A Thesis

entitled

Droplet Trajectory and Breakup Modeling with Comparisons to Previous Investigators’ Experimental Results for Slinger Atomizers

by

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Submitted to the Graduate Faculty as partial fulfillment of the requirements for the

Master of Science Degree in Mechanical Engineering

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An abstract of

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The Small Gas Turbine aircraft engine emerged in the 1940s as a distinct class of turbine engines. Following the late 1940s, the powerful, lightweight and reliable small gas turbine engine evolved as a major technology development leading to a new generation of aircraft, rotorcraft and missiles. The compact size of the small gas turbine engine inhibits the use of the fuel injection systems used in conventional gas turbine engines. Hence an effective and efficient fuel injection system which is also simple in design was required to produce high-quality atomization of fuel in the small gas turbine engine. A centrifugal atomizer or rotary fuel slinger is designed to work on simple principles of engineering mechanics, while providing fine atomization of liquid. This research is an attempt to provide insight into the mechanism of atomization produced by fuel slingers and develop a mathematical model to predict the droplet trajectories, based on the elements of droplet breakup dynamics. The results of the model were further evaluated to test for compatibility with existing theories of atomization and thereby assess the
applicability of the model to explain the complex mechanisms of liquid breakup from these atomizers. Mathematical expressions to predict the spatial and temporal positions of droplets were developed using appropriate relations for the coefficient of drag which accounted for the deformation of the droplets and reasonably accurate correspondence with experimentally visualized droplet trajectories has been observed. An analysis of the influence of hydrogen bonding in water, on the droplet breakup mechanism has also been made to account for the surface tension forces involved in atomization.
To my parents, brother and friends, who have been ever supportive of me, guiding and encouraging me through my endeavors.
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\( d_1 \) diameter of the jet at the wider section (m)

\( d_2 \) diameter of the jet at the narrower section (m)

\( P_1 \) pressure in the jet at the wider section (MPa)

\( P_2 \) pressure in the jet at the narrower section (MPa)

\( l \) length of the jet segment (m)

\( \sigma \) surface tension of the liquid (N/m)

\( D \) diameter of the drop resulting from jet breakup/characteristic length of the body (m)

\( d_0 \) diameter of the jet segment of length \( \lambda \) (m)

\( \rho \) density of the liquid/gas (kg/m\(^3\))

\( U \) velocity of the liquid/gas (m/sec)

\( \mu \) dynamic viscosity of the liquid/gas (Pa-sec)

\( \text{Re} \) Reynolds number, \( \text{Re} = \rho UD/\mu \)

\( \text{We} \) Weber number, \( \text{We} = \rho U^2 D/\sigma \)

\( \text{Oh} \) Ohnesorge number, \( \text{Oh} = \mu/ (\rho \sigma D)^{1/2} \)

\( C_D \) coefficient of drag of the droplet

\( \text{We}^* \) critical value of We corresponding to breakup

\( D_{pq} \) mean drop diameter based on different definitions

\( D_{32} \) Sauter Mean Diameter
D_{10}  \quad \text{Arithmetic Mean Diameter}

D_{30}  \quad \text{Volume Mean Diameter}

D_n  \quad \text{denotes that 10\% of the liquid volume consists of droplets with diameter smaller than } D_n

V'  \quad \text{volume of the drop, } V' = \frac{4}{3} \pi \left(\frac{D}{2}\right)^3 \text{ (m}^3\text{)}

W  \quad \text{relative velocity between the drop and air } W = \frac{dS}{dt} \text{ (m/sec)}

U_b  \quad \text{bulk velocity of the liquid at the exit of the slinger (m/sec)}

Q  \quad \text{volume flow rate of the liquid in the radial channel (m}^3\text{/sec)}

d_{\text{hole}}  \quad \text{diameter of the channel in the slinger (m)}

b  \quad \text{film thickness in the channel (m)}

W_r  \quad \text{radial component of the velocity, } W_r = U_b = \frac{q}{\pi b + d_{\text{hole}}} \text{ (m/sec)}

R  \quad \text{outer radius of the slinger (m)}

\omega  \quad \text{angular speed of the slinger (rad/sec)}

W_t  \quad \text{tangential component of the velocity, } W_t = R\omega \text{ (m/sec)},

\alpha  \quad \text{angle between } W_r \text{ and } W_t, \quad \alpha = \arctan \left(\frac{W_r}{W_t}\right) \text{ (rad)}

\rho_L/\rho_G  \quad \text{liquid-gas density ratio}

D_{\text{max}}  \quad \text{corresponds to the maximum drop diameter at the critical conditions (m)}

D_{\text{min}}  \quad \text{minimum stream wise diameter (m)}

C_{D_{\text{sp}}}  \quad \text{coefficient of drag of a sphere of diameter } D_0, \text{ at the Re corresponding to the local coefficient of drag of the deformed droplet}

\Delta p  \quad \text{pressure difference across a tensioned surface (MPa)}

R_x, R_y  \quad \text{radii of curvature in each of the axes that are parallel to the surface (m)}

\lambda_{\text{m}}  \quad \text{the most unstable wavelength of the wave disturbance on the jet surface (m)}

u_0  \quad \text{initial velocity of the jet (m/sec)}
\( Z^* \) modified Ohnesorge number, \( Z^* = (3\mu_L + \mu_{\text{air}})/(d_0\sigma_L)^{1/2} \)

d\( d_j \) diameter of the jet before breakup (m)

\( U_G \) velocity of the gas (m/sec)

\( t^* \) aerodynamic characteristics time, \( t^* = d_j(\rho_L/\rho_G)^{1/2}/U_G \) (sec)
CHAPTER 1

Introduction

1.1 Small gas turbines and purpose of development:

Since their development, gas turbines have seen many advances in the design, performance and extent of applications for commuting, military and space research purposes. One of these is the Small Gas Turbine. A Small Gas Turbine is a scaled-down version of the typical gas turbine, designed for lower cycle pressures and temperatures because of smaller blading and cooling limitations [1]. Consequently, they are used for low-power applications. In conventional gas turbine engines, the fuel is atomized with the help of a high-pressure fuel pump. Pressure-atomized sprays result in fine atomization since the pressure energy is converted to kinetic energy and the droplets thus formed have high velocities relative to the surrounding medium. Thus pressure atomizers are the most widely-used type of atomizers [2].

However, for compact applications, appropriately smaller pressure pumps cannot be used for small gas turbines. As a result, the small gas turbines used in corporate jets, Unmanned Air Vehicles (UAV), rotorcraft and cruise missile applications find the use of
pressure atomizers disadvantageous. The replacement of the typical fuel injection systems with a centrifugal atomizer design allows for high-speed operation, low-cost, reduced weight and simple design requirements for the purpose of fuel atomization.

1.2 Atomization in centrifugal atomizers:

In combustion applications, the purpose of atomization is to increase the surface area of the fuel and to achieve high rates of mixing and evaporation. Finer fuel droplet sizes result in higher volumetric heat release rates, easier ignition, a wider burning range, and lower exhaust concentrations of pollutant emissions [2]. Atomization in small gas turbines is achieved by centrifugal atomizers or rotary fuel slingers. Rotary fuel slingers produce fine fuel droplets by using centrifugal forces generated due to high-speed rotation. Fuel is supplied at low pressure along the hollow main shaft and is discharged in the radial direction due to centrifugal force through holes drilled in the shaft. Fuel discharges from the holes with high velocity, which corresponds to a pressure drop of 13-16 MPa [3].

A typical rotary slinger includes a multitude of radial holes on its periphery for even distribution of fuel from the center of rotation of the injector through channels of small diameter. These holes may possess different shapes and patterns and the channels may also follow different configurations (normal to the surface and inclined to the surface), accordingly affecting the atomization performance of the slinger [4]. The experimental study of the test slinger setup used for analysis included a slinger with a single hole only, for capturing the details of the liquid breakup more accurately and to
eliminate the inter-hole flow repartitions caused by manufacturing inaccuracies.

**1.3 Objective of thesis:**

Many attempts have been made to analyze the mechanism of liquid atomization under various physical and flow conditions and develop adequate models [5]. Some of these theories have been developed on an empirical basis as well as experimentally. Similar progress has been made with regards to a centrifugal atomizer and some of them [4], [6] have been successful in providing a partial understanding of the sophisticated processes. However, no specific rules or empirical relations have been arrived at, so far, describing and predicting the atomization performance of a slinger for different operating conditions and liquids. Some efforts have been made to correlate the expressions and governing principles of liquid jets atomized by different mechanisms with the case of atomization by rotary slingers, as current research is very limited.

This thesis is intended to substantiate the development of an analytical model which can reasonably explain a part of the liquid breakup mechanism (secondary breakup) and provide further insight into the complex phenomena involved in the disintegration of liquid jets exiting radially out of the holes of the slinger. The results of the model were compared with the data acquired experimentally by Sescu et al. [7] and sufficiently close correspondence was observed with regards to some important parameters. This analytical model was aimed at establishing mathematical expressions for the governing parameters of the liquid jet flow outside the slinger, such as velocity and displacement, which may define the trajectory of the liquid jet outside the slinger.
Further, an attempt was made to analyze the important types of liquid breakup, the primary jet breakup and the secondary breakup. These breakup types, whose spatial and temporal points/phases of occurrence depend on various conditions, are explained in forthcoming sections. The analysis on the breakup types was initiated by a classical approach and attempts were made to arrive at a reasonable base to predict the temporal behavior of breakup parameters corresponding to rotary slingers, by comparing with analyses and experimental data corresponding to straight jet/drop injectors whence, valuable observations and additions have been made with regards to the case of jet disintegration by a rotary slinger.

The results obtained from the analytical model were found to have reasonable consistency with the data interpreted from the images of the liquid jet obtained experimentally. Since the model developed was based on the properties of fluids such as density and viscosity, it may be considered fairly reasonable to extend its applicability to other liquids whose density and viscosity ratios are within the limits specified by analyses previously made, since past investigations have proven that liquid atomization in gases follow specific patterns of analysis for specific density ratios.
Figure 1.1 Photograph of the actual slinger setup in the lab.

Figure 1.2 Schematic of the experimental rig: 1-slinger; 2-slinger shield; 3-high-speed electric motor; 4-motor shield; 5-motor controller; 6-chassis; 7-liquid (fuel) tank; 8-metering pump; 9-feeding system.

Figure 1.2 Layout of the experimental rig [7].
1.4 Overview of slinger operation and experimental data acquisition:

The fuel slinger is the key component of the injection system of the small gas turbine. A generic model of a fuel slinger consists of a hollow axisymmetric shaft which is filled by fuel fed axially at a low pressure. Due to the high speeds of rotation, the fuel is slung radially outwards by centrifugal forces and is forced into small, radially-oriented channels. The fuel spreads evenly onto the periphery of the channels like a film and a limiting film thickness is reached when the shear stress at the circumferential boundary is balanced by the centrifugal force on the volume of the film in the channel [4]. The current analysis involves the use of water as the working fluid because of safety concerns over flammability.

Typical slingers operate at speeds of around 30000 rpm. The slinger, for which this model was developed, was tested at rotational speeds of 5100 rpm, 6000 rpm, 7200 rpm, 8100 rpm, 9000 rpm, 10200 rpm and 13200 rpm and flow rates of 14 mL/min and 32 mL/min. A fluid delivery and metering system was used to provide the desired liquid flow rate to the slinger.

Drops were measured after the breakup of the stream exiting from the slinger using an interferometric technique called Global Sizing Velocimetry [7]. This method involves the measurement of the intensity oscillation of scattered light from particles and it is used assuming the particles are spherical. Hence the drops are measured at a distance where they are nearly spherical. Images of the stream exiting the slinger were also taken to observe the characteristics of jet breakup under different operating conditions.
The mechanism of the stream breakup is complex and can be categorized into specific types due to unique characteristics displayed by each of them. These mechanisms are described in the next few sections.

Figure 1.3 Schematic of a slinger
1.5 Types of breakup:

As mentioned earlier, the process of liquid atomization has been widely known to follow the types of breakup described below.

1.5.1 Primary breakup:

This type of breakup pertains to the disintegration of the liquid stream/jet as it flows from the channel exit and is exposed to the ambient fluid, which may be air or other fluids, stationary or in motion, resulting in fragmentation of the stream into irregularly shaped globules of liquid. The disintegration of jets into drops depends on the velocities of discharge from the nozzle exit. The modes of primary jet breakup are classified into three types. The jets, whose velocities of discharge are of the order 1 m/sec, 10 m/sec and 100 m/sec, breakup due to axisymmetric waves, asymmetric waves and aerodynamic forces, respectively [3]. Axisymmetric waves cause narrow and wide bands in the jet, which correspond to high and low pressure regions, respectively, as shown in Figure 1.4. This approach is explained by the balance of pressure forces by the surface tension forces, assuming the binding forces are due to surface tension forces. The liquid is forced from the narrow bands to the wider bands, resulting in formation of drops.

\[ d_1 l P_1 = 2 l \sigma \]

\[ d_2 l P_2 = 2 l \sigma \]  

\[ \text{......Eqn. 1.1} \]

Further, the volume of a drop of diameter D, resulting from the jet breakup is equal to the volume of the jet segment of diameter \( d_0 \) and length \( \lambda \).
The surface energy is the surface tension force times the area on which it acts.
The lower the surface energy, the more stable the volume. Since the drop is more stable
than the jet segment, we get

\[ \frac{\pi D^3}{6} = \frac{\pi d_0^2}{4} \lambda \] ........Eqn. 1.2

\[ \pi D^2 \sigma \leq \pi d_0 \lambda \sigma \] ........Eqn. 1.3

Hence,

\[ D \geq 1.5d_0 \] ........Eqn. 1.4

Figure 1.4 Schematic of jet breakup due to axisymmetric, asymmetric and aerodynamic
wave disturbances, respectively [3].
Asymmetric waves may be referred to as waves caused by the ambient pressure forces acting on the jet resulting in convexities (low pressure) and concavities (high pressure). Since these are caused by aerodynamic forces, they are usually observed at higher discharge velocities compared to axisymmetric waves. Aerodynamic forces disintegrate the liquid jets when the discharge velocities are much higher and may reach supersonic speeds. The hierarchy of the order of size of the drops decreases with the largest drop-sizes by disintegration due to axisymmetric waves and the smallest drops by disintegration due to aerodynamic forces [3]. Also notable is that the length of the jet before breakup is the maximum for the case of breakup due to axisymmetric and decreases for the case due to aerodynamic forces.

A majority of the theories on the primary breakup of liquid jets at low discharge velocities are based on the concept of breakup due to the loss of stability of the jets after they exit from the nozzle of the injector [3]. The instability of these jets is explained by the development of wave disturbances on the surface of the jet, whose amplitudes grow rapidly. These disturbances grow in amplitude along the surface of the waves and eventually lead to instability of the fluid jet, resulting in drop formation. The disturbances could be caused by external or internal sources. External causes refer to the disturbances due to aerodynamic forces, which depend on the density of the medium and relative velocity between the jet and the surrounding medium. Internal causes may be due to disturbances within the injector nozzle, such as turbulence just at the injector exit, liquid expansion due to pressure drop, or vibration of the injector-needle.
The method of analysis of primary breakup of jets due to wave disturbances, called the method of small disturbances, is based on the concept of superposition of small vibrations and the motion of the liquid jet. The procedure involves iterating values of infinitesimally small incremental changes in pressure and velocity into the governing equations for the flow, namely the Navier-Stokes equations and the continuity equation. Linearization and integration of these equations lead to a system of equations that can be used to obtain the frequencies at which the amplitudes grow large enough to cause disintegration of the jet into drops. Solution to these equations was first obtained by Rayleigh [3].

Primary breakup may also be classified according to the flow regime obeyed by the stream exiting from the slinger hole into the cross-flow of the ambient medium. Non-turbulent primary breakup may occur when the Re, based on the velocity on the surface of the jet, at the exit of the slinger, is less than the characteristic Re defining turbulent flow for such flow configurations. Satisfactory discussion [4] suggests that, for typical rotary slingers, the critical value of the film Re for jet fuel, based on the film thickness and surface velocity in the radial channel is around 400, but since the working fluid in our case is water, this value may differ.

For primary breakup, different modes of breakup have been defined. These are similar to those classified for secondary breakup, namely “liquid column breakup”, “bag breakup”, “multimode breakup” and “shear breakup”. These modes apply to different regions on the We-Oh map [5]. Mazallon et al [8] observed, however, that for low Ohnesorge number, \( \text{Oh} = \mu_L/\left[\rho_L \sigma_d j\right]^{1/2} < 0.1 \) (where \( \mu_L \) is the liquid dynamic viscosity),
the breakup regime transitions are controlled by the cross flow Weber number, $W_e$ alone. Breakup regime transitions of the liquid jet are determined by the cross flow Weber number as follows: column breakup ($W_{eG} < 4$), bag breakup ($4 < W_{eG} < 30$) (modified by Sallam et al [9]), multimode breakup ($30 < W_{eG} < 110$), and shear breakup ($W_{eG} > 110$).

Studies made on primary breakup also explain the formation of column waves that occur along the stream wise direction of the liquid jet in a cross flow of air. Apart from the column waves, surface waves are also observed on the downwind region of the jet, which evolve into bag-like formations due to high pressure on the upwind side, hence called bag breakup [8]. The drops formed from the breakup can again be classified as bag-droplets or ring and node droplets, since the bag-like structure bloats with a rim/ring-like structure which has different number of nodes.

Some results show that the column waves of a non-turbulent liquid jet in cross flow within the bag breakup regime can be explained based on Rayleigh–Taylor instability. Even though the case of the rotary slinger assumes jet breakup in stationary air, some valuable correlations may be made with the one with gas cross flow.

1.5.2 Secondary breakup:

The drops that result from primary breakup are coarsely-shaped globules, which further break up into round droplets due to aerodynamic forces. The aerodynamic forces tend to develop a pressure distribution around the drop, as the relative velocity of the gas around it increases and deforms it into oblate spheroids. The force opposing such
deformation in the case of low viscosity liquids is the surface tension force and in addition to that, may include the force due to shear stress for the viscous fluids. Initial methods to analyze secondary breakup includes the consideration of equilibrium of the drag force and the surface tension force at the critical point of breakup, assuming the drops are rigid spheres [3]. An analysis of the secondary breakup of drops from the fuel slinger under consideration, based on this classical approach is described in Chapter 4.

\[ C_D \frac{\pi D^2 \rho g U^2}{2} = \pi D \sigma \] \hspace{1cm} \text{Eqn. 1.5}

The above equation can be written in terms of a non-dimensionalized parameter called Weber number, \( \text{We} \), which gives the ratio of the fluid’s inertia to the surface tension.

\[ \text{We} = \frac{\rho g U^2 D}{\sigma} \] \hspace{1cm} \text{Eqn. 1.6}

For critical conditions, Eqn. 1.7 gives the approximate maximum diameter the drop can reach before secondary breakup.

\[ D_{\text{max}} = \frac{\sigma \text{We}^*}{\rho g U^2} \] \hspace{1cm} \text{Eqn. 1.7}

Some basic models on secondary breakup assume the formation of boundary layers on the surface of the drops. Since the drops are assumed to be rigid spheres, the coefficient of drag (\( C_D \)) for these drops can be calculated from the relations between \( C_D \) and \( \text{Re} \) for spheres corresponding to the different regimes of flow; laminar, transition and turbulent. However, in actuality, the aerodynamic forces deform the drops in the transverse direction of the flow of the medium around the drops. Secondary breakup is
widely assumed to follow different modes depending on the We and Ohnesorge number, 
Oh, which gives the ratio of the viscous force to the surface tension force.

\[ \text{Oh} = \frac{\mu_L}{\sqrt{D\sigma_L\rho_L}} \]  \quad \text{Eqn. 1.8}

The different modes being oscillatory deformation, where the drops oscillate with 
a weakly damped amplitude, bag breakup, in which the drops evolve into bag-like 
structures, multimode breakup, which can be further classified into bag/plume breakup 
and plume/shear breakup and shear breakup, which occurs at very high velocities and 
corresponds to particles being sheared off the rim of the drops. These modes are 
represented on a We-Oh map for round liquid jets subjected to shockwave disturbances 
[5]. The map corresponds to Reynolds number, Re>100, 
We = \frac{\rhoGV_oD_b^2}{\sigma} \quad \text{and} \quad \text{Oh} = \frac{\mu_L}{\sqrt{D_o\sigma_L\rho_L}}, \quad \text{where “0” represents the initial conditions. Secondary} 
breakup may be treated as a rate process, rather than a spatially-governed process. 
Expressions have been developed to describe the time of drop breakup in terms of We 
and Oh for shockwave disturbances [10].
Figure 1.5 We-Oh map indicating different modes of secondary breakup for shockwave disturbances [8].

The above map shows that significant amounts of deformation and breakup occur when the drag force is comparable with the opposing surface tension force, with the liquid viscosity being low. Extensive research on the properties of secondary breakup relative to time has been covered for the case of bag and the multimode breakup type in the presence of shockwave disturbances [11], [12]. The study elaborates on the breakup involving water and ethanol drops, whose liquid/gas density ratios are greater than 500, Ohnesorge numbers less than 0.1 and Weber numbers of 15-150. In addition to this, some efforts have also been made to analyze the jet breakup and drop disintegration under steady disturbances [13]. Further analysis of secondary breakup has been made and elaborated on in the section pertaining to it.
CHAPTER 2

Description of the Experimental Setup and Data Acquisition Methods

The analytical model hence developed is based on and also verified by the experimental data acquired by Sescu et al [7]. The experimental study involved the analysis of the disintegration of a liquid jet ejected by a rotary slinger. As described earlier, the rotary slinger works on the principle of liquid atomization by centrifugal forces when run at high rotational speeds. The fuel is inducted into an axisymmetric cylindrical chamber at a low pressure and a prescribed flow rate and due to high rotational velocities; the fuel is discharged into small round holes on the inner periphery of the slinger by centrifugal forces. These holes extend as radial passages to the outer periphery, where the liquid is ejected as a jet. Different hole-patterns have been tested and various conclusions have been made regarding the shape and size of the holes [4]. In this experimental setup, the slinger was drilled with a single hole to avoid the flow interposition from different holes hindering clear observation of the breakup process.

In real cases of multi-holed slinger designs, the holes are required to be machined accurately, since the uniformity of flow inside the radial channels is heavily dependent on the quality of the surface finish of the flow passage. The flow rate in the experimental slinger was regulated to prevent the hole from getting completely filled by liquid at the corresponding speed of rotation. Some hole configurations include inclined passages.
from the inner periphery to the outer periphery of the slinger. For safety concerns regarding flammability, the liquid used was water and since the experiment was purported to mainly understand the atomization process in rotary slingers, some correlations can be made between the atomization and operating parameters for the test slinger and those for typical fuel slingers. The studies made by Dahm et al [4] confirm the observation of fuel spreading in the form of a film in the radial passages. An expression for the limiting film thickness has also been developed, which is related to the bulk average film velocity. A similar phenomenon has been presumed in the experimental study discussed here. However, an attempt to observe and measure the film thickness could not be made due to lack of appropriate equipment.

2.1 Experimental setup:

The slinger was run at speeds of 7200 rpm, 10200 rpm and 13200 rpm at flow rates of 14 mL/min and 32 mL/min. The test slinger was positioned and held by a chuck with a diameter of 5/8 in [7]. A hole of diameter $0.889 \times 10^{-3}$ m (0.035 in) was drilled perpendicular to the slinger axis and in the radial direction. The external diameter of the slinger was $36.195 \times 10^{-3}$ m (1.425 in), and the internal diameter was $28.575 \times 10^{-3}$ m (1.125 in). The slinger was powered by a high-speed electric motor that allowed rotational speeds in the range of 500-24,000 rpm. The required rotational velocity was regulated accurately by a digital controller. The liquid was supplied by a fluid delivery and metering system at the desired liquid flow rate to the slinger. A computer-controlled peristaltic pump was used to feed the slinger while providing good flow uniformity. The liquid flow rate could be adjusted with an accuracy of 0.5 mL/min.
2.2 Measurement of drop size:

A technique called Global Sizing Velocimetry (GSV) was used for measuring the droplet sizes. This interferometric method uses defocused images to record the intensity oscillation of scattered light from droplet particles. The size measurement is based on the angular oscillations of scattered light, which have a uniform spacing inversely proportional to the particle diameter. The basic theory of the method is based on the assumption that the particles are spherical. The TSI system was composed of an Nd: YAG pulse laser with vertical polarity, a camera linked to a synchronizer and a high-speed digital frame grabber. A cylindrical lens was used to generate the light sheet. A 4MP camera with a standard 105 mm lens was used to capture images. A 532 nm band pass, a 4X neutral density filter and a slit were placed on the camera lens. The system was driven by Insight software.

The optical setup was able to measure droplets within a range of diameters of 15-405 μm. The advantage of using GSV is that GSV is not sensitive to the droplet refractive index, meaning that measurements can be made on non-homogeneous droplets, reacting or burning droplets, and droplets of unknown refractive index. Similar to Particle Image Velocimetry, the GSV camera image capture mode also takes two consecutive images from each capture sequence [14]. The two images, taken using a known laser pulse separation time, were used to determine particle displacement in the field. The particle size was measured from the two images using the PM windowed algorithm. Using a unique tracking technique based on the particle size and particle motion information, the 2-D velocity of the particles was measured. An image correlation scheme applied to the
measured velocity gave sub-pixel displacement accuracy. Since the pulse separation of the two captured images could be varied, measurement of velocities ranging from a few mm/s to a few hundred m/s was possible [14].

2.3 Results and observations of the experimental run:

The test slinger was run at appropriate speeds and flow rates to produce atomization of satisfactory quality and this ensured that proper atomization was yielded within the dimensional limits of an actual combustion chamber. These iterative runs resulted in the slinger being run at 0.22 GPH (14 mL/min) and 0.5 GPH (32 mL/min), while the rotational speed was varied from 7200 to 13200 rpm.

The breakup phases were observed for the range of rotational speeds and volume flow rates mentioned above. Images were captured using the digital camera, such that the axis of the camera was parallel to the axis of rotation of the slinger. As described earlier, the atomization process included two phases of breakup, namely, primary and secondary breakup. Since primary breakup results in the jet breaking up into unevenly-shaped globules, this phase was easily observable for lower values of rpm, where the Rayleigh’s wave disturbances can be seen on the surface of the jet. The disturbances become progressively large with time and apparently disintegrate the jet into droplets larger than the diameter of the jet.

For the characteristic “subcritical breakup” observed for round-holed slingers, it has been observed that the diameter of the ligament produced after the film-collapse is much larger than the initial film thickness; hence the drop diameters produced from
primary breakup are found to be comparable to the ligament diameter [3]. Further, the jet and the drop diameters resulting from higher flow rates were observed to be larger. Figure 2.1 (a) shows the atomization process for the flow rate of 14 mL/min whereas Figure 2.1 (b) presents the primary breakup for the flow rate of 32 mL/min.

![Image of breakup visualization at n=7,200 RPM, Q = 14 mL/min](image)

Figure 2.1 (a) Breakup visualization at n=7,200 RPM, Q = 14 mL/min [7].
When the rotational speed was increased, the droplet diameters appeared to decrease, complying with the widely accepted fact that higher discharge velocities result in finer atomization. Moreover, the length of the jet breakup decreased with increase in the rotational speed. This can be visualized when the images are observed closely. At the exit of the slinger, the jet appears to be composed of a dominant ligament along with a few finer ligaments of the liquid. This can again be verified by similar liquid structures observed by Dahm et al. [4] in which the drop sizes decreased with increasing We, which is a measure of the ratio of the drag force to the surface tension force. The drag force component, which varies as the square of the relative velocity, increases at a faster rate as
velocity increases; compared to the corresponding rate of decrease in the diameter of the drop. The figures representing the decrease in the breakup length with increase in the rotational speed of the slinger are shown below.

![Breakup visualization at n=10,200 RPM, Q = 14 mL/min [7].](image)

The formation of multiple ligaments at higher rotational speeds may affect the estimation of the average diameter of the jet before breakup (explained in Chapter 3), since they apparently decrease the diameter of the dominant stream for increasing flow rates, while the effective jet diameter has been observed to increase with increasing flow rate at lower rotational speeds.
The droplet sizes were measured by setting the camera at an angle of 60° with respect to the forward scatter direction of the laser sheet. The particle sizes were measured at an average distance of about 100 mm from the slinger surface. The field of view of the camera was covering a linear distance of 15 mm (i.e., from 95 mm to 110 mm from the slinger surface in the radial direction). This particular distance was chosen because the particles in this area had spherical shapes for all the investigated rotational speeds, and the GSV method is applicable only to measurements of spherical droplets. The visualization results show that the secondary breakup of the particles has also taken place at smaller distances from the slinger. Particle distributions were presented in histo-
grams based on the number of droplets (in percent) as a function of diameter. The cumulative volume distribution curve represents the percentage of the total volume of the spray contained in drops below a given size.

Drop size distributions cannot always be used for calculative and analytical purposes as the amount of data handled may result in laborious computation and ineffective results. A more efficient representation of the set of drop-size data is the mean drop diameter. A mean drop diameter is a quantity that characterizes a set of uniform drops substituted for the real drop size spectrum which may contain a wide array of drop sizes [2]. Based on the mode of calculation and the type of application, the mean drop diameter has many definitions. These definitions determine characteristics such as the diameter, surface area and volume of drops. All definitions of mean drop diameter are based on the formula given below.

\[ D_{pq} = \sqrt[p-q]{\frac{\sum_{i=1}^{m} D_i^p \Delta n_i}{\sum_{i=1}^{m} D_i^q \Delta n_i}} \] ........Eqn.2.1

Based on the formula given above, the commonly used mean drop diameter definitions used are Sauter mean diameter, length (arithmetic) mean diameter and volume mean diameter. Sauter Mean Diameter or volume-to-surface-area mean diameter, SMD, is the diameter of the droplet whose ratio of volume to surface area is the same as that of the entire spray. For combustion applications the SMD is used to describe the atomization quality or drop penetration and also other important processes related to heat and mass transfer. The expression for SMD is:
The arithmetic mean diameter or the length mean diameter is the diameter of a uniform equivalent drop set with the same number of drops and sum of diameters as in the real set of non-uniform diameter drops. It is given by

\[
D_{10} = \frac{\sum_{i=1}^{m} D \Delta n_i}{\sum_{i=1}^{m} \Delta n_i}
\]  ……Eqn.2.3

The volume mean diameter, \(D_{30}\), is the diameter of a droplet whose volume, if multiplied by the number of droplets, equals the total volume of the sample.

\[
D_{30} = \sqrt[3]{\frac{\sum_{i=1}^{m} D^3 \Delta n_i}{\sum_{i=1}^{m} \Delta n_i}}
\]  ……Eqn.2.4

The diameters \(D_{0.1}\), \(D_{0.5}\) or \(D_{0.9}\) denote that 10%, 50% or 90% of the liquid volume consists of droplets with a diameter smaller than \(D_{0.1}\), \(D_{0.5}\) or \(D_{0.9}\), respectively. The measured particle diameters are reported in tables 1 and 2 for flow rates of 14 mL/min and 32 mL/min, respectively. The values are averaged over a large group of images (about 500 double images), acquired for each rotational speed. The corresponding histograms for some cases are shown in figures below.

Table 2.1 Diameters for flow rate of 14 mL/min [7].

<table>
<thead>
<tr>
<th>Rotational speed (rpm)</th>
<th>SMD (µm)</th>
<th>(D_{10}) (µm)</th>
<th>(D_{30}) (µm)</th>
<th>(D_{0.1}) (µm)</th>
<th>(D_{0.5}) (µm)</th>
<th>(D_{0.9}) (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7200</td>
<td>118</td>
<td>62</td>
<td>85</td>
<td>66</td>
<td>152</td>
<td>250</td>
</tr>
<tr>
<td>10200</td>
<td>85</td>
<td>52</td>
<td>66</td>
<td>51</td>
<td>94</td>
<td>180</td>
</tr>
<tr>
<td>13200</td>
<td>73</td>
<td>47</td>
<td>59</td>
<td>46</td>
<td>79</td>
<td>150</td>
</tr>
</tbody>
</table>
Table 2.2 Diameters for flow rate of 32 mL/min [7].

<table>
<thead>
<tr>
<th>Rotational speed (rpm)</th>
<th>SMD (µm)</th>
<th>D₁₀ (µm)</th>
<th>D₃₀ (µm)</th>
<th>D₀.₁ (µm)</th>
<th>D₀.₅ (µm)</th>
<th>D₀.₉ (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7200</td>
<td>119</td>
<td>66</td>
<td>86</td>
<td>70</td>
<td>142</td>
<td>240</td>
</tr>
<tr>
<td>10200</td>
<td>102</td>
<td>59</td>
<td>78</td>
<td>60</td>
<td>115</td>
<td>233</td>
</tr>
<tr>
<td>13200</td>
<td>92</td>
<td>56</td>
<td>72</td>
<td>57</td>
<td>103</td>
<td>197</td>
</tr>
</tbody>
</table>

Figure 2.4 Droplet size distribution at n=7200 rpm, Q = 14 mL/min [7].
Figure 2.5 Droplet size distribution at n=7200 rpm, Q = 32 mL/min [7].

Figure 2.6 Droplet size distribution at n=10200 rpm, Q = 14 mL/min [7].
It has been observed that for the same flow rate, the drop diameters reduce as the rotational speed increases. But, as the flow rates increase for a given rotational speed, the drop diameters increase. This can be seen from the table of SMD values as well as other definitions of characteristic diameters. Moreover, it can be observed from the histograms, for a given flow rate and lower rotational rates that the peak diameter has a larger magnitude and decreases as the rotational speed increases. Also, for a given rotational rate, for higher flow rates, the peak diameters themselves increase, but the frequency of drops having peak diameters decreases and the frequency of drop diameters is more distributed.

Therefore it can be inferred that, for a given flow rate and for higher rotational rates the frequency of larger drops decreases and the frequency of smaller drops also
increases, indicating finer atomization. Also, for a given rotational speed, increasing flow rates result in increased frequencies of all drop diameters, indicating coarser atomization. Based on the data obtained experimentally and an approximate estimation of the drop sizes, a model was developed to predict the trajectories for the jet exiting the slinger for various operating conditions, which will be discussed in the next chapter.
CHAPTER 3

An Analytical Model Based on the Equilibrium of Drop Inertia Force and Aerodynamic Drag Force

The disintegration of the liquid jet ejected from the slinger orifice results in the formation of coarsely-shaped globules and these globules are susceptible to secondary breakup to form smooth and spherical drops. Secondary breakup is a complex phenomenon which has been explored in the past and has been explained satisfactorily for straight jets. But in the case of a rotary slinger, the trajectories of droplets are observed to be curvilinear and require careful analysis. An attempt to predict the trajectories of droplets spatially and temporally has been made in this chapter by approximating the jet ejected from the slinger as a stream of droplets of uniform size.

An analytical model to predict droplet trajectories has been developed by analyzing the forces involved in the secondary breakup of a single drop and involves the balance of inertial force of the drop with the aerodynamic drag force acting on it. It is based on the case of a drop of a liquid exiting a nozzle with certain velocity and then suddenly exposed to air flowing coaxially with a different velocity [3], [15]. In this case the concept of relative velocity comes into the picture and a relevant expression has been
obtained for the displacement of the drop as a function of time in both X and Y directions, considering the two-dimensional motion of the drop. The $C_D$ in the expression for aerodynamic force is approximated experimentally as [3]

$$C_D = \pi \left( 0.128 + \frac{12.8}{Re} \right) \quad \ldots \ldots \text{Eqn. 3.1}$$

The above expression for $C_D$ holds good for $2<\text{Re}<1*10^4$, where Re is defined here as $\rho_{\text{air}} UD/\mu$ and the above expression approximates the regular curve for the relation between $C_D$ and Re for rigid spheres [16].

The following assumptions have been made to apply the above model to the analysis of the trajectory followed by the stream ejected by the fuel slinger [3]:

- The stream is approximated to be a very small spherical particle such as a drop, which has a finite mass and volume and no rotational momentum about its axis and is undeformable.
- The diameter of the drop is approximated for calculations by careful measurement of the average diameter of the stream exiting the slinger from the photographs taken experimentally, for different rotational speeds of the slinger.
- The buoyancy forces are neglected.
3.1 Mathematical formulation:

The force balance equation is shown below:

$$-\rho_{\text{water}} V'\frac{d^2 S}{dt^2} = C_d \left( \frac{1}{2} \rho_{\text{air}} * W^2 * A_{\text{ref}} \right) \quad \ldots \ldots \text{Eqn. 3.2}$$

The negative sign arises from the deceleration of the drop due to the drag force.

$W$ is the relative velocity between the drop and air and in our case is just the velocity of the drop, since air is assumed to be stationary. $A_{\text{ref}}$ is the reference area of the drop which in this case is for a blunt body and is the cross-sectional area of the drop.

On rearranging the terms and representing as a second order differential equation, one gets:

$$\frac{d^2 S}{dt^2} = -\left( \frac{0.096 \pi p_{\text{air}}}{\rho_{\text{water}} D} \right) * \left( \frac{dS}{dt} \right) + \left( \frac{9.6 \mu_{\text{air}}}{\rho_{\text{water}} D^2} \right) * \left( \frac{dS}{dt} \right) \quad \ldots \ldots \text{Eqn. 3.3}$$

Initial conditions in terms of velocity and displacement are applied to the equation.

As it can be observed, the velocity at the exit of the slinger consists of two components, namely, the radial and the tangential component.

The radial component of the drop/stream is obtained by solving the Navier-Stokes equations defined for the liquid film flowing in the radial channel of the slinger before exiting from the slinger [4]. An expression for the bulk velocity of the liquid film converging into a stream outside the slinger was developed [4] and is given below:
The bulk velocity of the liquid at the exit of the slinger is \( U_b \)

\[
U_b = W_r = \frac{q}{\pi b \cdot d_{hole}} \quad \ldots \ldots \text{Eqn. 3.4}
\]

The tangential component of the velocity is given by \( W_t \)

\[
W_t = R\omega \quad \ldots \ldots \text{Eqn. 3.5}
\]

The resultant velocity being

\[
\overline{W} = W_r + W_t \quad \ldots \ldots \text{Eqn. 3.6}
\]

Where

\[
|\overline{W}| = \sqrt{W_t^2 + W_r^2}
\]

On solving analytically, a solution to the above differential equation is obtained for different values of the drop diameters, subject to the following initial conditions: \( t=0, W_t = R\omega \) and \( W_r = \frac{q}{\pi b \cdot d_{hole}} \), where \( b \) is the thickness of the film, depending on \( \omega \) and \( q \) for different conditions. The solution for the differential equation is obtained for different rotational speeds and flow rates. The displacement \( S \) is resolved into X and Y components based on the angle \( \alpha \) between \( W_r \) and \( W_t \) and the components are plotted against each other to observe a linear relation. It is to be noted that the velocity \( W \) and each of its components are calculated relative to stationary air and all refer to the stationary frame of reference, i.e., with reference to a stationary observer away from the rotating slinger.
Sample plot:

Figure 3.1 Sample plot of Y displacement vs. X displacement from the slinger exit.

The graphs are plotted for other rotational speeds and are attached in the Appendix. These graphs are overlaid on a 1:1 scale drawing of the slinger cross-section with proper orientation at different angular positions corresponding to small time-steps of rotation of the order of 0.0001 sec. An approximate pictographic representation of the actual trajectory is shown below in Figure3.2:
Figure 3.2 Schematic of an approximate temporally-related trajectory of a drop.

From the above calculations it was observed that the obtained approximate trajectories matched almost exactly with the trajectories in the photographs of similar experimental situations, corresponding to different rotational speeds of the slinger and flow rates of the liquid. The comparisons were made for the following cases, namely 7200 rpm, 10200 rpm and 13200 rpm with flow rates of 14 mL/min and 32 mL/min for each case. The magnitude of error obtained compared with the experimental values was between 3 and 4%. The error could be attributed to the assumption that drops are ejected
out of the slinger, unlike the real case of a stream being ejected out and then the occurrence of the drop breakup. The above model holds well for Re pertaining to the range of slinger speeds under consideration.

Figure 3.3 Schematic of sample temporally-related trajectory plotted using developed equations.
Figure 3.3 shows a sample plot of the trajectory of different drops at a particular
time instant in AutoCAD. The instantaneous values of the resultant of the displacements
in the X and Y directions are plotted and a B-spline curve is fitted around those point
locations and the plot is scaled to the size of the photographs obtained experimentally.
The plots thus obtained were overlaid on the photographs and reasonably close
concurrence was observed for all cases of liquid flow rates and speeds of rotation used
for experimentation.

An attempt to develop another governing equation that fits the actual trajectory
more closely than the previous solution was also made. From the images showing the
calculated trajectory laid over the actual trajectory, one can see the lag in the calculated
trajectory, suggesting that the expression for the coefficient of drag assumes values
higher than the actual ones in the case of lower angular speeds. But this assumption could
be false due to the other assumption of drops coming out of the slinger, rather than a jet
coming first and then disintegrating into drops.

The Morsi and Alexander model for the particle trajectories in two phase flow
suggests an expression for the coefficient of drag for different ranges of Re. The Morsi
and Alexander model is the most complete, adjusting the function definition frequently
over a large range of Reynolds numbers [17]. The expression for the coefficient of drag is

\[ C_D = a_1 + \frac{a_2}{Re} + \frac{a_3}{Re^2} \] .......Eqn. 3.7
Where $a_1, a_2, a_3$ are given as:

- $0, 24, 0$ for $0 < Re < 0.1$
- $3.69, 22.73, 0.0903$ for $0.1 < Re < 1$
- $1.222, 29.1667, -3.8889$ for $1 < Re < 10$
- $0.6167, 46.5, -116.67$ for $10 < Re < 1000$
- $3644, 98.33, -2778$ for $100 < Re < 1000$
- $0.357, 148.62, -47500$ for $1000 < Re < 5000$
- $0.46, -490.546, 578700$ for $5000 < Re < 10000$
- $0.5191, -1662.5, 5416700$ for $Re \geq 10000$

The calculations are made by solving the differential equation obtained by the inertia force – drag force balance again, similar to the previous equation, which is subject to initial conditions. The values of spatial displacement obtained as a function of time are plotted using AutoCAD software and the trajectory is traced for different particles at the same time instant.

Comparisons of trajectories for other rotational speeds and flow rates corresponding to the two models and the MATLAB programs used to calculate the same are attached in the Appendix.
Figure 3.4 Comparison of trajectories for an angular speed of 7200 rpm and flow rates of 14 mL/min and 32 mL/min using expressions for $C_D$ from different models.
3.2 Results and inferences:

- From the above assumptions of rigid spheres (particles) being ejected out of the slinger holes, neglect of buoyancy forces and approximation of the drop diameters from the photographs visually, to calculate the trajectories for a particular liquid flow rate and speed of rotation, a differential equation governing the path of the particles/drops was obtained and on solving using the initial conditions a closed-from solution was easily achieved.

- In all results obtained from the calculations for different cases, it can be observed that the trajectories suggest a higher drag than in actuality, thus indicating that the drops are not totally rigid and the assumption of drops being ejected from the slinger holes is not entirely correct. In reality the liquid is ejected as a stream which breaks up into primary and secondary drops.

- The comparison of the results obtained by using two different expressions for the coefficient of drag indicates little difference in the drop trajectories. Even though the Morsi and Alexander model suggests a more specific expression covering greater ranges of Re, it has not been proved to be remarkably better than the previous expression chosen for the coefficient of drag.

Thus, a physical model has been developed, with reasonably close results that vary between 3% and 5% of the actual experimental data.
CHAPTER 4

Analysis of Secondary Breakup Based on the Equilibrium of Aerodynamic Drag Force and Force due to Surface Tension

This chapter elaborates on the mechanism of secondary breakup of the drops. In addition, a comparison of data corresponding to the point of onset of secondary breakup in the currently analyzed fuel slinger has been made with relevant studies made in the past for drop breakup in a cross-flow and subject to steady disturbances [19] and an attempt to predict the conditions and region of secondary breakup has been made.

The drops obtained after primary breakup are intrinsically unstable to secondary breakup. Various hypotheses for the globules disintegrating into finer and rounder drops have been put forward and different modes for secondary breakup have also been discussed [5]. Under some conditions, such as pressure-atomized sprays, the high pressure assists the process of atomization by reducing the surface tension, where the liquid approaches its thermodynamic critical point. Extensive research has been done on two types of well-defined disturbances: shock wave disturbances, which provide step changes in the ambient environment of the drop at the end of primary breakup; and steady disturbances, which may characterize the breakup mechanism in the case of centrifugal atomizers. But
detailed analysis of the features of secondary breakup has been made for the case of
shockwave disturbances, which predict similar breakup behavior in some aspects for
steady disturbances in dense sprays, whose liquid-gas density ratios ($\rho_L/\rho_G$) exceed a
value of 500.

4.1 Drop deformation and mechanism of breakup:

When effects of viscosity are small, the regime observed just before the breakup
phase has been categorized as bag breakup, wherein the drop deforms into a thin disc
normal to the flow direction and subsequently into a balloon-like shape. This bag/balloon
breaks up into drops which are much smaller than the parent drops. Important analysis of
the mechanism of secondary breakup by deformation of the drop into a disc and the
validation of the mechanism using the Drop Deformation and Breakup (DDB) model has
been made for drops undergoing bag, shear and catastrophic breakup [18]. The different
modes of breakup are shown in Figure 4.2.

![Figure 4.1 Schematic of the general mechanism of drop deformation and disintegration](image)

[3].
Each of these breakup modes is characterized by a specific range of a non-dimensionalized quantity known as the Weber number (We), within which they occur. These breakup modes have been described as oscillatory deformation (no breakup), bag breakup, multimode breakup and shear breakup. These breakup modes are shown on the We-Oh map; the length scale for the We and Oh is the initial drop diameter and the velocity scale is the relative velocity between the gas and the drop. The We and Oh numbers are based on initial conditions prior to the onset of secondary breakup [5]. Even though these experiments and visualizations correspond to drop breakup subject to shockwave disturbances, their applicability to the secondary breakup in the case of fuel
slingers may be considered with the We calculated based on the drop diameter [4], since extensive investigations have not been made in the past for drop breakup subject to steady disturbances.

![We-Oh map](image)

**Figure 4.3** The We-Oh map indicating different modes of secondary breakup for shockwave disturbances. [8].

On plotting the data corresponding to the We and Oh coordinates (Oh for the calculated data can be observed to be <0.1) based on calculations from the model developed, assuming the equilibrium condition of the drop’s inertial force and the aerodynamic drag force, it appears that the drops lie in the range of We corresponding to 10% < deformation < 20%, suggesting no breakup, which is contrary to the experimental
visualizations made for the rotary slinger, thus indicating the inappropriateness of comparison with some of the calculations for shockwave disturbances.

Further, the Oh-independence of the atomization process [4] at relatively low values of Oh, which is the ratio of the viscous effects to the surface tension forces, i.e., for Oh<=0.1 (found in our case), reflects the point that viscous effects are largely negligible. The Oh number represents viscous effects in high-speed atomization in the way that they dampen the disturbances that develop on the surface of the drop, thereby reducing the rate of deformation [5]. The values of Oh number for the operating conditions considered in this analysis have been found to be less than 0.1, which is typical of practical slingers, but again, this depends on the fuel chosen.

4.2 Critical conditions for secondary breakup:

Observation of the photographs of the liquid jet disintegration faintly suggests the occurrence of secondary breakup by the bag breakup mode. Since secondary breakup has been approximated as a rate process and not an instantaneous one [5]; and bag breakup is characterized by phases involving the drop first deforming into a disk-like shape. Further increase in drag effects results in a bag-like structure which eventually ruptures into smaller droplets at the critical point. The classical approach towards analyzing the forces involved during secondary breakup and obtaining critical values of quantities has been the consideration of the equilibrium condition of the aerodynamic drag force and surface tension forces for a rigid, spherical drop [3]. When the former is higher than the latter, the drop deforms and disintegrates. Accordingly,
The expression for the critical value of Weber number (We*) may be obtained as [3]:

$$\text{We}^* = \frac{\rho_{\text{air}} U^2 D_{\text{max}}}{\sigma} = \frac{8}{C_D} \quad \text{Eqn. 4.2}$$

The critical We represents the criterion for deformation that lead to secondary disintegration of a drop. $D_{\text{max}}$ corresponds to the maximum drop diameter at the critical conditions and any drop greater than $D_{\text{max}}$ would undergo further disintegration. A curve is plotted between the critical diameter and the velocity of the spherical drop at the point of the corresponding drop breakup for a given liquid (water, in this case) and ambient fluid (air, in this case) for the transition and turbulent regimes of flow. It can be observed that higher velocities are required to disintegrate smaller drops, since the coefficient of drag for rigid spheres decreases as Re increases (within the region of critical Re) [16].

Further analysis on the critical conditions for secondary breakup is shown in the later sections of this chapter by comparison with the curves for secondary breakup involving data from experimental investigation by Sescu et al [7] and results from the model discussed earlier.
Figure 4.4 Plot representing the relation between the drop diameters and their corresponding critical velocities needed for secondary breakup.

The above graph phenomena since it is known that primary drops are not rigid spheres and the surface is subject to deformation [3], [13], [15]. The calculations involved in the generation of this graph are included in Appendix B. A more realistic approach to analyze the mode of secondary breakup, considering the effects of deformation and the change in the drag coefficients, is described in the following sections.

4.3 Comparison of results of current model with past investigations:

Considerable studies have been made on atomization of jets subject to shockwave disturbances, which provide a step change in the ambient environment, but these conditions may not apply to the case of atomization in rotary slingers. Investigations have also been made for the case of atomization of drops exposed to steady disturbances, but a comprehensive understanding of the mechanism, as to whether the disturbances were due
to the injector needle or external factors, has not been reported [13]. Since the possibility of the presence of shockwave disturbances during the atomization process in a rotary slinger is ruled out, the research work pertaining to jet atomization with steady disturbances may be used as a basis to analyze the same with centrifugal atomizers pertaining to the aforementioned operating conditions. It has also been observed that the We for breakup are comparable for steady and shockwave disturbances, but again, as mentioned before, the applicability of correlations involving We to the secondary breakup in the case of fuel slingers rests on the universality of We and Oh, which are based on the drop diameter.

Since it has been observed that the drops that undergo disintegration are not rigid spheres, but have flexible surfaces resulting in internal circulation [3], the classical model for secondary breakup previously described, involving the assumption of equilibrium condition of the aerodynamic drag force and the surface tension forces, has been slightly modified to incorporate actual modes of breakup, notably, the deformation of the primary drop into oblate spheroids and flat discs.

Different modes of breakup have been described based on the range of We they lie within. Accurate information regarding the mode of secondary breakup obeyed by the analyzed jet was not available, hence it was assumed that the jet underwent bag breakup throughout the range of operating conditions. For the case of a drop ejected with a certain velocity into ambient gas moving at a different velocity, if the relative velocity between the drop and the ambient gas decreases, then the drop is subjected to various types of disintegration, one of them being the bag/parachute type [3]. This type of disintegration is
initiated with the drop deforming into an oblate spheroid and progressing into a flat disc-shape. As the aerodynamic drag force increases, the disc bulges into a bag with a thicker rim and finally ruptures at the critical value of the drag force, resulting in smaller droplets from the bag and slightly larger drops from the rim [10], [11] & [13].

Drops undergo significant deformation in the period prior to the onset of breakup and, as mentioned earlier, the drops obtain an oblate-spheroid shape, from which further deformation progresses. A relatively large database of the maximum drop deformations for steady disturbances in drop-gas environments has been summarized through past investigations. Phenomenological analyses have led to a reasonably good correlation of these results in terms of the maximum cross-stream diameter, $D_{\text{max}}$, and the minimum stream wise diameter, $D_{\text{min}}$, as shown below [13]:

$$\frac{D_{\text{max}}}{D_{\text{min}}} = (1 + 0.07\sqrt{\text{We}})^3, \text{We} < 20$$

.......Eqn. 4.3

Where, the second relation to find $D_{\text{max}}$ and $D_{\text{min}}$ is

$$D_{\text{min}}D_{\text{max}}^2 \approx D_0^3$$

.......Eqn. 4.4

$D_0$ is the initial drop diameter before secondary breakup.

As mentioned before, the values calculated from the analytical model developed were plotted on the We-Oh map describing the drop breakup modes when subjected to shockwave disturbances, to try and develop any correlations similar to existing ones, even though the calculated values correspond to steady disturbances. The results do not agree with the type of breakup expected for the case under consideration and could be attributed to the possible inappropriateness of comparison in terms of the type of
disturbance. Research on the effects of drop distortion on the mechanism of drop breakup shows that shockwave disturbances cause a significant amount of deformation in drops before breakup and hence the location of the points plotted on the We-Oh map may be displaced due to the plotting of data corresponding to under-deformed drops on a graph for highly-deformed drops [13].

The primary assumption for applying the equilibrium condition to the aerodynamic drag force and surface tension forces is that the volume of the spherical drop before deformation is approximately equal to the volume of the deformed disc-shape [12]. Based on this, the analysis presented currently is intended to add further ground to the existing concepts of mechanisms for secondary breakup for the rotary fuel slinger and an attempt to correlate the data obtained from current calculations with the data compiled by Hsiang, Faeth and Wu for drop deformations has been made. It has already been mentioned that for the liquid considered, which has a low Oh (Oh<0.1), the viscosity effects don’t have a significant role for steady disturbances and play a substantial role only in the case of shockwave disturbances, in reducing the rate of deformation.

Considering all available data for the drag coefficients of drops in liquids and gases, and for steady and shockwave disturbances, it appears that the coefficient of drag of the drop is mostly a function of the degree of deformation of the drop [13]. This behavior is shown below in the curve plotted between the ratio \( C_D/C_{Dsp} \) and the ratio \( D_{max}/D_{min} \) where \( C_D \) and \( C_{Dsp} \) are the local coefficient of drag of the drop and the coefficient of drag of a sphere at the same \( Re \), respectively. It can be clearly seen that the larger range of \( D_{max}/D_{min} \), just before breakup, corresponds to shockwave disturbances,
indicating the step change in the ambient gas flow around the drop. The above correlation has been used to evaluate the local drop drag coefficients in the current analysis of the classical model [3], involving the equilibrium condition of the aerodynamic drag force and the surface tension forces. The calculations were made for various initial drop diameters pertaining to different operating conditions of the slinger, considering the deformation of the spherical drop into a disc-like shape, which approximately assumes dimensions of a flat cylinder.

![Image](image.png)

**Figure 4.5** Experimental graph showing the variation of $C_D/C_{Dsp}$ with drop deformation [13].

The dimensions of the deformed drops in the tabular form shown below were estimated by conserving the volume of the drop and the subsequently formed discs. The values of the coefficient of drag for spheres at the corresponding Re [16] were used to
calculate the local drop drag coefficients. The equilibrium condition for the aerodynamic 
drag force and the force due to surface tension leads to the calculation of the values of 
\( D_{\text{max}} \) and \( D_{\text{min}} \) by solving the fifth order polynomial given below [3].

\[
C_D \left( \frac{1}{2} \rho_{\text{air}} U_0^2 \cdot \frac{\pi D_{\text{max}}^2}{4} \right) = \pi \cdot D_{\text{max}} \cdot \sigma \\
\text{.........Eqn. 4.5}
\]

The parameter \( C_d \) is substituted as a straight line function between \( C_{D_{\text{Csp}}} \) and the 
\( D_{\text{max}}/D_{\text{min}} \), is obtained from the above graph in Figure 4.6 as [13]

\[
\frac{C_D}{C_{D_{\text{Csp}}}} = 0.45 \cdot \left( \frac{D_{\text{max}}}{D_{\text{min}}} + 0.32 \right)  \\
\text{.........Eqn. 4.6}
\]

The above equilibrium condition using Eqn. 4.4, results in the following 
polynomial as developed in this thesis:
0.3194 \frac{C_{Dsp} \cdot U^2 \cdot D_{max}^5}{D_0^5} + 0.1514 \cdot C_{Dsp} \cdot U^2 \cdot D_{max}^2 - 0.2287 \cdot D_{max} = 0

\ldots\ldots\text{Eqn. 4.7}

\text{Figure 4.7} Plot of the drop drag coefficient as a function of We for steady disturbances [13].

The values of $D_0$ were approximated by averaging methods involving the flow rate and time for the drops to traverse a finite distance and $U_0$ was evaluated at the point of secondary breakup, which was visually estimated using the results of the analytical model previously described. The solutions to the polynomial consist of only one real and
positive value corresponding to every value of $D_0$, $C_{Dsp}$ and $U_0$. By using the principle of conservation of volume for the sphere and the disc, $D_{min}$ was evaluated and the values were plotted on the graph discussed in the results below.

Figure 4.7 shows the relation between the normalized coefficients of drag for drops at the point of maximum deformation. $C_{Dsp}$ is the drag coefficient of a sphere at the same Re at which $C_D$ is calculated. It may be observed that the normalized drop drag coefficients do not vary much for small We up to 10, which is the region of focus in this analysis. Using this idea, the unknown parameters ($D_{max}$ and $D_{min}$) of the equation obtained by the equilibrium condition (Eqn. 4.7) are established, as shown in Table 4.1 of this chapter.
<table>
<thead>
<tr>
<th>rpm</th>
<th>initial drop diameter in μm</th>
<th>diameter for drag force D_{drag}</th>
<th>velocity in m/sec</th>
<th>Re (initial dimensions)</th>
<th>We (initial dimensios)</th>
<th>C_{w}</th>
<th>Co</th>
<th>C_{p}(C_{w})</th>
<th>Drag force in N</th>
<th>D_{m, w} for surface tension</th>
<th>Surface Tension force in N</th>
<th>D_{m, w} / D_{w}</th>
<th>Volume of spherical drop</th>
<th>Volume of deformed drop</th>
<th>Surface tension/drag force</th>
<th>D_{m, w} / D_{w}</th>
<th>(D_{m, w} / D_{w})^{1/2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>5700</td>
<td>(14 μL/min)</td>
<td>294.81</td>
<td>0.0002</td>
<td>9.247</td>
<td>9</td>
<td>153.1</td>
<td>13</td>
<td>0.00684</td>
<td>0.41724</td>
<td>0.81</td>
<td>2</td>
<td>0.60</td>
<td>07</td>
<td>1.00</td>
<td>09</td>
<td>3.2E-06</td>
<td>0.0003</td>
</tr>
<tr>
<td>6000</td>
<td>(14 μL/min)</td>
<td>291.22</td>
<td>0.0002</td>
<td>7</td>
<td>10.75</td>
<td>9</td>
<td>158.7</td>
<td>60</td>
<td>0.00727</td>
<td>0.50053</td>
<td>0.75</td>
<td>73</td>
<td>0.64</td>
<td>62</td>
<td>1.05</td>
<td>06</td>
<td>3.4E-06</td>
</tr>
<tr>
<td>7200</td>
<td>(14 μL/min)</td>
<td>171.14</td>
<td>0.0001</td>
<td>8</td>
<td>12.6</td>
<td>6</td>
<td>144.6</td>
<td>33</td>
<td>0.00886</td>
<td>0.44976</td>
<td>0.90</td>
<td>16</td>
<td>0.99</td>
<td>06</td>
<td>1.06</td>
<td>06</td>
<td>2.3E-06</td>
</tr>
<tr>
<td>7500</td>
<td>(32 μL/min)</td>
<td>231.94</td>
<td>0.0002</td>
<td>4</td>
<td>12.99</td>
<td>8</td>
<td>202.5</td>
<td>33</td>
<td>0.00771</td>
<td>0.64566</td>
<td>0.76</td>
<td>44</td>
<td>0.51</td>
<td>01</td>
<td>0.61</td>
<td>06</td>
<td>3.7E-06</td>
</tr>
<tr>
<td>8100</td>
<td>(32 μL/min)</td>
<td>174.67</td>
<td>0.0001</td>
<td>8</td>
<td>12.99</td>
<td>8</td>
<td>152.6</td>
<td>95</td>
<td>0.00886</td>
<td>0.40060</td>
<td>0.90</td>
<td>55</td>
<td>0.96</td>
<td>02</td>
<td>1.05</td>
<td>04</td>
<td>2.5E-06</td>
</tr>
<tr>
<td>9000</td>
<td>(32 μL/min)</td>
<td>173.65</td>
<td>0.0001</td>
<td>8</td>
<td>12.99</td>
<td>8</td>
<td>151.6</td>
<td>96</td>
<td>0.00891</td>
<td>0.45654</td>
<td>0.90</td>
<td>55</td>
<td>0.96</td>
<td>04</td>
<td>1.05</td>
<td>04</td>
<td>2.5E-06</td>
</tr>
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<td>1200</td>
<td>(14 μL/min)</td>
<td>135.66</td>
<td>0.0001</td>
<td>4</td>
<td>17.18</td>
<td>8</td>
<td>161.9</td>
<td>64</td>
<td>0.01005</td>
<td>0.71</td>
<td>0.87</td>
<td>4</td>
<td>0.92</td>
<td>65</td>
<td>1.05</td>
<td>06</td>
<td>2.7E-06</td>
</tr>
<tr>
<td>1000</td>
<td>(22 μL/min)</td>
<td>194.36</td>
<td>0.0002</td>
<td>14</td>
<td>18.61</td>
<td>4</td>
<td>242.1</td>
<td>25</td>
<td>0.00842</td>
<td>1.10641</td>
<td>0.65</td>
<td>07</td>
<td>0.72</td>
<td>01</td>
<td>1.06</td>
<td>01</td>
<td>4.0E-06</td>
</tr>
<tr>
<td>1300</td>
<td>(14 μL/min)</td>
<td>122.53</td>
<td>0.0001</td>
<td>3</td>
<td>24.61</td>
<td>3</td>
<td>205.8</td>
<td>91</td>
<td>0.01007</td>
<td>1.259</td>
<td>0.75</td>
<td>68</td>
<td>0.90</td>
<td>23</td>
<td>1.05</td>
<td>05</td>
<td>3.6E-06</td>
</tr>
<tr>
<td>1300</td>
<td>(22 μL/min)</td>
<td>176.62</td>
<td>0.0001</td>
<td>8</td>
<td>23.64</td>
<td>8</td>
<td>282.8</td>
<td>8</td>
<td>0.00584</td>
<td>1.66153</td>
<td>0.62</td>
<td>50</td>
<td>0.66</td>
<td>31</td>
<td>1.06</td>
<td>04</td>
<td>5.9E-06</td>
</tr>
</tbody>
</table>
4.4 Results and correlation of results with other investigations:

Figure 4.8 showing the relation between drop deformation and We [13].

The ratio, $D_{\text{max}}/D_{\text{min}}$, was used to plot points (red squares) on the graph by Hsiang and Faeth [13], relating the drop deformation and We, for drops subject to steady disturbances, which have been deformed into nearly flat discs with relevant dimensions in the longitudinal and transverse directions. The disc progressively deforms into a bag-
like structure and due to instabilities on the surface, disintegrates to give finer droplets [13].

From the graph by Hsiang and Faeth [13] for the drop deformation vs. We in the case of drops subject to steady disturbances, after plotting the data points calculated using the above equilibrium condition, it can be observed that the calculated data points lie close to the majority of the points corresponding to different liquids and different continuous phases. But, comparing the trend followed by the points plotted using calculated data, with the pattern for water drops disintegrated in air (hollow circles), one can observe that the deformation follows a flat trend with We.

The trend offset of the calculated data from the original pattern for water drops (hollow circles) could be attributed to the fact that the data plotted by Hsiang and Faeth [13] corresponds to the secondary breakup of nearly spherical drops emitted initially by a drop generator, whereas, the rotary slinger involves the progressive disintegration of a round jet into coarsely-shaped globules and then further breakup into smaller and rounder droplets. The consideration of coarsely shaped drops for calculations may have lead to inappropriate comparison of initial drop conditions with that assumed by Hsiang and Faeth (such as drop diameter, coefficient of drag and velocity, which is used to calculate the coefficient of drag of a sphere based on the corresponding Re).

Another interesting investigation of critical conditions corresponding to secondary breakup was made by plotting the data obtained from calculations for the droplet dimensions discussed earlier in this chapter in the graph showing the critical conditions
for secondary breakup of rigid spheres using the condition of equilibrium between the aerodynamic drag force on the droplet and the force due to surface tension acting on it.

Figure 4.9 Graph showing data for critical conditions corresponding to secondary breakup for rigid spheres and calculations made in this chapter for deformed droplets.

The above graph plotted for various sizes of droplets (rigid spheres) includes droplets moving in the transition and turbulent range of Re, for which the $C_D$ values have been calculated from sources by L. Bayvel and Z. Orzechowski [3]. The considerable discrepancy between the two sets of data points may be attributed to the different expressions for calculation of $C_D$ and difference in the droplet dimensions due to
deformation, it may be inferred that the theoretical consideration of breakup of droplets, assuming rigid spheres may not apply thoroughly to the analysis of secondary breakup using the classical approach.

Thus, the data obtained from the analytical model discussed previously was analyzed for understanding secondary breakup of jets from rotary fuel slingers and obeys correlations established from past investigations of secondary breakup of droplets.
CHAPTER 5

Surface Tension and Hydrogen Bonding

As the classical model for secondary breakup involves assuming the equilibrium of the aerodynamic drag force and the force due to surface tension for the drop nearing the point of secondary breakup (described in Chapter 4), an attempt to analyze the significance of the components of surface tension force in the force balance has been made and the forces then compared with the actual value due to surface tension of water.

This chapter focuses on the exploration of the magnitude of forces due to hydrogen bonding, assuming that the major component of surface tension in water is the hydrogen bonding force [20]. To verify this, the hydrogen bond forces on the periphery of the droplet and the mid-plane of the spherical droplet are estimated and compared with the accepted value of surface tension of water, which is obtained experimentally.

It is to be analyzed, which of the two, the forces of hydrogen bonding at the periphery of the drop or the ones in the mid-plane contribute majorly to the atomization-resisting force. In short, the contribution of the force of surface tension (attributed to hydrogen bonding forces) to the resistance of atomization is estimated on each of these geometrical surfaces and compared with the force equivalent to the accepted value of...
surface tension of water. So the primary assumption is that, for water, the major contributing force to the surface tension is hydrogen bonding.

5.1 Surface tension:

Surface tension (σ) is defined as the energy required to reduce the surface to its minimum area [3]. Surface tension may also be defined as the force per unit length for the reduction of surface energy to its minimum. Its units are J/m² or N/m in the SI system. According to the theory of molecular attraction, the molecules of liquid below the surface are equally attracted in all directions. But the molecules on the surface are attracted by much stronger cohesive forces than those in the bulk of the liquid and the resistance offered by the surface film against this attraction is called surface tension [21].

The equilibrium condition of the two opposing forces mentioned earlier implies that the drop resists the deforming and disintegrating effects of the drag forces through surface tension and when the drag forces reach a critical value, the primary drop disintegrates to give secondary drops.

Surface tension is caused by intermolecular forces, which are weaker than ionic and covalent bond forces, but they have a visible effect on the chemical behavior of substances. Intermolecular forces are known to include the following components [22]:

- London dispersion forces or dipole-induced dipole interaction.
- Dipole-dipole interactions
- Hydrogen bonding
In the case of water, these intermolecular forces are mainly due to hydrogen bonds, as they are high energy bonds.

5.1.1 London dispersion forces:

The London dispersion forces are the weakest intermolecular forces and are temporary attractive forces that take effect when the electrons in two adjacent atoms occupy positions that result in the atoms forming temporary dipoles. Because of the constant motion of the electrons, an atom or molecule can develop a temporary (instantaneous) dipole when its electrons are distributed asymmetrically about the nucleus.

5.1.2 Dipole-dipole interactions:

Dipole-dipole interactions, also called Keesom interactions, arise from localized permanent dipoles. The two opposite charges are on the same molecules and, thus, neutralize each other so there is no net charge on the molecule, as there is with an ion. However, the individual charges can interact with the individual charges of opposite sign on another molecule, and the interaction can be extremely strong. The energy of dipole-dipole interactions can approach the energy of a weak chemical bond.

5.1.3 Hydrogen bonding:

Hydrogen bonds are a stronger form of dipole-dipole interactions, caused by highly electronegative atoms. They only occur between hydrogen and oxygen, fluorine or nitrogen, and are the strongest intermolecular force. Hydrogen bonding in water occurs
between a hydrogen atom of one water molecule and a highly electronegative oxygen atom of another water molecule. Intermolecular hydrogen bonding is responsible for the high boiling point of water (100 °C).

In general, the order of strength of the above three intermolecular forces is:

Hydrogen bonding > Dipole-dipole interactions > London dispersion forces

Surface tension is responsible for the shape of liquid droplets. Although easily deformed, droplets of water tend to be pulled into a spherical shape by the cohesive forces of the surface layer. As mentioned earlier, surface tension is explained by these intermolecular forces and focus is made on hydrogen bonding in water, since it is considered to be the major contributing factor to surface tension.

![Figure 5.1 Schematic showing the pressure difference acting across a tensioned surface.](image)

As shown in Figure 5.1, if no force acts normal to a tensioned surface, the surface must remain flat. But if the pressure on one side of the surface differs from pressure on the other side, the pressure difference times surface area results in a normal force. In
order for the surface tension forces to cancel the force due to pressure, the surface must be curved. The diagram shows how surface curvature of a tiny patch of surface leads to a net component of surface tension forces acting normal to the center of the patch. When all the forces are balanced, the resulting equation is known as the Young–Laplace equation:

\[ \Delta p = \sigma \left( \frac{1}{R_x} + \frac{1}{R_y} \right) \]  

…………Eqn.5.1

For a spherical drop, inside, the lower the radius, the higher is the excess pressure, \( \Delta p \).

For the surface energy,

\[ dE = 4\pi R^2 \Delta p \, dR \]  

…………Eqn.5.2

For a sphere, the surface area is \( A = \pi R^2 \)

\[ dA = 8\pi R \, dR \]

Hence,

\[ \sigma = \frac{\Delta p \cdot R}{2} \]  

…………Eqn.5.3

5.2 Estimation of forces due to hydrogen bonding:

As observed in chapter 4, the classical condition for analyzing the breakup of a primary drop into secondary drops was assumed to be the equilibrium between the aerodynamic drag force on the surface of the drop and the force due to surface tension.
An attempt is made to calculate the force due to surface tension theoretically, by assuming that the major contribution to surface tension in water is by the hydrogen bond force. Using the basic principles of atomic physics, an estimate of the hydrogen bond force on a peripheral edge was made, as the classical model for secondary breakup of a drop suggests that the force due to surface tension acts along the peripheral line, tangential to the curved surface but normal to the diametrical plane (see Figure 4.1, pg 43).

The value of the hydrogen bond energy for water (liquid state) has been obtained differently from different investigations [23] and the average value can be approximated at 2.7 kcal/mole of hydrogen bonds. Hence it was required to calculate the number of hydrogen bonds in the mid-plane of the spherical drop, which get sheared during atomization. The number of hydrogen bonds in the mid-plane of the drop can be calculated by estimating the number of hydrogen bonds a single water molecule can participate in. A single water molecule ideally participates in four hydrogen bonds in a tetrahedral structure as shown below in Figure 5.2. The solid blue lines represent covalent bonds and the dashed lines represent hydrogen bonds. The structure shown below is for ice which has a very orderly structure, compared to water. But water is also assumed to show a similar structure, for simplification, since it is a liquid and the bond orientation is expected to change interminably.
Figure 5.2 The structure of hydrogen bonds in water [24].

It can be seen from Figure 5.2, that each water molecule participates in four hydrogen bonds, two involving the oxygen atom of the analyzed molecule and two hydrogen atoms of two other water molecules and the other two involving the two hydrogen atoms of the analyzed molecule and two oxygen atoms of different water
molecules. In this way no water molecule may involve itself in multiple hydrogen bonds with another water molecule.

Hence, the number of hydrogen bonds in a plane can be calculated by estimating either the number of hydrogen atoms or oxygen atoms per water molecule involved in hydrogen bonding. The number of hydrogen atoms or oxygen atoms per water molecule involved in hydrogen bonding = 2. Hence, the number of molecules of water in the mid-plane can be calculated. Assuming a single V-shaped water molecule, having an effective diameter of 2.75 Å [25], the volume of the molecule is approximated as 1.08892*10^{-29} m^3.

Assuming a spherical droplet of diameter 80 µm, whose volume is 2.6808*10^{-13} m^3, we calculate the number of molecules in the spherical droplet.

It is also known that one mole of water molecules has an atomic mass of 18.01528 gm [26], and since the density of water is 998 kg/m^3 at 20^0C, the volume for one mole of water molecules is 18.016 cc. One mole of any substance contains an Avogadro number of molecules, which is 6.023*10^{23} molecules. Now this drop contains water molecules and voids in between the molecules [23]. Hence the number of molecules in the 80 µm diameter droplet, by using Avogadro’s concept is 8.96228*10^{15} molecules.

The calculation of volume occupied by 8.96228*10^{15} molecules using the diameter of a single water molecule is made, giving 9.759*10^{-14} m^3.

Compared with the volume of the spherical drop calculated knowing its diameter
(2.6808*10^{-13} \text{ m}^3), which contains 8.96228*10^{15} molecules and includes void spaces between the water molecules, the volume of water molecules only, calculated above (9.759*10^{-14} \text{ m}^3) is less.

The number of water molecules in the mid-plane can be approximated as:

\[
\frac{\text{Area of mid - plane of 80 \mu m diameter droplet}}{\text{area of circles formed by the 2.75 \text{\AA}\ diameter molecules}}
\]

Hence the number of water molecules in the mid-plane is 8.4628*10^{10}, which should actually be less, since the void spaces are not taken into account in this calculation.

Assuming hexagonal packing of the molecules, whose packing density is 0.9, the number of water molecules in a planar area can be estimated. A small fraction of hydrogen bonds are broken at any given temperature of water and it increases at higher temperatures [23]. At 20^0C, the fraction of intact bonds is approximately 0.47.

Now, since each oxygen atom participates in 2 unique hydrogen bonds with other water molecules, and each water molecule contains one oxygen atom and the total number of water molecules in the mid-plane is 8.4628*10^{10} or 1.26457*10^{-13} moles of water molecules. As mentioned earlier, the average value of the hydrogen bond energy in liquid water is approximated at 2.7 kcal/mole or 11.2968 kJ/mole of water molecules. Hence, the total hydrogen bond energy for 1.26457*10^{-13} moles of water molecules is 8.92853*10^{-9} \text{ J}. Also, the accepted value of surface tension of water is 0.0728 N/m or 0.0728 J/m^2 of unit surface area, which corresponds to 1.4637*10^{-9} \text{ J} for the droplet.
considered, of size 80 µm. If it is assumed that the hydrogen bonds resist the shearing of the water droplet in the mid-plane, then the energy required to shear the hydrogen bonds is $8.92 \times 10^9$ J, which is about six times the actual value of surface energy for the droplet corresponding to the surface tension of water. Hence there may be ambiguity in the assumption that atomization takes place due to shearing of hydrogen bonds exactly in the mid-plane of the droplet.

Another approach to estimating the contribution of hydrogen bonds to surface tension in the atomization of water droplets is to evaluate the number of hydrogen bonds broken at the periphery of the water drop during secondary breakup. Again, the number of hydrogen bonds depends on the number of water molecules at the periphery of the droplet. Considering the same droplet diameter of 80 µm, the circumference of the circle aligned with the diameters of the water molecules on the periphery would be $2.513265 \times 10^4$ m, which would accommodate 913914.7213 molecules of diameter 2.75 Å. Assuming each oxygen molecule to form two hydrogen bonds each, with other water molecules, there are about $1.51738 \times 10^{18}$ moles of water molecules on the periphery of the water droplet.

The bond energy corresponding to each hydrogen bond is about 2.7 kcal/mole of water molecules, resulting in a total hydrogen bond energy of $1.714 \times 10^{-14}$ J, which accounts for the hydrogen bonding energy for the water molecules on the circumference of the drop. Since surface tension is a phenomenon relevant to the surface area, it is difficult to transpose the value obtained above to one with suitable units (J/m² or N/m).
Hence it can be inferred that the surface tension of water may be attributed to the hydrogen bonds of water molecules but the value of the surface tension lies between the two extreme values calculated at the mid plane of the droplet and the circumference of the droplet, respectively. Hence the theoretical value of the surface tension of water is assumed to be a resultant of the hydrogen bonding of associated molecules, while the actual value, obtained experimentally, lies between two values estimated by different approaches, considering the disintegration of a droplet. A graph similar to Figure 4.9 shows the variation of diameter of the droplet with the velocity at critical conditions of secondary breakup.

![Graph](image)

**Figure 5.3** Graph showing data for critical conditions corresponding to secondary breakup of rigid spheres, secondary breakup due to hydrogen bond breaking in the mid plane and calculations made in Chapter 4 for deformed droplets.
The points based on the value of surface tension calculated considering the breaking of bonds on the circumference of the droplet could not be plotted since the equilibrium condition between the aerodynamic drag force and the force due to surface tension would not apply to the extremely low value of surface tension evaluated. Therefore, it may be stated that the classical approach to analyzing secondary breakup of a droplet, involving the equilibrium between the aerodynamic drag force and the force due to surface tension may be valid, but the significance of hydrogen bonding in the surface tension of water may be better approximated through advanced experimental or statistical techniques. Thus, a theoretical estimate of the surface tension of water has been made in this chapter by analyzing the secondary breakup of a water droplet in terms of the forces resisting further breakup.
CHAPTER 6

Time for Onset of Secondary Breakup

This chapter is on the attempts to describe the process of secondary breakup as a function of time and other physical properties characterizing the flow. Secondary breakup, as described earlier in Chapter 4, has been found to initiate as a flattened disc and progress into a bag or parachute, based on the regime it can be classified into, according to the We of the cross-flow. Since it is difficult to establish a relation for the length of the jet before breakup or the spatial location of the point/phase of onset of breakup for the case of a rotary fuel slinger, it is more appropriate and accurate to define the onset of secondary breakup as a function of time.

6.1 Correlation between results of current model and past investigations:

Attempts have been made in the past, to describe secondary breakup for pressure-atomized sprays subjected to shockwave disturbances, by temporal relations. Liang et al [27] summarized past measurements of breakup times, including the findings of P.Y. Liang, T. W. Eastes, & Gharakhari [28] and P. G. Simpkins and E. J. Bales [29] for shear breakup and those of A. A. Ranger and J. A. Nicholls, [30] and W. G. Reinecke and W. L. McKay [31] for catastrophic breakup - all for shock wave disturbances at large $\rho_L/\rho_G$
and low Oh. For these conditions, breakup times ($t_b$) could be normalized by the characteristic shear breakup time, $t^* = d_0 \left( \frac{\rho_l}{\rho_g} \right)^{1/2} U_0$ [27], finding that the normalized breakup time does not vary greatly over the large range of $We$ that includes both the shear and catastrophic breakup regimes. A remarkable feature of the breakup time results in Figure 6.1 for Oh<0.1, which is the case in the current analysis, is that $t_b/t^*$ varies very little even though $We$ varies widely and several breakup regimes are involved. In fact, the correlation developed for shear breakup is [21]:

$$t_b/t^* = 5.0 \quad \ldots\ldots\text{Eqn. 6.1}$$

The above expression provides a good representation of all measurements illustrated in the following figure for Oh<0.1. However at large Oh, $t_b/t^*$ has been found to increase due to effects of liquid viscosity retarding the rate of deformation, discussed in Chapter 4.
Figure 6.1 Experimental results illustrating the relation between the normalized secondary breakup time and the cross-flow We for shockwave disturbances [5].

Liang et al [27], however, have shown that breakup times are equal to 5.5t* for a wide range of breakup conditions, where t* is the characteristic secondary breakup time for shear breakup defined by Ranger and Nicholls [29]. Elaborate investigations have been made by Hsiang and Faeth on establishing temporal relations for breakup properties in different regimes [5], [10] [11], [12], [13] and [32]. t_b/t* was calculated for different rotational rates and flow rates for the fuel slinger currently analyzed and the following values were found as tabulated in table 6.1.
Table 6.1 Listings of the normalized time of breakup due to [29] using data from current analysis.

<table>
<thead>
<tr>
<th>rpm</th>
<th>Flow rate (mL/min)</th>
<th>initial drop diameter (m)</th>
<th>initial velocity (m/sec)</th>
<th>$t_b/t^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5100</td>
<td>14</td>
<td>0.00018598</td>
<td>9.932584425</td>
<td>3.891603</td>
</tr>
<tr>
<td>32</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6000</td>
<td>14</td>
<td>0.00016378</td>
<td>11.6535</td>
<td>4.444074</td>
</tr>
<tr>
<td>32</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7200</td>
<td>14</td>
<td>0.0001442</td>
<td>13.64551714</td>
<td>5.824942</td>
</tr>
<tr>
<td>32</td>
<td>0.0002404</td>
<td></td>
<td></td>
<td>3.146158</td>
</tr>
<tr>
<td>8100</td>
<td>14</td>
<td>0.00012235</td>
<td>15.6638</td>
<td>6.219199</td>
</tr>
<tr>
<td>32</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9000</td>
<td>14</td>
<td>0.0001003</td>
<td>17.38081462</td>
<td>7.215454</td>
</tr>
<tr>
<td>32</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10200</td>
<td>14</td>
<td>0.0001202</td>
<td>19.33091926</td>
<td>5.301331</td>
</tr>
<tr>
<td>32</td>
<td>0.00016827</td>
<td></td>
<td></td>
<td>4.192734</td>
</tr>
<tr>
<td>13200</td>
<td>14</td>
<td>0.00009615</td>
<td>25.3854</td>
<td>6.412775</td>
</tr>
<tr>
<td>32</td>
<td>0.0001322</td>
<td></td>
<td></td>
<td>5.140277</td>
</tr>
</tbody>
</table>

It can be observed from the above table, though the calculated values of the normalized time of secondary breakup cannot be mapped on the plot generated by using previous investigations, they appear to follow a reasonably close correspondence with the correlation ($t_b/t^* \approx 5.18$) specified by Liang et al [27]. Thus, adherence with the above correlation for the time of secondary breakup even at very low We, as found in the current work, is observed for data calculated for the current analysis.

Since, it is observed that the data obtained using the analytical model for rotary fuel slingers obeys existing correlations for pressure-atomized jets subject to shockwave disturbances, it may be theorized that the above correlation applies reasonably well to atomization using rotary fuel slingers. Thus, the secondary breakup process for atomization by rotary fuel slingers can be temporally described.
CHAPTER 7

Temporal Properties of Primary Breakup

7.1 Introduction to primary breakup and its causes:

As introduced earlier, primary breakup is defined as the breakup of the jet into droplets/coarsely-shaped globules, depending on the velocity of the jet. Theories have been laid to describe the atomization process for pressure-atomized jets since they are the most-widely used type of atomizers. Further, the jet is known to disintegrate due to wave disturbances on its surface which eventually reach critical amplitude to cause instability on the surface of the jet and result in the formation of droplets.

Causes of these disturbances have been mentioned in the chapter containing the introduction to this thesis. One of the earliest theories on the disruption of a liquid jet was offered by Rayleigh (1878-79) who made a mathematical analysis of the stability of a non-viscous jet. A laminar liquid flow with a velocity potential and with the jet only under the influence of surface tension forces was considered and it was found that a jet would be unstable and ready to disrupt if its length were greater than its circumference [15]. Although the analysis does not elaborate on the breakup details for different ranges
of velocities, it has been used as a base for other theories (Haenlein, Weber and Castleman) proposed on jet disintegration.

Investigations by Holfelder (1932) and Haenlein (1931) include observations of liquid jet disruption in the form of high-speed spark photography. The discharge velocities were gradually increased and the jet disintegration due to different types of disturbances (shown in Chapter 1) and similar to those described by [3], but by different names, are shown below.

Figure 7.1 The different types of wave disturbances seen at different jet velocities [15].
Figure 7.2 The relationship between jet breakup distance and jet velocity [15].

The first type of disturbance is seen at low velocities as shown in Figure 7.2, which is rotationally symmetric and droplet formation and is primarily due to the effect of these disturbances and surface tension forces. The second type of disturbance is seen for a higher range of velocities and droplet formation is due to the combined effect of the disturbance and air friction. The third type of disturbances, also known as asymmetric disturbances, are described in Chapter 1 and the droplets formed are due to a more pronounced effect of air friction along with the effect of these disturbances. The other type of disturbances shown in Chapter 1, due to atomization immediately downstream of the exit of the jet, is caused mainly by aerodynamic drag forces and is seen at very high velocities (~100 m/sec).
Figure 7.3 The stages of disintegration in relation to Reynolds number and Ohnesorge number [15].

Figure 7.2 shows that the jet Re \((\rho L U_0 D/\mu L)\) observed in the cases currently analyzed for rotary atomizers lie within the range for the first type of disturbance which suggests that breakup occurs only due to the wave disturbance and not air friction. But on visual observation of the jets, twisted ribbon-like shapes can be seen on the jets after they exit from the orifice.

The different modes of primary breakup identified by Mazallon et al [8], namely, non-turbulent liquid jets in cross-flow, can experience column breakup, bag, multimode and shear breakup, similar to those found in the secondary breakup of drops.
7.2 Effects of high pressure on surface tension:

High-pressure atomization of round liquid jets injected into N$_2$ gas was studied by L.P. Hsiang and G.M. Faeth [33] and it was found that as pressure increases, surface tension decreases and surface gas density increases, which make turbulent atomization, emerge at a lower jet speed. The atomization regime of a low-speed jet changes drastically at a certain pressure P$_t$. At pressure < P$_t$, hydrodynamic-assisted capillary instability emerges immediately downstream of the nozzle exit and the liquid jet is disintegrated at a wavelength much shorter than one of Rayleigh instability. The analysis of this breakup mechanism identifies a new hydrodynamic-assisted capillary instability [34], which has the maximum growth rate proportional to jet speed and, thereby, is more unstable at low jet speed than the well-known Taylor instability.

At high pressure, as in the case of pressure atomizers, the small value of surface tension of the near-critical mixing surface jet makes the hydrodynamic force effective enough to deform the liquid surface at a small relative velocity, and a hydrodynamic-assisted liquid breakup process will become a main mechanism to produce droplets from ligaments. Hence pressure atomizers use the high pressures to their advantage in efficient atomization of liquid jets.

In the case of rotary slingers, the liquid issues from the slinger holes into the ambient gas at 1 atm. [4] & [6], but with a kinetic energy equivalent to a pressure head of more than 10.34 MPa. Hence in rotary fuel slingers, at low exit pressure, surface tension is large and gas density is small, so the breakup of the liquid column obeys the Rayleigh
mechanism. Thus, Taylor (or Rayleigh) type of instability emerges immediately downstream of the nozzle exit where the shear layer is thin and the velocity difference between liquid and gas phases is large. As it moves downstream, this capillary instability couples with the aerodynamic forces (depending on the regime followed by the jet breakup, which is governed by the cross flow We and Oh for the liquid jet), leading to the disintegration of the helically or axisymmetrically deformed liquid jet.

### 7.3 Model for predicting drop size after primary breakup:

This model is for a low velocity liquid jet ejected into a viscous liquid or gas (between 0 and 2 m/sec) [35]. The flows of both, the jet and the ambient fluid are assumed laminar. As a result of capillary instability, the jet breaks up into discrete droplets. At low velocities, jet breakup results in a train of uniform-size droplets [36], [37], [38].

Further, at low jet velocities, surface deformation of the jet is controlled by the most unstable disturbance wave at the early stages of jet instability, but before the transition process, where higher harmonics are not seen. Therefore it is assumed that each most-unstable disturbance wave produces a single droplet with the fundamental frequency, hence,

\[ u_0 \Delta t = N \lambda_m \]  

\[ \text{Eqn. 7.1[35]} \]

A dimensionless most-unstable wave number is defined as \( \eta_m = \pi d_0 / \lambda_m \) and an expression to predict the diameter of the droplets relative to the jet diameter before breakup is given [35]:

81
An equation, based on Rayleigh’s maximum instability theory, for the most-unstable wave number to cause the jet surface to disrupt was given as [35]

\[
\eta_m = (2 + 2Z \ast)^{-1/2}
\] .......Eqn. 7.3

Substituting into the above equation, we get an expression which predicts the size of the drop generated due to the most unstable wave:

\[
\frac{d}{d_0} = \left(\frac{3\pi}{2\eta_m}\right)^{1/3} \sqrt{1 + Z \ast}^{1/2}
\] .......Eqn. 7.4

Where, \( Z \ast = (3\mu_L + \mu_{air})/(d_0\sigma_L)^{1/2} \) is a modified Ohnesorge number [35].

This equation may apply to low velocities (between 0 and 2 m/sec) for liquid-in-liquid or liquid-in-gas systems.

**7.4 Estimation of time of onset of primary breakup:**

Sallam et al. [9] observed that the breakup time for low Oh jets was independent of the Weber number for the three breakup regimes; bag, multimode and shear and was equal to \( 2.5t* \), where \( t* \) is the aerodynamic characteristics time \( (t* = d_i(\rho_L/\rho_G)^{1/2}/U_G) \) due to Ranger and Nicholls [29]. However, Mazallon et al. [8] found that, when viscous effects are small (Oh < 0.1), breakup regime transitions of the liquid jet are determined by the cross-flow Weber number as follows: column breakup (WeG < 4), bag breakup
(4 < We<sub>G</sub> < 30), multimode breakup (30 < We<sub>G</sub> < 110), and shear breakup (We<sub>G</sub> > 110).

Elaborate experimental investigation of the primary breakup of non-turbulent round liquid jets in gas cross flows made by Mazallon et al. [8] have resulted in the accumulation of data that may help in understanding jet primary breakup regimes better. Jet deformation properties, time of onset of primary breakup, and liquid column and liquid surface disturbance wavelengths were obtained for air cross-flows at normal temperature and pressure. This led to a comparison of data calculated from the analytical model previously described and visual data by Sescu et al [7], with the graphs plotted by Mazallon et al [6], who have compiled data from various satisfactory sources ([5], [11], [12]). The range of We worked on by Mazallon et al [8] cover the range of We in the current analysis partially. Since the jet Re calculated for the cases analyzed presently, are of the order of a few hundreds, which indicates flow in the non turbulent/transition range for cylinders/spheres, the results obtained by Mazallon et al may be considered for comparison with the data calculated from the analytical model to verify its applicability to rotary slingers. Further the Oh for the experimental investigation by Mazallon et al included low values corresponding to the present analysis.

Since, for low Oh, the jet breakup regime transitions are determined by the cross-flow We, which is given by ρ<sub>G</sub>U<sub>0</sub><sup>2</sup>d<sub>0</sub>/σ, where ρ<sub>G</sub> is the density of the ambient gas, U<sub>0</sub>, d<sub>0</sub> are the initial velocity and diameter of the jet, respectively, before breakup, it is shown in later calculations that the rotary slinger currently under analysis produced jets which lie in the column breakup regime. The column breakup regime is somewhat different from
the other breakup regimes described, such as bag breakup, multimode breakup, shear breakup and catastrophic breakup. These modes are similar to the secondary breakup modes described by [10], [13].

Mazallon et al established that the time of onset of primary breakup, normalized by the characteristic secondary breakup time by Ranger and Nicholls [29], is dependent on the We unlike the independence of the normalized time of onset of secondary breakup of We, described in Chapter 6.

Figure 7.4 Correlation between the normalized time of onset of primary breakup and We. Past measurements of data corresponding to the normalized time of onset of secondary breakup of droplets by [5], [10], [11], [12], [13] and [32] are also shown.
The normalized time of onset of primary breakup \( (t_i/t^*) \), where 
\[
t^* = d_0 (\rho_L/\rho_{\text{air}})^{1/2}/u_{\text{air}},
\]
was calculated using data obtained from the analytical model for various experimental operating conditions and the values (blue-colored dots) were plotted on the graph shown by Mazallon et al. This graph also plots data for the normalized time of onset of secondary breakup obtained from past investigations, for comparison, and they are found to be of the same order of magnitude, but constant, whereas the normalized time of onset of primary breakup varies with We.

The correlation between the normalized time of onset of primary breakup and the We was found to be

\[
t_i/t^* = 8.76*\text{We}^{-0.62} \quad \text{.........Eqn. 7.5 [8]}
\]

The plot of data points calculated using the analytical model described earlier shows that the liquid jets corresponding to the current analysis obey the liquid column breakup regime, which correspond to low We \((0.3<\text{We}<1.8)\). Although the actual graph shown by Mazallon et al does not include the range of \(\text{We}<1\), the correlation may be used to extend the plot to the desired range for verifying the regime of jet breakup for the calculated data points. Thus, it may be reasonably assumed that data obtained from the analytical model follows the same correlation described by Mazallon et al [8] for the time of onset of primary breakup of round non-turbulent liquid jets.
CHAPTER 8

Conclusions

As the objective of the thesis suggests, an analytical model which can reasonably explain the breakup processes in the disintegration of round liquid jets ejected from a rotary atomizer/slinger has been compared to Sescu et al [7]. This model predicts, within acceptable tolerances, the drop diameters and velocities of the droplets formed after the breakup of the liquid jets which are ejected from centrifugal atomizers. This model based on water-air combination, may be applied to other liquid-gas combinations with liquid-gas density ratios > 500 and Re > 100, since previous analyses have been made for liquid-gas density ratios and Re in this range and satisfactory observations and conclusions have been made.

8.1 Primary breakup:

An attempt was made to describe primary breakup of jets ejected by the centrifugal atomizer under analysis and temporal relations developed for round cylindrical jets in the past [8], [29] have been shown to be applicable to the case of atomization by rotary slingers. Due to the lack of data relevant to the wavelengths of
waves formed on the surface of the liquid jets, an elaborate analysis of the primary breakup of liquid jets from rotary fuel slingers could not be included in this model.

The reasonably close fit of data points calculated using results of the drop inertia force-aerodynamic drag force equilibrium (Chapter 4) demonstrates the validity of the approach and assumptions made in the development of relevant mathematical relations. Observation of the trend followed by the calculated data points in Figure 7.4 suggests that the type of primary breakup that occurred in the experiments could be the liquid column breakup, characterized by low We. Further, on visual observation of images of the liquid jets, signs of bag breakup or bag/shear breakup are seemingly absent.

8.2 Model for prediction of droplet trajectories:

The model involving the equilibrium between the inertial force of the drop and the aerodynamic drag force has been shown to sufficiently match the actual droplet trajectories for various operating speeds and flow rates of the slinger. Further, the use of accurate relations for the coefficient of drag for drops subject to deformation may have resulted in the close correspondence of the estimated trajectories with the actual drop trajectories. The expression for the average velocity of the film in the radial channels of the slinger, provided by Dahm et al [4] has been found to yield fairly accurate values of radial components of the velocity of the jet exiting the slinger orifice, which resulted in accurate drop trajectories.
Even though the Morsi and Alexander equation for the drag coefficients of droplets which describes the variation of $C_D$ with $Re$ for a complete range of $Re$ accurately, it may not have provided accurate results for the solution to the governing equation of the trajectories. But the equation for $C_D$ derived from L. Bayvel, and Z. Orzechowski [3], has apparently provided better results compared to the other expression for the coefficient of drag. Thus, the approach of analyzing the equilibrium between physical forces, namely the inertial force of the drop and the aerodynamic drag force and the approximation of the liquid jet as a stream of drops has resulted in the establishment of relations describing the position of drops spatially and temporally, with reasonable accuracy.

**8.3 Explanation of secondary breakup of drops from the rotary slinger:**

Relations to predict the secondary breakup of drops produced from the currently analyzed centrifugal atomizer could not be developed due to lack of accurate data pertaining to the dimensions of minute droplets, although some correlations between the data obtained, using the relations for drop trajectories, and data compiled from relevant past investigations were made possible and reasonable correspondence was observed. Temporal correlations between the time of onset of secondary breakup for the slinger under analysis (using data calculated from the analytical model) and data from past investigations were also successfully established in a similar fashion.

Thus, the analytical model used to compare with data from Sescu et al [7] as part of this thesis provides useful insight into the atomization of liquid jets ejected by rotary
fuel slingers and further augmentation can be made to describe completely, the complex mechanisms of liquid atomization using centrifugal atomizers.
REFERENCES


APPENDICES

APPENDIX A

Graphs of $X$ and $Y$ displacements, comparison of trajectories and photographs of actual trajectories
Graph of X component of displacement (X) in m. vs. time (t) in sec, at 5100 rpm and flow rate of 14 mL/min
Graph of Y component of displacement (Y) in m. vs. time (t) in sec, at 5100 rpm and flow rate of 14 mL/min
Graph of Y component of displacement (Y) in m. vs. X component of displacement (X) in m., at 5100 rpm and flow rate of 14 mL/min
Graph of X component of displacement (X) in m. vs. time (t) in sec, at 6000 rpm and flow rate of 14 mL/min
Graph of Y component of displacement (Y) vs. time (t) at 6000 rpm and flow rate of 14 mL/min
Graph of Y component of displacement (Y) vs. X component of displacement (X) at 6000 rpm and 14 mL/min.
Graph of X component of displacement (X) in m. vs. time (t) in sec, at 7200 rpm and flow rate of 14 mL/min
Graph of Y component of displacement (Y) in m. vs. time (t) in sec, at 7200 rpm and flow rate of 14 mL/min
Graph of Y component of displacement (Y) in m. vs. X component of displacement (X) at 7200 rpm and flow rate of 14 mL/min.
Graph of X component of displacement (X) in m. vs. time (t) in sec, at 7200 rpm and flow rate of 32 mL/min
Graph of Y component of displacement (Y) in m. vs. time (t) in sec, at 7200 rpm and 32 mL/min
Graph of Y component of displacement (Y) vs. X component of displacement (X) at 7200 rpm and 32 mL/min

Graph of Y component of displacement (Y) in m. vs. X component of displacement (X) in m., at 7200 rpm and flow rate of 32 mL/min
Graph of $X$ component of displacement ($X$) in m. vs. time ($t$) in sec, at 8100 rpm and flow rate of 14 mL/min
Graph of Y component of displacement (Y) in m. vs. time (t) in sec, at 8100 rpm and flow rate of 14 mL/min.
Graph of Y component of displacement (Y) in m. vs. X component of displacement (X) in m., at 8100 rpm and flow rate of 14 mL/min
Graph of X component of displacement (X) in m. vs. time (t) in sec, at 9000 rpm and flow rate of 14 mL/min
Graph of Y component of displacement (Y) in m. vs. time (t) in sec, at 9000 rpm and flow rate of 14 mL/min
Graph of Y component of displacement (Y) in m. vs. X component of displacement (X) at 9000 rpm and 14 mL/min
Graph of X component of displacement (X) in m. vs. time (t) in sec, at 10200 rpm and flow rate of 14 mL/min
Graph of $Y$ component of displacement ($Y$) in m. vs. time ($t$) in sec, at 10200 rpm and flow rate of 14 mL/min
Graph of Y component of displacement (Y) in m. vs. X component of displacement (X) in m., at 10200 rpm and flow rate of 14 mL/min
Graph of X component of displacement (X) in m. vs. time (t) in sec, at 10200 rpm and flow rate of 32 mL/min
Graph of Y component of displacement (Y) in m. vs. time (t) in sec, at 10200 rpm and flow rate of 32 mL/min
Graph of Y component of displacement (Y) in m. vs. X component of displacement (X) in m., at 10200 rpm and 32 mL/min.
Graph of X component of displacement (X) in m. vs. time (t) in sec, at 13200 rpm and flow rate of 14 mL/min
Graph of Y component of displacement (Y) in m. vs. time (t) in sec, at 13200 rpm and flow rate of 14 mL/min
Graph of Y component of displacement (Y) in m. vs. X component of displacement (X) in m., at 13200 rpm and flow rate of 14 mL/min
Graph of X component of displacement (X) in m. vs. time (t) in sec, at 13200 rpm and flow rate of 32 mL/min
Graph of Y component of displacement (Y) in m. vs. time (t) in sec, at 13200 rpm and flow rate of 32 mL/min
Graph of Y component of displacement (Y) in m. vs. X component of displacement (X) in m., at 13200 rpm and 32 mL/min
Comparison of actual photograph of trajectory and calculated trajectory at $n = 5100$ RPM, $Q = 14$ mL/min
Comparison of actual photograph of trajectory and calculated trajectory at n = 6000 RPM, Q = 14 mL/min
Comparison of actual photograph of trajectory and calculated trajectory at $n = 7200$ RPM, $Q = 14$ mL/min
Comparison of actual photograph of trajectory and calculated trajectory at n = 7200 RPM, Q = 32 mL/min
Comparison of actual photograph of trajectory and calculated trajectory at \( n = 8100 \) RPM, \( Q = 14 \) mL/min
Comparison of actual photograph of trajectory and calculated trajectory at $n = 9000$ RPM, $Q = 14$ mL/min
Comparison of actual photograph of trajectory and calculated trajectory at $n = 10200$ RPM, $Q = 14$ mL/min
Comparison of actual photograph of trajectory and calculated trajectory at n = 10200 RPM, Q = 32 mL/min
Comparison of actual photograph of trajectory and calculated trajectory at n = 13200 RPM, Q = 14 mL/min
Comparison of actual photograph of trajectory and calculated trajectory at $n = 13200$ RPM, $Q = 32$ mL/min
Breakup visualization at n = 5100 RPM, Q = 14 mL/min
Breakup visualization at $n = 6000$ RPM, $Q = 14$ mL/min
Breakup visualization at $n = 7200$ RPM, $Q = 14$ mL/min
Breakup visualization at n = 7200 RPM, Q = 32 mL/min
Breakup visualization at $n = 8100$ RPM, $Q = 14$ mL/min
Breakup visualization at $n = 9000$ RPM, $Q = 14$ mL/min
Breakup visualization at $n = 10200$ RPM, $Q = 14\ \text{mL/min}$
Breakup visualization at $n = 10200$ RPM, $Q = 32$ mL/min
Breakup visualization at $n = 13200$ RPM, $Q = 14$ mL/min
Breakup visualization at $n = 13200$ RPM, $Q = 32$ mL/min
MATLAB code corresponding to 5100 rpm and flow rate of 14 mL/min

format long

Rslinger = 0.0180975;
dhole = 0.000889;

q = 2.3333E-07;  % 14 mL/min, conversion from mL/min to m³/sec = 1/(60*10^6) %
omega = 534.07075;  % rpm = 5100 %
rhoair = 1.204;
rhowater = 1000;
dia = 0.00018598;
muair = 1.85*10^-5;
muwater = 0.001002;

thick = 0.984745*(muwater*q/rhowater/Rslinger/omega/omega/dhole)^(1/3)

Vtan = Rslinger*omega
Vrad = q/pi/dhole/thick
ratio = Vrad/Vtan
Vresult = sqrt(Vtan*Vtan + Vrad*Vrad)
alphaindegrees = atan(ratio)*180/pi
A = 3*π*ρ_{\text{air}}*0.128/4/\text{dia}/\rho_{\text{water}}

B = 3*π*μ_{\text{air}}*12.8/4/\text{dia}/\rho_{\text{water}}

α = \text{atan}(\text{ratio})

S_{\text{dis}} = \text{dsolve}'D2S+(1.9525*DS*DS)+(16.131*DS)','S(0)=0','DS(0) = 9.9326','t')

\text{pretty}(S_{\text{dis}})

t=1:0.0001:1.01;

\text{answer} = \text{eval}(S_{\text{dis}})

X = \text{eval}(S_{\text{dis}}*\cos(α));

Y = \text{eval}(S_{\text{dis}}*\sin(α));

\text{plot}(X,Y)
MATLAB code corresponding to 6000 rpm and flow rate of 14 mL/min

format long

Rslinger = 0.0180975;

dhole = 0.000889;

q = 2.3333E-07; % 14 mL/min, conversion from mL/min to m³/sec = 1/(60*10^6) %

omega = 628.318; % rpm = 6000 %

rhoair = 1.204;

rhowater = 1000;

dia = 0.00016378;

muair = 1.85*10^-5;

muwater = 0.001002;

thick = 0.984745*(muwater*q/rhowater/Rslinger/omega/omega/dhole)^(1/3)

Vtan = Rslinger*omega

Vrad = q/pi/dhole/thick

ratio = Vrad/Vtan

Vresult = sqrt(Vtan*Vtan + Vrad*Vrad)

alphaindegrees = atan(ratio)*180/pi
A = 3*\pi*rhoair*0.128/4/dia/rhowater

B = 3*\pi*muair*12.8/4/dia/dia/rhowater

alpha = atan(ratio)

Sdis = dsolve('D2S+(2.2171*DS*DS)+(20.8004*DS)','S(0)= 0','DS(0) = 11.6535','t')

pretty(Sdis)

t=0:0.0001:0.05;

answer = eval(Sdis)

X = eval(Sdis*cos(alpha));

Y = eval(Sdis*sin(alpha));

plot(X,Y)
MATLAB code corresponding to 7200 rpm and flow rate of 14 mL/min

format long

Rslinger = 0.0180975;

dhole = 0.000889;

q = 2.3333E-07; % 14 mL/min, conversion from mL/min to m^3/sec = 1/(60*10^6) %

omega = 753.9822369; % rpm = 7200 %

rhoair = 1.204;

rhowater = 1000;

dia = 0.0001442;

muair = 1.85*10^-5;

muwater = 0.001002;

thick = 0.984745*(muwater*q/rhowater/Rslinger/omega/omega/dhole)^(1/3)

Vtan = Rslinger*omega

Vrad = q/pi/dhole/thick

ratio = Vrad/Vtan

Vresult = sqrt(Vtan*Vtan + Vrad*Vrad)

alphaindegrees = atan(ratio)*180/pi
\[ A = 3\pi \rho_{\text{air}} \times 0.128/4/\text{dia/} \rho_{\text{water}} \]

\[ B = 3\pi \mu_{\text{air}} \times 12.8/4/\text{dia/} \rho_{\text{water}} \]

\[ \text{alpha} = \text{atan(ratio)} \]

\[ \text{Sdis} = \text{dsolve}'D2S+(2.518*DS*DS)+(26.8326*DS)','S(0)= 0','DS(0) = 13.6455','t') \]

\[ \text{pretty(Sdis)} \]

\[ t=0:0.0001:0.05; \]

\[ \text{answer} = \text{eval(Sdis)} \]

\[ X = \text{eval(Sdis*cos(alpha))}; \]

\[ Y = \text{eval(Sdis*sin(alpha))}; \]

\[ \text{plot(X,Y)} \]
MATLAB code corresponding to 7200 rpm and flow rate of 32 mL/min

format long

Rslinger = 0.0180975;

dhole = 0.000889;

q = 5.3333E-07;       % 32 mL/min, conversion from mL/min to m3/sec = 1/(60*10^6) %

omega = 753.9822369;     % rpm = 7200 %

rhoair = 1.204;

rhowater = 1000;

dia = 0.0002404;

muair = 1.85*10^-5;

muwater = 0.001002;

thick = 0.984745*(muwater*q/rhowater/Rslinger/omega/omega/dhole)^(1/3)

Vtan = Rslinger*omega

Vrad = q/pi/dhole/thick

ratio = Vrad/Vtan

Vresult = sqrt(Vtan*Vtan + Vrad*Vrad)

alphaindegrees = atan(ratio)*180/pi
A = 3*\pi*rho_{air}*0.128/4/dia/rho_{water}

B = 3*\pi*mu_{air}*12.8/4/dia/dia/rho_{water}

alpha = \arctan(ratio)

S_{dis} = \text{dsolve('D2S+(1.5105*DS*DS)+(9.6544*DS)','S(0)= 0','DS(0) = 14.5315','t')}

pretty(S_{dis})

t=0:0.0001:0.05;

answer = \text{eval}(S_{dis})

X = \text{eval}(S_{dis}*\cos(alpha));

Y = \text{eval}(S_{dis}*\sin(alpha));

\text{plot}(X,Y)
MATLAB code corresponding to 8100 rpm and flow rate of 14 mL/min

format long

Rslinger = 0.0180975;

dhole = 0.000889;

q = 2.3333E-07;  % 14 mL/min, conversion from mL/min to m³/sec = 1/(60*10^6) %

omega = 848.23;  % rpm = 8100 %

rhoair = 1.204;

rhowater = 1000;

dia = 0.00012235;

muair = 1.85*10^-5;

muwater = 0.001002;

thick = 0.984745*(muwater*q/rhowater/Rslinger/omega/omega/dhole)^(1/3)

Vtan = Rslinger*omega

Vrad = q/pi/dhole/thick

ratio = Vrad/Vtan

Vresult = sqrt(Vtan*Vtan + Vrad*Vrad)

alphaindegrees = atan(ratio)*180/pi
A = 3*pi*rhoair*0.128/4/dia/rhowater

B = 3*pi*muair*12.8/4/dia/dia/rhowater

alpha = atan(ratio)

Sdis = dsolve('D2S+(2.9679*DS*DS)+(37.2722*DS)','S(0)= 0','DS(0) = 15.6638','t')

pretty(Sdis)

t=0:0.0001:0.05;

answer = eval(Sdis)

X = eval(Sdis*cos(alpha));

Y = eval(Sdis*sin(alpha));

plot(X,Y);
MATLAB code corresponding to 9000 rpm and flow rate of 14 mL/min

format long

Rslinger = 0.0180975;

dhole = 0.000889;

q = 2.3333E-07;    % 14 mL/min, conversion from mL/min to m³/sec = 1/(60*10^6) %

omega = 942.4777961;    % rpm = 9000 %

rhoair = 1.204;

rhowater = 1000;

dia = 0.0001003;

muair = 1.85*10^-5;

muwater = 0.001002;

thick = 0.984745*(muwater*q/rhowater/Rslinger/omega/omega/dhole)^(1/3)

Vtan = Rslinger*omega

Vrad = q/pi/dhole/thick

ratio = Vrad/Vtan

Vresult = sqrt(Vtan*Vtan + Vrad*Vrad)

alphaindegrees = atan(ratio)*180/pi
A = 3*pi*rhoair*0.128/4/dia/rhowater

B = 3*pi*muair*12.8/4/dia/dia/rhowater

alpha = atan(ratio)

Sdis = dsolve('D2S+(3.6203*DS*DS)+(55.4614*DS)','S(0)= 0','DS(0) = 17.38','t')

pretty(Sdis)

t=0:0.0001:0.05;

answer = eval(Sdis)

X = eval(Sdis*cos(alpha));

Y = eval(Sdis*sin(alpha));

plot(X,Y);
MATLAB code corresponding to 10200 rpm and flow rate of 14 mL/min

format long

Rslinger = 0.0180975;

dhole = 0.000889;

q = 2.3333E-07; % 14 mL/min, conversion from mL/min to m³/sec = 1/(60*10^6) %

omega = 1068.141502; % rpm = 10200 %

rhoair = 1.204;

rhowater = 1000;

dia = 0.0001202;

muair = 1.85*10^-5;

muwater = 0.001002;

thick = 0.984745*(muwater*q/rhowater/Rslinger/omega/omega/dhole)^(1/3)

Vtan = Rslinger*omega

Vrad = q/pi/dhole/thick

ratio = Vrad/Vtan

Vresult = sqrt(Vtan*Vtan + Vrad*Vrad)

alphaindegrees = atan(ratio)*180/pi
\[ A = 3\pi\rho_{\text{air}} \cdot 0.128/4/\text{dia}/\rho_{\text{water}} \]

\[ B = 3\pi\mu_{\text{air}} \cdot 12.8/4/\text{dia}/\rho_{\text{water}} \]

\[ \alpha = \arctan(\text{ratio}) \]

\[ S_{\text{dis}} = \text{dsolve}'D2S+(3.0209*DS*DS)+(38.6175*DS)','S(0)=0','DS(0)=19.669','t' \]

\[ \text{pretty}(S_{\text{dis}}) \]

\[ t=0:0.0001:0.05; \]

\[ \text{answer} = \text{eval}(S_{\text{dis}}) \]

\[ X = \text{eval}(S_{\text{dis}} \cdot \cos(\alpha)); \]

\[ Y = \text{eval}(S_{\text{dis}} \cdot \sin(\alpha)); \]

\[ \text{plot}(X,Y) \]
MATLAB code corresponding to 10200 rpm and flow rate of 32 mL/min

format long

Rslinger = 0.0180975;

dhole = 0.000889;

q = 5.3333E-07;  % 32 mL/min, conversion from mL/min to m3/sec = 1/(60*10^6) %

omega = 1068.141502;   % rpm = 10200 %

rhoair = 1.204;

rhowater = 1000;

dia = 0.00016827;

muair = 1.85*10^-5;

muwater = 0.001002;

thick = 0.984745*(muwater*q/rhowater/Rslinger/omega/omega/dhole)^(1/3)

Vtan = Rslinger*omega

Vrad = q/pi/dhole/thick

ratio = Vrad/Vtan

Vresult =sqrt(Vtan*Vtan + Vrad*Vrad)

alphaindegrees = atan(ratio)*180/pi
A = 3*pi*rhoair*0.128/4/dia/rhowater

B = 3*pi*muair*12.8/4/dia/dia/rhowater

alpha = atan(ratio)

Sdis = dsolve('D2S+(2.1579*DS*DS)+(19.705*DS)','S(0)= 0','DS(0) = 20.3325','t')

pretty(Sdis)

t=0:0.0001:0.05;

answer = eval(Sdis)

X = eval(Sdis*cos(alpha));

Y = eval(Sdis*sin(alpha));

plot(X,Y);
MATLAB code corresponding to 13200 rpm and flow rate of 14 mL/min

format long

Rslinger = 0.0180975;

dhole = 0.000889;

q = 2.3333E-07;       % 14 mL/min, conversion from mL/min to m³/sec = 1/(60*10^6) %

omega = 1382.300768;      % rpm = 13200 %

rhoair = 1.204;

rhowater = 1000;

dia = 0.00009615;

muair = 1.85*10^-5;

muwater = 0.001002;

thick = 0.984745*(muwater*q/rhowater/Rslinger/omega/omega/dhole)^(1/3)

Vtan = Rslinger*omega

Vrad = q/pi/dhole/thick

ratio = Vrad/Vtan

Vresult = sqrt(Vtan*Vtan + Vrad*Vrad)

alphaindegrees = atan(ratio)*180/pi
A = 3*\text{pi}\cdot\rho_{\text{air}}\cdot0.128/4/\text{dia}/\rho_{\text{water}}

B = 3*\text{pi}\cdot\mu_{\text{air}}\cdot12.8/4/\text{dia}/\rho_{\text{water}}

\alpha = \text{atan}(\text{ratio})

S_{\text{dis}} = \text{dsolve}'(D^2S+(3.7766\cdot DS\cdot DS)+(60.35236\cdot DS)','S(0)= 0','DS(0) = 25.3854','t')

\text{pretty}(S_{\text{dis}})

t=0:0.0001:0.05;

\text{answer} = \text{eval}(S_{\text{dis}})

X = \text{eval}(S_{\text{dis}}\cdot\cos(\alpha));

Y = \text{eval}(S_{\text{dis}}\cdot\sin(\alpha));

\text{plot}(X,Y);
MATLAB code corresponding to 13200 rpm and flow rate of 32 mL/min

format long

Rslinger = 0.0180975;

dhole = 0.000889;

q = 5.3333E-07;      % 32 mL/min, conversion from mL/min to m³/sec = 1/(60*10^6) %

omega = 1382.300768;      % rpm = 13200 %

rhoair = 1.204;

rhowater = 1000;

dia = 0.0001322;

muair = 1.85*10^(-5);

muwater = 0.001002;

thick = 0.984745*(muwater*q/rhowater/Rslinger/omega/omega/dhole)^(1/3)

Vtan = Rslinger*omega

Vrad = q/pi/dhole/thick

ratio = Vrad/Vtan

Vresult = sqrt(Vtan*Vtan + Vrad*Vrad)

alphaindegrees = atan(ratio)*180/pi
\[ A = 3\pi \rho_{\text{air}} \cdot 0.128/4/\text{dia}/\rho_{\text{water}} \]

\[ B = 3\pi \mu_{\text{air}} \cdot 12.8/4/\text{dia}/\rho_{\text{water}} \]

\[ \alpha = \text{atan}(\text{ratio}) \]

\[ S_{\text{dis}} = \text{dsolve}'D2S+(2.74673 \cdot DS \cdot DS)+(31.92493 \cdot DS)' , 'S(0)= 0', 'DS(0) = 26.1122', 't' \]

\[ \text{pretty}(S_{\text{dis}}) \]

\[ t=0:0.0001:0.05; \]

\[ \text{answer} = \text{eval}(S_{\text{dis}}) \]

\[ X = \text{eval}(S_{\text{dis}} \cdot \cos(\alpha)); \]

\[ Y = \text{eval}(S_{\text{dis}} \cdot \sin(\alpha)); \]

\[ \text{plot}(X,Y) \]
APPENDIX B

Sample Calculations:

Sample calculation for graph in Figure 4.4:

The equilibrium between the aerodynamic drag force acting on a spherical drop and the force due to surface tension on the drop is assumed.

The aerodynamic drag force on a spherical droplet is given by

\[ F_D = C_D \left( \frac{1}{2} \rho \rho_{\text{air}} \cdot V^2 \cdot \frac{\pi D^2}{4} \right) \]

Where

\[ C_D \] is the coefficient of drag for the spherical droplet

\[ C_D = \frac{24}{Re} \text{ for } 1 \times 10^4 < Re < 0.4 \] (Laminar regime)

\[ = \frac{18.5}{Re^{0.6}} \text{ for } 0.4 < Re < 1000 \] (Transition regime)

\[ \approx 0.44 \text{ for } 1000 < Re < 2 \times 10^4 \] (Turbulent regime)

\[ Re = \frac{\rho \rho_{\text{air}} \cdot V \cdot D}{\mu \rho_{\text{air}}} \]
Where

\[ \rho_{\text{air}} \] is the density of ambient gas, which is air, in our case

\[ \rho_{\text{air}} = 1.205 \, \text{kg/m}^3 \text{ at } 25^\circ \text{C} \]

\[ \mu_{\text{air}} = 1.85 \times 10^{-5} \, \text{kg/m-sec} \]

\( V \) is the relative velocity between the droplet and the ambient air, since air is stationary in this case, \( V \) is taken as velocity of the droplet

\( D \) is the diameter of the drop

The force due to surface tension on a spherical droplet is given by 

\[ F_S = \pi \cdot D \cdot \sigma \]

Where

\( \sigma \) is the surface tension of the dispersed liquid phase, which is water

\( \sigma, \text{ for water} = 0.0728 \, \text{N/m at } 25^\circ \text{C} \)

For a sample droplet of size 144 \( \mu \text{m} \),

The aerodynamic drag force is given by 

\[ C_D \cdot \left( \frac{1}{2} \cdot \rho_{\text{air}} \cdot V^2 \cdot \frac{\pi D^2}{4} \right) \]

\[ C_D = 18.5/Re^{0.6}, \text{ since } Re = \rho_{\text{air}} \cdot V \cdot D/\mu_{\text{air}} = 1.205 \cdot V \cdot 144 \cdot 10^{-6}/1.85 \cdot 10^{-5} \]

The force due to surface tension on a spherical droplet is given by 

\[ \pi \cdot D \cdot \sigma \]
For equilibrium of the above two forces, the velocity, \( V = 107.5 \text{ m/sec} \), which is the maximum velocity at which a spherical drop of diameter 144 \( \mu \text{m} \) can remain without disintegration due to drag forces.

Hence, the value of \( \text{Re} \) at this velocity and diameter of the droplet = 1008.292 \( \mu \text{m} \), which indicates flow in the turbulent regime.

Therefore, \( C_D \approx 0.44 \) is used to calculate the aerodynamic drag force.

**Sample calculation for graph in Figure 4.9:**

Figure 4.8 shows that the variation between the normalized coefficient of drag of the actual droplets and \( \text{We} \).

\( C_{D_{sp}} \) is the coefficient of drag of a sphere having volume equivalent to that of the actual droplet at the same \( \text{Re} \) as for the actual droplet.

The equilibrium condition between the aerodynamic drag force acting on a spherical drop and the force due to surface tension on the drop is checked for the droplet sizes approximated in the model developed in Chapter 3. Parallely, a calculation is also made for the drop deformation ratio \( d_{\text{max}}/d_{\text{min}} \) using the condition of constancy of volume of the drop.

Since the \( C_D \) of the droplet is calculated in Chapter 3 by the expression, \( C_D = \pi \times \left( 0.128 + \frac{12.8}{\text{Re}} \right) \), which predicts the values for the coefficient of drag for all ranges of \( \text{Re} \), and since \( C_D/C_{D_{sp}} \) in Figure 4.8 shows a flat trend with a nearly-constant value of 1.06 for the range of \( \text{We} \) in our case, the equilibrium condition is modified slightly to
implicitly calculate the values of $d_{\text{max}}$ and $d_{\text{min}}$, which correspond to the maximum and minimum diameters when the droplet deforms into a disc-like shape.

The expression to calculate $d_{\text{min}}$, for a known $d_{\text{max}}$ is $d_{\text{max}}/d_{\text{min}} = (1+0.07 \text{ We}^{0.5})^3$

We is calculated by $\rho_{\text{air}} V_0^2 D_0 / \sigma$

Where

Subscript 0 in these calculations refer to initial conditions before secondary breakup

The value of $d_{\text{max}}$ is iterated while satisfying the condition of equilibrium of the aerodynamic drag force and the force due to surface tension on a deformed droplet (disc-like shape)

The aerodynamic drag force on a deformed droplet is given by

$$F_D = C_D \left( \frac{1}{2} \rho_{\text{air}} V^2 \pi \frac{D_{\text{max}}^2}{4} \right)$$

Where

$C_D$ is evaluated from Figure 4.8 with the $C_D/C_{\text{DSP}}$ ratio $\approx 1.06$

$C_{\text{DSP}}$ is obtained from the relation between the Re and coefficient of drag for a sphere.

The force due to surface tension on a deformed droplet is given by $F_S = \pi D_{\text{min}} \sigma$
The equilibrium condition for the above two forces on a deformed drop gives a fifth order polynomial in \( d_{\text{max}} \), as shown in Eqn. 4.7

This equation is solved only by random iteration of \( d_{\text{max}} \) so that the ratio \( C_D/C_{\text{Dsp}} \) ratio \( \approx 1.06 \) is also satisfied.

Thus, for operating conditions of 5100 rpm, flow rate of 14 mL/min and a droplet of diameter 294.81 \( \mu \text{m} \), the visually-estimated point of secondary breakup corresponds to a droplet velocity of 9.2478 m/sec, obtained from the model developed in Chapter 3.

It appears that the equilibrium condition of the two forces is not realized while maintaining the \( C_D/C_{\text{Dsp}} \) ratio \( \approx 1.06 \), invoking the concept of a third force(s) involved in the equilibrium.

Thus, for the above conditions, the We, based on parameters corresponding to initial conditions, is found to be 0.41734, which is low. The drag force from the iterative calculation is 0.0000032 N and the force due to surface tension is found to be 0.0000695 N, which is wide of the equilibrium condition, and the corresponding \( d_{\text{max}}/d_{\text{min}} \) ratio = 1.6446, which is of the order of magnitude for the deformation ratio for most droplets undergoing secondary breakup [13].

The parameter \( (d_{\text{max}}/d_{\text{min}})^{1/3} - 1 \) is found to be 0.18038 and is plotted on the graph as shown in Figure 4.9 for the corresponding We.
Sample calculation for graph in Figure 6.1 and Table 6.1:

The calculations for the time of onset of secondary breakup have been shown and compared with data compiled by Hsiang and Faeth from other investigations.

Secondary breakup times \( t_b \) could be normalized by the characteristic shear breakup time, \( \tau^* = d_o \left( \frac{\rho_L}{\rho_G} \right)^{1/2}/U_o \)

Where

\[
\begin{align*}
    d_o & \text{ in these calculations is the initial diameter of the droplet at the slinger exit} \\
    \rho_L & \text{ is the density of liquid, in our case, water} \\
    \rho_G & \text{ is the density of the ambient gas, air, in our case} \\
    U_o & \text{ is the velocity of the droplet at the exit of the slinger orifice}
\end{align*}
\]

The values of \( t_b/\tau^* \) have been calculated for droplets in various investigations, subject to shockwave disturbances and for different values of We, as shown in Figure 6.1. Liang et al suggested that breakup times are equal to 5.5\( \tau^* \) for a wide range of breakup conditions or \( t_b/\tau^* = 5.5 \).

The values of \( t_b \) are calculated from the model developed and shown in Chapter 3, by visual estimation of the point of onset of secondary breakup and normalized by \( \tau^* \). For a sample calculation, the parameters at an operating speed of 8100 rpm and flow rate of 14 ml/min are considered, the average diameter of the droplet before the onset of breakup
is evaluated by counting the number of drops in a given interval of time before secondary breakup and relating it to the volume flow rate.

The diameter of the droplet in the sample case was found to be 122.35 μm, the velocity of the droplet just before the onset of secondary breakup \( (U_0) \) was found to be 15.6638 m/sec and the time at which the onset of secondary breakup was visually observed was 0.0014 sec. The characteristic shear breakup time was evaluated to be 0.000225 sec, thus the \( t_b/t^* \) ratio was found to be 6.21919. Values for other conditions were estimated and plotted on the graph shown in Figure 6.1 for comparison with past investigations.

**Sample calculation for graph in Figure 7.4:**

The calculations based on expressions provided by Mazallon et al are explained below

The normalized time of onset of primary breakup \( (t_i/t^*) \), where \( t^* = d_0 \left( \frac{\rho_L}{\rho_{air}} \right)^{1/2}/u_{air} \) is plotted as a function of \( We \) in Figure 7.4

\[
    t^* = d_0 \left( \frac{\rho_L}{\rho_{air}} \right)^{1/2}/u_{air}
\]

Where

\( d_0 \) in these calculations is the initial diameter of the jet at the exit of the slinger orifice.

\( \rho_L \) is the density of dispersed phase, water in our case \( (\rho_L = 998 \text{ kg/m}^3) \)

\( u_{air} \) is the velocity of cross-stream continuous medium, which is air in our case.
Since ambient air is stationary in our case, $u_{\text{air}}$ is taken as the velocity of the droplet calculated in Chapter 3, due to the approximation of the continuous liquid stream as a train of droplets.

For an initial droplet diameter ($d_0$) of 144.2 $\mu$m, the velocity of liquid at the slinger exit ($u_{\text{air}}$) is 13.6455 m/sec, thus $t^* = 0.000538$ sec.

Mazallon et al compiled data for the times of breakup of round jets from various sources and established a relation between the normalized time of onset of primary breakup ($t/t^*$),

\begin{equation}
\frac{t_i}{t^*} = 8.76^*W e^{-0.62}
\end{equation}

Where

\[ We = \rho_{\text{air}}u_{\text{air}}^2d_0/\sigma \]

The We was found to be low, 0.4569, indicative of liquid column breakup in our case, as suggested by Mazallon et al.

t$_i$/t$^*$ for the above case is found to be 4.394 and is plotted on the graph in Figure 7.4