Model Building in the LHC Era:
Vector-like Leptons and SUSY GUTs

Dissertation

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By
Zijie Poh, M.S.
Department of Physics

The Ohio State University
2017

Dissertation Committee:
Stuart Raby, Advisor
Eric Braaten
Christopher Hill
John Beacom
ABSTRACT

In this thesis, we study the phenomenology of some models beyond the Standard Model (SM). First, we analyze a model in which the SM is extended by a family of vector-like (VL) leptons. We find that the strongest constraints are coming from the muon $g - 2$, $R_{\mu\mu} = \Gamma(h \rightarrow \mu\mu)/\Gamma(h \rightarrow \mu\mu)_{\text{SM}}$, $R_{\gamma\gamma}$, and $\text{BR}(\mu \rightarrow e\gamma)$. Although VL leptons couple to all three families of the SM leptons, electron-VL couplings are highly constrained. The second model that we consider is a three family SO(10) SUSY GUT with Yukawa unification. We perform a global $\chi^2$ analysis and show that SUSY effects do not decouple even though the best fit universal scalar mass parameter at the GUT scale is $m_{16} \approx 25 \text{ TeV}$. The fit to $\sin(2\beta)$, and up and down quark masses of this model, however, are not great. We show that the fits can be improved by choosing a Pati-Salam (PS) group as the gauge group and modifying the Yukawa sector. The best-fit point, consistent with the bound obtain from reinterpreting gluino simplified model analyses by the ATLAS and CMS collaborations, has gluino mass $M_\tilde{g} = 1.9 \text{ TeV}$ and $\chi^2/\text{dof} \approx 1.12$. Finally, we study the reheating and the baryogenesis (via leptogenesis) of this PS model extended by an inflation sector. This model reheats via instant preheating due to bosonic and fermionic broad parametric resonances. The lepton asymmetry is obtained from the CP asymmetric right-handed neutrino (RHN) decay. The leptogenesis analysis is performed by including all three families of the RHNs because, by fitting to the low-energy observables, the heaviest RHN decays to produce a lepton asymmetry with the correct sign, while the two lighter RHNs decay to produce the wrong sign.
This thesis is dedicated to my wife Luka Liu.
ACKNOWLEDGMENTS

I am profoundly grateful to my advisor, Stuart Raby, for his patience in teaching me particle physics and his guidance on the research. He made me understand the importance of a strong and solid foundation. A special thank to my committee members, Eric Braaten, Chris Hill, and John Beacom for their time, support, and guidance. I would also like to thank Brad Trees, my undergraduate research advisor, for introducing me to research in theoretical physics; and Barbara Andereck, my undergraduate academic advisor, for being a mentor in my academic path.

I would like to thank my fellow collaborators. In particular, sincere thanks to Chuck Bryant for his incessant guidance. My endeavor as a graduate student would have been much harder if it wasn’t for Chuck’s help. Although I never had the opportunity to work with Archana Anandakrishnan, I would like to thank her for introducing me to particle physics and assisting me in progressing during my first research project. Nevertheless, I would also like to thank Hong Zhang, Kuan-Hao Chen, Russell Colburn, Bowen Shi, Shaun Hampton, and Weifeng Ji for all the intellectually stimulating and interesting conversations.

I would not have made it without the love and support from my wife, Luka Liu. I am immensely grateful for my parents, Poh Swee Hiang and Wong Siew Lee for instilling critical and scientific thinking in me from a young age and giving me the opportunity to advance my studies in the United States. I am also thankful to my sisters, Poh Zhiling and Poh Zhini, and my brother, Poh Ziyang for all the encouragements. I am also very grateful to Luka’s parents, Liu Zhicheng and Leng Qi, for their unconditional support. I would also like to thank Poka and Ponyo, my two lovely four-legged furry family members, for cheering me up after a hard day at work.
VITA

May, 2013 .......................... B.A. Physics and Mathematics, Ohio Wesleyan University, Delaware, Ohio

May, 2016 .......................... M.S. Physics, The Ohio State University, Columbus, Ohio

Publications


Fields of Study

Major Field: Physics
# Table of Contents

<table>
<thead>
<tr>
<th>Chapters</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Abstract</strong></td>
<td>ii</td>
</tr>
<tr>
<td><strong>Dedication</strong></td>
<td>iii</td>
</tr>
<tr>
<td><strong>Acknowledgments</strong></td>
<td>iv</td>
</tr>
<tr>
<td><strong>Vita</strong></td>
<td>v</td>
</tr>
<tr>
<td><strong>List of Figures</strong></td>
<td>ix</td>
</tr>
<tr>
<td><strong>List of Tables</strong></td>
<td>xii</td>
</tr>
</tbody>
</table>

## Chapters

1 **Introduction** ................................................. 1  
   1.1 Standard Model of Particle Physics ................................. 2  
      1.1.1 Why Beyond the SM? ......................................... 8  
      1.1.2 SM augmented with Right-handed Neutrinos ....................... 12  
   1.2 Supersymmetry .................................................. 14  
      1.2.1 Minimal Supersymmetric Standard Model ....................... 18  
   1.3 Supersymmetric Grand Unified Theory ............................. 24  
      1.3.1 SU(5) .................................................... 26  
      1.3.2 SO(10) .................................................. 29  
      1.3.3 Pati-Salam Group .......................................... 31  
   1.4 Baryogenesis via Leptogenesis ................................... 32  
   1.5 Conclusion ..................................................... 40  

2 **Vector-like Leptons** ........................................... 42  
   2.1 Model ......................................................... 43  
   2.2 Analysis Procedure ............................................ 47  
   2.3 Results ....................................................... 50  
   2.4 Conclusions .................................................... 56  

3 **SO(10) SUSY GUT with Yukawa Unification** ....................... 58  
   3.1 Model ......................................................... 59  
   3.2 Analysis Procedure ............................................ 63  
      3.2.1 Renormalization Procedure ................................ 63  
      3.2.2 Global $\chi^2$ Analysis .................................. 66  
   3.3 Results ....................................................... 68  
      3.3.1 Inclusive vs. Exclusive $|V_{ub}|$ and $|V_{cb}|$ ............... 68
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 One-loop correction to the Higgs mass.</td>
<td>12</td>
</tr>
<tr>
<td>1.2 Plot of inverse gauge couplings, $\alpha_j^{-1}$, versus log$_{10}(Q/\text{GeV})$. The dashed lines are the inverse couplings for the SM while the solid lines are those for the MSSM with the superpartner mass taken to be 750 GeV (blue lines) or 2.5 TeV (red lines). This plot is obtained from Ref. [1].</td>
<td>25</td>
</tr>
<tr>
<td>1.3 Diagrams contributing to the CP asymmetry in the decay of right-handed neutrinos to lepton and Higgs boson. There are also diagrams where right-handed neutrinos decay to slepton and Higgsino. In addition, the decay of right-handed sneutrinos also contribute to the CP asymmetry.</td>
<td>35</td>
</tr>
<tr>
<td>2.1 Box diagrams contributing to $b \to s \tilde{e}_a \tilde{e}_a$.</td>
<td>47</td>
</tr>
<tr>
<td>2.2 Plots of the muon $g - 2$ discrepancy, $\Delta a_\mu$, versus $R_{\mu\mu} = \Gamma(h \to \mu\mu)/\Gamma(h \to \mu\mu)<em>{\text{SM}}$. The four plots have different ranges of $M_L$. The gray points are ruled out. The dashed lines show the 1σ and 2σ bounds of $\Delta a</em>\mu$ and the upper bound of $R_{\mu\mu}$. The solid lines show the central value of $\Delta a_\mu$ and $R_{\mu\mu} = 1$.</td>
<td>51</td>
</tr>
<tr>
<td>2.3 Plots of $\Delta a_\mu$ versus $M_L$. The two plots have different ranges of $\lambda$. The gray points are ruled out. The dashed lines are the 1σ and 2σ bounds of $\Delta a_\mu$ while the solid line is the central value of $\Delta a_\mu$.</td>
<td>52</td>
</tr>
<tr>
<td>2.4 Plot of $\Delta a_\mu$ versus $R_{\gamma\gamma}$. The lightly shaded points are ruled out. The dashed lines show the 1σ and 2σ bounds of $\Delta a_\mu$, and the 1σ bound of $R_{\gamma\gamma}$. The solid lines show the central value of $\Delta a_\mu$ and that of $R_{\gamma\gamma}$.</td>
<td>52</td>
</tr>
<tr>
<td>2.5 Plots of $\Delta a_\mu$ versus $R_{\gamma\gamma}$. The gray points are ruled out. The dashed lines show the 1σ and 2σ bounds of $\Delta a_\mu$, and the 1σ bound of $R_{\gamma\gamma}$. The solid lines show the central value of $\Delta a_\mu$ and that of $R_{\gamma\gamma}$. This model requires $</td>
<td></td>
</tr>
<tr>
<td>2.6 Plot of the muon-VL couplings, $\lambda_{\mu}^E v/M_E$ versus $\lambda_{\mu}^L v/M_L$. The solid and dashed black lines are the approximate empirical bounds on the muon-VL couplings. These bounds are not exact, but are obtained empirically (see text for more discussions).</td>
<td>54</td>
</tr>
</tbody>
</table>
2.7 Plot of $\Delta a_\mu$ versus $\text{BR}(\mu \to e\gamma)$, which gives the strongest LFV constraint. The lightly shaded points are ruled out. The dashed lines show the 1$\sigma$ and 2$\sigma$ bounds of $\Delta a_\mu$, and the upper bound of $\text{BR}(\mu \to e\gamma)$. The solid line shows the central value of $\Delta a_\mu$. Simultaneously satisfying $\text{BR}(\mu \to e\gamma)$ and $\Delta a_\mu$ to within 1$\sigma$ requires $\langle \lambda_e/\lambda_\mu \rangle \lesssim 10^{-4}$. ........................................... 55

3.1 Plot of $\chi^2$/dof versus $m_{16}$ for cases where the value of $|V_{ub}|$ and $|V_{cb}|$ are taken to be the inclusive values, the exclusive values, or the average of inclusive and exclusive values. Solid lines refer to the universal $M_{1/2}$ model while dashed lines refer to the mirage mediated $M_{1/2}$ model. This plot shows that our model favors the exclusive values of $|V_{ub}|$ and $|V_{cb}|$. ........................................... 69

3.2 Plot of $\chi^2$ versus $m_{16}$ for the universal $M_{1/2}$ model. The $\chi^2$ in this plot is due to the set of observables in Table 3.3 which contributes directly to a minimum of $\chi^2$ at $m_{16} \approx 25$ TeV. ........................................... 73

3.3 These plots show the contour of $\chi^2$/d.o.f versus $\tilde{M}_3$ and $m_{16}$. The 4$\sigma$ bound is also included in the plot. For $\alpha = 1.5$, the upper bound is within reach of the next run of LHC. In addition, we also see that our model favors $m_{16} \approx 25$ TeV. ........................................... 75

4.1 A plot of the $\chi^2$/dof contour lines as a function of gluino mass $\tilde{M}_3$ and the universal scalar mass at the GUT scale $m_{16}$. The numbers on the black contour lines are $\chi^2$/dof while the green dotted lines are the 1.0 and 1.2 $\sigma$ bound from the $\chi^2$ analysis with 27 dof. This plot has 27 dof because $m_{16}$ and $M_{1/2}$ are fixed as the $x$ and $y$ axis. The horizontal white dotted line is the current gluino mass bound of this model (see Sec. 4.3). The yellow star is the point with the lowest $\chi^2$ for gluino mass above the current bound. The black star is a benchmark point where its input parameters and low-energy fits are shown in the appendix. Since the global $\chi^2$ minimum is below the lower limit of the plot, this model prefers low gluino mass. However, this plot also shows that this model is not very sensitive to the gluino mass, because $\chi^2$ increases relatively slowly as the gluino mass increases. ........................................... 89

4.2 The validation plot for the Gtt simplified model with $m_{\chi_0^1} = 200$ GeV in the 0 lepton with large mass splitting signal region of the ATLAS analysis [2]. The vertical blue bars show the number of events passing all cuts while the horizontal red line is the 95% upper limit of the number of events allowed. The gluino mass bound obtained from this plot, $M_{\tilde{g}} \sim 1.875$ TeV, is in agreement with the gluino mass bound from the ATLAS collaboration, $M_{\tilde{g}} \sim 1.9$ TeV. ........................................... 93

4.3 Number of events passing all cuts for this model in the 0 lepton with large mass splitting signal region of the ATLAS analysis [2]. The scalar mass of all the points in this plot is $m_{16} = 20$ TeV. The gluino mass bound obtained from this plot, $M_{\tilde{g}} \sim 1.875$ TeV, is the same as that from the validation plot. Hence, we conclude that the gluino mass bound of this model is the same as that of the simplified model. ........................................... 94

5.1 Hartree approximation for $\alpha = 1$ in describing the transfer of energy between the inflaton and the up-type Higgs boson as the inflaton oscillates around its minimum. ........................................... 124
5.2 The inflaton value in this plot is normalized to the first oscillation amplitude, $2.74 \times 10^{-1} \text{M}_{\text{pl}}$, while the Higgs boson number densities are normalized to the up-type Higgs boson number densities created at the first zero-crossing, $7.23 \times 10^{-11} \text{M}_{\text{pl}}^3$. At every zero-crossing of the inflaton, the Higgs bosons are created and subsequently decay out of the system almost instantaneously.

5.3 The inflaton speed in this plot is normalized to the initial inflaton speed, $2.63 \times 10^{-6} \text{M}_{\text{pl}}^2$. Since the non-perturbative creation of the Higgs boson is instantaneous and the subsequent the Higgs boson decay are almost instantaneous, the inflaton experiences a drastic decrease in speed at every zero-crossing. The Higgs boson decays transfers energy out from the inflaton-Higgs system.

5.4 Since the Universe is radiation dominated after a couple inflaton oscillations, the reheating process of our model is very efficient.

5.5 This plot is a continuation of Fig. 5.2. The decrease in the inflaton oscillation amplitude increases the Higgs boson number densities by decreasing the Higgs boson decay rate. The increase in the Higgs boson number densities increases the inflaton oscillation frequency, which further decreases the inflaton oscillation amplitude by increasing the frequency of the inflaton non-perturbative creation of the Higgs boson.

C.1 The definition of $\theta_{K^*, \ell}$, and $\phi$ for $B^0 \rightarrow K^{*0}(\rightarrow K^+\pi^-)\mu^+\mu^-$.

D.1 One-loop Feynman diagrams that contribute to $C_7$ Wilson coefficient in the background-field method.
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>3</td>
</tr>
<tr>
<td>1.2</td>
<td>9</td>
</tr>
<tr>
<td>1.3</td>
<td>19</td>
</tr>
<tr>
<td>2.1</td>
<td>44</td>
</tr>
<tr>
<td>2.2</td>
<td>64</td>
</tr>
<tr>
<td>3.1</td>
<td>67</td>
</tr>
<tr>
<td>3.2</td>
<td>72</td>
</tr>
</tbody>
</table>

1.1 Fundamental particles of the SM and their corresponding quantum numbers. 
\[ i = 1, 2, 3 \text{ are the family index} \]

1.2 Experimental values of the 18 SM parameters \[4\].

1.3 Particle content of the MSSM. A Superpartner has the same charges as its SM counterpart. Right-handed neutrinos, \( \bar{\nu} \), are SM singlets with \((1,1,0)\) charge under \( SU(3)_C \times SU(2)_L \times U(1)_Y \).

2.1 The quantum numbers of leptonic sector particles relevant to this chapter.
\[ i = 1, 2, 3 \text{ is the SM family index.} \]

2.2 List of observables. \( \Delta a_\mu \) is the discrepancy of the measured muon \( g-2 \) and the SM prediction. \( A_{FB}^{(le)} \) is the electron, muon, and tau left-right asymmetry in Z decay. \( A_{FB}^{(0B)} \) is the electron, muon, and tau forward-backward asymmetry in Z decay. \( R_{\mu\mu} = \Gamma(h \rightarrow \mu\mu)/\Gamma(h \rightarrow \mu\mu)_{\text{SM}} \) and similarly for \( R_{\tau\tau} \) and \( R_{\gamma\gamma} \). \( R_{K} = \Gamma(B^+ \rightarrow K^+\mu\mu)/\Gamma(B^+ \rightarrow K^+ee) \) while \( R_{K^{*0}} = \Gamma(B^0 \rightarrow K^{*0}\mu\mu)/\Gamma(B^0 \rightarrow K^{*0}ee) \). Lepton nonuniversality experimental values are take from LHCb measurements \[5,6\] while the other experimental values are taken from PDG \[4\].

3.1 The universal \( M_{1/2} \) model has 24 arbitrary parameters while the mirage mediated \( M_{1/2} \) model has an additional parameter, \( \alpha \), which determines the amount of splitting of \( M_{1/2} \), at the GUT scale.

3.2 45 low-energy observables that are fitted in the global \( \chi^2 \) analysis (see text for more discussions).

3.3 The pull, theoretical value, and experimental measurements for the set of observables for universal \( M_{1/2} \) model with \( M_{\tilde{g}} \approx 1.2 \text{ TeV} \) that contribute directly to a minimum of \( \chi^2 \) at \( m_{16} \approx 25 \text{ TeV} \). In addition, the observable \( P_4' \) of \( B^0 \rightarrow K^{*0}\mu^+\mu^- \) for high \( q^2 \) bins is also included to illustrate that it is independent on \( m_{16} \). See text for more discussions.
3.4 Without fixing any ratios, the fine-tuning is one part in $10^5$. When the ratio of $m_{H_u,d}/m_{16}$ and $A_0/m_{16}$ are fixed, the fine-tuning is about one part in 500. This suggests that these ratios should be fixed in a more fundamental natural theory. In addition, fine-tuning increases as $m_{16}$ increases. Hence, in our model, small $m_{16}$ is favored by naturalness.

3.5 Predictions with $m_{16} = 25$ TeV for $M_{\tilde{g}} \approx 1.2$ and 1.6 TeV. All masses in the table are in TeV units. Our prediction for the branching ratio $\mu \rightarrow e\gamma$ is consistent with the current upper bound of $5.7 \times 10^{-13}$ \cite{7}. In addition, our prediction of the electron electric dipole moment is consistent with the current upper bound of $10.5 \times 10^{-28}$ e cm \cite{7}.

4.1 26 input parameters of the PS model.

4.2 51 low energy observables that are fitted in the global $\chi^2$ analysis.

4.3 Branching ratios of gluinos, charginos, and neutralinos for two benchmark points of this model. The gluino branching ratios of this model are not even close to that of a simplified model. Hence, it is important to reinterpret the ATLAS and CMS simplified model analyses to obtain the gluino mass bounds of this model. The chargino and neutralino masses and compositions are shown in Table 4.5.

4.4 SUSY mass spectrum from four benchmark points of this model. Benchmark points A and C have $m_{16} = 20$ TeV, while benchmark points B and D have $m_{16} = 25$ TeV. Benchmark points A and B have $M_{\tilde{\chi}} = 2.0$ TeV while benchmark points C and D have $M_{\tilde{\chi}} = 2.6$ TeV. The compositions of the charginos and neutralinos of benchmark points A and B are shown in Table 4.5. The input parameters of these benchmark points are in App. H.2. In addition, the prediction of the electron dipole moment, the branching ratio of $\mu \rightarrow e\gamma$ and the neutrino $C\bar{P}$ violating phase are also presented in this table.

4.5 Neutralino and chargino masses in GeV and their composition in percentage for the two benchmark points A and B in Tab. 4.4. The lightest neutralino is mainly the Bino. Hence, the lightest neutralino cannot be the dark matter candidate. However, as discussed in previous chapter, this problem can be alleviated if the gaugino masses are nonuniversal.

5.1 Quantum numbers of particles in the inflation model with $PS \times Z^R_4$ symmetry. The $D_3$ family symmetry indices are omitted.

B.1 Quantum numbers of SM particles in the Yukawa sector.
Physics is the study of natural physical phenomena using experimental and quantitative methods. One of the paths to the ultimate understanding of nature is through the understanding of fundamental particles and their interactions via fundamental forces. Particle physics is the study of physics along this direction.

One of the major unsolved problems in particle physics is the uncovering of the Theory of Everything, a self-consistent theory that encompasses all fundamental forces and fundamental particles. The first attempt that was made in this quest was the unification of electricity and magnetism as electromagnetism, which was first formulated in 1873 by James Clerk Maxwell [8]. The next fundamental force that was discovered is the weak force, which is required to explain beta decays. In 1960s, the weak and electromagnetic theories were unified as the electroweak theory by Sheldon Lee Glashow, Steven Weinberg, and Abdus Salam [9–11]. It was not until the late 1960s to early 1970s that Quantum Chromodynamics (QCD), the theory of strong force, was formulated, thus giving birth to the Standard Model (SM) of particle physics.

Although the SM describes the interactions of fundamental particles via the strong, weak, and electromagnetic forces, these forces are distinct and are not unified. A way to achieve unification of these forces is to extend the SM with supersymmetry (SUSY), thus giving rise to Supersymmetric Grand Unified Theories (SUSY GUTs). A major goal of this thesis is to make a baby step in increasing our understanding of SUSY GUTs.

This chapter aims to provide motivation and necessary backgrounds needed to under-
stand the thesis. In Sec. 1.1, a brief overview of the SM and its challenges are presented. Right-handed neutrinos are then introduced to the SM to naturally give the light neutrino masses via the seesaw mechanism. In Sec. 1.2, supersymmetry (SUSY), which ameliorates the hierarchy problem, is described. The Minimal Supersymmetric Standard Model (MSSM) is introduced next as a specific example of a supersymmetric extension of the SM. MSSM provides a natural dark matter candidate with the correct relic abundance and predicts gauge coupling unification, which is the topic of the next section. In particular, brief overviews of Supersymmetric Grand Unified Theories (SUSY GUTs) with SU(5), SO(10), and Pati-Salam (PS) gauge group, respectively, are presented. In addition to gauge coupling unification, SU(5) SUSY GUT enables $b - \tau$ Yukawa unification while SO(10) and PS SUSY GUTs enable $t - b - \tau - \nu_\tau$ Yukawa unification. A natural consequence of right-handed neutrino is leptogenesis, the generation of lepton asymmetry in the early universe, which is described in Sec. 1.4. Finally, the conclusion of this chapter and an overview of this thesis are given in Sec. 1.5.

1.1 Standard Model of Particle Physics

The SM describes three of the four fundamental forces of nature and the interactions of fundamental particles via these three forces. These three forces, the strong, weak, and electromagnetic forces, are described by three different gauge groups. The gauge group for the strong force is $SU(3)_C$. The gauge group for the electroweak force is $SU(2)_L \times U(1)_Y$.

The fundamental particles can be separated into fermions and bosons. Fermions can be further subdivided into quarks and leptons. The SM has three families of up-type quarks, down-type quarks, charged leptons, and left-handed neutrinos, respectively. The bosons, on the other hand, can be subdivided into the gauge and scalar bosons. The gauge bosons of the $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ gauge groups are eight gluons, three $W$ bosons, and a $B$ boson, respectively. The only fundamental scalar boson of the SM is the Higgs boson, which is responsible for giving mass to some of the above-mentioned particles. The fundamental
Table 1.1: Fundamental particles of the SM and their corresponding quantum numbers. $i = 1, 2, 3$ are the family index.

particles of the SM along with their quantum numbers are summarized in Table 1.1.

The Lagrangian of the SM is given by

$$L_{\text{SM}} = L_{\text{fermion}} + L_{\text{gauge}} + L_{\text{Higgs}} + L_{\text{Yukawa}}.$$  

In the following subsections, we will review each of these terms separately.

**Fermion Sector**

The kinetic terms of the fermion sector are

$$L_{\text{fermion}} = q^* i \bar{\sigma}^\mu D_\mu q + \bar{u}^* i \bar{\sigma}^\mu D_\mu \bar{u} + \bar{d}^* i \bar{\sigma}^\mu D_\mu \bar{d} + \ell^* i \bar{\sigma}^\mu D_\mu \ell + \bar{e}^* i \bar{\sigma}^\mu D_\mu \bar{e},$$  

The fermions in Table 1.1 and the rest of this chapter are written in terms of left-handed Weyl spinors. A four component Dirac spinor contains two left-handed Weyl spinors,

$$\psi_x = \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix} = \begin{pmatrix} \chi \\ (i \tau_2) \bar{\chi}^* \end{pmatrix},$$  

where $\tau_2$ is the second Pauli matrix.
where family indices are suppressed, $\bar{\sigma}^\mu = (1, -\tau^i)$, $\tau^i$ are Pauli matrices, and the gauge covariant derivative is

$$D_\mu = \partial_\mu + ig_s T^A G^A_\mu + ig T^a W^a_\mu + ig' \frac{Y}{2} B_\mu. \quad (1.4)$$

$g_s$, $g$, and $g'$ are the SU(3)$_C$, SU(2)$_L$, and U(1)$_Y$ gauge couplings, respectively. These couplings are arbitrary parameters of the SM. $T^A = \lambda^A/2$ for $A = 1, \ldots, 8$ are the SU(3)$_C$ generators, where $\lambda^A$ are the Gell-Mann matrices. $T^a = \tau^a/2$ for $a = 1, 2, 3$ are the SU(2)$_L$ generators, where $\tau^a$ are the Pauli matrices. $Y$ is the U(1)$_Y$ hypercharge. $G^A_\mu$, $W^a_\mu$, and $B_\mu$ are the gluons, $W$ bosons, and $B$ boson, respectively.

The mass eigenstates of $W^a_\mu$ and $B_\mu$ are

$$W^\pm_\mu = \frac{W^1_\mu \mp i W^2_\mu}{\sqrt{2}}, \quad (1.5)$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & -\sin \theta_W \\ \sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}. \quad (1.6)$$

Hence, the SU(2)$_L \times$ U(1)$_Y$ gauge covariant derivative in terms of these mass eigenstates is

$$D_\mu = \partial_\mu + i \frac{g}{\sqrt{2}} (\tau^+ W^+_\mu + \tau^- W^-_\mu) + i \frac{g}{\cos \theta_W} (T^3 - Q \sin^2 \theta_W) Z_\mu + ie A_\mu, \quad (1.7)$$

where

$$\tau^\pm = \frac{\tau_1 \pm i \tau_2}{2}, \quad (1.8)$$

the electric charge is

$$Q = T^3 + \frac{Y}{2}, \quad (1.9)$$

and

$$e = g \sin \theta_W. \quad (1.10)$$
Gauge Sector

The kinetic terms of the gauge sector are

\[ \mathcal{L}_{\text{gauge}} = -\frac{1}{4} G^A_{\mu \nu} G^{A;\mu \nu} - \frac{1}{4} W^a_{\mu \nu} W^{a;\mu \nu} - \frac{1}{4} B_{\mu \nu} B^{\mu \nu}, \]  

(1.11)

where the field strengths of the gauge bosons are

\[ G^A_{\mu \nu} = \partial_{\mu} G^A_{\nu} - \partial_{\nu} G^A_{\mu} - g_s f^{ABC} G^B_{\mu} G^C_{\nu}, \]  

(1.12)

\[ W^a_{\mu \nu} = \partial_{\mu} W^a_{\nu} - \partial_{\nu} W^a_{\mu} - g_s \epsilon^{abc} W^b_{\mu} W^c_{\nu}, \]  

(1.13)

\[ B_{\mu \nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu}. \]  

(1.14)

\( f^{ABC} \) are structure constants of the SU(3)_C algebra satisfying

\[ [T^A, T^B] = i f^{ABC} T^C, \]  

(1.15)

while \( \epsilon^{abc} \), the antisymmetric tensor, are structure constants of the SU(2)_L algebra satisfying

\[ [T^a, T^b] = i \epsilon^{abc} T^c. \]  

(1.16)

Higgs Sector

The Lagrangian of the Higgs sector is

\[ \mathcal{L}_{\text{Higgs}} = (D_{\mu} H)^\dagger (D^\mu H) - \frac{\lambda}{2} \left( \bar{H} H - \frac{v^2}{2} \right)^2, \]  

(1.17)

where the Higgs boson is a complex SU(2)_L doublet with four real degrees of freedom,

\[ H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_3 - i \phi_4 \\ \phi_1 - i \phi_2 \end{pmatrix}. \]  

(1.18)

From the Higgs potential, it is easy to see that the Higgs boson obtains a vacuum expectation value (VEV) proportional to the weak scale, \( v \), which is an arbitrary parameter of the SM. Thus, the electroweak theory with SU(2)_L \times U(1)_Y gauge group is spontaneously broken down to the electromagnetic theory with U(1)_Q gauge group. Using SU(2)_L \times U(1)_Y
symmetry, the VEV of $\phi_i$ for $i = 2, 3, 4$ can be rotated away giving

$$H = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \phi_3 - i\phi_4 \\ v + \phi_1 - i\phi_2 \end{array} \right),$$

(1.19)

where we have redefined the fields $\phi_i$ for $i = 2, 3, 4$. With this redefinition, the Higgs potential becomes

$$V(H) = \frac{\lambda}{8} \left[ \left( \sum_{i=1}^{4} \phi_i^2 \right)^2 + 4v\phi_1 \sum_{i=1}^{4} \phi_i^2 + 4v^2\phi_1^2 \right].$$

(1.20)

Hence, $\phi_1$ is identified as the physical Higgs boson, $h$, which has a mass of

$$m_h = \lambda v^2,$$

(1.21)

where $\lambda$ or equivalently, $m_h$ is another arbitrary parameter of the SM. On the other hand, $\phi_i$ for $i = 2, 3, 4$ are the would-be Nambu-Goldstone bosons that will be eaten by $W^\pm$ and $Z$ bosons.

To obtain masses of gauge bosons after electroweak symmetry breaking (EWSB), let

$$H = H_v + \bar{H} = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v \end{array} \right) + \frac{1}{\sqrt{2}} \left( \begin{array}{c} \phi^3 - i\phi^4 \\ \phi_1 - i\phi_2 \end{array} \right).$$

(1.22)

The kinetic term becomes

$$(D_\mu H)^\dagger(D^\mu H) \supset (D_\mu H_v)^\dagger(D^\mu H_v)$$

$$= \frac{1}{2} g^2 v^2 W^a_\mu W^{a\mu} - \frac{gg'v^2}{4} W^3_\mu B^{\mu} + \frac{g'^2 v^2}{8} B_\mu B^\mu.$$  

(1.23)

Rewriting $W^a_\mu$ and $B_\mu$ bosons in terms of mass eigenstates defined in Eq. 1.5 and Eq. 1.6, the mass term becomes

$$(D_\mu H_v)^\dagger(D^\mu H_v) = \frac{g^2 v^2}{4} W^+_\mu W^{-\mu} + \frac{(g^2 + g'^2)v^2}{8} Z_\mu Z^{\mu}.$$  

(1.24)
Hence,
\[ \tan \theta_W = \frac{g'}{g}, \quad (1.25) \]
\[ M_W = \frac{g v}{2}, \quad (1.26) \]
\[ M_Z = \frac{M_W}{\cos \theta_W}, \quad (1.27) \]

and as expected, the photon is massless.

**Yukawa Sector**

The Lagrangian of the Yukawa sector is
\[ \mathcal{L}_{\text{Yukawa}} = -q_i [y^u]_{ij} \bar{u}_j \tilde{h} - q_i [y^d]_{ij} \bar{d}_j \tilde{h} - \ell_i [y^e]_{ij} \bar{\ell}_j h + \text{h.c.}, \quad (1.28) \]

where \( y^{u,d,e} \) are \( 3 \times 3 \) Yukawa matrices and \( \tilde{h} = (i \tau_2) h^* \). After EWSB, the Higgs boson obtains a VEV and fermions get chiral symmetry breaking masses.
\[ m^{u,d,e} = y^{u,d,e} v. \quad (1.29) \]

In general, the mass matrices are not diagonal and can be diagonalized with a bi-unitary transformation.
\[ m^{u,\text{diag}} = U_q^\dagger m^u U_u, \quad (1.30) \]
\[ m^{d,\text{diag}} = U_q^\dagger m^d U_d, \quad (1.31) \]
\[ m^{e,\text{diag}} = U_\ell^\dagger m^e U_e. \quad (1.32) \]

The masses of quarks and leptons are arbitrary parameters of the SM. In the fermion mass basis, \( Z \) bosons and photon couplings are flavor diagonal while \( W^\pm \) bosons couplings are not flavor diagonal (see Eq. (1.7)). This misalignment leads to the Cabibbo-Kobayashi-Maskawa (CKM) matrix [12, 13].

\[ \mathcal{L}_{\text{fermion}} \supset -\frac{g}{\sqrt{2}} W^\mu_\pm \bar{u}^* \sigma_\mu V_{\text{CKM}} \tilde{d}, \quad (1.33) \]
where $\hat{u}$ and $\hat{d}$ are the mass bases and

$$V_{\text{CKM}} = U_q^\dagger U_d.$$  \hfill (1.34)

The CKM matrix is a complex $3 \times 3$ unitary matrix. In general, a $3 \times 3$ unitary matrix has nine real parameters: three Euler angles and six phases. However, five phases can be absorbed with a redefinition of $\hat{u}$ and $\hat{d}$. Hence, the CKM matrix only have four arbitrary real parameters: three angles and a $CP$ violating phase. The CKM matrix in the Wolfenstein parametrization is $^{[14]}$

$$V_{\text{CKM}} = \begin{pmatrix}
1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + \mathcal{O}(\lambda^4). \hfill (1.35)$$

As shown in Table $^{[12]}$, the diagonal elements of the CKM matrix are measured to be larger than the off-diagonal elements. Hence, flavor violations in the quark sector are generally small.

Since neutrinos are massless in the SM, by defining $U_\nu = U_\ell$, the lepton sector does not have the above misalignment as the $W^\pm$ bosons couplings are flavor diagonal.

In summary, the SM has 18 arbitrary parameters: three gauge couplings, a Higgs VEV, a Higgs mass, six quarks masses, three leptons masses, and four CKM matrix elements. $^2$ Experimental measurements of these parameters are given in Table $^{[12]}$. In this table $\alpha = g^2/4\pi$, $G_\mu$ is the Fermi coupling constant $G_F/\sqrt{2} = g^2/8M_W^2$ measured from the muon lifetime, $\alpha_{\text{EM}}^{-1}$ is measured at $Q^2 = 0$, charm and bottom quark masses are the running masses in the $\overline{\text{MS}}$ scheme, $\bar{\rho} = \rho(1 - \lambda^2/2)$, and $\bar{\eta} = \eta(1 - \lambda^2/2)$.

### 1.1.1 Why Beyond the SM?

The SM is an extremely successful model. For example, one of the most precise measurements in physics is the fine structure constant, which is measured to a higher precision than one part in $10^{10}$. However, there are some very clear needs for physics beyond the SM.

$^2$The SM has 19 arbitrary parameters if the QCD vacuum angle, $\theta_{\text{QCD}}$, is included.
<table>
<thead>
<tr>
<th>Sector</th>
<th>Parameter</th>
<th>Experimental Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauge</td>
<td>$\alpha_s(M_Z)$</td>
<td>0.1181(11)</td>
</tr>
<tr>
<td></td>
<td>$G_\mu$</td>
<td>$1.1663787(6) \times 10^{-5}$ GeV$^{-2}$</td>
</tr>
<tr>
<td></td>
<td>$\alpha_{EM}$</td>
<td>137.035999139(31)</td>
</tr>
<tr>
<td>Higgs</td>
<td>$v$</td>
<td>246 GeV</td>
</tr>
<tr>
<td></td>
<td>$m_h$</td>
<td>$125.09 \pm 0.24$ GeV</td>
</tr>
<tr>
<td>Quark</td>
<td>$m_u(2$ GeV)</td>
<td>$2.2^{+0.6}_{-0.4}$ MeV</td>
</tr>
<tr>
<td></td>
<td>$m_d(2$ GeV)</td>
<td>$4.7^{+0.5}_{-0.4}$ MeV</td>
</tr>
<tr>
<td></td>
<td>$m_s(2$ GeV)</td>
<td>$96^{+8}_{-4}$ MeV</td>
</tr>
<tr>
<td></td>
<td>$m_c(m_c)$</td>
<td>$1.28 \pm 0.03$ GeV</td>
</tr>
<tr>
<td></td>
<td>$m_b(m_b)$</td>
<td>$4.18^{+0.04}_{-0.03}$ GeV</td>
</tr>
<tr>
<td></td>
<td>$M_t$</td>
<td>$173.1 \pm 0.6$ GeV</td>
</tr>
<tr>
<td>Lepton</td>
<td>$M_e$</td>
<td>$0.5109989461(31)$ MeV</td>
</tr>
<tr>
<td></td>
<td>$M_\mu$</td>
<td>$105.6583745(24)$ MeV</td>
</tr>
<tr>
<td></td>
<td>$M_\tau$</td>
<td>$1776.86 \pm 0.12$ MeV</td>
</tr>
<tr>
<td>CKM</td>
<td>$\lambda$</td>
<td>$0.22506 \pm 0.00050$</td>
</tr>
<tr>
<td></td>
<td>$A$</td>
<td>$0.811 \pm 0.026$</td>
</tr>
<tr>
<td></td>
<td>$\bar{\rho}$</td>
<td>$0.124^{+0.019}_{-0.018}$</td>
</tr>
<tr>
<td></td>
<td>$\bar{\eta}$</td>
<td>$0.356 \pm 0.011$</td>
</tr>
</tbody>
</table>

Table 1.2: Experimental values of the 18 SM parameters [4].
1. **Arbitrary Parameters**: Perhaps the most obvious hint for physics beyond the SM is due to the 18 arbitrary parameters of the SM known only from experimental measurements. In addition to the arbitrariness of these parameters, the measured values raise some interesting questions. For example, why are fermion masses hierarchical? Why are flavor violations in the quark sector small? These questions can be answered by invoking the anthropic principle, which states that the observed Universe must be compatible with conscious life that observes it. Hence, the parameters have their observed values otherwise we will not be here to observe it. However, a more satisfying answer is to be able to explain the underlying reason.

2. **Neutrino Mass**: Neutrinos are massless in the SM but as will be explained in Sec. 1.1.2, neutrinos are observed to have tiny masses. For neutrinos to acquire Dirac masses, the SM needs right-handed neutrinos, which are absent in the SM. On the other hand, for neutrinos to acquire Majorana masses, fermion number has to be violated. To date, the type of neutrinos masses are not settled yet. This will be discussed further in Sec. 1.1.2.

3. **Muon anomalous magnetic moment**: The experimentally measured muon anomalous magnetic moment and the SM prediction are given by

\[
a_\mu^{\text{exp}} = 11659209.1(5.4)(3.3) \times 10^{-10},
\]

\[
a_\mu^{\text{SM}} = 11659180.3(0.1)(4.2)(2.6) \times 10^{-10}.
\]

The discrepancy between the experimental and theoretical values is

\[
\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 288(63)(49) \times 10^{-11},
\]

which corresponds to a 3.6\(\sigma\) deviation.

4. **Hierarchy problem**: In quantum field theory, masses of fundamental particles can receive loop corrections. Although fermion masses are protected from quadratic divergences by chiral symmetry and gauge boson masses are protected from large radiative corrections by gauge symmetry, the masses of fundamental scalars are not protected
from large radiative corrections by any symmetry. In particular, the Higgs boson re-
ceives mass correction from a fermion in the loop as shown in Fig. 1.1a. The quadratic
divergent part of this correction is
\[ \Delta m_h^2 = -\frac{\lambda_f^2}{8\pi^2} \Lambda_{UV}^2, \]
(1.39)
where \( \lambda_f \) is the Higgs-fermion coupling given by \( -\lambda_f h \bar{f} f \) and \( \Lambda_{UV} \) is the ultraviolet
cutoff scale that regulates the loop integral. This cutoff can be interpreted as the scale
of new physics (NP), which can be as high as the Planck scale, \( 10^{18} \) GeV. However,
the Higgs mass is measured to be \( m_h^2 = 125.90 \pm 0.24 \) GeV. The large cancellation
required to stabilize the Higgs mass at the weak scale is the Hierarchy problem. As will
be discussed in Sec. 1.2, SUSY provides a natural solution to the hierarchy problem.\(^3\)

5. **Matter-antimatter asymmetry**: The imbalance between matter and antimatter is
quantified by the baryon-to-entropy ratio, which is experimentally measured to be \[ n_B \]
\[ s \sim (8.679 \pm 0.054) \times 10^{-11}, \]
(1.40)
where \( n_B \) is the baryon number density minus the anti-baryon number density and \( s \)
is the entropy density of the Universe. This definition of baryon asymmetry is useful
because the entropy is conserved during the expansion of the Universe. As will be
explained in Sec. 1.4, the SM is unable to explain this asymmetry.

6. **Dark matter**: Another indisputable evidence of physics beyond the SM is the dark
matter. One of the first observations that revealed the existence of dark matter was
by Vera Rubin and Kent Ford \[17, 18\]. They measured the rotational speed of high
luminosity spiral galaxies as a function of radius and found that the rotation curves
remain flat up to large radii away from the center. Hence, there must be additional
non-luminous matter, or dark matter, in the outer regions of these galaxies.\(^4\) The

\(^3\)Another interesting potential solution to this problem, which is beyond the scope of this thesis, is to
construct the Higgs boson as a composite particle rather than a fundamental scalar. This is called the
composite Higgs paradigm. See Ref. \[15\] for a review.

\(^4\)There are attempts to explain this discrepancy with modified Newtonian dynamics (MOND). However,
MOND cannot completely eliminate the need for dark matter \[19\].
most recent measurements of the energy densities of baryons, dark matter, and dark energy are \[ 16 \]

\[
\begin{align*}
\Omega_b h^2 &= 0.02230 \pm 0.00014, \\
\Omega_{DM} h^2 &= 0.1188 \pm 0.0010, \\
\Omega_{\Lambda} h^2 &= 0.6911 \pm 0.0062,
\end{align*}
\]

where \( \Omega_i \) are the ratios of energy density of component \( i \) to the critical density,

\[
h = \frac{H_0}{100 \frac{\text{Mpc}}{\text{km}}} = 0.6774 \pm 0.0046,
\]

and \( H_0 \) is the present day Hubble parameter.

This list of hints for physics beyond the SM is by no means a complete list.\(^5\) Instead, this is the list of SM challenges that NP models in this thesis try to tackle.

### 1.1.2 SM augmented with Right-handed Neutrinos

The first hint that the neutrinos are massive is from Ray Davis’s experiment at Homestake Mine, South Dakota in the late 1960s \[ 20 \]. This experiment observed only about one-third of the number of predicted electron neutrinos coming from the Sun. This is the solar neutrino problem. To solve this problem, Mikheyev, Smirnov, and Wolfenstein proposed that neutrinos oscillate from electron neutrinos to muon and tau neutrinos. Hence, not all

\(^5\)Another obvious need for extending the SM is the lack of gravitational description in the SM. This is one of the biggest unsolved problems in physics.
of the electron neutrinos produced in the Sun are detected \cite{21, 22}. Neutrino oscillations are possible only if neutrinos are massive and there is a misalignment between neutrino flavor and mass eigenstates, that is

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\
U_{\tau 1} & U_{\tau 2} & U_{\tau 3}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix},
\]

(1.45)

where \(\nu_i\) for \(i = 1, 2, 3\) are the mass eigenstates and \(U_{PMNS}\) is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix \cite{23, 24}. Neutrino oscillations were first verified in 1998 by the Super-Kamiokande experiment in Japan, which found that muon neutrinos coming from below the Earth is only half the number of muon neutrinos coming directly from the Earth atmosphere \cite{25}. This is called the atmospheric neutrino problem. This deficit is due to muon neutrinos oscillating to tau neutrinos as muon neutrinos pass through the Earth. On the other hand, the solar neutrino problem is confirmed in 2002 by Sudbury Neutrino Observatory (SNO) in Canada where the flux of electron neutrinos and all neutrinos coming from the Sun are measured \cite{26, 27}.

Although neutrinos have tiny masses, unlike charged fermion masses, neutrino masses are less hierarchical. In addition, unlike CKM matrix, the mixing angles in the PMNS matrix are large. Since neutrinos are massive, lepton flavors are not conserved. Finally, neutrinos being massive add nine new arbitrary parameters into the SM. They are three neutrino masses, three mixing angles, and three phases, where we have assumed that neutrinos are Majorana particles. The current best fit values of these parameters are \cite{28}

\[
\Delta m^2_{21} = (7.21 - 7.54) \times 10^{-5} \text{ eV}^2, \quad (1.46)
\]

\[
|\Delta m^2_{31}| = (2.46 - 2.54) \times 10^{-3} \text{ eV}^2, \quad (1.47)
\]
\[
\sin^2 \theta_{12} = (2.81 - 3.14) \times 10^{-1}, \quad (1.48)
\]
\[
\sin^2 \theta_{13} = (2.05 - 2.25) \times 10^{-2}, \quad (1.49)
\]
\[
\sin^2 \theta_{23} = (4.17 - 4.70) \times 10^{-1}, \quad (1.50)
\]
\[
\delta = (1.13 - 1.64) \times \pi, \quad (1.51)
\]

where \( \Delta m_{ij}^2 = m_i^2 - m_j^2 \) for \( i, j = 1, 2, 3 \) and the numbers quoted are the 1\( \sigma \) ranges for the normal hierarchy where \( \nu_3 \) is the heaviest neutrino. Another possible ordering is the inverted hierarchy where \( \nu_3 \) is the lightest neutrino.

There are many models that can give neutrinos a tiny mass (for a review, see Ref. [29]). The model that we will consider in this thesis is the SM augmented with three heavy right-handed Majorana neutrinos, \( \bar{\nu}_i \) for \( i = 1, 2, 3 \). The light neutrino masses are obtained via the seesaw mechanism. \( \bar{\nu}_i \) are SM singlets and the Lagrangian involving \( \bar{\nu}_i \) is

\[
\mathcal{L}_{\text{RHN}} = \bar{\nu}^* i \tilde{\sigma}_\mu D^\mu \nu - \ell_i [y^\nu]_{ij} \bar{\nu}_j \tilde{h} + \frac{1}{2} \bar{\nu}_i [M_R]_{ij} \bar{\nu}_j + \text{h.c.}, \quad (1.52)
\]

where \( M_R \) is the right-handed neutrino Majorana mass matrix, which is a complex \( 3 \times 3 \) symmetric matrix. After EWSB, neutrinos get Dirac masses given by

\[
M^\nu = y^\nu v. \quad (1.53)
\]

After integrating out \( \bar{\nu}_i \), the light neutrino mass matrix is

\[
\bar{M}^\nu = M^\nu M_R^{-1} M^{\nu T}. \quad (1.54)
\]

Since the light neutrino mass matrix is suppressed by the heavy neutrino mass matrix, the light neutrinos are naturally light.

### 1.2 Supersymmetry

One approach to solve the hierarchy problem is to invoke a new symmetry that relates fermions and bosons. The quadratic divergent part of the one-loop correction for a scalar
mass involving a real scalar in the loop (see Fig. 1.1b) is

\[ \Delta m_h^2 = \frac{\lambda_S}{16\pi^2} \Lambda_{UV}^2, \]

(1.55)

where \( \lambda_S \) is the Higgs-scalar coupling given by \(-\lambda_S |h|^2 |S|^2\). Comparing Eq. 1.39 and Eq. 1.55 we see that if for every fermion, there are two real scalars, or one complex scalar, with coupling \( \lambda_S = |\lambda_f|^2 \), then the quadratic divergence is canceled naturally.

SUSY is a framework that solves the hierarchy problem by introducing a fermionic partner for every boson and a bosonic partner for every fermion (see Ref. [1] for an excellent review). In particular, SUSY introduces a supersymmetric generator, \( Q \), with the following transformation.

\[ Q|\text{Boson}\rangle = |\text{Fermion}\rangle, \]

\[ Q|\text{Fermion}\rangle = |\text{Boson}\rangle. \]

(1.56)

In SUSY, an irreducible representation of the SUSY algebra is called a supermultiplet, which consists of fermionic and bosonic states. These fermionic and bosonic states have equal masses and are called the superpartners of one another. In addition, one can show that each supermultiplet contains the same number of fermionic and bosonic degrees of freedom, hence naturally solving the hierarchy problem. The SUSY generator also commutes with the generators of gauge transformations. So, superpartners are in the same gauge group representation.

The simplest supermultiplet is a chiral supermultiplet, which contains a Weyl spin-1/2 fermion and a complex spin-0 scalar. A chiral superfield can be written as

\[ \Phi = \phi + \sqrt{2} \theta \psi + \theta \theta F, \]

(1.57)

where \( \phi \) is a complex spin-0 scalar, \( \psi \) is a Weyl spin-1/2 fermion, \( F \) is an auxiliary spin-0 scalar, and \( \theta \) is a Grassmann variable. The auxiliary field does not have a kinetic term, hence is unphysical. The Grassmann variable transforms as a left-handed Weyl spinor and has a mass dimension of \([\text{mass}]^{-1/2}\). The chiral superfield, on the other hand, has a mass.

\[ ^6 \text{Actually, SUSY does not solve the hierarchy problem completely. Instead, it replaces the hierarchy problem with the little hierarchy problem, which will be introduced later.} \]
The next simplest supermultiplet is a vector supermultiplet, which contains a spin-1 vector boson and a spin-1/2 fermion. In the Wess-Zumino (WZ) gauge, a vector supermultiplet is given by

\[ V^a_{\text{WZ}} = \theta^\dagger \bar{\sigma}^\mu \theta A^a_{\mu} + \theta^\dagger \theta \lambda^a + \frac{1}{2} \theta \theta \theta^\dagger \theta^\dagger D^a, \]  

(1.58)

where the superscript \( a \) is a gauge index, \( A^a_{\mu} \) is a spin-1 gauge boson, \( \lambda^a \) is a spin-1/2 fermion called gaugino, and \( D^a \) is an auxiliary field. On the other hand, the field strength superfield in the Wess-Zumino gauge is given by

\[ (W^a_{\alpha})_{\text{WZ}} = \lambda^a_{\alpha} + \theta_{\alpha} D^a + \frac{i}{2} (\sigma^\mu \bar{\sigma}^\nu \theta)_{\alpha} F^a_{\mu\nu} + i\theta \theta (\sigma^\mu \nabla^\mu \lambda^a)_{\alpha}, \]  

(1.59)

where \( \alpha \) is a spinor index and \( \sigma^\mu = (1, \tau^i)^T \).

In SUSY, the most general supersymmetric Lagrangian includes a Kähler potential, gauge kinetic functions, and a superpotential. The \( D \)-term of the Kähler potential, \( K \), gives the kinetic terms of the fermion sector (compare with Eq. 1.3)

\[ \mathcal{L}_{\text{fermion}} = [K]_D, \]  

(1.60)

where a \( D \)-term is defined by

\[ [\cdots]_D = \int d^2 \theta d^2 \theta^\dagger \cdots. \]  

(1.61)

A renormalizable Kähler potential is given by

\[ K = \Phi_i^* e^{2g_a T^a V^a} \Phi_i, \]  

(1.62)

where \( g_a \) is the gauge coupling. The Kähler potential is real and has mass dimension \([\text{mass}]^2\).

In terms of component fields, the kinetic terms of the fermion sector are

\[ \mathcal{L}_{\text{fermion}} = |F_i|^2 - |D_\mu \phi_i|^2 + i \bar{\psi}_i \bar{\sigma}^\mu D_\mu \psi_i + g_a \phi_i^* T^a \phi_i D^a - (\sqrt{2} g_a \phi_i^* T^a \psi_i \lambda^a + \text{h.c.}). \]  

(1.63)

\(^7\)SUSY also has a description for a spin-2 graviton, which is beyond the scope of this thesis. The superpartner of the graviton is a spin-3/2 gravitino.
The gauge kinetic term, \( f(\Phi_i) \), appears in the kinetic terms of the gauge sector (compare with Eq. 1.11)

\[
\mathcal{L}_{\text{gauge}} = \frac{1}{4} [f(\Phi_i) W^{a\alpha} W^a_{\alpha}] F + \text{h.c.},
\]

where a \( F \)-term is defined by

\[
[\cdots]_F = \int d^2 \theta \cdots.
\]

Assuming that \( f(\Phi_i) = 1 \), the Lagrangian in terms of component fields is

\[
\mathcal{L}_{\text{gauge}} = \frac{1}{2} D^a D^a + i \lambda^a \sigma^\mu D_\mu \lambda^\dagger a - \frac{1}{4} F^{a\mu\nu} F_{a\mu\nu},
\]

where we have dropped a total derivative term, which cannot be ignored in a non-abelian gauge theory.

The superpotential, \( W \), is a mass dimension \([\text{mass}]^3\) holomorphic function of chiral superfields. The \( F \)-term of a superpotential gives the Yukawa terms,

\[
\mathcal{L}_{\text{Yukawa}} = [W(\Phi_i)]_F.
\]

The Yukawa sector Lagrangian can be written in terms of \( W \) as

\[
\mathcal{L}_{\text{Yukawa}} = F_i \frac{\partial W}{\partial \Phi_i} \bigg|_{\theta=0} - \frac{1}{2} \psi_i \psi_j \frac{\partial^2 W}{\partial \Phi_i \partial \Phi_j} \bigg|_{\theta=0} + \text{h.c.}.
\]

From this Lagrangian, we see that the Yukawa terms always have two fermions. Other fields that multiply by these two fermions, if exists, must be scalars. For example, a dimension four operator has two fermions and a scalar while a dimension five operator has two fermions and two scalars.

The auxiliary fields, on the other hand, can be eliminated using their classical equation of motions,

\[
F^*_i = - \frac{\partial W}{\partial \Phi_i} \bigg|_{\theta=0},
\]

\[
D^a = - g_a \phi_i^a T^a \phi_i.
\]
and the scalar potential is given by

$$V(\phi_i) = \sum_i |F_i|^2 + \frac{1}{2} \sum_a (D^a)^2.$$  \hfill (1.71)

Equipped with these basic building blocks of a supersymmetric theory, in the next section, we discuss the simplest supersymmetric extension of the SM.

### 1.2.1 Minimal Supersymmetric Standard Model

The minimal supersymmetric extension of the SM is the MSSM. In the MSSM, each quark has a scalar superpartner called squark. Similarly, each lepton has a scalar superpartner called slepton. On the other hand, each gauge boson has a fermionic superpartner called gaugino. As for the Higgs boson, SUSY requires two Higgs doublets because Higgsino, the fermionic partner of the Higgs boson, contributes to an anomaly. Two Higgs doublets are needed to cancel this anomaly, which is required for a gauge theory to be the consistent. In addition, since the superpotential is holomorphic, an up-type Higgs boson is needed to couple to up-type quarks and a separate down-type Higgs boson is needed to couple to down-type quarks. Hence, the MSSM has an up-type Higgs, $H_u$, with hypercharge $Y = 1$ and a down-type Higgs, $H_d$, with the opposite hypercharge $Y = -1$. The particle content of the MSSM is given in Table 1.3.

In the MSSM, the SM kinetic terms of the fermion sector, Eq. 1.3, are replaced by

$$L_{\text{Kinetic}} = \left[ Q_i^\dagger e^{-2(g_e T^AV^A_g + g_T^AV^A_W + g_W^AV^A_{Wg})} Q_i \right]_D + \ldots ,$$  \hfill (1.72)

where the ellipsis is terms with $Q_i$ replaced by $\bar{U}_i$, $\bar{D}_i$, $\bar{L}_i$, $\bar{E}_i$, and $\bar{N}_i$, respectively. The SM kinetic terms of the gauge sector, Eq. 1.11, are replaced by

$$L_{\text{gauge}} = \frac{1}{4}[W^A_g W^A_g]_F + \frac{1}{4}[W^a_W W^a_W]_F + \frac{1}{4}[W_B W^A_B]_F + \text{h.c.},$$  \hfill (1.73)

while the SM Yukawa terms, Eq. 1.28, are replaced by the $F$-term of the following super-
Table 1.3: Particle content of the MSSM. A Superpartner has the same charges as its SM counterpart. The charges of the SM particles are given in Tab. 1.1. Right-handed neutrinos, $\bar{\nu}$, are SM singlets with $(1, 1, 0)$ charge under $SU(3)_C \times SU(2)_L \times U(1)_Y$.

The MSSM is also defined with a discrete $R$-parity symmetry defined by

$$P_R = (-1)^{3(B-L)+2s}, \quad (1.75)$$

The last term of this superpotential is $\mu H_u H_d$, where $\mu$ is a dimensionalful variable. Hence, there is no a priori reason for $\mu$ to be of the order of weak or SUSY scale, which is required for the Higgs VEV to be of the order of weak scale.\footnote{The SUSY scale is the geometric mean of stop masses.} This is the $\mu$ problem, which can be avoided by assuming that it does not exist at tree level and only arises from the VEV of some new fields.

The MSSM is also defined with a discrete $R$-parity symmetry defined by\footnote{$R$-parity is a phenomenological constraint that we impose on MSSM. The MSSM is not inconsistent even if $R$-parity is violated.}
where $B$ is the Baryon number, $L$ is the Lepton number, and $s$ is the spin. Under this symmetry, all SM particles have +1 charge while all superpartners have −1 charge. To conserve $R$-parity, every interaction vertex must have an even number of superpartners. Hence, the lightest supersymmetric particle (LSP) is stable and if it is neutral, it is a dark matter candidate.

**SUSY Breaking**

Since we do not observe any superpartner having the same mass as its SM counterpart, SUSY must be a broken symmetry at low energy. Hence, a complete supersymmetric model should include a SUSY breaking sector, which is beyond the scope of this thesis. Instead, the most common way of introducing SUSY breaking is to parametrize the SUSY breaking sector by introducing in the effective MSSM Lagrangian additional terms that explicitly break SUSY. These terms are

$$\mathcal{L}_{\text{soft}} = -\frac{1}{2}(M_3 \tilde{G} \tilde{G} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} + \text{h.c.})$$

$$- ([a_u]_{ij} \tilde{q}_i H_u \tilde{u}_j + [a_d]_{ij} \tilde{q}_i H_d \tilde{d}_j + [a_e]_{ij} \tilde{\ell}_i H_d \tilde{\nu}_j + [a_\nu]_{ij} \tilde{\ell}_i H_u \tilde{\nu}_j + \text{h.c.})$$

$$- [m_{\tilde{q}}^2]_{ij} \tilde{q}_i \tilde{q}_j - [m_{\tilde{u}}^2]_{ij} \tilde{u}_i \tilde{u}_j - [m_{\tilde{d}}^2]_{ij} \tilde{d}_i \tilde{d}_j$$

$$- [m_{\tilde{\ell}}^2]_{ij} \tilde{\ell}_i \tilde{\ell}_j - [m_{\tilde{\nu}}^2]_{ij} \tilde{\nu}_i \tilde{\nu}_j - [m_{\tilde{\nu}}^2]_{ij} \tilde{\nu}_i \tilde{\nu}_j$$

$$- [m_{H_u}^2] H_u H_u - [m_{H_d}^2] H_d H_d - (B \mu H_u H_d + \text{h.c.}) \] .$$

$M_j$ for $j = 1, 2, 3$ are gaugino mass terms. $a_u, a_d, a_e$, and $a_\nu$ are trilinear couplings, which are analogous to Yukawa couplings. $m_{\tilde{q}}, m_{\tilde{u}}, m_{\tilde{d}}, m_{\tilde{\ell}}, m_{\tilde{\nu}},$ and $m_{\tilde{\nu}}$ are scalar mass terms. $m_{H_u}$ and $m_{H_d}$ are Higgs mass terms. These additional arbitrary parameters are called the soft-SUSY breaking terms.

**Boundary Conditions**

The soft SUSY breaking terms introduce a huge number of arbitrary parameters into MSSM.\footnote{Since these parameters are soft-SUSY breaking terms, the arbitrariness of MSSM is due to our ignorance of the SUSY breaking mechanism and not due to SUSY itself.} To obtain a predictive theory, we have to reduce the number of arbitrary param-
eters to below the number of observables. This can be done by imposing renormalization group boundary conditions on these parameters.

Unless specified otherwise, the renormalization group boundary conditions that are assumed in this thesis are those of the so-called constrained MSSM (CMSSM) with non-universal Higgs masses (NUHM2). These boundary conditions are

1. Gaugino masses: Universal gaugino mass,

\[ M_{1/2} = M_1 = M_2 = M_3. \] (1.77)

2. Trilinear couplings: Universal trilinear coupling that aligns with Yukawa couplings,

\[ a_{u,d,e,\nu} = A_0 y_{u,d,e,\nu}, \] (1.78)

where \( A_0 \) is a constant.

3. Scalar masses: Universal scalar mass,

\[ m_{16} = m_{\tilde{q}} = m_{\tilde{u}} = m_{\tilde{d}} = m_{\tilde{\ell}} = m_{\tilde{\nu}}. \] (1.79)

4. Higgs masses\(^\text{11}\) Non-universal Higgs masses and different from scalar masses,

\[ m_{H_u} \neq m_{H_d} \neq m_{16}. \] (1.80)

**Mass Eigenstates**

Since there are two Higgs doublets in the MSSM, there are eight scalar Higgs bosons in which three of them are the would-be Nambu-Goldstone bosons, which will be eaten by \( W^\pm \) and \( Z \) bosons after EWSB. To see the Higgs bosons mass eigenstates explicitly, we can

\(^{11}\)In CMSSM, Higgs masses equal scalar masses.
rewrite Higgs bosons in the gauge eigenstates as

\[
\begin{pmatrix}
H_0^u \\
H_0^d
\end{pmatrix} = \begin{pmatrix}
v_u \\
v_d
\end{pmatrix} + \frac{1}{\sqrt{2}} R_\alpha \begin{pmatrix}
h^0 \\
H^0
\end{pmatrix} + i \frac{1}{\sqrt{2}} R_\beta \begin{pmatrix}
G^0 \\
A^0
\end{pmatrix},
\]

(1.81)

\[
\begin{pmatrix}
H_+^u \\
H_-^d
\end{pmatrix} = R_\beta \begin{pmatrix}
G^+ \\
H^+
\end{pmatrix},
\]

(1.82)

where we have assumed that \( v_u \) and \( v_d \) minimize the tree level Higgs potential and the rotation matrices

\[
R_\alpha = \begin{pmatrix}
\cos \alpha & \sin \alpha \\
-\sin \alpha & \cos \alpha
\end{pmatrix},
\]

(1.83)

\[
R_\beta = \begin{pmatrix}
\cos \beta & \sin \beta \\
-\sin \beta & \cos \beta
\end{pmatrix},
\]

(1.84)

are chosen such that the light Higgs, \( h^0 \), the heavy Higgs, \( H^0 \), the \( CP \)-odd Higgs, \( A^0 \), and the charged Higgs, \( H^\pm \), are mass eigenstates, while the would-be Nambu-Goldstone bosons \( G^0 \) and \( G^\pm \) are massless. In the decoupling limit, \( M_{A^0} \gg M_Z \), where \( \alpha \approx \beta - \pi/2 \), the masses of \( H^0, A^0, \) and \( H^\pm \) are nearly degenerate, and \( h^0 \) is SM-like. We will be focusing on this limit.

Neutralinos in the gauge eigenstates are \( \tilde{B}, \tilde{W}^0, \tilde{H}^0_u, \) and \( \tilde{H}^0_d \). In general, the neutralino mass matrix, \( M_\tilde{\chi} \), in this gauge eigenstate basis is non-diagonal. This mass matrix can be diagonalized with a unitary \( 4 \times 4 \) neutralino mixing matrix, \( N \), to obtain the neutralino mass eigenstates, \( \tilde{\chi}^0_i \) for \( i = 1, \ldots, 4 \).

\[
M_{\tilde{\chi}^0} = N^* M_\tilde{\chi} N^{-1}.
\]

(1.85)

The gauge eigenstates of the positively and negatively charged charginos are \( \psi^+ = (\tilde{W}^+, \tilde{H}^+_u) \) and \( \psi^- = (\tilde{W}^-, \tilde{H}^-_d) \), respectively. Since the chargino mass matrix, \( M_{\tilde{C}} \), is multiplied by oppositely charged charginos on both sides,

\[
\mathcal{L}_{\text{Chargino}} = -\frac{1}{2} \psi^- T M_{\tilde{C}} \psi^+ + \text{h.c.},
\]

(1.86)
\( M_{\tilde{C}} \) has to be diagonalized by two unitary \( 2 \times 2 \) matrices, \( U^* M_{\tilde{C}} V^{-1} \). The chargino mass eigenstates are denoted as \( \tilde{\chi}_i^\pm \) for \( i = 1, 2 \).

The gluinos, the superpartner of the gluon, is a color-octet fermion, which is unable to mix with any superparticles. Hence, the gluino gauge eigenstate is the mass eigenstate.

The up-type squark gauge eigenstates are \( \tilde{u}_L, \tilde{c}_L, \tilde{t}_L, \tilde{u}_R, \tilde{c}_R, \) and \( \tilde{t}_R \). Since these six squarks have the same quantum numbers, they can mix with another. In general, the first two family squarks have similar masses because their Yukawa couplings are very small. Hence, the mixing induced from renormalization group equation (RGE) running is negligible\(^{12}\). Hence, the gauge eigenstates of the first two family squarks are also the mass eigenstates. On the other hand, the first two family squarks can have very different masses from the third family squarks because the Yukawa coupling of the third family is much larger than that of the first two families. This larger Yukawa coupling induces a much larger mixing between \( \tilde{t}_L \) and \( \tilde{t}_R \). Hence, the stop mass matrix is not diagonal and can be diagonalized by a unitary \( 2 \times 2 \) stop mixing matrix to obtain the stop mass eigenstates, \( \tilde{t}_i \) for \( i = 1, 2 \), where \( \tilde{t}_1 < \tilde{t}_2 \). The mass eigenstates of the down-type squark, and charged sleptons are defined in a very similar fashion.

**Is SUSY Still Alive?**

SUSY is a very successful theory from the theoretical standpoint. For example, SUSY ameliorates the hierarchy problem and MSSM with \( R \)-parity provides a natural dark matter candidate with the correct thermal relic abundance. In addition, MSSM enables gauge coupling unification, which will be discussed in the next section. When SUSY is taken to be a local symmetry, it naturally incorporates general relativity leading to supergravity. Furthermore, a realistic string theory must have SUSY. One of the problems with MSSM is that, due to our ignorance of the SUSY breaking sector, MSSM has too many arbitrary parameters. In addition, no supersymmetric particles have been observed yet. It is important to emphasize that even if supersymmetric particles are not observed in the near future, SUSY is not in general ruled out. Instead, this simply implies that SUSY is not a

\(^{12}\)Recall that we have assumed CMSSM with NUHM2 boundary condition.
1.3 Supersymmetric Grand Unified Theory

Another motivation for SUSY is that gauge couplings are unified in the MSSM. The one-loop renormalization group equations (RGEs) for the SU(3)$_C \times$ SU(2)$_L \times$ U(1)$_Y$ gauge group are given by

$$\frac{d}{dt} \alpha_j^{-1} = -\frac{b_j}{2\pi},$$  \hspace{1cm} (1.87)

where $j = 1, 2, 3$, $t = \ln(Q/Q_0)$, $Q$ is the normalization scale, and the coefficients $b_j$ are

$$(b_1, b_2, b_3) = \left(\frac{41}{10}, -\frac{19}{6}, -7\right) \text{ for SM},$$

$$(b_1, b_2, b_3) = \left(\frac{33}{5}, 1, -3\right) \text{ for MSSM}. \hspace{1cm} (1.88)$$

Solving these RGEs shows that gauge couplings are not unified for the SM. However, MSSM happens to have the right particle content that SM gauge couplings are unified at the grand unified theory (GUT) scale, $M_G \approx 10^{16}$ GeV. Fig. 1.2 is a plot of $\alpha_j^{-1}$ for $j = 1, 2, 3$ versus $\log_{10}(Q/\text{GeV})$ for the SM and MSSM [1]. The dashed lines are the evolution of the inverse couplings for the SM while the solid lines are that for the MSSM. The plot is obtained by evolving two-loop RGEs. For the MSSM case, the superpartner masses are taken to be either 750 GeV or 2.5 TeV. In addition, the strong coupling $\alpha_s(M_Z)$ is allowed to vary between 0.117 and 0.120. This figure shows that the three couplings are unified at $M_G \approx 10^{16}$ GeV.

Hence, MSSM suggests that the three SM gauge groups are unified into a single gauge group above the GUT scale. To be more precise, the GUT scale is defined at the scale where $\alpha_G = \alpha_1(M_G) = \alpha_2(M_G)$. As we will show in this thesis, to fit low-energy observables, $\alpha_3(M_G) \neq \alpha_G$. Instead,

$$\epsilon_3 = \frac{\alpha_3(M_G) - \alpha_G}{\alpha_G} \approx -4\%.$$ \hspace{1cm} (1.89)

Hence, in a GUT theory, the three arbitrary SM parameters in the gauge sector are replaced by $\alpha_G$, $M_G$, and $\epsilon_3$. In the following subsections, we will discuss SU(5), SO(10), and PS.
Figure 1.2: Plot of inverse gauge couplings, $\alpha_j^{-1}$, versus $\log_{10}(Q/\text{GeV})$. The dashed lines are the inverse couplings for the SM while the solid lines are those for the MSSM with the superpartner mass taken to be 750 GeV (blue lines) or 2.5 TeV (red lines). This plot is obtained from Ref. \[1\].
SUSY GUTs. For a comprehensive review of SUSY GUTs, please refer to *Supersymmetric Grand Unified Theories: From Quarks to Strings via SUSY GUTs* by Stuart Raby \[31\].

### 1.3.1 SU(5)

The simplest GUT group is SU(5) first proposed by Georgi and Glashow \[32\]. The SM gauge groups can be unified under SU(5) because SU(3) × SU(2) × U(1) is a subgroup of SU(5). This can be easily seen from SU(5) generators. The first eight SU(5) generators can be identified with SU(3) generators,

\[
T_A = \begin{pmatrix} \frac{1}{2} \lambda_A & 0 \\ 0 & 0 \end{pmatrix}, \quad A = 1, \ldots, 8.
\]  

(1.90)

The 21\(^{\text{th}}\), 22\(^{\text{th}}\), and 23\(^{\text{th}}\) generators of SU(5) can be identify with SU(2) generators,

\[
T_A = \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{2} \tau_{A-20} \end{pmatrix}, \quad A = 21, 22, 23.
\]  

(1.91)

The 24\(^{\text{th}}\) generator of SU(5), which commutes with the generators of SU(3) and SU(2), can be identified with hypercharge,

\[
T_{24} = \sqrt{\frac{3}{5}} \begin{pmatrix} -1/3 & 0 & 0 & 0 \\ 0 & -1/3 & 0 & 0 \\ 0 & 0 & -1/3 & 0 \\ 0 & 0 & 0 & 1/2 \end{pmatrix}.
\]  

(1.92)

The remaining 12 generators of SU(5) are given by

\[
\frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \frac{1}{2} \begin{pmatrix} -i & 0 \\ 0 & 0 \end{pmatrix}, \quad \frac{1}{2} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \ldots, \quad \frac{1}{2} \begin{pmatrix} 0 & 0 \\ 0 & -i \end{pmatrix}.
\]  

(1.93)
The normalization in the generators are chosen so that

\[ \text{tr}(T_AT_B) = \frac{1}{2} \delta_{AB} . \]  

(1.94)

In SU(5), \( \bar{d} \) and \( \ell \) can be unified in a 5 representation,

\[
\bar{5} = \begin{pmatrix}
\bar{d}_1 \\
\bar{d}_2 \\
\bar{d}_3 \\
-e \\
\nu
\end{pmatrix} .
\]  

(1.95)

In addition, \( q, \bar{u} \), and \( \bar{e} \) can be unified in a 10 representation,

\[
10 = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 & \bar{u}_3 & -\bar{u}_2 & u^1 & d^1 \\
-\bar{u}_3 & 0 & \bar{u}_1 & u^2 & d^2 \\
\bar{u}_2 & -\bar{u}_1 & 0 & u^3 & d^3 \\
-u^2 & -u^2 & -u^3 & 0 & \bar{e} \\
-d^2 & -d^2 & -d^3 & -\bar{e} & 0
\end{pmatrix} ,
\]  

(1.96)

while right-handed neutrinos are singlets under SU(5).

The Higgs doublets are contained in 5 and \( \bar{5} \) representations, respectively,

\[
5_H = \begin{pmatrix}
T \\
H_u
\end{pmatrix} ,
\]

(1.97)

\[
\bar{5}_H = \begin{pmatrix}
\bar{T} \\
H'_d
\end{pmatrix} ,
\]

(1.98)

where \( H'_d = (-i\tau_2)H_d \), and \( T \) and \( \bar{T} \) are color-triplet Higgs bosons, which are needed to fill a complete five-dimensional representation. A complete SU(5) theory needs to have a mechanism to give these color-triplet Higgs mass larger than \( 10^{10} \text{GeV} \) to avoid proton decay \[33, 34\].

27
In SU(5), the kinetic terms of chiral superfields become (compare with Eq. 1.72)

\[ L_{\text{Kinetic}} = \left[ 10^i e^{-2g_5 T^A} V^A \right]_D + \left[ \bar{5}^i e^{-2g_5 T^A} V^A \bar{5}^i \right]_D + \left[ 
\bar{\bar{5}}^i e^{-2g_5 T^A} V^A \bar{\bar{5}}^i \right]_D + \left[ \bar{\bar{\bar{\bar{5}}}^i e^{-2g_5 T^A} V^A \bar{\bar{\bar{\bar{5}}}^i} \right]_D \]  

(1.99)

where \( V^A \) for \( A = 1, \ldots, 24 \) are vector superfields, and the gauge coupling at the GUT scale is

\[ g_5 = g_1 = g_2 = g_3. \]  

(1.100)

The kinetic terms in the gauge sector become (compare with Eq. 1.73)

\[ L_{\text{Gauge}} = \frac{1}{4} |W_G^A W_G^A|_F + \text{h.c.}, \]  

(1.101)

where \( W_G^A \) is the field strength superfield. On the other hand, the superpotential become (compare with Eq. 1.74)

\[ W_{\text{SU(5)}} \supset \left[ y_u \right]_{ij} 10_i 10_j 5_H + \left[ y_d \right]_{ij} 10_i \bar{5} j 5_H \]  

(1.102)

\[ + \left[ y_{\nu} \right]_{ij} \bar{\bar{5}}_i \bar{\bar{\bar{5}}} j 5_H - \frac{1}{2} [M_{R}]_{ij} \bar{N}_i \bar{N}_j, \]

In terms of MSSM superfields, the superpotential is

\[ W_{\text{SU(5)}} \supset [y_u]_{ij} Q_i U_j \mathcal{H}_u + [y_d]_{ij} (Q_i \bar{D}_j + \bar{E}_i L'_j) \mathcal{H}'_d \]  

(1.103)

\[ + [y_{\nu}]_{ij} L_i \bar{N}_j \mathcal{H}_u - \frac{1}{2} [M_{R}]_{ij} \bar{\bar{N}}_i \bar{\bar{N}}_j, \]

where \( L' = (-i\tau_2)L \). This superpotential shows that SU(5) unifies the Yukawa couplings of the down-type quark and charged leptons within a family. However, fitting to low-energy data shows that the first two family Yukawa couplings cannot be unified. Assuming Yukawa unification for the first two families gives

\[ \frac{\lambda_s}{\lambda_d} = \frac{\lambda_e}{\lambda_d}, \]

where \( \lambda \) denotes entries of Yukawa matrices. At one loop level, this ratio is approximately
invariant under RGE running, hence at low energy, this relation predicts

\[
\frac{m_s}{m_d} = \frac{m_\mu}{m_e},
\]

which is wrong. Hence, Yukawa unification can only occur for the third family. To solve this problem while still having $b - \tau$ Yukawa unification, Georgi and Jarlskog showed that if the down-quark and charged-lepton Yukawa matrices are

\[
Y^d = \lambda \begin{pmatrix}
0 & B & 0 \\
B & A & 0 \\
0 & 0 & 1
\end{pmatrix}, \\
Y^e = \lambda \begin{pmatrix}
0 & B & 0 \\
B & -3A & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

and $B \ll A$, then the eigenvalues of these matrices satisfy

\[
\lambda_b = \lambda_\tau, \\
\lambda_s = \frac{1}{3}\lambda_\mu, \\
\lambda_d = 3\lambda_e,
\]

and give a low energy first two families fermion mass relation that fits the observed values.

1.3.2 SO(10)

Another gauge group that can unify the SM gauge groups is SO(10) (see Ref. 36 for a comprehensive review of SO(10)). In SO(10), quarks and leptons (including the right-handed neutrino) of a single family are contained in a $16$ spinor representation. Hence, SO(10) naturally accommodates right-handed neutrinos. In terms of SU(5) representations, the spinor representation can be written as

\[
16 = 10 + 5 + 1.
\]
On the other hand, the two Higgs doublets are contained in a single ten-dimensional representation, where

\[ 10 = 5_H + \bar{5}_H. \]  

(1.110)

Hence, the superpotential becomes (compare with Eq. 1.74 and Eq. 1.103)

\[ W_{SO(10)} \supset \lambda 16_3 1016_3. \]  

(1.111)

Similar to SU(5), Yukawa unification can only occur for the third family. Hence, in SO(10) we have

\[ \lambda = \lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau}. \]  

(1.112)

One way to obtain the first two family fermion masses that agree with experimental values is to forbid the above tree level Yukawa term for the first two families using family symmetry. In such theories, first two family fermions couple to some heavy states, which are then integrated out to give effective Yukawa terms suppressed by the masses of the heavy states. Examples of such theories are studied in detail in Chapter 3 and Chapter 4.

A consequence of the \( t - b - \tau - \nu_\tau \) Yukawa unification is

\[ \frac{\lambda_t}{\lambda_b} \approx \frac{m_t}{m_b} \frac{v_d}{v_u} = \frac{1}{\tan \beta} \frac{m_t}{m_b}, \]  

(1.113)

where we have used the fact that this ratio is invariant under RGE running. This relation implies that \( \tan \beta \approx 50 \). Hence, Yukawa unification imposes an additional large \( \tan \beta \) constraint. With large \( \tan \beta \), there are significant threshold corrections to the bottom quark mass, charged lepton masses, and CKM matrix elements from supersymmetric loop calculations [37–39]. In addition, the branching ratios \( \text{BR}(b \to s\gamma) \) and \( \text{BR}(B_s \to \mu^+\mu^-) \) are also enhanced in the large \( \tan \beta \) limit [40–42].

SO(10) can be broken down to the SM through SU(5) or PS gauge group SU(4)_C \times
SU(2)\textsubscript{L} × SU(2)\textsubscript{R},

\[ \text{SO}(10) \rightarrow \text{SU}(5) \times U(1), \]  
\[ \rightarrow \text{SU}(4) \times SU(2) \times SU(2). \]  

The mechanism of GUT breaking is beyond the scope of this thesis.

### 1.3.3 Pati-Salam Group

In a PS gauge group, the lepton number is considered as the fourth color, hence the SU(4)\textsubscript{C} gauge group.

In general, models with PS gauge group are not GUTs because the PS group has three different gauge groups. However, Kobayashi et al. showed that a four-dimensional PS model can be obtained by orbifolding a higher dimensional GUT \cite{Kobayashi:1973ia, Kim:1979if}. Hence, in this thesis, we will consider models with PS gauge group as GUTs and assumed that they can be obtained from higher dimensional GUTs.

In PS, fermions are contained in \( Q \) and \( \bar{Q} \), which transform as \( (4, 2, 1) \) and \( (\bar{4}, 1, \bar{2}) \), respectively, under the PS group. In terms of SM fields,

\[ Q^{\alpha_i} = \left( q \ell \right)^{\alpha_i}, \]  
\[ \bar{Q}_{\beta j} = \left( \bar{u} \bar{\nu} \bar{d} \bar{e} \right)_{\beta j}, \]  

where upper \( \alpha = 1, \ldots, 4 \) are indices of 4 under SU(4)\textsubscript{C}, lower \( \beta = 1, \ldots, 4 \) are indices of \( \bar{4} \) under SU(4)\textsubscript{C}, upper \( i = 1, 2 \) are indices of \( 2 \) under SU(2)\textsubscript{L}, and lower \( j = 1, 2 \) are indices of \( \bar{2} \) under SU(2)\textsubscript{R}.

On the other hand, the two Higgs doublets are contained in \( H \), which transforms as \( (1, 2, \bar{2}) \) under the PS group,

\[ H^i_j = \left( H_u \quad H_d \right)^i_j. \]
\[ B - L \text{ acting on a 4 of the PS group is} \]

\[
B - L = \begin{pmatrix}
\frac{1}{3} & 0 & 0 & 0 \\
0 & \frac{1}{3} & 0 & 0 \\
0 & 0 & \frac{1}{3} & 0 \\
0 & 0 & 0 & -1
\end{pmatrix},
\]  \hspace{1cm} (1.119)

and the electric charge is

\[ Q = T_{3L} + T_{3R} + \frac{1}{2}(B - L), \]  \hspace{1cm} (1.120)

where \( T_{3L} \) is the third generator of SU(2)_L and similarly for \( T_{3R} \).

The Yukawa term in a PS model can be written as

\[ W_{PS} \supset \lambda \bar{Q}_3 H Q_3. \]  \hspace{1cm} (1.121)

Hence, PS naturally unifies Yukawa couplings. Similarly, to SU(5) and SO(10), the unification can only occur for the third family. In fact, a SUSY GUT with a PS gauge group, which will be discussed in Chapter 4, can be very similar to that with a SO(10) gauge group.

In summary, SUSY GUT is a class of models that are very well motivated theoretically. However, it is important to note that SUSY GUTs have some challenges also. For example, SUSY GUTs predict proton decay. The proton lifetime prediction from SUSY GUTs is the same order of magnitude as the current bounds. Hence, if four space-time dimension SUSY GUTs is correct, then it is crucial to observe proton decay in the near future. However, this bound can be avoided by higher dimension SUSY GUTs.

### 1.4 Baryogenesis via Leptogenesis

The observed baryon asymmetry cannot be due to initial conditions of the Universe because any primordial baryon asymmetry would have been diluted away during inflation. Hence, baryon asymmetry has to be generated dynamically after inflation. In this section, we will discuss why the SM is unable to explain baryogenesis, the dynamic generation of baryon asymmetry. Then we will discuss baryogenesis via leptogenesis in a SUSY GUT. For a
comprehensive review of leptogenesis, see Ref. [45–47].

In 1967, Sakharov showed that baryogenesis can occur only if the following three conditions are met [48].

1. **Baryon number violation**: The baryon number violation is necessary because without baryon number violating processes, baryon asymmetry cannot be generated from a baryon symmetric initial state.

2. **C and CP violation**: $C$ and $CP$ have to be violated so that processes involving baryons occur at a different rate from $C$ and $CP$ conjugate processes involving antibaryons.

3. **Out-of-equilibrium dynamics**: The above processes have to occur out of equilibrium otherwise the produced asymmetry would be driven to zero due to $CPT$ symmetry.

To understand why the SM is insufficient to explain baryogenesis, consider the SM contributions to these three conditions.

1. In the SM, baryon number is conserved at tree level but the chiral anomaly creates a $B + L$ violating current. This will be discussed in detail later.

2. In the SM, $C$ is violated in weak interactions while the only source of $CP$ asymmetry is the $CP$ violating phase in the CKM matrix. This $CP$ violation can be parametrized by the Jarlskog invariant, which is of the order of $10^{-20}$ [49]. This $CP$ violation is too small to generate the observed baryon asymmetry [50–52]. Hence, beyond the SM physics is needed to provide a larger source of $CP$ asymmetry.

3. In the SM, a departure from thermal equilibrium occurs at the electroweak phase transition [53 54]. However, this transition is not a strong first order transition, which is required by baryogenesis. Hence, a different out-of-equilibrium dynamic is required for baryogenesis.
There are many potential NP solutions to baryogenesis. In this thesis, we will discuss a specific kind called leptogenesis, which was first discovered by Fukugita and Yanagita [55]. Leptogenesis occurs through heavy right-handed neutrinos, which were discussed in Sec. 1.1.2. As will be discussed below, right-handed neutrino Yukawa couplings provide a new source of $CP$ asymmetry. In addition, since right-handed neutrinos are SM singlets and their Yukawa couplings can be small, a departure from thermal equilibrium can be achieved if their interaction rates are smaller than the expansion rate of the Universe. Since right-handed neutrinos have Majorana mass and lepton number cannot be consistently assigned for a Majorana particle, lepton number is naturally violated. Hence, lepton asymmetry is generated when right-handed neutrinos decay. The SM sphaleron process, which will be discussed later, then converts the lepton asymmetry to baryon asymmetry [56].

Leptogenesis is thermal if the right-handed neutrinos were in thermal equilibrium at some point in the history of the Universe. Non-thermal leptogenesis, on the other hand, occurs when right-handed neutrinos were never in thermal equilibrium. The main difference between thermal and non-thermal leptogenesis is that in thermal leptogenesis, the right-handed neutrino abundances are fixed by the thermal abundance.

**CP Violation**

Since a Majorana particle is its own anti-particle, right-handed neutrinos can decay to $\ell h_u$ and $\ell^\dagger h_u^\dagger$. Hence, a net lepton asymmetry is generated if the decay rates to these two channels are different. The $CP$ asymmetry parameter of right-handed neutrino decays is defined as

$$\epsilon_{\varphi_i} = \frac{\Gamma_{\varphi_i^\dagger \rightarrow \ell h_u} - \Gamma_{\varphi_i \rightarrow \ell^\dagger h_u^\dagger}}{\Gamma_{\varphi_i^\dagger \rightarrow \ell h_u} + \Gamma_{\varphi_i \rightarrow \ell^\dagger h_u^\dagger}},$$

where the family indices of the final state leptons are summed.

Diagrams contributing to the $CP$ asymmetry are shown in Fig. 1.3. The $CP$ asymmetry is due to the interference between tree level and one loop diagrams and is given by [45, 47, 57–].
Figure 1.3: Diagrams contributing to the CP asymmetry in the decay of right-handed neutrinos to lepton and Higgs boson. There are also diagrams where right-handed neutrinos decay to slepton and Higgsino. In addition, the decay of right-handed sneutrinos also contribute to the CP asymmetry.
\[ \epsilon_{\bar{\nu}_i} = \frac{1}{8\pi} \sum_{k \neq i} \text{Im}\{(y^\nu\dagger y^\nu\dagger)_{ki}\} g \left( \frac{M^2_{R_k}}{M^2_{R_i}} \right), \tag{1.123} \]

where \( y^\nu \) is defined in Eq. [1.52] and

\[ g(x) = -\sqrt{x} \left( \frac{2}{x-1} + \ln \frac{1+x}{x} \right). \tag{1.124} \]

The sign of this result agrees with the sign in Ref. [45, 57, 59], but disagrees with the sign in Ref. [47]. To ease the comparison between papers, a dictionary relating different definitions of Yukawa matrices is given in App. [F].

When \( M_{R_i} \ll M_{R_k} \), the \( CP \) asymmetry is

\[ \epsilon_{\bar{\nu}_i} \approx -\frac{3}{8\pi} \text{Im}\{(y^\nu\dagger y^\nu\dagger)_{ki}\} \frac{M_{R_i}}{M_{R_k}}. \tag{1.125} \]

This \( CP \) asymmetry is calculated when \( \bar{\nu}_k \) are in the loop. However, when \( \bar{\nu}_k \) decays, \( \bar{\nu}_k \) should already be integrated out of the theory to give the Weinberg operator,

\[ \eta_{ij} = [y^\nu]_{ik} \frac{1}{M_k} [y^{\nu^T}]_{kj}. \tag{1.126} \]

Hence, instead of the \( CP \) asymmetry above, the \( CP \) asymmetry should be

\[ \epsilon_{\bar{\nu}_i} \approx \frac{3}{8\pi} \text{Im}\{[y^\nu\dagger \eta y^{\nu^*}]_{ii}\} M_i. \tag{1.127} \]

**Thermal Leptogenesis**

To understand the basic idea of thermal leptogenesis, let’s ignore the two heavier right-handed neutrinos and consider only the lightest right-handed neutrino, \( \bar{\nu}_1 \), which has mass of \( M_1 \). We will explain later why this is a reasonable assumption. In general, thermal leptogenesis assumes that \( \bar{\nu}_1 \) is produced from the thermal bath when the temperature of the Universe is above \( M_1 \). Hence, the number density of \( \bar{\nu}_1 \) is given by the thermal

\[ ^{13} \epsilon_{\bar{\nu}_i} \text{ in Ref. [60] has a factor of } 16\pi \text{ instead of } 8\pi \text{ because the asymmetry in Ref. [60] is calculated for the SM.} \]
equilibrium number density,
\[ n_{\bar{\nu}_1}^{\text{eq}} = \frac{g}{2\pi^3} \int_{M_1}^\infty dE \sqrt{E^2 - M_1^2} e^{-E/T} = \frac{g}{2\pi^3} T^3 \frac{M_1^2}{T^2} K_2 \left( \frac{M_1}{T} \right), \tag{1.128} \]
where \( K_2 \) is the modified Bessel function of the second kind and \( g = 2 \) is the number of internal degrees of freedom of right-handed Majorana neutrinos. When the Universe temperature drops below \( M_1 \), the \( \bar{\nu}_1 \) number density becomes exponentially suppressed,
\[ n_{\bar{\nu}_1} \propto e^{-M_1/T}. \tag{1.129} \]

If \( \bar{\nu}_1 \) decays after it is out of thermal equilibrium, then the decay process satisfies all Sakharov conditions and a net lepton asymmetry will be produced.

The two heavier right-handed neutrinos are often neglected in thermal leptogenesis, because Buchmüller et al. showed that the inverse decay, the \( \Delta L = 1 \) scatterings, and the \( \Delta L = 2 \) processes of the lightest right-handed neutrino are able to quickly washout asymmetry up to \( 10^{10} \) orders of magnitude \cite{61}. These washout processes occur when \( \bar{\nu}_1 \) is in thermal equilibrium. Hence, the asymmetry produced before the temperature of the Universe drops below \( M_1 \) is washed out by thermal processes involving \( \bar{\nu}_1 \) and the final lepton asymmetry is only contributed by the out-of-equilibrium decay of \( \bar{\nu}_1 \). This is also why although the \( CP \) asymmetries in the production and decay of \( \bar{\nu}_1 \) are the same, a net asymmetry is produced when \( \bar{\nu}_1 \) decay out of equilibrium.

**Non-thermal Leptogenesis**

In non-thermal leptogenesis, the right-handed neutrinos are not created from the thermal bath. Instead, other production mechanisms are needed. In Chapter 5, we will describe a scenario where the Higgs boson has mass dependent on the VEV of the inflaton. Hence, as the inflaton oscillates after inflation ends, the Higgs boson can be heavier than the right-handed neutrinos. Thus, right-handed neutrinos are created non-thermally via the decay of the Higgs boson.

Since in non-thermal leptogenesis, right-handed neutrinos are never in thermal equilibrium, the washout effect for thermal leptogenesis is absent. The only washout effect
remaining is if the $CP$ asymmetries from the decays of the three right-handed neutrinos have different signs.

**Sphaleron**

In this subsection, we will briefly review the sphaleron that leads to $B + L$ violation in the SM. See Ref. [45–47, 62] for a more thorough review.

The SM $B$ and $L$ currents are

\[
J_B^\mu = \frac{1}{3} \sum_i q_i^* \gamma_\mu q_i - \bar{u}_i^* \gamma_\mu \bar{u}_i - \bar{d}_i^* \gamma_\mu \bar{d}_i , \tag{1.130}
\]

\[
J_L^\mu = \sum_i \ell_i^* \gamma_\mu \ell_i - \bar{e}_i^* \gamma_\mu \bar{e}_i . \tag{1.131}
\]

Classically, $B$ and $L$ are conserved,

\[
B = \int d^3 x J^B_0 (x) , \tag{1.132}
\]

\[
L = \int d^3 x J^L_0 (x) . \tag{1.133}
\]

However, due to the Adler-Bell-Jackiw triangle anomalies, $B$ and $L$ are not conserved beyond tree level [63, 64]. Instead,

\[
\partial^\mu J_B^\mu = \partial^\mu J_L^\mu = N_f \frac{32 \pi^2}{3} \epsilon_{\mu \nu \rho \sigma} (g^2 W^a_\mu W^{a \rho \sigma} - g'^2 B_\mu B^{\rho \sigma}) , \tag{1.134}
\]

where $N_f = 3$ is the number of families. Hence, in the SM, $B - L$ is conserved while $B + L$ is not [65, 66]. The changes in $B$ and $L$ are due to the changes in the topological charge of the gauge field,

\[
B(t_f) - B(t_i) = \int_{t_i}^{t_f} dt \int d^3 x \partial^\mu J_B^\mu
= N_f [N_{cs}(t_f) - N_{cs}(t_i)] , \tag{1.135}
\]

where $N_{cs}$ is the topological charge of the gauge field or the Chern-Simons number. For vacuum to vacuum transitions, $N_{cs}$ is an integer. Therefore, in a non-abelian gauge theory, there are infinitely many degenerate vacuum states. The vacuum states differ from one another by $\Delta N_{cs} = \pm 1, \pm 2, \ldots$, and are separated by a potential barrier with height $E_{sph}$. 

38
Since changes in vacuum with different $N_{cs}$ correspond to changes in $B$ and $L$, the smallest changes of $B$ and $L$ in the SM are

$$\Delta B = \Delta L = \pm 3.$$  \hfill (1.136)

In a semi-classical approximation, the tunneling between neighboring gauge vacua is given by the instanton configuration. Hence, in the SM, SU(2)$_L$ instanton leads to an effective 12-fermion interaction operator

$$O_{B+L} = \prod_{i=1,2,3} q_i q_i \ell_i.$$  \hfill (1.137)

At zero-temperature, the instanton transition rate is \[67, 68\]

$$\Gamma \approx e^{-S_{\text{int}}} = e^{2\pi/\alpha} = \mathcal{O}(e^{-165}).$$  \hfill (1.138)

Hence, at zero-temperature, $B + L$ violation is negligible. However, in a thermal bath, Kuzmin, Rubakov, and Shaposhnikov showed that thermal fluctuations enable transition from one gauge vacuum to another to occur over the potential barrier \[69\]. Such transition is given by the sphaleron configuration, which corresponds to the unstable solution at the top of the potential \[70\]. For temperatures approximately between the electroweak phase transition temperature and $10^{12}$ GeV, the sphaleron transition rate is not suppressed \[71–76\]. In equilibrium, the excess in $B - L$ implies a baryon excess of \[56\]

$$n_B \approx \begin{cases} -\frac{28}{79} n_L \text{ (in SM)}, \\ -\frac{8}{23} n_L \text{ (in MSSM)}, \end{cases}$$  \hfill (1.139)

where $n_L$ is the lepton number density minus anti-lepton number density and we have assumed that the sphaleron is out of equilibrium above the electroweak phase transition \[77\].

In summary, leptogenesis is a very interesting avenue to achieve baryogenesis because leptogenesis is a byproduct that one necessarily gets if light neutrino masses arise from heavy right-handed neutrinos via the seesaw mechanism.
1.5 Conclusion

In this chapter, we presented a brief overview of the SM of particle physics. Although the SM is very successful in many aspects, the needs for physics beyond the SM are present. Some hints for physics beyond the SM are 1) the arbitrariness of SM parameters, 2) the lightness of neutrinos masses, 3) the muon anomalous magnetic moment discrepancy, 4) the hierarchy problem, 5) the matter-antimatter asymmetry, and 6) the dark matter. In this thesis, models beyond the SM that can address these problems and be tested in the near future are studied.

In this chapter, we discussed a model that naturally gives light neutrino masses by enlarging the SM with three families of right-handed neutrinos. The light neutrino masses are obtained via the seesaw mechanism. We, then discuss SUSY, a framework for extending the SM that ameliorates the hierarchy problem. MSSM, the simplest supersymmetric extension of the SM is then discussed. In the MSSM, the lightest supersymmetric particle is a dark matter candidate. A consequence of the MSSM is gauge coupling unification at the GUT scale. Hence, in the section following the section on MSSM, we discussed SU(5), SO(10), and PS SUSY GUTs. In addition to gauge coupling unification, GUTs justify the boundary conditions of the MSSM soft SUSY breaking parameters. SO(10) and PS SUSY GUTs also naturally contain right-handed neutrinos and allow $t - b - \tau - \nu_\tau$ Yukawa unification. Finally, we discuss baryogenesis via leptogenesis, which is a natural byproduct of right-handed neutrinos with Majorana masses.

This thesis is organized as follows: In Chapter 2 we introduce a very simple extension of the SM in which the SM is extended with a family of vectorlike (VL) leptons that couple to all three families of the SM leptons. This model can explain the muon anomalous magnetic moment discrepancy while being consistent with current experimental bounds on heavy-charged leptons, precision electroweak measurements, Higgs boson decay measurements, and current lepton flavor violation bounds. In Chapter 3 we discuss a realistic SUSY GUT model with SO(10) gauge symmetry. In order to match the measured hierarchical fermion masses, we include a $D_3 \times [U(1) \times Z_3 \times Z_3]$ family symmetry. To determined the goodness
of low-energy observables fit of this model, we performed a global $\chi^2$ analysis. In addition, we also found that by fixing specific ratios of some GUT scale boundary conditions, the fine-tuning of this model can be reduced to one part in 500. This suggests that these ratios should exist naturally in a more fundamental theory, such as a string theory. A problem of this model is that the global $\chi^2$ fit for $\sin(2\beta)$, $m_u$, and $m_d$ are not very good. Hence, in Chapter 4 we showed that by changing the gauge group to PS group and modifying the Yukawa sector of the theory, the $\chi^2$ value is improved significantly. We also reinterpret ATLAS and CMS analyses to obtain the current gluino mass bound of this model. In Chapter 5 we performed detail reheating and leptogenesis analyses of a PS SUSY GUT inflation model proposed by Bryant and Raby [78]. This model reheats via instant preheating, a process where the inflaton decays quickly and non-perturbatively. As for leptogenesis, we found that by fitting the low-energy observables, the heaviest right-handed neutrino decays to produce asymmetry with the correct sign while the two lighter right-handed neutrinos decay to produce asymmetry of the wrong sign. Hence, to obtain the observed baryon asymmetry, the baryogenesis mechanism needs to be non-thermal leptogenesis.

The main goal of this thesis is to construct particle physics models beyond the SM in an attempt to further our understanding of the fundamental building blocks of the Universe. One approach of model building is to make minimal extension to the SM in an attempt to solve SM challenges one by one. An example of this approach is the VL leptons model studied in Chapter 2. Another diametrically different approach is to make huge and theoretically well motivated changes to the SM in an attempt to solve as many SM challenges as possible. Examples of this approach are SUSY GUTs studied in Chapter 3, 4, and 5. In addition to solving SM challenges, these models have predictions that can be tested and constrained by current experiments.
Chapter 2

Vector-like Leptons

As mentioned in the preceding section, one discrepancy between a SM prediction and experimental measurement that has been known for a long time is the muon anomalous magnetic moment. In this chapter, we study a simple extension of the SM that is able to explain this discrepancy. This model is the SM with one family of vector-like (VL) leptons. Dermišek et al. showed that such a model with VL leptons coupling exclusively to the muon is sufficient to explain this discrepancy [79]. In a more natural theory, however, the VL leptons would couple to all three families of the SM leptons, and that has been studied extensively in the literature [80–83]. Due to the lepton flavor violating nature of this model, the SM-VL couplings are known to be highly constrained.

In this chapter, we try to provide a holistic point of view of the model in which the SM is extended by one family of VL leptons and the VL leptons have nonzero couplings to all three families of the SM leptons. We are interested in the constraints on this model coming from satisfying the heavy-charged lepton mass bound, electroweak precision data, the muon $g - 2$, lepton flavor violation (LFV), Higgs boson decay constraints, and lepton nonuniversality observables, $R_K$ and $R_{K^0}$. We find that this model cannot simultaneously satisfy electroweak precision measurements and lepton nonuniversality discrepancies. As for
the other observables, we find that the more constraining observables are the muon $g - 2$, $R_{\mu\mu} = \Gamma(h \rightarrow \mu\mu)/\Gamma(h \rightarrow \mu\mu)_{\text{SM}}$, $R_{\gamma\gamma}$, and $\text{BR}(\mu \rightarrow e\gamma)$.

### 2.1 Model

The model that we study is the SM with one family of VL leptons. The particles in the leptonic sector and their corresponding quantum numbers are given in Table [2.1](#) and the leptonic sector Lagrangian is given by

\[
\mathcal{L} \supset -\bar{\ell}_L y_{ei}^e e_{Ri} H - \bar{\ell}_L \lambda_i^E E_R H - \bar{L}_L \lambda_i^L e_{Ri} H - \bar{E}_L \lambda E_R H^\dagger \\
- M_L \bar{L}_L R - M_E \bar{E}_L E_R + \text{h.c.},
\]

(2.1)

where $i = 1, 2, 3$ is the SM family index.

#### Lepton Mass Matrix

Without loss of generality, we assume that the SM lepton Yukawa matrix, $y^e$, is already diagonalized. Thus, the lepton mass matrix is

\[
\begin{pmatrix}
\bar{e}_{Li} & \bar{L}_L & \bar{E}_L
\end{pmatrix}
\begin{pmatrix}
y_{ei}^e & \lambda_i^E & 0 \\
\lambda_i^L & M_L & \lambda v \\
0 & \lambda v & M_E
\end{pmatrix}
\begin{pmatrix}
e_{Ri}
\end{pmatrix}
\equiv \bar{e}_L a M e_{Ra},
\]

(2.2)

where $a = 1, \ldots, 5$. Let $U_L$ and $U_R$ be unitary matrices that diagonalize the charged lepton mass matrix,

\[
U_L^\dagger M U_R = \begin{pmatrix}
M_{e_i} & 0 & 0 \\
0 & M_{e4} & 0 \\
0 & 0 & M_{e5}
\end{pmatrix} \equiv M^\text{diag},
\]

(2.3)

and the mass basis is

\[
[\bar{e}_{L,R}]_a = [U_{L,R}^\dagger]_{a,a'} [e_{L,R}]_{a'}. 
\]

(2.4)

---

**Notice that even though the notation for the VL leptons in this chapter is very similar to the notation for the superfield in the introduction chapter, they are different. In particular, we use $L_{L,R}$ for VL leptons while $L$ for lepton doublet superfield in Chapter [I].**
\[
\ell_{Li} = \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix}, \quad e_{Ri}, \quad H = \begin{pmatrix} \phi^+ \\ v + \frac{h + i\phi^0}{\sqrt{2}} \end{pmatrix}, \quad L_{L,R} = \begin{pmatrix} I^0_{L,R} \\ I^{-}_{L,R} \end{pmatrix}, \quad E_{L,R}
\]

<table>
<thead>
<tr>
<th></th>
<th>SM</th>
<th>VL</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU(2)_L</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>U(1)_Y</td>
<td>-1</td>
<td>-2</td>
</tr>
</tbody>
</table>

Table 2.1: The quantum numbers of leptonic sector particles relevant to this chapter. \(i = 1, 2, 3\) is the SM family index. The electric charge is given by \(Q = T_3 + Y/2\) and the Higgs vacuum expectation value is 174 GeV. The field \(h\) is the physical Higgs boson and the fields \(\phi^+\) and \(\phi^0\) are the would-be Nambu-Goldstone bosons that give mass to the \(W\) boson and \(Z\) boson, respectively.

In this model, neutrinos are assumed to only obtain a VL mass, \(M_L\).

**Z-lepton Couplings**

The Z-lepton couplings are given by

\[
\mathcal{L} \supset \frac{g}{c_W} Z_\mu \left[ \bar{\ell}_{La} \gamma^\mu (T^3_a + s^2_W) e_{La} + \bar{\ell}_{Ra} \gamma^\mu (T^3_a + s^2_W) e_{Ra} \right], \quad (2.5)
\]

where \(s_W = \sin \theta_W, c_W = \cos \theta_W\) and \(T^3_a\) is the SU(2) generator where

\[
T^3_a e_{La} = -\frac{1}{2} \text{diag}(1, 1, 1, 1, 0) e_{La} \equiv T^3_L e_{La} \\
T^3_a e_{Ra} = -\frac{1}{2} \text{diag}(0, 0, 0, 1, 0) e_{Ra} \equiv T^3_R e_{Ra}. \quad (2.6)
\]

Since these matrices are not proportional to the identity matrix, the Z-lepton couplings are not diagonal, when we rotate to the lepton mass basis,

\[
\mathcal{L} \supset Z_\mu \left[ \bar{\ell}_{La} \gamma^\mu g^Z_{Lab} \ell_{Lb} + \bar{\ell}_{Ra} \gamma^\mu g^Z_{Rab} \ell_{Rb} \right], \quad (2.8)
\]

where \(g^Z_{L,R} = (g/c_W) [U_{L,R} (T^3_{L,R} + s^2_W) U_{L,R}]\). Hence, this model has LFV Z-boson decays.
W-lepton Couplings

The W-lepton couplings are given by

\[ \mathcal{L} \supset \frac{g}{\sqrt{2}} W^+_\mu [\bar{\nu}_{La} \gamma^\mu e_{La} + \bar{\nu}_{Ra} \gamma^\mu e_{Ra}] + h.c., \]  

(2.9)

where

\[
\begin{pmatrix}
\nu_{La} \\
L_L^0 \\
0
\end{pmatrix}
\quad \quad \quad \quad \quad \quad \quad
\begin{pmatrix}
\nu_{Ra} \\
L_R^0 \\
0
\end{pmatrix}.
\]

(2.10)

Hence, in the charged lepton mass basis, we have

\[ \mathcal{L} \supset W^+_\mu [\bar{\nu}_{La} \gamma^\mu g^W g_{La} \hat{e}_{La} + \bar{\nu}_{Ra} \gamma^\mu g^W g_{Ra} \hat{e}_{Ra}] + h.c., \]  

(2.14)

where \( g^W_L = (g/\sqrt{2}) \text{diag}(1, 1, 1, 1, 0) U_L \) and \( g^W_R = (g/\sqrt{2}) \text{diag}(0, 0, 0, 1, 0) U_R \).

Higgs-lepton Couplings

The couplings between the physical Higgs boson and the leptons are

\[ \mathcal{L} \supset -\frac{1}{\sqrt{2}} h \bar{e}_{La} Y^e_{a} e_{Rb} + h.c., \]  

(2.12)

where

\[
Y^e = \begin{pmatrix}
y_{i}^e & 0 & \lambda_i^E \\
\lambda_i^L & 0 & \lambda \\
0 & \bar{\lambda} & 0
\end{pmatrix}.
\]

(2.13)

In the mass basis, we have

\[ \mathcal{L} \supset -\frac{1}{\sqrt{2}} h \bar{e}_{La} \hat{Y}^e_{a} \hat{e}_{Rb} + h.c., \]  

(2.14)

where

\[
\hat{Y}^e = U_L^\dagger Y^e U_R.
\]

(2.15)
This Yukawa matrix is nondiagonal because $Y^e_v = \mathcal{M} - \text{diag}(0, 0, 0, M_L, M_E)$. Hence,

$$\hat{Y}^e = \mathcal{M}^\text{diag}/v - U_L^\dagger \text{diag}(0, 0, 0, M_L, M_E)U_R/v,$$

where the second term is nondiagonal.

**Lepton Non-Universality**

To calculate the effect of this model on lepton nonuniversality, we consider the following Hamiltonian \[84, 85\]

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_{j=9,10} C_j O_j,$$

where

$$O_9 = (\bar{s}_L \gamma^\mu b_L)(\bar{e}_a \gamma_\mu \hat{e}_a),$$

$$O_{10} = (\bar{s}_L \gamma^\mu b_L)(\bar{e}_a \gamma_\mu \gamma_5 \hat{e}_a).$$

The NP contribution to these two Wilson coefficients are coming from the box diagrams in Fig. 2.1 (see App. A for calculation \[86\])

$$C^{\text{NP}}_9 = -\frac{1}{s_W^2} \frac{1}{4} \left[ U^+_1(x,y)g_1(x,y) + U^+_0(x,y)g_0(x,y) \right],$$

$$C^{\text{NP}}_{10} = \frac{1}{s_W^2} \frac{1}{4} \left[ U^-_1(x,y)g_1(x,y) + U^-_0(x,y)g_0(x,y) \right],$$

where $x = M_t^2/M_W^2$, $y = M_L^2/M_W^2$,
Figure 2.1: Box diagrams contributing to $b \rightarrow s \hat{e}_a \hat{e}_a$.

\[ g_1(x, y) = \frac{1}{x - y} \left[ \frac{x^2}{(x - 1)^2} \log x - \frac{y^2}{(y - 1)^2} \log y - \frac{1}{x - 1} + \frac{1}{y - 1} \right], \quad (2.23) \]
\[ g_0(x, y) = \frac{1}{x - y} \left[ \frac{x}{(x - 1)^2} \log x - \frac{y}{(y - 1)^2} \log y - \frac{1}{x - 1} + \frac{1}{y - 1} \right], \quad (2.24) \]

and

\[ U_1^\pm(x, y) = ||U_L||_{4a}^2 \pm ||U_R||_{4a}^2 + \frac{v^2}{4M_L^2}xy \left( ||Y^{\nu R}U_L||_{4a}^2 \pm ||Y^{\nu L}U_R||_{4a}^2 \right), \quad (2.25) \]
\[ U_0^\pm(x, y) = \frac{v}{M_L}xy \left( ||U_L||_{4a}[Y^{\nu R}U_L^*]_{4a} + ||U_R||_{4a}[Y^{\nu L}U_L]_{4a} \right) \pm (2.26) \]

\[ \pm \left[ U_R||_{4a}[Y^{\nu L}U_R^*]_{4a} \pm [U_L]_{4a}[Y^{\nu R}U_R]_{4a} \right]. \]

2.2 Analysis Procedure

The analysis of this chapter is similar to that in Ref. [79]. A new feature of this chapter is that VL leptons are not assumed to couple exclusively to muons. Instead, VL leptons couple to all three families of SM leptons and we are interested in the constraints of the 10 model parameters: VL Masses, $M_{L,E}$; VL-VL couplings, $\lambda, \tilde{\lambda}$; and SM-VL couplings, $\lambda^{L,E}_{e,\mu,\tau}$. The
Yukawa couplings $y_{e,\mu,\tau}$ are not free parameters because $y_{e,\mu,\tau}$ are chosen such that $m_{e,\mu,\tau}$ are the central values in Particle Data Group (PDG) \cite{PDG}. We considered $M_{L,E} \in (100, 1000)$ GeV and $\lambda, \bar{\lambda} \in (-1, 1)$. As for the SM-VL couplings, we considered

$$
\frac{\lambda_{e,\mu,\tau}^{L,E} v}{M_{L,E}} \in (-0.09, 0.09). \tag{2.27}
$$

The ranges of the SM-VL couplings are chosen to satisfy the electroweak constraints.\footnote{With our upper limit on $M_{L,E} = 1000$ GeV, this implies an upper bound on the dimensionless couplings $\lambda_{e,\mu,\tau}^{L,E} \lesssim 0.5$.}

The constraints that we consider in this chapter are from the heavy-charged lepton mass bound, precision electroweak data, the muon $g - 2$, LFV, Higgs boson decays, and lepton nonuniversality observables. See Table 2.2 for the complete list of observables. All of the experimental values, other than lepton nonuniversality observables, are taken from the PDG \cite{PDG}. The experimental value for $R_K$ is taken from Ref. \cite{R_K}, while $R_{K^*0}$ is recently measured by LHCb \cite{R_K*0}.

The heavy-charged lepton mass bound quoted by the PDG, $M > 100.8$ GeV, is from the LEP experiment. There are more recent bounds on the mass of VL leptons obtained from reinterpreting ATLAS and CMS analyses \cite{ATLAS,CMS}. If the lightest VL lepton is predominantly $E_{L,R}$, then the bound is similar to the LEP bound. However, if the lightest VL lepton is predominantly $L_{L,R}$, then the bound can be more stringent. For example, Falkowski et al. showed that if the VL lepton decays only to $e$ and $\mu$, then the bound is $M_{e4} \gtrsim 450$ GeV \cite{ATLAS}. On the other hand, Kumar et al. showed that if the VL lepton decays only to $\tau$, then the bound may only be $M_{e4} \gtrsim 275$ GeV \cite{CMS}. Due to the sampling method that we explain below, VL leptons in this model can either be predominantly $E_{L,R}$ or $L_{L,R}$, depending on model parameters. In addition, VL leptons of this model can decay to all three SM leptons. Hence, reinterpretations of the ATLAS and CMS analyses are needed to obtain the bound on this model. To be conservative, we have decided to use the LEP bound in this chapter while keeping in mind that more stringent bounds may exist.

Theoretical calculations are performed at leading order, that is all observables other than $\Delta a_\mu$, $BR(\ell \to \ell'\gamma)$, $R_{\gamma\gamma}$, $R_K$, $R_{K^*0}$ are calculated at tree level. The effect of one-loop
Muon $g - 2$

<table>
<thead>
<tr>
<th>Heavy Charged Leptons</th>
<th>$\mu$</th>
<th>$\Delta a_\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_4$</td>
<td>$M_{e_4}$</td>
<td></td>
</tr>
</tbody>
</table>

Precision Electroweak

<table>
<thead>
<tr>
<th>$Z$</th>
<th>$A_{e,\mu,\tau}$, $A_{FB}^{(0e),(0\mu),(0\tau)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\text{BR}(Z \rightarrow ee)$, $\text{BR}(Z \rightarrow \mu\mu)$, $\text{BR}(Z \rightarrow \tau\tau)$</td>
</tr>
<tr>
<td>$W$</td>
<td>$\text{BR}(W \rightarrow e\nu_e)$, $\text{BR}(W \rightarrow \mu\nu_\mu)$, $\text{BR}(W \rightarrow \tau\nu_\tau)$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$\text{BR}(\mu \rightarrow e\nu_\mu\nu_e)$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$\text{BR}(\tau \rightarrow e\nu_\tau\nu_e)$, $\text{BR}(\tau \rightarrow \mu\nu_\mu\nu_\mu)$</td>
</tr>
</tbody>
</table>

Lepton Flavor Violation

<table>
<thead>
<tr>
<th>$Z$</th>
<th>$\text{BR}(Z \rightarrow e\mu)$, $\text{BR}(Z \rightarrow e\tau)$, $\text{BR}(Z \rightarrow \tau\mu)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>$\text{BR}(\mu \rightarrow e\gamma)$, $\text{BR}(\mu \rightarrow 3e)$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$\text{BR}(\tau \rightarrow e\gamma)$, $\text{BR}(\tau \rightarrow \mu\gamma)$</td>
</tr>
<tr>
<td></td>
<td>$\text{BR}(\tau \rightarrow 3e)$, $\text{BR}(\tau \rightarrow 3\mu)$</td>
</tr>
</tbody>
</table>

Higgs

| $h$ | $R_{\mu\mu}$, $R_{\tau\tau}$, $R_{\gamma\gamma}$, $\text{BR}(h \rightarrow \mu\tau)$ |

Lepton Non-Universality

| $B$ meson | $R_K$, $R_{K^*\pi}$ |

Table 2.2: List of observables. $\Delta a_\mu$ is the discrepancy of the measured muon $g - 2$ and the SM prediction. $A_{e,\mu,\tau}$ is the electron, muon, and tau left-right asymmetry in $Z$ decay. $A_{FB}^{(0e),(0\mu),(0\tau)}$ is the electron, muon, and tau forward-backward asymmetry in $Z$ decay. $R_{\mu\mu} = \Gamma(h \rightarrow \mu\mu)/\Gamma(h \rightarrow \mu\mu)_{\text{SM}}$ and similarly for $R_{\tau\tau}$ and $R_{\gamma\gamma}$. $R_K = \Gamma(B^+ \rightarrow K^+\mu\mu)/\Gamma(B^+ \rightarrow K^+ee)$ while $R_{K^*\pi} = \Gamma(B^0 \rightarrow K^{*0}\mu\mu)/\Gamma(B^0 \rightarrow K^{*0}ee)$. Lepton nonuniversality experimental values are taken from LHCb measurements [5, 6] while the other experimental values are taken from PDG [4].

calculations are expected to be small. The theoretical calculation of the VL contribution to the muon $g - 2$ is taken from Ref. [79]. The calculation for $\text{BR}(\ell \rightarrow \ell'\gamma)$ and $R_{\gamma\gamma}$ are performed at one-loop [91, 92]. Since all calculations are performed at leading order, we have included a 1% theoretical error when ensuring that the calculated observables satisfy the current experimental bounds. As for the lepton nonuniversality analysis, we have used flavio, a very versatile program that calculates $b$-physics observables written by Straub et al. [93]. To calculate the NP effects of the observables implemented in flavio, one only has to specify the NP contributions to the Wilson coefficients.

In the analysis, we obtain scatter plots by sampling from the parameter space and checking to see if the sampled points satisfy the constraints mentioned above. To ensure that we cover all regions in this vast parameter space, we divide VL masses into four different regions: $M_{L,E} \in [100, 150), [150, 250), [250, 500), [500, 1000)$ GeV, and the VL-VL couplings
into two different regions\textsuperscript{16} $|\lambda|$, $|\bar{\lambda}| \in [0, 0.75), [0.75, 1)$. As for the muon-VL couplings, we considered $|\lambda_{\mu}^{L,E} v / M_{L,E}| \in [0, 0.06), [0.06, 0.09)$. For each of these regions, we sampled 10,000 points satisfying the heavy-charged lepton mass bound and the electroweak precision observables. The total number of simulated points is 2.56 millions points. The parameters $M_{L,E}, \lambda, \bar{\lambda}, \lambda_{\mu}^{L,E}$ are sampled from a uniform distribution while $|\lambda_{e,\tau}^{L,E} v / M_{L,E}| \in [10^{-10}, 0.09)$ are sampled from a log-uniform distribution. The electron-VL and tau-VL couplings are sampled from a log-uniform distribution because we expect these couplings to be highly constrained by LFV observables and we are interested in determining the degree of fine-tuning of these two parameters in order to be consistent with LFV constraints.

2.3 Results

In this section, we present the results of our numerical analysis. For all plots in this section, we have classified the simulated points into two groups. This classification is based on whether a point satisfies all observables listed in Table\textsuperscript{2.2} other than the plotted observables and the lepton nonuniversality observables. The lightly shaded points do not satisfy one or more of these observables while the solid colored points satisfy all these observables.

Figure\textsuperscript{2.2} shows plots of $\Delta a_\mu$ versus $R_{\mu\mu}$. The four plots have different ranges of $M_L$, which is a meaningful discriminator because the VL contribution to the muon $g - 2$ from the $W$ boson loop is due to the SU(2) doublet VL neutrinos, $L_{L,R}^0$, which have mass $M_L$\textsuperscript{79}. The dashed lines show the 1σ and 2σ bounds of $\Delta a_\mu$, and the upper bound of $R_{\mu\mu}$. The solid lines show the central value of $\Delta a_\mu$ and $R_{\mu\mu} = 1$\textsuperscript{17}. From this figure, we see that this model can be ruled out in the future if future measurements of the muon $g - 2$ and $R_{\mu\mu}$ have much smaller uncertainties, and $R_{\mu\mu}$ is measured to be SM-like while the muon $g - 2$ is measured to have a similar central value.

Figure\textsuperscript{2.2} also shows that there are no points with $250 \text{ GeV} < M_L < 400 \text{ GeV}$ that

\textsuperscript{16}These couplings can be positive or negative. The quoted ranges are the magnitude. Similarly for SM-VL couplings.

\textsuperscript{17}Notice that there is no measurement of $R_{\mu\mu}$ yet. There is only an upper bound.
fit the muon $g - 2$ within $1\sigma$.\footnote{The bounds on parameter space obtained are not strict because the analysis is performed by random sampling from the vast parameter space. Our sampling method attempts to cover the whole parameter space but there might still be regions that are missed by the sampling method.} This observation is further illustrated in Fig. 2.3 which shows plots of $\Delta a_{\mu}$ versus $M_L$. The two plots have different ranges of $\bar{\lambda}$. This figure shows that for $\bar{\lambda} < 0.25$, this model requires either $M_L < 250$ GeV or $M_L > 600$ GeV to fit the muon $g - 2$ within $1\sigma$. On the other hand, this model requires $M_L > 400$ GeV for $\bar{\lambda} > 0.25$. This plot also shows that the allowed parameter space for $M_L \lesssim 250$ GeV can potentially be eliminated by the upcoming Fermilab E989 experiment if the muon $g - 2$ central value stays the same while the uncertainties decrease by a couple factors \cite{94}.

In general, as the VL masses increase, the new physics effects should approach zero. However, Fig. 2.3 seems to violate this fact. The muon $g - 2$ does not approach zero as $M_L$ increases because other parameters, such as $M_E$, $\bar{\lambda}$, and $\lambda^{L,E}_{\mu}$, are not fixed. In fact, if all of the other parameters are fixed, then the muon $g - 2$ approaches zero as $M_L$ increases.

Figure 2.4 shows a plot of $\Delta a_{\mu}$ versus $R_{\gamma\gamma}$. The points in this plot are separated into different colors based on $M_{e4}$. As expected, for heavier VL mass eigenstates, $R_{\gamma\gamma}$ is
Figure 2.3: Plots of $\Delta a_\mu$ versus $M_L$. The two plots have different ranges of $\tilde{\lambda}$. The gray points are ruled out. The dashed lines are the 1$\sigma$ and 2$\sigma$ bounds of $\Delta a_\mu$ while the solid line is the central value of $\Delta a_\mu$.

Figure 2.4: Plot of $\Delta a_\mu$ versus $R_{\gamma\gamma}$. The lightly shaded points are ruled out. The dashed lines show the 1$\sigma$ and 2$\sigma$ bounds of $\Delta a_\mu$, and the 1$\sigma$ bound of $R_{\gamma\gamma}$. The solid lines show the central value of $\Delta a_\mu$ and that of $R_{\gamma\gamma}$.

clustered around one. This plot shows that $M_{e4} > 500$ GeV is a more robust region than regions with smaller $M_{e4}$ because a larger percentage of simulated points are within the experimental bounds. A very interesting scenario will arise if the central value of $R_{\gamma\gamma}$ stays and uncertainties in the measurement decrease as more data are collected. In this scenario, we will have the potential to place an upper bound on the mass of the lightest VL mass eigenstate because there are no points with $M_{e4} > 500$ GeV and $R_{\gamma\gamma} \gtrsim 1.1$.

In Fig. 2.5 which shows plots of $\Delta a_\mu$ versus $R_{\gamma\gamma}$, the simulated points are separated
Figure 2.5: Plots of $\Delta a_\mu$ versus $R_{\gamma\gamma}$. The gray points are ruled out. The dashed lines show the $1\sigma$ and $2\sigma$ bounds of $\Delta a_\mu$, and the $1\sigma$ bound of $R_{\gamma\gamma}$. The solid lines show the central value of $\Delta a_\mu$ and that of $R_{\gamma\gamma}$. This model requires $||\lambda_\mu|| > 0.03$ to fit $\Delta a_\mu$ and $||\lambda_\mu|| < 0.09$ to fit $R_{\gamma\gamma}$.

Into four different plots based on different values of

$$||\lambda_\mu|| \equiv \sqrt{\left(\frac{\lambda^L_\mu v}{M_L}\right)^2 + \left(\frac{\lambda^E_\mu v}{M_E}\right)^2}.$$  \hspace{1cm} (2.28)

$||\lambda_\mu||$ is a meaningful variable because muon-VL couplings play a significant role in fitting $\Delta a_\mu$ and this variable captures the norm of the muon-VL couplings normalized by the VL masses. From this figure, we see that this model requires $||\lambda_\mu|| > 0.03$ to fit $\Delta a_\mu$ within $1\sigma$ and $||\lambda_\mu|| < 0.09$ to fit $R_{\gamma\gamma}$.

Figure 2.6 shows a plot of $\lambda^L_\mu$ versus $\lambda^E_\mu$. The gray points are all simulated points. The red points satisfy $\Delta a_\mu$ within $1\sigma$ while the blue points are consistent with $\Delta a_\mu$ and $R_{\gamma\gamma}$\footnote{The square shape in Fig. 2.6 is unphysical and is due to our choice of sampling range.}. To satisfy $\Delta a_\mu$, the muon-VL couplings need to satisfy approximately the following condition,

$$\left|\frac{\lambda^E_\mu v \lambda^L_\mu v}{M_E M_L}\right| \gtrsim 7 \times 10^{-4},$$  \hspace{1cm} (2.29)

which is shown by the solid lines. This bound is not an exact bound, but is an empirically
obtained bound satisfied by most simulated points. On the other hand, to satisfy both $\Delta a_\mu$ and $R_{\gamma\gamma}$, the muon-VL couplings need to satisfy approximately the following condition,

$$
\left( \frac{\lambda_E^\mu v}{M_E} \right)^2 + \frac{1}{1.08} \left( \frac{\lambda_L^\mu v}{M_L} \right)^2 \lesssim 0.08^2,
$$

which is showed by the dashed black lines. Similarly, this is not an exact bound.

Figure 2.7 shows a plot of $\Delta a_\mu$ versus $\text{BR}(\mu \to e\gamma)$, which gives the strongest LFV constraint. The points in this plot are separated into four colors based on ranges of the ratio of electron-VL to muon-VL couplings,

$$
\left\langle \frac{\lambda_e}{\lambda_\mu} \right\rangle \equiv \frac{1}{2} \left( \frac{\lambda_L^e}{\lambda_L^\mu} + \frac{\lambda_E^e}{\lambda_E^\mu} \right).
$$

The dashed lines show the $1\sigma$ and $2\sigma$ bounds of $\Delta a_\mu$, and the upper bound of $\text{BR}(\mu \to e\gamma)$.
The solid line shows the central value of $\Delta a_\mu$. This figure shows that simultaneously satisfying $\text{BR}(\mu \to e\gamma)$ and $\Delta a_\mu$ to within $1\sigma$ requires $\langle \lambda_e/\lambda_\mu \rangle \lesssim 10^{-4}$. This figure shows that this model requires some level of fine-tuning.

The most stringent constraints for the tau-VL coupling is from electroweak observables. The sampling range for tau-VL couplings, $\lambda_{\tau E}^{L,E} v / M_{L,E} \in (-0.09, 0.09)$, is based on electroweak constraints. The next strongest constraint for the tau-VL coupling is $\text{BR}(\tau \to \mu\gamma)$. This constraint, however, does not rule out any value of the tau-VL couplings within the sampling range. Finally, $\text{BR}(h \to \mu\tau)$ does not constrain the parameter space at all. This is in agreement with a previous analysis by Falkowski et al., which shows that the constraint from LFV Higgs boson decays is at least four orders of magnitude smaller than that from $\text{BR}(\ell \to \ell'\gamma)$ [87].

As for the lepton nonuniversality measurements, the calculated values of $R_K$ and $R_{K^*0}$ do not deviate from the SM predictions because the Wilson coefficients contain the SM-VL mixing matrices squared, which are highly constrained by electroweak precision measure-

\footnote{Out of all simulated points, there are only four points that violate this bound with the largest violation being $\langle \lambda_e/\lambda_\mu \rangle = 2 \times 10^{-4}$.}
ments. The ranges of Wilson coefficients in this model are

\[-3.21 \times 10^{-11} \leq C_9^{NP}(e) \leq 7.26 \times 10^{-12},\]
\[-3.21 \times 10^{-11} \leq C_{10}^{NP}(e) \leq 7.26 \times 10^{-12},\]
\[-7.88 \times 10^{-3} \leq C_9^{NP}(\mu) \leq 2.21 \times 10^{-3},\]
\[-7.86 \times 10^{-3} \leq C_{10}^{NP}(\mu) \leq 2.18 \times 10^{-3}.\]

(2.32)

As a comparison, to fit all the CP-conserving \( b \to s\mu^+\mu^- \) anomalies along with \( R_K \) and \( R_{K^*0} \), the Wilson coefficients need to have values \( C_9^{NP}(\mu) = -1.20 \pm 0.20 \) and \( C_9^{NP}(e) = C_{10}^{NP}(e) = C_{10}^{NP}(\mu) = 0 \), or \( C_9^{NP}(\mu) = -C_{10}^{NP}(\mu) = -0.68 \pm 0.12 \) and \( C_9^{NP}(e) = C_{10}^{NP}(e) = 0 \) [95].

### 2.4 Conclusions

In this chapter, we considered a very simple extension of the SM in which the SM is extended with one family of VL leptons; where the VL leptons couple to all three families of the SM leptons. We studied the constraints on this model coming from the heavy-charged lepton mass bound, electroweak precision data, the muon \( g-2 \), lepton flavor violation, Higgs boson decays and lepton nonuniversality observables. See Table 2.2 for the complete list of observables considered in this chapter. All experimental values, other than lepton nonuniversality observables, are taken from the PDG [4]. The experimental value for \( R_K \) is taken from Ref. [5], while \( R_{K^*0} \) is recently measured by LHCb [6]. All theoretical calculations are performed at leading order while the lepton nonuniversality observables are calculated using flavio [93].

In this chapter, we showed that this model can fit all but the lepton nonuniversality measurements. The more constraining observables are the muon \( g-2 \), \( R_{\mu\mu}, R_{\gamma\gamma}, \) and \( \text{BR}(\mu \to e\gamma) \). We find that if \( R_{\mu\mu} \) is measured to be SM-like, then this model cannot simultaneously fit both the muon \( g-2 \) within 1\( \sigma \) and \( R_{\mu\mu} \) (see Fig. 2.2). In addition, we also find that the SU(2) doublet VL mass is required to satisfy \( M_L \lesssim 250 \text{ GeV} \) or \( M_L \gtrsim 400 \text{ GeV} \) in order to fit the muon \( g-2 \) within 1\( \sigma \) (see Fig. 2.3). If the heavy-charged
lepton mass bound increases to be above $M_L \gtrsim 250$ GeV, then the muon $g - 2$ can produce a stronger mass bound. Fitting to the muon $g - 2$ requires $||\lambda_\mu|| > 0.03$ while fitting to $R_{\gamma\gamma}$ requires $||\lambda_\mu|| < 0.09$. Hence, the muon-VL coupling is constrained to be within $0.03 < ||\lambda_\mu|| < 0.09$. Although we allow the VL leptons to couple to all three families of the SM leptons, by simultaneously fitting the muon $g - 2$ and BR($\mu \rightarrow e\gamma$), the ratio of the electron-VL coupling to muon-VL coupling is constrained to be $\langle \lambda_e/\lambda_\mu \rangle \lesssim 10^{-4}$. Hence, this model requires some level of fine-tuning. On the other hand, the strongest constraints on the tau-VL coupling is coming from electroweak precision observables. The recently measured BR($h \rightarrow \mu\tau$) is less constraining than the electroweak precision observables. We also find that this model cannot explain the $b$-physics lepton nonuniversality measurements.
Chapter 3

SO(10) SUSY GUT with Yukawa Unification

This chapter is based on the work in

In this chapter, we consider SUSY GUTS, another class of models beyond the SM. In particular, we focus on an SO(10) SUSY GUT with Yukawa unification and a $D_3 \times [U(1) \times Z_3 \times Z_3]$ family symmetry. Previous analyses showed that this model fits reasonably well to low-energy observables such as gauge couplings, gauge boson masses, fermion masses, Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, neutrino mass differences and mixing angles [96–98]. In this analysis we focus on 1) the exclusive versus inclusive measurements of $|V_{ub}|$ and $|V_{cb}|$, 2) the fits to observables that are coupled to SUSY, in particular some observables in the decay of $B^0 \rightarrow K^{*0}\mu^+\mu^-$, 3) the differences between the fit of two different gaugino mass GUT boundary conditions, and 4) the fine-tuning of the model.

This rest of the chapter is organized as follows. In Sec. 3.1 the model of the SUSY GUT with an SO(10) gauge symmetry is given. In Sec. 3.2 we explain the global $\chi^2$ analysis procedure. The results of the analysis is presented in Sec. 3.3. Finally, the discovery prospects and predictions of this model are discussed in Sec. 3.4 and we conclude this chapter in Sec. 3.5.
3.1 Model

In this section, we discuss a complete three family Yukawa unified SUSY GUT with SO(10) gauge symmetry and a $D_3 \times [U(1) \times Z_3 \times Z_3]$ family symmetry. This model has been extensively studied previously [96–100]. In this chapter we focus solely on the Yukawa sector as the other sectors of the model such as the right-handed neutrino sector of the model are identical to previous analyses.

Charged Fermion Sector

The superpotential in the charged fermion sector is given by

$$W_{SO(10)} = \lambda 16_3 \ 10 \ 16_3 + 16_a \ 10 \chi_a$$

$$+ \bar{\chi}_a \left( M_\chi \chi_a + 45 \frac{\phi_a}{M} 16_3 + 45 \frac{\tilde{\phi}_a}{M} 16_a + A 16_a \right),$$

where $16_i$ is the spinor representation of SO(10), which contains a family of fermions and their supersymmetric partners, and $i = 1, 2, 3$ is the family index. $16_3$ is a singlet under $D_3$ symmetry, while $16_a, a = 1, 2$ are doublets under $D_3$ symmetry. $10$ is the 10 dimensional representation of SO(10), which contains a pair of Higgs doublets. $45$ is the adjoint representation of SO(10) that is assumed to obtain VEV in the $B - L$ direction. $\chi_a$ and $\bar{\chi}_a$ for $a = 1, 2$ are Froggatt-Nielsen states [101] and are doublets under $D_3$ symmetry. $\tilde{M}$ is trivial under all groups while $M_\chi = M_0 (1 + \alpha X + \beta Y)$, where $X$ and $Y$ are generators of $SO(10)$, and $\alpha$ and $\beta$ are some constants. $A$ is an SO(10) singlet flavon field and a non-trivial singlet under $D_3$ symmetry. Finally, $\phi_a$ and $\tilde{\phi}_a$ are SO(10) singlet flavon fields, which are assumed to obtain VEVs of the form

$$\langle \phi_a \rangle = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \langle \tilde{\phi}_a \rangle = \begin{pmatrix} 0 \\ \tilde{\phi}_2 \end{pmatrix}. \quad (3.2)$$
After integrating out the Froggatt-Nielsen states, $\chi_a$ and $\bar{\chi}_a$, and defining
\[
G_{x,y} = \frac{M_0}{M_X} = \frac{1}{1 + \alpha x + \beta y},
\]
\[
G_{x_1,y_1,x_2,y_2}^\pm = G_{x_1,y_1} \pm G_{x_2,y_2},
\]
where $x$ and $y$ are the eigenvalues of $X$ and $Y$, we obtain the following effective superpotential
\[
W_{\text{eff}}^{\text{SO}(10)} \supset [y^u]_{ij} q_i H_u \bar{u}_j + [y^d]_{ij} q_i H_d \bar{d}_j + [y^e]_{ij} \ell_i H_u \bar{\nu}_j + [y^\nu]_{ij} \ell_i H_u \bar{\nu}_j,
\]
where $i,j = 1, 2, 3$ are the family indices and the Yukawa matrices are given by\(^{21}\) (see App. B for derivation)
\[
y^u = \begin{pmatrix}
0 & \epsilon' G_{1, -\frac{4}{3}; 1, \frac{1}{3}} & -\epsilon \xi G_{1, -\frac{4}{3}} \\
-\epsilon' G_{1, -\frac{4}{3}; 1, \frac{1}{3}} & \epsilon G_{1, -\frac{4}{3}; 1, \frac{1}{3}} & -\epsilon G_{1, -\frac{4}{3}} \\
\epsilon \xi G_{1, \frac{1}{3}} & \epsilon G_{1, \frac{1}{3}} & \lambda
\end{pmatrix},
\]
\[
y^d = \begin{pmatrix}
0 & \epsilon' G_{-\frac{2}{3}, \frac{1}{3}; 1, \frac{1}{3}} & -\epsilon \xi G_{-\frac{2}{3}, \frac{2}{3}} \\
-\epsilon' G_{-\frac{2}{3}, \frac{1}{3}; 1, \frac{1}{3}} & \epsilon G_{-\frac{2}{3}, \frac{1}{3}; 1, \frac{1}{3}} & -\epsilon G_{-\frac{2}{3}, \frac{2}{3}} \\
\epsilon \xi G_{1, \frac{1}{3}} & \epsilon G_{1, \frac{1}{3}} & \lambda
\end{pmatrix},
\]
\[
y^e = \begin{pmatrix}
0 & -\epsilon' G_{-\frac{3}{1}, -1; 1, 2} & 3\epsilon \xi G_{1, 2} \\
\epsilon' G_{-\frac{3}{1}, -1; 1, 2} & 3\epsilon G_{-\frac{3}{1}, -1; 1, 2} & 3\epsilon G_{1, 2} \\
-3\epsilon \xi G_{-\frac{3}{1}, -1} & -3\epsilon G_{-\frac{3}{1}, -1} & \lambda
\end{pmatrix},
\]
\[
y^\nu = \begin{pmatrix}
0 & -\epsilon' G_{-\frac{3}{1}, -5; 1, 0} & 3\epsilon \xi G_{5, 0} \\
\epsilon' G_{-\frac{3}{1}, -5; 1, 0} & 3\epsilon G_{-\frac{3}{1}, -5; 1, 0} & 3\epsilon G_{5, 0} \\
-3\epsilon \xi G_{-\frac{3}{1}, -1} & -3\epsilon G_{-\frac{3}{1}, -1} & \lambda
\end{pmatrix},
\]
\(^{21}\)Notice that the neutrino Yukawa matrix elements are of the same order of magnitude as that of the charged fermions. The light neutrino masses are a result of the seesaw mechanism with hierarchical right-handed neutrino masses\([96, 98]\). In our model $M_{R_1} \sim 10^9$ GeV, $M_{R_2} \sim 10^{11}$ GeV and $M_{R_3} \sim 10^{13}$ GeV.

60
where

\begin{align}
\epsilon &= -\frac{1}{6} \frac{M_G \phi_1}{M_0 M}, \\
\tilde{\epsilon} &= +\frac{1}{6} \frac{M_G \phi_2}{M_0 M}, \\
\epsilon' &= -\frac{1}{2} \frac{A}{M_0}, \\
\xi &= \frac{\phi_2}{\phi_1}.
\end{align}

(3.10, 3.11, 3.12, 3.13)

Taking the limits of \( \beta \ll \alpha \), the Yukawa matrices can be rewritten as (again, see App. [B] for derivation)

\begin{align}
y^u &= \begin{pmatrix} 0 & \tilde{\epsilon}' \rho & -\tilde{\epsilon} \xi \\ -\tilde{\epsilon}' \rho & \tilde{\epsilon} \rho & -\tilde{\epsilon} \\ \tilde{\epsilon} \xi & \tilde{\epsilon} & \lambda \end{pmatrix}, \\
y^d &= \begin{pmatrix} 0 & \tilde{\epsilon}' & -\tilde{\epsilon} \xi \sigma \\ -\tilde{\epsilon}' & \tilde{\epsilon} & -\tilde{\epsilon} \sigma \\ \tilde{\epsilon} \xi & \tilde{\epsilon} & \lambda \end{pmatrix}, \\
y^e &= \begin{pmatrix} 0 & -\tilde{\epsilon}' & 3\tilde{\epsilon} \xi \\ \tilde{\epsilon}' & 3\tilde{\epsilon} & 3\tilde{\epsilon} \\ -3\tilde{\epsilon} \xi \sigma & -3\tilde{\epsilon} \sigma & \lambda \end{pmatrix}, \\
y^\nu &= \begin{pmatrix} 0 & -\tilde{\epsilon}' \omega & 3\tilde{\epsilon} \xi \omega / 2 \\ \tilde{\epsilon}' \omega & 3\tilde{\epsilon} \omega & 3\tilde{\epsilon} \omega / 2 \\ -3\tilde{\epsilon} \xi \sigma & -3\tilde{\epsilon} \sigma & \lambda \end{pmatrix},
\end{align}

(3.14, 3.15, 3.16, 3.17)

where

\begin{align}
\tilde{\epsilon} &= \frac{\epsilon}{1 + \alpha}, \\
\tilde{\epsilon} &= \tilde{\epsilon}' = \frac{-\epsilon}{1 + \alpha}, \\
\xi &= \frac{-\phi_2}{\phi_1}.
\end{align}

(3.18-3.21)
\[ \sigma = \frac{1 + \alpha}{1 - 3\alpha} \]  \hspace{1cm} (3.22)

\[ \omega = \frac{2\sigma}{2\sigma - 1}. \]  \hspace{1cm} (3.23)

Of these parameters, \( \tilde{\epsilon}, \xi, \rho \) and \( \sigma \) are assumed to be complex while \( \tilde{\epsilon}, \tilde{\epsilon}' \) and \( \lambda \) are assumed to be real. Hence, the Yukawa sector of this model has 11 real parameters, four of which are phases. Also, the quark mass matrices accommodate the Georgi-Jarlskog mechanism described in Chapter 1. This is a result of the VEV of 45 in the \( B-L \) direction.

As can be seen from the Yukawa matrices, the benefit of taking the limit of \( \beta \ll \alpha \) is that the 12, 21 and 22 elements of \( y^u \) are the only Yukawa matrix elements that are dependent on \( \rho \). Since, \( \rho \propto \beta \ll \alpha \), the theory naturally provides hierarchical up and down quark masses. However, taking this limit does not reduces the number of arbitrary parameters in the Yukawa sector.

In summary, the charged-fermion sector of our model has 12 parameters - 11 Yukawa parameters and \( \tan \beta \). In comparison, the SM has 13 arbitrary parameters - six quark masses, three charged lepton masses, and four CKM matrix elements. Hence, our model has one prediction in the charged fermion sector. Including the neutrino sector, our model has three additional arbitrary parameters to fit six additional observables - two neutrino mass difference squared, three real mixing angles, and a \( CP \) violating phase. Hence, our model has four predictions in the fermion sector.

**Boundary Conditions**

The GUT scale boundary conditions of our model are shown in Table 3.1. They are the GUT scale \( M_G \) (defined where \( \alpha_1 = \alpha_2 \)), the gauge coupling \( \alpha_G(M_G) \), and a GUT scale threshold correction \( \epsilon_3 \) satisfying \( \alpha_3(M_G) = \alpha_G(1 + \epsilon_3) \). The GUT scale threshold correction in a complete theory would result from the massive states at the GUT scale, but here is parametrized to fit the experimentally measured value of \( \alpha_s(M_Z) \).

In addition, we have also parametrized the SUSY breaking sector with the soft SUSY breaking parameters. The soft SUSY breaking parameters of our model are universal scalar masses \( m_{16} \), a universal trilinear coupling \( A_0 \) and non-universal Higgs masses \( m_{H_u}, m_{H_d} \).
As for the boundary condition for the gaugino masses, $M_{1/2}$, we consider two different cases: 1) universal gaugino masses and 2) mirage mediated gaugino masses. The mirage mediated gaugino masses are defined as

$$M_i = \left( 1 + \frac{g_2^2 b_i \alpha}{16 \pi^2} \log \frac{M_{pl}}{m_{16}} \right) M_{1/2},$$

(3.24)

where $M_{1/2}$ and $\alpha$ are arbitrary parameters and $b_i = (33/5, 1, -3)$ for $i = 1, 2, 3$ are the $\beta$-function coefficients. Notice that the universal boundary condition corresponds to $\alpha = 0$. This set of soft SUSY breaking parameters represents a minimal set of parameters needed for a robust electroweak symmetry breaking.

Mirage boundary condition is interesting because $\alpha = 1.5$ is the optimal scenario for a well-tempered dark matter. Hence, in this chapter we set $\alpha = 1.5$ when the gaugino masses are mirage mediated. On the other hand, it has been shown that the LSP of universal boundary condition is predominantly Bino-like, which leads to an over-closed universe. However, this problem can be solved by introducing axions with mass lighter than the LSP into the model. Due to these consideration, we do not include dark matter phenomenology into the scope of this chapter.

Our model also has 11 Yukawa textures parameters, 3 right-handed neutrino masses, along with $\tan \beta$ and $\mu$. All the parameters other than $\tan \beta$ and $\mu$ are defined at the GUT scale.

To summarize, all the arbitrary parameters of our model are listed in Table 3.1. With universal $M_{1/2}$ boundary condition, our model has 24 arbitrary parameters while with mirage mediated $M_{1/2}$ boundary condition the number of arbitrary parameters increases to 25.

### 3.2 Analysis Procedure

#### 3.2.1 Renormalization Procedure

The main analysis program used is maton, which was initially written by Radovan Dermišek. This program renormalizes parameters from the GUT scale to low-energy scales where
Universal $M_{1/2}$ Model

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input Parameters</th>
<th>No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauge</td>
<td>$\alpha_G, M_G, \epsilon_3$</td>
<td>3</td>
</tr>
<tr>
<td>SUSY (GUT scale)</td>
<td>$m_{16}, M_{1/2}, A_0, m_{H_u}, m_{H_d}$</td>
<td>5</td>
</tr>
<tr>
<td>Yukawa Textures</td>
<td>$\lambda, \hat{\epsilon}, \hat{\epsilon}', \xi, \rho, \sigma, \phi_{\xi}, \phi_{\rho}, \phi_{\sigma}$</td>
<td>11</td>
</tr>
<tr>
<td>Neutrino</td>
<td>$M_{R_1}, M_{R_2}, M_{R_3}$</td>
<td>3</td>
</tr>
<tr>
<td>SUSY (EW Scale)</td>
<td>$\tan \beta, \mu$</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>24</td>
</tr>
</tbody>
</table>

Mirage Mediated $M_{1/2}$ Model

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input Parameters</th>
<th>No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauge</td>
<td>$\alpha_G, M_G, \epsilon_3$</td>
<td>3</td>
</tr>
<tr>
<td>SUSY (GUT scale)</td>
<td>$m_{16}, M_{1/2}, A_0, m_{H_u}, m_{H_d}, \alpha$</td>
<td>6</td>
</tr>
<tr>
<td>Yukawa Textures</td>
<td>$\lambda, \hat{\epsilon}, \hat{\epsilon}', \xi, \rho, \sigma, \phi_{\xi}, \phi_{\rho}, \phi_{\sigma}$</td>
<td>11</td>
</tr>
<tr>
<td>Neutrino</td>
<td>$M_{R_1}, M_{R_2}, M_{R_3}$</td>
<td>3</td>
</tr>
<tr>
<td>SUSY (EW Scale)</td>
<td>$\tan \beta, \mu$</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>25</td>
</tr>
</tbody>
</table>

Table 3.1: The universal $M_{1/2}$ model has 24 arbitrary parameters while the mirage mediated $M_{1/2}$ model has an additional parameter, $\alpha$, which determines the amount of splitting of $M_{1/2}$, at the GUT scale.
observables are calculated.

With the exception of $\tan \beta$ and $\mu$, all input parameters of maton are defined at the GUT scale and are run down to the heaviest right-handed neutrino mass scale. At this scale, all the right-handed neutrinos are integrated out of the theory. This is a reasonable approximation because the running between the three right-handed neutrinos mass scales are not significantly different. After integrating out the right-handed neutrinos, we use the two-loop MSSM RGE to run down to the weak scale. At the weak scale, we include the one-loop threshold corrections for the Yukawa and the gauge couplings, which was calculated by Pierce et al. [104]. The gluino mass, $M_{\tilde{g}}$, and $CP$-odd Higgs mass, $M_A$, are one-loop pole masses. However, we do not include one-loop threshold corrections for the other scalar masses. Instead, we estimated that these corrections are about 10%. Hence, the GUT scale parameters of our model have an inherent 10% theoretical error. Adding the one-loop threshold correction for the soft scalar masses can be a future project.

After including the one-loop threshold correction at the weak scale, we ensure radiative EWSB by requiring tadpole diagrams to vanish at one-loop level (see Ref. [104] for discussion and Ref. [98] for more details of our analysis). By imposing this condition, $M_Z$ is fit exactly and the magnitude of the parameter $\mu$ is also fixed. The detailed procedure of our calculation is presented in Ref. [98].

Observables

A new feature of our analysis as compared to previous analyses is that we have included additional $b$-physics observables. These observables are defined in App. [C]. In App. [D], we show the calculation for the $C_7$ Wilson coefficient at one-loop. In particular, we have included 4 $CP$-even $B^0 \rightarrow K^{*0}\mu^+\mu^-$ angular observables: $F_L$, $P_2$, $P'_4$, and $P'_5$. We did not include other $CP$-even observables because the theoretical uncertainties of those observables are much too big. Hence, they do not constrain our model. Each of these observables are measured in two different energy bins: a low $q^2$ bin with $q^2 \in [1,6]$ GeV$^2$ and a high $q^2$ bin with $q^2 \in [14,18,16]$ GeV$^2$. These angular observables are calculated by SuperIso v3.4 [105]. Since SuperIso assumes that all soft parameters are real and only takes the
diagonal entries of the trilinear couplings into account, we do not include $CP$-odd $B^0 \to K^{*0} \mu^+ \mu^-$ angular observables in our analysis.

Table 3.2 includes 45 observables that we include in the global $\chi^2$ analysis. Table 3.2 has multiple entries for $|V_{ub}|$ and $|V_{cb}|$ because to account for the inconsistencies in the inclusive and exclusive measurements of $|V_{ub}|$ and $|V_{cb}|$, we perform the global $\chi^2$ analysis using the inclusive and exclusive measurements separately. We also perform an additional analysis where the error bars of $|V_{ub}|$ and $|V_{cb}|$ cover both the inclusive and exclusive values.

Most experimental values of the observables are obtained from PDG [7]. The neutrinos related observables are obtained from a global analysis of solar, atmospheric, reactor and accelerator neutrino data by Gonzalez-Garcia et al. [106]. The $b$-physics observables are obtained from Heavy Flavor Averaging Group (HFAG) [107], CMS [108], and LHCb [109, 110].

The program that calculates most observables are maton, which is also the renormalization program mentioned in the previous subsection. The Higgs mass is calculated from SplitSuSpect [111] while the $b$-physics observables are calculated from SuperIso v3.4 and SUSY_FLAVOR v2.0 [105, 112].

With the approximate treatment of the threshold correction described in the previous subsection, we estimated 0.5% theoretical uncertainty for observables calculated in maton. The theoretical uncertainty for $G_\mu$ is estimated to be 1% because we have neglected the SUSY vertex and box diagrams calculation. The theoretical uncertainty for the $B^0 \to K^{*0} \mu^+ \mu^-$ observables are taken from the SuperIso manual. However, since SuperIso does not take into account the imaginary part of the soft parameters, we assigned an additional 15% theoretical errors to the calculation.

3.2.2 Global $\chi^2$ Analysis

To perform the global $\chi^2$ analysis, we construct a $\chi^2$ function

$$\chi^2 = \sum_i \frac{|x_i^{\text{th}} - x_i^{\exp}|^2}{\sigma_i^2},$$

(3.25)
<table>
<thead>
<tr>
<th>Observable</th>
<th>Exp. Value</th>
<th>Ref.</th>
<th>Program</th>
<th>Th. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_Z$</td>
<td>91.1876 ± 0.0021 GeV</td>
<td>107</td>
<td>Input</td>
<td>0.0%</td>
</tr>
<tr>
<td>$M_W$</td>
<td>80.385 ± 0.015 GeV</td>
<td>107</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\alpha_{em}$</td>
<td>1/137.03599967(44)</td>
<td>107</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$G_\mu$</td>
<td>1.1663787(6) x 10^{-5} GeV^{-2}</td>
<td>107</td>
<td>maton</td>
<td>1%</td>
</tr>
<tr>
<td>$\alpha_s(M_Z)$</td>
<td>0.1185 ± 0.0006</td>
<td>107</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$M_t$</td>
<td>173.21 ± 0.51 ± 0.71 GeV</td>
<td>107</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$m_t(m_t)$</td>
<td>4.18 ± 0.03 GeV</td>
<td>107</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$M_h$</td>
<td>1776.82 ± 0.16 MeV</td>
<td>107</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$m_h - m_t$</td>
<td>3.45 ± 0.05 GeV</td>
<td>107</td>
<td>maton</td>
<td>10%</td>
</tr>
<tr>
<td>$m_t(m_t)$</td>
<td>1.275 ± 0.025 GeV</td>
<td>107</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$m_t(2$ GeV)</td>
<td>95 ± 5 MeV</td>
<td>107</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$m_h/m_t(2$ GeV)</td>
<td>17 − 22</td>
<td>107</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$Q$</td>
<td>21 − 25</td>
<td>107</td>
<td>maton</td>
<td>5%</td>
</tr>
<tr>
<td>$M_{ee}$</td>
<td>105.6583715(35) MeV</td>
<td>107</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$M_{e_{11}}$</td>
<td>0.5109989289(11) MeV</td>
<td>107</td>
<td>maton</td>
<td>0.5%</td>
</tr>
</tbody>
</table>

| $|V_{ud}|$ | 0.2253 ± 0.0008 | 107  | maton | 0.5%  |
| $|V_{cd}|$ (Inclusive) | 0.0422 ± 0.0007 | 107  | maton | 0.5%  |
| $|V_{cd}|$ (Exclusive) | 0.0395 ± 0.0008 | 107  | maton | 0.5%  |
| $|V_{ub}|$ (Both) | 0.0408 ± 0.0021 | 107  | maton | 0.5%  |
| $|V_{cb}|$ (Inclusive) | 0.00411 ± 0.00024 | 107  | maton | 0.5%  |
| $|V_{cb}|$ (Exclusive) | 0.00328 ± 0.00029 | 107  | maton | 0.5%  |
| $|V_{ub}|$ (Both) | 0.00385 ± 0.00086 | 107  | maton | 0.5%  |
| $|V_{cd}|$ | 0.00840 ± 0.0006 | 107  | maton | 0.5%  |
| $|V_{cd}|$ | 0.00400 ± 0.0027 | 107  | maton | 0.5%  |
| sin(2β)    | 0.682 ± 0.019 | 107  | maton | 0.5%  |
| $\epsilon_K$ | (2.2325 ± 0.1555) x 10^{-3} | 112  | SUSY_FLAVOR | 10% |

| $\Delta m_{Q1}/\Delta m_{Q2}$ | 35.0345 ± 0.3884 | 112  | SUSY_FLAVOR | 20% |
| $\Delta m_{Q3}$ | (3.37 ± 0.03) x 10^{-10} MeV | 112  | SUSY_FLAVOR | 20% |

| $\Delta m_{Q3}$ | (7.02 − 8.09) x 10^{-5} eV | 106  | maton | 0.5%  |
| $\Delta m_{Q3}$ | (2.317 − 2.607) x 10^{-3} eV | 106  | maton | 0.5%  |
| sin$^2\theta_{23}$ | 0.270 − 0.344 (3σ range) | 106  | maton | 0.5%  |
| sin$^2\theta_{31}$ | 0.382 − 0.643 (3σ range) | 106  | maton | 0.5%  |
| sin$^2\theta_{32}$ | 0.0186 − 0.029 (3σ range) | 106  | maton | 0.5%  |

| $M_h$ | 125.7 ± 0.4 GeV | 113  | SplitSuSpect | 3 GeV |

| BR(h → sγ) | (3.45 ± 21 ± 7) x 10^{-6} | 107  | SuperIso | 40%  |
| BR(B^0 → μ^+μ^-) | (2.98 ± 0.7 ± 0.9) x 10^{-3} | 108  | SUSY_FLAVOR | 20% |
| BR(B^0 → μ^+μ^-) | (3.9 ± 0.7 ± 0.11) x 10^{-3} | 108  | SUSY_FLAVOR | 20% |
| BR(B^+ → τ^+ν) | (1.14 ± 22) x 10^{-6} | 107  | SUSY_FLAVOR | 50% |

| BR(B^0 → K^0μ^+μ^-)_low q^2 | 0.34 ± 0.03 ± 0.04 ± 0.02 x 10^{-7} | 109  | SuperIso | 105% |
| BR(B^0 → K^0μ^+μ^-)_high q^2 | 0.45 ± 0.06 ± 0.04 ± 0.02 x 10^{-7} | 109  | SuperIso | 105% |
| $\eta_q^2 A_{FB}(B^0 → K^0μ^+μ^-)$ | 4.9 ± 0.9 GeV | 109  | SuperIso | 25% |
| $F_1(A^0 → K^0μ^+μ^-)_low q^2$ | 0.65 ± 0.07 ± 0.03 | 109  | SuperIso | 45% |
| $F_1(A^0 → K^0μ^+μ^-)_high q^2$ | 0.33 ± 0.07 ± 0.03 | 109  | SuperIso | 80% |
| $-2P_3 = A^0_{FB}(B^0 → K^0μ^+μ^-)_low q^2$ | -0.60 ± 0.24 ± 0.04 | 109  | SuperIso | 95% |
| $-2P_3 = A^0_{FB}(B^0 → K^0μ^+μ^-)_high q^2$ | -0.62 ± 0.22 ± 0.01 | 109  | SuperIso | 95% |
| $P_1(B^0 → K^0μ^+μ^-)_low q^2$ | 1.90 ± 0.05 ± 0.02 | 109  | SuperIso | 45% |
| $P_1(B^0 → K^0μ^+μ^-)_high q^2$ | 0.58 ± 0.36 ± 0.02 | 110  | SuperIso | 30% |
| $P_1(B^0 → K^0μ^+μ^-)_low q^2$ | 0.21 ± 0.20 ± 0.03 | 110  | SuperIso | 45% |
| $P_1(B^0 → K^0μ^+μ^-)_high q^2$ | -0.79 ± 0.20 ± 0.18 | 110  | SuperIso | 60% |

Table 3.2: 45 low-energy observables that are fitted in the global $\chi^2$ analysis (see text for more discussions).
where \( x_i^{\text{th}} \) are the calculated values, \( x_i^{\exp} \) are the experimentally measured values, and \( \sigma_i^2 \) are the sum of the squares of the experimental and theoretical uncertainties. To find the minimum of this \( \chi^2 \) function, we use the Minuit package maintained by CERN \[113\]. As in most minimization problems, obtaining the true global minimum is not guaranteed. To increase the likelihood of obtaining the true global minimum, we iterate the minimization process with random initial guesses for the input parameters.

Since all the minimizations are performed with fixed value of \( m_{16} \) and \( M_{1/2} \), the degrees of freedom in the universal \( M_{1/2} \) model is \( 45 - 24 + 2 = 23 \). On the other hand, the degrees of freedom in the mirage mediated \( M_{1/2} \) model is \( 45 - 25 + 2 = 22 \). In calculating the degrees of freedom, we made a simplifying assumption that the observables are uncorrelated.

### 3.3 Results

Several benchmark points with the results of the global \( \chi^2 \) analysis are given in App. \[H.1\], a plot of \( \chi^2 \) versus \( m_{16} \) is given in Fig. \[3.1\] and \( \chi^2 \) contours in the two dimensional plane of \( \tilde{M}_3 \) versus \( m_{16} \) is given in Fig. \[3.3\]. Let us now discuss some features of the results.

#### 3.3.1 Inclusive vs. Exclusive \( |V_{ub}| \) and \( |V_{cb}| \)

Due to the discrepancy between the values of \( |V_{ub}| \) and \( |V_{cb}| \) determined from inclusive and exclusive semi-leptonic decay, we define three different \( \chi^2 \) functions:

1. \( |V_{ub}| \) and \( |V_{cb}| \) are taken to be the inclusive values,
2. \( |V_{ub}| \) and \( |V_{cb}| \) are taken to be the exclusive values, and
3. \( |V_{ub}| \) and \( |V_{cb}| \) are taken to be the average of inclusive and exclusive values with error bars overlapping with the error bars of both the inclusive and exclusive measurements.

The results of these three analyses are shown in Fig. \[3.1\]. We see that for both the universal boundary condition \( \alpha = 0 \) and mirage boundary condition with \( \alpha = 1.5 \), the...
Figure 3.1: Plot of $\chi^2$/d.o.f versus $m_{16}$ for cases where the value of $|V_{ub}|$ and $|V_{cb}|$ are taken to be the inclusive values, the exclusive values, or the average of inclusive and exclusive values. Solid lines refer to the universal $M_{1/2}$ model while dashed lines refer to the mirage mediated $M_{1/2}$ model. This plot shows that our model favors the exclusive values of $|V_{ub}|$ and $|V_{cb}|$.

$\chi^2$/d.o.f obtained by fitting to the inclusive values are the biggest. Hence, we predict that the exclusive values of $|V_{ub}|$ and $|V_{cb}|$ are the correct values for both universal and mirage gaugino masses.

Since the $\chi^2$ difference between case (2) and case (3) is small and to be conservative, the analyses of the rest of the chapter are done for case (3), where $|V_{ub}|$ and $|V_{cb}|$ are the average of the inclusive and exclusive values.
3.3.2 SUSY Non-decoupled observables

\textbf{b-physics observables}

Some of the measured angular observables of $B^0 \rightarrow K^*0 \mu^+\mu^-$ are in tension with the SM predictions. For example, $P_4'$ in the high $q^2$ bin has a $2.7\sigma$ discrepancy with the SM prediction, $P_5'$ in the low $q^2$ bin has a $2.5\sigma$ discrepancy with the SM prediction, and $P_2$ in the low $q^2$ bin has a $2\sigma$ discrepancy with the SM prediction \cite{109,110,114}. These observables are defined in App. C \cite{115,116}. In addition, previous analyses found that the tension in $P_4'$ of the high $q^2$ bin cannot be explained by the MSSM \cite{114,117}. On the other hand, the tension of $F_L$ and $P_5'$ of the low $q^2$ bin can be explained by the MSSM by having a negative contribution to the $C_7$ Wilson coefficient. In the standard model $C_7^{\text{SM}} \approx -0.32$. The tension in $F_L$ and $P_5'$ can be further reduced by making $C_7$ more negative \cite{114,118}. In the MSSM, chargino-stop and charged Higgs loops contribute to $C_7$. The $C_7$ contribution from the charged Higgs is always negative. The charged Higgs of our model has mass around 2 TeV. So, the charged Higgs contribution to $C_7$ is non-negligible and is in the correct direction. The chargino-stop loop contribution of $C_7^{\text{MSSM}}$ has the following form \cite{119,23}

$$C_7^{\text{MSSM}} = \frac{\mu A_t \tan \beta}{m_t^2} \text{sign}(C_7^{\text{SM}}). \quad (3.26)$$

Since sign($\mu A_t$) is negative in our model, this term contributes to $C_7$ in the wrong direction. Hence, to reduce the contribution of this term, our model favors large scalar masses.

From our global $\chi^2$ analysis, we see that the calculated value of $P_4'$ in the high $q^2$ bin does not depend on $m_{16}$, which is expected. In addition, the value of $P_4'$ calculated in our model is in agreement with the SM. Hence, our results are in agreement with previous analysis that the tension in $P_4'$ cannot be explained in the MSSM. As shown in Table 3.3, the tension of $F_L$ and $P_5'$ with the experimental values decreases as $m_{16}$ increases. This is again in agreement with our expectation as explained above.

\textsuperscript{23}Note, the equation for the chargino contribution to $C_7^{\text{MSSM}}$ given in Eq. 21, Ref. \cite{114} apparently has the wrong sign.
SUSY corrections to the $W$ mass

The SUSY correction for $M_W$ is given by \cite{120,122}

$$
\delta M_W \approx \frac{M_W}{2} \frac{c_W^2}{s_W^2} \Delta \rho, \quad (3.27)
$$

where the one-loop squark contribution is given by

$$
\Delta \rho_{\text{SUSY}}^1 = \frac{3G\mu}{8\sqrt{2\pi^2}} \left[ -s_W^2 c_W^2 F_0(m_{t_1}^2, m_{t_2}^2) - s_W^2 c_W^2 F_0(m_{b_1}^2, m_{b_2}^2) 
\quad + c_W^2 s_W^2 F_0(m_{t_1}^2, m_{b_1}^2) + c_W^2 s_W^2 F_0(m_{t_2}^2, m_{b_2}^2) 
\quad + s_W^2 c_W^2 F_0(m_{t_1}^2, m_{t_2}^2) + s_W^2 c_W^2 F_0(m_{b_1}^2, m_{b_2}^2) \right], \quad (3.28)
$$

where $s_W = \sin \theta_W$, $c_W = \cos \theta_W$, $s_{\tilde{q}} = \sin \theta_{\tilde{q}}$, $c_{\tilde{q}} = \cos \theta_{\tilde{q}}$ and

$$
F_0(x,y) = x + y - \frac{2xy}{x-y} \ln \frac{x}{y}. \quad (3.29)
$$

$F_0$ has properties of $F_0(x,x) = 0$ and $F_0(x,0) = x$. Hence, we see that when the mass splitting of the squarks is large, the SUSY contribution to the 1-loop $M_W$ can be significant. This is in agreement with our analysis which shows that the pull from $M_W$ increases as the value of $m_{16}$ increases above 20 TeV (see Table 3.3). Hence, SUSY corrections to $M_W$ are significant and they can go in the right direction.

SM Higgs Mass

Fitting to the Higgs mass also constrains the value of $m_{16}$. The dominant one-loop contribution to the Higgs mass is given by

$$
m_h^2 \approx m_Z^2 \cos^2 2\beta + \frac{3}{(4\pi^2)} \frac{m_t^4}{v^2} \left[ \ln \frac{M^2_{\text{SUSY}}}{m_t^2} + \frac{X_t^2}{M^2_{\text{SUSY}}} \left( 1 - \frac{X_t^2}{12M^2_{\text{SUSY}}} \right) \right], \quad (3.30)
$$

where $X_t = A_t - \mu / \tan \beta$ is the stop mixing parameter and $M^2_{\text{SUSY}} = m_{t_1} m_{t_2}$. In our model, $X_t < -\sqrt{6} M_{\text{SUSY}}$ and the ratio $X_t/M_{\text{SUSY}}$ becomes less negative as $m_{16}$ increases. Hence, as $m_{16}$ increases, the Higgs mass also increases. The pull in $\chi^2$ due to $M_h$ has a minimum around $m_{16} = 25$ TeV as shown in Table 3.3.

In summary, the observable $P'_4$ of $B^0 \to K^{*0}\mu^+\mu^-$ in the high $q^2$ bin does not depend
### Table 3.3: The pull, theoretical value, and experimental measurements for the set of observables for universal $M_{1/2}$ model with $M_{\tilde{g}} \approx 1.2$ TeV that contribute directly to a minimum of $\chi^2$ at $m_{16} \approx 25$ TeV.

In addition, the observable $P'_4$ of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ for high $q^2$ bins is also included to illustrate that it is independent on $m_{16}$. See text for more discussions.

<table>
<thead>
<tr>
<th>$m_{16}$/TeV</th>
<th>$M_W$/GeV</th>
<th>$M_h$/GeV</th>
<th>$BR(B \rightarrow \tau \nu)$</th>
<th>$F_L (B^0 \rightarrow K^{*0} \mu^+ \mu^-)_{low \ q^2}$</th>
<th>$P'<em>4 (B^0 \rightarrow K^{*0} \mu^+ \mu^-)</em>{low \ q^2}$</th>
<th>$P'<em>4 (B^0 \rightarrow K^{*0} \mu^+ \mu^-)</em>{high \ q^2}$</th>
<th>$P'<em>5 (B^0 \rightarrow K^{*0} \mu^+ \mu^-)</em>{low \ q^2}$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_{16}$/TeV</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_W$/GeV</td>
<td>80.470</td>
<td>80.461</td>
<td>80.460</td>
<td>80.478</td>
<td>80.545</td>
<td>80.385</td>
<td>11.400</td>
<td></td>
</tr>
<tr>
<td>$M_h$/GeV</td>
<td>117.990</td>
<td>122.130</td>
<td>124.655</td>
<td>126.270</td>
<td>127.692</td>
<td>125.700</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$BR(B \rightarrow \tau \nu) \times 10^5$</td>
<td>6.633</td>
<td>6.134</td>
<td>6.230</td>
<td>6.222</td>
<td>6.178</td>
<td>11.400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_L (B^0 \rightarrow K^{*0} \mu^+ \mu^-)_{low \ q^2}$</td>
<td>0.743</td>
<td>0.735</td>
<td>0.725</td>
<td>0.721</td>
<td>0.719</td>
<td>0.650</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P'<em>4 (B^0 \rightarrow K^{*0} \mu^+ \mu^-)</em>{low \ q^2}$</td>
<td>0.817</td>
<td>0.671</td>
<td>0.592</td>
<td>0.572</td>
<td>0.566</td>
<td>0.580</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P'<em>4 (B^0 \rightarrow K^{*0} \mu^+ \mu^-)</em>{high \ q^2}$</td>
<td>1.219</td>
<td>1.219</td>
<td>1.219</td>
<td>1.219</td>
<td>1.219</td>
<td>-0.180</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P'<em>5 (B^0 \rightarrow K^{*0} \mu^+ \mu^-)</em>{low \ q^2}$</td>
<td>-0.730</td>
<td>-0.553</td>
<td>-0.463</td>
<td>-0.434</td>
<td>-0.424</td>
<td>0.210</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.3: The pull, theoretical value, and experimental measurements for the set of observables for universal $M_{1/2}$ model with $M_{\tilde{g}} \approx 1.2$ TeV that contribute directly to a minimum of $\chi^2$ at $m_{16} \approx 25$ TeV. In addition, the observable $P'_4$ of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ for high $q^2$ bins is also included to illustrate that it is independent on $m_{16}$. See text for more discussions.

On $m_{16}$ while the observables $F_L$ and $P'_5$ of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ in the low $q^2$ bin favor large $m_{\tilde{t}}$, which is in agreement with Altmannshofer et al. [114]. On the other hand, fitting to $M_W$ and $M_h$ disfavors large $m_{\tilde{t}}$. These effects collectively contribute to having a minimum $\chi^2$ around $m_{16} \approx 25$ TeV. The plot of $\chi^2$ versus $m_{16}$ is shown in Fig. 3.2.

### 3.3.3 Bounds on $M_{\tilde{g}}$

To obtain a better picture for the favored value of gluino mass, we plotted two contour plots of $M_{\tilde{g}}$ versus $m_{16}$. One for the universal boundary condition $\alpha = 0$ and another for
Figure 3.2: Plot of $\chi^2$ versus $m_{16}$ for the universal $M_{1/2}$ model. The $\chi^2$ in this plot is due to the set of observables in Table 3.3, which contributes directly to a minimum of $\chi^2$ at $m_{16} \approx 25$ TeV.
the mirage boundary condition with $\alpha = 1.5$. The current bound on $M_{\tilde{g}}$ for our model is around 1.2 TeV [100]. The contour plots are created by calculating $\chi^2$/d.o.f for 25 equally distributed values of $m_{16}$ and $M_{1/2}$, which gives us $1 < M_{\tilde{g}} < 2$ TeV. We then use cubic interpolation to obtain the smooth contours of $\chi^2$/d.o.f.

In addition to the contour lines of the $\chi^2$/d.o.f, we also plotted a 4$\sigma$ contour line. From this, we see that for mirage boundary conditions $M_{\tilde{g}} \lesssim 1.8$ TeV. However, for universal boundary condition, the 4$\sigma$ $M_{\tilde{g}}$ bound can be as high as 3 TeV, which is not shown in the figure. Hence, for mirage boundary conditions, we expect the 4$\sigma$ bound on the gluino mass to be within reach in the next run of the LHC. As pointed out by Ref. [100], the dominant decay mode of the gluino in the universal gaugino mass boundary condition is $tb\tilde{\chi}_1^\pm$. The remaining decay modes are $tt\tilde{\chi}_i^0$ and $b\bar{b}\tilde{\chi}_i^0$ for $i = 1, 2$. On the other hand, the dominant mode for gluino decay in the mirage gaugino mass boundary condition is $tb\tilde{\chi}_i^\pm$ for $i = 1, 2$. In all cases, the dominant signature for gluinos in this model is given by $b$ jets, leptons and missing $E_T$ [100].

### 3.3.4 Fine-Tuning

We studied the fine-tuning of our model using the fine-tuning measure introduced by Ellis et al. [123], and studied in detail by Barbieri and Giudice [124]:

$$
\Delta_{BG} = \max \Delta_{a_i}, \quad \Delta_{a_i} = \left| \frac{\partial \ln M_Z^2}{\partial \ln a_i^2} \right|,
$$

(3.31)

where $a_i$s are input parameters of the model. This fine-tuning measure calculates the sensitivity of $M_Z$ due to a small variation of the input parameters defined at the GUT scale.

Electroweak symmetry is broken radiatively in our model. From radiative electroweak symmetry breaking, the $CP$-odd Higgs mass, $m_A$, and the $\mu$-term are calculated at one-loop [104]. This calculation requires the physical $Z$ pole mass, $M_Z$. Hence, in our model, $M_Z$ is fit precisely. To make sure that radiative electroweak symmetry breaking is consistent, $m_A$ and $\mu$ are calculated iteratively until they converge.

On the other hand, when we calculate fine-tuning using Eq. (3.31) we use the benchmark
Figure 3.3: These plots show the contour of $\chi^2$/d.o.f versus $M_{\tilde{g}}$ and $m_{16}$. The $4\sigma$ bound is also included in the plot. For $\alpha = 1.5$, the upper bound is within reach of the next run of LHC. In addition, we also see that our model favors $m_{16} \approx 25$ TeV.
points. The benchmark points are the inputs that produce minimum $\chi^2$ value for their respective values of $m_{16}$ and $M_{1/2}$. Hence, at each benchmark point, radiative electroweak symmetry breaking is consistent. Thus, instead of fixing $M_Z$ and calculating $m_A$ and $\mu$ iteratively, we then use the value of $m_A$ and $\mu$ to calculate $M_Z$. We then compare this value of $M_Z$ to the exact value to obtain the fine tuning parameter $\Delta_{BG}$.

The input parameters that we vary are $a_i = \{\mu, M_{1/2}, m_{16}, m_{H_u}, m_{H_d}, A_0\}$. The results of our calculations are summarized in Table 3.4. These results can be understood by the running of the GUT scale parameters that contribute to $Z$ mass. For $\tan \beta = 10$, $M_Z$ written in terms of the GUT scale parameters is [125–128]

$$M_Z^2 \approx -2.18\mu^2 + 4.22M_{1/2}^2 - 0.82M_{1/2}A_0 + 0.22A_0^2 - 1.27m_{H_u} - 0.053m_{H_d}^2 + 1.34m_{16}^2.$$ (3.32)

Although the above equation is derived for $\tan \beta = 10$, to the lowest order approximation, we do not expect this result to change drastically when $\tan \beta$ increases to $\approx 50$. The calculated fine tuning values shown in Table 3.4 are of the same order as the fine tuning predicted from this equation. As an example, by direct substitution of the $m_{16} = 20$ TeV benchmark points into the above equation, we find that if $m_{H_u,d}/m_{16}$ and $A_0/m_{16}$ are fixed, then $\Delta_{BG} \approx 2000$ is of the same order as our calculation.

From Table 3.4 we see that if there are no constraints on the input parameters (first five rows), then the fine-tuning is about one part in $10^5$. However, if the GUT scale parameters are constrained such that $m_{H_u,d}/m_{16} \approx \sqrt{2}$ and $A_0/m_{16} \approx -2$, then the fine-tuning of our theory is reduced to about one part in 500. This suggests that, in a more fundamental natural theory, the ratio of $m_{16}$ with $m_{H_u,d}$ and $A_0$ could be fixed naturally. In the context of the Baer et al., who argues that dependent terms should be combined into a single independent quantity before evaluating fine-tuning [125], we claim that $m_{16}, m_{H_u,d},$ and $A_0$ might be dependent quantities in a more fundamental theory. Hence, these quantities should be combined before calculating fine-tuning. We discuss one possible partial example of a more fundamental theory that naturally combines these quantities in App. E.
Fine-Tuning of Benchmark Points with $\alpha = 0$ and $M_\tilde{g} \approx 1.2$ TeV

<table>
<thead>
<tr>
<th>Varying Parameters</th>
<th>$m_{16}$</th>
<th>10 TeV</th>
<th>15 TeV</th>
<th>20 TeV</th>
<th>25 TeV</th>
<th>30 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>140</td>
<td>190</td>
<td>210</td>
<td>360</td>
<td>490</td>
<td></td>
</tr>
<tr>
<td>$M_{1/2}$</td>
<td>260</td>
<td>340</td>
<td>400</td>
<td>430</td>
<td>450</td>
<td></td>
</tr>
<tr>
<td>$m_{16}$</td>
<td>12000</td>
<td>27000</td>
<td>47000</td>
<td>74000</td>
<td>110000</td>
<td></td>
</tr>
<tr>
<td>$m_{H_d}$</td>
<td>760</td>
<td>1500</td>
<td>3900</td>
<td>6100</td>
<td>8700</td>
<td></td>
</tr>
<tr>
<td>$m_{H_u}$</td>
<td>10000</td>
<td>23000</td>
<td>40000</td>
<td>62000</td>
<td>89000</td>
<td></td>
</tr>
<tr>
<td>$A_0$</td>
<td>9300</td>
<td>21000</td>
<td>39000</td>
<td>61000</td>
<td>85000</td>
<td></td>
</tr>
<tr>
<td>$m_{16}$ with $A_0/m_{16}$ fixed</td>
<td>22000</td>
<td>49000</td>
<td>87000</td>
<td>130000</td>
<td>190000</td>
<td></td>
</tr>
<tr>
<td>$m_{16}$ with $m_{H_u,d}/m_{16}$ fixed</td>
<td>9500</td>
<td>22000</td>
<td>40000</td>
<td>62000</td>
<td>86000</td>
<td></td>
</tr>
<tr>
<td>$m_{16}$ with $m_{H_u,d}/m_{16}, A_0/m_{16}$ fixed</td>
<td>240</td>
<td>400</td>
<td>630</td>
<td>740</td>
<td>850</td>
<td></td>
</tr>
</tbody>
</table>

Fine-Tuning of Benchmark Points with $\alpha = 1.5$ and $M_\tilde{g} \approx 1.2$ TeV

<table>
<thead>
<tr>
<th>Varying Parameters</th>
<th>$m_{16}$</th>
<th>10 TeV</th>
<th>15 TeV</th>
<th>20 TeV</th>
<th>25 TeV</th>
<th>30 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>110</td>
<td>240</td>
<td>290</td>
<td>340</td>
<td>380</td>
<td></td>
</tr>
<tr>
<td>$M_{1/2}$</td>
<td>320</td>
<td>420</td>
<td>500</td>
<td>560</td>
<td>580</td>
<td></td>
</tr>
<tr>
<td>$m_{16}$</td>
<td>12000</td>
<td>28000</td>
<td>48000</td>
<td>74000</td>
<td>110000</td>
<td></td>
</tr>
<tr>
<td>$m_{H_d}$</td>
<td>750</td>
<td>1500</td>
<td>4600</td>
<td>6000</td>
<td>8500</td>
<td></td>
</tr>
<tr>
<td>$m_{H_u}$</td>
<td>10000</td>
<td>23000</td>
<td>39000</td>
<td>62000</td>
<td>89000</td>
<td></td>
</tr>
<tr>
<td>$A_0$</td>
<td>9200</td>
<td>21000</td>
<td>39000</td>
<td>60000</td>
<td>86000</td>
<td></td>
</tr>
<tr>
<td>$m_{16}$ with $A_0/m_{16}$ fixed</td>
<td>22000</td>
<td>49000</td>
<td>87000</td>
<td>130000</td>
<td>190000</td>
<td></td>
</tr>
<tr>
<td>$m_{16}$ with $m_{H_u,d}/m_{16}$ fixed</td>
<td>9600</td>
<td>21000</td>
<td>39000</td>
<td>61000</td>
<td>87000</td>
<td></td>
</tr>
<tr>
<td>$m_{16}$ with $m_{H_u,d}/m_{16}, A_0/m_{16}$ fixed</td>
<td>330</td>
<td>450</td>
<td>670</td>
<td>890</td>
<td>1100</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.4: Without fixing any ratios, the fine-tuning is one part in $10^5$. When the ratio of $m_{H_u,d}/m_{16}$ and $A_0/m_{16}$ are fixed, the fine-tuning is about one part in 500. This suggests that these ratios should be fixed in a more fundamental natural theory. In addition, fine-tuning increases as $m_{16}$ increases. Hence, in our model, small $m_{16}$ is favored by naturalness.
3.4 Predictions and Discovery Prospects

The mass spectrum of the benchmark point of $M_\tilde{g} \approx 1.2$ TeV and $m_{16} = 25$ TeV is shown in Table 3.5. From the $\chi^2$ analysis, we see that the scalar masses are predicted to be around 5 TeV, while the first and second generation scalars have mass around $m_{16} \approx 25$ TeV. With scalars in this mass range, the stop in our model does not completely decouple and can have non-negligible effects on flavor physics. In addition, our light Higgs is SM-like with the heavy Higgs with mass around 2 TeV.

In Table 3.5 we give the light sparticle masses, the $CP$ violating angle for neutrino oscillations $\delta$, $\text{BR}(\mu \rightarrow e\gamma)$, and the electric dipole moment of the electron for two different values of $M_\tilde{g}$ and for $\alpha = 0$ and 1.5. Note that, in general, the gauginos are the lightest sparticles. In addition, $\text{BR}(\mu \rightarrow e\gamma)$ and the electric dipole moment of the electron are within reach of future experiments.

3.5 Conclusion

We have analyzed a three family SO(10) SUSY GUT with Yukawa unification for the third family. The model gives reasonable fits to fermion masses and mixing angles, as well as many other low-energy observables. See App. H.1 for some benchmark points of the global $\chi^2$ analysis. A plot of $\chi^2$ versus $m_{16}$ is given in Fig. 3.1 and $\chi^2$ contours in the two dimensional plane of $M_\tilde{g}$ versus $m_{16}$ is given in Fig. 3.3.

We performed an analysis with universal gaugino masses and with non-universal gaugino mass with splitting determined by “mirage mediation” boundary conditions described by Eq. 3.24. For mirage mediated gaugino masses, we considered $\alpha = 1.5$, which is consistent with a well-tempered dark matter candidate [103]. In both cases the model favors $m_{16} \approx 25$ TeV. Nevertheless, due to RG running [129], stops and sbottoms have masses of the order 5 TeV, while the first two family scalar masses are of the order of $m_{16}$. With $m_{16}$ lying in this mass range, stops in our model do not completely decouple from low-energy flavor observables (see Sec. 3.3.2). Best fit points are found with a gluino mass less than 2 TeV. Our gluinos decay predominantly into third generation quarks [100]. Moreover, in
\[
m_{16} \quad 25 \quad 25 \quad 25 \quad 25
\]
\[
\alpha \quad 0 \quad 1.5 \quad 0 \quad 1.5
\]
\[
\chi^2/d.o.f \quad 2.158 \quad 2.275 \quad 2.220 \quad 2.505
\]
\[
m_{\tilde{t}_1} \quad 4.903 \quad 5.011 \quad 4.909 \quad 5.249
\]
\[
m_{\tilde{t}_2} \quad 6.021 \quad 6.120 \quad 6.033 \quad 6.301
\]
\[
m_{\tilde{b}_1} \quad 5.989 \quad 6.088 \quad 6.455 \quad 6.606
\]
\[
m_{\tilde{b}_2} \quad 6.454 \quad 6.541 \quad 6.445 \quad 6.267
\]
\[
m_{\tilde{\tau}_1} \quad 9.880 \quad 9.931 \quad 9.912 \quad 10.040
\]
\[
m_{\tilde{\tau}_2} \quad 15.369 \quad 15.365 \quad 15.393 \quad 15.516
\]
\[
M_{\tilde{g}} \quad 1.202 \quad 1.187 \quad 1.613 \quad 1.690
\]
\[
m_{\tilde{\chi}_1^0} \quad 0.203 \quad 0.551 \quad 0.279 \quad 0.900
\]
\[
m_{\tilde{\chi}_2^0} \quad 0.404 \quad 0.665 \quad 0.538 \quad 1.018
\]
\[
m_{\tilde{\chi}_1^+} \quad 0.404 \quad 0.665 \quad 0.538 \quad 1.018
\]
\[
m_{\tilde{\chi}_2^+} \quad 1.128 \quad 1.243 \quad 1.232 \quad 1.537
\]
\[
M_A \quad 2.194 \quad 2.082 \quad 2.477 \quad 3.352
\]
\[
\sin \delta \quad -0.289 \quad -0.482 \quad -0.520 \quad -0.576
\]
\[
BR(\mu \rightarrow e\gamma)/10^{-13} \quad 1.108 \quad 1.430 \quad 1.239 \quad 1.340
\]
\[
edm_{e}/10^{-30} \text{ e cm} \quad -1.403 \quad -3.305 \quad -1.763 \quad -5.886
\]

Table 3.5: Predictions with \( m_{16} = 25 \text{ TeV} \) for \( M_{\tilde{g}} \approx 1.2 \) and \( 1.6 \text{ TeV} \). All masses in the table are in TeV units. Our prediction for the branching ratio \( \mu \rightarrow e\gamma \) is consistent with the current upper bound of \( 5.7 \times 10^{-13} \) \[7\]. In addition, our prediction of the electron electric dipole moment is consistent with the current upper bound of \( 10.5 \times 10^{-28} \text{ e cm} \) \[7\].
a previous analysis [100] we showed that the dominant LHC signature for gluinos in the model is given by $b$-jets, leptons, and missing $E_T$. Note that, in general, the gauginos are the lightest sparticles. The $CP$-odd Higgs mass is of the order of 2 TeV, thus the light Higgs couplings are very much SM-like. In Table 3.5 we present additional predictions. We give the predictions for the $CP$ violating angle for neutrino oscillations $\delta$, $BR(\mu \rightarrow e\gamma)$, and the electric dipole moment of the electron for two different values of $M_{\tilde{g}}$ and for $\alpha = 0$ and 1.5. In addition, $BR(\mu \rightarrow e\gamma)$ and the electric dipole moment of the electron are within reach of future experiments. Thus this theory is eminently testable!

We evaluated the amount of high scale fine-tuning of our model. In general we find fine-tuning of the order one part in $10^5$. However we note that with particular boundary conditions at the GUT scale (when the ratio of $m_{16}$ to $A_0$ and $m_{H_{u,d}}$ are fixed at $A_0/m_{16} \approx -2$ and $m_{H_{u,d}}/m_{16} \approx \sqrt{2}$) the fine-tuning is reduced to one part in 500. We do not have a fundamental theory that gives these two ratios naturally, Nevertheless, in such a fundamental theory the amount of fine-tuning is reduced considerably.

Finally, with the large value of $m_{16} \sim 25$ TeV we expect the gravitino mass to be at least this large. Perhaps it is large enough to avoid a cosmological gravitino problem [130]. In addition, moduli may also be suitably heavy to avoid a cosmological moduli problem [131–134]. Hence the scalar masses are clearly in an intermediate range, i.e. too heavy to be “natural” and lighter than “Split SUSY.” We thus are positioned on the border between these two limiting cases, i.e. this is “SUSY on the Edge”.

80
In this chapter, we consider a SUSY GUT of a PS gauge group. Similar to previous chapter, this model has three families with a $D_3 \times [U(1) \times Z_3 \times Z_3]$ family symmetry and Yukawa unification. Previous analyses showed that this model extended by an inflation sector can fit the tensor-to-scalar ratio, the scalar spectral index, and the scalar power spectrum [78].

The primary goal of this chapter is to show that by modifying only the Yukawa sector of the model, low-energy fits can be improved significantly from a $\chi^2$/dof $\geq 2.2$ for $M_3 = 1.6$ TeV (see Chapter 3) to a $\chi^2$/dof $= 1.12$ for $M_3 = 1.9$ TeV. In addition, we also found that the most recent ATLAS and CMS data require that $M_3 \gtrsim 1.9$ TeV. For more details on the model, the analysis procedure and the phenomenology aspects of the model, please refer to the previous chapter and Refs. [96] [100].

This rest of the chapter is organized as follows. In Sec. 4.1, we modify the Yukawa sector of the theory in Chapter 3 and choose PS group as the gauge group in order to improve the fits to $\sin(2\beta)$, $m_u$, and $m_d$. In Sec. 4.2, we explain the global $\chi^2$ analysis procedure along with the results showing the improvements over the global $\chi^2$ fits in Chapter 3. In Sec. 4.3,
we reinterpret the ATLAS and CMS gluino simplified model analyses to obtain the current gluino mass bounds of this model. Finally, the discovery prospects and predictions of this model are discussed in Sec. 4.4 and we conclude this chapter in Sec. 4.5.

4.1 Model

In this chapter, we modify the Yukawa sector of the theory in order to improve the fits to \( \sin(2\beta) \), \( m_u \), and \( m_d \). Instead of writing the superpotential in SO(10) notation, Eq. 3.1, we can rewrite it using PS fields:

\[
W_{PS} = \lambda Q_3 H \bar{Q}_3 + Q_a \bar{F}_a + F_a \bar{Q}_a + F_c \phi_a \bar{Q}_a + 15 \frac{\phi_a}{M} Q_3 + 15 \frac{\tilde{\phi}_a}{M} \bar{Q}_a + 15 \tilde{\phi}_a \bar{Q}_a + A \bar{Q}_a
\]

(4.1)

where \( \{Q, F\} = (4, 2, 1) \), \( \{\bar{Q}, \bar{F}\} = (\bar{4}, 1, 2) \) and \( H = (1, 2, 2) \) under PS symmetry. 15 is the adjoint representation of SU(4)C that is assumed to obtain VEV in the \( B - L \) direction.

In addition, \( F_a^c \) and \( \bar{F}_a^c \) are the conjugate of \( F_a \) and \( \bar{F}_a \), and \( M_F = M_X \). By requiring a PS instead of a SO(10) gauge symmetry, we have more freedom in adding new terms to the superpotential.

To improve the fit of \( m_u \) and \( m_d \), we introduce the following terms to the superpotential\(^{24}\)

\[
F_a^c \Theta' \bar{Q}_a + F_a^c \Theta' Q_a + \bar{F}_a^c \tilde{\Theta}_a Q_a - F_a^c \tilde{\Theta}_a Q_a
\]

(4.2)

where \( \Theta' \) transforms as a trivial singlet and \( \tilde{\Theta}_a, a = 1, 2 \) transforms as doublets under \( D_3 \) symmetry. In addition, we assume that \( \tilde{\Theta}_a \) obtains a VEV of the form

\[
\langle \tilde{\Theta}_a \rangle = \begin{pmatrix} \tilde{\Theta}_1 \\ 0 \end{pmatrix}
\]

(4.3)

\(^{24}\)By adding only the \( \Theta' \) terms, we are able to fit \( m_u \) and modestly improve the fit of \( m_d \). Having both the \( \Theta' \) and \( \tilde{\Theta} \) terms significantly improves the fit of both \( m_u \) and \( m_d \).
and both $\Theta'$ and $\tilde{\Theta}_1$ are real parameters. With these terms, the superpotential of this model becomes

$$W_{\text{PS}} = \lambda Q_3 H\bar{Q}_3 + Q_a H\bar{F}_a + F_a H\bar{Q}_a$$

$$+ F_a^c \left( \frac{M_F}{M} \phi_a Q_3 + 15 \tilde{\phi}_a Q_a + AQ_a + \Theta' Q_a + \tilde{\Theta}_a Q_a \right)$$

$$+ F_a^c \left( \frac{M_F}{M} Q_3 + 15 \tilde{\phi}_a Q_a + AQ_a + \Theta' Q_a - \tilde{\Theta}_a Q_a \right) , \quad (4.4)$$

Notice that a PS gauge symmetry allows the last term in the last two lines of the above equation to have different sign. With SO(10) gauge symmetry, we are unable to make these two terms have opposite sign without introducing a VEV in the $B - L$ direction. We find that when we introduce such a VEV, we are able to fit the electron mass, but both $m_u$ and $m_d$ are too large as in our previous analysis.

With the new terms in the Yukawa sector, we obtain the following effective superpotential after integrating out the Froggatt-Nielsen states:

$$W^\text{eff}_{\text{PS}} \supset [y^u]_{ij} q_i H_u \bar{u}_j + [y^d]_{ij} q_i H_d \bar{d}_j + [y^e]_{ij} \ell_i H_e \bar{e}_j + [y^\nu]_{ij} \ell_i H_u \bar{\nu}_j , \quad (4.5)$$

where $i, j = 1, 2, 3$ are the family indices and the Yukawa matrices of this model become (see App. B for derivation)

$$y^u = \left( \begin{array}{ccc}
-\bar{\delta} G^-_{1,-\frac{4}{3};1,\frac{1}{3}} & \bar{\epsilon}' G^-_{1,-\frac{4}{3};1,\frac{1}{3}} + \theta' G^+_{1,-\frac{4}{3};1,\frac{1}{3}} & -\epsilon \xi G_{1,-\frac{4}{3}} \\
-\epsilon \delta G^-_{1,-\frac{4}{3};1,\frac{1}{3}} & \epsilon G^-_{1,-\frac{2}{3};1,\frac{1}{3}} & -\epsilon \xi G_{1,-\frac{4}{3}} \\
\epsilon \xi G_{1,\frac{1}{3}} & \epsilon G_{1,\frac{1}{3}} & \lambda
\end{array} \right) , \quad (4.6)$$

$$y^d = \left( \begin{array}{ccc}
-\bar{\delta} G^-_{3,-\frac{4}{3};1,\frac{1}{3}} & \bar{\epsilon}' G^-_{3,-\frac{4}{3};1,\frac{1}{3}} + \theta' G^+_{3,-\frac{4}{3};1,\frac{1}{3}} & -\epsilon \xi G_{3,-\frac{4}{3}} \\
-\epsilon \delta G^-_{3,-\frac{4}{3};1,\frac{1}{3}} & \epsilon G^-_{3,-\frac{2}{3};1,\frac{1}{3}} & -\epsilon \xi G_{3,-\frac{4}{3}} \\
\epsilon \xi G_{3,\frac{1}{3}} & \epsilon G_{3,\frac{1}{3}} & \lambda
\end{array} \right) , \quad (4.7)$$
\[
\begin{align*}
y^\nu &= \begin{pmatrix}
\tilde{\theta}G_{-3,-1;1,2} & -\epsilon'G_{-3,-1;1,2} + \theta'G_{+,-3,-1;1,2} & 3\epsilon\xi G_{1,2} \\
\epsilon'G_{-3,-1;1,2} + \theta'G_{+,-3,-1;1,2} & 3\epsilon G_{-3,-1;1,2} & 3\epsilon G_{1,2} \\
-3\epsilon G_{-3,-1} & -3\epsilon G_{-3,-1} & \lambda
\end{pmatrix}, & (4.8) \\
y^\nu &= \begin{pmatrix}
\tilde{\theta}G_{-3,-1;5,0} & -\epsilon'G_{-3,-1;5,0} + \theta'G_{+,-3,-1;5,0} & 3\epsilon\xi G_{5,0} \\
\epsilon'G_{-3,-1;5,0} + \theta'G_{+,-3,-1;5,0} & 3\epsilon G_{-3,-1;5,0} & 3\epsilon G_{5,0} \\
-3\epsilon G_{-3,-1} & -3\epsilon G_{-3,-1} & \lambda
\end{pmatrix}, & (4.9)
\end{align*}
\]

where

\[
\theta' = \frac{1}{2} \frac{\theta'}{M_0}, & \quad (4.10) \\
\tilde{\theta} = \frac{1}{2} \frac{M_G \tilde{\Theta}_1}{M_0 M}. & \quad (4.11)
\]

In Chapter 3, we took the limit of \( \beta \ll \alpha \). Although this limit simplifies the interpretation of the Yukawa matrices, it does not reduce the number of input parameters. Since previous model does not produce good fits to these observables, we have decided to include the full \( G_{x,y} \), defined in Eq. 3.3, in this analysis. Without taking this limit, the arbitrary parameters in the Yukawa sector are \( \lambda, \epsilon, \epsilon', \tilde{\epsilon}, \xi, \alpha, \beta, \theta' \), and \( \tilde{\theta} \). Of these parameters, \( \epsilon', \xi, \alpha, \beta, \theta' \), and \( \tilde{\theta} \) are complex while \( \lambda, \epsilon, \) and \( \tilde{\epsilon} \) are real. Comparing with Chapter 3, \( \tilde{\epsilon} \) is a complex parameter and \( \epsilon' \) is a real parameter. However, we found that taking \( \epsilon' \) as a complex parameter and \( \tilde{\epsilon} \) as a real parameter produces a better fit for \( \sin(2\beta) \).

Instead of 11 arbitrary parameters in the Yukawa sector of Chapter 3 to improve the fit to \( \sin(2\beta) \), \( m_u \), and \( m_d \), we have introduced two additional real parameters, \( \theta' \) and \( \tilde{\theta} \). Hence, the fermion sector now has 17 parameters - 13 Yukawa parameters, \( \tan \beta \) and three right-handed neutrino masses, while the SM has 19 observables - nine fermion masses, four CKM matrix elements, two neutrino mass differences, three real neutrino mixing angles, and one neutrino \( CP \) violating phase. Hence, this model has two predictions in the fermion sector.
Table 4.1: 26 input parameters of the PS model.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input Parameters</th>
<th>No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauge</td>
<td>$\alpha_G, M_G, \epsilon_3$</td>
<td>3</td>
</tr>
<tr>
<td>SUSY (GUT scale)</td>
<td>$m_{16}, M_{1/2}, A_0, m_{H_u}, m_{H_d}$</td>
<td>5</td>
</tr>
<tr>
<td>Yukawa Textures</td>
<td>$\lambda, \epsilon, \tilde{\epsilon}, \xi, \alpha, \beta, \theta', \tilde{\theta}, \phi_{\ell'}, \phi_{\xi}, \phi_{\alpha}, \phi_{\beta}$</td>
<td>13</td>
</tr>
<tr>
<td>Neutrino</td>
<td>$M_{R_1}, M_{R_2}, M_{R_3}$</td>
<td>3</td>
</tr>
<tr>
<td>SUSY (EW Scale)</td>
<td>$\tan \beta, \mu$</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>26</td>
</tr>
</tbody>
</table>

Boundary Conditions

All the boundary conditions in the gauge sector and SUSY sector are the same as the universal $M_{1/2}$ boundary condition in Chapter 3. In this chapter, we are mainly interested in showing that the global $\chi^2$ fit can be improved significantly with a slight modification to the superpotential of the charged fermion sector. Hence, instead of consider two different boundary conditions for gaugino masses, we will only consider universal gaugino mass boundary conditions. In addition, the results in Chapter 3 between universal $M_{1/2}$ boundary condition and mirage mediated $M_{1/2}$ boundary condition are very similar other than mirage mediated $M_{1/2}$ having a slightly larger $\chi^2$.

4.2 Global Chi-Squared Analysis

4.2.1 Analysis Procedure

The renormalization procedure in this chapter is identical to that of the previous chapter. However, the theoretical errors in this chapter are increased by a few percent due to the large scatter of SUSY particle masses between $m_{16}$ and $M_Z$. We run renormalization group equations from the lightest right-handed neutrino mass scale to the electroweak scale using two-loop renormalization group equations and remove the over-running by performing one-loop threshold corrections. The theoretical errors are obtained by performing threshold corrections in different orders; that is performing gauge threshold corrections before or
after Yukawa threshold corrections. These errors were neglected in Chapter 3.

In this chapter, we fit this model to 51 observables listed in Table 4.2. As a comparison with previous analyses, we have added the following observables: $V_{ud}$, $V_{cd}$, $V_{cs}$, $V_{tb}$, $m_u$, and $m_d$. In addition, we have updated all experimental values to the latest values available in the PDG and HFAG [41, 135]. The values of $|V_{cb}|$ and $|V_{ub}|$ used are the average of the inclusive and exclusive values from the PDG with error bars overlapping the inclusive and exclusive error bars. We also updated the publicly available software, SuperIso v3.5 and SUSY_FLAVOR v2.54 [105, 136]. Every $\chi^2$ minimization of this chapter is done by fixing $m_{16}$ and $M_{1/2}$ in a grid of points and then minimizing with respect to the other 24 parameters. Since we are fitting to 51 observables, for each fixed value of $m_{16}$ and $M_{1/2}$, we have 27 dof.

Similar to Chapter 3, a way to visualize the gluino mass that this model favors is to make a $\chi^2$/dof contour plot of the gluino mass as a function of the scalar mass at the GUT scale, $m_{16}$. To produce this plot, we perform the $\chi^2$ minimization for gluino mass ranges from 1.6 to 2.8 TeV with an increment of 0.2 TeV and scalar mass at the GUT scale ranging from 10 to 30 TeV with an increment of 5 TeV. We control the gluino mass by selecting the appropriate value of $M_{1/2}$ at the GUT scale. We then perform a two-dimensional cubic spline interpolation on these 30 points to obtain a two-dimensional surface of $\chi^2$/dof. To increase the likelihood that each of these 30 points is at the minimum, we perform minimization repeatedly until the change in $\chi^2$ after five repetitions is lower than 0.001. After the $\chi^2$ value settles down, we make a small shift in the values of input parameters other than $M_{1/2}$ and $m_{16}$ and reperform the minimization to make sure that the shifted parameters eventually return to the original value. With this procedure, we are confident that the points that we obtain are at least in a very deep local minimum, if not the global minimum.

4.2.2 Results

Figure 4.1 shows the $\chi^2$/dof contour plot with gluino mass ranging from 1.7 to 2.7 TeV and the scalar mass at the GUT scale ranging from 10 to 30 TeV. The values of $\chi^2$/dof ranges from 1.10 to 1.89. The black contour lines show that this model prefers small gluino
<table>
<thead>
<tr>
<th>Observable</th>
<th>Exp. Value</th>
<th>Ref.</th>
<th>Program</th>
<th>Th. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_Z$</td>
<td>91.1876 ± 0.0021 GeV</td>
<td>[3]</td>
<td>Input</td>
<td>0.6%</td>
</tr>
<tr>
<td>$M_W$</td>
<td>80.385 ± 0.015 GeV</td>
<td>[3]</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\alpha_{em}$</td>
<td>$1/137.035999139(31)$</td>
<td>[3]</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$G_{\mu}$</td>
<td>$1.1663787(6) \times 10^{-5}$ GeV$^{-2}$</td>
<td>[3]</td>
<td>maton</td>
<td>1.0%</td>
</tr>
<tr>
<td>$\alpha_1(M_Z)$</td>
<td>$0.1181 \pm 0.0006$</td>
<td>[3]</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$M_t$</td>
<td>173.21 ± 0.51 ± 0.71 GeV</td>
<td>[4]</td>
<td>maton</td>
<td>1.1%</td>
</tr>
<tr>
<td>$m_t(m_t)$</td>
<td>4.185 ± 0.035 GeV</td>
<td>[3]</td>
<td>maton</td>
<td>3.0%</td>
</tr>
<tr>
<td>$M_{1/2}$</td>
<td>1776.86 ± 0.12 MeV</td>
<td>[3]</td>
<td>maton</td>
<td>1.1%</td>
</tr>
<tr>
<td>$m_{b} - m_{c}$</td>
<td>3.45 ± 0.05 GeV</td>
<td>[4]</td>
<td>maton</td>
<td>10.8%</td>
</tr>
<tr>
<td>$m_{c}/m_{c}$</td>
<td>1.27 ± 0.03 GeV</td>
<td>[3]</td>
<td>maton</td>
<td>1.1%</td>
</tr>
<tr>
<td>$m_{s}/m_{c}$</td>
<td>$98 \pm 6$ MeV</td>
<td>[4]</td>
<td>maton</td>
<td>1.1%</td>
</tr>
<tr>
<td>$Q$</td>
<td>$19.5 \pm 2.5$</td>
<td>[3]</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$m_{s}/m_{c}$</td>
<td>$24 \pm 2$</td>
<td>[3]</td>
<td>maton</td>
<td>5.0%</td>
</tr>
<tr>
<td>$m_{s}/m_{c}$</td>
<td>$2.3 \pm 0.3$ MeV</td>
<td>[3]</td>
<td>maton</td>
<td>1.1%</td>
</tr>
<tr>
<td>$m_{s}/m_{c}$</td>
<td>$4.75 \pm 0.45$ MeV</td>
<td>[3]</td>
<td>maton</td>
<td>1.1%</td>
</tr>
<tr>
<td>$M_{1/2}$</td>
<td>105.6588745(24) MeV</td>
<td>[3]</td>
<td>maton</td>
<td>2.1%</td>
</tr>
<tr>
<td>$M_{1/2}$</td>
<td>0.5109989461(31) MeV</td>
<td>[3]</td>
<td>maton</td>
<td>1.1%</td>
</tr>
<tr>
<td>$V_{ud}$</td>
<td>$0.97417 \pm 0.00021$</td>
<td>[3]</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$V_{us}$</td>
<td>$0.2248 \pm 0.0006$</td>
<td>[3]</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$V_{ub}$</td>
<td>$(4.13 \pm 0.60) \times 10^{-3}$</td>
<td>[3]</td>
<td>maton</td>
<td>2.1%</td>
</tr>
<tr>
<td>$V_{cd}$</td>
<td>$0.220 \pm 0.005$</td>
<td>[3]</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$V_{cb}$</td>
<td>$0.995 \pm 0.016$</td>
<td>[3]</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$V_{ud}$</td>
<td>$(40.75 \pm 2.25) \times 10^{-3}$</td>
<td>[3]</td>
<td>maton</td>
<td>2.1%</td>
</tr>
<tr>
<td>$V_{cd}$</td>
<td>$(8.2 \pm 0.6) \times 10^{-3}$</td>
<td>[3]</td>
<td>maton</td>
<td>2.1%</td>
</tr>
<tr>
<td>$V_{cb}$</td>
<td>$(40.0 \pm 2.7) \times 10^{-3}$</td>
<td>[3]</td>
<td>maton</td>
<td>2.1%</td>
</tr>
<tr>
<td>$V_{cd}$</td>
<td>$1.009 \pm 0.031$</td>
<td>[3]</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\sin^2(2\beta)$</td>
<td>$0.691 \pm 0.017$</td>
<td>[4]</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\Delta m_{B_d}/\Delta m_{B_s}$</td>
<td>$(2.233 \pm 0.015) \times 10^{-3}$</td>
<td>[3]</td>
<td>SUSY_FLAVR [136]</td>
<td>10.0%</td>
</tr>
<tr>
<td>$\Delta m_{B_d}$</td>
<td>$34.8479 \pm 0.2324$</td>
<td>[3]</td>
<td>SUSY_FLAVR [136]</td>
<td>20.2%</td>
</tr>
<tr>
<td>$\Delta m_{B_s}$</td>
<td>$(3.354 \pm 0.022) \times 10^{-10}$ MeV$^2$</td>
<td>[4]</td>
<td>SUSY_FLAVR [136]</td>
<td>20.6%</td>
</tr>
<tr>
<td>$\Delta m_{B_d}$</td>
<td>$(7.375 \pm 0.165) \times 10^{-10}$ MeV$^2$</td>
<td>[3]</td>
<td>SUSY_FLAVR [136]</td>
<td>20.0%</td>
</tr>
<tr>
<td>$\Delta m_{B_s}^2$</td>
<td>$(2.50 \pm 0.04) \times 10^{-10}$ eV$^2$</td>
<td>[3]</td>
<td>SUSY_FLAVR [136]</td>
<td>5.0%</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>$0.2975 \pm 0.0165$</td>
<td>[28]</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>$0.4435 \pm 0.0265$</td>
<td>[28]</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>$0.0215 \pm 0.0010$</td>
<td>[28]</td>
<td>maton</td>
<td>0.5%</td>
</tr>
<tr>
<td>$M_A$</td>
<td>$125.90 \pm 0.24$ GeV</td>
<td>[3]</td>
<td>SplitSuept [111]</td>
<td>3.8%</td>
</tr>
<tr>
<td>$\text{BR}(b \to s \gamma)$</td>
<td>$(332 \pm 16) \times 10^{-6}$</td>
<td>[135]</td>
<td>SUSY_FLAVR [136]</td>
<td>47.3%</td>
</tr>
<tr>
<td>$\text{BR}(B_s \to \mu^+ \mu^-)$</td>
<td>$(2.94 \pm 0.65) \times 10^{-9}$</td>
<td>[135]</td>
<td>SUSY_FLAVR [136]</td>
<td>22.4%</td>
</tr>
<tr>
<td>$\text{BR}(B_d \to \mu^+ \mu^-)$</td>
<td>$(0.40 \pm 0.15) \times 10^{-9}$</td>
<td>[135]</td>
<td>SUSY_FLAVR [136]</td>
<td>21.5%</td>
</tr>
<tr>
<td>$\text{BR}(B \to \tau \nu)$</td>
<td>$(106 \pm 19) \times 10^{-6}$</td>
<td>[135]</td>
<td>SUSY_FLAVR [136]</td>
<td>50.4%</td>
</tr>
<tr>
<td>$\text{BR}(B \to K^+ \mu^+ \mu^-)_{low , \epsilon^2}$</td>
<td>$(0.34 \pm 0.06) \times 10^{-7}$</td>
<td>[100]</td>
<td>SuperIso [108]</td>
<td>105.1%</td>
</tr>
<tr>
<td>$\text{BR}(B \to K^+ \mu^+ \mu^-)_{high , \epsilon^2}$</td>
<td>$(0.56 \pm 0.10) \times 10^{-7}$</td>
<td>[100]</td>
<td>SuperIso [108]</td>
<td>190.0%</td>
</tr>
<tr>
<td>$\text{BR}(B \to K^+ \mu^+ \mu^-)_{low , \epsilon^2}$</td>
<td>$0.65 \pm 0.09$</td>
<td>[100]</td>
<td>SuperIso [108]</td>
<td>45.0%</td>
</tr>
<tr>
<td>$-2P_2 = A_B^{\mu \nu}(B \to K^+ \mu^+ \mu^-)_{low , \epsilon^2}$</td>
<td>$-0.66 \pm 0.24$</td>
<td>[100]</td>
<td>SuperIso [108]</td>
<td>192.8%</td>
</tr>
<tr>
<td>$-2P_2 = A_B^{\mu \nu}(B \to K^+ \mu^+ \mu^-)_{high , \epsilon^2}$</td>
<td>$0.50 \pm 0.03$</td>
<td>[100]</td>
<td>SuperIso [108]</td>
<td>45.0%</td>
</tr>
<tr>
<td>$P_0'(B \to K^+ \mu^+ \mu^-)_{low , \epsilon^2}$</td>
<td>$0.58 \pm 0.36$</td>
<td>[110]</td>
<td>SuperIso [108]</td>
<td>50.4%</td>
</tr>
<tr>
<td>$P_0'(B \to K^+ \mu^+ \mu^-)_{high , \epsilon^2}$</td>
<td>$-0.18 \pm 0.70$</td>
<td>[110]</td>
<td>SuperIso [108]</td>
<td>35.0%</td>
</tr>
<tr>
<td>$P_2'(B \to K^+ \mu^+ \mu^-)_{low , \epsilon^2}$</td>
<td>$0.21 \pm 0.21$</td>
<td>[110]</td>
<td>SuperIso [108]</td>
<td>45.9%</td>
</tr>
<tr>
<td>$P_2'(B \to K^+ \mu^+ \mu^-)_{high , \epsilon^2}$</td>
<td>$-0.79 \pm 0.27$</td>
<td>[110]</td>
<td>SuperIso [108]</td>
<td>60.0%</td>
</tr>
</tbody>
</table>

Table 4.2: 51 low energy observables that are fitted in the global $\chi^2$ analysis.
mass because the minimum $\chi^2$ value occurs at gluino mass below the lower limit of the plot. The green dotted lines are the 1.0 and 1.2 $\sigma$ bound. These lines show that even with $M_\tilde{g} = 2.7$ TeV, $\chi^2$/dof can still be as low as $\approx 1.15$, which is well within the 2$\sigma$ bound.\footnote{In fact, even with gluino mass as high as 3.1 TeV and $m_{16} = 25$ TeV, $\chi^2$/dof = 1.33, which is also below the 2$\sigma$ bound.}

Since the $\chi^2$ contour lines are very flat in the gluino mass direction, this model is not very sensitive to gluino mass. Hence, this model cannot be ruled out even if the gluino is not seen during the current run of the LHC.

Also shown in Fig. 4.1 are a horizontal white line, a black, and a yellow star. The horizontal white line is the current gluino mass bound obtained by reinterpreting the most recent gluino mass bound from the ATLAS and CMS collaborations (see the next section). The black star is a benchmark point. The input parameters and the low-energy fits of this benchmark point are shown in the appendix. The yellow star is the point with the minimum $\chi^2$ with gluino mass still allowed by current bound. Notice that this point is exactly on the current mass bound indicating that even though the $\chi^2$/dof is relatively small for large gluino mass, this model still prefers small gluino mass.

### 4.3 Current LHC Bounds

The typical search for supersymmetry is performed under the assumption of a simplified model, such as the T1tttt or Gtt model, in which the gluino decays 100% of the time to $t\bar{t}\tilde{\chi}^0_1$. On the other hand, gluinos in this model do not decay via a single channel (see Table 4.3 for typical branching ratios of gluino of this model). Hence, to obtain the current gluino bound for this model, the analyses performed by the ATLAS and CMS collaborations have to be reinterpreted.

The most stringent gluino mass bound is from ATLAS-CONF-2016-052, where the gluino mass bound of the Gtt simplified model is around 1.9 TeV with the lightest neutralino mass $m_{\tilde{\chi}^0_1} = 200$ GeV \cite{ATLAS-CONF-2016-052}. ATLAS-CONF-2016-052 considers the signal region with zero or more leptons, $b$-jets and missing transverse momentum. The most stringent gluino mass bound from the CMS collaboration is from CMS-SUS-16-014, where the gluino mass bound of the

In fact, even with gluino mass as high as 3.1 TeV and $m_{16} = 25$ TeV, $\chi^2$/dof = 1.33, which is also below the 2$\sigma$ bound.
Figure 4.1: A plot of the $\chi^2$/dof contour lines as a function of gluino mass $M_{\tilde{g}}$ and the universal scalar mass at the GUT scale $m_{16}$. The numbers on the black contour lines are $\chi^2$/dof while the green dotted lines are the 1.0 and 1.2 $\sigma$ bound from the $\chi^2$ analysis with 27 dof. This plot has 27 dof because $m_{16}$ and $M_{1/2}$ are fixed as the $x$ and $y$ axis. The horizontal white dotted line is the current gluino mass bound of this model (see Sec. 4.3). The yellow star is the point with the lowest $\chi^2$ for gluino mass above the current bound. The black star is a benchmark point where its input parameters and low-energy fits are shown in the appendix. Since the global $\chi^2$ minimum is below the lower limit of the plot, this model prefers low gluino mass. However, this plot also shows that this model is not very sensitive to the gluino mass, because $\chi^2$ increases relatively slowly as the gluino mass increases.
Table 4.3: Branching ratios of gluinos, charginos, and neutralinos for two benchmark points of this model. The gluino branching ratios of this model are not even close to that of a simplified model. Hence, it is important to reinterpret the ATLAS and CMS simplified model analyses to obtain the gluino mass bounds of this model. The chargino and neutralino masses and compositions are shown in Table 4.5.
T1tttt simplified model is 1.75 TeV for $m_{\tilde{t}^0} = 200$ GeV \cite{137}. CMS-SUS-16-014 considers the signal region with jets and missing transverse momentum. In this chapter, these two analyses are reinterpreted with this model.

In addition, a CMS analysis, CMS-SUS-16-021, which considers the signal region of two opposite-sign same-flavor leptons with jets and missing transverse momentum, found a $2.1(1.1)\sigma$ local(global) deviation in the number of observed events compared to the SM background \cite{138}. Since this model produces signal in this region, we include this analysis in this chapter. To be impartial, we also reinterpret a CMS analysis that considers signal region with same-sign dilepton events, CMS-SUS-16-020 \cite{139}.

The experimental data, for all the analyses mentioned above, are obtained at the center-of-mass energy $\sqrt{s} = 13$ TeV. The integrated luminosity for the ATLAS analysis is $14.8 \text{ fb}^{-1}$, while that of the CMS analyses is $12.9 \text{ fb}^{-1}$.

4.3.1 Analysis Procedure

In a nutshell, the analyses are reperformed by focusing on the 95% upper limit of the number of events allowed, $N_{UL}$, calculated from the SM background and the number of observed events. $N_{UL}$ is the 95% Bayesian upper limit for a Poisson parameter calculated using a uniform prior. By focusing on the number of events allowed, we do not need to perform background simulation. Instead, we only need to simulate events produced by the models in consideration, such as the simplified model and this model. To validate this analysis, we first ensure that the simplified mass bound obtained from this analysis matches those from the ATLAS and CMS analyses. Mass bounds are obtained by ruling out masses where the 95% lower limit on the number of events passing all cuts exceeds $N_{UL}$. Once the analysis is validated, we can reperform the analysis based on this model to obtain the gluino mass bound of this model.

For analysis of a simplified model, events are simulated by supplying PYTHIA 8.219 with an SLHA file that contains the SUSY spectrum, mixing angles and decay tables \cite{140}. For each mass point, 10,000 events are simulated. The simulated events are then passed to Delphes 3.4.0, a detector simulator that outputs events as recorded by the detector \cite{141}.
The card files of Delphes, which specifies various detector specific parameters such as the triggering and candidate selection requirements, are modified according to the selection criteria of the ATLAS and CMS analyses. The output of Delphes then goes through a cutflow code that we wrote. The number of events passing all cuts is then normalized by the ratio of the number of events produced at the LHC to the number of simulated events. The number of events produced at the LHC equals the product of the luminosity and the production cross section, which is obtained from the LHC SUSY Cross Section Working Group [142]. The normalized number of events passing all cuts is then compared to $N_{UL}$ to produce the mass bound.

The analysis of this model is almost identical to that of the simplified model. The only difference is that the SLHA file of the simplified model is simple and can be written directly by hand, while that of this model is very complicated. Luckily, maton is also a spectrum generator. After obtaining the SUSY spectrum along with all the mixing angles and couplings of the model, we use SUSY-HIT 1.5a to calculate the decay tables [143]. The output of SUSY-HIT is then used as input to PYTHIA and the procedure of the simplified model analysis outlined in the previous paragraph is repeated.

4.3.2 Results

Out of all the signal regions in the four analyses that we studied, the most constrained bound comes from the 0 lepton with large mass splitting signal region in the ATLAS analysis, ATLAS-CONF-2016-52. This signal region is called Gtt-0L-A. The events in this signal region are required to have $N_{signal\, lepton} = 0$, $N_{jet} \geq 8$, $N_{b-jet} \geq 3$, $p_T^{jet} > 30$ GeV, $E_T^{miss} > 400$ GeV, $\Delta \phi_{4j}^{min} > 0.4$ rad, $m_{b-jets}^{T,min} > 80$ GeV, $m_{incl}^{eff} > 2000$ GeV and $M_{\Sigma J}^{T} > 200$ GeV. These parameters are defined in Ref. [2]. Hence, in this section, we only show the results of this specific analysis.

Figure 4.2 is the validation plot from our analysis of the Gtt simplified model with $m_{\tilde{\chi}^0_1} = 200$ GeV. The red horizontal line is the 95% upper limit of the number of events allowed, $N_{UL} = 3.8$. The vertical blue bars are the normalized number of events passing all cuts. The error bars represent the 95% upper and lower limits of the number of events passing
Figure 4.2: The validation plot for the Gtt simplified model with $m_{\tilde{\chi}_1^0} = 200$ GeV in the 0 lepton with large mass splitting signal region of the ATLAS analysis [2]. The vertical blue bars show the number of events passing all cuts while the horizontal red line is the 95% upper limit of the number of events allowed. The gluino mass bound obtained from this plot, $M_{\tilde{g}} \sim 1.875$ TeV, is in agreement with the gluino mass bound from the ATLAS collaboration, $M_{\tilde{g}} \sim 1.9$ TeV.

all cuts. These limits are derived from the uncertainties in the gluino production cross section and the counting experiment. The size of the error bars shrinks as the gluino mass increases because the number of simulated events stays constant but the gluino production cross section decreases. Figure 4.2 shows that the gluino mass bound from our analysis is $M_{\tilde{g}} \sim 1.875$ TeV, which is well within 20% of the gluino mass bounds from the ATLAS analysis, $M_{\tilde{g}} \sim 1.9$ TeV. This is the expected precision because we do not have the state of the art analysis tools available to the ATLAS collaboration, such as the detector simulator. From this, we conclude that our analysis is in agreement with the ATLAS analysis.

On the other hand, Fig. 4.3 is the plot for this model. The scalar mass for all points in this plot is $m_{16} = 20$ TeV. We have checked that points with $m_{16} = 25$ TeV produce the same gluino mass bound, since the gluino mass bound of this model is very similar to that
Figure 4.3: Number of events passing all cuts for this model in the 0 lepton with large mass splitting signal region of the ATLAS analysis [2]. The scalar mass of all the points in this plot is $m_{16} = 20$ TeV. The gluino mass bound obtained from this plot, $M_{\tilde{g}} \sim 1.875$ TeV, is the same as that from the validation plot. Hence, we conclude that the gluino mass bound of this model is the same as that of the simplified model.
of the simplified model. We conclude that the current gluino mass bound of this model is $M_{\tilde{g}} \gtrsim 1.9$ TeV.

With this gluino mass bound, this model is unable to fit the excess CMS found in the two opposite-sign same-flavor leptons analysis [138]. Hence, we predict that this excess is a statistical fluctuation.

### 4.4 Predictions and Discovery Prospects

From the analysis in the previous section, the current gluino mass bound of this model is $M_{\tilde{g}} > 1.9$ TeV. Even with this mass bound, Fig. 4.1 shows that a wide range of parameter space still is $< 1.2\sigma$. Hence, this model is not ruled out by low-energy data and current LHC bounds. Since our $\chi^2$ analysis is well below $2\sigma$ even for a gluino as heavy as 2.7 TeV (see Fig. 4.1), this model will, unfortunately, not be ruled out, even if the gluino is not found in this run of the LHC.

The SUSY mass spectrum for four benchmark points is given in Table 4.4 and Table 4.5. Also in Table 4.5 are compositions of charginos and neutralinos of benchmark points A and B. The benchmark points have $m_{16} = 20$ and 25 TeV, and $M_{\tilde{g}} = 2.0$ and 2.6 TeV. The lightest scalar mass of this model, $m_{\tilde{t}_1}$ ranges from $3 - 5$ TeV while the first two family scalar masses are either around 20 or 25 TeV depending on the value of $m_{16}$ at the GUT scale. The scalars of the first two families are decoupled from the low-energy theory while the third family scalars are not. Hence, SUSY is not completely decoupled from the SM.

The $CP$-odd Higgs, $A$, the heavy Higgs, $H^0$, and the charged Higgs, $H^\pm$, all have masses around $5 - 6$ TeV showing that we are in the decoupling limit where the light Higgs boson behaves like a SM Higgs boson. Table 4.4 also shows predictions for the electron electric dipole moment $edm_e$, the branching ratio BR($\mu \to e\gamma$), and the $CP$ violating phase in the neutrino sector, $\sin\delta$. These values are consistent with current experiment bounds. Note, however, that these predictions differ significantly from our previous results. In particular, the electric dipole moment of the electron and BR($\mu \to e\gamma$) are significantly smaller than before. In addition, the $CP$ violating angle in the lepton sector is now of order $90^\circ$ for
\( m_{16} = 25 \text{ TeV.} \)

<table>
<thead>
<tr>
<th>Benchmark point</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_{16}/\text{TeV} )</td>
<td>20</td>
<td>25</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>( M_{\tilde{g}}/\text{TeV} )</td>
<td>2.00</td>
<td>2.00</td>
<td>2.60</td>
<td>2.60</td>
</tr>
<tr>
<td>( \chi^2/\text{dof} )</td>
<td>1.14</td>
<td>1.16</td>
<td>1.16</td>
<td>1.17</td>
</tr>
<tr>
<td>( m_{\tilde{t}_1}/\text{TeV} )</td>
<td>3.68</td>
<td>4.70</td>
<td>3.70</td>
<td>4.65</td>
</tr>
<tr>
<td>( m_{\tilde{t}_2}/\text{TeV} )</td>
<td>4.38</td>
<td>5.52</td>
<td>4.43</td>
<td>5.49</td>
</tr>
<tr>
<td>( m_{\tilde{b}_1}/\text{TeV} )</td>
<td>4.17</td>
<td>5.32</td>
<td>4.17</td>
<td>5.23</td>
</tr>
<tr>
<td>( m_{\tilde{b}_2}/\text{TeV} )</td>
<td>4.32</td>
<td>5.47</td>
<td>4.36</td>
<td>5.43</td>
</tr>
<tr>
<td>( m_{\tilde{\tau}_1}/\text{TeV} )</td>
<td>7.47</td>
<td>9.30</td>
<td>7.52</td>
<td>9.27</td>
</tr>
<tr>
<td>( m_{\tilde{\tau}_2}/\text{TeV} )</td>
<td>12.2</td>
<td>15.2</td>
<td>12.2</td>
<td>15.2</td>
</tr>
<tr>
<td>( m_{\tilde{\chi}^0_1}/\text{GeV} )</td>
<td>352</td>
<td>352</td>
<td>474</td>
<td>474</td>
</tr>
<tr>
<td>( m_{\tilde{\chi}^0_2}/\text{GeV} )</td>
<td>586</td>
<td>636</td>
<td>650</td>
<td>665</td>
</tr>
<tr>
<td>( m_{\tilde{\chi}^+}/\text{GeV} )</td>
<td>585</td>
<td>636</td>
<td>646</td>
<td>661</td>
</tr>
<tr>
<td>( m_{\tilde{\chi}^-}/\text{GeV} )</td>
<td>710</td>
<td>751</td>
<td>911</td>
<td>914</td>
</tr>
<tr>
<td>( (M_A \approx M_{H^0} \approx M_{H^\pm})/\text{TeV} )</td>
<td>5.18</td>
<td>6.39</td>
<td>5.39</td>
<td>6.67</td>
</tr>
<tr>
<td>( \text{edm}_e/10^{-32}\text{ e cm} )</td>
<td>-3.46</td>
<td>-1.77</td>
<td>-4.47</td>
<td>-2.28</td>
</tr>
<tr>
<td>( \text{BR}(\mu \to e\gamma)/10^{-17} )</td>
<td>2.08</td>
<td>0.922</td>
<td>1.84</td>
<td>0.869</td>
</tr>
<tr>
<td>( \sin \delta )</td>
<td>0.759</td>
<td>0.935</td>
<td>0.644</td>
<td>0.993</td>
</tr>
</tbody>
</table>

Table 4.4: SUSY mass spectrum from four benchmark points of this model. Benchmark points A and C have \( m_{16} = 20 \text{ TeV} \), while benchmark points B and D have \( m_{16} = 25 \text{ TeV} \). Benchmark points A and B have \( M_{\tilde{g}} = 2.0 \text{ TeV} \) while benchmark points C and D have \( M_{\tilde{g}} = 2.6 \text{ TeV} \). The compositions of the charginos and neutralinos of benchmark points A and B are shown in Table 4.5. The input parameters of these benchmark points are in App. II.2. In addition, the prediction of the electron dipole moment, the branching ratio of \( \mu \to e\gamma \) and the neutrino CP violating phase are also presented in this table.

### 4.5 Conclusion

In this chapter, we modified the Yukawa sector of the SUSY GUT with SO(10) gauge symmetry in Chapter 3. This chapter aimed to improve the fits to low-energy observables, such as \( \sin(2\beta) \), \( m_u \), and \( m_d \). By shifting the phase from one Yukawa texture to another, we were able to improve the fit to \( \sin(2\beta) \). On the other hand, to fit \( m_u \) and \( m_d \), we
<table>
<thead>
<tr>
<th>Mass in GeV</th>
<th>Composition in percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
</tr>
<tr>
<td>$\tilde{\chi}_1^0$</td>
<td>352</td>
</tr>
<tr>
<td>$\tilde{\chi}_2^0$</td>
<td>586</td>
</tr>
<tr>
<td>$\tilde{\chi}_3^0$</td>
<td>621</td>
</tr>
<tr>
<td>$\tilde{\chi}_4^0$</td>
<td>711</td>
</tr>
<tr>
<td>$\tilde{\chi}_1^+\chi_1$</td>
<td>585</td>
</tr>
<tr>
<td>$\tilde{\chi}_2^+\chi_2$</td>
<td>710</td>
</tr>
<tr>
<td>$\tilde{\chi}_I^{\pm}\chi_I$</td>
<td>585</td>
</tr>
</tbody>
</table>

Table 4.5: Neutralino and chargino masses in GeV and their composition in percentage for the two benchmark points A and B in Tab. 4.4. The lightest neutralino is mainly the Bino. Hence, the lightest neutralino cannot be the dark matter candidate. However, as discussed in previous chapter, this problem can be alleviated if the gaugino masses are nonuniversal.

choose PS gauge symmetry due to higher flexibility than SO(10), and we introduced two real parameters to the 11, 12, and 21 entries of the Yukawa matrices. This increased the number of parameters to 26 parameters, see Table 4.1.

By fitting to 51 low-energy observables, see Table 4.2, our global $\chi^2$ analysis has 27 dof. The modification to the Yukawa sector, see Sec. 4.1, improves the best fit from $\chi^2$/dof = 1.90 to $\chi^2$/dof = 1.12, see Fig. 4.1. Even for gluinos as heavy as $M_\tilde{g} = 2.7$ TeV, our analysis shows that $\chi^2$/dof $\approx$ 1.15. Thus, this model is not ruled out even if gluinos are not found during this LHC run. On the bright side, this model indicates that low-energy SUSY is still a viable model and the LHC might hopefully find gluinos in the near future.

In addition to the global $\chi^2$ analysis, we also reinterpreted ATLAS and CMS analyses in signal regions with high jet multiplicities and large missing transverse momentum. Since gluinos of this model do not decay via a single decay channel, see Table 4.3, the gluino mass bound of this model might be different from that of a simplified model. Gluinos of this model decay predominantly via $t\bar{b}\tilde{\chi}_{1,2,3,4}^+$, $t\bar{t}\tilde{\chi}_{1,2,3,4}^0$ and $b\bar{b}\tilde{\chi}_{1,2,3,4}^0$. However, we found that
the gluino mass bound of this model is very similar to that of a simplified model where $M_{\tilde{g}} \sim 1.9$ TeV. The most constraining signal region that we found is also the same as that of the simplified model, which is from the ATLAS analysis in the signal region with 0 lepton, large jet multiplicities and large missing transverse momentum (Gtt-0L-A) of ATLAS-CONF-2016-052 \cite{2}.

Previous analysis by Bryant et al. shows that this model can be extended to fit inflation observables measured by BICEP2/Keck and Planck joint collaboration via a subcritical hybrid inflation \cite{78}. Further studies of the consequences of this model for the early Universe are warranted.

Since SUSY particles have not been observed at the LHC and natural SUSY models prefer light superpartners, one might think that low-energy SUSY models are no longer attractive on the grounds of naturalness. However, Chapter \ref{sec:3} showed that the fine-tuning of this model can be of the order of one part in 500, assuming that some soft SUSY breaking boundary conditions defined at $M_{\text{GUT}}$ can be obtained from a more fundamental theory. Thus, although this model is not the most natural model, it is much more natural than the SM. In addition, by construction, this model uses small GUT representations, thus the model has the potential for a UV completion to a higher dimensional string theory.
Chapter 5

Reheating and Leptogenesis after Inflation in a Pati-Salam Model

This chapter is based on the work in

In this chapter, we consider a PS SUSY GUT with a discrete $\mathbb{Z}_4^R$ symmetry. Bryant et al. showed that by introducing a $\mathbb{Z}_4^R$ discrete symmetry to the PS gauge symmetry, one can obtain an inflation model that fits the tensor-to-scalar ratio, scalar spectral index, and scalar power spectrum [78]. Coupled with results from previous chapter, which showed that the matter sector of the model can fit low-energy observables, this model is a complete theory [98–100, 144–146]. In addition, previous analysis showed that $\mathbb{Z}_4^R$ symmetry forbids the SUSY $\mu$ term and dimension four and five proton decay operators to all orders in perturbation theory [147, 148]. Non-perturbative effects can then generate the $\mu$ term and suppress dimension five proton decay, while preserving R-parity. In this chapter, we provide a detailed discussion of the reheating process and the baryogenesis of this model.

Reheating of this model occurs via instant preheating [149, 151]. As a consequence of a broad parametric resonance, particle creation occurs in a discrete manner. A lepton asymmetry is induced when the inflaton decays to the Higgs boson and the Higgs boson
subsequently decays to right-handed neutrinos. Unlike most leptogenesis analyses in the literature, we do not have the privilege to integrate out the two heavier right-handed neutrinos \[45, 147, 152\]. By fitting to low-energy observables, the heaviest right-handed neutrino gives the correct sign for the lepton asymmetry while the two lighter right-handed neutrinos have the wrong sign.

In Sec. 5.1, we briefly review the inflation model in Ref. \[78\]. The mechanism of instant preheating is explained in Sec. 5.2. It is based on detailed discussions of bosonic and fermionic broad parametric resonances found in Ref. \[149–151\]. Also included in Sec. 5.2 are parts of the model relevant to reheating and baryogenesis such as the decay of the inflaton and waterfall field, and the generation of the lepton asymmetry. In Sec. 5.3, evolution equations for evolving the system from the end of inflation to the decay of the right-handed neutrinos are presented. Parameters of the model and the simulation procedure are outlined in Sec. 5.4, while the results and discussions are contained in Sec. 5.5.

5.1 Brief Review of the Model

5.1.1 Inflation Sector

In this section we briefly review the inflation model in Ref. \[78\]. Particles in the inflaton sector along with their quantum numbers are given in Table 5.1. The superpotential of the inflation sector is given by

\[
W_{\text{Inflation}} = \Phi \left( \kappa S \bar{S} + m_\phi Y + \frac{1}{\sqrt{2}} \alpha \mathcal{H} \mathcal{H} \right) + \lambda X \left( S \bar{S} - \frac{\nu_F S}{2} \right) + \bar{S} \Sigma \bar{S} + SS, \tag{5.1}
\]

while the Kähler potential is given by

\[
K = \frac{1}{2} (\Phi + \Phi^\dagger)^2 + \bar{S}^\dagger \bar{S} + S^\dagger S + Y^\dagger Y + X^\dagger X \left[ 1 - c_X \frac{X^\dagger X}{M^2_{\text{pl}}} + a_X \left( \frac{X^\dagger X}{M^2_{\text{pl}}} \right)^2 \right], \tag{5.2}
\]

where \( \Phi \) is the inflaton superfield, and \( S \) and \( \bar{S} \) are waterfall superfields. The Kähler potential has a shift symmetry, \( \text{Im}(\Phi) \rightarrow \text{Im}(\Phi) + \Theta \), where \( \Theta \) is a real constant.

Since the waterfall fields are assumed to obtain a VEV in the right-handed neutrino
direction during the waterfall transition, the PS gauge symmetry is broken to the SM gauge symmetry during inflation. The superfield, $\Sigma$, guarantees that the effective low-energy theory below the PS breaking scale is just the MSSM. Since PS is spontaneously broken during inflation, there is no monopole problem.

The singlet, $X$, is introduced to obtain $F$-term hybrid inflation in which the inflaton-waterfall fields coupling is independent of the waterfall fields self-coupling. The term with the singlet, $Y$, is added to obtain a supersymmetric vacuum after inflation. The parameter $m_\phi \sim 10^{-6} M_{pl}$, where $M_{pl} = 2.4 \times 10^{18}$ GeV is the reduced Planck scale, is smaller than in typical chaotic inflation models. The $F$-term of $Y$ lifts the flatness of the potential above the critical point. The term with the Higgs boson superfield, $\mathcal{H}$, is added to enable reheating. This will be discussed in Sec. 5.2.

The parameter $\kappa$ is chosen to satisfy $\kappa \ll \lambda$ such that the critical value $\phi_c$, where the sign of the waterfall field mass squared becomes negative, satisfies $\phi_c = \frac{\lambda v_{PS}}{\kappa} \gg M_{pl}$. This is the parameter regime where subcritical hybrid inflation occurs [153, 154].

Performing a $\chi^2$ fit to cosmological data, a best fit point was found with

$$\kappa \simeq 4.5 \times 10^{-4} ,$$
$$\lambda \simeq 0.8 ,$$
$$m_\phi = 10^{-6} M_{pl} ,$$
$$v_{PS} \simeq 1.25 \times 10^{-2} M_{pl} \simeq 3 \times 10^{16} \text{ GeV} .$$

Table 5.1: Quantum numbers of particles in the inflation model with PS$\times Z_4^R$ symmetry. The $D_3$ family symmetry indices are omitted.
With these parameter values, 60 e-foldings of inflation started at \( \phi_* = 14.5 M_{\text{pl}} \) and the cosmological observables are computed to be

\[
\begin{align*}
  r &= 0.084, \\
  n_s &= 0.963, \\
  A_s &= 2.21 \times 10^{-9}.
\end{align*}
\]  

(5.7) (5.8) (5.9)

5.1.2 Matter Sector

The matter sector of theory is given by

\[
W_{\text{Matter}} = W_{\text{Yukawa}} + W_{\text{Neutrino}}.
\]  

(5.10)

The superpotential of the Yukawa sector is identical to Eq. 4.1. Hence, the Yukawa matrices are given by Eq. 3.14, Eq. 3.15, Eq. 3.16, and Eq. 3.17. Since the Yukawa matrices are identical to those in Chapter 3, the results of the global \( \chi^2 \) analysis in fitting low-energy observables are directly applicable to this model.

The superpotential of the neutrino sector is given by

\[
W_{\text{Neutrino}} = S(\lambda_2 N_a \bar{Q}_a + \lambda_3 N_3 \bar{Q}_3) - \frac{1}{2}(\lambda'_2 Y N_a N_a + \tilde{\theta}_a N_a N_a + \lambda'_3 Y N_3 N_3).
\]  

(5.11)

Here \( Y \) is identified as a flavon field. The VEV of \( Y \) gives a heavy mass term for \( N_a \) and \( N_3 \). Integrating out \( N_a \) and \( N_3 \) gives

\[
W_{\text{Neutrino}}^{\text{eff}} = \frac{\lambda_2^2}{2M_1} (S\bar{Q}_1)^2 + \frac{\lambda_2^2}{2M_2} (S\bar{Q}_2)^2 + \frac{\lambda_3^2}{2M_3} (S\bar{Q}_3)^2,
\]  

(5.12)

where

\[
M_1 = \lambda'_2 Y,
\]  

(5.13)

\[
M_2 = \lambda'_2 Y + \tilde{\theta}_2,
\]  

(5.14)

\[
M_3 = \lambda'_3 Y,
\]  

(5.15)

and \( \tilde{\theta}_1 \) is assumed to be zero.
After expanding the waterfall field by its VEV, Eq. 5.12 becomes

\[
\frac{\lambda_i^2}{2M_i} \left( \frac{\sigma + i\tau + \sqrt{2} v_{PS}}{2} \right)^2 \bar{\nu}_i \nu_i \supset \frac{1}{2} M_{R_i} \bar{\nu}_i \nu_i + \frac{h_i}{2} (\sigma + i\tau) \bar{\nu}_i \nu_i,
\]

(5.16)

where we have neglected terms quadratic in \(\sigma\) and \(\tau\),

\[
M_{R_i} \equiv \frac{\lambda_i^2 v_{PS}^2}{2M_i},
\]

(5.17)

\[
h_i \equiv \frac{\lambda_i^2 v_{PS}}{\sqrt{2}M_i},
\]

(5.18)

and \(\lambda_1 = \lambda_2\).

### 5.2 Instant Preheating in Broad Strokes

After inflation the Universe must reheat. The reheating of this model occurs via the process of instant preheating. At the same time, a lepton asymmetry can be obtained. Let us now describe this process.

From the inflation model described in Sec. 5.1, the inflaton superfield couples to the Higgs boson superfield via

\[
W \supset \frac{1}{\sqrt{2}} \alpha \Phi \mathcal{H} \mathcal{H}.
\]

(5.19)

Including the Yukawa sector and the right-handed neutrino mass term, the superpotential is

\[
W \supset \sqrt{2} \alpha \Phi H_u H_d + [\lambda_u]_{ij} \bar{u}_i H_u q_j + [\lambda_\nu]_{ij} \bar{\nu}_i H_u \ell_j +
\]

\[
[\lambda_d]_{ij} \bar{d}_i H_d q_j + [\lambda_e]_{ij} \bar{e}_i H_d \ell_j + \frac{1}{2} M_{R_i} \bar{\nu}_i \nu_i.
\]

(5.20)

The Yukawa matrices, \(\lambda_{u,d,e,\nu}\), from this section onwards are defined in Weyl notation with doublets on the right. Comparing to the definition of Yukawa matrices in Chapter 3 and Chapter 4

\[
\lambda_{u,d,e,\nu} = (y^{u,d,e,\nu})^T.
\]

(5.21)

We switched notation to doublets on the right because the RGEs in our program maton are
written with doublets on the right.

Without loss of generality, we can work in a basis where $\lambda_{u,d,e}$ are diagonal. Since we have chosen to work in the right-handed neutrino mass basis, we cannot simultaneously diagonalize $\lambda_{\nu}$. In this basis, the superpotential becomes

$$W \supset \sqrt{2} \alpha \Phi H_u H_d + [\lambda_u]_{ii}\bar{u}_i H_u q_i + \lambda_{\nu} H_u \ell_j +$$

$$[\lambda_d]_{ii}\bar{d}_i H_d q_i + \lambda_{\nu} H_d \ell_i + \frac{1}{2} M_R \tilde{\nu}_i \tilde{\nu}_i.$$  \hspace{1cm} (5.22)

From the superpotential, the $F$-term of $H_u$ is

$$|F_{H_u}|^2 = |\sqrt{2}\alpha \phi h_d + [\lambda_u]_{ii}\bar{u}_i q_i + [\lambda_{\nu}]_{ij}\bar{\nu}_i \ell_j|^2$$

$$= 2\alpha^2 |\varphi|^2 |h_d|^2 + \sqrt{2}\alpha (\varphi h_d [\lambda_u^\dagger]_{ii}\bar{u}_i q_i^\dagger + \varphi h_d [\lambda_{\nu}^\dagger]_{ij}\bar{\nu}_i \ell_j^\dagger + \text{h.c.}) + \ldots ,$$  \hspace{1cm} (5.23)

where the ellipsis includes the quartic sfermion terms. The scalar component of the inflaton superfield is

$$\varphi = a + i\phi \sqrt{2},$$  \hspace{1cm} (5.24)

where $\phi$ is the inflaton. At the end of inflation, $a$ is stabilized at the origin and $\phi$ oscillates around $\phi = 0$ \hspace{1cm} [78]. Hence, the $F$-term of $H_u$ becomes

$$|F_{H_u}|^2 = \alpha^2 |\varphi|^2 |h_d|^2 + \alpha \phi (i[\lambda_u^\dagger]_{ii} h_d \bar{u}_i q_i^\dagger + i[\lambda_{\nu}^\dagger]_{ij} h_d \bar{\nu}_i \ell_j^\dagger + \text{h.c.}) + \ldots .$$  \hspace{1cm} (5.25)

Similarly, the $F$-term of $H_d$ is

$$|F_{H_d}|^2 = \alpha^2 |\varphi|^2 |h_u|^2 + \alpha \phi (i[\lambda_u^\dagger]_{ii} h_u \bar{d}_i q_i^\dagger + i[\lambda_{\nu}^\dagger]_{ij} h_u \bar{\nu}_i \ell_j^\dagger + \text{h.c.}) + \ldots .$$  \hspace{1cm} (5.26)

Another $F$-term that contributes to the production and the decay of the right-handed sneutrinos is

$$|F_{\tilde{\nu}_i}|^2 = \sum_{j=1}^{3} [\lambda_{\nu}]_{ij} h_u \ell_j + M_R \tilde{\nu}_i \supset \sum_{j=1}^{3} [\lambda_{\nu}^\dagger]_{ij} M_R \ell_j^\dagger \tilde{\nu}_i + \text{h.c.},$$  \hspace{1cm} (5.27)

where we have dropped quadratic and quartic scalar terms.
With these $F$-terms, the Lagrangian is given by

$$
\mathcal{L} > - \left( |F_{H_u}|^2 + |F_{H_d}|^2 + \sum_{i=1}^{3} |F_{\nu_i}|^2 \right) \\
- \left( \sqrt{2} \alpha \tilde{\phi} \bar{h}_u h_d + \sqrt{2} \alpha \tilde{\phi} h_d h_u + \alpha \bar{h}_u \tilde{h}_d \phi + \text{h.c.} \right) \\
- \left( [\lambda_u]_{ii} \bar{u}_i h_u q_i + [\lambda_u]_{ij} \bar{u}_i h_u \ell_j + [\lambda_d]_{ii} \bar{d}_i h_d q_i + [\lambda_d]_{ii} \bar{d}_i h_d \ell_i \\
+ [\lambda_u]_{ii} \bar{u}_i \tilde{h}_u q_i + [\lambda_u]_{ij} \bar{u}_i \tilde{h}_u \ell_j + [\lambda_d]_{ii} \bar{d}_i \tilde{h}_d q_i + [\lambda_d]_{ii} \bar{d}_i \tilde{h}_d \ell_i \\
+ [\lambda_u]_{ii} \bar{u}_i \tilde{h}_u q_i + [\lambda_u]_{ij} \bar{u}_i \tilde{h}_u \ell_j + [\lambda_d]_{ii} \bar{d}_i \tilde{h}_d q_i + [\lambda_d]_{ii} \bar{d}_i \tilde{h}_d \ell_i + \text{h.c.} \right) \right). \tag{5.28}
$$

The Lagrangian shows that the Higgs boson masses are universal and time dependent.

$$
m_h \equiv m_{h_u} = m_{h_d} = m_{\tilde{h}_u} = m_{\tilde{h}_d} = \alpha \langle \phi \rangle. \tag{5.29}
$$

Since the amplitude of the inflaton oscillations is of order the Planck scale, the Higgs boson can be heavier or lighter than the right-handed neutrinos depending on the value of the inflaton VEV. Hence, the Higgs boson can decay to the right-handed neutrinos and vice versa.

### 5.2.1 Non-perturbative Decay of the Inflaton

For a Lagrangian that includes

$$
\mathcal{L} > \frac{1}{2} \alpha^2 \phi^2 \chi^2, \tag{5.30}
$$

where $\chi$ is a real scalar field, Kofman et al. [149] showed that when $\phi$ oscillates around $\phi = 0$, $\phi$ creates $\chi$ very efficiently at every zero-crossing. The number density of $\chi$ created for a specific momentum $k$ is given by

$$
n_k = \exp \left( - \frac{\pi k^2}{\alpha |\dot{\phi}_0|} \right), \tag{5.31}
$$

where $|\dot{\phi}_0|$ is the speed of $\phi$ at zero-crossing. Hence, the number density of $\chi$ created is

$$
n_{\chi,0} = \int \frac{d^3 k}{(2\pi)^3} n_k = \frac{(\alpha |\dot{\phi}_0|)^{3/2}}{8\pi^3}, \tag{5.32}
$$

105
and the typical momentum of $\chi$ is

$$k_\chi = \frac{1}{n_{\chi,0}} \int \frac{d^3k}{(2\pi)^3} kn_k = \frac{2(\alpha|\dot{\phi}_0|)^{1/2}}{\pi}.$$  \hspace{1cm} (5.33)

In our model, the coupling between the scalar Higgs doublets and the inflaton is of this form. Therefore, Higgs bosons are created efficiently at each zero-crossing with number densities

$$n_{h_u,0} = n_{h_d,0} = 4n_{\chi,0}.$$  \hspace{1cm} (5.34)

The factor of 4 is because each Higgs doublet is complex and has four real degrees of freedom.

Similarly, for a Lagrangian that includes

$$\mathcal{L} \supset \alpha \phi \bar{\psi} \psi,$$  \hspace{1cm} (5.35)

where $\psi$ is a fermion, $\phi$ creates $\psi$ very efficiently at every zero-crossing. The number density of $\psi$ created is the same as that in the bosonic case \[151\]. Notice that this process does not violate Pauli exclusion principle because $n_k \leq 1$ as shown in Eq. \[5.31\]. The boson and fermion states are created very efficiently because they are created for a huge range of momenta (see Eq. \[5.32\]). Hence, in our model, the Higgsinos are also created efficiently at every zero-crossing with number densities

$$n_{\tilde{h}_u,0} = n_{\tilde{h}_d,0} = 2n_{\chi,0}.$$  \hspace{1cm} (5.36)

The factor of 2 is because the Higgsinos are doublets.

By conservation of energy, the speed of the inflaton is decreased by the following amount at every zero-crossing:

$$\Delta \dot{\phi}_0^2 = 2\Delta \rho_{\phi,0} = -2k_\chi [n_{h_u,0} + n_{h_d,0} + n_{\tilde{h}_u,0} + n_{\tilde{h}_d,0}] = -\frac{6\alpha^2|\dot{\phi}_0|^2}{\pi^4}.$$  \hspace{1cm} (5.37)

It is important to note that Eq. \[5.31\] is valid only if there are no broad parametric enhancement from background Higgs bosons \[149\]. Parametric enhancement occurs when the created Higgs bosons have the same momentum as the background Higgs bosons. Broad parametric resonance occurs when parametric enhancement occurs for a wide range of mo-
menta. Since thermalization rate of the Higgs boson

\[ \Gamma \sim n \sigma \sim n T^{-2} \sim (10^{-11} M_{\text{pl}}^3)(10^6 M_{\text{pl}}^{-1})^{-2} \sim 10^{-5} M_{\text{pl}}, \]

is larger than the inflaton oscillation frequency \( \sim 10^{-6} M_{\text{pl}} \), the background Higgs bosons, if they exist, are thermal (see Sec. 5.5 for the source of these estimated numbers). Hence, out of the whole momentum spectrum, only Higgs bosons with momenta close to the thermal temperature experience a parametric enhancement. To a good approximation, this parametric enhancement is ignored in this chapter.

In addition, the non-perturbative instant preheating occurs while

\[ q = \frac{\alpha^2 \phi_{\text{amp}}^2}{4 m_{\phi}^2} \gg 1, \]

and ends only when \( q \sim 1/3 \), where \( \phi_{\text{amp}} \) is the inflaton oscillation amplitude and \( m_{\phi} \) is the inflaton mass [339].

### 5.2.2 Perturbative Decay of the Inflaton

In addition to the non-perturbative decay of the inflaton described in the preceding section, the inflaton can also decay perturbatively to the Higgs boson. Although this effect is only significant long after the non-perturbative decay ends, we include this effect at all times.

The masses of the decay products are ignored in all decay rates presented in this chapter. Energy conservation is guaranteed by including a Heaviside step function in all calculations.

The perturbative decay of the inflaton to the scalar Higgs bosons is due to the \( F \)-terms of \( H_u \) and \( H_d \) in Eq. 5.25 and Eq. 5.26. The decay rate of this process is given by

\[ \Gamma_{\phi \rightarrow h} \equiv \Gamma_{\phi \rightarrow h_u} = \Gamma_{\phi \rightarrow h_d} = \frac{1}{16 \pi} \frac{(\alpha m_h)^2}{m_{\phi}} \Theta(m_{\phi} - 2m_h). \]

On the other hand, the perturbative decay of the inflaton to the Higgsinos is due to Yukawa-like terms in the Lagrangian in Eq. 5.28. The decay rate of this process is given by

\[ \Gamma_{\phi \rightarrow \tilde{h}} \equiv \Gamma_{\phi \rightarrow \tilde{h}_u^0 \tilde{h}_d^0} + \Gamma_{\phi \rightarrow \tilde{h}_u^+ \tilde{h}_d^-} = \frac{2}{8 \pi} \alpha^2 m_{\phi} \Theta(m_{\phi} - 2m_h). \]
5.2.3 Decay of the Higgs Bosons

A very interesting phenomena of our model is that the scalar Higgs bosons are massless when they are created from the non-perturbative decay of the inflaton. As the inflaton rolls up the potential, the Higgs bosons obtain masses proportional to the value of the inflaton VEV as shown in Eq. [5.29]. Before the Higgs bosons become heavier than the right-handed neutrinos, they can only decay to massless particles, which we refer to as radiation. Eventually, the Higgs bosons become massive enough and start decaying to the right-handed neutrinos. In addition, as we will see, the Higgs bosons decay rates are proportional to their masses, that is the decay rates increase as the inflaton rolls up the potential.

Up-type Higgs Boson

The decay channels of the up-type Higgs boson are

1. Right-handed neutrinos: $h_u \rightarrow \bar{\nu}_i^\dagger \ell_j^\dagger$ with decay rate

$$
\Gamma_{h_u \rightarrow \bar{\nu}_i^\dagger \ell_j^\dagger} = \sum_{j=1}^{3} \Gamma_{h_u \rightarrow \bar{\nu}_i^\dagger \ell_j^\dagger} + \Gamma_{h_u \rightarrow \bar{\nu}_j \ell_i^\dagger} \\
= \sum_{j=1}^{3} \frac{1}{8\pi} \lambda_{\nu;ij}^2 m_h \Theta(m_h - M_{R_i}) \\
= \frac{1}{8\pi} (\lambda_{\nu} \lambda_{\nu}^\dagger)_{ii} m_h \Theta(m_h - M_{R_i})
$$

The factor of 2 is due to the charged and neutral Higgs bosons.

2. Right-handed sneutrinos: $h_u \rightarrow \tilde{\nu}_i^\dagger \tilde{\nu}_j^\dagger$ with decay rate

$$
\Gamma_{h_u \rightarrow \tilde{\nu}_i^\dagger \tilde{\nu}_j^\dagger} = \sum_{j=1}^{3} \Gamma_{h_u \rightarrow \tilde{\nu}_i^\dagger \tilde{\nu}_j^\dagger} + \Gamma_{h_u \rightarrow \tilde{\nu}_j \tilde{\nu}_i^\dagger} \\
= \sum_{j=1}^{3} \frac{1}{16\pi} \lambda_{\nu;ij}^2 M_{R_i}^2 m_h \Theta(m_h - M_{R_i}) \\
= \frac{1}{16\pi} (\lambda_{\nu} \lambda_{\nu}^\dagger)_{ii} M_{R_i}^2 m_h \Theta(m_h - M_{R_i})
$$

108
3. Radiation: \( h_u \to \tilde{d}_i \tilde{q}_i, \tilde{e}_i \tilde{\ell}_i, \tilde{u}_i \tilde{q}_i^{\dagger} \) with decay rate

\[
\Gamma_{h_u \to R} = \sum_{i=1}^{3} \frac{1}{16\pi} \left( N_c |\lambda_{d;ii}|^2 + |\lambda_{e;ii}|^2 \right) m_h + 2 \frac{1}{8\pi} N_c |\lambda_{u;ii}|^2 m_h .
\]  

Down-type Higgs Boson

The decay channels of the down-type Higgs boson are

1. Right-handed sneutrinos: \( h_d \to \tilde{\nu}_i \tilde{\ell}_j \) with decay rate

\[
\Gamma_{h_d \to \tilde{\nu}_i \tilde{\ell}_j} = \sum_{j=1}^{3} \Gamma_{h_d \to \tilde{\nu}_i \tilde{\ell}_j} = \sum_{j=1}^{3} \frac{1}{16\pi} |\lambda_{\nu;ij}|^2 m_h \Theta(m_h - M_{R_j})
\]

\[
= 2 \frac{1}{16\pi} (\lambda_{\nu}^{\dagger} \lambda_{\nu})_{ii} m_h \Theta(m_h - M_{R_i}) .
\]  

2. Radiation: \( h_d \to \tilde{u}_i \tilde{q}_i, \tilde{d}_i \tilde{q}_i, \tilde{e}_i \tilde{\ell}_i \) with decay rate

\[
\Gamma_{h_d \to R} = \sum_{i=1}^{3} \frac{1}{16\pi} N_c |\lambda_{u;ii}|^2 m_h + 2 \frac{1}{8\pi} N_c |\lambda_{d;ii}|^2 + |\lambda_{e;ii}|^2 m_h .
\]

Up-type Higgsinos

The decay channels of the up-type Higgsinos are

1. Right-handed neutrinos: \( \tilde{h}_u \to \tilde{\nu}_i \tilde{\ell}_j \) with decay rate

\[
\Gamma_{\tilde{h}_u \to \tilde{\nu}_i \tilde{\ell}_j} = \sum_{j=1}^{3} \Gamma_{\tilde{h}_u \to \tilde{\nu}_i \tilde{\ell}_j} = \sum_{j=1}^{3} \frac{1}{16\pi} |\lambda_{\nu;ij}|^2 m_h \Theta(m_h - M_{R_j})
\]

\[
= 2 \frac{1}{16\pi} (\lambda_{\nu}^{\dagger} \lambda_{\nu})_{ii} m_h \Theta(m_h - M_{R_i}) .
\]
2. Right-handed sneutrinos: $\tilde{h}_u \rightarrow \tilde{\nu}^+_i \ell^+_j$ with decay rate

$$
\Gamma_{\tilde{h}_u \rightarrow \tilde{\nu}^+_i \ell^+_j} = \sum_{j=1}^{3} \Gamma_{\tilde{h}_u \rightarrow \tilde{\nu}^+_i \ell^+_j} + \Gamma_{\tilde{h}_u \rightarrow \tilde{\nu}^+_i \ell^+_j}
$$

$$
= \sum_{j=1}^{3} \frac{1}{16\pi} |\lambda_{\nu;ij}|^2 m_h \Theta(m_h - M_{R_i})
$$

$$
= \frac{1}{16\pi} (\lambda_{\nu;ii})^2 m_h \Theta(m_h - M_{R_i}).
$$

(5.48)

3. Radiation: $\tilde{h}_u \rightarrow \tilde{u}^+_i \tilde{q}^+_i, \tilde{\bar{u}}^+_i \tilde{q}^+_i$ with decay rate

$$
\Gamma_{\tilde{h}_u \rightarrow R} = \sum_{i=1}^{3} 4 \frac{1}{16\pi} N_c |\lambda_{u;ii}|^2 m_h.
$$

(5.49)

**Down-type Higgsinos**

The only decay channel of the down-type Higgsinos is

1. Radiation: $\tilde{h}_d \rightarrow \tilde{d}^+_i \tilde{q}^+_i, \tilde{\bar{d}}^+_i \tilde{q}^+_i, \tilde{e}^+_i \ell^+_i, \tilde{\bar{e}}^+_i \ell^+_i$ with decay rate

$$
\Gamma_{\tilde{h}_d \rightarrow R} = \sum_{i=1}^{3} 4 \frac{1}{16\pi} (N_c |\lambda_{d;ii}|^2 + |\lambda_{e;ii}|^2) m_h.
$$

(5.50)

This decay rate is multiplied by a factor of 4 because the decay to $\tilde{d}^+_i \tilde{q}^+_i$ and $\tilde{\bar{d}}^+_i \tilde{q}^+_i$ have the same coupling. Similarly for the other two decay products.

**5.2.4 Decay of the Right-handed Neutrinos and Sneutrinos**

Right-handed neutrinos can decay to the Higgs boson and lepton when they are heavier than the Higgs boson.

**Right-handed Neutrinos**

Right-handed neutrinos can decay to
1. Up-type Higgs bosons: $\tilde{\nu}_i \rightarrow \ell_j^\dagger h_u^\dagger$ with decay rate

$$\Gamma_{\tilde{\nu}_i \rightarrow h_u^\dagger} = \sum_{j=1}^{3} \Gamma_{\tilde{\nu}_i \rightarrow \ell_j^\dagger h_u^\dagger} + \Gamma_{\tilde{\nu}_i^\dagger \rightarrow \ell_j h_u}$$

$$= \sum_{j=1}^{3} 2 \frac{1}{16\pi} |\lambda_{\nu;i;j}|^2 M_{R_i} \Theta(M_{R_i} - m_h)$$

$$= 2 \frac{1}{16\pi} (\lambda_{\nu}^\dagger \lambda_{\nu})_{ii} M_{R_i} \Theta(M_{R_i} - m_h).$$  \hfill (5.51)

2. Up-type Higgsinos: $\tilde{\nu}_i \rightarrow \tilde{\ell}_j^\dagger h_u^\dagger$ with decay rate

$$\Gamma_{\tilde{\nu}_i \rightarrow h_u^\dagger} = \sum_{j=1}^{3} \Gamma_{\tilde{\nu}_i \rightarrow \tilde{\ell}_j^\dagger h_u^\dagger} + \Gamma_{\tilde{\nu}_i^\dagger \rightarrow \tilde{\ell}_j h_u}$$

$$= \sum_{j=1}^{3} 2 \frac{1}{16\pi} |\lambda_{\nu;i;j}|^2 M_{R_i} \Theta(M_{R_i} - m_h)$$

$$= 2 \frac{1}{16\pi} (\lambda_{\nu}^\dagger \lambda_{\nu})_{ii} M_{R_i} \Theta(M_{R_i} - m_h).$$  \hfill (5.52)

**Right-handed Sneutrinos**

Right-handed sneutrinos can decay to

1. Up-type Higgs bosons: $\tilde{\nu}_i \rightarrow h_u^\dagger \tilde{\ell}_j^\dagger$ with decay rate

$$\Gamma_{\tilde{\nu}_i \rightarrow h_u^\dagger} = \sum_{j=1}^{3} \Gamma_{\tilde{\nu}_i \rightarrow h_u^\dagger \tilde{\ell}_j^\dagger} + \Gamma_{\tilde{\nu}_i^\dagger \rightarrow h_u \tilde{\ell}_j}$$

$$= \sum_{j=1}^{3} 2 \frac{1}{16\pi} |\lambda_{\nu;i;j}|^2 M_{R_i} \Theta(M_{R_i} - m_h)$$

$$= 2 \frac{1}{16\pi} (\lambda_{\nu}^\dagger \lambda_{\nu})_{ii} M_{R_i} \Theta(M_{R_i} - m_h).$$  \hfill (5.53)
2. Down-type Higgs bosons: $\tilde{\nu}_i \rightarrow \ell^+_j h_d$ with decay rate

$$
\Gamma_{\tilde{\nu}_i \rightarrow h_d} = \sum_{j=1}^{3} \Gamma_{\tilde{\nu}_i \rightarrow \ell^+_j h_d} + \Gamma_{\tilde{\nu}_i \rightarrow \ell^+_j h_d} \nonumber \\
= \sum_{j=1}^{3} 2 \frac{1}{16\pi} |\lambda_{\nu_{ij}}|^2 \frac{m_h^2}{M_{R_{i}}} \Theta(M_{R_{i}} - m_h) \tag{5.54} \\
= 2 \frac{1}{16\pi} (\lambda_{\nu_{ij}}\lambda_{\nu_{jj}})_{ii} \frac{m_h^2}{M_{R_{i}}} \Theta(M_{R_{i}} - m_h) .
$$

3. Up-type Higgsinos: $\tilde{\nu}_i \rightarrow \ell^+_j h_u^*$ with decay rate

$$
\Gamma_{\tilde{\nu}_i \rightarrow h_u^*} = \sum_{j=1}^{3} \Gamma_{\tilde{\nu}_i \rightarrow \ell^+_j h_u} + \Gamma_{\tilde{\nu}_i \rightarrow \ell^+_j h_u} \nonumber \\
= \sum_{j=1}^{3} 2 \frac{1}{16\pi} |\lambda_{\nu_{ij}}|^2 M_{R_{i}} \Theta(M_{R_{i}} - m_h) \tag{5.55} \\
= 2 \frac{1}{16\pi} (\lambda_{\nu_{ij}}\lambda_{\nu_{jj}})_{ii} M_{R_{i}} \Theta(M_{R_{i}} - m_h) .
$$

5.2.5 Lepton Asymmetry

A net lepton asymmetry can be produced when we consider the decay of the Higgs bosons along with the subsequent decay of the right-handed neutrinos. Let the $CP$ asymmetry from the Higgs bosons decay be

$$
\epsilon_{\nu_{i}} \equiv \frac{\Gamma_{h_u \rightarrow \tilde{\nu}_{i} \ell} - \Gamma_{h_u \rightarrow \tilde{\nu}^*_i \ell^*}}{\Gamma_{h_u \rightarrow \tilde{\nu}_{i} \ell} + \Gamma_{h_u \rightarrow \tilde{\nu}^*_i \ell^*}} , \tag{5.56}
$$

and that from the right-handed neutrinos decay be

$$
\epsilon_{\nu_{i}} \equiv \frac{\Gamma_{\tilde{\nu}_{i} \rightarrow \ell h_u} - \Gamma_{\tilde{\nu}_{i} \rightarrow \ell h_u^*}}{\Gamma_{\tilde{\nu}_{i} \rightarrow \ell h_u} + \Gamma_{\tilde{\nu}_{i} \rightarrow \ell h_u^*}} , \tag{5.57}
$$
where the family indices of the leptons are summed. Then, for example, when an up-type Higgs decay, we have

\[
\begin{align*}
    h_u \rightarrow & \left\{ \begin{array}{l}
    \frac{1 + \epsilon_{h_i}}{2} \bar{\nu}_i \ell \\
    \frac{1 - \epsilon_{h_i}}{2} \bar{\nu}_i \ell^\dagger
    \end{array} \right\} \\
    \rightarrow & \left\{ \begin{array}{l}
    \frac{(1 + \epsilon_{h_i})(1 + \epsilon_{\bar{\nu}_i})}{4} h_u \ell \ell \\
    \frac{(1 + \epsilon_{h_i})(1 - \epsilon_{\bar{\nu}_i})}{4} h_u \ell^\dagger \ell \\
    \frac{(1 - \epsilon_{h_i})(1 + \epsilon_{\bar{\nu}_i})}{4} h_u \ell \ell^\dagger \\
    \frac{(1 - \epsilon_{h_i})(1 - \epsilon_{\bar{\nu}_i})}{4} h_u \ell^\dagger \ell^\dagger
    \end{array} \right\}, \tag{5.58}
\end{align*}
\]

where the \( \epsilon \) factors are the branching ratios. We see that only half of the decay channels have a net lepton asymmetry. Hence, the final lepton asymmetry is

\[
    n_L \equiv n_\ell - n_\bar{\ell} = 2 \frac{(1 + \epsilon_{h_i})(1 + \epsilon_{\bar{\nu}_i})}{4} n_{h_u} - 2 \frac{(1 - \epsilon_{h_i})(1 - \epsilon_{\bar{\nu}_i})}{4} n_{h_u} = \epsilon_{h_i} n_{h_u} + \epsilon_{\bar{\nu}_i} n_{\bar{\nu}_i}. \tag{5.59}
\]

In the last equality, we used \( n_{\bar{\nu}_i} = n_{h_u} \), which is true in this process because each up-type Higgs creates a right-handed neutrino. There is a factor of 2 multiplying the branching ratio of the lepton asymmetric final states because these states have either two leptons or two anti-leptons.\(^{26}\)

Using the results in Sec. 1.4 the \( CP \) asymmetry parameters for the heaviest right-handed neutrinos, \( \bar{\nu}_3 \), to the lightest right-handed neutrinos, \( \bar{\nu}_1 \), are given by

\[
    \epsilon_{\bar{\nu}_3} = \epsilon_{h_3} = - \frac{1}{8\pi} \sum_{j=1,2} \text{Im}\{[\lambda_{\nu}(\lambda_{\nu}^\dagger)_j^3]^2\} \frac{M_{R_j}}{M_{R_3}} g \left( \frac{M_{R_j}}{M_{R_3}} \right)
\]

\[
    \epsilon_{\bar{\nu}_2} = \epsilon_{h_2} = - \frac{1}{8\pi} \text{Im}\{[\lambda_{\nu}(\lambda_{\nu}^\dagger)_2^2]^2\} \frac{M_{R_2}}{M_{R_2}} + 3 \frac{3}{8\pi} \frac{\text{Im}[\lambda_{\nu}(\lambda_{\nu}^\dagger)_j^3 \lambda_{\nu}^\dagger_2^2]}{(\lambda_{\nu}(\lambda_{\nu}^\dagger)_j^2)^2} M_{R_2} \tag{5.60}
\]

\[
    \epsilon_{\bar{\nu}_1} = \epsilon_{h_1} = \frac{3}{8\pi} \frac{\text{Im}[\lambda_{\nu}(\lambda_{\nu}^\dagger)_j^2 \lambda_{\nu}^\dagger_1^1]}{(\lambda_{\nu}(\lambda_{\nu}^\dagger)_j^1)^2} M_{R_1}
\]

where we have made the assumption that the decay products are massless. With this assumption, \( CP \) asymmetries due to the Higgs bosons and the right-handed neutrinos decays are equal\(^{[155]}\). This assumption is made throughout the chapter.

\(^{26}\)In our evaluation of the net lepton asymmetry we follow the analysis of Ahn and Kolb\(^{[155]}\). We note here that, with regards to their formula equivalent to Eq. 5.59 they do not take into account that the final states have either two leptons or two anti-leptons, therefore our lepton asymmetry is a factor of 2 bigger than theirs.
5.2.6 Decay of the Waterfall Fields

In addition to the inflaton, there are waterfall fields, $\sigma$, after the inflation ends. The relevant superpotential is given by Eq. 5.12 with $\lambda_1 \equiv \lambda_2$. The $F$-term of $\bar{Q}$ is

$$|F_{Q_i}|^2 = \left| \frac{\lambda_i^2}{M_i} \left( \frac{\sigma + i\tau + \sqrt{2}v_{PS}}{2} \right)^2 \tilde{\nu}_i \right|^2$$

$$\supset \frac{\lambda_i^4}{M_i^2} v_{PS}^2 \tilde{\nu}_i \tilde{\nu}_i^\dagger \left( \frac{3}{4} \sigma^2 + \frac{1}{\sqrt{2}} v_{PS} \sigma + \frac{1}{4} v_{PS}^2 \right)$$

$$= \frac{3}{2} h_i^2 \sigma^2 \tilde{\nu}_i \tilde{\nu}_i^\dagger + \sqrt{2} h_i^2 v_{PS} \sigma \tilde{\nu}_i \tilde{\nu}_i^\dagger + M_{R_i}^2 \tilde{\nu}_i \tilde{\nu}_i^\dagger,$$

where $h_i$ and $M_{R_i}$ are given in Eq. 5.17 and Eq. 5.18. The first term in this $F$-term is similar to the broad parametric resonance term in Eq. 5.30. Hence, one would expect parametric resonance to occur in the waterfall field. However, the corresponding broad parametric resonance parameter is too small for broad parametric resonance to occur:

$$q = \frac{9 h_i^4 \sigma_{\text{amp}}}{4 m_{\sigma}^2} \ll 1.$$  \hfill (5.62)

The second term in the $F$-term above allows for the decay of the waterfall field to the right-handed sneutrinos, while the Yukawa-like terms from the superpotential in Eq. 5.16 allows for the decay of waterfall field to the right-handed neutrinos. Hence, the waterfall field perturbative decay rates are

$$\Gamma_{\sigma \to \tilde{\nu}_i \tilde{\nu}_i} = \frac{1}{64\pi} h_i^2 m_\sigma,$$  \hfill (5.63)

$$\Gamma_{\sigma \to \tilde{\nu}_i \tilde{\nu}_i} = \frac{1}{16\pi} \frac{h_i^4 v_{PS}^2}{m_\sigma}.$$  \hfill (5.64)

Moreover, since $h_i = \sqrt{2} M_{R_i}/v_{PS}$, the waterfall field decays predominantly to the heaviest right-handed neutrino.

5.3 Evolution Equations

To analyze the evolution of all particles after inflation ends, we follow the approach used by Ahn et al. \cite{155}. As a first approximation, we do not consider the momentum of particles.
5.3.1 Equation of Motion of the Inflaton

The equation of motion of the inflaton is given by [149]

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2\phi + \alpha^2(h_u^2 + h_d^2 + \tilde{h}_u^2 + \tilde{h}_d^2)\phi = -2\Gamma_{\phi\rightarrow h}\dot{\phi} - \Gamma_{\phi\rightarrow \tilde{h}}\dot{\phi}.$$  \hfill (5.65)

By the Hartree approximation defined in Ref. [149], the VEV of the Higgs bosons is of the form

$$\langle h^2 \rangle = \frac{n_h}{m_h} = \frac{n_h}{\alpha|\phi|}.$$  \hfill (5.66)

The factor of 2 multiplying the decay rate is because the inflaton can decay to both the up-type and the down-type Higgs bosons. Hence, the inflaton equation of motion can be written as

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2\phi + \alpha(n_h + n_{\tilde{h}} + n_{h_u} + n_{\tilde{h}_u} + n_{h_d} + n_{\tilde{h}_d})\text{sign}(\phi) = -2\Gamma_{\phi\rightarrow h}\dot{\phi} - \Gamma_{\phi\rightarrow \tilde{h}}\dot{\phi},$$  \hfill (5.67)

where $H$ is the Hubble parameter in units of the reduced Planck mass,

$$H^2 = \frac{1}{3} \left[ \rho_\phi + m_h(n_{h_u} + n_{h_d} + n_{\tilde{h}_u} + n_{\tilde{h}_d}) + \sum_i M_{R_i}(n_{\bar{\nu}_i} + n_{\tilde{\nu}_i}) + \rho_R \right].$$  \hfill (5.68)

5.3.2 Evolution Equations for the Number Density of the Higgs Bosons

To derive the evolution equation for the Higgs bosons, we start by considering just the interaction between the inflaton and the Higgs bosons. The inflaton energy density is defined as

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} m_\phi^2\phi^2.$$  \hfill (5.69)

With this definition, the rate of change of the inflaton energy density is

$$\dot{\rho}_\phi = \dot{\phi}(\ddot{\phi} + m_\phi^2\phi).$$  \hfill (5.70)
By multiplying the equation of motion with $\dot{\phi}$, we have

$$
\dot{\rho}_\phi + 3H\dot{\phi}^2 + \dot{m}_h(n_{h_u} + n_{h_d} + n_{\tilde{h}_u} + n_{\tilde{h}_d}) = -2\Gamma_{\phi \rightarrow h}\dot{\phi}^2 - \Gamma_{\phi \rightarrow \tilde{h}}\dot{\phi}^2 
$$

(5.71)

To conserve energy between the inflaton and the Higgs bosons, we have

$$
\dot{\rho}_{h_u} + 3H\rho_{h_u} - \dot{m}_{h_u}n_{h_u} - \Gamma_{\phi \rightarrow h}\dot{\phi}^2 = 0.
$$

(5.72)

With $\rho_h = E_h n_h$, we have

$$
E_h\dot{n}_{h_u} + \dot{E}_h n_{h_u} + 3HE_h n_{h_u} - \dot{m}_{h_u} n_{h_u} - \Gamma_{\phi \rightarrow h}\dot{\phi}^2 = 0
$$

(5.73)

$$
\dot{n}_{h_u} + 3Hn_{h_u} + \frac{\dot{E}_h - \dot{m}_{h_u}}{E_h} n_{h_u} - \Gamma_{\phi \rightarrow h}\dot{\phi}^2 \frac{\dot{\phi}^2}{E_h} = 0
$$

Since we are ignoring the momentum of the Higgs bosons, $\dot{E}_h = \dot{m}_{h_u}$. In addition, conservation of energy requires $m_\phi = 2E_h$. So, the evolution equation of the Higgs bosons that conserves energy with that of the inflaton is given by

$$
\dot{n}_{h_u} + 3Hn_{h_u} - 2\Gamma_{\phi \rightarrow h}\frac{\dot{\phi}^2}{m_\phi} = 0.
$$

(5.74)

The evolution equations for the three other Higgs bosons are very similar and we will omit the derivation.

Including the decay and the creation of the Higgs bosons, the full evolution equations for the Higgs bosons are given by

$$
\dot{n}_{h_u} + 3Hn_{h_u} = \sum_{i=1}^{3} \left[ -\gamma_1^{-1}\Gamma_{h_u \rightarrow \tilde{\nu}_i}(n_{h_u} - n_{eq}^i) - \gamma_1^{-1}\Gamma_{h_u \rightarrow \tilde{\nu}_i}(n_{h_u} - n_{eq}^i) 
+ \Gamma_{\tilde{\nu}_i \rightarrow h_u} n_{\tilde{\nu}_i} + \Gamma_{\tilde{\nu}_i \rightarrow h_u} n_{\tilde{\nu}_i} 
- \gamma_1^{-1}\Gamma_{h_u \rightarrow R}(n_{h_u} - n_{eq}^i) + 2\Gamma_{\phi \rightarrow h}\frac{\dot{\phi}^2}{m_\phi},
\right]
$$

(5.75)

$$
\dot{n}_{h_d} + 3Hn_{h_d} = \sum_{i=1}^{3} \left[ -\gamma_1^{-1}\Gamma_{h_d \rightarrow \tilde{\nu}_i}(n_{h_d} - n_{eq}^i) + \Gamma_{\tilde{\nu}_i \rightarrow h_d} n_{\tilde{\nu}_i} 
- \gamma_1^{-1}\Gamma_{h_d \rightarrow R}(n_{h_d} - n_{eq}^i) + 2\Gamma_{\phi \rightarrow h}\frac{\dot{\phi}^2}{m_\phi},
\right]
$$

(5.76)

116
\[
\dot{n}_h + 3Hn_h = \sum_{i=1}^{3} \left[ -\gamma_h^{-1} \Gamma_{\tilde{h}_u \rightarrow \tilde{\nu}_i} (n_{\tilde{h}_u} - n_{h}^{eq}) - \gamma_h^{-1} \Gamma_{h_u \rightarrow \tilde{\nu}_i} (n_{\tilde{h}_u} - n_{h}^{eq}) \\
+ \Gamma_{\tilde{\nu}_i \rightarrow \tilde{h}_u} n_{\tilde{\nu}_i} + \Gamma_{\tilde{\nu}_i \rightarrow \tilde{h}_u} n_{\tilde{\nu}_i} \right] \\
- \gamma_h^{-1} \Gamma_{\tilde{h}_u \rightarrow R} (n_{\tilde{h}_u} - n_{h}^{eq}) + 2\Gamma_{\phi \rightarrow \tilde{h}_u} \frac{\dot{\phi}^2}{m_\phi},
\]
(5.77)

\[
\dot{n}_d + 3Hn_d = -\gamma_h^{-1} \Gamma_{\tilde{h}_d \rightarrow R} (n_{\tilde{h}_d} - n_{h}^{eq}) + 2\Gamma_{\phi \rightarrow \tilde{h}_u} \frac{\dot{\phi}^2}{m_\phi}.
\]
(5.78)

Since these evolution equations do not take momentum into account, we need to explicitly enforce detailed balance. We thus replace \(n_h\) by \((n_h - n_h^{eq})\), where \(n_h^{eq}\) is thermal equilibrium number density of the Higgs bosons, because the Higgs boson number densities should always approach the equilibrium number. The thermal equilibrium number density of the Higgs bosons are given by

\[
n_h^{eq} = \frac{g^2}{2\pi^3} \int_{m_h}^{\infty} dE \ E \sqrt{E^2 - m_h^2} e^{-E/T} = \frac{g^2}{2\pi^3} T^3 \frac{m_h^2}{T^2} K_2 \left( \frac{m_h}{T} \right),
\]
(5.79)

where \(K_2\) is the modified Bessel function of the second kind, the internal degrees of freedom (the \(g\)-factor) of the Higgs bosons is \(g = 4\) and we have used Maxwell-Boltzmann statistics.

Since the masses and \(g\)-factor are the same among the Higgs bosons, they have the same thermal equilibrium number density. On the other hand, the neutrinos never have sufficient time to equilibrate.

The Higgs bosons are in kinetic equilibrium with the thermal bath when the temperature is larger than their corresponding masses. Thus, we have to take into account that the Higgs bosons are not decaying at rest by multiplying their decay rates by a dilation factor, \(\gamma\), where

\[
\gamma_h = \sqrt{m_h^2 + T^2/m_h}\quad^{27}
\]

\(27\)We would like to thank Gary Steigman for pointing this out.
5.3.3 Evolution Equations for the Number Density of the Right-handed Neutrinos and Sneutrinos

The evolution equations for the right-handed neutrinos are

\[
\dot{n}_{\bar{\nu}_i} + 3Hn_{\bar{\nu}_i} = -\Gamma_{\bar{\nu}_i \rightarrow h_u^\dagger} n_{\bar{\nu}_i} - \Gamma_{\bar{\nu}_i \rightarrow \tilde{h}_u} n_{\bar{\nu}_i} + \gamma_h^{-1} \Gamma_{h_u \rightarrow \bar{\nu}_i^\dagger} (n_{h_u} - n_{h_u}^{eq}) + \gamma_{\tilde{h}}^{-1} \Gamma_{\tilde{h}_u \rightarrow \bar{\nu}_i^\dagger} (n_{\tilde{h}_u} - n_{\tilde{h}_u}^{eq}) \]

\[+ 2\Gamma_{\sigma \rightarrow \bar{\nu}_i} \frac{\rho_{\sigma}}{m_{\sigma}}.\]  

(5.80)

\[
\dot{n}_{\tilde{\bar{\nu}}_i} + 3Hn_{\tilde{\bar{\nu}}_i} = -\Gamma_{\tilde{\bar{\nu}}_i \rightarrow h_u^\dagger} n_{\tilde{\bar{\nu}}_i} - \Gamma_{\tilde{\bar{\nu}}_i \rightarrow \tilde{h}_u^\dagger} n_{\tilde{\bar{\nu}}_i} - \Gamma_{\tilde{\bar{\nu}}_i \rightarrow \tilde{h}_u} n_{\tilde{\bar{\nu}}_i} + \gamma_h^{-1} \Gamma_{h_u \rightarrow \tilde{\bar{\nu}}_i^\dagger} (n_{h_u} - n_{h_u}^{eq}) + \gamma_{\tilde{h}}^{-1} \Gamma_{\tilde{h}_u \rightarrow \tilde{\bar{\nu}}_i^\dagger} (n_{\tilde{h}_u} - n_{\tilde{h}_u}^{eq}) \]

\[+ \gamma_h^{-1} \Gamma_{\tilde{h}_u \rightarrow \tilde{\bar{\nu}}_i} (n_{\tilde{h}_u} - n_{\tilde{h}_u}^{eq}) \]

\[+ 2\Gamma_{\sigma \rightarrow \tilde{\bar{\nu}}_i} \frac{\rho_{\sigma}}{m_{\sigma}}.\]  

(5.81)

5.3.4 Evolution Equation for the Energy Density of the Waterfall Fields

The evolution equation for the energy density of waterfall fields is

\[
\dot{\rho}_{\sigma} + 3H\rho_{\sigma} + \Gamma_{\sigma \rightarrow \bar{\nu}_i} \rho_{\sigma} + \Gamma_{\sigma \rightarrow \tilde{\bar{\nu}}_i} \rho_{\sigma} = 0.
\]

(5.82)
5.3.5 Evolution Equation for the Energy Density of Radiation

The evolution equation for the energy density of radiation is \[ \dot{\rho}_R + 4H\rho_R = \sum_{i=1}^{3} \left[ \frac{1}{2} \gamma^{-1}_h \Gamma_{h_u \to \bar{\nu}_i} \rho_h(n_{h_u} - n_{h}^\text{eq}) + \frac{1}{2} \gamma^{-1}_h \Gamma_{h_u \to \nu_i} \rho_h(n_{h_u} - n_{h}^\text{eq}) \right] + \gamma^{-1}_h \Gamma_{h_u \to R} \rho_h(n_{h_u} - n_{h}^\text{eq}) \]

\[ + \frac{1}{2} \gamma^{-1}_h \Gamma_{h_d \to \bar{\nu}_i} \rho_h(n_{h_d} - n_{h}^\text{eq}) + \gamma^{-1}_h \Gamma_{h_d \to R} \rho_h(n_{h_d} - n_{h}^\text{eq}) \]

\[ + \frac{1}{2} \gamma^{-1}_h \Gamma_{h_u \to \bar{\nu}_i} \rho_h(n_{h_u} - n_{h}^\text{eq}) + \frac{1}{2} \gamma^{-1}_h \Gamma_{h_u \to \nu_i} \rho_h(n_{h_u} - n_{h}^\text{eq}) \]

(5.83)

5.3.6 Evolution Equation for the Number Density of Lepton Asymmetry

The number density of the lepton asymmetry is defined by

\[ n_L = n_\ell - n_{\bar{\ell}} \]

(5.84)

and its evolution equation is

\[ \dot{n}_L + 3Hn_L = \sum_{i=1}^{3} \left[ \epsilon_{h_i} \gamma^{-1}_h \Gamma_{h_u \to \bar{\nu}_i} (n_{h_u} - n_{h}^\text{eq}) + \epsilon_{h_i} \gamma^{-1}_h \Gamma_{h_u \to \nu_i} (n_{h_u} - n_{h}^\text{eq}) \right] + \epsilon_{h_i} \gamma^{-1}_h \Gamma_{h_d \to \bar{\nu}_i} (n_{h_d} - n_{h}^\text{eq}) \]

\[ + \epsilon_{h_i} \gamma^{-1}_h \Gamma_{h_d \to \nu_i} (n_{h_d} - n_{h}^\text{eq}) + \epsilon_{h_i} \gamma^{-1}_h \Gamma_{h_u \to \bar{\nu}_i} (n_{h_u} - n_{h}^\text{eq}) + \epsilon_{h_i} \gamma^{-1}_h \Gamma_{h_u \to \nu_i} (n_{h_u} - n_{h}^\text{eq}) \]

(5.85)

\[ + \epsilon_{h_i} \Gamma_{\tilde{\nu}_i \to h_u} n_{\tilde{\nu}_i} + \epsilon_{h_i} \Gamma_{\tilde{\nu}_i \to h_d} n_{\tilde{\nu}_i} \]

\[ + \epsilon_{h_i} \Gamma_{\bar{\nu}_i \to h_u} n_{\bar{\nu}_i} + \epsilon_{h_i} \Gamma_{\bar{\nu}_i \to h_d} n_{\bar{\nu}_i} + \epsilon_{h_i} \Gamma_{\bar{\nu}_i \to h_u} n_{\bar{\nu}_i} + \epsilon_{h_i} \Gamma_{\bar{\nu}_i \to h_d} n_{\bar{\nu}_i} \]

where \( \epsilon_{h_i} \) is defined in Eq. 5.56.

Notice that we did not include in this equation, \( \Delta L = 1 \) scatterings and \( \Delta L = 2 \)

\(^{28}\)Recall, \( M_{R_i} \) is the mass of the heavy right-handed neutrinos. Until the inflaton field settles at its minimum \( \langle \phi \rangle = 0 \), the Higgs bosons are on average very heavy.
processes mediated by the right-handed neutrinos. These processes are important if the right-handed neutrinos are in thermal equilibrium \[61, 156\]. See Sec. 5.5.3 for more discussions.

### 5.4 Procedure

We start the analysis of the reheating and baryogenesis at the first zero-crossing after the last 60 e-foldings of inflation. The details of the inflationary epoch can be found in Ref. [78].

The parameters used to produce the results in the rest of the chapter are

\[
\begin{align*}
\phi(0) = 0, & \quad \dot{\phi}(0) = -2.72 \times 10^{-6} M^2_{\text{pl}}, \\
m_\phi = 5.80 \times 10^{-6} M_{\text{pl}}, & \quad m_\sigma = 9.80 \times 10^{-3} M_{\text{pl}}, \\
\rho_\sigma(0) = 5.18 \times 10^{-15} M^4_{\text{pl}}, & \quad h_{3,2,1}^2 = 2 \frac{M^2_{R_3}}{v_{PS}} ,
\end{align*}
\]

\[
\sum_{i=1}^3 |\lambda_{u,ii}|^2 = 0.4099, \quad \sum_{i=1}^3 |\lambda_{d,ii}|^2 = 0.4041, \quad \sum_{i=1}^3 |\lambda_{e,ii}|^2 = 0.3245 , \tag{5.86}
\]

\[
M_{R_{3,2,1}} = \{1.136 \times 10^{-5}, 2.337 \times 10^{-7}, 3.685 \times 10^{-9}\} M_{\text{pl}},
\]

\[
(\lambda_\nu \lambda_\nu^\dagger)_{33,22,11} = \{3.316 \times 10^{-1}, 1.255 \times 10^{-2}, 1.736 \times 10^{-4}\},
\]

\[
\epsilon_{3,2,1} = \{-4.429 \times 10^{-5}, 6.038 \times 10^{-4}, 6.044 \times 10^{-7}\}.
\]

The inflation parameters are obtained by fitting to inflation observables while the matter sector parameters are obtained by fitting to the low-energy observables [78]. A benchmark point of low-energy fits is given in App. H.3.

#### 5.4.1 Non-perturbative regime

Using these parameters, we start our analysis by evolving the following set of evolution equations: Eq. 5.67, Eq. 5.75 - Eq. 5.78, Eq. 5.80 - Eq. 5.83, Eq. 5.85 and

\[
\dot{a} = aH, \tag{5.87}
\]

where the Hubble parameter, $H$, is given by Eq. 5.68. This set of evolution equations is solved using eighth-order Runge-Kutta method.
Since the condition for the non-perturbative creation of the Higgs bosons, \( q \gg 1 \) in Eq. [5.39] is dependent on the oscillation amplitude of the inflaton, we have to accurately identify the amplitude of the inflaton. When the velocity of the inflaton, \( \dot{\phi} \), changes sign, we interpolate the values of \( \dot{\phi} \) from the previous zero-values using a cubic spline to determine the time when \( \dot{\phi}(t^0) = 0 \). Then we interpolate and shift all other outputs from the set of evolution equations back to time \( t^0 \) and compute \( q \) using the amplitude, \( \phi(t^0) \). If \( q \leq 1/3 \), we stop the non-perturbative creation of the Higgs bosons from the inflaton and enter the purely perturbative regime.

Similarly, the determination of the zero-crossing of \( \phi \) is just as important because the Higgs bosons are created non-perturbatively at \( \phi = 0 \). Hence, the steps described above are repeated when \( \phi \) changes sign. We then manually increase the number of the Higgs bosons by Eq. [5.34] and Eq. [5.36] and decrease the speed of the inflaton by Eq. [5.37] to take into account the instantaneous energy loss due to the creation of the Higgs bosons.

### 5.4.2 Perturbative regime

Although the inflaton can no longer create the Higgs bosons non-perturbatively, we still have to consider the oscillation of the inflaton because the Higgs mass depends on the value of \( \phi \). However, we are no longer interested in the precise time of zero-crossing of the inflaton. Hence, to simplify numerical calculations, we assume that the inflaton oscillates sinusoidally and convert the inflaton equation of motion to a first-order differential equation of the inflaton energy density[29]:

\[
\dot{\rho}_\phi + 3H\rho_\phi + 2\Gamma_{\phi \rightarrow h}\rho_\phi + \Gamma_{\phi \rightarrow h}\rho_\phi = 0. \tag{5.88}
\]

The matching condition for the non-perturbative and perturbative regimes is \( \rho_\phi = \frac{m_\phi^2 \phi_\text{amp}}{2} \), where \( \phi_\text{amp} \) is the value of the inflaton amplitude when the non-perturbative evolution ends.

With this approximation, we convert all quantities that depend on \( \phi \) to the corresponding averaged value. For example, the Higgs mass, which is varying between 0 to \( \alpha \phi_\text{amp} \)

---

[29] We have used the time averaged result that \( \rho_\phi^2 = \rho_\phi \).
becomes

\[ \langle m_h \rangle = \frac{2}{\pi} m_{h_{\text{max}}}^{\text{max}} = \frac{2}{\pi} \alpha \sqrt{\frac{2 \rho_\phi}{m_\phi}}. \] (5.89)

Similarly, all the decay rates are also converted to the appropriate average decay rate, which is shown explicitly in App. G.

In this regime, the following set of evolution equations is solved: Eq. 5.88, Eq. 5.75 - Eq. 5.78 with \( \phi^2 \) replaced by \( \rho_\phi \), Eq. 5.80 - Eq. 5.83, and Eq. 5.85. The calculation is continued until the inflaton and the Higgs boson number densities are smaller than 1% of the number density of the lepton asymmetry. After this point, the effect of the inflaton and the Higgs bosons on the lepton asymmetry is insignificant and the inflaton and the Higgs boson evolution equations are removed from the set of evolution equations to further simplify the calculation. The set of relevant evolution equations now reduces to Eq. 5.82 and

\[
\dot{n}_{\nu_i} + 3Hn_{\nu_i} = -\Gamma_{\bar{\nu}_i \rightarrow h_u^\dagger} n_{\bar{\nu}_i} - \Gamma_{\bar{\nu}_i \rightarrow \tilde{h}_u^\dagger} n_{\bar{\nu}_i} + 2\Gamma_{\sigma \rightarrow \bar{\nu}_i} \frac{\rho_\sigma}{m_\sigma}
\]

\[
\dot{n}_{\bar{\nu}_i} + 3Hn_{\bar{\nu}_i} = -\Gamma_{\bar{\nu}_i \rightarrow h_u^\dagger} n_{\bar{\nu}_i} - \Gamma_{\bar{\nu}_i \rightarrow h_d} n_{\bar{\nu}_i} - \Gamma_{\bar{\nu}_i \rightarrow \tilde{h}_u^\dagger} n_{\bar{\nu}_i} + 2\Gamma_{\sigma \rightarrow \bar{\nu}_i} \frac{\rho_\sigma}{m_\sigma}
\]

\[
\dot{\rho_R} + 4H\rho_R = \sum_{i=1}^{3} +\Gamma_{\bar{\nu}_i \rightarrow h_u^\dagger} M_R n_{\bar{\nu}_i} + \Gamma_{\bar{\nu}_i \rightarrow \tilde{h}_u^\dagger} M_R n_{\bar{\nu}_i} + \Gamma_{\bar{\nu}_i \rightarrow h_d} n_{\bar{\nu}_i} - \Gamma_{\bar{\nu}_i \rightarrow \tilde{h}_u^\dagger} M_R n_{\bar{\nu}_i}
\]

\[
\dot{n}_L + 3Hn_L = \sum_{i=1}^{3} +\epsilon_{\nu_i} \Gamma_{\bar{\nu}_i \rightarrow h_u^\dagger} n_{\bar{\nu}_i} + \epsilon_{\nu_i} \Gamma_{\bar{\nu}_i \rightarrow \tilde{h}_u^\dagger} n_{\bar{\nu}_i} + \epsilon_{\nu_i} \Gamma_{\bar{\nu}_i \rightarrow h_d} n_{\bar{\nu}_i} - \epsilon_{\nu_i} \Gamma_{\bar{\nu}_i \rightarrow \tilde{h}_u^\dagger} M_R n_{\bar{\nu}_i}
\]

Notice that the right-handed sneutrinos can no longer decay to the down-type Higgs bosons because the inflaton has decayed out of the system and all remaining terms in the \( F \)-term of the Higgs bosons are quartic sfermion terms (see Eq. 5.25 and Eq. 5.26).

This set of evolution equations is evolved until the two heavier right-handed neutrinos decayed and the asymmetry contribution of the lightest right-handed neutrino is less than

\[ m_h = \alpha \langle \phi \rangle = 0 \] and we neglect three-body decays of the sneutrinos.

\[ \text{Note, this is because} \]

122
Finally, the baryon-to-entropy ratio is calculated from the lepton asymmetry 1%.

\[
\frac{n_B}{s} = -\frac{8}{23} \frac{n_L}{s}.
\]  

(5.91)

5.4.3 Sources of Uncertainties

It is important to note that there are multiple sources of uncertainties in our calculation. An obvious and main source of theoretical uncertainty is the neglect of momentum distributions and the exclusion of the $\Delta L = 1$ scatterings and $\Delta L = 2$ processes mediated by the right-handed neutrinos from our simulation. Hence, there are implicit error bars in each of the plots shown in this chapter. An interesting follow-up project would be to include the full Boltzmann equations treatment and the $\Delta L = 1$ scatterings and $\Delta L = 2$ processes to remove this theoretical uncertainty.

We also want to point out that the set of evolution equations are a set of non-linear, highly coupled, and stiff differential equations. Numerical solutions to differential equations with this characteristic are unstable, unless the time step is taken to be very small. However, with too small of a time step, numerical error will become significant. To tackle this problem, we have chosen the largest time step such that a smaller time step only changes the final energy density by a negligible amount.

5.5 Results and Discussions

5.5.1 Understanding the Hartree Approximation

To better understand the Hartee approximation, consider a simple case with only the up-type Higgs boson and the inflaton [149]. In addition, we assume that the non-perturbative creation of the Higgs bosons occurs only at the first zero-crossing, and the inflaton and the Higgs bosons do not decay perturbatively. We also ignore the expansion of the Universe. With these assumptions, the relevant evolution equations are

\[
\ddot{\phi} + m_\phi^2 \phi + \alpha n_h_u \text{sign}(\phi) = 0
\]

\[
\dot{n}_h_u = 0.
\]

(5.92)
The solution of this set of evolution equations with $\alpha = 1$ is plotted in Fig. 5.1, which shows the energy density of the inflaton and that of the up-type Higgs boson as a function of time. The energy densities are normalized to the initial energy density in the inflaton field. From this figure, we see that the up-type Higgs bosons are created with zero mass and do not contribute to the initial energy density. As the inflaton rolls up the potential, its energy density is transferred to the up-type Higgs boson while the total energy density of the inflaton-Higgs system stays constant. Similarly, the Higgs boson energy density is transferred back to the inflaton as the inflaton rolls down the potential. This simple case shows that the Higgs boson number density term in the inflaton equation of motion describes the transfer of energy between the inflaton and the Higgs boson as the inflaton oscillates.

5.5.2 Reheating

At early times, the non-perturbative creation of the Higgs bosons is very efficient. When the number densities of the created Higgs bosons exceed its thermal equilibrium number density, the Higgs bosons decay to radiation and to the right-handed neutrinos. This occurs
Figure 5.2: The inflaton value in this plot is normalized to the first oscillation amplitude, $10^{-1} M_{pl}$, while the Higgs boson number densities are normalized to the up-type Higgs boson number densities created at the first zero-crossing, $7.23 \times 10^{-11} M_{pl}^3$. At every zero-crossing of the inflaton, the Higgs bosons are created and subsequently decay out of the system almost instantaneously.

almost instantaneously because the oscillation amplitude of the inflaton is of order Planck scale while the Higgs boson decay rate is proportional to the Higgs boson mass, which is proportional to the inflaton VEV. This effect is shown in Fig. 5.2 which shows the magnitude of the inflaton oscillations and the Higgs boson number densities as a function of time for $\alpha = 1$. The magnitude of the inflaton oscillations is normalized to the first oscillation amplitude, $10^{-1} M_{pl}$, while the Higgs boson number densities are normalized to the number of the up-type Higgs bosons created at the first zero-crossing, $7.23 \times 10^{-11} M_{pl}^3$.

Figure 5.3 shows the inflaton oscillation speed as a function of time for $\alpha = 1$. From this plot, we see that the inflaton experiences a drastic decrease in speed at every zero-crossing due to the almost instantaneous decay of the Higgs bosons. When the Higgs bosons decay, energy is transferred out from the inflaton-Higgs system to the decay products. In addition, this drastic decrease does not occur when the inflaton speed is at its maximum because the inflaton oscillation is damped. The sub-figure, which is the zoomed-in version of the
Figure 5.3: The inflaton speed in this plot is normalized to the initial inflaton speed, $2.63 \times 10^{-6} M_{\text{pl}}^2$. Since the non-perturbative creation of the Higgs boson is instantaneous and the subsequent the Higgs boson decay are almost instantaneous, the inflaton experiences a drastic decrease in speed at every zero-crossing. The Higgs boson decays transfers energy out from the inflaton-Higgs system.

plot, shows the typical behavior of the inflaton speed at zero-crossing. As shown in the sub-figure, the inflaton speed has a discontinuous drop, which is due to the energy lost in the non-perturbative creation of the Higgs bosons. The sub-figure also shows that the inflaton speed increases momentarily before the zero-crossing. As the inflaton rolls down the potential, the Higgs boson mass decreases. Once the Higgs bosons become lighter than the right-handed neutrinos, the right-handed neutrinos start to decay to the Higgs bosons, increasing the energy in the inflaton-Higgs system. However, as shown in Sec. 5.5.1, as the inflaton rolls down the potential, energy is transferred from the Higgs bosons to the inflaton. Hence, the inflaton speed increases for a short period of time before reaching the bottom of the potential.

The dynamics in the non-perturbative regime with $q \gg 1$ is dominated by a broad parametric resonance. When $q \leq 1/3$, and the inflaton continues to oscillate, there is only a narrow region in momentum space where parametric resonance can occur. At every passage
of $\phi$ through zero, the Higgs bosons are produced, however they quickly thermalize on an oscillation time scale. Thus these Higgs bosons acquire momentum outside the region of the parametric resonance, which in effect suppresses the additive effect. It is at this point that instant preheating ceases to dominate.

Figure 5.4 shows the energy densities of the inflaton, the waterfall fields, radiation, and the right-handed neutrinos as a function of time for $\alpha = 1$. From this figure, we see that after a couple of inflaton oscillations, radiation energy density dominates over all other energy densities. The associated reheat temperature is of order $T_{\text{reheat}} \sim 10^{15}$ GeV, i.e. less than the GUT scale. This shows that the reheating process of our model is very efficient.

The epoch of radiation domination occurs later as $\alpha$ decreases because the non-perturbative and the perturbative decay rates of the inflaton decrease as $\alpha$ decreases. In addition, we clearly see from this figure that the energy densities of the inflaton and the right-handed neutrinos increase or decrease like a step-like function. This step-like function behavior is due to the non-perturbative creation of the Higgs bosons that only occurs at zero-crossing.

As the inflaton oscillation amplitude decreases, the Higgs boson decay occurs slower, which increases the Higgs boson number densities in the system (see Fig. 5.5). This effect increases the inflaton oscillation frequency because, from the Hartree approximation in Eq. 5.67, the oscillation frequency is given by

$$\omega = m_\phi^2 + \alpha^2 \langle h^2 \rangle = m_\phi^2 + \alpha \frac{m_h}{|\phi|}. \quad (5.93)$$

The increase in the inflaton oscillation frequency further increases the Higgs boson number densities because the non-perturbative creation now occurs more frequently. In addition, the increase in the Higgs boson number densities decreases the inflaton oscillation amplitude because the Higgs bosons are taking away more energy from the inflaton. Notice that this effect produces a feedback effect that decreases the amplitude of the inflaton oscillation at a faster rate. This effect can be seen in Fig. 5.5, which shows the inflaton oscillation amplitude and the Higgs boson number densities as a function of time with $\alpha = 1$. This figure is the continuation of Fig. 5.2.

This will have consequences for gravitino bounds which we discuss in the conclusion.
Figure 5.4: Since the Universe is radiation dominated after a couple inflaton oscillations, the reheating process of our model is very efficient.

Figure 5.5: This plot is a continuation of Fig. 5.2. The decrease in the inflaton oscillation amplitude increases the Higgs boson number densities by decreasing the Higgs boson decay rate. The increase in the Higgs boson number densities increases the inflaton oscillation frequency, which further decreases the inflaton oscillation amplitude by increasing the frequency of the inflaton non-perturbative creation of the Higgs boson.
We define the reheating temperature only when the energy density is dominated by radiation energy density. In Fig. 5.4, this corresponds to $m_\phi t \approx 14$. From this, we see that the reheating temperature, $T_{RH} \approx 10^{15}$ GeV. The reheating temperature of this model is so high because the inflaton creates the Higgs bosons very efficiently at every zero-crossing. Hence, a large amount of energy is transferred from the inflaton to the Higgs bosons. Since the Higgs bosons decays to radiation quickly, the Universe reheats before the energy density is able to be diluted.

5.5.3 Leptogenesis

An interesting feature of our model is that the phases in the right-handed neutrino Yukawa matrix are fixed by fitting to the low-energy data. With these phases, the decay of the heaviest right-handed neutrinos produce more anti-leptons than leptons, while the decay of the two lighter right-handed neutrinos produce more leptons than anti-leptons. Hence, unlike most models in the literature, we cannot integrate out any right-handed neutrinos. Most of our baryon asymmetry is created from the heaviest right-handed neutrinos, while the two lighter right-handed neutrinos wash out a portion of the asymmetry. To obtain the correct baryon asymmetry, our model requires some level of fine-tuning between the asymmetry created from the heaviest right-handed neutrinos and the washout by the two lighter right-handed neutrinos.

From Sec. 5.5.2, we see that the reheating temperature is larger than all the three right-handed neutrino masses, the right-handed neutrinos in this model are thermal. As described in Sec. 1.4, in a thermal leptogenesis, the lightest right-handed neutrino washes out all asymmetry created by two heavier right-handed neutrinos. Since the lightest right-handed neutrino does not produce the asymmetry with the correct sign, this model cannot fit the observed baryon asymmetry.

Potential Solution

There are two approaches to solve this problem.
1. Modify the neutrino Yukawa matrix such that the lightest right-handed neutrinos produce a lepton asymmetry with the correct sign.

2. Modify the theory such that the reheating temperature is below the lightest right-handed neutrino mass.

In this subsection, we will briefly discuss the second approach.

The reheating temperature of this model is very high because the inflaton creates the Higgs bosons very efficiently and the Higgs bosons decay to radiation quickly. One method to circumvent this problem is to introduce a new field and instead of coupling the inflaton to the Higgs boson, we couple the inflaton to this new field. Since the inflation dynamics does not depend on the inflaton-Higgs term, Eq. 5.19, we can modify this part of theory without reperforming the inflation analysis.

In order for instant reheating to occur, this new field needs to have zero mass at some point in the inflaton oscillation. With instant reheating, most of the inflaton energy density will be transferred to this new field after a couple oscillations. This new field can potentially resolve the reheating temperature issue if we do not allow it to decay until much of its energy density is diluted by expansion. However, we still need the right-handed neutrinos to exist in the early Universe. So, this new field needs to couple to the right-handed neutrinos and eventually has a mass larger than the right-handed neutrinos masses. The reheating process then occurs as the right-handed neutrinos are decaying.

Another challenging aspect of this theory is that since this new field does not decay, it does not thermalize. We cannot ignore the backreaction of this new field to its non-perturbative production from the inflaton \[149\]. Hence, our current treatment in the non-perturbative regime will be insufficient.

5.6 Conclusion

This chapter is an extension of the previous Pati-Salam subcritical hybrid inflation paper by Bryant et al., which was shown to successfully reproduce inflation observables \[78\]. In this chapter, we studied the reheating process and the baryogenesis via leptogenesis of this
In the instant preheating process, the coupling of the inflaton to the Higgs bosons causes the inflaton to non-perturbatively decay to the Higgs bosons efficiently as it oscillates around its minimum. The produced Higgs bosons then decay to radiation and reheat the Universe. We find that the reheating temperature $T_{\text{reheat}} \sim 10^{15} \text{ GeV}$. This can, in principle, create a cosmological problem with gravitinos. However, for gravitino masses greater than $\sim 40 \text{ TeV}$, the only problem concerns the over-closure of the Universe by an LSP with mass of order 100 GeV [158]. This suggests that the LSP in our model would need to be a light axino in conjunction with an axion dark matter candidate. This then ties in interestingly to the scenario of Ref. [159].

As for baryogenesis, fitting to low-energy observables forces the $CP$ asymmetry parameter of the heaviest right-handed neutrinos to have the correct sign, while that of the two lighter right-handed neutrinos to have the wrong sign. Hence, it is important to include all three right-handed neutrinos in our analysis. This model, however, cannot produce the correct baryon asymmetry because the right-handed neutrinos are thermal and the lightest right-handed neutrinos washout the asymmetry produced by the heaviest right-handed neutrinos. A potential solution to this problem is discussed in Sec. 5.5.3.
In a broad sense, model building beyond the SM can be separated into two general approaches. One approach to model building is to make minimal extensions to the SM in an attempt to solve SM challenges one by one. An example of this approach is the vector-like leptons model studied in Chapter 2. Another diametrical different approach is to make huge and theoretically well-motivated changes to the SM in an attempt to solve as many SM challenges as possible. Examples of this approach are the SUSY GUTs studied in Chapter 3, 4, and 5.

In Chapter 2, we studied a model in which a family of VL leptons is added to the SM. We identified regions of parameter space that fit the discrepancy between the experimentally measured muon anomalous magnetic moment and the SM prediction while satisfying current mass bound on heavy charged lepton, precision electroweak measurements, LFV constraints, and Higgs decay constraints. This analysis showed that the more stringent measurements are muon $g - 2$ discrepancy, the deviation from the SM of Higgs bosons decaying to muons $R_{\mu\mu}$, the deviation from the SM of Higgs bosons decaying to photons $R_{\gamma\gamma}$, and the branching ratio of muon decaying to electron $BR(\mu \rightarrow e\gamma)$. We found that although VL leptons couple to all three families of the SM leptons, the ratio of the electron-VL couplings to the muon-VL couplings is highly constrained. We also found that this model cannot fit lepton non-universality measurements, $R_K$ and $R_{K^*}$.

In Chapter 3, we studied a three family SO(10) SUSY GUT with Yukawa unification. We found that this model has a lower $\chi^2$ when fitting to exclusive than inclusive measurements.
of $|V_{ub}|$ and $|V_{cb}|$. Hence, this model predicts that exclusive measurements are the correct measurements. In addition, this model favors scalar masses that are in the intermediate range of $m_{16} \approx 25$ TeV, which is neither a split nor natural SUSY. We also found that universal gaugino masses and mirage mediated gaugino masses with $\alpha = 1.5$ produce similar gluino mass bound. Finally, the fine-tuning of this model can be reduced from one part in $10^5$ to one part in 500 when the ratio of the Higgs mass to the scalar mass at the GUT scale and the ratio of trilinear couplings to the scalar mass at the GUT scale are fixed. This suggests that these two ratios occur naturally in a more fundamental theory.

The SO(10) SUSY GUT analyzed in Chapter 3 only has reasonable global $\chi^2$ fit to low-energy observables. In Chapter 4, we changed the gauge group from SO(10) to PS group in order to improve the fit. We showed that by introducing two additional parameters to the first two families, the global $\chi^2$ fit is improved significantly. In addition, we also reinterpret ATLAS and CMS simplified model analyses to obtain the current gluino mass bound of this model. From the analysis, we found that the $2\sigma$ upper bound of the gluino mass of this model is beyond the reach of LHC run 2. Hence, this model will not be ruled out even if gluinos are not found in LHC run 2. On the bright side, this model indicates that low-energy SUSY is still viable and LHC might find gluinos in the near future.

In Chapter 5, we studied the reheating and leptogenesis of an inflation model with PS gauge symmetry and a $Z_4^R$ discrete symmetry. This model reheats via instant preheating in which the inflaton decays non-perturbatively to Higgs bosons as the inflaton oscillate after inflation ends. This decay is very efficient. Hence, the inflaton loses most of its energy density in the first couple oscillations and the Universe reheats almost instantaneously. Since the Universe reheats before much of the energy density in the Universe is diluted, the reheating temperature is very high. The leptogenesis of this model occurs via $CP$ asymmetric decays of right-handed neutrinos. By fitting to low-energy observables, the heaviest right-handed neutrino produces asymmetry with the correct sign while the two lighter right-handed neutrinos produce asymmetry with the wrong sign. Since the reheating temperature is higher than the right-handed neutrinos masses, the asymmetry created by the two heavier right-handed neutrinos is completely washed out by the lightest right-
handed neutrinos. Hence, unfortunately, this model does not produce the correct baryon asymmetry. Potential solutions to this problem are outlined in Sec. 5.5.3.

With the upcoming muon anomalous magnetic moment experiment at Fermilab scheduled to release its first result in Spring 2018, we will soon find out if the VL leptons model can still explain the muon $g - 2$ discrepancy. It is also interesting to see if this model can withstand Higgs decay constraints as more Higgs decay data are collected at the LHC. On the other hand, although gluinos are still not discovered, the High Luminosity LHC, which targets to collect $3000 \text{ fb}^{-1}$ at $14\text{ TeV}$ by the end of 2035, has $5\sigma$ discovery potential for gluino mass up to $2.35\text{ TeV}$ and can exclude gluino mass up to $2.9\text{ TeV}$ [160, 161]. Hence, in the near future, we should be able to determine if the SUSY GUTs considered in this thesis are part of nature.
Bibliography


136


[86] T. Inami and C. S. Lim, “Effects of Superheavy Quarks and Leptons in Low-Energy Weak Processes $K_L \to \mu\bar{\mu}, K^+ \to \pi^+\nu\bar{\nu}$ and $K^0 \leftrightarrow \bar{K}^0$,” *Prog. Theor. Phys.* **65** (1981) 297. [Erratum: Prog. Theor. Phys.65,1772(1981)].


[108] CMS, LHCb Collaboration, V. Khachatryan *et al.*, “Observation of the rare $B_s^0 \rightarrow \mu^+ \mu^-$ decay from the combined analysis of CMS and LHCb data,” [1411.4413](https://arxiv.org/abs/1411.4413).


[116] J. Matias, F. Mescia, M. Ramon, and J. Virto, “Complete Anatomy of $\bar{B}_d\to\bar{K}^{*0}(\to K\pi)l^+l^-$ and its angular distribution,” *JHEP 1204* (2012) 104, [1202.4266](1202.4266).


[165] C. Bobeth, A. J. Buras, F. Kruger, and J. Urban, “QCD corrections to $\bar{B} \to X_{d,s} \nu \bar{\nu}$, $\bar{B}_{d,s} \to \ell^+ \ell^-$, $K \to \pi \nu \bar{\nu}$ and $K_L \to \mu^+ \mu^-$ in the MSSM,” *Nucl. Phys.* **B630** (2002) 87–131, [hep-ph/0112305](https://arxiv.org/abs/hep-ph/0112305).


[169] L. Widhalm, *A study of the rare decay $K^0_L \to \pi^0 \pi^\pm e^\mp \nu_e$ (Ke4) at the CERN CP-Violation experiment NA48*. PhD thesis, Vienna, OAW, 2001.


Appendix A
Box Diagram Calculation

In this appendix, we calculate four box diagrams that have new physics (NP) contributions to the decay of $b \to s\ell\ell$ in an extension of the Standard Model with vector-like (VL) leptons. As shown in Fig. 2.1, the NP contributions to these diagrams are due to VL leptons in the loop. The NP contributions enter via Wilson coefficients $C_9$ and $C_{10}$ considered in Chapter 2. The calculation in this appendix is done in the ’t Hooft-Feynman gauge.

A.1 Feynman Rules

To see all the Feynman rules explicitly, we start by rewriting the Lagrangian that is relevant to our calculation. The definition of the fields and their corresponding quantum numbers are given in Table 2.1. The Lagrangian of the leptonic sector is given in Eq. 2.1.

**W-lepton Couplings**

From Eq. 2.11, we have

$$
\mathcal{L} \supset \frac{g}{\sqrt{2}} \left[ W^+_{\mu} \bar{\nu}_a \gamma^\mu (\tilde{U}_L)_{ab} P_L + [\tilde{U}_R]_{ab} P_R \right] \bar{e}_b \gamma^\mu (\tilde{U}_L^*)_{ab} P_L + [\tilde{U}_R^*]_{ab} P_R \nu_a \right] , \tag{A.1}
$$

where $P_{L,R}$ are projection operators and

$$
\tilde{U}_L = \text{diag}(1,1,1,1,0) U_L ,
$$

$$
\tilde{U}_R = \text{diag}(0,0,0,1,0) U_R .
$$

Notice that $[U_L]_{4a} = [\tilde{U}_L]_{4a}$, where $a = 1, \ldots, 5$. Similarly for $U_R$. 

153
**Higgs-lepton Couplings**

From Eq. 2.1, the Lagrangian containing Higgs-lepton couplings is

$$\mathcal{L} \supset -\bar{\ell}_LiH - \bar{\ell}_Li\lambda_i^E e_RH - \bar{\ell}_L\lambda'_i e_RH - \bar{\ell}_L\lambda^\pounds e_RH - \bar{E}_L\lambda L_RH^\dagger + \text{h.c.} \quad (A.3)$$

Rewriting the Lagrangian in terms of the physical Higgs and the would-be Nambu-Goldstone bosons gives

$$\mathcal{L} \supset -\left( v + \frac{h}{\sqrt{2}} \right) \bar{e}_L Y^e_{ab} e_{R_b} - \frac{i\phi^0}{\sqrt{2}} \bar{e}_L Y^{e\phi^0}_{ab} e_{R_b}$$

$$- \phi^+ \nu_L Y^\nu \nu_{ba} e_{R_a} - \phi^- \bar{e}_L Y^{\nu R\dagger}_{ab} \nu_{R_b} + \text{h.c.} \quad (A.4)$$

where

$$Y^e \equiv \begin{pmatrix} y^e_{ii} & 0 & \lambda^E_i \\ \lambda'_i & 0 & \lambda \\ 0 & -\bar{\lambda} & 0 \end{pmatrix}, \quad (A.5)$$

$$Y^{e\phi^0} \equiv \begin{pmatrix} y^e_{ii} & 0 & \lambda^E_i \\ \lambda'_i & 0 & \lambda \\ 0 & \bar{\lambda} & 0 \end{pmatrix}, \quad (A.6)$$

$$Y^\nu \equiv \begin{pmatrix} y^\nu_{ii} & 0 & \lambda^E_i \\ \lambda'_i & 0 & \lambda \\ 0 & 0 & 0 \end{pmatrix}, \quad (A.7)$$

$$Y^{\nu R\dagger} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \bar{\lambda} & 0 \end{pmatrix}. \quad (A.8)$$

In the charged-lepton mass basis, we have

$$\mathcal{L} \supset -\left( v + \frac{h}{\sqrt{2}} \right) \bar{e}_L \tilde{Y}^e_{ab} \hat{e}_{R_b} - \frac{i\phi^0}{\sqrt{2}} \bar{e}_L [U^\dagger_L Y^{e\phi^0} U_R]_{ab} \hat{e}_{R_b}$$

$$- \phi^+ \nu_L [Y^\nu \nu_R]_{ba} \hat{e}_{R_a} - \phi^- \bar{e}_L [U^\dagger_L Y^{\nu R\dagger}]_{ab} \nu_{R_b} + \text{h.c.} \quad (A.9)$$
So, the Feynman rules for the coupling in diagrams (b)-(d) in Fig. 2.1 involving the charged would-be Nambu-Goldstone bosons are

\[ \phi^+ : -i([Y^{\nu \nu}_L]_{4a} P_R + [Y^{\nu \nu}_R U_L]_{4a} P_L), \]  
\[ \phi^- : -i([Y^{\nu \nu*}_R U_R]_{4a} P_L + [Y^{\nu \nu*}_L U_L]_{4a} P_R). \]  

(A.10)  

(A.11)

Since all the calculations are performed in the charged-lepton mass basis, to simplify notation, we will drop \(^\wedge\) in the rest of this appendix.

### A.2 Loop Diagrams Calculations

Before we start to evaluate the four diagrams in Fig. 2.1 let’s consider two loop integrals that we will be using. These loop integrals are performed easily with Package-X developed by Patel [162].

\[ A_{\alpha \beta}(M_i, M_L) \equiv \int \frac{d^4 q}{(2\pi)^4 (q^2 - M_i^2)^2 (q^2 - M_L^2)} \left( \frac{g_{\alpha \beta}}{(q^2 - M_i^2)(q^2 - M_L^2)} \right) \]
\[ = - \frac{i}{64\pi^2 M_W} g_1(x_i, y) g_{\alpha \beta}, \]  

(A.12)

\[ B(M_i, M_L) \equiv \int \frac{d^4 q}{(2\pi)^4 (q^2 - M_W^2)^2 (q^2 - M_i^2)(q^2 - M_L^2)} \]
\[ = - \frac{i}{16\pi^2 M_W} g_0(x_i, y), \]  

(A.13)

where \( x_i = M_i^2/M_W^2, \ y = M_L^2/M_W^2 \) and

\[ g_1(x, y) = \frac{1}{x - y} \left[ \frac{x^2}{(x - 1)^2} \log x - \frac{y^2}{(y - 1)^2} \log y - \frac{1}{x - 1} + \frac{1}{y - 1} \right], \]
\[ g_0(x, y) = \frac{1}{x - y} \left[ \frac{x}{(x - 1)^2} \log x - \frac{y}{(y - 1)^2} \log y - \frac{1}{x - 1} + \frac{1}{y - 1} \right]. \]  

(A.14)  

(A.15)
Diagram (a)

\[ \square^{(a)} = \left( \frac{g}{\sqrt{2}} \right)^4 \sum_{i=u,c,t} V_{ib} V_{is}^* \int \frac{d^4 q}{(2\pi)^4} \left( \frac{-i}{q^2 - M_W^2} \right)^2 \left[ \bar{s} \gamma^\mu P_L \frac{i(q + M_i)}{q^2 - M_i^2} \gamma^\nu P_L b \right] \]

\[ \bar{e}_a ([U_L]_{4a} \gamma_{\nu} P_L + [U_R]_{4a} \gamma_{\nu} P_R) i(q + M_L) \frac{([U_L]_{4a} \gamma_{\mu} P_L + [U_R]_{4a} \gamma_{\mu} P_R) e_a}{q^2 - M_L^2} \]

\[ = \frac{g^4}{4} \sum_{i=u,c,t} V_{ib} V_{is}^* \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2 - M_W^2)^2(q^2 - M_i^2)(q^2 - M_L^2)} \]

\[ [\bar{s} \gamma^\mu P_L (q + M_i) \gamma^\nu P_L b] \]

\[ [\bar{e}_a ([U_L]_{4a} \gamma_{\nu} P_L + [U_R]_{4a} \gamma_{\nu} P_R)(q + M_L) ([U_L]_{4a} \gamma_{\mu} P_L + [U_R]_{4a} \gamma_{\mu} P_R) e_a] . \]

The last two square brackets can be written as

\[ q_{\alpha} q_{\beta} [\bar{s} \gamma^\mu \epsilon^\alpha \gamma^\nu P_L b][\bar{e}_a \gamma_{\nu} \gamma_{\mu} ([U_L]_{4a} \gamma_{\mu} P_L + [U_R]_{4a} \gamma_{\mu} P_R) e_a] , \]

where we have dropped terms linear in \( q \). Using Eq. A.12

\[ \square^{(a)} = \frac{g^4}{4} \sum_{i=u,c,t} V_{ib} V_{is}^* A_{\alpha \beta}(M_i, M_L) [\bar{s} \gamma^\mu \epsilon^\alpha \gamma^\nu P_L b] \]

\[ [\bar{e}_a \gamma_{\nu} \gamma_{\mu} ([U_L]_{4a} \gamma_{\mu} P_L + [U_R]_{4a} \gamma_{\mu} P_R) e_a] . \]

Using \( g_{\alpha \beta} \) from \( A_{\alpha \beta} \), the last two square brackets can be written as

\[ [\bar{s} \gamma^\mu \epsilon^\alpha \gamma^\nu P_L b][\bar{e}_a \gamma_{\nu} \gamma_{\mu} ([U_L]_{4a} \gamma_{\mu} P_L + [U_R]_{4a} \gamma_{\mu} P_R) e_a] . \]

Using the following Dirac matrix identity,

\[ \gamma^\mu \gamma^\alpha \gamma^\nu = g^{\alpha \nu} \gamma^\mu + g^{\alpha \mu} \gamma^\nu - g^{\mu \nu} \gamma^\alpha - i \epsilon^{\beta \mu \nu \alpha \nu} \gamma_\beta \gamma^5 , \]

we can rewrite the last two square brackets as

\[ 4 [\bar{s} \gamma^\mu P_L b][\bar{e}_a \gamma_{\mu} ([U_L]_{4a} \gamma_{\mu} P_L + [U_R]_{4a} \gamma_{\mu} P_R) e_a] . \]
Putting all these together, we have

\[ \Box^{(a)} = -i \frac{4GE}{\sqrt{2}} \sum \limits_{i, u, c, t} V_{ib} V_{is}^* e^2 \frac{1}{16\pi^2} \frac{1}{s_W^2} \frac{1}{2} g_1(x_i, y) \]

\[ [\bar{s} \gamma^\mu P_L b][\bar{e}_a \gamma_\mu ([|U_L|_{4a}]^2 P_L + [|U_R|_{4a}]^2 P_R) e_a]. \]

Hence, the contributions of this diagram to the Wilson coefficients are

\[ C_9^{NP(a)} = -\frac{1}{s_W^2} \frac{1}{4} ([[U_L|_{4a}]^2 + [|U_R|_{4a}]^2) g_1(x_i, y), \]

\[ C_{10}^{NP(a)} = \frac{1}{s_W^2} \frac{1}{4} ([[U_L|_{4a}]^2 - [|U_R|_{4a}]^2 g_1(x_i, y). \]  

(A.16)

Diagram (b)

\[ \Box^{(b)} = \left( \frac{g}{\sqrt{2}} \right)^4 \sum \limits_{i, u, c, t} V_{ib} V_{is}^* \int \frac{d^4q}{(2\pi)^4} \left( \frac{-i}{q^2 - M_W^2} \right)^2 [\bar{s} \gamma^\mu P_L \frac{i(q + M_i)}{q^2 - M_i^2} P_L b \right] \]

\[ \left[ \bar{c}_a - v([Y^{\nu L*} U_R^*]_{4a} P_L + [Y^{\nu R*} U_L^*]_{4a} P_R) \frac{i(q + M_L)}{q^2 - M_L^2} ([U_L]_{4a} \gamma_\mu P_L + [U_R]_{4a} \gamma_\mu P_R) e_a \right] \]

\[ = \frac{g^4}{4} \sum \limits_{i, u, c, t} V_{ib} V_{is}^* \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - M_W^2)(q^2 - M_i^2)(q^2 - M_L^2)} \]

\[ \left[ \bar{s} \gamma^\mu P_L (q + M_i) \frac{M_i}{M_W} P_L b \right] \]

\[ \left[ \bar{c}_a - v([Y^{\nu L*} U_R^*]_{4a} P_L + [Y^{\nu R*} U_L^*]_{4a} P_R) \frac{(q + M_L)}{M_W} ([U_L]_{4a} \gamma_\mu P_L + [U_R]_{4a} \gamma_\mu P_R) e_a \right]. \]

The last two square brackets can be written as

\[ -v \frac{M_L^2 M_i}{M_W^2} [\bar{s} \gamma^\mu P_L b][\bar{e}_a \gamma_\mu ([|U_L|_{4a}]^2 [Y^{\nu R*} U_L^*]_{4a} P_L + [U_R]_{4a} [Y^{\nu L*} U_R^*]_{4a} P_R) e_a], \]

where we have dropped terms linear in \( q \). Using Eq. (A.13)

\[ \Box^{(b)} = -\frac{g^4}{4} \sum \limits_{i, u, c, t} V_{ib} V_{is}^* B(M_i, M_L) \frac{v M_L^2 M_i}{M_W^2} [\bar{s} \gamma^\mu P_L b] \]

\[ [\bar{e}_a \gamma_\mu ([U_L]_{4a} [Y^{\nu R*} U_L^*]_{4a} P_L + [U_R]_{4a} [Y^{\nu L*} U_R^*]_{4a} P_R) e_a]. \]
Putting all these together, we have

\[
\square^{(b)} = \frac{AG_F}{\sqrt{2}} \sum_{i=u,c,t} V_{ib} V_{is}^* \frac{e^2}{16\pi^2} \frac{1}{s_W^2} \frac{1}{2 M_L} x_i y q_0(x_i, y)
\]

\[
[\bar{s} \gamma^\mu P_L b][\bar{e}_a \gamma_\mu ([U_L]_{4a} [Y^{\nu R} U_L^*]_{4a} P_L + [U_R]_{4a} [Y^{\nu L} U_R^*]_{4a} P_R) e_a].
\]

Hence, the contributions of this diagram to the Wilson coefficients are

\[
C^{NP(b)}_9 = \frac{1}{4} \frac{1}{s_W^2} M_L x_i y ([U_L]_{4a} [Y^{\nu R} U_L^*]_{4a} + [U_R]_{4a} [Y^{\nu L} U_R^*]_{4a}) q_0(x_i, y),
\]

\[
C^{NP(b)}_{10} = -\frac{1}{4} \frac{1}{s_W^2} M_L x_i y ([U_L]_{4a} [Y^{\nu R} U_L^*]_{4a} - [U_R]_{4a} [Y^{\nu L} U_R^*]_{4a}) q_0(x_i, y).
\]

Diagram (c)

\[
\square^{(c)} = \left( \frac{g}{\sqrt{2}} \right)^4 \sum_{i=u,c,t} V_{ib} V_{is}^* \int \frac{d^4 q}{(2\pi)^4} \left( \frac{-i}{p^2 - M_W^2} \right) \left[ \bar{s} \left( \frac{M_i}{M_W} P_R \frac{i(\not{q} + M_L)}{q^2 - M_L^2} \gamma^\mu P_L b \right) \right]
\]

\[
\left[ \bar{e}_a ([U_L^*]_{4a} \gamma_\mu P_L + [U_R^*]_{4a} \gamma_\mu P_R) \frac{i(\not{q} + M_L)}{q^2 - M_L^2} - \nu ([Y^{\nu L} U_R]_{4a} P_R + [Y^{\nu R} U_L]_{4a} P_L) e_a \right].
\]

\[
= \frac{g^2}{4} \sum_{i=u,c,t} V_{ib} V_{is}^* \int \frac{d^4 q}{(2\pi)^4} \left( \frac{1}{(q^2 - M_W^2)^2(q^2 - M_L^2)} \right)
\]

\[
\left[ \bar{s} \left( \frac{M_i}{M_W} P_R (\not{q} + M_L) \gamma^\mu P_L b \right) \right]
\]

\[
\left[ \bar{e}_a ([U_L^*]_{4a} \gamma_\mu P_L + [U_R^*]_{4a} \gamma_\mu P_R) (\not{q} + M_L) - \nu ([Y^{\nu L} U_R]_{4a} P_R + [Y^{\nu R} U_L]_{4a} P_L) e_a \right].
\]

The last two square brackets can be written as

\[
- \frac{v M^2 M_L}{M_W^2} \left[ \bar{s} \gamma^\mu P_L b \right] [\bar{e}_a \gamma_\mu ([U_L^*]_{4a} [Y^{\nu R} U_L]_{4a} P_L + [U_R^*]_{4a} [Y^{\nu L} U_R]_{4a} P_R) e_a],
\]

where we have dropped terms linear in \( q \). Using Eq. A.13

\[
\square^{(c)} = -\frac{g^4}{4} \sum_{i=u,c,t} V_{ib} V_{is}^* B(M_i, M_L) \frac{v M^2 M_L}{M_W^2} \left[ \bar{s} \gamma^\mu P_L b \right]
\]

\[
[\bar{e}_a \gamma_\mu ([U_L^*]_{4a} [Y^{\nu R} U_L]_{4a} P_L + [U_R^*]_{4a} [Y^{\nu L} U_R]_{4a} P_R) e_a].
\]
Putting all these together, we have

\[
\Box^{(c)} = \frac{AG_F}{\sqrt{2}} \sum_{i=u,c,t} V_{ib}V_{is}^* \frac{e^2}{16\pi^2} \frac{1}{s_W^2} \frac{1}{2M_L} x_i y_0(x_i, y)
\]

\[s^\gamma_\mu P_L b [\bar{e}_a \gamma_\mu ([U_L^*]_{4a}[Y_{\nu R} U_L]_{4a} P_L + [U_R^*]_{4a}[Y_{\nu L} U_R]_{4a} P_R) e_a].\]

Hence, the contributions of this diagram to the Wilson coefficients are

\[
C_{9}^{NP(c)} = \frac{1}{s_W^2} \frac{1}{4M_L} x_i y_0([U_L^*]_{4a}[Y_{\nu R} U_L]_{4a} + [U_R^*]_{4a}[Y_{\nu L} U_R]_{4a}) g_0(x_i, y),
\]

\[
C_{10}^{NP(c)} = -\frac{1}{s_W^2} \frac{1}{4M_L} x_i y_0([U_L^*]_{4a}[Y_{\nu R} U_L]_{4a} - [U_R^*]_{4a}[Y_{\nu L} U_R]_{4a}) g_0(x_i, y). \tag{A.18}
\]

Diagram (d)

\[
\Box^{(d)} = \left(\frac{g}{\sqrt{2}}\right)^4 \sum_{i=u,c,t} V_{ib}V_{is}^* \int \frac{d^4q}{(2\pi)^4} \left(\frac{-i}{q^2 - M_W^2}\right)^2 \left[\frac{M_i}{M_W} P_R \frac{i(\not{q} + M_i) M_i}{q^2 - M_i^2} P_L b\right]
\]

\[\bar{e}_a v([Y_{\nu L}^* U_R^*]_{4a} P_L + [Y_{\nu R}^* U_L^*]_{4a} P_R) i(\not{q} + M_L)\frac{v([Y_{\nu L} U_R]_{4a} P_R + [Y_{\nu R} U_L]_{4a} P_L)}{M_W}\]

\[\frac{q_0 q_\beta}{M_W^2 s_W^2 s_\alpha P_L b [\bar{e}_a \gamma_\beta ([Y_{\nu R} U_L]_{4a} [Y_{\nu L} U_L]_{4a} P_L + [Y_{\nu L} U_R]_{4a} [Y_{\nu R} U_R]_{4a} P_R) e_a].}\]

The last two square brackets can be written as

\[q_0 q_\beta \frac{v^2 M_L^2}{M_W^4} [s_\gamma^\alpha P_L b [\bar{e}_a \gamma_\beta ([Y_{\nu R} U_L]_{4a} [Y_{\nu L} U_L]_{4a} P_L + [Y_{\nu L} U_R]_{4a} [Y_{\nu R} U_R]_{4a} P_R) e_a].\]

where we have dropped terms linear in \(q\). Using Eq. [A.12]

\[
\Box^{(d)} = \frac{g^4}{4} \sum_{i=u,c,t} V_{ib}V_{is}^* A_{\alpha \beta}(M_i, M_L) \frac{v^2 M_L^2}{M_W^4} [s_\gamma^\alpha P_L b [\bar{e}_a \gamma_\beta ([Y_{\nu R} U_L]_{4a} [Y_{\nu L} U_L]_{4a} P_L + [Y_{\nu L} U_R]_{4a} [Y_{\nu R} U_R]_{4a} P_R) e_a].\]
Putting all these together, we have

\[
\square^{(d)} = -i \frac{4G_F}{\sqrt{2}} \sum_{i=u,c,t} V_{ib} V_{ib}^* \frac{e^2}{8\pi^2} \frac{1}{s_W} \frac{1}{8 M_L^2} x_i y g_1(x_i, y)
\]

\[
[\bar{s} \gamma^\mu P_L b][\bar{e}_a \gamma_\mu (|Y^{\nu \tau} U_L|_{4a})^2 P_L + |Y^{\nu \mu} U_R|_{4a}|^2 P_R) e_a] .
\]

Hence, the contributions of this diagram to the Wilson coefficients are

\[
C_{9}^{\text{NP}(d)} = -\frac{1}{s_W^2} \frac{1}{16 M_L^2} x_i \gamma (|Y^{\nu \tau} U_L|_{4a})^2 + |Y^{\nu \mu} U_R|_{4a}|^2) g_1(x_i, y),
\]

\[
C_{10}^{\text{NP}(d)} = \frac{1}{s_W^2} \frac{1}{16 M_L^2} x_i \gamma (|Y^{\nu \tau} U_L|_{4a})^2 - |Y^{\nu \mu} U_R|_{4a}|^2) g_1(x_i, y) .
\]

(A.19)

The sum of the Wilson coefficients from these four diagrams (Eq. A.16 - Eq. A.19) is the total NP Wilson coefficients

\[
C_{9}^{\text{NP}} = -\frac{1}{s_W^2} \frac{1}{4} [U_1^+(x, y) g_1(x, y) + U_0^+(x, y) g_0(x, y)],
\]

\[
C_{10}^{\text{NP}} = \frac{1}{s_W^2} \frac{1}{4} [U_1^-(x, y) g_1(x, y) + U_0^-(x, y) g_0(x, y)],
\]

(A.20)

where \(x = M_t^2 / M_W^2\), \(y = M_L^2 / M_W^2\),

\[
g_1(x, y) = \frac{1}{x - y} \left[ \frac{x^2}{(x - 1)^2} \log x - \frac{y^2}{(y - 1)^2} \log y - \frac{1}{x - 1} + \frac{1}{y - 1} \right],
\]

(A.21)

\[
g_0(x, y) = \frac{1}{x - y} \left[ \frac{x}{(x - 1)^2} \log x - \frac{y}{(y - 1)^2} \log y - \frac{1}{x - 1} + \frac{1}{y - 1} \right],
\]

(A.22)

and

\[
U_1^+(x, y) = |U_L|_{4a}|^2 \pm |U_R|_{4a}|^2 + \frac{1}{4 M_L^2} x y \left( |Y^{\nu \tau} U_L|_{4a}|^2 \pm |Y^{\nu \mu} U_R|_{4a}|^2 \right),
\]

(A.23)

\[
U_0^+(x, y) = -\frac{v}{M_L} x y \left( |U_L|_{4a}[Y^{\nu \tau} U_L^*]_{4a} + |U_L^*|_{4a}[Y^{\nu \mu} U_R]_{4a}
\]

\[
\pm [U_R]_{4a}[Y^{\nu \mu} U_L^*]_{4a} \pm [U_R^*]_{4a}[Y^{\nu \tau} U_L]_{4a} \right). \]
Appendix B

Derivation of the Yukawa Matrices

In this appendix, we derive the Yukawa matrices of the SO(10) and PS SUSY GUTs in Chapter 3 and Chapter 4.

B.1 SO(10) SUSY GUT

In this section, we derive the Yukawa matrices for the SO(10) SUSY GUT in Chapter 3. The quantum numbers of the particles in the Yukawa sector are given in Table B.1 and the superpotential of the Yukawa sector is given in Eq. 3.1.

\[ W_{\text{SO(10)}} = \lambda_{16} 16_3 10 16_3 + 16_a 10 \chi_a + \bar{\chi}_a \left( M_\chi \chi_a + 45 \frac{\phi_a}{M} 16_3 + 45 \frac{\tilde{\phi}_a}{M} 16_a + A_{16_a} \right), \]  

(B.1)

where \( \chi_a \) and \( \bar{\chi}_a \) are Froggatt-Nielsen states, \( M_\chi = M_0 (1 + \alpha X + \beta Y) \), 45 is assumed to obtain a VEV in the B − L direction, A is a non-trivial \( D_3 \) singlet, and \( \phi^a \) and \( \tilde{\phi}^a \) are flavon fields, which are assumed to obtain VEVs

\[ \langle \phi_a \rangle = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \langle \tilde{\phi}_a \rangle = \begin{pmatrix} 0 \\ \tilde{\phi}_2 \end{pmatrix}. \]  

(B.2)
First, we start by integrating out the Froggatt-Nielsen states.

\[
\frac{dW_{SO(10)}}{d\chi_a} = M_\chi \chi_a + \frac{45}{M} \phi_a M_3 + \frac{45}{M} \phi_a M_5 + A_{16} \chi_a = 0
\]

\[
\chi_a = -M_\chi^{-1} \left( \frac{45}{M} \phi_a M_3 + \frac{45}{M} \phi_a M_5 + A_{16} \chi_a \right), \quad (B.3)
\]

and

\[
\frac{dW_{SO(10)}}{d\bar{\chi}_a} = 16_a 10 + \bar{\chi}_a M_\chi = 0
\]

\[
\bar{\chi}_a = -16_a 10 M_\chi^{-1}. \quad (B.4)
\]

The effective superpotential after integrating our the Froggatt-Nielsen states becomes

\[
W_{SO(10)}^{\text{eff}} = \lambda 16_3 10 16_3 - 16_a 10 M_\chi^{-1} \left( \frac{45}{M} \phi_a M_3 + \frac{45}{M} \phi_a M_5 + A_{16} \chi_a \right). \quad (B.5)
\]

Using the definition in Eq. 3.3

\[
G_{x,y} = \frac{M_0}{M_x} = \frac{1}{1 + \alpha x + \beta y}, \quad (B.6)
\]

where \(x\) and \(y\) are the eigenvalues of \(X\) and \(Y\).
Up Quark Sector

First consider the up-quark sector. We can rewrite the Yukawa terms in the superpotential as

$$W_{\text{SO}(10)}^{\text{eff}} \supset \lambda (q_3 H_u \bar{u}_3 + \bar{u}_3 H_u q_3)$$

Expanding the $D_3$ doublet indices, we have

$$W_{\text{SO}(10)}^{\text{eff}} \supset \lambda (q_3 H_u \bar{u}_3 + \bar{u}_3 H_u q_3)$$

Adopting the notation of doublets on the left, we have

$$W_{\text{SO}(10)}^{\text{eff}} \supset [y^u]_{ij} q_i H_u \bar{u}_j ,$$

where the up-quark Yukawa matrix is given by Eq. 3.6

$$y^u = 
\begin{pmatrix}
0 & \epsilon' G_{1, -\frac{4}{3}; -\frac{1}{3}} & -\epsilon \xi G_{1, -\frac{4}{3}} \\
-\epsilon' G_{1, -\frac{4}{3}; -\frac{1}{3}} & \epsilon G_{1, -\frac{4}{3}; -\frac{1}{3}} & -\epsilon G_{1, -\frac{4}{3}} \\
\epsilon \xi G_{1, -\frac{4}{3}} & \epsilon G_{1, -\frac{4}{3}} & \lambda
\end{pmatrix} ,$$

163
\( \epsilon, \tilde{\epsilon}, \epsilon' \) and \( \xi \) are given by Eq. 3.10 - Eq. 3.13 and \( G_{x1, y1; x2, y2}^\pm \) is given by Eq. 3.4

\[
\begin{align*}
\epsilon &= -\frac{1}{6} \frac{M \phi_1}{M_0 M}, \\
\tilde{\epsilon} &= +\frac{1}{6} \frac{M \phi_2}{M_0 M}, \\
\epsilon' &= -\frac{1}{2} \frac{A}{M_0}, \\
\xi &= \frac{\phi_2}{\phi_1}, \\
G_{x1, y1; x2, y2}^\pm &= G_{x1, y1} \pm G_{x2, y2}.
\end{align*}
\] (B.11) (B.12) (B.13) (B.14) (B.15)

Notice that we have rescaled the matrix such that the 33 element is \( \lambda \) instead of \( 2\lambda \). We can rescale the matrix because the parameters are arbitrary.

Taking the limit of \( \beta \ll \alpha \) gives

\[
\begin{align*}
G_{1, 1/3}^1 &\approx \frac{1}{1 + \alpha}, \\
G_{1, -4/3}^1 &\approx \frac{1}{1 + \alpha}, \\
G_{1, -4/3; 1, 1/3}^- &\approx \frac{5}{3(1 + \alpha)^2}.
\end{align*}
\] (B.16)

Define

\[
\begin{align*}
\rho &= \frac{5}{12} \frac{\beta}{\alpha \sigma}, \\
\sigma &= \frac{1 + \alpha}{1 - 3\alpha}, \\
\hat{\epsilon} &= \frac{\epsilon}{1 + \alpha}, \\
\tilde{\hat{\epsilon}} &= \frac{\epsilon \sigma - 1}{1 + \alpha}, \\
\hat{\epsilon}' &= \frac{\epsilon' \sigma - 1}{1 + \alpha},
\end{align*}
\] (B.17) (B.18) (B.19) (B.20) (B.21)

the up-quark Yukawa matrix is given by Eq. 3.14

\[
y^u = \begin{pmatrix}
0 & \epsilon' \rho & -\tilde{\epsilon} \\
-\epsilon' \rho & \tilde{\epsilon} \rho & -\hat{\epsilon} \\
\tilde{\epsilon} \xi & \hat{\epsilon} & \lambda
\end{pmatrix},
\] (B.22)
Down Quark Sector

The Yukawa sector superpotential of the down-quark sector can be obtained from that of the up-quark sector by 1) changing the quantum numbers of \( X \) and \( Y \), and 2) replacing \( H_u \) with \( H_d \). Hence, the down-quark Yukawa matrix can be obtained from the up-quark Yukawa matrix, Eq. \[\text{B.10}\], by replacing \( G_{1,-4/3} \) with \( G_{-3,2/3} \).

\[
y^d = \begin{pmatrix}
0 & \epsilon' G^{-3,2/3,1,1}\frac{2}{3} & -\epsilon G^{-3,2/3,1,1}\frac{2}{3} \\
-\epsilon' G^{-3,2/3,1,1}\frac{4}{3} & \tilde{\epsilon} G^{-3,2/3,1,1}\frac{2}{3} & -\epsilon G^{-3,2/3,1,1}\frac{2}{3} \\
\epsilon G^{-3,2/3,1,1}\frac{1}{3} & \epsilon G^{-3,2/3,1,1}\frac{1}{3} & \lambda
\end{pmatrix}.
\] (B.23)

In the limit of \( \beta \ll \alpha \), we have

\[
G^{-3,2/3} \approx \frac{1}{1 - 3\alpha} = \frac{\sigma}{1 + \alpha},
\] (B.24)

\[
G^{-3,2/3,1,1}\frac{4}{3} \approx \frac{\sigma - 1}{1 + \alpha},
\] (B.25)

and the down-quark Yukawa matrix is given by Eq. \[3.15\]

\[
y^d = \begin{pmatrix}
0 & \epsilon' & -\tilde{\epsilon} \sigma \\
-\epsilon' & \tilde{\epsilon} & -\epsilon \sigma \\
\tilde{\epsilon} \sigma & \tilde{\epsilon} & \lambda
\end{pmatrix}.
\] (B.26)

Charged-Lepton Sector

The Yukawa sector superpotential of the charged-lepton sector can be obtained from that of the up-quark sector by 1) changing the quantum numbers of \( X, Y, \) and \( B - L \), and 2) replacing \( H_u \) with \( H_d \). Hence, the charged-lepton Yukawa matrix can be obtained from the up-quark Yukawa matrix, Eq. \[\text{B.10}\], by 1) replacing \( G_{1,-4/3} \) with \( G_{1,2} \), 2) replacing \( G_{1,1/3} \) with \( G_{-3,-1} \), and 3) multiplying the \( B - L \) terms with \(-3\).

\[
y^e = \begin{pmatrix}
0 & -\epsilon' G^{-3,-1,1,2} & 3\epsilon \xi G_{1,2} \\
\epsilon' G^{-3,-1,1,2} & 3\epsilon \xi G^{-3,-1,1,2} & 3\epsilon G_{1,2} \\
-3\epsilon \xi G_{-3,-1} & -3\epsilon G_{-3,-1} & \lambda
\end{pmatrix},
\] (B.27)
In the limit of $\beta \ll \alpha$, we have

$$G_{1,2} \approx \frac{1}{1 + \alpha}, \quad \text{(B.28)}$$

$$G_{-3,-1} \approx \frac{1}{1 - 3\alpha} = \frac{\sigma}{1 + \alpha}, \quad \text{(B.29)}$$

$$G_{-3,-1;1,2} \approx \frac{\sigma - 1}{1 + \alpha}, \quad \text{(B.30)}$$

and the charged-lepton Yukawa matrix is given by Eq. 3.16:

$${y^e} = \begin{pmatrix} 0 & -\tilde{\epsilon} & 3\tilde{\epsilon}\tilde{\xi} \\ \epsilon' & 3\tilde{\epsilon} & 3\tilde{\epsilon} \\ -3\tilde{\epsilon}\xi\sigma & -3\tilde{\epsilon}\sigma & \lambda \end{pmatrix}. \quad \text{(B.31)}$$

**Neutrino Sector**

The Yukawa sector superpotential of the neutrino sector can be obtained from that of the up-quark sector by changing the quantum numbers of $X, Y,$ and $B - L$. Hence, the neutrino Yukawa matrix can be obtained from the up-quark Yukawa matrix, Eq. B.10 by 1) replacing $G_{1,2}$ with $G_{5,0}$, 2) replacing $G_{1,\frac{1}{3}}$ with $G_{-3,-1}$, and 3) multiplying the $B - L$ terms with $-3$.  

$${y^\nu} = \begin{pmatrix} 0 & -\epsilon'G_{-3,-1;5,0} & 3\epsilon\xi G_{5,0} \\ \epsilon'G_{-3,-1;5,0} & 3\epsilon\xi G_{-3,-1;5,0} & 3\epsilon G_{5,0} \\ -3\epsilon\xi G_{-3,-1} & -3\epsilon G_{-3,-1} & \lambda \end{pmatrix}, \quad \text{(B.32)}$$

In the limit of $\beta \ll \alpha$, we have

$$G_{5,0} \approx \frac{1}{1 + 5\alpha} = \omega \frac{1}{2(1 + \alpha)}, \quad \text{(B.33)}$$

$$G_{-3,-1;5,0} \approx \omega\frac{\sigma - 1}{1 + \alpha}, \quad \text{(B.34)}$$

where

$$\omega = \frac{2\sigma}{2\sigma - 1}. \quad \text{(B.35)}$$
Hence, the neutrino Yukawa matrix is given by Eq. 3.17:

\[
y'' = \begin{pmatrix}
0 & -\tilde{\epsilon}'\omega & 3\tilde{\epsilon}\xi\omega/2 \\
\tilde{\epsilon}'\omega & 3\tilde{\epsilon}\omega & 3\tilde{\epsilon}\omega/2 \\
-3\tilde{\epsilon}\xi\sigma & -3\tilde{\epsilon}\sigma & \lambda 
\end{pmatrix}.
\]

(B.36)

B.2 Pati-Salam SUSY GUT

In this section, we derive the Yukawa matrices for the PS Model in Chapter 4. The super-potential of the Yukawa sector is given in Eq. 4.4

\[
W_{\text{PS}} = \lambda Q_3 H Q_3 + Q_a H \bar{F}_a + F_a H \bar{Q}_a \\
+ \bar{F}_a^c \left( M_F \bar{F}_a + 15 \frac{\phi_a}{M} \bar{Q}_3 + 15 \frac{\tilde{\delta}_a}{M} \bar{Q}_a + A \bar{Q}_a + \Theta' \bar{Q}_a + \bar{\Theta} a \bar{Q}_a \right) \\
+ F_a^c \left( M_F F_a + 15 \frac{\phi_a}{M} Q_3 + 15 \frac{\tilde{\delta}_a}{M} Q_a + A Q_a + \Theta' Q_a - \bar{\Theta}_a Q_a \right),
\]

(B.37)

where \{Q_i, F_a\} = (4, 2, 1), \{\bar{Q}_i, \bar{F}_a\} = (\bar{4}, 1, 2) and \(H = (1, 2, 2)\) under PS symmetry. \(15\) is the adjoint representation of SU(4)_C that is assumed to obtain a VEV in the \(B-L\) direction. In addition, \(F_a^c\) and \(\bar{F}_a^c\) are the conjugate of \(F_a\) and \(\bar{F}_a\), respectively, and \(M_F = M_\chi\).

The differences between this superpotential and that of the SO(10) model, Eq. [B.1] are 1) instead of SO(10), the superfields are written using PS fields, and 2) the addition of the following terms, which are absent from the SO(10) model.

\[
\bar{F}_a^c \Theta' \bar{Q}_a + F_a^c \Theta' Q_a + \bar{F}_a^c \bar{\Theta} a \bar{Q}_a - F_a^c \bar{\Theta}_a Q_a.
\]

(B.38)

Hence, in this section, we will only derive the Yukawa matrix contributions from these additional terms.

After integrating out the Froggatt-Nielsen states, the effective superpotential is given by

\[
W_{\text{PS}}^{\text{eff}} \supset -Q_a H M_F^{-1} (\Theta' \bar{Q}_a + \bar{\Theta} a \bar{Q}_a) - Q_a H M_F^{-1} (\Theta' Q_a - \bar{\Theta}_a Q_a).
\]

(B.39)
Up Quark Sector

Again, we start by considering the up-quark sector. The Yukawa terms in the superpotential can be rewritten as

\[
W_{\text{eff}}^{\text{PS}} \supset -q_a H_u \frac{\Theta'}{M_0} G_{1,-\frac{1}{3}} \bar{u}_a - \bar{u}_a H_u \frac{\Theta'}{M_0} G_{1,\frac{1}{3}} q_a \\
- q_a H_u \frac{\tilde{\Theta}_a}{M_0} G_{1,-\frac{1}{3}} \bar{u}_a + \bar{u}_a H_u \frac{\tilde{\Theta}_a}{M_0} G_{1,\frac{1}{3}} q_a.
\]

(B.40)

Expanding the $D_3$ doublet indices, we have

\[
W_{\text{eff}}^{\text{PS}} \supset -q_1 H_u \frac{\Theta'}{M_0} G_{1,-\frac{1}{3}} \bar{u}_2 - \bar{u}_1 H_u \frac{\Theta'}{M_0} G_{1,\frac{1}{3}} q_2 \\
- q_2 H_u \frac{\Theta'}{M_0} G_{1,-\frac{1}{3}} \bar{u}_1 - \bar{u}_2 H_u \frac{\Theta'}{M_0} G_{1,\frac{1}{3}} q_1 \\
- q_1 H_u \frac{\tilde{\Theta}_1}{M_0} G_{1,-\frac{1}{3}} \bar{u}_1 + \bar{u}_1 H_u \frac{\tilde{\Theta}_1}{M_0} G_{1,\frac{1}{3}} q_1,
\]

(B.41)

where we have assumed that $\tilde{\Theta}_a$ has a VEV of

\[
\langle \tilde{\Theta}_a \rangle = \begin{pmatrix} \tilde{\Theta}_1 \\ 0 \end{pmatrix}.
\]

(B.42)

Adopting the doublets on the left notation, the addition contributions to the up-quark Yukawa matrix is

\[
\tilde{y}^u = \begin{pmatrix} -\tilde{\theta} G^+_{1,-\frac{1}{3}; \frac{1}{3}} & \theta' G^+_{1,-\frac{1}{3}; \frac{1}{3}} & 0 \\ \theta' G^+_{1,-\frac{1}{3}; \frac{1}{3}} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},
\]

(B.43)

where

\[
\theta' = -\frac{1}{2} \frac{\Theta'}{M_0},
\]

(B.44)

\[
\tilde{\theta} = +\frac{1}{2} \frac{\tilde{\Theta}_1}{M_0}.
\]

(B.45)

Similar to previous section, we have rescaled the matrix by multiplying the parameters by $1/2$.

Adding the new contribution (Eq. B.43) to the up-quark Yukawa matrix in the previous
section (Eq. B.10) gives the up-quark Yukawa matrix in Eq. 4.6.

\[
y_u = \begin{pmatrix}
-\tilde{\theta} G^{-}_{1,-\frac{2}{3};1,\frac{1}{3}} & \epsilon' G^{-}_{1,-\frac{4}{3};1,\frac{1}{3}} + \theta' G^{+}_{1,-\frac{4}{3};1,\frac{1}{3}} & -\epsilon G_{1,-\frac{4}{3}} \\
-\epsilon' G^{-}_{1,-\frac{2}{3};1,\frac{1}{3}} + \theta' G^{+}_{1,-\frac{2}{3};1,\frac{1}{3}} & \tilde{\epsilon} G^{-}_{1,-\frac{4}{3};1,\frac{1}{3}} & -\epsilon G_{1,-\frac{4}{3}} \\
\epsilon' G_{1,\frac{1}{3}} & \epsilon G_{1,\frac{1}{3}} & \lambda
\end{pmatrix}
\] (B.46)

Down Quark Sector

Similar to the previous section, the down-quark Yukawa matrix can be obtained from the up-quark Yukawa matrix, Eq. B.10, by replacing \( G_{1,-\frac{4}{3}} \) with \( G_{-3,\frac{2}{3}} \). Hence, the down-quark Yukawa matrix is given by Eq. 4.7.

\[
y_d = \begin{pmatrix}
-\tilde{\theta} G^{-}_{-3,\frac{2}{3};1,\frac{1}{3}} & \epsilon' G^{-}_{-3,\frac{4}{3};1,\frac{1}{3}} + \theta' G^{+}_{-3,\frac{4}{3};1,\frac{1}{3}} & -\epsilon G_{-3,\frac{2}{3}} \\
-\epsilon' G^{-}_{-3,\frac{2}{3};1,\frac{1}{3}} + \theta' G^{+}_{-3,\frac{2}{3};1,\frac{1}{3}} & \tilde{\epsilon} G^{-}_{-3,\frac{4}{3};1,\frac{1}{3}} & -\epsilon G_{-3,\frac{2}{3}} \\
\epsilon' G_{1,\frac{1}{3}} & \epsilon G_{1,\frac{1}{3}} & \lambda
\end{pmatrix}
\] (B.47)

Charged-Lepton Sector

Similar to the previous section, the charged-lepton Yukawa matrix can be obtained from the up-quark Yukawa matrix, Eq. B.10, by 1) replacing \( G_{1,-\frac{4}{3}} \) with \( G_{1,\frac{2}{3}} \), 2) replacing \( G_{1,\frac{1}{3}} \) with \( G_{-3,-\frac{1}{3}} \), and 3) multiplying the \( B - L \) terms with \(-3\). Hence, the charged-lepton Yukawa matrix is given by Eq. 4.8.

\[
y_e = \begin{pmatrix}
\tilde{\theta} G^{-}_{-3,-\frac{1}{3};1,2} & -\epsilon' G^{-}_{-3,-1;1,2} + \theta' G^{+}_{-3,-1;1,2} & 3\epsilon G_{1,2} \\
\epsilon' G^{-}_{-3,-1;1,2} + \theta' G^{+}_{-3,-1;1,2} & 3\tilde{\epsilon} G^{-}_{-3,-1;1,2} & 3\epsilon G_{1,2} \\
-3\epsilon G_{-3,-1} & -3\epsilon G_{-3,-1} & \lambda
\end{pmatrix}
\] (B.48)

Neutrino Sector

Similar to the previous section, the neutrino Yukawa matrix can be obtained from the up-quark Yukawa matrix, Eq. B.10, by 1) replacing \( G_{1,\frac{2}{3}} \) with \( G_{5,0} \), 2) replacing \( G_{1,\frac{4}{3}} \) with \( G_{-3,-\frac{1}{3}} \), and 3) multiplying the \( B - L \) terms with \(-3\). Hence, the neutrino Yukawa matrix
is given by Eq. 4.9

\[
y'' = \begin{pmatrix}
\tilde{\theta}G_{-3,-1;5,0} & -e'G_{-3,-1;5,0} + \theta'G_{-3,-1;5,0} & 3\xi G_{5,0} \\
e'G_{-3,-1;5,0} + \theta'G_{-3,-1;5,0} & 3\bar{\epsilon}G_{-3,-1;5,0} & 3\epsilon G_{5,0} \\
-3\epsilon\xi G_{-3,-1} & -3\epsilon G_{-3,-1} & \lambda
\end{pmatrix}.
\]  (B.49)
Appendix C

\( B^0 \rightarrow K^{*0} \mu^+ \mu^- \) OBSERVABLES

In this appendix, we define \( B^0 \rightarrow K^{*0} \mu^+ \mu^- \) observables used in the global \( \chi^2 \) analyses of the SUSY GUTs in Chapter 3 and Chapter 4. This appendix mainly follows Ref. [163, 164].

C.1 Effective Hamiltonian

The effective Hamiltonian for \( b \rightarrow s\mu^+ \mu^- \) is given by [85, 165]

\[
H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \mathcal{H}^{(t)}_{\text{eff}}
\]  

where

\[
\mathcal{H}^{(t)}_{\text{eff}} = \sum_{i=1}^{2} C_i \mathcal{O}_i + \sum_{i=3}^{6} C_i \mathcal{O}_i + \sum_{i=7,8,9,10,P,S} \left( C_i \mathcal{O}_i + C'_i \mathcal{O}'_i \right).
\]

We have dropped Cabibbo-suppressed contributions from \( u \) and \( c \) quarks, which are only relevant for observables sensitive to complex phases. The operators are defined as [85, 163]

\[
\mathcal{O}_1 = (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L),
\]

\[
\mathcal{O}_2 = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L),
\]

\[
\mathcal{O}_3 = (\bar{s}_L \gamma_\mu b_L) \sum_q (\bar{q} \gamma^\mu q),
\]

\[
\mathcal{O}_4 = (\bar{s}_L \gamma_\mu T^a b_L) \sum_q (\bar{q} \gamma^\mu T^a q),
\]
The primed operators, \( O'_7, 8, 9, 10, P, S \), have opposite chirality to the unprimed operators. The primed operators and the scalar and pseudoscalar operators, \( O_S, P \), are either highly suppressed or absent in the SM.

The Wilson coefficients are calculated at \( \mu = M_W \) and in a perturbative expansion of \( \alpha_s(M_W) \).

\[
C_i = C_i^{(0)} + \frac{\alpha_s}{4\pi} C_i^{(1)} + \left( \frac{\alpha_s}{4\pi} \right)^2 C_i^{(2)} + O(\alpha_s^3),
\]

where \( C_i^{(0,1,2)} \) are tree-level, one-loop, and two-loop contributions. In the SM, all tree-level contributions vanish but \( C_2^{(0)} = -1 \). The non-vanishing SM Wilson coefficients at one loop are \[85]\]

\[
\begin{align*}
C_4^{(1)} &= E_0(x), \\
C_7^{(1)} &= -\frac{1}{2} A_0(x), \\
C_8^{(1)} &= -\frac{1}{2} F_0(x), \\
C_9^{(1)} &= \frac{1 - 4s_W^2}{s_W^2} C_0(x) - \frac{1}{s_W^2} B_0(x) - D_0(x), \\
C_{10}^{(1)} &= \frac{1}{s_W^2} B_0(x) - \frac{1}{s_W^2} C_0(x),
\end{align*}
\]
where \( x = (m_t^{\overline{\text{MS}}} / M_W)^2 \) and

\[
A_0(x) = \frac{-3x^2 + 2x^2}{2(1 - x)^4} \ln x + \frac{22x^3 - 153x^2 + 159x - 46}{36(1 - x)^3},
\]
\[
B_0(x) = \frac{x}{4(1 - x)^2} \ln x + \frac{1}{4(1 - x)},
\]
\[
C_0(x) = \frac{3x^2 + 2x}{8(1 - x)^2} \ln x + \frac{-x^2 + 6x}{8(1 - x)},
\]
\[
D_0(x) = \frac{-3x^4 + 30x^3 - 54x^2 + 32x - 8}{18(1 - x)^4} \ln x + \frac{-47x^3 + 237x^2 - 312x + 104}{108(1 - x)^3},
\]
\[
E_0(x) = \frac{-9x^2 + 16x - 4}{6(1 - x)^4} \ln x + \frac{-7x^3 - 21x^2 + 42x + 4}{36(1 - x)^3},
\]
\[
F_0(x) = \frac{3x^2 + 2x}{2(1 - x)^4} \ln x + \frac{5x^3 - 9x^2 + 30x - 8}{12(1 - x)^3}.
\]

\( A_0 \) is obtained from the magnetic dipole moment term of \( b \to s \gamma \) as shown in App. D. \( B_0 \) is obtained from box diagrams with two \( W^\pm \) bosons, \( C_0 \) is obtained from penguin diagrams where a \( Z \) boson couples to a dimuon pair, and \( D_0 \) is obtained from penguin diagrams where a photon couples to a dimuon pair.

After obtaining these coefficients at \( \mu = M_W \), they are run down to \( m_b \) with SM RGEs, which mix these coefficients. The mixing matrix of these Wilson coefficients is given in Ref. \[85, 166-168\].

As we will see in Sec. C.2, the Wilson coefficients in the matrix element of \( B^0 \to K^{*0} \mu \mu \) always appears in the following combinations.

\[
C_7^{\text{eff}} = \frac{4\pi}{\alpha_s} C_7 - \frac{1}{3} C_3 - \frac{4}{9} C_4 - \frac{20}{3} C_5 - \frac{80}{9} C_6,
\]
\[
C_8^{\text{eff}} = \frac{4\pi}{\alpha_s} C_8 + C_3 - \frac{1}{6} C_4 + 20C_5 - \frac{10}{3} C_6,
\]
\[
C_9^{\text{eff}} = \frac{4\pi}{\alpha_s} C_9 + Y(q^2),
\]
\[
C_{10}^{\text{eff}} = \frac{4\pi}{\alpha_s} C_{10},
\]
\[
C_{7,8,9,10}^{\text{eff}} = \frac{4\pi}{\alpha_s} C_{7,8,9,10}^{\prime},
\]

where \( Y(q^2) \) is a linear combination of \( C_{1,2,3,4,5,6} \) defined in Eq. 2.10 of Ref. [163].

173
C.2 Differential Decay Distributions

The decay observed in experiment is \( B^0 \to K^{*0}(\to K^{-}\pi^+)\mu^+\mu^- \) instead of \( B^0 \to K^{*0}\mu^+\mu^- \). Hence, the observed decay is a four-body decay. See App. B of Ref. [169] for a detail discussion of a four-body decay. Similarly, the \( CP \)-conjugated mode is \( B^0 \to K^{*0}(\to K^+\pi^-)\mu^+\mu^- \).

Assuming that \( K^{*0} \) is on-shell, the differential decay distribution can be defined in terms of four parameters [3]. Three of the four parameters are angles, which uniquely describe the decay (see Fig. C.1).

1. \( q^2 \): The dimuon mass square defined as

\[
q^2 = (p_{\mu^+} + p_{\mu^-})^2,
\]

where \( p_{\mu^+} \) and \( p_{\mu^-} \) are the four-momenta of the final states \( \mu^+ \) and \( \mu^- \), respectively. The range of \( q^2 \) is

\[
4m^2_{\ell} \leq s \leq (m_B - m_{K^{*}})^2.
\]

2. \( \theta_{K^*} \): For the decay of \( B^0 \), \( \theta_{K^*} \) is defined as the angle between the direction of \( K^- \) in

---

Figure C.1: The definition of \( \theta_{K^*}, \theta_\ell, \) and \( \phi \) for \( B^0 \to K^{*0}(\to K^+\pi^-)\mu^+\mu^- \) [3].
the $K^{*0}$ rest frame and the direction of $\bar{K}^{*0}$ in the $\bar{B}^0$ rest frame.

$$\cos \theta_{K^*} = \hat{p}_{K^-} \cdot \hat{p}_{K^{*0}},$$  \hspace{1cm} (C.34)

where $\hat{p}_X^{(Y)}$ is the unit vector in the direction of the four-momentum of the final state $X$ in the $Y$ rest frame.

For the decay of $B^0$, $\theta_{K^*}$ is defined as the angle between the direction of $K^+$ in the $K^{*0}$ rest frame and the direction of $K^{*0}$ in the $B^0$ rest frame.

$$\cos \theta_{K^*} = \hat{p}_{K^+} \cdot \hat{p}_{K^{*0}}.$$  \hspace{1cm} (C.35)

The range of $\theta_{K^*}$ is

$$0 \leq \theta_{K^*} \leq \pi.$$  \hspace{1cm} (C.36)

3. $\theta_{\ell}$: For the decay of $\bar{B}^0$ and $B^0$, $\theta_{\ell}$ is defined as the angle between the direction of the $\mu^-$ in the dimuon rest frame and the direction between the dimuon in the $\bar{B}^0$ or $B^0$ rest frame, respectively.

$$\cos \theta_{\ell} = \hat{p}_{\mu^-} (\mu^+) \cdot \hat{p}_{\mu^-} (\mu^-).$$  \hspace{1cm} (C.37)

The range of $\theta_{\ell}$ is

$$0 \leq \theta_{\ell} \leq \pi.$$  \hspace{1cm} (C.38)

This definition is different from the definition used by the LHCb collaboration [109, 110].

4. $\phi$: For the decay of $\bar{B}^0$ and $B^0$, $\phi$ is defined as the angle between the plane formed by $K^+\pi$ in the $B$ rest frame and the plane formed by the dimuon in the $B$ rest frame.

$$\cos \phi = (\hat{p}_{\mu^-} (\mu^+) \times \hat{p}_{K^+}) \cdot (\hat{p}_{\mu^-} (\mu^-) \times \hat{p}_{\pi^+}),$$  \hspace{1cm} (C.39)
for the decay of $\bar{B}^0$ and

$$\cos \phi = (\hat{p}_{\mu^-}^{(B^0)} \times \hat{p}_{\mu^+}^{(B^0)}) \cdot (\hat{p}_{K^+}^{(\bar{B}^0)} \times \hat{p}_{\pi^-}^{(B^0)})$$, \hspace{1cm} (C.40)

for the decay of $B^0$. The range of $\phi$ is

$$0 \leq \phi \leq 2\pi.$$ \hspace{1cm} (C.41)

The full angular distribution of the decay, $\bar{B}^0 \rightarrow K^{*0}(\rightarrow K^-\pi^+)\mu^+\mu^-$, is given by

$$\frac{d^4\Gamma}{dq^2 \, d\cos \theta \, d\cos \theta_{K^*} \, d\phi} = \frac{9}{32\pi} \bar{I}(q^2, \theta_\ell, \theta_{K^*}, \phi),$$ \hspace{1cm} (C.42)

where

$$I(q^2, \theta_\ell, \theta_{K^*}, \phi) = I_1^s \sin^2 \theta_{K^*} + I_1^c \cos^2 \theta_{K^*} + (I_2^s \sin^2 \theta_{K^*} + I_2^c \cos^2 \theta_{K^*}) \cos 2\theta_\ell + I_3 \sin^2 \theta_{K^*} \sin^2 \theta_\ell \cos 2\phi + I_4 \sin 2\theta_{K^*} \sin 2\theta_\ell \cos \phi + I_5 \sin 2\theta_{K^*} \sin \theta_\ell \cos \phi + (I_6^s \sin^2 \theta_{K^*} + I_6^c \cos^2 \theta_{K^*}) \cos \theta_\ell + I_7 \sin 2\theta_{K^*} \sin \theta_\ell \sin \phi + I_8 \sin 2\theta_{K^*} \sin \theta_\ell \sin \phi + I_9 \sin^2 \theta_{K^*} \sin^2 \theta_\ell \sin 2\phi.$$ \hspace{1cm} (C.43)

The full angular distribution for the $CP$-conjugated decay, $B^0 \rightarrow K^{*0}(\rightarrow K^+\pi^-)\mu^+\mu^-$, is

$$\frac{d^4\Gamma}{dq^2 \, d\cos \theta \, d\cos \theta_{K^*} \, d\phi} = \frac{9}{32\pi} \bar{I}(q^2, \theta_\ell, \theta_{K^*}, \phi),$$ \hspace{1cm} (C.44)

where $\bar{I}(q^2, \theta_\ell, \theta_{K^*}, \phi)$ can be obtained from $I(q^2, \theta_\ell, \theta_{K^*}, \phi)$ by replacing $I_{1,2,3,4,7}$ with $\bar{I}_{1,2,3,4,7}$ and $I_{5,6,8,9}$ with $-\bar{I}_{5,6,8,9}$. $\bar{I}_i$ equals $I_i$ but with all weak phases conjugated. The sign for some of the angular coefficients changes because under $CP$ transformation, $\theta_\ell \rightarrow \theta_\ell - \pi$ and $\phi \rightarrow -\phi$.

From an experimental perspective, these angular coefficients can be obtained directly by performing a full angular fit [170].
C.3 Transversity Amplitudes

In naive factorization, the matrix element of \( B \to K^*(\to K\pi)\mu^+\mu^- \) is given by \cite{163}

\[
\mathcal{M} = G_F \alpha \sqrt{2\pi} V_{tb} V_{ts}^* \left\{ \langle K\pi|\bar{s}\gamma^\mu(C_{9}^{\text{eff}} P_L + C_{9}^{\prime\text{eff}} P_R)b|\bar{B}\rangle \\
- \frac{2m_b}{q^2} \langle K\pi|\bar{s}\sigma^{\mu\nu}q_\nu(C_{7}^{\text{eff}} P_R + C_{7}^{\prime\text{eff}} b)|\bar{B}\rangle (\mu\gamma_\mu) \\
+ \langle K\pi|\bar{s}\gamma^\mu(C_{10}^{\text{eff}} P_L + C_{10}^{\prime\text{eff}} P_R)b|\bar{B}\rangle (\mu\gamma_\mu) \\
+ \langle K\pi|\bar{s}(C_S P_R + C_{10}^{\prime}\text{eff} P_L)b|\bar{B}\rangle (\mu\gamma_\mu) \right\},
\]  
(C.45)

where \( P_{L,R} \) are projector operators.

To discuss the transversity amplitudes, notice that we can consider \( B \to K^*(\to K\pi)\mu^+\mu^- \) as the decay of a B meson to an on-shell \( K^* \) and a virtual vector boson, \( V^* \), which subsequently decays to a pair of muons \cite{163}

\[
\mathcal{M}_{(m,n)}(B \to K^*V^*(\to \mu^+\mu^-)) \propto \epsilon_{K^*}^{\mu}(m)M_{\mu\nu} \sum_{n,n'} \epsilon_{V^*}^{\nu}(n)\epsilon_{V^*}^{\rho}(n')g_{nn'}(\mu\gamma_{\rho}P_{L,R}\mu),
\]  
(C.46)

where \( g_{nn'} = \text{diag}(+, -, -, -) \), and \( \epsilon_{K^*}^{\mu} \) and \( \epsilon_{V^*}^{\nu} \) are the polarization vectors of \( K^* \) and \( V^* \), respectively. \( m, n, n' \) are the polarization of the vectors. \( K^* \) can be polarized transversely, \( m = \pm \), and longitudinally, \( m = 0 \). In addition to transverse and longitudinal polarization, \( V^* \) can also be timelike, \( n = 0 \). The transversity amplitudes can be defined by contracting the indices of the polarization vectors

\[
A_{\perp,\parallel} = \frac{1}{\sqrt{2}}(H_+ \mp H_-) \\
A_0 = H_0 \\
A_t = \mathcal{M}_{(0,0)}(B \to K^*V^*(\to \mu^+\mu^-)),
\]  
(C.47)

(C.48)

(C.49)

where the helicity amplitudes are

\[
H_m = \mathcal{M}_{(m,m)}(B \to K^*V^*(\to \mu^+\mu^-)) \quad \text{for} \quad m = \pm, 0.
\]  
(C.50)

In addition to the polarization of the vectors, the muon pair can have different chirality also. Hence, \( A_{\perp,\parallel,0} \) can be separated into six terms, \( A_{\perp,\parallel,0}^{L,R} \). The seven transversity amplitudes
above cover all the effective Wilson coefficients in the matrix elements other than those for
the scalar operators, $O_S$ and $O'_S$. The pseudoscalar operator, $O_P - O'_P$, is included because
it couples to axial-vector currents. Hence, it can be absorbed into the $A_t$. To include the
scalar operator, the scalar transversity amplitude, $A_S$, is added as an additional transversity
amplitude.

At leading order, the transversity amplitudes are given by [3] 163

\[
A_{L,R}^L = N_\perp [C^9_{+10} V(q^2) + C^7_{+1} T_1(q^2)] , \tag{C.51}
\]

\[
A_{L,R}^R = N_\parallel [C^9_{-10} A_1(q^2) + C^7_{-2} T_2(q^2)] , \tag{C.52}
\]

\[
A_{0,L,R}^L = N_0 [C^9_{-10} A_{12}(q^2) + C^7_{-2} T_{12}(q^2)] , \tag{C.53}
\]

\[
A_t = N_t C^7_{10+P} A_0(q^2) , \tag{C.54}
\]

\[
A_S = N_S C^7_S A_0(q^2) , \tag{C.55}
\]

where

\[
C^\pm_{9+10} = \frac{1}{m_B \pm m_{K^*}} [(C^\text{eff} \pm C'^\text{eff}) \mp (C^\text{eff} \pm C'^\text{eff})] , \tag{C.56}
\]

\[
C^\pm_7 = \frac{2m_b}{q^2} (C^\text{eff} \pm C'^\text{eff}) , \tag{C.57}
\]

\[
C^-_{10+P} = 2(C^\text{eff} - C'^\text{eff}) + \frac{q^2}{m_\mu} (C_P - C'_P) , \tag{C.58}
\]

\[
C^-_S = C_S - C'_S . \tag{C.59}
\]

The normalization factors are

\[
N_\perp = N \sqrt{2} \lambda^2 , \tag{C.60}
\]

\[
N_\parallel = -N \sqrt{2}(m_B^2 - m_{K^*}^2) , \tag{C.61}
\]

\[
N_0 = -\frac{N}{2m_{K^*} \sqrt{q^2}} , \tag{C.62}
\]

\[
N_t = \frac{N}{\sqrt{q^2}} \lambda^{1/2} , \tag{C.63}
\]

\[
N_S = -2N \lambda^{1/2} , \tag{C.64}
\]
where

\[ N = V_{tb}V_{ts}^* \left( \frac{G_F^2 \alpha^2}{3 \times 2^{10} \pi^5 m_B^3} q^2 \lambda^{1/2} \beta_\mu \right)^{1/2}, \quad (C.65) \]

\[ \lambda = m_B^4 + m_{K^*}^4 + q^4 - 2(m_B^2 m_{K^*}^2 + m_{K^*}^2 q^2 + m_B^2 q^2), \quad (C.66) \]

\[ \beta_\mu = \sqrt{1 - 4m_\mu^2/q^2}. \quad (C.67) \]

\[ V(q^2), A_{0,1,2}(q^2), \text{ and } T_{1,2,3}(q^2) \] are seven independent form factors defined in Eq. 2.12 and Eq. 2.14 of Ref. [163], and

\[ A_{12}(q^2) = (m_B^2 - m_{K^*}^2 - q^2)(m_B + m_{K^*})A_1(q^2) - \frac{\lambda}{m_B + m_{K^*}} A_2(q^2), \quad (C.68) \]

\[ T_{23}(q^2) = 2m_b(m_B^2 + 3m_{K^*}^2 - q^2)T_2(q^2) - \frac{2\lambda m_b}{m_B - m_{K^*}} T_3(q^2). \quad (C.69) \]

At leading order, \[ [171, 172] \]

\[ \frac{T_1}{V} \approx 1, \quad (C.70) \]

\[ \frac{T_2}{A_1} \approx 1, \quad (C.71) \]

\[ \frac{T_{23}}{A_{12}} \approx \frac{q^2}{m_B^2}. \quad (C.72) \]

Hence, the transversity amplitudes, at leading order, can be written as

\[ A_{\perp}^{L,R} = N_{\perp}(C_{9,10}^- + C_7^+)V(q^2), \quad (C.73) \]

\[ A_{\parallel}^{L,R} = N_{\parallel}(C_{9,10}^- + C_7^+)A_1(q^2), \quad (C.74) \]

\[ A_0^{L,R} = N_0 \left( C_{9,10}^- + \frac{q^2}{m_B^2} C_7^- \right) A_{12}(q^2). \quad (C.75) \]

Detailed discussions of form factors are beyond the scope of this thesis. For more extensive discussions, see Ref. [163, 164].

In the next section, we will define observables in terms of angular coefficients of the differential decay distribution. Here we provide the relationship between the angular coef-
ficients and the transversity amplitudes:

\[ I_1^s = \frac{2 + \beta^2}{4} \mu \left[ |A_L^L|^2 + |A_L^R|^2 + (L \rightarrow R) \right] + \frac{4m^2}{q^2} \text{Re}(A_L^L A_R^R + A_L^R A_R^L), \]  
(\text{C.76})

\[ I_1^c = |A_L^L|^2 + |A_R^R|^2 + \frac{4m^2}{q^2} \left[ |A_L|^2 + 2\text{Re}(A_L^L A_R^R) \right] + \beta_\mu^2 |A_S|^2, \]  
(\text{C.77})

\[ I_2^s = \frac{\beta^2}{4} \mu \left[ |A_L^L|^2 + |A_L^R|^2 + (L \rightarrow R) \right], \]  
(\text{C.78})

\[ I_2^c = -\beta_\mu^2 |A_L^L|^2 + (L \rightarrow R), \]  
(\text{C.79})

\[ I_3^s = \frac{1}{2} \beta_\mu^2 |A_L^L|^2 - |A_L^L|^2 + (L \rightarrow R), \]  
(\text{C.80})

\[ I_3^c = \frac{1}{\sqrt{2}} \beta_\mu^2 [\text{Re}(A_L^L A_L^R) + (L \rightarrow R)], \]  
(\text{C.81})

\[ I_5^s = \sqrt{2} \beta_\mu \left[ \text{Re}(A_L^L A_L^R) - (L \rightarrow R) - \frac{m_\mu}{\sqrt{q^2}} \text{Re}(A_L^L A_S^* + A_L^R A_S^*) \right], \]  
(\text{C.82})

\[ I_5^c = 2 \beta_\mu [\text{Re}(A_L^L A_L^R) - (L \rightarrow R)], \]  
(\text{C.83})

\[ I_6^c = 4 \beta_\mu \frac{m_\mu}{\sqrt{q^2}} \text{Re}[A_L^L A_S^* + (L \rightarrow R)], \]  
(\text{C.84})

\[ I_7^s = \sqrt{2} \beta_\mu \left[ \text{Im}(A_L^L A_L^R) - (L \rightarrow R) + \frac{m_\mu}{\sqrt{q^2}} \text{Im}(A_L^L A_S^* + A_L^R A_S^*) \right], \]  
(\text{C.85})

\[ I_7^c = \frac{1}{\sqrt{2}} \beta_\mu [\text{Im}(A_L^L A_L^R) + (L \rightarrow R)], \]  
(\text{C.86})

\[ I_8^c = \beta_\mu^2 [\text{Im}(A_L^L A_L^R) + (L \rightarrow R)]. \]  
(\text{C.87})
C.4 Observables

The CP-averaged angular observables are defined as

\[ \langle P_1 \rangle_{\text{bin}} = \frac{1}{2} \int_{\text{bin}} dq^2 [I_3 + \bar{I}_3], \quad (C.88) \]

\[ \langle P_2 \rangle_{\text{bin}} = \frac{1}{8} \int_{\text{bin}} dq^2 [I_6^s + I_6^\bar{s}], \quad (C.89) \]

\[ \langle P_3 \rangle_{\text{bin}} = -\frac{1}{4} \int_{\text{bin}} dq^2 [I_9 + \bar{I}_9], \quad (C.90) \]

\[ \langle P_4' \rangle_{\text{bin}} = \frac{1}{N_{\text{bin}}} \int_{\text{bin}} dq^2 [I_4 + \bar{I}_4], \quad (C.91) \]

\[ \langle P_5' \rangle_{\text{bin}} = \frac{1}{2N_{\text{bin}}} \int_{\text{bin}} dq^2 [I_5 + \bar{I}_5], \quad (C.92) \]

\[ \langle P_6' \rangle_{\text{bin}} = -\frac{1}{2N_{\text{bin}}} \int_{\text{bin}} dq^2 [I_7 + \bar{I}_7], \quad (C.93) \]

\[ \langle P_8' \rangle_{\text{bin}} = -\frac{1}{2N_{\text{bin}}} \int_{\text{bin}} dq^2 [I_8 + \bar{I}_8], \quad (C.94) \]

where

\[ N_{\text{bin}}^2 = -\int_{\text{bin}} dq^2 [I_2^s + \bar{I}_2^s] \int_{\text{bin}} dq^2 [I_2^\bar{s} + \bar{I}_2^\bar{s}]. \quad (C.95) \]

For each of the above angular observables, there is a CP asymmetric counterpart where the plus sign in the numerator is replaced by a minus sign. The observables are ratios of angular coefficients so that the form factors at leading order cancel \[164\]. Thus, the observables are clean theoretically.

In addition to these angular observables, the more traditional observables can also be defined in terms of angular coefficients

\[ \langle A_{FB} \rangle = -\frac{3}{4} \frac{\int dq^2 [I_6^s + \bar{I}_6^\bar{s}]}{\langle d\Gamma/dq^2 \rangle + \langle \bar{d}\Gamma/dq^2 \rangle}, \quad (C.96) \]

\[ \langle F_L \rangle = -\frac{\int dq^2 [I_2^s + I_2^\bar{s}]}{\langle d\Gamma/dq^2 \rangle + \langle \bar{d}\Gamma/dq^2 \rangle}, \quad (C.97) \]

\[ \left\langle \frac{d\text{BR}}{dq^2} \right\rangle = \tau_B \frac{\langle d\Gamma/dq^2 \rangle + \langle \bar{d}\Gamma/dq^2 \rangle}{2}, \quad (C.98) \]

\[ \langle A_{CP} \rangle = \frac{\langle d\Gamma/dq^2 \rangle - \langle \bar{d}\Gamma/dq^2 \rangle}{\langle d\Gamma/dq^2 \rangle + \langle \bar{d}\Gamma/dq^2 \rangle}, \quad (C.99) \]
where \( \tau_B \) is the lifetime of the \( B \) meson,

\[
\langle d\Gamma/dq^2 \rangle = \frac{1}{4} \int dq^2 [3I_1^c + 6I_1^s - I_2^c - 2I_2^s],
\]

(C.100)

and similarly for \( \langle d\Gamma/dq^2 \rangle \). Similarly, one can define \( CP \)-violating observables \( A_{FB} \) and \( F_L \).
Appendix D

$C_7$ Wilson Coefficient Calculations

In this appendix, we calculate the $C_7$ Wilson coefficient for the operator $O_7$ defined in Eq. C.9. The calculation is performed at one-loop level with a top quark in the loop. We have used the background-field version of the 't Hooft-Feynman Gauge.

The background-field method is a way of quantizing gauge field theories without losing explicit gauge invariance. For a review of the background-field method, please refer to Ref. [173] or Chapter 16.6 of Ref. [174]. The general idea of quantizing the gauge field in the background-field method is to split the gauge field into a classical background field, $\hat{V}$, and a quantum field, $V$:

$$\mathcal{L}(\hat{V}) \rightarrow \mathcal{L}(\hat{V} + V).$$

The background field is a fixed field configuration while the quantum field is a fluctuation field that is the integration variable in the functional integral. Hence, the fields in loops are quantum fields while the external fields are background fields. In the background-field method, the gauge-fixing term only breaks the gauge invariance of the quantum fields. The effective action is gauge invariant with respect to the background field. Since fermions do not appear in the gauge-fixing term, the Feynman rules for fermions are identical to those obtained from the conventional quantization method. One benefit of the background-field method is that some vertices in the $R_\xi$ gauge calculated from the conventional method do not appear in the background-field method. For example, the $\phi^\pm W^\pm \gamma$ vertex, where $\phi^\pm$
is the would-be Goldstone boson, is absent in the background-field method. Hence, the number of Feynman diagrams is reduced in the background-field method.

The Feynman diagrams in the background-field method that contribute to $C_7$ are shown in Fig. D.1. Please refer to the Appendix A of Ref. [175] for the complete list of Feynman rules. To obtain $C_7$, we only need to extract the magnetic dipole moment term of these diagrams. To extract the magnetic dipole moment term, notice that these diagrams have the following form

$$i\mathcal{M} = ib(\Gamma^\mu + \Gamma_5^\mu \gamma_5) s \epsilon_\mu.$$  \hspace{1cm} (D.1)

We will ignore the $\Gamma_5^\mu$ term because it does not contribute to the magnetic dipole moment. Using Lorentz invariance, we can rewrite $\Gamma^\mu$ as

$$\Gamma^\mu = \Gamma_\gamma \gamma^\mu + \Gamma_p p^\mu + \Gamma_{p'} p'^\mu$$

$$= \Gamma_\gamma \gamma^\mu + \frac{\Gamma_p + \Gamma_{p'}}{2} (p^\mu + p'^\mu) + \frac{\Gamma_p - \Gamma_{p'}}{2} q^\mu.$$  \hspace{1cm} (D.2)
The last term equals zero by applying the Ward identity. Using the Gordan identity,

\[ m_s \gamma^\mu + m_b \gamma^\mu = (p^\mu + p'^\mu) + i\sigma^{\mu\nu}q_\nu, \tag{D.3} \]

where

\[ \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]. \tag{D.4} \]

We can write Eq. D.2 as

\[ \Gamma^\mu = \Gamma_\gamma^\gamma^\mu + \frac{\Gamma_p + \Gamma_p'}{2} (m_s \gamma^\mu + m_b \gamma^\mu - i\sigma^{\mu\nu}q_\nu) \tag{D.5} \]

\[ = \left[ \Gamma_\gamma + \frac{\Gamma_p + \Gamma_p'}{2} (m_s + m_b) \right] \gamma^\mu + \left[ -\frac{\Gamma_p + \Gamma_p'}{2} \right] i\sigma^{\mu\nu}q_\nu. \tag{D.6} \]

Hence, to extract the magnetic dipole moment term, we only need to evaluate

\[ -\frac{\Gamma_p + \Gamma_p'}{2}. \tag{D.7} \]

In the rest of this appendix, we will evaluate the above quantity for the four diagrams shown in Fig. D.1.

**Diagram (a)**

\[ \Gamma_{(a)}^\mu = \int \frac{d^4k}{(2\pi)^4} \frac{i\gamma_\nu P_L}{\sqrt{2s_W}} \frac{i(p' - \sl{k} + m_t)}{(p' - k)^2 - m_t^2} \left( -\frac{i}{3}\gamma^\mu \right) \frac{i(p - \sl{k} + m_t)}{(p - k)^2 - m_t^2} \]

\[ = -\frac{e^3}{2s_W^{3/2}} \int \frac{d^4k}{(2\pi)^4} \frac{\gamma_\mu P_L (p' - \sl{k} + m_t) \gamma_\nu (p - \sl{k} + m_t)}{\gamma_\nu P_L (p' - \sl{k} + m_t) \gamma_\mu (p - \sl{k} + m_t) \gamma_\nu P_L} \frac{1}{[k^2 - m_W^2]}, \]

Using Feynman parameters, the denominator can be written as

\[ \frac{1}{[(p' - k)^2 - m_t^2][(p - k)^2 - m_t^2][k^2 - m_W^2]} = 2 \int_0^1 dz \int_0^{1-z} dy \frac{1}{(\ell^2 - \Delta)^3}, \]

185
\[ \ell = k - (p' x + py), \]
\[ \Delta = m_t^2(y + z) + m_W^2(1 - y - z). \]

We have dropped the \( m_b^2, m_s^2, \) and \( q^2 \) terms in \( \Delta \). With this definition, diagram (a) becomes

\[ \Gamma^{(a)}_\mu = -\frac{e^3}{2s_W^2} \frac{4}{3} \int_0^1 dz \int_0^{1-z} dy \int \frac{d^4 \ell}{(2\pi)^4 \ell^2} \frac{\text{Numerator}}{(\ell^2 - \Delta)^3}, \]

where

\[ \text{Numerator} = \gamma_\nu P_L(-\ell - \gamma^i(z - 1) - \gamma p + m_t)\gamma_\mu(-\ell - \gamma^i z - \gamma (y - 1) + m_t)\gamma^\nu P_L \]
\[ = P_R \gamma_\nu \ell \gamma_\mu \gamma^\nu P_L + P_R \gamma_\nu \left[z(z - 1)\gamma^i \gamma_\nu \gamma^\nu + (y - 1)(z - 1)\gamma^i \gamma_\nu \gamma^\nu + yz \gamma_\nu \gamma^\nu \gamma^\nu \gamma^\nu \right] + y(y - 1)\gamma_\nu \gamma^\nu \gamma^\nu \gamma^\nu \gamma^\nu \gamma^\nu P_L. \]

Terms linear in \( m_t \) vanish due to \( P_L \) and \( P_R \). To extract the magnetic dipole moment, we only need to consider \( \gamma_\nu \gamma^i \gamma_\mu \gamma^\nu \gamma^\nu \) and \( \gamma_\nu \gamma^i \gamma^\nu \gamma^\nu \gamma^\nu \gamma^\nu \) because, as shown above, we only need terms that are multiplied by \( p^\mu \) and \( p'^\mu \). In addition, terms proportional to \( m_s \) are dropped. With some gamma matrix manipulation, we can rewrite these two terms as

\[ \gamma_\nu \gamma^i \gamma_\mu \gamma^\nu \gamma^\nu = -4\gamma^i p_\mu + 4(p' \cdot p) \gamma_\mu - 4\gamma^i p'_\mu + 2\gamma^i \gamma_\mu \gamma^\nu, \]
\[ \gamma_\nu \gamma^i \gamma^\nu \gamma^\nu \gamma^\nu \gamma^\nu = -4\gamma^i p_\mu + 2\gamma_\mu \gamma^\nu. \]

Using the on-shell condition for \( b \), \( \Gamma_p \) and \( \Gamma_p' \) defined above are given by

\[ \Gamma_p = -\frac{e^3}{2s_W^2} \frac{2}{3} 2 P_R m_b \int_0^1 dz \int_0^{1-z} dy \int \frac{d^4 \ell}{(2\pi)^4 \ell^2} \frac{-4(y - 1)(z - 1)}{(\ell^2 - \Delta)^3}, \]
\[ \Gamma_p' = -\frac{e^3}{2s_W^2} \frac{2}{3} 2 P_R m_b \int_0^1 dz \int_0^{1-z} dy \int \frac{d^4 \ell}{(2\pi)^4 \ell^2} \frac{-4y(y - 1)}{(\ell^2 - \Delta)^3}. \]

The momentum integral can now be evaluated to give

\[ \int \frac{d^4 \ell}{(2\pi)^4 \ell^2} \frac{1}{(\ell^2 - \Delta)^3} = i(-1)^3 \frac{1}{4(4\pi)^2} \frac{1}{2\Delta}. \]
So,
\[
\Gamma_p = i \frac{e^3}{2s_W^2} \frac{2}{3} \frac{1}{(4\pi)^2} P_R m_b \int_0^1 dz \int_0^{1-z} dy \frac{-4(y-1)(z-1)}{m_t^2(y+z) + m_W^2(1-y-z)} ,
\]
\[
\Gamma_{p'} = i \frac{e^3}{2s_W^2} \frac{2}{3} \frac{1}{(4\pi)^2} P_R m_b \int_0^1 dz \int_0^{1-z} dy \frac{-4y(y-1)}{m_t^2(y+z) + m_W^2(1-y-z)} .
\]

Now, the contribution to \( C_7 \) is
\[
-\frac{\Gamma_p + \Gamma_{p'}}{2} = \frac{i}{(4\pi)^2} \frac{e^3}{s_W^2} \frac{2}{3} P_R m_b \int_0^1 dz \int_0^{1-z} dy \frac{(y-1)(z-1) + y(y-1)}{x(y+z) + (1-y-z)} ,
\]
where \( x = m_t^2/m_W^2 \). Solving the integral gives
\[
-\frac{\Gamma_p + \Gamma_{p'}}{2} = -i \frac{g^2}{(4\pi)^2} e \frac{4G_F}{\sqrt{2}} P_R m_b \left[ \frac{-5x^2 - 5x + 4}{9(x-1)^3} + \frac{4x^2 - 2x}{3(x-1)^4} \log x \right].
\]

Comparing to Eq. C.9, the contribution of this diagram to \( C_7 \) is
\[
C_7^{(a)} = -\frac{1}{2} \left[ \frac{-5x^2 - 5x + 4}{9(x-1)^3} + \frac{4x^2 - 2x}{3(x-1)^4} \log x \right]. \tag{D.8}
\]

The factor of \( 1/2 \) is coming from converting \( \epsilon_{\mu q} \) to \( F_{\mu\nu} \).

**Diagram (b)**

\[
\Gamma^{(b)} = \int \frac{d^4k}{(2\pi)^4} \frac{ie}{\sqrt{2}S_W} \gamma_\nu P_L \frac{i(k + m_t)}{k^2 - m_t^2} \frac{ie}{\sqrt{2}s_W} \gamma_\mu P_L \frac{i}{(p - k)^2 - m_W^2} \frac{i}{(p' - k)^2 - m_W^2} \left[ -ie \right] \left[ g^{\mu\nu}(2k - p' - p)^\mu + g^{\mu\nu}(p + q - p')^{\nu'} + g^{\mu\nu}(p' - p - q)^\nu \right].
\]

A calculation very similar to that of diagram (a) can be performed to obtain the contribution of this diagram to \( C_7 \), which is given by
\[
C_7^{(b)} = -\frac{1}{2} \left[ \frac{20x^2 - 19x + 5}{6(x-1)^3} + \frac{-2x^3 + x^2}{(x-1)^4} \log x \right]. \tag{D.9}
\]
Diagram (c)

\[ \Gamma^{(c)}_{\mu} = \int \frac{d^4k}{(2\pi)^4} ie \left[ -\frac{1}{\sqrt{2}s_W m_W} P_L + \frac{1}{\sqrt{2}c_W m_W} P_R \right] \frac{i(p' - k + m_t)}{(p' - k)^2 - m_t^2} \left( -\frac{2}{3} \gamma^\mu \right) \frac{i(p - k + m_t)}{(p - k)^2 - m_t^2} \]

\[ \left. \frac{1}{\sqrt{2}s_W m_W} P_L - \frac{1}{\sqrt{2}c_W m_W} P_R \right] \frac{i}{k^2 - m_W^2}. \]

A calculation very similar to that of diagram (a) can be performed to obtain the contribution of this diagram to \( C_7 \), which is given by

\[ C_7^{(c)} = -\frac{1}{2} \left[ \frac{-5x^3 + 19x^2 - 20x}{18(x - 1)^3} + \frac{-x^2 + 2x}{3(x - 1)^4} \log x \right]. \] (D.10)

Diagram (d)

\[ \Gamma^{(d)}_{\mu} = \int \frac{d^4k}{(2\pi)^4} ie \left[ -\frac{1}{\sqrt{2}s_W m_W} P_L + \frac{1}{\sqrt{2}c_W m_W} P_R \right] \frac{i(k + m_t)}{k^2 - m_t^2} \]

\[ \left. \frac{1}{\sqrt{2}s_W m_W} P_L - \frac{1}{\sqrt{2}c_W m_W} P_R \right] \frac{i}{(p - k)^2 - m_W^2} \frac{i}{(p' - k)^2 - m_W^2} \frac{i}{k^2 - m_W^2} \]

\[ (-ie)(p + p' - 2k)_\mu. \]

A calculation very similar to that of diagram (a) can be performed to obtain the contribution of this diagram to \( C_7 \), which is given by

\[ C_7^{(d)} = -\frac{1}{2} \left[ \frac{-4x^3 + 5x^2 + 5x}{12(x - 1)^3} + \frac{x^3 - 2x^2}{2(x - 1)^4} \log x \right]. \] (D.11)

The total contribution of \( C_7 \) due to top quark running in the loop is

\[ C_7 = -\frac{1}{2} \left[ \frac{-22x^3 + 153x^2 - 159x + 46}{36(x - 1)^3} + \frac{-3x^3 + 2x^2}{2(x - 1)^4} \log x \right]. \] (D.12)
Appendix E

Natural SUSY Breaking

In Section 3.3.4 we showed that if GUT scale boundary conditions are constrained such that $m_{H_u,d}/m_{16} \approx \sqrt{2}$ and $A_0/m_{16} \approx -2$, then the fine-tuning of the theory is reduced to the order of 1 part in 500. In this appendix, we provide an example of an attempt to obtain these ratios naturally from a more fundamental theory.

Consider heterotic orbifold models with dilaton/moduli SUSY breaking as discussed in Ref. [176]. Following Eq. 60 and Eq. 61 in Ref. [176], the scalar masses for sparticle $\alpha$, the trilinear coupling, and the gaugino masses are given by

$$m_{\alpha}^2 = m_{3/2}^2 (1 + 3C^2 \cos^2 \theta \vec{n}_{\alpha} \cdot \vec{\Theta}^2) + V_0$$ \hfill (E.1)

$$A_0 = -\sqrt{3}Cm_{3/2} \left( \sin \theta e^{i\gamma s} + \sum_{i=1}^{6} e^{-i\gamma \Theta_i[1 + n_{\alpha}^T_i + n_{\beta}^T_i + n_{\gamma}^T_i - (T_i + T_i^\dagger) \partial_i \log Y_{161016}]}, \right)$$ \hfill (E.2)

$$M_\alpha = \sqrt{3}m_{3/2} \sin \theta e^{-i\gamma s},$$ \hfill (E.3)

where

$$C^2 = 1 + \frac{V_0}{3m_{10}^2}. \hfill (E.4)$$

$V_0$ is the tree-level cosmological constant, which we will set to be zero (i.e. $C = 1$), and $\theta$ determines the amount of SUSY breaking of the dilaton sector versus the moduli sector. $\cos \theta = 0$ means all the SUSY breaking is due to the dilaton. $\vec{n}_{\alpha}$ are modular weights of matter fields and $\vec{\Theta}$ gives the probability for the SUSY breaking contribution of each
modulus, such that $\sum_{i=1}^{6} \Theta_i^2 = 1$. We then let $\theta = 0$ such that at tree level gauginos are massless. They would then obtain one-loop-suppressed masses due to moduli SUSY breaking. We also assume that the Yukawa couplings, $Y_{\alpha\beta\gamma}$, are independent of the moduli.

As a particular example, consider a $\mathbb{Z}_2 \times \mathbb{Z}_6'$ orbifold with twist vectors given by $(1/2, 1/2, 0)$ and $(1/6, 2/3, 1/6)$ in the three two-tori \cite{177}. There are three Kahler moduli in this example. Then we have

$$m_{10}^2 = m_{3/2}^2 (1 + 3(n_{10} T_1 (\Theta_1^2 + n_{10} T_3 (\Theta_3^2)),$$  \hspace{1cm} (E.5)

where we have assumed that $\Theta_2 = 0$.

We now assume that the Higgs 10 multiplet comes from the bulk on the second two torus. Thus, $n_{10} T_1 = n_{10} T_3 = 0$ and $n_{10} T_2 = -1$. Hence, $m_{10} = m_{3/2}$ and therefore,

$$m_{16}^2 = m_{10}^2 (1 + 3(n_{10} T_1 (\Theta_1^2 + n_{10} T_3 (\Theta_3^2)).$$  \hspace{1cm} (E.6)

If the 16 multiplet lives in the fifth twisted sector with modular weights, $n_{16} T_1 = n_{16} T_3 = -1/6$ and $n_{16} T_2 = -2/3$, then we have

$$m_{10} = \sqrt{2} m_{16}.$$  \hspace{1cm} (E.7)

This is our first constraint, that

$$m_{10} \equiv (m_{H_u} + m_{H_d})/2 \approx \sqrt{2} m_{16}.$$  \hspace{1cm} (E.8)

With this, we have

$$A_{16 10 16} = -\sqrt{6} m_{16} (e^{-i\gamma_1} \Theta_1 [1 + 2 n_{10} T_1 + n_{10} T_3] + e^{-i\gamma_3} \Theta_3 [1 + 2 n_{16} T_3 + n_{16} T_3])$$  \hspace{1cm} (E.9)

and we need $A_{16 10 16} = A_0 = -2 m_{16}$ where the last equality is the boundary condition at
the sweet spot. This can be solved with

\[ e^{-i\gamma_1} \Theta_1 = \frac{1}{\sqrt{2}} e^{-i\gamma}, \]  
\[ e^{-i\gamma_3} \Theta_3 = \frac{1}{\sqrt{2}} e^{+i\gamma}, \]  
\[ \gamma = \pi/6. \]

Of course, it would require the dynamics of stabilizing moduli and SUSY breaking to fix these particular values of $\Theta_1$. 

191
Appendix F

YUKAWA MATRICES CONVENTIONS

The definition of Yukawa matrices in different references regarding CP asymmetry are different. In this appendix, we provide a dictionary on the definition of Yukawa matrices to ease the comparison between different references.

In Chapter 1 and Sec. 5.1 the Yukawa matrices are defined in Weyl notation with doublets on the left. The Yukawa matrices with this definition are denoted as $y$.

From Sec. 5.2 onwards, the Yukawa matrices are defined in Weyl notation with doublets on the right. The Yukawa matrices with this definition are denoted as $\lambda$. The reason for this switch in definition is that the RGEs in the global $\chi^2$ analysis program, maton, is written with doublets on the right.

We denote Yukawa matrices in different references as

\[
\begin{align*}
\text{Covi et al. [57]} & : \quad \lambda_{\text{CRV}}, \\
\text{Giudice et al. [59]} & : \quad y_{\text{GNRRS}}, \\
\text{Buchmuller et al. [45]} & : \quad h_{\text{BPY}}, \\
\text{Davidson et al. [47]} & : \quad \lambda_{\text{DNN}}.
\end{align*}
\]

The relationship between these definitions are

\[
\lambda = y^T = \lambda_{\text{CRV}}^T = y_{\text{GNRRS}}^T = h_{\text{BPY}} = \lambda_{\text{DNN}}^T. \quad \text{(F.1)}
\]
Appendix G
AVERAGING INFATONS OSCILLATIONS

After the inflaton stops decaying non-perturbatively, the simulation in Chapter 5 can be simplified by assuming that the inflaton oscillates sinusoidally. In this appendix, we calculate average decay rates with the assumption that the inflaton’s sinusoidal oscillations have a period $2\pi$ and amplitude $\phi_{\text{amp}}$:

$$\phi(t) = \phi_{\text{max}} \sin(m\phi t) .$$  \hspace{1cm} (G.1)

Since the oscillation is symmetrical, only the first quarter of the oscillation, which is from $m\phi t = 0$ to $m\phi t = \pi/2$, is considered.

G.1 Perturbative Decays of Inflatons

Perturbative Decays of Inflatons to Higgs Bosons

The decay rate of inflatons to Higgs bosons in Eq. 5.40 is

$$\Gamma_{\phi\rightarrow h} \propto \frac{\phi^2}{m_\phi} \Theta(m_\phi - 2\alpha\phi) .$$  \hspace{1cm} (G.2)

Due to the Heaviside Theta function, this decay rate is non-zero only when $\phi < m_\phi/2\alpha$ or from $m_\phi t = 0$ to

$$m_\phi t_c = \sin^{-1}\left(\frac{m_\phi}{2\alpha\phi_{\text{max}}}\right) .$$  \hspace{1cm} (G.3)
Hence, the average decay rate is
\begin{equation}
\langle \Gamma_{\phi \to \tilde{h}} \rangle \propto \frac{2}{\pi} \int_0^{m_{\phi} t_c} d(m_{\phi} t) \frac{\phi_{\text{max}}^2}{m_{\phi}} \sin^2(m_{\phi} t) \nonumber \\
= \frac{2\phi_{\text{max}}^2}{m_{\phi} \pi} \left( \frac{m_{\phi} t_c}{2} - \frac{1}{4} \sin(2m_{\phi} t_c) \right). \tag{G.4}
\end{equation}

Perturbative Decays of Inflatons to Higgsinos

The decay rate of inflatons to Higgsinos in Eq. 5.41 is
\begin{equation}
\Gamma_{\phi \to \tilde{h}} \propto m_{\phi} \Theta(m_{\phi} - 2\alpha_{\phi}). \tag{G.5}
\end{equation}

Similar to the decay to Higgs bosons, this decay only occur from \(m_{\phi} t = 0\) to \(m_{\phi} t = m_{\phi} t_c\). Hence, the average decay rate is
\begin{equation}
\langle \Gamma_{\phi \to \tilde{h}} \rangle \propto \frac{2}{\pi} \int_0^{m_{\phi} t_c} d(m_{\phi} t) m_{\phi} \nonumber \\
= \frac{2m_{\phi} t_c}{\pi}. \tag{G.6}
\end{equation}

G.2 Decays of Higgs Bosons

Decays of Higgs Bosons to Right-Handed (s)Neutrinos

The decay rates of up-type Higgses to right-handed neutrinos given in Eq. 5.42, down-type Higgses to right-handed sneutrinos in Eq. 5.45, and up-type Higgsinos to right-handed (s)neutrinos in Eq. 5.47 and Eq. 5.48 have the following form:
\begin{equation}
\{ \Gamma_{h_u \to \tilde{\nu}_i}, \Gamma_{h_d \to \tilde{\nu}_i}, \Gamma_{\tilde{h}_u \to \tilde{\nu}_i}, \Gamma_{\tilde{h}_u \to \tilde{\nu}_i} \} \propto \phi \Theta(\alpha_{\phi} - M_{R_i}). \tag{G.7}
\end{equation}

These decay rates are non-zero only from
\begin{equation}
m_{\phi} t_c = \sin^{-1} \left( \frac{M_{R_i}}{\alpha_{\phi_{\text{max}}}} \right), \tag{G.8}
\end{equation}

194
to $m_\phi t = \pi/2$. Hence, the average decay rates are

$$\langle \{ \Gamma_{h_u \to \tilde{\nu}_i^1}, \Gamma_{h_d \to \tilde{\nu}_i^1}, \Gamma_{\tilde{h}_u \to \tilde{\nu}_i^1}, \Gamma_{\tilde{h}_d \to \tilde{\nu}_i^1} \} \rangle \propto \frac{2}{\pi} \int_{m_\phi t_c}^{\pi/2} d(m_\phi t) \phi_{\text{max}} \sin(m_\phi t)$$

$$= \frac{2}{\pi} \phi_{\text{max}} \sqrt{1 - \frac{M_{R_i}^2}{\alpha^2 \phi_{\text{max}}^2}}.$$  \hspace{1cm} (G.9)

On the other hand, the decay rate of up-type Higgs bosons to right-handed sneutrinos in Eq. 5.43 is

$$\Gamma_{h_u \to \tilde{\nu}_i^1} \propto \frac{M_{R_i}^2}{\phi} \Theta(\alpha \phi - M_{R_i}).$$ \hspace{1cm} (G.10)

Similar to above, this decay rate is non-zero only from $m_\phi t = m_\phi t_c$ to $m_\phi t = \pi/2$. Hence, the average decay rate is

$$\langle \Gamma_{h_u \to \tilde{\nu}_i^1} \rangle \propto \frac{2}{\pi} \int_{m_\phi t_c}^{\pi/2} d(m_\phi t) \frac{M_{R_i}^2}{\phi_{\text{max}} \sin(m_\phi t)}$$

$$= -\frac{2}{\pi} \frac{M_{R_i}^2}{\phi_{\text{max}}} \ln \tan \frac{m_\phi t_c}{2}.$$ \hspace{1cm} (G.11)

**Decays of Higgs Bosons to Radiations**

The decay rates of Higgs bosons to radiation (i.e. massless particles) in Eq. 5.44, Eq. 5.46, Eq. 5.49, and Eq. 5.50 have the following form

$$\Gamma_{\{h_u, h_d, \tilde{h}_u, \tilde{h}_d\} \to R} \propto \phi.$$ \hspace{1cm} (G.12)

Hence, the average decay rates are

$$\langle \Gamma_{\{h_u, h_d, \tilde{h}_u, \tilde{h}_d\} \to R} \rangle \propto \frac{2}{\pi} \phi_{\text{max}}.$$ \hspace{1cm} (G.13)
**G.3 Decays of Right-Handed (s)Neutrinos**

**Decays of Right-Handed (s)Neutrinos to Up-Type Higgs Bosons**

The decay rates of right-handed (s)neutrinos to up-type Higgses in Eq. 5.51, Eq. 5.52 and Eq. 5.53 have the following form

\[
\{\Gamma_{\bar{\nu}_i \rightarrow h_u^\dagger}, \Gamma_{\bar{\nu}_i \rightarrow \tilde{h}_u^\dagger}, \Gamma_{\tilde{\nu}_i \rightarrow h_u^\dagger}\} \propto M_{R_i} \Theta(M_{R_i} - \alpha\phi). \tag{G.14}
\]

These decays only occur between \(m_\phi t = 0\) and \(m_\phi t = m_\phi t_c = \sin^{-1}\left(\frac{M_{R_i}}{\alpha\phi_{\text{max}}}\right)\). \tag{G.15}

Hence, the average decay rates are

\[
\langle\{\Gamma_{\bar{\nu}_i \rightarrow h_u^\dagger}, \Gamma_{\bar{\nu}_i \rightarrow \tilde{h}_u^\dagger}, \Gamma_{\tilde{\nu}_i \rightarrow h_u^\dagger}\}\rangle \propto \frac{2}{\pi} \int_0^{m_\phi t_c} d(m_\phi t) M_{R_i} \frac{\phi^2}{M_{R_i}^2} \Theta(M_{R_i} - \alpha\phi) \tag{G.16}
\]

**Decays of Right-Handed (s)Neutrinos to Down-Type Higgs Bosons**

The decay rate of the right-handed sneutrinos to the down-type Higgses in Eq. 5.54 is

\[
\Gamma_{\tilde{\nu}_i \rightarrow h_d} \propto \frac{\phi^2}{M_{R_i}^2} \Theta(M_{R_i} - \alpha\phi). \tag{G.17}
\]

Similar to above, this decay only occur between \(m_\phi t = 0\) and \(m_\phi t = m_\phi t_c\). Hence, the average decay rate is

\[
\langle\Gamma_{\tilde{\nu}_i \rightarrow h_d}\rangle \propto \frac{2}{\pi} \int_0^{m_\phi t_c} d(m_\phi t) \frac{\phi^2_{\text{max}}}{M_{R_i}^2} \sin^2(m_\phi t) \tag{G.18}
\]

\[
= \frac{2\phi^2_{\text{max}}}{\pi M_{R_i}} \left[\frac{m_\phi t_c}{2} - \frac{1}{4} \sin(2m_\phi t_c)\right].
\]
## Appendix H

**Benchmark Points**

### H.1 SO(10) SUSY GUT Benchmark Points

In this appendix, we list the benchmark points for the SO(10) SUSY GUT model considered in Chapter 3.

Input parameters of benchmark point A: $M_\tilde{g} = 1.20$ TeV

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input Param.</th>
<th>Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gauge</strong></td>
<td>$1/\alpha_G$</td>
<td>26.0</td>
</tr>
<tr>
<td></td>
<td>$M_G/10^{16}$ GeV</td>
<td>2.55</td>
</tr>
<tr>
<td></td>
<td>$\epsilon_3/%$</td>
<td>-1.30</td>
</tr>
<tr>
<td><strong>SUSY (GUT scale)</strong></td>
<td>$m_{16}$/TeV</td>
<td>25.0</td>
</tr>
<tr>
<td></td>
<td>$M_{1/2}$/GeV</td>
<td>280</td>
</tr>
<tr>
<td></td>
<td>$A_0$/TeV</td>
<td>-51.4</td>
</tr>
<tr>
<td></td>
<td>$(m_{H_d}/m_{16})^2$</td>
<td>1.86</td>
</tr>
<tr>
<td></td>
<td>$(m_{H_u}/m_{16})^2$</td>
<td>1.61</td>
</tr>
<tr>
<td><strong>Neutrino</strong></td>
<td>$M_{R_1}/10^9$ GeV</td>
<td>9.19</td>
</tr>
<tr>
<td></td>
<td>$M_{R_2}/10^{11}$ GeV</td>
<td>5.79</td>
</tr>
<tr>
<td></td>
<td>$M_{R_3}/10^{13}$ GeV</td>
<td>3.51</td>
</tr>
<tr>
<td><strong>SUSY (EW Scale)</strong></td>
<td>$\tan \beta$</td>
<td>50.3</td>
</tr>
<tr>
<td></td>
<td>$\mu$/GeV</td>
<td>1210</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input Param.</th>
<th>Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Yukawa Textures</strong></td>
<td>$\lambda$</td>
<td>0.610</td>
</tr>
<tr>
<td></td>
<td>$\lambda\tilde{e}$</td>
<td>0.0308</td>
</tr>
<tr>
<td></td>
<td>$\lambda\tilde{\epsilon}$</td>
<td>0.00491</td>
</tr>
<tr>
<td></td>
<td>$\lambda\tilde{e}'$</td>
<td>-0.00192</td>
</tr>
<tr>
<td></td>
<td>$\lambda\tilde{\epsilon}$</td>
<td>0.00365</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>$\rho$</td>
<td>0.0698</td>
</tr>
<tr>
<td></td>
<td>$\phi_\gamma$/rad</td>
<td>0.581</td>
</tr>
<tr>
<td></td>
<td>$\phi_\epsilon$/rad</td>
<td>3.47</td>
</tr>
<tr>
<td></td>
<td>$\phi_\sigma$/rad</td>
<td>0.523</td>
</tr>
<tr>
<td></td>
<td>$\phi_\rho$/rad</td>
<td>3.95</td>
</tr>
<tr>
<td><strong>Mirage mediation</strong></td>
<td>$\alpha$</td>
<td>0</td>
</tr>
</tbody>
</table>
Low energy observables fit of benchmark point A

<table>
<thead>
<tr>
<th>Observable</th>
<th>Fit</th>
<th>Exp.</th>
<th>Pull</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_Z$/GeV</td>
<td>91.1876</td>
<td>91.1876</td>
<td>0.0000</td>
<td>0.4541</td>
</tr>
<tr>
<td>$M_W$/GeV</td>
<td>80.4784</td>
<td>80.3850</td>
<td>0.2320</td>
<td>0.4027</td>
</tr>
<tr>
<td>$1/\alpha_{em}$</td>
<td>137.2810</td>
<td>0.0073</td>
<td>0.3569</td>
<td>0.6864</td>
</tr>
<tr>
<td>$G_\mu/10^{-4}$ GeV$^{-2}$</td>
<td>1.1789</td>
<td>1.1664</td>
<td>1.0598</td>
<td>0.0118</td>
</tr>
<tr>
<td>$\alpha_3(M_Z)$</td>
<td>0.1192</td>
<td>0.1185</td>
<td>0.8199</td>
<td>0.0008</td>
</tr>
<tr>
<td>$M_t$/GeV</td>
<td>174.0947</td>
<td>173.2100</td>
<td>0.7171</td>
<td>1.2337</td>
</tr>
<tr>
<td>$m_t(m_t)$/GeV</td>
<td>4.1986</td>
<td>4.1800</td>
<td>0.5092</td>
<td>0.0366</td>
</tr>
<tr>
<td>$M_s$/GeV</td>
<td>1.7772</td>
<td>1.7768</td>
<td>0.0428</td>
<td>0.0089</td>
</tr>
<tr>
<td>$(M_s - M_c)$/GeV</td>
<td>3.1701</td>
<td>3.4500</td>
<td>0.8720</td>
<td>0.3209</td>
</tr>
<tr>
<td>$m_s(m_s)$/GeV</td>
<td>1.2509</td>
<td>1.2750</td>
<td>0.9333</td>
<td>0.0258</td>
</tr>
<tr>
<td>$m_s(2\text{GeV})$/GeV</td>
<td>0.0953</td>
<td>0.0950</td>
<td>0.0609</td>
<td>0.0050</td>
</tr>
<tr>
<td>$m_s/m_s(2\text{GeV})$</td>
<td>0.0702</td>
<td>0.0513</td>
<td>2.8247</td>
<td>0.0067</td>
</tr>
<tr>
<td>$1/Q^2$</td>
<td>0.0018</td>
<td>0.0019</td>
<td>0.4528</td>
<td>0.0001</td>
</tr>
<tr>
<td>$M_s$/GeV</td>
<td>0.1056</td>
<td>0.1057</td>
<td>0.0578</td>
<td>0.0005</td>
</tr>
<tr>
<td>$M_s/10^{-4}$ GeV</td>
<td>5.1143</td>
<td>5.1100</td>
<td>0.1674</td>
<td>0.0256</td>
</tr>
<tr>
<td>$</td>
<td>V_{us}</td>
<td>$</td>
<td>0.2245</td>
<td>0.2253</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>$</td>
<td>0.0404</td>
<td>0.0408</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>/10^{-3}$</td>
<td>3.1235</td>
<td>3.8500</td>
</tr>
<tr>
<td>$</td>
<td>V_{us}</td>
<td>/10^{-3}$</td>
<td>8.8463</td>
<td>8.4000</td>
</tr>
<tr>
<td>$</td>
<td>V_{us}</td>
<td>$</td>
<td>0.0396</td>
<td>0.0400</td>
</tr>
<tr>
<td>$\sin(2\beta)$</td>
<td>0.6296</td>
<td>0.6280</td>
<td>2.7214</td>
<td>0.0193</td>
</tr>
<tr>
<td>$\epsilon_K$</td>
<td>0.0022</td>
<td>0.0022</td>
<td>0.0022</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\Delta M_{S_{2}}/\Delta M_{B_{d}}$</td>
<td>34.8195</td>
<td>35.0345</td>
<td>0.0308</td>
<td>6.9747</td>
</tr>
<tr>
<td>$\Delta M_{S_{2}}/10^{-10}$ MeV</td>
<td>3.9946</td>
<td>3.3370</td>
<td>0.8224</td>
<td>0.7996</td>
</tr>
<tr>
<td>$m_b^2/10^{-5}$ eV$^2$</td>
<td>7.5883</td>
<td>7.5550</td>
<td>0.0621</td>
<td>0.5363</td>
</tr>
<tr>
<td>$m_b^2/10^{-3}$ eV$^2$</td>
<td>2.4649</td>
<td>2.4620</td>
<td>0.0197</td>
<td>0.1455</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>0.3028</td>
<td>0.3070</td>
<td>0.1125</td>
<td>0.0370</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>0.6600</td>
<td>0.5125</td>
<td>1.1300</td>
<td>0.1305</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>0.0162</td>
<td>0.0218</td>
<td>1.7510</td>
<td>0.0032</td>
</tr>
<tr>
<td>$M_5$/GeV</td>
<td>126.2697</td>
<td>125.7000</td>
<td>0.1882</td>
<td>3.0265</td>
</tr>
<tr>
<td>$\text{BR}(B \rightarrow s\gamma)/10^{-4}$</td>
<td>2.7220</td>
<td>3.4300</td>
<td>0.5419</td>
<td>1.3064</td>
</tr>
<tr>
<td>$\text{BR}(B_s \rightarrow \mu^+\mu^-)/10^{-9}$</td>
<td>2.7213</td>
<td>2.8000</td>
<td>0.0888</td>
<td>0.8676</td>
</tr>
<tr>
<td>$\text{BR}(B_s \rightarrow \mu^+\mu^-)/10^{-10}$</td>
<td>1.0734</td>
<td>3.9000</td>
<td>1.7509</td>
<td>1.6143</td>
</tr>
<tr>
<td>$\text{BR}(B \rightarrow \tau\nu)/10^{-5}$</td>
<td>6.2223</td>
<td>11.4000</td>
<td>1.3588</td>
<td>3.8104</td>
</tr>
<tr>
<td>$\text{BR}(B \rightarrow K^+\mu^-\mu^-)_{\text{low }q^2}/10^{-8}$</td>
<td>4.7860</td>
<td>3.4000</td>
<td>0.2739</td>
<td>5.0610</td>
</tr>
<tr>
<td>$\text{BR}(B \rightarrow K^+\mu^-\mu^-)_{\text{high }q^2}/10^{-8}$</td>
<td>7.5495</td>
<td>5.6000</td>
<td>0.1356</td>
<td>14.3788</td>
</tr>
<tr>
<td>$q_0^2(\text{Arg}(B \rightarrow K^+\mu^-\mu^-))/\text{GeV}^2$</td>
<td>3.7120</td>
<td>4.9000</td>
<td>0.9190</td>
<td>1.2927</td>
</tr>
<tr>
<td>$F_L(B \rightarrow K^+\mu^-\mu^-)_{\text{low }q^2}$</td>
<td>0.7207</td>
<td>0.6500</td>
<td>0.2101</td>
<td>0.3366</td>
</tr>
<tr>
<td>$F_L(B \rightarrow K^+\mu^-\mu^-)_{\text{high }q^2}$</td>
<td>0.3108</td>
<td>0.3300</td>
<td>0.0726</td>
<td>0.2644</td>
</tr>
<tr>
<td>$P_L(B \rightarrow K^+\mu^-\mu^-)_{\text{low }q^2}$</td>
<td>0.0331</td>
<td>0.3300</td>
<td>2.3939</td>
<td>0.1240</td>
</tr>
<tr>
<td>$P_L(B \rightarrow K^+\mu^-\mu^-)_{\text{high }q^2}$</td>
<td>-0.4336</td>
<td>-0.5000</td>
<td>0.3364</td>
<td>0.1974</td>
</tr>
<tr>
<td>$P_L'(B \rightarrow K^+\mu^-\mu^-)_{\text{low }q^2}$</td>
<td>0.5717</td>
<td>0.5800</td>
<td>0.0208</td>
<td>0.3988</td>
</tr>
<tr>
<td>$P_L'(B \rightarrow K^+\mu^-\mu^-)_{\text{high }q^2}$</td>
<td>1.2190</td>
<td>-0.1800</td>
<td>1.7066</td>
<td>0.8198</td>
</tr>
<tr>
<td>$P_L'(B \rightarrow K^+\mu^-\mu^-)_{\text{low }q^2}$</td>
<td>-0.4335</td>
<td>0.2100</td>
<td>2.2451</td>
<td>0.2866</td>
</tr>
<tr>
<td>$P_L'(B \rightarrow K^+\mu^-\mu^-)_{\text{high }q^2}$</td>
<td>-0.7116</td>
<td>-0.7900</td>
<td>0.1552</td>
<td>0.5052</td>
</tr>
<tr>
<td>Total $\chi^2$</td>
<td>46.7692</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Input parameters of benchmark point B: $M_\tilde{g} = 1.19$ TeV

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input Param.</th>
<th>Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauge</td>
<td>$1/\alpha_G$</td>
<td>26.0</td>
</tr>
<tr>
<td></td>
<td>$M_G/10^{16}$ GeV</td>
<td>2.60</td>
</tr>
<tr>
<td></td>
<td>$\epsilon_3/%$</td>
<td>-1.50</td>
</tr>
<tr>
<td>SUSY (GUT scale)</td>
<td>$m_{16}/\text{TeV}$</td>
<td>25.0</td>
</tr>
<tr>
<td></td>
<td>$M_{1/2}/\text{GeV}$</td>
<td>520</td>
</tr>
<tr>
<td></td>
<td>$A_0/\text{TeV}$</td>
<td>-51.2</td>
</tr>
<tr>
<td></td>
<td>$(m_{H_d}/m_{16})^2$</td>
<td>1.85</td>
</tr>
<tr>
<td></td>
<td>$(m_{H_u}/m_{16})^2$</td>
<td>1.61</td>
</tr>
<tr>
<td>Neutrino</td>
<td>$M_{R_1}/10^9$ GeV</td>
<td>9.06</td>
</tr>
<tr>
<td></td>
<td>$M_{R_2}/10^{11}$ GeV</td>
<td>5.67</td>
</tr>
<tr>
<td></td>
<td>$M_{R_3}/10^{13}$ GeV</td>
<td>3.24</td>
</tr>
<tr>
<td>SUSY (EW Scale)</td>
<td>$\tan \beta$</td>
<td>50.2</td>
</tr>
<tr>
<td></td>
<td>$\mu/\text{GeV}$</td>
<td>1240</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input Param.</th>
<th>Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yukawa Textures</td>
<td>$\lambda$</td>
<td>0.610</td>
</tr>
<tr>
<td></td>
<td>$\lambda \tilde{c}$</td>
<td>0.0310</td>
</tr>
<tr>
<td></td>
<td>$\lambda \tilde{e}$</td>
<td>0.00487</td>
</tr>
<tr>
<td></td>
<td>$\lambda \tilde{c}'$</td>
<td>-0.00191</td>
</tr>
<tr>
<td></td>
<td>$\lambda \tilde{\epsilon} \xi$</td>
<td>0.00375</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>$\rho$</td>
<td>0.0708</td>
</tr>
<tr>
<td></td>
<td>$\phi_i/\text{rad}$</td>
<td>0.567</td>
</tr>
<tr>
<td></td>
<td>$\phi_e/\text{rad}$</td>
<td>3.49</td>
</tr>
<tr>
<td></td>
<td>$\phi_\sigma/\text{rad}$</td>
<td>0.530</td>
</tr>
<tr>
<td></td>
<td>$\phi_\rho/\text{rad}$</td>
<td>3.94</td>
</tr>
<tr>
<td>Mirage mediation</td>
<td>$\alpha$</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Low energy observables fit of benchmark point B

<table>
<thead>
<tr>
<th>Observable</th>
<th>Fit</th>
<th>Exp.</th>
<th>Pull</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_Z/\text{GeV}$</td>
<td>91.1876</td>
<td>91.1876</td>
<td>0.0000</td>
<td>0.4540</td>
</tr>
<tr>
<td>$M_W/\text{GeV}$</td>
<td>80.5197</td>
<td>80.3850</td>
<td>0.3344</td>
<td>0.4029</td>
</tr>
<tr>
<td>$1/\alpha_{em}$</td>
<td>137.1416</td>
<td>0.0073</td>
<td>0.1540</td>
<td>0.6857</td>
</tr>
<tr>
<td>$G_\mu/10^{-5}\text{GeV}^2$</td>
<td>1.1829</td>
<td>1.1664</td>
<td>1.3978</td>
<td>0.0118</td>
</tr>
<tr>
<td>$\alpha_3(M_Z)$</td>
<td>0.1189</td>
<td>0.1185</td>
<td>0.4798</td>
<td>0.0008</td>
</tr>
<tr>
<td>$M_t/\text{GeV}$</td>
<td>173.8449</td>
<td>173.2100</td>
<td>0.5150</td>
<td>1.2328</td>
</tr>
<tr>
<td>$m_t(m_t)/\text{GeV}$</td>
<td>4.2023</td>
<td>4.1800</td>
<td>0.6094</td>
<td>0.0366</td>
</tr>
<tr>
<td>$M_\mu/\text{GeV}$</td>
<td>1.7772</td>
<td>1.7768</td>
<td>0.0450</td>
<td>0.0089</td>
</tr>
<tr>
<td>$(M_B - M_t)/\text{GeV}$</td>
<td>3.1680</td>
<td>3.4500</td>
<td>0.8791</td>
<td>0.3207</td>
</tr>
<tr>
<td>$m_e(m_e)/\text{GeV}$</td>
<td>1.2570</td>
<td>1.2750</td>
<td>0.6979</td>
<td>0.0258</td>
</tr>
<tr>
<td>$m_\mu(10\text{GeV})/\text{GeV}$</td>
<td>0.0947</td>
<td>0.0950</td>
<td>0.0671</td>
<td>0.0050</td>
</tr>
<tr>
<td>$m_\tau(m_\mu)/\text{GeV}$</td>
<td>0.0700</td>
<td>0.0513</td>
<td>2.7901</td>
<td>0.0067</td>
</tr>
<tr>
<td>$1/Q^2$</td>
<td>0.0018</td>
<td>0.0019</td>
<td>0.5027</td>
<td>0.0001</td>
</tr>
<tr>
<td>$M_\nu/\text{GeV}$</td>
<td>0.1056</td>
<td>0.1057</td>
<td>0.1457</td>
<td>0.0005</td>
</tr>
<tr>
<td>$M_t/10^{-5}\text{GeV}$</td>
<td>5.1145</td>
<td>5.1100</td>
<td>0.1775</td>
<td>0.0256</td>
</tr>
<tr>
<td>$[V_{us}]$</td>
<td>0.2244</td>
<td>0.2253</td>
<td>0.6440</td>
<td>0.0014</td>
</tr>
<tr>
<td>$[V_{cb}]$</td>
<td>0.0407</td>
<td>0.0408</td>
<td>0.0584</td>
<td>0.0021</td>
</tr>
<tr>
<td>$[V_{us}]/10^{-3}$</td>
<td>3.1307</td>
<td>3.8500</td>
<td>0.8363</td>
<td>0.8601</td>
</tr>
<tr>
<td>$[V_{td}]/10^{-3}$</td>
<td>8.8596</td>
<td>8.4000</td>
<td>0.7639</td>
<td>0.6016</td>
</tr>
<tr>
<td>$[V_{ts}]$</td>
<td>0.0398</td>
<td>0.0400</td>
<td>0.0652</td>
<td>0.0027</td>
</tr>
<tr>
<td>$\sin(2\beta)$</td>
<td>0.6285</td>
<td>0.6820</td>
<td>2.7790</td>
<td>0.0193</td>
</tr>
<tr>
<td>$\epsilon_K$</td>
<td>0.0023</td>
<td>0.0022</td>
<td>0.1149</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\Delta M_B / \Delta M_{B_{ud}}$</td>
<td>35.5946</td>
<td>35.0345</td>
<td>0.0786</td>
<td>7.1295</td>
</tr>
<tr>
<td>$\Delta M_{B_{ud}} / 10^{-10}\text{MeV}$</td>
<td>3.9756</td>
<td>3.3370</td>
<td>0.8025</td>
<td>0.7958</td>
</tr>
<tr>
<td>$m_\mu^2 / 10^{-5}\text{eV}^2$</td>
<td>7.6111</td>
<td>7.5550</td>
<td>0.1046</td>
<td>0.5364</td>
</tr>
<tr>
<td>$m_\tau^2 / 10^{-3}\text{eV}^2$</td>
<td>2.4657</td>
<td>2.4620</td>
<td>0.0255</td>
<td>0.1455</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>0.3134</td>
<td>0.3070</td>
<td>0.1724</td>
<td>0.0370</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>0.6319</td>
<td>0.5125</td>
<td>0.9146</td>
<td>0.1305</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>0.0153</td>
<td>0.0218</td>
<td>2.0337</td>
<td>0.0092</td>
</tr>
<tr>
<td>$M_t/\text{GeV}$</td>
<td>124.5455</td>
<td>125.7000</td>
<td>0.3814</td>
<td>3.0265</td>
</tr>
<tr>
<td>$BR(B \to s\gamma)/10^{-4}$</td>
<td>2.7270</td>
<td>3.4300</td>
<td>0.5372</td>
<td>1.3087</td>
</tr>
<tr>
<td>$BR(B_s \to \mu^+\mu^-)/10^{-9}$</td>
<td>2.5215</td>
<td>2.6800</td>
<td>0.3228</td>
<td>0.8627</td>
</tr>
<tr>
<td>$BR(H_d \to \mu^+\mu^-)/10^{-10}$</td>
<td>1.0192</td>
<td>3.9000</td>
<td>1.7861</td>
<td>1.6129</td>
</tr>
<tr>
<td>$BR(B \to \tau\nu)/10^{-5}$</td>
<td>6.2272</td>
<td>11.4000</td>
<td>1.3568</td>
<td>3.8124</td>
</tr>
<tr>
<td>$BR(B \to K^\ast\mu^+\mu^-)_{\text{low } q^2}/10^{-8}$</td>
<td>4.8580</td>
<td>3.4000</td>
<td>0.2839</td>
<td>5.1361</td>
</tr>
<tr>
<td>$BR(B \to K^\ast\mu^+\mu^-)_{\text{high } q^2}/10^{-8}$</td>
<td>7.6648</td>
<td>5.6000</td>
<td>0.1415</td>
<td>14.5975</td>
</tr>
<tr>
<td>$q^2_0(AR_B(B \to K^\ast\mu^+\mu^-))/\text{GeV}^2$</td>
<td>3.7150</td>
<td>4.9000</td>
<td>0.9163</td>
<td>1.2903</td>
</tr>
<tr>
<td>$F_2(B \to K^\ast\mu^+\mu^-)_{\text{low } q^2}$</td>
<td>0.7208</td>
<td>0.6500</td>
<td>0.2103</td>
<td>0.3366</td>
</tr>
<tr>
<td>$F_2(B \to K^\ast\mu^+\mu^-)_{\text{high } q^2}$</td>
<td>0.3108</td>
<td>0.3300</td>
<td>0.0726</td>
<td>0.2644</td>
</tr>
<tr>
<td>$P_2(B \to K^\ast\mu^+\mu^-)_{\text{low } q^2}$</td>
<td>0.0335</td>
<td>0.3300</td>
<td>2.3879</td>
<td>0.1242</td>
</tr>
<tr>
<td>$P_2(B \to K^\ast\mu^+\mu^-)_{\text{high } q^2}$</td>
<td>-0.4336</td>
<td>-0.5000</td>
<td>0.3364</td>
<td>0.1974</td>
</tr>
<tr>
<td>$P_3(B \to K^\ast\mu^+\mu^-)_{\text{low } q^2}$</td>
<td>0.5697</td>
<td>0.5800</td>
<td>0.0258</td>
<td>0.3985</td>
</tr>
<tr>
<td>$P_3(B \to K^\ast\mu^+\mu^-)_{\text{high } q^2}$</td>
<td>-1.2190</td>
<td>-0.1800</td>
<td>1.7066</td>
<td>0.8198</td>
</tr>
<tr>
<td>$P_5(B \to K^\ast\mu^+\mu^-)_{\text{low } q^2}$</td>
<td>-0.4334</td>
<td>0.2100</td>
<td>2.2450</td>
<td>0.2866</td>
</tr>
<tr>
<td>$P_5(B \to K^\ast\mu^+\mu^-)_{\text{high } q^2}$</td>
<td>-0.7117</td>
<td>-0.7900</td>
<td>0.1550</td>
<td>0.5052</td>
</tr>
<tr>
<td>Total $\chi^2$</td>
<td>47.7692</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Input parameters of benchmark point C: $M_{\tilde{g}} = 1.61$ TeV

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input Param.</th>
<th>Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauge</td>
<td>$1/\alpha_G$</td>
<td>26.2</td>
</tr>
<tr>
<td></td>
<td>$M_G/10^{16}$ GeV</td>
<td>2.32</td>
</tr>
<tr>
<td></td>
<td>$\epsilon_3/%$</td>
<td>-0.651</td>
</tr>
<tr>
<td>SUSY (GUT scale)</td>
<td>$m_{16}/$TeV</td>
<td>25.0</td>
</tr>
<tr>
<td></td>
<td>$M_{1/2}/$GeV</td>
<td>450</td>
</tr>
<tr>
<td></td>
<td>$A_0/$TeV</td>
<td>-51.3</td>
</tr>
<tr>
<td></td>
<td>$(m_{H_d}/m_{16})^2$</td>
<td>1.86</td>
</tr>
<tr>
<td></td>
<td>$(m_{H_u}/m_{16})^2$</td>
<td>1.61</td>
</tr>
<tr>
<td>Neutrino</td>
<td>$M_{R_1}/10^9$ GeV</td>
<td>9.09</td>
</tr>
<tr>
<td></td>
<td>$M_{R_2}/10^{11}$ GeV</td>
<td>5.72</td>
</tr>
<tr>
<td></td>
<td>$M_{R_3}/10^{13}$ GeV</td>
<td>3.23</td>
</tr>
<tr>
<td>SUSY (EW Scale)</td>
<td>$\tan\beta$</td>
<td>50.3</td>
</tr>
<tr>
<td></td>
<td>$\mu/$GeV</td>
<td>1230</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input Param.</th>
<th>Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yukawa Textures</td>
<td>$\lambda$</td>
<td>0.610</td>
</tr>
<tr>
<td></td>
<td>$\lambda\tilde{e}$</td>
<td>0.0311</td>
</tr>
<tr>
<td></td>
<td>$\lambda\tilde{\epsilon}$</td>
<td>0.00486</td>
</tr>
<tr>
<td></td>
<td>$\lambda\tilde{e}'$</td>
<td>-0.00191</td>
</tr>
<tr>
<td></td>
<td>$\lambda\tilde{\epsilon}$</td>
<td>0.00375</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>$\rho$</td>
<td>0.0710</td>
</tr>
<tr>
<td></td>
<td>$\phi_i$/rad</td>
<td>0.563</td>
</tr>
<tr>
<td></td>
<td>$\phi_i$/rad</td>
<td>3.49</td>
</tr>
<tr>
<td></td>
<td>$\phi_i$/rad</td>
<td>0.526</td>
</tr>
<tr>
<td></td>
<td>$\phi_i$/rad</td>
<td>3.95</td>
</tr>
<tr>
<td>Mirage mediation</td>
<td>$\alpha$</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Low energy observables fit of benchmark point C

<table>
<thead>
<tr>
<th>Observable</th>
<th>Fit</th>
<th>Exp.</th>
<th>Pull</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_Z/\text{GeV}$</td>
<td>91.1876</td>
<td>91.1876</td>
<td>0.0000</td>
<td>0.4535</td>
</tr>
<tr>
<td>$M_W/\text{GeV}$</td>
<td>80.4507</td>
<td>80.3850</td>
<td>0.1633</td>
<td>0.4025</td>
</tr>
<tr>
<td>$v_{\mu}/\text{GeV}$</td>
<td>137.7125</td>
<td>0.0073</td>
<td>0.9825</td>
<td>0.6886</td>
</tr>
<tr>
<td>$G_\mu/10^{-5}\text{ GeV}^2$</td>
<td>1.1732</td>
<td>1.1664</td>
<td>0.5798</td>
<td>0.0117</td>
</tr>
<tr>
<td>$\alpha_s(M_Z)$</td>
<td>0.1188</td>
<td>0.1185</td>
<td>0.4140</td>
<td>0.0008</td>
</tr>
<tr>
<td>$M_t/\text{GeV}$</td>
<td>174.1882</td>
<td>173.2100</td>
<td>0.7927</td>
<td>1.2340</td>
</tr>
<tr>
<td>$m_t(m_t)/\text{GeV}$</td>
<td>4.1954</td>
<td>4.1800</td>
<td>0.4220</td>
<td>0.0366</td>
</tr>
<tr>
<td>$M_s/\text{GeV}$</td>
<td>1.7781</td>
<td>1.7768</td>
<td>0.1417</td>
<td>0.0089</td>
</tr>
<tr>
<td>$(M_s - M_t)/\text{GeV}$</td>
<td>3.1568</td>
<td>3.4500</td>
<td>0.9175</td>
<td>0.3196</td>
</tr>
<tr>
<td>$m_e(m_e)/\text{GeV}$</td>
<td>1.2595</td>
<td>1.2750</td>
<td>0.5993</td>
<td>0.0258</td>
</tr>
<tr>
<td>$m_e(2\text{GeV})/\text{GeV}$</td>
<td>0.0939</td>
<td>0.0950</td>
<td>0.2147</td>
<td>0.0050</td>
</tr>
<tr>
<td>$m_d(m_s(2\text{GeV})$</td>
<td>0.0701</td>
<td>0.0513</td>
<td>2.8052</td>
<td>0.0067</td>
</tr>
<tr>
<td>$1/Q^2$</td>
<td>0.0018</td>
<td>0.0019</td>
<td>0.5319</td>
<td>0.0001</td>
</tr>
<tr>
<td>$M_s/\text{GeV}$</td>
<td>0.1056</td>
<td>0.1057</td>
<td>0.1818</td>
<td>0.0005</td>
</tr>
<tr>
<td>$M_t/10^{-4}\text{GeV}$</td>
<td>5.1145</td>
<td>5.1100</td>
<td>0.1749</td>
<td>0.0256</td>
</tr>
<tr>
<td>$</td>
<td>V_{us}</td>
<td>$</td>
<td>0.2244</td>
<td>0.2253</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>$</td>
<td>0.0404</td>
<td>0.0408</td>
</tr>
<tr>
<td>$</td>
<td>V_{cb}</td>
<td>/10^{-3}$</td>
<td>3.1033</td>
<td>3.8500</td>
</tr>
<tr>
<td>$</td>
<td>V_{td}</td>
<td>/10^{-3}$</td>
<td>8.8101</td>
<td>8.4000</td>
</tr>
<tr>
<td>$</td>
<td>V_{ts}</td>
<td>$</td>
<td>0.0396</td>
<td>0.0400</td>
</tr>
<tr>
<td>$\sin(2\beta)$</td>
<td>0.6270</td>
<td>0.6820</td>
<td>2.8562</td>
<td>0.0193</td>
</tr>
<tr>
<td>c_K</td>
<td>0.0022</td>
<td>0.0022</td>
<td>0.2052</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\Delta M_{B_s}/\Delta M_{B_d}$</td>
<td>35.3739</td>
<td>35.0345</td>
<td>0.0479</td>
<td>7.0854</td>
</tr>
<tr>
<td>$\Delta M_{B_s}/10^{-10}\text{MeV}$</td>
<td>3.9433</td>
<td>3.3370</td>
<td>0.7681</td>
<td>0.7894</td>
</tr>
<tr>
<td>$m_b^2/10^{-5}\text{ eV}^2$</td>
<td>7.6562</td>
<td>7.5550</td>
<td>0.1886</td>
<td>0.5364</td>
</tr>
<tr>
<td>$m_d^2/10^{-3}\text{ eV}^2$</td>
<td>2.4631</td>
<td>2.4620</td>
<td>0.0077</td>
<td>0.1455</td>
</tr>
<tr>
<td>$\sin^2\theta_{12}$</td>
<td>0.3170</td>
<td>0.3070</td>
<td>0.2689</td>
<td>0.0370</td>
</tr>
<tr>
<td>$\sin^2\theta_{23}$</td>
<td>0.6264</td>
<td>0.5125</td>
<td>0.8722</td>
<td>0.1305</td>
</tr>
<tr>
<td>$\sin^2\theta_{13}$</td>
<td>0.0149</td>
<td>0.0218</td>
<td>2.1658</td>
<td>0.0032</td>
</tr>
<tr>
<td>$M_s/\text{GeV}$</td>
<td>124.5054</td>
<td>125.7000</td>
<td>0.3947</td>
<td>3.0265</td>
</tr>
<tr>
<td>$BR(B \to s\gamma)/10^{-4}$</td>
<td>2.6840</td>
<td>3.4300</td>
<td>0.5789</td>
<td>1.2887</td>
</tr>
<tr>
<td>$BR(B_s \to \mu^+\mu^-)/10^{-9}$</td>
<td>3.0247</td>
<td>2.8000</td>
<td>0.2429</td>
<td>0.9522</td>
</tr>
<tr>
<td>$BR(B_d \to \mu^+\mu^-)/10^{-10}$</td>
<td>1.1022</td>
<td>3.9000</td>
<td>1.7323</td>
<td>1.6151</td>
</tr>
<tr>
<td>$BR(B \to \tau\nu)/10^{-5}$</td>
<td>6.1884</td>
<td>11.4000</td>
<td>1.3727</td>
<td>3.7966</td>
</tr>
<tr>
<td>$BR(B \to K^+\mu^+\mu^-)/10^{-8}$</td>
<td>4.7640</td>
<td>3.4000</td>
<td>0.2707</td>
<td>5.0381</td>
</tr>
<tr>
<td>$BR(B \to K^0\mu^+\mu^-)/10^{-8}$</td>
<td>7.5110</td>
<td>5.6000</td>
<td>0.1336</td>
<td>14.3059</td>
</tr>
<tr>
<td>$q_B^0(B \to K^+\mu^+\mu^-)/\text{GeV}^2$</td>
<td>3.6690</td>
<td>4.9000</td>
<td>0.9579</td>
<td>1.2850</td>
</tr>
<tr>
<td>$F_L(B \to K^0\mu^+\mu^-)/\text{GeV}^2$</td>
<td>0.7225</td>
<td>0.6500</td>
<td>0.2149</td>
<td>0.3374</td>
</tr>
<tr>
<td>$F_L(B \to K^+\mu^+\mu^-)/\text{GeV}^2$</td>
<td>0.3108</td>
<td>0.3300</td>
<td>0.0726</td>
<td>0.2644</td>
</tr>
<tr>
<td>$F_0(B \to K^0\mu^+\mu^-)/\text{GeV}^2$</td>
<td>0.0228</td>
<td>0.3300</td>
<td>2.5196</td>
<td>0.1219</td>
</tr>
<tr>
<td>$F_0(B \to K^+\mu^+\mu^-)/\text{GeV}^2$</td>
<td>-0.4336</td>
<td>-0.5000</td>
<td>0.3364</td>
<td>0.1974</td>
</tr>
<tr>
<td>$F_0'(B \to K^0\mu^+\mu^-)/\text{GeV}^2$</td>
<td>0.5820</td>
<td>0.5800</td>
<td>0.0050</td>
<td>0.4001</td>
</tr>
<tr>
<td>$F_0'(B \to K^+\mu^+\mu^-)/\text{GeV}^2$</td>
<td>1.2190</td>
<td>-0.1800</td>
<td>1.7066</td>
<td>0.8198</td>
</tr>
<tr>
<td>$F_3(B \to K^0\mu^+\mu^-)/\text{GeV}^2$</td>
<td>-0.4455</td>
<td>0.2100</td>
<td>2.2578</td>
<td>0.2903</td>
</tr>
<tr>
<td>$F_3(B \to K^+\mu^+\mu^-)/\text{GeV}^2$</td>
<td>-0.7116</td>
<td>-0.7900</td>
<td>0.1552</td>
<td>0.5052</td>
</tr>
<tr>
<td>Total $\chi^2$</td>
<td>48.8413</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

202
Input parameters of benchmark point D: $M_g = 1.69 \text{TeV}$

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input Param.</th>
<th>Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauge</td>
<td>$1/\alpha_G$</td>
<td>26.4</td>
</tr>
<tr>
<td></td>
<td>$M_G/10^{16}\text{GeV}$</td>
<td>2.09</td>
</tr>
<tr>
<td></td>
<td>$\epsilon_3/%$</td>
<td>0.0193</td>
</tr>
<tr>
<td>SUSY (GUT scale)</td>
<td>$m_{16}/\text{TeV}$</td>
<td>25.0</td>
</tr>
<tr>
<td></td>
<td>$M_{1/2}/\text{GeV}$</td>
<td>900</td>
</tr>
<tr>
<td></td>
<td>$A_0/\text{TeV}$</td>
<td>-50.8</td>
</tr>
<tr>
<td></td>
<td>$(m_{H_d}/m_{16})^2$</td>
<td>1.86</td>
</tr>
<tr>
<td></td>
<td>$(m_{H_u}/m_{16})^2$</td>
<td>1.60</td>
</tr>
<tr>
<td>Neutrino</td>
<td>$M_{R_1}/10^9\text{GeV}$</td>
<td>9.12</td>
</tr>
<tr>
<td></td>
<td>$M_{R_2}/10^{11}\text{GeV}$</td>
<td>5.79</td>
</tr>
<tr>
<td></td>
<td>$M_{R_3}/10^{13}\text{GeV}$</td>
<td>3.24</td>
</tr>
<tr>
<td>SUSY (EW Scale)</td>
<td>$\tan\beta$</td>
<td>50.3</td>
</tr>
<tr>
<td></td>
<td>$\mu/\text{GeV}$</td>
<td>1530</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input Param.</th>
<th>Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yukawa Textures</td>
<td>$\lambda$</td>
<td>0.610</td>
</tr>
<tr>
<td></td>
<td>$\lambda\epsilon\hat{ }$</td>
<td>0.0311</td>
</tr>
<tr>
<td></td>
<td>$\lambda\hat{ }$</td>
<td>0.00486</td>
</tr>
<tr>
<td></td>
<td>$\lambda\epsilon\hat{ }$</td>
<td>-0.00192</td>
</tr>
<tr>
<td></td>
<td>$\lambda\epsilon\xi$</td>
<td>0.00373</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>$\rho$</td>
<td>0.0708</td>
</tr>
<tr>
<td></td>
<td>$\phi_z/\text{rad}$</td>
<td>0.567</td>
</tr>
<tr>
<td></td>
<td>$\phi_z/\text{rad}$</td>
<td>3.49</td>
</tr>
<tr>
<td></td>
<td>$\phi_A/\text{rad}$</td>
<td>0.522</td>
</tr>
<tr>
<td></td>
<td>$\phi_A/\text{rad}$</td>
<td>3.96</td>
</tr>
<tr>
<td>Mirage mediation</td>
<td>$\alpha$</td>
<td>1.5</td>
</tr>
</tbody>
</table>

203
Low energy observables fit of benchmark point D

<table>
<thead>
<tr>
<th>Observable</th>
<th>Fit</th>
<th>Exp.</th>
<th>Pull</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_Z$/GeV</td>
<td>91.1876</td>
<td>91.1876</td>
<td>0.0000</td>
<td>0.4540</td>
</tr>
<tr>
<td>$M_W$/GeV</td>
<td>80.4655</td>
<td>80.3850</td>
<td>0.2000</td>
<td>0.4026</td>
</tr>
<tr>
<td>$1/\alpha_{em}$</td>
<td>137.7323</td>
<td>0.0073</td>
<td>1.0111</td>
<td>0.6887</td>
</tr>
<tr>
<td>$G_\mu/10^{-5}$ GeV$^2$</td>
<td>1.1740</td>
<td>1.1664</td>
<td>0.6469</td>
<td>0.0117</td>
</tr>
<tr>
<td>$\alpha_3(M_Z)$</td>
<td>0.1188</td>
<td>0.1185</td>
<td>0.2597</td>
<td>0.0008</td>
</tr>
<tr>
<td>$M_t$/GeV</td>
<td>174.3427</td>
<td>173.2100</td>
<td>0.9175</td>
<td>1.2345</td>
</tr>
<tr>
<td>$m_0(m_0)$/GeV</td>
<td>4.2001</td>
<td>4.1800</td>
<td>0.5479</td>
<td>0.0366</td>
</tr>
<tr>
<td>$M_0$/GeV</td>
<td>1.7774</td>
<td>1.7768</td>
<td>0.0644</td>
<td>0.0089</td>
</tr>
<tr>
<td>$(M_b-M_L)$/GeV</td>
<td>3.1659</td>
<td>3.4500</td>
<td>0.8863</td>
<td>0.3205</td>
</tr>
<tr>
<td>$m_e(m_e)$/GeV</td>
<td>1.2574</td>
<td>1.2750</td>
<td>0.6825</td>
<td>0.0258</td>
</tr>
<tr>
<td>$m_0(2$GeV$)/GeV$</td>
<td>0.0936</td>
<td>0.0950</td>
<td>0.2741</td>
<td>0.0050</td>
</tr>
<tr>
<td>$m_0(2$GeV$)/m_0$</td>
<td>0.0701</td>
<td>0.0513</td>
<td>2.8082</td>
<td>0.0067</td>
</tr>
<tr>
<td>$1/Q^2$</td>
<td>0.0018</td>
<td>0.0019</td>
<td>0.5170</td>
<td>0.0001</td>
</tr>
<tr>
<td>$M_0$/GeV</td>
<td>0.1656</td>
<td>0.1057</td>
<td>0.1571</td>
<td>0.0005</td>
</tr>
<tr>
<td>$M_0/10^{-4}$GeV</td>
<td>5.1139</td>
<td>5.1100</td>
<td>0.1545</td>
<td>0.0256</td>
</tr>
<tr>
<td>$[V_{es}]$</td>
<td>0.2244</td>
<td>0.2253</td>
<td>0.6688</td>
<td>0.0014</td>
</tr>
<tr>
<td>$[V_{es}]$</td>
<td>0.0400</td>
<td>0.0408</td>
<td>0.3609</td>
<td>0.0021</td>
</tr>
<tr>
<td>$[V_{es}]/10^{-3}$</td>
<td>3.0662</td>
<td>3.8500</td>
<td>0.9113</td>
<td>0.8601</td>
</tr>
<tr>
<td>$[V_{us}]$</td>
<td>8.7156</td>
<td>8.4000</td>
<td>0.5247</td>
<td>0.6016</td>
</tr>
<tr>
<td>$[V_{us}]$</td>
<td>0.0392</td>
<td>0.0400</td>
<td>0.2960</td>
<td>0.0027</td>
</tr>
<tr>
<td>$\sin(2\beta)$</td>
<td>0.6259</td>
<td>0.6820</td>
<td>2.9122</td>
<td>0.0193</td>
</tr>
<tr>
<td>$\sin^2\theta_{12}$</td>
<td>0.0022</td>
<td>0.0022</td>
<td>0.0034</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\sin^2\theta_{23}$</td>
<td>0.6197</td>
<td>0.5125</td>
<td>0.8210</td>
<td>0.1305</td>
</tr>
<tr>
<td>$\sin^2\theta_{13}$</td>
<td>0.0146</td>
<td>0.0218</td>
<td>2.2520</td>
<td>0.0032</td>
</tr>
<tr>
<td>$M_0$/GeV</td>
<td>122.0502</td>
<td>125.7000</td>
<td>1.2059</td>
<td>3.0265</td>
</tr>
<tr>
<td>$BR(B\to s\gamma)/10^{-4}$</td>
<td>2.6310</td>
<td>3.4300</td>
<td>0.6321</td>
<td>1.2640</td>
</tr>
<tr>
<td>$BR(B_s\to \mu^+\mu^-)/10^{-9}$</td>
<td>3.5145</td>
<td>2.8000</td>
<td>0.7203</td>
<td>0.9920</td>
</tr>
<tr>
<td>$BR(B_d\to \mu^+\mu^-)/10^{-10}$</td>
<td>1.0522</td>
<td>3.9000</td>
<td>1.7647</td>
<td>1.6138</td>
</tr>
<tr>
<td>$BR(B\to \tau\nu)/10^{-5}$</td>
<td>6.1099</td>
<td>11.4000</td>
<td>1.4090</td>
<td>3.7610</td>
</tr>
<tr>
<td>$BR(B\to K^0\mu^+\mu^-)</td>
<td>_{low q^2}/10^{-8}$</td>
<td>4.6780</td>
<td>3.4000</td>
<td>0.2583</td>
</tr>
<tr>
<td>$BR(B\to K^0\mu^+\mu^-)</td>
<td>_{high q^2}/10^{-8}$</td>
<td>7.4066</td>
<td>5.6000</td>
<td>0.1281</td>
</tr>
<tr>
<td>$q^2(B\to K\mu^+\mu^-)/GeV^2$</td>
<td>3.6290</td>
<td>4.9000</td>
<td>0.9946</td>
<td>1.2779</td>
</tr>
<tr>
<td>$F_0(B\to K^0\mu^+\mu^-)</td>
<td>_{low q^2}$</td>
<td>0.7240</td>
<td>0.6500</td>
<td>0.2189</td>
</tr>
<tr>
<td>$P_0(B\to K^0\mu^+\mu^-)</td>
<td>_{low q^2}$</td>
<td>0.3108</td>
<td>0.3300</td>
<td>0.0726</td>
</tr>
<tr>
<td>$P_0(B\to K^0\mu^+\mu^-)</td>
<td>_{low q^2}$</td>
<td>0.0132</td>
<td>0.3300</td>
<td>2.6254</td>
</tr>
<tr>
<td>$P_0(B\to K^0\mu^+\mu^-)</td>
<td>_{high q^2}$</td>
<td>-0.4337</td>
<td>-0.5000</td>
<td>0.3358</td>
</tr>
<tr>
<td>$P_0(B\to K^0\mu^+\mu^-)</td>
<td>_{high q^2}$</td>
<td>0.5918</td>
<td>0.5800</td>
<td>0.0294</td>
</tr>
<tr>
<td>$P_0(B\to K^0\mu^+\mu^-)</td>
<td>_{high q^2}$</td>
<td>1.2190</td>
<td>-0.1800</td>
<td>1.7066</td>
</tr>
<tr>
<td>$P_0(B\to K^0\mu^+\mu^-)</td>
<td>_{high q^2}$</td>
<td>-0.4562</td>
<td>0.2100</td>
<td>2.2685</td>
</tr>
<tr>
<td>$P_0(B\to K^0\mu^+\mu^-)</td>
<td>_{high q^2}$</td>
<td>-0.7117</td>
<td>-0.7900</td>
<td>0.1550</td>
</tr>
<tr>
<td>Total $\chi^2$</td>
<td>52.6056</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
H.2 Pati-Salam SUSY GUT Benchmark Points

In this appendix, we list the benchmark points for the PS SUSY GUT model considered in Chapter [4].

Input parameters of benchmark point A: $M_g = 2.00$ TeV

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input Param.</th>
<th>Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauge</td>
<td>$1/\alpha_G$</td>
<td>26.0</td>
</tr>
<tr>
<td></td>
<td>$M_G/10^{16}$ GeV</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>$\epsilon_3/%$</td>
<td>-1.68</td>
</tr>
<tr>
<td>SUSY (GUT scale)</td>
<td>$m_{16}/$TeV</td>
<td>20.0</td>
</tr>
<tr>
<td></td>
<td>$m_{1/2}/$GeV</td>
<td>660</td>
</tr>
<tr>
<td></td>
<td>$A_0$/TeV</td>
<td>-40.6</td>
</tr>
<tr>
<td></td>
<td>$(m_{H_d}/m_{16})^2$</td>
<td>1.98</td>
</tr>
<tr>
<td></td>
<td>$(m_{H_u}/m_{16})^2$</td>
<td>1.61</td>
</tr>
<tr>
<td>Neutrino</td>
<td>$M_{R_1}/10^9$ GeV</td>
<td>4.62</td>
</tr>
<tr>
<td></td>
<td>$M_{R_2}/10^{11}$ GeV</td>
<td>8.32</td>
</tr>
<tr>
<td></td>
<td>$M_{R_3}/10^{13}$ GeV</td>
<td>4.71</td>
</tr>
<tr>
<td>SUSY (EW Scale)</td>
<td>$\tan \beta$</td>
<td>50.4</td>
</tr>
<tr>
<td></td>
<td>$\mu$/GeV</td>
<td>630</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input Param.</th>
<th>Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yukawa Textures</td>
<td>$\lambda$</td>
<td>0.617</td>
</tr>
<tr>
<td></td>
<td>$\lambda \epsilon$</td>
<td>0.0326</td>
</tr>
<tr>
<td></td>
<td>$\lambda \tilde{\epsilon}$</td>
<td>0.0100</td>
</tr>
<tr>
<td></td>
<td>$\lambda \epsilon'$</td>
<td>-0.00300</td>
</tr>
<tr>
<td></td>
<td>$\lambda \xi$</td>
<td>0.00201</td>
</tr>
<tr>
<td></td>
<td>$\alpha$</td>
<td>0.138</td>
</tr>
<tr>
<td></td>
<td>$\theta'/10^{-5}$</td>
<td>5.03</td>
</tr>
<tr>
<td></td>
<td>$\theta/10^{-5}$</td>
<td>2.92</td>
</tr>
<tr>
<td></td>
<td>$\phi_{\epsilon}/$rad</td>
<td>-0.277</td>
</tr>
<tr>
<td></td>
<td>$\phi_{\xi}$/rad</td>
<td>3.41</td>
</tr>
<tr>
<td></td>
<td>$\phi_{\alpha}$/rad</td>
<td>0.963</td>
</tr>
<tr>
<td></td>
<td>$\phi_{\beta}$/rad</td>
<td>-1.26</td>
</tr>
</tbody>
</table>
Low energy observables fit of benchmark point A

<table>
<thead>
<tr>
<th>Observable</th>
<th>Fit</th>
<th>Exp.</th>
<th>Pull</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{B}/\text{GeV}$</td>
<td>91.1876</td>
<td>91.1876</td>
<td>0.0000</td>
<td>0.4514</td>
</tr>
<tr>
<td>$M_{B}/\text{GeV}$</td>
<td>80.4734</td>
<td>80.3850</td>
<td>0.2238</td>
<td>0.3949</td>
</tr>
<tr>
<td>$V_{ub}$/GeV</td>
<td>137.3435</td>
<td>137.0360</td>
<td>0.4478</td>
<td>0.6867</td>
</tr>
<tr>
<td>$G_{B}/10^{-5}\text{ GeV}^{-2}$</td>
<td>1.1761</td>
<td>1.1664</td>
<td>0.8264</td>
<td>0.0118</td>
</tr>
<tr>
<td>$a_{s}(M_{Z})$</td>
<td>0.1177</td>
<td>0.1181</td>
<td>0.4791</td>
<td>0.0008</td>
</tr>
<tr>
<td>$M_{t}/\text{GeV}$</td>
<td>174.6978</td>
<td>173.2100</td>
<td>0.4161</td>
<td>2.1338</td>
</tr>
<tr>
<td>$m_{b}(m_{b})/\text{GeV}$</td>
<td>4.3264</td>
<td>4.1850</td>
<td>1.0388</td>
<td>0.1362</td>
</tr>
<tr>
<td>$M_{t}/\text{MeV}$</td>
<td>1776.0100</td>
<td>1776.8600</td>
<td>0.0428</td>
<td>19.8568</td>
</tr>
<tr>
<td>$(M_{B} - M_{S})/\text{GeV}$</td>
<td>3.3028</td>
<td>3.4500</td>
<td>0.4098</td>
<td>0.3592</td>
</tr>
<tr>
<td>$m_{c}(m_{c})/\text{GeV}$</td>
<td>1.2685</td>
<td>1.2700</td>
<td>0.0142</td>
<td>0.0332</td>
</tr>
<tr>
<td>$m_{s}(2\text{GeV})/\text{MeV}$</td>
<td>97.7602</td>
<td>98.0000</td>
<td>0.0393</td>
<td>0.6987</td>
</tr>
<tr>
<td>$m_{s}(m_{s})/2\text{GeV}$</td>
<td>18.5602</td>
<td>19.5000</td>
<td>0.3843</td>
<td>2.0519</td>
</tr>
<tr>
<td>$Q$</td>
<td>21.5785</td>
<td>23.0000</td>
<td>0.6256</td>
<td>2.2725</td>
</tr>
<tr>
<td>$m_{s}(2\text{GeV})/\text{MeV}$</td>
<td>2.6880</td>
<td>2.3000</td>
<td>0.7758</td>
<td>0.5002</td>
</tr>
<tr>
<td>$m_{s}(2\text{GeV})/\text{MeV}$</td>
<td>5.2646</td>
<td>4.7500</td>
<td>1.1417</td>
<td>0.4508</td>
</tr>
<tr>
<td>$M_{S}/\text{MeV}$</td>
<td>105.2131</td>
<td>105.6584</td>
<td>0.2053</td>
<td>2.1690</td>
</tr>
<tr>
<td>$M_{S}/\text{MeV}$</td>
<td>0.5108</td>
<td>0.5110</td>
<td>0.0278</td>
<td>0.0057</td>
</tr>
<tr>
<td>$</td>
<td>V_{us}</td>
<td>$</td>
<td>0.9745</td>
<td>0.9742</td>
</tr>
<tr>
<td>$</td>
<td>V_{us}</td>
<td>$</td>
<td>0.2245</td>
<td>0.2248</td>
</tr>
<tr>
<td>$</td>
<td>V_{us}</td>
<td>/10^{-3}$</td>
<td>3.9904</td>
<td>4.1300</td>
</tr>
<tr>
<td>$</td>
<td>V_{us}</td>
<td>$</td>
<td>0.2244</td>
<td>0.2200</td>
</tr>
<tr>
<td>$</td>
<td>V_{us}</td>
<td>$</td>
<td>0.9735</td>
<td>0.9950</td>
</tr>
<tr>
<td>$</td>
<td>V_{us}</td>
<td>/10^{-3}$</td>
<td>44.1574</td>
<td>40.7500</td>
</tr>
<tr>
<td>$</td>
<td>V_{us}</td>
<td>/10^{-3}$</td>
<td>7.9898</td>
<td>8.2000</td>
</tr>
<tr>
<td>$</td>
<td>V_{us}</td>
<td>/10^{-3}$</td>
<td>43.6115</td>
<td>40.0000</td>
</tr>
<tr>
<td>$</td>
<td>V_{us}</td>
<td>$</td>
<td>0.9990</td>
<td>1.0090</td>
</tr>
<tr>
<td>$\sin(2\beta)$</td>
<td>0.6922</td>
<td>0.6910</td>
<td>0.0672</td>
<td>0.0173</td>
</tr>
<tr>
<td>$\rho_K/10^{-3}$</td>
<td>2.0225</td>
<td>2.2330</td>
<td>1.0379</td>
<td>0.2028</td>
</tr>
<tr>
<td>$\Delta M_{B_0}/\Delta M_{B_0}$</td>
<td>43.7269</td>
<td>34.8479</td>
<td>1.0037</td>
<td>8.8463</td>
</tr>
<tr>
<td>$\Delta M_{B_0}/10^{-10}\text{MeV}$</td>
<td>2.9005</td>
<td>3.3540</td>
<td>0.7802</td>
<td>0.5812</td>
</tr>
<tr>
<td>$m_s^{2}/10^{-3}\text{eV}^2$</td>
<td>7.3484</td>
<td>7.3750</td>
<td>0.6568</td>
<td>0.4044</td>
</tr>
<tr>
<td>$m_d^{2}/10^{-3}\text{eV}^2$</td>
<td>2.0596</td>
<td>2.5000</td>
<td>0.0726</td>
<td>0.1323</td>
</tr>
<tr>
<td>$\sin^2\theta_{12}$</td>
<td>0.2960</td>
<td>0.2975</td>
<td>0.0915</td>
<td>0.0166</td>
</tr>
<tr>
<td>$\sin^2\theta_{23}$</td>
<td>0.4419</td>
<td>0.4435</td>
<td>0.0599</td>
<td>0.0266</td>
</tr>
<tr>
<td>$\sin^2\theta_{13}$</td>
<td>0.0217</td>
<td>0.0215</td>
<td>0.1493</td>
<td>0.0010</td>
</tr>
<tr>
<td>$M_{B}/\text{GeV}$</td>
<td>122.7975</td>
<td>125.0900</td>
<td>0.4854</td>
<td>4.7225</td>
</tr>
<tr>
<td>$BR(B \rightarrow \tau \tau)/10^{-6}$</td>
<td>299.9500</td>
<td>332.0000</td>
<td>0.2243</td>
<td>142.9017</td>
</tr>
<tr>
<td>$BR(B_s \rightarrow \mu^+ \mu^-)/10^{-9}$</td>
<td>5.1836</td>
<td>2.9500</td>
<td>1.6808</td>
<td>1.3289</td>
</tr>
<tr>
<td>$BR(B_s \rightarrow \mu^+ \mu^-)/10^{-9}$</td>
<td>0.1223</td>
<td>0.4000</td>
<td>1.8234</td>
<td>0.1523</td>
</tr>
<tr>
<td>$BR(B \rightarrow \tau \tau)/10^{-6}$</td>
<td>96.4905</td>
<td>106.0000</td>
<td>0.1822</td>
<td>52.1761</td>
</tr>
<tr>
<td>$BR(B \rightarrow K^\ast \mu^+ \mu^-)/10^{-7}$</td>
<td>0.5456</td>
<td>0.3400</td>
<td>0.3567</td>
<td>0.5765</td>
</tr>
<tr>
<td>$BR(B \rightarrow K^\ast \mu^+ \mu^-)/10^{-7}$</td>
<td>0.7904</td>
<td>0.5600</td>
<td>0.1531</td>
<td>1.5055</td>
</tr>
<tr>
<td>$\eta_{K}(K \rightarrow \mu^+ \mu^-))/\text{GeV}^2$</td>
<td>3.8492</td>
<td>4.9000</td>
<td>0.7921</td>
<td>3.2458</td>
</tr>
<tr>
<td>$F_{L}(B \rightarrow K^\ast \mu^+ \mu^-)/\text{GeV}^2$</td>
<td>0.7522</td>
<td>0.6500</td>
<td>0.2917</td>
<td>0.3503</td>
</tr>
<tr>
<td>$F_{L}(B \rightarrow K^\ast \mu^+ \mu^-)/\text{GeV}^2$</td>
<td>0.3514</td>
<td>0.3300</td>
<td>0.0725</td>
<td>0.2952</td>
</tr>
<tr>
<td>$F_{K}(B \rightarrow K^\ast \mu^+ \mu^-)/\text{GeV}^2$</td>
<td>0.0679</td>
<td>0.3300</td>
<td>1.4536</td>
<td>0.1803</td>
</tr>
<tr>
<td>$P_{K}(B \rightarrow K^\ast \mu^+ \mu^-)/\text{GeV}^2$</td>
<td>-0.4333</td>
<td>-0.5000</td>
<td>0.3381</td>
<td>0.1973</td>
</tr>
<tr>
<td>$P_{K}(B \rightarrow K^\ast \mu^+ \mu^-)/\text{GeV}^2$</td>
<td>0.5788</td>
<td>0.5800</td>
<td>0.0029</td>
<td>0.4007</td>
</tr>
<tr>
<td>$P_{K}(B \rightarrow K^\ast \mu^+ \mu^-)/\text{GeV}^2$</td>
<td>1.2177</td>
<td>-0.1800</td>
<td>1.7055</td>
<td>0.8195</td>
</tr>
<tr>
<td>$P_{K}(B \rightarrow K^\ast \mu^+ \mu^-)/\text{GeV}^2$</td>
<td>-0.3221</td>
<td>0.2100</td>
<td>2.0721</td>
<td>0.2568</td>
</tr>
<tr>
<td>$P_{K}(B \rightarrow K^\ast \mu^+ \mu^-)/\text{GeV}^2$</td>
<td>-0.7119</td>
<td>-0.7900</td>
<td>0.1545</td>
<td>0.5053</td>
</tr>
<tr>
<td>Total $\chi^2$</td>
<td>30.9601</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Input parameters of benchmark point B: $M_g = 2.00 \text{ TeV}$

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input Param.</th>
<th>Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauge</td>
<td>$1/\alpha_G$</td>
<td>26.0</td>
</tr>
<tr>
<td></td>
<td>$M_G/10^{16} \text{ GeV}$</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>$\epsilon_3/%$</td>
<td>-1.70</td>
</tr>
<tr>
<td>SUSY (GUT scale)</td>
<td>$m_{16}/\text{TeV}$</td>
<td>25.0</td>
</tr>
<tr>
<td></td>
<td>$m_{1/2}/\text{GeV}$</td>
<td>620</td>
</tr>
<tr>
<td></td>
<td>$A_0/\text{TeV}$</td>
<td>-51.0</td>
</tr>
<tr>
<td></td>
<td>$(m_{H_d}/m_{16})^2$</td>
<td>1.97</td>
</tr>
<tr>
<td></td>
<td>$(m_{H_u}/m_{16})^2$</td>
<td>1.61</td>
</tr>
<tr>
<td>Neutrino</td>
<td>$M_{R_1}/10^9 \text{ GeV}$</td>
<td>4.66</td>
</tr>
<tr>
<td></td>
<td>$M_{R_2}/10^{11} \text{ GeV}$</td>
<td>8.28</td>
</tr>
<tr>
<td></td>
<td>$M_{R_3}/10^{13} \text{ GeV}$</td>
<td>4.70</td>
</tr>
<tr>
<td>SUSY (EW Scale)</td>
<td>$\tan \beta$</td>
<td>50.5</td>
</tr>
<tr>
<td></td>
<td>$\mu/\text{GeV}$</td>
<td>704</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input Param.</th>
<th>Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yukawa Textures</td>
<td>$\lambda$</td>
<td>0.618</td>
</tr>
<tr>
<td></td>
<td>$\lambda \epsilon$</td>
<td>0.0326</td>
</tr>
<tr>
<td></td>
<td>$\lambda \epsilon'$</td>
<td>-0.00300</td>
</tr>
<tr>
<td></td>
<td>$\lambda \xi$</td>
<td>0.00202</td>
</tr>
<tr>
<td></td>
<td>$\alpha$</td>
<td>0.138</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>0.0280</td>
</tr>
<tr>
<td></td>
<td>$\theta'/10^{-5}$</td>
<td>4.99</td>
</tr>
<tr>
<td></td>
<td>$\theta/10^{-5}$</td>
<td>2.89</td>
</tr>
<tr>
<td></td>
<td>$\phi_\epsilon/\text{rad}$</td>
<td>-0.266</td>
</tr>
<tr>
<td></td>
<td>$\phi_\xi/\text{rad}$</td>
<td>3.40</td>
</tr>
<tr>
<td></td>
<td>$\phi_\alpha/\text{rad}$</td>
<td>0.962</td>
</tr>
<tr>
<td></td>
<td>$\phi_\beta/\text{rad}$</td>
<td>-1.27</td>
</tr>
</tbody>
</table>
### Input parameters of benchmark point C: $M_{\tilde{g}} = 2.60$ TeV

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input Param.</th>
<th>Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauge</td>
<td>$1/\alpha_G$</td>
<td>26.1</td>
</tr>
<tr>
<td></td>
<td>$M_G/10^{16}$ GeV</td>
<td>2.16</td>
</tr>
<tr>
<td></td>
<td>$\epsilon_3/%$</td>
<td>-1.65</td>
</tr>
<tr>
<td>SUSY (GUT scale)</td>
<td>$m_{16}/$TeV</td>
<td>20.0</td>
</tr>
<tr>
<td></td>
<td>$m_{1/2}/$GeV</td>
<td>940</td>
</tr>
<tr>
<td></td>
<td>$A_0/$TeV</td>
<td>-40.5</td>
</tr>
<tr>
<td></td>
<td>$(m_{H_d}/m_{16})^2$</td>
<td>1.99</td>
</tr>
<tr>
<td></td>
<td>$(m_{H_u}/m_{16})^2$</td>
<td>1.61</td>
</tr>
<tr>
<td>Neutrino</td>
<td>$M_{R_i}/10^9$ GeV</td>
<td>4.65</td>
</tr>
<tr>
<td></td>
<td>$M_{R_2}/10^{11}$ GeV</td>
<td>8.42</td>
</tr>
<tr>
<td></td>
<td>$M_{R_3}/10^{13}$ GeV</td>
<td>4.75</td>
</tr>
<tr>
<td>SUSY (EW Scale)</td>
<td>$\tan \beta$</td>
<td>50.3</td>
</tr>
<tr>
<td></td>
<td>$\mu$/GeV</td>
<td>650</td>
</tr>
</tbody>
</table>

### Input parameters of benchmark point D: $M_{\tilde{g}} = 2.60$ TeV

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input Param.</th>
<th>Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauge</td>
<td>$1/\alpha_G$</td>
<td>26.2</td>
</tr>
<tr>
<td></td>
<td>$M_G/10^{16}$ GeV</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>$\epsilon_3/%$</td>
<td>-1.21</td>
</tr>
<tr>
<td>SUSY (GUT scale)</td>
<td>$m_{16}/$TeV</td>
<td>25.0</td>
</tr>
<tr>
<td></td>
<td>$m_{1/2}/$GeV</td>
<td>895</td>
</tr>
<tr>
<td></td>
<td>$A_0/$TeV</td>
<td>-50.9</td>
</tr>
<tr>
<td></td>
<td>$(m_{H_d}/m_{16})^2$</td>
<td>1.98</td>
</tr>
<tr>
<td></td>
<td>$(m_{H_u}/m_{16})^2$</td>
<td>1.61</td>
</tr>
<tr>
<td>Neutrino</td>
<td>$M_{R_i}/10^9$ GeV</td>
<td>4.83</td>
</tr>
<tr>
<td></td>
<td>$M_{R_2}/10^{11}$ GeV</td>
<td>8.40</td>
</tr>
<tr>
<td></td>
<td>$M_{R_3}/10^{13}$ GeV</td>
<td>4.79</td>
</tr>
<tr>
<td>SUSY (EW Scale)</td>
<td>$\tan \beta$</td>
<td>50.5</td>
</tr>
<tr>
<td></td>
<td>$\mu$/GeV</td>
<td>673</td>
</tr>
</tbody>
</table>
H.3 Reheating and Leptogenesis Benchmark Point

In this appendix, we list the benchmark point for the reheating and leptogenesis analysis considered in Chapter 5.

Input parameters: $\tilde{M}_{\tilde{g}} = 1.20 \text{ TeV}$

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input Param.</th>
<th>Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauge</td>
<td>$1/\alpha_G$</td>
<td>26.2</td>
</tr>
<tr>
<td></td>
<td>$M_G/10^{16} \text{ GeV}$</td>
<td>2.31</td>
</tr>
<tr>
<td></td>
<td>$\epsilon_3/%$</td>
<td>-0.680</td>
</tr>
<tr>
<td>SUSY (GUT scale)</td>
<td>$m_{16}/\text{TeV}$</td>
<td>25.0</td>
</tr>
<tr>
<td></td>
<td>$m_{1/2}/\text{GeV}$</td>
<td>400</td>
</tr>
<tr>
<td></td>
<td>$A_0/\text{TeV}$</td>
<td>-51.4</td>
</tr>
<tr>
<td></td>
<td>$(m_{H_d}/m_{16})^2$</td>
<td>1.89</td>
</tr>
<tr>
<td></td>
<td>$(m_{H_u}/m_{16})^2$</td>
<td>1.61</td>
</tr>
<tr>
<td>Neutrino</td>
<td>$M_{R_1}/10^9 \text{ GeV}$</td>
<td>9.03</td>
</tr>
<tr>
<td></td>
<td>$M_{R_2}/10^{11} \text{ GeV}$</td>
<td>5.74</td>
</tr>
<tr>
<td></td>
<td>$M_{R_3}/10^{13} \text{ GeV}$</td>
<td>2.95</td>
</tr>
<tr>
<td>SUSY (EW Scale)</td>
<td>$\tan \beta$</td>
<td>50.3</td>
</tr>
<tr>
<td></td>
<td>$\mu/\text{GeV}$</td>
<td>994</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input Param.</th>
<th>Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yukawa Textures</td>
<td>$\lambda$</td>
<td>0.606</td>
</tr>
<tr>
<td></td>
<td>$\lambda\tilde{c}$</td>
<td>0.0306</td>
</tr>
<tr>
<td></td>
<td>$\lambda\tilde{\epsilon}$</td>
<td>0.00442</td>
</tr>
<tr>
<td></td>
<td>$\lambda\tilde{c}'$</td>
<td>-0.00184</td>
</tr>
<tr>
<td></td>
<td>$\lambda\xi$</td>
<td>0.00364</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>$\rho$</td>
<td>0.0741</td>
</tr>
<tr>
<td></td>
<td>$\phi\xi/\text{rad}$</td>
<td>0.531</td>
</tr>
<tr>
<td></td>
<td>$\phi\xi/\text{rad}$</td>
<td>3.46</td>
</tr>
<tr>
<td></td>
<td>$\phi\sigma/\text{rad}$</td>
<td>0.503</td>
</tr>
<tr>
<td></td>
<td>$\phi\rho/\text{rad}$</td>
<td>3.99</td>
</tr>
</tbody>
</table>
Low energy observables fit

<table>
<thead>
<tr>
<th>Observable</th>
<th>Fit</th>
<th>Exp.</th>
<th>Pull</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_Z/\text{GeV}$</td>
<td>91.1876</td>
<td>91.1876</td>
<td>0.0000</td>
<td>0.4559</td>
</tr>
<tr>
<td>$M_W/\text{GeV}$</td>
<td>80.4468</td>
<td>80.3850</td>
<td>0.1537</td>
<td>0.4022</td>
</tr>
<tr>
<td>$1/\alpha_{em}$</td>
<td>137.6063</td>
<td>137.0360</td>
<td>0.8324</td>
<td>0.6852</td>
</tr>
<tr>
<td>$G_F/10^{-5}\text{ GeV}^2$</td>
<td>1.1739</td>
<td>1.1664</td>
<td>0.6461</td>
<td>0.0117</td>
</tr>
<tr>
<td>$\alpha(M_Z)$</td>
<td>0.1191</td>
<td>0.1185</td>
<td>0.7247</td>
<td>0.0008</td>
</tr>
<tr>
<td>$M_t/\text{GeV}$</td>
<td>173.9468</td>
<td>173.2100</td>
<td>0.3468</td>
<td>2.1247</td>
</tr>
<tr>
<td>$m_t(m_t)/\text{GeV}$</td>
<td>4.3095</td>
<td>4.1800</td>
<td>0.9916</td>
<td>0.1306</td>
</tr>
<tr>
<td>$M_\chi/\text{GeV}$</td>
<td>1.7745</td>
<td>1.7769</td>
<td>0.1207</td>
<td>0.0199</td>
</tr>
<tr>
<td>$(M_\chi - M_L)/\text{GeV}$</td>
<td>3.3159</td>
<td>3.4500</td>
<td>0.3578</td>
<td>0.3749</td>
</tr>
<tr>
<td>$m_t(m_t)/\text{GeV}$</td>
<td>1.2408</td>
<td>1.2750</td>
<td>1.1894</td>
<td>0.0288</td>
</tr>
<tr>
<td>$m_t(2\text{GeV})/\text{GeV}$</td>
<td>0.0899</td>
<td>0.0950</td>
<td>0.9886</td>
<td>0.0515</td>
</tr>
<tr>
<td>$m_t(m_t)/\text{GeV}$</td>
<td>0.0716</td>
<td>0.0513</td>
<td>3.0436</td>
<td>0.0067</td>
</tr>
<tr>
<td>$Q$</td>
<td>25.9397</td>
<td>23.0000</td>
<td>1.2742</td>
<td>2.3071</td>
</tr>
<tr>
<td>$M_{\chi}/\text{GeV}$</td>
<td>0.1048</td>
<td>0.1057</td>
<td>0.4139</td>
<td>0.0022</td>
</tr>
<tr>
<td>$M_{\chi}/10^{-4}\text{ GeV}$</td>
<td>5.1422</td>
<td>5.1100</td>
<td>0.5634</td>
<td>0.0571</td>
</tr>
<tr>
<td>$[V_{ud}]$</td>
<td>0.9745</td>
<td>0.9742</td>
<td>0.0508</td>
<td>0.0049</td>
</tr>
<tr>
<td>$[V_{us}]$</td>
<td>0.2244</td>
<td>0.2253</td>
<td>0.6695</td>
<td>0.0014</td>
</tr>
<tr>
<td>$[V_{ub}]/10^{-3}$</td>
<td>3.1730</td>
<td>3.8500</td>
<td>0.7838</td>
<td>0.8657</td>
</tr>
<tr>
<td>$[V_{cb}]$</td>
<td>0.2242</td>
<td>0.2250</td>
<td>0.9967</td>
<td>0.0081</td>
</tr>
<tr>
<td>$[V_{ts}]$</td>
<td>0.9737</td>
<td>0.9860</td>
<td>0.7365</td>
<td>0.0167</td>
</tr>
<tr>
<td>$[V_{tb}]/10^{-3}$</td>
<td>41.1696</td>
<td>40.8000</td>
<td>0.1634</td>
<td>2.2622</td>
</tr>
<tr>
<td>$[V_{td}]/10^{-3}$</td>
<td>8.9578</td>
<td>8.4000</td>
<td>0.8932</td>
<td>0.6245</td>
</tr>
<tr>
<td>$[V_{ts}]/10^{-3}$</td>
<td>40.3083</td>
<td>40.0000</td>
<td>0.1092</td>
<td>2.8231</td>
</tr>
<tr>
<td>$[V_{ts}]$</td>
<td>0.9991</td>
<td>1.0210</td>
<td>0.6744</td>
<td>0.0324</td>
</tr>
<tr>
<td>$\sin(2\tilde{\beta})$</td>
<td>0.6295</td>
<td>0.6820</td>
<td>2.7223</td>
<td>0.0193</td>
</tr>
<tr>
<td>$\sin(2\tilde{\beta})$</td>
<td>0.0019</td>
<td>0.0022</td>
<td>1.2750</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\Delta M_{\mu}/\Delta M_{B_d}$</td>
<td>29.7609</td>
<td>34.8272</td>
<td>0.7181</td>
<td>7.0465</td>
</tr>
<tr>
<td>$\Delta M_{B_d}/10^{-10}\text{ MeV}$</td>
<td>3.6386</td>
<td>3.3560</td>
<td>0.4203</td>
<td>0.6723</td>
</tr>
<tr>
<td>$m_{\chi}^2/10^{-3}\text{ GeV}^2$</td>
<td>7.4085</td>
<td>7.3750</td>
<td>0.0826</td>
<td>0.4057</td>
</tr>
<tr>
<td>$m_{\chi}^2/10^{-3}\text{ GeV}^2$</td>
<td>2.6028</td>
<td>2.5000</td>
<td>0.2818</td>
<td>0.1318</td>
</tr>
<tr>
<td>$\sin^2\theta_{12}$</td>
<td>0.2940</td>
<td>0.2975</td>
<td>0.2107</td>
<td>0.0166</td>
</tr>
<tr>
<td>$\sin^2\theta_{23}$</td>
<td>0.4573</td>
<td>0.4435</td>
<td>0.5195</td>
<td>0.0266</td>
</tr>
<tr>
<td>$\sin^2\theta_{13}$</td>
<td>0.0206</td>
<td>0.0215</td>
<td>0.8706</td>
<td>0.0010</td>
</tr>
<tr>
<td>$M_\chi/\text{GeV}$</td>
<td>125.7055</td>
<td>125.0900</td>
<td>0.1279</td>
<td>4.8104</td>
</tr>
<tr>
<td>$BR(B \to s\gamma)/10^{-8}$</td>
<td>3.2107</td>
<td>3.4300</td>
<td>0.1338</td>
<td>1.6389</td>
</tr>
<tr>
<td>$BR(B_s \to \mu^+\mu^-)/10^{-9}$</td>
<td>4.6979</td>
<td>2.8500</td>
<td>2.0300</td>
<td>0.9103</td>
</tr>
<tr>
<td>$BR(B_d \to \mu^+\mu^-)/10^{-10}$</td>
<td>1.5821</td>
<td>4.0000</td>
<td>1.3977</td>
<td>1.7299</td>
</tr>
<tr>
<td>$BR(B \to \tau\nu)/10^{-5}$</td>
<td>6.0704</td>
<td>11.4000</td>
<td>0.8669</td>
<td>6.1480</td>
</tr>
<tr>
<td>$BR(B \to K^\mu^+\mu^-)_\text{low }q^2/10^{-8}$</td>
<td>4.6258</td>
<td>3.4000</td>
<td>0.3384</td>
<td>3.6226</td>
</tr>
<tr>
<td>$BR(B \to K^\mu^+\mu^-)_\text{high }q^2/10^{-8}$</td>
<td>6.6955</td>
<td>5.6000</td>
<td>0.1025</td>
<td>10.6892</td>
</tr>
<tr>
<td>$q^2_{(B \to K^\mu^+\mu^-)}/(\text{GeV})^2$</td>
<td>3.8434</td>
<td>4.9000</td>
<td>0.6894</td>
<td>1.5327</td>
</tr>
<tr>
<td>$F_L(B \to K^\mu^+\mu^-)_\text{low }q^2$</td>
<td>0.7524</td>
<td>0.6500</td>
<td>0.3345</td>
<td>0.3061</td>
</tr>
<tr>
<td>$F_L(B \to K^\mu^+\mu^-)_\text{high }q^2$</td>
<td>0.3514</td>
<td>0.3300</td>
<td>0.0767</td>
<td>0.2789</td>
</tr>
<tr>
<td>$P_L(B \to K^\mu^+\mu^-)_\text{low }q^2$</td>
<td>0.0660</td>
<td>0.3300</td>
<td>0.3969</td>
<td>0.6651</td>
</tr>
<tr>
<td>$P_L(B \to K^\mu^+\mu^-)_\text{high }q^2$</td>
<td>-0.4334</td>
<td>-0.5000</td>
<td>0.2933</td>
<td>0.2270</td>
</tr>
<tr>
<td>$P_L(B \to K^\mu^+\mu^-)_\text{low }q^2$</td>
<td>0.5807</td>
<td>0.5800</td>
<td>0.0018</td>
<td>0.4009</td>
</tr>
<tr>
<td>$P_L(B \to K^\mu^+\mu^-)_\text{high }q^2$</td>
<td>1.1276</td>
<td>-0.1800</td>
<td>1.9886</td>
<td>0.7028</td>
</tr>
<tr>
<td>$P_L(B \to K^\mu^+\mu^-)_\text{low }q^2$</td>
<td>-0.3244</td>
<td>0.2100</td>
<td>2.3130</td>
<td>0.2311</td>
</tr>
<tr>
<td>$P_L(B \to K^\mu^+\mu^-)_\text{high }q^2$</td>
<td>-0.7122</td>
<td>-0.7900</td>
<td>0.1426</td>
<td>0.5455</td>
</tr>
<tr>
<td>Total $\chi^2$</td>
<td>47.6151</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>