A Dissertation entitled

Understanding Pre-service Teachers’ Self-assessment: The Case of Fraction Division

by

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Submitted to the Graduate Faculty as partial fulfillment of the requirements for the

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An Abstract of

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The purpose of this dissertation was to examine how pre-service teachers self-assessed their understanding of fraction division. Towards this goal, I examined how pre-service teachers identified what they knew and what they struggled with (i.e., self-knowledge), and how they used this self-knowledge to predict their actual performance (i.e., actual performance prediction). Thirteen pre-service teachers were individually interviewed. First, without written computation, they discussed their knowledge of 12 fraction division problems and accordingly predicted what score they would achieve on these problems. Next, they solved the problems. Their actual scores were compared to their predicted scores in order to assess prediction accuracy. Findings indicate that pre-service teachers displayed high self-knowledge accuracy of fraction division; however, they were unable to use self-knowledge effectively to predict their actual performance. Factors that influenced the relationship between prediction scores and actual performance scores will be described.
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Chapter 1

Introduction

1.1 Rationale

During the past several decades, research has shown that teacher subject matter knowledge directly impacts student learning (Ball, 1991; Ma, 1999). Ball, Thames, and Phelps (2008) stated, “teachers who do not themselves know a subject well are not likely to have the knowledge they need to help students learn this content” (p. 404). Teachers’ knowledge of their subject matter influences the ways in which teachers formulate and present instructions to make it comprehensible for their students. Developing a solid knowledge of subject matter is essential to student learning and would be a solid addition to current moves to improve teachers’ ability to teach.

The development of teacher’s knowledge of subject matter occurs in a systematic process. According to Ma (1999), for teachers, subject matter knowledge develops during three periods of time: schooling, teacher preparation programs, and teaching careers. While still students in school, teachers achieve mathematical competence. During teaching preparation programs, their competence shifts to focusing on teaching and
learning school mathematics. As they teach in schools they empower students with mathematical competence.

For teachers, the development of subject matter knowledge never stops. In fact, “to be a teacher [in such a rapidly changing world] is a lifelong commitment” (Ryan & Cooper, 2012, p. 520). To keep up with all the new information in teaching and learning, teachers need to continuously study how and what they will teach to students. According to Schifter (1996a), “even experienced teachers can and should continue learning in their own classrooms” (p. 163). The rapid and continuous change in education demands teachers participate as lifelong learners. It is not surprising, then, that pre-service teachers need to learn lifelong skills during teaching preparation programs that enable them to modify their teaching in the future and gradually improve over time (Morris, Hiebert, & Spitzer, 2009; Ryan & Cooper, 2012). To become lifelong learners, pre-service teachers need to learn self-assessment skills in conjunction with teaching skills. Ryan and Cooper depict this combination as follows:

Although it is important to prepare … teachers for initial practice, it is even more important to help them develop the attitudes and skills that will enable them to become lifelong students of teaching. Ideally, rather than relying on authority… they will continually examine and evaluate their practice, effectiveness, and accomplishments (p.164).

Typically in teacher preparation programs, pre-service teachers acquire knowledge of their subject matter, students, and instructional practices and how to put those practices to effective use in their classrooms. Combining these kinds of knowledge with learning self-assessment skills, where pre-service teachers can evaluate their own work and monitor their progress over time, would empower pre-service teachers and put them on the rich path towards being effective teachers.
One argument for teaching self-assessment skills in teaching preparation programs is based on theories of how people learn. One branch of these theories contains metacognitive theories. Flavell (1979) initially invented the term metacognition in the late 1970s to mean “cognition about cognitive phenomena,” or more simply “thinking about thinking” (p. 906). Metacognitive theories refer to one’s knowledge about one’s own cognition (i.e., self-knowledge) and how to use this knowledge to regulate cognition (i.e., self-regulation).

Research suggests that learners who are able to identify their strengths and weaknesses can regulate their own cognition and facilitate learning (Schraw & Moshman, 1995; Pintrich, 2010). Therefore, pre-service teachers’ ability to improve their own learning would be cognitively supported by having adequate self-assessment skills. This suggests that designing assessment tools that can help pre-service teachers develop self-assessment skills would be extremely beneficial to teachers.

One suggested self-assessment tool is a knowledge survey (Nuhfer & Knipp, 2003; Wirth & Perkins, 2005). A knowledge survey consists of topic or course learning objectives framed as a large number of questions (e.g., 100 questions); questions that are usually used in exams. Learners do not answer the survey questions. Rather, they only provide perception of their performance on the survey questions. In other words, they read the questions, determine what they know and can do, and accordingly predict (using a rating scale) their confidence levels in competently answering each question if they were to appear on an actual test. Clauss and Greedey (2010) employed a knowledge survey with senior college students in the disciplines of biology, ecology, evolution, and mathematics. They found “knowledge surveys to be an effective way to help students
assess and monitor their current abilities and levels of understanding” (p. 16). Students utilized the knowledge survey to identify and keep track of strengths and weaknesses of their own knowledge.

Based on this review of research, it seemed clear that pre-service teachers could use a knowledge survey as a self-assessment tool. More specifically, self-assessment here includes identifying what they know and can do as well as using this identification to predict their performance. But, what is not clear is to what extent pre-service teachers might be accurate in their self-assessment and what factors might influence self-assessment accuracy. These two issues are a significant concern when employing a knowledge survey: a concern that is addressed in this dissertation.

In an effort to address this concern, I designed a knowledge survey and examined closely pre-service teachers’ self-assessment accuracy and the factors that influenced it. Since fraction division is one of the most complex topics for pre-service teachers (e.g., Ball, 1990; Simon, 1993; Lubinski, Fox, & Thomason, 1998; Tirosh, 2000; Nillas, 2003; Li & Smith, 2007), I examined pre-service teachers’ self-assessment of fraction division to elicit an understanding of what kinds of self-assessment would be helpful in what is considered to be a complex topic. My next step was to design a knowledge survey for this study. In the next section, I discuss the knowledge survey concept and designing a knowledge survey for this study.

1.1.1 The Knowledge Survey Concept

Rapid changes in education demand that learners show independence in acquiring methods for learning throughout their life. This notion has roots in learning theories such as metacognitive theories. From a metacognitive view:
Students who know their strengths and weaknesses can adjust their own cognition, and, thus facilitate learning . . . [while those who] lack knowledge of their strengths and weaknesses will be less likely to adapt to different situations and regulate their own learning in them (Pintrich, 2010, p. 222–223).

Learners with adequate self-assessment skills are often able to take control of their own learning and improve it. Wirth and Perkins (2005) believe that traditional assessment tools (e.g., exams) do not support the development of self-assessment. During exams, learners are asked to answer certain questions. A teacher or instructor assesses these answers. Learners are not involved in the assessment process of their own work in this situation. Rejecting traditional methods, Wirth and Perkins suggest using exam questions in ways that help learners develop self-assessment skills. They suggest the self-assessment tool of a knowledge survey. As described above, a knowledge survey is a survey consisting of a number of exam questions. In contrast to exams, students do not answer the survey questions. Rather, they self-assess their confidence levels (using a rating scale) in competently answering each question if they were to appear on an actual test.

This focus on self-assessment is the main distinction between using questions in traditional assessment tools and the knowledge survey tool; however, there are two other significant differences in regards to how the survey’s questions are structured and used. First, the questions used in exams are often designed based on the lower thinking levels of Bloom’s Taxonomy (i.e., knowledge, comprehension, application) and rarely on higher thinking levels (i.e., analysis, synthesis, and evaluation). In a knowledge survey, questions are designed using all of Bloom’s Taxonomy levels. Second, exams can only use a limited number of questions due to time constraints. A limited number of questions, however, cover a narrow range of content. Conversely, a knowledge survey uses a large
number of questions (e.g., 100 questions) and can assess a much larger content.

The process of designing a knowledge survey consists of two main steps. The first step is identifying clear and detailed learning objectives. The second step is transforming these objectives into questions based on all of Bloom levels (Nuhfer & Knipp, 2003).

1.1.2 Developing a Knowledge Survey for This Study

Although developing a well-designed knowledge survey is important for effective results, it is even more important to understand how learners might use it. The goal of the study is to locate and understand those factors that help learners to provide accurate self-assessment and those factors that hinder them from providing accurate self-assessment. Understanding these factors would help make adjustments to improve self-assessment accuracy. Inspired by both Nuhfer and Knipp (2003) and Wirth and Perkins (2005), I designed a knowledge survey for fraction division and utilized it to understand how pre-service teachers self-assess their understanding.

In designing this survey, I first defined a detailed framework for fraction division proficiency of eight components (discussed in the literature review). I then developed fraction division problems (or selected them from other sources) for each of the framework components. Each component had, at least, 6 problems. Each one of the six problems was developed or selected based on one of the 6 Bloom’s Taxonomy levels. This process resulted in a knowledge survey with a high volume of fraction division items (i.e., 48 items).

One of the challenges of implementing the knowledge survey was deciding how much of the survey to use given the limited time I could expect to spend with busy pre-
service teachers. A pilot study revealed that using more than 12 items could be both time-consuming and unfeasible. Since the purpose of the study was to examine how pre-service teachers self-assessed their knowledge on a knowledge survey not to assess the knowledge survey per se, a representative sample of the designed knowledge survey (i.e., 12 items) would be sufficient to serve the purpose of the study.

1.2 The Purpose of the Study

Rapid changes and increased complexity in education present the need for preparing pre-service teachers as independent learners. Ryan and McCrae (2005) state “pre-service teachers who can confront their own mathematical errors, misconceptions, and strategies in order to reorganize their subject matter knowledge have an opportunity to develop rich pedagogical content knowledge” (p. 641). Self-assessment skills can help pre-service teachers, in the future, identify the strengths and weaknesses of their teaching and improve it over time.

Knowledge surveys are a tool that can be used by pre-service teachers to self-assess what they know and do not know. Yet, it is unknown to what extent pre-service teachers can be accurate in their self-assessment, and what factors influence self-assessment accuracy. Investigating how pre-service teachers self-assess their understandings of their own knowledge would provide a better picture of the successes and challenges they encounter during the self-assessment process. Plans can then be developed to improve pre-service teachers’ skills in identifying their strengths and weaknesses.
1.3 Research Questions

In this dissertation, my goal was to examine how pre-service teachers self-assess their understanding with respect to fraction division. The two following two questions guided the research:

1. How accurate are pre-service teachers in self-assessing their own understandings of fraction division? Specifically, I examined:
   - Pre-service teachers’ accuracy in identifying what they know and do not know (i.e., self-knowledge accuracy)
   - Pre-service teachers’ accuracy in predicting their performance (i.e., actual performance prediction accuracy)

2. What types of successes and challenges do pre-service teachers encounter as they self-assess their own understandings of fraction division? Specifically, I examined:
   - Factors influencing identifying pre-service teachers’ self-knowledge.
   - Factors influencing pre-service teachers’ actual performance prediction accuracy.

To address these questions, I designed a mixed method study using quantitative data and qualitative data. For the first question, I compared and contrasted the pre-service teachers’ written work and verbal explanation to examine how accurate pre-service teachers were in identifying strengths and weaknesses. To examine prediction accuracy, I used pre-service teachers’ performance as an internal criterion. That is, the actual performance scores were compared to the predicted scores (i.e., rating scores) using two different approaches. Detailed data of each approach will be reported. For the second question, I interviewed the pre-service teachers to examine the factors that influence the accuracy of their self-assessment. The nature of these factors and how they influenced accuracy will be shared.
Chapter 2

Literature Review

Teacher subject matter knowledge directly impacts student learning. To keep up with all the new information in teaching and learning, teachers need to continuously study how and what they will teach to students. That is, teachers should be equipped with adequate self-assessment skills that enable them to modify their teaching and gradually improve over time. It is not surprising, then, that pre-service teachers need to learn such skills during teaching preparation programs. This study aims to explore how pre-service teachers self-assess their understanding with respect to fraction division. In this chapter, I begin with an overview of the importance of teacher subject matter knowledge and follow that with a discussion of how self-assessment (e.g. metacognition) can be helpful in enhancing teacher subject matter knowledge.

Next, I describe the self-assessment tool, a knowledge survey, used in this study to examine pre-service teachers’ self-assessment. Finally, I will discuss the fraction division framework I used to design the knowledge survey. I will review research on fraction division and, as a result, define a detailed eight component framework for fraction division proficiency.
2.1 Teacher Subject Matter Knowledge

Shulman (1986) classified teacher content knowledge into three categories: subject matter, pedagogical knowledge, and curriculum knowledge. These categories are intertwined in practice, as teachers with strong subject matter knowledge can transform that knowledge into strong pedagogical knowledge. Ball, Thames, and Phelps (2008) explain this notion as follows:

Teachers must know the subject they teach. Indeed, there may be nothing more foundational to teacher competency. The reason is simple: teachers who do not themselves know a subject well are not likely to have the knowledge they need to help students learn this content. (p. 404)

Building on the concept of teacher content knowledge by Shulman (1986), Ball (2008) further divided the category of subject matter content knowledge into two subcategories: common content knowledge (CCK) and specialized content knowledge (SCK). CCK is the knowledge needed to solve mathematical problems. This type of knowledge is not unique to teaching. Teachers need to be able to produce correct solutions to the mathematical problems that they ask students to solve. However, this ability entails mathematical knowledge that others possess as well. For example, Ball argues that any individual who knows mathematics can solve problems that require knowing that a square is a rectangle or that zero divided by a number equals zero. In short, CCK is the knowledge that any educated adult (not only a teacher) needs to solve mathematical problems.

In contrast, SCK is the mathematical knowledge needed for teaching. This unique type of knowledge allows teachers to perform mathematical functions for teaching, such as linking representations to underlying ideas and to other representations, responding to
students’ why questions, or appraising and adapting the mathematical content of textbooks (Ball, 2008). SCK involves specific mathematical understanding and reasoning.

Ball used the problem provided in Figure 2-1 to illustrate what SCK entails. SCK allows teachers to recognize that the first word problem requires division by 2 rather than 1/2; the second requires multiplication by 2 rather than division by 1/2; and the third correctly represents the calculation by using a measurement interpretation of division (i.e., how many times a quantity can be contained in another quantity). More specifically, “figuring out which story problems fit with which calculations, and vice versa, is a task engaged in teaching this content, not something done in solving problems with this content” (Ball, 2008, p. 400).

<table>
<thead>
<tr>
<th>Which of the following story problems can be used to represent 1 1/4 divided by 1/2?</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) You want to split 1 1/4 pies evenly between two families. How much should each family receive?</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>b) You have $1.25 and may soon double your money. How much money would you end up with?</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>c) You are making some homemade taffy, and the recipe calls for 1 1/4 cups of butter. How many sticks of butter (each stick = 1/2 cup) will you need?</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 2-1: SCK example.

Similarly, Ma (1999) introduced subject matter knowledge as Profound Understanding of Fundamental Mathematics (PUFM). PUFM refers to the deep understanding of mathematical structures that enables teachers to connect various pieces of mathematics. Ma stated the following:

The teaching of a teacher with PUFM has connectedness, promotes multiple approaches to solving a given problem, revisits and reinforces basic ideas, and has longitudinal coherence. A teacher with PUFM is able to reveal and present connections among mathematical concepts and procedures to students. He or she appreciates different facets of an idea and various approaches to a solution . . . and is able to provide explanations . . . for these facets and approaches. (p. 124)
PUFM presents an ideal structure for teachers’ conceptual and procedural knowledge. Ma pointed out that a teacher with PUFM can compute an answer to the problem $1 \frac{3}{4} \div \frac{1}{2}$ by using different computational approaches and can pose story problems to represent the different meanings of fraction division. The depth of teachers’ profound understanding of fraction division is measured by their ability to make connections between entire fraction division concepts and procedures.

Hiebert and Lefevre (1986) described procedural knowledge as “being made up of two distinct parts. One part is composed of the formal language, or symbol representation system, of mathematics. The other part consists of the algorithms, or rules, for completing mathematical tasks” (p. 5). Teachers who exhibit procedural knowledge know how to use a specific algorithm rule to solve a mathematical problem. Procedural knowledge refers to the understanding of how to solve problems (Hiebert & Lefevre, 1986). In other words, procedural knowledge involves algorithms and the use of such algorithms to solve mathematical problems.

Conversely, conceptual knowledge is defined as knowledge that is rich in relationships. Teachers who exhibit conceptual knowledge know why a specific rule works to solve a mathematical problem. Conceptual knowledge refers to the understanding of why certain problems are solved in a particular way (Hiebert & Lefevre, 1986). This knowledge includes concepts, ideas, and the relationships among them that provide the meaning behind algorithms.

The National Council of Teachers of Mathematics (NCTM, 2000) reports that both procedural and conceptual knowledge are required for teaching mathematics. Teachers should have solid procedural and conceptual knowledge of the subjects that
they teach. They should be competent in implementing effective and various ways of teaching students how to compute mathematical problems properly. This competence entails a solid knowledge of the procedures, rules, and algorithms used in mathematics. Teachers must also be competent in helping students comprehend why these procedures, rules, and algorithms work.

2.2 Metacognition

The term metacognition was originally introduced in the late 1970s by the physiologist John Flavell (1979) as thinking about one’s own thinking. The definition of metacognition has been developed to include two aspects: self-knowledge—what one knows about cognition; and self-regulation—how one uses that knowledge to regulate cognition. In short, metacognition is knowledge and regulation of one’s thinking processes.

Self-knowledge refers to understanding ones’ own strengths and weaknesses. One understands what one knows and can do, as well as what one does not know and cannot do. Lack of self-knowledge can obstruct learning (Pintritch, 2010). For example, if a learner believes he understands a text after reading it, but in reality he does not, it is less likely he will re-read it to check understanding. Conversely, if a learner realizes that he struggles when understanding a text, he is more likely to re-read it and apply different strategies to help him understand it.

Self-regulation refers to employing strategies to enhance performance, tracking the progress of this performance, and adjusting strategies as needed. For instance, if a student struggles to understand a text, he is more likely to re-read it and apply different strategies to help him understand it. Self-regulation involves checking how the new
method is working to see whether it is helping improve reading or not. If the new method is not working as expected, it needs to be adjusted or replaced with another method to ensure better reading.

Teaching metacognitive strategies in context has been shown to improve student learning. Bransford, Brown, and Cocking (2004) argue, “the integration of metacognitive instruction with discipline-based learning can enhance student achievement and develop in students the ability to learn independently” (p. 21). Research has shown that applying metacognitive strategies helped improve understanding in different disciplines such as writing (Flower & Hayes, 1981; Palincsar & Brown, 1984) and mathematics (Schoenfeld, 1983, 1985, 1991).

2.3 Knowledge Survey

2.3.1 Why to Use a Knowledge Survey

Metacognition has been used in education to assist in understanding learning for the past few decades. Although defining metacognition was controversial, most definitions were faithful and kept the core meaning: to help learners learn on their own for life. With this definition in mind, traditional assessment and metacognition do not go hand-in-hand. Traditional assessment comes in different forms, such as quizzes and exams. Although they are some of the most widely used assessment tools, they have limitations. According to Wirth and Perkins (2005), traditional assessment tools are often summative (i.e., the assessment takes place at the end of a learning event). To a large extent, they only cover a limited portion of the learning continuum. Furthermore, they typically assess the lower levels of thinking instead of the higher ones, such as analysis, synthesis, and evaluation. Accordingly, Wirth and Perkins suggested that there is need for
an assessment tool that (a) can be used to provide formative assessments (i.e., occurring while knowledge is being learned) of student understanding; (b) can provide a more comprehensive assessment of student learning; and (c) can be readily employed by students to monitor their own learning and to develop skills of self-assessment (p. 4).

Nuhfer and Knipp (2003) introduced the knowledge survey as a tool that overcomes the limitations of traditional assessment tools. A knowledge survey consists of the full breadth of learning objectives, which are presented as a large collection of questions drawn from the course quizzes, exams, and other tools. These questions are designed according to Bloom’s Taxonomy (i.e., knowledge, comprehension, application, analysis, synthesis, and evaluation). These knowledge surveys do not actually require learners to answer questions; rather, they use a rating scale to assess their confidence levels in their abilities to answer each question with competence if the question were to appear on an actual test. In other words, what is being surveyed is the confidence teachers have in their own judgment compared to actual performance.

Comparing confidence judgment to actual performance is called calibration. Calibration has been defined as “the process of matching perception of performance with actual level of performance (Nietfeld, Cao, & Osborne, 2006, p 161). Calibration can be examined in different approaches. According to Schraw (2009) “the most common approach is to make a continuous confidence judgment that ranges from no confidence to complete confidence. A second approach is to make a dichotomous prediction whether performance will be successful or unsuccessful.” (p. 34). The dichotomous calibration is measured as either 100% accuracy or 0% accuracy; however, the continuous calibration is measured differently. Continuous calibration is measured as the difference between
predicted and actual performance (absolute accuracy) (Hacker, Bol, & Bahbahani, 2008; Schraw, 2009; Bol, Hacker, Walck, & Nunnery, 2012). The smaller the difference between predicted performance and actual performance, the more calibrated the learner is. Conversely, the larger the difference between predicted performance and actual performance, the less calibrated the learner is.

Nuhfer and Knipp (2003) presented an excerpt of six items from a knowledge survey (see Table 2.1). The six items were taken from a 200-item knowledge survey for an introductory course. They were designed for a unit lesson on asbestos, ranging from the lowest Bloom level (i.e., knowledge) to highest Bloom level (i.e., evaluation). Students responded to the 200-item knowledge survey, including the six items, using a bubble sheet or a web-based format in accordance with the instructions in Figure 2-2.

Table 2.1: Excerpt of six items from a 200-item knowledge survey.
(Taken from Nuhfer and Knipp’s, 2003, p.2).

<table>
<thead>
<tr>
<th>Item #</th>
<th>Bloom Level</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>Knowledge</td>
<td>What is asbestos?</td>
</tr>
<tr>
<td></td>
<td>Recalling facts, terms, basic concepts and answers</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>Comprehension</td>
<td>Explain how the characteristics of amphibole asbestos make it more conducive to producing lung damage than other fibrous minerals.</td>
</tr>
<tr>
<td></td>
<td>Demonstrative understanding of facts and ideas</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>Application</td>
<td>Given the formula Mg3Si2O5(OH)4, calculate the weight percent of magnesium in Chrysotile.</td>
</tr>
<tr>
<td></td>
<td>Solving problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>Analysis</td>
<td>Two controversies surround the asbestos hazard: (1) it's nothing more than a very expensive bureaucratic creation, or (2) it is a hazard that accounts for tens of thousands of deaths annually. What is the basis for each argument?</td>
</tr>
<tr>
<td></td>
<td>Examine and break information into parts.</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>Synthesis</td>
<td>Develop a plan for the kind of study needed to prove that asbestos poses a danger to the general populace.</td>
</tr>
<tr>
<td></td>
<td>Compile information together in a different way</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>Evaluation</td>
<td>Which of the two controversies expressed in item 24 above has the best current scientific support?</td>
</tr>
<tr>
<td></td>
<td>Making judgments about</td>
<td></td>
</tr>
</tbody>
</table>
This is a knowledge survey, not a test... In this knowledge survey, don't actually try to answer any of the questions provided. Instead rate (on a three point scale) your confidence to answer the questions with your present knowledge. Read each question and then fill in an A, B or C...
Mark an "A" as response if you feel confident that you can now answer the question sufficiently...
Mark a "B" response to the question if you can now answer at least 50% of it...
Mark a "C" as response to the question if you are not confident that you could adequately answer the question for graded test purposes at this time

Figure 2-2: Instructions from a knowledge survey.

2.3.2 How to Use Knowledge Survey

Knowledge surveys benefit both instructors and students. Wirth and Perkins (2005) described one benefit for the instructor:

Results of pre-course surveys provide... information about knowledge and understanding of students as they enter a course. If prerequisite knowledge is poorly developed, or if some introductory content has already been mastered, [instructors] modify course activities accordingly. (p.3)

From another perspective, a pre-course survey can provide insight into each student’s comfort level regarding the course materials. Such data makes the instructor aware of the students who may need extra assistance or those who truly have mastered the prerequisites necessary to engage the challenges encompassed in the course (Nuhfer, 2003). In brief, pre-course knowledge surveys give the instructor a sense of what the students already know and need to learn. In contrast, mid- and post-course knowledge surveys provide information about learning gains. They provide a sense of what students have learned and what they still struggle with. Another benefit is that a knowledge survey is a tool for course preparation in which the instructor identifies learning objectives and organizes content and activities before the course begins.
For students, knowledge surveys provide a rich study guide for the entire course. Students can use these surveys to prepare for each lecture or exam, to monitor their learning as the semester progresses, and to foster metacognition. As students self-assess their understanding of the facts and concepts presented in the surveys, they can identify the gaps in their understanding. Thus, they can devise and implement plans to enhance their learning and to close the gaps. They may re-study the survey materials that they have struggled with, either on their own or with assistance from others (Wirth & Perkins, 2005). In addition, Clauss and Geedey (2010) stated that “it is clear that the surveys foster metacognition—students’ abilities to predict their performances on various tasks . . . and monitor their current levels of mastery and understanding” (p. 22).

2.4 Framework For Fraction Division Proficiency

2.4.1 General Framework For Fraction Operations

A comprehensive framework for a certain mathematics topic should contain the full range of content knowledge and processes (i.e., reasoning and representation skills) that must be possessed to master that topic. In this context, several studies have suggested various general and specific frameworks as well as the most significant aspects of mathematics content knowledge and processes needed to help students successfully learn mathematics.

Huinker (2002) discussed examples of fifth-grade students’ work from a large urban school system with the purpose of characterizing fraction operation sense. She argued that there are seven dimensions (see Figure 2-3) that students must meet to be proficient in fraction operations.
The first dimension is the understanding of the interpretations of operations and their models. To obtain a solid understanding of operations, students must mainly understand their interpretations. They must understand addition as the combination of quantities into a larger single quantity, subtraction as the removal of a quantity from another quantity, multiplication as the combination of equal quantities, and division as the separation of a quantity into equal quantities (or the determination of how many quantities can be contained within other quantities). Students should also be able to model those operations by using a variety of models, such as area models, set models, or number lines.

The second dimension is the understanding of the effects of an operation on pairs of numbers or fractions. Students should comprehend the reasoning behind the answers that they obtain when applying mathematical operations. That is, they should understand

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<tbody>
<tr>
<td>1.</td>
<td>Understanding the meanings and models of fraction operations</td>
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<td></td>
<td>- Ability to interpret the four basic operations (i.e., addition,</td>
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<td></td>
<td>subtraction, multiplication, division) and demonstrate them</td>
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<td></td>
<td>using models (e.g. area model)</td>
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<td>2.</td>
<td>Understanding the effects of an fraction operation on a pair of</td>
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<td>numbers</td>
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<td></td>
<td>- Ability to justify the reasonableness of the product of all</td>
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<td>fraction operations</td>
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<tr>
<td>3.</td>
<td>Real-world applications</td>
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<td>- Ability to recognize and generate real-world situations for</td>
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<td></td>
<td>fraction operations</td>
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<td>4.</td>
<td>Meaning and mathematical language associated with operations</td>
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<td>symbols</td>
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<td></td>
<td>- Ability to connect operations symbols (e.g. +) to their</td>
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<td></td>
<td>meanings using proper mathematical language</td>
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<td>5.</td>
<td>Ability to translate easily across the basic presentations of</td>
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<td>operations (i.e., real-world, oral language, concrete,</td>
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<td>pictorial, symbolic)</td>
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<td>6.</td>
<td>Understanding relationships between operations</td>
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<td>- Ability to interpret how the four basic operations interact</td>
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<td>with each other</td>
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<td>7.</td>
<td>Ability to compose and decompose numbers, and use properties</td>
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<td></td>
<td>of operations, to solve mathematical problems</td>
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Figure 2-3: Huinker’s seven dimensions of fraction operation sense.
as well as be able to explain what happens to the answer when adding, subtracting, multiplying, or dividing either whole numbers or fractions.

The third dimension is a real-world application. This component includes the ability to recognize real-world situations for specific operations and to “regularly describe or pose …situations for [these] specific operations” (Huinker, 2002, p.73). In other words, students should not only be able to recognize which real-world situations can be solved by addition, subtraction, multiplication, or division of whole numbers and fractions, but also be able to create real-world situations for these operations.

The fourth dimension is the ability to derive meaning from symbols and formal mathematical language. This component includes the ability to use symbols as tools to denote actions and things students already know. It is the ability, for example, to connect the symbol “+” to the act of combining quantities into a larger single quantity (i.e., addition), the symbol “−” to the removal of a quantity from another quantity (i.e., subtraction), the symbol “×” to the act of combining of equal quantities (i.e., multiplication), and the symbol “÷” to as the act separating a quantity into equal quantities (i.e., division).

This type of connection between symbols and their meanings develops as students move from informal mathematical language to formal mathematical language (e.g., from splitting to dividing and the division symbol “÷”). Students use symbols as tools to present things that they know and understand. Without this connection, “students manipulate symbols without meaning, rather than thinking of symbols as quantities and actions to be performed or records of actions already performed” (Huinker, 2002, p.73). The fifth dimension is the ability to translate with ease the basic presentations of
operations (i.e., real-world, spoken language into concrete, pictorial, and symbolic presentations). As mentioned earlier in the real-world applications component, students should be able to create appropriate real-world problems for the mathematical operations they use. Creating an appropriate real-world problem for a specific operation is only one presentation, and they should also be able to generate the other basic representations of the problem by manipulating classroom materials, pictures or drawings and formal mathematical language. The reason is that “students who cannot easily connect these representations lack the power to make sense of fractional concepts and operations and to see the usefulness of fractions in the world around them” (Huinker, 2002, p.73).

The sixth dimension is the understanding of the relationships between operations. Understanding of the inverse relationship between addition and subtraction \((a + b = c, b + a = c, c - b = a \text{ or } c - a = b)\) as well as multiplication and division \((a \div b = c, a \div c = b, c \div b = a \text{ and } b \cdot c = a)\) contributes to the development of thinking strategies and student-generated algorithms for computation.

The seventh dimension is the ability to compose and decompose numbers and to use mathematical operations. Understanding the properties of operations help students use different computational strategies in a more flexible manner. For example, understanding the distributive property \((a(b + c) = ab + ac)\) allows students to analyze a problem and to create an equivalent problem.

Although the seven dimension framework do not explicitly emphasize computational proficiency, Huinker has argued that computational proficiency emerges naturally as a result of the development of the seven dimensions. She explains: “students with richly connected knowledge of fraction operations are able to develop flexible
student-generated strategies for computation and work with problem situations meaningfully” (p. 78). In other words, students should not be rushed to master computational approaches for fraction operations until they establish a solid foundation of fraction operation sense. However, both fraction operation sense and computational proficiency are essential for learning fraction operations.

This notion is in alignment with the National Council of Teachers of Mathematics (NCTM) recommendations. NCTM (2000) states:

Developing fluency requires a balance and connection between conceptual understanding and computational proficiency. On the one hand, computational methods that are over-practiced without understanding are often forgotten or remembered incorrectly (Hiebert 1999; Kamii, Lewis, & Livingston 1993; Hiebert & Lindquist 1990). On the other hand, understanding without fluency can inhibit the problem solving process (Thornton, 1990, p. 5). (p. 35)

This argument suggests that another dimension should be added to the framework. This dimension would contain the ability to use a variety of computational approaches for operations and the ability to provide a conceptually sound explanation for why they work. Students should able to come up with different computational approaches to solve a single mathematical problem. $\frac{1}{2} + \frac{1}{4}$, for example, can be solved by different approaches including, but not limited to, the common denominator rule $\frac{1}{2} + \frac{1}{4} = \frac{6}{4} + \frac{1}{4}$ and the associative property of addition $(1 + \frac{1}{2}) + \frac{1}{4} = 1 + (\frac{1}{4} + \frac{1}{4})$. Students who conceptually grasp the meaning of addition should be able to use such computational approaches and reason why they work to obtain the correct solution.

In the following section, I review the literature to synthesize a framework for fraction division. In the review, I integrate teachers’ and pre-service teachers’ subject matter knowledge of fraction division with the framework for fraction division. I address
the essential parts that constitute this framework and review the performances of teachers and pre-service teachers in these parts.

2.4.2 Specific Framework For Fraction Division

Huinker’s fraction operation sense dimensions were well addressed in the studies that examined teachers’ and pre-service teachers’ fraction division knowledge (e.g., Ball, 1990; Simon, 1993; Lubinski, Fox, & Thomason, 1998; Ma, 1999; Tirosh, 2000; Nillas, 2003; Li & Smith, 2007; Orrill, Araujo & Jacobson, 2010). In light of these dimensions, each study discussed one or more of the dimensions and suggested specialized fraction division concepts and procedures (see Figure 2-4) that should be acquired to help enhance pre-service teachers’ knowledge in the fraction division area. In this review, I will use the term “components” instead of “dimensions”

2.4.2.1 Understanding the Meanings and Models of Fraction Division

Nillas’s (2003) research examined pre-service teachers’ conceptual and procedural understandings of the division of fractions. In her study, ten elementary pre-service teachers solved several fraction division problems that included the following: (a) a contextual measurement problem, (b) a contextual inverse of a multiplication problem, and (c) a contextual inverse of a Cartesian product problem.

Measurement Interpretation

The contextual measurement problem was as follows:

You have 3 1/4 cups of grapefruit juice. You take medicine each day, and your doctor wants you to limit the amount of grapefruit juice you drink when you take your medicine. You are only allowed to drink 3/4 of a cup of grapefruit juice each day with your breakfast. Can you drink the exact amount of grapefruit juice
allowed without having to buy more?

<table>
<thead>
<tr>
<th>1. Understanding the meaning and models of fraction division, including the following:</th>
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<tbody>
<tr>
<td>- Measurement interpretation.</td>
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<td>- Sharing interpretation.</td>
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<td>- Unit rate interpretation.</td>
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<tr>
<td>- Division as the inverse of multiplication.</td>
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<tr>
<td>- Division as the inverse of a Cartesian product interpretation.</td>
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<tr>
<th>2. Understanding the effects of fraction division on a pair of numbers where fractions are involved:</th>
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<tbody>
<tr>
<td>- Understanding that the answer does not always become smaller when dividing.</td>
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<tr>
<td>- Understanding that the answer does not always become larger when multiplying.</td>
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<tr>
<th>3. Ability to perform real-world applications:</th>
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<tbody>
<tr>
<td>- Recognizing real-world situations for fraction division operations.</td>
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<tr>
<td>- Posing (designing) real-world situations for fraction division operations.</td>
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<tr>
<th>4. Understanding meanings and mathematical language associated with fraction division symbols:</th>
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<tr>
<td>- Understanding that a number of different meanings can be derived from the same fraction division symbols.</td>
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<td>- Describing the different meanings attached to fraction division symbols by using the language of mathematics.</td>
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<th>5. Ability to translate across various modes of interpretation (i.e., real-world situations, oral language, and concrete, pictorial, and symbolic interpretations):</th>
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<tr>
<td>- Solving fraction division problems involving real-world contexts.</td>
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<td>- Fluently discussing fraction division problems and their solutions by using oral language and concrete, pictorial, and symbolic representations.</td>
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<tr>
<th>6. Understanding the relationships between fraction division and other operations:</th>
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<tr>
<td>- The relationships between division and multiplication.</td>
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<tr>
<td>- The relationships between division and subtraction.</td>
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<th>7. Ability to compose and decompose numbers and to use the properties of operations to solve fraction division problems:</th>
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<tr>
<td>- Commutative law.</td>
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<td>- Distributive law.</td>
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<tr>
<th>8.) Ability to use a variety of computational approaches for fraction division and to provide a conceptually sound explanation for why they work:</th>
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<tr>
<td>- Invert-and-multiply algorithm.</td>
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<td>- Decimal division algorithm.</td>
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<tr>
<td>- “Maintain the value of a quotient” rule.</td>
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<tr>
<td>- Common denominator algorithm.</td>
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**Figure 2-4: Fraction division framework proficiency components.**

In this problem, all ten participants solved it correctly and demonstrated
considerable understanding of the measurement interpretation. They showed correctly
that they could drink a total of 3 servings of 3/4 cups, with 1/4 of a cup left over. For
example, Angela showed that the measurement interpretation could be executed using
repeated subtraction. She explained that she started by subtracting 3/4 on the first day
from the total amount given (3 1/4). Then she repeatedly subtracted 3/4 from the
remainder until there was not enough left to subtract 3/4. In other words, 3 1/4 – 3/4 = 2
1/2, 2 1/2 – 3/4 = 1 3/4, 1 3/4 – 3/4 = 1, 1 – 3/4 = 1/4. She concluded that there were 3
complete quantities of 3/4 in 3 1/4, with 1/4 leftover. In short, she applied the
measurement interpretation by showing how many times 3/4 fits within 3 1/4.

Inverse of Multiplication Interpretation

The contextual inverse of a multiplication problem was as follows: “In a student
survey on class time preferences, 55 students said they prefer morning sessions. This is
two-and-a-half times the number of students who prefer the afternoon sessions. How
many students responded to the survey?” Only four pre-service teachers were able to
solve this problem correctly. They set up the equation as follows: 2 1/2 × the number of
students who preferred the afternoon sessions = 55. To determine the number of students
who preferred the afternoon sessions (i.e., 22), they divided 55 by 2 1/2. Next, they added
22 (i.e., the number of students who preferred the afternoon sessions) to 55 (i.e., the
number of students who preferred the morning sessions) to arrive at 77, which denotes
the total number of students who responded to the survey. In short, the teachers
demonstrated that division can be procedurally interpreted as the inverse of
multiplication; however, they failed to present it in a pictorial form. They were unable to
connect the values in the problem or to provide a correct answer by using pictures.
**The Inverse of a Cartesian product**

The following represented the inverse of a Cartesian product problem: “Are three meters of fencing enough to enclose a rectangular vegetable plot that has a length of 3/4 of a meter and an area of 6/20 square meters?” In this problem, all ten pre-service teachers realized the need to apply the rectangular area formula to determine the width given the area; however, the values of the problem (i.e., 3/4 and 6/20) were obstacles. As fractions, the values in the problem made it challenging for the participants to provide complete, correct answers to the problem pictorially and (for 6 of them) to solve it symbolically. Only four were able to use the area formula to provide the correct answers, and none of those could solve the problem by using pictures.

**Sharing Interpretation**

Tirosh (2000) used a word problem in a diagnostic questionnaire (pre-test and post-test division items) to explore 30 pre-service elementary school teachers’ understandings of sharing interpretation. The word problem was as follows: “Four friends bought 1/4 of a kilogram of chocolate and shared it equally. How much chocolate did each person receive?” With the problem 1/4 ÷ 4, the pre-service elementary school teachers were not asked to calculate the problem. Instead, they were asked to write an expression that represented the problem without making any calculations, to write common incorrect responses, and to describe possible sources of the incorrect responses. Following this query, 29 of the 30 pre-service elementary school teachers provided correct expressions (i.e., 1/4 ÷ 4), and 19 were able to list common incorrect responses (i.e., 1/4 × 4) to the problem. However, they could not explain that the sources of
incorrect responses referred to the constraints (i.e., the divisor must be less than the dividend).

In 2002, Siebert raised the issue of whole number division and its extension to the division of fractions. He argued that the sharing interpretation of whole number division could not be extended directly to the division of fractions without any modifications. For example, from a sharing perspective, $6 \div 2$ can be thought of as partitioning six dollars between two people to determine the amount that each person receives. The following question may then be asked: “how many dollars should each person receive?” However, attempts to visualize fraction division problems by using a similar sharing perspective result in complexity. According to Siebert (2002), the reason is that “we can no longer write a story problem for $\frac{1}{2} \div \frac{1}{4}$, because how can $\frac{1}{2}$ of [a dollar] be shared with $\frac{1}{4}$ of a person?” (p. 251). This problem can be successfully interpreted using a different method of sharing. This solution involves two actions: sharing and duplicating. For example, the problem $\frac{1}{2} \div \frac{1}{4}$ can be thought of as follows: if $\frac{1}{4}$ of a group shares $\frac{1}{2}$, how much does the whole group share? Because each $\frac{1}{4}$ of the group receives $\frac{1}{2}$, the whole group (four $\frac{1}{4}$s) receives two portions (four $\frac{1}{2}$s). We duplicate $\frac{1}{2}$ four times to determine how much the whole group receives. The answer can be expressed as two dollars per group. Ball (1990) introduced this sharing division interpretation by representing the expression $100 \div \frac{1}{2}$ with the following example: “Imagine that you are packing apples in crates. You have 100 apples and find that they fill exactly half a crate. How many apples will it take to fill a crate?” (p. 452). In short, the following question should be asked in sharing situations with fraction division: “How much is the whole?”
This interpretation of the division problem uses proportional reasoning to determine the size of one group. Determining the size of one group is equivalent to determining the unit rate because the rate is expressed in *per unit* form (i.e., three miles per hour). This interpretation can be called *the determination of a unit rate*. However, it falls under the sharing interpretation because its goal is to determine the amount within the whole (i.e., how much the whole group receives). For example, Siebert (2002) argued that the goal is to determine how much of the dividend should be associated with one unit of the divisor, which is the same goal of the sharing interpretation of whole numbers (i.e., how much a single person receives).

Gregg and Gregg (2007) explored the unit rate interpretation of fraction division with pre-service teachers in mathematics methods courses. They explored problems using both whole numbers and fractions. One such problem was as follows: I have 2/3 of a whole cake. I want to divide it equally into 2 containers. How much cake will be in each container? Pre-service teachers drew a whole rectangle and divided it vertically into three parts. Next, they shaded two of the three parts to separate 2/3 of the cake. Then they cut the 2/3 horizontally into three equal parts and determined what fraction of the whole cake each of the three parts had. A pre-service teacher wrote: \[ \frac{2}{3} = \frac{4}{9} = \frac{2}{9} + \frac{2}{9}. \] Each container has \( \frac{2}{9} \) of the cake. Gregg and Gregg found that a few students had difficulty interpreting the problem as \( \frac{2}{3} \div 2 \).

The pre-service teachers moved toward solving problems involving fractions such as the following: I have 2/3 of a whole cake. It fills up exactly 1/4 of my container. How much cake will fit in 1 whole container? Many students applied repeated addition or multiplication. They explained that because 1/4 of the container had 2/3 of the cake, the
whole container had \(2/3 + 2/3 + 2/3 + 2/3\) or \(4 \times 2/3\). No division was involved in the pre-service teachers’ solutions. Thus, Gregg and Gregg worked with the pre-service teachers to connect the problem with division. They compared the problem with whole numbers to the problem with fractions as follows:

\[
\frac{2}{3} \text{ cake} \div 2 \text{ containers} = \text{the amount of cake per container}
\]

\[
\frac{2}{3} \text{ cake} \div 1/4 \text{ container} = \text{the amount of cake per container}
\]

In both problems, a certain amount of cake fits into a certain space, and the aim is to determine how much 1 container will contain.

2.4.2.2 Understanding the Quantitative Effects of an Operation on a Pair of Numbers Where Fractions Are Involved

The Answer Does Not Always Get Smaller When Dividing Or Larger When Multiplying

Tirosh (2000) discussed learners’ struggles with the sharing interpretation. Based on the literature concerning the mistakes learners make when dividing fractions, she explained that learners tend to apply the properties of operations with natural numbers directly to fractions and to interpret division initially by using the sharing interpretation. The sharing interpretation of division imposes three constraints on the operation of division. First, the divisor must be a whole number; second, the divisor must be less than the dividend; and finally, the quotient must be less than the dividend. Regarding the first constraint, the research indicates that providing students with unrealistic sharing in real-world situations while teaching fraction division reinforces the idea that the divisor must be a whole number. Thus, it becomes impossible for the students to accept the idea of dividing (i.e., sharing) two candies with 1/2 of a person. For the latter two constraints, the research has shown that relying on the sharing interpretation limits learners’
understandings of the effects of division operations on a pair of numbers. Some learners still believe that every division situation makes things smaller and that both the divisor and quotient must therefore be less than the dividend (i.e., $6 \div 2 = 3$). This belief affects learners’ abilities to learn fraction division because they compare the structures of such expressions with those of fraction division problems (i.e., $\frac{1}{2} \div \frac{1}{4} = 2$). Thus, consistently teaching the sharing interpretation of division with whole numbers in conjunction with fraction division may result in serious limitations in students’ abilities to perform fraction division problems.

Li and Smith (2007) asked pre-service middle school teachers how they would explain to their students the sharing situation in the following problem: $\frac{2}{3} \div 2 = \frac{1}{3}$. The divisor in this problem is a whole number. As mentioned earlier, one of the three constraints imposed by the sharing interpretation of division is that some learners always believe that the divisor must be a whole number. Although this problem did not violate this constraint, it violated the other two constraints, suggesting that both the divisor and quotient must be less than the dividend. Of the 46 pre-service middle school teachers, 46% were unable to provide complete explanations of the computation, whereas 22% explained that the computation is equivalent to multiplying by its reciprocal ($\frac{2}{3} \times \frac{1}{2}$) without giving an explanation of why it works; 26% provided pictorial representations demonstrating how to divide $\frac{2}{3}$ by 2 to obtain the answer ($\frac{1}{3}$).

Tirosh (2000) used a word problem in a diagnostic questionnaire (pre-test and post-test division items) to explore 30 pre-service elementary school teachers’ understandings of the sharing interpretation. The word problem was as follows: “Four friends bought $\frac{1}{4}$ kilogram of chocolate and shared it equally. How much chocolate did
each person receive?” The divisor in this word problem is a whole number. Again, the problem did not violate the constraint stating that the divisor must be a whole number, but it violated the other two constraints, which suggested that both the divisor and the quotient must be less than the dividend.

In the problem $\frac{1}{4} \div 4$, the pre-service elementary school teachers were not asked to calculate the answer to the problem. Instead, they were asked to write an expression representing the problem, to write common incorrect responses, and to describe possible sources of the incorrect responses. Following this query, 29 of the 30 pre-service elementary school teachers provided the correct expression (i.e., $\frac{1}{4} \div 4$), and 19 were able to list common incorrect responses (i.e., $\frac{1}{4} \times 4$); however, they could not explain that the sources of incorrect responses were related to the constraints of the sharing interpretation (i.e., the divisor must be less than the dividend). In short, pre-service teachers should understand that the answer does not always become smaller when dividing or larger when multiplying.

2.4.2.3 Real-world Applications

**Recognize real-world Situations for Fraction Division Operations**

Nillas (2003) pointed out that even though all ten elementary pre-service teachers solved the measurement problem correctly and demonstrated considerable understanding, “all of them did not recognize right away that this problem was a division problem” (p. 103). This lack of recognition applies to the inverse of multiplication problems as well. Only four elementary pre-service teachers recognized them as division problems.

**Pose (Design) Real-world Situations for Fraction Division Operations**

Simon (1993) investigated 33 pre-service elementary teachers’ abilities to write a
story problem for the expression $\frac{3}{4} \div \frac{1}{4}$. Twenty-three of the 33 were unable to develop an appropriate story problem; these twenty-three teachers generated sharing situations that restricted their abilities to present $\frac{3}{4} \div \frac{1}{4}$ in a proper story problem. Most of them thought in terms of sharing situations and could not reverse their answers once they were proven wrong. In contrast, the ten pre-service elementary teachers who were successful at this task generated measurement situations. Simon claimed: “the most accessible word problems are [measurement] ones because making groups of $\frac{1}{4}$ is more easily visualized than making 1/4 groups ($\frac{1}{4}$ of a group)” (p. 247). The study reinforced the notion that pre-service elementary teachers find it more difficult to think flexibly and consciously about fraction division while using the sharing interpretation than while applying the measurement interpretation.

2.4.2.4 Meaning and Mathematical Language Associated with Fraction Division Symbols

Ma (1999) examined American teachers' procedural and conceptual knowledge of fraction division and compared it with that of their Chinese peers. She found that sixty-five of the 72 Chinese teachers who participated in the study created more than 80 story problems addressing the problem $1 \frac{3}{4} \div \frac{1}{4}$. Twelve teachers were able to create more than one story and derive different meanings of the division symbol in a fraction division problem. In other words, they understood that the division symbol in a fraction division problem does not only mean splitting a quantity into equal quantities (i.e., sharing); rather, it could represent a measurement, the unit rate, or the inverse of a Cartesian product situation. In contrast, one of 21 American teachers was able to generate a conceptually appropriate representation of the meaning of fraction division.
2.4.2.5 Ability to Translate Easily Across Various Modes of Interpretation

Ball (1990) found that all 19 elementary and secondary pre-service teachers who participated in a study had trouble grasping the real-life meaning of fraction division. She investigated how these teachers related the number sentence $1 \frac{3}{4} \div \frac{1}{2}$ to a real-life situation. Seventeen of the subjects were able to calculate the problem correctly, but only five were able to provide appropriate real-life situations corresponding to the problem. Typically, the elementary and secondary pre-service teachers had difficulty with the expression $1 \frac{3}{4} \div \frac{1}{2}$ because they considered division in sharing terms only (i.e., as forming a certain number of equal parts). Ball indicated that the sharing interpretation “corresponds less easily to division with fractions than does the measurement interpretation of division” (p. 141). Reliance on the sharing interpretation made it difficult for the prospective teachers to make sense of and to interpret $1 \frac{3}{4} \div \frac{1}{2}$ appropriately. The study concluded that elementary and secondary pre-service teachers had significant difficulty with the meaning of fraction division. Their understandings were restricted to remembering rules and were weakly attached to other ideas about division.

2.4.2.6 Understanding the Relationships Between Fraction Division and other Operations

The Relationship Between Division and Subtraction

Revisiting the measurement problem ($3 \ 1/4 \div 3/4$) in Nillas’s study discussed earlier Angela not only demonstrated the measurement interpretation but also showed her understanding of the relationship between division and subtraction. She repeatedly subtracted $3/4$ from $3 \ 1/4$ until she arrived at the correct answer. That is, $3 \ 1/4 – 3/4 = 2 \ 1/2$, $2 \ 1/2 – 3/4 = 1 \ 3/4$, $1 \ 3/4 – 3/4 = 1$, $1 – 3/4 = 1/4$. She showed that division may be
viewed as repeated subtraction.

**The Relationship Between Division and Multiplication**

In reference to the inverse of multiplication interpretation in Nillas’s study, pre-service teachers showed evidence of understanding. After they set up the equation \((2 \frac{1}{2} \times \text{the number of students who prefer the afternoon sessions} = 55)\), they divided 55 by \(2\frac{1}{2}\) to determine the number of students who preferred the afternoon sessions (i.e., 22). In short, they demonstrated their understandings of the inverse relationship between division and multiplication.

**2.4.2.7 Ability to Compose and Decompose Numbers and to Use the Properties of Operations to Solve Fraction Division Problems**

**Commutative Law**

In Ma’s study (1999), Chinese teachers have argued: “dividing by a number is equivalent to multiplying by its reciprocal.”. This principle can be simplified using the commutative law. Ma explained this idea as follows:

We can use students’ knowledge to support the rule that dividing by a fraction is equivalent to multiplying by its reciprocal. They have learned the commutative law and how to remove parentheses. They have learned that a fraction is equivalent to the result of division; for example, \(1/2 = 1 \div 2\). Now, using this knowledge, we can rewrite the equation as follows:

\[
1 \frac{3}{4} \div \frac{1}{2} = 1 \frac{3}{4} \div (1 \div 2) \\
= 1 \frac{3}{4} \div 1 \times 2 \quad \text{after removing parentheses} \\
= 1 \frac{3}{4} \times 2 \div 1 \\
= 1 \frac{3}{4} \times (2 \div 1) \\
= 1 \frac{3}{4} \times 2
\]

(p. 59)

In this case, Chinese teachers are elaborating on how algorithms make sense by using the commutative law to convert the operation with fractions into an operation with whole numbers.
**Distributive Law**

They argued that the distributive law (i.e., \(a(b+c)=ab+ac\)) is another means of computing \(1\frac{3}{4} \div \frac{1}{2}\). This is done by converting \(1\frac{3}{4}\) from its actual form (i.e., a mixed number) into an improper fraction \((1 + \frac{3}{4})\). They then followed two steps: 1) divide 1 and \(\frac{3}{4}\) by \(\frac{1}{2}\) and 2) add the two quotients together to obtain the final answer.

The solution Ma (1999) derives is as follows:

\[
1 \frac{3}{4} \div \frac{1}{2} = (1 + \frac{3}{4}) \div \frac{1}{2} \\
= (1 + \frac{3}{4}) \times \frac{2}{1} \\
= (1 \times 2) + (\frac{3}{4} \times 2) \\
= 2 + 1 \frac{1}{2} \\
= 3 \frac{1}{2}
\]

or

\[
1 \frac{3}{4} \div \frac{1}{2} = (1 + \frac{3}{4}) \div \frac{1}{2} \\
= (\frac{1}{2} + \frac{3}{4} \div \frac{1}{2}) \\
= 2 + 1 \frac{1}{2} \\
= 3 \frac{1}{2}
\]

(p.62)

2.4.2.8 Ability to Use a Variety of Computational Approaches for Fraction Division and to Provide a Conceptually Sound Explanation for Why They Work

The 72 Chinese teachers in Ma’s study employed the standard algorithm invert-and-multiply strategy (IM) and proposed two other strategies:

**Decimal Division Algorithms**

More than one third of the 72 Chinese teachers demonstrated a solid procedural knowledge of fraction division by using decimals as an alternative way of computing \(1 \frac{3}{4} \div \frac{1}{2}\). They converted the fractions into decimals and solved the problem: \((1.75 \div 0.5 = 3.5)\).

**Maintain the Value of a Quotient Rule**

In contrast, nine of the 21 American teachers were able to reach a complete answer for the problem \(1 \frac{3}{4} \div \frac{1}{2}\), though they were unable to explain why IM worked.
Although IM is the most widely used method for solving fraction division problems, it is also the least understood. The Texas Mathematics Teacher (2005) provides a series of examples explaining the logic behind the IM algorithm. They utilize the measurement interpretation of fraction division with Cuisenaire Rods (i.e., a set of colored rods whose sizes range from 1 to 10 centimeters) to construct a framework of reasoning behind the IM algorithm. For example, the comparison between the whole unit and the tenth unit appears as:

<table>
<thead>
<tr>
<th>1 Whole Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/10</td>
</tr>
</tbody>
</table>

Statement in words: How many groups of 1/10 are in one whole?

Statement in numbers: $1 \div \frac{1}{10} = 10$.

In the case of $2 \div \frac{1}{10}$, we have two whole units:

<table>
<thead>
<tr>
<th>1 Whole Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1 Whole Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/10</td>
</tr>
</tbody>
</table>

$(1 \div \frac{1}{10}) + (1 \div \frac{1}{10}) = 10 + 10.$

$2 \div \frac{1}{10} = 2 \times 10.$

They then progress to more complex examples, such as the following:

$3 \div \frac{2}{3}$.

Statement in words: How many groups of $\frac{2}{3}$ are in three whole units?
Starting with one whole unit (1 ÷ 2/3), the representation below shows that one whole unit contains 1 complete unit of 2/3 in addition to 1/2 of it (or 3/2 of 2/3).

<table>
<thead>
<tr>
<th>1 Whole Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>1/3</td>
</tr>
</tbody>
</table>

1 ÷ 2/3 = 3/2

3 ÷ 2/3 = (1 ÷ 2/3) + (1 ÷ 2/3) + (1 ÷ 2/3)

3 ÷ 2/3 = 2/3 + 2/3 + 2/3 = 3 × 2/3

**Common Denominator Algorithm**

Flores, Turner, and Bachmann (2005) described how two teachers, Elizabeth and Carolyn, posed problems to develop their own conceptual understandings of fraction division in terms that would also be meaningful for their students. During a three-hour session in a graduate methods course, the two teachers used a fraction kit while posing fraction division problems. Elizabeth posed sixty-four of her own problems, whereas Carolyn posed forty-five problems. After approximately twenty problems, Carolyn worked with the problem 2/3 ÷ 1/4. She converted the dividend and divisor into equivalent fractions with a common denominator: 8/12 ÷ 3/12. Next, she crossed out the common denominator and wrote 8 ÷ 3 and 2 2/3. She then realized that when dividing fractions with the same denominator, the denominator is irrelevant and does not become part of the answer.

### 2.5 Summary

This chapter provided an overview of: 1) teacher subject matter knowledge, 2) self-assessment (e.g. metacognition), 3) the Knowledge Survey Tool, and 4) a detailed
framework for fraction division proficiency. Chapter Three will present the design of this study and methods for collecting and analyzing data.
Chapter 3

Methodology

3.1 Sample

The sample consisted of 13 pre-service teachers at a Midwestern University. The subjects selected for this study met two predetermined criteria. First, a subject had to be majoring in a licensure program that leads to certification in middle mathematics teaching. Two of the 13 subjects were working toward certification for grades 7-12 and 11 were working toward certification for grades 4-9. Second, they must have studied fraction division in a mathematics methods course during their education program.

Two sampling methods were used to select this sample: criterion sampling and convenience sampling. Criterion sampling involves selecting subjects who meet some predetermined criterion of importance. Convenience sampling involves selecting subjects that are available and willing to participate in the research study. Two faculty members from the University identified potential subjects for the study. I contacted these potential subjects and discussed their willingness to participate in the study.

3.2 Instrument
The interview instrument used for data collection was a knowledge survey. The process of creating the knowledge survey for this study underwent four stages:

1. Defining a framework for fraction division proficiency (see Figure 2-4).
2. Developing fraction division problems for each of the framework’s eight components.
3. Classifying the developed fraction division problems in accordance with the 6 Bloom’s Taxonomy levels for each of the framework’s eight components.
4. Selecting fraction division problems to create the instrument based on two criteria: 1) select an equal number of problems per Bloom level, and 2) select at least one problem from each component.

The final knowledge survey included 12 fraction division problems (See Table 3.1) distributed evenly across the 6 Bloom levels (i.e., two problems per Bloom level).

Twelve problems were identified with the following considerations in mind:

1. Time constraints: A pilot study provided evidence that rating and solving more than 12 fraction division problems within the allocated time for the study (i.e., 1 hour) can be very challenging.
2. Primary purpose of study: This study aimed to investigate pre-service teachers’ self-assessment of fraction division for all Bloom cognitive levels collectively as well as at each level individually. Thus, having an even distribution of problems for each Bloom level was pre-determined in order to examine the influence on self-assessment.
Table 3.1: Fraction division knowledge survey.

<table>
<thead>
<tr>
<th>Bloom Level</th>
<th>Fraction Division Knowledge Survey Problems</th>
<th>Fraction Division Proficiency Component(s) Involved</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge</td>
<td>1. Write a definition for the term fraction. 2. Write a definition for the term division.</td>
<td>Understand the meaning and models of fraction division (#1). Meanings and mathematical language associated with fraction division symbols (#4).</td>
</tr>
<tr>
<td>Comprehension</td>
<td>3. You are making up word problems for your students. Which of the following word problem(s) can be used to represent $3 \div 1/4$? (Circle all that apply.)</td>
<td>Understand the meaning and models of fraction division (#1). Recognize real-world situations for fraction division operation (#3). Understand that a number of different meanings can be derived from the same fraction division symbol (#4). Understand the relationship between division and multiplication as well as between division and subtraction (#6).</td>
</tr>
<tr>
<td></td>
<td>A. 3 cups of orange juice fill up exactly $1/4$ of a container. How many cups will fill the whole container?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B. 3 friends each have $1/4$ of a cookie. How many cookies would they have if they put them all together?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C. In a survey, 3 students said they prefer pizza. These three students represent $1/4$ of the number of students who prefer a hamburger. How many prefer the hamburger?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>D. How many lengths of $1/4$ yard can be cut from $3$ yards of cloth?</td>
<td></td>
</tr>
<tr>
<td>Application</td>
<td>4. Explain in your own words how to find the length of a rectangle with an area of $2 \ 3/5$ square meters and a width of $2/3$ meters.</td>
<td>Ability to compose and decompose numbers and to use the properties of operations to solve fraction division problems (#7). Ability to use a variety of computational approaches for</td>
</tr>
<tr>
<td></td>
<td>$2/3$</td>
<td>$2 \ 3/5 \ m^2$</td>
</tr>
</tbody>
</table>
## Analysis

**Examine and break information into parts.**

<table>
<thead>
<tr>
<th>Task</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.</td>
<td>One of Mr. Smith’s students created an incorrect real-world problem to represent $5 \div 1/2$ as follows: Sara has 5 pizzas and she wants to give half of them to her friend. How much pizza will her friend get? Examine the student’s misconception.</td>
</tr>
<tr>
<td>8.</td>
<td>One of Mr. Smith’s students solved the problem $3/4 \div 1/4$ by dividing the numerators and denominators: $\frac{3}{4} \div \frac{1}{4} = \frac{3}{1} \div \frac{1}{4}$. He got 3, which is the correct answer. Explain if his method is always correct.</td>
</tr>
</tbody>
</table>

## Synthesis

**Compile information together in a different way.**

<table>
<thead>
<tr>
<th>Task</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.</td>
<td>Create a story problem for $5/8 \div 3$.</td>
</tr>
</tbody>
</table>

## Evaluation

**Making judgments about information, validity of ideas or quality of work based on a set of criteria.**

<table>
<thead>
<tr>
<th>Task</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.</td>
<td>Discuss the validity of following statements: The answer does not always get smaller when I divide.</td>
</tr>
<tr>
<td>12.</td>
<td>Dividing by a number is always equivalent to multiplying by its reciprocal.</td>
</tr>
</tbody>
</table>

In preparation for formal data collection, I conducted a pilot study. The pilot study was conducted with two pre-service teachers to ensure that the items were clearly
written and the time needed to complete data collection with each individual was reasonable and appropriate. During the pilot study, I presented each pre-service teacher with the knowledge survey individually. I asked them to rate their confidence level for solving each of the 12 fraction division problems on the knowledge survey, discuss why they selected a particular confidence level for each problem, and lastly solve the 12 fraction division problems. After the pilot study session was finished, I discussed with each pre-service teacher the clarity of the items and the time allocated for the session (i.e., one hour). The pilot study revealed that the items were clearly written and the time limit for the session was reasonable to rate and solve 12 items.

The formal data collection for this research took place in a college of education. I met individually with each participant in a quiet room. The research session was conducted in three stages. During the first stage, participants were provided with the knowledge survey and asked not to solve any of the fraction division problems. Instead, the participant was asked to self-assess his or her confidence level in solving each one of the twelve fraction division problems using a rating scale. The rating scale ranged from 0 to 3 (0 = I cannot begin to answer this problem; 1 = I can partially answer this problem; 2 = I can answer most of the problem; and 3 = I can answer the entire problem with full confidence). The participant’s rating scores, referred to as a prediction score, were recorded on a sheet answer (see Appendix A). During the rating process, the participant was asked not to use written mathematics. When the participant was rating their confidence level for solving each problem, they were not allowed to perform written calculations.

During the second stage, a Think Aloud Protocol (TAP) was applied. That is, the
participant was asked to think aloud and verbalize aloud their thought processes for how they rated each problem. If the participant’s thoughts did not reveal insight into factors that influenced how they rated their confidence, the participant was asked to answer two specific questions about the rating selections: Why did you pick this particular rating of your confidence level to solve this problem? and Can you discuss specific aspects of the problem that led to the rating you chose? In the final stage, the participant was asked to solve the twelve fraction division problems he or she originally rated.

Each research session lasted approximately one hour. For data analysis purposes, the research session was videotaped to document what took place. The video camera was aimed only on the participant’s written work. The video camera captured written work and the exchange between the researcher and the participant.

3.4 Data Analysis

Data were collected from the knowledge survey results, actual performance results (i.e., solutions to the fraction division problems), TAP recordings, and interview recordings. Analyzing the data took place in two stages. The first stage involved examining self-assessment accuracy. The second stage involved examining factors that influenced self-assessment accuracy.

Self-assessment Accuracy

The analysis of self-assessment accuracy involved examining (a) how accurate the participants were in identifying what did and did not know about fraction division and (b) how accurate they were in using this to predict their actual performance.

Pre-service Teachers’ Self-knowledge Accuracy

To examine pre-service teachers’ accuracy in identifying what they did and did
not know, I reviewed the data from the TAP recordings, interviews recordings, and written solutions to the fraction division problems. In the following three examples, I address how I examined the pre-service teachers’ accuracy in identifying what they did and did not know about fraction division. The first two examples discuss self-assessment accuracy. The last example discusses self-assessment inaccuracy.

In the first example, a pre-service teacher indicated, as he was thinking aloud or responding to interview questions for Problem 2 in the knowledge survey that he was able to provide a definition for division. He defined division in the following way: the process of finding how many times one number or quantity is contained in another. This is a correct answer. Thus, I could verify that he was accurate in identifying what he knew.

In a second example, a pre-service teacher indicated while responding to an interview question for Problem 10 in the knowledge survey, that she could not develop a story problem to represent a fraction number division sentence (i.e., \( \frac{3}{4} \div \frac{1}{3} \)). In this case, I had to check her written answer since there was no verbal answer. In her written answer, she did not provide any story problem. Thus, she was accurate in identifying what she did not know.

In a third example, to discuss the validity of the statement of Problem 11: the quotient does not always become smaller when you divide, a pre-service teacher’s answer was: “if you divide 12 by 3 then the quotient is always going to be smaller than 12. I do not think that is a valid statement.” Clearly, she was unaware that her answer was incorrect.

Following the process discussed above, I examined the self-knowledge accuracy for each pre-service teacher of each of the 12 fraction division problems. I created a table
with two columns. The first column was labeled correct self-knowledge. In this column, I recorded the number of situations where the pre-service teachers accurately identified what they knew for each problem. The second was labeled incorrect self-knowledge. In this column, I recorded the number of situations where the pre-service teachers inaccurately identified what he/she knew on the fraction division problems. At the end of the table, I recorded the sum of the number of situations for both accurate self-knowledge and inaccurate self-knowledge.

**Pre-service Teachers’ Actual Performance Prediction Accuracy**

To examine pre-service teacher’s accuracy in predicting their performance, I compared the rating scores (i.e., predicted scores) on the knowledge survey with the actual performance results (i.e., actual performance scores) when solving the 12 problems. Before discussing how I compared the predicted scores to the actual performance scores, I will discuss how I obtained each one.

On the knowledge survey, participants recorded their predicted scores for each fraction division problem using a scale that ranged from 0 to 3 (0 = I cannot begin to answer this problem; 1 = I can partially answer this problem; 2 = I can answer most of the problem; and 3 = I can answer the entire problem with full confidence). All the predicted scores were collected and organized in a chart for comparing predicted scores to actual performance scores.

For the actual performance, I evaluated the participants’ written solutions to the problems for correctness using an actual performance scale (see Appendix B). The actual performance scale ranged from 0 to 3 (0 = no answer; 1 = partial answer; 2 = almost complete answer; and 3 = complete answer). The actual performance scale was
developed to be in alignment with the rating scale (see Table 3.2).

I compared the predicted scores with the actual performance scores using two following approaches suggested in the literature:

1. the dichotomous calibration (accurate or inaccurate prediction) and,
2. the continuous calibration (how far pre-service teachers were from accurate prediction-measured by taking the absolute difference between predicted and actual performance scores)

Table 3.2: Comparison between knowledge survey scale and actual performance scale.

<table>
<thead>
<tr>
<th>Knowledge</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Survey Scale</td>
<td>I cannot begin to answer this problem</td>
<td>I can partially answer this problem</td>
<td>I can answer most of the problem</td>
<td>I can answer the entire problem with full confidence</td>
</tr>
<tr>
<td>Actual</td>
<td>No Answer</td>
<td>Partial Answer</td>
<td>Almost Complete Answer</td>
<td>Complete Answer</td>
</tr>
<tr>
<td>Performance Scale</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To be confident with my scoring of results, three of my dissertation committee members scored a random sample from the fraction division problems using the same rubric. Each faculty member scored 3 pre-service teacher’s solutions on 2 different fraction division problems. In total, they scored 18 solutions. My scoring and the committee members’ scoring produced agreement on 14 out the 18 solutions. That is, my scoring matched theirs on these 14 solutions. The discrepancy on the remaining 4 solutions is shown in Table 3.3.

The difference between my score and the committee members’ scores was one point on two problems and two points on the other two problems. With Problem 1, the scoring disagreement did not change the pre-service teachers’ accuracy status. The pre-service teacher scored less than predicted based on both scorings, which indicates he
Table 3.3: Scoring discrepancy.

<table>
<thead>
<tr>
<th>Problem</th>
<th>The pre-service teacher’s predicted score</th>
<th>The researcher’s scoring</th>
<th>The committee member’s scoring</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

overestimated his ability to solve the problem. With the remaining 3 problems, the accuracy status changed from underestimated to accurate, accurate to overestimated, and overestimated to underestimated respectfully.

The Dichotomous Calibration Approach

Each participant rated his or her confidence level for solving 12 fraction division problems. Then, they solved the same problems. The 12 problems were organized as 2 problems for each one of the 6 Bloom levels. In total, the 13 participants provided 156 responses to the 12 problems, 26 responses per Bloom level. The result of the matches and mismatches between the participants’ predicted scores (rating scores) and their actual performance scores (actual performance results) was summarized to analyze whether the participants accurately estimated, underestimated, or, overestimated their prediction. The dichotomous calibration approach of analysis was done to examine prediction accuracy in general (i.e., all the 6 Bloom levels together) as well as in particular (i.e., each Bloom level separately).

In order to determine prediction accuracy, I compared each participant’s predicted score to the actual performance score. I did this for each fraction division problem. I recorded each comparison as a match (100% accuracy) or a mismatch (0% accuracy). A match indicated that the participant predicted his or her confidence level for solving the problem accurately. For example, if the participant chose a confidence score of 2 for a
certain problem on the knowledge survey and then scored 2 when solving the problem, this was recorded as a match. A mismatch, on the other hand, indicated that the participant failed to predict his or her confidence level for solving the problem accurately. Any mismatch was recorded as either underestimation or overestimation.

Underestimation was recorded if the participant’s actual performance score was greater than his/her predicted score. Namely, the participant failed to predict his or her confidence level for solving the problem accurately but did better than predicted. Overestimation was recorded if the participant’s actual performance score was less than his/her predicted score. That is, the participant failed to predict his or her confidence level for solving the problem accurately by performing worse than predicted.

**The Continuous Calibration Approach**

The dichotomous calibration is measured as either 100% accuracy or 0% accuracy; however, the continuous calibration is measured differently. Continuous calibration is measured as the difference between predicted and actual performance (absolute accuracy) (Hacker, Bol, and Bahbahani, 2008; Schraw, 2009; Bol, Hacker, Walck, & Nunnery, 2102).

The continuous calibration is typically measured at the global and local levels. At the global level, one makes a single judgment about the overall performance on a test. Global calibration is measured by taking the absolute difference between the overall predicted test performance and the overall actual test performance. The smaller the difference between predicted performance and actual performance, the more calibrated the learner is. Calibration scores can be confusing in some cases. In order to make calibration scores intuitively clear, Hacker, Bol, and Bahbahani (2008) provided a
They used this formula to measure global calibration by asking students to predict how many of test questions they would get correct. For example, a test includes 15 questions in total. As a student predicted he would get 11 questions correct, but actually got 14 correct. Then the difference is 11 (predicted) minus 14 (actual) equals –3. The negative sign “–” indicates that the student was underconfident. Conversely, if the sign is positive this indicates the student was overconfident. However, the focus is on the absolute difference between the predicted performance and the actual performance, which is 3. As a result, the student would have a calibration score of 80%. This student is considered to have a high ability to predict his/her actual performance.

At the local level, one makes a single judgment about the performance on a single question on a test. Local calibration is measured by taking absolute difference between the predicted performance and the actual performance of a single question on a test. The same formula is used to measure local calibration, however, the denominator of the formula (the total items) would be preplaced by: the maximum points the question is worth:

$$1 - \left( \frac{|Prediction - Performance|}{Total\ of\ Items} \right) \times 100$$

For example, a question is worth 3 points in total. If student predicted score of 3 but actually scored 2, the difference is 3 (predicted) minus 2 (actual) equals 1. The absolute difference is 1. The calibration score is calculated as $(1 - 1/3)$ times 100. As a result, the student would have a calibration score of 67%. Another student predicted a
score of 3 on the same question but, in fact, scored 1. His/her calibration score would be 34%. The student who earned the 67% is the better calibrated of the two. In other words, this student has higher ability to predict his/her actual performance than the student who errand the 34%. The higher calibration score the higher ability to predict actual performance.

In this study, I measured pre-service teachers’ calibration at the local level. I measured each pre-service teacher’s score calibration for each fraction division problem and added up all calibration scores to get one value. To illustrate, for each pre-service teacher, I calculated the absolute difference between their predicted score and actual performance for each one of the 12 fraction division problems. I ended up with 12 absolute difference values. Next, I added up the total for these values and inserted it into the formula to determine how calibrated the pre-service teachers was. In Table 3.4, I addressed how I calculated calibration for each pre-service.

Table 3.4: Pre-service teacher absolute difference accuracy example.

<table>
<thead>
<tr>
<th>Fraction Division Problem</th>
<th>Absolute Difference (Predicted Score-Actual Score)</th>
<th>Total Points of the Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3-3 = 0</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>3-3 = 0</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3-1 = 2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>3-3 = 0</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>2-0 = 2</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>2-3 = 1</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>3-3 = 0</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>3-3 = 0</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>2-3 = 1</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>2-3 = 1</td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>2-1 = 1</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>3-0 = 3</td>
<td>3</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>11</strong></td>
<td><strong>36</strong></td>
</tr>
</tbody>
</table>
This pre-service teacher has a local calibration score of \((1 - 11/36) \times 100 = 69.5\%\).

At the end of these calculations, the mean was calculated for all pre-service teachers.

**Factors Influencing Self-assessment Accuracy**

This part of the analysis examined factors that influenced self-knowledge and actual performance prediction accuracy within each cognitive level of Bloom’s Taxonomy. I used Glaser and Strauss’s (1965) constant comparison analysis. It is a method of analyzing data in order to develop a grounded theory. This method aims to understand a phenomenon and generate a theory that explains how the phenomenon works in contrast to applying an existing theory to the phenomenon to be studied. Constant comparison analysis process involves collecting data, coding, grouping codes, revisiting data for new codes, and creating a theory.

The primary data sources were TAP and interviews data. After transcribing the entire data, I repeatedly searched the data for factors that influenced pre-service teachers’ self-knowledge. I examined each pre-service teacher’s statement to determine what helped pre-service teachers identify what they did and did not know for each fraction division problem. Factors were labeled and analyzed in light of the full corpus of data.

To search for factors that influenced pre-service teachers’ actual performance prediction accuracy, I broke the transcript into six sections, one section for each Bloom level. For example, the first section was for Bloom’s Knowledge level (Problem 1 and Problem 2). This section included a collection of the 13 pre-service teachers’ statements on why they selected a particular confidence level for Problem 1 and Problem 2. I organized this collection of statements in a table with three columns. The first column was labeled accuracy. It included statements from the pre-service teachers whose
predicted score matched their actual performance score. The second column was labeled underestimation. It included statements from the pre-service teachers whose predicted score was less than their actual performance score. The third column was labeled overestimation. It included statements from the pre-service teachers whose predicted score was greater than their actual performance score. This process was done for each Boom level. Once I organized all the pre-service teachers’ statements as described, I read through the data multiple times. I began to identify factors that influenced accuracy, underestimation, and overestimation for each Bloom level.

In total, I coded 156 statements made by the pre-service teachers. I used two coding types: In Vivo coding which entails using the interviewee’s exact words to name codes (Strauss, 1987) and descriptive coding (i.e., academic term). Table 3.5 shows examples of the coding process. The coding process was executed in two stages. In the first stage, I used the pre-service teachers’ own language for the initial coding. That is, I labeled each meaningful phrase found in the pre-service teachers’ statements and

<table>
<thead>
<tr>
<th>Pre-service Teacher’s Statement</th>
<th>Initial Code In Vivo Coding</th>
<th>Final Code Descriptive Coding</th>
</tr>
</thead>
<tbody>
<tr>
<td>I am not positive about the distributive law. ¹ I do not know how to solve the problem using the distributive law… ² But I know how to solve it with the invert and multiply rule.</td>
<td>¹“I do not know how to solve the problem using the distributive law” ²“I know how to solve it the invert and multiply rule”</td>
<td>Knowledge base</td>
</tr>
<tr>
<td>It is easy to define division…There are several definitions. ² Division means to break up a whole into equal parts.</td>
<td>¹“Easy to define division” ²“Break up a whole into equal parts”</td>
<td>Item difficulty</td>
</tr>
</tbody>
</table>
assigned it a code. Since each statement usually included more than one phrase, two codes, at least, were assigned for each statement. In the second stage, I combined all the In Vivo codes in each statement into one descriptive code.

At one point, combining the In Vivo codes into one descriptive code was challenging. Overlap occurred between two close descriptive codes. In 82 statements, I assigned both descriptive codes—namely, item difficulty and knowledge base—for each statement. However, after looking carefully into the pre-service teachers’ statements, I found specific common characteristics among the statements. These characteristics helped develop a clear definition for each descriptive code. As a result, I selected one code over the other.

In 21 statements, I found that the pre-service teachers briefly discussed the difficulty level of the problem and focused more on their knowledge base. In other words, they indicated that they were unable to solve the problem using the assigned method, although they could discuss alternative approaches to solve it and applied one of these approaches. Accordingly, I selected the knowledge base code for these statements.

In 61 statements, I found that the pre-service teachers discussed the difficulty level of the problem in depth and focused less on their knowledge base. They indicated that the problem was easy to solve and discussed why. Next, they provided a solution. In cases where the problem was difficult, the pre-service teachers indicated that they could not solve the problem and discussed why they were unable to provide a solution. Accordingly, I selected the item difficulty code for these statements.

As result, I only selected one descriptive code for each pre-service teacher’s statement, which resulted in 156 codes (see Table 3.6). The table includes a list of the
codes generated from the data as well as their definition. These factors were analyzed in light of all research data. The analysis included examining the factors and their relationships as the basis for arriving at a theoretical understanding of pre-service teachers’ self-assessment of fraction division.

Table 3.6: Definitions and frequency of factors influencing actual performance prediction accuracy.

<table>
<thead>
<tr>
<th>Code</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item Difficulty</td>
<td></td>
</tr>
<tr>
<td>The pre-service teachers indicated that the problem was easy to solve and provided a solution.</td>
<td></td>
</tr>
<tr>
<td>The pre-service teachers indicated that the problem was difficult to solve and did not provide a solution.</td>
<td>61</td>
</tr>
<tr>
<td>Problem Solving Skills</td>
<td></td>
</tr>
<tr>
<td>The pre-service teachers applied problem-solving techniques to solve the problem.</td>
<td>27</td>
</tr>
<tr>
<td>Knowledge Base</td>
<td></td>
</tr>
<tr>
<td>The pre-service teachers indicated that they were unable to solve the problem using the assigned method. Instead, they solved it using a different method.</td>
<td>21</td>
</tr>
<tr>
<td>Cautiousness</td>
<td></td>
</tr>
<tr>
<td>The pre-service teachers had the knowledge to solve the problem but they conservatively underestimated their ability.</td>
<td>21</td>
</tr>
<tr>
<td>No Response</td>
<td></td>
</tr>
<tr>
<td>The pre-service teachers did not discuss any areas of strength or weakness on the problem.</td>
<td>17</td>
</tr>
<tr>
<td>Incorrect Self-knowledge</td>
<td></td>
</tr>
<tr>
<td>The pre-service teachers believed they knew how to solve the problem but in fact did not.</td>
<td>9</td>
</tr>
<tr>
<td>Total</td>
<td></td>
</tr>
<tr>
<td></td>
<td>156</td>
</tr>
</tbody>
</table>
Chapter 4

Results

In this section, I report results from the data analysis. The section is organized according the two research questions. The first question is: How accurate are pre-service teachers in self-assessing their own understandings of fraction division? The results for this question will be reported under two sub-sections: (a) pre-service teachers’ self-knowledge accuracy (b) pre-service teachers’ actual performance prediction accuracy. The second question is: What types of successes and challenges do pre-service teachers encounter as they self-assess their own understandings of fraction division? The results for this question are reported in two sub-sections: (a) factors influencing pre-teachers’ self-knowledge: (b) factors influencing actual performance prediction accuracy.

4.1 Research Question 1

4.1.1 Pre-service Teachers’ Self-Knowledge Accuracy

In total, pre-service teachers provided 156 responses to the 12 fraction division problems when they completed the knowledge survey. Pre-service teachers accurately identified what they did and did not know in 130 out of 156 responses to the 12 fraction division problems (see Table 4.1). According to the TAP and interview recording, pre-
service teachers did not perform mental arithmetic. This occurred only in 17 out of 156 responses.

Without mental arithmetic, pre-service teachers did not discuss any areas of strength or weakness on certain fraction division problems. Instead, they said they needed more time or paper and pencil to solve a problem. Thus, it was not possible to examine what they knew or did not know about fraction division in relation to that problem.

Table 4.1: Frequency of pre-service teachers’ self-knowledge accuracy.

<table>
<thead>
<tr>
<th>Total solutions to the 12 fraction division problems</th>
<th>156</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response</td>
<td>139</td>
</tr>
<tr>
<td>Accurate Self-knowledge</td>
<td>130</td>
</tr>
<tr>
<td>Inaccurate Self-knowledge</td>
<td>9</td>
</tr>
<tr>
<td>No Response</td>
<td>17</td>
</tr>
</tbody>
</table>

With mental arithmetic, the pre-service teachers were able to discuss what they knew and could do. This occurred in the remaining 139 out of 156 responses. Pre-service teachers who performed arithmetic mathematics accurately identified what they knew and did not know in 130 responses (i.e., accurate self-knowledge). In the remaining 9 responses, pre-service teachers believed they were accurate in identifying what they knew and did not know but in reality they were not (i.e., inaccurate self-knowledge). Ultimately, pre-service teachers correctly identified what they knew and did not know in 130 of the 156 responses.

4.1.2 Pre-service Teachers’ Actual Performance Accuracy

To answer the first research question, I used the two approaches suggested by the literature: the dichotomous calibration (accurate or inaccurate prediction) and the
continuous calibration (prediction accuracy measured by taking the absolute difference between predicted and actual performance score).

4.1.2.1 The Dichotomous Calibration Approach

Each one of the 13 pre-service teachers predicted their confidence level in solving each one of the 12 fraction division problems and then solved these problems. In total, pre-service teachers made 156 predictions. In order to examine prediction accuracy, I first performed item-by-item level calibration to determine whether a prediction was accurate or inaccurate (a dichotomous prediction). In other words, the predicted score was compared to the actual performance score of each problem to determine if there was a match (100% accuracy) or a mismatch (0% accuracy). For example, if a pre-service teacher chose a confidence score of 2 on a certain problem on the knowledge survey and actually scored 2, then this was a match and his accuracy was 100%. Conversely, if he chose a confidence score of 2 on another problem and actually scored 1 or 3 on that problem, then this was a mismatch and his accuracy was 0%.

Table 4.2 shows detailed data of the accurate and inaccurate predictions each pre-service teacher made on the 12 fraction division problems. This data are summarized later in Table 4.3 to examine prediction accuracy in general (all Bloom levels) as well as in Table 4.4 to examine prediction accuracy for each Bloom level in particular. In Table 4.2 the comparison between the predicted score (i.e., rating score) and the actual performance score of each problem is recorded in the format: (predicted score, actual performance score). The comparison (2,2), for example, denotes a match while (1,2) indicates a mismatch. Matches are labeled as √. Mismatches are labeled either as underestimation (U) or overestimation (O). Underestimation means that pre-service teachers failed to
Table 4.2: Pre-service teachers’ predicted and actual score by item and Bloom level.

<table>
<thead>
<tr>
<th>Participant</th>
<th>Problem</th>
<th>Knowledge Level</th>
<th>Comprehension Level</th>
<th>Application Level</th>
<th>Analysis Level</th>
<th>Synthesis Level</th>
<th>Evaluation Level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>(\checkmark)</td>
<td>(3,3)</td>
<td>(\checkmark)</td>
<td>(3,3)</td>
<td>(\checkmark)</td>
<td>(3,3)</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>(\checkmark)</td>
<td>(3,3)</td>
<td>(\checkmark)</td>
<td>(3,3)</td>
<td>(\checkmark)</td>
<td>(3,3)</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>(\checkmark)</td>
<td>(2,1)</td>
<td>(\checkmark)</td>
<td>(1,1)</td>
<td>(\checkmark)</td>
<td>(1,1)</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>(\checkmark)</td>
<td>(2,3)</td>
<td>(\checkmark)</td>
<td>(1,3)</td>
<td>(\checkmark)</td>
<td>(1,1)</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>(\checkmark)</td>
<td>(2,3)</td>
<td>(\checkmark)</td>
<td>(3,3)</td>
<td>(\checkmark)</td>
<td>(1,1)</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>(\checkmark)</td>
<td>(3,2)</td>
<td>(\checkmark)</td>
<td>(2,2)</td>
<td>(\checkmark)</td>
<td>(1,1)</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>(\checkmark)</td>
<td>(3,3)</td>
<td>(\checkmark)</td>
<td>(3,3)</td>
<td>(\checkmark)</td>
<td>(1,1)</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>(\checkmark)</td>
<td>(2,3)</td>
<td>(\checkmark)</td>
<td>(3,3)</td>
<td>(\checkmark)</td>
<td>(1,1)</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>(\checkmark)</td>
<td>(3,3)</td>
<td>(\checkmark)</td>
<td>(3,3)</td>
<td>(\checkmark)</td>
<td>(1,1)</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>(\checkmark)</td>
<td>(3,3)</td>
<td>(\checkmark)</td>
<td>(3,3)</td>
<td>(\checkmark)</td>
<td>(1,1)</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>(\checkmark)</td>
<td>(2,2)</td>
<td>(\checkmark)</td>
<td>(3,3)</td>
<td>(\checkmark)</td>
<td>(1,1)</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>(\checkmark)</td>
<td>(3,3)</td>
<td>(\checkmark)</td>
<td>(3,3)</td>
<td>(\checkmark)</td>
<td>(1,1)</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>(\checkmark)</td>
<td>(2,2)</td>
<td>(\checkmark)</td>
<td>(2,3)</td>
<td>(\checkmark)</td>
<td>(1,1)</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>(\checkmark),</td>
<td>(3,3)</td>
<td>(\checkmark),</td>
<td>(3,3)</td>
<td>(\checkmark),</td>
<td>(3,3)</td>
</tr>
</tbody>
</table>

Total (\(\checkmark\),U,O) \(\checkmark\), (9,4,0) (9,3,1) (4,3,6) (9,4,0) (4,2,7) (9,4,0) (8,1,4) (6,5,2) (5,5,3) (8,4,1) (9,1,3) (8,0,5)
predict their confidence level for solving the problem correctly, however, they performed better than predicted. For example, a pre-service teacher chose a confidence level of 1, but in fact scored 2 or 3. Overestimation means that pre-service teachers failed to predict their confidence level for solving the problem correctly by scoring worse than predicted. For example, a pre-service teacher chose a confidence level of 3 but in fact scored 2, 1, or zero.

Out of 156 predictions from the knowledge survey, the results in Table 4.3 show that pre-service teachers made 88 accurate predictions when the rating score from the knowledge survey was compared to their actual performance score after solving the problems. They scored an average accuracy rate of 56.4% and 68 inaccurate predictions scoring an average inaccuracy rate of 43.6%. The inaccurate 68 predictions (43.6%) were distributed as underestimation with 36 predictions (23.1%) and overestimation with 32 predictions (20.5%).

Table 4.3: Frequency of pre-service teachers’ accurate and inaccurate predictions for problems 1-12.

<table>
<thead>
<tr>
<th>Participant</th>
<th>Accurate Prediction (100% Accuracy)</th>
<th>Inaccurate Prediction (0% Accuracy)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency</td>
<td>Percent</td>
</tr>
<tr>
<td>1</td>
<td>11</td>
<td>91.6%</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>100%</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>25%</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>58.3%</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>58.3%</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>58.3%</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>58.3%</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>41.6%</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
<td>58.3%</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
<td>50%</td>
</tr>
<tr>
<td>11</td>
<td>5</td>
<td>41.6%</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
<td>50%</td>
</tr>
<tr>
<td>13</td>
<td>5</td>
<td>41.6%</td>
</tr>
<tr>
<td>Total</td>
<td>88</td>
<td>56.4%</td>
</tr>
</tbody>
</table>
An analysis of the inaccurate 68 predictions shows that most pre-service teachers did not miss, in general, accurate prediction (100% accuracy) by a large margin. Out of these total inaccurate predictions, 44 (64.7%) were documented as pre-service teachers missing the accurate prediction by one level (e.g., predicted level 2 but scored 1 or 3), 23 (33.8%) by two levels, and only 1 (1.5%) by three levels. This result suggests that pre-service accuracy teachers are not poor predictors.

Table 4.4 shows the frequency of accurate and inaccurate predictions for each Bloom level. Looking carefully at these frequencies, one can see that the accuracy of each Bloom level differed from the others; however, this difference was not guided or affected by the hierarchy of Bloom’s Taxonomy. Pre-service teachers’ ability to self-assess did not always decrease as they moved from lower Bloom levels to more complex Bloom levels. The data revealed that pre-service teachers had evidently higher self-assessment accuracy at the least complex Bloom level (i.e., knowledge level) with 18 (69.2%) accurate predictions out of 26 predictions as well as the most complex Bloom level.

Table 4.4: Frequency of accurate and inaccurate predictions for each Bloom level.

<table>
<thead>
<tr>
<th>Item</th>
<th>Accurate Prediction (100% Accuracy)</th>
<th>Inaccurate Prediction (100% Accuracy)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency</td>
<td>Percent</td>
</tr>
<tr>
<td>Knowledge Level Items 1 &amp; 2</td>
<td>18</td>
<td>69.2%</td>
</tr>
<tr>
<td>Comprehension Level Item 3 &amp; 4</td>
<td>13</td>
<td>50%</td>
</tr>
<tr>
<td>Application Level Items 5 &amp; 6</td>
<td>13</td>
<td>50%</td>
</tr>
<tr>
<td>Analysis level Items 7 &amp; 8</td>
<td>14</td>
<td>54%</td>
</tr>
<tr>
<td>Synthesis level Items 9 &amp; 10</td>
<td>13</td>
<td>50%</td>
</tr>
<tr>
<td>Evaluation level Items 11 &amp; 12</td>
<td>17</td>
<td>65.4%</td>
</tr>
<tr>
<td>Total</td>
<td>88</td>
<td>56.4%</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
level (i.e., evaluation level) with 17 (65.4%) accurate predictions out of 26 predictions.

Table 4.5 shows the accuracy, underestimation, and overestimation rates for Bloom levels from the highest to the lowest. In a general sense, pre-service teachers were equally likely to underestimate and overestimate their ability to self-assess. The 43.6% percent inaccuracy was distributed as 23.1% underconfidence (pre-service teachers did better than predicted) and 20.5% overconfidence (pre-service teachers did worse than predicted).

Table 4.5: Accuracy, underestimation, and overestimation rates for Bloom levels from highest to the lowest.

<table>
<thead>
<tr>
<th></th>
<th>Accuracy</th>
<th>Inaccuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Knowledge (69.2%)</td>
<td>Synthesis (34.6%)</td>
</tr>
<tr>
<td>2</td>
<td>Evaluation (65.4%)</td>
<td>Knowledge (27%)</td>
</tr>
<tr>
<td>3</td>
<td>Analysis (54%),</td>
<td>Comprehension (27%)</td>
</tr>
<tr>
<td>4</td>
<td>Comprehension (50%)</td>
<td>Application (23%)</td>
</tr>
<tr>
<td>5</td>
<td>Application (50%)</td>
<td>Analysis (23%)</td>
</tr>
<tr>
<td>6</td>
<td>Synthesis (50%)</td>
<td>Evaluation (3.8%)</td>
</tr>
<tr>
<td></td>
<td>Average 56.4%</td>
<td>23.1%</td>
</tr>
</tbody>
</table>

4.1.2.2 The Continuous Calibration Approach

I examined the accuracy by calculating the absolute difference between pre-service teachers’ predicted scores and actual performance scores (a continuous calibration). Table 4.6 shows the accuracy for each pre-service teacher. The data showed that pre-service teachers were very close to the correct prediction in 44 (64.7%) out of the 67 incorrect predictions. To illustrate, pre-service teachers drifted from the correct prediction by one level only (e.g., they chose a confidence level of 2 but they scored 1 or 3). As a result of this close prediction, the lowest accuracy rate was 69.5% for pre-service teacher 3,12, and 13. With the dichotomous calibration, these three pre-service teachers scored accuracy rates of 25%, 50%, and 41.6% respectfully. In fact, the accuracy rate was
higher for each pre-service teacher except for pre-service teacher 2 (she scored 100% in both calibrations) when calculating the continuous calibration. With the continuous calibration, pre-service teachers scored a mean accuracy rate of 80.15%.

Table 4.6: Pre-service teacher absolute difference accuracy.

<table>
<thead>
<tr>
<th>Participant</th>
<th>Sum of Absolute Difference (Predicted Score-Actual Score)</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>94.4%</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>100%</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>69.5%</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>83.3%</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>80.6%</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>80.6%</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>77.8%</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>77.8%</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>86.1%</td>
</tr>
<tr>
<td>10</td>
<td>7</td>
<td>80.6%</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td>72.2%</td>
</tr>
<tr>
<td>12</td>
<td>11</td>
<td>69.5%</td>
</tr>
<tr>
<td>13</td>
<td>11</td>
<td>69.5%</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>80.15%</td>
</tr>
</tbody>
</table>

### 4.2 Research Question 2

#### 4.2.1 Factors Influencing Pre-service Teachers’ Self-Knowledge

The main factor that influenced pre-service teachers’ self-knowledge was mental arithmetic. Pre-service teachers correctly identified what they did and did not know in 130 out of 156 responses to the 12 fraction division problems. Mental arithmetic was a significant factor. Pre-service teachers who performed mental arithmetic were able to manipulate numbers, operations, and their relationships in their heads. This helped them identify their areas of strengths and weaknesses and thus they were able to determine how much of the problem they could solve. Those who did not perform any mental Arithmetic
often picked a confidence level they believed was correct without sufficiently examining the problem. When asked about the reason for selecting this particular level, they could not discuss any areas of strength or weakness in their fraction division knowledge. Instead, they claimed the need for more time or paper and pencil to solve a problem.

4.2.2 Factors Influencing Actual Performance Prediction Accuracy

The qualitative analysis of the data revealed that six factors influenced pre-service teachers’ actual performance prediction when solving the problems in the knowledge survey (see Figure 4-1). In general, two of these factors (i.e., item difficulty and problem-solving skills) led to accuracy. The remaining four factors (i.e., knowledge base, cautiousness, no response, and incorrect self-knowledge) led to inaccuracy (i.e., underestimation or overestimation). The six factors are reported by Bloom level. I will discuss the factors that led to accuracy, underestimation, and overestimation for each Bloom level.

1. Item Difficulty
   - The pre-service teachers indicated that the problem was easy to solve and provided a solution.
   - The pre-service teachers indicated that the problem was difficult to solve and did not provide a solution.
2. Problem Solving Skills
   - The pre-service teachers applied problem-solving techniques to solve the problem.
3. Knowledgebase
   - The pre-service teachers indicated that they were unable to solve the problem using the assigned method. Instead, they solve it using a different method.
4. Cautiousness
   - The pre-service teachers had the knowledge to solve the problem but they conservatively underestimated their ability.
5. No Response
   - The pre-service teachers did not discuss any areas of strength or weakness on the problem.
6. Incorrect self-knowledge
   - The pre-service teachers believed they knew how to solve the problem but in fact did not.

Figure 4-1: Factors influencing actual performance prediction accuracy.
4.2.2.1 Bloom’s Knowledge Level

Items 1 and 2 in the knowledge survey were designed expressly to represent the level of knowledge by the pre-service teachers taking the survey. The participants were asked to write a definition for the term fraction in item 1 and a definition for the term division in item 2.

Accuracy

The accuracy here refers to the correctness of pre-service teachers’ prediction, not to the correctness of their work when solving the problems. For example, if a pre-service teacher predicted he would score a zero on a certain problem and then, in fact, solved the problem incorrectly, his prediction was considered correct, although his work was not.

In this study, 156 predictions were made in total, that is, 26 predictions per Bloom level. For Bloom’s knowledge level, the results showed that 18 out of 26 predictions made by pre-service teachers were recorded as accuracy. The factor for these predictions was coded as item difficulty. In their statements, pre-service teachers clearly specified whether defining the terms fraction and division were easy or hard.

In defining fractions, a representative statement made by the pre-service teachers was: “It is very straightforward...a fraction is a representation of a part of a whole.” Similarly, pre-service teachers explained their knowledge of division stating: “it is something that I practice a lot...division is the process of an amount into groups of equal size.”

None of the pre-service teachers claimed they had difficulty defining the term fraction, however, a small number did have trouble in regard to defining the term division. One statement by a pre-service teacher showed a clear recognition of difficulty
in defining division. That teacher wrote: “defining division is a little harder...it is kind of gray area for me.” As a result, he did not provide a definition for division.

**Underestimation**

Seven predictions were recorded as underestimation. Pre-service teachers’ statements on these predictions were coded as cautiousness. In this code, pre-service teachers claimed they knew how to define fraction and division; however, they selected a lower confidence level in case they missed any keyword of the definition. A representative statement made by these teachers for this situation was “I have a definition [for fraction] that I have gone over in-class… but I think there is still probably a better way of saying it” or “I know what a fraction is but… there are a lot of potential definitions for fraction.”

**Overestimation**

Only one prediction was recorded as overestimation. The pre-service teacher’s statement come up with a complete definition. He stated, “It is obviously dividing, it is separating something into smaller portions.” His definition was good but not complete. It was missing the part of the definition concerning the necessity of the equality of the divided portions. He selected level 3 but in fact, he scored a 2: a lower score than he predicted.

**4.2.2.2 Bloom’s Comprehension Level**

Items 3 and 4 were designed to represent Bloom’s comprehension level. Problem 3 states: “You are making up word problems for your students. Which of the following word problem(s) can be used to represent 3 ÷ 1/4?” Pre-service teachers were provided with four real world situations to choose from: 3 different fraction division situations (i.e., measurement, a unit rate, and inverse of multiplication) and one multiplication
situation. Problem 4 read as follows: “Explain in your own words how to find the length of a rectangle with an area of 2 3/5 square meters and a width of 2/3 meters.”

**Accuracy**

The study indicated 13 out of 26 of the pre-service teacher predictions were recorded as accuracy. The statements on the 13 accurate predictions were coded as problem solving skills. These teachers were able to demonstrate an understanding of concepts involved and devised a clear plan to solve the problem assigned in details. With Problem 3, a pre-service teacher explained his plan for solving the problem as: “When I look at 3 divided by 1/4 already I am thinking of a type of a word problem in my head that would apply. So I can apply that knowledge to these 4 answers and pick up the correct ones.” Another pre-service teacher explained: “It was difficult but to solve it I did not think in terms of story problem first. I solved it by dividing 3 by 1/4, and I knew what the answer was. So I looked at the options to see which one would give me the correct answer.”

With Problem 4, one pre-service teacher devised a plan to show his understanding of how to connect the rectangle area and fraction division as follows: “I can set up an equation. I know the area of a rectangle; it is the length times the width. I have the area; it's 2 and 3/5. So I would write that in the left of my equation equals the length times the width. I have the width and I just find the length. Yet a different participant summarized the plan as “knowing the formula for area... you take 2/3 times x equals 2 3/5. You take the area and divide it by the width to get the length.”

**Underestimation**

Seven predictions were recorded as underestimation. The pre-service teachers’ statements
were coded as no response. Pre-service teachers’ explained that they took a cursory glance at the problems and did not perform mental computation. With Problem 3, a pre-service teacher stated: “I did not read through all of [the answer choices]. I just read the question, so they may be tricky.” When asked why she selected a lower confidence level, she said, “It just looks more complicated…I have to think about it”

**Overestimation**

None of the pre-service teachers overestimated their ability to use the area of a rectangle in problem 4. With problem 3, six pre-service teachers scored worse than they predicted. These pre-service teachers’ statements were coded as knowledge base. From their statements, one can indicate that these pre-service teachers’ understanding of fraction division was restricted to the measurement interpretation. Although there were three different fraction division situations (i.e., measurement, a unit rate, and inverse of multiplication) that can represent $3 \div 1/4$, they only selected the measurement situation.

**4.2.2.3 Bloom’s Application Level**

Items 5 and 6 were designed to represent the level of difficulty pre-service teachers might recognize in solving a problem in fraction division. Problem 5 was: “solve the problem $1 \frac{3}{4} \div 1/4$ using the distributive law” while Problem 6 was “solve the problem $4/5 \div 1/4$ using decimals”

**Accuracy**

The study showed that 13 out 26 predictions were recorded as accuracy. Pre-service teachers’ statements on these predictions varied. Out of the 13 statements, 8 statements were coded as item difficulty. Pre-service teachers clearly specified whether they considered applying the distributive law and decimals was easy for them or hard.
With Problem 5, a pre-service teacher indicated he was able to solve the problem using the distributive law: “it is not difficult for me to solve the problem. $1 \frac{3}{4} \div \frac{1}{4} = (1 + \frac{3}{4}) \div \frac{1}{4}$.” Conversely, other pre-service teachers indicated that determining what the distributive law is was an area of weakness for them that would affect their performance in solving the problem correctly. A representative statement was “I could solve the problem but not with the distributive law…I don't know if I ever learned [the distributive law] or just maybe under a different name.”

With Problem 6, the pre-service teachers clearly stated that they were able to use decimals to solve fraction division problems. Evidence for this was found in statements such as: “it is easy for me to turn fractions into decimals and…how to do division using decimals…it is 0.8 divided by 0.25.”

The remaining 5 statements by pre-service teachers were coded as problem solving skills. Pre-service teachers were able to mentally understand the problem, devise a plan, and carry out that plan to solve Problem 5 and use decimals to solve Problem 6 before they used any written computation. A representative response for Problem 5 was: “I would convert $1 \frac{3}{4}$ to $1 + \frac{3}{4}$ and divide each quantity by $\frac{1}{4}$”. For problem 6, a pre-service teacher explained that she “would convert $\frac{4}{5}$ to 0.8 and $\frac{1}{4}$ to 0.25 and then use the long division to [divide 0.8 by 0.25].”

**Underestimation**

Six predictions were recorded as underestimation. The six pre-service teachers’ statements were coded as cautiousness. With Problem 5 one pre-service teacher stated: “I know what the distributive law is but I'm not entirely sure how to solve the division problem using that. I haven't done that for a long time.” With Problem 6, a representative
statement was “to be honest it gets complicated when you divide decimals . . . [There is] more room for error.”

Two statements were coded as Knowledge in Problem 6. In the statements, the pre-service teachers claimed that they were confident they would be able to solve the fraction division problem using decimals, however, they selected a lower confidence level because this is not the standard way they were taught to solve fraction division problems… “I chose level 2… because I would not usually solve it with decimals but I am fairly confident that I can figure it out.”

Overestimation

None of the pre-service teachers overestimated their ability to use decimals in problem 6, however, some pre-service teachers did in problem 5. Six predictions were recorded as overestimation and the pre-service teachers’ statements on these predictions were coded as knowledge base. In these statements, the pre-service teachers clearly indicated that they could not use the distributive law to solve Problem 5. Consequently, they should have selected confidence level zero, however, they did not. They assigned themselves a confidence level of one or two because they believed that what mattered was only solving the problem the way they were taught.

4.2.2.4 Bloom’s Analysis Level

Items 7 and 8 were designed to represent Bloom’s analysis level. Problem 7 was: One of Mr. Smith’s students created an incorrect real-world problem to represent $5 \div \frac{1}{2}$ as follows: Sara has 5 pizzas and she wants to give half of them to her friend. How much pizza will her friend get? Explain the student’s misconception. Problem 8 was: One of Mr. Smith’s students solved the problem $\frac{3}{4} \div \frac{1}{4}$ by dividing the numerators and
denominators: \( \frac{3+1}{4+4} \). He got 3, which is the correct answer. Explain if his method is always correct.

**Accuracy**

The data showed that 14 out of 26 predictions were recorded as accuracy for these problems. All the statements made on these predictions were coded as problem solving skills. Pre-service teachers read the problem more than one time in an attempt to understand the misconception presented in Problem 7 as well as to discover whether the method Problem 8 would always work to solve fraction division problems. They devised a clear plan to solve the problems assigned in details.

A representative response for Problem 7 was: “If there were 5 pizzas and she gave half of them to her friends she is really taking five and divide it by two, which is different than dividing by a half. I can see that.” Another pre-service teacher elaborated upon the misconception saying that, “dividing 5 by 1/2 is going to end with 10 if she is giving half of 5 she's going to be getting 2 1/2. So the misconception is that she is dividing by 2 rather than dividing by 1/2.”

With Problem 8, participants had a challenging mental computation in determining if dividing the numerator by the denominator is always a correct method to solve fraction division problems. Since this method is not one of the typical methods for solving fraction division problems, some participants understood what the problem was asking and explained they planned to plug in different numbers to further prove the correctness of this method. A pre-service teacher explained, “I will have to plug in a few numbers to see if this method is always correct…1/2 ÷ 1/4 equals 1/2 times 4. The answer is 2. Using this method, the problem will be 1 ÷ 2/4 and that equals to 1 times 4/2.”
The answer is 2 as well. I think this [method] is always going to be correct because dividing by a fraction is the same to multiplying by its reciprocal.”

**Underestimation**

Six predictions were coded as underestimation. All the six pre-service teachers’ statements were coded as knowledge base. Some pre-service teachers claimed there were accustomed to solve fraction division problem using a certain rule (e.g., Invert and Multiply) thus they did not try any other methods. With Problem 7, a pre-service teacher explained, “I looked at the equation and I solved it. I knew how to flip and multiply but [can] I truly explain why she was incorrect? I don't think I can fully do it.”

With Problem 8, pre-service teachers statements included the following, “I never thought of that procedure before and I was never taught so” and “I am not sure I can solve it. This is not the way that I usually solve fractions. I have to try out my way and see if he [is] in fact doing the same thing.” Theses pre-service teachers were unable to examine the method assigned in the problem. Instead, they used the Invert-and Multiply to solve the problem

**Overestimation**

Six predictions were recorded as overestimation. All the six pre-service teachers’ statements were recorded as no response. Pre-service teachers did not provide an explanation to the misconception presented in Problem 7 as well as to whether the method used is always correct in Problem 8. Some statements by these six pre-service teachers included: “I have to read the question a few times to understand and to find the misconception” and “I will just have to take a little longer to figure out what was she thinking.”
4.2.2.5 Bloom’s Synthesis Level

Item 9 and items 10 were designed to represent the level of Bloom’s synthesis the participants possessed. Problem 9 asked the participants to create a story problem for: 5/8 ÷ 3 while Problem 10 asked the participants to create a story problem for: 3/4 ÷ 1/3.

Accuracy

The study showed that 13 out of 26 predictions were recorded as accuracy. Pre-service teachers’ statements on 9 predictions of out of the 13 predictions were coded as item difficulty. On the one hand, some pre-service teachers were able to discuss the creation of story problems that represents 5/8 ÷ 3 in statements such as: “I had a lot of practice creating story problems for faction division… you got 5/8 of a pound of sugar and you need to divide that into three equal parts.” For 3/4 ÷ 1/3, a pre-service teacher explained, “if you have three quarters of a pound of beef [and] I want to make patties that weight one third of a pound each, how many patties can I make?” On the hand, others claimed difficulty with the creation of story problems. One pre-service teacher claimed that, “creating story problems is not one of [her] strong suits. Similarly, Another one stated: “crating a story problem is hard for me. I'm trying to visualize it in my head a little. But it looks difficult to do conceptually rather than procedurally.”

Underestimation

Nine predictions were recorded as underestimation. Pre-service teachers’ statements on the nine predictions were coded as cautiousness. In this group, pre-service teachers claimed they knew how to create story problems for fraction division; however, they selected a lower confidence level for different reasons. Some claimed they could not mentally think of any story at that time and they needed more time to process.
Representative statement was “I can truly do it but I could not come up with a story off the top of my head. That's why I gave it a 1.” Other pre-service teachers were concerned about using the appropriate mathematical language. One stated, “I'm confident that I can create a story problem. I would hope that I don't get tripped up on certain words that I used to the story problem and that can be an issue”

**Overestimation**

Ten predications were recorded as overestimation. These pre-service teachers were not able to accurately predict what score they would get for creating fraction division story problems and this group scored worse than they predicted. All the six pre-service teachers’ statements were coded as no response. A representative statement was: “I did not think the problem through. The numbers are a little friendlier, so I may be able to do that”

**4.2.2.6 Bloom’s Evaluation Level**

Items 11 and 12 were designed to represent the Bloom’s evaluation level possessed by pre-service teachers. They were asked to discuss the validity of the statement: “the quotient does not always become smaller when I divide” and the statement “dividing by a number is always equivalent to multiplying by its reciprocal” for Problems of 11 and 12 respectively.

**Accuracy**

The study showed that 17 out of 26 predictions were recorded as accuracy. All the statements given by pre-service teachers on these 17 predictions were coded as item difficulty. In discussing the statement “the quotient doesn’t always get smaller when dividing,” one pre-service teacher said: “I definitely understood division… if you divide
by a number less than one it's going to be bigger.” Conversely, another pre-service teacher explained “if you divide a whole number by a fraction the quotient actually increases…I know that procedurally but I cannot explain it conceptually.”

In discussing the statement “dividing by a number is always equivalent to multiplying by its reciprocal,” pre-service teachers who predicted their score accurately responded in different forms. Some believed that “flip and multiply is more ingrained” because “that is how [we] always solve fractions so it is true.” This group concluded, “it is true that dividing by a fraction is equivalent to multiplying by its reciprocal, $a \div b = \frac{a}{b} = a \times \frac{1}{b}$. Only exception would be dividing by zero.”

Other pre-service teaches showed a clear recognition of weakness in their ability to validate the statement. One stated, “I could show through examples that [it] is [a] valid statement but to discuss why I surely can't.”

**Underestimation**

Only one prediction was recorded as underestimation for this section. This pre-service teacher’s statement was coded as cautiousness. The pre-service teacher indicated that the quotient does not always get smaller when dividing, however, he thought he could have difficulty explaining that. He said, “it's true because the quotient doesn't always become smaller [when] you divide . . . But it might be a little of trouble to explain”

**Overestimation**

Eight predictions were recorded as overestimation. All the eight statements on theses predictions were coded as incorrect self-knowledge. These pre-service teachers identified what they could actually do incorrectly. In other words, they explained how
they could solve the problem though their solution was incorrect. Some pre-service teachers still carried out one of the constraints caused by the sharing interpretation of fraction division: the quotient must be less than the dividend. Others believed that they understood how invert-and-multiply works since it is the standard algorithm they usually used for fraction division but, in fact, they did not.

4.2.2.7 Summary

Two of the 6 factors helped pre-service teachers predict their actual performance accurately. The first factor was item difficulty. Pre-service teachers made accurate predictions where the problem was easy or difficult regardless the level of Bloom’s Taxonomy. When a pre-service teacher encountered an easy fraction division problem, pre-service teachers expressed verbally how easy the problem was and provided a solution as well. When encountered a difficult fraction division problem, they declared how difficult the problem was and did not provide a solution. In each situation, they were able to accurately predict how much of the problems they could solve.

The second factor was problem solving skills. Pre-service teachers with good problem solving skills were more likely to make accurate predictions of their performance. It was evident that some pre-service teachers were following problem solving techniques such as: understand the problem, devise a plan, carry out the plan, and look back. By using these techniques, pre-service teachers were able to develop a sense of what score they would get if they solve the problems using written computation.

There were four factors that hindered pre-service teachers from predicting their actual performance accurately. The first factor was knowledge base. Some pre-service teachers’ understanding was limited to the invert-and-multiply method. When they
encountered a non-standard method that they could not apply, they resorted to the standard method they had learned. Since they were able to solve the problem using invert-and-multiply method, they ignored the method assigned in the problem and rated their ability higher than it was. The second of factors was cautiousness. Some pre-service teachers rated their confidence level as less than what they were capable of. The third factor was no response. In a number of occasions, the pre-service teachers did not test any approaches to solve the problem in their heads and thus they were unable to predict how much of a problem they could solve. Forth factor is incorrect self-knowledge. On certain fraction division problems, the pre-service teaches believed they knew the correct solution but in fact they did not. Accordingly, they rated their ability as higher than it was.
Chapter 5

Discussion

In this dissertation, my goal was to examine how pre-service teachers self-assessed their understanding with respect to fraction division. The two following questions guided the research:

1. How accurate are pre-service teachers in self-assessing their own understandings of fraction division?

2. What types of successes and challenges do pre-service teachers encounter as they self-assess their own understandings of fraction division?

5.1 Research Question 1

In general, the results of this study indicate that pre-service teachers often provided accurate self-knowledge of fraction division. Pre-service teachers displayed a high level of understanding of what they did and did not know on the fraction division items before they attempted to solve them. Flavell (1979) indicated that self-knowledge is an important component of effective learning. Pintritch (2010) further argued that having self-knowledge is important to learning but it is much more important is to have accurate self-knowledge. The fact that pre-service teachers had a high self-knowledge accuracy
rate is a promising finding because it suggests that pre-service teachers tend to have an important skill for lifelong learning.

One interesting finding of this study is that self-knowledge accuracy did not always lead to accurate predictions of actual performance. Some might argue that pre-service teachers would likely be able to predict their actual performance since they could accurately identify what they knew and what they struggled with; however, this view was not supported by this study’s findings. On a number of occasions, the pre-service teachers accurately determined how many of the problems they could solve; however, they were unable to predict what score they would get on these problems.

I found that examining pre-service teachers’ prediction accuracy was not simple. The results indicated that the degree of self-assessment accuracy differed markedly between dichotomous calibration and continuous calibration. With the dichotomous calibration, on an item-by-item comparison (100% accuracy or 0% accuracy), the average of pre-service teachers’ accuracy rate was not promising. Interestingly, pre-service teachers were very close to the correct prediction as they often drifted by one level only (e.g., they predicted a score of 2 but scored a 1 or 3). As a result of this close prediction, the accuracy rate was much higher when calculating the continuous calibration. This suggests that pre-service teachers can provide a significant data of what they already know and need to learn.

Similar to findings by Clauss and Geedey (2010), this study’s results showed that pre-service teachers had higher prediction accuracy at the least complex Bloom level (i.e., knowledge level) as well as the most complex Bloom level (i.e., evaluation) compared to the rest of Bloom levels. Clauss and Geedey examined 309 survey/exam
result pairs performed by junior and senior college students in the disciplines of biology, ecology, evolution, and mathematics. They concluded that students were better self-assessors at the lowest Bloom level (i.e., knowledge) and highest Bloom levels (i.e., analysis, synthesis, and evaluation), and poorer self-assessors at the intermediate levels (i.e., comprehension, application).

5.2 Research Question 2

Self-knowledge accuracy is found to be positively linked with mental arithmetic. Being able to manipulate numbers, operations, and relationships in their heads, pre-service teachers were able to identify their areas of strength and weakness and thus determine how much of the problem they could solve. This suggests that pre-service teachers tend to appreciate the use of mental arithmetic and utilize it to evaluate their knowledge as much as they do with written calculations. NCTM (2000) points out that the ability to perform mental arithmetic is essential to learning. Mental arithmetic provides both tools for solving mathematical problems and filters for evaluating solutions. Seen in this light, I believe that development of pre-service teachers’ mental arithmetic skills would enhance not only their mathematical abilities but also their self-assessment skills.

In this study, the high prediction accuracy at Bloom knowledge level and Bloom evaluation level was strongly related to item difficulty. Overall, all pre-service teachers considered knowledge level items to be very easy items and thus predicted that they would be able to solve them. The pre-service teachers were often accurate in their prediction. With the evaluation-level items, pre-service teachers whose understanding of fraction division was strong perceived the items as easy and predicted that they would be
able to provide a solution. Pre-service teachers with less understanding of fraction
division claimed that the items were very difficult to solve and thus predicted that they
would not be able to solve them. Both were often accurate in their predictions. This
further suggests that prediction accuracy of actual performance is not always related to
strong understanding of fraction division.

The study also showed that prediction inaccuracy of actual performance is linked
to knowledge base. Flores (2002) argued that “in the United States, division of fractions
has been taught for so long by giving a procedures with no explanation why it works that
some today’s teachers and prospective teachers only know that method” (p. 246). This
study revealed that some pre-service teachers’ ability to solve fraction division problems
was often limited to the standard method (i.e., invert and multiply). Their inability to use
a variety of computational approaches for fraction division affected their prediction
accuracy. When encountering a non-standard method that they cannot apply, pre-service
teachers spontaneously resorted to the standard methods they had learned. Since they are
able to solve the problem using invert-and multiply method, they ignore the method
assigned in the problem and rate their ability higher than it is.

Ma (1999) pointed out that teachers with Profound Understanding of Fundamental
Mathematics (PUFM) can compute an answer to a given fraction division problem by
using different computational approaches and can pose story problems to represent the
different interpretations of fraction division. The results of this study revealed that some
pre-service teachers’ fraction division understanding was restricted to the measurement
interpretation. When asked to select all story problems that could be used to exemplify a
given numerical fraction division problem, they often looked for measurement situations
and neglected the others. Pre-service teachers believed that they had provided a complete answer, but in reality they did not. Again, they rate their ability as higher than it is. This suggests that pre-service teachers without PUFM tend to be less accurate in predicting their actual performance than those with PUFM. This is consistent with the findings of Bol and Hacker (2001), who concluded that lower-achieving students were less accurate in their prediction accuracy than higher-achieving students.

5.3 Significance of the Study and Conclusion

In conclusion, although pre-service teachers often displayed accurate self-knowledge of fraction division, they seemed to struggle using their accurate self-knowledge to predict their actual performance. However, the fact that they accurately identified their own strong and weak points on most of their responses to the fraction division problems and predicted their actual performance on more than half of them indicates that self-assessment could provide significant data to decision makers in education and to the pre-service teachers themselves.

Decision makers in education can utilize this data to develop plans to improve pre-service teachers’ subject matter knowledge of fraction division. Pre-service teachers themselves, on the other hand, can utilize this data to control their own learning of fraction division and thereby become academically and professionally successful. Ryan and McCrae (2005) state “pre-service teachers who can confront their own mathematical errors, misconceptions, and strategies in order to reorganize their subject matter knowledge have an opportunity to develop rich pedagogical content knowledge” (p. 641). Pre-service teachers should learn how to learn and become independent learners. To also
help their students learn in the future, pre-service teachers should be equipped with self-assessment skills that enable them to improve their subject matter knowledge and thus their instruction. Raising the awareness of the importance of self-assessment in the mathematical development of pre-service teachers is essential for mathematics education.

These findings suggest that teacher preparation programs could integrate self-assessment into the courses offered in their programs. I hope that the findings of this study will encourage teacher preparation programs to direct their instruction towards facilitating the understanding and meaningful use of self-assessment.

5.4 Limitations and Recommendations

A limitation of this study was that the data resulted from studying pre-service teachers in one school. It is also acknowledged that this is a small-scale investigation that included 13 pre-service teachers and a 12-item instrument. The data of the study need to be interpreted with this limitation in mind.

Another limitation was the coding process. In this study, I developed a single code scheme. I only assigned one descriptive code for each statement made by the pre-service teachers during the interviews. Since coding is an interpretive act, further studies could be applied using different types of coding schemes. This may provide additional insight into issue of pre-service teachers’ self-assessment.

This study examined how pre-service teachers self-assessed their understanding of fraction division using a knowledge survey. The study showed that the knowledge survey helped pre-service teachers identify areas of strengths and weaknesses of their fraction division knowledge. Metacognition includes self-knowledge—what one knows about cognition; and self-regulation—how one uses that knowledge to regulate cognition,
Thus, further investigation should address how pre-service teacher utilize information they obtain from a knowledge survey to enhance their fraction division knowledge over time.
References


Appendix A

Fraction Division Self-assessment Questionnaire Answer Sheet

Please read each problem 1 through 12 in the questionnaire and rate your confidence level for solving each one. Do not actually solve the problem. When rating your confidence level use the following scale:

- Rating of 0: “I cannot begin to answer this problem.”
- Rating of 1: “I can partially answer this problem.”
- Rating of 2: “I can solve most of the problem.”
- Rating of 3: “I can solve the entire problem with full confidence.”

Please record your ratings into the chart below.

<table>
<thead>
<tr>
<th>Rating</th>
<th>0: I cannot begin to answer this problem</th>
<th>1: I can partially answer this problem</th>
<th>2: I can solve most of the problem</th>
<th>3: I can solve the entire problem with full confidence</th>
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<tbody>
<tr>
<td>Problem</td>
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## Appendix B

### Rubric

<table>
<thead>
<tr>
<th>Item</th>
<th>Score of 0 - No Answer</th>
<th>Score of 1 - Partial Answer</th>
<th>Score of 2 - Almost Complete Answer</th>
<th>Score of 3 - Complete Answer</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>There is no definition, or the definition has no relation to fraction.</td>
<td>The definition addresses some, but not all the mathematical components of fraction definition.</td>
<td>The definition addresses most, but not all the mathematical components of fraction definition.</td>
<td>The definition addresses all the mathematical components of fraction definition.</td>
</tr>
<tr>
<td>2</td>
<td>There is no definition, or the definition has no relation to division.</td>
<td>The definition addresses some, but not all the mathematical components of division definition.</td>
<td>The definition addresses most, but not all the mathematical components of division definition.</td>
<td>The definition addresses all the mathematical components of division definition.</td>
</tr>
<tr>
<td>3</td>
<td>None of the three correct answers A, C, and D is circled</td>
<td>One of the three correct answers A, C, and D is circled.</td>
<td>Two of the three correct answers A, C, and D are circled.</td>
<td>The three correct answers A, C, and D are circled.</td>
</tr>
<tr>
<td>4</td>
<td>There is no explanation of the solution.</td>
<td>Some, but not all the mathematical components of finding the length of a rectangle with a known area are addressed.</td>
<td>Most, but not all the mathematical components of finding the length of a rectangle with a known area are addressed.</td>
<td>All the mathematical components of finding the length of a rectangle with a known area are addressed.</td>
</tr>
<tr>
<td>5</td>
<td>There is no or incorrect solution.</td>
<td>Some part of the answers are correct</td>
<td>Most parts of the answers are correct</td>
<td>All parts of the answer are correct.</td>
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<td>6</td>
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<tr>
<td><strong>6</strong></td>
<td>There is no or incorrect solution.</td>
<td>Some parts of the answer are correct.</td>
<td>Most parts of the answer are correct.</td>
<td>All parts of the answer are correct.</td>
</tr>
<tr>
<td><strong>8</strong></td>
<td>There is no or incorrect solution.</td>
<td>There is a correct indication whether the method works or not, but no explanation why.</td>
<td>There is a correct indication whether the method works or not, but the explanation is not clearly presented.</td>
<td>There is a correct indication whether the method works or not, and the explanation is clearly presented.</td>
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<tr>
<td><strong>10</strong></td>
<td>There is no or incorrect story problem.</td>
<td>The story problem addresses some, but not all the underlying mathematical concepts in the task.</td>
<td>The story problem addresses most, but not all the underlying mathematical concepts in the task.</td>
<td>The story problem addresses all the underlying mathematical concepts in the task.</td>
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<tr>
<td><strong>12</strong></td>
<td>There is no or incorrect evaluation of the statement.</td>
<td>There is a correct indication whether the statement is valid or not, but no explanation why or incorrect explanation.</td>
<td>There is a correct indication whether the statement is valid or not, but the explanation is not clearly presented.</td>
<td>There is a correct indication whether the statement is valid or not and the explanation is clearly presented.</td>
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Appendix C

Instrument Key Answer

1) Write a definition for the term fraction.

One of the following is an acceptable answer:

a) Fraction is a rational number written as \( \frac{a}{b} \) where the integers \( a \) and \( b \) are called the numerator and the denominator, respectively. The numerator represents a number of equal parts and the denominator, which cannot be zero, indicates how many of those parts make up a unit or a whole.

Alternative interpretations:

b) Operator: an operator is a transformer that stretches or shrinks another value. Example: \( \frac{2}{3} \) of a number can be found by multiplying the amount by 2 and then dividing it by 3 or visa versa. That is, \( \frac{2}{3} \) of 6 = 4

c) Ratio: comparing quantities with like units. Example: \( \frac{2}{3} \) represents 2 parts per 3 parts.

d) Rate: comparing quantities with unlike units. Example: \( \frac{2}{3} \) represents 2 dollars per 3 candy bars.

e) Quotient: dividing the numerator by the denominator. Example: \( \frac{2}{3} \) of a pizza is the amount each person gets when 3 people share 2 pizzas. \( \frac{2}{3} \times \frac{2}{3} = \frac{2}{3} \times \frac{2}{3} \).

f) Measure: a unit fraction with numerator of 1 repeated the times given in the numerator. Example: \( \frac{2}{3} \) represents 2 measures of \( \frac{1}{3} \) each.

2) Write a definition for the term division.

One of the following is an acceptable answer:

a) The process of separating an amount into equal parts.

b) The process of finding how many times one number or quantity is contained in another.

3) You are making up word problems for your students. Which of the following
word problem(s) can be used to represent $3 \div \frac{1}{4}$? (Circle all that apply.)

A. 3 cups of orange juice fill up exactly $\frac{1}{4}$ of a container. How many cups will fit in the whole container?
B. 3 friends each have $\frac{1}{4}$ of a cookie. How many cookies would they have if they put them all together?
C. In a survey, 3 students said they prefer pizza. These three students represent $\frac{1}{4}$ of the number of students who prefer burger. How many prefer the burger?
D. How many lengths of $\frac{1}{4}$ yard can be cut from 3 yards cloth?

A, C, and D are the correct answers.

4) Explain in your own words how to find the length of a rectangle with an area of $2 \frac{3}{5}$ square meters and a width of $\frac{2}{3}$ meters.

\[
\text{Area} = \text{Length} \times \text{Width}
\]

To find the area of a rectangle, we multiply the length by the width. The formula is:

\[A (\text{area}) = L (\text{length}) \times W (\text{width})\]

To find the length, we divide the area by the width:

\[L = A \div W\]

\[L = 2 \frac{3}{5} \div \frac{2}{3} = 3 \frac{9}{10}\]

5) Solve the problem $1 \frac{3}{4} \div \frac{1}{4}$ using the distributive law.

One of the problem is an acceptable answer:

a) \[
1 \frac{3}{4} \div \frac{1}{4} = (1 + \frac{3}{4}) \div \frac{1}{4}
\]
\[
= (1 + \frac{3}{4}) \times 4/1
\]
\[
= (1 \times 4) + (\frac{3}{4} \times 4)
\]
\[
= 4 + 3
\]
\[
= 7
\]

b) \[
1 \frac{3}{4} \div \frac{1}{4} = (1 + \frac{3}{4}) \div \frac{1}{4}
\]
\[
= (1 \div 4) + (\frac{3}{4} \div 1/4)
\]
\[
= 4 + 3
\]
\[
= 7
\]

6) Solve the problem $\frac{5}{4} \div \frac{1}{4}$ using decimals.

\[0.8 \div 0.25 = 3.2\]
7) One of Mr. Smith’s students created an incorrect real-world problem to represent $5 \div 1/2$ as follows:
Sara has 5 pizzas and she wants to give half of them to her friend. How much pizza will her friend get?
Explain the student’s misconception.

The student confused division with $1/2$ with division with 2. She created a story about dividing the quantity 5 evenly into two parts.

8) One of Mr. Smith’s students solved the problem $3/4 \div 1/4$ by dividing the numerators and denominators: \[
\frac{3+1}{4+4}. \text{ He got 3, which is the correct answer.}
\]
Explain if his method is always correct.

This method always works, however, for many numbers it would require more work to achieve the same results obtained by the standard “invert – and – multiply” algorithm. For example,

\[
\begin{align*}
3/5 \div 4/7 &= \frac{3+4}{5+7} = \frac{3}{4} \\
&= \frac{5+7}{5+7} \\
3/4 \times 4/4 &= \frac{3}{20/7} \\
5/7 \times 7/7 &= 21/20
\end{align*}
\]

Or

\[
\begin{align*}
3/5 \div 4/7 &= \frac{3+4}{5+7} = \frac{3}{4} \\
&= \frac{3}{5/7} \\
3/4 \div 5/7 &= 3/4 \times 7/5 = \frac{21}{20}
\end{align*}
\]

With Invert and Multiply algorithm:

\[
\begin{align*}
3/5 \div 4/7 &= 3/5 \times 7/4 = 21/20
\end{align*}
\]

9) Create a story problem for $5/8 \div 3$

Example: Three friends equally share $5/8$ of pizza. How much pizza will each one get?

10) Create a story problem for $3/4 \div 1/3$

Example: John has $3/4$ of a whole cake. It fills up exactly $1/3$ of his container. How much cake will fit in the whole container?
Discuss the validity of following statements:

11) The answer does not always get smaller when I divide.

True statement. If the divisor is a whole number, the quotient gets smaller. Example: $6 \div 2 = 3$. If the divisor is a fraction, the quotient gets larger. Example: $6 \div \frac{1}{2} = 12$.

12) Dividing by a number is always equivalent to multiplying by its reciprocal.

True statement. There are many methods to explain the equivalence of “dividing by a number” and “multiplying by its reciprocal”. These methods include, but not limited to, the following: (One method is sufficient as an answer)

a) $1 \frac{3}{4} \div \frac{1}{2}$ means that $\frac{1}{2}$ of a number is $1 \frac{3}{4}$, what is that number? The answer is $3 \frac{1}{2}$, which is exactly the same as the answer for $1 \frac{3}{4} \times 2$. The divisor 2 is the reciprocal of $\frac{1}{2}$.

b) $3 \div \frac{2}{3}$ means how many groups of $\frac{2}{3}$ are in three whole units? Starting with one whole unit ($1 \div \frac{2}{3}$), the representation below shows that one whole unit contains 1 complete unit of $\frac{2}{3}$ in addition to $\frac{1}{2}$ of it (or $\frac{3}{2}$ of $\frac{2}{3}$).

<table>
<thead>
<tr>
<th>1 Whole Unit</th>
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<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>$\frac{1}{3}$</td>
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</table>

$1 \div \frac{2}{3} = 3/2$

$3 \div \frac{2}{3} = (1 \div \frac{2}{3}) + (1 \div \frac{2}{3}) + (1 \div \frac{2}{3})$

$3 \div \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = 3 \times \frac{2}{3}$

c) $1 \frac{3}{4} \div \frac{1}{2} = 1 \frac{3}{4} \div (1 \div 2)$

$= 1 \frac{3}{4} \div 1 \times 2$ after removing parentheses

$= 1 \frac{3}{4} \times 2 \div 1$

$= 1 \frac{3}{4} \times (2 \div 1)$

$= 1 \frac{3}{4} \times 2$