Flow and Acoustic Characterization of Complex Supersonic Jets

Dissertation

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By

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Abstract

Jet noise has been a major source of concern for commercial and military aviation sectors alike. The need to assuage the adverse impact of jet noise on human health has led to increased interest in jet noise source identification and noise level minimization/mitigation. Most previous works on common round supersonic jets have primarily explored the ideal case of simple, perfectly expanded jet configurations. In real world scenarios however, many of these simplifications do not hold. Two such considerations are examined in this work. The first is a simple configuration operating at complex operating conditions, specifically an imperfect expansion i.e. where the jets are operating at off-design conditions. The second concerns a complex configuration at simple conditions: specifically two jets (twin-jets such as those on fighter aircraft) operating in close proximity to each other. In this work, Large Eddy Simulation (LES) based high-fidelity computations are used to understand the dynamics of imperfectly expanded and twin-jets respectively, with the following objectives: 1) Identify the impact of active flow control techniques on the plume dynamics and acoustic characteristics of underexpanded
jets, and 2) Investigate the interaction dynamics of the twin-jet plumes and study its associated sound field which exhibits complex radiation characteristics.

To meet the first objective, a single-jet with a fully expanded Mach 1.3 jet, issuing from a converging nozzle operating at underexpanded conditions is considered. After selecting the appropriate grid based on mesh resolution studies, flow validation is conducted which indicates an excellent qualitative and quantitative agreement of the computed jet plume characteristics with the experimental observations. The analysis of underexpanded flow-field at two different Reynolds numbers indicates a relative independence of the jet flow characteristics and downstream plume evolution to the variation in Reynolds number. A detailed investigation of the near-field pressure fluctuations at different polar angles confirms the presence of three distinct noise sources i.e. downstream directed mixing noise, side-line directed broadband shock associated noise, and upstream directed screech tone noise. The frequency of the screech tone, generated as a result of a feedback process, is observed to be consistent with theoretical and previous experimental works. To investigate the effect of active flow control on the flow and acoustic features of the underexpanded jet, Localized Arc Filament Plasma Actuators (LAF-PAs) are employed, which are modeled using a semi-empirical surface heating technique. For the control simulations, axisymmetric mode pulsing is considered at two different Strouhal numbers of $St = 0.3$ and $St = 0.9$. These simulations show that the response of the jet to flow control is a strong function of the actuation frequency. Relative to the baseline uncontrolled case, actuating at the column
mode instability frequency ($St = 0.3$) results in an increase in the rate of spreading of the shear-layer. Furthermore, analysis of the phase-averaged results reveals the formation of large-scale toroidal structures that are generated due to the excitation of jet column instabilities at this actuation frequency. As a consequence of the formation of these large-scale features, the $St = 0.3$ case exhibits increased noise levels, relative to the uncontrolled case. On the other hand, the higher frequency actuation affects the initial shear-layer instability and interferes with the formation of the toroidal events, that are observed for the $St = 0.3$ actuation case, and further appears to weaken even the naturally occurring turbulent structures. As a result, noise level reduction is observed, relative to the uncontrolled case. In spite of the absence of axisymmetric toroidal events, detailed integral azimuthal length scale analyses reveal the dominance of the axisymmetric ($m = 0$) mode, even at large distances from the nozzle exit. This behavior indicates that flow control methods need not always have a visual signature of their influence on the system.

To achieve the second objective of this work of computationally investigating the flow and acoustic characteristics of complex propulsion systems, a Mach 1.23 circular twin-jet configuration, at perfectly expanded conditions, with an inter-nozzle spacing of two jet diameters is considered. The validity of the simulations is established by comparing the jet flow structure and the near-field linear-array noise levels to the experimental results, which exhibit a good match. A qualitative investigation of the jet plume structure reveals the dominance of a helical ($m = 1$) mode on both the jet plumes, which is manifested in the form of cork-screw type
features encompassing the supersonic jet core of each of the plumes. Analysis of the inter-nozzle region using mean velocity profiles indicates the formation of a secondary flow, which extends a significant influence on the jet plume shear-layers in its proximity. However, this impact is confined to small azimuthal angles and as a result, many of the potential core properties are almost similar to those of a single-jet at identical flow conditions. Analysis of the near-field pressure fluctuations along the plane containing the jet centers, at various polar angles, reveals the presence of jet noise shielding, where the noise levels imposed by a twin-jet is less than the sum of two incoherent single-jets. This shielding effect is observed to diminish with increasing polar angles. On the other hand, no such shielding is observed along the plane perpendicular ($\phi = 90^\circ$) to that containing the jet axes and instead noise level amplification, relative to the sum of two incoherent single-jets is noticed. This is attributed to the unabated sound radiation of the twin-jet plume in this direction.

To aid in the identification and understanding of the noise generation mechanisms, in addition to predicting the radiated noise levels and directivity, a decomposition of the flow-field into constituent fluid-thermal (FT) modes based on Momentum Potential Theory (MPT) is performed. The decomposed hydrodynamic mode is observed to be a true representation of the unsteady turbulence, which is evident from the identification of $m = 1$ helical mode patterns in the iso-levels of this variable. The noise signature associated with these turbulent features is observed to be contained within the decomposed acoustic mode, which exhibits
a pronounced wavepacket structure in the jet core with an apparent \( m = 0 \) axisymmetric toroidal mode dominance. Consistent with recent works on perfectly expanded supersonic jets, the acoustic wavepacket in the core of the jet exhibits significant temporal and axial modulation, which play a major role in determining acoustic radiation characteristics such as directivity and intermittency. Comparing the near-field noise level predicted by the acoustic mode with that of the overall pressure fluctuations, an excellent match is obtained both qualitatively and quantitatively. The decomposed acoustic mode correctly predicts the shielding and magnification phenomena, which signifies its ability to predict acoustic behavior of even severely complex configurations.
Dedication

To my parents for their constant support and encouragement through all the highs and lows. To my brother Tejaswi and for all the things he has done for me. To my wife Apoorva, for being my motivation and source of strength, and for all her love.
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Chapter 1

Introduction and Objectives

For decades, engines based on jet propulsion have been the workhorses of commercial and military aviation sectors. The thrust generated by such jet engines is a reactionary response to the emission of a high velocity jet into ambient air. The interaction between the exhaust jet and the ambient air results in instabilities which manifest into flow turbulence, consisting of eddies of various spatio-temporal scales. These eddies, especially those with larger length scales, generate an acoustic signature that is radiated into the far-field as mixing noise, which is more generally referred to as the jet noise. This jet noise is a significant component of engine noise, which in turn is a major contributor to the overall aircraft noise.

For the commercial aviation industry, the proximity of human habitats to airports poses a significant problem. Several research works [22, 26] which documented the impact of aircraft noise on human health report increased risk of heart disease, stroke and high blood pressure among people living in these communities. The ever increasing air traffic only aggravates the situation. For the military avia-
tion sector on the other hand, there is an immediate need to assuage the working conditions of personnel who work in the vicinity of advanced supersonic capable military aircraft, which have an increased number of noise sources (discussed further below), relative to their subsonic counterparts. The use of twin-jet configurations, to meet challenging mission requirements, further elevates the noise problem. In order to curb the sound levels, aviation authorities around the world, both commercial and military, have introduced stringent restrictions on maximum average day and night noise levels, which has led to an increased interest in the field of jet noise with an aim of identifying noise sources and subsequently reducing noise levels.

Most jet noise studies performed to date, especially for supersonic case, consider simple, ideally expanded jet configurations. Though a significant insight into jet flow behavior, noise sources and sound radiation characteristics has been obtained, there is a need to extend these efforts towards incorporating complex real world conditions, such as off-design engine operation and complex twin-jet configurations, where two engines are placed in close proximity to each other.

This work aims to bridge this gap by performing a detailed investigation of the aforementioned real world problems using high-fidelity numerical simulations with the following objectives: 1) Identification of the impact of active flow control techniques on the plume dynamics and acoustic characteristics of underexpanded jets, and 2) Investigation of the interaction dynamics of the twin-jet plumes and examination of its associated sound field, which has been shown to exhibit complex
radiation characteristics. In the following sections, the fundamental features of the chosen problems are first presented, followed by a review of previous efforts that address the inherent issues/concerns associated with these flows. Finally, the need for conducting the current research and targeted goals for each of the problems are established.

1.1 Underexpanded Supersonic Jet Flow

To achieve the first objective of analyzing the impact of active flow control on the flow and acoustic features of an underexpanded jet, it is first necessary to identify the fundamental aspects of underexpanded jets and their noise sources.

1.1.1 Supersonic Jet Noise Sources

The operating flow regime of the jet engine has a significant influence on the characteristics of the exhaust jet and consequently on the jet noise. In subsonic flows, where the Mach number of the jet at the nozzle exit is less than the speed of sound ($M_e < 1$), the only source of jet noise is mixing noise. For jet exit Mach numbers greater than unity, i.e. supersonic jet flows, two additional sound sources, Broadband Shock Associated Noise (BBSAN) and screech noise, exist depending on the engine exit jet conditions. A detailed description of these supersonic jet noise sources is presented below.
1.1.1.1 Jet Mixing Noise

A typical flow-field associated with a jet exiting a circular engine nozzle is shown in Fig. 1.1. By rearranging the classic Navier-Stokes equations into a non-homogeneous wave equation, the seminal theoretical work of Lighthill [47, 48], identified three distinct source mechanisms which are responsible for the aerodynamically generated sound i.e. monopole, dipole, and quadrupole, and classified the jet turbulence as a quadrupole source. His results showed good agreement in the side-line direction i.e. the direction perpendicular to the jet flow, but failed to accurately capture the downstream/aft noise radiation characteristics. This is attributed to the assumption of stochastic jet turbulence. With the discovery of large-scale coherent structures by Crow and Champagne [23] and Brown and Roshko [16], it
was understood that jet turbulence is not stochastic but consists of structures with significant spatio-temporal coherence. The narrowband aft directed radiation is a direct consequence of generation and propagation of these large-scale features. Based on the characteristic features of these large-scale turbulent structures, their classification as stochastic quadrupole sources in the Lighthill’s analogy is erroneous and consequently results in the failure of prediction of the noise levels in the downstream direction.

Once the presence of the large-scale structures within the shear-layer was confirmed, efforts such as those of Tam and Chen[86] and Tam and Burton[84, 85] proposed theories to predict the noise radiated by these coherent features. By identifying that jets operating at high nozzle exit velocities exhibit low radial spreading rates, these efforts were able to model the large-scale structures as instability waves. Approximating the physical structure of these instability waves to that of a wavy wall, they showed that supersonic propagation of this wavy wall, with respect to the ambient surrounding, results in generation of Mach waves. These Mach waves were observed to radiate over a range of frequencies primarily in the downstream direction, which was attributed to the growth and decay of large-scale structures.

Jet noise measurements collected over many years, at various flow conditions, suggest two distinct noise spectra: 1) A peaky narrowbanded spectrum at shallow polar angles, with respect to the jet flow direction and 2) A relatively broadband spectrum in the side-line and upstream direction, with lower sound amplitudes
compared to the shallow angle radiation. The genesis of these distinct directivity patterns was explained by Tam\[83\] who proposed a two source model with the sources being fine scale turbulence and large-scale structures. His work associates the downstream, narrowbanded, super directed noise radiation to the dynamics of the large-scale structures, which was discussed earlier. On the other hand, the broadband, lower amplitude side-line radiation is attributed to the fine scale structures, which are omnipresent in the shear-layer and which can be considered isotropic and incoherent. Various studies, both experimental\[57, 58, 88\] and computational\[9\], using different techniques lend credit to this two source theory.

On par with Tam’s representation of large-scale structures as linear instability waves, more recent works \[19, 35, 38, 82\] approximate these features as wavepackets. By applying appropriate filters to high resolution near-field data, these efforts educed wavepackets in subsonic flows. Success has also been achieved in modeling these wavepackets and their radiative characteristics with linear stability theory. The work of Sinha et al. \[78\] successfully extended wavepacket modeling and eduction techniques to supersonic flows and reported excellent match of the features of the predicted noise field with previously established results.

1.1.1.2 Broadband Shock Associated Noise

To understand BBSAN, it is necessary to understand the behavior of supersonic jets at off-design conditions. A supersonic jet a nozzle operating at design conditions has the same static pressure as the surrounding fluid. However, over the course of
the flight, the engine is usually required to operate at off-design conditions, where a static pressure mismatch occurs between engine exit and the surrounding air. If the exit pressure is less than the ambient pressure i.e., if the jet is overexpanded, shock waves are initiated at the lip of the engine nozzle. The static pressure of jet increases and matches with the surrounding as it passes through these shock waves. If the exit pressure is greater than the ambient pressure i.e. if the jet is underexpanded, expansion waves are generated at the nozzle lip. As the jet traverses these expansion waves, its static pressure decreases and matches with that of the ambient fluid. For both the overexpanded and the underexpanded case, the generated shocks or expansion fans travel across the jet and reflect off the shear-layer on the other side. These waves undergo multiple such reflections which results in the formation of shock-cell train. An illustration of an imperfectly expanded jet containing these shock-cells is shown in Fig. 1.2. The result of the interaction of these shock-cells with the large-scale structures results in the generation of BBSAN. The characteristic feature of BBSAN is its radiation primarily in the side-line and the upstream angles [83], with respect to the jet flow direction. The frequency and amplitude of BBSAN depends on various parameters such as shock-cell spacing, convection velocity of the large-scale structures, observer distance and polar angle. The theoretical models of Harper-Bourne and Fisher [36], and Tam[89] have been successfully able to predict BBSAN.
1.1.1.3 Screech Noise

The discovery of screech noise is credited to the pioneering work of Powell [59]. Screech noise is a narrowband phenomenon generated due to the formation of a feedback loop. The acoustic waves created as a result of the instability wave - shock-cell interaction travel in the upstream direction and impinge at the nozzle exit, where incipient shear-layer is highly sensitive to external disturbances. As a result, instabilities are excited which convect downstream, grow in amplitude and interact with the shock-cells resulting in upstream directed radiation, thus creating a feedback loop. Experimentally [39, 43], it has been observed that the initiation of the feedback loop occurs near third or fourth shock-cell from the nozzle exit. The
generation of screech tone is a highly dynamic phenomenon and need not occur for all imperfectly expanded jet conditions [62]. However, when present, screech noise outweighs all other components of jet noise. The seminal work of Seiner [71] classified jet screech into different modes as a function of jet exit Mach number. It was reported that at low Mach numbers \((\approx M_e < 1.3)\) axisymmetric/toroidal instabilities dominate the jet flow, whereas at high Mach numbers, helical mode instability assumes dominance. Furthermore, it was reported that formation of jet screech results in an oscillatory response of the jet plume. For high enough Mach numbers, flow visualizations reported a flapping mode behavior of the jet due to simultaneous existence of both clockwise and anti-clockwise helical instabilities. Other experiments [55, 87] also reported the presence of screech harmonics which have different radiation characteristics compared to the fundamental screech tone. Tam [87] extended his weakest link theory of screech tone generation to “account for the non-linearity of the instability wave when the screech is intense”. As a result, his theory was successful in corroborating the experimental results of Norum [55] with respect to directivity and strength of the screech harmonics.

1.1.2 Noise Control Techniques

For imperfectly expanded jets, as discussed above, the formation of shock-cells and the consequent existence of two additional noise generation mechanisms is a significant source of concern with respect to the near and far-field noise levels.
In order to adhere to the noise level regulations, effective flow control techniques need to be developed which target the sound generation mechanisms and result in noise minimization/mitigation. Over the years, numerous flow control techniques have been developed which can be broadly classified into two major categories: 1) Passive and 2) Active.

Passive control methods are those in which the geometry of the engine nozzle is physically altered to influence the noise sources. Chevrons and tabs[17, 44, 70, 94, 102] have been the most widely used passive control devices. These have successfully mitigated large-structure jet noise by generating counter-rotating streamwise vortices to enhance shear-layer mixing and breakdown. Passive approaches can however engender inefficiencies under cruise conditions, which has led to exploration of active techniques using self-excited nozzles[96], synthetic or microjet[2, 18], acoustic[63], and plasma-based techniques [68, 76]. The advantage of active control techniques over passive control techniques is the ability for the former to alter the control parameters such as frequency and amplitude as and when required. As a result, the above mentioned efforts have shown that active control brings a new dynamic into the picture by being able to target any key instability frequencies in the jet, on demand. Depending on the Strouhal number \( St = fL/U \), where \( f \) is the frequency, \( L \) is a length scale and \( U \) is a velocity scale) of excitation, either enhancement in shear-layer mixing or a reduction in near and far-field noise signature can be achieved.
1.1.3 Motivation and Objectives: Imperfectly Expanded Jets

The efforts employing control, discussed in the previous section, have mostly been on perfectly expanded jet flows. There are relatively fewer studies on controlled imperfectly jets, most of which are experimental. Among passive control efforts is the work of Murugappan et al. [54], which implements a self-resonant approach to instigate enhanced shear-layer spreading rates at preferred or sub-preferred mode frequencies, and a reduction of the same at initial flow instability frequencies. A significant reduction in core length due to increased jet mixing was observed by Feng and McGuirk [28] in their control studies using nozzle tabs. Effect of active control explored using microjets [42] as well as pulsed tabs [6] report similar observations. An initial parametric assessment by Samimy and Adamovich [66] on various imperfectly expanded supersonic jets, reveals a reduced influence of the Localized Arc Filament Plasma Actuators (LAFPAs) based active flow control technique, relative to their influence on perfectly expanded jets. This was attributed to the competition of energy between those structures which are naturally amplified and those that are generated as a result of actuation.

Numerical works are scarcer still and there is a significant need to address this gap. For this purpose, highly validated Large Eddy Simulation (LES) method with an established LAFPA modeling [31, 80] technique is employed to explore the impact of flow control on an underexpanded jet. This work can be viewed as a natural extension to previous works [30, 31] where a LAFPA based approach was em-
ployed to examine the control of a perfectly expanded jet subjected to excitation at different modes (axisymmetric, azimuthal and mixed) and frequencies. Results from these works agree fairly well with the experimental observations, thereby validating the employed methodology to model LAFPAs. Mixing enhancement is observed to occur at lower frequencies ($St \approx 0.3$), that are associated with the column mode instability, whereas peak noise mitigation is observed at higher frequencies, that are associated with initial shear-layer instability ($St \approx 2$). The simulations of [30], connect this difference to the excitation or suppression of different instability mechanisms, depending on the actuation frequency.

The objective of this part of the work is to determine the modification of fundamental flow characteristics of underexpanded jets caused by LAFPAs. Additionally, the influence of this flow control technique on the features of acoustic radiation of the underexpanded jet is also examined. The results from simulations conducted to achieve the above-mentioned objectives are presented in detail in chapter 3.

1.2 Supersonic Twin-jet Configuration.

The second objective of this work is to investigate a more complex configuration. Specifically the interaction dynamics of two closely spaced supersonic jets in a twin-jet configuration and its associated noise field. To identify effective techniques for analyzing flow-field of such complex geometries, an understanding of
the established knowledge of the configuration is required, key aspects of which are discussed below.

1.2.1 Features of a Twin-jet Configuration

A common feature of supersonic military aircraft, such as the F-15 and B-1, is the placement of two jet engines in relatively close proximity to each other, referred to as the twin-jet configuration. In addition to improved propulsive power, such a configuration provides redundancy in case of failure of one of the engines. However, the closely spaced jets interact with each other, resulting in a very complicated flow-field compared to a single-jet configuration at similar conditions.

The fundamental features of such a flow-field can be broadly classified into three major categories, based on the impact of the plume interaction on a) inter-nozzle fluid dynamics, b) shear-layer development, and c) noise levels. With regard to the impact on the inter-nozzle region, previous works such as those by Lin et al. [49] and Moustafa [52] have described the classification of a twin-jet flow domain into three regions based on the degree of interaction of the evolving jet plumes: a) Converging, b) Merging, and c) Combined. These regions are illustrated in the schematic of Fig. 1.3.

In the following discussion, to better visualize the flow features, following established notation, the shear-layers of the jet plumes growing along the inter-nozzle region will be referred to as the “inner” shear-layers and those growing
along the quiescent outer region as the “outer” shear-layers. After exiting the nozzles, the inner shear-layers from each plume grow and eventually begin to merge along the mid-plane of the twin-jet configuration. The point of first interaction between the shear-layers from adjacent jets is the merging point and the region between the nozzle exit plane and the merging point is called the converging region. This region can extend between $5D$ to $15D$, where $D$ is nozzle exit diameter, depending on various factors such as the lateral spacing between the jets and nozzle exit Mach number [52]. As will be discussed in detail in chapter 4, though no direct kinematic interaction is evident between the jets in this region, they engender a mutual influence on each other and result in significantly higher levels of induced velocity relative to an equivalent single-jet. Beyond the merging point, the inner shear-layers undergo vigorous intermixing along the mid-plane of the twin-
jet configuration, leading to strong non-linear local velocity growth. The saturation of this non-linear growth signifies the end of the inter-mixing process between the shear-layers. This point of saturation is the combined point and the region between the merging and combined points is called the merging region. The above discussed inter-mixing results in the evolution of an elliptic cross-sectional profile in this region [56]. Beyond the combined point, the combined flow-field of the two jets gradually transitions from an elliptical to a circular cross-sectional distribution, with the region of maximum velocity shifting from the jet plume centers to the mid-plane i.e. symmetry plane of the configuration. From the standpoint of the impact of a twin-jet configuration on noise levels, prior studies have established the presence of various features that are exclusive to a twin-jet flow, which are now discussed.

With respect to the influence of plume interactions on the noise field, early experimental efforts in the supersonic regime reported structural damage to aircraft components resulting from the presence of very high levels of pressure fluctuations in the inter-nozzle region [7]. The extensive pioneering work by Seiner et al. [72] revealed that these dynamic pressure fluctuations are a result of the formation of screech tones, and their amplitude amplification due to phase coupling between the jet plumes. They also showed that the coupling between the jet plumes is a strong function of the Mach number. In the lower Mach number range \((1 < M < 1.2)\), the plumes show relatively weak coupling but as the Mach number increases \((M > 1.3)\), the jet plumes exhibit significant coupling behav-
ior, resulting in a symmetric flapping of the plumes along the plane containing the jet centers. Another consequence of the twin-jet arrangement is the existence of an azimuthally asymmetric noise field [13]. The closely spaced interacting jets radiate without restriction in the direction perpendicular to the plane containing the jet centers. This results in enhanced noise levels even beyond those obtained with two linearly interacting incoherent jets [14], referred to as noise magnification, which is attributed to the non-linear interaction between the twin-jet plumes. In the plane containing the jets however, particularly in the downstream direction, one jet can shield the radiation from the other. This shielding phenomenon can result in a noise benefit of up to 3dB [14], relative to the noise level of two linear, incoherent single-jets. Shielding is a result of several mechanisms including reflection, refraction, diffraction, and scattering [75]. Stronger shielding intensity is observed for larger lateral separation distance between the nozzles [40] which is attributed to the presence of a longer converging region and thus enhanced rates of reflection and refraction of acoustic energy. Though noise reduction is observed in the plane containing the jet centers, increased noise levels in other azimuthal directions offset achieved benefits of shielding.

1.2.2 Motivation and Objectives: Twin-jet Configuration

Although there have been several experimental efforts, there have been far fewer computational studies on the twin-jet configuration. Gao et al. [32] performed
Large Eddy Simulations (LES) of a Mach 1.34 twin-jet and identified the flapping mode instability of the jet plumes. They also conducted a Dynamic Mode Decomposition (DMD) based study to analyze the jet plume coupling mechanism. Results of this study indicate an upstream shift of the first and second dynamic modes of the twin-jet, relative to a single-jet, which results in an interaction of the instability waves with stronger shock-cells leading to increased screech tone noise and BBSAN in the twin-jet configuration. Bres et al [15] conducted an LES investigation of a heated overexpanded twin-jet configuration. Their simulations successfully capture the azimuthal asymmetry of the radiated noise field and the effects of jet noise shielding. In context of these features, they report good agreement with experimental measurements. Though significant insight into the sound field of the twin-jet configuration is gained from these simulations and other experimental efforts, there is still a lack of understanding of the impact of interaction between the jets, on the near-field hydrodynamic features such as shear-layer and potential core characteristics.

The objectives of this part of the work are: 1) Investigate the dynamics of twin-jet plume interactions on the hydrodynamic features of the flow-field such as jet plume structure and shear-layer characteristics, 2) Qualitative and quantitative study of acoustic features of the twin-jet noise field such as jet noise shielding, magnification, and azimuthal asymmetry of the noise radiation, and 3) Analysis of the constituent fluid-thermodynamic (FT) components of the flow-field, extracted from the simulation data using a novel decomposition approach, with special em-
phasis on investigating the radiative characteristics of the decomposed acoustic mode.

The need for including the last part of the above-mentioned objectives stems from the desire to isolate the pure acoustic features of a jet’s flow-field with an aim to identify the fundamental noise generation mechanisms and the radiative acoustic behavior of a jet. As previously discussed, recent research works have explored wavepacket modeling approaches to predict the noise characteristics of a jet. However, such methods can be cumbersome to implement on complex flow-fields such as twin-jet configurations, where significant non-linear interactions dominate the near-field of the jet plumes. In such cases, analysis of the pure acoustic nature of the configuration requires special decomposition tools which split the pressure fluctuations into contributions from different modes. Different techniques have been employed to perform this decomposition primarily based on the knowledge of the propagation velocities of the contribution mechanisms. Tinney and Jordan [91] explored the use of a wave speed based Fourier filtering technique. At high supersonic speeds, this technique can effectively identify the contribution of different mechanisms based on the propagation speeds of the generated waves. However, for low supersonic flows, the applicability of this technique is limited as all the contributing processes have similar speed of propagation. Grizzi and Camussi [34] sought an improvement to this technique by applying filtering based on wavelets. However, the segregation of contribution of individual modes still requires information regarding the velocities of each of the modes. More recently,
Unnikrishnan and Gaitonde [92] employed Momentum Potential Theory (MPT) proposed by Doak [24], to extract the pure acoustic mode, in addition to other fluid-thermodynamic modes i.e. hydrodynamic and thermal, of a perfectly expanded Mach 1.3 jet. Their results show that the MPT based acoustic mode can be used to study the pure acoustic features at any location. In addition to characterizing the radiative features of the jet, they were able to gain significant insight into the noise generation mechanisms. Considering the complex flow-field of the twin-jet configuration, segregation of the FT modes offers a unique opportunity to examine the dynamics in a different manner.
Large Eddy Simulations (LES) are employed to computationally explore the two problems of interest outlined in the previous section. Algorithmic details along with the numerical setup of the problems is discussed below. These are encompassed in the FDL3DI code.

2.1 Governing Equations

The strong conservation form of unsteady, compressible, full three dimensional Navier-Stokes equations are solved in curvilinear coordinates [81, 97].

\[
\frac{\partial}{\partial \tau} \left( \frac{\vec{U}}{J} \right) + \frac{\partial \hat{F}}{\partial \xi} + \frac{\partial \hat{G}}{\partial \eta} + \frac{\partial \hat{H}}{\partial \zeta} = \frac{1}{Re} \left[ \frac{\partial \hat{F}_v}{\partial \xi} + \frac{\partial \hat{G}_v}{\partial \eta} + \frac{\partial \hat{H}_v}{\partial \zeta} \right]
\]  

(2.1)

where \( \vec{U} = \{ \rho, \rho u, \rho v, \rho w, \rho E \} \) is the solution vector, \( J = \partial (\xi, \eta, \zeta, \tau) / \partial (x, y, z, t) \) is the transformation Jacobian, and \( \hat{F}, \hat{G} \) and \( \hat{H} \) are the inviscid fluxes. These can be written as:
\[ \hat{F} = \begin{bmatrix} \rho \hat{U} \\
\rho u \hat{U} + \hat{\xi}_x p \\
\rho v \hat{U} + \hat{\xi}_y p \\
\rho w \hat{U} + \hat{\xi}_z p \\
(\rho E + p) \hat{U} - \hat{\xi}_t p \end{bmatrix} \]

\[ \hat{G} = \begin{bmatrix} \rho \hat{V} \\
\rho u \hat{V} + \hat{\eta}_x p \\
\rho v \hat{V} + \hat{\eta}_y p \\
\rho w \hat{V} + \hat{\eta}_z p \\
(\rho E + p) \hat{V} - \hat{\eta}_t p \end{bmatrix} \]

\[ \hat{H} = \begin{bmatrix} \rho \hat{W} \\
\rho u \hat{W} + \hat{\zeta}_x p \\
\rho v \hat{W} + \hat{\zeta}_y p \\
\rho w \hat{W} + \hat{\zeta}_z p \\
(\rho E + p) \hat{W} - \hat{\zeta}_t p \end{bmatrix} \]  

(2.2)

The contravariant velocity components can be written as:

\[ \hat{U} = \hat{\xi}_t + \hat{\xi}_x u + \hat{\xi}_y v + \hat{\xi}_z w \]  

(2.3)

\[ \hat{V} = \hat{\eta}_t + \hat{\eta}_x u + \hat{\eta}_y v + \hat{\eta}_z w \]  

(2.4)

\[ \hat{W} = \hat{\zeta}_t + \hat{\zeta}_x u + \hat{\zeta}_y v + \hat{\zeta}_z w \]  

(2.5)

Here, \( \hat{\xi}_x = J^{-1} \frac{\partial \xi}{\partial x} \), with similar definitions for the other metric quantities.

The specific total energy is given by

\[ E = \frac{T}{(\gamma - 1) M^2} + \frac{1}{2} (u^2 + v^2 + w^2). \]  

(2.6)

where \( u, v \) and \( w \) are the Cartesian velocity components. All quantities are non-dimensionalized by their respective reference values (specified later) except pressure, which is normalized by the reference dynamic head i.e. \( \rho_{ref} U_{ref}^2 \). The perfect
gas relationship is also assumed.

### 2.2 Spatial Discretization Schemes

The discretization of the inviscid terms is performed using the Roe Flux-Difference splitting scheme \[65\]. In this method, an exact solution to an approximate Reimann problem is sought.

\[
\frac{\partial \hat{Q}}{\partial t} + \bar{A} \frac{\partial \hat{Q}}{\partial n} = 0
\]  

(2.7)

where \( \hat{Q} = U/J, \bar{A} = \partial \bar{F}/\partial \hat{Q} = \hat{Q}\Lambda\hat{Q}^{-1} \), \( n \) is the direction normal to the cell face.

The sum of viscous and the convective fluxes normal to the cell face is given by \( \bar{F} \). The Jacobian matrix, \( \bar{A} \), is defined as \( \bar{F}_R - \bar{F}_L = \bar{A}(\hat{Q}_R - \hat{Q}_L) \). The flux at the interface can be approximated as:

\[
\bar{F}_{j+1/2} = 0.5[\bar{F}(U^L) + \bar{F}(U^R)] - 0.5\hat{Q}\Lambda|\hat{Q}|^{-1}(U^R - U^L)
\]  

(2.8)

where \( \hat{Q}\Lambda|\hat{Q}|^{-1} = \partial F/\partial U \) and \( \hat{} \) refers to the Roe averaged state between \( U^L \) and \( U^R \). As an example, the density and velocity at the Roe averaged state are written as:

\[
\hat{\rho} = \sqrt{\rho^L\rho^R}
\]  

(2.9)
\[ \hat{u} = \frac{\sqrt{\rho^L u^L} + \sqrt{\rho^R u^R}}{\sqrt{\rho^L} + \sqrt{\rho^R}} \quad (2.10) \]

where \( L \) and \( R \) are the reconstructed values of the variables at the left and right sides of a cell face, respectively. Specific equations for all the Roe averaged quantities are given in Ref. [51].

A third-order upwind biased method encapsulating the MUSCL (Monotone Upstream-centered Scheme for Conservation Laws) approach of Van Leer [95] is employed. Using this approach, the quantities \( U^L \) and \( U^R \) from Eqn. 2.8 can be written as:

\[ U^R_{j+\frac{1}{2}} = U_{j+1} - \frac{1}{4} \left[ (1 - \eta_M) \tilde{\Delta}_{j+\frac{1}{2}} + (1 + \eta_M) \tilde{\Delta}_{j+\frac{1}{2}} \right] \quad (2.11) \]

\[ U^L_{j+\frac{1}{2}} = U_j + \frac{1}{4} \left[ (1 + \eta_M) \tilde{\Delta}_{j+\frac{1}{2}} + (1 - \eta_M) \tilde{\Delta}_{j-\frac{1}{2}} \right] \quad (2.12) \]

Here, \( \eta_M = 1/3 \) and \( \tilde{\Delta}_{j-\frac{1}{2}} = L(\Delta_{j+\frac{1}{2}}, \Delta_{j-\frac{1}{2}}) \) where \( L \) is the applied limiting function which limits the slope of the reconstruction thereby ensuring Total Variation Diminishing (TVD). \( \Delta_{j-\frac{1}{2}} \) is defined as \( U_j - U_{j-1} \). The choice of the limiting function is crucial as it needs to provide required accuracy and robustness, and further prevent unwanted oscillatory behavior of the solution. In this work, the van Leer
harmonic limiter is employed which is defined as:

\[ L(\Delta_1, \Delta_2) = \frac{\Delta_1 \Delta_2 + |\Delta_1 \Delta_2|}{\Delta_1 + \Delta_2 + \epsilon} \]  

(2.13)

where \( \epsilon \) is a small number to ensure non-zero denominator. The viscous terms are solved using a second order central difference scheme. The subgrid closure is obtained without an explicit model consistent with the approach of implicit LES (ILES). For free jets, the efficacy of this approach has been discussed extensively by Grinstein et al. [33].

2.3 Time Integration

The approximately factored Beam-Warming method [5] with second-order diagonalization [61] is used for time integration. To counter the errors due to linearization, factorization and application of explicit boundary conditions, a sub-iteration strategy is employed. The second order accuracy is achieved by using two Newton sub-iterations as shown below.
\[
\left( \frac{1}{J} \right)^{p+1} + \phi^i \Delta \tau \delta^{(2)}_\xi \left( \frac{\partial F^p}{\partial U} - \frac{1}{\text{Re}} \frac{\partial F^p_v}{\partial U} \right) \right] J^{p+1} \times
\]
\[
\left( \frac{1}{J} \right)^{p+1} + \phi^i \Delta \tau \delta^{(2)}_\eta \left( \frac{\partial G^p}{\partial U} - \frac{1}{\text{Re}} \frac{\partial G^p_v}{\partial U} \right) \right] J^{p+1} \times
\]
\[
\left( \frac{1}{J} \right)^{p+1} + \phi^i \Delta \tau \delta^{(2)}_\zeta \left( \frac{\partial G^p}{\partial U} - \frac{1}{\text{Re}} \frac{\partial H^p_v}{\partial U} \right) \right] \Delta U \times
\]
\[= -\phi^i \Delta \tau \left[ \left( \frac{1}{J} \right)^{p+1} \frac{(1 + \phi)U^p - (1 + 2\phi)U^n + \phi U^{n-1}}{\Delta \tau} + U^p \left( \frac{1}{J} \right)^p \right] \times
\]
\[\delta_\xi \left( F^p - \frac{1}{\text{Re}} F^p_v \right) + \delta_\eta \left( G^p - \frac{1}{\text{Re}} G^p_v \right) + \delta_\zeta \left( H^p - \frac{1}{\text{Re}} H^p_v \right) \]

The inviscid flux Jacobians i.e. \( \hat{A} = \partial \hat{F} / \partial \hat{Q}, \hat{B} = \partial \hat{G} / \partial \hat{Q}, \hat{C} = \partial \hat{H} / \partial \hat{Q} \) have the generic form
here $c_1 = \gamma (E/\rho) \phi^2 = 0.5(\gamma - 1)(u^2 + v^2 + w^2)$ and $\theta = k_x u + k_y v + k_z w$. In the above equation, $\hat{A}$, $\hat{B}$ and $\hat{C}$ are represented by $k = \xi, \eta$ and $\zeta$ respectively. By using similarity transforms as shown in Eqn. 2.16, the Jacobian matrices can be diagonalized.

$$\hat{A} = T_\xi \hat{\Lambda}_\xi T^{-1}_\xi, \hat{B} = T_\eta \hat{\Lambda}_\eta T^{-1}_\eta, \hat{C} = T_\zeta \hat{\Lambda}_\zeta T^{-1}_\zeta$$  \hspace{1cm} (2.16)

where $\Lambda_\xi, \Lambda_\eta, \Lambda_\zeta$ are the diagonal matrices of the Jacobians. The elements of the principal diagonal of each of these matrices represent the eigen values of their corresponding Jacobian matrices. These diagonal matrices take the following form:
\[
\hat{\Lambda}_\xi = \begin{bmatrix}
U & 0 & 0 & 0 & 0 \\
0 & U & 0 & 0 & 0 \\
0 & 0 & U & 0 & 0 \\
0 & 0 & 0 & U + a\sqrt{\xi_x^2 + \xi_y^2 + \xi_z^2} & 0 \\
0 & 0 & 0 & 0 & U - a\sqrt{\xi_x^2 + \xi_y^2 + \xi_z^2}
\end{bmatrix}
\] (2.17)

The compact form of the above equation along with those for the other two
diagonal matrices is given in Eqn. 2.18

\[
\hat{\Lambda}_\xi = D \left[ U, U, U + a\sqrt{\xi_x^2 + \xi_y^2 + \xi_z^2}, U - a\sqrt{\xi_x^2 + \xi_y^2 + \xi_z^2} \right]
\]

\[
\hat{\Lambda}_\eta = D \left[ V, V, V + a\sqrt{\eta_x^2 + \eta_y^2 + \eta_z^2}, V - a\sqrt{\eta_x^2 + \eta_y^2 + \eta_z^2} \right]
\] (2.18)

\[
\hat{\Lambda}_\zeta = D \left[ W, W, W + a\sqrt{\zeta_x^2 + \zeta_y^2 + \zeta_z^2}, W - a\sqrt{\zeta_x^2 + \zeta_y^2 + \zeta_z^2} \right]
\]
The left and right eigenvectors are given by:

\[
T_k = \begin{bmatrix}
\tilde{k}_x & \tilde{k}_y & \tilde{k}_z \\
\tilde{k}_x u & \tilde{k}_y u - \tilde{k}_z \rho & \tilde{k}_z u + \tilde{k}_y \rho \\
\tilde{k}_x v + \tilde{k}_z \rho & \tilde{k}_y v & \tilde{k}_z v - \tilde{k}_x \rho \\
\tilde{k}_x w - \tilde{k}_y \rho & \tilde{k}_y w + \tilde{k}_x \rho & \tilde{k}_z w \\
\tilde{k}_x \frac{\phi^2}{\gamma - 1} + \rho(\tilde{k}_z v - \tilde{k}_y w) & \tilde{k}_y \frac{\phi^2}{\gamma - 1} + \rho(\tilde{k}_x w - \tilde{k}_z u) & \tilde{k}_w \frac{\phi^2}{\gamma - 1} + \rho(\tilde{k}_y u - \tilde{k}_x v)
\end{bmatrix}
\]

(2.19)
\[
T_k^{-1} = \begin{bmatrix}
\tilde{k}_x \left( 1 - \frac{\phi^2}{\alpha^2} \right) - \rho^{-1} (\tilde{k}_x v - \tilde{k}_y w) & \tilde{k}_x (\gamma - 1) u a^{-2} & \tilde{k}_z \rho^{-1} + \tilde{k}_x (\gamma - 1) v a^{-2} \\
\tilde{k}_y \left( 1 - \frac{\phi^2}{\alpha^2} \right) - \rho^{-1} (\tilde{k}_x w - \tilde{k}_z u) & -\tilde{k}_z \rho^{-1} + \tilde{k}_y (\gamma - 1) u a^{-2} & \tilde{k}_y (\gamma - 1) v a^{-2} \\
\tilde{k}_z \left( 1 - \frac{\phi^2}{\alpha^2} \right) - \rho^{-1} (\tilde{k}_y u - \tilde{k}_x v) & \tilde{k}_y \rho^{-1} + \tilde{k}_z (\gamma - 1) u a^{-2} & -\tilde{k}_z \rho^{-1} + \tilde{k}_z (\gamma - 1) v a^{-2} \\
\beta (\phi^2 - a \tilde{\theta}) & \beta [\tilde{k}_x a - (\gamma - 1) u] & \beta [\tilde{k}_y a - (\gamma - 1) v] \\
\beta (\phi^2 + a \tilde{\theta}) & -\beta [\tilde{k}_x a + (\gamma - 1) u] & -\beta [\tilde{k}_y a + (\gamma - 1) v]
\end{bmatrix}
\]

(2.20)

where \( \tilde{\theta} = \tilde{k}_x u + \tilde{k}_y v + \tilde{k}_z w, \beta = 1/(\sqrt{2} \rho \alpha) \), \( \alpha = \rho/(\sqrt{2} \alpha) \), \( \tilde{k}_x = k_x/\sqrt{k_x^2 + k_y^2 + k_z^2} \)

and \( k = \xi, \eta, \zeta \) for \( \hat{A}, \hat{B}, \hat{C} \) respectively. The eigen values of the viscous Jacobian matrices are approximated by the diagonal elements of the Jacobian matrices:

\[
\lambda_v(\xi) = \frac{\mu}{\rho} (\xi_x^2 + \xi_y^2 + \xi_z^2) J^{-1} \\
\lambda_v(\eta) = \frac{\mu}{\rho} (\eta_x^2 + \eta_y^2 + \eta_z^2) J^{-1} \\
\lambda_v(\zeta) = \frac{\mu}{\rho} (\zeta_x^2 + \zeta_y^2 + \zeta_z^2) J^{-1}
\]

(2.21)

The uncoupled system of the implicit approximate factorization scheme is fi-
nally written as:

\[
T_{\bar{\xi}} - \phi^i \Delta \tau \left[ \left( \frac{1}{J} \right)^{p+1} + \phi^i \Delta \tau \delta_{\bar{\xi}} \Lambda_{\bar{\xi}} - \delta_{\bar{\xi}\bar{\xi}} \lambda_v(\bar{\xi}) \right] \Delta U_2 =
\]

\[
\delta_{\bar{\xi}} \left( F^p - \frac{1}{Re} F^p_v \right) + \delta_{\eta} \left( G^p - \frac{1}{Re} G^p_v \right) + \delta_{\zeta} \left( H^p - \frac{1}{Re} H^p_v \right) \]

\[
\left[ \left( \frac{1}{J} \right)^{p+1} + \phi^i \Delta \tau \delta_{\eta} \Lambda_{\eta} - \delta_{\eta\eta} \lambda_v(\eta) \right] \Delta U_2 = \left( \frac{1}{J} \right)^{p+1} N^{-1} \Delta U_2 \tag{2.23}
\]

\[
\left[ \left( \frac{1}{J} \right)^{p+1} + \phi^i \Delta \tau \delta_{\zeta} \Lambda_{\zeta} - \delta_{\zeta\zeta} \lambda_v(\zeta) \right] \Delta U_0 = \left( \frac{1}{J} \right)^{p+1} P^{-1} \Delta U_1 \tag{2.24}
\]

\[
\Delta U = T_{\bar{\zeta}}^{-1} \Delta U_0 \tag{2.25}
\]

### 2.4 Mesh Structure and Boundary Conditions

Considering the two problems of interest in this study, different numerical setup is required to facilitate the calculation of an accurate solution. In chapter 2.5, the setup for the underexpanded case is described and the same for the twin-jet is outlined in chapter 2.6

#### 2.5 Underexpanded Supersonic Jet Flow - Setup

The computational domain employed to compute the underexpanded jet is shown in Fig. 2.1. This domain is discretized with a cylindrical grid on a converging noz-
Figure 2.1: Computational domain including the boundary conditions

where the $\xi, \eta, \zeta$ planes are along the streamwise, radial and azimuthal directions respectively. The mesh is clustered towards the nozzle wall and nozzle exit, and is gradually stretched out in the far-field. The computational domain extends to $20D$ in the streamwise direction (measured from the nozzle inlet) and $10D$ in the radial direction (measured from the nozzle centerline).

Characteristic boundary conditions are applied on the upstream boundary, outside of the nozzle. At quiescent ambient conditions, the velocity at the upstream boundary is subsonic and corresponding characteristic boundary conditions, shown in Eqn.2.26, are chosen.
\[ P_{\xi=1} = 0.5 \left( P_\infty + P_{\xi=2} - \rho_{\xi=2} \frac{P_{\xi=2}}{\rho_{\xi=2}} \right)^{1/2} \left[ \tilde{\xi}_x (u_\infty - u_{\xi=2}) + (v_\infty - v_{\xi=2}) + (w_\infty - w_{\xi=2}) \right] \]

\[ \rho_{\xi=1} = \rho_\infty + \frac{P_{\xi=1} - P_\infty}{\gamma \left( \frac{P_{\xi=2}}{\rho_{\xi=2}} \right)} \]

\[ u_{\xi=1} = u_\infty - \tilde{\xi}_x \frac{P_\infty - P_{\xi=1}}{\rho_{\xi=2} \left( \frac{P_{\xi=2}}{\rho_{\xi=2}} \right)^{1/2}} \]

\[ v_{\xi=1} = v_\infty - \tilde{\xi}_y \frac{P_\infty - P_{\xi=1}}{\rho_{\xi=2} \left( \frac{P_{\xi=2}}{\rho_{\xi=2}} \right)^{1/2}} \]

\[ w_{\xi=1} = w_\infty - \tilde{\xi}_z \frac{P_\infty - P_{\xi=1}}{\rho_{\xi=2} \left( \frac{P_{\xi=2}}{\rho_{\xi=2}} \right)^{1/2}} \]

(2.26)

Non-reflecting conditions are applied on the downstream and far-field boundaries of the domain. For subsonic outflow, the pressure at the boundary is specified to be ambient \((P_\infty)\). This value, along with values of the three velocity components and density recorded at one node point interior to the outflow boundary, are used to compute the values of the primitive variables along the outflow boundary. The expressions used are given in Eqn. 2.27.
\[ P_{\xi_{\text{max}}} = P_\infty \]

\[ \rho_{\xi_{\text{max}}} = \rho_{\xi_{\text{max}}-1} + \frac{(P_\infty - P_{\xi_{\text{max}}-1}) \rho_{\xi_{\text{max}}-1}}{\gamma P_{\xi_{\text{max}}-1}} \]

\[ u_{\xi_{\text{max}}} = u_{\xi_{\text{max}}-1} - \bar{\xi}_x \frac{P_{\xi_{\text{max}}-1} - P_\infty}{\rho_{\xi_{\text{max}}-1} \left( \frac{\gamma P_{\xi_{\text{max}}-1}}{\rho_{\xi_{\text{max}}-1}} \right)^{1/2}} \]

\[ v_{\xi_{\text{max}}} = v_{\xi_{\text{max}}-1} - \bar{\xi}_y \frac{P_{\xi_{\text{max}}-1} - P_\infty}{\rho_{\xi_{\text{max}}-1} \left( \frac{\gamma P_{\xi_{\text{max}}-1}}{\rho_{\xi_{\text{max}}-1}} \right)^{1/2}} \]

\[ w_{\xi_{\text{max}}} = w_{\xi_{\text{max}}-1} - \bar{\xi}_z \frac{P_{\xi_{\text{max}}-1} - P_\infty}{\rho_{\xi_{\text{max}}-1} \left( \frac{\gamma P_{\xi_{\text{max}}-1}}{\rho_{\xi_{\text{max}}-1}} \right)^{1/2}} \]

If the outflow is supersonic, the boundary conditions are switched to a simple zero gradient condition which are given in Eqn. 2.28.

\[ P_{\xi_{\text{max}}} = P_{\xi_{\text{max}}-1} \]

\[ \rho_{\xi_{\text{max}}} = \rho_{\xi_{\text{max}}-1} \]

\[ u_{\xi_{\text{max}}} = u_{\xi_{\text{max}}-1} \]

\[ v_{\xi_{\text{max}}} = v_{\xi_{\text{max}}-1} \]

\[ w_{\xi_{\text{max}}} = w_{\xi_{\text{max}}-1} \] (2.28)

The singularity on the jet centerline is treated by assuming solution continuity. The branch cut in the \( \zeta \) direction is treated with a five point overlap to provide a complete (five-point) stencil on all points in the \( \zeta \) direction. The operating conditions of the jet are chosen to be representative of the Syracuse Anechoic facility[74]
where the corresponding underexpanded jet experiments were performed. The stagnation pressure, set to $P_0 = 295.2 kPa$, is chosen to give a fully expanded Mach number of 1.3. A length scale of $D = 0.0508 m$ corresponding to the nozzle exit diameter, $T_{ref} = 298 K$, $u_{ref} = 346 m/s$, $\rho_{ref} = 1.3 kg/m^3$, are chosen to be the reference values used for non-dimensionalization. Non-dimensional time is employed by defining characteristic time which is calculated as $T_c = T u_{ref} / D$. For all the underexpanded single-jet simulations, a non-dimensional timestep of 0.001 is employed which corresponds to a physical timestep of $1.46 \times 10^{-7} s$. The simulations are run for approximately 150 characteristic times to isolate the impact of initial transience. Once a statistically stationary state is reached, the simulations are run for a further 150 characteristic time to collect mean flow and other statistics.

### 2.5.1 Grid Resolution Study

Three grids ranging from 3 million to 54 million points are used to study the effect of mesh resolution. Table 2.1 lists the values of the grid point distribution along $\xi$, $\eta$ and $\zeta$ directions and mesh resolution near the nozzle wall and nozzle exit. The development of the Mach number and fluctuating axial velocity along the centerline

<table>
<thead>
<tr>
<th>Grid Name</th>
<th>Mesh Distribution</th>
<th>$\Delta \xi$ at nozzle exit</th>
<th>$\Delta \eta$ at lip-line</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>214 x 140 x 104</td>
<td>0.01$D$</td>
<td>0.005$D$</td>
</tr>
<tr>
<td>G2</td>
<td>801 x 441 x 104</td>
<td>0.0015$D$</td>
<td>0.0009$D$</td>
</tr>
<tr>
<td>G3</td>
<td>1011x 521 x 104</td>
<td>0.001$D$</td>
<td>0.0008$D$</td>
</tr>
</tbody>
</table>

Table 2.1: Grid distribution in $\xi, \eta, \zeta$ directions employed for the underexpanded case
Figure 2.2: Comparison of centerline mean Mach number among different grids of the jet are plotted in Figs. 2.2 and 2.3 respectively. The undulations observed in the mean are indicators of shock-cells, whose intensity is proportional to the amplitude of the oscillations. From Fig. 2.2, it is observed that both the strength and number of shock-cells are underpredicted on G1. G2 and G3 however predict similar levels of strength and number of shock-cells. The turbulence intensity (Fig. 2.3) grows in a monotonic manner as jet breaks down. Here too, G2 and G3 show similar rates of turbulence growth, thus indicating that grid resolution of G2 is sufficient for analyzing the dynamics of the controlled underexpanded jet.
2.5.2 Actuator Model

For the simulations employing flow control, as mentioned in chapter 1, the active control method considered is based on LAFPAs [67]. Eight actuators are considered, arranged on the inner periphery of the nozzle lip, as shown in Fig. 2.4 (bright red spots). Each actuator consists of a pair of electrodes which generate an arc causing ionization of the local fluid and thus producing plasma. This yields a rectangular on-off pulsed signal, which is different from conventional continuously varying acoustic drivers as noted in Ref. [68]. The construction and working of the plasma actuators is presented in more detail in Ref. [93].

To mimic the effect of LAFPAs, the semi-empirical approach of Gaitonde and
Samimy [31] is adopted. In this approach, the effect of the arc is recreated by imposing a surface heating condition. The numerical actuators cover a circumferential length of $3\text{mm}$, or a circular extent of about $13^\circ$ which corresponds to the distance between each pair of electrodes, and cover a streamwise length of $1\text{mm}$ which corresponds to the width of the groove used to mount the actuators. The nozzle wall temperature is assumed to be $1.12T_\infty$. For this work, the excitation of all the actuators is done simultaneously i.e. an axisymmetric ($m = 0$) excitation mode is employed. Two actuation frequencies, $St = 0.3$ and $St = 0.9$, are imposed at a duty cycle of 50%. Duty cycle if defined as percentage of “on-time” of the actuators relative to the time for one complete actuation cycle, at a given excitation frequency. For the $St = 0.3$ case, a complete actuation cycle requires 3333 steps and
with a 50% duty cycle, the actuators are on for 1666 time steps. Similarly, for the 
$St = 0.9$ case, a complete actuation cycle requires 1111 steps and with a 50% duty 
cycle, the actuators are on for 555 time steps. The response of the jet to variation 
in the excitation frequency is discussed in detail in chapter 3.2. The switching on 
of the actuators (i.e. activating the boundary condition) raises the temperature of 
actuation region. For this work, a temperature ratio of $T/T_\infty = 5$ corresponding to 
a local surface temperature of about 1500$K$ is employed.

This approach to model the effect of the actuators is appropriate for the current 
speed regime. The model has been extensively tested (see [31, 80]) for different 
modes and excitation frequencies. Key experimental data including mean flow, 
fluctuating quantities, and visualization have been reproduced accurately. The 
mean flow has further been favorably compared for different excitation modes 
with the analytical formulation of Cohen and Wygnanski [21].

### 2.6 Supersonic Twin-jet Configuration - Setup

Due to the geometrical features of the twin-jet configuration (see Fig. 1.3), it is 
difficult to construct a conformal structured mesh. To overcome this challenge, an 
overset grid technique based on overlapping structured meshes is employed. In 
this method, finer resolution meshes around regions of interest are overset onto 
coarser background meshes. This technique helps in maintaining structured grid 
topology even around complicated geometries. The discretized domain employed
Figure 2.5: Illustration of the computational domain discretized using an overset meshing technique to compute the twin-jet configuration is shown in Fig. 2.5, with the nozzle region in the inset. The domain extends to $25D$ in the downstream direction, $15D$ in the sideline direction (measured from the symmetry plane between the two jets), and $16.15D$ in the direction perpendicular to the plane containing the jet centers.

All simulations performed in this study employ a circular twin-jet geometry with a center-to-center spacing of $2D$, where $D$ is diameter at the nozzle exit and is taken to be $0.01905m$. The current work is part of a joint computational and experimental campaign. Henceforth, any usage of ‘corresponding/concurrent experimental work/efforts’ refers to the equivalent experimental work [45] performed at the Aerospace Research Center at the The Ohio State University. These experiments employ a conical converging-diverging nozzle geometry with a design
Mach number of approximately $M = 1.23$. However, to minimize computational costs, a small sleeve region of length $0.5D$ is employed instead of the entire nozzle interior. Henceforth, any reference to the exit of the nozzle implies the exit of the sleeve region.

The effect of the boundary layer on the development of the shear-layers can be significant [10, 79]. Thicker boundary layers, though delayed in their initial breakdown in the free shear region, exhibit a faster growth rate thereafter, than compared to the thinner ones. The boundary layer properties of the companion experiments are unknown, though visualizations indicate that they are very thin. In such thin boundary layer jet flows, the results of Gaitonde and Samimy [31] indicate that the major features of the breakdown process are captured quite accurately by assuming a laminar profile. Hence, all simulations are performed by employing a compressible laminar boundary layer profile with finite boundary layer thickness of $0.1r_0$, where $r_0$ is the radius of the nozzle exit. The equation employed is similar to that used by Bogey et al. [10] and is given in Eqn. 2.29 The resulting velocity profile relative to a uniform condition is shown in Fig. 2.6. The Crocco-Busemann relation is employed to obtain the temperature profile and the density profile is subsequently obtained from the equation of state [98].

$$u_x(r) = u_j \frac{r_0 - r}{\delta} \left[ 2 - 2\left(\frac{r_0 - r}{\delta}\right)^2 + \left(\frac{r_0 - r}{\delta}\right)^3 \right], \text{if } r \geq r_0 - \delta$$

$$= 1, \text{if } r < r_0 - \delta$$

(2.29)
Figure 2.6: Comparison of the nozzle exit boundary layer profile employed in the current work, with that of a slug uniform flow profile.
A centerline Mach number of 1.234 is employed at the nozzle exit at a Reynolds number of 775,000. The nozzle exit jet conditions, i.e., $T_{ref} = 220.76K$, $U_{ref} = 367.52 m/s$ and $\rho_{ref} = 1.599 kg/m^3$, are considered as the reference quantities to non-dimensionalize the governing equations. These conditions are based on the nominal values of accompanying experiments [45]. A single-jet simulation at the same conditions is also performed to provide a reference with which to estimate the effects of one plume in the twin-jet on the other.

### 2.6.1 Grid Resolution Study

A grid dependence study is performed by comparing results obtained from two grids, Grid-1 with approximately $42 M$ cells and Grid-2 with approximately $80 M$ cells. For both grids, the mean $u$-velocity contours on the plane containing the two jets are plotted in Fig. 2.7. A qualitative comparison of these contours indicates no evident differences. A more quantitative comparison is facilitated by analyzing the axial and radial trends of mean streamwise velocity, as shown in Fig. 2.8. The plots of axial variation of the mean streamwise velocity along the centerline of one of the twin-jet nozzles is shown in Fig. 2.8(a), where both grids predict an almost identical length for the potential cores, i.e., the point on the axis where the velocity starts to decay appreciably. Likewise, the velocity decay rate beyond the collapse of the cores is also similar. This suggests that the growth rate of the shear-layers predicted by both the grids is nearly identical. This observation is further
Figure 2.7: Comparison of mean axial velocity contours obtained from Grid-1 and Grid-2 bolstered by analyzing the radial variation of the streamwise velocity, along the plane containing the jets, at two different axial locations, plotted in Fig. 2.8(b). The profiles at each of the axial locations exhibit almost identical distribution. The observations from Figs. 2.8(a) and 2.8(b) suggest that the resolved features on both the grids are characteristically similar.

To investigate the efficiency of the two grids in resolving the dynamic features of the shear-layer, in Fig. 2.9(a) we plot the mean velocity along the outer lip-line (entraining from the quiescent surrounding) for one of the nozzles. In addition to similar growth rates, both the profiles also show a similar location of saturation of the mean axial velocity. Furthermore, the profiles of $rms$ of pressure fluctuations along this location, shown in Fig. 2.9(b), also exhibit similar character-
(a) Streamwise variation of mean axial velocity along the centerline of one of the nozzles of the twin-jet

(b) Radial variation of mean axial velocity for one of the nozzles of the twin-jet, at different axial locations ($z/D = -1$ is the center of the nozzle and $z/D = 0$ is the symmetry plane between the two nozzles)

Figure 2.8: Impact of grid resolution on streamwise and radial distribution of mean axial velocity
(a) Streamwise variation of mean axial velocity along the outer lip-line of one of the nozzles of the twin-jet

(b) Streamwise variation of $rms$ of pressure fluctuations along the outer lip-line of one of the nozzles of the twin-jet

Figure 2.9: Impact on grid resolution on flow characteristics along the lip-line of the nozzle
istics. The phenomena of laminar-to-turbulent transition of the jet shear-layer in addition to the formation of roll-up structures and their subsequent pairing results in the formation of the characteristic double-hump observed in both the profiles. This feature of initially laminar jets has been previously reported by both experimental[101] and computational [12] efforts. Overall, a very good agreement is noticed between the two profiles indicating that the resolution of the dynamic features of the shear-layer in both the grids is almost identical. From Figs. 2.7, 2.8 and 2.9, it is evident that both grids exhibit equal fidelity in resolving the twin-jet flow features and hence it is concluded that a grid independence is achieved on Grid-1. For the purpose of this work, results obtained from Grid-2 are discussed in detail.
Chapter 3

Flow and Acoustic Characteristics of an

Underexpanded Jet

Supersonic jet noise has been an active area of research for quite a few decades and significant insight is already available with regard to the basic flow characteristics and noise generating features of the supersonic jet flows. This knowledge of the fundamental mechanisms has eventually led to the implementation of flow control techniques with an overarching aim of noise mitigation/minimization. However, as discussed in chapter 1, much of the research on the applicability of control has been primarily on perfectly expanded flows, and the little research performed on controlled underexpanded jets is mostly experimental in nature. The primary goal of the work reported in this chapter is the characterization of the response of a Mach 1.3 underexpanded jet to flow control employing the use of LAFPAs. To establish the fidelity of the present simulations in capturing the crucial features of the underexpanded jet, in addition to providing a reference for the identification
of impact of the control technique, simulations of uncontrolled jet are first performed and are discussed in chapter 3.1. Control simulations are then carried out by employing the phenomenological model for modeling the LAFPAs (discussed in chapter 2). These simulations are conducted for an axisymmetric excitation of the actuators at two different excitation frequencies of $St = 0.3$ and $St = 0.9$ which reveals insight into the varied response of the underexpanded jet to flow excitation at different frequencies. These results in the context of the hydrodynamic features of the jet plume are discussed in chapter 3.2. Finally, a study of the influence of LAFPAs on the near-field mode dominance, and noise levels, compared to the uncontrolled simulations, is performed and discussed in Secs. 3.3 and 3.4.

3.1 Uncontrolled Supersonic Underexpanded Jet Flow

In this section, the fundamental behavior of an underexpanded jet is studied in detail, with specific emphasis on shock-cell structure and its influence on the jet centerline velocity profiles. Results from the simulations conducted to determine the variation of shock-cell features to degree of underexpansion are discussed first.

3.1.1 Experimental Validation and the Effect of Degree of Under-expansion

To study the dependency of the shock-train on the intensity of underexpansion, two values of upstream stagnation pressure i.e. $P_0 = 246kPa$ and $P_0 = 295.2kPa$
are considered, which correspond to a Nozzle Pressure Ratio (NPR) of 2.42 and 2.91, respectively. Validation studies are performed by comparing the results obtained from the above simulations with those obtained experimentally at the Syracuse University. For easier reference, the $P_0 = 246kPa$ will henceforth be referred as *LTP* or the Lower Total Pressure case and the $P_0 = 295.2kPa$ will be referred to as *HTP* or the Higher Total Pressure case.

The jet flow issued from imperfectly expanded jets is required to traverse the shock-cell features created due to the mismatch of conditions between the nozzle exit and the ambient. As a result, the jet flow experiences significant velocity variations, the features of which can be studied in detail by considering the profiles of jet centerline velocity. In Fig. 3.1, the streamwise variation of mean axial velocity profiles along the centerline of the jet, for both values of upstream stagnation pressure, are plotted. The profile of the same from the corresponding experimental work is also included for comparison. As expected, the lower intensity underexpansion case i.e. the LTP case, predicts weaker and much more closely placed shock-cells compared to its higher intensity counterpart. Between the two underexpansion conditions, the HTP case exhibits a very good correlation with the experimental observations. A qualitative comparison of the shock-cell structure obtained from the HTP case and the experiments is shown in Figure 3.2, where 2-D contours of the mean axial velocity are plotted. Significant similarity is observed between simulations and experiments, particularly in the development of the shock-cell structures. Small variations in the width of the shear-layer are associated with the difficulty in
Figure 3.1: Comparison of experimental and numerical profiles of mean axial centerline velocity

Figure 3.2: Comparison of numerical and experimental contours of mean axial velocity
reproducing the nozzle exit boundary layer, which depends on tunnel conditions (see Ref. [8] for a detailed discussion). This in turn can influence the growth of the free shear-layer as reported by Bogey and Bailly [10]. Nonetheless, the overall comparison reflected in Figs. 3.1 and 3.2 provides a degree of confidence in the accuracy of the numerical scheme. In the following sections, the focus will be on the results obtained from cases employing higher upstream pressure. However, a few significant results from the lower pressure case are also reported for comparison.

3.1.2 Effect of Reynolds number

The nozzle exit boundary layer thickness and state (level of turbulence) can play a significant role in the development of the flow-field evolution of a jet [10, 46, 64, 80]. For relatively thin boundary layers encountered in this work however, the results of Ref. [31] suggest that the primary factor dictating the evolution of the plume is the initial shear-layer instability, which is a direct function of the flow Reynolds number. To explore this issue for the current underexpanded jet, simulations are conducted at two values of Reynolds numbers i.e. \( \text{Re}=1.2 \times 10^6 \) and \( \text{Re}=100,000 \). Results from these simulations indicate no significant qualitative effect on the overall flow structure. For example, Fig. 3.3 plots the contours of instantaneous Mach number for both the high Reynolds number (Fig. 3.3(a)) and the low Reynolds number (Fig. 3.3(b)) cases. The periodic shock-cell train generated as a result of underexpansion of the supersonic jet is clearly visible in both, and exhibits no sig-
Figure 3.3: Contours of instantaneous Mach number for both Reynolds number cases
significant differences. A quantitative comparison is performed by plotting the mean axial velocity along the centerline of the jet, as shown in Fig. 3.4. Both cases predict similar number and intensity of shock-cells. No significant differences in the predicted value of the potential core length, defined as the distance between the nozzle exit and the axial location where the jet centerline velocity diminishes to 95% of the nozzle exit velocity, or the decay rate of the velocity beyond the core collapse location is observed. In Fig. 3.5, the radial variation of the mean velocity at the nozzle exit is plotted for both the cases. Consistent with boundary layer theory, the lower Reynolds number case exhibits thicker boundary layer at the nozzle exit compared to the higher Reynolds number case. However, this difference in
Figure 3.5: Comparison of radial variation of mean axial velocity for both the Reynolds number cases at the nozzle exit
Figure 3.6: Comparison of axial variation of jet width between the Reynolds number cases.

Efflux boundary layer thickness does not influence the initial shear-layer characteristics. This is observed in the plot of axial variation of jet halfwidth shown in Fig. 3.6. The jet halfwidth is defined as the distance between the jet centerline and the radial location where the velocity is 50% of the local maximum (centerline) velocity. The profiles for both cases plotted in Fig. 3.6 exhibit hardly any variation compared to each other. The visible lack of significant differences between the two Reynolds number cases is attributed to the exit boundary layer turbulence levels. Zaman [100] classified boundary layers as nominally laminar, if the magnitude of nozzle exit boundary layer velocity fluctuations is greater than but close to 1% of nozzle exit velocity, and nominally turbulent, for turbulent intensity values around
10% of the nozzle exit velocity. Both Reynolds number cases, based on this classification, are observed to be laminar since the turbulence intensities are less than 1%. Furthermore, the boundary layers remain relatively thin, which suggests that the dominant mechanism in the plume is the shear-layer instability, similar to the observations of Gaitonde and Samimy [31]. Thus, for the laminar boundary layers encountered in the current work, the Reynolds number does not significantly impact the overall development of the supersonic jet flow. For this reason, the high Reynolds number case is used for all the subsequent simulations.

3.2 Controlled Supersonic Underexpanded Jet Flow

With the knowledge of the fundamental jet plume features of the uncontrolled underexpanded jet, described in chapter 3.1, simulations are performed to determine the impact of flow control on the modification of these features. For control simulations, active flow control technique through the use of LAFPAs (see chapter 2) is employed at an axisymmetric ($m = 0$) mode, at two different excitation frequencies corresponding to $St = 0.3$ and $St = 0.9$, corresponding to physical frequencies of 2043.3 Hz and 6129 Hz, respectively. These actuation frequencies are chosen to target excitation of specific instabilities of the flow. The $St = 0.3$ actuation amplifies the instability associated with the jet-column and the $St = 0.9$ actuation frequency targets the excitation of the near-nozzle shear-layer instabilities. Though $St = 0.9$ is a higher harmonic of $St = 0.3$, control cases are performed at this higher Strouhal
number to facilitate isolation of the effects of this higher frequency.

### 3.2.1 Instantaneous Flow Features

To qualitatively establish the response of the underexpanded jet to flow control, the contours of instantaneous Mach number on the $z = 0$ plane for both control cases are first considered. These contours are plotted in Fig. 3.7. The contours indicate a fundamental distinction in the response of the jet to the excitation Strouhal number. The shock-cell features exhibit substantially different structure for the two control cases. The lower Strouhal number case, shown in Fig. 3.7(a), exhibits very few shock-cells compared to the higher Strouhal number case (Fig. 3.7(b)). Furthermore, contours for the $St = 0.3$ case qualitatively exhibit higher rates of jet spreading after the initial jet development, compared to the $St = 0.9$ excitation case. A qualitative comparison of Mach number contours plotted in Fig. 3.7 with the same for the uncontrolled jet (Fig. 3.3) indicates a longer core length for both control cases, the quantification of which is presented in chapter 3.2.2.

As explained above, the excitation frequencies employed target the amplification of specific instabilities of the flow. Such amplification, if successful, often exhibits visual traces in the form of large-scale structures, the dynamics of which can be studied by considering the iso-levels of Q-criterion. For both the control cases, the instantaneous Q-criterion iso-levels are plotted in Fig. 3.8. The iso-levels for the lower Strouhal number excitation case indicate the presence of large axisymmetric
Figure 3.7: Contours of instantaneous Mach number for the control cases
Figure 3.8: Iso-levels of instantaneous Q-criterion for the control cases colored by vorticity magnitude
toroidal events. It is also observed that successive toroids are connected through longitudinal hairpin like vortices. The amplification of the jet column mode instability due to the actuation at $St = 0.3$ results in the generation of these structures. On the other hand, excitation at $St = 0.9$ does not appear to instigate the generation of toroidal structures. A more detailed discussion of the impact of control on the mean characteristics of the jet is presented in the next section.

### 3.2.2 Time-averaged flow features

Although the instantaneous flow provides a good indication of the qualitative features of response of the jet to flow control, a quantitative study is required to establish the statistical impact. In this section, we examine the mean flow for both the control cases, obtained by averaging over $\approx 150$ characteristic times. In a mean sense, the actuators have relatively little effect on boundary layer at the nozzle exit where the actuation is applied. This is evident from Fig. 3.9, where the radial variation of mean velocity at the nozzle exit for the control cases is plotted, along with same for the uncontrolled case. The profiles for all the three cases are very similar and exhibit similar values of boundary layer thickness ($0.00175D$ for baseline, $0.00165D$ for $St = 0.3$, $0.00166D$ for $St = 0.9$). This confirms that the effect of the actuators is manifested by the manner in which they manipulate the free shear-layer that forms downstream of the nozzle lip. Following Zaman’s classification [100](discussed in chapter 3.1.2), the exit boundary layers for the control
cases are classified as nominally laminar due to peak turbulence intensity levels of 2.5% of the nozzle exit velocity observed for both the cases. These levels are observed in the plot of radial variation of turbulence intensity at the nozzle exit shown in Fig. 3.10. The distinctive feature of this plot is the noticeable peaking of the turbulence intensities for the control cases relative to the baseline uncontrolled case, which is attributed to the influence of flow control.

The variation of mean centerline velocity with axial distance is plotted in Fig. 3.11 for the different cases. The effects of enhancement and inhibition respectively, of large-scale entrainment at $St = 0.3$ and $0.9$ are clearly evident. Although both the control cases continue to show shock-cells, there is a significant difference in the number and features of the shock-cells, consistent with the qualitative obser-
Figure 3.10: Comparison of radial variation of turbulence intensity between the baseline and the control cases at the nozzle exit

Figure 3.11: Comparison of centerline mean axial velocity for control cases
vations of Fig. 3.7. The higher Strouhal number case predicts larger number of shock-cells, even relative to the baseline case, whereas the lower Strouhal number case exhibits fewer such features. With regard to core length, Fig. 3.11 indicates that the higher Strouhal number case does indeed exhibit longer core length corroborating the qualitative inferences obtained from Fig. 3.7. However, the lower Strouhal number case is characterized by an early drop in the mean centerline velocity around \( x/D = 4 \). This drop is attributed to the enhanced levels of entrainment owing to the formation the axisymmetric toroidal structures. The fewer shock-cells however yield a longer region of relatively more uniform velocity. After the collapse location, the rate of decay of the mean velocity is the highest for the low Strouhal number case whereas the higher Strouhal number case exhibits a subdued decay rate.

These observations are further substantiated by analyzing the jet width as shown in Fig. 3.12 as a function of axial distance, for all cases. At \( St = 0.3 \), the formation of strong axisymmetric toroids results in increased levels of entrainment which leads to a higher spreading rate of the jet. The enhanced entrainment is not sustained due to the weakening of the toroidal structures as they convect downstream, which effects a drop in the spreading rate at downstream locations, resulting in a narrower jet width beyond \( x/D = 5 \), as observed in Fig. 3.12. Beyond this location, the natural evolution of the jet results in a gradual rate of increase in jet width till about \( x/D = 8 \) where the potential core collapses resulting in an immediate increase in the growth rate. This increased growth rate beyond the collapse of the
Figure 3.12: Comparison of variation of jet width with axial distance

potential core is observed to be proportional to the decay rate of the mean centerline velocity as seen in Fig. 3.11.

The $St = 0.9$ case exhibits lower rates of spreading relative to the $St = 0.3$ case which is attributed to the inhibition of the generation of the large-scale structures. Additionally, the jet spreading rates for the $St = 0.9$ case appear to be subdued even in comparison to the uncontrolled case. Since jet mixing is a significant function of the entrainment behavior of the jet plume, which in turn is dictated by the strength of the large-scale features of the shear-layer, the preceding observation indicates a suppression of the naturally occurring large-scale structures due to actuation at $St = 0.9$. As a result, a longer potential core ($x/D = 8$ compared to $x/D = 5.5$ for the baseline case) is observed (Fig. 3.11). Similar to the lower
Figure 3.13: Comparison of development of Turbulent Kinetic Energy along the jet centerline

Strouhal number case, the collapse of the potential core results in an immediate increase in the growth rate which is observed to be proportional to the rate of decay of the mean centerline velocity. At a further downstream location of $x/D = 10$, computations predict lower values of jet width for both the control cases compared to the baseline no control case. A similar behavior was observed by Samimy et al[68] in their experiments employing control at a high Strouhal number on a Mach 0.9 jet.

The variation of Turbulent Kinetic Energy (TKE) with axial distance along the centerline of the jet is plotted for all cases in Fig. 3.13. The cells formed as a result of underexpansion yield oscillations in these profiles. The values rise through the
shocks, while they fall through expansions. For the $St = 0.3$ case, an increase in the TKE levels is observed compared to the uncontrolled case, as a result of the formation of toroidal rollers and their subsequent interaction with the shock-cell structures. However, the location of peak TKE is observed to be much further upstream ($x/D = 4$) for $St = 0.3$ case relative to the baseline no control ($x/D = 8$) and $St = 0.9$ cases ($x/D = 10$), which is attributed to peaking of amplitude of the generated axisymmetric structures at this location. The consequence of suppression of naturally occurring large-scale turbulent features, in addition to inhibition of toroidal structure formation results in a lowering of the peak levels of TKE for the $St = 0.9$ case, relative to both the baseline and the $St = 0.3$ cases.

3.2.3 Phase-averaged flow features

To obtain a deeper insight into the dynamics of the axisymmetric toroids that are created due to excitation at $St = 0.3$, phase-averaged flow-fields are examined. Unlike time averaging where all the resolved timesteps are considered, phase-averaging is performed by averaging the flow at a specific point in each actuation cycle, over numerous actuation cycles. Figures 3.14 and 3.15 show the phase averaged contours of vorticity magnitude at two different phases for both the control cases. The contours for the $St = 0.3$ case are characterized by the presence of large vortical structures (as discussed in chapter 3.2.1). The symmetric nature of these vortices is consistent with the axisymmetric quality of the generated large-scale
Figure 3.14: Contours of phase averaged vorticity magnitude at phase-1 for both control cases
Figure 3.15: Contours of phase averaged vorticity magnitude at phase-4 for both control cases
structures. Compared to the $St = 0.9$ case, the structures observed for the lower Strouhal number case are stronger and more coherent to longer downstream distances. Further analysis of these structures is performed by plotting the iso-levels of Q-criterion at different phases, as shown in Fig. 3.16 and Fig. 3.17. For the lower Strouhal number case, successive phases clearly show the formation, propagation and destruction of the toroidal structures as they convect downstream. The rollers in their incipient stage are comprised of azimuthal variations (observed in the iso-levels at $\phi = 0.22$ and $\phi = 0.35$) which interact with each other resulting in the formation of the large-scale vortex rings. The successively shed rings are observed to be connected by longitudinal rib like structures which extend from the outer part of one ring to the inner region of the earlier shed ring. A difference in the break-up pattern of the successive structures is also observed. From the Fig. 3.16, the roll-up marked ‘A’ is observed to grow and finally “explode” in a direction pointing away from the jet axis. On the other hand, the structure marked ‘B’, as it convects downstream, is observed to shrink in size and break-up in a direction pointing towards the jet axis. This difference may be associated with the effect of “on” versus “off” events that arise when the actuator arc is struck. In contrast to the lower Strouhal number case, the $St = 0.9$ case (Fig. 3.17) does not exhibit the formation of any large-scale toroidal events, though it is still characterized by the azimuthal variations. The formation of the toroids in $St = 0.3$ case is due to the amplification of the instabilities. Since $St = 0.3$ corresponds to the most unstable column mode frequency of the jet, actuation at this frequency results in the ampli-
Figure 3.16: Iso-levels of phase averaged Q-criterion for the $St = 0.3$ case
Figure 3.17: Iso-levels of phase averaged Q-criterion for the $St = 0.9$ case

This response of the jet is absent when excited at higher frequencies and hence does not result in the formation of the rollers. For the same reason, the $St = 0.9$ simulation does not show any significant dynamical variations at different phases. This is observed from the contours of vorticity magnitude for the $St = 0.9$ case shown in Figs. 3.14 and 3.15 and is also re-established from the iso-levels of Q-criterion plotted in Fig. 3.17 (only a two phases are plotted for brevity).

3.3 Azimuthal length scales and mode domination

As discussed above, for the axisymmetric excitation mode, the effect of control depends on the frequency of actuation. For cases excited at $St = 0.3$, the effect is visually observable due to the generation of toroidal roll-up structures. However, at higher Strouhal numbers ($St = 0.9$), such structures are not prominent. In order to further quantify the impact of control in the near-field, the azimuthal interaction length scale is calculated for both the control cases and is compared with baseline
Figure 3.18: Comparison of axial variation of L-parameter among different cases

no control case. Figure 3.18 plots the variation of azimuthal integral length scale
($L_{11}^{\theta}$) with axial distance for both the control cases, and the uncontrolled case. The
azimuthal integral length scale is defined as follows

$$ L_{11}^{\theta} = \frac{1}{r_0} \int_0^\pi R_{11}^{\theta}(r_0 \delta \theta) d(r_0 \delta \theta) $$  \hspace{1cm} (3.1) 

where $R_{11}^{\theta}$ is the cross-correlation function:

$$ R_{11}^{\theta}(\delta \theta) = \frac{\langle u'_z(r, \theta, z)u'_z(r, \theta + \delta \theta, z) \rangle}{\langle u'^2_z(r, \theta, z) \rangle^{1/2}\langle u'^2_z(r, \theta + \delta \theta, z) \rangle^{1/2}} $$  \hspace{1cm} (3.2) 

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Based on the definition of the length scale, for an axisymmetric mode, \( L_{11}^{(\theta)} = \pi \). In this study, all the length scales are calculated along the lip-line i.e. \( r = D/2 \) from jet centerline and are normalized by \( \pi \) so as to obtain a maximum value of \( L_{11}^{(\theta)} = 1 \) for the axisymmetric mode. Figure 3.18 indicates that the shock-cells also have a substantial impact on the integral length scale which is evident due to the undulations in the profiles. Thus, for a shock-expansion wave dominant flow, though the effect of control is significant, it is overshadowed by the influence exerted by the shock structures. As a result of actuation at \( m = 0 \) mode, both the control cases show very high values of length scales (\( \approx 0.9 \)) for longer downstream distance relative to the baseline no control case (\( \approx 0.7 \)). Bogey and Bailly\cite{10} reported values of \( \approx 0.53 \) for a Mach 0.9 jet at a Reynolds number of 100,000 for a non-dimensional nozzle exit shear-layer momentum thickness of 0.005. They also reported a decrease in the interaction length scale in the shear-layer as a result of introduction of pressure perturbations into the boundary layer. Based on the values observed in this work, we conclude that exciting at \( m = 0 \) increases the dominance of axisymmetric mode but does not inhibit the formation of other modes which is evident from the non unity values of length scales for the control cases (as mentioned earlier, for a pure axisymmetric case, \( L_{11}^{(\theta)} = 1 \)).

This is further corroborated by the plots of Sound Pressure Level (SPL), though at the chosen location (\( x/D = 10 \)), there may be a hydrodynamic component. The
SPL is calculated as shown in Eqn. 3.3,

\[ SPL = 10\log_{10}\left(\frac{PSD}{P_{ref}^2}\right) \]  

(3.3)

where PSD stands for the Power Spectral Density of the pressure fluctuations. Figures 3.19, 3.20 and 3.21 show the SPL values for different pressure modes obtained using azimuthal mode decomposition. The fundamental idea of the process is similar to that described in Ref. [77] which employs the decomposition method on experimental data. Application of an alternative azimuthal mode-decomposition method on numerically obtained data for perfectly expanded jets is reported in Ref. [30], which shows a good correlation with the experimental data. The same technique is now applied to the current underexpanded results by considering pressure readings from eight probes located azimuthally along the lip-line at \(x/D = 10\), which are processed to obtain four modes corresponding to \(m = 0, 1, 2, 3\). Figure 3.19 plots the spectral decomposition for the uncontrolled case. It is observed that all the modes have relatively similar levels of contribution as the total signal. A closer examination indicates that \(m = 1\) mode has the highest contribution in the lower Strouhal number range compared to the other modes. However, from Fig. 3.20 which plots the spectral decomposition for the \(St = 0.3\) case, it is observed that the \(m = 0\) is the most substantial contributor to the total signal in the lower Strouhal number range (\(St < 1\)). The total signal is significantly influenced by the \(m = 0\) mode, evident from the peak observed around \(St = 0.3\).
Figure 3.19: Azimuthal decomposition for the baseline case

Figure 3.20: Azimuthal decomposition for the $St = 0.3$ case
which corresponds to the actuation frequency and also the preferred column mode frequency. Though not as significant as the axisymmetric mode, other modes also contribute to the total signal which is evident from their individual SPL levels. At higher Strouhal numbers \((St > 1)\), no mode shows any particular dominance. In contrast, for the \(St = 0.9\) case, though \(m = 0\) is still the dominant contributor to the total levels, the contribution is not as significant as the \(St = 0.3\) case. This is observed in the Fig. 3.21 which plots the spectral decomposition for the \(St = 0.9\) case. The decreased dominance of the \(m = 0\) mode is attributed to the absence of strong toroidal events which cause significant pressure fluctuations as observed for the \(St = 0.3\) case. Comparing Fig. 3.19, Fig. 3.20 and Fig. 3.21, it is also observed that the total SPL for \(St = 0.3\) case is significantly higher compared to both the no control and the \(St = 0.9\) case. This behavior is attributed to the generation or large-scale structures which [1, 37, 69], as discussed in chapter 1, which have a significant influence on the radiative noise characteristics, especially in the downstream polar angle region. In addition to restricting the formation of newer large-scale features like the lower frequency actuation case, excitation at \(St = 0.9\) has a significant weakening effect on the naturally occurring turbulent structures themselves which results in lower levels of pressure signature for this actuation condition.
Figure 3.21: Azimuthal decomposition for the $St = 0.9$ case

3.4 Near-field Noise Characteristics

From the discussion presented chapter 3.2, it is clear that actuation at lower frequencies, especially at those that are close to the frequency of jet column instability ($St = 0.3$) excites shear-layer instabilities which manifest as large-scale toroidal features. The role of large-scale structures as efficient sound sources has been long established (see chapter 1) and intuitively, it is expected that further enhancement in such large-scale features, like for the $St = 0.3$ excitation, leads to higher levels of noise radiation. To investigate this aspect, the noise radiation characteristics of the uncontrolled case are first studied and later, the modification to these properties as
Both qualitative and quantitative approaches are considered to analyze the radiative features of the jet. A frequently used qualitative technique [29] to identify noise radiation patterns is to examine velocity dilatation ($\nabla \cdot \vec{V}$) contours, which act as a surrogate for pressure fluctuations. These contours overlaid over those of vorticity magnitude for the uncontrolled case are plotted in Figs. 3.22. Three primary directions of radiation associated with the three sources of supersonic jet noise (as discussed in chapter 1) are clearly observed. The downstream directivity (red arrow) is attributed to large-scale structures and their associated mixing noise. The side-line (green) and upstream (yellow) directivity is a result of both BBSAN and screech generation due to the interaction of the large-scale structures with shock-cells. To quantify the specific radiation characteristics in these direc-
Figure 3.23: Illustration of probe locations at different polar angles

tions, pressure fluctuations are recorded at three points belonging to the three regions i.e. downstream, side-line and upstream. Representative locations of these points is illustrated in Fig. 3.23 and the variation of SPL with Strouhal number for each of the point probes is plotted in Fig. 3.24. The profile at the downstream location i.e. $\theta = 30^\circ$, exhibits a “peaky” spectrum with maximum amplitude in the $St = 0.3 – 0.4$ range. This observation is consistent with established results, for example Tam et al. [88], which reported the super directive noise in the low polar angle zone to be predominantly in the $St = 0.3 – 0.5$ range, with a reported decrease in noise levels at high polar angles. Comparing the SPL profiles at $\theta = 30^\circ$ and $\theta = 75^\circ$, the sound amplitude at the higher polar angle is indeed lower. Furthermore, consistent with the prediction of Tam [88], among other works, the spectrum exhibits a significant broadband behavior.
An interesting feature of the profile at $\theta = 75^\circ$, is the presence of a conspicuous “hump” centered around $St = 0.9 - 1$. This is attributed to the BBSAN which exhibits peak directivity in the side-line direction [83]. On closer observation, though not as pronounced, this feature is also observed in the spectrum at $\theta = 30^\circ$. As reported by Tam [83], this behavior is expected as the radiative characteristics of the BBSAN deteriorate with decreasing polar angle i.e. in the downstream direction. Comparing the profile at $\theta = 100^\circ$ with the other two, a distinct spectral tone at $St = 0.45$ with high SPL magnitudes is observed. This peak is associated with the formation of screech, which is directed primarily in the upstream direction, generated due to the formation of a feedback loop as described in chapter 1. The screech frequency observed for this case almost identically matches the theoretical value.
calculated from the screech frequency estimation equation proposed by Tam et al. [90] and the experimental measurements of Seiner et al. [72]. The characteristic “hump” associated with the BBSAN is also noticed at this location.

A major goal of flow control is noise mitigation. To identify the efficacy of LAFPAs and different forcing frequencies employed in this study to achieve this goal, a qualitative study is first conducted by analyzing the contours of velocity dilatation. These contours are plotted in Fig. 3.25 for the $St = 0.3$ case and in Fig. 3.26 for the $St = 0.9$ case. Comparing with the directivity patterns of the uncontrolled case (Fig. 3.22), it is evident that excitation at jet column instability frequency ($St = 0.3$) instigates a noise amplification in all the directions. This behavior is apparent from the increased strength of the dilatation contours in Fig. 3.25 when compared to Fig. 3.22. The excitation of instabilities leading to the formation...
Figure 3.26: Contours of velocity dilatation along $z = 0$ plane for the $St = 0.9$ case of large toroidal structures, that are essentially stronger than the naturally occurring features, results in the increased noise levels in the downstream direction. As a result of generation of stronger large-scale structures for the $St = 0.3$ case, the shock-structure interaction is also strengthened leading to increased radiation even in the upstream direction. In stark contrast, the contours for the $St = 0.9$ case, plotted in Fig. 3.26, indicate reduction in sound amplitudes, which is evident from the weaker and less coherent dilatation patterns. As discussed earlier in the context of Fig. 3.17, excitation at $St = 0.9$, in addition to hindering the roll-up of the toroidal events, weakens the naturally forming shear-layer turbulent structures which results in noise level reduction. The quantitative analysis presented below further corroborates these observations.

To perform a direct comparison of the noise levels for all cases, Overall Sound
Pressure Levels (OASPL) are calculated using the \textit{rms} of pressure fluctuations, recorded at locations shown in Fig. 3.23, by employing the relation shown in Eqn. 3.4.

\[
OASPL = 20 \log_{10} \left( \frac{p_{\text{rms}}}{p_{\text{ref}}} \right) \tag{3.4}
\]

where \( p_{\text{ref}} = 20 \mu Pa \), which corresponds to the smallest pressure fluctuation recognizable by the human ear. From these OASPL values, \( \Delta \text{OASPL} \) defined as the difference between the OASPL amplitudes of the uncontrolled case and the control case, is calculated for each of the excitation frequencies, and plotted in Fig. 3.27. Reinforcing the observations inferred from the qualitative analysis presented above, excitation at \( St = 0.3 \) results in a noise amplification at all the probe locations, which is evident from the negative values of \( \Delta \text{OASPL} \) calculated at these probes. Previous efforts on perfectly expanded jets \cite{69} reported similar observations for downstream polar angles \((\theta < 50)\), however, they did not observe significant amplification of noise levels relative to the baseline case for the upstream direction. The amplification observed at higher polar angles in Fig. 3.27 is attributed to the strengthening of the shock-structure as reported earlier. Considering the \( \Delta \text{OASPL} \) between the uncontrolled and the \( St = 0.9 \) case, a consistent reduction of noise levels, in accordance with qualitative findings reported earlier, is also observed here.
Figure 3.27: Values of $\Delta$OASPL between the uncontrolled and $St = 0.3$, and uncontrolled and $St = 0.9$
3.5 Summary

In this chapter, results obtained from high-fidelity LES performed to investigate the effect of active control on an underexpanded supersonic jet are analyzed. For this purpose, a Mach 1.3 underexpanded jet issuing from a converging nozzle is considered. The shock-cell train a characteristic feature of such imperfectly jets is effectively captured, as evident from the good match obtained with experimental observations. The shock-train exhibits a significant influence on the flow, the signature of which is readily visible as the presence of undulations in the mean velocity and jet width profiles. For these shock dominated flows, simulations performed to identify the impact of Reynolds number indicates relative independence of the jet flow evolution and overall flow-field development to variations in Reynolds number. Analysis of near-field noise levels and directivity patterns indicate the existence of three distinct noise components: turbulent mixing noise, broadband shock associated noise, and screech tone noise, all of which have different radiation characteristics. The peak magnitudes of turbulent mixing noise occur in the downstream direction, in the range of $St = 0.3 - 0.4$, which is consistent with previous efforts. The side-line direction is dominated by relatively broadband spectrum, with a characteristic “hump” at high frequencies ($St \approx 1$), which is attributed to the shock-structure interaction resulting in BBSAN. High amplitude screech tone at a distinct narrow band frequency of $St \approx 0.45$ is recorded at the upstream po-
lar angles. This screech frequency agrees with the previous theoretical predictions and experimental observations. To facilitate the study of active control techniques on jet flow properties, eight LAFPAs are modeled using a semi-empirical surface heating technique and placed along the circumference at the exit of the nozzle. Axisymmetric mode pulsing ($m = 0$) is considered at two different Strouhal numbers of 0.3 (corresponding to preferred column mode frequency of the jet) and 0.9. Application of control affects the plume properties such as core length and spreading rates, depending on the actuation frequency. The $St = 0.3$ case shows enhanced levels of mixing compared to uncontrolled and $St = 0.9$ cases. This behavior is attributed to the formation of large toroidal structures which increase the spreading rate of jet plume by enhancing the rate of entrainment. The formation of these rollers is attributed to the amplification of jet column mode instabilities, when actuated at $St = 0.3$. The life cycle of these structures is clearly visible from the the analysis of the iso-levels of Q-criterion, which reveal the formation of two different types of rollers corresponding to the switching-on and switching-off of the actuators. The formation of large-scale features is shown to enhance the noise levels in the near-field, relative to uncontrolled case. On the other hand, actuating at $St = 0.9$ does not result in the formation of toroids. Furthermore, a suppression of naturally occurring structures is evident from the diminished levels of $TKE$ for this case even when compared to the uncontrolled case. As a result, the near-field noise levels experience a reduction in magnitude relative to the uncontrolled case. Although there are significant differences in coherent structures at the two excitation
frequencies, the integral length scale studies show modest differences in azimuthal correlations only near the nozzle exit. Further downstream, the control cases exhibit similar length scales indicating that the influence of control does not always have a visual signature.
Chapter 4

Flow Characteristics of a Twin-jet Configuration

One of the objectives of the current work is to explore the flow and acoustic behavior of complex propulsive systems, specifically the twin-jet configuration. As mentioned in Chapter 1, research works establishing the flow physics of real world scenarios such as the twin-jet configuration are far fewer, relative to those which study the behavior of comparatively simpler configurations such as a single-jet. As a result, there exists a significant scope for advancing the knowledge of the behavior of such complex flow-fields. The work performed here aims to further the understanding of the complex flow-field characteristics of a specific real world case i.e. a twin-jet configuration. The objectives of this thrust are: 1) Investigate the dynamics of twin-jet plume interactions on the hydrodynamic features of the flow-field such as jet plume structure and shear-layer characteristics, 2) Qualitative and quantitative study of acoustic features of the twin-jet noise field such as
jet noise shielding and magnification and azimuthal asymmetry of the noise radiation, and 3) Analysis of the constituent fluid-thermal (FT) components of the flow-field, extracted from the simulation data using a novel decomposition approach, with special emphasis on investigating the radiative characteristics of the decomposed acoustic mode. For this purpose, a circular twin-jet system operating at a perfectly expanded Mach number of 1.23 is considered. After validating the simulations by comparing with corresponding experimental results (chapter 4.1), the fundamental properties of jet plumes are discussed in detail. This analysis, presented in chapter 4.2, emphasizes the influence of the plume interaction on the characteristics of the jets such as the potential core properties and shear-layer characteristics.

4.1 Experimental Validation

In this section, a validation of the simulations is performed by first examining the evolution and propagation of dominant modes of the twin-jet configuration. As shown by Seiner et al. [72] and later corroborated by Kuo et al. [45], the primary instability modes of the twin-jet plumes and their coupling mechanism are a function of the flow Mach number. The jet plumes in lower Mach number range ($1 < M < 1.2$) are dominated by the axisymmetric mode ($m = 0$) instability resulting in the formation of toroidal structures. In this regime, there is no significant plume interaction. With an increase in Mach number, a helical mode ($m = 1$) in-
stability dictates the jet plume structure. At Mach numbers above $M \sim 1.3$, the helical modes of the plumes exhibit a strong interaction, resulting in a symmetric flapping mode coupling between the plumes. To understand the nature of jet plume structure for the Mach number of focus in this work ($M = 1.23$), iso-levels of Q-criterion, colored by u-velocity showing in Fig. 4.1, are examined. The hairpin-like vortices (marked in circles), arising out of azimuthal instabilities, with heads in the outer shear-layer and legs oriented towards the jet axis, are clearly evident. As the vortices propagate downstream, the legs of these vortices, which extend into the higher-speed core, travel faster than their heads, resulting in their elongation. The complex turbulent interactions form helical mode ($m = 1$) patterns (marked in rectangular boxes). These helical patterns exhibit symmetry about the mid-plane of the system i.e. they possess opposite directionality. Looking downstream, the
Figure 4.2: Experimental Schlieren image depicting the presence of an \( m = 1 \) helical mode instability

left nozzle contains a right-handed screw type helix whereas the right nozzle contains a left-handed screw type helix. The emergence of the helical mode agrees with the measurements of Seiner et al. [72], who reported the presence of \( m = 1 \) helical mode patterns for each of the jet plumes at an operating Mach number of \( M = 1.23 \). Indeed, concurrent experiments [45] have also observed a dominant \( m = 1 \) helical mode, as seen in the Schlieren image of Fig. 4.2, taken from the work of Kuo et al. [45].

Further validation is performed by comparing the computationally obtained near-field OASPL values, with those obtained from the experiments. For this purpose, pressure fluctuations are recorded along a linear-array as illustrated in Fig. 4.3. The array, designated A1, contains twelve probes chosen to match with the microphone locations of the concurrent experimental effort. The first probe is located at a distance of \( \sim 1.5D \) from the centerline of one of the nozzles of the
twin-jet and $1.5D$ from the nozzle exit plane. The twelve probes are separated from each other by a distance of $1D$ measured along the array. The array is located on the $\phi = 0^\circ$ azimuthal plane, which is the plane containing the jet centers, at an inclination of $\sim 10^\circ$ to the nozzle centerline. This angle is chosen so as to approximately match with the outer shear-layer spreading rate of the jet plume. Care is taken to ensure the placement of the probes outside shear-layer of the jet plume to reduce the impact of the vortical features of the jet plume. In addition to probes along array A1, pressure fluctuations are also recorded at probes along two other arrays, designated A2 and A3, whose probe locations are obtained by radially shifting the probes of A1 by $0.5D$ and $1.5D$ respectively.
In Fig. 4.4, we plot the OASPLs obtained along the three array locations. As expected, sound levels decrease with increasing radial distance from the jet. Comparing the values along A2 with those obtained from the experiment, we observe good qualitative agreement, and modest quantitative differences. The profiles peak at similar axial distance, approximately $6D$, from the nozzle exit. For reference, the core collapse occurs also at $\sim 6D$ (discussed in detail in chapter 4.2). The simulated values also exhibit similar decay rate beyond the maximum location. It is interesting that better comparison with experiment is obtained for OASPLs along A2 than A1. Though the profile suggests similar characteristic trends, values along A1 exhibit higher levels when compared to the experiment. This may possibly be attributed to the laminar-to-turbulent transition of computational boundary layers. These layers, being laminar at the nozzle exit, transition after exiting the nozzle (discussed further below) and consequently result in an increase in pressure fluctuations being recorded along the closer (A1) array. This behavior is consistent with numerous studies on single-jet configurations that have related the influence of the boundary layer thickness and state to near and far-field noise levels. For example, Barré et al. [4] and Bogey et al. [10] reported increased noise levels for jets with laminar exit boundary layers, compared to those of turbulent exit boundary layers.

Since experimental mean flow data is not available for this configuration, direct quantitative validation is difficult. Thus, we compare our results with previously published single-jet experimental data [68] obtained at similar operating Mach
Figure 4.4: Comparison between the experimental and computationally obtained OASPL values along the three near-field arrays numbers ($M = 1.3$). Figure 4.5 shows the profile of axial variation of the mean streamwise centerline velocity for the twin-jet case, along with measurements of Samimy et al [68]. Results for a corresponding single-jet computed for comparison purposes is also included. In addition to these, a representative curve of centerline decay rate calculated using Witze’s theory [99] is also included. Following common practice [8], the computational profiles have been axially shifted to collocate the core collapse location with that obtained using Witze’s theory ($\approx 8D$) to directly compare the centerline delay rate among all the profiles, which are clearly in good agreement. The minor differences between the experimental and the other profiles are ascribed to the slightly higher operating Mach number of the experiments. Other modest differences relative to the theoretical profile are consistent with the results of Bogey et al.[12] and relate to the state of the nozzle exit boundary layer.
4.2 Interaction dynamics of twin-jet plumes

As mentioned in chapter 1, the interaction between the closely placed jet plumes results in a very complicated flow-field. The features of jet plume interaction depend on the jet exit conditions, some of which were already discussed in the context of the $m = 1$ mode of Fig. 4.1. Here we focus primarily on the mean features. Where appropriate, the results are contrasted to those of a single-jet to highlight the impact of coupling between the twin-jet plumes. For reference, Fig. 4.6 shows the important locations of data extraction on the plane containing the two jets and also the angular convention used in this paper. It is noted here that $\phi$ in the context of results in this section refer to the azimuthal angle. In chapter 3, this variable was
used to denote the different phases in the actuation cycle.

Figure 4.7 shows the contours of mean u-velocity along the $\phi = 0^\circ$ plane. The plumes are symmetric about the mid-plane as expected and show similar features. Thus, key aspects such as the potential core length and the shear-layer spreading patterns remain similar for each. Also, the left and right-handedness of the $m = 1$ mode on the left and right plumes (looking downstream, see Fig. 4.1) do not result in any perceptible differences in mean characteristics. The most striking features in Fig. 4.7 lie between the two axes. To investigate these features, we analyze the axial variation of the streamwise velocity along the symmetry plane of the twin-jet system, as shown in Fig. 4.8. The u-velocity profile imposed by a single-jet system at an identical radial location (i.e. $1D$ from the centerline of the single-jet) is also included.

As noted earlier, the twin-jet flow-field can be classified into three main regions based on the interaction behavior of the jets [49]: a) converging, b) merging, and c) combined. In the converging region, the shear-layers from both the jet plumes grow through entrainment. This induces a local region of relatively higher streamwise velocity as clearly evident from Fig. 4.8. In the region $x/D < 4$, the twin-jet configuration exhibits a positive u-velocity of about 10% of the nozzle exit velocity, which is much larger than observed at a corresponding location with a single-jet. Similar observations have been made by Marsters [50] who also reported formation of a secondary jet, between two parallel plane jets, with a magnitude of around 10% of nozzle exit velocity. The end of the converging region corresponds to the
Figure 4.6: Illustration of different locations of data extraction and angular convention
Figure 4.7: Contours of mean axial velocity along the plane containing the two jets ($\phi = 0$)

Figure 4.8: Axial variation of mean streamwise velocity along the symmetry plane for the twin-jet, and along an identical radial location for the single-jet
first direct interaction between the inner jet shear-layers at the merging point. In this region, we observe the onset of non-linear velocity growth along the symmetry region, owing to the physical interaction between the adjacent jets. Figure 4.8 also indicates that the merging point for the twin-jet configuration is at $x/D \sim 3.3$. On the other hand, because of the natural growth and spreading, the single-jet shear-layer also exhibits a substantial velocity growth initiated further downstream i.e. $x/D \sim 3.8$.

This difference in the location of initiation can be explained by analyzing the bending of jet plumes. The phenomenon is illustrated by tracking the boundary of the potential core of the single-jet plume, and one of the twin-jet plumes as shown in Fig. 4.9. The potential core boundary can be tracked by plotting a single velocity contour line at 95% of the nozzle exit velocity. As observed in the zoomed in view of the end of the potential core region, shown in Fig. 4.9, the twin-jet plume boundary does not overlap exactly with single-jet plume boundary but instead exhibits a deviation towards the symmetry plane of the system, away from the centerline. A more quantitative assessment suggests that the twin-jet case exhibits a bending of about 3% of the nozzle diameter towards the symmetry plane. These observations also hold for the other twin-jet plume. In contrast, the single-jet plume does not exhibit any bending. This is evident from the radial position of the potential core collapse location, which remains along the centerline of the plume. For these reasons, the phenomenon of bending appears to be responsible for the earlier initiation of non-linear velocity growth along the inter-nozzle region of the twin-jet,
Figure 4.9: Zoomed in view of the potential core boundary near the core collapse location

relative to the location of onset of natural growth of the single-jet (at identical radial distance).

Some prior works [49] on two turbulent planar jets have reported the formation of a recirculation zone along the converging region. However, as seen in Fig. 4.8, no negative velocities are recorded in the present simulations along the symmetry plane in the converging region, indicating the absence of any recirculation. This difference in behavior is attributed to the geometrical construct of the current twin-jet configuration, where ambient air is allowed to be entrained into the inter-nozzle region. The current results are in agreement with the observations of Marsters [50] and Elbanna [25] who used “ventilated” configurations, where the jets exhaust from free standing nozzles and report the presence of a high velocity region along
the symmetry plane of the twin-jet configuration. In contrast, efforts [49] observing a recirculation region employed “unventilated” i.e. slotted jet configurations, where the entrainment of the ambient air is restricted. In such configurations, the genesis of a recirculation region is similar to that responsible for the formation of a recirculation zone in cases such as flow over a backward facing step. Beyond the merging point, a strong non-linear velocity growth is observed till \( x/D \approx 10 \) (see Fig. 4.8), where the mean velocity attains its peak and shear-layers completely merge with each other. This merging process continues into the combined region where the mixing of the plumes ultimately results in a single-jet type structure.

The three regions introduced above, converging, merging and combined show very distinct features. To illustrate these differences, profiles of spanwise variation of axial velocity along the plane containing the jets (i.e. \( \phi = 0 \)), at various streamwise locations (belonging to the different regions), are analyzed and plotted in Fig. 4.10(a). The lack of physical interaction between the jet plumes manifests as a double-square-pulse shape for the cross-flow profile of u-velocity in the converging region \( (x/D = 0.7) \). In agreement with Fig. 4.8, the cross-flow profile in the converging region exhibits a very high axial velocity component along the symmetry plane \( (z/D = 0 \text{ in Fig. 4.10(a)}) \). In the merging region, as the shear-layers grow and interact, the velocity profile depicts a double-bell shape \( (x/D = 7) \). Moving into the combined region, the dissipation of the velocity along the jet centers is clearly observed \( (x/D = 14) \). Eventually, the peak velocity region shifts from the centerline of the jets to the symmetry plane of the system. As a result, the
cross-flow profile of axial velocity resembles a single-jet type single-bell profile \( (x/D = 21) \). Comparing the combined region twin-jet profile with that of an actual single-jet profile at identical axial location, as shown in Fig. 4.10(b), we observe a very similar qualitative trend. At this location, the twin-jet profile does not retain features of the original double-pulse configuration. This indicates the completion of the merging process between the jet plumes and the subsequent transformation into a single-jet type configuration. The quantitative differences between the two profiles is attributed to the variation in the flow dynamics of each of the cases that are responsible for the evolution of the respective profiles.

To further study the 3-D mean flow development in these three regions, we analyze cross-flow contours of axial velocity in each of these regions. Previous efforts [56] exploring turbulent circular twin-jets at much larger separation distance (center-to-center spacing of \( 5D \)) reported a negligible induced velocity component along the symmetry plane. As a consequence, they reported the formation of a symmetric velocity contour pattern about the centerline of the individual jets, in the converging region. In addition to the above observation, they also reported the evolution of an elliptic cross-sectional profile in the merging region. Despite the presence of a secondary flow in the converging region, the cross-flow contours of streamwise velocity obtained from the current simulations agree with the observations of Okamoto et al. [56]. These contours exhibit symmetry about the mid-plane of twin-jet configuration, as observed from Fig. 4.11. Also in agreement with their observations in the merging region, the cross-flow axial velocity contours depict
(a) Comparison of the u-velocity profiles at different axial locations belonging to different regions of the twin-jet

(b) Comparison of u-velocity profiles between the single-jet and the twin-jet cases at $x/D = 21$ belonging to the combined region of the twin-jet case

Figure 4.10: Illustration of profiles of radial variation of the streamwise velocity
an elliptical cross-sectional distribution. This behavior is attributed to the intense interaction between jet plumes resulting in the above discussed non-linear velocity growth. As reported earlier, beyond the merging region, the twin-jet flow-field gradually transitions to assume a single-jet type structure. As a result, the cross-flow contours in the combined region exhibit an almost circular contour distribution.

The secondary flow of relatively high magnitude discussed in the context of Fig. 4.8 was attributed to the interaction of the induced velocity fields of the two twin-jet plumes, and the geometrical construction of the twin-jet configuration affecting entrainment of the ambient air. The characteristics of the inner shear-layers of the two jet plumes are directly affected by this secondary flow. Considering the geometry of the twin-jet, it is evident that the outer shear-layer entrains from a

Figure 4.11: Cross-flow contours of axial velocity at different axial locations for the twin-jet case
quiescent surrounding whereas, as discussed above, the inner shear-layer entrains from the inter-nozzle region with an induced secondary flow. As a result, the inner lip-line experiences relatively smaller velocity gradients and consequently the azimuthal development of the twin-jet shear-layer loses its azimuthal symmetry. This behavior is clearly visible in Fig. 4.12(a) where we plot the axial variation of the radial derivative of mean velocity i.e. $\partial u/\partial z$, along the inner and and outer lip-lines of the twin-jet configuration. In the region upstream of $x/D \sim 3.5$, as expected, the profile along the inner lip-line exhibits lower radial velocity gradients relative to the outer lip-line. Since turbulence can be considered to be an instability generated due to shear, the presence of lower levels of shear directly implies the existence of lower turbulence intensity levels. This observation is corroborated by plots of axial variation of turbulence intensity ($u_{rms}$), along both the inner and outer lip-lines, as shown in Fig. 4.12(b). As theorized earlier, beyond the initial flow settling region ($x/D < 2$), the profile along the inner lip-line exhibits lower levels of turbulence intensity relative to the other profiles. However, a common feature of both the profiles plotted in Fig. 4.12(b) is the presence of a double-hump profile. As discussed in chapter 2, the formation of roll-up structures and their subsequent pairing, along with the laminar-to-turbulent transition of the shear-layer is responsible for this characteristic feature, consistent with the literature. [12, 101]

Another consequence of entrainment of high velocity fluid along the inner lip-line is the observation of lower mixing levels. This phenomenon can be analyzed with plots of axial variation of streamwise velocity along the inner and outer lip-
Figure 4.12: Illustration of axial variation of shear and turbulence intensities along the inner and outer lip-lines for the twin-jet case

(a) Axial variation of the radial derivative of u-velocity along the inner and outer lip-lines for the twin-jet case

(b) Axial variation of $\text{rms}$ of u-velocity along both the lip-lines for the twin-jet case
lines, as shown in Fig. 4.13. Due to entrainment of relatively higher velocity fluid and the resulting lower mixing levels in the region between the plumes, the inner lip-line profile exhibits larger mean velocity compared to both the outer lip-line and the single-jet. The outer lip-line, however, exhibits similar mean velocity profile compared to the single-jet, since the local conditions are similar.

A more in-depth idea of the differences in entrainment between the inner and outer lip-lines of the twin-jet case is obtained by studying the distribution of radial velocity along the $\phi = 0$ plane. Filled contours of this variable are plotted in Fig. 4.14 and for comparison, the same is plotted for the single-jet in Fig. 4.15. The single-jet contours clearly show a symmetric distribution of the radial velocity.
Figure 4.14: Filled contours of $w$-velocity for the twin-jet case along the plane containing the jet centers i.e $\phi = 0$

Figure 4.15: Filled contours of $w$-velocity for the single-jet case along $y = 0$ plane
parameter, with respect to the jet centerline. They also indicate the presence of radial velocity inversion [27], where the radial velocity profile exhibits an inflection point. This is more evident from the plots of radial variation of the w-velocity at an axial location of $x/D = 3.5$ shown in Fig. 4.16(a). The profile exhibits a “double-s” shape with an inflection point located near the lip-lines. Between the centerline and the inflection point, the velocity distribution indicates the presence of a flow directed away from the jet centerline indicating the spreading of the jet. Beyond the inflection point however, an inward directed flow, which is representative of the local entrainment patterns, is observed, as evident from the change in sign of the radial velocity. Along the outer lip-line, the w-velocity distribution for the twin-jet case (Fig. 4.14) exhibits very similar behavior to that of the corresponding single-jet case. Along the inner lip-line however, the contours show significant differences. Visually, as seen from Fig. 4.14, no discernible radial velocity inversion location can be identified along the inner lip-line. This observation is corroborated by the plots of radial variation of $w$-velocity at $x = 3.5$, along the inner and outer lip-lines, as shown in Fig. 4.16(b). Along the inner lip-line, the profile does not indicate the presence of any velocity inversion. This is again consistent with the effect of the confined inter-nozzle area on entrainment.

The above discussion clearly describes the impact of the secondary flow on the properties of shear-layer development. It is already known that the average location of the collapse of the potential core of the jet is a strong function of the shear-layer characteristics. To identify the impact of the azimuthally varying shear-layer
Figure 4.16: Radial variation of the w-velocity for both the single-jet and twin-jet cases, at $x/D = 3.5$, illustrating the phenomenon of radial variation inversion
profiles on the potential core length, we plot the axial variation of the mean centerline u-velocity for one of the jets in the twin-jet configuration, along with the single-jet in Fig. 4.17. The twin-jet profile does not show any significant variation in the length of the potential core compared to the single-jet case but does exhibit a lower subsequent decay rate. Though the inner and the outer shear-layers of the twin-jet configuration show substantial differences in their development characteristics (Figs. 4.12, 4.13), these differences do not appear to influence the collapse of the jet plume. This behavior may be attributed to the confined azimuthal influence of the inter-nozzle region. To illustrate this, we plot the axial variation of the mean velocity along the lip-line of one of the twin-jet nozzles, at multiple azimuthal locations in Fig. 4.18. From this figure, it is evident that except along the inner lip-line, all the other locations show similar velocity profiles compared to the single-jet profile. Since the inner lip-line profile is influenced by the inter-nozzle region, it is hypothesized that this influence is confined to small azimuthal angles and hence the profiles along other azimuthal angles are relatively unaffected. As a result, the mean location of the collapse of the potential core remains unaffected. The higher decay rate for the single-jet profile is due to the enhanced spreading of the single-jet shear-layers beyond the collapse location, which is discussed in detail in chapter 5.1
Figure 4.17: Comparison of axial variation of mean u-velocity along the centerline of the single-jet case and that of one of the nozzles of the twin-jet.

Figure 4.18: Profiles of axial variation of u-velocity at different azimuthally located lip-lines for the twin-jet case. Angles are measured counter clockwise with respect to nozzle center.
4.3 Summary

The primary aim of this chapter was to analyze the impact of twin-jet plume interactions on the dynamics of the inter-nozzle region, jet shear-layer characteristics, and jet plume properties. For this purpose, computations based on an LES approach are conducted on a Mach 1.23 circular twin-jet system with a center-to-center spacing of two nozzle diameters. An overset meshing technique is employed to discretize the complex flow domain. In agreement with concurrent and previous experiments at similar flow conditions, a Q-criterion based qualitative analysis of the flow structure reveals the presence and dominance of an $m = 1$ helical mode. Detailed analysis of the mean flow indicates the formation of a high magnitude secondary flow along the inter-nozzle region, in the converging section. In addition to the interaction of the mutually induced velocity fields, the formation of secondary flow is attributed to the “ventilated” construction of the twin-jet geometry allowing for the entrainment of ambient air into the inter-nozzle region. The effect of this secondary flow on shear-layers in the inter-nozzle region, relative to those entraining from the quiescent surrounding, is characterized. Analysis of shear-layers at different azimuthal angles indicates that the impact of the secondary flow on the shear-layer properties is confined to very small azimuthal angles, with respect to the centerline of the jet. As a result, the potential core of the twin-jet system is generally similar to that of a single-jet configuration, at identical
flow conditions.
Chapter 5

Acoustic Characteristics of the Twin-jet Configuration

The modification of jet plume properties, relative to a conventional single-jet plume, was explored in detail in the previous section. Such modifications, in addition to the non-axisymmetric nature of the twin-jet configuration, can have a significant impact on the sound levels in the near and far-fields. A consequence of the twin-jet arrangement is the existence of an azimuthally asymmetric noise field [13]. The closely spaced interacting jets radiate without restriction in the direction perpendicular to the plane containing the jet centers. This results in enhanced noise levels even beyond those obtained with two linearly interacting incoherent jets [14], referred to as noise magnification, which is attributed to the non-linear interaction between the twin-jet plumes. In the plane containing the jets however, particularly in the downstream direction, one jet can shield the radiation from the other. Though noise reduction is observed in the plane containing the jet centers,
increased noise levels in other azimuthal directions offset the benefits of shielding.

In this section, this complex sound field generated by the twin-jet configuration is analyzed in detail. First, traditional diagnostic techniques such as dilatation contours and Sound Pressure Levels (SPL) will be used to identify the fundamental characteristics such as radiated frequencies and directivity in the acoustic near-field. Next, a novel decomposition technique based on Momentum Potential Theory (MPT) is employed (chapter 5.2) to extract the fundamental fluid-thermodynamic (FT) components of the flow i.e. hydrodynamic, acoustic, and thermal. The aim of this decomposition is to examine the efficiency of pure acoustic mode in reproducing the complex features of twin-jets' sound field in addition to investigating the core dynamics of the acoustic mode which are responsible for its radiative behavior. These aspects are discussed in detail in chapters 5.2.1 and 5.2.2.

5.1 Impact of twin-jet configuration on the near-field

As described in chapter 3.4, a common method to visualize the directivity and intensity of radiating pressure waves is by studying the velocity dilatation $(\nabla \cdot \bar{V})$ contours [29], that act as a surrogate for pressure fluctuations. The instantaneous contours of velocity dilatation calculated on the twin-jet flow-field, along the $\phi = 0^\circ$ plane are plotted in Fig. 5.1. These contours exhibit several compression waves with wave fronts directed towards the downstream direction. These waves are
consistent with the Mach wave radiation emanating from supersonically traveling large-scale structures in the flow. As discussed in chapter 1, with regard to high-speed single-jet flows, research [9, 88] has shown the existence of two primary radiation directions: a) A radially coherent downstream directed sound field attributed to the large-scale structures and b) A spatially decorrelated stochastic field in the side-line direction attributed to the fine-scale turbulence. An illustration of this behavior can be observed in Fig. 5.2 where the dilatation contours of the single-jet case, simulated in this work as a reference, are plotted. Significant similarities are readily noticed when comparing the dilatation contours of the single-jet (Fig. 5.2) with those of the twin-jet (Fig. 5.1). Both figures clearly show strong and coherent downstream directed radiation, whereas, the radiation in the side-line di-
Figure 5.2: Contours of divergence of velocity (dilatation) on the \( y = 0 \) plane for the single-jet case

Correction is not as large in magnitude and does not exhibit any high levels of spatial coherence.

As mentioned earlier, the twin-jet flow-field shows complex acoustic behavior such as jet noise shielding and azimuthal asymmetry. In the analysis presented below, various locations in near-field of the twin-jet, and equivalent locations for the corresponding single-jet are probed to highlight the quantitative differences between the near-field sound levels of the two configurations and to also identify the underlying mechanisms responsible for such differences.

First, the sound levels obtained from both configurations, at the probe locations along the array-A1 (see Fig. 4.3) are compared. For this purpose, the pressure fluctuations recorded along array-A1 are processed to calculated the correspond-
ing OASPLs. For both the cases, the variation of OASPL with the probe location is plotted in Fig. 5.3. From this figure, slightly lower values of the OASPL are observed for the twin-jet case, when compared to the corresponding single-jet.

Though interesting at first, this result is consistent with concurrent experimental measurements [45] and is explained by jet plume bending. As discussed in reference to Fig. 4.9, the twin-jet plumes bend towards each other. In effect, this places the probes in a less perturbed location at the edge of the plume and results in the recording of lower levels of pressure fluctuations. A related reason is associated with the spreading rates for both the twin-jet and the single-jet cases, as shown in Fig. 5.4. Spreading rate is calculated as the distance between the centerline of the nozzle to the radial location where the velocity is half of the local
Figure 5.4 shows that the outer shear-layer for the twin-jet case exhibits lower spreading rates, especially beyond $x/D \sim 6$, compared to the single-jet case. This reduction in spreading rate of the twin-jet is in agreement with the work of Moustafa [53] who also reported lower spreading rates for a twin-jet relative to a single-jet. Considering the location of the point probes (Fig. 4.3), this behavior of the outer twin-jet shear-layer, like bending, results in an increased distance between the point probes and the twin-jet plume, thereby resulting in lower levels of pressure fluctuations.

The differences in the spreading characteristics between the two cases is also observed qualitatively from the contours of mean axial velocity, plotted at a value of 10% of the nozzle exit velocity, as shown in Fig. 5.5. The contours for one of twin-jet plumes, and the single-jet plume are overlaid on top of each other to aid a one to one comparison. These contours corroborate the observations of Fig. 5.4. After the initial flow settling region ($x/D \approx 2$) where both cases exhibit nearly identical spreading rates, the twin-jet outer shear-layer appears to spread at a slower rate compared to the single-jet case. In addition to impacting the pressure fluctuations, the suppression of the spreading rate beyond the core collapse region for the twin-jet case is also responsible for the reduced decay rate of axial velocity observed in Fig. 4.17.

Next, to investigate jet noise shielding for the twin-jet case, pressure fluctuations are recorded at different points (corresponding to different polar angles $\theta$) on the $\phi = 0^\circ$ plane, along a $13D$ arc measured from symmetry plane of the
Figure 5.4: Comparison of spreading rates of the single-jet case and the outer shear-layer for the twin-jet case

Figure 5.5: Comparison of $u = 0.1 \times U_{ref}$ contour line for the twin-jet and the single-jet cases illustrating the suppression of spreading for the twin-jet case
twin-jet, centered at the nozzle exit plane. A representative sketch of the setup is shown in Fig. 5.6. Equivalent probe locations are also considered for the corresponding single-jet. As previously discussed, the phenomenon of shielding plays a crucial role along the plane containing the jet centers i.e. $\phi = 0^\circ$ plane, especially in the lower polar angle region. To isolate the impact of shielding, pressure fluctuations recorded at the probes are processed to calculate OASPL as well as to extract $\Delta$OASPLs, defined as the difference between the OASPL of twin-jet and
Figure 5.7: Variation of $\Delta\text{OASPL}$ at the probe locations shown in Fig. 5.6, on the $\phi = 0^\circ$ plane of the twin-jet configuration

$\text{OASPL}+3dB$ values of the single-jet. The $3dB$ increment to the single-jet sound levels represents the doubling of acoustic power due to the presence of two incoherent non-interacting jets. Therefore, a negative value of $\Delta\text{OASPL}$ indicates the presence of shielding, a value close to zero indicates weak plume interaction, and a higher positive value indicates a strong non-linear interaction between the twin-jet plumes [14]. In Fig. 5.7, the variation of $\Delta\text{OASPL}$ versus polar angle is plotted. For low polar angles i.e. in the downstream direction, negative values of $\Delta\text{OASPL}$ are recorded indicating a strong shielding effect which is most dominant around $40^\circ$ polar angle. As the polar angle increases, the intensity of shielding steadily decreases and $\Delta\text{OASPL}$s become positive. However, the magnitude of the positive values is relatively close to zero indicating a linear interaction between the
Figure 5.8: Schematic of the probe locations chosen on the $\phi = 90^\circ$ plane of the twin-jet configuration to investigate the presence of jet shielding pressure waves from both the jet plumes.

The noise field of a twin-jet is azimuthally asymmetric and for that reason, the jet plume interaction can have a significantly different effect on the sound levels along different azimuthal angles. To demonstrate this behavior, similar probe locations as Fig. 5.6 are considered but on the $\phi = 90^\circ$ plane i.e the plane perpendicular to the plane containing the jet centers. A schematic of these probe locations is shown in Fig. 5.8. Equivalent probe locations are considered for the corresponding single-jet. The variation of $\Delta$OASPL with polar angle on the $\phi = 90^\circ$ plane is plotted in Fig. 5.9. Unlike the observations along the $\phi = 0^\circ$ plane (Fig. 5.7), the
Figure 5.9: Variation of $\Delta OASPL$ at the probe locations shown in Fig. 5.8, on the $\phi = 90^\circ$ plane of the twin-jet configuration

profile along the $\phi = 90^\circ$ plane exhibits negligible traces of shielding. Furthermore, the large amplitudes of $\Delta OASPL$ in the higher polar angle region indicate a strong acoustic interaction between the jets. This enhancement of sound levels in the $\phi = 90^\circ$ plane is attributed to the absence of any mechanism to curtail the radiation of pressure waves, from both the jets, in this direction.

5.2 Flow and Acoustic Field Characterization using Momentum Potential Theory

From the above discussion, it is evident that the twin-jet configuration exhibits complex acoustic behavior. In order to investigate the fundamental mechanisms
that are responsible for such complexities, it is necessary to employ a decomposi-
tion that can efficiently segregate the flow-field into contributions from different
constituent modes. Recently, Unnikrishnan and Gaitonde [92] successfully imple-
mented a decomposition based on Momentum Potential Theory (MPT) which was
originally proposed by Doak [24], to extract the constituent fluid-thermodynamic
modes of a flow-field. Considering the complex flow-field of the twin-jet which
was discussed in great detail in chapter 4, segregation of the FT modes offers an op-
portunity to examine the dynamics in a different manner. For this purpose, the de-
composition based on MPT is performed, following the implementation methodo-
logy of Ref. [92]. A brief description of MPT and its implementation are provided
below, for completeness.

The extraction of the constituent fluid-thermodynamic (FT) modes of a flow-
field using MPT is based on the decomposition of the momentum-density ($\rho \bar{u}$),
which can be written as:

$$
\rho \bar{u} = \bar{B} + B' - \nabla \psi', \nabla . \bar{B} = 0, \nabla . B' = 0 \quad (5.1)
$$

where $\bar{B}$ is the mean solenoidal component, $B'$ is the fluctuating solenoidal com-
ponent and $\psi'$ is the fluctuating scalar component, the negative gradient of which
forms the irrotational component of $\rho \bar{u}$. The mean scalar potential ($\bar{\psi}$) is assumed
to be zero based on the irrotational and solenoidal nature of the parameter for
statistically converged flows. In conjunction with mass conservation equation for
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time stationary flows, the decomposition expressed in Eqn. 5.1 results in a Poisson equation for $\psi'$ given in Eqn. 5.2.

$$\nabla^2 \psi' = \frac{\partial \rho'}{\partial t}$$  \hspace{1cm} (5.2)

The irrotational field consists of both the acoustic and thermal components of the flow. Due to the inherent linearity of Eqn. 5.2, equivalent equations for extracting the thermal and acoustic modes are easily obtained and are given in Eqn. 5.3

$$\psi' = \psi'_A + \psi'_T, \nabla^2 \psi'_A = \frac{1}{c^2} \frac{\partial p'}{\partial t}, \nabla^2 \psi'_T = \frac{\partial \rho}{\partial S} \frac{\partial S'}{\partial t}$$  \hspace{1cm} (5.3)

where $\psi'_A$ and $\psi'_T$ are the acoustic and thermal components of the irrotational momentum-density, respectively, $c$ is the local speed of sound, and $S$ is entropy. Full 3-D versions of the equations are solved in a generalized coordinate system, employing second order accurate schemes. By implementing appropriate boundary conditions and utilizing the output of LES to formulate the source terms, the equations for the irrotational component i.e. $\psi'$ (Eqn. 5.2) and the acoustic component i.e. $\psi'_A$ (Eqn. 5.3) are first solved. Using the identity $\psi'_T = \psi' - \psi'_A$, the thermal component is computed and the solenoidal component is then calculated from Eqn. 5.1.

In the analysis presented below, the qualitative features of the constituent modes extracted using the methodology outlined above are first explored. Next, the quan-
tification of dynamics of these modes in the core of the jet is performed. Finally, analysis of the near-field, similar to the one presented in chapter 5.1, is performed to investigate the efficiency of the acoustic mode to predict the noise radiation and directivity characteristics of the twin-jet configuration.

5.2.1 Qualitative features of the FT Modes

Qualitative insight into the structural features of the decomposed modes is obtained by studying the iso-levels of each of these modes. The hydrodynamic mode is considered first, the iso-levels of which are plotted in Fig. 5.10. It is noted here that, owing to their dominance in the core, the iso-levels are plotted using the axial components of each the decomposed modes. For comparison purposes, the iso-levels for the decomposed hydrodynamic mode for the corresponding single-jet are plotted in Fig. 5.11. In each of these figures, two different iso-levels for each mode are plotted to obtain a deeper understanding. The hydrodynamic mode, which essentially represents the turbulent character of the flow, exhibits different qualitative features for the two different iso-levels. At the higher iso-level (Fig. 5.10(a)), the structure of the hydrodynamic mode exhibits axially varying features. Closer to the nozzle exit plane, the iso-levels indicate a significantly chaotic zone with very little spatial coherence. However, further downstream, relatively higher levels of coherence are noticed which is evident from the pronounced spatial features. On the other hand, the iso-levels plotted at a lower mag-
Figure 5.10: Iso levels of the axial component of the hydrodynamic mode for the twin-jet case
Figure 5.11: Iso levels of the axial component of the hydrodynamic mode for the single-jet case
nitude (Fig. 5.10(b)) display markedly different features that form coherent patterns around the jet plume. Closer to the nozzle exit plane i.e. in the converging region, the hydrodynamic mode features patterns with significantly high spatial coherence, especially in the azimuthal direction. The structure of these patterns is reminiscent of the structure displayed by the iso-levels of Q-criterion, plotted in Fig. 4.1. Looking downstream, the mode shape on the jet to the left of the symmetry plane is similar to a clock-wise helix, winding around the circumference of the jet plume. Alternatively, the pattern encompassing the jet on the right side of the symmetry plane exhibits counter clock-wise helical structure. Further downstream, the helices grow in size and eventually appear to lose their helicity around $x/D = 4$ and exhibit an axisymmetric $m = 0$ toroidal structure. As the jets merge, these axisymmetric structures interact with each other, creating a larger toroidal feature that surrounds both the jets.

The dynamic behavior of the hydrodynamic mode discussed thus far is a direct consequence of the roll-ups initiated by the K-H instability of the jet plumes. Closer to the nozzle exit, these roll-up structures are small in size and relatively less coherent resulting in the seemingly unorganized behavior observed in the converging region noticed in Fig. 5.10(a). The generation of these structures also influences their immediate neighborhood resulting in a footprint in the form of coherent helical patterns observed in Fig. 5.10(b). Being outside the bounds of the shear-layer, the footprint of the structures is devoid of any turbulent flow activity that interferes and breaks up the flow coherence. As a result, the helices exhibit high levels
of spatial coherence. The K-H structures grow in size as they convect downstream which consequently leads to the widening of helical rollers, as seen in the merging region of Fig. 5.10(b), with the maximum size observed just near the location of the collapse of the potential core ($x/D \approx 6$). Comparing the hydrodynamic mode structure of the twin-jet with that of the equivalent single-jet (Fig. 5.11), no helical pattern is observed for the latter. The single-jet is instead dominated by toroidal $m = 0$ axisymmetric structures. This difference in behavior is attributed to the influence of the adjoining jet in the twin-jet configuration, the presence of which results in the establishment of the $m = 1$ helical mode as the dominant instability.

The acoustic mode ($-\frac{\partial \psi A}{\partial x}$) for the twin-jet is plotted in Fig. 5.12. The acoustic mode can be considered as the irrotational auditory signature of turbulent structures, that were discussed in detail in the context of the hydrodynamic mode. The acoustic mode is not contaminated by the flow turbulence and so the iso-levels contain no visual traces of the underlying flow features that are responsible for its genesis, irrespective of the magnitude of the plotted iso-level. Instead, it exhibits organized features with high spatio-temporal coherence, which together form a wavepacket structure with a maximum amplification around $x/D \approx 5$.

This acoustic wavepacket is better visualized considering the the 2-D contours of $-\frac{\partial \psi A}{\partial x}$ on the $\phi = 0^\circ$ plane plotted in Fig. 5.13. The contours clearly show the acoustic signature of the incipient K-H roll-up features close to the nozzle exit. At this location the wavepacket has limited influence on the surroundings due to its smaller spatial coherence. Further downstream however, as the structures grow
Figure 5.12: Iso levels of the axial component of the acoustic mode for the twin-jet case.
Figure 5.13: Contours of the axial component of the acoustic mode on $\phi = 0^\circ$ plane
and convect, the acoustic foot-print of these structures also grows simultaneously and extends an influence on a larger region. The location of maximum amplification is in the range of \(4 < x/D < 6\), which agrees with the inference obtained from Fig. 5.12. This location is noticed to be slightly upstream relative to the peak amplification location of the hydrodynamic mode. Much further downstream, the break-up of the turbulent structures results in the disintegration of the wavepacket feature.

The radiation of the noise to the near and far-field is a manifestation of the dynamics of the wavepacket. Unlike the hydrodynamic mode, the acoustic mode wavepackets appears to signify the dominance of an \(m = 0\) toroidal instability. In other words, the turbulent flow structures which exhibit \(m = 1\) helical instability engender a \(m = 0\) acoustic response. Though interesting at first, this behavior can be explained considering the propagation speed of the acoustic mode. The acoustic field generated around a turbulent structure convects in all spatial directions at the local speed of sound. The convection of turbulent structures, however, is influenced by the jet flow and is principally in one direction. As a result, at the moment of initiation, the acoustic field associated with the K-H roll-ups instantaneously spreads in all the directions at the local sound speed, thereby exhibiting an axisymmetric structure. Consistent with the features of the acoustic mode, the thermal mode \((-\frac{\partial \psi'}{\partial x})\) for the twin-jet, 2D contours of which are plotted in Fig. 5.14, also exhibits a wavepacket structure. However, it is restricted to the jet core and since the present simulations do not employ heated jets, the thermal mode appears
Figure 5.14: Contours of the axial component of the thermal mode on $\phi = 0^\circ$ plane relatively dormant and may not play a significant role in defining the overall characteristics of the jet flow. For that reason, the dynamics of the thermal mode are not presented in detail and emphasis is placed on understanding the solenoidal and acoustic components of the twin-jet.

As discussed above, the hydrodynamic and the acoustic modes represent the turbulence and sound features of the flow respectively. Consequently, this dictates that the acoustic mode should be the sole contributor to the radiative features of the flow. To establish this aspect, 2-D contours of $||B'||$ (in color) overlaid with $||\nabla\psi'||$ (in grayscale), on the $\phi = 0^\circ$ plane are plotted in Fig. 5.15. The unsteady turbulence of the flow contributes significantly to the strength of the solenoidal fluctuations whereas only a part of the turbulent kinetic energy is manifested in
Figure 5.15: Comparison of 2-D contours of $||B'||$ (in color) overlaid with $||\nabla \psi'||$ (in grayscale), on the $\phi = 0^o$ for the twin-jet configuration.
the radiating acoustic component [60]. For this reason the acoustic fluctuations are rather small and hence the contours for the two modes are plotted using different scales. From Fig. 5.15, it is evident that the hydrodynamic fluctuations faithfully adhere to boundaries of the shear-layer. Comparing to the 2-D contours of vorticity plotted in Fig. 5.16, the location of peak solenoidal fluctuations is observed to be consistent with the locations of high vorticity further consolidating the characteristic feature of hydrodynamic mode as the representation of the turbulent flow features. The acoustic mode on the other hand is not representative of the turbulent features of the flow but instead is associated with the radiative acoustic component. The patterns of the acoustic mode are reminiscent of those observed in the
velocity dilatation contours that were discussed in the context of Fig. 5.1. Similar to the dilatation contours, the acoustic fluctuations exhibit a marked downstream directivity evidenced by the wave fronts propagating in this direction. These observations establish the efficacy of the acoustic mode in reproducing the qualitative features of the radiative sound field of the twin-jet flow-field.

5.2.2 Quantitative features of the FT modes

The qualitative features of the decomposed modes identified in the previous section are now augmented with quantitative features of the flow modes.

First, characteristic features of the modes in the core of the jet are explored in detail. For this purpose, chronograms which are defined as the contours of spatio-temporal variation of variable are considered. The chronograms of $B'_x$ along the inner and outer lip-lines (Fig. 4.6) are plotted in Fig. 5.17 and the same for $-\frac{\partial \psi'}{\partial x}$ are plotted in Fig. 5.18. A common feature of all the contours plotted in both the figures, is the presence of axial and temporal modulation of coherent structures. At locations close to the nozzle exit plane and tracing a vertical path parallel to the y-axis, all the contours indicate the presence of high frequency recurring events. This is a direct signature of the formation of K-H roll-up structures in the shear-layer near this axial location. At further downstream locations, as a result of growth, interaction and merging of the K-H structures, the frequency is expected to drop. Substantiating this claim, at downstream axial locations, all the contours exhibit
Figure 5.17: Chronograms of the hydrodynamic mode along the inner and outer lip-lines of one of the nozzles of the twin-jet.
Figure 5.18: Chronograms of the acoustic mode along the inner and outer lip-lines of one of the nozzles of the twin-jet
lower frequency features which is evident from the reduction in the number of temporal events. However, qualitatively comparing the chronograms of the individual modes along their respective outer and the inner lip-lines, at a downstream location for example at $x/D = 6$, the contours along the inner lip-line appear to indicate a higher frequency of temporal events relative to the outer lip-line. This behavior is a direct consequence of the geometrical construct of the twin-jet configuration. Due to the close proximity of the jets, the shear-layers from the jets directly interact with each other, the dynamics of which have been discussed in detail in chapter 4. Beyond the merging point of the jets ($x/D \approx 3.3$), the turbulent features of the individual shear-layers mix with each other, which results in the break-up of the large structures into smaller structures. As a result of this behavior, the chronograms of two modes along the inner lip-line (Figs. 5.17(a) and 5.18(a)), which is directly affected by the merging of jet plumes, exhibits larger number of temporal events.

In addition to temporal variation, the contours for the acoustic mode exhibit significant axial modulation or “jittering”. This behavior is a direct consequence of unsteady core collapse, which modulates the acoustic wavepacket. This phenomenon has been found to be a crucial component in predicting the noise levels using a wavepacket modeling approach, especially in subsonic jets. In order to obtain a match with experimental/simulated data, most works [20, 73] artificially enforce jittering when modeling the wavepacket dynamics. The observation of jittering in the decomposed acoustic mode analyzed in this work, lends further
credit to such procedures. Furthermore, as will be shown later, the natural occurrence of jittering in the acoustic mode, directly contributes towards acquiring an excellent match of the acoustic characteristics predicted by the acoustic mode, to those extracted from the overall pressure fluctuations obtained directly from the LES.

As discussed above, different dominant frequencies are observed along the axial extent of both modes. To quantify these frequencies, PSD of the time signals at multiple axial points, on both the inner and outer lip-lines, are performed and gathered together in form of space-Strouhal number contours. These contours for the hydrodynamic mode are shown in Fig. 5.19 and in Fig. 5.20 for the acoustic mode. The phenomena of cascading due to structure interaction, as discussed above, is vividly observed in all the contour plots. Closer to the nozzle exit, the K-H instabilities roll-up at a very high frequency which is approximately $St = 1$. With increasing axial distance, the dominant frequency gradually reduces, which is attributed to the coalescence of the turbulent structures. Comparing the contours at different lip-lines for the individual modes, the features along the inner lip-line exhibit lower amplitude relative to their counterparts along the outer lip-line. This behavior is consistent with the observations of Fig. 4.12, where the reduction in shear and consequently the strength of the roll-up structures is observed due to formation of secondary flow along the inter-nozzle region. Analyzing the contours of the hydrodynamic mode along the outer lip-line, in the range of $3.5 < x/D < 6$, significant features centered around $St \approx 0.2$ are observed. The hydrodynamic
Figure 5.19: PSD of the hydrodynamic mode at various locations along the inner and outer lip-lines of one of the nozzles of the twin-jet
Figure 5.20: PSD of the acoustic mode at various locations along the inner and outer lip-lines of one of the nozzles of the twin-jet
mode attains a peak magnitude in the region $3.5 < x/D < 6$, which contributes the most to this frequency. This is consistent with the qualitative observations inferred from Fig. 5.10. The jet-column mode instability frequency, usually representative of the dynamic core collapse frequency, has been historically established to be in the range of $St = 0.2 - 0.5$, depending on the flow conditions. This instability frequency is essentially determined by the dynamics of the unsteady turbulent features which intrude into the core of the jet thereby causing a core collapse. For the current case, the identification of $St \approx 0.2$ as the most amplified frequency in the proximity of core collapse location establishes jet column mode instability frequency of the twin-jet plumes. This frequency is also visible along the inner lip-line of the configuration, but however, is relatively weak and is restricted to a very small axial region ($3 < x/D < 4$). Beyond this region, the contours do not indicate significant contribution from lower frequencies along the inner lip-line. This is consistent with the process of break-up of the large-scale structures (lower frequency events) due to the chaotic merging of the twin-jet shear-layers, as discussed above. Conversely, the contours along the outer lip-line indicate that large-scale features persist for a longer spatial extent, uninfluenced by the merging process, which is evident from the very low frequencies observed beyond $x/D = 6$. However, the natural collapse of the potential core results in a disintegration of these features.

The contours of the acoustic mode (Fig. 5.20) exhibit two significant features with frequencies centered around $St \approx 0.6$ and $St \approx 0.35$. The higher frequency
represents the fundamental or the “internal” frequency of the wavepacket [20, 92].
The acoustic wavepacket attains its peak magnitude in the region $3 < x/D < 4$, which contributes the most to this frequency. This observation agrees with the findings discussed with respect to Figs. 5.12 and 5.13, where it was noted that the peak amplification of the acoustic mode occurs relatively upstream, compared to the hydrodynamic mode (Fig. 5.10). The lower peak frequency observed in Fig. 5.20, in the range of $St = 0.3 – 0.4$, about the axial region of $4 < x/D < 5$ corresponds to range of acoustic radiation frequencies in the near and far-fields. The modulation of the acoustic wavepacket due to the unsteady core collapse phenomena in the axial region of $4 < x/D < 6$ results in the radiation of the dominant frequency of this region i.e. $0.3 < St < 0.4$. The observation of the average wavepacket frequency being higher than the frequency of peak noise radiation is consistent with the findings of Refs. [20, 92].

The above discussion presents in detail the dynamic features of the hydrodynamic and the acoustic mode, in the core of jet. A major observation reported in chapter 5.2.1 is regarding the radiative characteristics of the modes. To summarize, the hydrodynamic mode is non radiative and is contained within the shear-layer, whereas the acoustic mode is radiative and is the sole contributor to the noise levels in the near and far-field. The analysis of the acoustic mode in the core of the jet provided insight into its radiation characteristics, specifically the frequencies of radiation. To corroborate these observations, two probes located at a distance of $D = 5$ and 12 are considered along a $30^\circ$ ray from the nozzle exit plane, on the
Figure 5.21: Representative locations of probes at \( D = 5 \) and \( D = 12 \), located on a 30° ray centered at the nozzle exit along the symmetry plane of the twin-jet configuration plane containing the two jets i.e. \( \phi = 0^\circ \) plane. An illustration of these probes is shown in Fig. 5.21. In addition to reconciling the observed radiative frequencies of the acoustic mode in the jet core with those actually measured in the near-field, the two selected locations are also used to demonstrate the diminishing influence of the solenoidal fluctuations with increasing distance. The frequency content of each of \( p', B'_x \) and \( -\frac{\partial \psi'}{\partial x} \) is analyzed by calculating the PSD. The spectral profiles obtained for each of these modes is plotted in Fig. 5.22 for probe at \( D = 5 \) and in Fig. 5.23 for the probe at \( D = 12 \). From both the figures, the similarities between the profiles of pressure fluctuations and the acoustic fluctuations is striking. Both exhibit a narrow banded behavior with a spectral peak around \( St \approx 0.35 \). Furthermore, the two profiles exhibit a significant overlap over a major portion of
Figure 5.22: Comparison of PSD of the hydrodynamic and acoustic modes with that of the overall pressure fluctuations at the $D = 5$ probe shown in Fig. 5.21

Figure 5.23: Comparison of PSD of the hydrodynamic and acoustic modes with that of the overall pressure fluctuations at the $D = 12$ probe shown in Fig. 5.21
the frequency spectrum, which increases with increasing distance. This behavior essentially indicates that farther away from the jet, pressure fluctuations are primarily acoustic in nature. On the other hand, observing the magnitudes of the hydrodynamic mode, it is easily inferred that the effect of solenoidal fluctuations abates rapidly with increasing distance from the jet. At the closer probe location i.e. \( D = 5 \) (Fig. 5.22), it is observed that \( B'_x \) extends a significant influence on the pressure fluctuations up to a Strouhal number of \( \approx 0.7 \). For Strouhal numbers beyond this value, the contribution from the solenoidal fluctuations is negligible and the acoustic fluctuations make up the pressure field. However, as the distance increases to \( D = 12 \) (Fig. 5.23), the influence of the solenoidal fluctuations is considerably reduced and is restricted to low Strouhal number range i.e., below \( St = 0.08 \). This low frequency contribution from non-acoustic components has been previously observed by Arndt et al. [3] and more recently by Bogey et al. [11], who classify this component as aerodynamic in nature. The present analysis confirms their observations by associating the low frequency contribution to the non-radiating hydrodynamic component of the flow-field.

In addition to characterizing the rapidly decaying behavior of the solenoidal fluctuations, the above analysis confirms the increasing role of the acoustic fluctuations in influencing the overall pressure fluctuations, with increasing distance from the jet. Previous studies [41, 92] have described in detail the role of intermittency in the jet noise. In order to identify this behavior for the present twin-jet case, the overall pressure fluctuations recorded at \( D = 8 \) probe along the \( \theta = 30^o \)
Figure 5.24: Contours of wavelet coefficient of the pressure fluctuations at $D = 8$ probe along the $\theta = 30^\circ$ ray on the $\phi = 0^\circ$ plane

ray on the $\phi = 0^\circ$ plane (see Fig. 5.21 for the polar angle representation) are processed using wavelet analysis and the corresponding time-frequency characteristics are extracted. The contours of the wavelet coefficients (scalogram) plotted against non-dimensional time on $x$-axis and Strouhal number on $y$-axis, for each of the modes, along with the pressure fluctuations, are plotted in Figs. 5.24, 5.25 and 5.26. Consistent with the observations of Kearney-Fischer et al. [41] and Unnikrishnan and Gaitonde [92] among others, the scalogram of the pressure fluctuations exhibits significant intermittency (marked in circles) in the peak radiative band of sound frequencies i.e., around $St \approx 0.35$ as also observed from Fig. 5.24. The scalogram of the acoustic mode, plotted in Fig. 5.25, also exhibits intermittent
Figure 5.25: Contours of wavelet coefficient of the acoustic mode at $D = 8$ probe along the $\theta = 30^\circ$ ray on the $\phi = 0^\circ$ plane

Figure 5.26: Contours of wavelet coefficient of the hydrodynamic mode at $D = 8$ probe along the $\theta = 30^\circ$ ray on the $\phi = 0^\circ$ plane
activity in the Strouhal number range between $0.3 < St < 0.4$. This observation, in conjunction with that of Fig. 5.23, show that the acoustic mode captures not only the radiation frequencies of the flow-field but also temporally intermittent events, which have been shown to play a key role in defining the far-field jet noise characteristics. In addition to the intermittent events, the scalogram of pressure fluctuations (Fig. 5.24) indicate intermittent features with high temporal coherence for very low Strouhal number ($St < 0.1$). These features are not readily detectable in the scalogram of the acoustic mode. However, the low frequency features are distinctly visible when in the scalogram of the hydrodynamic mode, plotted in Fig. 5.26. Furthermore, no contribution is apparent over the remaining spectral range, in agreement with the observations of Fig. 5.23.

The above discussion, thus identifies the pure acoustic mode as the sole contributor to radiating noise levels, though it is influenced by the hydrodynamic mode which is the source of its energy. A final check on the efficacy of the acoustic mode in fully describing even complex sound fields is investigated. As shown earlier (see chapter 5.1), the twin-jet configuration exhibits azimuthally varying noise fields and phenomena such as shielding and magnification. To determine the capability of the decomposed acoustic mode to even predict these phenomena, the values of $-\frac{\partial \psi' \Delta}{\partial x}$ are extracted at locations shown in Fig. 5.6 and Fig. 5.8, that lie along the $\phi = 0^\circ$ and $\phi = 90^\circ$ plane, respectively. The values of this parameter, at equivalent locations, are also extracted for the corresponding single-jet case. Since MPT is based on decomposition of momentum-density and not pressure,
the fluctuations recorded for the acoustic mode in the near-field cannot be directly compared to the pressure fluctuations. Following the proposal of Unnikrishnan and Gaitonde. [92], the conversion of acoustic momentum-density fluctuations to equivalent pressure fluctuations is performed by scaling the former with the ambient speed of sound. From these scaled values, ΔOASPL is calculated and its the variation with polar angle on both the aforementioned azimuthal planes is shown in Figs. 5.27. Comparing with the trends of ΔOASPL obtained from pressure fluctuations along both the probed planes (Fig. 5.7 and Fig. 5.9), the profiles of the acoustic mode exhibit almost identical behavior, both qualitatively and quantitatively. Along the plane containing the jet centers (ϕ = 0°), the ΔOASPL profiles of the acoustic mode (Fig. 5.27) exhibit high levels of shielding in the lower polar
angle region ($\theta < 45$), which gradually diminishes with increasing $\theta$. For sideline polar angles ($\theta > 60$), the trends of Fig. 5.27 even indicate small amounts of magnification i.e. higher noise levels for the twin-jet greater than the combined noise levels of two incoherent jets, agreeing with the observations of Fig. 5.7. For the probe locations along the perpendicular plane ($\phi = 90^\circ$), the profiles plotted in Fig. 5.28 exhibit negligible amounts of shielding, which is evident only at the low polar angle region. With increase in polar angle, the profiles indicate significant levels of magnification which, as discussed in reference to Fig. 5.9, is attributed to the unrestricted radiation of the sound waves in the perpendicular direction. Thus, the acoustic mode obtained from decomposition using MPT completely characterizes the acoustic field of complex configurations such as a twin-jet. The insight
gained from all the analysis and observations discussed in this section, especially regarding the acoustic mode, can be very useful in extending the state of the art wavepacket modeling based noise predictive technique to complex flow configurations.

5.3 Summary

In this chapter, the near-field acoustic characteristics of the twin-jet configuration are explored in detail. This work serves as a natural extension to the work presented in chapter 4. First, a qualitative investigation of the sound field is conducted by considered the contours of velocity dilatation on the plane containing the jet centers ($\phi = 0^\circ$). These contours indicate similar sound directivity patterns, with maximum directivity in the downstream polar angles, when compared to the dilatation contours of the equivalent single-jet. A quantification of the influence of a twin-jet configuration the near-field noise levels is performed by considering the overall sound pressure level amplitudes. On the plane containing the jet centers, the values of $\Delta$OASPL between the twin-jet and single-jet+3dB, the latter representing the contribution of noise levels from two incoherently interacting jets, indicate presence of jet noise shielding. Shielding is defined as noise level reduction of for the twin-jet configuration relative to the single-jet+3dB levels, due to the restriction of propagation of pressure waves issued by one jet due to the presence of the other jet. On the $\phi = 0^\circ$ plane, results show high levels of shielding with
noise reduction up to 3dB at low polar angles (downstream direction). With increasing polar angle however, the intensity of shielding is observed to be reduced and for highest polar angles considered here, a noise level magnification, where the twin-jet exhibits higher noise levels than the the sum of two incoherently interaction jets. This increase in noise levels is attributed to the interaction between the jet plumes. Similar analysis on the perpendicular plane indicates significant levels of noise amplification of up to 2.5dB, especially in the upstream direction. This behavior on the perpendicular plane is attributed to the non-linear interaction of sound waves that radiate in this direction unabated. These observations concerning shielding and magnification are in agreement with previous and concurrent experimental efforts.

Results are also reported from the analysis of the fluid-thermodynamic modes extracted using the Momentum Potential Theory (MPT). The decomposed hydrodynamic mode is observed to represent the unsteady turbulent features of the flow. It is non-radiative in nature and decays rapidly outside the shear-layer. In addition to depicting the underlying incoherent small scale flow turbulence, analysis of the iso-levels of this mode indicates coherent helical $m = 1$ mode dominance on both the twin-jet plumes. This result is in agreement with those reported in chapter 4, where the Q-criterion iso-levels exhibit similar behavior. The acoustic mode is a pure representation of the acoustic signature of the turbulent features of the flow. This mode is responsible for radiative acoustic characteristics of the flow-field and analysis indicates a wavepacket nature for this mode in the jet core. As previously
established, the dynamics of these wavepackets, specifically the temporal and axial modulation of its envelope, results in the intermittent bursts of acoustic energy leading to acoustic radiation. A near identical match is observed when comparing the near-field noise levels and trends predicted by the acoustic mode with those calculated using the overall pressure fluctuations. The acoustic mode efficiently predicts the phenomena of jet noise shielding, noise amplification, and azimuthal asymmetry, including quantitative trends. The insight gained from all the analysis and observations discussed in this section, especially regarding the acoustic mode, can be very useful in extending the state-of-the-art wavepacket modeling based noise predictive technique to complex flow configurations.
Conclusions and Future Directions

To address the concerns associated with high jet noise levels, a significant amount of research has been performed towards identifying the noise sources and implementing flow control techniques to reduce the jets’ acoustic signature. However, most of these efforts employ simplifying assumptions such as simple nozzle configurations and ideally expanded flow conditions. Real world situations are often more complex and do not admit such simplifications. The primary objective of this work was to explore the characteristics of two such real world situations i.e. a simple supersonic nozzle operating at imperfectly expanded conditions and a complex twin-jet configuration operating at perfectly expanded conditions. For this purpose, thoroughly validated high-fidelity large eddy simulations were performed. The insight obtained from these simulations on both of the investigated problems is presented below.

1. Underexpanded Circular Supersonic Jet
The primary goal of this part of the work was to explore the impact of flow control on imperfectly expanded round jets and identify the subsequent modifications to the radiated acoustic field. Uncontrolled simulations first conducted to explore the nature of the underexpanded jet clearly identify the shock-train structures contained within the jet plume, which are formed due to the off-design operating conditions. The shock-cells extend a significant dominance towards characterizing the jet plume dynamics, which was evident from the undulations observed in the jet centerline and jet spreading rate profiles. After analysis the jet plume features, the study of noise characteristics of the underexpanded was performed. The analysis of the near-field pressure fluctuations recorded at different polar angles exhibit the presence of three distinct noise sources. The spectrum at the downstream probe indicates a peak around frequencies of $St = 0.3 - 0.4$, which is in agreement with the peak radiative frequencies in this direction established by previous works. The dominant source mechanism for radiation in the downstream direction is associated with the large-scale turbulent structures and their convective behavior. The spectrum in the side-line direction exhibits a relatively more broadband nature, with a characteristic “hump” centered around $St \approx 1$, which is associated with the BBSAN, generated due to shock-structure interaction. The upstream probe records a screech tone at $St \approx 0.45$, which is consistent with the findings of previous theoretical and experimental works.
After identifying the flow and acoustic characteristics of underexpanded jet considered in this work, simulations were conducted to identify the effect of active control technique on such jets. For this purpose, actuation through the use of LAFPAs was considered, which were modeled using a semi-empirical surface heating technique. Axisymmetric mode pulsing ($m = 0$) was considered at two different Strouhal numbers of 0.3 (corresponding to preferred column mode frequency of the jet) and 0.9. Results from these simulations reveal a strong dependence of the flow response to the actuation frequency. Actuation at $St = 0.3$ targets the column mode instability of the jet and as a result, the flow responds by generating large-scale toroidal structures with high spatio-temporal coherence. These structures significantly enhance the mixing characteristics compared to uncontrolled jet. In agreement with results from previous works on flow control of perfectly expanded jets, the generation of large-scale structures was observed to increase the mixing noise levels in the lower polar angle region, compared to the uncontrolled jet. Furthermore, enhancement of noise levels in the side-line and upstream directions was observed which was attributed to the strengthening of interaction between the generated toroidal structures and the shock-cells.

On the other hand, actuation at $St = 0.9$ results in a relatively muted response of the jet, where the generated structures do not exhibit any significant levels of coherence. Furthermore, in comparison to the uncontrolled
case, analysis indicates a suppression of naturally occurring structures. As a result, actuation at this frequency results in an overall decrease in the noise levels along all the aft, side-line and upstream directions. Based on the results and observations of from all the investigated cases, the main contribution of this work is the establishment of control authority on shock-dominated flows. The work identifies different ranges of excitation frequencies based on the purpose of excitation: 1) Frequencies near the jet column instability frequency \( (St = 0.3) \) are recommended for mixing enhancement and 2) Frequencies around \( St = 0.9 \) are recommended for noise reduction.

2. Closely spaced perfectly expanded twin supersonic jets

The aim of this part of the work was to investigate the complex flow and acoustic fields of a twin-jet configuration. For this purpose, a Mach 1.23 circular twin-jet configuration with a center-to-center spacing of two nozzle diameters was considered. An overset meshing technique was employed to discretize the complex geometrical construct of the twin-jet geometry.

Analysis performed to identify the flow structure revealed the dominance of a helical \( m = 1 \) mode, which visually manifests in the form of cork-screw type roll-up structures surrounding the plume of each of the jets, with opposite orientation. This observation is consistent with concurrent and previous experimental works at similar twin-jet operating conditions. The interaction of the jet plumes owing to their close proximity, results in the a high magni-
tude secondary flow along the inter-nozzle region, in the converging section. Analysis of shear-layers at different azimuthal angles indicated that the impact of this secondary flow on the shear-layer properties is confined to very small azimuthal angles, with respect to the centerline of the jet. As a result, the characteristics of the potential core of the twin-jet system is generally similar to that of a single-jet configuration, at identical flow conditions.

One of the major aims of this work was to computationally study the acoustic characteristics of the twin-jet configuration. For this purpose, an indepth analysis was performed, using near-field pressure fluctuations, to extract the trends of the noise levels. The study of the pressure fluctuations at various polar angles, on the plane containing the jet centers ($\phi = 0^\circ$), revealed the existence of jet noise shielding, which was attributed to the action of one jet interfering in the propagation of the sound waves issued from the other jet. For downstream polar angles, a peak shielding intensity of up to $3dB$ was observed, which was observed to gradually diminish with increasing polar angle. For higher polar angles along the plane containing the jets, a noise amplification of up to $1dB$ was noticed, due to the interaction of the closely spaced jets. Conversely, noise levels along the $\phi = 90^\circ$ plane, i.e. the perpendicular to the plane containing the jet centers, indicated negligible levels of shielding. At most of the measured angles on this plane, noise magnification was observed with a peak increase in noise levels of up to $2.5dB$ at the largest
polar angle probed. This amplification is a direct result of the non-linear interaction of the unhindered sound waves in this direction. In agreement with previous experimental observations, the simulations successfully establish the unique noise characteristics of the twin-jet configuration i.e. shielding, magnification, and azimuthal asymmetry.

In order to gain a deeper insight into the fundamental noise generation and radiation mechanisms, Momentum Potential Theory (MPT) was employed to extract the pure acoustic component (representing the noise field), the pure hydrodynamic mode (representing the unsteady turbulence) and the pure thermal mode (representing the entropic fluctuations due to thermal gradients). In addition to defining the incoherent, smaller turbulent scales, the hydrodynamic mode was observed to successfully reproduce the coherent $m = 1$ helical mode of each of the jet plumes. Analysis of the features of the acoustic mode, in the core of the jet, indicated a wavepacket structure with a clear $m = 0$ dominance. This wavepacket structure exhibit significant spatio-temporal modulations and directly impact the radiative acoustic characteristics. The trends of the noise levels extracted from the imposed fluctuations of the decomposed acoustic mode in the near-field indicate excellent agreement with those of the overall pressure fluctuations. The pure acoustic mode, both qualitatively and quantitatively, successfully identified the intensity of jet shielding, noise amplification, and azimuthal asymmetry.
of the twin-jet configuration.

Through this work, significant insight was obtained into the dynamics of closely spaced twin-jet plumes and their impact of near-field noise levels. Furthermore, a significant contribution of this work was to show that the pure acoustic mode obtained from decomposition using MPT completely characterizes the acoustic field of configurations such as a twin-jet, where complex dynamics like wavepacket merging occur. The insight gained from all the analysis and observations, especially regarding the acoustic mode, can be very useful in extending the state of the art wavepacket modeling based noise prediction techniques for identification of noise characteristics of complex flow configurations.

### 6.1 Future Directions

1. Underexpanded circular supersonic jet

- The present analysis is restricted to only one mode of actuation and two forcing frequencies. The effect of employing higher modes i.e. \( m = 1, -1, \pm 1, 3 \) forced at higher Strouhal numbers (\( St > 1 \)) can be explored.

- The FT mode decomposition based on MPT can be applied to the uncontrolled and controlled jets: 1) To identify the fidelity of the decomposed acoustic mode in capturing multiple noise source mechanisms i.e. BB-
SAN and screech noise, in addition to mixing noise, 2) To examine the
dynamic characteristics of the acoustic wavepacket structure and devise
efficient control strategies for noise reduction and 3) To eventually for-
mulate noise prediction tools based on the fundamental features of the
decomposed acoustic mode.

2. Closely spaced perfectly expanded twin supersonic jets

- With the present perfectly expanded twin-jet simulation as a reference
  scale, simulations can be performed towards incorporating the effects of
  imperfect expansion. These simulations can reveal a great insight into
  phenomena such as coupling of jet plumes and screech tone formation
  and noise level amplification.

- The fundamental noise generation mechanisms in complex configura-
tions can be fully investigated by examining the energy sources and
sinks of each of the FT modes by solving the Total Fluctuating Enthalpy
(TFE) equation.

- Control techniques using LAFPAs can be implemented to study the re-
sponse of the jet plumes to external forcing and the subsequent modifi-
cations to the sound field.
Bibliography


