A Thesis

entitled

Identification, Simulation and Control of an Ankle Foot Orthosis

by

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Submitted to the Graduate Faculty as partial fulfillment for

The Master of Science in Electrical Engineering

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An Abstract of

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Drop foot is a neuromuscular condition affecting many patients with abnormality in the walking pattern. The most common treatment to address this condition is ankle foot orthoses. This research presents a study on development of an identified model for a torsional shape memory alloy-based ankle foot orthosis for drop foot disorder. The model has been developed in stages. First, a dynamic model of ankle in walking is developed using identification method. A dynamic model of torsional shape memory materials is then developed. In the final stage, these two models are combined. This final model is employed to evaluate performance of the proposed ankle foot orthosis in mitigating drop foot. To this end, a control system is developed to match dynamic behavior of the patient with normal ankle behavior. This research provides a computation approach to design and develop advance assistive devices for ankle such as orthosis, prosthesis and bionic.
Dedication

To my great family, Mariana and Graça!
Acknowledgements

I would like to thank my adviser Dr. Mohammad Elahinia. I would also like to thank my committee members Dr. Mansoor Alam and Dr. Richard Molyet.

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Chapter One
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*Design of AFO for Drop Foot Patients—Technologies*

**2.1. Introduction**

Dorsiflexion is the upward motion of the foot. Drop foot is a significant weakness of ankle dorsiflexion recognized as a motor deficiency. Foot drop can occur when there is any injury to the dorsiflexors or to any point along the neural pathway that supply them. Most of the time, foot drop occurs due to the neurological disorder [1]. Rarely, diseased or non-functional muscles cause foot drop. Designated muscle to provide dorsiflexion is tibialis anterior which is shown in Fig. 1.

![Tibialis anterior muscle](image)

Fig. 1. Tibialis anterior muscle [2].
Many patients with gait abnormality are diagnosed with drop foot disorder. One of the common treatment approaches toward a drop foot is ankle foot orthosis (AFO). Although, there has been quite a good progress in designing AFOs, but there is still need to a small size, wearable device that can resolve drop foot, effectively.

In general, AFOs can be categorized in two groups as follows:

1) Passive, which have fixed and articulated joints,

2) Active, which have active actuators.

However, one can add another group of AFOs as semi-active devices.

Since developed passive AFOs were not fully helpful for some drop foot patients, active AFOs (AAFOs) were developed offering full controllable assistive device. But, current AAFOs have various limitations to get into market such as 1) impractical size, 2) heavy weight, and 3) high electrical power need.

Drop foot patients are a group of patients with ankle disorder that have capability for plantarflexion and suffer from lack of dorsiflexion. Thus, drop foot patients need an AFO which can enhance ankle dorsiflexion.

In the next section, drop foot and its impact on gait are analyzed. Section 1.3 presents various AFO technologies. Section 1.4 discusses shape memory alloys and their potential in AFO applications.
2.2. Drop Foot Effects on Gait

Drop foot is a motor deficiency caused by total or partial central paralysis of the muscles innervated by the common peroneal nerve, i.e. the anterior tibial muscle and the peroneal group [3]. Stroke is an often cause of drop foot [4]-[5]. However, many other causes are reported for spinal cord injury that results in drop foot [4].

Drop foot effects gait, considerably. A healthy gait or a normal walking pattern depends on biomechanical and nervous system features [5]. Stability and specific metabolic energy expenditure are properties of a healthy gait [6]. Drop foot can cause unstable and higher energy expenditure gait. Several researchers have performed comprehensive biomechanical measurements to analyze gait patterns for basic understanding and rehabilitation purposes [4], [7]. Drop foot effects kinematics, kinetics, power, energy and stability of gait. Speed of walking has reported as the simplest methods to diagnose ankle disorder. It should be noted that an abnormal gait caused by drop foot might also cause

Thus, treatment of this disorder is necessary to restore stability and other features of a normal gait.

1.2.1 Normal Gait

A gait consists of two main parts, stance and swing. Stance starts with foot strike and ends with toe off. Afterwards, swing starts and ends with next hill strike. Fig. 2
displays these two phases of a gait cycle. Furthermore, states of foot with their respective needed assistance are shown in this figure.

Despite a healthy subject, a drop foot patient cannot accomplish dorsiflexion, properly. As it will be discussed, abnormality in dorsiflexion also affects other parts of gait and characteristics of other joints such as knee and hip. Thus, the main purpose of drop foot treatment is to provide enough dorsiflexion assistance so that patient can achieve a normal gait pattern.

![Fig. 2. Phases of a gait cycle [8].](image)

Behavior of ankle is studied by various tools such as kinetics and kinematics of ankle. Torque and angle of the ankle in a normal gait is shown in Fig. 3. A wide line is used to display these graphs since these characteristics might vary from one to another.
In a normal gait, at hill strike, dorsiflexor causes a controlled plantarflexion to prevent foot slap. In fact, in this state, dorsiflexor acts like a linear spring [9]. Likely at toe off, dorsiflexor provides dorsiflexion to prevent foot drag. In this state and during swing phase, a position control is needed which provides external torque to push foot upward.

Fig. 3. Angle and moment of ankle of a normal subject.
Another characteristic of ankle which is interesting to be studied is ankle stiffness. Fig. 4 shows stiffness characteristic of an ankle in a normal gait. Note that at foot flat, an ankle dorsiflexion occurs which does not need tibialis anterior muscle to function. However, this dorsiflexion is due to movement of center of mass.

![Diagram of ankle stiffness](image)

**Fig. 4.** Stiffness characteristic of ankle of a normal gait. HS: Hill strike, FF: Flat foot, MD: Max-dorsiflexion, TO: Toe off.

At hill strike, although an ankle plantarflexion occurs, but it is dorsiflexor muscle that engages and prevents foot slap. Fig. 5 shows muscle activation pattern of a normal
gait. It includes all involved muscles from hip to ankle. It can be seen that tibialis anterior is activated at hill strike and toe off.

Furthermore, it can be seen that from foot flat to max-dorsiflexion, as the body passes over the fixed foot, ankle goes to dorsiflexion without tibialis anterior muscle contribution. At max-dorsiflexion, ankle goes to plantarflexion to prepare for lift off. One can divide swing phase into three periods: Initial swing (toe off to foot clearance), mid swing (foot clearance to vertical tibia) and late swing (vertical tibia to foot strike).

Fig. 5. Activation of muscles in a normal gait [10].
It is clear that a dorsiflexor controls stability when the first contact of foot occurs until flat foot. Furthermore, a dorsiflexor controls stability in swing phase.

Dorsiflexor performance effects overall gait dynamics; and thus, affects energy and power throughout the gait cycle.

1.2.2 Abnormality in Gait due to Drop Foot

A foot drop disorder disturbs the normal walking pattern, in various levels. Therefore, it was hypothesized that the drop foot would cause a redistribution of net joint moments about the ankle, knee and hip joints in which case the knee and/or the hip joints could be overloaded and probably lead to joint degradation over many years [3].

In [3], it is shown that drop foot subject has an incomplete swing phase. Hip and knee were also adversely affected. During the gait cycle, besides excessive hip flexion (about 13 degrees), no hip extension was observed; knee flexion for affected leg shows one peak trend instead of normal double peak behavior [3]. Furthermore, it has been observed that unaffected leg motion has 20% late heel strike and the subject lifts that leg to only half of the normal value [3].

In fact, when a person with drop foot walks, the affected foot slaps down onto the floor. Thus inherently, patient would try to compensate by raising the thigh excessively, as if walking upstairs. Fig. 6 shows how a drop foot patient changes its plantarflexion on ankle to prevent toe drag and slap foot.
Fig. 6. Ankle angle of the unaffected foot in a drop foot patient and its comparison with normal; focus on excessive dorsiflexion of ankle made by excessive extending hip.

In fact, the patient extends the hip excessively which results in excessive dorsiflexion in ankle. Moment of ankle in drop foot patient (Fig. 7) is also showing that ankle does not have required torque to provide dorsiflexion control.
1.2.3 Drop Foot Treatment

A drop foot patient might use excessive energy to prevent drop foot consequences. This is not only waist of energy and tiring, but also might cause other biomechanical injuries for parts involved in walking. Thus, the first step is to diagnose drop foot. Walking speed is a well-known indicator of overall walking performance [11]. This is rather simple to assess and is widely used, but this measure alone gives no information about which factors cause changes in walking performance [3]. Therefore, several researchers have performed more comprehensive biomechanical measurements to analyze gait patterns for basic understanding and rehabilitation purposes [8].

After its detection, drop foot is treated by various methods such as follows:

1- Functional electrical stimulation (FES),
2- Tendon transfer from the posterior tibial muscle,

3- Ankle foot orthosis (AFO).

An important feature of upper motor neuron lesion (UMNLs) is that electrical excitability of the associated peripheral nerves is still intact; thus, facilitating the use of functional electrical stimulation (FES) to restore or enhance gait for some of these cases [13]. A history of FES evolution is presented in [13]. Basically, FES is based on shorts bursts of electrical pulses to generate muscle contraction. This method in a controlled fashion is employed for drop foot correction. Hybrid FES and walker systems are currently used for paraplegic subjects to walk [14]. But, there are still challenges associated with FES usage such as raise of heart rate and energy consumption [15].

2.3. AFO Technologies

AFO technologies can be categorized based on their design method. AFOs are mostly considered to be either 1) Passive, or 2) Active. In this section, a review of these technologies is presented. Recent studies to employ shape memory alloys in AFO applications are also discussed.
1.3.1 Passive AFO Technologies

Passive AFOs are the first kind of AFOs developed to address ankle abnormality. Since wearable property of an assistive device is necessary for daily usage, passive AFOs are the most common AFOs in the market. Fig. 8 shows three different designs of passive AFO.

Fig. 8. Various passive AFO designs. A: Hybrid AFO with a linear spring and thermoplastic AFO [16-17]. B: Polypropylene AFO. C: one of different designs reported in [18].

A design guideline of passive AFOs has been presented in [19]. It has been shown that how various disorders related to ankle can be considered in the design of a passive AFO.
AFOs are mostly prescribed to assist in the following functions: 1) toe clearance and prevent excessive inversion of ankle joint during swing phase, 2) absorb body weight impact at initial contact, and 3) support forward propulsion of the body during mid to late stance phase [20]. Direction of muscle moment and desired AFO moment are shown in Fig. 9.

![Fig. 9. Muscle moment and desired AFO moment [19].](image-url)

Dorsiflexion assistant in a passive AFO is sometimes provided by a spring. Spring has been shown a good performance in providing toe off assistance. There are several issues that should be considered in design of a spring for AFO applications. Influence of spring-like AFO in energy cost of walking is discussed in [20]. It is shown that stiffness of used spring affects various parameters of a gait such as energy cost of walking. An optimum design of spring for AFO applications is proposed in [20].

Some research have compared and discussed performance of various AFO designs. One of these studies is on dynamic and hinged AFOs [21]. It is shown that
hinged AFO prevented toe drag and a heel-toe gait pattern was achieved; however, dynamic AFO did not show such a performance [21]. In Fig. 8, A shows a hinged AFO and B shows a dynamic AFO.

1.3.2 Active AFO Technologies

Active AFOs (AAFOs) started to be considered since passive AFOs did not show a fully controllable and flexible assistance. Limitation of passive AFOs was the main reason to move toward an active AFO. It has been shown that AFOs do not prevent toe drag in some patients or do not prevent both toe drag and slap foot [22]-[23]. In fact, developed passive AFOs could not enhance gait in severe drop foot cases. The main reason was that a passive AFO was not able to provide enough external torque to push toe upward in the swing phase. However, an active AFO could provide external torque using an actuator such a motor. Another application of active AFOs is in diagnosis and rehabilitation. Various AAFO designs are developed using different types of actuators. Fig. 10 shows two different active AFOs.
Impact of AAFO with compressed air actuator on individuals with spinal cord injury has been tested by Sugisaka et al. It has shown that this AAFO can improve push-off kinematics without large decreases in muscle activation amplitudes [22]. This study shows this AAFO might need to be improved by automatic triggering (e.g. a footswitch) and higher-level controllers to reduce patient cognitive effort [22].

Footswitch control method has been recently attracted attention of control researchers to improve AAFO’s performance. Developed AAFO by Sugisaka [23] uses this control method for an artificial muscle activated by compressed air. Footswitch control method developed in [19] is shown in Fig. 11. In this control scheme, sensors are set under foot to recognize hill’s and toe’s state in order to make a decision.
detected state of foot, control signal commands for pressurization or decompression. This type of footswitch control method is an open loop control since there is no control of the error signal.

Fig. 11. Footswitch control developed in [23].

AAFO developed by Blaya and Herr [21] uses series elastic actuator and has shown a good performance in control of drop foot. This AAFO uses variable impedance (stiffness) control scheme. Stiffness control scheme adjusts stiffness of ankle and provides external torque using a spring in series with a linear motor. This AAFO effectively has improved gait of drop foot patients. Stiffness control and usage of elastic material as actuator have been proved to be more close to biological behavior of a muscle [24]. It has been highlighted that important characteristics of AAFO such as wearable and
portable can be difficult to be achieved by traditional robotic designs; and a spring-based linear actuator is needed [24].

Stiffness control strategy is a promising approach to address drop foot problem by AFO. However, current designs lack practical properties to be used in a daily manner. Next section discusses new trends on the employment of smart material in design of AFOs. Smart material offers interesting properties that are interesting to be studied in AFO applications.

2.4. Torsional Shape Memory Alloy

SMAs present features that are not shown in materials traditionally used in engineering. Thus, they are highly considered for innovative applications in various fields. Torsional shape memory alloy is a kind of SMA which its torsion characteristic is exploited. Torsional SMAs can be used for active hinges in AFO applications. To analyze effect of torsional SMA and examine different parameters that effect behavior of this SMA, a model of this actuator is needed. Developed models of SMA were dto examine a hysteresis behavior and find a static model which fits to the experimental results. For the purpose of this research, a model over time is needed to get time dependent input and provide time independent output torque.
Superelasticity property of SMAs takes place when the material changes between martensite and austenite upon loading and unloading of the material accommodating large deformations. These transformations allow for recoverable axial strains of up to 8% which is far greater than the recoverable strains in typical metals [25]-[26]. A one-dimensional (1D) model is developed for SMAs in [27] by Brinson. Based on Brinson’s model, a model is developed for torsional SMAs assuming the SMA undergoes pure torsion [28]-[29]. In this model, the axial model is modified to capture shear stresses and strains as a function of applied torque. This model is used in this research as it is simple and accurate enough.

Hysteresis behavior of SMAs is an interesting property for various applications. In this research, torsional SMAs are modeled and used as actuator in an AFO.

1.4.1 Shape Memory Alloys in AFOs

SMAs can be used as active of passive actuators for AFO applications. Their distinguished characteristics such as high energy/mass ratio and controllable features are interesting in design of AFOs for drop feet. In the passive mode, SMA can absorb energy during plantarflexion and release it in dorsiflexion. It has been discussed that Nitinol holds much promise to be employed in prosthetic and orthotic applications, exploiting both their shape memory and superelastic capabilities within the correct context [30]-[31].

28
SMA wires have been initially studied for lower limb orthoses [30]-[31]. Stirling et al. developed a knee ankle foot prosthesis (KAFP) using SMA wires [32]. In this work, SMA wires are tested for prosthetic applications as it is shown in Fig. 12.

![SMA wires for ankle prosthetic application](image)

**Fig. 12. SMA wires for ankle prosthetic application [32].**

Shape memory property of SMA wires is considered in [28]; meaning that a power supply was used to heat SMA wires in order to provide torque. In another work, Applicability of SMA wires as actuators in active AFOs is discussed in [31]. It has been discovered that SMAs provide the ability to embed actuators into a soft material [31]. However, there were couple of challenges such as too low response time and need for a significant power supply [31]. These research conclude that SMA wires employed as
active actuators may not be appropriate for prosthetic applications. Recently, SMA wires have been used as active actuator to design AFOs for ankle rehabilitation purposes. Fig. 13 shows one an active AFO developed using SMA wires by Pittaccio for rehabilitation purposes of drop foot patients [32].

This rehabilitation device has shown the expected performance which are needed forces and displacements; a dorsiflexion from $-4.5^\circ$ to $+20^\circ$ was achieved. It was
believed that the use of such a device could be beneficial particularly in the early phases of post-stroke care, when it is vital to stop the insurgence of contractures and spasticity [33]. The main problem with the use of SMA in this context appears to be dealing with lengthy and hardly controllable cooling rates [33]. Another rehabilitation device using SMA is developed in [34]. This rehabilitation AAFO device adds electromyographic (EMG) signals to control scheme for more adaptivity.

These studies developed active actuators using SMA wires. A controller was designed to regulate electrical input and thus, heating of wires. Although. These studies reveal that SMAs have interesting features to be considered as actuator in design of AAFOs, but there are some challenges mostly related to response speed and electric power. In this research, we focused on superelastic property of torsional SMA rods. Thus, it is desired that SMA rod absorbs energy during plantarflexion and provides enough dorsiflexion to avoid toe drag in swing phase. Furthermore, it is desired to exploit

2.5. Modeling of Ankle Biomechanics

There are various works on the modeling of a biomechanical system in the literature. Human leg has been considered as a damped, linear and translational spring to understand its function during the stance phase of walking [34]-[36]. Human foot during stance has been modeled using several independent damped spring systems [37].
Modeling of ankle behavior in walking is needed to design and simulate any ankle prosthesis or orthosis. Non-disabled human ankle has been modeled with a passive mechanical device; however, it seems that stiffness (slope of torque-angle characteristic of ankle) may be necessary [38]-[39]. Slope of torque-angle curve is viewed as the dynamic stiffness or quasi-stiffness of the ankle joint as it is not measured at equilibrium [40]. Stiffness of ankle is found to be varying and thus, a dynamic model would be necessary to replicate the ankle behavior [37].

A hysteresis behavior appears in waking for ankle moment-angle function. Dingwell modeled stance phase of walking in [39]. In this work, controlled plantarflexion and dorsiflexion are modeled as linear and non-linear springs, respectively. However, this work only studied stance phase of walking and swing phase was not considered.

A comprehensive model based on stiffness characteristics of ankle was presented in [42]. This model uses a motor in series with a spring to simulate ankle dynamics in whole walking cycle.

In this research, system identification method is developed to model ankle biomechanics. This identification uses a combination of linear and nonlinear functions to describe ankle behavior during walking. System identification theory provides various tools to identify physical systems [43]. In this research, one of various identification methods is used to model human ankle is walking.
2.6. **Problem Description**

Drop foot patients need an effective AFO which can be wearable for daily activities. An effective AFO ameliorates patient’s gait toward a normal gait. Current passive AFOs are not useful for severe drop foot cases and active AFOs are also not practical and wearable. Thus, a novel design of AFO is necessary in order to address this problem. It seems traditionally used materials in engineering would not be a solution for this problem. Thus, shape memory alloys are considered in this research. Superelasticity of SMAs (loading and unloading) seems more effective than shape memory property, since time response of these material is not high enough. The purpose of this study is to examine applicability of torsional SMAs in AFOs for drop foot. There are a lot of parameters involved in this design such as geometry of SMAs and initial angle. Thus, a simulation software is needed in order to examine various designs of torsional SMA-based AFOs for drop foot.

2.7. **Outline**

Next chapter represents the following parts:

- Dynamic modeling of ankle in walking for both normal and drop foot patients,
- Model of torsional SMAs
- Incorporation of ankle model and SMA rod.
Chapter III presents results and simulations. Effects of SMA rods and various parameters involved in the system performance are discussed. This chapter presents an approach to address drop foot problem by an effective design of SMA rod in an AFO.

2.8. Contribution

First, a dynamic model of ankle is developed using system identification theory. Models of ankle joint in both normal and drop foot patient are developed using system identification tool of MATLAB. This model of ankle in walking is important for various reasons such as follows: 1) it enables coupling of any actuator in simulation, and 2) it enables design and test of various controllers.

Second, a dynamic model of torsional shape memory actuators is developed and incorporated to ankle dynamics. This model is employed in analyzing impact of a torsional SMA in an AFO. Furthermore, this model is used to test various designs of torsional SMA in AFO applications.
Chapter Three

Dynamic Modeling of Ankle Biomechanics

3.1. Dynamic Model of Ankle Biomechanics

Stiffness of ankle during level walking is the slope of torque-angle curve. Moment-angle characteristic is unique for every individual; however, it has couple of properties such as follows: 1) hysteresis loop, and 2) speed dependent. Torque-angle characteristic of a healthy individual is shown in Fig. 14. Experimental test to obtain these data are carried out in Motion Analysis Laboratory of The University of Toledo.
Fig. 14. Stiffness characteristic of ankle of a normal subject. HS: Hill strike, FF: Flat foot, MD: Max-dorsiflexion, TO: Toe off.

In Fig. 14, it can be seen that stiffness of ankle is almost constant in every event of a gait cycle. This characteristic was the main reason toward stiffness control scheme. However, stiffness controls might vary in details. Stiffness control developed in [21] employs a linear motor which regulates stiffness of a spring (by changing its length) and provides external torque if needed. This idea has been also developed in active prosthesis control.

Since the purpose of this research was to evaluate performance of a torsional SMA-based AFO, a model of ankle in time domain was needed. In fact, information of
state of foot in a gait cycle was not enough to examine impact of a torsional SMA at every moment.

Therefore, system identification theory was employed in order to find out a dynamic model of ankle during walking. This model is to get input torque in time domain and provide output angle. This is shown in Fig. 15.

![Ankle Model Diagram](image)

**Fig. 15.** Ankle model needed to evaluate a novel AFO design.

Next section discusses the theory employed in this research to identify ankle model.

### 3.2. Data for Identification

Experimental data are collected from normal and drop foot individuals in order to model both normal and abnormal ankle dynamics in walking. Experiments results are collected in motion analysis laboratory of the University of Toledo. Torque and angle of normal and abnormal ankle in a gait cycle for normal speed of walking are shown in Fig. 16. Angle and torque are obtained as 100 time series. Time step size of obtained data based on the speed of walking is presented in Table I. It is
asked individuals to walk in their normal speed and three levels faster and slower of their normal speed. In Table I, fast2 is faster than fast1 and slow2 is slower than slow1.

![Fig. 16. Torque and angle of ankle in a cycle and normal speed.](image)

<table>
<thead>
<tr>
<th>Abnormal Ankle</th>
<th>time step</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slow 2</td>
<td>0.01262</td>
</tr>
<tr>
<td>Slow 1</td>
<td>0.01184</td>
</tr>
<tr>
<td>Normal</td>
<td>0.01137</td>
</tr>
<tr>
<td>Fast 1</td>
<td>0.01062</td>
</tr>
<tr>
<td>Fast 2</td>
<td>0.01014</td>
</tr>
</tbody>
</table>

Table I. Time steps for various speeds of normal ankle and one single speed for abnormal ankle.
Normal speed data is used to identify the model of ankle in walking. Data for various speeds and another individual’s normal speed of walking data are used to validate this model.

### 3.3. Identification of Ankle Model in Walking

Various linear and nonlinear methods have been tested in this research in order to find the most accurate model. Linear method such as linear parametric methods and nonlinear methods such as nonlinear ARX did not show satisfactory prediction (fitness less than 60%). Among all methods tried, nonlinear method called Hemmerstein had the most accurate model describing dynamics of both abnormal and normal ankles. This means that nonlinearities in the system have the character of a static nonlinearity at the input and output side, while the dynamics itself is linear. Thus, ankle model can be depicted as in Fig. 17 in which \( u(t) \) and \( y(t) \) represent torque and angle, respectively.

![Identification method of ankle model](image)

Fig. 17. Identification method of ankle model.
Linear dynamics of a system can be described as follows:

\[ y(t) = G(q)w(t), \quad (1) \]

where, \( y \) and \( u \) are output and input, respectively. Function \( G(q) \) is the discrete time representation of Laplace transfer function \( G(s) \). A generalized model structure of \( G(q) \) can be represented as follows:

\[ G(q) = \frac{B(q)}{F(q)}, \quad (2) \]

\[ B(q) = 1 + b_1 q^{-1} + \cdots + b_{n_b} q^{-n_b} \]

\[ F(q) = 1 + f_1 q^{-1} + \cdots + f_{n_f} q^{-n_f}. \]

Based on (2), one may rewrite (1) as follows:

\[ y(t) + f_1 y(t - 1) + \cdots + f_{n_f} y(t - n_f) = b_1 w(t - 1) + \cdots + b_{n_b} w(t - n_b). \quad (3) \]

Coefficients in (3) are usually considered in a matrix \( \theta \):

\[ \theta = [b_1 \ b_2 \ \cdots \ b_{n_b} f_1 \ f_2 \ \cdots \ f_{n_f}]^T. \quad (4) \]
Thus, one may write (1) as follows:

\[ y(t) = G(q, \theta)w(t), \quad (5) \]

\[ w(t) = f(u(t)). \quad (6) \]

Black-box identification of the system is to determine \( \theta \). Function \( f \) in (6) can be chosen appropriately to model nonlinearities accurately. There are various functions that can be used such as a polynomial, piecewise linear, saturation, sigmoid network and so on. Various nonlinear functions are tested in order to model nonlinearities such as sigmoid and saturation functions. It is been observed that a piecewise linear function has the most accurate answer.

Various algorithms are developed to identify this mathematically modeled system such as Gauss-Newton. Here, predictive error method (PEM) is employed to identify parameters of ankle model. This model is presented as follows:

\[ Ay(t) = \frac{B}{F}u(t - Nk). \quad (7) \]

Various orders for linear block parameters \( B \) and \( F \) are examined. Order of \( B \) and \( F \) are adjusted by a theoretical discussion as follows: a second order model has been used.
in the literature to model dynamics of a muscle. However, in a walking cycle, dorsiflexion and plantarflexion must be modeled; thus, a forth order differential equation for $F$ seem reasonable. Furthermore, always one of dorsiflexion or plantarflexion occurs; thus, it seems reasonable to have a second order $B$ in nominator. Computational also shows that a system of $B$ with an order of 2 and $F$ with an order of 4 has the most accurate prediction. Looking at angle and torque data, it is clear that their optimum points do not occur at the same time. Thus a delay might be necessary to predict torque-angle relationship. A delay is employed in input which increased the accuracy by 5%, meaning that $Nk = 1$.

10 break points are introduced for piecewise linear function to structure nonlinear block of the model.

Obtained linear function is as follows:

$$T = \frac{0.6078z^2 - 0.6078z}{z^4 - 3.61z^3 + 5.011z^2 - 3.159z + 0.7623} \quad (8)$$

where, $z$ is the Laplace operator in the discrete-time.
For abnormal ankle during walking, the same approach has been processed.

\[
T = \frac{7.895z^2 - 7.78z}{z^4 - 1.41z^3 - 0.4491z^2 + 1.4159z - 0.5456} \quad (8)
\]

It can be seen from (8) and (9) that these two models have different dynamics. They both have two zeros at zero and about 1. However, their poles are at different points. Validation of these models is an important part of identification. Obtained normal ankle model is validated using two other speeds of walking (slower and faster). However, only fitness of identified model for abnormal ankle was considered as validation of this model.

Fig. 18 shows how the validation was accomplished. Obtained torque from experiment test was given as input to the predicted model and its output is the angle that the model predicts. This output is compared with angle which is obtained from experiment test for validation.
Fig. 18. Validation of ankle model obtained from identification.

In summary, a model of ankle during walking is presented. System identification approach is employed to obtain a model which can predict ankle dynamics. The model has a linear block which constructs dominant dynamics of ankle. Furthermore, there is a nonlinear block which models nonlinearities in the input.

### 3.4. Modeling of Torsional SMA

Wire SMAs have been considered for lower limb orthoses applications. There are serious challenges associated with wire SMAs with current designs. Response speed and power supply to control actuation of these materials are these challenges. However, shape memory alloys have got considerable benefits that are encouraging for medical applications.
Since response speed and power problems have not been fully resolved yet, passive designs using SMAs are more realistic at this point. Furthermore, a lightweight and wearable design is needed in order to get into daily life of patients. All of these reasons initiated this research to examine applicability of torsional SMA for application in AFOs for drop foot patients. This chapter presents computational results of incorporating torsional SMA into a hinged AFO. Next section discusses modeling of torsional SMA. Section 4.3 presents incorporation of torsional SMA to a hinged AFO.

The shear strain $\gamma$ is expressed as a function of the angular deflection $\theta$ and the radial location $r$ as follows:

$$\gamma(r) = \frac{\theta r}{l}, \quad (9)$$

where, $l$ is the length of the uniform rod. From the classical one-dimensional torsion theory, the shear strains and linear strains can be related as follows:

$$\epsilon(r) = \frac{\gamma(r)}{2}, \quad (10)$$

The equivalent linear strains are obtained from each radial location for every angular displacement. The linear stresses are then calculated from the linear strains using uniaxial SMA (in this case, Brinson’s) model as follows:
\[ \sigma(r) = \sigma(\varepsilon, T, r), \quad (11) \]

where \( T \) is the temperature of the rod and \( r \) is the radial location.

The axial stresses at each radial station are related to the equivalent shear stresses using Poisson’s effect as follows:

\[ \tau(r) = \frac{\sigma(r)}{1 + \nu}, \quad (12) \]

where \( \nu \) is the Poisson’s ratio for the material. Thus, torque at every angle is found by integration of the shear stresses over the radial locations as follows:

\[ T = \int_0^{r_1} 2\pi r^n \tau(r) \, dr, \quad (13) \]

where \( \tau \) is the shear stress for each radial, \( r_1 \) is the radius of the rod and \( T \) is the total torque that is applied to cause the angular deflection of \( \theta \).

Therefore, model of SMA is to calculate total torque at every moment based on varying angular deflection given as input.
Function $\sigma$ in (11) is calculated in various ways in the literature. However, a simple method is to obtain this function from Fig. 19.

![Stress strain behavior](image)

Fig. 19. Stress strain behavior of a superelastic shape memory alloy material.

In Fig. 19, $\sigma_{Mf}$, $\sigma_{Ms}$, $\sigma_{As}$ and $\sigma_{Af}$ are critical stresses. After $\sigma_{Af}$, material undergoes phase transformation to martensite. Equations describing Fig. 19 are obtained as follows:
Critical stresses are calculated using material properties and temperature using phase plot diagram of a shape memory alloy material presented in Fig. 20.
Fig. 20. Typical phase plot diagram of a shape memory alloy material.

Therefore, developed model of SMA has a diagram which is shown in Fig. 21. First block constructs critical stresses and second block finds torque for every angular deflection.
This model is validated by experimental results [26].

3.5. Incorporating Torsional SMA into AFP

Ankle diagram is shown in Fig. 22. A hinged AFO can be designed and exploit SMA in a way that can rotate with ankle angle, then the total torque is as follows:

\[
\tau_{net} = \tau_{SMA} - \tau_{ankle} , \quad (16)
\]

where \(\tau_{ankle}\) and \(\tau_{SMA}\) are ankle and SMA torque, respectively.

Such a design can be mechanically controlled in a way to engage or disengage rotation of SMA rod.
For drop foot patient, it is desired that SMA provides torque after toe off and increases stiffness to prevent slap foot. Thus, desired mechanical functions for SMA rod are shown in Table II.

Table II. Mechanical desired structure to incorporate SMA rod in AFO.

<table>
<thead>
<tr>
<th>hill strike-foot flat</th>
<th>foot flat-max dorsiflexion</th>
<th>max dorsiflexion-toe off</th>
<th>toe off-hill strike</th>
</tr>
</thead>
<tbody>
<tr>
<td>engage</td>
<td>disengage</td>
<td>engage</td>
<td>disengage</td>
</tr>
</tbody>
</table>

Table II illustrates how SMA loads during plantarflexions in a gait cycle. Thus, from max-dorsiflexion to toe off, SMA loads; and then, it unloads and provides torque to push toe upward. To prevent slap foot from hill strike to flat foot, SMA adds stiffness to ankle and acts like a spring.
A system is developed to incorporate SMA model to ankle model in order to analyze whole system behavior. Diagram of this system is shown in Fig. 23.

Fig. 23. Diagram of system to examine impact of torsional SMA on ankle dynamics.

In Fig. 23, mechanical structure block is programmed in order to apply engage and disengage functions. Ankle model is used to get input as angle and as torque. Ankle model that gets angle as output has initial values to start the model.

There are three parameters that SMA rod can be controlled to display desired function, 1) Geometry of rod, 2) Initial angle, and 3) Temperature. In this research, the effects of geometry and initial angle of rod are considered.
Chapter Four

Results and Discussion

First, a dynamic model of ankle in walking was identified. Fig. 24 shows the prediction of normal ankle model with 95% fitness. Prediction of abnormal ankle model with 94% is shown in Fig. 25.

Fig. 24. Prediction using PEM method for normal ankle, 95% fit.
Fig. 25. Prediction using PEM method for abnormal ankle, 94% fit.

Obtained output angle of ankle model is compared with experimentally obtained angle in Fig. 26 for a speed of little lower than normal speed of walking. Fig. 27 displays the comparison of predicted angle and experimental angle for a speed of a little higher than normal speed. Prediction error for lower speed was 10% and for higher speed was 22%.
Fig. 26. Comparison of predicted angle and experimental angle, lower speed.

Fig. 27. Comparison of predicted angle and experimental angle, higher speed.
Torsional behavior of SMA is modeled and validated by experimental data. Fig. 28, Fig. 29 and Fig. 30 show the model prediction for rods with 0.023", 0.020" and 0.018" diameters, respectively. Length of all rods is 0.4" length. Material properties of rods are shown in Table II.

Table II. Material properties of rods.

<table>
<thead>
<tr>
<th>$M_f$</th>
<th>$M_s$</th>
<th>$A_s$</th>
<th>$A_f$</th>
<th>$C_m$</th>
<th>$C_a$</th>
<th>$S_x$</th>
<th>$S_f$</th>
<th>$E_a$</th>
<th>$E_m$</th>
<th>$e_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-26</td>
<td>-19</td>
<td>-18</td>
<td>-2</td>
<td>7.5</td>
<td>7</td>
<td>10</td>
<td>10</td>
<td>39000</td>
<td>31000</td>
<td>0.032</td>
</tr>
</tbody>
</table>

Fig. 28. Prediction of model for 0.023" diameter rod.
Fig. 29. Prediction of model for 0.020" diameter rod.

Fig. 30. Prediction of model for 0.018" diameter rod.
Then, these two models were coupled in order to examine impact of SMA rod on gait cycle. Results of final system are presented in Fig. 31 to Fig. 34. In first scenario, a rod with 40 mm length and 2 mm diameter was considered. Fig. 31 shows torque-angle relationship of this rod.

![Fig. 31. Torque-angle relationship of a rod with 40 mm length and 2 mm.](image)

With initial angle at zero degree, this rod will act like a linear spring since maximum angular deflection will be on linear part of Fig. 31. Performance of whole system with this SMA is shown in Fig. 32.
Fig. 32. Performance of AFO with an SMA acting in linear area.

It can be seen that SMA acting like a linear spring improved plantarflexion between hill strike and flat foot. However, it could only slightly improve dorsiflexion after toe off. Developed software is able to be used in design of a suitable SMA with different geometries and initial angles.

Hysteresis behavior of SMA is of much interest in pushing toe upward. This is because to push toe upward, a higher torque is needed than when ankle angle is zero. This requirement can be completely fulfilled by hysteresis behavior of SMA. Increasing diameter of rod or decreasing its length results in higher torque that SMA can provide.
Among various tests, it has been found that a rod with 20 mm length and 2 mm shows a much better performance. Fig. 30 compares proposed design with patient ankle behavior.

![Graph showing ankle angle for different gaits](image)

Fig. 33. Performance of an AFO with 20 mm length and 2 mm SMA rod.

This design was more improved to allow a more normal plantarflexion and also better toe off. This time, a rod with 10 mm length and 2 mm was tested. Fig. 34 shows ankle angle for two normal gaits, one abnormal gait and one with proposed design of SMA in AFO.
Fig. 34. Performance of AFO with proposed SMA design is shown as black line.

This reveals that desired performance can be achieved by controlling geometry of SMA rod.

In summary, results display how SMA acted as a linear spring at hill strike and a controlled plantarflexion was obtained. This could effectively solve slap foot problem. It also was shown that at toe off, as SMA unloads it provides assistive torque to push foot upward. Due to hysteresis behavior of SMA, this torque was of higher magnitude in the beginning than later. This behavior is desired since higher torque is needed to dorsiflex foot in the beginning of toe off.
A comparison between steal hinged and SMA hinged AFOs is informative in order to study effectiveness of this novel structure. Fig. 35 displays this comparison for a 15 mm length and 2 mm diameter SMA rod.

![Graph showing comparison between simple hinged AFO and SMA AFO](image)

Fig. 35. Comparison of steal hinged AFO and SMA AFO.

SMA AFO is more active during the gait than simple hinged AFO. It can be seen that during hill strike to foot flat, SMA AFO provides more resistance to prevent slap food. This makes a more plantarflexion than simple hinged AFO. Furthermore, it can be seen that excessive plantarflexion during max-dorsiflexion to toe off is more treated in SMA AFO.
It should be noted that the results shown in Fig. 35 can be improved by increasing the diameter and decreasing the length of SMA rod. In fact, the ideal prescription of this novel AFO for patient needs a computational software. This software is to get gait data of the patient and suggest a suitable diameter and length of SMA rod. This was the main purpose of this research that is to provide a computer design software for physician to design the right AFO for every patient.
Chapter Five

Future Work

This research has presented a hinged AFO using torsional shape memory alloy. For my most knowledge, this design has not been developed before. This is an initial study that evaluates potential of torsional SMA to be used in AFO applications.

This research has presented an initial study on the development of torsional SMA for AFO applications. Model of ankle was developed to be used in coupling with a torsional SMA rod and study its impacts on the human gait. Since developed models for ankle were not useful to model ankle dynamics in walking, system identification was used to obtain a model that described model of ankle. A dynamic model of torsional SMA was also introduced. Finally, these two models were coupled together to display whole system performance. It has been shown that performance of proposed AFO depends on design of torsional SMA.

Future work can be on different aspect of this study such as optimization of design and its implementation. If response speed of SMA not a challenge, then this structure can also be used in an active scheme.
Works Cited


