Imaging Methods for Passive Radar

DISSERTATION

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Abstract

Passive radar systems typically piggyback on illuminators of opportunity, whose bandwidths are typically smaller and whose frequencies are significantly lower than that of typical active radar imaging platforms, such as synthetic aperture radar (SAR) and inverse SAR (ISAR). This work investigates the potential of such systems to produce useful radar imagery given these spectral constraints, along with other challenges associated with passive radar.

A narrowband Doppler imaging technique is demonstrated, which generates range resolution in the absence of bandwidth, purely using stripmap aperture synthesis. A monostatic sensor configuration is used to demonstrate the feasibility of the concept, both in simulation and with experimental measurements at X-band; the limits on range and cross-range resolution are also derived. These ideas are then applied to the passive imaging scenario using a set of FM radio and digital television transmitters.

The design of a multistatic digital television passive radar is presented, along with architecture design considerations faced by wideband passive radar receivers. A new passive radar downconversion architecture is examined as a means for gathering wideband data without sacrificing back-end ADC bandwidth. As passive radar processing is computationally complex, advancements to range-Doppler map calculation and filtering methods for direct signal interference suppression are discussed. Several
unique micro-Doppler measurements and distributed multipath phenomenology are also investigated.

Finally, a framework for passive ISAR (P-ISAR) imaging of non-cooperative air targets is developed to aid in target classification, building on the previous chapters’ algorithmic advances. This single-illuminator framework is developed to lay the groundwork which enables multistatic imaging for future passive radar systems. The unique challenges of motion-compensation and image formation under the narrowband passive radar constraints are investigated; traditional autofocus algorithms and focal metrics are shown to be inappropriate for narrowband P-ISAR systems, and target tracking is used to aid in rotational motion estimation which enables more sophisticated image formation procedures. Simulated results validate the P-ISAR framework, and experimental results demonstrate the ability to differentiate between large and small aircraft using a single 6 MHz digital television illuminator.
Dedication

To my parents, siblings, and my wife, Brittany... their unwavering love and support made this work possible. Anyone reading this document should also thank my Father for his meticulous efforts as editor-in-chief!
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List of Abbreviations

ADC  Analog to Digital Conversion (Converter)
ADS-B  Automatic Dependent Surveillance-Broadcast
ATSC  Advanced Television Systems Committee
BPF  Band Pass Filter
CAD  Computer Aided Design
CAF  Cross Ambiguity Function
CFAR  Constant False Alarm Rate
CLEAN  An iterative deconvolution algorithm for identifying point sources in imagery
COTS  Commercial Off-The Shelf
CPI  Coherent Processing Interval
CT  Computed Tomography
CW  Continuous Wave
DAB  Digital Audio Broadcasting
DDC  Digital Down-Converter (Conversion)
DFT  Discrete Fourier Transform
DOA  Direction of Arrival
DR   Dynamic Range
DSI  Direct Signal Interference
DSP  Digital Signal Processing
DTV  Digital TeleVision
DVB-T Digital Video Broadcasting-Terrestrial
ECA  Extensive Cancellation Algorithm
EIRP Equivalent Isotropically Radiated Power
ENOB Effective Number Of Bits
ESL  ElectroScience Laboratory (at The Ohio State University)
FBLMS Fast Block Least Mean Squares
FFT  Fast Fourier Transform
FIR  Finite Impulse Response
FISC Fast Illinois Solver Code
FM   Frequency Modulation
FPGA Field Programmable Gate Array
GEO  Geostationary Orbital
GLONASS  Russian GNSS system

GMTI  Ground Moving Target Indicator(tion)

GNSS  Global Navigation Satellite System

GPS   Global Positioning System

GSM   Global System or Mobile Communication

IF    Intermediate Frequency

IFFT  Inverse Fast Fourier Transform

IIP3  Input 3rd order Intercept Point

INR   Interference to Noise Ratio

ISAR  Inverse Synthetic Aperture Radar

JEM   Jet Engine Modulation

LFM   Linear Frequency Modulation

LMS   Least Mean Squares

LNA   Low Noise Amplifier

LO    Local Oscillator

LOS   Line of Sight

LPF   Low Pass Filter

LPI   Low Probability of Intercept
LS  Least Squares

LTE  Long Term Evolution, a 4G wireless broadband technology

MATLAB  MATrix LABoratory, a numeric computing environment

MoM  Method of Moments

MSa/s  Mega SAmles per Second

MSym/s  Mega SYMbols per Second

MTI  Moving Target Indication

MuTeRa  MUltistatic digital Television passive RAdar

NLMS  Normalized Least Mean Squares

OFDM  Orthogonal Frequency Division Multiplexing

PCIe  Peripheral Component Interconnect Express

PCL  Passive Coherent Location

PGA  Phase Gradient Autofocus

PLL  Phase-Locked Loop

PPP  Prominent Point Processing

PR  Passive Radar

PSF  Point Spread Function

RAID  Redundant Array of Independent Discs
RCS  Radar Cross Section
RD   Range-Doppler (map or bin)
RF   Radio Frequency
RLS  Recursive Least Squares
RRCS Root-Raised CoSine (filter)
SAR  Synthetic Aperture Radar
SINR Signal to Interference plus Noise Ratio
SNR  Signal to Noise Ratio
SSD  Solid State Drive (or Device)
STFT Short Time Fourier Transform
SWAP Size, Weight, And Power
UAS  Unmanned Airspace Systems
UHF  Ultra High Frequency
VHF  Very High Frequency
VSWR Voltage Standing Wave Ratio
WiFi Wireless FIdelity
WiMax Worldwide Interoperability for Microwave Access
Chapter 1: Introduction

This thesis investigates the potentials of imaging with passive radar systems using broadcast illuminators of opportunity. The low frequencies and narrow bandwidth of such systems present an atypical parameter space when compared to conventional radar imaging. In addition to the core processing algorithms particular to digital television based passive radar, methods of narrowband imaging are investigated within these constraints for both stationary and moving targets using single and multiple transmitters. A systems-based approach, which emphasizes experimental validation of the proposed concepts, is stressed throughout the work to ensure the models and methods are applicable in typical real-world scenarios. This approach results in a comprehensive framework for realizing passive radar imaging, while simultaneously addressing most of the challenges faced by such systems.

1.1 The Passive Radar Concept

Passive radar systems, in contrast to conventional active radar systems, do not have a dedicated transmitter but instead rely on existing sources of electromagnetic radiation as their illuminating source. Because of this, passive radar systems never utilize the
same receive aperture as the transmitter and are therefore a class of bistatic radar sys-
tems in the strictest sense. As a result, the same geometric and processing challenges 
of bistatic systems must be contended with for a passive radar system with appreci-
ciable transmit-receive separation. The opportunistic transmitter (often referred to 
as the *illuminator of opportunity*) can range from broadcast or point-to-point com-
munications systems, sensor systems or other non-cooperative radar systems. Since 
many of these transmissions were not designed for radar purposes, additional signal 
processing is often required to achieve a similar level of performance to a dedicated 
active radar installation.

There are a number of benefits to passive radar which make it particularly attrac-
tive for both military and commercial purposes. The lack of transmitter results in 
a system that is low cost, uses significantly less power, and is lightweight for imple-
mentation on a number of small platforms. Due to the proliferation of high-powered 
communication and radar systems, passive radar systems can be quickly ’dropped’ 
in place for situations in need of rapid deployment. Because no radiation is emit-
ted, the system is inherently stealthy and possesses a low probability of intercept 
(LPI). Passive radar is also bandwidth free, which is particularly favorable due to 
continually increasing spectral congestion and spectrum licensing costs. The low fre-
quencies of many broadcast illuminators and bistatic geometries also provide some of 
the necessary ingredients to counter stealth technologies. A wideband passive radar 
receiver can process multiple illuminators simultaneously, for multiple (redundant) 
looks on targets of interest which reduce the radar system’s fade vulnerability and 
track performance.

The most rudimentary passive radar system consists of two channels, the reference
and surveillance, as demonstrated in Figure 1.1. Although passive radar installations can be mounted on moving platforms, the discussion and works of this dissertation will be primarily focused on a fixed, ground-based passive radar observing moving targets. The reference channel records the direct-path signal, a delayed copy of the transmitted waveform from the illuminator of opportunity. The second channel, commonly referred to as the surveillance channel, records target echoes from the area of interest. Conventional processing calculates Doppler shifted cross-correlations to generate a range-Doppler map, which is demonstrated in Figure 1.2. From this two-dimensional image, detection and tracking can be performed directly [1]. Targets can be seen as the spikes in the range-Doppler surface away from the zero velocity clutter ridge. The continuous nature of most broadcast illuminators, in contrast to many low duty-cycle pulsed radar systems, results in the presence of strong direct signal interference (DSI) in the surveillance channel, which degrades the sensitivity of the receiver system and must be mitigated for good performance. The strong peak in the map at the \((0,0)\) coordinate represents the residual direct path leakage in the surveillance waveform after imperfect subtraction.
Figure 1.2: Sample range-Doppler map

The bistatic range and (bistatic) velocity axes of the range-Doppler map allow the radar to detect a target along an iso-range ellipse and its corresponding bistatic velocity component. This measurement alone corresponds to an infinite number of Cartesian target positions along the isorange ellipsoid, but when combined with the direction of arrival (DOA) results from a receive antenna array or trilateration with multiple transmitters, a target’s Cartesian position can be estimated. Using this information only, the target’s RCS and track mobility are the only discriminating features that can be used to learn more about the target of interest. These parameters, for instance, can be used to estimate the approximate size of the target, or differentiate a highly maneuvering jet from a commercial airliner, but offer little more than this in regards to additional information about the target features or potential methods of classification or identification.
1.2 Imaging with Passive Radar

Passive radar imaging, on the other hand, is a more advanced modality of operation that extracts further target information for classification purposes: measurement of the bulk length, width, or possibly details of shape or motions of the target’s sub-structures. Imagery can either be in the form of a two-dimensional target reflectivity function, or micro-Doppler analysis of target motions. For air targets, passive radar imagery could be used to classify based upon the nature of the propulsion mechanism (propeller-based aircraft, jet aircraft, or a helicopter). Differentiation between small private airliners and drones would be extremely useful information for next generation unmanned airspace systems (UAS). For military purposes, these system outputs could mean discrimination of friend or foe along with related threat level.

As with any conventional radar system, the range resolution of a passive radar is proportional to the bandwidth of the transmitted signals, and the cross-range resolution is a function of the surveillance antenna’s beam pattern and range to the target. Due to the low frequencies of favorable illuminators, this cross-range resolution or localization performance is typically coarse because of limitations on tractable antenna dimensions. In addition, the bandwidth of most passive radar illuminators is much smaller than many radar systems, which degrades the range resolution. Therefore, conventional radar imaging techniques such as synthetic aperture radar (SAR) and inverse SAR (ISAR) would result in coarse imagery. However, passive radar has a particular advantage due to the number of available illuminators which permit new and more advanced methods of forming multistatic imagery which have not been investigated fully.
The VHF and UHF bands generally contain the best suited illuminators of opportunity for most scenarios involving a passive radar observing air targets, because of the proliferation of available transmissions and high radiated powers in these bands. In North America, frequency-modulated (FM) radio is situated in the 88-108 MHz VHF band, and Digital Television (DTV) broadcast signals span the 470-700 MHz segment of the UHF band.

A number of authors have shown that fine two-dimensional imagery is theoretically possible from narrowband, multistatic passive ISAR systems in the UHF and VHF bands [2–7] over wide angles. However, there still remain many significant system limitations that prevent these results from being realizable, which can be seen in the lack of literature with experimental passive SAR or ISAR results. Those which have been published only utilize one physical transmit site, usually with multiple adjacent frequency channels [8–11], which rarely occurs in practice. Although this situation arises occasionally in Europe (due to the single frequency network scheme of DVB-T) a set of contiguous channels on a single common transmitter mast are almost nonexistent in the North American region. Due to the physical separation in phase centers and disjointed carrier frequencies, these signals cannot be treated as a single wideband waveform and thus alternative imaging approaches investigated herein must be employed.

This dissertation explores the feasibility of inverse imaging techniques for passive radar, with a fixed passive radar installation observing an airborne target. Resolutions and imagery for wide-angle imaging are developed, and applied to the typical passive radar scenario of multistatic, multi-band operation. The design of a passive radar to experimentally realize the discussed concepts is also presented, which allows for
exploration of practical limitations not fully investigated or neglected in the literature.

1.3 Outline and Contributions

This work seeks to develop a framework for passive radar imaging, investigating the theoretical performance limitations in parallel with the practical challenges faced by experimental passive radar systems. Chapter 2 reviews the well-established, foundation theory on which this work is built. More recent developments in the literature regarding narrowband imaging and passive radar are surveyed in Chapter 3.

Chapter 4 develops a method of narrowband imaging, implemented in a stripmap fashion. Monochromatic waveforms with aperture synthesis are demonstrated to generate not only cross-range (as with conventional SAR) but also down-range resolution. Resolution limits for a monostatic imaging system are derived using a k-space analysis, as a function of angular span and frequency. Experimental results establish the validity of the narrowband approach. Chapter 5 extends these concepts to a passive radar scenario with bistatic geometries and multiple transmitters. Practical limitations, such as forward scatter geometries, target coherence, and system limitations are discussed and directly examined via simulation results.

The design of a wideband, multi-transmitter passive radar system is presented in Chapter 6. A new passive radar downconversion scheme is developed, which circumvents bandwidth limitations of state-of-the-art high dynamic range analog to digital converters (ADCs). Design considerations and practical limitations for wideband systems to support passive radar imaging are discussed. Core signal processing algorithms are developed, and a new fast algorithm for calculating the range-Doppler
surface is developed which is much more computationally efficient than existing algorithms. A theoretical and experimental treatment of direct signal interference (DSI) suppression, a crucial stage in passive radar signal processing, is examined in Chapter 7. The first known experimental observations of jet engine modulation and distributed multipath using passive radar are shown in Chapter 8. Micro-Doppler signatures of propeller aircraft and a helicopter are also experimentally demonstrated, and shown to enable classification based upon the micro-Doppler signature of the range-Doppler surface.

Chapter 9 develops the framework for processing inverse synthetic aperture radar (ISAR) imagery from the standard range-Doppler output of most passive radar systems. Particular considerations for translational and rotational motion compensation for narrowband systems are investigated. The benefits of 3D target tracking using a network of passive radar transmitters are utilized to solve the problem of rotational motion compensation. This enables image scaling to a common reference plane, necessary for realizing simultaneous imaging across multiple illuminators of opportunity as a means of increasing image resolution. The developed passive imaging approach is validated by successful experimental generation of P-ISAR imagery with air targets, demonstrating results which begin to permit methods of target classification based upon estimation of the cross-range width.

The novel contributions of this thesis can be summarized as follows:

- Stripmap Doppler imaging was experimentally demonstrated, along with limits on down-range and cross-range resolution. A journal paper was published as a result of this work [12]
• Multistatic narrowband imaging approaches were investigated using a simulated analysis, and the limitations of such techniques were discussed

• A wideband passive radar testbed was designed and constructed, using a new passive radar RF tuning architecture

• A fast algorithm for approximating range Doppler map generation was developed, which circumvents range- and Doppler limitations of conventional pulsed radar systems

• Adaptive filtering algorithms for DSI mitigation were thoroughly investigated, along with an analysis of parameters which affect passive radar performance. A journal paper was published as a result of this work [13]

• Jet engine modulation (JEM) was observed experimentally for the first time using a passive radar

• The phenomena of distributed multipath was discovered, showing excellent agreement between experimental results and a simple model-based approach

• A comprehensive framework for passive ISAR imaging was developed to enable multiple transmitter fusion

• Conventional ISAR metrics and autofocus algorithms were shown to be inadequate to properly focus narrowband ISAR imagery. The Doppler centroid parametric method of translational motion compensation was shown to be sufficient narrowband passive ISAR
• The first fully-scaled, experimental passive ISAR image using a single transmitter was developed
Chapter 2: Basic Theory

Imaging with passive radar invokes a number of different sub-disciplines of radar theory. This chapter gives a brief overview of the fundamental theory behind these areas: bistatic radar, passive radar, and established radar imaging techniques. The sections pertaining to bistatic and passive radar address complexities regarding separation between the transmitter and receiver of a particular radar system (the bistatic geometry) and the basics of passive radar architectures and signal processing. There are two predominant modes in which a radar can form a high resolution two-dimensional image: SAR, and ISAR. These fundamental concepts underpin the development of Doppler and passive radar imaging techniques, described in Chapters 4-9.

This chapter assumes the reader is familiar with basic radar theory. For readers unfamiliar with the area, a textbook introduction to radar along with imaging (SAR) principles is given in *Introduction to Airborne Radar* [14] and *Principles of Modern Radar* [15]. Skolnik’s *Radar Handbook* [16] is a more in-depth, comprehensive overview of a variety of radar theory. All of these texts establish the foundation for basic radar principles which underlay any coherent radar imaging system.
2.1 Bistatic Radar

Bistatic radar, in the most generic sense, is a radar system whose transmitter and receiver are not collocated. Many radars often employ separate transmit and receive antennas in close proximity (to reduce transmit-receive coupling and avoid use of a microwave circulator), but this is usually regarded as a quasi-monostatic setup and the antenna center can be considered the physical midpoint between the two antennas. Skolnik [17] defines bistatic radar as one whose transmit and receive antennas are separated by a significant amount, “comparable with the target distance.” The general radar principles remain: delay of the transmitted waveform measured at the receiver results in the range estimate, and the Doppler shift of the received waveform results in the estimate of a target’s velocity. However, the geometry of bistatic systems has consequential differences in how these parameters are observed when compared with monostatic systems.

Bistatic radar theory is fairly well established, and a number of texts have been published on the subject [1,18–20]. Separate transmit and receive sites cause the iso-range contours to be ellipsoidal rather than spherical, thus the range and Doppler resolution varies as a function of bistatic angle [17,21]. These bistatic geometries present additional complications for imaging, which was originally set in a monostatic context and predominantly implemented as such.

Figure 2.1 shows a sample geometry for bistatic radar. Any of the components (transmitter, receiver, or target) can either be stationary or moving, but a fixed bistatic radar observing an air target is shown to most closely represent the developments of this manuscript. Note that three ranges are required to fully portray a
bistatic radar target scenario, in contrast to the single range of a monostatic system. The bistatic baseline, $R_L$, is the distance between the transmitter and receiver. The distance from the transmitter to target, or transmitter range, is denoted $R_T$, and the distance to the receiver from the target, $R_R$, are unique and vary as a function of target position. The sum of the transmit and receive ranges is known as the bistatic range, $R_B = R_T + R_R$. A final important characteristic of bistatic systems is the bistatic angle, $\beta$, between the line-of-sight paths to the transmitter and receiver from the target; effects of this on a bistatic radar’s range and Doppler sensitivity will be examined shortly.

The propagation paths of the transmit, $R_T$, and receive ranges, $R_R$, are unique, requiring the conventional radar range equation to be modified from the monostatic case. The resulting expressions deriving the received power, $P_r$, is shown in (2.1) and
the signal to noise ratio (SNR) in (2.2).

\[
P_r = \frac{P_t G_t G_r \lambda^2 \sigma_b}{(4\pi)^3 R_T^2 R_R^2 L}
\]

\[
\text{SNR} = \frac{P_t G_t G_r \lambda^2 \sigma_b}{(4\pi)^3 R_T^2 R_R^2 kTBF L}
\]

Where \(P_t\) is the power into the transmitting antenna (Watts), \(G_t\) and \(G_r\) are the antenna gain of the transmitter and receiver (unit-less), \(\sigma_b\) is the bistatic target RCS \((m^2)\), \(\lambda\) is the wavelength of operation \((m)\), \(L\) is the system losses (unit-less, \(L > 1\)), \(k\) is Boltzmann’s constant \((1.38 \times 10^{-23})\), \(T\) is the noise temperature of the receive antenna \((^\circ K)\), \(B\) is the bandwidth of the receiver (Hz), and \(F\) is the noise factor of the receiver chain (unit-less). Although the range equation appears to be very similar to the monostatic case, the different path lengths results in significant implications on contours of constant SNR. Contours of constant SNR occur when the product of the transmit and receive range \((R_T \times R_R = \alpha)\) is constant, such that the received power, \(P_r\), is constant assuming all other parameters are unchanged. These contours are known as Ovals of Cassini [18].

When a bistatic radar measures the bistatic delay, \(t_d\), to a particular target, it corresponds to a bistatic range value, \(R_B = t_d \times c\), where \(c\) is the speed of light \((m/s)\). Note that there are many points in space that correspond to the same bistatic range position, forming an iso-range contour. If beam pattern effects are neglected, an echo for a particular delay time can lie at any point along this surface. The iso-range surface for bistatic radar (where \(R_B = R_T + R_R = R_o\)) is an ellipsoid, compared with the spherical isorange contours of monostatic radar. Fig. 2.2 illustrates these contours in the x-y plane for a transmitter/receiver pair with a normalized baseline.
length of unity. These isorange contours, if rotated around the x-axis, form the ellipsoid surfaces of constant range in three dimensions.

The Doppler shift, observed by the radar, fundamentally arises from changes in the bistatic range due to target motion; this is manifest as a phase change over time at the receiver. This can be written as

$$f_D = \frac{1}{\lambda} \frac{dR_B}{dt} = \frac{f_c}{c} \frac{d(R_T + R_R)}{dt}, \quad (2.3)$$

where $f_c$ is the center frequency of the illuminating signal. The change in bistatic range is known as the bistatic velocity, $V_B = dR_B/dt$, defined here as a scalar. When a target’s velocity vector, $\hat{v}$, results in a time-variant bistatic range, a Doppler shift proportional to this change is measured at the receiver. Contrary to this, if a target is traveling along the isorange contours of Figure 2.2, there is no apparent bistatic range change, and the target will appear stationary. Equation (2.3) can be recast in terms of the local target geometry to reveal other properties of influence of the
bistatic geometry on the Doppler shift, as follows:

\[ f_D = \frac{2V}{\lambda} \cos(\phi) \cos(\beta/2). \] (2.4)

The speed of the target, in \( m/s \), is defined as the absolute value of the velocity vector, \( V = ||\hat{v}|| \). The magnitude of the Doppler shift also depends on the bistatic angle at the target, \( \beta \), as well as the angle between the target’s velocity vector, and the bistatic bisector, \( \phi \), depicted in Fig. 2.3. Note that the bistatic bisector is always normal to the iso-range surface, and can be calculated from the transmit and receive unit vectors as \( \hat{u}_B = (\hat{u}_T + \hat{u}_R) / ||\hat{u}_T + \hat{u}_R|| \), where \( ||\cdot|| \) is the Euclidean norm. Notice that when \( \beta = 0 \) the Doppler shift expression reduces to the monostatic case. In addition, when the target is along the bistatic baseline (\( \beta = 180 \)) no Doppler shift is observed.

There are two ways of computing the target velocity (once again, scalars) resulting from the observed Doppler shift, as either the bistatic velocity and the equivalent...
monostatic velocity, respectively, as follows:

\[ V_B = f_D \lambda \]  \hspace{1cm} (2.5)
\[ V_M = \frac{f_D \lambda}{2} \]  \hspace{1cm} (2.6)

The bistatic velocity, the rate of change in the bistatic range, is most often used in practice but has a range of \( 0 < V_B < 2V \). This can be misleading, as the bistatic velocity can be twice as large as the true target speed. To avoid misinterpretation, the equivalent monostatic velocity calculation, shown in Equation (2.6), yields a more intuitive result bounded by the target velocity, although it is not linked directly to a change in radial range, as is the case for monostatic radar. In either case, the corresponding velocity resolutions can be calculated by setting \( f_D = 1/T \) in (2.5) and (2.6), where \( T \) is the length of the processing interval determining the Doppler resolution.

The bistatic delay resolution of a bistatic radar is the inverse of the illuminating signal’s bandwidth, assuming matched filtering or pulse compression is employed at the receiver. This implies a bistatic range resolution of

\[ \delta_B = \frac{c}{B} \]  \hspace{1cm} (2.7)

meters. Therefore, two targets can generally be differentiated by a particular radar system as long as the difference in their corresponding bistatic ranges is greater than \( \delta_B \). In a manner similar to that of the preceding Doppler analysis, the effects of this should be examined for the case of local target geometry to be fully appreciated.
The effective monostatic range resolution for a bistatic system is the true Cartesian separability along the bistatic bisector for a certain target position, perpendicular to the isorange contours. Although the bistatic range resolution, $\delta_B$, is constant, the effective monostatic range resolution, $\delta_R$, varies as a function of target position. The bistatic range resolution is defined as (2.8)

$$\delta_R = \frac{c}{2B \cos(\beta/2)}.$$  \hspace{1cm} (2.8)

Notice that this definition approaches the monostatic resolution criteria, $c/2B$, as $\beta \to 0$. Since both velocity resolution and range resolution are maximum for these geometries, this represents the best-case mode of operation for bistatic radars, and is known as the “over-the-shoulder” geometry when the target is opposite the transmitter direction at the receiver.

### 2.2 Passive Radar

Passive radar, being inherently bistatic, faces the exact same geometrical intricacies facing its active bistatic counterparts, but with a few additional complications. The fundamental difference between passive and bistatic radar systems is that the transmitted waveform is out of control of the radar engineer and typically not known a-priori. Because many illuminators of opportunity are designed for communication systems, rather than radar, the signals often contain features for synchronization which degrade radar performance if not mitigated. These signals are typically 100% duty cycle (in stark contrast to the low duty cycle of most radars’ pulsed waveforms), resulting in a strong interfering signal at the receiver, but also requiring
non-conventional signal processing algorithms to generate a range-Doppler map to be used for target detection.

As described in Section 1.1 and shown in Figure 2.4, a passive radar system generally consists of two channels: reference and surveillance. The reference antenna, $s_r(t)$, collects a delayed copy of the transmitted signal, shown in (2.9). The self ambiguity surface [22] of this waveform defines the delay and Doppler resolution of a passive radar system for a particular illuminator, consistent with the bistatic radar theory Section 2.1. An example of a self ambiguity function for experimentally collected high definition FM (HDFM) waveform data is shown in Figure 2.5 [23]. A simple model for the reference waveform is shown in (2.9).

$$s_r(t) = A_r s_{tx} \left(t - \frac{R_L}{c}\right) + \nu_r(t) \quad (2.9)$$

Note that the reference waveform consists of a scaled version of the transmitted signal, $s_{tx}(t)$, delayed by the bistatic baseline range at the speed of light, $R_L/c$, as well as additive white Gaussian noise, $\nu_r(t)$. This distance, as well as the transmit and receive locations, is generally assumed to be known and fixed for most passive radar systems.
Figure 2.5: Self ambiguity function of HDFM waveform

\( A_r \) is the complex amplitude term representing the propagation magnitude and phase effects. In practice, \( s_r(t) \) is gathered using a highly directive reference antenna to maximize the SNR of the reference waveform, 

\[
E \left[ \left| A_r s_{tx} \right|^2 \right] / E \left[ |\nu_r|^2 \right],
\]

where \( E[\cdot] \) is the expectation operator.

Due to practical lower bounds on realizable antenna beamwidths, the reference signal also contains some level of multipath and clutter in practice, but this has been omitted for simplicity. The presence of clutter in the reference signal results in false peaks in the RD surface at closer ranges than the true distance to the scatterer (see Section 7.3.1.2 for further discussion). If the effects of such components are detrimental to performance, remodulation of the reference signal can be implemented [24–26], which can remove clutter, multipath, and noise from \( s_r(t) \). Note that although these methods have only been demonstrated for digital video broadcast (DVB-T) and Advanced Television Systems Committee (ATSC) A/53 waveforms, they can be extended to any digitally modulated waveforms whose signal structure is known, such as GSM, DAB, WiFi, or WiMax. Unfortunately, this procedure adds computational overhead and care must be taken such that the carrier phase effects are properly accounted for to avoid degradation in the matched filtering process [26]. The signal must also have
adequate SNR to be properly demodulated without significant errors such that the reconstruction is accurate.

The surveillance waveform of a passive radar system consists of DSI (direct path, multipath and clutter), Doppler-shifted target echoes, and thermal noise. The general form for the surveillance waveform, including these three components, can be written as follows:

\[
s(t) = s_{dsi}(t) + s_{tar}(t) + \nu_s(t)
\]  

(2.10)

where \( s_{dsi}(t) \) is the direct signal component, \( s_{tar}(t) \) is the target signal, and \( \nu_s(t) \) is thermal noise. The DSI and target components are also made up of scattered signals of \( s_{tx}(t) \); an in-depth treatment of DSI and its mitigation is detailed in Section 7.1.

The first stage of passive radar processing involves some form of signal conditioning, often in the form of removing repetitive synchronization symbols, or pilot tones used for receiver phase-locked-loop (PLL) tracking. These features usually result in range- or Doppler-ambiguities in the self ambiguity surface of the transmitted waveform if left untreated. Experimental demonstration of this is shown in Section 6.3.3. DSI suppression is then performed to remove the strong direct path and clutter components, \( s_{dsi}(t) \). An overview of the related theory and algorithms for performing DSI mitigation is shown in Chapter 7.

The fundamental stage of passive radar processing is computation of the RD map, from which target detection and tracking can be carried out directly. This surface is calculated via Doppler-shifted cross-correlations of the reference and surveillance waveforms in the form of the crossambiguity function, as
\[ \Psi(R_D, f) = \int_{t=0}^{T} s_s(t) s_r^* (t - R_D/c) e^{-j2\pi ft} \, dt, \] (2.11)

calculated for hypothesized targets at bistatic range past baseline, \( R_D \), and Doppler frequency \( f \). \( R_D \) sets the correlation delay time, \( t_d = R_D/c \). In physical bistatic range terms this can be viewed as \( R_D = R_B - R_L \); rearranging terms shows that the true bistatic range is \( R_B = R_D + R_L \). The Doppler frequency can also be used to calculate a target’s bistatic velocity, \( V_B = f \times \lambda \), given the known center frequency of operation.

Examples of RD maps for experimental passive radar data with DTV waveforms are shown in Figure 1.2 and the figures of Section 6.4. This stage, if implemented exactly, is very computationally complex, and approximations are generally made to reduce the runtime for experimental implementation. A new algorithm which minimizes the runtime while minimizing degradation in correlation gain is presented in Section 6.3.5.

Although the bistatic Doppler and range estimates from a single transmitter are ambiguous, consistent with bistatic radar theory presented previously, angle of arrival estimates from a single transmitter or trilateration with multiple illuminators (or multiple receivers) can be employed to estimate and track a targets’ three dimensional position. This information can then be used for subsequent imaging operations, demonstrated in the following section.

2.3 Imaging

2.3.1 SAR

Synthetic aperture radar (SAR) imaging is the most established radar imaging mode, and many texts and fundamental papers are devoted to the subject, e.g. [6,27–31].
Conventional stationary ground-based radar systems can have excellent resolution in range by using large bandwidths, but have relatively coarse cross range resolution due to limitations on aperture size. Large apertures are often required to achieve acceptable imaging resolutions in cross-range which worsens with increasing range from the radar. SAR imaging overcomes these limitations by placing the radar system on a moving platform and gathering spatially distributed measurements to synthesize a large aperture via digital signal processing.

The most simple mode of operation of a SAR platform is to fly a straight path with a fixed side-looking real aperture, as shown in Fig. 2.6 (a). The aperture is synthesized as the real beam is “dragged” past a target. Other forms of SAR imaging include spotlight and circular SAR, shown in (b) and (c) of Fig. 2.6. Spotlight SAR uses a steered real aperture to illuminate a fixed patch on the ground for a longer period, to attain higher cross range resolution. Circular SAR involves flying a circular path around a central location to provide continuous surveillance and higher cross range resolution, as in Spotlight SAR. The cross-range resolution arises due to the unique Doppler profiles of the differing cross range locations.

As the platform traverses the flight path, pulses are transmitted and echoes collected at a pulse repetition frequency (PRF) satisfying the Nyquist criteria for the
cross-range compression operations. One-dimensional range profiles can be generated from each pulse; by exploiting the phase differences from pulse to pulse, high cross range resolution can be generated. Data collection for an airborne stripmap SAR platform is illustrated in Fig. 2.7.

Down range resolution is in the direction perpendicular to the flight path for stripmap SAR, while cross range is along the direction of the synthetic aperture, or flight path. For the top-down illustration of Fig. 2.7, down range would correspond to the horizontal axis and cross range to the vertical axis. The down range resolution can be calculated as

$$
\delta_{dr} = \frac{c}{2B \cos(\psi)},
$$

where $B$ bandwidth of the transmitted pulse and $\psi$ is the elevation angle of the sensor from the ground patch of interest. The cross range resolution for SAR

$$
(2.12)
$$
is determined by the frequency of illumination $f_c$ and the angle of integration, $\theta$, as

$$\delta_{cr} = \frac{c}{2f_c \theta}. \quad (2.13)$$

For higher frequencies and wider angles of observation, the cross-range resolution increases. It is evident from Fig. 2.6 (a) that $\theta$ is limited to the beamwidth of the real aperture ($\theta_{ant} = \lambda/d$, where $d$ is the antenna diameter) for a side-looking SAR system. Substituting the expression for antenna beamwidth into (2.13), the resolution can be rewritten as $\delta_{cr} = d/2$. Notice that the cross range resolution is range-independent; if the range to the target increases, a collection over the same range of angles allows a larger aperture to be formed, which keeps the cross range resolution constant.

### 2.3.2 ISAR

For both SAR and ISAR imaging, range resolution is formed through high bandwidth waveforms, and cross range resolution is formed via aperture synthesis due to relative motion between the radar and target. The difference between the two techniques lies in the motion which generates the synthetic aperture. For SAR, a moving radar platform collects pulses over a stationary scene or target and then applies the proper range and cross range compression to form the imagery. For the case of ISAR, the radar is fixed, and the target’s motion generates the cross range resolution, demonstrated in Figure 2.8. It is important to make the distinction that it is the change in aspect angles over which a target is viewed that permits some degree of cross range resolution, rather than simply recording a target’s Doppler shifts – and certainly in some special cases (e.g., a target is flying directly towards or away from the radar).
there is no cross range resolution [31].

One may think that the principles for SAR and ISAR should be the same for similar relative motions between the transmitter and receiver. However, the techniques are fundamentally different because the target is almost always non-cooperative for ISAR [32]. Due to this non-cooperative nature, the trajectory is not known during the acquisition, but must be estimated from the radar returns followed by imaging operations. This requires additional translational and rotational motion compensation not required with SAR, which can rely on precise inertial motion compensation measurements for the platform trajectory.

The first step of ISAR processing involves removal of the translational motion component of the target, such that scatterers fall in the same range bins throughout the coherent processing interval (CPI). Following this, phase compensation is applied to remove residual translational phase components from the measurements, leaving the rotational motion component as the only phase change from pulse to pulse in the ideal case.
Once the effects of translational motion are removed from the data, an image can be formed with a fast Fourier transform (FFT) operation in the slow time. This method of image formation assumes the effective target rotation is uniform throughout the CPI, and that the total angular observation is sufficiently small to avoid smearing [33], Chapter 7. However, in such a scenario it is not possible to know the scaling of the cross-range axis, and the units of the image are in down range (m) and cross range (Hz). If the target’s rotational motion can be estimated, the imagery can be improved, and the cross range axis can be scaled to a spatial dimension matching that of the down range resolution.

To this date, there have been two texts published solely on ISAR [32,34] while others have substantial sections devoted to ISAR and focusing with SAR systems [6,27,33,35]. The basic principles of ISAR and techniques for autofocus and range migration compensation are covered. Much of the ISAR literature surrounds methods of fine motion compensation, which can be grouped into parametric and non-parametric algorithms. The most common non-parametric techniques include prominent point processing (PPP) and the phase gradient algorithm (PGA) while image contrast and entropy based techniques make up the parametric methods for autofocus [36]. The theory underpinning ISAR, motion compensation, and image reconstruction is vast; the interested reader should refer to the references in this paragraph as well as those presented in Chapter 3.
Chapter 3: Literature Review

This chapter serves to review and discuss the contemporary literature relevant to imaging with passive radar systems. Recent developments in passive radar signal processing will be reviewed first, as they form the foundation which is common to most radar operations and all experimental results illustrated in this manuscript. Doppler radar imaging techniques are then discussed, whose application would be useful to passive radar, because many illuminators of opportunity are inherently narrowband. The Doppler imaging references serve as the foundation underpinning the narrowband imaging investigations of Chapters 4 and 5. Bistatic imaging literature, recent passive imaging developments, both SAR and ISAR techniques, are then discussed and form the basis of Chapter 9.

3.1 Passive radar

An overview of passive radar systems and their fundamental processing techniques can be found in a few recent texts [16,19,37–39]. A majority of the work deals with waveforms of various illuminators, coverage and performance prediction, processing methodologies for direct signal interference (DSI), and correlation methods for range-Doppler processing. A number of commercial passive systems have been developed
to date, some of which include Silent Sentry, Celldar and Homeland Alerter [40]. Most of these systems are only capable of detecting and tracking targets; hence the motivation for the investigations of this manuscript toward more advanced imaging capabilities.

### 3.1.1 Signals of Opportunity

Because most illuminators of opportunity are primarily designed for communications purposes, they often are encumbered with features assisting receiver synchronization and channel estimation. These portions of the signal, e.g. pilot tones and synchronization symbols, often degrade radar performance. Luckily, a-priori knowledge of these aspects enable development of techniques to mitigate these unwanted effects [23, 25, 41–43]. The characteristics affect the passive radar’s signal processing chain, influence the range and velocity resolution, and ultimately determine the reliability and availability of acceptable radar performance. For example, digitally modulated waveforms typically present a stable power spectrum and bandwidth – thus constant opportunity for radar exploitation. In comparison, FM radio – an analog modulation scheme – possesses time-varying bandwidth due to the modulating signal content which directly affects both range resolution and SNR [19].

### 3.1.2 Processing Methods

The continuous nature of most illuminators for passive radar, compared with the low duty cycles of most pulsed radar systems, results in a persistent and strong interfering signal in the passive radar’s surveillance channel. A number of spatial nulling [44–47] and time-domain adaptive methods [44, 48–52] have been proposed for mitigating
these effects. Ultimately, as will be shown in Chapter 7, the presence of the direct signal adds an additional 5-10 dB to the effective noise figure of a typical passive radar receiver even with state of the art mitigation strategies.

There have been significant investigations to increase SNR by extending the CPI [25, 53–56]. Extended CPIs not only improve detection performance, but also increase Doppler resolution. For imaging applications, this results in finer image resolution with increased dwell time. Work to join multiple collocated illuminators separated in frequency was developed by Olsen [57–59], by down-converting the various illuminator channels and applying phase and Doppler frequency corrections to a particular target of interest. These techniques could prove valuable for passive imaging scenarios utilizing a single transmit site and spectrally diverse illuminators.

3.2 Narrowband Imaging

When significant bandwidth isn’t available to perform conventional SAR or ISAR imaging, alternative techniques may be employed. The beneficial quality of long, narrowband signals is their excellent Doppler resolution, which can be exploited in a slightly different manner from that of traditional, high bandwidth radar imaging techniques to yield two-dimensional resolution. While tomography is limited to the application of many sensors surrounding an object and a small scene, Doppler imaging expands on the concepts of two-dimensional target resolution through exploitation of the frequency shift in a more general manner. This permits imaging of large scenes using a single sensor flown along an arbitrary flight path.
3.2.1 Doppler Imaging

The concept of Doppler imaging with radio waves was originally used to produce reflectivity maps of lunar surfaces with relaxed hardware constraints [60, 61] compared to the SAR-like range-Doppler techniques of Ryle and Hewish [62]. The geometrical assumptions of such work only hold for certain scenarios involving particular lunar orbital geometries and constant iso-Doppler contours. The narrowband imaging technique was then further generalized to enable accurate reconstruction when the iso-Doppler lines exhibit appreciable curvature [63], as observed from low orbiting satellites with repeated passes over polar regions.

A surge of interest in narrowband imaging was seen between 2000 and 2010 from the mathematical community [64–67]. Nolan first developed the mathematical framework [64] describing the forward model and image formation for an arbitrary SAR-type imaging scenario. This framework was then adapted to Doppler imaging by backprojecting short-time correlation windows along hyperbolic iso-Doppler contours for the case of imaging over flat earth [65, 68]. The case of a circular aperture and Doppler central slices using a short time Fourier transform (STFT) window was presented by Chua [69]. A computationally efficient image formation procedure using the Radon-Hough transform was developed to filter out various Doppler slopes at different points in the received signal’s STFT. The sub-aperture approach inherent in the image formation is more robust to phase errors (either due to positional inaccuracies, hardware effects, or target persistence) at the expense of final image resolution.

Cheney et al. attempts to link spatial, temporal, and spectral aspects of the proposed imaging techniques to provide a unified vision of the various radar imaging
methods [66]. The authors propose that SAR and ISAR are orthogonal to Doppler imaging, although their cross-range resolving mechanism is enabled with Doppler processing of a pulse train. The flat-earth and straight line assumptions of [68] are generalized for arbitrary flight trajectories and bistatic transmit-receive geometries by Yarman and Wang et al. [70–72]; the approach applies the standard back-propagation algorithm to arbitrary flight paths. In [73], a passive bistatic imaging architecture is propose by correlating between two mobile receivers with a fixed, ground-based illuminator. Sego et al. developed an engineering-focused hybrid method of Doppler-based projections with a slight bandwidth, similar to that seen with passive radar systems [74]. The slight bandwidth increase beyond that of a purely monochromatic system allows for improved sidelobe control. Subaperture processing is then computed and summed over a larger complete aperture. Varying the subaperture size results in a tradeoff between increased 3 dB target response width and sidelobe control.

### 3.2.2 Narrowband RF Tomography

Tomography spawned from the medical community in the early 1970’s as a noninvasive means of generating 2D cross-section imagery of the human body [75]. X-ray CT (computed tomography) scanners usually use a radiation source paired with a detector array to gather projections of the tissue’s scattering density over a variety of angles. Using the Radon transform, a collection of these projections can be combined and processed to generate a 2D representation of the body to be imaged. Long after SAR’s initial development, it was realized that the signal processing algorithms and resolution analysis would benefit from a tomographic treatment [76, 77], with some
important distinctions: first, radar imaging is a coherent technique where both magnitude and phase of scattering centers are measured; second, the projection axis is perpendicular to the radial direction from the radar, such that the measured response is the integral of all scatterers in each range bin.

Narrowband tomography, on the other hand, resolves solely on the basis of Doppler or change in range between multiple sensors. A full 2D image can be generated with two monochromatic, phase coherent rows of sensors, centered around a single scene. As long as the apertures are aligned in orthogonal planes, apertures reveal range and cross-range information. Mensa et al. first developed coherent, narrowband RF tomography for an equivalent geometry of a rotating turntable scene and stationary sensor under the far-field plane wave assumption [2], demonstrating an upper bound on resolution of $0.2\lambda$ with a full 360° planar aperture and -8 dB peak sidelobe levels. Rather than sampling an annulus of the target’s k-space representation [6,76], a long but thin circular ring in the spatial frequency domain is recorded. The bistatic equivalence theorem [78] is used as a means of changing the effective illuminating wavelength [79], with close match in simulation and experimental results. The sidelobe levels and alternative inverse Fourier transformation algorithms were also investigated [80]. A thorough treatment of his work, including limitations on DFT image formation, can be found in *High Resolution Radar Imaging* [81]. Further analysis of the point spread function and its sidelobes from both multi-tone operation and incoherent subaperture approaches were then analyzed [3,4]. Investigations of extension of the previous ideas to networks of sensors, either wide or narrowband, are presented in a series of publications [82–84].

Ultimately, the narrowband imaging concept has been theoretically validated as
a means of producing 2D imagery by simply exploiting Doppler, or spatial diversity, without the overhead of costly, high-bandwidth radar sensors. Note that these techniques all assume perfect knowledge of the relative motions between the target and sensor. In addition, an experimental trial demonstrating a proof-of-concept for a stripmap collection, rather than a controlled turntable experiment, still remains to be seen, with the exception of the author’s publication [12] and the results of Chapter 4.

3.3 Bistatic Imaging

The general theory of bistatic SAR imaging is closely related to that of monostatic SAR, but implementation of such systems requires overcoming some additional unique challenges, often unnecessary in monostatic systems [85]. The inherent separation in the transmit and receive portions of the bistatic radar system requires synchronization, both in time and frequency, such that the collected data is coherent throughout the processing interval. These means of synchronization affect the design and performance achievable in terms of resolution and the maximum integration period. The same physical separation results in ellipsoidal isorange contours and therefore spatially-variant range and Doppler resolution, which must be compensated for in the image formation procedure to avoid degradation of the final image product.

Fundamental theory, algorithms, and geometric considerations of bistatic SAR are described in Bistatic Radar: Emerging Technology [19] and PhD theses by Rigling [86] and Yates [87]. Rigling’s thesis developed image formation procedures for bistatic SAR, including Backprojection and the Polar Format Algorithm [88]. Scattering models and volumetric reconstruction techniques were also investigated. Yates also adapted range migration algorithm to bistatic geometries and conducted experimental
flight trials using a pair of QuinetiQ radar systems. Some of the challenges of bistatic SAR are addressed with Krieger’s summary paper [85] on bi- and multistatic-SAR which include: timing and synchronization, antenna coordination, and measurement of the physical displacement of the transmit and receive platforms.

As the principal algorithms surrounding backprojection and its approximate variants are fairly well established, much of the recent literature on bistatic SAR addresses issues of time and phase (or frequency) synchronization inherent resulting from separate clocks driving the local oscillators for the transmit and receive platforms. The two predominant methods of coping with these issues are to either use a direct path reference signal, or a GPS controlled timebase [89–94]. Luckily, with passive radar systems, the presence of a reference channel for correlation processing inherently performs the function of time and phase synchronization, as long as the relative geometry between the transmitter and passive radar receiver is known.

To date, there have been few fielded bistatic SAR systems outside of preliminary experimental trials. TandemX, a pair of cooperative imaging satellites in parallel orbits [85,95,96], are commonly found in the literature as a bistatic imaging system, but the separation of the platforms is much less than the distance to the scene, which results in operation closer to interferometric SAR than a true bistatic system. A more recent project, DIFFERENT, considers a more extensive sensor network of low-cost, low System Weight and Power (SWAP) SAR modules with more appreciable transmit/receive separation [97].
3.4 Passive Imaging

Being relevant to the focus of this thesis, recent publications relating to passive imaging (both SAR and ISAR) will be reviewed and discussed more thoroughly than the preceding sections. The contemporary literature will be separated into SAR and ISAR, as there is significant overlap between the techniques, although Chapters 5-9 primarily focus on the ISAR scenario.

3.4.1 Passive SAR

Recent publications surrounding passive SAR primarily fall into one of three categories: investigations of suitability and performance of different illumination sources, mathematical analyses and simulations of the passive imaging, and finally experimental proof-of-concept trials. These three publication areas make up the subjects of the next three subsections. Most of the imaging theory remains the same as that of bistatic radar, whereas the approach to the signal processing may differ due to the factors discussed in Section 2.2 and Section 3.1.

Much of the passive radar literature has focused on various digital television standards and FM radio (primarily due to their abundance and high transmit powers); this trend persists throughout the imaging subset of the passive radar community. Here we consider some preliminary studies and feasibility of various alternative illuminators of opportunity when used for SAR imaging.
3.4.1.1 Satellite Illuminators

Satellite illuminators have a number of advantages when compared to their terrestrial counterparts with passive radar. Because the signal impinges on any targets from above, there are limited shadow regions due to terrain features [98], and the difficulties of detecting high altitude targets with ground-based illuminators are eliminated [99]. This also results in a clean, line-of-sight reference signal no matter where the receiver is sited. In addition, the long standoff range of the satellite helps mitigate the direct signal interference. Assuming a satellite is continually available, a single mobile receiver can continue to utilize the same transmission over a much broader area than a land-based system.

A good overview of the types of satellites and their feasibility for imaging and target detection was conducted by Cristallini [100], showing that high powered geostationary orbital (GEO) satellites, such as digital television with Eutelsat W2A or broadband networking with Inmarsat I-4 EMEA, should be capable of detecting air targets at medium range. The first successful detections of aircraft with a passive satellite system were conducted by DSTO using DVB-S [40]. Soon thereafter, Barott demonstrated similar results using a dish antenna and XM radio satellites [98].

There has been significant research into using navigation (GNSS) satellites for SAR imaging, due to the number of available satellites and deterministic waveform structure, which ease some of the signal processing burden. Because the power levels are comparatively low, the long dwell time required does not allow detection of moving targets but is permissible for imaging stationary targets, as is the case with SAR imaging. Most GNSS satellites sit in a low medium earth orbit of around 20,000
km in altitude. Cherniakov first conducted mock satellite SAR trials by transmitting GLONASS waveforms from a cooperative receiver [101], demonstrating the hardware and signal processing architecture along with preliminary experimental imagery. Later, full-scale experimental SAR trials with a GLONASS satellite show improved imagery and the ability to detect buildings with a stationary receiver and relying on satellite motion for aperture synthesis [102, 103]. Subsequent experimental trials demonstrate the ability to image wider areas with higher powered (but still GNSS) Galileo satellites and a moving receiver [104].

Radar satellites typically possess much higher power levels and, of course, possess ideal pulsed waveforms for radar purposes. At first glance these would be optimal for an space-surface radar or imaging system, but the low revisit rate of such platforms prohibits all but intermittent operation. Nonetheless, there have been investigations into such systems demonstrating excellent results. The first passive bistatic SAR imaging trials with a radar satellite were conducted using Envisat and a ground-based receiver [105] demonstrating the proof-of-concept. Additional experimental trials were conducted using TerraSAR-X, exhibiting a high level of resolution and comparable imagery to the monostatic TerraSAR-X results [106]. An extension of this project tested the abilities of multiple receive channels on the receiving hardware for digital elevation mapping with an interferometric approach and a cooperative transmitter [107]. The Warsaw University of Technology investigated Envisat-1 for passive SAR [108] using the CLEAN algorithm for DSI suppression and experimental results showing coarse imagery [109]. Later trials demonstrated much higher quality imagery using the wider bandwidth and higher center frequency of TerraSAR-X [110, 111].
3.4.1.2 Wireless Internet

WiMax is a form of broadcast wireless internet, defined by the IEEE 802.16 standard, which aims to provide broadband internet coverage to multiple users over a wide area, exhibiting coverage similar to cellular networks. Gutiérrez and Jackson present an overview of the physical layer and signal structure of WiMax networks [43]. Demodulation of the OFDM subcarriers allows the frequency domain of the target scene to be estimated and allows for reconstruction of the scene. Maximum bandwidth for WiMax is 20 MHz, which is significantly greater than other previously considered signals of interest, yielding a down-range resolution of approximately 7.5 m. Potential range ambiguities can be seen when using the preamble portion of the signal, caused by repetitive features which manifest as multiple peaks from the correlation response after matched filtering. Another potential challenge using WiMax and cellular signals is the sectorized transmission structure which broadcasts on different frequencies at different locations.

An investigation into WiFi signals and cancellation methods to obtain a clean reference signal is presented by Falcone [112, 113]. Cross range profiles from a passive ISAR experiment are presented along with the cancellation methods, and show increased preservation of the target response over previous cancellation methods.

3.4.1.3 Digital Television

The previous few years have seen incredible advances in DTV Passive SAR using low-cost, airborne platforms. The Warsaw University of Technology demonstrated the first SAR imagery using Range-Doppler imaging on board an aircraft with an 8
MHz DVB-T transmitter [114], further refined and shown alongside ISAR imagery of nautical targets in [115]. These results were further focused and show a high correlation to airborne optical imagery, further validating the technique [116]. A well-focused SAR image was also produced by the Swedish Defense Research Agency using a DVB-T transmitter and an airborne, modified CARABAS-II receiver [117]. This paper also shows results of a bistatic dual-airborne imaging platform operating in the VHF regime.

### 3.4.1.4 Other Waveforms

This subsection focuses on other experiments conducted in a controlled environment using waveforms similar to that of many passive radar illuminators of opportunity. In 2010, Maslikowski and Kulpa presented a bistatic quasi-passive SAR using commercial off-the-shelf (COTS) parts and a linear rail [118]. The results show modest quality imagery using 36 MHz of bandwidth and digital pseudonoise waveforms. A number of experiments demonstrated Doppler beam-sharpened imagery at close range using cooperative bandwidth- and frequency-scaled version of WiMAX, LTE, DAB, and DVB-T signals [43, 119, 120].

### 3.4.1.5 Mathematical Methods.

Much of the mathematical treatment for passive SAR stems from a generalization of the Doppler imaging papers presented in Section 3.2.1 to bistatic geometries. Yarman and Yazici propose a passive SAR concept from the Doppler imaging approach investigated in Section 3.2.1, but with a static transmitter and two moving receivers [70, 71, 121–123]. The received signals from the two airborne platforms are
cross-correlated, and imagery is formed through standard filtered backprojection. The authors do not make any attempt to quantify performance compared to a more conventional architecture consisting of a single receiver with dedicated reference and surveillance channels. The model also neglects any direct signal interference terms, which dominate any receiver in practice and would likely wash out the image results if the technique were to be applied directly. In addition, the overhead of twice the positional measurement equipment, the communication link between platforms, and double the number of airborne receivers renders the concept very impractical, and was not further investigated. A more standard approach was presented by Wang for the case of a mobile transmit/receive pair in *Bistatic SAR Imaging using UltraNarrow-Band Continuous Waveforms* [72].

### 3.4.2 Passive ISAR

As will be demonstrated, there have been a number of investigations into passive ISAR imaging, but very few experimental results with airborne targets. Initial simulated investigations into ISAR imaging will be reviewed first, followed by an overview of experimental trials to date.

Some of the initial studies regarding passive ISAR were performed by Lanterman, Munson and Wu [124–129]. These papers demonstrate the theoretical feasibility of leveraging multiple narrowband signals over wide angles to create high resolution 2D imagery, allowing for classification, and possibly identification, of mid-size jet aircraft. The data were generated using the Method of Moments (MoM) by the Fast Illinois Solver Code (FISC [130]) and a CAD model of an X29 aircraft. These simulations were implemented by assuming that the bistatic equivalence theorem [78] holds
for bistatic angles exceeding 90°. This assumption is a significant limitation since bistatic equivalence is only valid for moderately small bistatic angles with complex targets [131]. The authors also include geometrically adverse situations which, in practice, would prohibit passive radar operation due to overwhelming direct signal interference, e.g., when a target flies through the baseline separating the transmitter and receiver. The papers consider an extremely large number (up to 21) different signals of opportunity, far beyond the number of simultaneous receptions of current state of the art passive radar systems. In addition, the exact target trajectory and any phases imparted by the receiving hardware are assumed to be known, thus allowing all signals to be used coherently with one another. Unfortunately, this will never be the case in practice for non-cooperative targets whose trajectory must be estimated. A variety of other factors, such as propagation, time-varying multipath, and typical broadcast transmitters mounted on tall masts (often experiencing significant sway due to wind loading), further perturb the phase of the measurements. Existing autofocus algorithms can only focus a single transmitter for coherent processing intervals which are orders of magnitude shorter than the flight paths assumed by the authors. The approach to the fundamental passive radar processing (to retrieve the k-space samples) are also ignored, which makes up a very significant part of any experimental system. However, the work is a valuable contribution in the demonstrative sense that multistatic passive ISAR is theoretically possible given a moderately realistic target model (MoM simulation).

The early works of Wu and Lanterman et al. present reconstruction based on a subaperture approach and alternative time-frequency reconstruction techniques, such as the smoothed pseudo Wigner-Ville distribution (SPWVD) [124, 125, 127].
Although these approaches appear to increase the clarity of the imagery, they cannot be implemented without first successfully implementing autofocus and cohering the various transmitters. The CLEAN algorithm was also proposed as a form of super resolution and means of revealing low-lying target features [126]. A method of region-enhanced image formation was investigated by solving an optimization problem with an iterative algorithm based on half-quadratic regularization [129]. There have been a few publications that consider receiver positioning for multistatic imaging [132], considering resolution, sidelobe levels, and SNR, along with analyses optimizing ISAR imagery due to ISAR motions [132–134]. Some simulated analyses of isotropic and non-isotropic scatterers observed with a high number (25 and 103) of FM transmitters show different results for coherent and non-coherent image formation procedures, although the final image is not recognizable [135].

There are very few successful experimental results of passive ISAR imaging of airborne targets. The first published result by Suwa [9] was produced using a six-channel transmitter with 36 MHz aggregate bandwidth – far wider than what is typically found for ATSC or DVB-T broadcast sites. The stages of processing are not described in detail and the final image result appears to simply be a range-aligned, extended time, range-Doppler map, with a target extent of 200 m, far longer than the reported target length of 73 m, suggesting that the image formation processing was not properly applied. Further work was performed by Nakamura [136] with an aggregate bandwidth of 48 MHz, but the final image is still not recognizable, as the autofocus method was simply a coarse line fit to the target’s range-time history. A performance prediction and simulation for passive ISAR using geostationary satellites was performed by Pastina in [137].
Olivadese et al. demonstrates the feasibility of passive ISAR, using DVB-T signals with a signal level simulation and point scatterers in the shape of a large ship [138]. The approach was then extended to full-scale experimental trials using a set of three contiguous DVB-T transmitters on a single mast, for an aggregate bandwidth of 24 MHz [8]. Although this situation arises rather frequently in Europe, due to the single frequency network scheme of DVB-T, multiple adjacent frequency channels on a single common transmitter mast are very rare in the American regions for similar resolutions. The authors show successful imagery of both large freight ships and aircraft, with varying levels of resolution. Martorella describes a thorough theoretical framework for a single transmitter and receiver closely spaced in angle along with experimental results of imaging sea and air targets [10]. This represents the most comprehensive treatment of ISAR imaging by far, but does not include using multiple transmitters for target tracking and thus cross-range scaling from Doppler measurements. These results are promising, but the bandwidth available for these successful trials is not representative of passive radar coverage throughout most of the world.

Bacyzk shows the result of a jet imaged with a single DVB-T transmitter, tracked using the intersection of three separate illuminators [11]. These results demonstrate a coarse image using a single 6 MHz illuminator, but it is not clear if any true target features are visible. Garry et al. demonstrated that by imagery jointly across multiple illuminators, a wider synthetic aperture is formed, which can simultaneously increase the cross-range and down-range resolution [139]. It is clear that the multistatic approach to passive ISAR imaging requires a wideband system capable of multiple reference and surveillance beams. To date, no experimental results have been published that demonstrate passive multistatic imagery from multiple illuminators of
opportunity, neither from one location with significant spectral gaps nor various geometrical locations. With this additional information, the apparent rotation vector can be better estimated and used to scale the cross-range axis from Doppler to a physical cross range dimension, further enabling classification. These methods will be investigated as the topic of Chapter 9.

3.4.2.1 Motion Compensation

ISAR is fundamentally different from SAR in the sense that the target is non-cooperative, i.e. the relative motions between the sensor and target are completely unknown and must be estimated directly from the radar data itself. It would therefore be an oversight not to reference some of the contemporary literature which represent recent advances in motion compensation, due to its important role in ISAR image formation. Motion compensation algorithms generally fall into two categories: the methods which compensate for radial motion (generally applied first), followed by the algorithms to correct for rotational motion.

Two autofocus algorithms for multistatic passive ISAR imaging are presented by Liu and Munson [140]. An entropy minimization approach as well as a parametric autofocus method are applied simultaneously across multiple illuminators to maximize the final image clarity. The authors fail to take into account the additional phase effects of the receiver as well as target coherence, similar to the issues discussed in the previous section, so it would likely not be applicable in practice.

Brisken et al. recently developed a comprehensive approach for motion compensation with a multistatic ISAR scenario [141,142], the first of which [141] deals with a
single transmitter, multiple receiver scenario. The work addresses many of the challenges of an ISAR system, including: correlation, clutter removal, range and velocity estimation, orientation estimation, followed by image formation and autofocus. An experimental 800 MHz testbed at X-band was used with a single transmitter and five distributed receiver nodes. The multilateration from the various bistatic pairs is used to estimate the target trajectory, which can be used (assuming the target orientation tracks the velocity vector) to estimate the apparent rotation. The phase gradient algorithm is then applied to each tx-rx pair to generate independent imagery for each receiver using various motion information.

A similar approach to motion compensation was investigated using a product of each image's entropy, using the same dataset [142]. These experiments and the theoretical developments are valuable approaches for the wideband multistatic case where each bistatic pair can be used to generate high resolution imagery, but the technique is not directly applicable to a spectrally constrained scenario commonly found with passive radar imaging, due to limitations of the translational motion compensation algorithms as shown in Chapter 9.

3.5 Summary

Passive radar is receiving significant attention from the radar community, with a trend towards implementation of advanced methods such as imaging. Significant challenges with passive radar remain due to its narrowband nature and bistatic geometries. In Chapter 4, the theory of Doppler imaging is shown along with the first experimental results of such a system. Extension of this technique to passive imaging in an ISAR context is then investigated in Chapter 5. Chapter 6 shows the development of a
hardware architecture capable of supporting next-generation passive radar operations, and advances core passive processing algorithms. Chapter 7 provides an extensive evaluation for DSI mitigation algorithms. Chapter 8 discusses the phenomenology of distributed multipath and micro-Doppler, stemming from empirical observations using the developed passive radar testbed. The framework for passive ISAR imaging of air targets is the focus of Chapter 9, demonstrating useful imagery of air targets using a single ATSC illuminator.
Narrowband methods can provide an alternative to conventional wideband approaches for radar imaging. Traditionally, narrowband imaging has been constrained to tomographic imaging which has limited application due to the geometric constraints of the technique. However, as will be discussed here, Doppler imaging using a conventional strip map SAR geometry can produce fine resolutions without the need for wide bandwidths, which is a potentially promising technique for passive radar that often utilizes moderately low bandwidth illuminators of opportunity.

Consider a traditional SAR sensor with a cross-range resolution of 15 cm, which implies a bandwidth of 1 GHz or greater, a sizeable portion of the available electromagnetic spectrum in any one radar band. This requirement for wider bandwidths increasingly has to compete with communications and navigation applications that also are demanding a greater and greater share of the RF portion of the electromagnetic spectrum. As a result, there is increasing pressure for radar systems to share, or even relinquish, their current allocations [143]. With this background of increasing spectral congestion, techniques that have the potential to offer similar image resolutions without having to employ wide bandwidth waveforms are extremely attractive. Such narrowband techniques could significantly improve the prospects for more efficient overall utilization of the electromagnetic spectrum.
The approach examined in this chapter uses Doppler (or phase change over time) as the basis for achieving finer down-range resolution. The Doppler imaging technique makes no requirement on the bandwidth of the transmitted signal and in principle is able to work with monochromatic waveforms. Such an approach can also be embedded within a traditional SAR or ISAR collection geometry. The combination of being inherently narrowband and having a simple collection geometry makes high resolution Doppler imaging a potentially attractive technique for passive radar, and such an implementation will be examined in Chapter 5.

A practical implementation of Doppler imaging will be emphasized, rather than its mathematical framework. The essential construct of the Doppler imaging approach is first introduced in order to explain the engineering principles underlying the concept and to show how this leads to a simple implementation. This provides a foundation for simulation and experimentation as well as a means of exploring the limits of performance. Results from experiments are presented which show remarkable agreement with theory while also highlighting further technical challenges of the approach.

4.1 Doppler Imaging Concept

Consider a traditional sideways looking swath-mapping SAR collection geometry, illustrated in Figure 4.1. This demonstrates how an aperture is synthesized as the radar system, carried by an aircraft, effectively “drags” its side-looking real beam past two targets separated in range. The synthetic aperture in the swath-mapping mode extends for the duration over which the targets remain in the beam. In this example it is set by positions at times $t_1$ and $t_4$, for target 1. As is well known, this length sets the limits on cross-range (along-track) resolution, which is approximately
Figure 4.1: Conventional aperture synthesis showing that targets further in range enter the beam before target nearer in range and exit the beam later equal to half the physical size of the real beam antenna [144]. Target 2 remains in the beam for a shorter duration and the corresponding aperture is formed from positions $t_2$ and $t_3$. Although the synthetic aperture is shorter, the reduced slant range compensates, and the along-track resolution is nearly identical for both targets. In traditional swath-mapping SAR, the synthesized aperture provides no resolution in the radar line-of-sight direction. Downrange resolution is typically achieved via wide band waveforms and pulse compression.

However, there are some important differences between the echo responses from the two targets. By simple geometry, the Doppler frequency response from a target is determined by the aircraft’s radial velocity and radar carrier frequency. For a side-looking sensor shown in Figure 4.1, the Doppler frequency, $f_D$, is

$$f_D = \frac{2v}{\lambda} \sin (\phi) \quad (4.1)$$
Where:

\[ v = \text{the sensor velocity} \]
\[ \lambda = \text{the wavelength} \]
\[ \phi = \text{the angle measured from the side-looking direction, seen in Figure 4.1} \]

Because the real antenna of the imaging radar is fixed and side-looking for the swath-mapping case, this restricts the radial velocity as a target’s response is measured, and thus limits the measured Doppler frequency. For a real antenna of beamwidth \( \theta_{BW} \) degrees, as in Figure 4.1, the extent of \( \phi \) is limited to \( \pm \theta_{BW}/2 \). Therefore, the maximum Doppler frequency measured by the radar is

\[
f_{D_{Max}} = \frac{2v}{\lambda} \sin \left( \frac{\theta_{BW}}{2} \right)
\]

(4.2)

When a target enters (or exits) the radar beam, the Doppler frequency is \( f_{D_{Max}} \) (or \( -f_{D_{Max}} \)). However, due to the different range positions of the two targets shown in Figure 4.1, and the slanted real beam edges of the radar, the two targets will remain inside the -3 dB beam pattern of the antenna for different durations. Therefore, the frequency response of each target will have equal Doppler extent, determined by the beamwidth as in Equation (4.2), but expanded or compressed in time due to the down-range position. This results in two linear frequency, chirp-like functions, with different frequency slopes, as seen in Figure 4.2. In other words, this separation results in a time dilation of the Doppler response of one target with respect to the other as a direct function of range and provides the basis for resolving the two targets in range. The Doppler response is well approximated using the typical linear function for
Figure 4.2: The time dilation of the FM Doppler echoes for two targets separated in down-range as shown by the solid and dashed lines. The Doppler span is the same for both targets and is determined by the beamwidth, frequency and velocity swath-map SAR, and can be realized by approximation of the trigonometric function of $\sin(\alpha) \approx \alpha$ for $\phi$ in Equation (4.2) and relating $\phi$ to sensor position over time.

The Doppler slope is related to the target’s phase response as the differential of the instantaneous phase. Consider the furthest range target, shown by the solid line. The measured phase response takes the usual quadratic form and it is this that is used to form a synthetic aperture. The phase response from target 2, which is nearer in range, is also a quadratic but differs in its parameterization. In terms of the imaging geometry, this comes about due to the dependence of the rate of phase change on the different distances between the illuminating radar system and the target. In other words, the phase values measured at each point in the synthesized array will be progressively different solely due to the two targets being at different ranges.

The degree of time dilation between the two FM responses corresponds to the difference in radial range of the two targets. This difference in the Doppler frequency-
time slope enables resolution of the two targets in range. This can be accomplished by collecting data in a normal swath-mapping SAR mode and processing separate time-dilated synthetic apertures each centered on the two targets. In this way, the combination of differential aperture synthesis specific to different target ranges forms the basis of the Doppler imaging technique. It also provides a relatively straightforward means of implementation that simply exploits current SAR processing approaches.

4.2 Doppler Imaging Theory

4.2.1 Derivation of the Technique

Having described Doppler imaging in conceptual terms, the generation of resolution in range and cross-range is now examined in more detail. To illustrate the principles, the geometry is restricted to the two-dimensional x-y plane, although the following constructs can be extended to three dimensions. Implications of this are highlighted in the resolution analysis of Section 4.3.

Assume a radar system is traversing a path along the x-axis, i.e., in the positive \( \hat{x} \) direction with a velocity, \( v \). This is illustrated in Figure 4.3. The sensor position is denoted \( s(t) = (vt, 0) \). The real beam, of beamwidth \( \theta_{BW} \), points in the \( \hat{y} \) direction.

To investigate the down-range dependence of the Doppler response, consider a target located along the y-axis. This target, at position \( r(y) = (0, y) \), will therefore lie within the 3 dB beam pattern, approximately, in the interval given by:

\[
-\frac{y}{v} \tan \left( \frac{\theta_{BW}}{2} \right) < t < \frac{y}{v} \tan \left( \frac{\theta_{BW}}{2} \right)
\] (4.3)
Figure 4.3: Doppler imaging geometry

Here only targets at zero cross range are considered. However, the technique can be readily generalized to targets with an arbitrary cross-range coordinate, $x$, which will be delayed in time by $x/v$ assuming a full stripmap collection geometry. From (4.3), it can be seen that the duration over which the target, $r(y)$, is illuminated is a linear function that increases with down-range position, $y$. The target entering and exiting the real beam delimits the total extent of the Doppler spread as given by (4.2). The instantaneous Doppler frequency, $f_D$, for a point $r(y)$ is given by (4.4), by substituting $\phi = \tan^{-1}\left(\frac{vt}{y}\right)$ into (4.1).

$$f_D(t, y) = \frac{2v}{\lambda} \sin \left(\tan^{-1}\left(\frac{vt}{y}\right)\right)$$  \hspace{1cm} (4.4)

In order to satisfy the Nyquist criteria, the echo signal must be sampled at a PRF of at least twice the maximum Doppler given by (4.2). A set of matched filters, $m(t, y)$, each matched to the phase response of a target at each down-range position,
where \( k \) is the wavenumber, \( 2\pi/\lambda \). The duration of each matched filter will be restricted to the time that a point at a given down-range position, \( r(y) \), remains in the beam as determined by (4.3). The phase of the complex exponential resulting from the changing range to the target, \(-2k \| r(y) - s(t)\|\), can be well approximated as a quadratic function [5]. The derivative of this quadratic phase over time yields the linear Doppler slope for each down-range position \( y \), given by (4.4).

The reflectivity function of the scene is then constructed by the correlation against the baseband received waveform, \( w_{rx}(t) \), inherent in matched filter processing. The received signal results from the convolution of the transmitted narrowband waveform, \( w_{tx}(t) \), with the illuminated portion of the complex scene reflectivity function \( \Psi(x,y) \). Mathematically this can be represented as

\[
\psi(x,y) = \int_{\tau_1}^{\tau_0} w_{rx}(t) m^*(t - \frac{x}{v}, y) \mathrm{d}t
\]

where \( d(t,x,y) = \|s(t) - (x,y)\| \) is the instantaneous range to point \((x,y)\) within the target scene, and \( c \) is the speed of light. To reconstruct an estimate of the scene’s reflectivity function, the matched filters are correlated against the received waveform from (4.6). The time delay of this cross correlation, \( x/v \), determines the cross range position as follows:

\[
\psi'(x,y) = \int_{\tau_0}^{\tau_1} w_{rx}(t) m^*(t - \frac{x}{v}, y) \mathrm{d}t
\]
where:

\[ \Psi'(x, y) = \text{estimated scene reflectivity at point } (x, y) \text{ on the ground} \]

\[ w_{rx}(t) = \text{the baseband received signal over the synthetic aperture formation} \]

\[ m^*(\cdot) = \text{complex conjugate of matched filter } m(t, y) \]

\[ \tau_o = \text{time at which point } (x, y) \text{ enters the beam} \]

\[ \tau_1 = \text{time at which point } (x, y) \text{ exits the beam} \]

In this way the time-varying Doppler echo is integrated for a single location on the ground in both range and cross-range thus forming a single pixel. By varying the filter \( m(t, y) \) and correlating against \( w(t) \), a line of pixels in down-range can be generated. By concatenating multiple, sequential synthetic apertures over a single flight trajectory, each formed for a different down-range position, a complete two-dimensional image may be created. This final image is the result of the true scene reflectivity \( \Psi(x, y) \) convolved with the two-dimensional point spread function of the Doppler imaging response, determined by the frequency and beamwidth of the illuminating antenna.

It is important to note that no assumptions have been made about the bandwidth of the illuminating signal. In principle a pure tone, or CW waveform, could be used — (4.3) through (4.7) solely depend on the Doppler or phase of the echo signal. Alternatively, the technique may also be considered as a method to obtain radar image resolutions finer than those implied by the signal bandwidth. The method described above would be applied to sections of the received signal with extents up
to the width of the pulse. Targets with spacing less than the $c/2B$ bandwidth-limit could then be resolved.

### 4.2.2 Simulation

Simulation results are presented here to examine imaging performance using the narrowband Doppler technique and to investigate the imaging process in greater detail. In the simulation, two point targets are located at the same cross-range coordinate but at different down-range positions as shown in Fig. 4.4. The two targets are separated by just 60 cm in the down-range direction. A radar system with an operating frequency of 8.9 GHz transmits a sequence of unmodulated pulses, 0.17 $\mu$s in duration. This renders the effective down-range resolution to be 25 m, much coarser than the separation of the two targets in down-range. The radar moves along a straight trajectory in order to synthesize a 2 m long aperture, at a range of 2 m from the nearest target, using a swath-mapping SAR geometry. The seemingly short range was chosen for the simulation to match the measurements described in the next section.

Although the target lies in the far field of the antenna throughout the collection, the substantial size of the scene relative to the radar range will result in significant wavefront curvature across the target scene. The results of this will be discussed in Section 4.2.3 regarding the resolution analysis. The 3 dB beamwidth of the radar antenna is just over 50°, sufficiently wide to ensure that the closest target is continuously illuminated over the length of the synthetic aperture. The illumination frequency and the real beamwidth determine the minimum sampling distance required at which the radar sends and receives signals to avoid aliasing (identical to the slow-time sampling requirements of swath-mapping SAR). For the simulation reported here, the radar
Figure 4.4: Delta target locations for Doppler imaging simulation

sends and receives signals every 1 cm along the synthetic aperture path, well within the Nyquist limit ($\lambda/2\theta$) of approximately 2 cm.

In order to resolve the targets spaced by 60 cm, the Doppler imaging technique must resolve in down-range much below the 25 m range resolution implied by the pulse length. Each returned echo within the synthetic aperture is matched filtered and sampled at the time a signal would take to propagate the two-way path from the radar to the scene center. The radar pulse is sufficiently long such that the sampled value consists of overlapping returns from all illuminated targets within the scene. As a consequence, the Doppler information for all targets is contained in a single complex sample at each radar position. The simulation proceeds by synthesizing an aperture that uses the entire $50^\circ$ real beamwidth followed by processing which focuses the aperture on a range-dependent basis.

More specifically, the simulation first generates a library of reference functions for the filters, $m(t, y)$, matched to each down-range location. The radar moves along
the “flight path” and collects the received waveform, $w(t)$, by matched filtering and sampling the reflected copies of the transmitted pulse for each pulse location along the synthetic aperture. Once $w(t)$ has been collected, the received data is correlated against the bank of matched filters to generate the final image.

The matched filter characteristics for the different range positions are now examined in more detail to provide further insight into the way in which the Doppler imaging algorithm operates. The filter characteristics that are matched to the target’s phase response of positions at 2 m and 2.6 m in down-range, are shown in Figure 4.5.

Figure 4.5 shows how the effective time dilation of the phase of the echo returns varies with down-range position and thus manifests as series of nested quadratics. This difference in the quadratic phase functions enables the two targets to be resolved in down-range to a much finer degree than implied by the pulse bandwidth. The solid curve represents the echo response focused to the 2 m range position and the dashed
Figure 4.6: The phase and magnitude response history for the two targets trace the 2.6 m position. The closer of the two targets exhibits a more rapidly varying phase response and therefore a steeper Doppler slope, consistent with that shown in Figure 4.5. Over the full synthetic aperture, with a SNR of 10 dB, the two targets yield the combined magnitude and phase response shown in Figure 4.6.

The differing positions of the two targets cause the received echoes to go in and out of phase over the length of the synthetic aperture and hence exhibit constructive and destructive interference. This is manifested as envelope fluctuations that are evident in the magnitude response seen in Figure 4.6. The series of time dilated Doppler filters, $m(t, y)$, for each down-range position are next correlated against this combined phase history to form the final image as shown in Figure 4.7.

Figure 4.7 (a) shows the two targets to be correctly located in down-range and clearly differentiable, thus demonstrating that the Doppler imaging technique as a method of achieving down-range resolution from multiple narrowband measurements.
Figure 4.7: (a) Reconstructed image using the Doppler imaging algorithm (b) Down-range cut of target scene (c), (d) Cross-range cuts of targets at 2 and 2.6 m positions, respectively
However, as is also clear from Figure 4.7, there are a number of imaging characteristics that differ from the more traditional wideband imaging case. The filtering process yields a point spread function (PSF) response with significant sidelobes, cuts of which are shown in Figure 4.7 (b)-(d). These sidelobes can be seen jointly in cross-range and down-range in a diagonal direction, tapering off less quickly than those of a conventional SAR imaging system. This is seen in Figure 4.7 (c) and (d), where the sinc-like single target response is superimposed with the opposing target’s diagonal sidelobes, resulting in the cross-range shelf-like structure. This sidelobe behavior is a function of the imaging process operating over such an extended angular range. This will be re-examined in the next section, when the limits on down-range resolution are presented. In principle, these sidelobes can be reduced using appropriately designs weighting functions, multiple discrete frequencies, or alternate image reconstruction techniques. However, here we concentrate on the basic principles underlying the Doppler imaging technique and next use experimentation to verify the results of the simulation.

### 4.2.3 Resolution Analysis

In this section, the limits on range resolution are identified and are found to be dependent solely on the collection’s angular span and carrier frequency. The resolving capability of the Doppler technique can be examined by investigating the representation in the two-dimensional Fourier domain [7], or k-space, as is often done for SAR and tomography. This approach not only helps to gain a better understanding of the imaging process but also allows prediction of imaging performance as a function of illumination parameters.
Each narrowband echo measurement represents a single sample of k-space at a radius of $2/\lambda$ and angle determined by the position of the sensor [6]. For example, a finite aperture in swath-mapping mode will sample an arc in k-space. The length of the arc is proportional to the angular span of the synthetic aperture. For a fixed standoff range, the longer the arc, the longer the synthetic aperture and the higher the cross-range resolution [76]. The radial width of the arc is directly related to the bandwidth of the illuminating signal, and conventionally, the larger the bandwidth the finer the down-range resolution.

This is illustrated schematically in Figure 4.8 for the case of a large synthetic aperture and a narrowband illumination signal, representative of a typical Doppler imaging case. In wideband SAR, the angular length of the synthesized aperture tends to be much smaller than that shown in Figure 4.8 and the radial width of the arc much wider. That is to say, the shape of the area in k-space is very different and this highlights the most apparent difference between swath-mapping SAR and
Doppler imaging. For both SAR and Doppler imaging, the cross-range resolution results from the length of the synthetic aperture in $k_x$. Conversely, unlike SAR where the down-range resolution is a product of the bandwidth causing extent in $k_y$, in Doppler imaging the extent in $k_y$ is a product of the Fourier curvature of the synthesized aperture. This gives the narrowband sampling pattern a component in the down-range direction. The dependency of both dimensions on the synthetic aperture accounts for the sidelobes exhibiting a coupling in down- and cross-range.

By representing the imaging data in the Fourier domain, equations for cross-range and down-range resolution can be determined based upon the geometry of Figure 4.8. The sampled span of a target’s two-dimensional Fourier response, for the in-plane sensor depicted in Figure 4.1, is shown in Equations 4.8 and 4.9. For many scenarios, a sensor may be required to be located outside the x-y plane, and the depression angle, $\psi$, will cause a compression of $\Delta Y$ by $\cos \psi$, due to the geometrical projection of the three dimensional k-space representation onto the $k_x$-$k_y$ plane [6].

$$\Delta Y = \frac{2}{\lambda} \left( 1 - \cos \left( \frac{\theta_{BW}}{2} \right) \right)$$ (4.8)

$$\Delta X = \frac{4}{\lambda} \sin \left( \frac{\theta_{BW}}{2} \right)$$ (4.9)

Where:

$\Delta Y$ = the $k_y$ occupancy in k-space, inversely related to down-range resolution ($m^{-1}$)
\( \Delta X \) = the \( k_x \) occupancy in k-space, inversely proportional to cross-range resolution (m\(^{-1} \))

\( \lambda \) = the wavelength of the illuminating radiation (m)

\( \theta_{BW} \) = the angular extent over which imaging takes place (rads)

Equations (4.8) and (4.9) can be inverted to yield the final expressions for down-range and cross-range resolutions. These quantities can be thought of as a form of spatial bandwidth in both the range and cross range dimensions, consistent with the analysis of Wang [145]. The cross-range resolution specifies the peak-to-null width, while the down-range resolution represents that of the -4 dB Rayleigh resolution. This is a result of the band-pass characteristic of Fourier space sampling in a given dimension. For completeness, the expressions for resolution in the down-range and cross-range dimensions are given in (4.10) and (4.11).

\[
\delta_r \approx \frac{1}{\Delta Y} = \frac{\lambda}{2 \left(1 - \cos \left( \frac{\theta_{BW}}{2} \right) \right)} \tag{4.10}
\]

\[
\delta_{cr} \approx \frac{1}{\Delta X} = \frac{\lambda}{4 \sin \left( \frac{\theta_{BW}}{2} \right)} \tag{4.11}
\]

Where:

\( \delta_r \) = down-range resolution (m)

\( \delta_{cr} \) = cross-range resolution (m)

Equations (4.10) and (4.11) readily demonstrate the behavior of Doppler imaging as a function of the radar’s parameters. First, resolution becomes finer in both dimensions.
as the angle increases. Second, both the down-range resolution and cross-range resolution scale linearly with wavelength. Essentially, the longer the wavelength, or the lower the frequency, the lower the resolution. Clearly (4.10) and (4.11) suggest that the drop in resolution that comes from longer wavelengths could be compensated for by increasing the angular extent of the imaging process. However, such an increase may not always be viable.

A keen observer may note the lack of the down-range distance to the target in expressions (4.10) and (4.11), which seems to contradict the Doppler slope explanation of range resolution given in Section 4.1. Clearly, as the range increases, two targets with a fixed down-range separation will have converging Doppler responses. However, the time over which the Doppler response is observed will increase proportionately, resulting in finer frequency discrimination between the two targets’ responses. This causes the range resolution to be independent of range, much as the cross-range resolution of SAR is invariant to this effect.

The expressions are only accurate as long as the target has an echo response whose underlying radar cross-section persists over the entirety of the angular span of the data collection. As such, there is a dependency on the coherency properties of the target that will affect imaging performance. This is true of all SAR systems and the resolution achieved in traditional SAR will vary throughout the image based on the dominant target scattering mechanisms. This is a characteristic that has only tended to be revealed in wide angle SAR imaging [146]. Without target persistence, processing over wider angles will cause image degradation.

Note that the Fourier domain analysis assumes plane waves across the scene of interest. This will clearly not be the case for the simulated results of the previous
Figure 4.9: Down-range resolution vs. angular span for an illuminating frequency of 8.9 GHz

section, a 2 m range to the scene center and 4 m² image. However, these expressions are being used to examine the resolution of a point spread function for a single target, which is confined to an area much smaller than the entire scene. At this scope, the plane wave approximation can be made with much less deviation from the true case. For this reason, the k-space analysis is a valid technique to investigate resolutions even when the plane wave assumption does not hold. As the portion of the image being considered expands, the curvature of the wavefronts will have an increasing effect on the result, e.g. the broader complex sidelobe response. Comparison between the theoretical resolution limits and those measured in simulation will further verify this concept.

Figure 4.9 shows a plot of down-range resolution as a function of angular span for an illumination frequency of 8.9 GHz. The two curves represent the range resolution measured from the Doppler imaging simulation and that which is predicted by (4.10).
This shows that very fine resolutions can be achieved over large angular spans, but this will be limited by the target’s response under such scenarios. Figure 4.9 also shows that resolutions as fine as 1 m can be achieved with more modest angular spans.

4.3 Experimental Results and Analysis

4.3.1 Setup

A series of experiments have been performed to validate the Doppler imaging concept using parameters closely matching the previous simulation. Three separate experiments were performed: (a) single target at 2m range (b) single target at 2.6 m range (c) two targets at 2 and 2.6 m range, at the same cross-range position.

The single target experiments demonstrate the point spread function and range discrimination capabilities of the Doppler imaging technique, while the two target collection exhibits the resolving potential of the method. The targets were trihedrals with a side length of 22 cm. A stationary X-band horn antenna with a 3 dB beamwidth of 50° was placed approximately 2.3 m from the center of a linear positioner holding the target structure. The positioner moved the two targets a total distance of 2 m in cross-range to form the (inverse) synthetic aperture in 1 cm increments, the same spatial sampling used in the previous simulation. A schematic of the experimental setup is shown in Figure 4.10.

Although this is the inverse of the simulated result above, the relative motion between targets and antenna is the same. Performing the experiment in this manner eliminated phase errors due to moving cables and simplified background subtraction.
The measurements were taken using a single tone generated at a frequency of 8.9 GHz. The reported measurements are shown after the background (a DC value) has been removed.

### 4.3.2 Results

The measured phase response for experiments (a) and (b), single targets at 2 and 2.6 m down-range, are shown in Figure 4.11. The graph shows both the measured target phase response vs. the ideal simulated response of the previous section in Figure 4.5.

Figure 4.11 shows a striking similarity between the measured and simulated phase responses from the two trihedral targets. The measured curves begin to diverge slightly from the ideal matched filter versions at the extremes of the synthesized aperture. This most likely occurs as a result of the trihedral RCS pattern differing from a perfect point scatterer at larger angles. However, the target response persists...
Figure 4.11: Simulated vs. experimental phase response for target at 2 m and 2.6 m with little reduction in amplitude (approximately 3 dB) over the large angular span of approximately plus and minus 25°. It is this large angular span that allows the Doppler imaging technique to resolve between the two targets in down-range.

The simulated and measured phase responses can be converted to instantaneous frequency plots as shown in Figure 4.12. The Doppler response was calculated as the differential of the phase response shown in Figure 4.11. Increased dispersion from the simulation is shown at the extents of the linear rail. This effect was most likely due to a combination of the signal power being reduced at the edges of the horn antenna’s radiation pattern and the reduction in target RCS at a large aspect angles. To reduce this effect, the data was low-pass filtered as shown in Figure 4.13. Figure 4.11, Figure 4.12 and Figure 4.13 show that the phase and Doppler responses closely matched those simulated.

The raw data collected from both the 2 m and 2.6 m targets of Experiments (a)
Figure 4.12: Instantaneous target Doppler response

Figure 4.13: Instantaneous target Doppler, after running average
Figure 4.14: (a) Image of trihedral at range of 2 m (b) Down-range cut through target response (c) Cross-range cut at 2 m (d) Cross-range cut at 2.6 m

and (b) was next used to form imagery. The image formation process was identical to that used to produce the image shown in Figure 4.7.

Figure 4.14 shows the result of Experiment (a), an image of the trihedral target located at a range of 2 m, along with down-range and cross-range cuts through the peak target response. The figure shows that most of the target energy was, as expected, concentrated at a range of 2 m. The down range cut also shows the lack of nulls in the range profile for the target, contrary to the sync response which would be seen from a typical SAR platform which generates range resolution through pulse compression. Note that the cross range resolution does exhibit a sync-like response,
shown in Figure 4.14 (c), while the 2.6 m range of Figure 4.14 (d) shows a cut through the diagonal sidelobes, at a moderately high level of -10 dB relative to the peak. This high level sidelobe response could mask weaker targets and is a limiting factor for Doppler imaging’s effective dynamic range, methods of which will be discussed in the conclusions of Chapter 10.

As seen in the simulated result of Section 4.2, there was a joint cross- and down-range distribution of the sidelobe response. However, there are differences in detail from the simulated case with energy being “smeared” over a broader area. The cause behavior is thought to have arisen due to complex EM interactions between the trihedral target and the positioner. These structured phase perturbations, seen in Figure 4.12, result in phase mismatch between the matched filter from an ideal point target, which cause a less defined sidelobe pattern when compared to the simulated case, assuming a point target and additive Gaussian noise. The achieved range resolution was obtained by measuring the 3 dB points of the target response and was 22 cm for the case where the target was located at the 2 m down-range position.

The image resulting from Experiment (b) is shown in Figure 4.15, with a target located at a down-range position of 2.6 m. Figure 4.15 shows a similar response to that of Figure 4.14 and most energy was again concentrated at the target range with sidelobes smeared in both spatial dimensions. The 3 dB range resolution for this image was 25 cm, slightly coarser than the 22 cm measured for the 2 m target. This result was expected, since the nearer range target was illuminated over a slightly larger angular span and hence was expected to have a slightly finer range resolution. This particular result is a direct consequence of a finite synthetic aperture length imposed by the linear target positioner for the experimental setup. For a continuous,
Figure 4.15: (a) Image of trihedral at range of 2.6 m (b) Down-range cut through target response (c) Cross-range cut at 2 m (d) Cross-range cut at 2.6 m
or unlimited, collection where all down range positions are illuminated over the full antenna beam pattern, as shown in Figure 4.1, the down range resolution would be equal.

In Experiment (c) both trihedral targets were simultaneously imaged, one located at 2 m and the other at 2.6 m down-range from the radar. The rear target was positioned slightly above the front target to avoid any shadowing over the formation of the inverse synthetic aperture. Figure 4.16 shows the resulting image with the two targets clearly observable and resolved. The 3 dB range resolution of the target positioned at 2 m was 23 cm and for the target at 2.6 m was 35 cm. Again, the 3 dB resolution of the far target’s response was found to be slightly broader than that of the closer target. Figure 4.16 shows good agreement with the simulated results depicted in Figure 4.7, although as for the earlier experiments the sidelobe pattern was less structured. Comparing the cross-range cuts for the two target case with Figures 4.14-4.15, it is now clear that the cross-range cut behavior seen in Figure 4.16 (c) and (d) consists of both the sync response of the target at that position as well as the opposing target’s diagonal sidelobes. Overall, it can be concluded that the Doppler imaging method can be used to construct imagery with a range resolution that vastly exceeds that implied by the pulse bandwidth if the target motion is known or can be well estimated.

4.3.3 Variations in observed resolution

Consider Experiments (a) and (b) of the two single point targets at down-ranges of 2 and 2.6 m. The range resolution predicted for the 2 m target case was governed by the angular span of 53° which had a predicted range resolution of 16.1 cm, using
Figure 4.16: (a) Image of two targets at 2 and 2.6 m down-range (b) Down-range cut of target scene (c) Cross-range cut at 2 m (d) Cross-range cut at 2.6 m
The target positioned at 2.6 m had an angular span of 42° and therefore should exhibit a down-range resolution of 24.3 cm. These predicted range resolutions are both finer than that experimentally measured values of 24 and 28 cm. This was attributed primarily to the natural windowing in the collected data, caused by the tapering of power at the edges of the antenna beam pattern. This “windowing” in the synthetic aperture dimension broadened the main lobe of the target response in both the cross- and down-range directions, while simultaneously reducing the sidelobe levels.

4.4 Monostatic Narrowband Imaging Conclusions

Overall, Doppler imaging is a potentially promising technique for achieving down-range resolution from narrowband measurements. For situations in which the target reflectivity remains correlated over a large angular span, the conventional $c/2B$ expression for down-range resolution can be significantly improved upon—by an order of magnitude in the examples presented. A basic theoretical model and explanation have been presented to demonstrate the key underlying mechanisms that provide range resolution beyond that of the conventional bandwidth limitation. The resolution for the technique is proportional to the wavelength of the signal, with longer wavelengths giving coarser resolutions, and inversely proportional to the angular span of the observation, with larger spans yielding finer resolutions.

Simulations have been used to demonstrate that the narrowband technique is able to resolve in both down-range and cross-range, and to identify the resolution limits in both domains. The theoretical expressions for resolution exhibited close correspondence with simulated results. Experimental results verified the validity of
the concept and showed excellent agreement with the theory. Through utilization of
the advocated techniques there is potential to use inexpensive, compact hardware for
radar imaging on any number of moving platforms.
Chapter 5: Multistatic Narrowband Imaging

Now that the narrowband Doppler imaging technique has been established for mono-
static high frequency systems, the technique can be evaluated under circumstances
more analogous to a typical passive radar ISAR imaging scenario, where a set of fixed
transmitters illuminate an airborne target. The effects of the multiple illuminators
and bistatic geometries, along with their effects on image formation and resulting
features can then be examined.

A majority of the current literature for passive ISAR imaging, both theoretical
and experimental systems, only exploit a single channel or transmit site [8,9,103,109].
Work has been done by Olsen to concatenate multiple channels from a single location
into one contiguous band [147], but the concepts are only developed to focus to a single
point on the target and thus are not suitable for imaging. The authors in [129] show
that high resolution imaging is possible using multiple analog television illuminators,
but the number of signals simultaneously exploited is far greater than any current
systems’ capabilities. Full coherence and synchronization between all channels is
also assumed, which requires system calibration and geometrical knowledge to within
a fraction of a wavelength, and therefore may not be a realistic basis of predicted
performance. Overall, simultaneous exploitation of multiple transmit sites for imaging
has not been sufficiently addressed.
A typical set of illuminators in urban North America, well suited for ISAR processing, is presented in Section 5.1 for FM and ATSC transmissions. The basic signal model for imaging using a segmented, short-time cross correlation is developed, and the spatial frequency (k-space) representation of the set of transmitters for the Columbus, OH area is also shown. The difficulties associated with accurately joining different arcs of k-space, i.e. data from different transmitters, are then discussed, followed by the implications of coherency on the resulting imagery. Simulation results for the point spread function, as a function of illuminator and coherency, are then shown and analyzed.

5.1 Related Theory

The vast number of illuminators in both the FM and DTV bands allows the radar engineer many design freedoms to tailor passive radar system performance. The aim of this section is to develop a model that will allow image quality to be explored in terms of its variance as a result of different transmitter selections, flight trajectory, and coherence.

The approximate locations of existing high powered UHF and VHF band transmissions around the Columbus region are illustrated in Figure 5.1. All visible DTV (ATSC) transmissions shown exhibit equivalent isotropic radiated powers (EIRPs) of at least 100 kW, and 30 kW for FM. Note that some locations have been slightly dispersed to show the existence of multiple channel transmissions in one location. The radome icon represents the location of the ElectroScience Laboratory at The Ohio State University, the location of the receiver for the subsequent analysis. To
the East, John Glenn International Airport is a source of the majority of air traffic in the Columbus, Ohio area.

5.2 Signal Model

Imaging, related algorithms, and theory are well understood from a monostatic context, where the illuminating waveforms are often well-behaved linear frequency modulated (LFM) signals. For a multistatic, non-cooperative imaging system such as passive radar, the theory is less formally developed. Presented here are the fundamental theoretical constructs representing a narrowband, wide angle method of processing the multistatic passive radar data to form imagery. A brief summary of the theoretical signal model will be presented here; for a more detailed development refer to Appendix C.1.
As mentioned in Section 2.2, a basic passive radar system records two data streams, in the form of a reference channel and surveillance channel. The analytic representation of the illuminating transmitted waveform is $s_{tx}(t)$; the passive radar’s reference signal $s_r(t)$; and surveillance signal $s_s(t)$. Figure 5.2 illustrates these designations for the single transmitter case observing an air target. $R_L$ represents the bistatic baseline, or the straight-line distance between the illuminator and receiver’s reference channel. $R_T$ is the transmitter range, and $R_R$ is receiver range.

Neglecting a scaling term, the reference channel is simply a delayed, phase shifted version of the transmitted baseband waveform, as shown in (5.1) and derived in Appendix C.1:

$$s_r(t) = s_{tx} \left( t - \frac{R_L}{c} \right) e^{-j(kR_L+\phi)} + \nu_r(t) \quad (5.1)$$

where $k$ is the wavenumber for the transmitter, $2\pi/\lambda$. The unknown phase term, $\phi$, represents the random phase offset resulting from the fact that the receiver’s local
oscillator is not locked with that of the transmitter, and $\nu_r(t)$ is the thermal noise in the reference channel.

The scattered surveillance signal results from the integral of the point scatterers on the target. Target motion results in time-varying range (delay) and phase terms (Doppler), but also RCS fluctuations due to interactions of target scatterers as the observation geometry progresses, as shown in (5.2).

$$s_s(t) = \sigma(f; \hat{u}_\beta; \beta) s_{tx}(t - \frac{R_T(t) + R_R(t)}{c}) e^{-j(k[R_T(t) + R_R(t)] + \phi)} + \nu_s(t)$$ (5.2)

The RCS, $\sigma$, and delay terms are modeled as singular values corresponding to the integral of the target reflectivity (see Appendix C.2), a valid model for the case of narrowband imaging where the target response is confined to a single resolution cell. The effects of this integral are incorporated via dependence on frequency, bistatic bisector, $\hat{u}_\beta$, and bistatic angle, $\beta$, further corrupted by thermal noise, $\nu_s(t)$. Extending this model to $N$ transmitters of different carrier frequencies, we have $N$ simultaneous surveillance and reference channels, each with different bistatic paths. Substituting bistatic range $R_B(t) = R_T(t) + R_R(t)$, and indexing to the $n^{th}$ transmitter yields the following models for the surveillance and reference signals, respectively (note that all $(n)$ exponents represent transmitter indices, rather than an exponential operator), ignoring the previous noise terms.

$$s^n_s(t) = \sigma^n(f^n; \hat{u}_\beta^n; \beta^n) s^n_{tx}(t - \frac{R^n_B(t)}{c}) e^{-j(k^n R^n_B + \phi^n)}$$ (5.3)
\[
\begin{align*}
 s^n_r(t) &= s^n_{tx}(t - \frac{R^n_L}{c}) e^{-j(k^n R^n + \phi^n)}. \\
 \text{(5.4)}
\end{align*}
\]

The cross correlation between the waveforms is calculated over a short coherent-interval, so that the stop and hop approximation can be used [16]. If the phase of the target response in the reference channel is restricted to less than \(\pi/2\) radians, or \(\lambda/4V_B\) where the bistatic velocity \(V_B = dR_B/dt\), the degradation in correlation gain is limited to 1 dB, as derived in Appendix A. The time dependence of the bistatic range can then be described as the range from the \(m^{th}\) processing interval with \(t = mT_{seg} + t'\), where \(T_{seg}\) is the time between correlations, can be written as follows in (5.5). Details of this derivation can be found in Appendix C, with the phase terms resulting from the demodulation process at the receiver:

\[
 s^n_s(m; t') = \sigma \left( f^n; \hat{u}^n_{\beta}; \beta^n \right) s^n_{tx}(mT_{seg} + t' - \frac{R^n_B[m]}{c}) e^{-j(k^n R^n[m] + \phi^n)} \quad \text{(5.5)}
\]

The cross correlation output is then calculated at the delay associated with the target over length \(T_{int}\), which satisfies the stop and hop condition. This operation, for the \(m^{th}\) correlation segment and \(n^{th}\) transmitter is represented as follows:

\[
\begin{align*}
 \gamma^n[m] &= \int_{-\frac{T_{int}}{2}}^{\frac{T_{int}}{2}} s^n_s(t, m) s^n_r(t - \tau^n(m)) \, dt \\
 \gamma^n[m] &= \sigma \left( f^n; \hat{u}^n_{\beta}; \beta^n \right) e^{-j k_o (R_T[m] + R_R[m] - R_L)} \int_{T_{int}} |s_{tx}(t)|^2 \, dt \\
 \text{(5.6)}
\end{align*}
\]

This output can, in theory, then be stripped of the residual phase terms due to the baseline and bistatic range, and normalized to remove the scaling inherent in propagation and the energy of the correlation segment. This phase and amplitude,
after correction, represents direct samples of the target’s two-dimensional frequency response. This is determined by the frequency, location of the bistatic bisector, and the bistatic angle.

5.3 Bistatic K-Space Theory

The k-space, or spatial frequency domain, representation of a target’s electromagnetic reflectivity is referenced to a coordinate system centered on the target. This, for the context of a single illuminator passive radar geometry, is shown in Figure 5.3. As the target moves along its trajectory, different samples of the target’s frequency distribution are collected, defined by the illumination frequency, bandwidth, and bistatic geometry. A derivation of bistatic k-space sampling theory is given in Appendix C.2.

Figure 5.3 highlights the geometrical parameters of interest, where $\hat{u}_T$, $\hat{u}_R$, and $\hat{u}_\beta$ are unit vectors in the direction of the transmitter, receiver and bistatic bisector. The target scattering density function is represented by $\tilde{\sigma}$, and the bistatic angle can be calculated as $\beta = \cos^{-1}(\hat{u}_T \cdot \hat{u}_R)$, where the $(\cdot)$ operator represents the vector dot product. The angle of the bistatic bisector, $\hat{u}_\beta = (\hat{u}_T + \hat{u}_R) / \|\hat{u}_T + \hat{u}_R\|$, determines
the angle of the k-space sampling point. Both the frequency, \( f \), and bistatic angle, \( \beta \), affect the radius of the k-space sampling point of the target’s frequency domain representation. This point is defined by vector \( K \), shown in (5.7).

\[
K = \hat{u}_\beta \frac{2f}{c} \cos \left( \frac{\beta}{2} \right)
\]  

(5.7)

The Fourier transform of the target’s scattering density is expressed as \( F_\sigma (k_x, k_y, k_z) \), which, when evaluated corresponding to point \( K \), determines the target’s instantaneous RCS value, as follows in (5.8).

\[
\sigma (f, \hat{u}_\beta, \beta) = F_\sigma (\hat{x} \cdot K, \hat{y} \cdot K, \hat{z} \cdot K) \\
= \iiint \tilde{\sigma} (x, y, z) e^{-jK \cdot (\hat{x}x + \hat{y}y + \hat{z}z)} \, dx \, dy \, dz
\]  

(5.8)

Note that this description, similar to that of (5.3)-(5.6), is unique to each transmitter’s illumination frequency and geometry. This representation assumes the following: the plane wave assumption (see Appendix C.2); the bistatic equivalence theorem holds [78]; and the scattering density, \( \tilde{\sigma} \), remains constant as a function of bisector angle. The limitations of these assumptions have yet to be extensively quantified, but will have significant impact on ultimate image quality and the chosen image formation procedure.

5.4 Columbus Passive Imaging Analysis

A simulation has been developed with the transmitters located in Fig. 5.1, and can be used to map out the sampling of k-space as a target flies into John Glenn airport.
A 3D representation of k-space is used within the simulation, but to simplify the analysis we show the two-dimensional projection onto the 2D $F_\tilde{\sigma}(k_x, k_y, 0)$ plane.

The flight geometry and location of FM and DTV transmitters is shown in Figure 5.4. Figure 5.5 (a) shows the mapping into k-space for the scenario shown in Figure 5.4. The cluster of arcs in the bottom of Figure 5.5 results from a single mast to the south of the ElectroScience Lab containing multiple transmitters at differing frequencies. Curves that cross the origin are mapped from the transmitters in the upper right quadrant of Figure 5.4. The origin crossing occurs when the target passes across the baseline between the transmitter and receiver. In such cases, no range or Doppler information is available. Those transmitters would be ill-suited for selection in imaging tasks, and thus Figure 5.4 begins to show the utility of the k-space analysis for complex illuminator sets. From a practical standpoint, the surveillance channel should avoid strong direct signal interference from any single transmitter to maintain adequate dynamic range.

Figure 5.5 (b) demonstrates the same k-space principles for the DTV transmitters in the Columbus area shown in Figure 5.4. The frequency of these emitters is significantly higher than FM, approximately 470-700 MHz compared with FM, 100 MHz. Note the difference in scales of Fig. 5.5 (a) and (b). The higher center frequencies of DTV transmissions allow for finer Doppler resolution, and thus higher image resolutions. The bandwidth is 6 MHz, compared to 150 kHz in the FM case. These correspond with monostatic range resolutions of 25 m for DTV and 1 km for FM. The larger bandwidth is demonstrated in Figure 5.5 (b) as thicker lines in k-space. Contrary to the typical SAR imaging theory, where bandwidth defines down range resolution and Doppler defines cross range resolution, there is a blurring of dimensions.
when bistatic geometries and large viewing angles are considered.

Although neither signal’s bandwidth alone is enough to resolve a typical air target sufficiently, the concatenation of multiple illuminators using the type of techniques presented in [147] with a single tower, along with other spatially separate transmitters, may be adequate to image an average sized airborne target.

SAR image reconstruction algorithms assume full coherency in the collected data, such that a two-dimensional inverse fast Fourier transform (IFFT) is sufficient to form a high resolution image. This is valid for traditional monostatic SAR, because the sensor uses a single high performance oscillator and illuminates targets at a single center frequency and over a relatively small angular range.

Target coherence, where the scattering centers locations’ and phase response remain approximately constant, may not apply outside an angular range of approximately 10°. For a full collection, as shown in previously, this is clearly exceeded,
Figure 5.5: K-space sampling patterns (a) FM transmitters (b) DTV transmitters with angles reaching spans of 180°. If the target is not coherently integrable over this extended angular range, IFFT processing may not be the optimal method of image formation and other techniques should be investigated.

Two target model hypotheses are presented in [135]. The first assumes the target consists of a set of isotropic point scatterers, and images resulting from each transmitter can be summed coherently. The second involves non-isotropic points, such that each transmitter’s image must be formed separately and then summed incoherently. The first model should be the theoretical upper bound in terms of resolution, but will not often hold true in practice because of the reasons discussed previously.

Excluding target coherency effects, coherence of the radar sensor itself merits additional attention. Although physically separate illuminators are not phase locked to the receiver’s LO or sampling clock, there typically exists a common timebase between the receive channels of a passive radar. Therefore, we can assume the signals are coherent-on-receive, and any phase drift between the transmitter and receiver hardware will be removed in the cross correlation processing. Due to these processes the coherency of the radar is then limited solely by the receiving system’s phase
stability over the bistatic delay time. It is worth noting, however, that many broadcast illuminators of opportunity employ a GPS disciplined reference to reduce phase noise, which if combined with a high quality crystal oscillator can exhibit phase stability on the order of picoseconds [89]. If similar equipment is available at the receiver, phase drift can be minimized to a fraction of a degree. However, one can expect significant of phase degradation due to time-varying multipath at the receiver; this is difficult to determine for a single illuminator but even more difficult for a multistatic system where each channel is unique must be continually estimated.

Likely the most challenging aspect of forming a coherent multistatic system is the fact that ISAR targets are fundamentally non-cooperative, in contrast to SAR systems where the ground targets are assumed stationary and platform motions are carefully measured via an inertial measurement unit (IMU) to track precise antennas motions. Neglecting the previously discussed challenges, and focusing on an idealized point target and fully-synchronized phase-stable radar system, the unknown motions and turbulence of the target results in a defocused image for a single transmitter. This is exacerbated in a multiple illuminator system. Data driven autofocus algorithms such as Map Drift, Prominent Point Processing (PPP), and the Phase Gradient Autofocus (PGA), developed [6,27] originally for high resolution SAR systems to focus residual motions outside the accuracy of the on-board IMU, have been applied to single transmitter ISAR systems [36]. Unfortunately, these techniques often rely on high range resolution and are developed for a single transmitter, and techniques for cohering separate transmissions have not yet been developed. Clearly, by realizing wideband operation – enabled by reception of multiple transmissions – passive radar and imaging operations can be improved; finer resolutions, improved feature extrac-
tion, fade vulnerability, and robustness to adverse geometrical effects near the bistatic baseline.

5.5 Image Reconstruction and PSF Analysis

The following simulated images show the point spread function of a fully coherent model consisting of an isotropic point target traversing the flight path shown in Figure 5.4. The simulation was repeated for both FM and DTV transmissions in the Columbus area. The results are shown in Figure 5.6 (a) and (b), respectively. Notice that the resolution is much finer with the higher frequency DTV illuminators.

Both images demonstrate that the -3 dB width of the target response is much finer than that which is implied by the bandwidth of the signals alone. For FM, the -3 dB width is approximately 0.5 m in the cross range direction, and 1 m in the down range direction. This asymmetry is caused by the greater coverage along the kx axis of k-space. For DTV the width of the PSF is less than 20 cm in both dimensions.

Both of the preceding figures assume that the target consists of a single point scatterer, whose response is unchanged with frequency, illumination angle, or bistatic angle. This allows us to see a rough upper bound in terms of expected performance for a fully coherent system using a number of illuminators set in a realistic geometrical context. Processing the signals in this fashion requires precise knowledge of the baselines $R_L^n$ and bistatic ranges $R_B^n$, which implies that highly accurate tracking data is available. This may not be achievable in practice, and the incoherent target model presented in [135] may be required because of phase inaccuracies in the receiving equipment as well as target aspect dependence.
The data was then processed for each channel, and the resulting images summed incoherently as a result. Figure 5.7 (a) and (b) show the result after this processing was applied, for FM and DTV transmitters. Note that the collection flight path and illuminating geometry are identical to that used to create Figure 5.6.

The incoherent target models show similar trends to the coherent cases, but at a cost of increased sidelobe levels. The width of the main peak has also increased, effectively degrading the resolving power of the instrument. Figure 5.7 (a) shows large fan-like projections in the sidelobe structure, which could mask adjacent scattering
Figure 5.8: PSF using FM and DTV (a) Coherent combination (b) Incoherent combination

centers. Nevertheless, the main response is only marginally broader than the coherent case, and other incoherent reconstruction algorithms may be able to decrease the sidelobe structure and thus increase resulting dynamic range.

Simultaneous wideband operation of passive radar systems would also increase the number of angles over which the target can be viewed and imaged. Both FM and DTV illuminators were simulated and the point spread function generated for the geometry of Figure 5.4. The coherent case, shown in Figure 5.8 (b), shows a more regular, symmetrical sidelobe response. This is likely from the increase in angular and frequency diversity over which the target is viewed.

The incoherent case for FM and DTV illuminators, shown in Figure 5.8 (b), is the same as would result from an incoherent summation of Figure 5.7 (a) and (b). The broad response from FM illuminators can be seen underlying the DTV response. The apparent weakness of the FM response relative to DTV is due to the radial filtering function, similar to filtered backprojection [75], necessary to prevent the low frequency signals from dominating the image response. The degree of influence on the
final image is due to the total k-space span of the signals, smaller for FM than DTV. Otherwise, the resulting image appears very similar to Figure 5.7 (a) with a degraded mainlobe response. From these figures, we see a significant improvement in the fully coherent case for both FM and DTV, but marginal improvement when operating incoherently. However, the width of the PSF still shows great promise for realizing high resolution imagery even when subject to experimental coherence limitations.

5.6 Multistatic Narrowband Imaging Conclusions

Passive radars, using FM and DTV signals, have been shown capable of generating high resolution imagery using a limited number of transmissions. The incoherent method, although more robust to phase errors, exhibits higher sidelobe levels when compared to the fully coherent method of image formation. Simulation results for the point target response are encouraging, but further developments are necessary to realize such results with experimental passive radar systems. The first step in solving the remaining challenges involves addressing the gaps in the literature, particularly: pre-processing stages, limitations of target and system coherency, and unkown flight geometries. In addition, the propagation channels, should be properly modeled and accounted for in the approach. The following Chapters in this manuscript address many of these challenges, culminating in the experimental passive ISAR imagery of Chapter 9.
Chapter 6: Passive Radar Development

Much of the potential for passive radar systems resides in the fact that multiple illuminators can be used simultaneously with only slight incremental cost to the receiver system. Such operation allows for redundant observation of surveillance regions, thus improving system performance when compared to a single transmitter scenario. In addition, some tasks not achievable with a real-beam single transmitter passive system become viable with multistatic operation, such as direct 3D Cartesian detection and tracking. Estimation of a target’s path is a necessary component for realizing ISAR imagery as demonstrated in Chapter 5. In addition, a passive ISAR system piggybacking on narrow bandwidth illuminators requires the resolution improvement of multistatic operation and wide integration angles for useful imagery of typical airborne targets.

Although the architecture of many passive systems is known, none of the current literature investigates the design considerations necessary for simultaneous wideband operation. Here I examine these aspects in regards to a wideband digital television (DTV) passive radar, consisting of multiple receive channels, to support a variety of passive radar research activities. These design considerations warrant attention because the architecture of multistatic passive radar systems, which utilize geometrically diverse illuminators at widely different radiating power levels, varies considerably
from traditional active, monostatic systems.

Both receiver hardware and signal processing algorithms require particular consideration to efficiently achieve the full potential of a multistatic system. The spectral diversity of the various transmissions necessitates a wideband system to collect, store, and process the number of available illuminating signals within the band of operation. In addition, transmitters often possess a wide range of effective isotropic radiated powers (EIRPs) in addition to different bistatic baselines relative to the passive radar receiver. This further exacerbates the already large dynamic range requirement for a single illuminator passive radar, which must detect weak target echoes in the presence of strong direct signal interference. A vast number of parallel data streams, due to the many transmitters and receive channels, each must undergo substantial digital processing prior to RD map formation and further operations. Therefore, any improvements in computational costs associated with these fundamental processing stages can substantially reduce system latency while simultaneously reducing the cost and power requirements of hardware used for digital signal processing (DSP).

Section 6.1 is an overview and introduction to the MUltistatic digital TElevision passive RAdar (MuTeRa) testbed at a high level, followed by details of hardware (antenna configurations, RF architecture, digital backend, air truth recording, etc) in Section 6.2. A new passive radar hybrid RF downconversion scheme is shown which maximizes the number of illuminators without sacrificing receiver linearity or digital backend dynamic range. Following this, the fundamental signal processing stages of passive radar are presented in Section 6.3, with a focus on North American DTV waveforms and the related signal structure. A new algorithm for calculation of the range-Doppler surface using the cross-ambiguity approximation is developed,
which preserves integration gain over range while circumventing the unambiguous range/Doppler tradeoff (common to conventional pulsed radar systems) of existing algorithms. Finally, Section 6.4 summarizes some key accomplishments with the system not investigated elsewhere in this manuscript but which merit mentioning.

6.1 MuTeRa System Overview

The MuTeRa testbed is a flexible passive radar system consisting of multiple transmitters and a single receive site, designed for observing air targets at close receiver range (<1 km) to mid range (>100 km). The MuTeRa system is situated on the rooftop of the ElectroScience Laboratory (ESL), in Columbus, Ohio. The radar collects a wideband, 150 MHz portion of the DTV spectrum on eight separate receive channels. Each channel comprises a corresponding antenna, RF downconversion chain and analog to digital converter (ADC), as shown in Figure 6.1. The wide passband allows the system to collect up to eight simultaneous DTV transmissions from the Columbus area (not to be confused with the eight receiver channels) from five distinct transmit sites, allowing for multistatic observations of a single target over a range of diverse geometries (bistatic baseline and bistatic angle), frequencies (spectral diversity), and power levels (effective SNR). Thus, the system represents a powerful testbed for a number of various research topics including: passive radar detection, localization and tracking, array processing, imaging, polarimetry, ground moving target indication (GMTI), etc.

MuTeRa is also equipped with a GPS receiver for precise timestamping of the ADC data, or alternatively for positioning data to support dynamic experiments with a moving receiver. An automatic dependent surveillance-broadcast (ADS-B)
receiver is integrated with the system to provide air truth data, which can be used to validate the various radar operations. The system is primarily controlled with a TALON 2746 data recorder, produced by Pentek Inc., as shown in Figure 6.2. This device is a turnkey recorder system centered around Windows PC, which is equipped with two PCI express (PCIe) cards for ADC capability and an additional PCIe card as the GPS receiver. The ADS-B receiver, a Kinetic Avionics SBS-3, is connected via USB and processes the 1090 MHz extended squitter data before transferring to the TALON 2746 for cataloging.

6.2 Hardware Design Considerations

Unfortunately, multistatic passive operation places additional hardware challenges on a system compared with a single illuminator setup, primarily with regards to linearity, dynamic range, and data rate which adds to the processing overhead. To examine the design methodology, one must first consider the fundamental goal of
the MuTeRa system: to detect targets over a desired surveillance area on multiple wideband receive channels, either with real-beam or phased array operation. This results in the following requirements:

- Each receive channel must consist of a separate antenna, RF downconversion, and ADC with storage capability.

- Each channel should be wideband, to collect multiple illuminating signals concurrently each of which with the capability to detect airborne targets.

- Reference channels should have adequate line-of-sight to the transmitter to provide a delayed copy of the transmit waveform to use in direct signal interference (DSI) suppression and RD map processing.
• Surveillance channels should have sufficient dynamic range to receive, downconvert, and sample to preserve weak target echoes in the presence of strong DSI terms.

To support these requirements, the various aspects of passive radar system design with a focus on the MuTeRa will be considered in the following subsections.

6.2.1 Choice of Illuminator

In addition to the exact hardware configuration of a passive radar, illuminator selection is the first and most crucial step in designing a PR system. The center frequency and bandwidth determine the lower bound on range and velocity resolution. The cumulative effects of various transmitters along with the receiver siting should be carefully considered in the preliminary system design phase to avoid unwanted effects. As an example, it is clear that an ideal receiver for a distant illuminator (high gain and low noise figure) would be saturated if located within close proximity to strong in-band transmitter. If designed for a strong nearby transmitter, a low-gain system with a moderate noise figure would perform poorly for distant transmitters.

Illuminator selection can be thought of as a two-step process, first determining the type of illuminator, such as FM, DTV, GSM, etc., followed by the selection individual illuminator channels if multiple sources of a particular type exist. Location, frequency, and transmitted power all affect the system performance and must be evaluated.

Illuminator selection should made considering the following factors:

• Illuminating signal bandwidth (range resolution) and modulation (temporal stability and Woodward ambiguity function structure)
• Transmit parameters of beampattern (coverage in azimuth and elevation) center frequency/CPI (velocity resolution) and power (SNR and DSI effects)

• Spatial deployment of available transmit towers (coverage region of a multiple illuminator system)

• Frequency mapping and reuse (single frequency network or multi frequency network can cause co-channel interference)

The advanced television systems committee (ATSC) DTV band, from 470-700 MHz, was chosen for MuTeRa because it met many of the factors listed above. The high transmit powers of many broadcast ATSC stations (with many EIRPs in excess of 500 kW) result in high SNRs of both the reference waveform and target signals compared with other, lower powered, broadcast illuminators. Table 6.1 shows the power levels according to the current FCC database of ATSC transmitters around Columbus, OH [148]. The ATSC signal is also digitally modulated, details of which are explained in Section 6.3.1. Note that these high transmit powers also increase the DSI level, which prompted the investigations of Chapter 7. This modulation structure results in stable range and Doppler performance compared with analog signals of opportunity [23], with an ideal noise-like thumbtack ambiguity surface after removal of the pilot tone, as demonstrated in Section 6.3. The range resolution resulting from ATSC’s 6 MHz bandwidth (50 m bistatic range resolution) is far greater than FM radio’s 150 kHz channel bandwidth (2 km bistatic range resolution). Other illuminators, such as LTE and WiMax, sometimes possess greater bandwidths than DTV, but the frequency reuse pattern and low transmit power prohibit operation in all but short
<table>
<thead>
<tr>
<th>Callsign</th>
<th>Fc (MHz)</th>
<th>EIRP (kW)</th>
<th>PRX (dBm)</th>
<th>Baseline (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WCMH</td>
<td>473</td>
<td>902</td>
<td>-7</td>
<td>3.4</td>
</tr>
<tr>
<td>WCLL</td>
<td>503</td>
<td>15</td>
<td>-29.6</td>
<td>3.4</td>
</tr>
<tr>
<td>WBNS</td>
<td>515</td>
<td>1000</td>
<td>-7.3</td>
<td>3.4</td>
</tr>
<tr>
<td>W23BZ</td>
<td>527</td>
<td>15</td>
<td>-39.4</td>
<td>11.7</td>
</tr>
<tr>
<td>WSFJ</td>
<td>533</td>
<td>1000</td>
<td>-36.3</td>
<td>31.1</td>
</tr>
<tr>
<td>WCSN</td>
<td>587</td>
<td>15</td>
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<td>605</td>
<td>1000</td>
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<td>7.1</td>
</tr>
<tr>
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<td>W44DC</td>
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<td>15</td>
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<td>WWWHO</td>
<td>665</td>
<td>1000</td>
<td>-34.4</td>
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<tr>
<td>WSYX</td>
<td>677</td>
<td>1000</td>
<td>-16.1</td>
<td>7.1</td>
</tr>
</tbody>
</table>

Table 6.1: DTV transmitters in the Columbus, OH area for a receiver at the Electro-Science Laboratory. Highlighted call-signs fall within the passband of MuTeRa’s RF front end.

range scenarios due to low SNR and co-channel interference in addition to the typical DSI.

The varied spatial distribution and low frequency reuse patterns of ATSC illuminators results in a wide coverage area for target observations, allowing for ample opportunities where valuable data can be collected. The location of the MuTeRa testbed and illuminator locations, center frequencies, and transmit power levels are shown in Figure 6.3. Only illuminators whose center frequencies are within the passband and roll-off of the system’s bandpass filter from 530-680 MHz are shown in the map, corresponding to the illuminators highlighted in Table 6.1. The current set of eight illuminators is far less than the 25 available 6 MHz ATSC channels, and the number of simultaneous channels available to the system will increase as broadcasts are added to the area. Notice that the transmitters are located north, east and south of the MuTeRa site, indicated by the radome icon. This site was chosen for the following reasons. First, the vast set of baseline angles to the illuminators allow for many
Figure 6.3: RF transmitter map of the MuTeRa testbed. Radome icon indicates MuTeRa position
varied bistatic observations with a single target, aiding investigation of how bistatic geometries influence passive radar performance. Second, the spread in positions allow for multiple observations of various elevation angles to a particular target, permitting investigation of target altitude (or elevation angle) on passive radar performance. Third, a variety of illuminator positions also minimizes blind spots due to baseline degradation that occurs when targets are on the bisector between the transmitter and receiver. Such scenarios allow for forward scatter investigation for many flight paths in comparison to a central cluster of transmitters. Parameters of these illuminators specific to a receiver located at the ESL are listed in Table 6.1, along with the bistatic baselines and estimated receive power levels for the direct path. The received powers were calculated by the Longley-Rice algorithm [149], which incorporates knowledge of the transmitter height and EIRP, terrain between the transmitter and receiver, as well as receive antenna height.

The design of passive radar systems must consider the DSI power and SNR of the reference waveform, rather than simply being limited by thermal noise. Strong DSI in the surveillance channel, in addition to saturating the receive chain, leads to masked targets in the RD surface and drastically reduces the effective detection range. A degraded reference signal with high noise or multipath can also degrade performance. As a starting point for a design, propagation modeling tools or simply the Friis transmission formula [150] and the bistatic radar equation can be used, introduced in Chapter 2. Note that these results are also heavily dependent on local terrain features, such as trees or buildings, which are not generally well modeled and require measurements for accurate data. Further examination of the received signal quality with regards to the DSI masking and performance degradation is examined in
Sections 7.1-7.3. Null steering, physical shielding, and adaptive digital cancellation algorithms can be employed to compensate for DSI, but their effectiveness is limited. Selecting transmitters or a receiver site with a greater baseline generally decreases the masking effect of DSI. This method has limits, however, as the quality of the reference signal will degraded from noise and multipath to a level which proves unusable. At such point demodulation-remodulation techniques (only available for digitally modulated systems) can be employed to overcome reference channel multipath and signal degradation [25,26] if SNR remains high enough, or a secondary receiver and communication link can be established to provide reference signal information if additional cost and complexity are permitted.

6.2.2 Antennas

The choice of receive antenna significantly impacts the coverage and ultimate interference-to-noise ratio (INR) of a PR system. Unlike dedicated radar systems, which often employ high gain antennas on both transmit and receive, PR systems usually piggy-back off nearly omnidirectional broadcast apertures, such that only receiving aperture SNR improvement is possible. Although this limits the achievable SNR and generally increases the DSI levels, it enables persistent surveillance over wide areas and permits extended CPIs for increased processing gain. The added cost of such configuration, however, is the requirement of a number of antennas, each paired with multiple, independent RF receive channels. These can be in the form of separate high gain, real-beam antennas which span a particular angular region of interest, or a set of wide-angle antennas closely spaced to form a digitally steered array. This results in flexible beamforming for target tracking and null steering for DSI reduction, but
a more extensive calibration is required than for a real-beam system. For a multi-
static system with illuminators at various angular positions, a proportionate number
of receive antennas or digitally steered beams are required to gather the various ref-
ence signals. In cases where line of sight to the transmitter is available, a narrow
beamwidth antenna is desired to minimize the multipath and noise contributions of
the reference waveform, whose presence can degrade the effective SNR of the radar
system.

Each type of antenna configuration and PR architecture exhibits particular ad-
vantages. Advanced array systems with a bank of tunable RF receive chains would
be well suited to situations such as air traffic control or military situational aware-
ness, where precise multi-target tracking is necessary. However, when low size, weight
and power (SWAP) is a driving requirement, such as collision avoidance for the next
generation unmanned aircraft systems (UAS) [151], a single channel receive aperture
would be ideal. With such a configuration, signal processing can be used to distin-
guish the reference and surveillance signals from a composite waveform. The most
typical configuration found in basic passive radars is comprised of two physical receive
apertures (reference and surveillance) per transmitter, as described in Chapter 2. To
support investigation into any of the above designs, MuTeRa’s RF and digital backend
design was developed to handle the range of power levels from the various antennas
and transmitters in the Columbus, OH area. Mounts for both circular and linear
arrays were fabricated, for 360° coverage with a moderate beamwidth and a limited
angular sector with finder beamwidth, respectively. These were populated with small
bow-tie elements operating in the UHF bands. In addition, a number of high-gain
Yagi antennas were acquired for testing real-beam passive radar performance, both
in the H-pol (co-pol for ATSC) and V-pol (cross-pol) states. The Yagi antennas are usually used for reference channels, with one co-polarized antenna pointing south and another towards the northwest for each group of illuminators. The antenna configuration of MuTeRa is shown in Figure 6.4. All antennas are off-the-shelf, low-cost, commercially available DTV antennas. The Yagis can be configured either as reference or surveillance antennas, and the linear array of low-gain bow-ties can be used to test phased array concepts, adaptive beamforming, nulling, and direction of arrival for tracking purposes.

Note that DTV antennas are designed for cable TV coaxial cable with a characteristic impedance of 75 $\Omega$, in contrast to 50 $\Omega$ of most RF equipment. Impedance matching pads or additional transformer baluns can be used to improve the reflection at this impedance mismatch, but these solutions are costly and unnecessary for a low-power receiving system. The mismatch loss experienced by this is 0.177 dB, calculated as follows:

$$L_{\text{mismatch}}^{\text{dB}} = -10 \log_{10} \left( 1 - \Gamma^2 \right) = 10 \log_{10} \left( 1 - \frac{(Z_2 - Z_1)^2}{Z_2 + Z_1} \right)$$

$$= -10 \log_{10} \left( 1 - \left( \frac{50 - 75}{50 + 75} \right)^2 \right) = 0.177 \text{ dB.} \quad (6.1)$$

where $\Gamma$ is the reflection coefficient, $Z_1$ is the source (antenna) impedance, and $Z_2$ is the load (transmission line) impedance. It is important that this interface be as close to the antenna as possible, to minimize the ringing effect. If placed directly at the antenna, any reflections from mismatch are simply re-radiated out of the receiving antennas. Since power levels are low for a receive-only passive radar, these reflections will not damage any RF components, which would be possible with a similar antenna.
Figure 6.4: MuTeRa antenna configuration. Left: high gain Yagi-Uda array antennas for reference signal or real-beam surveillance antennas. Right: 2D array of bow-tie elements for beam and null steering.

configuration for a high-powered transmit system.

Polarization of the receiving antennas also deserves particular attention. To maximize the SNR of the line of sight (LOS) reference signal, the reference antenna is typically co-polarized. This simultaneously minimizes the strength of any multipath bounces, whose polarization state is likely altered upon reflection off nearby objects and clutter around the bistatic baseline. Note that the reference and surveillance antennas are not required to be of the same polarization. For the surveillance channel, a cross-polarized configuration is often used in the hope of reducing direct signal interference, as discussed in Chapter 17 of [39]. The methodology behind this approach is as follows: even though directive antennas, oriented away from the bistatic baseline, may be used for surveillance, the direct signal is often still the strongest receive signal. By orienting the antenna in such a manner, the physical coupling to the co-polarized direct signal is significantly reduced. This reduces the average power level into the RF front end, which permits designs with lower noise figures and thus more linear operation. The target signal experiences different polarimetric behavior
due to the often complex scattering mechanisms based upon their structure and orientation. Thus the cross-pol antenna should see, on average, a similar target power for the cross-pol orientation compared with a co-pol. Experimental tests of the power spectrum showed up to 5 dB improvement in DSI level for a Yagi facing away from the illuminators, and around 15-20 dB of signal strength reduction when the direct path is closer to the main lobe of the surveillance antenna.

The general architecture of the receiving antenna system typically falls into two categories: real-beam systems and array based systems. This decision flows down into the other aspects of the receiver system, as the number of antennas and their parameters (gain, beamwidth, front-back lobe ratio) required dictates the number of RF chains, their input power levels and required gain, and the ADCs of the digital backend. For a single transmitter system, a real beam system can be implemented with two digitizers (or one if demod-remod techniques are used, see Section 7.3.1.2). Array based systems, if full beam steering capability is to be preserved in post-processing, require an equal number of ADCs for each array element. In general, an array configuration opens up more advanced radar processing and applications, as nulls can be placed or adapted in the direction of the DSI, and direction of arrival (DOA) estimates of target position are achievable with greater accuracy than with a real-beam system. The beam can be simultaneously steered towards the transmitter to receive a direct reference signal to alleviate the need for a dedicated reference, if the polarization permits such capability.

Along with the additional RF and backend hardware of an array, the challenges of in-situ array calibration and mutual coupling compound the complexity of such systems. For wideband operation using multiple transmitters, as is the case with
the MuTeRa testbed, these procedures must be individually measured for each illuminator, as the electromagnetic parameters of each antenna and the RF chain are unique for each transmitter frequency and bandwidth. The additional computational requirements of arrays can be mitigated using parallel processing and algorithmic optimization. For example, fixed beamforming coefficients which comprise a set of staring sectors for surveillance before detection and tracking significantly reduce the overhead of the core signal processing operations described in Section 6.3.

The structure of the array also impacts what is achievable with the system. Circular arrangements allow for maximum flexibility and beam directions in any azimuth position, but the number of real apertures which can be used for a particular look direction is limited unless isotropic elements are used, adversely affecting sensitivity due to increased DSI leakage. The circular arrangement allows complete angular beam agility for any number of surveillance and reference beams, but at a cost of receiver noise figure and gain (to be discussed in Section 6.2.3). Details of such an architecture were proposed by Inggs [152]. Note that if this arrangement is used for both surveillance and reference signals, the polarization must be chosen to either increase DSI (co-pol) or degrade the reference signal quality (cross-pol). For situations where surveillance is restricted to a finite sector, a linear array can be used with moderate gain apertures to increase the total effective receive antenna gain, thus increasing detection performance from increased SNR. Assuming the target zone is located away from the angle towards the illuminator of opportunity, this gain also helps reduce direct signal breakthrough in the sidelobes of the receive array.
6.2.3 RF Front End

The RF front end of the radar system has the important task of amplifying and translating the RF waveforms received by the antennas to a lower intermediate frequency scaled such that the ADC can properly sample the output signal. The design of the RF front end should be such that the desired signal (strong direct path signal for reference channels, and weaker target signals for surveillance channels) is preserved throughout this process. This requires image rejection during the mixing process, and proper anti-aliasing filtering prior to digitization with the ADC.

Dynamic range requirements for any passive radar system are high, because the receivers must be able to discriminate very weak target echoes in the presence of strong continuous transmissions. Ratios exceeding 90 dB between the DSI and target levels are fairly common. However, these requirements are made even more extreme for wideband systems for the reasons discussed in the introduction to this chapter. This wide variation of direct signal power and target powers requires the RF front end to be highly linear to preserve the small signal content. The back-end ADC also requires a wide dynamic range, details of which are given in Section 6.2.4. In addition to this, the receiver should have a low noise figure, because the effective DSI suppression is capped by the surveillance channel's interference to noise ratio, shown in Chapter 7. Unfortunately, there is a fundamental trade-off between the linearity and noise figure of a particular system. For a fixed set of amplifiers and mixers in a receiver chain, insertion of attenuators prior to the nonlinear components not only improves the VSWR due to reflections but also increases the input third order intercept point (IIP3). However, additional attenuation increases the noise figure of
Figure 6.5: Power spectrum recorded on the rooftop of the ElectroScience Laboratory

the receiver due to the Friis cascade noise equation. Ideally, this attenuation should only be inserted prior to the first LNA so that the noise contribution is minimized, which requires a LNA with a moderately high 1 dB compression point (1dBc) to avoid saturation prior to linearizing pads.

The RF design should accommodate the maximum expected power level at the antenna without saturating. This is generally set by the DSI, which is often orders of magnitude greater than the desired target signal, in contrast to conventional system design where target echoes are stronger with respect to the ground clutter. As mentioned previously, the bistatic range equation and propagation modeling tools can facilitate this portion of the design process. However, accurate power levels from local multipath (buildings, roadways, foliage) is very difficult to predict; thus a site survey, with the antennas in the proper orientation, is recommended for accurate data to be used in the design of the RF front end.

A snapshot of the DTV power spectral density is shown in Figure 6.5, recorded
with a roof-mounted Yagi antenna and a digital oscilloscope. This spectrum demonstrates the multiple signals of opportunity with their 6 MHz bandwidth and varying power levels. To calculate the power into the RF front end, the power spectral density can be integrated across the desired frequency range of the passive radar receiver.

After the preliminary spectral survey of the available transmissions for the proposed receiver site was completed, it was found that the nearby transmitters, WCMH and WBNS in Table 6.1, were prohibitively strong and would severely hinder the sensitivity level, or the minimum detectable signal, of the passive radar receiver. Most commercially available LNA’s with 20 dB gain would be saturated beyond the 1 dBc point for even moderately directive reference antennas. Other transmitters with greater baselines result in significantly less power into the RF chain. As such, these signals would be much closer to the noise and ADC quantization floor if received using an RF front end designed to accommodate the strong signals from WCMH and WBNS.

It is possible to design an RF system with a low noise figure and minimal amplification to avoid saturating the ADCs while receiving the high-powered, nearby transmitters. However, the quantization noise of the ADC will then be much higher than the RF front end’s output thermal noise floor, which results in a high effective noise figure for the system. To circumvent this, a bandpass filter was designed to significantly attenuate these high-powered signals and pass the 530-680 MHz portion of the RF spectrum prior to amplification with the low noise amplifier (LNA). Figure 6.2 shows the general block diagram and filtering scheme of the RF downconversion chain for the MuTeRa system. Note that the architecture resembles a standard superheterodyne design, but possesses an atypical hybrid high side/low side
architecture, to be discussed shortly. A photograph of the assembled RF chain is shown in Figure 6.7. Notice that additional attenuators are shown with the assembled hardware and can be manually adjusted depending on the signal power from the feed antenna, its pointing direction and polarization. Eight such RF chains were fabricated and the output connected to the ADCs of the TALON recorder system. A local oscillator (LO) distribution network was also constructed, which consists of a tunable Agilent RF signal generator, high-powered amplifier and eight-way splitter to feed the mixers of each channel at the proper drive level.

As will be explained in the next section on the digital backend, the effective number of bits of the ADC broadly determines the effective dynamic range of the receiver. Unfortunately, ADCs with higher bit counts are limited to reduced sampling frequencies. An approach to circumvent these restrictions was implemented on the MuTeRa front end, using a hybrid tuning scheme to condense 150 MHz of RF spectrum into a 16 bit, 200 MSa/s with approximately 80 MHz of usable analog bandwidth. This allows for a maximum number of potential illuminating signals to be recorded con-
currently without compromising the state-of-the-art backend dynamic range. This architecture is possible due to the sparsely populated DTV band, shown in Figure 6.5. The tradeoff for this architecture is a 3 dB increase in noise power. However, for the illuminator configuration in Columbus, OH area the system is primarily DSI limited and the sacrifice in noise figure was deemed acceptable given the additional capability for testing multistatic imaging algorithms.

The spectral investigation into how the illuminators are affected at each point in the receive chain is demonstrated in Figure 6.8. Subplot (a) shows the spectrum at the input of the RF front end, followed by (b) the output of the first RF bandpass filter from 530-680 MHz. Note that significant power in WBNS remains, and must be accounted for in the power budget of the passive radar. The color coding of the plots denotes the illuminator frequencies greater than the LO frequency, indicated by the black vertical arrow. Signals which are high-side converted (when $f_c < f_{LO}$ in red) will result in a flipped spectrum, which if uncompensated manifests as a mirrored RD surface around the zero Doppler axis. This is easily correctable in software by simply inverting the Doppler axis of the final RD map, or by conjugating the baseband complex data stream.

The IF spectrum at the output of the receiver, showing seven separate IF channels within the 530-680 MHz passband, is presented in Figure 6.8 (c). The IF center frequencies can be calculated with $f_{IF} = |f_c - f_{LO}|$. By searching the available LO frequencies between the DTV channels, the sparsely populated transmitters could be interleaved without overlapping and interfering with adjacent channels. Using this process, the 608 MHz LO frequency was chosen for the system. Notice that the downconverted illuminators just outside the RF passband are at least 80 MHz, which
Figure 6.8: Passive radar spectrum (a) RF spectrum at antenna feed (b) RF spectrum after first BPF (c) IF spectrum after hybrid downconversion technique
can easily be filtered out with DSP operations. Other tuning schemes using the same
design of Figure 6.6 can also be used to collect only the lower or upper portions of
the RF passband, by using a LO frequency of approximately 530 or 680 MHz. In
practice, slightly lower and higher values (respectively) were found to perform better
due to the AC coupling of the ADCs in the TALON backend. With this alternate
tuning configuration, 4-5 simultaneous signals can be captured with a 3 dB reduction
in thermal noise power as no fold-over occurs.

6.2.4 Digital Backend and Software Control

The ADCs which comprise the digital backend typically have two driving requirements
for passive radar: possess a sampling frequency satisfying the Nyquist sampling cri-
teria for the signal of interest, and have a sufficient number of bits that the dynamic
range can sample the strong DSI as well as the target echoes. The dynamic range of
a particular ADC is set by the effective number of bits (ENOB) as follows [153]

\[ DR_{dB} = 1.76 + 6.02 \times ENOB. \]

This dynamic range, \( DR \), is measured in dB relative from the maximum input level
of the ADC, such that \( P^dB_{\text{min}} = P^dB_{\text{fullscale}} - DR_{dB} \), but can be exceeded assuming the
signal of interest is dithered with a stronger signal, as is the case of the target signal
and passive radar. To maximize the passive radar’s dynamic range, a 16 bit ADC
was chosen with an effective bit count of approximately 12.5. Using the typical 6 dB
per bit calculation, the dynamic range of the back-end is approximately 77 dB. It is
important to note that this dynamic range is not the ultimate dynamic range for the
system, which must include the effects of the signal processing and any non-linearities present in the RF front end.

The 16 bit ADCs are packaged in a PCIe card on the TALON data recorder, with a 700 MHz analog passband and a full-scale input of +8 dBm. This large passband enables bandpass sampling architectures, which can be used for sub-sampling illuminators of interest to avoid analog downconversion. Four ADCs are integrated in each of the two PCIe Pentek Boards, model 78661 shown in Figure 6.9. Each ADC board’s sample clock is disciplined from the 10 MHz reference output of the Agilent RF signal generator used for local oscillator synthesis. The eight ADCs shared between the two boards receive the output of the eight RF chains which comprise the MuTeRa testbed. Each card is also equipped with a Xilinx Virtex-6 FPGA capable of
routing the ADC signals and performing parallel digital downconversion of the data. Alternatively, the raw samples can be passed through at the full rate of 200 MSa/s to a 7.6 TB array of sixteen 512 GB SSD drives in a RAID-0 configuration. The 2-byte (16 bit) samples streamed at the full rate of 200 MSa/s across eight channels results in a data throughput of about 3.2 GB per second, which can be sustained by the TALON recorder until the drives are at full capacity (about 40 minutes of 200 MSa/s data across the eight channels.) After saving the data to memory, it can be loaded by additional software for post processing.

Figure 6.9 also shows the GPS receiver board, produced by Symmetricom. The primary purpose of the GPS receiver is in the form of a time reference to set the clock of the TALON receiver system to the GPS time standard for a one-to-one correspondence with the air truth data, explained in the next section. The positioning data output by this board can also be used in moving receiver experiments, such as passive
GMTI or SAR, but the current installation is fixed with a known position which does not require GPS location knowledge. The GPS functionality is directly integrated with the PENTEK record software, SystemFlow, which configures the ADC sampling parameters, channel configuration, and FPGA channel mapping as well as the DDC operations with the bundled IP core. It also controls the arming of record channels and supplies a software trigger to begin collections. In addition, SystemFlow is equipped with a signal viewer tool which allows for real-time monitoring of the input spectrum. Figure 6.10 shows the time and frequency domains of the sampled DTV signals at IF after downconversion with the RF front ends described in Section 6.2.3 and a 608 MHz LO. One thousand samples are used for the FFT shown in the right spectral data. Notice that the some channels exhibit a peak at the left or right part of the band because of the pilot tone, which is an indicator of high-side or low-side downconversion.

### 6.2.5 Air Truth Data

In order to validate the radar operations and output of the MuTeRa system, knowledge about the data being observed is required. Information regarding targets shape and size, position, and velocity is extremely valuable when evaluating the performance of any radar functions. The Automatic Dependent Surveillance-Broadcast (ADS-B) system is the natural solution to provide this data, as it is installed on a significant percentage of commercial and private aircraft. The system utilizes a GPS receiver on board the aircraft, then reports the plane’s callsign along with latitude and longitude positioning information and flight altitude level. The ADS-B system is a component of the Next Generation Air Transportation System, which aims to implement GPS
as the primary means of navigation in contrast to air traffic control radar. By 2020, it is mandated that ADS-B Out equipment be in use for the majority of controlled airspaces.

The Kinetic Avionics SBS-3 receiver system, shown in Figure 6.11, records extended squitter ADS-B transmissions in the 1090 MHz band shared with secondary surveillance radar systems. This device is fed by a monopole antenna and parses the ADS-B messages ready for transfer to the TALON PC via a USB link. Basestation software, bundled with the SBS-3, aggregates these packets to provide a display similar to air traffic control, as shown in Figure 6.12. Notice the number of aircraft visible by the system in the map of the Ohio area, as well as the list of aircraft in the rightmost columns of the software. The various columns represent the various datapoints broadcast with the ADS-B packets, including Mode-S callsign, altitude, latitude, longitude, speed, track and vertical rate. Basestation can also log this information, which can also be used to log all received ADS-B packets. The callsign data can be used in conjunction with aircraft registries to determine the type of aircraft for dimensions and features (such as propulsion mechanism) to compare with radar
results. Appendix B details the conversion operations required for translating the raw ADS-B data into this convenient format for data display.

6.3 Core Signal Processing

Figure 6.13 demonstrates the high-level diagram of a passive radar instantiation with $M$ transmitters and a passive radar receiver with $N$ receive antennas as a phased array. The $N$ wideband analog signals received from the RF front end are then sampled by the ADC followed by digital downconversion either with on-board FPGAs, in the case of the MuTeRa testbed, or via post processing in MATLAB. Digital beam-forming operations then form reference and surveillance beams for each of the $M$ transmitters, resulting in $2M$ separate data streams, assuming only one surveillance beam is formed for each transmitter. Signal conditioning, in the form of pilot tone suppression or mismatch filtering and DSI suppression, then takes place. Following this, cross-correlation between each of the $M$ transmitters’ reference and surveillance waveforms results in $M$ separate RD maps, which can be fused for higher-level functions such as multistatic detection, tracking, or imaging. The discussions of the following subsections detail the more specific operations of these various stages for a single transmitter. Although these procedures are somewhat standard, significant contributions in the areas of DSI cancellation and cross correlation have been made and are detailed below.
Figure 6.12: Kinetic Avionics SBS-3 control software and Ohio air picture
6.3.1 DTV Signal Specification

Specific Advanced Television Systems Committee (ATSC) signal features will be discussed in subsequent sections, as most published literature focuses solely on DVB-T waveforms [8,42,54], which are prevalent in the European and Asian regions. A brief introduction to the autocorrelation properties of ATSC waveforms was presented by Barott [25]. This DTV broadcast signal structure is specified in the ATSC’s A/53 standard [154]. The waveform consists of an 8-level amplitude modulated signal with three-bit symbols of $\pm[1, 3, 5, 7]$ at a rate of 10.76 MSyms/s. The pilot is at DC level of 1.25 relative to the modulating symbols, which makes up 7% of the total signal power. The resulting double-sided modulation is then filtered with a root-raised cosine filter (RRCS) with shape parameter $\alpha = 0.1152$ at the Nyquist rate to a 3 dB bandwidth of 5.38 MHz and bandwidth between cutoff points of 6 MHz, situated to
pass the upper sideband, pilot and a vestige of the lower sideband to assist with receiver synchronization. Although most bits are randomized the signal also includes a synchronization sequence of four bits which repeats every 77.3 μs, resulting in a -44.8 dB ambiguity at 23.19 km bistatic range. This is not of significant concern for moving target detection, since the ambiguity will be nestled inside the zero-Doppler clutter ridge. Apart from this, suppression of the pilot tone is also required to eliminate a zero-Doppler ambiguity ridge, which will be addressed in Section 6.3.3.

6.3.2 Digital Downconversion

Single channel receiver systems often do not require digital downconversion (DDC) procedures, but employ an I/Q sampling architecture and analog downconversion using a bandwidth matched to the ADC. Under such circumstances, a complex zero-centered frequency representation is immediately available to the receiver for further processing. For a wideband system consisting of multiple signals of interest at various center frequencies, the ADC samples multiple illuminators of opportunity simultaneously. These individual channels must be extracted with digital downconversion techniques individually to generate the complex analytic representation of the signal required for further processing.

Figure 6.14 illustrates the operations for extracting the baseband signal for a particular illuminator, $s_{BB}[n]$ from the wideband intermediate frequency waveform, $s_{IF}[m]$. The sample indices are different for the IF and baseband representations due to the downsampling operation, such that $t = mT_{IF} = nT_{BB}$. $s_{IF}[m]$ is first mixed with a complex sinusoid at the center frequency of the signal of interest, $f_{IF}$, in Hz. A finite impulse response (FIR) filter with impulse response, $h_{LP}[m]$, is then used
to bandlimit the signal to $-B/2 < f < +B/2$. The signal can then be downsampled by an integer factor, $F_{ds}$, to output the complex analytic signal representation at a sampling frequency of approximately $1.25 \times B$. A polyphase filterbank implementation was used to significantly reduce the computational requirements of this operation [155]. The overlap-save batch processing methodology was also used to reduce memory requirements for large datasets, often on the tens of seconds at 200 MSa/s.

### 6.3.3 Waveform Conditioning

Waveform conditioning, also known as ‘mismatch filtering’, consists of applying an equalization filter or blanking to the baseband waveforms to improve the correlation performance [25, 26, 156, 157]. The characteristics and degree of these operations depend on the signal of interest. Often, this procedure involves spectral whitening or removal of a pilot tone(s) used for synchronization of communication devices. If not accounted for, the continuous wave pilot tone(s) can cause ambiguities in the range-Doppler map, shown in Fig. 6.15.

For the case of ATSC DTV signals, the presence of a pilot tone results in a zero-Doppler ridge 22 dB below the autocorrelation peak. A spectral mask was applied to
Figure 6.15: (a) Power spectral density (PSD – Bartlett method) prior to pilot tone suppression (b) PAD after pilot tone suppression (c) Range-Doppler map prior to pilot tone suppression (d) Range-Doppler map after pilot tone suppression
the data by finding the frequency bin corresponding to the maximum power spectral density of the received signal, then zeroing all frequency bins ±1 kHz. After application of the spectral mask, an inverse Fourier transform is then calculated for further time-domain signal processing. Note that the batch processing methodology for DDC operations cannot be applied, due to phase mismatch of adjacent FFT batches. This waveform conditioning also improves the eigenvalue spread of the waveforms’ correlation matrices [19], and without such operations many of the filtering methods for DSI suppression fail to converge properly which has a detrimental effect on the system noise floor. Further discussion of this is provided in Chapter 7.

6.3.4 DSI Cancellation

As discussed previously, the greatest source of interference for passive radar systems is often the direct and scattered signals from stationary objects in the environment received by the surveillance antenna. A thorough treatment of the problem is provided in Chapter 7 with practical analysis of time-domain filtering techniques, both theoretical and experimental considerations. Results of this study showed that the Extensive Cancellation Algorithm (ECA) generally exhibited the best suppression performance, but with significant runtimes. In practice, and for most results in this manuscript, either the Fast Block Least Mean Squares (FBLMS) or Least Squares (LS) algorithms were employed, due to their generally good performance with very low computational costs and thus fast runtimes. Of these two algorithms, the LS filter was most often used when targets of interest were observed under situations where the bistatic geometries produced relatively low bistatic velocities.
6.3.5 Short-Time Cross-Correlation Method

As mentioned in Section 2.2, calculation of the cross ambiguity function (CAF) is fundamental to operation of passive radar systems. Because the signals are stochastic in nature, efficient implementation of the CAF must be done via digital methods post digitization. The equivalent form of (2.11) converted to discrete time follows in (6.2), such that $t = nT_s$:

$$\Psi [l, m] = \sum_{n=0}^{N-1} s_s [n] s_r^* [n - l] e^{-j2\pi mn/N} \tag{6.2}$$

where $N$ corresponds to the number of samples in a CPI. The indices $l$ and $m$ correspond to the discrete time delay and Doppler shift, such that the bistatic range past the baseline can be calculated as $R_D = cT_s \times l$. The true Doppler shift can also be calculated as $f_D = mF_s/N$. Although the CAF can be calculated over $N$ unique Doppler bins and $N - 1$ delay values, in practice the dimensionality of this surface is substantially smaller due to limits of realistic target velocities and the finite detection range of a particular passive radar system. Note that this expression is optimal only if a given target exhibits a constant bistatic velocity throughout the CPI, when in reality there is always some finite amount of apparent acceleration whether due to the explicit target motion or the time-varying bistatic geometries. However, under short observation periods where target acceleration is sufficiently small this expression is well matched [158].

Exact implementation of (6.2) can be achieved with two primary methods: correlation FFT and the direct FFT method of [39], Chapter 17. The correlation FFT method implements the CAF sequentially for each Doppler shift by first Doppler shift-
ing the reference or surveillance waveform, followed by cross-correlation implemented as frequency domain multiplication. The Doppler shift operation is also implemented via a circular shift in the frequency domain (rather than time-domain multiplication) to ease the computational burden. The second, direct FFT approach fixes the delay, \( l \), then multiplies the waveforms in the time domain. The remaining expression to calculate the CAF is in the form of the FFT, to generate the desired Doppler points. Although both approaches leverage the efficiency of the FFT, both algorithms are fairly computationally costly and would prove difficult to implement in real time with modern DSP technology. Notice that both algorithms involve calculation of \( N \) correlation points for each Doppler- or Range-bin, most of which are discarded because the values are far greater than the effective detection range or expected target velocities.

In order to significantly reduce the computational burden, an approximation to the cross correlation can be made, to both simplify the calculation and reduce the points which are subsequently discarded as they lie outside the range and Doppler values of interest. (6.2) can be rewritten as follows:

\[
\Psi [l, m] = \sum_{q=0}^{Q-1} e^{-j2\pi mq/Q} \sum_{n=0}^{L-1} s_s[n + qL] s_r^*[n + qL - l] e^{-j2\pi mn/QL} \tag{6.3}
\]

where \( Q \) short-time cross correlations of length \( L \), such that \( L \cdot Q = N \), are calculated. The correlation repetition rate should be more than twice the maximum Doppler frequency, or \( (L \cdot T_s)^{-1} \geq 2 \cdot f_d^{max} \). If the Doppler index is limited to \( Q \) Doppler bins, such that \( m = \{-Q/2, -Q/2 + 1, ..., Q/2 - 1\} \), then the final phase term can be ignored with minimal impact on the range-Doppler map. The approximation can
then be written as shown in (6.4).

\[ \Psi'[l, m] = \sum_{q=0}^{Q-1} e^{-j2\pi mq/Q} \sum_{n=0}^{L-1} s_s[n + qL] s_r^*[n + qL - l] \]  

(6.4)

This approximation is analogous to the stop-and-hop approximation [159] made by every pulsed Doppler radar system. For most conventional pulsed systems, the pulse length is far shorter than the than time for a single cycle of target phase \( \tau \ll 1/f_d \). Because of this, the approximation typically has negligible impact on the matched filter performance. For passive radar, the approximation has a more significant effect due to the 100% duty cycle transmissions and computational considerations of the implementation. Notice that for stationary targets at zero Doppler, the short-time correlation approximation is exactly (6.2), but as the Doppler index increases to \( Q/2 \) the correlation gain performance degrades to 1 dB, as shown in Appendix A. For situations where this degradation in correlation performance is unacceptable, \( Q \) can be increased to improve the correlation gain at the cost of increased computational complexity.

Pulsed radar systems suffer from an inherent maximum range/Doppler ambiguity problem which can be circumvented due to the continuous nature of most signals of opportunity. Effectively, any range and Doppler target position can be calculated upon inspection of (6.2). In reality, there is a bound on the maximum Doppler frequency of \( Fs/2 \) and range from the length of the record interval and the CPI, but for most passive radars these bounds lie far beyond that of expected target velocities and detection ranges.
There are two predominant methods in the literature for implementing the ambiguity approximation of (6.4): the batches algorithm and the channelization technique. A similar approach can be used to extend the coherent integration time of a passive radar system without increasing computational complexity [55]. The batches algorithm multiplies the rightmost summation in the fast time domain for each correlation lag for each batch, followed by application of the Fourier transform across the $Q$ batches. The channelization technique calculates the DFT of each batch and piece-wise multiples in the frequency domain to approximate the correlation of each short time sequence. This is followed by Doppler processing via a second DFT across the $Q$ batches, and finally an IFFT along each batch to the range-Doppler domain. The channelization technique is significantly less costly than the batches algorithm, but makes an additional approximation with the batch correlation implemented as a circular convolution, so that the correlation efficiency decreases with both Doppler and range.

A new algorithm, the batch FFT, for implementing (6.4) without further approximations but with significantly less computational requirements than the batches algorithm, was developed. The correlation of each batch is implemented in the frequency domain, similar to the channelization algorithm, but utilizes zero padding such that the circular correlation output is identical to that of linear correlation.

The batch FFT method begins by constructing the reference and surveillance batch matrices, $S_r$ and $S_s$, for $\Delta$ delay bins and $Q$ Doppler bins. The reference matrix is constructed from $Q$ non-overlapping reference batches of length $L$ zero padded by $\Delta$ samples, shown in (6.5). The surveillance channel is constructed from batch segments of $L_\Delta = L + \Delta$ samples, overlapping by $\Delta$ points.
\[ S_r^{L \times Q} = \begin{pmatrix} S_r^{L \times Q} \\ 0^{\Delta \times Q} \end{pmatrix} = \begin{pmatrix} s_r[0] & s_r[L] & \cdots & s_r[(Q-1)L] \\ s_r[1] & s_r[L+1] & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ s_r[L-1] & s_r[2L-1] & \cdots & s_r(QL) \end{pmatrix} \] (6.5)

\[ S_s^{L \times Q} = \begin{pmatrix} s_s[0] & s_s[L] & \cdots & s_s[(Q-1)L] \\ s_s[1] & s_s[L+1] & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ s_s[L+\Delta-1] & s_s[2L+\Delta-1] & \cdots & s_s[QL+\Delta] \end{pmatrix} \] (6.6)

constructed in this manner, the first \( \Delta \) points of interest will be equivalent to the linear correlation of each batch comprising the cross-ambiguity approximation calculation. The range Doppler map can be calculated simply through column-wise FFT’s, piece-wise multiplication via the Hadamard product \([\circ]\), followed by an IFFT along the columns.

\[ R_r^{L \times Q} = F^{-1} (F s_r \circ F s_s), \] (6.7)

where \( F \) and \( F^{-1} \) are the Fourier transform and inverse Fourier transform matrices. The matrix \( R \) represents the range-slow time for \( L + \Delta \) batch correlation points, the first \( \Delta \) representing the linear correlation segment of each batch. This is then
Algorithm | Multiplications Required | $N = 1 \times 10^6 \quad Q=200$
|----------------|--------------------------|-------------------------|
| Correlation FFT | $2N\log_2(N) + Q[N + N\log_2(N)]$ | $4.23 \times 10^9$
| Direct FFT | $\Delta[N + N\log_2(N)]$ | $8.37 \times 10^{10}$
| Batches | $\Delta[N + Q\log_2(Q)]$ | $4.01 \times 10^9 \ (Q = 200, \text{loss} \leq 3.92 \text{ dB})$
| | | $4.01 \times 10^9 \ (Q = 400, \text{loss} \leq 0.91 \text{ dB})$
| Batch FFT | $3Q\Delta\log_2(L\Delta) + L\Delta Q + L\Delta Q\log_2(Q)$ | $8.65 \times 10^9 \ (Q=200, \text{loss} \leq 3.92 \text{ dB})$
| | | $1.24 \times 10^8 \ (Q=400, \text{loss} \leq 0.91 \text{ dB})$

Table 6.2: Computational cost for the various range-Doppler map algorithms

truncated to the first $\Delta$ points as shown in (6.8).

\[
R' = \begin{pmatrix}
R(1,1) & R(1,2) & \ldots & R(1,Q) \\
R(2,1) & R(1,1) & & \vdots \\
& \vdots & \ddots & \vdots \\
R(\Delta,1) & R(\Delta,2) & & R(\Delta,Q)
\end{pmatrix}
\tag{6.8}
\]

The range-slow time matrix is then multiplied by the Fourier transform matrix along the rows (slow-time), to generate the range-Doppler map, as follows

\[
\Psi' = R'F.
\]

The various algorithms’ computational costs are compared in Table 6.2, for an example DTV passive radar system at 10 MHz sampling frequency, 600 MHz center frequency, 100 ms CPI, maximum bistatic range of 120 km past the baseline, and equivalent monostatic velocity of 250 m/s. The corresponding parameters for the various range-Doppler algorithms are shown in the third column of Table 6.2. Notice
that the batch FFT method is 32 times less computationally costly than the batches technique, even when $Q$ is increased such that there is less than 1 dB in correlation gain loss compared with the exact calculation of (6.2).

### 6.4 Performance Summary

A number of additional experimental data have been gathered by the MuTeRa testbed which are worthy of mention:

- The system has revealed a number of first-time observations of various phenomenologies, as discussed in Chapter 8.

- The six element bow-tie array of Fig. 6.4 has been proven to yield RMS angular (direction of arrival – DOA) errors of less than $0.5^\circ$ [160].

- Aircraft have been detected at bistatic ranges exceeding 100 km, and at cruising altitudes exceeding 30,000 ft. DTV passive radar systems are often not thought to be of great utility at long range and high altitudes, due to the relatively narrow transmit antenna beampatterns. As such, such scenarios merit further investigation in future work.
Chapter 7: DSI Suppression Techniques

This chapter serves to predict the performance of, and provide a benchmark for, a number of time-domain DSI mitigation techniques. Furthermore, it provides an extensive overview of the factors affecting suppression performance. Both classical adaptive filtering methods [161] and more recent algorithmic developments in DSI suppression [48,162] will be evaluated using extensive theoretical and experimental verification. These include: suppression performance in terms of mean, variance over time, RD clutter null width, and runtime, along with additional remarks on the implementation considerations. All predominant factors that affect suppression performance will be discussed, which include: signal purity, passive radar processing effects, and the DSI suppression algorithms and related parameters. Signal purity describes the presence of noise, interference, and clutter in both channels of a given passive radar system, while the processing primarily concerns the choice of coherent processing intervals. All of these parameters vary widely between various passive radar instantiations, and this in turn potentially requires adjustments in the choice of suppression algorithm and its tunable parameters to deal with changing scenarios and operational goals.

The power-related equations for modeling the signal levels, DSI suppression, and the effects of the processing will also be developed such that a system’s effective noise
level can be calculated – one of the fundamental radar parameters, directly influencing almost every radar operation. Analysis of these parameters not only clarifies the importance of DSI suppression, but gives a strategy for predicting performance for a particular scenario based upon the experimental benchmark results of the various algorithms.

Due to the importance of DSI mitigation techniques, there has been an appreciable amount of investigation in recent years. Predominant among these developments is the Extensive Cancellation Algorithm (ECA) by Colone [48], essentially a least squares (LS) technique generalized to incorporate Doppler-shifts of the reference waveform. In addition, variants of the CLEAN algorithm originally used in radio astronomy have also been proposed for passive radar DSI suppression [50,162]. These methods, along with the typical LS algorithm, assume the interference terms are fixed throughout the CPI, effects of which will be examined in Section IV. Many classical adaptive filtering techniques can also be adapted to remove interference, such as the normalized least mean squares (NLMS) algorithm, fast block LMS (FBLMS) and recursive least squares (RLS) [49,51,52,161]. These adaptive algorithms will be shown to be less susceptible to degradation in time-varying clutter environments due to their continually adaptive nature.

A few surveys to date have compared the effectiveness of various DSI suppression algorithms [49,51,52,163]. Cardinali compared least mean squares (LMS), NLMS, RLS, and ECA algorithms against simulated analog FM waveforms [163], but these results were not verified with experimental datasets. Suppression of the DSI was found to be approximately 50 dB. The ideal fixed clutter coefficients and signal structure of the analog FM results in suppression benchmarks which are not applicable to
most modern digital signals in a realistic clutter and multipath environment. Palmer [52] conducted a survey comparing LS (Wiener) filtering, LMS, and RLS for digital video broadcast (DVB-T). The results show only 12-15 dB of reduction in the noise floor, far worse than the authors realized by any suppression algorithms under any circumstance. No insight is given to why the filters were performing in such a manner. The study also did not include the FBLMS nor ECA algorithm, both of which will be shown to be high performing algorithms with excellent suppression capability. In particular, FBLMS has received very little attention outside of the work of Garry [49] and Meller [164], but it will be shown that it is the most promising candidate among all compared algorithms for realizing real-time passive radar operation without sacrificing significant suppression performance.

This chapter serves to unite the seemingly contradictory reports of performance in the literature for DSI suppression, and to highlight the various underlying factors which can degrade suppression performance. This comparison provides an overview and coverage of time-domain suppression algorithms and allows for an informed decision when selecting a DSI suppression strategy in passive radar system design. The following algorithms will be compared on simulated and experimental digital television (DTV) data:

1. Extensive Cancellation Algorithm (ECA)

2. Least Squares (LS)

3. CLEAN

4. Normalized Least Mean Squares (NLMS)
5. Fast Block Least Mean Squares (FBLMS)

6. Recursive Least Squares (RLS)

Although ATSC waveforms (described in Section 6.3.1) are used here, the results are broadly applicable to any digitally modulated waveform which has been processed to eliminate undesirable artefacts in the ambiguity function. The pilot tone was suppressed with a notch filter in the frequency domain prior to DSI suppression filtering to eliminate a zero-Doppler ridge in the ambiguity surface. If these operations are properly applied, the behavior of the algorithms and radar performance will be very similar to that of a random noise signal [50]. As such, the performance of the various suppression algorithms and related benchmarks should be generally consistent across other digital transmissions under similar conditions.

Section 7.1 is devoted to the fundamentals of the passive radar signal model, with relevance to the direct signal and clutter components. Range equations for the target, DSI, and thermal noise level of the surveillance channel are then developed. Section 7.2 is devoted to fundamentals of DSI filtering methods within the passive radar context. Section 7.3 is an extensive evaluation of the various DSI suppression algorithms. An analysis of the primary factors affecting DSI suppression performance through simulated and experimental comparisons is first presented, followed by an extensive experimental validation across a wide range of DTV passive radar scenarios.

### 7.1 Passive Radar Signal Model

The basic signal models and passive radar range-Doppler processing will be explained here as an introduction to the problem of DSI suppression. Methods for calculating
the effective DSI suppression can then be discussed, followed by range equations governing the magnitude of various components in the signal model and their relative strengths.

### 7.1.1 Received Signal Model

Expanding upon the signal model of Section 2.2, the surveillance waveform consists of three components, the DSI, target signal, and thermal noise, as

\[ s_s(t) = s_{dsi}(t) + s_{tar}(t) + \nu_s(t). \]  

(7.1)

Assuming that the waveform is observed over a sufficiently small interval such that the Doppler-shifts can be treated as a constant, (7.1) can be expanded as

\[ s_s(t) = \left[ A^d(t) s_{tx}(t - \frac{R_L}{c}) + \sum_{p=1}^{P} A^p(t) s_{tx}(t - \tau^p) e^{j2\pi f_d^p t} \right] + \sum_{q=1}^{Q} A^q s_{tx}(t - \tau^q) e^{j2\pi f_d^q t} + \nu_s(t) \]  

(7.2)

where \( A^{(n)} \) represents the complex amplitude and phase terms for each component, representing both propagation effects and clutter/target RCS values, where the superscript \((n)\) is \(d, p,\) and \(q\) for the direct path, clutter and target terms respectively. The \( f_d \) terms represent the various Doppler shifts, proportional to their velocity relative to the bistatic bisector for the local target geometry [18]. Variables \( \tau \) are the delay for each component, set by the bistatic range divided by the speed of light, \((R_T + R_R)/c)\).
The first two bracketed terms of (7.2) comprise the DSI: direct path/multipath breakthrough (generally the strongest signal) and $P$ significant ground clutter responses, respectively. Note that the direct path breakthrough has the same delay as the reference, $s_r(t)$, with a known delay but unknown amplitude. In contrast, both the delay and amplitude of various clutter discretes is unknown and may possess a slight Doppler component, due to slight movement in the environment (e.g. trees swaying due to wind.) Typically this Doppler shift is less than the desired target Doppler, $f^p < f^q$.

Broadly, DSI suppression aims to estimate the first two terms of (7.2) in some fashion using knowledge of the transmitted waveform acquired through $s_r(t)$. This estimate of the DSI component can then be coherently subtracted from the surveillance waveform such that the residual leaves only the target response and noise.

### 7.1.2 Power Calculations

The relative strengths of the various terms in (7.1) determine the degree to which they impact passive radar system operations and performance. Modeling the power from a hypothesized target can be done by adopting the bistatic range equation, shown in (7.3) below.

\[
P_{\text{tar}} = \frac{P_{\text{av}} G_{\text{tar}} G_{\text{r}}^* \lambda^2 \sigma_b}{(4\pi)^3 R_T^2 R_R^2 L_t}
\]  

(7.3)

Where the factors are as follows:

- $P_{\text{av}}^t$: Average transmitted power of illuminator of opportunity
- $G_{\text{tar}}^t$: Gain of transmit antenna towards target zone
\( G_{t_{ar}} \)  Gain of surveillance antenna main beam

\( \lambda \)  Wavelength

\( \sigma_b \)  Bistatic radar cross section (RCS)

\( R_T \)  Transmit range (transmitter to target)

\( R_R \)  Receiver range (receiver to target)

\( L_t \)  Losses (propagation and system) along target path

Note that the bistatic RCS, for any particular target, is a complex parameter which depends on RF center frequency, bistatic bisector and bistatic angle and can vary significantly, much as is the case for a monostatic RCS. Unfortunately, there is a lack of data in the open literature for realistic values of this parameter, but for complex targets at wide bistatic angles the expected cross section may be significantly smaller, up to 10 dB, than that of the monostatic value [1]. The thermal noise power in the surveillance channel is the standard formulation of

\[
P_n = kTBF, \quad (7.4)
\]

where \( k \) is Boltzmann’s constant, \( T \) is the effective noise temperature of the receive antenna, \( B \) is the noise bandwidth, and \( F \) the noise factor.

To adequately model the DSI terms, a combination of the Friis transmission formula and a summation of bistatic radar range equations is required. This is due to the fact that DSI, in the most general form, consists of both direct path breakthrough into the surveillance antenna’s sidelobes (one-way propagation) and strong multipath
and clutter from the environment around the receiver (two-way propagation over various range and angles.) These two terms, respectively, are shown in (7.5), and set the level of the amplitude coefficients of the bracketed term in (7.2).

\[
P_{d_{si}} = \frac{P_{av}^t G_t^r G_p^r \lambda^2}{(4 \pi)^2 R_d^2 L_d} + \sum_{p=1}^{P} \frac{P^p v G_t^p G_p^p \lambda^2 \sigma_b^p}{(4 \pi)^3 R_{T,p}^2 R_{R,p}^2 L_b}
\]  

(7.5)

Where:

- \(G_t^r\): Transmit antenna gain towards the receiver
- \(G_t^t\): Surveillance antenna gain towards transmitter
- \(G_t^p, G_p^p\): Transmit and receive antenna gain towards p\(^{th}\) clutter cell
- \(\sigma_b^p\): Bistatic radar cross section of the p\(^{th}\) clutter cell
- \(R_{T,p}, R_{R,p}\): Transmit and receive range of the p\(^{th}\) clutter cell

Generally the first term (direct path) of (7.5) dominates the power of the surveillance waveform. For passive radar systems with a fixed or adaptive antenna null in the direction of the transmitter, the direct path breakthrough may be weaker than some strong clutter components.

### 7.1.3 Passive Radar Processing

Now that the theory behind the signal model and power related calculations of a passive radar have been developed, the effects of signal processing can be examined. The fundamental stage of passive radar processing is computation of the RD map, from which target detection and tracking can be carried out directly, as shown in Section 2.2.
DSI suppression is typically performed prior to RD processing, which reduces the power of the DSI term in the surveillance waveform by $R_{\text{dsi}}$ dB – the focus of the remaining sections. In addition, the Doppler processing inherent in RD map formation can further separate moving target echoes from the stationary clutter and direct signal components. This is due to the thumbtack autocorrelation floor of noise-like digital waveforms at $10\log_{10}(\tau B)$ dB down from the zero-delay and Doppler peak. This has been demonstrated for noise radar waveforms, ATSC, and DVB-T transmitters [50, 157, 165]. Therefore, the difference in the fully coherent integration of the target signal compared with the incoherent integration of the DSI (when mismatched at different delay and Doppler shifts of the target) and noise terms results in the following effective power levels relative to the target strength, shown in (7.6) and (7.7).

$$P_{\text{dsi, eff}}^{\text{dB}} = P_{\text{dsi}}^{\text{dB}} - R_{\text{dsi}} - 10\log_{10}(\tau B)$$  \hspace{1cm} (7.6)

$$P_{\text{n, eff}}^{\text{dB}} = P_{\text{n}}^{\text{dB}} - 10\log_{10}(\tau B)$$  \hspace{1cm} (7.7)

Note that for (7.6) and (7.7) to hold, the integration time, $\tau$, must be sufficiently short such that coherent integration of the desired target signal is maintained. This depends highly on the target kinematics, and integration time for a target with bistatic velocity $V$ and radial acceleration $A_r$ should be restricted to $\tau < c/VB$ or $\tau < \sqrt{\lambda/A_r}$ which confines range and Doppler migration, respectively, to less than a resolution cell [166]. Effects of quantization noise can also be considered, but are typically negligible with state of the art analog to digital converters (ADC’s) with a large number of bits [165]. Quantization can be modeled as an additional noise term.
\[ P_q = \frac{P_{\text{max}}}{(1.5 \times 4^N_b)} , \]
where \( N_b \) is the effective number of bits in the ADC, and \( P_{\text{max}} \) is the maximum input power of the ADC. If an IF sampling architecture with oversampling is employed, the noise power in a particular signal bandwidth is further reduced by the factor \( 2B/F_s \), where \( F_s \) is the ADC’s sampling frequency. After digitization, this value is further suppressed by the processing gain inherent in the cross correlation operation, and thus gets reduced by the same \( 10\log_{10}T_B \) factor as the thermal noise. The ratio of the target power to interference terms can be used to calculate the surveillance antenna’s signal to interference plus noise ratio (SINR), which reveals the expected radar performance for a particular scenario (7.8).

\[ \text{SINR} = \frac{P_{\text{tar}}}{P_{\text{dsi\_eff}} + P_{\text{n\_eff}}} \] (7.8)

The goal of DSI subtraction is to reduce the effective DSI contribution by a sufficient amount such that the in-band thermal noise floor remains the limiting factor, which can only be suppressed via matched filtering of the reference waveform. At such point, increasing the SNR in the radar must be gained by alternative methods of increasing the receive antenna gain, minimizing the noise figure and losses in the receive chain, or by extending the length of the coherent integration gain if possible.

Figure 7.1 illustrates the general relative power levels of the direct signal, noise and target signal in the surveillance waveform. The power levels are described in the preceding equations, the arrows represent stages of processing and shielding, and the dashed lines illustrate the range of power levels often seen by passive radar systems. The top of the first column, representing the direct signal power, begins with the power in the reference signal, \( P_{\text{ref}} \), often with a directive antenna towards the
illuminator. Physical isolation (null-steering or sidelobes oriented towards the transmitter, polarization mismatch, or shielding) further decrease the DSI power received by the surveillance channel relative to the reference. The level of $P_{dsi}$, $P_n$, and $P_{tar}$ represent the power levels then digitized by the passive radar receiver. The DSI is then further mitigated by the DSI suppression factor, $R_{dsi}$, due to the time-domain processing. When the RD map correlation is matched to a particular target’s range and Doppler position, the target signal experiences a large coherent integration gain compared with the unmatched terms: noise and DSI (which is only integrated at the zero Doppler position). The net effect of the correlation processing is therefore a relative reduction in the DSI and noise terms by the time-bandwidth product compared with the full integration experienced by the target signal.

The maximum possible DSI suppression, if measured from the reduction in the RD map noise floor, corresponding to complete elimination of the direct signal terms
such that only the target signal and noise remain, is given by the interference to noise ratio (INR = \( \frac{P_{dsi}}{P_n} \)).

\[
\text{INR} = \frac{P_{dsi}}{P_n}
\]  

(7.9)

### 7.1.4 Practical Calculation of DSI Suppression

The effectiveness of DSI suppression must be established by proper choice of metric. Some authors [163, 167] have evaluated the DSI suppression by the ratio of powers in the surveillance waveform before and after suppression. However, it was found through experimental testing and many empirical observations, conducted in the presented research, that this method of calculation is not reliable for estimating the effective reduction of the noise floor in the RD map; it results in over-estimation of suppression by 3-5 dB. By computing the RD surface twice (prior to and after DSI suppression), the effective DSI reduction can be directly calculated, allowing for accurate estimation of \( R_{dsi} \).

The portion of the RD map isolated for the DSI suppression value is shown in Figure 7.2 (a) and (b) bounded by the rectangular box. The region of the RD map selected excludes any significant clutter contributions, as well as that of strong target responses, seen in Figure 7.2 (b) after the application of DSI suppression. All powers shown in the various subplots are consistent and measured relative to the direct signal power of the reference waveform. The effective value of DSI suppression is found by computing the average of the dB values between Figure 7.2 (a) and (b), which minimizes the estimate of the floor level to erroneous datapoints, such as a target above the noise floor. Notice the strong target circle in Figure 7.2 (b) was completely
Figure 7.2: Illustration of LS DSI suppression on experimentally collected DTV data. (a) RD map prior to suppression (b) RD map after suppression (c) zero-Doppler cut of subplot a (D) zero-Doppler cut of subplot b.
masked by the sidelobes of the DSI terms on the zero-Doppler line of Figure 7.2 (a) prior to subtraction. The SIR for a given target can be calculated as the target’s peak response divided by the average power in the rectangular border.

Figure 7.2(c) and (d) show the zero-Doppler cut through the RD maps shown in Figure 7.2 (a) and (b). The 1000 filter taps of the LS filter used for suppression show excellent cancellation of the clutter to 30 km, suppressing the direct signal breakthrough and clutter by up to 80 dB. The effective noise floor is also reduced by about 35 dB, far less than the reduction in the zero-Doppler clutter. Notice that the cancellation of 1,000 coefficients revealed not only the target of Figure 7.2 (b), but also residual clutter components beyond 30 km which also previously masked by the direct signal breakthrough.

7.2 DSI Suppression Fundamentals

This section focuses on the basic model for DSI suppression, a brief overview of the various algorithms to be compared in Section IV, and practical implementation considerations.

7.2.1 Framework for Adaptive Filtering

The general model for adaptive filtering, tailored to the DSI suppression problem, is shown in Figure 7.3. Because these algorithms are implemented via digital signal processing after digitization with an ADC, the terms are shown here with square brackets to indicate discrete time, sampled at interval $T_s$ such that $t = nT_s$. The reference
Figure 7.3: Block diagram for DSI suppression

waveform passes through an FIR filter with impulse response $h[i]$, representing the delay and complex scattering coefficients of the DSI.

$$s_{dsi}[n] = \sum_{i=0}^{M-1} h^*[i] s_r[n-i], \quad \text{(7.10)}$$

where $M$ is the number of discrete time delay coefficients to properly model the maximum bistatic range of the DSI components greater than the noise power, $M = \lceil R_{D}^{max}/cT_s \rceil$, with $\lceil \cdot \rceil$ as the ceiling operator. The various discrete samples of $h[i]$ represent a continuum of clutter responses, and as such it is inevitable that many clutter responses will arrive at a time delay corresponding to a fractional number of ADC sampling periods. Typically, a response for a given clutter discrete will be spread across neighboring coefficients of $h[i]$ with reduced amplitude [168].

The DSI removal process first estimates the unknown clutter and direct path coefficients, $\hat{h}[i]$, then convolves the result with the reference channel waveform to estimate $s_{dsi}[n]$. This output is subtracted from $s_s[n]$ to leave only target responses and thermal noise in the surveillance signal. The final output after these operations is
represented by $s_c[n]$, a DSI-free ‘clean’ surveillance channel consisting of the target response with additive noise.

While relatively weak clutter responses (compared with the direct path leakage $A_d$) may have a negligible impact on the total DSI power in the surveillance waveform prior to suppression (7.5), their power is often significantly greater than thermal noise and will thus degrade the SINR if not accounted for. The effectiveness of the DSI subtraction process degrades if $M$ is insufficient to properly model the clutter response, but computational requirements are relaxed as $M$ decreases. Effects of varying the number of coefficients of $\hat{h}$ will be further examined in Section 7.3.

### 7.2.2 DSI Mitigation Algorithms

Methods of digital filtering for DSI suppression can be classified by two primary categories, adaptive filtering and fixed coefficient filtering. Conventional classification for adaptive filtering approaches groups schemes into two primary groups: block and fully adaptive methods. Fully adaptive methods update the filter coefficients $\hat{h}[n]$ for each fast time sample $n$ throughout the processed CPI, while methods which update the coefficients every $N$ samples are referred to as block methods. Typically $N = M$, the same length as the number of filter coefficients in $\hat{h}$, significantly smaller than the number of samples in a typical CPI for passive radar that is usually at least 100 ms [54], requiring upwards of one million samples. For this work, a filter whose coefficients are updated *within a CPI* (both fully adaptive and block methods) shall be classified as adaptive, whereas filters which select a single set of coefficients for the entire CPI shall be referred to as fixed coefficient filtering methods.
Of the compared algorithms, LS, ECA and CLEAN are categorized as fixed co-
efficient methods, while NLMS, RLS and FBLMS are adaptive filtering techniques.
Fixed coefficient filtering techniques assume that the joint statistics of the reference
and surveillance waveforms are wide-sense-stationary over the CPI, which may not
hold in practice due to time varying clutter environments and receiver system antenna
motion and instabilities. The adaptive techniques, in contrast, can automatically ad-
just to time-varying signal statistics which can cause degradation of the suppression
performance if not properly accounted for. Investigations into this will be discussed
in Section 7.3.

Details of the algorithms are well documented and thus outside the scope of this
manuscript; the reader should refer to the following references for the detailed theory
of the following algorithms: the CLEAN algorithm was developed by Kulpa [50,162],
the ECA by Colone [169], while the procedures for LS, NLMS, RLS, and FBLMS are
derived in detail by Haykin [161].

Remarks on Implementation

For this manuscript the ECA has been implemented in a single stage by di-
rectly solving for the optimal clutter coefficients to project the surveillance signal
into the orthogonal subspace to the clutter subspace, as in Section III of [169],
\[ \hat{h} = (X^H X)^{-1} X^H s_s, \] where \( X \) is a matrix of time-delayed, Doppler-shifted repli-
cas of the reference waveform. Although there exist batch methods and iterative
subtraction algorithms to remove strong clutter and targets, direct implementation
was used to permit more straightforward comparison with other algorithms. Regard-
ing implementation complexity, calculating the autocorrelation matrix \( (X^H X) \) and
cross correlation vector \( X^H s_s \) directly with FFT correlation methods was found to
significantly reduce memory and computational requirements.

The CLEAN algorithm was also implemented using only a cross correlation of the reference and surveillance, without compensating for Doppler shifted target echoes, as proposed by Kulpa [50]. A similar approach which suppressed zero-Doppler clutter has been proposed for through-wall sensing [170]. This method used interpolation in range for the subtraction stage, but it was found that this additional step was an unnecessary addition as it did not improve suppression significantly. It will be shown in the following sections that without interpolation, much improved suppression values were achieved compared with the average 19 dB of suppression [170].

The NLMS algorithm was chosen rather than that LMS algorithm due to the normalization factor in the adaptive algorithm, which eliminates hand-tuning the adaptation parameter, µ, for each illuminator due to the various signal strengths. An alternative would be to normalize the energy of reference signals prior to DSI processing, which would permit a single optimized parameter for all signals.

Difficulty in optimizing implementation varies considerably between the algorithms, primarily due to the different number of parameters for each method. The LS algorithm finds the filter coefficients \( \hat{h}[i] \) which minimize the squared error signal, \( s_c[n] \). LS has no tunable parameters and thus gives a single solution for a particular CPI, which makes it straightforward to implement. ECA generalizes this operation for Doppler-shifted versions of the reference waveform, but still assumes that the magnitude and phase remains constant throughout the processing interval. However, the ECA has the additional choice of the number of Doppler bins (and delays) to suppress outside the zero-Doppler clutter region.

The NLMS and RLS algorithms have only one single adaptive parameter, which
controls the coefficient update rate. FBLMS, on the other hand, has two tunable parameters which affect the convergence and performance of filter [161]. It was found that setting the frequency domain parameter $\gamma = 1$ for FBLMS allowed for convergence to be achieved without too much difficulty. An additional energy normalization factor\(^1\) was also included, taking inspiration from NLMS, which allowed for a single common adaptive parameter to be used across all experimental and simulated datasets. This approach, developed during the experimental analyses of Section 7.3.2, was not found in the literature but was found to work well in practice.

Assuming the CPI is fixed to a particular length, all filtering schemes require common input in terms of the number of coefficients of the estimated direct path and clutter coefficients, $\hat{h}^i$. The number of suppression iterations for CLEAN algorithm can be viewed as the counterpart to the number of filter taps for the other methods. The number of filter taps for optimal performance must be estimated based upon experimentally collected data, due to the dependence on the clutter distribution seen by the surveillance antenna. In all cases, there is a definite tradeoff between the number of filter taps and computation requirements for all algorithms, as will be investigated in Section 7.3.1.

\(^1\) The normalized version of the FBLMS, where the true update coefficient, consistent with [161] is: $\alpha = \mu \cdot 4000E \left[s^2_n(n) \right]$. This normalization constant typically converges within the range of $0 < \mu < 1$, a similar range to the behavior of the NLMS algorithm.
7.3 Evaluation of DSI Suppression

7.3.1 Factors Affecting Suppression

Aside from the choice of suppression algorithm, there are numerous factors which influence suppression performance. From extensive simulation and experimental analyses conducted here, the primary factors can be classified as external effects and intrinsic processing parameters. External factors affecting suppression have to do with the signal purity (amount of multipath and SNR) of the reference and surveillance waveforms, as well as the physical clutter environment and its stationarity. The choice of processing parameters involves not only the choice of suppression algorithm and its tunable coefficients, but also the CPI over which the data is to be processed.

7.3.1.1 Channel Length

Most broadcast illuminators for passive radar exhibit high power levels and near omnidirectional beampatterns slanted toward primary users on the ground. This results in a large number of strong clutter components over a wide swath of bistatic ranges and angles. Thus, a given DSI filter must have an adequate number of taps to sufficiently suppress the zero-Doppler clutter ridge in bistatic range until the clutter power decreases past the thermal noise, the theoretical maximum effective suppression of (7.9). Results of suppression versus bistatic range are shown in Figure 7.4, plotted for many experimentally measured DTV illuminators using the LS algorithm. Details of the experimental setup for these results are given in Section 7.3.2. Notice that the suppression of the effective noise floor stabilizes after approximately 30 km in bistatic range, although clutter coefficients remain above the noise floor shown in Figure 7.2.
However, the shapes of the curves vary due to clutter distribution between each transmitter. Notice that significant clutter is seen beyond 30 km in Figure 7.2, but suppressing these causes no further reduction of the effective noise floor in the RD map. The cause of this effect can either be attributed to residual components of the stronger direct path breakthrough due to imperfections in the DSI mitigation algorithm, or the thermal noise floor in the surveillance channel, as will be further investigated in Section 7.3.2.

### 7.3.1.2 Signal Purity

Three predominant factors comprise signal purity – SNR of the reference waveform, INR (interference to noise ratio) of the surveillance waveform, and the presence of clutter or multipath in the reference channel. For passive radar systems with minimal reference clutter (using moderately directive reference antennas and line of sight to the illuminating source) the effective suppression level is bounded by the lower of either the reference SNR or the surveillance INR. In practice the reference SNR is typically higher due to the directive antenna beam pointed straight towards the transmitter.
Therefore, the thermal noise in the surveillance channel is most often the limiting factor for performance.

Figure 7.5 shows the results of simulation analysis demonstrating suppression performance as a function of both INR and SNR. A 6 MHz band-limited noise waveform was generated to represent the transmitted waveform. The direct path and clutter coefficients, $h[i]$, were modeled as complex Gaussian random variables whose variance decreased by factor $(1/\text{index})^{0.25}$ to represent the $1/R^4$ spreading losses of the radar. This clutter vector is shown in Figure 7.5 (d). These coefficients were convolved with the reference waveform to generate an ideal surveillance signal representing a stationary clutter environment throughout the CPI.

Results of Figure 7.5 (a) show suppression as a function of surveillance INR, with a noise free reference signal. For this ideal case, the DSI can be very well estimated, but the noise remains after filtering and sets the noise floor of the RD map. In Figure 7.5 (b), the reference SNR was varied and suppression applied to a noise free surveillance signal. In this case, the noise is added to the surveillance waveform as a side-effect of the DSI subtraction process, which is shown to limit the suppression performance in a matter somewhat analogous to the surveillance INR case. For the case where noise was added to both the reference and surveillance simultaneously of Figure 7.5 (c), the mitigation performance degrades further by approximately 2 to 3 dB due to the additive noise effects between both channels. These results also show an upper limit of the LS algorithm of around 64 dB of suppression for an ideal linear and stationary clutter environment.

A complete treatment of reference signal clutter is outside the scope of this manuscript. The presence of strong specular scatterers with additional time delays
of $t_c$ relative to the direct path results in noncausal ambiguities from the direct path and false targets at a range of $R_{tar} - c t_c$, where $R_{tar}$ is the true bistatic range to the target. As mentioned previously, these disturbances can be removed for digital modulations through remodulation procedures at a slight increase in computational requirements. Another method for mitigating noncausal clutter ambiguities is to delay the surveillance waveform prior to DSI suppression [168]. It was found that 0.5 dB improvement could be achieved in practice for FBLMS with ATSC by delaying the surveillance waveform up to 1 $\mu$s (100 samples at 10 MSa/s), but no significant improvement was seen for the other algorithms. This was attributed to the FBLMS’s adaptive component operating in the frequency domain, rather than the time domain of the other algorithms.

In addition, target echoes intended for the surveillance channel may appear in the reference channel causing additional range- and Doppler-shifted RD peaks. For a target at range and Doppler coordinates of $(R_{tar}, f_{tar})$, a secondary peak will appear at $(-R_{tar}, -f_{tar})$ due to correlation of the leaked target echo with the direct path. A more unfortunate consequence of this phenomena is the disturbance in the target signal which occurs due to the subtraction processes inherent in DSI suppression, where the worst case is complete cancellation of the target signal. Although unlikely, this would occur when the relative amplitude and phase of the leaked target signal between the reference and surveillance channels was equal to that of the first, direct-path leakage coefficient in the estimated DSI channel. For geometrical scenarios where target responses are strongly received by the reference channel (which will most likely occur in the direction of the bistatic baseline), the demodulation operations above would be necessary for reliable operation and estimates of the target’s echo strength.
unless a secondary remote receiver is available for reference sampling.

Close, strong Doppler-shifted target responses greater than the noise floor in the surveillance channel behave as an additional interference terms and can also limit suppression performance. A number of methods have been proposed to mitigate these effects [48, 50, 168, 171]. However, the focus of this manuscript is on clutter and direct path suppression, and as such all results are presented in the absence of strong target responses which limit the radar’s sensitivity.

7.3.1.3 CPI Length and Clutter Stability

Long CPI’s often result in degradation of DSI suppression performance with fixed-coefficient filtering methods, even for a stationary passive system and illuminator. This behavior is attributed to the non-stationary behavior of the clutter coefficients, which vary slowly due to moving clutter (trees swaying) and antenna motion. The resulting phase changes throughout the CPI will cause mismatch in the true and estimated channel coefficients, resulting in impartial subtraction of the DSI component. Although a fixed coefficient method (LS or ECA) may return the optimal solution for minimizing the squared error signal for the entire CPI, the result is not guaranteed to be the exact channel coefficient when localized to a particular instant in time.

Figure 7.6 demonstrates the degradation in suppression of up to 8 dB for extended CPIs, due to the real-world phase variations of the channel coefficients. These results were computed using a single experimental ATSC illuminator, consistent with experimental trials of Section 7.3.2. Notice that the adaptive algorithms (FBLMS and NLMS) experience constant performance or improve with CPI length (due to longer training sequences and better coefficient estimation), whereas the fixed coefficient
Figure 7.5: (a) Suppression vs surveillance INR (b) Suppression vs reference SNR (c) Suppression vs SNR and INR (d) Simulated random channel coefficients
Figure 7.6: Suppression vs. CPI Length with 1000 filter taps, or 30 km. (a) Suppression of the RD noise floor (b) Runtime in seconds vs. CPI length.

Methods degrade as the processing interval lengthens. In order to sufficiently suppress these time-varying components, adaptive algorithms must be selected and tuned to properly track the slowly moving objects and clutter which are not of interest to the radar.

The runtimes of Figure 7.6 (b) show that all algorithms have runtimes exceeding the CPI length when computed with MATLAB on an Intel Xeon E5-2687W octa-core processor. Notice that all runtimes are proportional to the CPI length, except for LS, where the runtime remains high for short CPIs. This is due to the inversion stage.
of the cross-correlation matrix set by $M$, rather than the processing interval. Significant effort was made to reduce the runtime of each algorithm within the MATLAB framework where possible. Runtimes for the adaptive algorithms do not include the training period to converge to locally optimum coefficients at the start of the CPI. Since all algorithms exhibit runtimes greater than the CPI length, a simple serial implementation cannot provide uninterrupted update rates without further effort to decrease the computation time, such as implementation on an FPGA or dedicated DSP board. Allowing for processing latency, a similar system could provide continuous update rates at the same interval as the CPI assuming $Q$ parallel processing chains can be employed, where $Q = \lceil T_{Proc}/T_{CPI} \rceil$.

### 7.3.1.4 Adaptive Parameters and RD Null Width

Adaptive filters are typically designed to minimize the mean-square error signal, $s_c[n]$, and therefore high performing (fast convergence) filters such as RLS generally achieve very low error. However, because of the convergence properties of the filter, an appreciable Doppler null will be created in the RD map. The adaptation parameter and behavior determines the width of the low-pass filter characteristic of the clutter estimation, $s_{dsi}$, which is then subtracted to generate the cleaned surveillance signal \[168\]. Because the filter is not intelligent enough to discriminate targets of interest from clutter, any target signals will also be removed. The highly adaptive nature of the filter results in subtraction of the $s_{tar}[n]$ component of $s_s[h]$ of Figure 7.3. Careful consideration of the DSI suppression scheme’s adaptive coefficients should be taken to ensure the Doppler null is sufficiently wide to remove the dominant stationary and slow-moving clutter but retain signals from targets at low velocity.
Figure 7.7: Null width of fixed-coefficient and adaptive algorithms (a) LS filter RD map (b) FBLMS filter RD map (c) LS mean zero-Doppler cut through null (d) FBLMS mean zero-Doppler cut through null
The application should also be considered when selecting and tuning an adaptive algorithm, due to the varying widths of the Doppler null in the RD map. Using the fixed coefficient algorithms applied full CPI (rather than the batch processing method of [169]), only a single line of Doppler pixels will be suppressed because the filter taps are fixed over the entire processing interval. For the case where slow-moving targets are of interest, this would be desired. However, if only fast-moving targets are of concern, a fully adaptive algorithm would be a preferential subtraction scheme due to the adaptation to time varying clutter effects. Figure 7.7 shows the RD map and the Doppler cuts of the LS and FBLMS suppression schemes. The cuts of Figure 7.7 (c) and (d) were the averaged over a number of range bins where the DSI component was suppressed. Increasing the adaptive parameter of the various adaptive algorithms (NLMS, FBLMS, and RLS) results in faster convergence but also a wider LPF bandwidth for the filter coefficient estimation [168], which creates a larger null in the RD map after subtraction, inside of which detection performance would suffer.

By tuning the adaptive filter parameter to track the Doppler spread from the time varying direct path and clutter coefficients, optimal suppression can be achieved. However, the adaptive parameter also sets the rate of initial convergence at the start of the CPI in which only partial clutter cancellation occurs unless the DSI filter coefficients, $h[i]$, are initialized close to the true values of $h[i]$. For a fixed data length, this limits the coherent integration time as the initial data must be discarded to avoid a higher interference level due to the presence of DSI terms. For implementation with PR systems generating sequential RD snapshots where continuous operation of the adaptive filter is prohibited, there are two primary methods of channel filter
initialization: first, the final estimate for the channel coefficients from the previous CPI can be used [52]; second, an initial training period can be performed by iterating through the first few milliseconds of a CPI with a highly adaptive filter coefficient until the clutter coefficients have stabilized. The resulting estimate of $\hat{h}[i]$ is then stored and applied to the full CPI, with a reduced adaptation parameter set to the desired width of the Doppler null in the RD map. This procedure yields a good estimate of the initial channel coefficients, can operate on a single CPI and minimizes training time (due to the increased adaptation rate). Therefore, it was implemented to generate the results of the adaptive filters in this manuscript.

7.3.1.5 Other Factors

Recent publications have proposed methods for dealing with fractional delays resulting from clutter delays for non-integer multiples of the sampling frequency [168, 172]. Due to natural displacement of the reference and surveillance antenna, as well as the continuum of clutter discretes, any experimental system will contain many fractionally delayed components. However, it was found that the impact of this behavior resulted in negligible degradation in DSI subtraction performance for all algorithms (less than 1 dB). In the time domain, a fractional delay manifests as two clutter coefficients in the adjacent range positions [168]. The FBLMS performs the correlation in the frequency domain directly, which can also directly compensate for fractional shifts of the clutter distribution.

Distortions of the signals can arise through reflection, propagation, and reception of the signals in both the reference and surveillance channels. These distortions,
if not equivalent for both the reference and surveillance channels, will cause a mismatch between the true and estimated DSI components, resulting in residual direct signal interference prior to subtraction operations. RF and ADC design for passive radar systems should prioritize linearity to minimize these effects. Unfortunately, this behavior is difficult to measure or predict in practice.

### 7.3.2 Experimental Validation

This section investigates suppression performance for the DSI suppression algorithms discussed in this paper on experimental ATSC passive radar data, using a variety of illuminators and passive radar geometries. The data was collected in five different measurements spaced throughout three days. Each measurement consisted of a five second record interval. Results in this section reveal the behavior of suppression performance over system operational times, and validate some of the factors affecting DSI suppression presented in the previous subsection. Results herein are applicable to any digital illuminator of opportunity which possess similar noiselike ambiguity performance.

#### 7.3.2.1 Experimental Setup

All experimental data contained herein were gathered with the MUltistatic digital TElevision passive RAdar (MuTeRa) at The Ohio State University’s ElectroScience Laboratory, located in Columbus, Ohio [157]. This wideband UHF system collects the DTV spectrum from 580-680 MHz and measures a number of different transmitters and real antenna beams, as illustrated in Figure 7.8. Each transmission was digitally downconverted separately with a final 10 MHz sampling frequency, to satisfy Nyquist
with sufficient margin for optimal performance with slight additional computational overhead. The geographical spread in transmit sites for the collections is illustrated in Figure 7.8 (a), and the transmit parameters are listed in Table 7.1, all of which broadcast with horizontal linear polarization. The wide range of bistatic baseline distances, center frequencies, angular spread, and transmit power result in a rich dataset that can be analyzed to thoroughly evaluate the DSI suppression performance of the various algorithms from Section 7.1.

Figure 7.8 (b) demonstrates the antenna configuration, consisting of two dedicated reference Yagi antennas, co-polarized to the reference signal. The surveillance antenna was facing West, roughly 90° away from the transmitters of opportunity, polarized vertically (cross-pol) for reduction in the DSI breakthrough.

7.3.2.2 Suppression Performance with 100 ms CPI

This section analyzes the DSI suppression performance of the various algorithms for a general passive radar scenario observing air targets. A 100 ms CPI is used here,
<table>
<thead>
<tr>
<th>Transmitter Index</th>
<th>Center Frequency [MHz]</th>
<th>Baseline [km]</th>
<th>Power [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>587</td>
<td>2.9</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>605</td>
<td>7.2</td>
<td>1,000</td>
</tr>
<tr>
<td>3</td>
<td>617</td>
<td>20.6</td>
<td>503</td>
</tr>
<tr>
<td>4</td>
<td>665</td>
<td>45.9</td>
<td>1,000</td>
</tr>
<tr>
<td>5</td>
<td>677</td>
<td>7.2</td>
<td>1,000</td>
</tr>
</tbody>
</table>

Table 7.1: Transmitter parameters for Figure 7.8

corresponding to the maximum length for coherent integration gain for air target velocities up to 250 m/s with the 25 m range resolution of ATSC waveforms. The data is sampled at 10 MSa/s after digital downconversion, resulting in 1 MSa data per processing interval. To directly compare the results of the various DSI mitigation techniques, the parameters of each algorithm were individually tuned to maximize the effective suppression between the various illuminators. The adaptive filtering methods (NLMS, FBLMS, and RLS) were set to maintain a -3 dB Doppler null, relative to the noise floor after subtraction, within ±100 Hz, or 10 Doppler bins for the CPI and sampling parameters. This null corresponds to an equivalent monostatic target velocity of less than 25 m/s at 600 MHz. The optimized parameters for each filter are shown in Table 7.2. All filters used 1,000 coefficients, equivalent to 30 km bistatic range past the baseline, where the suppression stabilized as a function of filter length across the various illuminators and algorithms (see Figure 7.4).

The runtimes for the one million sample CPI and 1000 estimated channel coefficients, $\hat{h}[i]$, are shown in the final column, calculated with MATLAB on the Intel Xeon E5-2687W octa-core processor, as in the discussion of Figure 7.6 (b). The results of the runtime tests show that both the FBLMS and LS algorithms represent good candidates for real-time implementation, with runtimes of less than 1 s. It may also possible to further improve the speed of LS filtering by exploiting the Toeplitz
<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Parameters</th>
<th>Runtime (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Suppression</td>
<td>N/A</td>
<td>0.0</td>
</tr>
<tr>
<td>ECA</td>
<td>3 Doppler bins 50 range bins</td>
<td>12.5</td>
</tr>
<tr>
<td>LS</td>
<td>N/A</td>
<td>0.64</td>
</tr>
<tr>
<td>FBLMS*</td>
<td>$\mu = 0.1, \gamma = 1$</td>
<td>0.38</td>
</tr>
<tr>
<td>CLEAN</td>
<td>1,000 iterations, $\alpha = 0.95$</td>
<td>72.5</td>
</tr>
<tr>
<td>NLMS</td>
<td>$\mu = 0.09$</td>
<td>15.2</td>
</tr>
<tr>
<td>RLS</td>
<td>$\mu = 1$</td>
<td>$&gt;1$ day</td>
</tr>
</tbody>
</table>

Table 7.2: DSI filter parameters and runtimes

nature of the autocorrelation matrix, $X^H X$, for the matrix inversion procedure [161], which was not performed here. Due to the excessive runtime of RLS and no apparent benefit over the other adaptive algorithms, the filter will be discarded from further comparisons. Faster (albeit more complex) implementations do exist, but the computational requirements still remain significantly higher than NLMS, which had a runtime 150 times that of the CPI. In addition, the fast convergence rate of RLS forced the adaptive parameter to unity, $\mu = 1$, to avoid a large null in the RD map effectively attenuating targets at all Doppler frequencies in the RD map. Compared with the other algorithms under such conditions, the RLS filter did not demonstrate any benefit for any other metric.

Suppression results for all remaining algorithms (ECA, LS, CLEAN, NLMS and FBLMS) across the various illuminators and collections are shown in Figure 7.9. The five measurements of five seconds each were processed at intervals of 0.25 s, such that the start of each CPI for all measurements was $(0, 0.25, 0.5, \ldots, 4.75)$ s, respectively, for a total of 20 suppression datapoints for each filter across the five simultaneous received ATSC signals of Figure 7.8. In total, 100 values of suppression were tallied for each algorithm and illuminator pairing, but the data is shown downsampled by a
factor of two for illustration purposes. In addition, the tabulated mean and variance (of the dB suppression values) are shown in Table 7.3, sorted by algorithm and illuminator center frequency. Notice that the second and third rows of this table shows the reference signal SNR and the surveillance INR, measured by replacing the antenna feed with a matched load and calculating the power spectral density of the in-band noise signal. The ratio of the signal power to this noise power calculates the SNR and INR values for the reference and the surveillance channel.

From analysis of Figure 7.9, it is clear that there is a general trend for some filters to outperform the others. In general, the adaptive algorithms of FBLMS and NLMS and the ECA exhibited the highest levels of suppression. The two lower performing algorithms, purely based upon suppression performance, are the LS method and the CLEAN algorithm. Because of high computational cost of the CLEAN algorithm, coupled with the fact that the suppression was comparatively poor, it will not be discussed further here as there is no distinct advantage of the technique. The LS algorithm, on the other hand, was the second fastest algorithm of all tested, which merits further discussion and investigation.

From inspection of Table 7.3, note that the reference SNR is higher than the surveillance INR. This is due to the cross-polarized surveillance antenna and different RF architecture of the reference and surveillance channels. In general, the effective suppression is approximately 10-20 dB less than the surveillance channel INR. It would appear that as the INR increases, the gap between the suppression and the INR widens slightly. Notice that the 665 MHz results of the furthest illuminator, where the reference signal was likely not as pure as the closer range transmitters resulting in the lower SNR and INR, exhibited close to the same performance for
all algorithms, and an effective noise floor only 6 dB away from the thermal noise level. The stairstep function which transitions every 20 CPIs (for each measurement) shows that some unknown environmental effects, impacting the effective suppression value, occurred in the hours between collections. This could be caused by changes in propagation due to weather or varying quality of the reference signal purity.

These results show that the state of the art of passive radar systems are not capable of reaching the receiver’s thermal noise floor, solely relying on a time domain DSI approach and real-beam antennas. The cause of this could either be attributed to limitations of the suppression algorithms or to receiver system effects causing some mismatch in the surveillance and reference signals. However, assuming either were the case, one would not expect to see such deep reduction of the zero-Doppler terms for the various algorithms as illustrated in Figure 7.2. In general, it is clear that regardless of the mitigation algorithm that is chosen, effective suppression values of 30-45 dB can be expected for typical passive radar scenarios with high INR values. Knowledge of the surveillance channel noise floor can provide a good indicator for the effective noise or interference floor of a passive radar system with a good reference signal quality. A carefully chosen DSI suppression algorithm can provide an effective noise figure of 6-20 dB plus the true RF noise figure.

An ideal DSI mitigation algorithm would exhibit stable suppression performance, such that the variance in suppression when analyzed over time (between multiple CPIs) was zero. Of all algorithms, the FBLMS resulted in the most stable temporal performance shown in Table 7.3 across all illuminators. The other high performing algorithms (NLMS and the ECA) experienced a fair amount of CPIs where the result was unstable and suppression performance dropped significantly, the root cause of
Figure 7.9: DSI suppression results for 5 illuminators across 5 collections, each with 20 CPIs
<table>
<thead>
<tr>
<th>Fc (MHz)</th>
<th>587</th>
<th>605</th>
<th>617</th>
<th>665</th>
<th>677</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference SNR (dB)</td>
<td>50.7</td>
<td>66.4</td>
<td>60.3</td>
<td>36.6</td>
<td>65.2</td>
</tr>
<tr>
<td>Surveillance INR (dB)</td>
<td>41.3</td>
<td>61.2</td>
<td>56.1</td>
<td>34.1</td>
<td>62.5</td>
</tr>
<tr>
<td>ECA</td>
<td>33.3</td>
<td>3.5</td>
<td>41.8</td>
<td>3.6</td>
<td>47.2</td>
</tr>
<tr>
<td>LS</td>
<td>31.5</td>
<td>2.7</td>
<td>38.2</td>
<td>5.2</td>
<td>43.6</td>
</tr>
<tr>
<td>FBLMS</td>
<td>33.9</td>
<td>0.6</td>
<td>42.3</td>
<td>0.5</td>
<td>47.9</td>
</tr>
<tr>
<td>NLMS</td>
<td>33.8</td>
<td>0.9</td>
<td>42.0</td>
<td>7.3</td>
<td>47.4</td>
</tr>
<tr>
<td>CLEAN</td>
<td>31.0</td>
<td>1.0</td>
<td>34.9</td>
<td>0.4</td>
<td>42.5</td>
</tr>
</tbody>
</table>

Table 7.3: Mean (left) and variance (right) in suppression values for all tests (dB)
which has not yet been determined. In particular, the performance of the NLMS algorithm varied throughout the measurements of the 605 MHz illuminator shown in Figure 7.9. The LS algorithm was found to be the worst for consistency and in many cases dropped to a few dB of suppression, which resulted in the very high variances for the 605 and 617 MHz illuminators shown in Table 7.3. The behavior of the various algorithms was found to be different for each illuminator, and could not be tied to a particular physical parameter or attribute relating to the passive radar setup. In short, in-situ measurements and experimental validation is necessary to maximize the effective dynamic range of a particular passive radar receiver.

Overall, the high levels of suppression and stable performance of the FBLMS filter, combined with its minimal runtime of all compared algorithms, resulted a very appealing candidate algorithm for DSI suppression in most passive radar systems with particular emphasis on real-time implementation. NLMS and the ECA algorithm did occasionally achieve better suppression values for certain CPIs, but their unstable nature would result in less predictable tracking performance (and potentially updates with varying frequency if certain CPIs are deemed unusable due to the poor performance of the mitigation algorithm). However, FBLMS is one of the fully adaptive algorithms with a clutter notch of ±100 Hz, and therefore for situations where slow moving targets are the primary interest, a system designer may want to choose one of the fixed coefficient algorithms, such as the ECA or LS methods to enable operation in these regions. In particular, the LS algorithm is very attractive due to its simplicity and fast runtimes, requiring no inputs for operation other than the number of filter coefficients over which to operate.
7.4 Conclusions on DSI Suppression

A practical method of estimating effective DSI suppression was developed, along with the nuances behind the passive radar signal model and their impact on suppression. Runtime, mean and variance in suppression performance, and selection of each algorithm’s tunable parameters must be evaluated and are inherently linked to the radar operational modality and objectives. The primary factors affecting suppression performance, aside from the choice of algorithm, were shown to be: the physical clutter and multipath channel, purity of the reference and surveillance waveforms, and stationarity of the clutter coefficients throughout the CPI over which the adaptive filter is implemented. It was shown that it is often not necessary to suppress all visible clutter coefficients in the range Doppler surface to minimize the effective noise floor, as long as the sidelobes from the clutter lie below the thermal noise floor. For the experimental results shown here, this was approximately 30 km beyond the bistatic baseline for the various transmitters, which reduces the required filter taps and therefore computational complexity. For extended CPIs generally longer than 100 ms, it was found that the suppression performance of the fixed coefficient algorithms (ECA, LS, and CLEAN) degraded significantly due to slowly time varying clutter coefficients, which suggest using an adaptive filter (FBLMS and NLMS). These algorithms require careful tuning of the adaptation parameters to simultaneously optimize suppression performance while maintaining a sufficiently narrow Doppler clutter null as to not blind the system to slow moving targets.

Practical implementation considerations and the configuration of many algorithms for successful operation on simulated and experimental data were given to aid the
practicing engineer attempting to integrate these methods on a system. Extensive experimental results evaluating all algorithms' DSI suppression over time and frequency show that the noise floor of a passive radar cannot be reached in practice, and a combination of techniques (adaptive array antennas, analog suppression, reference signal remodulation, etc) may be required to achieve this maximum dynamic range from passive radar systems. The effective floor after application of the DSI algorithms was shown to be approximately 6-20 dB above the thermal noise floor in the ADC, such that a large effective noise figure should be used when predicting passive radar performance. Results of the individual algorithms demonstrate that FBLMS and LS both represent good candidates for near real-time implementation, with runtimes under 1 s for a typical CPI of 100 ms at 10 MSa/s. The FBLMS algorithm, which has not received significant attention in the literature, was shown to be among the most favorable of all algorithms, due to the high suppression performance, and minimal computational complexity and variance in suppression over time. However, the significant Doppler null width of FBLMS inhibits its use for slow moving target scenarios, where LS or the ECA perform better algorithms. Overall, DSI suppression was shown to be a critical stage of passive radar processing, and selection of an algorithm is an important stage in passive radar system design, requiring careful consideration of a number of factors.
Chapter 8: Passive Radar Phenomenology

This chapter demonstrates the high level of performance of the MuTeRa testbed (seen in Chapter 6) by highlighting unique experimental datasets gathered by the system. The newly observed phenomena of distributed multipath is shown in Section 8.1, along with a variety of unique micro-Doppler phenomenology in Section 8.2.

8.1 Distributed Multipath

The high SINRs achievable with the judicious design of hardware, and signal processing algorithms are capable of revealing low level phenomena. An example of this is shown in Figure 8.1, where a Boeing 737 was detected using WSFJ, the 533 MHz ATSC illuminator. An extended coherent processing interval (CPI) of 0.6 s and its increased velocity resolution emphasize the micro-Doppler response due to jet engine modulation (JEM), offset from the target peak at approximately 10 m/s spacing. In addition, the image demonstrates the pronounced distributed multipath response, at increasing range values and falling off to zero Doppler near 5 km. The SINR of the target in Figure 8.1 is close to 60 dB, further enhanced by the fact that the target is fairly close to the receiver, at approximately 1.5 km receiver range. It will be shown,
The existence of multipath in radar systems is well known, typically causing constructive and destructive interference of the incident energy impinging on a target [16]. This is due to superposition of the direct path and a reflected surface wave. However, this effect is typically confined to a single range-Doppler (RD) bin, and thus simply acts as a spatial (or time) variant modulation of the target’s RCS. Note that this is significantly different from the multipath response exhibited in Figure 8.1.

Although multipath effects are apparent in most radar systems to some degree, the magnitude of the contributions is far greater for passive radar systems. This is due to the fact that illuminators of opportunity possess antenna beampatterns that are close to isotropic and directed towards the ground, where the systems’ primary users are located. This is in stark contrast to conventional radar system antenna design where directive antennas are used which maximize the SINR and minimize
competing clutter or multipath responses. The distributed multipath effect presented here will likely have a significant impact on both air and ground passive radar systems operating in the short- to mid-range regions.

Most recent literature regarding multipath concerns DSI of passive radar systems, which consists of the direct path signal and multipath off fixed, ground based objects surrounding the passive radar [163,169]. No multipath component is considered in the target path. There has been limited recent work surrounding multipath scattering of a target’s response [173–175]. A technique for improving localization performance with through-wall radar systems is demonstrated by Leigsnering [174]. In [173], a positioning and processing procedure is suggested for maximizing information from a sensor network observing a target embedded in a multipath environment. A detection method using adaptive OFDM waveforms for targets in multipath scenarios is proposed by Sen [175], but the authors make a number of assumptions which do not hold for most systems: perfect knowledge of the environment, a single possible set of multipath components per range bin position, and equal time delays for the target return as well as all multipath components. The trail effect resulting from distributed multipath, demonstrated in Figure 8.1, has not been addressed in open literature.

8.1.1 First observations

During a set of passive radar trials, the trail phenomena demonstrated in Figure 8.1 was observed. The cause of this was originally hypothesized as either a type of wake turbulence effect, possibly due to high levels of atmospheric or exhaust particulate, or a distributed multi-bounce off of the Earth’s surface. The size of exhaust and
Figure 8.2: Experimental jet trail with negative velocity target

moisture particles relative to UHF wavelengths indicates that the wake effect hypothesis is unlikely. Further testing revealed a negative velocity target exhibiting the trail phenomenon, shown in Figure 8.2, which indicates that the trail could not be attributed to wake turbulence or particulate. For a receding target where the range is increasing over time, a wake effect would manifest itself at shorter distances than the true target range.

The illuminator of opportunity for the results of Figure 8.1 was a 1 MW EIRP DTV transmitter approximately 30 km away from the MuTeRa passive radar system. The Boeing 737 target was approaching John Glenn International airport, at an altitude of approximately 700 m. An over-the-shoulder geometry was used, such that the bistatic angle of the target’s direct signal was relatively small. Figure 8.2 used the same transmitter but for an airliner taking off from the airport.
8.1.2 Multipath Theory

Conventional radar theory typically invokes the Born, or single scatter, approximation [30]. However, as demonstrated in Figure 8.1, the multiple bounce paths often contain a significant amount of energy and must be accounted for to accurately model many high-dynamic range passive radar scenarios. The received signal model for distributed multipath, which accounts for the range and apparent velocity of different ground scatterers, will be developed here. This model extends the Born approximation to include the direct path as well as the two possible double-bounce ground paths, as illustrated in Figure 8.3.

The transmitted waveform emitted by the illuminator of opportunity, $s_{tx}(t)$, propagates out into the environment and is reflected off all objects within the beampattern of the broadcast antenna. The passive radar’s surveillance antenna, pointing towards the target area of interest, receives a combination of DSI, Doppler-shifted target echoes and thermal noise, as follows

$$s_s(t) = s_{tar}(t) + s_{dsi}(t) + s_n(t)$$  \hspace{1cm} (8.1)

where $s_{tar}(t)$ is the target response, $s_{dsi}(t)$ is the DSI response, and $s_n(t)$ is thermal
noise. For a stationary receiver, the DSI contributions originate from direct signal leakage into the surveillance beam as well as ground clutter and multipath, which can be reduced through digital filtering. The target response consists of the direct target echo but also the multipath scattered echoes off the terrain from the two different ground paths, as follows:

$$s_{\text{tar}}(t) = \alpha \sigma_t s_{tx} \left( t - \frac{R_B}{c} \right) e^{j2\pi f_D^t \tau} + \sum_{n=1}^{N} \alpha^n \sigma_t^n \sigma_g^n s_{tx} \left( t - \frac{R_d^n}{c} \right) e^{j2\pi f_D^n} + \sum_{m=1}^{M} \alpha^m \sigma_g^m \sigma_t^m s_{tx} \left( t - \frac{R_d^m}{c} \right) e^{j2\pi f_D^m} \quad (8.2)$$

The three terms of Equation (8.2) correspond to the direct path, $N$ significant target-ground multipath scatterers, and $M$ ground-target multipath terms, respectively. These three paths directly correspond to the three paths illustrated in Fig. 3. Spreading losses and antenna gain are grouped into the $\alpha$ terms. The target and ground RCS values are represented by $\sigma_t$ and $\sigma_g$, while the bistatic and distributed multipath range terms are $R_B$ and $R_d$, respectively. The Doppler frequencies, $f_D$, for each term depend on the portion of the path that is changing over time, the bistatic
angle, and the relative angle to the target’s velocity vector, as in (8.3) and illustrated in Figure 8.4.

\[ f'_D = \cos\left(\frac{\beta_i}{2}\right) \cos (\phi_i) \frac{2v}{\lambda} \]  

(8.3)

Symbol \( \beta \) is the bistatic angle, \( \phi \) is the angle relative to the target’s velocity vector, \( v \) is the target velocity, and \( \lambda \) is the wavelength at the center frequency of the illuminator of opportunity. Note that the incident and reflected angles are substantially different for each of the three paths of Figure 8.3, but also vary between each ground scatterer. This will result in a spread of various Doppler frequencies for a significant number of terrain double bounce components. The different range values shown in Equation (8.2) for each ground scatterer and the various paths will cause a dispersion in range greater than \( R_b \).

### 8.1.3 Simulation Analysis

A simulation was created to evaluate the mapping of distributed ground clutter into the bistatic range and Doppler domain. This was used to validate multipath as the cause of the empirical observations and to investigate the different phenomenologies that can arise as a result of the multipath effect. A number of different experimentally observed distributed multipath scenarios were simulated, using the transmitter, receiver and target positions from the ADS-B air truth gathered during the data collection. From this, a one-to-one comparison can be made to validate the accuracy of the multipath simulations.
Figure 8.5: Sample geometry for case A in Tables 8.1-8.2

The simulation first sets up the distributed multipath scenario by defining transmitter, receiver and ground clutter positions, as well as the target’s location and velocity as shown in Tables 8.1 and 8.2. The ground scatterers are placed at a rectangular grid situated around the target, as shown in Figure 8.5. The ground clutter discrete are spaced by 50 m in the x- and y- axes, then randomly distributed with a variance equal to the step size in all three dimensions. Notice that the transmitter is not shown in Figure 8.5, because it is situated 31 km away from the receiver, well outside the limits of the plot. The target’s bistatic RCS, $\sigma_t$, is set to 0 dBsm for all paths for simplicity, and the noise floor is set to -30 dB from this target power. The normalized ground clutter RCS, $\sigma_{go}$, is set to 0 dBsm, such that an individual ground scatterer’s RCS is $\sigma_g = \sigma_{go} \cdot A$, where $A$ is the area of the ground patch. The phase of each ground clutter return is also random and uniformly distributed. All three paths of Figure 8.1 are then simulated, and mapped to the proper location in the RD map through calculation of the bistatic range and Doppler shift, using (8.3). Spreading losses for each leg of the various paths are assumed to be that of free space, $1/R^2$. 

184
<table>
<thead>
<tr>
<th>Geom ID</th>
<th>Target Parameters</th>
<th>Geom ID</th>
<th>Transmitter Parameters</th>
<th>Multipath Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Boeing 737</td>
<td>[x, y, z] (m)</td>
<td>Position</td>
<td>Frequency</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Aircraft</td>
<td>[x, y] (km)</td>
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<tr>
<td>A</td>
<td>-1312, 309, 669</td>
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<td>A &amp; B</td>
<td>[-2.5 to 0.5, -1 to 1.5]</td>
</tr>
<tr>
<td>B</td>
<td>Boeing 737</td>
<td>89.3</td>
<td>C</td>
<td>[3 to 7, -1 to 1.5]</td>
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<tr>
<td>C</td>
<td>Boeing 787</td>
<td>121.6</td>
<td>D</td>
<td>[3 to 10, -2 to 2]</td>
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<td>D</td>
<td>Challenger 300</td>
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<td></td>
<td>50</td>
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<td></td>
<td></td>
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<td></td>
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<td>253.6</td>
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<td>50</td>
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Table 8.1: Experimental Target Parameters

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<thead>
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<th>Geom ID</th>
<th>Transmitter Parameters</th>
<th>Multipath Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Position [x, y, z] (km)</td>
<td>Frequency (MHz)</td>
</tr>
<tr>
<td>A &amp; B</td>
<td>29.9, 8.9, 0.1</td>
<td>533</td>
</tr>
<tr>
<td>C</td>
<td>2.0, -6.9, 0.3</td>
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</tbody>
</table>

Table 8.2: Experimental Parameters

### 8.1.4 Simulation Validation

Geometries A and B of Tables 8.1-8.2 were intended to establish the validity of the multipath simulation and compare the effects with the experimentally collected data.

Results of Geometry A, shown in Figure 8.6, show strong agreement for the multipath effect between the experimental and simulated data, in (a) and (b), respectively. The extent of the multipath distribution varies as the shape of the bounding box for the ground clutter is changed, but the general contour remains the same. The region of strong ground multipath responses could be more accurately predicted with propagation modeling software; however, the results obtained here are sufficient to confirm that the experimental observations can be attributed to multipath effects. The intensity of the trail for the experimental result is approximately 30-40 dB below the peak.
target response. This multipath scattering level corresponds to approximately 25 dB SINR. This power can be expected to fall off at or near the square of the receiver range, in contrast to monostatic radar’s $1/R^4$, for scenarios where the target is near the receiver, but distant relative to the transmitter.

Figure 8.7 (a) shows a different multipath scattering profile of the experimental target in the previous figure at a later instant in time, once again close match to the simulated result of Figure 8.7 (b). The dropoff in Doppler frequency is more gradual than that observed in the first experiment, simply due to the different multipath geometry and the resulting apparent bistatic angle. Interestingly, the multipath response extends beyond zero-Doppler, a counter-intuitive result. This can be visualized by examining the third path of Figure 8.3 for a target over the receiver, which lengthens (negative Doppler) in contrast to the reduction in paths 1 and 2 (positive Doppler.) This Doppler inversion, as well as Doppler expansion – where the multipath response occurs at greater Doppler frequencies than the target itself- are also demonstrated in the simulated result of Figure 8.7 (b). Although the Doppler expansion cannot be
seen for this particular experimental result, it is demonstrated in the next section.

8.1.5 Target-Ground and Ground-Target Paths

As illustrated in Figure 8.3, in addition to the direct target path there exist two distinct ground scattering paths. Since broadcast DTV transmitters are designed to support primary communications receivers on the ground, a majority of the energy is directed in that region through a directive transmit antenna pattern. Because of this, a natural assumption would be that the ground-target path (or path #3 of Figure 8.3) would typically dominate in magnitude. However, experiments have demonstrated that this is not always the case. Here, the response of the two paths is analyzed separately and compared with experimental data, using aircraft at further ranges and lower SINRs.

Experimental and simulated results of Geometry C are shown in Figure 8.8. The experimental result demonstrates two trails extending from the target response, at
both greater and lesser Doppler frequencies. This result confirms the Doppler gain phenomena demonstrated in the previous simulated case. Notice the vastly different responses of the target-ground path and the ground-target path. Comparing these results to the experimental case, it is clear that the ground-target path is the dominant scattering mechanism for this particular scenario.

Results of Geometry D demonstrate a dominant target-ground path, in contrast to that of Geometry C. From analysis of Table 8.1, the two aircraft were in similar x-y coordinate locations, but the aircraft in Geometry D was flying at a significantly higher altitude. This altitude, and also the pitch and roll of the aircraft, likely have an influence on which multipath scattering is more pronounced. This particular result also demonstrates the great extent of multipath scattering in the RD map, as the target at 7 km range exhibits a response extending past 15 km. In addition, the intensity of the multipath response at 12 km is 0.5 dB stronger than the direct target response. Note that this is a clear example of Doppler inversion and the magnitude of the multipath scattering for a realistic target situation.

8.1.6 Implications of Distributed Multipath

Implications of the distributed multipath effect are numerous. First, the multipath response provides a resistance to target fades, where the direct target response would not be detected by a conventional constant false alarm rate (CFAR) algorithm. Figure 8.10 demonstrates an experimental case where the target cannot be detected above the noise floor, but the double bounce phenomena is clearly visible. Second, knowledge of the behavior of the multipath effect could also be incorporated into detection and tracking algorithms to improve performance. Identification of the multipath trail
Figure 8.8: Multipath results for geometry C
Figure 8.9: Multipath results for geometry C
could also improve the probability of detection while simultaneously reducing the probability of false positives in the jet trail region of the RD map. Third, exploitation of the distributed multipath effect may provide a counter to stealth targets, where a majority of this energy is reflected off in some specular direction (towards the ground in some cases.) Fourth, multipath also raises the possibility of detecting targets outside of the surveillance antenna’s 3 dB beamwidth – which could either be desired or unwanted, depending on the scenario. Finally, if the radar system has knowledge of the local terrain surrounding the receiver and target, an inverse mapping from the shape of the jet trail to a point in space could provide coarse localization or bearing information for a target, or possibly reveal information about the target’s orientation or scattering mechanisms for classification purposes.

It was shown that the distributed ground multipath can often be observed using DTV passive radar systems for targets at moderate ranges. This phenomena is very pronounced for close targets, and the shape of the multipath response can be easily
predicted by mapping the ground terrain double bounce responses into the bistatic RD domain. Results were also shown with multipath responses equal in amplitude and also exceeding the direct target response. Interesting effects, such as Doppler inversion and expansion, result from different multipath propagation methods and were examined both experimentally and through simulation. Various ground bounce paths, both ground-target and target-ground, were found to dominate in different cases. Future work will investigate methods of extracting information and utilizing knowledge of distributed multipath to improve system performance.

8.2 Micro-Doppler Observations

Not only has the radar demonstrated detection at high altitude and long range, it has recorded a number of target echoes which posses characteristic micro-Doppler signatures. These unique signatures can be used for target classification, or possibly identification, provided a significant database or certain distinguishing features are recognizable. These types of features are also radar imagery and thus directly relevant to the topic of the dissertation, although not in a 2D spatial dimension as is typically thought of when discussing radar imagery output from typical SAR or ISAR systems.

8.2.1 Helicopter

Figure 8.11 shows a subsection of a processed RD map of a helicopter and its micro-Doppler signature. Unfortunately, ADS-B data were not available for this particular target of interest, and therefore the exact make and model of the helicopter were unknown because the range to the target prohibited visual identification. However,
the RD map of subplot (a) shows the immense number of micro-Doppler sidebands from the periodic blade flashes offset from the bulk target velocity at approximately 30 m/s equivalent monostatic velocity. The bulk target velocity and its offset are determined by the changing bistatic range, but the spacing in the micro-Doppler spectral lines is set solely by the blade flash frequency. This frequency is determined by the rotation rate and number of blades on each rotor of the helicopter, which typically possess a main rotor and tail rotor. Figure 8.11 (b) shows the Doppler cut through the center of the target position, after shifting the bulk Doppler response to 0 Hz.

Assuming the blades are visible, the separation in frequency of the micro-Doppler signature can be written as

\[
\Delta F = k \times \frac{RPM}{60} \times N
\]

(8.4)

where \( \Delta F \) is the location of each micro-Doppler spectral component relative to the
<table>
<thead>
<tr>
<th>Rotor</th>
<th>$\Delta F_B$ (Hz)</th>
<th>Blades</th>
<th>RPM (Hz)</th>
<th>Gear Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main</td>
<td>39.6</td>
<td>4</td>
<td>600</td>
<td>1:1</td>
</tr>
<tr>
<td>Tail</td>
<td>73.8</td>
<td>2</td>
<td>2160</td>
<td>3.6:1</td>
</tr>
</tbody>
</table>

Table 8.3: Helicopter parameters measured from Doppler cut of Figure 8.11

bulk target velocity; $k$ is an integer corresponding to the harmonic number of the micro-Doppler response, $RPM$ is the rotation of the rotor in revolutions per minute, and $N$ is the number of blades on the rotor. From analyzing the Doppler cut of the target response, it was clear that two fundamental frequencies and their harmonics were present at 39.6 and 73.8 Hz, respectively. From analysis of typical helicopter motor rotation speeds and blade counts, the RPM of the main rotor was estimated to be 600 Hz with a blade count of four, shown in Table 8.3. The tail rotor was found to have two blades with a 2160 RPM rotation, which would indicate a gear ratio between the two rotors of 3.6, well within the typical range of parameters commonly found on commercial helicopters.

### 8.2.2 Propeller-Driven Aircraft

A propeller-based aircraft was also observed, and upon analysis of the resulting data, strong micro-Doppler responses were seen. These are shown in Fig. 8.12. The magnitude of these harmonics was surprisingly found to be near that of the bulk plane response; the harmonic frequency spacing was also similar to the Doppler shift produced by the true aircraft speed. The presence of an ADS-B transponder indicated that it was a Cirrus SR22T. Compared with the helicopter, where the intensity of the harmonic micro-Doppler components fell off rapidly, the propeller micro-Doppler response is very strong; theoretically, the roll-off in amplitude should be proportional
to the blade length [176]. This is as expected, as helicopter blades are much longer than the propeller of small commercial aircraft. The SR22T is designed with constant speed throttle at 2500 RPM, and changes the thrust by altering the pitch of the propeller blades. Using the known parameters of the three rotor blades in (8.4), we obtained $\Delta\beta = k \cdot (2500/60) \cdot 3 = k \cdot 125$ Hz. The micro-Doppler peaks at multiples of 125 Hz are exactly the same as those measured from the Doppler cut, Fig. 8.12 (b). This result instills great confidence in the accuracy and validity of using the micro-Doppler signature as a significant indicator for target classification.

### 8.2.3 Jet Engine Modulation

Finally, a Boeing 737 was observed upon approach to John Glenn International airport, and the RD map exhibited Doppler sidebands shown in Fig. 8.13. This jet engine modulation follows similar principles to the previous micro-Doppler signatures, but results from slight periodic modulation of the turbine response rather than
strong intermittent blade flashes seen with the helicopter and Cirrus targets. To my knowledge, this is the first reported observation of jet engine modulation seen with a passive radar system. Figure 8.13 (b) shows the spectrogram of the peak target response computed over the 500 ms record interval. Although the target experiences range migration over the extended CPI, the elongated processing interval allows for finer frequency resolution and improved Doppler estimates. Because of the low frequencies of DTV illuminators, the wavelength is not small enough to penetrate into the further rotor stages, which makes classification more difficult for these types of datapoints.
Chapter 9: Passive ISAR Imaging

9.1 Introduction and Background

Many operational scenarios demand tasks beyond that of detection and tracking to reveal the nature of a particular target of interest. Such concepts date back to the second World War with the advent of identification, friend or foe (IFF) transponder systems. Often this data is of utmost importance, directly influencing decision making in military scenarios and helping enforce commercially controlled airspaces. Successful target classification, or placing targets into predetermined “categories,” often reveals the nature of a potential threat or disturbance. Simple methods of target classification include RCS estimation and analysis of targets’ dynamics, but with new low-observables and highly circumstantial target motion these methods of target classification are only moderately reliable. In addition, under no circumstances do they permit target identification – the assignment of a particular radar signature to a specific vehicle or aircraft.

The two primary methods of radar-based target classification are: a) micro-Doppler exploitation, and b) high resolution measurements, in either down- or cross-range (or both) [176]. Some examples of micro-Doppler signatures were given in Section 8.2, and Chapter 5 investigated multistatic narrowband imaging using aperture
synthesis to generate 2D imagery assuming a known target trajectory. This chapter investigates potentialities of high-resolution imagery with passive inverse synthetic aperture radar (P-ISAR) from a high-level algorithmic perspective, developing a comprehensive framework highlighting considerations particular to P-ISAR. The resulting high resolution imagery could allow estimation of a target’s length or width for classification purposes, with all the benefits of passive radar operation: low probability of intercept due to stealthy operation, low cost receiver systems, and bandwidth-free utilization of the EM spectrum.

P-ISAR is significantly different from conventional ISAR in a few main respects: inherent bistatic geometries, various communications waveforms, and the frequencies and bandwidths are generally low compared with dedicated ISAR systems – all of which affect image quality and processing algorithms. As the literature review of Section 3.4 indicates, P-ISAR has generated significant attention in recent years. Many of the investigations show that high resolution 2D imagery is theoretically possible from a multistatic network of passive radar transmitters and receivers [124–129,139], but these works generally neglect many significant problems common to any ISAR system. The stages of data processing, system calibration, target motion-compensation (mocomp), and image co-registration/fusion are assumed to be known – each representing problems which are far from trivial for any experimental system. As such, experimental generation of similar results is a formidable task, highlighted by the sparsity of successful experimental results in the literature. This chapter investigates a practical experimental approach to P-ISAR, accounting for common ISAR challenges but also highlighting many additional challenges particular to narrowband P-ISAR systems. A multi-channel P-ISAR has a distinct advantage for ISAR imaging, which relies
heavily on target motion to generate the rotation necessary for cross-range resolution: the multiple transmitters and thus bistatic geometries result in more frequent opportunities where the target geometry is favorable for image formation.

Existing experimental-ISAR imagery of air targets do not exhibit significant features which appear useful for target classification [8,10,136]. Most of these results were from a collection of frequency contiguous, collocated illuminators which can be coherently exploited as a single wideband transmission. This is in contrast to typical arrangements of opportunistic transmissions throughout the world where spatial and spectral separation are common. A rare 48 MHz bandwidth scenario consisting of eight consecutive Integrated Services Digital Broadcasting-Television (ISDB-T) transmitters from the Tokyo Tower demonstrated the first experimental ISAR image [136], but the lack of comparison to an expected result and absence of distinguishing features leaves questions on the efficacy of the image reconstruction. More recently, ISAR results of a commercial airliner and large freight ships have been shown [8,10], which also utilize an uncommon scenario with three collocated and spectrally-contiguous DVB-T illuminators for an aggregate 24 MHz bandwidth. Similar ships at the same transmit site were observed by a group from Warsaw, along with the first airborne passive SAR imagery [115]. Once again, the images offer no conclusive evidence of proper aperture synthesis, and no information beyond scatters along the range extent is demonstrated; note that this is achievable simply through analysis of a range-Doppler map. To date there is only one publication showing single channel P-ISAR experimental results [11], but it is also unclear if the image provides any valuable target classification information. Colone et al. investigated a short range imaging technique for cross-range profiling of automobiles using Wi-Fi signals [177]. The ap-
The approach is limited to particular geometries and, as will be demonstrated later, the entropy minimization procedure for image autofocus often does not result in properly focused P-ISAR imagery.

The approach developed here pertains to P-ISAR of non-cooperative targets with a stationary passive radar receiver utilizing a distributed ATSC network. The image formation procedure is presented as an extension to the general passive radar processing, developed in Chapters 6 and 7, for each illuminator separately. The 3D target track is used to aid in estimation of the non-cooperative target’s rotational motion, which enables more sophisticated image formation procedures than the conventional range-Doppler ISAR approach. This has numerous benefits:

- It prevents Doppler-defocusing of the imagery over wide angles (which are required for high cross-range resolution at low frequencies)
- It addresses the cross-range Doppler scale factor for target width estimation
- It permits imaging to a known projection plane
- The previous two points enable fusion of multistatic data [124–129,139]

It will be shown that limitations of the motion compensation techniques, and apparent target motion, generally dictate the ultimate image resolution, and thus prohibit narrowband techniques of enhancing range resolution beyond that of the signal bandwidth (see [7] and Chapter 4).

This chapter consists of three primary sections. First, the core algorithmic framework for P-ISAR imaging is developed in tandem with a P-ISAR simulation using a (relatively wide) 60 MHz signal bandwidth. The simulation is then used to generate
simulated 6 MHz narrowband ISAR results for the case of a single ATSC transmitter, and the effects of narrowband operation on motion compensation are investigated. Following this analysis, the developed approach for P-ISAR is demonstrated with experimentally collected data from the MuTeRa testbed described in Chapter 6.

### 9.2 Passive ISAR Framework

The high-level algorithmic framework illustrating P-ISAR image formation is shown in Figure 9.1 (a). Any data products, such as Raw Data In are represented by the gray shaded ovals, whereas any functional blocks are denoted with white rectangles. Notice that breakdowns for the Core PR Processing and P-ISAR Processing functional blocks are illustrated in Figure 9.1 (b) and (c), respectively. The Tracking block is not a key part of this text, and therefore it will be only briefly discussed.

The Core PR Processing block comprises the necessary processing to generate useful range-Doppler detection surfaces from raw ADC waveforms. The block represents the standard PR algorithms as developed in Chapter 6 and 7: digital downconversion, mismatch filtering (pilot tone suppression in the case of ATSC waveforms), followed by DSI suppression and then cross-correlation for range-Doppler (RD) map formation. Because many targets often maneuver near zero-Doppler, a DSI suppression algorithm which minimizes the Doppler null width is desired, such as the Lease Squares filter [13,177]. Note that in the general case for \( N_T \) transmitters of opportunity and \( N_R \) surveillance channels, there may be up to \( N_T \times N_R \) total such surfaces.

These surfaces pass to the Tracker which performs detection and tracking. The detection stage is easily accomplished using a conventional 2D CFAR algorithm, whose
results are then passed to the tracker. The tracker must then perform data associ-ation between the various detections, followed by localization to transform the data from the bistatic range and possibly angle domain to 3D Cartesian space. A host of different localization methods can be used for this stage: tri-lateration between various illuminators [178], direction of arrival (DOA) and range estimates if an appropriate surveillance array is available [160], or direct localization techniques based upon generalized likelihood ratio test methods [179]. In any case, the tracker must utilize the multiple RD maps produced by the Core PR Processing block. Note that the tacking performance is assumed to be on the order of the range resolution at best, at least 25 meters for ATSC for equivalent monostatic geometry of over-the-shoulder operation. Optionally, the Tracker can feed its results back to the Core PR Processor for better informed operations, such as more efficient transmitter utilization or to vary
range/Doppler limits for reducing the RD map generation. Tracking data is also sent to the *P-ISAR Processor*, the primary focus of this chapter, for imaging operations.

The job of the *P-ISAR Processor* is to turn the received RD map radar data into a 2D image, with the aid of supplementary information from the *tracker*. As with conventional SAR imaging, observation of a target over a set of angles permits high cross-range resolution to be synthesized through digital processing of the received radar echoes. With ISAR, target motion – in contrast to sensor motion for SAR – generates the synthetic aperture, but precise information regarding target motion is unknown and therefore must be estimated from the radar data. These stages of target motion compensation, usually unnecessary with conventional SAR, represent extremely important stages in ISAR image formation, and the image quality directly depends on their effectiveness. The motion compensation is of two primary formats: translational motion compensation (TMC) and rotational motion compensation (RMC). The first stage of motion compensation (TMC) is, for most cases, required to form acceptable imagery. Range-Doppler image formation can then be used to create an image whose axes are down-range and Doppler, because the true rotation of the target is often unknown. This method of image formation assumes that the target’s apparent angular velocity is constant and limited to moderately small angles, otherwise the image suffers defocusing. Application of RMC allows for targets’ apparent nonuniform rotations – often exaggerated by changing bistatic geometries and arbitrary flight paths – to be compensated and the Doppler axis scaled to a physical dimension.

Finally, the *image formation processor* (IFP) uses the motion-compensated radar data to form a 2D image. In the simple case, as described before, this image forma-
tion is similar to RD map formation, with a Fourier transform along the slow-time dimension of a range-time data matrix. If RMC is available, the Doppler axis can then be scaled to cross-range, assuming the target rotation and size are small. More sophisticated methods of image formation are required for wider angles, such as the polar formatting algorithm or variants of the backprojection technique. Note that up to \( N_T \) separate images can be formed with this framework. Fusion for multistatic imaging will be briefly discussed in Section 9.4.2 but is left as an extension to this work.

9.3 Passive ISAR Theory

This section introduces P-ISAR theory, expanding on the architecture previously introduced. In conjunction, the P-ISAR simulation is introduced for an ideal case to investigate the effect of the processing algorithms. The same algorithms will be applied to the narrowband investigations of Sections 9.4-9.4.2. Here we evaluate the following stages of P-ISAR image formation:

- Target windowing

- Translational mo-comp
  - Coarse range alignment
  - Fine range alignment
  - Autofocus

- Rotational mo-comp via tracking

- Image formation
9.3.1 Received Data Format

The received surveillance channel waveform, $s_s(t)$, for a particular transmitter within the P-ISAR framework can be expressed as a spatial interval over the target’s spatial reflectivity, $\tilde{\sigma}$, as follows:

$$s_s(t) = A(t) \iiint \tilde{\sigma}(x) s_{tx} \left( t - \frac{R_B(x,t)}{c} \right) e^{-jkR_B(x,t)} dx \quad (9.1)$$

The transmitted signal, $s_{tx}(t)$, is delayed and phase-shifted based upon the target reflectivity, where $k = 2\pi/\lambda$ and $R_B$ is the bistatic range measured from transmitter-target-receiver, with dependence on $x$ – the 3D position vector for the coordinate system centered on the target – and $t$ due to target motion in the global coordinate frame. The amplitude constant, $A(t)$, in front of the integral represents any losses (spreading, atmospheric attenuation, receiver losses, beampattern effects, etc) seen by the system. The direct signal and clutter terms are neglected here for simplicity and because they are significantly suppressed via DSI mitigation, although we will show that the residual is a DC clutter term which is accounted for with Doppler windowing, the topic of the next subsection.

Note that the target can have translational motion, $\mathbf{v}$, and rotational motion, $\mathbf{\Omega}$, in the global reference frame, each of which results in unique motions when projected into the bistatic observation of the passive radar. This is illustrated in Fig. 9.2, along with the bistatic angle $\beta$ and baseline $R_B$.

For the simulated P-ISAR results shown here, the amplitude term is considered
constant and therefore neglected. The transmitted signal is 100% duty cycle, with a flat power spectral density; this is a good approximation of the true ATSC power spectrum after filtering and pilot tone suppression, demonstrated in Section 6.3.3.

The RD map into the P-ISAR processor is calculated with the typical cross-correlation method or an approximation thereof (see Section 6.3.5) between the surveillance and the reference waveforms, \( s_r(t) \):

\[
\Psi (R_D, f_D) = \int_{t=0}^{T} s_s(t) s_r^*(t - R_D/c) e^{-j2\pi f_D t} \, dt
\]  

(9.2)

where \( R_D \) is the bistatic delay range, or bistatic range past baseline, and \( f_D \) is the Doppler frequency. The RD map, after an inverse Fourier transform along the Doppler axis, generates a data-cube similar to range/pulse data – or range time (RT) map – of conventional ISAR and SAR processing [10].

To illustrate this, along with the remaining P-ISAR operations, a simulation was developed with a target flying a straight path through \((1, 0)\) km in the x-y plane, with a velocity vector 60° from the x axis. The nose of the 40x40 m aircraft is aligned with the flight path illustrated in Figure 9.3 (a) and (b). The plane is flying with a velocity
of 200 m/s, observed with a 2 second CPI. The receiver is located at the origin with the transmitter at \((0, -1)\) km. The illuminator possesses a 600 MHz center frequency and 60 MHz bandwidth; while uncommon, it is not unfounded [136]. Although not broadly applicable to many PR installations due to the large bandwidth and target size, the parameters were chosen to illustrate the effectiveness of the proposed algorithmic framework and to investigate phenomena particular to bistatic ISAR imaging. Implications of more narrowband operations – or, equivalently, moderate bandwidth with small targets – are investigated in Section 9.3.2. The apparent rotation spanned by the target is approximately 6°.

### 9.3.2 Target Windowing

Although the time domain DSI suppression methods serve to suppress the direct path signal and clutter via filtering, the suppression process leaves some vestige of strong clutter discretes which can be removed via windowing in the range-Doppler domain. Clutter cancellation is often performed in ISAR systems [32,141], but the issue is more severe with passive radar illuminators, which often use transmit apertures whose
main beams point towards the ground, in the area of the systems’ primary users.

In addition, noise in Doppler bins outside of the target’s Doppler track should be suppressed to increase SNR of the RT maps used in subsequent imaging operations, such as motion compensation. The windowing process uses data provided by the tracker to determine bounds of the target position (bistatic range and frequency) throughout the CPI, plus an additional buffer. The windowed RD map can then be generated from the full RD surface as follows:

$$\Psi_W(R_D, f_D) = \begin{cases} 
\Psi(R_D, f_D) & \text{for } f_W^{\text{min}} < f_D < f_W^{\text{max}} \\
& \text{and } R_W^{\text{min}} < R_D < R_W^{\text{max}} \\
0 & \text{otherwise}
\end{cases}$$

This windowing operation is equivalent to convolution with a bandpass filter in the time dimension of the RT map, passing the target response while rejecting broadband noise and the clutter residuals post-DSI processing. Range bins outside the range of interest are discarded, reducing the computational load for following operations. The new effective Doppler bandwidth of the data is reduced by factor

$$Q = 2 \left[ f_W^{\text{max}} - f_W^{\text{min}} \right] / f_D^{\text{max}},$$

increasing SNR of the RT map by $10 \log_{10} (Q)$ dB in addition to improvement due to clutter suppression. Note that windowing as a method of further clutter suppression is only possible provided the target does not cross the zero-Doppler line over the imaging interval.

Results for the simulated case shown in Figure 9.3 are shown in Figure 9.4. Notice that the presence of residual zero-Doppler clutter, shown in Fig. 9.4 (c) manifested as vertical “stripes” in the RT map, Fig. 9.4 (a). After application of the RD window,
Figure 9.4: Target windowing illustration. (a) Initial RT map (b) RT map after windowing (c) Initial RD map (d) Windowed RD map with limits shown in Fig. 9.4 (d), these components are no longer visible in the RT map of Fig. 9.4 (b), in addition to a reduction of the background noise level.

### 9.3.3 TMC - Range Alignment

The first step of TMC is known as coarse motion compensation or range alignment; a number of methods have been developed since the advent of ISAR. Some of the techniques include cross-correlation [180], range centroid [181], Keystone transform [182], and entropy-based methods [183]. The range centroid method was chosen due to its simplicity and fast implementation, in addition to reduced glint susceptibility when compared to the cross correlation technique, which can result in spurious “spikes” in
the alignment process, and therefore poor estimates of target translation. A threshold is applied to each pseudo-pulse in the RT domain, discarding values less than 20 dB below the peak, resulting in a windowed range profile for the $i^{th}$ pulse, $\rho^i_w(R)$. The range centroid is then calculated as follows:

$$R^i_C = \frac{\int R |\rho^i_w(R)|^2 dR}{\int |\rho^i_w(R)|^2 dR} \quad (9.4)$$

Because the target motion must be physically constrained to a relatively smooth profile over a short observation, an additional least squares fit to a parametric motion model reduces the impact of any spurious centroid estimates. A second-order polynomial is then fit to the centroid data, to estimate the target range over time as

$$R_C(t) = \alpha_0 + \alpha_1 t + \frac{\alpha_2 t^2}{2} \quad (9.5)$$

The range data is then aligned to the smoothed target centroid at the center of the processing interval, $\alpha_0$. To avoid quantization of the range shifts, the realignment should be performed via a phase ramp in the Frequency domain using the Fourier time shift property, $s(t-a) = \int S(f)e^{-j2\pi fa}e^{j2\pi ft} df$. The phase due to propagation, which directly varies with target range, should also be compensated for at this stage as $\phi = -2\pi f R_s/c$, where $R_s$ is the range shift required to align the centroid fit, $R_C(t)$, to the range midpoint $\alpha_0$.

These processing steps are illustrated in Figure 9.5. Notice that the range centroid exhibits significant deviation from the LS fit in Figure 9.5 (a) and (b), which is solely due to target scintillation due to prior knowledge of the target’s linear flight path. Because the range centroid method only uses magnitude of the range data, it is of
limited precision; however, it is more robust to degradation due to noisy data than coherent techniques using the additional phase information. In order to better focus the image, phase information must be used, in the form of the Doppler centroid, examined next. Notice that the phase compensation centers the target around zero Doppler, which permits decimation in the Doppler domain to reduce computational burden of subsequent operations. This, however, should be done with caution to avoid Doppler aliasing and phase unwrapping issues of the Doppler centroid.

Figure 9.5: TMC - range alignment. (a) RT map with range centroid estimates (b) Close-up of centroid values and LS fit (c) RT map after alignment (d) RD map after range alignment and phase compensation
9.3.4 TMC - Phase Compensation

After range alignment, fine TMC must be applied, using the phase information from pulse-to-pulse for improved accuracy. The goal of TMC in ISAR imaging is to remove any translational motion, such that the target’s center point remains stationary; any remaining phase changes are due to apparent rotation. Therefore, a target which has been properly range-aligned and phase compensated will exhibit a spectrogram whose frequency response straddles zero-Doppler. The Doppler centroid method \[181\] computes the spectrogram of a prominent target scatterer after coarse range alignment, as follows:

\[
R_{\text{max}} = \arg \max_R \sum_{m=1}^{M} |\rho^m_a(R)|^2
\]  \hspace{1cm} (9.6)

\[
\zeta(f, \tau, a) = \mathcal{F} \left\{ \Pi \left( \frac{t - \tau}{a} \right) \rho^m_a(R_{\text{max}}) \right\}
\]

where \( \Pi(t) = \begin{cases} 
1 & \text{if } |t| \leq \frac{1}{2} \\
0 & \text{if } |t| > \frac{1}{2}
\end{cases} \)  \hspace{1cm} (9.7)

Where \( R_{\text{max}} \) is the mean target centroid of the \( m^{th} \) aligned range profile, \( \rho^m_a(R) \).

The Doppler spectrogram, \( \zeta \), is computed by first applying the sliding window \( \Pi \) across all pulses at range \( R_{\text{max}} \), followed by the Fourier transform, \( \mathcal{F} \). Note that the spectrogram is also controlled by changing the window’s scale factor, \( a \), controlling the tradeoff between time- and Doppler-sensitivity set by the Heisenberg uncertainty principle. The resulting Doppler trend is fit in the least squares sense, by using the same approach as the Range Centroid method. Compensation then is also in the same
Figure 9.6: Doppler centroid phase compensation. (a) Initial spectrogram (b) LS fit to Doppler centroid and phase response (c) Spectrogram after compensation (d) RD map after compensation

fashion, with both range and phase corrections applied to the data. The frequency response is fit using an $n^{th}$ order polynomial to Doppler centroid, for the case of $n = 3$, resulting in the following:

$$f_C(t) = \omega_0 + \omega_1 t + \frac{\omega_2 t^2}{2} + \frac{\omega_3 t^3}{3!}$$  \hspace{1cm} (9.8)

$$\hat{\phi}(t) = 2\pi \int f_C(t) \, dt$$

$$= 2\pi \left[ \omega_0 t + \frac{\omega_1 t^2}{2!} + \frac{\omega_2 t^3}{3!} + \frac{\omega_3 t^4}{4!} \right]$$  \hspace{1cm} (9.9)

The estimated Doppler centroid contour is then integrated to find the phase contour, shown in (9.9). Analysis of this equation demonstrates that an $n^{th}$ order poly-
nomial is able to correct for $n + 1$ different components of the range migration, or defocusing. Results of the Doppler centroid estimation and application on the simulated dataset are shown in Figure 9.6. Note that the aircraft shape is now visible, although the individual point scatterers illustrated in Figure 9.3 are not easily distinguishable.

9.3.5 TMC - Autofocus

In order to fully focus the image, an appropriate autofocusing algorithm must be used as a secondary stage of TMC. The well-known phase gradient algorithm (PGA) [6,184] was chosen as a benchmark due to its non-parametric approach, compensating for higher order or random motions than the previous centroid implementations. PGA consists of the following primary steps:

1. Beginning with the RD map, shift strongest scatter in each range bin to zero-Doppler

2. Apply Doppler window and IFFT

3. Estimate the phase gradient across the $N$ range bins: $\hat{\phi}_i = \angle \left[ \sum_{n=1}^{N} \rho_i \rho_{i-1}^* \right]$

4. Integrate the gradient to update the TMC phase estimate

5. Is RMS phase residual from step 4 below the threshold?
   
   *No*: Reduce window size and return to 1
   
   *Yes*: Stop

This procedure was implemented on the simulated data, with a starting window size of 50%, which shrinks by 20% for each iteration. The results of the PGA focusing are

\[ \text{214} \]
Figure 9.7: PGA iteration and results

illustrated in Figure 9.7, for a RMS phase stop criterion of $1^\circ$. Notice that the number of pulses is now decreased from previous results, due to significant downsampling along the slow-time axis of the RT map. Any linear components of the phase estimate are also discarded, which would not improve image focus but only shift the target’s cross range position.

Figure 9.8 compares the P-ISAR imagery computed with simple range-Doppler focusing before (using only the centroid methods) and after application of the PGA. Notice that the autofocus technique improves the clarity of the image significantly, but the outer edge scatterers are slightly blurred, due to Doppler walk of the target scatterers throughout the CPI.
For small angular rotations, the residual Doppler shifts after TMC are proportional to cross-range position, demonstrated in Appendix C.3. This allows ISAR imagery to be formed, in many cases, by applying a Fourier transformation in the slow-time axis of the RST map – in essence, a compensated RD map is analogous to an ISAR image.

This method has the benefit of producing imagery when the apparent target rotation is not known exactly, at the cost of the ability to scale the Doppler axis to a physical cross-range dimension [32]. The efficacy of the RD approach relies on two primary assumptions: target rotation is constant throughout the CPI, and the total rotation satisfies the focusing criterion. The total rotation, $\eta$, results from the integral of the target’s apparent rotation vector, $\Omega(t)$, as follows:

$$\eta = \int^T \Omega(t) \, dt$$ (9.10)

For the ideal case of uniform motion, the angular limits of RD focusing can be
found by setting the two-way phase deviation of the the closest target point from the linear Doppler response to $\pi/8$ radians [33]. For bistatic and passive imaging, this limit becomes

$$\eta_{\max}^{\pi/8} = \frac{1}{2} \sqrt{\frac{\lambda}{r \cos (\beta/2)}} = \int_{T_{\max}}^{\Omega(t)} \Omega(t) \, dt,$$

(9.11)

for a target of radius $r$. Note that the total angle spanned by the target sets the limit on the CPI of $T_{\max}$, with the integral of the apparent angular velocity seen by the radar $\Omega(t)$.

As Fig. 9.8 (b) demonstrates, even though the simulated results are well approximated as a linear rotation, the points near the front and rear of the plane are blurred due to deviation from the linear Doppler response. The simulated case experienced a rotation of $6.1^\circ$, slightly exceeding $\eta_{\max}$ of $6.0^\circ$. To correct for the smearing, alternative image formation procedures – which require RMC – must be implemented. The benefits of this are many: the cross-range scaling problem is eliminated, non-uniform target motions can be used for imaging, apparent rotation beyond $\eta_{\max}$ can be used without image quality degradation, and the radial dependence on focal quality can be bypassed.

One of the primary techniques for RMC is the Prominent Point Processing (PPP) algorithm [27]. This method isolates strong scatterers with minimal variance (i.e. lowest likelihood of multiple scatterers) to first perform TMC, correct to constant rotation, then compute the cross-range scale factor. The narrowband nature of most PR installations prohibits this approach, since points cannot be isolated sufficiently for reliable RMC. An alternative method for RMC is based upon target tracking using two netted monostatic radars [185,186]. Similar concepts were recently demonstrated
Figure 9.9: P-ISAR target tracking

for a multistatic set of high-resolution radars at X-band and close range for ground moving vehicles [141, 187].

Figure 9.9 illustrates the proposed tracking approach for P-ISAR, using some form of PR localization estimate [178, 179], often followed by smoothing via a Kalman filtering process [141]. The target position vector, \( \vec{x} \), represents the position of scatters in the target coordinate system, which tracks the translation and rotational motion of the target. The Tracker subsystem outputs a 3D Cartesian track as estimates of the target’s origin over time, represented as \( \vec{p}(t) \). This positional estimate enables calculation of both the bistatic angle, \( \beta \), as well as the bistatic bisector, \( \hat{u}_\beta \), at all points within the CPI. The effects of both of these bistatic geometrical parameters can be analyzed succinctly as follows:

\[
\vec{k}_\beta(t) = \frac{2\pi}{\lambda_c} \left( \hat{u}_{Rx}(t) + \hat{u}_{Tx}(t) \right) \tag{9.12}
\]

where \( \hat{u}_{Tx} \) and \( \hat{u}_{Rx} \) are unit vectors towards the transmitter and receiver, respectively. \( \vec{k}_\beta \) represents an equivalent monostatic wave-vector oriented along the bisector \( (\vec{k}_\beta \parallel \hat{u}_\beta) \) with a magnitude equivalent to the wavenumber scaled by the bistatic half-angle \( (\| \vec{k}_\beta \| = k \cos(\beta/2)) \). It is important to note that \( \vec{k}_\beta \) is defined relative

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to the target coordinate system.

The target velocity profile, \( \vec{v}(t) \) can either be estimated from the Kalman velocity track, or as a derivative of the global track over time, \( \vec{v}(t) = \frac{d\vec{p}(t)}{dt} \). The path and velocity vector are used to estimate the rotational motion of the target, similar to the ground moving target imaging (GMTI) approach by Brisken [141]. This approach can also be used to estimate the 3D orientation of an aircraft, provided it is not maneuvering aggressively. For a cruising aircraft both yaw and pitch should be fairly well estimated from the path history, but roll motion in gentle banked curves may prove more difficult to predict.

Once the position and orientation of the target’s coordinate system have been estimated, the bistatic wave-vector’s profile can be reconstructed. The transmitter’s bandwidth and angular span determine the down-range and cross-range resolutions, scaled by the bistatic half-angle shown in (9.13). The derivation of this can be found in Appendix C.

\[
\begin{align*}
\delta_{dr} &= \frac{c}{2B \cos(\beta/2)} \\
\delta_{bi} &= \frac{\lambda_c}{4\eta \cos(\beta/2)}
\end{align*}
\] (9.13)

The inherent bistatic geometries of P-ISAR operation cause not only the cross-range resolution to be spatially variant but also the down-range resolution. This dependence requires some form of localization to estimate even the most basic radial target estimate, in contrast to P-ISAR. The target’s monostatic range axis can be estimated from the bistatic delay and bistatic angle, as \( R_{dr}^{mono} = \frac{R_{bi}^{bi}}{2 \cos(\beta/2)} \). Coarse tracking accuracy can suffice to generate a reasonably good estimate of the range scale at small to moderate bistatic angles, due to the \( \cos(\cdot) \) behavior in this range. It is clear that the accuracy of the tracking data and assumptions made in estimating the ap-
parent target rotation are more sensitive to track quality, but an extensive evaluation of this is outside the scope of this work. For a target with a flat and level nominal flight path, constant track errors (such as a height or range offset) will be manifest as very slight estimation error of the apparent rotation, whereas more intricate target motions are much more difficult to represent as with any ISAR system.

9.3.7 Image Formation

As mentioned previously, the most common method of P-ISAR image formation is with the simple method of RD processing. However, as mentioned in Section 9.2, this approach suffers from a host of limitations. If rotational information is available, the Doppler axis can be scaled to cross-range, but this comes with added complications when compared with the monostatic case. Alternative methods of image formation include polar reformatting and backprojection [27]. The methods of RD imaging and backprojection are investigated as the primary methods of image formation due to their computational efficiency and broad applicability, respectively.

9.3.7.1 RD Image Formation Limitations

The RD method of image formation is the staple of the ISAR community, with many autofocus algorithms (such as PGA) relying on the assumptions inherent in its application. However, the low frequencies common to PR illuminators inherently limit the number of cross-range bins for a focused image beyond that of a higher frequency system at $\eta_{max}$. By equating $\eta$ of (9.13) with (9.11), the maximum cross range resolution of the RD algorithm can be evaluated as a function of the target
Figure 9.10: RD resolution limitations for 20 m target radius. (a) Best case cross-range resolution for RD image formation (b) Number of cross-range bins

radius, illuminating frequency and geometry; this also allows computation of the number of cross range bins.

\[
\delta_{\text{cr, Max}}^{\text{RD}} = \sqrt{\frac{rc}{\lambda c \cos (\beta/2)}} \quad (9.14)
\]

\[
N_{\text{dr, Max}}^{\text{RD}} \approx \frac{2r}{\delta_{\text{cr}}} = \sqrt{\frac{f_r c \cos (\beta/2)}{\lambda}} \quad (9.15)
\]

Note that the cross-range resolution degrades with decreasing frequency and increasing bistatic angle, thus decreasing the number of cross-range bins available for focused imagery with the RD algorithm. A graph of this, for a 40 m radius target is shown in Figure 9.10, for a set of bistatic angles and geometries. Notice that the resolution is coarser at low frequencies and wider bistatic angles, thus restricting the total number of cross-range bins on a fully focused target.

The slight changes in the bistatic angle over an imaging interval cause a skew in the sampled tomographic annulus of the target’s k-space representation, which causes distortions in the RD map even if the range and cross-range axes are scaled properly.
Figure 9.11: K-space skew due to bistatic geometrical distortion

This phenomenon is similar to the skew present in squint-mode SAR [188]. This image skew can be partially corrected, to a first order, with a cross-range shift of the RD image as a function of range position. To compute the magnitude of this shift, the skew factor must be estimated via analysis of the k-space sampling pattern, as the ratio of the radial change relative to the angular width of the sampled tomographic annulus, shown in (9.16). This skew is also illustrated in the Figure 9.11, for the previous simulated P-ISAR case, for which a skew factor of approximately -0.27 was calculated.

\[
S_f = \frac{d \| \mathbf{k} \|}{d \Omega} \approx \frac{\| \mathbf{k}(0) \| - \| \mathbf{k}(T) \|}{E[\| \mathbf{k}(t) \|] \eta}
\]  

(9.16)

The data is then skewed via a cross-range shift by introducing a shift in the cross range direction whose magnitude depends on the down-range positions in the image, as follows:
\[
\Psi_{\text{RD}}'(R_{\text{dr}}, R_{\text{cr}}) = \Psi_{\text{RD}}(R_{\text{dr}}, R_{\text{cr}} + S_f R_{\text{dr}})
\]  
(9.17)

where the initial scaled RD map is \( \Psi_{\text{RD}} \), and the skew-corrected RD map is \( \Psi_{\text{RD}}' \).

### 9.3.7.2 Backprojection and IFP Comparison

Backprojection is essentially a spatial matched filter of the target, which requires the previously mentioned TMC and RMC stages to be implemented. Although not required, results shown here invoke the far-field plane wave assumption, a good approximation when the transmit and receive ranges fulfill the Fraunhofer condition of \( R_T, R_R \ll 2D^2/\lambda \) (typically satisfied in most ISAR imaging scenarios). Note that this plane wave approximation can still result in defocusing away from the target center, but is much less restrictive than the constant Doppler approximation of the RD imaging algorithm. The data is weighted relative to the density of samples in the \( k_X - k_Y \) Fourier data representation for accurate reconstruction. An additional challenge with the backprojection operation is assignment of the zero-range position corresponding to the TMC corrections. In theory, correctly applied TMC computes the phase to a particular scatterer or point within the target’s reference frame. However, the precise location of this point is not generally output by the autofocus or TMC procedure and therefore must be estimated prior to backprojection. Our approach, which agreed well with simulated results, is to sum the intensities of each pulse prior to TMC to compute the global range profile, followed by calculation of the centroid position. This global range centroid then becomes the zero position for subsequent backprojection applications.
### Table 9.1: Estimated target dimensions

<table>
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<tr>
<th>IFP</th>
<th>Estimated Width [m]</th>
<th>Estimated Length [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scaled RD</td>
<td>35</td>
<td>45</td>
</tr>
<tr>
<td>RD with Skew Correction</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Backprojection</td>
<td>40</td>
<td>40</td>
</tr>
</tbody>
</table>

The results of three IFPs are shown in Figure 9.12 for (a) scaled RD map image formation, (b) skew-corrected RD processing, and (c) backprojection. The target scatterers are shown with equal scale for direct comparison to the resulting imagery, with the bistatic bisector shown relative to the target origin. The 40x40m target’s dimensions were manually estimated from the images by drawing a line from the peak of the dominant wing and nose/tail scatterers, and tabulated in Table 9.1. Notice that without the skew factor, but with down-range and cross-range scaling of the range axes, the target dimensions are distorted. In both (a) and (b) (RD image formation) the targets near the nose and tail of the plane are slightly distorted, and the target’s orientation is different from that of (d) due to the alignment of the bisector with the down-range axis. The backprojected image exhibits none of the distortions present in the RD images, and the scatterers appear at the proper locations with the correct orientation.

#### 9.3.8 Framework Contributions

An overview of the necessary theory and algorithms for P-ISAR imaging have been developed. The principle components of *Core PR Processing* and *tracking* were identified as critical operations enabling useful P-ISAR image formation under many scenarios. The importance of TMC was highlighted as a crucial step for realizing focused imagery, while track-based-RMC was introduced as a means for proper scaling
Figure 9.12: Comparison of P-ISAR IFPs. (a) Scaled RD image (b) De-skewed RD image (c) Backprojected image (d) Target scatterers
of both the down- and cross-range axes of ISAR imagery. A de-skew method was
developed for RD imaging under the general case of bistatic angle variation within
the CPI. Improved image formation procedures over standard range-Doppler imaging,
primarily backprojection, have demonstrated that target dimensions can be reliably
estimated from P-ISAR processing, under the ideal case of a wideband contiguous
bandwidth. The following section will investigate the effects of lower bandwidth on
image formation.

9.4 Narrowband P-ISAR

This section investigates the implications of a narrow bandwidth – typically found
among PR illuminators – on P-ISAR imaging algorithms and the image formation
procedure. What constitutes a narrow bandwidth system is not well defined, but
clearly is application specific. For example, the 6 MHz bandwidth of ATSC transmis-
sions is comparatively wideband versus a typical air surveillance radar’s 1 \( \mu \)s pulse
(1 MHz equivalent bandwidth). ISAR for man-made target imaging is typically of
much higher bandwidth, such that the range resolution is significantly smaller than
the target length, \( \delta_{dr} < D \), which offers many independent range samples of the
target’s response. For this section, we assume that only one or two range samples of
the target are available with a single illuminator of opportunity (IoO).

At this point, it is important to revisit the fact that a large majority of the liter-
ature regarding P-ISAR assumes the ability to use a large number of transmitters to
form fully coherent imagery [124–129,139]. There have been some methods which at-
tempt to fuse multi-band transmissions from co-located IoO’s [58,59], but the concept
is only focused on a single target point and thus not suitable for ISAR imagery. There
are no existing techniques in the open literature to generate simultaneous imagery from collections of multistatic, frequency-diverse ISAR transmissions. In practice, to implement multistatic P-ISAR, each transmitter must be used individually to generate a set of bistatic images on a common plane, to then be co-registered and combined – using the framework of Section 9.2 and Section 9.3. This section investigates the first stage of such processing, using narrowband PR illuminators treated as separate bistatic pairs.

Previous simulated results were confined to the 2D x-y plane for simplicity in explanation, but almost all experimental systems must consider the true 3D imaging geometry. Within the confines of the P-ISAR framework, the steps of TMC and PR processing are independent of this geometry, but target tracking and image formation operations must consider it. To analyze the predicted resolutions as well as rotation-informed image formation procedure, k-space samples as demonstrated in (9.12), are projected onto the x-y plane.

\[
\mathbf{K}_{\beta}^{XY}(t) = \left[\mathbf{K}_{\beta}(t) \cdot \hat{x}\right] \hat{x} + \left[\mathbf{K}_{\beta}(t) \cdot \hat{y}\right] \hat{y} \tag{9.18}
\]

The expected resolutions for the x-y projection can be predicted by inverting the span of the k-space sampling patterns [76], or directly computing the effective parameters from the projection and then applying the formulas of (9.13). Note that unless the resulting image is co-planar with the natural imaging plane, a loss in resolution results as a side-effect of this projection. For the IFP, each pulse must be dilated relative to the depression angle prior to backprojection to avoid unwanted image plane distortions. If imaging is to be extended to multiple transmitters as a means
of increasing resolution, these stages are vital operations to perform prior to image fusion.

9.4.1 Simulation Analysis

The narrowband implementation example, shown in Section 9.3, was developed using the parameters of Table 9.2. Notice that the bandwidth has now been reduced to 6 MHz, consistent with a single ATSC channel, most typically found around the North American continent. The simulation parameters result in effective down-range and cross-range resolutions of 26.8 m and 2.2 m, accounting for all geometrical effects using (9.13). In theory, P-ISAR imaging over moderate angles with ATSC is potentially a viable technique for generating cross-range profiles, assuming proper focusing using TMC and RMC.

<table>
<thead>
<tr>
<th>Radar Parameters</th>
<th>Geometrical Parameters</th>
<th>Target Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_c$</td>
<td>665 MHz</td>
<td>TX Position</td>
</tr>
<tr>
<td>$B$</td>
<td>6 MHz</td>
<td>RX Position</td>
</tr>
<tr>
<td>CPI</td>
<td>1.4 s</td>
<td>Target Position</td>
</tr>
<tr>
<td>$\delta_{DR}$</td>
<td>26.8 m</td>
<td>$\beta$</td>
</tr>
<tr>
<td>$\delta_{CR}$</td>
<td>2.2 m</td>
<td>$\eta_XY$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Length</td>
</tr>
<tr>
<td></td>
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<td>Velocity</td>
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<tr>
<td></td>
<td></td>
<td>Motion</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Linear</td>
</tr>
</tbody>
</table>

Table 9.2: Narrowband P-ISAR Simulated Setup

The target is shown in Figure 9.13 (a), with the right half of the aircraft outlined with point scatterers representing the side of direct illumination – thus more pronounced point-like scattering. The back-wing reflector, indicated in red, is set 6 dB stronger than the other points to resemble stronger corner-like backscatter. Notice that the 17 m range extent of the half-target is less than a single range resolution cell, so a sole independent sample of the target’s response is obtained at any instant.

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The 40 m length of the target matches that of a Boeing 737-800. The target is now offset in altitude by 300 m, and therefore the estimated rotation is measured from the $k_x - k_y$ plane.

![Diagram](image-url)

Figure 9.13: Narrowband simulation target geometry. (a) Target scatterers (b) Flight path (c) K-Space sampling

The k-space sampling pattern and initial windowed portion of the RD map, after application of the *Core PR Processing* algorithms, is shown in Figure 9.13 (c). Notice that the narrow radial extent of the k-space pattern relative to the angular extent is indicative of the finer cross-range resolution than down-range. Proper estimation of the target’s rotational and translational motion is necessary to accurately reconstruct this sampling pattern; for this simulated case, target motion is purely linear, with the nose of the aircraft aligned with the velocity vector. The RD and RST maps of Figure 9.14 show the target response in both domains, while it is evident there is significant glint seen to the modulating response of the multiple point scatterers at the same range bin. This behavior should also be expected anytime the motion is suitable for P-ISAR imaging and the target consists of multiple point-like responses, and the mo-comp procedure must be effective across this behavior to form an image.
of sufficient cross range resolution. The simulation has also implemented with a post-processed image 30 dB SNR (prior to this, such as the RD map, will exhibit a lower SNR) to provide a more realistic imaging environment, evident in the images.

![Figure 9.14: (a) Range-Slow Time map (b) Range-Doppler map](image)

### 9.4.1.1 Narrowband TMC Analysis

The P-ISAR framework, described in Section 9.3, was applied using a narrowband configuration using three variants of mo-comp:

1. Clairvoyant image formation – target motion (TMC and RMC) is known exactly.

2. TMC: range and Doppler centroid only (no PGA). RMC: estimated via 3D target tracking as in Section 9.3.6.

3. TMC: centroid methods and PGA. RMC: estimated via 3D target tracking as in Section 9.3.6.

In all cases, the target track is known exactly which only influences the step of RMC. All TMC methods, with the exception of the clairvoyant case, were performed
solely on the basis of the radar returns. Standard backprojection using a plane-wave assumption is used as the IFP to the X-Y plane, generally well aligned with the primary aircraft dimensions. In cases of aggressive maneuvering, tracking data in conjunction with target kinematic modeling can provide a better informed image projection plane from which to extract useful target features.

The resulting images for each variant of TMC are shown in Figure 9.15, for both rectangular and Kaiser windows (with shape parameter $\beta = 4$) in Doppler prior to backprojection. It was found that aggressive windowing, e.g. Hamming, increased the Doppler resolution such that the target features were unrecognizable. To aid in the quantitative analysis of these results, the commonly accepted entropy [189] and contrast [190] image focal quality metrics are shown above each subplot in Figure 9.15; these metrics are defined as follows:

\[
\text{Entropy} \equiv \frac{E\{ -I^2(x,y) \ln[I^2(x,y)] \}}{E\{I^2(x,y)\}} \tag{9.19}
\]

\[
\text{Contrast} \equiv \sqrt{\frac{E\{[I^2(x,y) - E\{I^2(x,y)\}]^2\}}{E\{I^2(x,y)\}} \tag{9.20}
\]

where $I(x,y)$ is the amplitude (absolute value) of the complex P-ISAR 2D image after backprojection. The entropy metric measures the degree of “randomness” in the image; as such, a well-focused image typically possesses a minimum image entropy. Many TMC autofocus techniques, both parametric and non-parametric [187, 189, 191–193], are based upon this approach. The contrast method computes the RMS standard deviation of the image; as such, a well focused image image should have maximum image contrast. This has also been widely accepted as a standard method
of TMC for ISAR processing [190,193–197].

Figure 9.15: Narrowband P-ISAR simulation with 6 MHz bandwidth
The clairvoyant method of TMC, shown in Figure 9.15 (a), demonstrates the best-case image using conventional IFP techniques, representing an upper bound in P-ISAR image formation for the simulation’s particular parameterization. Notice that the radial direction is relatively coarse, with a range direction oriented along the bistatic bisector direction of approximately 27 m, as predicted by the formula. The width of the dominant scatterers correspond to the true 40 m target length, as represented with the target overlay. The various target scatterers along the length of the plane (the cross-range direction) are visible, with the center wing scatterer and tail response slightly stronger in magnitude, consistent with the +6 dB scatterer.

Application of the Kaiser window, shown in Figure 9.15 (b), enables discrimination of the target shape with lowered sidelobes at a slight cost in resolution.

The images of the centroid (range and Doppler) techniques for TMC of Figure 9.15 (c)-(d) demonstrate results which are very similar in shape to the Clairvoyant simulation, with slightly different sidelobe patterns due to minor imperfections of the focusing. Algorithmic parameters of the motion compensation were unchanged from the developments of Section 9.3. The entropy and contrast metrics of the images match closely to the clairvoyant case, with a slight decrease in focus for both parameters.

Conventional wisdom for ISAR would imply that application of an autofocus algorithm would improve the image quality. A simple visual comparison of Figure 9.15 (e)-(f) with (c)-(d) shows that this is not the case. One of the most important findings of this chapter is the following: application of conventional autofocus algorithms for narrowband ISAR result in degraded image quality – likely for both the PGA and metrics-based (entropy and contrast) autofocus techniques. Both entropy and contrast
metrics wrongly indicate that the image using PGA (Figure 9.15 (e)-(f)) is “better” focused when compared to the clairvoyant case (Figure 9.15 (a)-(b)), even though the image quality is clearly degraded after application of the PGA. Therefore, the clairvoyant case is not a global minimum – thus, the cost function is non-convex and convergence to the properly focused condition cannot be guaranteed.

This behavior can be explained with a simple examination of the PGA algorithm: Prominent scatterers are first isolated and used to compute a phase response, which is used to estimate the translational motion across the aperture (and removed with multiplying by the conjugate response of the observed phase pattern). For the initial iterations of the PGA, a wide window captures a set of the scatterers, if not all. At this point in the algorithm, all phase change due to target rotation and scatterer interaction is removed, such that the resulting image consists predominantly of the amplitude fluctuations evident in the data. This behavior results in the single dominant scatter with arbitrary sidelobe patterns, as evidenced in the results of Figure 9.15 (e)-(f). Without multiple range bins to provide independent estimates of the target’s translational motion (due to the narrow bandwidth), effects of target glint are indistinguishable from translational motion. Thus, a parametric model for the target must be developed without explicitly relying on the contrast and entropy techniques.

9.4.1.2 Effects of Track Error

To investigate the claims of robust RMC and thus image formation in the presence of track errors, as explained in Section 9.3.6, a DC path error of (50, −25, 100) is introduced in the simulation. This error is approximately twice the range resolution,
combined with the relatively short standoff range of approximately 1 km, will result in fairly significant rotational distortion when compared to expected errors in a realistic system [160]. This path and the RD map with an overlaid track are demonstrated in Figure 9.16 (a). Figure 9.16 (b) and (c) compare the image formed with centroid TMC and a Kaiser window with and without track error, respectively. Although there are some slight dissimilarities between these results, the general shape and dimensions of the target are unaffected. Most of the significant differences result from cross-range bins containing multiple scatterers.

(a) Track offset and RD map illustration

(b) Centroid, Kaiser window, no path error  (c) Centroid, Kaiser window, with path error

Figure 9.16: Simulated P-ISAR imagery with track error
### Table 9.3: Experiment #1 parameters

<table>
<thead>
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<th>Geometrical Parameters</th>
<th>Target Parameters</th>
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</tr>
<tr>
<td>$B$</td>
<td>6 MHz</td>
<td>RX Pos. (2, -7, 0.3) km</td>
</tr>
<tr>
<td>CPI</td>
<td>6 s</td>
<td>Target Pos. (0.7, 1.3, 0.6) km</td>
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<td>$\delta_{DR}$</td>
<td>25.7 m</td>
<td>$\beta$ 25.7°</td>
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<tr>
<td>$\delta_{CR}$</td>
<td>1.1 m</td>
<td>$\eta$ 12.0°</td>
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</table>

#### 9.4.2 Experimental Trials

The P-ISAR framework was tested against experimental data gathered by the MuTeRa system, described in Chapter 6. Although most of the operations do not change from the previous simulated examples, the tracking subsystem is provided by an ADS-B receiver. There is no conclusive source to the accuracy of such devices, where the position measurements heavily depend on the accuracy of the GPS receiver, current satellites constellation, and the resulting dilution of precision (DOP). In addition, conversion of the ADS-B data-stream to a radar track suffers from timestamping inaccuracies and the assumptions inherent in conversion of barometric pressure altitude (see Appendix B) to true target height. Bistatic range and velocities from such systems are comparable to that which might be experienced by a multistatic tracking system using PR illuminators.

#### 9.4.2.1 Experiment #1

Parameters of the first P-ISAR experiment are shown in Table 9.3. The ERJ-175LR aircraft was observed using a 665 MHz ATSC transmitter located approximately 46 km to the south of the ElectroScience laboratory. The approach path to the John Glenn International airport is shown in Figure 9.17 (a), from the ADS-B readout.
The positions of blue circles indicate packets transmitted from the extended squitter system, while the red line indicates the portion of the flight path used for subsequent P-ISAR imaging. The k-space sampling pattern, projected on the $k_x - k_y$ plane is shown in Figure 9.17 (b), for the 6 MHz bandwidth and 12 degree target rotation. Notice that the angular length of the pattern is significantly larger than its radial dimension, such that a high level of cross-range resolution is expected compared to the down-range resolution, in agreement with the previous narrowband simulations.

Figure 9.17: Experiment #1 geometry analysis

The RD and RST maps before and after windowing are shown in Figure 9.18.
Figure 9.18: Experiment #1 Radar Data. (a) RST prior to windowing (b) RST after windowing (c) RD prior to windowing (d) RD after windowing
(a)(c) and (b)(d), respectively. The high noise floor and clutter from Fig. 9.18(a) is significantly suppressed when compared to Fig. 9.18(b). The ADS-B target track, interpolated to the midpoint of each range profile of the RST data, is overlaid with the dashed red line; this curve demonstrates slight mismatch between the air-truth and the target response. The discrepancy appears to be primarily in Doppler, most likely due to errors in the reported velocity and bearing information. Notice that target scintillation along the peak of the target response in the RT map of Figure 9.18 (b) is similar to the previous narrowband simulated results of Figure 9.14 (a), using a point target model.

Figure 9.19 (a) and (b), for rectangular and Kaiser windowing ($\beta = 4$) in Doppler, shows the experimental P-ISAR images after applying the narrowband approach. The range and Doppler centroid methods were used for TMC, with target tracking via ADS-B and image formation with backprojection onto the x-y plane. To develop a sense of what might be expected based upon this simulated geometry, a clairvoyant simulation was implemented using the experimental target track and the previous target, shown in Figure 9.19 (c) and (d). Note that at low frequencies and wide angles, it is not clear how point-like the phenomenology of the target scattering may be, or the precise locations of the scattering centers, so these images are primarily shown to illustrate the target orientation and point-spread-function of an ideal point scatterer. The target appears to have a -10 dB contour extending to $\pm 15$ m, agreeing well with the true target length of 30 m. Figure 9.19 (a) and (b) represent the first single channel P-ISAR image of an air target which has been scaled in cross-range – in essence, the first potential candidate which can be used to test methods of multistatic image fusion, to be considered in future work.
Figure 9.19: Test #1 - Embraer ERJ-175LR
Radar Parameters | Geometrical Parameters | Target Parameters
---|---|---
$f_c$ | 665 MHz | TX Pos. | (−6, −46, 0.3) km | Manuf. | Embraer
$B$ | 6 MHz | RX Pos. | (0, 0, 0) km | Model | Phenom 300
CPI | 6 s | Target Pos. | (−0.1, 1.7, 0.5) km | Length | 16 m
$\delta_{DR}^{XY}$ | 25.4 m | $\beta$ | 20.7° | Width | 16 m
$\delta_{CR}^{XY}$ | 1.5 m | $\eta^{XY}$ | 9.0° | Speed | 96 m/s

Table 9.4: Experiment #2 parameters

### 9.4.2.2 Experiment #2

The second experiment’s parameters are tabulated in Table 9.4. The smaller Phenom 300 aircraft has both a length and wingspan of 16 m, observed with the same transmitter used in the previous test. The flight path is shown in Figure 9.20 (a), still to the north of the receiver but with a much straighter path than the previous test. The k-space sampling pattern of Figure 9.17 (b) is also similar to the previous case, with a slightly smaller total rotation of 9°.

The RD and RST maps for experiment 2 are shown in Figure 9.21. It is clear that in this case the ADS-B target track is a much closer match to the received radar data.

Figure 9.22 (a) and (b) demonstrate the experimental ISAR images, with (c) and (d) provided for simulated comparison. Notice that the area of dominant scatter corresponds roughly to the target length of 16 m, and is significantly smaller than the results of Figure 9.19. These results demonstrate the ability of P-ISAR to form estimates of a target’s width based upon high synthetic cross-range resolution. To fully evaluate the quality of these images, electromagnetic modeling is necessary to determine the true scattering centers of the observed targets versus frequency and bistatic transmit/receive angle positioning throughout the processing interval. In
Figure 9.20: Experiment #2 geometry analysis

reality, these effects are difficult to predict, but the results of this chapter represent a significant advancement towards enabling target classification with passive radar systems.

9.5 Conclusions

A comprehensive framework for P-ISAR imaging has been developed as a means of estimating the cross-range span of targets of interest, in a context which enables
Figure 9.21: Experiment #2 Radar Data. (a) RST prior to windowing (b) RST after windowing (c) RD prior to windowing (d) RD after windowing
Figure 9.22: Test #2 - Embraer ERJ-175LR
multistatic image fusion for future work. The importance of TMC was shown to have a drastic impact on image quality, and the conventional methods of autofocus- ing do not converge to a well-focused image, but instead minimize metrics which do not apply in the narrowband imaging case. Utilizing available tracking data enables cross-range scaling and backprojection operations to circumvent the many limitations of the range-Doppler imaging approach. The requirement of a parametric model for target focusing may limit the approach to targets at fairly close range, such that a large enough angle is spanned during the imaging interval within the limits of the Doppler centroid approach. This investigations of this Chapter have the potential for significant image fidelity improvement in future work: extension to multiple transmitters has the potential to increase both cross- and down- range resolution for certain transmitter geometries, but this requires additional operations of co-registration and appropriate fusion operations. Future work should also investigate improved narrow-band TMC algorithms, which could leverage multiple illuminators (intrinsically available with systems utilizing trilateration to develop a target track) for more robust and accurate motion corrections. These processes should account for the electromagnetic target phenomenology to ensure that the imaging operations appropriately model to the complex scattering phenomenology at low frequencies which span wide viewing angles.
Chapter 10: Summary and Conclusions

A suite of various imaging techniques, particularly tailored to passive radar systems, has been developed in this thesis with an emphasis on experimental validation. Doppler imaging approaches which apply matched filters to the range-dependent phase response were developed in Chapters 4 and 5, when the imaging geometry is known precisely as is generally the case with SAR. The Doppler imaging technique was experimentally evaluated with a simple stripmap data collection geometry for application on a wide variety of platforms, followed by a simulated investigation using passive radar parameters in Chapter 5. It was shown that the Doppler imaging approach is a feasible method of generating fine range resolution in both the cross-range and down-range dimensions. Future work should investigate strategies for mitigating some of the challenges inherent in this approach: higher sidelobe levels, different resolutions in both cross- and down-range, and methods for compensating for target persistence under wide angle imaging scenarios.

Chapter 6 documents the design and development of a next-generation passive radar testbed able to gather an unprecedented suite of radar data simultaneously from a number of different illuminators of opportunity. The core passive radar processing algorithms necessary range-Doppler surfaces were developed particular to ATSC transmissions and their mismatch filtering. A fast implementation of the
cross-correlation approximation for range-Doppler map generation was developed, along with bounds on the SNR degradation. A set of experimental results highlights the phenomenology of multiple-bounce scattering and its prevalence in passive radar, manifest differently with passive radar than with conventional radar systems. In addition, a number of different target responses were shown to exhibit micro-Doppler phenomena as a means of target classification. Chapter 7 then evaluates a number of adaptive filtering algorithms for mitigating the direct signal interference present in most passive radar systems. It was shown that the fast block LMS algorithms and least-squares represent good candidates both in terms of their excellent suppression performance and low computational cost.

A framework for passive ISAR imaging of air targets with a passive radar network was then developed in Chapter 9. All challenges of ISAR image formation are addressed: data pre-processing, translational motion compensation, target tracking for rotational motion compensation, and backprojection operations to a common image plane. It was shown that conventional methods of autofocusing have an adverse effect on image quality when applied to narrowband data, and a parameterized form of Doppler centroid phase compensation for translational motion compensation was proposed. Experimental validation of the technique demonstrated the ability to estimate the cross-range span of a target, within the limits of its scattering phenomenology. Future work will extend these single-transmitter approaches to a multistatic passive radar network to form multiple simultaneous images, and investigate methods of fusion for improved target estimation extending to simultaneous high-resolution 2D imaging for improved classification performance.
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Appendix A: Loss due to Cross-Correlation Approximation

Here we present the derivation for the loss in correlation gain for the batch correlation methods by evaluating the expression in the continuous time domain, followed by the equivalent discrete time formulations. Consistent with the notation of Section 6.3.5 the approximation to the exact cross ambiguity function can be written as

\[
\Psi'[l,m] = \sum_{q=0}^{Q-1} e^{-j2\pi mq/Q} \sum_{n=0}^{L-1} s_s[n + qL] s^*_r[n + qL - l]
\]  

(A.1)

The loss in signal processing gain arises due to the fact that the rightmost summation involves computation of the correlation between the reference and surveillance waveform, which will not be matched for targets with a nonzero Doppler shift. The reference signal will be assumed to have a constant envelope of unity, such that \(|s_r(t)| = 1\). The surveillance signal consists of a Doppler-shifted version of the reference signal, \(s_s(t) = s_r(t)e^{j2\pi ft}\) The correlation output for a single short-time correlation batch, assuming the Doppler shift is accounted for, can be written as a function of the Doppler frequency of the matched filter, and the correlation batch length \(T_b\) as follows

\[
\zeta (f, T_b) = \int_0^{T_b} s_s(t) s^*_r(t) e^{-j2\pi ft} dt.
\]
Plugging in the expression for the surveillance waveform written as a Doppler-shifted copy of the reference, this expression can be easily simplified and reduces to $T_b$

$$\zeta (T_b) = \int_0^{T_b} s_\tau (t) e^{j2\pi ft} s^*_\tau(t) e^{-j2\pi ft} dt = \int_0^{T_b} |s_\tau(t)|^2 dt = T_b$$

The cross-correlation approximation method does not account for the phase change within a single batch due to the Doppler shift, such that the correlation output becomes

$$\zeta' (f, T_b) = \int_0^{T_b} s_s (t) s_\tau (t) dt = \int_0^{T_b} s_\tau (t) e^{j2\pi ft} s^*_\tau (t) dt = \int_0^{T_b} e^{j2\pi ft} dt$$

$$= \frac{e^{j2\pi ft}}{j 2\pi f} \bigg|_{0}^{T_b} = -\frac{j}{2\pi f} [e^{j2\pi fT_b} - 1]$$

The loss in correlation gain can be calculated in decibels through the following expression

$$L (f, T_b) = 20 \log_{10} \left( \frac{|\zeta (T_b)|}{|\zeta' (f, T_b)|} \right)$$

A more insightful expression for the loss function can be derived by applying trigonometric identities to $|\zeta'|$ as follows.

$$|\zeta' (f, T_b)| = \left| \frac{-j}{2\pi f} [e^{j2\pi fT_b} - 1] \right| = \frac{1}{2\pi f} |\cos (2\pi fT_b) + j\sin (2\pi fT_B) - 1|$$

$$= \frac{1}{2\pi f} \sqrt{(\cos (2\pi fT_b) - 1)^2 + \sin^2 (2\pi fT_B)} = \frac{1}{2\pi f} \sqrt{2 - 2\cos (2\pi fT_b)}$$

$$= \frac{1}{\sqrt{2\pi f}} \sqrt{1 - \cos (2\pi fT_B)} = \frac{1}{\pi f} \sin (\pi fT_b)$$
The correlation loss function can be computed directly from

\[ L'(f, F_b) = L\left( f, \frac{1}{F_b} \right) = 20\log_{10} \left( \frac{\pi f}{F_b} \csc \left( \frac{\pi f}{F_b} \right) \right) \]

The greatest correlation loss due to this approximation occurs when computing the response from a target at the maximum unambiguous Doppler, \( f = F_b / 2 \), resulting in a 3.92 dB loss. In order to reduce the correlation loss to less than 1 dB, the batch repetition rate should be \( F_b \geq 4 \times f_{\text{max}} \). The correlation loss as a function of normalized target frequency relative to \( F_b \) is shown in Figure A.1.

The loss function can also be represented in discrete time by substituting \( F_b = 1/T_b = Q/NT_s \) and \( f = m/NT_s \) such that the expression can be represented as:

\[ L'[m, Q] = 20\log_{10} \left( \frac{\pi m}{Q} \csc \left( \frac{\pi m}{Q} \right) \right) \]

Where \( Q \) is the total number of Doppler bins resulting from the slow time DFT and \( m \in \left\{ -\frac{Q}{2}, -\frac{Q}{2} + 1, \ldots, \frac{Q}{2} - 2, \frac{Q}{2} - 1 \right\} \).
Figure A.1: Correlation Loss vs. Doppler Frequency
Appendix B: ADS-B Data Transformations

Mode-S extended squitters send out periodic location data of an aircraft at a rate of approximately 2 Hz, in packets containing the GPS referenced latitude, longitude, and barometric pressure altitude. One may be tempted to simply implement the spherical earth approximation to convert these measurements into a Cartesian space, but over the significant transmitter and target distances these approximations result in spatially varying bistatic range errors in excess of 100 m. To provide precise truth information for DTV radar data, range accuracy finer than 50 m (a bistatic resolution cell at 6 MHz bandwidth) is desired. Therefore, utmost care should be taken to preserve the fidelity of the truth data to avoid introducing additional mismatch in the results of the radar data and the prediction supplied by the ADS-B recordings.

To understand how the data can be transformed into a local Cartesian representation, the fundamentals of various earth height models must be presented. Although the spherical earth model is convenient and can be easily calculated and transformed, the shape of the earth’s surface is much better approximated as an ellipsoid which is slightly fatter around the equator. The World Geodetic System (WGS) is a standard which establishes Earth’s coordinate system with this ellipsoidal reference, with less deviation from Earth’s surface at the expense of additional computations. The WGS-84 system is also used as the altitude reference point for the GPS coordinate system,
while some sophisticated receivers automatically compute the height above mean sea level. A number of conversion utilities exist to translate WGS-84 coordinates into a Cartesian coordinate system centered on Earth’s origin. Unfortunately, other altitude standards and map data are required to transform the aircraft and the transmitter altitudes into a common reference frame. These various models, each with common latitude/longitude positions but different heights, are illustrated in Figure B.1.

In addition to the WGS-84 ellipsoid mentioned previously, there is an earth Geoid surface (EGM96) of equal gravitational pull which represents an approximate mean sea level. The EGM96 model can significantly deviate from the WGS-84 ellipsoid as a result of large landmasses up to approximately 100 m. Note that the WGS-84 altitude reference represents the best-fit ellipsoid for the EGM96 geoid model. The advanced GPS receivers which report height above mean sea level do so relative to this EGM96 reference frame.

The information about the true earth’s surface or terrain, also known as Topographic data, is measured relative to the height above mean sea level, or the EGM96 Geoid. FCC database generally provide data on the transmitter towers as height above mean sea level (AMSL), or height above average terrain (HAAT) depending on
Table B.1: Examination of various altitude measurements

<table>
<thead>
<tr>
<th>Param.</th>
<th>Description</th>
<th>Data Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>Height above ground/terrain level</td>
<td>Physical site measurements</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FCC antenna height above ground</td>
</tr>
<tr>
<td>$h$</td>
<td>Height above WGS-84 ellipsoid (Geodetic height)</td>
<td>Basic GPS systems</td>
</tr>
<tr>
<td>$H$</td>
<td>Height above EGM-96 geoid</td>
<td>Topographic data</td>
</tr>
<tr>
<td></td>
<td>Height above mean sea level (orthometric height)</td>
<td>FCC transmitter HAMSL data</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GPS equipped with EGM-96 data</td>
</tr>
<tr>
<td>$N$</td>
<td>EGM-96 geoid undulation from WGS-84 ellipsoid</td>
<td>Table lookup for EGM-96 data</td>
</tr>
</tbody>
</table>

the record. These various height measurements and their data sources are summarized in Table B.1. The most convenient common reference frame is relative to the WGS-84 ellipsoid, or the Geodetic height, for the most straightforward conversion to a common Cartesian reference frame. Either the HAAT ($T$) or height AMSL ($T + H$) can be used for this calculation. If the height AMSL is given directly, the terrain height at the transmitters ($H$) does not need to be known explicitly. From this, the Geodetic height can then be calculated as $h = T + H + N$.

Unfortunately, the conversion of the altitude reported by ADS-B is not as straightforward. The altitude of the fifth column in Figure 6.12 is not a direct measurement of any of the listed parameters in Table B.1. The reported altitude is actually a flight level, derived from the aircraft’s on-board altimeter. The pressure variation over height is generally 1 in/Hg for 1,000 ft change in altitude, with a standard reference pressure for altimeter systems of 29.92 in/Hg. Although aircraft altimeters calibrate the altimeter to the local terrain height via the Kollsman window reference pressure, the ADS-B transponder systems report the flight level (in ft) directly calculated from the measured pressure relative to standard pressure. However, due to weather condi-
Figure B.2: Aircraft flight path derived from ADS-B data. Receiver centered at the origin of the Cartesian coordinate system.

tions, the true pressure at MSL can fluctuate significantly from this standard pressure level. Therefore, in order to calculate the true aircraft altitude AMSL, the barometric pressure at the aircraft’s location must be known from weather data. This can then be used to compute the true HAMSL using the following formula:

\[ H = PA + 1,000(B - 29.92) \]  

(B.1)

where \( H \) is HAMSL, \( PA \) is the reported pressure altitude referenced to 29.92 in/Hg (data reported in ADS-B), and \( B \) is the current barometric pressure at mean sea level (in/Hg). This can then be corrected to the Geodetic height with knowledge of the EGM96 geoid for subsequent conversion to the Cartesian reference frame.

The WGS-84 to Cartesian conversion yields global positions relative to the Earth’s origin. Assuming the transmitter, target, and receiver positions (and velocity vector) undergo the same transformation, they reside in a new and common coordinate system. This allows straightforward calculation of useful benchmark parameters, such as bistatic range and velocity of the RD map. However, these points will lie on a
slant-plane with a large offset from the origin due to the Earth’s curvature and radius. To translate these measurements to a receiver-centric coordinate system, a set of two rotation matrices around the z-axis and x-axis are constructed based upon the receiver’s Cartesian coordinate position. These rotations align the x-axis with an East pointing vector at the receiver, the y-axis in the North direction, and the z-axis in the vertical direction. After the rotation is applied, the z-offset of the Earth radius is subtracted to translate the receiver position vector to the origin. These same transformations, derived from the receiver’s location, are then applied to the transmitters, target locations and velocity vectors for convenient calculation and data display. This also allows for azimuth and elevation angles relative to the receiver and receiver range can be computed directly as the angle from the primary axes and the Euclidean norm, respectively. The results after all such operations were applied to a flight trajectory of a commercial airliner are shown in Figure B.2.
Appendix C: Passive Radar Signal Model

This appendix develops the radar signal models necessary for analyzing passive bistatic imaging scenarios. In C.1, a simple point target model is first developed to determine the phase of the complex target response output from cross correlation processing. C.2 generalizes this derivation to bistatic observation of an extended target and its k-space sampling theory, used in the investigations of Chapters 4, 5 and 9. The model for a rotating target’s Doppler response and cross-range resolution for ISAR scenarios is then developed in C.3.

C.1 Point Target Correlation Phase Response

The general form of a transmitted RF waveform from a single illuminator can be represented as follows,

$$s_{tx}^r(t) = Re \left[ s_{tx}(t) e^{j\omega_c t} \right] = |s_{tx}(t)| \cos (\omega_c t + \angle s_{tx}(t))$$

(C.1)

where $s_{tx}(t)$ is a band-limited complex modulating signal with center frequency of $\omega_c$ rad/s, and whose the amplitude varies with $|s_{tx}(t)|$ and phase $\angle s_{tx}(t)$. For frequency modulated signals the amplitude would be constant, while the phase term is proportional to the integral of the underlying data signal. ATSC waveforms, which
are 8-VSB modulated as described in Chapter 6, the phase term is either 0 or \( \pi \), and \(|s_{tx}(t)|\) modulates the envelope prior to vestigial bandpass filtering. Note that this nomenclature applies to more complex digital waveforms (such as OFDM), but its format more naturally represents amplitude and phase/frequency modulation.

The passive radar’s reference channel, \( s_r(t) \), captures a scaled and delayed replica from the transmitter. Weaker multipath and noise terms are ignored here because the focus is to investigate the target correlation response. The RF signal received by the radar can be written as

\[
s_{rf}^r(t) = A_r s_{tx}^r \left( t - \frac{R_L}{c} \right) = A_r \left| s_{tx} \left( t - \frac{R_L}{c} \right) \right| \cos \left[ \omega_o \left( t - \frac{R_L}{c} \right) + \angle s_{tx} \left( t - \frac{R_L}{c} \right) \right].
\]

(C.2)

\( A_r \) is the scalar term representing cumulative attenuation (propagation, atmospheric loss) and system gains (antenna, receiver gain, etc) along the baseline. This baseline distance also determines the time delay at the speed of light \( t_D = \frac{R_L}{c} \), where \( R_L = \|x_{tx} - x_{rx}\| \) is the Euclidean distance from the transmitter to the receiver.

The complex, baseband representation of the reference waveform, \( s_r(t) \), after downconversion (either analog or digital) can then be written as

\[
s_r(t) = A_r s_{tx} \left( t - \frac{R_L}{c} \right) e^{-j(k_c R_L + \phi)}
\]

(C.3)

with phase terms at the carrier frequency removed through demodulation. The additional terms due to propagation are reflected in the exponential term, where \( k_c \) is the wavenumber at the center frequency \( \frac{2\pi}{\lambda_c} = \frac{\omega_c}{c} \), and the residual \( \phi \) term due to the fact that only frequency (not phase) of the receiver is locked with the transmit
site. In reality, (C.3) also contains a thermal noise term. Because the focus is to investigate the target response to the illuminating waveform, the noise is neglected in this analysis.

The scattered surveillance waveform from the ideal point target is also an amplitude-scaled, phase shifted copy of the transmitted waveform due to propagation. The time delay of the scattered signal is given by the bistatic range divided by the speed of light, \( \frac{R_T + R_R}{c} \), where the transmit and receive ranges are \( R_T = \| \mathbf{x}_{\text{tar}} - \mathbf{x}_{\text{tx}} \| \), and \( R_R = \| \mathbf{x}_{\text{tar}} - \mathbf{x}_{\text{rx}} \| \), respectively. The signal phase is also modified by the complex target reflectivity, \( \sigma = |\sigma| \exp(j \angle \sigma) \). After downconversion with the same phase-consistent LO used in the reference channel downconversion, the surveillance signal becomes

\[
s_s(t) = A_s \sigma s_{tx} \left( t - \frac{R_T + R_R}{c} \right) e^{-j[k_c(R_T(t) + R_R(t)) + \phi]}. \tag{C.4}
\]

The time varying phase term results in a Doppler shift at the receiver of \( f_D(t) = \frac{d}{dt} \left( \frac{1}{\lambda} [R_T(t) + R_R(t)] \right) \). Conventional passive radar processing calculates the cross ambiguity surface over extended CPIs, as shown in Section 2.2, which generally requires compensation for the Doppler frequency. Alternatively, correlation can be computed over a number of short intervals of length \( T_{\text{int}} \) where phase due to target motion is insignificant, satisfying \( \tau f_D \ll 1 \), as shown in (C.5):

\[
\gamma(\tau) = \int_{-\frac{T_{\text{int}}}{2}}^{\frac{T_{\text{int}}}{2}} s_s(t) s^*_r(t - \tau) \, dt. \tag{C.5}
\]

This phase condition is analogous to pulse-Doppler radar processing (often referred to as the stop-and-hop or point-and-shoot approximation) and can be applied to segment processing of continuous passive signals (see Section 6.3.5). Discretizing target motion
into intervals of $T_{seg}$, such that $t = mT_{seg}$, allows the target motion to be represented
as a discrete function $R_T[m] = R_T(mT_{seg})$. The time corresponding to the signal
content within a segment can then be represented $t = mT_{seg} + t'$, the surveillance
signal for the $m^{th}$ segment then becomes

$$s_s(m; t') = A_s \sigma s_{tx} \left( mT_{seg} + t' - \frac{R_T[m] + R_R[m]}{c} \right) e^{-j[kc(R_T[m] + R_R[m]) + \phi]}$$

for $-\frac{T_{seg}}{2} < t' < \frac{T_{seg}}{2}$. (C.6)

The cross correlation output for the $m^{th}$ segment at correlation lag $\tau$ can then be
written as

$$\gamma(m; \tau) = \int_{-\frac{T_{int}}{2}}^{\frac{T_{int}}{2}} s_s(m; t') s^*_r(m; t' - \tau) \, dt'.$$ (C.7)

The correlation integral is matched to the peak target response when evaluating at the
delay corresponding to the bistatic range past the baseline, $\tau_{tar}[m] = (R_T[m] + R_R[m] - R_L)/c$.

To examine this, we will substitute (C.6) and (C.3) into (C.7):

$$\gamma(m; \tau_{tar}) = \int_{-\frac{T_{int}}{2}}^{\frac{T_{int}}{2}} A_s \sigma s_{tx} \left( mT_{seg} + t' - \frac{R_T[m] + R_R[m]}{c} \right) e^{-j[kc(R_T[m] + R_R[m]) + \phi]}$$

$$\cdot A_r s^*_x \left( mT_{seg} + t' - \frac{R_T[m] + R_R[m]}{c} \right) e^{j[kcR_L + \phi]} \, dt'$$

$$= A_s A_r \sigma e^{-j[kc(R_T[m] + R_R[m] - R_L)] \int_{-\frac{T_{int}}{2}}^{\frac{T_{int}}{2}} \left| s_{tx} \left( mT_{seg} + t' - \frac{R_T[m] + R_R[m]}{c} \right) \right|^2 \, dt'$$ (C.8)

The phase between each correlation sequence can then be used to estimate the target’s
Doppler shift and over the unambiguous range of $-\frac{F_{seg}}{2} < f_D < \frac{F_{seg}}{2}$. Assuming
the gain factors of (C.9) can be removed using propagation modeling tools and the
the energy of the reference signal is normalized such that \( \int_{T_{int}} |s_{tx}(t)|^2 dt = 1 \), the correlation output reduces to

\[
\gamma[m] \propto \sigma e^{-j\alpha(R_R[m]+R_R[m]-R_L)}.
\] (C.10)

If the precise location of the target is known to a fraction of a wavelength, the phase terms of (C.10) can be stripped from the correlation output, so that the resulting sample simply represents the complex reflectivity of the underlying point target response.

\[
\gamma'[m] = \sigma
\] (C.11)

The output of the cross correlation process for a single delta target is a constant complex value corresponding of the target reflectivity, independent of angle and target position in the global reference frame. This is independent of the frequency and position, when the phase terms of propagation are accurately accounted for. When applied to an image reconstruction algorithm, such as backprojection or polar formatting, this would result in a peak at the origin with cancellation elsewhere in the image. If the target consists of multiple scatterers, the RCS would vary as a function of angle. By exploiting these angular modulations, the locations of the scatterers can be estimated within the diffraction limits of the reconstruction technique.

Unfortunately, the true location of a target (required for simplifying (C.10) to (C.11)) is unknown in practice, and error terms of multiple wavelengths are common due to limited accuracy of range measurements. Estimation of these terms across the coherent processing interval is the goal of motion compensation algorithms, as introduced in Section 3.4.2.1. Note that these algorithms are generally only able to
estimate the phase progression (between the \(m^{th}\) and \(m^{th} + 1\) correlation segment) and further adjustment is necessary to fuse image data across multiple transmitters with different bistatic range terms.

### C.2 Extended Target Model and Bistatic Equivalence

Although real targets often exhibit some degree of point-like response, a more accurate model is to assign a continuous distribution of electromagnetic reflectivity, \(\tilde{\sigma}(x, y)\), where \(x\) and \(y\) represent the location on orthogonal axes of a concentric target coordinate system. The target response can be thought of as a collection of distributed point scatterers, not dissimilar to that developed in Appendix C.1, or a more continuous reflectivity which depends on the target shape and material properties. Estimation of this function is the goal of traditional radar imaging systems, which is enabled by first employing a forward model to describe the received data of a radar prior to image formation. Note that although a 2D representation is shown here for ease of illustration, the model can be readily extended to three dimensions. A more realistic model should also incorporate scattering as a function of transmit and receive angles (or, equivalently, the bisector and bistatic angle).

Dropping the slow time correlation index \((m)\) for simplicity, and incorporating the two-dimensional target reflectivity allows the surveillance waveform to be modeled as a 2D integral over the target surface, as follows

\[
s_s(t) = A_s \int \int \tilde{\sigma}(x, y) s_{tx} \left( t - \frac{R_T(x, y) + R_R(x, y)}{c} \right) \times e^{-j[k_c(R_T(x, y) + R_R(x, y) + \phi)]} \, dx \, dy. \tag{C.12}
\]
When the reflectivity is a delta function at the origin, \( \tilde{\sigma}(x, y) = \sigma_o \delta^2(x, y) \), the integral reduces to the form shown in Equation (C.4). A sample target reflectivity function, with coordinate system is illustrated in Figure C.1a. Unit vectors \( \hat{u}_T \) and \( \hat{u}_R \) point towards the transmitter and receiver locations, respectively, separated by the bistatic angle, \( \beta = \hat{u}_T \cdot \hat{u}_R \), whose angular midpoint aligns with the bistatic bisector, \( \hat{u}_\beta \). As the target maneuvers, the directions of these unit vectors (and therefore the bistatic angle) change.

![Figure C.1: (a) Extended target coordinate system (b) Rotated target coordinate system](image)

A coordinate transformation can then be applied rotate points within the original \((x, y)\) reference frame by \( \theta \) degrees to \((\bar{x}, \bar{y})\), as illustrated in Figure C.1a. The rotation is chosen to align the bistatic bisector with the new negative \( \bar{y} \) axis. The unit vector representing the bistatic bisector is calculated as follows:

\[
\hat{u}_\beta = \frac{\hat{u}_R + \hat{u}_T}{\|\hat{u}_R + \hat{u}_T\|}.
\]

This rotation results in symmetry of the transmit and receive directions, \( \hat{u}_R \) and \( \hat{u}_T \),
shown in Figure C.1b. The orthonormal rotation matrix describing this transformation can be calculated simply as
\[
\begin{bmatrix}
\bar{x} \\
\bar{y}
\end{bmatrix} = \begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix}.
\]
The form of the scattered surveillance signal remains effectively unchanged by this transformation, but the range terms are now expressed as a function of \(\bar{x}\) and \(\bar{y}\), as indicated in the (C.13).

\[
s_s(t) = A_s \int\int \tilde{\sigma}(\bar{x}, \bar{y}) s_{tx} \left( t - \frac{R_T(\bar{x}, \bar{y}) + R_R(\bar{x}, \bar{y})}{c} \right) \times \nonumber e^{-j[k_c(R_T(\bar{x}, \bar{y})+R_R(\bar{x}, \bar{y}))+\phi]} \, d\bar{x} \, d\bar{y}. \quad (C.13)
\]

Figure C.1b also illustrates the relative angles from the transmit and receive unit vectors to point \((\bar{x}, \bar{y})\) on the target, as \(\theta_T\) and \(\theta_R\). For most imaging scenarios, the target diameter is small relative to the transmit and receive ranges, \(D \ll R_T\) and \(D \ll R_R\), allowing us to invoke the far-field plane wave assumption. This allows the transmit and receive ranges associated with point \((\bar{x}, \bar{y})\) to be written as
\[
R_T(\bar{x}, \bar{y}) \simeq R_{To} - r \cos \theta_T 
\]
\[
R_R(\bar{x}, \bar{y}) \simeq R_{Ro} - r \cos \theta_R
\]

Where \(r = \sqrt{\bar{x}^2 + \bar{y}^2}\), \(R_{To}\) is the range from the transmitter to origin \((0,0)\), and \(R_{Ro}\) is the range from the receiver to the origin. We’ll substitute the individual ranges for
the complete surveillance bistatic range as length, \( R_B(\bar{x}, \bar{y}) \) as

\[
R_B(\bar{x}, \bar{y}) = R_T(\bar{x}, \bar{y}) + R_R(\bar{x}, \bar{y}) \\
\simeq R_{To} + R_{Ro} - r (\cos \theta_T + \cos \theta_R).
\]

(C.15)

Using the identity: \( \cos u + \cos v = 2 \cos \left( \frac{u+v}{2} \right) \cos \left( \frac{u-v}{2} \right) \), and substituting \( R_{Bo} \) for \( R_{To} + R_{Ro} \), allows the previous expression to be expressed as follows:

\[
R_B(\bar{x}, \bar{y}) \simeq R_{Bo} - 2r \cos \left( \frac{\theta_T + \theta_R}{2} \right) \cos \left( \frac{\theta_T - \theta_R}{2} \right).
\]

(C.16)

Notice that the angle \( (\theta_T + \theta_R)/2 \) represents the angle from the negative \( \bar{y} \) axis to \((\bar{x}, \bar{y})\) in the rotated coordinate system. Therefore, \( r \cos ((\theta_T + \theta_R)/2) \) in the rotated coordinate system is a projection from point \((\bar{x}, \bar{y})\) onto the \(-\bar{y}\) axis. This expression can be further simplified using the fact that \( \beta = \theta_T - \theta_R \), reducing \( R_B \) to a function of \( \bar{y} \) and bistatic angle \( \beta \) for a fixed \( R_{Bo} \).

\[
R'_{B}(\beta, \bar{y}) \simeq R_{Bo} + 2\bar{y} \cos \left( \frac{\beta}{2} \right)
\]

(C.17)

Substituting this expression into Equation (C.13) results in a simplified expression of the surveillance waveform,

\[
s_s(t) = A_s \int \left[ \int \tilde{\sigma}(\bar{x}, \bar{y})d\bar{x} \right] s_{tx} \left( t - \frac{R_{Bo} + 2\bar{y} \cos \left( \frac{\beta}{2} \right)}{c} \right) \times \]

\[
e^{-j[kc(R_{Bo} + 2\bar{y} \cos \left( \frac{\beta}{2} \right)) + \phi]} \, d\bar{x} \, d\bar{y}. \]  

(C.18)

In this form, the only dependence on \( \bar{x} \) is \( \tilde{\sigma} \), and by integrating the scattering
centers along this isorange axis yields \( p_{\vec{\sigma}} (\theta; \vec{y}) = \int_{-D/2}^{D/2} \vec{\sigma} (\vec{x}, \vec{y}) d\vec{x} \). This represents the fact that all isorange scatterers within the beampattern of the antenna are integrated and indistinguishable from a single observation angle. Note that this is a projection of the RCS onto the \( \vec{y} \) axis, represented at a particular angle \( \theta \) relative to the target’s original coordinate system. This allows the received surveillance signal to be expressed as a one-dimensional integral along the target projection axis, inline with the bistatic bisector.

\[
s_s(t) = A_s e^{-j(k_o R_{Bo} + \phi)} \int_{-D/2}^{D/2} p_{\vec{\sigma}} (\theta; \vec{y}) e^{-j2k_o \cos(\vec{\beta}/2)\vec{y}} s_{tx} \left( t - \frac{R_{Bo} + 2\vec{y} \cos(\vec{\beta}/2)}{c} \right) d\vec{y}
\]

\hspace{1in} (C.19)

![Bistatic Delay Illustration](image)

**Figure C.2: Bistatic Delay Illustration**

As illustrated in Fig. C.2, moving along to the bistatic bisector by increment \( \Delta \vec{y} \) results in an increase in both the transmit and receive ranges by \( \Delta r \cos(\vec{\beta}/2) \), such that the propagation delay time relative to the origin can be written \( t_d = \frac{2\vec{y} \cos(\vec{\beta}/2)}{c} \). The previous integral can be expressed in terms of the delay time with this relationship:

\[
s_s(t) = A_s e^{-j(k_c R_{Bo} + \phi)} \int p_{\vec{\sigma}} \left( \theta; \frac{cl_d}{2 \cos(\vec{\beta}/2)} \right) e^{-j\omega cl_d s_{tx}} \left( t - \frac{R_{Bo}}{c} - t_d \right) dt_d
\]

\hspace{1in} (C.20)
Where \( t_d \) is the time delay corresponding to a particular location on the target projection, and \( R_{Bo} / c \) is the bistatic path delay to the target center. The new constant \( A_o \) incorporates the scaling factor that results from the change of integration variable as well as the previous \( A_s \) term. The cross-range integrated target reflectivity function, \( p_{\sigma} \), modified by the phase delay term, can be expressed as \( \tilde{p}_{\sigma} \), where the spatial projection point depends on the delay time and the bistatic angle, with the phase term at the carrier frequency:

\[
\tilde{p}_{\sigma} (\theta; t_d) = p_{\sigma} \left( \theta; \frac{c t_d}{2 \cos \left( \frac{\theta}{2} \right)} \right) e^{-j \omega_c t_d} (C.21)
\]

This allows the received signal, \( s_m(t) \), to be expressed as the convolution of \( \tilde{p}_{\sigma} (\theta; t_d) \) with a delayed copy of the transmitted waveform, \( s_{tx}(t) \).

\[
s_s(t) = A_o \tilde{p}_{\sigma} (\theta; t_d) \otimes s_{tx}(t - \frac{R_{Bo}}{c}) (C.22)
\]

Using the Fourier analogue, the time domain convolution can be expressed as a spectral multiplication of the two frequency domain relationships, \( F(\omega) = \int f(t) e^{-j \omega t} dt \), as follows:

\[
S_s(\omega) = \tilde{P}_{\sigma}(\theta; \omega) \cdot S_{tx}(\omega) e^{-j \omega_c R_{Bo} / c} (C.23)
\]

Where the phase-modulated projection function’s transform is \( \tilde{P}_{\sigma}(\theta; \omega) \). The time scaling, \( \mathcal{F} \{ f(at) \} = F(\omega / a) / |a| \), and phase shift, \( \mathcal{F} \{ f(t)e^{jat} \} = F(\omega - a) \), properties allow \( \tilde{P}_{\sigma}(\theta; \omega) \) to be written as a function of the original rotated projection.
function’s transform, \( P_{\sigma}(\theta; \omega) \) as follows:

\[
\bar{P}_{\sigma}(\theta; \omega) = P_{\sigma}\left(\theta; \frac{2 \cos (\beta/2)}{c} (\omega + \omega_c)\right).
\] (C.24)

Substituting this relationship into Equation (C.23), the final expression for the frequency domain representation of the received signal then becomes

\[
S_{s}(\omega) = P_{\sigma}\left(\theta; \frac{2}{c} \cos \left(\frac{\beta}{2}\right) (\omega + \omega_c)\right) S_{tx}(\omega) e^{-j\omega_c R_{Bo}/c}.
\] (C.25)

After correlation with the reference signal, \( S_r = S_{tx}(\omega) e^{-j\omega_c R_L/c} \), the correlation output is

\[
\gamma(\omega; \theta; \beta) \propto P_{\sigma}\left(\theta; \frac{2}{c} \cos \left(\frac{\beta}{2}\right) (\omega + \omega_c)\right) |S_{tx}(\omega)|^2 e^{-j\omega_c (R_{Bo} - R_L)/c}
\] (C.26)

\[
= P_{\sigma}\left(\theta; \frac{2}{c} \cos \left(\frac{\beta}{2}\right) (\omega + \omega_c)\right) e^{-j\omega_c (R_{Bo} - R_L)/c}
\] (C.27)

where the transmitted spectrum term is simplified by assuming it is unity over the region of support. Equation (C.25) demonstrates that the received signal after multiplication is a convolution of the transmitted signal’s autocorrelation (sync-function for a rectangular spectrum) with the target’s projected RCS at a frequency of \( \omega_c \cos (\beta/2) \) rad/s, equivalent to a spatial frequency of \( \frac{2\omega_c}{c} \cos (\beta/2) \) rads/m. This relationship shows that the resulting signal is equivalent to one which would be received by a radar operating along the bistatic bisector, at a reduced center frequency by factor \( \cos (\beta/2) \), known as the bistatic equivalence theorem [78, 198].
The projection-slice theorem states that the Fourier transform of a projection of a function is a radial slice of its frequency domain representation, defined in (C.28).

\[ F(k_x, k_y) = \mathcal{F}(f(x, y)) = \int \int f(x, y) e^{-j(k_xx + k_y y)} dx dy \]  

For a projection at angle \( \theta \) (the direction of the bisector) at the equivalent bistatic angular frequency of \( \frac{2\omega_c}{c} \cos(\beta/2) \) radians/m, the correlation response after phase and amplitude compensation represents a single point of the target’s spatial frequency representation, or \( F_\tilde{\sigma}(k_x, k_y) \). Note that this value also represents the RCS of the target when observed with the radar system.

\[ \gamma(\omega; \theta; \beta) \propto F_\tilde{\sigma} \left( \frac{2\omega_c}{c} \cos \left( \frac{\beta}{2} \right) \cos(\theta), \frac{2\omega_c}{c} \cos \left( \frac{\beta}{2} \right) \sin(\theta) \right) \]  

\[ = \sigma(\omega; \theta; \beta) \]

### C.3 Doppler Response of a Rotating Target

This section serves to derive the cross-range resolution for the case of a rotating target and range-Doppler image formation, as well as the angular bounds of such a technique. For more in-depth treatment, the reader should refer to the following: [33, 79, 199].

**Monostatic Cross-Range Resolution**

Consider the case of a rotating target at angular velocity \( \Omega \) rad/s and diameter \( D \), illuminated by a radar at a standoff range of \( R_o \), as illustrated in Figure C.3. Point
$(x, y)$, located at radius $r_t$ from the target center, is moving at velocity $\vec{v}$ due to the target rotation. Using the far field assumption, $R_o \gg D$, the range to the point of interest can be expressed as $R = R_o + y_o$. The point’s instantaneous velocity and radial velocity seen by the radar can be expressed, respectively, as

\begin{align*}
  v &= |\vec{v}| = \Omega r_t \quad \text{(C.30)} \\
  v_{rad} &= \frac{dR}{dt} = \Omega r_t \sin(\phi) = \Omega x. \quad \text{(C.31)}
\end{align*}

Where $\phi$ is defined clockwise from the positive $y$ axis. Notice that the radial velocity is simply a function of the rotation rate and the scatterer’s cross range position. Scatterers close to the center of rotation at a small cross range offset exhibit the same velocity as a scatterer with a large radius, because most of the larger radius target’s motion is tangential to the radar’s line of sight. The Doppler of this point can be expressed as $f_D = \frac{2v_{rad}}{\lambda}$, resulting in the following Doppler shift versus cross range position:

![Rotated target coordinate system](image-url)
This Doppler relationship can be used to find the Nyquist frequency associated with the target rotation for a scatterer at the maximum cross-range position of $D/2$:

$$f_{Nyq} = 2f_D^{max} = 2\Omega D/\lambda.$$  
Assuming the target motion is uniform throughout an observation interval and scatterers do not migrate range bins, the cross range resolution can be calculated based upon the frequency resolution criteria, $\Delta f = 1/T$, which results the following expressions for cross range resolution, in meters:

$$\delta_{cr} = \frac{\lambda}{2\Omega T} = \frac{\lambda}{2\eta} = \frac{c}{2f_c\eta}. \quad \text{(C.32)}$$

Where the total angular observation interval, in radians, is represented by $\eta = \Omega T$.

With conventional imaging methods, such as ISAR, the range resolution due to signal bandwidth generates the down range resolution, the cross-range position can be estimated with typical range-Doppler techniques. This only holds true for moderately small observation angles with range-Doppler techniques, because as the target rotates the scattering centers eventually migrate range and cross-range resolution cells, resulting in defocusing and decreased image resolutions. To circumvent these limitations, backprojection or polar formatting can be used to further increase target resolution beyond range-Doppler limitations.

The limits of range-Doppler focusing can be derived by setting a tolerance on the phase deviation due to rotation of the nearest point on the target, at zero cross range (focused by complex summing of all radar returns). Setting the two-way phase deviation equal to 22.5°, as used for deriving the Fraunhofer distance, the angular
limit for a target of radius \( r_t \) becomes

\[
\eta_{max}^{22.5^\circ} \approx \frac{1}{2} \sqrt{\frac{\lambda}{r_t}}.
\]  

(C.33)

**Bistatic Cross-Range Resolution**

Bistatic resolution limits are the same as the previously derived expressions, with an additional \( \cos(\beta/2) \) term which accounts for the far-field projection of the bistatic range increase onto the bistatic bisector, as illustrated in Fig. C.2. The Doppler shift is also scaled by the same term, as shown in (C.34).

\[
f_{\text{bi}}^D = \frac{2\Omega x \cos(\beta/2)}{\lambda}
\]  

(C.34)

Due to the reduced Doppler due to the bistatic angle, the cross-range resolution degrades inversely proportional to the Doppler contraction, as shown in (C.35).

\[
\delta_{\text{bi}}^{\text{cr}} = \frac{c}{2f_c \eta \cos(\beta/2)}
\]  

(C.35)

The down-range resolution is also affected by the bistatic geometry, for a signal of bandwidth \( B \) is as follows:

\[
\delta_{\text{dr}} = \frac{c}{2B \cos(\beta/2)}.
\]