Refractivity Inversion Utilizing X-Band Array Measurement System

DISSERTATION

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Abstract

Variations in the refractive properties of the marine atmospheric boundary layer (MABL) can lead to non-standard propagation of radiowaves. An ability to quickly assess the influence of the atmosphere on shipboard surveillance and communication systems is required to avoid unwanted extended signal transmissions as well as poor functionality of these systems. While refractive conditions can be determined in numerous ways, methods utilizing radio frequency propagation measurements can directly determine the impact of the atmosphere on these systems.

A novel transmit-receive array system called the X-band Beacon-Receiver array (XBBR) was developed with the purpose of determining MABL evaporation duct height (EDH) values. An experiment campaign was conducted to deploy the multichannel array system and corresponding beacon transmitters to investigate their ability to characterize MABL refractivity utilizing both the amplitude and phase of recorded signals. The method proposed compares propagation loss and phase values given by the Variable Terrain Radio Parabolic Equation (VTRPE, Ryan, 1991) modeling software for various propagation environments with measurements obtained by the XBBR array. Meteorological data was also recorded to act as input to the Navy Atmospheric Vertical Surface Layer Model (NAVSLaM); this allows for determination of the evaporation duct height from in-situ meteorological data to serve as the ground truth for comparison with our evaporation duct height estimation.
Furthermore, this dissertation investigates the temporal and spatial fluctuations of radio frequency transmissions in a turbulent atmosphere. The multiple receive channels of the XBBR allow for the covariance of signals measured at each receiver to be compared with a model and atmospheric turbulence parameters to be extracted. This model is then used to simulate possible transmit and receive array configurations in an attempt to optimize system performance and minimize ambiguities in EDH inversion.

This dissertation discusses campaign measurements, EDH inversion results, the effectiveness of coherently measuring phase to decrease inversion ambiguity, and through simulations studies provides insight into the physical geometry of future propagation measurement systems with improved inversion accuracy.
For my friends, family, and fellow scientists
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Chapter 1: INTRODUCTION

1.1 Motivation

The performance of wireless communication and radar systems can be greatly impacted by the medium in which they operate. Radio equipment attempting to transmit data through or gather information about a maritime environment in particular can be affected by variations in atmospheric refractivity. Knowledge of the local propagation environment could help to provide system performance information. This dissertation discusses the development of a real-time method to estimate refractive conditions in the marine atmospheric boundary layer (MABL) for these purposes.

1.2 Atmospheric Refractivity

The refractive index, represented as $n = c/v$, describes the influence of a medium on how light, or other radiation, propagates through the medium. Here, $n$ is the index of refraction, $c$ is the speed of light in a vacuum, and $v$ is the phase velocity of light in the medium. When electromagnetic radiation passes between materials with different values of $n$, the waves bend, or refract, according to Snell’s law:
Figure 1.1: For an electromagnetic wave traveling from a medium with a higher index of refraction with incidence angle $\theta_1$ to a medium with a lower index of refraction ($n_1 > n_2$), Snell’s law predicts that the transmitted wave will refract away from the normal at angle $\theta_2$.

$$\sin(\theta_1)n_1 = \sin(\theta_2)n_2$$  \hspace{1cm} (1.1)

where $\theta_1$ is the incidence angle of the wave, $\theta_2$ is the angle of refraction (both measured with respect to normal), and $n_1$ and $n_2$ are the indices of refraction for the two media. A simple example illustrating Snell’s law for a wave propagating through the boundary between a medium with a higher index of refraction to one with a lower index of refraction ($n_1 > n_2$) is shown in Fig. 1.1.

Values of $n$ in the atmosphere are typically very close to one, around 1.00035. Small changes in $n$ can have a significant impact on radio waves and $N$, the “refractivity”, is typically used, given by:

$$N = (n - 1) \times 10^6$$  \hspace{1cm} (1.2)
Values of $N$ in the atmosphere typically vary between 200 and 400 $N$-units and can be calculated from atmospheric properties using:

$$N = 77.6\frac{P}{T} - 5.6\frac{e}{T} + 3.75 \times 10^5\frac{e}{T^2}$$  \hfill (1.3)$$

Here, $P$ is the barometric pressure (millibars), $e$ is the partial pressure of water vapor (millibars), and $T$ is the absolute temperature ($K$) [6]. In the MABL, many factors, including evaporation from the body of water, lead to changing values of $N$ with respect to height above the water. For example, the air just above the ocean surface can have a slightly higher value of $e$ than the air above. According to Eq. 1.3, while holding all other variables constant, this negative $\frac{\partial e}{\partial z}$ (where $z$ is altitude), results in negative $\frac{\partial N}{\partial z}$. While in reality this gradient would appear continuous, by modeling the atmosphere as discretized layers having a constant value of $N$ (or $n$) in each layer, the effect of $-\frac{\partial N}{\partial z}$ on a propagating wave can be visualized; see Fig. 1.2.

The gradual refraction of radiation, exemplified in Fig. 1.2, can greatly affect the propagation of radio waves transmitted from shipboard communications and sensing systems. Because these systems operate over long ranges (on the order of km), refractivity is often discussed in terms of modified refractivity, in $M$-units, which takes into consideration the curvature of the Earth and is defined as:

$$M = N + \frac{h}{a} \times 10^6 \simeq N + 0.157h$$ \hfill (1.4)$$

Here, $N$ is the refractivity, $h$ is the height above the Earth’s surface (m), and $a$ is the radius of the Earth (km).

The degree to which a wave is refracted can be categorized as experiencing either sub-refraction, standard refraction, super-refraction, or ducting. Of these categories,
Figure 1.2: For electromagnetic radiation traveling through discretized atmospheric layers in which $n_1 > n_2 > n_3 > n_4 > n_5$, the incident wave is refracted further and further away from the normal.

which are defined by their refractivity gradients and are shown in Fig. 1.3, the case that is of interest for this dissertation is ducting. As can be seen in Fig. 1.3, when $dM/dz = 0$, the ray curvature equals the curvature of the Earth. Ducting then occurs when the M-gradient is negative which causes the transmitted wave to bend toward the Earth faster than the curvature of the Earth causes the propagating wave to bend away from the Earth. The wave then reflects off the surface of the Earth, refracts back toward the surface again, and remains trapped in a "duct" [7].

Ducts in the MABL can be one of three different types: Evaporation Ducts, Surface Based Ducts, and Elevated Ducts. These three types of ducts are defined by their $M$-profiles ($M$-units vs. altitude) and are shown in Fig. 1.4. Evaporation ducts are the most common type, typically present because of oceanic evaporation.
Figure 1.3: Refractivity conditions include subrefraction, standard, superrefraction, and ducting [1].

This duct type is caused by a rapid decrease in humidity just above the surface of the water body over which the duct exists. As can be seen on the right side of Fig. 1.4, the presence of a duct can lead to propagation well past the horizon and to the existence of “radio holes” (atmospheric regions in which, due to the refraction of waves propagating at certain angles, radio waves will not propagate). In an evaporation duct, the point at which $dM/dz = 0$ is known as the evaporation duct height (EDH) and is important in describing the evaporation duct. Values of EDH typically range between 0 and 40 meters and will be the focus of this dissertation because of their prevalence and strong effect on propagation.

Refractivity conditions can vary greatly around the world. Details pertaining to the refractive properties of the atmosphere off the coast of Southern California,
Figure 1.4: Left: M-profiles typical for Evaporation Ducts (a), Surface Based Ducts (b), and Elevated ducts (c). Right: Typical EM propagation paths caused by the corresponding M-profiles. [1].
Northern California, Wallops Island, Virginia, and in the Persian Gulf, are provided in [8–11], respectively.

1.3 M-profile Models

Multiple models exist for relating vertical M-profiles to atmospheric variables. These include the Musson-Genon Gauthier Bruth (MGB) model [12], a model proposed by Babin [13], the Paulus-Jeske (PJ) model [14], and the Navy Atmospheric Vertical Surface Layer Model (NAVSLaM) [15–17]. The two models that will be discussed here are the PJ model and NAVSLaM.

1.3.1 Paulus-Jeske

The PJ evaporation duct model is commonly used to define the M-profile from specific atmospheric conditions and its derivation can be found in [18]. Following Paulus, \( N \) can be defined as:

\[
N = \frac{77.6P}{T} + 3.73 \times 10^5 \frac{e}{T^2} \tag{1.5}
\]

In the atmospheric surface layer (ASL) (which is generally regarded as the region of the atmosphere adjacent to the ocean (or other body of water) surface and up to 100 m or higher), the potential refractivity can be defined as

\[
\phi = \frac{77.6P_0}{\theta} + 3.73 \times 10^5 \frac{e_p}{\theta^2} \tag{1.6}
\]

where \( \theta \) is the potential temperature, \( e_p \) is the potential waver vapor pressure, and \( P_0 \) is the pressure (set to 1000 mb). The potential refractivity \( \phi \) relative to the
potential refractivity $\phi_0$ at the sea surface can be used as a formulation for profiles of conservative properties in the ASL. According to [19], this is given by

$$\phi - \phi_0 = \frac{\phi^*}{\kappa} \left[ \ln \left( \frac{z + z_0}{z_0} \right) - \psi_\phi \left( \frac{z + z_0}{z_0} \right) \right]$$

(1.7)

were $\phi_0$ is calculated from the sea surface temperature and the saturation water vapor pressure at that temperature, $\phi^*$ is a scaling factor, $\kappa$ is von Karman’s constant, $z$ is the altitude, $z_0$ is the aerodynamic roughness length, and $\psi_\phi$, which is a stability function, is zero for neutral conditions. By taking the derivative of Eq. 1.7 with respect to $z$, Paulus derives:

$$\frac{d\phi}{dz} = \frac{\phi^*}{\kappa(z + z_0)}$$

(1.8)

As given in Fig. 1.3, the gradient required for ducting is

$$\frac{dN}{dz} = -\frac{10^6}{a} = -0.157 \text{ N/m}$$

(1.9)

where $a$ is the Earth radius in meters. Looking at these derivatives and assuming that $\theta \simeq T$ and $e_p \simeq e$, the following can be written

$$\frac{d\phi}{dz} = \frac{dN}{dz} - \frac{\partial N}{\partial P} \frac{dP}{dz}$$

(1.10)

According to Paulus, the partial derivative $\frac{\partial N}{\partial P} = \frac{77.6}{T}$ varies between 0.28 and 0.26 mb$^{-1}$ between 0°C and 30°C. $\frac{dP}{dz}$ can then be evaluated as

$$\frac{dP}{dz} = -\rho g = -\frac{P g}{RT}$$

(1.11)
where: $\rho$ is the density, $g$ is the acceleration due to gravity, $P = P_0$, $R$ is the individual gas constant for dry air, and $T$ is the temperature. This leads to a variation of $\frac{dP}{dz}$ between -0.12 and -0.11 mb/m. By assuming the standard temperature (15°C), the critical gradient for trapping in terms of $\phi$ is -0.125m$^{-1}$ which leads to Eq. 1.10 becoming

$$-0.125 = \frac{\phi^*}{\kappa(\delta + z_0)}$$  \hspace{1cm} (1.12)

Here, $\delta$ is the height at which the trapping layer occurs (EDH) and the aerodynamic roughness length, $z_0$, is taken to be $1.5 \times 10^{-4}$ m. If neutral conditions are assumed, when the air-sea temperature difference (ASTD) is zero, this can be simplified to a one-parameter, log-linear formula as a function of height above the sea surface ($z$) given by:

$$M(z) = M_0 + c_0 \left( z - \delta \ln \frac{z + z_0}{z_0} \right)$$  \hspace{1cm} (1.13)

where $M_0$ is the modified refractivity at the sea surface (typically around 330 M-units, this does not directly effect the propagation as it is the derivative of $M$ which determines how refractivity affects propagation [20]), $c_0$ corresponds to the neutral refractivity profile described by Paulus [18] as $c_0 = 0.125$ M-unit/m. For EDHs of 0, 5, 10, and 20 m, and an $M_0$ value of 300 M-units, the M-profiles calculated using Eq. 1.13 are shown in Fig. 1.5.

While other, multi-parameter, models are capable of providing M-profiles that are more representative of atmospheric conditions, Eq. 1.13 has been shown to be a relatively accurate and reasonable approximation for the M-profile consisting of an evaporation duct. This dissertation addresses an inversion method, discussed later,
which is greatly simplified by the fact that Eq. 1.13 is a function of a single parameter, \( \delta \). For this reason, the PJ model is used in forward modeling computations in what follows.

### 1.3.2 NAVSLaM

The Navy Atmospheric Vertical Surface Layer Model (NAVSLaM) is an evaporation duct model used by the Navy and is part of the U.S. Navy’s Oceanographic and Atmospheric Library (OAML). NAVSLaM can use limited atmospheric data to characterize near-surface radio-frequency refractivity and output M-profiles above the ocean by employing the Monin-Obukhov similarity theory (MOST) [16]. Atmospheric data acting as input to NAVSLaM typically comes from numerical weather model outputs, satellite measurements, and *in-situ* observations.

The MOST is a relationship that allows for vertical mean flow and turbulence properties in the ASL to be expressed as universal dimensionless functions [17].
Figure 1.6: PJ and NAVSLaM M-profiles for an EDH of 9.6 m. The NAVSLaM profile was created with an air temperature of 22.2 °C water temperature of 24.0 °C, a wind speed of 4.5 m/s, relative humidity of 75.5%, and atmospheric pressure of 1009.1 hPa. The profiles, with M-units on the x-axis and altitude on the y-axis, are compared with (left) $M_0 = 0$ (so the differences in M-deficit are apparent), and (right) $\min(M\text{-profile}) = 0$ (to better see the differences M-profile slopes above and below the duct).

The application of MOST requires the assumption that atmospheric conditions are horizontally homogeneous and stationary and that turbulent fluxes within the ASL are nearly constant. NAVSLaM uses a modified version of the COARE (Coupled Ocean Atmosphere Response Experiment, see [21,22]) bulk algorithm to compute refractivity profiles. Some uncertainties in M-profile creation arise when the Air-Sea Temperature Difference (ASTD) is between 0.5 and 3 degrees Celsius [23]. NAVSLaM acts as the “Ground Truth” for comparison with the inversion results that are discussed in this dissertation. Fig. 1.6 shows PJ and NAVSLaM M-profiles for an EDH of 9.6 m for comparison.
1.4 Refractivity Estimation Methods

Numerous methods have been developed to measure or predict atmospheric refractivity. The strengths and weaknesses of several methods are discussed in what follows.

1.4.1 Bulk Measurements

Direct refractivity estimation can be performed using \textit{in situ} data from radiosonde measurements. The use of either balloons or rockets to measure vertical temperature, humidity, and pressure profiles makes this possible by launching a sonde from various measurement platforms \cite{24-26}. Refractivity can then be calculated using Eq. 1.2. However, for the purpose of getting refractivity data quickly to help make decisions in real-time, this method is expensive, slow to retrieve the information needed, and usually undersamples spatial and temporal atmospheric variability.

1.4.2 Refractivity from Ground Based GPS Measurements

Another technique to measure refractivity involves using the signals from Global Positioning System (GPS) satellites to infer information about the environment. Anderson \cite{27,28} estimates the refractive index of the troposphere by comparing observed interference patterns from satellite-borne beacons at low elevation angles to patterns from a family of assumed refractivity profiles. Lowry \cite{29} performs further investigation into this method. Because signals emitted from satellites passing near the horizon pass through the lower atmosphere, the signal is distorted. By comparing ray propagation models with a least squares metric to the measured tropospheric delay and phase difference in the GPS signal the refractivity can be estimated. Lin \cite{30} and
Wang [31] have also developed methods to infer the refractivity profiles by measuring signals from GPS satellites. Lin obtains the N-profiles through the use of the zenith delay of a single ground-based GPS receiver, and Wang uses both direct and reflected signals of the GPS-L1 to invert for the EDH. These methods show promise because of their spatial coverage, but are still under development.

1.4.3 Radio Frequency Measurements

An early method of utilizing radio field-strength measurements to determine the M-profile was developed by Macfarlane [32]. However, Green [33] deduced that his method required accurate field-strength measurements on the order of $\pm 0.1 dB$ and was therefore impractical for actual application. Hitney [34] compares long-term UHF signal strength along a path in the southern California coastal area to the Naval Ocean Systems Radio Physical Optics model to invert for the refractivity profile. Rogers [35, 36] discusses the effects of variability in the atmospheric refractivity for similar radio frequency (RF) measurements taken during the 1993-1994 Coastal Atmospheric Refractivity (VOCAR) experiment. More recently, Jicha [37] attempts to estimate radio refractivity through the use of diffraction loss measurements in shadowed, over-the-radio-horizon environments. RF measurements show promise for investigation into refractivity due to the large impact a refractive environment can have on propagation.

1.4.4 Microwave Refractometry

Direct measurement of the refractivity of the atmosphere can be done using a microwave refractometer [38–40]. This device uses two cavities, one filled with the atmosphere to be sampled and the other is a sealed sample of standard atmosphere.
which will act as a reference. Both are exposed to the same microwave source and the resonance frequencies of each cavity can be compared to allow for direct measurement of the index of refraction [25]. While accurate, this method has high equipment cost and requires a large sampling time. Because of the large sampling time, temporal variability of the atmosphere (during the sawtooth flight pattern of a helicopter for example) can be an issue.

1.4.5 Lidar Refractivity Measurements

Using Lidar to characterize the evaporation duct location and scattering properties is also possible [41–43]. This is due to the ability for lidar to measure the gradients in density and specific humidity, using the rotational Raman temperature profile, and vibrational Raman scattered returns, respectively. While this method is both faster than radiosondes for gathering vertical variations in atmospheric properties and capable of measuring horizontal variations, it is high-cost and limited by daytime background radiation.

1.4.6 Refractivity from Clutter

Refractivity from clutter (RFC) refers to estimating the atmospheric refractivity profile through the use of radar clutter returns [20,44–55]. This is accomplished by determining the environment whose simulated clutter patterns match radar measurements. This has promise because it can use the radar returns that are already available to most ships. However, it is limited in range for EDH estimation by the limitations of the pre-existing navigational radar’s transmitting parameters.
1.4.7 COAMPS

Atmospheric prediction models, like the Coupled Ocean/Atmosphere Mesoscale Prediction System (COAMPS) developed by the Naval Research Laboratory (NRL), can also be used, in conjunction with NAVSLaM, to predict ducting conditions [56–60]. This method is very good for large-scale estimations, but is not particularly accurate for small-scale situations and has difficulties predicting the M-profile at low altitudes.

Of these methods for determining the refractivity of the atmosphere, bulk parameter measurements, COAMPS, ground based GPS measurements, and microwave refractometry are all somewhat limited either spatially or temporally in determining instantaneous refractivity. RF propagation measurements, however, with the advent of faster computers and efficient modeling algorithms, can be utilized to quickly determine the prevailing refractive conditions. It is for these reasons, and the fact that little to no investigation into the effects of a refractive environment on signal phase has occurred, that the focus of this dissertation will be the utilization of complex RF propagation measurements to determine refractive conditions.

1.5 Dissertation Overview

The main objective of this research is to improve the current ability to estimate evaporation duct heights in the MABL. This has been accomplished by developing a measurement system capable of fitting a vertically sampled, normalized, array of one-way transmission loss measurements with parabolic equation models. Chapter 2 discusses the strengths and weaknesses of multiple approaches to modeling the effects of refractivity on radiowave propagation. Chapter 3 explains the inversion technique
used to estimate the EDH and describes the X-Band Beacon Receiver (XBBR) array system that is used to obtain the required information. Simulation studies that were performed in the system design process are also discussed. Chapter 4 describes measurement campaigns in which the methods described in this dissertation were applied, and discusses the results that followed. Chapter 5 attempts to model the log-amplitude and phase fluctuations of a plane wave in a turbulent atmosphere and also shows that if phase information of transmitted signals can be accurately measured, it can improve the inversion technique discussed in Chapter 3. In Chapter 6, statistical methods are applied in an attempt to optimize XBBR array parameters for the purposes of decreasing inversion ambiguity. The dissertation then concludes with Chapter 7, which addresses future improvements that can be made to the work discussed here.
Chapter 2: MODELING NON-STANDARD PROPAGATION

2.1 Introduction

Numerous techniques have been developed and utilized over the years to model radiowave propagation in both standard and non-standard atmospheres. Some of these methods include mode theory, geometric optics (GO) and parabolic wave equation (PWE) techniques. This chapter discusses these approaches to modeling radiowave propagation.

2.2 Mode Theory

Watson [61] first introduced mode theory in 1918 to solve for the diffraction of radio waves by the Earth. Later, the approach was expanded upon by Budden, 1961 [62] and applied to both terrestrial and tropospheric waveguides by Wait [63, 64]. While most mode theory algorithms have been replaced by parabolic equation methods, the MLAYER waveguide model [65] still provides a reliable reference solution for the other methods discussed here.

When a small number of modes exist, mode theory provides a very efficient solution through an eigenfunction decomposition of solutions to the wave equation.
Difficulties arise, however, as more modes exist, which is typical when an elevated duct is present. Mode theory also struggles when dealing with range-dependent refractivity environments [66]. Basic principles of mode theory are discussed in Chapter 5 of Levy [3].

Modeling propagation in a ducting atmosphere in terms of propagation in a dielectric slab waveguide is one approach that can be taken while considering mode theory. Balanis [67] discusses solving for the electric and magnetic fields in a dielectric-covered ground plane. For the purposes of ducting atmospheres, the ocean acts as the ground plane, the portion of the atmosphere above the ocean and below the plane that refracts radiowave propagation back toward the ocean acts as the dielectric, and the atmosphere above is treated as air. The refractivity profiles in this model can be described by a piece-wise linear function which allows for the index of refraction at each horizontal layer of the waveguide to be defined.

Extending the structure to include more than two interfaces increases the complexity of solving for the modes of propagation, but could be helpful for the case of ducting above an ocean because of how the refractivity profile can act as multiple dielectric layers (evaporation duct and elevated duct, for example). One method of approaching a multilayer dielectric waveguide is covered by Lee in [68]. He begins by solving for the modes and dispersion characteristics for an N-layer lossless planar waveguide (Lee’s derivation is for optical applications, but the theory can be applied to the case of electromagnetic trapping).

Other methods of solving for the fields within a multilayer dielectric waveguide are discussed in [69, 70], among others. Hunsperger follows a similar derivation on
Anemogiannis [69] approaches the problem using a “transfer-matrix formulation” to find the modes that propagate within a waveguide. As can be seen in these works, small variations in layer depth, index of refraction, frequency, etc., can greatly affect which modes will exist and where they will exist. Solutions of this type with emphasis on a structure similar to that of an evaporation duct and/or an elevated duct may be extremely complicated because of the variability of the structure. Each layer, because of its individual index of refraction, will treat each frequency slightly differently in such ways that what modes end up as evanescent waves and what end up trapped in the next layer, which, for a variable environment, can be a quite intensive derivation.

Solutions to the waveguide problem provide answers similar to those given by the parabolic wave equation. They both allow for the computation of an electric or magnetic field due to some source, at some distance from the source. The waveguide solution, however, is less flexible for problems in which the structure is extremely variable and irregular, as in the case of evaporation ducts above the ocean. That being said, the waveguide can still provide insight into the general behavior of radio waves above the ocean.

For the purposes of using measured RF signals at a distance from a known receiver to estimate the evaporation duct height (EDH), the model for a waveguide approximation for this structure will not provide a more complete answer than either the parabolic wave equation or a hybrid theory like the Advanced Propagation Model (APM). However, using the waveguide model for various possible scenarios (various evaporation duct heights, for example) could allow for quick calculations of what modes will propagate and at what angle they will propagate with respect to the
horizontal. This could be useful in determining if there are more optimal transmit or receive heights to reduce the possible group delay of the different modes in the signal at the receiver. This method could also be helpful in showing what frequencies might be close to the cutoff frequencies for particular modes and might be more likely to have dispersion errors caused by group delay.

### 2.3 Geometric Optics

Geometric optics (GO) and ray tracing methods, while not representative of the diffractive effects of the environment on radio transmission, are often used as part of hybrid propagation models [72, 73]. Along with quickly providing potentially important information regarding the behavior of wavefronts, ray-tracing methods efficiently provide accurate solutions for propagation angles where refractive effects are less severe, typically greater than $5^\circ$ from the main direction of propagation [3]. Levy describes ray-tracing methods suitable for a range-independent environment, and more general information regarding geometric optics can be found in [74–77].

### 2.4 Parabolic Wave Equation

The parabolic approximation of the wave equation, which models energy propagating in a cone centered in a particular direction known as the paraxial direction (described in section 2.4.2), was first developed by Leontovich and Fock beginning in the 1940s [78, 79] with the goal of solving for the diffraction of radio waves at long ranges. While some powerful theories of diffraction by obstacles were developed by Malyuzhinets [80] in the 1950s, it was not until the processing power provided by digital computers that the parabolic equation (PE) underwent further investigation [3].
Rather than looking for closed-form expressions, Hardin and Tappert [81, 82] developed an efficient numerical solution to the PE, known as the split-step/Fourier solution, for application to underwater acoustic problems, and Claerbout developed a finite difference method for geophysics applications [83]. Developments like these lead to the PE’s application in many fields including waterwave propagation [84], optics [85], and seismic wave propagation [86]. The focus here will remain on the application of the PE to tropospheric radiowave propagation to determine field strength at long ranges in anomalous propagation environments. This idea was introduced by Ko et al. in 1983 [87].

2.4.1 PWE Derivation

The PWE can be derived by starting with Maxwell’s equations in a sourceless environment, and following Kuttler [2]

\[
\nabla \times \mathbf{E} = -j\omega \mu \mathbf{H} \quad (2.1)
\]

\[
\nabla \times \mathbf{H} = j\omega \varepsilon \mathbf{E} \quad (2.2)
\]

Taking the curl of Eq. (2.2) and using Eq. (2.1) appropriately, it follows that

\[
\nabla \times (\nabla \times \mathbf{H}) = \nabla \times (j\omega \varepsilon \mathbf{E})
\]

\[
\nabla \times \nabla \times \mathbf{H} = j\omega (\nabla \varepsilon \times \mathbf{E} + \varepsilon \nabla \times \mathbf{E})
\]

\[
\nabla \times \nabla \times \mathbf{H} = j\omega (\nabla \varepsilon \times \mathbf{E} + \varepsilon (-j\omega \mu \mathbf{H})) \quad (2.3)
\]

\[
\nabla \times \nabla \times \mathbf{H} = j\omega (\nabla \varepsilon \times \left( \frac{\nabla \times \mathbf{H}}{j\omega \varepsilon} - j\omega \mu \varepsilon \mathbf{H} \right))
\]

\[
\nabla \times \nabla \times \mathbf{H} - \frac{\nabla \varepsilon}{\varepsilon} \times \nabla \times \mathbf{H} - \mu \varepsilon \omega^2 \mathbf{H} = 0
\]
where \( \omega \) is the angular radiation frequency, \( \mu \) is the magnetic permeability, and \( \varepsilon \) is the electric permittivity. Spherical coordinates, with origin in the center of the earth as seen in Fig. 2.1, are used for this portion of the derivation.

Here the goal is to describe the electric and magnetic fields in the far-field from a source located at \( \theta = 0, r = r_s \) in the region \( r \geq a_e \), where \( a_e \) is the radius of the earth. Focus will remain on the propagation case of a vertical electric dipole for which the existing, non-zero fields include: \( E_r, E_\theta \), and \( H_\phi \). It follows that \( H = H_\phi \hat{\phi} \) [2].

The first assumption made in this derivation is that all quantities will be independent of \( \phi \). This assumption works if \( \varepsilon \) varies slowly in the \( \phi \) direction (\( \varepsilon \) remains a function of \( r \)). Eq. 2.3 can now be expanded to get the scalar wave equation for \( H_\phi \) using

\[
\nabla \times \nabla \times H = \nabla (\nabla \cdot H) - \nabla^2 H
\n
\]
\[ \nabla \times \nabla \times \mathbf{H} - \frac{\nabla \varepsilon}{\varepsilon} \times \nabla \times \mathbf{H} - \mu \varepsilon \omega^2 \mathbf{H} = \] (2.4a)

\[ \nabla (\nabla \cdot \mathbf{H}) - \nabla^2 \mathbf{H} - \frac{\nabla \varepsilon}{\varepsilon} \times \nabla \times \mathbf{H} - \mu \varepsilon \omega^2 \mathbf{H} = 0 \] (2.4b)

\[ (\nabla \cdot \mathbf{H}) = \nabla (\frac{1}{r^2} \frac{\partial}{\partial r}(r^2 H_r)) + \frac{1}{r} \frac{\partial}{\partial \theta} (\sin \theta H_\theta) + \frac{1}{r} \frac{\partial}{\partial \phi} \partial H_\phi \rightarrow 0 \] (2.4c)

\[ \nabla^2 \mathbf{H} = \frac{\partial^2 H_\phi}{\partial r^2} + \frac{2}{r} \frac{\partial H_\phi}{\partial r} - \frac{1}{r^2 \sin^2 \theta} + \frac{1}{r^2} \frac{\partial^2 H_\phi}{\partial \theta^2} + \frac{\cot \theta \partial H_\phi}{r^2} \] (2.4d)

\[ \nabla \times \mathbf{H} = \hat{r} \left( \frac{H_\phi \cos \theta}{\sin \theta} = \frac{\partial H_\phi}{\partial \theta} \right) - \hat{\theta} \left( \frac{H_\phi}{r} + \frac{\partial H_\phi}{\partial r} \right) \] (2.4e)

\[ \hat{u} = \frac{\nabla \varepsilon}{\varepsilon} = \frac{1}{r} \frac{\partial}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial \varepsilon}{\partial \theta} \hat{\theta} + \hat{\phi} \] (2.4f)

\[ \hat{v} = \left( \frac{H_\phi \cot \theta}{r} + \frac{1}{r} \frac{\partial H_\phi}{\partial \theta} \right) \hat{r} - \left( \frac{H_\phi}{r} + \frac{\partial H_\phi}{\partial r} \right) \] (2.4g)

\[ \hat{u} \times \hat{v} = (u_2 v_3 - u_3 v_2) \hat{r} + (u_3 v_1 - u_1 v_3) \hat{\theta} + (u_1 v_2 - u_2 v_1) \hat{\phi} \] (2.4h)

\[ \frac{1}{r} \frac{\partial}{\partial r} \left[ \frac{1}{\varepsilon} \frac{\partial}{\partial r}(r H_\phi) \right] + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left[ \frac{1}{\varepsilon \sin \theta} \frac{\partial}{\partial \theta} \sin \theta H_\phi \right] + \mu \varepsilon \omega^2 H_\phi = 0 \] (2.4i)

Determining a substitution for \( H_\phi \) that results in a relatively simple wave equation is then accomplished by choosing

\[ H = \varepsilon^\beta \sin \theta^\gamma r^v U(r, \theta) \] (2.5)

where \( \beta, \gamma, \) and \( v \) are yet to be determined constants. Eq. (2.5) can then be combined with Eq. (2.4) to get

\[ \frac{1}{r^{v+1}} \left[ \frac{\partial}{\partial r^2} (r^{v+1} U) + (2\beta - 1) \frac{1}{\varepsilon} \frac{\partial}{\partial r} \frac{\partial}{\partial r} (r^{v+1} U) \right] + \frac{1}{r^2} \left( \frac{\partial^2}{\partial \theta^2} U + \left[ (2\beta - 1) \frac{1}{\varepsilon} \frac{\partial}{\partial \theta} \varepsilon + (2\gamma + 1) \cot \theta \right] \frac{\partial}{\partial \theta} U \right) + (g + \mu \varepsilon \omega^2) U = 0 \] (2.6)
with
\[
g = -\sqrt{\varepsilon} \sin \theta \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) \frac{1}{\sqrt{\varepsilon} \sin \theta} \tag{2.7}
\]
Different values for $\beta$, $\gamma$, and $\nu$ are possible, but following [2], $\beta = \frac{1}{2}$, $\gamma = -\frac{1}{2}$ will be chosen, and $\nu = -\frac{1}{2}$. These choices cause the derivatives of $U(r, \theta)$ to be uncoupled from the terms involving $\varepsilon$ and $\sin \theta$ which is permissible with the assumptions made up to this point including the assumption that these describe the far-field. It then follows that
\[
\frac{1}{r^{\nu+1}} \left( \frac{\partial^2}{\partial r^2} (r^{\nu+1} U) \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} U + (g + \mu \varepsilon \omega^2) U = 0 \tag{2.8a}
\]
\[
\frac{1}{\sqrt{r}} \left( -\frac{U}{4 r^{3/2}} + \frac{1}{\sqrt{r}} \frac{\partial}{\partial r} U + \sqrt{r} \frac{\partial^2}{\partial r^2} U \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} U + U g + U \mu \varepsilon \omega^2 = 0 \tag{2.8b}
\]
\[
\left(-\frac{1}{4 r^2} + g + \mu \varepsilon \omega^2 \right) U + \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) U = 0 \tag{2.8c}
\]
where $\mu \varepsilon \omega^2 = k^2 n^2$ if $k = \omega / c$ and $n = \sqrt{\frac{\mu}{\mu_0 \varepsilon_0}}$. Eq. (2.5) then becomes
\[
H = \sqrt{\frac{\varepsilon}{r \sin \theta}} U \tag{2.9}
\]
Next, a transform into the rectangular coordinate system occurs in which the $z$- and $x$-coordinates may be thought of as altitude and range, respectively. The angle-preserving transformation function given in [2] is used:
\[
\xi = 2 a_e \frac{a_e + i \zeta}{\zeta + i a_e} \tag{2.10}
\]
where $a_e$ is the radius of the earth, $\zeta = r \sin \theta + ir \cos \theta$, and $\xi = x + iz$. Because the mapping is conformal, one can write
\[ \nabla_{r,\theta} = \left| \frac{d\xi}{d\zeta} \right|^2 \nabla_{x,z} \]  

(2.11)

Eq. (2.8c) can now be written as

\[ (\nabla_{r,\theta} + h_{r,\theta})U(r, \theta) = 0 \]  

(2.12)

where \( h_{r,\theta} = g - 1/4r^2 + k^2n^2 \). The transformed Helmholtz equation is then given by:

\[ \left( \nabla_{x,z} + \left| \frac{d\xi}{d\zeta} \right|^2 h_{x,z} \right) U(x, z) = 0 \]  

(2.13)

Rapid fluctuations for near-horizontal propagation is then done using

\[ U(x, z) = e^{jkx}u(x, z) \]  

(2.14)

which when combined with Eq. (2.13) yields

\[ \frac{\partial^2 u}{\partial x^2} + 2jk \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial z^2} + \left( h_{x,z} \left| \frac{d\xi}{d\zeta} \right|^2 - k^2 \right) u = 0 \]  

(2.15)

The next assumption to be made is the parabolic approximation. Assuming that

\[ \left| \frac{\partial^2 u}{\partial x^2} \right| \ll k \left| \frac{\partial u}{\partial x} \right| \]  

(2.16)

leads to the assumption that \( u \) and \( \varepsilon \) vary smoothly with range. This approximation limits good representation of the field to “nearly horizontal” propagation directions [2]: this approximation “has been found to result in significant modal phase velocity errors for propagation in directions that are more than \( \approx 15^\circ \).” Ignoring the \( \frac{\partial^2 u}{\partial x^2} \) neglects backscattered fields and the PWE can now be written as:
\[
\frac{\partial u(x, z)}{\partial x} = \frac{j}{2k} \frac{\partial^2 u(x, z)}{\partial z^2} + \frac{jk}{2} (m^2(x, z) - 1) u(x, z) \tag{2.17}
\]

where \( m = M \times 10^{-6} + 1 \) is the refractive index. If the refractive index varies too greatly with \( x \), then the paraxial approximation won’t hold and this version of the PWE should not be used. The PWE should not be used if permittivity varies quickly in the \( \phi \) direction, the backscattered field is of interest, propagation angles of interest are greater than \( \approx 15^\circ \) (although some methods of solving the PWE are capable of accommodating larger propagation angles which will be discussed later), or if the refractive index varies greatly in the direction of propagation.

Another approach to deriving the parabolic wave equation is found in [3] chapters 2, 3, and 4. Here, begin with the two-dimensional scalar wave equation

\[
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} + k^2 n^2 \psi = 0 \tag{2.18}
\]

where \( k \) is the wavenumber and \( n \) is the refractive index. This only remains a good approximation if \( n \) varies slowly on the scale of a wavelength. Next the reduced function in the paraxial, \( x \)-direction, is introduced

\[
u(x, z) = e^{-jkx} \psi(x, z) \tag{2.19}\]

Now writing Eq. (2.18) in terms of \( u \) provides

\[
\frac{\partial^2 u}{\partial x^2} + 2jk \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial z^2} + k^2 (n^2 - 1) u = 0 \tag{2.20}
\]

which for \( h_{x,z} \left| \frac{dk}{dx} \right|^2 / k^2 = n^2 \) is comparable to Eq. 2.15. This can be factored into

26
\[
\left[ \frac{\partial}{\partial x} + jk(1 - Q) \right] \left[ \frac{\partial}{\partial x} + jk(1 + Q) \right]
\]  
(2.21)

where what Levy calls the “pseudo-differential operator” \( Q \) is defined as

\[
Q = \sqrt{\frac{1}{k^2} \frac{\partial}{\partial z^2} + n^2(x, z)}
\]  
(2.22)

The two terms here represent the forward and backward propagating waves. These can be solved independently if the paraxial approximation is made to which the outgoing parabolic wave equation has the solution

\[
u(x + \Delta x, .) = e^{jk\Delta x(-1+Q)}u(x, .)
\]  
(2.23)

Different approximations of \( Q \) can be made that can lead to either narrow-angle or wide-angle parabolic equation solutions.

### 2.4.2 Paraxial Angle Approximation

Due to the nature of the problem, for the purposes of modeling radiowave propagation over long distances, many are “interested in solving problems where energy propagates at small angles from a preferred direction, called the paraxial direction” [3]. The positive \( x \)-direction is often chosen as this direction of propagation, as can be seen in Fig. 2.2, and the paraxial angle is the angle between the horizontal (or the desired direction of propagation) and the actual angle of propagation.

The paraxial approximation can be defined as

\[
\left| \frac{\partial^2 u}{\partial x^2} \right| \ll k \left| \frac{\partial u}{\partial x} \right|
\]  
(2.24)
which allows the $\frac{\partial^2}{\partial x^2}$ term to be ignored. The paraxial approximation, assuming the radiation will stay within the paraxial angle from the horizontal, allows for the split-step FFT-based formulation to be accurate within $\pm 15^\circ$ of the paraxial angle [3]. According to [2], modal phase velocity errors for propagation occur in directions of more than $15^\circ$ for acoustic problems. For a plane wave traveling at angle $\alpha$ from the paraxial direction, the first neglected term that arises in the Taylor expansion is proportional to

$$\frac{1}{k^2} \left| \frac{\partial^2 u}{\partial z^2} \right| = \sin^2 \alpha$$

(2.25)

where according to [3] error goes from $10^{-7}$ for $\alpha = 1^\circ$, to $10^{-3}$ for $\alpha = 10^\circ$, and over $10^{-2}$ for $\alpha = 20^\circ$. Application of the PWE for the purposes of this dissertation will occur for cases in which the range in the $x$-direction, $R$, is much greater than the height, $z$, above the paraxial direction ($R \gg z$). For this case, where $\arctan(\frac{R}{z}) = \alpha$, $\alpha$ remains small and the paraxial approximation holds.
2.4.3 PWE Algorithms

There exist three main families of PWE algorithms: finite-difference methods [3, 88], finite-element methods [89–93], and the split-step Fourier Parabolic Equation (SSF PE) method [2, 3, 30, 73, 81, 82, 94–99]. While all three methods have been proven to be both efficient and accurate solutions, the focus here will be on the SSF PE methods.

Split-Step Algorithm

In tropospheric propagation applications, the most common algorithm used in conjunction with the PWE is known as the Split-Step Fourier method. Following the derivation given by Kuttler et al. [2], the SSF PE algorithm is described in what follows. Beginning with the standard PWE:

$$\frac{\partial u(x, z)}{\partial x} = \frac{j}{2k} \frac{\partial^2 u(x, z)}{\partial z^2} + \frac{jk}{2} \left( m(x, z)^2 - 1 \right) u(x, z)$$  \hspace{1cm} (2.26)

where $u$ is of the form $e^{-jwt}$ and propagation is in the x-direction. The split-step (Fourier) technique uses the solution of the Fourier transform of Eq. (2.26). The transform is defined as

$$U(x, p) = \mathcal{F} u(x, z) \equiv \int_{-\infty}^{\infty} u(x, z) e^{-jpz} dz$$  \hspace{1cm} (2.27)

$$U(x + \Delta x, p) = U(x, p) e^{j(-p^2/2k + kM10^{-6})\Delta x}$$  \hspace{1cm} (2.28)

where $p$ is the transform variable which will act as the vertical wave number and Eq. (2.28) is the transformed equation after a step in $x$ of $\Delta x$. The inverse Fourier transform is then applied to Eq. (2.28) to get $u(x + \Delta x, z)$ which can be written as
\[ u(x + \Delta x, z) = e^{jkm(z)10^{-6}\Delta x} \mathcal{F}^{-1}\{e^{-j(p^2/2k)\Delta x} \mathcal{F}\{u(x, z)\}\} \tag{2.29} \]

in which the first term can be looked at as the environment propagator and the second term as the free space propagator. Variations in \( m \) introduce error when compared with Eq. (2.26) [100]. The PWE can now be written as

\[
\frac{\partial u(x, z)}{\partial x} = j(\mathcal{A}(x, z) + \mathcal{B}(z)) u(x, z) \tag{2.30}
\]

where

\[
\mathcal{A}(x, z) = \frac{k}{2}(m^2(x, z) - 1) \\
\mathcal{B}(z) = \frac{1}{2k} \frac{\partial^2}{\partial z^2} \tag{2.31}
\]

If \( m \) is constant, \( \mathcal{A} \) and \( \mathcal{B} \) commute and one can solve for \( u \) at \( x + \Delta x \) from \( x \)

\[ u(x + \Delta x, z) = e^{j\Delta x(\mathcal{A} + \mathcal{B})} u(x, z) \tag{2.32} \]

where

\[ e^{j\Delta x(\mathcal{A} + \mathcal{B})} = e^{j\Delta x\mathcal{A}} + e^{j\Delta x\mathcal{B}} \tag{2.33} \]

Now, the Fourier transform can be used to integrate across \( z \) using

\[
U(x, p) \equiv \mathcal{F}(u(x, z)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} u(x, z) e^{-jpx} dz \tag{2.34}
\]

\[
u(x, z) = \mathcal{F}^{-1}(U(x, p)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} U(x, p) e^{jpx} dp \tag{2.35}\]

where \( p = k \sin \theta \) and \( \theta \) is the angle above the horizontal. It follows that
\[ u(x + \Delta x, z) = e^{j \Delta x (m^2 - 1)/2} \mathcal{F}^{-1} \left( e^{-j \Delta x p^2 / 2k} \mathcal{F}(u(x, z)) \right) \]  
(2.36)

For a constant \( m \), this solution is easy, but varying \( m \) requires some approximations. The first being that \( \frac{\partial m}{\partial z} \) is small such that \( \mathcal{A} \) and \( \mathcal{B} \) commute, or

\[ (\mathcal{A} + \mathcal{B}) \left( \int (\mathcal{A} + \mathcal{B}) dx \right) \simeq \left( \int (\mathcal{A} + \mathcal{B}) dx \right) (\mathcal{A} + \mathcal{B}) \]  
(2.37)

Eq. (2.30) can now be rewritten as

\[ u(x + \Delta x, z) = e^{[i \int_x^{x+\Delta x} \mathcal{A}(x, z) dx + j \int_x^{x+\Delta x} \mathcal{B}(z) dx]} u(x, y) \]  
(2.38)

Assuming that \( m \) varies slowly with range it can be approximated that

\[ \int_x^{x+\Delta x} \mathcal{A}(x, z) dx \equiv \mathcal{A}(x + \frac{1}{2} \Delta x, z) \Delta x \]  
(2.39)

Lastly, because \( \mathcal{A} \) and \( \mathcal{B} \) no longer commute, the separation in Eq. (2.30) can no longer be exact. An alternative separation of \( \mathcal{A} \) and \( \mathcal{B} \) is

\[ \frac{\partial u(x, z)}{\partial x} = e^{j \Delta x \mathcal{B}/2} e^{j \Delta x \mathcal{A}} e^{j \Delta x \mathcal{B}/2} \]  
(2.40)

The error associated with using this separation method rather than Eq. (2.33) can be estimated by evaluating \( \frac{\partial u}{\partial x} \) from Eq. (2.32) with Eq. (2.33). The difference between the two can be shown to be [97]

\[ \Delta x [jm \frac{\partial m}{\partial x} u + m \frac{\partial m}{\partial z} \frac{\partial u}{\partial z} + \frac{m u}{2} \frac{\partial^2 m}{\partial z^2} + \frac{u}{2} \left( \frac{\partial m}{\partial z} \right)^2] + O[(\Delta x)^2] \]  
(2.41)

This error can be reduced by having a small \( \Delta x \) so the variation in the refractive index is small.
Attempting to solve the PWE numerically can lead to issues for overly complicated environments. There are many assumptions that go into creating the parabolic wave equation (paraxial approximation, constant in the $\phi$-direction, etc.) that will lead to errors if the environment is rapidly varying. Issues also arise when there is enough back-scatter interference to affect the propagation because the backward-propagation term is neglected before solving numerically. Choice of step size that doesn’t correctly capture the variation in the atmosphere will also affect the outcome of numerical solutions.

An alternative approach to the split-step formulation of the parabolic wave equation is given in [3]. Levy starts with the PWE

$$\frac{\partial u}{\partial x} = \frac{jk}{2} \left[ \frac{1}{k^2} \frac{\partial^2}{\partial z^2} + (n^2(x, z) - 1) \right] u \quad (2.42)$$

For the range-independent case, when permittivity does not vary with range, the solution can be written as

$$u(x, \Delta x), z = e^{\delta(A+B)}u(x, z) \quad (2.43)$$

where

$$A = \frac{1}{k^2} \frac{\partial^2}{\partial z^2} \quad (2.44)$$

$$B = n^2(z) - 1 \quad (2.45)$$

$$\delta = \frac{jk\Delta x}{2} \quad (2.46)$$
The goal of this approach is to split the equation into two terms to separate the diffractive effects (A) from the refractive effects (B). More information on the derivation by Levy can be found in chapter 3 of [3]. Solving the PWE numerically can be very computationally intensive if the assumptions named above (slowly varying permittivity in the x-direction and constant refractivity in the direction of propagation) are not followed. Also, because the split-step solution is usually implemented using the fast Fourier Transform (FFT), care must be shown to prevent aliasing caused by the fact that the FFT approximates the Fourier transform by a circular Fourier sum. This leads to maximum height values for specific frequencies. According to Craig [97]: “For a given FFT size, the Nyquist theorem implies that there is a maximum height in z-space ($z_{\text{max}}$) and a maximum $p = (k \sin \theta)$ in p-space that can be presented. Thus a cut-off in the maximum angle of propagation must be applied, and for a given FFT size, this cut-off will be smaller at higher frequencies.” Solutions to the split-step approach are also less suitable than finite-difference approaches for terrain modeling.

Another derivation of the split-step solution can be found in [2]. Here, the exact solution, for a constant $m$ is given by

$$u(x, z) = e^{jk^m(x-x_0)} \mathcal{F}^{-1} \left[ e^{-j \frac{\rho^2}{2k} \left( x-x_0 \right)} \mathcal{F}(u(x_0, z)) \right]$$  \hspace{1cm} (2.47)

where $u(x_0, z)$ is the initial field.

Variations of the aforementioned methods have been investigated thoroughly over the years. Levy et al. developed a horizontal Parabolic equation (HPE) method which they show to have substantial computing gains. Their HPE quickly provides a full-wave solution at each horizontal layer from an initial horizontal field that is
found using the standard PWE method [101]. Dockery applies a mixed-Fourier transform (MFT) to improve impedance boundary problems when using the SSF PE algorithm [95]. An investigation into the differences between the wide-angle and narrow-angle propagators is discussed by Kuttler et al. in [96]. Donohue et al. [102] applies a “shift-map” coordinate transform, developed originally by Tappert, to flatten terrain/boundaries for the wide-angle PWE solution. This method improved solutions for complicated and steeply sloped terrain. More recently, Lin et al. presents a higher order solution of the SSF PE method. Although this solution required smaller steps in range than the standard SSF PE method and therefore more computation time, it does provide the ability to quantify errors seen in the standard, lower order solution.

2.4.4 Applications of the PWE

Due to the flexibility of application and wide range of effects that are taken into consideration by the PWE, it has been adapted as the preferred technique for solving for tropospheric propagation [100]. Multiple thoroughly validated programs including TPEM (Terrain Parabolic Equation Model) [103], TEMPER (Tropospheric Electromagnetic Parabolic Equation Routine) [94], APM (Advanced Propagation Model) [104] which is a combination of TPEM and RPO (Radio Physical Optics), and VTRPE (Variable Terrain Radiowave Parabolic Equation) [105], implement the PWE to solve tropospheric propagation problems. The focus here will remain on the VTRPE model due to ease of use and availability.
VTRPE

The VTRPE model is a range-independent, computationally efficient computer program that applies the split-step Fourier method to solve the parabolic wave equation for the complex electric and magnetic fields as a function of height and range from a given radio frequency source [105]. Developed as an extension to the Radio Parabolic Equation (RPE) model, also developed by Ryan [106], VTRPE incorporates full-wave propagation physics to account for multi-path interference from surface reflections, the effects of diffraction phenomena, and the atmospheric refraction of radio waves.

VTRPE uses information regarding transmitter properties, atmospheric refractivity or meteorological data, and parameters including surface elevation, dielectric properties, ionospheric data, and rough sea and land surface parameters for its calculations. It provides the user with the power pattern propagation factor ($PF$), which can be converted to propagation loss ($PL$) using $PL = 20 \log(2k_0R) − PF$, along a vertical slice of the atmosphere on a pre-determined grid. Here, $k_0$ is the wave number and $R$ is the range. Example PWE calculated PL grids for an 11 GHz transmitter located 5 m above MSL, with a 45 degree half-power beamwidth for (a) 0 m EDH and (b) 20 m EDH, are shown in Fig. 2.3. The input file that was used to create the propagation loss grid with a 20 m EDH grid is given in Appendix B.

The only difference between the input files used for these two outputs (Fig. 2.3) is the refractivity profile, or M-profile which for both (a) and (b), was created using the Paulus-Jeske formula, Eq. 1.13, and by only varying the $\delta$ term (EDH). By modeling an evaporation duct using the PWE, it is apparent in Fig. 2.3 that varying that single parameter can lead to a significant decrease in PL on the order of 50 dB at a
range of 50 km. While this one parameter does not capture the complexity of the environment and problem at hand, it is precisely this variation in PL that will allow for the inversion technique in what follows.

In addition to providing the user with $PF$ and $PL$, VTRPE can also provide the user with the complex PWE field, $\psi$. The complex propagation factor is provided in the form:

$$F(z, r) = \frac{R}{\sqrt{r}} e^{j k_0 (r-R)} \psi(z, r)$$

(2.48)

where $z$ is the height above the surface, $r$ is the range, $R$ is the transmitter to receiver slant range given by $R = \sqrt{r^2 + (z - z_0)^2}$ (where $z_0$ is the transmitter height), and $k_0$ is the wavenumber. The scaling factor $\frac{R}{\sqrt{r}} e^{j k_0 (r-R)}$ arises from the conversion between
the Earth-centered spherical coordinates that the PWE calculations are performed under and cylindrical coordinates.

2.5 Conclusion

Mode theory, geometric optics, and parabolic wave equation techniques are all capable of providing insight into the effects of a refractive media on radio waves. While algorithms such as MLAYER and APM still take advantage of mode theory and geometric optics properties, many attempts to predict electromagnetic behavior in a refractive environment utilize the efficiency and accuracy of the PWE approach described here. The following chapter describes a measurement system and inversion technique that utilizes VTRPE to implement the PWE for the purposes of EDH estimation.
3.1 Introduction

An inverse problem one in which observations are used to determine the causal factors that produced them. The VTRPEs implementation of the PWE can provide a model with which to compare RF propagation measurements and allow for refractivity inversion. If the PWE is provided the necessary parameters for a certain measurement, the PWE can yield expected values of that measurement for different refractive conditions. While a single observation may lead to ambiguous inversion results, multiple measurements that take advantage of the spatial or frequency diversity capable of being modeled by the PWE can reduce these ambiguities. Measuring the signal strength from a single transmitter at one range but multiple heights can lead to different observed values, which can be seen in Fig. 2.3. Similarly, frequency diversity among transmitters can lead to unique propagation measurements because a refractive atmosphere affects different frequencies differently. Frequency diversity however, while an effective way to “probe” the propagation differences, requires a larger bandwidth and potentially higher cost system. With these considerations in
mind, a measurement system was designed to take advantage of the spatial diversity of RF propagation measurements to invert for the refractive conditions.

The goal of the X-Band Beacon Receiver (XBBR) array system is to characterize the evaporation duct height by measuring the propagation loss from a vertical array of transmitters at a vertical array of receivers for comparison with computer simulated propagation loss. X-band was chosen as the frequency of operation because a system back-end was readily available, the components are modest in size, initial tests showed that there would be little interference from other sources at test sights, and X-band propagation is strongly influenced by the effects of atmospheric ducting above the ocean. The effects of atmospheric ducting can be compared to how a waveguide can be used to focus radio waves in a given direction. For this reason, the idea behind transmitting and receiving from various heights was to try to sample the vertical extent of an evaporation duct.

3.2 Inversion Method

The inversion technique implemented by the XBBR array compares measured values of signal loss to the signal loss calculated by the PWE for different refractive conditions (varying EDH for simplification purposes). Conditions that lead to PWE signal loss values that are close to the measured values are taken as the estimated refractive conditions. In other words, the goal is to determine the parameter value (EDH) which maximizes the probability of matching the observed data given that a certain distribution around the “true” value is assumed. This is known as maximum likelihood estimation (MLE). To minimize the difference between the measurements and model, derivation of the maximum likelihood estimator begins with the expected
distribution of a variable. These measurements are expected to have a normal or Gaussian distribution which has a probability density function given by:

\[ p(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \] (3.1)

where \( \sigma \) is the standard deviation of the variable, and \( \mu \) is its mean. The likelihood function is then given by the product of \( n \) possible measurements adhering to this PDF which can be written as

\[ L(\mu, \sigma^2; x_1, ..., x_n) = (2\pi\sigma^2)^{-n/2} \exp \left( -\frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2 \right) \] (3.2)

This can be simplified by taking the log of the likelihood function which gives

\[ \ell(\mu, \sigma^2; x_1, ..., x_n) = -\frac{n}{2} \log(\sigma^2) - \frac{n}{2} \log(2\pi) - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2 \] (3.3)

The MLE of \( \sigma^2 \) can then be found by taking the derivative with respect to \( \sigma^2 \) and setting it equal to zero. The MLE of the variance is given by:

\[ \hat{\sigma}^2 = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n} \] (3.4)

For the purposes of refractivity inversion, \( \mu \) represents the PWE modeled signal loss and \( x_i \) represents a measured value of signal loss. Therefore, the PWE modeled value (and its corresponding EDH) which minimizes \( \hat{\sigma}^2 \), for a given measurement, returns the EDH which is most statistically similar to that measurement. The objective function, for a single measurement (\( n = 1 \)), that is minimized to invert for the EDH is then given by
\[ \Phi_{i,j}(EDH) = \left( L_{i,j,dB}^{obs} - L_{i,j,dB}^{sim}(EDH) \right)^2 \]  

(3.5)

where \( L_{i,j,dB}^{obs} \) is the observed (or simulated) signal loss value, and \( L_{i,j,dB}^{sim}(EDH) \) is the signal loss calculated using the PWE for varying EDHs. The same MLE derivation can be followed for a multivariate normal distribution in which the PDF is given by:

\[
p(x|\mu, \Sigma) = \frac{1}{(2\pi)^{N/2} |\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x - \mu)^T \Sigma^{-1} (x - \mu)\right)
\]  

(3.6)

where \( x \) is a vector of length \( N = N_r \times N_t \) (where \( N_r \) is the number of receivers and \( N_t \) is the number of transmitters), \( \mu \) (also a vector of length \( N \)) represents the PWE library values, and \( \Sigma \) is the covariance matrix of the measured paths. Following a similar approach as before, the derivative of the log-likelihood function (which is the product of \( n \) Gaussian distributions) can be found which leads to a maximum likelihood estimator for the variance, or covariance matrix \( \Sigma \) in this case, of

\[
\hat{\Sigma} = \frac{1}{n} \sum_{i=1}^{n} (x - \mu)(x - \mu)^T
\]  

(3.7)

The library which is most statistically similar to a given measurement is then found by finding the minimum of \( \hat{\Sigma} \). Which, for a single collection \( (n = 1) \), can be written in terms of measured and modeled signal loss values as:

\[
\Phi(EDH) = \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} \left( L_{i,j,dB}^{obs} - L_{i,j,dB}^{sim}(EDH) \right)^2
\]  

(3.8)

For a more in depth derivation of the MLE for multivariate Gaussian distributions, see [107].
3.3 System Design Studies

The investigation began with a design study to assess the utility of a multiple transmitter/receiver system for determining refractive properties in the lower atmosphere. Typical expectations for propagation over the sea surface would indicate significant multipath interference when in the line-of-sight, with the interference between the direct and sea-reflected paths eventually producing a rapid power decay in range on the order of \( R^4 \) or more, where \( R \) is the range between the transmitter and receiver. In the presence of ducting, the signal levels can be much higher due to the trapping of wave energy in the duct.

Supplying the PWE model with proposed transmit and receive heights, range, operational frequency, antenna patterns, and a modified refractivity profile produces predictions of propagation loss. As an example, the predicted propagation loss for a transmitter and receiver placed at 4.5 and 13.6 meters, respectively, above the surface of the Earth at a range of 15 km is illustrated in Fig. 3.1 as a function of the EDH. These results show that for varying EDH, the propagation loss to the receiver varies by about 30 dB. This propagation loss profile was created using a Gaussian transmit antenna with a 45-degree half-power beamwidth. For the given conditions, measuring a propagation loss of \( \sim 132.5 \) dB would indicate an EDH of 10 m, for example. However, ambiguities arise in this single transmitter-receiver case; a measured value of 140 dB could indicate an EDH of 0, 19, or 31 meters.

Given the complexity of the multiple possible “modes” in the MABL waveguide, which can interfere with each other, as well as other complex propagation behaviors, the use of a single transmit-receive path at a fixed range observed at a single frequency is insufficient to retrieve atmospheric information robustly. Some design
Figure 3.1: Using the PWE model, a library of propagation loss for specific transmitter height, receiver height, frequency, and for various evaporation duct heights was created. This figure shows the propagation loss for an 11.0487 GHz transmitter, 4.5m above the ocean and a 13.6m high receiver 15km from the transmitter.

Considerations for improving the information observed include the use of multiple frequencies, the use of phase information in the measurement, measuring loss at multiple ranges, or vertically sampling the environment at fixed range. The latter possibility was the focus of this investigation and was studied initially using a simulation of expected retrieval performance in a ducting atmosphere. While a single transmitter and receiver capable of predetermined vertical motion could similarly probe different propagation paths, an experiment designed in this manner would not facilitate the possibility of coherent phase measurements across the receive array. The design study was constrained to consider only transmit/receive array configurations that would be realizable at modest cost and complexity. A maximum of 3 transmitters and 5 receivers were examined.
3.4 Array Simulations

The method investigated for decreasing the ambiguity seen in Fig. 3.1 is to produce arrays of both transmitters and receivers to create a matrix of waveguide sampling points. By using fixed frequency beacons that are slightly detuned from one another, a “matrix” of propagation paths can be observed since the transmission of each beacon can be distinguished at each receiver. Note that the arrays considered here are “widely spaced” since the vertical separation between elements is very large compared to the electromagnetic wavelength. The wide spacing of elements is due to the benefits of spatially sampling received powers in the waveguide rather than seeking to beamform the received array in a particular direction. A complete PWE library for a possible set of system parameters is given in Fig. 3.2.

Figure 3.2: PWE values for signal loss from transmitters operating at 11.0455, 11.0479, and 11.0487 GHz, located 1, 2.7, and 4.5 meters above the water, measured by receivers located 3.8, 6.2, 8.6, 11.2, and 13.6 meters above the water at a range of 15 km.
A Monte Carlo simulation was run to examine the relationship between the number of measured propagation paths and duct height estimation accuracy. The transmitters used in this simulation operated at 11.0455, 11.0479, and 11.0487 GHz at 1, 2.7, and 4.5 m above mean sea level (MSL). The receive array was placed 15 km away and had possible receiver heights of 3.8, 6.2, 8.6, 11.2, and 13.6 m. Given the true propagation loss, as seen in Fig. 3.2, 1000 realizations of simulated measured propagation loss were created assuming log-normal noise on the received power with a standard deviation of 5 dB. Log-normal fading was assumed based on the expected signal fluctuation caused by shadowing in the MABL environment. These noise values were generated independently for each transmit-receive element pair because at this time it was uncertain whether or not the signals from each transmitter would remain correlated at the receive array. Investigation into correlated signal fluctuation will be discussed in Chapter 5. An estimated duct height is then obtained for all 1000 realizations in the simulation. Histograms comparing the actual EDH and the inversion result for different numbers of transmit-receive paths can be seen in Fig. 3.3. It is apparent from these histograms that there is much less ambiguity with a configuration that utilizes 15 transmit-receive paths than with one that uses only two.

The bias and standard deviation from the actual EDH is then calculated for each evaporation duct height. By performing Monte Carlo trials on multiple configurations for each number of transmit and receive paths and averaging the results, the relationship between estimation accuracy and the number of array elements is determined. The individual bias and standard deviation for varying path numbers can be seen
Figure 3.3: Histogram of evaporation duct height estimations based on 1000 Monte Carlo simulations for (a) 2 propagation paths, and (b) 15 propagation paths. The correct EDH is marked by the red vertical line.
Figure 3.4: The difference between the expected value and the actual value (a), and the standard deviation of the inverted EDH (b), for 1, 3, 5, 10, and 15 propagation paths.

in Fig. 3.4. Again, using this inversion technique, it is apparent that increasing the number of paths decreases the error and ambiguity.

Fig. 3.5 shows the relationship between the average bias and standard deviation for all EDHs and number of paths. The results show that significant ambiguities can occur with an insufficient number of transmit/receive paths. Thus, the larger the number of propagation paths that are measured, the greater the ability to determine evaporation duct height. From this study, it was decided to construct a prototype measurement system utilizing the maximum number of transmit and receive elements that could be operated within project budget constraints and pre-existing system limitations. While the simple log-normal noise assumed may not capture the full complexity of MABL environment propagation, it is expected that conclusions reached regarding the relative performance of differing transmit/receive combinations should be indicative of the relative performance of these combinations in practice.
Figure 3.5: The average bias (a), and average standard deviation (b), over all EDHs compared to number of paths for 1000 Monte Carlo trials.

### 3.5 Transmit System

The transmit beacons were designed to be low-power (in case deployment occurred on a buoy with limited power supply) and to be omni-directional in azimuth (in an attempt to negate the effects of possible platform motion on signal loss). An omni-directional antenna was purchased and mounted on watertight housing for the x-band oscillators as seen in Fig. 3.6. An early version of the transmit beacons utilized a voltage-controlled oscillator to allow for tuning of the beacons to avoid possible interference. These proved, however, to be very unstable, drifting on the order of multiple MHz/hour. The current iteration of beacons are excited using a dielectric resonator oscillator (which is more stable in frequency than the previous iteration, drifting on the order of only KHz) purchased from MITEQ Inc. with an output power of +13 dBm. The beacons were pre-tuned to the fixed frequencies of
Figure 3.6: Three transmitting beacons containing coaxial resonator oscillators driving omindirectional antennae at 13 dBm.

11.0455, 11.0463, 11.0471, 11.0479, 11.0487, and 11.0495 GHz. This slight separation in frequency allows for the beacons to be distinguished in post-processing.

Implementation of a phase-locked loop (PLL) control system was investigated as a potential transmit signal source. The ADF41020 from Analog Devices is a PLL frequency synthesizer capable of outputting frequencies between 4 and 18 GHz. In conjunction with the evaluation board (Analog Devices UG-405), it is speculated that a PLL transmit beacon may provide a programmable, wide-band, stable (in both frequency and amplitude) transmit source for a future measurement system capable of wide-band measurements.

3.6 Receive System

The receive end for this system captures signals from the various transmit beacons on five separate channels simultaneously. The signals are received by high-gain X-band horn antennas and then amplified and mixed with an 11-GHz signal generated with a local oscillator; distinct local oscillators for each channel are phase-locked using a 10-MHz reference. The signal then passes through bandpass filters and is sampled using three, two-channel, 16-bit ADCs operating at 80 MSPS. Data acquisition and
processing is performed by FPGAs that are synchronized by a common clock and trigger mechanism. Acquisition length, while adjustable within the FPGA software, was chosen to be one million samples per capture, or approximately 12.5 msec (this length was variable but chosen as a balance between recording enough data and allowing for the data to be saved to the drive in a reasonable amount of time for the next acquisition to begin). Raw data is then archived on the host computer for post-processing and the system is retriggered to record another 12.5 msec of data. A simplified block diagram of the XBBR can be seen in Fig. 3.7.

3.7 Post-Collection Processing

After storage of raw A/D samples, a ∼1 million point FFT is applied to the entire 12.5 ms acquisition. The FFT operation provides a coherent processing gain so that the data signal-to-noise ratio (SNR) is improved. By examining the received power over multiple acquisitions, a spectrogram can be created and the separate beacon tones can be identified and their amplitude vs. time extracted. This can be seen in Fig. 3.8.

In practice, direct comparison of measured SNR and the PWE library requires a robust calibration of measured receive powers as well as knowledge of the transmit powers, transmit antenna gains, etc. Given this knowledge, measured received powers could then be compared directly to those predicted by the model. In order to reduce calibration uncertainties caused by any variations in the measurement set-up, an alternate approach was used. In this method, model predictions of the powers received at each of $N_r$ receivers for each of $N_t$ transmitters (in dBm) are combined into an array of length $N_rN_t$. The data is then normalized. The mean of the array is subtracted
Figure 3.7: Simple block diagram of the XBBR system. The X-band horn antennas provide 20dB gain. The down-converter assemblies were purchased from Quinstar Technologies and consist of an amplifier, mixer, and local oscillator. Each provides 20dB conversion gain, has a noise figures of 3dB, and outputs an intermediate frequency of 10-100 MHz. The bandpass filters consisted of a low-pass filter (Mini-Circuits part number: ZX75LP-70+) and high pass filter (Mini-Circuits part number: SHP-48+) in series to reduce out of band noise. Analog to digital conversion is performed using Xilinx ML605 boards sampling at 80 MSPS. The maximum system bandwidth is then, based on Nyquist Theory, 40 MHz.
Figure 3.8: Example spectrogram of data from the SoCal 2013 campaign composed of 11,495, 12.5 msec measurements. There are ~3 seconds between each measurement. After applying the FFT the signal-to-noise ratio for each of the 3 visible transmitters, around 48, 55, and 77 MHz in this case, can be extracted.

from itself, and the result is divided by its standard deviation to create an array of standardized values relative to the average. The same operation is performed on measured data, and the standardized power values are used in the retrieval process. Standardizing allows both the measured and library values to have zero mean and unit variance which allows for an unbiased comparison. Doing so removes the influence of the beacon transmit power or other environmental effects (e.g. rain) that would impact all paths similarly, making the retrieval less dependent on such variations. This is under the assumption that all beacons transmit identical powers. The inversion could be expanded to invert for individual transmit powers if this is found to not be the case.
3.7.1 Analysis of Standardization Problem

First, the received power, $P_{r(i,j)}$ (for a given transmitter $i$ and receiver $j$), is the measured radio signal power given by the Friis transmission formula [108]:

$$P_{r(i,j)} = \frac{P_{tj}G_{tj}\lambda^2G_{ri}F_{i,j}}{(4\pi R)^2}$$

(3.9)

Here, $P_{tj}$ is the transmitter power, $G_{tj}$ is the transmitter antenna gain, $\lambda$ is the wavelength, $G_{ri}$ is the receiver gain, $F_{i,j}$ is the propagation factor for a specific transmit-receive path, and $R$ is the distance between the transmitter and receiver. The signal-to-noise ratio ($SNR$) is then defined as

$$SNR_{i,j} = \frac{P_{tj}G_{tj}\lambda^2G_{ri}F_{i,j}}{(4\pi R)^2(N_f r_j)(kB)} = \frac{P_{signal}}{P_{noise}}$$

(3.10)

where the noise term is composed of the receiver noise figure ($N_f r_j$), Boltzmann’s constant ($k_B$), and bandwidth ($B$), and $P_{signal}$ and $P_{noise}$ are the measured signal power, and measured noise power, respectively. The $SNR$ is used to negate the differences in signal levels caused by differences in receiver noise-floors. To remove the effects of signal variation from unknown variables in the $SNR$ calculation, this can first be simplified as

$$SNR_{i,j} = CF_{i,j}$$

(3.11)

The measured values are then converted to dB to match the output of the modeling software:

$$10\log_{10}SNR_{i,j} = 10\log_{10}C + 10\log_{10}F_{i,j}$$

(3.12)
The average $\text{SNR}_{i,j}$ across the array of paths is then subtracted to remove $C$

\[
\sum_{i=1}^{N_r} \sum_{j=1}^{N_t} 10 \log_{10} \text{SNR}_{i,j} = N_r N_t 10 \log_{10} C + \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} 10 \log_{10} F_{i,j} \quad (3.13)
\]

\[
\frac{1}{N_r N_t} \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} 10 \log_{10} \text{SNR}_{i,j} = 10 \log_{10} C + \frac{1}{N_r N_t} \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} 10 \log_{10} F_{i,j} \quad (3.14)
\]

By subtracting Eq. 3.14 from Eq. 3.12 it follows that:

\[
10 \log_{10} \text{SNR}_{i,j} - \frac{1}{N_r N_t} \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} 10 \log_{10} \text{SNR}_{i,j} =
\]

\[
10 \log_{10} F_{i,j} - \frac{1}{N_r N_t} \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} 10 \log_{10} F_{i,j} \quad (3.15)
\]

One then can write:

\[
\mu_{\text{obs}} = \frac{1}{N_r N_t} \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} 10 \log_{10} \text{SNR}_{i,j} \quad (3.16)
\]

\[
\mu_{\text{sim}} = \frac{1}{N_r N_t} \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} L_{\text{sim}}^{(i,j),dB} \quad (3.17)
\]

Where $L_{\text{sim}}^{(i,j),dB}$ is the PWE simulated signal loss. The standardized signal loss values for observed and simulated data, $L_{s,(i,j),dB}^{\text{obs, sim}}$ are then given by:

\[
L_{s,(i,j),dB}^{\text{obs}} = \frac{10 \log_{10}(\text{SNR}_{i,j}) - \mu_{\text{obs}}}{\sigma_{\text{obs}}} \quad (3.18)
\]

\[
L_{s,(i,j),dB}^{\text{sim}} = \frac{L_{\text{sim}}^{(i,j),dB} - \mu_{\text{sim}}}{\sigma_{\text{sim}}} \quad (3.18)
\]

where $\sigma_{\text{sim,obs}}$ represent the standard deviation of the array of simulated or observed signal loss values. The PWE values shown in Fig. 3.2 were standardized using this
method and the result is shown in Fig. 3.9. These standardized values are compared with measurements and the EDH that leads to the smallest “distance” between the two is determined to be the correct EDH. This is found using the objective function seen in Eq. 3.8.

3.8 Conclusion

This chapter discussed the XBBR array system and inversion technique that is proposed to estimate atmospheric refractivity using propagation loss measurements gathered from an array of transmitters and an array of receivers. The system measures X-band signals and compares the relative SNR values across the array to values
that are expected for different refractive conditions calculated using the VTRPEs application of the PWE. The refractive conditions that minimize the difference between measured values and PWE-modeled values is extracted as the correct conditions.
Chapter 4: MEASUREMENT CAMPAIGNS

4.1 Introduction

This chapter discusses radiowave propagation measurements gathered during three different campaigns between July 2013 and October 2015. The first was the Trident Warrior 2013 (TW13) campaign, in which transmissions from shipboard radios and radars were recorded passively from the shore in an attempt to observe the effects of ducting. Following TW13, SoCal 2013 saw deployment of transmitters and receivers similar to those described in Chapter 3 aboard two research vessels off the coast of San Diego. The final measurements covered in this dissertation were taken between May and October of 2015 from the Scripps Institution of Oceanography Pier in La Jolla, California. This chapter examines the measurements taken during these campaigns and discusses the results and information gathered.

4.2 Trident Warrior 2013 (July 13-18, 2013)

4.2.1 Overview

The TW13 campaign provided the Ohio State University (OSU) an opportunity to passively measure RF signals from multiple transmitters with the goal of better
understanding propagation in the MABL. TW13 was a Navy fleet experiment coupled with atmospheric and oceanic sensing equipment that took place off the coast of Norfolk, VA between July 13th and 18th, 2013. An overarching goal of the experiment was to validate mesoscale numerical weather prediction models with in-situ measurement systems.

Methods of data collection used throughout the campaign to validate these models included: 48 ScanEagle UAV Flight Hours (Day/Night, Nearshore, Offshore, 3 payloads), UHF/VHF, X, C, S Band Signal Strength Measurements, 3 Waveglider USVs with Met and Ocean Payloads, 5 Scripps drifting wave buoys, 2 NPS Flux Buoys, 8 SLOCUM and Seaglider UUVs, and 4 P-3 Flights/250 AXBTs.

Representatives from multiple institutions carried out measurements including: the Naval Research Laboratory, the Naval Surface Warfare Center Dahlgren, Space and Naval Warfare Systems Center Pacific (SPAWAR), Naval Postgraduate School, Oregon State University, OSU, and the Woods Hole Oceanographic Institute. Personnel from SPAWAR San Diego deployed a spectrum analyzer on board the R/V Knorr that was capable of measuring power from shore-based sources of opportunity at frequencies less than 2 GHz. Along with these measurements, two RT 1944 C-band radio antennas, one on the shore near Dam Neck, VA, and one on board the Knorr, recorded received power levels as a function of time.

OSU contributed shore-based measurements of transmissions from sources on the Knorr from the 5th floor balcony (≈ 50 feet above sea level) of a hotel located in Virginia Beach, VA. X-band transmissions from the R/V Knorr navigation radar were recorded using a high gain (≈ 1.8° beamwidth) X-band Faruno antenna and an existing X-band radar receiver system that is capable of capturing and recording time
domain radar transmission with 16 bits of resolution at 80 MSPS. During TW13, the system record 60 msec continuously (i.e. 4.8 million samples), every $\approx 3.2$ seconds, leading to a duty cycle of 1.87%. These C- and S-band transmissions from the Knorr (RT 1944 radio and on-board S-band navigation radar) were recorded using a C-band antenna, and a standard-gain, wideband horn antenna, respectively, in conjunction with an E4407B Agilent Spectrum Analyzer. The setup of OSU operations during TW13 can be seen in Figs. 4.1 and 4.2.

Additional measurements were conducted for transit of a smaller vessel known as the Stiletto. This vessel carried an X-band navigation radar and transited back-and-forth to locations near the Va. Beach Hilton. Such measurements provided observations of a signal at shorter ranges with a well-known path of travel. These measurements were conducted on July 15th, between 1 p.m. and 3 p.m. local time; Fig. 4.3 illustrates the Stiletto path during this period.
(a) Simultaneously recording X-band signals from Stiletto and C-band from the Knorr.

(b) Using a telescope to visually line up receive X-band antenna with Knorr location.

Figure 4.2: Receiver setup on balcony of hotel room approximately 50 feet above sea level.

Figure 4.3: Path taken by Stiletto during TW2013 on 7/15/2013.
4.2.2 Measurements

The times during which data was collected summarized in Fig. 4.4 and detailed measurement information, including tables of the specific collection times, can be found in Appendix A.

Sample spectrograms of C- and S-band data collected during TW13 can be seen in Fig. 4.5. An example of X-band data, in the time-domain, is shown in Fig. 4.6.

4.2.3 Discussion

A quick analysis of the measurements shows successful shore-based data acquisition. However, many challenges arose in the analysis of different frequency data sets. Due to the fact that both S-band and X-band radars are commonly utilized devices in the maritime environment, separation of signals of interest and interference from other radars proved to be difficult. An on-line ship tracking tool, which may not have been all-inclusive, showed ships other than the Knorr in the vicinity that could have provided unwanted signals at our receivers. Furthermore, these commonly used ship-based radar systems often operate by transmitting pulses while rotating about a vertical axis. This can result in fluctuation of measured signal amplitude due to the...
(a) Spectrogram of shore-based spectrum (b) S-band collection on July 13th, 2013. Analyzer measurements of C-band R/T Data collected with E4407B spectrum analyzer and 2-18 GHz horn antenna. The Knorr navigation radar as well as other radars in the area are observed.

Figure 4.5: Sample C- and S-band data collected during TW13.

Figure 4.6: Digitally sampled radar pulse potentially from the R/V Knorr X-band navigation radar.
azimuthal transmit antenna pattern variations. It was difficult to extract information from the C-band measurements due to uncertainties in whether signals were being received from the RT 1944 radio aboard the Knorr, or a similar radio on the shore. Limited radio transmit time also reduced the utility of these measurements.

An attempt at comparing the received signal powers, by taking the maximum of each spectrum analyzer sweep in the C-and S-bands, to the distance between the transmitters and receivers is shown in Fig. 4.7. Due to limited data and the previously mentioned issues, it is difficult to determine the cause of increased power levels seen at some of the longer ranges or find any correlation between signal level and range.

Considering the X-band data, numerous different pulse types were found upon post-collection processing. Fig. 4.8 depicts a time series concatenation of several different pulse types that were found within the data. Of the different sources found within the data, none were with any certainty identified as transmissions from the X-band navigation radar of the Knorr to allow for investigation into the effects of ducting.

Figure 4.7: Comparison between the maximum value given by the spectrum analyzer for each sweep and distance between the shore based receivers and signals potentially from transmitters aboard the R/V Knorr.
Figure 4.8: X-band sources that were received with the X-band system. This figure includes just 10 of the different sources that were visible during the duration of the Trident Warrior campaign.
4.2.4 Summary

TW13 provided an opportunity to passively measure S-, C-, and X-band RF signals propagating in the MABL. However, due to insufficient knowledge of transmitter properties (operating frequency, location, antenna patterns, etc.), the effect of atmospheric refractivity on these signals was impossible to examine. From these inconclusive results, it was determined that to better study non-standard radiowave propagation, future measurement campaigns would measure the signals from well-known sources with well known properties. The first of two measurement campaigns to implement transmitters was known as SoCal 2013.

4.3 SoCal 2013 (November 9-22, 2013)

4.3.1 Overview

SoCal 2013, was located off the coast of San Diego, CA, from November 9-22, 2013. For this campaign, the XBBR was deployed aboard the Scripps Institution of Oceanography’s R/P FLIP (FLoating Instrument Platform). The multiple receivers of the system were separated in height and observed the transmissions of multiple beacons located aboard the R/V Melville. Fig. 4.9 illustrates the approximate locations of transmit and receive antennas. Fig. 4.10 shows a transmitter and receiver used during the experiment, along with a spectrum analyzer displaying the tuned frequency differences between the transmitters.

4.3.2 Measurements

Approximately 147 hours of X-band propagation measurements were collected during which the distance between the R/P FLIP (receive array) and R/V Melville
Figure 4.9: SoCal 2013 concept of operations: multiple receive array channels separated vertically observe vertically separated transmit beacons, each at differing frequency.

Figure 4.10: (Upper left) Photograph of packaged beacon transmitter and antenna that was deployed during SoCal 2013. (Upper right) Spectrum analyzer measurement of 5 simultaneous beacon transmissions showing separation in frequency. (Lower) One of the five antenna/down-conversion modules for the SoCal 2013 receive array.
Figure 4.11: Distance between the FLIP and Melville during SoCal 2013. The darker portion of the line represents times during which the system was recording and the transmitter were within main beamwidth of the receiving antennas.

(Transmit beacons) varied between 0.5 km and 7 km. Fig. 4.11 shows the distance between the Melville and FLIP and during which times the receivers were recording the transmitters within the 3dB beamwidth of the receivers. Of the 6 transmit beacons that were provided during SoCal 2013, only 2 beacons operated from the duration of the campaign from the R/V Melville. The third beacon was damaged during shipping and not used, and beacon 4 was located on a quad copter during a small portion of the campaign. Beacons 5 and 6 were tuned to the same frequency and indistinguishable in post-processing (with the original plan being to deploy beacon 5 or 6 on the quadcopter and turn the other off during copter deployments). Of the planned 5 receiver channels, 4 recorded measurements throughout the campaign and one did not appear to record anything (possibly due to a cabling issue). As a result, a total of 8 paths were measured. The fast Fourier transform (FFT) was applied to the data records, and the amplitude over time was extracted for each distinguishable frequency.
4.3.3 Discussion

Due to the fact that measurements were never made for distances greater than 7 km, and at these ranges the effects of ducting do not act as the major contributing factor to signal-strength determination, the direct plus reflected Friis Formula (Eq. 4.1, [108]) was calculated and compared with the collected data as a performance validation. Based on these ranges, it was expected that the SNR would fluctuate with range according to the constructive and destructive interference patterns determined by the direct signal and the signal reflected from the ocean surface. Taking into consideration the transmit power of the beacons \( P_t \), the gain of the receiving and transmitting antennas \( G_r \) and \( G_t \), respectively, the wavelength \( \lambda \), system noise \( F \), temperature \( T \), Boltzmann’s constant \( K \), bandwidth \( B \), range \( R \), transmit and receive heights \( h_1 \) and \( h_2 \) and the reflection coefficient \( \Gamma \), the received signal-to-noise ratio was approximated as

\[
SNR = \frac{P_t G_t G_r \lambda^2}{(4\pi R)^2 F K T B} \left| 1 + \Gamma e^{-\frac{j(4\pi h_1 h_2)}{\lambda R}} \right|^2
\]  

(4.1)

Where \( \Gamma \), the reflection coefficient [108], is defined as

\[
\Gamma = \frac{\varepsilon_2}{\varepsilon_1} \cos(\theta) - \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2(\theta)} \left[ 1 + e^{-\frac{j(4\pi h_1 h_2)}{\lambda R}} \right]
\]  

(4.2)

In Eq. 4.2, \( \varepsilon_1 \) is the permittivity of the air, \( \varepsilon_2 \) is the permittivity of water, \( \theta \) is the angle of incidence taking into consideration transmitter and receiver height and the distance between them, \( k \) is the wavenumber, and \( H_{\text{rms}} \) is the mean wave height of the ocean surface, approximated to be 0.3m (because the mean wave height was not recorded, multiple values were implemented and 0.3m lead to the closest match...
between collected and simulated data). Fig. 4.12 shows the comparison between the measured signal loss data and the Friis formula given by Eq. 4.1 (the heights were adjusted from their approximate locations to better fit the data).

Data gathered during this campaign was limited to ranges less than 7 km, but was able to show strong similarities between the collected received power profiles and the SNR found using Eq. 4.1, as seen in Fig. 4.12. Very few datasets exist with the proper antenna alignment, and those that do show similar results to those seen in Fig. 4.12.

Based on the data gathered and difficulties faced in this experiment, it was concluded that improvements to beacon and receive array platforms would improve future measurements. Transmitting and receiving from stationary locations would reduce variations in power caused by antenna motion and would allow for better atmospheric studies. Furthermore, the oscillators that were used drifted on the order of MHz in frequency. For this reason, to keep them separated and distinguishable, the beacons were tuned with at least 5 MHz between them, forcing our system to capture 40 MHz of bandwidth. More stable beacons may have allowed for closer spacing in frequency, and therefore recorded data to be decimated by up to a factor of eight, reducing the time to write the data to disk and improving the data rate of the system.

4.3.4 Summary

SoCal 2013 provided an opportunity to deploy both transmitters and receivers for with the goal of investigating the refractive effects of the atmosphere on X-band radio frequencies. However, due to path length limitations, data gathered during this campaign was only able to act as proof of concept for recording propagation loss with
Figure 4.12: The measured signal loss during on outbound transit of the Melville. Transmit beacons 1 and 2 (f1 and f2) were located ≈ 16.5 and 7.5 meters above the water. The receivers (c1, c2, c4, and c6) were ≈ 3.6, 8, 17, and 30 m above the water.
the XBBR. These results did provide motivation for the next measurement campaign and experiment design aspects gathered from this campaign included utilizing more stable beacons and measuring propagation paths past the radio horizon over longer periods of time.

4.4 Scripps Pier Test (August 2014 - October 2015)

4.4.1 Overview

Based on the conclusions from previous measurement campaigns, the next goal was to perform long-term measurements from stationary platforms using the vertical arrays of transmitter and receivers as described in Chapter 3. The Scripps Pier test consisted of two parts: system validation and long-term measurement.

The long-term deployment would utilize a small buoy for the transmit platform. Power supply limitations aboard the buoy required low power transmitters thus limiting the possible maximum range between transmit and receive arrays. To determine these limitations before measurements were carried out and because the goal was to measure propagation paths further than those that were measured in SoCal 2013, simple link budget calculations were performed using Eq. 4.1 with a reflection coefficient $\Gamma = 1$. The results of these calculations for possible transmit and receive heights are shown in Fig. 4.13.

Based on the link budgeted calculations, for a transmitter height of 1m and receiver height of 3m, the SNR remains positive (other than nulls caused by reflected signal interference) to approximately 5km. For the higher transmitter and receiver case, 5m and 15m respectively, the SNR remains positive out to approximately 20 km. All of these cases assume standard propagation and it was expected that the presence
Figure 4.13: The SNR (dB) vs range (km) link budget calculations for (top) a low (1m) transmitter and low (3m) receiver and (bottom) a high (5m) transmitter and high (15m) receiver. The radio horizon based on the transmitter and receiver heights is shown in red. Both calculations were performed assuming a transmit power of 13dBm, transmit gain of 0dB, frequency of 11GHz, receiver gain of 20dB, receiver noise figure of 2dB, and receiver bandwidth of 5MHz.
of atmospheric ducting would allow for higher SNR values at longer ranges. Based on these link budget results, the distance between the transmit array and receive array was chosen to be 15km. This distance would allow for some measurements of propagation paths past the horizon and others to remain above 0 SNR during the case of standard propagation.

4.4.2 System Validation Measurement

Initial testing of the XBBR array occurred on September 3rd, 2014. After system setup on the Scripps Pier in La Jolla, CA, three transmitters were operated at heights of approximately 0.9, 2.8, and 4.8 m aboard a vessel that transited from near the pier to approximately 25 km in range. The receive channels were located at approximately 3.6, 6, 7.4, 9.8, 12.3, and 14.5 meters above the mean sea level (MSL). The receiver setup on the pier is seen in Fig. 4.14 and receiver heights relative to the pier are given in Fig. 4.15.

The received powers for each transmit-receive combination were then compared to a PWE code for various evaporation duct heights. The beacon tones were identified and their amplitude vs. time values were extracted. Comparison with the PWE with an 8m EDH showed a strong correlation, as can be seen in Fig. 4.16. Similar to the SoCal 2013 conclusion, the results validated that the system was providing reasonable propagation observations, in this case to ranges up to 25 km, for the transmit and receive configuration and for the given atmospheric conditions.
Figure 4.14: Five high gain horns and their respective down-converter boxes are mounted on the Scripps Pier in La Jolla, CA to act as receivers. Three are mounted below pier level, and two are mounted above pier level.

Figure 4.15: Receiver mounting heights with respect to each other and the wave height measuring device on the end of the Scripps Pier.

- All heights measure from top of pier
- Average distance from top of pier to water is: $h = -9\text{m} (?)$

Heights From Water (Used in simulation)

- $h_f = 0.8\text{m}$; Transmit height (m)
- $h_r = h_r + 2.3\text{m}$; Receive height (m)
- $h_r = h_r + 2.44\text{m}$
- $h_r = h_r + 4.57\text{m}$
- $h_r = h_r + 5.16\text{m}$
Figure 4.16: Normalized at 9 km, a comparison of measured data and a TPEM simulation, with an evaporation duct height of 8 m, shows strong correlation.
4.4.3 Long-Term Measurement

Long-term deployment of the XBBR array system began at Scripps Institution of Oceanography on May 22nd, 2015, and multiple months of measurements were recorded. Transmitters were mounted on a buoy located 15 km west of the Scripps pier, where the receivers were located. The design of the buoy can be seen in Fig. 4.17. Along with radiowave propagation measurements, meteorological sensors gathered air temperature, water temperature, wind speed, humidity, and pressure data at the buoy. Using the NAVSLaM and the measured wind speed, air temperature, sea surface temperature, relative humidity, and air pressure, refractivity profiles were created to act as a ground truth for comparison with the XBBR array EDH estimation. The MOST that is implemented by NAVSLaM to calculate a vertical refractivity profile from the single point of atmospheric data (gathered at the buoy) assumes conditions to be horizontally homogeneous. Therefore, both the inversion technique and NAVSLaM calculations assume a constant refractivity profile along the propagation path.

4.4.4 Amplitude Measurements

Similarly to the method of evaporation duct height retrieval seen in Sect. 3.3, measured XBBR data, an array of fifteen values for relative signal strength across the different transmit-receive combinations, were fitted to PWE libraries to estimate the EDH. The library entry that is “closest” to the measured signals is used as the EDH estimation. The closes library entry was found using MLE. Unlike in Sect. 3.3, however, signal fluctuations appear to be correlated in time and This minimum difference is found using the Eq. 3.8 where $L_{i,j,dB}(EDH)$ represents the standardized,
Figure 4.17: Buoy acting as platform for 3 transmit beacons during the Scripps Pier long-term measurement. (left) Beacons were mounted at heights of 1.5, 2.7 and 4.5 meters above the water while atmospheric measurements were gathered at a height of 3m. (right) Illustration of buoy and its tether.
simulated values (for various evaporation duct heights) and $L_{i,j,dB}^{obs}$ represents the standardized, observed propagation loss, both in dB, with the sum taken over the $N_r \times N_t$ measurements available from a system having $N_t$ transmitters and $N_r$ receivers.

Fig. 4.18 provides an example of measured XBBR standardized values as a function of time; 15 curves corresponding to each TX/RX pair comprised of 3 transmitters and 5 receivers are included. The EDH was estimated by comparing these values, gathered every 10 seconds during the last 10-minutes of each hour, to the PWE library using the objective function defined in Eq. 3.8. The estimated EDH is then found by minimizing the objective function; examples of which are shown in Fig. 4.19. The average estimated EDH, $\mu_{EDH}$, and standard deviation, $\sigma_{EDH}$, was then found for each 10-minute window, and an upper and lower bound for a 95% confidence interval, $CI$, was calculated using Eq. 4.3 [4]. Here, $n$ is the number of samples (typically between 45 and 50 for each 10 minute collection window), and the $z^*$ multiplier is 1.96, which is typically associated with a 95% CI.

$$CI = \mu_{EDH} = \pm z^* \times \frac{\sigma_{EDH}}{\sqrt{n}}$$ (4.3)

Although the system was deployed between May 22nd and October 10th, 2015, data was not collected during various time periods due to weather complications and system maintenance. The days with sufficient measurements (both RF and atmospheric data) that will be discussed further were between May 22nd and June 17th, and between August 19th and October 10th, 2015. Fig. 4.20 and 4.21 show the inversion results, and corresponding confidence intervals, compared with NAVSLaM calculations during these collection periods. Also shown in these figures is the atmospheric data that was collected on the buoy by Scripps instruments mounted 10ft
Figure 4.18: Sample of recorded and post-processed, standardized, XBBR data. Because there are 3 transmitter and 5 receivers, there are a total of 15 propagation loss values at each sampling time. Measured data was recovered every 10 seconds, for approximately 10 minutes of every hour. It is speculated that relative signal variability over these time periods is due to buoy motion.

above MSL during these X-band measurements. Relative humidity, wind speed, and ASTD are important factors for refractivity calculation using NAVSLaM, while pressure has a lesser effect [15]. It is apparent that there is a strong correlation between these two methods of evaporation duct height calculation. However, while there is a general trend shared between NAVSLaM and the inversion results, an EDH offset existed during portions of the experiment. The difference between the estimated EDH and ground truth for these time periods is shown in Fig. 4.23.
Figure 4.19: Average objective functions over the course of 2, 10-minute collections. Lower values correlate to a better match between measured and PWE library values. The plot on the left has a well-defined minimum compared to the plot on the right which leads to a lower standard deviation of estimated EDH values (95% CI spanning 0.3m as opposed to 1.7m). However, the plot on the right shows a lower value of the objective function at the minimum which means a better match between measured and PWE calculations (EDH estimation error of 3.9m on the left and 2.7m on the right).
Figure 4.20: From top to bottom, the PWE inversion and NAVSLaM comparison (the dark blue represents a 95% confidence interval [4] calculated using the mean EDH, and the standard deviation of the estimation during each 10 minute collection), ASTD (°C), atmospheric pressure (hPa), relative humidity (%), and wind speed (m/s), during the time period between May 22nd, and June 17th, 2015.
Figure 4.21: From top to bottom, the PWE inversion and NAVSLaM comparison (the dark blue represents a 95% confidence interval [4] calculated using the mean EDH, and the standard deviation of the estimation during each 10 minute collection), ASTD (°C), atmospheric pressure (hPa), relative humidity (%), and wind speed (m/s), during the time period between August 19th, and October 10th, 2015.
Figure 4.22: Correlation between NAVSLaM and PWE inversion for the sets of data seen in Figs. 4.20 (left) (correlation coefficient = 0.885) and 4.21 (right) (correlation coefficient = 0.397 (0.735 without outliers)).

Discussion

Collections between May 22nd and June 17th, 2015 had an RMSE of $\sim$4.0m. If the average bias that exists in this subset of data is removed ($\sim$3.85m), this RMSE reduces to $\sim$0.9m. Collections between August 19th, and October 10th, 2015 had an RMSE of $\sim$4.4m. If, however, the errors that are greater than two standard deviations away from ground truth are ignored, the RMSE for the second set reduces to $\sim$1.9m. The correlation between EDHs estimated using XBBR measurements and PWE libraries and EDHs calculated using NAVSLaM is seen in Fig. 4.22. While there is strong correlation between the ground truth and the inverted results, a large bias exists during the earlier portion of the measurement campaign. This large bias, along with other errors, are thought to exist for multiple reasons:
1. The Paulus-Jeske refractivity model used to create the PWE libraries assumes neutral stability (ASTD ≈ 0 [18]) which allows for the refractivity to be defined by a single parameter (EDH) and greatly simplifies the inversion. However, this does not capture the complexity of possible modified refractivity profiles in the MABL. Furthermore, as can be seen in Figs. 4.20 and 4.21, a majority of measurements were made when the ASTD was less than zero and neutral stability should not be assumed.

2. The current inversion technique assumes a range independent profile between the transmitters and receivers (that a single M-profile can describe the entirety of the atmosphere between the arrays).

3. In reality, due to the buoy geometry, transmitter mounting, platform motion, and rough sea surface conditions, the measured signal loss may vary due to parameters other than the EDH; PWE libraries do not currently account for these parameters.

4. The current inversion only assumes the presence of an evaporation duct and neglects the possible presence of other duct types.

5. Lastly, it should be noted that NAVSLaM, which is acting as the ground truth for this dissertation, is a model and it is possible that it has errors of its own. For this reason, it is recommended that future measurements occur alongside radiosonde measurements to verify results with more certainty.

On September 10th, 2015, NAVSLaM provided an EDH 9.5 meters higher than the inverted EDH. In Fig. 4.24, the ASTD during the collections which lead to an
Figure 4.23: The difference between the estimated EDH and ground truth for the collections between (top) May 22nd and June 17th, 2015 and (bottom) August 19th, and October 10th, 2015. The red denotes times when the EDH inversion led to an EDH much greater (greater than two times the standard deviation of error) than the ground truth.

inverted EDH much larger than NAVSLaM calculations (the large positive values seen in the bottom of Fig. 4.23) are highlighted by red Xs. These errors are thought to exist due to both: larger NAVSLaM EDH error due to a positive ASTD, and the neutral stability assumption made in utilizing the simple PJ refractivity model.

For a complete time-series of M-profiles during the long-term measurement, including those determined by the inversion process and those calculated using NAVSLaM, the reader is directed toward Fig. 4.25. From this figure, it is apparent that differences exist between the PJ and NAVSLaM models and those differences are greater in the M-profiles below the EDH. While the inversion technique discussed here relies on duct height as the sole parameter for M-profile generation, duct strength, or
Figure 4.24: ASTD between August 19th, and October 10th, 2015. The red Xs mark times when the EDH inversion led to an EDH much greater (greater than two times the standard deviation of error) than the ground truth.

M-deficit (denoted by $\delta M$ in the left side of Fig. 1.4 and defined as the difference between the minimum value of modified refractivity and the modified refractivity at the surface), also plays a large role in how a refractivity profile can effect radiowave propagation and the output of PWE calculations. The PJ model (Eq. 1.13) leads to a fixed $\delta M$ for a given duct height. Refractivity profiles created using NAVSLaM are capable of multiple values of $\delta M$ for a single duct height. This can be seen in Fig. 4.26 which shows PJ and NAVSLaM-modeled refractivity profiles during portions of the long-term experiment when an 8 m EDH was present. Here, all profiles are normalized with a modified refractivity of 0 at the surface to more easily visualize M-deficit differences.

Looking at the relationship between M-deficit and EDH throughout the long-term experiment, Fig. 4.27, one can see the nearly exactly linear relationship for the PJ model, while the relationship for NAVSLaM profiles is much more variable. An example of how much of an effect M-deficit can have on PWE output is provided in
Figure 4.25: Time-series of M-profiles during the long-term measurement campaign. The date is represented on the x-axis with the left column being from the first half of the experiment and the second half in the right column. In all plots, the color-scale represents the modified refractivity in M-units with the minimum value of the profile (EDH) equal to 0. Row 1 (a,b) shows M-profiles created using the PJ model, row 2 (c,d) shows the NAVSLaM calculations, and row 3 (e,f) shows the absolute value of the difference between the two.
Figure 4.26: During portions of the long-term measurement when an 8 meter EDH was present, the blue line represents the M-profile created using Eq. 1.13 and the red line represents M-profiles created using NAVSLaM.

Fig. 4.28. The XBBR measurement taken on August 19th, 2015, at 21:39:47 leads to an inverted EDH of 8.9 m, and NAVSLaM, using atmospheric conditions at the time, also provided an EDH of 8.9 m. However, the M-deficit for a PJ profile with this EDH is 11.6 M-units, while refractivity profile calculated by NAVSLaM at the time has an M-deficit of 40.7 M-units. The difference between the M-profiles is shown on the left side of Fig. 4.28, while the right side provides the standardized signal loss values expected at each of the receivers given by the PWE model for each M-profile shown. For these reasons, it is suggested that future inversions utilize a refractivity model defined by multiple parameters.

In order to investigate the validity of the range independence assumption, atmospheric data taken on the pier was used as input to NAVSLaM to approximate the refractivity near the shore. Wind speed, air temperature, and pressure measurements were taken and available at the pier. However, humidity and sea surface temperature were only available at the buoy and therefore used for pier refractivity calculations as
Figure 4.27: Relationship between EDH and M-deficit for (blue) PJ model and (red) NAVSLaM profiles during the long-term measurement.

Figure 4.28: (left) M-profiles created using the PJ and NAVSLaM refractivity model for a measurement taken at Scripps pier on 19 August, 2015, at 21:39:47 that estimated an EDH of 8.9 m. (right) The PWE modeled relative signal loss values for each of the refractivity profiles shown on the left.
well as buoy refractivity calculations. As can be seen in Fig. 4.29, although there are only small differences between the pier-measured and buoy-measured atmospheric values, these small differences can lead to NAVSLaM calculated EDH values that differ on the order of a few meters.

In order to further investigate the potential causes of error, the differences between $L_{i,j,dB}^{sim}$ and $L_{i,j,dB}^{obs}$ were examined for each transmit receive path. These relative signal loss values over the course of the Scripps Pier long-term measurement are provided in Fig. 4.30. In this figure it is apparent that although the right column is comprised of the PWE values which minimized the objective function, the bottom row of figures shows that certain transmit-receive combinations (Tx1Rx1 for example) were on average less alike the expected values than were other combinations. This could be due to poor system representation in the PWE library creation. Further investigation to reduce these errors could be accomplished by increasing the number of inversion parameters to include certain transmit and receive antenna parameters along with EDH.

Along with evaporation ducts, it is possible for a surface-based duct (SBD) to exist under certain atmospheric conditions. The presence of a SBD could have potentially affected the signal loss experienced by the transmitted signals during the Scripps Pier measurement. To investigate the potential affect of a SBD on signal loss, the PWE libraries (of standardized signal loss) were created with a modified refractivity profile that contained both an evaporation duct and a SBD. This profile can be described by an evaporation duct (defined by the PJ model) and a tri-linear model for a SBD given by [109]:
Figure 4.29: NAVSLaM calculated EDH using the atmospheric parameters as measured at the buoy (transmit location, blue) and as measured at the pier (receive location, red)
Figure 4.30: (top row) The measured relative signal loss values for each of the 15 transmit and receive paths (y-axis) over the course of the long-term measurement (x-axis), (middle row) the PWE library standardized signal loss values which minimized the objective function when compared with the measured values, and (bottom row) $|L^\text{sim}_{i,j,dB} - L^\text{obs}_{i,j,dB}|$. The color scale represents the standardized values of signal loss.
\[ M(z) = M_0 + \begin{cases} 
  c_0 \left( z - \delta \ln \frac{z + \delta u}{z_0} \right) & \text{if } z \leq h_1 \\
  c_1 h_1 + c_2 (z - h_1) & \text{if } h_1 \leq z \leq h_2 \\
  c_1 h_1 + c_2 h_2 + 0.118(z - h_1 - h_2) & \text{if } z \geq h_2 
\end{cases} \]  

(4.4)

where \( c_1 \) and \( h_1 \) represent the slope and thickness of the base layer of the SBD, and \( c_2 \) and \( h_2 \) represent the slope and thickness of the inversion layer, \( \delta \) is the EDH, \( M_0 \) is the refractivity at the sea surface, and the slope of the profile above both ducts is commonly defined as 0.118 M-units/m [109]. The layer below the SBD is the PJ profile defined previously in Eq. 1.13. The aforementioned inversion was performed using PWE library values created using M-profiles defined by Eq. 4.4 with \( c_1 = 0.118 \) M-units/m, \( h_1 = 75 \) m, \( c_2 = -0.2 \) M-units/m, and \( h_2 = 100 \) m (values typical for a tri-linear SBD [109]). The M-profile in question is shown in Fig. 4.31. While these are just estimates for possible SBD parameters in the MABL, by performing the inversion with libraries created with a SBD present (as opposed to solely an evaporation duct), the RMSE from the NAVSLaM calculated EDH during the first half of the experiment (05/22-6/17) reduced from \( \sim 4.0 \) m to \( \sim 3.5 \) m. This does not prove that a SBD defined by these parameters existed during the measurements, but does show that including a SBD in PWE calculations can affect the expected signal loss values and neglecting them may have been a factor in the large bias that existed during the first half of the experiment.
Figure 4.31: A modified refractivity profile created using Eq. 4.4 for an EDH of 20m and $c_1 = 0.118$ M-units/m, $h_1 = 75m$, $c_2 = -0.2$ M-units/m, and $h_2 = 100m$ representing the presence of both an evaporation duct and surface based duct.
4.4.5 Phase Measurements

As discussed in Chapter 3, the XBBR array system was designed with the intent to coherently measure the relative phase across the array of receivers. Due to the fact that the transmitted signals are continuous and there is no timing synchronization between the transmitters and receivers, the absolute phase at each receiver during a particular collection is irrelevant. However, the measured signal at receiver $i$ with phase relative to the signal measured at receiver $j$ is potentially useful and is defined as

$$L'_{i,j} = \frac{L_i L_j^*}{|L_j|}$$

(4.5)

where $\cdot^*$ is the complex conjugate and $|\cdot|$ is the absolute value. In Fig. 4.32, $L'_{i,1}$ can be seen over the course of multiple 10-minute collection periods (approximately 45 collections). Note that $L'_{2,1}$ remains relatively constant over time when compared with $L'_{3,1}$, $L'_{4,1}$, and $L'_{5,1}$ which seem to fluctuate, somewhat similarly, but separately from $L'_{2,1}$. Because receivers 1 and 2 are on the same ADC board, the “Master” board, they record simultaneously, which is consistent with $L'_{2,1}$ remaining relatively constant over time. Receivers 3 and 4, however, are on a separate board, “Slave1”, which is triggered by the Master board. This trend is seen throughout the first part of the long-term collection between May and July and is thought to be a result of inconsistent FPGA trigger timing.

Between the two large datasets that were gathered from the Scripps Pier, the system was rebooted and it appears that some of the inconsistency in relative phase measurements was corrected. This can be seen in Fig. 4.33.
Figure 4.32: The relative phase (relative to receiver 1) measured across the 5 element array from a single transmitter over 2, 10 minute collection periods of the long-term measurement. Collections occurred during the first large dataset between May and July.

Figure 4.33: The relative phase (relative to receiver 1) measured across the 5 element array from a single transmitter over 2, 10 minute collection periods of the long-term measurement. Collections occurred during the second large dataset between August and October.
Figure 4.34: The PWE modeled relative phase (relative to receiver 1) across the 5 element array from a single transmitter for EDHs ranging from 0 to 40 meters and a system configuration similar to what was used to produce the library seen in Fig. 3.2

If the system did measure this relative phase accurately, and the complex PWE library accurately represents the relative phase for the measurement parameters during this time, the $L'_{i,1}$ measured and modeled should resemble each other. Using the same input parameters as the amplitude inversion above, the complex PWE library was created and absolute phase is converted into relative phase using Eq. 4.5. An example PWE library for the relative phases of 5 receivers, from a single transmitter, for varying EDH can be seen in Fig. 4.34.

The measurements in Fig. 4.33 were obtained during a period of time when NAVS-LaM calculated a 5.1m (left) and 6m (right) EDH. The PWE modeled $L'_{i,1}$ are added as the dotted lines to Fig. 4.33 to produce Fig. 4.35.
Figure 4.35: The measured (solid) relative phase compared with the PWE modeled (dotted) relative phase for a NAVSLaM calculated 5.1m (left) and 6m (right) EDH.
Figure 4.36: (a) PWE calculated \( L'_{i,1} \) for a range (distance between transmitter and receiver) of 14.24km (blue) and a range of 13.74km (red). (b) PWE calculated \( L'_{i,1} \) for a receive array center height (height of Rx3 above MSL) of 8.55m (blue) and a height of 9.55m (red).

Discussion

The exact cause of the differences between measured and modeled phase is difficult to determine. Assuming that the PWE accurately models the phase progression, it seems unlikely that the cause for these differences is due to errors in complex library creation, as the relative phase found using the PWE is fairly stable with respect to changing parameters such as range and receiver height (see Fig. 4.36). It is apparent that varying the range and receive-array height, on the order of 0.5km and 1m respectively, during complex PWE library creation does not lead to large differences in relative phase. While this is not a complete sensitivity analysis of PWE modeling, this does show that errors in PWE library creation do not lead to the fluctuations in relative phase of the same magnitude that were seen during the long-term measurement.
Along with possible receiver inconsistencies leading to inaccurate phase measurements, transmitter instability may also have lead to unaccounted-for errors. To investigate how the transmitter may affect the relative phase, the XBBR array was set up indoors and measurements were taken in a stable environment. Using one of the transmitters, over the course of 9 minutes, 140 measurements were taken (each one million samples, or 12.5 ms). Then, $L'_{i,j}$ was calculated for each of the receivers using Eq. 4.5. Fig. 4.37 shows the phase that was measured for $j = 2$ (the signal measured at receiver 2). Here, it is apparent that the relative phase both drifts slowly over time (in the case of $L'_{1,2}$ and $L'_{5,2}$) and jumps a few radians between collections (in the case of $L'_{3,2}$).

The frequency stability of the oscillators used in the beacons is very reliant on operating temperature. After powering on, the oscillators require sufficient time to
reach operating temperature. The measurements shown in Fig. 4.37 were taken after the beacon had been transmitting for approximately 30 minutes and was thought to be up to operating temperature. However, as can be seen in Fig. 4.38, the transmitter still drifted 1.2 KHz in frequency over the course of 9 minutes. The beacons were exposed to varying atmospheric temperatures during the long-term experiment which would affect the operating temperature, and, in turn, operating frequency, which might contribute to fluctuations in measured relative phase between collections. For this reason, among others previously mentioned, it was determined that the phase measurements gathered during the long-term measurement are incapable of providing information about the presence of ducting through comparison with PWE calculations. Continuing simulation studies nevertheless will assume that use of phase information is possible through revised system design.
4.5 Conclusion

This chapter discussed measurement campaigns that were carried out in an attempt to better understand the effects of evaporation ducts on radiowave propagation. TW13 provided an opportunity to measure signal strength from transmitters of opportunity in the S-, C-, and X-band radio frequency ranges from a shore-based location. However, examination of the effects of atmospheric refractivity on these signals was impeded by uncertainties in the source locations, and interference from other sources.

The XBBR array was then deployed off the coast of Southern California as part of SoCal 2013. Measurements of signal loss from an array of transmitters aboard the R/V Melville were recorded by an array of receivers aboard the R/P FLIP and compared to expected values given by the direct+reflected Friis formula. The good correlation between modeled and measured values led to the conclusion that the system was capable of measuring propagation. Refractivity inversion was not completed using these measurements because it was concluded that the effects of ducting on the propagation loss would be difficult to discern over the much more dominant effects of line-of-sight propagation for the short range paths used in the campaign.

Between May and October of 2015, the XBBR array system recorded propagation loss from an array of transmitters at an array of receivers on the Scripps Pier in La Jolla, California. The data gathered during this campaign was processed using an inversion technique which compared measurements to PWE generated libraries of possible signal loss values for varying EDHs. While XBBR array measurements of phase were shown to be inconsistent with complex PWE calculations, inversion results utilizing solely the measured amplitude were highly correlated with NAVSLaM-generated
EDHs. It was suggested, however, that future inversions could have better results if performed using a refractivity profile model defined by multiple parameters. Future measurements could also be improved by reducing the amount of unaccounted-for signal fluctuations and performing a calibration routine that would allow for direct comparison of measured data and PWE libraries as opposed to the standardized versions. Other possible improvements include: implementing a more stable transmit platform (pier to pier instead of buoy to pier), increasing transmit power to ensure signals are visible at all times, and better transmitter and receiver placement. To address this last point, simulations were performed with the goal of finding optimal system parameters. This study is discussed in Chapter 6.
Chapter 5: SIGNAL FLUCTUATION IN A TURBULENT ATMOSPHERE

5.1 Introduction

All previous measurements and simulations utilizing the aforementioned inversion technique have been carried out exclusively taking the amplitude of the propagating wave into consideration. However, it is hypothesized that if:

- The relative phase across the array of receivers from a transmitted signal can be measured accurately,

- PWE calculations can be performed that accurately describe the phase of a transmitted signal, and

- The fluctuations in the index of refraction of the atmosphere do not render the relative phase information incomparable to the PWE library,

then including relative phase information for EDH inversion would improve this method of refractivity estimation. However, as discussed in the previous chapter, the XBBR was unable to measure relative phase accurately and therefore these first two points could not be investigated here. Instead, using a model developed by Akira
Ishimaru [5] this chapter examines RF signal amplitude and phase fluctuations that are caused by a fluctuating index of refraction in a turbulent atmosphere.

Physically, it is thought that turbulence may impact the amplitude and phase of a transmitted signal by effectively changing the propagation path length that is experienced by that signal. It is expected that small changes in path length may have a large impact on the measured phase; a path length difference of only 1.35cm is half of a wavelength at 11GHz. This may lead to constructive and destructive interference that can occur in the waveguide-like environment that exists when an evaporation duct is present. Although the phase wasn’t measured accurately during the Scripps Pier measurements, it will be shown that fluctuations of measured signal amplitude (while not completely caused by the turbulent atmosphere but also by platform motion and a temporal changes in the rough ocean surface) compare fairly well with modeled fluctuations. This comparison allows for certain turbulence parameters to be estimated during the long-term experiment and leads to a possible model for relative phase fluctuations. This chapter then concludes with a discussion of whether measuring the relative phase of the signal across the receive array would improve the inversion technique discussed previously.

5.2 Propagation in a Turbulent Atmosphere

Ishimaru [5] statistically examines the effects of a fluctuating index of refraction on a propagating plane wave. His discussion begins with Maxwell’s Equations in terms of the index of refraction $n$, wavenumber $k_0$, and electric field at a given range $E(r)$:
\[ \nabla^2 \mathbf{E}(\mathbf{r}) + k_0^2n^2 \mathbf{E}(\mathbf{r}) - 2\nabla \left( \frac{\nabla n}{n} \cdot \mathbf{E} \right) = 0 \] (5.1)

Considering a wave propagating in the \( x \)-direction, with a \( y \)-component of the electric field, ignoring the last term (which can be done as long as the wavelength, \( \lambda \ll l \) where \( l \) is the correlation distance of the medium) one can write, for \( U(\mathbf{r}) = E_y(\mathbf{r}) \)

\[ (\nabla^2 + k_0^2n^2) U(\mathbf{r}) = 0 \] (5.2)

With interest in a fluctuating medium, or when \( k^2 = k_0^2 \langle n \rangle^2 \), where \( \langle n \rangle \) is the average index of refraction and \( n_1 \) is the fluctuation of the index of refraction, one can write

\[ [\nabla^2 + k^2(1 + n_1)^2] U(\mathbf{r}) = 0 \] (5.3)

Ishimaru then derives an approximate solution for weak fluctuation using the Rytov transformation in which

\[ U = \exp(\psi_0 + \psi_1 + \psi_2 + \ldots) \] (5.4)

It follows, in [5], that the first Rytov solution, for a weakly turbulent case, is given by

\[ U(\mathbf{r}) = U_0(\mathbf{r}) \exp(\psi_1(\mathbf{r})) \] (5.5)

where

\[ \psi_1(\mathbf{r}) = \int_V h(\mathbf{r}, \mathbf{r}') n_1(\mathbf{r}) dV \] (5.6)
\[ h(r, r') = 2k^2 G(r - r')U_0(r')/U_0(r) \]  

(5.7)

Here, \( U_0 \) is the field in the absence of fluctuation (\( \delta n = 0 \), where \( \delta n = 2n_1 + n_1^2 \simeq 2n_1 \)) and

\[ G(r, r') = \frac{\exp(jk |r - r'|)}{4\pi |r - r'|} \]  

(5.8)

The amplitude \( A \) and phase \( S \) can then be examined by writing

\[ U(r) = A(r)e^{jS(r)} \]

(5.9)

\[ U_0(r) = A_0(r) \exp(jS_0(r)) \]

and the phase and amplitude fluctuation are given by

\[ \psi_1(r) = \chi + jS_1 = \ln(A/A_0) + j(S - S_0) \]  

(5.10)

here \( \chi \) is the fluctuation of the logarithm of the amplitude, \( S \) is the phase fluctuation, and \( A_0, S_0, \) and \( U_0 \) are the values of amplitude, phase, and field in the absence of fluctuating \( n \). Following Ishimaru’s spectral approach, where \( L \) is the range from the source in the \( x \)-direction and \( \rho = y\hat{y} + z\hat{z} \), one can define

\[ \chi(L, \rho) = \frac{1}{2} [\psi_1(L, \rho) + \psi_1^*(L, \rho)] \]

\[ S(L, \rho) = \frac{1}{2i} [\psi_1(L, \rho) - \psi_1^*(L, \rho)] \]  

(5.11)

which can be done if \( \psi_1 \)'s two-dimensional characteristics are more dependent on the covariance of the index of refraction in the \( y, z \)-plane than its covariance in the \( x \)-direction. In the spectral domain, the spectral density of the index of refraction, \( \Phi_n(\kappa) \), is used to incorporate a fluctuating index of refraction into correlation calculations. Typically, \( \Phi_n(\kappa) \) is characterized by the inner scale of turbulence, \( l_0 \), the outer
scale of turbulence, \( L_0 \), and the structure constant, \( C_n \), as described by Tatarski [110].

One definition, the von Karman spectrum, defines \( \Phi_n(\kappa) \) as

\[
\Phi_n(\kappa) = 0.033C_n^2(\kappa^2 + \kappa_L^2)^{-11/6} \exp(-\kappa^2/\kappa_m^2)
\]

where \( \kappa_m = 5.92/l_0 \), \( \kappa_L = 1/L_0 \). According to Ishimaru, \( C_n \) is on the order of \( 10^{-7}m^{-1/3} \) for strong turbulence and \( 10^{-9}m^{-1/3} \) for weak turbulence, \( L_0 \) is on the order of 10-100 m, and \( l_0 \) is on the order of millimeters. It follows that the amplitude and phase covariance functions, respectively, at the plane \( x = L \), are given by

\[
B_{\chi}(L, \rho_1, \rho_2) = \langle \chi(L, \rho_1)\chi(L, \rho_2) \rangle = \text{cov}(\chi(L, \rho_1), \chi(L, \rho_2))
\]
\[
B_S(L, \rho_1, \rho_2) = \langle S_1(L, \rho_1)S_1(L, \rho_2) \rangle = \text{cov}(S_1(L, \rho_1), S_1(L, \rho_2))
\]

If \( \Phi_n(\kappa) \) does not vary laterally over the distance \( \sqrt{\lambda L} \), then these covariance functions reduce to

\[
B(L, \rho) = 2\pi^2k^2L \int_0^\infty \kappa d\kappa J_0(\kappa \rho)f(\kappa)\Phi_n(\kappa)
\]

where \( J_0(\kappa \rho) \) is the Bessel function of order zero, and \( f(\kappa) \) is the spectral filter function give for amplitude fluctuation by

\[
f_\chi(\kappa) = 1 - \frac{\sin(\kappa^2L/k)}{\kappa^2L/k}
\]

for phase fluctuation by

\[
f_S(\kappa) = 1 + \frac{\sin(\kappa^2L/k)}{\kappa^2L/k}
\]

and for the cross-correlation between amplitude and phase by
Figure 5.1: Filter functions and spectral density $\Phi_n$ in the region $L \gg l^2/\lambda$. (taken from [5])

$$f_{\chi S}(\kappa) = \sin^2 \left( \frac{\kappa^2 L}{2k} \right) \kappa^2 \frac{L}{2k} \frac{\kappa^2 L}{2k} (5.17)$$

In the region where $L \gg l^2/\lambda$, (where $l$ is the correlation distance, or scale of turbulence describing the average size of turbulent "blobs" or eddies [5]) the filter functions compared with $\Phi_n$ as a function of $\kappa$ are shown in Fig. 5.1. It is apparent from Fig. 5.1 that in this region

$$f_{\chi}(\kappa) \simeq f_{S}(\kappa) \simeq 1$$

which simplifies the covariance functions for amplitude and phase fluctuations to Eq. 5.18. The cross-correlation filter function approaches zero in this region. Examples of covariance as a function of $\rho$ for various turbulence parameters are shown in Fig. 5.2.

$$B_{\chi}(L, \rho) = B_{S}(L, \rho) = 2\pi^2k^2L \int_0^\infty \kappa d\kappa J_0(\kappa \rho) \Phi_n(\kappa) \quad (5.18)$$
Figure 5.2: Covariance found using Eq. 5.18 for a single value of $C_n$ and (a) varying values of $L_0$ and (b) varying values of $C_n$. 
5.3 Amplitude Fluctuation

During the Scripps Pier long-term measurement campaign, the XBBR array captured approximately 45 collections of $1 \times 10^6$ samples at 80 MSPS, every hour, over the course of multiple months. Because the XBBR measured the signal loss from transmitters at five receivers simultaneously, the data gathered provides insight into the covariance of amplitude fluctuation as a function of $\rho$.

In order to determine how well Eq. 5.18 models the expected variation in a turbulent atmosphere, it was compared with the covariance among the signals measured at each of the receivers during the long-term XBBR deployment. The vertical array provided measurements of amplitude at values of $\rho$ (seen in Table 5.1), at a range, $L$, of approximately 15 km.

<table>
<thead>
<tr>
<th>$\rho (m)$</th>
<th>Rx1</th>
<th>Rx2</th>
<th>Rx3</th>
<th>Rx4</th>
<th>Rx5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rx1</td>
<td>0.0</td>
<td>2.6</td>
<td>5.2</td>
<td>7.9</td>
<td>10.2</td>
</tr>
<tr>
<td>Rx2</td>
<td>2.6</td>
<td>0.0</td>
<td>2.6</td>
<td>5.3</td>
<td>7.6</td>
</tr>
<tr>
<td>Rx3</td>
<td>5.2</td>
<td>2.6</td>
<td>0.0</td>
<td>2.7</td>
<td>5.0</td>
</tr>
<tr>
<td>Rx4</td>
<td>7.9</td>
<td>5.3</td>
<td>2.7</td>
<td>0.0</td>
<td>2.3</td>
</tr>
<tr>
<td>Rx5</td>
<td>10.2</td>
<td>7.6</td>
<td>5.0</td>
<td>2.3</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 5.1: Distances between receivers (m) on the Scripps Pier.

The covariance of measured signal strength was found using Eq. 5.13 after converting to log-amplitude fluctuation using Eq. 5.10. Here, $A_0$ is defined as the median amplitude. The covariance matrix, $\Sigma$, between the receivers was found in which the entries of $\Sigma$ are defined by
Figure 5.3: The measured (blue), and modeled (red), covariance as a function of $\rho$ at a plane 15 km from a source through an turbulent atmosphere with an inner scale of turbulence $l_0 = 0.05 m$, $L_0 = 57.7136 m$, frequency of 11 GHz, and structure constant $C_n = 6 \times 10^{-7} m^{-1/3}$.

\[
\Sigma_{i,j} = \text{cov}(\chi_{Rx,i}, \chi_{Rx,j})
\]  

(5.19)

where $i$ and $j$ refer the receiver, 1 through 5, in question. The measured covariance was then compared with modeled covariance using Eq. 5.18 for varying values of $C_n$ and $L_0$. It was found that given our range of 15 km and frequency of 11 GHz, $l_0$ had very little impact on covariance as a function of $\rho$. Fig. 5.3 compares the measured $\Sigma$, averaged over both the duration of the campaign (between May 22nd and June 17th, and between August 19th and October 8th, 2015) and all transmitters, and modeled $\Sigma$. By implementing the least squares approach, and minimizing the difference between measured and modeled covariances, values of $C_n$ and $L_0$, $6 \times 10^{-7} m^{-1/3}$ and 75m, respectively, were extracted. These values are within the ranges of practical values provided by Ishimaru.
It is speculated that the large variance of the signal strength ($\Sigma_{i,j}$ where $i = j$, or where $\rho = 0$) is due to fluctuations of the signal as a result of factors other than a turbulent atmosphere. These factors include, but are not limited to, transmitter platform motion (and buoy geometry affecting transmit antenna patterns) and receive amplifier instability. As can be seen in Fig. 4.17, the lowest transmitter (Tx1) was mounted within the structure of the buoy. Therefore, the effective antenna pattern, although designed to be omni-directional, was greatly impacted by the buoy itself when compared to the other transmitters. Measured fluctuations of the signals from Tx1 were greater than fluctuations of signals from the other transmitters. This is apparent in Fig. 5.4 which shows the standard deviation, $\sigma_{x,i}$, for $i = 1, 2, 3$ corresponding to Tx1, Tx2, and Tx3, of the measured log-amplitude fluctuations. The $\sigma_{x,i}$ shown is the standard deviation of a transmitted signal during each of the 10 minute collection periods. A single value of $\sigma_{x,i}$ is found for each transmitter by averaging the signal’s standard deviation as observed by each of the receivers. Eq. 5.18 does not account for these factors but can provide an estimate for turbulence parameters to model the phase fluctuation. While the model derived by Ishimaru does not match up perfectly with measured fluctuations, in particular with the variance of the signals measured at each of the receivers individually, it will act as the model with which to predict amplitude and phase fluctuations. Values of $L_0 = 75\text{m}$, and $C_n = 6 \times 10^{-7}m^{-1/3}$, as seen in Fig. 5.3, will be used in what follows.
Figure 5.4: The standard deviation, $\sigma_{\chi,i}$, for $i = 1, 2, 3$ corresponding to Tx1, Tx2, and Tx3, of the measured log-amplitude fluctuations, averaged across the receive array, during the main collection periods of the campaign.
5.4 Improvement due to Phase

Under the assumption that a future array measurement system is capable of measuring relative phase accurately, simulations were run in an attempt to understand exactly how much improvement to inversion accuracy would be seen for different amounts of phase variation. A similar objective function is used as before (Eq. 3.8), however, instead of standardized values of signal loss, $L_{s,(i,j),dB}^{obs,sim}$, simulated values are defined to have phase relative to the phase at one of the receivers. Here, propagation loss is defined as:

$$PL = 20 \log(2k_0 R) - 20 \log |F|$$

(5.20)

where $R$ is the range, $k_0$ is the wave number, and $F$ is the complex output of VTRPE defined in Section 2. The complex portion of the PWE, $\phi$, is given by:

$$\phi = \text{atan2}(\text{imag}(\psi), \text{real}(\psi))$$

(5.21)

where the complex PWE field, $\psi$, of the VTRPE output is given in Eq. 2.48. Before normalization to the relative phase values, the PWE library values, $Sim_{tx,rx}$, are defined for each transmit-receive path as

$$Sim_{tx,rx} = PLe^{i\phi}$$

(5.22)

and the “observed” values, $Obs_{tx,rx}$, are defined as

$$Obs_{tx,rx} = PLe^{i\phi}e^{ip_n} + a_n$$

(5.23)
Figure 5.5: Distribution of recorded amplitude fluctuations \((10 \log_{10}(SNR_{tx,rx}) - \text{avg}(10 \log_{10}(SNR_{tx,rx})))\) over the course of the long-term measurement depicted here is shown to be well described by a normal distribution with a mean of zero and standard deviation of 1dB

where, \(p_n\) and \(a_n\) are phase and amplitude noise, respectively, generated such that the amplitude and phase fluctuate according to the covariances defined in the previous section.

Based on the fact that measured amplitude fluctuations can be described well by a normal distribution, see Fig. 5.5, and under the assumption that the covariance of the amplitude and phase fluctuations can be described by Eq. 5.18, the amplitude and phase fluctuations will be modeled as a multivariate normal distribution in which the PDF is defined as [111]

\[
f(x, \mu, \Sigma) = \frac{1}{\sqrt{|\Sigma|(2\pi)^d}} e^{-1/2[(x-\mu)^T \Sigma^{-1}(x-\mu)]} \tag{5.24}
\]

Here, \(x\) is the vector of correlated random variables \((x = \ln\left(\frac{|\text{Obs}_{tx,rx}|}{\text{mean}(|\text{Obs}_{tx,rx}|)}\right)\) when simulating the correlated amplitude fluctuations measured across the receive array from a specific transmitter, and \(x = (\text{angle}(\text{Obs}_{tx,rx}) - \text{mean}(\text{angle}(\text{Obs}_{tx,rx})))\) when
simulating the correlated phase fluctuations), \( \mu \) is the vector containing the corresponding means of those variables \( \mu = \ln \left( \frac{|Sim_{tx,rx}|}{\text{mean}(|Sim_{tx,rx}|)} \right) \) when simulating the correlated amplitude fluctuations measured across the receive array from a certain transmitter, and \( \mu = \text{angle}(Sim_{tx,rx}) - \text{mean} \left( \text{angle}(Sim_{tx,rx}) \right) \) when simulating the correlated phase fluctuations), \( \Sigma \) is the covariance matrix, \( \cdot^T \) is the vector transpose, and \( d \) is the number of random variables, or length of vector \( \mathbf{x} \). For application here, \( d \) is equal to the number of receivers and the covariance matrix \( \Sigma \) is determined by Eq. 5.18.

The relative simulated values for use in the complex objective function are then

\[
Sim_{tx,rx}' = Sim_{tx,rx} \frac{Sim_{tx,1}^*}{|Sim_{tx,1}|}
\] (5.25)

and similarly, the relative observed values are given by

\[
Obs_{tx,rx}' = Obs_{tx,rx} \frac{Obs_{tx,1}^*}{|Obs_{tx,1}|}
\] (5.26)

The complex objective function to be minimized in this case is given by

\[
\Phi_c(EDH) = \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} |Obs_{tx,rx}' - Sim_{tx,rx}'(EDH)|^2
\] (5.27)

Monte Carlo trials were performed to test the hypothesis that inclusion of phase information in EDH inversion improves estimation accuracy. Due to uncertainties in the validity of the covariance model derived previously, simulations of complex inversions were carried out for both correlated and uncorrelated phase fluctuations. For correlated fluctuations, simulated measurements were created using Eq. 5.24 where \( \Sigma \) is found using Eq. 5.18 and turbulence parameters which were found by comparing the
measured and modeled amplitude fluctuations. For uncorrelated phase fluctuations, \( \Sigma \) is a diagonal matrix populated by the desired standard deviation.

For each value of EDH (0:0.4:40 m), 1000 instances of \( \text{Obs}_{tx,rx} \) were compared to \( \text{Sim}_{tx,rx} \) for a PWE library created using the parameters from the long-term XBBR deployment. \( \text{Obs}_{tx,rx} \) was simulated for correlated phase with values of \( C_n \) between \( 10^{-9}m^{-1/3} \) (weak turbulence) and \( 10^{-7}m^{-1/3} \) (strong turbulence), and for uncorrelated phase (independent noise added on each receiver) with values of standard deviation from 1 to 25 degrees. Because the inversion method utilizes the relative phase between receivers, fluctuations in the atmosphere that lead to completely correlated variations in measured phase ultimately do not negatively impact EDH inversion and this case was not investigated.

5.4.1 Results

The standard deviation from the expected value of EDH was found for both correlated and uncorrelated phase fluctuations; see Fig. 5.6. In Fig. 5.6 (a), the standard deviation from the correct EDH is always decreased if the phase is included and the phase fluctuates at each of the receivers coherently. In Fig. 5.6 (b), the average standard deviation (averaged over all EDH) from the correct EDH for amplitude only inversions with values of \( C_n \) between \( 10^{-9}m^{-1/3} \) (weak turbulence) and \( 10^{-7}m^{-1/3} \) (strong turbulence) is shown in the solid lines. The dashed line represents the average standard deviation (averaged over all EDH) from the correct EDH for inversions including uncorrelated phase fluctuations and varying noise standard deviation. For a weakly turbulent atmosphere (blue line) including the relative phase only improves
the inversion up to an uncorrelated phase standard deviation of about 4 degrees. However, for strong turbulence (purple line), including the phase information reduces the average standard deviation from the correct EDH during inversion for uncorrelated phase noise with a standard deviation up to approximately 19 degrees.
Figure 5.6: (a) Standard deviation form correct EDH values for 1000 Monte Carlo trials utilizing amplitude (solid) and phase (dashed) information. Both amplitude and phase are correlated with $\Sigma$ found using Eq. 5.18 with $L_0 = 75m$ and varying $Cn$ values) and (b) (solid) The average standard deviation (averaged over all EDH) from the correct EDH for amplitude only inversions with values of $C_n$ between $10^{-9}m^{-1/3}$ (weak turbulence) and $10^{-7}m^{-1/3}$ (strong turbulence) and (dashed) the average standard deviation (averaged over all EDH) from the correct EDH for inversions including uncorrelated phase fluctuations with a standard deviation between 0.5 and 25 degrees.
5.5 Conclusion

This chapter discussed modeling the amplitude and phase fluctuations of a plane wave incident on a turbulent atmosphere, with the goal to determine whether measuring the phase at an array of receivers would be viable. It has been shown that measured signal amplitude exhibited similar fluctuation to that given by Eq. 5.18. This comparison allowed for an approximation of the average outer scale of turbulence and average structure constant which are thought to describe the turbulence of the atmosphere during the long-term measurement.

Improvement to EDH estimation by including phase information was also investigated. For the cases in which both amplitude and phase fluctuations were correlated according to Eq. 5.18, the standard deviation of the estimated EDH from the correct EDH was reduced by including the phase information. Monte Carlo trials were also performed with correlated amplitude fluctuations but uncorrelated phase fluctuations. It was found that for small, uncorrelated fluctuations in phase, using the relative phase as part of the inversion still leads to a reduced variance in estimation, while larger standard deviations from the expected value of phase can lead to larger errors in estimation than amplitude only inversions.

Conclusions that can be drawn from these results are limited by the fact that the fluctuation of phase in a turbulent atmosphere was not accurately measured by the XBBR array. This also limits the ability to determine how accurately the PWE models the phase of transmitted signal in the MABL. However, based on the simulations performed, it seems as though including the relative phase would likely have improved the EDH estimation during the Scripps long-term measurement campaign. These results are limited to the system parameters and physical geometry of the
XBBR during the measurements. To further investigate the benefits of measuring phase, a system parameter optimization study was performed and is discussed in what follows.
Chapter 6: XBBR ARRAY OPTIMIZATION

6.1 Introduction

Initial results obtained by the XBBR array encourage further design studies to improve EDH estimation accuracy for future systems. The goal was to investigate how system parameters that had previously been chosen due to budgetary constraints, receiver limitations, or other constraints, could be chosen to increase inversion reliability. Parameters that are used as inputs to the PWE and are of interest for optimizing the XBBR include transmit frequency, transmitter height, and receiver height. Two methods for investigating system optimization, utilizing Monte Carlo analysis and the Cramér-Rao lower bound, are discussed in what follows.

6.2 Monte Carlo Analysis

The optimal system parameters are defined as those that led to the most unique set of signal loss and relative phase values for each possible EDH in question, $Sim_{tx,rx,t}$. The PWE library with the most variation as the EDH varies will allow for measurements to match well with one, and only one, entry. Ideally, for any given measurement, the objective function used to invert for the EDH would have a clearly defined minimum and the EDH could be estimated with great certainty. To determine the
quality of a given PWE library, a multi-case confusion matrix was computed using Monte Carlo trials.

For these simulation studies, correlated noise was added to values of signal loss and phase for each transmit-receive pair given by the PWE library in question for a given evaporation duct height. Then, the objective function, Eq. 5.27, is used to classify each of the simulated measurements by comparing the noisy data to values expected for each EDH. The result of each classification then populates a confusion matrix, or error matrix. In this case, each column represents a predicted EDH, and each row represents the estimated EDH. For an ideal library, after 1000 Monte Carlo trials, the minimum of the objective function would occur at the EDH from which the noisy library values were created. This would lead to a confusion matrix with a value of 1000 at each of the diagonal elements. When an objective function returns a minimum at an EDH other than what is predicted, the confusion matrix obtains values off the diagonal. The precision, $P$, and recall, $R$ can then be calculated for each EDH using the following:

$$P = \frac{TP}{TP + FP} \quad (6.1)$$

$$R = \frac{TP}{TP + FN} \quad (6.2)$$

where $TP$ represents true positives, $FP$ represents false positives, and $FN$ represents false negatives. In other words, the precision can be defined as: “When a certain EDH is predicted, how often is it the correct EDH?” Similarly, recall can be described as: “When the actual EDH is a certain value, how often does the inversion predict that value?” Using Eq. 6.1 and Eq. 6.2, the $F_1$ score can then be calculated:
Figure 6.1: The $F_1$ as a function of EDH for the XBBR parameters while it was deployed on the Scripps Pier. (Frequencies = 11, 11, 11 GHz, Transmitter heights = 2, 3, 4.5 m, and Receiver heights = 4, 6, 8.5, 11, 13.5m)

$$F_1 = 2 \frac{(P)(R)}{P + R}$$  \hspace{1cm} (6.3)

The $F_1$ is the harmonic mean of precision and recall which provides a measure of accuracy for a confusion matrix. A value close to unity means that for a given EDH, the objective function over the chosen number of trials tends to return the correct EDH. Meanwhile, for values closer to zero, the opposite is true. In Fig. 6.1, $F_1$ is shown for the XBBR parameters during long term deployment. As noted in the previous chapter, there is definite improvement if the phase is included in the inversion.

Both amplitude only and complex inversions for the example shown in Fig. 6.1 lead to a relatively low $F_1$ for EDHs lower than 20 meters. This is due to the fact that the PWE library that exists with this setup has very little variability for these
EDHs. The amplitude only library is shown in Fig. 3.2 while the relative phase for this system setup is shown in Fig. 4.34. Both of these libraries have little variability among signal loss and relative phase as EDH varies between 0 and 20 meters which leads to the relatively low $F_1$.

The parameters that lead to a confusion matrix with the highest average $F_1$ will be defined as an “optimal” parameter set. General trends of higher $F_1$ with respect to certain parameters are also of interest. Note that measurements taken using the XBBR array were not capable of providing absolute signals-loss values for comparison with PWE outputs, so the standardized values for both were used for the inversion. Here, however, the absolute signal loss provided by the PWE is utilized.

Determining these confusion matrices and their corresponding $F_1$’s using Monte Carlo trials was computationally expensive. In an attempt to reduce the computation time, implementing the Cramér-Rao Lower Bound to determine which system parameters will lead to a lower variance from the actual EDH during inversion was also investigated.

### 6.3 Cramér-Rao Lower Bound

The Cramér-Rao lower bound (CRLB) provides the minimum variance of an unbiased estimator [111]. Given measurements $x$, distributed according to a probability density function (PDF), $f(x; \theta)$, where $\theta$ is an unknown, deterministic parameter, the CRLB provides the minimum variance of an unbiased estimator, $\hat{\theta}$. It is defined as the inverse of the Fisher Information, $I$,

$$CRLB = var(\hat{\theta}) \geq \frac{1}{I(\theta)} \quad (6.4)$$
where \( I(\theta) \) is defined as:

\[
I(\theta) = -E \left[ \frac{\partial^2 \ell(x; \theta)}{\partial \theta^2} \right]
\]  

(6.5)

Here, \( \ell(x; \theta) = \log(p(x; \mu(\theta))) \) is the log-likelihood function and \( E \) is the expected value operator. For the purposes of system optimization, the CRLB can provide insight into the relative “quality” of a PWE library. Each library, which is made up of an array of signal loss values (where array length is the number of transmit-receive pairs) at each discretized value of EDH, can be defined statistically by a probability density function (PDF), \( f(x; \mu(\theta)) \). When a PDF is viewed as a function of an unknown parameter, \( \theta \) in this case, it can be termed as the likelihood function, \( p(x; \mu(\theta)) \approx f(x; \mu(\theta)) \). Here, \( x \) represents distributed measurements around the mean, \( \mu(\theta) \), which correspond to the array of signal loss values in the PWE library at each value of EDH (\( \theta \)). The CRLB then provides a minimum variance of an unbiased estimator for each value of EDH. The CRLB for each possible value of EDH in a given PWE library are calculated. The average CRLB can then be found and will act as the metric for determining how optimal a setup is when compared with other setups. It is hypothesized that PWE libraries with lower average CRLBs will lead to system setups with decreased inversion ambiguity.

### 6.3.1 Normal Distribution for Uncorrelated Model

For PWE libraries defined by a single transmit-receive pair, simulated measurements would be described by the likelihood function of a normal distribution with mean \( \mu \) and standard deviation \( \sigma \) given by:
\[ p(x; \mu(\theta)) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left( \frac{(x - \mu(\theta))^2}{2\sigma^2} \right) \quad (6.6) \]

The XBBR operates by measuring 15 values of signal loss simultaneously. This can be described statistically by a joint-likelihood function. For uncorrelated fluctuations between the 15 propagation paths, similar to the simulation studies earlier, the joint-likelihood function is then given by:

\[ p(x; \mu(\theta)) = \frac{1}{\sqrt{2\pi\sigma^2}} 15 \prod_{n=1}^{15} \exp \left( \frac{(x_n - \mu_n(\theta))^2}{2\sigma^2} \right) \quad (6.7) \]

where \( x_n \) is the measured value for each of the transmit-receive paths and \( \mu_n \) is the expected signal loss for that same transmit-receive path. The log-likelihood function is then given by:

\[ \ell(x; \mu(\theta)) = \ln(p(x; \mu(\theta))) = \frac{-15}{2} \ln(2\pi\sigma^2) - \sum_{n=1}^{15} \frac{(x_n - \mu_n(\theta))^2}{2\sigma^2} \quad (6.8) \]

The first derivative of this log-likelihood function is

\[ \frac{\partial \ell(x; \mu(\theta))}{\partial \theta} = \frac{1}{\sigma^2} \sum_{n=1}^{15} \mu_n'(\theta) (x_n - \mu_n(\theta)) \quad (6.9) \]

The second derivative is then:

\[ \frac{\partial^2 \ell(x; \mu(\theta))}{\partial \theta^2} = \frac{1}{\sigma^2} \sum_{n=1}^{15} \mu_n''(\theta) (x_n - \mu_n(\theta)) - \mu_n'(\theta)^2 \quad (6.10) \]

The Fisher Information as a function of EDH can then be written as

\[ I(\theta) = -E \left[ \frac{\partial^2 \ell}{\partial \theta^2} \right] = \frac{1}{\sigma^2} \sum_{n=1}^{15} \mu_n'(\theta)^2 \quad (6.11) \]
Figure 6.2: \( \sqrt{CRLB} \) as a function of EDH for (a) the XBBR long-term measurement parameters, and (b) parameters that lead to a relatively low average CRLB (Frequencies = 13, 15, 17 GHz, Transmitter heights = 21, 22, 23 m, and Receiver heights = 39, 42, 45, 48, 51 m)

It is clear that a PWE library with more variability as EDH varies will lead to a lower CRLB. For optimization purposes, it is of interest to find the PWE library that leads to the lowest average minimum variance (CRLB) for all EDH. Examples of the CRLB as a function of EDH can be seen in Fig. 6.2 for both the XBBR setup parameters and parameters which lead to a much lower CRLB. The XBBR parameters resulted in a much higher CRLB than the setup that transmitted frequencies of 13, 15, and 17 GHz, all from a height of 22m, while the receivers were located at 39, 42, 45, 48, and 51m. Note also that the CRLB was much higher for lower EDHs when calculated using the Scripps Pier measurement parameters. This result only holds true if the libraries were used to invert for the EDH using data that fluctuates incoherently.
6.3.2 Multivariate Normal Distribution for Correlated Model

If the covariance matrix is known for all transmit and receive paths, a multivariate normal distribution can be used to calculate the CRLB and a minimum variance can be found for correlated fluctuations. With three transmitters and five receivers, the measurements made by the XBBR would have a $N \times N$ ($N = 15$) covariance matrix ($\Sigma$) during the long-term measurement, and the PWE libraries would be described by the multivariate normal distribution likelihood function given by

$$p(x; \mu(\theta)) = (2\pi)^{-N/2} |\Sigma|^{1/2} \exp \left((-1/2)(x - \mu(\theta))^T \Sigma^{-1} (x - \mu(\theta))\right)$$  \hspace{1cm} (6.12)$$

where $.^T$ is the transpose, $x$ is the measured/simulated value for each of the transmit-receive pairs, and $\mu$ is the vector of PWE library values for a given EDH. The log-likelihood function is then given by

$$\ell(x; \mu(\theta)) = -\frac{1}{2} \left(k \ln(2\pi) + \ln(|\Sigma|) + (x - \mu(\theta))^T \Sigma^{-1} (x - \mu(\theta))\right)$$ \hspace{1cm} (6.13)$$

For simplification purposes, this can be rewritten, ignoring constants (because the goal is to find the derivative on which the constants have no effect) and as a summation instead of vector multiplication as:

$$\ell(x; \mu(\theta)) = -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \Sigma^{-1}_{i,j} (x_i - \mu_i(\theta))(x_j - \mu_j(\theta))$$ \hspace{1cm} (6.14)$$

Note that $\Sigma$ is assumed to be independent of duct height $\Theta$. The second derivative can then be easily found as
\[
\frac{\partial^2 \ell}{\partial \theta^2} = -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \Sigma_{i,j}^{-1} \left( -x_i \mu''_j(\theta) + x_j \mu''_i(\theta) + \mu''_i(\theta) \mu_j(\theta) + 2 \mu'_i(\theta) \mu'_j(\theta) + \mu_i(\theta) \mu''_j(\theta) \right)
\]  
(6.15)

The Fisher information for a given \( \theta \) is then the negative expectation value of the above equation which simplifies to:

\[
I(\theta) = -E \left[ \frac{\partial^2 \ell}{\partial \theta^2} \right] = \sum_{i=1}^{N} \sum_{j=1}^{N} \mu'_i(\theta) \Sigma_{i,j}^{-1} \mu'_j(\theta)
\]  
(6.16)

After derivation it was apparent that the CRLB simply takes into consideration the curvature of the PWE library. Maximizing this curvature, or minimizing the inverse, occurs when a library has outputs that vary greatly as a function of EDH, which, according to the CRLB, will lead to the lowest variance. However, the CRLB does not take into consideration the similarities that are possible between PWE library values for two different EDHs. A particular setup up might lead to a library that produces a very low average CRLB because the signal loss and relative phase values vary greatly with EDH, but also might, for instance, have library entries for a 5m and 30m EDH that are similar enough to cause ambiguities during an inversion. Although the CRLB can allow for much faster calculation of minimum variance for a particular system setup, the goal of this study was to reduce ambiguities for EDH inversion, which the CRLB does not account for. It is for this reason that the CRLB was not used further in the following parameter optimization studies.

### 6.4 Parameter Space

In the search for optimizing the XBBR system, the possible values for transmit height, transmit frequency, and receive height are summarized in Tables 6.1. Other
parameters, such as range and transmitting antenna properties that can also affect measured/simulated signal values, are not discussed here and will be left to future studies.

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<th>Step Size</th>
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<td>0-8 GHz</td>
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<td>0-15 m</td>
</tr>
<tr>
<td>Receive Height</td>
<td>2-29 m</td>
<td>0-7.5 m</td>
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Table 6.1: Parameter space for optimization study

A total of 92 different transmit array configurations, 90 different frequency combinations, and 98 different receive array configurations lead to a total of 811,440 possible XBBR setups for Monte Carlo Trials. All configurations adhere to the following limitations which are shown in Fig. 6.3:

- The range is fixed at 15 km.
- The number of Transmitters is fixed at 3, and the number of Receivers is fixed at 5.
- Rx 3 is located at the height defined as “Receive Array Center Height” (m).
- Rx 2 is located one times the “Receive Array Step Size” (m) below Rx 3.
- Rx 4 is located one times the “Receive Array Step Size” (m) above Rx 3.
- Rx 1 is located two times the “Receive Array Step Size” (m) below Rx 3.
- Rx 5 is located two times the “Receive Array Step Size” (m) above Rx 3.
- Tx 2 is located at the height defined as “Transmit Array Center Height” (m).
- Tx 1 is located one times the “Transmit Array Step Size” (m) below Tx2.
- Tx 3 is located one times the “Transmit Array Step Size” (m) above Tx 2.
Figure 6.3: System description for varying parameters during parameter optimization study of XBBR Array. Possible setups are described by these parameters to limit the optimization search space.

- Tx 2 operates at the frequency defined as "Center Frequency" (GHz)
- Tx 1 operates at one times the "Frequency Step Size" (GHz) below Tx2.
- Tx 3 operates at one times the "Frequency Step Size" (GHz) above Tx2.

6.5 Results

$F_1$ was found for each EDH for all possible parameter combinations. The PWE library with the highest average $F_1$ is considered optimal. However, because many different combinations of parameters led to relatively high $F_1$'s, discussing trends in library quality as a function of parameters was more useful than defining a single best library. In an attempt to visualize the results, histograms were created counting the number of setups that utilized each parameter. Fig. 6.4 shows which parameters appeared in the top and bottom 1% of configurations when sorted according to $F_1$. 

133
Figure 6.4: Histograms showing which system parameters were used in the top and bottom 1% of configurations when sorted by average $F_1$ for both amplitude only calculations [(a) and (c)], and calculations including the relative phase [(b) and (d)].
Fig. 6.4 does show that higher frequencies and transmit arrays tend to reduce ambiguity in inversion. In order to better visualize the effects of each parameter on the quality of a given library, the average $F_1$ for all setups that contain each parameter was found. These trends can be seen in Figs. 6.5, 6.6, and 6.7.

From Fig. 6.5 (a), one should gather that both amplitude-only and complex PWE libraries that utilize higher frequencies tend to produce better results. In (b), it can be seen that increasing the bandwidth of the system also improves inversions, but to a lesser degree than center frequency. In Fig. 6.6 (a), it can be gathered that increasing the transmit array height can improve inversion results greatly. However, as can be seen in Fig. 6.6 (b), transmitter spacing has little effect on the $F_1$ score. These results are promising if this type of system (widely spaced receive arrays) is to be deployed among a fleet of ships utilizing the navigation radars as transmitters of opportunity; these radars typically exist on most ships at relatively high altitudes (on
Figure 6.6: The average $F_1$ for all setups that utilized transmit array heights (the height of the center of the array) ranging between 2 and 30 m (a), and transmit array spacing between 0 and 14 m (b)

Figure 6.7: The average $F_1$ for all setups that utilized receive array heights (the height of the center of the array) ranging between 2 and 30 m (a), and receive array spacing between 0 and 14 m (b)
Figure 6.8: (a) The $F_1$ at each EDH for the system setup that led to the highest average $F_1$ for an amplitude only inversion (Frequencies = 13.5, 15, 16.5 GHz, Transmitter height = 21.5, 22, 22.5m, and Receiver heights = 23, 24.5, 26, 27.5, 29m). (b) The $F_1$ at each EDH for the system setup that led to the highest average $F_1$ for a complex inversion (Frequencies = 16.5, 17, 17.5 GHz, Transmitter height = 29, 29.5, 30m, and Receiver heights = 27, 27.5, 28, 28.5, 29m) the order of 15 to 20m). The simple trends that were seen in the previous figures are difficult to discern in Fig. 6.7 due to the nature of the parameter limitations. These limitations led to more possible setups with receive arrays near the center (15m) because those configurations allow for more receiver spacing possibilities (a receive array with a center height of 5m can’t have greater than 3m array spacing because the lowest receiver possible in this study was 2m above MSL).

The $F_1$ score for the optimal libraries (for (a) amplitude only, and (b) phase included) are shown in Fig. 6.8. Although the parameters used to determine Fig. 6.8 ((a) Frequencies = 13.5, 15, 16.5 GHz, Transmitter height = 21.5, 22, 22.5m, and
Figure 6.9: (a) The $F_1$ at each EDH for a reworked set of XBBR parameters that led to a relatively high average $F_1$ for an amplitude only inversion (Frequencies = 8, 10, 12 GHz, Transmitter height = 20, 20, 20m, and Receiver heights = 4, 7, 10, 13, 16m). (b) The $F_1$ at each EDH for a reworked set of XBBR parameters that led to a relatively high average $F_1$ for a complex inversion (Frequencies = 9.5, 10, 10.5 GHz, Transmitter height = 15, 15, 15, and Receiver heights = 9, 12, 15, 18, 21m).

In Fig. 6.9, (a) Frequencies = 8, 10, 12 GHz, Transmitter height = 20, 20, 20m, and Receiver heights = 4, 7, 10, 13, 16m (b) Frequencies = 9.5, 10, 10.5 GHz, Transmitter height = 15, 15, 15, and Receiver heights = 9, 12, 15, 18, 21m), it can be seen that

Receiver heights = 23, 24.5, 26, 27.5, 29m. and (b) Frequencies = 16.5, 17, 17.5 GHz, Transmitter height = 29, 29.5, 30m, and Receiver heights = 27, 27.5, 28, 28.5, 29m) produce nearly perfect $F_1$ scores, they are difficult to realize as a system. By tweaking the parameters that were used in the XBBR measurements, the inversions can be greatly improved without the need for extremely high frequencies or extremely high transmitters and receivers.
inversion accuracy could be greatly improved by increasing the transmitter height and slightly increasing the bandwidth to measure across a larger portion of the X-band. It can be noted that these $F_1$ values are attainable with a single transmitter (at a single height) if it is capable of transmitting with a large bandwidth ($\approx 4$ GHz) and the receive array is large enough. If the phase information can be measured appropriately, and it can be shown that the PWE can accurately model the phase, similar inversion accuracy can be achieved at a smaller bandwidth and lower transmitter height.

6.6 Conclusion

Two methods to optimize XBBR system parameters for the purposes of reducing ambiguity in EDH inversion have been investigated: Monte Carlo Trials and application of the CRLB theory. Monte Carlo results are expected to be a better method for system optimization but are more limited by computation. It does appear that the CRLB method does allow for comparable trends to be understood. However, because the CRLB is determined solely on the derivative of the PWE library and does not take into consideration exactly how similar different portions of the library might be, further investigation using this method was not pursued. Other methods for determining the potential ambiguity of a particular setup and PWE library will be left for future studies.

Furthermore, it was shown that while optimizing all parameters might reduce EDH estimation error almost completely (for the given range and measured signal fluctuation), much improvement can be gained by only changing some of the measurement parameters. Two system setups were proposed that might reduce the errors
seen in EDH estimation (one if only the amplitude is to be measured and another if
the phase can be accurately measured) for future measurement campaigns.

For a measurement system with the goal of inverting for refractivity using only
signal loss, it is recommended that propagation path diversity is attained by imple-
menting either a widely spaced receive array or by transmitting multiple different
frequencies (with frequency differences on the order of GHz between each transmit-
ted signal). It is also recommended that transmitters be placed higher rather than
lower (on the order of 20m above MSL). If a system can be designed and deployed
that meets these recommendations, sufficiently good inversion accuracy can be at-
tained for an amplitude-only system. On the other hand, if a system capable of
measuring phase is available, the improvement in inversion accuracy through phase
inclusion would allow for a much smaller receive array and transmitters to operate
over a smaller bandwidth and lead to similar results.

It should be noted that the results shown in this chapter assume a propagation
range of 15 km. While the exact $F_1$ scores will change for different ranges and each
system setup, it is expected that the general trends between decreased inversion am-
biguity and system parameters will hold for propagation ranges around 15km (±5km
approximately). However, this can not be assumed with any certainty based on these
results.

Furthermore, the methods shown here can also be used to determine the relative
quality of a system configuration if limited deployment options are available. If large
frequency differences are unavailable, which might occur if the goal is to utilize trans-
mitters of opportunity, for example, then receive array height and spacing should be
optimized.
Chapter 7: CONCLUSIONS AND FUTURE WORK

This dissertation explored the application of a novel transmit-receive array system capable of determining the EDH in the MABL. This approach shows promise as a method of estimating the current refractive environment in an attempt to increase radiowave propagation awareness.

Chapter 1 provided essential background information regarding atmospheric refractivity and briefly discusses current methods for refractivity estimation. Chapter 2 discussed methods for modeling non-standard radiowave propagation in the MABL. Many modeling approaches exist, including geometric optics, mode theory, and application of the PWE. Most of the widely accepted and used software suites implement, whether in conjunction with other methods or alone, the PWE to solve for electromagnetic fields in the face of variable refractivity and complex geometries.

Chapter 3 described the inversion methods used for estimating the EDH and provided information regarding XBBR array specifications. Based on Monte Carlo trials implementing the objective function used for inversion and 5 dB log-normal noise added independently to each channel, it was shown that error and standard deviation from the expected EDH value are reduced as the number of transmitters and receivers for the system in question are increased. These results lead to the design
and deployment of a three-transmitter, five-receiver system. There is much room for improvement for the inversion technique described here and future work may include:

1. The use of a multi-parameter definition of refractivity profile rather than using EDH as a single parameter for PWE library creation. An objective function that compares measurements to PWE libraries that incorporate M-profile creation based on the total range of possible atmospheric conditions (rather than M-profiles based on EDH alone) could greatly improve inversion results. This method may require the use of a genetic algorithm to reduce inversion time.

2. Investigation into the efficacy of different objective functions.

3. A reduction in the number of approximations made in PWE library creation. The results given in this paper assume a 45-degree HPBW for the transmitting antenna, a smooth sea surface, and a constant $M_0$ value (value of M-profile at height $z = 0$) at all times.

4. Performing radiosonde measurement along with future propagation loss measurements. This would provide a more accurate ground truth to compare inversion results with.

In Chapter 4 the results of measurements from three measurement campaigns were discussed: Trident Warrior 2013, SoCal 2013, and the Scripps Pier Measurement. Little information regarding atmospheric refractivity was gathered from TW2013 due to uncertainty in transmitter information. SoCal 2013 provided the opportunity to deploy an array of transmitters and receivers as described in Chapter 3, but, due to range limitations, it mostly acted as a proof of functionality for the XBBR array system. This was accomplished through comparison of measured signal loss and a direct
plus reflected propagation model. Long-term deployment of the XBBR array system at Scripps Pier allowed for inversion of EDH using the methods discussed in Chapter 3, and the results compared fairly well with “ground truth” values provided by NAVSLaM. Differences between NAVSLaM calculations and XBBR inversion results are thought to be the result of multiple uncertainties and approximations, as well as possible FPGA timing issues. These FPGA timing issues, along with transmitter instability, were also thought to be the reason phase measurements were inconsistent with PWE modeled values. The block diagram of a potential future version of the XBBR array, designed to improve inversion results, is shown in Fig. 7.1. Improvements include a stable frequency synthesizer on both the transmit and receive end (allowing for a wider operating bandwidth), a GPS satellite synchronized clock with a stable reference clock (for phase coherency on the receive end and frequency stable transmissions from the transmitters), and a multichannel analog-to-digital converter with a better duty cycle than the current version.

Chapter 5 discussed the amplitude and phase fluctuations for a plane wave in the presence of a turbulent atmosphere and varying refractive index. Measured amplitude fluctuations were compared with modeled covariance values. This allowed for an approximation of turbulence parameters during the long-term measurement campaign and provided a model for correlated phase fluctuation. It then was shown that for certain amounts of phase variance, both correlated and uncorrelated, EDH inversion could be improved using the previously mentioned inversion technique and including the relative phase measured across an array of receivers. Future work includes:

1. Performing complex RF measurements for comparison with both fluctuation models and complex PWE models for improved inversion accuracy.
2. Improving the model for amplitude and phase variance and covariance based on comparison with accurately measured phase and amplitude fluctuation.

Chapter 6 discussed two methods for optimizing the XBBR array system through Monte Carlo simulation and CRLB application. Monte Carlo trials were chosen for study parameter optimization due to the fact that the CRLB only accounts for PWE library curvature while ignoring possible library ambiguity. Through Monte Carlo trials, it was shown that potential inversion errors can be reduced by increasing transmit array height, receive array height and spacing, and by implementing a system with a wider bandwidth than the current system. Other parameters, such as range and variable array spacing, were not considered and left for future investigation. Future work for attempts to optimize a system like the XBBR include:
1. Comparing various other methods of PWE library quality such as implementing the Kullback-Leibler divergence for investigating the statistical difference between library entries.

2. Reducing restrictions on system parameters. For certain EDHs, ranges, atmospheric conditions, etc., variable array spacing may lead to fewer ambiguities.
Appendix A: TRIDENT WARRIOR COLLECTION
INFORMATION

1. S-band: \( \approx 16 \) hours (Wallclock time) of spectrum analyzer data, 7/13 and 7/14
   - Power vs. Frequency swept over 3010-3090 MHz in 401 frequencies
   - Transferred via serial interface to control and time stamping computer
   - Low duty cycle: one sweep recorded every \( \approx 25-50 \) seconds
   - 1561 sweeps in all

2. C-band: \( \approx 5 \) hours (Wallclock time) of spectrum analyzer, 7/14, 15, and 16
   - Power vs. Frequency swept over 4875-4925 MHz in 401 frequencies
   - Transferred via serial interface to control and time stamping computer
   - Low duty cycle: one sweep recorded every \( \approx 12-25 \) seconds
   - 1459 sweeps in all

3. X-band: \( \approx 23 \) hours (Wallclock time) of time domain captures, 7/13-17
   - Also 1.75 hours of Stiletto data on 7/15
   - Very low duty cycle at 80 MSPS, 16 bits resolution
   - One mode 60 msec every \( \approx 3.2 \) seconds (1.87% duty cycle)
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Table A.3: Collection times for X-band data. Note distances in miles.

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**Total Time** 1:48:15

Table A.4: Collection times for X-band data from Stiletto.
Appendix B: VTRPE INPUT FILE

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IDEBUG = 1
IPLLOT = 2
IGRAPH = 0
IBATCH = 1
AGLFLAG = F
ICOMPLEX = 0
ILOSS = 1
MEGAHZ = 11000
IPOLAR = 1
ZR = 1, -500, 0.1984
IRUNIT = 0
RFIRST = 1
RNGINC = 0.1
RLAST = 50
ZTRANS = 5
TXLAT = 32.9021
TXLON = -117.4036
IROUGH = 0
&END
&TXDATA
ISOURCE = 0
BMWIDTH = 45
BMELEV = 0
TXAGLFLAG = T
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&SECTOR
PRANGE = 0
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NVP = 301
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