Search for the Θ+ Pentaquark at CLAS Using the Minimum Momentum Spectator Approximation

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This thesis titled
Search for the Θ+ Pentaquark at CLAS Using the Minimum Momentum Spectator Approximation

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DEDICATION

This thesis is dedicated to the wonderful Richard and Sally Vogt
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Abstract

In the following, the minimum momentum spectator approximation (MMSA) as proposed in [1] will be applied to CLAS g10 data in an attempt to search for a peak at 1.524 MeV/c^2 for the invariant mass distribution of nK^+ as seen by the LEPS Collaboration [1]. Prior to this report, the CLAS g10 data has been analyzed for this peak returning a null result. The current analysis is different by the application of the MMSA to this data, providing a parameter-free way to correct for the Fermi momentum. An upper limit on the Θ^+ cross section of 1-7 nb with 90% CL has been obtained in the mass range 1.5-1.59 GeV/c^2.

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1 Motivation

The goal of this paper is to search for the $\Theta^+$ claimed to be seen by the LEPS collaboration [1]. Back in 2004 the g10 data was used to look for the $\Theta^+$ but no positive evidence was found. The search for the $\Theta^+$ was at a stand still until 2009 when LEPS published a narrow peak at $1524\text{MeV}/c^2$ for the reaction $\gamma d \rightarrow \Theta^+ K^- (p) \rightarrow K^+ K^- (np)$ using a newly developed approximation method called the Minimum Momentum Spectator Approximation (MMSA). [1] The current analysis adopts the MMSA method to the CLAS g10 data at similar kinematics to the LEPS measurement.

2 Setup

The data being discussed were taken at the Thomas Jefferson National Accelerator Laboratory (TJNAF) using the CLAS detector. This detector measures the momentum and the time of flight for charged particles. Using the gflux method (discussed in Section 4.4.1), the number of incoming photons are counted for each data run. The photon beam for the g10 experiment is made by electrons from the accelerator, with beam energy 3.767 GeV, striking a metal target giving off Brehmsstrahlung radiation where the photons in the range of 0.8-3.6 GeV were recorded. The energy of the photon was found by measuring the electron after the collision in a tagging spectrometer. The tagged photon then enters the liquid deuterium target which measures 24 cm in length with a 4 cm diameter. The target center was positioned 25 cm upstream from the CLAS center. The g10 data run was split up into two different equal configurations: one had the torus magnet set at 2250 Amps and the other set at 3375 Amps [2]. This analysis only focuses on the events that occurred at 2250 Amps, since this configuration had better acceptance for forward-going kaons.
3 Analysis

The reaction being studied here is $\gamma d \rightarrow K^+ K^- (pn)$ where only the $K^+$ and $K^-$ are required to be detected by CLAS. The data set is from the CLAS g10 run. Here, only the runs for the low-field torus setting are used. While it is possible to analyze the runs for the high-field torus setting, it was found that the additional statistics are not needed to obtain a reasonable value of the cross section upper limit.

There are several reaction mechanisms that can lead to this final state. One being, $\phi$-meson production, $\gamma X \rightarrow \phi X$ where $X$ is either a nucleon or a deuteron followed by $\phi \rightarrow K^+ K^-$. This reaction can be seen in the invariant mass $M(K^+, K^-)$, as a peak at 1.020 GeV. The natural width of the $\phi$ is 4.3 MeV, which is roughly half of the CLAS resolution. A second reaction is production of the $\Lambda(1520)$ baryon, via $\gamma p \rightarrow K^+ \Lambda^*$ followed by $\Lambda^* \rightarrow K^- p$. This reaction can be seen in the missing mass, $MM(\gamma p, K^+)$, corrected for Fermi momentum using the MMSA technique (Section 4.2), at a mass of 1.520 GeV, with a width of 15 MeV. Thirdly, production of a $\Theta^+$ pentaquark, via $\gamma n \rightarrow K^- \Theta^+$, followed by $\Theta^+ \rightarrow K^+ n$. This reaction, if it exists, could be seen as a peak in the missing mass, $MM(\gamma n, K^-)$, corrected using the MMSA, at a mass between about 1.52 and 1.54 GeV. If the $\Theta^+$ exists, it has a natural width of $\Gamma < 1$ MeV, and hence would be seen in the data with the resolution of CLAS, $\sim 10$ MeV.

The analysis is presented below as follows: first, an overview of the analysis cuts is presented, followed by sections describing each cut. When comparisons with Monte Carlo (MC) are shown, the generator for the MC is 4-body phase space is used (note: a different generator is used for for individual reactions such as $\Lambda(1520)$ production).
3.1 Initial Event Selection

Kaon/anti-kaon events are not as plentiful as pion/anti-pion events, and in total are a very small piece of the data set. To reduce file sizes and code run times, a few wide selection/filtering cuts were placed on the data and simulation. These filters (the data skim) are applied to every plot in this analysis note.

3.1.1 Kaon Pair Requirement

An applied timing cut used the measured time of flight and a calculated time of flight using measured or assumed quantities. The calculated difference below gives a window of time for the cut,

$$\Delta t = t_{flight} - \frac{d_{path} \times E}{p \times c}$$

where \(t_{flight}\) is the difference in time measured by the TOF paddles and the trigger time, \(E = \sqrt{p^2 + m^2}\) is calculated with the assumed kaon mass and the measured momentum \(p\), \(d_{path}\) measured path-length of the particle, and \(c\) speed of light.

This analysis requires a detection of a \(K^-\) and \(K^+\). The events of interest do not contain any pions and therefore rough timing boundaries of \(\pm 1\) ns were placed to select particles as kaons, protons, or neither given those assumed masses. This criteria, combined with the requirement that the event has at least one valid photon, was applied to every part of this analysis.

3.1.2 Photon Selection

Once an event was selected to possibly contain two kaons (one \(K^+\) and one \(K^-\)), a valid photon was searched for. This photon had to be a good geometrical hit, and therefore required to be in taggoodhit. These events typically had few (one to five) photons assigned to this bank. Three requirements were put on this photon to label this a valid event. The
first requirement, required that the timing of the photon had to be within one nanosecond of the event (using tr_time and vertex_time). The second requirement, checked if each photon formed an event within $\pm 0.2\text{GeV}$ of the desired missing mass. The desired missing mass for the reaction $\gamma d \rightarrow \phi (d)$ was that of a deuteron ($MM(\gamma d, K^- K^+)$).

The reactions $\gamma d \rightarrow K^+ \Lambda (1520)(n)$ and $\gamma d \rightarrow K^- \Theta^+ (p)$ required a missing mass of a nucleon, where the target nucleon is assumed to be at rest ($MM(\gamma n, K^+ K^-)$). This is demonstrated in Equation 2-3 and Figure 1. The third and final requirement, required that there only be one valid photon for the event. This multiplicity cut was estimated to be a 5% effect, meaning only 5% of the events had two or more photons after the above selection criteria.

$$MM(\gamma d, K^- K^+) = \sqrt{(P_\gamma + P_d - P_{K^-} - P_{K^+})^2}$$

(2)

$$MM(\gamma n, K^- K^+) = \sqrt{(P_\gamma + P_n - P_{K^-} - P_{K^+})^2}$$

(3)

Where $P_\gamma$ is the 4-momentum of the incoming photon, $P_d$ is the 4-momentum of an at rest deuteron, $P_n$ is the 4-momentum of an at rest neutron, and $P_{K^+} & P_{K^-}$ are the 4-momenta of $K^+ & K^-$ respectively.
(a) The missing mass distribution found by equation 2 for $\gamma d \rightarrow \phi(d)$ The following figure came from files requiring a detection of $K^+ & K^-$ with a veto on proton detection

(b) The missing mass distribution found by equation 3 for $\gamma d \rightarrow \Lambda(1520)K^+ (n)$ The following figure came from files requiring a detection of $K^+ & K^-$ with no veto on proton detection

(c) The missing mass distribution found by equation 3 for $\gamma d \rightarrow \Phi^+ K^- (p)$ The following figure came from files requiring a detection of $K^+ & K^-$ with a veto on proton detection

Figure 1

3.2 Cuts

The event selection, used for this analysis is presented in Table 1. These cuts will be referenced throughout the text and in figure captions to clarify which events are being plotted. The cuts and symbols used are described in the following sections. It should be noted that all figures in this analysis have satisfied the conditions outlined in section 3.1. Further reference to cuts is in relation to cuts outlined in Table 1 or is directly specified. Furthermore, we removed events that were known to have beam trips in the run, taken from the beam trip files.
Table 1: The following table displays the final cuts used for this analysis. The number of the cut in this table corresponds to the order in which that particular cut was implemented into the analysis code.

<table>
<thead>
<tr>
<th>Number</th>
<th>Cut Type</th>
<th>Cut Made</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cut1</td>
<td>Start Counter</td>
<td>$\Delta t_{st-tr} &lt; 1.11$ ns (data)</td>
</tr>
<tr>
<td>Cut2</td>
<td>Vertex</td>
<td>$-36 &lt; z - vertex &lt; -16$</td>
</tr>
<tr>
<td>Cut3</td>
<td>Timing</td>
<td>$t_{radius} &lt; 0.54$ ns (data)</td>
</tr>
<tr>
<td>Cut4</td>
<td>Missing Mass</td>
<td>MM($\gamma, \pi^+, \pi^-$) $&gt; 1.0$ GeV/c$^2$</td>
</tr>
<tr>
<td>Cut5</td>
<td>Fiducial</td>
<td>Kinematic Constraints</td>
</tr>
<tr>
<td>Cut6</td>
<td>$\phi$ meson</td>
<td>$1.01 &lt; M(K^+, K^-) &lt; 1.03$ GeV/c$^2$</td>
</tr>
<tr>
<td>Cut7</td>
<td>$p_{min}$</td>
<td>$</td>
</tr>
</tbody>
</table>

3.3 Vertex

This section gives the spectrum of the $z$-vertex of the $K^+, K^-$. The liquid deuterium target resides in the beam axis from -37 cm to -13 cm. Figures 2 and 3 show the $Z$-vertex spectrum of both the $K^+$ and $K^-$ for the data and Monte Carlo respectively. In the analysis all counts that reside outside of $-36 < V_z < -16$ are thrown out for either $K^+$ or $K^-$. The same cut was used in the published analysis of the $\Theta^+$ search [3].
Figure 2: This plot shows the number of counts for the value of the z-vertex for $K^+$ and $K^-$ of the data. (Cut 2 has been satisfied)

Figure 3: This plot shows the number of counts for the value of the z-vertex for $K^+$ and $K^-$ of the 4-body phase space Monte Carlo. (Cut 2 has been satisfied)

It is assumed the particles start directly from the beam line. This should be a rea-
sonable assumption for both the $\Theta^+$ and the $\Lambda(1520)$, since the $\Lambda(1520)$ decays via the strong interaction (to $pK^-$), the distance it travels from the beamline is negligible. This means there is no need to take a distance of closest approach approximation when looking at the $z$-vertices.

### 3.4 Particle Identification

To better identify an event of interest, a timing coincidence cut for $K^+$ & $K^-$ was employed. As can be seen in Figure 4 there is a circle about $\Delta t = 0$ which $\Delta t$ for the kaons is concentrated. The counts in the corner are due to background pions being assigned the incorrect mass. A circle cut was used centered at zero, defined as follows:

$$x^2 + y^2 = r^2$$

where $x = \Delta t(K^+)$ is the timing difference calculated in equation (1) for the $K^+$ and similarly for $y = \Delta t(K^-)$. Also $r = t_{\text{radius}}$, where $t_{\text{radius}}$ is the radius extending from the center circle found in figure 4 in units of nanoseconds.
Figure 4: Time difference between the kaon and trigger (units of ns). The bottom left corner of counts is due to background pions. The black lines indicate the bin at which a projection was made, as described in section 3.5 (No Cuts)
Figure 5: Monte Carlo time difference between the kaon and trigger (units of ns). The black lines indicate the bin at which a projection was made, as described in section 3.5 (No Cuts)
(a) Data timing versus momentum plots before any cuts.

(b) Timing versus momentum for Monte Carlo of phase space of $\gamma d \rightarrow K^+K^- n$ production, before any cuts are applied.

(c) Data timing versus momentum plots after cuts 3&4. The vertical bands seen disappear when an additional cut for the in the start counter is applied (see section 3.6).
3.5 Time Cut

In figure 7 the x and y projections are centered at zero of figure 4. Most events are within $|\Delta t| < 0.5$ ns.

![Projections of $\Delta t$ Data](image)

Figure 7: Data projections for both X and Y are taken for the central strips of figure 4. The black lines in figure 4 indicate the bin at which a projection was made. The projections are then fit to a gaussian over a 1st order polynomial. From this we are able to extract the standard deviation of $\Delta t(K^-)$ (left) and $\Delta t(K^+)$ (right). The legends display the variable $p_2$ for the standard deviation in units of ns. (No Cuts)
Figure 8: MC projections for both X and Y are taken for the central strips of figure 5. The black lines in figure 5 indicate the bin at which a projection was made. The projections are then fit to a gaussian over a 1st order polynomial. From this we are able to extract the standard deviation of $\Delta t (K^-)$ (left) and $\Delta t (K^+)$ (right). The legends display the variable p2 for the standard deviation in units of ns. (No Cuts)

In figure 7 the standard deviations are shown as p2, and are different by about 5%. Averaging the two in figure 7 gives the variable $\sigma_{\text{radius}}$. A 3 standard deviation cut was used for both data and Monte Carlo ($t_{\text{radius}} = 3\sigma_{\text{radius}} \sim 0.54$ ns for the data and $t_{\text{radius}} = 3\sigma_{\text{radius}} \sim 0.75$ ns for the MC).

### 3.6 Start Counter Cut

In figures 6a & 6c, there are vertical lines in the plot even after pion cuts have been applied. These streaks can be explained by the difference between the time given by the
start counter and the trigger time. In order to make sure that the time value taken by the
start counter is valid, it is required that one of the tracks went through the start counter.
Once this condition is met, a time difference is taken between the start counter and the
trigger for $K^+ & K^-$. As can be seen in equation 5

$$\Delta t_{st} = t_{st} - t_{tr} - C$$

(5)

where $t_{st}$ is the time given by the start counter, $t_{tr}$ is the time given by the trigger, and
$C = 0.8$ ns is a constant.

(a) Along the x-axis is the $\Delta t_{st}$ for $K^+$ and along the y-axis $\Delta t_{st}$ for $K^-$(b) Shown is the loss of events made by
the cut as explained below.

Figure 9: The time difference is centered around the origin. No cuts have been imple-
mented for (a). Cut 1 is applied for (b).

In order to achieve the difference found between figures 9a & 9b a radius for $\Delta t_{st} K^+ & \Delta t_{st} K^-$
is made (similar to that in section 3.4) with the following equation

$$x^2 + y^2 = r^2$$

(6)

where $x = \Delta t_{st} K^+$, $y = \Delta t_{st} K^-$, and $r = \Delta t_{st} - t_{tr}$. Following the same method as section
3.5, projections are made over the central strips for each axis, which are shown in figure
10.

**Figure 10:** The projections for both X and Y are taken over the central strips of the 2D plot, similar to figures 7&8. In the legends the variable p2 displays the value of the standard deviation. (Cuts 3&4)

In figure 10, the standard deviations for the X and Y projections and are different by about 4%. Averaging these gives $\sigma_{stradius}$, and a 3-$\sigma$ cut was used for the data ($\Delta t_{st-tr} = 3\sigma_{stradius} \sim 1.11$ ns). When this cut is implemented the vertical bands seen in figure 6c are gone.
Figure 11: Same as figure 6c except that cut 1 has been added

3.7 Pion Background Cut

The data has been skimmed for events with candidate $K^+K^-$ pairs. This skim had the possibility of allowing misidentified particles (particularly pions) into the data. When calculating the missing mass, $MM(γN, K^+K^-)$, for the skimmed data, the histogram in Figure 12 is obtained. The missing mass is found using the following approximate method. The four momentum of the incoming photon is added to the four momentum
of a stationary nucleon to give a total incoming momentum, $P_{tot}$, subtracting off the momentum of the $K^-$ and $K^+$, gives

$$MM(\gamma n, K^+ K^-)^2 = (P_{tot} - P_{k^-} - P_{k^+})^2,$$  \hfill (7)

Figure 12 shows a peak centered near 0.75 GeV, suggesting that there is some pion background. To determine whether the additional peak can be attributed to the misidentified particles, a 2D plot is created comparing the missing mass assuming kaons to that of assuming pions.
To remove the particle misidentifications, it is required that $MM(\gamma n, \pi^+\pi^-) > 1.0 GeV$, shown by the black line in Figure 13. Note that this cut removes most of the two-pion production from the nucleon, but does not remove other backgrounds, such as three-pion production.

3.8 Energy Loss Correction

When traveling through the detector, particles naturally lose energy. The distance the particles travel through the detector is known. We used the CLAS standard energy loss package [5]. This program uses information from the g10a geometry, the particle four momentum, the vertex position, and the target type. The output of this program is a
corrected four momentum of the particle. After the correction, a 0.2% shift in the mass spectrum is seen, as shown in Figures 14&15.

Figure 14: Shown is the distribution of $MM(\gamma n, K^+K^-)$ from the data before and after energy loss. These plots are the same as Figure 12 but includes cuts 1-7. On the left is the $MM(\gamma n, K^+K^-)$ before the energy loss corrections, and on the right, after.
Figure 15: The distribution of $MM(\gamma n, K^+ K^-)$ from the simulation before and after energy loss. These plots are the same as Figure 12 but includes cuts 1-7. On the left is the $MM(\gamma n, K^+ K^-)$ before the energy loss corrections, and on the right, after.

## 3.9 Fiducial Cuts

Figures 16 and 17 show $\phi$ and $\theta$ distribution for the $K^+$ and $K^-$ respectively. The procedure to cut out the events in unreliable fiducial regions, was copied from a similar analysis of the g10 data [3]. Shown are the fiducial plots for the data. The same cuts were implemented for all simulations and reactions under study in this analysis.
(a) Shown is all the events before fiducial cuts (Cut 5) were implemented. No other cuts have been implemented as well.

(b) Shown is all the events after fiducial cuts (Cut 5) were implemented. No other cuts have been implemented

Figure 16

(a) The events before fiducial cuts were implemented. (No Cuts)

(b) The events after fiducial cuts (Cut 5) are implemented.

Figure 17

3.10 Bad TOF paddle cuts

The final detector performance cut is the TOF paddles that were bad in each sector. There are six sectors, and for each sector there are 48 paddles. In order to analyze what paddles are bad and good, a 2-D histogram was made for each sector. Figure
Figure 18: Plot of $\Delta t$ versus TOF paddle number for $K^+$ tracks. No Cuts
4 Calibrations with Known Peaks

4.1 Photon Energy

LEPS published their data in the range of $2.0 \text{ GeV} < E_\gamma < 2.4 \text{ GeV}$. CLAS has a range of $0.8 \text{ GeV} < E_\gamma < 3.6 \text{ GeV}$, as shown in Figure 19.

![Figure 19: CLAS photon energy (GeV) for the skimmed data. (No cuts)](image)

The vast majority of the photons hitting the target are for $E_\gamma > 2.0 \text{ GeV}$. In order to match the kinematics of LEPS, later a cut is chosen on $2.0 \text{ GeV} < E_\gamma < 2.4 \text{ GeV}$. For the next sections, however, the full photon energy range is kept.
4.2 Minimum Momentum Spectator Approximation

In the previous sections, the initial momenta of target nuclei are assumed to be zero. These initial momenta values, Fermi momenta, are unmeasured yet known to be non-zero. Neglecting them can cause noticeable broadening of peaks, as the target nucleon moves with respect to the incoming photon and the assumed spectator nucleon. In each of the processes seen, \( \gamma p \rightarrow K^+ \Lambda(1520) \rightarrow K^+K^- p \) and \( \gamma n \rightarrow K^- \Theta^+ \rightarrow K^-K^+ n \), the spectator nucleon of the deuteron has a Fermi momentum of about 80 MeV/c. Because the relative size of this value to the momenta of the photon and kaons is quite small, a first approach is to neglect the existence of the spectator. The current approach will improve this approximation by assuming that a spectator will have the minimum amount of momentum necessary for the total energy momentum of the deuteron. In the currently studied reaction, this can be written as

\[
p_{pn} = p_{\text{miss}} = p_\gamma + p_d - p_{K^+} - p_{K^-},
\]

where each of the momentum vectors are 4-vectors. With the exception of the deuteron, each of the quantities on the right hand side are measured quantities. Note that \( p_{pn} \) is derived solely from measured quantities and the deuteron mass \( (p_d = (m_d, 0)) \). Next, the nucleon momentum is written in the center-of-mass frame as

\[
p_{CM} = \sqrt{\frac{(M_{pn} + m_p + m_n)(M_{pn} - m_p + m_n)(M_{pn} + m_p - m_n)(M_{pn} - m_p - m_n)}{2M_{pn}}},
\]

where \( m_p \) is the PDG value of the proton mass, \( m_n \) is the PDG value of the neutron mass, and \( M_{pn} \) is the missing mass (invariant mass from the four vector, \( p_{pn} \)). The above equation returns the magnitude of the nucleon 3-momentum and thus \( p_{CM} \) is a scalar. The same will be true for \( p_{\text{min}} \) and \( p_{\text{res}} \) in the next equations. The notation used
is context-dependent, in which symbols without the vector arrow can be either a scalar or a 4-Vector, whereas $\vec{p}$ is used to represent the associated 3-Vector.

As mentioned in [1], distinguishing between $m_p$ and $m_n$ will affect later calculations by less than 1 MeV and thus the current analysis imposes the approximation $m_p \approx m_n$. Additionally necessary at this point is a check on the kinematics. Due to finite detector resolutions it is possible that the measurement of $M_{pn}$ is calculated to be less than $m_p + m_n$. In this case, the above calculation for the momentum in the center of mass frame does not return a number. To avoid these possible fluctuations, any value recorded which is less than $m_p + m_n$ is set to be $m_p + m_n$. It is immediately seen that this is equivalent to setting $p_{CM} = 0$ and thus mathematically simplifying the Equation 10 to $p_{min} \approx |\vec{p}_{miss}|/2$. This simplification is consistent with assigning the momentum of the spectator to the minimum possible value. In order to mathematically achieve the assumption that the spectator has the minimum momentum in the case that $M_{pn} > m_p + m_n$, following [1] the direction of the momentum is antiparallel to the total missing momentum by writing the corresponding component. The minimum momentum can be found by

$$p_{min} = \frac{-p_{CM}E_{miss}}{M_{pn}} + \frac{\sqrt{p_{CM}^2 + m_N^2 |\vec{p}_{miss}|}}{M_{pn}},$$

(10)

where $p_{CM}$ is the magnitude of the combined nucleon 3-momentum, $E_{miss}$ is the missing energy (or the energy component of $p_{pn}$), $m_N$ is the spectator nucleon mass, and both $M_{pn}$ and $\vec{p}_{miss}$ are described above. The variables $\vec{p}_{miss}$ and $E_{miss}$ construct the 4-Vector $p_{miss} = (E_{miss}, \vec{p}_{miss})$. To conserve momentum in the direction of $p_{miss}$, the momentum component of the interacting nucleon can then be written as

$$p_{res} = |\vec{p}_{miss}| - p_{min},$$

(11)

For the interaction with the neutron ($\gamma n \rightarrow K^-\Theta^+ \rightarrow K^-K^+n$) the final momentum of
Figure 20: The $p_{\text{min}}$ distribution is shown for $K^+K^-$ events. (Cuts 2-5)

The neutron is written as

$$\vec{p}_n = \frac{\vec{p}_{\text{res}} + \vec{p}_{\text{miss}}}{|\vec{p}_{\text{miss}}|}$$  \hspace{1cm} (12)$$

The invariant mass, $M(nK^+)$, comes from the sum of the 4-Vectors of the neutron and $K^+$ where $n = (m_n, \vec{p}_n)$. Performing the calculation in this way [$M(nK^+)$] allows the known information of the neutron mass to be inserted as additional information.

In figure 20, the quasi-free processes are found within the peak $p_{\text{min}} = 0$ GeV/c.
Figure 21: The following plot shows the effect that coherent processes are contributing, to the distribution of $p_{\text{min}}$. The red line, is the exact same as figure 20a with cuts 2-5 implemented. The blue line, after cutting out the mass range $1.85 GeV/c^2 < MM(\gamma d, K^+ K^-) < 1.9 GeV/c^2$.
Figure 22: The relationship between $p_{\text{min}}$ and the missing mass. Some coherent events are left over for positive values of $p_{\text{min}}$. The amount is small as can be seen in figure 21. (Cuts 2-5)

When $\text{MM}(\gamma, K^+K^-)$ is equal to the nucleon mass, as would be expected from a free nucleon approximation, then $p_{\text{min}} = 0$.

Note the $\sigma$ parameter ($p_2$) in both cases of figure 20. This will be denoted as $\sigma_{p_{\text{min}}}$ to select the cut value. The quasi-free processes are selectively chosen by requiring the data and Monte Carlo to have values of $|p_{\text{min}}| < 2\sigma_{p_{\text{min}}}$ in order to minimize contributions from inelastic and coherent reactions.

4.3 $\Lambda(1520)$

In order to see the application of the MMSA to a known peak, first look at the $\Lambda(1520)$ produced within the reaction $\gamma p \rightarrow K^+\Lambda(1520) \rightarrow K^+K^-p$. An important aspect of this
analysis lies in the dependence found between the values of $p_{\text{min}}$ and $\text{MM}(\gamma p, K^+ K^-)$. In Figure 23, a two dimensional histogram is plotted to visualize the relationship.

Figure 23: 2D histogram showing correlation of $p_{\text{min}}$ to $M(pK^-)$. (Cuts 2-7)

Before using the MMSA, a phenomenological correction using $p_{\text{min}}$ is explored to give a more intuitive understanding of the MMSA. A rotation is implemented by a linear correction factor using $p_{\text{min}}$. This rotation serves to correct for the correlation which exists between these two values, essentially a first-order correction to the Fermi momentum spread, as shown in Figure 24.
Figure 24: The rotated 2D histogram showing correction of correlation of $p_{\text{min}}$ to $M(pK^-)$. (Cuts 2-7)

Figure 25 shows the $M(pK^-)$ spectra before (blue) and after (green) applying this phenomenological rotation using $p_{\text{min}}$. The peak for the $\Lambda(1520)$ is clearly visible only after the correction. Without the rotation, Fermi motion of the target nucleon has smeared the resolution of the peak. This demonstrates that the $p_{\text{min}}$ variable can be used as a first-order correction to remove the effects of Fermi motion from deuterium.
Figure 25: Comparison of $M(pK^-)$ before and after application of the rotation. The green histogram is after the rotation using $p_{min}$. (Cuts 2-7)

It should be noted that this way of finding the $\Lambda(1520)$ peak was done using the missing 4-Vector and calculating its invariant mass. The degree of rotation of Figure 24 is a free parameter, chosen to straighten the line near $M(p,K^-) = 1520MeV/c^2$. When the mass of $\Lambda(1520)$ is directly calculated using MMSA this phenomenological correction is not needed, as shown in figures 26 and 27. The MMSA also works well for the simulated events as shown in figure 28.
Figure 26: Dependence on $p_{min}$ when $M(pK^-)$ was directly calculated using MMSA, without any free parameters. (Cuts 2-7)
Figure 27: Comparison of original $M(pK^-)$ (blue) and a calculation using MMSA (red) without any free parameters. (Cuts 2-7)
Figure 28: Monte Carlo comparison of original M(\(pK^-\)) (blue) and a calculation using MMSA (red) without any free parameters. (Cuts 2-7)

The peak of M(pK\(^-\)) shows both an increase in height as well as a narrowing in width, as expected from this analysis since events which had originally been spread out due to Fermi momentum are shifted back into the \(\Lambda(1520)\) peak. In the analysis of M(nK\(^+\)), to search for a possible \(\Theta^+\) peak, the MMSA approximation is used to get a similar correction for the Fermi momentum without any phenomenological correction.
4.4 Differential Cross Section of $\Lambda(1520)$

This section shows a preliminary look at the differential cross section of a known photoproduction reaction. The differential cross sections are calculated by:

$$\frac{d\sigma}{d\cos \theta} = \frac{\text{Yield}}{(\delta \cos \theta)AL}$$  \hspace{1cm} (13)

where $A$ is the acceptance and $L$ is the luminosity (see below). The plots in this section do not include systematic errors, as they are merely a consistency check since the cross section of $\Lambda(1520)$ is known [8].

4.4.1 Gflux

The number of photons is calculated using the gflux method [6]. Only gflux files corresponding to good data files were kept. In each gflux file the number of photons are binned over a beam energy range of $0.5 < E_\gamma < 3.0$ (GeV). There are 100 rows in each gflux file between $E_i$ and $E_i + 0.025 GeV$ (for $i=1,100$) with the number of photons (column 1) for that range, followed by the uncertainty of that number (column 2). These gflux files have been shown to be reliable for calculating cross sections for previous g10 analyses [4].

4.4.2 Luminosity

The Luminosity is calculated by

$$L(E_\gamma) = \frac{N_{\gamma}(E_\gamma)\rho_lN_A}{d_{MM}},$$  \hspace{1cm} (14)

where $N_{\gamma}$ is the number of photons, $\rho_d = 0.169 (g/cm^3)$ is the density of deuterium, $l = 24$ cm is the length of target, $N_A$ is Avogadro’s number and $d_{MM} = 2.014$ (g/mole)
is the molar mass of deuterium.

4.4.3 Fitting the Peak

In order to extract the yield for the \( \Lambda(1520) \) peak, a Gaussian curve on top of a polynomial background was fit to the data. The same method is used for the Monte Carlo acceptance. The data is binned into energy and \( \cos(\theta_{CM}) \) as shown in Fig. 29.

The Gaussian is parameterized as:

\[
f(x) = a \exp(-\frac{(x - b)^2}{2c^2}),
\]

where \( b \) and \( c \) are held fixed (at \( b = 1.520 \) and \( c = 0.015 \)), and \( a \) is a fitting parameter.

To extract the total number of counts (the yield) for each interval, the gaussian was integrated using

\[
yield = a \cdot c \cdot \sqrt{2\pi} \cdot \frac{90}{0.32}.
\]

The final term in the equation is due the binning of the histogram, which has 90 bins over a mass range of 0.32 GeV/c^2.

4.4.4 Monte Carlo Simulation

Several Monte Carlo simulations were done for the reactions \( \Lambda(1520) \rightarrow pK^- \) and \( \Theta^+ \rightarrow nK^+ \). Each simulation consisted of over 2,000,000 generated events. The photon energy used a Bremsstrahlung distribution and the initial angle of the particles is from a pure phase space distribution. A t-slope of 3 was used in order to be consistent with previous analysis of \( \Lambda(1520) \) [8]. The \( \Theta^+ \) simulation had other t-slopes generated as well.
Figure 29: These plots shows how the peak in $M(pK^-)$, the $\Lambda(1520)$, is changing with respect to energy and $\cos(\theta)^{K^+}_{CM}$. Each beam energy range is divided up into six $\cos(\theta)^{CM}$ sections. The plots are grouped (going left to right) in six equal angle bins from $0.0 < \cos(\theta)^{K^+}_{CM} < 0.9$, and continuing in energy. (Cuts 2-7)
Figure 30: Similar to figures 23 & 26 except the above shows simulated $\gamma p \rightarrow \Lambda(1520) K^+ \rightarrow p K^- K^+$ events. (Cuts 2-7)

4.4.5 Branching Ratio

In order to generate simulated data more efficiently, each specific decay of interest was generated. Then the branching ratios of each decay was folded into the acceptance to obtain the correct cross section for each reaction. In the case of $\Lambda(1520) \rightarrow p K^-$, the branching ratio is 22.5%, since $\Lambda(1520) \rightarrow N\bar{K}$ has a branching ratio of 45% [7].

4.4.6 Acceptance

The acceptance is calculated from the Monte Carlo (MC) yield by:

$$ A = \frac{\text{yield}_{MC}}{N_{\text{events}}} \times \frac{\Gamma_{\Lambda \rightarrow pK^-}}{\Gamma_{\Lambda}} $$

(17)

where $\text{yield}_{MC}$ is the reconstructed events from simulation, $N$ is the number of generated events, and $\frac{\Gamma_{\Lambda \rightarrow pK^-}}{\Gamma_{\Lambda}}$ is the branching fraction. The acceptance values for the $\Lambda(1520)$ are shown in Figure 31 for each angle and energy bin. See Table 4 for values.

In addition to the acceptance of the detector, the yield extraction of $\Lambda(1520)$ after MMSA has been questioned. In order to show the efficiency of extracting the events
Figure 31: The $\Lambda(1520)$ acceptance as a function $\cos(\theta)_{CM}$ for a given range of beam energy and a $t$-slope of $-3$. The bins are spaced evenly from $0.0 < \cos(\theta)_{CM} < 0.9$. See also Table 4 for numerical values.
(a) The simulated signal for $\Lambda(1520)$ with no background.

(b) The same signal except with a phase space background.

Figure 32

of $\Lambda(1520)$ on top of the noise, two simulations were carried out: $\gamma p \rightarrow \Lambda(1520)K^+ \rightarrow pK^-K^+$ and $\gamma n \rightarrow nK^+K^-$ which is the phase space distribution of $K^+ K^-$ off the bound neutron. The phase space simulation is used to model the background in figure 27, as shown in figure 32b.

When applying the same procedure of section 4.4.3 to extract the counts in figure 32b, a yield of $30243 \pm 177$ is found. In figure 32a the peak with no background, fit using the same procedure, gives $30027 \pm 276$ events. This gives an a fitting uncertainty over a smooth background of $\approx 1-2\%$.

4.4.7 Comparison

Using Eq. (13), the resulting differential cross sections are shown in Fig. 33.
Figure 33: The differential cross-section of the $\Lambda(1520)$ as a function of the beam energy and $\cos(\theta)_{CM}$. Each beam energy is divided up into $\cos(\theta)_{CM}$ bins. This compares the cross section of $\Lambda(1520)$ obtained from the MMSA to the results from [8].

For unknown reasons, our cross sections do not match those measured from the proton target using CLAS [8]. While there are possible physics effects (such as re-interactions with the spectator nucleon coupled with the MMSA correction) that could possibly explain this discrepancy, we decided to pursue a different approach to checking our normalizations, as explained in the next section.

4.5 Coherent Phi production

In addition to using the MMSA on the $\Lambda(1520)$ to look at the consistency of our results, we can make a comparison with the published CLAS cross sections on coherent phi
photoproduction[9]. Given that our data was selected to have only $K^+K^-$ events, we looked for events at the $\phi$ invariant mass that had a missing mass (off the deuteron target) to also have a peak at the deuteron mass. Such a skim gave fewer events as can be seen in the figure 34. Figure 35 shows the same plot, but for the Monte Carlo with many more events.

Figure 34: Shown is the correlation between the missing mass ($MM(\gamma d, K^+K^-)$) near the deuteron and possible reconstructed $\phi$ events ($M(K^+K^-)$). (Cuts 2-3)
Figure 35: Shown is the simulated correlation between the missing mass \( MM(\gamma d, K^+K^-) \) near the deuteron and possible reconstructed \( \phi \) events \( M(K^+K^-) \). (Cuts 2-3)

In order to isolate the \( \phi \) production events cuts 2-3 were implemented as well as the energy window of \( 1.6 < E_\gamma < 2.6 \). An important caveat is that the \( \phi \) could be produced from \( \gamma d \rightarrow \phi d \) or \( \gamma N \rightarrow \phi N \). These incoherent events will not give a missing mass at the deuteron mass. However, they do create a missing mass peak at the nucleon mass.

To separate coherent \( \phi \) production from incoherent events, another cut was made. The quantity \( MM(\gamma p, K^+K^-p) \) (assuming the target proton was at rest and using the MMSA generated using the proton’s 4-vector) should be nearly zero if the event is not a coherent event off the deuteron. As shown in figure 36c a cut was placed to remove incoherent events with a mass \( (M(P_{\gamma} - P_{nucleon} - P_{K^+} - P_{K^-}) < -0.2 GeV) \). Another cut was made on the missing mass assuming a deuteron target \( 1.86 \, GeV/c^2 < MM(K^+, K^-) < 1.9 \, GeV/c^2 \).
(a) Coherent $\phi$ simulations.

(b) Incoherent $\phi$ simulations.

(c) Data for coherent $\phi$ production, shown with cuts to remove incoherent events.
Figures 36a through 36c show these two cuts. In Figure 36a these events are simulated $\gamma d \rightarrow \phi (d) \rightarrow K^+ K^- (d)$ with a veto on proton detection. The red lines show the cut of $1.86 \text{ GeV/c}^2 < MM(K^+, K^-) < 1.9 \text{ GeV/c}^2$ and the green line shows the cut $(M(P_{\gamma nuc} - P_{nucleon} - P_{K^+} - P_{K^-}) < -0.2 \text{ GeV})$. Figure 36b display the simulated events of $\gamma d \rightarrow \phi pn \rightarrow K^+ K^- pn$ with the same cuts. Figures 36a&36b show that the cuts chosen have little effect coherent events (estimated at $\approx 6\%$ lost) as can be seen in figure 37, where simulated coherent $\phi$ events are shown with and without these cuts. These cuts were then implemented in the data where the coherent events signal was weak as can be seen in figure 36c.

![Missing Mass K+K- MC Coherent Events Before and After Cut](image_url)

**Figure 37:** On the left is the number of coherent events before the cuts $(M(P_{\gamma nuc} - P_{nucleon} - P_{K^+} - P_{K^-}) < -0.2 \text{ GeV})$ & $1.86 \text{ GeV/c}^2 < MM(K^+, K^-) < 1.9 \text{ GeV/c}^2$ and on the right after.
Figure 38: Shown are sample figures of one bin in $t$ used to extract the yield for $\gamma d \rightarrow \phi d$.

Figure 38 shows the simulated coherent $\phi$ production with a $t$ bin of $-0.7 \text{ GeV}^2/c^2 < t < -0.6 \text{ GeV}^2/c^2$. Using this information, the same $4\sigma$ cuts were used in the data which are indicated by the redlines in figure 38b. This same process was done for all $t$ bins as all the figures in the data have weak signals. This weak signal is then integrated. The green lines in figure 38b indicate the sidebands. The length of each sideband is exactly half the range that the signal was integrated over. The acceptance was found by integrating between the black lines and dividing by the number of events generated. The luminosity for the range of $1.6 < E_\gamma < 2.6$ is $12440 \text{ nb}^{-1}$. Using these variables the cross-section was calculated. Counting these events and correcting for acceptance in a given range of $t$, we obtained cross sections that are in good agreement with Ref. [9] (see figure 39). Values for the cross sections are given in Table 5.
Although these result from comparison to coherent phi production do not explain our discrepancy with the results on $\Lambda(1520)$ production, it does give us confidence in our normalization procedures.

4.6 Background

The data was skimmed for tracks on two oppositely charged kaons. At the energies available here, the two kaons can come from a $\phi$, a $\Lambda(1520)$, and possibly from a $\Theta^+$. Since CLAS has little access to forward angles the background from $\phi$ events is reduced. Figure 40 shows the $\phi$ spectrum for beam energies of 1.4-3.0 GeV. Here, if $1.01 < M(K^+K^-) < 1.03$ GeV/c$^2$ then that event is excluded (cut 6). Figure 40 also shows the remaining events after the cut.

To see whether $\Lambda(1520)$ events have any effect on the number of $\phi$ events found, figure 41 shows that the two are not correlated. Also, after cutting on $\phi$ there is minimal change to the $\Lambda(1520)$ peak, as shown in figure 42.
Figure 40: Invariant mass $M(K^+K^-)$ spectrum, showing a peak at the φ meson mass at 1.02 GeV/c² before the cut (a) and the remaining events after cut 6 is applied (b).

Figure 41: If it were the case that Λ(1520) events were contributing to the φ spectrum we would expect to see some vertical trend line at $M(pK^-) = 1.52 GeV/c^2$. Cuts 2-5 & 7
Figure 42: Above shows directly how the spectrum of $M(pK^-)$ is changed when $\phi$ events are cut. The red line indicates events before a cut of $M(K^+,K^-) > 1.05\text{GeV}/c^2$ and blue after. In addition cuts 2-5 & 7 are used.

5 Results

The MMSA is now applied to the reaction $\gamma n \rightarrow K^-\Theta^+ \rightarrow K^-K^+n$. $M(nK^+)$ is calculated as discussed in section 4.2 using 4-Vectors of the neutron and the measured $K^+$. Figure 43 shows the correlation between $p_{min}$ and $M(nK^+)$ before and after the MMSA correction. The events appear to fill the phase space. Unlike the case of the $M(pK^-)$ for the $\Lambda(1520)$, no visible line of correlation is directly apparent in this Figure. Therefore if the $\Theta^+$ exists, then its signal within this data is not resolvable through the background using the current set of cuts. Figure 44 shows that the MMSA works correctly for the $\Theta^+$ simulations.
Figure 43: Correlation of the calculated mass $M(nK^+)$ with $p_{min}$ before (left) and after (right) the MMSA correction. These plot can be compared with similar plots for the $\Lambda(1520)$ shown in Figure 26. (Cuts 2-7)
Figure 44: The simulated correlation of the calculated mass $M(nK^+)$ with $p_{min}$ before (left) and after (right) the MMSA correction. (Cuts 2-7)
The projection of the $M(nK^+)$ spectrum is shown in Figure 45 before (blue) and after (red) the MMSA is applied. This analysis has thus far been insufficient to resolve any peak around the reported mass of 1.524 GeV/c$^2$ using the same methods as used for the LEPS publication [1]. Therefore instead of a direct cross section, an upper limit can be calculated.

![M(nk+) Using MMSA](image)

Figure 45: Invariant mass $M(nK^+)$ spectrum before (blue) and after (red) the MMSA has been applied. (Cuts 1-7)

6 Upper Limit of $\Theta^+$

Since there is no $\Theta^+$ signal in the data, we proceed to extract an upper limit on the cross-section. This was done assuming different assumed masses for the $\Theta^+$. Each assumed mass (of $\Theta^+$) used the cross section calculation of

$$\sigma = \frac{Yield}{A \times L}, \quad (18)$$

54
where \textit{Yield} is the extracted possible signal (at a 90\% CL) given a specific background, \( A \) is the acceptance, and \( L \) is the luminosity. The calculation of luminosity was described in Section 4.4.2. Since the upper limit is being calculated for the integrated energy range of 2.0-2.4 GeV (the same range as the LEPS2 paper), the luminosity in all calculations of the upper limit was 4.700 pb\(^{-1}\), with a normalization uncertainty of 5\% based on Mibe’s paper [9]. The total counts (over all accepted angles) are extracted as the yield. Although we are missing counts from extreme forward angles due to the beam hole, by varying the simulated t-slope to realistic values, an approximation of the events lost is folded into the acceptance.

6.1 Yield Extraction

The background is modeled in the region which a "signal" could be resting. This is shown in figure 46 for a 4\(^{th}\) order polynomial fit.

Once the parameters of the polynomial are determined, an additional Gaussian can be fit on top of this background, as seen in Figure 46. A Gaussian with a fixed \( \sigma \) of 10 MeV and a given mean (corresponding to the signal mass) was fit to this histogram, where only the height was left as a free parameter. The width was verified to resemble distributions in the simulation as seen by Section 6.2. The mass of the \( \Theta^+ \) was varied for the Gaussian fits. The Gaussian yields associated for each assumed mass are given in table 6.
Figure 46: Fit with the polynomial curve (red) on the quantity $M(nK^+)$. The signal fit is hard to see in conjunction with the fit, and therefore is also drawn separately (green). (Cuts 1-7)

Working backwards from the cross section LEPS quotes ($\sim$12nb) at 1524 $MeV/c^2$, with the luminostiy, acceptance, and resolution of CLAS a curve representing this signal can be drawn (Figure 47).

Figure 47: Fit with a polynomial curve (red) for the $M(nK^+)$ spectrum. The black dotted curve in this figure is the peak we would expect to see assuming a 12 nb cross-section and a mass of 1524$MeV/c^2$. The spectrum below shows the background of events where $(K^+K^-p)$ were detected. (Cuts 1-7)
6.2 $\Theta^+$ Acceptance

The acceptance of $\Theta^+$ was done similar to the $\Lambda(1520)$. The acceptance for the $\Theta^+$ is integrated over all angles of the CLAS acceptance, for bins in the beam energy range of $2.0 \, GeV < E_\gamma < 2.4 \, GeV$. See Table 6 for detailed acceptance values. The branching ratio for $\Theta^+ \rightarrow nK^+$ is assumed to be 50% [1] and is folded into the calculated cross section. An example fit for the accepted event from the simulation can be seen in Figure 48.

![Figure 48: A fit to the $\Theta^+$ signal. This fit is used to extract the counts to get the acceptance. (Cuts 1-7)](image)

The results for the $\Theta^+$ acceptance as a function of mass and t-slope are shown in Fig. 51.
Figure 49: The acceptance as a function of assumed mass is fairly linear given a specific t-slope. As expected, when the t-slope increases the acceptance decreases.

For the final result a t-slope of -3 was used for the acceptance. Additional Monte Carlo events with a t-slope of -3 for assumed masses of 1515, 1530, 1550, 1570 MeV/c$^2$ were also generated. Table 6 shows the acceptance for all assumed masses of the $\Theta^+$. The acceptances for masses not directly generated were found by interpolation.

## 6.3 Upper Limit as a Function of Mass

Again, the cross section is calculated by $\sigma = Y/(A \cdot L)$ where $L$ is the Luminosity, and $A$ is the acceptance. As noted above, $Y$ in this case is the Gaussian yield. In the case of $2.0 < E_\gamma < 2.4$ GeV the luminosity was found to be $4700.8 \pm 0.1 nb^{-1}$ with a normalization uncertainty of 5%. A fourth order polynomial was fit over the background with a gaussian fit on top for each assumed $\Theta^+$ mass. From this, the total statistical error can be calculated using:

$$\Delta\sigma_{stat}^{M_{\Theta^+}} = |\sigma| \sqrt{\left(\frac{\Delta Y}{Y}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta A}{A}\right)^2}$$

(19)
where $\Delta \sigma_{\Theta^+}^{M_{\Theta^+}}$ is the statistical uncertainty for each fit of an assumed $\Theta^+$ mass, $\Delta Y$ is the uncertainty in the yield, $\Delta L$ is the uncertainty in the luminosity ($\approx 0.1nb$), and $\Delta A$ is the uncertainty in the acceptance. In more detail,

$$\Delta Y = |Y| \ast \frac{\Delta a}{a}$$  \hspace{1cm} (20)

where $a$ and $\Delta a$ are the amplitude of the gaussian and error in the amplitude, respectively, and

$$\Delta A = A \sqrt{\frac{1-A}{N \ast A}}$$  \hspace{1cm} (21)

where $A$ is the acceptance (without the branching ratio), and $N$ is the number of events.

In figure 50 the contribution of the statistical uncertainty to the upper limit is shown by plotting the cross section ($\sigma$) and the cross section plus the statistical uncertainty ($\sigma + \Delta \sigma_{\Theta^+}^{M_{\Theta^+}}$).

![Cross-Section + Statistical Uncertainty](image)

Figure 50: Shown in black is the cross section, and in red the sum of cross section plus statistical uncertainty.
6.4 Systematic Uncertainties

In this section, we show how the individual cuts effect the final uncertainty in the analysis on $\Theta^+$. This was done by varying cuts 1-7. The variation of each cut is specified in Table 2.

<table>
<thead>
<tr>
<th>Cut Type</th>
<th>Cut#</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start Counter</td>
<td>Cut1</td>
<td>$2.5 \sigma_{stradius}$</td>
</tr>
<tr>
<td>Vertex</td>
<td>Cut2</td>
<td>$-35.0 &gt; z &lt; -17.0$</td>
</tr>
<tr>
<td>Timing</td>
<td>Cut3</td>
<td>$2.5 \sigma_{stradius}$</td>
</tr>
<tr>
<td>Missing Nucleon</td>
<td>Cut4</td>
<td>$MM(\gamma, \pi^+, \pi^-) &gt; 1.2 \text{ GeV}/c^2$</td>
</tr>
<tr>
<td>Fiducial $\phi$</td>
<td>Cut5</td>
<td>no cut</td>
</tr>
<tr>
<td>$p_{min}$</td>
<td>Cut7</td>
<td>$2.5 \sigma_{p_{min}}$</td>
</tr>
</tbody>
</table>

After the cut was varied, the cross section was recalculated. This was done for each cut and each assumed mass of $\Theta^+$. The variations on the cross section are shown in figure 51.

![Systematic Plot](image.png)

Figure 51: The following plot shows for each assumed mass how the varied cut changes the cross section.

The systematic uncertainty had a different value for each assumed mass. The uncertainty was taken as the max deviation of the cross section for a varied cut from the base
cross section for the cuts specified in Table 1 as shown below:

\[
\Delta\sigma_{\text{sys}}^{M_{\Theta^+}} = \max\{\sigma_o - \sigma_{1,2,3,4,5,6,7}\}
\]  

(22)

where \(\Delta\sigma_{\text{sys}}^{M_{\Theta^+}}\) is the systematic uncertainty for each assumed mass of the \(\Theta^+\), and \(\sigma_o\) is the cross section calculated using the cuts in table 1. The \(\sigma_{1,2,3,4,5,6,7}\) are the cross sections for the varied cuts specified table 2. These systematic differences for each assumed mass can be seen figure 52, where the black points represent \(\sigma_o\). Some points give larger yields for the assumed \(\Theta^+\) mass, and others give yields which are smaller than \(\sigma_o\). Since all of the variations are of the same order-of-magnitude, we choose the one with the largest deviation from \(\sigma_o\) to represent the systematic uncertainty associated with our choice of cuts.

Figure 52: The black dots indicate the same cross section seen in figure 50, now with the attached statistical error. The pink triangles indicate the systematic uncertainty on top of the statistical uncertainty. It can be seen the statistical difference dominates the systematic.

In addition to the systematic uncertainty estimate from variations of the cuts, we have two additional uncertainties. The first is a systematic uncertainty for the luminosity from events with multiple photons (see section 3), measured at 3-5%. The second is
an uncertainty for variations in the polynomial background, which is estimated at 5% based on using higher-order polynomial fits. These are folded in quadrature with the other systematic uncertainties.

Figures 50 & 52 show the upper limit, calculated using statistical and systematic uncertainties. In more detail, the total uncertainty was calculated from the following equation:

$$\Delta \sigma^{M_{\Theta^+}}_{tot} = \sqrt{(\Delta \sigma^{M_{\Theta^+}}_{sys})^2 + (\Delta \sigma^{M_{\Theta^+}}_{stat})^2}$$ (23)

where $\Delta \sigma^{M_{\Theta^+}}_{tot}$ is the total uncertainty for each assumed mass of the $\Theta^+$. In order to calculate a 90% confidence level $1.64 \times \Delta \sigma^{M_{\Theta^+}}_{tot}$ is added on top of the base cross section. The final distribution with the 90% CL is shown in figure 53. This is our final result.

![Upper Limit of $\Theta^+$ cross section for a given assumed mass.](image)

Figure 53: $\Theta^+$ cross section for a given assumed mass.
Below are tables with details of the cross section calculations.

### Table 3: Values of Differential Cross Section of $\Lambda(1520)$

<table>
<thead>
<tr>
<th>Energy</th>
<th>$\cos\theta$</th>
<th>$\frac{d\sigma}{d\cos\theta}(\mu b)$</th>
<th>Error</th>
<th>Yield</th>
<th>Error</th>
<th>Luminosity$(\mu b^{-1})$</th>
<th>Error</th>
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<td>0.125</td>
<td>0.012</td>
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<td>4.29e+06</td>
<td>102</td>
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<td>0.00605</td>
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<td>0.0125</td>
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Table 4: Acceptance for the Differential Cross Section of $\Lambda(1520)$

<table>
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<th>$\cos\theta$</th>
<th>Acceptance</th>
<th>Error</th>
<th>$Yield_{mc}$</th>
<th>#Events</th>
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<tbody>
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The acceptance values in this table have assumed a branching ratio 0.225

Table 5: Cross-Section of Coherent Phi Production off Deuteron

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<tr>
<th>t-slope</th>
<th>Signal</th>
<th>Sideband</th>
<th>Counts</th>
<th>Acceptance</th>
<th>$\frac{d\sigma}{dt}$ (nb)</th>
<th>Mibe $\frac{d\sigma}{dt}$ (nb)</th>
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</thead>
<tbody>
<tr>
<td>-1.7</td>
<td>4</td>
<td>12</td>
<td>16</td>
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<td>-1.3</td>
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<td>16</td>
<td>25</td>
<td>0.007996</td>
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<td>-1.1</td>
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<td>20</td>
<td>34</td>
<td>0.01019</td>
<td>0.5523</td>
<td>0.57</td>
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<tr>
<td>-0.9</td>
<td>27</td>
<td>24</td>
<td>51</td>
<td>0.01197</td>
<td>0.9069</td>
<td>0.94</td>
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<tr>
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<td>20</td>
<td>41</td>
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<tr>
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<td>16</td>
<td>43</td>
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<td>2.17</td>
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Table 6: Yield of M(nK+) For Different Mass Values

<table>
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<tr>
<th>Mass GeV/c²</th>
<th>Yield</th>
<th>Error</th>
<th>Acceptance</th>
<th>Error</th>
<th>σ(nb)</th>
<th>Error</th>
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<tbody>
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<td>36.797</td>
<td>0.00557867</td>
<td>5.6352e-05</td>
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<td>37.683</td>
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<td>5.57415e-05</td>
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<td>-37.9053</td>
<td>38.520</td>
<td>0.00552521</td>
<td>5.5131e-05</td>
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<td>1.48318</td>
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In this table is listed the values used to find the values in figure 53. For the acceptance
an assumed branching ratio of 50% and t-slope = -3 is used with a luminosity of 4700.8 ± 0.1(nb⁻¹).

7.1 Coherent φ Plots

![Figure 54](image-url)
Figure 55

Figure 56

Figure 57
Figure 58

Figure 59
References


[4] N. Compton, K. Hicks and M. Camp, “Cross Sections of \( \gamma d \rightarrow K^0\Lambda \) from g10”, CLAS-NOTE 2014, http://www.jlab.org/Hall-B/shifts/admin/paper_reviews/2014/k0lamv4-9309054-2014-11-18-v8.0.PDF.


[10] D. Carman “Search for the Θ+ Pentaquark in the CLAS g10 Data via the Exclusive Production Reaction \( \gamma + d \rightarrow K^+ + K^- + p + n \)”, CLAS-NOTE 2006,