Unpacking Mathematics Teacher Educators’ Decision Making When Designing Online Professional Development Programs

DISSEPTION

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Abstract

Reliance on online media in providing educational opportunities has gained tremendous momentum in mathematics education. Genre of studies that explore pedagogical practices of mathematics teacher educators in the online environment is still in infancy. Using survey method and three in-depth case studies in this work I examined pedagogical practices of mathematics teacher educators who provide online professional education programs for prospective and practicing mathematics teachers to unpack the nature of their decision making.

Over a hundred mathematics teacher educators in the US completed the survey. Individual case analysis followed by the interviews using Schoenfeld’s (2010) decision making framework with special attention to the MTEs’ goals, orientations, and knowledge. Boyd’s (1993) eight decision dimensions of distance learning was used to identify types of decisions that MTEs made and Perks and Prestages’ (2008) mathematics teacher educator knowledge tetrahedron was used to analyze domains of knowledge that MTEs relied on when they made pedagogical decisions. Results indicated MTEs’ epistemological perspectives about the nature of mathematics and mathematics learning shaped their practices and decisions. Additionally, two knowledge domains that were specific for MTEs’ decisions about online mathematics teacher educating were identified: knowledge of mathematics teachers and teachers’ needs (KMT) and knowledge of the online environment for mathematics teacher educating (KOMTE). KMT was drawn
when MTEs made decisions about the PD goals and KOMTE was drawn when they designed and organized learning activities for their teachers. The study conceptualized the relationship among MTEs’ goals, orientations, and different domains of knowledge utilized when they make decisions about implementing online mathematics teacher professional development programs.
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Chapter 1: Introduction

With the recent advances in information communication technology, interest in the use of online distant learning has grown exponentially. In teacher education, using hybrid or purely online approaches to improve teachers’ professional knowledge has also gained popularity in the past twenty years (Crockett, 2010). Although it remains unclear when online professional development was first introduced in the educational setting, it is known that as early as the mid-1900s, school systems across the world have relied on technology to educate and train teachers, staff, and administrators (Crockett, 2010). The use of virtual technologies to provide distant teacher education depicts a leap forward in the history of educational technology, assuming teacher training can overcome temporal and spatial constraints. Yet, with regard to discussions about virtual distant education, practitioners and researchers have agreed that online distant education is not meant to replace the face-to-face system of education; instead, it is meant to complement it as an additional resource to provide extra learning opportunities (Keegan, 1993).

The online learning environment is now widely utilized in mathematics teacher education with the promise that it provides mathematics teachers with high-quality professional development opportunities, whilst accommodating for their busy schedules and geographic location (Dede, Jass Ketelhut, Whitehouse, Breit, & McCloskey, 2009; Whitehouse, Breit, McCloskey, Ketelhut, & Dede, 2006). Engelbrecht and Harding (2005) noted a number of potential benefits of online mathematics courses. These
benefits include: (1) the potential for student access to a wide range of educational resources online, (2) the potential for providing convenience and flexibility for training opportunities with regard to both time and place, (3) the potential for providing opportunities for a dynamic learning environment, (4) the potential for communication and collaboration beyond physical and regional boundaries, (5) the potential for independent learning, and (6) the potential for providing an environment in which students familiar with the digital world can integrate naturally.

A search of keywords, “virtual”, “online”, “mathematics teacher”, in major journals and relevant databases resulted in about 25 groups of mathematics teacher educators around the world who had published studies on online mathematics teacher education within the past five years. Collectively, these studies concerned the design of online courses or programs for in-service or prospective mathematics teachers. Among the different types of online mathematics teacher education courses or programs studied, some addressed the design for an entire graduate study program including multiple online courses; some were about the design of a professional development program; and others were about the design of an online mathematics teacher educational course including methods courses, technological integration courses, (Science, technology, engineering, and science, technology, engineering, and mathematics (STEM) integration courses, etc. Each online course/program has its own unique features and focus regarding the course/program’s design, and the teacher educators who designed these courses or programs shared invaluable insight into the design of their online courses or programs.
Pape et al. (2015) shared a yearlong, asynchronous online teacher education program for mathematics teachers with a focus on mathematical knowledge for teaching Grades 3 to 5. Their program was organized around week long modules that included activities across 35 weeks. Each module consisted of an introduction, anticipatory activity, content and discussion, and classroom connections (Pape et al., 2015, p. 20). The activities were presented in video-recorded slideshow presentations. The teacher educators placed emphasis on social presence in the online environment where norms of participant-to-participant interactions were purposefully established. In addition, they emphasized cognitive presence in their online program where they ensured that participants had multiple opportunities to do mathematics as well as to engage their colleagues in mathematical conversations.

Similarly, Larkin and Jamieson-Proctor (2015) redesigned their face-to-face mathematics education course for elementary pre-service teachers for online delivery. Their online course had components of lecture presentations as well as synchronous virtual classrooms. Their course design was guided by a theory about the relationship of course structure, course dialogue and student autonomy. Course structure referred to the rigidity or flexibility of the course in response to individual students’ learning needs; course dialogue referred to interactions between the instructor and the learners; and learner autonomy recognized the importance of the learner to determine their learning experiences. After two rounds of implementations and modifications of the design, the teacher educators found that structural elements, such as video lectures, and dialogue fostered by synchronous virtual classrooms, did not necessarily work in opposition to
each other in the online learning environment as they so often do in the face-to-face environment, where more lectures often meant less dialogues. In fact, their students’ feedback reflected that the structure and dialogue elements enabled more learner autonomy and supported their development of the use of virtual manipulatives for better understanding.

Teacher collaboration, engagement, and autonomy are the themes that are generally addressed in the literature about online mathematics teacher education (Chieu & Herbst, 2016; Clay, Silverman, & Fischer, 2012; Hjalmars, 2017; Kastberg, Lynch-Davis, & D’Ambrosio, 2014; Larkin & Jamieson-Proctor, 2015; Pape et al., 2015). Chieu and Herbst (2016) used video records of teaching to create a series of animations of cartoon characters in classroom situations to engage teachers in an asynchronous online forum discussion about teaching. Their design emphasized social presence in their online meetings where the teachers were encouraged to discuss teaching events and reflect on important teaching decisions that occurred during those events. The researchers developed codes to study the participants’ conversations and identified evaluation, reflection and alternativity as three desired outcomes of online conversation. Hjalmars (2017) asserted that interaction was not necessarily difficult due to the online environment. Instead, communication difficulties might arise when teachers were forced to work with other teachers. As an online course designer, she felt that what was necessary was to understand how to facilitate communication and conversations for the teachers using online learning technology.
Kastberg et al. (2014) reflecting on design and implementation of an asynchronous course on proportional reasoning for in-service teachers, emphasized the importance of establishing a relationship between the online teacher educators and the course participants. The authors shared that their failure in getting to know their teachers beyond the first session restricted them in addressing the teachers’ context-based needs. The authors suggested constructivist listening was critical for teaching in the asynchronous environment. Ways of listening might include prompt response to the participants’ discussion posts or willingness to be helped by the participants with technological issues.

When talking about the design of online mathematics teacher education courses or programs, the most exiting literature outline optimal organizational structures for the environment. One study reported on the type of mathematical tasks suitable for online implementation. Schwartz (2012) compared implementations of a problem, “I can count to 20 before you can!”, in a face-to-face K-2 mathematics methods course versus in an online K-2 mathematics methods course. In the face-to-face class, the instructor first modeled the game; then students played with a partner to develop strategies for winning the game. The instructor encouraged the students to list conjectures on the board and then discussed the strategies in a class group format. In the online course, students were given written directions about the game; they then worked in teams to figure out how to win the game. Students in the online class were required to post to the discussion board on at least three different days during the week to provide meaningful comments to group members. During the second week, the students posted their strategies on a whole class
discussion board. Schwartz (2012) reported that the change in how the task was implemented resulted in different learning outcomes; the online students were more detailed in their mathematical representations and systematic in their conjecture and thinking while the face-to-face class had more opportunities to discuss the pedagogical implications of the game. After reflecting on the experience, the author commented that the online environment could not model the same teaching methods as the face-to-face environment due to the fact that there were pedagogical decisions that might need to be made in the course of the face-to-face setting that were not present in the online setting.

Pedagogical issues related to the online environment are generally raised as an issue of concern by scholars at large when reporting on the online design models. Were these issues exclusively about the online environment or were there larger concerns associated with mathematics teaching and learning in the online environment? With the increased interest in utilizing the virtual distance learning model in mathematics teacher education, mathematics teacher educators (MTEs) are involved in designing online courses and programs for mathematics teachers. Their knowledge, values, and personalities influence their design. In fact, research shows that MTEs depend largely on their personal experiences as mathematics teachers, their own research interests, and their perspectives and orientations about the field to structure courses or professional development sessions (Zollinger, 2014). J. Chauvot and Lee (2015) mentioned that both teacher educators who designed their online program insisted on aligning their instruction with their learning philosophies even if such instruction would be difficult to implement in the online environment. These deeply rooted personal beliefs highlight the importance
of learning more about how MTEs think and make decisions about their practices. Yet, research relative to practices of mathematics teacher educators has been limited (Even, 2014). Furthermore, even less is known about how mathematics teacher educators go about designing online programs for mathematics teachers. In this dissertation study, I explored the pedagogical practices of mathematics teacher educators who are responsible for providing online content-based professional development in an effort to unpack the nature of their decision making as they carried out that task. This work contributes to the larger agenda of unpacking teacher decision making model whilst informing the mathematics teacher education research community about the role of information communication technology, particularly with regard to the online learning environment in mathematics teacher education.

The overall research question that guided this study concerned how the online environment interacts with mathematics teacher educators’ decision making as they design and implement online mathematics teacher preparation or professional development. The three sub-questions were:

1. What dimensions of decisions do MTEs make when they design online professional development for mathematics teachers?
2. What are MTEs’ personal beliefs, perspectives and domains of knowledge that are influential in their decisions?
3. How do MTEs’ goals, personal beliefs and perspectives, and knowledge influence their decisions regarding the online PD content?
Definition of Terms

The following terms appear throughout this report. Definitions are provided to ensure that information is clearly presented from the researcher to the readers.

Online learning: “Online teaching and learning is faculty-delivered instruction via the internet. Online instruction includes real-time (synchronous) and anytime, anywhere (asynchronous) interactions” (Poe & Stassen, 2002, p. 5).

Distance education: Hoey (2012) described distance education as a form of education that takes place between the instruction and student, and the students with each other, when they are separated by distance and time. Distance education includes older technologies such as mail-delivered correspondence education and interactive television. In the context of this study, distance education is interchangeably used with the terms online learning and online distance learning. The term virtual environment, online environment, or virtual learning environment (VLE), or the Internet are also used interchangeably to describe the medium used to support distance learning.

Decision making: Decision making is a thinking process as described in Schoenfeld (2010). However, the term decision or decisions, specifically observable decision, are the actions followed by the process of deciding. This notion of decision is consistent with Herbst and Chazan’s (2012) notion of teacher action, in the sense that teachers’ decisions are manifested in their actions or practices of teaching.
Chapter 2: Literature Review

The purpose of this dissertation study is to examine how mathematics teacher educators utilize the online environment when planning and implementing professional learning for pre-service and in-service teachers of mathematics. Essentially, this dissertation examines the teaching and teacher decision making that takes place in the online environment. This chapter offers a summary of the literature on existing studies pertaining to: (1) online teaching, (2) practices of mathematics teacher educating and knowledge of mathematics teacher educators, and (3) knowledge for using technology in teaching. Last, an overview of the theoretical framework guiding the study will be offered.

Research on Online Teaching

The online environment in education has been utilized as a communication medium or a social platform for interactions by individuals. To date, research on how the online environment affords teaching and learning has consistently been framed and theorized in terms of social interaction, which was originally derived from research on distance education. In the following, I set out to trace the development of research on distance education with regard to the issues of social interaction, teacher’s role, and teaching in online education.

Distance education theories. A mechanics view of interaction and dialogue was one of the earliest theoretical perspectives of distance learning (Gunawardena & McIsaac,
2004). Such a view draws on an industrial model of production where technology was used to reproduce teaching materials and learning occurs in individual self-study. Researchers emphasized dialogues and interactions between the learner and the text in building knowledge (Holmberg, 1991). Students creating simulated conversation in the educational materials was encouraged as a method of learning from a distant (Holmberg, 1991). Though the emphasis of distance learning is still between the learner and the text, ideas of two-way reciprocal communication emerged in early distance education research.

A more recent theory of distant learning is credited to Moore’s (1993) model of transactional distance, which “connotes the interplay among the environment, the individuals and the patterns of behaviors in a situation” (Boyd & Apps, 1980, p. 5, cited in Moore, 1993). Moore (1993) highlighted the importance of the transactional view in distance education as the temporal and spatial separation of the teacher from the learner creates a special psychological and communication space to be bridged. Grounded in this perspective, Moore (1993) emphasized that distance education must include multi-layered dialogues between students and the teacher. He identified three key interactive components for a meaningful learning experience: (1) dialog (i.e. interaction between learners and teachers); (2) structure (i.e. the design of the program or learning materials); and (3) autonomy (i.e. the degree of learners’ self-directing in their own learning). The combinations of the first two components aim to reduce the transitional distance to achieve the third component (i.e. the autonomy of the learner). Moore’s theory implies that teachers may either engage in more explicit interaction with learners through telecommunication technologies but with more open structured programs; or engage in
tightly structured programs where maximum guidance and direction and advice are provided by course designers. He argued that learners have to exercise more autonomy when (1) greater structure and lower dialogue or (2) less structure and more dialogue are provided in the program. As reviewed in the introduction chapter, Larkin and Jamieson-Proctor (2015) used Moore’s transactional distance theory to design an online mathematics teacher education program. They reported that online distance learning did not necessarily encompass the reciprocal relationship between structure and dialogue. Rather, their learners had additional autonomy when more structure and more dialogue were provided.

In the mechanics view, the teacher’s role was not considered as critical. Moore’s theory suggested teachers’ intervention would greatly reduce learner’s autonomy. Moore’s perspective has been challenged by those who espouse a constructivist view of teaching and learning. Constructivist-based theories of distance education place a greater emphasis on the role of community. Indeed, the community of inquiry framework has gained increasing prominence when the online environment becomes the dominant medium of learning for distance learning (Anderson, 2008; Garrison, Anderson, & Archer, 2001, 2010; Lehman & Conceição, 2010).

Anderson and colleagues (Anderson, Rourke, Garrison, & Archer, 2001) in a study of teaching in the online environment theorized the online community of inquiry as a construct that integrates cognitive presence, social presence, and teaching presence. Among these three presences, one emphasizes the specific content development (cognitive); one focuses on the collaborative context and presence of real and functional
human beings instead of machines (social); and the other one accounts for a supportive or facilitative role for the above two (teaching). The diagram of community of inquiry model of online teaching is depicted in Figure 1.

Figure 1. Community of inquiry by Anderson (2008)

**Online teaching and teacher’s role.** Traditionally, teachers have been assumed responsible for educating and guiding students towards acquisition of subject matter knowledge through instructing, mentoring, evaluating, assessing progress, and performing research (Clark, 1989). Has the teacher’s role changed in the online education
environment? Have “space” and “time” constraints redefined the teacher’s role in the learning environment?

Some researchers have argued that as multimedia technologies become prevalent in education, the teacher is no longer the sole source of knowledge and gradually transforms into being a knowledge facilitator (Gunawardena & McIsaac, 2004). Vertecchi (1993) proposed an organizational structure for characterizing the role of a teacher in the distance education system. He argued that unlike traditional education, where different functions of teaching (e.g. planning the teaching methods, preparing content, communicating the content to student, assessing student learning results) are accomplished by a single teacher, different teaching functions are separated out and specialized by individual teachers in their specific roles. These functions are:

- Designing: outlining and defining the learning goals and the audience of the education, and also selecting technological tools and communications systems;
- Development: preparing all the content materials needed; and
- Implementation: communicating with students about the material such as distributing the study materials, and grading assignments (Vertecchi, 1993).

Vertecchi (1993) argued that the success of the online learning project depended on the logical design of the project and the internal coordination of each specialization part. Vertecchi further proposed that the role of the teacher in distance learning has become so specific that a single teacher can no longer play a defining part in one’s learning.
In the community of inquiry model of online education, Anderson and colleagues regard teaching presence as the most important element (Anderson, 2008; Anderson, Rourke, Garrison, & Archer, 2001). They posit teaching in the online environment involves “devising and implementing activities to encourage discourse between and among students, between the teacher and the student, and between individual students, groups of students, and content resources” (Anderson, 2008, p. 345). In other words, the teacher’s role in online teaching is to devise content-based discourse among online participants so communities of inquiry can be established. The researchers emphasized that teaching in the virtual learning environment must go beyond moderating, and that the online teacher must add subject-matter expertise through a variety of forms of instructional actions. They opposed a laissez-faire “guide-on-the-side” approach in online teaching, but rather, emphasized the provision of rigorous instruction. As such, the online teacher should support and contribute to the learning discourse community by “injecting comments, referring students to information resources, and organizing activities that allow the students to construct the content in their own minds and person contexts” (Anderson et al., 2001, p. 9).

Anderson (2008), in a later theoretical paper, further elaborated on the importance of teaching presence in the online learning community. He studied the teacher’s choice between the synchronous and asynchronous models of delivery, and assessment models used in teaching. He introduced two protocols for online assessment. One concerns the quality of content of the students’ postings. The other one is more of a prescriptive of recommendations for postings, such as how long to write, what words to avoid, and how
to cite. For the latter one, evaluation criteria were attached. Both protocols were designed for the assessment of asynchronous graduate level online courses. Anderson (2008) suggested that the rubrics for assessment could also provide students with clear directions of desired participation, which is a critical component of developing teaching presence.

**Interaction and online teaching.** As teaching presence is emphasized in online education, organizing, initiating and moderating interactions become key elements. Virtual technologies force teachers to organize and present the course materials in a completely different manner (Cyrs & Conway, 1997). To begin, communication is mostly in written form. Absence of physical presence adds complexity to teaching and learning. For instance, in mathematics learning, multiple representations drawn or produced using physical manipulatives, gestures or body language are not as rich as in the face-to-face environment. Also, written communication, unlike drawings, is limited to a linear representation of thoughts. Shea, Pelz, Fredericksen, and Pickett (2002) reported that the asynchronous dimension of the online environment forces teachers to plan ahead for the sequence and the content of materials intended to be delivered. In the synchronous dimension (e.g. chat room), multiple, simultaneous dialogues in the conversation are likely, especially in instances where students’ dialogue could be related to personal experiences and does not reflect reasoning (Angeli & Valanides, 2009; Borba & Llinares, 2012). Bower (2001) reported the problem of teachers becoming overwhelmed with the demands of reading online postings when teaching in a large class where there could a large number of postings.
Despite the complexities associated with communication in the online environment, online technologies are developing towards more customizable features for educators so that educators may organize the environment in desired forms. In a recent study, my colleagues and I examined different patterns of student-teacher interaction in synchronous mathematics classrooms as the teacher organized the live classroom using different chat pods for interactions (Huang, Kellert, & Manouchehri, 2015). We reported an increase in the amount of student-student discussion and participation as the teacher introduced two chat pods: one designated for questions and the other designed for responses; and the increased student-student discussion provided greater opportunities for the teacher to address students’ issues.

**Teachers’ experiences with online teaching.** A review of the existing literature about online teaching reveals two major theories that guided conceptualization of online teaching by scholars: theories of learning and theories of distance education. To some extent, these theories are idealized and deterministic in nature. Some studies reported that some teachers are more reluctant to adopt online technology in their instruction than others (Grant, 2004). Moreover, teachers react to their experiences of teaching online. They may enjoy it and continue to be present; or they may become frustrated and avoid doing it. Ultimately, teachers must choose whether or not to adopt online teaching based on their own experiences. Walter Gudea (2008) developed a theory based on the online teaching experiences after interviewing teachers about the practice of online teaching. The research intended to unpack factors that may impact teachers’ online teaching experiences. Findings identified ten emerging themes related to online teaching
experiences: adjustments, choice, differences among modalities, issues, teacher needs, teaching, teaching demands, teaching dimensions, teaching with technology and technology. A diagram depicting these factors is shown in Figure 2.

Figure 2. Main categories and relationships by Walter Gudea (2008)

In summary, the literature surrounding online teaching raises issues that teaching in the online environment is complicated and teacher’s roles are redefined in this environment. However, discussions about teaching practices cannot be separated from the
subject of teaching and teachers of the subject matter. This dissertation studies online
teaching practices pertaining to mathematics teacher educators who provide content-
based mathematics teachers professional development online. The next section of the
literature review focuses on mathematics teacher educators and their practices of
mathematics teacher educating in the online environment.

**Research on Mathematics Teacher Educators**

Mathematics teacher educators’ (MTEs) work is complex; they need to meet
multiple goals that address various conceptual dimensions. They tend to incorporate both
mathematical content and methodologies that would elucidate effective mathematics
teaching practices based upon an understanding of what mathematics teachers require for
their professional learning. Researchers have examined MTEs’ practices as a disciplinary
group. Goldsmith and colleagues (Goldsmith, Doerr, & Lewis, 2014), in a recent review
of literature of mathematics teacher education, synthesized 106 articles published in
major databases between 1985 and 2008, which examine mathematics teachers’ learning.
They reported six major categories of the kind of work that MTEs engaged in for the
professional learning of the mathematics teachers. These are: (1) mathematics content
knowledge, (2) teachers’ identity, beliefs, and dispositions, (3) teachers’ instructional
practice, (4) teachers’ collaboration and community building, (5) attending to student
thinking, (6) curriculum, and others. Some of these categories concern the content
specific for teachers to learn such as mathematics content knowledge, student thinking,
instructional methods, which might be explicitly stated in the course syllabus. Some of
these categories are about the kind of teacher learning environment that MTEs intend to
establish during professional development practice, such as teachers’ collaboration and community building, and identifying changes. Other categories, such as teachers’ beliefs and dispositions, maybe indicators for MTEs about teachers’ learning which show changes accompanied with teachers’ engagement in the content and practices that are organized and structured by MTEs.

In a similar manner, Zollinger (2014) identified seven areas that seemingly exerted influence on teacher educators’ practices including: (1) reflective practice, (2) teacher learning, (3) teacher beliefs and attitudes, (4) technology, (5) teacher practices, 6) social justice, and (7) children’s mathematics. The two distinct categories that are not in Goldsmith et al.’s synthesis are technology and social justice, which have become important elements in mathematics teachers’ professional learning in recent years.

Studies of practices of mathematics teacher educating or the work of MTEs are plenty in current literature but studies about MTEs are rare. I searched key words “mathematics teacher educators”, “teacher educators”, “didacticians” in major journals, conference proceedings, book chapters and dissertation databases in mathematics education. After selections based on abstract and relevance, 29 published articles and 7 dissertations since 2005 were selected for the following review. In the following sections, I will first define mathematics teacher educators as a professional community and then I will review the reports that were about MTEs’ journeys of becoming teacher educators and reflections on their practices of working with mathematics teachers. Then, I devote the rest of the sections to the review of studies focusing on MTEs’ knowledge, beliefs and practices.
Defining mathematics teacher educators (MTEs). Different terms have been used to describe mathematics educators who work with teachers in various contexts and roles (Even, 2008); for example, mathematics teacher educator (MTE), and didactician (Jaworski & Huang, 2014). The label of mathematics teacher educator (MTE) can be applied to a broad range of individuals according to who they are and what they do. Jaworski and Huang (2014) defined didacticians as “mathematics (teacher-) educators who work with practicing teachers to promote developments in teaching and learning mathematics: the term includes university faculty, teaching, researchers, curriculum development coordinators, master teachers, mathematics coaches, and so on.” This definition of didacticians implies individuals who are recognized as MTEs may do vastly different work from one another; for instance, how a university faculty supports the learning of preservice or in-service teachers is not the same as the how a mathematics coach or a curriculum development coordinator would provide support. Nevertheless, in Jaworski and Huang’s (2014) notion of didacticians or teacher educators, they emphasize that the work of MTEs involves “a transformation of theoretical ideas and research findings into modes of teaching that are informed by theory and research” (p.174). The transformation takes place at two levels: between teachers and teacher educators, and between teachers and learners of mathematics.

Rino (2015), in his dissertation study, proposed that the definition of mathematics teacher educator (MTE) should not only include a description of who they are and what they do, but also what they believe (Rino, 2015). The author studied the beliefs of 16 MTEs from across the United States, who were full-time university faculty members
supporting the learning and development of preservice and in-service mathematics teachers, including those who teach content courses and those who teach methods, those who teach elementary teachers and those who teach secondary, those housed in education departments and those who teach in mathematics departments, etc. The author looked for common beliefs that MTEs shared and learned that despite some exceptions, MTEs are likely to believe: “(a) deep content knowledge enables PSTs to make better decisions as mathematics teachers, (b) an understanding of students is a critical aspect of mathematics teaching, (c) PSTs should experience learning using the same methods that they will be expected to use, (d) MTEs need to understand the contexts for which they are preparing PSTs, and (2) MTEs should have a vision of PSTs as reflective practitioners who learn and develop and push the field of mathematics education forward” (Rino, 2015, p. 100).

Together with Jaworski and Huang (2014) and Rino’s (2015) definitions, MTEs, regardless of whether they are practice-based (e.g. mathematics coaches), or university-based (e.g. university professors), shoulder the responsibility of transforming research into the practice of teaching and should share the vision about how mathematics teachers should learn. The following review will focus on the practices of mathematics teacher educating among university-based mathematics teacher educators.

**Reflecting one’s own developmental journey.** Earlier published papers on mathematics teacher educators placed a strong emphasis on the professional growth of MTEs. Researchers reported reflective self-studies of their own journeys of becoming teacher educators. Some of these reports represent views offered by the general population of teacher educators and some studies are more specific about mathematics
teacher education. All of these studies emphasize reflection on practice as a key for MTEs’ professional growth (Even, 2008; Zaslavsky, 2008). This notion is consistent with the constructivist’s views of learning in the sense that learning is a mental process. One’s thinking may be advanced by making connections to one’s activities and the consequences of those activities through the means of reflection (Goos, 2009).

Zeichner (2005) reflected upon his transition from being a classroom teacher to a cooperating teacher and later to a university teacher educator. He suggested a few important experiences for the preparation of future teacher educators. These experiences include: 1) teaching courses, 2) supervising preservice teachers while completing field placements, and 3) conducting self-studies gauging how to improve their own practices as teacher educators. Additionally, Zeichner emphasized the importance of knowing the literature on teacher education. He explained:

Ignorance of the literature in teacher education prevents one from potentially seeing one’s practice as a teacher educator in new ways that challenge one’s existing frameworks and cuts one off from what has been learned by teacher educators in other programs about particular aspects of teacher education such as instructional strategies to accomplish particular purposes. It reduces teacher education to a commonsense activity and is inconsistent with the scholarly norms that universities claim to embrace. (Zeichner, 2005, pp. 122-123).

Tzur (2001, 2008) traced his history of learning mathematics, learning to teach mathematics, learning to teach teachers, and learning to mentor MTEs. Specifically, he reported that as he reflected on his journey of becoming MTE, he noticed that his engagement in writing his thesis and in contemplating his practices with teachers allowed him to see a deeper relationship “between specific teacher intervention and students’
learning” (Tzur, 2001, p. 269) based on education theories. He conceptualized his journey as a MTE into a four-foci model:

(1) Learning mathematics as a student
(2) Learning to teach mathematics as a teacher
(3) Learning to teach mathematics teachers as a teacher educator, and
(4) Learning to teach MTEs as a mentor.

For each focus, Tzur (2001) emphasizes specific reflective questions for professional growth. He notes three reflective questions for MTEs to consider when engaged in the process of teacher educating, including: “(3a) what it means to teach mathematics. (3b) how someone comes to know how to teach mathematics and (3c) how someone’s activities promote others’ learning mathematics teacher” (p.273).

Similarly, Krainer (2008) reflected on his own development trajectory as a mathematics learner, mathematics teacher, and mathematics teacher educator. He used reflection-on-action as a construct to analyze his own practices following a classroom event in a methods course. The author argued that his engagement in teaching and research had broadened his initial theoretical ideas and his knowledge grew as he developed as a MTE. The author emphasized the importance of research and theory in mathematics teaching and learning for MTEs’ development. He suggested that MTEs’ practices should reflect what is learned in research and theory.

**Reflecting on joint practices with teachers.** Chapman (2009) argued that reflecting on one’s own practice can provoke professional growth but it may be constrained by one’s own perspective. He suggested that this situation can be “minimized
if reflection is viewed also as a collective activity” (Chapman, 2009, p. 126). He argued that the genuine discourse necessary for in-depth change in action could be absent without the medium of collaborative relationships. This notion is consistent with what Jaworski (2014) reported in her synthesis of literature on MTEs practices. Jaworski (2014) noticed that collaborative critical inquiry between teachers and didacticians emerges as a critical force for teaching and MTEs’ own development. Coles (2014) relied on an activist perspective to examine teachers and MTEs’ learning from using videos. While he was investigating how mathematics teachers learn from using videos, he also examined how MTEs may support such professional learning of mathematics teachers. He concluded that in order to facilitate a productive discussion of videos MTEs need to listen in a particular manner (i.e. “heightened listening”) so as to discern what teachers say and what kind of things teachers say. Coles argued that MTEs need to deliberately engage in heightened listening to establish a discussion norm when videos are used.

Rowland, Turner, and Thwaites (2014) applied the Knowledge Quartet (KQ) theory developed by their own team to understand mathematics teacher knowledge. The authors posited that while they were documenting and reflecting on their own practices with elementary mathematics teachers based on the KQ theory, they learned more about their roles as mathematics teacher educators. They documented that as the researchers learned more about teachers’ mathematical knowledge that was revealed through the KQ analysis of their practice, they modified the content in the methods course, realizing the importance of including examples and representations of teaching as vehicles for motivating learning about teaching. They also engaged colleagues from around the world
to use the KQ framework to learn about teachers’ mathematical knowledge and they constantly discussed how they understood individual codes and instances. They realized that the KQ research into teaching and learning in the classroom provided a foundation for professional development of teacher educators.

Goodchild (2014) reflected on his experiences of providing professional development for secondary teachers to reveal challenges and tensions of teacher educating based on what teachers pursued or received. Goodchild analyzed his work with teachers using the perspective of community of practice. He emphasized teachers and MTEs are equal partners in the process of learning, despite differences in their functions and responsibilities.

The studies reported above reveal that MTEs’ own learning resided in designing and revising their practices with teachers through constant reflection upon theories and literature. A common factor for MTEs’ learning is partnership with colleagues and respecting the knowledge that teachers bring to the process. Jaworski and Huang (2014) suggested:

Collaboration and mutual engagement in communities of inquiry can enable development of new forms of practice and provide support where risky new practices are involved. However, it can also raise issues and tensions which have to be addressed. Through such mutual inquiry and addressing of issues, it is possible to achieve complementary goals of didacticians and teachers for the benefit of students who learn mathematics in their classrooms (p.184).

**MTEs’ knowledge.** Studies of mathematics teacher educators’ knowledge structure also emerge in the literature. Sowder (2007) identified three types of knowledge that are critical in training teachers in mathematics, and MTEs are potentially helping teachers to develop one or more types of this knowledge. They are: (1) knowledge-for-
practice; (2) knowledge-in-practice; and (3) knowledge-of-practice. Knowledge-for-practice refers to the kind of knowledge acquired from formal professional development programs and university course work required before teachers are able to teach mathematics. Examples are Cognitively Guided Instruction (CGI) and Integrating Mathematics Assessment (IMA). These are professional development models that have particular focuses, such as student thinking, which have shown successes with regard to enhanced teachers’ instructional practice. Knowledge-in-practice refers to the knowledge that emerges as teachers’ reflect upon their teaching practices. Professional communities of practice and professional development models, like lesson study, are geared towards the development of this type of teacher knowledge. Lastly, knowledge-of-practice refers to the kind of knowledge learned when teachers use their own classroom teaching experiences to investigate the practices of teaching and learning. This type of knowledge often relates to the work of practitioner researchers. Both prospective teachers and practicing teachers may engage in developing any of the above three types of knowledge in their professional journey, which also become the work of MTEs’ in providing and facilitating teacher learning.

Some researchers conceptualize MTEs’ knowledge as an extension of knowledge for teaching (Perks & Prestage, 2008; Zaslavsky & Leikin, 2004). For example, Perks and Prestage (2008) proposed a teacher-knowledge tetrahedron model to conceptualize teacher educators’ knowledge. The three components of the tetrahedron are: (1) practical wisdom refers to the past experiences of successful or less successful design and implementation of teacher training activities and sessions (e.g. what teachers need to
know and how the sessions are structured to engage them in learning); (2) professional traditions refer to research on mathematics teaching, existing ways of working in the field or in the institution, or national or local standards of teacher training; and (3) learner-knowledge refers to teachers’ educators experiences of being a teacher, which constitutes all three components in the teacher-knowledge tetrahedron including mathematics teacher educators’ knowledge that is gained from being a mathematics learner at school or college, practical wisdom gleaned from experiences of being a mathematics teacher, and the knowledge acquired from professional traditions of curriculum, standardized assessment, etc. As such, the teacher knowledge tetrahedron is within the vertex of learner knowledge in teacher-educator knowledge.

J. B. Chauvot (2009) studied her own content knowledge, structure, and growth from the time she was a novice mathematics teacher educator during her doctoral program until her tenure-track position at a large institution. Relying on multiple frameworks on teacher knowledge, including Shulman’s PCK, Grossman’s (1990) four areas of teacher knowledge, Murray and Male’s (2005) notion of pragmatic knowledge for teacher educators, she analyzed her teaching journals, teaching artifacts collected over the years as well as other related narratives. She depicted a map of her “perception of the knowledge [she has] sought in [her] role as a mathematics teacher educator-researcher” (p.362). The map contained three elements: (a) knowledge of context (i.e., teaching university courses or mentoring doctoral students); (b) three categories of knowledge as articulated by Shulman (1986) (i.e., subject matter content knowledge, PCK, and curricular knowledge); and (c) knowledge of research, which serves as a unifier to
Shulman’s categories (i.e., conducting, interpreting and writing). What is valuable in her study is that she applied Shuman’s model to document what MTE’s knowledge looks like in different contexts of her work. For example, in Figure 3, Chauvot (2009) documented what subject matter content knowledge for a MTE is like for proportional reasoning in teaching university courses versus in mentoring doctoral students.

![SMCK for a MTE-R](image)

**Figure 3. Subject matter knowledge for a mathematics teacher educator-researcher**

She wrote:

As I created the syllabus for the proportional reasoning course, questions related to subject matter content knowledge were: What is proportional reasoning? What concepts and skills constitute proportional reasoning? How do the concepts and
skills relate to one another? How does proportional reasoning connect across the discipline? How does proportional reasoning connect to other disciplines? How do children develop proportional reasoning skills? What are appropriate instructional strategies to facilitate the development of proportional reasoning? What resources and materials are available to teachers to foster development in proportional reasoning? (journal & syllabi analysis). I found myself recognizing the importance of understanding and connecting concepts and topics such as additive and multiplicative reasoning, unitizing, rational number, rate, percentage, ratio, proportion, covariance, invariance, slope, similarity, measurement and probability. (J. B. Chauvot, 2009, p. 365)

In contrast to teaching university courses, Chauvot identified knowledge of history of mathematics education as a disciplined field, knowledge of the role of theory, and knowledge of research methodology as important subject matter knowledge for herself as MTE-Research when she mentored doctoral students. Chauvot’s (2009) study has exemplified Shulman’s PCK for a teachers’ teacher and a doctoral students’ teacher.

Though Chauvot’s (2009) self-study adopted the teacher knowledge framework as her analytical lens, her study provides us with evidence that mathematics teacher educators’ knowledge may not necessarily be the same as mathematics teachers’ knowledge (Beswick, Goos, & Chapman, 2014; Jaworski, 2008). For example, mathematics teachers need to be knowledgeable about school curriculum in considerable detail in order to teach it and to report students’ attainment in relation to it. Mathematics teacher educators, however, only need to know about school curricula but not in the detail needed by teachers who work with it on a daily basis and are accountable for outcomes it specifies. Rather, they need to know about curricula in a more general way, including how and to what extent research is reflected in them, and the overall expectations of the content teachers at different grade levels should be able to teach. Mathematics teacher
educators’ knowledge might, therefore, be seen as overlapping with mathematics teachers’ knowledge but not entirely containing it (Beswick and Chapman, 2012, p.3 as cited in Jaworski, 2014). Though MTE knowledge and teacher knowledge may not be the same, they do overlap. Figure 4 depicts the relationship between MTE knowledge domain and teacher’s knowledge domain as shown in Jaworski’s (2008) paper.

Similarly, Zollinger (2014) studied three mathematics teacher educators’ knowledge domains as they provided professional development for practicing teachers or conducted methods courses for preservice teachers. In addition to identifying which knowledge domains each MTE drew from when working with teachers, he also investigated how different teacher knowledge domains might have influenced MTEs’ practices. He categorized MTEs’ knowledge domains based on Shulman’s model, Ball
and colleagues’ model of mathematics knowledge for teaching, as well as Elbaz’s (1983) notion of practical knowledge of teachers, and analyzed the frequencies of each knowledge domain exhibited in MTEs’ practices. His findings reveal that pedagogical content knowledge was the primary source of knowledge while subject matter knowledge and curriculum knowledge were the secondary sources in the three MTEs’ practices. The author stresses that mathematical content knowledge, though not the focus of their work with teachers, was the key factor influencing MTEs’ interactions with mathematics teachers. Indeed, the discussion of MTE’s practice and knowledge cannot be separated from mathematics content knowledge. What kind of mathematical content knowledge is needed for MTEs to work with teachers? To response question, Zazkis and Zazkis (2011) studied this issue specifically for MTEs who work closely with elementary teachers.

Zazkis and Zazkis (2011) interviewed five mathematics teacher educators who had experience with teaching methods used for courses taught by elementary preservice teachers. The teachers discussed the values of their mathematical knowledge in relation to their teaching of an elementary mathematics methods course. The researchers asked the five MTEs when they had the opportunity to use, or to take advantage of their advanced knowledge in mathematics. Participants provided different insights in response to this question. One MTE shared that his goals for the methods course were to engage teachers in appreciating the process of doing mathematics and in developing the kind of mathematical thinking that goes beyond school mathematics. One MTE shared that her advanced knowledge of mathematics helped her design and facilitate mathematical
problem solving activities with teachers. Another MTE shared an account of a classroom event where a teacher asked about division by zero. It was her advanced mathematical knowledge that helped her recognize her students’ confusion and allowed her to facilitate the conversation using limits in calculus. The authors noted that when all five MTEs discussed their use of mathematical knowledge, they tried to introduce their students (i.e. elementary school mathematics teachers), either implicitly or explicitly, to disciplinary mathematical ideas. Yet, the authors also noted that all participating MTEs expressed that mathematical knowledge is “not much about the content” but as “an extended experience with the content that contributes to acquiring what Cory (2001) alludes to as ‘image of mathematics’, that is, ‘knowledge about that discipline’, in addition to the ‘body of mathematics’ ” (Zazkis & Zazkis, 2011, p. 261). In other words, mathematical knowledge is the knowledge about mathematics.

The studies reported above concerned MTEs’ knowledge structure. Most reports offered the researchers conceptualization of teacher educating based on their own reflection as MTEs. Zollinger’s (2014) and Zazkis & Zazkis’ (2011) studies appear to be methodologically different as they conceptualized MTEs’ knowledge structure based on interviews or observations.

MTEs’ beliefs. Some recent dissertation studies have examined mathematics teacher educators’ practices, knowledge and/or beliefs. Taylor (2011) investigated the actions and purposes of MTEs in development of preservice teachers’ PCK in an elementary mathematics methods/content course. Based on a longitudinal study of the MTEs, the author documented the MTEs’ 10 core purposes for the course based on the
four components of PCK (knowledge of instructional strategies, knowledge of curriculum, knowledge of student understanding, knowledge of assessment) and 34 corresponding actions. The author learned that the MTEs’ 34 identified actions were aligned to their respective categories of core purposes, which provided evidence that the MTEs’ actions were goal-oriented. The author also learned that the MTEs’ actions were interdependent and closely connected to the mathematics teaching in grades 1-6. However, I wonder if the interdependence of the MTEs’ actions was due to the nature of the four PCK components, which closely relate to each other when teaching grades 1-6 mathematics. Nevertheless, this paper was written in 2011 and it was one of the first studies that was not categorized as a self-study, and additionally, utilized the methods of analyzing videos of a MTE’s practice.

In a more recent dissertation study, K. R. Johnson (2013) considered the issue of positionality and MTEs’ practices. She argued that illuminating the beliefs, knowledge, assumption, and values could inform/constrain/skew the decisions that MTEs make about what learning opportunities to provide teachers. Reflecting on her own practice, she posited that questioning herself/identify/positions over time helped her to develop and deepen her own understanding of her positionality as a mathematics teacher educator. She referred to these questions as the heuristic. These questions include:

1. What is the discussion/moment that stands out to me the most about today’s interactions between myself and the mathematics teachers with whom I work? For what reason(s) is this discussion/moment salient?
2. When I am interacting with mathematics teachers, what do they say or do that “disturbs” me? What does this tell me about what I believe, assume, or know to be true?
3. Considering the mathematics teachers I am working with, what “sticks” (Anderson, 2009) with me about who they are collectively, as individuals, and as
mathematics teachers as we have worked together over time? Why are these the ideas that stick with me and how do they shape our continued interactions? (K. R. Johnson, 2013, p. 43)

One cannot deny that MTEs’ beliefs or perspectives influence their practices. As mentioned at the beginning of the paper, the label of MTEs could apply to a broad range of individuals if the moniker of MTE is defined according to what they do. Rino (2015) studied common beliefs that MTEs hold in order to help define mathematics teacher educators. Yet, how MTEs’ beliefs influence their practices is another area for investigation. Toth (2014) in his dissertation studied mathematics teacher educators’ personal theories about instructional strategies with regard to technology use and examined how they incorporated their personal theories in classrooms. He interviewed and observed three teacher educators’ practices and conducted regression analysis. The results showed that the three MTEs did not incorporate technology in their content or pedagogy or pedagogical content activities more than 45% in their instructional time. Moreover, how the three MTEs viewed the concept of mathematics (mathematics as activity, content, or way of thinking) governed the way they ultimately conceptualized and utilized technology. The author suggested that identifying the functionality of a technology with respect to their perspectives about mathematics teaching might allow the instructors to utilize tools in a more meaningful way.

Summary. Goos’ (2009) argument for using a sociocultural approach to research about mathematics teacher educators could best conclude the existing literature surrounding mathematics teacher educators. Goos proposed adapting Valsiner’s Zone Theory to the teacher educator as a learner. Valsiner’s Zone Theory has been adapted in
teaching-learning interactions where the Zone of Free Movement (ZFM) and the Zone of Promoted Action (ZPA) are two additional zones to accompany the Zone of Proximal Development (ZPD) to account for context of learning (ZPM) as well as opportunities of learning (ZPA) (Figure 5). In the context of teaching and learning mathematics, for example, researchers studied how teachers might create the apparent environment for promoting discussions (i.e. ZFM) as well as how their teaching actions might allow students to have exposure to these experiences (i.e. ZPA).

In a similar manner, Goos (2009) suggested that zone theory can be extended to mathematics teacher educators as learners. She explained: “This theoretical extension of the zone model opens up the possibility for investigation of how mathematics teacher educators’ knowledge and beliefs define a set of possibilities for their continuing development (ZPD), how their professional contexts constrain their actions (ZFM), and
how they experience and benefit from different opportunities to learn (ZPA)” (Goos, 2009, p. 212). The literature reviewed in this section included a number of studies on MTEs’ knowledge and beliefs, which is the ZPD. Some studies have considered the ZFM of mathematics teacher educators – contexts. For example, J. B. Chauvot (2009) studied her own knowledge when she was teaching courses versus when was mentoring doctoral students. Zazkis and Zazkis (2011) studied mathematics teacher educators’ mathematical knowledge when conducting elementary teachers’ methods courses. MTEs have more professional autonomy than school teachers; their work may involve pre-service, in-service or possibly future MTEs’ education. In each of the contexts, there could exist constraints or challenges. Last but not least, some researchers investigated learning opportunities, that is, ZPA of mathematics teacher educators. For example, Tzur (2008) identified four levels of MTEs’ development: learning mathematics, learning to teach mathematics, learning to teach mathematics teachers and learning to mentor future MTEs. The zone model provides a rather cohesive view to examine what exists in the current literature. Yet, as Goos also identified in the paper, this is a young field. Further research needs to be developed to validate a theory for mathematics teacher educator development.

**Research on Knowledge for Using Technology in Teaching**

In the previous two sections, I reviewed research on online teaching and research on mathematics teacher educators as these two bodies of research closely relate to the study of mathematics teacher educators’ practices of designing online teacher education. In this section of literature review, I will review research studies that conceptualize
knowledge for using technology in teaching. As digital technology was first introduced for subject learning, teachers were encouraged to integrate technology into their teaching practices. A number of studies identified teachers’ knowledge of technology as a key factor in whether or not technology is integrated into their practices. This phenomenon was common around the world. Ursini and Sacristán (2006) found that, in Mexico, teachers’ resistance towards using technology was due to their absence of technological knowledge, and the digital divide between students’ and teachers’ knowledge about computers. Goos and Bennison (2004) in a survey of 485 secondary mathematics teachers in Australia reported that absence of technology knowledge generally yielded doubts about the value of technology in helping students’ mathematics learning.

The discussion about the type of teacher knowledge needed for effective integration of technology in teaching involves not only the use of a digital tool, but also the epistemological issue relative to the transformation of mathematical knowledge as a result of the presence of technology, also known as, the didactical transposition. Assude, Buteau, and Forgasz (2010) claimed that the changes in mathematical knowledge and mathematical practices that emerge inform that the presence of digital tools are challenging to transpire in classrooms because this new “form” of mathematics has not been clearly established or understood by teachers. Manouchehri (1999) reported that only a third of the 120 middle and high school teachers she surveyed had training in the use of computers in content specific areas, and more than half of the participants did not feel that they had sufficient knowledge about how educational software may be used for teaching mathematics. Although the survey was conducted almost two decades ago, the
survey results highlighted the fact that the teachers needed to acquire specific pedagogical knowledge about the nature and purpose of exploratory activities to advance the use of computers in mathematics education. Knowledge about integrating technology in teaching is more complicated than knowing how to use the technological tools. To address this fact, the technological pedagogical content knowledge (TPACK) framework was proposed to highlight the types of knowledge important for teachers when using technology for teaching subject matter (Mishra & Koehler, 2006).

**Technological pedagogical content knowledge (TPACK).** Mishra and Koehler (2006) outlined the TPACK (originally formulated as TPCK) framework to identify knowledge domains essential for teaching with technology. TPACK has been widely used for describing teachers’ knowledge for designing, implementing, and evaluating curriculum and instruction with technology (Kimmons, 2015).

TPACK builds upon Shulman’s (1986) description of pedagogical content knowledge (PCK) to describe how teachers’ understanding of educational technologies and PCK interact with each other to inform their work with technology. The TPACK framework, first, identifies three distinct domains of knowledge that are related to teachers’ instructional activities with technology: content knowledge (i.e. subject-matter knowledge), pedagogical knowledge (i.e. knowledge of teaching); and technological knowledge (i.e. knowledge of how to use technological tools). Second, similar to how Shulman described pedagogical content knowledge (PCK), as a complex interaction between content knowledge and pedagogical knowledge, Mishra and colleagues (2006) delineate three *hybrid* knowledge domains: between pedagogical knowledge and content
knowledge, *pedagogical content knowledge* (PCK); between technological knowledge and pedagogical knowledge, *technological pedagogical knowledge* (TPK); and between technological knowledge and content knowledge and *technological content knowledge* (TCK). Finally, the authors described the integration of PCK, TPK, and TCP as Technological Pedagogical Content Knowledge (TPCK), which is the knowledge for fully integrating technology for teaching specific subject matter. The overall framework is depicted in Figure 6. The authors argue that these knowledge domains are not static; in other words, teachers are constantly engaged in the process of establishing, maintaining and re-establishing “a dynamic equilibrium among all components” (Koehler & Mishra, 2009, p. 25).
Research instruments have been developed and used by teacher educators, administrators, and policy makers to assess teachers’ proficiency with using technology based on the TPACK framework (Schmidt et al., 2009). However, some scholars have also criticized the model suggesting that the framework creates an illusion of a complex model (Kimmons, 2015). Kimmons (2015) contended that distinguishing between the different types of knowledge in the TPACK model is difficult, especially in practice. For instance, in a survey administered to 596 K-12 online teachers by Archambault and
Crippen (2009), high correlations between TCK and TPK, TPK and TPACK, and TCK and TPACK were reported. This result raised questions regarding the adequacy of the assessment tool, and further questioned whether TCK, TPK, and TPACK were distinguished in practice (Archambault & Barnett, 2010). Because of the fuzziness of the boundaries of knowledge presented in the model, Kimmons (2015) cautioned against using the framework to evaluate teachers’ performance. These criticisms do not intend to devalue the TPACK framework or any studies that utilize it. Instead, they encourage further conceptualizations of the TPACK structure.

**TPACK in mathematics education.** Niess et al. (2009) used the TPACK framework to model mathematics teachers’ development of TPACK. Rather than distinguishing each category of knowledge domain as depicted in the original framework, Niess et al. (2009) assumed the TPCK framework as the ultimate form of knowledge for teachers when integrating technology, and modeled progressions of mathematics teachers’ thinking and understanding toward the complexities integrated in TPACK (Figure 7). Niess and her colleagues’ documented changes in a mathematics teacher’s technology integration practices over four years. They proposed five stages of development pertaining to TPACK including: (1) recognizing (i.e. teachers can use the technology and recognize how they can align with mathematics content); (2) accepting (i.e. teachers have developed some attitude toward using or not using technology in mathematics classroom); (3) adapting (i.e. teachers engage in activities that lead to a choice to adopt or reject teaching mathematics with technology); (4) exploring (i.e. teachers actively design and implement teaching and learning mathematics with
appropriate technology; and (5) *advancing* (i.e. teachers reflect on the results of decision of integrating technology in mathematics classroom to inform future technology use).

Figure 7. Developmental progressions of TPCK (Niess et al., 2009).

Niess et al. (2009) identified four major themes specific to mathematics teachers’ practices and used the five developmental stages to propose a learning progression of teacher knowledge development in these four areas. The stages include: (1) curriculum and assessment, which examine how teachers integrate technology in curriculum materials, the treatment of the subject matter and assessing student understanding; (2) learning, which looks at how teachers utilize technological tools to advance student cognition; (3) teaching, which considers how teachers use technology to develop mathematical concepts, and the extent to which teachers recognize the importance of technology related professional development; and (4) access, which focuses on the extent
to which teachers allow students access to technology tools. The progression of learning themes is described in Table 1.

Table 1. Progression of Learning Themes

<table>
<thead>
<tr>
<th>Progression</th>
<th>Mathematics Learning Descriptor</th>
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<tbody>
<tr>
<td>Recognizing</td>
<td>View mathematics as being learned in specific ways and technology often gets in the way of learning</td>
</tr>
<tr>
<td>Accepting</td>
<td>Has concerns about students’ attention being diverted from the learning of appropriate mathematics to a focus on the technology in the activity</td>
</tr>
<tr>
<td>Adapting</td>
<td>Begins to explore, experiment and practice integrating technologies as mathematics learning tools</td>
</tr>
<tr>
<td>Exploring</td>
<td>Uses technologies as tools to facilitate the learning of specific topics in the mathematics curriculum</td>
</tr>
<tr>
<td>Advancing</td>
<td>Plans, implements, and reflects on teaching and learning with concern and personal conviction for student thinking and understanding of mathematics through integration of the appropriate technologies</td>
</tr>
</tbody>
</table>

**Pedagogical fidelity.** Knowledge of technology and TPACK is necessary but insufficient to describe why and how teachers use technology in their practices. The phrase *Pedagogy fidelity* was coined to describe teachers’ decisions and practices when utilizing digital technology in their teaching practices in mathematics education literature (Zbiek, Heid, Blume, & Dick, 2007). It refers to the extent to which mathematics teachers allow the use of technological tools to be in accordance with the nature of mathematics learning that the tools afford. By way of explanation, if the teacher associates high pedagogical fidelity to a tool, then he or she is more likely to use it to plan and design activities for students to experience the mathematics learning that the tool underlies. For example, Zbiek (1995) reported on a mathematics teacher, LeAnne, who learned to use the computer algebra system (CAS) to solve mathematical problems and to design
curriculum materials for use in the classroom. She had demonstrated a high level of ability in using CAS to produce rules, tables and graphs of different functions, and to explore properties of families of functions. Zbiek (1995) described that LeAnne could use “the tool fluently to achieve her goals, occasionally editing her previous work or ideas.” However, her own experience with CAS did not transfer to her classroom. At the beginning of the academic year when students met in the computer lab and worked through exploration, LeAnne found it challenging to work in a noisy situation where students asked many questions about not only mathematical tasks but also how to use the technology. She desired an orderly classroom; hence, by the end of the year, though explorations were still encouraged by LeAnne, she resorted to the use of demonstrations of exploration in class while her students took notes. In LeAnne’s case, the technology (i.e. the computer algebra system) supported her views on the value of mathematical investigations but contradicted her views on how students should behave in the classroom. The mismatch between her mathematical and pedagogical preferences suggested a low degree of pedagogical fidelity in LeAnne’s decisions about using CAS explorations (Zbiek et al., 2007).

The review of literature on teacher decision making about the use of technology in instruction sheds light on the importance of teachers’ knowledge of technology and the subject matter. Yet, studies also show discrepancies between teacher knowledge and practice, and pedagogical fidelity aimed to address this gap in teachers’ practices. The study of mathematics teacher educators’ practices of online teacher educating concerns MTEs’ use of technology for their online instruction as they are working in a fully
technologized environment. Research on knowledge for teaching in the online environment is still in its infancy. This study will draw on theoretical frameworks pertaining to decision making, online distance learning, and mathematics teacher educator knowledge to address MTEs’ work in and with the online environment.

**Theoretical Framework**

The study is informed by three theoretical models including Schoenfeld (2010) theory of teacher decision making, Boyd (1993) distance learning decision dimensions, and Perks and Prestage (2008) mathematics teacher educator knowledge tetrahedron. Each part of the framework supports the analysis of mathematics teacher educators’ (MTEs’) decision making for conducting online teacher education. These three models augmented each other to frame the analysis of the study. Boyd’s (1993) distance learning decision dimensions guided the understanding of the types of decisions that mathematics teacher educators might make when designing online professional development courses. Schoenfeld’s (2010) decision making theory provided a framework to analyze MTEs’ decision making from the lens of goal, orientation and knowledge. The analysis of the third component of Schoenfeld’s decision making, knowledge was supported by Perks and Prestage’s (2008) teacher educator knowledge tetrahedron. The following sections describe each model and the analytical structure they provided in this study.

**Schoenfeld’s decision making.** Schoenfeld (2010) and his research team examined decision making through the lens of human thinking. He modeled mathematics teachers’ in-the-moment decision making process through the lens of problem solving. He viewed teaching as a goal oriented activity, and likewise, he considered that teachers
encounter problems and solve problems constantly during their teaching. He framed his study of teacher decision making using the mathematical problem solving framework that he developed in the 1980s. The framework proposes four essential elements for explaining people’s decision making when problem solving. They are: (1) knowledge base (what they know), (2) heuristics (tools and techniques), (3) monitoring and self-regulation (reflective aspect), and (4) beliefs (individuals’ sense of mathematics, content, and others that shape what they do). Grounded in these four elements, Schoenfeld (2010) outlined an in-the-moment decision making process as follows:

An individual enters into a particular context with a specific body of resources, goals, and orientations. The individual takes in and orients to the situation. Certain pieces of information and knowledge become salient and are activated. Goals are established (or reinforced if they pre-existed). Decisions consistent with these goals are made, consciously or unconsciously, regarding what directions to pursue and what resources to use…… Implementation begins. Monitoring (whether effective or not) takes place on an ongoing basis. This process is iterative, down to individual utterances or action: Routines aimed at particular goals have subroutines, which have their own subgoals. If a subgoal is satisfied, the individual proceeds to another goal or subgoal. If a goal is achieved, other goals take priority via decision making. If the process is interrupted or things don’t seem to be going well, decision making kicks into action once again. This may or may not result in a change of goals and/or the pathways used to try to achieve them. (Schoenfeld, 2010, p. 18)

In this model, individuals’ goals, resources, and orientations are the key components of the mechanism, which are shaping or being shaped by or refined by constant monitoring and self-regulating during the entire process. Goals refers to objectives that an individual wishes to accomplish. Goals can be different in size according to the degree of importance each individual places on them. They could be short-term goals, medium-term goals, or long-term goals. Teaching is a goal-oriented activity, and furthermore, teachers always design and plan lessons with an agenda of
goals in mind. *Resources*, refers to the kinds of knowledge individuals possess. This construct relates to the assets of individuals’ instead of resources in general. Schoenfeld (2010) identified different types of knowledge, including facts or isolated knowledge, procedural knowledge (how to do things), conceptual knowledge (the intellectual rationale that explains why things work), and problem-solving strategies. *Orientations*, refers to the attitudes and beliefs of individuals about the objects that they interact with, also known as, beliefs, dispositions, values, preferences. Orientations are considered to have strong explanatory power with regard to people’s interpretation or reaction to situations.

Goals, resources and orientations are three key elements to be explored in this dissertation study to understand mathematics teacher educators’ (MTEs) decision making. For mathematics teacher educators’ knowledge, I will draw from Perks and Prestage’s (2008) teacher educator knowledge tetrahedron to support framing the analysis of MTEs’ knowledge structure.

**Teacher education knowledge tetrahedron.** Perks and Prestage’s (2008) teacher educator tetrahedron was briefly mentioned earlier in the previous chapter. The teacher educator tetrahedron is grounded in the notion that mathematics teacher learning is a reflective practice (Schön, 1983). The model was developed from their work with prospective mathematics teachers. They identified three aspects of knowledge that were central to how they navigated teacher learning. They were: “(1) practical wisdom – knowledge from being in the classroom; (2) professional traditions – knowledge from existing school curriculum and practices and research; and (3) learner knowledge – the
perspective teachers own knowledge” (Perks & Prestage, 2008, p. 269). Learner knowledge refers to teachers’ knowledge gained from being a mathematics student at school or in college. The three components inform each other and form a tetrahedron for mathematics teachers (Figure 8).

Figure 8. Teacher-knowledge tetrahedron (Perks & Prestage, 2008)

Perks and Prestage (2008) adapted this teacher-knowledge tetrahedron to propose aspects of knowledge important to teacher educators. The three vertices of the tetrahedron are redefined to capture the work of mathematics teacher educators. So the three vertices became: (1) professional traditions – knowledge of research about mathematics teaching, knowledge of existing ways of working in the field, and knowledge of national or local standards of teacher training; (2) practical wisdom – knowledge of designing and conducting mathematics teacher education sessions; and (3) learner-knowledge – knowledge gained from being a teacher, which constitutes all three
components in the teacher-knowledge tetrahedron. As such, the teacher knowledge tetrahedron is within the vertex of learner knowledge in the teacher-educator knowledge tetrahedron (Figure 8).

Figure 9. Teacher-educator knowledge tetrahedron

Among the three components of the teacher-educator knowledge tetrahedron, teacher knowledge for mathematics teaching (that is, mathematical knowledge for teaching) acts as learner knowledge for teacher educators. According to Hill, Ball, and Schilling (2008), teacher knowledge for mathematics teaching includes not only subject matter knowledge (i.e. common content knowledge, knowledge at the mathematical horizon and specialized content knowledge), it also includes pedagogical content knowledge (i.e. knowledge of students, knowledge of teaching, and knowledge of curriculum). Professional traditions develop from personal experiences, research and education about mathematics teacher education. Practical wisdom emerges from
considering what teachers need to know and how the sessions are structured so as to engage them in learning. Perks and Prestage (2008) suggest that these three components of teacher educators’ knowledge interact with each other to inform mathematics teacher educators’ decisions when they design and conduct teacher education sessions.

**Instructional system theory for cyberspace.** The third component of the framework draws from Boyd’s (1993) distance learning theory on decision dimensions, situating the study in the context of online education. Indeed, Boyd (1993) proposed a theory for the cyberspace. The theory is prescriptive in nature in that it aims to account for all stakeholders, after hearing all of the arguments and non-dominative discourses/voices agree upon. Boyd (1993) restructured Stafford Beer’s Viable System Theory to incorporate five vital functions for distance education institution. These are five discourse spaces that structure the virtual distance education (Boyd referred it as the cyberspace):

- **System V discourse spaces:** a constitutive discourse space to set the organization up; and a judiciary discourse space to deal with infringements of regulations.
- **System IV discourse spaces:** an anticipatory-intelligence discourse space to look outside and into the future.
- **System III discourse spaces:** a task allocation and monitoring system; a recruitment/marketing system to recruit students and staff, and market courseware; and a resource acquisition and waste disposal system.
- **System II discourse spaces:** the resource allocation, monitoring and balancing system.
- **System I discourse spaces:** instructional design and production systems; teaching broadcasting/publication distribution systems; learning-teaching conversation discourse-space systems; and learner support discourse sub-systems (Boyd, 1993, p. 246).

Boyd (1993) depicted how the system functions with all of the above discourse spaces in cyberspace education. All five systems account for the activities of the
participants in system, but the current study focuses exclusively on System I discourse spaces where instructional design and production is situated. Within System I, Boyd (1993) further outlined eight descriptive and prescriptive dimensions or variables that are essential in understanding this system. In Boyd’s description, these eight dimensions are four pairs of corresponding dimensions, which include: (1) agreed goals (i.e. the purposes of the course goal) and corresponding rules and mechanism that make sure goals are achieved; (2) subject matter (i.e. content) and corresponding metaphors and views that illustrate the content; (3) choices of real and virtual spaces and corresponding media used in the space; and (4) participants and corresponding sociostructure of the participants (Figure 10).
Figure 10. Eight necessary focal system dimensions

Since all eight dimensions were developed specifically for cyberspace distance education research or development project, they frame the design of this study to capture the essential elements constituted in mathematics teacher educators’ practices of designing and/or implementing content-specific online teacher education.

**Summary.** The theoretical framework proposed in this work is a hybrid model. Schoenfeld’s (2010) teacher decision making model which encompasses three elements
(goals, resources, and orientations) was used to frame MTEs’ decision making. The resource/knowledge construct under Schoenfeld’s model for MTEs was framed by the model pertaining to MTEs’ knowledge using Perks and Prestage’s (2008) knowledge tetrahedron. Boyd’s eight necessary focal system dimensions were utilized to identify MTEs’ online instructional design decisions. The hybrid model provides analytic power to understand mathematics teacher educators’ work with the online technology and in the online environment.
Chapter 3: Methodology

This study seeks to unpack mathematics teacher educators’ practices when conducting teacher education efforts in the online environment with a focus on their decision making processes. Yet, research in both online distance mathematics teacher education and mathematics teacher educating is only now emerging as a prominent area with interest residing in better understanding knowledge domains and epistemologies that inform practice. Indeed, the number of studies that aim to shed light on teacher educators’ field of practice are currently limited. Because the focus of this work was on gaining a better understanding the “how” of practice and decision making, a qualitative inquiry was most suitable, because it allows for development of insights into subjects’ thinking and work (Merriam, 1998). However, due to a lack of studies related to online mathematics teacher education practices, a large-scale study was needed to provide a status report of the range of practices of online mathematics teacher education to identify case study subjects. To this end, I employed mixed methods for this study. A self-report survey and case-based interviews provided multiple data sources to complement each other as well as to inform the development of another to provide insight into aspects of MTEs’ pedagogical actions and decisions with virtual technology (Greene, Caracelli, & Graham, 1989).
Mixed-Model Design

R. B. Johnson and Onwuegbuzie (2004) classified mixed methods into two designs: *mixed-method design* (that is, quantitative analysis methods are used for quantitative data and qualitative analysis methods are used for qualitative data independently) and *mixed-model design* (that is, mixing qualitative and quantitative approaches within or across different types of data). This study is a mixed-model design (see Figure 11) with an overall qualitative objective to answer a “how” research question (i.e. how mathematics teacher educators make decisions about the design and implementation of online teacher education). Both quantitative (from surveys) and qualitative data (from interviews) was collected following a sequential design where quantitative data was first collected and analyzed to inform qualitative data collection (Creswell & Plano Clark, 2011). Quantitative and qualitative methods of analysis were utilized in survey analysis to identify interview participants and design interview protocols. Then interview data was collected, transcribed and analyzed qualitatively.
Figure 11. Mixed-model design

**Procedure**

Adopting the mixed model design, the study involved the development, administration and analysis of a nationally distributed survey, and then the completion of in-depth case studies of selected participants identified through analysis of the survey results. The procedure started in January 2016 and analysis was completed in May 2017. The timeline of this study is described in Table 2.

**Table 2. Timeline of the procedure of the study**

<table>
<thead>
<tr>
<th>Timeline</th>
<th>Task</th>
<th>Summary of the task</th>
</tr>
</thead>
<tbody>
<tr>
<td>January – April, 2016</td>
<td>Survey development</td>
<td>The survey was designed. Five cognitive interviews were conducted to receive feedback on the content, clarity, and length of the survey. The survey was modified accordingly.</td>
</tr>
<tr>
<td>May 2016</td>
<td>Survey piloting</td>
<td>The survey was administered via Qualtrics online survey software to a small sample of mathematics teacher educators.</td>
</tr>
</tbody>
</table>
Table 2 continued

<table>
<thead>
<tr>
<th>Timeline</th>
<th>Task</th>
<th>Summary of the task</th>
</tr>
</thead>
<tbody>
<tr>
<td>June – August 2016</td>
<td>Pilot survey analysis</td>
<td>Findings from the pilot survey were analyzed. Some items were modified.</td>
</tr>
<tr>
<td>September 2016</td>
<td>Survey administration</td>
<td>The survey was administered via Qualtrics online survey software to the mathematics teacher educator community. The survey took approximately 20 minutes to complete.</td>
</tr>
<tr>
<td>October – November, 2016</td>
<td>Follow-up interviews</td>
<td>Three interview participants were identified from the survey analysis. One-on-one interviews were conducted via Skype. Each interview was about an hour long.</td>
</tr>
<tr>
<td>December—May, 2016</td>
<td>Data analysis</td>
<td>Interview and survey data analysis were analyzed. Mathematics teacher educators’ pedagogical practices for online mathematics teacher education were conceptualized.</td>
</tr>
</tbody>
</table>

**Survey Design**

The survey aimed to capture mathematics teacher educators’ (MTEs) practices relative to designing and conducting teacher education in the online learning environment. The development of the survey took approximately six months and followed a design experiment methodology. From January 2016 to April 2016, an initial draft of the survey was prepared to be piloted among a group of mathematics teacher educators. Five cognitive interviews with a volunteered sample were also conducted to receive feedback on the content, clarity, and length of the survey. The initial survey was piloted in June 2016 to a small sample of 26 mathematics teacher educators. Responses obtained on the initial survey were then utilized to modify and finalize the instrument. In particular, some of the open-response questions on the initial survey were modified to be presented in the choice format. The options offered on these multiple choice items were
constructed based on results from the pilot. The final survey was administered via Qualtrics in September 2016.

The survey consisted of four sections. The first section captured general information about the participants’ professional backgrounds including the types of institutions for which they worked, numbers of years of experience working with teachers, and years of experience conducting mathematics teacher education online. The questions were presented in multiple choice format (see Appendix A).

The second part of the survey asked participating MTEs to share information about an online mathematics teacher education course or professional development (PD) program that they had designed and taught. The participants were asked to consider only those courses that offered more than 50 percent of the learning activities online. The items in this section of the survey were informed by Boyd’s (1993) framework which encompasses eight necessary dimensions of design in distance learning, and included the following five dimensions: (1) the course objectives and foci, (2) characteristics of the course participants (e.g. number of participants, grade level), (3) online formats of the course or PD (e.g. synchronous, asynchronous, hybrid), (4) mathematical and pedagogical content activities, and (5) organization of the content activities.

The third section of the survey obtained information about how MTEs designed the course and what adjustments they felt were necessary to be made when organizing the course or PD content activities for online delivery. They were also asked to report the type of difficulties they encountered when they designed and implemented the online course or PD.
The last section of the survey asked MTEs to share their perspective on their experiences of conducting online mathematics teacher education sessions in comparison to a face-to-face delivery format. This last section provided evidence of how the online environment had impacted MTEs’ practices.

**Survey Participants**

The online survey was administered to mathematics teacher educators in the US. More than 800 email addresses were obtained from a major national organization whose members are mathematics teacher educators. Email soliciting consent to participate in the study were sent to all 800 candidates. Of that number, 114 mathematics teacher educators responded to the invitation and agreed to complete the survey. Among them, only 68 MTEs, who had conducted online courses with more than 50% of learning activities occurring online, were identified as qualified candidates for the study.

The majority of the qualified candidates were faculty members working in either research focused institutions or teaching focused institutions. Five participants identified themselves as graduate students, two as education consultants, and four as mathematics or educational specialists. 38% of the participants had more than fifteen years of experience working with mathematics teachers and only 16% of the participants had less than five years of experience. The majority of the participants were new to online teacher education with more than two thirds of the sample stating that it was their first time developing online mathematics teacher education courses or PD programs within the past five years.
Interview Participants

The three candidates for the case study portion of the study consisted of Dr. Abby, Dr. Beth and Dr. Colin, who were selected based on their survey responses as well as their willingness to participate in the case study phase of the research. All three participants had designed and implemented mathematics teacher professional development programs offering 100% of the learning activities online. In an effort to capture a variety of mathematics teacher educators’ practices of online teacher education, the criteria used to identify follow-up interview participants involved their experiences working with mathematics teachers, their experiences with online teacher education, their current positions, their online PD focus, and their design logics.

To begin, the three interview participants had different backgrounds and experiences. Dr. Abby and Dr. Colin were faculty members working at research institutions. Both had over 15 years of experience working with mathematics teachers. Dr. Colin had five years of experience conducting online mathematics teacher education while Dr. Abby only had two years at the time when they completed the survey. Dr. Beth identified herself as an outreach specialist for a research institution. She had about five years of experience working with mathematics teachers. During those years, she had five years of experience designing and conducting teacher education online. Hence, the interview participants consisted of one experienced MTE with more experiences of online teacher education (i.e. Dr. Colin), one experienced MTE with fewer experiences of online teacher education (i.e. Dr. Abby), and one less experienced MTE with rich experiences of online teacher education (i.e. Dr. Beth).
Since the survey responses indicated that a variety of online teacher education courses and programs were in existence, ranging from research courses, theory courses, history courses to content specific courses, an effort was made to identify MTEs whose online programs focused on supporting teachers’ learning of mathematics. Dr. Abby’s online course concerned teaching and learning statistics; Dr. Beth’s program was about teacher content knowledge development in numbers and algebra; and Dr. Colin’s program was about learning mathematics via discourse with a focus on dynamic geometry. All three participants focused on teachers’ mathematics learning and teaching practices but each addressed a different content strand.

The last criteria for interview participant selection was based on their claimed design logics as noted on the survey. In one of the survey items, participants were asked to identify one of the design sequences best describing how they designed the online environment and the PD. Three sequences were provided (see Figure 12, Figure 13, and Figure 14). 50% of the survey participants selected Sequence A as their design logic. Dr. Beth fell under this category. 43% of the sample selected Sequence C, so did Dr. Abby. The least selected sequence was Sequence B, which depicted analysis of the online technology prior to PD content design, and Dr. Colin identified this sequence as his own approach. The three design sequences were used as a key reference for analyzing the three interview cases.
Figure 12. Design logic sequence A

Figure 13. Design logic sequence B
Interview Procedures

The purpose of the interview was to provide an understanding of mathematics teacher educators’ pedagogical practices within the online environment. I conducted forty-minute long semi-structured interviews with the three selected MTEs. The interview protocol was designed to complement the survey in order to capture how each MTE went about designing online professional development programs for mathematics teachers (see Appendices B, C, and D). Schoenfeld’s (2010) decision making theory framed the design of the interview questions. Questions aimed to reveal the subjects’ goals, orientations, and knowledge bases that they utilized in their practices. The interview consisted of two parts. The first part solicited background information; particularly, the core principles and beliefs that guided the participants’ work. The second part solicited detailed information about their online professional development programs.
that they had identified in the survey. The interview questions in this part were based on each individual’s responses and the questions varied from one participant to the other. In general, I asked about their goals and motives, their choices of technology, their choices of the PD content, as well as their rationale for their respective designs. I also asked them to compare their practices within the online teacher education model with their practices within the face-to-face teacher education model. Each MTE provided specific information about their online programs, including resources they used or particular activities they had designed along with any published articles or reports relative to this work. The list of materials provided by each participant are described in Table 3.

Table 3. List of data collected from each MTE

<table>
<thead>
<tr>
<th>Participants</th>
<th>Data collected</th>
<th>Interview date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dr. Abby</td>
<td>Survey, interview, publications, Massive Open Online Course (MOOC) program website</td>
<td>October 2016</td>
</tr>
<tr>
<td>Dr. Beth</td>
<td>Survey, interview, publications, a sample mini-lecture video</td>
<td>April, 2016 and September 2016</td>
</tr>
<tr>
<td>Dr. Colin</td>
<td>Survey, interview, publications</td>
<td>November 2016</td>
</tr>
</tbody>
</table>

**Data Analysis**

Data analysis consisted of four stages. The first two stages concerned examination of the survey data analysis. In the first stage, some descriptive statistics and a rough summary of responses to the open-response items were generated. Based on the first round of analysis, I identified interview participants as described earlier in this chapter and produced interview protocol for each candidate. The second stage of data analysis involved summarizing the survey data to produce an exhaustive report of the MTEs’
concerns and considerations when they designed and implemented online teacher education sessions. The third stage of data analysis involved analyzing interview data and artifacts collected from the case subjects. The last stage consisted of cross-case analysis.

Survey data. The analysis of the survey was divided into four parts. In Part I, I used descriptive statistics to provide background information on the survey participants’ backgrounds, and their preferred logic maps when designing online programs.

Part II intended to capture the characteristics of online mathematics teacher education courses or professional development programs (PDs) that were reported by the survey participants. In order to do so, I first categorized the types of online mathematics teacher education courses according to MTEs’ descriptions. I then used descriptive statistics to report the characteristics of the online courses/PDs in each category. Such characteristics included: (1) mathematics content strands that were offered; (2) the targeted course/PD participants; (3) grade bands; (4) online delivery formats; (5) the foci of the course/PD; (6) MTEs’ choice of mathematics-related activities; (7) MTEs’ choice of pedagogical related activities; and (8) methods of participant online interaction.

In considering information for Part III, I first analyzed the multiple-choice options concerning how the MTEs designed the online teacher education sessions using descriptive statistics. I then used the grounded theory approach (Corbin & Strauss, 2007) to analyze the open-response items in an effort to categorize the kind of considerations and adjustments that the MTEs had made for the online delivery. All survey participants’ responses to each question were recorded. Clusters that emerged were labeled. I generated six categories of codes, which were adjustments about: (1) tasks
and activities design, (2) technological tools, (3) online platforms, (4) content illustration methods, (5) control system, and (6) groupings. Then, I coded terms and phrases using the six codes for all responses. Frequencies of the codes were then calculated.

Part IV provided text-based open-response data. Analysis of this data set followed the same procedure as utilized in Part III. This part captured the mathematics teacher educators’ perspectives about the use of the online environment to provide mathematics teacher education, “what are the advantages of the online mathematics teacher education (in comparison to the traditional face-to-face environment).” All MTE’s responses to this question were first read. Key terms and phrases about: (1) mathematics, (2) mathematics teaching and learning, (3) mathematics teachers’ learning, and (4) mathematics teachers’ learning in the virtual environment were coded. The texts that showed emphasis such as capitalized texts and exclamation marks were highlighted to capture items the MTEs considered to be important. After an iteration process of comparing and contrasting codes in all of the cases, three different roles of the online environment emerged which were labeled as "the online environment as a learning tool", "the online environment promoting diversity" and "the online environment as a teaching or research tool."

Finally, the analysis of survey data was summarized in a status report which will be presented in the next chapter. The summary was guided by the following questions:

- What types of courses or PD programs were provided for mathematics teachers online? What were the formats of the online courses or PD programs?
- What were the activities that were conducted online?
• How did MTEs organize these activities in comparison to the face-to-face environment?
• What difficulties did MTEs encounter when they designed and implemented online teacher education?
• What adjustments did they make to the PD content design to accommodate the online environment?
• How did MTEs view the efficacy of the online environment for providing mathematics teacher education?

A combination of qualitative and quantitative data results was organized according to the above questions.

**Interview and other collected data.** Interview and other data were collected from each case-study participant, and all of the feedback was transcribed. The data was first used to analyze each participant as an individual case. After each participant’s case was constructed, then a cross-case analysis was conducted. The outcome of the cross-case analysis was used to build a working model for MTEs’ decision making within the online mathematics teacher educating practices.

**Individual case analysis.** Each individual case analysis followed three phases. The first phase of analysis was intended to construct a case profile for each MTE with information about their backgrounds, orientations, and goals. Background information with respect to their experiences of working with mathematics teachers, experiences of conducting online mathematics teacher education, and experiences of research in mathematics teaching and learning was drawn from each subject’s survey and interview
responses. Information with respect to MTEs’ goals and orientations were drawn from the interviews, as a part of the interview questions specifically asked the subjects to articulate about their beliefs and orientations towards mathematics teacher education, and goals for their online PD.

I complemented their own accounts with information about their online design, including the topics that they chose for the teachers, the tasks and activities that they used, and the approaches of online organization for the activities. For example, Dr. Beth incorporated students’ mathematical representations in all of her online modules. Likewise, she placed a high priority on student thinking as a venue for teacher learning. She organized the examinations and discussions of children’s mathematics to support teachers to learn about structural connections between arithmetic and algebra. So in her case, I identified that her orientations toward mathematics and mathematics teaching were: mathematics as a set of structural concepts and listening to and making sense of children’s mathematics critical to effective teaching practice.

In the second phase of each case analysis, a design logic map was produced that highlighted what each MTE thought about the entire design of their online PDs. First, I wrote a summary of all of the activities that each case subject designed and how those activities were organized in their online PD programs. I then identified each MTEs’ key considerations and pedagogical practices with regard to the PD’s goals, topics, activities and the use of the online environment. Lastly, I used their selected design logics as the skeleton to build Detailed Design Logic maps for each interview subject.
In the third phase of interview analysis, I used the Detailed Design Logic maps to identify knowledge domains that the MTEs tended to exercise in their practices. Perks and Prestage’s (2008) mathematics teacher educator knowledge tetrahedron and mathematical knowledge for teaching by Hill et al. (2008) were used to describe the types of knowledge that each MTE drew from at each stage of their design practices as in the Detailed Design Logic maps. See Table 4 for the list of knowledge types and indicators of knowledge domains for each interview participant.

Table 4. Codes and indicators of knowledge domains for MTEs

<table>
<thead>
<tr>
<th>Categories of knowledge</th>
<th>Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Professional traditions</td>
<td>Knowledge of research on mathematics teaching, knowledge of existing ways of working in the field, and knowledge of national or local standards of teacher training</td>
</tr>
<tr>
<td>Practical wisdom</td>
<td>Knowledge of designing and conducting mathematics teacher education sessions</td>
</tr>
<tr>
<td>Learner knowledge</td>
<td>Knowledge gained from being a teacher</td>
</tr>
<tr>
<td>Common Content Knowledge (CCK)</td>
<td>Mathematics knowledge and skills that are not unique to teaching but are also found in many other professions or occupations that also use mathematics</td>
</tr>
<tr>
<td>Specialized Content Knowledge (SCK)</td>
<td>Mathematics knowledge for teaching, including representing mathematical ideas, providing mathematical explanations, examining and understanding unusual solution methods to problems</td>
</tr>
<tr>
<td>Knowledge at the mathematical horizon</td>
<td>Knowledge of how mathematical concepts and ideas are connected</td>
</tr>
<tr>
<td>Knowledge of content and students (KCS)</td>
<td>Knowledge of student mathematics; being able to predict and interpret students’ mathematical thinking; knowledge of conceptions and misconceptions</td>
</tr>
</tbody>
</table>

Continued
Table 4 continued

<table>
<thead>
<tr>
<th>Categories of knowledge</th>
<th>Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge of content and teaching (KCT)</td>
<td>Knowledge of choosing activities, sequencing content for instruction, etc.</td>
</tr>
<tr>
<td>Knowledge of curriculum</td>
<td>Knowledge of programs, textbooks, instructional materials available, and knowledge of how topics, subjects, are presented in these materials</td>
</tr>
</tbody>
</table>

For example, in Dr. Beth’s case, in the third stage of her online PD design as in her Detailed Design Logic map, she extensively used artifacts of school learners’ mathematical work and findings of research on student learning of topics in number and algebra. Knowledge of content and student (KCS) and specialized content knowledge (SCK) was identified as domains that Dr. Beth relied on for her practices in this stage. In this way, I identified knowledge domains that the MTEs exhibited in each stage of their design practices. The knowledge domains were summarized and described using the teacher educator knowledge tetrahedron by Perks and Prestages (2008).

Cross-case analysis. The last stage of data analysis was cross-case analysis, which was conducted to compare the decision making practices of the three subjects. To begin, I used Boyd’s (1993) model of decision dimensions to identify and summarize decisions that were common to all three subjects. I then compared their perspectives and orientations in relation to their decisions and identified knowledge domains that were critical to these decisions. The cross-case analysis allowed me to generalize the relationship among goals, orientations, knowledge bases and the MTEs’ decision making when designing online teacher education sessions and programs.
Validity

To enhance the internal validity of the study, data triangulation were applied (Merriam, 1998). First, both survey data and interview data were collected that complemented each other. The survey data provided an overview of MTEs’ practices while case studies provided an insight into how MTEs implemented their online programs, including each step they took in the process and details of their teacher learning activities that they designed. Second, each interview subject represented a distinct background and experience with mathematics teacher educating and online teacher educating. This variety provided a comprehensive understanding of MTEs’ practices. Samples of learning activities and published papers about the online programs were collected from each participant for data triangulation purposes. The process of data analysis was constantly examined and reviewed by another expert in mathematics teacher education to reach a consensus on the results and modeling MTEs’ decision making. Lastly, the technique of member checking was also used to enhance the validity of the analysis. Initial drafts of the case analysis were shared with each interview candidates. One candidate responded and provided feedback on the content of the analysis.
Chapter 4: Survey Report

This chapter provides a status report of MTEs’ online mathematics teacher educating practices in the US. The analysis of the survey was based on 64 mathematics teacher educators’ complete responses to the survey. Each MTE reported on an online teacher education course or professional development program that they had designed and implemented. In this report, I will first provide an overview of the type of courses and PD programs offered for mathematics teachers and ways that MTEs organized these courses or PD via an online delivery format. I then will compare MTEs’ practices in the online environment versus in the face-to-face environment, and report on the difficulties that they encountered along with adjustments that they made to overcome these difficulties.

An Overview

The reported online mathematics teacher education programs ranged from traditional college education courses to Massive Open Online Courses (MOOCs) with over 200 participants. A third of the courses were hybrid, meaning they involved some level of in-person sessions. About half of the online programs met a synchronous-only format and one half of the programs offered all online activities asynchronously; even so, all of these online courses or PD programs had more than 50% of learning activities occur online. About two thirds of the reported online mathematics teacher education
programs were professional development for practicing teachers, while 19% of the programs were university courses for prospective teachers. The remaining 19% of the courses were courses for both prospective and practicing teachers.

**Content foci.** One survey asked the participants to identify three main foci of their online mathematics teacher education programs. A frequency diagram (Figure 15) was produced to summarize the foci. The vertical axis represents the topics and the horizontal axis depicts the frequency of the counts of the participants who identified the topics. According to the figure, “mathematics and teaching methods”, “mathematics and learners’ mathematical thinking”, and “teachers’ mathematical knowledge” were the three top focusing themes. Other topics, such as curriculum, research and theory, mathematics teacher leadership, were also reported by the participants.

![Frequency Diagram of Content Foci](image)

**Figure 15. Foci of online mathematics teacher educational programs**
In terms of mathematical content areas, the online programs were not exclusive to a single mathematical strand. In fact, only six MTEs reported that their online program focused on a particular topic such as geometry, statistics, discrete mathematics, etc. These programs targeted middle school curriculum or higher levels of mathematics. Others reported that multiple content strands were involved in their programs. Some of these programs were designed for early childhood or elementary school teachers, and some focused on mathematical modeling, problem solving, or mathematical reasoning, where by the nature of their topics, multiple mathematical concepts could be discussed. One MTE expressed that mathematical content was the “backdrop” for their discussions of teaching methods, curriculum, etc. A few MTEs indicated mathematical content areas did not apply to their online programs where discussions of history, theory or research methods were the targeted content. In general, the survey results indicated that a variety of professional learning opportunities were offered online for mathematics teachers. Furthermore, mathematics teacher educators’ responsibilities did not only cover teachers’ development in mathematics teaching and learning, but also teachers’ development as professionals, including development of knowledge of theory and research and capabilities of working with other professionals.

**Online activities and organizing methods.** Table 5 ranks the popularity of the activities that mathematics teacher educators used in their online courses or PD programs. The ranking was derived from the number of counts where the participants indicated devoting a considerable amount of time to the activities. As shown in the table,
“discussing multiple representations”, “analyzing teaching practices”, “connecting different mathematical ideas”, “analyzing learners’ mathematical ideas” and “discussing specific mathematical thinking process” were the top five popular activities in the online teacher education sessions. The total counts exceeded 25 for these five categories and they were not particularly different. The results are consistent with the results of content foci where mathematics teaching methods, learners’ mathematics, and teachers’ mathematical knowledge were indicated to be the most common in the online teacher education programs.

Table 5. Ranks of popularity of course/PD activities

<table>
<thead>
<tr>
<th>Rank</th>
<th>Online course/PD activities</th>
<th>Counts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Discussing multiple representations and interpretations of central ideas</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>Analyzing teaching practices (e.g. questioning, feedback)</td>
<td>29</td>
</tr>
<tr>
<td>2</td>
<td>Connecting different mathematical ideas</td>
<td>29</td>
</tr>
<tr>
<td>4</td>
<td>Analyzing learners’ mathematical ideas</td>
<td>27</td>
</tr>
<tr>
<td>5</td>
<td>Discussing specific mathematical thinking processes (e.g. proving)</td>
<td>26</td>
</tr>
<tr>
<td>6</td>
<td>Designing tasks</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>Solving complex or unfamiliar problems</td>
<td>19</td>
</tr>
<tr>
<td>8</td>
<td>Assessing student learning</td>
<td>18</td>
</tr>
<tr>
<td>9</td>
<td>Discussing research in mathematics education</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td>Reviewing curriculum materials</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>Discussing skills and methods for computations</td>
<td>9</td>
</tr>
<tr>
<td>12</td>
<td>Investigating technology-based problems</td>
<td>5</td>
</tr>
<tr>
<td>13</td>
<td>Discussing social justice issues</td>
<td>4</td>
</tr>
</tbody>
</table>

**Online vs. face-to-face.** To see whether MTEs would select different activities for online delivery compared to face-to-face delivery, the survey participants were asked to indicate whether they conducted a similar PD or taught in a similar course prior to the
online delivery. 13 MTEs indicated they had conducted a similar course or PD prior to the online format. The survey asked these MTEs to rate the extent to which they used the list of Table 5 activities in the face-to-face environment. Seven of them rated differently in some of the activities for the frequencies of use in the face-to-face environment. Due to the fact that the number of participants was small, no general pattern was found with regard to what activities were more frequently or less frequently used in the face-to-face rather than in the online environment.

Yet, all participating MTEs, regardless of whether they had taught a similar course or PD in the face-to-face environment or not, expressed that there were certain courses that were much easier to implement in the face-to-face environment than in the online environment. The bar graph in Figure 16 summarizes the MTEs’ perspectives on the kind of course that they believed to be easier to deliver online. Certain courses had a much higher number of votes for face-to-face delivery than online delivery. These courses are mathematics content courses/PD, mathematics teaching methods courses/PD, interdisciplinary courses/PD, or technology integration course/PD. Mathematics content weighs heavily in the delivery in these courses or PD programs. Courses like mathematics educational theory, curriculum, or assessment courses had a weaker preference for face-to-face delivery over online delivery. This phenomenon seems to indicate intense mathematics content courses were more challenging to deliver in the online environment. This result could be the issue with the nature of having mathematics or mathematics related discussions in these courses or PD sessions. Before moving onto the difficulties, also note that despite certain courses or PD sessions that were preferred to
be delivered face-to-face than online, there were still a small number of MTEs who preferred online delivery. Ultimately, there was no absolute preferable method of delivery.

Figure 16. MTEs’ opinions on courses easier to deliver online versus face-to-face

In the next section, I will report on the difficulties that MTEs encountered as they designed and delivered their online courses or PD sessions.

**Difficulties & Adjustments**

The survey had a multiple-choice question that asked the participants to identify difficulties they encountered when implementing the online teacher education program.
Table 6 provides the percentage of selections. As the summary indicates, the first two difficulties identified were about task design and task selection for online delivery. More precisely, the difficulties were about designing tasks that could generate online interaction and tasks that were compatible with online technology. Some MTEs in the “others” selection expressed that modifying content that had been used in the face-to-face environment into a web-based presentation was a key challenge. These difficulties were both technological (as they were specifically related to the online environment) and pedagogical (as they were about task design).

Table 6. Difficulties when designing and implementing online courses

<table>
<thead>
<tr>
<th>Difficulties encountered</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Posing tasks or discussion prompts for desired online interaction</td>
<td>57%</td>
</tr>
<tr>
<td>Selecting tasks that are compatible with the online environment or other technology used</td>
<td>43%</td>
</tr>
<tr>
<td>Following and providing feedback to participants' posts</td>
<td>41%</td>
</tr>
<tr>
<td>Organizing online grouping</td>
<td>32%</td>
</tr>
<tr>
<td>Engaging participants from different backgrounds</td>
<td>25%</td>
</tr>
<tr>
<td>Selecting appropriate online tools and/or mathematics specific technology</td>
<td>25%</td>
</tr>
<tr>
<td>Getting participants to use online technology or other selected technology</td>
<td>23%</td>
</tr>
<tr>
<td>Others. Please specify:</td>
<td>14%</td>
</tr>
</tbody>
</table>

The third and fourth most frequently identified difficulties were about facilitating social interaction in the online environment, as they were following posts, providing feedback or organizing grouping. Again, these difficulties were also pedagogical and technological. Some of the challenges were not listed in the survey but expressed in the “others” selection, including knowing if participants were actively and consistently
engaged. Additionally, the time needed to plan online sessions was another issue. One MTE said he or she spent “at least 6 times as much work as face-to-face teaching.” Indeed, conducting mathematics teacher education in the online environment exerted new pedagogical challenges for MTEs. They had to adjust their goals and content activities to the online environment which was different from the face-to-face environment that they had used previously. In the next section, I will report on the kind of adjustments MTEs to make to deliver their content online.

**Adjustment made.** An open-response question asked the participants to identify and rate the importance of the revision they had made to the PD content and content organization. 57 participants responded to this question. Using the grounded theory approach (Corbin & Strauss, 2007), seven categories of important adjustments for delivering online content emerged. The total counts of these types were calculated and examples were listed as illustrated in Table 7. These six types of adjustments included: (1) tasks and activities design, (2) technological tools, (3) online platforms, (4) content illustration methods, (5) control system, and (6) groupings.
Table 7. Types of adjustments made by MTEs

<table>
<thead>
<tr>
<th>Adjustments</th>
<th>Count</th>
<th>Examples</th>
</tr>
</thead>
</table>
| Tasks and activities design  | 32    | “Most activities had a product for students to produce - a short reflection, a video, an outline, discussion board participation, a Wiki, etc.”
|                              |       | “How to engage participants in authoring representations (rather than just consume)” |
|                              |       | “I structured the content in the form of the performance-based assignments for the course that paralleled what they were learning about in each session.” |
|                              |       | “I included more readings than in a face to face course because students were not able have discussions of topics covered.” |
|                              |       | “Had to abandon all the face-to-face activities like game playing, modeling group instruction, and debates.” |
| Technological tools          | 14    | “I substituted technology tools for a lot of the geometry explorations which I usually do with pencil and paper.” |
|                              |       | “I used many more online resources, like applets and video, that students were expected to investigate ahead of time.” |
| Online platforms             | 12    | “I thought about the organization of the course and which environments to use in each year also - to support teachers' discussion, collaboration, and dissemination of what they learned and created.” |
| Content illustration methods | 8     | “Content was organized into discrete modules, each with a variety of short activities.” |
| Control system               | 9     | “I created rubrics, adjusted assignments, and tried to create more opportunities for students to meet with me via skype in response to students' feedback and to support students meeting the demands of assignments.” |
|                              |       | “My colleagues and I developed several protocols to follow (both synchronously and asynchronously) to help facilitate conversation - deep conversation about teaching - between participants.” |
| Grouping                     | 1     | “how I grouped students”                                                |
Seventeen of the fifty-seven MTEs mentioned that task design or activity design for online delivery was a key adjustment they had to make. The adjustments involved consideration of the type of tasks that were suitable and most effective in the online environment. Such considerations fall into two camps, in general. Some MTEs expressed that they spent more time designing tasks that were product oriented. As shown in the example column in Table 7, one MTE said that they used performance-based tasks where the teachers would produce a video, a short reflection, a Wiki, etc. parallel to what they learned in each session, and another MTE reported that course activities engaged the teachers in “authoring representation” rather just consuming. Some MTEs mentioned that they designed activities that would give teachers more learning autonomy with little interference from the facilitator (such as more reading, independent work). Four MTEs expressed that they had included more lecture style or presentation work in the online platform. A couple of MTEs said that they had to abandon certain activities involving physical modeling in the online environment or that they had to incorporate more focused activities. Another group of MTEs’ adjustments were about designing tasks and learning activities that were appropriate with various online platforms. These MTEs reported that they had reorganized many of the in-class activities suitable to an online environment including adding discussion forums, synchronous chat rooms, and independent work. One MTE claimed to design new and unique activities that were meant just for online delivery. In this camp, MTEs’ task design was not independent of the consideration about the online platforms and technological tools. That explains why the counts of adjustments to the technological tools and to online platforms were high. Yet, no identical approach
towards adjustments was used. Some adjusted the tasks and activities to fit in the environment or tools available; some did not change the tasks but re-organized with suitable online tools or platforms; and others adjusted both.

Nine MTEs mentioned their major adjustment was designing a control system that accounted for online learning, such as rubrics for feedback or protocols for online discussions. These control systems captured desired learning processes or outcomes.

Four MTEs’ important adjustments concerned how to structure the content to be presented to their participants. Three of the MTEs mentioned they divided the content into small discrete modules. The other MTE said he/she used videos as a presentation method. Despite these unique cases, these adjustments often occurred concurrently. For instance, task selection considerations were made together with considerations of the online platform or online tools available, or how to build in a control system for effective delivery. Tasks, technology, online environment and delivery were not considered separately, as evident in the MTEs’ responses.

The Online Environment for Mathematics Teacher Education

In the last section of the survey, participants views about the online learning environment were elicited in an open-response question. 67 teacher educators responded to the question. Analysis revealed three different roles of the online environment for providing mathematics teacher education, consisting of: (1) the online environment as a learning tool, (2) the online environment to promote diversity, and (3) the online environment as a teaching or research tool.
As a learning tool. The most popular view of the virtual learning environment reported by MTEs was the view of it as a tool to engage teacher learning. Some MTEs viewed that the online environment mediated teacher reflection; some viewed it as a platform to discuss technology integration; some saw that it supporting building teacher communities of learning. In the following, I will offer descriptions of the above mentioned functions of the online learning environment as a learning tool, illustrated with data from MTEs' responses.

The online environment mediates teacher reflection. The most popular perspective of the virtual environment was that it is a tool that mediates learning. This perspective provides an environment for teachers (i.e. participants) to think and reflect thoroughly. Such a view values teachers' learning through reflection. One teacher educator expressed:

It provides teachers time to think privately about mathematics before sharing their ideas with the group. I think this important given the culture around mathematics teachers that they are supposed to know the answers to everything, so when teachers are faced with difficult concepts, time to think on their own is crucial.

Another teacher educator reflected:

Online learning allows students to pause and reflect on the content when needed. While there are timing guidelines in online instruction (i.e. "Finish lesson 1 by Friday"), the students are still self paced in the course. If they need to stop and walk away from the lesson for some time, then they can do so and then come back where they left off. They can also complete the material when they are most alert and motivated. And, if they get confused with the material, they can stop, rewind the video, re-read a section, go back through an assignment, etc. before they continue with the course.

In first quote, the MTE viewed that teachers are always faced with a culture that demanded immediate responses, and teachers were always the ones who were required to
provide such quick responses, especially in mathematics. This MTE used “think privately about mathematics”, “think”, “mathematics” several times in the response. This MTE placed a great deal of value upon the time and opportunities for teachers to think about the mathematics. He/she valued the asynchronous feature of online learning which did not pressure teachers to provide responses quickly. In a similar regard, the MTE in the second quote, used the term “self-paced” to bring up the kind of desired learning that allowed teachers to think independently and monitor their own learning process, which might not be available in the face-to-face learning environment.

In order to promote learning, some MTEs specifically talked about approaches that they used in their online teacher education courses supporting teachers’ reflection, such as video viewing, as one MTE wrote, “because participants can view videos at their own pace and watch them multiple times if desired”.

*The online environment supports technology integration.* Another advantage of the online environment acting as a learning tool in teacher education was its ability to promote technological integration for mathematics teaching and learning. The survey responses evidenced that a number of MTEs viewed and utilized the online environment as a valuable space in which to invite teachers to explore a variety of mathematics teaching and learning related digital resources, such as mathematics simulations, mathematics software, and/or a range of online resources, such as mathematics learning videos; and to discuss affordances and limitations of each specific technological tool for the learning and teaching of mathematics. One MTE stated that, “we also used many applets that forced the teachers to engage with mathematics content”. This MTE focused
on mathematics content knowledge for teachers. The use of the term “forced” expresses how this individual experienced the effect of implementing mathematics related online applets on teachers’ mathematical learning. The term “forced” interpreted as “cannot be resisted by the subject” or “have to do” implies also that this MTE was satisfied with how the online environment enabled her/his teachers to use many applets yielding teachers’ learning of mathematics content.

Similarly, another teacher educator, presented his/her view of the potential benefit that the virtual learning environment that might provide for teachers’ learning, “I believe it (the virtual learning environment) could help students learn more deeply so long as there are appropriate applets/interactive lessons for students to explore.” The statement cautions about the kind of digital tools to integrate into the online learning as the respondent used “so long as” and “appropriate.” Careful analysis of affordances and limitations of the different types of mathematical digital learning tools were under this MTE’s consideration.

*The online environment establishes communities of learning for teachers.* Another perspective was that the online environment served as a learning tool in mathematics teacher education. This perspective was reflected in claims about its efficacy to promote a collaborative learning environment. MTEs who expressed such a view highlighted virtual collaboration as an affordance of online technology. Community of inquiry appeared as a major theme among the MTEs. One MTE expressed, “collaborate and learn from teachers with similar interests who may not live close by and thus with whom collaboration would be difficult without technology.” This MTE used the phrase “teachers with similar
interests” implying common inquiries into practice were present but the distance between teachers might have prohibited such an inquiry community to establish without the online space. In other words, this MTE viewed that the online learning environment mediated teachers’ learning by providing an inquiry community.

*The online environment facilitates other forms of teacher learning.* Another perspective was that the online environment serves as a learning tool in teacher education associated within its role in supporting teachers to “create” their own knowledge. One MTE posited that the environment allowed teachers not just to listen and receive information, but also to create knowledge. This MTE viewed that the online learning environment is an exploring tool for teachers. He or she expressed, “it's a different way of thinking. You must keep students interactive and involved. It isn't a lecture tool, but discovery tool. Let students do their own exploring…using the technology.” This MTE did not explicitly mention the kind of explorations that the online technology supported with regard to teacher learning, but the responses did reflect this teacher educator viewed that the online technology as a way for promoting teacher learning, which might take the form of allowing teachers to actively and creatively construct their knowledge or to explore mathematics with technology.

Most the mathematics teacher educators who responded to the survey expressed a view of the virtual learning environment as a learning tool but they viewed the affordance of online learning differently. Some saw that the environment promoted teachers’ learning because it allowed teachers to think and reflect freely, deeply, and independently before presenting their ideas publicly. Some saw that the virtual environment afforded
learning because integrating mathematics learning technology came naturally in such an environment. Others viewed that the virtual tools as a communication tools allowed for building communities of inquiry among mathematics teachers. Lastly, we have one educator whose view was that the environment allowed teachers to create and explore knowledge. Each MTE perceived the role of the online environment differently in terms of how it promoted teachers’ learning.

**As a tool for promoting diversity.** The second major role of the online environment that teacher education educators embraced was its unique function for connecting people without spatial and temporal constraints. I called this perspective “the virtual learning environment as a tool for promoting diversity.” Such a role was manifested in two major functions.

*Reaching the unreachable.* 39 MTEs mentioned the virtual delivery helped with reaching out to a diverse population of teachers, going beyond spatial and temporal constraints. One MTE said,

> It's potentially immensely helpful to rural teachers who may not have access to "traditional" delivery as well as to schools who may not have funds to pay for PD, but may be able to access technology through grants and federal funds.

This MTE used a couple of adverbs, “potentially” and “immensely” to express his/her strong hope of seeing professional development provided to rural teachers becoming possible via online courses. It seems that this teacher educator felt for the teachers who could not be reached in the traditionally form of teacher education.

A few teacher educators reported the online environment enabled teachers to collaborate or form networks of professional relationship. This is what one educator
mentioned, “collaborate and learn from teachers with similar interests who may not live close by and thus with whom collaboration would be difficult without technology.” Indeed, the use of online technology allows teachers from various locations to share and collaborate when it is structured purposefully. In this MTE’s point of view, online collaboration broke the barriers of distance or diverse background, which might be difficult to achieve in the face-to-face teacher education courses.

**Supporting multiple voices.** The second function of the online environment in promoting diversity was that it made it possible for multiple voices to participate, which allowed a forum for an open sharing of ideas. 15 MTEs reported the virtual technology encouraged teachers from diverse perspectives to share their voices. For example, one MTE expressed, “there is opportunity to hear ALL voices in an online learning environment, which is not always feasible in "traditional" course settings.” This MTE capitalized the word “all.” His/her capitalization highlights the opportunities and openness of the virtual learning environment in providing mathematics teachers a forum in which to voice their opinions. He/she even compared it to the “traditional” course setting to highlight the opportunity for openness. To elaborate on this element, another MTE mentioned that it was easy for one participant to dominate the conversation in the face-to-face environment, and due to time constraints in the traditional setting, it was not always possible for everyone to contribute their thoughts. Instead, the virtual learning environment had made that possible.

**As a teaching or research tool.** A couple of MTEs outlined the advantage of online teacher education from an instructor or a researcher perspective. One educator
reflected that the ability to “go back to what you ‘said’ before and what others have shared” informed instructors in deducing their subsequent moves. Like the previous view that the virtual environment allowed students time to think and reflect before sharing, this educator saw that the same benefit also applied to the teaching side that he/she might reflect upon teachers’ discussions or contributions before posing further instructions. He/she saw this particular functionality of the environment played a significant role in teaching online. In either response, the teacher educator viewed the virtual environment from his or her own perspectives – supporting their teaching or researching practices. Both realized the functionality of record keeping of the virtual technology in delivering instructions.

**Summary.** This section reported on MTEs’ views on the efficacy of the online environment for providing mathematics teacher education. Each view was drawn heavily from their own knowledge of the online environment and experiences of conducting online mathematics teacher education. The three different roles of the online environment as reported – as a learning tool, promoting diversity, and as a teaching/research tool, captured differences of MTEs’ pedagogical practices of providing online mathematics teacher education. It is important to note that most of MTEs did not necessarily attach to a single view about the online environment, but instead, held multiple views to inform their practices.

**Survey Report Summary**

This chapter provided a status report on the practices of online mathematics teacher education in this country. The report highlighted the fact that many MTEs’
experiences were involved in developing online programs for mathematics teachers. Topics, such as methods of mathematics teaching, student mathematics, multiple representation, etc., that are commonly discussed in the mathematics teacher educators’ community, had also been the focus of mathematics teacher education in the online environment. Despite the fact that there were common practices that MTEs shared in the community, the report shows that no MTEs shared identical practices, especially when they were practicing online teacher education. Rather, each MTE faced different difficulties and made various adjustments for the online environment. Indeed, MTEs’ goals, beliefs, and knowledge bases for their online teacher education programs vary. For instance, their beliefs and knowledge of the online environment were different, as the online environment was a relatively new platform of learning for many MTEs. The following case study will analyze what beliefs influenced the MTEs’ design thinking, what knowledge domains the MTEs would draw upon for their practices, and unpack how their goals, knowledge and beliefs had influenced their design decisions for online teacher educating.
Chapter 5: Case Studies

This chapter presents an analysis of three selected mathematics teacher educators’ (MTEs’) design decision making for their online professional development programs. I used Dr. Abby, Dr. Beth, and Dr. Colin as their pseudonyms for the analysis. My goal in the case analysis was to understand these three MTEs’ pedagogical practices and design thinking when providing online professional development (PD) for mathematics teachers.

For each case, I began with the MTE’s background and the context of their online PD programs. Then I described their goals for the online programs and their orientations toward different areas of education that might have influenced their design thinking. The case analysis then proceeds with descriptions of the PD programs, including key content and content activities that the MTEs designed. The descriptions contribute to modeling the MTEs’ Detailed Design Logic maps. Then, I analyzed the MTEs’ knowledge exercised in each stage of their design logics. Lastly, I synthesized each subject’s goals, orientations, knowledge and their practices to provide an overall perspective of their online PD design activities and the decision making processes involved.

The Case of Dr. Abby

Dr. Abby was a faculty member in a research-focused university. In the survey, she indicated that she had over 15 years of experience of working with mathematics teachers. The first time she was involved in developing and teaching an online mathematics teacher education course was two years ago. Dr. Abby indicated that she
was a secondary school mathematics teacher prior to her doctoral study. Her curiosity about student learning with educational technology such as graphing calculators drove her to pursue a doctoral degree in mathematics education. During her graduate study, she had an opportunity to teach elementary school students mathematics, particularly, probability using technology. This experience challenged her to think deeper about educational technology, which became the focus of her later research career. She had published books, chapters, and papers in probability and statistical reasoning with technology.

**Background of the PD program.** The online PD program that Dr. Abby reported was a massive open online course (MOOC) for teachers of statistics. It was an asynchronous, free, and open access online course and it was of interest to teachers in the United States and around the world, including middle school, high school, and introductory college teachers of statistics. Almost 800 participants registered in the PD program when it first started. Due to the nature of MOOCs, which are free and open, there were approximately 180 active participants through the final units in the course, as Dr. Abby reported. Lastly, Dr. Abby was the main person who designed the PD.

**Dr. Abby’s goals.** As an MTE, Dr. Abby always intended to design and create opportunities for teachers to achieve in their career as mathematics teachers. She carried such expectations for the teachers she worked with, so they could also become designers of learning, who actively orchestrate various resources to provide learning opportunities for their own students. In the interview, Dr. Abby expressed,

Everything that I do with my students who are often teachers is situating them in learning experiences so that they can achieve…. One of my overarching goals is
for them to actually, in the end, be able to see themselves as designers for their own students.

Indeed, in Dr. Abby’s work with mathematics teachers, including deepening their understanding of content and their understanding of different pedagogy strategies, she provided tools and resources for the teachers so they could use them critically to enhance the learning experience for their own students. Guided by such expectation, supporting teachers to enhance statistical learning experiences for their students was the overarching goal for the MOOC program.

Under this overarching goal, Dr. Abby had subgoals specifically for the PD. Her subgoals consisted of two aspects, the technology and the content. The technological aspect was to engage a diverse online audience afforded by the MOOC learning environment, as she described:

Because I was teaching an open access free course, I had to be very aware that teachers’ backgrounds would vary significantly and that teachers would come from different contexts, teach different grade levels, and even be from different countries with different standards. Thus, I had to purposely build a focus on teaching ideas and students work on statistics tasks that would be interesting and accessible to many teachers as well as be focused on statistics content that would enhance understanding of key ideas.

Dr. Abby considered the ability to reach a diverse audience as one of the affordances of the online learning environment. As described in the above quote, she had specifically considered the PD audience in setting the goals for the PD so that the content would be appealing to this particular audience.

Aside from the technological consideration, Dr. Abby’s subgoals were also about the content of the PD. Her content subgoals could be summarized as the following: (a)
advancing teachers’ knowledge of statistics and (b) understanding how children or students think statistically.

The following lists Dr. Abby’s goals for the design of the PD include overarching goals, subgoals for the PD, and online delivery goals. The online delivery goals were related to the MOOC design principles for Dr. Abby to organize learning activities. The overarching goal was to:

- G1: Support teachers in enhancing statistical learning experiences for their students.

The subgoals for the MOOC were to:

- G2: Engage teachers from various backgrounds across the world in the online learning of statistics content.
- G3: Change teachers’ views of statistics to see it as an investigative process rather than a computational subject.
- G4: Help teachers to understand how children learn statistics from a developmental perspective.

The online delivery goals were to:

- G5: Create an environment for teachers to learn from different voices.
- G6: Design materials to connect to teachers’ jobs.
- G7: Enhance statistical learning with technological resources.

Dr. Abby’s orientations. In the interview, Dr. Abby was specifically asked how this framework aligned with her orientations along five critical dimensions: (a) mathematics and statistics, (b) statistics learning, (c) statistics teaching, (d) mathematics
teacher education, and (e) the online teacher education. An account of each of these orientations is set out in the following.

*Orientations toward mathematics and statistics.* Dr. Abby viewed both mathematics and statistics as sense-making activities. She included habits of mind of productive statistical thinking as the anchor of her course and statistical problem-solving activities were a key component of the PD. Such practices reflect her preference for statistical thinking and the importance of sense-making in learning statistics.

Dr. Abby viewed statistics and mathematics as different subjects, despite the fact that they share many common tools in solving problems. She emphasized that statistics requires data, context, and probabilistic thinking while mathematics may not. She specifically pointed out the differences of measurement in statistics and mathematics, where measurement in statistics (e.g., intelligence) may not be as accurate as measurement concepts in mathematics (e.g., volume and area). She said that mathematical reasoning might help solve statistical problems, but that cannot represent the whole picture of statistical thinking.

*Orientations toward statistics learning.* Dr. Abby’s orientation toward statistics learning was consistent with her views about statistics. She stated that statistical learning could not be treated the same as traditional ways of learning mathematics—computing. She criticized current practices of teaching statistics by many mathematics teachers,

“they (teachers) put off teaching statistics until the end of the year as long as possible because they don’t want to do it. When they use data, they use data that doesn’t make any sense. It doesn’t have a context. They just give their students numbers, and they want them to do mathematics with it. They want them to compute statistics.”
Instead, she emphasized that statistics learning should always involve an investigational process and making sense of the context of the problem and the data.

Dr. Abby viewed statistics learning as a developmental process. In other words, students’ approaches to statistical reasoning could be described by levels of statistical sophistication. She emphasized that these levels of sophistication did not necessarily correspond to grade levels. Regardless of age or grade level, students could grow in their statistical sophistication toward productive statistical habits of mind as they learned more about conducting statistical investigations.

**Orientations toward statistics teaching.** Consistent with her orientations toward statistics learning, Dr. Abby gave a high priority to pedagogical strategies that focus on students’ reasoning in a statistical investigation. When describing the rationale for PD design, she indicated that attending to students’ learning of specific statistics content was one of the five design principles for the MOOC course. Three of the five units of the PD emphasized how students reason about data and make decisions about data.

Further, Dr. Abby viewed teaching statistics as a way to provide children/students learning opportunities about authentic statistics. She viewed that teachers of statistics were designers of learning. They gathered resources and tools, and were critical about what resources were suitable to use, and how to use them to maximize the learning opportunities for their students.

**Orientations toward mathematics teacher education.** Dr. Abby viewed teaching mathematics was to provide learning opportunities to students. She insisted providing multiple perspectives for teachers would support them in creating learning opportunities.
Orientations toward online teacher education. Dr. Abby embraced and encouraged the openness of the online learning environment. She allowed teachers from various backgrounds to contribute and build a learning community around the content of statistics, even though this was one of the challenges that their team faced when designing the free online course. In terms of online teaching, she did not view online teaching as inferior to face-to-face teaching though they should not be equivalent. She viewed the implementation of the pedagogical choices in an online course as dissimilar to that in a face-to-face environment.

Summary. Table 8 summarizes Dr. Abby’s orientations, which are used in describing Dr. Abby’s design decision-making process in a later section.

Table 8. Summary of Dr. Abby’s orientations

| Orientations toward mathematics and statistics | O1: Both mathematics and statistics are sense-making activities. They are both content and processes.  
O2: Statistical reasoning differs from mathematical reasoning. |
|---|---|
| Orientations toward statistical learning | O3: Statistical learning is investigational not computational.  
O4: Learning statistical reasoning is a developmental process.  
O5: Content-specific technologies play a major role in supporting statistical learning. |
| Orientations toward statistical teaching | O6: Teaching statistics should consider students’ reasoning in the statistical investigation.  
O7: Teachers of statistics are to provide meaningful statistics learning opportunities for their students. |
<table>
<thead>
<tr>
<th>Orientations toward mathematics teacher education</th>
<th>O8: Teachers’ perspectives and orientations toward the subject are fundamental to their teaching practices.</th>
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</table>
| Orientations toward online teacher education | O9: The online environment provides opportunities for teachers in different settings to access and participate in professional development and content-rich online resources.  
O10: Online teaching is not inferior to face-to-face teaching though they should not be considered equivalent. |

**What happened in the online PD.** Dr. Abby’s MOOC PD could be summarized as “one central concept, three approaches, and multiple forms of online learning”.

*One central idea.* Due to the MOOC environment where Dr. Abby faced a diverse population of teachers as her participants, she designed the program around big ideas in statistics so as to address the needs of teachers of every background. She chose the focusing concept to be data investigation in statistics, which included a cycle of four stages: question posing, data collection, data analysis, and result interpretation. The entire PD was designed around this concept, including introducing teachers to the statistical investigation cycle, solving problems using the statistical investigation cycle, discussing tasks for statistical investigations, and analyzing student reasoning based on the statistical investigation cycle. Dr. Abby anticipated this way of viewing, encountering, or learning statistics would be novel to many teachers who registered in the PD so it would provide the teachers with another perspective to think about statistics and statistics learning. She expressed:
I’m betting on…. they (the teachers) just give their students numbers and they want to do mathematics… They don’t really engage them in a statistical investigation. And they don’t really engage them in thinking about that how statistics is different than mathematics. It’s very different than mathematics. But because math teachers are responsible for teaching it, they teach it like math… So I wanted to put opportunities to perturb that in my participants in the MOOC for them to start realizing, “Oh, what I’ve been doing is not really statistics.

The choice of statistical investigation cycle was intended for the MOOC participants to encounter a different statistics learning experience to how they learned statistics in schools. The idea of “perturbing the teachers” was Dr. Abby’s approach to engage teachers’ learning. She anticipated her PD program acting as a springboard for the participants in their professional learning journey in a sense that the teachers would continue to seek learning more about statistical investigation and further professional development in the area after attending the PD.

*Three approaches.* Dr. Abby designed five units around the central idea of statistical investigation. The five units could be summarized as three approaches to understanding statistical investigations for teachers. First, Dr. Abby engaged the teachers in working with real-world data, with an intention of distinguishing mathematical problems and statistical problems. She chose tasks where data were messy and involved different contexts to consider for teachers to investigate in each unit. She wanted the teachers to experience statistical investigation in a way that was not always taught in school. In the interview, she specifically mentioned a task in unit 1 where she assigned the participants to work with a Gapminder task. She said:

I chose to highlight Gapminder because it’s a technological tool that allows you to look at five variables at the same time and that’s way more than most teachers ever let their students experience and I wanted to let them … could see the power of using this real data.
Most tasks that Dr. Abby chose for the teachers to experience were like the Gapminder task, which were distinct from the usual curriculum tasks. The tasks required the teachers to reason about the context, to make associations among the variables, and to use technological tools to analyze the data. Dr. Abby expected the teachers to experience habits of mind of statistics via solving these problems, rather than just compute statistics the way most teachers experienced. She used the term “habits of mind” from mathematics education to describe individuals’ heuristics when they approach an investigation problem. However, these habits of mind were different from those of those in mathematics education. She and her colleagues described statistical habits of mind according to each stage of the investigation cycle. For instance, when posing questions, one habit of mind was to “ask contextually-based questions that call for the use of data to answer”; when analyzing data, one habit of mind was to “consider context of your question to identify measurement issues such as missing data, outliers.”

The second approach to experience the investigation cycle was understanding students’ statistical reasoning in data investigation. She invited the participants to consider sophistication in levels of thinking about the four stages of the investigation cycle. She and her colleague recalibrated the GAISE framework to include the four-stage investigation cycle with an intention of making the GAISE user-friendly for teachers. Meanwhile, the teachers were experiencing the four-stage statistical investigation cycle from the perspective of student reasoning.

The third approach to understanding the investigation cycle in Dr. Abby’s design was via analyzing statistics tasks. She and her colleague wrote a summary of criteria for
statistical investigation tasks. These criteria were written in the form of questions and organized around the four stages of the investigation cycle. For instance, for the question posing stage, the teachers were asked to examine the task by considering whether it allowed students to pose statistical questions based on their interests; for the data collection stage, they were invited to examine the task to see if it allowed students to plan to collect data—from sampling to choosing measurements. By examining the tasks, the teachers encountered the statistical investigation cycle as they were expected to use the cycle to analyze their curriculum tasks when they plan and teach statistics lessons.

In short, the content of the entire PD was organized around the four-stage statistical investigation cycle (i.e., question posing, data collection, data analysis, and interpretation). In Dr. Abby’s design, the teachers could encounter the investigation cycle from three different experiences: (a) solving problems with real data, (b) learning about how students reason about data, and (c) analyzing statistical tasks. Each of these experiences was closely relevant to the teachers’ pedagogical content knowledge development in statistics and statistics teaching and learning. Indeed, the choice of focusing on one big idea, with three different approaches to the big idea, allowed us to see how Dr. Abby organized the subject matter of statistics to engage the teachers in developing their content knowledge and pedagogical content knowledge in statistics. In the following, I will describe how Dr. Abby organized these learning experiences in the MOOC environment.

*Multiple forms of online learning.* Dr. Abby’s MOOC design was characterized by multiple types of materials and resources represented by multimedia. In the PD, the
teachers had access to a critical research summary of current statistics education, and an arrangement of classroom-ready materials (e.g., lesson plans, statistics investigation tasks, assessment items). Some of these resources were open online resources carefully selected by Dr. Abby and her team. Some of these resources were drawn from research in statistics education or materials originally developed by Dr. Abby. These resources were utilized for the teachers to encounter the big idea—the statistical investigation cycle, and they were materials or resources designed for the teachers to experience statistics investigation, to understand statistical reasoning, or to analyze authentic investigation tasks.

Dr. Abby also utilized multimedia or online communication tools for the participants to engage in the materials in the PD, including videos of classroom instruction, animations of children’s reasoning, statistics technological tools, videos of statisticians, and statistics educators’ discussion panels and discussion forums. For instance, for the teachers to experience the investigation cycle with real-world data, they could solve tasks with integrated statistical technology, watch a video of how the task was implemented in a classroom, and discuss the task with colleagues in the discussion forum. Dr. Abby’s intention was to promote a *self-directed learning* environment in the PD, where the participants could interact with multiple resources and people, not the instructor alone. Unlike a traditional face-to-face learning environment, where interactions are largely between the instructor and the participants, sometimes between the participants, the PD was designed such that the nodes of interaction were between the participants and multiple resources. This approach was unusual among other MTEs, as
will be revealed in later case studies, and such a design approach was partly required by
the MOOC design team and guided by Dr. Abby’s views on the role of a teacher educator
whose role was to provide knowledge, resources, and opportunities for teachers to design
their own teaching practices.

**The Detailed Design Logic map.** To connect Dr. Abby’s pedagogical decisions
over the statistics concept focus, the learning activities for teachers to experience the
chosen concept, and the organization of learning activities in the online environment, I
incorporated Dr. Abby’s design logic—sequence C as indicated in the survey—into
consideration. The sequence C design logic in the survey (Figure 17) describes a design
progression where the decisions of the content and activities in the online environment
designed for mathematics teachers’ professional learning were the results of an integrated
consideration of both *content objectives* (i.e., either mathematics or mathematics
teaching) and *analysis of online technological tools*. In other words, neither the content
objectives nor the features of the online tools have a determining role in the MTEs’
decisions over what content will be delivered online. This was the one of most popular
choices of design logic sequence among the three given in the survey.

In Dr. Abby’s case, the broad *content* consideration for the PD was teaching
statistics, and the *online technology analysis* involved two things: (a) the MOOC learning
environment and (b) the MOOC design principles endorsed by her institution. This was
because the PD program was funded by the institution, so she had to follow the
institution’s design principles. Though these design principles were content-related (e.g.,
job-related learning, and community building), I considered them as part of Dr. Abby’s
technological analysis because they were about the MOOC environment, a technological-learning environment. She had to account for these technological-environment-related considerations in her choices of the PD content and the organization of the content, though she had endorsed these principles.

Figure 17. Design logic sequence C

Hence, I used the sequence C design logic as the foundation, then added specific practices and considerations taken by Dr. Abby in the design process to model a detailed design logic map for Dr. Abby (Figure 18).
Figure 18. Dr. Abby’s Detailed Design Logic map

How does it work? The detailed design logic map is an organization of Dr. Abby’s pedagogical practices for the design and development of the PD as identified
earlier in this section. The map consists of rectangles, labeled C1 to C5, representing the considerations that Dr. Abby had taken in developing the PD, and ovals, labeled P1 to P10, representing the pedagogical practices that Dr. Abby endorsed or enacted in the PD. The pedagogical practices were evident in Dr. Abby’s description of the PD.

Instead of having two parallel flows of logic as in sequence C, I modeled Dr. Abby’s detailed design logic in a single progression. This is because her consideration of the content of the professional development program and her analysis of the technological environment (i.e., the MOOC environment) were fully integrated. The PD content and the analysis of the MOOC environment influence each other in every stage of the design. To make clear how the logic works, I used four different stages to describe Dr. Abby’s design logic and each stage involved both the content and the technological environment considerations. The stages are: stage 1—deciding PD objectives, stage 2—determining content (“the big idea”), stage 3—deciding topics (approaches to “the big idea”), and stage 4—designing for online delivery. I will outline each stage in the following.

Stage 1: Deciding objectives. Dr. Abby was given the task of developing a MOOC for teacher professional development in statistics teaching by a funded institution. As a PD program developer, she made decisions about the objectives of the PD, subject to some institutional and technological constraints. For objective decisions, she considered elements of effective professional development, elements of effective online professional development and also the MOOC design principles endorsed by the institution that funded the program. She described:
So first, I really had to think a lot about effective professional development. Whether it is face-to-face or online. So what works for teachers and for an effective professional development. And then also in an online environment, how should you be designing things for that type of medium.

These two considerations are labeled C1 and C2 in Figure 18. C1 was PD content-related and C2 was technological-related. Juggling between C1 and C2, Dr. Abby decided the objective for the MOOC professional development was to perturb teachers’ current views of statistics, learning statistics, and teaching statistics, which was identified as the first pedagogical practice that Dr. Abby used (P1). She expressed:

I don’t want to try to solve a particular problem. I want to perturb the teachers enough so that they would be able to have the confidence to seek further professional development, to further develop their ideas around teaching statistics.

Dr. Abby intended to disturb the teachers’ current beliefs and orientations about statistics in the hope of influencing their teaching practices. Yet, she also needed to account for the diversity of the population of teachers that the MOOC environment allowed, which is C2. To address the MOOC environment consideration, she limited the PD audience to those who practiced teaching statistics in middle school, high school and introductory college levels, where she was able to draw on knowledge from research and her own practical experiences to identify the audience’s needs and beliefs about statistics and statistics learning. By setting the PD audience range, she was confident that perturbing the audience’s views and beliefs of statistics and statistics teaching and learning was an effective professional development objective for the MOOC.

Stage 2: Determining content. The second stage of the design logic for Dr. Abby was to determine an overall content for the PD that serves the PD objective (P1), which runs from C3 to P2 in the detailed design logic map. C3 is a consideration leading to the
pedagogical practice P2. As labeled in the detailed design logic map, C3 is about “what statistical content would be effective for changing the teachers’ views”. For this consideration, Dr. Abby thought not only about the statistical content but also the MOOC design principles that the funded institutions had imposed on the course. Both the course content and the MOOC conditions were important for this consideration, and the MOOC conditions were the technological constraints imposed on her content decision making.

Dr. Abby mentioned that she was required to develop a course that would closely connect to the teachers’ jobs. So when deciding the statistics content for the MOOC, she specifically had to think over the concepts in statistics that were closely connected to the teachers’ daily teaching, which included teaching statistics in middle schools, high schools and colleges around the world. Because of the impossibility of meeting every audience’s teaching needs, Dr. Abby drew from her knowledge of effective statistics teaching to identify the “real” content need for teaching statistics. She said:

I try to center that (statistics content) around the typical kind of statistical concepts that would be typical in this context (middle school to intro college statistics courses), and to think about that anybody’s job, the context that they work in might be different, but the concepts of their teaching should be the same, should be very similar. And so, to me, I had not to think about job-related being—they’re middle school teachers, or they are high school teachers or they are college teachers, but that they teach this type of statistical idea. So, the content really became to the fuller wherever I had to think about the connected to their job.

The narrative above evidences that Dr. Abby considered the MOOC audience’s job needs and identified the commonality of their content needs. She decided she would focus on a common idea—a “big idea” that every statistics teaching job might encounter.
Indeed, for this decision, she took into consideration both the subject and the environment. She also said:

I consider anybody who is trying to help others to learn statistical content that their job is to create experiences for their learners in whatever context that they work in. And so I wanted them to able to experience…. getting opportunities for students to kind of learn statistics ideas in ways that I was anticipating would most likely be different than what the majority of people coming into my course have done before.

Dr. Abby also felt that what she identified, the “big idea”—the four-stage statistical investigation cycle—would be an unfamiliar way of approaching statistics to many in the audience, which would be effective to perturb their thinking about statistics (P1). Therefore, we see that in stage 2, from C3 to P2, the MTE made decisions about the PD content and these decisions were based on the MTE’s careful considerations of both the objective (P1) and the analysis of the MOOC design principles (i.e., job-related principles).

**Stage 3: Deciding topics.** The third stage of the logic map is decisions regarding specific approaches to deliver the “big idea”. These approaches were specific activities to engage the learners in interacting with the core content. As described earlier in this section, Dr. Abby had designed three approaches for the MOOC participants to experience the statistical investigation cycle. These were through: (a) developing habits of mind when solving statistics investigation tasks, (b) a developmental perspective of statistical learning, and (c) analyzing and designing statistics investigation tasks. These approaches were labeled as P3, P4 and P5 respectively in the detailed design logic map, as they were three pedagogical practices that Dr. Abby used, and they were decisions specifically about how statistics should be delivered and experienced by the MOOC.
participants. Moreover, these three pedagogical practices (P3 to P5) are parallel to each other in the detailed design logic map since they were all about the big idea and no single one was more superior to another in Dr. Abby’s design.

Stage 4: Designing for online delivery. The last stage of Dr. Abby’s MOOC design logic was about structuring and organizing the content activities as identified in stage 3 in the MOOC environment. Ultimately, stage 4 decisions determined how the MOOC participants would experience the professional development online. At this stage, technological consideration was critical, and this was C5. Dr. Abby needed to follow the technological features of the MOOC, such as the online asynchronous nature, the design principles endorsed by the funded institution, and the laws and regulations on conducting an open online course. For example, Dr. Abby needed to make sure the design of the participants’ online experiences was community and cooperative focused, as endorsed by the funded institution. Also, she needed to follow the copyright regulations for readings. She mentioned that she could not upload publications that were not open to the public. So, she summarized the literature and uploaded the summary for the participants. The same issue also applied to the use of videos. She needed to make animations of student reasoning or teaching practices instead of using the actual videos from her previous research work. Taking into consideration all of the constraints imposed by the MOOC environment on the content activities that she wanted the teachers to experience (i.e., P3, P4, and P5), Dr. Abby designed various activities and online learning experiences for the participants, which are outlined as pedagogical practices P6 to P10: read and discuss research summary; watch videos and animations of teaching practices and student
reasoning; use online technological tools to explore statistics; watch expert panel
discussions; and discuss issues and practices with online colleagues. At this stage, the
MTE’s pedagogical practices were closely technologically content-related.

Summary. In the detailed design logic map, I modeled Dr. Abby’s design and
pedagogical practices according to design logic sequence C—the integrated design logic.
The logic was integrated because, at each stage of the design, considerations and
practices were both about the content of the PD and the MOOC environment. At some
stages, the considerations or decisions were more about the PD content, such as stage 3,
while at other stages, such as stage 4, the considerations or decisions were more about the
MOOC environment. Yet, each stage, either the content considerations or the MOOC
environment considerations, were influenced by the previous stage’s practices. They all
related to each other in an integrated model. In Figure 19 and Figure 20, the stages that
were about the PD content are highlighted in pink and the stages that were more about the
considerations of the MOOC environment are marked in blue. Yet, they were all
interdependent with each other—that is why some stages were highlighted both in pink
and blue.
Figure 19. Dr. Abby’s Detailed Design Logic map with content-related considerations and practices highlighted
Figure 20. Dr. Abby’s Detailed Design Logic map with MOOC environment-related considerations and practices highlighted

**Characteristics of Dr. Abby’s pedagogical practices.** Based on the analysis of Dr. Abby’s design logic, we see in particular the immensely strong commitment to her...
chosen field—teaching statistics—based on her personal motivation developed from her teaching and research career. She intended to capture a broad PD audience range, which was unprecedentedly large in the field of mathematics teacher education, using the advances of online communication technology. She expressed that she went outside of her comfort zone to develop this professional development course. Beyond her personal characteristics, there are some useful observations to be made about the methods and approaches in Dr. Abby’s practices that relate to her design thinking.

First, Dr. Abby’s personal perspective of statistics teacher education played a dominant role in the entire MOOC design, particularly, her views about the needs of statistics teachers, and her knowledge of how best teachers learn (in other words, knowledge of effective professional development for statistics teachers). This reflects a teacher and teaching-oriented characteristic of her PD design thinking; her orientations toward teaching and teacher education set up the entire tone of the PD, even though she needed to follow certain MOOC design principles from the funded institution. She was able to adopt and reinterpret the principles according to her perspectives in the design process. One of the design principles that Dr. Abby needed to follow was to connect to the MOOC participants’ job. The problem she faced was a diverse population of participants who might not necessarily share the same kind of teaching job. She worked from her own knowledge of what statistics teaching should look like from her perspective of statistics education. She interpreted the teachers’ needs from her understanding of common statistics teaching practices and research in statistics education. She grounded the PD objective in this interpretation and understanding. She also added to the objective
based on her knowledge of effective teacher learning. She said, “I want to perturb the teachers enough so that they would be able to have the confidence to seek further professional developing around teaching statistics.” The choice “to perturb the teachers” over the choice of “I don’t want to solve a particular problem” as a way of providing teacher professional development was not random but theoretically grounded, reflecting a certain perspective of teacher education in the MTE’s design thinking.

The second characteristic of Dr. Abby’s design thinking was the strong influence of her own philosophy of teaching over the MOOC content design and organization. As a mathematics teacher, Dr. Abby saw her role as a designer who utilized tools and gathered resources to provide learning opportunities for students. As an MTE, Dr. Abby expected her students—who were always teachers—to see themselves also as designers in providing learning opportunities for their students. This was the core philosophy of her work with teachers and students. Dr. Abby modeled this philosophy in her MOOC course. She not only provided the teachers with different approaches to encounter the big idea but also provided them with various online resources and methods for teaching and learning statistics. These resources included short research articles, teaching videos, a range of statistical tasks with real-world data, and technological use. In such a way, she, as the teacher educator, by carefully selecting and revising resources, provided learning opportunities for her students. In return, she expected the teachers to experience, be familiar with, and be critical toward a range of resources to provide and maximize learning opportunities for their own students. This approach to the content of the PD was not common among other MTEs as reported in the survey; yet, it was highly consistent
with Dr. Abby’s beliefs about her own role in providing professional development for teachers.

The third characteristic of Dr. Abby’s design thinking is content-technology integrated design thinking, as indicated in her detailed design logic map. From deciding the PD objectives to determining the methods of online delivery, Dr. Abby was bounded by the MOOC environment as well as the design principles for MOOC professional development by the funded institution. Instead of viewing the design principles required by the institution, Dr. Abby had endorsed these principles as a part of her own perspective of the MOOC course design. Though she struggled to provide some features in the PD that could fully meet the principles, such as the use of a discussion forum, she worked with these MOOC design principles when deciding the PD content objectives and choices of tasks. This characteristic is reflected in the different colors in Figure 19 and Figure 20.

In summary, based on the analysis of the logic of Dr. Abby’s design and pedagogical practices in the MOOC course, three characteristics of her design thinking were identified. First, her objectives were framed by her knowledge of teacher education—what teachers of statistics need to change in their practices of teaching the subject and how teachers learn in professional development. Second, her choice of tasks, as well as the organization of the PD activities in the MOOC environment, was very consistent with her perspectives on learning and the role of a teacher or teacher educator in providing mathematics education or mathematics teacher education. Finally, the fully integrated PD content considerations and technological considerations reflect
instrumental genesis in Dr. Abby’s practices. In the analysis above, Dr. Abby’s knowledge of teacher education had become salient in influencing her practice, in particular, framing the objectives of the PD. What are other knowledge domains that the MTE might have drawn from in developing the MOOC course? In the following section, I will go deeper into the knowledge structure of Dr. Abby’s practices to understand better the entire design decision-making process.

**Knowledge involved in Dr. Abby’s design thinking.** In Dr. Abby’s Detailed Design Logic map, I identified 10 pedagogical practices (P1-P10) exhibited in designing the online course. Each of these practices was an important decision after careful consideration of the entire design of the PD. These practices were driven by her personal perspectives of mathematics education and mathematics teacher education teaching and learning, as well as her knowledge as a mathematics teacher educator. Per Schoenfeld’s decision-making model, goal, orientation, and knowledge were three key elements in professionals’ decision making. From this perspective, various types of knowledge were exercised with Dr. Abby’s pedagogical practices from P1 to P10. In this section, I will identify the type of knowledge and knowledge domain at each stage of her design activity.

In the first and second stages of her design logic, when goals and objectives were considered, Dr. Abby relied on her knowledge of teachers and knowledge of effective mathematics teacher professional development to frame the overall professional development goals. She drew knowledge from mathematics teacher education about what constituted effective professional development, including research on mathematics
teaching and existing ways of working in the face-to-face environment and the online environment. This represented *professional traditions* as in Perks and Prestage’s (2008) teacher educator knowledge tetrahedron. Such knowledge was informed by her work with mathematics teachers as a teacher educator over the years and her own experiences as a mathematics teacher for both middle school and high school.

As indicated in Figure 20, in the first two stages of Dr. Abby’s design logic, her decisions about the PD objectives were in conjunction with considerations of the online environment. She considered the potential audience of the MOOC environment and what design principles she needed to follow. Such considerations were technological, because it was about what the MOOC environment could afford. So *knowledge of the online environment* was drawn upon to inform her decisions. Such considerations were about content, because she had to consider a statistical concept or concepts that address teachers’ needs for participants of various teaching and learning backgrounds. *Horizon knowledge* or *knowledge at the mathematical horizon* was drawn upon for her decisions. She said:

I wanted my job to distil it down into some core ideas … so that they would walk away with an expanded toolkit of how to think about statistics teaching more holistically rather than, “Oh, I have to teach this objective on this day and I’m just gonna teach it using some dry numbers and doing some analysis of it.

Having a holistic viewpoint of statistics and statistical learning was the core of Dr. Abby’s purpose for the chosen content. This knowledge came from the MTE’s own knowledge of statistics as well as research in statistics reasoning, which belongs to *learner knowledge* in the teacher educator knowledge tetrahedron.
In stage 3—deciding content approaches to the “big idea”, Dr. Abby chose to use three practices (P3, P4, and P5) to engage teachers in the selected content—the statistical investigation cycle: (a) problem-solving with real data, (b) understanding statistics reasoning levels, and (c) analyzing and designing investigation tasks. These three practices were about teaching and learning statistics and highly informed by research in statistics education. Dr. Abby mentioned that she did an extensive review of recent research in how students learn statistics to inform the tasks she chose and to produce a teacher-friendly version of the student reasoning framework for the MOOC participants. This is knowledge of research in statistics education that Dr. Abby drew from, which belongs to the professional traditions in statistics education in the learner knowledge vertex of the teacher educator knowledge tetrahedron. Some of the tasks were from Dr. Abby’s own research on statistics reasoning in the past.

In the final stage of design logic, Dr. Abby organized the selected learning activities in the MOOC environment. As shown in Figure 18, these activities included watching videos or animations of teaching practices, and exploring statistics using online statistical software. Dr. Abby was very purposeful in how these activities were organized online. She mentioned certain tasks should be placed in the first unit to capture online participants’ attention and interests and certain tasks were placed in a later unit where the emphasis was on student reasoning followed by problem-solving. How she sequenced online modules and tasks reflects her careful consideration of the content of the learning activities as well as the MOOC online learning environment for each learning activity. The knowledge domains she relied on were the knowledge of the online environment,
knowledge of mathematics teachers (i.e., *professional traditions*), and *specialized content knowledge* for teaching and learning statistics. One of her pedagogical practices was exploring statistics using technological tools (P8). This practice was informed by the knowledge of teaching and learning statistics with technological tools. Using Koehler and Mishra’s (2006) knowledge model, such knowledge is *technological pedagogical content knowledge* (TPCK or TPACK), where technology refers to specific technologies for advancing the learning of statistical content. Her TPACK came from her research experiences. Dr. Abby had been a researcher who specialized in developing and understanding technology in statistics to support students’ statistical reasoning. In her PD task design, she extensively engaged teachers to explore different statistical technological tools for learning and teaching statistics investigation.

Following is a summary of the knowledge domains in Dr. Abby’s online teacher education practices:

- **K1**: Professional traditions in mathematics teacher education.
- **K2**: Knowledge of the MOOC environment including MOOC design principles from the funded institution.
- **K3**: Learner knowledge — knowledge of students and statistics.
- **K4**: Learner knowledge – knowledge at the mathematical horizon.
- **K5**: Learner knowledge – specialized content knowledge.
- **K6**: Learner knowledge – TPACK for statistics teaching and learning.
- **K7**: Practical wisdom of working with mathematics teachers,
Using the teacher educator knowledge tetrahedron (Figure 21) helps to identify the domains of knowledge from which Dr. Abby drew. These various kinds of knowledge were gained from either knowledge of research or personal experiences of teaching and learning statistics or working with mathematics teachers. Knowledge from research, either her own research or research knowledge from the community, had heavy weightings in all knowledge domains listed above. Even her practical experiences of working with mathematics teachers were a part of her research. The distinction between professional knowledge and practical knowledge became less clear and indistinguishable.

Figure 21. Teacher educator knowledge tetrahedron for Dr. Abby

**Case synthesis.** In Table 9, I combined four stages of Dr. Abby’s pedagogical practices (Figure 18) with her goals, orientations, and resources identified for the design
and delivery of the PD. The table intends to describe the types of decisions that the MTE made and how goals, orientations, and resources influenced this decision making.

Table 9. Dr. Abby’s goals, orientations, knowledge, and decision making

<table>
<thead>
<tr>
<th>Stages</th>
<th>Decisions</th>
<th>Guiding goals</th>
<th>Guiding Orientations</th>
<th>Guiding Resources/Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 1: Deciding objectives</td>
<td>What for? Overarching goal (G1) and subgoals (G2, G3, G4)</td>
<td></td>
<td>• Orientations toward mathematics teacher education (O8)</td>
<td>• Professional traditions in mathematics teacher education (K1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Orientations toward statistical teaching (O7)</td>
<td>• Knowledge of online environment (K2)</td>
</tr>
<tr>
<td></td>
<td>Who? Participants</td>
<td></td>
<td>• Orientations toward online teacher education (O10, O11)</td>
<td></td>
</tr>
<tr>
<td>Stage 2: Determining the big idea</td>
<td>What? Choice of subject matter</td>
<td>Overarching goal (G1) and subgoals (G2, G3, G4)</td>
<td>• Orientations toward statistics and statistics learning (O1, O2, O3)</td>
<td>• Learner knowledge – knowledge at the mathematical horizon (K4)</td>
</tr>
</tbody>
</table>

Continued
Table 9 continued

<table>
<thead>
<tr>
<th>Stages</th>
<th>Decisions</th>
<th>Guiding goals</th>
<th>Guiding Orientations</th>
<th>Guiding Resources/Knowledge</th>
</tr>
</thead>
</table>
| Stage 3: Deciding approaches to the big idea | How to illustrate? Choices of approaches to the subject matter | Overarching goal (G1) and subgoals (G2, G3, G4) | • Orientations toward statistics and statistics learning (O1, O2, O3, O4)  
• Orientations toward statistics teaching (O6) | • Learner knowledge — knowledge of students and statistics (K4)  
• Learner knowledge — specialized content knowledge (K5)  
• Learner knowledge — TPACK (K6) |
| Stage 4: Designing online delivery | Through which? Choice of media | Online delivery goals (G5, G7) | • Orientations toward statistics learning (O5) | • Knowledge of online environment (K2)  
• Knowledge of online environment (K2) and knowledge of mathematics teachers (K1) |
| | With whom? (e.g., social grouping) | Online delivery goals (G5, G7) | • Orientations toward mathematics teacher education (O9)  
• Orientations toward online teacher education (O10) | |

*What decisions were made?* The second column of Table 9 indicates the dimensions of decisions that Dr. Abby made in the four stages of the MOOC design. These decisions were based on Boyd’s (1993) eight dimensions for distance education design. According to the Table 8 analysis, Dr. Abby’s decisions throughout the PD
design and implementation were about six of the eight dimensions: (a) decisions for goal setting, (b) decisions for participants, (c) decisions for subject matter, (d) decisions for illustration of the subject matter (i.e., approaches to the subject matter), (e) decisions for choice of media, and (f) decisions for online interaction (i.e., social grouping). These dimensions are circled in red in Figure 22. Dr. Abby’s MOOC project started with an invitation to design a course on MOOC for teachers by the funded institution. In her situation, where to conduct the professional development as in Boyd’s dimensions was a fixed factor for Dr. Abby, as was the “how to control—rules and mechanisms, etc.” dimension. This was because Dr. Abby was required to follow certain rules and principles in designing the PD program, instead of setting up the design principles and rules herself. Other than these two factors, she made decisions about the target audience, objectives, content, content illustration, delivery, and social interaction methods within the MOOC environment. Each of the decisions was distributed in the four stages of design logic I modeled.

In stage 1, which is the stage of deciding objectives, decisions about the audience and goals (both overarching goals G1 and subgoals G2, G3, and G4) were made. In stage 2, which was about determining what statistics content to focus on, decisions over the choice of subject matter were made. Stage 3 was about deciding approaches to the big idea. Decisions over how the subject matter was to be approached by the audience were made. Stage 4 was the stage for organizing the topics and activities in stage 3 in the MOOC environment, where decisions over how these activities were presented in the online media as well as decisions over how to organize online interaction among
participants were made. According to the detailed design logic map, all of these decisions were made in a logical progression in Dr. Abby’s design thinking, which was influenced by integrated considerations of both the content (i.e., statistics) and the technological factors.

Figure 22. Dimensions of Dr. Abby’s decisions circled

**Goals, orientations, knowledge, and decisions.** Depending on the types of decisions that Dr. Abby made at each stage, the goals, orientations, and knowledge that
influenced her decisions were different. In the first stage, as the MTE determined the objectives of the course, she set up for herself the overarching goal (G1) and subgoals (G2, G3, and G4) for the MOOC course. Goals here were not factors for decision making as in Schoenfeld’s model, but they were decisions to be made for the MTE, because unlike students’ mathematical problem-solving situations or in-the-moment classroom decisions for teachers, where individuals usually enter the situation with some sort of goals in mind, Dr. Abby did not make in-the-moment decisions in the entire design activity. Instead, she needed to frame a problem (i.e., an objective), given resources and constraints, and resolve the problem through her design. Her epistemological perspective of statistics guided her design, including what she thought about the needs of teachers for setting up PD goals, as shown in the Table 9 analysis. However, her design was constrained/afforded by the type of environment where she provided the PD. Her knowledge and orientations toward what the online learning environment could afford influenced her decisions, particularly, whom to offer the course for and subsequently what would be effective learning for the target audience.

In stages 2 and 3, where the decisions were related to the subject—teaching and learning statistics—the guiding orientations and knowledge for Dr. Abby’s decisions were dominated by her research knowledge and perspectives in statistics teaching and learning. She was able to identify her MOOC audience’s common statistics teaching practices, and was confident about how to change their perspectives on statistics teaching via the subject of statistics during her decision making. Her knowledge of the subject of statistics and teaching statistics played a critical role.
In the last stage, decisions on the organization of the content activities in the MOOC environment (i.e., choice of media and choice of social grouping for interaction and learning) were made. At this stage, Dr. Abby’s decisions, first, were guided by the online delivery principles of the funded institution for teachers’ online learning experiences. These principles were the online delivery goals for Dr. Abby at this stage of design. Since this was the first time that she designed an online course, her decisions on content activities were also experiential, or trial and error based on professional traditions in MOOC teacher education. Further, in this stage of decisions, she drew extensively on knowledge from her own research in statistics and technology to organize the online statistical problem-solving tasks. Learning statistics with technology was one of the core activities in the MOOC PD.

Throughout the entire MOOC design and delivery activity, Dr. Abby made several key decisions in areas including the objectives, content, content activities, and online delivery methods. For each decision, certain knowledge and orientations were more salient than others. Schoenfeld (2010) also discussed whether decisions were under certainty or not. For decisions under uncertainty, one may go back and reevaluate the decisions after implementation. For Dr. Abby, her decisions on the objectives, PD content, and choice of learning activities were certain. She showed confidence in her choice of the content and content activities for the PD, as she had rather stable orientations and knowledge of statistics teaching and learning after many years of research experience in the field, but she was uncertain about her decisions of certain online features for teachers’ learning such as discussion boards. She expressed in both the
survey and the interview that if anything needed to be revised or changed for future
MOOC, it would be the discussion forum (i.e., social interaction) and some assessment
methods for online participants. Pedagogical knowledge of the online environment was
rather new to her and her decision making.

The Case of Dr. Beth

Dr. Beth was a professional development facilitator affiliated with a research-based
university. She taught high school mathematics for two years prior to starting her
graduate program. She obtained her doctoral degree in the field of geophysics. She was
then invited by a mathematics education professor to implement a state mandated
professional development program for all teachers who possessed a kindergarten through
grade 8, special education, or secondary mathematics certification, and all those who
might potentially teach mathematics, including administrators. The professional
development (PD) program was focused on early number concepts, rational numbers,
operations and algebra. It offered three sections (Grades K-3, Grades 4-8, and Grades 6-
12) and Dr. Beth was the leading facilitator for the Grades 6-12 section. The PD program
was originally administered in a traditional face-to-face format. A year after Dr. Beth
joined the program, the PD program transitioned to an online delivery format and Dr.
Beth was responsible for developing the online model for the Grades 6-12 section. She
served in that capacity for four years. At the time of data collection, Dr. Beth had three
years of experience administering an online teacher education program.

I met with Dr. Beth at a conference where she presented a session on a hybrid
mathematics teacher professional development program that she and her colleagues had
designed and implemented. I asked if she would be willing to participate in my study and when she agreed, I scheduled an hour long interview with her via Adobe Connect. The analysis presented in this chapter is based on the survey responses she provided, the one-hour interview, her conference presentation notes, a paper that she and her colleagues published about the professional development program, and a sample Voice Thread mini lecture produced by her. All interview data was transcribed, coded, and analyzed according to the decision making framework.

**Goals.** As I have noted in the decision making framework, MTEs often have different levels of goals (e.g. overarching, medium-term, and session objectives) or different types of goals (e.g. content goals, social goals, delivery goals), simultaneously. Overarching goals represent intentions that MTEs have for the overall professional development program. Medium-term content goals may include the “big ideas” in a particular unit or across multiple units. Whereas, more refined content goals may derive from the learning objectives in a specific session.

When designing and delivering the online Grades 6-12 PD sessions, Dr. Beth was expected to meet certain learning goals that had already been established for the previous professional development program. These were goals that had been implemented over the course of many years in the traditional in-person environment. She mentioned in the interview, “…because it is a state-mandated course, so we need to have really solid consistency across. So you can’t really have somebody take an online version and do completely different problems than the people who are doing the in-person version…” Consequently, Dr. Beth incorporated overarching goals into her online PD; for example,
her goals included advancing mathematics teachers’ mathematical content knowledge in early number concepts, rational numbers, operations and algebra, and changing teachers’ beliefs about mathematics and self-efficacy in teaching mathematics. Based off the overarching PD goals, Dr. Beth had more specific online implementation goals, which specified the purposes for conducting PD sessions in the online environment (that is, for teachers to interact with the content) and important content elements to be included in the online sessions. Dr. Beth’s local goals were learning objectives for specific online sessions/modules. They included expectations for the ways in which the participants would interact with the content and the problems during the online sessions.

The course goals, major content and learning goals are separated from her online PD module design goals that are listed in Table 10.

Table 10. Dr. Beth’s goals

<table>
<thead>
<tr>
<th>Overarching PD goals</th>
<th>G1: Advance mathematics teachers’ mathematical content knowledge in early number concepts, rational numbers, operations and algebra.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G2: Change teachers’ beliefs about mathematics and self-efficacy in teaching mathematics.</td>
</tr>
<tr>
<td>Online PD modules</td>
<td>G3: Choose appropriate online delivery methods and organize learning activities using these delivery methods so participants can fully interact with the materials.</td>
</tr>
<tr>
<td>design and</td>
<td>G4: Organize online learning activities that include the following elements: (1) taking students’ ideas seriously, (2) engaging students conceptually, (3) encouraging multiple strategies and models, (4) focusing on the structure of mathematics, and (5) addressing misconceptions.</td>
</tr>
<tr>
<td>implementation goals</td>
<td>Continued</td>
</tr>
</tbody>
</table>

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**Orientations.** During the interview, I focused my questions on Dr. Beth’s orientation towards mathematics teacher education and her perspectives on the utility of online mathematics teacher education. In presenting her views, I will offer an account of Dr. Beth’s orientations along four critical dimensions: (1) mathematics, (2) mathematics learning, (3) mathematics teaching, and (4) mathematics teacher education. Each of these orientations, either implicitly or explicitly, permeates Dr. Beth’s design and delivery of the online PD modules.

*Orientation towards mathematics.* Dr. Beth viewed the subject of mathematics as a set of structural concepts that embody connected and interrelated ideas. She mentioned during the interview that she usually pressed teachers to think about common strategies or methods for teaching different mathematical concepts; for example, rational numbers, variables, etc. She also viewed mathematics as an outcome of the human thinking process; in other words, a human’s capability to develop and progress in their understanding of concepts. She cited realistic mathematics education to solidify her view of mathematics.
as a progression of cognition from informal (contextualize mathematics) to formal (abstract and decontextualized).

**Orientation towards mathematics learning.** Dr. Beth’s orientation towards mathematics learning was consistent with her view of mathematics. She expressed that mathematics learning should include developing a deep and connected understanding of mathematical concepts. Such learning should start with informal and contextually relevant ideas and methods, and then move towards more sophisticated and abstract forms of those concepts.

Dr. Beth viewed the process of learning mathematics as a sense-making activity. To her, understanding mathematics meant more than memorizing a set of unrelated algorithms and facts; rather, she viewed the process of learning mathematics as a route to understanding mathematics through experience and inquiry. She devoted a couple of modules to focus on fact fluency where she challenged teachers to consider the conceptual meanings behind each fact and standard algorithm. However, she also contended that one should press onto more efficient methods/procedures of performing mathematics. This technique was visible in the sample mini lecture where she asked participants to compare the efficiency of different student methods.

Dr. Beth believed that all students were capable of learning mathematics. She said during the interview, “Because for me, that’s a big equity issue that we assume all students are able to do mathematics and that we can use their thumbs of knowledge.” This perspective of learning is consistent with socio-cultural learning theories where students always arrive at school with prior and informal mathematical knowledge and the
teachers’ role is to incorporate their prior knowledge into instruction and press for formal and higher levels of mathematics understanding to be achieved.

*Orientation towards mathematics teaching.* Dr. Beth endorsed student centered teaching. In the online mini lecture, she shared different students’ representations and asked teachers to reflect further on their own students’ methods and strategies. As a key part of the PD framework, she cited Cognitively Guided Instruction where the focus is primarily on student understanding and making sense of students’ beliefs about mathematics.

*Orientation towards mathematics teacher education.* Dr. Beth placed a high priority on student thinking as a venue for teachers’ learning. She viewed making sense of children’s mathematics as a critical element to effective practice. In her online PD design, examples of children’s mathematical work were used as a foundation for teachers to learn about multiple representations of students’ ideas. She organized the examination of children’s mathematics in a way that enabled teachers to see mathematical structure, and hence, to advance teachers’ knowledge of arithmetic and algebra. During the interview, Dr. Beth mentioned that students’ mathematics had become the core of her work with teachers, “that’s something that goes into all of my teacher education that we always look at how do students do. All of those kinds ideas go into my work.”

Dr. Beth also believed that teacher learning is best supported through participation in a learning community. Features of a learning community, according to her, include: (1) producing and sharing mathematical ideas and thinking, (2) listening to others’ mathematical ideas, and (3) examining from a critical standpoint each other’s
mathematical ideas. Developing an interactive community was one of the key elements that Dr. Beth attempted to establish in the online learning environment. She purposefully selected Voice Thread for teachers to use as a tool to narrate and share their ideas with their cohorts for discussion. The functionality of such online community required that the participants articulate their thoughts, both orally and in writing. The overall purpose of this community was for teachers to encounter multiple representations of mathematical ideas. Therefore, interaction between the content and the teachers was at the heart of this community.

Lastly, Dr. Beth valued the process of reflection. This is revealed in her online mini lecture design, where she insisted that teachers pause and contemplate the content that she presented in the video. She also designed the assignment for the purposes of teacher reflection upon the content presented by the instructor (herself), the mathematics produced by the other participants, and the mathematics produced by the participants’ students. She was deliberate in engaging teachers in reflecting upon content they already knew and all additional information they received during the sessions.

*Orientation toward online learning.* Dr. Beth mentioned during the interview that she previously disliked online learning due to her prior experiences with the environment. Yet, her experiences with conducting the online PD and feedback she received from participants changed her view on how the online environment functioned in providing learning opportunities for mathematics teachers. In other words, she viewed the online environment as a technology tool to serve learning purposes. She expressed, “there are advantages here because they [participants] can post their work and really think about it,
and then they can come to somebody else’s work and go ‘oh wait.’ They will say like, ‘I watched that two or three times before I understood what you were doing. But now I get it and now I understand algebra much better’…”

Summary. Dr. Beth’s orientations towards mathematics, mathematics teaching and learning, and mathematics teacher education were reflected in the professional development program that her team developed. Since her own training as a teacher educator was grounded in the same PD model, the theoretical perspectives guiding the PD program were largely influenced Dr. Beth’s work with teachers. Although the online sections that she developed were required to be theoretically consistent with the in-person PD program, they reflected Dr. Beth’s own perspectives about mathematics, and mathematics teaching and learning, and mathematics teacher education. Table 11 provides a summarized list of her orientations.

Table 11. Dr. Beth’s orientations

<table>
<thead>
<tr>
<th>Orientation towards mathematics</th>
<th>O1: Mathematics is a set of structural concepts with connected and interrelated ideas.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>O2: Mathematics is an outcome of the human developmental process of thinking.</td>
</tr>
<tr>
<td>Orientation towards mathematics learning</td>
<td>O3: Mathematics learning includes developing a deep and connected understanding of the structure of mathematical concepts.</td>
</tr>
<tr>
<td></td>
<td>O4: Mathematics learning includes pressing onto more efficient methods/procedures of doing mathematics.</td>
</tr>
<tr>
<td></td>
<td>O5: All students are able to learn mathematics.</td>
</tr>
</tbody>
</table>

Continued
Table 11 continued

| Orientation toward mathematics teaching | O6: Mathematics teaching should be student-centered. The key role of mathematics teachers is to make sense of students’ mathematical abilities and steer students towards higher levels of mathematics learning. |
| Orientation towards mathematics teacher education | O7: Student mathematics and student thinking are the key methods to develop teachers’ content knowledge. |
| Orientation towards the online learning environment | O8: It is a communication platform for online participants including PD facilitators and PD participants. |

**What occurred in the online PD.** During the interview, Dr. Beth described to me the content and the structure of the modules that she developed. Each module focused on different mathematical concepts, and was uniquely organized for the online participants. Dr. Beth used several types of problems in these online modules.

One problem type used was context-based tasks. An example of such a task is: “Maren has four bags of dirt and each garden takes \( \frac{3}{4} \) of the bag. So how many gardens can she make and how much would have left over?” For context-based problems, Dr. Beth expected teachers to encounter students who employed multiple methods to solve these problems. She presented them with six different pieces of work and asked them to make connections between any three of the representations.

The second type of problem that Dr. Beth used in her sessions was computational tasks. Examples of such problems were 389-269, and 4x7. She used these tasks for developing teachers’ fact fluency. Dr. Beth first asked teachers to solve the problems without using a standard algorithm. Then, she asked teachers to watch a mini lecture that
highlighted the different approaches children used to solve the tasks. Next, she assigned teachers to use ideas from the mini lecture to tackle similar computation problems using multiple approaches. Once the tasks were completed, the teachers would upload a video of their approaches for others to review and discuss. Dr. Beth shared that the decisions she made when designing the computation problem modules were: (1) how to support teachers’ conceptual understanding of computation problems, and (2) how to ensure teachers would encounter multiple strategies in the online sessions. The first decision aligned with Dr. Beth’s content goal for the course. She chose to use multiple representations as the approach to support teachers’ conceptual understanding, which was the practice adopted from the previous PD program. The second decision was about how to operationalize such a goal in the online learning environment. She showed the teachers selected student representations of these problems and tasked the teachers to use multiple representations to solve the computation problems.

Note that for both types of problems – context-based problems and computation problems, they were problems that appeared frequently in the curriculum materials familiar to the teachers. Dr. Beth’s approach to deepening the teachers’ conceptual understanding of mathematical concepts was by employing multiple representations. She was deliberate in leading the teachers to distinguish among certain types of representations. In her mini lectures, different representations such as number lines, ratio tables, and area models were repeatedly used and presented to the teachers; additionally, teachers were assigned to reflect upon these representations. The extension to both
context-based and computation problems was an effort to delve into the mathematical structure of multiple structures, as she expressed:

We’re really asking them to think about what’s the deeper mathematics behind that problem and how might students solve it and how do we press them towards more formal and efficient strategies.

Lastly, a small component of the PD involved pedagogical discussion. In summary, the outline of Dr. Beth’s online PD content reveals the intentionality of her practices. These include: (1) using problems that were familiar to the teachers, (2) focusing on multiple representations, (3) connecting informal strategies and formal algorithms, (4) selecting purposeful student models of representations and using them across multiple concepts, and (5) assigning teachers to make connections between the representations presented in order to reinforce teachers’ learning. Most of these practices were also mentioned in the interview with Dr. Beth. The mini lecture video provides evidence of these practices at the operational level. Each practice reflects Dr. Beth’s specific pedagogical purpose. They were not random acts, but goal-driven practices; in fact, each practice appeared to be connected and guided by her knowledge of cognitive psychology and the implications of this perspective in accounting for mathematics learning.

Sample mini lecture observation. Another observation that was made was a mini lecture of a module provided by Dr. Beth as an example of her online design. The sample mini lecture was a self-paced video consisting of 21 slides. Each slide is approximately one-minute long and corresponded with Dr. Beth’s recorded voice dialogue. The mini
lecture was delivered asynchronously; the teachers could go through the slide at their own pace.

The sample mini lecture was a module on the concept of division. The problem used in the mini lecture was context-based. I parsed the mini lecture into four major sections (I, II, III & IV) according to the sequence of her slides. Section I is the introduction slide, where the title of the module was presented. Section II, which contained slides 2–13, is the main part of the mini lecture. In these slides, a context-based division problem was presented and six different student strategies for solving the same division task were discussed. Section III, which contained slides 14-18, included some key ideas about realistic mathematics education movement and the importance of this methodology to mathematics teaching and learning. Following Section III, Dr. Beth referred back to division problem in Section IV, which contained slides 19-21, and posed two extended problems for the teachers to solve as their assignment. One problem was about how problem situations might influence the type of representations used by the solver. The other problem was a standard computational problem involving division. Both problems intended to connect back to the representation models presented in the earlier slides. In the following section of this paper, I will offer detailed descriptions and analysis of the Sections II, III, and IV of the mini lesson, where I will identify key practices that Dr. Beth used when implementing this online module.

In Section II (slides 2-13) of the lesson, Dr. Beth described the problem situation to the teachers (How many classrooms that sit 29 students would be needed for a school of 609 students?). The problem was a simple division problem, $609 \div 29$. Dr. Beth
proceeded by showing students’ informal strategies of adding up and subtracting down to solve the problem. In later slides, she illustrated for the teachers how ratio tables were used by students when solving 609÷29. She showed two ways that ratio tables were used – one was doubling and the other was in multiples of ten. The method implemented involved adding up multiples of 2’s or 10’s to reach to 609. On one slide after another, Dr. Beth presented to the teachers the area model to solve the division problem, then partial quotient, and then long division. The order of the representations reflected the progression of the sophistication of these strategies. Such intension became explicit in slide 13, where “division progression” was discussed. We see how Dr. Beth organized Section II slides to be clearly goal-oriented. As indicated during the interview, connecting informal strategies and pressing onto more formal strategies was one of her goals for these modules. She purposely sequenced the student representations so that formal strategies would only be revealed after first discussing the informal strategies.

In discussing the use of the area model to solve the division problem, Dr. Beth reminded the teachers that such a model was used in the multiplication module, and therefore, she expected the teachers to be familiar with the model. By repeatedly using the same model across multiple concepts, it was revealed that Dr. Beth’s choice of representations was deliberate. The use of the area model in this module served two purposes: First, it is one of the models of division that is different from the standard division algorithm; second, the area division model was intended to help the teachers connect multiplication and division.

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The Section II slides also show that Dr. Beth valued efficiency in mathematics learning. She presented different methods for that particular strategy, then compared and contrasted which methods were more efficient for the strategy. For instance, she discussed two different ways of doing partial quotients. She identified chunking in 20s as more efficient than chunking in 10s. Similarly, she did the same thing with ratio tables. She showed the teachers that using a table of multiples of 10 was more efficient than the one that included multiples of 2. Efficiency was not identified as one of the elements in the PD framework; yet, Dr. Beth’s repeated emphasis on the efficiency of various strategies provided clear evidence of the importance of mathematical efficiency in Dr. Beth’s orientations toward mathematics and mathematics learning.

In Section III of the mini lecture, Dr. Beth introduced the teachers to the history of mathematics and Realistic Mathematics Education, which was the foundation of the entire PD program. By introducing the theoretical background to the teachers, Dr. Beth provided a context to help teachers understand why connecting informal strategies and formal strategies was so important. Her discussion of utilizing realistic mathematics education is closely tied to her own experiences of learning mathematics as well as the subject matter that she was presenting on the topic of division. The combination of division progression and presentation of theoretical foundation for such practices reinforced the importance of connection making, which was the major goal for Dr. Beth in this module.

The last section (Section IV) in the mini lecture contained the assignment slides 19-21. The assignment slides were important to Dr. Beth in her online module design, as
she mentioned during the interview that figuring out how to ask questions to ensure that the key ideas would come across to the teachers was a major component of her online design. This objective aligns with her orientation towards online education and her orientation towards teaching learning in that her desired online PD design would be interactive and engaging for teachers. Module assignment was the method she used to interact with the teachers whilst optimizing the participants’ engagement with the content.

The assignment consisted of two division tasks. The first task consisted of two parts: one was about dividing a given number of students into a fixed number of classrooms; the other problem was a carpeting problem—an area exercise. She asked the teachers to compare the two problem contexts and to think about how each problem situation might lead to certain representations that could be used to solve the problems. In the slide, she also had images of a ratio table, an area model and partial division, as references for teachers. The task was attached to pictures of strategy representations. Such an approach, nonetheless, was purposeful because Dr. Beth expected the teachers to study, think about, and be able to use these representations. By attaching the pictures and assigning teachers to think about these representations, it provided her with evidence of how the teachers comprehended these division strategies, so ultimately, her goal of implementing this online module would be achieved. From presenting the selected six division strategies to the teachers to assigning the teachers to reflect upon three of the strategies again confirms that Dr. Beth’s choice of student representations was not random, but purposeful.
The second task was a computation task (426 ÷ 30). Three different representations—a ratio table, an area model and partial division, were also listed alongside the computation task. Dr. Beth asked the teachers, first, to solve the computation problem using two of the representations and then discuss connections between the representations. She then asked the teachers to comment on the advantages and disadvantages of using long division for solving the task. This computation task served a different purpose than the first problem. The first problem was intended to help the teachers realize where different representations might occur. The second problem (i.e. the computation task) engaged the teachers in making connections among different representations (informal vs. formal), and eventually, encouraged them to compare these approaches with the standard long division algorithm (formal). This computation problem made it clear that Dr. Beth was looking for evidence from the online participants that they could make such connections. Her assignment was designed in a way to reassure that the content presented in the mini lecture was internalized by the online participants.

Summary. The analysis of this sample mini lecture reveals the intentionality of Dr. Beth’s practices. These include: (1) using problems that the teachers were familiar with, (2) focusing on multiple representations, (3) connecting informal strategies and formal algorithm, (4) selecting purposeful student models of representations and using them across multiple concepts, and (5) assigning the teachers to make connections between representations presented in order to reinforce the teachers’ learning. Most of these practices were also mentioned in the interview with Dr. Beth. The mini lecture video provides evidence of these practices at the operational level. As the detailed analysis
unfolds, each of the practices reflects Dr. Beth’s specific pedagogical purpose. They were not random but goal-driven; each practice appeared to be connected and guided by a few key decisions undertaken by the MTE.

**Putting it all together – the Detailed Design Logic map.** To better understand Dr. Beth’s pedagogical practices, that is, how she organized the online PD for mathematics, I carefully examined her design logic – Sequence A as indicated in the survey was used to compose a Detailed Design Logic map. Sequence A (Figure 23) briefly describes a design progression capturing MTEs’ design logic: identifying objectives – selecting corresponding topics – selecting appropriate online delivery methods for the chosen topics. Dr. Beth followed this design logic for her online PD. She expressed that she was advised to, “let go of the format you think you need and start from the other side. Start from what you want people to get out of the course then work backwards to the technology that you’re going to meet within some sort of the boundaries of what we have available to us.” Following this expert recommendation, she decided to allow the learning objectives to drive the model. I used the Sequence A design logic as the skeleton, then added specific PD objectives, learning topics, and online delivery methods according to the interview and mini lecture data to provide a map that delineates the various relationships between Dr. Beth’s decisions and practices of her online mathematics teacher education practices (Figure 24).
Figure 23. Design logic sequence A
Each of the rectangles in Figure 24, labeled C1 through C4, represents a point at which the MTE might make a consideration for the content of the PD. Each of the ovals, labeled P1 through P10, represents a possible pedagogical practice by the MTE.

As indicated on the right side of the figure, the Detailed Design Logic is exhibited along three stages. In the first stage, Dr. Beth identified the online PD objectives. These objectives, together with theoretical foundation of the PD, informed the work outlined in
the second stage – selecting problems and topics for the PD. These first two stages were selected by Dr. Beth and her team. They were consistent across multiple sessions in the PD program. These two stages also determined how Dr. Beth designed and delivered the online sessions to assure that the online sessions remained consistent with the in-person sessions. On the left-hand side of Figure 24, these two stages were described as “adopt PD objectives” and “adopt PD activities” to specify that Dr. Abby’s practices in these two stages had been adopted from the original PD program. The last stage, described as “design online delivery” on the left-hand side of the map, represents the specific practices that Dr. Beth designed to engage the participants in the specific online sessions. In the following, I will break down each stage to analyze Dr. Beth’s design logic.

**Stage 1: Adopting PD objectives.** In the Detailed Design Logic map, this stage starts from C1 and moves along to the P1, P2 and P3 nodes (Figure 24). C1 is a consideration for Dr. Beth, which aligned with the overarching PD content goals as previously implemented in the face-to-face PD. This consideration consists of two parts (i.e. two sub-goals): (1) supporting teachers’ development of mathematical content knowledge in number and algebra, and (2) structuring and delivering the PD online. The first part is the PD content goal, which needed to be consistent across the in-person PD program that had been developed several years before. The second part is the implementation goal for Dr. Beth – the person who transitioned the in-person PD to an online platform. Dr. Beth’s design logic begins with the content goal. Practices P1, P2 and P3 (Figure 24) are three core pedagogical practices that she adopted from the face-to-face PD as the key objectives of the entire online PD. These practices aimed to support
the teachers’ development of mathematical content knowledge in number and algebra as well as to address their beliefs about learning mathematics. These practices guided the entire online PD.

P1 focused on the conceptual understanding of individual concepts such as number, operation or algebra. P2 emphasized the mathematical structure of number and algebra concepts, for example, decomposing numbers (e.g. 5 equals 3+2) and its relationship with algebraic operations (5x = 3x + 2x). Dr. Beth described it as a “progression of concepts.” Another pedagogical practice used by Dr. Beth was discussing the theoretical and pedagogical rationales behind P1 and P2 that aimed to better support the teachers’ knowledge of mathematics and the intricacies of their learning. This approach is identified as P3 on the Detailed Design Logic map. Dr. Beth mentioned that although explicit pedagogical or theoretical discussions were not included in every PD module, they were present throughout the PD supporting the entire PD content and practices. In the sample mini lecture provided, Dr. Beth introduced the history of mathematics education and discussed the rationale of Realistic Mathematics Education to support the importance of connection making in mathematics learning. Dr. Beth considered this inclusion to play a critical role in altering teacher beliefs towards mathematics and mathematics learning.

Stage 2: Adopting PD activities. The second stage of the Detailed Design Logic map is the breakdown consideration and corresponding practices associated with P1. It starts from P1 to C2, and then proceeds to P4, P5, and P6 (see Figure 24). This is a breakdown associated with the implementation focused on developing conceptual
understanding for a concept in number and algebra. As such, the key consideration for this stage is how to support the teachers’ conceptual understanding in number and algebra. The pedagogical practices that Dr. Beth endorsed and operated for this consideration include using problems with which the teachers were familiar (P4), focusing on multiple representations (P5), and connecting different representations and transitioning to formal algorithms (P6).

As I described in the previous section, the mathematical problems that Dr. Beth used in the PD were either context-based problems or computation problems. These problems were textbook problems that the teachers routinely encountered in their curriculum (P4). Dr. Beth indicated that the use of these problems was deliberate because the teachers were so familiar with these problems that they would benefit from cognitive dissonance by (1) showing them multiple points to these problems (that is, P5) and (2) extending the problem to the discussion of mathematical structure (that is, P6). P5 and P6 are practices responding to the consideration of how to structure the PD for the teachers’ conceptual understanding of the concepts examined. In the sample mini lecture on division, these practices progressed in the sequence of P4, P5, P6 and then P3, which was the order she followed in the face-to-face sessions as well as in the online sessions.

Stage 3: Designing online delivery. The last stage of Dr. Beth’s design practices (starting with C3 and C4) is the implementation stage. So far, for stages 1 and 2, the PD objectives and topics were pre-determined as they were included in the face-to-face PD sessions. Included in Stage 3 were the practices associated with how Dr. Beth designed
the online PD for effectively achieving the program objectives-related practices in stage 1 (P1, P2 and P3) and successfully implemented the activities in stage 2 (P4, P5, and P6).

If the PD practices were implemented in the face-to-face environment, then Dr. Beth expressed would ask the teachers to solve the tasks in groups. She would then draw on the teachers’ own work to highlight the “big ideas” behind the tasks. She had previously tried using the same approach in her first attempt at online PD delivery, which was unsuccessful. During the interview, Dr. Beth used the phrase “less interactive” to describe her first attempt at the online PD. In the face-to-face environment, the presence of immediate communication feedback and physical proximity helped her learn about the teachers’ reactions to the content being explored, which was a feature that was unavailable to Dr. Beth in the online environment. Bearing such technological constraints in consideration, Dr. Beth had to contemplate: (1) how to ensure that the teachers would encounter multiple strategies or representations in the online delivery (C3), and (2) how to ensure that the teachers could make connections among these representations in the online delivery (C4). These two considerations concerned two aspects of Dr. Beth’s design intensions: One was to ensure that the provision of required content for the teachers was present in the online sessions and the other was her intension to receive feedback from the teachers’ learning outcomes, addressing the lack of communication cues constraint.

For C3, where she needed to decide on how to enable the teachers to see multiple representations of the discussed concept, say, division, in the mini lecture, Dr. Beth chose to illustrate six different students’ strategies of division, which included adding up,
subtracting down, ratio table, area model, partial division, and long division algorithm. She carefully selected these strategies and illustrated all of them in the video, slide by slide in a particular order (P7). By doing so, Dr. Beth ensured that the teachers viewed all of the important strategies and the representations were emphasized. This was one way that Dr. Beth responded to C3. Dr. Beth also mentioned during the interview that she sometimes asked the teachers to share their students’ strategies but this was more likely to be placed as an assignment in the module rather than discussed in the mini lecture.

For C4, Dr. Beth needed to determine how to assure that the “big idea” would emerge in the online sessions. First, she presented to the teachers the connections and progressions of the representations as well as the progressions of the concepts (when appropriate), right after presenting them with all of the strategies and representations (P8). These representations and strategies were purposefully selected to reveal the connections or the “big ideas” (P9). The online participants were then assigned to complete a similar task including a reflection upon the connections (P10). In the division mini lecture, the assigned task was identical to the division problem Dr. Beth had illustrated in the previous slides. The online participants were asked to apply what they had learned from the mini lecture. Her assignment was intended for the purpose of receiving feedback on whether the teachers had completed the mini lecture and could then apply the knowledge. This was one of the goals of the assignment so that Dr. Beth could attain evidence of learning. She also posed questions that attempted to promote reflection by the teachers on their own practices and knowledge of mathematics learning.
From the top down. All of Dr. Beth’s online delivery actions (P7-P10) were purposeful and driven by the overall PD objectives. She worked through what was required to be delivered online (i.e. objectives and topics) as well as the constraints imposed by the online learning environment. Her final actions (P7-P10) aligned closely with her goals and orientations toward mathematics, mathematics teaching and learning and mathematics teacher education. Looking at the Detailed Design Logic map vertically, I depicted three paths of thinking based on the PD content. First, starting from the overall decision (D1), moving along P1, C2, P4 and P5, then C3, and finally P7 (Figure 25). This path concerns engaging the teachers in the analysis of students’ multiple strategies and representations. The second path also starts from C1, then to P2, together with P6, then C4, and finally P8, P9 and P10 (Figure 26) emphasizing the connection between representations and the structure of mathematics emerged from the connection. The third path is C1 to P3 (Figure 27), which emphasizes discussing the theoretical and pedagogical rationales behind the PD, providing a context for the content analysis teachers were expected to complete.
Figure 25. Dr. Beth’s Detailed Design Logic vertical path I: C1-P7
Figure 26. Dr. Beth’s Detailed Design Logic vertical path II: C1-P8, P9 and P10
Figure 27. Dr. Beth’s Detailed Design Logic vertical path III: C1-P3

**Characteristics of Dr. Beth’s pedagogical practices.** The Detailed Design Logic map connects the *practices* (i.e. P1-P10) that Dr. Beth undertook as she designed and delivered the online PD sessions with the specific *considerations* (i.e. C1-C4) that she might have encountered. The map includes specific learning objectives, topics and delivery methods. Four characteristics of Dr. Beth’s pedagogical practices are revealed from the Detailed Design Logic map: (1) goal-driven throughout her practices; (2) a
“system” approach to adopt content and concepts from others’ practices; (3) awareness of technological constraints provided by the online delivery method; and (4) constantly seeking to resolve the constraints.

Firstly, as clearly indicated in her Detailed Design Logic map, Dr. Beth’s pedagogical practices were goal-driven. She had clear objectives for each online module. The sample mini lecture was as an example of an online module where pressing teachers to make connections between different division representations and strategies was an objective that was intentionally incorporated into the design. She purposefully selected student strategies and then presented them to the teachers in a particular order (P7), and explicitly made connections between these strategies (P8). Furthermore, she used historical and theoretical discussion (P3) to explain why connection making was important, which supported her objectives and actions of P7 and P8.

Secondly, Dr. Beth adopted concepts, orientations and professional development activities from the original in-person professional development program, but she did so by taking a “system” approach. She selected specific problems with a keen awareness of what type of interaction she would pursue around them. As such, she was flexible and goal driven. The online modules were systematically built and they augmented each other. This is the second characteristic of the MTEs’ design thinking.

Thirdly, based on Dr. Beth’s comments, I came to believe that she was aware of the constraints and limitations associated with online media and knew of the challenges she would have to overcome when developing her online program. She repeatedly compared her experience of the face-to-face learning environment with the challenges
encountered in the online format. She identified the limitations associated with sharing ideas during the sessions; for instance, she found it challenging to gather teachers’ work in the online setting to discuss as was done in the face-to-face PD. She also identified additional limitations of online interaction where she could not easily gather evidence of learning from her online participants due to the loss of physical cues. The choice of using Voice Thread mini lecture was to resolve the issues accompanying the lack of physical interaction. As she realized the difficulty of learning via synchronous peer-to-peer interaction or facilitator-participant interaction, she relied on a different technique for teachers to learn – mini lectures. She designed the mini lectures to directly show the teachers all the representations and strategies that she wanted the teachers to comprehend. In this way, her participants learned via receiving, processing and reflecting on the information presented. She said this approach was direct instruction but she tried to make it as succinct as possible by carefully selecting representations and presenting them in a particular fashion. I characterize such a practice as an attempt at resolving conflicts between technology constraints and content delivery.

**Knowledge involved in Dr. Beth’s design thinking.** According to Schoenfeld’s decision making model, the professionals’ thought process is closely aligned with their goals, orientations and knowledge. In the previous section, I analyzed Dr. Beth’s thought process as an online educational designer and instructor. In this section, I will focus on the type of mathematics teacher educator (MTE) knowledge that was involved at each stage of her practices. Before proceeding, it must be emphasized that the knowledge derived here is the knowledge relevant to Dr. Beth’s practices of conducting this state-
mandated online PD for mathematics teachers of grades 9-12. The analysis focuses on Dr. Beth’s practices of one PD. However, like many accomplished mathematics teacher educators, Dr. Beth has a wide range of knowledge that might be employed elsewhere at different times.

In the objective and topic stages (i.e. stages 1 and 2) of Dr. Beth’s Detailed Design Logic map, she adopted practices from the face-to-face professional development program. As such, she followed the acceptable methods for working in the profession in her own practices, which is knowledge of professional traditions in Perks & Prestage’s (2008) teacher educator knowledge tetrahedron framework. To be more specific, the types of knowledge that were involved in Dr. Beth’s stages 1 and 2 practices were: (1) knowledge of mathematics teacher, including teacher knowledge, teacher belief and teacher change; (2) knowledge of content and student (KCS); (3) specialized content knowledge (SCK) in numbers and algebra; (4) knowledge at the mathematics horizon; and (5) knowledge of history of mathematics education. Knowledge from research on mathematics teacher knowledge, teacher belief and teacher change grounded Dr. Beth’s work in the PD program. The original PD program relied on the constructivist’s theory to inform how teacher learning was organized. From this perspective, Dr. Beth’s PD design was to redefine teachers’ existing knowledge of mathematics and perceptions about mathematics by providing the teachers with examples of students’ multiple strategies and representations of mathematical concepts.

Knowledge of content and student (KCS), and specialized content knowledge (SCK) were evident in all of the choices of PD activities that she incorporated into the
program. In all of the PD modules, she extensively used student mathematics and research in students’ learning about numbers and algebra. KCS and SCK was the knowledge base for such practice. Furthermore, the student representations were used to promote discussions of concept connections where knowledge at the mathematics horizon was exercised. KCS and SCK are knowledge domains for MTEs as well as for mathematics teachers, so they are learner knowledge in the teacher educator knowledge tetrahedron. Learner knowledge was defined as the type of knowledge that MTEs are required to know as a mathematics teacher for teacher education.

Another type of knowledge that was evident in Dr. Beth’s practice was the knowledge of history of mathematics education. She used the history of mathematics education in the US to discuss the importance of connection making with the teachers. Such knowledge belongs to professional traditions in mathematics education, located in the learner knowledge vertex in the teacher educator knowledge tetrahedron.

In the delivery stage (i.e. stage 3), Dr. Beth’s practices were identified in P7 to P10. In this stage, Dr. Beth systematically organized the activities into individual modules and presented them to her participants using mini lectures. Knowledge of the online environment as well as pedagogical knowledge of using the online environment for mathematics teachers’ learning was exercised. She received advice from experts on the choice of online tools (i.e. knowledge of the online environment) and she structured the Voice Thread using mini lecture and video sharing formats for the teachers to interact with the PD learning content and with each other in order to achieve desire learning outcome (i.e. pedagogical knowledge of the online environment for mathematics teachers’
learning). Such knowledge was informed by experts in online education (i.e. professional traditions) as well as from experimenting in practices (i.e. practical wisdom of conducting online teacher education sessions).

In the following, I summarize knowledge domains in Dr. Beth’s online teacher education practices. Figure 28 illustrates the relationship of these knowledge domains using the teacher educator knowledge tetrahedron.

- K1: Professional traditions in mathematics teacher education adopted from existing PD practices: knowledge of teachers’ content, knowledge of mathematics and beliefs towards mathematics
- K2: Learner knowledge – knowledge of content and student (KCS)
- K3: Learner knowledge – specialized content knowledge in number and algebra (KCS)
- K4: Learner knowledge – knowledge at the mathematical horizon
- K5: Learner knowledge – history of mathematics education
- K6: Practical wisdom – knowledge and experiences of conducting online mathematics teacher education sessions
- K7: Practical wisdom – Pedagogical knowledge of online mathematics teacher educating
Case synthesis. The previous analysis of Dr. Beth’s goals, orientations, knowledge and pedagogical practices in the online professional development program examines the analysis of Dr. Beth’s decision making as an online PD designer and instructor. To synthesize Dr. Beth’s case, I will first identify the dimensions of decisions involved in her practice according to Boyd (1993). Then, I will model her design thought process based on Schoenfeld’s (2010) human decision making framework.

What decisions were made? Dr. Beth’s decisions throughout the PD design and implementation were about five of the eight dimensions: (a) decisions for choice of virtual platform, (b) decisions for media, (c) decisions for illustration of the subject matter (i.e., approaches to the subject matters), (d) decisions for online sociostructure
(e.g. groupings, interaction), and (e) decisions for mechanisms of learning outcome control. These dimensions are cycled in Figure 29. Dr. Beth was assigned the task of designing the state-wide professional development program that had previously only taken place in the face-to-face environment. Decisions about goals, participants, and content were predetermined. Other than these three factors, she made decisions about the online delivery of the PD including choices of platforms, and organization of the learning activities in the online platforms. Each of these dimensions of decisions is reflected in her Detailed Design Logic map.

![Diagram showing Dr. Beth’s decision dimensions](image)

Figure 29. Dimensions of Dr. Beth’s decisions circled
Modeling design process. To connect decisions with Dr. Beth’s goals, orientations and knowledge, I modeled her design thought process as follows:

1. As a mathematics teacher educator, Dr. Beth was given responsibilities to design and implement an online professional development program for mathematics teachers of grades 9 to 12. She started this process with a body of goals, orientations and knowledge of mathematics education and mathematics teacher education.

2. Dr. Beth then analyzed the situation. Her main responsibility was to design and deliver the identical aspects of the original professional development content to the Grades 9-12 section in the online learning environment. To accomplish this objective, she had fixed PD goals (G1 and G2), and content organization goals (G4).

3. She adopted existing PD resources and PD perspectives of mathematics teacher education and mathematics teaching and learning. At this point, she exercised her knowledge of both mathematics and mathematics teachers’ needs.

4. She established an implementation goal (G3), and specific local goals for online module designs (G5 – G8). Thereupon, her knowledge of the online environment and pedagogical knowledge of organizing mathematics teacher education sessions online was exercised.

5. She had two key decisions to make in regard to the design:
o Decision 1: what online delivery tools should be used to accomplish G3? This was an unfamiliar situation for Dr. Beth. She had multiple types of online technologies to choose from. Such technological tools included technology for online communication (e.g. synchronous versus asynchronous platforms, online management system, etc.), and technology for mathematics learning (e.g. online simulations, mathematical programs etc.). Her choice of online technological tools was evaluated by her knowledge of the online environment. Her final choice for online technology was using Voice Thread mini lectures, which was an online tool for interaction and content presentation, consistent with Dr. Beth’s orientations towards the online environment for learning (O11 and O12).

o Decision 2: how should learning activities be organized with the chosen technology? This decision includes sub-decisions about what content to present, in what order, how to pose tasks for participant-teacher interaction or participant-participant interaction, and how to assess teacher learning using the online delivery tools that she chose. Her choices for the above decisions relied on her knowledge of the online platform (i.e. Voice Thread and Blackboard), her knowledge of mathematics and students, and pedagogical knowledge of conducting mathematics teacher education.

6. She implemented the online PD (P7 to P10).
7. She monitored the effectiveness and success of the implementation, which informed future decisions of the online modules design (Step 5). It is an iterative process.

Figure 30. Modeling Dr. Beth’s decision making process

The above diagram (Figure 30) models the above decision making process with color-coded goals, orientations, knowledge and practices identified so far. From the diagram, one can see that the role of goals, orientations, and knowledge come into play at different stages of the entire decision making process. A couple of features are identified with this decision making model. First, the diagram indicates Dr. Beth’s degree of freedom of decision making is within her choice of online delivery technology and the organization of the learning activities is within her choice of delivery technology,
represented in the loop from “Make decisions” to “Monitor.” The top row of the diagram, that is, the processes of “Identify fixed learning goals” and “Adopt existing PD content and orientations” did not require the MTE to make a decision with regard to the online PD. Rather, what was adopted from the face-to-face PD might inform her implementation of the online modules and sessions. Second, given the decision making cycle, Dr. Beth’s knowledge of online technology played a critical role in the decision making process. Her knowledge of the online environment supported her ability to identify implementation goals and her pedagogical knowledge of online education supported her organization of the online learning experiences for teachers. The technological knowledge is not simply her knowledge of using the online tools, but is also her understanding of the PD and the implementation of the content using the online tools.

The Case of Dr. Colin

Dr. Colin was a professor of mathematics and affiliated with a research-based university. In this capacity, he has extensively researched many areas of mathematics education, including ethnomathematics, development of mathematical ideas, reasoning and heuristics, writing and mathematics learning, and collaborative mathematical problem solving with technology. He indicated that he had over fifteen years of experience working with mathematics teachers. Likewise, he had extensive experience working with elementary teachers at the university, and later he supported his university by developing a secondary mathematics teacher education program. In recent years, he was involved in a funded project that supported students and teachers of mathematics in developing their mathematical reasoning through online collaboration. By the time of the
Dr. Colin had five years of experience developing an online mathematics teacher professional development program.

Dr. Colin indicated a willingness to participate in a follow-up interview based on his survey responses. After the initial survey analysis, I contacted Dr. Colin to determine whether he would be willing to participate in the interview and after receiving his approval, I organized a 30 to 40-minute-long interview with him via Skype. The analysis presented in this chapter is based on the survey response he provided, the 40-minute Skype interview, and a conference proceeding that he authored about the online professional development program shared in the survey. All interview data was transcribed, coded, and analyzed according to the decision making framework.

**Goals.** The online professional development project reported by Dr. Colin for this study was one activity of a larger research project for which he and his team received funding. As indicated by Dr. Colin during the interview, the project’s goal was to understand how small groups of individuals extend their learning in productive mathematical discourse. This online PD program was designed to support teachers to implement a set of mathematical tasks that would promote productive mathematical discussions in their classrooms.

An important characteristic of Dr. Colin’s goal setting was that the entirety of the PD goals, from overarching goals to local goals, were all guided by his epistemological perspective of mathematics and mathematics learning (I will discuss this perspective in a later section of the paper) such that he was very specific about the design of the
environment and the tasks for teachers. Table 12 lists Dr. Colin’s goals for the design of the PD.

Table 12. Dr. Colin’s goals

<table>
<thead>
<tr>
<th>Overarching goal</th>
<th>G1: Develop productive mathematical discourse among teachers and students.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two sub-goals</td>
<td>G2: Provide an environment that allows teachers to engage in productive mathematical discussions.</td>
</tr>
<tr>
<td></td>
<td>G3: Support teachers to reflect upon the discursive learning process.</td>
</tr>
<tr>
<td>Local goals for mathematical tasks design (in the context of a dynamic geometry environment)</td>
<td>G4: The mathematical tasks need “to stimulate creativity.”</td>
</tr>
<tr>
<td></td>
<td>G5: The tasks need “to encourage collaboration.”</td>
</tr>
<tr>
<td></td>
<td>G6: The tasks need to allow the participants to realize each other’s “emergent ideas.”</td>
</tr>
<tr>
<td>Local goals for pedagogical task design</td>
<td>G7: Design tasks and assignments that have theoretical substance to support teachers to reflect upon the process and content of mathematical discourse.</td>
</tr>
</tbody>
</table>

**Orientations.** The initial survey asked the participants to identify the guiding theories and framework of the online course, and ways the MTEs conceptualized their content goals. Dr. Colin’s responses to the survey questions allowed me to become familiar with those theories and the principles that guided his work. Similar to the previous cases, I will offer an account of Dr. Colin’s orientations along five critical dimensions: (1) mathematics, (2) mathematics learning, (3) mathematics teaching, (4) mathematics teacher education, and (5) online teacher education.
Orientation towards mathematics. Dr. Colin inherited a dialogic notion of mathematics from a mathematician, Caleb Gattegno, who viewed mathematics as the result of one talking to oneself and to others about specific perceived objects, relations and relations of relations (or dynamics involved with relations): “No one doubts that mathematics stands by itself, is the clearest of the dialogues of the mind with itself. Mathematics is created by mathematicians conversing first with themselves and with one another” (Gattegno, 1987, p.13). In Dr. Colin’s paper, he emphasized that mathematical meaning arises through discussions and interactions and such discussions and interactions should be purposeful – about a mathematical object, or relations, or dynamics of relations, and the interactions should be genuinely engaged in by learners. He gave examples of such purposeful mathematical discussions and interactions (or mathematical discourse), including “questioning, affirming, reasoning, justifying, and generalizing.”

Further, Dr. Colin distinguished mathematical content into two categories. One category is about mathematical notations, labels, and notations, such as Cartesian axes, notations of coordinates. He characterized these mathematical “cultural and historical” conventions as arbitrary; meaning, they cannot be constructed through memorization or association. The other category of mathematical content is relations of mathematical objects and the dynamics or properties among these relations. He posited that this category is essential for learning as they can be noticed and learned. This is evident in the geometric tasks used in the PD which were designed to allow for the participants to interact with the specific mathematical objects and to engage them in dialogues about the relations of the objects and/or the dynamic relations of the objects.
**Orientation towards mathematics learning.** Dr. Colin’s orientation towards mathematics learning was consistent with his view of mathematics. He believed children learn mathematics akin to learning a language. Children develop learning power before they enter school, which is evident from them learning their native languages during earlier stages of life. The various cognitive abilities that children possess in learning their native languages apply to their learning of mathematics since mathematical meanings arise through discursive discussions and interactions, which is similar to learning a language.

Further, he believed that children are sense makers of mathematics but in order to increase the possibilities of productive mathematical meaning making, he encouraged tools to be provided for individual learners to employ, or a mathematical-content focused, collective environment be established. From Dr. Colin’s perspective, such tools could be either an epistemic tool or a technological tool. Epistemic tools are tools that allow mathematical learners to communicate about the mathematical objectives, the relations or the dynamic of the relations. In his PD design, he and his team encouraged participants to ask themselves or their partners “what they perceive” (i.e. what do you notice), “how it connects to what they already know” (i.e. what does it mean to you), and “what they want to know more about it” (i.e. what do you wonder about). Technological tools could be a mathematical technology such as a dragging tool in the dynamic geometry environment or a collaborative virtual environment. According to Dr. Colin, learners make sense of mathematical objects and their relations through their interactions with such tools.
Orientation towards mathematics teaching. Dr. Colin had a reformed mind for mathematics teaching. He rejected the traditional types of teaching based upon rote and procedures. Rather, his view was one that any teaching approaches should intimately connect with how children learn mathematics. From this perspective, he emphasized the mathematical content that children are aware of and the mathematical content that children are not aware of, and the role of teachers in providing such awareness to children. He rejected the idea of teachers providing cultural tools that merely require students to memorize or make associations. Instead, he saw teachers’ roles in providing support through careful task designs that allow students to engage in productive mathematical discourse such as questioning, reasoning, justifying, etc.

Orientation towards mathematics teacher education. Dr. Colin’s orientations towards mathematics, and mathematics teaching and learning guided his views about how mathematics teacher education should be conducted. He placed a greater emphasis on student mathematics learning as the foundation of the mathematics teacher education:

If we understand how children learn then connecting that with an approach to teaching mathematics which is more in line with the learning powers of students. And part of the perspective that I have, not only working with these elementary perspective teachers but later on, as I helped our department build a secondary education program, had to do with understanding the learning powers that children develop before they enter school.

From my conversation with Dr. Colin, I learned that his work with mathematics teachers was grounded in the perspective of children’s cognitive power of learning mathematics and much of his practice was focused on changing teachers’ experiences with mathematical learning, and hence, their beliefs about mathematics learning.
Dr. Colin valued reflection in teacher learning. This concept is seen in his pedagogical task design, where he and his team designed a set of tasks encouraging teachers to reflect on “the process and content of mathematical discourses” occurring in their mathematical problem solving sessions. They called such tasks “meta-tasks.” Again, collaborative reflective practices among teachers were emphasized in his online PD, which is consistent with Dr. Colin’s general beliefs about learning or meaning making.

Lastly, Dr. Colin was of the opinion that mathematics teacher education should be theoretically grounded. One of his pedagogical task design goals was to incorporate theoretical substance in teacher’s reflection. His tasks included exercises in which teachers reflect upon readings about mathematical teaching and learning to guide teachers’ reflection upon their own problem solving experiences that occurred in the PD and that will occur in their future thinking about teaching mathematics.

*Orientation towards online learning.* Dr. Colin’s perspectives of the online learning environment were formulated by its role and affordance as a collaborative communication technological tool. As mentioned earlier, he adopted a sociocultural perspective of learning to view the role of such technology in children’s or teachers’ mathematical learning, meaning that the collaborative e-learning environment is “to initiate and foster productive mathematical discursive activity.” In other words, the technological tool could potentially be viewed as an instrument mediating learners’ encounters with mathematics if the mathematical tasks were appropriately designed and implemented in the environment. In both the interview and survey responses, he mentioned the theory of instrumental genesis as the guide for his design of the online
environment. This theory emphasizes the interactions between the individual learner and the technology in the process of mathematical learning. Similarly, Dr. Colin and his team redesigned the virtual learning environment to encourage such interactions. Using the survey summary categories about mathematics teacher educators’ beliefs regarding the role of the online environment in teacher learning, Dr. Colin’s orientation belongs in the category of the environment “as a learning tool” that affords teacher collaboration and reflection as well as integration with mathematical programs to promote specific mathematical learning.

_Summary._ Dr. Colin posited a strong social perspective towards mathematics, mathematics teaching and learning, mathematics teacher education and online learning. Although I described Dr. Colin’s orientations in separate categories, they all relate to one another. His orientations towards mathematics informed his beliefs about mathematics learning, and hence, mathematics teaching. Likewise, his orientations formulated the basis for how he conducted mathematics teacher education. Essentially, his beliefs span three primary points: (1) individuals have the power of learning; (2) productive learning occurs when genuine discussions and interactions take place; and (3) such productive learning could be fostered or mediated by technology or appropriate learning tools. The following table (Table 13) provides a summarized list of his orientations, which will be used for describing Dr. Colin’s design decision making process in a later section.
Table 13. Dr. Colin’s orientations

| Orientation toward mathematics | O1: Mathematics consists of a set of objects, relations and relations of relations.  
O2: Mathematical meaning arises as one engages in self-talk or dialogs with others.  
O3: Mathematics is a sense-making activity. |
|-------------------|----------------------------------------------------------------------------------------------------------------------------------|
| Orientation toward mathematics learning | O4: Learning mathematics is akin to learning native languages. Children develop the necessary learning power for understanding mathematics before they enter school.  
O5: Productive mathematical learning could be mediated by appropriate tools including cognitive tools and technological tools. |
| Orientation toward mathematics teaching | O6: Mathematics teaching should be student-centered. Teaching approaches should intimately connect with how children learn mathematics.  
O7: The teachers’ role in providing productive mathematical learning should be conducted through careful task design that allows students to engage in productive mathematical discourse such as questioning, reasoning, justifying, etc. |
| Orientation toward mathematics teacher education | O8: Mathematics learning is the foundation of mathematics teacher education.  
O9: Mathematics teacher education should include theoretical substance. |
| Orientation toward the online learning environment | O10: The online environment plays an instrumental role in mediating learners’ encounters with mathematics. |

**What happened in the online PD.** During the interview, I asked Dr. Colin to describe for me specifically how the online professional development program was run and his role in designing and implementing the PD sessions. Dr. Colin emphasized that the entire design and operation of the professional development program was grounded in the epistemological perspective of mathematics and mathematics learning that productive
mathematics discourse was the key for learning. In order to promote this philosophy of mathematical learning, he and his team were involved in two key design practices: (1) developing a technological environment that afforded genuine participant interactions and (2) designing tasks that facilitated productive mathematical discourse.

In the first stage, Dr. Colin and his team explored the use of an online portal for engaging students in a synchronous virtual environment – the chat room. In the chat room, the participants were provided with a chat space and a whiteboard where some elementary tools such as pencils and erasers, and straight line and basic shape tools, were provided. Dr. Colin shared during the interview that in this initial stage, he and his project team used the basic chat room environment to examine how the participants used the environment, how they interacted with the tools provided and how the tools shaped the discourse within the environment. From this experience, the team learned what this environment could afford and what constraints it had. One affordance they identified was that groups of three or four worked effectively for group collaboration in the chat room. Yet, they also learned that the chat dialog was too difficult to follow and the mathematical tools were limited for solving mathematical problems. In order to overcome these limitations, the project team decided to program in some mathematical tools to support mathematical problem solving and learning. Specifically, they added a multi-user, dynamic version of GeoGebra into the chat room. They also designed the chat room environment with referencing features enabling the participants to refer back to previous dialogues or objects on the whiteboard or on the GeoGebra screen. These two moves
served the very same goal for promoting productive mathematics discourse for the learning of mathematics.

The analysis of affordance and constraints of the environment not only informed the project team about the design of the online environment, but also enabled Dr. Colin to reflect on the differences between traditional face-to-face classrooms and virtual environments, and the role of an instructor or a facilitator in these environments in order to promote the desired discourse for participants’ learning of mathematics. Dr. Colin expressed:

The salient different is that in present classroom environments the teacher is physically present, whereas in a virtual learning environment the teacher is artificially present; that is, the teacher exists largely as an artifact of digital tools. Consequently, the design of the tasks that are to be objectives of learners’ activities in virtual environment need to be constructed in ways that support particular learning goals such as productive mathematical discourse.

Dr. Colin did not believe that the interventions of an instructor or a facilitator in the virtual chat room encouraged more genuine discussions or productive mathematical conversations among the participants. Rather, he believed that the role of an instructor or a facilitator was no more than posing questions to engage productive discussions and interactions. This could be accomplished by carefully designing tasks or posing further questions after reading the participants’ group summary. As such, Dr. Colin considered task design to be a major and critical part of the PD design to support the particular learning goals – productive mathematical discourse. This was also where Dr. Colin had devoted most of his effort in the PD development, since how these tasks were constructed for the online participants represent the expectations of the MTE for his participants.
Thus far, we can summarize some features of the online professional development program that Dr. Colin was involved in designing. First, the professional development program utilized the synchronous feature of online tools, specifically, the chat room, for groups of three to four participants to collaborate on assigned tasks in each session. Second, the chat room was programmed in a multi-user version of GeoGebra – mathematical tools for engaging in mathematical problem solving. Third, no facilitator or “teacher” was assigned to the chat room to directly interact with the participants. Instead, tasks assigned to the participants facilitated chat room activities and conversations. Based on the design of the PD sessions, Dr. Colin was not involved in “facilitating” the PD session in the same sense as he was in a face-to-face environment. Rather, the majority of the “facilitating” was accomplished beforehand through careful task design. In other words, there were no direct interactions between the MTE and the PD participants in the virtual sessions as there had been in the face-to-face environment. In the following section, I will describe specifically the types of tasks used in facilitating the professional development.

**PD tasks.** Dr. Colin described the two types of tasks designed for the mathematics teachers. One type was mathematical investigation tasks using GeoGebra. Specifically, these were geometric investigation tasks. The other types of tasks used in the PD sessions were called “meta-tasks” where the teachers were invited to reflect on the content and the process of mathematical discourse that occurred in the previous geometric investigation task. According to Dr. Colin, the teachers met for two hours, twice a week synchronously in their small-group chat rooms. The first meeting was
devoted to the geometric investigation tasks and the second meeting was devoted to the
discourse reflection tasks – the meta-tasks.

*Geometric investigation tasks.* For mathematics teachers to use the problems in
their classrooms to engage students in productive mathematics discussions and
interactions, Dr. Colin believed that the teachers needed to experience the kind of group
discussions and interactions that the tasks generated and the kind of mathematical
discourse that the teachers themselves produced. For this purpose, the teachers were
invited to solve the mathematical tasks in intimate groups in the PD sessions. Dr. Colin
emphasized that these mathematical tasks served as the “vehicle” for teachers
experiencing productive mathematical discourse. He insisted that solving the problem
was not the primary purpose during these problem solving sessions; rather, conversely,
the process of solving the problems, that is, the discussions and interactions involved was
the main purpose. To achieve such results, Dr. Colin had specific design goals for these
tasks: first, they needed “to stimulate creativity”; second, they needed “to encourage
collaboration”; and third, they needed to allow the participants to encourage each other’s
“emergent ideas” (G7-G9). Guided by these three local design goals, Dr. Colin and his
design team outlined specific principles for the mathematical tasks. A brief summary of
these design principles is listed below:

- The tasks would allow teachers to notice the behaviors of mathematical
  objectives, relations of the objectives, and further dynamics of the objectives
  by utilizing the dragging tools in the dynamic geometry environment.
- The tasks would invite participants (or teachers) to pose questions about what they notice, and to reflect on the mathematical meaning of what they notice and encounter.

- The tasks would prompt individuals to notice certain features of the objectives or relations.

- The tasks would provide formal mathematical languages that support participants’/teachers’ discussions.

- The tasks would allow facilitators to respond to the participants with feedback based on the participants’ work to include extensions for generalizing relationships, or noticing other relations or attributes of the mathematical objects, or posing further questions.

Based on the above principles, these mathematical tasks were open-ended and investigational in nature. Two weighty emphases were placed on these mathematical tasks. One was about the investigation of mathematical objects, relations, and relations of relations. Such emphasis required the tasks afford all of these features of mathematics, not just about a particular mathematical object, but also its relations within the object and its relations with other mathematical objects. Dr. Colin chose to use geometry as the mathematics content for these tasks as they had programmed a dynamic geometry environment into the chat room. Dr. Colin shared that they constructed 55 tasks, which were divided into clusters. Within each cluster, there was a connected sequence of activities in order to develop a particular geometric idea. Geometric constructions were one of the heavily investigated topics. Other topics included congruence, similarities and
some others involving the teachers using particular theories to problem solve, such as exploring special points of triangles, centroids, etc. The following is an example of a cluster of construction tasks that Dr. Colin shared during the interview. The sequence of tasks explored the following topics in order for the teachers to understand how to use the construction of perpendicular lines to construct quadrilaterals with 90 degrees. These topics included:

(1) Given two congruent circles and the circumference of each circle goes through the center of the other, construct two symmetric equilateral triangles and explore the diagonals of the rhombus that is created.

(2) Given a straight line, explore the construction of a line perpendicular to it.

(3) Given a straight line and a point outside the line, explore the construction of a perpendicular line going through the point that is not on the line.

(4) Construct squares or rectangles.

(5) Given a quadrilateral, construct angle bisectors and explore the resulting quadrilateral formed by the intersection of the angle bisectors: What happened when you moved them? What kind of quadrilateral is it? Prove it.

Tasks based on (1) – (4) were construction tasks while tasks based on (5) were investigational tasks requiring the participants to explore, conjecture and use particular theorems to prove their conjectures. In designing the tasks, Dr. Colin purposely asked teachers in their small groups to chat about and be attentive to the constructed dependency relations among the geometric objects that they constructed (i.e. the resulting figure could be dragged around without falling apart), as they are fundamental to
geometric construction. Dr. Colin shared that the cluster consisted of about ten tasks around these five topics in the cluster. The teachers were typically given two to four tasks in each synchronous two-hour problem solving session. They would produce a summary of the geometric responses to the tasks with justifications or relations that they noticed among the objects, or attempts to a proof, or questions they had. The summary was then read by the facilitators (i.e. Dr. Colin and his team), who might pose extension questions for further investigations.

The other emphasis in the principles of the mathematical task design was placed on the online participants’ experiences with these features of mathematics, such as inviting the participants to pose questions about what they observed, or inviting participants to generalize relationships or other attributes of the mathematical objects. In order to achieve these goals, Dr. Colin and his team designed the tasks to include specific epistemic tools, named by Dr. Colin, to engage the participants. These epistemic tools were three questions: (1) What do you notice? (2) What does it mean to you? (3) What do you wonder about? These questions were posed in order to prompt the teachers to identify what they perceived and to make connections between what they knew and what they wanted to know. Dr. Colin and his team also established group norms for the teachers in solving mathematical problems during the online synchronous interactions with each other.

Meta-tasks. The other type of task that Dr. Colin and his team focused on was the meta-tasks. The purpose of the meta-tasks was for the teachers to reflect on the content and the process of their own experiences when solving mathematical problems in the
previous sessions. In particular, the teachers used the logs of their conversations from the previous problem solving sessions to examine and reflect on the specific issues regarding encouraging productive discourse. Dr. Colin shared during the interview that the teachers would discuss issues of norms of collaboration, accountable talk, Common Core Mathematical Practice, and technological pedagogical content knowledge (TPACK). These were pedagogical issues in mathematics education with a focus on collaboration, promoting productive mathematical discourse and technology in mathematics teaching and learning. In comparison to the mathematical tasks, these tasks were exclusively aimed at teaching. The mathematics tasks were embedded in these meta-tasks for examinations, reflections and further discussions, which was why these tasks were called meta-tasks. The design of the meta-tasks provided evidence of the MTE’s emphasis on investigational and collaborative experiences that the teachers had during the problem solving process.

Another goal for Dr. Colin with these meta-tasks was to incorporate theoretical substance into the teachers’ reflections. In order to do so, he assigned homework for the teachers, asking them to read about these issues before they logged into the chat room for log analysis and discussions with their teams.

Summary. The analysis of the two types of tasks reveals Dr. Colin’s intentionality of the tasks for teachers to engage in the online sessions and how he implemented these tasks with the teachers. For the mathematical problem solving tasks, his design intensions included: (1) focusing on dynamic geometry investigational problems, (2) using clusters and carefully sequenced tasks to build mathematical ideas such as constructing 90 degree
quadrilaterals in the example Dr. Colin provided, and (3) posing questions that invite the teachers to notice, reflect and wonder about the mathematical objects and their relations. For the meta-tasks, he asked the teachers to (1) examine group problem solving logs and reflect on specific pedagogical issues about on their own experiences of discursive learning, and (2) read about research on specific pedagogical issues related to group interactions and learning, and technology mediated mathematical learning. The listed design practices by Dr. Colin were specific about the content, including both the mathematical content and the pedagogical content of the online professional development. Though he, as the PD facilitator, did not directly interact with the teachers during their synchronous sessions, these tasks or specific questions of the tasks acted as his “voice” as to how he would interact with the teachers to support their learning of mathematics or to support their understanding of the importance of the discursive mathematical learning. As the detailed analysis unfolds, each of the task design practices reflects Dr. Colin’s epistemological beliefs about mathematics and mathematics learning. They were not random, but instead, appeared to be connected and guided by a dialogic notion of mathematics and a social view of learning mathematics.

**Putting it all together.** In order to better understand Dr. Colin’s pedagogical practices of providing the online PD, including how he designed the environment, selected the subject and topics, and organized the learning activities in the online environment similar to other cases, I used a logic map to connect the different pedagogical practices involved in designing the PD as reported by Dr. Colin in the survey
and the interview. The logic map also aimed to model Dr. Colin’s design thought process for understanding his ultimate decision making.

The logic map is built from design logic progressions in the survey. Dr. Colin reported his design thinking was associated with Sequence B (Figure 31), which captures a technological-driven design logic: The teacher educator analyzes the affordances and constraints of the technological environment, which informs the teacher educator about the objective(s), specific topics and content activities for the professional development. Based on this design logic sequence, I added specific practices and considerations that Dr. Colin had undertaken in the design process, as reported during the interview, and built a Detailed Design Logic map for Dr. Colin (Figure 32).

Figure 31. Design logic sequence B
Figure 32. Dr. Colin’s Detailed Design Logic map
**How does it work?** As in the previous two cases, each of the rectangles, labeled C1 through C4, represents a point at which Dr. Colin considered the options for the PD design. Each of the ovals, labeled P1 through P9, represents a possible pedagogical practice by Dr. Colin. As shown on the left-hand side of the logic map, Dr. Colin’s detailed design logic was modeled in three stages. The first stage is designing the technological learning environment, which includes the considerations of the affordances and limitations of the virtual and technological environment for the learning of mathematics and subsequent pedagogical design practices adopted based on the analysis. The second stage, labeled “deciding objectives,” is about the professional development objectives for teachers using the technological environment designed. The third stage, labeled “choosing topics,” is about considerations for specific content activities and how they were organized to serve the objectives. In the following section, I will break down each practice level to analyze Dr. Colin’s design logic.

*Stage 1: Designing the technological environment.* Designing the technological environment was the most substantial aspect of Dr. Colin’s PD design. To Dr. Colin, the technological environment was not just about the use of the virtual environment or the technological tools used to deliver any content activities, instead it was about creating an environment to promote a specific kind of mathematics learning that was anchored in his epistemological beliefs about mathematics and mathematics learning. As such, this environment consisted of not only the technological tools to use, but also the corresponding content and content activities specifically designed for this technological environment. Technology and learning content was aimed to be integrated to promote
mathematics learning in this environment. Thus, as Dr. Colin checked the option of
design logic Sequence B, the technological-driven design logic, in the survey, the
technology, as described in the sequence, was more than the use of tools but also
included the corresponding learning accomplished with the tools. This was because Dr.
Colin’s design thinking was all about promoting discursive mathematical learning guided
by his epistemological beliefs. He said:

I think what comes even before either the tasks or the environment is our
epistemological perspective, our view of mathematics learning. It is a view that
takes mathematics learning to be at its core, a dialogic process of individuals
talking about objects and relations among objects. And it’s the relation among
objects which leads to the mathematics, and so our design of the environment had
to do with – yes we want groups of individuals to be in dialogue with each other
and so we’re designing an online environment that provides tools to support the
dialogue.

The emphasis the Dr. Colin placed on the epistemological perspective of
mathematics learning shows that any of his design practices for the online professional
development program were neither ad hoc nor involuntary, but highly motivated by his
beliefs on mathematics and mathematics learning. In fact, this professional development
program was part of Dr. Colin’s larger research project, in which he was seeking to glean
insights into the dialogical process individuals undergo while learning mathematics.

Accordingly, the first, and guiding consideration, C1, as shown in Figure 32, for
Dr. Colin was about providing an environment to support the dialogic process of
mathematics learning. Under this consideration, there were four sub-considerations.
These were: what technological tools were needed to support collaborative interactions
(C1.1); what technological tools were needed to support working on mathematics (C1.2);
what learning activities best support collaborative interactions (C1.3); and what role did
the facilitators have in the environment (C1.4). Each one of the considerations for the
technology design was guided by the overall purpose, supporting a dialogic process of
mathematical learning (C1). The first two considerations were about the provision of the
technological tools that support dialogic learning and working with mathematics. The last
two considerations were pedagogical considerations for participants’ (i.e. the teachers)
learning experiences, particularly, participants’ interactions with the content and
interactions with facilitators in the environment. For the first two considerations, Dr.
Colin and his team programmed a multi-modal environment that integrated a virtual chat
room and a dynamic geometry environment – GeoGebra so participants could use
mathematical tools in GeoGebra to work collaboratively on mathematical problems in the
virtual chat room. This is labeled P1, a practice, in design logic. The decision for this
practice was based on Dr. Colin’s previous experiences of conducting small group virtual
problem solving sessions, which informed him and his team about the needs of
 technological tools for working with mathematics. For considerations C1.3 and C1.4, Dr.
Colin chose to conduct small-group problem solving sessions for the virtual participants
(P2). Again, from previous research experiences, he learned that conducting small-group
problem solving sessions promoted productive collaborative interactions, and to further
maximize the effectiveness of collaboration, the facilitator should play a minimum role
during the session.

Thus far, from C1 to P2 the considerations and practices are about the
environment design, which was based on the analysis of the affordances and the
constraints of the environment from previous experiences of Dr. Colin when conducting virtual problem solving groups. Yet, the environment design did not stop there, since the environment went beyond the choice of technology and establishment of the environment. To Dr. Colin, the most important part of the environment was the content or tasks that fit into the environment, as he viewed the technological tools, the organizations of people, and the content as an integral of the environment. This leads to the second consideration, C2, what tasks should be used and how they should be organized. Based on his epistemological perspective of mathematics and mathematics learning that were mentioned earlier, he chose to focus on dynamic geometric investigation tasks (P3), since these tasks were supported by GeoGebra. (He mentioned that he and his team would also implement algebra and other content areas using this environment at a later date). These geometric tasks were carefully selected, clustered, and sequenced for the participants (P4). Specific questions were posed to invite teachers to notice, reflect and wonder about the mathematical objects and relations (P5). These were three pedagogical practices that Dr. Colin endorsed during the task design. Together with these carefully designed tasks and the purposefully designed technological environment (from C1 to P5), the virtual learning environment was established to support the intended discursive mathematical learning. Dr. Colin and his team operated this environment with students as well as teachers to better understand the dialogic process of learning in the environment. According to Dr. Colin, this stage, designing the environment, was a project that attempted to understand how small groups of individuals extend their abilities to engage in productive mathematical discourse.
**Stage 2: Deciding PD objectives.** The second stage, deciding objectives, as indicated in Figure 32 was specific for the professional development reported in Dr. Colin’s survey. Dr. Colin’s intention for designing the technological learning environment was to promote the discursive learning process in classrooms, that is, C3. In order to do so, he and his team conducted the professional development for teachers and set the objective so as the participating teachers were able to use the environment (including the integrated virtual chat room and GeoGebra environment and the tasks) for their students to promote productive mathematical discourse (i.e. P6). As such, this overarching objective for the professional development follows the environment design stage in Dr. Colin’s Detailed Design Logic Map, which is Stage 2 design thinking. In order for the teachers to implement the technological environment and the tasks, Dr. Colin engaged teachers in two practices: one was to have them, as students, solve geometric problems in small groups in the environment (P6); and the other was to engage teachers to reflect upon their own experiences of the discursive learning process (P7). These two objectives were the sub-goals for the PD.

**Stage 3: Choosing PD activities.** The last stage was deciding PD activities for the participants, which was the consideration C4. At this stage, Dr. Colin practiced three activities: one was for the teachers to engage in small-group geometric investigation sessions in the virtual chat rooms (P7); another activity was for the teachers to reflect upon their small group experiences (P8); and the third activity was to assign the teachers to read and reflect about their virtual discursive learning experiences in the literature (P9) in order to support their classroom teaching practices. For P9, Dr. Colin chose topics
relevant to discursive learning as well as technology and mathematics teaching and
learning for teachers’ reflection. These were the “meta-tasks” mentioned earlier.

From the top down. Similar to other cases, Dr. Colin’s detailed design logic map
could be understood vertically, as shown in Figure 33 and Figure 34. I identified two
paths in this logic. These two lines of thinking represent the two PD objectives, yet both
paths were built upon the stage of the learning environment design (i.e., stage 1). The first
path is about using the environment to engage the teachers in the dialogic process of
problem solving. The path starts from C1 to P7, covering the entire stage 1, stage 2, and
P7 as shown in Figure 33. This is the path that includes the considerations and practices
in designing the online professional development sessions for mathematics teachers to
experience the dialogic process of mathematical learning. That is, the aim was for the
mathematics teachers to engage in the learning process as mathematics students. The
second path is a line of design thinking specific for advancing teaching practices. This
path also covered the entire stage 1, stage 2 and then P8 and P9, as shown in Figure 34.
Similarly, this path also covered stage 1 because Dr. Colin’s considerations and practices
for this objective were built upon the use of the environment. Instead of the experiences
of learning the mathematical content, this path was about the learning of the
mathematical learning processes, that is, reflections about the mathematical learning
experiences.
Figure 33. Dr. Colin’s Detailed Content Design Logic vertical path I
Figure 34: Dr. Colin’s Detailed Content Design Logic vertical path II
Characteristics of Dr. Colin’s pedagogical practices. In one dimension (vertical), the Detailed Design Logic map modeled the perceptive flow of Dr. Colin’s design practices and pedagogical practices (i.e. P1 through P9) with specific considerations (i.e. C1 through C4) that he might have encountered. In the other dimension (horizontal), that is, from Stage 1 to Stage 3, the map clusters the MTE’s practices or considerations to describe the type of design activities involved. Based on the detailed design logic map, like in other cases, I identified three characteristics of Dr. Colin’s design thinking in the process of developing and implementing the online professional development. These characteristics include: (1) framing or approaching the “problem” from a particular perspective; (2) having a system approach of content design; and (3) identifying and resolving technological constraints proactively to meet research perspectives.

First, as indicated in the detailed design logic map, the MTE, Dr. Colin, in his practice of developing the professional development program, from framing the “problem,” designing the program, to implementing the program, was driven by a particular perspective of mathematics education. This was mentioned during the interview. He and his team developed the PD program in order to stimulate or enable the emergence of a particular perspective of mathematics learning, which was the discursive mathematical learning or productive mathematical discourse. In this case, the perspective that was taken was a personal one as it mirrored his beliefs about mathematics and mathematics learning. This perspective provided a holistic view of the online PD program. From how they set up the virtual chat room (e.g. small groups of 3-4
participants without facilitators) to how they designed the tasks for the participants (e.g. provided epistemic tools for discussions and interactions) these steps were guided by this perspective, though several considerations and adjustments were taken according to analysis of the environment. As such, though the professional development program was established to meet research purposes of the funding agent, the program development was strongly influenced by the MTE’s personal motivations to understand specific phenomenon of mathematics education.

Second, similar to Dr. Beth’s case, with regard to task design, I also came to see a “system” approach in Dr. Colin’s thinking. The “system” was manifested in Dr. Colin’s task design thinking, particularly: (a) how tasks relate to each other utilizing the affordance of the virtual learning environment; and (b) how tasks related to different areas of teacher development. As mentioned earlier, the participants were engaged in two types of tasks in the virtual sessions. First, they engaged in small-group mathematical problem solving tasks. Second, they examined their problem solving session logs to reflect upon specific pedagogical issues in mathematics teaching and learning. These were the meta-tasks in which the participants were engaged. The meta-tasks utilized an affordance of the virtual learning environment – the use of discussion logs, which were not easily available in the face-to-face environment. They were tasks developed from the problem solving tasks. In Dr. Colin’s design, the first “tasks” were the reflection tasks, and the second “tasks” were the mathematical tasks. These two types of tasks related to each other in a systematic way in Dr. Colin’s design thought process. Further, the design of the meta-tasks was aimed to incorporate: (a) the teachers’ mathematical problem
solving experiences, (b) the research about collaborative norms, mathematical practices, accountability, technology and mathematics education, and (c) the teachers’ reflections on their own experiences and practices of teaching mathematics. These meta-tasks systematically captured multiple areas of teacher development. Based on these two characteristics, the design of the meta-tasks reflects a “system” approach of the MTE’s thinking. It is a “system” because the PD content, purposes, and technological use were all related, incorporated, and reflected in the task design.

Lastly, with regard to the online learning environment, similar to Dr. Beth’s case, Dr. Colin and his team proactively identified the affordances and the constraints, especially the constraints of the environment in providing learning experiences for mathematics teachers. Yet, how Dr. Colin resolved the online technological constraints was very different from Dr. Beth’s case. With Dr. Beth’s case, she found the virtual learning environment was unable to provide the same level of interactions as in the face-to-face environment. Therefore, she adopted the technology, VoiceThread, and conducted mini lectures to resolve the lack of physical interaction problem, despite the fact that lecturing was not her ideal way of interacting with the participants. Her online PD design was developed around what the technology allowed and did not allow. In contrast, to resolve the environment constraints, Dr. Colin and his team redesigned the technological environment in order to meet their own perspectives of learning. They programmed the referencing tools to the chat room to enhance interactions and incorporated GeoGebra for working with mathematics and they also designed tasks specifically to be used in the environment. The redesigning process allowed for the desired learning process to emerge
in the environment without the cost of forgoing the pedagogical values that the MTE held. Dr. Colin took a much more proactive approach than Dr. Beth with regard to technology. I call it “proactively identifying and resolving technological constraints to meet research perspective.”

It is important to note that the comparisons of Dr. Beth and Dr. Colin’s practices for the third point were not to devalue or praise either MTE’s practices. Given the time, resources, and purposes of the PD programs (Dr. Colin’s PD development was a research project), there could have been limitations to redesigning the online environment to serve any desired goals. The emphasis of the comparison was to identify the differences in the design thought processes involved: one chose to use existing online delivery tools for the PD and worked around it; and the other redesigned the online environment, which was rather proactive in his thinking and practice considering the constraints encountered by the online environment for mathematics teachers’ learning.

**MTE knowledge involved in Dr. Colin’s design logic.** In the previous sections, I modeled Dr. Colin’s design thought process logic used to develop the online professional development program for middle and high school mathematics teachers. I highlighted several characteristics of the design thought process that were unique to Dr. Colin in the context of the PD development. In this section, I will focus on the type of mathematics teacher educator (MTE) knowledge involved at each level of the design logic in his design pedagogical practices. Before proceeding, I would like to disclaim that the knowledge structure derived here was unique to the context of developing this specific online professional development program. Like any accomplished mathematics teacher
educators, Dr. Colin has a wide range of knowledge that might be employed elsewhere at different times.

According to Figure 32, Dr. Colin’s design logic was modeled in three stages: designing the technological learning environment, deciding the PD objectives, and choosing topics for the objectives. In the technological learning environment design stage the thought process included: acquiring knowledge for choosing what technological tools to use for collaboration (C1.1), determining what technological tools to use for working on mathematics (C1.2), identifying the most effective type of mathematics learning activities in the online environment (C1.3), and understanding the role of facilitators in the online environment (C1.4). To a large extent, the knowledge of technology was specifically about the knowledge of the online environment for supporting learning experiences in general as well as mathematics learning experiences. As such, the types of mathematics teacher educator (MTE) knowledge that Dr. Colin relied on at this level of PD development were knowledge of the online environment and pedagogical knowledge of using the online environment for mathematics teacher educating practices. In Dr. Colin’s design thinking and practices, such knowledge was informed by his knowledge of mathematics teachers, knowledge of teaching and learning with mathematics technology, and knowledge of online learning. These knowledge sources were drawn from Dr. Colin’s own research experiences in mathematics education, including conducting research projects and knowledge from the research community (that is, professional traditions). He mentioned during the interview that prior to developing the online PD for mathematics teachers he was involved in another research project that investigated small
groups of students who were tasked with problem solving in virtual chat rooms with basic online tools. Consequently, his past empirical research and experiences of theorizing about small group problem solving in the online environment had prepared him tremendously for the analysis of the technological environment for the online PD.

In the same stage, Dr. Colin’s practices were identified by P1 through P5. Practices P1 and P2 (that is, integrating the chat room and the dynamic geometry environment, and engaging teachers in small group problem solving without facilitators in the chat room) were practices implemented after the technological environment analysis was conducted. As such, the knowledge involved in these two practices was the same as mentioned in the previous discussion.

For P3, P4 and P5, these were practices for mathematical tasks design: focusing on dynamic geometric investigation (P3), sequencing and clustering in a precise manner in order to build certain mathematical ideas (P4), and inviting the teachers to notice, reflect and wonder about the mathematical objects and relations (P5). The knowledge domains that the MTE relied on to develop these tasks were about the content of geometry and the technological environment for geometry, GeoGebra. Using Mishra and Koehler’s (2006) construct, the relevant knowledge domain is Technological Pedagogical Content Knowledge (TPCK) in geometry. According to Hill, Ball and Schilling’s (2008) model of mathematical knowledge for teaching: the specific content knowledge domains involved in practices P3 and P4 were specialized content knowledge (SCK) in geometry and knowledge of mathematics horizon. The clustering and sequencing practice required the MTE to extensively draw from the knowledge of
mathematics horizon since it was about constructing mathematical ideas. Indeed, both knowledge domains that the MTE relied on in practices P3 and P4 were knowledge for mathematics teaching. In other words, these were knowledge domains that a mathematics teacher was supposed to rely on if he or she was teaching children geometry in the way Dr. Colin was engaging the participating teachers in geometry learning in the PD sessions. As such, these knowledge domains are learner knowledge as in Perks and Prestage’s (2008) teacher educator knowledge tetrahedron. For practice P5—inviting the participants to notice, reflect and wonder about mathematical objects and relations through careful problem posing, it is a similar method to the knowledge of mathematics and teaching (KCT) in Hill et al.’s (2008) knowledge model for mathematics teachers. Dr. Colin’s approach to inviting mathematics learners (he called these questions epistemic tools for dialogues about mathematics) was guided by his epistemological perspective of mathematics.

In stages 2 and 3, the design focus was on developing the PD for the teachers. Particularly, in stage 3, when the MTE made decisions about the PD activities, several knowledge domains were salient in his pedagogical practices. For the three pedagogical practices in stage 3, one was about developing the teachers to become learners of mathematics by engaging them in small-group problem solving sessions (P7), and another was about encouraging the teachers to reflect on the discursive learning experiences that took place in the problem solving sessions (P8). Knowledge domains for these two practices were about how teachers learn, which is knowledge of mathematics teachers and their needs. Engaging teachers in problem solving and reflecting on learning
experiences are conventional practices in teacher education, which have been widely utilized and extensively researched for their effectiveness. The decision to incorporate these two practices into the online PD relied on the MTE’s knowledge of professional tradition in mathematics teacher education, as in the teacher educator knowledge tetrahedron. Moreover, for P8, examining group problem solving logs and reflecting upon specific pedagogical issues, such a practice utilized the affordance of the virtual learning environment – records of logs, for teacher reflection purposes. The MTE utilized practical wisdom involving technological use as well as teacher education approach. It is the pedagogical knowledge for online mathematics teacher educating.

Lastly, for practice P9, assigning reading on specific pedagogical issues, knowledge that the MTE relied on was also a conventional practice among the mathematics teacher education community (i.e. professional traditions in mathematics teacher education).

In the following, I summarize the knowledge domains involved in Dr. Colin’s practices and his considerations in developing the online PD program for mathematics teachers:

- **K1**: Professional traditions in mathematics teacher education – teacher learning through reflection on practices and reflection on theories
- **K2**: Learner knowledge – knowledge of teaching and learning mathematics with technology, that is, technological pedagogical content knowledge (TPCK or TPACK) in geometry
- **K3**: Learner knowledge – specialized content knowledge (SCK) in geometry
• K4: Learner knowledge – knowledge of mathematical horizon
• K5: Professional traditions – knowledge of research in mathematics teaching (e.g. collaboration norms, accountability, Common Core Mathematical Practices, technology and mathematics teaching)
• K6: Practical wisdom – conducting mathematics learning sessions in the virtual environment
• K7: Practical wisdom – Knowledge of the online environment, and pedagogical knowledge of conducting mathematics teacher education sessions in the online environment

These knowledge domains were not drawn independently for each practice or consideration that the MTE undertook. For example, K2 – knowledge of dynamic geometry environment and students’ geometry learning, and K7 – knowledge of the online environment was drawn together for the design of geometric problem solving tasks in the PD. Knowledge domains of K1 (teacher reflection on theory and practices), K5 (research in pedagogical issues) and K7 (knowledge of affording of the virtual environment in providing resources for teacher learning) were drawn together for the design of the meta-tasks for teachers. The relationship of the knowledge domains that Dr. Colin drew from in his practices is illustrated in Figure 35 using Perks and Prestage’s (2008) teacher educator knowledge tetrahedron.
Case synthesis. In the following Table 14, I combined the three stages of Dr. Colin’s PD design activities with his goals, orientations and resources to identify the types of decisions he made. Based on the table, I will synthesize how the role of goals, orientations, and knowledge come into play at different stages of Dr. Colin’s design process.
Table 14. Dr. Colin’s goals, orientations, knowledge, and decisions

<table>
<thead>
<tr>
<th>Stages</th>
<th>Decisions</th>
<th>Guiding goals</th>
<th>Guiding Orientations</th>
<th>Guiding Resources/Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 1: Designing environment</td>
<td>What for? Subgoal (G2)</td>
<td></td>
<td>• Orientations toward mathematics (O1, O2)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Orientations toward mathematics learning (O4)</td>
<td></td>
</tr>
<tr>
<td>Where? Virtual chat room</td>
<td>Subgoal (G2)</td>
<td>• Orientation toward the online learning environment (O10)</td>
<td>• Practical wisdom of conducting online teacher education (K6)</td>
<td></td>
</tr>
<tr>
<td>Through which? Virtual chat room integrated with GeoGebra</td>
<td>Subgoal (G2)</td>
<td>• Orientations toward mathematics learning and technology (O5)</td>
<td>• Practical wisdom of conducting online teacher education (K6)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Learner knowledge – knowledge of teaching and learning mathematics with technology (K2)</td>
<td>Continued</td>
</tr>
</tbody>
</table>
Table 14 continued

<table>
<thead>
<tr>
<th>Stages</th>
<th>Decisions</th>
<th>Guiding goals</th>
<th>Guiding Orientations</th>
<th>Guiding Resources/Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 1: Designing environment</td>
<td><strong>What? Geometry</strong></td>
<td>Subgoal (G2)</td>
<td></td>
<td>Learner knowledge – knowledge of teaching and learning mathematics with technology (K2)</td>
</tr>
<tr>
<td></td>
<td><strong>How to illustrate?</strong></td>
<td></td>
<td>Orientations toward mathematics (O3)</td>
<td>Learner knowledge – knowledge of teaching and learning mathematics with technology (K2)</td>
</tr>
<tr>
<td></td>
<td><strong>With whom?</strong></td>
<td>Subgoal (G2)</td>
<td></td>
<td>Practical wisdom of conducting online teacher education (K6)</td>
</tr>
<tr>
<td></td>
<td><strong>How control?</strong></td>
<td>Local goals (G4, G5, G6)</td>
<td>• Orientations toward mathematics (O1, O2) • Orientations toward mathematics learning (O4)</td>
<td>• Learner knowledge – knowledge of teaching and learning mathematics with technology (K2) • Learner knowledge - specialized content knowledge (SCK) in geometry (K3) • Learner knowledge – knowledge of mathematics at the horizon (K4) • Practical wisdom of research in mathematics teaching and learning (K6)</td>
</tr>
</tbody>
</table>

Continued
Table 14 continued

<table>
<thead>
<tr>
<th>Stages</th>
<th>Decisions</th>
<th>Guiding goals</th>
<th>Guiding Orientations</th>
<th>Guiding Resources/Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 2:</td>
<td>Deciding PD objectives</td>
<td>What for? Overarching PD goal and subgoals <em>(G1, G2, G3)</em></td>
<td>• Orientation toward mathematics teaching <em>(O6, O7)</em> • Orientation toward mathematics teacher education <em>(O8)</em></td>
<td>• Professional traditions in mathematics teacher education <em>(K1)</em></td>
</tr>
<tr>
<td>Who?</td>
<td>Middle school and high school mathematics teachers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stage 3:</td>
<td>Choosing PD activities</td>
<td>What? Dialogic learning process</td>
<td>Overarching goal <em>(G1)</em> and subgoals <em>(G2, G3, G4)</em></td>
<td>• Orientation toward mathematics teaching <em>(O6, O7)</em> Orientation toward mathematics teacher education <em>(O8)</em></td>
</tr>
<tr>
<td>How illustrate?</td>
<td>Choices of approaches to the subject matter</td>
<td>Overarching goal <em>(G1)</em> and subgoals <em>(G2, G3, G4)</em></td>
<td>• Orientation toward mathematics teacher education <em>(O9)</em></td>
<td>Professional traditions in mathematics teacher education <em>(K1)</em></td>
</tr>
<tr>
<td>How control?</td>
<td>Task design</td>
<td>Overarching goal <em>(G1)</em> and subgoal <em>(G7)</em></td>
<td>• Orientation toward mathematics teacher education <em>(O9)</em></td>
<td>Professional traditions in mathematics teacher education <em>(K1)</em></td>
</tr>
</tbody>
</table>
What decisions were made? The second column of the table describes the types of decisions that Dr. Colin made during the three stages of PD design. According to Boyd’s (1993) dimensions of distance education decisions, Dr. Colin’s decisions involved all eight dimensions, including: (a) decisions for the PD goals (i.e. what for?), (b) decisions for the subject matter (i.e. what?), (c) decisions for learning activities (i.e. how illustrate), (d) decisions for the use of virtual space (i.e. where?), (e) decisions for the use of technological media for the subject matter (i.e. through which?), (f) decisions for the participants ((i.e. who?), (g) decisions for social groupings and interactions ((i.e. with whom?), and decisions for rules and mechanisms of the PD (i.e. how control?).

Examining the decision dimensions, neither Dr. Abby nor Dr. Beth’s design activities involved all eight dimensions of decisions. This professional development strategy was a part of Dr. Colin’s research project into discursive mathematical learning. His design activities and thinking were akin to a research process involving a long and careful design process: he framed research goals based on his epistemological beliefs towards mathematics and mathematics learning; he and his team adopted and redesigned technological tools to serve the research goals including redesigning the virtual chat rooms to have reference functions and GeoGebra tools; and they also carefully designed geometry investigation tasks specifically for individuals’ learning in the integrated virtual environment they designed. The professional development program was like the data collection phase from which he could study individuals’ learning process in the environment. The nature of such design activities determined the MTE to be purposeful in every dimension of the design activities as every dimension served an inquiry. If one
dimension was not controlled, the outcome of the inquiry might not be controlled for either dimension. Therefore, unlike the previous two cases, Dr. Colin’s design decisions involved all dimensions of Boyd’s model (Figure 36), according to my analysis.

Figure 36. Dimensions of Dr. Colin’s decisions circled in red

Another feature of Dr. Colin’s decisions was the nested nature of his online professional development program. Nested means this online PD was a portion of the
larger research project, so part of the PD design (which is stage 1, designing the technological environment) was drawn from the larger project which essentially was a study about mathematics learning. Only from stage 2 was the design specifically for the PD project, which was about teacher learning. Since mathematics learning and teacher learning are two highly overlapping areas in mathematics teacher education, the MTE used teachers’ mathematical learning as one of the objectives for teachers’ professional development (i.e., P6). He also added another objective related to teaching practices for the participants (i.e., P7). Both objectives operated under the same environment yet had different content. Consequently, some of the decisions, such as decisions about “where?” and “through which?”, were made in stage 1; and some of the decisions, such as “what for?”, “what?”, “how to illustrate?”, were made in stage 1 as well as stages 2 and 3 because the content was different so the design was different. Specifically, as shown in Table 14, Dr. Colin made decisions about the subject matter in stage 1 and the subject matter was determined to be dynamic geometry investigation in this stage. He also made decisions about the subject matter in stage 3; however, the subject matter was determined to be about the dialogic learning process in that stage. Similarly, he made decisions about task design (that is, decisions about “how to control”) in stage 1 and these tasks were geometric problem solving tasks. Likewise, he also made decisions about task design in stage 3 and these tasks were the meta-tasks, the pedagogical tasks for teacher reflection. Since the MTE had two objectives for the teachers, the decisions for content-related dimensions were made twice in his design logic.
Goals, orientations, knowledge, and decisions. The orientations and knowledge that Dr. Colin drew from in his decision making were highly dependent on the stages of his design logic. In stage 1, his decisions were about designing the technological learning environment that supports the dialogic learning process. The MTE’s thinking was around mathematics, mathematics learning and technology. As such, the orientations and knowledge domains that he relied upon were research in mathematics learning, and mathematics learning and technology. Another uniqueness of this stage was the integrated virtual chat room and GeoGebra design. Such thinking was largely from the MTE’s research experiences of learning with digital environments. In contrast to stage 1, Dr. Colin’s thinking was related to teachers’ professional learning in stages 2 and 3. As such, the knowledge domains and orientations that Dr. Colin drew from were mathematics teacher education related, including teachers’ content knowledge and teaching practices. All in all, regardless of the stages of the design, Dr. Colin’s design thinking and decision making were motivated by his epistemological view about mathematics as a cultural product, which was the original source of the research project, and hence, the impetus for his online professional development project.

Cross-Case Analysis

In the previous three sections, I presented detailed accounts of three MTEs’ online PD design and their rationales for the choices they made for their online programs based on survey information, interviews, and artifacts of their practices. This section offers a cross- analysis of these three cases to provide an account of the differences and similarities among the three subjects’ decision making processes and their design
thinking models. The cross-case analysis serves as foundation for proposing a model of MTEs’ decision making.

**Comparing design logic stages.** In each case study, Detailed Design Logic maps were produced to delineate each subject’s logic when designing the online PD programs (see Figure 18, Figure 24, and Figure 32 for reference). Each Detailed Design Logic map was summarized in stages to provide each MTE’s pedagogical practices that were endorsed with regard to considerations made pertaining to the PD goals and objectives, focusing concepts and activities, and online organization of the activities. Table 15 compares the stages across the three subjects.

Table 15. Three MTEs’ stages of their design logics

<table>
<thead>
<tr>
<th>Stage</th>
<th>Dr. Abby</th>
<th>Dr. Beth</th>
<th>Dr. Colin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 1</td>
<td><em>Deciding</em> PD goals and objectives</td>
<td><em>Identifying</em> PD goals and objectives</td>
<td><em>Designing</em> online learning environment</td>
</tr>
<tr>
<td>Stage 2</td>
<td><em>Deciding</em> focusing concepts</td>
<td><em>Adopting</em> focusing concepts and concept-related topics and activities</td>
<td><em>Deciding</em> PD objectives</td>
</tr>
<tr>
<td>Stage 3</td>
<td><em>Deciding</em> concept-related topics and activities</td>
<td><em>Designing</em> online delivery</td>
<td><em>Choosing</em> PD content activities</td>
</tr>
<tr>
<td>Stage 4</td>
<td><em>Designing</em> online delivery</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Each PD program consisted of three broad components in each of the three subjects’ design. The components were *disciplinary concepts* (mathematical concepts) addressed, such as number and algebra, *concept-related topics and activities*, such as mathematics reasoning and student representations, and *online delivery/organization of the topics and activities*, for example, small group problem solving, watching and
reflecting on video presentations, posting in the discussion forum and so on. MTEs’
design practices were identifying, deciding, designing or adopting these components,
which were the verbs shown in italic in Table 15. “Deciding” and “designing” signify
that the pedagogical choices were initiated by the MTEs themselves. This was the case
for Dr. Abby and Dr. Colin. “Identifying” and “adopting” indicate that the pedagogical
choices were adopted from other sources although implementation choices were
autonomously made by the MTEs. This was the case for Dr. Beth as her learning
objectives, mathematical content and content activities were predetermined by a larger
PD program she was to implement online.

The shaded cells in the table indicate occasions in which the online environment
interacted with the MTEs’ practices and decisions. As shown in the table, the online
environment interacted with Dr. Abby’s design practices in stages 1, 2, and 4, while
technology considerations were identified in only one stage for both Dr. Beth and Dr.
Colin. The technological environment occurred in the content delivery stage (i.e. stage 3)
for Dr. Beth, and in the environment design stage (i.e. stage 1) for Dr. Colin. In Dr.
Colin’s case, choices about technological tools, structure of online problem solving
sessions, mathematics concepts of interest, and task design were all considered in the
same stage.

It is important to note that the design logic stages are abbreviated versions of each
MTE’s Detailed Design Logic maps. The stages do not necessarily imply a linear
progression of thinking on the part of the MTEs, rather, they identify the relationships of
the key phases of practices associated with their design. For example, objective decisions
guided Dr. Abby’s choices on focusing concepts; meanwhile, the environment design stage guided Dr. Colin’s decisions on the PD session objectives. Comparison of the three MTEs’ design logic stages revealed similarities between Dr. Abby’s and Dr. Beth’s PD content decisions. Both of their design logic maps contained three distinct phases of content design. The first phase is about the choice of the mathematical strands or concepts for the PD participants. The second phase is about decisions regarding how mathematics may need to be organized through sequencing coherent activities for the participants. The third phase is about how these PD activities need to be delivered via the online medium. In Dr. Colin’s case, these three phases were not so obvious. A major part of his content decisions was incorporated in the environment design stage. As such, his choice of focusing concepts, activities, and tasks were specifically designed for the environment where his desired learning experiences for the teachers would take place.

In the following section, I will elaborate on each stage of the MTEs’ Detailed Design Logic maps to compare their design thought process regarding the online PD’s focus, choice of mathematical concepts, topics and activities related to the concepts and online organization of the activities.

Comparing MTEs’ PD content and content activities. Table 16 illustrates the MTEs’ content decisions with regard to the PD focus and selected disciplinary concepts. Table 12 describes their decisions about the PD activities according to their choice of focusing concept or concepts, and online organization of PD activities as extracted from the Detailed Design Logic maps. The numbers in parenthesis in Table 17 refer to their pedagogical practices as described in their Detailed Design Logic maps.
Table 16. Three MTEs’ PD content decisions breakdown

<table>
<thead>
<tr>
<th>PD focus</th>
<th>Disciplinary concepts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dr. Abby</td>
<td>Teaching statistics</td>
</tr>
<tr>
<td></td>
<td>Statistical investigation cycle</td>
</tr>
<tr>
<td>Dr. Beth</td>
<td>Teachers’ mathematical knowledge in number and algebra</td>
</tr>
<tr>
<td></td>
<td>Early number, rational number, operations and algebra</td>
</tr>
<tr>
<td>Dr. Colin</td>
<td>Discursive mathematics learning</td>
</tr>
<tr>
<td></td>
<td>Dynamic geometry</td>
</tr>
</tbody>
</table>

Table 17. Three MTEs’ PD activities and online delivery methods

<table>
<thead>
<tr>
<th>PD topics and activities</th>
<th>Online delivery methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dr. Abby</td>
<td>Multiple online resources including:</td>
</tr>
<tr>
<td>• Developing habits of mind of statistical reasoning (P3)</td>
<td></td>
</tr>
<tr>
<td>• Solving real-world statistical investigation problems (P3)</td>
<td></td>
</tr>
<tr>
<td>• Statistical learning stages (P4)</td>
<td></td>
</tr>
<tr>
<td>• Analyzing and designing statistics investigation tasks (P5)</td>
<td></td>
</tr>
<tr>
<td>Dr. Beth</td>
<td>Mini lecture presentation including:</td>
</tr>
<tr>
<td>• Revisiting problems that teachers are familiar with (P4)</td>
<td></td>
</tr>
<tr>
<td>• Examining multiple representations from children’s mathematics (P5)</td>
<td></td>
</tr>
<tr>
<td>• Making connections of children’s mathematics (P6)</td>
<td></td>
</tr>
<tr>
<td>Dr. Colin</td>
<td>Small group solving dynamic geometry investigational tasks (P7)</td>
</tr>
<tr>
<td>• Understanding productive mathematical discourse, the tasks and the environment (P6)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Reflecting discursive learning experiences (P8)</td>
</tr>
<tr>
<td></td>
<td>Literature reading reflection (P9)</td>
</tr>
</tbody>
</table>
In Dr. Abby’s PD program, teaching and learning statistics was the focus. Most specifically, she targeted teachers’ statistics teaching practices with a goal of helping them develop a better understanding of the methodologies they could use to enhance learning of the subject. As such, she chose the statistics investigation cycle as a model to engage her teachers to make sense of statistics, and to understand how statistics should be taught and exposed to students. She used activities such as solving statistical explorations, analyzing habits of mind, statistical thinking, and designing statistical investigations problems in the MOOC sessions. All activities were centered around the statistical investigation cycle.

Dr. Beth’s online PD focus was on developing mathematics teachers’ content knowledge about number concepts and early algebra. Number operations, rational number, and connections were among the concepts that were addressed in the online PD. Related PD topics included children’s concept representations, and learning progressions within and across various number and algebra concepts. In Dr. Abby’s and Dr. Beth’s respective designs, developmental perspective regarding how children learn mathematics (such as learning progression), guided their concept selection. They used examples of school learners’ reasoning as the means to help teachers develop an understanding of a variety of ways that children solved problems.

Dr. Colin’s PD’s focus concerned developing productive mathematical discourse in classrooms. Geometry was a mathematical strand that he used in the online PD. Even so, central to the PD was not investigating topics from geometry, but rather, the central focus of the PD was delving into the discursive learning experiences pertaining to
learning geometry. The PD was designed so the teachers could understand the intricacies of productive mathematical discourse through the use of tasks. Geometry was a vehicle for generating productive discourse.

The second column of Table 12 shows how these activities were organized in the online environment. In Dr. Abby’s MOOC PD design, for every module, she organized a variety of online learning resources including multi-media presentations (e.g. videos, animations) for teachers, students and researchers, multiple statistics investigation websites, discussion forums and others alike. In Dr. Beth’s online design, mini lectures were utilized as the main media where all content activities were delivered. She organized modules around these mini lectures to include concepts of number and early algebra as covered in the larger PD. She posted mini videos to be viewed and the PD participants were also required to post mini videos to interact with each other. Lastly, in Dr. Colin’s design, the participants engaged in small-group problem solving activities and were asked to reflect on those activities in an effort to experience discursive mathematical learning. Unlike Dr. Abby’s and Dr. Beth’s online delivery methods, Dr. Colin did not only choose online delivery methods, but he also redesigned the online learning environment to incorporate a dynamic mathematics environment for the purposes of achieving desired learning experiences for the teachers.

The three MTEs’ PD designs maintained unique features regarding their choices of focusing concepts to methods of online delivery. Yet, despite this diversity in features, their methodologies relied on some common approaches such as analyzing students’ mathematics or solving investigation problems. Each MTE imparted themselves – their
knowledge, beliefs and orientations in their PD design. The following comparative analysis will focus on the three MTEs’ goals, and orientations that guided their design and the knowledge domains that they exercised during the process.

**Comparing MTEs’ goals, orientations and knowledge.** To understand how MTEs made their decisions, I grounded my comparison using the three constructs in Schoenfeld’s (2010) decision making theory: goals, orientations, and resources/knowledge of each MTE for their pedagogical practices of conducting online PD programs.

**Goals.** In the case analysis, I distinguished between overarching goals and subgoals for each of the MTEs. Overarching goals could be classified as those goals associated with the entire project, while subgoals concerned specific learning objectives for the PD participants or design goals for the MTEs. The overall goals guided the PD design and the subgoals guided the specific online session design. For Dr. Abby, the overarching goal was enhancing teachers’ teaching of statistics. Her subgoals included providing statistics learning opportunities for the participating teachers according to what they needed for effective practice. Dr. Beth’s overarching goal was advancing mathematics teachers’ content knowledge in the areas of early number concepts, rational number, operations and algebra. Her subgoals were set to ensure the participants fully encountered students’ various mathematics representations along with the progressions of their representations during the online PD. Though both MTEs aimed to address mathematics content learning for teaching, Dr. Abby’s goals lean more towards development of pedagogical skills pertaining to the subject (i.e. pedagogical content) by
providing a wealth of online teaching and learning resources, while Dr. Beth’s goals lean more towards the development of teachers’ mathematical content knowledge with a focus on children’s mathematics. In the case of Dr. Colin, the overarching goal was to foster productive mathematical discourse among teachers. His subgoals included providing a productive environment (both technological and mathematical) for development of generative discourse. While Dr. Abby’s goals emphasized teaching statistics and Dr. Beth’s goals emphasized learning mathematics, Dr. Colin’s goals focused on the environment design where technology, mathematics and tasks were to be incorporated in an effort to advance productive mathematical discourse among teachers.

Each MTE’s goals were reflected in what they identified as the PD foci and learning activities they included in their design. Their goals were all unique as the MTEs either incorporated their personal orientations and perspectives or honored goals and objectives set by others when planning. Indeed, one distinguishing feature among the MTEs was whether they had set the goals themselves.

In Dr. Abby’s case, I listed six sub-goals under one overarching goal. Three of these subgoals pertained to the MOOC content, and three of the subgoals addressed the MOOC learning environment design. The overarching goal and the three MOOC content subgoals were related to statistics teaching, set by Dr. Abby herself, while the three MOOC learning environment design subgoals were adopted from the funding agent. Dr. Abby’s decisions over the PD content were guided by a combination of her own goals as well as those goals mandated by the funding agent. In other words, she had to consolidate
the external demands with her own agenda when she made decisions about the PD
content and content activities.

Similar to Dr. Abby, some of the goals and objectives that guided Dr. Beth’s
design decisions were not set by her. I identified eight goals for Dr. Beth: two
overarching goals, two subgoals for online implementation design, and four local goals
for designing one specific online module. The overarching PD goals and the four local
goals were related to the PD subject-matter and content activities, which were adopted
from the original PD program. Dr. Beth was not the goal setter for the PD content and
content activities. Rather, her online implementation design was meant to meet the
preconceived goals of the face-to-face program.

In Dr. Colin’s case, I identified seven goals for his online design decisions. One
overarching research goal, two subgoals for the PD activities and four subgoals for
designing PD tasks. All of these goals, set by him and his team, derived from his
epistemological views on the nature of mathematics learning.

For Dr. Abby, her online PD design was to develop a program that met the
requirement of a funding agent; for Dr. Beth, the online PD design was about
implementing an already existing PD; and for Dr. Colin, his online PD design was a
research project where he intended to fulfill his own program of research. To this extent,
the degree of freedom for setting the PD goals and making decisions for the PD varied.
As evidenced from the above analysis, if the goals were set by the MTEs themselves,
then they exerted greater influence over the topics as well as how those topics were
covered in the PD.
Orientations. Orientations that guided each MTE’s PD content decisions were summarized in Table 18. Each MTE’s orientations toward mathematics, mathematics teaching and learning, and teacher education were reflected in their practices. As shown in the table, their orientations toward mathematics or mathematics learning permeated all of their practices. Unlike the other two MTEs, Dr. Abby’s decisions about the targeted focusing concept were geared toward her considerations of teachers’ needs in their teaching; consequently, her views on teachers’ needs played a major part in how she organized her sessions.

The MTEs’ orientations reflected their epistemological perspectives on the nature of mathematics and mathematics learning. Specifically, the perspective that guided Dr. Abby’s entire design, including her PD goals, choices of focusing concepts, tasks and activities was her epistemological perspective on the nature of statistics and statistics learning. Dr. Beth’s online PD content design was grounded in her knowledge of cognitive psychology and how mathematics learning was accounted for through that lens. Her PD stressed multiple representations, mental images that children associated with different mathematical concepts and links among various ideas they processed. Although Dr. Beth’s decisions about the PD’s mathematics content and activities were determined by others, her own orientations and perspectives aligned with them; hence, she found it easy to adopt and implement the content in the online PD sessions. In Dr. Colin’s case, his epistemological views on the nature of mathematics as a cultural product and epistemologies of mathematics learning guided his content decisions so in his case, the
PD focus was about ways in which the teachers could experience the evolution of mathematical knowledge through interaction.

Table 18. Three MTEs’ guiding orientations for PD content decisions

<table>
<thead>
<tr>
<th></th>
<th>Decisions about disciplinary concepts guided by</th>
<th>Decisions about concept-related topics and activities guided by</th>
<th>Decisions about online organization of content activities guided by</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dr. Abby</td>
<td>Orientations toward: • statistics • statistics learning • teachers and teacher needs</td>
<td>Orientations toward: • statistics • statistics learning • statistics teaching</td>
<td>Orientations toward: • statistics learning • teacher education • the utility of the online environment</td>
</tr>
<tr>
<td>Dr. Beth</td>
<td>Orientations toward: • mathematics learning</td>
<td>Orientations toward: • mathematics learning</td>
<td>Orientations toward: • mathematics learning</td>
</tr>
<tr>
<td>Dr. Colin</td>
<td>Orientations toward: • mathematics • mathematics learning</td>
<td>Orientations toward: • mathematics • mathematics learning</td>
<td>Orientations toward: • mathematics • mathematics learning</td>
</tr>
</tbody>
</table>

Knowledge. The analysis of the three MTEs’ Detailed Design Logic maps revealed differences in the type of knowledge domain each selected and addressed during their PD sessions. In this section, I will identify and compare the knowledge domains that the three MTEs relied on in the process of developing their PD designs. The comparative analysis of the MTEs’ knowledge is grounded in the teacher educator knowledge tetrahedron proposed by Perks and Prestage (2008), which was discussed in the literature review chapter of this dissertation. In addition to the knowledge tetrahedron, I used Hill, Ball & Schilling’ (2008) categories of mathematical knowledge for teaching (MKT) to specify particular areas in which the MTEs drew from in their work. The intent to combine the two knowledge models was to provide a better understanding of what
domains of knowledge of mathematics teachers’ were present in mathematics teacher educators’ knowledge. Table 19 summarizes the knowledge domains that the MTEs drew from as they designed and implemented the online PD sessions. As shown in the first column of the table, *knowledge of mathematics teachers and teachers’ needs (KMT)* was the primary domain for all three of the MTEs’ decisions about the PD’s focus. With respect to the teacher educator tetrahedron, KMT pertains to practical wisdom at the teacher educator knowledge level as it is a portion of designing teacher education sessions (Perks & Prestage, 2008). Indeed, addressing teachers’ needs was the driving force for all three MTEs, but as revealed in each MTEs’ Detailed Design Logic maps, what they actually identified as teachers’ needs were different. Specifically, Dr. Abby focused on a precise piece of statistics and ways it should be taught. Her *knowledge of statistics and teaching* (Hill et al., 2008) was a key source that informed her that this element was important for her PD participants, and this piece knowledge belongs to learner knowledge of the teacher educator knowledge tetrahedron. Additionally, Dr. Abby drew from her *knowledge of MOOC* to identify the population of her participants so as to identify their common needs. Dr. Beth’s PD focused on mathematics teachers’ content knowledge development using children’s mathematics. She drew from the *knowledge of students and mathematics* for her PD focus and this piece of knowledge also belongs to learner knowledge of the teacher educator tetrahedron. Dr. Colin identified that teachers’ need to experience mathematics learning as a cultural practice. He drew from the knowledge of research and theory of mathematics learning to inform his decision and this knowledge also belongs to the learner knowledge of the teacher
educator tetrahedron. With all three cases, the MTEs’ learner knowledge was the key source that influenced their knowledge of mathematics teachers and teachers’ needs, which informed how they focused the development of their PD program. The interaction between learner knowledge and practical wisdom confirms that knowledge domains do influence each other to support MTEs’ decision making as suggested by Perks and Prestage (2008).

Table 19. Knowledge domains that the three MTEs relied on for PD content decisions

<table>
<thead>
<tr>
<th>Dr. Abby</th>
<th>Decisions about PD focus and disciplinary concepts drew from</th>
<th>Decisions about concept-related topics and activities drew from</th>
<th>Decisions about online organization of content activities drew from</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Knowledge of mathematics teachers’ needs and effective teacher education</td>
<td>• Knowledge of student statistical reasoning</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Knowledge of statistics and teaching</td>
<td>• Knowledge of statistics learning with technology</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Knowledge of MOOC</td>
<td>• Knowledge of tasks and statistics learning</td>
<td>• Knowledge of MOOC</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Knowledge of online environment</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Knowledge of statistics online resources</td>
<td></td>
</tr>
</tbody>
</table>

Continued
Table 19 continued

<table>
<thead>
<tr>
<th>Decisions about PD focus and disciplinary concepts drew from</th>
<th>Decisions about concept-related topics and activities drew from</th>
<th>Decisions about online organization of content activities drew from</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dr. Beth</td>
<td>• Knowledge of mathematics teacher change</td>
<td>• Knowledge of online environment</td>
</tr>
<tr>
<td></td>
<td>• Knowledge of students and mathematics learning</td>
<td>• Knowledge of online environment</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Knowledge at the mathematics horizon</td>
</tr>
<tr>
<td>Dr. Colin</td>
<td>• Knowledge of mathematics teacher needs</td>
<td>• Knowledge of online environment</td>
</tr>
<tr>
<td></td>
<td>• Knowledge of theories of mathematics learning</td>
<td>• Knowledge at the mathematics horizon</td>
</tr>
<tr>
<td></td>
<td>• Knowledge of virtual chat room</td>
<td></td>
</tr>
</tbody>
</table>

Table 20 lists the functions of the online environment utilized by each mathematics teacher educator. As illustrated, the online environment as a medium for generating interaction was utilized by all three MTEs but in different ways: Dr. Abby used the discussion forum; Dr. Beth used Voice Thread (a video-based interaction platform); and Dr. Colin used a virtual chat room. These choices were informed by their knowledge of the online environment and such knowledge was developed either from their personal experiences with online education or research reported on online education. In Dr. Abby’s case, MOOC design principles from the funding agent were the source for her knowledge of what could be done and what could not be done in the MOOC environment. In Dr. Beth’s case, her previous experiences of conducting an online PD
influenced her knowledge of the online environment; in particular, it helped her know how participants interacted in the online environment. In Dr. Colin’s case, his previous research experience of utilizing virtual chat rooms for student problem solving enabled him to identify the limitations of mathematical symbols in the virtual chat room for mathematical problem solving.

Table 20. Functions of the online environment utilized by the three MTEs

<table>
<thead>
<tr>
<th>MTEs</th>
<th>Functions of the online environment utilized to organize PD content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dr. Abby</td>
<td>Providing a wealth of online learning resources, including subject-specific simulations for learning</td>
</tr>
<tr>
<td></td>
<td>Multi-media learning</td>
</tr>
<tr>
<td></td>
<td>Interaction medium for multiple voices</td>
</tr>
<tr>
<td>Dr. Beth</td>
<td>Asynchronous interaction medium</td>
</tr>
<tr>
<td></td>
<td>Information presentation media</td>
</tr>
<tr>
<td>Dr. Colin</td>
<td>Synchronous interaction medium</td>
</tr>
<tr>
<td></td>
<td>Integrated mathematical learning technology</td>
</tr>
</tbody>
</table>

Another key domain common to all three MTEs when organizing PD activities in the online environment was the knowledge of the efficacy of the online environment for mathematics teacher education. This knowledge requires both knowledge of the online environment and knowledge of mathematics teachers. For instance, Dr. Abby drew from her knowledge of online resources of statistics learning to provide a range of online resources for her participants to access. Her research experiences of using simulations for statistics learning enabled her to integrate the online tools and statistics tools as additional types of learning resources in her design. Dr. Colin designed the problem solving tasks to be used in the virtual chat room without facilitators in order to foster discourse-related mathematics learning experiences. It was his knowledge of the virtual chat room and
knowledge of what teachers’ need that encouraged him to design an environment where mathematical tools, learner interactive features, and mathematical tasks were all incorporated to induce productive mathematical discourse.

Summary. In this section, I compared the three MTEs’ PD content decision making with regard to their goals, orientations and knowledge. As the analysis unfolded, each MTEs’ knowledge and orientation toward mathematics education, mathematics teacher education and online education were particularly important when they designed and implemented the online PD. Their knowledge and orientation was grounded in their experiences with teaching and learning in each of these areas and/or knowledge of research and theory in each of these areas. Based on the above comparative analysis, knowledge of mathematics teachers’ needs, knowledge of the online environment, and knowledge of the online environment for mathematics teacher educating were three key domains of knowledge for their online teacher educating practices. Each of these domains of knowledge was informed by various domains of knowledge, such as, mathematical knowledge for teaching (MKT) (i.e. learner knowledge, as in the teacher educator knowledge tetrahedron), technological pedagogical content knowledge (TPACK), knowledge of research and theory, and their own experiences of mathematics teaching and learning, working with mathematics and online education.
As discussed in Chapter 1, the current trend in mathematics teacher education encourages that professional programs be offered in an online format. This trend is due to various reasons, ranging from institutional pressure for larger enrollment, additional opportunities for funding and the teacher educators’ own commitment to reaching out to a larger teacher population. Yet, research with respect to practices of mathematics teacher educating within the online delivery model is scarce. MTEs’ work is complex; in fact, they need to meet multiple goals that address various conceptual domains. As such, they tend to focus on both mathematical content and methodologies that would elucidate effective mathematics teaching practices based upon an understanding of what mathematics teachers need in their professional learning. With regard to online teacher education, MTEs also conduct the work of effectively organizing and delivering professional learning for mathematics teachers. Further, since MTEs have strong backgrounds in research, their work is grounded in theoretical perspectives on mathematics teaching and learning as well as epistemological views on the nature of mathematics itself. Therefore, it is reasonable to assume that this complexity might be manifested in the pedagogical choices they make when designing and delivering online teacher education programs. As such, the purpose of the current study was to understand the conceptual considerations that MTEs take into account when organizing online
teacher education. The aim was to provide a theoretical model of how and why MTEs do what they do as they engage in online-based teacher educating activities. To this end, I examined the practices of MTEs who provided online mathematics teacher education sessions for either perspective teachers or experienced teachers using surveys, interviews and in-depth case study methods.

In chapter 4, I provided a status report on current trends in online mathematics teacher education practices in the US. In chapter 5, I presented detailed accounts of three MTEs’ online professional development program designs, practices, and rationalizations of decisions they made using survey information, interview data, and artifacts from their online programs. This chapter offers conceptualization of the findings based on the survey and case-study results and addresses the research questions: How does the online environment interact with MTEs’ decision making as they design and implement online mathematics teacher preparation or professional development? The three sub-questions were:

1. What dimensions of decisions do MTEs make when they design online professional development for mathematics teachers?

2. What are MTEs’ personal beliefs, perspectives and domains of knowledge that are influential in their decisions?

3. How do MTEs’ goals, personal beliefs and perspectives, and knowledge influence their decisions regarding the online PD content?

In crafting responses to each of the research questions, I drew from the analysis presented in the previous chapters and proposed a hypothetical model for studying MTEs’
decision making when they design and implement online teacher education. The proposed model is grounded in decision making theory by Schoenfeld (2010) in combination with the teacher educator knowledge tetrahedron proposed by Perks and Prestage (2008) and the eight dimensions of instructional design for distance learning by Boyd (1993). Each model provides different facets for this study. Schoenfeld’s model provides the basis for decision making; Perks and Prestage’s model of teacher educator knowledge tetrahedron supports understanding the work of mathematics teacher educators; and Boyd’s eight dimensions aid in understanding the dimensions of instructional design pertaining to the online distance learning environment.

**Decision making.** Schoenfeld (2010) described individuals’ decision making, particularly, in-the-moment decision making to be shaped by *goals, orientations* and *resources*. *Goals* refer to something that an individual hopes to accomplish. Goals can be different in size according to the degree of importance each individual places on them. They could be short-term goals, medium-term goals, or long-term goals. Teaching is a goal-oriented activity, in that teachers always design and plan lessons with an agenda of goals in mind. *Orientations* refer to the attitudes and beliefs of individuals about the objects that they interact with, also known as, beliefs, dispositions, values, and preferences. Orientations are considered to have strong explanatory power over people’s interpretations or reactions to situations. *Resources* refer to the kinds of knowledge individuals possess. This construct relates to the assets of individuals instead of resources in general. In particular, Schoenfeld (2010) identified four forms of knowledge: facts or isolated pieces of knowledge, procedural knowledge (how to do things), conceptual
knowledge (the intellectual rationale that explains why things work), and problem-solving strategies. Using knowledge of teaching as an example, it could be depicted in all four forms. It is factual as it encompasses a multitude of domains including curricular, content knowledge as well as knowledge of students. It is procedural, as it includes control over many procedurals territories, the “how to” chores such as how to manage discussions, how to organize classrooms, and how to grade papers, to list a few. It is conceptual when teacher decisions are grounded in the subject matter and learning theories. It also involves command of strategies and heuristics such as using real-life examples to motivate students.

**Teacher educator knowledge tetrahedron.** Grounded in the notion that mathematics teacher learning is a reflective practice (Schön, 1983), Perks and Prestage (2008) identified three aspects of knowledge that were central to mathematics teachers from their own work with prospective mathematics teachers. The three aspects are: “(1) practical wisdom – knowledge from being in the classroom; (2) professional traditions – knowledge from existing school curriculum and practices and research; and (3) learner knowledge – the prospective teachers own knowledge” (Perks & Prestage, 2008, p. 269). Learner knowledge refers to teachers’ knowledge gained from being a mathematics student at school or in college. The three components inform each other and form a tetrahedron for mathematics teachers (Figure 37).
Perks and Prestage (2008) adapted this teacher-knowledge tetrahedron to propose aspects of knowledge important to teacher educators. The three vertices of the tetrahedron are redefined to capture the work of MTEs. So, the three vertices became: (1) professional traditions – knowledge of research about mathematics teaching, knowledge of existing ways of working in the field, and knowledge of national or local standards of teacher training; (2) practical wisdom – knowledge of designing and conducting mathematics teacher education sessions; and (3) learner-knowledge – knowledge gained from being a teacher, which constitutes all three components in the teacher-knowledge tetrahedron. As such, the teacher knowledge tetrahedron is within the vertex of learner knowledge in the teacher-educator knowledge (Figure 38).
Among the three components of teacher educator knowledge, teacher knowledge for mathematics teaching (that is, mathematical knowledge for teaching) acts as learner knowledge for teacher educators. According to Hill et al.’s (2008) category, it includes not only subject matter knowledge (i.e. common content knowledge, knowledge at the mathematical horizon and specialized content knowledge), but also pedagogical content knowledge (i.e. knowledge of students, knowledge of teaching, and knowledge of curriculum). Professional traditions develop from personal experiences, research and education about mathematics teacher education. Practical wisdom emerges from considering what teachers need to know and how the sessions are structured so as to engage them in learning. Perks and Prestage (2008) suggest that these three components of teacher educators’ knowledge interact with each other to inform MTEs’ decisions when they design and conduct teacher education sessions.
**Instructional system theory for cyberspace.** Boyd (1993) identified five spaces that structure the virtual distance learning environment. The space relevant to this study is *instructional design and production system*, which contains eight necessary dimensions of decisions for instructional design consideration (Figure 39). In Boyd’s description, these eight dimensions are four pairs of corresponding dimensions, which include: (1) agreed goals (i.e. the purposes of the course goal) and corresponding rules and mechanism that make sure goals are achieved; (2) subject matter (i.e. content) and corresponding metaphors and views that illustrate the content; (3) choices of real and virtual spaces and corresponding media used in the space; and (4) participants and the corresponding sociostructure of the participants.
Research Question #1

To answer the first research question, "what decisions are made in MTEs’ practices," analysis drew from a comprehensive perspective on the kinds of decisions involved in MTEs’ design practices. The case study results showed all of the dimensions were present in each MTE’s online PD, even though some dimensions were predetermined — they were not decisions that the MTEs’ had to consider. For example,
in Dr. Abby’s case, the dimension of “real and virtual place” was predetermined as she was assigned to conduct the PD as MOOC. This was also the case for Dr. Beth as the PD goals, content, and participants were not presented as choices for her. Her decisions concerned the dimensions of media, social groupings, and rules and mechanisms. Dr. Colin was the only MTE among the three participants who autonomously determined all eight dimensions of the decisions for his online PD.

As the three MTEs’ Detailed Design Logics maps reveal, some dimensions were considered in relationship with one another in one stage of their design logic. For example, when Dr. Abby determined the PD objectives, she also made decisions about the participants (i.e. the “who?” dimension in Figure 39) and the focusing concepts to address gaps in teachers’ knowledge (i.e. the “what?” dimension in Figure 39). Similarly, for all three MTEs, as they organized the PD activities, they made decisions about: how content may need to be illustrated (i.e. the “how illustrate?” dimension), what media to use (i.e. the “through which?” dimension), and how to structure participant interactions (i.e. the “with whom” dimension), simultaneously. Hence, instead of considering eight dimensions as in Boyd’s model, I organized these eight dimensions into two groups of decisions that were key to MTEs practices when implementing the online PD. They are: (1) setting the PD goals, and (2) operationalizing the PD goals. The first group of decisions (PD goals) is about addressing the needs of the participants. In terms of Boyd’s (1993) framework, this group of decisions involved dimensions of “agreed goals”, “subject matter/focusing concepts”, “virtual places”, and “participants.” The second group of decisions is about the choice of activities, and structuring the online
environment to organize the activities, including online format, media to use, and participation structure. In terms of Boyd’s framework, this group of decisions involves “rules and mechanisms”, “metaphors, views and topics”, “media” and “social groupings.” The second group of decisions are the enactment of the first group of decisions. In Boyd’s framework, as in Figure 39, the second group of decisions are on the opposite side of the first group of decisions.

**Research Question #2**

The second research question was about MTEs’ personal beliefs and domains of knowledge that are influential to their decisions. I identified the MTEs’ orientations in the case studies including orientations toward mathematics, mathematics teaching and learning, and teacher education. As the cross-case analysis revealed, each MTE’s epistemological perspectives on the nature of mathematics and mathematics learning for both students and teachers were the connections among the different orientations, and indeed, their epistemologies guided every online pedagogical practice, including how they set and operationalize the PD goals. However, what was prominent in the MTEs’ decision making was the knowledge exercised in their online design and teaching practices. Multiple levels and domains of knowledge were identified in the study results, and different domains of knowledge were drawn upon for different decisions. Two types of knowledge were prominent for decisions about setting the PD goals and decisions about operationalizing the PD goals: One is knowledge of mathematics teachers and teachers’ needs for setting the PD goals; and the other is knowledge of the online environment for mathematics teacher educating for operationalize the PD goals.
Drawing from the three case studies, the knowledge common to all three MTEs when determining the online PD goals was their knowledge of mathematics teachers and teachers’ needs (I will call it knowledge of mathematics teachers or KMT throughout the following text). Regarding the knowledge or resources, the construct in Schoenfeld’s (2010) decision making theory, knowledge of mathematics teacher can be regarded as conceptual knowledge as it is grounded in theories and has intellectual explanations for why mathematics teachers need this training or others. With reference to Perks and Prestage’s (2008) teacher educator knowledge tetrahedron, KMT relates MTEs’ practical wisdom as it is the knowledge for considering what mathematics teachers need to know when conducting teacher education sessions. As evident in all three cases, the MTEs’ KMT was informed by their knowledge of professional traditions (including personal experiences of working with mathematics teachers and research in mathematics teachers and teacher education) and mathematical knowledge for teaching (that is, learner knowledge for MTEs). Mathematical knowledge for teaching includes knowledge of mathematics content, knowledge of students, knowledge of teaching, and curriculum knowledge (Hill et al., 2008). For example, in Dr. Abby’s case, addressing teachers’ needs from a wide range of backgrounds was incorporated into her PD goals. She relied on her knowledge at the mathematical horizon to specify a core concept that was suitable for all of her PD participants as the PD objective. In short, KMT, as a type of conceptual knowledge, was utilized for setting PD goals. This knowledge was informed by research in mathematics teacher education, experiences of working with mathematics teachers, and knowledge of mathematics teaching and learning.
To operationalize the PD goals, all three MTEs relied on their knowledge of mathematics teachers and knowledge of the online environment. More precisely, it was the knowledge of the online environment for mathematics teacher educating (KOMTE) that each MTE drew from when they designed and organized learning activities for their teachers. KOMTE is a type of pedagogical knowledge that takes into consideration the efficacy of the online environment for mathematics teachers’ learning. According to Schoenfeld’s (2010) resource/knowledge construct in the decision making framework, KOMTE is similar to knowledge of teaching that embodies knowledge of facts, procedures and heuristics that are grounded in research, theories and practices as in Perks and Prestage’s (2008) knowledge tetrahedron. As such, “mathematics teacher educating” is used in KMOTE, rather than “mathematics teacher education” to emphasize that such knowledge draws from a continuum of research on mathematics teaching and learning as well as experiences of working with mathematics teachers. Relationally, KOMTE encompasses knowledge of mathematics teachers (KMT) and knowledge of the online environment (KO). KO is about knowledge of features of technology, which can be factual or procedural (Schoenfeld, 2010) and informs MTEs about what is possible to do within the online environment. For example, Dr. Colin knew that the online chat room failed to provide a sufficient tool for mathematical problem solving. Consequently, he and his team programmed GeoGebra-based simulations in the environment for small groups discussions about mathematics in the chat room. All three MTEs organized learning activities that were appropriate for use in the online environment, but they also tailored these activities to the needs of their teachers. KOMTE relates to a number
knowledge domains that MTEs exercise in their decision making. KOMTE also relates to the conceptual construct of technological pedagogical content knowledge (TPACK).

Detailed descriptions of KOMTE and its relationship with other knowledge domains will be addressed in responses to research question #3. Conceptual meaning of KOMTE will be addressed in the discussion section.

In summary, as in research question #1, the MTEs made two sets of decisions: (1) they set the PD goals and (2) they operationalized the PD goals in the online environment. As in research question #2, *knowledge of mathematics teachers and their needs* (KMT) was the domain of knowledge that they drew from when they set the PD goals, and *knowledge of the online environment for mathematics teacher educating* (KOMTE) was the knowledge that they drew from when they operationalized the PD goals in the online environment. These two knowledge domains serve at the forefront of MTEs’ practices of online mathematics teacher educating, and they directly relate to the two sets of decisions associated with the PD goals; however, these two knowledge domains encompass multiple knowledge domains that are relevant to MTEs’ decision making serving at the backdrop. The relationship of MTEs’ goals, orientation and multiple knowledge domains is addressed in research question #3.

**Research Question #3**

To address the third research question about how MTEs goals, beliefs and knowledge influence their decisions regarding the online PD design, I proposed a model that captures the relationship among goals, orientations and various domains of knowledge relevant to MTEs’ pedagogical practices of online mathematics teacher
educating based on examples from the case studies as well as constructs that seemed central and yet not present in every case study or the survey results, for instance, MTEs’ knowledge of research and theory in online education. The model was then utilized to investigate three interview subjects’ knowledge structure to provide specific indicators of MTEs’ knowledge domains involved in their decisions.

As shown in Figure 40, the model describes MTEs’ decision making for designing and implementing online mathematics teacher education programs in four levels. First, the model describes the role of MTEs’ epistemological perspectives (i.e. orientations) when making decisions. Second, KMT and KOMTE are described in bold, as they are directly linked to the PD goals. Third, the model specifies domains of knowledge related to KMT and KOMTE to explain what they are and their relationships to other domains of knowledge, which is referred to in the Venn diagram in the center of the figure. Lastly, the model describes the sources of the knowledge domains with respect to Perks and Prestage’s (2008) constructs. In this way, the model connects goals, orientations and relationship of knowledge domains of MTEs’ decision making. In the following, I will describe the knowledge domains and their relationships in detail as they are central to MTEs’ decision making in the context of online mathematics teacher education.
Figure 40. A model of MTEs’ decision making for online mathematics teacher educating
In the middle of the Venn diagram, three knowledge domains were identified that are related to KMT and KOMTE. These three knowledge domains are: mathematical knowledge for teaching (MKT), knowledge of teacher educating (KTE) in general, and knowledge of the online environment (KO).

KTE is knowledge about the methods of providing teacher education yielding teacher change in knowledge and practices of teaching. This is a generic form of knowledge that involves teachers’ needs across disciplines, such as teacher knowledge of general pedagogy, teacher collaboration, leadership, etc. KTE was evident among the three case subjects and MTEs who participated in the survey. For example, all three MTEs and many survey respondents expressed ways they organized teacher interactions in the online environment and monitored teacher participation in the online courses. The MTEs exercised their knowledge of teacher educating to engage their teachers in professional learning and such knowledge was not restricted to mathematics teacher education only.

MKT is knowledge about mathematics teaching and learning, as defined by Hill, et al. (2008). It involves three types of mathematical content knowledge (common content knowledge, specialized content knowledge, and knowledge of mathematical horizon) and three types of pedagogical content knowledge for mathematics teaching (knowledge of content and students, knowledge of content and teaching, and knowledge of curriculum). Evidence of MKT was clear in all three MTEs in their decisions and practices. Dr. Abby extensively utilized her knowledge of research in student statistics learning with technology in the MOOC PD, and horizon knowledge to determine the
statistical content for her participants. Similarly, Dr. Beth utilized her knowledge of mathematical horizon to connect different concepts in number and algebra for her teachers. Dr. Colin drew from his knowledge of teaching and learning geometry in the dynamic environment to design his tasks. In general, MKT was the knowledge that MTEs drew from to design their PD content activities as MKT was the target knowledge for teacher development. In Perks and Prestage’s (2008) teacher educator knowledge tetrahedron, MKT is learner knowledge for teacher educators. For all three MTEs, such knowledge came from research (i.e. professional traditions) and particularly research they had conducted on their own.

Knowledge of the online environment (KO) is knowledge about the functions of the online technology such as chat room collaboration, digital tools, whiteboard presentations, online animations, etc. KO involves skills required to operate online tools or any online platforms, that is, knowing what the online tools or online platforms can do and cannot do. Basically, it is technological knowledge or knowledge of the technology. For example, Dr. Beth’s knowledge of the online environment includes her knowledge of the affordances and the constraints of different collaboration tools such as Adobe Connect, Voice Tread and Blackboard Collaboration from a user’s standpoint. She eventually chose Voice Thread as the main platform for the online PD for the facilitator (herself) to present information and the teachers to collaborate after the evaluation of different online platforms. Dr. Abby’s knowledge of the online environment pertains to the MOOC environment as an asynchronous learning platform and what this platform could afford to do technologically such as video uploading, etc. Dr. Colin’s KO was
about knowing how to use the virtual chat room platform and tools in the chat room such as whiteboard, chat space, etc. With reference to Perks and Prestage’s (2008) teacher educator knowledge tetrahedron, KO is not a construct in the tetrahedron but similar ideas could apply for all three MTEs, as their KO came from previous experiences of using the online technologies (i.e. practical wisdom).

MKT, KTE, and KO are intertwined yielding three areas of intersection: knowledge of mathematics teachers and teachers’ needs (KMT), knowledge of online teacher educating (KOTE), and knowledge of online mathematics teaching and learning (KOMTL). Knowledge of mathematics teachers and their needs (KMT) is at the intersection of MKT and KTE. KMT is concerned with professional learning of mathematics teachers, their development of mathematical knowledge, their beliefs towards mathematics and mathematics learning, and community building. Moreover, these learning opportunities were not general learning opportunities but were specifically about learning particular piece of mathematics – statistical investigation. Thus, KMT goes beyond a simple consideration of the needs of teachers in general; however, it is strongly connected with mathematics teaching and learning. Moreover, certain domains of MKT have a stronger influence on MTEs’ decisions than others. For instance, knowledge of mathematics and students was the main knowledge domain for Dr. Beth when establishing her online module objectives.

Knowledge of online teacher educating (KOTE) is knowledge about how to offer general teacher education sessions online. At the intersection of KTE and KO, for example, depicts knowledge of how to structure an effective online course for teachers.
In the case studies, Dr. Abby relied on the MOOC design principles, which were principles for conducting online teacher education in general, to organize her MOOC for statistics. Her knowledge of the design principles is an example of KOTE informing her practices. Dr. Beth used both Voice Thread and Blackboard Collaborate as different ways to engage teachers’ participation. Dr. Colin used chat logs to engage teacher discussions. These multiple ways of engaging teachers online reflect KOTE in practice. Using Perks and Prestage’s (2008) teacher educator knowledge tetrahedron constructs, both Drs. Abby and Beth’s KOTE was knowledge of professional traditions, while Dr. Colin’s KOTE was practical wisdom as it was developed from his prior practice of conducting online teacher education sessions.

KOMTL is knowledge about learning or teaching mathematics in the online environment, at the intersection of MKT and KO. This is knowledge about how mathematics learning takes place in the online environment and/or how to teach mathematics in the online environment. The online environment affords as well as constrains mathematics representations. As MTEs plan to design online mathematics learning sessions for teachers, they need to know how mathematics and learning of mathematics may need to be navigated or manipulated when utilizing the online platforms or online tools. This knowledge is evident in practices of all three case-candidates. Dr. Abby and Dr. Colin incorporated mathematics-specific tools for teachers to interact with mathematics content in the design. Dr. Beth took the traditional route; she designed slides with mathematical problems for teachers to access online. With reference to the teacher educator knowledge tetrahedron, the three MTEs’ KOMTL were either
drawn from professional traditions in mathematics education and/or practical wisdom of designing and conducting learning mathematics sessions online.

It is worth noting that unlike KMT, both KOTE and KOMTL are relevant to MTEs’ practices but may not be the primary knowledge domains for MTEs’ pedagogical decisions. In the case studies, all three MTEs’ professional development programs were directly related to teachers’ mathematics learning, so KOMTL was essential. Yet, the survey results indicate that teaching and learning mathematics might not be the focus for every online mathematics teacher course or PD program. Instead, MTEs also designed online courses for mathematics teacher leadership, or mathematics history and research methods. In those cases, KOMTL may not be as central as KOTE. Further studies on other types of subject-focused online teacher educating practices need to be conducted to confirm how KOMTL might be exercised in non-mathematical content focused online courses.

Last but not least, knowledge of the online environment for mathematics teacher educating (KOMTE) resides at the center of the Venn diagram. KMOTE is a type of pedagogical knowledge that supports MTEs to operationalize PD goals in a particular online platform with online tools, after knowing what mathematics teachers’ needs are to be addressed. KOMTE is the manner in which teachers’ needs for teaching mathematics is addressed in the online environment. Further discussions on KOMTE are presented in detail in the following section.

Thus, six domains of knowledge are identified to be relevant in MTEs’ online mathematics teacher educating practices, as shown in the Venn diagram. The two
knowledge domains, KMT and KOMTE, which are in bold, are the knowledge at the forefront of MTEs’ decisions and practices. Other knowledge domains are relevant and inform these two domains of knowledge. With reference to the teacher educator knowledge tetrahedron, these knowledge domains were informed by professional traditions especially knowledge of research in mathematics education, mathematics teacher education and online education, and practical wisdom of teaching and learning of mathematics, such as experiences working with mathematics education and experiences of online teaching and learning. This is shown outside the Venn diagram. The knowledge domains and the relationship among the knowledge domains in the Venn diagram is a transpose of the teacher educator knowledge tetrahedron with an extension of online education elements added to the framework.

Putting it all together, from the top, epistemological perspectives about mathematics and mathematics learning guide the MTEs knowledge and decisions. As the cross-case analysis revealed, at the heart of the MTEs’ decision making was their epistemological perspectives on the nature of mathematics and mathematics learning that guided their choices; informed their practices; and even oriented their knowledge about mathematics teachers and teacher educating in the online environment. In the middle resides two layers of knowledge domains with an outer layer informing the inner layer. The primary knowledge domains--knowledge of mathematics teachers and teachers’ needs (KMT), and knowledge of the online environment for mathematics teacher education (KOMTE) directly influenced MTEs’ decisions about the PD goals as well as the enactment of the PD goals. Knowledge of teacher educating (KT), mathematical
knowledge for teaching (MKT), knowledge of the online environment (KO), knowledge of online mathematics teaching and learning (KOMTL), and knowledge of online teacher education (KOTE) are domains of knowledge that are relevant to and inform the two primary knowledge domains. Lastly, MTEs’ knowledge of research and theory, and their personal experiences in each area are the sources of their knowledge, which are represented at the outer layer of the Venn diagram.

Discussions

Mishra and Koehler (2006) developed the notion of technological pedagogical content knowledge (TPCK or TPACK) to capture the relationships between and among content knowledge, pedagogical knowledge, and technological knowledge for integrating technology into teaching and learning. Prior to the development of the TPACK model, Koehler et al. (2004) described a transactional model among content, pedagogy and technology to understand the role of technology in relation to content and pedagogy when faculty designed online courses. As shown in Figure 41, the transactional model was simply a triangle with vertices consisting of content, pedagogy and technology. In this model, the content refers to a specific subject to be learned or taught, which would be the focusing ideas, knowledge, procedures and also representations of the subject matter. Technology refers to the specific technological tools used for learning which could include books, pens and paper, or digital tools such as the online environment, applets, simulations, etc. Pedagogy refers to the practices, or methods of teaching. The authors posited that these three elements are inseparable when it comes to designing an online course. Indeed, what drives pedagogical decisions could be different from face-to-face
courses because technologies could potentially determine the content and the kinds of representations of it, and hence, define or constrain other pedagogical decisions. This was evident in the case studies as well as the data extracted from the survey responses in this dissertation study; however, how the technology interacted with content and pedagogy in each case study subject’s pedagogical decisions was different. The Detailed Design Logic maps and the survey responses about the adjustments made provide insights into the transactional relationship among technology, content, and pedagogy in the MTEs’ decisions.

In Dr. Abby’s case, the online environment (i.e. technology) interacted with her practices at all three stages in her design logic. In stage 1, when she set PD goals, though she had examined the affordances of the MOOC environment, her decisions were based on her knowledge of mathematics teachers. In stage 2, when she decided on focusing content, this was the stage when technology influenced her decisions; particularly, it

Figure 41. Content, pedagogy and technology (Koehler et al., 2004)
interfered with her content decisions regarding the mathematics/statistics topics she
chosen. The online environment welcomed teachers with diverse mathematical
backgrounds, so she had to choose a concept that would accommodate all teachers and
her subsequent pedagogical decisions were based on the focusing content. In stage 3,
when she organized the learning activities in the MOOC environment, technology
interfered with her decisions again. This time, the online environment constrained the
representations of her activities because of legal rights and other issues. As a result, she
redirected the participants to other online resources where they could access teaching
videos. Lastly, she made her own animations, and summarized research articles for her
participants. In this situation, the interaction of technology and content activities resulted
in pedagogical decisions that were made to accommodate the interaction.

In Dr. Beth’s case, the online environment interacted with her decisions only at
stage 3 when she considered delivery methods. In her case, the online environment
constrained the desired interactions that she would expect to have as in the face-to-face
environment. Therefore, she relied on a lecture format to organize her content activities
so that all representations of mathematics would be revealed and teachers would consider
connecting these representations according to her expectations. This is the case when
technology interacted with pedagogy so the subsequence decisions on content activities
had to be made.

In Dr. Colin’s case, although the online environment interacted with his design
decisions only at stage 1 when he was designing the environment, the technology
component in fact interfered with Dr. Colin’s planning regarding both his content and his
pedagogical decisions simultaneously. Because the online environment that Dr. Colin and his team designed featured GeoGebra, a dynamic geometry and algebra learning tool, his mathematical content choice centered around dynamic geometry. Moreover, the online environment that they designed also featured a virtual synchronous chat room, so, their pedagogical decisions on task design were based on what the virtual chat room could afford so as to achieve the desired interactions among participants. Dr. Colin’s case provided evidence of the technology interactions with both content decisions as well as pedagogical decisions.

The three cases offered four situations where technology interacts with both content and pedagogical decisions: (1) technology determined content choice, so as subsequent pedagogical decision (i.e. Dr. Abby’s stage 2 design); (2) technology constrained content representations, thereby influencing pedagogical decisions on how content activities could be reorganized (i.e. Dr. Abby’s stage 4 design); (3) technology constrained desired pedagogical outcomes, so content activities and presentations were altered (i.e. Dr. Beth’s stage 3 design); and (4) technological environment defined both content choice and pedagogical practices on task design (i.e. Dr. Colin’s stage 1 design). The analysis of the three cases showed evidence of pedagogical practices resulting from interactions between the online environment (i.e. technology) and content (including content activities) or the impact on content organization as a result of the interaction between technology and pedagogy. In fact, it was technology that drove the other two elements of the triangle (either content or pedagogy), so the third element was rearranged
in the three subjects’ cases. Ultimately, decisions about any element in the triangle had implications for the other two elements.

Survey results also provided further evidence of how MTEs’ pedagogical practices were impacted by the online environment. Among the 58 collected responses from the open-response question on adjustments made for the online environment, 25 reported that the adjustments made were related to choices of content or content activities due to various affordances and limitations of the online environment that had impacted content. Furthermore, 10 MTEs indicated that they had to modify their choice of focusing concepts or considerations for special content activities to accommodate the diverse needs of mathematics teachers who participated in online courses or PD programs. Nine MTEs indicated that they adjusted content activities with increasing uses of online mathematics applications and simulations due to the representations of the technology imposed on mathematics. A few of the MTEs mentioned incorporating multiple online resources relevant to their course/PD content. Each pedagogical practice related to content adjustments was made based on the analysis of the affordance and constraints of the online environment for the subject matter. Among the 58 responses, 24 were coded as adjustments made that were purely pedagogical. In other words, these adjustments were not necessarily relevant to the course or PD content; for example, they included methods of interaction, collaboration, different modes of communicating/participating online, feedback, etc. Yet, it remains unclear whether MTEs’ content organization was altered as a result of adjustments to pedagogy, like the case of Dr. Beth, since the survey data were too limited to provide such insights.
In general, both the case study results and the survey results confirmed the possible transactional relationship between content, pedagogy and technology as proposed by Koehler and colleagues (2004), in the context of online mathematics teacher education. If all three elements in the transactional relationship were evident in MTEs’ practices, their technological pedagogical content knowledge would be exercised. However, as shown in the above discussion, technology, pedagogy and content was integrated differently for each MTE because each MTE had their unique technological environment within which to work. Moreover, each MTE’s goals and expectations for their teacher education courses or programs varied. Even though MTEs’ TPACK was involved in their online mathematics teacher educating practices, knowledge domains that encompassed TPACK were intricately different. The relationship of knowledge domains as presented in the working model of MTEs’ decision making for online mathematics teacher educating practices (Figure 40) offers some details of MTEs’ TPACK in the context of designing online mathematics teacher education sessions. In fact, I argue that the knowledge domain (Knowledge of the Online Environment for Mathematics Teacher Educating, KOMTE) represents TPACK in the context of online mathematics teacher education.

KOMTE. KOMTE is the integration of three different domains of knowledge: knowledge of the teacher educating (KTE), knowledge of the online environment (KO), and mathematical knowledge for teaching (MKT). I argue that when MTEs’ online design practices actively involved consideration of the relationship among technology, pedagogy and content, they drew knowledge from KOMTE. In the other words, when
TPACK was exercised in MTEs’ online design practices, KOMTE was the knowledge domain from which they drew.

In the case of Dr. Abby, technology influenced her decisions about PD content in two instances. The first instance occurred during the beginning two stages of her design logic when she defined and refined her PD objectives and focusing concept because the MOOC environment allowed diverse teacher populations to enroll in the course. The domains of knowledge from which Dr. Abby drew included knowledge of the online environment (KO) as she realized what the MOOC environment allowed for in terms of participant population, and knowledge of mathematics teachers and teachers’ needs (KMT) as she had to identify the needs of teachers from a variety backgrounds to design the PD. In Figure 42, KMT is shaded by the dashed lines and KO is shaded in grey. Her subsequent pedagogical practices such as designing learning activities related to the focusing concept (including modifying the GAISE report for mathematics teachers of all backgrounds) were based on her knowledge of these two domains. Therefore, the pedagogical knowledge to operationalize the chosen focusing concept is the knowledge in the middle. Her mathematics content knowledge, especially knowledge of mathematics at the horizon, was particularly important in her operation as she had to address to the concept of statistics to teachers at all levels.
The second instance of how technology shaped Dr. Abby’s pedagogical decisions occurred in stage 4 when she organized the MOOC activities for the participants to access the learning of statistics. The knowledge domains from which she drew included knowledge of statistics teaching and learning (i.e. MKT for statistics) and knowledge of online teacher educating (KOTE) as all assigned activities were about teaching and learning statistics. She needed to conceptualize these activities in ways that teachers might fully experience peer collaboration opportunities, self-paced learning opportunities, multi-media learning opportunities, and many other learning opportunities provided by the Internet. Knowledge of providing teachers with multiple learning opportunities via the online environment belongs to knowledge of online teacher educating, that is, KOTE. As shown in Figure 43, the dashed area represents KOTE and the shaded area represents...
MKT for statistics. The knowledge at the intersecting area represents the pedagogical knowledge needed to organize teachers’ learning of statistics pedagogy in the MOOC environment.

Figure 43. MKT intersecting KOTE

In the case of Dr. Beth, the online environment interacted with her decisions when she considered online delivery methods to motivate teachers’ learning of number and algebra concepts by relying on children’s representations of these concepts as learning
media. The knowledge domains that influenced her pedagogical practices included knowledge of mathematics teachers and teachers’ needs (KMT), which was gained from previous experiences with facilitating the original face-to-face PD sessions, and knowledge of online teacher education informed by experts in online teacher education. Figure 44 shows two intersecting dashed areas representing these two domains of knowledge. The intersecting area of KMT and KOTE (i.e. the small area in the middle) indicates the pedagogical knowledge for organizing the learning activities using the Voice Thread mini lecture.

![Figure 44. KMT intersecting KOTE](image)

Figure 44. KMT intersecting KOTE
In the case of Dr. Colin, the online environment influenced his decisions in his design logic stage 1 when he was designing the environment. He relied on his previous experiences of conducting problem solving sessions in the virtual chat rooms with students to realize what possibilities the virtual chat room environment afforded in terms of learning mathematics (i.e. knowledge of online mathematics teaching and learning KOMTL). His epistemological perspectives on the nature of mathematics and mathematics learning informed his beliefs about what teachers needed to know to experience how mathematics should be learned. As illustrated in Figure 45, the knowledge domains that he relied on included knowledge of online mathematics teaching and learning (KOMTL) and knowledge of mathematics teachers (KMT). The intersecting area of KMT and KOMTL, which is the small area in the middle of the Venn diagram, represents the pedagogical knowledge for organizing teachers’ learning experiences in the virtual chat rooms, for example, selecting and sequencing dynamic geometry problem solving tasks, and designing meta-tasks for teachers to reflect on their experiences of mathematical discussions using chat logs.
The three cases demonstrated that when MTEs’ decisions and practices involved consideration of content, technology, and pedagogy, they drew knowledge from domains of teacher educating, mathematics teaching and learning and the online environment, and operationalized from knowledge at the intersecting area of the three domains, which is the knowledge of the online environment for mathematics teacher educating (KOMTE). KOMTE is TPACK as it concerns with the pedagogical content knowledge of online mathematics teacher educating. KOMTE is more than TPACK as it gives specifications of domains of knowledge that MTEs exercise in the context of online mathematics teacher education for better understanding of TPACK and MTEs’ knowledge for online teacher educating practices.

Further, the different ways that knowledge domains intersect with one another to manifest KOMTE as illustrated in Figure 42, Figure 43, Figure 44, and Figure 45 reveal
how the MTEs utilized the online environment for advancing teachers’ learning. For
instance, KOMTE in Dr. Abby’s case is represented by the interaction of MKT (i.e.
mathematical knowledge for teaching) and KOTE (i.e. knowledge of teacher educating)
as in Figure 43. She extensively utilized statistics simulations to promote teachers’
estatistics learning drawing knowledge from MKT and she designed a variety of online
learning opportunities (e.g. peer collaboration, expert panels, video discussions) for her
teachers drawing knowledge from KOTE. How she drew knowledge domains to inform
her pedagogical practices revealed that the online environment was a platform to provide
learning opportunities for teachers to Dr. Abby. Such opportunities included
opportunities of collaboration and discussions, opportunities of encountering experts, and
opportunities to utilize technologies to solve statistics investigation problems.

Whereas, KOMTE in Dr. Beth’s case is represented by the intersection of
knowledge of mathematics teachers (KMT) and knowledge of online teacher educating
(KOTE). Her knowledge of mathematics teachers’ needs (i.e. KMT) for this particular
PD program was adopted from the original face-to-face PD for which she had to comply.
Her choices regarding how to utilize the online environment to design the PD were
mainly reliant on KOTE that the specific online tools could be used to engage teachers to
interact with learning materials and interact with the facilitators or peer participants. As
such, the online environment mainly served as a presentation tool and a platform for
facilitating teacher interaction.

KOMTE in Dr. Colin’s case is represented by the intersection of knowledge of
mathematics teachers (KMT) and knowledge of online mathematics teaching and
learning (KOMTL). How he utilized the online environment was informed by KMT that the teachers needed to experience mathematical discourse and understand mathematics as a cultural product. As such, the online environment with a specific use of the virtual chat room served as a *discourse promoter* for teachers’ learning.

The above three examples support the finding that knowledge of the online environment for mathematics teacher educating (i.e. KOMTE) was different for the three MTEs, hence, the ways that the online environment was used by each to advance mathematics teachers’ learning were different. Identifying how KOMTE was manifested using the six domains of knowledge in the Venn diagram helped pinpoint which knowledge domains were more influential than others in the MTEs’ decisions, and hence, provided explanatory power to the differences in practices.

**Reflections on Decision Making**

In behavioral sciences, decision making has played a critical part in theorizing human behavior. The use of decision making to account for human behavior falls under the assumption that individuals’ actions are not just passive responses to environmental stimuli, and therefore, that individuals can think. Decision making takes place when individuals engage in problem solving activities (Schoenfeld, 2010), which can be rational (i.e. actions consistent with one’s reasoning) or irrational, and can be based on explicit knowledge or tacit knowledge. In education, researchers also seek to understand behavior of learners and teachers through the lens of decision making. Considering the behaviors of teachers, for instance, it is all about choices, where individuals’ roles, positions or relationships become possible (Herbst & Chazan, 2012). A statement was
made by Buchmann (1986): “what teachers do is neither natural nor necessary but based on choice” (p.529). Decision making is the crux of teaching in that it provides a lens to understand teachers and teaching in general, as every decision teachers make is motivated by different important factors including: (1) what teachers know; (2) what they believe; and (3) a number of factors pertaining to the environment in which teaching takes place.

Mathematics education research has taken two routes to examine teachers’ decision making: (1) through the context of individuals’ knowledge, preferences, needs and values; and (2) through the process of individuals interacting with the environment. Schoenfeld’s work in problem solving took the first route where he extensively examined individual student or mathematics teacher’s in-the-moment decision making processes as they engaged in problem solving activities (Schoenfeld, 1985, 2010). Schoenfeld (1992) studied students’ mathematical problem solving and decision making. He extended the problem solving framework to study mathematics teachers’ in-the-moment decision making as he suggested teachers are constantly engaging in problem solving activities during teaching (Schoenfeld, 2010, 2011). Schoenfeld’s work highlighted a cognitive process of decision making. Herbst and colleagues (Herbst & Chazan, 2003, 2011, 2012; Herbst, Chazan, Kosko, Dimmel, & Erickson, 2016) took the second route. They examined mathematics teachers’ decision making but they grounded their analysis in the instructional system and examined tacit norms and cultures that influence teachers’ choices. They highlighted that teachers’ choices are not context-free; consequently, certain types of contexts influenced the type of decisions that teachers made.
This dissertation considered teacher decision making in a different context than what was captured in Schoenfeld’s (2010) framework. MTEs were the subject of this study. Conducting online teacher education sessions was the context. Unlike Schoenfeld’s study where in-the-moment decisions were studied, my study focused on MTEs’ design decisions, which occurred during planning for the online delivery. In fact, in-the-moment decisions become less important in the context of online education as many decisions were made ahead of time. This was true for all three MTEs in my case studies. Both Dr. Abby and Dr. Beth’s online PD programs were 100% asynchronous where everything was planned ahead of time and uploaded to the website for participants to access in their own time. Despite the fact that Dr. Colin’s program was 100% synchronous using chat rooms, facilitators played a little role during the live sessions; rather, Dr. Colin and his team carefully structured the tasks in a way that participants could engage in live sessions with each other without the presence of facilitators. A few MTEs in the survey expressed that they spent more time planning for online courses than for face-to-face courses. It is not that MTEs did not make in-the-moment decisions in their online sessions, but rather, in-the-moment decision making became less important as the majority of decisions were made prior to the implementation of the online sessions.

Due to the fact that decisions were made prior to implementation, the type of decisions that MTEs made became more predictable than those that might reveal or be enacted in the in-the-moment phases. In fact, using Boyd’s (1993) framework of decisions dimensions, I summarized two groups of decisions that MTEs made during their online mathematics teacher educating practices. As my study results showed, these

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two groups of decisions were about choices of content and online organization of content activities to engage participating teachers, rather than in-the-moment reactions to a student’s question or a response. One group of decisions were made for setting PD/course goals and the other group of decisions were made for operationalizing the goals. As such, the type of decisions were more predictable and largely influenced by instructional contexts, as suggested by Herbst et al. (2016). One group of decisions were specifically related to the context of course/PD goal setting and the other group of decisions were related to the enactment of goals in the online environment context.

Although the study of MTEs’ decisions in the context of online mathematics teacher education was not the same as the studies of teachers’ in-the-moment decision making, the three cognitive indicators – goals, orientations and resources as in Schoenfeld’s (2010) framework were still applicable for studying MTEs’ decision making. By extending Schoenfeld’s framework to online mathematics teacher education, my study further provides insight into how these three constructs played differently in their roles in the online environment and specifies knowledge domains that influence MTEs’ decisions. Schoenfeld (2010) suggested that among the three constructs, goals specify that teaching is a goal orientated activity in every moment; knowledge is a type of resource that teachers draw from when make decisions; and orientation determines how teachers interpret or react to situations. In the practice of online mathematics teacher educating, my case studies showed MTEs’ epistemological perspectives guided every stage of their pedagogical decisions. It explains why each MTE identified mathematics teachers’ needs differently, though knowledge of mathematics teachers and teachers’
needs (KMT) was a common knowledge domain that all three MTEs in the case study drew from. It also explains why each MTE’s decisions of online learning activities were different, yet knowledge of the online environment for mathematics teacher education (KOMTE) was another common knowledge domain that was particularly important for all three MTEs’ practices. Further, my study provides a detailed account of MTEs’ knowledge and relationship of knowledge domains for their decision making using Perks and Prestage’s (2008) teacher educator knowledge tetrahedron framework. My analysis showed that the use of tetrahedron as the representation for MTEs’ knowledge structure has become insufficient as the work of MTEs encompassed multiple areas of knowledge and the knowledge structure became more complex as the online environment was incorporated as part of their decision making. As a consequence, in Figure 40, I used a Venn diagram to describe the knowledge domains and the relationship among these knowledge domains, and I also highlighted the two types of integrated knowledge (KMT and KOMTE) utilized directly in MTEs’ decision making while others were support knowledge domains in MTEs’ decision making. However, using Perks and Prestage’s (2008) tetrahedron framework did highlight the importance of research knowledge and practical knowledge in MTEs’ knowledge structure. As evident in my case study, the MTEs’ research and practice fell on a continuum where research informed their practice and their practice informed research.

Limitations of the Study

In this dissertation study, a mixed-method research design was used to examine MTEs’ decision making during designing and implementing online mathematics teacher
education programs. The goal was to conceptualize MTEs’ pedagogical practices in the online environment and to advance the field’s knowledge in this regard. The quantitative survey data collection followed by three in-depth case studies provided an insight into MTEs’ decisions. As a result of using the mixed-method design, there were limitations associated with the participants and data.

First, the survey intended to provide a status report of MTEs’ practices in the US. In the survey, I asked participants to share only one online course or PD they had designed and implemented. However, many MTEs had designed multiple online courses or programs. One could assume that the survey participants might have reported on what they considered to be their most successful online course or the most recent one that they had implemented, which might not reflect their pedagogical practices of other online courses/programs that they implemented. To this end, the status report might not capture the overall MTEs’ pedagogical practices because there were potentially other online courses they had instructed and their practices might be drastically different there.

The survey captured a number of different online courses designed and offered by MTEs including courses in mathematics and those with an emphasis on mathematics teaching and learning such as methods courses, content development courses or PDs. Some courses addressed mathematical content as the background, such as history courses, mathematics teacher leadership training, etc. The selection of in-depth case study participants only included the MTEs who provided mathematical content related online courses. The model proposed in this study was based on MTEs’ pedagogical practices for online courses or programs with heavy mathematical content. MTEs’ decision making
might potentially be different if they were involved in designing online courses where mathematics only served as a background.

Another limitation of the methodology is that data on the MTEs’ design processes were collected in interviews by asking them about what they did rather than observing them as they made those decisions. The interview subjects described their online programs and briefly summarized the design processes that they encountered. As such, my Detailed Design Logic maps were constructed based on recall data rather than observation. Potential for data for key decisions about technology, content, or pedagogy in the design processes might have failed to capture. However, observing MTEs’ design process will be methodologically challenging as the online course design may take a long time to plan before implementation. Dr. Abby indicated that she had spent two years planning the MOOC program, and many teacher educators indicated in the survey that planning online classes took much longer than planning face-to-face classes. Though observations of MTEs’ designing processes might provide more precise data of MTEs’ thinking and decision making processes than interviews, it requires sufficient time and opportunities to do observation and collect data.

**Implication for Future Research**

Limitations of the study naturally lend themselves for future research. Future research on MTEs’ decision making in the online environment could be improved and further extended methodologically as well as conceptually.

The theoretical framework proposed in this study could be used to study MTEs’ decision making for teaching non-mathematical content focused online courses such as
mathematics leadership training, mathematics educational history and theory courses, equity and diversity courses, student teacher supervision, or to study MTEs’ decisions making for teaching other online courses such as technological integration courses, STEM integration courses, etc. where mathematics was a component of the course but not always the primary focus of the course. My survey results showed that MTEs’ teaching involves not only teaching and learning of mathematics but many other aspects of mathematics education. Future study of MTEs’ decision making in other online courses is needed to confirm (or modify if not) if the model could capture the generalization of the work of MTEs’ practices of online mathematics teacher educating.

This study provided a few indicators where the online environment had impacted MTEs’ course/PD content decisions and subsequent pedagogical adjustments. However, limited data in both the case studies, and survey report could not provide an in-depth understanding of how the online environment impacted MTEs’ content decisions – whether the decisions were due to the nature of the subject matter or MTEs’ knowledge or MTEs’ epistemological perspective on the subject matter. Future studies comparing MTEs’ face-to-face practices and online practices with the same course could potentially provide more evidence on what had changed and what had not changed for the online environment, and explain why MTEs would or would not make the adjustments for the online delivery. Such conceptualization is important for the community as it leads to epistemological discussions on whether the online environment has impacted the nature of mathematics teachers’ learning or not. This dissertation study started the discussions, but more studies in this regard are needed.
The findings from this dissertation study capture the various knowledge domains upon which the MTEs rely, and the relationship among these knowledge domains, goals and epistemological perspectives when making decisions for the online courses or PD programs. An extension of this study is to explore indicators of different behaviors of MTEs’ knowledge domains to establish a trajectory of MTEs’ knowledge. Such studies would potentially benefit the community to provide training for future MTEs to design and deliver online teacher education courses.

Implication for Practice

Results from the survey reported that MTEs faced a number of difficulties when designing online teacher education courses. This study offers a framework to guide MTEs to support their online practices.

The model from this study will help MTEs to identify the type of decisions they need to make. I consolidated Boyd’s (1993) eight dimensions of decisions into two groups of decisions. The first group of decisions relates to identifying the course or PD goals including identifying participants and needs of participants, and the second group of decisions relates to choices of content activities and online organization of content activities, including choices of online media and social grouping for content activities. In practice, these two groups of decisions could be regarded as two design stages. As MTEs designed the online program, it is helpful to first consider the needs of the teachers’ and set PD goals with references to Boyd’s (1993) eight dimensions and identify which dimensions are fixed and which ones are not. Then, it is helpful to consider the organization of online activities with reference to the eight dimensions, and identify
which ones are fixed and which ones are not for decision making. Grouping decisions may ease the design process as the type of decisions to be made were made clear.

The model from this study shows that knowledge of the online environment for mathematics teacher education (KOMTE) is a critical pedagogical knowledge for MTEs when operationalizing online courses. Such knowledge involves elements of content, technology and pedagogy and encompasses six domains of knowledge. The implication is that for an effective online mathematics teacher education course or program, MTEs with expertise in mathematical content knowledge and knowledge of teaching and learning of mathematics need to take a hand in the design of the technology to support their pedagogical purposes as the consideration of content and technology is not separate. For institutional support, support in professional knowledge of online education may be helpful for MTEs but it may not be effective to require mathematics teacher education courses to fit into a fixed format of a universal online course because pedagogical practices would be different when content is taken into consideration with the online environment as indicated in this study.

Moreover, as indicated from my research, as MTEs moved their values in mathematics education into the online environment, a new learning environment in contrast to the traditional face-to-face environment, they were forced to think about the relationship of the content, pedagogical practices and the online environment, and to exercise their knowledge in every domain and their knowledge of the online environment for mathematics teacher education (KOMTE). MTEs make decisions on the fly when they teach in the face-to-face environment because the face-to-face environment is more
natural in their teaching experiences. Meanwhile, they need to think through all of their pedagogical decisions during the design and planning of the online sessions. Both the institution and the MTE community need to empower MTEs to have a greater degree of flexibility, knowledge and sufficient time to design the online course.

Conclusion

Online mathematics teacher education is gradually becoming a trend in mathematics teacher education. However, this particular genre of studies that focus on pedagogical practices of MTEs is still in its infancy. This study offered an overview of MTEs’ practices in the US and in-depth case studies on MTEs’ practices focusing on decision making. Results of the study showed that when granted freedom to take control of designing an online course, MTEs incorporated their values, beliefs, knowledge, and experiences to make pedagogical decisions with regard to the content and the technology. A model of MTEs’ decision making with regard to goals, orientations and knowledge was built to provide information that can guide the community as well as institutions on the kind of support that can provide for teacher educators. This study contributes to research on the development of MTEs and provides information for MTEs to examine their own pedagogical practices of online teacher educating or support future MTEs to develop the knowledge necessary for designing and conducting online teacher education.
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Poe, M., & Stassen, M. (2002). *Teaching and learning online: Communication, community, and assessment*


Zollinger, S. (2014). *Examining the knowledge domains used in the practice of mathematics teacher educating.* (Doctor of Philosophy), The Ohio State University, Columbus, Ohio.
Appendix A: Survey

Part I: Background Information

*This section contains four questions about your professional background.*

1. What is your current position?
   a. Faculty
   b. Graduate student
   c. District teacher leader
   d. Education consultant
   e. Other. Please specify: ________________________

2. Years of experiences of working with mathematics teachers
   a. 0-5
   b. 6-10
   c. 10-15
   d. >15

3. What is your current affiliation?
   a. Research focused university
   b. Teaching focused university
   c. K-12 school districts
   d. Other affiliation. Please specify: ______________

4. When was your first time teaching/developing an *online* mathematics teacher education course or professional development (PD)?
   a. Within the past 2 years
   b. 2 years ago
   c. 3 years ago
   d. 4 years ago
   e. 5 years ago
   f. More than 5 years ago

Part II: One Online Course
Questions in this section will ask you to share one online mathematics teacher education course or a PD program you designed and/or implemented. The selected course/PD should have **more than 50%** of the learning activities occur online.

- Yes, I can share an online course/PD that meets the requirement. (Continue)
- Sorry, I don’t have an online course/PD that meets the requirement. (End of the survey.)

5. Please provide the title or a brief description of the online course/PD you selected.

6. It is a __________________________. Select one that applies.
   a. course for mathematics teachers
   b. professional development for mathematics teachers
   c. Both

**Note:** For the rest of the survey, we will use “online course” for either a course or a PD program.

7. Which grade bands were the focus of the course? Select all that apply.
   a. Early childhood
   b. Elementary
   c. Middle grades
   d. High school

8. How many participants were in the course? (If you had multiple sessions, please select how many participants **per session**?)
   a. 10 or fewer
   b. 11-20
   c. 21-30
   d. 31-40
   e. 41-50
   f. 51-75
   g. 75-100
   h. 101-200
   i. More than 200

9. Did you meet in person? Yes/No
   Yes, then answer the following question
10. What was the percentage of your in-person sessions?
   Completely face-to-face         completely
   online
   | 0% | 20% | 40% | 60% | 80% | 100% |

11. Did you meet the participants synchronously? Yes/No
   Yes, then answer the following question

12. What was the percentage of your synchronous sessions?
   | 0% | 20% | 40% | 60% | 80% | 100% |

13. What communication methods did you use in your online course? Please select all that apply.
   a. Discussion board
   b. Interactive multimedia (e.g. Voice Thread)
   c. Presentation videos
   d. Chat rooms
   e. Collaboration websites (e.g. wiki)
   f. Social media
   g. Assessment rubrics to provide feedback
   h. Other. Please specify: ________________________

Part III: Course Content

_Questions in this section will ask you to share content activities of your online course and some important decisions you made for the content activities._

14. What were the top three foci of your course?
   a. Teachers’ mathematical knowledge
   b. Mathematics and learners’ mathematical thinking
   c. Mathematics and teaching methods
   d. Curriculum analysis and curriculum standards
   e. Technology integration
   f. Equity and social justice
   g. Research and theories in mathematics education
   h. History of mathematics and mathematics teaching
   i. Interdisciplinary and/or STEM connections
   j. Other. Please specify: ________________________

15. What mathematical content areas were addressed in your online course? Select all that apply.
   a. Number and Quantity
   b. Algebra and Functions
   c. Geometry
   d. Probability and Statistics
e. Other: ______
f. Does not apply

16. How often did you use the following approaches in delivering the online sessions? Please drag following items in the appropriate boxes.
a. Discussing skills and methods for computations  
b. Discussing multiple representations and interpretations of central ideas  
c. Solving complex or unfamiliar problems  
d. Investigating technology-based problems  
e. Connecting different mathematical ideas  
f. Discussing specific mathematical thinking processes (e.g. proving)  
g. Analyzing learners’ mathematics  
h. Designing tasks  
i. Reviewing curriculum materials  
j. Analyzing specific teaching practices (e.g. questioning)’  
k. Discussing social justice issues  
l. Discussing research in mathematics education  
m. Other. Please specify: ___________________

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17. Had you taught this course in the face-to-face environment prior to teaching it online?  
Yes/No

*If yes, answer next question.*

18. How often did you use the following approaches in delivering the face-to-face course? Please drag following items in the appropriate boxes.
a. Discussing skills and methods for computations  
b. Discussing multiple representations and interpretations of central ideas  
c. Solving complex or unfamiliar problems
d. Investigating technology-based problems
e. Connecting different mathematical ideas
f. Discussing specific mathematical thinking processes (e.g. proving)
g. Analyzing learners’ mathematics
h. Designing tasks
i. Reviewing curriculum materials
j. Analyzing specific teaching practices (e.g. questioning)
k. Discussing social justice issues
l. Discussing research in mathematics education
m. Other. Please specify: ___________________

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19. Who designed the online course?
   a. Mostly myself
   b. Some myself and some others
   c. Mostly others. I made some adjustments.
   d. All others. I just used the materials.

   *If Q19a, b or c is selected, answer the following:*

20. What informed your decisions? You may list the guided principles/theories/experiences.

   *If Q19a or b is selected, answer the following:*

21. Which of the following sequences best describe your design process?
   a. Sequence A: 1. Setting up the learning objectives; 2. Identifying the topics to be discussed; 3. Deciding how and what technology to use.
b. Sequence B: 1. Analyzing the affordances and constraints of technology; 2. Setting up learning objectives in light of the analysis of Step 1; 3. Deciding topics to be discussed.

c. Sequence C: 1. Setting up the learning objectives; 2. Identifying topics based on the learning objectives; 3. Analyzing the affordances and constraints of technology; 4. Identifying topics based on the analysis of technology; and 5. Deciding topics to be discussed based on Steps 2 and 4. (Note: Steps 1 and 2, and steps 3 and 4 are parallel.)
d. None above

*If d is selected, answer the following:*

Could you briefly describe your design process?

---

22. What were some important adjustments you made regarding the **choice of content to discuss** (both mathematics and teaching mathematics) or **content organization** for online delivery?

23. How important were the following resources to you when you designed/delivered the online course? Please drag following items in the appropriate boxes.

   a. Common practices and research in mathematics teacher education
   b. Common practices and research in mathematics teaching and learning
   c. Common practices and research in online education
   d. Experts in the department, college or field
   e. Experiences of teaching mathematics
   f. Experiences of working with mathematics teachers
   g. Past online teaching experiences
   h. Past online learning experiences
   i. Own program of research
   j. Personal mathematics knowledge
24. How many times have you taught this online course?
   a. None. I was involved in designing only.
   b. Once
   c. Twice
   d. Three times
   e. More than three times

25. What difficulties did you encounter when you designed/delivered the course online? Select all that apply.
   a. Engaging participants from different backgrounds
   b. Selecting appropriate online tools and/or mathematics specific technology
   c. Selecting tasks that are compatible with the online environment or other technology used
   d. Posing tasks or discussion prompts for desired online interaction
   e. Organizing online grouping
   f. Following and providing feedback to participants’ posts
   g. Getting participants to use online technology or other selected technology
   h. Others. Please specify: ______________________________________

Part IV. Perspectives of Online Teacher Education

*This is the last section. It has three questions about your opinions of online mathematics teacher education.*
26. In your opinion, what are the advantages of online mathematics teacher education? You can select maximum two that are relevant and important to you.
   a. Professional development opportunities for teachers from different locations
   b. Time for teachers to reflect. Deeper conversations and reflection.
   c. Opportunities to explore interactive mathematics educational technology
   d. Opportunities for teacher collaboration and building learning communities
   e. Opportunities to hear all teachers' voices
   f. Archiving teachers’ ideas
   g. Others: ________________________

27. In your opinion, which of the following courses/PD are easier to deliver online and which ones are easier to deliver face-to-face? Please drag following items in the appropriate box.
   a. Mathematics content course/PD
   b. Mathematics teaching methods course/PD
   c. Mathematics educational theory course/PD
   d. Technology integration course/PD
   e. Mathematics curriculum or assessment course/PD
   f. Interdisciplinary course/PD
   g. Large scale mathematics teacher PD
   h. Small scale mathematics teacher PD
   i. Teachers of different grade levels and/or background
   j. Teachers of similar grade levels and/or background

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28. Lastly, in your opinion, are there any mathematical topics that are more difficult to teach online than others? Please briefly provide reasons.
Thank you for participating in the online survey. We will donate $10 to AMTE for your effort of completion.

If you are willing to participate in a follow-up interview for more information about your online mathematics teacher training course or the online professional development program, please leave your name and contact information. Otherwise, click "submit" button to submit your response.

Last name:
First name:
Email:
Confirm email:
Contact number:
Appendix B: Interview Protocol (Dr. Abby)

Thank you for indicating your willingness to share your experiences of conducting online teacher education with me. Here are some follow-up questions for me to learn more about some key design decisions you made for the online course for teachers.

1. Before going to talk about your MOOC design experiences, I want to learn about you as a mathematics teacher educator. What are some core principles that guide your work as a mathematics teacher educator?

2. I want to learn more about the participants in your MOOC PD. Could you share more about the participants’ backgrounds? What was your original motive for offering such a course? How did your participants’ backgrounds affect your design and implementation of the course?

3. You mentioned in the survey, the objective of the MOOC PD was to advance content understanding and pedagogical strategies for teaching statistics. You followed a design framework that emphasize: (a) self-directed learning, (b) peer-supported learning, (c) job-centered learning, and (d) learning from multiple voices. Could you share how this framework guided your choice of pedagogical activities you used in the course?

4. You mentioned in the survey as well as in your paper that you built a new framework based on GAISE. Could you share how this framework guided your choice of mathematical activities you used in the course?
5. How many modules were in the course? How did each module progress through the course? In the survey, you mentioned you focused on a few big ideas in the course. What were they?

6. Could you share an example of one module – how did you organize content activities, how did you engage participants in the content, how did you interact with participants, and how did you assess their learning?

7. You mentioned you had taught the same course in the face-to-face environment. Could you share how different your face-to-face course was compared to the online course, in terms of the content design?

8. Did you encounter any constraints or limitations when you designed the MOOC PD? At what stage did you encounter the constraints? How did you overcome those constraints or limitations?

9. Did you have any experiences of online teaching prior to developing this course? If yes, how did your previous experiences of online teaching inform your design and implementation of this course?

10. I learned that you have done extensive research on probability and statistics teaching and learning. How did your work with teachers and students of statistics informed your design this MOOC PD?
11. Lastly, have you considered teaching other mathematical content in the online sessions? In your opinion, are there any mathematical topics that are more difficult to teach online than others?
Appendix C: Interview protocol (Dr. Beth)

Thank you for participating in this research. Today, I will ask about your online mathematics teacher educating experiences. Then we will explore your online courses.

1. Could you tell me about your teaching experience at the K-12 and/or other universities?

2. Could you tell me about your face-to-face teaching experiences in your current institution? When and what courses did you teach?

3. Could you tell me about your online teaching experiences in this institution? When and what courses did you teach?

4. Tell me about your online mathematics teacher education course.
   a. What was the title?
   b. What were the goals/objectives?
   c. Who are the audience? Pre-service or inservice
   d. Which grade band?
   e. How many participants?
f. What were the main objectives?

g. What mathematical content activities?

h. What were the practices?

i. What were the instructional strategies, group work, assessments, participation, etc.?

5. How was this different from the face-to-face course(s) you have taught in the past? (e.g., choice of content, instructional strategies, group work, assignments, participation, grading and so on)

6. How was this different from the online course(s) you have taught in the past? If it is different, why did you change it?

7. How and why did you design the current course the way you did?

   a. Could you provide me with an overview of the process of designing your online course?

      i. What were your goals for the course?

      ii. What principles or theories were guiding you in the process (selecting content, facilitating content for the right setting, structuring interactions)?
iii. What resources or information did you use?

b. Did you design the course by yourself, or were you expected to follow a course design created by a previous instructor or institution?

8. What would be the most difficult element for designing/teaching this course? Why do you think so? What is your plan?

9. What do you see the advantages of your online course? In what ways does it best support the learning and development of mathematics teachers?

10. What are some parts of your course that you believe need improvement? Why do you view that as a necessary improvement?

11. Could you tell me about one of the most effective lessons you have taught via online? Could you tell me about it in terms of course content, teaching experience, technology skills, online teaching skills, and so on? How did you design the lesson?

12. Do you happen to change and make certain design decision while your course is in session? (e.g. Feedback from your students, cadre group, online learning coordinator, etc.)?
Appendix D: Interview Protocol (Dr. Colin)

Thank you for indicating your willingness to share your experiences of conducting online teacher education with me. Before going to talk about your online course, I want to learn about you as a mathematics teacher educator.

1. What are some core principles that guide your work as a mathematics teacher educator? What kind of teaching/research experiences you had have influenced your work with teachers?

My understanding of the professional development was: 1) situated in a virtual learning environment (synchronous chatroom) integrated with dynamic geometry environment; 2) for middle and high school mathematics teachers who had little experience of dynamic geometry nor experiences of online collaboration; and 3) promoting productive discourse as one of the focuses. Please feel free to correct me if I’m wrong.

2. I wonder what your original motive for offering such an online professional development was.

You called the environment of your professional development program “online collaborative dynamic geometry environment”. Seemingly, you integrated two environments -- the virtual learning environment and the dynamic geometry environment.

3. What motivated you to integrate these two environments as you/your team designed the PD?

4. How do you see the role of each environment plays in advancing teachers’ learning?

5. Particularly, you used the instrument-mediated model as one of your design principles. What instrument did you refer to – the online learning environment or the dynamic geometry environment?
You mentioned you had two types of tasks. One type was mathematical explorations using a multi-user, dynamic version of GeoGebra.

6. Were they all GeoGebra-based mathematical exploration tasks? Did you have other mathematical tasks apart from GeoGebra-based mathematical exploration tasks? Why or why not?

7. How did you organize these tasks?

8. You mentioned teachers worked collaboratively on 55 tasks. Were there any specific mathematics/mathematics learning themes you wanted to address among these 55 tasks?

The other type of tasks was what you called “meta-tasks” where teachers reflected on their own process and content of mathematical discourse occurred in prior tasks.

9. What motivated you to have these meta-tasks? (How did you come up with these meta-tasks?)

10. The example you provided was reflecting on their collaboration norms. What were other themes/topics that teachers were ask to reflect upon in these meta-tasks?

11. How did you organize these meta-tasks? Did you/facilitators provide feedback to these tasks?

You mentioned in the survey that you taught a similar course in the face-to-face environment.
12. How did your previous experiences of face-to-face work with teachers inform your task design?

Lastly, have you considered teaching other mathematical content using the same environment you have designed, or using a different virtual platform? What advantages and limitations do you foresee if you teach other mathematics topics in this environment? Please specify the mathematics topics.