Beamforming Techniques for Frequency-Selective and Millimeter-Wave Indoor Broadcast Channels

Dissertation

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Abstract

Wireless communication networks have become ubiquitous in recent years. Current wireless applications are possible thanks to small WiFi cells that provide high-speed indoor coverage and outdoor macro-cells that support user mobility. Next generation wireless networks will use similar architectures to enable new applications such as augmented and virtual reality, the internet of things, ultra-high definition video streaming, and massive data transmission and storage. However, these applications require unprecedented high-speed data transfer capabilities enabled by large frequency bandwidths. Motivated by spectrum scarcity in bands below 6 GHz, previously unused millimeter-wave (mmWave) bands, where large bandwidths are available, are now considered for future wireless networks. The necessity for efficient communication techniques for such large bandwidths and mmWave frequencies is the main motivation for this dissertation, with a focus on the complex radiowave propagation conditions found in indoor environments.

Propagation mechanisms such as multiple reflections, diffractions, and transmissions through walls are commonly found in indoor wireless communications, which cause variations in the received signal along its bandwidth (wideband or frequency-selective channels). Traditionally, antenna arrays have been used together with beamforming (linear processing) techniques to improve the system’s performance. However, those techniques were designed for narrowband systems (e.g., zero-forcing or matched
filtering) and their application to wideband systems requires additional processing that increases system's complexity.

In the first part of this dissertation, we tackle the problem of beamforming in frequency-selective channels with two approaches: 

* i) we use the electromagnetic time-reversal (TR) effect to directly design novel wideband beamformers, and
* ii) we generalize the block-diagonalization (BD) procedure used in narrowband channels to the frequency-selective case. For both approaches, we derive theoretical performance bounds under different propagation conditions and provide numerical simulations for various operation parameters. We found that these frequency-selective beamforming solutions require low-complexity receivers and can efficiently address problems such as inter-symbol interference and inter-user interference.

In the second part of this dissertation, we focus on the design of beamforming procedures for wideband mmWave systems considering their specific hardware constraints and propagation characteristics. In this case, antenna arrays with tens of elements are commonly used to compensate for large propagation losses. Hybrid analog/digital beamforming, that combines phase-shifters in the RF domain with digital baseband processing, has been proposed to reduce hardware complexity. Thus, we introduce a novel hybrid beamforming algorithm for multiuser wideband mmWave systems. The algorithm accounts for hardware constraints and realistic antenna array effects such as beam squint, antenna coupling, and individual element radiation patterns. We provide numerical evaluations of the algorithm with both statistical and ray-tracing channel models. Results show that the algorithm enables multi-Gbps connectivity to multiple users in real-life scenarios with only a 3 dB performance loss with respect to ideal fully-digital beamforming with much simpler hardware.
To my mother
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Wireless communication networks have become ubiquitous in recent years. People access the internet through a myriad of portable or mobile devices and regularly use data-intensive applications such as high-definition video streaming. Moreover, smart devices like home appliances, sensors, and actuators also connect to the internet to exchange information. This level of connectivity has become possible thanks to small WiFi cells that provide high-speed indoor coverage and outdoor macro-cells that support user mobility. Next generation wireless networks will use similar architectures to enable new applications such as augmented and virtual reality, the internet of things, ultra-high definition video streaming, and massive data transmission and storage [2]. However, these new applications require further evolution in the network’s architecture and also in the components used at the base stations, access points (AP), and user equipment (UE) [3].

One of the main challenges in next generation networks is the design of communication techniques that perform well under the complex radiowave propagation conditions found in indoor environments. Propagation mechanisms such as multiple reflections, diffractions, and transmissions through walls are commonly found in indoor wireless communications, which cause variations in the received signal along its bandwidth (frequency-selective channels). Traditionally, antenna arrays have been used
together with beamforming (linear processing) techniques to improve the system’s performance. However, those techniques are designed for narrowband (frequency-flat) systems (e.g., zero-forcing [4] or matched filtering [5]) and their application to wideband systems require additional processing that increase system’s complexity. These factors motivate the work in Chapters 2 - 4, where we propose beamforming methods specifically designed for frequency-selective channels. In particular, we take advantage of the electromagnetic time-reversal (TR) effect to design efficient beamformers for such scenarios [6].

Considering the radio channel as a linear system, TR is a wideband beamforming technique that uses the time-reversed channel impulse response (CIR) as a linear filter applied to the transmitted signal [7]. Such pre-filter enables spatial (3D) focusing of the signal at the receiver given that all multipath components add coherently at the receiver’s location, while they combine incoherently in other positions in space [8, 9]. The main advantage of TR with respect to current techniques for frequency-selective channels is the reduced computational complexity at the transmitter and, especially, at the receiver [10].

Another feature of future wireless networks is the requirement of large bandwidths to support unprecedented high-speed data transfer demanded by new applications. Such bandwidths are not available in conventional microwave frequency bands below 6 GHz. Thus, researchers have turned their attention to previously unused millimeter-wave (mmWave) bands (e.g., 28, 38, 60, and 72 GHz). However, many challenges remain open to fully exploit the potential of mmWave systems. In particular, beamforming procedures that consider the specific hardware constraints and propagation characteristics of mmWave are an active research area [11]. High-gain
antennas, which are commonly implemented with planar arrays with tens of antenna elements, are required to compensate for large propagation losses at mmWave. Thus, fully digital beamforming, which requires one RF chain for every antenna, is unfeasible due to hardware complexity and power consumption constraints. Several hybrid analog/digital beamforming architectures (which use phase shifters, power splitters/combiners, and switches in the RF domain together with digital baseband processing) have been proposed as practical solutions for mmWave terminals [12–15]. The hybrid beamforming design problem refers to the selection of phase-shifter configurations and digital processing matrices that maximize a given performance criteria under the hardware constraints. In Chapter 5, we focus on the hybrid beamforming design problem for wideband multiuser mmWave systems.

1.1 Related Work

Beamforming for Frequency-Selective Channels

A number of works have addressed different aspects of TR beamforming, with particular focus on single user systems [16–20]. In these references, the spatial focusing and equalization properties of TR are analyzed, and both theoretical and empirical characterizations of its bit error rate (BER) have been made under specific scenarios and channel models. A common finding in the literature is that inter-symbol interference (ISI) is the main limiting factor of TR because it imposes a lower bound in the BER at high signal-to-noise ratios (SNR) [20]. The challenge of mitigating ISI in TR has received increasing attention. Different equalization solutions have been proposed in [21,22] for single-user systems. Although the empirical performance of these solutions has been proven, their theoretical performance bounds have not been
addressed. For example, the influence of the channel power-delay profile (PDP) and other propagation conditions on the TR beamforming system is unknown.

For multiuser systems, different TR techniques for multiple access in the downlink are proposed in [6,23–25]. Inter-user interference (IUI) is recognized as the main limiting factor on the performance of multiuser TR instead of ISI. An important feature identified in these works is that the signal-to-interference-plus-noise ratio (SINR) increases linearly with the number of antennas, which shows a promising research direction for large antenna array systems.

The extension of narrowband beamforming techniques to the frequency-selective case is another important research direction to consider. In particular, block diagonalization (BD) [4] is of significant importance given that, under certain conditions, achieves the sum capacity of broadcast channels [26]. BD uses linear processing at the transmitter to set IUI to zero, forcing a block-diagonal structure in the precoder-channel matrix product. The generalization of BD to frequency-selective channels is not trivial since the channel matrix incorporates space-time information from each transmitter/receiver antenna pair instead of the space-only information found in frequency-flat channels. This generalization has not been explored in the past and is the motivation of our work in Chapter 4.

**Beamforming for mmWave**

The hybrid beamforming problem for single-user narrowband mmWave systems was first studied in [12] and [13]. The approach in [12] is to assume perfect channel state information (CSI) to find hybrid analog/digital beamformers that best approximate the optimal digital precoder (maximizing the spectral efficiency in the downlink).
This solution takes advantage of channel sparsity in the angular domain and considers phase shifters with quantized states. In [13], the mmWave channel estimation is formulated as a sparse reconstruction problem that uses hierarchical beamforming codebooks for training transmissions over increasingly narrow angular regions in space. The use of these codebooks has become a design principle for hybrid beamforming in mmWave. For example, the IEEE 802.11ad 60 GHz WLAN standard uses beam training over a hierarchical codebook to establish wireless links [1, 27, 28]. Also, reference [29] presents a codebook construction methodology for hybrid beamforming with planar antenna arrays in single-user narrowband systems.

For multiuser mmWave systems, similar approaches of training over hierarchical codebooks are used to design hybrid beamformers that approximate narrowband multiuser beamformers such as block diagonalization or zero forcing [30–32]. Common assumptions in these works are perfect CSI, total power constraints at the transmitter, and analog beamforming with infinite resolution phase shifters.

Antenna-specific aspects of wideband mmWave beamforming have also been studied recently, with focus on the array response’s frequency dependence (beam squint effect). References [33] and [34] study the capacity loss in mmWave systems due to beam squint and propose a beamforming design to compensate for this effect. Other antenna aspects of beamforming for mmWave terminals are addressed in [35].

1.2 Contributions

In Chapter 2, we present two contributions on time-reversal (TR) beamforming for single-user indoor wideband MISO systems. First, we provide novel analyses of a baseband TR system using two commonly used indoor propagation channel models.
We derive closed-form approximations for the inter-symbol interference (ISI) with these channel models in order to characterize the influence of propagation conditions (such as the power-delay profile, delay spread, and bandwidth) on TR performance metrics. In particular, we analyze spatial focusing and time compression performance of TR beamforming and their impact on the bit error rate (BER). As a second contribution, we introduce an equalized TR (ETR) technique that mitigates the ISI of conventional TR. ETR utilizes a zero-forcing pre-equalizer at the transmitter in a cascade configuration with the TR pre-filter. Unlike previous approaches to ISI mitigation in TR, we derive theoretical performance bounds for ETR and show that it greatly enhances the BER performance of conventional TR with minimal impact to its beamforming capabilities. By means of numerical simulations, we verify our closed-form approximations and show that ETR outperforms conventional TR with respect to the BER under any SNR.

In Chapter 3, we analyze a baseband TR beamforming system for mm-wave multiuser massive MIMO. We verify that, as the number of antennas increases, TR yields good equalization and interference mitigation properties, but inter-user interference (IUI) remains a main impairment. Thus, we propose a novel technique called interference-nulling TR (INTR) to minimize IUI. We evaluate numerically the performance of INTR and compare it with conventional TR and equalized TR beamforming. We use a 60 GHz MIMO channel model with spatial correlation based on the IEEE 802.11ad SISO NLoS model to demonstrate that INTR outperforms conventional TR with respect to average BER per user and achievable sum rate under diverse propagation conditions.
In Chapter 4, we generalize the concept of block diagonalization (BD) to frequency-selective channels. We demonstrate that BD is possible in frequency-selective MIMO broadcast channels to eliminate IUI and derive the conditions on the number of transmit antennas and the transmission block length (as functions of the number of users and channel delay spread) for the existence of BD beamformers. We also propose three different approaches to mitigate/eliminate ISI in block transmissions: time-reversal-based BD (TRBD), equalized BD (EBD), and joint processing BD (JPBD). We demonstrate that JPBD, which uses linear processing at the transmitter and the receiver, approximates full multiplexing gain for a sufficiently large transmit block length, and show its diversity-multiplexing trade-off. Extensive numerical simulations show that the achievable rate and probability of error performance of all the proposed techniques improve that of conventional time-reversal beamforming. Moreover, JPBD provides the highest achievable rate region for frequency-selective MIMO broadcast channels.

In Chapter 5, we present a new beamforming algorithm for multiuser wideband mmWave systems where one access point uses hybrid analog/digital beamforming while the user terminals have phased-array beamforming only. For these hardware configurations, we describe: i) the construction of novel beamformer sets (codebooks) with wide sector beams and narrow beams based on the orthogonality property of beamformer vectors, ii) a new hybrid beamforming algorithm that uses training transmissions over the codebooks to select the beamformers that maximize the received sum-power along the bandwidth, and iii) a numerical validation of the algorithm in standard indoor scenarios for mmWave WLANs using channels obtained with both statistical and ray-tracing models. Our algorithm is designed to serve multiple users.
in a wideband OFDM system and does not assume perfect channel knowledge or a channel structure. Moreover, we consider antenna-specific aspects, such as antenna coupling, element radiation pattern, and beam squint that are not addressed in current literature. We characterize the algorithm’s achievable rate and show that attains more than 74% of the spectral efficiency (only a 3 dB SNR loss) with respect to an ideal fully-digital beamforming solution in the analyzed scenarios.
Chapter 2: Equalized Time Reversal Beamforming for Frequency-Selective Indoor MISO Channels

2.1 Introduction

Very short-range wireless architectures, such as pico and femtocells, are becoming ubiquitous as data volume increases and spectrum scarcity makes high-density deployments more feasible economically [36]. Short-range solutions can be used to offload cellular network traffic to wireless local area networks (WLAN), as seen with the proliferation of indoor WiFi hotspots. Because of their smaller size and increasing operating frequencies, these architectures, as well as future types of indoor networks, may adopt access points (AP) that employ irregularly-spaced or other unconventional antenna arrays instead of the arrays in use today. New beamforming techniques that perform well in such scenarios are thus highly desirable.

One of the techniques with potential to provide advanced beamforming capabilities in rich scattering scenarios is time-reversal (TR) [6], which allows a transmitter antenna array to focus the electric field at specific points in space using knowledge

\footnote{The term beamforming is traditionally used to denote phased array techniques for beam-steering in flat-fading channels, i.e. operating in the 2D manifold spanned by the azimuth and elevation angles. In this chapter, we use the term beamforming in a broader sense to denote signal processing techniques for frequency-selective multipath channels, that allow spatial focusing of RF power in co-range as well (3D), or even in time (4D space-time beamforming) [9,16,37].}
of the scattering environment. TR originated in acoustics [7, 38] and was later generalized and studied in the context of electromagnetic imaging and tracking applications [39–44]. An early more primitive notion of (narrowband) TR was also introduced in electromagnetics under in the form of the retrodicreative or Van Atta reflector array [45].

In the context of communication systems, TR is a signal transmission technique that uses the time-reversed channel impulse response (CIR) as a linear filter applied to the transmitted signal. Such pre-filter enables spatial focusing of the signal at the receiver and compression of the CIR in the time domain [8, 9, 16, 18, 46]. First, spatial focusing in TR occurs because all multipath components add coherently at the receiver’s location, while they combine incoherently in other positions in space; this is allowed by the spatial signature contained in the CIR. Second, in-phase addition of multipath components takes place at specific sampling instants. This effect is due to the matched filter behavior of the TR pre-filter, which also has partial equalization properties that reduce inter-symbol interference (ISI) [47]. Due to this appealing characteristics, TR beamforming is particularly attractive for indoor pico and femtocells, where the channel is typically slow-varying and rich scattering is prevalent. In such scenarios, spatial focusing can be maintained without requiring a fast update of the channel state information (CSI). In addition, the main advantage of TR with respect to conventional multi-carrier systems in use today is the reduced computational complexity at the transmitter and, specially, at the receiver [10]. General advantages stemming from beamforming towards green wireless systems [48, 49] also exist, with TR receiving special attention for its potential use to improve energy efficiency in future wireless networks [19, 50].
Because of their high temporal resolution, most of the work in TR has focused on ultrawideband (UWB) systems, although the suitability of this technique in conventional wideband systems has been verified as well [18,47]. The performance of TR, in terms of bit error rate (BER) and focusing capability, has been addressed by means of empirical and theoretical approaches. In [16], the authors study the space-time focusing of a single-input single-output (SISO) TR system in two scattering scenarios; they define performance metrics and find empirical formulas for them. References [19] and [25] present a theoretical analysis on space-time focusing under single user SISO and multi-user multiple-input single-output (MISO) systems, respectively. The probability of bit error in TR systems has been investigated both theoretically [18,19,25], and empirically (BER) [17]. These works focus on separating the received signal components into desired signal power and ISI power (inter-user interference is also characterized in some cases) in order to obtain approximations to the signal-to-interference-plus-noise-ratio (SINR). However, the error of these approximations and their sensitivity to propagation conditions have not been analyzed. For example, the influence of the channel power-delay profile (PDP) on the TR beamforming system is unknown.

This motivates the first contribution of this chapter, which is the performance characterization of conventional TR beamforming for single-user indoor MISO channels in typical pico and femtocells. Our analysis is based on two statistical channel models [51] with different PDPs that are well suited for such scenarios. We use these two indoor channel models to provide a novel performance comparison of TR beamforming techniques under different propagation conditions, viz. delay spread,
sampling time (bandwidth), and CIR duration. We derive closed-form approximations to the probability of bit error and space-time focusing performance parameters. We find that performance is highly dependent on the propagation conditions and, hence, the relevance of the presented analysis.

In the second part of this chapter, we propose an equalized TR technique based on a previous contribution by our group [6]. We focus on a single-user MISO frequency-selective channel scenario, operating in conventional wideband systems with low complexity receivers. A number of works have addressed the problem of mitigating ISI in TR. For example, in [17] the authors propose the joint design of a TR pre-filter and a zero-forcing (ZF) pre-equalizer by finding the pre-filter closest to TR that sets the ISI power to zero. A similar approach is presented in [52], where the TR pre-filter is used in cascade configuration with a pre-equalizer, which is found by minimizing the ISI power through an semi-definite relaxation approximation. A multiuser TR equalization approach is found in [53], where the equalizer design is constrained to solutions that null the interference to other users. Reference [54] shows a TR waveform design that maximizes the sum rate in multiuser systems, and using a rate back-off strategy to reduce ISI. An equalized spatial multiplexing TR scheme for single-input multiple-output (SIMO) systems is presented in [22] for UWB.

However, previous approaches have not address the following aspects: i) theoretical performance with respect to focusing capability or BER is not characterized, ii) the behavior of previous solutions is not analyzed with respect to changes in propagation conditions, iii) the required up-sampling for rate back-off in some solutions demands costly high-speed hardware and/or decreasing the transmission rate, and iv) other solutions increase the receiver’s computational complexity versus conventional
TR by either using multiple receiving antennas or costly receiver equalizers. Thus, in
the proposed ETR scheme, we use a discrete ZF pre-equalizer in cascade with a TR
pre-filter at the transmitter in order to eliminate the ISI component in the received
signal while preserving the spatial focusing of conventional TR beamforming. Our
improvements with respect to previous works that have dealt with mitigating ISI in
TR systems are:

- The proposed ETR technique adds computational complexity to the transmitter
  only, maintaining the simplicity of the conventional TR receiver. Such ad-
  ditional complexity compared to conventional TR is limited by using a single
  equalizer shared by all of the transmit antennas.

- Unlike previous solutions, we derive theoretical performance bounds for both
  the probability of bit error and the beamforming capability of the proposed
  ETR technique. We also compare these bounds with those of conventional TR
  under different propagation conditions.

Our model is based on the assumption of a static (block fading) channel with
perfect CSI at the transmitter. This assumption is particularly appropriate for indoor
wireless communications, where the channel is slow-varying compared to the frame
structure in upper layers. Therefore, we do not focus on the influence of channel
estimation errors in our analysis. However, specific works regarding imperfect CSI
at the transmitter in TR systems can be found in the literature [55–59]. By means
of numerical simulations, we validate the results and derived bounds herein under
the assumed conditions. We also demonstrate that the proposed ETR technique
outperforms conventional TR in terms of BER without a significant impact on the beamforming capability.

The remainder of this chapter is organized as follows. Section 2.2 describes the conventional TR and ETR system models. In Section 2.3, we present the performance analysis of both techniques based on the power components of the received signal. We also define performance parameters and derive closed-form expressions for them. Section 2.4 presents numerical simulation results for the performance parameters and a comparison against the theoretical approximations. This is followed by concluding remarks in Section 2.5.

2.2 System Model

In this section we introduce the discrete signal model for conventional TR and the proposed ETR. We also present the corresponding radio channel models that will be used in the next section to characterize the performance of those techniques. The general idea behind TR is to use the time-reversed CIR from every antenna to the receiver as a pre-filter for the transmitted signal. Such pre-filter acts as a beamformer in the spatial domain, focusing the RF power around the receiver. For the ETR case, we propose a TR pre-filter in cascade with a ZF pre-equalizer in order to mitigate the ISI of conventional TR. The system model for conventional TR and ETR is depicted in Fig. 2.1.

2.2.1 Conventional TR Signal Model

Consider a digital MISO baseband wireless communication system with $M$ transmit antennas and one single-antenna user. Let $s[n]$ be the complex transmitted signal representing arbitrarily modulated symbols, where $n \in \mathbb{Z}^+$ is the discrete time index.
Figure 2.1: Single-user MISO system model for conventional TR (up) and ETR (down). In conventional TR one pre-filter is used in each antenna. In ETR, an additional pre-equalizer is introduced to the transmitted in order to mitigate the ISI. Note that a minimum complexity receiver is used.

This signal is assumed to have unit power (i.e. $\mathbb{E}[|s[n]|^2] = 1 \ \forall n$) regardless of the modulation. In conventional TR, the discrete time transmitted signal from the $i$-th antenna is

$$x_{tr}^i[n] = \sqrt{\rho} \ s[n] \otimes h_i^*[L - 1 - n] \sqrt{P_h}, \quad i = 1, \ldots, M,$$

where $\rho$ is the total average transmitted power, $\otimes$ denotes convolution; $h_i[n], \ n = 0, \ldots, L - 1$, is the complex CIR from the $i$-th transmit antenna to the receiver; and $P_h$ is a normalization factor introduced to ensure that the total transmitted power...
remains constant in every realization. This factor is defined as

\[
P_h = \sum_{i=1}^{M} \sum_{l=0}^{L-1} |h_i[l]|^2. \quad (2.1)
\]

Then, \( h_i^*[L - 1 - n] \) is the complex-conjugated time-reversed CIR applied as a pre-filter to the transmitted sequence. When perfect CSI is available at the transmitter and the channel is static, the received baseband signal is

\[
y_{tr}[n] = \frac{1}{\sqrt{P_h}} \sum_{i=1}^{M} s[n] \otimes h_i^*[L - 1 - n] \otimes h_i[n] + z[n] \\
= \sum_{i=1}^{M} s[n] \otimes h_{tr,i}[n] + z[n]
\]

where \( z[n] \) represents additive white Gaussian noise with variance \( \sigma_z^2 \), and we have defined the equivalent time-reversed CIR (TR-CIR) for the \( i \)-th antenna as

\[
h_{tr,i}[n] = \frac{1}{\sqrt{P_h}} h_i^*[L - 1 - n] \otimes h_i[n] \\
= \frac{1}{\sqrt{P_h}} \sum_{l=0}^{L-1} h_i[l] h_i^*[L - 1 - n + l] \\
\]

\( n = 0, \ldots, 2L - 2. \)

The effect of the TR filter is thus to replace the original CIR with the TR-CIR, whose properties will be analyzed hereinafter. Notice that we can rewrite the received signal in order to separate the desired symbol, the ISI, and the noise as

\[
y_{tr}[n] = \sqrt{\rho} \left( \sum_{i=1}^{M} \sum_{l=0}^{L-1} |h_i[l]|^2 s[n - L + 1] \right) + \sqrt{\rho} \sum_{i=1}^{M} \sum_{l=0}^{2L-2} h_{tr,i}[l] s[n - l] + z[n]. \quad (2.2)
\]

This separation in (2.2) can be interpreted in the following way. First, note that \( h_{tr,i}[n] \) is a scaled autocorrelation function of \( h_i[n] \), whose peak amplitude is

\[
\max_n |h_{tr,i}[n]| = |h_{tr,i}[L - 1]| = \frac{1}{\sqrt{P_h}} \sum_{l=0}^{L-1} |h_i[l]|^2.
\]
Thus, the focusing time effected by TR occurs at sample $L-1$ in the TR-CIR. At that instant, the multipath components corresponding to the desired symbol add in phase, so its coefficient is real and positive. Moreover, the ISI components add incoherently. The net result is an increase in the desired signal power and a reduction in the ISI. Note that $h_{tr,i}[k]$ has $2L-1$ non-zero samples, so the ISI spans across $2L-2$ symbols².

### 2.2.2 Proposed Equalized TR Signal Model

A main challenge in conventional TR beamforming is to mitigate the ISI component of the received signal. As seen in (2.2), and depending on the specific channel realization, the ISI can represent a significant percentage of the total received power, thus affecting detection. Typically this problem can be solved with equalization at the receiver, RAKE receivers or OFDM [60], but this would increase the low computational complexity enabled by TR. Thus, we propose an equalizer $g[n]$ of length $L_E$ (i.e. $n = 0, \ldots, L_E - 1$) cascaded with the TR pre-filters, with the goal of minimizing the ISI power at the receiver [6], as shown in Fig. 2.1. A single equalizer is shared by all the transmit antennas in order to reduce the required computational complexity. We refer to this approach as ETR. Following the model and notation in Section 2.2.1, the ETR transmitted signal in the $i$-th antenna is

$$x_{i}^{eq}[n] = \sqrt{\rho} s[n] \otimes \frac{h_{i}^{*}[L-1-n] \otimes g[n]}{\sqrt{P_g}}$$  \hspace{1cm} (2.3)

where the normalization factor $P_g$ is defined as

$$P_g = \sum_{i=1}^{M} \sum_{n=0}^{L+L_E-2} |h_{i}^{*}[L-1-n] \otimes g[n]|^2.$$  

²Note that, being an scaled autocorrelation function of $h_{i}[n]$, the equivalent time-reversed CIR has more samples than $h_{i}[n]$, but it has a lower delay spread (i.e. most of its energy is compressed in a number of samples less than $L$). We take into account all of the non-zero samples of $h_{tr,i}[n]$ in order to fully characterize the residual ISI of TR beamforming.
Note that $P_g$ accounts for the number of antennas, so $\rho$ is not explicitly divided by $M$ in (2.3). When perfect CSI is available at the transmitter and the channel is static, the received signal in ETR is

$$y_{eq}[n] = \sqrt{\frac{\rho}{P_g}} s[n] \otimes g[n] \otimes \sum_{i=1}^{M} h_{tr,i}[n] + z[n].$$

We propose a ZF pre-equalizer design for $g[n]$ whose objective is to completely eliminate the ISI component in the received signal. Although ZF is vulnerable to noise when used at the receiver [61], this problem is not of concern at the transmitter. The ZF criterion for the equalizer design is

$$g_{zf}[n] \otimes \sum_{i=1}^{M} h_{tr,i}[n] = \delta[n - n_0],$$

(2.4)

where $g[n] = g_{zf}[n]$ is the ZF equalizer solution, $\delta[n]$ is the unitary impulse function, and $n_0 \in [0, \ldots, 2L + L_E - 3]$ is an arbitrarily selected delay. Note that (2.4) is an overdetermined system of linear equations with $L_E$ unknowns and $2L + L_E - 2$ equations, which can be represented in matrix form as

$$\begin{bmatrix}
\sum_{i=1}^{M} h_{tr,i}[0] \\
\sum_{i=1}^{M} h_{tr,i}[2L-2] \\
0 \\
\vdots \\
0 \\
\end{bmatrix}
H
\begin{bmatrix}
g_{zf}[0] \\
g_{zf}[L_E - 1] \\
g_{zf}[L_E - 2] \\
\vdots \\
0 \\
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
1 \\
0 \\
0 \\
\delta_{n_0}
\end{bmatrix},$$

where $H \in \mathbb{C}^{(2L+L_E-2) \times L_E}$ is a banded Toeplitz (convolution) matrix. Thus, the equalizer has only a least-squares solution $g_{zf} = (H^H H)^{-1} H^H \delta_{n_0}$, where (2.4) is only satisfied when $L_E \to \infty$ [62]. We now take the ZF criterion to the frequency domain in order to facilitate the analysis in Section 2.3. Let $G_{zf}[k]$ and $H_i[k]$ denote
Table 2.1: Channel Model Parameters

| Tap Separation ($T_s$) [ns] | 1 cluster $\sigma$ [ns] $L$ $\gamma$ $\sigma_1$ [ns] $\sigma_2$ [ns] 2 clusters $L_1$ $L_2$ $L$ |
|---------------------------|-----------------|-------|-----|-----|-----|-------|-------|-------|
| 2.5                       | 8               | 33    | 0.4786 | 8    | 14   | 8     | 17    | 33    |
| 5                         | 8               | 17    | 0.4786 | 8    | 14   | 4     | 9     | 17    |
| 10                        | 8               | 9     | 0.4786 | 8    | 14   | 2     | 5     | 9     |

the discrete Fourier transforms (DFT) of $g_{zf}[n]$ and $h_i[n]$, respectively, with $n, k = 0, \ldots, 2L + L_E - 3$ (zero padding is used in order to represent the linear convolution). After applying the DFT to (2.4), the ZF equalizer in the frequency domain is

$$G_{zf}[k] = e^{-j\frac{2\pi(n_0-L+1)}{2L+L_E-2}k} \sum_{i=1}^{M} |H_i[k]|^2.$$ (2.5)

In the next section, we use the frequency domain representation given by (2.5) in order to obtain performance bounds for ETR. We also analyze the effect of equalizer’s length $L_E$ over the ISI power. Using the ZF equalizer, the received signal is then

$$y_{eq}[n] \approx \rho \sqrt{\frac{P_g}{P_s}} s[n - n_0] + z[n],$$ (2.6)

where the ISI term is neglected by assuming a sufficiently large $L_E$. This approximation is also analyzed in Section 2.4.

2.2.3 Wideband Radio Channel Model

As mentioned above, TR benefits from rich scattering, so it can be conveniently applied for indoor wireless communications. We selected two statistical baseband channel models suitable for such scenarios to make the performance analysis. The first one is a simple single-cluster CIR model with exponential power decay in time. The second model is a more general case with two propagation clusters, each one
of them with exponential power decay. Even though the first model is a particular case of the second, we consider it here separately in order to illustrate the derivation process and to facilitate interpretation of the results in Section 2.3. In addition, as demonstrated in Section 2.3, the performance of TR is strongly dependent on propagation conditions, i.e. PDP and delay spread. Most of the results of current literature use only the single-cluster channel model for the analysis of TR techniques. However, by using a second PDP, we show that the analyses of TR performance are model-dependent and should not be generalized.

For simplicity, we only take into account here the case where each CIR tap represents the contribution from several unresolvable multipath components with the same average amplitudes. Thus, diffuse scattering is assumed and both channel models have Rayleigh distributions. The common features of the two models are that the CIR $h_i[n]$ is modeled as a circular symmetric complex Gaussian random variable with zero mean $\forall i, n$. We assume that the transmit array elements have sufficient separation (e.g. irregular array). The system operates in a rich scattering environment, so $h_i[n]$ and $h_i'[n']$ are independent and uncorrelated if $i \neq i'$ or $n \neq n'$ (i.e. uncorrelated scattering). We also define the following constraint on the CIR total power:

$$\sum_{l=0}^{L-1} \mathbb{E} \left[ |h_i[l]|^2 \right] = \Gamma, \forall i,$$

where $\Gamma \ll 1$ is a constant accounting for the channel induced propagation losses. This constraint implies that the channels between each transmit antenna and the receiver have the same average power. The variance of $h_i[n]$ is specified by the power delay profile (PDP) model, as follows:
Model 1

This is the standard reference PDP model for indoor wireless communications [51]. The power in the CIR decreases exponentially in time with a single scattering cluster:

\[
E[ |h_i[n]|^2 ] = \begin{cases} 
A e^{-\frac{n T_s}{\sigma}} & \text{if } n = 0, \ldots, L - 1, \\
0 & \text{otherwise,}
\end{cases}
\]

where \( T_s \) is the sampling period or tap spacing, \( \sigma \) is the delay spread parameter, and \( A \) is selected to satisfy (2.7).

Model 2

The PDP matches common indoor propagation models, such as the IEEE 802.11n/ac Channel B in [63] and [64]. This is an exponential decay model with two scattering clusters. This is valid for indoor WLANs with operating frequencies around 2.4 GHz and 5 GHz, and bandwidths of up to 1.28 GHz:

\[
E[ |h_i[n]|^2 ] = \begin{cases} 
A e^{-\frac{n T_s}{\sigma_1}} & \text{if } 0 \leq n \leq L_1 - 1, \\
A e^{-\frac{n T_s}{\sigma_1}} + \gamma A e^{-\frac{(n-L_1) T_s}{\sigma_2}} & \text{if } L_1 \leq n \leq L_2 - 1, \\
\gamma A e^{-\frac{(n-L_1) T_s}{\sigma_2}} & \text{if } L_2 \leq n \leq L - 1, \\
0 & \text{otherwise,}
\end{cases}
\]

where \( \sigma_1 \) and \( \sigma_2 \) are the delay spread parameters, \( L_1 \) is the starting sample for the second cluster, \( L_2 \) is the number of samples in the first cluster, \( \gamma \) is the relative power of the second cluster, and \( A \) is the normalization constant selected such that (2.7) is satisfied.

Note that both models correspond to Rayleigh channels, with a duration of \( L \) samples in the CIR. However, Model 2 has a higher delay spread due to the strong delayed power contribution from the second scattering cluster. Table 2.1 shows the parameter values of each channel model under different CIR lengths, selected according to the standard [63], [64]. Parameters for Model 1 are the same as those for...
the first cluster in Model 2. These parameters are used for comparison purposes in Section 2.4.

2.3 Performance Analysis of Conventional TR and ETR

We now characterize the performance of conventional TR and the proposed ETR technique with respect to the probability of bit error and the spatial focusing capability. In conventional TR, as stated Section 2.2.1, the received signal (2.2) has three components: desired symbol, ISI, and noise. Individual components in the ISI sum have a complex double gaussian distribution [65], and they are dependent random variables. Thus, the ISI sum does not meet the assumptions of the conventional central limit theorem, and its distribution does not necessarily converges to a Gaussian distribution when the number of terms goes to infinity [66]. Nevertheless, an approximation to the probability of bit error in conventional TR systems assuming that ISI is Gaussian has been found to be sufficiently close to the numerical results in Section 2.4. For BPSK and QPSK modulations, this approximation is

\[ P_{e,BPSK}^{tr} \approx Q\left( \sqrt{\frac{2P_S}{P_{ISI} + \sigma_z^2}} \right) \quad \text{and} \quad P_{e,QPSK}^{tr} \approx Q\left( \sqrt{\frac{P_S}{P_{ISI} + \sigma_z^2}} \right), \quad (2.8) \]

respectively, where \( Q(\cdot) \) is the complementary cumulative distribution function of a standard Gaussian random variable, \( P_S \) is the desired signal power, \( P_{ISI} \) is the intersymbol interference power, and \( \sigma_z^2 \) is the noise power. Note that in conventional TR the performance is limited by ISI at high SNR. In the case of ETR, we assume that we can neglect the ISI term in the received signal due to equalization, which is true for a sufficiently large \( L_E \) (as analyzed next). Thus, a lower bound on the probability of bit error in BPSK and QPSK [60] using ETR are, respectively,
\[ P_{e,BPSK}^{eq} \geq Q\left( \sqrt{\frac{P_{eq}}{\sigma^2}} \right) \quad \text{and} \quad P_{e,QPSK}^{eq} \geq Q\left( \sqrt{\frac{P_{eq}}{\sigma^2}} \right), \]

where \( P_{eq} \) is the received signal power in (2.6). Similar expressions for other modulations can be found in [60]. In this section, we derive the expressions for the power of each of those components in terms of the channel PDP, which are necessary for the performance characterization of TR and ETR. These expressions have not been compared previously across different channel models, so they constitute one of the contributions of this chapter. We also study the influence of the equalizer’s length over its ISI suppression capability. In addition, we define parameters to measure the TR space-time focusing performance, and then present closed-form approximations for them using the indoor channel models introduced above.

2.3.1 Desired Signal Power

Conventional TR

The desired signal power in (2.2) is

\[ P_S = \mathbb{E} \left[ \rho \sum_{i=1}^{M} \sum_{l=0}^{L-1} |h_i[l]|^2 \right] = \rho M \Gamma. \quad (2.9) \]

which can be obtained from the channel power constraint (2.7). Note that this signal power is independent of the channel model and is directly proportional to the number of antennas.

ETR

According to (2.6), the received signal power is

\[ P_{eq} = \rho \mathbb{E} \left[ \frac{1}{P_g} \right]. \]
As shown in Appendix A, an upper bound on the received power (which causes a lower bound in the probability of bit error) is

\[ P_{eq} \leq \rho M \Gamma. \]  

(2.10)

Thus, the received power in ETR is at best the desired power in conventional TR and a reduction in the beamforming capability is expected. We analyze this issue later. However, the probability of bit error is lower in ETR due to the elimination of the ISI. We verify this bound numerically in Section 2.4.

### 2.3.2 Intersymbol Interference Power in Conventional TR

The ISI power \( P_{ISI} \), is derived here from (2.2) as the sum of the power in the TR-CIR at instants other than the focusing time (i.e., \( l \in \{0, \ldots, 2L-2\}, l \neq L-1 \)):

\[
P_{ISI} = \rho \mathbb{E} \left[ \sum_{i=1}^{M} \sum_{l=0, l \neq L-1}^{2L-2} h_{tr,i}[l] \right]^2 = \rho \mathbb{E} \left[ \sum_{i=1}^{M} \sum_{l=0, l \neq L-1}^{2L-2} \frac{h^*_i[L - 1 - l] \otimes h_i[l]}{\sqrt{P_h}} \right]^2.
\]  

(2.11)

Note that \( P_h \) is a random variable that depends on the CIR, as given by (2.1), so the calculation of (2.11) is not straightforward. As shown in B, we use an expansion for the expectation of the ratio of correlated random variables \([67] [68]\) in order to derive the following approximation for this equation:

\[
\hat{P}_{ISI} = \frac{\rho}{M \Gamma} \sum_{l=0}^{2L-2} \sum_{i=1}^{M} \sum_{n=0}^{L-1} \mathbb{E} \left[ |h_i[n]|^2 \right] \mathbb{E} \left[ |h_i[L - 1 - l + n]|^2 \right].
\]  

(2.12)

The approximation error reduces with a small number of antennas, larger delay spreads, and/or larger bandwidths (see Appendix B). Notice that the received ISI

\[
\hat{P}_{ISI} = \frac{\rho}{M \Gamma} \sum_{l=0}^{2L-2} \sum_{i=1}^{M} \sum_{n=0}^{L-1} \mathbb{E} \left[ |h_i[n]|^2 \right] \mathbb{E} \left[ |h_i[L - 1 - l + n]|^2 \right].
\]  

(2.12)

The approximation error reduces with a small number of antennas, larger delay spreads, and/or larger bandwidths (see Appendix B). Notice that the received ISI
power depends on the PDP model. Therefore, we evaluate (2.12) using the channel models described in Section 2.2.3. From now on, let the superscripts \((1)\) and \((2)\) denote variables calculated using Model 1 and Model 2, respectively, and the symbol \(\hat{\cdot}\) denote the corresponding variable approximation. Then, the results for \(P_{ISI}\) are

\[
\hat{P}_{ISI}^{(1)} = \rho \Gamma \left( \frac{1 - e^{-\frac{T_s}{\tau}}}{1 - e^{-\frac{LT_s}{\sigma}}} \right)^2 \sum_{l=0}^{2L-2} \sum_{n=0}^{L-1} \sum_{n \in [l-L+1, n \geq l]} e^{-\frac{(n-L_1)T_s}{\sigma_1}} C[l, n] + \gamma \sum_{n=L_1}^{L-1} \sum_{n \in [l-L+1, n \geq l]} e^{-\frac{(n-L_1)T_s}{\sigma_2}} C[l, n],
\]

(2.13)

\[
\hat{P}_{ISI}^{(2)} = \rho \Gamma \frac{2 \sum_{l=0}^{2L-2} \sum_{n=0}^{L-1} e^{-\frac{nT_s}{\sigma_1}} C[l, n] + \gamma \sum_{n=L_1}^{L-1} e^{-\frac{(n-L_1)T_s}{\sigma_2}} C[l, n]}{\left( \sum_{n=0}^{L-1} e^{-\frac{nT_s}{\sigma_1}} + \gamma \sum_{n=L_1}^{L-1} e^{-\frac{(n-L_1)T_s}{\sigma_2}} \right)^2},
\]

(2.14)

where

\[
C[l, n] = \begin{cases} 
    e^{-\frac{(L-1-l+n)T_s}{\sigma_1}} & \text{if } l - L + 1 \leq n \leq l - L + L_1, \\
    e^{-\frac{(L-1-l+n-L_1)T_s}{\sigma_1}} + \gamma e^{-\frac{(L-1-l+n-L_1)T_s}{\sigma_2}} & \text{if } l - L + L_1 + 1 \leq n \leq l - L + L_2, \\
    e^{-\frac{(L-1-l+n-L_1)T_s}{\sigma_2}} & \text{if } l - L + L_2 + 1 \leq n \leq l, \\
    0 & \text{otherwise.}
\end{cases}
\]

There are two interesting remarks about the power components in conventional TR that we found through the proposed approximation. First, the ISI power does not depend on the number of antennas, but the desired signal power is directly proportional to it. Hence, from the probability of error (2.8), an increase in \(M\) would increase the ratio between \(P_S\) and \(P_{ISI}\) and, consequently, it would improve the BER at high SNR. This phenomena could be harnessed in the context of massive MIMO systems [69].

Second, there are three parameters that can affect the ISI power: the tap separation \(T_s\) (or, equivalently, the bandwidth), the channel delay spread \(\sigma\), and the CIR duration \(L\). Thus, ISI power is strongly dependent on the propagation environment.
In order to obtain a better insight on the impact of these three parameters on the BER performance of TR beamforming, we next define the usable power ratio relating desired signal power and ISI power.

### 2.3.3 Usable Power and Time Compression in Conventional TR

The usable power ratio is a parameter that will help to compare different scenarios (characterized by their channel models) through a single metric. From the received signal in (2.2), we know that the total received power with conventional TR is \( P_R = P_S + P_{ISI} \). Note that, according to the probability of error (2.8), conventional TR performance is limited by the ratio between \( P_S \) and \( P_{ISI} \) in the high SNR regime. Thus, the usable power ratio is defined as \( U \triangleq P_S/P_{ISI} \), which measures the fraction of the received power that can be effectively used at the detector and determines a lower bound to (2.8). Using the expressions for Model 1 and Model 2, the usable power ratio approximations are, respectively,

\[
\hat{U}^{(1)} = M \left( 1 - e^{-\frac{LT_s}{\sigma}} \right)^2 \left( 1 - e^{-\frac{T_s}{\sigma}} \right)^2 \sum_{\substack{n=0 \atop l \neq L-1}}^{L-2} \sum_{\substack{n=0 \atop n \leq l \neq L-1 \atop n \geq -L+1}}^{L-1} e^{-\frac{(L_1+l+2n)T_s}{\sigma}} e^{-\frac{(L_2+l+2n)T_s}{\sigma}}
\]

\[
\hat{U}^{(2)} = M \left( \sum_{n=0}^{L_2-1} e^{-\frac{nT_s}{\sigma_1}} \gamma \sum_{n=L_1}^{L-1} e^{-\frac{(n-L_2)T_s}{\sigma_2}} \right)^2 \sum_{\substack{n=0 \atop l \neq L-1}}^{L-2} \sum_{\substack{n=0 \atop n \leq l \neq L-1 \atop n \geq -L+1}}^{L_2-1} e^{-\frac{nT_s}{\sigma_1}} C[l,n] + \gamma \sum_{\substack{n=L_1 \atop n \leq l \neq L-1 \atop n \geq -L+1}}^{L-1} e^{-\frac{(n-L_2)T_s}{\sigma_2}} C[l,n]
\]

This particular parameter has no relevance for ETR, since we assume the equalizer completely eliminates ISI. We analyze numerically the impact of propagation conditions (namely, parameters \( T_s, \sigma \), and \( L \)) over \( U \) in Section 2.4.
2.3.4 Interference Mitigation and Spatial Focusing

The spatial focusing capability of conventional TR has important interference mitigation applications in wireless communications. In this subsection we analyze the signal power at points in the space different than the receiver’s location by considering an unintended receiver with uncorrelated CIR. Physically, in the frequencies where the employed channel models are valid, uncorrelated CIRs are obtained with just a few wavelengths of separation (e.g. see [70]). We use this analysis to determine the power ratio between the targeted receiver’s and nearby locations as a measure of the spatial focusing, and compare conventional TR with our proposed ETR technique.

Consider an unintended receiver with CIR denoted by $h_{u,i}[n]$ from the $i$-th transmit antenna, where $h_{u,i}[n]$ and $h_p[l]$ are identically distributed and uncorrelated for all $i$, $p$, $n$, and $l$. More specifically, $h_{u,i}[n]$ has the same power delay profiles and power constraints described in Section 2.2 for $h_i[n]$. In conventional TR, the signal at the unintended receiver is given by

$$y_{tr}^u[n] = \sqrt{\rho} \sum_{i=1}^{M} s[n] \otimes h_i^*[L-1-n] \sqrt{P_h} \otimes h_{u,i}[n] + z[n].$$

The desired signal power captured by the unintended receiver is equal to the power of the sample at instant $L-1$ in its equivalent TR-CIR. Then, we define that interference power as

$$P_{tr}^{int} = \mathbb{E} \left[ \left| \sum_{i=1}^{M} \frac{h_i^*[L-1-n]}{\sqrt{P_h}} \otimes h_{u,i}[n] \right|^2 \right]_{n=L-1}.$$

Using the same procedure that we used in the derivation of $P_{IST}$, which can be found in Appendix B, the interference power becomes
\[
\hat{P}_{tr} = \frac{\rho}{\Gamma} \sum_{l=0}^{L-1} \mathbb{E} [ |h_{u,i}[l]|^2 ] \mathbb{E} [ |h_l[l]|^2 ].
\]

Again, this expression depends on the user PDP and the unintended receiver PDP, which are assumed to be identical. Thus, using the defined models, we get

\[
\hat{P}_{tr}^{(1)} = \rho \Gamma \left( \frac{1 + e^{-\frac{LT_s}{\sigma_1}}}{1 + e^{-\frac{T_s}{\sigma_1}}} \right) \left( 1 - e^{-\frac{T_s}{\sigma_2}} \right),
\]

(2.17)

\[
\hat{P}_{tr}^{(2)} = \rho \Gamma \frac{\left( \sum_{n=0}^{L_2-1} e^{-\frac{2nT_s}{\sigma_1}} + \gamma^2 \sum_{n=L_1}^{L-1} e^{-\frac{2(n-L_1)T_s}{\sigma_2}} + 2\gamma \sum_{n=L_1}^{L_2-1} e^{-\frac{nT_s}{\sigma_1}} e^{-\frac{(n-L_1)T_s}{\sigma_2}} \right)^2}{\left( \sum_{n=0}^{L_2-1} e^{-\frac{nT_s}{\sigma_1}} + \gamma \sum_{n=L_1}^{L_1} e^{-\frac{(n-L_1)T_s}{\sigma_2}} \right)^2}. 
\]

(2.18)

In ETR, the signal at an unintended receiver is

\[
y_{eq}^{u}[n] = \sqrt{\rho} s[n] \otimes \frac{g[n]}{\sqrt{T_g}} \otimes \sum_{i=1}^{M_T} \hat{h}_i^{u}[n] \otimes h_{u,i}[n] + z[n].
\]

(2.19)

In this case, the equalizer does not match the CIR to the unintended receiver, so the signal has a desired signal component and an ISI component due to imperfect equalization. This total received power can be approximated as (see Appendix C)

\[
\hat{P}_{eq}^{int} = \rho \Gamma.
\]

(2.20)

Note that both the received power and the interference power are independent of the channel model in ETR, as long as the power constraint (2.7) is satisfied. We define the effective spatial focusing parameter as the ratio between the usable power at the receiver and the usable power at the unintended receiver (without considering the ISI in the signal). This parameter has been used previously in related literature, e.g. [18].

Then, for conventional TR and ETR this parameter is, respectively,

\[
\eta_{tr} = \frac{P_S}{P_{tr}^{int}} \quad \text{and} \quad \eta_{eq} = \frac{P_{eq}}{P_{eq}^{int}},
\]

28
and measures the ability of the beamformer to focus the signal power on a specific point in space, i.e. the power that can be used effectively at the detector. In the case of conventional TR, we use the expressions (2.9), (2.17), and (2.18) to obtain the following closed-form approximations to $\eta_{tr}$:

\[
\hat{\eta}_{tr}^{(1)} = M \frac{1 + e^{-\frac{T_s}{\sigma}}} {1 + e^{-\frac{LT_s}{\sigma}}} \frac{1 - e^{-\frac{LT_s}{\sigma}}} {1 - e^{-\frac{T_s}{\sigma}}},
\]

(2.21)

\[
\hat{\eta}_{tr}^{(2)} = M \left( \frac{\sum_{n=0}^{L_2-1} e^{-\frac{nT_s}{\sigma_1}} + \gamma \sum_{n=L_1}^{L-1} e^{-\frac{(n-L_1)T_s}{\sigma_2}}}{\sum_{n=0}^{L_2-1} e^{-\frac{2nT_s}{\sigma_1}} + \gamma^2 \sum_{n=L_1}^{L-1} e^{-\frac{2(n-L_1)T_s}{\sigma_2}} + 2\gamma \sum_{n=L_1}^{L-1} e^{-\frac{nT_S}{\sigma_1}} e^{-\frac{(n-L_1)T_S}{\sigma_2}}} \right)^2.
\]

(2.22)

It is clear that the spatial focusing in TR increases with the number of antennas in a similar way as in a conventional phased array. Nevertheless, TR allows a 3D focusing of the signal using the information in the CIR, instead of the 2D beam-steering performed by phased arrays, i.e. TR can achieve full array gain in multipath environments. A numerical analysis of the behavior of $\eta_{tr}$ is given in Section 2.4 with respect to the channel model parameters. In the case of ETR, from (2.10) and (2.20), an upper bound on the spatial focusing parameter $\eta_{eq}$ is around $M$.

We also define an alternate measure of spatial focusing that we call \emph{apparent power focusing}. This measures the total spatial focusing of the signal in conventional TR, including the presence of ISI. The definition is

\[
\hat{\eta}_{tr}' = \frac{P_S + P_{ISI}}{P_{int} + P_{ISI}},
\]

where the ISI power is the same at the unintended receiver, due to the fact that $h_{u,i}[n]$ and $h_i[n]$ have the same PDP. In previous works, the difference between the \emph{effective power focusing} and the \emph{apparent power focusing} has not been clearly defined. Thus,
we introduce this parameter in order to make a distinction between the total power present in the focusing point (which includes desired signal power and ISI), and the power that can be actually used at the detector (only the desired signal power).

A detailed analysis of the parameters calculated in this section is provided next.

2.4 Numerical Results and Discussion

In this section, we illustrate the time compression property of TR and ETR by analyzing their equivalent CIRs. Then, we present numerical results for the performance parameters defined in Section 2.3.

2.4.1 Time Compression and Pre-Equalization

Fig. 2.2 shows the time compression property of conventional TR and ETR. The original CIRs (one per transmit antenna) have power contributions from all the multipath components at different times. TR beamforming focuses the all those contributions in a single sampling instant, but there is a significant residual ISI power. ETR mitigates the ISI at the cost of a reduced focusing on the desired sampling instant, so the equivalent CIR approaches a delta function. Moreover, the ISI power is diminishingly small as $L_E \to \infty$. Fig. 2.3 shows the behavior of desired signal power and ISI power as a function of equalizer length $L_E$. These results were obtained by averaging those powers over 1000 channel realization using Model 2 with $M = 4$, $L = 33$, and $T_s = 2.5$ ns. Both signal power and ISI power decay exponentially as $L_E$ increases, until no significant variation is observed. This occurs when $L_E \approx L$, which corresponds to a ratio of approximately 30 dB between signal power and ISI power in the worst case. These results indicate that near-cancellation of ISI is achieved with a finite equalizer’s length. If the number of antennas is increased, the required
Figure 2.2: (a) one CIR realization for antenna 1 generated according to Model 2 with $L = 33$. (b) equivalent TR-CIR obtained with conventional TR, i.e. as observed by the receiver; note the time focusing capability at the 32-th sample. (c) ETR equivalent CIR: a ZF pre-equalizer with length $L_E = 33$ is cascaded with the TR pre-filters. ISI is greatly reduced with this approach at the cost of a reduced focusing. The equivalent CIR approaches a delta function. Results with $M = 4$ antennas.

The equalizer’s length decreases proportionally, as can be concluded from the usable power parameter $U$ definition in Section 2.3. Therefore, we set $L_E = L$ for the following simulations in this section, noting that such equalizer’s length allows the system to be noise limited rather than ISI limited.

### 2.4.2 Beamforming Performance Parameters

We analyze numerically the expressions found in the previous section for $\hat{U}$, $\hat{\eta}_{tr}$ and $\hat{\eta}'_{tr}$ in conventional TR. Fig. 2.4 shows these results in terms of the ratio between the symbol time and the channel delay spread $T_s/\sigma$ (we use $T_s/\sigma_1$ for Model 2). We set the remaining parameters so they approximate Channel Model B in [63] (i.e., $L = 33$, $L_1 = 8$, $L_2 = 17$, $\gamma = 0.4786$ and $\sigma_2 = 1.75\sigma_1$). The number of antennas was set to $M = 4$. The ratio between the tap spacing $T_s$ and the delay spread parameter
Figure 2.3: (a) Desired signal power, and (b) ISI power as a function of equalizer’s length $L_E$. Note that ISI power is larger when CIR length and/or delay spread increases (i.e. Model 2). Both powers decay by increasing $L_E$, and nearly perfect ISI suppression can be achieved at the cost of a marginal decrease in desired signal power. Results with $M = 4$.

$\sigma$ determines the frequency selectivity of the channel: smaller values of $T_s/\sigma$ imply larger signal bandwidths or stronger scattering in the channel.

Fig. 2.4a shows that the usable power ratio for Model 1 $\hat{U}^{(1)}$ increases when the channel $T_s/\sigma$. However, the same behavior is not observed for Model 2, where the variations of $\hat{U}^{(2)}$ are not significant. Thus, no general conclusions on the ISI power behavior can be drawn, given its nonlinear dependence on several propagation parameters (see (2.13) and (2.14)). Typical wideband channels, which are characterized by $T_s/\sigma < 1$, have a usable power ratio ranging from 5 dB to 15 dB in the simulated scenarios, which will limit the BER performance at high SNR.

Fig. 2.4b and Fig. 2.4c show the results for the effective spatial focusing and the total power focusing parameters. In both cases, an increase in the spatial focusing
(beamforming capability) of conventional TR is observed for scenarios with stronger scattering and/or larger bandwidths (small $T_s/\sigma$). Also, $\hat{\eta}_{tr} > \hat{\eta}^\prime_{tr}$ in all cases, which can be interpreted in the following way. Even though the received signal power at the desired user is between 6 dB and 8 dB (approximately) stronger than the signal power at the unintended receiver, an important fraction of these powers are composed of ISI. However, the usable power at the user’s detector is actually significantly larger than the usable power at the unintended receiver (it can reach up to 25 dB in the simulated conditions). This is because the TR pre-filter is matched only to the desired user’s CIR, and does not offer partial equalization at other spatial locations. It is also worth noting that an approximate upper bound on $\hat{\eta}_{eq}$ is the number of antennas (6 dB under the conditions described on Fig. 2.4) regardless of the channel model. We return to this issue later.

We also performed Monte Carlo simulations of the described conventional TR and ETR systems under tap separations of 2.5 ns, and 10 ns, consistent with current WLAN models as specified in [63] and [64]. We calculated the performance parameters presented in Section 2.3 for 1000 channel realizations, with the transmission of $10^4$ frames of 10 symbols in each one of them. The number of transmit antennas was $M = 4$ and the channel parameters were selected according to Table 2.1.

In concordance with the results in Fig. 2.4, the simulation shows that the total focusing performance improves by decreasing the tap separation, as presented in Table 2.2. This is due to the increasing number of resolvable multipath components in the CIR, which are all coherently combined at the receiver thanks to the TR pre-filter. Also, the results are consistent with the closed form approximations (2.21) and (2.22). In the case of ETR, the approximate upperbound of 6 dB for the spatial focusing is
Figure 2.4: Performance parameters introduced in Section 2.3 for conventional TR, calculated for both channel models as a function of the ratio between the symbol duration and the channel delay spread $T_s/\sigma$. Other parameters are: $L = 33$, $L_1 \approx 8$, $L_2 \approx 17$, $\gamma = 0.4786$, $\sigma_1 = 8$ ns, and $\sigma_2 = 1.75\sigma_1$. (a) ISI power and usable power ratio, (b) Interference power and effective spatial focusing, and (c) apparent spatial focusing.
satisfied in these scenarios, and a loss of between 1dB and 2dB is observed with respect to conventional TR. This is caused by the effect of the equalizer over the desired signal and ISI power: larger delay spreads, smaller tap separations, or larger $L$ decrease the total received power under a constant $L_E$, as seen in Fig. 2.3. These results clearly demonstrate the potential of TR techniques for beamforming.

### 2.4.3 BER Performance

We calculated the BER of both conventional TR and ETR as a function of the signal to noise ratio defined as $SNR = \frac{\rho \Gamma}{\sigma_z}$. First, in Fig. 2.5 we verify our approximation to the probability of error in conventional TR using the closed form expressions (2.9), (2.13) and (2.14). BPSK and QPSK modulations were used. The difference between our theoretical $P_e$ approximation and the simulated BER results improves in Model 2. This is due to the smaller variance of the normalization factor in Model 2, as explained in Appendix B. It is worth noting that the approximation accuracy to the probability of error is highly dependent on the specific set of parameters describing the propagation conditions.

It is observed that Model 1 (weaker scattering) has a better BER performance, as expected from the usable power ratio results in Fig. 2.4a. In addition, it is
clear that the BER in both modulations is too high to be of practical use in the scenarios considered here; this is because the ISI power causes a lower bound on the probability of bit error, as stated in Section 2.3. Thus, the relevance of the proposed ETR technique to overcome this problem is evident.

Fig. 2.6 shows the simulated BER performance for the TR and ETR and the lower bound for the probability of error using BPSK. The number of antennas is $M = 4$. The equalizer’s length is the same as the CIR length, i.e. $L_E = L$. Again it is noted that conventional TR has a lower bound on the BER caused by ISI, and that the performance deteriorates by increasing the delay spread (Model 2) or the bandwidth due to stronger ISI power. Variations of the BER in ETR are not significant with respect to changes in model parameters. ETR outperforms conventional TR under
any SNR by mitigating the ISI, so its BER performance approaches that of the AWGN channel.

Fig. 2.7 shows the simulated BER of conventional TR and ETR under different channel models, and number of antennas. BER performance variations in ETR are significant with respect to the channel models. However, when increasing the number of antennas from 4 to 8, conventional TR performance improves significantly, approaching that of ETR. Specifically, lower BER can be achieved at high SNR when increasing the number of antennas due to the linear dependence of the usable power ratio on \( M \) (as seen in (2.15) and (2.16)). This phenomena can be harnessed in systems with a large number of transmit antennas, where the equalization properties of TR can allow sufficiently low BER without further processing [69].
2.5 Conclusions

We have analyzed a baseband TR beamforming system using two propagation models commonly used in indoor wireless communications. In particular, this analysis is relevant for pico and femtocells in conventional wideband systems such as WiFi networks. We derived a novel closed-form approximation for the ISI power in such scenarios in single-user wideband systems without rate back-off, and compare the probability of bit error obtained under different propagation conditions.

We analyzed parameters for the spatial focusing and time compression properties of TR beamforming and found closed-form approximations for them. By analyzing this parameters under two channel models, we found that TR performance is strongly dependent on propagation conditions. Specifically, there are significant variations on
the ISI power depending on the power delay profile, the symbol duration (bandwidth), and the channel delay spread. Hence, no general conclusions can be extracted for the ISI for specific propagation conditions.

We then proposed an equalized TR technique as a solution to mitigate ISI. ETR uses a single ZF pre-equalizer at the transmitter in cascade configuration with the TR pre-filters. Unlike previous approaches, we analytically showed that the proposed technique greatly enhances the performance of conventional TR with low impact to its beamforming capability. An upper bound on the received power of ETR was also derived, which corresponds to a lower bound on the probability of bit error.

The spatial focusing performance of conventional TR and ETR was analyzed by calculating the signal power at an unintended receiver with uncorrelated CIR. We showed that the effective power ratio and the apparent power ratio between the receiver and the interfered user increase with either the channel delay spread or the signal bandwidth. Moreover, it was shown that the use of ETR has a small impact over this spatial focusing parameters.

By means of numerical simulations, we verified that the proposed ETR technique outperforms conventional TR with respect to the BER under any SNR, even though the total received power is greater for conventional TR. We also verified the accuracy of the approximation to the ISI power in conventional TR and found that it improves for channel model with stronger delayed components (Model 2 defined here).
Chapter 3: Interference-Nulling Time-Reversal Beamforming for mm-Wave Massive MIMO in Multi-User Frequency-Selective Indoor Channels

3.1 Introduction

Massive MIMO systems have been recently recognized as one of the technologies that can bring unprecedented performance gains for next generation wireless communications [71]. Among its potential benefits, noise, fading and inter-user interference (IUI) effects have been shown to progressively reduce as the number of antennas in the system increases [72]. Thus, a large number of antennas simplifies the multiple access layer and increases the system’s capacity [73]. However, many challenges remain for the implementation of massive MIMO systems such as: channel estimation and reciprocity issues, large pilot overheads, hardware cost, size and power limitations, network architecture adaptations, antennas and propagation aspects [74].

Recently, millimeter wave (mm-wave) and massive MIMO have been proposed in tandem for next generation systems [75,76]. This can be easily justified because a large number of antennas operating at mm-wave frequencies (e.g. 28, 38, 60 and 73 GHz) can be used in compact devices due to the small wavelength (4 to 10 mm approx.) and (hence) small antenna sizes. In addition, it has been shown that mm-wave networks
are suitable for dense small cells (especially in indoor environments), as inter-cell interference is naturally mitigated due to high propagation losses at those frequencies\cite{3,77}. Another benefit of using mm-wave is the huge bandwidth availability, with some standards planning to operate with more than 2 GHz bandwidth, e.g.\cite{78}.

Nevertheless, a number of problems arise for mm-wave massive MIMO systems. In particular, their performance is highly dependent on the antenna array configuration and the propagation environment. Hence, factors such as the coupling between antennas and channel spatial correlation play a significant role on the actual capacity and diversity gain that this kind of systems can achieve\cite{74}.

Beamforming is needed in mm-wave systems because of large propagation losses. Traditionally, antenna arrays use beamsteering techniques in order to increase the received power in specific directions\cite{79} and, consequently, achieve diversity gain. This is usually performed with either analog (RF) or digital (baseband) phase shifters in each antenna. Recent approaches to beamforming\footnote{The term beamforming is traditionally used to denote phased array techniques for beam steering, i.e. operating in the 2D manifold spanned by the azimuth and elevation angles. In this chapter, we shall use the term beamforming in a broader sense to denote signal processing techniques that allow spatial focusing of RF power in co-range as well (3D) or even in time (4D space-time beamforming) \cite{9,16,37}.} in mm-wave massive MIMO use hybrid analog beamsteering combined with digital precoding techniques assuming narrowband fading channels\cite{12,13,75,80,81}. This hybrid analog/digital solutions are necessary given that fully digital solutions requires one digital to analog converter per antenna, which is extremely constly in terms of power. However, conventional beamsteering is not always appropriate in multipath channels with frequency selective fading, such as those found in indoor environments. In those cases, more sophisticated
techniques are required to take full advantage of the number of elements in the array and also multipath propagation.

The specific propagation aspects of mm-wave systems have been recently studied. Statistical models for mm-wave channels have been developed in [82,83], where extremely narrow antenna radiation patterns are considered using massive MIMO. These models provide characterization of scattering clusters in the angular and delay domains, power-delay profiles (PDPs), and propagation losses in outdoor scenarios. Similar models can be found for indoor scenarios [84]. Another popular SISO channel model for indoor mm-wave systems is the IEEE 802.11ad [85], which considers extremely narrow radiation patterns and analog beamsteering. However, there are only few studies on the spatial correlation in mm-wave MIMO channels. An interesting work is [86], where it is demonstrated that correlation at 60 GHz can be very high due to the small number of multipath components (MPCs). It has also been recognized that the specific structure of spatial correlation is highly dependent on the scattering environment. Given these conclusions, it is not clear yet whether diversity and/or multiplexing schemes should be used in order to maximize the system’s gain [87].

In this chapter we propose a novel time-reversal (TR) based [6] solution to the multi-user beamforming problem for indoor scenarios in mm-wave massive MIMO, which provides both diversity and multiplexing gains. TR is a transmission technique that enables spatial focusing of the signal at the receiver by using the time-reversed channel impulse response (CIR) as a linear filter applied to the transmitted signal [9,16]. TR is considered a promising technique for future massive MIMO systems [74]. By using TR, all multipath components are added in phase at the receiver at a specific instant providing i) an increase in the signal power in the surroundings
of the receiver (commonly referred to as spatial focusing), and \( ii \) a partial equalization effect (commonly known as time focusing) that reduces inter-symbol interference (ISI) caused by the channel’s frequency selectivity \([18,88]\). This features enable low computational complexity receivers, which is a key advantage of TR with respect to multicarrier (OFDM-like) systems \([10]\). Moreover, multipath components add incoherently at regions in space away from the receiver, mitigating interference to other users \([19,20]\).

A number of works have addressed different aspects of TR beamforming, with particular focus on single user systems \([16–20]\). In these references, the spatial and temporal focusing properties of TR have been considered, and both theoretical and empirical characterizations of bit error rate (BER) have been made under specific scenarios and channel models. A common finding in the literature is that ISI is the main limiting factor of TR. This is because ISI imposes a lower bound in the achievable BER at high signal to noise ratios (SNR) in single user systems \([20]\).

The challenge of mitigating ISI in TR has also received increasing attention. Different equalizing solutions have been proposed in \([20–22]\) for single-user systems. An important result of \([20]\) is that the ratio between the desired signal power and the ISI power in TR increases linearly with the number of antennas. Thus, BER performance can potentially have a significant improvement when TR is applied in massive MIMO, without additional equalization.

For multiuser systems, TR for multiple access in the downlink was proposed in \([23]\) and \([24]\), where IUI is recognized as the main limiting factor of BER performance. Also, \([6]\) proposes several multi-user TR techniques. A multiple access TR technique
that uses rate-backoff is proposed in [25], where an approximation for the signal-to-interference-plus-noise-ratio (SINR) is given, showing that it increases with the number of antennas.

However, previous works have not addressed the following aspects:

- Proposed beamsteering techniques in mm-wave massive MIMO are narrowband (for flat fading channels), and do not take full advantage of multipath propagation to increase diversity gain. Thus, these techniques may not be appropriate for frequency selective channels found in indoor scenarios.

- TR beamforming techniques, which have been thoughtfully analyzed in other scenarios and take advantage of rich scattering, have not been studied in the context of mm-wave massive MIMO. More specifically, [25] and [20] suggest that SINR in conventional TR grows linearly with the number of antennas, enabling low complexity receivers.

In this context, the contributions of this chapter are the following:

- We introduce a simple channel model for 60 GHz massive-MIMO, which is based on the IEEE 802.11ad model [85]. We define the probability distribution of the channel taps, their PDP, and spatial correlation.

- We study the performance of conventional TR in multi-user systems when the number of antennas at the transmitter is very large. Moreover, we generalize the ETR [20] to multi-user systems, and compare its performance with conventional TR. We demonstrate that, provided a sufficiently large number of transmit antennas, TR does not need further equalization, becoming an attractive beamforming alternative.
• Using the previous analysis, where we find that conventional TR performance is IUI-limited, we propose a novel TR multi-user beamforming technique that minimizes IUI and exploits rich multipath commonly found in indoor environments. We call this technique interference-nulling time-reversal (INTR).

• We analyze and compare numerically the performance of these TR techniques using the proposed statistical MIMO channel model for 60 GHz.

Commonly Used Acronyms in this Chapter

AP - Access Point; CB - cubicle scenario; CIR - channel impulse response; CR - conference room scenario; ETR equalized timer-reversal; INTR - interference-nulling time-reversal; ISI - inter-symbol interference; IUI - inter-user interference; LR - living room scenario; MPC - multipath component; PDP - power-delay profile; TR - time-reversal; US - uncorrelated scattering.

Notation

Lower and upper case symbols represent signals in the time and frequency domains, respectively. Boldface symbols represent vectors or matrices, whose dimensions are specified explicitly. \( \otimes \) is the convolution operator between two signals. \( \mathbb{E} [\cdot] \) represents expectation over a random variable. The operators \((\cdot)^T\), \((\cdot)^*\), \((\cdot)^H\) and \((\cdot)^{-1}\) represent transpose, complex conjugate, Hermitian transpose, and matrix inverse, respectively. The norm of the vector \(\mathbf{a}\) is denoted as \(\|\mathbf{a}\| = \sqrt{\langle \mathbf{a}, \mathbf{a} \rangle}\), where \(\langle \mathbf{a}, \mathbf{b} \rangle = \mathbf{b}^H \mathbf{a}\) represents the complex inner product of vectors \(\mathbf{a}\) and \(\mathbf{b}\). The superscripts \((\cdot)^{tr}\), \((\cdot)^{eq}\), and \((\cdot)^{in}\) denote variables calculated using time-reversal, equalized time-reversal, and interference-nulling time-reversal pre-filters, respectively.
3.2 Time-Reversal Beamforming System Model

In this section, we present the general discrete signal model for TR beamforming. We first generalize to the multi-user case two TR techniques for single-user scenarios [89]. These techniques serve as a baseline comparison for the novel INTR introduced in Section 3.3.

3.2.1 General TR Signal Model

Consider a digital baseband downlink wireless communication system, consisting of one Access Point (AP) with $M$ transmit antennas and $N$ single-antenna user terminals as depicted in Fig. 3.1. The transmitter has a very large number of antennas, so $M \gg N$. We denote the transmit antenna set as $\mathcal{M} = \{1, 2, \ldots, M\}$ and the user set as $\mathcal{N} = \{1, 2, \ldots, N\}$. Also, let $m, m' \in \mathcal{M}$ and $n, n' \in \mathcal{N}$ be arbitrary elements in those sets. The AP transmits simultaneously an independent data stream to each user. Let $s_n(t)$ be the complex random signal transmitted to the $n$-th user, where $t \in \mathbb{Z}_+$ is the discrete time index. These transmitted signals are assumed to have unit average power, i.e. $\mathbb{E}[|s_n(t)|^2] = 1$, $\forall n, t$, regardless of the modulation. In a TR multi-user system, the transmitter sends independent signals simultaneously to the users using different pre-filters for each one of them. Thus, the baseband transmitted signal from the $m$-th antenna is

$$x_m(t) = \sqrt{\rho} \sum_{n=1}^{N} s_n(t) \otimes p_{m,n}^*(t),$$

(3.1)

where $\rho$ is the total average transmitted power in the AP, $p_{m,n}(t)$ is the power-normalized pre-filter from the $m$-th transmit antenna to the $n$-th user (with a duration of $L_p$ samples, i.e. $t = 0, \ldots, L_p - 1$), and $h_{m,n}(t)$ is the random channel impulse.
Figure 3.1: System model. An AP with $M$ transmit antennas sends simultaneously an independent data stream to $N$ single antenna users using time-reversed pre-filters $p_{m,n}^*(-t)$.

response (CIR) from the $m$-th transmit antenna to the $n$-th user (with a length of $L$ samples). The random CIR vector to the $n$-th user is defined as

$$h_n(t) = [h_{1,n}(t), \ldots, h_{M,n}(t)]^T \in \mathbb{C}^M. \quad (3.2)$$

In Section 3.4, we introduce the statistical characterization of $h_{m,n}(t)$ for mm-wave channels. Let $H_{m,n}(f)$ be the discrete Fourier transform (DFT) of $h_{m,n}(t)$. In an analogous way to the time domain representation, the steering vector to the $n$-th user is

$$H_n(f) = [H_{1,n}(f), \ldots, H_{M,n}(f)]^T \in \mathbb{C}^M. \quad (3.3)$$

The selection of $p_{m,n}(t)$ depends on the particular TR technique, as discussed later.
in this section. We define the pre-filter vector to the $n$-th user as
\[
p_n(t) = [p_{1,n}(t), \ldots, p_{M,n}(t)]^T \in \mathbb{C}^M. \tag{3.4}
\]
Let $P_{m,n}(f)$ be the DFT of $p_{m,n}(t)$. Then, we define the frequency domain pre-filter vector to the $n$-th user as
\[
P_n(f) = [P_{1,n}(f), \ldots, P_{M,n}(f)]^T \in \mathbb{C}^M. \tag{3.5}
\]
The received baseband signal at user $n$ is
\[
y_n(t) = \sqrt{\rho} s_n(t) \otimes \sum_{m=1}^{M} p_{m,n}^*(t) \otimes h_{m,n}(t)
\]
\[+ \sqrt{\rho} \sum_{n' = 1}^{N} \sum_{m = 1}^{M} s_{n'}(t) \otimes p_{m,n'}^*(t) \otimes h_{m,n}(t)
\]
\[+ z_n(t), \quad \text{(3.6)}
\]
where $z_n(t)$ represents additive white Gaussian noise (AWGN) with variance $\sigma_z^2$. Next, we extend the conventional TR and equalized TR single-user formulation in [89] to multi-user scenarios by explicitly defining the pre-filter $p_{m,n}(t)$ in terms of the CIR. Note that $p_{m,n}(t)$ is properly normalized so the transmitted power $\rho$ regardless of the number of antennas or users.

### 3.2.2 Multiuser Conventional TR Beamforming

The general idea behind TR is to use the time-reversed CIR from every antenna to the receiver as a pre-filter for the transmitted signal. Such pre-filter acts as a beamformer in the spatial domain, focusing the RF signal around the receiver. In conventional TR, assuming perfect channel state information (CSI) at the transmitter,
the pre-filter vector is
\[ p^{tr}_n(t) = \frac{h_n(L - 1 + t)}{\sqrt{P^{tr}_h}}, \]  
(3.7)
where \( P^{tr}_h \) is a normalization factor introduced to ensure that the total transmitted power remains constant in every realization, this is
\[ P^{tr}_h = \sum_{n=1}^{N} \sum_{l=0}^{L-1} \|h_n(l)\|^2. \]  
(3.8)
Note that, in this case, the pre-filter’s length is equal to the CIR length, i.e. \( L^{tr}_p = L \).
Replacing the conventional TR pre-filter into (3.6) and using the definitions in Section 3.2.1, the time domain received signal in conventional TR is
\[
y^{tr}_n(t) = \sqrt{\frac{\rho}{P_h}} \sum_{l=0}^{L-1} \|h_n(l)\|^2 s_n(t - L + 1)
+ \sqrt{\frac{\rho}{P_h}} \sum_{l \neq L-1} \sum_{m=1}^{M} \sum_{l'=0}^{L-1} h_{m,n}(l') h^*_{m,n}(L - 1 - l + l') s_n(t - l)
+ \sqrt{\frac{\rho}{P_h}} \sum_{n' \neq n} \sum_{m=1}^{M} h^*_{m,n'}(L - 1 - t) \otimes h_{m,n}(t) \otimes s'_{n'}(t)
+ z_n(t), \]  
(3.9)
This received signal is composed of four terms: i) the desired symbol multiplied by a real factor resulting from coherent combination of multipath components in the CIR, ii) ISI caused by incoherent addition of CIR components, iii) IUI caused by the signals directed to other users (whose TR pre-filters do not match the CIR to the \( n \)-th user), and iv) AWGN. Thus, in a conventional multiuser TR beamforming system, ISI and IUI are important problems that hamper detection. In the single-user
scenario, ETR was proposed before as a solution to mitigate the ISI component in the received signal [89]. We extend ETR to the multiuser case next.

3.2.3 Multi-user Equalized TR Beamforming

ETR uses the TR pre-filter in cascade with a ZF pre-equalizer in order to mitigate the ISI of conventional TR. In [89], it is demonstrated that ETR outperforms conventional TR with respect to BER in a single-user scenario, with a marginal loss in the spatial focusing capability. We now extend this technique to the multi-user scenario by defining the pre-filter vector components for the \( n \)-th user as

\[
p_{m,n}^{eq}(t) = \frac{h_{m,n}(L - 1 + t) \otimes g_{n}^*(t)}{\sqrt{P_{h}^{eq}}},
\]

where \( g_{n}(t) \) represents a ZF linear equalizer with length \( L_E \). Thus, we have \( L_p^{eq} = L + L_E - 1 \). The normalization factor is

\[
P_{h}^{eq} = \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{l=0}^{L + L_E - 2} |h_{m,n}^*(L - 1 - l) \otimes g_{n}(l)|^2.
\]

One equalizer is required for each user, with the \( n \)-th equalizer designed to satisfy

\[
g_{n}(t) \otimes \sum_{m=1}^{M} h_{m,n}^*(L - 1 - t) \otimes h_{m,n}(t) = \delta(t - t_0),
\]

where \( t_0 \) is an arbitrary delay. Equation (3.12) can be written as an over-determined system of linear equations on \( g_{n}(t), t = 0, \ldots, L_E - 1 \). Thus, perfect ZF equalization is not possible with a finite equalizer’s length [62], but a good approximation can be achieved with a sufficiently large \( L_E \), eliminating the second term in (2.2). A detailed discussion on this subject is provided in [89]. Using the ETR pilot, and assuming perfect equalization, the time domain received signal at user \( n \) is
\[ y_{eq}^n(t) = \sqrt{\frac{\rho}{P_h}} s_n(t - t_0) \]

signal directed to the \( n \)-th user

\[ + \sqrt{\frac{\rho}{P_h} P_{eq}} \sum_{n' = 1}^{N} \sum_{m=1}^{M} s_{n'}(t) \otimes g_{n'}(t) \otimes h_{m,n'}^*(L - 1 - t) \otimes h_{m,n}(t) \]

\[ + z_n(t), \quad \text{noise} \] (3.13)

which has no ISI term, but still contains IUI. Thus, both conventional TR and ETR performance is limited by ISI and/or IUI, as detailed next.

### 3.2.4 Performance Analysis of TR and ETR

We now turn our attention to the power components in (2.2) and (3.13), following the same procedure as in [89]. The fundamental assumptions are that the system operates under uncorrelated scattering with uncorrelated channels between users, and normalized channel power, as stated in Section 3.4. In the derivations below, we also employ the approximation \( \mathbb{E}[a/b] \approx \mathbb{E}[a]/\mathbb{E}[b] \) for two random variables \( a \) and \( b \), as analyzed in [25, 89] for TR systems. Complete derivations are not shown due to space constraints. Here, we verify that TR is a suitable technique for massive MIMO systems, where \( M \gg 1 \). Let \( P_{tr}^{isi} \), \( P_{tr}^{iui} \), and \( P_{tr}^{tr} \) represent the power in the first, second, and third terms in (2.2), respectively. Then, the average desired signal power is

\[
\mathbb{E} [P_{tr}^{tr}] = \mathbb{E} \left[ \frac{\rho}{P_h} \sum_{l=0}^{L-1} \| h_n(l) \|^2 s_n(t - L + 1) \right]^{2} \approx \frac{M \rho \Gamma}{N}. \quad (3.14)
\]
The average ISI power in (2.2) can be approximated as

\[
\mathbb{E} \left[ P_{\text{isi}}^{\text{tr}} \right] = \mathbb{E} \left[ \frac{\rho}{P_{\text{tr}}} \left| \sum_{l=0}^{2L-2} \sum_{m=1}^{M} \sum_{m' = 1}^{M} \sum_{l' = 0}^{L-1} h_{m,n}(l')h_{m',n}(L - 1 - l + l')s_n(t - l) \right|^2 \right]
\]

\[
\approx \frac{\rho}{MNT} \sum_{l=0}^{2L-2} \sum_{l'' = 0}^{L-1} \sum_{m=1}^{M} \sum_{m' = 1}^{M} \sum_{l' = 0}^{L-1} \mathbb{E} \left[ h_{m,n}(l')h_{m',n}(l') \right] \times \mathbb{E} \left[ h_{m,n}^*(L - 1 - l + l')h_{m',n}(L - 1 - l + l') \right].
\] (3.15)

An approximation to the average IUI power is

\[
\mathbb{E} \left[ P_{\text{iui}}^{\text{tr}} \right] = \mathbb{E} \left[ \frac{\rho}{P_{\text{tr}}} \left| \sum_{l=0}^{2L-2} \sum_{m=1}^{M} \sum_{n' = 1}^{N} \sum_{n'' = 1}^{N} \sum_{m' = 1}^{M} \sum_{n'' = 1}^{N} \mathbb{E} \left[ h_{m,n}(l')h_{m',n'}(l') \right] \times \mathbb{E} \left[ h_{m,n'}^*(L - 1 - l + l')h_{m',n}(L - 1 - l + l') \right] \right|^2 \right]
\]

\[
\approx \frac{\rho}{MNT} \sum_{l=0}^{2L-2} \sum_{l'' = 0}^{L-1} \sum_{m=1}^{M} \sum_{n' = 1}^{N} \sum_{n'' = 1}^{N} \mathbb{E} \left[ h_{m,n}(l')h_{m',n'}(l') \right] \times \mathbb{E} \left[ h_{m,n'}^*(L - 1 - l + l')h_{m',n}(L - 1 - l + l') \right].
\] (3.16)

For ETR, the average desired signal power is bounded by

\[
\mathbb{E} \left[ P_{s}^{\text{eq}} \right] = \mathbb{E} \left[ \frac{\rho}{P_{h}^{\text{eq}}} \right] \leq \frac{M\rho\Gamma}{N}.
\] (3.17)

The average IUI power in ETR has a similar form to (3.16), but it is not shown here since it is not the focus of this work. However, we analyze it numerically in Section 3.5. From (3.14)-(3.16), we can make the following remarks with respect to TR beamforming in massive MIMO systems:

- Both ISI and IUI powers are highly dependent on the propagation conditions. More specifically, power delay profiles and correlation between antennas are present in the terms of the form \( \mathbb{E}[h_{m,n}(l)h_{m',n}^*(l)] \). Thus, increasing spatial correlation would increase both ISI and IUI, degrading performance.
• In the case of uncorrelated antennas, \( \mathbb{E}[h_{m,n}(l)h_{m',n'}^*(l)] = 0 \) if \( m \neq m' \). Hence, the sums would only depend on the power delay profile (which is the same for all antennas), and both ISI and IUI powers would be independent of \( M \).

• Desired signal power increases linearly with \( M \). Thus, in uncorrelated channels \( \mathbb{E}[P_{tr}^{sp}/P_{tr}^{isi}] \to \infty \) and \( \mathbb{E}[P_{tr}^{sp}/P_{tr}^{iui}] \to \infty \) as \( M \to \infty \). This implies that, with a sufficiently large number of antennas, a conventional TR beamforming system is noise limited instead of interference limited.

• However, if channels are spatially correlated (as in realistic scenarios), equalization and interference mitigation provided by TR reduce.

• Note that, given that CIR statistics for users \( n \) and \( n' \) are the same, i.e. \( \mathbb{E}[h_{m,n}^*(l)h_{m,n'}(l)] = \mathbb{E}[h_{m,n'}^*(l)h_{m',n}(l)] \) \( \forall l \), then \( P_{iui}^{tr} \) is larger than \( P_{isi}^{tr} \) by a factor on the order of the number of users. Thus, IUI mitigation should be given priority over equalization when proposing improvements over conventional TR.

Given this characteristics of TR beamforming in massive MIMO, we now propose a novel TR extension to overcome the problems of IUI, even under highly correlated channels.

### 3.3 Interference-Nulling Time-Reversal Beamforming

We are now concerned with the design of pre-filter vectors that combine the spatial focusing properties of conventional TR, while also providing additional IUI mitigation. We start from the frequency representation of the received signal, and formulate
an optimization problem for the design of the pre-filters. The frequency domain equivalent of (3.6) is

\[ Y_n(f) = \sqrt{\rho} \langle H_n(f), P_n(f) \rangle S_n(f) + \sqrt{\rho} \sum_{n' \neq n}^{N} \langle H_n(f), P_{n'}(f) \rangle S_{n'}(f) + Z_n(f), \quad (3.18) \]

where \( S_n(f) \) is the DFT of \( s_n(t) \), and \( Z_n(f) \) is the DFT of \( z_n(t) \). Appropriate zero padding is used in the time domain in order to represent linear convolution as a product in the frequency domain. The complex inner product defined above allows a convenient simplification in (3.18) with respect to (3.6), which is useful for the problem formulation. Let \( H(f) = [H_1(f) \ldots H_N(f)] \in \mathbb{C}^{M \times N} \) be the matrix with columns given by the steering vectors to all users. Also, let \( H_{-n}(f) \in \mathbb{C}^{M \times N-1} \) be the matrix formed by removing the \( n \)-th column from \( H(f) \), i.e. removing the steering vector to user \( n \). For notational simplicity, we drop the frequency dependence in the remainder of this section. Note that the IUI power in (3.18) is proportional to \( \sum_{n' \neq n} |\langle H_n, P_{n'} \rangle|^2 \). Thus, our objective is to find the pre-filter \( P_{*,n} \) which is closest to the conventional TR solution in the frequency domain (providing partial equalization of the received signal), and such that the IUI is set to zero. Formally, this optimization problem can be formulated as

\[
P_{*,n} = \arg \min_{P_n} \left\| P_{tr} - P_n \right\|^2
\]

subject to \( H_{-n}^H P_n = 0 \), \( \forall n \in \mathcal{N}, \forall f \in [0, \ldots, L + L_p - 1] \),

\[
(3.19)
\]

whose solution is

\[
P_{*,n} = \left( I_M - H_{-n} \left( H_{-n}^H H_{-n} \right)^{-1} H_{-n}^H \right) P_{tr}, \quad \forall n, f.
\]

\[
(3.20)
\]
where $I_M$ is the $M \times M$ identity matrix. Thus, we call $P_{*,n}$ the interference-nulling time-reversal (INTR) pre-filter in the frequency domain. Geometrically, the constraint in the problem ensures that the vector $P_{*,n} \in \text{null}\{H_n^H\}$, and the solution is the projection of $P_{tr}^n$ into that null space. This is illustrated in Fig. 3.2.

### 3.4 Channel Model for 60 GHz Massive MIMO

As mentioned above, TR actually benefits from rich scattering, so it can be conveniently applied for indoor wireless communications. In this section, we briefly describe the IEEE 802.11ad model for 60 GHz SISO systems in such scenarios [85], and extend it to the correlated multi-user massive MIMO case. In the following, we use a statistical description of $h_{m,n}(t)$, given by its probability distribution, power delay profile (PDP), and spatial correlation in the context of massive MIMO systems.
3.4.1 Channel Tap Distribution

The most popular channel model for mm-wave propagation is the IEEE 802.11ad. This is a SISO double directional statistical channel model based on a limited set of measurements, complemented with ray-tracing simulations. This model is defined for three indoor scenarios: conference room (CR), living room (LR), and cubicle environment (CB). Some important model features include: support of two types of antennas (isotropic and basic steerable antenna array), support of polarization, wideband and pathloss modeling under LoS and NLoS situations.

The IEEE 802.11ad channel model follows a scattering cluster structure, both in time and angular domains. Thus, several multipath components (MPCs) observed in the CIR have similar propagation delays and angles of departure/arrival. More specifically, each central ray arriving at the receiver has pre-cursor rays (which arrive earlier) and post-cursor rays (which arrive later). This is due to irregular scattering objects and geometrical features which are large compared to the wavelength. Both pre-cursor and post-cursor rays have less amplitude than the central ray. The resolvability of those MPCs depend exclusively on the system’s sampling time (bandwidth). When those MPCs are not resolvable, they contribute to the same tap in the CIR. Given this propagation characteristics, we assume that \( h_{m,n}(t) \) has zero mean and that \( |h_{m,n}(t)| \) is Nakagami distributed, with parameters \( m \) and \( \Omega \), for all \( m, n \) and \( t \) [90]. Recall that the \( m \) parameter in the Nakagami distribution is analogous to the \( K \) factor in the Rician distribution, and that a larger \( m \) implies a large power ratio between the central ray (specular component) and the other rays (diffuse components). The parameter \( \Omega \) depends also on the amplitudes of the specular and diffuse components, and on the channel PDP (tap average power) [90]. Table 3.1 shows the
Table 3.1: Nakagami $m$ parameter and RMS delay spread of IEEE 802.11ad scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Nakagami $m$ parameter</th>
<th>RMS delay spread [ns]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CB</td>
<td>4.34</td>
<td>3.47</td>
</tr>
<tr>
<td>CR</td>
<td>2.56</td>
<td>4.82</td>
</tr>
<tr>
<td>LR</td>
<td>1.74</td>
<td>7.81</td>
</tr>
</tbody>
</table>

values of $m$ and RMS delay spread in the IEEE 802.11ad scenarios. The larger value of $m$ in the CB scenario is due to the reduced scattering within the cubicles, which reduces the number and power of diffuse components contributing to each channel tap.

3.4.2 Power Delay Profile

We are particularly interested in the PDP, a second order statistic defined as

$$A_h(t) = \mathbb{E} \left[ |h_{m,n}(t)|^2 \right], \quad \forall m, n,$$

where the expectation is calculated over CIRs that are subject to the same large-scale fading [91]. Signal power components depend on the PDP and spatial correlation, as seen in Section 3.2.4. We assume that all CIR in the system have the same PDP. This is valid for mm-wave indoor environments, where APs are usually positioned on or close to the ceiling and similar shadowing affects all elements in the transmit array.

We also define the following constraint on the CIR total power:

$$\sum_{t=0}^{L-1} \mathbb{E} \left[ |h_{m,n}(t)|^2 \right] = \sum_{t=0}^{L-1} A_h(t) = \Gamma,$$

where $\Gamma \ll 1$ is a constant accounting for channel induced propagation losses. This constraint implies that all channels between the transmit antennas and each receiver have same average power. Fig. 3.3 shows PDPs obtained over $10^6$ realizations of
the IEEE 802.11ad model for the three scenarios simulated under NLoS and isotropic antennas. We do not consider LoS situations since they correspond to flat-fading channels, which are of no interest here. Isotropic antennas are assumed so the system can take advantage of all MPCs in the channel. In practice, planar omni-directional antennas (e.g. [92]) would be a good alternative for implementation. We observe that RMS delay spread is minimum for the CB scenario, where an AP is located in the ceiling of an office populated with cubicles. In that case, scattering is confined within the cubicle’s structure and other delayed paths (e.g. reflections from outer walls) are obstructed. On the other hand, CR and LR scenarios correspond to more open spaces, where first and second order reflections from walls are considered. Those reflections cause long tails in their PDPs, increasing their delay spread.

3.4.3 Spatial Correlation Model

Consider $m, m' \in M, n, n' \in N$, and $t, t' \in \{0, \ldots, L - 1\}$. We make the following assumptions with respect to CIRs in the systems:

- CIR are correlated across transmit antennas, i.e. the spatial channel autocorrelation function is $R_h(\Delta d) \neq 0$, where $\Delta d$ is the distance between two measured CIRs. The specific correlation structure depends on the array configuration, but it is assumed that the process in wide sense stationary with respect to the space. This implies that $\mathbb{E}[h_{m,n}(t)h_{m',n}^*(t)] \neq 0$.

- Different users have uncorrelated CIRs to the AP, i.e. $h_{m,n}(t)$ and $h_{m,n'}(t)$ are uncorrelated if $n \neq n', \forall m, t$. This is due to the fact that MPCs are independent for different users. This can be clearly seen in the CB environment, where each user is assumed to be in its own cubicle.
• CIR taps are uncorrelated, i.e. \( h_{m,n}(t) \) and \( h_{m,n}(t') \) are uncorrelated if \( t \neq t' \), \( \forall m, n \). This is the conventional uncorrelated scattering (US) assumption widely used in the literature [60], and implies that contributions to different taps come from different scatterers.

Nakagami correlated variables (across antennas) are generated according to the method described in [90], as follows. Consider the setting in Fig. 3.4. A planar randomly-oriented array with \( M \) isotropic elements is located in the environment according to the standard [85]. An isotropic receiving antenna is randomly located in the environment as well. Each tap is assumed to have specular and diffuse contributions from an irregular scatterer (located according to the corresponding delay), whose amplitudes depend on the PDP and the desired \( m \) parameter. All contributions to a fixed tap in a given CIR come from the same scatterer, with different taps corresponding to different scatterers. Using this procedure, the resulting normalized spatial correlation function \( R_h(\Delta d) \) is shown in Fig. 3.5. These results are consistent with measured and simulated spatial correlations in 60 GHz channels, e.g. [86]. High correlation values are caused by the reduced number of dominant MPC contributing to each tap. The specific correlation between transmit antenna elements depends only on the geometry of the array. For the numerical validation shown in Section 3.5, we use rectangular arrays with 32 (8 \( \times \) 4), 64 (8 \( \times \) 8), or 128 (16 \( \times \) 8) elements with a uniform separation of 20 mm.

3.5 Numerical Results and Discussion

In this section, we present numerical results for the performance analysis of the multiuser TR, ETR and INTR techniques, as described in Sections 2.2 and 3.3.
Figure 3.3: Power delay profile of IEEE 802.11ad channel model scenarios with isotropic antennas. RMS delay spreads are 3.47 ns for the CB scenario, 4.82 ns for CR and 7.81 ns for LR.

Figure 3.4: Method to generate correlated Nakagami CIR. Different taps are assumed to have contributions from specular and diffuse reflections from different objects. The transmit array is planar (rectangular) with uniformly distributed elements.

### 3.5.1 Pilot Length and Channel Correlation

First, we analyze the impact of pre-filter’s length $L_p$ and spatial correlation on the signal power components. We calculate the values of $P_s$, $P_{isi}$, and $P_{iui}$ for the
three techniques over 1000 channel realizations with and without spatial correlation. Results are shown in Table 3.2 for 2 and 10 users in the CB scenario with 64 antennas. For conventional TR beamforming, it is clear that IUI power is the main problem for multi-user communications, as it can be up to an order of magnitude greater than ISI power. It is also observed that channel correlation decreases desired signal power and increases interference, affecting the overall system performance. Also, note that when using spatially uncorrelated channels, both ISI and IUI suffer small or no change when increasing the number of antennas. However, when the CIRs are correlated both types of interference suffer a small increase. For ETR, increasing pre-filter’s length improves ISI suppression, but IUI remains the same as in conventional TR. ISI reduction in ETR by increasing $L_p$ is a typical consequence of zero-forcing equalization [89]. However, ETR is not designed to mitigate IUI. Thus, BER performance of TR and ETR are expected to be very similar since the scenarios we consider are clearly IUI limited.
For INTR, IUI mitigation improves by increasing $L_p$. This is due to the discarding of $L - 1$ time samples when performing the transformation between the frequency domain prefilter (of length $L + L_p - 1$) and the time domain prefilter (of length $L_p$). Such discarding is necessary due to the circular convolution theorem. Thus, the time domain prefilter is a least squares projection of the optimum frequency domain solution. The error in the projection reduces as $L_p$ increases.

We observe the impact of signal power components over the BER performance in Fig. 3.6, where the influence of channel spatial correlation is also shown. Signal to noise ratio is defined as $\text{SNR} = \rho \Gamma / \sigma_z^2$, where $\sigma_z^2$ is the variance of $z_n(t) \forall n, t$. These results were obtained for 5 users and 32 antennas in the CB scenario, with a transmission of $10^6$ BPSK symbols over 1000 channel realizations. Performance of both TR and ETR is limited by IUI, which causes a lower bound in the BER. We notice that ETR does not provide a significant improvement over conventional TR in the case of multi-user massive MIMO systems. Thus, ETR does not offer any advantage for such scenarios, given its greater computational complexity with respect to TR. In the case of INTR, IUI is successfully mitigated and hence INTR outperforms the other techniques. We also observe that channel correlation degrades system performance in all cases.

3.5.2 Number of Antennas and Number of Users

Fig. 3.8 shows the average BER performance results per user for varying number of antennas and users. These results where obtained with the transmission of $10^6$ BPSK symbols over 1000 spatially correlated channel realizations. We used a fixed pre-filter length $L_p = 90$ and the CB scenario PDP. Conventional TR performance results are
Figure 3.6: Prefilter length ($L_p$) vs (a) ISI power in ETR, and (b) IUI power in INTR. These results were obtained with $L = 60$ and $N = 10$ in the CB scenario. Other signal components in each technique remained approximately constant vs. $L_p$. It is noted that increasing $L_p$ reduces ISI power in ETR and IUI in INTR. This is due to the discarding of $L - 1$ time samples when performing the transformation between the frequency domain prefilter (of length $L + L_p - 1$) and the time domain prefilter (of length $L_p$). Such discarding is necessary due to the circular convolution theorem. Thus, the time domain prefilter is a least squares projection of the optimum frequency domain solution. The error in the projection reduces as $L_p$ increases.

consistent with the analysis made in Section 3.2.4. The desired signal power increases linearly with the number of antennas while interference components remain constant. Thus, the minimum achievable BER per user improves by increasing the number of antennas, providing diversity gain. On the other hand, increasing the number of users with a fixed number of antennas decreases the desired signal power and increases IUI. This is reflected in a higher BER for larger $N$. INTR outperforms conventional TR in every simulated scenario. Nevertheless, the performance improvement provided by INTR is more evident with a large number of users or a limited number of antennas.
Table 3.2: Received signal power components for $M = 64$. Values are normalized $/(\rho \Gamma)$.

<table>
<thead>
<tr>
<th>N. of users $N$</th>
<th>Technique</th>
<th>$L_p = L = 60$</th>
<th>$L_p = 90$</th>
<th>$L_p = 120$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$P_s$ $P_{isi}$ $P_{iui}$</td>
<td>$P_s$ $P_{isi}$ $P_{iui}$</td>
<td>$P_s$ $P_{isi}$ $P_{iui}$</td>
</tr>
<tr>
<td>Uncorrelated channel</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>TR</td>
<td>32.00 0.15 0.51</td>
<td>- - -</td>
<td>- - -</td>
</tr>
<tr>
<td></td>
<td>ETR</td>
<td>- - -</td>
<td>31.9 0.01 0.52</td>
<td>31.9 0.001 0.52</td>
</tr>
<tr>
<td></td>
<td>INTR</td>
<td>31.7 0.15 0.09</td>
<td>31.6 0.16 0.01</td>
<td>31.6 0.15 0.002</td>
</tr>
<tr>
<td>10</td>
<td>TR</td>
<td>6.40 0.03 0.90</td>
<td>- - -</td>
<td>- - -</td>
</tr>
<tr>
<td></td>
<td>ETR</td>
<td>- - -</td>
<td>6.37 0.002 0.90</td>
<td>6.37 0.0003 0.90</td>
</tr>
<tr>
<td></td>
<td>INTR</td>
<td>5.77 0.04 0.17</td>
<td>5.58 0.04 0.02</td>
<td>5.55 0.04 0.004</td>
</tr>
<tr>
<td>Correlated channel</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>TR</td>
<td>31.4 0.6 0.88</td>
<td>- - -</td>
<td>- - -</td>
</tr>
<tr>
<td></td>
<td>ETR</td>
<td>- - -</td>
<td>30.3 0.02 0.88</td>
<td>30.3 0.003 0.88</td>
</tr>
<tr>
<td></td>
<td>INTR</td>
<td>30.8 0.61 0.15</td>
<td>30.6 0.6 0.02</td>
<td>30.6 0.6 0.003</td>
</tr>
<tr>
<td>10</td>
<td>TR</td>
<td>6.39 0.15 1.48</td>
<td>- - -</td>
<td>- - -</td>
</tr>
<tr>
<td></td>
<td>ETR</td>
<td>- - -</td>
<td>6.07 0.004 1.47</td>
<td>6.07 0.0005 1.47</td>
</tr>
<tr>
<td></td>
<td>INTR</td>
<td>5.32 0.14 0.25</td>
<td>5.06 0.14 0.04</td>
<td>5.02 0.14 0.006</td>
</tr>
</tbody>
</table>

3.5.3 Average Achievable Sum Rate

The achievable sum rate measures the downlink spectral efficiency in a multiple-access system. Assuming that each user treats ISI and IUI as Gaussian interferences, and according to the model defined in Sections 2.2 and 3.3, the average achievable sum rate for a multi-user TR system is

$$R = \mathbb{E} \left[ \sum_{n=1}^{N} \log_2 \left( 1 + \frac{P_{s,n}}{P_{isi,n} + P_{iui,n} + \sigma_z^2} \right) \right],$$

(3.23)

where $P_{s,n}$, $P_{isi,n}$, and $P_{iui,n}$ are the desired signal power, ISI power, and IUI power, respectively, calculated at user $n$ for a given realization. For simplicity, it is also assumed that the channel is used for downlink transmission all the time. A proper reduction factor can be used to account for uplink time in a TDD system or channel estimation overheads. Numerical results for the average achievable sum rate are shown in Fig. 3.9. These results were obtained in the LR scenario with correlated channels.
Figure 3.7: Average BER per user comparison of TR, ETR, and INTR, under correlated and uncorrelated channels (across antenna elements) with $M = 32$, and $N = 5$. Results are shown for different pilot lengths ($L_p$). It is observed that spatial correlation increases ISI and IUI, degrading performance. Also, increasing prefilter’s length improves IUI mitigation in INTR.

and $L_p = 90$. As seen, INTR offers a significant improvement over conventional TR, doubling its rate in some cases and providing a remarkable multiplexing gain. In addition, we simulated a more extreme case with $N = 30$, and $N = 50$ and 128 antennas, with the purpose of further demonstrate the capabilities of TR to handle IUI. Results are shown Fig. 3.9c. Even though the assumption of uncorrelated CIR between users is hardly met when $N$ is that large, results show that an outstanding efficiency of more than 170 bps/Hz can be achieved with INTR. In all the simulated scenarios our proposed INTR technique outperforms conventional TR, as it can better withstand an increase in user load.
Figure 3.8: Average BER per user for TR and INTR. (a) Different number of antennas $M$ with $L_p = 90$ and $N = 5$. (b) Different number of users $N$ with $L_p = 90$ and $M = 64$. An important diversity gain is achieved even in spatially correlated channels. The effect of IUI is mitigated by increasing the number of antennas.

Figure 3.9: Achievable rate of TR and INTR in the LR scenario. (a) $M = 32$ antennas, (b) $M = 128$ antennas, (c) $M = 128$ antennas with an extreme number of users. The multiplexing gain increases with the number of antennas.

3.6 Conclusion

We have analyzed a baseband TR beamforming system for mm-wave multi-user massive MIMO. We studied conventional TR and equalized TR and found that their
performance is IUI limited. We also noticed that, when the number of antennas is large, the ratio between the desired signal power and ISI or IUI power increases. Thus, we confirm the potential of TR as a beamforming technology for massive MIMO. We also note that equalizing solutions such as ETR are not necessary when the number of transmit antennas is large. After identifying IUI as the main detection impairment for TR systems, we propose a modified technique called INTR. This technique calculates the transmit pre-filters in the frequency domain that set the IUI to zero and are closest to the original TR solution. We proposed a 60 GHz MIMO channel model, where CIR taps are modeled with Nakagami distributed amplitudes. In addition, we use PDPs given by the IEEE 802.11ad SISO NLoS model, and generate spatial correlation in the CIRs according to a geometrical model. By means of numerical simulations, we verified that the proposed INTR outperforms conventional TR with respect to average BER and achievable sum rate. In particular, we note that INTR performance is extremely tolerant to increases in the number of users, and provides both diversity and multiplexing gains simultaneously.
Chapter 4: Space-Time Block Diagonalization for Frequency-Selective MIMO Broadcast Channels

4.1 Introduction

Wireless multiuser-MIMO (MU-MIMO) systems, which are composed of one multiple-antenna base station and a set of user terminals sharing time and frequency resources, have been intensively studied in the past decade. These systems are modeled as MIMO broadcast channels in the downlink, where each user receives a linear combination of the signals directed to all the users. Thus, the main characteristic of these systems is the presence of inter-user interference (IUI) and, as a result, processing techniques at the transmitter and/or receivers are required so that every user can detect the signal directed to it. A number of such methods exist, which operate on different principles depending on the channel being frequency-flat or frequency-selective.

Dirty paper coding (DPC), a nonlinear method, achieves the capacity in frequency-flat MIMO broadcast channels [93,94]. However, linear processing techniques are still of great interest since they offer reduced computational complexity compared to DPC [4,95,96]. In particular, block diagonalization (BD) [4] is of significant importance given that, under certain conditions, it achieves the DPC sum capacity [26].
frequency-flat channels, the channel matrix has only space information (the complex channel coefficients between each transmitter/receiver antenna pair). Hence, BD uses a linear precoder to force a block-diagonal structure in the precoder-channel matrix product, which sets the IUI to zero.

For frequency-selective MIMO broadcast channels, the capacity region is unknown in terms of the channel statistics, even in the SISO scenario [97]. In this case, the channel matrix incorporates space-time information since a channel impulse response (CIR) characterizes the propagation between each transmitter/receiver antenna pair. The CIR spreads the transmitted signal in the time-domain, causing inter-symbol interference (ISI) in the received signal. Thus, the transmitter and/or the receivers must use equalization in order to mitigate performance losses generated by ISI. For this reason, frequency-flat linear processing techniques are not easily extended to the frequency-selective case.

Time-reversal (TR) based pre-filters [6, 25, 98, 99] have been extensively used for frequency-selective MIMO broadcast channels, because they improve the system’s energy efficiency and reduce its computational complexity with respect to multicarrier (frequency-flat) systems [10]. TR uses the time-reversed complex-conjugated CIR as a linear pre-filter applied at the transmitter, and uses simple single-tap receivers. This pre-filter focuses the electric field around the receiving antennas [100] and also provides partial equalization due to its matched-filter properties, compressing the equivalent CIR in the time-domain [20]. However, TR performance is limited by both ISI and IUI [69], so the design of linear processing techniques in frequency-selective MIMO broadcast channels is still an open problem.
In this chapter, we generalize BD linear precoding to frequency-selective MIMO broadcast channels. Current literature on the subject focus on linear FIR pre-filters and provides no connection with linear techniques for frequency-flat channels. As a first contribution, we formulate the linear transceiver design problem for frequency-selective channels using general linear combiners instead of FIR filters, and representing the channels as Toeplitz matrices. This provides a bridge between the frequency-flat and frequency-selective cases. Then, we demonstrate that BD is possible in frequency-selective channels if the transmitted block length is sufficiently large and if the number of transmit antennas is greater than the number of users (or equal to, in some cases). We give specific conditions on the transmitted block length as a function of the channel delay spread, the number of user, and the number of transmit antennas. The processing in frequency-selective channels involves space-time information, and we show that any BD precoder in this case acts as a space-time block coder that eliminates IUI. In addition, we propose three approaches to mitigate or eliminate ISI in the received signal, which work in cascade configuration with the BD precoder. The first two approaches, time-reversal-based BD (TRBD) and equalized BD (EBD) use channel state information (CSI) at the transmitter only to design linear precoders and use low complexity sample-drop receivers. The third approach, joint processing BD (JPBD) uses CSI at the transmitter and the receivers to jointly calculate linear precoders and receiver combiners.

In the novel TRBD, IUI is eliminated and ISI is mitigated by using a linear combiner that approximates the TR pre-filter. The general linear combiner structure of TRBD allows the complete elimination of IUI, which is not possible using conventional finite impulse response (FIR) filters in TR techniques such as [54]. Moreover,
unlike similar TR formulations (e.g. [69]), TRBD reduces the precoding computational complexity, since it does not require frequency-domain operations.

The second approach, EBD, acts explicitly as a pre-equalizer [60] over each block-diagonalized channel, giving a minimum squared error solution for the precoder. In contrast with previous works (e.g., [17, 25, 52]), EBD completely eliminates IUI.

JPBD uses the singular value decomposition (SVD) of the block-diagonalized channel to eliminate ISI and provides perfect equalization in the received signal. We propose novel linear transmitter and receiver structures for joint interference suppression and channel equalization. As opposed to BD in frequency-flat channels, JPBD uses the singular values of the block-diagonalized channel to perform amplitude equalization across the received signal samples and not for power allocation. Thus, we also propose procedures for maximum sum-rate power allocation for the three techniques. For each of the proposed approaches, we theoretically analyze:

1. The optimization problems related to the precoder design, which have closed-form solutions in each case.

2. The ergodic achievable rate region.

3. The high SNR performance, evaluated in terms of the diversity and multiplexing gains.

4. The effective signal to interference plus noise ratio (SINR) for low SNR.

5. The power allocation scheme that maximizes the achievable sum-rate.

Extensive numerical simulations show that the achievable rate regions of the proposed techniques improve those of conventional TR beamforming. Moreover, we
demonstrate that any linear precoding technique (processing at the transmitter only, including TRBD and EBD) cannot eliminate ISI completely, implying zero diversity and multiplexing gains in the high SNR regime. JPBD achieves full multiplexing gain (equal to the number of users) in the limit when the transmitted block size goes to infinity, and its diversity gain improves with larger channel delay spreads or larger time-domain redundancy added at the transmitter. With these characteristics, JPBD provides the highest achievable rate region for frequency-selective MIMO broadcast channels. We also analyze the behavior of each design versus different system parameters (e.g. number of antennas, number of users, SNRs) and show good agreement between simulated and theoretical results.

4.2 System Model

Consider a MIMO baseband downlink wireless communication system consisting of one transmitter (base station or access point) equipped with $M$ transmit antennas and $K$ single-antenna users, as depicted in Fig. 4.1. The system operates over a MU-MIMO fading channel, where the transmitter sends a block of $B$ complex symbols to each user, followed by a guard interval of $L + L_p - 2$ symbols, where $L$ is the...
delay spread in the channel and \( L_p \) is the time-domain redundancy added by the precoder. This redundancy is used to reduce the probability of error (as in any error correcting code), but it also improves the performance when the number of users in the system increases (as discussed in Section 4.5). This guard interval is analogous to the cyclic prefix used in multi-carrier systems, and it is also required in finite impulse response (FIR) pre-filters, such as TR beamforming. The received signal at user \( k \) is represented as

\[
y_k = G_k \left[ H_k P_k s_k + \sum_{k'=1, k' \neq k}^{K} H_k P_{k'} s_{k'} + z_k \right], \tag{4.1}
\]

where \( G_k \) is the receiver filter, \( H_k \) is the channel matrix, \( P_k \) is the transmitter precoder, \( s_k \) is the transmitted signal, and \( z_k \) is Gaussian noise. In this section, we describe this system model in detail.\(^4\)

### 4.2.1 Transmitter

Let \( s_k = [s_k(1), \ldots, s_k(B)]^T \in \mathbb{C}^B \) denote the random vector of complex time-domain transmitted symbols, where \( s_k(t) \) is the symbol directed to user \( k \) at time \( t \) with average power \( \mathbb{E} \{ |s_k(t)|^2 \} = \rho_k, \forall t \). These time domain symbols are i.i.d. random variables selected from an arbitrary alphabet. As shown in Fig. 4.1, the precoding matrix \( P \) maps the stacked transmitted signal vector \( s = [s_1^T, \ldots, s_K^T]^T \in \mathbb{C}^{BK} \) to the transmit antennas. The total transmitted power constraint is \( \sum_k \rho_k = P_{\text{max}} \), and the precoding matrix \( P \in \mathbb{C}^{M(B+L_p-1) \times BK} \) is

\(^4\)We use the following notation. \((\cdot)^T\), \((\cdot)^*\), \((\cdot)^H\), \((\cdot)^{-1}\), \((\cdot)^+\), and \(\| \cdot \|_F\) represent transpose, complex conjugate, conjugate transpose, inverse, pseudoinverse, and Frobenius norm of a matrix, respectively. \([A]_{ij}\) is the element in the \( i \)-th row and \( j \)-th column of matrix \( A \). \(\|a\|_2 = \sqrt{\mathbf{a}^H \mathbf{a}}\) is the \(\ell_2\) norm of the vector \( a \). \(\mathbb{E} \{ \cdot \} \) denotes expected value. We use the definitions in [101] for complex matrix differentiation.
\[
P = \begin{bmatrix}
P_{1,1} & P_{1,2} & \cdots & P_{1,K} \\
P_{2,1} & P_{2,2} & \cdots & P_{2,K} \\
\vdots & \vdots & \ddots & \vdots \\
P_{M,1} & P_{M,2} & \cdots & P_{M,K}
\end{bmatrix}
\]

where \( P_{m,k} \in \mathbb{C}^{(B+L_p-1) \times B} \) is the linear combiner which maps the time-domain block \((B \text{ symbols})\) directed to user \( k \) to a time-domain block transmitted from antenna \( m \) \((B_t = B + L_p - 1 \text{ symbols})\). Thus, the precoders add \( L_p - 1 \) time-domain redundancy symbols. Note that, when the precoder is a FIR filter of length \( L_p \), \( P_{m,k} \) is a banded Toeplitz matrix representing the convolution between the filter and the transmitted block [62]. We define \( P_k = [P_{1,k}^T \cdots P_{M,k}^T]^T \in \mathbb{C}^{B_tM \times B} \) as the stacking of all the precoders directed to user \( k \), such that \( P = [P_1 \cdots P_K] \). We also set \( \|P_k\|_F^2 = 1, \forall k \), so the combiner does not alter the average power of \( s_k \). Given the previous definitions, \( P \) is a linear space-time block coder. The time-domain signal vector transmitted from antenna \( m \) is

\[
x_m = \sum_{k=1}^{K} P_{m,k} s_k \in \mathbb{C}^{B_t}.
\]

### 4.2.2 Channel

We focus on quasi-static channels, where the channel matrix remains invariant over a block of \( B + L + L_p - 2 \) time samples. The frequency-selective MIMO broadcast channel matrix \( H \in \mathbb{C}^{K(B_t+L-1) \times B_tM} \) is

\[
H = \begin{bmatrix}
H_{1,1} & H_{1,2} & \cdots & H_{1,M} \\
H_{2,1} & H_{2,2} & \cdots & H_{2,M} \\
\vdots & \vdots & \ddots & \vdots \\
H_{K,1} & H_{K,2} & \cdots & H_{K,M}
\end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_K \end{bmatrix},
\]
where $H_{k,m} \in \mathbb{C}^{(B_t + L - 1) \times B_t}$ is a banded Toeplitz convolution matrix with the CIR coefficients from transmit antenna $m$ to user $k$ given by

$$
H_{k,m} = \begin{bmatrix}
h_{k,m}(1) & 0 & \cdots & 0 \\
\vdots & h_{k,m}(1) & \ddots & \vdots \\
h_{k,m}(L) & \vdots & \ddots & 0 \\
0 & h_{k,m}(L) & \cdots & h_{k,m}(1) \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & h_{k,m}(L)
\end{bmatrix}
$$

That is, $H_{k,m}$ is constructed with the CIR vector $h_{k,m} = [h_{k,m}(1), \ldots, h_{k,m}(L)]^T \in \mathbb{C}^L$, where $L$ is the finite CIR duration. We also define the channel matrix to user $k$ as $H_k = [H_{k,1} \cdots H_{k,M}] \in \mathbb{C}^{B_r \times B_t \times M}$, i.e. the stacking of the channels matrices between all transmitter antennas and user $k$. Note that the received signal is spread in the time domain ($B_t$ transmitted symbols are spread across $B_r = B_t + L - 1$ received samples).

The CIR time samples $\{h_{k,m}(t)\}$ are zero-mean complex circularly-symmetric Gaussian random variables with diagonal covariance matrices $Q_h = E\{h_{k,m}h_{k,m}^H\} \in \mathbb{C}^{L \times L}$, $\forall k, m$. A common model for the diagonal elements of $Q_h$ (the channel power delay profile) is [63]

$$
[q_h]_l = \left(1 - e^{\frac{lt_s}{\sigma_h}}\right) e^{-\frac{(l-1)t_s}{\sigma_h}}, \quad (4.2)
$$

where $t_s$ is the sampling time, and $\sigma_h$ is the mean channel delay spread. The factor in parenthesis in (4.2) normalizes the channel power to satisfy the constraint $\text{Tr}(Q_h) = 1$. The diagonal structure of $Q_h$ ensures that the CIRs are uncorrelated across users, antennas, and time. This assumption is made in order to determine fundamental limits on the performance of frequency-selective MIMO broadcast channels, which are achieved under such uncorrelated scattering conditions.
4.2.3 Receivers

One of the main advantages of single-carrier frequency-selective techniques over their multi-carrier counterparts is the reduced complexity at the receiver. We consider simple linear receiver structures, where $G_k \in \mathbb{C}^{B \times B_r}$ represents a time-domain linear combiner at user $k$. In this work, we use two types of receivers. The first is a simple receiver that discards the first $\lceil (L + L_p - 2)/2 \rceil$ and the last $\lfloor (L + L_p - 2)/2 \rfloor$ time samples of the received block. This is the most common receiver in TR systems, since the discarded samples are ISI only [69]. This filter has the form

$$G_k = g_k \bar{G} = g_k [0 \ I_{B_r} \ 0],$$

where $g_k \in \mathbb{R}^+$ represents an arbitrary gain control, $\bar{G} \triangleq [0 \ I_{B_r} \ 0]$ is the sample drop matrix, and $0$ is a zero matrix. We describe the second linear receiver in Section 4.3.3, where we exploit channel knowledge to improve the system performance.

The last component in the receiver signal in (3.6) is $z_k \in \mathbb{C}^{B_r}$, which is the vector of time-domain noise samples. We assume $z_k$ is a complex circularly-symmetric Gaussian random vector with covariance matrix $\eta I_{B_r}$, $\forall k$, where $\eta$ is the average noise power per sample. According to (3.6), the desired symbol block $s_k$ is subject to a linear transformation induced by the matrix $G_k H_k P_k$, with its diagonal elements representing the desired signal, while the off-diagonal elements correspond to ISI. IUI is determined by the matrices $G_k H_k P_{k'}$ with $k' \neq k$. We define the desired signal, ISI, IUI, and noise power gains as
\[ \alpha_{D,k} = \| (G_k H_k P_k) \circ I_B \|_F^2, \]
\[ \alpha_{ISI,k} = \| G_k H_k P_k \|_F^2 - \alpha_{D,k}, \]
\[ \alpha_{IUI,k} = \sum_{k' \neq k} \rho_{k'} \| G_k H_k P_{k'} \|_F^2, \]
\[ \alpha_{N,k} = \| G_k \|_F^2, \]
respectively, where \( \circ \) denotes Hadamard product. Thus, the effective signal to interference plus noise ratio at receiver \( k \) is

\[ \text{SINR}_k = \frac{\rho_k \alpha_{D,k}}{\rho_k \alpha_{ISI,k} + \alpha_{IUI,k} + \eta \alpha_{N,k}}, \quad (4.3) \]

Note that, in frequency-selective MU-MIMO systems, both ISI and IUI are significant impairments for signal detection.

### 4.3 Block Diagonalization for Frequency-Selective Channels

BD was first proposed for frequency-flat MU-MIMO channels in [4]. The idea is to design a precoder such that the equivalent channel matrix \( HP \) has a block diagonal structure. Thus, BD sets the IUI at every receiver to zero and the received signal in (3.6) has only the first and third terms. This allows a per-user precoder design since (3.6) depends only on the user index \( k \). In the original formulation, BD is performed over a channel matrix with only spatial information between transmitter and receiver. However, the frequency-selective channel matrix comprises both space and time channel information. In this section, we analyze the particular structure of BD for frequency-selective channels, and propose three techniques to tackle its specific challenges. For the first two techniques, we assume a sample drop receiver \( G_k = \bar{G} \) and focus on the precoder design. We also assume perfect channel state information (CSI) at the transmitter. For the third technique we jointly design \( P_k \),
and $G_k$ assuming CIS is also available at the receiver. From the received signal (3.6), IUI is set to zero when $H_k P_{k'} = 0$, if $k \neq k'$. If we define the interference matrix for user $k$ as the stacking:

$$
\tilde{H}_k = \begin{bmatrix} H_1^T & \cdots & H_{k-1}^T & H_{k+1}^T & \cdots & H_K^T \end{bmatrix}^T,
$$

(4.4)

the condition for BD is $\tilde{H}_k P_k = 0$, $\forall k$, i.e. the columns of $P_k$ must lie in the null space of $\tilde{H}_k$. Thus, as the first step to design the precoder $P_k$, we perform the singular value decomposition (SVD) of $\tilde{H}_k$, in order to obtain a basis for null $\left( \tilde{H}_k \right)$. This SVD can be written as

$$
\tilde{H}_k = \tilde{U}_k \Sigma_k \begin{bmatrix} \tilde{V}_k^{(1)} & \tilde{V}_k^{(0)} \end{bmatrix}^H,
$$

where $\begin{bmatrix} \tilde{V}_k^{(1)} & \tilde{V}_k^{(0)} \end{bmatrix}^H$ is the matrix formed with the right singular vectors of $\tilde{H}_k \in \mathbb{C}^{B_r(K-1) \times B_t M}$. More specifically, the columns of $\tilde{V}_k^{(0)}$ form a basis for the null space of $\tilde{H}_k$. Note that this matrix defined in (4.4) is a column stacking of matrices taken from the set $\{H_k\}$, so it is almost surely full row rank. Thus, unlike BD in frequency-flat channels, the dimension of $\tilde{V}_k^{(0)}$ in frequency-selective channels is known to be $B_t M \times B_v$, where $B_v = B_t M - B_r (K-1)$, independent of the propagation conditions. Thus, we establish $B_v > 0$ as a condition for the existence of $\tilde{V}_k^{(0)}$ (a stronger condition is required as detailed next). A BD precoder for the frequency-selective channel $H_k$ must then have the form

$$
P_k = \tilde{V}_k^{(0)} \bar{P}_k,
$$

(4.5)

where $\bar{P}_k \in \mathbb{C}^{B_v \times B}$ maps the transmitted block to user $k$ to the domain of $\tilde{V}_k^{(0)}$. Consequently, the search space for a BD precoder increases by using a larger number of antennas $M$ or reducing number of users $K$. The linear transformation $G_k H_k \tilde{V}_k^{(0)} \bar{P}_k$
must be full rank so that the transmitted symbol block $s_k$ can be recovered at the receiver. Therefore, both $\text{rank}(\tilde{V}_k^{(0)}) \geq B$ and $\text{rank}(\tilde{P}_k) \geq B$ must hold, implying that $B_v = B_t M - B_r (K - 1) \geq B$. By enforcing this constraint, we can obtain the following conditions on the block size $B$ and the redundancy length $L_p$ for BD to be possible:

$$B \left( \frac{M - K}{M - K + 1} \right) + L_p - 1 \geq \frac{(K - 1)(L - 1)}{M - K + 1},$$

$$M \geq K.$$ (4.6)

The first inequality can be used as a design criteria by either fixing $B$ or $L_p$, and then calculating the requirements on the other parameter. The second inequality states that the number of antennas must be greater than or equal to the number of users. The design problem then corresponds to finding the best matrix $\tilde{P}_k$ to satisfy given performance optimization criteria for the desired signal transformation $G_k H_k \tilde{V}_k^{(0)} \tilde{P}_k$.

An intuitive approach is to design $\tilde{P}_k$ to provide some form of equalization (ISI mitigation), since IUI is already set to zero by using $\tilde{V}_k^{(0)}$. In the following, we propose two approaches to find $\tilde{P}_k$, namely time-reversal-based BD and equalized BD, which use a simple receiver of the form $G_k = \tilde{G}$. We also present a third technique to jointly design $P_k$ and $G_k$, using channel knowledge at the receiver. We present the solutions to the proposed optimization problems in the Appendices.

### 4.3.1 Time-Reversal-Based Block Diagonalization

TR beamforming is an emerging technique for SDMA over frequency-selective MU-MIMO channels. TR uses the complex-conjugate time-reversed CIR as a FIR filter at the transmitter, and yields space-time focusing of the signal at each receiver [25, 69].
In TR, the precoder $P_{m,k}$ is a (banded Toeplitz) convolution matrix constructed from the vector $h_{k,m}^{TR} = [h_{k,m}^*(L), \ldots, h_{k,m}^*(1)]^T$ as

$$P_{m,k}^{TR} = \begin{bmatrix}
h_{k,m}^*(L) & 0 & \cdots & 0 \\
\vdots & h_{k,m}^*(L) & \ddots & \vdots \\
h_{k,m}^*(1) & \vdots & \ddots & 0 \\
0 & h_{k,m}^*(1) & \cdots & h_{k,m}^*(L) \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & h_{k,m}^*(1)
\end{bmatrix},$$

where the first factor ensures the precoder normalization. Note also that the redundancy is the same as the CIR length ($L_p = L$). We denote the TR precoder for user $k$ as

$$\bar{H}_k = \left( B_t \sum_{m=1}^{M} \|h_{k,m}^{TR}\|_2^2 \right)^{-\frac{1}{2}} \begin{bmatrix} P_{1,k}^{TR T} & \cdots & P_{M,k}^{TR T} \end{bmatrix}^T \in \mathbb{C}^{M(B+L-1) \times B}.$$

TR maximizes the desired signal power at the receiver by acting as a matched-filter, but its performance is limited by both ISI and IUI. We propose time-reversal based BD (TRBD) to take advantage of those properties of TR while eliminating IUI. Unlike the approach in [69], TRBD does not require a Fourier transform of the channel impulse responses, thus reducing the precoder computational complexity. Also, its general linear combiner structure allows the complete elimination of IUI, which is not possible using conventional FIR pre-filter approaches (e.g., [54]). The idea of TRBD is to obtain the closest precoder (in the minimum squared error sense) to the TR pre-filter such that BD is achieved, which can be found by solving

$$\min_{P_k} \| \tilde{V}_k^{(0)} \bar{P}_k - \bar{H}_k \|_F^2, \quad \text{s.t.} \quad \| \tilde{V}_k^{(0)} \bar{P}_k \|_F^2 = 1. \quad (4.7)$$

This problem has a closed-form solution (see Appendix D) such that the TRBD precoder is given by

$$80$$
\[ \mathbf{P}_{k}^{\text{TR}} = \mathbf{\hat{V}}_{k}^{(0)} \frac{\mathbf{\hat{V}}_{k}^{(0)H} \mathbf{H}_{k}}{\left\| \mathbf{\hat{V}}_{k}^{(0)H} \mathbf{H}_{k} \right\|_{F}} \in \mathbb{C}^{M(B+L-1) \times B}, \quad (4.8) \]

### 4.3.2 Equalized Block Diagonalization

The performance of TR-based techniques is limited by ISI since TR pre-filters act only as partial equalizers: they maximize the desired signal power in (3.6) but they do not mitigate ISI explicitly. Henceforth, we propose a second strategy for precoder design, which aims to diagonalize the desired signal transformation, i.e. \( \bar{\mathbf{G}} \mathbf{H}_{k} \mathbf{P}_{k} \approx \mathbf{I}_{B} \). This design criteria is equivalent to maximize the desired signal to ISI power ratio. Note that this novel approach improves other pre-equalization solutions (e.g., [17,25,52]), given that IUI is completely eliminated from the received signal. A complete diagonalization of the form \( \bar{\mathbf{G}} \mathbf{H}_{k} \mathbf{P}_{k} = \mathbf{I}_{B} \) is not attainable since an overdetermined system of linear equation results for the precoder. However, ISI can still be minimized by a least squares solution. In our particular BD model, the problem can be stated as

\[
\min_{\mathbf{P}_{k}} \left\| \mathbf{C}_{k} \mathbf{\hat{P}}_{k} - \mathbf{I}_{B} \right\|_{F}^{2}, \quad \text{s.t.} \quad \left\| \mathbf{\hat{V}}_{k}^{(0)} \mathbf{\hat{P}}_{k} \right\|_{F}^{2} = 1. \quad (4.9)
\]

where \( \mathbf{C}_{k} = \bar{\mathbf{G}} \mathbf{H}_{k} \mathbf{\hat{V}}_{k}^{(0)} \in \mathbb{C}^{B \times B} \). We refer to this approach as equalized block diagonalization (EBD). The solution for the precoder (see Appendix E) is

\[
\mathbf{P}_{k}^{\text{eq}} = \mathbf{\hat{V}}_{k}^{(0)} \left( \mathbf{C}_{k}^{H} \mathbf{C}_{k} + \mu_{k} \mathbf{I}_{B_c} \right)^{-1} \mathbf{C}_{k}^{H}, \quad (4.10)
\]

where \( L_{p} > 0 \) is arbitrarily chosen, \( \mu_{k} \in \mathbb{R} \) is a Lagrange multiplier satisfying the first-order necessary condition

\[
\sum_{i=1}^{B_{c}} \frac{\lambda_{C_{k,i}}}{(\lambda_{C_{k,i}} + \mu_{k})^{2}} = 1, \quad (4.11)
\]
and \( \{ \lambda_{C_k,i} \}_{i=1}^{B_v} \) is the set of eigenvalues of the positive definite matrix \( C_k^H C_k \). The left-hand side in (4.11) is a monotonically decreasing function of \( \mu_k \), so the unique solution can be easily found numerically by using any line search algorithm.

### 4.3.3 Joint Transmitter/Receiver Processing in BD

Both TRBD and EBD assume a sample drop receiver \( \bar{G} \), but cannot eliminate ISI in the received signal. Thus, we propose a joint precoder/receiver design for BD when CSI is available at both the transmitter and the receiver. Thus, CSI of all the system is required at all users (this is also a constraint for frequency-flat BD).

Even though the CSI requirement is an inherent practical limitation of BD, we show that perfect equalization is possible using joint processing, such that both ISI and IUI are completely eliminated. The idea is to design both \( G_k \) and \( \bar{P}_k \) such that

\[
G_k H_k \tilde{V}_k^{(0)} \bar{P}_k = I_B.
\]

We refer to this approach as joint processing block diagonalization (JPBD). Unlike frequency-flat BD, JPBD jointly suppresses the IUI and equalizes the received signal, operating as a linear space-time block code. We begin with the SVD of the equivalent block diagonalized channel

\[
H_k \tilde{V}_k^{(0)} = U_k \Sigma_k V_k^H \in \mathbb{C}^{B_r \times B_v},
\]

(4.12)

where \( U_k \) and \( V_k \) are unitary matrices, \( \Sigma_k \) is a rectangular diagonal matrix, and we assume \( B_v \geq B_r \) so that the pseudo-inverse of \( \Sigma_k \) satisfies \( \Sigma_k \Sigma_k^+ = I_{B_v} \). This assumption holds if

\[
B_t \geq \frac{K(L - 1)}{M - K} \quad \text{and} \quad M > K.
\]

(4.13)

Note that (4.13) is a stronger condition on the transmitter block length than (4.6), viz. the number of transmit antennas must be strictly greater than the number of
users. The system achieves a complete channel diagonalization if the precoder and receiver filter matrices are designed as $P_k^{JP} = \tilde{V}_k^{(0)} V_k^* \Sigma_k^+ \hat{P}_k$ and $G_k^{JP} = \tilde{G}_k U_k^H$, where $\hat{P}_k \in \mathbb{C}^{B_r \times B}$ projects the transmitted block of size $B$ to the received signal space of dimension $B_r = B + L + L_p - 2$ and $\tilde{G}_k \in \mathbb{C}^{B \times B_r}$ reverses this operation. Using these matrices, the linear transformation corresponding to the desired signal in (3.6) is $G_k^{JP} H_k P_k^{JP} = \tilde{G}_k \hat{P}_k$. The final step in the design is to find the matrices $\tilde{G}_k$ and $\hat{P}_k$ that satisfy $\tilde{G}_k \hat{P}_k \propto I_B$. A possible approach to this problem is to set $\tilde{G}_k$ and $\hat{P}_k$ such that $\text{rank}(\tilde{G}_k) = B$ and $\hat{P}_k = \tilde{G}_k^+ / \| \Sigma_k^+ \tilde{G}_k^+ \|_F^2$ (the precoder is normalized). The first condition ensures that $\tilde{G} \tilde{G}^+ = I_B$, so ISI is completely eliminated. Using this approach the SINR at user $k$ is

$$\text{SINR}_k^{JP} = \frac{\rho_k \alpha_{D,k}}{\eta \alpha_{N,k}} = \frac{\rho_k}{\eta} \left( \| \Sigma_k^+ \tilde{G}_k^+ \|_F^2 \right) \left( \| \tilde{G}_k \|_F^2 \right)^{-2}.$$  

(4.14)

Hence, we can select the matrix $\tilde{G}_k$ that maximizes the SNR by solving

$$\min_{\tilde{G}_k} \left( \Sigma_k^+ \tilde{G}_k^+ \right)_F^2 \left( \tilde{G}_k \right)_F^2.$$  

(4.15)

Optimality conditions for this problem lead to a nonlinear matrix equation with no general closed-form solution (see Appendix F). However, if we assume that $\tilde{G}_k$ is a rectangular diagonal matrix with real positive entries, a closed-form solution to this problem exists and is given by

$$\left[ \tilde{G}_k \right]_{ii} = \sqrt{\frac{1}{\sigma_{k,i}}},$$  

(4.16)

where $\sigma_{k,i}$ is the $i$-th singular value of $H_k \tilde{V}_k^{(0)}$. Hence, the precoder and receiver filter in JPBD are
\[ P_k^{ JP } = \tilde{V}_k^{(0)} V_k \frac{\Sigma_k^+ \tilde{G}_k^+}{\| \Sigma_k^+ \tilde{G}_k^+ \|_F}, \quad (4.17) \]
\[ G_k^{ JP } = \tilde{G}_k U_k^H. \quad (4.18) \]

The JPBD precoder and receiver filter resemble the conventional BD solution in [4] by using: 

i) the interference suppression provided by \( \tilde{V}_k^{(0)} \),

ii) the eigenbeamformers \( U_k \) and \( V_k \), which share the role of eliminating ISI, and

iii) the amplitude equalizers \( \Sigma_k^+ \tilde{G}_k^+ \) and \( \tilde{G}_k \), which ensure that all symbols in the received block have the same average power. Note that \( H_k \tilde{V}_k^{(0)} \) has \( B_r \) singular values, but only \( B \) of them are used to calculate the JPBD solution. The influence of these singular values on the performance of JPBD is analyzed in Section 4.4. In addition, using (4.14) and (4.16), the SINR in terms of the singular values of \( H_k \tilde{V}_k^{(0)} \) is

\[ \text{SINR}_{JP}^k = \frac{\rho_k}{\eta} \left( \sum_{i=1}^{B} \frac{1}{\sigma_{k,i}} \right)^{-2}. \quad (4.19) \]

### 4.3.4 Power Allocation for Sum-Rate Maximization

In the previous section, we presented three linear processing techniques for the frequency-selective MIMO broadcast channel. Both TRBD and EBD do not eliminate ISI in the received signal, so conventional waterfilling [102] cannot be applied for power allocation. Thus, in this section we propose a power allocation scheme for sum-rate maximization in TRBD and EBD, which takes into account ISI in the received signal. Maximizing the sum-rate in the downlink subject to a maximum power constraint can be stated as

\[ \max_{\rho} \sum_{k=1}^{K} \rho_k \log_2 (1 + \text{SINR}_k), \quad \text{s.t. } \| \rho \|_1 \leq P_{\text{max}}, \ \rho \geq 0, \quad (4.20) \]
where $\rho = [\rho_1, \ldots, \rho_K]^T$ is the vector of transmitted powers, and $\| \cdot \|_1$ denotes $\ell_1$ vector norm. Using the Lagrange multiplier method (see Appendix G), the optimal power allocation in this case is

$$
\rho_k = \sqrt{\frac{\eta \alpha_{D,k}^2 \alpha_{N,k}^2}{2 \alpha_{ISI,k} (\alpha_{D,k} + \alpha_{ISI,k})} + \frac{4 \eta \alpha_{D,k} \alpha_{ISI,k} \alpha_{N,k}}{\lambda \ln(2)}} (\alpha_{D,k} + \alpha_{ISI,k}) - \frac{\eta \alpha_{N,k} (\alpha_{D,k} + 2 \alpha_{ISI,k})}{2 \alpha_{ISI,k} (\alpha_{D,k} + \alpha_{ISI,k})},
$$

(4.21)

where $\lambda$ is a Lagrange multiplier satisfying

$$
\left| \sum_{k=1}^{K} \sqrt{\frac{\eta^2 \alpha_{D,k}^2 \alpha_{N,k}^2}{2 \alpha_{ISI,k} (\alpha_{D,k} + \alpha_{ISI,k})} + \frac{4 \eta \alpha_{D,k} \alpha_{ISI,k} \alpha_{N,k}}{\lambda \ln(2)}} (\alpha_{D,k} + \alpha_{ISI,k})} \right| - \frac{\sum_{k=1}^{K} \eta \alpha_{N,k} (\alpha_{D,k} + 2 \alpha_{ISI,k})}{2 \alpha_{ISI,k} (\alpha_{D,k} + \alpha_{ISI,k})} = P_{\text{max}}.
$$

(4.22)

Note that the left hand side in (4.22) is a monotonically decreasing function of $\lambda$, so its unique value satisfying the constraint can be found by using a line search algorithm. This search should be limited to the interval $0 < \lambda \leq \min_{k} (\alpha_{D,k} / [\alpha_{N,k} \ln(2)])$ so that $\rho \geq 0$ holds. In the case of JPBD, since ISI is completely eliminated, conventional waterfilling can be applied for power allocation using the signal to noise ratio in (4.19).

### 4.4 Performance Analysis of Frequency-Selective BD techniques

In this section, we analyze the performance of BD methods for frequency-selective channels under different SNR regimes. For high SNR, the system is characterized by $\rho_k/\eta \to \infty$, $\forall k$, which implies $P_{\text{max}}/\eta \to \infty$ given the power constraint $\sum_k \rho_k = P_{\text{max}}$. In this case, we analyze the diversity and the multiplexing gains for each BD method.
When the system operates at low SNR, the term associated to noise dominates the denominator in (4.3), i.e. \( \eta \alpha_{N,k} \gg \rho_k \alpha_{ISI,k} \), and we obtain a technique-independent upper bound for the SINR.

### 4.4.1 Multiplexing Gain

Assuming the receivers treat interference as Gaussian noise, we define the ergodic achievable rate for user \( k \) as

\[
R_k (\text{SINR}_k) = \left( \frac{B}{B + L + L_p - 2} \right) \mathbb{E} \{ \log_2 (1 + \text{SINR}_k) \},
\]

\[
= \left( \frac{B}{B + L + L_p - 2} \right) \mathbb{E} \left\{ \log_2 \left( 1 + \frac{\rho_k \alpha_{D,k}}{\rho_k \alpha_{ISI,k} + \eta \alpha_{N,k}} \right) \right\},
\]

(4.23)

where the factor outside the expectation accounts for the guard interval, the expectation is taken over the channel matrix \( H \), and \( \text{SINR}_k \) is given in (4.3) with \( \alpha_{IUL,k} = 0 \) since any BD technique eliminates IUI. The multiplexing gain for user \( k \) is defined as

\[
r_k = \lim_{\frac{\rho_k}{\eta} \rightarrow \infty} \frac{R_k (\text{SINR}_k)}{\log_2 \left( \frac{\rho_k}{\eta} \right)},
\]

(4.24)

and the system multiplexing gain is

\[
r = \sum_{k=1}^{K} r_k.
\]

(4.25)

Thus, \( r \) is the slope in the achievable sum-rate \( R = \sum_k R_k (\text{SINR}_k) \) at high SNR when plotted against \( P_{\text{max}}/\eta \) (since \( P_{\text{max}}/\eta \rightarrow \infty \) implies \( \rho_k/\eta \rightarrow \infty, \forall k \)). Note that, if \( \alpha_{ISI,k} \neq 0 \), \( \text{SINR}_k \rightarrow \alpha_{D,k}/\alpha_{ISI,k} \) when \( \rho_k/\eta \rightarrow \infty \). Consequently, since TRBD and EBD cannot (completely) eliminate ISI, their system multiplexing gains are
\( r^{\text{TR}} = r^{\text{eq}} = 0, \)

respectively. In contrast, JPBD eliminates ISI and using L’Hôpital’s rule with \( \alpha_{\text{ISI},k} = 0 \) on (4.23)-(4.25), JPBD achieves a system multiplexing gain

\[
 r^{\text{JP}} = \frac{BK}{B + L + L_p - 2},
\]

(4.26)

where we have assumed that the channels for different users have the same statistics. Note that \( \lim_{B \to \infty} r^{\text{JP}} = K \), i.e. JPBD has full multiplexing gain (equal to the number of users) when the transmitted block size goes to infinity. Thus, JPBD outperforms other techniques in the high SNR regime.

### 4.4.2 Diversity Gain

The diversity gain for user \( k \) is defined as

\[
 d_k = - \lim_{\rho_k \eta \to \infty} \frac{\mathbb{E}\{\log(P_e(\text{SINR}_k))\}}{\log(\rho_k \eta)},
\]

where \( P_e(\text{SINR}_k) \) is the probability of error at user \( k \). Assume that the symbols in \( s_k \) are taken from a QAM constellation. Then, the error probability at high SNR is approximately [103, Sec. 9.1.2]

\[
 P_e(\text{SINR}_k) \approx \frac{1}{\text{SINR}^{1-r_k}},
\]

which assumes the QAM rate increases continuously with SNR (this cannot be attained in practice, where discrete modulation orders are used). The diversity gain for user \( k \) is then

\[
 d_k = (1 - r_k) \lim_{\rho_k \eta \to \infty} \frac{\mathbb{E}\{\log(\text{SINR}_k)\}}{\log(\rho_k \eta)}.
\]

(4.27)
The fact that $\text{SINR}_k \rightarrow \alpha_{D,k}/\alpha_{\text{ISI},k}$ if $\alpha_{\text{ISI},k} \neq 0$ implies that the diversity gain for TRBD and EBD is

$$d_k^{\text{TR}} = d_k^{\text{eq}} = 0,$$  \hspace{1cm} (4.28)

respectively. In contrast, replacing (4.19) into (4.27) gives the following diversity gain for JPBD

$$d_k^{\text{JP}} = 1 - r_k^{\text{JP}} = \frac{L + L_p - 2}{B + L + L_p - 2}.$$  \hspace{1cm} (4.29)

Thus, the diversity-multiplexing tradeoff is clearly observed [103, 104]. According to (4.26) and (4.29), for a fixed block length $B$ a larger channel delay spread $L$ or a larger precoder redundancy $L_p$ improve the diversity gain but deteriorate the multiplexing gain. In contrast, for fixed $L$ and $L_p$, a larger block length improves the multiplexing gain but deteriorates the diversity gain.

### 4.4.3 Low SNR Characterization

Now, we derive a bound for the SINR at low SNR (i.e. $\eta \alpha_{N,k} \gg \rho_k \alpha_{\text{ISI},k}$), and demonstrate that it is proportional to the the number of transmit antennas $M$ and the transmitted block length $B_t$. We assume the best case scenario where the equalization provided by any technique is such that $\alpha_{\text{ISI},k} \approx 0$ and $\alpha_{D,k} = \|G_kH_kP_k\|_F^2$. Under those conditions, the SINR is

$$\text{SINR}_k = \frac{\rho_k \|G_kH_kP_k\|_F^2}{\eta \|G_k\|_F^2} \leq \frac{\rho_k}{\eta} \|H_k\|_F^2,$$  \hspace{1cm} (4.30)

where we used the submultiplicative property of Frobenius norms ($\|AB\|_F \leq \|A\|_F \|B\|_F$ for any matrices $A$ and $B$) [105], and the precoder normalization $\|P_k\|_F^2 = 1$. Taking the expectation of (4.30) with respect to the channel yields
Figure 4.2: Achievable rate regions of the proposed techniques with SNR = $P_{\text{max}}/\eta = 20$ dB (left) and 50 dB. $P_{\text{max}}$ is the total transmitter power and $\eta$ is the noise power at each receiver. The system has $M = 8$ antennas.

$$\mathbb{E}\{\text{SINR}_k\} \leq \frac{\rho_k}{\eta} M B_t. \quad (4.31)$$

Henceforth, the number of antennas on the frequency-selective MU-MIMO downlink provides a multiplicative gain on the low SNR regime, rather than the conventional improvement on the high SNR diversity and multiplexing gains of the frequency-flat case.

### 4.5 Numerical Results and Discussion

We performed extensive simulations of the three proposed BD techniques for frequency-selective channels using parameters as shown in Table 4.1 (unless indicated explicitly in each figure). We selected these values to approximate those of common
Figure 4.3: Achievable sum rate for $K = 2$ (left), and $K = 6$ users versus SNR = $P_{\text{max}}/\eta$ for $M = 8$ antennas. The theoretical reference is a line with a slope equal to the multiplexing gain.

Table 4.1: Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean delay spread ($\sigma_h$)</td>
<td>15 ns</td>
</tr>
<tr>
<td>Sampling time ($t_s$)</td>
<td>10 ns</td>
</tr>
<tr>
<td>Block length ($B$)</td>
<td>30 symbols</td>
</tr>
<tr>
<td>CIR duration ($L$)</td>
<td>9 samples</td>
</tr>
<tr>
<td>Precoder redundancy† ($L_p$)</td>
<td>1 sample</td>
</tr>
<tr>
<td>Number of transmit antennas ($M$)</td>
<td>8</td>
</tr>
<tr>
<td>Number of channel realizations</td>
<td>$10^3$</td>
</tr>
</tbody>
</table>

† For EBD and JPBD. $L_p = 1$ implies that the precoder does not add time-domain redundancy. TRBD uses $L = L_p$.

WLAN channel models such as [63], and we assume the system operates over a 100 MHz bandwidth in a typical indoor scenario. Each random channel matrix realization was generated to match the model described in Section 4.2.2.
4.5.1 Achievable Rate Regions

Fig. 4.2 shows the limits of the achievable rate region for $K = 2$ users under the power constraint $\rho_1 + \rho_2 = P_{\text{max}}$. The plot shows that TR, TRBD, and EBD improve slightly with a higher $P_{\text{max}}/\eta$, since they are limited by ISI (as well as IUI in TR) and not by noise. JPBD capacity region expands when increasing $P_{\text{max}}/\eta$ since it eliminates ISI and IUI completely. The achievable rate regions are close to squared in all BD techniques given that IUI is set to zero, which implies that increasing the transmitted power to a given user does not increase interference to the others.

4.5.2 Achievable Sum Rate and Multiplexing Gain

Fig. 4.3 (left and center) shows the maximum achievable sum rate as a function of $P_{\text{max}}/\eta$ (using the power allocation scheme described in Section 4.3.4). The figure shows that TR, TRBD, and EBD have a bound on the maximum sum rate when $P_{\text{max}}/\eta \to \infty$ since they do not eliminate ISI completely (this corroborates the fact that their multiplexing gain is $r = 0$). It is also observed that JPBD has the best performance at high SNR and the simulated multiplexing gain shows good agreement with the theoretical results. Note that, when the number of users increases, higher SNR is required to achieve the same rate since less power is allocated per user.

4.5.3 Bit Error Rate and Diversity Gain

We analyze the average bit error rate (BER) per user performance of the proposed methods with the transmission of $10^6$ bits using QAM constellations of different orders. Fig. 4.4 shows the BER with different number of antennas and different modulation orders. An approximate 6 dB gain is observed on the required $P_{\text{max}}/\eta$ for JPBD when doubling the number of antennas, which is consistent with the bound in...
(4.31) for two users (it translates to a 3 dB gain on $\rho_k/\eta$ for each user). It is also clear that TR, TRBD, and EBD cannot eliminate ISI, inducing a lower bound on the BER at high SNR. However, ISI can be mitigated by using a larger number of antennas, so a lower BER at high SNR is observed when increasing $M$. This characteristic of TR based systems has been also observed in other works [69]. Fig. 4.4 (right) shows the JPBD performance when increasing the QAM constellation size. Note that the diversity gain in (4.29) assumes that the rate (constellation size) increases continuously with SNR, so $d_k^{JP}$ gives a bound on the BER slope for increasing modulation order at high SNR. Thus, the diversity gain slope is better observed when the modulation order is increased with the SNR, e.g., Fig. 4.4 (right) shows an adaptive-rate modulation where the modulation rate is $2^{R_{QAM}}$ and $R_{QAM}$ is the largest even integer smaller than or equal to $r_k \log_2 (P_{\text{max}}/\eta)$ (this ensures a rectangular QAM constellation if $R_{QAM} \geq 4$). This adaptive modulation scheme shows good agreement with the diversity gain, according to the plot.

4.5.4 Impact of the Number of Users

Fig. 4.5 (left) shows the maximum achievable sum rate as a function of the number of users $K$, with all other system parameters kept constant. We used the power allocation in Section 4.3.4. The figure shows that JPBD has the best performance again, followed by EBD, and TRBD. The sum rate in JPBD increases linearly until the number of users approaches the number of antennas (16 in this example) and then drops markedly when $L_p = 1$ (no time-domain redundancy is added at the precoder). This behavior is caused by the SINR dependence on the first $B$ singular values of
Figure 4.4: Bit error rate performance with $K = 2$ users and $L_p = 1$ (no time-domain redundancy) for the transmission of $10^6$ symbols. On the left, 16-QAM BER with $B = 100$ symbols, different number of antennas and techniques. On the right, JPBD BER performance with different QAM orders, adaptive modulation rate, and the theoretical reference (a line with a slope equal to the diversity gain).

$H_k \tilde{V}_k^{(0)}$ as given by (4.19). As discussed in Section 4.3, $H_k \tilde{V}_k^{(0)} \in \mathbb{C}^{B_r \times B_v}$ has $B_r$ non-zero singular values. Thus, a smaller $B_v = B_rM - B_r(K - 1)$ (caused by increasing number of users), decreases the amplitude of those singular values and also the SINR in JPBD. A practical solution to this problem is to increase the precoder redundancy $L_p$, which increases both $B_v$ and the SINR enabling an almost linear growth in the sum rate when the number of users approaches the number of antennas. We observe this effect in Fig. 4.5 (center and right). However, increasing $L_p$ has a small impact on the sum rate when the number of users is low compared to the number of antennas.
Figure 4.5: System analysis when the number of users increases with fixed $B = 125$ and $M = 16$, and different time-domain redundancies. (Up) Maximum achievable sum rate as a function of $K$. (Center) Singular values of $\mathbf{H}_k \tilde{\mathbf{V}}_k^{(0)}$. (Down) SINR coefficient for JPBD ($\alpha_{D,k}/\alpha_{N,k}$) as given by (4.19). The maximum sum-rate collapses when the number of users increases due to the singular values of $\mathbf{H}_k \tilde{\mathbf{V}}_k^{(0)}$ approaching zero.
4.6 Conclusion

We explored the generalization of BD precoding techniques, originally proposed for frequency-flat MIMO broadcast channels, to the frequency-selective case. Such generalization is not straightforward since the channel matrix has a space-time structure constructed from the channel impulse responses. We derived the conditions under which BD is feasible for block transmissions in frequency-selective MIMO broadcast channels: the transmitted block length should be sufficiently large and the number of transmit antennas should be greater than or equal to the number of users (see inequality (4.6)).

Even though any BD eliminates IUI, frequency selectivity induces ISI in the received signal. Thus, we proposed three approaches to mitigate or suppress ISI. The first approach, TRBD, finds the BD precoder matrix which is closest (in the minimum squared error sense) to the TR pre-filter; although it improves the performance of conventional TR, it is still limited by ISI. EBD is the second approach, which explicitly minimizes ISI using an equalizer at the transmitter; EBD outperforms TR based solutions but cannot suppress ISI completely. Moreover, we showed that any precoding-only scheme which do not eliminate ISI has zero diversity and multiplexing gains (their achievable sum rates are bounded at high SNR). Thus, we propose a joint transmitter/receiver design called JPBD, which is based on the SVD of the equivalent block-diagonalized channel. We demonstrated that, for an infinite block length, JPBD achieves full multiplexing gain (equal to the number of users). We showed that the diversity gain in JPBD improves with larger channel delay spread or larger time-domain precoder redundancy, but decreases with larger block length $B$ (see eq. (4.29)).
Extensive numerical simulations show that all the proposed BD solutions for frequency-selective MIMO broadcast channels outperform conventional TR beamforming. Moreover, numerical results show good agreement with the theoretical results derived in this chapter. We also examined the performance of each technique under different operation parameters, e.g. number of antennas, number of users, block length, and precoder redundancy.
Chapter 5: Hybrid Beamforming Algorithm for Multiuser Wideband Millimeter-Wave Systems

5.1 Introduction

Millimeter wave (mmWave) systems are emerging technologies for future wireless communication networks. These systems use carrier frequencies around 28, 38, 60, and 72 GHz, where large bandwidths are available to alleviate spectrum scarcity affecting current cellular networks [3]. Propagation at mmWave frequencies is characterized by large free-space losses and strong atmospheric attenuation [106]. These features, initially perceived as unfavorable, were later recognized as advantages for short-range (<100 meters) communications. For example, large propagation losses can be used to increase frequency reuse by decreasing interference in dense small-cell networks. Furthermore, high-gain directional antennas can be implemented in compact terminals (given the small wavelength) to compensate for propagation losses and improve link security by reducing radiation to undesired directions [107]. Implementation of high-gain antennas is feasible in mmWave systems either as uniform linear arrays (ULAs) [108] or planar arrays [35], which have sharp pencil-like radiation patterns. Thus, beamforming techniques are indispensable to align the radiation patterns according to the specific scenario.
A number of works have explored the propagation characteristics of mmWave frequencies and their application to 5G cellular networks (e.g., see [109] for an overview of this topic). In particular, 60 GHz bands have been studied for indoor wireless communications since atmospheric absorption losses at these frequencies are too strong for longer (outdoor) links [107]. Current and developing standards for indoor WLAN use carriers around 60 GHz, where the available unlicensed bandwidth is approximately 5 GHz [1,110]. Applications such as ultra-high-definition video streaming, augmented and virtual reality, mass data distribution, and indoor wireless backhauling will be enabled by this unprecedented bandwidth availability [110].

Despite the advantages of mmWave systems, many challenges remain open to fully exploit their potential. One of the most active research areas in mmWave is the design of beamforming procedures that take into account the specific hardware constraints and propagation characteristics at these frequencies [11]. High-gain antennas are required to compensate for large propagation losses, which are commonly implemented with arrays of tens (or even hundreds) of antenna elements. The implementation of these large antenna arrays is feasible in compact devices due to the small wavelength. Thus, fully digital beamforming, which requires one RF chain for every antenna element, is unfeasible due to hardware complexity and power consumption constraints. For this reason, several hybrid analog/digital beamforming architectures (which use phase shifters, power splitters/combiners, and switches in the RF domain together with digital baseband processing) have been proposed as practical solutions for mmWave terminals [12–15]. The hybrid beamforming design problem, which is the focus of this chapter, refers to the selection of phase-shifter configurations and digital
processing matrices that maximize a given performance criteria under the hardware constraints.

5.1.1 Related Work

The hybrid beamforming problem for single-user narrowband mmWave systems was first studied in [12] and [13]. In [12], assuming perfect channel state information (CSI), the authors design hybrid transmit beamformers to approximate the linear precoder that maximizes the spectral efficiency (which are known to be the right singular vectors of the channel matrix). This is done by leveraging the spatially sparse nature of the mmWave channel and considering phase shifters with quantized states. This work is expanded in [13], where channel estimation is formulated as a sparse reconstruction problem. This approach uses sets of hybrid beamforming configurations, called hierarchical beamforming codebooks, that provide main beams of different widths. These codebooks are used to perform training transmissions over increasingly narrow angular regions in space, converging to the sharpest beam that maximizes the received power. The use of hierarchical beamforming codebooks has become a design principle for hybrid beamforming in mmWave for different applications. For example, the IEEE 802.11ad 60 GHz WLAN standard uses beam training over a hierarchical codebook [1,27,28]. Also, reference [29] presents a codebook construction methodology for hybrid beamforming with planar antenna arrays in single-user narrowband systems.
A widely used design criteria for hybrid beamformers in mmWave systems is the spectral efficiency (mutual information) maximization in the system. In the single-user frequency-selective case, [14] proposes a greedy optimization technique to maximize mutual information under perfect CSI. Reference [111] obtains closed-form solutions to a relaxed mutual information maximization problem and examine subarray structures for single-user frequency-selective hybrid beamforming assuming perfect knowledge of the channel sample covariance matrix.

A multiuser hybrid beamforming design solution for narrowband mmWave is presented in [30], where the analog beamformer is obtained by training over a hierarchical codebook and the digital beamformer is calculated in a conventional zero-forcing approach. For multiuser frequency-selective hybrid beamforming, [31] formulates the design problem such that the hybrid beamformer approximates the fully digital solution found with block-diagonalization (BD) [4]. The work in [31] assumes perfect CSI with a total power constraint and analog beamforming with infinite resolution phase shifters. Reference [32] also investigates hybrid beamforming in the multiuser frequency-selective case assuming perfect CSI and infinite resolution phase shifters; its approach is to simplify the problem to a frequency-flat equivalent by using an average of the channel matrices along subcarriers.

Antenna-specific aspects of wideband mmWave beamforming have also been studied recently, with focus on the array response’s frequency dependence (beam squint effect). References [33] and [34] study the capacity loss in mmWave systems due to beam squint and propose a beamforming design to compensate for this effect. Other antenna aspects of beamforming for mmWave terminals are addressed in [35].
5.1.2 Contributions

In this chapter, we tackle the hybrid beamforming design problem for wideband (OFDM) mmWave systems consisting of one access point (AP) and multiple user equipments (UEs). We assume that the AP has a fully-connected hybrid beamforming architecture with multiple RF chains and one uniform antenna array (the number of antenna elements in the array is much larger than the number of RF chains). We assume each UE has one RF chain and one uniform antenna array with analog beamforming capabilities. Furthermore, we assume that all the terminals use phase shifters with quantized resolution. The specific contributions of this chapter with respect to previous works are the following:

- Novel analog hierarchical beamforming codebooks for the AP and the UEs specifically designed for their hardware configurations. The codebooks provide sector beams and narrow beams to allow training transmissions over increasingly narrow regions in the angular domain. The codebooks are based on the orthogonality of beamforming vectors in uniform antenna arrays, which preserves the hierarchical codebook structure over the bandwidth and compensates for beam squint effects.

- We propose a hybrid beamforming algorithm that decouples the design process into two stages: the analog beamforming beam selection procedure and the digital beamforming. First, in the beam selection procedure, the algorithm obtains analog beamformers that maximize the estimated sum-power across subcarriers for each user by using orthogonal beamforming codebooks for uplink and downlink training transmissions. In the second part, after the optimal
analog beamformers are obtained for all UEs, the AP calculates a BD digital beamformer that maps the signal directed to the users to the available RF chains. This algorithm does not assume any structure in the channel, it does not assume perfect CSI (it actually provides channel estimates), and it takes into account beam squint, element radiation patterns, and antenna coupling effects present in wideband mmWave systems. The algorithm assumes that the channel is reciprocal and quasi-static; consequently, it is suitable for time division duplex systems.

- We provide a performance evaluation of our algorithm in terms of the achievable sum-rate in the downlink under two channel models:  
  i) a statistical channel model with a discrete number of paths and uniformly distributed angles of departure and arrival, and  
  ii) channel matrices obtained with ray-tracing simulations of a scenario taken from the IEEE 802.11ay channel models [110]. These simulations include diffuse scattering at surfaces to model mmWave propagation characteristics. Moreover, we compare our algorithm versus fully-digital BD precoding (with one RF chain per antenna), which is the linear beamforming technique that best approximates the channel capacity [112]. We show that our algorithm’s achievable sum-rate performance has only approximately a 3 dB loss with respect to the fully-digital BD solution.

Throughout the chapter, we show application examples for ULAs and parameters taken from the IEEE 802.11ad WLAN standard. However, the principles discussed
here are also applicable to 2D uniform planar arrays and other types of mmWave networks. We begin by describing the system model in next section\(^5\).

### 5.2 System Model

In this section, we introduce wideband (OFDM) mmWave system models for the two stages of our algorithm. We begin by defining the hardware configurations at the AP and the UEs, and then we present baseband signal models for two stages of hybrid beamforming design. In the *beam selection* procedure, the algorithm searches for analog beamformers that maximize the estimated received power for each UEs, alternating between uplink and downlink training transmissions. We define single-user uplink and downlink OFDM system models for this stage. In the *digital beamforming* stage, the algorithm uses the analog beamformers found in the first stage to calculate a digital beamformer that maps the transmitted signals (directed to all users) to the available RF chains. Hence, we define an OFDM multiuser MIMO downlink model with hybrid beamforming. Importantly, the models presented here include wideband effects relevant in mmWave systems that are traditionally ignored in literature. For example, we include detailed frequency-dependent array response vectors that account for antenna coupling, element radiation patterns, and beam squint effects.

\(^5\) *Notation*: \(a\) and \(A\) are scalars, \(\mathbf{a}\) is a vector, and \(\mathbf{A}\) is a matrix. Vector and matrix sizes are defined explicitly for every variable. \((\cdot)^T\), \((\cdot)^*\), \((\cdot)^H\), and \(\|\cdot\|_F\) represent transpose, complex conjugate, conjugate transpose, and Frobenius norm of a matrix, respectively. \([\mathbf{A}]_{m,n}\) is the element in the \(m\)-th row and \(n\)-th column of \(\mathbf{A}\). \([\mathbf{A}]_{:,n}\) is the \(n\)-th column of \(\mathbf{A}\). \(\|\mathbf{a}\|_2 = \sqrt{\mathbf{a}^H\mathbf{a}}\) is the \(\ell_2\) norm of \(\mathbf{a}\). \(\mathbf{I}_N\) is the \(N \times N\) identity matrix. \(\mathbb{E}\{\cdot\}\) denotes expected value and \(\mathcal{CN}(\mathbf{0}, \mathbf{R})\) is the zero-mean circularly-symmetric complex Gaussian distribution with covariance matrix \(\mathbf{R}\).
Figure 5.1: AP hardware configuration. Each RF chain is connected to a group of $M_{\text{ap}}$ phase shifters (p.s.) through one splitter/combiner (S/C). One phase shifter from each RF chain is connected to every antenna through other S/C.

5.2.1 Hardware Configuration

We consider a mmWave system consisting of one AP and $U$ user equipment (UEs). The AP has $M_{\text{ap}}$ antennas and $N_{\text{rf}}$ RF chains, which are fully connected as depicted in Fig. 5.1. Since the AP has multiple RF chains, it can also perform digital processing in addition to the analog beamforming, an operation known as hybrid beamforming or precoding. In the UE side, where there are usually very strict size and power constraints, we use a less demanding hardware configuration with $M_{\text{ue}}$ antennas and a single RF chain, as shown in Fig. 5.2. This is a conventional analog phased-array configuration, with one phase shifter per antenna. However, we propose to employ a switching network so that the UE can also use a subarray of $M_{\text{sub}}$ antennas to provide wider beams at the expense of less beamforming gain. The UEs can use either the
Figure 5.2: User equipment configuration. Each one of the $M_{ue}$ antennas is connected to a single phase shifter (p.s.). An RF switch allows the antenna reconfiguration to use the full array with $M_{ue}$ antennas or a subarray with $M_{sub}$ antennas.

full array or the subarray configurations by controlling the RF switching network denoted by SW in Fig. 5.2. We assume that all of the phase shifters have digital control. Therefore, there is a discrete (quantized) set of phases and beamformers available at the terminals. In Section 5.4, we describe how to leverage this hardware configuration to construct discrete beamforming codebooks with both wide sector beams and narrow beams.

5.2.2 Single-User mmWave Uplink - Beam Selection

Consider a wideband wireless communication system represented in complex baseband. The system uses an OFDM waveform with $K$ subcarriers, and we assume that the OFDM cyclic-prefix removes all of the inter-symbol interference. In the beam selection procedure, the UE transmit a training sequence $x[k] \in \mathbb{C}^{1 \times T}$ in the uplink,
which spans $T$ time-domain OFDM symbols (each row represents different time samples). The received uplink vector signal, which represents the output of all RF chains at the AP in subcarrier $k$, is

$$
y_{ul}[k] = \sqrt{\frac{\rho}{K}} P_a^T H^T[k] g^*[k] + z_{ul}[k] \in \mathbb{C}^{N_{rf} \times T}, \quad (5.1)$$

where $\rho$ is the transmitted power, $g \in \mathbb{C}^{M_{ue} \times 1}$ is the column vector of beamforming coefficients (steering vector) at the UE, $H^T[k] \in \mathbb{C}^{M_{ue} \times M_{ap}}$ is the uplink MIMO channel matrix between the UE and AP arrays, $z_{ul}[k]$ is a complex white Gaussian noise matrix whose elements have zero-mean and variance $\sigma_z^2/N_{rf}$, and $P_a \in \mathbb{C}^{M_{ap} \times N_{rf}}$ is the analog beamforming matrix defined as

$$P_a = [p_{a,1}, \ldots, p_{a,N_{rf}}],$$

where the column vector $p_{a,n}$ represents the $n$-th RF chain beamforming coefficients. The time-domain sequence received at RF chain $n$ and subcarrier $k$ is

$$y_{ul,n}[k] = v_n[k] x[k] + z_{ul,n}[k], \quad (5.2)$$

where

$$v_n[k] = \sqrt{\frac{\rho}{K}} P_{a,n}^T H^T[k] g^*$$

is the complex channel coefficient after beamforming. We assume the power constraints $\|p_{a,n}\|_2^2 = N_{rf}^{-1} \forall n$, $\|P_a\|_F^2 = 1$, and $\|g\|_2^2 = 1$, which take into account the power splitters/combiners in the system. This model, depicted in Fig. 5.3a, assumes that the transmitted power is uniformly distributed across subcarriers. Note that the analog beamformers $P_a$ and $g$, which represent the phase shifters configuration, are independent of frequency (we elaborate on this property in Section 5.2.7).
Figure 5.3: Systems model. (a) Training uplink and downlink, and (b) Downlink for user $u$. 
5.2.3 Single-User mmWave Downlink - Beam Selection

For the downlink, the AP sends a training signal $x[k]$ with equal power through all the RF chains and subcarriers. Using the hardware described above, and assuming a reciprocal channel, the received signal at the UE for subcarrier $k$ is

$$y_{dl}[k] = w[k]x[k] + z_{dl}[k] \in \mathbb{C}^{1 \times T},$$

where $z_{dl}[k] \sim \mathcal{CN}(0, \sigma_z^2 I_T)$ is complex white Gaussian noise, and the complex downlink channel coefficient is

$$w[k] = \sqrt{\frac{\rho}{KN_rf}} g^H H[k] p_a 1,$$

$$= \sqrt{\frac{\rho}{KN_rf}} g^H H[k] \left( \sum_{n=1}^{N_{rf}} p_{a,n} \right).$$

where $1$ is a $N_{rf} \times 1$ vector whose entries are all $1$. For convenience, we use the same total transmit power $\rho$ in the uplink and downlink, and we define the signal-to-noise ratio in the system as

$$\text{SNR} = \frac{\rho}{\sigma_z^2}. \quad (5.6)$$

Fig. 5.3a shows the downlink model and its relationship with the uplink. In the special case where all RF chains use the same beamformer $p$, the downlink channel coefficient reduces to

$$w[k] = \sqrt{\frac{N_{rf} \rho}{K}} g^H H[k] p.$$

If we set $p_{a,n} = p$ in (5.3), the relationship $v[k] = N_{rf}^{-\frac{1}{2}} w[k]$ between uplink and downlink channel coefficients holds due to channel reciprocity. The factor $N_{rf}^{-\frac{1}{2}}$ accounts
for the power loss caused by a single RF chain receiving with a given beamformer in the uplink, whereas all RF chains transmit the same training signal with the same beamformer in the downlink.

### 5.2.4 Multiuser Downlink - Hybrid Beamforming

When the AP sends independent signals to $U$ different users, the downlink is represented by a broadcast channel where the signal at user $u$ is

$$y_u[k] = h_{eq,u}[k]P_d[k]s[k] + z_{dl,u}[k], \quad (5.8)$$

where $g_u[k]$ and $H_u[k]$ are the beamformer and the channel matrix for user $u$, respectively, $s[k] = [s_1[k], \ldots, s_U[k]]^T \in \mathbb{C}^{U \times 1}$ is the vector of complex symbols transmitted to the users, $P_a$ is defined as in (5.2), the digital precoder $P_d[k] \in \mathbb{C}^{N_{rf} \times U}$ maps the $U$ independent transmitted signals to the available RF chains, and the equivalent MISO channel after analog beamforming for user $u$ is

$$h_{eq,u}[k] = g_u^H H_u[k]P_a \in \mathbb{C}^{1 \times N_{rf}}, \quad (5.9)$$

which represents the channel coefficients between each RF chain and user $u$. The transmitted signals satisfy the total transmitted power constraint $\sum_u \rho_u \leq \rho$, where $\rho_u = \mathbb{E}[|s_u[k]|^2]$. In this model, the AP applies both analog and digital beamformers to the signal, so the total transmitter processing is represented by a hybrid precoding matrix $P_a P_d[k] \in \mathbb{C}^{M_{ap} \times U}$. Thus, the maximum number of users the AP can support is limited by the number of RF chains ($U \leq N_{rf}$). In addition, we use the power constraint $\|P_a P_d[k]\|_F = 1$, $\forall k$, which allows a fair comparison with the benchmark
BD technique. In Section 5.3, we formulate the problem of selecting analog beamformers $P_a$ and $\{g_u[k]\}$ that maximize the received SNR at every user and a digital beamformer $P_d[k]$ that eliminates the remaining inter-user interference.

5.2.5 Channel Model

The hybrid beamforming algorithm presented in Section 5.5 is independent of the channel structure but is specifically designed for uniform (linear or planar) antenna arrays. In this section we describe a simple channel model for this type of antenna arrays, which help us characterize the algorithm’s performance. Assuming there are $L$ propagation paths between the AP and the UE (with angles of arrival and departure for path $\ell$ given by $\theta_{ue,\ell}$ and $\theta_{ap,\ell}$, respectively), the downlink channel matrix has the form

$$
H[k] = (I_{M_{ue}} + S_{ue}[k]) \left( \sum_{\ell=1}^{L} \alpha_{\ell} a_{ue}(k, \theta_{ue,\ell}) a_{ap}^H(k, \theta_{ap,\ell}) \right) \\
\times (I_{M_{ap}} + S_{ap}[k]) \in \mathbb{C}^{M_{ue} \times M_{ap}},
$$

(5.10)

where $S_{ap}[k]$ and $S_{ue}[k]$ are the frequency-dependent S-parameter matrices of the antenna arrays at the AP and the UE, respectively, $a_{ap}(k, \theta_{ap,\ell}) \in \mathbb{C}^{M_{ap} \times 1}$ is the array response vector at the AP in the angle of departure $\theta_{ap,\ell}$ at subcarrier $k$, $a_{ue}(k, \theta_{ue,\ell}) \in \mathbb{C}^{M_{ue} \times 1}$ is the array response vector at the user terminal in the angle of arrival $\theta_{ue,\ell}$ at subcarrier $k$, and $\alpha_{\ell} \in \mathbb{C}$ is the coefficient of path $\ell$. This multipath model is adapted from [113] to consider antenna coupling effects by using the array’s S-parameters as in [114, Ch. 2]. We define the following model for the S matrix elements in ULAs assuming perfect impedance matching:
\[
[S[k]]_{m,m'} = \begin{cases} 
0 & \text{if } m = m', \\
\frac{c \exp(-j2\pi d f_k |m-m'|)}{d|m-m'|} & \text{if } m \neq m', 
\end{cases}
\]

where \( c \) is a scalar that determines the coupling amplitude, which is typically below \(-15\) dB for adjacent antennas [115]. The idea behind this model is to obtain coupling coefficients whose power decay with the squared of the distance separating antenna elements. This is a conservative model for coupling, especially for adjacent elements that may observe other power components of higher order. However, the model can be easily configured using the constant \( c \) to match measurements of implemented antenna arrays. The array response vectors depend exclusively on the array geometry. Note that the model given by (5.10) assumes ULAs but it can also be extended to planar antenna arrays as pointed out in [29].

### 5.2.6 Antenna Array Response Vector and Radiation Pattern

One of the features of OFDM mmWave systems operating with the hardware configuration described above is that the array response vectors are frequency-dependent. This is caused by changes in the electrical length of element spacing at different frequencies. As a consequence of this feature, a fixed phase shifter configuration has different maximum radiation directions for different subcarriers. This effect is commonly known as \textit{beam squint} and can severely impact the performance of mmWave communications [33,34]. We develop here a frequency-dependent antenna array response vector model. To analyze its impact on the array’s radiation pattern, let \( f_0 \) denote a reference frequency in the band of interest, such that the array inter-element spacing is referenced to it. Let \( f_k \) denote the frequency of subcarrier \( k \). As
an example, we present a ULA model for the AP, where $\theta$ represents the angle with respect to the array axis (matching the azimuth angle). The elements in the array response vector $a_{ap}(k, \theta, \phi)$ represent the phase of an incoming/outgoing plane wave received/generated by the array in the far field [116]. For a ULA, the $m$-th element in $a_{ap}(k, \theta, \phi)$ is

$$a_{ap,m}(k, \theta, \phi) = F(k, \theta, \phi) \exp \left( j2\pi df \frac{k}{f_0} \left[ m - \frac{M+1}{2} \right] \cos \theta \right),$$

where $m = 1, \ldots, M_{ap}$, $d$ is the antenna element spacing (normalized to the central frequency wavelength), and $F(k, \theta, \phi)$. As an example, we use antenna elements with radiation patterns

$$F(k, \theta, \phi) = \begin{cases} 
2\sin \theta \sin \phi & \text{if } \theta \in [0, \pi] \text{ and } \phi \in [0, \pi], \\
10^{-2} & \text{otherwise.}
\end{cases}$$

This model approximates patch antennas used at mmWave found in literature, e.g. [117–119], which are typically mounted over reflecting chasis. The arrays are able to scan a half-space only, with a small power leakage to the back. The array’s electric field pattern at subcarrier $k$ is

$$\Psi_{ap}(k, \theta, \phi) = a_{ap}^H(k, \theta, \phi) (I + S_{ap}[k]) p.$$ 

Analogous definitions apply to the array’s radiation pattern at the UE $\Psi_{ue}(k, \theta, \phi)$.

### 5.2.7 RF Phase Shifters

Phase shifters commonly have a frequency-independent group delay $\tau$, which translates into a quasi-linear phase change with respect to frequency. A typical group delay value for mmWave phase shifters is around $10\text{ps}–100\text{ps}$ [120,121]. Thus,
we can model the frequency response of a phase shifter as 

\[ b(k) = e^{-j2\pi \tau (f_k - f_0)}e^{j\beta_0}, \]

where \( \beta_0 \) is the phase shift at the reference frequency \( f_0 \). However, the phase difference between phase shifters connected to different antenna elements is independent of \( e^{-j2\pi \tau (f_k - f_0)} \). Thus, we ignore this term when designing the beamforming vectors so that they are independent of frequency. Fig. 5.4 shows an example of two radiation patterns at different frequencies in the 60 GHz band obtained with the models described in this Section and parameters described in the figure caption.

5.3 Problem Statement

After defining the system model, we now formally state our approach to hybrid beamforming in the wideband multiuser downlink in (5.8). We propose to decouple
the design of $P_a$ and $P_d[k]$. The general idea is to first find analog beamformer vectors that maximize the SNR for each UE and construct the analog matrix $P_a$ using those vectors. Then, the AP can design $P_d[k]$ with conventional digital beamforming techniques to eliminate the remaining inter-user interference. To construct $P_a$, we aim to solve the following optimization problem for each user:

$$\{p^*_u, g^*_u\} = \arg \max_{p \in \mathcal{B}(M_{ap}), g \in \mathcal{B}(M_{ue})} \sum_k |g^H H_u[k] p|^2,$$

(5.11)

where $v_u = [v_u[1], \ldots, v_u[K]]^T$ is the channel coefficient vector, $v_u[k]$ is the channel coefficient as given in (5.3) for user $u$, $\mathcal{B}(M_{ap})$ and $\mathcal{B}(M_{ue})$ are discrete beamformer sets for the AP and UEs, respectively. These sets are known as codebooks and their cardinality depends on the number of antennas and the quantized phase-shifter configurations. The optimization problem above can be understood as a sum-power maximization across all subcarriers, where the system selects the best beamformers available at the AP and the UE. Solving this optimization problem with exhaustive search requires perfect knowledge of the objective function (perfect CSI) for every combination of feasible values of $p$ and $g$. Since this is unattainable in practice, we relax the problem in (5.11) by using an channel vector estimate $\hat{v}_u$ to get

$$\{p^*_u, g^*_u\} = \arg \max_{p \in \mathcal{B}(M_{ap}), g \in \mathcal{B}(M_{ue})} \|\hat{v}_u\|^2.$$

(5.12)

To solve this problem, we propose a heuristic algorithm to reduce the number required of training transmissions with respect to an exhaustive search over $\mathcal{B}(M_{ap}) \times \mathcal{B}(M_{ue})$. Our approach is based on hierarchical codebooks that we describe in Section 5.4. Note that training transmissions provide maximum likelihood (ML) channel estimates for
every user. However, using ML estimates, the solutions to (5.11) and (5.12) are equal only when the SNR is asymptotically large. This fact, combined with the algorithm operation, causes solutions to (5.11) and (5.12) to be different with a probability different than zero at finite SNR. If those solutions differ, we declare a beam selection error, and we use the beam selection error rate (BSER) as an algorithm performance metric. Once (5.12) is solved for every \( u \), \( P_a \) is constructed using \( \{p^*_u\} \) as columns. Estimates of the equivalent MISO channel given by (5.9), and denoted by \( \hat{h}_{eq,u}[k] \) for user \( u \), are obtained for every UE. Finally, the AP calculates the \( P_d[k] \) using block-diagonalization to eliminate the residual inter-user interference after analog beamforming. This algorithm is detailed in Section 5.5.

5.4 Codebook Design

In this section, we describe a method to design beamforming codebooks for the analog beamformers \( P_a \) and \( \{g_u\} \) based on the orthogonality of beamforming vectors. We leverage the hardware architectures described in Section 5.2 to construct hierarchical codebooks whose elements provide main beams of two widths. The first codebook at the AP, denoted as \( P_s \), scans wide sectors of the angular domain in the uplink by using different (adjacent) narrow beams in each RF chain. At the UEs, \( G_s \) is the codebook with sector beamformers obtained using the subarray configuration. The codebooks with narrow (pencil) beams are denoted by \( P \) and \( G \) at the AP and the UEs, respectively. The algorithm in Section 5.5 uses these codebooks to solve (5.12).
Figure 5.5: Power radiation patterns (in dBi) generated by orthogonal beamformers for a uniform linear array with (a) 8 antennas (B(8)) with \( b_7(8) \) highlighted in black, and (b) 16 antennas (B(16)) with \( b_{13}(16) \) highlighted in black, both operating at the central frequency. Parameters: \( d = 0.5 \), \( F(k, \theta, 90^\circ) = 2 \sin \theta \) (elevation angle is 90\(^\circ\)). The first beamformer is symmetric with one grating lobe. The sector spanned by \( b_7(8) \) (dotted line) overlaps with beams generated by \( b_{12}(16) \), \( b_{13}(16) \), and \( b_{14}(16) \).

### 5.4.1 Orthogonal Beamformers for ULAs

We base our beamforming codebook design on the orthogonality of beamforming vectors. We use this feature to construct an analog beamforming matrix \( P_a \) such that beamformers pointing to different users are orthogonal. This simplifies the digital precoder design and facilitates the hierarchical structure design. The orthogonality property holds independently of frequency, so the alignment between narrow beams and sector beams is not vulnerable to the beam squint effect. With the goal of simplifying the codebook description, we assume that the AP and the UEs have 1D ULAs printed over a planar substrate with element spacing of half-wavelength at the frequency \( f_0 \), i.e. \( d = 0.5 \). This configuration is common in mmWave systems [122],
and it allows us to describe the radiation pattern for \( \theta \in [0, \pi) \) only. The orthogonality property also exists for uniform 2D antenna arrays, so the principles described here can be easily extended to other uniform array geometries.

Let \( \mathcal{B}(M) \) denote a set of orthogonal beamforming vectors for a ULAs with \( M \) antennas defined as

\[
\mathcal{B}(M) = \{ b_m(M) \in \mathbb{C}^M \mid m = 1, \ldots, M \},
\]

where

\[
b_m(M) = \frac{1}{\sqrt{M}} [1, e^{j \beta_m(M)}, \ldots, e^{j (M-1) \beta_m(M)}].
\]  

(5.13)

The entries in the beamforming vector represent the phase shift applied to each antenna element. We ignore the beamforming vector’s frequency dependence as justified in Section 5.2.7. The phase difference between elements is

\[
\beta_m(M) = \pi \left( 1 - 2 \frac{(m - 1)}{M} \right).
\]

The elements in \( \mathcal{B}(M) \) are orthogonal, since they satisfy

\[
b_m^H(M) b_{m'}(M) = \delta_{m,m'}, \; \forall m \neq m', \; \forall M,
\]

where \( \delta_{m,m'} \) is the Kronecker delta function. Fig. 5.5 shows radiation patterns generated by two orthogonal beamformer sets, \( \mathcal{B}(8) \) and \( \mathcal{B}(16) \). Some important properties of the beamformers in \( \mathcal{B}(M) \) and their radiation patterns are:

1. \( \|b_m(M)\|_2 = 1, \forall m, M \) to ensure power normalization.

2. The set always contains a broadside beam (with direction of maximum radiation at 90° with respect to the array axis) when \( m = \frac{M}{2} + 1 \) independently of frequency and element spacing.
3. Their directions of maximum radiation are not uniformly distributed in the interval $[0, \pi)$. Beams which are closer to the array’s axis are wider than beams close to the array’s broadside.

4. Orthogonality of beamforming vectors is independent of frequency, thus it is preserved even under beam squint.

5. The number of orthogonal beams is the same as the number of antennas. However, they do not provide the same gain or number of sidelobes since these features depend on the element pattern and element spacing.

6. If the element spacing at the operating frequency is larger than half-wavelength, grating lobes exist in the radiation pattern for one or more beams in the set. However, orthogonality ensures that the maximum radiation directions of a given beamformer (including grating lobes) coincide with nulls in the radiation patterns of orthogonal beamformers.

7. Let $M$ and $N$ be two integers such that $M > N$ and $\frac{M}{N}$ is even. Then, main beams generated by $\mathcal{B}(N)$ point in the same directions as some of the beams in $\mathcal{B}(M)$ (beams in $\mathcal{B}(M)$ are narrower). Moreover, there are precisely $\frac{M}{N} + 1$ beams in $\mathcal{B}(M)$ that overlap a wider beam in $\mathcal{B}(N)$. This is observed in the sectors marked in Fig. 5.5.

We use the characteristics above to construct hierarchical codebooks for the AP and the UEs in the following subsections.
5.4.2 Sector Codebook with Hybrid Beamforming

In this section, we describe how to construct analog beamforming matrices that scan wide sectors in the angular domain by leveraging the hybrid architecture at the AP. These matrices form a codebook that contains possible configurations for $P_a$ in the uplink given by (5.1). We begin by defining the orthogonal $M_{ap} \times N_{rf}$ beamforming matrices as

$$B_m = \begin{bmatrix} b_{l_1(m)}(M_{ap}) & b_{l_2(m)}(M_{ap}) & \cdots & b_{l_{N_{rf}}(m)}(M_{ap}) \end{bmatrix},$$

where

$$l_n(m) = (m-1)N_{rf} + n,$$

$$m = 1, \ldots, M_{ap}/N_{rf},$$

$$n = 1, \ldots, N_{rf}. \quad (5.14)$$

where the columns are $N_{rf}$ elements in $B(M_{ap})$ with adjacent beams. The codebook of sector beamforming matrices is then constructed as

$$\mathcal{P}_s = \{P^{(m)} \mid m = 1, \ldots, M_{ap}\},$$

where

$$P^{(m)} = \frac{1}{\sqrt{N_{rf}}}B_m, \quad (5.15)$$

where the first factor accounts for the $N_{rf}$-port power splitter at each antenna. From the point of view of hardware configuration, each RF chain in the AP uses a different beamformer, such that different angular domains are seen at the uplink outputs. Fig. 5.6a shows an example of the radiation patterns for the sector codebook with $M_{ap} = 16$ antennas and $N_{rf} = 4$ chains.

5.4.3 Narrow Beam Codebook with Hybrid Beamforming

The codebook $\mathcal{P}$ has configurations for $P_a$ that provide narrow main beams for the downlink in (5.4). Such beamformers are constructed by configuring the same
beamforming vector in all RF chains, which focuses the radiated power in a narrow angular region. Codeword \( n \) within sector \( m \) is

\[
P^{(m,n)} = \frac{1}{\sqrt{N_{\text{rf}}}} [b^{(m)}_{l_{n}(m)}(M_{\text{ap}}) \cdots b^{(m)}_{l_{n}(m)}(M_{\text{ap}})],
\]

(5.16)

where \( l_{n}(m) \) is as defined in (5.14). This implies that the same beamforming vector \( b_{l_{n}(m)}(M_{\text{ap}}) \) is assigned to all RF chains. The second-level codebook is then

\[
\mathcal{P} = \{P^{(m,n)} \mid (m, n) \in [1, \ldots, M_{\text{ap}}] \times [1, \ldots, N_{\text{rf}}]\}.
\]

Fig. 5.6b shows a set of narrow beamformers \( \{P^{(3,n)}\} \), whose main beams point in directions within the sector beamformer \( P^{(3)} \). Note the gain improvement by a \( N_{\text{rf}} \) factor with respect to the sector beamformers.

Figure 5.6: Power radiation patterns (in dBi) generated by (a) sector beams — RF chains in the uplink use adjacent narrow beam, and (b) narrow beams — all RF chains in the downlink use the same beamforming vector, thus increasing the array gain by an \( N_{\text{rf}} \) factor. In this example, \( M_{\text{ap}} = 16 \) antennas and \( N_{\text{rf}} = 4 \).
Figure 5.7: (a) Sector beams constructed from orthogonal beamformers when only the subarray is active ($\mathbf{g}^{(3)}$ is highlighted). (b) Narrow beams (full array configuration) that overlap with the sector $\mathbf{g}^{(3)}$. In this example, $M_{\text{ue}} = 16$ and $M_{\text{sub}} = 8$.

### 5.4.4 Sector Codebook with a Subarray

We denote the sector codebook at the UEs as $\mathcal{G}_s$, which is used to select a $\mathbf{g}$ to operate both in the uplink and downlink ((5.1) and (5.4), respectively). The sector beamformers are available only when the $M_{\text{sub}}$-antenna subarray is active at the UE.

We use the orthogonal beamformer set $\mathcal{B}(M_{\text{sub}})$ to construct the sector codebook as

$$\mathcal{G}_s = \left\{ \mathbf{g}^{(m)} \mid m = 1, \ldots, M_{\text{sub}} \right\}.$$
with sector beamformers are defined as

\[ \mathbf{g}^{(m)} = \sqrt{\frac{M_{\text{sub}}}{M_{\text{ue}}}} \left[ \mathbf{b}_m (M_{\text{sub}}) \right] \]

(5.17)

where the first factor accounts for the power loss in the switching network that activates the subarray.

### 5.4.5 Narrow Beam Codebook with a Subarray

The narrow beam codebook \( \mathcal{G} \) is also used to select the \( \mathbf{g} \) at the UE, and is constructed using narrow orthogonal beams from \( \mathcal{B}(M_{\text{ue}}) \). This means that the UE uses the whole antenna array. We define the codebook as

\[ \mathcal{G} = \left\{ \mathbf{g}^{(m,n)} \mid (m, n) \in [1, \ldots, M_{\text{ap}}] \times \left[ 1, \ldots, \frac{M_{\text{ue}}}{M_{\text{sub}} + 1} \right] \right\}, \]

where the codewords are selected as

\[ l(m, n) = \left[ \frac{M_{\text{ue}}}{M_{\text{sub}}} (m - 1) - \frac{M_{\text{ue}}}{2M_{\text{sub}}} + n \right] \mod M_{\text{ue}}, \]

(5.18)

and we define the function

\[ [n] \mod M = \begin{cases} M, & \text{if } n \mod M = 0, \\ n \mod M & \text{if } n \mod M \neq 0. \end{cases} \]

This definition guarantees that, for a fixed \( m \), beamformers in the set \( \left\{ \mathbf{g}^{(m,n)} \right\}_{n=1}^{M_{\text{ue}}/M_{\text{sub}} + 1} \) point their main beams in directions overlapping the sector beam generated by \( \mathbf{g}^{(m)} \).

The function \([\cdot] \mod M\) is required so that the adjacency between the main beams generated by \( \mathbf{b}_1(M_{\text{ue}}) \) and \( \mathbf{b}_{M_{\text{ue}}}(M_{\text{ue}}) \) is taken into consideration. Fig. 5.7 shows the construction of codebooks \( \mathcal{G}_s \) and \( \mathcal{G} \) with \( M_{\text{ue}} = 16 \) antennas and \( M_{\text{sub}} = 8 \) antennas.

Note that the 3 beamformers in \( \left\{ \mathbf{g}^{(3,n)} \right\} \) overlap with the sector from \( \mathbf{g}^{(3)} \).
5.5 **mmWave OFDM Beamforming Algorithm**

In this section, we present an algorithm for the independent construction of $P_a$ and $P_d[k]$ in the multiuser downlink given by (5.8). The algorithm also finds beamformers $\{g_u\}$ for every UE and estimates their channel coefficients.

### 5.5.1 General Description of the Algorithm

Our algorithm for hybrid beamforming is split into two procedures: i) the analog beamforming design procedure (or *beam selection*) to solve (5.12), and ii) the digital beamformer design based on block-diagonalization (BD). *Beam selection* is applied to every UE independently and consists of 3 training transmission alternating between uplink (first) and downlink (second and third). This is equivalent to an alternating optimization of the problem in (5.12) where, starting with a fixed $g$ (Stage 1), the algorithm solves for $p$ using an exhaustive search over $P_s$. Given the hierarchical codebook structure and the fact that $P_s$ contains all the beamformers in $B(M_{ap})$ distributed over different RF chains, a search over $P_s$ is equivalent to a search over $B(M_{ap})$. Once an optimum configuration for $P_a$ is found, the AP begins training transmissions in the downlink while the UE performs an exhaustive search over $G_s$ (Stage 2) and then a limited search over $G$ (Stage 3). The limited search uses the superposition property of hierarchical orthogonal beamformers $G_s$ and $G$. When the training transmissions are finished and beamformers for all UEs are selected, we describe how to construct $P_a$ using the beamformers to each UE as columns. The digital beamforming design requires channel estimates $\{\hat{h}_{eq,u}\}$ obtained with training transmissions from each UE, where the AP uses $P_a$ as constructed in the previous
Algorithm 1 Analog Beam Selection - Stage 1 (Uplink)

**Input:** $\mathcal{P}_s$ known at the AP and $\mathcal{G}_s$ known at the UE. Training signal $\mathbf{x}[k]$.

**Output:** Beamformers $\mathbf{P}^\star$, $\mathbf{p}^\star$ known at the AP.

1. for $m' = 1$ to $M_{\text{sub}}$
   2. UE: set $\mathbf{g} = \mathbf{g}^{(m')}$.
   3. for $m = 1$ to $\frac{M_{\text{ap}}}{N_{\text{rf}}}$
      4. UE: Transmit $\mathbf{x}[k]$.
      5. AP: set $\mathbf{P}_a = \mathbf{P}^{(m)}$ and receive the signal.
      6. for every $k \in \mathcal{K}$
         7. AP: estimate $v^{m,m'}_n[k]$ using (5.19).
      8. end for
   9. end for
10. end for
11. AP: obtain $\mathbf{P}^\star$ and $\mathbf{p}^\star$ using (5.20) and (5.21).
12. return $\mathbf{P}^\star$ and $\mathbf{p}^\star$ known at the AP.

stage. Once these estimates are available, the AP applies BD to calculate $\mathbf{P}_d[k]$ as described in [4]. The details of each stage are presented in the following.

### 5.5.2 Beam Selection - Stage 1 (Uplink)

This stage is summarized in Algorithm 1 and Fig. 5.8 shows an implementation example. This stage’s goal is to find beamformer $\mathbf{p}^\star \in \mathcal{B}(M_{\text{ap}})$ that solves (5.12). The detailed steps are the following:

1. The UE uses every sector beamformer in $\mathcal{G}_s$ to transmit $\frac{M_{\text{ap}}}{N_{\text{rf}}}$ training sequences sequentially. For each sector beamformer at the UE, the AP receives the training signals by sequentially sweeping through its $\frac{M_{\text{ap}}}{N_{\text{rf}}}$ sector beams.

2. Let $\mathbf{y}^{m,m'}_{ul,n}[k]$ be the uplink received signal at RF chain $n$ when $\mathbf{g} = \mathbf{g}^{(m')}$ and $\mathbf{P}_a = \mathbf{P}^{(m)}$ (see (5.2)). We denote the corresponding channel coefficient as $v^{m,m'}_n[k]$. Since $\mathbf{y}^{m,m'}_{ul,n}[k]$ is a $T$-dimensional Gaussian row vector, we can obtain an ML channel coefficient estimation as [123, Sec. 4.4]
Figure 5.8: Beam selection procedure - First stage (uplink). The UE transmits the training signal using every sector beamformer in $\mathcal{G}_s$. For each UE beamformer, the AP tries every sector beamformer in $\mathcal{P}_s$. In this example, $M_{ap} = M_{ue} = 16$ antennas, $M_{sub} = 8$ antennas, and $N_{rf} = 4$ chains.

\[ \hat{v}_{m,m'}^{n}[k] = \frac{y_{nl}^{m,m'}[k] x^H[k]}{\|x[k]\|_2^2}. \]  

(5.19)

To reduce the computational complexity, this channel estimation can also be performed for a reduced subcarrier subset $\mathcal{K} \subseteq \{1, \ldots, K\}$. The subcarriers used for channel estimation are commonly known as pilot subcarriers and they can be used to track changes in the channel while other subcarriers are used for data transmission. We denote the number of pilot subcarriers as $K_t = |\mathcal{K}|$.
and analyze the algorithm’s performance with respect to $K_i$ in Section 5.6. The vector of channel estimates is $\hat{v}_n^{m,m'} = [\hat{v}_n^{m,m'}[1], \ldots, \hat{v}_n^{m,m'}[K]]^T$.

3. After trying all the possible AP/user sector beam combinations, the AP obtains the beamformers that maximizes the received power across subcarriers as

$$\left\{ \mathbf{P}^{(m*)}, \mathbf{g}^{(m*)}, n_* \right\} = \arg \max_{\mathbf{P}^{(m)} \in \mathcal{P}_s \atop \mathbf{g}^{(m')} \in \mathcal{G}_s \atop n \in \{1, \ldots, N_{rf}\}} \left\| \hat{v}_n^{m,m'} \right\|_2^2,$$

which is equivalent to an exhaustive search over $\mathcal{B}(M_{ap}) \times \mathcal{G}_s$, given that all of the elements in $\mathcal{B}(M_{ap})$ are used in $\mathcal{P}_s$. The optimal narrow AP beamformer solving (5.12) is in column (RF chain) $n_*$ within $\mathbf{P}^{(m*)}$. Due to the codebook structure in (5.14) and (5.15), this search leads to

$$\mathbf{p}^* = \mathbf{b}_{ln_*(m*)}(M_{ap}).$$

Note that multiple RF chains are used to reduce the number of required training transmissions by testing simultaneously $N_{rf}$ distinct beamformers using the codebook $\mathcal{P}_s$.

4. We use the notation $\mathbf{P}^* = \mathbf{P}^{(m*,n*)}$, which is the beamforming matrix with $\mathbf{p}^*$ in all its RF chains.

5. A total of $\frac{M_{ap}}{N_{rf}} \times M_{ue}$ training transmissions are required in this stage.

### 5.5.3 Beam Selection - Stage 2 (Downlink)

In this stage, summarized in Algorithm 2, the UE finds the sector beam $\mathbf{g}^{m*}$ that maximizes the estimated downlink received sum-power across subcarriers. Fig. 5.9 shows an implementation example. The procedure is the following:
Figure 5.9: Beam selection procedure - Second stage (downlink). The AP uses the optimal beamformer found in stage 1 to transmit training signals. The UE finds its best sector beamformer by sweeping through beamformers in $G_s$. In this example, $M_{ap} = M_{ue} = 16$ antennas, $M_{sub} = 8$ antennas, and $N_{rf} = 4$ chains. The sector where $g^{(2)}$ provides maximum received power (under a single-path model) is highlighted.

1. The AP sends training signals using $P_a = P^*$. 

2. The UE receives the training signals by sequentially using all the sector beams in $G_s$. 

3. Using (5.4) with $g = g^{(m')}$ and $P_a = P^*$, the received signal in this downlink is $y_{dl}^{(m')}[k]$. The corresponding channel coefficient is $w_{m'}[k]$. 

4. Following the same procedure as in Stage 1 the UE obtains an ML downlink channel coefficient estimate as 

$$
\hat{w}_{m'}[k] = \frac{y_{dl}^{(m')}[k]x^H[k]}{\|x[k]\|_2^2}.
$$

We define the vector of downlink channel estimates for UE beamformer $m'$ as 

$$
\mathbf{w}_{m'} = [\hat{w}_{m'}[1], \ldots, \hat{w}_{m'}[K]]^T.
$$
Algorithm 2 Analog Beam Selection - Stage 2 (Downlink)

Input: \( P^\star \) known at the AP. \( G_s \) known at the UE. Training signal \( x[k] \).
Output: Beamformer \( g^{(m'_\star)} \) known at the UE.

1: AP: set \( P_a = P^\star \)
2: for \( m' = 1 \) to \( \frac{M_{\text{sub}}}{N_{\text{sub}}} \) do
3: AP: transmit \( x[k] \).
4: UE: set \( g = g^{(m')} \) and receive the signal.
5: for every \( k \in K \) do
6: UE: estimate \( \hat{w}_{m'}[k] \) using (5.22).
7: end for
8: end for
9: UE: obtain \( g^{(m'_\star)} \) from (5.23).
10: return \( g^{(m'_\star)} \) known at the UE.

5. The UE finds its best sector beamformer as

\[
g^{(m'_\star)} = \arg \max_{g^{(m')} \in G_s} ||\hat{w}_{m'}||_2^2,
\]

(5.23)

this is, the UE selects the beam that maximizes the total estimated sum-power across subcarriers using exhaustive search within \( G_s \) for a fixed AP beamformer.

6. The number of required training sequence transmissions for this stage is \( M_{\text{sub}} \).

5.5.4 Beam Selection - Stage 3 (Downlink)

In this stage, summarized in Algorithm 3, the UE obtains a narrow beam that maximizes the estimated downlink sum-power. Fig. 5.10 shows an implementation example. The procedure is the following.

1. The AP sends training signals using \( P_a = P^\star \).

2. The UE receives the training signal by sequentially using all the narrow beams that overlap the sector generated by \( g^{(m'_\star)} \), i.e., the set \( \{g^{(m'_\star,n')}\} \subset G, n = 1, \ldots, \frac{M_{\text{ue}}}{M_{\text{sub}}} + 1 \).
Figure 5.10: Beam selection procedure - Third stage (downlink). The AP uses the optimal beamformer found in stage 1 to transmit training signals. The UE finds its best narrow beamformer by sweeping through beamformers in \( \{ g^{(m', n')} \} \), i.e. narrow beams that overlap with the optimal sector found in stage 2 (highlighted). In this example, \( M_{\text{ap}} = M_{\text{ue}} = 16 \) antennas, \( M_{\text{sub}} = 8 \) antennas, and \( N_{\text{rf}} = 4 \) chains.

3. The received signal in this downlink is denoted as \( y_{\text{dl}}^{(m', n')} [k] \), which has the form in (5.4) with \( g = g^{(m', n')} \) and \( P_a = P^* \). The corresponding channel coefficient is \( w_{m', n'} [k] \).

4. Following the same procedure as in Stages 1 and 2, the UE obtains an ML downlink channel coefficient estimate denoted \( \hat{w}_{m', n'} [k] \). The vector of downlink channel estimates for all subcarriers is \( \hat{w}_{m', n'} = [\hat{w}_{m', n'}[1], \ldots, \hat{w}_{m', n'}[K]]^T \).

5. The terminal finds its best narrow beamformer as

\[
g^* = \arg \max_{g^{(m', n')} \in G} \| \hat{w}_{m', n'} \|^2_2. \tag{5.24}
\]

Note that this is an heuristic algorithm since it assumes that a solution for \( g^* \) found using (5.23) and (5.24) is equivalent to an exhaustive search in (5.12). However, this is only true when the channel has one path. When multiple paths
Algorithm 3 Analog Beam Selection - Stage 3 (Downlink)

**Input:** $P^*$ known at the AP, $g^{(m',n')}$ and $G$ known at the UE. Training signal $x[k]$.

**Output:** Beamformer $g^*$ known at the terminal.

1. AP: set $P_a = P^*$.
2. for $n' = 1$ to $M_{ap} + 1$ do
3. AP: transmit $x[k]$.
4. UE: set $g = g^{(m',n')}$ and receive the signal.
5. for every $k \in \mathcal{K}$ do
6. UE: estimate $w_{m',n'}[k]$.
7. end for
8. end for
9. UE: find $g^*$ using (5.24).
10. return $g^*$ known at the UE.

If $U = N_{rf}$ the analog beamforming matrix is constructed as $P_a = [p_1^* \ p_2^* \cdots p_U^*]$.

Hence, every RF chain points to a different UE.

exist, our algorithm might converge to a suboptimal solution even at high SNR.

However, the performance loss due to this misalignment is negligible in the scenarios analyzed in Section 5.6.

6. The number of required training sequence transmissions for this stage is $M_{ap} + 1$.

At this point in the algorithm, both the AP and the UE know the beamformers that maximize the received power.

5.5.5 Beam Selection - Stage 4 (Analog Beamforming Matrix Construction)

Let $p_u^*$ and $g_u^*$ denote the optimum AP and UE beamformers for user $u$, respectively, obtained by applying the previous stages to all the UEs. We construct the analog beamforming matrix for the multiuser downlink in (5.4) depending on the number of RF chains and the number of users:

- If $U = N_{rf}$ the analog beamforming matrix is constructed as $P_a = [p_1^* \ p_2^* \cdots p_U^*]$. Hence, every RF chain points to a different UE.
• If $U < N_{\text{rf}}$, the beamforming vector for any given user should appear at least once as a column in the analog beamforming matrix. For example, if $N_{\text{rf}} = 4$ and $U = 2$, $P_a = [p_1^* \ p_1^* \ p_2^* \ p_2^*]$ is an acceptable matrix. The number of times that a beamforming vector appears in the matrix has no influence in our algorithm.

• If $U > N_{\text{rf}}$, simultaneous communications with all the UEs are not feasible using linear precoding only since $\text{rank}(P_aP_d[k]) < U$. This is also the case when two or more UEs share the same AP beamformer vector. In such cases, other multiple access techniques should be combined with hybrid precoding to communicate with all the users. The algorithm could also be repeated for some UEs to find alternative beams to form feasible analog beamforming matrices.

### 5.5.6 Digital Beamformer Design

After obtaining a suitable $P_a$, the AP calculates a digital beamformer $P_d[k]$. We assume that the analog precoding matrix is able to resolve signals to different users such that $\text{rank}(P_a[k]) = U$, and we use block-diagonalization (BD) to eliminate residual inter-user interference after analog beamforming [4]. The first step is to obtain equivalent MISO channel estimates (as given by (5.9)) using the following procedure, which is executed for every UE:

• User $u$ sends the training signal in the uplink using $g_u^*$.

• The AP uses $P_a$ as constructed from the beam selection procedure.

---

\(^6\)Further algorithm extensions could consider power constraints per RF chain. In that case, the multiplicity of a vector in the analog beamforming matrix might be used as a design parameter.
• The AP obtains a maximum likelihood channel coefficient estimation at each RF chain. Then the equivalent MISO channel estimate for user $u$ is $\hat{h}_{eq,u}[k] = [\hat{v}_{u,1}[k], \ldots, \hat{v}_{u,N_{rf}}[k]]$, where $\hat{v}_{u,n}[k]$ denotes the estimate for RF chain $n$.

• $U$ total training transmissions are required in this stage.

BD enforces the zero-interference constraints

$$\hat{h}_{eq,u}[k] [P_d[k]]_{:,u'} = 0, \forall u \neq u', \forall k,$$

and then maximizes the received signal power at each user, as shown in [4].

5.6 Numerical Results and Discussion

In this section, we describe numerical experiments that validate the proposed algorithm under standard operating conditions in indoor environments.

5.6.1 Statistical Algorithm Characterization

First, we provide an algorithm performance characterization under a statistical single-path channel model with angles of departure and arrival uniformly distributed over the interval $[0, \pi]$. This is represented as $L = 1$ and $\alpha_1 \sim \mathcal{CN}(0, 1)$ in (5.10), for which we also used a $-15$ dB coupling between adjacent antenna elements as described in Section 5.2. Under these conditions, the channel power constraint is $\mathbb{E} \{||H[k]||_2^2\} = M_{ap} M_{ue}, \forall k$. ULAs with 16 and 32 antennas were analyzed since they fit typical form factors for indoor terminals. Other parameters are summarized in Table 5.1 and were taken from [1, p. 446].

The algorithm performance is analyzed in terms of two metrics. The first metric is the rate at which the algorithm fails to find the global solution to (5.11) given a
large number of channel realizations. We call this parameter the beam selection error rate (BSER), and we obtained it for $10^5$ channel realizations as shown in Fig. 5.11 for different numbers of pilot subcarriers. If $\text{rank}(P_a) < U$ for a particular realization, the result is not considered for the calculation. There is a linear decay in BSER by increasing the SNR due to the reduction in the ML channel estimation error. In addition, we observe that lower BSER is obtained with fewer pilot subcarriers at low SNR, but more pilot subcarriers perform better at high SNR. This is because having more pilot subcarriers helps to better capture beam squint effects at large SNR. In contrast, low SNR requires focusing the power on fewer pilot subcarriers to obtain better channels estimations. For comparison, we also used a channel model with 3 paths (uniformly distributed AoD and AoA with relative powers 0, $-10$, and $-10$ dB, preserving the channel power constraint). The algorithm has performance lower bound at $\text{BSER} \approx 7\%$ for the 3-path channel, and errors are made mostly when there
Figure 5.11: Beam selection error rate for single-path and 3-path channel models with $M_{ap} = M_{ue} = 16$. Plots for different number of pilot subcarriers.

are 2 dominant paths with similar total sum-power. In these cases, the algorithm converges to local optima of (5.11) without significant power losses.

The second performance metric is the achievable sum-rate $R$ (spectral efficiency) defined as

$$R = \frac{1}{K} \sum_{k=1}^{K} \sum_{u=1}^{U} \mathbb{E} \left[ \log_2 \left( 1 + \frac{\eta_u}{\eta_{int} + \sigma_z^2} \right) \right], \quad (5.25)$$

where $\eta_u$ and $\eta_{int}^u$ are the desired signal and the interference powers given by

$$\eta_u = \frac{\rho_u}{K} \left| g_u^H H_u[k] P_a [P_{d[k]}]_{:,u} \right|^2,$$

$$\eta_{int}^u = \sum_{u' \neq u} \frac{\rho_{u'}}{K} \left| g_u^H H_u[k] P_a [P_{d[k]}]_{:,u'} \right|^2,$$

respectively. BD guarantees that $\eta_{int}^u$ is set to zero under perfect CSI. We evaluated the achievable sum-rate of our algorithm in the single-path channel model. The results
are shown in Fig. 5.12. As a reference, we used an ideal fully-digital beamforming architecture (one RF chain per antenna) with BD at the transmitter, eigenbeamforming at the receivers, and perfect CSI. The beam selection procedure was made with $K_t = 16$ pilot subcarriers in each case. Fig. 5.12a shows the achievable sum-rate for 2 and 4 users with 16 antennas at the AP and each UE. Fig. 5.12b shows the achievable sum rate for 16 and 32 antennas at every terminal when there are 4 users in the system. The algorithm’s performance is approximately only 3 dB below that of fully-digital BD regardless of the number of antennas and users. This small difference is remarkable taking into account the hardware simplification from the fully-digital to the hybrid beamforming architecture. Sum-rate results with the 3-path channel model have negligible variations with respect to the single-path model and are thus omitted here. This indicates that selection of suboptimal beamformers does not have significant impact over the performance in terms of achievable sum-rate.

5.6.2 Ray-Tracing Validation

We conducted ray-tracing simulations in scenarios specified in the IEEE 802.11ay channel models for 60 GHz WLAN [110]. Our goal is to examine the algorithm operation under real-life conditions and determine potential SNR regimes, propagation features, and algorithm accuracy. We selected the conference room evaluation scenario depicted in Fig. 5.13, where users will access wireless services such as ultra-high-definition video streaming, augmented and virtual reality, and mass-data distribution in dense hot-spots [110]. Other scenarios, such as living rooms and enterprise cubicle offices, have similar geometric characteristics and have the same use cases. We set up four UEs in the conference room, with their antenna arrays located 10 centimeters
Figure 5.12: Achievable sum rate $R$ as a function of SNR $= \frac{\rho}{\sigma^2}$ for the hybrid beamforming algorithm and ideal fully-digital BD beamforming. (a) Different number of users with $M_{ap} = M_{ue} = 16$ antennas. (b) Different number of antennas ($M_{ap} = M_{ue}$) with $U = 4$ users.
Table 5.2: Algorithm Achievable Rates [bits/s/Hz] vs. BD in the Conference Room Scenario

<table>
<thead>
<tr>
<th>Number of Antennas</th>
<th>User 1</th>
<th>User 2</th>
<th>User 3</th>
<th>User 4</th>
<th>Average per User</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_{\text{ap}} = M_{\text{ue}} = 16)</td>
<td>5.10 (6.83)</td>
<td>5.10 (6.83)</td>
<td>5.10 (6.83)</td>
<td>3.21 (3.30)</td>
<td>1.63 (1.68)</td>
<td>5.10 (6.83)</td>
</tr>
<tr>
<td></td>
<td>1.63 (4.71)</td>
<td>0.44 (0.95)</td>
<td>1.59 (1.59)</td>
<td>1.77 (2.23)</td>
<td>7.09 (8.93)</td>
<td></td>
</tr>
<tr>
<td>(M_{\text{ap}} = 32, M_{\text{ue}} = 16)</td>
<td>5.92 (6.87)</td>
<td>5.92 (6.87)</td>
<td>5.92 (6.87)</td>
<td>3.99 (4.24)</td>
<td>2.25 (2.45)</td>
<td>5.92 (6.87)</td>
</tr>
<tr>
<td></td>
<td>7.42 (7.94)</td>
<td>1.17 (1.71)</td>
<td>2.23 (2.41)</td>
<td>2.77 (3.13)</td>
<td>11.09 (12.52)</td>
<td></td>
</tr>
<tr>
<td>(M_{\text{ap}} = M_{\text{ue}} = 32)</td>
<td>6.84 (7.15)</td>
<td>6.84 (7.15)</td>
<td>6.84 (7.15)</td>
<td>4.87 (5.18)</td>
<td>3.01 (3.30)</td>
<td>6.84 (7.15)</td>
</tr>
<tr>
<td></td>
<td>8.13 (8.93)</td>
<td>1.17 (1.71)</td>
<td>2.99 (3.25)</td>
<td>3.38 (3.98)</td>
<td>13.52 (15.92)</td>
<td></td>
</tr>
</tbody>
</table>

* All the values are rounded. Values in parenthesis are the reference achievable rates with fully-digital BD.

above the central table. The AP array height is 2.9 meters (10 centimeters under the ceiling), which is a typical deployment in WiFi networks. We modeled this conference room scenario using the commercial ray-tracing software Wireless Insite® developed by REMCOM®, as shown in Fig. 5.13. This simulator uses a diffuse scattering model to simulate scattering from surfaces at mmWave frequencies [124,125]. This approach approximates measured channel features [126,127]. Under the diffuse scattering model, each interaction of a ray at a dielectric boundary is a source for multiple (scattered) rays, whose powers depend on a configurable angular distribution. We set the diffuse scattering parameters following the developer’s recommendations to approximate measured channel impulse responses [126]. The AP and UEs use planar ULAs with 16 or 32 antennas and beamforming codebooks designed with the method described in Section 5.4. This type of arrays are suited for common terminals such as laptops, tablets, smartphones, TVs, and video projectors. We set up the arrays such that the beam codebook sweeps along the azimuth angle. Antenna arrays at UEs have maximum gain at 0° elevation angle, while the AP are tilted −45° in the elevation angle, pointing to the middle of the conference room.
Figure 5.13: Conference room scenario model in Wireless Insite®.

Ray-tracing simulations are used to obtain frequency-dependent channel matrices for every user in the room with the parameters in Table 5.1. The algorithm is then applied to this scenario, where global optima for (5.11) are found with BSER = 0 after $10^3$ noise realizations. This outstanding no-error performance is due to the presence of strong line of sight or reflections and a well-oriented antenna array at the AP. Fig. 5.14 shows the frequency response between RF chain 1 and UE 1 (whose beams are paired) and also its equivalent power-delay profile obtained via inverse discrete Fourier transform. It is clear that, after beamforming, the channel impulse response is sparse. Received power varies with frequency mainly due to beam squint, but also due to weak multipath contributions.

Table 5.2 shows the achievable rates in this scenario contrasting our hybrid beamforming algorithm and an ideal fully-digital BD beamforming solution. Results are shown with a system load of 1, 2, and 4 UEs, and different antenna array sizes. A
maximum spectral efficiency of 13.52 bits/s/Hz is achieved in this conference room scenario, which is sufficient to provide multi-Gbps links to all users. The algorithm achieves sum-rates of more than 74% of those obtained with fully-digital BD, with a much simpler hardware configuration.

5.6.3 Implementation Considerations

Some further practical considerations for the algorithm operation are listed below:

- Phase shifters with \( \log_2 M_{ap} \) control bits are required at the AP and \( \log_2 M_{ue} \) bits at the UE since the number of orthogonal beamformers is equal to the number of antennas.
• The algorithm requires the transmission of a total of \( \frac{M_{\text{ap}}}{N_{\text{rf}}} M_{\text{sub}} + M_{\text{sub}} + \frac{M_{\text{ue}}}{M_{\text{sub}}} + 1 \) training sequences. Guard intervals are required between training transmissions so the phase shifters and switches can reconfigure.

• Using the parameters in Table 5.1 with \( M_{\text{ap}} = M_{\text{ue}} = 32 \), \( M_{\text{sub}} = 8 \), and \( N_{\text{rf}} = 4 \), the total number of training transmissions would be 77 with a required time of approximately 3.75 \( \mu \)s. Similarly, if we consider a settling time of 50 ns to reconfigure switches and phase shifters [121], the minimum time required for guard intervals would be 3.85 \( \mu \)s. Thus, a total time in the order of tens of microseconds would be required to perform the multiuser beamforming procedure in the analyzed indoor scenarios.

Given the considerations above, the training procedure can efficiently track changes in the channel with periodic repetitions controlled by higher protocol layers.

5.7 Conclusions

We presented an hybrid analog/digital beamforming algorithm for multiuser wideband mmWave systems. The algorithm is designed to work with a fully-connected hybrid architecture at the AP consisting one antenna array, multiple RF chains, and one set of phase shifters for each RF chain. The UEs are equipped with one antenna array, one RF chain, and a switching networks with two operation modes: full-array or subarray. We propose a novel design of beamforming codebooks with sector beams and narrow beams using these hardware configurations. The codebooks are based on the orthogonality principle of beamforming (steering) vectors of uniform arrays, which preserve the hierarchical codebook structure along the operation bandwidth.
We presented system models that account for realistic antenna array effects such as beam squint, antenna coupling, and individual element radiation patterns.

The algorithm decouples the design of analog and digital beamformers by first finding analog beamformers that maximize the received power at each UE (beam selection procedure) and then calculating a digital beamformer that eliminates inter-user interference. For beam selection, the algorithm uses the orthogonal codebooks to transmit training signals (alternating between uplink and downlink) with the objective of finding beamformers at the AP and UE that maximize the received sum-power across subcarriers. The digital beamformer is designed using block-diagonalization to suppress inter-user interference using multiple RF chains at the AP.

We provided numerical evaluations of the algorithm in both statistical (Monte Carlo) and real-life scenarios. First, algorithm performance in terms of beam selection error rate (BSER) and achievable sum-rate was analyzed for a single-path channel model with uniformly distributed angles of departure and arrival. Results show that the BSER decreases linearly by increasing SNR due to better channel estimates. In addition, errors in the analog beam selection do not significantly impact the achievable sum-rate, where the algorithm’s performance is approximately only 3 dB below of that obtained with ideal fully-digital BD. Furthermore, we evaluated the algorithm performance in the conference room scenario specified in the IEEE 802.11ay mmWave WLAN standard. We obtained channel matrices for this scenario using a commercial ray-tracing simulator that uses diffuse scattering to approximate measured mmWave channels. The algorithm achieves 13.52 bits/s/Hz spectral efficiency under common operating condition in this scenario when the AP and UEs have arrays with 32 antennas each. This efficiency is close to the 15.92 bits/s/Hz obtained with
ideal fully-digital BD, which would require one RF chain for each antenna element in the arrays. Remarkably, the algorithm enables multi-Gbps connectivity to multiple users in this mmWave wideband system, with antenna and hardware limitations included.
Chapter 6: Conclusion

In this dissertation, we introduced several beamforming (linear processing) solutions for frequency-selective and mmWave channels. For frequency-selective channels we focused on two approaches: i) using the electromagnetic time-reversal (TR) effect to directly design wideband beamformers, and ii) generalize the block-diagonalization procedure used in narrowband channels to the frequency-selective case. For the mmWave beamforming design, we focused on specific hardware limitations of mmWave technologies. In particular, we focused on hybrid analog/digital solutions including antenna-specific impairments.

In Chapter 2, we introduced an equalized TR (ETR) technique as a solution to mitigate ISI. ETR uses a single zero-forcing equalizer at the transmitter in cascade with the TR pre-filters. Unlike previous approaches, we analytically showed that ETR greatly enhances the performance of conventional TR with low impact to its beamforming capability. An upper bound on the received power of ETR was also derived, which corresponds to a lower bound on the probability of bit error. The spatial focusing performance of conventional TR and ETR was analyzed by calculating the signal power at an unintended receiver with uncorrelated CIR. We showed that the equalization and spatial focusing properties of TR improve with either the channel
delay spread or the signal bandwidth. Moreover, it was shown that the use of ETR has a small impact over the spatial focusing parameters.

In Chapter 3, we analyzed a baseband TR beamforming system for mm-wave multi-user massive MIMO. We studied conventional TR and ETR in multiuser scenarios and found that their performance is IUI limited. We also noticed that, when the number of antennas is large, the ratio between the desired signal power and ISI or IUI power increases. Thus, we confirm the potential of TR as a beamforming technology for massive MIMO. After identifying IUI as the main detection impairment for TR systems, we proposed a modified technique called interference-nulling TR (INTR). This technique calculates transmit pre-filters in the frequency domain that set the IUI to zero and that are closest to the conventional TR solution. By means of numerical simulations, we verified that INTR outperforms conventional TR with respect to average BER and achievable sum rate. In particular, we note that INTR performance is extremely tolerant to increases in the number of users.

In Chapter 4, we explored the generalization of BD precoding, originally proposed for frequency-flat MIMO broadcast channels, to the frequency-selective case. We derived the conditions under which BD is feasible for block transmissions in frequency-selective MIMO broadcast channels: the transmitted block length should be sufficiently large and the number of transmit antennas should be greater than or equal to the number of users. Even though BD eliminates IUI, frequency selectivity induces ISI in the received signal. Thus, we proposed different approaches to mitigate or suppress ISI. Extensive numerical simulations showed that all the proposed BD solutions for frequency-selective MIMO broadcast channels outperform conventional TR beamforming. We showed that joint processing at the transmitter and the
receiver (JPBD) eliminates ISI and IUI, and that its multiplexing gain is equal to the number of users for an infinite block length. Remarkably, we showed that using larger block lengths (increasing time-domain redundancy) actually increases the achievable rate when the number of users increases in the system. This is due to the reduction in the number of singular values close to zero in the equivalent channel matrices.

Finally, Chapter 5 presented an hybrid analog/digital beamforming algorithm for multiuser wideband mmWave systems. The algorithm accounts for hardware constraints (limited number of RF chains) and realistic antenna array effects such as beam squint, antenna coupling, and individual element radiation patterns. We propose novel beamforming codebooks based on the orthogonality principle of beamforming vectors in uniform arrays, which preserve the hierarchical codebook structure along the operation bandwidth. The algorithm decouples the design of analog and digital beamformers by first finding analog beamformers that maximize the received power at each UE (beam selection procedure) and then calculating a digital beamformer that eliminates inter-user interference. For beam selection, the algorithm uses the orthogonal codebooks to transmit training signals alternating between uplink and downlink. The digital beamformer is designed using block-diagonalization to suppress inter-user interference using multiple RF chains at the AP. We provided numerical evaluations of the algorithm in both statistical (Monte Carlo) and real-life scenarios. Results show that the beam selection error decreases linearly by increasing SNR due to better channel estimates. In addition, errors in the analog beam selection do not significantly impact the achievable sum-rate, where the algorithm’s performance is approximately only 3 dB below of that obtained with ideal fully-digital BD. For a realistic conference room scenario modeled using ray-tracing, the algorithm achieves
13.52 bits/s/Hz spectral efficiency under common operating condition when the AP and UEs have arrays with 32 antennas each, with 4 RF chains at the AP and 1 at each UE. This efficiency is close to the 15.92 bits/s/Hz obtained with ideal fully-digital BD, which would require one RF chain for each antenna element in the arrays. Remarkably, the algorithm enables multi-Gbps connectivity to multiple users in this mmWave wideband system, with antenna and hardware limitations included.
Appendix A: Upper Bound on the ETR Received Power

In this Appendix, we derive an upper bound for the received power with the proposed ETR technique. From (2.4), (2.5) and Parseval’s theorem, it follows that

\[
P_g = \frac{1}{2L + L_E - 2} \sum_{i=1}^{M_T} \sum_{k=0}^{2L + L_E - 3} \left| H_i^*[k] G_{zf}[k] e^{-j \frac{2 \pi (L-1)}{2L + L_E - 7} k} \right|^2
\]

where we have used zero padding in order to represent the linear convolution. Now, the received signal power using ETR beamforming is

\[
P_{eq} = \rho \mathbb{E} \left[ P_g \right] = \rho \mathbb{E} \left[ \frac{2L + L_E - 2}{\sum_{k=0}^{2L + L_E - 3} \frac{1}{\sum_{i=1}^{M_T} |H_i[k]|^2}} \right].
\]  

(A.1)

Note that the expression inside the expectation operator in (A.1) is a concave function of \(|H_i[k]|^2\) (i.e. it is a double composition of an affine function and its reciprocal) [128, Sec. 3.2.4]. Hence, from Jensen’s inequality we get

\[
P_{eq} \leq \frac{\rho (2L + L_E - 2)}{\sum_{k=0}^{2L + L_E - 3} \frac{1}{\sum_{i=1}^{M_T} \mathbb{E}[|H_i[k]|^2]}}.
\]  

(A.2)

By using (2.7), uncorrelated scattering and the DFT definition, we also have
\[
\mathbb{E}[|H_i[k]|^2] = \mathbb{E}\left[\sum_{m=0}^{2L+L_E-3} \sum_{n=0}^{2L+L_E-3} h_i[m] h_i^*[n] \times e^{-j \frac{2\pi m}{2L+L_E-2} k} e^{j \frac{2\pi n}{2L+L_E-2} k}\right]
\]

\[
= \mathbb{E}\left[\sum_{n=0}^{2L+L_E-3} |h_i[n]|^2\right] = \Gamma. \tag{A.3}
\]

Replacing (A.3) in (A.2):

\[P_{eq} \leq \rho M \Gamma.\]
Appendix B: Approximation to the ISI power in conventional TR

In this appendix, we derive an approximation to the ISI power in conventional TR systems and analyze the approximation error using the variance of the normalization factor. From (2.1) and (2.11), the ISI power is

\[
P_{ISI} = \rho \frac{\left[ \sum_{i=1}^{M} \sum_{l=0}^{L-1} \sum_{n=0}^{L-1} h_i[n] h_i^*[L - 1 - l + n] \right]^2}{\sum_{i=1}^{M} \sum_{l=0}^{L-1} |h_i[l]|^2}.
\] (B.1)

Let \(a\) and \(b\) be two correlated random variables. According to [67] and [68], an expansion for the expectation of the ratio of \(a\) and \(b\) is

\[
E \left[ \frac{a}{b} \mid b \neq 0 \right] = \frac{E[a]}{E[b]} + \sum_{i=1}^{\infty} (-1)^i \frac{E[a] \langle^i b \rangle + \langle a,^i b \rangle}{E[b]^{i+1}},
\] (B.2)

where \(\langle^i b \rangle = E[(b - E[b])^i]\) is the \(i\)-th central moment of \(b\) and \(\langle a,^i b \rangle = E[(a - E[a])(b - E[b])^i]\) is the \(i\)-th mixed central moment of \(b\) and \(a\). Thus, if we only consider the first term in the expansion, we can approximate (B.1) as

\[
\hat{P}_{ISI} = \rho \frac{E \left[ \sum_{i=1}^{M} \sum_{l=0}^{L-1} \sum_{n=0}^{L-1} h_i[n] h_i^*[L - 1 - l + n] \right]^2}{E \left[ \sum_{i=1}^{M} \sum_{l=0}^{L-1} |h_i[l]|^2 \right]}.
\] (B.3)
We will analyze the approximation error later in this appendix. Since the channel has uncorrelated scattering and \( h_i[n] \) has zero mean, we note that

\[
\mathbb{E} [h_i[n]h_i^*[L - 1 - l + n]h_i^*[n']h_i'[L - 1 - l' + n']] = 0
\]

if \( i \neq i' \) or \( n \neq n' \).

Thus, the only non-zero terms in the numerator of (B.3) are of the form

\[
\mathbb{E} [\|h_i[n]\|^2] \mathbb{E} [\|h_i[L - 1 - l + n]\|^2].
\]

Using these results we get:

\[
\hat{P}_{ISI} = \rho \frac{M \Gamma^2}{M} \sum_{l=0}^{2L-2} \sum_{i=1}^{M} \sum_{n=0}^{L-1} \frac{\mathbb{E} [\|h_i[n]\|^2] \mathbb{E} [\|h_i[L - 1 - l + n]\|^2]}{L - l + n},
\]

where the constraints in the sum over \( n \) come from the definition of the PDP for \( n \in \{0, \ldots, L - 1\} \), so \( 0 \leq L - 1 - l + n \leq L - 1 \) must hold. Now, notice that if we only consider the first term in the expansion (B.2), the equality holds if the variance of the denominator is vanishingly small. Thus, we use this variance (denoted \( \text{Var}[\hat{P}_h] \)) as a measure of the approximation error. Using the given channel models, this variance is

\[
\text{Var}[\hat{P}_h]^{(1)} = M \Gamma^2 \left( \frac{1 + e^{-\frac{LT_s}{\sigma}}}{1 + e^{-\frac{T_s}{\sigma}}} \right) \left( \frac{1 - e^{-\frac{L}{T_s}}}{1 - e^{-\frac{T_s}{\sigma}}} \right),
\]

\[
\text{Var}[\hat{P}_h]^{(2)} = M \Gamma^2 \left( \frac{\sum_{n=0}^{L-1} e^{-\frac{2nT_s}{\sigma_1}} + \gamma^2 \sum_{n=L_1}^{L-1} e^{-\frac{2(n-L_1)T_s}{\sigma_2}} + 2\gamma \sum_{n=L_1}^{L-1} e^{-\frac{nT_s}{\sigma_1}} e^{-\frac{(n-L_1)T_s}{\sigma_2}}}{\left( \sum_{n=0}^{L-1} e^{-\frac{nT_s}{\sigma_1}} + \gamma \sum_{n=L_1}^{L-1} e^{-\frac{(n-L_1)T_s}{\sigma_2}} \right)^2} \right).
\]

Fig. B.1 shows the variance of the normalization factor as a function of the CIR length for different values of \( T_s/\sigma \) or \( T_s/\sigma_1 \), and according to the channel model. The variance approaches zero for decreasing values of \( T_s/\sigma \). This means that the approximation given by (B.4) improves in scenarios with larger delay spread, or smaller tap
Figure B.1: Variance of the normalization factor for Model 1 - eq. (B.5), and Model 2 - eq. (B.6). Note that the variance is diminishingly small when the ratio $T_s/\sigma$ is small (richer scattering or large bandwidth). Thus, a smaller approximation error between $P_{ISI}$ and $\hat{P}_{ISI}$ is expected in Model 2, and also for smaller tap separations and/or larger delay spreads. Other parameters are: $L = 33$, $L_1 = 8$, $L_2 = 17$, $\gamma = 0.4786$, $\sigma_1 = 8$ ns, and $\sigma_2 = 1.75\sigma_1$.

separation. For example, it is observed that a better approximation is achieved for Model 2 due to the stronger scattering (delayed components with larger power) under same CIR length.
Appendix C: Total Power at an Unintended Receiver using ETR

In this appendix we derive a closed-form approximation for the total power at an unintended receiver when our proposed ETR technique is used. For this, we use the same procedure as in B. From (2.19), the total power at an unintended receiver is

\[
P_{\text{eq}}^{\text{int}} = \rho E \left[ \frac{2^{L+L_{E}-3} \sum_{n=0}^{M_{T}} \sum_{i=1}^{M_{T}} |g[n] \otimes h_{i}^*[L - 1 - n] \otimes h_{u,i}[n]|^{2}}{2^{L+L_{E}-3} \sum_{n=0}^{M_{T}} \sum_{i=1}^{M_{T}} |g[n] \otimes h_{i}^*[L - 1 - n]|^{2}} \right],
\]

(C.1)

which takes into account the desired signal power and the ISI power. Using Parseval’s theorem,

\[
P_{\text{eq}}^{\text{int}} = \rho E \left[ \frac{2^{L+L_{E}-3} \sum_{k=0}^{M_{T}} \sum_{i=1}^{M_{T}} |G[k]H_{i}^*[k]H_{u,i}[k]e^{-j2\pi(L-1)/2^{L+L_{E}-2}k}|^{2}}{2^{L+L_{E}-3} \sum_{k=0}^{M_{T}} \sum_{i=1}^{M_{T}} |G[k]|^{2}|H_{i}[k]|^{2}} \right],
\]

(C.2)

where \( H_{u,i}[k] \) is the DFT of \( h_{u,i}[n] \). Using the same expansion as in B, (C.2) can be approximated as
where $H_i[k]$ and $H_{u,i'}[k']$ have zero mean and are uncorrelated for all $i$, $k$, $i'$ and $k'$. Also, $H_{u,i}[k]$ and $H_{u,i}[k]$ are uncorrelated $\forall k$ if $i \neq i'$. Then, (C.3) becomes

\[
\hat{P}_{\text{int}}^{eq} = \rho \frac{\mathbb{E} \left[ \sum_{k=0}^{2L+L_E-3} \sum_{i=1}^{MT} \left| G[k] \right|^2 \left| H_i[k] \right|^2 \left| H_{u,i}[k] \right|^2 \right]}{\mathbb{E} \left[ \sum_{k=0}^{2L+L_E-3} \sum_{i=1}^{MT} \left| G[k] \right|^2 \left| H_i[k] \right|^2 \right]^{2}}.
\]

Finally, from the channel power normalization, $\mathbb{E} \left[ \left| H_{u,i}[k] \right|^2 \right] = \Gamma$, so

\[
\hat{P}_{\text{int}}^{eq} = \rho \Gamma.
\]
Appendix D: TRBD Precoder Solution

We obtain the TRBD precoder design by solving

$$\min_{\bar{P}_k} \left\| \bar{V}_k^{(0)} \bar{P}_k - \bar{H}_k \right\|_F^2, \quad \text{s.t.} \quad \left\| \bar{V}_k^{(0)} \bar{P}_k \right\|_F^2 = 1,$$

(D.1)

The Lagrangian of (D.1) is

$$\mathcal{L}_{\text{TR}} (\bar{P}_k, \lambda_k) = \left\| \bar{V}_k^{(0)} \bar{P}_k - \bar{H}_k \right\|_F^2 + \lambda_k \left( \left\| \bar{V}_k^{(0)} \bar{P}_k \right\|_F^2 - 1 \right)
\begin{equation}
= \text{Tr} \left( \bar{V}_k^{(0)} \bar{P}_k \bar{P}_k^H \bar{V}_k^{(0)H} \right) + \text{Tr} \left( \bar{H}_k \bar{H}_k^H \right) - \text{Tr} \left( \bar{V}_k^{(0)} \bar{P}_k \bar{H}_k^H \right)
\end{equation}
\begin{equation}
- \text{Tr} \left( \bar{H}_k \bar{P}_k^H \bar{V}_k^{(0)H} \right) + \lambda_k \left[ \text{Tr} \left( \bar{V}_k^{(0)} \bar{P}_k \bar{V}_k^{(0)H} \right) - 1 \right]
\end{equation}
\begin{equation}
= \text{Tr} \left( \bar{P}_k \bar{P}_k^H \right) + 1 - \text{Tr} \left( \bar{V}_k^{(0)} \bar{P}_k \bar{H}_k^H \right) - \text{Tr} \left( \bar{H}_k \bar{P}_k^H \bar{V}_k^{(0)H} \right)
\end{equation}
\begin{equation}
+ \lambda_k \left[ \text{Tr} \left( \bar{P}_k \bar{P}_k^H \right) - 1 \right],
\end{equation}

where we used the cyclic permutation invariance of the trace, the TR precoder normalization \( \text{Tr} \left( \bar{H}_k \bar{H}_k^H \right) = 1 \), and the fact that \( \bar{V}_k^{(0)H} \bar{V}_k^{(0)} = \mathbf{I}_{B_v} \) (the columns of \( \bar{V}_k^{(0)} \) are orthonormal). \( \lambda_k \in \mathbb{R} \) is a Lagrange multiplier. Taking the Lagrangian derivative with respect to \( \bar{P}_k \) yields the Karush-Kuhn-Tucker (KKT) condition [129]

$$\frac{\partial \mathcal{L}_{\text{TR}} (\bar{P}_k, \lambda_k)}{\partial \bar{P}_k} = \bar{P}_k^* - \bar{V}_k^{(0)T} \bar{H}_k^* + \lambda_k \bar{P}_k^* = 0.$$

(D.2)

Using the complex-conjugate of (D.2) and applying the constraint \( \left\| \bar{V}_k^{(0)} \bar{P}_k \right\|_F^2 = \text{Tr}\{\bar{P}_k \bar{P}_k^H\} = 1 \) we have

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Replacing (D.3) in (4.5) yields

\[
\mathbf{P}_{TR}^{k} = \frac{\mathbf{V}_{k}^{(0)H} \mathbf{H}_{k}}{\| \mathbf{V}_{k}^{(0)H} \mathbf{H}_{k} \|_{F}}. 
\]  

(D.4)
Appendix E: EBD Precoder Solution

The EBD precoder, which operates as an equalizer at the transmitter, is found by solving

\[
\min_{\bar{P}_k} \left\| C_k \bar{P}_k - I_B \right\|_F^2, \quad \text{s.t.} \quad \left\| \bar{V}_k^{(0)} \bar{P}_k \right\|_F^2 = 1, \quad (E.1)
\]

whose Lagrangian is

\[
\mathcal{L}_{eq}(\bar{P}_k, \mu_k) = \text{Tr} \left( C_k \bar{P}_k \bar{P}^H_k C^H_k \right) - \text{Tr} \left( C_k \bar{P}_k \right) - \text{Tr} \left( \bar{P}^H_k C^H_k \right) + B \\
+ \mu_k \left[ \text{Tr} \left( \tilde{V}_k^{(0)} \bar{P}_k \bar{P}^H_k \tilde{V}_k^{(0)H} \right) - 1 \right].
\]

Using the cyclic permutation invariance of the trace and \( \tilde{V}_k^{(0)H} \tilde{V}_k^{(0)} = I_{Bc} \), the KKT condition for (E.1) is

\[
\frac{\partial \mathcal{L}_{eq}(\bar{P}_k, \mu_k)}{\partial \bar{P}_k} = C_k^T C_k^* \bar{P}_k^* - C_k^T + \mu_k \bar{P}_k^* = 0. \quad (E.2)
\]

Using the complex-conjugate of (E.2) we get \( \bar{P}_k^{eq} = (C_k^H C_k + \mu_k I_{Bc})^{-1} C_k^H \), which replacing into (4.5) yields the EBD precoder

\[
\bar{P}_k^{eq} = \bar{V}_k^{(0)} (C_k^H C_k + \mu_k I_{Bc})^{-1} C_k^H. \quad (E.3)
\]

Note that, since \( C_k^H C_k \) is Hermitian, the eigendecomposition \( C_k^H C_k = U_{C_k} \Lambda_{C_k} U_{C_k}^H \) is possible, where \( U_{C_k} \) is a unitary matrix and \( \Lambda_{C_k} = \text{diag} (\lambda_{C_k,1}, \ldots, \lambda_{C_k,Bc}) \) is the
diagonal matrix with the (positive real) eigenvalues of $C_k^H C_k$. By enforcing the constraint $\text{Tr} \left( P_k P_k^H \right) = 1$, we get

\[
\text{Tr} \left( P_k^{\text{eq}} P_k^{\text{eq} H} \right) = \text{Tr} \left( (C_k^H C_k + \mu_k I_{B_c})^{-1} C_k^H C_k (C_k^H C_k + \mu_k I_{B_c})^{-1} \right) \\
= \text{Tr} \left( (\Lambda_{C_k} + \mu_k I_{B_c})^{-1} \Lambda_{C_k} (\Lambda_{C_k} + \mu_k I_{B_c})^{-1} \right) \\
= \sum_{i=1}^{B_c} \frac{\lambda_{C_k,i}}{(\lambda_{C_k,i} + \mu_k)^2} = 1. \tag{E.4}
\]

Note that (E.4) has multiple solutions for $\mu_k$, but its left hand side is monotonically decreasing when $\mu_k \geq 0$. Thus, the unique solution for $\mu_k$ can be found by using any line search algorithm.
Appendix F: JPBD Precoder Solution

We calculate the matrix $\tilde{G}_k$ in (4.17) by maximizing the SNR at the receiver, which can be equivalently stated as

$$\min_{\tilde{G}_k} \| \Sigma_k^{+} \tilde{G}_k^+ \|_F^2 \| \tilde{G}_k \|_F^2,$$  \hspace{1cm} (F.1)

with no constraints, since the precoder is already normalized. The first order necessary condition for this problem is

$$\frac{d}{d \tilde{G}_k} \left( \| \Sigma_k^{+} \tilde{G}_k^+ \|_F^2 \| \tilde{G}_k \|_F^2 \right) = \| \tilde{G}_k \|_F^2 \left( -\tilde{G}_k^{+T} \Sigma_k^{+T} \Sigma_k^{+} \tilde{G}_k^{+*} \tilde{G}_k^+ + \tilde{G}_k^{+T} \tilde{G}_k^{+*} \tilde{G}_k^{+T} \Sigma_k^{+T} \Sigma_k^{+} \right) - \tilde{G}_k^{+T} \tilde{G}_k^{+*} \tilde{G}_k^+ \Sigma_k^{+T} \Sigma_k^{+} \tilde{G}_k^{+*} + \| \Sigma_k^{+} \tilde{G}_k^+ \|_F^2 \tilde{G}_k^+ = 0,$$  \hspace{1cm} (F.2)

where we have used the complex matrix differentials defined in [101]. Applying complex-conjugate, using $\tilde{G}_k^+ = \tilde{G}_k^H \left( \tilde{G}_k \tilde{G}_k^H \right)^{-1}$, and rearranging (F.2) gives

$$\| \Sigma_k^{+} \tilde{G}_k^+ \|_F^2 \tilde{G}_k \tilde{G}_k^H = \| \tilde{G}_k \|_F^2 \tilde{G}_k^{+H} \Sigma_k^{+H} \Sigma_k^{+} \tilde{G}_k^+, \hspace{1cm} (F.3)$$

which is a nonlinear matrix equation with multiple stationary points for the objective function in (F.1). A general closed-form solution for this equation does not exist.
Thus, for simplicity, assume $\tilde{G}_k$ is rectangular diagonal with real positive entries $\tilde{g}_{k,i} = [\tilde{G}_k]_{ii}$, $i = 1, \ldots, B$. In such case, the objective function has the form

$$\left\| \sum_k^+ \tilde{G}_k \right\|^2 F \left\| \tilde{G}_k \right\|^2 F = \left( \sum_{i=1}^B \frac{1}{\sigma_{k,i}^2} \tilde{g}_{k,i}^2 \right) \left( \sum_{i=1}^B \tilde{g}_{k,i}^2 \right) \geq \left( \sum_{i=1}^B \frac{1}{\sigma_{k,i}} \right)^2,$$

where we applied the Cauchy-Schwarz inequality. Therefore, the objective function achieves its minimum when $\tilde{g}_{k,i} \propto 1/(\sigma_{k,i} \tilde{g}_{k,i})$ and we can define the closed-form solution

$$\tilde{g}_{k,i} = \sqrt{\frac{1}{\sigma_{k,i}}}.$$
Appendix G: Sum-Rate Maximization Solution

In this section, we show the solution to the power allocation problem for sum-rate maximization:

$$\max_{\mathbf{\rho}} \sum_{k'=1}^{K} \log_2 (1 + \text{SINR}_{k'}) \quad \text{s.t.} \quad \|\mathbf{\rho}\|_1 \leq P_{\text{max}}, \quad \mathbf{\rho} \geq 0. \quad (G.1)$$

The Lagrangian of (G.1) is

$$\mathcal{L}(\mathbf{\rho}, \lambda) = -\sum_{k'=1}^{K} \log_2 \left[ \frac{(\alpha_{D,k'} + \alpha_{\text{ISI},k'})\rho_{k'} + \alpha_{N,k'}}{\alpha_{\text{ISI},k'}\rho_{k'} + \alpha_{N,k'}} \right] + \lambda \left( \sum_{k'=1}^{K} \rho_{k'} - P_{\text{max}} \right),$$

where $\lambda \geq 0$ is a Lagrange multiplier. The KKT condition for this problem is

$$\frac{\partial \mathcal{L}(\mathbf{\rho}, \lambda)}{\partial \rho_k} = -\frac{\alpha_{D,k}\alpha_{N,k}}{\ln(2) [(\alpha_{D,k} + \alpha_{\text{ISI},k})\rho_k + \alpha_{N,k]} [\alpha_{\text{ISI},k}\rho_k + \alpha_{N,k}] + \lambda = 0,$$

which results in the following quadratic equation for $\rho_k$:

$$\rho_k = \pm \sqrt{\frac{\eta\alpha_{D,k}^2\alpha_{N,k}^2 + 4\eta\alpha_{D,k}\alpha_{\text{ISI},k}\alpha_{N,k}(\alpha_{D,k} + \alpha_{\text{ISI},k})}{2\alpha_{\text{ISI},k}(\alpha_{D,k} + \alpha_{\text{ISI},k})}} - \frac{\eta\alpha_{N,k}(\alpha_{D,k} + 2\alpha_{\text{ISI},k})}{2\alpha_{\text{ISI},k}(\alpha_{D,k} + \alpha_{\text{ISI},k})}, \quad (G.2)$$

Note that the above equation has two solutions for every $k$, so we select the positive sign in the first factor, which gives a positive power. Thus, enforcing both $\sum_k \rho_k \leq P_{\text{max}}$ and $\mathbf{\rho} \geq 0$ gives
where $0 \leq \lambda \leq \alpha_{D,k}/[\alpha_{N,k} \ln(2)]$, $\forall k$, must hold so the power allocated to each user is positive. Note that the right hand side of (G.3) is monotonically decreasing on $\lambda$, and hence a unique solution to (G.3) can be found through a line search over the interval

$$0 \leq \lambda \leq \min_{k} \frac{\alpha_{D,k}}{\alpha_{N,k} \ln(2)}.$$  

(G.4)
Bibliography


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