Essays on Production-Based Exchange Rates and Uncertainty

Dissertation

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Abstract

This dissertation examines both the determinants and the effects of exchange rate dynamics. The work is both theoretical and empirical and examines exchange rates from the perspective of investment and production activity. The dissertation is divided into three chapters.

The first chapter starts by documenting a novel empirical fact, foreign currencies appreciate in the future (with respect to the U.S. dollar) upon larger capital investment rates of foreign subsidiaries of U.S. firms. I examine the link between exchange rates and the real economy in an investment-based asset pricing model. In the model, real investment adjustment costs abroad and at home are key determinants of investment return differentials. Higher foreign affiliate investment rates capture higher underlying investment opportunities abroad, and in turn higher Tobin’s Q-differentials between the foreign affiliate and the U.S. headquarter. Going forward, the foreign currency appreciates against the U.S. dollar to offset the higher marginal benefits of production abroad relative to at home. I explore the testable implications of the model in a structural estimation framework using Generalized Method of Moments (GMM). The model does well in accounting for mean excess returns to currency portfolios sorted on the basis of investment rate differentials and interest rate differentials.
The second chapter takes exchange rates as given and examines how uncertainty in their underlying prices influences firm decision making. More broadly, the chapter shows both theoretically and empirically how real and financial frictions amplify the impact of uncertainty shocks on firms' investment, employment, debt (term structure of debt growth), and cash holding. We start by building a model with real and financial frictions, alongside uncertainty shocks, and show how adding financial frictions to the model roughly doubles the negative impact of uncertainty shocks on investment and hiring. The reason is higher uncertainty induces the standard negative real-options effects on the demand for capital and labor, but also leads firms to hoard cash and cut debt to hedge against future shocks, further reducing investment and hiring. We then test the model using a panel of US firms and a novel instrumentation strategy for uncertainty exploiting differential firm exposure to exchange rate and factor price volatility. We find that higher uncertainty reduces real investment and hiring, while also leading firms to increase cash holdings by cutting debt, dividends and stock-buy backs, and these effects are strongest in periods of higher financial frictions and for the most financially constrained firms. This highlights why in periods with greater financial frictions – like during the global-financial-crisis – uncertainty can be particularly damaging.

The third chapter examines exchange rate dynamics and their link to uncertainty in international markets. In particular, recent models with Kreps-Porteus (1978) recursive preferences highlight the role of long-run mean risk in accounting for many
stylized facts in international finance and international macroeconomics. These recursive preferences are a special case of the generalized disappointment-aversion preferences (GDA) recently introduced by Routledge-Zin (2010). I examine GDA preferences in a setting that exhibits both country-specific and global economic uncertainty in a fully specified production economy. By nature of the GDA preferences, economic uncertainty plays a more important role than long-run mean risk and is the main source behind time-variation in the probability of disappointing economic outcomes. This time-varying probability drives productivities, risk-sharing arrangements between representative agents, and exchange rate movements. The model highlights how domestic and international uncertainty is relevant for asset prices and cross-border flows. I present the model with two countries, two goods, and asymmetric investment frictions.
I thank everyone who has supported me throughout my academic and professional career. This dissertation is dedicated to God, my parents, my wife, and my grandmother.
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Chapter 1: Production-Based Exchange Rates

The chapter starts by documenting a novel empirical fact, foreign currencies appreciate in the future (with respect to the U.S. dollar) upon larger capital investment rates of foreign subsidiaries of U.S. firms. Sorting countries into portfolios based on foreign affiliate investment rates yields a currency excess return spread of roughly 3.6 percent per year. This affiliate investment rate effect subsists after controlling for the carry trade and momentum effects in currency markets. I examine the link between exchange rates and the real economy in an investment-based asset pricing model. In the model, real investment adjustment costs abroad and at home are key determinants of investment return differentials. Higher foreign affiliate investment rates capture higher underlying investment opportunities abroad, and in turn higher Tobin’s $q$-differentials between the foreign affiliate and the U.S. headquarter. Going forward, the foreign currency appreciates against the U.S. dollar to offset the higher marginal benefits of production abroad relative to at home. I explore the testable implications of the model in a structural estimation framework using Generalized Method of Moments (GMM). The model does well in accounting for mean excess returns to currency portfolios sorted on the basis of investment rate differentials and interest rate differentials.
1 Introduction

This paper studies theoretically and empirically the real determinants of exchange rate dynamics. I show that real investment frictions are crucial to understand the relation between firms’ investment and bilateral exchange rate movements. The paper starts by documenting a novel empirical fact, foreign currencies appreciate in the future (relative to the U.S. dollar) upon larger investment rates of foreign subsidiaries of U.S. firms. In particular, the larger the differential in investment rates is between foreign affiliates and the aggregate U.S. domestic investment rate, the higher the appreciation of the foreign currency is. In a panel of aggregate foreign affiliate investment data for over 50 countries in 30 years, I find that annual real investment rate differentials are positively associated with real exchange growth rates one and two years going forward. A 10 percentage point differential between the foreign affiliate investment rate and the aggregate U.S. domestic investment rate is roughly associated with a 2.3 percent appreciation of the foreign currency the following year. Moreover, sorting countries into portfolios based on foreign affiliate investment rates yields a currency excess return spread of roughly 3.6 percent per year. This affiliate investment rate effect is robust to controlling for the carry trade and momentum effects in currency markets.

I examine the link between exchange rates and the real economy in an international production-based asset pricing model. It features real investment frictions both abroad and at home. More specifically, representative firms in each country operate both domestically and internationally through fully owned representative affiliates. Both the headquarters and the affiliates - henceforth business units - contribute to the production of the local national real economy and choose capital investment optimally to maximize overall firm equity value. Capital investment is costly to adjust both at home and abroad, which captures various frictions associated with investment process, e.g., installation costs, training costs for multinational firms, etc. Moreover, I assume
marginal costs of investment are heterogenous across affiliates. The key prediction of the model is that under constant returns to scale, the stock return of the firm is equal to a Tobin’s $Q$-weighted average of investment returns at home and abroad adjusted for exchange rates. This result directly ties firms’ real decisions to real exchange rate dynamics, and nests in the special (closed-economy) case of the equivalence between stock and (domestic) investment returns, a relation first established in the aggregate by Cochrane (1991) and further examined in the cross-section of U.S. stock returns (e.g., Liu, Whited, Zhang (2009)).

The central insight of the model is that fluctuations in relative currency prices are determined by marginal rates of transformation of capital at home and abroad. In particular, I show that equilibrium exchange rates are tied to both the production side of the economy (by means of marginal rates of transformation of foreign and domestic capital) and to the more common consumption side (by means of marginal rates of substitution, or risk sharing implied by consumption bundles). As such, the model produces testable implications over exchange rates from both sides of the economy, both of which are equally important in equilibrium. Throughout, however, I exploit the production-based implication on exchange rate fluctuations and abstain from having to impose additional economic structure necessary for consumption.

In particular, I examine bilateral exchange rates by looking at foreign affiliate and U.S. domestic investment rates. These rates are the key components of foreign and domestic investment returns (or marginal rates of transformation of capital) to which exchange rates are tied to. From the time $t$ perspective of the multinational firm headquartered in the U.S., higher investment opportunities abroad (as captured by higher foreign affiliate Tobin’s $Q$) imply that the firm forgoes domestic investment in favor of foreign investment. It continues to do so while facing increasingly stiff adjustment costs and only up to the point where the marginal benefits of production abroad exceed the marginal benefits at home. The exchange rate (defined as the
relative price of the foreign final good in terms of the home good) appreciates into
the following period just enough to account for the balancing of equilibrium marginal
benefits of production at home and abroad. That is, as in the data, the foreign
currency appreciates relative to the U.S. dollar after high investment rates of foreign
subsidiaries of U.S. multinationals.

Moreover, I explore the testable implications of the model in a structural
estimation framework (e.g., Liu, Whited, and Zhang 2009). Generalized Method of
Moments (GMM) is employed to estimate affiliate-level model parameters and obtain
predicted values of bilateral exchange rate growths. The fit of the model is examined
at the portfolio-level of currency excess returns. The testing portfolios are quintiles of
annual excess returns on currencies sorted on the basis of investment rate differentials
and interest rate differentials (e.g., the carry trade in Lustig and Verdelhan (2007)).

When the model is first used to match the currency excess returns across the carry
trade quintiles, the model predicts an annual excess return spread of 8.24%, which is
close to the 7.73% observed in the sample of 48 countries (with affiliate investment
data of U.S. parent companies) from 1983 to 2013. A test of joint significance of excess
return errors jointly equal to zero fails to be rejected, with p-value 0.81. Implied
adjustment costs largely fall within the high range of plausible costs documented in
the literature (e.g., Hamermesh and Pfann 1993, Merz and Yashiv 2007, and Bloom
2009).

Furthermore, the model also implies a good fit in the investment differential
portfolios. In the data, the average currency excess returns on the investment
differential quintiles rise monotonically from 2.74% to 6.31% per year, for an annual
spread of 3.57%. More importantly, the model captures this increasing excess return
pattern well and predicts a spread of 6.08%. A Chi-square test of joint significance
of the excess return errors also fails to reject the model for these portfolios, with a
p-value of 0.52.
I also investigate the joint link between investment differentials, interest differentials, and exchange rates. In particular, double sorting is examined, as sorting on investment and interest rate differentials involves some degree of common sorting information (their cross-sectional correlation is roughly 10%). Specifically, I construct 9 portfolios sequentially sorted on both investment differentials and interest rate differentials, and then collapse to 3 portfolios across one dimension to control for the other at a time. I find that both effects are not subsumed by one another. After controlling for interest rate differentials, the investment differential effect yields an average currency excess return spread across tercile portfolios of 2.20% per year. Similarly, after controlling for the investment differential effect, the spread across carry trade terciles is of 5.18% per year. Moreover, the model also does a reasonably good job in predicting the mean excess returns of those 3 double-sorted portfolios, with p-values of Chi-square tests of 0.31 and 0.53, respectively.

On the quantity side, \( q \)-differentials also pin down bilateral differences in investment costs across countries. By varying the parameters of the capital adjustment technologies, a cross-section of marginal costs of investing abroad is produced. Indeed, the GMM estimates shed light on the size of adjustment costs faced by foreign affiliates of U.S. parents, which have little guidance in the literature. Disaggregating the total adjustment costs between home and foreign costs indicates that most of the costs necessary to account for mean currency excess returns are due to affiliate technology (70% on average), as opposed to the home technology (U.S. economy).

Lastly, the positive relation between investment differentials and exchange rate growth in the model arises endogenously due to the interaction between business unit productivities, adjustment costs, and marginal \( q \)s at home and abroad. The model mechanisms rely on two main channels. First, heterogenous non-linear capital adjustment costs across affiliates is crucial. Affiliate-level estimation of the foreign
adjustment technology parameters are necessary to produce meaningful bilateral exchange rates dynamics. If capital is modeled to be freely adjustable, or, in general, suffers no cross-affiliate differences in its marginal cost of investment, then the model fails to match mean exchange rate growths. For example, fixing the capital adjustment cost parameters at arbitrary fixed estimates across bilateral currencies produces statistically large mean excess return errors and even larger implied volatilities.

Second, cross-affiliate variations in investment rate differentials are the key component for currency excess return portfolios. Eliminating its cross-sectional variation by fixing investment differentials at mean cross-sectional values produces large currency excess return errors. Taken together, the model highlights the role of real frictions in generating the cross-sectional variations in exchange rate dynamics and the relations between investment and interest rate differentials and exchange growth portfolios.

2 Related Literature and Motivation

This paper approaches two central research topics in international finance. One, the lack of correlation between consumption growth differentials and exchange rate movements, an anomaly first found by Backus and Smith (1993) and documented to have become more severe over time in Colacito and Croce (2013)). Specifically, standard consumption-based models imply that consumption growth differentials across countries are closely tied (if not perfectly under time-additive preferences) to exchange rate growths. However, this prediction is well at odds with empirical examinations, which show low to no correlation in the data. Two, the tendency of high interest rate currencies to appreciate over time (the forward-premium anomaly, Fama (1984)). This empirical finding is at odds with the uncovered interest rate parity (UIP) condition, which states that when the foreign interest rate is higher than the U.S. interest rate, risk neutral and rational U.S. investors should expect the foreign
currency to depreciate against the dollar by the difference between the two interest rates. This way, borrowing at home and lending abroad, or vice versa, produces a zero return in excess of the U.S. short-term interest rate. Lustig and Verdelhan (2007) examine the cross-section of currency portfolios sorted on interest rate differentials (i.e., carry trade portfolios), and conclude that low interest rate currencies provide domestic investors with a hedge against domestic aggregate consumption growth risk.

This paper, instead, offers an alternative production-based view to exchange rates and currency excess returns. First, I show that equilibrium exchange rates are tied to both the production side of the economy (by means of marginal rates of transformation of foreign and domestic production of capital) and to the consumption side (by means of marginal rates of substitution, or risk sharing implied by consumption bundles). As such, the model produces testable implications over exchange rates from both sides of the economy, none of which is more important than the other and jointly coexist in equilibrium. Thus, instead of contributing to the extensive literature on how exchange rates are linked to consumption bundles, I present novel testable implications on exchange rates derived from the first-order conditions of producers. This approach frees me from needing to impose economic structure on consumer preferences, and instead I focus on technological production functions and how capital adjustment costs influence equilibrium bilateral exchange rates. In testing the model I forego examine correlations of exchange rates with consumption and rather evaluate how key factors of production (such as investment-to-capital and output-to-capital ratios) are linked to exchange rate movements.

Broadly, this paper contributes to the growing literature on production-based asset pricing, a field initiated by Cochrane (1991), who approaches U.S. stock returns from a production-based model - which "is explicitly analogous to the consumption-based model" of stock returns. It is connected to the literature that estimates investment Euler equations using aggregate (or firm-level) investment data (e.g.,
Shapiro (1986) and Whited (1992)). More recently, Liu, Whited, and Zhang (2009) explore the cross-section implications on U.S levered stock returns, while Belo, Xue, and Zhang (2013) examine the cross-sectional valuation implications of a similar investment-based model. In general, this \( q \)-theory of asset pricing builds on the early work of Brainard and Tobin (1968) and Tobin (1969). This paper expands this broad investment-based literature by examining how exchange rates are influenced by domestic and foreign investment decisions of multinational firms.

The paper also relates to the literature that tackles exchange rate predictability. Since Meese and Rogoff (1983a, 1983b, 1988), it has been well known that exchange rates are very difficult to predict in the time-series using economic models. In particular, the consensus in the literature remains that the toughest benchmark for model performance comparison is the non-theoretical model of a random walk without drift. That is, the best predictor for tomorrow’s exchange rate is today’s exchange rate. Rossi (2013) provides a comprehensive review of models of exchange rate predictability existing in the literature. This paper adds to this literature by documenting novel evidence of predictability of exchange rates arising from investment rate differentials.

The rest of the paper is organized as follows. Section 3 presents the empirical results that examine the link between investment rate differentials and currency excess returns. Section 4 presents the production-based model that links exchange rate growths to foreign and domestic investment rates. Section 5 describes the econometric methodology and approach used to test the model. Section 6 reports the model results, and section 7 concludes.
3 Empirical Results

The goal of this section is to examine the empirical link between investment rate differentials and exchange rates. Section 3.1 reports summary statistics and regression results of bilateral exchange rate growths on investment rate differentials abroad and at home. Section 3.2 presents currency excess returns on portfolios sorted on investment differentials, interest rate differentials, and momentum.

3.1 Summary Statistics and Key Correlations

Investment data on aggregate U.S domestic and foreign affiliates is from the Bureau of Economic Analysis (BEA). All empirical examinations are from the perspective of a U.S investor. The BEA offers aggregate-level data for domestic and foreign operations of US multinational firms. Currently, no other government-agency or private institution in the world publicly provides detailed foreign operations data as the BEA. Thus, choosing the U.S as the home country is not only for economic relevance, but necessary. By law, all U.S headquartered firms (including U.S. individuals and legal entities ("U.S. persons")) who own at least one affiliate and own 10 percent or more of a foreign business abroad, and foreign individuals and legal entities ("foreign persons") who are owned 10 percent or more by a U.S. person are mandated to respond to BEA’s multinational survey data. The sample is annual from 1983 to 2013. The aggregate affiliate data at the country-level of operations is of majority-owned affiliates (referred to as 'MOFAs' by the BEA). Although an unbalanced panel of more than 75 countries is available, I restrict the sample to countries with 12 or more years of BEA data, for a total of 52 countries. The empirical results are robust and generally stronger to including more countries, yet I restrict to countries with 12 years of affiliate data for consistency with the GMM estimation and tests of the model in section 3 (which drop countries with few data points). Bilateral
nominal exchange rate data is converted to real terms using consumer price indexes (CPI) of the U.S and each counterpart country. Annual aggregate data for investment flows and capital stock is in real terms, deflated using implicit price deflators from the BEA.

Annual real currency excess (FX) returns (at the country-level) are defined following Lustig and Verdelhan (2007). Define the nominal exchange rate \( s_{f,t} \) as the December of year \( t \) average of daily exchange rates, where exchange rates are relative currency prices of one unit of foreign currency in U.S dollar terms. A positive growth in the exchange rate implies an appreciation of the foreign currency with respect to the U.S dollar. Thus, the annual real excess return is defined as

\[
R_{t+1}^{FX,e} = \left( i_{t} + 1 \right) \left( \frac{s_{f,t+1}}{s_{f,t}} \right) - \left( i_{US,t} + 1 \right) \frac{\pi_{t+1}^{f}}{\pi_{t+1}^{US}}
\]

where \( \pi_{t+1}^{f} \) and \( \pi_{t+1}^{US} \) are inflations abroad and in the US from December \( t \) to \( t+1 \), and \( i_{t} \) is the average December \( t \) interest rate in country \( j \). Equivalently, replacing the nominal exchange rate with the real exchange rate, \( e_{f,t} \), the real currency excess (FX) returns is

\[
R_{t+1}^{FX,e} = \left( i_{t} + 1 \right) \frac{e_{f,t+1}}{e_{f,t}} - \left( i_{US,t} + 1 \right) \frac{\pi_{t+1}^{f}}{\pi_{t+1}^{US}}
\]

(1)

Table 1 presents summary statistics of exchange rates and real activity at home and abroad. Exchange rates are volatile. The standard deviation of real exchange rates is 16.38%, and 16.43% for nominal exchange rates. Investment rates of affiliates abroad are, on average, substantially higher than U.S domestic investment. The average real investment rate abroad is 23.26%, while only 10.81% in the U.S during the sample period. However, the standard deviation is much larger for foreign investment rates, 13.73%, and only 1.02% for domestic rates. Real output ratios are also on average higher and more volatile than domestic output. The median output ratio of foreign affiliates is 1.45 while 1.20 at home. These results are also reported as differentials in percentage points. The mean investment rate differential between foreign and domestic investment is 12 percentage points over the sample period, with a
standard deviation of 11 points, and skewness of 1.81. The mean output differential is of 61 percentage points, standard deviation of 91 points and kurtosis of 4.03. Interest rate differentials have a median of 42 percentage points and a standard deviation of 91 points. Differences in GDP and consumption growth are linked theoretically to exchange rate growths. Both have a mean differential of 1 percentage point and a standard deviation of 4.3 and 4.6 points, respectively.

Differences in investment and output at home and abroad are important drivers in the investment-based model of section 4. As examined below, contemporaneous and lagged investment rates enter the return on investment equations, and their relative contribution to the overall stock return determine equilibrium movements in exchange rates. As a broad motivation to the link between exchange rate growths and affiliate investment and output-to-capital differentials, Figures 1 and 2 plot for a sample of countries exchange rate growths with investment and output differentials, respectively. The plots are standardized, z-score. The plots confirm the sharp period of dollar depreciation of the late 1980’s (as tracked by positive changes in the real exchange rate), and also the general sharp appreciation of the dollar against foreign currencies during the great recession of 2008 (e.g., see the U.K). These and other large shifts in the relative price of the dollar generally coincide with rises and falls in investment and output differentials. Which, again, according to the model of 4 are central ingredients in the oscillations of exchange rates (per their contribution to the investment returns at home and abroad).

Table 2 reports regression results of exchange rates growths on investment rate differentials. Panel A runs time-series cross-sectional panel regressions (with country and time fixed effects and clustering of standard errors), while Panel B runs Fama-MacBeth (1973) cross-sectional regressions (which consist of running $T$ cross-sectional regressions and averaging the coefficients across time, with Newey-West adjusted standard errors to account for autocorrelation of up to 3 years)). Coefficients
are standardized to ease interpretation of magnitudes. In both panels, higher foreign affiliate investment is associated with 1-year ahead appreciation of the foreign currency relative to the U.S dollar. This result is robust to controlling for differentials in interest rate, consumption, and GDP. Moreover, interest rate differentials enter with the correct sign reported in the literature, implying that foreign currencies indeed tend to appreciate (instead of depreciate as stated by the UIP condition) upon rises in foreign interest rates. However, this interest rate effect is statistically weak relative to the predictability arising from investment rate differentials. In addition, the last two columns in Panel A indicate that the foreign affiliate investment effect is robust to constraining the sample to only the largest U.S trading partners (33 countries as reported by the U.S Fed) and to a smaller sample of only 13 developed countries with major currencies (e.g., Menkhoff et al. (2012)).

Table 3 examines how exchange rates are associated with investment differentials at different timing horizons, in particular up to 2 years prior to the growth in exchange rate and up to 1 year after the growth. Univariate results are reported on the left side, while multivariate results on the right. The univariate results indicate that exchange rates are predictable up to two years ahead by foreign affiliate investment rates. However, the positive association flips sign and becomes negative both contemporaneously and 1-year after the growth. That is, following an appreciation of the foreign currency, foreign affiliate investment tends to drop. Conversely, after a negative shock to the foreign currency (e.g., after the British Pound depreciates relative to the U.S dollar), U.S-owned affiliates operating within the U.K increase their capital expenditures. This latter effect is consistent with Erel et al. (2013) that examine firm-level Foreign Direct Investment (FDI) flows after a merger and acquisition of a foreign affiliate takes place. They do not examine, however, how affiliate real investment is associated with future exchange rate movements (which is at the heart of this Q—theoretic paper). In particular, my analysis focuses on real
investment flows rather than FDI flows (which can be mostly of financial nature, i.e., acquisition of full ownership of foreign firms). Distinguishing between these two series is important as they can potentially behave quite differently in nature. For example, one can think of firms looking for tax-haven countries in which to invest in stock (e.g., M&A in Ireland), and see large financial FDI flows towards that country, while only observing relatively small real investment rates for production. Figure 3 plots the lead and lag investment differential effects on exchange growth reported in Table 3. The triangles indicate statistical significance at the 10%. The strongest association is lagged at $t - 1$, followed by $t + 1$. In short, foreign affiliate investment rates predict an appreciation of the foreign currency going forward 1 and 2 years, but drop contemporaneously and 1 year later. In terms of magnitudes, a standard deviation increase in foreign investment rates are positively associated with a future 2.5% appreciation of the foreign currency, while negatively associated with 0.7% appreciation in the previous year.

Table 4 examines whether currency excess returns are also predictable by foreign investment rates. In particular, it reports panel regression results of real currency excess returns (equation 1) on investment rate differentials, interest differentials, and momentum in currency excess returns (defined as the 1-year lagged currency excess return). Coefficients are standardized to ease interpretation of magnitudes. The results indicate that foreign affiliate investment rate is associated with a positive currency excess return 1-year into the future. In particular, a standard deviation increase in foreign affiliate investment rates yields a currency excess return premium of 1.3% per year. This finding is novel to this paper. It is robust to controlling for the well-known carry-trade and momentum effects in currency markets. Moreover, the 12-month currency momentum effect is subsumed by the foreign investment rate differential effect. This analysis is further extended below at the portfolio level.

In particular, the next section shifts the focus from individual currencies to high
versus low differentials at the portfolio level. Because investment rate differentials fluctuate over time, the variation in average excess returns across portfolios is much larger than the spread in average excess returns across individual currencies. Moreover, portfolio analysis provides a setting that helps attenuate the measurement error to which aggregate investment data is subject - which is much less accurately measured than interest rate and momentum differential data. Moreover, building currency portfolios allows the filtering away of currency changes that are orthogonal to changes in investment rate and interest rate differentials.

3.2 Portfolio-level Currency Excess Returns

Currency excess return portfolios are formed on the basis of investment rate differentials, interest rate differentials (i.e., carry trade), and momentum in currency excess returns. Here I abstract from the end-of-year to end-of-year currency excess return timing of Lustig and Verdelhan (2007) and adapt to mid-year to mid-year annual timing familiar to the investment-based asset pricing literature (e.g., Cochrane (1991, 1996) and Liu, Whited, Zhang (2009)). However, results are robust and stronger if excess returns are measured from year-end to year-end. Portfolios are defined as follows. Every June of year $t$, countries are classified into 5 portfolios based on the differentials of year $t - 1$. For the carry trade, the percentiles used as cutoffs are based on the December $t - 1$ interest rate differential $(\hat{i}_{t-1} - \hat{i}_{t-1}^{US})$. Interest rates are the nominal December $t - 1$ monthly average of daily interest rates. The investment rate portfolios are defined on the basis of real investment rate differentials $(\hat{I}_{A,f,t-1}/K_{A,t-2} - \hat{I}_{H\ell,Q,t-1}/K_{H\ell,Q,t-2})$, where investment flow is the aggregate over the course of year $t - 1$, and capital is the stock defined at the end of year $t - 2$. Portfolios are rebalanced every June. Real currency excess (FX) returns are defined as in equation (1), from July of year $t$ to June of year $t + 1$. Note that nominal risk-

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1A maximum of five portfolios is chosen to avoid having too few countries in each portfolio - (specially at the beginning of the sample).
free rates $i_t^f$ and $i_t^{US}$ are known at June of time $t$. To be consistent with the GMM model estimates of expected currency excess returns, I use the June exchange rate (in place of end of year), because the timing matching of stock returns to the weighted average of investment returns, involves investment returns that go (approximately) from the middle of the year $t$ to the middle of the year $t+1$. Lastly, annual momentum portfolios are formed on the basis of past annual winner currencies (high) and short in annual loser currencies (low).

### 3.2.1 Univariate Sorts

Table 5 reports the currency excess returns to the 5 portfolios sorted on investment rate differentials. The returns are from the perspective of a U.S investor. Panel A reports the average excess return (percent per year), t-statistic, standard deviation (in percentages) and Sharpe ratio. The portfolios show a monotonically increasing excess return from 2.74% for the lowest investment differential countries to 6.31% for the highest. The return spread between the extreme quintile portfolios is of 3.57% per year, with a t-stat of 2.98 (Newey-West adjusted standard errors for 3 covariances). The highest standard deviation across the five portfolios is 9.72%, and the lowest 6.09%. Portfolio 5 has the highest Sharpe ratio, 0.78, and portfolio 2 the lowest, 0.3. Descriptive statistics of the portfolios are also reported. The average number of countries in each portfolio is roughly 8. The investment rate differential is 2.25 (percentage points) for the lowest portfolio, 14.54 for the 4th portfolio, and 25.6 for the highest. Moreover, the portfolios show a monotonic increase in the interest rate differential, from 3.09 in the lowest, to 5 percentage points in the highest. This overlap in both increasing interest rate and investment rate differential is examined further below.

Panel B of Table 5 presents realized currency excess returns for portfolios sorted on the basis of interest rate differentials. The five portfolios show a monotonically
increasing mean excess return from 0.80% in portfolio 1 to 8.53% in portfolio 5. The spread is statistically significant at 7.73%. This is comparable to near 7% carry trade spread originally documented in Lustig and Verdelhan (2007), for a somewhat overlapping period 1971-2002. The Sharpe ratios range from 0.1 in the low portfolio to 1.03 in the fourth portfolio. The average number of countries in each portfolio is about 6. There are fewer countries in these portfolios, relative to the investment differential sorts (with an average of 8), due to less availability of t-bill data at the monthly frequency. The interest rate differential is increasing from -1.34 (low) to 12.64 (high). The investment rate differential, although not monotonically, increases from 10.67 (low), 9.20 (portfolio 2), up to 14.58 (high).

Finally, Panel C of Table 5 presents realized currency excess returns for momentum portfolios. The high minus low currency excess return spread across quintiles is 3.15% per year (t-stat of 1.64%). This spread is comparable to the 12-month formation and holding period returns reported in Menkhoff et al (2012), using a sample of 48 countries from 1976 to 2010. Moreover, the momentum sorts seem to share common sorting information from investment rate differentials. In particular, the momentum sorts have investment rate differentials characteristics that rise almost monotonically from quintile 1 to 5 (from a IK differential of 9.89 in past currency losers to 15.10 in past winners). In contrast, the portfolios show a U-shaped interest rate differential characteristic.

### 3.2.2 Multivariate sorts

The characteristics of investment and interest rate differential portfolios indicate that these sorting criteria are related to an extent. Interest differentials rise across the investment differential portfolios, and vice versa. Thus, Table 6 examines the question of whether the excess return spread disappears after controlling for the second sorting dimension. The technique is as follows. To control for interest rate differentials, at
every June of year $t$, countries are sorted into three portfolios based on year $t - 1$ interest rate differential. Then within each of these portfolios, countries are sorted into 3 additional portfolios on the basis of year $t - 1$ investment rate differentials. This gives a total of 9 sequentially sorted portfolios. Lastly, excess returns are averaged across the second dimension of sorting (i.e., investment differentials) to create 3 final portfolios. These portfolios allow some degree of control of the first dimension of sorting (interest rate differentials). A three-by-three sorting criteria is used to avoid having some portfolios running too thin in their number of countries.

Panel A of table 6 reports investment differential portfolios that control for interest rate differentials. First, the collapsed sequential sorts successfully controls for interest rate differential across the investment differential portfolios. The interest rate differential is not monotonically increasing, 3.35 (low), 3.90 (medium), 3.66 (high). Whereas the investment differential rises from 5.36 (low), 11.61 (medium), 21.68 (high). The realized excess return spread is of 2.20% per year (with Newey-West adjusted errors, 3 lags). This is down from 3.57% in the univariate sorts. Thus, interest rate and investment rate differentials share common information, but the investment differential effect exists above and beyond the interest differential. Analogously, Panel B of table 6 reports the carry trade portfolios controlling for investment rate differentials. The average investment differential is similar across the three carry trade portfolios, while the interest differential rises from -0.31 (low) to 9.83 (high). The realized excess return is 5.18% per year, t-stat 3.67, (down from 7.73% in univariate sorts). In short, the key takeaway from Table 6 is that both the interest rate differential effect and the investment rate differential effect exist above and beyond controlling for each other. This evidence is consistent with the panel multivariate regressions at the currency-level in section 3.1. However, the portfolio-level currency spreads are larger than the average excess returns across individual currencies because they exploit the fact that investment rate differentials fluctuate
over time and that they filter out changes in excess returns orthogonal to investment rate differentials.

Finally, Panel C of Table 6 reports results for portfolios created from the intersection of the two sorting dimensions. In particular, every June of year $t$, countries are independently classified into groups of 3 on the basis of breakpoints defined at the 33 and 66 percentiles (of year $t-1$ carry trade and the investment differentials). Then, 9 portfolios are created on the combination of the two separate classifications. Both the average excess returns to each of the 9 portfolios and 7 spreads across terciles are presented. From the 7 spread portfolios only two are statistically significant at the 1 percent. One, the spread between the low I/K + low interest portfolio and low I/K + high interest portfolio yields an excess return spread of 6.06% per year (t-stat 3.70). Two, the spread between the low I/K+low interest and high I/K+high interest yields and excess return spread of 5.23% (t-stat 2.94).

Table 7 repeats the conditional sort analysis above but for investment and momentum portfolios. As seen in the univariate momentum sorts (Panel C of Table 5), the portfolios are almost perfectly monotonic in investment rate differentials. Panel A of Table 7 indicates that the investment differential effect is not subsumed by controlling for the momentum effect in currency markets (spread of 2.04% per year, t-stat of 3.42%). However, Panel B indicates that the momentum effects is no longer significant after controlling for the investment rate differential effect (spread of 1.59%, t-stat of 1.11). In both cases, the sorting technique succeeds in controlling for one effect at time (as seen by the IK and momentum characteristics reported across portfolios). Moreover, the portfolio results described here are, again, consistent with the panel regression results described at the currency-level. In that setting the result is the same: the momentum effect is subsumed by the investment differential effect, but not the other way around.

Having documented the empirical evidence of the link between exchange rates and
investment rates at home and abroad, I develop in the next section an investment-based asset pricing model that sheds light on the mechanisms behind the link.

4 The Model

I study the link between exchange-rates and production in an international asset-pricing model of domestic and foreign affiliate production. The representative firm in each country operates both domestically and internationally through fully owned representative affiliates. Management can forego investing domestically in favor of increasing capital stock of affiliates abroad. These affiliates have their own technologies and produce using domestic inputs to maximize overall firm equity value. The representative multinational firms are owned by representative foreign and domestic shareholders. These have claims to firm dividends. Real exchange rates are necessary both for the conversion of local profits to home-country consumption units and for the conversion of dividends paid out domestically to foreign agents. In equilibrium, exchange rates are tied both to the production side (by means of marginal rates of transformation of foreign and domestic production) and to the consumption side (by means of marginal rates of substitution, or risk sharing implied by consumption bundles). As such, the model produces testable implications on exchange rates from both sides of the economy. I focus primarily on the equally-important former - and avoid the need to impose additional economic structure on preferences.

To ease presentation the model is presented first in a two-country world economy setting and later extended to \( n \geq 3 \) countries. Moreover, the simple consumption-side of the economy is presented first.
4.1 Two-Country World Economy

4.1.1 The problem of the household

There are two countries, home $h$ and foreign $f$, each with a representative representative agent $j = \{h, f\}$ and a multinational firm $i = \{h, f\}$. The firms issue equity shares $S_{i,t}$ with ex-dividend stock price $p_{i,t}$. The representative agent $j$ purchases equity shares of firm $i$, $S_{i,t}^j$. Therefore subscripts of shares indicate the origin of equity issuance and superscripts ownership. Shareholders are entitled to receive dividends $d_{i,t}$ per unit of $S_{i,t}^j$. Agent $j$ earns the competitive domestic real wage, $w_{j,t}$ for providing total domestic labor $N_{j,t}$ (where there is no labor mobility across countries).

Therefore the home agent faces a budget constraint at time $t$ as follows:

$$p_{h,t}(S_{h,t+1}^h - S_{h,t}^h) + e_t p_{f,t}(S_{f,t+1}^h - S_{f,t}^h) \leq w_{h,t} N_{h,t} + d_{h,t} S_{h,t}^h + e_t d_{f,t} S_{f,t}^h - C_{h,t}$$  \hspace{1cm} (2)

where $e_t$ is the real exchange rate defined as the relative price of one unit of the foreign final good (available for foreign consumption $C_{f,t}$) in terms of the home final good (available for domestic consumption $C_{h,t}$). The term $e_t p_{f,t}(S_{f,t+1}^h - S_{f,t}^h)$ denotes the purchase of additional shares of the foreign firm at the prevailing stock price, $p_{f,t}$, and converted back to home consumption good units via the exchange rate $e_t$.

Moreover, the agent maximizes expected lifetime utility

$$\max_{\{C_{h,t+s}, S_{h,t+s+1}^h, S_{f,t+s+1}^h\}} E_t \sum_{s=0}^{\infty} \beta^s u(C_{h,t+s})$$  \hspace{1cm} (3)

subject to equation (2) and the initial number of shares $S_{h,0}$ and $S_{f,0}$. The first order condition with respect to the purchase of next period equity shares in the foreign firm $S_{f,t+1}^h$ is
1 = E_t\left[M_{h,t+1}\frac{e_{t+1}}{e_t}R_{f,t+1}^S\right]  \quad (4)

where \( R_{f,t+1}^S = \frac{p_{f,t+1}^{f} + d_{f,t+1}^{f}}{p_{f,t}^{f}} \) is the foreign firm stock return and \( M_{h,t+1} \) is the stochastic discount factor (SDF) of the home agent\(^2\).

Similarly, the foreign agent in country \( j \) faces an analogue budget constraint at time \( t \):

\[
p_{f,t}(S_{f,t+1}^{f} - S_{f,t}^{f}) + \frac{1}{e_t}p_{h,t}(S_{h,t+1}^{f} - S_{h,t}^{f}) \leq w_{f,t}N_{f,t} + d_{f,t}S_{f,t}^{f} + \frac{1}{e_t}d_{h,t}S_{h,t}^{f} - C_{f,t} \quad (5)
\]

and also chooses to buy additional shares optimally. Her first order condition with respect to the purchase of next period equity shares in the foreign firm \( S_{f,t+1}^{f} \) is

\[
1 = E_t\left[M_{f,t+1}\frac{e_{t+1}}{e_t}R_{f,t+1}^S\right]  \quad (6)
\]

In equilibrium and in the presence of no arbitrage both (4) and (6) must hold. This produces a standard consumption risk-sharing implication between agents, i.e., an \textit{equivalence} between exchange rate growth and the ratio of marginal utilities

\[
\frac{e_{t+1}}{e_t} = \frac{M_{f,t+1}}{M_{h,t+1}} \quad (7)
\]

This equivalence result is long standing in the international literature, and implies high (or perfect - under time additive preferences) correlations of real exchange rates growth and consumption growth differentials. A result that is well known to be at odds with the data - which reveals very low to no correlation whatsoever (an anomaly first identified by Bakus and Smith (1993), and documented to have become more severe over time in Colacito and Croce (2013)).

\(^2\)Specifically, \( M_{h,t+1} = \beta \frac{\Lambda_{h,t+1}}{\Lambda_{h,t}} \), where \( \Lambda_{j,t} \) is the Lagrangean multiplier on the budget constraint of agent \( j \) at time \( t \).
4.1.2 The problem of the multinational Firm

Each representative firm headquartered in country \( i = \{h, f\} \) operates both domestically and abroad. Aside from producing locally, the firm contributes to foreign consumption demand by operating a representative affiliate in country \( l \neq i \).

Both the headquarters and the affiliate business units use local capital stock and labor inputs to produce the domestic consumption good. The home firm finances its aggregate domestic and foreign investment, \( I_h = I_{HQ,h} + I_{A,h} \), by issuing new equities \( p_{h,t}[(S_{h,t+1}^h - S_{h,t}^h) + (S_{f,t+1}^f - S_{f,t}^f)] \) - where the subscripts HQ and A denote the headquarter and affiliate business units, respectively.

Each business unit has a Cobb-Douglas production function with constant returns to scale. The home firm’s total production is

\[
\Pi^h_t = \Pi_{HQ,h}^h(K_{HQ,h}^{h}, N_{h,t}^{HQ,h}, Z_{HQ,h}) + e_t \Pi_{A,t}^h(K_{A,t}^{h}, N_{f,t}^{A,h}, Z_{A,t})
\]

Here, labor is supplied locally by the domestic agent, such that \( N_{h,t}^{HQ,h} \) is the labor supplied by the home agent to the headquarter of the home firm and \( N_{f,t}^{A,h} \) is the labor supplied by the foreign agent to the foreign affiliate of the home firm.

Also, \( Z_{HQ,t}^h \) and \( Z_{A,t}^h \) are vectors of exogenous aggregate and business-unit shocks.

Each business unit has a constant returns to scale production function, with standard homogeneity and marginal product of capital implications. For example, constant returns to scale implies that the production function of the home headquarter business is

\[
\Pi_{HQ,h}^h(K_{HQ,h}^{h}, N_{h,t}^{HQ,h}, Z_{HQ,h}) = K_{HQ,h}^{h} N_{h,t}^{HQ,h} Z_{HQ,h}
\]

and that the marginal product of capital is given by

\[
\frac{\partial K_{HQ,h}^{h}}{\partial N_{h,t}^{HQ,h}} = \kappa_{HQ}^{h} Y_{HQ,h}^{h}/K_{HQ,h}^{h}
\]

in which \( \kappa_{HQ}^{h} \) is the capital’s share in real output as defined by \( Y_{HQ,h}^{h} \) (and as explained below, measured as real value added at the business unit level).

Moreover, even though neither the home nor the foreign agent value leisure, they split their full labor supply \( N_{j,t} \) between all business units operating within their country, i.e., \( N_{j,t} = N_{HQ,j}^{h} + N_{f,t}^{A,j} \) where \( l \neq i \). In addition, the set of producer first order conditions with respect to labor determine a single real wage, \( w_{j,t} \), in each country.
Capital depreciates at an exogenous rate of $\delta_t$, which is technology-specific and time-varying, $K_{HQ,t+1}^h = I_{HQ,t}^h + (1 - \delta_{HQ,t}^h)K_{HQ,t}^h$ and $K_{A,t+1}^h = I_{A,t}^h + (1 - \delta_{A,t}^h)K_{A,t}^h$.

Each business unit incurs adjustment costs when investing in its capital stock. The adjustment cost function, denoted $\Phi$, is increasing and convex in $I_t$, is decreasing in $K_t$, and has constant returns to scale in $I_t$ and $K_t$. Moreover, functional parameters allow heterogenous non-linear adjustment costs across technologies:

$$
\Phi_{HQ,t}^i(I_{HQ,t}^i, K_{HQ,t}^i) = \frac{\theta_{HQ}^i}{\nu_{HQ}^i} \left( \frac{I_{HQ,t}^i}{K_{HQ,t}^i} \right)^{\nu_{HQ}^i} K_{HQ,t}^i
$$

$$
\Phi_{A,t}^i(I_{A,t}^i, K_{A,t}^i) = \frac{\theta_{A}^i}{\nu_{A}^i} \left( \frac{I_{A,t}^i}{K_{A,t}^i} \right)^{\nu_{A}^i} K_{A,t}^i
$$

where $\theta_{HQ}^i > 0$ and $\nu_{HQ}^i > 1$ are the slope and curvature parameters for the headquarter of the home firm, which in turn may differ from the counterpart adjustment cost $\theta_{A}^i > 0$ and curvature $\nu_{A}^i > 1$ parameters of its representative affiliate operating abroad. The estimation of the adjustment cost parameters are central to the GMM tests in section 5. They play a key role in the tests that aim at tracking movements in the bilateral real exchange rate. Moreover, the magnitude of the affiliate slope parameter $\theta_{A}^i$ will vary widely across bilateral exchange rates, and in general is inversely proportional to the size of the affiliate capital stock $K_{A,t}^i$ in each country.

The payout of the multinational home firm is:

$$
D_t^h = \Pi_{HQ,t}^h(K_{HQ,t}^h, N_{HQ,t}^h, Z_{HQ,t}^h) - \Phi_{HQ,t}^h(I_{HQ,t}^h, K_{HQ,t}^h) - I_{HQ,t}^h + e_t\{\Pi_{A,t}^h(K_{A,t}^h, N_{A,t}^h, Z_{A,t}^h) - \Phi_{A,t}^h(I_{A,t}^h, K_{A,t}^h) - I_{A,t}^h\}
$$

where an analogue equation applies to the foreign firm, but using the reciprocal of the exchange rate.

Moreover firm $i$ chooses optimal capital investment at home and abroad and local labor input to maximize the cum-dividend market value of equity:
\[ V_i^t = \max_{\{ \delta_{HQ,t+s}, \delta_{A,t+s}, K_{HQ,t+s+1}, K_{A,t+s+1}, N_{HQ,t+s+1}, N_{A,t+s+1} \}_{s=0}^\infty} E_t \left[ \sum_{s=0}^\infty M_{i,t+s} D_{i,t+s}^t \right] \]  

(11)

where the firm discounts payoffs using the domestic agent’s marginal rate of substitution \( M_{i,t+1} \), which is correlated with the productivity shocks \( Z_{HQ,t}^i \) and \( Z_{A,t}^i \).

**Proposition 1** Home firm equity value maximization implies that \( E_t[M_{h,t+1} R_{HQ,t+1}^{I,h}] = 1 \) and \( E_t[M_{h,t+1} \frac{\epsilon_{t+1}}{\epsilon_t} R_{A,t+1}^{I,h}] = 1 \) in which the investment returns at home

\[
R_{HQ,t+1}^{I,h} = \frac{\kappa_{HQ}^h V_{HQ,t+1}^h - \Phi_{HQ,t+1}^h(I_{HQ,t+1}^h, K_{HQ,t+1}^h)}{q_{HQ,t}^h K_{HQ,t+1}^h} 
\]  

(12)

and abroad:

\[
R_{A,t+1}^{I,h} = \frac{\kappa_{A}^h V_{A,t+1}^h - \Phi_{A,t+1}^h(I_{A,t+1}^h, K_{A,t+1}^h) + \frac{\partial \Phi_{HQ,t+1}^h}{\partial I_{HQ,t+1}^h} I_{HQ,t+1}^h + q_{A,t+1}^h (1 - \delta_{HQ,t+1}^h) K_{HQ,t+1}^h}{q_{A,t}^h K_{A,t+1}^h} 
\]  

(13)

jointly determine relative rates of transformation. Further, define headquarter’s Tobin’s \( q \) as \( q_{HQ,t}^h = \left[ \frac{\rho_{HQ}}{\kappa_{HQ}^h} \right]^{(\phi_{HQ}^h-1)} + 1 \) and affiliate’s Tobin’s \( q \) as \( q_{A,t}^h = \left[ \frac{\rho_{A}}{\kappa_{A,t}^h} \right]^{(\phi_{A}^h-1)} + 1 \), and \( P_{t}^h \equiv V_{t}^h - D_{t}^h \equiv p_{h,t}(S_{h,t+1}^h + S_{f,t+1}^h) \equiv q_{HQ,t}^h K_{HQ,t+1}^h + e_t q_{A,t}^h K_{A,t+1}^h \) as the ex-cash-dividend equity value, where the last two terms are the equity values of the ongoing business concerns at home and abroad, then the stock return of the home firm is the (exchange-rate adjusted) weighted average of investment returns at home and abroad:

\[
R_{t+1}^{S,h} = \frac{R_{HQ,t+1}^{I,h}(q_{HQ,t}^h K_{HQ,t+1}^h) + e_t R_{A,t+1}^{I,h}(q_{A,t}^h K_{A,t+1}^h)}{(q_{HQ,t}^h K_{HQ,t+1}^h) + e_t(q_{A,t}^h K_{A,t+1}^h)} 
\]  

(14)

or, equivalently, next period’s bilateral exchange rate is linked non-linearly to aggregate stock and investment returns:
The stock return equation (14) is a generalization of the well-known (closed-economy) result of the equivalence between stock and domestic investment returns, a relation first established in the aggregate by Cochrane (1991) and explored further in the cross-section of U.S stock returns (e.g., Liu, Whited, Zhang (2009)). In the lack of capital stock abroad, the equity value of the ongoing business concern abroad is zero \( (q_{A,t}^h K_{A,t+1}^h) = 0 \), the second term in both the numerator and denominator cancel out, and (14) reduces to the classical equivalence between the returns on stock and domestic investment in a single-country world economy, i.e., \( R_{S,h}^{t+1} = R_{I,HQ,t+1}^{t,h} \). However, in the more general open-economy case developed here, the stock return in equation (14) is now also an increasing function of the return on investment abroad \( R_{I,A,t+1}^{t,h} \). This foreign investment return is in units of the foreign final good, and thus is converted back to units of the domestic final good by means of the exchange rate at time \( t+1 : e_{t+1} \).

This exchange rate can be solved for and examined in equation (15) from the lense of production. In particular, the bilateral exchange rate will fluctuate in accordance to three return components: aggregate stock return, \( R_{S,h}^{t+1} \), return on domestic investment, \( R_{I,HQ,t+1}^{t,h} \), and return on foreign investment, \( R_{I,A,t+1}^{t,h} \). To facilitate the intuition it is easier to examine the equation first under a two-period setting:

\[
 e_{t+1} = \frac{R_{S,h}^{t+1}[(q_{HQ,t}^h K_{HQ,t+1}^h) + e_t(q_{A,t}^h K_{A,t+1}^h)] - R_{I,HQ,t+1}^{t,h}(q_{HQ,t}^h K_{HQ,t+1}^h)}{R_{I,A,t+1}^{t,h}(q_{A,t}^h K_{A,t+1}^h)}
\]

(15)

\[^4\text{Note that the investment return equations in (12) and (13) can be decomposed in specific economic terms. In particular, the investment returns are ratios of marginal benefits of investment at period } t+1 \text{ to the marginal costs of investment at } t. \text{ The denominators are the marginal costs of investment, including the marginal purchasing costs (unity) and the marginal adjustment costs } \left[ \theta(\frac{K_t}{Y_t})^{(v-1)} + 1 \right], \text{ scaled by size of the associated capital stocks } K_{t+1}. \text{ In the numerator, } \kappa Y_{t+1}, \text{ is associated with the marginal product of one additional unit of capital, the second and third terms are tied to the reduction in installation costs of having the additional unit of capital, and the fourth term is the marginal continuation value of the extra unit of capital.} \]
\[ e_{t+1} = \frac{R_{t+1}^S h \left[ (q_{t+1}^h K_{t+1}^h) + e_t(q_{t+1}^h K_{t+1}^h + 1) \right] - \kappa_{t+1}^h Y^h_{t+1}}{\kappa_{t+1}^h Y^h_{t+1}} \] (16)

In this case, the exchange rate is a function of the time \( t + 1 \) stock return, \( R_{t+1}^S h \), market value at time \( t \) of the ongoing business concern at home, \( (q_{t+1}^h K_{t+1}^h) \), market value at time \( t \) of the ongoing business concern abroad (converted to home final good units), \( e_t(q_{t+1}^h K_{t+1}^h) \), time \( t + 1 \) marginal product of home capital, \( \kappa_{t+1}^h Y^h_{t+1} \), and time \( t + 1 \) marginal product of foreign capital, \( \kappa_{t+1}^h Y^h_{t+1} \). From the perspective of time \( t \), it’s immediate that the higher investment opportunities abroad (as captured by \( q_{t+1}^h \)) imply an appreciation of the relative price of the foreign final good going forward (i.e., the exchange rate rises at time \( t + 1 \)). Because, at the margin, the benefits to the firm of producing abroad next period outweigh the benefits of producing at home, the firm forgoes domestic investment and pursues the higher foreign marginal benefits at time \( t + 1 \) by increasing investment abroad at time \( t \) (but not indefinitely as it faces increasingly stiff adjustment costs abroad and only up to point where the marginal product of foreign capital exceeds the marginal product of home capital). All in all, the model states that the higher investment rate today of foreign affiliates are in equilibrium a manifestation of higher investment opportunities abroad, which in turn are followed by an appreciation of the foreign currency (as in the data).

This predictive implication still follows in the multi-period setting, where after expanding the exchange rate equation in (15) by using equations (12) and (13), we still have that investment rate abroad at time \( t \) only appears in the numerator of the exchange rate (as part of the foreign affiliates’ Tobin’s \( q : q_{t+1}^h \)). However, the marginal benefits of capital at time \( t + 1 \) are no longer only a function of the marginal product of capital (rather they involve all the terms in the numerators of the return on investment equations at home and abroad in (12) and (13), respectively). Moreover, the foreign affiliate investment rate at time \( t + 1 \) has an opposite reducing effect on the exchange rate at time \( t + 1 \) (as in the data). Intuitively, as the foreign investment
rate at time $t+1$ increases (because of high investment opportunities abroad at time
$t+2$, as captured by $q_{A,t+1}^h$), the foreign affiliate starts to demand more investment
resources available to satisfy both consumption and investment at home at time $t+1$.
As the foreign affiliate gobbles up the limited home good resources it increases their
relative price (which by definition is a decrease in the exchange rate $e_{t+1}$, again,
defined as the relative price of one unit of the foreign final good in terms of the home
final good).

Separately, and more broadly, the exchange rate equation 15 is an equilibrium
implication that makes a bold statement: the marginal rates of transformation of
capital pin down equilibrium exchange rate movements. Just as the intertemporal
substitution effects of consumption pin down exchange rates, the production side of
the economy also does so by means of marginal rates of transformation of capital.
In general, agents’ budget constraints are affected by the equilibrium movements in
exchange rates implied by the production model. For instance, a reduction in the
exchange rate means that the US$ appreciates going forward, which in turn implies
that the agent in the U.S sees a reduction in his income at time $t$ from his equity stake
in the foreign firm, yet finds it simultaneously cheaper, ceteris paribus, to buy more of
the foreign stock going into the future $t+1$. These intertemporal substitution effects,
aris from the equilibrium intertemporal and intratemporal equation of production
in 15.

In addition to delivering a testable implication on exchange rates in 15, the
model also has a statement on both the level and return on the ex-dividend value
of the firm (i.e., aggregate multinational valuation implications). Again, exploiting
$P^h_t \equiv V^h_t - D^h_t \equiv p_{h,t}(S^h_{h,t+1} + S^f_{h,t+1}) \equiv q_{HQ,t}^h K^h_{HQ,t+1} + e_{t+1} q_{A,t}^h K^h_{A,t+1}$ as the ex-cash-
dividend equity value, we have

$$\frac{P_{t+1}^h}{P_t^h} = \frac{q_{HQ,t+1}^h K_{HQ,t+2}^h + e_{t+1} q_{A,t+1}^h K_{A,t+2}^h}{q_{HQ,t}^h K_{HQ,t+1}^h + e_{t} q_{A,t}^h K_{A,t+1}^h} \quad (17)$$
is the *gross* ex-dividend stock return on the home firm. Moreover, the model also delivers a testable dividend-price ratio implication:

\[
\frac{D_{ht+1}^h}{P_t^h} = \frac{\Pi_{HQ,t+1}^h - \Phi_{HQ,t+1}^h - I_{HQ,t+1}^h + e_{t+1}\{\Pi_{A,t+1}^h - \Phi_{A,t+1}^h - I_{A,t+1}^h\}}{q_{HQ,t}^h K_{HQ,t+1}^h + e_{t}q_{A,t}^h K_{A,t+1}^h}
\]

(18)

### 4.2 Three-Country World Economy

The world-economy is extended to have three representative agents \( j = \{h, f, g\} \) and three representative multinational firms \( i = \{h, f, g\} \). As before, each firm helps satisfy foreign consumption demand by operating a fully owned affiliate in country \( l \neq i \). Moreover, each firm is public and potentially owned by all agents through the purchase of shares.

To ease presentation the home firm is emphasized. Let \( e_{j,t} \) be the exchange rate or relative price of 1 unit of the final good in country \( j \) in terms of the good in \( h \).

Then home agent’s budget constraint in equation (2) is extended to

\[
p_{h,t}(S_{h,t+1}^h - S_{h,t}^h) + e_{f,t}p_{f,t}(S_{f,t+1}^h - S_{f,t}^h) + e_{g,t}p_{g,t}(S_{g,t+1}^h - S_{g,t}^h) \\
\leq w_{h,t}N_{h,t} + d_{h,t}S_{h,t}^h + e_{f,t}d_{f,t}S_{f,t}^h + e_{g,t}d_{g,t}S_{g,t}^h - C_{h,t}
\]

(19)

where \( e_{g,t}p_{g,t}(S_{g,t+1}^h - S_{g,t}^h) \) represents agent \( j = h \)'s purchase of additional shares in firm \( i = g \) at the prevailing stock price \( p_{g,t} \), which entitles him to dividend payments \( e_{g,t+1}d_{g,t+1}S_{f,t+1}^h \) in the following period (already converted to the home good units using the spot relative price \( e_{g,t+1} \)).

The home agent maximizes lifetime utility

\[
\max_{\{C_{h,t+s}, S_{h,t+s+1}^h, S_{f,t+s+1}^h, S_{g,t+s+1}^h\}_{s=0}^\infty} E_t \sum_{s=0}^\infty \beta^s u(C_{h,t+s})
\]

(20)

subject to [19] Agents in country \( f \) and \( g \) also maximize analogue lifetime utilities.
As before, in the presence of no arbitrage, the set of optimality conditions of agents purchasing equity stakes in foreign firms give links between exchange growths and ratios of marginal utilities

\[ \frac{e_{f,t+1}}{e_{f,t}} = \frac{M_{f,t+1}}{M_{h,t+1}} \]  
\[ \frac{e_{g,t+1}}{e_{g,t}} = \frac{M_{g,t+1}}{M_{h,t+1}} \]  

(21)  

(22)  

where the information contained in exchange rates inform about all possible combinations of relative marginal utilities across countries, e.g., \( \frac{e_{g,t+1}}{e_{f,t}} = \frac{M_{f,t+1}}{M_{g,t+1}} \). Thus, indicating that exchange rates suffice in understanding differences in marginal utilities or pricing kernels.

Proceeding as before, the home firm chooses optimal capital investment at home and abroad and local labor input to maximize the cum-dividend market value of equity:

\[
V^h_t = \max \left\{ I^h_{HQ,t}, I^h_{A,f,t}, I^h_{A,g,t} \right\} \left[ E_t \left[ \sum_{s=0}^{\infty} M_{h,t+s} D^h_{t+s} \right] \right]
\]  

(23)  

where the payout is now

\[
D^h_t = \Pi^h_{HQ,t} (K^h_{HQ,t}, N^h_{HQ,t}, Z^h_{HQ,t}) - \Phi^h_{HQ,t} (I^h_{HQ,t}, K^h_{HQ,t}) - I^h_{HQ,t} + e_{f,t} \left\{ \Pi^h_{A,f,t} (K^h_{A,f,t}, N^h_{A,f,t}, Z^h_{A,f,t}) - \Phi^h_{A,f,t} (I^h_{A,f,t}, K^h_{A,f,t}) - I^h_{A,f,t} \right\} + e_{g,t} \left\{ \Pi^h_{A,g,t} (K^h_{A,g,t}, N^h_{A,g,t}, Z^h_{A,g,t}) - \Phi^h_{A,g,t} (I^h_{A,g,t}, K^h_{A,g,t}) - I^h_{A,g,t} \right\}
\]  

(24)  

and affiliate subscripts now further denote the location of the business unit, e.g., \( I^h_{A,g,t} \), is the capital investment done by the subsidiary located in country \( g \) and owned by the home firm.
Home firm equity value maximization implies that the stock return on the home multinational firm is a weighted average of the three investment returns

\[ R_{t+1}^{S,h} = \frac{R_{HQ,t+1}^{I,h}(q_{HQ,t}^{h}K_{HQ,t+1}^{h}) + e_{f,t+1}R_{A,f,t+1}^{I,h}(q_{A,f,t}^{h}K_{A,f,t+1}^{h})}{(q_{HQ,t}^{h}K_{HQ,t+1}^{h}) + e_{f,t}(q_{A,f,t}^{h}K_{A,f,t+1}^{h}) + e_{g,t}(q_{A,g,t}^{h}K_{A,g,t+1}^{h})} \] (25)

or, equivalently, as a model of investment-based exchange rates, then next period's bilateral exchange rate between countries \( h \) and \( f \) is linked non-linearly to aggregate stock and investment returns:

\[
e_{f,t+1} = \frac{R_{t+1}^{S,h}(q_{HQ,t}^{h}K_{HQ,t+1}^{h}) + e_{f,t}(q_{A,f,t}^{h}K_{A,f,t+1}^{h}) + e_{g,t}(q_{A,g,t}^{h}K_{A,g,t+1}^{h})}{R_{A,f,t+1}^{I,h}(q_{A,f,t}^{h}K_{A,f,t+1}^{h})} \] (26)

Moreover, the ex-dividend equity value is the sum of the ongoing business concerns in the three countries, as follows \( P_t^h \equiv V_t^h - D_t^h \equiv p_{h,t}(S_{h,t+1}^h + S_{f,t+1}^f + S_{g,t+1}^g) \) and

\[
P_t^h = (q_{HQ,t}^{h}K_{HQ,t+1}^{h}) + e_{f,t}(q_{A,f,t}^{h}K_{A,f,t+1}^{h}) + e_{g,t}(q_{A,g,t}^{h}K_{A,g,t+1}^{h}) \] (27)

Lastly, the weighted-average of investment return implications carry through to the more general case of a world economy with a countable set \( W \) of countries. The numerator and denominator in equation (25) are simply expanded by the inclusion of additional terms that enter additively, namely the returns on investment and corresponding equity values of ongoing business concerns in other countries.
5 Econometric Methodology

The main asset pricing tests are at the portfolio level of currency excess returns. In particular, the tests involve portfolios sorted on the basis of interest rate differentials (i.e., the carry trade) and investment rate differentials. The implementation is done in two broad steps. First, the currency-level implication of the investment-based model is exploited in GMM. Using moment conditions obtained from the model, affiliate-level parameters are estimated, and an implied series of bilateral real exchange rate growths is obtained. Second, these implied series are employed to construct country- and portfolio-level predicted currency excess returns. The implied returns are compared against the average portfolio currency excess returns, and the overall fit of the model is evaluated on the basis of a Chi-square test.

5.0.1 GMM Moment Conditions

Although the two-country world economy version of the model suffices for the estimation of bilateral exchange rates, a more complete, yet estimable, version is chosen. In particular, to preserve completeness and to avoid an explosion in the number of affiliate-level parameters requiring estimation, the three-country world economy model is pursued. All tests are from the perspective of the U.S as the home country. The second country $f$ is always the one for which the exchange rate is being estimated in equation (26), and a proxy for the rest-of-the-world (ROW) is implemented as country $g = \text{ROW}$. This aggregate ROW economy excludes capital investment within the U.S and that of the affiliate in country $f$.

Based on equation (26), the first moment condition tests whether the average real exchange rate growth observed in the data, $\bar{e}_{f,t+1}$, equals the average real exchange rate growth implied by the three-country model:
The real exchange of the ROW economy $e_{ROW,t}$ is constructed as a geometrically weighted average of bilateral real exchange rates in the set $ROW$. The definition that follows is a practice implemented by the U.S Federal Reserve Board and several other central banks, international organizations, and private-sector financial institutions.

Let the real exchange rate index at time $t$, $e_{ROW,t}$

$$e_{ROW,t} = e_{ROW,t-1} \times \prod_j^{ROW(t)} \left( \frac{e_{j,t+1}}{e_{j,t}} \right)^{w_{j,t}}$$

(29)
where $e_{j,t+1}$ is the real exchange rate of country $j$ at time $t$, and $w_{j,t}$ is the weight of currency $j$ in the index at time $t$, and $ROW(t)$ is the set of foreign currencies in the index at time $t$, and $\sum_j w_{j,t} = 1$. The set of countries in $ROW$ and its weights are chosen so that it accounts for a substantial portion (about 90%) of relevant commercial partners of the U.S. The set has 35 countries.

The second moment condition exploits the valuation implication of the model, and tests whether the average ex-dividend U.S stock return observed in data equals the average ex-dividend return of the firm predicted by the model:

$$E_t \left[ \frac{P_{t-1} - P_t}{P_t} \right] - \frac{\left( \left[ \theta_{HQ}^{h} \left( \frac{t_{HQ,t+1}^{h}}{K_{HQ,t+1}^{h}} \right)^{(\nu_{HQ}^{h}-1)} \right] + 1 \right) K_{HQ,t+2}^{h} }{ \left( \left[ \theta_{A,f}^{h} \left( \frac{t_{A,f,t+1}^{h}}{K_{A,f,t+1}^{h}} \right)^{(\nu_{A,f}^{h}-1)} \right] + 1 \right) K_{A,f,t+2}^{h} + e_{f,t+1} } - 1 \right] = 0 \quad (30)$$

The third moment condition tests whether the average aggregate U.S cash-
The dividend-price ratio equals the average dividend-price ratio predicted by the model:

\[
E_t \left[ \frac{D_{t+1}^h}{P_t^h} - \frac{\theta_H^h}{\nu_H^h} \left( \frac{R_{HQ,t+1}^h}{K_{HQ,t+1}^h} \right) \nu_H^h K_{HQ,t+1}^h - I_{HQ,t+1}^h \right] + e_{f,t} \left( \kappa_{A,f}^h \nu_{A,f,t+1}^h - \frac{\theta_{A,f}^h}{\nu_{A,f}^h} \left( \frac{R_{A,f,t+1}^h}{K_{A,f,t+1}^h} \right) \nu_{A,f}^h K_{A,f,t+1}^h \right) - I_{A,f,t+1}^h + e_{ROW,t} \left( \kappa_{A,ROW}^h \nu_{A,ROW,t+1}^h K_{A,ROW,t+1}^h - I_{A,ROW,t+1}^h \right) \right] = 0
\]

Following the concerns stated in Belo, Xue, and Zhang (2013), the estimation is done only unconditionally. Instruments are not used to estimate the conditional version of the model. Concerns over measurement errors (including specification errors) being correlated with lagged instruments invalidate the scaling of conditional moments with instruments. These concerns are likely to apply in the specification and measurement of the levels of key variables, such as \( e_{ROW,t}; D_{t+1}^h; P_t^h \).

The three moment conditions in (28) (30) (31) are employed in estimating the parameters of each bilateral currency \( e_{f,t+1} \). One-step GMM estimation with the identity weighting matrix is implemented in all cases. Use of this matrix preserves the economic structure of the model across currency specifications. The parameters that govern the moment conditions are business-level triplets of the slope and curvature of the capital adjustment costs and the capital share in output, i.e., \( (\theta_{HQ}^h, \nu_{HQ}^h, \kappa_{HQ}^h, \theta_{A,f}^h, \nu_{A,f}^h, \kappa_{A,f}^h, \theta_{A,ROW}^h, \nu_{A,ROW}^h, \kappa_{A,ROW}^h) \). The ideal estimation setting would identify these 9 parameters using at least as many moment conditions. However, the concerns over the invalidity of using instruments to increase the number of moment conditions makes the system underdetermined. Thus, having more
parameters than moment conditions, I follow Cochrane (1996) in calibrating a subset of parameters that have extensive guidance in the literature and focus on estimating the rest (with special focus on the ones that have little to no guidance). In short, however, the main parameters of focus are the foreign affiliate curvature and slope adjustment cost parameters, \( b \equiv (\nu^h, \theta^h) \). The results are less sensitive to all other parameter estimates. Moreover, the affiliate slope parameter \( \theta^h \) is key in all tests and varies widely across currencies.

Lastly, although the estimation is done at the exchange rate level, the asset pricing test of interest is ultimately the cross-section of portfolio currency excess returns. First, implied country-level currency excess returns are subject to both measurement errors in affiliate-level data and specification errors. The use of portfolios excess returns reduces the influence of these errors (such as unobserved affiliate-level fixed effects and, more importantly, country-specific frictions not captured in the Euler equations of investment and trade). Second, the carry trade (which builds on the cross-section of currency excess returns) has not been examined under the lense of production and investment rate differences of multinationals. It is central to this study.

6 Model Results

This section present the model results. Subsection 6.1 presents the model implied expected excess returns to portfolios of currencies and subsection 6.2 performs comparative static analysis to shed light on the mechanisms at work in the model.
6.1 Portfolio Expected Currency Excess Returns

6.1.1 Univariate Sorts

Table 8 reports the currency excess returns to the 5 portfolios sorted on investment rate differentials. Panel A presents the realized excess returns. Panel B reports the implied excess returns from the GMM estimation of the investment-based model. As with average excess returns, the implied excess returns show a monotonic increase from 2.04% in the lowest portfolio to 8.12% in the highest. The implied return spread is 6.08% (2.51 points higher than in the data). As reported by the mean excess return error, the higher implied spread comes from both an underprediction in excess return for the lowest portfolio (-0.73) and an overprediction (1.83) in the highest. In addition, the implied standard deviations are roughly two-fold or more compared to the realized deviations. The highest portfolio reports a standard deviation of 20.98%, vs only 8.08% in the data. Although on average the model is able to produce low excess return errors, it tends to overpredict both gains and losses to the portfolios. Overprediction has been documented in models of currency change forecastability. Even though a model on average might capture the direction of growth, it can also consistently overpredict the growth. However, the p-value of a $\chi^2$ statistic that tests that the excess returns errors are jointly zero is presented (p-value of 0.51). Therefore, the model is not rejected in accounting for the average excess returns across the five investment differential portfolios.

Panel C presents economic implications of the GMM estimates. Average magnitudes of slope coefficients of capital adjustment are shown for both the headquarter $\phi_{HQ}$ and ROW $\phi_{ROW}$ business units. Magnitude ranges are $\phi_{HQ} \in (20.59, 32)$, $\phi_{ROW} \in (9.98, 14.14)$. Moreover, the average estimate of the curvature of the country $j$ affiliate is on average 1.01. This implies almost linear capital adjustment costs for the country of the exchange rate of interest. More importantly, however,
is to analyze the economic implications of the slope and curvature estimates - as these are meaningless in themselves unless analyzed on their contribution to the total implied adjustment costs. In this end, the table also presents the implied total cost for the aggregate home multinational and individual adjustment costs arising from each business unit. Using the definition in equation (9), total adjustment cost is estimated as follows:

$$ \Phi_{t}^{total} = \Phi_{HQ,t}^{h} + e_{f,t} \Phi_{A,f,t}^{h} + e_{ROW,t} \Phi_{A,ROW,t}^{h} $$

To interpret the magnitude of the implied adjustment costs, two measures are reported. One with respect to total output measured as gross output (i.e., value added) and another with respect to total sales. Which denominator is used in equation (32) is non-trivial, as it can lead to quite different inferences on adjustment cost estimates. As stated by the BEA, gross product is used commonly because it is free of double counting, unlike sales or receipts data, which reflect not only value added within the firm, but also the value of intermediate inputs purchased from the outside of the firm. Fortunately, BEA has affiliate-level data on both. For all countries the average ratio of value added to sales is never higher than 31% and only 23.08% in recent 2013. The implied total adjustment costs for affiliates presented here use those ratios and proxy the headquarter value added to sales ratio at 0.3.

Using sales as denominator in equation (32), the model implies a monotonically increasing total adjustment cost for the investment differential portfolios from 20.37% (low) to 34.29% (high). These estimates state that countries with high investment rate differentials are positively associated with currency excess returns. Table 8 further reports how much of the total costs arises from each business unit. Headquarters and ROW adjustment costs range from less than 3% to 5%. In sharp contrast, the average adjustment cost at the affiliate j is much larger, ranging from 15% to 30%.

The total costs with respect to sales reported in 8 fall within estimates surveyed
in the literature. For instance, Merz and Yashiv 2007 survey total adjustment costs documented in the literature ranging from 1% up to 77% or more (see also Hamermesh and Pfann 1993 for similar examinations of U.S total adjustment cost estimates). Bloom (2009, Table IV) estimates adjustment costs at the firm-level between zero and 20%. At the portfolio level in the cross-section of U.S Stock returns, and using sales as a measure of output, Belo, Xue, and Zhang (2013) report total cost estimates ranging from small <1% to 15.01% (Table 5), and as high as 19.42% at the industry level (Table 11).

Panel A of Table 9 presents realized currency excess returns for portfolios sorted on the basis of interest rate differentials. Panel B presents the model excess returns. The implied spread between portfolio 5 and 1 is of 8.24%. As in the data, the implied excess returns monotonically increase from 1.31% (low) to 9.55% (high). The standard deviation is, again, much higher than that in data, twofold for portfolio 1 and short of threefold of portfolios 4 and 5. The implied Sharpe ratios range from 0.09 (low) to 0.44 (high). The mean errors are small and statistically insignificant for all 5 portfolios. Thus, the model successfully pins down the first moment of carry trade excess returns. Moreover, the $\chi^2$ statistic that tests that the excess returns errors are jointly zero has p-value of 0.81). Therefore, the model is not rejected in accounting for the average excess returns across the five interest differential portfolios.

Panel C of of Table 9 states that most of the adjustment costs arise from the country of affiliate $j$. In particular, accounting for a maximum of 23.76% of total output. Headquarters and ROW affiliates account for the rest of adjustment costs, for a maximum total adjustment cost of 28.14% (for portfolio 3). Moreover, the curvature estimate of affiliate $j$ is near 1, and as with the investment differential sorted portfolios, implies nearly linear adjustment costs.

Figures 4 illustrates the fit of the model by plotting the predicted currency excess returns, of both investment and interest rate differential portfolios, against the
realized excess returns. If the model’s fit is perfect, all the scattered points should lie exactly on the 45-degree line. The figure shows the monotonically increasing pattern of excess returns in both cases. Thus, it seems to do a reasonable job in matching excess returns patterns. Yet, clearly, the pattern is better captured for the larger spread of the carry trade than the investment differentials.

6.1.2 Multivariate sorts

Panel A of Table 10 reports investment differential portfolios that control for interest rate differentials. The model implied results capture the increasing mean excess return pattern across the 3 investment differential portfolios. The implied spread is of 3.39% (t-stat of 2.54) compared to 2.20% (t-stat 2.83) in the data. Moreover, the model is not rejected by a $\chi^2$ test, p-value of 0.32. Analogously, Panel B reports the carry trade portfolios controlling for investment rate differentials. The realized spread across the three interest rate portfolios is 5.18% per year (t-stat 3.67), while the model overpredicts the carry trade spread 6.26% (t-stat 2.57). Nonetheless, the model captures the increasing excess return pattern across the three portfolios, and the null of excess return errors jointly equal to zero also fails to be rejected ($\chi^2$ test p-value of 0.537).

Figure 5 illustrate the fit of the model of the two portfolios that control for one another effect. The plot shows a reasonably good fit for the interest rate differential portfolios that control for investment differentials. The model tends to overpredict the mean excess returns, but the increasing pattern in returns is aligned. In terms of the investment portfolios that control for the carry trade, the model also overpredicts the excess return in each of the 3, and misprices the highest investment differential portfolio the most.

Table 11 reports results for portfolios created from the intersection of the two sorting dimensions. Overall, the model does a good job in accounting for the mean
excess return seen in the data. The model largely captures the magnitude of excess return errors. The model yields an implied spread from the high I/K-high interest minus low I/K-low interest of 6.91% (t-stat of 2.53), compared to 5.23% (2.94) in the data. The null of joint equality of zero excess return errors is not rejected (χ² test p-value of 0.78). Tables 12 and 13 present the realized and model implied currency excess returns for all 9 sequentially sorted portfolios used in estimating the collapsed sequentially-sorted portfolios that control for one effect at a time. Overall, the model also does a good job in accounting for the mean excess returns seen in the data. The model fails to be rejected in both sets of 9 portfolios.

Finally, Panel A of Figure 6 plots the 9 sequentially sorted portfolios. The increasing patterns in returns seem to be well captured, with the most mispricing coming from the portfolios that earn the highest excess returns. Panel B of 6 plots the average versus predicted excess returns to the 9 intersection portfolios. The model seems to perform well in the pricing of most portfolios, with the largest mispricing arising from the portfolios earning the highest excess returns.

6.2 Inspecting the Mechanism

Comparative static experiments shed light on the key components of the investment returns driving the GMM estimation results. Cross-affiliate variations in the main components of investment returns abroad (gross product-to-capital Y/K and investment-to-capital I/K) can play quantitatively different roles in accounting for cross-sectional currency excess returns. By eliminating their cross-sectional variations one can examine their quantitative importance in matching average portfolio currency excess returns. A large change in the magnitude of the expected currency excess return errors suggests that the component in question is quantitatively important.

In particular, this section performs the following accounting exercise. First, every year eliminate the cross-affiliate variation in investment rates by setting each
individual affiliate investment rate equal to either a) the cross-sectional average of all affiliate investment rates, i.e., \( I/K^h_{A,f,t} = \overline{I/K^h_{A,j,t}}, \) or b) the U.S domestic investment rate, i.e., \( I/K^h_{A,f,t} = I/K^h_{US,t}. \) Second, use the parameter estimates of the benchmark model to reconstruct implied portfolio currency excess returns. Third, examine the magnitude of the implied excess return errors relative to the benchmark model. Similar steps are implemented for output-to-capital ratios. The accounting exercise implies that the differentials in investment rates, \( (I/K^h_{A,f,t} - I/K^h_{US,t}) \), and output-to-capital ratios, \( (Y/K^h_{A,f,t} - Y/K^h_{US,t}) \), are equal across affiliates every year.

Table 14 presents the comparative static results for both the carry trade and investment-rate differential portfolios. Panel A presents the benchmark model results for the carry trade. Panels B and C present the accounting results when the cross-affiliate variation in investment rates is eliminated, while Panel D eliminates the cross-affiliate variation in output-to-capital ratios. Between the two components, clearly, variation in investment rate is the most important. When fixing investment rate differentials, the average excess return errors are up to an order of magnitude larger than that of the benchmark, e.g., the excess return on the highest interest rate differential portfolio is overpredicted by 11.4 percent per year (compared to 1.04 percent of the benchmark model). However, the errors are larger when the affiliate investment is set to equal to cross-sectional average affiliate investment (Panel B) than when set to equal the U.S domestic investment rate (Panel C). Moreover, the implied volatilities of the portfolios are also much larger when the affiliate investment rate is fixed at the cross-sectional average. In contrast, the model is much less sensitive to fixing the output-to-capital ratios, as seen in Panel D where the implied volatilities and excess return errors are closer to the benchmark model.

Similarly, Panels F, G, and H present the comparative static results for the investment rate differential currency portfolios. The conclusions are similar to the carry trade analysis. First, the portfolios are much more sensitive to cross-affiliate
variations in investment rates than output-to-capital ratios (with excess return errors 10 times as big as the benchmark model). This quantitative results says that changes to the marginal product of capital are less important than changes to both the marginal costs of investment and the investment opportunities abroad (as captured by $q$s). By fixing the investment rate differentials, the implied $q$ differentials are distorted. This in turn means that the cross-sectional rankings of the investment opportunities abroad are distorted and inhibit the model from exploiting the positive correlation between investment rate differentials and exchange rate growths (discussed in section 3.1). Second, the model performs much worse when affiliate investment rates are fixed and set to equal the cross-affiliate average investment rate than when fixed to equal the U.S domestic investment rate. This is likely attributed to the fact that the mean affiliate investment rate is on average more than two times as large as the mean U.S domestic investment rate (0.23 and 0.11, respective in Table 1), thus not only distorting the ranking of $q$s but also shifting the mean of the implied $q$s even farther away from their benchmark values. Moreover, the higher average cross-sectional affiliate investment rates also make the returns on investment abroad much more volatile and increases the excess return errors.

7 Conclusion

The paper examines the theoretical and empirical link between exchange rates and capital investment at home and abroad. I first document novel empirical evidence on the positive cross-sectional association between foreign currency appreciation (with respect to the U.S. dollar) and foreign affiliate aggregate investment of U.S. firms. I then develop an international asset-pricing model to understand the economic mechanisms driving the empirical results. The model ties investment differentials abroad and at home to $q$-differentials. The higher investment opportunities abroad,
as captured by the higher $q$-differentials, imply an appreciation of the foreign currency.

The exchange rate predictions of the model are taken to the data in a GMM structural estimation framework of cross-sectional currency excess returns. The $q$-theory of foreign and domestic investment does a reasonably good job in accounting for mean excess returns to portfolios sorted on the basis of interest and investment differentials abroad and at home. On the quantity side, the model also sheds light into the size of capital adjustment costs at the affiliate level, which have little guidance in the literature. Most of the costs necessary to account for mean currency excess returns are due to affiliate technology (about 70% or more), as opposed to the home U.S. technology. Moreover, the mean excess returns to the testing portfolios are increasing in total adjustment costs.

The work above is a first stab at integrating exchange rates, and more broadly international asset pricing, with production at home and abroad. The mean excess returns to currency portfolio results are encouraging. However, moving forward more complete models with, say, international trade, intangible foreign and domestic assets, debt, taxes, and other components will allow tackling other important stylized facts in international finance. Presently, the undersigned is working on extending the model to include intangible investment across affiliates, and also at work on taking the model to the firm level in the cross-section of U.S firms.
References


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This figure plots real exchange rate growths and investment rate differentials (between aggregate foreign affiliate and the U.S domestic investment rate) over time for a sample of countries. The lines are standardized z-score.
Figure 2 Real Exchange Rate Growth and Output Differentials

This figure plots real exchange rate growths and output-to-capital differentials (between aggregate foreign affiliate and the U.S domestic output-to-capital ratios) over time for a sample of countries. The lines are standardized z-score.
Table 1
Descriptive Statistics: Differentials in Aggregate Real Activity Abroad (Affiliates) and at Home (U.S Headquarters)

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev</th>
<th>Median</th>
<th>P5</th>
<th>P25</th>
<th>P75</th>
<th>P95</th>
</tr>
</thead>
<tbody>
<tr>
<td>real (e_{t+1}/e_t)</td>
<td>1.01</td>
<td>0.17</td>
<td>0.00</td>
<td>-0.18</td>
<td>-0.06</td>
<td>0.07</td>
<td>0.21</td>
</tr>
<tr>
<td>nominal (s_{t+1}/s_t)</td>
<td>-1.05</td>
<td>0.16</td>
<td>-0.01</td>
<td>-0.37</td>
<td>-0.10</td>
<td>0.02</td>
<td>0.14</td>
</tr>
<tr>
<td>Affiliate (I/K_{Af,t})</td>
<td>0.23</td>
<td>0.14</td>
<td>0.20</td>
<td>0.09</td>
<td>0.15</td>
<td>0.27</td>
<td>0.44</td>
</tr>
<tr>
<td>U.S (I/K_{US,t})</td>
<td>0.11</td>
<td>0.01</td>
<td>0.11</td>
<td>0.09</td>
<td>0.10</td>
<td>0.11</td>
<td>0.13</td>
</tr>
<tr>
<td>Affiliate (Y/Y_{Af,t})</td>
<td>1.75</td>
<td>1.13</td>
<td>1.45</td>
<td>0.56</td>
<td>0.97</td>
<td>2.16</td>
<td>4.00</td>
</tr>
<tr>
<td>U.S (Y/Y_{US,t})</td>
<td>1.15</td>
<td>0.21</td>
<td>1.20</td>
<td>0.82</td>
<td>0.97</td>
<td>1.27</td>
<td>1.52</td>
</tr>
<tr>
<td>((I/K_{Af,t}) - (I/K_{US,t}))</td>
<td>0.12</td>
<td>0.11</td>
<td>0.10</td>
<td>-0.01</td>
<td>0.05</td>
<td>0.17</td>
<td>0.33</td>
</tr>
<tr>
<td>((i_t^f - i_t^{US}))</td>
<td>0.05</td>
<td>0.13</td>
<td>0.02</td>
<td>-0.04</td>
<td>0.00</td>
<td>0.05</td>
<td>0.20</td>
</tr>
<tr>
<td>((\Delta GDP_t^f - \Delta GDP_t^{US}))</td>
<td>0.01</td>
<td>0.04</td>
<td>0.01</td>
<td>-0.06</td>
<td>-0.01</td>
<td>0.03</td>
<td>0.08</td>
</tr>
<tr>
<td>((\Delta C_t^{US} - \Delta C_t^f))</td>
<td>0.01</td>
<td>0.05</td>
<td>0.01</td>
<td>-0.07</td>
<td>-0.01</td>
<td>0.03</td>
<td>0.08</td>
</tr>
</tbody>
</table>

This table presents summary statistics of exchange rates (defined as the U.S dollar price of 1 unit of foreign currency), interest rates, and investment rates at home and abroad of firms headquartered in the U.S. Nomenclature is as follows, \(e_{t+1}\) is the real exchange rate, \(s_{t+1}\) is nominal, Affiliate \((I/K_{Af,t})\) is the annual investment rate of foreign affiliates of U.S parents operating in country \(f\), U.S \((I/K_{US,t})\) is the aggregate domestic non-residential investment rate of firms headquartered in the U.S, and U.S \((Y/Y_{US,t})\) is the gross domestic output of these firms. Moreover, \(i_t^f\) and \(i_t^{US}\) are the nominal T-bill rates issued in countries \(f\) and the U.S. \(\Delta GDP_t^f\) and \(\Delta C_t^f\) are the net growth in aggregate gross domestic product and consumption in country \(f\), respectively. The sample is an unbalanced annual panel of 52 countries, from 1983 to 2013.
Table 2
Exchange Rate Growth Panel and Cross-Sectional Regressions

<table>
<thead>
<tr>
<th>Panel A</th>
<th>Panel Regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual LHS:</td>
<td>Real Exchange Rate Growth</td>
</tr>
<tr>
<td>$(I/K_{A,f,t-1} - I/K_{US,t-1})$</td>
<td>0.017</td>
</tr>
<tr>
<td>3.27</td>
<td>3.33</td>
</tr>
<tr>
<td>$(i_{t-1}^f - i_{t-1}^{US})$</td>
<td>0.011</td>
</tr>
<tr>
<td>1.42</td>
<td>1.39</td>
</tr>
<tr>
<td>$(\Delta C_{t-1}^{US} - \Delta C_{t-1}^{f})$</td>
<td>0.002</td>
</tr>
<tr>
<td>0.48</td>
<td>1.99</td>
</tr>
<tr>
<td>$(\Delta GDP_{t-1}^{f} - \Delta GDP_{t-1}^{US})$</td>
<td>0.0125</td>
</tr>
<tr>
<td>2.23</td>
<td>1.61</td>
</tr>
<tr>
<td>$Adj \ R^2$</td>
<td>0.274</td>
</tr>
<tr>
<td># obs</td>
<td>1,031</td>
</tr>
<tr>
<td>Country-Time FE&amp;SE</td>
<td>yes</td>
</tr>
<tr>
<td>33 largest trading</td>
<td>yes</td>
</tr>
<tr>
<td>13 developed &amp; major</td>
<td>yes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B</th>
<th>Fama-MacBeth Cross-Sectional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual LHS:</td>
<td>Real Exchange Rate Growth</td>
</tr>
<tr>
<td>$(I/K_{A,f,t-1} - I/K_{US,t-1})$</td>
<td>0.017</td>
</tr>
<tr>
<td>4.00</td>
<td>3.85</td>
</tr>
<tr>
<td>$(i_{t-1}^f - i_{t-1}^{US})$</td>
<td>0.010</td>
</tr>
<tr>
<td>2.23</td>
<td>1.99</td>
</tr>
<tr>
<td>$(\Delta C_{t-1}^{US} - \Delta C_{t-1}^{f})$</td>
<td>0.001</td>
</tr>
<tr>
<td>0.06</td>
<td>1.17</td>
</tr>
<tr>
<td>$(\Delta GDP_{t-1}^{f} - \Delta GDP_{t-1}^{US})$</td>
<td>0.012</td>
</tr>
<tr>
<td>1.59</td>
<td></td>
</tr>
<tr>
<td>$Adj \ R^2$</td>
<td>0.041</td>
</tr>
<tr>
<td># obs</td>
<td>1,031</td>
</tr>
<tr>
<td>Newey-West SEs (3 lags)</td>
<td>yes</td>
</tr>
<tr>
<td># Cross-Sections</td>
<td>30</td>
</tr>
</tbody>
</table>

This table reports regression results of exchange rates growths on investment rate differentials and controls. Panel A runs time-series cross-sectional panel regressions (with country and time fixed effects and clustering of standard errors), while Panel B runs Fama-MacBeth (1973) cross-sectional regressions (which run $T$ cross-sectional regressions and coefficients are averaged across time, with Newey-West (1987) adjusted standard errors to account for autocorrelation of up to 3 years)). Coefficients are standardized to ease interpretation of magnitudes. The investment rate differential $(I/K_{A,f,t} - I/K_{US,t})$ is the annual difference at time $t$ in the aggregate investment rate of foreign affiliates in country $f$ of U.S parents and the aggregate U.S rate.

50
### Table 3

Exchange Rate Growth and Affiliate Investment Rate (Lead and Lags)

<table>
<thead>
<tr>
<th>Real Exchange Rate Growth at $t$</th>
<th>Univariate Adj $R^2$</th>
<th>Multivariate Adj $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(I/K_{A,f,t-2} - I/K_{US,t-2})$</td>
<td>0.013 0.096 0.76</td>
<td>0.004 0.76</td>
</tr>
<tr>
<td>$(I/K_{A,f,t-1} - I/K_{US,t-1})$</td>
<td>0.025 0.109 1.68</td>
<td>0.028 1.68</td>
</tr>
<tr>
<td>$(I/K_{A,f,t} - I/K_{US,t})$</td>
<td>-0.001 -0.009 -0.77</td>
<td>-0.009 -0.77</td>
</tr>
<tr>
<td>$(I/K_{A,f,t+1} - I/K_{US,t+1})$</td>
<td>-0.007 -0.092 -1.81</td>
<td>-0.009 -1.81 0.112</td>
</tr>
</tbody>
</table>

Country & Time FE & SE | Yes | Yes | Yes | Yes |
# obs | 1,299 | 1,299 | 1,299 | 1,299 |
# Time Periods | 27 | 27 | 27 | 27 |
# Countries | 52 | 52 | 52 | 52 |

This table examines how exchange rates are associated with investment differentials at different timing horizons, in particular up to 2 years prior to the growth in exchange rate and up to 1 year after the growth. Univariate results are reported on the left side, while multivariate results on the right. Coefficients are standardized to ease interpretation of magnitudes. Panel regression includes both country and time fixed effects and clustering of standard errors. The investment rate differential $(I/K_{A,f,t} - I/K_{US,t})$ is the annual difference at time $t$ in the aggregate investment rate of foreign affiliates in country $f$ of U.S parents and the aggregate U.S domestic investment rate of firms headquartered in the U.S. The sample is annual and includes affiliate data in up to 52 countries, from 1983 to 2013.
Table 4
Real Currency Excess Return $FX_t$, Panel Regressions

<table>
<thead>
<tr>
<th></th>
<th>$I/K_{A,f,t-1}- I/K_{US,t-1}$</th>
<th>$(i_{t-1}^f - i_{t-1}^{US})$</th>
<th>$FX_{t-1}$</th>
<th>Country &amp; Time FE</th>
<th>Adj $R^2$</th>
<th># obs</th>
<th># Time Periods</th>
<th>Max. # Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.013</td>
<td>0.011</td>
<td>0.012</td>
<td>Yes</td>
<td>0.166</td>
<td>1008</td>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>0.011</td>
<td>0.011</td>
<td>0.012</td>
<td>Yes</td>
<td>0.166</td>
<td>1008</td>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>0.012</td>
<td>0.011</td>
<td>0.012</td>
<td>Yes</td>
<td>0.166</td>
<td>1008</td>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>0.010</td>
<td>0.011</td>
<td>0.012</td>
<td>Yes</td>
<td>0.166</td>
<td>1008</td>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>2.25</td>
<td>2.15</td>
<td>2.09</td>
<td>Yes</td>
<td>0.166</td>
<td>1008</td>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>2.15</td>
<td>2.09</td>
<td>2.00</td>
<td>Yes</td>
<td>0.166</td>
<td>1008</td>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>3.2</td>
<td>3.19</td>
<td>3.17</td>
<td>Yes</td>
<td>0.166</td>
<td>1008</td>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>0.034</td>
<td>0.034</td>
<td>0.033</td>
<td>Yes</td>
<td>0.166</td>
<td>1008</td>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>0.034</td>
<td>0.034</td>
<td>0.033</td>
<td>Yes</td>
<td>0.166</td>
<td>1008</td>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>0.033</td>
<td>0.033</td>
<td>0.033</td>
<td>Yes</td>
<td>0.166</td>
<td>1008</td>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>0.020</td>
<td>0.019</td>
<td>0.018</td>
<td>Yes</td>
<td>0.166</td>
<td>1008</td>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>0.020</td>
<td>0.019</td>
<td>0.018</td>
<td>Yes</td>
<td>0.166</td>
<td>1008</td>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>0.020</td>
<td>0.019</td>
<td>0.018</td>
<td>Yes</td>
<td>0.166</td>
<td>1008</td>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>0.020</td>
<td>0.019</td>
<td>0.018</td>
<td>Yes</td>
<td>0.166</td>
<td>1008</td>
<td>30</td>
<td>45</td>
</tr>
</tbody>
</table>

This table examines whether currency excess returns (defined in equation (1)) are predictable by foreign investment rates. In particular, it reports panel regression results of real currency excess returns on investment rate differentials, interest differentials, and momentum in currency excess returns (defined as the 1-year lagged currency excess return). Coefficients are standardized. Panel regression includes both country and time fixed effects and clustering of standard errors. The investment rate differential ($I/K_{A,f,t-1}- I/K_{US,t-1}$) is the annual difference at time $t$ in the aggregate investment rate of foreign affiliates in country $f$ of U.S parents and the aggregate U.S domestic investment rate of firms headquartered in the U.S. ($i_{t-1}^f - i_{t-1}^{US}$) is the interest rate differential (T-bills) at the end of year $t$. The sample is annual from 1983 to 2013.
This figure plots the exchange rate growth at year $t = 0$ associated with investment rate differentials realized at year $t = -2$ to year $t = +1$. The exchange rate growth is in response to a standard deviation increase in the investment rate differential ($I/K_{A,f,t+j} - I/K_{U,S,t+j}$) during those years. The results are from the multivariate panel regression results presented in Table 3. The x-axis indicates the time of the investment rate differential, and the y-axis the magnitude of the change to the real exchange rate growth. Triangles indicate significance at the 10% level.
Table 5
Realized Currency Excess Returns (FX)

A: Investment Rate differentials Sorts

<table>
<thead>
<tr>
<th>Realized Returns</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
<th>H-L</th>
</tr>
</thead>
<tbody>
<tr>
<td>FX Mean Excess return</td>
<td>2.74%</td>
<td>2.87%</td>
<td>3.75%</td>
<td>4.38%</td>
<td>6.31%</td>
<td>3.57%</td>
</tr>
<tr>
<td>[t]</td>
<td>2.34</td>
<td>1.54</td>
<td>2.00</td>
<td>2.81</td>
<td>4.06</td>
<td>2.98</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>6.09%</td>
<td>9.72%</td>
<td>9.73%</td>
<td>8.09%</td>
<td>8.08%</td>
<td>5.92%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.45</td>
<td>0.30</td>
<td>0.39</td>
<td>0.54</td>
<td>0.78</td>
<td></td>
</tr>
<tr>
<td>avg. n countries</td>
<td>8.30</td>
<td>8.00</td>
<td>8.00</td>
<td>8.00</td>
<td>7.85</td>
<td></td>
</tr>
<tr>
<td>avg. ((I/K_{A,f,t} - I/K_{US,t}))</td>
<td>2.25</td>
<td>7.05</td>
<td>10.31</td>
<td>14.54</td>
<td>25.60</td>
<td></td>
</tr>
<tr>
<td>avg. ((i_t^f - i_t^{US}))</td>
<td>3.09</td>
<td>2.43</td>
<td>2.88</td>
<td>4.81</td>
<td>5.00</td>
<td></td>
</tr>
</tbody>
</table>

B: Interest Rate differentials Sorts

<table>
<thead>
<tr>
<th>Realized Returns</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
<th>H-L</th>
</tr>
</thead>
<tbody>
<tr>
<td>FX Mean Excess return</td>
<td>0.80%</td>
<td>2.81%</td>
<td>3.71%</td>
<td>4.25%</td>
<td>8.53%</td>
<td>7.73%</td>
</tr>
<tr>
<td>[t]</td>
<td>0.55</td>
<td>1.38</td>
<td>2.34</td>
<td>2.46</td>
<td>5.34</td>
<td>5.83</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>7.63%</td>
<td>10.60%</td>
<td>8.23%</td>
<td>8.97%</td>
<td>8.31%</td>
<td>8.55%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.10</td>
<td>0.26</td>
<td>0.45</td>
<td>0.47</td>
<td>1.03</td>
<td>0.90</td>
</tr>
<tr>
<td>avg. n countries</td>
<td>6.77</td>
<td>6.45</td>
<td>6.21</td>
<td>6.45</td>
<td>5.81</td>
<td></td>
</tr>
<tr>
<td>avg. ((I/K_{A,f,t} - I/K_{US,t}))</td>
<td>10.67</td>
<td>9.20</td>
<td>11.88</td>
<td>13.29</td>
<td>14.58</td>
<td></td>
</tr>
<tr>
<td>avg. ((i_t^f - i_t^{US}))</td>
<td>-1.34</td>
<td>0.57</td>
<td>2.12</td>
<td>5.17</td>
<td>12.64</td>
<td></td>
</tr>
</tbody>
</table>

C: Momentum Sorts (\(FX_{t-1}\))

<table>
<thead>
<tr>
<th>Realized Returns</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
<th>H-L</th>
</tr>
</thead>
<tbody>
<tr>
<td>FX Mean Excess return</td>
<td>3.02%</td>
<td>3.15%</td>
<td>4.01%</td>
<td>2.77%</td>
<td>6.17%</td>
<td>3.15%</td>
</tr>
<tr>
<td>[t]</td>
<td>1.91</td>
<td>1.03</td>
<td>2.22</td>
<td>1.40</td>
<td>4.21</td>
<td>1.64</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>8.20%</td>
<td>8.92%</td>
<td>9.39%</td>
<td>10.25%</td>
<td>7.62%</td>
<td>9.78%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.37</td>
<td>0.35</td>
<td>0.43</td>
<td>0.27</td>
<td>0.81</td>
<td>0.32</td>
</tr>
<tr>
<td>avg. n countries</td>
<td>6.74</td>
<td>6.33</td>
<td>6.30</td>
<td>6.33</td>
<td>5.89</td>
<td></td>
</tr>
<tr>
<td>avg. ((I/K_{A,f,t-1} - I/K_{US,t}))</td>
<td>9.89</td>
<td>11.42</td>
<td>11.08</td>
<td>12.13</td>
<td>15.10</td>
<td></td>
</tr>
<tr>
<td>avg. ((i_t^{f-1} - i_t^{US}))</td>
<td>3.27</td>
<td>2.67</td>
<td>1.98</td>
<td>2.97</td>
<td>6.61</td>
<td></td>
</tr>
<tr>
<td>avg. ((FX_{t-1}))</td>
<td>-9.36</td>
<td>0.02</td>
<td>5.38</td>
<td>8.96</td>
<td>19.12</td>
<td></td>
</tr>
</tbody>
</table>

This table reports currency excess returns to portfolios sorted on investment rate and interest rate differentials and momentum. Each panel reports the average excess return (% per year), t-statistic, standard deviation (in percentages), Sharpe ratio, and descriptive statistics. Standard errors of the t-statistics are Newey-West (for 3 years of autocorrelation adjustment). The returns are from the perspective of a US investor. Every June of year \(t\), countries are classified into 5 portfolios based on the differentials of year \(t-1\). For the interest rate differentials, the percentiles used as cutoffs are based on the December \(t-1\) interest rate differential \((i_t^{f-1} - i_t^{US})\). Interest rates are the nominal December \(t-1\) monthly average of daily interest rates.
### Table 6
Realized Currency Excess Returns (FX), Collapsed Sequential and Intersection

#### Investment differential portfolios, Control for Interest rate Diff.
Sort first on $i_t$ diff, second on $I/K$ diff, and collapse across $I/K$ diff

<table>
<thead>
<tr>
<th>Panel A</th>
<th>Low $I/K$ diff</th>
<th>2</th>
<th>High $I/K$ diff</th>
<th>H-L (year%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FX Mean Excess return</td>
<td>2.58%</td>
<td>4.71%</td>
<td>4.78%</td>
<td>2.20%</td>
</tr>
<tr>
<td>$[t]$</td>
<td>1.73</td>
<td>3.07</td>
<td>3.21</td>
<td>2.83</td>
</tr>
<tr>
<td>avg. $(I/K_{A,f,t} - I/K_{US,t})$</td>
<td>5.36</td>
<td>11.61</td>
<td>21.68</td>
<td></td>
</tr>
<tr>
<td>avg. $(i_t^f - i_t^US)$</td>
<td>3.35</td>
<td>3.90</td>
<td>3.66</td>
<td></td>
</tr>
</tbody>
</table>

#### Interest rate differentials portfolios, Control for Investment Diff.
Sort first on $I/K$ diff, second on $i_t$ diff, and collapse across $i_t$ diff

<table>
<thead>
<tr>
<th>Panel B</th>
<th>Low $i_t$ diff</th>
<th>2</th>
<th>High $i_t$ diff</th>
<th>H-L (year%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FX Mean Excess return</td>
<td>1.63%</td>
<td>4.20%</td>
<td>6.81%</td>
<td>5.18%</td>
</tr>
<tr>
<td>$[t]$</td>
<td>1.07</td>
<td>2.49</td>
<td>4.44</td>
<td>3.67</td>
</tr>
<tr>
<td>avg. $(I/K_{A,f,t} - I/K_{US,t})$</td>
<td>11.57</td>
<td>12.29</td>
<td>12.16</td>
<td></td>
</tr>
<tr>
<td>avg. $(i_t^f - i_t^US)$</td>
<td>-0.31</td>
<td>3.01</td>
<td>9.83</td>
<td></td>
</tr>
</tbody>
</table>

#### Intersection of Investment and Interest Rate Differentials

<table>
<thead>
<tr>
<th>Panel C</th>
<th>Low $I/K$ diff</th>
<th>2</th>
<th>High $I/K$ diff</th>
<th>H-L (y%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low $i_t$ diff</td>
<td>FX Ex. Ret.</td>
<td>0.90%</td>
<td>2.05%</td>
<td>2.47%</td>
</tr>
<tr>
<td>$[t]$</td>
<td>0.64</td>
<td>1.02</td>
<td>1.32</td>
<td>1.22</td>
</tr>
<tr>
<td>2</td>
<td>FX Ex. Ret.</td>
<td>2.60%</td>
<td>4.58%</td>
<td>4.67%</td>
</tr>
<tr>
<td>$[t]$</td>
<td>1.46</td>
<td>2.66</td>
<td>2.21</td>
<td>1.16</td>
</tr>
<tr>
<td>High $i_t$ diff</td>
<td>FX Ex. Ret.</td>
<td>6.77%</td>
<td>4.22%</td>
<td>6.13%</td>
</tr>
<tr>
<td>$[t]$</td>
<td>3.53</td>
<td>1.53</td>
<td>4.24</td>
<td>-0.77</td>
</tr>
<tr>
<td>H-L per year</td>
<td>6.06%</td>
<td>1.58%</td>
<td>3.66%</td>
<td>5.23%</td>
</tr>
<tr>
<td>$[t]$</td>
<td>3.70</td>
<td>0.72</td>
<td>1.89</td>
<td>2.94</td>
</tr>
</tbody>
</table>

This table examines two aspects. One, the question of whether the excess return spreads of the investment differential and of the interest rate differential disappear after controlling for one another. To control for interest rate differentials, at every June of year $t$, countries are sorted into three portfolios based on year $t-1$ interest rate differential. Then within each of these portfolios, countries are sorted into 3 additional portfolios on the basis of year $t-1$ investment rate differentials. This gives a total of 9 sequentially sorted portfolios. Lastly, excess returns are averaged across the second dimension of sorting (i.e., investment differentials) to create 3 final portfolios. These portfolios allow some degree of control of the first dimension of sorting (interest rate differentials). A three-by-three sorting criteria is used to avoid having portfolios running too thin in their number of countries. Panel A reports investment differential portfolios that control for interest rate differentials, and Panel B reports interest rate differential portfolios that control for investment rate differentials. Two, Panel C reports portfolios created from the intersection.
## Table 7
Realized Currency Excess Returns (FX), Collapsed Sequential and Intersection

### Investment differential portfolios, Control for Momentum

Sort first on $FX_{t-1}$, second on $I/K$ diff, and collapse across $I/K$ diff.

<table>
<thead>
<tr>
<th>Panel A</th>
<th>Low $I/K$ diff</th>
<th>2</th>
<th>High $I/K$ diff</th>
<th>H-L (year%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FX Mean Excess return</td>
<td>2.74%</td>
<td>4.12%</td>
<td>4.78%</td>
<td>2.04%</td>
</tr>
<tr>
<td>$[t]$</td>
<td>1.76</td>
<td>2.53</td>
<td>3.40</td>
<td>3.42</td>
</tr>
<tr>
<td>$(I/K_{A,t-1} - I/K_{US,t-1})$</td>
<td>5.42</td>
<td>11.51</td>
<td>21.21</td>
<td></td>
</tr>
<tr>
<td>$(FX_{t-1})$</td>
<td>5.00</td>
<td>5.02</td>
<td>5.27</td>
<td></td>
</tr>
</tbody>
</table>

### Momentum differential portfolios, Control for $I/K$ Differentials

Sort first on $I/K$ diff, second on $FX_{t-1}$, and collapse across $FX_{t-1}$.

<table>
<thead>
<tr>
<th>Panel B</th>
<th>Low $FX_{t-1}$</th>
<th>2</th>
<th>High $FX_{t-1}$</th>
<th>H-L (year%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FX Mean Excess return</td>
<td>3.45%</td>
<td>3.26%</td>
<td>5.04%</td>
<td>1.59%</td>
</tr>
<tr>
<td>$[t]$</td>
<td>2.32</td>
<td>1.97</td>
<td>3.20</td>
<td>1.11</td>
</tr>
<tr>
<td>$(I/K_{A,t-1} - I/K_{US,t-1})$</td>
<td>11.71</td>
<td>11.90</td>
<td>12.04</td>
<td></td>
</tr>
<tr>
<td>$(FX_{t-1})$</td>
<td>-4.44</td>
<td>5.74</td>
<td>15.02</td>
<td></td>
</tr>
</tbody>
</table>

### Intersection of Investment Differentials and Momentum

<table>
<thead>
<tr>
<th>Panel C</th>
<th>Low $I/K$ diff</th>
<th>2</th>
<th>High $I/K$ diff</th>
<th>H-L (y%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low $FX_{t-1}$</td>
<td>FX Ex. Ret.</td>
<td>1.32%</td>
<td>3.70%</td>
<td>4.62%</td>
</tr>
<tr>
<td>$[t]$</td>
<td>0.88</td>
<td>1.91</td>
<td>2.51</td>
<td>2.03</td>
</tr>
<tr>
<td>2 FX Ex. Ret.</td>
<td>3.74%</td>
<td>2.50%</td>
<td>4.59%</td>
<td>0.55%</td>
</tr>
<tr>
<td>$[t]$</td>
<td>2.14</td>
<td>1.18</td>
<td>2.17</td>
<td>0.47</td>
</tr>
<tr>
<td>High $FX_{t-1}$</td>
<td>FX Ex. Ret.</td>
<td>4.73%</td>
<td>4.27%</td>
<td>5.86%</td>
</tr>
<tr>
<td>$[t]$</td>
<td>1.98</td>
<td>2.22</td>
<td>3.54</td>
<td>1.01</td>
</tr>
<tr>
<td>H-L per year</td>
<td>3.96%</td>
<td>0.57%</td>
<td>1.08%</td>
<td>4.55%</td>
</tr>
<tr>
<td>$[t]$</td>
<td>1.78</td>
<td>0.31</td>
<td>0.59</td>
<td>2.75</td>
</tr>
</tbody>
</table>

This table examines two aspects. One, the question of whether the excess return spreads of the investment differential and momentum in currency excess returns disappear after controlling for one another. To control for momentum, at every June of year $t$, countries are sorted into three portfolios based on year $t-1$ FX currency excess returns. Then within each of these portfolios, countries are sorted into 3 additional portfolios on the basis of year $t-1$ investment rate differentials. This gives a total of 9 sequentially sorted portfolios. Lastly, excess returns are averaged across the second dimension of sorting (i.e., investment differentials) to create 3 final portfolios. These portfolios allow some degree of control of the first dimension of sorting (momentum). A three-by-three sorting criteria is used to avoid having portfolios running too thin in their number of countries. Panel A reports investment differential portfolios that control for momentum, and Panel B reports momentum portfolios that control for investment rate differentials. Two, Panel C reports portfolios created from the intersection.
Table 8
Realized vs Implied Currency Excess Returns (FX), Investment Rate differentials

<table>
<thead>
<tr>
<th>Currency Excess Returns (FX), Investment Rate differentials</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
<th>H-L</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Realized Returns</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FX Mean Excess return</td>
<td>2.74%</td>
<td>2.87%</td>
<td>3.75%</td>
<td>4.38%</td>
<td>6.31%</td>
<td>3.57%</td>
</tr>
<tr>
<td>[t]</td>
<td>2.34</td>
<td>1.54</td>
<td>2.00</td>
<td>2.81</td>
<td>4.06</td>
<td>2.98</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>6.09%</td>
<td>9.72%</td>
<td>9.73%</td>
<td>8.09%</td>
<td>8.08%</td>
<td>5.92%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.45</td>
<td>0.30</td>
<td>0.39</td>
<td>0.54</td>
<td>0.78</td>
<td></td>
</tr>
<tr>
<td><strong>Implied Returns</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FX Mean Excess return</td>
<td>2.04%</td>
<td>4.28%</td>
<td>5.04%</td>
<td>5.24%</td>
<td>8.12%</td>
<td>6.08%</td>
</tr>
<tr>
<td>[t]</td>
<td>0.62</td>
<td>1.24</td>
<td>1.38</td>
<td>1.39</td>
<td>2.01</td>
<td>3.53</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>17.04%</td>
<td>17.89%</td>
<td>18.93%</td>
<td>19.66%</td>
<td>20.98%</td>
<td>9.53%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.12</td>
<td>0.24</td>
<td>0.27</td>
<td>0.27</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>pχ² (χ² joint signif.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>avg. e_{ft+1}/e_{ft} error</td>
<td>-0.73</td>
<td>1.03</td>
<td>1.36</td>
<td>0.96</td>
<td>1.83</td>
<td></td>
</tr>
<tr>
<td>[t]</td>
<td>-0.27</td>
<td>0.36</td>
<td>0.50</td>
<td>0.31</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>Total Φ/Y (sales)</td>
<td>20.37</td>
<td>22.65</td>
<td>24.54</td>
<td>25.08</td>
<td>34.29</td>
<td></td>
</tr>
<tr>
<td>Headq. Φ_{HQ}/Y (sales)</td>
<td>3.99</td>
<td>4.68</td>
<td>4.5</td>
<td>3.72</td>
<td>3.03</td>
<td></td>
</tr>
<tr>
<td>Affiliate Φ_{f}/Y (sales)</td>
<td>15.33</td>
<td>16.8</td>
<td>18.78</td>
<td>20.25</td>
<td>30.39</td>
<td></td>
</tr>
<tr>
<td>ROW Φ_{ROW}/Y (sales)</td>
<td>1.05</td>
<td>1.17</td>
<td>1.26</td>
<td>1.11</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>Headq. adj. slope Φ_{HQ}</td>
<td>27.71</td>
<td>32.00</td>
<td>29.90</td>
<td>25.32</td>
<td>20.59</td>
<td></td>
</tr>
<tr>
<td>Affiliate curvature η_{f}</td>
<td>1.011</td>
<td>1.016</td>
<td>1.014</td>
<td>1.012</td>
<td>1.011</td>
<td></td>
</tr>
</tbody>
</table>

This table reports model-implied currency excess returns for the 5 portfolios sorted on investment rate differentials. Panel A reports the average excess return (percent per year), t-statistic, standard deviation (in percentages) and Sharpe ratio. Panel B reports the implied excess returns from the GMM estimation of the investment-based model in section 4. t-statistics of the mean errors are Newey-West with adjustment for autocorrelation of 3 covariances. pχ² is the p-value of a χ² test of joint significance of model errors equal to zero. Mean errors are reported in percent per year. Panel C presents the economic implications of the capital adjustment costs of the GMM estimates, where the total adjustment cost to the representative firm is measured with respect to total output sales (Φ/Y).
Table 9
Realized vs Implied Currency Excess Returns (FX), Interest Rate differentials

<table>
<thead>
<tr>
<th>Currency Excess Returns (FX), Interest Rate differentials</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Realized Returns</strong></td>
</tr>
<tr>
<td>FX Mean Excess return</td>
</tr>
<tr>
<td>[t]</td>
</tr>
<tr>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
</tr>
<tr>
<td><strong>Implied Returns</strong></td>
</tr>
<tr>
<td>FX Mean Excess return</td>
</tr>
<tr>
<td>[t]</td>
</tr>
<tr>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
</tr>
<tr>
<td>$p_{\chi^2}$ (χ² joint signif.)</td>
</tr>
<tr>
<td>avg. $e_{f,t+1}/e_{f,t}$</td>
</tr>
<tr>
<td>[t]</td>
</tr>
<tr>
<td>Total $\Phi/Y$ (sales)</td>
</tr>
<tr>
<td>Headq. $\Phi_{HQ}/Y$ (sales)</td>
</tr>
<tr>
<td>Affiliate $\Phi_{f}/Y$ (sales)</td>
</tr>
<tr>
<td>ROW $\Phi_{ROW}/Y$ (sales)</td>
</tr>
<tr>
<td>Headq. adj. slope $\phi_{HQ}$</td>
</tr>
<tr>
<td>ROW adj. slope $\phi_{ROW}$</td>
</tr>
<tr>
<td>Affiliate curvature $\eta_{f}$</td>
</tr>
</tbody>
</table>

This table reports model-implied currency excess returns for the 5 portfolios sorted on interest rate differentials. Panel A reports the average excess return (percent per year), t-statistic, standard deviation (in percentages) and Sharpe ratio. Panel B reports the implied excess returns from the GMM estimation of the investment-based model in section 4. t-statistics of the mean errors are Newey-West with adjustment for autocorrelation of 3 covariances. $p_{\chi^2}$ is the p-value of a χ² test of joint significance of model errors equal to zero. Mean errors are reported in percent per year. Panel C presents the economic implications of the capital adjustment costs of the GMM estimates, where the total adjustment cost to the representative firm is measured with respect to total output sales ($\Phi/Y$). Moreover, implied total adjustment costs are presented separately for headquarters (HQ), affiliate in country $j$, and the rest of the world (ROW). Lastly, mean capital adjustment cost parameter estimates of the slope $\phi$ and curvature $\eta$ are also presented.
This figure illustrates the fit of the model by plotting the predicted currency excess returns of both investment and interest rate differential portfolios against the realized excess returns. If the model’s fit is perfect, all the scattered points should lie exactly on the 45-degree line. The model predicted excess returns are from Tables 8 and 9.
### Table 10
**Realized vs Implied Currency Excess Returns (FX), Collapsed Sequential Sorts**

#### Panel A

**Investment differential portfolios, Control for Interest rate Diff.**

Sort first on $i_t$ diff, second on $I/K$ diff, and collapse across $I/K$ diff

<table>
<thead>
<tr>
<th>Realized Returns</th>
<th>Low $I/K$ diff</th>
<th>2</th>
<th>High $I/K$ diff</th>
<th>H-L (year%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FX Mean Ex. return</td>
<td>2.58%</td>
<td>4.71%</td>
<td>4.78%</td>
<td>2.20%</td>
</tr>
<tr>
<td>[$t$]</td>
<td>1.73</td>
<td>3.07</td>
<td>3.21</td>
<td>2.83</td>
</tr>
<tr>
<td>Implied Returns</td>
<td>Low $I/K$ diff</td>
<td>2</td>
<td>High $I/K$ diff</td>
<td>H-L (year%)</td>
</tr>
<tr>
<td>FX Mean Ex. return</td>
<td>3.29%</td>
<td>5.19%</td>
<td>6.67%</td>
<td>3.39%</td>
</tr>
<tr>
<td>[$t$]</td>
<td>1.01</td>
<td>1.51</td>
<td>1.66</td>
<td>2.54</td>
</tr>
<tr>
<td>$p_{\chi^2}$ (joint signif.)</td>
<td>0.3189</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(I/K_{A,t} - I/K_{US,t})$</td>
<td>5.36</td>
<td>11.61</td>
<td>21.68</td>
<td></td>
</tr>
<tr>
<td>$(i_t^I - i_t^US)$</td>
<td>3.35</td>
<td>3.90</td>
<td>3.66</td>
<td></td>
</tr>
</tbody>
</table>

#### Panel B

**Interest rate differentials portfolios, Control for Investment Diff.**

Sort first on $I/K$ diff, second on $i_t$ diff, and collapse across $i_t$ diff

<table>
<thead>
<tr>
<th>Realized Returns</th>
<th>Low $i_t$ diff</th>
<th>2</th>
<th>High $i_t$ diff</th>
<th>H-L (year%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FX Mean Ex. return</td>
<td>1.63%</td>
<td>4.20%</td>
<td>6.81%</td>
<td>5.18%</td>
</tr>
<tr>
<td>[$t$]</td>
<td>1.07</td>
<td>2.49</td>
<td>4.44</td>
<td>3.67</td>
</tr>
<tr>
<td>Implied Returns</td>
<td>Low $i_t$ diff</td>
<td>2</td>
<td>High $i_t$ diff</td>
<td>H-L (year%)</td>
</tr>
<tr>
<td>FX Mean Ex. return</td>
<td>2.19%</td>
<td>5.04%</td>
<td>8.45%</td>
<td>6.26%</td>
</tr>
<tr>
<td>[$t$]</td>
<td>0.73</td>
<td>1.27</td>
<td>2.07</td>
<td>2.57</td>
</tr>
<tr>
<td>$p_{\chi^2}$ (joint signif.)</td>
<td>0.537</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(I/K_{A,t} - I/K_{US,t})$</td>
<td>11.57</td>
<td>12.29</td>
<td>12.16</td>
<td></td>
</tr>
<tr>
<td>$(i_t^I - i_t^US)$</td>
<td>-0.31</td>
<td>3.01</td>
<td>9.83</td>
<td></td>
</tr>
</tbody>
</table>

This table presents the model-implied currency excess returns for sequentially sorted portfolios that control for one dimension at a time. The descriptive text in Table 6 provides details on these portfolios. Panel A reports investment differential portfolios that control for interest rate differentials, and Panel B reports interest rate differential portfolios that control for investment rate differentials. Standard errors of t-statistics are Newey-West with adjustment for autocorrelation of 3 covariances. $p_{\chi^2}$ is the p-value of a $\chi^2$ test of joint significance of model errors equal to zero.
This figure illustrates the fit of the model by plotting the predicted currency excess returns of the portfolios that control for one dimension at a time against the realized excess returns. If the model’s fit is perfect, all the scattered points should lie exactly on the 45-degree line. The model predicted excess returns are from Table 10.
### Realized vs Implied Currency Excess Returns (FX), Intersected Sorts

<table>
<thead>
<tr>
<th>Intersection of Investment and Interest Rate Differentials</th>
<th>Realized Returns</th>
<th>Low $I/K$ diff</th>
<th>2</th>
<th>High $I/K$ diff</th>
<th>H-L %y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low $i_t$ diff</td>
<td>FX Ex. Ret.</td>
<td>0.90%</td>
<td>2.05%</td>
<td>2.47%</td>
<td>1.57%</td>
</tr>
<tr>
<td></td>
<td>$t$</td>
<td>0.64</td>
<td>1.02</td>
<td>1.32</td>
<td>1.22</td>
</tr>
<tr>
<td>2</td>
<td>FX Ex. Ret.</td>
<td>2.60%</td>
<td>4.58%</td>
<td>4.67%</td>
<td>2.30%</td>
</tr>
<tr>
<td></td>
<td>$t$</td>
<td>1.46</td>
<td>2.66</td>
<td>2.21</td>
<td>1.16</td>
</tr>
<tr>
<td>High $i_t$ diff</td>
<td>FX Ex. Ret.</td>
<td>6.77%</td>
<td>4.22%</td>
<td>6.13%</td>
<td>-1.40%</td>
</tr>
<tr>
<td></td>
<td>$t$</td>
<td>3.53</td>
<td>1.53</td>
<td>4.24</td>
<td>-0.77</td>
</tr>
<tr>
<td>H-L per year</td>
<td></td>
<td>6.06%</td>
<td>1.58%</td>
<td>3.66%</td>
<td>5.23%</td>
</tr>
<tr>
<td></td>
<td>$t[τ]$</td>
<td>3.70</td>
<td>0.72</td>
<td>1.89</td>
<td>2.94</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Implied Returns</th>
<th>Low $I/K$ diff</th>
<th>2</th>
<th>High $I/K$ diff</th>
<th>H-L %y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low $i_t$ diff</td>
<td>FX Ex. Ret.</td>
<td>1.44%</td>
<td>2.04%</td>
<td>2.43%</td>
</tr>
<tr>
<td></td>
<td>$t$</td>
<td>0.45</td>
<td>0.68</td>
<td>0.66</td>
</tr>
<tr>
<td>2</td>
<td>FX Ex. Ret.</td>
<td>3.86%</td>
<td>4.37%</td>
<td>7.67%</td>
</tr>
<tr>
<td></td>
<td>$t$</td>
<td>1.06</td>
<td>1.15</td>
<td>1.51</td>
</tr>
<tr>
<td>High $i_t$ diff</td>
<td>FX Ex. Ret.</td>
<td>6.59%</td>
<td>4.90%</td>
<td>8.35%</td>
</tr>
<tr>
<td></td>
<td>$t$</td>
<td>1.34</td>
<td>1.09</td>
<td>1.89</td>
</tr>
<tr>
<td>H-L per year</td>
<td></td>
<td>5.94%</td>
<td>2.44%</td>
<td>5.91%</td>
</tr>
<tr>
<td></td>
<td>$t$</td>
<td>1.92</td>
<td>0.67</td>
<td>2.22</td>
</tr>
</tbody>
</table>

$p_{χ^2}$ (joint sig.)                                     0.78

This table reports the model-implied currency excess returns for portfolios created from the intersection of the two main sorting dimensions (interest and investment rate differentials). The descriptive text in Table 6 provides details on these portfolios. Standard errors of $t$-statistics are Newey-West with adjustment for autocorrelation of 3 covariances. $p_{χ^2}$ is the p-value of a $χ^2$ test of joint significance of model errors equal to zero. The average excess returns and spreads to these portfolios are presented in Panel A (realized) and Panel B (implied).
Figure 6 Sequentially Sorted and Intersected Portfolios

The figures illustrate the fit of the model by plotting the predicted currency excess returns of sequentially and intersected portfolios against the realized excess returns. If the model’s fit is perfect, all the scattered points should lie exactly on the 45-degree line. The model predicted excess returns are from Tables 11, 12, and 13.
Table 12
Realized vs Implied Currency Excess Returns (FX), Sequential Sorts 1

<table>
<thead>
<tr>
<th>Sort first on $i_t$ diff and second on $I/K$ diff</th>
<th>Realized Returns</th>
<th>Low $i_t$ diff</th>
<th>2</th>
<th>High $i_t$ diff</th>
<th>H-L</th>
</tr>
</thead>
<tbody>
<tr>
<td>FX Ex. Ret. Low $I/K$ diff</td>
<td>-0.02%</td>
<td>2.75%</td>
<td>4.88%</td>
<td>4.76%</td>
<td></td>
</tr>
<tr>
<td>t</td>
<td>-0.03</td>
<td>2.44</td>
<td>3.01</td>
<td>2.93</td>
<td></td>
</tr>
<tr>
<td>FX Ex. Ret. 2</td>
<td>1.83%</td>
<td>3.93%</td>
<td>7.40%</td>
<td>5.75%</td>
<td></td>
</tr>
<tr>
<td>t</td>
<td>1.71</td>
<td>4.10</td>
<td>4.87</td>
<td>2.96</td>
<td></td>
</tr>
<tr>
<td>FX Ex. Ret. High $I/K$ diff</td>
<td>2.09%</td>
<td>4.80%</td>
<td>6.56%</td>
<td>4.98%</td>
<td></td>
</tr>
<tr>
<td>t</td>
<td>1.84</td>
<td>3.60</td>
<td>5.31</td>
<td>2.63</td>
<td></td>
</tr>
<tr>
<td>H-L per year</td>
<td>2.12%</td>
<td>2.34%</td>
<td>2.34%</td>
<td>7.10%</td>
<td></td>
</tr>
<tr>
<td>t-value</td>
<td>1.82</td>
<td>1.60</td>
<td>1.07</td>
<td>3.69</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Implied Returns</th>
<th>Low $i_t$ diff</th>
<th>2</th>
<th>High $i_t$ diff</th>
<th>H-L</th>
</tr>
</thead>
<tbody>
<tr>
<td>FX Ex. Ret. Low $I/K$ diff</td>
<td>0.56%</td>
<td>2.81%</td>
<td>6.33%</td>
<td>6.24%</td>
</tr>
<tr>
<td>t</td>
<td>0.31</td>
<td>1.37</td>
<td>2.40</td>
<td>2.16</td>
</tr>
<tr>
<td>FX Ex. Ret. 2</td>
<td>2.21%</td>
<td>4.16%</td>
<td>9.12%</td>
<td>6.35%</td>
</tr>
<tr>
<td>t</td>
<td>1.26</td>
<td>2.04</td>
<td>3.71</td>
<td>1.87</td>
</tr>
<tr>
<td>FX Ex. Ret. High $I/K$ diff</td>
<td>3.42%</td>
<td>7.47%</td>
<td>7.87%</td>
<td>5.90%</td>
</tr>
<tr>
<td>t</td>
<td>1.53</td>
<td>2.41</td>
<td>2.85</td>
<td>2.41</td>
</tr>
<tr>
<td>H-L per year</td>
<td>2.76%</td>
<td>5.38%</td>
<td>2.42%</td>
<td>8.66%</td>
</tr>
<tr>
<td>$[t]$</td>
<td>1.53</td>
<td>2.41</td>
<td>2.85</td>
<td>2.41</td>
</tr>
<tr>
<td>$p_{\chi^2}$ (test of joint signif.)</td>
<td></td>
<td></td>
<td></td>
<td>0.99</td>
</tr>
</tbody>
</table>

This table reports the model-implied currency excess returns for portfolios sequentially sorted first on interest rate differential and second on investment rate differential. The descriptive text in Table 6 provides details on these portfolios. Standard errors of t-statistics are Newey-West with adjustment for autocorrelation of 3 covariances. $p_{\chi^2}$ is the p-value of a $\chi^2$ test of joint significance of model errors equal to zero. The average excess returns and spreads to these portfolios are presented in Panel A (realized) and Panel B (implied).
### Table 13
Realized vs Implied Currency Excess Returns (FX), Sequential Sorts 2

<table>
<thead>
<tr>
<th>Sort first on $I/K$ diff and second on $i_t$ diff</th>
<th>Realized Returns</th>
<th>Low $I/K$ diff</th>
<th>2</th>
<th>High $I/K$ diff</th>
<th>H-L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low $i_t$ diff</td>
<td>FX Excess Mean</td>
<td>0.59%</td>
<td>1.25%</td>
<td>2.15%</td>
<td>1.77%</td>
</tr>
<tr>
<td></td>
<td>[t]</td>
<td>0.69</td>
<td>1.21</td>
<td>2.06</td>
<td>1.27</td>
</tr>
<tr>
<td>2</td>
<td>FX Excess Mean</td>
<td>2.24%</td>
<td>3.92%</td>
<td>6.04%</td>
<td>4.30%</td>
</tr>
<tr>
<td></td>
<td>[t]</td>
<td>1.66</td>
<td>3.96</td>
<td>5.43</td>
<td>2.64</td>
</tr>
<tr>
<td>High $i_t$ diff</td>
<td>FX Excess Mean</td>
<td>6.88%</td>
<td>4.82%</td>
<td>7.52%</td>
<td>1.24%</td>
</tr>
<tr>
<td></td>
<td>[t]</td>
<td>4.84</td>
<td>2.79</td>
<td>4.42</td>
<td>0.54</td>
</tr>
<tr>
<td>H-L per year</td>
<td></td>
<td>6.53%</td>
<td>3.13%</td>
<td>6.01%</td>
<td>7.78%</td>
</tr>
<tr>
<td>t-value</td>
<td></td>
<td>4.08</td>
<td>1.75</td>
<td>2.38</td>
<td>3.39</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Implied Returns</th>
<th>Low $I/K$ diff</th>
<th>2</th>
<th>High $I/K$ diff</th>
<th>H-L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low $i_t$ diff</td>
<td>FX Excess Mean</td>
<td>1.76%</td>
<td>0.98%</td>
<td>3.28%</td>
</tr>
<tr>
<td></td>
<td>[t]</td>
<td>0.99</td>
<td>0.57</td>
<td>1.57</td>
</tr>
<tr>
<td>2</td>
<td>FX Excess Mean</td>
<td>2.86%</td>
<td>3.45%</td>
<td>8.85%</td>
</tr>
<tr>
<td></td>
<td>[t]</td>
<td>1.22</td>
<td>1.76</td>
<td>3.32</td>
</tr>
<tr>
<td>High $i_t$ diff</td>
<td>FX Excess Mean</td>
<td>9.24%</td>
<td>4.45%</td>
<td>10.80%</td>
</tr>
<tr>
<td></td>
<td>[t]</td>
<td>3.24</td>
<td>1.83</td>
<td>3.45</td>
</tr>
<tr>
<td>H-L per year</td>
<td></td>
<td>7.95%</td>
<td>3.49%</td>
<td>7.45%</td>
</tr>
<tr>
<td></td>
<td>[t]</td>
<td>2.61</td>
<td>1.21</td>
<td>2.29</td>
</tr>
<tr>
<td>$p_{\chi^2}$ (test of joint signif.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This table reports the model-implied currency excess returns for portfolios sequentially sorted first on investment rate differential and second on interest rate differential. The descriptive text in Table [6](#) provides details on these portfolios. Standard errors of t-statistics are Newey-West with adjustment for autocorrelation of 3 covariances. $p_{\chi^2}$ is the p-value of a $\chi^2$ test of joint significance of model errors equal to zero. The average excess returns and spreads to these portfolios are presented in Panel A (realized) and Panel B (implied).
Table 14
Expected Return Errors and Statistics from Comparative Static Experiments

<table>
<thead>
<tr>
<th>Interest Rate differentials</th>
<th>Currency portfolios (FX)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low $i_t$ diff</td>
</tr>
<tr>
<td>Panel A Benchmark Model Results</td>
<td></td>
</tr>
<tr>
<td>$e_{f,t+1}/e_{f,t}$ error</td>
<td>0.53</td>
</tr>
<tr>
<td>Panel B No cross-affiliate variation in $I/K$: $I/K_{A,f,t}^h = I/K_{A,j,t}^h$</td>
<td></td>
</tr>
<tr>
<td>$e_{f,t+1}/e_{f,t}$ error</td>
<td>7.83</td>
</tr>
<tr>
<td>Standard Dev</td>
<td>24.73</td>
</tr>
<tr>
<td>Panel C No cross-affiliate variation in $I/K$: $I/K_{A,f,t}^h = I/K_{US,t}^h$</td>
<td></td>
</tr>
<tr>
<td>$e_{f,t+1}/e_{f,t}$ error</td>
<td>6.14</td>
</tr>
<tr>
<td>Standard Dev</td>
<td>17.04</td>
</tr>
<tr>
<td>Panel D No cross-affiliate variation in $Y/K$: $Y/K_{A,f,t}^h = Y/K_{A,j,t}^h$</td>
<td></td>
</tr>
<tr>
<td>$e_{f,t+1}/e_{f,t}$ error</td>
<td>1.54</td>
</tr>
</tbody>
</table>

This table presents comparative static experiments that shed light on the key components of the investment returns driving the GMM model estimation results. Cross-affiliate variations in the main components of investment returns abroad (output-to-capital $Y/K$ and investment-to-capital $I/K$) can play quantitatively different roles in accounting for cross-sectional currency excess returns. First, every year the cross-affiliate variation in investment rate is eliminated by setting each individual affiliate investment rate equal to either a) the cross-sectional average of all affiliate investment rates, i.e., $I/K_{A,f,t}^h = I/K_{A,j,t}^h$, or b) the US domestic investment rate, i.e., $I/K_{A,f,t}^h = I/K_{US,t}^h$. Second, use the parameter estimates of the benchmark model to reconstruct implied portfolio currency excess returns. Similarly for $Y/K$. Panels A through D report comparative static results for interest rate differential portfolios.
Chapter 2: The Finance-Uncertainty Multiplier

We show theoretically and empirically how real and financial frictions amplify the impact of uncertainty shocks on firms’ investment, employment, debt (term structure of debt growth), and cash holding. We start by building a model with real and financial frictions, alongside uncertainty shocks, and show how adding financial frictions to the model roughly doubles the negative impact of uncertainty shocks on investment and hiring. The reason is higher uncertainty induces the standard negative real-options effects on the demand for capital and labor, but also leads firms to hoard cash and cut debt to hedge against future shocks, further reducing investment and hiring. We then test the model using a panel of US firms and a novel instrumentation strategy for uncertainty exploiting differential firm exposure to exchange rate and factor price volatility. We find that higher uncertainty reduces real investment and hiring, while also leading firms to increase cash holdings by cutting debt, dividends and stock-buy backs, and these effects are strongest in periods of higher financial frictions and for the most financially constrained firms. This highlights why in periods with greater financial frictions – like during the global-financial-crisis – uncertainty can be particularly damaging.

1This is a co-authored paper with Nick Bloom (Stanford University, Department of Economics) and Xiaoji Lin (The Ohio State University, Department of Finance, Fisher College of Business). I am grateful to them for their encouragment, support, and learning experience.
1 Introduction

We study theoretically and empirically the impact of uncertainty shocks on firms’ real and financial activity including investment, employment, debt, payout, and cash holding. We start by building a model with real and financial frictions, alongside uncertainty shocks, and show how adding financial frictions to the model almost doubles the negative impact of uncertainty shocks on investment and hiring. The reason is higher uncertainty induces the standard negative real-options effects on the demand for capital and labor, but also leads firms to hoard cash and cut debt to hedge against future shocks, further reducing investment and hiring. Furthermore, firms cut short-term debt more than long-term debt, causing a negative relation between uncertainty and the term structure of debt issuance.

We then test this model of real and financial frictions using a panel of US firms and a novel instrumentation strategy for uncertainty exploiting differential firm exposure to exchange rate, factor price and policy uncertainty. We find that higher uncertainty significantly reduces real investment and hiring, while also leading firms to take a more cautious financial position by increasing cash holdings and cutting debt, dividends and stock-buy backs. These findings highlight not only the importance of financial frictions for amplifying the impact of uncertainty shocks, but also how in periods with greater financial frictions – like during the global-financial-crisis – how uncertainty can be particularly damaging.

To understand the effects of uncertainty shocks on real activity and financial flows, we build a dynamic structural model including both real and financial frictions that amplify the impact of uncertainty shocks. It features a large cross section of firms where heterogeneity is driven by firm-specific productivity. Uncertainty is time-varying, so the model includes shocks to both the level of firms’ productivity (the first moment) and its conditional volatility (the second moment). On the real side, investment incurs fixed cost and is partially irreversible (e.g. Cooper and Haltiwanger
while employment is frictionless adjusted. On the financing side, building on the existing literature on capital structure models (e.g., Hennessy and Whited 2005), firms issue both short-term and long-term debt to finance investment, both of which are costly to issue. Long-term debt has a longer maturity than short-term debt and pays out a periodic coupon. The adjustment costs on short-term and long-term debt captures the cost of liquidity risk on short-term debt (e.g., Diamond 1991) and the agency costs (asset substitution) associated with long-term debt (e.g., Myers 1977, Barclay and Smith 1995, Hoven Stohs and Mauer 1996). In addition, both short-term and long-term debt are subject to collateral constraints which limit firms’ debt capacity. Firms also issue external equity in addition to debt to finance investment. External equity is assumed to be costly to capture equity flotation cost and information cost on equity issuance. Firms make investment, long-term debt, and short-term debt issuance/cash saving decisions to maximize the market value of equity. In the model, firms face the trade-off between the liquidation and issuance cost and the tax benefit of short-term debt and long-term debt in presence of time-varying uncertainty. Additionally, firms also manage the trade-off between total debt, equity, and cash in financing capital investment.

The model highlights the endogenous interactions between uncertainty shocks, investment, short-term and long-term debt issuance, and cash saving decisions. Intuitively, when uncertainty is high, firms reduce capital investment and employment, a standard prediction implied by investment models with fixed cost and partial irreversibility on investment. Furthermore, when financial frictions exist firms also want to build up cash to hedge against future potential shocks. This provide an additional impetus to cut investment and equity payout to cut cash outflows. Firms also want to reduce debt to increase their flexibility to respond to shocks, so lower debt levels, particularly short-term debt which is the most restrictive.

As a broad motivation for the model Figure 7 shows the correlations of the
quarterly VIX index which proxies for aggregate uncertainty and the aggregate real and financial variables. The top two panels show that times of high aggregate real investment rate and employment growth are negatively correlated with the low VIX. The middle two panels show that the total debt (sum of the short-term and long-term debt) growth and the term structure of the debt growth (short-term debt growth to long-term debt growth ratio) are negatively related with the VIX, implying firms cut the short-term debt more than the long-term debt when uncertainty shock is high. The bottom two panels show that cash holding is positively while dividend payout and equity repurchase are negatively related to the VIX.

We also test these predictions on a panel of US firms. One issue with estimating investment-uncertainty regressions is endogeneity, highlighted by theoretical and empirical evidence arguing for reverse causality from growth to uncertainty\(^1\). Our view is that both channels likely operate - uncertainty both reduces growth (an impact mechanism) and lower growth increases uncertainty (an amplification mechanism). Hence, the approach in this paper is to start by investigating the causal channel of uncertainty on growth, leaving the reverse causality amplification mechanism to explore in other papers.

To investigate the causal impact of uncertainty we develop an instrumentation strategy that exploits firm’s differential exposure to energy and currency uncertainty (as measured by at-the-money forward call options for oil and 7 widely traded currencies) as well as policy uncertainty (proxied by the industry level exposure measure from Baker, Bloom and Davis (2016). This identification strategy works well delivering a first-stage F-statistic of between 20 to 40 and passing the Hansen over-identification test. Our second stage results reveal that higher uncertainty significantly reduces real activity (proxied by investment, hiring, input and sales

\(^1\)See, for example, Van Nieuwerburgh and Veldkamp 2006; Bachman and Moscarini 2012; Pastor and Veronesi 2012, Orlik and Veldkamp 2015, Berger, Dew-Becker and Giglio 2016, and Fajgelbaum et al. 2016, for models and empirics on reverse causality with uncertainty and growth.
growth), while also leading firms to take a more cautious financial position (by increasing cash holdings, and cutting debt, dividends and stock-buy backs). Furthermore, high uncertainty also cause firms to adjust the term structure of debt growth towards the long-term debt, i.e., firms cut the long-term debt than the short-term debt facing high uncertainty shocks. These results are consistent with the model predictions.

**Related literature** Our paper relates to the investment literature that studies the impact of real frictions on investment dynamics (e.g., Doms and Dunne (1998), Davis and Haltiwanger (1992), Caballero, Engel, and Haltiwanger (1995), Cooper and Haltiwanger (2006)). In particular, we are complementary to Abel and Eberly (1996) who show that costly reversibility is important to explain the real investment dynamics. We differ in that we show that time-vary uncertainty and financial frictions also play important roles in capturing firm-level real investment activity in addition to real frictions.

This paper also contributes to the growing literature that studies the impact of uncertainty shocks on firms real decisions and business cycle fluctuations. For example, Leahy and Whited (1996), Gusio and Parigi (1999) and Bloom, Bond, and Van Reenen (2007) all provide evidence suggesting that firm-level uncertainty shocks reduces firms’ investment and labor hiring, while Ramey and Ramey (1995), Bloom (2009), Fernandez-Villaverde et al. (2011) provide evidence suggesting macro uncertainty shocks appear to drive business cycle fluctuations.\(^2\)

Our empirical analysis also relates to the empirical corporate finance literature that studies the determinants of capital structure choice, e.g., Rajan and Zingalas (1995), Welch (2004), etc. We are complementary to these studies by showing that uncertainty shocks have significant impact on firms’ financial flows. Our paper is closely related to Chen et. al. (2014) who also look at the impact of uncertainty shocks

\(^2\)A more extensive literature review is contained in survey paper Bloom (2014).
on firms’ financing decisions. We differ in that empirically we use the instrumentation approach to study the causal effect of past uncertainty on the future changes of firms’ financing flows and also build a structural model to interpret our results, while Chen et. al. (2014) focus on the contemporaneous relations between realized volatility and capital structure.

This paper also relates to the literature that examines the impact of financial frictions on corporate investment and financial flows. Hennessy and Whited (2005) study firms’ leverage choice and investment decisions in the presence of financial distress costs and equity flotation costs. Bolton, Chen, and Wang (2013) study firms’ investment, financing, and cash management decisions in a dynamic q-theoretic framework in which, external financing conditions are stochastic. Rampini and Viswanathan (2013) examine the relationship between investment, capital structure, leasing and risk management and show that collateral is a key determinant of firms’ capital structure. Our analysis is complementary to these studies in that we focus on the impact of uncertainty shocks on firms’ capital structure choice, a dimension that is not studied in these papers.

On the macro side we have similarities to Alessandri and Mumtaz (2016) and Lhuissier and Tripier (2016) who show in VAR estimates a strong interaction effect of financial constraints on uncertainty. More generally Arellano, Bai and Kehoe (2012) and Christiano, Motto and Rostagno (2014) both focus on the macro impact of changes in micro-uncertainty through the lens of financial constraints via costs of default. The former does this via the financing costs needed by entrepreneurs to hire labor in advance, while the latter focuses on the capital accelerator channel of entrepreneurs requiring external finance for capital investment.

The papers closest to ours are: Gilchrist, Sim and Zakrajsek (2014) who examine the interaction between uncertainty and financial constraints, focusing on the importance of credit conditions in channeling the impact of uncertainty shocks;
and Chen (2016) who investigates how firms manage their joint financing and investment when aggregate uncertainty is time-varying, noting positive interactions. We differ in that we study the impact of firms’ uncertainty on the debt and the term-structure choice in addition to liquidity management, we empirically use the instrumentation approach to identify the causal effect of uncertainty on real and financial additivity, and focus on the multiplicative effect of financial constraints and uncertainty, particularly during recessions.

The rest of the paper is laid out as follows. In section 2 we write down the model. In section 3 we present the main quantitative results of the model. In section 4 we describe the international data that we use in the paper. In section 5 we present the empirical findings on uncertainty shocks and financial flows. Section 6 concludes.

2 Model

The model features a continuum of heterogeneous firms facing uncertainty shocks and financial frictions. Firms take on both short-term debt (or save in cash) and long-term debt and trade off the tax benefit of debt and liquidation cost. Firms choose optimal levels of physical capital investment, labor, and short-term debt (cash) and long-term debt each period to maximize the market value of equity.

2.1 Technology

Firms use physical capital ($K_t$) and labor ($H_t$) to produce a homogeneous good ($Y_t$). To save on notation, we omit firm index whenever possible. The production function is given by

$$Y_t = \tilde{Z}_t K_t^\alpha H_t^{1-\alpha},$$

in which $\tilde{Z}_t$ is firms’ productivity. The firm faces an isoelastic demand curve with elasticity ($\varepsilon$),
\[ Q_t = XP_t^{-\varepsilon}, \]

where \( X \) is a demand shifter. These can be combined into a revenue function

\[ R(Z_t, X, K_t, H_t) = Z_t^{1-\varepsilon} X^{1/\varepsilon} K_t^{\alpha(1-1/\varepsilon)} (H_t)^{(1-\alpha)(1-1/\varepsilon)}. \]

For analytical tractability we define \( a = \alpha (1 - 1/\varepsilon) \) and \( b = (1 - \alpha) (1 - 1/\varepsilon) \), and substitute \( Z_t^{1-a-b} = Z_t^{1-1/\varepsilon} X^{1/\varepsilon} \). With these redefinitions we have

\[ S(Z, K, H) = Z_t^{1-a-b} K_t^a H_t^b. \]

Wages are normalized to 1 denoted as \( \bar{W} \). Given employment is flexible, we can obtain optimal labor.\(^3\) Note that labor can be pre-optimized out even with financial frictions which will be discussed later.

Productivity is defined as a combination of a firm and aggregate productivity process, both following an AR(1) process

\[ z_{t+1} = \begin{cases} z_{t+1}^f + z_{t+1}^m & (2) \\ z_{t+1}^f = \bar{z}^f (1 - \rho_z) + \rho_z z_{t+1}^f + \sigma_z^f \varepsilon_{t+1}^f & (3) \\ z_{t+1}^m = \bar{z}^m (1 - \rho_z) + \rho_z z_{t+1}^m + \sigma_z^m \varepsilon_{t+1}^m & (4) \end{cases} \]

in which (dropping the firm and macro superscript for simplicity) \( z_{t+1} = \log(Z_{t+1}) \), \( \varepsilon_{t+1} \) is an i.i.d. standard normal shock (drawn independently across firms and at the macro level), and \( \bar{z} \), \( \rho_z \), and \( \sigma_{t}^{f,m} \) are the mean, autocorrelation, and conditional volatility of the productivity processes.

The firm and macro stochastic volatility processes are both assumed for simplicity

\(^3\)Pre-optimized labor is given by \((\frac{b}{\bar{W}} Z_t^{1-a-b} K_t^a)^{1-\varepsilon} \).
to follow the same two-point Markov chains
\[
\sigma_t^{\{f,m\}} \in \left\{ \sigma_L^{\{f,m\}}, \sigma_H^{\{f,m\}} \right\}, \text{ where } \Pr \left( \sigma_{t+1}^{\{f,m\}} = \sigma_j^{\{f,m\}} | \sigma_t^{\{f,m\}} = \sigma_k^{\{f,m\}} \right) = \pi_{k,j}^\sigma, \quad (5)
\]
where we assume that the firm and macro volatility processes are uncorrelated.

Physical capital accumulation is given by
\[
K_{t+1} = (1 - \delta)K_t + I_t, \quad (6)
\]
where \(I_t\) represents investment and \(\delta\) denotes the capital depreciation rate.

We assume that capital investment is costly reversible and entails nonconvex adjustment costs, denoted as \(G_t\), which are given by:
\[
G_t = I_t^+ + \left( 1 - c_k^P \right) I_t^- + b_k K_t 1_{\{I_t \neq 0\}} \quad (7)
\]
in which \(c_k^P, b_k > 0\) are constants. \(I_t^+\) and \(I_t^-\) are positive and negative investment, respectively. The capital adjustment costs include planning and installation costs, learning to use the new equipment, or the fact that production is temporarily interrupted. The nonconvex costs \(b_k K_t 1_{\{I_t \neq 0\}}\) capture the costs of adjusting capital that are independent of the size of the investment. The costly reversibility can arise because of resale losses due to transaction costs or the market for lemons phenomenon. The resale loss of capital is labelled \(c_k^P\) and is denominated as a fraction of the relative purchase price of capital.

2.2 Long-term and short-term debt and collateral constraint

Firms use equity as well as short-term (one period) debt and long-term (multi-period) debt to finance investment. At the beginning of time \(t\), firms can issue an amount of short-term debt, denoted as \(B_t^S\), which must be repaid at the beginning of period
Following Hackbarth, Miao, and Morellec (2006) we model long-term debt with finite maturity via sinking funds provisions. We denote by $B^L_t$ the book value of long-term debt that firms have outstanding at time $t$. Long-term corporate bonds pay a fixed coupon $c$ in every period and a fraction $\theta$ is paid back each period (after payment of the coupon) with $0 < \theta < 1$. The average maturity of these long-term bonds then corresponds to $1/\theta$ periods. Denoting new long-term bond issuance by $N_t$, the amount of long-term corporate bonds evolves as

$$B^L_{t+1} = (1 - \theta) B^L_t + N_t$$ \hfill (8)

The firm’s ability to borrow is bounded by the limited enforceability as firms can default on their obligations. Following Hennessy and Whited (2005), we assume that the only asset available for liquidation is the physical capital $K_t$. In particular, we require that the respective liquidation values of capital is greater than or equal to the short and the long-term bonds, and that the sum of the short-term and long-term bonds cannot exceed the liquidation value of capital either. It follows that the collateral constraints are given by

$$B^S_{t+1} \leq \varphi^S K_t.$$ \hfill (9)

$$B^L_{t+1} \leq \varphi^L K_t.$$ \hfill (10)

The parameters $\varphi^S$, $\varphi^L$ are constants satisfying the constraints $0 < \varphi^S < \varphi^L < 1$, $0 < \varphi^S + \varphi^L \leq 1$ which affect the tightness of the collateral constraints, and therefore, the borrowing capacity of the firm. Due to the collateral constraints, the interest rate, denoted by $r_f$, is the risk-free rate which is assumed constant.

Firms can also save in cash when the short-term debt $B^S_t$ takes on negative values. Firms also incur adjustment costs on debt, denoted by $\Phi_t$ when changing the amount...
of short-term debt and long-term debt outstanding,

\[ \Phi_t = b^S K_t \mathbf{1}_{\{\Delta B_t^S \neq 0 \text{ and } B_t^S > 0\}} + b^L K_t \mathbf{1}_{\{N_t \neq 0\}} \]  

(11)

where \( \Delta B_t^S = B_t^S - B_{t-1}^S \), and \( b^S, b^L > 0 \). The debt adjustment costs capture the fact that adjusting capital structure is costly in terms of both managerial time and also issuance costs. For short-term debt, it captures the cost associated with liquidity risk, e.g., borrowers are forced into inefficient liquidation because refinancing is not available, thus prefer long-term contract (Diamond 1991). For long-term debt, it captures the agency costs associated with long-term debt (Myers 1977), e.g., the under-investment problem associated with debt overhang in the long-term debt contract. It also captures the information cost associated with long-term contract as borrowers with favorable inside information may avoid locking in their financing cost with long-term debt contracts. Lastly, cash is freely adjusted.

2.3 Costly external equity financing

Taxable corporate profits are equal to output less capital depreciation and interest expenses: \( S_t - \bar{W} H_t - \delta K_t - r_f B_t^S \mathbf{1}_{\{B_t^S > 0\}} - c B_t^L \). It follows that the firm’s budget constraint can be written as

\[
E_t = (1 - \tau) \left( S_t - \bar{W} H_t \right) + \tau \delta K_t + \tau r_f B_t^S \mathbf{1}_{\{B_t^S \geq 0\}} + \tau c B_t^L - I_t - G_t \\
+ B_{t+1}^S - (1 + r_s) B_t^S + N_t - (c + \theta) B_t^L - \Phi_t,
\]

(12)

in which \( \tau \) is the corporate tax rate, \( \tau \delta K_t \) is the depreciation tax shield, \( \tau r_f B_t \mathbf{1}_{\{B_t^S > 0\}} \) and \( \tau c B_t^L \) are the interest tax shields where \( c \) is the coupon rate, and \( E_t \) is the firm’s payout. When \( B_t^S > 0 \), short-term debt interest rate is \( r_s = r_f \); when short-term debt is negative, cash saving rate is assumed to be \( r_s = 0 \).
When the sum of investment, capital, and debt adjustment costs exceeds the sum of after tax operating profits and debt financing, firms can take external funds by means of seasoned equity offerings. External equity $O_t$ is given by

$$O_t = \max (-E_t, 0).$$  \hspace{1cm} (13)

In practice, firms face external equity financing costs, which involve both direct and indirect costs. We do not explicitly model the sources of these costs. Rather, we attempt to capture the effect of the costs in a reduced-form fashion. The external equity costs are similarly to debt assumed to scale with firm size as measured by the capital stock:

$$\Psi (O_t) = \eta K_t 1_{\{O_t > 0\}}.$$

Finally, firms do not incur costs when paying dividends or repurchasing shares. The effective cash flow $D_t$ distributed to shareholders is given by

$$D_t = E_t - \Psi_t.$$

\hspace{1cm} (15)

2.4 Firm’s problem

Firms solve the maximization problem by choosing capital investment, labor, short-term debt/cash and long-term debt optimally:

$$V_t = \max_{I_t, H_t, B_{t+1}^{ST}, B_{t+1}^{LT}} D_t + \beta \mathbb{E} V_{t+1},$$

\hspace{1cm} (16)

subject to firms’ capital accumulation equation (Eq. 6), collateral constraints (Eq. 9 and 10), budget constraint (Eq. 12), and cash flow equation (Eq. 15).

\hspace{1cm} 4In reality, firms tend to smooth dividends payout. We don’t model dividend adjustment cost because that would introduce another state variable which would further complicate the problem.
3 Main results

This section presents the model solution and the main results. We first calibrate the model, then we simulate the model and study the quantitative implications of model for the relationship between uncertainty shocks and firms’ real activity and financial flows.

3.1 Calibration

The model is solved at a monthly frequency. Because all the firm-level accounting variables in the data are only available at an annual frequency, we time-aggregate the simulated accounting data to make the model-implied moments comparable with those in the data.

Table 15 reports the parameter values used in the baseline calibration of the model. The model is calibrated using parameter values reported in previous studies, whenever possible, or by matching the selected moments in the data reported in Table 16. To evaluate the model fit, the table reports the target moments in both the data and the model. To generate the model’s implied moments, we simulate 3,000 firms for 1,000 monthly periods. We drop the first 400 months to neutralize the impact of the initial condition. The remaining 600 months of simulated data are treated as those from the economy’s stationary distribution. We then simulate 100 artificial samples and report the cross-sample average results as model moments.

[Insert Table 15 here]

[Insert Table 16 here]

We split the parameters into two groups. The first group includes those parameters which are based on the estimates in the previous literature including \( \{\alpha, \varepsilon, \delta, \beta, c, \theta, \eta, \rho_z, \sigma^f, \sigma^m, \sigma^L, \sigma^H, \sigma_{LH}, \sigma_{HH}, \varepsilon \} \). We set the share of capital the production function at 1/3, and the elasticity of demand \( \varepsilon \) to 4 which implies a
markup of 33%, consistent with Hall (1988). The capital depreciation rate $\delta$ is set to be 1% per month. The discount factor $\beta$ is set so that the real firms’ discount rate $r_f = 4\%$ per annum, close the average of the real annual S&P index return in the data. This implies $\beta = 0.9967$ monthly. The rate of retirement of the long-term debt $\theta = 1/120$, implying the length of the long-term contract is 10 years, close to the empirical estimate in Guedes and Opler (1996). The monthly coupon rate $c$ is set to 5% per year, implying that the average term premium is 1% per annum, close to the average in the U.S. We set the calibrate equity issuance cost parameters so that on average it costs 8% of the total level of issuance, consistent with the estimates in Altinkilic and Hansen (2000) and the estimates in Hennessy and Whited (2007). The fraction of equity issuance implied by the model is 23% close to the data at 17% estimated in Belo, Lin, and Yang (2016). We set the persistence of firms’ micro and macro productivity as $\rho_z = 0.97$ following Khan and Thomas (2008). We set the baseline firm volatility as $\left\{ \sigma_L^f, \sigma^m_L \right\} = \{0.10, 0.02\}$ and the high uncertainty state $\sigma_H^{f,m} = 2 \times \sigma_L^{f,m}$, close to the level in Bloom (2009). We set transition probabilities of $\pi_{L,H}^H = 1/36$, and $\pi_{H,H}^H = 1 - 1/36$, consistent with one uncertainty shock every three years. The long-run average level of firm-specific and macro productivity, $\bar{z}^f$ and $\bar{z}^m$ are arbitrary scaling variables which we set to unity.

The second group contains the six parameters calibrated to match some moments in the data, including $\left\{ c^P_k, b_k, b^S, b^L, \varphi^S, \varphi^L \right\}$. We calibrate the capital adjustment cost parameters to match several cross-sectional and time-series moments of firms’ investment rates. Table 16 shows that this calibration of the model matches reasonably well the volatility of firm-level investment rate. The investment resale loss parameter $c^P_k$ is set at 2.5% to match the inaction region in investment rate. Investment fixed cost parameter $b_k$ is set at 0.01. The implied volatility of investment rate is 24%, close to the data moment at 23%. We calibrate the short-term and long-term debt adjustment cost parameters $b^S = b^L = 0.03\%$ and the tightness of
the collateral constraint for short-term and long-term debt $\varphi^s = 0.3$ and $\varphi^l = 0.55$
to match the average level of financial leverage at 0.55 and the short-term debt to
long-term ratio at 0.27 in the data. The model implied average leverage is 0.50 and
the implied short-term debt to long-term ratio is 0.19, close to the data moment.

3.2 Uncertainty shocks, real and financing flows

In this subsection, we conduct the panel regression analysis using the artificial data
obtained from the simulation of the model. For the real data we regress the rates
of investment, employment growth, short-term debt and long-debt growth, the term
structure of debt growth (the ratio of the short-term debt growth to the long-term
debt growth), cash holding growth, and payout (dividend plus share repurchase)
growth on one-year lagged stock return volatilities, alongside a full set of firm and
year fixed-effects. To align simulated results with these real data results we aggregate
monthly simulated data to annual values summing flow variables like sales over the
year and taking year end values for stock variables like capital, and then use the same
lag and fixed-effect structure in the regressions. Hence, we construct our simulated
accounting data regressions to exactly mimic the process for the real data regressions.

Panel A in Table 17 presents the benchmark calibration result while panel E
presents the data moments which will be discussed in detail in section 5. The
model predicts a negative relation between past return volatility and investment
rate, and employment growth in the univariate regressions. The model implied
univariate regression slopes of investment and employment growth are -0.012 and
-0.011, reasonably close to their respective data moments of -0.020 and -0.022 (noting
that we did not calibrate our parameters to meet these moments). Turning to financial
flows, the model also predicts a negative relation between past return volatility and
short-term debt growth and a negative relation between past return volatility and
the term structure of debt growth. The model implied slope coefficients on debt
growth and the term structure of debt growth are -0.017 and -0.238, respectively, again reasonably close to the data moments of -0.045 and -0.103. Furthermore, cash holding growth and past return volatility are positively correlated; the model implied slope is 0.229, somewhat higher than the data at 0.078. Dividend payout growth is negatively correlated with past return volatility; the model implied moment is -0.109, smaller than the data slope at -0.257. So, overall these six qualitative predictions from the model fit the data.

[Insert Table 17 here]

3.3 Inspecting the mechanism

In this section we first study the impulse responses of the real and financial variables in the benchmark model and then compare them to a model without financial frictions.

To simulate the impulse response, we run our model for 400 periods and then in month zero kick every simulation level of uncertainty up to its high level, and then let the model to continue to run as before. Hence, we are simulating a one period increase in uncertainty on the ergodic distribution. We perform this analysis 100 times and take the average (to average across macro shocks) for the benchmark model and a model without financial adjustment costs, i.e., debt and equity issuance costs are zero. Figure 8 plots the impulse responses of the main real and financial variables. For the real variables we see upon impact capital and employment levels drop and slowly recover, while debt drops, particularly short-run debt, as does equity payout, so that cash holdings rise. We also see that compared to the benchmark, the simulation with no financial frictions has a smaller and less persistent impact on real variables. This highlights the role of financial frictions in multiplying the impact of uncertainty shocks on investment and hiring - if the future is uncertain and it is expensive to tap debt or equity funding, firms increase their cash holdings.
We also see that without financial frictions there is no impact of uncertainty on cash. The reason is that without financial frictions firms do not hold cash as it pays no interest and it has no liquidity value since debt and equity are perfectly liquid. Since firms do not hold cash the impact of uncertainty on other financial variables is muted (they observed impact is entirely driven by the financing requirements of investment and hiring).

Lastly figure 9 plots the impulse responses of output in the benchmark model and the model without financial frictions. Upon impact, output falls with similar magnitudes when volatility is high in both two cases. However, after the impact, the response of output in the model without financial frictions reverts to the steady state level immediately whereas the response of output in the benchmark model with financial frictions persists for more than 12 months before reverting to the long-run level. Taken together, financial frictions clearly amplify the impact of uncertainty shocks on real and financial variables.

Next we perform several comparative statics analyses to show the economic forces driving the overall good fit of the model. Panels B and C in Table 17 present the results. We consider two specifications:

- A model without real frictions (no partial irreversibility $c_k^P = 0$ and fixed cost is zero $b_k = 0$, and

- A model without financial frictions (no debt and equity issuance costs $b^S = b^L = \eta = 0$).

The results without real frictions are reported in panel B of Table 17. We see the responses of investment rate, employment and cash growth drop substantially relative to the benchmark. For example, the slope on investment drops from -0.012 in the benchmark to -0.002, employment growth from -0.011 to -0.009, and cash growth from 0.229 to 0.079. Furthermore, the term structure of debt growth loads positively, 0.010
compared to -0.238 in the benchmark and -0.103 in the data, which is counterfactual to the data. The slope on dividend growth does not change significantly (-0.109 in the benchmark compared to -0.108 in Panel A). Hence, real-frictions are needed to get reasonable real - and in this case financial - variable responses to uncertainty shocks.

When we shut down the financial frictions in panel C (i.e., both short-term and long-term debt and equity issuances are free), the slope coefficients on investment rate and employment growth drop by around half (from -0.012 in the benchmark to -0.007 for investment and from -0.011 to -0.007 for employment growth). This finding shows that financial frictions play an important role amplifying the effect of uncertainty shocks on real quantities. In addition, the coefficient on debt growth falls by more than two thirds from -0.017 to -0.005. The term structure of debt growth becomes unresponsive to the volatility shock, the slopes drops to zero, compared to -0.238 in the benchmark. Turning to cash, because all marginal sources of external financing are free now (debt up to the collateral constraints), firms do not save precautionarily, thus the equilibrium cash holding is zero. Similar to Panel C, dividend growth does not drop significantly, from -0.109 in the benchmark to -0.094. Taken together, these comparative analyses show that both real frictions and financial frictions amplify the impact of the uncertainty shocks and are jointly important for the model to capture the quantitative effect of uncertainty shocks on real and financial activity.

Lastly, we study the impact of uncertainty shock for real and financial activity in recessions. To simulate a recession in the model, we let the model run for 400 months, then induce an aggregate productivity shock in month 0 and then let firms productivity evolve again following the standard transition process. The productivity shock moves all firms down two productivity levels if possible - so firms at productivity level 5 (the highest level in our 5 point firm grid) move to 3, those at 4 to 2 and those at 3, 2 (or 1) move to 1. Panel D in Table [17] reports the result. Interestingly, the responses of both real and financial variables are much stronger than those in the
benchmark calibration during the recession, because financial and real constraints become far more binding. For example, the slope coefficients on investment and employment growth are -0.031 and -0.030, respectively, about 50% bigger in absolute magnitude than the benchmark. The slope coefficients on financial variables including debt, term structure of debt growth, cash growth and payout are -0.043, -0.468, 0.438, and -0.243, respectively, about twice as big in magnitudes as those implied in the benchmark calibration. Hence, we in summary, we find that the triple interaction of real adjustment costs, financial adjustment costs and a recession leads to a dramatic amplification of the impact of uncertainty shocks on firms real and financial behavior.

3.4 General equilibrium type analysis

Currently the model is in a particular equilibrium setting, with a general equilibrium set-up requiring flexibility in four prices: wages, good prices, interest rates and the term-structure (long vs short rates). We are currently working on this extension, but adding these four state variables is complex so it not yet complete.

We should note, however, that in US data wages and inflation rate are acyclical (King and Rebelo 1999) and so any well specified general equilibrium (GE) effects should be second order through the wage and good price channel. Interest rates do vary over the cycle, and in particular are negatively correlated with the VIX. Finally, the term structure also appears to be broadly acyclical (and, in particular, is uncorrelated with the VIX). Hence, our main pricing variable we need to consider in GE is interest-rates. As a short-cut to a full GE model we are also testing models assuming interest-rates drops ranging from 1% to 4% after uncertainty shocks (noting the zero-lower bound of a 4% drop). So far we find broad robustness of our results on the impact of uncertainty shocks, with the intuition being that higher uncertainty moves the Ss inaction bands outwards, making firms temporarily unresponsive to prices changes.
{Note: in the long sample of 1963 to 2014, term spread and the VIX have a correlation at 0.06, and this correlation is remain low except during the Financial Crisis period, in which the term spread and the VIX have a correlation is 0.25}.

4  Data and Instruments

We first describe the data and variable construction, then the identification strategy.

4.1  Data

Stock returns are from CRSP and annual accounting variables are from Compustat. The sample period is from January 1963 through December 2014. Financial, utilities and public sector firms are excluded (i.e., SIC between 6000 and 6999, 4900 and 4999, and above 9000). Compustat variables are at the annual frequency. Our main empirical tests involve regressions of changes in real and financial variables on changes in lagged uncertainty, where the lag is both to reduce concerns about endogeneity and because of natural time to build delays. Thus, our sample requires firms to have at least 3 consecutive non-missing data values. To ensure that the changes are indeed annual, we require a 12 month distance between fiscal-year end dates of accounting reports from one year to the next.

In measuring firm-level uncertainty we employ both realized annual uncertainty from CRSP stock returns and option-implied uncertainty from OptionMetrics. Uncertainty measured from stock-returns is the standard-deviation of returns over the accounting year (which typically spans about 252 days). OptionMetrics provides daily implied volatility data for underlying securities from January 1996 through December 2014, with our principal measure being the "at-the-money" "365-day" implied volatility. Additional information about OptionMetrics, Compustat, and CRSP data is provided in Appendix (??).
For all variables growth is defined following Davis and Haltiwanger (1992), where for any variable $x_t$ this is $\Delta x_t = (x_t - x_{t-1})/(\frac{1}{2}x_t + \frac{1}{2}x_{t-1})$, which yields growth rates bounded between -2 and 2. The only exceptions are CRSP stock returns and capital formation. For the latter investment rate (implicitly the change in gross capital stock) is defined as $I_{i,t} = \frac{CAPEX_{i,t}}{K_{i,t-1}}$ where $K$ is net property plant and equipment, and $CAPEX$ is capital expenditures. The changes and ratios of real and financial variables are then all winsorized at the 1 and 99 percentiles.

### 4.2 Identification Strategy

Our identification strategy exploits firms’ differential exposure to energy, currency and policy exposure to generate exogenous changes in firm-level uncertainty. The idea is that some firms are very sensitive to, for example, oil prices (e.g. energy intensive manufacturing and mining firms) while others are not (e.g. retailers and business service firms), so that when oil-price volatility rises this shifts up firm-level volatility in the former group relative to the latter group. Likewise, some industries have different trading intensity with Europe versus Mexico (e.g. industrial machinery versus agricultural produce firms), so changes in bilateral exchange rate volatility generates differential moves in firm-level uncertainty. Finally, some industries - like defense, health care and construction - are more reliant on the Government, so when aggregate policy uncertainty rises (for example, because of elections or government shutdowns) firms in these industries experience greater increases in uncertainty.

This approach is conceptually similar to the classic Bartik (1991) identification strategy which exploits different regions exposure to different industry level shocks, and builds on the paper by Stone and Stein (2013).

The sensitivities to energy and currency prices are estimated at the industry as the factor loadings of a regression of a firm’s stock return on energy and currency price changes. That is, for firm $i$ in industry $j$, $sensitivity_i^c = \beta_j^c$ is estimated as
follows

\[ r_{i,t}^{\text{risk adj}} = \alpha_i + \sum_c \beta_{j,c} \cdot r_{i,c}^c + \epsilon_{i,t} \]  

(17)

where \( r_{i,t}^{\text{risk adj}} \) is the daily risk-adjusted return on firm \( i \) (explained below), \( r_{i,c}^c \) is the change in the price of commodity \( c \), and \( \alpha_i \) is firm \( i \)'s fixed effect. The sensitivities are estimated using daily price data from the twenty years (Jan. 1985 to Dec. 2004) prior to the main two stage least squares (2SLS) estimation period. This estimation is run at the SIC 3-digit industry level to yield sufficient sample size to identify the crucial \( \beta_{j,c} \) coefficients.

The risk-adjusted return is estimated from the residuals obtained from firm-level regressions on the Carhart (1997) four-factor asset pricing model. In particular, we define the daily risk-adjusted return as the residuals obtained from the time series regression of each firm’s excess return on the four factors over the full pre-estimation sample period (1985 to 2004):

\[ r_{i,t}^{\text{EXCESS}} = \alpha^c + \beta_{i,mkt} \cdot MKT_t + \beta_{i,HML} \cdot HML_t + \beta_{i,SMB} \cdot SMB_t + \beta_{i,UMD} \cdot UMD_t + \epsilon_{i,t} \]  

(18)

where \( r_{i,t}^{\text{EXCESS}} \) is firm \( i \)'s daily CRSP stock return (including dividends and adjusted for delisting) in excess of the t-bill rate, \( MKT \) is the CRSP value-weighted index in excess of the risk free rate, \( HML \) is the book-to-market factor, \( SMB \) is the size factor, \( UMD \) is the momentum factor. These factor data are obtained from CRSP.

We adjust returns for risk to address concerns over whether the sensitivities to energy and currencies (\( \beta_{j,c} \) in equation 17) are largely capturing market-wide risks instead of exposure to energy and currency shocks. Nonetheless, we report that our main results are largely similar if we skip the risk-adjustment of returns and estimate the sensitivities to energy and currency prices using the raw CRSP returns directly.
in equation 17.

For energy we use the crude-oil price, and for exchange rates we select the 7 "major" currencies used by the Federal Board in constructing the nominal and real trade-weighted U.S. Dollar Index of Major Currencies. For these eight market prices (the oil price and the seven exchange rate prices) we need not only their daily levels (for calculating the sensitivities $\beta_j^c$ in equation 17) but also their annual implied volatilities $\sigma_{j,t}^c$ as a measure of their uncertainty. The composite of these two terms - $|\beta_j^c| \cdot \sigma_{j,t}^c$ - is then an industry-by-year instrument for uncertainty, where the first term is the absolute value of the sensitivity estimated in equation 17 at the industry level. Our instrumental variables estimation thus uses nine instruments - the oil price exposure term, the seven currencies exposure terms, and the policy-uncertainty exposure term (which is defined as industry average share of total revenue from Federal Government contracts times the policy uncertainty index, all of which comes from Baker, Bloom and Davis (2016)).

Finally, to control for any correlations between returns levels and volatilities of these 9 instrumental variables we also include as controls in the IV regressions the exposure times their returns. That is, in the regressions we also include $\beta_j^c \cdot r_t^c$ for the 7 currencies, the oil price and the EPU index (where for the EPU index the $r_t^c$ is the level of government expenditure as a share of GDP).

5 Empirical findings

We start by examining how volatility relates to investment, then other real outcomes finally followed by financial variables.

\(^3\)see http://www.federalreserve.gov/pubs/bulletin/2005/winter05_index.pdf. These include: the euro, Canadian dollar, Japanese yen, British pound, Swiss franc, Australian dollar, and Swedish krona. Each one of these trades widely in currency markets outside their respective home areas, and (along with the U.S. dollar) are referred to by the Board staff as major currencies.
5.1 Investment results

Table 18 examines how uncertainty influences future capital investment. Column 1 presents the univariate Ordinary Least Squares regression results of investment rate on lagged annual realized stock return volatility. We observe highly statistically significant coefficients on return volatility, showing that firms tend to invest more when their firm-specific uncertainty is low. Column 2 proxies uncertainty with implied volatility, which yields a larger coefficient, although on a much smaller sample because of the limited availability of implied volatility data. While these results are consistent with the model these estimations suffer from endogeneity concerns - for example, changes in firms investment plans could change stock-prices, leading to a causal impact from investment on realized volatility.

We address these identification concerns by using our energy, currency and policy exposure instruments. Column 3 shows the univariate 2SLS results when we instrument lagged realized volatility. We see the point estimate of the coefficient on uncertainty is very near the OLS coefficient in column 1 (while remaining highly statistically significant), possibly because the upward bias from reverse causality roughly offsets the downward attenuation bias from measurement error in realized volatility as a proxy for firm-by-year uncertainty.

A more rigorous test is run in columns 4 and 5, where we include a full set of controls based on the prior literature (e.g. Welch 2004). In particular, we include controls for Tobin’s Q, sales and stock-returns to control for firm moment shocks, as well as book leverage, profitability (return on assets) and tangibility to control for financial conditions. Column 4 presents the OLS multivariate results while column 5 shows the 2SLS results. In both cases, rises in uncertainty remains a strong predictor.

\[\text{Of course there are several other factors changing between these columns, including the sample size (the IV sample is post 2007 onwards due to the lack of implied volatility data for oil and currency prices before 2006) and local average treatment effects (the instrumental variables estimation obtains identification from changes in macro energy, currency and policy risk).}\]
of future reductions in capital investment even after controlling for lagged changes in Tobin’s Q and changes in various measures of a firm’s status (such as firm return on assets and sales). Interestingly, the point estimates for OLS in column 4 and IV in column 5 are similar in magnitude. Finally, in column (6) we include the IV for implied volatility and again find a significant negative impact of uncertainty on investment.

In terms of magnitudes these results imply that a two-standard deviation increase in realized volatility would reduce investment by between 4% to 6% (using the results from our preferred specifications in column (5) and (6)). This is moderate in comparison to firm-level investment fluctuations (which have a standard deviation of 24.7%), but is large when considering that annual investment rates drop about 2% or 3% during recessions as show in figure [7].

[Insert Table 18 here]

5.1.1 First stage results

The instrumental investment results are only valid if the first-stage regressions have been predictive power and meet the exclusion restriction, which we jointly examine in table [19] which plots the first stage for the investment estimations. In columns (1) and (2) we report the first stages for the univariate and multivariate IV columns (3) and (5) from table [18]. We see that, first, the F-statistics indicate a well identified first stage with values of 27 and 26. We also find reassuringly the Hansen overidentifying test does not reject the validity of our instruments with p-values of 0.494 and 0.354.

As another check of our identification strategy we would like to see our instruments are all positive and significant. Indeed, we see in columns (1) and (2) that any significant instrument in the first stage is positive. The negative coefficients are for insignificant instruments, which presumably arises because of the multicollinearity between the exchange rate instruments. To confirm this in columns (3) and (4) we
re-run the first stage specifications but including only each instrument one-by-one and report in the cells for each instrument the results from this regressions. So, for example, the values 0.278 and 12.51 at the top of column (3) are the point-estimate and t-statistic for a first stage like column (1) except when the only instrument was exposure to oil volatility (using the same sample). We now see that every instrument is positive and strongly significant, suggesting the reason that some instruments in columns (1) and (2) were not significant is due to multicollinearity.

[Inset Table 19 here]

5.2 Employment, intangible capital and sales

Table 20 examines the predictive implications of uncertainty on other important real outcomes. Panel A examines employment changes, Panel B changes in the investment in intangible capital (as measured by expenditure on sales, general and administration and R&D, extending the approach of Eisfeldt and Papanikolaou (2013)) and Panel C changes in sales. In each panel we present the same 6 columns of regression results as we did for investment. The specifications in each column follow the regression specifications described in the investment rate Table 18. Moreover, to preserve space we only report the point coefficient estimates on lagged changes in uncertainty (the results include the exact same set on control variables as for investment).

The three panels show that realized and implied volatility is negatively related to real future outcomes in employment, investment in intangible capital, and sales. These results are largely confirmed in specifications 3 and 6 where we instrument realized and implied volatility by volatility exposure to commodity markets. In particular, the negative coefficient estimates are quite pronounced and 5% statistically significant for both intangible capital and sales, and weakly (10% or 15%) significant for employment. Moreover, as with investment these regressions show a strong first-stage with F-statistics above 20 in all specifications except for the implied volatility
where smaller sample sizes cut the F-statistics to around 8 to 10.

Hence, in summary this confirms the robustness of the causal impact of uncertainty shocks on real firm activity across other inputs (employment and intangible capital investment) and output (sales) - even in the presence of extensive first-moment and financial conditions controls - plus an extensive instrumentation strategy for uncertainty.

### 5.3 Financial variables

Table 21 examines how firm uncertainty affects future total debt and the debt maturity structure. Panel A shows that increases in uncertainty reduce the willingness of firm’s to increases their overall debt. The correlations are strong and significant in both the OLS and instrumental variable regressions. Panels B examines how uncertainty affects corporate payout. Consistent with a precautionary saving motive rises in a firm’s uncertainty associates with a large reduction in equity payouts. Panel C examines the firms cash holding policies, and finds weaker evidence for an impact of uncertainty leading firms to accumulate cash reserves, again as part of the an increase in cautionary behavior. Finally, in Panel D we regresses changes in the ratio of short to long term debt. Uncertainty has a strong negative sign in the basic OLS regressions in columns (1) and (2), indicating that firm’s short-term debt ratio is lower when uncertainty is higher. While the IV results show a similar result in terms of the point-estimate they have large standard-errors so are not statistically significant.
5.4 Instrument and credit supply robustness

In table 22 we investigate the main multivariate investment results dropping each instrument one-by-one in columns (2) to (10) (compared to the baseline results in column (1)). As we see across the columns the results are impressively robust - the F-test and is 20 or above in all specifications and the Hansen over-identifying test does not reject at the 5% level in any specification. Hence, our results are not driven by one particular instrument, but instead are driven by the combined identification of energy, exchange rate and policy uncertainty driving firm-level uncertainty fluctuations. This suggests this identification strategy will be broadly useful for a wide-range of models of the causal impact of uncertainty on firm behavior.

In Appendix table ?? we investigate the robustness of the results to including firm-level controls for financial constraints. One concern could be that uncertainty reduces financial supply - for example, banks are unwilling to lend in periods of high uncertainty - which causes the results we observe. So to try to address this we include a variety of different controls for firms financial conditions and show our results are robust to this. In particular for both the realized and implied volatility we include controls for the firms: CAPM-beta (defined as the covariance of the firms daily returns with the market returns in that year) in columns 2 and 2A, the firms Kaplan and Zingales (1997) and Whited and Wu (2006) financial conditions measures in columns 3 and 3A, the firms credit rating (a full set of dummies including an indicator for no rating) in columns 4 and 4A, and all these measures combined in columns 5 and 5A. In summary, as we can see from table (Table:2SLSRobustness) including these financial supply variables does not notably change our results. So while these are not perfect controls for financial conditions, the robustness of our results to their inclusion suggests that financial supply conditions are unlikely to be the main driver.

\footnote{In the final column the Hansen test is significant at the 10% level, which given this table runs 10 tests across the instruments, is within the distribution of the random sampling.}
of our results.

5.5 The finance-uncertainty multiplier

Finally, table 23 shows the results from running a series of finance-uncertainty interactions on the data during the 2008-2010 period of the financial crisis. In panel A we measure firms financial constraints by their S&P credit rating in 2005. Following Duchin et al. (2010) firms with positive debt and no S&P credit rating are defined as constrained, while those with either no-debt or a credit rating are defined as unconstrained. We then interact this measure of financial constraints with uncertainty in our preferred specification for uncertainty, which uses realized volatility with a full set of controls and instrumental variables (e.g. the column (5) specification in tables 4 and 6).

In the top panel of table 23 we see that the interaction of financial constraints (proxied by the S&P credit rating) with uncertainty is significantly negative for investment, intangible capital investment and sales (although insignificant for employment). This suggests that for financially constrained firms uncertainty has a significantly more negative effect of tangible and intangible investments, and hence also on sales. In the middle and bottom panels we proxy financial constraints using the 2005 level of firm size measured by employment and assets, and again find a similar result - smaller firms (who are typically more financially constrained) are significantly more responsive to increased uncertainty in terms of cutting investment and sales. Hence, overall this provides important evidence for an interactive effect of financial constraints and uncertainty in deterring firms investment activities during the 2008-2010 period of the financial crisis.

To show this graphically figure 10 plots investment rates for financially constrained

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82005 is chosen as the year before our regression data starts, since we examine 2008-2010, with the explanatory variables lagged one year (2007-2009) and require another lag to generate the first differences.
and unconstrained firms from 2003 to 2013. We normalize the investment rates of both groups of firms to their respective values of investment rates in 2006. Financial constraints are defined as a firm having short or long-term debt but no public bond rating (see, e.g., Faulkender and Petersen 2006 and Duchin, Ozbas and Sensoy 2010). Volatility is the annual realized stock return volatility. It is clear that constrained and unconstrained firms’ investment rates track each other closely until the Great Recession and the spike in uncertainty, at which point the constrained firms’ investment drop substantially more than unconstrained firms. As uncertainty recedes post 2012 the gaps start to recede again as the investment rates begin to converge.

6 Conclusion

This paper studies the impact of uncertainty shocks on firms’ real and financial activity both theoretically and empirically. We build a dynamic capital structure model which highlights the interactions between the time-varying uncertainty and the external financial frictions and the real frictions. The model generates the links between uncertainty shocks and real and financial activity observed in the data. We show that both real and financial frictions significantly amplify the impact of uncertainty shocks on firms’s real and financing decisions. Empirically, we test the model and show that uncertainty shocks cause firms to reduce investment and employment on real side and furthermore, reduce their total debt and the term structure of debt, while increase the cash holding and cut dividend payout on financial side. Taken together, our theoretical and empirical analyses show that real and financial frictions are quantitatively crucial to amplify the impact of uncertainty shocks on firms’ activity.
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This figure plots the aggregate stock market volatility and the selected real and financial variables. Stock market volatility is the quarterly average of the monthly VIX. We construct quarterly series of the aggregate investment rate following Bachmann et al. (2011) using quarterly investment and capital data from the national account and fixed asset tables, available from the Bureau of Economic Analysis (BEA). Employment is the quarterly average of seasonally adjusted total private employment from BLS with the ID of CES0500000025. Short-term debt, long-term debt, and cash are from the NIPA Integrated Macroeconomic Accounts Table S.5.q nonfinancial corporate business. Short-term debt is the sum of open market paper (line 123) and short-term loans (line 127). Long-term debt is the sum of bonds (line 125) and mortgages (line 130). Cash is the sum of currency and transferable deposits (line 97) and time and savings deposits (line 98). Dividend is the quarterly average of the aggregate real dividend from the stock market data on Robert Shiller’s webpage http://www.econ.yale.edu/~shiller/data.htm.
Figure 8 Impulse responses of real and financial flows

This figure plots the impulse responses of the real and financial variables from the low volatility state to high volatility state while fixing the level of productivity at the long-run average level. There are two model specifications: i) the benchmark model (solid line) and ii) a model without financial frictions (no debt costs $b^S = b^L = \eta = 0$, dash line).
This figure plots the impulse responses of output from the low volatility state to high volatility state while fixing the level of productivity at the long-run average level. There are two model specifications: i) the benchmark model (solid line) and ii) a model without financial frictions (no debt and equity issuance costs $b^S = b^L = \eta = 0$, dash line).
Figure 10 Investment rates of constrained and unconstrained firms

This figure plots the average quarterly investment rates of the constrained (the red line) and unconstrained firms (the blue line) normalized to their respective values of the first quarter of 2006. Financial constraints are defined as a firm having short or long term debt but no bond rating (see Faulkender and Petersen 2006 and Duchin, Ozbas and Sensoy 2010). Volatility is the annual realized stock return volatility (the green line).
<table>
<thead>
<tr>
<th>Description</th>
<th>Notation</th>
<th>Value</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Predetermined parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subjective discount factor</td>
<td>$\beta$</td>
<td>0.996</td>
<td>Long-run average for U.S. firm-level discount rate at 4% annually</td>
</tr>
<tr>
<td>Share on capital</td>
<td>$\alpha$</td>
<td>0.33</td>
<td>Capital share in output is one-third, labor share is two-thirds</td>
</tr>
<tr>
<td>Markup</td>
<td>$\varepsilon$</td>
<td>4</td>
<td>33% markup (Hall 1988)</td>
</tr>
<tr>
<td>Wage</td>
<td>$w$</td>
<td>1</td>
<td>Wage rate normalized to 1</td>
</tr>
<tr>
<td>Rate of depreciation for capital</td>
<td>$\delta$</td>
<td>0.01</td>
<td>Capital depreciation rate assumed 1% per month (~10% annually)</td>
</tr>
<tr>
<td>Linear equity issuance cost</td>
<td>$\eta$</td>
<td>0.08</td>
<td>Linear equity issuance cost (Hennessy and Whited 2005)</td>
</tr>
<tr>
<td>Coupon rate of long-term debt</td>
<td>$c$</td>
<td>0.05/12</td>
<td>Coupon rate of long-term debt (term premium on Corp.+ T. Bills)</td>
</tr>
<tr>
<td>Maturity of long-term debt</td>
<td>$1/\theta$</td>
<td>120</td>
<td>Avg. maturity of long-term debt as 10 years (Guedes-Opler 1996)</td>
</tr>
<tr>
<td>Monthly cond. vol. of idio. productivity</td>
<td>$\sigma_L$</td>
<td>0.10</td>
<td>Baseline uncertainty (Bloom 2009)</td>
</tr>
<tr>
<td>Monthly cond. vol. in high idio. vol state</td>
<td>$\sigma_H$</td>
<td>0.20</td>
<td>Uncertainty shocks 2 times baseline uncertainty (Bloom 2009)</td>
</tr>
<tr>
<td>Monthly trans. prob. from low to high vol</td>
<td>$\pi_{L,H}$</td>
<td>0.0278</td>
<td>Uncertainty shocks expected every 3 years (Bloom 2009)</td>
</tr>
<tr>
<td>Monthly trans. prob. of staying in high vol</td>
<td>$\pi_{H,H}$</td>
<td>0.9722</td>
<td>Probability of high uncertainty state remaining at the high state</td>
</tr>
<tr>
<td>Persistence of logged idio. productivity</td>
<td>$\rho_z$</td>
<td>0.97</td>
<td>Persistence of logged idiosyncratic productivity (Belo et al 2014)</td>
</tr>
<tr>
<td>Average of logged idio. productivity</td>
<td>$\bar{\varepsilon}$</td>
<td>-3</td>
<td>Scalar; average long-term debt is 0.5</td>
</tr>
<tr>
<td><strong>Calibrated parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed cost of investment</td>
<td>$b$</td>
<td>0.01</td>
<td>Fixed cost of investment</td>
</tr>
<tr>
<td>Partial irreversibility on investment</td>
<td>$c^P_k$</td>
<td>0.025</td>
<td>Investment resale loss</td>
</tr>
<tr>
<td>Adj. cost parameters in short-term debt</td>
<td>$b^S$</td>
<td>0.3%</td>
<td>Adj. cost parameters in short-term debt</td>
</tr>
<tr>
<td>Adj. cost parameters in long-term debt</td>
<td>$b^L$</td>
<td>0.3%</td>
<td>Adj. cost parameters in long-term debt</td>
</tr>
<tr>
<td>Tightness of collateral constraint ST debt</td>
<td>$\phi^S$</td>
<td>0.30</td>
<td>Tightness of collateral constraint short-term debt</td>
</tr>
<tr>
<td>Tightness of collateral constraint LT debt</td>
<td>$\phi^L$</td>
<td>0.55</td>
<td>Tightness of collateral constraint long-term debt (match S/L ratio)</td>
</tr>
</tbody>
</table>

This table presents the predetermined and the calibrated parameter values of the benchmark model.
Table 16

Unconditional moments under the benchmark calibration

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. dev. of investment rate</td>
<td>0.25</td>
<td>0.19</td>
</tr>
<tr>
<td>Std. dev. of net hiring rate</td>
<td>0.23</td>
<td>0.24</td>
</tr>
<tr>
<td>Mean of financial leverage</td>
<td>0.56</td>
<td>0.55</td>
</tr>
<tr>
<td>Average fraction of the firms holding cash</td>
<td>0.50</td>
<td>0.49</td>
</tr>
<tr>
<td>Short term to long term debt ratio</td>
<td>0.27</td>
<td>0.23</td>
</tr>
<tr>
<td>Average fraction of the firms issuing equity</td>
<td>0.17</td>
<td>0.16</td>
</tr>
</tbody>
</table>

This table presents the selected moments in the data and implied by the model under the benchmark calibration. The reported statistics in the model are averages from 100 samples of simulated data, each with 3000 firms and 600 monthly observations (50 years). We report the cross-simulation averaged annual moments.

Table 17

Coefficient on lagged changes in volatility for real and financial variables.

<table>
<thead>
<tr>
<th>Real</th>
<th>Financial</th>
</tr>
</thead>
<tbody>
<tr>
<td>I/K</td>
<td>dEmp</td>
</tr>
<tr>
<td>A: Benchmark model</td>
<td>Volatility</td>
</tr>
<tr>
<td>B: No real frictions</td>
<td>Volatility</td>
</tr>
<tr>
<td>C: No financial frictions</td>
<td>Volatility</td>
</tr>
<tr>
<td>D: Recessions</td>
<td>Volatility</td>
</tr>
<tr>
<td>E: Data</td>
<td>Volatility</td>
</tr>
</tbody>
</table>

This table reports the model regression results of real and financial variables on lagged stock return volatility. The reported statistics in the model are averages from 100 samples of simulated data, each with 3000 firms and 600 monthly observations. We report the cross-simulation averaged annual moments. I/K is the investment rate, dEmp is the employment growth, dDebt is the total debt growth, ST/LT is the short-term debt to long-term debt growth, dCash is the cash growth rate, and dDiv the dividend growth in the model and cash dividend plus repurchase growth in the data.
<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Investment</strong></td>
<td>OLS</td>
<td>OLS</td>
<td>IV</td>
<td>OLS</td>
<td>IV</td>
<td>IV</td>
</tr>
<tr>
<td></td>
<td>Realized</td>
<td>Implied</td>
<td>Realized</td>
<td>Realized</td>
<td>Realized</td>
<td>Realized</td>
</tr>
<tr>
<td><strong>Volatility</strong></td>
<td>-0.029***</td>
<td>-0.060***</td>
<td>-0.056*</td>
<td>-0.023***</td>
<td>-0.062**</td>
<td>-0.098*</td>
</tr>
<tr>
<td></td>
<td>(-13.694)</td>
<td>(-11.192)</td>
<td>(-1.905)</td>
<td>(-4.322)</td>
<td>(-1.968)</td>
<td>(-1.699)</td>
</tr>
<tr>
<td><strong>Book Lev.</strong></td>
<td></td>
<td></td>
<td></td>
<td>-0.011***</td>
<td>-0.012***</td>
<td>-0.009***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-4.869)</td>
<td>(-4.935)</td>
<td>(-2.791)</td>
</tr>
<tr>
<td><strong>Stock Return</strong></td>
<td>0.031***</td>
<td>0.031***</td>
<td>0.021**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7.647)</td>
<td>(6.720)</td>
<td>(2.162)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Sales</strong></td>
<td>0.041***</td>
<td>0.040***</td>
<td>0.028**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.208)</td>
<td>(4.899)</td>
<td>(2.229)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Return Assets</strong></td>
<td>-0.000</td>
<td>-0.000</td>
<td>-0.001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.540)</td>
<td>(-0.624)</td>
<td>(-1.291)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Tangibility</strong></td>
<td>-0.104***</td>
<td>-0.103***</td>
<td>-0.082***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-11.530)</td>
<td>(-10.796)</td>
<td>(-6.146)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Tobin’s Q</strong></td>
<td>0.024**</td>
<td>0.018*</td>
<td>0.014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.427)</td>
<td>(1.655)</td>
<td>(0.920)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st moment</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Firm-Time FE’s</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>148,729</td>
<td>26,215</td>
<td>21,153</td>
<td>19,434</td>
<td>17,310</td>
<td>10,590</td>
</tr>
<tr>
<td>F-test 1st stage</td>
<td>29.18</td>
<td>25.59</td>
<td>7.939</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hansen J P-val</td>
<td>0.249</td>
<td>0.354</td>
<td>0.200</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This table presents OLS and 2SLS regression results for investment rate on lagged changes in firm-level volatility and firm-level controls. Investment defined as $I_t / K_{t-1}$ (Capex/Lagged Capital). Sample period is annual from 1963 to 2014. Specifications 1, 2, and 4 are OLS regressions, while 4, 5, and 6 are 2SLS regressions. The latter instrument lagged changes in realized volatility by lagged changes in volatility exposure to energy and currency markets (measures by at-the-money implied volatility of oil and 7 widely traded currencies) and economic policy uncertainty from Baker, Bloom, and Davis (2016). We measure firm-level uncertainty in two ways: realized and implied volatility. Realized volatility is the annual volatility of the firm’s stock return, estimated as the 12-month fiscal-year standard deviation of daily CRSP returns. We annualize this standard deviation by multiplying by the square root of 252 (average trading days in a year). Implied volatility is proxied by using OptionMetrics’ 365-day implied volatility of at-the-money-forward call options. In all regressions specifications we include both firm and time fixed effects, where time dummies are defined at the fiscal-semester basis. Standard errors are clustered at the firm level. All regressors are in changes from fiscal year $t-2$ to $t-1$. In addition to our main set of controls (book leverage, stock return, sales, return on assets, tangibility, and Tobin’s Q) our main multivariate regressions include sensitivity to oil and currency prices as controls (i.e., 1st moment controls). Data availability on implied volatility of oil and currencies restrict the start of the 2SLS sample to fiscal year 2007. Statistical significance: *** $p<0.01$, ** $p<0.05$, * $p<0.1$, † $p<0.15$. See sections 4 and 5 for the details on the construction of variables and data.
Table 19
Investment rate - 2SLS 1st Stage Results

<table>
<thead>
<tr>
<th>Specif.: Realized vol</th>
<th>Univariate Multivariate</th>
<th>Univariate Multivariate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set-up</td>
<td>All instruments together</td>
<td>Instruments individually</td>
</tr>
<tr>
<td>change vol exposure Oil</td>
<td>0.182*** 0.194***</td>
<td>0.278*** 0.295***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(6.010) (5.880)</td>
<td>(12.510) (11.720)</td>
</tr>
<tr>
<td>change vol exposure Cad</td>
<td>-0.074 -0.080</td>
<td>0.313*** 0.298***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(-1.080) (-1.100)</td>
<td>(7.680) (6.740)</td>
</tr>
<tr>
<td>change vol exposure Euro</td>
<td>-0.071 -0.067</td>
<td>0.359*** 0.381***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(-1.180) (-1.030)</td>
<td>(12.870) (12.560)</td>
</tr>
<tr>
<td>change vol exposure Jpy</td>
<td>0.182*** 0.202***</td>
<td>0.421*** 0.444***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(3.530) (3.630)</td>
<td>(13.070) (12.250)</td>
</tr>
<tr>
<td>change vol exposure Aud</td>
<td>-0.021 -0.018</td>
<td>0.357*** 0.361***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(-0.340) (-0.260)</td>
<td>(11.190) (10.230)</td>
</tr>
<tr>
<td>change vol exposure Sek</td>
<td>0.245*** 0.240***</td>
<td>0.455*** 0.449***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(3.180) (2.770)</td>
<td>(13.520) (12.020)</td>
</tr>
<tr>
<td>change vol exposure Chf</td>
<td>0.073 0.083*</td>
<td>0.373*** 0.394***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(1.420) (1.500)</td>
<td>(13.490) (12.830)</td>
</tr>
<tr>
<td>change vol exposure Gbp</td>
<td>0.139** 0.143**</td>
<td>0.444*** 0.462***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(2.220) (2.010)</td>
<td>(13.640) (12.610)</td>
</tr>
<tr>
<td>change vol exposure Policy</td>
<td>0.510*** 0.466**</td>
<td>0.351*** 0.340***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(3.110) (2.620)</td>
<td>(5.370) (4.820)</td>
</tr>
</tbody>
</table>

Observations 21,153 17,310
F-test 27.7 25.59
Hansen J Chi-sq(8) P-val 0.4944 0.3538

This table presents the first stage results of the univariate and multivariate 2SLS regression of investment rate on lagged change in realized volatility and main set of controls. Columns 1 and 2 instrument lagged changes in volatility with the benchmark set of 9 instruments (i.e., lagged changes in sensitivity to volatility of oil, 7 widely traded currencies, and economic policy uncertainty). Columns 3 and 4 instrument lagged change in volatility using only one the 9 instruments at a time. When instrumenting firm-level volatility with sensitivities to oil and/or currencies we include the price sensitivity to that corresponding commodity as control. Realized volatility is the annual volatility of the firm’s stock return, estimated as the 12-month fiscal-year standard deviation of daily CRSP returns. We annualize this standard deviation by multiplying by the square root of 252 (average trading days in a year). Statistical significance: *** p<0.01, ** p<0.05, * p<0.1, † p<0.15. See sections 4 and 5 for the details on the construction of variables and data.
## Table 20
### Additional Real Quantities

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<td>Realized</td>
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<tr>
<td><strong>Implied</strong></td>
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**Panel A: Employment**

<table>
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<tr>
<th>Volatility</th>
<th>-0.028***</th>
<th>-0.078***</th>
<th>-0.056</th>
<th>-0.019***</th>
<th>-0.070†</th>
<th>-0.151*</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>(-11.186)</td>
<td>(-12.172)</td>
<td>(-1.417)</td>
<td>(-2.802)</td>
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<td>21,152</td>
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</tr>
<tr>
<td>F-test 1st stage</td>
<td>29.18</td>
<td>25.60</td>
<td>7.939</td>
<td>10,590</td>
<td>7.939</td>
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<tr>
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<td>0.171</td>
<td>0.357</td>
<td>0.0565</td>
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</table>

**Panel B: Intangible Capital Investment**

<table>
<thead>
<tr>
<th>Volatility</th>
<th>-0.043***</th>
<th>-0.077***</th>
<th>-0.083**</th>
<th>-0.035***</th>
<th>-0.076**</th>
<th>-0.210***</th>
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</thead>
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<tr>
<td></td>
<td>(-17.472)</td>
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<td>(-2.316)</td>
<td>(-5.813)</td>
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<td>Observations</td>
<td>148,729</td>
<td>26,215</td>
<td>21,153</td>
<td>19,434</td>
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<td>F-test 1st stage</td>
<td>29.18</td>
<td>25.59</td>
<td>7.939</td>
<td>10,590</td>
<td>7.939</td>
<td></td>
</tr>
<tr>
<td>Hansen J $\chi^2(8)$</td>
<td>0.249</td>
<td>0.354</td>
<td>0.200</td>
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**Panel C: Sales**

<table>
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<tr>
<th>Volatility</th>
<th>-0.025***</th>
<th>-0.079***</th>
<th>-0.154***</th>
<th>-0.028***</th>
<th>-0.135**</th>
<th>-0.173**</th>
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<tr>
<td></td>
<td>(-8.071)</td>
<td>(-9.277)</td>
<td>(-3.120)</td>
<td>(-3.173)</td>
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</tr>
<tr>
<td>Observations</td>
<td>151,187</td>
<td>26,392</td>
<td>21,164</td>
<td>19,459</td>
<td>17,318</td>
<td>10,592</td>
</tr>
<tr>
<td>F-test 1st stage</td>
<td>29.19</td>
<td>25.37</td>
<td>7.931</td>
<td>10,592</td>
<td>7.931</td>
<td></td>
</tr>
<tr>
<td>Hansen J $\chi^2(8)$</td>
<td>0.00154</td>
<td>0.0146</td>
<td>0.409</td>
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<td></td>
</tr>
</tbody>
</table>

This table reports regression results of changes in employment (Panel A), changes in intangible capital investment (SG&A+R&D) (Panel B), and changes in Sales (Panel C), where growth rates defined as $(x_t - x_{t-1})/(0.5 * x_t + 0.5 * x_{t-1})$. Specifications 1 through 6 follow the same setup of the investment rate specifications described in Table 18. To preserve space we do not report the coefficients and t-statistics on controls. The sample period is annual from 1963 to 2014. Specifications 1, 2, and 4 are OLS regressions, while 4, 5, and 6 are 2SLS regressions. The latter instrument lagged changes in realized volatility by lagged changes in volatility exposure to energy and currency markets (measures by at-the-money implied volatility of oil and 7 widely traded currencies) and economic policy uncertainty from Baker, Bloom, and Davis (2016). We measure firm-level uncertainty in two ways: realized and implied volatility. Realized volatility is the annual volatility of the firm’s stock return, estimated as the 12-month fiscal-year standard deviation of daily CRSP returns. We annualize this standard deviation by multiplying by the square root of 252 (average trading days in a year). Implied volatility is proxied by using OptionMetrics’ 365-day implied volatility of at-the-money-forward call options. In all regressions specifications we include both firm and time fixed effects, where time dummies are defined at the fiscal-semester basis. Standard errors are clustered at the firm level. All regressors are in changes from fiscal year $t - 2$ to $t - 1$. In addition to our main set of controls (book leverage, stock return, sales, return on assets, tangibility, and Tobin’s Q) our main multivariate regressions include sensitivity to oil and currency prices as controls (i.e., 1st moment controls). Data availability on implied volatility of oil and currencies restrict the start of the 2SLS sample to fiscal year 2007. Statistical significance: *** $p<0.01$, ** $p<0.05$, * $p<0.1$, † $p<0.15$. See sections 4 and 5 for the details on the construction of variables and data.
<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
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<tr>
<td></td>
<td>OLS OLS IV</td>
<td>OLS IV IV IV</td>
<td>OLS OLS IV</td>
<td>OLS IV IV IV</td>
<td>OLS OLS IV</td>
<td>OLS IV IV IV</td>
</tr>
<tr>
<td></td>
<td>Realized</td>
<td>Implied</td>
<td>Realized</td>
<td>Realized</td>
<td>Realized</td>
<td>Realized</td>
</tr>
<tr>
<td>A: Total Debt</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volatility</td>
<td>-0.054***</td>
<td>-0.061***</td>
<td>-0.219*</td>
<td>-0.068***</td>
<td>-0.496***</td>
<td>-0.621**</td>
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<tr>
<td></td>
<td>(-7.762)</td>
<td>(-2.915)</td>
<td>(-1.737)</td>
<td>(-3.259)</td>
<td>(-3.588)</td>
<td>(-2.465)</td>
</tr>
<tr>
<td>Observations</td>
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<td>26,217</td>
<td>21,034</td>
<td>19,426</td>
<td>17,289</td>
<td>10,575</td>
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<td>F-test 1st stage</td>
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<td>7.949</td>
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</tr>
<tr>
<td>Hansen J (\chi(8))</td>
<td>0.445</td>
<td>0.347</td>
<td>0.336</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B: Payout</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volatility</td>
<td>-0.133***</td>
<td>-0.285***</td>
<td>-0.916***</td>
<td>-0.211***</td>
<td>-0.533**</td>
<td>-1.382***</td>
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<tr>
<td></td>
<td>(-14.272)</td>
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<td>(-4.429)</td>
<td>(-7.055)</td>
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<tr>
<td>Observations</td>
<td>151,465</td>
<td>26,397</td>
<td>21,164</td>
<td>19,461</td>
<td>17,318</td>
<td>10,592</td>
</tr>
<tr>
<td>F-test 1st stage</td>
<td>29.19</td>
<td>25.37</td>
<td>7.931</td>
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<td>Hansen J (\chi(8))</td>
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<td>0.00487</td>
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<tr>
<td>C: Cash holding</td>
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<td></td>
</tr>
<tr>
<td>Volatility</td>
<td>0.022***</td>
<td>-0.004</td>
<td>0.107</td>
<td>0.042**</td>
<td>0.195</td>
<td>0.406</td>
</tr>
<tr>
<td></td>
<td>(2.877)</td>
<td>(-0.192)</td>
<td>(0.853)</td>
<td>(1.994)</td>
<td>(1.298)</td>
<td>(1.271)</td>
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<td>21,108</td>
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<td>0.0902</td>
<td>0.00456</td>
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<tr>
<td>D: Short term/Long term</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volatility</td>
<td>-0.056***</td>
<td>-0.078**</td>
<td>0.196</td>
<td>-0.053</td>
<td>0.220</td>
<td>-0.029</td>
</tr>
<tr>
<td></td>
<td>(-5.021)</td>
<td>(-2.003)</td>
<td>(0.879)</td>
<td>(-1.437)</td>
<td>(0.877)</td>
<td>(-0.054)</td>
</tr>
<tr>
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<td>16,376</td>
<td>14,595</td>
<td>9,439</td>
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<tr>
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<td>0.0265</td>
<td>0.816</td>
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</table>

This table reports regression results of changes in total debt (Panel A), changes in firm payout (sum of cash dividend and share repurchase; Panel B) and changes in cash holdings (Panel C), where growth rates are defined as \((x_t - x_{t-1})/(0.5 * x_t + 0.5 * x_{t-1})\), and changes in the ratio of short- to long-term debt (Panel D). Specifications 1 through 6 follow the same setup of the investment rate specifications described in Table 18. To preserve space we do not report the coefficients and t-statistics on controls. The sample period is annual from 1963 to 2014. Specifications 1, 2, and 4 are OLS regressions, while 4, 5, and 6 are 2SLS regressions. The latter instrument lagged changes in realized volatility by lagged changes in volatility exposure to energy and currency markets (measures by at-the-money implied volatility of oil and 7 widely traded currencies) and economic policy uncertainty from Baker, Bloom, and Davis (2016). We measure firm-level uncertainty in two ways: realized and implied volatility. Realized volatility is the annual volatility of the firm’s stock return, estimated as the 12-month fiscal-year standard deviation of daily CRSP returns. We annualize this standard deviation by multiplying by the square root of 252 (average trading days in a year). Implied volatility is proxied by using OptionMetrics’ 365-day implied volatility of at-the-money-forward call options. In all regressions specifications we include both firm and time fixed effects, where time dummies are defined at the fiscal-semester basis. Standard errors are clustered at the firm level. All regressors are in changes from fiscal year \(t - 2\) to \(t - 1\).
Table 22
2SLS Sensitivity to Individual Instruments

<table>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
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</thead>
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<td></td>
<td>None</td>
<td>Oil</td>
<td>Cad</td>
<td>Euro</td>
<td>Jpy</td>
<td>Aud</td>
<td>Sek</td>
<td>Chf</td>
<td>Gbp</td>
<td>Policy</td>
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<td>Real Variables</td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>Investment Rate</td>
<td>-0.062**</td>
<td>-0.077**</td>
<td>-0.065**</td>
<td>-0.064**</td>
<td>-0.057*</td>
<td>-0.061*</td>
<td>-0.071**</td>
<td>-0.060*</td>
<td>-0.059*</td>
<td>-0.063**</td>
</tr>
<tr>
<td>Employment</td>
<td>-0.070†</td>
<td>-0.103**</td>
<td>-0.068†</td>
<td>-0.074†</td>
<td>-0.063</td>
<td>-0.070†</td>
<td>-0.072†</td>
<td>-0.071†</td>
<td>-0.077*</td>
<td>-0.042</td>
</tr>
<tr>
<td>Intangible Capital</td>
<td>-0.076**</td>
<td>-0.084*</td>
<td>-0.079**</td>
<td>-0.077**</td>
<td>-0.063†</td>
<td>-0.074*</td>
<td>-0.083**</td>
<td>-0.078**</td>
<td>-0.077**</td>
<td>-0.103**</td>
</tr>
<tr>
<td>Sales</td>
<td>-0.135**</td>
<td>-0.159***</td>
<td>-0.135**</td>
<td>-0.137**</td>
<td>-0.126**</td>
<td>-0.134**</td>
<td>-0.114**</td>
<td>-0.141**</td>
<td>-0.142**</td>
<td>-0.179***</td>
</tr>
<tr>
<td>Financial Variables</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Debt Total</td>
<td>-0.496***</td>
<td>-0.493***</td>
<td>-0.513***</td>
<td>-0.498***</td>
<td>-0.485***</td>
<td>-0.491***</td>
<td>-0.477***</td>
<td>-0.504***</td>
<td>-0.500***</td>
<td>-0.461***</td>
</tr>
<tr>
<td>Payout</td>
<td>-0.533**</td>
<td>-0.781***</td>
<td>-0.531**</td>
<td>-0.531**</td>
<td>-0.489**</td>
<td>-0.535**</td>
<td>-0.512**</td>
<td>-0.521**</td>
<td>-0.581**</td>
<td>-0.488**</td>
</tr>
<tr>
<td>Cash Holdings</td>
<td>0.195</td>
<td>0.204</td>
<td>0.210</td>
<td>0.201</td>
<td>0.217</td>
<td>0.189</td>
<td>0.211</td>
<td>0.179</td>
<td>0.175</td>
<td>0.272*</td>
</tr>
<tr>
<td>Short/Long Term</td>
<td>0.220</td>
<td>0.267</td>
<td>0.216</td>
<td>0.221</td>
<td>0.210</td>
<td>0.217</td>
<td>0.199</td>
<td>0.165</td>
<td>0.279</td>
<td>0.199</td>
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<td>Investment Stats</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>17,310</td>
<td>17,610</td>
<td>17,310</td>
<td>17,310</td>
<td>17,310</td>
<td>17,310</td>
<td>17,310</td>
<td>17,310</td>
<td>17,310</td>
<td>19,081</td>
</tr>
<tr>
<td>F-test of 1st stage</td>
<td>25.59</td>
<td>24.38</td>
<td>28.45</td>
<td>28.71</td>
<td>27.54</td>
<td>28.83</td>
<td>28.09</td>
<td>28.50</td>
<td>27.80</td>
<td>29.46</td>
</tr>
<tr>
<td>Hansen J Chi-sq P</td>
<td>0.354</td>
<td>0.373</td>
<td>0.469</td>
<td>0.346</td>
<td>0.288</td>
<td>0.383</td>
<td>0.573</td>
<td>0.328</td>
<td>0.301</td>
<td>0.0739</td>
</tr>
</tbody>
</table>

This table presents 2SLS regression results using all main controls but dropping individual instruments one at a time from the benchmark set of 9 instrumental variables (IVs). Results in (1) are the benchmark 2SLS multivariate results (on realized volatility) presented in column (5) in Tables 18 and 21. The sample is annual from 2007 to 2014. The statistics under "Investment Stats" correspond to the 1st stage results of the multivariate 2SLS regression of investment rate on lagged change in volatility and main set of controls. 1st Stage statistics for other real and financial estimations are largely comparable to their benchmark specifications with the full set of instruments. In all regressions specifications we include both firm and time fixed effects, where time dummies are defined at the fiscal-semester basis. Standard errors are clustered at the firm level. Statistical significance: *** p<0.01, ** p<0.05, * p<0.1, † p<0.15. See sections 4 and 5 for the details on the construction of variables and data.
Table 23
Vol. Impact on Financially Constrained and Unconstrained Firms

<table>
<thead>
<tr>
<th>2008-2010</th>
<th>Investment</th>
<th>Employm.</th>
<th>Intang. Capital</th>
<th>Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: S&amp;P Credit Ratings</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volatility</td>
<td>-0.025</td>
<td>-0.106**</td>
<td>-0.071†</td>
<td>-0.119†</td>
</tr>
<tr>
<td>(0.628)</td>
<td>(-2.368)</td>
<td>(-1.517)</td>
<td>(-1.441)</td>
<td></td>
</tr>
<tr>
<td>Vol.*Finan. Constraint</td>
<td>-0.198***</td>
<td>0.034</td>
<td>-0.154**</td>
<td>-0.340***</td>
</tr>
<tr>
<td>(-2.886)</td>
<td>(0.462)</td>
<td>(-2.264)</td>
<td>(-2.801)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>2,857</td>
<td>2,829</td>
<td>2,602</td>
<td>2,857</td>
</tr>
<tr>
<td>F-test of 1st stage</td>
<td>14.26</td>
<td>13.42</td>
<td>13.28</td>
<td>14.26</td>
</tr>
<tr>
<td>Hansen J Chi-sq P-val</td>
<td>0.653</td>
<td>0.150</td>
<td>0.0729</td>
<td>0.0393</td>
</tr>
<tr>
<td>B: Employment</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volatility</td>
<td>-0.089**</td>
<td>-0.107**</td>
<td>-0.108**</td>
<td>-0.230**</td>
</tr>
<tr>
<td>(-2.030)</td>
<td>(-2.070)</td>
<td>(-2.075)</td>
<td>(-2.465)</td>
<td></td>
</tr>
<tr>
<td>Vol.*Finan. Constraint</td>
<td>-0.111***</td>
<td>-0.001</td>
<td>-0.064*</td>
<td>-0.192***</td>
</tr>
<tr>
<td>(-3.207)</td>
<td>(-0.022)</td>
<td>(-1.717)</td>
<td>(-2.814)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>2,857</td>
<td>2,829</td>
<td>2,602</td>
<td>2,602</td>
</tr>
<tr>
<td>F-test of 1st stage</td>
<td>14.26</td>
<td>13.42</td>
<td>13.28</td>
<td>13.28</td>
</tr>
<tr>
<td>Hansen J Chi-sq P-val</td>
<td>0.807</td>
<td>0.123</td>
<td>0.0934</td>
<td>0.0386</td>
</tr>
<tr>
<td>C: Assets</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volatility</td>
<td>-0.059</td>
<td>-0.083†</td>
<td>-0.104**</td>
<td>-0.113</td>
</tr>
<tr>
<td>(-1.353)</td>
<td>(-1.631)</td>
<td>(-2.125)</td>
<td>(-1.251)</td>
<td></td>
</tr>
<tr>
<td>Vol.*Finan. Constraint</td>
<td>-0.105**</td>
<td>0.042</td>
<td>-0.100**</td>
<td>-0.074</td>
</tr>
<tr>
<td>(-2.541)</td>
<td>(0.893)</td>
<td>(-2.221)</td>
<td>(-0.886)</td>
<td></td>
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<tr>
<td>Observations</td>
<td>2,081</td>
<td>2,062</td>
<td>1,919</td>
<td>2,081</td>
</tr>
<tr>
<td>F-test of 1st stage</td>
<td>18.31</td>
<td>17.91</td>
<td>18.72</td>
<td>18.31</td>
</tr>
<tr>
<td>Hansen J Chi-sq P-val</td>
<td>0.627</td>
<td>0.517</td>
<td>0.369</td>
<td>0.0889</td>
</tr>
</tbody>
</table>

This table presents multivariate 2SLS estimates from panel regressions explaining investment rate, changes in employment, changes in intangible capital investment, and changes in sales for financially constrained and unconstrained firms, during the financial crisis of 2008. Sample period is for fiscal years 2008 to 2010. In addition to instrumenting changes in firm-level annual realized volatility, we instrument the interaction of this volatility with a dummy that proxies for whether a firm was ex-ante financially constrained or not in fiscal year 2005. We define this dummy in terms of credit ratings and firm size. For bond ratings, we use S&P rating on long-term debt, and consider a firm to have been ex-ante financially constrained if it had positive debt and no bond rating in 2005 and unconstrained otherwise (which includes firms with zero debt and no debt rating). In terms of size, a firm is considered to have been financially constrained if it was a small firm and unconstrained if a big firm in 2005, were small are firms in tercile 1 and big are firms in tercile 3 with respect to 2005 sample size percentiles (where size is measured by employees and total assets). Firms in the median tercile are ignored. In all regressions specifications we include both firm and time fixed effects, where time dummies are defined at the fiscal-semester basis. Standard errors are clustered at the firm level. All regressors are in changes from fiscal year \( t - 2 \) to \( t - 1 \). In addition to our main set of controls we further include sensitivity to oil and currency prices.
Recent models with (Kreps and Porteus, 1978) recursive preferences highlight the role of long-run mean risk in accounting for many stylized facts in international finance and international macroeconomics. These recursive preferences are a special case of the generalized disappointment-aversion preferences (GDA) recently introduced by Routledge and Zin, 2010. I examine GDA preferences in a setting that exhibits both country-specific and global economic uncertainty in a fully specified production economy. By nature of the GDA preferences, economic uncertainty plays a more important role than long-run mean risk and is the main source behind time-variation in the probability of disappointing economic outcomes. This time-varying probability drives productivities, risk-sharing arrangements between representative agents, and exchange rate movements. The model highlights how domestic and international uncertainty is relevant for asset prices and cross-border flows. I present the model with two countries, two goods, and asymmetric investment frictions.
1. Introduction

Recent models involving long-run risk and recursive preferences have advanced our understanding on some of the most long-standing empirical facts in international finance. The models in Colacito and Croce 2013 and Colacito, Croce, Ho, and Howard 2013 with recursive Kreps-Porteus preferences (Kreps and Porteus 1978), as in Epstein and Zin 1989, highlight the role of long-run risk for international finance puzzles but are silent about the role of economic uncertainty. Moreover, by virtue of the Kreps-Porteus preferences, the results of the aforementioned models are sensitive to particular parameter values, some of which have been a source of lively debate in the literature. This paper aims at circumventing these limitations by extending the recursive preferences to represent the generalized disappointment-aversion preferences (GDA) recently introduced by Routledge and Zin 2010 and studied in a closed economy model with long-run risks in Bonomo, Garcia, Meddahi, and Tedongap 2011. By nature of the GDA preferences, economic uncertainty plays a more important role than long-run mean risk and is the main source behind time-variation in the probability of disappointing economic outcomes. This time-varying probability drives productivities, risk-sharing arrangements between representative agents, and exchange rate movements. The model highlights how domestic and international uncertainty is relevant for asset prices and cross-border flows. A two-country and a two-good model with GDA preferences, production, country-specific economic uncertainty in productivity growth, and asymmetric investment frictions is proposed.

With Kreps-Porteus preferences, progress has been made on the front of long-standing questions such as why do high interest rate currencies tend to appreciate over time? Known as the forward premium anomaly Fama 1984, which contradicts the uncovered interest rate parity condition (UIP). Why is risk sharing so low across

\[1\text{The logic behind the UIP condition is that when the foreign interest rate is higher than the U.S interest rate, risk-neutral and rational U.S investors should expect the foreign currency to depreciate against the dollar by the difference between the two interest rates. This way, borrowing at home and} \]
nations? That is, why are consumption growth correlations not higher among nations? Why are exchange rate differences not correlated with consumption growth differences across countries (Backus and Smith, 1993)? The latter two questions arise from a canonical model with complete markets and standard time-additive preferences.

Colacito and Croce (2013) successfully tackle these stylized facts in an endowment economy model with Kreps-Porteus preferences, as in the recursive utility in Epstein and Zin (1989) that separate risk aversion from intertemporal substitution. Building on the long-run risk literature (Bansal and Yaron, 2004), these preferences are combined with a small long-run component driving country-specific endowment growth. While with standard time-additive preferences only short-run risk is compensated, with Kreps-Porteus preferences long-run risks earn a positive risk premium provided investors prefer early resolution of uncertainty. In this setting, agents in the two countries look for a risk-sharing arrangement that allows them to smooth future utility in addition to short term consumption. Moreover, agents care not only about future expected utility but also about future utility variance. This mean-variance tradeoff drives international allocations and exchange rate adjustments, and ultimately provide an explanation to some of the long-standing questions in finance that originated from standard preferences. More recently, Colacito et al., 2013 extend the model in Colacito and Croce, 2013 to a fully specified two-country production economy. They show that country-specific long-run components in the mean of productivity growth, combined with Kreps-Porteus preferences, are able to also address the quantity anomaly documented by Kehoe and Kydland, 1992 (a long-standing puzzle in international macroeconomics). As in the data, Colacito et al., 2013 show that the model is able to produce a smaller correlation of consumption across countries than that of their outputs. The mechanisms at work explaining the aforementioned ques-

lending abroad, or vice versa, produces a zero return in excess of the U.S short-term interest rate. This is known as the UIP condition, and it is violated in the data, except in the case of very high inflations currencies (for an extensive examination of currencies, see Lustig and Verdelhan (2007)).
tions arise from the combination of Kreps-Porteus preferences and long-run components in the mean of output growth.

[Routledge and Zin, 2010] recently introduced preferences that generalize the disappointment aversion preferences of [Gul, 1991] and embedded them in the recursive utility framework of [Epstein and Zin, 1989]. Outcomes that lie below a disappointment threshold represent a utility cost. Compared to expected utility, GDA overweighs outcomes below this disappointment threshold, which is set at a fraction of the certainty equivalent of future utility. Moreover, GDA preferences introduce an endogenous variation in the probability of disappointment. As such, counter-cyclical risk aversion arises endogenously when the probability of disappointing outcomes in recessions is higher than in booms. In a simplified [Mehra and Prescott, 1985] two-state endowment setting, [Routledge and Zin, 2010] show that recursive utility with GDA preferences generates a large equity premium with counter-cyclical risk aversion. As in [Bansal and Yaron, 2004], [Bonomo et al., 2011] extend the GDA investigation to a closed-economy endowment setting with long-run risks in the mean and volatility of consumption and dividend growth. More recently, [Liu and Miao, 2014] incorporate GDA preferences in a closed-economy with production and regime switching in the conditional mean and volatility of productivity growth. They show that volatility risk in productivity growth carries a positive and sizable risk premium, albeit a procyclical dividend process is also necessary to attain an equity premium commensurate with U.S. historical data.

This study examines international asset prices in a world production economy with GDA preferences and economic uncertainty (as measured by the fluctuating conditional volatility of country-specific productivity growth). Examining the implications of these two features on international asset prices and economic flows is novel to this paper. GDA preferences incorporate a risk-premium component not present in the Kreps-Porteus preferences. This additional term in the GDA stochastic dis-
count factor captures time-varying disappointment aversion. It has two key features important for this study: First, it does not depend on the relation between the EIS and the relative risk aversion parameter. A relation that is crucial in the recent work of [Colacito and Croce 2013]. Particularly, they show that a relatively large preference for early resolution of uncertainty is important in order to produce quantitative results consistent with (any) stylized facts in international finance. Second, economic uncertainty is the primary driving source determining the time-varying probabilities of disappointing outcomes (the main mechanism in GDA preferences). Thus, in contrast to models with Kreps-Porteus preferences, it is not long-run risk in the mean of productivity growth that drive the key mechanisms able to explain international stylized facts but rather the long-lasting effects of economic uncertainty. This is not to say that long-run risk in the mean does not play a role, the extent that it does in the presence of economic uncertainty will be examined, but that it no longer is sufficient (or even necessary) under GDA preferences.

This study also departs from recent related work in how home and foreign investment is modeled. Investment in physical capital is subject to convex adjustment costs. The representative firm in each country finds it increasingly difficult to build up capital. Moreover, general intuition would say that a firm deciding to adjust its capital stock would find it more costly to use foreign investment goods than locally-available investment goods. Thus, in the model the firm finds it more costly to build up capital using foreign resources than local ones. In addition, this study also pursues an appropriate and far more accurate solution method than that used in recent related work. Specifically, utility under GDA preferences is not-differentiable at the point where disappointment sets in. Therefore, one cannot rely on log-linear approximations or perturbation methods to find an approximate solution to the model (as done in [Colacito and Croce 2013] and [Colacito et al., 2013]). Thus, to solve the model here, the numerically stable and accurate simulation-version of the parameterized
expectations approach (PEA) proposed recently by Judd, Maliar, and Maliar [2011] is pursued. Judd et al. [2011] show that their generalized stochastic simulation algorithm (GSSA) delivers accuracy levels comparable to the best accuracy attained in the literature, while being tractable in high-dimension applications.

The rest of the paper is organized as follows. Section 2 presents the model. A subsection is devoted to the details of the GDA preferences. Section 3 discusses the model solution. It presents optimality conditions and asset pricing equations. A subsection there explains the numerical method proposed to solve the model. Section 4 concludes and highlights the subsequent steps of this working paper.

2. The Model

Time in this model is discrete. The world economy consists of two countries, home ($h$) and foreign ($f$), each populated by identical infinitively lived agents. Each country produces a different good with its own technology, using labor and physical capital as factors of production. Markets are complete in state-contingent claims. As such, the general economic environment is similar to the workhorse world economy model of Backus, Kehoe, and Kydland [1994] (hereafter BKK), which in turn is a two-country extension of the closed economy of Kydland and Prescott [1982].

The details of the model are presented as follows. First, the generalized disappointment aversion preferences (GDA) and associated stochastic discount factors are introduced. Second, the consumption bundle and the technologies underlying the production of the two goods are described. Here economic uncertainty affecting productivity and the investment frictions faced by the representative firms are discussed. Third, the world resource constraints and national accounts are presented. Finally, the relation between the real exchange rates and the stochastic discount factors in each country are explained. This relation follows from the complete markets assump-
2.1. **Generalized Disappointment Aversion**

[Routledge and Zin, 2010](#) generalize the disappointment-aversion preferences of [Gul, 1991](#) and embed them in the recursive utility framework with a representative agent of [Epstein and Zin, 1989](#). It is well known that the latter recursive preferences (that build on [Kreps and Porteus, 1978](#)) have an expected utility certainty equivalent that disentangles the degree of risk aversion from the elasticity of intertemporal substitution. This certainty equivalent is modified when one incorporates the generalized disappointment aversion (GDA) preferences of [Routledge and Zin, 2010](#). Formally, for \( i = \{ h, f \} \) let \( U_{i,t} \) represent the recursive intertemporal utility functional:

\[
U_{i,t} = \begin{cases} 
(1 - \beta)C_{i,t}^{-\frac{1}{\psi}} + \beta [R_{i,t}(U_{i,t+1})]^{-\frac{1}{1-\gamma}} & \text{if } \psi \neq 1 \\
C_{i,t}^{1-\beta} [R_{i,t}(U_{i,t+1})]^\beta & \text{if } \psi = 1
\end{cases}
\]

(1)

in which \( C_{i,t} \) is the current consumption bundle (described below) of the corresponding representative agent, \( \beta \in (0, 1) \) is the subjective discount factor, \( \psi > 0 \) is the elasticity of intertemporal substitution (EIS), and \( R_{i,t}(U_{i,t+1}) \) is the certainty equivalent of random future utility conditional on the current period information set. This certainty equivalent can be understood as a risk adjustment to the date \( t + 1 \) continuation value of a consumption plan, and is specified as follows (e.g., [Heaton and Li, 2008](#)):

\[
R_{i,t}(U_{i,t+1}) \equiv E_t[(U_{i,t+1})^{1-\gamma} \mid \mathcal{F}_t]^{1/(1-\gamma)}
\]

(2)

where \( \mathcal{F}_t \) is the information set available at time \( t \) and \( U_{i,t+1} \) is the Kreps-Porteus preference continuation value of utility.

---

2Whereas, standard time-additive preferences create an inversely proportional link between risk aversion and intertemporal substitution.
With GDA preferences, however, the certainty equivalent \( R_{i,t} \) is defined implicitly as:

\[
\frac{R_{i,t}(U_{i,t+1})^{1-\gamma}}{1-\gamma} = E_t \left[ \frac{U_{i,t+1}^{1-\gamma}}{1-\gamma} \right] - (\alpha^{-1} - 1) \\
\times E_t \left[ I \left( \frac{U_{i,t+1}}{R_{i,t}(U_{i,t+1})} < \nu \right) \left[ \frac{(\nu R_{i,t}(U_{i,t+1}))^{1-\gamma}}{1-\gamma} - \frac{U_{i,t+1}^{1-\gamma}}{1-\gamma} \right] \right]
\]

where \( I(\cdot) \) is an indicator function that takes a value of one when the underlying condition is satisfied (0 otherwise), and the parameter \( \nu \in (0,1] \) captures the percentage of the certainty equivalent \( R \) at which all outcomes below this threshold are considered disappointing. The special case of disappointment aversion in Gul, 1991 occurs when \( \nu = 1 \). The parameter \( \alpha \in (0,1] \) measures the degree of penalty the agent imposes on the disappointing outcomes. Thus, when \( \alpha \) is unity no penalty is imposed and \( R \) is equivalent to the certainty equivalent of expected utility in (2).

When \( \alpha < 1 \), there is a utility cost for all outcomes below the threshold \( \nu R \). These outcomes receive an extra weight \( (\alpha^{-1} - 1) \) and decrease the certainty equivalent \( R \). Moreover, given the recursive nature of the intertemporal utility, it is clear that both the disappointment threshold, \( \nu R \), and the certainty equivalent are time-varying. Lastly, the implicit function for \( R \) represents a root-finding problem, or equivalently a fixed-point problem.

Following Heaton and Li, 2008, the Kreps-Porteus preferences give rise to the following stochastic discount factor (SDF) in each country \( i = \{h,f\} \):

\[
M_{i,t,t+1} = \beta \left( \frac{C_{i,t+1}}{C_{i,t}} \right)^{-\frac{1}{\psi}} \left( \frac{U_{i,t+1}}{R_{i,t}(U_{i,t+1})} \right)^{\frac{1}{\psi} - \gamma}
\]

The familiar and special case of time-separable constant relative risk aversion (RRA) occurs when \( \gamma = \frac{1}{\psi} \). In which case risk is short-run and directly proportional to consumption growth, as in the asset pricing models of Lucas, 1978 and Breeden.
The continuation value relative to its risk adjustment, \( \left( \frac{U_{t+1}}{R_{t+1}(U_{t+1})} \right) \), triggers a premium when \( \gamma \geq \frac{1}{\psi} \). This risk premium is long-run in nature as it depends on the forward looking continuation value of a consumption plan. Thus, the second term in (4) plays a key role in the production economy of Colacito et al., 2013 as it picks up the "long-run risk" in innovations to their assumed highly persistent productivity growth process, where lifetime utility measures the long-run. The long-run risks are key in their international asset pricing implications. Note that when the difference between \( \gamma \) and \( \frac{1}{\psi} \) is small, the long-run component in the SDF (the second term) becomes small in magnitude, making its premium negligible. The difference between \( \gamma \) and \( \frac{1}{\psi} \) determines the preference for early resolution of uncertainty about continuation values. When \( \gamma \) is (substantially) larger than \( \frac{1}{\psi} \), any variation in the long-run risk component will produce a non-negligible effect over the pricing kernel \( M_{i,(t,t+1)} \) in (4). This is where the assumptions on the parameter values in Colacito and Croce, 2013 are key. Their choices of \( \gamma (=8) \) and \( \frac{1}{\psi} (=1/1.5) \) are such that the premium captures long-run risk in the mean of productivity growth. Reducing \( \gamma \) or slightly varying \( \psi \) breaks down the ability of the model in reproducing moments commensurate with historical data. For example, when the EIS is lowered to around 1 or below 1, their equity premium declines monotonically and becomes drastically negative, the risk-free rates become too high and too volatile, and the model no longer produces results consistent with any of the stylized fact in international finance.

Moreover, one may ask what value is appropriate for EIS. The answer is inconclusive and subject to a lively debate in the literature. Hansen, Heaton, Lee, and Roussanov, 2007 and Bansal, Kiku, and Yaron, 2009 report empirical evidence in favor of a value greater than 1. However, Beeler and Campbell, 2009, Hall, 1988 and Campbell, 1999 estimate an elasticity below 1.

\(^3\)For example, specification (5) with an EIS of 0.67 produces first moments of the risk free rate and equity premium of 5.40% and -8.34% per annum, respectively, and a negative correlation of 41% between consumption growth differences and exchange rate growth.
Moving along, the stochastic discount factor that ultimately interest this paper arises from the more general GDA preferences. Thus, building on the exposition made in the closed economy setting of Bonomo et al. 2011, the pricing kernels for the home and foreign countries \( i = \{ h, f \} \) are:

\[
M_{i,t,t+1} = \beta \left( \frac{C_{i,t+1}}{C_{i,t}} \right)^{-\frac{1}{\psi}} \left( \frac{U_{i,t+1}}{R_{i,t}(U_{i,t+1})} \right)^{\frac{1}{\psi} - \gamma} \\
\times \left( \frac{1 + (\alpha^{-1} - 1)I\left( \frac{U_{i,t+1}}{R_{i,t}(U_{i,t+1})} < \nu \right)}{1 + \nu^{1-\gamma}(\alpha^{-1} - 1)E_t[I\left( \frac{U_{i,t+1}}{R_{i,t}(U_{i,t+1})} < \nu \right)]} \right),
\]

where the risk adjustment, \( R_{i,t} \), is defined in (3). Thus, relative to the pricing kernels of the Kreps-Porteus preferences in (4) we now have an additional long-run risk premium component (the third term) that spawns whenever the ratio of the continuation value of utility to its risk adjustment is less than the disappointment threshold reference point \( \nu \). In the special case of the disappointment aversion of Gul 1991 this threshold point is one (i.e., \( \nu = 1 \)). In addition, when the penalty parameter \( \alpha = 1 \), there is no penalty on disappointing outcomes, the new long-run risk premium term is shut down (constant and equal to 1), and the GDA SDF reduces to the familiar Kreps-Porteus SDF, as in the recursive utility of Epstein and Zin 1989.

Routledge and Zin 2010 refer to the new long-run term as a ”decision weight”, which in the case of expected utility these weights are simply the (conditional) probabilities. With GDA preferences, however, decision weights behave as probabilities tilted toward disappointing outcomes. Recall from the analysis done on (3), the GDA certainty equivalent \( R \) is time varying, and so is the disappointment threshold \( \nu R \). Thus, time variation in the decision weights in (5) arises from time variation in the probabilities of disappointing outcomes (i.e., from the variation in the disappointment threshold \( \nu R \)). Thus, GDA preferences have an additional channel through which counter-cyclical risk aversion may be induced. In particular, effective counter-cyclical
risk aversion can arise endogenously when the probability of disappointing outcomes in recessions is higher than in boom times.

Time-varying effective risk aversion highlights a mechanism at work. From the GDA SDF in (5) note that an increase in the volatility of future utility $U_{i,t+1}$ will make disappointing outcomes more probable, which in turn increase the variation in the third term in (5). Consequently, the SDF becomes more volatile as well. Bonomo et al., 2011 employ this mechanism to study asset pricing implications in a closed endowment economy. They induce a higher volatility of future utility by proposing a model that exhibits persistent time-varying volatilities of consumption and dividend growth (i.e., long-run risk in volatility). This persistent economic uncertainty (as measured by persistent volatilities in consumption and dividend growth) combined with generalized disappointment-averse investors are sufficient to generate an equity premium commensurate with historical data, as well as a low and stable risk-free rate and reasonable first and second moments of the price-dividend ratio.

For the purposes of this study, it is important to note that if the difference between $\gamma$ and $1/\psi$ is small, the effect of the second-term on the GDA SDF will be small (thus hindering the results in Colacito et al., 2013). In this case, risk-sharing allocations will depend on the third term, which as show in Bonomo et al., 2011 is more sensitive to the long-lasting effect of economic uncertainty than the long-run risk in the mean of productivity.

To close this section, throughout the paper, the representative agents of both home and foreign countries are assumed to have the same $\gamma$, $\psi$, subjective discount factor, $\beta$, and generalized disappointment aversion parameters, $\alpha$ and $\kappa$.

2.2. Consumption Bundle

The recursive utility $U_t$, in (1), shows that agents $i = \{h, f\}$ value time $t$ consumption bundles, $C_{i,t}$. These consumption bundles are composite goods of the home and
foreign produced goods, $X_t$ and $Y_t$, respectively. As in BKK, the consumption bundle aggregate takes the form of a constant elasticity of substitution (CES) composite good that is standard in equilibrium trade models:

$$
C_{h,t} = \left[ \lambda x_{h,t}^{1 - \frac{1}{\eta}} + (1 - \lambda) y_{h,t}^{1 - \frac{1}{\eta}} \right]^{\frac{1}{1 - \frac{1}{\eta}}}, \quad C_{f,t} = \left[ (1 - \lambda)x_{f,t}^{1 - \frac{1}{\eta}} + \lambda y_{f,t}^{1 - \frac{1}{\eta}} \right]^{\frac{1}{1 - \frac{1}{\eta}}}
$$

(6)

where $x_{i,t}$ and $y_{i,t}$ denote the consumption of good $X_t$ and good $Y_t$ in country $i = \{h, f\}$ at date $t$. The consumption elasticity of substitution between foreign and domestic goods is specified by $\eta$. The consumption aggregate also specifies a preference for the consumption of the local good over the foreign consumption good. Calibrating $\lambda$ to be larger than $\frac{1}{2}$ implies a consumption aggregate preference in favor of the local consumption good. Colacito et al., 2013 argue that this consumption aggregate preference represents a home bias in consumption (and is crucial for their results). However, from a general equilibrium perspective, true home bias in consumption should arise from the time-series consumption choices made by the representative agents. As such, equilibrium home bias in consumption for the home consumer would rise when, on average, the agent chooses to consume more of the home consumption good, $x_{h,t}$, than the foreign good, $y_{h,t}$. Showing the mechanisms behind an equilibrium home bias in consumption would be novel in the literature. It is an objective of this paper.

BKK specify a local consumption preference of around $\lambda = 88\%$, while Colacito et al., 2013 use a $\lambda = 97\%$ to attain their best asset pricing results. Moreover, Colacito et al., 2013 differ from BKK in allowing the degree of preference for local consumption to differ from the degree of technological preference (explained below) for the use of local investment goods over foreign investment goods (which is otherwise equal in BKK). As for empirical evidence, Gust, 2008 find that the U.S levels of home-bias in consumption and in investment differ vastly. Specifically, they document that foreign
consumption goods represent only 3% to 5% of the U.S consumption bundle, while foreign investment goods represent about 40% of U.S aggregate investment.

2.3. Aggregate Productivity

To capture time-varying macroeconomic uncertainty, the productivity process of each country exhibits stochastic volatility. Specifically, the home productivity process, $A_{h,t}$ follows:

$$
\log A_{h,t} = \rho_A \log A_{h,t-1} + \sigma_{h,t} \varepsilon_{h,a,t} \\
\sigma_{h,t}^2 = (1 - \rho_\sigma)\mu_\sigma + \rho_\sigma \sigma_{h,t-1}^2 + \varepsilon_{h,\sigma,t}
$$

and the foreign productivity process, $A_{f,t}$, follows:

$$
\log A_{f,t} = \rho_A \log A_{f,t-1} + \sigma_{f,t} \varepsilon_{f,a,t} \\
\sigma_{f,t}^2 = (1 - \rho_\sigma)\mu_\sigma + \rho_\sigma \sigma_{f,t-1}^2 + \varepsilon_{f,\sigma,t}
$$

where $\varepsilon_{i,a,t}$ is an innovation to the production technology of country $i = \{h, f\}$, and $\varepsilon_{i,\sigma,t}$ is an innovation to the standard deviation of the same technology. The two productivity processes are stationary (i.e., they exhibit no deterministic or stochastic growth trend). To the extent of international models with recursive preferences and production, examining the role of country-specific stochastic volatility, $\sigma_{i,t}$, is novel to this paper. It is modeled as a slowly reverting persistent AR1 process (i.e., $\rho_\sigma$ smaller but close to one).

The productivity processes have two features worth highlighting. First, shocks in the volatility of productivity, $\varepsilon_{i,\sigma,t}$, have enduring effects on economic uncertainty into the future. Increases in uncertainty, in turn, raise the volatility of future utility as a consequence of a now more volatile consumption profile into the future. At the
same time, the probability of disappointing future outcomes is larger. Consequently, the variation of the third term in the SDF of the GDA preferences (5) increases, and along with it also the variation of the SDF itself. A sufficiently volatile SDF helps tackle asset pricing stylized facts. Second, short-run shocks, $\varepsilon_{i,a,t}$, have a short-lived effect on country-specific productivity growth, and thus short-lasting effects on the growth rates of the two goods, $X_t$ and $Y_t$. Impulse-response analyses to each of the different shocks will be central to this paper. They will shed light on the mechanisms at work influencing both international asset prices and international macroeconomic quantities.

Shocks are jointly log-normally distributed. Their characteristics are as follows:

$$\varepsilon_t = [\varepsilon_{h,a,t} \ \varepsilon_{f,a,t} \ \varepsilon_{h,\sigma,t} \ \varepsilon_{f,\sigma,t}] \sim i.i.d \ N(0, \Sigma),$$

with covariance matrix:

$$\Sigma = \begin{bmatrix}
\sigma_{sr}^2 & \rho_{sr}\sigma_{sr}^2 & 0 & 0 \\
\rho_{sr}\sigma_{sr}^2 & \sigma_{sr}^2 & 0 & 0 \\
0 & 0 & \sigma_{sv}^2 & \rho_{sv}\sigma_{sv}^2 \\
0 & 0 & \rho_{sv}\sigma_{sv}^2 & \sigma_{sv}^2
\end{bmatrix},$$

where the subscripts $sr$ and $sv$ denote the short run component of productivity and stochastic volatility, respectively.

Economic uncertainty is pervasive in the world economy. As such, $\rho_{sv} \in (0, 1)$ is set to a large value to capture a large correlation between the stochastic volatility of each country. Following BKK, the world economy exhibits low correlations of short-run shocks across countries (i.e., low $\rho_{sr}$).

?? extends the productivity process of both countries to include country-specific long-run risk components present in Colacito et al., 2013. As such, the processes discussed above in (7) and (8) are special cases. Recall that the quantitative results
in [Colacito et al., 2013] depend heavily on combining long-run risks with [Epstein and Zin, 1989] recursive utility. Thus, a natural goal of this study is twofold: i) show that in the presence of GDA recursive preferences and stochastic volatility, long-run risks are no longer necessary to address international asset pricing facts and ii) provide new economic insights behind these stylized facts.

2.4. Aggregate Investment and Capital Formation

Investment at time $t$ is a bundle of domestic and foreign produced investment goods. Similar to the consumption bundles in (6), the investment bundles take the form of a CES composite good. Specifically, total aggregate investment, $I_{i,t}$, in each country $i = \{h, f\}$ is:

$$I_{h,t} = \left[ \nu I_{h,h,t}^{1-\frac{1}{\xi}} + (1 - \nu) I_{f,h,t}^{1-\frac{1}{\xi}} \right]^{\frac{1}{1-\frac{1}{\xi}}}$$

$$I_{f,t} = \left[ (1 - \nu) I_{h,f,t}^{1-\frac{1}{\xi}} + \nu I_{f,f,t}^{1-\frac{1}{\xi}} \right]^{\frac{1}{1-\frac{1}{\xi}}}$$

(11)

where from a home (foreign) country perspective, $I_{h,h,t}(I_{f,f,t})$ measures real local investment, while $I_{h,f,t}(I_{f,h,t})$ measures real investment abroad. The investment bundle elasticity of substitution between foreign and domestic investment is captured by $\xi$.

From the perspective of a domestic firm, technological preference for the use of local investment over foreign investment goods is captured by the parameter $\nu$. As such, calibrating $\nu$ to be larger than $\frac{1}{2}$ implies a technological investment preference in favor of local investment goods. [Colacito et al., 2013] argue that this technological preference represents a home bias in investment. However, from a general equilibrium perspective, home bias in investment should arise from the time-series investment choices made by a firm. As such, equilibrium home bias in investment for the home firm would rise when, on average, the firm chooses to utilize more of the home investment good, $I_{h,h,t}$, than the foreign good, $I_{f,h,t}$. As with the earlier discussion on
consumption home-bias, showing the mechanisms behind an equilibrium home bias in investment would be novel in the literature. It is an objective of this paper.

The stock of physical capital in each country evolves according to the following laws of motion:

\[
K_{h,t+1} = (1 - \delta)K_{h,t} + I_{h,t} \quad K_{f,t+1} = (1 - \delta)K_{f,t} + I_{f,t}
\]  

(12)

where \(\delta\) is the depreciation rate of existing capital, and aggregate investment in each country, \(I_{i,t}\), is specified in (11).

?? modifies the investment bundles to be subject to capital adjustment costs. As such, the costs imply that investment expenditures do not produce additional capital one-to-one. Instead, the representative firm in each country finds it increasingly difficult to build up capital. The investment bundles, however, incorporate a novel feature in exhibiting asymmetric investment frictions. The intuition is that a firm deciding to adjust its capital stock would find it more costly to make use of foreign investment goods than locally-available investment goods. Thus, the firm finds it more difficult to build up capital using foreign resources than local ones.

[Raffo] 2008 incorporates symmetric investment frictions in a two country economy, but focuses primarily on international macroeconomic features and not on asset pricing implications. BKK embody the time-to-build structure of [Kydland and Prescott] 1982 in their world economy. [Colacito et al.] 2013 do not model capital adjustment. However, their study incorporates a heterogeneous exposure of capital vintages to aggregate productivity risk, which behaves as a friction in investment. They show that such friction is crucial to their quantitative asset pricing results (e.g., increase the equity premium, where otherwise the model produces an equity premium of less than 1% and a volatility slightly above 1% per year without capital vintages). Although their heterogeneous exposure of capital vintages can be incorporated to the
asymmetric capital adjustment costs considered in ??, a natural goal is to show that such friction is not necessary in the present model. Specifically, show that GDA preferences, exogenous economic uncertainty, and asymmetric capital adjustment costs suffice to address international quantities and prices. If these three aspects of the model are not sufficient for a large equity premium, then the model could exhibit financial leverage, as in Jermann 1998 and Liu and Miao 2014. Financial leverage can raise dividend volatility and hence raise the levered equity premium. Moreover, the model could also exhibit a procyclical dividend process as in Ju and Miao 2012. Liu and Miao 2014 find that a procyclical dividend process (consistent with the data) is key for their equity premium results in a closed economy model with GDA preferences. Lastly, equity premium could also be raised by having the productivity levels of each country further subject to large negative low-probability shocks (rare disasters, as in Barro 2009).

2.5. Production Functions and Resource Constraints

As is standard in international real business cycle (RBC) models, output in each country is a Cobb-Douglas aggregation of country-specific capital and labor. Output can be used at home or abroad for consumption or investment. Recall that $X_t$ and $Y_t$ denote the time $t$ output for the goods in the home and foreign countries, respectively. Thus, the resource constraints in the two countries are:

$$X_t = K_{h,t}^{\tilde{\alpha}}(A_{h,t}N_{h,t})^{1-\tilde{\alpha}} = x_{h,t} + x_{f,t} + I_{h,h,t} + I_{h,f,t}$$

$$Y_t = K_{f,t}^{\tilde{\alpha}}(A_{f,t}N_{f,t})^{1-\tilde{\alpha}} = y_{f,t} + y_{h,t} + I_{f,f,t} + I_{f,h,t}$$

where the parameter $\tilde{\alpha}$ denotes the capital-share of output, which should not be confused with the penalty parameter $\alpha$ of the generalized disappointment aversion preferences discussed above. Given that the representative agent in each country does
not value leisure, in equilibrium total labor supply equals unity in both countries, i.e.,\( N_{i,t} = 1, \forall t \) and \( i = \{h, f\} \). Assuming that leisure is not valued is due to the focus of the paper on asset prices.\(^4\)

Following the principles in BKK, but employing the local good as the numeraire, the resource constraints can be used to re-express output in each country as follows:

\[
X_t = (x_{h,t} + P_t y_{h,t}) + (I_{h,h,t} + P_t I_{f,h,t}) + (x_{f,t} + I_{h,f,t}) - P_t(y_{h,t} + I_{f,h,t})
\]

\[
Y_t = (y_{f,t} + x_{f,t}/P_t) + (I_{f,f,t} + I_{h,f,t}/P_t) + (y_{h,t} + I_{f,h,t}) - (x_{f,t} + I_{h,f,t})/P_t
\]

(14)

where \( P_t \) denotes the terms of trade (i.e., ratio of prices) of the two goods evaluated at equilibrium quantities:

\[
P_t = \frac{\left\{ \frac{\partial C_{h,t}(x_{h,t}, y_{h,t})}{\partial y_{h,t}} \right\}}{\left\{ \frac{\partial C_{h,t}(x_{h,t}, y_{h,t})}{\partial x_{h,t}} \right\}} = \frac{(1 - \lambda)}{\lambda} \left( \frac{x_{h,t}}{y_{h,t}} \right)^\frac{1}{\eta}
\]

(15)

Output in (14) specifies how the resource constraints are linked to the national accounts of each country:

\[
X_t = \underbrace{(x_{h,t} + P_t y_{h,t})}_{C_{m,h,t}} + \underbrace{(I_{h,h,t} + P_t I_{f,h,t})}_{I_{m,h,t}} + \underbrace{(x_{f,t} + I_{h,f,t})}_{\text{Exports}_{m,h,t}} - \underbrace{P_t(y_{h,t} + I_{f,h,t})}_{\text{Imports}_{m,h,t}}
\]

\[
Y_t = \underbrace{(y_{f,t} + x_{f,t}/P_t)}_{C_{m,f,t}} + \underbrace{(I_{f,f,t} + I_{h,f,t}/P_t)}_{I_{m,f,t}} + \underbrace{(y_{h,t} + I_{f,h,t})}_{\text{Exports}_{m,f,t}} - \underbrace{(x_{f,t} + I_{h,f,t})/P_t}_{\text{Imports}_{m,f,t}}
\]

(16)

where the subscripts \( m \) indicate that the quantities are accounting aggregates measured in local units. As in Colacito et al. 2013, measuring quantities in this fashion facilitates the comparison of the model’s quantitative results with that of readably available data sources (e.g., data from the Organization for Economic Co-operation

\(^4\)This assumption could be relaxed later, as to show consistency also with international labor aggregates.
and Development (OECD)).

2.6. Real Exchange Rates and Currency Risk Premium

2.6.1. Real Exchange Rates

Complete markets and no arbitrage opportunities give a relationship between the real exchange rates and the stochastic discount factors of each country. Following Backus, Foresi, and Telmer [2001], this relationship is obtained from a general setting using the fundamental equation of asset pricing, $E_t[M_{t,t+1}R_{t+1}]$.

The Euler equation for a domestic investor buying a domestic bond with return $R_{h,t+1}$ is $E_t[M_{h_{t,t+1}}R_{h,t+1}]$. Similarly, the Euler equation for a foreign investor buying a foreign bond with return $R_{f,t+1}$ is $E_t[M_{f_{t,t+1}}R_{f,t+1}]$. Complete markets imply that the pricing kernels in the two previous equations are unique. Thus, denoting $e_t$ as the real exchange rate expressed in domestic consumption goods per foreign goods at time $t$, the Euler equation of the domestic investor buying the foreign bond is $E_t[M_{h_{t,t+1}}] = e_t + e_{t+1}R_{f,t+1}$. Given that this third equation must hold along with the two other Euler equations, it must be that the following relation is satisfied:

$$
\frac{M_{f_{t,t+1}}}{M_{h_{t,t+1}}} = \frac{e_{t+1}}{e_t}
$$

Thus, complete markets imply that the change in the real exchange rate equals the ratio of the two SDFs at home and abroad. In this paper, international asset pricing and macroeconomic implications are examined using the SDFs associated with the GDA preferences in [5]. In logs, the relationship in (17) states that the log-growth of the real exchange rate in consumption units is equal to the difference in the logs of the stochastic discount factors:

$$
\Delta e_{t+1} = m_{f_{t,t+1}} - m_{h_{t,t+1}}
$$

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Note that with incomplete markets the choices of $m_{h,t,t+1}$ and $m_{f,t,t+1}$ are not unique, which causes the previous relationship to no longer behave as a function but rather as a correspondence. A different strand in the literature examines the implications of incomplete markets. For instance [Kehoe and Perri, 2002] have an influential paper highlighting the role of financial frictions. The role of incomplete markets are also examined in [Bai and Zhang, 2010] and [Petrosky-Nadeau, 2011].

2.6.2. Currency Risk Premium

The forward premium anomaly naturally gave rise to an investment strategy called carry trade. This investment strategy consists in going long in the high interest rate currencies and short in the low interest rate currencies. From the perspective of a U.S investor, [Lustig and Verdelhan, 2007] analyze returns to portfolios of currencies sorted by interest rate differences and find that a U.S investor earns an average annual spread of 500 basis points. The model in this study analyzes the exchange rate risk premium. As stated in [Verdelhan, 2010] the exchange rate risk premium is the excess return of a domestic investor who borrows funds at home, converts them to a foreign currency, lends at the foreign risk-free rate, and then converts earnings to the original currency. Thus, in logs, the currency excess return $r_{t+1}$ is

$$ r_{t+1} = \Delta e_t + r_{f,t} - r_{h,t} $$

(19)

The domestic investor gains the foreign interest rate $r_{f,t}$ and pays the domestic interest rate $r_{h,t}$. Therefore, she exhibits a loss if the domestic currency (say, the dollar) appreciates in real terms (i.e., $e_t$ decreases) when holding assets abroad.
3. Model Solution

3.1. International Risk Sharing

The decentralized market equilibrium with complete asset markets is equivalent to the solution of a corresponding social planner problem, where each country has a Negishi weight $\mu_i$ in the planner’s objective. Therefore, efficient allocations can be computed as the solution to the planner’s problem. From a market economy perspective, the Negishi weights can be viewed to indicate the relative size of the endowments in each country. Specifically, the planner attaches nonnegative Pareto weights $\mu_h = \mu$ and $\mu_f = 1 - \mu$ on the representative agents and chooses allocations $\{x_{i,t}, y_{i,t}, N_{i,t}, K_{i,t+1}, I_{i,j,t}\}_{t=0}^{+\infty}$ for $i, j \in \{h, f\}$ to maximize

$$\Omega_0 = \mu_h U_{h,0} + \mu_f U_{f,0}$$

subject to a sequence of two economy-wide resource constraints and two laws of motion for physical capital:

$$x_{h,t} + x_{f,t} + I_{h,h,t} + I_{h,f,t} \leq K_{h,t}^{\tilde{\alpha}} (A_{h,t} N_{h,t})^{1-\tilde{\alpha}}$$

$$y_{f,t} + y_{h,t} + I_{f,f,t} + I_{f,h,t} \leq K_{f,t}^{\tilde{\alpha}} (A_{f,t} N_{f,t})^{1-\tilde{\alpha}}$$

$$K_{h,t+1} \leq (1-\delta)K_{h,t} + I_{h,t}$$

$$K_{f,t+1} \leq (1-\delta)K_{f,t} + I_{f,t}$$

[Anderson 2005] theoretically suggests a recursive way to characterize problems of this type, with recursive preferences, in a one good endowment economy. [Croce 2011] extends this technique to multiple good endowment economies and [Colacito et al. 2013] apply it to a two-country two-good economy with production. The insight is that the solution of consumption allocations can be conveniently cast in terms of a
time-varying vector of Pareto weights. Let $S_t = \mu_{h,t}/\mu_{f,t}$ denote the time $t$ ratio of Pareto weights. With non-recursive utility, standard international RBC models do not have time-varying weights. The planner simply assigns equal weights to the countries in every period (i.e., $\mu_f = \mu_h$ and $S_t = 1 \forall t$), thus giving each country equal importance in her objective.

The optimal consumption allocation rules are characterized from the first-order necessary conditions (with respect to consumption):

$$
S_t \frac{\partial C_{h,t}}{\partial x_{h,t}} \bigg\rvert_{C_{h,t}} \frac{1}{C_{h,t}} = \frac{\partial C_{f,t}}{\partial x_{f,t}} \bigg\rvert_{C_{f,t}} \frac{1}{C_{f,t}}
$$

$$
S_t \frac{\partial C_{h,t}}{\partial y_{h,t}} \bigg\rvert_{C_{h,t}} \frac{1}{C_{h,t}} = \frac{\partial C_{f,t}}{\partial y_{f,t}} \bigg\rvert_{C_{f,t}} \frac{1}{C_{f,t}}
$$

(22)

where the ratio of time $t$ Pareto weights, $S_t$, is a state variable. The innovation to the ratio of Pareto weights follows from the ratio of previous period GDA stochastic discount factors, defined in (5), and the contemporaneous ratio of consumption bundle growth of the two countries:

$$
S_t = S_{t-1} \frac{M_{h,\{t-1,t\}}}{M_{f,\{t-1,t\}}} \frac{C_{h,t}}{C_{f,t}} \frac{C_{h,t-1}}{C_{f,t-1}}, \forall t \geq 1
$$

(23)

with $S_0 = \mu/(1 - \mu)$. Recall, that the GDA stochastic discount factors, $M_{i,\{t-1,t\}}$, are expressed in units of the corresponding local consumption aggregate, $C_{i,t}$. However, as explained in subsection 2.5, economy aggregates are expressed in local output units. Thus, for consistency, the GDA stochastic discount factors in (5) can be expressed in local output units as follows:

---

$^5$ $S_t$ is shown in [Croce 2011](#) to have a well-defined ergodic distribution.
\[ M^X_{h,\{t,t+1\}} = \left( \frac{x_{h,t}}{x_{h,t+1}} \frac{C_{h,t+1}}{C_{h,t}} \right)^{\frac{1}{\eta}} M_{h,\{t,t+1\}} \]

\[ M^Y_{f,\{t,t+1\}} = \left( \frac{y_{f,t}}{y_{f,t+1}} \frac{C_{f,t+1}}{C_{f,t}} \right)^{\frac{1}{\eta}} M_{f,\{t,t+1\}} \]  

Equations (22) are the optimal or efficient risk-sharing conditions. Every period, marginal utilities of consumption are equalized across agents. The equations can be further expressed as follows:

\[ S_t \lambda \left( \frac{C_{h,t}}{x_{h,t}} \right)^{\frac{1}{\eta}} \frac{1}{C_{h,t}} = (1 - \lambda) \left( \frac{C_{f,t}}{x_{f,t}} \right)^{\frac{1}{\eta}} \frac{1}{C_{f,t}} \]

\[ S_t (1 - \lambda) \left( \frac{C_{h,t}}{y_{h,t}} \right)^{\frac{1}{\eta}} \frac{1}{C_{h,t}} = \lambda \left( \frac{C_{f,t}}{y_{f,t}} \right)^{\frac{1}{\eta}} \frac{1}{C_{f,t}} \]  

(25)

3.2. Investment and Capital Optimality Conditions, Asset Prices

The price of capital can be characterized from the first-order and envelope conditions of the social planner’s problem. Let \( P_{i,K,t} \) denote the time \( t \) cum-dividend price of capital in countries \( i = \{h, f\} \). Optimal investment of each agent in her own country, \( I_{h,h,t} \) and \( I_{f,f,t} \), satisfies the following conditions:

\[ P_{h,K,t} = \frac{1}{\left( \frac{\partial I_{h,t}}{\partial I_{h,h,t}} \right)_{I_{h,h,t}}} \equiv \frac{1}{I_{h,t}} \left( I_{h,h,t} \right)^{\frac{1}{\eta}} P_{f,K,t} = \frac{1}{\left( \frac{\partial I_{f,t}}{\partial I_{f,f,t}} \right)_{I_{f,f,t}}} \equiv \frac{1}{I_{f,t}} \left( I_{f,f,t} \right)^{\frac{1}{\eta}} \]  

(26)

or

\[ P_{h,K,t} = \frac{1}{\nu} \left( I_{h,h,t} \right)^{\frac{1}{\tau}} P_{f,K,t} = \frac{1}{\nu} \left( I_{f,f,t} \right)^{\frac{1}{\tau}} \]  

(27)

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Optimal investment abroad for each agent, $I_{h,f,t}$ and $I_{f,h,t}$, satisfies the following conditions:

$$P_{f,K,t} = \frac{1}{\left(\frac{\partial I_{f,t}}{\partial I_{h,f,t}}\right)_{P_t}} = \frac{1}{I_{f,t}(I_{h,f,t})} P_t , \quad P_{h,K,t} = \frac{1}{\left(\frac{\partial I_{h,t}}{\partial I_{f,h,t}}\right)_{P_t}} = \frac{1}{I_{h,t}(I_{f,h,t})} P_t$$

(28)

or

$$P_{f,K,t} = \frac{1}{P_t (1 - \nu)} \left( \frac{I_{h,f,t}}{I_{f,t}} \right)^{\frac{\tilde{\alpha}}{\tilde{\alpha} - 1}} , \quad P_{h,K,t} = \frac{P_t}{(1 - \nu)} \left( \frac{I_{f,h,t}}{I_{h,t}} \right)^{\frac{\tilde{\alpha}}{\tilde{\alpha} - 1}}$$

(29)

where the prices of capital take into account, every period, the real exchange rate risk of investing abroad through the terms of trade, $P_t$ defined in (15). Thus, provided the cum-dividend prices of capital, the four investment optimality conditions (27) and (29) alone do not determine optimal investment. For this, the optimal mix of home and foreign consumption goods has to also be determined (jointly).

The optimal decision with respect to the capital stock of next period in the home country, $K_{h,t+1}$, satisfies:

$$P_{h,K,t} = E_t[M^X_{h,t,t+1} (\tilde{\alpha} K_{h,t+1}^{\tilde{\alpha} - 1} (A_{h,t+1} N_{h,t+1})^{1 - \tilde{\alpha}} + P_{h,K,t+1}(1 - \delta) )]$$

(30)

this equation states that the current cum-dividend price of home capital must equal the expected discounted future payoff from an additional unit of home capital. This reward has two components: the increased output as given by the marginal product of capital $\tilde{\alpha} K_{h,t+1}^{\tilde{\alpha} - 1} (A_{h,t+1} N_{h,t+1})^{1 - \tilde{\alpha}} = \tilde{\alpha} \frac{X_{t+1}}{K_{h,t+1}}$, and the residual value of the remaining unit of capital $P_{h,K,t+1}(1 - \delta) = \frac{P_{t+1}}{(1 - \nu)} \left( \frac{I_{f,h,t+1}}{I_{h,t+1}} \right)^{\frac{\tilde{\alpha}}{\tilde{\alpha} - 1}} (1 - \delta)$ which takes exchange rate risk into account through the next period terms of trade, $P_{t+1}$.

Similarly, the optimal decision with respect to the foreign capital stock of next period, $K_{f,t+1}$, satisfies:

$$P_{f,K,t} = E_t[M^Y_{f,t,t+1} (\tilde{\alpha} K_{f,t+1}^{\tilde{\alpha} - 1} (A_{f,t+1} N_{f,t+1})^{1 - \tilde{\alpha}} + P_{f,K,t+1}(1 - \delta) )]$$

(31)
Let $Q_{i,K,t}$ denote the time $t$ ex-dividend price of capital in countries $i = \{h, f\}$:

$$
Q_{h,K,t} = E_t[M_{h,t,t+1}^X P_{h,K,t+1}] \quad , \quad Q_{f,K,t} = E_t[M_{f,t,t+1}^Y P_{f,K,t+1}]
$$

(32)

The cum- and ex-dividend prices of physical capital give rise to the return on capital $R_{i,t+1}$ in each country. Prices and returns are summarized in the following proposition:

**Proposition 1** (Prices and Returns). *Capital is priced in the home and foreign countries, $i = \{h, f\}$, as the cum- and ex-dividend prices, $P_{i,K,t}$ and $Q_{i,K,t}$:

$$
P_{h,K,t} = E_t\left[M_{h,t,t+1}^X \left(\alpha \frac{X_{t+1}}{K_{h,t+1}}\right)\right] + (1 - \delta)Q_{h,K,t}
$$

$$
Q_{h,K,t} = E_t[M_{h,t,t+1}^X P_{h,K,t+1}]
$$

$$
P_{f,K,t} = E_t\left[M_{f,t,t+1}^Y \left(\alpha \frac{Y_{t+1}}{K_{f,t+1}}\right)\right] + (1 - \delta)Q_{f,K,t}
$$

$$
Q_{f,K,t} = E_t[M_{f,t,t+1}^Y P_{f,K,t+1}]
$$

(33)

The return on capital in the home and foreign country satisfy:

$$
R_{h,K,t} = \frac{P_{h,K,t+1}}{Q_{h,K,t}} \quad , \quad R_{f,K,t} = \frac{P_{f,K,t+1}}{Q_{f,K,t}}
$$

(34)

The risk-free rates in the home and foreign countries are:

$$
\frac{1}{R_{h,t}^f} = E_t[M_{h,t,t+1}^X] \quad , \quad \frac{1}{R_{f,t}^f} = E_t[M_{f,t,t+1}^Y]
$$

(35)
The log currency excess return of a home investor satisfies:

\[
    r_{t+1}^e = \Delta e_{t+1} + r_{f,t}^f - r_{h,t}^f.
\]  

This presents asset prices and optimality conditions for investment and capital that follow from the extended model with asymmetric capital adjustment costs (described in ??).

3.3. Computation Technique

This subsection discusses the technique proposed to solve the model. The introduction of GDA preferences and stochastic volatility into the recursive utility framework makes the model exhibit increased non-linearities. GDA utility is non-differentiable at the point where disappointment sets in, thus one cannot rely on log-linear approximations or perturbation methods to find an approximate solution to the model. Moreover, the model also presents stochastic volatility, which increases the dimensionality of the state-space of the two-country model. To solve the endowment economy model in Bonomo et al. 2011 with GDA preferences and economic uncertainty, they recur to a Markov switching process for consumption and dividends that matches the long-run risk and stochastic volatility specifications. They also derive analytical formulas for the population moments of equity premia, as well as the coefficients and $R^2$ of predictability regressions. To solve the model in Coiacito et al. 2013, with recursive utility and production, they recur to perturbation methods, which are known to suffer from accuracy limitations when the approximate solution is far from the steady state of the model (e.g., see Acedanski 2010).\footnote{However, Caldara, Fernandez-Villaverde, Rubio-Ramirez, and Yao 2012 conclude that perturbation methods are an attractive approach due to their low computing cost. Moreover, perturbed solutions can be useful as an initial guess for more accurate solution methods.} This paper pursues a far more accurate solution method. Specifically, the numer-
ically stable and accurate simulation-version of the parameterized expectations approach (PEA) recently introduced by Judd et al. 2011 is employed. It is well known that PEA is a non-finite state space method for computing equilibria in stochastic dynamic models (e.g., Wright and Williams, 1984, Miranda and Helmberger, 1988, Christiano and Fisher, 2000). The simulation-based version of PEA is used in Marcet, 1988, Den-Hann and Marcet, 1990, Marcet and Lorenzoni, 1999 and Maliar and Maliar, 2003. This technique is a particular projection method (see Christiano and Fisher, 2000). However, in contrast to projection methods, simulation-based PEA does not suffer as easily from the curse of dimensionality and can be applied to models with many endogenous state variables and many approximated conditional expectation functions. To overcome numerical stability issues of conventional simulation-based PEA, Judd et al. 2011 propose the generalized stochastic simulation algorithm (GSSA) that is both numerically stable and tractable in high-dimensional applications. The GSSA delivers high-degree polynomial approximations and is able to attain accuracy levels comparable to the best accuracy attained in the economics numerical methods literature. Judd et al. 2011 show the good approximation quality of the GSSA in a multicountry one-sector growth model with up to 20 countries and 40 state variables (i.e., 20 endogenous capital state variables and 20 exogenous productivities). The reasonably high dimensionality of the problem considered in the present study makes the use of a GSSA technique appropriate. It is also able to deal with the kinks of GDA preferences.

The state space of the model consists of seven variables: capital in each country, $K_{h,t}$, $K_{f,t}$, the ratio of Pareto weights, $S_t$, the productivity levels in each country, $A_{h,t}$, $A_{f,t}$, and the volatility in each country, $\sigma_{h,t}$, $\sigma_{f,t}$. Thus, the state-space $X$ of the model is a subspace of $\mathbb{R}^7$, given by $X := [\bar{K}_h, \bar{K}_f] \times [\bar{A}_h, \bar{A}_f] \times [\bar{\sigma}_h, \bar{\sigma}_f] \times [\bar{S}, \bar{S}]$, where the grids for the non-shock state variables are determined from a set of points realized in equilibrium over simulations of the
model. Unlike conventional projection methods, solving the model on a set of points realized in equilibrium avoids the cost of finding a solution in areas of the state space that are effectively never visited in equilibrium. As such, the domain over which the stochastic simulation operates grows far less rapidly with the dimensionality of the problem.

The discretization approach used by Caldara et al. (2012) in a one country model is extended here to discretize the stochastic processes for the volatility and productivity of each country. As such, the Tauchen (1986) method is used as follows. First, for each country \( i = \{h, f\} \) a grid of \( M \) points is created \( G_\sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_M\} \) for \( \sigma_{i,t} \) and a transition matrix \( \Pi^M \) with generic element \( \Pi^M_{a,b} = \text{Prob}(\sigma_{i,t+1} = \sigma_{b,t} | \sigma_{i,t} = \sigma_{a,t}) \). The grid is large enough to cover 3 standard deviations of the process of each country in each direction. Then, for each \( M \) point, create a grid with \( J \) points \( G_A = \{A_{1}^{m}, A_{2}^{m}, \ldots, A_{J}^{m}\} \) for \( A_{i,t} \) with transition matrices \( \Pi^{L_{m}} \) with generic element \( \Pi^{L_{m}}_{a,b} = \text{Prob}(A_{i,t+1}^{m} = A_{b}^{m} | A_{i,t}^{m} = A_{a}^{m}) \). Where, conditional on each countries’ \( \sigma_{m} \) the grid covers 3 standard deviations in each direction. The grids for \( A_{i,t} \) are made to depend on the level of volatility \( m \) to adapt the accuracy of Tauchen’s procedure to each conditional variance. Judd et al. (2011) propose the use of Gauss-Hermite quadrature integration (product) in solving a model with up to twenty countries. Thus, Gauss-Hermite discretization could also be employed here. In addition, the discrete state space method of Rouwenhorst (1995) can also be considered. As suggested in Kopecky and Suen (2010), this latter method is more reliable than others in the presence of highly persistent processes.

3.4. Computation Implementation

To solve the model, the GSSA method will parameterize the functional equations for the cum-dividend prices in (30) and (31), and the indirect utility functions in (1). In addition, the terms of trade in (15) will also be parameterized. This last
parametrization allows the method to avoid having to solve a rather large system of equations each period. For convenience the equations are rewritten here:

\[ P_{h,K,t} = E_t \left[ M_{h,(t,t+1)}^X \left( \frac{X_{t+1}}{K_{h,t+1}} + P_{h,K,t+1} (1 - \delta) \right) \right] \] (37a)

\[ P_{f,K,t} = E_t \left[ M_{f,(t,t+1)}^Y \left( \frac{Y_{t+1}}{K_{f,t+1}} + P_{f,K,t+1} (1 - \delta) \right) \right] \] (37b)

\[ U_{h,t} = \{(1 - \beta)C_{h,t}^{1 - \frac{1}{\psi}} + \beta \left[ R_{h,t}(U_{h,t+1}) \right]^{1 - \frac{1}{\psi}} \}^{1 - \frac{1}{\psi}} \text{ if } \psi \neq 1 \] (37c)

\[ U_{f,t} = \{(1 - \beta)C_{f,t}^{1 - \frac{1}{\psi}} + \beta \left[ R_{f,t}(U_{f,t+1}) \right]^{1 - \frac{1}{\psi}} \}^{1 - \frac{1}{\psi}} \text{ if } \psi \neq 1 \] (37d)

\[ P_t = \frac{(1 - \lambda)}{\lambda} \left( \frac{x_{h,t}}{y_{h,t}} \right)^{\frac{1}{\eta}} \] (37e)

Thus, five orthogonal polynomials \( \psi^n(\gamma^n, K_h, K_f, A_h, A_f, \sigma_h, \sigma_f, S) \) for \( n = 1, 2, 3, 4, 5 \) will separately approximate each of the right-hand sides of the equations in (37). The parameters of these functions are collected in the yet to be determined coefficient vectors \( \gamma^n, n = 1, 2, 3, 4, 5 \). Although solving the model with GDA preferences is the ultimate goal of this implementation. The outline of the algorithm below is for the simpler case of the Kreps-Porteus preferences. The GDA preferences make the algorithm more involved and require additional care.

1. Given a heptuple \((K_h, K_f, A_h, A_f, \sigma_h, \sigma_f, S)\), compute

   \[ P_{h,K,t} = \psi^1(\gamma^1, K_h, K_f, A_h, A_f, \sigma_h, \sigma_f, S), \]

   \[ P_{f,K,t} = \psi^2(\gamma^2, K_h, K_f, A_h, A_f, \sigma_h, \sigma_f, S) \] and

   \[ P_t = \psi^5(\gamma^5, K_h, K_f, A_h, A_f, \sigma_h, \sigma_f, S). \]

2. These polynomials are then used in equations (27) and (29) to compute the time \( t \) values of home and abroad investment \( I_{h,h,t}, I_{h,f,t}, I_{f,h,t}, \) and \( I_{f,f,t} \). Here, investment is determined using the definition of investment bundles in (11), and the values of current period capital stocks and cum-dividend prices of capital. A system of 4 equations in 4 unknowns is solved.
3. Using these four investment values, compute investment bundle values $I_{h,t}$ and $I_{f,t}$, as defined in (11).

4. Using capital and productivities, $(K_h, K_f, A_h, A_f)$, and the optimal values of labor, $(N_{h,t} = 1, N_{f,t} = 1)$, compute output in each country $(X_t, Y_t)$ as defined by the Cobb-Douglas functional forms in (13).

5. The two resource constrains in (13), the two efficient risk-sharing conditions in (25), the values of output $(X_t, Y_t)$, the values of home and abroad investment $I_{h,h,t}, I_{h,f,t}, I_{f,h,t}, I_{f,f,t}$ and the ratio of Pareto weights $S$, give the home and abroad consumption values, $x_{h,t}, x_{f,t}, y_{h,t}$, and $y_{f,t}$. Again, 4 equations and 4 unknowns.

6. Obtain consumption bundle values, $C_{h,t}$ and $C_{f,t}$, as defined in (6).

7. Consumption, investment, and the terms of trade give the national accounts of net exports, consumption, and exports in local output units, as specified in (16).

8. The capital accumulation equations (12) give next period capital stocks $K_{h,t+1}$ and $K_{f,t+1}$.

9. Using the dynamics for aggregate productivity in (7) and (8), compute $\sigma_{h,t+1}$, $\sigma_{f,t+1}$, $A_{h,t+1}$, and $A_{f,t+1}$ using the discretization method described above.

10. The ratio of Pareto weights of next period $S_{t+1}$ follows from equation (23). However, unlike capital tomorrow, this state variable is not a predetermined endogenous state variable. In other words, the ratio of weights tomorrow is not known today because it depends on consumption tomorrow, which is endogenous and itself depends on $S_{t+1}$. This makes solving for the ratio of tomorrow rather cumbersome. Specifically, solving for the ratio of tomorrow, $S_{t+1}$, requires an additional layer of non-linear equation solving for every possible volatility level.
every possible productivity level, and every capital stock tomorrow. Moreover, this non-linear equation solve for $S_{t+1}$ is done for every period within the outer iteration loop that determines the coefficients of each of the five parameterized equations in (37). All in all, solving the model is very computationally intensive.

Moreover, computing $S_{t+1}$ requires the use of the third and fourth polynomials, $U_{h,t} = \psi^3(\gamma^3, K_h, K_f, A_h, A_f, \sigma_h, \sigma_f, S)$ and $U_{f,t} = \psi^4(\gamma^4, K_h, K_f, A_h, A_f, \sigma_h, \sigma_f, S)$, and the consumption bundles computed in step 6 to compute the intertemporal pricing kernels $M_{h,\{t,t+1\}}$ and $M_{f,\{t,t+1\}}$ in (4). Note that the SDFs in this step require either the use of Tauchen or Gauss-Hermite integration to approximate the conditional expectations on the second term of the utility functionals.

11. The SDFs in local units $M_{h,\{t,t+1\}}^X$ and $M_{f,\{t,t+1\}}^Y$ in (24) follow from the consumption bundles and SDFs computed above.

Following the suggestions in Maliar and Valero, 2013 a low-cost fixed-point iteration is used to determine the unknown coefficients of the approximated functions of the GSSA method. Accuracy of the solution is increased gradually by using the polynomial solution of lower degree (e.g., $p - 1$) as starting points to compute the solution of higher degree (e.g., $p$). Upon convergence, the residual functions $R^1$, $R^2$, $R^3$, $R^4$, and $R^5$ determine the accuracy of the solution method. These follow from the equations in (37).

4. Conclusion

Recent models with Kreps and Porteus, 1978 recursive preferences and long-run mean risk account for many stylized facts in international finance and international macroeconomics. These models, however, are reliant on assumptions placed on cal-
ibrated parameter values. The quantitative results of models such as Colacito and Croce 2013 and Colacito et al. 2013 are hindered upon slight changes in values. To overcome these limitations, the recursive preferences are extended to represent the generalized disappointment-aversion preferences (GDA) recently introduced by Routledge and Zin 2010. GDA preferences incorporate an additional risk-premium term not present under Kreps-Porteus preferences. Moreover, they exhibit an endogenous time-variation in: (i) country-specific probability of disappointing outcomes, and (ii) effective risk-aversion. Both are influenced by economic uncertainty, as measured by stochastic volatility in productivity growth.

All in all, this study examines international asset prices in a world production economy with GDA preferences and economic uncertainty. Examining the implications of these two features on international asset prices and economic flows is novel to this paper. The characteristics of the model are: GDA preferences, two countries, two goods, production, asymmetric investment frictions, and economic uncertainty. The solution to the model is being implemented.
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