Clinical Interviews of Shape Understanding in a Dynamic Geometry Environment

THESIS

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By

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Abstract

When given the opportunity to design my own research project, I immediately thought of designing a study around student that I have worked with for almost two years. The reasoning he was displaying in class was both fascinating and frustrating and between the constraints of class time period, class size, and school requirements, I found it difficult to dig deeper into this student’s reasoning, particularly surrounding proof. What was originally intended to be a teaching experiment about proof evolved into a study of a particular student’s responses to problem tasks through clinical interviews. Clinical interviews focus on the competencies of students, not the deficiencies, and validates student thinking as worthwhile and noteworthy (Ginsburg, 1997). Using Battista’s (2012) Learning Progression of Shape Understanding, I was able to identify student competencies and convert traditional paper tasks to manipulative tasks in a dynamic geometry environment (DGE) in an effort to assist him in advancing through the progressions. With the frameworks of clinical interviews, Schoenfeld’s (1985) research on problem solving, and Battista’s (2012) Learning Progressions, I was able to learn more about my student’s geometric thinking and problem solving skills and determined that shape characterization was accelerated in a DGE (Ginsburg, 1997; Battista, 2012).
Vita

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Fields of Study

Major Field: Education Teaching & Learning
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Rationale

Every few years, teachers are given a new or modified list of standards that students are expected to meet. One concept that has traditionally been a required component of high school geometry courses is the writing of formal deductive proofs, usually concerning triangles, quadrilaterals, or angle systems. However, there is a progression of the development of the understanding of shapes that generally determines whether or not students are developmentally ready to carefully sequence and bridge the given and deduced properties to write a complete formal proof (Heinze, 2000; Battista, 2012). These Learning Progressions, measured on a scale from 0-4, culminate with the ability to understand and write formal deductive proofs (Battista, 2012). Thus, if a student is not at the concluding level of the Learning Progression, the proof will likely be incomplete, incorrect, or proven with only empirical or numerical (Marrades & Gutierrez, 2000).

In this study, I will focus on the Shape Understanding Learning Progression because of its importance to the geometry curriculum that I currently teach and because I had noticed that a particular student of mine seemed to have difficulties with proof even more so than his peers. When I would ask this student in class for more clarification on the choices he was making, the time necessary for me to further examine his reasoning and how his ideas were connected was unavailable during regular class time. This particular student often seemed defeated and even occasionally mentioned feeling “dumb”. I wanted to find additional time to work with this particular student one on one
to dig deeper into his thinking of proof using the method of the clinical interview (Ginsburg, 1997). Using clinical interviews to give a subject the entire attention of a researcher or teacher not only focuses on the competencies of a student but also leads the researcher to truly unearth the brilliance of the mathematical thoughts of the subject. Having more directed, focused, uninterrupted, one on one time with this particular student had the potential to gain insight into what competencies he possessed and where he was reasoning on the Learning Progressions (Ginsburg, 1997; Battista, 2012).

In order to reach the fourth level of the Shape Understanding Learning Progression, students should first be able to identify shapes as visual-wholes, describe the parts and properties of shapes, interrelate properties and categories of shapes, and finally after consistently reasoning at each of the previous cognitive milestones, they should be developmentally prepared to understand and create formal deductive proofs (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010; Battista, 2012). However, Battista’s (2012) research shows that the Learning Progressions are not strictly linear. Students may demonstrate reasoning at different levels of understanding depending on the context of the task. If a teacher is unaware of where a student generally reasons on the Learning Progression spectrum, he or she may not be able to fully understand why a student appears unable to master certain grade level expectations. This may cause frustration for teachers, students, and parents.

As a teacher, I greatly value problem solving and mathematical modeling and recognize that facilitating the development of proof is my weakest area as a teacher. I recognize the importance of proof but admittedly preferred to spend more of my planning
time developing three-dimensional modeling and trigonometry tasks. Thus, when I decided to take on this project, I wanted to focus specifically on proof tasks to better understand a particular student’s shape reasoning and to develop my skills as an educator. The original purpose of this study was to investigate a specific student’s proof reasoning on the Learning Progressions and then to examine how interactions in a dynamic geometry environment (DGE) may affect reasoning (Battista, 2012). However, when I began investigating my student’s reasoning, I determined that what I was learning about my student was much more about problem solving strategies through a proof task than the proof product.

This project will focus on two primary research questions:

1) How might a Dynamic Geometry Environment assist students in progressing through the shape understanding Learning Progression?

2) What problem solving strategies do children use when working on a task above their shape understanding Learning Progression level?

These research questions will be explored through clinical interviews with the student (Ginsburg, 1997). The focus of these interviews is to diagnose the student’s Learning Progression level and uncover potential differences in student thinking on paper and in the DGE. In the style of a clinical interview, it is important to note that all student thinking that did not take place as a physical action (partitioning, dragging of a vertex in the DGE, etc.) was verbally reported by the student. Because of this, a complete reveal of all processing factors is impossible (Swanson et al., 1981). Limitations aside, a clinical interview with a single student provides to the teacher or researcher important
information about not just one student’s thinking but possible insight into how many students may generally think about a topic (Ginsburg, 1997). As a teacher, the takeaways from these interviews with this single student were intended to be generalized and applied to my future practice. Although the study focuses on a unique student’s reasoning and learning, the true student in this study was myself, the teacher. Lesh and Kelly (1997) note “a mathematics classroom in which a teacher pays close attention to how students represent mathematics can become ‘a learning environment for both teacher and students’” (p. 403). My overarching goal of this study was to learn about the reasoning of a specific student to better understand him as a learner and mathematics practitioner.

Implementing a researcher point of view can be quite difficult for a practicing teacher, especially when the subject involved is a current student. Limiting corrective feedback and becoming an observer and hypothesis generator is a shift that requires practice and patience (Ginsburg, 1997). Paying careful attention to student mathematical representation does not mean corrective feedback should be given. Rather, teacher researchers should try to make sense of student thinking in a positive way and to regard their ideas in ways that are not corrective or demeaning (Selter, 1998, p. 22). During clinical interviews, teachers can ensure that interactions with students are positive and supportive by focusing on student competencies (Ginsburg, 1997). Battista’s (2012) Learning Progressions are designed to highlight student competencies by describing what students are able to do at a specific developmental level (not grade level) of reasoning. For any teacher of any subject, student competencies should be the starting place for all teacher-student interactions and instruction. The shortcomings of students often become
the focus of teacher-student-parent interaction and the Learning Progressions provides a framework for students to be recognized for what they can do, not what they cannot do (Battista, 2012).

Students at a high school level may have some sort of understanding of shape properties, which can be identified using the Learning Progressions, even if they are not reasoning at the desired grade level expectation of level four of the Learning Progression (Battista, 2012). Building student inter-relational understanding of shape properties by selecting tasks that will help them make these connections personally, not by giving them a list of properties to memorize, will help students move toward a comprehensive understanding of proof in which their resources are a bank of recognizable and connectable properties (Battista, 2002). Students’ progress through levels of the Learning Progressions by moving from empirical evidence of shape properties to more generalized characteristics of shape properties (Battista, 2012). Because of the ability to quickly create cases of a shape characteristic in a DGE, the DGE provides opportunities for students to make endless empirical cases to move toward generalization. The tasks in these interviews will examine the differences in student reasoning on paper and pencil tasks and dynamic geometry tasks.

Although there is a Learning Progression alignment with high school geometry standards (the expectation being that students are reasoning at level four), this paper provides a space for examining how interactions in a DGE might increase the number of competencies a student has while moving through the Learning Progressions. Since this is my first clinical interview project, my large scale generalization goal was to gain
insight into student thinking by practicing listening closely and interpreting a unique
student’s thinking and problem solving decisions through the framework of the Learning
Progressions (Battista, 2012).
Theoretical Framework

Learning Progressions

The development of geometric and spatial thinking is generally recognized by teachers and researchers to be somewhat accurately described by the van Hiele levels of geometric thought (Battista, 2007). These five levels are the building blocks from informal identification of basic shape characteristics to formal deductive proofs and progress in a linear fashion (Battista, 2007). However, through his research, Battista (2007) discovered that students do not always move linearly through the levels and that the stepping-stones of the van Hiele levels are often too broad to truly describe the student thinking that occurs in between levels. To remedy this disconnect between the linear van Hiele levels and student demonstrated reasoning, Battista’s (2012) Learning Progressions include descriptive interrelated sublevels to more accurately describe the dynamic spectrum of student thought (Table 1).

Theoretically, the van Hiele levels describe the development of geometric thought if it were to progress in an organized and linear fashion (Battista, 2007). Because of this, the van Hiele levels are useful in developing macro-scale curricula or grade band expectations. However, on a micro-scale level with individual students, the more nuanced descriptions of the Learning Progressions levels and sublevels may better help teachers and researchers determine from where student thought is derived and where it may lead (Battista, 2007). Similarly to the van Hiele levels, the Learning Progressions also follow a
five level progression but have multiple sublevels to show the smaller cognitive milestones that students make in progressing from one main level to the next (Table 1, Battista, 2012).

To determine the particular level of student reasoning on the Learning Progressions, the Cognition-Based Assessments (CBA) were created (Battista, 2012). Students who exhibit specific indicators of reasoning can be determined to be at any level on the spectrum, depending on the context of the CBA task. The CBA tasks were designed to be facilitated by teachers and researchers via clinical interviews with the goal of uncovering student reasoning (Battista, 2012). This experiment will use CBA tasks both on paper and in a DGE to identify at which Learning Progression level the subject is reasoning.

For the purpose of this project, the focus will be on levels 2.3, 3, and 4. Descriptions of these levels are shown below in Table 1 with levels 2.3 through 4 highlighted to show where high school students would typically reason in alignment with the content standards.

<table>
<thead>
<tr>
<th>Level</th>
<th>Sublevel</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>Student identifies shapes as visual-wholes.</td>
</tr>
<tr>
<td>1.1</td>
<td></td>
<td>Student incorrectly identifies shapes as visual-wholes</td>
</tr>
<tr>
<td>1.2</td>
<td></td>
<td>Student correctly identifies shapes as visual-wholes.</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>Student describes parts and properties of shapes</td>
</tr>
</tbody>
</table>

Table 1. Shape Understanding Learning Progression, Battista (2012)
Dynamic Geometry Environments

In this project, the DGE will be used as a vehicle to dive deeper into uncovering student reasoning in the Shape Understanding Learning Progression. When students are first introduced to formal deductive proof, they are likely in a typical high school geometry class. Traditionally, students would be asked to memorize properties and definitions and then expected to use them later in proof. If the students do not recall the accurate properties, the teacher will most likely tell them directly or write them on the board. This may be because teachers can be inclined to teach the students in the exact same way in which they were taught (Sinclair & Bruce, 2014). The use of a DGE allows
for teachers and students to explore the properties of shapes in a non-traditional and more hands-on approach. Within most DGEs, students can move vertices, segments, or even reveal measurements to determine the value of angles or segments. Working in a DGE allows for students to build their own mental models about shapes and properties and to “move to higher levels of geometric thinking instead of having to memorize a laundry list of shape properties” (Battista, 2002, p. 339). Students then become active doers and creators of mathematics and not passive consumers or sponges of information (Battista, 2002). Working in a DGE fosters the development of increasingly sophisticated schemas about properties of shape (Battista, 2002). The manipulative environment allows students to explore a figure at a deeper level than if they were given the same figure on paper (Battista, 2002). Although there are many DGEs available to students and educators, the DGEs used in this experiment are the Individualized Dynamic Geometry Instruction (iDGi) Parallelogram Maker (Battista) and Geogebra.

Although Battista argues that the use of a DGE is beneficial to property development, Chazan describes the worries that researchers and teachers initially had regarding the potential dangers of students using a DGE to replace deductive reasoning with empirical measurement reasoning (Chazan, 1993). Because of the ability to “turn on measurements” or recreate a given proof in a DGE, students can use measurement to reach conclusions rather than using deduction and the chaining of properties (Chazan, 1993). However, this early 1990’s fear seems to have lasted only briefly or to have not been heeded. A Google search of the terms “dynamic geometry programs high school mathematics” returned over fifty DGE programs in both two a three dimensional
representations. Thus, the software is available in abundance to teachers and students alike at the click of a button. Because of the available quantity and the potential benefits to using the software, training students to use the software appropriately and efficiently may benefit both students and teachers (Battista, 2002). Although there are different forms of proof, in these clinical interviews, deductive synthetic proof is “prioritized” as the ultimate end goal for the student because of the grade level expectations and the alignment of those expectations to the language of the Learning Progressions (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010; Battista, 2012). From the way my current district’s scope and sequence is developed, students first have experience with empirical proofs using transformations as evidence. From there, students are expected to bridge transformation based proofs to triangle congruence proofs (deductive) and then to quadrilateral proofs. Because of this sequence, I place equal emphasis on both deductive proof and empirical proof but at different times in the sequence of the curriculum. Deductive proofs are the focus of this project because it became evident in my classroom that many of my students struggled to make the cognitive leap from empirical proof to deductive proof.

**Justification of Student Decision Making**

Marrades and Gutierrez’s (2000) Types of Justification model (Figure 1) allows teachers and researchers to dive deeper into the evidence students give during Learning Progressions CBA tasks with the end goal of level four, formal deductive proof (Battista, 2012).
For the purposes of this experiment, I will focus mainly on the empirical type of justifications. Students who struggle with deductive proof are likely to be reasoning at level three of the Learning Progression and will use empirical justifications (Battista, 2012; Marrades and Gutierrez, 2000). These empirical justifications, although not meeting the mandated geometry standards of formal deductive proof, should be given merit because they still provide insight into student understanding of shape properties.

Students who use the “Naive Empiricism” type of justification and use perceptual evidence to prove a case would apply visual or tactile perceptions (measurement, visual estimation) (Marrades & Gutierrez, 2000). These students would likely use evidence from a DGE to prove their thoughts or visual intuitions or assumptions. Students who use previous examples from their mathematical knowledge bank to make connections would be using the “inductive type” of naive empiricism justification (Marrades & Gutierrez, 2000). This is not to say that this type of empirical evidence is without merit. It simply is
a strategy that a student who reasons at level 2 or 3 of the Learning Progressions will use to eventually deduce properties.

This brings the discussion back to the fear that DGEs prevent students from following the deductive path of proof justification (or prevents them from seeing the value in a deductive proof) (Chazan, 1993). When analyzing the Marrades and Gutierrez (2000) justification pathways as parallel to the Learning Progressions, we can see that it may not be because of the availability of the DGE that forces students down the empirical path, but that it may be due to their Learning Progression level. Students who are consistently reasoning at level four of shape understanding Learning Progression may not need (and should not need) to empirically verify properties for a specific task in a DGE. Their understanding of properties should be deep enough that additional empirical evidence is unnecessary and they will exclusively take the deductive justification type pathway (Marrades and Gutierrez, 2000). However, a student who reasons at level 3.1 in the Learning Progressions may not have regular access to a generalized property because those properties have not yet been discovered or truly internalized as a problem solving resource (Schoenfeld, 1985; Battista, 2012). Those students would be forced to rely on empirical evidence and thus will use the DGE as a problem-solving tool to build those connections and create evidence.

**Problem Solving**

When students are faced with a task in which a solution is not readily available, they default to different problem solving strategies (Schoenfeld, 1985). In response to the primary research question of “what strategies do children use when working on a task
above their shape understanding Learning Progression level?”, three aspects of Schoenfeld’s (1985) research on problem solving will be the main focus of analyzing the results of this experiment: solver resources, solver heuristics, and solver control. A task is considered a problem, and not an exercise, if the solver does not have a viable solution path (Schoenfeld, 1985). Thus, if a geometry student is given a proof task and does not immediately recognize the deductions that need to be made in order to bridge the elements to the goal, the task can no longer be defined as an “exercise” and can now be defined as a “problem” (Schoenfeld, 1985; Heinze et al., 2008). Schoenfeld (1985) defines resources as “mathematical knowledge possessed by the individual that can be brought to bear on the problem at hand” (p. 15). Examining problem solving through the lens of the shape understanding Learning Progression, typical knowledge (resources) brought to the problem by the solver would mainly involve the properties of shapes and informal empirical knowledge and evidence (Schoenfeld, 1985). Some of these resources might also include non-mathematical knowledge or real world experiences that influence spatial reasoning and orientation (Schoenfeld, 1985). Schoenfeld (1985) defines heuristics as “strategies and techniques for making progress on unfamiliar or nonstandard problems” (p. 15). Again examining problem solving through the Learning Progressions, this would involve such strategies as “testing and verification,” “drawing figures,” “exploiting related problems,” much of which can take place in a DGE to build empirical evidence for students who reason at level three (Schoenfeld, 1985, p. 15; Battista, 2012). If a student is not reasoning at level four on the shapes understanding Learning Progression, he or she may default to non-deductive reasoning strategies in order to
complete the proof with some sort of evidence, even if deductive evidence is expected by the problem poser (textbook, teacher, standardized test, etc.) (Battista, 2012). Although deductive proof is the desired end goal for high school geometry students, non-deductive reasoning strategies should still be codified and recognized as valid progress toward a solution for students who are not yet ready to write formal deductive proofs. The results of these clinical interviews will attempt to do exactly that: recognize, codify, and analyze a single student’s reasoning and problem solving strategies on paper and in a DGE.
Method

The experiment took place over a three-week time period with one-on-one clinical interview settings ranging in length from two minute to forty-five minute sessions. The student was given access to a DGE and a calculator. The ultimate goal of the experiment was to determine the subject’s Learning Progression level, prompt the subject to give more in depth reasoning to decision making, and to determine how interactions in a DGE impacted his reasoning. Comparisons between student reasoning on paper and in a DGE were made by presenting the student with the same task both on paper and in the DGE through three phases of data collection: pre-test, experimental, and post-test.

Participant

The participating student, Carl, is an active fifteen-year-old sophomore concurrently studying Algebra II and Geometry. He has been on an Individual Education Plan (IEP) since he was ten years old but does not receive specific support services for mathematics. His IEP specifies small group testing accommodations, extended time on assignments and assessments, and executive functioning goals such as organization and note taking. He has stated that he felt confident in his freshman Algebra I class but does not feel as confident in his Algebra II and Geometry course work. Carl plays on the soccer team, enjoys playing video games, and during the data collection period was eagerly awaiting his sixteenth birthday and the opportunity to pass his driver’s test. In addition to being Carl’s Geometry teacher, I was also his Algebra I teacher. During the
data collection time period, I had Carl as a student for a total of eighteen months. I will be continuing on with Carl for his junior and senior years as his computer science teacher. By the time he graduates, Carl will have had me as a teacher for all of his four high school years. Carl was selected for this project because of his hunger for understanding, his desire to find ways to make math “make sense”, and his openness to trying new strategies. His family shares this openness and is greatly supportive of his academic decisions and is actively involved in his scheduling and IEP progress monitoring meetings. From personal experience, Carl thrives in a one-on-one setting in which he feels that time is not a constraint and that the pressure of peers succeeding more quickly is a nonissue.

**Author as Instrument Protocol**

The goal of this experiment was to learn about the specific mathematical thinking for a specific child. Qualitative data gathered and analyzed by the researcher as a human instrument calls for the researcher to be fully aware of biases, past research, and biographical experiences (Greenbank, 2003, p. 799). As a student researcher, at the time of these interviews, I had been enrolled in the Masters of Arts in STEM Education at The Ohio State University for eighteen months. My elective courses centered around technology and digital tools in the classroom, mathematical modeling, mathematical problem solving, and Learning Progressions and processes in mathematics. Because of my passion of combining mathematical problem solving and technology, I have done a great deal of personal research and exploration in DGEs such as Desmos and Geogebra. My required courses in the Master’s program at Ohio State gave me the time and space to
learn about global issues such as diversity and equity in education, research methods, balanced assessment, and the theories and history of STEM education. I have previously conducted clinical interviews with the CBA Learning Progressions tasks but in general have limited experience with research. I have been teaching for a total of six school years and have taught Geometry for the past two years. I feel confident in my understanding of the language of the Common Core Standards required by my district and their connections to past and future standards expected to be met by my students. Bringing my personal and academic experiences to the experiment as well as my personal experiences with Carl as a student, allowed me to make decisions and inferences from the nuances of Carl’s produced or verbalized thinking. Having taught Carl geometry, I recognize when he applies a strategy to a situation having seen that strategy before in a different and non-mathematically equivalent context. In class, Carl can easily lose interest in tasks that he feels are above his developmental level. He gets discouraged by the pace and ease at which some of his peers work and occasionally turns that frustration towards me and “shuts down.” This is frustrating to Carl, myself, and his intervention specialist. “The full shut down” happens when Carl does not want to engage any further in a task. He focuses on his own deficiencies in comparisons to his peers and essentially gives up for the remainder of the class period. Admittedly, I often do most of the cognitive lifting for him to get him out of his shut down. I simply become a crutch to get him through the lesson and those feelings of inadequacy continue to build within Carl and myself as a teacher. These types of interactions are what caused me to immediately think of Carl when designing this experiment. I wanted to learn how to think of Carl outside of his deficits.
and focus on how he makes connections with the resources he already possesses. I wanted to be a better teacher for him and for his family and I wanted to learn how to understand his thinking and help prepare him for successful mathematical, career, and social futures. Knowing his family and personal academic history, combined with my experiences at Ohio State, I am able to make well informed and research based conclusions about Carl’s thinking as an individual learner and as a general producer and consumer of mathematics.

**Data Collection**

Data was collected in three phases with each session length shown in Table 2.

<table>
<thead>
<tr>
<th>Task Type</th>
<th>Phase 1: Pre Test</th>
<th>Phase 2: Exploration</th>
<th>Phase 3: Post Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper Tasks</td>
<td>Figure 2 (5 minutes)</td>
<td>Figure 4 (3 minutes)</td>
<td>Figure 2 (2 minutes)</td>
</tr>
<tr>
<td>DGE Tasks</td>
<td>Figure 3 (5 minutes)</td>
<td>Figure 5 (45 minutes)</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Data Collection

Data was collected from these phases in four forms: screencast audio and visual recordings of the Carl’s actions in the DGE, written student work, audio files of the interviews, and researcher field notes.
Selected Tasks

The purpose of the first phase of data collection was to diagnose Carl’s pre-experiment level of understanding using Battista’s (2012) Learning Progression of Understanding Shapes levels (Table 1). In this initial phase, two tasks were presented to determine Carl’s initial Learning Progression level (Battista, 2007). The first task (Figure 2) presented Carl with six quadrilaterals (Battista).

He was asked, “which of these are rectangles?” and “describe exactly how you determine if something is a rectangle.” The goal of this task was to gain information about his understanding of the hierarchical relationship between, rectangles, and squares (Battista 2012). The second task was presented in the DGE called iDGi (Figure 3), developed specifically for assessing student reasoning on the Learning Progressions (Battista, 2011).
Carl was given a dynamic “parallelogram maker” where he was able to adjust the angles and sides of a quadrilateral that will only create parallelograms. He was asked, “what kinds of shapes can you make with a parallelogram maker?” These two tasks were presented back to back during the same interview session to gather data to answer the research question of “how might a Dynamic Geometry Environment assist students in progressing through the shape understanding Learning Progression?”

The second phase of the data collection was a proof about a parallelogram, which was first presented to Carl on paper (Figure 4) and then in Geogebra (Figure 5) (Great Minds, 2015).
Suppose that ABCD is a parallelogram and that M and N are the midpoints of segment AB and segment CD, respectively. Prove that AMCN is a parallelogram.

Figure 4. Phase 2 Paper Proof Task, Great Minds (2015)

The task presented a parallelogram (given) and two segments connecting the midpoint of the two longest sides to the opposite angle, creating a second quadrilateral inside the larger. Carl was first asked to read and make a conclusion about the proof on paper and
then was given the same proof in the DGE. Within the DGE, Carl was given the opportunity to turn on the value of any angle or length that he wanted and was given the ability to move the vertices to change those values. The goal of this task was to see if Carl could return to the paper version of the task and write a proof using the evidence he gathered from exploring the dynamic image in the DGE.

The third and final phase of data collection was post-experiment diagnostic using Battista’s (2012) Learning Progressions to determine Carl’s reasoning of shape understanding after his experiences in the DGE. Carl was given the same task from the pre-experimental diagnosis (Figure 2) but this time was asked to “determine which of these are parallelograms. Describe exactly how you decide if a shape is a parallelogram” (Battista). This was asked differently than the pre-experimental interview to determine if Carl had created new hierarchical connections between quadrilaterals (Battista, 2012).

**Clinical Interview**

All tasks were presented and guided in the style of a clinical interview. The goal was to first discover Carl’s cognitive activities, second, to identify the cognitive activities in his actions and mathematical decisions, and finally to evaluate the levels of competency about those selected decisions (Ginsburg, 1981). These tasks were also selected in an attempt to answer the primary research question of “what problem solving strategies do children use when working on a task above their shape understanding Learning Progression level?” Since I am Carl’s teacher, and have been for two school years and will continue on with him for a third and then fourth year, my goal was to better understand and then predict the nature of his thought (Ginsburg, 1981). So many of
Carl’s decisions that I saw on a daily basis were fascinating yet divergent from the ideas of his classmates. During the time constraints of our class period, I was not able to spend the time to dig deeper into his thoughts and discover the root of their nature. Using the clinical interview questioning style in an attempt to “explicate the nature of” Carl’s thoughts, I created this project to become a better teacher for Carl and to see where those seemingly random thoughts were born (Ginsburg, 1981, p. 4). Because of Carl’s placement on an IEP, he has experienced years of testing and retesting to measure performance and to update his goals with his changing course schedule. This constant cycle of performance measurement neglects the competencies that are truly present in Carl’s thinking. Because of this constant comparison and measurement of performance, Swanson et al. (1981), state that “the exhibition of competence is generally reserved for out-of-school contexts,” in Carl’s case, such as soccer or driving (p. 37). The true goal of these interviews was to validate Carl’s thinking as useful, creative, and interesting and to generate hypotheses about Carl’s mathematical reasoning (Swanson et al., 1981). The style of these interviews followed Ginsburg’s (1997) “first commandment” of clinical interviews: “not to belittle the child’s ideas [because] they make some sense from the child’s point of view” (p. 141). Because of the extended amount of time I spend with Carl in math class and the amount of time I will still spend with him in future classes, my goal was to see from this point of view and better understand his mathematical problem solving resources to help guide and improve his mathematical future (Schoenfeld, 1985).

Data Analysis
The goal of the data analysis was to identify and codify Carl’s strategies and
decisions to better understand his reasoning and discover new competencies. This was an
attempt to answer the research question of “what problem solving strategies do children
use when working on a task above their shape understanding Learning Progression
level?” After codifying the strategies, decisions and reasoning were compared from paper
to DGE tasks and evaluated on the Learning Progressions levels in an attempt to answer
the research question of “how might a Dynamic Geometry Environment assist students in
progressing through the shape understanding Learning Progression?” (Battista, 2012).
After collecting the data, I listened to and watched each of the recordings and transcribed
student and interviewer statements. I also used Battista’s Learning Progressions to
determine the level of Carl’s understanding of shapes for each task (Table 1) (Battista,
2012).

After analyzing the transcription for the second phase of data collection, I noted
anything that may have been hypothesis generating of Carl’s competencies and decisions
(Swanson et. al, 2000). From these notations, I found that Carl regularly relied on two
mathematical strategies in an attempt to solve a problem or make a generalization:
triangle or quadrilateral interior angle sum, or approximation and assumption of a value
based on visual characteristics. Both of these strategies are of the empirical justification
type of the Marrades and Gutierrez (2000) framework. Of these two strategies, when
asked to explain his reasoning of using the strategy, one of three things occurred: 1) Carl
correctly solved and described an explanation that matched his strategy, 2) Carl
incorrectly applied a strategy that was inappropriate for a situation (what Tall (2004)
would call a “met-before”) or 3) Carl correctly applied a strategy but inaccurately described why the strategy was used. Tall (2004) describes a “met-before” as an inappropriate application of a previously encountered strategy or concept to a newly encountered concept. Schoenfeld (1985) would call this a problem solving resource but identifying this specific type of resource as a “met-before” implies that the selected resource is not likely to lead to a correct or logical conclusion. There was occasional overlap in strategies and reasoning.
Results

Phase 1: Pre-Experiment

The first task (Figure 2) was presented to determine Carl’s existing hierarchical knowledge of quadrilaterals. The transcription below demonstrates Carl’s application of the “visual as reality” strategy empirical justification type (Battista, 2012; Marrades & Gutierrez, 2000). Carl exhibits this strategy in his assumption of an angle as a right angle in the transcript below and in Figure 6 (Battista, 2012):

Carl: E is a rectangle.
Teacher: Why?
C: Because it has four sides and all are right angles. And A, and D. That’s it though.
T: Ok so how did you decide that those were rectangles?
C: Rectangles are always bigger than squares. I knew that because C is a square.
T: How do you tell the difference between a rectangle and a square?
C: Squares all the sides are equal. Rectangles is two and two are equal. They all equal each other.
T: So in order for a shape to be a rectangle what does it have to have?
C: Two of the same length and width and the length, the bottom and sides are different.
T: It looks like in shape B their lengths are the same.
C: Yeah.
T: Why didn’t you choose B as a rectangle?
C: Could this be a right angle (referencing the top rightmost angle, Figure 6)? Because these two are right (draws in right angle marker in upper right and bottom left corners).
Because of Carl’s formal description of the angles and opposite sides of a rectangle, Carl is exhibiting signs of a student who reasons at level 2.3 in the Learning Progression (Battista, 2012). However, in the second task given to Carl, which took place in the DGE, we see Carl exhibiting signs of a student who reasons at level 3.1 (Battista, 2012). In the parallelogram maker task, Carl states that a rectangle is “a rectangle looking parallelogram.” Because of this loose description of a specific type of parallelogram (one with four right angles), Carl is exhibiting signs of a student in level 3.1 (Battista, 2012). He uses the specific instance of a specific rectangle to create a type of parallelogram but does not generalize. He sees the exact parallelogram he has created as one instance but does not move outside of his empirical data to broaden his definition of a parallelogram. The use of the DGE allowed him to make specific cases of parallelograms and build his empirical evidence bank allowing what he thought was two different shapes (rectangles and parallelograms) to become versions of each other. The tasks in Figures 2 and 3 were presented in that order on the same day. Thus, Carl’s reasoning was at a higher level of the Learning Progressions in the DGE than on paper, in the same interview session with the same context. Because of this timeline, I interpret my finding to show that in a DGE,
Carl reasons at level 3.1 of shapes understanding Learning Progression, but on paper tasks, he is mostly reasoning at level 2.3 (Battista, 2012).

**Phase 2: Experimental Tasks**

The table below shows the number of occurrences of each mathematical strategy and the reasoning that occurred with the strategy.

<table>
<thead>
<tr>
<th>Mathematical Strategy</th>
<th>Met Before: Incorrect Value, Unmatched Explanation</th>
<th>Correct Value, Unmatched Explanation</th>
<th>Correct Value and Matched Explanation</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of Interior Angles</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(51%)</td>
<td></td>
</tr>
<tr>
<td>Visual as Reality Approximation</td>
<td>7</td>
<td>6</td>
<td>7</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(49%)</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>15 (37%)</td>
<td>13 (31%)</td>
<td>13 (31%)</td>
<td>41</td>
</tr>
</tbody>
</table>

Table 3. Carl’s Reasoning Strategies and Explanations

Most of Carl’s strategies (37%) were selected as a result of a “met before” (Tall, 2004). Some of Carl’s “met-befores” allowed him to arrive at a correct value but the explanation of the selected strategy does not match what took place or should result in a different value. If Carl is getting the correct value and explanation about 30% of the time, and the other 30% of the time he gets the correct value (if not asked to further explain, his misconception may not have surfaced), he would average approximately a 60% on a summative assessment, allowing him, with homework grades and non-assessment grades, to pass a course carrying on the same “met-befores” to the next course.
in a sequence (Tall, 2004). This analysis matches his historical grades in math classes, of which he has averaged 70%. Some of these “met-befores” that Carl applied to the interview tasks include the strategies in Table 4 (Tall, 2004).

<table>
<thead>
<tr>
<th>Pythagorean Theorem</th>
<th>Alternate Interior Angles</th>
<th>Vertical Angles</th>
<th>Acute/Obluse Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive Sides and Angles</td>
<td>Angle Bisectors</td>
<td>Perpendicular Lines</td>
<td>Parallel Lines</td>
</tr>
</tbody>
</table>

Table 4. Met-Befores Used by Carl

On problems that Carl is unsure of a strategy, he throws anything that he has in his most recent resources to see what might fit the visual approximations he has created (Ginsburg, 1997; Schoenfeld, 1985). Schoenfeld (1985) would argue that just mentioning these resources (such as “Pythagorean Theorem”) but not applying them is a justified problem solving strategy because it demonstrates that Carl exhibits “control” over his bank of resources and only selects resources and heuristics that may prove fruitful.

In this phase, Carl was given a paper proof task and then the same proof task in Geogebra (paper: Figure 4, Geogebra: Figure 5). Using his “visual as reality” strategy on the paper version, Carl “proves” that the interior parallelogram is a parallelogram because the sides appear the same and parallel and the opposite angles appear the same. This shows that he does know the definition of a parallelogram, which is something he struggled with during the Diagnostic Phase. This formal understanding of a parallelogram demonstrates reasoning at level 2.3 on the Learning Progressions because he still does not
demonstrate understanding the relationship of the shared and interrelated properties between the inner parallelogram and the larger parallelogram (Battista, 2012). When asked if he thought that he had enough information and if he was satisfied with his proof, he seemed confident and said he felt as though he had done what the problem had asked. When given the digital version of the same proof in Geogebra, the first thing he did was confirm his angle sums (that their sum is 360 degrees). This makes sense as to why finding the interior angles of a polygon is his most used strategy: he feels comfortable with his knowledge of the sum of interior angles and applies this strategy whenever possible. It is his greatest competency and he did not make a calculation error in finding missing angle sum in the entire experiment. Because there are so many angles in this problem, Carl uses his “visual as reality” strategy to assume that all obtuse angles in the problem are congruent because of “alternate interior angles”. When asked what “alternate interior angle” means, Carl said, “angles on the inside have to equal angles on the outside”. He recalls that alternate interior angles are congruent and uses this term as a way to describe any pair of congruent angles. His definition of congruence comes up again when he states that the two interior sides of the smaller parallelogram “are definitely congruent … but are they the same lengths?” This is a perfect example of a “met-before” in which Carl is applying a frequently used vocabulary word but does not know how to appropriately use the concept for a deductive proof, thus, Carl is still reasoning at a level 3.1 on the Learning Progressions in an attempt to solve a level four task (Battista, 2012). For Marrades and Gutierrez (2000) justification types, in an attempt
to write a deductive proof, Carl uses visual evidence or “naïve justification” to make a conclusion.

Consistent use of the empirical justification type does slightly frustrate Carl in the next transcript section, stating that he “just doesn’t know how to prove it”. Here, he moves away from angle sums and tries to apply the same thought process to side lengths:

Teacher: You proved that the top part of the parallelograms are congruent, how about the sides? Are they congruent?
Carl: Yes but don’t know how to prove it.
T: Are the triangles congruent?
C: They have the same length measures
T: Can you prove if the triangles are congruent?
C: Yes.
T: How?
C: They will have the same lengths. I just don’t know how to prove it.
T: (puts angle measure of triangles back in) Even though you already solved for the angles let’s put them back on. Now what?
C: I don’t know what these lines are supposed to add up to (EC and AF).
T: Like how angles are supposed to add up to 180?
C: Yeah.
T: I don’t know.
C: Me either that’s why I can’t solve it.
T: Are these triangles the same?
C: Yes. Because the angles are congruent. And the sides. But I don’t know all of the sides.
T: Is that enough information?
C: I think so but I want to find the exact number for those sides.
T: If you could find the number, do you think they would be the same or different?
C: Same.
T: (Turns on lengths) Were your right?
C: Yes they are congruent.
T: How could you figure that out without Geogebra?
C: I just really don’t know.
T: But you believe they are the same?
C: Yes but if I don’t have Geogebra I can’t prove it.

In this “met-before” example, Carl has extended his angle sum strategy to a “side sum strategy” (Tall, 2004). He is fully competent and confident of the bounds of the
angles of a triangle and quadrilateral. He wants that same structure for finding the sides of a triangle and is not satisfied to only rely on the information from the DGE. Finding this structure would build more empirical evidence toward proving that the inner quadrilateral is also a parallelogram, still allowing him to reason at level 3.1. Toward the end of the interview, Carl was truly sitting in cognitive dissonance and conveyed that he was disappointed that he would have to rely on technology just to be sure. This is not to say that he does not trust the evidence given by the technology. Because of his confidence with the Angle Sum Theorem, he expressed a desire to use some type of generalized theorem, not dissimilar from the Pythagorean Theorem, to solve for a missing side length in a non-right triangle. In this task, Carl uses the “visual as reality” strategy when partitioning and angle sums are not enough to reach the requested level of proof about congruent segments. Ultimately, proof was achieved through the route of empirical justification types. Carl exclusively used the “perceptual type” of “naïve empiricism” to complete his proof (Marrades & Gutierrez, 2000).

Phase 3: Post-Experiment

In the final task, Carl was asked to determine which of the six quadrilaterals were parallelograms and he selected A-E (Figure 2). When asked why, he stated that opposite sides are congruent and quadrilateral F was not a parallelogram because one of the sides is “slanted”. This demonstrates reasoning at level 3.3 of Battista’s Shapes Understanding Learning Progression (Battista, 2012). Carl uses logical (but not formal) deduction of the shapes and uses a minimal literal definition of parallelograms. This interview was the first time that Carl exhibited the understanding of the hierarchical nature of a
quadrilateral type. He stated that the parallelograms have congruent sides and recognized that the slope of a side of F is not the same as its opposite side. Placing rectangles and squares under the hierarchy of parallelograms is a step up in the progression from where he began the experiment (Battista, 2007). Although he has still not reached level four in completing formal deductive proofs, in these interview sessions, the DGE allowed Carl to move more fluidly from level 2 to level 3 making progress toward controlling his resources to apply to a proof (Battista, 2007; Schoenfeld, 1985).
Discussion

I was fortunate enough to have Carl as my subject, a student who was comfortable sharing his thoughts and decisions, honestly report what he was able to articulate of his processing activities (Swanson et al., 1981, p. 32). Although “what a subject reports will always involve selectivity and interpretation,” I plan to focus on Carl’s thoughts and decisions that may lead to possible answers or more questions from the primary research questions designed for this study (Swanson et al., 1981, p. 32). From the interview results, I determined that two conclusions can be made about Carl’s reasoning. First, Carl’s reasoning on the Learning Progressions was accelerated in a DGE and second, clinical interviews revealed a specific child’s unique problem solving strategies were often non-property based in an attempt to create proof.

Conclusion: Student Reasoning Accelerated in a Dynamic Geometry Environment

In response to the research question, “how might a Dynamic Geometry Environment assist students in progressing through the shape understanding Learning Progression?”, Carl’s reasoning during the diagnostic phase was higher on the DGE task than on the paper task. Carl demonstrated these differences in reasoning abilities both in phase one and in phase two. The descriptors of Battista’s framework (Table 1) allows researchers and teachers to categorize student thinking based on reasoning that students displayed. Carl made hierarchical connections (level 3 descriptors) after he was able to manipulate the parallelogram in the iDGi environment when minutes before he was
unable to make property based connections between squares, rectangles, and parallelograms on paper. This likely because Carl sees what is on paper is the only specific instance of a phenomenon or set of properties occurring. If a parallelogram has an acute angle of 68 degrees, he only sees that exact parallelogram and does not see it as a part of the greater realm of all possible parallelograms. This fits the Learning Progression level 2.3 description because Carl formally describes the properties of a single visible shape but does not describe the shape as an interrelation of properties of all other possible shapes with those qualities in common (Battista, 2012). However, in the DGE, Carl was able to quickly generate multiple empirical cases with the software, thus expanding his visual reality to include multiple cases of shared properties. In seeing the greater possibilities of a parallelogram in the parallelogram maker from Figure 3, Carl began to build an empirical evidence bank of other parallelograms and create his own constraints, rules, and properties about parallelograms as a general case (Battista, 2002).

In the tasks from phase two, Carl used the DGE to confirm his predictions about congruent segments or angles, thus verifying his “visual as reality” construct that he applies by intuition, estimation, or use of a previous example. By allowing him to “turn on” or reveal any distances or angle measures in Geogebra during the tasks, Carl was able to make more definitive and tangible connections to what it means for angles and segments to be “congruent.” In the parallelogram proof, although Carl was unable to use formal properties to deduce that the quadrilateral inside the parallelogram was also a parallelogram, he was able to use the values given by Geogebra to confirm his visual intuitions (Chazan, 1993). The ability to recreate a proof from paper to a DGE allowed
Carl to use empirical, physical, verified evidence to analyze shape construction (Battista, 2002; Battista, 2012). With more time in the DGE and the capabilities it provides to deepen his empirical knowledge bank, he will begin to logically, not just intuitively understand the properties of shapes, moving him to reason more consistently at level 3.3 on the Learning Progressions (Battista, 2012; Battista 2002).

In this experiment, shape characterization appears to have been accelerated in the DGE. On the same day with a similar task, Carl demonstrated the reasoning of a student at level 2.3 on paper and then at level 3.1 in the DGE (Battista, 2012). This jump, although only one level, is significant in reasoning abilities. On paper, Carl was unable to create a hierarchical connection between squares and rectangles, but in the DGE, he began to loosely make those hierarchical connections (Battista, 2002). In line with Battista’s (2007) evidence, this also contradicts the van Hiele theory that students progress through levels in a linear fashion and are immobilized from jumping back and forth between levels once they have “passed through” the previous levels (Battista, 2007). In Phase 1, the DGE gave Carl the confidence and tools to begin to form definitions and interrelate properties (Level 3) when just minutes before he was still focused on the description of a single shape without being able to interrelate (Level 2) (Battista, 2007; Battista, 2002). Although this is only one student and one DGE, this leads to a new research question of, “does working in a dynamic geometry environment actually lead to different math learning progression level?” Carl’s ability to generate multiple empirical cases of a property may have given him the insight into the generalization that he was unable to discover with a single case on paper (Battista, 2002).
Conclusion 2: Clinical Interviews Reveal Student Problem Solving Strategies

The second primary research question this paper attempts to address is, “what problem solving strategies do children use when faced with a task that is above their developmental understanding of shapes?” As previously stated, the original intent of this project was to examine student progress toward deductive proof. However, Carl’s exclusive use of empirical justification types in the clinical interviews lead to deeper connections to a problem solving framework rather than a proof framework (Marrades and Gutierrez, 2000). Carl used multiple strategies that were outside of the typical reasoning strategies within the Shape Understanding Learning Progressions descriptions, yet he was still able to use empirical evidence through various problem-solving resources to reach “proof.” These resources and the brilliance in their connections to other resources would never have come to my attention, or been recognized as competencies, had I not been able to spend time with Carl in the clinical interviews. In the array of tasks given in this experiment, Carl’s strategies can be summarized in one of two ways: listing of potential resources and application of a high competency strategy. Schoenfeld describes a problem as “a task in which a solution is not readily available with a known procedure” (Schoenfeld, 1985). Because Carl is not regularly reasoning in the upper levels of Battista’s level 3 of the shapes understanding Learning Progression, it can be assumed that tasks in which deductive proof is expected (as expected in the task from Figure 4) can be considered a problem solving task. Thus, typical high school geometry proof tasks are a “problem” for him or any other student who is asked to write a proof without truly having an understanding of the definitions and the interrelated properties of
shapes (Battista, 2012; Schoenfeld 1985). Because students who are not ready to tackle level four tasks are often given them because of their grade level and not their developmental level, students must select a strategy from their mathematical knowledge resources attempt to progress through the problem (Schoenfeld, 1985). Although Carl’s listing of possible strategies at once had seemed random, it is truly his way of running through his resource bank and exhibiting control over those resources by occasionally judging the reasonableness when deciding not to apply the strategy (Schoenfeld, 1985).

Students who lack confidence on tasks in which a solution is not readily available or accessible may list through their resources to try and see what fits (Schoenfeld, 1985). In time constraints of the classroom, Carl’s listing of strategies to me had seemed random. But in the clinical interview setting, I discovered that this listing of strategies is a brilliant coping mechanism for someone who has been given a problem that is above his developmental understanding of shapes. Although he listed the resources he felt comfortable using aloud, he did not apply all of these resources and used his control (Schoenfeld, 1985). Heeding Ginsburg’s strategy of “following the child’s response wherever it leads” led to the discovery of Carl’s exhibition of control. The listing of resources strategy is a reasonable way for students who are having difficulty with a task to search for an entry point into making progress toward a solution.

The second common strategy that Carl used in these tasks was to apply a strategy that has a high competency for past tasks. Selter (1998) states that “making sense of children’s mathematics in a positive way appears to be an attitude that can be extremely helpful not only for researchers and teachers, but, more importantly, for the pupils as well.
whose creativity and intelligence much too often is often not recognized” (p. 22). Because we can assume that Carl’s competencies are not always given credit or evaluated (rather, his performance is evaluated), he often applies a high yield strategy to build confidence or make headway on unknown quantities (Swanson et al., 1981). In this experiment, Carl’s top competency was the Angle Sum Theorem. Any time there was a triangle or a quadrilateral, Carl immediately solved for all missing angles, even if it was unnecessary. He did not make one calculation error in finding missing angles throughout the entire experiment. Because he felt so comfortable with this strategy, he tried to apply the strategy as a “side sum theorem” in the last task of phase 2 (Figure 5). Students who have constantly been told what they are doing wrong rather than what they are doing right may use their problem solving control to apply a high competency strategy, even if it will not be fruitful in progressing through the problem (Selter, 1998; Schoenfeld, 1985). A student who exhibits this strategy should be celebrated by teachers because for them it is likely that “failure is the norm” and this strategy builds confidence and grounds students in where their competencies lie, rather than where their skills are deficient (Swanson et al., 1981, p. 37). This high competency strategy may be the student’s only entry into the problem.

For Carl, these problem solving heuristics are typically applied in the “met-before” fashion, meaning that he has previously met a seemingly similar context, which is then inappropriately applied to a new situation (Tall, 2004; Schoenfeld, 1985). All of his strategies have proven to be successful in a specific case in the past and thus, that case is a part of his bank of empirical knowledge and might prove fruitful again. A “met-
before” is not necessarily a misconception. For example, if a child is expected to find the distance between two points and rather than use the expected Pythagorean Theorem or distance formula the student uses the Angle Sum Theorem, this does not necessarily mean that the student has a misconception about the Pythagorean Theorem. It simply means that the child applied a strategy from a prior experience to fit a new experience. Carl, like most students, has a deep bank of empirical knowledge from which he can retrieve. Years of homework problems, classroom example problems, and pages in a math book have all been “met-before” (Tall, 2004). Thus, to him, any of those strategies are realistic when it comes to a task that is above his current developmental level of shape understanding in which he does not recognize a viable solution path (Schoenfeld, 1985). Lesh and Kelly (1997) note that many problems have an overwhelming amount of information that is both relevant and available to students. Because these problems present large quantities of information (in particular proof problems, where a written list of givens is presented in addition to an image), “inappropriate meanings are often projected on the situation on the basis of expectations from similar prior experience” (Lesh & Kelly, 1997, p. 400). Students who typically struggle in mathematics will look deep into their empirical bank of knowledge to find a strategy that looks like it might fit. Thus, a “met-before” strategy of high competency may be frequently applied (Tall, 2004).

Through the clinical interview framework, analyzing Carl’s decisions on paper and in the DGE with the Learning Progressions allowed for me to better understand his decisions, problem solving skills, and reasoning. Since I will have the opportunity to
work with Carl until he leaves the world of high school and goes on to tackle college and career, I plan to continue to focus on his competencies and use technology when purposeful, appropriate, and accessible, to build upon those competencies and help develop his reasoning skills, justification of evidence, and confidence in mathematics.
Conclusions and Suggestions for Future Research

Although this is a single teaching experiment with a single child, conclusions can be made that prompt future research. When working in a DGE, Carl exhibited characteristics of a student who reasons at a higher shape understanding Learning Progression level than when he was working on paper (Battista, 2012). Since he is not yet reasoning at level four of the Learning Progressions, he did not use deductive justifications or property based strategies in an attempt to complete a proof. Carl instead used perceptual empirical justifications as a problem solving strategy (Marrades & Gutierrez, 2000; Schoenfeld, 1985). With this single subject experiment, and with the goal in mind to better understand his thinking through clinical interviews, this student was able to think at a higher level of Learning Progression while in a DGE. He was able to cite more empirical evidence as he moved toward property generation. When given a proof task above his developmental Learning Progression level, the subject used “met-before” problem solving resources to make progress toward a task goal (Tall, 2004; Schoenfeld, 1985). Large scale generalization would assume that students who are not developmentally ready for proof, but who will encounter proof tasks due to their grade level, will use empirical strategies and non-property based problem solving resources to produce evidence. This gets to a greater systemic issue of mandating standards before students are developmentally ready, but that is a topic for another and entirely different social experiment. In this project, the DGE provided a space for the student to gather
evidence and make conclusions with his current reasoning abilities and competencies more quickly than on paper. The use of a DGE accelerated the process of shape property generalization.

As a geometry teacher with the goal to learn more about the thinking of a singular student, the use of clinical interviews proved incredibly insightful. Other geometry teachers who are able to find the time to use clinical interviews to build trust with students and gain deeper understanding of student reasoning will greatly benefit. Using the Learning Progressions to more appropriately diagnose student understanding allowed for me to focus on the competencies of my subject and ultimately give value to his reasoning that had at once seemed random in whole group class sessions. All students’ reasoning should be recognized as valid and I am privileged as an educator to gain insight into a student’s thinking. Using clinical interviews to better understand student reasoning and to work on in a one-on-one setting removes the constraints of the bell, the pressure from peers, and allows for teachers and researchers to get to know the deep connections of student thinking.
References


