemology grounded on probability is remarkably different from that of the Received View. For Meeker, there’s just one type of knowledge. Anything we think we might know is capable of being doubted, even the apparent necessary nature of mathematical truth. This seems strange, however, given the fact that at least some mathematical demonstrations are quite certainly free from error. For how could simple arithmetical propositions, for example, not demonstrate mathematical necessity? Meeker is quick to point out that “it is plausible to read Hume as simply saying that the rules of the demonstrative sciences are necessary truths even if we do not perfectly grasp them.”55 Like Coleman, then, he views mathematics as necessary, but not necessary in the sense that it is analytic. For both authors, we can still hold a view

55 Ibid., pp. 231-232.
while at the same time maintaining that our knowledge of mathematics is derived from our probable inferences of experience. However, (at least for Meeker), there is still just one type of knowledge, namely, probable knowledge. Relations of ideas and matters of fact alike are encompassed by it.\textsuperscript{56}

While all of these authors borrow heavily from Hume’s texts when putting forth their arguments, it seems to me that the Received View most closely expresses Hume’s epistemology. Nevertheless, my goal is not so much to question any one of these interpretations as it to express what any such interpretation needs to avoid. For even if the Received View’s interpretation of Hume as bifurcating knowledge is the correct one, it is misguided to focus exclusively on Hume’s justificatory project in lieu of his more constructive naturalistic one. Any interpretation of Hume’s epistemology with such an exclusive focus will ultimately fail to account for Hume’s actual epistemological goals to provide a naturalistic etiology of our beliefs.

For this reason I object to Meeker’s analysis. His argument – based on section 1.4.1 of the \textit{Treatise} – for denying our ability to have certain knowledge of even relations of ideas fundamentally misrepresents Hume’s epistemological goals. Indeed, there is overwhelming textual support elsewhere in both the \textit{Treatise} and \textit{Enquiry} suggesting that Hume outright denies that mathematical demonstrations are capable of being doubted.\textsuperscript{57}

For example, in T 1.2.2.6 Hume states the following:


\textsuperscript{57} We might, for a period of time, entertain such doubts (as Hume does); but Hume’s point is that this is not a position anyone can consistently and universally maintain.
But here we may observe, that nothing can be more absurd, than this custom of calling a difficulty what pretends to be a demonstration, and endeavouring by that means to elude its force and evidence. ‘Tis not in demonstrations as in probabilities, that difficulties can take place, and one argument counterbalance another, and diminish its authority. A demonstration, if just, admits of no opposite difficulty; and if not just, ‘tis a mere sophism, and consequently can never be a difficulty. ‘Tis either irresistible, or has no manner of force… Demonstrations may be difficult to be comprehended, because of the abstractedness of the subject; but can never have any such difficulties as will weaken their authority, when once they are comprehended.⁵⁸

Hume’s claim here is that (probable) matters of fact are fundamentally different from (demonstratively certain) relations of ideas. While it is true that Treatise 1.4.1 seems to challenge this distinction, especially its claim that “all knowledge degenerates into probability,” Hume’s point here is that nature prevents us from maintaining these skeptical conclusions. Indeed, even if we grant Meeker’s conclusion that knowledge can only ever be probable – even as regards relations of ideas – Hume’s goal is not simply to erode our confidence in rationality through skepticism, but rather to explain how rationality operates. As he explains later on in that section:

Shoul’d it here be ask’d me, whether I sincerely assent to this argument, which I seem to take such pains to inculcate, and whether I be really one of those sceptics, who hold that all is uncertain, and that our judgment is not in any thing possesst of any measures of truth and falsehood; I shoul’d reply, that this question is entirely superfluous, and that neither I, nor any other person was ever sincerely and constantly of that opinion.⁵⁹

Hume’s overall motivation is to show why skepticism – even in regard to relations of ideas – is an impossible position to consistently uphold. Thus, he wants to replace what our reasoning might skeptically conclude about our beliefs with a more naturalistic account of our beliefs and the operations by which we acquire them.

⁵⁸ T 1.2.2.6
⁵⁹ T 1.4.1.7
1.4 Hume’s Naturalism

As the preceding section indicates, a more realistic epistemological undertaking, and one that I believe is manifest within the work of Hume, is not to understand reason as a justification of belief, but instead to understand belief as already having justified that which we reason about. This view reads Hume as a naturalistic thinker as opposed to a pyrrhonist skeptic. We cannot properly pursue epistemology solely by focusing on the question of how knowledge – regardless of whether or not it is bifurcated – is justified. As I argue in upcoming chapters, such a move will always appeal to ultimately unsuccessful foundationalist tendencies. Instead, then, we should follow Hume and begin with the fact that belief is already naturally preconfigured within us through our experiences and interactions. In the case of causal reasoning, for example, it is not epistemically detrimental that rational inquiry is incapable of justifying such claims. As H.O. Mounce explains:

Thus our reasoning about matters of fact can proceed only when the mind already takes the world in the form of causality, only when it is already adjusted to the causal process. The adjustment itself is prior to reason. It follows that our understanding of the world is based on relations which arise from the workings of nature, not from those of our understanding.\(^{60}\)

In other words, we only face the skeptical problems brought on by foundational appeals to justification when we read Hume as carrying out the sole project of justifying knowledge through reason. If we are to properly understand how our beliefs are justifiable, we must expand our epistemological project to include an account of why we have these beliefs in the first place. For our belief in, for example, the success of inductive and deductive inferences is already naturally set up within us. The belief is

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therefore impervious to anything rational inquiry might try to skeptically conclude about it. Reasoning as a means to justify beliefs only works when it derives its power from how those beliefs are naturally set up within us.

Understanding Hume as a naturalist avoids the skeptical issues brought about by reason’s attempt to justify our beliefs. Norman Kemp Smith is a central exponent of this interpretation of Hume. As he explains:

We cannot by means of our reason explain any of the ultimate characteristics of our experience – the origin of our sensations, the true “secret” nature of causal connection, apprehension of external reality, appreciation of beauty, judgment of an action as good or bad. And the alternative is not scepticism, but the practical test of human validity. Certain beliefs or judgments… can be shown to be “natural”, “inevitable”, “indispensable,” and are thus removed beyond the reach of our sceptical doubts.61

Hume draws the limits to human reason in terms of our natural beliefs in things like causation, the existence of an independent world, an underlying persistent self, and so on. Rational inquiry itself proves to be incapable of justifying our belief in such things, but because our belief in them is naturally within us prior to reason’s intervention, this need not be as problematic as it seems. Evidence for this interpretation of Hume can be found in the following passages from the Treatise and Enquiry:

Nature has not left this to [our] choice, and has doubtless esteem’d it an affair of too great importance to be trusted to our uncertain reasonings and speculation (T 1.4.2.1).
Nature, by an absolute and uncontrollable necessity, has determined us to judge as well as to breathe and feel (T 1.4.1.7).
[Human beings must inevitably] act and reason and believe; though they are not able by their most diligent enquiry, to satisfy themselves concerning the foundation of these operations, or to remove the objections, which may be raised against them (E XII.2).

Most fortunately, it happens, that since reason is incapable of dispelling these clouds [of dissatisfaction], nature herself suffices to that purpose (T 1.4.7.9).

Thus, skeptical reasoning about our beliefs leads to the conclusion that we’re incapable of rationally believing anything, even our own rationality; however, knowledge is saved from this fate by displacing these skeptical considerations “with whatever can be found to be true of human beings by the cautious observation of the natural processes by which they arrive at the responses, judgments, and actions by which they actually live.”


The benefit of pursuing this naturalistic approach to epistemology has been overlooked by more recent justificatory projects. In the two chapters that follow I examine those arguments focused exclusively on determining how either epistemological class can be justified in its knowledge claims. My point in this examination will be to show that these projects, which focus exclusively on how our knowledge claims are justified, fail to establish the justifications they seek. Again, as the above analysis of Hume’s epistemology should make clear, restricting epistemology to a justificatory project cannot succeed on its own, for we will only come to understand how our beliefs are justified by expanding epistemology to include a naturalistic understanding of the etiology of our beliefs.
Chapter 2

Inductive Reasoning

Perhaps the most referenced aspect of Hume’s epistemological legacy is the skeptical problem of induction. Indeed, because such skepticism seems obviously wrong, finding a justification of induction that avoids skepticism is a quest that has preoccupied much of twentieth century analytic philosophy. As Laurence Bonjour explains, this aversion to the skeptical response has created “an equally strong intuitive reason for thinking that a satisfactory justification for inductive reasoning ought to be available and making it seem intellectually scandalous if none can be found.”63 But as it was Hume’s “great merit to have shown that a justification of induction, if possible at all, is by no means easy to provide,”64 analytic philosophers continue to struggle to provide the sought after justification, despite their many attempts. This section offers a review of the merits and faults of a specific class of these attempts. I restrict this section to a specific class in order to show how these solutions in particular try to establish as an a priori premise the claim that nature exhibits sufficient uniformity by which to justify inductive inferences. Some are better than others, but I argue that in the end all fall victim to Hume’s original

characterization of the problem (i.e., specifically by failing to avoid issues of circularity).

In short, I argue that attempts to establish such a premise as a foundation for inductive inferences still fail to provide a justification of induction and therefore are likewise incapable of avoiding Hume’s skeptical conclusion. This is because these foundationalist attempts at justification place undue emphasis on Hume’s skeptical conclusions and overlook his more constructive, naturalistic account of why we possess beliefs about induction in the first place.

Since the class of attempts to be reviewed consistently fails to provide a justification of induction, one suspects that the skeptical problem of induction may very well be interminable and without logical solution. This conclusion has led me to the view that perhaps this epistemological project is misguided. In other words, I believe that the correct project is not to exclusively look for a rational/logical justification of induction by appeal to some sort of foundational premise, but rather to explain why it is we have beliefs about the success of inductive inferences in the first place. Indeed, this is the project I read Hume as undertaking, for it is evident from both the Treatise and Hume’s own personal letters that he never questions the truth of the causal maxim. Rather, as Smith makes clear:

[Hume’s] discussions concern only the grounds, or causes, upon which our belief in [the causal maxim], our opinion or judgment regarding it, really

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65 See Smith, The Philosophy of David Hume, pp. 412–413. Here Smith includes a letter from Hume addressed to John Stewart, Professor of Philosophy at the University of Edinburgh, who had accused Hume of asserting that something could begin to exist without having a cause. Hume asserts therein that “I never asserted so absurd a Proposition, as that any thing might arise without a Cause: I only maintain’d, that our Certainty of the Falsehood of that Proposition proceeded neither from Intuition nor Demonstration; but from another source.” Ibid., p. 413. Emphasis in original.
rests. These, he consistently maintains, are sheerly natural, and allow of no kind of absolute or metaphysical justification.\textsuperscript{66}

To be sure, commentators on Hume have frequently taken him to be questioning the validity of the causal maxim. But the close exegetical reading of Hume presented above should make it clear that this is not his epistemological project. His point is simply that the causal maxim cannot be proved by appeal to intuition, demonstration, or probability. This absence of proof does not, however, suggest that the maxim’s truth must therefore be dismissed, for it is a principle we are naturally disposed to believe.

We should begin by characterizing Hume’s problem of induction by way of more contemporary terminology. In so doing I borrow heavily from Wesley Salmon’s portrayal of the problem in his book \textit{The Foundations of Scientific Inference}. Again, the problem centers on how it is we are \textit{justified} in asserting any conclusion concerning unobserved matters of fact. In Salmon’s words:

\begin{quote}
Given some conclusion, however arrived at, regarding unobserved facts, and given some alleged evidence to support that conclusion, the question remains whether that conclusion is, indeed, supported by the evidence offered in support of it… As a first answer to this question we may point out that the inference does conform to an accepted inductive principle, a principle saying roughly that observed instances conforming to a generalization constitute evidence for it. It is, however, a very small step to the next question: What grounds have we for accepting this or any other inductive principle? Is there any reason or justification for placing confidence in the conclusions of inferences of this type?\textsuperscript{67}
\end{quote}

The problem, then, is to find a justification for inductive inferences that does not itself invoke inductive principles, for to appeal to any such principle would be “viciously circular.” In contemporary terms, we must find a justification that is both \textit{demonstrative} in that the argument’s premises \textit{necessitate} its conclusion, as well as \textit{ampliative} in that it

\textsuperscript{66} Ibid., p. 407.

\textsuperscript{67} Salmon, \textit{The Foundations of Scientific Inference}, p. 7.
allows us to generate broader conclusions about the world beyond what is explicitly stated or implied by the argument’s premises. In short, we need a synthetic a priori principle, but – again – one that does not depend on inductive inference and generate circularity. Hume’s profound conclusion was that there are no such synthetic a priori principles by which we could ground inductive inferences. Thus, as Salmon summarizes Hume’s verdict:

We cannot justify any kind of ampliative inference. If it could be justified deductively it would not be ampliative. It cannot be justified nondemonstratively because that would be viciously circular. It seems then, that there is no way in which we can extend our knowledge to the unobserved. We have, to be sure, many beliefs about the unobserved, and in some of them we place great confidence. Nevertheless, they are without rational justification of any kind!68

The consequences of this problem are not hard to discern: any inference we might draw about the future, whether made by scientists or – to use Hume’s phrase – “the vulgar,” lacks rational justification.69 We cannot rationally or logically determine that any inference about future matters of fact will be the case, for any such inference would have to be both ampliative and necessarily truth-preserving. And this combination, according to Hume, is simply impossible.

Nevertheless, Hume’s skeptical conclusion concerning inductive inferences has not deterred analytic philosophers from attempting to logically resolve the problem. Since

68 Ibid., p. 11.
69 Indeed, as Bonjour explains, “Not only does [Hume’s skeptical conclusion] render epistemically unjustified all inductively supported beliefs in laws or regularities in the world, but since even the beliefs in a world of enduring objects and, via memory, in one’s own past history seem to rely ultimately on such regularities, the unjustifiability of induction arguably leads to perhaps the most radical form of skepticism imaginable: a solipsism in which my epistemically justified beliefs are restricted entirely to my own present experience.” Laurence Bonjour, In Defense of Pure Reason: A Rationalist Account of A Priori Justification (New York: Cambridge University Press, 1998), p. 191.
an exhaustive account of these responses is well beyond the scope of this paper, I restrict my analysis to only those responses which attempt to establish, as a “foundational premise,” that the uniformity of nature is a synthetic a priori principle that justifies inductive inferences. In so doing I hope to show that any such response fails because it is unable to avoid the issues of circularity that are symptomatic of arguments that restrict their analysis to the skeptical implications of Hume’s doctrine of knowledge and typically ignore the constructive naturalistic aspects of Hume’s doctrine of belief.

The paradigmatic example of these responses is Kant’s appeal to the synthetic a priori. Having awoken Kant from his “dogmatic slumbers,” Hume gave Kant the powerful insight that it is impossible to do metaphysics empirically. For as soon as we look to experience for metaphysical knowledge (e.g., knowledge of causal necessity), we become flummoxed by skepticism. Instead, Kant saw synthetic a priori truths and synthetic a priori reasoning as the only possible source of such knowledge. He asserted that the propositions of mathematics and geometry are synthetic and a priori. They are a priori in that they are necessary truths: such propositions are true of any possible world and are knowable independent of experience. Moreover, they are synthetic in that they

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70 For a much more elaborate discussion of the variety of responses to the problem of induction, see Salmon, *The Foundations of Scientific Inference*. In general, these attempted solutions can be classified into two categories: 1) those that directly attack Hume’s arguments, and 2) those that dismiss the problem as a pseudo-problem arising out of linguistic confusion, in some cases even denying that inductive inferences are necessary for making conclusions about the unobserved, either in science or otherwise. Included within the first class are the synthetic a priori responses to be reviewed, as well as those that attempt to supply an empirical justification of induction. For a contemporary example of the empirical response, see Max Black, “Self Supporting Inductive Arguments,” *The Journal of Philosophy* 55.17 (August, 1948), pp. 718-725. For examples of the second category of responses, see Karl Popper, *Objective Knowledge* (Oxford: Clarendon Press, 1972), Chap. 1; Hans Reichenbach, *Experience and Prediction* (Chicago: University of Chicago Press, 1938), Chap. 5.
are ampliative and non-analytic: such propositions expand our knowledge beyond what is merely contained in the concepts in question.\footnote{As Kant explains, “The concept of twelve is in no way already thought by merely thinking this unification of seven and five, and though I analyze my concept of such a possible sum as long as I please, I shall never find the twelve in it” (Kant, Prolegomena, p. 19).} Similarly, Kant maintained that the “principle of universal causation” is a synthetic a priori truth. This synthetic a priori principle asserts that everything that happens presupposes something from which it follows according to a rule.\footnote{CPR A190/B240. Kant’s discussion of this principle is taken up in the Second Analogy.} According to Kant this principle lies outside of and beyond experience as a concept – in his terminology – of the pure understanding. Like the other concepts of the pure understanding, the principle of universal causation converts our subjectively valid perceptions into objectively valid ones that are necessary and universally valid. Our inductive inferences, then, are justified because of this synthetic a priori principle of the pure understanding.\footnote{Importantly, Kant restricts the validity of the concepts of the pure understanding to possible experiences. That is, they have no jurisdiction within the “noumenal realm” of things in themselves. They only apply to possible experiences, and since any possible experience can only be of phenomena, or “mere appearances,” these concepts cannot be said to hold for instances we could not possibly experience. This entails that the principle of universal causation cannot be used to establish that things outside the scope of any possible experience have a cause (e.g., the origin of the universe) – much to the chagrin of traditional rationalist attempts to use such a principle to prove, for instance, the existence of God.}

Kant’s resolution of the problem of induction may be found in the following quote from the Prolegomena:

To put to a test Hume’s problematic concept (which was his crux metaphysicorum), namely the concept of cause: first, the form of a conditional judgement in general, namely using a given piece of knowledge as ground and another as consequence, is given to me a priori by means of logic. But it is possible that in perception a rule of relation will be encountered, which will say: that a certain appearance is followed
constantly by another (though not conversely); and this will be a case for me to use the hypothetical judgement, and for example to say, if the sun shines long enough on a body, it grows warm. Admittedly there is as yet no necessity in the connection, and no concept of cause. But I continue and say: if the above proposition which is merely a subjective connection of perceptions is to be a proposition of experience, it must be regarded as necessary and universally valid. Such a proposition would be: the sun through its light is the cause of heat. The above empirical rule is now regarded as a law, and as one not valid merely of appearances, but valid of them on behalf of a possible experience, which needs comprehensively and therefore necessarily valid rules. Thus I do very well have insight into the concept of cause, as a concept necessarily belonging to the mere form of experience, and into its possibility as synthetic unification of perceptions in a consciousness in general.74

Again, Kant sees Hume’s quest for the concept of cause strictly in experience as mistaken. Both authors do agree, however, in that they do not believe that the causal principle is analytic. They also agree in that they don’t think the causal principle can be demonstrated by manipulating general concepts like existence, beginning of existence, event, cause, etc.75 Moreover, both start their analyses of causation from the subjective observation of two objects or events being constantly conjoined. Their disagreement, however, lies in the fact that Kant believes this subjective experience can be converted into a “necessary and universally valid” objective experience via the validity of the synthetic a priori concept of universal causation. He attempts to establish the validity of this concept via what he calls a “transcendental argument,” which purports to establish the truth of the principle by taking it as a necessary condition of experience. That is, he thinks that the causal principle must be true a priori for all possible experiences, for if it were not we would be incapable of discriminating between, on the one hand, our perception of constantly conjoined events, and, on the other hand, our merely successive

74 Kant, Prolegomena, pp. 72-73. My emphasis.
75 Dicker, Hume’s Epistemology and Metaphysics, p. 143.
perceptions (i.e., those perceptions we do not take as constantly conjoined). Our perception of constantly conjoined events *presupposes* the causal principle. Thus, because we do discriminate between the two kinds of successive perceptions, it follows that we have the synthetic a priori concept of causality. The concept is justified as such, because without it no such discrimination between perceptions would be possible. And because this concept is justified, it follows that our inductive inferences are justified.

Kant’s important insight here centers on the question of how it is we are able to distinguish two or more constantly conjoined events from enduring states of affairs. To illustrate this distinction he describes our observation of a ship moving downstream as characteristic of a constantly conjoined event.\(^\text{76}\) An enduring state of affairs, on the other hand, may be illustrated by observing the different parts of a house.\(^\text{77}\) Interestingly, in both situations our perceptions are successive: just as we observe the ship move from upstream to downstream in a successive manner, we likewise observe the different parts of a house in a successive manner (e.g., the roof, walls, basement, etc). This order entails that observation of successive perceptions alone is incapable of indicating why we take the one observation as a causal event and the other as an enduring state of affairs.\(^\text{78}\)

Succession on its own is incapable of supplying the distinction. Instead, what supplies the distinction is whether or not our successive perceptions could have occurred in the opposite order to that in which they in fact occurred. In other words, the distinction between events and enduring states of affairs centers on the fact that, in the case of the

\(^{76}\) CPR A192/B237

\(^{77}\) CPR A190-191/B235-236

\(^{78}\) Hume himself admits this point at T 1.3.6.3: “Contiguity and succession are not sufficient to make us pronounce any two objects to be cause and effect, unless we perceive, that these two relations are preserv’d in several instances.”
former, there is a necessary order of succession: our successive perceptions could not have occurred in any other order to that in which they actually occurred. This is the criterion of distinction according to Kant. Therefore, because we do indeed distinguish the two kinds of successive perceptions based on whether or not they are characterized by “order-indifference,” Kant concludes that the principle of universal causality must be true – and true synthetic a priori. It is the synthetic a priori rule by which we are capable of distinguishing constantly conjoined events from enduring states of affairs. In Kant’s language, taken from the Second Analogy of the *Critique of Pure Reason*:

If, then, we experience that something happens, we in so doing always presuppose that something precedes it, on which it follows according to a rule. Otherwise I should not say of the object that it follows. For mere succession in my apprehension, if there be no rule determining the succession in relation to something that precedes, does not justify me in assuming any succession in the object. I render my subjective synthesis of apprehension objective only by reference to a rule in accordance with which the appearances in their succession, that is, as they happen, are determined by the preceding state. The experience of an event [i.e., of anything as *happening*] is itself possible only on this assumption.  

In other words, we can only take two successive perceptions as representative of constantly conjoined *objective* events if we presuppose as a causal premise that there are “necessary and universally valid” rules that govern and organize our experience, rules which are not derived from experience but nevertheless directly apply to experience as “the ground of experience itself.” Kant agrees with Hume, then, in that he believes that

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80 CPR A195/B240. Translator’s emphasis.
81 CPR A196/B241. There is an important ambiguity here worth mentioning within Kant’s corpus over the use of the term “rule” in relation to causality. In the *Critique of Pure Reason* he seems to suggest that the only rule for ascribing causal relations to objects/events is the principle of universal causation, whereas in the *Prolegomena* he seems to suggest that there are other “derivative” rules that serve the same purpose (e.g.,
this necessary order – or what Hume calls necessary connection – cannot be an idea derived explicitly through experience or an idea that can be accounted for by the observed contiguity and succession of events. But unlike Hume, Kant argues that we perceive this necessary order within our experience of constantly conjoined events because of our recognition that there exists a valid rule which governs any such perception, namely, the synthetic a priori principle of universal causation.

Kant believes that he has resolved Hume’s problem of induction by casting the concept of cause in terms of a priori necessity, as well as casting it as synthetic in that it is ampliative and non-analytic. This general feature of our perceptions, “that everything that happens presupposes something from which it follows according to a rule,” is established by pure reason alone and without the aid of experiential evidence.\(^82\) It is a rationally and logically justified principle, according to Kant, because without it we could not discriminate the succession involved in causal inferences from the succession involved in our more rudimentary perceptions of, say, a house. Thus, our concept of causal necessity is not, as Hume says, the product of the imagination, but rather an a priori synthetic principle that is both valid on its own and validly justifies inductive inferences in general.

Unfortunately for Kant, the synthetic a priori status of this principle, as well as of the propositions of geometry and arithmetic, is rather suspicious. Indeed, Kant’s explanation of the possibility of synthetic a priori propositions in general has become

\(^{82}\) However, this is not to say that experience plays no role whatsoever in our understanding of the concept. Again, its a priori status means only that it is knowable without experience sufficing to prove it; this does not rules out the fact that experience is often necessary in order to direct our attention to the truth of such an a priori concept.
suspect. For starters, the development of non-Euclidean geometries following Kant’s
death overturned his account of Euclidean geometry as necessarily true and the only way
reality could be structured, for it became clear that other geometric systems may work
just as well, both in theory and in practice. Moreover, it has similarly become generally
accepted that the propositions of pure arithmetic lack synthetic content. These
developments have made it difficult to see how any synthetic a priori truths are possible,
including the principle of universal causation, for it’s unclear that one could really
establish the truth of such a principle without getting caught in the circularity mentioned
above.84

Similar to Kant’s argument is that of Bertrand Russell in his The Problems of
Philosophy. According to Russell, it is a synthetic a priori truth that nature is uniform.
His “Principle of Induction” asserts that if we associate two phenomena as being causally
connected to one another, then – granted we’ve never observed a case in which they have
not been so associated – the probability that they will be connected in the future increases
with the greater number of past cases in which we’ve observed their connection.
Furthermore, upon having observed a sufficient number of cases we’ll eventually
ascertain with confidence that it is nearly certain that such objects will always be
connected in the future.85 An example of such a conclusion would be the belief that there

83 Salmon, The Foundations of Scientific Inference, p. 39. For further elaboration on the
analyticity of mathematical propositions, see Chapter 3 below.
84 For a discursive objection to Kant’s transcendental argument for the principle of
universal causation, see Dicker, Hume’s Epistemology and Metaphysics, pp. 147-150.
85 Of course, such an inference is qualified as nearly certain because it’s conceivable that
the uniformity will fail to hold in future cases. As Russell explains, “The most we can
hope is that the oftener things are found together, the more probable it becomes that they
will be found together another time”. Bertrand Russell, The Problems of Philosophy
(New York: Barnes & Noble, 2004), p. 44. My emphasis.
is a sun and that it will rise tomorrow. Russell agrees with Hume in that this causal principle underlying our inductive inferences cannot be justified or proved based on experience, for to do so would be circular. Nor can experience disprove my belief that my future observations of some phenomenon will probably be consistent with my past ones. But Russell adds the further claim that the principle is known a priori. It falls within a certain class of other inferences we habitually make and have confidence in, but which are capable of being proved independently of experience (e.g., the principles of logic). He calls all such principles a priori, and then argues that “All a priori knowledge deals exclusively with the relations of universals.” We have immediate “knowledge by acquaintance” of universals. Therefore, we have immediate knowledge of the universal a priori principle of induction. Furthermore, it is a synthetic principle because it is ampliative and non-analytic.

Having established why it is that the principle of induction is synthetic a priori knowledge, Russell believes he has explained how it is we can deductively justify inductive arguments. That is, he believes when we take the principle of induction to be a priori knowledge, this allows us to deductively infer the likelihood that an event A will be connected with an event B, granted we have experience of their being so connected in the past. Moreover, our inference of their connection is more probable given the more times we’ve experienced such a connection. This general principle, which is known without

86 For example, my assertion that all dogs have four legs is based on my observing in all past instances dogs with four legs. The fact that there might be a dog with three legs does not disprove my probable assertion that the next dog I see will have four legs, for my original belief was simply based on my own experience of observing dogs.

experience sufficing to prove it,\textsuperscript{88} validly leads us to conclude that particular inductive claims are justified, provided they adhere to the conditions of the principle.

Objections to the synthetic a priori response center on a rejection of the synthetic a priori status of a principle like the uniformity of nature, the principle of universal causation, the principle of induction, etc. Hume himself offers strong criticisms against the possibility of any such principle. For example, in the 	extit{Enquiry} we find:

I shall venture to affirm... that the knowledge of this relation [i.e., of cause and effect] is not, in any instance, attained by reasonings \textit{a priori}... Let an object be presented to a man of ever so strong natural reason and abilities; if that object is entirely new to him, he will not be able, by the most accurate examination of its sensible qualities, to discover any of its causes or effects. Adam, though his rational faculties are supposed entirely perfect at the very first, could not have inferred from the fluidity and transparency of water that it would suffocate him...\textsuperscript{89}

Hume rather satirically admits that he is in total ignorance of how causal relationships could be derived from the pure understanding alone, without any reliance on the constant conjunctions one finds in experience. (But, of course, this appeal to past observations of constant conjunction is itself problematic). He reinforces this position throughout Part IV of the 	extit{Enquiry}, and he confronts the defender of causation’s a priori status with the challenge to show him how the pure understanding could possibly elicit causal relationships without experience. Hume’s challenge to Kant and Russell, then, is to show how a principle like the uniformity of nature is capable of being established without an inference from the experience of constant conjunction. To be sure, both Kant and Russell agree with Hume in that such a principle cannot be inferred from the experience of

\textsuperscript{88} Again, this does not suggest that experience may not help us in realizing the truth of the principle. Obviously the principle is not independent from our experiencing some event that might reinforce what it claims, but that experience itself cannot suffice to prove (or disprove) those claims.

\textsuperscript{89} Hume, 	extit{Enquiry}, pp. 337-338.
constant conjunction, “since all these arguments are founded on the supposition of that resemblance” [i.e., on the supposition that future and present instances of an event will resemble past instances]. Thus, Kant and Russell must seek recourse elsewhere, by attempting to establish the principle as a synthetic a priori truth. That is, to establish it as a synthetic a priori premise that necessitates any correct inductive inference to be both ampliative and necessarily truth preserving. If we could establish such a premise, then our inferences from the observed to the unobserved would be justified.

I do not believe that Kant and Russell are successful in their formulations of this synthetic a priori principle. Like Hume, I do not see how this principle is even conceivable without first having experience of the operations of the world and the objects therein. Even if we could assert, as Kant does, that our perceptions of the world necessitate there being causal regularities that organize our experience, there is no contradiction in assuming that those regularities will not hold in present and future cases. Indeed, it’s conceivable the world could radically depart from heretofore uniformities, which seems to rule out any chance of positing a necessary causal order onto our experiences of constant conjunction.

Moreover, as Strawson points out, Kant’s argument for why the truth of the causal principle is a necessary condition of experience appears to be founded upon a fallacy of equivocation. As he explains in his influential book The Bounds of Sense, Kant’s argument “not only shifts the application of the word ‘necessary,’ but also changes its sense, substituting one type of necessity for another.” The premise of Kant’s argument involves a conceptual or analytic sense of necessity: it is conceptually necessary that

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90 Ibid., p. 342.
one’s perceptions necessarily have the order that they in fact have, it being impossible that they could have taken place in the opposite order because of the “order-indifference” criterion. But Kant’s conclusion does not involve the conceptual type of necessity; rather, it refers to the causal necessity of the change in one’s perceptions that is in fact occurring. That is, the argument moves from a conceptual sense of necessity, where it is an analytic truth that our perceptions necessarily occur in a certain order, to a causal sense of necessity, where it is an objective truth that the objects of perception necessarily occur in a certain order. Furthermore, the premise and conclusion involve different applications of the notion of necessity: the premise applies the notion of necessity to the necessary correspondence of the order of our perceptions to the order of the perceived event, whereas the conclusion applies the notion of necessity to the order of the event itself. Strawson nicely summarizes: “It is a very curious contortion indeed whereby a conceptual necessity based on the fact of a change is equated with the causal necessity of that very change.”

If Strawson’s analysis of Kant’s argument is correct, which I believe it is, then we’re still unable to show how a synthetic a priori justification of the principle of universal causation is possible. Our inductive inferences from the observed to the unobserved thus still lack the rational or logical foundation we’d like them to have.

Russell’s justification fairs no better. Again, his characterization of the principle of induction as a synthetic a priori truth does not escape Hume’s original claim that it is not contradictory to suppose our inductive inferences to be false in the future. Indeed, the fact that Russell places the qualification of “near certainty” on the principle of induction

92 Ibid.
suggests that he was quite aware of the objection that nature could very well fail to be uniform in the future. He readily admits that the principle of induction is incapable of being proved or confuted by sense-experience, and instead claims that it is justified as an a priori general principle of the pure understanding. But it’s unclear how this premise’s a priori status is really anything other than an empirical generalization concerning observations of past uniformities. In other words, it’s unclear how a principle stating that 1) when two objects A and B have been found to be constantly connected and never dissociated, the greater the probability they will be associated in the future, and 2) under the same circumstances a sufficient number of cases of association between A and B will make it nearly certain that A and B will be associated in present and future instances, is not itself an empirical generalization derived from experience. And if this is the case, Russell fairs no better than Kant in escaping the aforementioned circularity.

The above objections to the synthetic a priori response to the problem of induction represent attacks on the particular arguments of Kant and Russell. There are, however, more general objections confronting any such synthetic a priori response. First, an a priori justification seems to proceed under the assumption that, if one could establish a principle like the uniformity of nature as an a priori truth, one could thereby essentially turn an inductive argument into a deductive one. The idea is that such an inductive a priori premise would establish that the truth of the inductive conclusion of any inductive argument follows necessarily, with deductive certainty, from the truth of the premises. Kant’s principle of universal causation, for example, when added as an additional premise to any (correct) inductive argument, purports to guarantee the truth of the inductive conclusion. We’ve already touched on why this is impossible: inductive
conclusions occasionally turn out to be false, so such an argument would either be invalid or contain at least one false premise. Since such inductive failures could conceivably take place in future instances, the inductive premise lacks the a priori necessity we’d like it to have. Thus the skeptical question of how inductive inferences are justified given a certain body of evidence in favor of the conclusion is still unanswered.

Secondly – although this criterion has been (correctly) conceded as untenable even among defenders of an a priori approach to justification\textsuperscript{93} – an a priori justification has often construed the uniformity principle as a \textit{self-evident} a priori truth. This self-evidence carries with it the implication that a non-uniform, chaotic universe is ruled out as an a priori possibility, for the self-evident nature of the principle would rule out the possibility of a contradictory statement regarding the uniformity of nature. But for all we might know on an a priori basis, this contradictory principle seems like it could very well be a possibility! That is, nature could very well be chaotic, knowledge of which could be classed epistemically in terms of an a priori principle (a non-uniformity principle, if you will). This possibility thus dismisses any chance of identifying the uniformity principle as a self-evident a priori truth. Again, the skeptical question of induction is left unresolved.

A third objection to the a priori response argues that any conclusion we infer from an a priori premise must somehow be already contained in the premises. Similar to the deductivist objection considered above, it amounts to the claim that any kind of inference from an a priori premise must be non-ampliative: the conclusion can say no more than what is already contained in the premises. But inductive conclusions by definition are ampliative inferences. When we make inductive conclusions about the unobserved based

\textsuperscript{93} For example, see Bonjour, \textit{A Reconsideration of the Problem of Induction}, p. 108.
upon the favorable evidence of similar events having occurred in past instances, we by
definition go beyond what is contained in those premises. An inductive premise
amounting to something like the uniformity of nature cannot therefore provide the
justification for inductive inferences about the unobserved.

As a final note concerning the justificatory status of a synthetic a priori principle
like the uniformity of nature, I bring up the work of Laurence Bonjour, who attempts to
revive the synthetic a priori response to the problem of induction and defends the
possibility of such a justification. He argues that by eliminating those aspects of the priori
justification which were so easily attacked by the above objections, we can still maintain
an a priori uniformity principle that justifies induction. In short, he argues that such a
principle need not turn induction into deduction, be a self-evident truth, or be dismissed
because of analytic “containment” issues. Moreover, he rejects the empiricist contention
that synthetic a priori knowledge is an impossible, empty class of knowable propositions.
Empiricism generally rejects synthetic a priori knowledge because it considers all a priori
knowledge as analytic or – in the case of Quinean radical empiricism – even outright
denies the possibility of a priori knowledge in general. But Bonjour’s point is that
analyticity is just as mysterious a designation of knowledge as that of the synthetic a
priori.

According to Bonjour, the moderate empiricist view – i.e., the view that all a
priori knowledge is restricted to analytic (tautologous) propositions and that the
justification of any such proposition need not therefore appeal to rationalistic a priori
insight – offers no univocal account of analyticity “capable of explaining all instances of

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94 E.g., see W.V.O Quine, “Two Dogmas of Empiricism,” in From a Logical Point of
a priori justification which does not itself ultimately depend on rationalistic a priori insight.” Indeed, the central claim of moderate empiricism that, “All a priori knowable propositions are analytic” is itself neither empirically justifiable nor an analytic proposition. This central claim of moderate empiricism is therefore incapable of justification based upon the very criteria of justification embraced by moderate empiricism. Secondly, radical empiricism – i.e., the view that outright denies the possibility of (even analytically) justified a priori knowledge – is similarly self-refuting. Its central claim that, “There are no a priori justifiable or knowable propositions” cannot itself be justified based upon the very criteria of direct observation embraced by radical empiricism. In short, Bonjour’s point is that, seeing as it is an entirely open question whether all a priori knowledge can be classed as analytic or can be denied outright as an epistemic possibility, there is therefore no reason to reject outright the possibility that at least some a priori knowledge could be classified as synthetic, including a principle amounting to the uniformity of nature.

Thus, following these compelling objections to empiricist accounts of a priori knowledge, Bonjour offers up a defense of synthetic a priori knowledge and its justification. He argues that moderate rationalism – the view maintaining that “rational a priori insight can provide us with genuine a priori justification and knowledge which

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96 Bonjour summarizes: “the moderate empiricist attempt to reconcile a priori justification with empiricism by invoking the concept of analyticity does not succeed, indeed does not really get off the ground; and the radical empiricist attempt to dispense entirely with such justification ends in a nearly total skepticism.” Bonjour, In Defense of Pure Reason: A Rationalist Account of A Priori Justification (New York: Cambridge University Press, 1998), p. 98.
extends beyond mere analytic truths to fundamental truths about the nature of reality, but grants that such justification is fallible and defeasible, and may even be defeated by empirical considerations” – is the only viable non-skeptical account not only of a priori justification and knowledge, but of knowledge generally.\(^97\) He uses the support he finds in favor of moderate rationalism in order to advance the possibility of a synthetic a priori solution to the problem of induction.

However, even if one accepts Bonjour’s criticism of standard empiricist accounts of analyticity, the burden of proof is still on Bonjour to provide the sought after synthetic a priori principle. Admittedly, he offers a sketch of what such a principle will need to look like, but he concedes that this sketch is far from complete.\(^98\) Indeed, this sketch is perhaps left incomplete because of the fact that any explicit formulation of such a principle is bound to face serious objections similar to the ones presented above. Moreover, even supposing we did have an explicitly formulated principle capable of justifying inductive inferences, it’s conceivable and perhaps even likely that this principle, upon sufficient examination, would turn out to be non-classifiable as synthetic a priori. As Salmon elegantly explains:

Moreover, even if a recalcitrant example were given – one that seemed to defy all analysis as either analytic or a posteriori – it might still be reasonable to suppose that we had not exercised sufficient penetration in dealing with it. If we are left with a total epistemological mystery on the question of how synthetic a priori propositions are possible, it might be

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\(^98\) Indeed, Bonjour concludes in the last chapter of *In Defense of Pure Reason*: “Though there is clearly much more to be done in this area, I believe that the present discussion is enough to show that an a priori justification of induction, in addition to being the only approach that can hope to genuinely solve the problem of induction and avoid the “coal pit” of extreme inductive skepticism, is far more defensible than it has usually been taken to be.” Bonjour, *In Defense of Pure Reason*, p. 216.
wise to suppose it more likely that our analytic acumen is deficient than that an epistemological miracle had occurred.99

It “might be reasonable,” then, given the historical baggage of synthetic a priori principles, to suppose that a newly conceived synthetic a priori principle will eventually turn out to be analyzable as analytic or as derivable from experience, therefore dismissing any chance it might have to resolve Hume’s problem of induction.100

Thus, because historically it appears that “no convincing example of a synthetic a priori proposition has yet been produced,”101 I argue that we should shift our epistemological focus away from trying to logically resolve the (interminable) skepticism found in Hume’s problem of induction and instead focus on those more constructive naturalistic aspects of Hume’s doctrine of belief. I believe this latter project has been overlooked by many responses to the problem of induction, much to the detriment of their arguments. By isolating their epistemological projects to a synthetic a priori justification of inductive inferences and deemphasizing more naturalistic explanations for how it is we have such beliefs, the above responses limit what epistemology is capable of accomplishing.102

100 This of course implies that we need to possess an account of analyticity that is not subject to the objections raised by Bonjour. Until such an account is offered, Bonjour’s argument represents a very promising avenue for further exploration into the possibility of an a priori justification of induction, and one that demands serious consideration by those more inclined towards empiricist accounts of a priori knowledge. As Engel concludes: “While [Bonjour’s] positive account does not provide us with all the answers, after reading his book we have a much better idea of the direction in which those answers must lie.” Engel, *In Defense of Pure Reason*, p. 166.
102 This criticism is less directed at Bonjour’s a priori justification of induction, which, to Bonjour’s credit, appears more focused on providing an etiological *explanation* for the success of inductive knowledge as opposed to a more restricted focus on the logical justification of an a priori principle like the uniformity of nature. This move is perhaps
Chapter 3

Deductive Reasoning

The pursuit of establishing an epistemic foundation in the form of some sort of "foundational premise" is not peculiar to inductive reasoning, but also manifests itself in deductive reasoning, specifically mathematics. We should begin by saying that the question here will be somewhat different from that concerning our inductive inferences. Here, the main concern for those who seek an epistemic foundation for mathematical knowledge will be to account for the apparent a priori and necessary nature of the deductive systems of mathematics and logic, and to do so without invoking the problematic notion of Kantian intuition. Instead of Kantian intuition, it is necessary to establish these deductive systems in terms of their analyticity. This preliminary assertion that mathematics and logic are analytic descends directly from the Received View of Hume, where these deductive systems are classed as relations of ideas. Like the argument I made earlier against restricting epistemology to a justificatory project of inductive inferences, I argue analogously against pursuing such an exclusive project for why Bonjour’s argument in favor of a synthetic a priori response is as compelling as it is. Much like Hume, he is concerned to show why it is we (enthymatically) presuppose the truth of a principle like the uniformity of nature, (as well as to sketch out what such a principle will need to look like in order to be both justified on its own and capable of justifying inductive inferences generally).

103 Shapiro, Thinking about Mathematics, p. 107.
mathematical knowledge. That is, just as we can only properly understand the nature of our beliefs concerning induction by way of an epistemology that includes both the process of justifying beliefs as well as a naturalistic account of how we come to have beliefs in the first place, likewise we can only properly understand the nature of mathematical knowledge by pursuing an epistemology that does not consider mathematics as an isolated faculty of the human mind. I argue against those who consider the nature of mathematical knowledge to be cut off from non-deductive influences, specifically psychology and biology. These naturalistic, non-deductive influences are inherently embedded within the nature of mathematical knowledge, which entails that the nature of mathematical knowledge cannot be properly understood without a thorough appreciation for how these non-deductive factors influence and motivate our knowledge of mathematics.

The prevailing view of mathematics regards it as epistemically independent from psychology and other naturalistic influences. For example, Alexander George and Daniel J. Velleman write in their book *Philosophies of Mathematics* that, “mathematics is the purest product of conceptual thought” which “sets it apart from all else.” Accordingly, these authors assume that the nature of mathematics can be described in isolation from other branches of knowledge like psychology and biology. Consequently, they argue that the question of the nature of mathematical knowledge can be approached without needing

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to take into account the cultural, educational, and neurological processes that make mathematical knowledge possible.\textsuperscript{105} As they explain:

We are likewise not going to pursue any kind of \textit{psychological} inquiry into mathematical thinking or development...[Such research] asks such questions as “What brain, or neural activity, or cognitive architecture makes mathematical thought possible?” or “What kind of environment is needed to facilitate the development of the capacity for such thought?” Again, while of great interest, such studies focus on phenomena that are really extraneous to the nature of mathematical thought itself.\textsuperscript{106}

Implicit within this view is the idea that mathematics as a \textit{justificatory} project can only be effectively pursued in isolation from other branches of knowledge. Moreover it assumes that the question of the nature of mathematical knowledge can be approached without taking into account what cognitive architecture makes mathematical thought possible.\textsuperscript{107}

Thus, in order to supply a justification for mathematical systems (e.g., arithmetic, the natural numbers, etc), one need not rely on non-deductive factors like psychology.

As Carlo Cellucci makes clear, the current prevailing view in the philosophy of mathematics is that discovery and justification are two separate and distinct enterprises. Mathematical discovery is considered an irrational process in that it is “purely subjective and psychological,” whereas the processes involved in mathematical justification are objective in that they are based on deductive logic.\textsuperscript{108} This prejudice is particularly evident in the logicist and neo-logicist programs that attempt to provide a foundation for arithmetic in the principles of logic. By establishing a justificatory foundation for

\textsuperscript{105} Note, George and Velleman also explicitly label history, sociology, and culture as “extraneous influences to the nature of mathematical thought itself” (ibid., p. 2). In my assessment of their claims I focus predominantly on their dismissal of the influence of psychology and biology.

\textsuperscript{106} Ibid. Emphasis in original.


\textsuperscript{108} Ibid., p. 7.
arithmetic in logical principles and assumptions that are not themselves subject to justification,109 logicists argue that arithmetic’s justification need not appeal whatsoever to human psychology or biology. These mathematical truths and their justification are true and justified regardless of whether or not there are any mathematicians to think about them. In other words, they are a priori, necessary, objective and mind-independent.

Frege is historically recognized as the first philosopher to advance the logicist doctrine, a movement in the philosophy of mathematics that understands mathematical truths to be equivalent or reducible to logical truths. In other words, logicism is a project maintaining that mathematical truths are reducible to basic logical axioms. Following Frege’s lead, Bertrand Russell furthered the project. He summarizes the motivations behind the movement in his work *The History of Western Philosophy*:

> From Frege’s work it followed that arithmetic, and pure mathematics generally, is nothing but a prolongation of deductive logic. This disproved Kant’s theory that arithmetical propositions are “synthetic” and involve a reference to time. The development of pure mathematics from logic was set forth in detail in *Principia Mathematica*, by Whitehead and myself.110

The central motivation of Frege, Russell and Whitehead was to find a foundation for mathematics – specifically arithmetic and “pure mathematics generally.” They believed they could generate such a foundation through logic.

109 Logic itself is not subject to justification because any such justification would require the use of those very same logical principles. As George and Velleman explain: “The laws of logic are what make it possible to infer one sentence from another, to justify one claim by appeal to another. Consequently, there would be something misplaced in a request for a wholesale justification of logic itself. To engage in reasoning is to rely on certain forms of argument, which already requires acceptance of a system of logic.” George and Velleman, *Philosophies of Mathematics*, p. 18. My emphasis.

One might ask why such a foundation is necessary for the truths of mathematics. Indeed, why can’t these truths stand by themselves without having to appeal to logic or any other sort of foundational edifice? One answer to this question is our (psychological) need to explain the apparent necessary and a priori nature of mathematical truth. Providing a foundation is at least one way of accounting for this intuition. According to logicism, general logical laws and definitions provide the foundation that allows us to account for the necessary and a priori nature of mathematics, a foundation that is not itself subject to questions of justification. Thus, mathematics is analytic according to logicism. The truths of mathematics are not known via experience: rather, they are known a priori because they can be derived from the basic laws and definitions of logic. Mathematical truths are both necessary and a priori because mathematics itself is analytic (i.e., derivable from general logical laws and definitions).

However, in order to erect this foundation, the logicist first needs to establish exactly how mathematics is analytically deducible from logic. To begin, then, Frege argues that a symbolic language is the only means by which we can perspicuously account for the meaning of mathematical statements. As he explains, “We need a system of symbols from which every ambiguity is banned, which has a strict logical form from

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112 Shapiro, Thinking about Mathematics, p. 109. Note, this is exactly the Received View’s understanding of Hume’s relations of ideas. However, Hume differs from logicism in that he is not exclusively concerned to show how such knowledge is justifiable, but rather is also concerned to show how such knowledge is psychologically, biologically and naturalistically possible.
which the content cannot escape.”  He thus proposes his symbolic language of “conceptual notation.” With this notation we can now begin to discuss how mathematics is deducible from logic. Now comes the need to formulate non-mathematical (i.e., logical) principles from which we can derive basic mathematical propositions (specifically those of arithmetic). Frege is able to accomplish this formulation by proposing what has since been called “Hume’s Principle.” This principle is essentially the thesis that two concepts are “equinumerous” so long as there is a one-to-one correspondence between each set of objects belonging to the concept classes. As Shapiro explains:

> Despite the particle ‘numerosity’ in the name, Frege showed how to define equinumerosity using only the resources of (so-called ‘high-order’) logic, without presupposing natural numbers or the notion of number generally.

In other words, Hume’s Principle provides the epistemic groundwork for the derivation of all the basic principles of arithmetic. Moreover, the principle itself is epistemically justified based on Frege’s definitions for objects, the concepts of those objects and the extensions of those concepts. Thus, according to Frege, we have a complete, consistent, and epistemically justified account of arithmetic’s derivability from basic logical principles.

Like Frege, Russell was also convinced of the derivability of mathematics from logical principles. Indeed, he was so committed to the logicist program that it led him to

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114 As Frege himself admits, however, this symbolic system is nonetheless still an imperfect system. But he thinks it is the best we can do.


116 Ibid., p. 112.
the discovery of an inherent flaw in Basic Law V, one of Frege’s basic axioms for the derivability of arithmetic. “Russell’s Paradox,” as it has since been called, was the discovery of an inconsistency within the axiom that leads to contradiction for the system as a whole. While for Frege this discovery led to his abandonment of logicism, Russell was not dismayed. He continued to pursue the logicist project in the work *Principia Mathematica*, co-written with Whitehead. In this *corpus magnus*, Russell and Whitehead seek to provide a complete and consistent foundation for arithmetic and pure mathematics generally, void of any unjustified implicit assumptions. In order to avoid the paradox entailed by Frege’s Basic Law V, Russell and Whitehead move the discussion away from concepts and towards a discussion of classes. However, in order to derive mathematics they have to propose certain principles whose epistemic justification is somewhat questionable (e.g., the principles of infinity and reducibility). While the authors admit that these principles are not justified in the same sense as the principles of logic, with them they are nevertheless able to derive “just about every branch of pure mathematics short of set theory.”

What is immediately apparent from these efforts to carry out logicism is the desire to avoid any kind of unjustified assumptions, especially if they happen to result in paradox. Interestingly, however, it seems like such a goal is never exactly fulfilled. In the case of Frege, his project flounders with the discovery of Russell’s Paradox. In the case of Russell himself, he is unable to avoid particular assumptions, like the principle of

117 Ibid., p. 123.
infinity, that are not on par with the justification of logical principles. Thus, despite their rigorous quest for foundation, these authors are unable to achieve their goal of founding mathematics on basic logical principles alone.

Regardless of its ultimate failure, however, logicism has had a significant influence on twentieth century analytic philosophy, most notably in the philosophical movement of logical positivism. This philosophical school adopted a fundamental logicist principle of conceiving of mathematics as an analytic enterprise. These philosophers, however, sought to establish the necessary and a priori nature of mathematics in *language* itself, as opposed to strictly in logic. Mathematical truth, then, is necessary because linguistically speaking it is true by definition. Likewise, it is a priori in that it represents an act of reflecting on language use. Ayer notably summarizes this position in his book *Language, Truth and Logic*, where he states:

> The a priori propositions of logic and pure mathematics… [are] necessary and certain only because they are analytic. That is, I maintain that the reason why these propositions cannot be confuted in experience is that they do not make any assertion about the empirical world, but simply record our determination to use symbols in a certain fashion.

Exactly like logicism, the positivists view mathematics as necessary and a priori because it is analytic. Empirical propositions, on the other hand, are probable but never possess the certainty of pure mathematics and logic. In order to assess any given proposition, we must discover the actual meanings of the terms involved. So in the case of pure mathematics and logic, upon analyzing such propositions and discovering what the terms

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118 Moreover, even more detrimental to Russell and his logicist project were Gödel’s incompleteness theorems, which essentially debunked the idea that arithmetic can be strictly conceived as purely analytic. This debunking is further explained below.  
involved actually mean we’ll find that they are true by definition. In other words, mathematics and logic are analytic because their “validity depends solely on the definitions of the symbols,”121 while (merely probable) empirical propositions are synthetic (and a posteriori) because their validity depends on empirical observation of the facts of experience. These latter propositions thus lack the secure certainty of truths by definition.122

Rudolph Carnap, another positivist and member of the Vienna Circle, was strongly influenced by the logicist work of Frege, Russell and Whitehead. However, like Ayer, Carnap is more concerned with the linguistic meaning of empirical (i.e., synthetic) versus mathematical (i.e., analytic) propositions than he is with proving the reducibility of mathematics to logic. In so doing, Carnap develops the idea of a “linguistic framework.” In such a framework, the rules are stipulated so as to govern the grammar and proper use of sentences. These rules will in some cases be synthetic, governing the proper use of sentences based upon one’s sense-experience, whereas other rules will be analytic, governing what logical inferences and assertions are allowed regardless of one’s sense-experience.123 The clear and explicit explication of these rules will determine what sentences are allowed within the framework and the relationships that hold between those sentences. Any sentence not in accord with the rules will not be accepted into the framework. Based on the rules of the framework such a sentence would be meaningless.

121 Ibid., p. 78.
122 Shapiro nicely sums up, “This is at least in the spirit of logicism, even if, strictly speaking, the truths of mathematics do not end up being true on logical grounds alone.” Shapiro, Thinking about Mathematics, p. 125.
123 Ibid., pp. 126-127.
By positing mathematical and logical truth as analytic and void of empirical content, Carnap’s empiricism accounts for the necessary and a priori nature of such truth.\(^{124}\)

Overall, then, the positivist project ventured away from the original goal of Frege and Russell to ground pure mathematics upon strictly logical foundations. The two schools were consistent, however, in that they both viewed mathematics as analytic. For both, mathematical truth does not depend upon any kind of empirical experience. Moreover, its validity reflects the way we use, define and manipulate symbols. For a time this core tenet of both logicism and positivism was highly endorsed and appeared to conclusively explain the necessary nature of mathematical propositions. However, Kurt Gödel eventually undermined this central claim when he successfully showed with his incompleteness theorems that any formal deductive system strong enough to model arithmetic theorems is incapable of being both complete and consistent. Following Gödel, it became evident that any logicist or positivist attempt to justify mathematical propositions in terms of their analyticity would ultimately fail.

But while logicism has encountered significant blows to the tenability of its overall project – including the difficulties expressed above concerning Russell’s paradox, its attack on the consistency of Frege’s Basic Law V, and Gödel’s incompleteness theorems – contemporary philosophers have nevertheless attempted to revive some of logicism’s fundamental doctrines. Aptly called neo-logicism, contemporary proponents of this view is that a “significant core” of mathematical truth is knowable a priori because it is capable of being derived from analytic rules. It is thus less ambitious than traditional logicism in that it does not seek to classify all of mathematics – or in some cases even

\(^{124}\) In Ayer’s terminology, these propositions are *tautologies.*
just arithmetic — purely in terms of logic. However, they do believe that at least some mathematical concepts are best described analytically. Our representative source of this approach can be found in the work of Crispin Wright. Wright seeks to rescue Frege’s project to make mathematics — specifically arithmetic — reducible to general laws and definitions. His results have generated considerable enthusiasm for the project of neo-logicism in recent years.\(^\text{125}\) According to Demopoulos, Wright shows in his work *Frege’s Conception of Numbers as Objects*, “that it is possible to extract from Frege’s *Grundlagen* a valid second-order proof of the Dedekind infinity of natural numbers using only a suitable formalization of...[a] ‘partial contextual definition’ for numerical identity.”\(^\text{126}\) This “partial contextual definition,” briefly referenced in the review of Frege above, is called “Hume’s Principle,” and is stated as follows:

For any concepts F, G, the number of F is identical to the number of G if and only if F and G are equinumerous.\(^\text{127}\)

Again, Frege himself used this principle in order to get his project of providing a logical foundation for arithmetic up and running; Wright adopts this principle from him. But whereas Frege’s logicism collapsed in light of Russell’s Paradox, Wright believes that he can still effectively carry out the original logicist mission by utilizing Hume’s Principle. Wright’s own argument for the derivation of arithmetic from Hume’s Principle is now known as “Frege’s theorem.” It is summarized as follows:


\(^{127}\) Shapiro, *Thinking about Mathematics*, p. 110.
The proof, prefigured in *Grundlagen*... [is] that second-order logic plus Hume’s Principle as [the] sole additional axiom suffices for a derivation of second-order arithmetic – or, more cautiously, for the derivation of a theory which allows of interpretation as second-order arithmetic.\textsuperscript{128}

In other words, Wright believes that the combination of Hume’s Principle and second-order logic is sufficient for the derivation of arithmetic.

However, Wright’s use of Hume’s Principle differs somewhat from Frege’s in that he calls it a stipulation. Its role is “to explain, if not explicitly define, the general notion of identity of cardinal number.”\textsuperscript{129} Hume’s Principle, as a stipulative definition for what constitutes the truth or falsity of a statement, can only refer to a restricted class of statements of numerical identity. Regardless, however, it still results in a sufficiently complete concept of number such that from it we can derive the basic laws of arithmetic.\textsuperscript{130} The explanation offered by Hume’s Principle of our concept of number can be defined in terms of second-order logic, which entails a complete and analytic account of arithmetic as derived from second-order logic with Hume’s Principle serving as its lone axiom.\textsuperscript{131}

Hence, because Wright views Hume’s Principle as a stipulation that explains how we may use the concept of number, he does not make the claim Frege does in conceiving of it as a general logical principle that explicitly defines the concept of number in light of our analysis of that concept. In this way, Wright subordinates the question of the

\textsuperscript{130} Demopoulos, “On the Origin and Status of Our Conception of Number,” p. 221.
\textsuperscript{131} Ibid.
analyticity of Hume’s Principle by showing that we cannot make any arguments concerning the principle’s analyticity unless we first account for its status as a stipulative definition that both introduces and governs the concept of number. Additionally, however, the stipulative nature of the principle under Wright’s account means that using any kind of traditional approach for determining analyticity is mistaken. The stipulation introduces the concept of number, as well as governs it. In other words, a traditional approach like Frege’s where we analyze our preexisting concept of number in order to derive the foundational principle can’t work. Our concept of number is a stipulation of Hume’s Principle combined with second-order logic and thus can’t be used as a means of establishing whether or not the principle itself is analytic.

Despite these concerns regarding the analytic nature of Hume’s Principle in Wright’s work, let us focus on the fact that Wright continues to pursue the logicist goal of reducing mathematics to logical laws and definitions. Thus, what we still possess in this account is a notion of foundational a priori premises, (which again seems to be a necessary outcome of the Received View’s division of knowledge). Here, however, the goal of Wright and others like him is to establish how the deductive inferences of mathematics can be justified. Their solution is to assert what seem to represent putative, a priori foundational premises.

While the neo-logicist program is alive and well, their views regarding the analyticity of arithmetic are still controversial. As Shapiro points out, the key to the program’s argument is whether or not it can successfully establish that Hume’s Principle or second-order logic do not have significant mathematical presuppositions built into

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132 Ibid., p. 211.  
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them. As he explains, “it is essential to the project that when attempting to establish basic arithmetic truth, we need not invoke Kantian intuition, empirical fruitfulness, and so on”.

In other words, neo-logicism must demonstrate that a foundational premise like Hume’s Principle does not itself imply mathematical concepts, for otherwise it will beg the question and arithmetic will fail to be grounded upon it and pure logic. Again, it is still contested whether or not Hume’s Principle or second-order logic possess this characteristic. In short, it is an open question as to whether or not a foundational premise like Hume’s Principle and second-order logic can demonstratively justify arithmetic in terms of analyticity.

More generally, however, the neo-logicist project is hindered by the fact that it does not include any kind of reference to the influence of the cognitive architecture involved in our understanding of arithmetical truth. As an exclusively justificatory project, it does not take into account the significant contribution our psychology and biology have on our understanding of mathematics and how it can be considered reliable demonstrative knowledge. Even if its project is eventually deemed successful on its own terms, it will significantly limit what we can know about mathematical knowledge if it does not increase the scope of its project to include the fact that our mathematical beliefs and their justification are just as dependent upon our psychology, biology and other naturalistic influences as they are upon deductive systems of logic and core axiomatic principles.

133 Shapiro, *Thinking about Mathematics*, p. 139.
As Cellucci explains, “the solution of a mathematical problem is both a process of
discovery and a process of justification.”\textsuperscript{135} His proposed “analytic method,” discussed
below, stresses the importance of the naturalistic influences which impact our
understanding of the nature of mathematical truth and its justification. It is his view that I
propose as a more effective alternative to the “axiomatic method” of demonstration for
mathematical problem solving.

According to the axiomatic method – the “current prevailing view” –
“[mathematical] demonstration consists in a deduction of a statement from basic axioms
which are supposed to be consistent.”\textsuperscript{136} For the axiomatic method, the goal of
demonstration is to provide a justification for the statement in question. Moreover, the
axiomatic method excludes the influences of psychology, biology and the nature of
mathematical discovery because they are viewed as irrelevant to the justification of
mathematical truths. However, this view – an inherent aspect of logicism and neo-
logicism – is untenable in light of Gödel’s incompleteness theorems. The deduction of a
statement from basic axioms cannot sufficiently explain the nature of mathematical
justification, for Gödel established in his first incompleteness theorem that for any
consistent and sufficiently strong axiom system there are truths within that system that
cannot be deduced from its axioms.\textsuperscript{137} The result is that any such axiomatic system
cannot be considered a complete and analytic system. Furthermore, Gödel established in
his second incompleteness theorem that for any consistent and sufficiently strong axiom
system it is impossible to know whether its basic axioms are consistent, for the statement

\textsuperscript{135} Cellucci, “Philosophy of Mathematics,” p. 8.
\textsuperscript{136} Ibid., p. 3.
\textsuperscript{137} Ibid.
by which that consistency is expressed cannot be reliably demonstrated within the system itself.\textsuperscript{138}

Thus, because Gödel conclusively established that mathematical claims cannot be expressed simply as analytical propositions, as well as other pervasive difficulties inherent within the axiomatic method,\textsuperscript{139} Cellucci instead proposes the analytic method of mathematical demonstration as what constitutes the actual method of mathematical problem solving and justification. As he explains:

The analytic method is the method according to which, to solve a problem, one looks for some hypothesis that is a sufficient condition for its solution. The hypothesis is obtained from the problem, and possibly other data, by some non-deductive rule, and must be plausible. But the hypothesis is in its turn a problem that must be solved, and is solved in the same way. That is, one looks for another hypothesis that is a sufficient condition for the solution of the problem posed by the previous hypothesis, it is obtained from the latter, and possibly other data, by some non-deductive rule, and must be plausible. And so on, \textit{ad infinitum}. Thus the solution of a problem is a potentially infinite process.\textsuperscript{140}

This method avoids the issues of analyticity raised by Gödel, on the one hand, because the chosen hypothesis serving as a mathematical solution to a problem need not belong to or be derivable from the system in which the problem itself occurs. On the other hand, analyticity constraints are avoided because this method requires only that the hypothesis be plausible as opposed to being capable of demonstration by absolutely reliable means.\textsuperscript{141} And since all that is required of the hypothesis is that it be plausible, it’s not the case that any such hypothesis is ever absolutely justified. The analytic method demands

\textsuperscript{138} Ibid., p. 4.
\textsuperscript{139} For further clarification of these difficulties, see ibid.
\textsuperscript{140} Ibid., p. 5.
\textsuperscript{141} Ibid., p. 7. By plausibility Cellucci means that “the arguments for the hypothesis must be stronger than those against it on the basis of experience, so, for the moment, the hypothesis can be approved.” Ibid., p. 5.
that any hypothesis proposed as a solution to a problem be time and again revisited in light of new experiential evidence.

Furthermore, the analytic method resolves the dichotomy between mathematical discovery and mathematical justification implicit within logicism and the axiomatic method, for it involves discovering new hypotheses via non-deductive rules as well as a process of justifying why one hypothesis is more plausible than another and should therefore be adopted as a tentative solution to the problem at hand. Unlike the axiomatic method, then, mathematical discovery is not considered to be an irrational, purely subjective and psychological process. It opposes the axiomatic method’s view that mathematical justification represents the sole aspect of mathematics that is a rational and objective process.\(^{142}\) As Cellucci explains:

The analytic method is a counterexample to this view because, on the one hand, it provides a basis for a logic of discovery, and, on the other hand, has nothing subjective or psychological about it, being based on non-deductive rules which are as objective as deductive rules. Such method cancels the separation between discovery and justification because, according to it, the solution of a problem is both a process of discovery and a process of justification.\(^ {143}\)

The dissolution of the disjunction said to hold between mathematical discovery and mathematical justification is exactly the type of maneuver that is necessary if we are to properly understand the nature of mathematical knowledge and its justification. This is especially clear in light of the failures of logicism and the insecure footing of neo-

\(^{142}\) Frege is perhaps the best example for authors who hold this view of mathematical discovery and mathematical justification as separate and distinct enterprises. This is especially evidenced by his claim that logic must concern itself “not with the way in which” mathematical propositions “are discovered but with the kind of ground on which their” justification “rests.” Frege, \textit{The Foundations of Arithmetic}, p. 36. Quoted in Cellucci, p. 7.

\(^{143}\) Ibid., p 8.
logicism in the pursuit of their exclusive justificatory projects. By divorcing the nature of mathematical discovery from the nature of mathematical justification, these projects either fail to realize their justificatory principles or unduly limit what epistemology is capable of accomplishing.

Like Cellucci, then, I argue that we give up on the axiomatic method of mathematical demonstration in favor of the analytic method because it more accurately depicts the nature of mathematical problem solving and justification. The analytic method is not subject to the limitations of the axiomatic method because it is able to account for the non-deductive factors that do, in fact, influence both the nature of mathematical knowledge and our understanding of this knowledge. Thus the analytic method is in the spirit of Hume and the naturalistic account of human belief he endorses. Admittedly, because of Gödel’s incompleteness theorems, Hume’s classification of relations of ideas as analytic is subject to the same issues as logicism, positivism and the axiomatic method in general. But Hume’s revelation that epistemology is best pursued when it supplements the project of justification with a naturalistic project that explains how we come to acquire beliefs – whether mathematical or inductive – lessens this blow. Our understanding of knowledge ought to be reconceived in light of new mathematical and scientific hypotheses which can better account for new experiential evidence concerning our nature as knowers.
Chapter 4

Concluding Remarks

What I have shown in this paper is the inefficacy of epistemology undertaken solely in the pursuit of answers to the question, “how is knowledge justifiable?” As Hume impressively foresaw, if we do not supplement this epistemological pursuit with one that additionally asks how we come to discover and develop knowledge in the first place, we will not be able to escape the clutches of skepticism. By not asking both questions, we will be left to nervously fret over the fact that knowledge, considered as an abstract product, appears incapable of any kind of justifiable foundation whatsoever. On the other hand, when we expand epistemology to consider the concrete processes by which knowledge is acquired, we can avoid complete skepticism. In particular, when we attempt to explain scientifically the cognitive architecture by which we acquire beliefs, we can mitigate the skeptical implications that result from a strictly justificatory pursuit of epistemic foundations.

I conclude this paper with a brief of account of why I think Quine’s project to naturalize epistemology gets at the kind of goals I have in mind. While Hume was hesitant to say much of anything about the “secret causes” governing our natural beliefs, Quine is able to say much more given the scientific advancements of the twentieth century. His
project to naturalize epistemology was preceded by a similar project of Otto Neurath, who consistently argued against the epistemological ideal of absolute certainty. Rather than looking for absolute certain knowledge through the justificatory project of epistemological foundationalism, Neurath’s conception of epistemology centered on our holistic body of beliefs about the world. This transition paved the way for Quine’s holism and his efforts to naturalize epistemology. As Dirk Koppelberg explains:

Looking back to the debate between Neurath and Schlick we now recognize the greatness of the legacy which Quine received from Neurath. We have seen that Neurath emphatically rejected the Cartesian quest for certainty, putting philosophy and science together into the often quoted ship on the open sea in its place as a substitute… We learnt that Neurath emphasized that within a consistent physicalism there is no room for a traditionally understood epistemology; although there might be room to deal with epistemic problems so as to transform them into a form which could be acceptably pursued within science. And finally it is well known that Neurath did not like the word ‘epistemology’ at all because he was well aware of the significance of his break with the tradition in which this term was deeply entrenched.144

Quine’s restructuring of epistemology into a project of scientific inquiry was in large part an outcome of Neurath’s rejection of foundationalism as an exclusive epistemological enterprise. Thus, much like the central argument of this paper, both Quine and Neurath desired to restructure epistemology in order for it to fall directly within the purview of science and avoid the problematic skepticism inherent within foundationalism.

In naturalizing epistemology Quine essentially groups it within the study of empirical psychology. Naturalistically conceived, epistemology now,

has to deal with the question of how we acquire our theory of the world; it has to explain language acquisition, neural perception and other cognitive

activities by exclusively scientific methods which, in turn, are justified on the ground of their survival value according to evolutionary theory.\textsuperscript{145}

These are the types of naturalistic influences I believe can allow us to better understand the nature of knowledge, both in terms of its justification and its acquisition as a system of beliefs.

By including these influences within our epistemological project we are able to explain how it is we acquire beliefs – and this explanation rescues us from the skeptical consequences of an epistemological project strictly concerned with the foundational justification of knowledge. Skepticism need not pervade our epistemology. As Hume inimitably expresses in the Conclusion to Book One of the \textit{Treatise}:

\begin{quote}
The \textit{intense} view these manifold contradictions and imperfections in human reason has so wrought upon me, and heated my brain, that I am ready to reject all belief and reasoning, and can look upon no opinion even as more probable or likely than another... [But] Most fortunately it happens, that since reason is incapable of dispelling these clouds, nature herself suffices to that purpose, and cures me of this philosophical melancholy and delirium, either by relaxing this bent of mind, or by some avocation, and lively impression of my senses, which obliterate all these chimeras. I dine, I play a game of back-gammon, I converse, and am merry with my friends.\textsuperscript{146}
\end{quote}

Rational inquiry into the epistemic foundations of knowledge is perhaps a spellbound, skeptical affair, but only when pursued in isolation from an empirical \textit{enquiry} into the myriad processes which contribute to our human nature.

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References


