Behavioral Economic Theory and Experimental Investigation

Dissertation

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By

Jeevant Rampal, M.A.

Graduate Program in Economics

The Ohio State University

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Dissertation Committee:

James Peck, Advisor

John Kagel

Paul J Healy

Dan Levin
Abstract

This dissertation investigates how and why individuals’ limitations in understanding decision problems, or behavioral biases, affect their optimal choices. The first chapter defines the Limited Foresight Equilibrium, henceforth referred to as LFE. LFE provides an equilibrium assessment for a model where players can possess limited foresight and they are uncertain about the opponents' foresight while playing a finite dynamic game of perfect information. We show the existence of LFE. The LFE entails limited foresight players updating their beliefs about the opponents' foresights within the play of a game. LFE implies that the higher the foresight of a player, the more accurate his beliefs about the opponents' foresights; further, if a low foresight player finds himself at an “unexpected” position in the game, he believes that one of his opponents has higher foresight than him. Thus, high foresight types, in LFE, take reputation effects into account. In applications, LFE is shown to rationalize experimental findings on the Bargaining game and the Centipede game.

The second chapter provides experimental evidence for the LFE's novel predictions in the context of a modified Race game. This experimental study investigates how and why the behavior of experienced players, who understand the “sure-win” strategy in a “winner-take-all” sequential move game, varies systematically based on two types of information about the opponent’s expertise. Treatment one: experienced subjects are told their opponent's experience-level in the game. Treatment two: a different set of experienced subjects are only shown their opponent's play against a computer. We find that both exogenous information and endogenous inference about the opponent's inexperience increase the probability with which experienced players abandon the
“sure-win” strategy and try for a higher payoff attainable only by winning from a losing position, that is, a position from which one wins only if the opponent makes a mistake. A maximum likelihood analysis shows that the LFE explains the data better than the Dynamic Level-k and Agent Quantal Response Equilibrium models.

The third chapter reports and models the discrepancy between the full-bidding and endow-and-upgrade findings from a willingness-to-pay (WTP) elicitation Becker-Degroot-Marschak (BDM) experiment for an improved food, conducted in rural India. We found that the distribution of the WTP for exchanging 1kg local pearl millet (LPM) for 1 kg of bio-fortified high-iron pearl millet (HIPM) dominated the distribution of the difference between the WTPs for 1kg HIPM and 1kg LPM. Thus the data rejects preferences that are standard or have status quo reference points, in favor of an expectations-based reference dependence model of loss aversion for the new product. The data is used to identify and estimate the loss aversion parameter and latent consumer valuations for HIPM in the consumer model, which point to a significant downward bias in conventional WTP estimates of HIPM using the BDM procedure, suggesting caution when one is using standard incentive compatible mechanisms for value elicitation.
Dedicated to my parents, Alka and Rakesh, to my wife, Sikta, my brother, Daksh, his wife, Anumeha, and my niece, Aria
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Vita

2007.................................................................B.A. Economics, Delhi University

2011.................................................................M.A. Economics, Delhi University

2013.................................................................M.A. Economics, The Ohio State University

2013 to present ..............................................Graduate Teaching Associate, Department of Economics, The Ohio State University

Fields of Study

Major Field: Economics
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Chapter 1: Limited Foresight Equilibrium

This chapter defines the Limited Foresight Equilibrium (LFE). LFE provides an equilibrium assessment for a model where players can possess limited foresight and they are uncertain about the opponents’ foresight while playing a finite dynamic game of perfect information. We show the existence of LFE. LFE entails limited foresight players belief updating about the opponents’ foresight within the play of a game. LFE implies that the higher the foresight of a player, the more accurate his beliefs about the opponents’ foresights; further, if a low foresight player finds himself at an “unexpected” position in the game, he knows that one of his opponents has higher foresight than him. Thus, high foresight types, in LFE, take reputation effects into account. In applications, LFE is shown to rationalize experimental findings on the Bargaining game, the Centipede game. The LFE’s novel predictions are corroborated by data on a modified Race game.\(^1\)

Introduction

Consider the setting of a finite set of players playing a sequential move game where each player knows all prior actions played at each move. That is, consider the setting of a finite, perfect information dynamic game. To solve for her optimal action at each move, each player should think ahead to the last stage and reason backwards using backward induction. Her optimal strategy should be calculated using the payoff possibilities in the game.

\(^1\)I would like to thank James Peck for many important comments in both conceptualizing and writing this paper. I also want to acknowledge Yaron Azrieli, Paul J Healy, Abhijit Banerji, and Anirban Kar for helpful comments and discussion. Last, I would like to thank the seminar participants at the Spring 2015 Midwest Economic Theory and International Trade Meetings, and the Winter School conference 2015 at DSE for their insightful comments.
and her belief about the actions of her opponents in the future stages of the game. However, there is ample experimental proof showing that even when this backward induction can reveal a dominant strategy for the player, a high proportion of players are unable to perform such backward induction (cf. Rampal (2017), Mantovani (2014), Levitt, List and Sadoff (2011)). It is notable that it is impossible to explain this failure to play a dominant strategy using Dynamic Level-k models (Ho and Su (2013), Kawagoe and Takizawa (2012)), which say that players are rational but their chosen strategies are determined by their subjective beliefs about their opponent’s cognitive-level/strategy, precisely because a dominant strategy is a best response irrespective of the players’ subjective beliefs. Further, as Johnson et al (2002) show, a sizable proportion of players ignore payoff-relevant information about the future stages of a game even when it is available upon browsing on their decision screen. Therefore, the limited ability to think/look ahead in a multi-stage game appears to be a specific form of bounded rationality which generates patterns of behavioral data not captured by alternative explanations.

We model this “limited ability to think/look ahead in a multi-stage game,” or what we call limited foresight, as one of the two main components of our theory. Foresight is defined to be the number of stages that a player can look ahead in a multi-stage dynamic game. If the foresight of a player does not extend to the last stage of the game from each of her moves, she is said to have limited foresight.

The second, novel, feature of this chapter is that we also model the scenario where players are uncertain about their opponents’ foresight. Several experimental studies (cf. Rampal (2017), Levitt, List and Sadoff (2011), and Palacios-Huerta and Volij (2009)) have combined players with different degrees of expertise in dynamic games of perfect information and found that the information about the opponents’ “expertise” has a significant

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2For example, in Rampal (2017), in one of the treatments, about 40 percent of the subjects fail to play the dominant strategy in a “winner-take-all” dynamic game against a computer which plays perfectly.

3Unless one resorts to a very specific strategy for the Level-0 player which involves failing to play the dominant strategy at the first few stages of a dynamic game but playing perfectly towards the last few stages.

4We do not model or investigate why players have limited foresight. For example, does limited foresight occur because calculations are harder with more stages in the game? Or is it because players think that they don’t need to consider future stages? Instead we focus on modeling: (i) only the implication of these primitives, i.e., limited foresight; and (ii) uncertainty about opponents’ foresights.

5Other, independent, studies that have modeled limited foresight similar to this chapter are Ke (2017), Mantovani (2014), and Roomets (2010).
impact on behavior. We model the scenario where this degree of expertise is captured by the level of foresight. That is, we model the scenario where players are playing a “seemingly” perfect information game, which means that the players can observe all prior actions every time they move, but the players may actually be uncertain about the level of foresight of their opponents. That is, players appear to be playing a game of perfect information, but they are actually playing a game of imperfect information with uncertainty about the opponents’ foresight. As a result, in our model, a limited foresight player’s “optimal” choice depends on both his foresight and his belief about the opponent’s foresight.

In our model, in addition to being uncertain about the opponents’ foresight, players can also update about the opponents’ foresight as they observe more moves of their opponents. The Limited Foresight Equilibrium (henceforth LFE) that we define and apply in this chapter formalizes the meaning of “optimal” choices given “consistent” beliefs in this framework of limited foresight and uncertainty about the opponent’s foresight.

The summary of the model and LFE is as follows. We start with an arbitrary game of “seemingly” perfect information. For example, consider Ann and Bob playing a Sequential Bargaining game (Rubinstein (1982) and Ståhl (1973)) where all prior choices are displayed. We map this game of perfect information to a game of imperfect information, called an Interaction game. In our example, the Interaction game is a scenario where there are multiple possible types of Ann and multiple possible types of Bob, and each type of Ann (respectively Bob) is uncertain about which type of Bob (Ann) she (he) is bargaining with. A type denotes a particular level of foresight.

Mapping the game of perfect information to its corresponding Interaction game models the uncertainty about the opponent’s type/foresight-level. However, this doesn’t model limited foresight. In particular, to solve for the optimal actions of a limited-foresight type of Ann, we cannot use the whole interaction game because the limited-foresight type cannot observe all the stages of the interaction game. Therefore we must consider curtailed

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6While Rampal (2017) induces different degrees of expertise by varying the degree of experience of the players in the game tested there, Levitt, List and Sadoff (2011) and Palacios-Huerta and Volij (2009) do so by mixing expert chess players with student subjects.

7Optimal choices also depend on his beliefs about the opponents’ beliefs about his foresight, and so on, but we make the assumption that the prior distribution over foresight levels is common knowledge, which lets us end this line of thinking at the first order beliefs.
versions of the interaction game to model both limited foresight and uncertainty about the opponent’s foresight simultaneously. We call such curtailed versions of the interaction game as *Curtailed Games*.

The LFE is solved and defined recursively. We start with the shortest possible curtailed version of the interaction game: the 1-staged Curtailed Game, named $CG(1)$, in which we curtail the interaction game after the first stage actions. If Ann starts the bargaining game, then $CG(1)$ is the game observed by the 0-foresight type of Ann, or $0_{Ann}$. $CG(1)$ models both: (i) the fact that $0_{Ann}$ has a foresight-level of 0; and (ii) that $0_{Ann}$ is uncertain about Bob’s type. We solve for the Sequential Equilibrium (SE) (Kreps and Wilson (1982)) of $CG(1)$ to obtain $0'_{Ann}$’s first stage LFE action and beliefs.\(^8\) Next, taking $0'_{Ann}$’s first stage LFE action and belief as given as Nature’s moves in $CG(2)$, we solve for the SE of $CG(2)$ solving for the LFE actions and beliefs of $1_{Ann}$ at her stage-1 information set, and $0_{Bob}$ at his stage-2 information set; as they both observe $CG(2)$ at those information sets. We proceed step-wise to obtain the LFE actions and beliefs for all the information sets of the Interaction Game. Therefore, in LFE, all foresight-types do the best they can within the bound of their foresight, given their belief about the probability distribution on opponents’ types.

Defining LFE as above provides us with the following properties. First, the LFE exists and it is upperhemicontinuous. Second, higher foresight types correctly anticipate lower types’ moves. This property is approximately mirrored in a finding from Reynolds’ (1992): testing recognition of opponent’s expertise among chess players, he found that “Higher rated players consistently made lower estimation errors” (of chess players’ ELO ratings). Third, when high foresight-types are estimating which lower foresight opponent-type they

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\(^8\)To make $CG(1)$ a well defined game we need to define payoff profiles for its terminal histories/nodes. In particular, if we are curtailling an interaction game with more than two stages after the first stage actions, then how do we construct payoffs after the first stage actions in $CG(1)$? In this chapter, in each curtailed game, after each last-stage action of that curtailed game, we construct a payoff profile such that the payoff to each player-type in the curtailed game is equal to the $(\min + \max)/2$ of the payoffs possible for that player-type from that action in the interaction game. For example, if $0_{Ann}$ asks for the whole first stage pie (worth a 100 units) in the first stage, then curtailing the bargaining interaction game after her demand of 100 implies a payoff of $(100 + 0)/2$ in $CG(1)$. This is because the minimum Ann can get after making any offer is 0 (when there is no agreement among Ann and Bob at any stage). We explain this “curtailment rule” in more detail in the model section. Mantovani (2014) also uses this same “curtailment rule” to map payoffs from a dynamic game to payoffs for its curtailed version. Ke (2017) formulates the axiomatic foundations of different possible “curtailment rules.”
are playing against, their belief becomes more accurate as they observe more moves of the opponent. This property is also mirrored in Rampal (2017) and Reynolds (1992); both studies found that the estimation error (about the opponent’s expertise) decreased as a function of the number of moves revealed. Fourth, due to the recursive definition of LFE, if a low foresight-type observes actions that were not part of the LFE strategies of opponent-types with lower foresight, then he discovers that he is playing against some opponent-type with a higher foresight than himself. The attempt by lower types to recognize opponent type and adjust behavior implies the fifth feature of our model: reputation effects. High foresight-types have to decide on what’s optimal: pretending to be a low type or revealing their high foresight-type. Importantly, this belief updating happens within a play of the dynamic game.

As a direct experimental evaluation of the LFE model, we direct the reader to Rampal (2017) which uses “race games” to illustrate the novel predictions of LFE. We summarize the findings of Rampal (2017) here. The “race game” begins with a box containing 9 items. Players move alternately, removing 1, 2, or 3 items from the box at each move. The player who removes the last item loses. There is a second mover advantage in this game. The prize from winning as the first-mover (second-mover) is 500 (200) experimental currency. Before the game begins, both players in a pair simultaneously choose if they want to be the first mover or the second mover in the game. One of their choices is selected with 50 percent chance each. Note that the prize from winning as the first mover is greater than the prize from winning as the second mover, but winning as the first mover is possible only if the second mover makes a mistake. Rampal (2017) found that if an experienced player was told that his opponent was inexperienced, or, in a different experiment, if an experienced player observed that his opponent failed to play a dominant strategy against a computer in a similar race game, the experienced player was significantly more likely to choose to be the “first mover” against such an opponent compared to an experienced opponent or an opponent who displayed perfect play. That is, Rampal (2017) finds that both (i) exogenous information, and (ii) endogenous inference about the opponent’s inexperience increase the probability with which experienced players abandon the “sure-win” strategy (of being the second mover) and try for a higher payoff attainable only by winning from a losing position.

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9The second mover should remove \((4 - \text{opponent’s previous choice})\) at each move to win.
i.e., a position from which one wins only if the opponent makes a mistake. A maximum likelihood analysis shows that the LFE model explains the data better than the Dynamic Level-k (Ho and Su (2013)) and AQRE (McKelvey and Palfrey (1998)) models. Therefore, three key aspects of the LFE model are established by Rampal (2017): first, the salience of the beliefs about the opponent’s ability to do backward induction; second, that experienced players update their beliefs about the opponent’s expertise based on the opponent’s past moves in the game; third, that the LFE model has a significantly higher likelihood, with respect to this data, when compared to the AQRE model and the Dynamic Level-k model. Therefore the interpretation of expertise/experience as the level of foresight is robust.

In the first of two applications of LFE described in this chapter, LFE is shown to have the ability to explain several qualitative findings on the Sequential bargaining game. We show that the LFE can simultaneously explain delays, near equal splits, disadvantageous counterproposals and subgame inconsistency. In particular, according to the LFE, disadvantageous counterproposals (Ochs and Roth (1989)) can be caused in a three period bargaining game because the second player, when of specific limited foresight-type, can fail to take into account that he has no bargaining power in the third period and that the pie shrinks after he rejects the first period offer. Thus, he rejects the first period offer he receives, but when he has to make a counterproposal, he faces a shrunk second period pie and the lack of last period bargaining power, which make a disadvantageous counterproposal sequentially rational. In the second application, we show that the LFE implies passing until the last few stages in a Centipede game with more than 4 stages, for arbitrary probability on limited foresight.

Related Literature

The closest works to ours in the area of dynamic games are the working papers of Ke (2017), Mantovani (2014), and Roomets (2010). These papers, in independent projects, model limited foresight in a similar fashion to ours. Ke (2017) describes the axiomatic foundations of curtailment rules to model one player, multi-stage, decision problems as observed by a player with limited foresight. Mantovani (2014) endogenizes the choice of foresight. He also demonstrates the existence of limited foresight using an experiment on a different
race game. The Valuation Equilibrium by Jehiel and Samet (2007) models how cognitively constrained players may group nodes of a sequential move game into exogenously given similarity classes, where each similarity class has a given valuation. Although related, Valuation Equilibrium does not deal with limited foresight specifically. Jehiel (1995) defines the Limited Forecast Equilibrium, where each player, at each of his moves, chooses his strategy to maximize his average payoff within his foresight horizon, given his forecast about the upcoming moves within that horizon. The forecasts are constrained to be consistent with the equilibrium strategies. Jehiel (1998a) provides a learning justification for these forecasts. Jehiel (1998b) and Jehiel (2001) extend the Limited Forecast Equilibrium to repeated games. The key difference among these papers and our work is that the second feature of our model, the uncertainty about the opponents’ foresight type, is absent from all the papers mentioned above. Consequently, updating belief about one’s opponents’ types within the game, strategic adjustments after updating beliefs about opponents’ foresight and reputation effects do not feature in these papers.

Most of the related Level-k literature deals with simultaneous move games (Stahl and Wilson (1995); Nagel (1995); Stahl (1996); Ho et al. (1998); Costa-Gomes et al. (2001); Camerer et al. (2004); Costa-Gomes and Crawford (2006); Crawford and Iriberri (2007a, b)), but it does capture the uncertainty about the opponent’s expertise. The paper from this literature that is closest in spirit to LFE is Alaoui and Penta (2016) which endogenizes the choice of level in a Level-k framework. In their model, the choice of the level of a player is a function of his maximum possible level, which is endogenously determined by his incentives and costs of thinking about the game at hand, his belief about the opponent’s maximum possible level and his belief about his opponent’s belief regarding himself. They disentangle the effect of these factors using a novel experimental design. While we don’t model the analogous question of how a player’s maximum possible foresight is determined, the LFE model does have the feature that a particular foresight type considers his first order beliefs (beliefs about opponents’ foresight) and second order beliefs (his belief about the opponents’ beliefs about his foresight) in choosing his optimal strategy. Further as we are dealing with dynamic games, we also study how these beliefs evolve across the stages of the game. Experimental studies of the relation between the opponent’s cognitive level and
a player’s choices in simultaneous Level-k settings include Agranov et al (2012), Gill and Prowse (2014), and Slonim (2005), among others.

Ho and Su (2013) and Kawagoe and Takizawa (2012) have adapted the Level-k model for sequential move games. They allow for updating about the opponent’s level/expertise across repetitions of play of the same game as opposed to within the play of a game as in here. The AQRE model of McKelvey and Palfrey (1998) defines a dynamic game analogue of the QRE model where players are playing error prone strategies. As discussed above, these theories do not model the specific form of bounded rationality that we model in LFE, i.e., limited foresight. The distinction is important because limited foresight fits particular patterns of observed data significantly better than these models (Rampal (2017)). Three examples of such data patterns are: first, failure to play dominant strategies in a dynamic game; second, a significant decline in the proportion of dominated choices as the game comes to an end; third, evolution of players’ beliefs about the opponent’s expertise (hence strategies) within a play of the game based on the opponent’s past moves in the game.

The reputation effects in our model are similar to the crazy type literature started by Kreps, Wilson, Milgrom and Roberts in 1982, yet there are important differences. Their crazy types’ behavior is exogenous, and their crazy types, whose counterparts in the LFE model would be the player-types with low foresight, have no incentive to discover whom they are playing against.

**Model**

The model that we define here seeks to capture the scenario where a finite set of players are playing what “seems to be” a finite dynamic game of perfect information. Specifically, all prior actions taken in the game are observed by every player at his move, but every player has a particular level of expertise/experience in the game and every player is uncertain about each opponent’s level of expertise/experience in the game. In particular, we will focus on the case where this level of expertise/experience translates into a level of foresight. That is we will model the scenario where every player can have one of various possible levels of foresight and every player is uncertain about the level of foresight of each of his
The foresight level of a player is defined as the number of subsequent stages that a player can observe from any given move.

To model this scenario we start with the game that the players “seem to be” playing, i.e. a finite dynamic game of perfect information called $\Gamma_0$. It is helpful to think of $\Gamma_0$ as the game of perfect information that the experimenter sets up for a set of players to play. We map this game to the game that is “actually” being played, i.e., a standard Bayesian game of imperfect information called $\Gamma$ where every player in $\Gamma_0$ can have one of several possible types where each type denotes a particular level of foresight. For example, in the experiment in Rampal (2017), players with different levels of experience in a “race game” were playing in pairs. Subjects were aware that there was variation in the experience-level across subjects. Rampal (2017) found that both endogenous inference and exogenous information about the opponent’s experience-level affected the observed optimal choices.

In $\Gamma$, each player-type is uncertain about each opponent’s level of foresight or, equivalently, each opponent’s type. The Limited Foresight Equilibrium (henceforth LFE) provides a strategy and a belief profile for $\Gamma$. However, we cannot solve for the LFE actions and beliefs of player-types who have limited foresight using $\Gamma$ because player-types with limited foresight cannot observe $\Gamma$ at their moves. Therefore we will consider appropriately curtailed versions of $\Gamma$, which is what the limited foresight player-types observe, to solve for the LFE strategy and belief profile of $\Gamma$. In the next subsection we define the underlying perfect information game $\Gamma_0$. After that, we construct $\Gamma$, the game of imperfect information, from $\Gamma_0$. Last, we construct the curtailed versions of $\Gamma$ which are observed by player-types possessing limited foresight.

**The Underlying Perfect Information Game**

We use the standard notation from Osborne and Rubinstein (1994), with minor modifications, to define an extensive game with perfect information and perfect recall\(^{11}\) called $\Gamma_0$. In particular, $\Gamma_0$ is defined as a collection of the following components.

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\(^{10}\)We don’t model how the expertise/experience translates into a particular level of foresight. This is left for future research.

\(^{11}\)We don’t specify the conditions for perfect recall.
Figure 1. An Underlying Game of Perfect Information: Three-Staged Centipede Game

- A set of players \( N_0 \).
- A set \( H_0 \) of finite sequences or histories such that:
  - The empty sequence \( \emptyset \) is a member of \( H_0 \). We refer to \( \emptyset \) as the initial history.
  - If \( (a^k_0)_{k=1}^K \in H_0 \) and \( L < K \) then \( (a^L_0)_{k=1}^L \in H_0 \). Where the \( k^{th} \) action, \( a^k_0 \), is said to be taken at the \( k^{th} \) stage of the game. The set of terminal histories, denoted as \( Z_0 \), is defined as the set of histories \( (a^k_0)_{k=1}^K \in H_0 \) such that there is no \( (K+1) \) such that \( (a^K_0)_{k=1}^{K+1} \in H_0 \).
- A set of possible actions in the game, \( A_0 \), and an action correspondence \( A_0(\cdot) \) which maps \( h_0 \in H_0 \) to a set \( A_0(h_0) \equiv \{a_0 : (h_0, a_0) \in H_0\} \).
- A function \( P_0(\cdot) \), called the player function, which maps each element of \( H_0 \) to an element in \( N_0 \). That is, \( P_0 \) assigns a player \( P_0(h_0) \) to each history \( h_0 \).
- For each player \( i \in N_0 \), a Bernouli utility function \( u_i \) which maps terminal histories to real numbers, i.e., \( u_i \) maps a terminal history \( z_0 \in Z_0 \) to a payoff \( u_i(z_0) \in \mathbb{R} \).

Thus, \( \Gamma_0 \) is defined as \( \{N_0, H_0, P_0, A_0, \{u_i\}_{i \in N_0}\} \). Let the maximum number of stages in \( \Gamma_0 \) be \( S \).\(^{12}\) For example, consider the three-staged centipede game as an example of an underlying game of perfect information.

\(^{12}\)Formally, \( S \equiv \max \{K: (a^K_0)_{k=1}^K \in H_0 \} \).
Constructing a Game of Imperfect Information from the Underlying Game of Perfect Information

We now construct an extensive game of imperfect information called $\Gamma$ from the extensive game of perfect information, $\Gamma_0$. The only form of imperfection in information we allow in $\Gamma$ relative to $\Gamma_0$ comes from the feature that we seek to model a scenario where each player $i \in N_0$ has several possible types and each player’s type is his private information. In particular except “Nature’s” move determining the probability distribution on players’ types, all other prior actions will be known at each move. For example, consider the case where $N_0 = \{\text{Ann}, \text{Bob}\}$ are playing $\Gamma_0$ in the underlying game. To define $\Gamma$, in the very first stage of $\Gamma$, we will introduce “Nature’s” move which specifies the probability distribution over the possible combinations of types of the players in $N_0$. Roughly speaking, after Nature moves, Ann and Bob will play $\Gamma_0$ knowing his/her own type, but without knowing his/her opponent’s type. For example, suppose $\Gamma_0$ is tic-tac-toe. Suppose that Nature specifies that “type $t_{\text{Ann}}$ of Ann has a 30 percent chance of occurring and type $t'_{\text{Ann}}$ of Ann has a 70 percent chance of occurring. Independently, type $t_{\text{Bob}}$ of Bob has a 60 percent chance of occurring and type $t'_{\text{Bob}}$ of Bob has a 40 percent chance of occurring.”

Then, in $\Gamma$, after this Nature’s move, Ann will play tic-tac-toe with Bob knowing her type, $t_{\text{Ann}}$ or $t'_{\text{Ann}}$, whichever it may be, but without knowing if Bob’s type is $t_{\text{Bob}}$ or $t'_{\text{Bob}}$. We now proceed to formally defining the construction of $\Gamma$ from $\Gamma_0$. The extensive game of imperfect information $\Gamma$, constructed from $\Gamma_0$, has the following components.

- A set of player-types termed $N$. To construct $N$ from $N_0$, each $i \in N_0$ generates a corresponding set of $i$-types in $\Gamma$ called $T_i$. Let $s_i$ denote the first stage of $\Gamma_0$ at which $i$ moves, then the set of possible types of player $i$ in $\Gamma$ is given by $T_i \equiv \{0_i, 1_i, \ldots, (S - s_i)_i\}$. Further, a particular combination of player-types of $\Gamma$ is $t \in T = \times_{i \in N_0} T_i$. Then $N = \bigcup_{i \in N_0} T_i$. A player-type, for example $1_i$, should be interpreted as follows. $1_i$ is type 1 of player $i \in N_0$. $1_i$ is that type of player $i$ who has foresight level of 1, i.e., given his move in $\Gamma$, $1_i$ can observe one more stage after his move. We model limited foresight more comprehensively in the next subsection.

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Formally, $s_i \equiv \min\{K: (a^0_k)_{k=1}^{K-1} \in H_0 \text{ and } P_0((a^0_k)_{k=1}^{K-1}) \in i\}$
• A set of sequences $H$, and a set of terminal sequences $Z$. The sets $H$ and $Z$ are generated from $H_0$ using a set valued mapping $\text{Seq}: H_0 \implies H$. Each sequence $(a^k_0)_{k=1,\ldots,K} \in H_0$ corresponds to a set of sequences in $H$, and $H = \bigcup_{h_0 \in H_0} \text{Seq}(h_0)$. The elements of $\text{Seq}((a^k_0)_{k=1,\ldots,K})$ comprise of all possible combinations of types of each of the players of $\Gamma_0$ chosen by Nature at the $\emptyset$ history before $(a^k_0)_{k=1,\ldots,K}$. That is, $\text{Seq}((a^k_0)_{k=1,\ldots,K}) = \{(t, (a^k_0)_{k=1,\ldots,K}) : t \in T\}$.\(^{14}\)

• A player function $P$, which maps each element of $H \setminus Z$ to an element in $N$. The function $P$ has the following properties.


  – If player $i$ moved after $(a^k_0)_{k=1,\ldots,K} \in H_0$ then some $t_i \in T_i$ moves after $((t_i, t_{-i}), (a^k_0)_{k=1,\ldots,K}) \in H$. That is, consider an arbitrary $((t_i, t_{-i}), (a^k_0)_{k=1,\ldots,K}) \in H$. If $P_0((a^k_0)_{k=1,\ldots,K}) = i$, then $P(((t_i, t_{-i}), (a^k_0)_{k=1,\ldots,K})) = t_i$.

• Nature’s move which specifies a probability distribution on $T$. This distribution, denoted by $\rho$, is assumed to be common knowledge.\(^{15}\) $\rho(t) \in [0, 1]$ for all $t \in T$, and $\sum_{t \in T} \rho(t) = 1$.

• A set of possible actions in the game, $A$, and an action correspondence $A(\cdot)$.

  – $A(\cdot)$ maps $h \in H$ to a set $A(h) \equiv \{a : (h, a) \in H\}$.

  – The set of possible actions, or action set, after a sequence $h \in H$ is the same as the action set after the corresponding $h_0 \in H_0$ that generated $h$. That is, consider an arbitrary $h \in H$ such that $h \neq \emptyset$. Let $h = (t, (a^k_0)_{k=1,\ldots,K})$. Then $A((t, (a^k_0)_{k=1,\ldots,K})) = A_0((a^k_0)_{k=1,\ldots,K})$.

• For each player type $t_i$, a partition $\mathcal{I}(t_i)$ of $\{h \in H : P(h) = t_i\}$. $I(t_i) \in \mathcal{I}(t_i)$ is an information set of $t_i$. These information sets obey the usual restriction that the actions available from and the player moving at all histories of an information set must be the

\(^{14}\) We say that the Nature’s $\emptyset$ history action is taken at the $0^{th}$ stage.

\(^{15}\) The common knowledge assumption helps simplify a lot of the following analysis. We discuss the effect of weakening of this assumption later.
The construction of $\Gamma$ from $\Gamma_0$ gives us more structure. Consider an arbitrary history $h_0$ of $\Gamma_0$. Suppose $i$ is the player moving after $h_0$, that is, $P_0(h_0) = i$. Then $h_0$ will map to $\text{Seq}(h_0) = \{(t, h_0) : t \in T\}$ in $\Gamma$. The set $\{(t, h_0) : t \in T\}$ will be subdivided into $|T_i|$ information sets in $\Gamma$, one information set for each $t_i \in T_i$. Further, for each $t_i$, his information set $I(t_i)$ such that $\text{Seq}^{-1}(I(t_i)) = h_0$ is given by $\{((t_i, t_{-i}), h_0) : t_{-i} \in T_{-i}\}$. That is, at each such information set of $\Gamma$, the player-type moving there, $t_i$, is aware about all prior actions, but he is uncertain about which combination of opponents’ types, $t_{-i}$, he is playing against. As $\bigcup_{h_0 \in H_0} \text{Seq}(h_0) = H$, all the information sets of $\Gamma$ obey this structure.

- For each player-type $t_i \in N$, a Bernouli utility function $u_{t_i}$ which maps terminal histories $Z$ to real numbers. Additionally, for each $z \in Z$, the utility derived by an arbitrary player-type $t_i$ at $z$, denoted as $u_{t_i}(z)$, is equal to the utility derived by $i$ at the corresponding $z_0 \in Z_0$. That is, $u_{t_i}(z) = u_i(\text{Seq}^{-1}(z))$, $\forall t_i \in T_i$, $\forall i \in N_0$, and for all $z \in Z$.

Thus, $\Gamma = \{N, H, \{I(t_i)\}_{t_i \in N}, P, A, \{u_{t_i}\}_{t_i \in N}\}$, corresponding to $\Gamma_0$, is defined by its construction using the $\text{Seq}(\cdot)$ correspondence. As an example, consider the conversion of the Centipede game in Figure 1 to an Interaction game depicted in Figure 2.

**What Limited Foresight Types Observe: Curtailed Games**

The Limited Foresight Equilibrium (LFE) provides an outcome prediction for the Interaction game $\Gamma$ by specifying an “equilibrium” strategy profile and the associated belief profile for it. We put “equilibrium” in quotes because LFE cannot be solved using the Interaction game. We can’t use a solution concept directly on the Interaction game because limited foresight player-types cannot observe the Interaction game at all of their information sets, hence cannot be optimizing based on it. Limited-foresight player-types, at each move, optimize based on a curtailed version of the Interaction Game that they observe from that move given their limited foresight level. That is, player-types use their move specific curtailed version of the Interaction game to optimize. These curtailed versions of the Interaction

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16Formally, $P(h) = t_i \forall h \in I(i_i)$, and $A(h) = A(h') \forall h, h' \in I(t_i)$. 

13
Figure 2. The Centipede Game Converted to an Interaction Game

Notes. The figure shows the conversion of the \( \Gamma_0 \) (the Centipede game depicted in Figure 1) into an Interaction game, \( \Gamma \), depicted here. As there are 3 stages in the Centipede game in Figure 1, so there are 3 types of player 1 possible, \( \{0_1, 1_1, 2_1\} \), and two types of player 2 possible, \( \{0_2, 1_2\} \). For each combination of player 1’s type and player 2’s type (who could be playing \( \Gamma_0 \) with each other) we redraw \( \Gamma_0 \) to generate \( \Gamma \). We construct the information sets so that each player 1 type, at each of his moves, observes the sequence of prior actions played, but he doesn’t know which player 2 type (\( 0_2 \) or \( 1_2 \)) he is playing against. Similarly, each player 2 type, at each of his moves, observes the sequence of prior actions played, but he doesn’t know which player 1 type (\( 0_1, 1_1 \) or \( 2_1 \)) he is playing against.

game are said to be the Curtailed games generated from an Interaction game.

As the name suggests, a Curtailed game is defined by curtailing the Interaction game at a particular stage. Consider an Interaction Game with \( S \)-stages, \( \Gamma \). An \( n \) staged Curtailed Game constructed from \( \Gamma \) will be labeled as \( CG(n) \). Let

\[
CG(n) = \{N, H^n, \{T^n(t_i)\}_{t_i \in N}, P^n, A^n, \{u^n_{t_i}\}_{t_i \in N}\}.
\]

The components of \( CG(n) \) are defined as follows.

- \( CG(n) \) is an exact replica of \( \Gamma \) until (and including) stage \( (n - 1) \). The player set \( N \) of \( CG(n) \) is the same as the player set \( N \) of \( \Gamma \). \( H^n \), the set of histories of \( \Gamma^n \), is defined as \( (t, (a_{0i}^k)_{k=1,...,K}) \in H \) such that \( K \leq n \). Further, \( H^n \) is partitioned
The set of terminal histories of $CG(n)$, denoted as $Z^n$, contains two kinds of terminal histories. First, the terminal histories of $\Gamma$ which end at or before an $n^{th}$ stage action; formally, let $Z^n(1) = \{(t, (a^{K}_{0})_{k=1,\ldots,K}) \in Z \mid K \leq n\}$. Second, those sequences/histories $(t, (a^{K}_{0})_{k=1,\ldots,K}) \in H$ with $K > n$, which are curtailed at $(t, (a^{K}_{0})_{k=1,\ldots,n})$ and are converted to terminal histories, $Z^n(2)$, in $CG(n)$. Formally, $Z^n(2) = \{(t, (a^{K}_{0})_{k=1,\ldots,n}) \in H \setminus Z\}$. The set of terminal histories of $CG(n)$ is denoted as $Z^n = Z^n(1) \cup Z^n(2)$.

- For those terminal histories of $CG(n)$ which are also the terminal histories of $\Gamma$, the payoffs of each player-type remain the same. That is, for all $z^n \in Z \cap Z^n$, $u^n_{i}(z^n) = u_{i}(z^n)$. The “controversial” choice that must be made in curtailling the Interaction Game is that “what is the payoff profile associated with a terminal history of $CG(n)$ which is not a terminal history of $\Gamma$?” Any payoff numbers placed at such synthetic terminal histories, $Z^n(2)$, of $CG(n)$, will have to follow some ad-hoc rule.\textsuperscript{18} We use the $[(\min + \max) ÷ 2]$ rule of Mantovani (2014).\textsuperscript{19} The $[(\min + \max) ÷ 2]$ rule implies that each player-type’s payoff after $h$ in $CG(n)$ is the average of the minimum and the maximum that that player-type could achieve in $\Gamma$ following all possible terminal action sequences after $h$. That is, for each $h \in Z^n(2)$, let $Z(h)$ be the set of terminal histories of $\Gamma$ where the actions in the first $n$ stages are played as specified in $h$. Formally, let $Z(h) = \{z \in Z : z = (h, (a^{K}_{0})_{k=n+1,\ldots,K})\}$. Then, for

\textsuperscript{17}The common knowledge assumption helps in reducing the number of possible different ways the various limited foresight players can observe a curtailed version of the Interaction Game. If every player-type had a different subjective belief over opponents’ types, $\lambda_{n-1}$, then we would have to construct a move specific Curtailed Game for each individual player-type. The common knowledge prior distribution lets us consider only $S$ possible curtailed versions of $\Gamma$ for the purpose of solving for the strategies and beliefs of all the player-types. This will become clearer when we define LFE.

\textsuperscript{18}Shaowei Ke (2017) has a working paper that justifies his rule using axiomatic foundations. To be clear, our contribution is to model limited foresight with uncertainty and updating about the opponent’s foresight within a play of the game.

\textsuperscript{19}In an older version of this chapter we used a “mean of stage-wise means” rule explained there, which didn’t change any of the results that follow. The interested reader can access the paper by an emailed request at rampal.5@osu.edu.
Notes. The figure shows the conversion of $\Gamma$, the Interaction game depicted in Figure 2, into its shortest curtailed game, $CG(1)$, depicted here. $CG(1)$ is identical to $\Gamma$ in all respects except that $CG(1)$ ends after the first stage action. If any type of player 1 chooses $T1$ (take in stage 1) in the first stage then the associated $CG(1)$ payoff profile is $(4, 1)$, as in the Interaction game. However, if some player 1 type chooses $P1$ (pass in stage 1), then in the construction of $CG(1)$, we must curtail the Interaction game after $P1$ and use the $\min + \max \over 2$ rule for payoffs. For example, after playing $P1$, the maximum a player 1 type can get in $\Gamma$ is 16 and the minimum he can get is 2, thus, his payoff from choosing $P1$ in $CG(1)$ is $\min + \max \over 2 = 9$. We mark $0_1$’s first stage information set as $D1$. This is because $CG(1)$ is exactly what $0_1$ observes at his first stage information set. Thus, $CG(1)$ is decisive for $0_1$ at stage 1. This will be made precise when we define LFE.

As an example of a curtailed game, consider the one-staged curtailed game, $CG(1)$ depicted in Figure 3, constructed from the Interaction game in Figure 2.

**Limited Foresight Equilibrium**

We now proceed to defining the LFE. First we need to define total foresight. Let a limited foresight type’s total foresight be the sum of (i) the stage number that the limited foresight type is moving at, and (ii) the level of foresight of that limited foresight type. Suppose we are trying to solve for the LFE action of the limited foresight player-type $t_i$ moving at some information set $I(t_i)$. The definition of LFE boils down to three rules of thumb: (a) $t_i$ knows the LFE actions of all the player types with lesser total foresight than him; (b) $t_i$ assumes that equal or higher total foresight types, including $t_i$ himself, together choose a
strategy profile for the curtailed game that he observes at $I(t_i)$. This strategy profile must
be sequentially rational for each player type in this curtailed game given (a), the rest of
the strategy profile, and the beliefs of these player types. (c) $t_i$’s beliefs, and the beliefs of
all other player-types in this curtailed game are calculated using the Bayes’ rule, given the
strategy profile in (b) and (a). Formally, we solve for $t_i$’s LFE action and belief at $I(t_i)$
be solving for the Sequential Equilibrium of the curtailed game he observes at $I(t_i)$, after
taking the LFE actions in (a) given as Nature’s moves.

We know that $t_i$ at $I(t_i)$ observed a particular curtailed version of the Interaction game.
To save us effort, we look for all other player-types, say $t_j$, $t_k$ and their information sets
$I(t_j)$ and $I(t_k)$, such that $t_j$, $t_k$ also observed exactly the same curtailed game as $t_i$ at $I(t_i)$.
We note the actions of $t_i$, $t_j$, and $t_k$ at $I(t_i)$, $I(t_j)$, and $I(t_k)$ solved for in (b) and their
beliefs solved for in (c) as the LFE actions and beliefs at these information sets. Proceeding
from the shortest curtailed game to the Interaction game as above gives us the LFE for an
Interaction game. Note that in solving for the Sequential Equilibrium of a curtailed game,
we are also solving the LFE actions and beliefs of player-types other than $t_i$, $t_j$, and $t_k$ at
$I(t_i)$, $I(t_j)$, and $I(t_k)$. However, we do not count them as LFE actions or beliefs. They are
simply needed to calculate the LFE actions and beliefs of $t_i$, $t_j$, and $t_k$ at $I(t_i)$, $I(t_j)$, and
$I(t_k)$.

**Defining the Limited Foresight Equilibrium**

To define the LFE, we will first need two definitions: *total foresight* and *decisive informa-
tion sets*. To define total foresight, let the foresight level of player-type $t_i$ be denoted as $t_i$
itself. For example, the player-type $3_i$ has a foresight level of 3. We denote the foresight
level of $3_i$ as $3_i$, it is understood that the foresight level is actually 3$^{20}$.

*Definition 1 (Total Foresight):* Consider a sequence $h = (t_i, (a_k^h)_{k=1,...,s-1}) \in H$, and
the player-type $P(h)$ moving at the $s^{th}$ stage of $\Gamma$. Let $P(h) = t_i$. The *total foresight* of
player-type $t_i$ at stage $s$ is $(t_i + s)$.

*Definition 2 (Decisive Information Sets):* Let $\Gamma$ be an $S$-staged Interaction Game. An
$n$-staged Curtailed Game, $CG(n)$, where $n < S$ is said to be *decisive* for the information

\[20\] So $3_i + 4 = 7$. Believe us, this abuse of notation helps simplify the notation.
sets \( D^n \) of \( CG(n) \), iff for all \( I(t_i) \in D^n \), if \( I(t_i) \) occurs at stage \( s \) of \( CG(n) \), then we must have that the total foresight of \( t_i \) at stage \( s \) is equal to \( n \), that is, \( (t_i + s = n) \) should hold true. \( CG(S) = \Gamma \) is decisive for \( D^S = \{I(t_i)\}_{t_i \in N} - \bigcup_{n=1}^{S-1} D^n \). We also say that the information sets in \( D^n \) are decisive for \( CG(n) \).

Consider an \( S \)-staged Interaction Game \( \Gamma \). \( \Gamma \) generates \( S \) distinct Curtailed Games given the assumption that Nature’s distribution over \( T \) is common knowledge in each Curtailed Game. Let \( M \) be the number of player-types in \( \Gamma \). Denote a strategy profile of \( \Gamma \) as \( \pi \). Where \( \pi = ((\pi_{t_i})_{t_i \in N}) \). Denote a belief system of \( \Gamma \) as \( \mu \). Where \( \mu = ((\mu_{t_i})_{t_i \in N}) \).

For each player-type \( t_i \), \( (\pi_{t_i}, \mu_{t_i}) \) specifies the action choice and belief of \( t_i \) at all the information sets of \( \Gamma \) where \( t_i \) moves. Formally, consider an arbitrary information set \( I(t_i) \) of \( t_i \). Let \( I(t_i) \) be generated by \( h_0 \in H_0 \). That is, \( Seq^{-1}(I(t_i)) = h_0 \). Then \( \pi_{t_i} \colon I(t_i) \mapsto \Delta(A(I(t_i))), \) and \( \mu_{t_i} \colon I(t_i) \mapsto \Delta\{(t_i, t_{-i}), h_0) : t_{-i} \in T_{-i}\} \).

Consider an \( n \in \{1, \ldots, S\} \). Let \( (\sigma^n, b^n) \) denote an assessment for \( CG(n) \). That is, \( \sigma^n = ((\sigma^n_{t_i})_{t_i \in N}) \) and \( b^n = ((b^n_{t_i})_{t_i \in N}) \) denote a strategy and belief profile for \( CG(n) \) respectively. For each player-type \( t_i \), \( (\sigma^n_{t_i}, b^n_{t_i}) \) specifies the action choice and belief of player-type \( t_i \) at all the information sets of \( CG(n) \) where \( t_i \) moves. Let the set of sequential equilibria of any game \( G \) be denoted as \( \Psi(G) \). Let \( D^n \) denote the decisive information sets of \( CG(n) \). The Limited Foresight Equilibrium of \( \Gamma \) will be an assessment \( (\pi, \mu) \) for \( \Gamma \) that we will construct below. We need one more definition before defining an LFE.

**Definition 3 (Modified Curtailed Games):** Consider a curtailed game \( CG(n) \) for some \( n \in \{2, \ldots, S\} \). Suppose \( \pi(.) \) provides the LFE strategy profile for all the decisive information sets of \( CG(1) \) through \( CG(n - 1) \), \( \bigcup_{k=1}^{n-1} D^k \). Then \( MCG(n) \) is defined by its construction from \( CG(n) \), given \( \pi \), by making two modifications. First, modify the player function of \( CG(n) \), \( P^n \), to \( mP^n \) so that in \( MCG(n) \), for all the decisive information sets of \( CG(1) \) through \( CG(n - 1) \), the player-type moving there is replaced by Nature. That is, in \( MCG(n) \), \( mP^n(I) = Nature \) for all the information sets \( I \in \bigcup_{k=1}^{n-1} D^k \). For all \( I \notin \bigcup_{k=1}^{n-1} D^k \), \( mP^n(I) = P^n(I) \). Second, we specify how Nature moves at these information sets using \( \rho^n \), which is an augmented version of the Nature’s move in\( CG(n) \), given by \( \rho \). In particular, in \( MCG(n) \), the initial prior distribution is the same as \( CG(n) \) and the Interaction Game \( \Gamma \), that is, \( \rho^n(\emptyset) = \rho \). Further, for all \( I \in \bigcup_{k=1}^{n-1} D^k \), \( \rho^n(I) = \pi(I) \), that
is, for all the decisive information sets of $CG(1)$ through $CG(n - 1)$, Nature moves exactly as specified by $\pi$.

Given the notation and definitions above, we have the following definition of LFE:

**Definition 4:** $(\pi, \mu)$ is a **Limited Foresight Equilibrium** of an $S$-staged Interaction Game $\Gamma$ if it is constructed in the following $S$ steps:

**Step 1:** Select a Sequential Equilibrium assessment $(\sigma^1, b^1) \in \Psi(CG(1))$. Set $(\pi(I), \mu(I)) = (\sigma^1(I), b^1(I)) \forall I \in D^1$.

**Step 2:** Convert $CG(2)$ to $MCG(2)$ using $\pi(D^1)$ obtained from Step 1. Select an assessment $(\sigma^2, b^2) \in \Psi(MCG(2))$. Set $(\pi(I), \mu(I)) = (\sigma^2(I), b^2(I)) \forall I \in D^2$.

**Step n:** Convert $CG(n)$ to $MCG(n)$ using $\pi(\bigcup_{k=1}^{n-1} D_k)$ obtained from Step 1 through Step $(n-1)$. Select an assessment $(\sigma^n, b^n) \in \Psi(MCG(n))$. Set $(\pi(I), \mu(I)) = (\sigma^n(I), b^n(I)) \forall I \in D^n$. Repeat Step n until $n = S$.\(^{21}\)

At step $n \in \{1, .., S\}$, given $MCG(n)$, the SE $(\sigma^n, b^n)$, is said to be the belief about the assessment (strategy and belief profile) in $MCG(n)$, as calculated by the player-types when moving at their respective information sets in $D^n$. Figure 4 below shows the construction of $MCG(2)$, the notes below the figure specify how to solve for the LFE for our Centipede game example.

**Limited Foresight Equilibrium Properties**

**Remark 1:** The interaction game nests the underlying finite sequential move game of perfect information. In particular, suppose an Interaction Game $\Gamma$ has the following common knowledge prior distribution chosen by Nature: the combination of player-types such that none of the player-types has any limitation to their foresight has a probability of 1, i.e.

\(^{21}\)Note 1: In the construction of an LFE in definition 2, we are assuming that a player type observing a longer $CG$ correctly anticipates which one of the many possible SE was selected at each of the shorter $CGs$. For example, for constructing $MCG(2)$, we need a selection from the Sequential Equilibria of $CG(1)$. We are assuming that $0_2$ at stage 2 and $1_1$ at stage 1 correctly guess which one of the many possible optimal choices is chosen by $0_1$ at stage 1. This assumption is significant in general, but it has no bearing on our Centipede game and Sequential Bargaining game results, as all $CGs \neq \Gamma$ have a unique SE there.
Figure 4. Modified Curtailed Game (2)

Notes. Solving for LFE: From CG(1), depicted in Figure 3, we know that in LFE, irrespective of beliefs, 01 chooses P1 at the information set D1 for which CG(1) is decisive. We construct MCG(2) by taking 01's LFE action at D1 as Nature’s move and marking it with the superscript S (denoting “solved”). This converts CG(2) to MCG(2). MCG(2) is identical to Γ in all respects except that (i) MCG(2) curtails Γ after the second stage action and (ii) in MCG(2), we take 01's LFE action at D1 as Nature’s move. MCG(2) is decisive for information sets denoted by D2: 11's information set at stage 1, and 02's information set at stage 2. In any SE of MCG(2), and therefore in LFE, 11 chooses P1 in stage 1, and 02 chooses P2 in stage 2. This is also irrespective of belief because in any SE of MCG(2), all player 2 types choose P2 at stage 2. Note that we mark the SE actions at even the non-decisive information sets by underlining them. So we underline P2 for 12 at stage 2 and T1 for 21 at stage 1. This is because the SE actions at non-decisive information sets are needed to calculate the LFE actions and beliefs at decisive information sets. To complete the LFE strategy profile, we need to solve for the LFE actions at the remaining information sets, D3, using the Interaction game in Figure 2. Now, Nature’s initial distribution is important. Suppose Nature chooses an independent uniform distribution on player 1’s types and player 2’s types. At stage 3, all types of player 1 will choose T3, irrespective of beliefs. Thus, at stage 2, 12 will choose T2 irrespective of beliefs. In any SE of MCG(3), 21's beliefs must be derived from Nature’s prior distribution using Bayes’ rule. Thus, 21's belief that his opponent is 12 or 02 is 0.5 each. The former chooses T2 in stage 2, while the latter chooses P2 in stage 2, thus 21's expected payoff from P1 is \( \frac{16+2}{2} = 9 \). So 21 chooses P1 in stage 1.
$\rho(t_1, ..., t_M) = 1$ if $t_i = (N - s_i)_i$ for all $i = 1, ..., M$. Then $\Gamma$ is equivalent to $\Gamma_0$, the game of perfect information that generated $\Gamma$. Therefore, in that case, the set of LFE of $\Gamma$ is equal to the set of Sequential Equilibria of $\Gamma$ which is identical to the set of SPNE of $\Gamma_0$.

Proposition 1(a) - Existence: for every finite Interaction Game, there exists at least one Limited Foresight Equilibrium.

1(b) - Upper Hemicontinuity: Given the extensive form, $\{N, H, \{I(t_i)\}_{t_i \in N}, P, A\}$, for an Interaction Game, the correspondence from pairs $(\rho, u)$ of initial probability distributions and payoff profiles to the set of Limited Foresight Equilibria for the game so defined is upper hemicontinuous.

The proof of existence and upper hemicontinuity of LFE follows from the existence and upperhemicontinuity of the Sequential Equilibrium (Kreps and Wilson (1982)). The details are given in Appendix A.

Proposition 2 and corollary tells us that calculating the SE of a given $MCG(n)$ can be quite easy. Proposition 2 tells us that in any $MCG(n)$, the beliefs of all types of a particular player at corresponding information sets are identical. Further, as the beliefs of different types of a given player are identical (that is, beliefs over the opponents’ types) at corresponding information sets, in equilibrium, if a player-type’s strategy from a given information set is strictly better than the next-best alternative, then in equilibrium, the strategies of all types of that player at the corresponding information sets are also identical. That is, all types of a particular player can be treated identically in an $MCG(n)$ up to the case of indifference. The case of indifference doesn’t arise in any $MCG$ shorter than the Interaction game in the Bargaining game and the Centipede game applications. My conjecture is that even in the case of indifference identical treatment of different player-types in any

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22For example, Palacios-Huerta and Volij (2009) may have been able to establish this condition in their experiment when expert chess players played other expert chess players in a Centipede game.
$MCG(n)$ is without loss of generality. However, I have not been able to prove this conjecture formally. To the best of my knowledge, there does not exist an application where this identical treatment of the different types of a particular player fails to produce a SE of some $MCG(n)$.

**Proposition 2:** Consider an arbitrary modified curtailed game, $MCG(n)$. Within $MCG(n)$, consider any two information sets of $t_i$ and $t'_i$ such that the sequence of prior actions, excluding Nature’s initial move, is the same. That is, consider $I(t_i)$ and $I(t'_i)$ such that $Seq^{-1}(I(t_i)) = Seq^{-1}(I(t'_i)) = h_0 \in H_0$. Further, let Nature’s initial probability distribution, $\rho$, over player-types be pairwise independent across the types of different players. That is, $\text{Prob}(t_i, t_j | \rho) = \text{Prob}(t_i | \rho).\text{Prob}(t_j | \rho)$ for any $t_i \in T_i$ and $t_j \in T_j$ and for any $i, j \in N_0$. Then for any totally mixed strategy profile of $MCG(n)$, denoted as $\sigma^n(.)$, the beliefs of $t_i$ and $t'_i$ over the histories in $I(t_i)$ and $I(t'_i)$ are identical if these beliefs are calculated using Bayes’ law given $\sigma^n(.)$. That is, if $Seq^{-1}(I(t_i)) = Seq^{-1}(I(t'_i)) = h_0$, then

$$b^n((t_{-i}, h_0) | t_i, \sigma^n) = b^n((t_{-i}, h_0) | t'_i, \sigma^n) \forall t_{-i} \in T_{-i} \tag{2}$$

Therefore, for any $MCG(n)$ and for any of its Sequential Equilibria, the equilibrium beliefs of all types of player $i$ are identical following action sequences that are identical up to Nature’s initial move.

Proof sketch. We provide an intuitive explanation of why proposition 2 is true. Let $Pr$ be short for “probability” for the purpose of this proof. Suppose the precedent of proposition 2 holds. Then

$$b^n((t_{-i}, h_0) | t_i, \sigma^n) = \frac{Pr((t_i, t_{-i}, h_0) | \sigma^n)}{\sum_{t_{-i} \in T_{-i}}[Pr((t_i, t_{-i}, h_0) | \sigma^n)]} \tag{3}$$
We have to show that for all $t_i, t'_i \in T_i$, for any given $t_{-i} \in T_{-i}$,

$$\frac{Pr((t'_i, t_{-i}, h_0) | \sigma^n)}{\sum_{t_{-i} \in T_{-i}}[Pr((t'_i, t_{-i}, h_0) | \sigma^n)]} = \frac{Pr((t_i, t_{-i}, h_0) | \sigma^n)}{\sum_{t_{-i} \in T_{-i}}[Pr((t_i, t_{-i}, h_0) | \sigma^n)]} \quad \text{(4)}$$

When we calculate $Pr((t_i, t_{-i}, h_0) | \sigma^n)$, the manner in which this term is affected by $t_i$ is captured by a multiplicative term that varies based on $t_i$, but given $i$’s type, it doesn’t vary as $t_{-i}$ varies (in the denominator of (4)). This is due to two main features of the structure of $MCG(n)$. First, the independence property of $\rho$ and, second, the construction of the information sets of $MCG(n)$. That is, given $i$’s type, say $t_i$, if he is moving after a sequence of moves given by $h_0$, he cannot distinguish among possible $t_{-i}$ that preceded $h_0$, because for all $t_{-i} \in T_{-i}$, the history given by a different $(t_{-i}, h_0)$ belong in the same information set for $t_i$. Similarly for each $j \neq i$, $j$’s types’ cannot distinguish among $t_{-j}$ for a given sequence of actions. Thus $j$’s types’ strategy cannot specify actions which are conditional on $i$’s type. Thus, the multiplicative term that is specific to $t_i$, but common to any $t_{-i}$, given $t_i$, cancels out from the numerator and denominator, leaving the same term on the RHS and LHS of (4), i.e. a term that is unaffected by $i$’s type. Details of this are given in Appendix A.

The corollary below gives us an important implication of proposition 2. Consider an arbitrary $MCG(n)$ corresponding to an interaction game $\Gamma$ and an underlying game $\Gamma_0$. Consider any SE, $(\sigma^n, b^n)$, of $MCG(n)$. If $\sigma^n$ specifies a strategy for $t_i$ following an action sequence $h_0$ such that it is strictly better than all alternatives. Then $\sigma^n$ must specify the same strategy for all other types of player $i$ at corresponding information sets.

**Corollary:** Suppose the conditions of proposition 2 hold. Consider an arbitrary action sequence $h_0$ of the underlying game. Let $\mathcal{I}^{h_0}(t_i)$ be the collection of information sets of $t_i$ which follow after the action sequence $h_0$. Let $U_t_i(\sigma^n_{t_i}(\mathcal{I}^{h_0}(t_i)) | \sigma^n_{-t_i}, b^n)$ be the expected payoff of $t_i$ from following the strategy $\sigma^n_{t_i}(\mathcal{I}^{h_0}(t_i))$ over the information sets $\mathcal{I}^{h_0}(t_i)$, given the belief profile $b^n$, and the strategies of all other player types given by $\sigma^n_{-t_i}$. If $U_t_i(\sigma^n_{t_i}(\mathcal{I}^{h_0}(t_i)) | \sigma^n_{-t_i}, b^n) > U^*_t_i(s^n_{t_i}(\mathcal{I}^{h_0}(t_i)) | \sigma^n_{-t_i}, b^n)$ for all possible strategies.
Prob \left( \text{from the LFE strategy profile using Bayes' rule wherever possible} \right). That is, 
\[ \forall \pi, \mu \]
the LFE strategy profile \( \pi, \mu \) over \( L^\text{ho} (t_i) \), then in equilibrium, the strategies of all other types of player \( i \), for example \( t_i' \), must be such that \( \sigma^n_{t_i'} (L^\text{ho} (t_i')) = \sigma^n_{t_i} (L^\text{ho} (t_i)) \), where for each \( I(t_i') \in L^\text{ho} (t_i') \), there exists a unique \( I(t_i) \in L^\text{ho} (t_i) \) such that \( \text{Seq}^{-1} (I(t_i)) = \text{Seq}^{-1} (I(t_i')) \). (The proof is given in the Appendix A).

**Definition 5 (lower types and higher types):** consider an arbitrary information stage-
K information set \( I(t_i) \) of \( \Gamma \). Let \( I(t_i) = \{(t, (a^k_0)_{k=1,...,K-1}) : t_{-i} \in T_{-i}\}. Define \( L(I(t_i)) \) as the subset of \( I(t_i) \) such that \( h = (t, (a^k_0)_{k=1,...,K-1}) \in L(I(t_i)) \) if and only if for any subsequence \( \hat{h} = (t, (a^k_0)_{k=1,...,r-1}) \) of \( h \) it is true that if \( P(\hat{h}) = t_j \) then \( (t_j + r) < (t_i + K) \). Note that \( \hat{h} \) is a stage \( r \) history of \( \Gamma \) and \( t_j \) denotes player-type \( t_j \)'s foresight level, so \( (t_j + r) \) is \( t_j \)'s total foresight at \( \hat{h} \), and similarly \( (t_i + K) \) is \( t_i \)'s foresight level at \( h \). For all \( h \in L(I(t_i)) \), \( t_i \) is said to be playing against lower opponent types, that is, at \( h \), \( t_i \) has greater total foresight that all of his opponents did in their respective prior moves in the sequence \( h \). Define \( L^c(I(t_i)) = I(t_i) - L(I(t_i)) \).

Proposition 3 states a consistency condition on the LFE belief \( \mu \). It says that in any LFE \( (\pi, \mu) \), given an information set \( I \), the belief distribution over \( L(I) \), should be derived from the LFE strategy profile \( \pi \) using Bayes' rule wherever possible.

**Proposition 3:** Let \( I \) be a stage-K information set of \( \Gamma \), such that \( P(I) = t_i \). In any LFE \( (\pi, \mu), t_i \)'s belief distribution over the set of nodes \( L(I) \), conditional on \( I \), must be derived from the LFE strategy profile using Bayes’ rule wherever possible. That is, \( \forall h \in L(I) \), if \( \text{Prob}(L(I) \mid \rho^{K+t_i}) = \text{Prob}(L(I) \mid \pi) > 0 \) we must have:

\[ \mu_{t_i} (h \mid L(I)) = \frac{\text{Prob}(h \mid \pi)}{\text{Prob}(L(I) \mid \pi)} = \frac{\text{Prob}(h \mid \rho^{K+t_i})}{\text{Prob}(L(I) \mid \rho^{K+t_i})} \]  \hspace{1cm} (5)
A sketch of the proof is stated here. In constructing any LFE \((\pi, \mu)\), when we get to step \((K + t_i)\) of the construction, we construct \(MCG(K + t_i)\) using steps 1 through \((K + t_i - 1)\). We have already solved for the LFE strategies for all shorter \(MCGs\). We consider the strategies of lower opponent types of \(t_i\) at stage \(K\) as Nature’s moves, which are common knowledge in \(MCG(K + t_i)\). Therefore, conditional on \(t_i\) moving after some sequence such that all prior moves are those of some lower opponent type, that is, conditional on \(t_i\) being at \(L(I)\), the probability belief on each such individual sequence in \(L(I)\) is calculated using Nature’s moves which are common knowledge in \(MCG(K + t_i)\). By definition, these Nature’s moves are given by the LFE strategy profile, \(\pi\). Technical details are stated in Appendix A.

Corollary: Consider two stage-s information sets \(I(t_i)\) and \(I(t'_i)\) of \(\Gamma\) such that they occur after the same history of actions. That is, \(I(t_i) = T_{-i}\) and \(I(t'_i) = T_{-i}\). If \(t_i < t'_i\) then we must have that \(|L(I(t_i))| \leq |L(I(t'_i))|\). Therefore, by proposition 3, the LFE conditional belief distribution of the higher foresight-level type is accurate (satisfies (5)) on a larger subset of \(I(t'_i)\) as compared to the subset of \(I(t_i)\) for which the lower foresight-level type’s conditional belief distribution is accurate (satisfies (5)). (The proof to this corollary is given in Appendix A.)

The corollary to proposition 3 (stated above) approximately captures the findings from Reynolds (1992). Reynolds (1992), while testing recognition of opponent’s expertise among chess players, found that “Higher rated players consistently made lower estimation errors” (of other chess players’ ELO ratings). If one proxies for foresight using experience-level or ELO ratings, then the corollary to proposition 3 approximately captures this. The reasons for only approximate similarity to Reynolds’ (1992) findings are that first, the proxying of foresight using ELO ratings is a leap of faith; second, in an LFE, the total believed probability on “lower ” types, \(\mu_{t_i}(L(I(t_i))) I(t_i)\), need not be derived from the LFE strategy
profile using Bayes’ rule (Proposition 4). However, as proposition 3 says, conditional on $L(I(t_i))$, the distribution of $\mu_{t_i}(L(I(t_i)) | I(t_i))$ among the various sequences of $L(I(t_i))$, i.e. the distribution of $\mu_{t_i}(L(I(t_i)) | I(t_i))$ among lower types, is derived from the LFE strategy profile using Bayes’ rule. It is notable that starting from the same common knowledge belief over opponents’ types, the belief of higher foresight-level types becomes “more accurate” (at least in the sense of proposition 3 and its corollary) after the same sequence of actions.

**Proposition 4:** If the total foresight of $t_i$ at the stage-$s$ information set $I(t_i)$ is less than $S$, i.e. $s + t_i < S$, then his belief distribution conditional on the histories in $I(t_i)$ need not be derived from the LFE strategy profile using Bayes’ rule. Thus, it need not be true that $\mu_{t_i}(h | I(t_i)) = \frac{\text{Prob}(h | \pi)}{\text{Prob}(I(t_i) | \pi)} \forall h \in I(t_i)$ and for all information sets $I(t_i)$ of $\Gamma$.

Proof by counterexample. Consider the example in Figures 1-4 again. Suppose Nature’s distribution on \{0, 1, 2\} is (0.5, 0, 0.5), and independently, Nature’s distribution on \{0, 1\} is (0.1, 0.9). To solve for the LFE, we start with $CG(1)$, depicted in Figure 3, where we see that $0_1$ chooses $P_1$ irrespective of beliefs. Next, taking this as given, we construct $MCG(2)$, depicted in Figure 4, where we see that in any SE of $MCG(2)$, all types of player-1 choose $P_1$ and all types of player-2 choose $P_2$, irrespective of beliefs. Thus, at his information set in the second stage, $0_2$’s LFE belief is (0.5, 0, 0.5) on \{0, 1, 2\}. However, this is not consistent with the LFE strategy profile because in any SE of $MCG(3)$, $2_1$ chooses $T_1$ in the first stage. Therefore, according to the LFE strategy profile, conditional on reaching the second stage, $0_2$ faces $0_1$ with a probability equal to one, unlike $0_2$’s LFE belief of (0.5, 0, 0.5) on \{0, 1, 2\}.

Proposition 4 follows because at the information sets of $MCG(s + t_i)$, the strategy pro-

\[23\] $2_1$ knows that he faces $1_2$ with probability 0.9 who will choose $T_2$ in the second stage. Thus $2_2$’s expected payoff from choosing $P_1$ is $(16 \times 0.1 + 2 \times 0.9)$, while his expected payoff from choosing $T_1$ is 8.
file for $MCG(s + t_i)$, $\sigma^{s+ti}$, may stipulate different optimal actions compared to the LFE strategy profile, $\pi$. Although $\sigma^{s+ti}$ can be different from the LFE profile $\pi$, it provides an optimal strategy with respect to $MCG(s + t_i)$ for each player type in $MCG(s + t_i)$ given the strategy and belief profiles $(\sigma^{(s+n)}, b^{(s+n)})$.

Proposition 5 (below) tells us that if a low foresight type observes a sequence of moves that cannot occur when playing against “lower” opponent types, then he discovers that he is playing against some “higher” type, and must use his total foresight at that move to optimize. At any information set, $I(t_i)$, where a limited foresight player-type $t_i$ moves, there is a certain subset of nodes, $L(I(t_i)) \subset I(t_i)$, which represent the cases where $t_i$ is playing against “lower” opponents’ types who, at all preceding moves leading to that information set, had a strictly lower total foresight than $t_i$ does at $t_i$. Proposition 5 (below) reflects the fact that $t_i$ knows these “lower” opponents’ types’ prior moves coming into these nodes of $L(I(t_i))$. If $t_i$ knows that the moves of these lower opponents’ types’ imply a zero probability of reaching any node in $L(I(t_i))$, and yet finds himself at the information set $I(t_i)$, then he knows that he is at a node of $I(t_i)$ where at least one of the opponent-type was not a “lower” type at some preceding move. In the two player case it means that one knows that one’s opponent had a higher total foresight at some preceding move. Recognition of the higher opponent type implies that the LFE actions of the lower types don’t matter for the calculation of the sequentially rational action at $I(t_i)$; $t_i$ must use his total foresight to optimize.

**Proposition 5**: If in the construction of an LFE, Nature’s moves in $MCG(s + t_i)$, denoted by $\rho^{s+ti}$, imply that the probability of reaching $I(t_i)$, a stage-$s$ information set, via only Nature’s moves is 0, then the LFE belief of $t_i$, conditional on $I(t_i)$, must put probability 1 on those nodes of $I(t_i)$ where at some preceding node, the player type moving there had total foresight at least $(s + t_i)$. That is, for all $t_i \in N$, for all $I(t_i) \in \mathcal{I}(t_i)$:


\[ \text{Prob}(L(I(t_i)) \mid \rho^{s+t_i}) = 0 \implies \mu_t_i([L(I(t_i))]^c \mid I(t_i)) = \frac{\text{Prob}([L(I(t_i))]^c \mid \pi)}{\text{Prob}(I(t_i) \mid \pi)} = 1 \] (6)

Proof: proposition 5 follows from the step-wise definition of LFE. By the LFE definition, all the nodes preceding the nodes in \( L(I(t_i)) \) have Nature as the player moving there and the actions taken by Nature are given by \( \rho^{s+t_i} \). \( \rho^{s+t_i} \) is common knowledge in \( MCG(s + t_i) \) and hence also known to \( t_i \) at \( I(t_i) \). Thus, the probability of reaching \( I(t_i) \) via only Nature’s moves, can be calculated using \( \rho^{s+t_i} \) by \( t_i \) at \( I(t_i) \). Thus,

\[
\mu_t_i(L(I(t_i)) \mid I(t_i)) = \frac{\text{Prob}(L(I(t_i)) \mid \rho^{s+t_i})}{\mu_t_i(I(t_i))}
\]

Therefore, if \( \text{Prob}(I(t_i)) \mid \rho^{s+t_i}) = 0 \), then \( \mu_t_i(L(I(t_i)) \mid I(t_i)) = 0 \), further, as \( \mu_t_i([L(I(t_i))]^c \mid I(t_i)) + \mu_t_i(L(I(t_i)) \mid I(t_i)) = 1 \), we must have that \( \mu_t_i([L(I(t_i))]^c \mid I(t_i)) = 1 \). Q.E.D.

Remark 2: Suppose \( t_i \) moves at two information sets \( I(t_i) \) and \( I'(t_i) \), which occur at stage \( s \) and \( s' \), respectively, of \( \Gamma \). By the construction of LFE, in step \( (s + t_i) \), \( t_i \) at stage \( s \) knows \( \rho^{(s+t_i)} \), and therefore he knows the LFE action choices of player-types at all the decisive information sets of \( MCG(1) \) through \( MCG(s + t_i - 1) \). If \( s' > s \) then \( t_i \) at \( s' \), by step \( (s' + t_i) \) of the construction of LFE, knows \( \rho^{(s'+t_i)} \), and therefore \( t_i \) at \( s' \) knows the LFE action choices of player-types at more information sets \( \bigcup_{n=1}^{(s'+t_i-1)} D^n \) of \( \Gamma \bigcup_{n=1}^{(s'+t_i-1)} D^n \supset \bigcup_{n=1}^{(s+t_i-1)} D^n \).

Remark 2 approximately mirrors another finding from Reynolds (1992), and the finding
from Rampal (2017). Rampal (2017) found that the more moves of the opponent observed by an expert “race game” player, the better his guess about the opponent’s experience level. In the same token, Reynolds (1992) found that the estimation error decreased as a function of number of moves revealed. Remark 2 suggests that in an LFE this can happen, as at a higher stage number, the same player type has a higher total foresight and hence observes a longer Curtailed Game. Thus, LFE actions are given as Nature’s move for a larger subset of the set of information sets of the Interaction Game.

Applications

A key aim for developing the LFE apparatus is to obtain general applicability in solving various existing puzzles observed in the experimental data collected on perfect information games. In this section we apply the LFE model to the Centipede game introduced by Rosenthal (1981) and the Sequential Bargaining game analyzed by Rubinstein (1982) and Ståhl (1973).

Sequential Bargaining

The Sequential Bargaining game (Rubinstein (1982) and Ståhl (1973)) has been studied extensively in the literature (c.f. Binmore et al (1985), Neelin et al (1988), Guth and Tietz (1987,1990), Ochs and Roth (1989), Johnson et al (2002), and Binmore et al (2002)). The game consists of two players bargaining over a pie of size $X$ over multiple periods. In each period one player makes a proposal on how to split the pie, and the other player accepts or rejects this proposal. If a proposal is accepted then the game ends and that proposal is implemented. If a proposal is rejected then the game proceeds to the next period where the player who rejected the last proposal now makes an offer but from a smaller pie as the pie gets multiplied by a “common” discount factor, $\delta \in [0, 1]$. In the finite period case, if no proposal is accepted, then after a rejection in the last period, both players get 0 payoff. The SPNE prediction is that in a $K$ period bargaining game, when $K$ is odd, the first proposal which offers the first mover/proposer $X \left[ (1 - \delta) \frac{(1-\delta^{K-1})}{1-\delta^2} + \delta^{K-1} \right]$ will be accepted.

Four stylized data trends, which are incongruent to the SPNE outcomes, have emerged
Figure 5. Sequential Bargaining Game and Associated Curtailed Payoffs Without Uncertainty

**Notes:** The figure shows curtailed payoff profiles being calculated using the (min+max)/2 method. The curtailed payoffs are depicted in blue above the game. The pies are 1000, 600 and 360 in period 1, 2 and 3 respectively. $x_i$ is player 1’s offer (to himself) in period i. $y_2$ is player 2’s offer (to player 1) in period 2. R implies “reject” and A implies “accept.”

in the experimental study of the Sequential Bargaining game. First, a tendency for first offers proposing equal split (Guth and Tietz (1987); Ochs and Roth (1989)) or offering the second round pie (Neelin et al (1988)) to the second mover. Second, offers made in the first period are often rejected (Ochs and Roth (1989)). Third, and perhaps the most surprising finding is that the first period offers are very often succeeded by disadvantageous counteroffers (Ochs and Roth (1989) found that 81 percent of counteroffers were disadvantageous). Fourth, subgame consistency is violated in that observed outcomes of a subgame tested as a separate game are different from the outcomes of this subgame when it is the strict subgame of a game (Binmore et al (2002)). In this subsection, we show that we can rationalize all these stylized data facts simultaneously by just utilizing the general model of limited foresight and uncertainty about the opponent’s foresight that we developed earlier. In particular, our rationalization does not use altruistic preferences, or preferences specific to the Bargaining game.

We consider the three period bargaining game with $\delta = 0.6$ as $\Gamma_0$. These specifications are used by Ochs and Roth (1989) in one of their treatments. Neelin et al (1988) and Johnson et al (2002) use $\delta = 0.5$, which doesn’t change the features of the LFE outcome we discuss below. We make the initial size of pie 1000 for simplicity. We convert $\Gamma_0$ into the Interaction Game, $\Gamma$, and present the features of its LFE. The Figure 5 below depicts the Curtailed payoffs associated with $\Gamma_0$, without showing the informational uncertainty.
As the three period bargaining game given in the Figure 5 above has 6 stages, we have six player-1 types \((0_1, 1_1, 2_1, 3_1, 4_1, \text{ and } 5_1)\) and five player-2 types \((0_2, 1_2, 2_2, 3_2, 4_2)\). We assume independent uniform distributions on both players' types. The LFE strategies for this uniform case are calculated in Appendix A and detailed in Table 15 there.

The following outcomes observed by the studies on the Bargaining game (mentioned in brackets) are observed in the LFE that we detail in Appendix A.

1. First round offer rejection (c.f. Ochs and Roth (1989)): \(0_1\) overestimates his bargaining position and thus demands the whole first period pie. This demand is rejected by all player-2 types. The offer of \(1_1\) and \(2_1\) is rejected by \(2_2\) and \(3_2\). This occurs because \(2_2\) and \(3_2\) fail to take into account that player-1 has absolute bargaining power in the last period and that the pie will shrink in the next period when they have to make a counterproposal.

2. First offers with near equal split or an offer equal to the second round pie (c.f. Neelin et al (1988); Guth and Tietz (1987); Ochs and Roth (1989)): \(3_1, 4_1, \text{ and } 5_1\) propose \((580, 420)\) in the first period. \(3_1, \text{ and } 4_1\) choose this proposal because they cannot foresee that they will have the bargaining advantage in the last (third) period, \(5_1\) chooses this proposal because he gets immediate acceptance with this generous offer. If \(5_1\) were to make a higher offer, his offer would be rejected by the limited foresight types of player-2 who fail to foresee player-1’s absolute bargaining power in the last period.

3. Disadvantageous counter proposals (c.f. Ochs and Roth (1989)): \(1_1\) and \(2_1\) make a proposal of \((700, 300)\) in the first period. However, \(2_2\) and \(3_2\) reject anything that gives them less than 420 because, due to their limited foresight, they think that all player-1 types will accept a proposal of \((180, 420)\) in the second period. However, \(3_2\) becomes rational in period 2 and observes the absolute bargaining power of his opponent in the last period. Thus, it is sequentially rational for \(3_2\) to make a disadvantageous counterproposal of \((360, 240)\). A theoretical prediction of the LFE model is that this feature should disappear if we change the extensive form and make player-2 think about the acceptance/rejection decision simultaneously with the coun-

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terproposal decision. Thus, one should take great care in matching the specification of moves in the game to the foresight of the players.

4. Subgame consistency violation (c.f. Binmore et al (2002)): Consider the 2-period Bargaining game with the starting pie of 600 being tested separately and its data being compared to the data generated from the last two periods of a 3-period Bargaining game. Binmore et al (2002) find that the results of the former do not match the data generated from the latter. According to LFE, this is to be expected if these seemingly perfect information games are in fact Interaction games. This because in the 3-period game, the outcome of the last two periods depends on what happened in the first period. For example if the first proposal was \((1000, 0)\), then player-1’s type is \(0_1\), and if the first proposal was \((700, 300)\) then player-1’s type can be \(1_1\) or \(2_1\) with equal probability. These different player-1 types have different optimal choices in the third period. Further player-2’s types \(3_2\) and \(4_2\) update about their opponent’s type based on the first period proposal and adjust their optimal actions in the second period. However, if two players are beginning a two-period Bargaining game, then their optimal choices only depend on their prior belief about the opponent’s foresight, which may well be different from their updated belief after observing the opponent’s choice in the first period of a 3-period Bargaining game.

Thus the LFE concept provides us several channels to explain several qualitative features of the data on Sequential Bargaining experiments. Fitting experimental data using this model is left as future work.

The Centipede Game

The Centipede game describes a situation in which two players alternately decide whether to *take* or *pass* an increasing pile of money. Consider an \(S\)-staged Centipede game. First, player 1 decides whether to *take* or *pass* a pile of money; if the player moving at stage \(i\) decides to *take* at stage \(i\) then he gets \(a_i\), the larger share of the existing pile of money, \(a_i + b_i\). If that player passes, the pile of money grows and \(a_i + b_i < a_{i+1} + b_{i+1}\). If a player *passes*, but his opponent *takes* in the next stage, he gets a payoff \(b_{i+1} < a_i\). However
if his opponent passes too, then the pile grows again and the player has a chance to take again and achieve a higher payoff $a_{i+2} > a_i$, $b_{S+1}$ denotes the payoff of the player moving at stage $S$, if he chooses pass at stage $S$. His opponent gets $a_{S+1} > b_{S+1}$. The unique SPNE prediction is that the first player should take in the very first stage, regardless of the number of stages that the pile can be passed and grown. The logic is that in the last stage, as $a_S > b_{S+1}$, the player moving there should take; but given this, one should take in the second-last stage, and this optimality of taking given one’s opponent is going to take in the next stage continues inexorably backwards, and leads to the SPNE prediction: take in the first stage. This is highly unintuitive and various experiments, eg. McKelvey and Palfrey (1992, 1998) reject the SPNE prediction.

Consider an $S$-staged Centipede game as $\Gamma_0$. We restrict our analysis to the Centipede games with the following payoff structure.

Definition 3: An $S$-staged Centipede game is said to have the payoff structure $P$ if for all $i \in \{1, \ldots, S+1\}$: (i) $b_i < b_{i+1} < a_i < b_{i+3} < a_{i+2}$ (ii) $a_i < \frac{b_{i+1} + a_{i+2}}{2}$ (iii) $\frac{a_i - b_{i+1}}{a_{i+2} - b_{i+1}} = \eta_i < \frac{1}{3}$.

Consider the six staged Centipede game used by McKelvey and Palfrey (1992) in Figure 6. This also has the payoff structure $P$ with $\eta_i = \frac{1}{7}$ for all $i$. If a term, for example $b_{i+3}$, does not exist then any condition on that term is satisfied vacuously. $b_{i+1} < a_i < a_{i+2}$ follows from $\Gamma_0$ being a Centipede game. Condition (iii) of definition 3 will be used in proving proposition 7 below. An $S$-staged perfect information Centipede game $\Gamma_0$ generates an Interaction Game $\Gamma$ with the player set $N = \{0_1, 1_1, \ldots, (S-1)_1, 0_2, 1_2, \ldots, (S-2)_2\}$. The following proposition says that given a certain form of initial probability distribution
on foresight types, even with arbitrary positive total probability on limited foresight types, all LFE outcomes entail *pass* being played with strictly positive probability by all foresight types until the end stages of a Centipede game. This result reflects the fact that rational the rational type player pretends to be low foresight types and attain a higher payoff by *passing* because his opponent cannot tell if he is rational or a limited foresight type, and thus the rational opponent *passes* with positive probability too.

**Proposition 7:** Consider an S-staged Centipede game $\Gamma_0$ with payoff structure $P$. $\Gamma_0$ generates a S-staged Interaction Game $\Gamma$. Let $\rho$, the probability distribution on $N$ be such that $\text{Prob}(j_1) = \text{Prob}(k_2) = q \in [0, 1], \forall j = 0, 1, ..., S - 2$ and $\forall k = 0, 1, 2, ..., S - 3,$ and $\text{Prob}((S - 1)_1) = 1 - (S - 1)q$, and $\text{Prob}((S - 2)_2) = 1 - (S - 2)q$. Further suppose the distribution on 1's types is independent of the distribution on 2's types. For all $q > 0$ such that $\sum_{j=0}^{S-1} \text{Prob}(j_1) = \sum_{k=0}^{S-2} \text{Prob}(k_2) = 1$, in any LFE of $\Gamma$, all types of both players “pass” with strictly positive probability from stage 1 through stage (S-3).

Proof sketch: first we show that any limited foresight type, at any stage at which his total foresight is strictly less than $S$, plays *pass* with probability 1. The proof proceeds to show that this fact implies that if the rational player-types all *stop* at a particular stage, say $s$, between 1 and $(S - 3)$, then in the next stage, the rational player-types face limited-foresight opponent types who played *pass* at stage $s$, and out of which only one will turn rational in stage $(s + 2)$. Thus, if the rational player-types all *stop* at stage $s$, then in stage $(s + 1)$, the rational player-types know that their opponent will pass with a high probability in stage $(s + 2)$, which implies that all player-types *pass* in stage $(s + 1)$, but that means that stopping at stage $s$ is not sequentially rational for the rational player types at stage $s$, and therefore not an LFE. The technical details are given in Appendix A.

This analysis is almost parallel to the McKelvey and Palfrey (1992) model without the errors in actions, heterogeneous beliefs and learning components. Both these analyses are in the same vein as the reputation literature of Kreps, Wilson, Milgrom and Roberts (1982).
particular, if we have a $\rho$ such that $\text{Prob}(0_1) = \text{Prob}(0_2) = 1 - q$ and $\text{Prob}((S - 1)_1) = \text{Prob}((S - 2)_2) = q$ then we can use McKelvey and Palfrey (1992) to characterize the unique LFE. The only difference would be that we would have to replace $S$ by $S - 1$ in their analysis as their altruist type (corresponding to $0_1$, $0_2$), who occurs with probability $(1 - q)$ chooses pass in all stages, while even the lowest foresight types in our analysis, $0_1$ and $0_2$, take in the $S^{th}$ stage.

**Conclusion**

This chapter defines the Limited Foresight Equilibrium (LFE). The LFE is defined for general applicability in the class of finite sequential move games with perfect information. In seeking to make more intuitive and experimentally justifiable predictions, we model the case where players are interested in maximizing own payoff, but each player possesses one of different levels of foresight. Further, players are uncertain about their opponents’ foresight. The LFE model nests the perfect information case. We prove the existence, upperhemicontinuity and other properties of LFE that seeks to capture the dynamics of real life finite sequential move games with “seemingly” perfect information. These properties are: (a) The higher the foresight-level of a player, the better he can estimate his opponents’ foresight. (b) The more moves any player-type observes, the better he becomes at guessing the opponent’s foresight level. (c) If a low foresight type is surprised by a sequence of moves impossible against lower types, he discovers that he is playing against some higher type, and must use his total foresight at that move to optimize. From (a), (b), and (c) we obtain: (d) The high foresight type must choose between revealing his type or pretending to be a low type. We show the applicability of LFE in two existing puzzles in the class of finite, two player alternate move games, namely, the Centipede Game and the Sequential Bargaining game. In the Centipede Game, LFE unleashes reputation effects, as in Kreps, Wilson, Milgrom and Roberts (1982), and McKelvey and Palfrey (1992), which lead to cooperative behavior even among rational players. In the Sequential Bargaining application, these features of the LFE help rationalize the disparate findings from the study of bargaining: namely, LFE produces outcomes that show (i) first round offer rejection (ii) first round
offer of near equal split (iii) disadvantageous counter proposals (iv) subgame inconsistency. These LFE results for Sequential Bargaining are parallel to several qualitative results in different experimental studies on bargaining.
Chapter 2: Opponent’s Foresight and Optimal Choices

This experimental study investigates how and why the behavior of experienced players, who understand the “sure-win” strategy in a “winner-take-all” sequential move game, varies systematically based on two types of information about the opponent’s expertise. Treatment (1): experienced subjects are told their opponent’s experience-level in the game. Treatment (2): a different set of experienced subjects are only shown their opponent’s play against a computer. We find that both (i) exogenous information, and (ii) endogenous inference about the opponent’s inexperience increase the probability with which experienced players abandon the “sure-win” strategy and try for a higher payoff attainable only by winning from a losing position, i.e., a position from which one wins only if the opponent makes a mistake. A maximum likelihood analysis shows that a model of limited foresight and uncertainty about the opponent’s foresight (Rampal (2016)) explains the data better than the Dynamic Level-k (Ho and Su (2013)) and AQRE (McKelvey and Palfrey (1998)) models.  

Introduction

There are countless examples where economic players participate in multi-stage interactions with each other. We focus on finite perfect information games where each player is aware of all prior decisions made in the interaction. The standard game-theoretic method

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for making outcome predictions about such interactions is to solve for the Subgame Perfect Nash Equilibrium (henceforth SPNE). The experimental literature has studied two key aspects of the SPNE model: first, do real players perform backward induction in solving their strategies (see Johnson et al (2002) and Binmore et al (2002))? Second, how does the belief about the opponent’s expertise in backward induction affect strategies? In two studies that use expert chess players, Palacios-Huerta and Volij (2009), and Levitt, List and Sadoff (2009) found contrary results in the context of the Centipede game. The former found that expert chess players were more likely to play SPNE strategies against other expert chess players, but not against student subjects. The latter did not find such a systematic difference in behavior. However in a zero-sum “race” game they found high incidence of SPNE play by expert chess players.

This chapter contributes to the second aspect of the debate. We investigate how and why the optimal strategy of expert players, in a perfect information “winner take all” dynamic game, is affected by the opponent’s given or perceived experience-level in the game. We add to the debate by investigating the question of whether subjects change behavior based on their inference about their opponent’s experience-level, where the inference is drawn by observing the opponent’s previous play during the game, without being explicitly told the opponent’s level of experience/expertise. We induce different levels of expertise by providing subjects with different levels of experience. In particular, the sequential move game we use is a “winner-take-all” game which has a “sure-win” strategy, similar to the “race” game. The data confirms that the experienced subjects learn the “sure-win” strategy of the game.

26Not necessarily, according to Johnson et al (2002), who monitored subjects’ “look-ahead” in a bargaining game, and found that the amount of stages subjects look forward was different across subjects. Binmore et al (2002) also study bargaining and find evidence showing that subjects’ behavior violates subgame and truncation consistency.

27Palacios-Huerta and Volij (2009) found that expert chess players stop a one-shot Centipede game at the first node (the SPNE prediction) more often when matched with other chess players as opposed to when they are matched with a student subject. They attribute this result to common knowledge of rationality among capable players, and thus conclude that it is the level of rationality and information about opponent’s rationality that determines outcomes rather than altruism or social preference. Levitt, List and Sadoff (2011) found that contrary to Palacios-Huerta and Volij (2009), expert chess players, playing with each other, play like student subjects in the Centipede game and cooperate in the beginning of the game. Levitt et al (2011) also test a zero-sum game called the “race to 100 game,” where each player has a dominant strategy. The “race to 100 game” is a zero-sum game, and there are no benefits from cooperation. They find a high incidence of SPNE play by expert chess players. Their results suggest that “failure to stop at the first node in centipede has little to do with an inability to reason backward.” They find that the “best inductors” in the race game had low stoppage rates in the centipede game at any node.
in their training session. Our key findings are: (i) experienced subjects are more likely to risk a loss and attempt to attain a higher payoff (higher than the payoff from winning using the “sure-win” strategy), which is attainable only if the opponent makes a mistake (plays a dominated strategy), when they are informed that their opponent is inexperienced. In a different treatment with different subjects we find that: (ii) experienced subjects’ accuracy of beliefs about the opponent’s experience level improves upon observing more moves of their opponent, and (iii) experienced subjects use the inference about the opponent’s inexperience, and increase the likelihood of risking a loss to try to attain the higher payoff against the opponents who play a dominated strategy in the earlier part of the interaction.

Therefore, finding (i) confirms the Palacios-Huerta and Volij (2009) result in the context similar to the “race to 100” game used by Levitt, List and Sadoff (2011). That is, our findings point to a systematic failure of SPNE within the paradigm of selfish and rational behavior. We can be sure that the behavior is indeed selfish because the game is “winner-take-all” and thus there is no possibility of a “fair” split of the payoffs. There is also no possibility of co-operation among players to increase joint payoffs like in the Centipede game. We know that experienced players are acting rationally in choosing to deviate from the SPNE strategy, and risking losing, because we can observe that they converge to SPNE behavior and play the “sure-win” strategy when playing another experienced opponent. In all previous literature and in our first treatment, each player is informed about his and the opponent’s expertise. One can argue that subjects being told their opponent’s experience-level/expertise reduces the relevance of these findings to dynamic games in the real world. But in treatment 2, our finding is that experienced subjects infer the experience-level of the opponent by observing his/her past behavior, and then act on this endogenous inference to take advantage of the opponent’s perceived inexperience. We also add to the literature by exploring the “why,” i.e., by trying to distinguish among which model explains such data the best. A model with limited foresight and uncertainty about the opponent’s foresight (Rampal (2016)) fits the data better than the dynamic Level-k model (Ho and Su (2013)) and the quantal response equilibrium for extensive games (McKelvey and Palfrey (1998)). The Ho and Su (2013) Level-k model relies on subjective beliefs about the opponent’s cognitive “level” to explain the data. The reason that the Ho and Su (2013) Level-k model does
worse is that it doesn’t allow for a failure to play weakly dominant strategies. But this is often the case in our data because a “high” level of foresight is required for understanding the weakly dominant strategy in our sequential move game.

This experimental study uses two modified versions of a “race game.” The race game is a two player, alternate move, perfect information, zero sum game. One version of the race game we use has the following specifications: two players move alternately, choosing numbers of “items to remove” from a box containing 9 items. At every move one can remove 1, 2 or 3 items. The player who removes the last item, loses, while his opponent wins. We call this game the Avoid 9 game. Note that if the second mover plays (4 minus the opponent’s previous choice) at each of his moves, he is guaranteed to win. In effect, it is a race to remove the 8th item. Similar race games have been extensively tested in the lab; c.f. Gneezy et al (2010), Dufwenberg et al (2010), Levitt et al (2011), Mantovani (2014).

Some of the reasons why the literature studies race games are: (a) they are zero sum games, which gets rid of explanations of the data originating from other-regarding preferences; (b) the SPNE of a race game is an equilibrium in weakly dominant strategies, and therefore the SPNE strategy remains a best response to the other player’s strategy no matter what one might believe about the opponent (c) the SPNE is easy to understand (once known) and apply once understood, and therefore deviations from SPNE strategies are easy to spot; (d) a player needs to reason backwards to understand the optimal strategies, and therefore the game serves as a good test of foresight.

The Avoid 9 game we use is different from the standard race game in the following ways. First, to decide the first and second mover in the game, we ask both players their first mover/second mover choices. After both players make their simultaneous choices, one of the choices is selected with 50 percent chance each. Second, we make the prize for winning as the first mover equal to 500 experimental currency units (henceforth, ECUs), and the prize for winning as the second mover as 200 ECUs. The first/second mover choices of the experienced subjects tell us if they want to play the “sure-win” strategy or if they want to try for the higher payoff, attainable only if the opponent makes a mistake as the second mover.

In treatment 1, we provide experience in the Avoid 9 game to half the subjects. Like
Gneezy et al. (2010), the experience leads to convergence towards SPNE play. That is, the experienced subjects learn the second mover advantage as they play 12 repetitions of the Avoid 9 game. Then we run a combined sub-session of experienced and inexperienced subjects stranger matched with each other. In the combined sub-session each player is told if the opponent is “experienced” or not. We find that experienced subjects are significantly more likely to choose to be the first mover, and put themselves in a losing position to try and win the bigger prize, if their opponent is inexperienced than if their opponent is experienced. We also observe that inexperienced subjects display no such difference in behavior when playing against inexperienced or experienced subjects.

In another treatment, treatment 2, we endogenize the process of learning about the opponent’s experience-level. Each subject in each round goes through two parts of the round. First, he plays an “avoid removing the 13th item” game with a perfectly playing computer. Label this game as C13. In C13: (i) the box contains 13 items and the player who avoids removing the 13th item wins; (ii) the human subject decides who the first mover is, him or the computer; (iii) there is only one prize, 500ECUs to win. In each round, each subject first finishes C13 and then plays his matched human opponent in an Avoid 13 game, with rules exactly like treatment 1, except that the box contains 13 items. That is, the Avoid 13 game played with a human subject contains the extra incentive to win as the first mover like the Avoid 9 game, and the second mover advantage is also the same. The earnings from the round are a sum of the earnings from the two parts of the round: C13 with a computer and Avoid 13 with a human subject. Similar to the first treatment, we provide experience to half the subjects, and make them do 8 repetitions of this two part round. In each two-part round, the experienced subject is shown his opponent’s moves versus the computer. After half the subjects are experienced, we mix experienced and inexperienced subjects together who are then randomly re-matched every round. We observe that experienced subjects are more likely to choose to be the first mover in their Avoid 13 interaction with a human opponent, if that human opponent lost his C13 game, compared to the case where the opponent won his C13 game. That is, replacing explicit information about the opponent’s experienced level with information about the opponent’s behavior in the first part of the round also produced a similar effect to actually telling the experienced player about the level of experience of his
opponent. Further, we ask the experienced subject about his opponent’s level of experience at two points of his opponent’s play in C13, and find that the accuracy of the experienced subjects’ answer improves after observing more moves of his opponent.

**Related Literature**

Other studies of race games include Dufwenberg et al (2010) and Gneezy et al (2010) who study different aspects of learning in the context of these games. A modified version of a race game is used by Mantovani (2014) to also show that subjects indeed display limited foresight. Mantovani (2014) observes evidence of a “sophisticated type” who puts himself in a losing position to attain a higher prize. But, based on their own data analysis, they rule out such behavior. The Mantovani (2014) model deals with limited foresight, but doesn’t model uncertainty about the opponent’s foresight. Thus, in their model, exogenous/endogenous information about the opponent’s foresight would not make a difference to the model’s prediction, contrary to our data results. Our design has two new elements to the race games studied in the literature: (a) in treatment 1, the simultaneous first/second mover decision stage to explore the effect of the belief about the opponent’s foresight on the experienced player’s optimal strategy; and (b) in treatment 2, the opportunity for endogenous learning of the opponent’s foresight/level, based on observation of the opponent’s prior play.

The effect of mixing experienced and inexperienced subjects in dynamic games has been studied in different contexts in the literature. Coq and Sturluson (2012) study the effect of exogenous information about the opponent’s experience in the context of the quantity precommitment dynamic game. They show that when experienced subjects play against an inexperienced opponent, the former choose higher capacities than when playing against an experienced opponent. They explain their findings using the AQRE model of McKelvey and Palfrey (1998). Dufwenberg et al (2005) study the effect of mixing inexperienced subjects among experienced subjects in multi-stage asset pricing games. There are some important differences in these studies compared to this chapter. First, the games are not “winner take all” and therefore the loss from a suboptimal strategy, if the opponent plays optimally, is not as stark; and second, they have no treatment with endogenous learning of the opponent’s experience level.
The effects of combining experienced and inexperienced subjects in simultaneous move games have been studied extensively.\textsuperscript{28} We refer the interested reader to Agranov et al (2012), Aloui and Penta (2016), Gill and Prowse (2014) and Slonim (2005).

We compare the Rampal (2016) model with the dynamic Level-k model of Ho and Su (2013), and the AQRE model of McKelvey and Palfrey (1998), to explain the data. Kawagoe and Takizawa (2012) also have a sequential level-k model that they apply to data on the Centipede game. We test their model with respect to our data as a robustness check of our results.\textsuperscript{29} Levin and Zhang (2016) is a working paper which brings together the Nash Equilibrium and the Level-k model. They extend their model to sequential move games. An application of their model to our data is left for future work. Also see Crawford, Costa-Gomes and Iriberri (2013) for a thorough survey of the work using the Level-k theory, most of which has been in simultaneous move games, unlike the sequential move game we study here.

**Experimental Design: Treatment 1**

The experiment for treatment 1 was conducted at The Ohio State University’s experimental economics laboratory using the laboratory’s subject pool.

\textsuperscript{28}Agranov et al (2012) study the effect of manipulating their subjects’ beliefs about their opponent’s cognitive levels in a simultaneous move 2/3 guessing game. Aloui and Penta (2016) endogenize the choice of level in a Level-k framework by modeling and studying the incentives and costs of choosing a certain cognitive level. In a simultaneous move setting, these incentives and costs are shown to depend on the payoffs and the opponent’s level. They disentangle the effect of one’s own cognitive limitation from the effect of one’s beliefs about the opponent’s cognition. Gill and Prowse (2014) study convergence of play towards equilibrium across repetitions of a simultaneous move p-beauty contest based on their measure of cognitive ability and character skills. Slonim (2005) tested the effects of varying the experience levels of the players in simultaneous move games similar to the beauty contest game. They find that only experienced subjects, and not inexperienced subjects, condition their behavior on the opponent’s experience level. The also find that introduction of new players interrupted the convergence towards equilibrium.

\textsuperscript{29}Kawagoe and Takizawa (2012) use a model of level-k where each level plays best responses with logit errors like the AQRE model of McKelvey and Palfrey (1998). We use their model as a robustness check of our results. However, we argue that a comparison with a logit Level-k model like theirs is not a reasonable model to compare. This is because, unlike the logit level-k model, when we compare the Ho and Su (2013) sequential level-k model to the LFE model, both bounded rationality models have error-less strategies (although both of them need an error term on the outcomes to keep the likelihood function finite), and therefore the comparison is among similar, though non-nested, models.
Exogenous Information Treatment

Treatment 1 comprised of 13 sessions, with a total of 154 subjects. The sessions were conducted using zTree (Fischbacher (2007)). Each session contained between 8 and 18 subjects. A session lasted 62 minutes on average with an average payment of USD 14.45. Each session used two games. First, a game called Avoid Removing the 9th Item. We refer to this game as the Avoid 9 game. The other game used was a three period sequential bargaining game (Rubinstein (1982)) with the common discount factor = 0.6. The bargaining game and its results are not relevant to this chapter. We now explain the rules of the Avoid 9 game.

The Avoid 9 game is played by 2 players. There are 9 items in a box. At every move, each player can choose to remove 1, 2, or 3 items from the box. If, at a move, the number of items left in the box is 2 or 1 then the maximum number of items that a player can remove at that move is 2 or 1, respectively. Before the game begins, both players simultaneously choose between “First Mover” and “Second Mover”, i.e., if they want to be the first mover/second mover in the game. One of the two players’ choices is implemented with 50 percent chance each. After it has been decided who the first mover is, the two players choose alternately. All prior choices are displayed to both players. If a player removes the 9th item, he loses, and his partner wins. If a player wins as the first mover, his payoff for the round is 500 ECUs (Experimental Currency Units). If a player wins as the second mover, his payoff for the round is 200 ECUs. If a player loses, he gets 50ECUs. The conversion rate used was 60ECUs = 1USD.

This game is a “race” game (a term used by Gneezy et al (2010), Levitt et al (2011), Mantovani (2014); Dufwenberg et al (2010) refer to their “race” game as “game to 21”) with the first mover/second mover decision stage (henceforth F/S decision stage) added to it. The game is called a “race” game because a player has to remove the 8th item to win, i.e., it is a race to 8. Define a “position” $n$ of the Avoid 9 game as the set of nodes such that for any node in that position, the sum of items removed at all nodes preceding that node is $n$. A particular position can contain several nodes. A node is in a winning position if the player moving at that node can choose a strategy (in the subgame with that node as
the root) that guarantees a win, regardless of the opponent’s strategy. A node is in a losing position if the opponent of the player moving at that node can choose a strategy (in the subgame with that node as the root) that guarantees a win to the opponent, regardless of the player’s strategy. The winning positions are \( W_9 = \{1, 2, 3, 5, 6, 7\} \), and losing positions are \( L_9 = \{0, 4, 8\} \). A position of 0 is the set of decision nodes of the selected first mover immediately succeeding the \( F/S \) decision stage. In the Avoid 9 game, the second mover can always win. As the second mover, one can choose \((4 \text{ minus the opponent’s previous choice})\) at each move to remove the 8th item. However, if the second mover fails to put the opponent in \( L_9 \) after one of his moves, then the first mover is guaranteed a win by playing a strategy which puts the second mover at a losing position at all of the second mover’s subsequent moves. Note that in the Avoid 9 game, the SPNE is for both players to choose “Second Mover” (henceforth \( S \)) and then choose 3, 2, and 1 from positions in \{1, 5\}, \{2, 6\}, and \{3, 7\}, respectively. The SPNE strategy places no restriction on actions from a position in \( L_9 \). We call this SPNE strategy as the “perfect” strategy.

The perfect strategy in Avoid 9:

\[
\text{perfect action} = \begin{cases} 
S & \text{at } \{F/S\} \text{ decision stage} \\
3 & \text{if } \text{Position } \in \{1, 5\} \\
2 & \text{if } \text{Position } \in \{2, 6\} \\
1 & \text{if } \text{Position } \in \{3, 7\} \\
\text{Arbitrary} & \text{if } \text{Position } \in \{0, 4\}
\end{cases} \quad (7)
\]

Note that if one’s \( F/S \) decision is implemented, the perfect strategy is a “sure-win” strategy. However, due to the extra incentive to win as the first mover, the perfect strategy is \textit{not} a weakly dominant strategy. If a risk neutral rational player believes with probability at least \( \frac{1}{3} \) that his opponent will make a mistake, then choosing “First Mover” (henceforth \( F \)) is optimal. Let any subgame with its root at a node in position 0 be labeled as \( A9_{\text{sub}} \). Note that in \( A9_{\text{sub}} \), the perfect strategy \textit{is} weakly dominant because the payoff from winning/losing is already decided, and the perfect strategy is a “sure-win” strategy from a winning position.

The design of treatment 1 is as follows:
1. In each session each subject went through 2 sub-sessions. First, the subjects were split into two types: **Experienced (Exp)** and **Inexperienced (Inexp)**, with Inexp subjects being at least 50 percent of the total subjects in any session. In the first sub-session (training sub-session):

   (a) **Exp subjects** (74 total subjects) **trained.** Exp subjects played 12 rounds of the Avoid 9 game among themselves. One round was randomly drawn as the round determining earning from the first sub-session.

   (b) **Inexp subjects** (80 total subjects) were not told about the Avoid 9 Game. They played between 5-8 rounds of a three period bargaining game (Rubinstein (1982)) among themselves. One round was randomly drawn as the round determining earning from the first sub-session.

2. In the second sub-session (combined sub-session) Exp and Inexp subjects were mixed. They played 8 rounds (round numbers 13-20 of the session) of the Avoid 9 Game together. The subjects were stranger matched into pairs before the beginning of each round. At all points during the Avoid 9 Game, each player was told if their opponent was Exp (if the other was Exp, we told the subject that the other was of type $S$, i.e., a subject who had played the same Avoid 9 game in the first sub-session) or Inexp (if the other was Inexp, we told the subject that the other was of type $D$, i.e., a subject who had played a different game in the first sub-session).

3. One round from the second sub-session was drawn at random for payment and added to the payment due from the first sub-session. All payments were made at the end of the session.

Subjects moved sequentially, with each subject, at each move being given a clock with 45 seconds on it to remind them to move. The clock could only flash if the time taken was more than 45 seconds, and the game did not proceed without the subject’s choice.

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30 The bargaining data will be the subject of another paper, not included in this dissertation.
Data Results for Treatment 1

In our analysis of the data, the following aspects of the subjects’ behavior will be important:

1. How did the Exp subjects’ “First/Second Mover” choices vary based on the opponent’s experience level.

2. Was there any systematic pattern of imperfect play?

The main results of treatment 1 can be observed in Figure 7. Figure 7 plots the average proportion of the Exp subjects choosing “First Mover” by round and opponent type. Figure 7 shows the choices of experienced subjects who had understood the “sure-win” perfect strategy in the training sub-session. We can see that in the combined sub-session (rounds 13-20), when the experienced subjects were playing against an inexperienced opponent (red triangles), as opposed to an experienced opponent (black dots), they were significantly more likely to choose “First Mover” and put themselves in a losing position in order to attain a higher winning prize.

**Result 1(a): Training Successful:** in the training sub-session (rounds 1-12) there was a significant increase in the proportion of Exp subjects playing the Avoid 9 game perfectly. The proportion of Exp subjects playing the imperfect strategy almost converged to zero.

Recall that the selected second mover cannot lose the Avoid 9 game if he plays perfectly. The reward for winning as the first mover (second mover) is 500 ECUs (200ECUs). This tempts the subjects to choose $F$ until they gain understanding of the perfect strategy of the game and until they are sufficiently convinced about their opponent’s understanding of the perfect strategy. In round 1, 78.4 percent of Exp subjects had chosen $F$. This percentage declined steadily (see Figure 7) through the rounds, and in round 12, only 2.7 percent of the Exp subjects chose $F$. This decline in proportions is highly significant with a p-value of approximately 0. The 2.7 percent of Exp subjects choosing $F$ in round 12 is a positive but insignificant. Recall that once the first and second movers are decided, i.e., in the subgame $A_{9_{sub}}$, it is a weakly dominant strategy to play perfectly. The percentage of pairs of subjects who displayed imperfect play in $A_{9_{sub}}$, reduced from 36.5 percent in the first two rounds, to 5.4 percent in the last two rounds of the training sub-session. Although 5.4 percent is
Figure 7. Experienced Subjects’ Behavior Based on Opponent’s Experience

Notes: The figure depicts the round-wise proportion of experienced subjects matched with an experienced opponent who chose “First Mover” (black dots) and the proportion of experienced subjects matched with an inexperienced opponent who chose “First Mover” (red triangles). The first mover loses if the second mover plays perfectly. The latter proportion is significantly higher, with a p-value < 0.05, for each of the rounds 13 through 16. This shows that the experienced subjects were more likely to risk losing to try for the higher prize when matched with an inexperienced opponent. The training sub-session was the first 12 rounds, and the combined sub-session was rounds 13-20.

positive and significant, the reduction in imperfect play in $A_{9_{sub}}$ is highly significant with a p-value of approximately 0.31.

**Result 2(a): Opponent’s experience level has a significant effect:** in the combined sub-session (rounds 13-20), experienced subjects were more likely to choose $F$ (“First mover”) against an inexperienced opponent than an experienced opponent.

Figure 7 illustrates result 2(a). Round-wise tests for difference in proportions show that for each of rounds 13-16, the Exp subjects matched with an Inexp opponent chose $F$ at a significantly higher rate (p-value $\leq 0.015$ for each round) than the Exp subjects matched with an Inexp opponent (see Table 16 in Appendix A).

Next, consider the probit results for 74 Exp subjects and 80 Inexp subjects’ choice data from rounds 13-20, the combined sub-session. Table 1, model (1), marked with Exp,
Table 1. Factors Influencing Probability of Choosing “First Mover”

<table>
<thead>
<tr>
<th></th>
<th>Exp</th>
<th>Inexp</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Opponent is Inexp</td>
<td>1.18*** (0.33)</td>
<td>0.12 (0.13)</td>
</tr>
<tr>
<td>Round</td>
<td>-0.31*** (0.07)</td>
<td>-0.43*** (0.04)</td>
</tr>
<tr>
<td>Constant</td>
<td>3.25*** (0.87)</td>
<td>6.2 (0.67)</td>
</tr>
<tr>
<td>No. of Obs.</td>
<td>544</td>
<td>640</td>
</tr>
<tr>
<td>Pseudo $R^2$</td>
<td>0.3342</td>
<td>0.3273</td>
</tr>
<tr>
<td>Session Dummies</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 1. Factors Influencing Probability of Choosing “First Mover”

Notes: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Numbers in parenthesis are robust standard errors clustered at subject-level. For model (1), the dependent variable is Exp players’ choice of first (takes value 1) or second mover (takes value 0). The sample size is 68 Exp subjects (after dropping session 8) across rounds 13-20. For model (2), the dependent variable is Inexp players’ probability of choice of first (takes value 1) or second mover (takes value 0). The sample size is 80 Inexp subjects across rounds 13-20.

Reports the results from a probit estimation with the Exp subjects’ choice of F/S in the Avoid 9 game as the dependent variable. The dependent variable took value 1 if an Exp subject chose F, and 0 if he chose S. The independent variables are: (i) a dummy for if the opponent was Inexp (value 1) or Exp (value 0); (ii) Round variable; (iii) Constant term; (iv) Session dummies. Table 1, column 3, marked with Inexp, reports the results from the same probit estimation done with the Inexp subjects’ choice of F/S in the Avoid 9 game as the dependent variable. The definition of the dependent variables implies that a positive coefficient on an independent variable means that a higher value of the independent variable increased the probability of the player choosing F.

Focusing on model (1), which reports the probit results for the Exp subjects,\textsuperscript{32} we observe the following. The highly significant coefficient on the dummy variable for “opponent is Inexp” (p-value < 0.001) suggests that Exp subjects were significantly more likely to choose F when playing against an Inexp opponent than an Exp opponent. The highly significant negative coefficient of the round variable (reflected in the downward sloping lines

\textsuperscript{32}In addition to the results in Table 1, for the probit estimates with the Exp subjects’ F/S choices, session numbers 2, 4, 5, 7, 9, 11 and 12 had a significantly negative coefficient, while session 8 perfectly predicted the choice S by Exp players in rounds 13-20, and thus was omitted without affecting the likelihood or the estimates of the other coefficients. For the probit estimation with the Inexp subjects’ F/S choices, only sessions 5 and 9 had a significantly positive effect (p-value < 0.1) on the probability to choose F.

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in Figure 7) can be caused by a combination of two factors. As subjects play more rounds, two things happen: (a) more of them learn the perfect strategy; (b) there is an increase in the subjects’ belief that the opponent (Exp or Inexp) understands the “sure-win” perfect strategy in $A9_{sub}$. As almost all the Exp subjects had learnt the perfect strategy after the training sub-session (result 1(a)), (b) appears to be the reason for the significant reduction in the proportion of Exp subjects choosing $F$ as the rounds progressed.

The positive but statistically insignificant coefficient of the “Opponent is Inexp” dummy in model (2) shows that while Inexp subjects also increased the likelihood of choosing $F$ when they faced an Inexp opponent, this increase was statistically insignificant. The round variable’s highly significant negative coefficient (p-value $< 0.001$) implies that as the Inexp subjects played more rounds of the Avoid 9 game, there was a significant increase in the proportion of Inexp subjects who understood the perfect strategy and believed that their opponent also understood the perfect strategy.

**Result 3(a): Faster learning speed of Inexp subjects:** Inexp subjects, playing their first 8 rounds of the Avoid 9 game in the combined sub-session, learned the “sure-win” perfect strategy of the second mover significantly faster than Exp subjects in their first 8 rounds of playing the Avoid 9 game.

Learning can be observed in two aspects of the data: (i) the $F/S$ decision of the players, and (ii) perfect/imperfect play as the selected second mover in $A9_{sub}$.\footnote{Looking at all possible positions where one can distinguish perfect play from imperfect play of a player does not change our results. For simplicity, we just look at the selected second mover’s perfect/imperfect play at his first decision.} Note that while for (i) the belief about the opponent’s understanding of the perfect strategy matters, for (ii), there is no possible belief than incentivizes imperfect play. Both aspects of the data lead us to conclude result 3(a).

Figure 8 depicts the learning speed difference based on the $F/S$ choices. Figure 8 depicts the Exp players’ average probability of choosing $F$ against other Exp players in their first 8 rounds (black dots), i.e., when the Exp subjects were gaining experience in the training sub-session. It also depicts the Inexp players’ average probability of choosing $F$ against another Inexp player in rounds 13-20 (red triangles), which are the Inexp subjects’
Figure 8. Learning Speed Comparison

Notes: The figure depicts the behavior of experienced subjects in rounds 1-8 of the session and the inexperienced subjects’ behavior in rounds 13-20 of the session (which are the inexperienced subjects’ first 8 rounds). The figure depicts the round-wise proportion of experienced subjects matched with an experienced opponent who chose “First Mover” (black dots) and the proportion of inexperienced subjects matched with an inexperienced opponent who chose “First Mover” (red triangles). The first mover loses if the second mover plays perfectly. The latter proportion is significantly lower, with a p-value < 0.011, for each of the rounds 5, 6 and 7. This shows the faster learning speed of the inexperienced subjects.

Note how both Inexp and Exp start out with the same rate of choosing $F$, but after their respective 4 rounds, the Inexp subjects play $F$ at a significantly lower rate. In their first three rounds, both Exp and Inexp subjects behave similarly, but for their rounds 5, 6, and 7 (session round numbers 17, 18 and 19), Inexp subjects choose $F$ in a significantly smaller percentage of cases against other Inexp players, as compared to the Exp players in their rounds 5, 6 and 7 of playing the Avoid 9 game. These differences in proportions have p-values ≤ 0.011 for each of the three rounds. These results are also confirmed by a probit estimation (details in Tables 17 and 18 in Appendix B).

As noted above, findings on the $F/S$ decision could just be a function of the subjective beliefs of the players. However, the data on aspect (ii) (perfect/imperfect play as the selected
second mover in $A_{9_{sub}}$ also supports the result 3(a): a significantly higher proportion of Inexp subjects (87.3 percent) played perfectly as the selected second mover in their first 8 rounds compared to Exp subjects as the selected second mover in their first 8 rounds (80.7 percent). The difference has a two-tailed p-value of 0.031. This is also confirmed by a probit estimation (Table 18 in Appendix B).

The reason for result 3(a) is not clear from the data. One would expect the reason to be that the Inexp subjects were playing with a combination of Inexp and Exp subjects in their first 8 rounds. This is in contrast to Exp subjects in their first 8 rounds, who just played other subjects new to the Avoid 9 game. A test of this rationale is not confirmed by the data.\footnote{We divide Inexp subjects into two categories: first, those who were matched with Exp opponents in at most 2 rounds out of the first four rounds of the combined sub-session. Second, those who were matched with Exp opponents in at least 3 rounds out of the first four rounds of the combined sub-session. Contrary to intuition, we find that the proportion of Inexp subjects who choose $F$ against an Inexp opponent in rounds 5-8 of the combined sub-session is lesser in the first category (6.1\%) compared to the second category (7.5\%). Also, the proportion of Inexp subjects who play perfectly as the second mover in rounds 5-8 of the combined sub-session is more in the first category (96\%) compared to the second category (92\%). But both these differences are statistically insignificant. This finding is robust to other such divisions of the Inexp subjects based on how often they encountered an Exp subject in the first four rounds of the combined sub-session.}

The next reason shows the presence of limited foresight among subjects.\footnote{The foresight of a player is defined as the number of subsequent stages that the player can observe from a given decision node (Rampal (2016)).} Result 4(a) reports that as subjects got closer to the end of the game, the rate of imperfect play declined significantly. Limited foresight would imply this because if subjects have limited foresight, then as subjects get closer to the end of the game, even the lower foresight subjects have enough foresight to understand the weakly dominant strategy and play perfectly. Thus, the rate of imperfect play declines as subjects get closer to the end of the game. This result is in line with the proof of limited foresight given by Mantovani (2014). Why limited foresight implies this pattern of imperfect play is discussed more explicitly in the model comparison section.

**Result 4(a):** The rate of imperfect play declined significantly closer to the end of the game. That is, as the position increased, or as the number of items left reduced, the rate of imperfect actions reduced. In particular, a player lost from a winning position with 4 or lesser items left in only 13 (1.13\%) of the 1060 Avoid 9 games played in treatment 1.
Table 2. Rate of Imperfect Play by Position

Notes: The figures are in percentage. The rate of perfect play at each position is 100 minus the rate of imperfect play given. The figures are reported from positions from which a perfect action is distinguishable from an imperfect action. A perfect action makes the position 4 (8) from a position 1, 2, or 3 (5, 6, or 7).

Table 2 shows the observed rates of imperfect play by position. We show the rates separately for three parts of the data: (i) the first four rounds of the Exp players; (ii) first four rounds of the Inexp players; and (iii) session (training and combined sub-sessions together). Recall that a position is a particular sum of items removed. From position 1-3 (5-7), the perfect action makes the subsequent node’s position as 4 (8). We don’t report imperfect play rates from positions 0 and 4 because any action at those positions is a perfect action. The rate of imperfect play is significantly more (p-values $\approx 0$) from a position in 1-3 (6-8 items remaining) as compared to 5-7 (2-4 items remaining) in each of (i), (ii), and (iii).

The next result shows that at losing positions, i.e., at positions where SPNE is silent on which action should be chosen, there was a systematic pattern of the actions chosen: subjects were significantly more likely to remove 1 item as compared to the next most chosen alternative. This was also true for experienced subjects. This indicates that subjects were uncertain about the foresight of their opponent, because if there is some chance that the opponent can have limited foresight, then that provides an extra incentive to remove just 1 item to keep the opponent as far away from the end of the game as possible and maximize the probability that he makes a mistake. This is discussed more explicitly in the model comparison sections.\(^{37}\)

\(^{37}\)The model with limited foresight and uncertainty about the opponent’s foresight (Rampal (2016)) partially captures this: the prediction for the full foresight player’s choice at position 0 is “remove 1 item.” This is because removing 1 item keeps an opponent with a certain level of limited foresight far enough from the end of the game to make a mistake. But at position 4, any action brings the opponent with any foresight level too close to the end of the game to make a mistake, hence there is no such prediction. The Dynamic Level-k model predicts that the Level-1 player, who believes that her opponent is Level-0 (who uniformly randomizes among all available actions at each decision node), would be indifferent among removing 1 and 2 items only at position 4. However, there is no predicted bias towards removing 1 item.
Table 3. Proportions of Choices from Losing Positions

Notes: The figures are in percentages. The p-values are two tailed p-values comparing the proportion of times 1 was chosen, as compared to the next most chosen alternative.

**Result 5(a): Choice proportions at losing positions.** At position 0 and 4, subjects were significantly more likely (with p-values of 0.001 and approximately 0, respectively) to choose to remove 1 item than next most likely alternative. This was also true for Exp subjects in the combined sub-session (see Table 3 below).

The next result, 6(a), tells us that it was ex-post weakly optimal for an Experienced subject to choose “First Mover” against an Inexperienced opponent when the Inexperienced subjects were “truly” inexperienced, i.e., in the first two rounds of the combined sub-session.

**Result 6(a): Ex-post optimality of being “First Mover” against a “truly” Inexp opponent.**

In the first two rounds of the combined sub-session, when the Inexp opponent was “truly” inexperienced it was weakly ex-post optimal for an Exp player to choose $F$. This can be seen in Table 4 below. In rounds 13-14, the first two rounds of the combined sub-session, if an Exp subject was the selected first mover, his average earnings were weakly more than his average earnings if he were the selected second mover. However, for all later rounds, enough of the Inexp subjects had understood the perfect strategy so that it was not ex-post optimal for an Exp subject to choose $F$ against an Inexp subject: for each round after round 14, the average earnings of the Exp subjects selected as the first mover against an Inexp opponent was significantly lesser that the average earnings of the Exp subjects selected as the second mover against an Inexp opponent. This finding agrees with the faster learning speed of the Inexp subjects (result 3(a)).
Table 4. Ex-post Earnings of Experienced Subjects against Inexperienced Opponents

<table>
<thead>
<tr>
<th>Round</th>
<th>Earning as Second Mover</th>
<th>Earning as First Mover</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>13-14</td>
<td>194 (16.5)</td>
<td>216 (31.6)</td>
<td>0.5</td>
</tr>
<tr>
<td>15-16</td>
<td>193 (15.8)</td>
<td>107 (23.2)</td>
<td>0</td>
</tr>
<tr>
<td>17-18</td>
<td>193 (11.5)</td>
<td>85 (17)</td>
<td>0</td>
</tr>
<tr>
<td>19-20</td>
<td>200 (7.3)</td>
<td>64 (11.4)</td>
<td>0</td>
</tr>
</tbody>
</table>

Experimental Design: Treatment 2

Within Game Learning Treatment

Treatment 2 comprised of 8 sessions. Each session contained between 8 and 18 subjects. A session lasted 64 minutes on average. The average payment made to the subjects was USD14.80. Treatment 2 was designed to ask the following question: “Do Exp subjects endogenously learn about and respond to their opponent’s experience-level by observing the opponent’s prior play in the round, without being explicitly informed about the opponent’s Exp/Inexp type?” Each session used three games. First, a game called the Computer 13 game, henceforth shortened to C13. Second, the Avoid Removing the 13th Item game, which we refer to as the Human 13 game (henceforth, H13 for short). Last, a three period sequential bargaining game (Rubinstein (1982)) with a common discount factor of 0.6.

The rules of the H13 game are the same as the Avoid 9 game except that to begin, the total number of items in the box is 13, so the player who removes the 13th item loses, while his opponent wins. The H13 game also begins with both players choosing F/S, and either player’s choice being selected with 50 percent chance each. In the H13 game, like in the Avoid 9 game, all prior choices in the game are displayed to both players, except at the simultaneous F/S decision stage. The payoffs of the H13 game were also the same as the Avoid 9 game: 500 ECUs for winning as the first mover 200 ECUs for winning as the second mover, and 50 ECUs for losing. We use 13 items instead of 9 to further widen the gap between subjects who understood the perfect strategy and those who did not understand it. Using 13 items also mean that this gap persisted for more rounds. These features help in distinguishing among competing theoretical models described later.

The winning positions in H13 are $W_{13} = \{1, 2, 3, 5, 6, 7, 9, 10, 11\}$, and losing posi-
tions are \( L_{13} = \{0, 4, 8, 12\} \). In the H13 game, the second mover can always win. As the second mover, one can choose (4 minus the opponent’s previous choice) at each move to remove the 12th item. However, if the second mover fails to put the opponent in \( L_{13} \) after one of his moves, then the first mover is guaranteed a win by playing a strategy which puts the second mover at a losing position at all of the second mover’s subsequent moves. Note that in the H13 game, the SPNE is for both players to choose “Second Mover” (henceforth \( S \)) and then choose 3, 2, and 1 from positions \{1, 5, 9\} , \{2, 6, 10\}, and \{3, 7, 11\}, respectively. The SPNE strategy places no restriction on actions from a position in \( L_{13} \). We call this SPNE strategy as the “perfect” strategy in H13.

The perfect strategy in H13:

\[
\text{perfect action} = \begin{cases} 
S & \text{at \{F/S\} decision stage} \\
3 & \text{if Position} \in \{1, 5, 9\} \\
2 & \text{if Position} \in \{2, 6, 10\} \\
1 & \text{if Position} \in \{3, 7, 11\} \\
\text{Arbitrary} & \text{if Position} \in \{0, 4, 8\}
\end{cases}
\] (8)

Note that if one’s \( F/S \) decision is implemented, the perfect strategy is a “sure-win” strategy in H13. However, due to the extra incentive to win as the first mover, the perfect strategy is not a weakly dominant strategy in H13. If a risk neutral rational player believes with probability at least \( \frac{1}{3} \) that his opponent will make a mistake, then choosing “First Mover” (henceforth \( F \)) is optimal. Let any subgame with its root at a node in position 0 be labeled as \( H_{13_{\text{sub}}} \). Note that in \( H_{13_{\text{sub}}} \), the perfect strategy is weakly dominant because the payoff from winning/losing is already decided, and the perfect strategy is a “sure-win” strategy from a winning position.

The C13 (Computer 13) game was played by a human subject, individually, against a perfectly playing computer. In treatment 2, the opponent’s performance in C13 is shown to the Exp player before the H13 game begins. The information about the opponent’s performance in the C13 game is used to replace treatment 1’s method of informing the Exp player if the opponent is Exp/Inexp. In the C13 game, the subjects were told that “the computer
plays perfectly to win.⁴⁸ There are two key differences in the C13 game compared to the H13 game. First, in the C13 game, the human subject decides who the first mover will be: him or the computer. After the first mover is decided by the human subject, he and the computer move alternately, choosing 1, 2 or 3 items to be removed from a box containing 13 items. The player who removes the last item loses, and his opponent wins. The second key difference in the C13 game is that a player earns 50 ECUs for a loss, and 500 ECUs for a win. That is, there is no extra-incentive to win as the first mover (which is also not possible as the computer plays perfectly). Thus, in C13, it is weakly dominant and an SPNE strategy to play the perfect strategy for H13 described above.

The design of treatment 2 was as follows:

1. In each session each subject went through 2 sub-sessions. First, the subjects were split into two types: Exp and Inexp, with Inexp subjects being at least 50 percent of the total subjects in the session. In the first sub-session (training sub-session):

   (a) Exp subjects trained. Each round of this sub-session comprised of two parts. At the start of each round, each Exp player, for example X, was randomly matched with another Exp player, say Y. In the first part of the round, both X and Y separately played C13 against their respective perfectly playing computer. In the second part of the round, X and Y played the H13 game with each other. In the training sub-session, Exp subjects played 8 rounds of C13-H13 among themselves, where each round had two parts: play the computer and then play the human opponent. The earning from a round was the sum of the earnings from its two parts. For C13, a subject was given 500ECUs for a win, and 50ECUs for a loss. For H13, a subject was given 500ECUs for winning as the first mover, 200ECUs for winning as the second mover, and 50ECUs for a loss. One round was randomly drawn as the round determining earning from the first sub-session.

   • At two junctures in the C13 part of each round, X and Y were asked about

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⁴⁸ The subjects were told that “The computer plays perfectly to win. That is, if there is a choice or a sequence of choices that the computer can make to win against you, the computer will make that choice or sequence of choices without making any mistake.”
the type (Exp or Inexp ?) of their opponent. First, after X made his F/S decision in C13, he was shown Y’s F/S decision vs the computer and then (this is the first juncture) X was asked about the type of his opponent. Second, X was shown Y’s actions and outcome versus the computer in Y’s C13 and then (this is the second juncture) X was asked about the type of his opponent. For each correct answer about the opponent’s type, 100 ECU's were added to the total earnings of the round for X. Similarly, Y was also shown X’s history of moves versus the computer in C13 and asked about the X’s type at two junctures. Note that in this training sub session, with only Exp subjects playing each other, the correct answer about the opponent’s type was always Exp. The Exp subjects were informed that the Inexp subjects were playing a different game, so in the training sub-session these answers were a source of “free money”.39

(b) Inexp subjects were not told about the H13 or C13 games. They played between 5-8 rounds of a three period bargaining game with the common discount factor $= 0.6$ (Rubinstein (1982)) among themselves. One round was randomly drawn as the round determining earning from the first sub-session.

2. In the second sub-session (combined sub-session) Exp and Inexp subjects were mixed and were “absolute stranger” matched into pairs before every round. They played 6 rounds as described in 1(a), i.e., play the C13 with one’s respective perfectly playing computer and then play the H13 game with the human opponent (C13-H13), with two modifications. First, any pair of types, i.e., (Inexp, Inexp), (Exp, Exp) or (Exp, Inexp) was possible. Second, Inexp subjects, unlike the Exp subjects, were not shown the history of their opponent’s moves versus the computer in C13 or the outcome of the opponent in the opponent’s C13 game versus the computer. Notably the Exp

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39 Despite this information, the answer about the opponent’s type was wrong in 12.76% of the cases at the first juncture, and 7.55% of the cases at the second juncture. These error percentages increased if the opponent lost his C13 part, which might have led a subject to believe that his opponent is an Inexp type. This confusion may be stemming from the fact that the question about opponent’s type seemed counter-intuitive given the information provided that the opponent cannot be Inexp. Further, the “free money” being offered to the subjects may also have made them suspicious. Several subjects asked the experimenter “how can my opponent be Inexp?” We stuck to this design to keep the rounds the same for Exp subjects in both sub-sessions of the experiment.
subjects were told the fact that the Inexp subjects would not be shown the opponent’s history or outcome of play in the C13 part of any round.\textsuperscript{40} The Inexp subjects were also not asked about the type of their opponent. Exp subjects were asked about their opponent’s type like in 1(a), however now the answer could be either Exp or Inexp. The reason for not showing the opponent’s history from C13 to the Inexp subjects was that we wanted the gap of understanding of the perfect strategy between the Inexp and Exp subjects to persist for as many rounds as possible in the combined sub-session.

3. One round from the second sub-session was drawn at random for payment and added to the payment due from the first sub-session. The conversion rate used in the training and combined sub-sessions was 120ECUs=1USD. The payment due to the Inexp subjects at the end of the two sub-sessions was paid to the Inexp subjects who then exited the experiment.

4. After the end of the two sub-sessions, Exp subjects participated in a risky choice study using the DOSE procedure (Wang, Filiba, and Camerer (2010)). Exp subjects made 20 binary choices (see Figure 17 in Appendix B), where each choice was between a lottery and a deterministic payment. The earnings/losses from the DOSE procedure were scaled by a factor of $\frac{1}{5}$. One of the 20 choices was selected at random and implemented. The resultant earning/loss was added/subtracted to the earning due to the Exp subjects from the training and combined sub-sessions. The Exp subjects were then paid, which concluded the experimental session.

Subjects moved sequentially in the training and combined sub-sessions. Each subject was given a clock with 30 seconds at each move to remind them to decide. The clock could only flash if the time taken was more than 30 seconds, and the game did not proceed without the subject’s choice. Note that the SPNE prediction of the C13-H13 game (combination of the two games) is that outcomes in C13 should not affect the outcomes in H13. In particular, both players should play the perfect strategy in C13 and then again in H13.

The key difference in treatment 2 as opposed to treatment 1 is that in treatment 1, in the

\textsuperscript{40}In fact all aspects of the design were common knowledge to all subjects with only one exception: the Inexp and Exp subjects did not know what games the Exp and Inexp, respectively, were playing in the training sub-session. They did know that these games were unrelated.
combined sub-session, where Exp and Inexp subjects were mixed, each player was told if their opponent is Exp or Inexp. In treatment 2, only the Exp subjects are shown the history of their opponent’s moves versus the computer, but not told their opponent’s type.

**Data Results for Treatment 2**

As per the analysis for treatment 1, we shall again focus on Exp subjects’ “First/Second Mover” choices and patterns of imperfect play.

**Result 1(b): Training successful:** In the training sub-session (rounds 1-8) the proportion of Exp subjects winning the C13 game against the computer increased significantly. The round 1 proportion was 17 percent, while the round 8 proportion was 79 percent, which is a highly significant increase with a p-value of approximately 0. Recall that the computer plays perfectly, and any deviation from the perfect strategy by the human player leads to a loss. Further, in $H_{13_{sub}}$, the proportion of Exp subjects playing perfectly as the selected second movers increased significantly in round 8 (75%) compared to round 1 (25%). This increase is also highly significant with a p-value of approximately 0.41

**Result 2(b): Opponent’s loss versus the computer has a significant effect on the Exp subjects’ behavior in the combined sub-session:** In the combined sub-sessions (rounds 9-14), in the H13 game, Exp subjects were more likely to choose $F$ against an opponent who lost his C13 game against the computer, as opposed to an opponent who won his C13 game.42

Figure 9 shows that the Exp subjects whose opponent lost in C13 (red triangles) chose $F$ at a significantly higher rate than the Exp subjects whose opponent won C13 (black dots).

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41To study the success/failure of training, we focus on the play of the Exp subjects in (i) the C13 part and (ii) $H_{13_{sub}}$. This is because in both (i) and (ii), playing perfectly is a weakly dominant strategy. In contrast, the optimal choice among $F/S$ in the H13 game is also affected by the belief about the opponent’s probability of playing imperfectly in the subsequent stages of the H13 game. Despite this interpretation problem, if we look at the F/S choice data by opponent’s outcome we have the following results. In rounds 1 and 2, the Exp subjects chose $S$ in 34.4% out of 96 cases, while the same for rounds 7 and 8 was 80.2%. The difference in proportion of $S$ choices is highly significant with a p-value of approximately 0. Further, in the training sub-session, $F/S$ choice in the H13 game weren’t affected by the opponent’s outcome in C13 (discussed below).

42We don’t observe any such significant difference in the training sub-session. The knowledge that one’s opponent is similarly experienced, may have led the Exp subjects to not put too much weight on the opponent’s performance in the C13 part of one round that the subject observes. Further, learning the perfect strategy is one skill, learning how to benefit from the opponent’s lack of understanding of the game is another skill. The latter might have taken additional practice to acquire.
Figure 9. Experienced Subjects’ Behavior Based on Opponent’s Performance

Notes: The figure depicts the round-wise proportion of experienced subjects who chose “First Mover” when matched with an opponent who won C13 (black dots) and the proportion of experienced subjects who chose “First Mover” when matched with an opponent who lost C13 (red triangles). The first mover loses if the second mover plays perfectly. The latter proportion is significantly higher, with a p-value < 0.05, for rounds 10 and 13. This shows that the experienced subjects were more likely to risk losing to try for the higher prize when matched with an opponent who lost C13. The training sub-session was the first 8 rounds, and the combined sub-session was rounds 9-14.

The p-values of this difference in proportions is less than 0.05 for rounds 10, and 13, and the one-tailed p-value of this difference is 0.062 for round 12 (see Table 19 in Appendix B).

Next, consider Table 5 which reports the probit estimation results for the data from the H13 games in the combined sub-session. The dependent variable is the probability that a subject chooses $F$ in the $F/S$ decision stage of H13 in the combined sub-session. The definition of the dependent variable implies that a positive coefficient on an independent variable means that a higher value of the independent variable increased the probability of the player choosing $F$. Estimations (1)-(3) have the Exp player’s $F/S$ choice in H13 as the dependent variable. The highly significant coefficient of “Opponent Lost C13” dummy in the probit estimations (1) and (2) implies that, in the combined sub-session, if the average Exp subject observed that his opponent in H13 had lost against the computer in the C13 part of that round, then he was significantly more likely to choose $F$ in the $F/S$ decision.
stage of H13. The coefficient of the same variable is statistically insignificant for the Inexp subjects (estimation (4)), which is expected because Inexp subjects were not shown the moves or outcome of the opponent in the opponent’s C13 part of the round. The round variable is negative for all the models, (1)-(4), because an increase in the round variable captures the learning about how to play the perfect strategy, and the increase in the belief that the opponent understands the perfect strategy, which makes choosing $S$ a worse option. In model (2) we also add the loss aversion and risk aversion parameters as measured using the DOSE (Wang et al (2010)) estimation technique.

In model (2) we find that both risk aversion and loss aversion have a negative effect on the probability that an Exp player chooses $F$. The coefficient of risk aversion is statistically insignificant, but the coefficient of loss aversion is significant with a p-value of 0.02. The sign of these coefficients is as expected. For the Exp subject, conditional on his $F/S$ choice being selected, choosing $F$ entails a gamble between winning (500ECUs) or losing (50ECUs) while choosing $S$ guarantees a sure win worth 200ECUs.

The result that Exp players were more likely to choose $F$ in the H13 part of the round when they observed that the opponent lost in C13 is driven by the more “skilled” Exp players. To verify this, we create a dummy variable, $High$, which takes value 1 if an Exp subject won the C13 game in at least 3 of the last 4 rounds of the training sub session (rounds 5-8).

43The two tailed p-value of this coefficient in model (1) and (2) is 0.005 and 0.004 respectively. For the model (1), if we replace the dummy for “Opponent lost C13” by a dummy for the “Opponent is Inexp”, the coefficient for that latter equals to 0.304 (robust standard error 0.15), which is significant with a p-value < 0.05. Recall that the opponent’s experience level is not known to any subject, and this result reflects that Exp subjects were able to spot Inexp opponents just by their play against the computer. We discuss this further below.

44Recall from the design that only Exp subjects participated in the risky choice study. Only Exp subjects participated in the risky choice study because of budgetary constraints and because we are more interested in how and why players who understand the perfect strategy (most of the Exp players) respond to information about the opponent’s performance in the C13 part of the round.

45Assuming the expected utility framework, the sign of the risk aversion is expected to be negative because in choosing $F$ an Exp subject is choosing a lottery where, conditional on his choice being implemented, and according to his subjective belief, there is some probability that his opponent will play imperfectly as the selected second mover in $H13_{sub}$, and give him 500ECUs for the H13 part of the round, and with remaining probability his opponent will play perfectly, and give him 50ECUs for the H13 part of the round. Thus the greater a subject’s risk aversion, the lesser likelihood of him not taking the 200ECUs for sure (conditional on his choice being selected), and choosing a lottery with an expected payment dependent on his subjective belief. The loss aversion term captures how much more a subject values losses compared to gains. In our design even if a player loses in H13, he still receives 50ECUs. But if an Exp player uses the 200ECUs he could have received by winning as a reference point (Koszegi and Rabin (2006) for example use rational expectations based reference points), and considers the reduction in earnings by 150ECUs due to choosing $F$ as a “loss”, one would expect the significant decrease in the propensity to choose $F$ due to a higher loss aversion term, as observed in model (2) and (3). The sign of the risk aversion coefficient is the opposite of expected in model (3), but the coefficient is again statistically insignificant.
### Table 5. Factors Influencing Probability of choosing “First Mover”

**Notes:** These results are for rounds 9-14 of 8 sessions. The errors were clustered by subject and reported in parenthesis. Probit (1) reports the results for 48 Exp subjects, while probits (2) and (3) report the results for 47 Exp subjects (risk aversion and loss aversion data for 1 Exp subject was lost). Probit (4) reports the results for 54 Inexp subjects.

The *High* variable indicates that the Exp subject has a high level of understanding of the perfect strategy for the H13 and C13 games. We add the *High* variable and an interaction term of *High* × *Opponent Lost in C13* to the model (2) and estimate model (3). The coefficient of *High* is negative and significant (p-value of 0.003), which means that given that the opponent won the C13 part of the round, being the *High* Exp player significantly reduced the probability of choosing *F*, which agrees with the definition of *High*. The coefficient of *High* × *Opponent Lost C13* is positive and significant (p-value of 0.029). This means that being a *High* Exp type who faced an opponent who lost in the C13 part of the average round of the combined sub-session, significantly increased a subject’s likelihood of choosing *F* in that round.

Results 3(b), 4(b), 5(b) and 6(b) approximately replicate the findings from the treatment 1 results 3(a), 4(a), 5(a), and 6(a), respectively. That is, in treatment 2, we find the following: (i) the learning speed of Inexp subjects was faster; (ii) the rate of imperfect play declined significantly closer to the end of the H13 game; (iii) subjects, including trained Exp subjects, were more likely to remove 1 item from losing positions in the H13 game; and (iv) in the combined sub-session, it was ex-post optimal for an Exp subject to be the first

<table>
<thead>
<tr>
<th>Dependent Variable: Prob(First Mover)</th>
<th>Exp (1)</th>
<th>Exp (2)</th>
<th>Exp (3)</th>
<th>Inexp (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opponent Lost C13</td>
<td>0.58*** (0.2)</td>
<td>0.59*** (0.21)</td>
<td>0.15 (0.26)</td>
<td>-0.19 (0.18)</td>
</tr>
<tr>
<td>Round</td>
<td>-0.14*** (0.05)</td>
<td>-0.17*** (0.05)</td>
<td>-0.16*** (0.05)</td>
<td>-0.40*** (0.05)</td>
</tr>
<tr>
<td>Risk Aversion</td>
<td>-0.14 (0.53)</td>
<td>0.17 (0.54)</td>
<td>0.17 (0.54)</td>
<td>0.17 (0.54)</td>
</tr>
<tr>
<td>Loss Aversion</td>
<td>-0.5*** (0.22)</td>
<td>-0.67*** (0.25)</td>
<td>-0.67*** (0.25)</td>
<td>-0.67*** (0.25)</td>
</tr>
<tr>
<td>High</td>
<td>-1.32*** (0.37)</td>
<td>1.04*** (0.46)</td>
<td>1.04*** (0.46)</td>
<td>1.04*** (0.46)</td>
</tr>
<tr>
<td>High × Opp Lost C13</td>
<td>0.53 (0.67)</td>
<td>2.04 (1.32)</td>
<td>3.13*** (1.41)</td>
<td>4.54*** (0.78)</td>
</tr>
<tr>
<td>No. of Obs.</td>
<td>278</td>
<td>272</td>
<td>272</td>
<td>312</td>
</tr>
<tr>
<td>Pseudo $R^2$</td>
<td>0.1368</td>
<td>0.1940</td>
<td>0.2586</td>
<td>0.1793</td>
</tr>
<tr>
<td>Session Dummies</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
mover in H13 if the opponent lost C13. Therefore, these results are described in Appendix C.

**Result 7:** Updating opponent’s type: In the combined sub-session (rounds 9-14), Exp subjects’ accuracy while answering the questions about their opponent’s Exp/Inexp type improved after they observed more actions of their opponent. When asked the opponent’s type at the first juncture, i.e., immediately after observing the opponent’s first/second mover decision against the computer in C13, the accuracy was 45 percent. At the second juncture, immediately after observing the opponent’s history of play and the consequent outcome versus the computer in the C13 part, the accuracy increased to 62 percent. The difference in proportions is highly significant with a p-value of approximately 0.

**Model Comparison**

In this section, we use the Maximum Likelihood Estimation method to distinguish among which theoretical explanation fits our data the best. The first of the three theoretical models we consider is “Limited Foresight Equilibrium” (Rampal (2017), described in chapter 1), henceforth LFE. The LFE model allows for players to have different levels of limited foresight (foresight implies the number of subsequent stages one can observe from a given decision stage) and uncertainty about the opponent’s foresight level. The uncertainty about the opponent’s foresight level implies that the LFE also entails belief updating about the opponent’s foresight level within the play of a game.

The second model we consider is the “Quantal Response Equilibria for extensive form games” (henceforth AQRE) by McKelvey and Palfrey (1998). This model posits that each player in the game makes errors in assessing his expected payoff from the actions following any given decision node. This implies that players best respond to a given strategy profile with errors and play totally mixed strategies. The AQRE model typically assumes a logit error structure which implies that the higher the expected payoff of an action relative to

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46The only notable difference is in results 6(a) and 6(b). In particular, for treatment 2, in the whole combined sub-session (rounds 9-14), when the opponent lost C13, the average earnings of the Exp player was weakly more when the Exp player was the selected first mover in H13 compared to when he was the selected second mover. In contrast, the ex-post optimality of an Exp player being the selected first mover against an Inexp opponent in the Avoid 9 game was only true for the first two rounds of treatment 1.
other actions, the higher the probability of that action being chosen.

The third model we consider is “A Dynamic Level-k Model for Sequential Games” by Ho and Su (2013). This model posits that players are rational, however, they have subjective beliefs about the “level” of rationality of the opponent. Thus, players choose strategies that are optimal given their beliefs about the opponent’s “level”, but these strategies may not be perfect strategies.

The key difference among the Dynamic Level-k and LFE models with respect to the data is the following. The LFE explains the pattern of imperfect play observed (results 4(a) and 4(b): the rate of imperfect play declined as the number of items left reduced) using limited foresight; in contrast, the Dynamic Level-k model, which assumes rationality and relies solely on subjective beliefs to explain the data, does not allow for imperfect play in \(A9_{sub}, H13_{sub}\) or C13. This is because playing imperfectly after the first/second movers have been decided is a weakly dominated strategy, which implies that no subjective belief about the opponent’s “level” can justify such a strategy being chosen. The AQRE model is also not able to simultaneously match the pattern of \(F/S\) decisions and the proportion of imperfect play we observe in \(A9_{sub}, H13_{sub}\) and C13. This is because the expected payoff from perfect actions, which lead to a win, is higher relative to imperfect actions in \(A9_{sub}, H13_{sub}\) and C13. Therefore, the predicted probability with which imperfect actions ought to be chosen according to the AQRE model is much lower compared to the data. Thus, we find that the LFE model performs better than the Dynamic Level-k model and the AQRE model in the MLE comparison.

The Limited Foresight Equilibrium

The LFE is a limited foresight equilibrium concept (see Limited Foresight Equilibrium (Rampal (2017), or chapter 1 of this dissertation) for a detailed theoretical discussion). The LFE is applied to finite dynamic games with perfect information.\(^\text{47}\) Fix an arbitrary perfect information game \(G\). Given \(G\), and the parameters of the LFE model, the LFE generates outcome predictions for \(G\) which are testable against the data on \(G\). The LFE model

\(^{47}\)Avoid 9 and Human 13 games have one stage of imperfect information, i.e., the simultaneous moves for the \(F/S\) decision. Extending the LFE model to account for this is straightforward.
incorporates two key features: limited foresight and uncertainty about the opponent’s foresight. To model the uncertainty, the LFE model converts $G$ into a standard Bayesian game of imperfect information, say $\Gamma$, where each player in $G$ can be one of a set of possible types. The limited foresight feature is captured by the definition of a type. The type of a player denotes his foresight-level. In a multi-stage game, a player’s foresight level (or type) is equal to the number of subsequent stages that that player can observe from any given move. For example, in the Avoid 9 game, a player with foresight level 1 (or a player with type-1), can observe the decision nodes and action sets in the current stage and the next stage, but cannot observe what the subsequent stages are going to be.

For simplicity, the prior joint distribution on types is assumed to be common knowledge. We allow for one exception to this assumption, which is discussed below. Each player’s own type is private information. Given the common prior, and the game $G$, the LFE provides a strategy and belief profile for the resulting $\Gamma$. The LFE strategy profile and the common prior imply a distribution over the outcomes observed when testing $G$. Note that $\Gamma$ nests $G$ as a special case where all players have “full foresight” with probability 1. To solve for the LFE where players have limited foresight, we have to use curtailed versions of $\Gamma$. This is because if a limited foresight type does in fact have limited foresight, then he must be choosing optimal decisions on the basis of a curtailed version of $\Gamma$. We use Appendix B to explain the procedure to calculate LFE in the games used in our experiment. We now describe the LFE of the Avoid 9, Computer 13, and Human 13 games described above.

**LFE of the Avoid 9 Game**

The free parameter of the LFE model is the distribution over foresight levels. For simplicity, we model four possible levels of foresight (four types) for each player. Foresight levels 0, 1, 2, and the ex-ante full foresight type, denoted by $f$. Let $(p_0, p_1, p_2, p_f)$ be the common knowledge prior distribution over the foresight levels (or types). We describe the solution of the LFE below. As a tie-breaker, we state the LFE where every player-type plays actions that give equal payoff with equal probability. At any given decision node, let a *Position*, denoted by $P$, be a particular sum of items removed in all preceding nodes. Let a *perfect strategy* imply the actions $a_4^* = (4 - P)$ for $0 < P < 4$; $a_8^* = (8 - P)$ for $4 < P < 8$. 

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and a uniform distribution over available actions at $P \in \{0, 4, 8\}$. Recall that the perfect strategy is the weakly dominant SPNE strategy in $A_{9_{\text{sub}}}$. 

LFE of the Avoid 9 Game:

- The foresight level 0 type randomizes uniformly among available actions at the $F/S$ decision stage and for $P < 5$; at $P \geq 5$, he plays perfectly. His beliefs don’t affect strategy.

- The foresight level 1 type chooses $F$ at the $F/S$ decision stage. He randomizes uniformly among available actions for $P < 2$; at $P = 2$, he randomizes uniformly among removing 1 or 2 items. For $P > 2$, he plays perfectly. His beliefs don’t affect strategy.

- The foresight level 2 type chooses $F$ at the $F/S$ decision stage. He plays perfectly in $A_{9_{\text{sub}}}$, except that he removes 3 items at $P = 0$ if he has a positive prior belief on a type-0 opponent, i.e., if $p_0 > 0$.

- The full foresight type chooses $F$ at the $F/S$ decision stage iff $(p_0 + p_1) \geq 0.5$, else, he chooses $S$. He removes 1 item at $P = 0$ if he has a positive prior belief that the opponent has foresight level 1, i.e., if $p_1 > 0$. For $P > 0$, he plays perfectly in $A_{9_{\text{sub}}}$.

We now explain the LFE given above. At any move of the type-0 (foresight level of 0) player in the Avoid 9 game, he cannot observe the stages of the game after his action at that move. But from a given move, say at the F/S decision stage, to provide a basis to choose among the available actions, $F$ or $S$ in this case, the type-0 player must observe payoffs after each of those actions. But the Avoid 9 game doesn’t have a terminal node after the F/S decision stage. Therefore, if an action taken at a limited foresight player’s foresight horizon does not lead to a terminal node of the underlying game $G$, our limited foresight model needs to create a synthetic payoff profile for that action. We call this synthetic payoff profile as the curtailed payoff profile, as it is constructed after curtailing the underlying game after the foresight horizon of a limited foresight player. We use the following \( \frac{(\text{min} + \text{max})}{2} \) curtailment rule used in Rampal (2016) and Mantovani (2014): from each action at the
foresight horizon of each limited foresight type, the *curtailed payoff profile* observed by him provides each player with \((\min + \max) / 2\) of the set of payoffs possible for that player from that action. Applying this rule at all possible moves of the type-0 player, one can understand the LFE strategy of the type-0 player. At the first stage, both choices, \(F\) and \(S\), can lead to him being the first or the second mover, based on the opponent and Nature’s decision; further, he can win or lose in either case in the subsequent stages. Thus his *curtailed payoff* after both \(F\) and \(S\) is \(\frac{500 + 50}{2}\), which, given our tie-breaking assumption, means that the type-0 player chooses \(F\) or \(S\) with probability \(\frac{1}{2}\) each. Next, note that for any \(P < 5\), for any action from that \(P\), the set of possible payoffs is the same, and therefore the curtailed payoff profile is the same: \((275, 125)\), where we write the payoff of the selected first mover (second mover) first (second). This implies that the type-0 player randomizes among available actions if \(P < 5\). But from \(P \in \{5, 6, 7\}\), \(\alpha_8^*\) gives a curtailed payoff equal to the winner’s payoff. Thus, from any \(P \geq 5\), the type-0 player plays perfectly. Notice that the opponent’s type does not matter for the type-0’s behavior; each of his moves is the last move of the “curtailed game” he observes from that move.

Proceeding similarly we can solve for the LFE actions of each foresight level at each position. This gives us a LFE strategy for each of the four foresight levels we model here. The details of how we solve for the LFE are provided in the Appendix B. It is worth noting here that the \(F/S\) decision of only the full foresight type (type-\(f\)) player depends on his prior beliefs. That is, if the type-\(f\) player believes with high enough probability that his opponent has a foresight level of 0 or 1, then he chooses \(F\) to maximize expected payoff (assuming risk neutrality). Otherwise he chooses \(S\). Prior beliefs of each player-type are assumed to be the same as the common knowledge prior distribution over the foresight levels in all but the following case. When applying the LFE model over the data from (Exp, Inexp) pairs, we use MLE to estimate a separate distribution \(p_{\text{Exp}}\) for the Exp players, and a separate distribution for the Inexp players \(p_{\text{Inexp}}\). This is because either type was playing an opponent of a different type, and the players were informed about it. In particular, in \(p_{\text{Exp}}\), we allow the Exp players to be only foresight level-0 or full foresight type. Further, we add a parameter \(\delta \in [0, 1]\) which captures the proportion of type-\(f\) Exp subjects who have a subjective belief that his Inexp opponent is foresight level-0 or 1 with probability at
least 0.5, and therefore choose $F$ in the $F/S$ decision stage. Therefore the LFE strategy of the type-$f$ Exp player in an (Exp, Inexp) pair becomes: choose $F$ with probability $\delta$, and $S$ with probability $(1 - \delta)$; subsequently, play as per the LFE strategy given above. Although LFE uses beliefs based on the common-knowledge prior distribution, we have to use subjective beliefs here because the Exp subjects observe that the opponent is from a different Inexp population.\footnote{Our conclusions do not change even if we estimate only a single common knowledge prior distribution over the foresight levels for the (Exp, Inexp) groups and impose subjective prior beliefs to be the same as the prior distribution over foresight levels that we estimate using maximum likelihood.}

**LFE of Human 13 and Computer 13 Games.**

For the game in treatment 2, C13-H13, for simplicity, we model only four foresight levels: foresight levels 0, 1, 4 and the ex-ante full foresight player, denoted by $f$. Let $p = (p_0, p_1, p_4, p_f)$ be the common knowledge prior distribution on these foresight levels. We solve for the LFE of H13 and C13 separately. This is because after C13 ends, at the beginning of H13, except the type-$f$ player, the beliefs of the other types do not affect the LFE in H13. Therefore the history of moves in C13 does not matter for the types other than $f$. For the type-$f$ player, let the updated belief on the four possible types of the opponent be $u = (u_0, u_1, u_4, u_f)$. Note that for the Inexp subjects, who get not information about the opponent’s play in C13, $p = u$. The LFE strategy profile for the H13 game and the C13 game is given below. The details are given in Appendix B. Let a *perfect* strategy imply the actions $a_{4}^* = (4 - P)$ for $0 < P < 4$; $a_{8}^* = (8 - P)$ for $4 < P < 8$; $a_{12}^* = (12 - P)$ for $8 < P < 12$, and a uniform distribution over available actions at $P \in \{0, 4, 8, 12\}$.

**LFE of the Human 13 Game:**

- The foresight level 0 type randomizes uniformly among available actions at the $F/S$ decision stage and for $P < 9$; at $P \geq 9$, he plays perfectly. His beliefs don’t affect strategy.

- The foresight level 1 type chooses $F$ at the $F/S$ decision stage. He randomizes uniformly among available actions for $P < 6$; at $P = 6$, he randomizes uniformly...
among removing 1 or 2 items. For \( P > 6 \), he plays perfectly. His beliefs don’t affect strategy.

- The foresight level 4 type chooses \( F \) at the \( F/S \) decision stage. He plays perfectly in \( H_{13_{sub}} \). He removes 1 item from \( P = 4 \) if he has a positive belief on the opponent being type-1, i.e., if \( u_1 > 0 \).

- The full foresight type chooses \( F \) at the \( F/S \) decision stage iff \( (u_0 + u_1) \geq 0.375 \), else, he chooses \( S \). He plays perfectly in \( H_{13_{sub}} \). He removes 1 item at \( P = 4 \) if he has a positive prior belief on the opponent being type-1, i.e., if \( u_1 > 0 \).

**LFE of C13:** The LFE of C13 is identical to the LFE of H13 except that: (a) The types with foresight levels 1 and 4 randomize uniformly among \( \{F, S\} \) at the \( F/S \) stage, while the full foresight type chooses \( S \) at this stage; (b) the full foresight type and the foresight level 4 type randomize uniformly among removing 1, 2 or 3 items even when \( P = 4 \) in the C13 game.

**Maximum Likelihood Estimation using LFE.** The data analysis using MLE is done separately for the two treatments. For each treatment, an observation is the observed choices of a pair of subjects in a round. Consider an observation from treatment 1. Denote the observation \( i \) by \( o_i \). Suppose \( o_i \) can be observed if the strategy profiles in \( S^2(o_i) \ni (s_1, s_2) \) are played, where the selected first mover chooses the strategy \( s_1 \), and the selected second mover chooses the strategy \( s_2 \).

\[ \text{Prob}(o_i) = \sum_{(s_1, s_2) \in S^2(o_i)} \left[ \sum_{j,k \in \{0, 1, 2, f\}} \text{Prob}(s_1 | \text{type } j, p_j) p_j \text{Prob}(s_2 | \text{type } k, p_k) \right] \tag{9} \]

\(^{49}\) A strategy maps each history of play to an action. However, in the Avoid 9 game, beliefs play a role in determining the LFE actions at only three information sets: (i) the \( F/S \) decision of the type-\( f \); (ii) \& (iii): the choice of types 2 and \( f \) at \( P = 0 \). We take this into account in the LFE stated above for Avoid 9, H13 and C13 games.
Where $\text{Prob}(s_1 | \text{type (foresight-level)} j)$ can be calculated using the LFE of the Avoid 9 game. In some (1.4%) of the observations in treatment 1’s combined sub-session we find that subjects play imperfectly at position 5 or more (4 or lesser items left). Such outcomes have 0 probability according to LFE as even the foresight level 0 plays perfectly at position 5 or more. In treatment 2’s combined sub-session, there is no such observation. To make the likelihood function finite, we deal with the 1.4% outcomes where $\text{Prob}(o_i)$ equals to 0 by using the “uniform error rate” $\epsilon$ used by Ho and Su (2013) (the Dynamic Level-k model discussed below) and Costa-Gomes et al (2001), among others. Let the number of possible outcomes be $N$. Then $\epsilon \in (0, \frac{1}{N})$ denotes the error probability that each of the possible $N$ outcomes will occur. With remaining probability $[1 - \epsilon N]$, the model’s prediction holds. Then $\text{Prob}^{LFE}(o_i)$ in application to the data becomes:

$$\text{Prob}^{LFE}(o_i) = \epsilon + (1 - \epsilon N)\text{Prob}(o_i)$$  \hspace{1cm} (10)

Suppose each observation $o_i$ in treatment 1 has frequency $f_i$ in the data. In treatment 1, $N$ (the total number of different outcomes possible) is 447. Then the log likelihood function for the LFE model in treatment 1 is given by:

$$LogL(LFE|p) = \sum_{i=1}^{447} f_i \log(\text{Prob}^{LFE}(o_i))$$  \hspace{1cm} (11)

We can similarly calculate the log likelihood for treatment 2 data using the LFE of the H13 and C13 games, and $p_{T2} = (p_0, p_1, p_4, p_f)$. In general, maximizing the likelihood using LFE implies searching over $(p, \epsilon)$ to maximize (5). In treatment 1, we estimate one $p$ for observations with (Exp, Exp) groups and another $p'$ for observations with (Inexp, Inexp). However, for the (Exp, Inexp) pairs, as discussed above, we estimate a separate distribution $p_{Exp}$ for the Exp players, a separate distribution $p_{Inexp}$ for the Inexp players, and the parameter $\delta \in [0, 1]$ which is the probability that a type-$f$ Exp player chooses $F$ in the $F/S$ stage (and plays perfectly in the subsequent stages).\footnote{Recall that the rationale behind this is that we allow $\delta$ proportion of type-$f$ Exp players in (Exp, Inexp) pairs to have the subjective belief that the probability that the Inexp opponent’s foresight level is 0 or 1 is more than $\frac{1}{2}$.} We estimate only a single error term $\epsilon$ for all treatment 1 data.
In treatment 2’s combined sub-session, we don’t include any error term as no 0-probability outcome was observed. We estimate only one \( p_{T2} \) for the treatment 2 data as no information about the opponent’s Exp/Inexp status was provided to any subject. According to the LFE model, at the beginning of the H13 part, the distribution on the foresight levels changes based on the outcome in C13. Table 24 in Appendix B describes the change in distribution on the foresight levels due to the outcome of win (\( W \)) or loss (\( L \)) in the C13 part of a round. According to the LFE model, the Exp players observe these outcomes and update their belief. However, beliefs change the subsequent strategy in H13 only for the full foresight type (type-\( f \)) Exp player who wins in his own C13 part. Intuitively, when an Exp subject observes that the opponent lost (won) C13, he updates and puts more (less) weight on the opponent being foresight level-0 or 1. Therefore, given our estimate of the prior distribution over foresight levels, if the opponent loses (wins), then the full foresight type player chooses \( F(S) \) in H13.

**Ho and Su (2013): The Dynamic Level-k model in Sequential Move Games**

The Ho and Su (2013) (henceforth HS) model applies the Level-k model to sequential move games. The version of level-k they apply has the following properties. The level-0 strategy is to randomize uniformly among the available actions at each move. The Level-1 strategy is a sequentially rational best response to the Level-0 strategy at every move. The Level-2 strategy is a sequentially rational best response to the Level-1 strategy at every move, and so on. Given these Level strategies, rational agents pick what they deem is the optimal strategy given their subjective belief about their opponent’s Level.

In their words: “A player \( i \) chooses the optimal rule \( L_k^* \) in round \( (t + 1) \) from the rule hierarchy \( \{L_0, L_1, ..., L_S\} \) based on belief \( B^i(t) \) to maximize expected payoffs.” This modeling implies that the probability of level-0 being chosen in a game is 0. This assumption of theirs agrees with the findings of Costa-Gomes and Crawford (2006) and Crawford and Iriberri (2007a 2007b). In particular, they state that “Because all players best respond given their beliefs, \( L_0 \) will not be chosen by any player and only occurs in the minds of the higher-level players.”

In the Avoid 9 and H13 (Human 13) games: (i) the Level-0 strategy is to randomize
uniformly among all available actions at all moves; (ii) therefore the Level-1 strategy is to choose $F$ and subsequently play perfectly; (iii) the Level-$\geq 2$ strategies are all the same: choose $S$ and play perfectly. In the C13 (Computer 13) game: (i) the Level-0 strategy leads to a loss with probability $53/54$, due to uniform randomization among available actions at each move; (ii) the Level-1 strategy is to choose $S$ and then play perfectly, as there is no extra incentive to win as the first mover, and even the level-0 player plays perfectly as the second mover with probability $1/27$. (iii) The Level-2 strategy is to choose $S$ and then play perfectly, as the Level-1 strategy implies perfect play. We argue that in C13, no subject could have had subjective beliefs that the computer is following the level-0 strategy because the subjects were told that: “the computer plays perfectly to win. That is, if there is a choice or a sequence of choices that the computer can make to win against you, the computer will make that choice or sequence of choices without making any mistake.” Given that all players, including those who choose Level-1, are assumed to be rational, and therefore know how to play the weakly dominant perfect strategy, i.e., they know how to “play perfectly to win,” it is implausible that some of these players would form subjective beliefs that the computer randomizes among available actions at any given decision node.\textsuperscript{51}

Thus, a distribution over Level-1 and Level-2 strategies generates a distribution over the outcomes of the Avoid 9 game in treatment 1, and the C13-H13 round in treatment 2. We search for the distributions that maximize the likelihood of the treatment 1 and treatment 2 data separately. Note that we need only the Level-1 and Level-2 strategies because all strategies above Level-2 are the same: choose $S$ and play perfectly. The fact that the level-k model assumes that the Level-0 does not exist, implies that the Level-k model puts 0 probability on several observations of the data.

Recall that an observation is the observed choices of a pair of subjects in a round. Because of the level strategies described above, in treatment 1 or treatment 2, any observation where the selected second mover fails to play perfectly in $A_{9,sub}$ or $H_{13,sub}$ gets 0 probability according to the HS model. Further, any observation in treatment 2 where one of the subjects loses in C13 also gets 0 probability. They tackle this 0 probability problem by intro-\textsuperscript{51}Our treatment 2 MLE results for the comparison with the Ho and Su (2013) model of dynamic Level-k are robust to the scenario where subjects still chose the Level-1 strategy, and believed that the computer plays the level-0 strategy.
ducing an error probability, $\epsilon$, which is the minimum probability of each possible outcome (a sequence of choices of a pair of subjects) that can occur given the game being tested. Consider again the outcome $o_i$ which can be observed only if the selected first mover and the selected second mover choose strategies $s_1$, $s_2$ respectively such that $(s_1, s_2) \in S^2(o_i)$. Let the probability that players choose the $L_1$ strategy be $P(L_1)$, and $P(L_2) = 1 - P(L_1)$. If there are a total of $N$ (for example $N = 447$ in the Avoid 9 game) possible choice pairs, then the Level-k model’s probability that outcome $o_i$ occurs is:

$$
Prob^{L_k}(o_i) = \epsilon + (1 - \epsilon N) \sum_{(s_1, s_2) \in S^2(o_i)} \left[ \sum_{j,k \in \{1,2\}} Prob(s_1|L_j)P(L_j)Prob(s_2|L_k)P(L_k) \right]
$$

(12)

Then the likelihood function of the dynamic Level-k model is given by:

$$
LogL(\text{Level}_k|P(L_1)) = \sum_{i=1}^{N} f_i \log(Prob^{L_k}(o_i))
$$

(13)

Thus maximizing the likelihood implies searching over $P(L_1)$ and the error term $\epsilon$. The HS (2013) Level-k model allows for belief $B^i(t)$ to change as a function of rounds (captured by $t$), based on either past experiences of a player (their “closed-loop” approach) or based on mental simulation of all possible experiences (their “open-loop” approach) by a player. They do not allow beliefs to change within a round. As beliefs change so does the distribution over the chosen levels. But it is the proportions of the chosen levels, $P(L_1)$, $P(L_{\geq 2})$, that generates the distribution over observed outcomes (as $P(L_{\geq 2}) = 1 - P(L_1)$, the $P(L_1)$ estimate captures the level distribution completely). We do not use their learning model in our MLE exercise, but account for this learning using other methods.

In the MLE for treatment 1, we estimate one $P(L_1)$ for observations with (Exp, Exp) groups and another $P'(L_1)$ distribution for observations with (Inexp, Inexp). However, for the (Exp, Inexp) pairs, we estimate a separate distribution $P(L_1)_{Exp}$ for the Exp players, and a separate $P(L_1)_{Inexp}$ for the Inexp players because either type was informed that they are playing an opponent of a different type. Thus, their subjective beliefs may have been
affected by this information.\textsuperscript{52} We estimate only a single error term \( \epsilon \) for all treatment 1 data.

In the MLE for treatment 2, we allow for a different distribution over levels in each round, which is the strongest possible effect learning can have, further, we also allow the level distribution to change between the C13 and H13 parts of a round. Our conclusions do not change upon doing these robustness checks. The HS Level-k model assumes that no one chooses the level-0 strategy. Therefore, without the error term, the Level-k model puts 0 probability on the outcome in which one of the group members lost in C13, which is the case for 57 percent of the combined sub-session data. However their error term covers for the latter eventuality and allows us to perform an MLE using the HS model.

**The Agent Quantal Response Equilibrium**

The Agent Quantal Response Equilibrium (AQRE) model of McKelvey and Palfrey (1998) extends the Quantal Response Equilibrium to extensive form games. The AQRE model introduces a separate additive payoff disturbance error term to the expected payoff associated with each action for each player, from each possible move of that player. In particular, let \( \pi_{ija}(b) \) is the expected payoff of player \( i \) from playing action \( a \) at the information set \( h^i_j \), given that the behavioral strategy profile for all players is \( b \). Then the player’s “actual” payoff in their model is \( \hat{u}_{ija} = \pi_{ija}(b) + \epsilon_{ija} \). They assume that \( \epsilon_{ija} \) is i.i.d. according to type I extreme value distribution with c.d.f. \( F(\epsilon_{ija}) = e^{-e^{-\lambda \epsilon_{ija}}} \). Let \( A(h^i_j) \) be the action set at the information set \( h^i_j \). Then in an AQRE \( b \), the probability of \( i \) choosing action \( a \) at \( h^i_j \) is \( b^i_j(a) = [e^{\lambda \pi_{ija}(b)}]/[\sum_{a' \in A(h^i_j)} e^{\lambda \pi_{ija}(b)}] \). The parameter \( \lambda \) generates a certain AQRE \( b \).

The behavioral strategy profile \( b \) in turn implies a probability distribution over the observed outcomes.

In the MLE for treatment 1, we estimate one \( \lambda \) for observations with (Exp, Exp) groups and another \( \lambda' \) for observations with (Inexp, Inexp). For the (Exp, Inexp) pairs, we estimate...
a separate $\lambda_{Exp}$ for the Exp players, and a separate $\lambda_{Inexp}$ for the Inexp players. In the MLE for treatment 2, we estimate a single $\lambda$ parameter. The AQRE model does not model updating/learning, however, the probability of the outcome observed in a round of C13-H13 is a product of the probabilities of the observed outcomes in the C13 part and the H13 part of that round. Each outcome in the C13 part has a probability calculated using the win (in C13) and loss (in C13) probability, determined by $\lambda$. The effect of win or loss in C13, by model is summarized in Table 24 in Appendix B.

In terms of our earlier discussion, let $o_i$ be an observed outcome in the data, and let $f_i$ be its observed frequency. Then:

\[
Prob^{AQRE}(o_i) = \sum_{(s_1, s_2) \in S^2(o_i)} [Prob(s_1|b(\lambda))Prob(s_2|b(\lambda))Prob(b(\lambda))] \tag{14}
\]

Then the likelihood function of the AQRE model is given by:

\[
LogL(AQRE|\lambda) = \sum_{i=1}^{N} f_i \log(Prob^{AQRE}(o_i)) \tag{15}
\]

**Comparative Data Analysis of Behavioral Models**

We report MLE results of the three models discussed in the previous section with respect to data. We focus only on the data from the combined sub-sessions of the two treatments. In total 56 different types of outcomes were observed in treatment 1, and 245 types of outcomes were observed in treatment 2, not counting outcomes as different even if the sequence of moves in C13 was different.

**Treatment 1: The Avoid 9 Game**

In Table 6, we report the MLE results from the first 3 rounds of the combined sub-session in treatment 1.\footnote{This is because those are the rounds where there is a clear difference between the level of understanding of Inexp and Exp subjects, which is the focus of this chapter. The order of the three models, and the significance of the differences remains unchanged if we take the whole combined sub-session data into account.}
**Treatment 1 MLE Result** - The likelihood rankings of the three models using pairwise Vuong (1989) test is: \(LFE > **AQRE; LFE > **HS; AQRE > HS.\)

Table 6, reports the likelihood and parameters of the three different models discussed above: (i) Limited Foresight Equilibrium (LFE), (ii) The Dynamic Level-k Model in Sequential Games of HS (2013), and (iii) The AQRE model. We separately estimate the parameters for each model for the three types of pairs that were possible in the combined sub-session: (Exp, Exp), (Exp, Inexp) and (Inexp, Inexp). The LFE model has the highest likelihood for each type of pair, and therefore also overall. AQRE comes in second, and the Dynamic Level-k model is third. To investigate the statistical significance of these results, we conduct pairwise Vuong (1989) tests. The difference between the likelihood of the LFE model and the AQRE model is significant with a p-value of 0.03 and a z-statistic of 2.16. The difference between the likelihood of the LFE model compared to the Dynamic Level-k model is significant with a z-statistic of 3.15, and a p-value < 0.01. The difference between the likelihood of the AQRE model compared to the Dynamic Level-k model is statistically insignificant.

The Kawagoe and Takizawa (2012), Level-k model uses a logit error structure similar to the AQRE model. The LFE and the HS model both use error-less strategies and the same error structure. Comparing the logit Level-k model to LFE is comparing the LFE model to a level-k model with different error structures. However, if one insists on this comparison, we find that the likelihood of the logit Level-k model is \(-899.45\), which is insignificantly lower than the LFE. The Akaike Information Criterion comparison also favors the LFE model, but the Bayesian Information Criterion favors the logit level-k model.

Thus, we conclude that the results from treatment 1 points towards LFE as the weakly more likely explanation of the observed outcomes. The key driver of the difference among the likelihoods of the LFE model and the Level-k model is that without the error term, the LFE model puts zero probability on 2.2 percent of the data, whereas the Level-k model (both logit and HS) puts zero probability on 13.4 percent of the data. This is because the latter model can’t explain imperfect play after the first/second mover has been decided,

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54For the whole combined sub-session the likelihoods are -2245, -2290, and -2294 for the LFE, HS and AQRE models. The LFE model has a significantly higher likelihood (p-value < 0.01).

55We don’t use the logit error term for the LFE model because for the logit model it doesn’t just matter which action gives more expected payoff. The amount of difference also matters. Thus the logit prediction for the LFE model is not robust to the “curtailment rule” used.
<table>
<thead>
<tr>
<th>Model</th>
<th>Inexp Inexp</th>
<th>Inexp Exp</th>
<th>Exp Exp</th>
<th>Ln(L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LFE</td>
<td>-260.9</td>
<td>-487.8</td>
<td>-141.9</td>
<td>-890.6</td>
</tr>
<tr>
<td>$P(0, 1, 2, f)$</td>
<td>(.25, .23, .31, .2)</td>
<td>Exp: (.05, 0, 0, .95); $\delta = .31$</td>
<td>Inexp: (.48, 0, .32, .2)</td>
<td>(0, 0, .05, .95)</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>8 × 10^{-5}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level-k</td>
<td>-285.3</td>
<td>-503.4</td>
<td>-148.3</td>
<td>-937.1</td>
</tr>
<tr>
<td>$P(L_1)$</td>
<td>0.65</td>
<td>Exp: 0.31; Inexp: 0.60</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>$\epsilon$</td>
<td></td>
<td></td>
<td>3 × 10^{-4}</td>
<td></td>
</tr>
<tr>
<td>AQRE</td>
<td>-272.8</td>
<td>-505.3</td>
<td>-143.8</td>
<td>-921.9</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.015</td>
<td>Exp: 0.017; Inexp: 0.019</td>
<td>0.028</td>
<td></td>
</tr>
</tbody>
</table>

Table 6. Relative MLE Performance of Theoretical Models

Notes: The table reports the log likelihood and parameters of the LFE, HS Level-k, and AQRE models for the first three rounds of the combined sub-session. The sample size is 231.

without an error term.

**Treatment 2**

There are 1591 different sequences of moves possible in the H13 game alone. Therefore, the total number of different sequences of moves possible in a round of treatment 2 is more than 10 million. Thus, to give the Ho and Su model a fair chance (which has an error term where each sequence must have the same minimum error probability of occurring), and for analytical tractability, we divide the observations from treatment 2 into 81 broad categories. The details of this categorization are in Appendix B. We categorize the observations on the following basis: (i) Exp/Inexp combination of a pair; (ii) the outcomes of a pair in their respective C13 game against the computer (iii) perfect/imperfect play by the selected first and second movers in H13.

**Treatment 2 MLE Result** The *likelihood rankings of the three models using pairwise Vuong (1989) test* is: LFE >** AQRE; LFE >** HS; HS >** AQRE.

Table 7 states the MLE results and parameters for each of LFE, AQRE, and HS models with respect to the data from the combined sub-session of treatment 2 (rounds 9-14). The LFE model has the highest likelihood, followed by the Dynamic Level-k model, which in turn has a significantly higher likelihood than the AQRE model.

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56No such division was done for treatment 1 as the number of possible outcomes was only 447.
Table 7. MLE Results for Treatment 2
Notes: The table reports the log likelihood and parameters of the LFE, HS Level-k, and AQRE models for the combined sub-session. The sample size is 295.

Vuong’s test for comparing the likelihoods of non-nested models shows that the likelihood of the LFE is significantly greater than the likelihood of the Level-k model (test statistic 7.95, \( p - value \approx 0 \)) or the AQRE model (test statistic 12.06, \( p - value \approx 0 \)). With a test-statistic of 7.95, the likelihood of the Dynamic Level-k model is also significantly greater than the likelihood of the AQRE model with a p-value of approximately 0. Note that the results in Table 7 are for the LFE model without an error term. While for the Dynamic Level-k model of Ho and Su (2013), 57% of the data would have 0 probability sans the error term.

The Ho and Su (2013) model allows for changes in level distribution across repetitions of a game. This level distribution changes because of changes in beliefs. One can argue that the two parts of the C13-H13 round are two repetitions of a very similar game and that the Exp subjects could have formed different subjective belief based on observing the opponent’s play in C13. Therefore in the results given in Table 7, we have allowed for players to choose different levels in H13 based on the information they get from their opponent’s behavior in C13. In C13, all players choose Level-2 or above because they are rational and they are explained that the computer plays perfectly (the MLE results on Ho and Su (2013) and resulting comparisons do not change if we allow subjects to choose the Level-1 strategy in C13). After C13 ends, one of three cases happen (i) an Exp subject observes his opponent lost in C13 (ii) an Exp subject observes his opponent won in C13 (iii) a subject observes nothing. Case (i) implies that the HS model puts 0 probability on that observation. We allow players to mix differently between Level-1 and Level-2 (the two possible levels) in H13 for cases (ii) and (iii).
As a robustness check, in the HS model, we allow for learning across rounds, i.e., we allow for a separate distribution on levels (and separate additional parameters), in each of rounds 9 through 14. The log likelihood of the data as per the sequential level-k model with such a specification is -993.96, which doesn’t change our conclusions.\(^5\) As a last robustness check we try the KT model of Level-k which uses a logit error structure. The likelihood of the KT model is -904.68, which is also significantly lesser (z-statistic of 3.55, and a p-value of 0.0004) than the likelihood of the LFE.\(^5\)

**Discussion**

The data suggests that in the first treatment, 13.4 percent of the times, subjects played imperfectly after the first and the second mover had been decided. Thus, most deviations from the perfect strategy occur at the \(F/S\) decision stage, which can also be accounted for by the Level-k model, which uses the lack of iterative reasoning among subjects as the reason for this finding. The MLE results favor the LFE model, which puts 0 probability on only 2.2 percent of the data without the error term, as opposed to 13.4 for the Level-k models. However, the logit Level-k model is a very close competitor, even though the comparison among different error structures is arguably inappropriate. Thus, the treatment 1 data does not show a high enough incidence of a failure to play the weakly dominated strategy in \(A_{9,sub}\) to conclusively distinguish among models.

The data from the second treatment shows that there was a high incidence of limited foresight, as 41.4 percent of subjects lost their interaction with the computer in C13 when there was no incentive to do so, and it was a weakly dominated strategy (given that it was announced and explained that the computer plays perfectly, playing imperfectly in C13 was

\(^5\)We also test a Cognitive Hierarchy (Camerer, Ho and Chong (2004)) model adapted to sequential move games. Their model includes the empirical existence of a level-0 player who randomizes uniformly. Further, they constrain beliefs to be “partial rational expectations” using prior beliefs. As a robustness check we relax this condition and search over the distribution of possible Levels that maximizes likelihood. The resulting maximum likelihood is -873.56. The LFE model has a significantly higher likelihood (z-statistic of 2.91 and p-value of 0.004) than even this “free” distribution cognitive hierarchy model. The difference becomes higher if we impose the “partial rational expectations” condition.

\(^5\)This result is for the case where subjects cannot choose Level-1 in C13 (because of them being rational and it being explained to them that the computer plays perfectly), however all subjects are allowed to switch to any level once C13 ends and before H13 begins. Even if 13% of the Inexp subjects are allowed to choose Level-1 in C13 (that is, allowed to hold beliefs that the computer randomizes uniformly) in the KT model, its log-likelihood is -863.9, which is significantly lower, with a z-statistic of 2.01 (two tailed p-value of 0.045) than the log likelihood of the LFE model.
a strictly dominated strategy). Thus, limited foresight and uncertainty about the opponent’s foresight played a primary role in generating the data in treatment 2. The MLE results indicate that the LFE model fits the treatment 2 data the best.

Another indicator of limited foresight is the comparison between the percentage of cases in which imperfect play was observed in $A_{9_{sub}}$ with the percentage of such cases in the similar, but longer, $H_{13_{sub}}$. For the $A_{9_{sub}}$ this percentage was 10.57 percent, for the latter it increased to 24.23 percent, which is a significant increase with a p-value of approximately 0. Further, there also seems to be evidence of uncertainty about the opponent’s foresight. Notice that the LFE implies that at $P = 0$ in the Avoid 9 game, the type-$f$ player removes 1 item if he believes that an opponent with foresight level of 1 exists. And similarly at $P = 4$ in the H13 game, the type-$f$ player removes 1 item for the same belief. Now notice that according to the MLE estimates of LFE, the type-$f$ is the most likely type in both treatments 1 and 2. Thus, the LFE can partially explain results 5(a) and 5(b) which show that players choose to remove 1 item from losing positions. According to LFE, the uncertainty about the opponent’s type is what makes the type-$f$ player remove 1 item, as this maximizes the probability of the opponent making a mistake.

**Conclusion**

We report results from an experiment using a sequential move “winner take all” game which we constructed by adding a First/Second mover decision stage to a “race game”. In the game we constructed, one can attain a prize of 200 ECUs by choosing a sure-shot winning strategy. One can attain a higher prize in this game by winning from a losing position. Winning from a losing position is possible only if the opponent makes a mistake and doesn’t choose his weakly dominant strategy. The results in treatment 1 indicate that experienced subjects, who understand how to win the “winner take all” game, are more likely to risk losing to try to attain the higher prize when they are told that their opponent is inexperienced. In treatment 2 we replace the exogenous information about the opponent’s experience level with information about the play of the opponent in a closely related zero-sum race game against the computer. We found that if the experienced subjects were shown that their opponent
lost the race game versus the computer, then the experienced subjects were more likely to risk losing to attain the higher prize in the “winner take all” game against that opponent. The results in treatment 2 indicate that experienced subjects become better at guessing their opponent’s level of experience after observing more moves of the opponent.

These findings point to a systematic failure of SPNE within the paradigm of selfish and rational behavior. We can be sure that the behavior paradigm is indeed selfish because the game is “winner-take-all.” We know that experienced players are acting rationally in choosing to deviate from the SPNE strategy, and risking a loss, because we can observe that they converge to SPNE behavior and play the “sure-win” strategy when playing another experienced opponent. Further, even if one argues that subjects being told their opponent’s experience-level is not very relevant to dynamic games in the real world, the finding that subjects observe past behavior and infer the experience-level of the opponent and then act on this inference suggests a need for theoretical investigation of the data reported here.

We test the relative performance of three models in explaining this data: (a) the Limited Foresight Equilibrium (LFE) (b) the Agent Quantal Response Equilibrium for extensive form games (AQRE), and (c) the Dynamic Level-k Model for Sequential Games (Ho and Su (2013)). Comparing the likelihoods of these models with respect to the data, we find that the data from both treatments favors the LFE explanation. The reason for this finding is that the Level-k model, without errors, puts zero probability on weakly dominated play. But weakly dominated play is observed in 13.4 percent of the treatment 1 data and 57 percent of treatment 2 data. In contrast, the LFE model, which explains weakly dominated play in dynamic games using a model of limited foresight and uncertainty about the opponent’s foresight, explains the observed patterns of play well.
Chapter 3: Loss Aversion and Willingness to Pay for New Products

Co-authored with Professor A. Banerji

This chapter reports and models the discrepancy between the full bidding and endow and upgrade findings from a willingness-to-pay (WTP) elicitation Becker-Degroot-Marschak (BDM) experiment for an improved food, conducted in rural India. We found that the distribution of the WTP for exchanging 1kg local pearl millet (LPM) for 1 kg of biofortified high-iron pearl millet (HIPM) first-order stochastically dominated the distribution of the difference between the WTPs for 1kg HIPM and 1kg LPM. Thus the data (i) rejects preferences that are standard or have status quo reference points, in favor of an expectations-based reference dependence model of loss aversion for the new product; and (ii) is used to identify and estimate the loss aversion parameter and latent consumer valuations for HIPM in the consumer model. These point to a significant downward bias in conventional WTP estimates of HIPM using the BDM procedure, suggesting caution when one is using standard incentive compatible mechanisms for value elicitation.\textsuperscript{59}

\textbf{Introduction}

This chapter reports, and provides an explanation and model for the discrepancy between the full bidding and endow and upgrade findings from a willingness-to-pay (WTP) elicitation

\textsuperscript{59}We are grateful to HarvestPlus, Bhushana Karandikar, Binu Cherian and students from the University of Pune for helping to implement the experiment; to Deepti Goel and J.V. Meenakshi for useful discussions; and to Ekin Birol for valuable inputs in the experimental design and analysis.
tion Becker-Degroot-Marschak (BDM) experiment conducted in rural Maharashtra, India. The objective was to understand consumers’ valuations of an improved food (biofortified high-iron pearl millet, or HIPM for short) in comparison with their valuations of locally available pearl millet (LPM), at a time when the HIPM was not yet available on the market. Our explanation of the discrepancy is based on consumer loss aversion for the novel HIPM, relative to expectations-based reference points, and optimal bidding by such loss averse consumers in a BDM mechanism.

Estimation of consumers’ valuations for new products using incentive compatible mechanisms is a fairly common practice in economics. Colson and Rousu (2013) summarize findings from 100 studies eliciting WTP for genetically-modified (GM) foods in over 20 countries. Lusk and Shogren (2007) show that more than a hundred academic studies have utilized experimental auctions for the purpose of preference elicitation. However, the presence of consumer loss aversion for a novel product, relative to expectations-based reference points, as evidenced by this chapter, suggests caution while interpreting reported values as true values.

Biofortification is a strategy of significantly enhancing micronutrient concentration in staple food crops using conventional breeding techniques, with the aim of helping to eliminate micronutrient deficiencies in vulnerable populations in developing countries. This study was conducted in 2012 in three major pearl millet growing districts of Maharashtra, the second-largest pearl millet producing state in India. The sample consisted of farmers or household members who were producers and consumers of pearl millet; the HIPM used was provided by HarvestPlus, and was developed by HarvestPlus in partnership with the International Crop Research Institute for the Semi-Arid Tropics (ICRISAT).

The BDM experiment was conducted over two rounds. In the first round, we elicited the WTP for 1 kg bags of biofortified high-iron pearl millet (HIPM) and local pearl millet (LPM) without informing participants about the identities of the two pearl millets or of the notion of a biofortified pearl millet. If a participant won the HIPM in the first round

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60 India has a large segment of population that is affected by iron-deficiency anemia (e.g. Gragnaloti et al. (2005) estimates that 75 percent of children in 2005 were anemic).

61 HarvestPlus (www.harvestplus.org) is the global leader in biofortification and a part of the CGIAR Research Program on Agriculture for Nutrition and Health (A4NH, http://www.a4nh.cgiar.org/).
he or she exited the experiment; otherwise they proceeded to round 2. Before round 2, participants watched an “infomercial,” which communicated the benefits of adequate iron in the diet, the nature of iron-deficiency anemia, and the existence of a new, high-iron form of pearl millet, branded and certified by an international agency.

For round 2, the identities of the two pearl millet varieties were distinguished by labeling the HIPM with the brand of the international agency mentioned in the infomercial. The participants who had won nothing in round 1 participated in a BDM WTP elicitation where they bid separately for LPM and HIPM, as they had done in round 1. This constituted Treatment 1. The participants who had won the LPM in round 1 participated in a BDM WTP elicitation where they bid to exchange the 1 kg of LPM they had won in round 1 for 1 kg of the HIPM. This was Treatment 2.

The BDM mechanism is incentive compatible, so it is optimal for participants with standard preferences to bid their true values/WTP. However, contrary to standard consumer theory we find that the distribution of the reported bids for exchanging 1 kg of LPM for 1 kg of HIPM in Treatment 2, first-order stochastically dominates the distribution of the difference between the reported WTPs for HIPM and LPM in Treatment 1. For short, we term the difference in HIPM and LPM bids as the bid premium for HIPM over LPM.

The experiment also included an additional arm to provide a robustness check and a different randomized allocation to the treatments. Thus initially, participants were randomly assigned to one of two groups, $A$ or $B$. Group $A$ participants participated in the experiment as described so far, getting subdivided into Treatments 1 and 2 following the first BDM round as above. Group $B$ participants were first provided information through the infomercial, and then participated in BDM elicitation, bidding separately for LPM and HIPM. There was only one round of bidding. Group $B$ thus provides a different estimate for the bid premium to compare with the endow and upgrade bids from Treatment 2. The Treatment 2 distribution stochastically dominates the distribution of the premium from Group $B$ as well.

Loss aversion theory with status quo reference points (eg. Kahneman, Knetsch and Thaler (1991), Tversky and Kahneman (1991)) is also contrary to our findings. That literature would expect the inequality to be the reverse, as subjects in Treatment 2, possessing the LPM, would be loss averse with respect to it, and thus be less willing to exchange it for
HIPM. We also show that a variety of possible confounds cannot explain the findings.

To explain the data anomaly, we use an adaptation of the expectations-based reference dependent loss aversion model of Koszegi and Rabin (2006) to the context of a BDM mechanism.\textsuperscript{62} Thus this chapter follows an active recent literature modeling reference dependence in auction settings, including Lange and Ratan (2010), Ratan (2014), Banerji and Gupta (2014), Ehrhart and Ott (2014) and Rosato (2014). In particular, we provide a loss aversion parameter with respect to the loss experienced upon not being able to purchase the HIPM, as the HIPM is a novel and beneficial form of pearl millet.

In this model, the optimal bid for HIPM depends on the agent’s value for it, the loss aversion parameter, and the support of the BDM price distribution. For consumer values for HIPM that are relatively low compared to the BDM support, bids are shaded below value; while for values which are high compared to the BDM support, bids are marked up above the values. Intuitively, for the former case, a subject bids lower than value to minimize her expectation of winning the HIPM, in order to minimize losses relative to the expectation if she does not win. For the latter case, in which a subject’s value is high relative to the BDM distribution, she bids higher than value to reduce the probability of expecting to purchase the highly-valued product but not being able to.

The experiment implies different reference points in the two treatments, and the model’s prediction is consistent with the data. Suppose an agent informed about the benefits of HIPM has values $v_H$, $v_L$ for the bags of HIPM and LPM respectively, and $v_E$ for exchanging a bag of LPM for one of HIPM. Then $v_H - v_L = v_E$, but for a large range of such values, the difference in bids for HIPM and LPM is less than the exchange bid. Loss aversion for HIPM leads to shading of bids relative to the values $v_H$ and $v_E$ (over a large interval of values), but no shading for the locally available LPM.\textsuperscript{63} While the $b_H$ for HIPM is a markdown of the entire value $v_H$, $b_E$ is only a shading of the incremental value $v_E$. Thus the difference in the full bids $b_H - b_L$ for HIPM and LPM can be squeezed below the exchange bid $b_E$. Moreover, in Treatment 1, an expectations-based reference point has the possibility of the agent winning the LPM when she expects to win the HIPM: the additional loss sensation

\textsuperscript{62}See also Shalev (2000) which presents a related idea in a game-theoretic context.

\textsuperscript{63}In fact, we only require that the consumer be more loss averse for HIPM than for LPM.
from this is minimized, for relatively low value agents, by shading their HIPM bids further.

We use the discrepancy between the full bidding and exchange bidding treatments to identify a loss aversion parameter in the data, and use it to identify the latent distribution of HIPM. This structural estimation suggests that the conventional estimate of average WTP for HIPM (that assumes BDM bids equal WTP) is biased downward by 12 percent.

**Related Literature**

The way the exchange bids (Treatment 2) and differences in full bids (Treatment 1) compare in our data is somewhat related to the findings of Elbakidze and Nayga (2015) and Corrigan and Rousu (2006) from their tests that adapt the “adding up test;” these papers also use the BDM mechanism. The “adding up test,” (in addition to a scope sensitivity test), was used by Diamond and Hausman (1994), Diamond (1996), and Hausman (2012) to evaluate contingent valuation and conclude that “contingent valuation is hopeless.” Elbakidze and Nayga (2015) tested if the WTP for two units of an item was equal to the sum of the WTP for the first unit and the WTP for the second unit. In one treatment, after eliciting the WTP for the first unit, they provided the first unit for free. And in another treatment the participants paid an amount equal to the BDM price drawn for the first unit. The WTP for the second unit was higher for the participants who got the first unit for free as compared to the subjects who had to pay for the first unit. The authors gave two possible explanations for this result: first, reciprocity effect, in that subjects wanted to reciprocate for receiving the first item for free by paying more for the second unit, or second, income effect, in that by paying for the first item subjects had suffered a reduction in their income set aside for activities like spending on experiments and therefore were willing to pay less for the second unit. Moreover, in both treatments, the sum of the two WTPs for the two units was greater than the WTP for two units of the good. Thus even taking into account lower income from paying for the first item, the WTP elicitation could not pass the adding up test. Corrigan and Rousu (2006) found that the WTP to pay for an additional unit of a good, after being endowed with the first unit for free, was 75 percent higher than the difference in WTP for two units minus the WTP for the first unit. They also attributed the result to a reciprocity effect.
Our design and objective are different: we do not vary the quantity of the good; we compare the WTP for an additional attribute, higher iron content, measured through the full bidding approach versus the endow-and-upgrade approach. Additionally, the participants pay the BDM price drawn for the LPM if they do win it. Thus, our endow and upgrade design is also different from the traditional method of actually endowing the subject with an object for free. This removes the reciprocity motive for higher bidding for exchanging LPM for HIPM. The variation across participants in LPM bid minus the BDM price paid also enables us to test if the income effect is causing the exchange bids to be higher; we find this is not the case (Table 11). Lusk et al (2004) use the "free endow" method and measure the WTP to exchange an generic steak (provided as free endowment) for 4 types of higher quality steak in 4 between subject treatments. They compare the WTP to exchange with the appropriate difference of the WTPs of the generic and the higher quality steak. Similar to our findings, they report a significant "reverse endowment effect" in the Vickrey auction, i.e. subjects in possession of the generic steak value it comparatively lesser when in possession of it, than when they have to bid for it. The BDM bids also displayed a negative, but insignificant, endowment effect. They do not explore the reason for this effect.

Several studies have analyzed the performance of the BDM mechanism as a value elicitation mechanism: (Irwin et al (1998), Noussair, Robin, and Ruffieux (2004), Lusk et al (2004), Rutstrom (1997)); but since they use induced values they are not comparable to settings that have unobserved consumer values for commodities, such as ours. A few papers have looked at how bids relate to the BDM distribution, in different contexts (Bohm et al. (1997), Lusk et al (2007), Banerji and Gupta (2014), Urbancic (2011)). Lusk et al (2007) analyzed how the distribution of the BDM affects the shape of the payoff function and the cost of deviating from truthful bidding.

There is a fast-growing literature on experimental evidence for expectations-based reference points, much of it based on manipulating such reference points in a variety of decision-
making (as well as a few game-theoretic) contexts, such as exchange, real effort, consumer choice and auction experiments. Ericson and Fuster (2011), Abeler, Falk, Goette and Huffman (2011), Crawford and Meng (2011), Gill and Prowse (2012), Bartling, Brandes and Schunk (2015), Karle, Kirchsteiger and Peitz (2015) and Rosato and Tymula (2016) find evidence in favor of expectations as a source of reference dependence. On the other hand, several papers have found that not all predictions of Koszegi and Rabin (2006) square up to lab evidence, or that alternative explanations/models may explain some data regularities better. These include Heffetz and List (2014), Goette, Harms and Sprenger (2014), Wenner (2015) and Gneezy, Goette, Sprenger and Zimmermann (2016).

**Experiment and Evidence**

**The Field Experiment**

We use data from a framed field experiment that was conducted to study consumer acceptance of a new variety of pearl millet among consumers in rural Maharashtra, a state in western India. At the core of the experiment is the use of the well known Becker-de Groot-Marschak (1964; henceforth BDM) mechanism to elicit a consumer’s true value or willingness to pay for this new variety.

Twelve central locations were chosen across three major pearl millet growing districts; two villages were randomly selected from a radius of 10 kilometers around each location; from each village, 9 or 10 households were randomly selected; one participant was selected from each household. The selection, and invitation to participate, were made on the day before the experiment was conducted at the corresponding central location close to the village. Participants were told that they would have the opportunity to taste, evaluate and possibly purchase 1 kg of some variety of pearl millet. They were advised to carry adequate cash to be able to avail of the purchase opportunity.

Participants in the experiment were randomly assigned to 2 groups, $A$ and $B$.\footnote{Group $B$ had about half the number of participants as Group $A$. This was done to make the number of participants more comparable across Treatments 1, 2 and Group $B$.

At each central location, all participants filled up a short socioeconomic survey. Apart from
information on demographics (including that on production and consumption of pearl millet), the survey had questions on prior awareness of the need for iron in the diet and of iron-deficiency anemia, as well as several other information areas.

Group A participants then evaluated two varieties of pearl millet for sensory traits, following the food science literature (Tomlins et al (2007)): grain traits (color and size), and the taste, color and other characteristics of the bhakri (the form of bread in which pearl millet is consumed) made from the two varieties. They assigned a score between 1 and 5 on a Likert scale for each characteristic, as well as an overall score, for each of the two varieties. One variety was the local pearl millet available in shops; the other was the new variety whose consumer acceptance motivated the study. At this point, participants did not know that this was a new variety not available on the market, and the visible traits of the two varieties were close enough for the the new one not to appear unusual.

Following this, a participant’s willingness to pay (WTP) for the two different varieties was elicited by applying the BDM mechanism. Participants were introduced and trained in this mechanism (see the instructions in the Appendix C). The training was standard and included explaining the incentive to optimally bid one’s value or WTP; note though that this optimality may not hold outside of “classical” preferences. Following the training, the participant put down his or her BDM bid for each variety. One of the two varieties was then randomly chosen as the “binding” one, and a sale price for 1 kg of this variety was randomly drawn from a uniform distribution between INR 5 and INR 30. If the participant’s bid for the variety was greater than or equal to this sale price, he or she “won” and purchased the grain, paying the randomly determined sale price; else the participant did not win the grain.

We term the above part of the experiment on Group A as round 1. Subsequently, the participants who obtained the new variety left the venue. Those that won the local variety (henceforth LPM or local pearl millet), or did not win anything, were requested to stay on and watch an infomercial video. This infomercial explained the importance of sufficient iron in the diet, including its importance for vulnerable household members (e.g. women of child-bearing age). It explained that compared to the LPM, the new variety could provide the household significantly higher levels of iron, and was branded and certified by an international agency. The new variety was in fact a “high-iron pearl millet” (HIPM) variety.
developed through conventional breeding techniques together known as biofortification.

Following the infomercial, there was another BDM round (round 2). In this round, the HIPM was labeled as a brand of the international agency. Participants that had not won anything bid once more for the two varieties in the same BDM setting as in round 1. This was Treatment 1. Those participants that, on the other hand, had won and purchased the LPM were asked in this BDM round to bid to exchange their LPM with a 1 kg bag of the biofortified HIPM; after they bid, a sale price was drawn randomly from the interval of INR 0 to INR 20 and if the bid was at least as high as the sale price, they exchanged their LPM for a 1 kg bag of HIPM, and paid the realized sale price. This was Treatment 2.

Group B participants watched the infomercial first, and then they evaluated the two pearl millet varieties for sensory characteristics; this was followed by BDM elicitation of bids for the LPM and the labeled HIPM variety.

**The Evidence**

The standard benchmark to compare results with is this. Consider an individual whose willingness to pay, or value, for the 1 kg bag of LPM is \( v_L \). Let \( V(q, w), q = 0, H, L \) be her utility if she has wealth \( w \) and gets 1 unit of \( H \) or \( L \) or none of either at the experiment. So,

\[
V(0, w) = V(L, w − v_L)
\]  

(16)

Following the infomercial, for an individual who did not win any pearl millet, his or her willingness to pay \( v_H \) for HIPM in round 2 satisfies

\[
V(0, w) = V(H, w − v_H)
\]  

(17)

On the other hand, if the individual won and purchased LPM in round 1, at a price \( p_L \), then following the infomercial his/her willingness to pay \( v_E \) to exchange the LPM bag of pearl millet for an HIPM bag satisfies

\[
V(L, w − p_L) = V(H, w − p_L − v_E)
\]  

(18)
Since the expenditure on 1 k.g. of pearl millet, at about INR 15-18, is very small relative to wealth, even for the poorest households in the sample, it would be a negligible part of annual income. Therefore, the willingness to pay for it may not be significantly affected by small changes in wealth; then the $v_E$ in equation 9 would be the same if wealth equaled $w - v_L$ rather than $w - p_L$. Substituting $v_L$ for $p_L$ in equation (18), it would follow from equations (16) and (17) that

$$v_H = v_L + v_E$$

With quasilinear utility ($V(q, w) = u(q) + w$), the result $v_H = v_L + v_E$ follows directly from equations (16) - (18). The intuition is straightforward in the absence of wealth effects: if an agent’s willingness to pay for LPM is $v_L$, and her willingness to pay for HIPM, after watching the infomercial is $v_H$, then upon endowing her with a 1 k.g. bag of the LPM, she is willing to pay the difference, $v_H - v_L$ to exchange the LPM bag for an HIPM bag. So, $v_H - v_L = v_E$.

Due to the incentive compatibility of the BDM mechanism in the presence of standard preferences, the benchmark case is then that a participant’s bid for LPM, $b_L$ equals $v_L$; his bid for HIPM in round 2, following the infomercial, $b_H$ equals $v_H$, and the post-infomercial exchange bid $b_E$ equals $v_E$. Therefore, $b_H - b_L = b_E$.

We test the equality $b_H - b_L = b_E$ for group A of our experiment by comparing the (round 2, post-infomercial) $b_H, b_L$ bids from Treatment 1 with the $b_E$ bids from Treatment 2. We compare (i) the mean and (ii) the distributions of $b_H - b_L$ from Treatment 1 with the those for $b_E$ from Treatment 2. This amounts to a comparison across the treatments of the premia that people are willing to pay for the HIPM relative to the LPM. With standard preferences, negligible wealth effects, and successful randomized allocation to treatment, both these comparisons should give insignificant differences.

The data rejects equality of $b_H - b_L$ (Treatment 1 (round 2)) and $b_E$ (Treatment 2 (round 2)). The Kolmogorov-Smirnov test statistic of 0.2607 (P-value 0.003) instead favors the alternate of $b_E$ stochastically dominating $b_H - b_L$. The stochastic dominance over a large interval (possibly excluding very high premia) is evident from Figure 10. Table 8
<table>
<thead>
<tr>
<th>Treatment</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
<th>Sample Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment 1</td>
<td>4.51</td>
<td>3.30</td>
<td>-5</td>
<td>15</td>
<td>101</td>
</tr>
<tr>
<td>Treatment 2</td>
<td>6.87</td>
<td>3.93</td>
<td>0</td>
<td>16</td>
<td>69</td>
</tr>
</tbody>
</table>

Table 8. Summary: HIPM-LPM Premia across Treatments

Notes: (i) Treatment 1 premium: HIPM bid - LPM bid in round 2 \((b_H - b_L)\). Treatment 2 premium: Exchange bid \(b_E\). (ii) t-statistic for difference in means = 4.10. (iii) 1 observation of \(b_E = 0\) (possible censoring); adjusting for this does not affect the rejection of equality of means.

indicates that the mean of \(b_E\) is significantly higher than that of \(b_H - b_L\). We discuss the comparison of Treatment 2 and Group B later in this section.

The randomized allocation of participants to the two Treatments appears to have worked. The socioeconomic characteristics of landownership, years of schooling, and proportion of women are not significantly different across Treatments; however, Treatment 2 participants are 3 years older on average (significant at 10 percent; Table 9). But as illustrated in the regression in Table 11, for instance, variations in these characteristics do not explain any of the variation in stated WTP. Most importantly for the randomization, in round 1, prior to the infomercial, the \(b_H - b_L\) premium is not significantly different across the Treatments.

In the remainder of this section, we provide evidence that the allocation to treatment is not compromised by any confounding factors, and rule out several potentially competing explanations.

Table 9 shows that the socioeconomic characteristics of landownership, years of schooling, and proportion of women are not significantly different across Treatments 1, 2 and Group B. While it is true that Treatment 2 participants are 3 years older on average than those in Treatment 1, a regression below (see Table 11) illustrates that variations in socioeconomic characteristics (such as age) do not explain any of the variation in stated WTP. All other comparisons of means between Treatment 1 and 2, and Group B and Treatment 2, are insignificant anyway; so the assignment to the treatments resulted in similar subject characteristics across them.

As mentioned earlier, the \(b_H - b_L\) premium from Group B will provide an alternative comparison (relative to Treatment 1), with the exchange bids in Treatment 2; and it will be shown that the data anomaly remains. So, we consider Treatment 1 versus Treatment 2
Next, since this chapter compares the premia for HIPM relative to LPM in the two treatments 1 and 2, we check whether this premium \((b_H - b_L)\) differs across participants in these treatments in round 1 (prior to the infomercial). It’s apparent from the baseline comparison in a difference-in-differences regression that this is not the case. The dependent variable is this premium, for Treatment 1 (both rounds) and Treatment 2, round 1; while for Treatment 2, round 2, it is the equivalent, exchange bid \(b_E\). We have the regression:
<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>std. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.79</td>
<td>0.02***</td>
</tr>
<tr>
<td>Round 2</td>
<td>3.71</td>
<td>0.00***</td>
</tr>
<tr>
<td>Treatment 2</td>
<td>-0.04</td>
<td>0.94</td>
</tr>
<tr>
<td>Round 2 * Treatment 2</td>
<td>2.40</td>
<td>0.00***</td>
</tr>
</tbody>
</table>

Table 10. HIPM-LPM Bid Differences: Difference-in-Differences Regression

Notes: Significance: *: 10 percent. **: 5 percent. ***: 1 percent. The dependent variable is HIPM-LPM bid difference $b_H - b_L$ for Round 1 and Treatment 1, Round 2; and HIPM exchange bid, $b_E$ for Treatment 2, Round 2.

\[
\text{premium} = \beta_1 + \beta_2 \times \text{Round 2} + \beta_3 \times \text{Treatment 2} + \beta_4 \times \text{interaction} \tag{20}
\]

The coefficient on Treatment 2 (Table 10 below) is the estimated difference in premia in round 1, across the treatments. It is very small and insignificant. The regression estimates also summarize the information that HIPM is valued higher than LPM in the absence of information (constant term); that information on HIPM adds INR 3.71 to the premium in Treatment 1, and an additional INR 2.4 in Treatment 2.

Third, we check whether Treatment 2 $b_E$ bids are significantly higher compared to Treatment 1 $b_H - b_L$ premia, owing to the presence of a wealth effect. The subjects in Treatment 2 typically paid a price $p_L$ for LPM that was less than their (Round 1) bid $b_L$ for it. This bid equals $v_L$, their value or WTP for LPM in the classical interpretation; on this interpretation, $b_L - p_L$ is the compensating variation. The discussion around equation 4 above suggests that the attendant wealth effects may be very small; however, the literature has several studies that argue that in an experimental context, people may have a higher propensity to spend out of windfall income (the evidence is mixed: For example, Thaler and Johnson (1990), Harrison (2006) have provided evidence of such a “house money” effect, whereas other studies like Clark (2002), Weber and Zuchel (2006) have provided evidence for the lack of such an effect). We examine whether such a “house money” effect may explain part of the Treatment effect.

In particular, we examine whether variations in the $b_L - p_L$ wealth term for Treatment 2 participants can explain variations in their $b_E$ exchange bids. The regression in Table 11
<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>4.73</td>
<td>2.02**</td>
</tr>
<tr>
<td>Nashik</td>
<td>-3.40</td>
<td>-1.82*</td>
</tr>
<tr>
<td>Solapur</td>
<td>-0.70</td>
<td>-0.67</td>
</tr>
<tr>
<td>Taste Var1</td>
<td>0.47</td>
<td>1.15</td>
</tr>
<tr>
<td>Taste Var2</td>
<td>0.67</td>
<td>1.11</td>
</tr>
<tr>
<td>Round 1 HIMP bid</td>
<td>0.08</td>
<td>0.57</td>
</tr>
<tr>
<td>Female</td>
<td>0.73</td>
<td>0.72</td>
</tr>
<tr>
<td>Land</td>
<td>0.03</td>
<td>0.15</td>
</tr>
<tr>
<td>Age</td>
<td>0.00</td>
<td>0.10</td>
</tr>
<tr>
<td>Income Effect</td>
<td>0.14</td>
<td>1.34</td>
</tr>
<tr>
<td>Sample Size</td>
<td>69</td>
<td></td>
</tr>
</tbody>
</table>

Table 11. OLS Regression: HIMP Exchange Bids (Treatment 2 - Round 2)

shows that controlling for other factors that may explain part of the variation in $b_E$, namely, consumers’ tastes as revealed in sensory evaluation (the first two principal components of the matrix of grain and bhakri sensory characteristic variables), round 1 bids for HIMP, gender, land owned, age, and district dummies, compensating variations in wealth are not significant. An alternative to this would be including data from both Treatments as in the difference in differences regression, with a variable for compensating variation taking the value 0 for Treatment 1 and $b_L - p_L$ for round 2 - Treatment 2 observations. But this variable introduces collinearity and is not usable.67

We have argued that even though selection into Treatment 2 implies that the participant bid, on average, more for LPM in round 1, relative to round 1 LPM bids of Treatment 1 participants, the HIMP - LPM bid premia are the same across treatments and this is what matters. As a fourth check, we consider whether participants could have overbid: if yes, then due to the selection process, the proportion of such participants could be greater in Treatment 2. This would imply that the Treatment 2 $b_E$ greater than Treatment 1 $b_H - b_L$ result may be on account of selection of more overbidders in Treatment 2.

On this point, we note first that there is no evidence in the literature on overbidding in BDM experiments in which the subjects are trained (Rutstrom (1997), Irwin et al (1998), 67This variable is too highly correlated with the interaction dummy (0.96) in the DID regression, as it shares the large number of zeros. This is a more general problem in this dataset, because the interaction dummy (in Table 10) has a large number of zeros relative to ones (271 vs. 69). To illustrate this, we constructed a variable that shared the zeros of the interaction dummy, and had random numbers from the uniform [0,1] distribution elsewhere. This variable had spurious explanatory power in the difference in differences regression. 96
Noussair et al (2004), Lusk et al (2004)). Cason and Plott (2014) found that the number of subjects who overbid were more than those who bid the induced value in the first round of BDM. However, they elicited the WTP using a card containing brief written instructions about how the BDM mechanism worked. This is in contrast to our elicitation procedure. We provided each subject an individual enumerator, who provided verbal instructions and explained why bidding true value was in the best interest of the subject. The difference in procedure comes from the different aim of our study compared to Cason and Plott (2014). While they were studying misconceptions among subjects, we were aiming to elicit true valuations. Overbidding in our experiment is unlikely for this reason, as well as the fact that participants had to pay from their own pocket if they won; there was no participation fee to fall back on if they overbid. It is germane to note that the average bid for the LPM, which was available on the market, was lower than its market price.

Notwithstanding this, suppose one insists that round 1 bids of Treatment 2 participants were higher than those of Treatment 1 participants on account not just of their values being higher, but of overbidding as well. The difference between the average round 1 HIPM bid for these two treatments is about INR 1.5. Table 12 below considers 3 benchmarks: that the extent of overbidding by Treatment 2 participants is, on average, 1/3rd, 2/3rd or all of 1.5, with the rest of the difference arising out of different valuations for HIPM. The mean difference between Treatments 1 and 2 (Exchange bid $b_E$ minus premium $b_H - b_L$) is adjusted downward by the amount of this benchmark for overbidding. We observe that this still preserves a positive difference between the 2 treatments; the difference is significant for the 1/3rd and 2/3rd overbid benchmarks. Even for the extreme benchmark of a INR 1.5 overbid, the remaining difference of 0.96 is close to 10 percent significance in a one-tailed test.

We consider a second, alternative, robustness check with respect to the possibility of more overbidders being selected into Treatment 2: we compare it with a subset of Treatment

---

68Rutstrom (1997) found that the average bids in the BDM were significantly lesser than the Vickrey auction bids. Irwin et al (1998) found that (in their experiment 1) when the BDM task details were made clear to the subjects, the BDM mechanism was incentive compatible. Noussair et al (2004) study the performance of the Vickrey auction and the BDM mechanism with induced values. They found that the latter is more prone to bias, but the bias was in the direction of underbidding. Lusk et al (2004) found that the BDM bids were statistically equivalent to the English auction (which has been shown, e.g. Kagel et al (1987), to be an accurate elicitation method of WTP) bids, and lower than Vickrey auction bids.
1 participants that matches it in terms of the distribution of round 1 HIPM bids. The set of Treatment 1 participants who bid INR 12 and above for HIPM in round 1 has a mean HIPM bid of INR 14.76 in round 1. This is insignificantly higher (by INR 0.41) than the corresponding round 1 HIPM mean for Treatment 2 (p-value = 0.55). The K-S test statistic has a p-value of 0.55, suggesting very similar distributions.

The mean $b_{H} - b_{L}$ premium for this truncated Treatment 1 sample is INR 2.16 smaller than the mean exchange bid $b_{E}$ for Treatment 2. This difference, and the difference in distributions, are both highly significant (p-values of 0.00 up to 2 decimal places).

A fifth possible confound is that when they come into round 2, Treatment 1 participants have a history of not winning either variety of the good in round 1. Negative emotions have been shown to reduce or even reverse the endowment effect (Lerner et al. (2004), Lin et al. (2006)). Could there be some pessimism or related emotion that keeps them from bidding higher for HIPM in round 2 (thereby reducing the premium for it, relative to Treatment 2)? To control for this framing possibility, we had an additional arm in the experiment, namely Group B, in which participants are first provided the nutrition information regarding HIPM, and then bid for HIPM and LPM in a BDM mechanism. For Group B, there is therefore a single round of bidding, and we consider the first difference (as can be seen from Table 8 above): the mean $b_{H} - b_{L}$ for Group B, at INR 3.56, is INR 3.4 lower than the mean exchange bid in Treatment 2 (p-value 0.00); the distribution of premia is also stochastically dominated by the exchange bid distribution. In fact, this difference is larger than the difference-in-difference effect in the comparison of Treatments 1 and 2. Thus if we assume that subjects in group A Treatment 1 were sadder or more disgusted than group B subjects, then we get a result contrary to the findings of Lin et al (2006) and Lerner et al (2004). This indicates that this particular reasoning for our finding may not be appropriate.

Following Kahneman, Knetsch and Thaler (1991), there is a literature that explores

<table>
<thead>
<tr>
<th>Overbid benchmark</th>
<th>Adjusted treatment effect</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-third</td>
<td>1.91</td>
<td>0.00</td>
</tr>
<tr>
<td>Two-third</td>
<td>1.45</td>
<td>0.02</td>
</tr>
<tr>
<td>Full</td>
<td>0.99</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Table 12. Treatment Effect using Overbid Benchmarks
whether endowing subjects with a good makes the good a part of a “status quo” reference point. One effect of a “status quo” reference point is a ratio of WTA (minimum price required to sell the endowed good) to WTP ratio of more than 1 (see Horowitz and McConnell (2002) for a review of WTA/WTP experimental studies). In contrast, in the present experiment, participants who won the LPM first, bid higher to exchange it for HIPM. Moreover, they were not “endowed with” with LPM; they had to pay for it out of their own pocket. So the “status quo” reference point is not an explanation for the phenomenon we observe here.

A final explanation for high bids in an endow and upgrade experiment that has been given in the literature (Elbakidze and Nayga (2015), Corrigan and Rousu (2006)) is reciprocity feelings towards the experimenter: that the experimenter has endowed the subject with a free good and the subject feels obliged to reciprocate. But in our experiment, as noted above, Treatment 2 subjects had to purchase the LPM that they won; and that with their own money, as there was no participation fee. Thus their higher bids are not explained by reciprocity.

The stylized fact of Treatment 2 HIPM exchange bids ($b_E$) being higher than Treatment 1 premia ($b_H - b_L$) over a large interval is, however, consistent with participants who, after watching the infomercial on the HIPM, are loss averse for this biofortified variety, relative to expected reference outcomes (Koszegi and Rabin (2006)). We now describe this model and its data implications.

**The Anticipated Loss Aversion Model**

**The Basic Model**

Koszegi and Rabin (2006) and the subsequent literature on (expectations-based) reference dependent preferences assumes that the utility from good $t$, $u_t(c_t|r_t)$, depends on the consumption level $c_t$ and the reference level of consumption, or reference point, $r_t$. It is a sum of consumption utility, $v_t(c_t) = u_t(c_t|c_t)$, and utility from gains and losses with respect to the reference level of consumption. Using the simplification in Lange and Ratan (2010), we normalize gains to zero. So,
\[ u_t(c_t|r_t) = v_t(c_t) - \theta_t\{0, v_t(r_t) - v_t(c_t)\} \]

where \( \theta_t > 0 \) implies the agent is loss averse for good \( t \): if her consumption \( c_t \) is less than the reference level \( r_t \), she suffers a loss sensation. If the consumption and reference levels are random (with distributions \( F_c \) and \( F_r \)), then the agent gets an expected utility \[ \int \int u(c|r) dF_r(r) dF_c(c) \]; which is a weighted average of utilities from all possible pairs of consumption and reference levels for consumption.

The reference point itself is determined by expectations of consumption. Consider a BDM setting with a single good, for which the agent bids; after which, a sale price is selected from a distribution \( F \) on an interval \([a, K]\). Since the sale price is random, the bid induces a distribution over consumption outcomes (that is outcomes consisting of obtaining the good and paying the sale price). In the models here, as in the literature, the reference level of consumption is random and its distribution coincides with that of the consumption distribution; moreover, the consumption outcome and reference outcomes are independent.

For instance, suppose the good is 1 bag of HIPM following the infomercial. We assume that due to the novelty of the good, and its unavailability on the market, the agent is loss averse towards it, (but not towards money). If her intrinsic value for the HIPM bag of grain is \( v \) (for cleaner notation, drop the subscript “H” from \( v_H \) for the purposes of Proposition 1), her utility from a bid of \( b \) is given by

\[ U(v, b, \theta) = \int_a^b (v - p) f(p) dp - \theta v F(b)(1 - F(b)) \] (21)

This expression can be derived from the Koszegi-Rabin form \[ \int \int u(c|r) dF_r(r) dF_c(c) \] by accounting for all configurations of consumption and reference levels and then taking an expectation (see e.g. Banerji and Gupta (2014)). The first term gives the expected consumption utility. The second term corresponds to loss sensations: for all pairs of actual sale price \( p \) and reference sale price \( r \) s.t. \( r < b < p \), the agent gets the good in the reference outcome but does not do so in the actual outcome, thus suffering a loss sensation of \( \theta v \). The second term integrates over all such \((r, p)\) pairs.

Proposition 1 gives us the optimal bid function for an agent with a loss aversion param-
eter in the interval \((0, (K - a)/(K + a))\).

**Proposition 1** Suppose \(0 < \theta < (K - a)/2K\). The optimal bid function, \(b^*(v)\), for all \(v \leq K\) is given by:

\[
b^*(v, \theta) = \begin{cases} 
a & \text{if } v \leq (a/(1 - \theta)) \\
\frac{v(K - a) - \theta v(K + a)}{(K - a) - 2\theta v} & \text{if } v \in (\frac{a}{1 - \theta}, \frac{K}{1 + \theta}) \\
K & \text{if } v \geq (K/(1 + \theta))
\end{cases}
\]

**Proof.**

Differentiating the utility function w.r.t. \(b\), we get the interior first-order condition:

\[
f(b) [(1 - \theta)v + 2\theta vF(b) - b] = 0
\]

Since \(f(b) = 1/(K - a)\), we can solve for \(b\) to get

\[
b = \frac{v(K - a) - \theta v(K + a)}{(K - a) - 2\theta v}.
\]

Differentiating the utility function twice w.r.t. \(b\) gives the second derivative \(D_{22}u(v, b, \theta) = f(b)(2\theta vF(b) - 1)\), since \(f'(b) = 0\) for the uniform density \(f\). If \(\theta \leq (K - a)/2K\), this is negative for all values \(v \in [0, K]\), so we have concave utility.

Finally, if bid \(b = a\), utility \(u(v, a, \theta) = 0\). And the derivative of \(u\) w.r.t. \(b\), at \(b = a\),

\[
D_2(u, a, \theta) = f(a)((1 - \theta)v - a) > (>)0 \text{ as } v > (>)a/(1 - \theta).
\]

On the other hand, choosing bid \(b = K\) wins the object with probability 1 \((F(K) = 1)\), and \(D_2(v, K, \theta) = f(K)((1 - \theta)v + 2\theta v - K) > 0\) if \(v > K/(1 + \theta)\). *Q.E.D.*

In the intermediate interval \((a/(1 - \theta), K/(1 + \theta))\) of values, the optimal bid function is strictly increasing and convex, and cuts the 45-degree line at \(v = (K + a)/2\); (Figure 11). From equation (6), the loss sensation from not winning the good when one expects to do so can be reduced by either reducing \(b\) (and therefore \(F(b)\)) (thus reducing reference expectations of winning), or by increasing \(b\) and reducing the chances of not winning. For lower values, it is optimal to reduce \(b\), shading it below value \(v\), and for higher values, to increase \(b\) and mark it up above \(v\).

We use Proposition 1 as a model of optimal bidding in Treatment 2. The participant
is already endowed with LPM, which she values at $v_L$. If her value for HIPM, post the infomercial, is $v_H$, then she assigns a value premium of $v_E = v_H - v_L$ for it. Thus if $v_E \in \left( \frac{a}{1-\theta}, \frac{K}{1+\theta} \right)$, her optimal bid $b^*(v_E, \theta) = \frac{v_E(K-a) - \theta v_E(K+a)}{(K-a) - 2\theta v_E}$.

The BDM elicitation in Treatment 1 is different in that the participant bids for both LPM and HIPM, and then one of these is randomly selected, following which a random sale price is drawn. We assume he/she is loss averse for HIPM, but not for LPM that is available on the market.\footnote{In order for our model to explain the data, we only need that participants are more loss averse for HIPM than for LPM; however, the data can only identify a single loss aversion parameter: so we associate this with HIPM.} For Treatment 1, a reference outcome consists of a pair $(L, r)$ or $(H, r)$ of a selected good, $H$ (HIPM) or $L$ (LPM), and a reference sale price $r$. A consumption outcome includes the actual selection of $H$ or $L$ and an actual sale price $p$. With probability $1/4$ each, the reference variety - actual selected variety pairs are $(H, H)$, $(L, L)$, $(L, H)$, $(H, L)$. In the last case, the participant, expecting HIPM to be selected, ends up with LPM being selected, and a corresponding loss sensation.
Suppose the participant bids \((b_H, b_L)\) for the two varieties. Then her utility is given by

\[
U(v_H, v_L, b_H, b_L, \theta) = \frac{1}{2} \int_a^{b_L} (v_L - p) dF(p) + \frac{1}{2} \left[ \int_a^{b_H} (v_H - p) f(p) dp - \theta v_H F(b_H)(1 - F(b_H)) \right] - \frac{1}{4} \theta v_H F(b_H) \tag{22}
\]

With probability 1/2 each, utility is derived from consumption utility for LPM or consumption utility and gain-loss utility from HIPM, and with probability 1/4, there is a loss of \(\theta v_H F(b_H)\) on account of expecting to have HIPM selected and win with probability \(F(b_H)\). As we can see from this expression, it is still optimal to choose \(b_L = v_L\) for LPM.

**Proposition 2:** Suppose \(0 < \theta < \frac{(K-a)}{2K}\). The optimal bid function, \(b_H^*(v_H, \theta)\) for \(v_H \in [0, K]\) is given by:

\[
b_H^*(v_H, \theta) = \begin{cases} 
  a & \text{if } v_H < \frac{a}{1 - \frac{3}{2} \theta} \\
  \frac{[K(1 - \frac{3}{2} \theta) - a(1 + \frac{1}{2} \theta)]v_H}{K - 2\theta v_H - a} & \text{if } \frac{a}{1 - \frac{3}{2} \theta} \leq v_H \leq \frac{K}{1 + \frac{2}{\theta}} \\
  K & \text{if } v_H > \frac{K}{1 + \frac{2}{\theta}}
\end{cases}
\]

**Proof (Sketch).** As in Proposition 1, the utility function in (22) is strictly concave (convex) in the bid according to whether \(\theta\) satisfies \(\theta < (>) \frac{(K-a)}{2v_H}\). Assuming that the highest possible \(v_H\) in the population was \(K\), we get the boundary value of \(\frac{(K-a)}{2K}\), below which the utility function in (22) is strictly concave. *Q.E.D.*

**Data implications of the anticipated-loss aversion model**

The interior \(b_H^*(v_H, \theta)\) given in Proposition 2 is convex in \(v_H\) for all \(\theta \in (0, \frac{(K-a)}{2K})\). Consider the value \(\hat{v}(\theta)\) at which the bid is equal to the value, that is, \(b_H^*(\hat{v}, \theta) = \hat{v}\). Simplifying, we get \(\hat{v} = \frac{3K+a}{4} = 23.75\), which is not dependent on \(\theta\). Note that at \(\hat{v}\), the optimal bid function will cross the 45 degree line, irrespective of \(\theta\); (see Figure 11, where the optimal bid function is drawn for \(\theta = 0.25\)). Similarly, \(b^*(v, \theta)\) (Proposition 1) is also convex, and solving \(b^*(v', \theta) = v'\) we get \(v' = \frac{(K+a)}{2}\) (independent of \(\theta\)).
A hypothetical agent who is loss-averse for HIPM but not for LPM would have bids $b_L$, $b_E$ and $b_H$ that satisfy $b_L = v_L$, $b_E = b^*(v_E, \theta)$ and $b_H = b^*_H(v_H, \theta)$, where $v_L + v_E = v_H$. First, suppose $b_H < \hat{v} = 23.75$ and $b_E \geq 10$. Then $b_H$ must correspond to a value $v_H < \hat{v}$ and be shaded below it; whereas $b_E > v_E \geq 10$.

So, we have $b_H < v_H = v_L + v_E \leq b_L + b_E$. By continuity of $b^*$, for $b_H < 23.75$, we have $b_H < b_L + b_E$, or $b_H - b_L < b_E$, for all $b_E \geq \bar{b}$, for some $\bar{b} < 10$. We can view this stochastic dominance of $b_E$ over $b_H - b_L$ as follows: since $v_H = v_L + v_E$, and $b_L = v_L$ (no loss aversion for LPM), in terms of bids, $b_L + b_E$ marks down only a part of $v_L + v_E$, whereas $b_H$ marks down the entirety if $v_H$ (see Figure 12, drawn for a value $v_L = b_L = 10$ and $\theta = 0.25$).

To summarize thus far, there exists $0 < \bar{b} < 10$ such that for any hypothetical agent with any $b_H < 23.75$ and $b_E \geq \bar{b}$ (and $v_H - v_L = v_E$), $b_H - b_L < b_E$. Additionally, for the specific design of our experiment, the model predicts that $b_H - b_L < b_E$ for all hypothetical agents with $v_E \in (0, 10)$ and $v_H < 23$ (with $v_H = v_L + v_E$ and $\theta \in (0, 0.4)$). This follows from a comparison of bid shading in the two Treatments. We have:

For all $v_H = v_L + v_E$, with $v_H < 23$ and $v_E \in (0, 10)$, and $\theta \in (0, 0.4)$,

$$v_E - b^*(v_E, \theta) < v_H - b^*_H(v_H, \theta)$$  \hspace{1cm} (23)

Numerical optimization yields that the maximum value for the bid shading on the left-hand side of equation (14), over all $v_E \in (0, 10)$ and all $\theta \in (0, 0.4)$ equals about 0.025; and the minimum value for the bid shading on the right-hand side over all admissible $\theta$ and $v_H \leq 23$ equals 0.06. Replacing $v_H$ by $v_L + v_E$ above, noting $b_L = v_L$, and rearranging, equation (8) yields $b^*_H(v_H, \theta) - b_L < b^*(v_E, \theta)$.

The model predicts greater bid shading in Treatment 1 (right-hand of equation (14)) for two reasons: (i) there is the additional possibility of loss sensations from winning LPM when one expects to win HIPM; for which reason people shade more if their $v_H$ value is low; this is confirmed by a direct comparison of $b^*_H(v, \theta)$ and $b^*(v, \theta)$ for a given $v$. (ii) The BDM interval in Treatment 1 is larger ([5, 30]) compared with [0, 20] for Treatment 2.

Loss averse agents shade more if the BDM interval is larger (Banerji and Gupta (2014),
Proposition 3).

Collecting the implications for comparing $b_H - b_L$ and $b_E$ from equation (8) and the discussion preceding it, we have:

**Implication 1** The distribution of $b_E$, obtained from Treatment 2 first-order stochastically dominates the distribution of $b_H - b_L$, obtained from the subsample of Treatment 1 satisfying $b_H < 23$.

A secondary implication (Implication 2) is that the stochastic dominance can reverse for bids involving $b_H$ much larger than 23.75. For example $b_H^*(v_H, \theta) > v_H = v_L + v_E \geq b_L + b^*(v_E, \theta)$ if $v_H > 23.75$ and $v_E < 10$.

The first implication is borne out by the evidence, as discussed earlier (Table 10, Figure 10). Implication 2 is a possibility, but it is difficult in the present context of an improved food for a participant to have a high full value $v_H$ for it, and yet a relatively low exchange value $v_E$. Figure 10 does suggest however that at some high bid less than the maximum, the cumulative distributions of $b_E$ and $b_H - b_L$ come close together.
Estimation of loss aversion and HIPM distribution

The optimal bid functions, \( b^*_{H}(v_H, \theta) \) and \( b^*_{E}(v_E, \theta) \), are increasing in the value \( v_H \) over an interval; but we can’t use them to invert bids to get values since we do not know the loss aversion parameter \( \theta \). In keeping with the tradition of identifying risk aversion from auction data, we assume that the agents share a common, true loss aversion parameter \( \theta \). Note first that if \( \theta \) is known, then we can invert bids in the interior of the BDM distributions to get values as follows:

**Proposition 3** (i) If \( b_E \in (a, K) \), using proposition 1, the inverse is given by:

\[
\phi_E(b_E, a, K, \theta) = \frac{b_E(K - a)}{2\theta b_E - \theta(K + a) + (K - a)}, \quad \text{where } \theta < \min\{\frac{K - a}{K + a}, \frac{K - a}{2K}\}
\]

(ii) If \( b_H \in (a, K) \), using proposition 2, the inverse is given by:

\[
\phi_H(b_H, a, K, \theta) = \frac{b_H(K - a)}{K(1 - \frac{3}{2}\theta) - a(1 + \frac{1}{2}\theta) + 2\theta b_H}, \quad \text{where } \theta < \min\{\frac{K - a}{K + a}, \frac{K - a}{2K}\}
\]

We first estimate \( \theta \) from the data and then use Proposition 3 to invert bids to values. Our estimation is based on the fact that prior to the allocation to the two Treatments, the pre-nutrition-information premia (i.e. round 1 differences between bids for HIPM and LPM) are not significantly different in the two groups. So it is reasonable to expect that brand and nutrition information enhances values \( v_H \) by similar amounts in the two Treatments. Thus if we invert bids to get values using the “true” \( \theta \), there should not be any significant difference between the distribution \( F_1(\theta) \) of \( \phi_H(b_H, \theta) - b_L \) from Treatment 1 and the distribution \( F_2(\theta) \) of \( \phi_E(v_E, \theta) \) from Treatment 2.

Our estimate \( \theta^* \) therefore solves

\[
\theta^* \in \arg\min \left\{ d_1(F_1(\theta), F_2(\theta)) \mid \theta \in \left[ 0, \frac{K - a}{K} \right] \right\}
\]

\[(24)\]
where \( a = 5, K = 30, F_1(\theta), F_2(\theta) \) are the empirical distribution functions of \( \phi_H(b_H, \theta) - b_L \) and \( \phi_E(b_E, \theta) \), and \( d_1 \) measures the absolute difference between the two distributions on a grid of 4000 points in the interval \([-5, 20]\).\(^{70}\)

We estimate \( \theta^* = 0.25 \) with a 95 percent confidence interval of \([0.18, 0.35]\) obtained by bootstrapping.\(^{71}\) Figure 13 shows that the empirical distribution functions \( F_1(\theta^*) \) and \( F_2(\theta^*) \) almost overlap.

**Discussion**

We use the loss aversion parameter estimate \( \theta^* = 0.25 \), the bidding data for \( b_H \) in Treatment 1 (round 2) and the inverse formula \( \phi_H(b_H, a, K, \theta^*) \) from Proposition 3 to estimate the participants’ values \( v_H \) for HIPM following the infomercial. Figure 14 plots (i) the density of these latent values; it also plots (ii) the densities of the bids, and of (iii) the first round

\(^{70}\)The estimate of \( \theta \) is robust to the choice of grid points.

\(^{71}\)The estimates are rounded off.
bids for HIPM, prior to the infomercial (labeled in Figure 14 as \( v_H, b_H, b_{H1} \) respectively. Under our benchmark assumption that participants were not loss averse to HIPM in round 1, and their bids reflected true values, the difference between densities (i) and (iii) captures the true effect of brand and nutrition information on participants’ valuations for HIPM.

Table 13 summarizes the effect: the latent valuations for HIPM are on average INR 2.31 higher than the Treatment 1 bids for HIPM post nutritional information. As nutritional information raises average bids by INR 3.96 (column 2), factoring in loss aversion along with information raises bids by INR 6.27 (column 3) (all differences are significant at 1 percent). On a base of INR 12.99, (mean WTP for HIPM before information), the conventional measure of increase in WTP (the BDM bid in Treatment 2) has a downward bias of about 12 percent.

It is evident from the model that the mean of the latent values could in principle be higher or lower than the mean of the BDM bids, depending on the location of the BDM interval in comparison to the probability mass of these (unknown) values. For an individ-
Comparison | $v_H - b_H$ | $b_H - b_L^1_H$ | $v_H - b_L^1_H$
---|---|---|---
Difference in means | 2.31*** | 3.96*** | 6.27***
t-value | 3.76 | 6.32 | 12.12

Table 13. Treatment 1: Estimated values versus bids for HIPM
Notes: $b_H, v_H$ are bids and estimated values for HIPM in round 2. The superscript 1 refers to round 1.

| $v_E - b_E$ | $v_E - (b_H^1 - b_L^2)^{T2}$ | $v_E - (b_H^1 - b_L^1)^{T1}$ | $v_E - (v_H - b_L^2)^{T1}$ |
---|---|---|---|
Difference in means | 0.2343 | 6.41 *** | 6.36 *** | 0.4217 |
t-value | 0.38 | 11.27 | 13.19 | 0.89 |

Table 14. Estimated Exchange Values, Bids, and Premium Comparisons
Notes: $v_E, v_H$ are estimated exchange value and full value for HIPM. $T1, T2$ are Treatments 1 and 2. $b_L^1, b_L^2, b_H^1$ are LPM bids in rounds 1 and 2 and HIPM bid in round 1. $b_E$ is exchange bid in Treatment 2.

Prior to the brand and nutrition information, we have seen that the difference between the mean bids for HIPM and LPM is about the same across the two treatments; each of these is significantly lower than the mean post-information exchange bid or value, $b_E$ or $v_E$ (Table 14 columns 2 and 3). There is no significant difference between the latent, mean
$v_H - v_L$ premium from Treatment 1 (round 2) and the latent $v_E$ from Treatment 2.

The estimate for $\theta$ here is almost the same as that obtained by Banerji and Gupta (2014) in a different WTP elicitation design. That paper also studies implications of loss aversion of this magnitude for first- and second-price auction revenues.

This chapter shows that expectations-based reference dependence predicts the discrepancy that we observe between our full bidding and endow and upgrade treatments for the new product, HIPM. This discrepancy is an "adding-up test" failure in the context of an additional qualitative feature in a new product. We rule out an income effect, using a robustness check, and a reciprocity effect, due to the experimental design. The model predicts the discrepancy even if an individual is loss averse for both old and new products, provided she is more loss averse for the new product: however, the 2-treatment experimental design can identify a single loss aversion parameter. Standard, incentive-compatible elicitation procedures result in biased WTP estimates in the presence of loss aversion: this chapter contributes a design that can identify loss aversion and correct this bias.
References


Appendix A: Proofs and Applications for Chapter One

Proof of proposition 1(a). (Existence of LFE): Consider an arbitrary finite Interaction Game \( \Gamma \). The \( CG(1) \) derived from \( \Gamma \) is also finite. Due to proposition 1 of Kreps and Wilson (1982), there exists a SE of \( CG(1) \). We can select an arbitrary SE(1) of \( CG(1) \) to construct \( MCG(2) \). \( MCG(2) \) is also finite. Thus the SE(2) of \( MCG(2) \) also exists. Proceeding thus, given the existence of SE of each of \( CG(1), MCG(2),..., MCG(n-1) \), we can construct \( MCG(n) \) in step \( n \) of definition 2. As \( MCG(n) \) is finite, there exists a SE of \( MCG(n) \). As this holds for \( n = 2, ..., N \) each of the steps in definition 2 can be carried out as defined, and thus there exists at least one LFE of \( \Gamma \). Q.E.D.

An inductive argument for Upperhemicontinuity of LFE. Consider an arbitrary \( S \)-stage Interaction Game \( \Gamma \). For a given extensive form, \( \{N, H, \{I(t_i)\}_{t_i \in N}, P, A\} \), let the correspondence \( f : \Delta T \times R^N \rightarrow \Pi \times M \) be the set valued function, mapping initial conditions and payoffs \((\rho, u)\), to the set containing all associated LFE assessments. An element of the set \( f(\rho, u) \) is an LFE, denoted as \((\pi, \mu)\). Fix a sequence \((\rho_k, u_k) \rightarrow (\rho, u)\) and an associated sequence \((\pi_k, \mu_k) \in f(\rho_k, u_k)\), such that \((\pi_k, \mu_k) \rightarrow (\pi, \mu)\). To show upperhemicontinuity, we need to show that \((\pi, \mu) \in f(\rho, u)\).

Given an arbitrary extensive form \( \{N', H', \{I'(t_i)\}_{t_i \in N'}, P', A'\} \), let \( \Psi : \Delta T \times R^{N'} \rightarrow \Sigma \times B \) be the Upper Hemi Continuous (UHC) correspondence mapping initial conditions and payoffs, \((\rho', u')\), to the set \( \Psi(\rho', u') \), which contains all the sequential equilibrium strategies and beliefs, \((\sigma, b)\), of the game so defined.

Let \( \pi(\tilde{H}), \mu(\tilde{H}) \) denote the vectors \( \pi \) and \( \mu \) restricted to the coordinates corresponding to the information sets contained in \( \tilde{H} \). Let \( f(\rho, u)(\tilde{H}) \) also represent each element of \( f(\rho, u) \) restricted to \( \tilde{H} \). We prove upperhemicontinuity by induction. Step 1: we show that \((\pi(D^1), \mu(D^1)) \in f(\rho, u)(D^1)\). Consider \( CG(1) = MCG(1) \). Corresponding to \((\rho_k, u_k)\)
we have $(\rho_k^n, u_k^n)$ for each element of the sequence $k = 1, 2, \ldots$. The superscript denotes the length of the $CG$. The construction of $u_k^1$ using the curtail and $\min^{+\max\over 2}$ method was described earlier. As the function which maps a finite set of real numbers to their $\min^{+\max\over 2}$ is a continuous function, $u_k \to u$ implies $u_k^1 \to u^1$. Also, $\rho_k^1 = \rho_k$, so $(\rho_k, u_k) \to (\rho, u)$ implies that $(\rho_k^1, u_k^1) \to (\rho^1, u^1)$. Now note that for each $k$ in the sequence, $(\pi_k(D^1), \mu_k(D^1)) = (\sigma_k^1(D^1), b_k^1(D^1))$, and $(\sigma_k^1(D^1), b_k^1(D^1)) \in \Psi(\rho_k^1, u_k^1)(D^1)$. We know that $\Psi(.)$ is UHC. Thus if $(\sigma_k^1(D^1), b_k^1(D^1)) \to (\sigma^1(D^1), b^1(D^1))$ then $(\sigma^1(D^1), b^1(D^1)) \in \Psi(\rho^1, u^1)(D^1)$. Given that $(\pi_k(D^1), \mu_k(D^1)) \to (\pi(D^1), \mu(D^1))$, and given that $(\pi_k(D^1), \mu_k(D^1)) = (\sigma_k^1(D^1), b_k^1(D^1)) \to (\sigma^1(D^1), b^1(D^1))$ it follows from the uniqueness of a limit that $(\sigma^1(D^1), b^1(D^1)) = (\pi(D^1), \mu(D^1)) \in \Psi(\rho^1, u^1)(D^1) \subset f(\rho, \mu)(D^1)$. Therefore $(\pi(D^1), \mu(D^1)) \in f(\rho, \mu)(D^1)$.

**Step 2:** Consider $MCG(n)$, where $n \in \{2, \ldots, N\}$. Let $(\pi(\bigcup_{i=1}^{i=n-1} D^i), \mu(\bigcup_{i=1}^{i=n-1} D^i)) \in f(\rho, u)(\bigcup_{i=1}^{i=n-1} D^i)$. We will show that $(\pi(\bigcup_{i=1}^{i=n} D^i), \mu(\bigcup_{i=1}^{i=n} D^i)) \in f(\rho, u)(\bigcup_{i=1}^{i=n} D^i)$. Given step 1, this will complete the proof.

Corresponding to $(\rho_k, u_k)$ we have $(\rho_k^n, u_k^n)$ for each $k = 1, 2, \ldots$. Using $\pi_k(\bigcup_{i=1}^{i=n-1} D^i)$ and $\rho_k$ we generate $\rho_k^n$ as detailed earlier. By continuity, $u_k^n \to u^n$. As $\pi_k(\bigcup_{i=1}^{i=n-1} D^i) \to \pi(\bigcup_{i=1}^{i=n-1} D^i)$ by assumption, thus: (i) $(\rho_k^n, u_k^n) \to (\rho^n, u^n)$ and (ii) it will suffice to show $(\pi(D^n), \mu(D^n)) \in f(\rho, u)(D^n)$. Now note that for each $k$, $(\pi_k(D^n), \mu_k(D^n)) = (\sigma_k^n(D^n), b_k^n(D^n))$, and $(\sigma_k^n(D^n), b_k^n(D^n)) \in \Psi(\rho_k^n, u_k^n)(D^n)$. We know that $\Psi(.)$ is UHC. Thus, if $(\sigma_k^n(D^n), b_k^n(D^n)) \to (\sigma^n(D^n), b^n(D^n))$, then $(\sigma^n(D^n), b^n(D^n)) \in \Psi(\rho^n, u^n)(D^n)$. Given that $(\pi_k(D^n), \mu_k(D^n)) \to (\pi(D^n), \mu(D^n))$, and given that

$$(\pi_k(D^n), \mu_k(D^n)) = (\sigma_k^n(D^n), b_k^n(D^n)) \to (\sigma^n(D^n), b^n(D^n)),$$

by the uniqueness of a limit, it follows that $(\sigma^n(D^n), b^n(D^n)) = (\pi(D^n), \mu(D^n)) \in \Psi(\rho^n, u^n)(D^n) \subset f(\rho, \mu)(D^n)$. Therefore, $(\pi(D^n), \mu(D^n)) \in f(\rho, u)(D^n)$. Q.E.D.
Proof of proposition 2. Let $Pr$ be short for “probability” for the purpose of this proof.

Suppose the precedent of proposition 2 holds. Then

$$b^n((t_{-i}, h_0) | t_i, I(t_i), \sigma^n) = \frac{Pr((t_i, t_{-i}, h_0) | \sigma^n)}{\sum_{t_{-i} \in T_{-i}} [Pr((t_i, t_{-i}, h_0) | \sigma^n)]}$$

(25)

We have to show that

$$\frac{Pr((t'_i, t_{-i}, h_0) | \sigma^n)}{\sum_{t_{-i} \in T_{-i}} [Pr((t'_i, t_{-i}, h_0) | \sigma^n)]} = \frac{Pr((t_i, t_{-i}, h_0) | \sigma^n)}{\sum_{t_{-i} \in T_{-i}} [Pr((t_i, t_{-i}, h_0) | \sigma^n)]}$$

(26)

When we calculate $Pr((t_i, t_{-i}, h_0) | \sigma^n)$, the manner in which this term is affected by $t_i$ is captured by a multiplicative term that varies based on $t_i$, but given $i$’s type, $t_i$ or $t'_i$, it doesn’t vary as $t_{-i}$ varies (in the denominator of (4)). Thus, the multiplicative term that is specific to $t_i$, but common to any $t_{-i}$, given $t_i$, cancels out from the numerator and denominator, leaving a term that is unaffected by $i$’s type.

Without loss of generality, let $h_0 = (a^k_0)_{k=1,\ldots,K}$. Let the information set containing the history $(t, (a^k_0)_{k=1,\ldots,r})$ be denoted as $I(t, (a^k_0)_{k=1,\ldots,r})$. Using the independence of $\rho$ we get the following.

$$Pr((t_i, t_{-i}, h_0) | \sigma^n) = Pr(t_i | \rho).Pr(t_{-i} | \rho).\sigma^n(a^1_0 | I(t)).\sigma^n(a^2_0 | I(t, a^1_0)) \ldots \sigma^n(a^K_0 | I(t, (a^k_0)_{k=1,\ldots,K-1}))$$

(27)

Define a subsequence or a subhistory of $(t, h_0)$ as $(t, (a^k_0)_{k=1,\ldots,r})$, such that $r \in \{1,\ldots,(K-1)\}$ and there exists a unique sequence of actions $(a^k_0)_{k=r+1,\ldots,K}$ such that $(t, (a^k_0)_{k=1,\ldots,r}, (a^k_0)_{k=r+1,\ldots,K}) = (t, h_0)$. For any $(t_i, t_{-i}, h_0)$ such that $t_{-i} \in T_{-i}$, let $R(t_i)$ be the collection of natural numbers $r(t_i) \in \{1,\ldots,K\}$ such that the player-type moving at the subhistory $(t, (a^k_0)_{k=1,\ldots,r(t_i)})$ of $(t, h_0)$ is $t_i$. Further, let $R^c(t_i) = \{1,\ldots,K\} - R(t_i)$. The set $R(t_i)$ does not depend on $t_{-i}$. It only depends on the $h_0$ component of $(t, h_0)$. It is also worth noting that by construction, for any $t_{-i}, t'_{-i} \in T_{-i}, I((t_i, t_{-i}), (a^k_0)_{k=1,\ldots,r(t_i)}) = I((t_i, t'_{-i}), (a^k_0)_{k=1,\ldots,r(t_i)})$ because by construction, information sets in $MCG(n)$ only reflect the uncertainty about the opponents’ types, that is,
uncertainty about $t_{-i}$. Therefore, given an action sequence $h_0$, changing the profile of opponents’ types preceding $h_0$ leaves us in the same information set.

We can rewrite (5), we get that

$$
Pr((t_i, t_{-i}, h_0) | \sigma^n) = Pr(t_{-i} | \rho) \Pi_{s \in R^n(t_i)} [\sigma^n(a_0^{s+1} | I(t, (a_0^k)_{k=1,\ldots,s}))] \times Pr(t_i | \rho) \Pi_{r \in R(t_i)} [\sigma^n(a_0^{r+1} | I(t, (a_0^k)_{k=1,\ldots,r}))]
$$

(28)

$$
\frac{Pr((t_i, t_{-i}, h_0) | \sigma^n)}{\sum_{t_{-i} \in T_{-i}} [Pr((t_i, t_{-i}, h_0) | \sigma^n)]} = \frac{Pr(t_{-i} | \rho) \Pi_{s \in R^n(t_i)} [\sigma^n(a_0^{s+1} | I(t, (a_0^k)_{k=1,\ldots,s}))]}{\sum_{t_{-i} \in T_{-i}} Pr(t_{-i} | \rho) \Pi_{s \in R^n(t_i)} [\sigma^n(a_0^{s+1} | I(t, (a_0^k)_{k=1,\ldots,s}))]}
$$

(29)

The proof will be complete if we show that the RHS of (7) does not depend on the type of player $i$, $t_i$. Consider some $s \in R^n(t_i)$. Suppose $P^n(t, (a_0^k)_{k=1,\ldots,s-1}) = t_j$. Then $I((t_i, t_j, t_{-(i,j)}), (a_0^k)_{k=1,\ldots,s})) = I((t_i', t_j, t_{-(i,j)}), (a_0^k)_{k=1,\ldots,s}))$ for any $t_i, t_i' \in T_i$, by the construction of information sets in $MCG(n)$. Therefore, by the definition of an information set,

$$
\sigma^n(a_0^{s+1} | I((t_i, t_j, t_{-(i,j)}), (a_0^k)_{k=1,\ldots,s})) = \sigma^n(a_0^{s+1} | I((t_i', t_j, t_{-(i,j)}), (a_0^k)_{k=1,\ldots,s})�)
$$

Thus, the RHS of (7) does not depend on $t_i$, and remains constant across $t_i, t_i' \in T_i$.

Q.E.D.

**Proof of corollary to proposition 2.** By proposition 2, if $Seq^{-1}(I(t_i)) = Seq^{-1}(I(t_i'))$, then $b^n(I(t_i)) = b^n(I(t_i'))$. Further, for corresponding information sets, other players’ ($j \neq i$) types cannot choose different actions for different types of player $i$. Thus, $t_i$ and $t_i'$ face the same strategy profile $((\sigma_{t_j}^n)_{t_j \in T_j})_{j \neq i}$. Therefore $U_{t_i}(s(T^{ho}(t_i))|\sigma_{-t_i}^n, b^n) = U_{t_i'}(s(T^{ho}(t_i'))|\sigma_{-t_i'}^n, b^n)$ for all strategies $s(.)$. Then as
Suppose Proof of the proposition on the Centipede Game.

Lemma 1: Consider any stage \( k < S \), therefore that

\[
U_{t_i}(\sigma^n_{t_i}(\mathcal{I}^{ho}(t'_i)))|\sigma^n_{-t_i}, b^n) > U_{t_i}(\sigma^n_{t_i}(\mathcal{I}^{ho}(t'_i)))|\sigma^n_{-t_i}, b^n)
\]

for all possible strategies \( s^n_{t_i}(\mathcal{I}^{ho}(t'_i)) \) over \( \mathcal{I}^{ho}(t'_i) \) and as \( \sigma^n \) is a SE strategy profile, we must have that \( \sigma^n_{t_i}(\mathcal{I}^{ho}(t'_i)) = \sigma^n_{t_i}(\mathcal{I}^{ho}(t_i)) \) when \( Seq^{-1}(\mathcal{I}^{ho}(t_i)) = Seq^{-1}(\mathcal{I}(t'_i)) \). Q.E.D.

Proof of proposition 3. Consider an arbitrary LFE, \((\pi, \mu), \) of \( \Gamma \). To calculate \( \mu_{t_i}(h | L(I)) \) within \((\pi, \mu) \), we need to complete steps 1 through \((t_i + K - 1) \) of the construction of the LFE \((\pi, \mu) \). In step \((K + t_i) \), we construct \( MCG(K + t_i) \). Consider an arbitrary \( h \in L(I) \). Let, without loss of generality, \( h \) be of the form \( h = (t, (a^k_0)_{k=1,...,K-1}) \). In constructing \( MCG(t_i + K) \), for any such \( h \in L(I) \), using the definition of \( L(I) \), it follows that for all subsequences of \( h \) of the form \( \hat{h} = (t, (a^k_0)_{k=1,...,r}) \) such that \( r \leq (K - 1) \),

\[
P(K+t_i)(\hat{h}) = Nature \text{ because } \hat{h} \in \bigcup_{n=1}^{K+t_i-1} D^n.
\]

We know \( \pi(\bigcup_{n=1}^{K+t_i-1} D^n) \) by steps 1 though \((K + t_i - 1) \) of the construction of an LFE. Thus, in constructing \( MCG(s + n) \), we set \( \rho^{K+t_i}(\hat{h}) = \pi(\hat{h}) \) for all subsequences \( \hat{h} \) of each \( h \in L(I) \). So \( \pi(\bigcup_{n=1}^{K+t_i-1} D^n) = \rho^{K+t_i}(\bigcup_{n=1}^{K+t_i-1} D^n) \), and therefore \( \mu_{t_i}(h | L(I)) \) is calculated using \( \rho^{K+t_i} \) and Bayes’ rule wherever \( Prob(L(I) | \rho^{K+t_i}) = Prob(L(I) | \pi) > 0 \). Q.E.D.

Proof of corollary to proposition 3. Suppose \( h = ((t_i, t_{-i}), (a^k_0)_{k=1,...,s-1}) \in L(I(t_i)) \), then we will show that \( h' = ((t'_i, t_{-i}), (a^k_0)_{k=1,...,s-1}) \in L(I(t'_i)) \) to complete the proof. All subsequences of \( h \) can be written as \( ((t_i, t_{-i}), (a^k_0)_{k=1,...,r-1}) \) for some \( r < s \). Fix an arbitrary subsequence of \( h \) of the form \( \hat{h} = ((t_i, t_{-i}), (a^k_0)_{k=1,...,r-1}) \), it must be true that if \( P(\hat{h}) = t_j \) then \( r + t_j < s + t_i \). By the construction of \( \Gamma \) using the \( Seq(.) \) function, we must have that for the same \( r \), the subsequence of \( h' \) given by \( \hat{h}' = ((t'_i, t_{-i}), (a^k_0)_{k=1,...,r-1}) \) is such that \( P(\hat{h}') = t_j \). Given that \( t_i < t'_i \), we must have that \( r + t_j < s + t_i < s + t'_i \). Repeating this argument for every \( r < s \), we have that for every subsequence \( \hat{h}' = ((t'_i, t_{-i}), (a^k_0)_{k=1,...,r-1}) \), for some \( r < s \), if \( P(\hat{h}') = t_k \) then \( r + t_k < r + t'_i \) and therefore that \( h' \in L(I(t'_i)) \). Q.E.D.

Proof of the proposition on the Centipede Game. Lemma 1: Consider any stage \( k < S \). Suppose \( \Gamma_0 \) has the payoff structure \( P \) and we replace the payoff profile after “pass” at
stage $k$ with $(x_{k+1}, y_{k+1})$; where $(x_{k+1}, y_{k+1})$ is the payoff profile calculated by taking the mean of the range of the payoff profiles at the terminal histories following from the stage $k$ action “pass”. Then $\min\{x_{k+1}, y_{k+1}\} > \max\{(a_i)_{i \leq k}, (b_i)_{i \leq k}\}$.

Lemma 1 follows straightforwardly due to the properties of payoff structure $P$. For example, curtailing Figure 6 at stage 3, we get $(x_4, y_4) = (132, 80)$, the minimum of which, 80, is higher than the maximum number in $\{(4,1), (2,8), (16,4)\}$, 16. Due to lemma 1, in any Curtailed Game not equal to $\Gamma$ (shorter than $\Gamma$), the highest payoff for both players occurs after pass at the last stage. So, irrespective of $\rho$, Nature’s initial probability distribution on $N$, for all $CG(1), MCG(2), ..., MCG(S - 1)$, there is a unique sequential equilibrium consisting of all player types playing pass with probability 1, and estimating that all other player types do the same. Thus, any limited foresight type, at any stage where he cannot observe $\Gamma$, chooses pass with probability 1. This implies that in $MCG(S)$, according to $\rho^S$, any Nature’s move, at any non initial node, specifies the pure action: pass.

The decisive information sets of $MCG(S)$ are those where the player type moving there is rational (has full total foresight) and thus can observe $MCG(S)$. Now, we analyze the condition for the rational player types, i.e. player types who can observe $\Gamma$, or $MCG(S)$ to pass with strictly positive probability from stages 1 through $(S - 3)$. We will prove this by contradiction. That is, we will show that it cannot be a SE of $MCG(S)$, and hence cannot be an LFE, for all the player types who turn rational (attain total foresight at least $K$) at some stage of $MCG(S)$ to choose strategies that imply that all rational types choose take with probability 1 at a stage before stage $(S - 2)$. Using the homogeneity of beliefs as detailed in proposition 2, let $r_i$ denote the identical probability belief of every rational player type at stage $i$ that at stage $(i + 1)$ the opponent will be Nature playing (and, by lemma 1, choosing pass) on behalf of a limited foresight opponent type, conditional on play reaching stage $i$. Let $p_i$ denote the identical probability put on pass by every rational player type at stage $i$. To show the contradiction, we only need to show that in any SE of $MCG(S)$, it cannot be the case that $p_i = 0$, where $i = 1, ..., (N - 3)$. Lemma 1 also holds with the “mean of stage-wise means” rule followed in an earlier version of this paper. The $\frac{\min + \max}{2}$ rule is only significant for lemma 1. Therefore it follows that proposition 7 also holds with the “mean of stage-wise means” rule.
Lemma 2: For any sequential equilibrium \((\sigma^S, b^S)\) of \(MCG(S)\) : (a) If \(\sigma^S\) implies that \(p_i = 1\), then \(\sigma^S\) must imply that \(p_j = 1\), for \(j \leq i\), where \(i = 1, \ldots, S\). (b) If \(b^S\) is such that at stage \(i\), \(r_i > \eta_i\), then \(\sigma^S\) must imply that \(p_i = 1\), for \(i = 1, \ldots, S - 2\).

Proof: (a) Let \((\sigma^S, b^S)\) imply that \(p_i = 1\). That is, all rational player types \textit{pass} with probability 1 at stage \(i\). Then, sequential rationality implies that according to \(\sigma^S\), irrespective of beliefs, \(p_{i-1} = 1\) because any rational type’s choice to \textit{pass} at stage \((i - 1)\) is going to be reciprocated by \textit{pass} with probability 1 at stage \(i\). Therefore, the payoff from \textit{pass} at stage \((i - 1)\) is at least \(a_{i+1}\). And \(a_{i+1} > a_{i-1}\), where \(a_{i-1}\) is the payoff from \textit{take} at stage \((i - 1)\). Similarly, \(p_{i-1} = 1\) implies that due to sequential rationality of \(\sigma^S\), we must have that \(p_{i-2} = 1\), and so on for all \(j \leq i\).

(b) Let \(i \in \{1, \ldots, (S - 2)\}\). Let \(v_i\) be the value to the rational types moving at stage \(i\) given sequential equilibrium play from stage \(i\) on in the \(MCG(S)\). By sequential rationality of \(\sigma^S\), it follows that \(v_i \geq a_i\). The payoff from \textit{take} at stage \(i\) is \(a_i\), the expected payoff from \textit{pass} at stage \(i\) is at least \(r_i v_{i+2} + [1 - r_i] b_{i+1}\). This is because, by lemma 1, Nature chooses \textit{pass} with probability 1 at stage \((i + 1)\). If \(r_i > \eta_i\) then the expected payoff from \textit{pass} is at least \(r_i v_{i+2} + [1 - r_i] b_{i+1} \geq r_i a_{i+2} + [1 - r_i] b_{i+1} > \eta_i a_{i+2} + [1 - \eta_i] b_{i+1} = a_i\), where the last equality follows by the definition of \(\eta_i\). So, by sequential rationality of \(\sigma^S\), \(p_i = 1\).

Now we can prove proposition 7 for rational player types by showing that for any SE of \(MCG(S)\), \((\sigma^S, b^S)\), it cannot be the case that \(\sigma^S\) implies \(p_i = 0\) at some \(i \leq S - 3\). Suppose \(\sigma^S\) implies \(p_i = 0\) at some \(i \leq S - 3\). Then, because two equiprobable limited foresight types being represented by Nature in stage \(i\) turn rational in stage \(i + 2\), \(r_{i+1} = \frac{S-i-2}{S-i} \geq \frac{1}{3} > \eta_{i+1} = \frac{1}{3}\). So, by lemma 2(b), we have \(p_{i+1} = 1\). But then lemma 2(a) implies \(p_i = 1\), a contradiction. \textit{Q.E.D.}

**LFE calculation for the 3 period bargaining game.** We assume the prior to be such that 

\[
Prob(t_1) = \frac{1}{6} \text{ for } t_1 \in \{01, 11, 21, 31, 41, 51\}, \text{ and independently, } Prob(t_2) = \frac{1}{5} \text{ for } t_2 \in \{02, 12, 22, 32, 42\}. \]

Let \(x_1\) (respectively \(x_3\)) denote the first mover’s (player 1)’s, denoted as P1, demand for himself in the first stage (fifth stage), when the period number is one (three) and the size of pie is 1000 (respectively 360). Thus, \((1000 - x_1)\) (respectively \((360 - x_3)\))
Table 15. LFE Strategies for the Sequential Bargaining Game

Notes: $x_1$, $x_3$ are the first and third period demands, respectively, of the first mover. $X_1$, $X_3$ are the maximum first and third period demands, respectively, of the first mover that are acceptable to the second mover in those periods. $y_2$ is the maximum offer of the second mover to the first mover from the second period pie. $Y_2$ is the first mover’s minimum acceptable amount from second period pie.

is the share of the first stage (fifth stage) pie offered to player 2, denoted P2. $y_2$ denotes P2’s offer to player 1, in the third stage, when the period number is two and the size of pie is 600. Thus $(600 - y_2)$ is the share of the third stage pie demanded by P2 for himself.

We summarize the LFE strategies in Table 15. $X_1$ (respectively $X_3$) denotes the maximum share of P1 out of the first (third) period pie, such that $(1000 - X_1)$ (respectively $(360 - X_3)$) is acceptable to P2. $Y_2$ denotes the minimum share of the second period pie offered by P2 to P1, such that it is acceptable to P1. As per the definition of LFE, we construct the LFE starting with the SE of $CG(1)$. In what follows, we specify the SE and LFE beliefs only when needed to determine optimal actions.

Step 1: in the unique\(^\text{73}\) SE(1) of $CG(1)$, all P1 types choose $x_1 = 1000$, and believe the prior distribution on P2 types. As $D^1$ consists of only 0₁’s move at stage I, the LFE action of 0₁ at stage I is $x_1 = 1000$.

\(^{73}\)This uniqueness is only of the SE strategy profile, not the belief profile. We mean the same thing by uniqueness in what follows.
the unique $SE(2)$ of $MCG(2)$, all P2 types at stage II accept if $x_1 \leq 700$, regardless of belief on P1 types. Thus, in $SE(2)$, all P1 types other than 0, choose $x_1 = 700$, regardless of belief. We note LFE actions at $D^2$, which contains $t'_1 s$ information set at stage I and $o'_2 s$ information set at stage II.

Step 3: fix the LFE actions at $D^1, D^2$ as Nature’s moves to convert $CG(2)$ to $MCG(3)$. In the unique $SE(3)$ of $MCG(3)$, all P2 types at stage III choose $y_2 = 0$, irrespective of belief. Thus, the stage II $SE(3)$ action for all P2 types is to accept if $x_1 \leq 700$, regardless of belief on P1 types. Thus, in $SE(3)$, all P1 types offer $x_1 = 700$, regardless of beliefs. We note the LFE actions at $D^3$, which contains $2'_1 s$ information set at stage I, $1'_2 s$ information set at stage II, and $0'_2 s$ information set at stage III.

Step 4: fix the LFE actions at $\bigcup_{n=1}^{3} D^n$, solved above, as Nature’s moves to convert $CG(4)$ to $MCG(4)$. In the unique $SE(4)$ of $MCG(4)$, all P1 types at stage IV accept if $y_2 \geq 180$, irrespective of belief. Therefore, the stage III $SE(3)$ action is for all P2 types to choose $y_2 = 180$, regardless of belief on P1 types. Therefore, the stage II $SE(4)$ action is for all P2 types to accept if $x_1 \leq (1000 - (600 - 180)) = 580$, regardless of belief on P1 types. Therefore, in $MCG(4)$, at stage I, 3, 1, 4, and 5 (others replaced by Nature) face an expected payoff of $388 (= \frac{2 \times 700}{5} + \frac{3 \times 180}{5})$ from choosing $x_1 = 700$ versus an expected payoff of 580 from choosing $x_1 = 580$, given that they must have belief as per the prior distribution in $SE(4)$. Therefore, in $SE(4)$, at stage I, 3, 1, 4, and 5 choose $x_1 = 580$. We note the LFE actions at $D^4$, which contains $3'_1 s$ information set at stage I, $2'_2 s$ information set at stage II, and $1'_2 s$ information set at stage III, and $0'_4 s$ information set at stage IV.

Step 5: fix the LFE actions at $\bigcup_{n=1}^{4} D^n$, solved above, as Nature’s moves to convert $CG(5)$ to $MCG(5)$. We now describe the unique $SE(5)$ of $MCG(5)$. All P1 types at stage V choose $x_3 = 360$, irrespective of belief. Therefore, the stage IV $SE(5)$ action is for all P1 types to accept if $y_2 \geq 180$, regardless of belief on P2 types. Given this, the stage III $SE(5)$ action is for all P2 types to choose $y_2 = 180$. Therefore, the stage II $SE(5)$ action for all P2 types is to accept if $x_1 \leq 580$. Therefore, in $SE(5)$, all P1 types choose $x_1 = 580$ given beliefs determined by the prior distribution on P2 types. We note the LFE actions at $D^5$, which contains $4'_1 s$ information set at stage I, $3'_2 s$ information set at stage II, $2'_2 s$ information set at stage III, $1'_1 s$ information set at stage IV, and $0'_1 s$ information set in
stage V.

Last step: fix the LFE actions at $\bigcup^{5}_{n=1} D^n$, solved above, as Nature’s moves to convert $\Gamma$ to $MCG(6)$. We now describe the unique $SE(6)$ of $MCG(6)$. In $SE(6)$, all P2 types at stage VI accept $x_3 \leq 360$, irrespective of belief. Given this, the stage V $SE(6)$ action is for all P1 types to choose $x_3 = 360$, regardless of belief on P2 types. Therefore, the stage IV $SE(6)$ action is for all P1 types to accept $y_2 \geq 360$. Note that $3_2$ and $4_2$, at stage III, know that if $x_1 = 1000$, then with probability one, P1’s type is 01, who will accept $y_2 = 180$ in stage IV, thus conditional on $x_1 = 1000$, 32 and 42 offer $y_2 = 180$.

Conditional on $x_1 = 700$, 32 and 42, at stage III, know that P1’s type is 11 or 21 with equal probability. However, the offer of 180 in that case will be rejected with probability $\frac{1}{2}$, and lead to an expected payoff of $210 \left(= \frac{(600-180)}{2} + \frac{0}{2}\right)$, which is less than the payoff from offering 360, and getting a payoff of 240 for sure. Thus, conditional on $x_1 = 700$, 32 and 42 offer $y_2 = 360$. Therefore in $SE(6)$, in stage II, 42 accepts $x_1 \leq 760$. Off equilibrium, 42 believes P1’s type must be 51. In stage I, 51 evaluates the expected payoff from $x_1 = 580$, or 700, or 760, given that his beliefs on P2 types are determined by the prior distribution, he chooses $x_1 = 580$ in $SE(6)$. We have now solved for the LFE actions of all the information sets of $\Gamma$, which completes the solution stated in Table 15 below.
Appendix B: Data Analysis for Chapter Two

Treatment 1 Data Analysis

Difference in Proportion of “First Mover” Choice by Round and Opponent’s Type.  
We analyze the round-by-round behavior of Exp subjects between rounds 13-20 in Table 16. For example, in round thirteen, 42 percent of the Exp subjects who faced an Inexp subjects chose $F$, while 8 percent of the Exp subjects who faced an Inexp opponent chose $F$. This difference is significant at 1 percent, with a p-value of 0.001. The difference in proportion of Exp subjects choosing $F$ based on opponent being Exp or Inexp remains significant from round 13 through round 17. The disappearance of this difference after round 17 suggests that by round 18, almost all the Inexp subjects had acquired experience and learnt the “sure-win” perfect strategy of the second mover in $A9_{sub}$, and that the Exp subjects were aware of this.

Learning Speed Difference.  
Table 17, second row, reports the percentage of $F$ choices of Exp players playing other Exp players in their first 8 rounds of playing the Avoid 9 game. The fourth row of Table 17 reports the percentage of $F$ choices made by Inexp players when facing another Inexp player in their first 8 rounds, which are rounds 13-20 of the session, of playing the Avoid 9 game. The bottom row reports the p-value comparing the second row to the fourth row, for each column. In their first three rounds, both Exp and Inexp subjects behave similarly, but for their rounds 5, 6, and 7 (session round numbers 17,18 and 19), Inexp subjects choose $F$ in a significantly smaller percentage of cases against other Inexp players, as compared to the Exp players in their rounds 5, 6 and 7 of playing the Avoid 9 game.

The findings in Table 17 are confirmed by a probit estimation in Table 18. We set the
Table 16. Percentage of “First Mover” choices of Exp Subjects by Round, Opponent’s Experience

<table>
<thead>
<tr>
<th>Round (Exp)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp against Exp</td>
<td>78%</td>
<td>64%</td>
<td>55%</td>
<td>47%</td>
<td>37%</td>
<td>36%</td>
<td>18%</td>
<td>10%</td>
</tr>
<tr>
<td>Round (Inexp)</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>Inexp against Inexp</td>
<td>82%</td>
<td>70%</td>
<td>53%</td>
<td>34%</td>
<td>10%</td>
<td>14%</td>
<td>0%</td>
<td>4.5%</td>
</tr>
<tr>
<td>p-value of difference</td>
<td>0.65</td>
<td>0.5</td>
<td>0.39</td>
<td>0.19</td>
<td>0.002</td>
<td>0.011</td>
<td>0.007</td>
<td>0.237</td>
</tr>
</tbody>
</table>

Table 17. Percentage of “First Mover” choices of Exp against Exp, Inexp against Inexp

Notes: The p-values are two-tailed p-values of difference in proportions between the proportions in the second row and the proportions in the fourth row.

<table>
<thead>
<tr>
<th>Subject is Exp</th>
<th>(1): Perfect play</th>
<th>(2): choosing F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.27*(0.14)</td>
<td>0.24**(0.12)</td>
</tr>
<tr>
<td>Controls</td>
<td>0.84***(0.25)</td>
<td>0.36(0.25)</td>
</tr>
</tbody>
</table>

Table 18. Treatment 1 Learning Speed Difference

Notes: Model (1) dependent variable takes value 1 when the selected second mover plays perfectly. Model (2) dependent variable takes value 1 when the player chooses F. Figures in parenthesis are robust standard errors. Column 2 has 460 observations with 147 clusters, and column 3 has 920 observations with 154 clusters.

F/S decisions of Exp against Exp and Inexp against Inexp in their respective first 8 rounds as the dependent variable. We use session dummies as controls, and one other independent variable: a dummy for “the player is Exp.” The probit result is the following. We find that the dummy for “the player is Exp” significantly increases the probability (p-value 0.037) of choosing F compared to the baseline of “the player is Inexp.”

Treatment 2 Data Analysis

Difference in Proportion of “First Mover” Choice by Round and Opponent’s Performance in C13. We report the round-by-round behavior of Exp subjects based on oppo-
Table 19. Proportion of “First Mover” Choice of Exp Subjects in H13 by Round and Opponent’s Performance

<table>
<thead>
<tr>
<th>Round</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opponent lost C13</td>
<td>40%</td>
<td>62%</td>
<td>37%</td>
<td>41%</td>
<td>36%</td>
<td>28%</td>
</tr>
<tr>
<td>Opponent won C13</td>
<td>39%</td>
<td>26%</td>
<td>21%</td>
<td>20%</td>
<td>11%</td>
<td>12%</td>
</tr>
<tr>
<td>p-value of two-tail test</td>
<td>0.94</td>
<td>0.015**</td>
<td>0.25</td>
<td>0.124</td>
<td>0.046**</td>
<td>0.305</td>
</tr>
</tbody>
</table>

nent’s performance for rounds 9-14 in Table 19. For example, in round 10, Exp subjects chose $F$ 62 percent of the times against an opponent who lost C13 versus the computer, and 26 percent of the times against an opponent who won C13 versus the computer. This difference is significant at LOS 5 percent with a two tailed p-value of 0.015.

**Result 3(b): Faster learning speed of Inexp subjects:** During their 6 rounds in the combined sub-session, the Inexp subjects learned the perfect strategy in the Computer 13 game faster than Exp subjects in their first 6 rounds in the training sub-session.74

Figure 15 depicts the win rate of Inexp and Exp subjects in the C13 part of their respective first 6 rounds. The win rate is defined as the number of wins of type $i$ subjects in the C13 part divided by total number of type $i$ subjects, where $i \in \{\text{Inexp, Exp}\}$. Table 20 (below), second row, reports the percentage of wins of Inexp players playing the computer in the C13 part of their first 6 rounds (rounds 9-14 of the session), by round. The third row reports the same for Exp subjects in their first six rounds (rounds 1-6 of the session). In their first 4 rounds, both Exp and Inexp subjects perform statistically equivalently (see Table 20). But in their rounds 5 and 6 (rounds 13 and 14 of the session), Inexp subjects win a significantly higher proportion of their C13 games, as compared to the Exp players in the rounds 5 and 6 of the training sub-session. The p-values of these differences are 0.035 and 0.017, respectively. Recall that Inexp subjects, unlike the Exp subjects, do not get information about how their opponent played the C13 part of a round. So Inexp subjects have lesser information in their rounds 2 (round 10 of the session) through 6 (round 14 of the session),

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74We focus on the C13 game because it provides a clean measure of the proportion of subjects who understood the perfect strategy. It is a weakly dominant strategy to play perfectly (strictly dominant given that subjects were told that the computer plays perfectly) in this game. The $F/S$ decision in C13 was not dependent on the belief about the ability of the human opponent to play perfectly. Further, the only way to win against the computer was to play perfectly at each decision node, including the $F/S$ decision stage.
Figure 15. Learning Speed Comparison (Treatment 2)

Notes: The figure depicts the behavior of experienced subjects in rounds 1-6 of the session and the inexperienced subjects' behavior in rounds 9-14 of the session (which are the inexperienced subjects' first 6 rounds). The figure depicts the round-wise proportion of experienced subjects who won C13 (black dots) and the proportion of inexperienced subjects who lost C13 (red triangles). It is a dominant strategy to play perfectly in C13 and win. The latter proportion is significantly higher, with a p-value < 0.05, for rounds 5 and 6. This shows the faster learning speed of the inexperienced subjects.

<table>
<thead>
<tr>
<th>Round (Inexp)</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inexp vs Computer</td>
<td>7.4%</td>
<td>22.2%</td>
<td>44.4%</td>
<td>65%</td>
<td>76%</td>
<td>83.3%</td>
</tr>
<tr>
<td>Round (Exp)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Exp vs Computer</td>
<td>16.7%</td>
<td>23%</td>
<td>37.5%</td>
<td>52%</td>
<td>56%</td>
<td>60.4%</td>
</tr>
<tr>
<td>p-value of two-tail test</td>
<td>0.147</td>
<td>0.933</td>
<td>0.477</td>
<td>0.192</td>
<td>0.035**</td>
<td>0.017**</td>
</tr>
</tbody>
</table>

Table 20. Exp and Inexp Subjects’ Percentage of Wins in the C13 Part of Respective First Six Rounds

compared to Exp subjects in their rounds 2 through 6. However, like for result 3(a), the data does not provide a clear answer. That is, separating Inexp subjects by how often they played Exp subjects produces no systematic pattern of learning difference.

Result 4(b): The rate of imperfect play declined significantly as the players got closer to the end of the H13 game. That is, as the position increased, or as the number of items left reduced, the rate of imperfect actions reduced. In particular, no subject lost from a winning position with 4 or lesser items left in the H13 games played in the combined sub-session.
Table 21. Rate of Imperfect Play by Position

<table>
<thead>
<tr>
<th>Position</th>
<th>1-3</th>
<th>5-7</th>
<th>9-11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session</td>
<td>21</td>
<td>5</td>
<td>0.8</td>
</tr>
<tr>
<td>Exp: first 3 rounds</td>
<td>45.5</td>
<td>21.7</td>
<td>3.5</td>
</tr>
<tr>
<td>Inexp: first 3 rounds</td>
<td>34.8</td>
<td>12.2</td>
<td>0</td>
</tr>
</tbody>
</table>

Notes: The figures are in percentage. The figures are reported from positions from which a perfect action is distinguishable from an imperfect action. A perfect action makes the position 4, 8, and 12 from a position in \{1, 2, 3\}, \{5, 6, 7\}, and \{9, 10, 11\} respectively.

Table 22. Proportions of Choices from Losing Positions

<table>
<thead>
<tr>
<th>Positions</th>
<th>Session</th>
<th>Exp in Combined sub-session</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Items Removed</td>
<td>Items Removed</td>
</tr>
<tr>
<td>0</td>
<td>40.7</td>
<td>41.1</td>
</tr>
<tr>
<td>4</td>
<td>48</td>
<td>43.4</td>
</tr>
<tr>
<td>8</td>
<td>53.3</td>
<td>53.3</td>
</tr>
</tbody>
</table>

Notes: The figures are in percentages. The p-values are two tailed p-values comparing the proportion of times “1” was chosen to the proportion of the next most chose alternative.

Table 21 shows the observed rates of imperfect play by position in the H13 games played in treatment 2. We show the rates separately for three parts of the data: (i) session (training and combined sub-sessions together); (ii) the first three rounds of Exp subjects in their training sub-session; and (iii) the first three rounds of Inexp subjects in the combined sub-session. Recall that a position is a particular sum of items removed. For each of (i), (ii), and (iii), a test of difference in proportions reveals that the rate of imperfect play is significantly more (p-values < 0.01) from a position in 1-3 (10-12 items remaining) as compared to 5-7 (6-8 items remaining), and from the latter compared to a position in 9-11 (2-4 items remaining).

**Result 5(b):** Choice proportions at losing positions in H13. At the losing positions 0, 4, and 8 in H13, subjects were significantly more likely (with p-values of approximately 0 for each) to choose to remove 1 item than the next most likely alternative. This results also holds true for Exp subjects in the combined sub-session. These results can be seen in Table 22 below.
Table 23. Ex-post Mean Earnings of Exp Subjects in H13 vs. Opponent who Lost C13

<table>
<thead>
<tr>
<th>Round</th>
<th>Earning as Second Mover</th>
<th>Earning as First Mover</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>9-14</td>
<td>187.7 (18)</td>
<td>193.2 (27.8)</td>
<td>0.8</td>
</tr>
<tr>
<td>9-10</td>
<td>181.8 (23.2)</td>
<td>163.9 (35)</td>
<td>0.427</td>
</tr>
<tr>
<td>11-12</td>
<td>200 (32.2)</td>
<td>320 (53.9)</td>
<td>0.035**</td>
</tr>
<tr>
<td>13-14</td>
<td>185 (45)</td>
<td>162.5 (67.5)</td>
<td>0.743</td>
</tr>
</tbody>
</table>

**Result 6(b):** Ex-post optimality of being “First Mover” in H13 against an opponent who lost C13.

In the combined sub-session (rounds 9-14), when the opponent lost C13, the average earnings of the Exp player was weakly more when the Exp player was the selected first mover in H13 rather than when he was the selected second mover. Thus, in the combined sub-session, it was ex-post optimal for an Exp player to choose $F$ in the H13 game when facing an opponent who lost his/her C13 game with the computer. This can be seen in the Table 23 below.

**LFE Details**

**Avoid 9 Game**

We first complete our description of the LFE for the Avoid 9 game.

The **type-1** player has foresight level of 1. Let $h$ be an arbitrary move of the type-1 player. Let $a^0$ be an action available at $h$, and $a^1$ be an action available in the next stage following $h \rightarrow a^0$. Then from $h$, the type-1 player observes all sequences \{h $\rightarrow a^0 \rightarrow a^1 \rightarrow \text{curtailed payoff profile}\}. At the F/S decision stage, the type 1 player observes all actions of the selected first mover, each action with an associated curtailed payoff profile. The curtailed payoff profile for all observable sequences is $(\frac{500+50}{2}, \frac{200+50}{2})$. Thus, the type-1 player chooses $F$. Similarly, from $P \leq 1$, the type-1 player observes that for all observable sequences, the curtailed payoff profile is $(\frac{500+50}{2}, \frac{200+50}{2})$. Thus he randomizes among available actions at $P \leq 1$. Both these decisions are regardless of the opponent’s type or the succeeding action.

From $P = 2$, the opponent’s move in the next stage matters for the type-1 player: if
he thinks that his opponent will play perfectly if the sum of items removed is 5 or more, then he should choose an action such that \( a < (5 - P) \). How does the LFE deal with the limited foresight type’s perception about other players’ types’ actions? To answer this, we first define total foresight. Let a limited foresight type’s total foresight be the sum of (i) the stage number that the limited foresight type is moving at, and (ii) the level of foresight of that limited foresight type. Consider a player-type, X, at a move \( h \). X has a certain total foresight. X is observing a curtailed version of the underlying game of perfect information. For our purposes, the LFE definition boils down to three **rules of thumb**: (a) X knows the LFE actions of the player types with lesser total foresight than him; (b) X assumes that equal or higher total foresight types play a sequentially rational strategy based on the curtailed version of the underlying game he observes at \( h \), and X chooses an action at \( h \) that is a sequentially rational best response to such a strategy profile; and (c) X’s perceived beliefs of equal or higher total foresight types, used to determine their sequentially rational strategy in (b), are Bayes’ rule consistent with the strategy profile that obeys (a) and (b).

By rule-of-thumb (b), the type-1 player assumes that any type of his opponent will play perfectly to attain the winner’s payoff given a move at \( P \geq 5 \). Thus, for \( 1 < P < 4 \), the type-1 player randomizes over the action set \( \{ a : P + a < 5 \} \). From \( P \geq 4 \), the type-1 player has enough total foresight to play perfectly.

From any move \( h \), the **type-2** player observes all sequences \( \{ h \rightarrow a^0 \rightarrow a^1 \rightarrow a^2 \rightarrow \text{curtailed payoff profile} \} \), where \( a^1 \) is an action available at the next stage after \( h \rightarrow a^0 \), and \( a^2 \) is an action available at \( h \rightarrow a^0 \rightarrow a^1 \). At the F/S decision stage, the type 2 player chooses \( F \) regardless of beliefs because after every observable sequence, the curtailed payoff profile is \( \left( \frac{500+50}{2}, \frac{200+50}{2} \right) \). As the selected first mover with \( P = 0 \), by rule (b), the type-2 player chooses \( a^0 = 3 \). From \( P \geq 1 \), the type-2 player plays perfectly observing that choosing \( a^*_4 \) and \( a^*_8 \) guarantees a winner’s payoff.

From \( P \geq 1 \), for identical reasons as the type-2 player, the type-\( f \) player also plays

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\( ^{75} \)This is because, given type-0’s LFE action at \( P \leq 3 \), removing 3 items maximizes the probability that his opponent will choose \( a^1 \) such that \( a^0 + a^1 > 4 \). And if \( a^0 + a^1 > 4 \), then the type-2 player can choose \( a^2 = a^*_4 \) to obtain the winner’s payoff as the curtailed payoff. Due to his limited foresight, this is the only sequence observable to him in which he wins as the first mover. Notice that the only position at which the belief about the opponent’s type matters for type-2 is at \( P = 0 \). If the type-2 player puts positive probability on the opponent being type-0, he removes 3 items as the first mover at \( P = 0 \). This belief is consistent with the type-0 player randomizing at the F/S decision stage.
perfectly. As the first mover at stage two (i.e., at $P = 0$), the type-$f$ player removes 1 item to maximize the probability that the opponent (type-0 or type-1) will not play $a^*_1$. The maximized probability is $(p_0 + p_1) \times \frac{2}{3}$. Thus, the type-$f$ player chooses F iff his belief on the opponent being type-0 or type-1 is high enough.

**Human 13 and Computer 13 Games**

At the F/S decision stage, the **type-0** player chooses F/S with probability $\frac{1}{2}$ each, as he observes that the curtailed payoff profile is (275,275) after both F and S. From any move with $P < 9$, the type-0 player observes the curtailed payoff profile (275,125) after every available action. Therefore from any such move, the type-0 player randomizes uniformly among 1, 2, 3. The threshold position after which all foresight types, including the type-0 player, play perfectly is 9, as opposed to 5 for the Avoid 9 game. This is because in H13, if $P \geq 9$, then the curtailed payoff after $a^*_1 = (12 - P)$ is the winner’s payoff.

The **type-1** player, from any move $h$, observes all sequences $\{h \to a^0 \to a^1 \rightarrow \text{curtailed payoff profile}\}$. The type-1 player chooses F at the F/S decision stage, as he observes only (275,125) as the possible curtailed payoff profile for all observable sequences from the first stage. For any move where $P \leq 5$, the type-1 player observes the curtailed payoff profile (275,125) after any observable sequence of actions. Therefore if $P \leq 5$, the type-1 player randomizes uniformly among $\{1, 2, 3\}$. From $P \in \{6, 7\}$, the type-1 player puts equal weight on $a^0$ such that $P + a^0 < 9$, because he assumes (by rule (b)) that if $P + a^0 \geq 9$, any type of his opponent will choose $a^1 = a^*_1$, giving him the loser’s payoff. At $P = 8$, the type-1 player is assured a loss, and thus he randomizes uniformly among $\{1, 2, 3\}$. From $P \geq 9$, the type-1 player plays perfectly.

The **type-4** player has a high enough foresight level such that, except at $P = 4$, and $P = 0$, he plays perfectly to attain the winner’s payoff. At $P = 0$, i.e., as the first mover beginning the game, the type-4 player removes 3 items, and at $P = 4$, he removes 1

---

6The choice at $P = 0$ is sequentially rational because the type-4 player’s limited foresight at stage two allows him to observe only one sequence where he wins as the first mover: if the second mover makes the sum of items removed more than 4 after the third stage move.
At the F/S decision stage, the type-4 player observes that if the first mover chooses 1 item at each move, then the curtailed payoff profile after the fifth stage action can only be (275, 125). As 275 > 200, the maximum possible for the second mover, the type-4 player chooses F.

The type-\( f \) player plays exactly as the type-4 from \( P \geq 1 \) for identical reasons. As the first mover, the type-\( f \) player randomizes because he knows (by rule (a)) that the probability of an opponent’s mistake is the same from \( P = 1, 2 \) or 3. At the F/S decision stage, the type-\( f \) player chooses F iff his updated belief (updating happens only if he observes opponent’s actions in C13; the updating uses Bayes’ rule and the LFE of C13) on the opponent being type-0 or type-1 (belief on type-0 and type-1 opponent is denoted as \( u_0 \) and \( u_1 \), respectively) is high enough. The probability that the opponent makes a mistake as the second mover is \((u_0 + u_1)(\frac{2}{3} + \frac{1}{3} \cdot \frac{2}{3})\). Thus the threshold for \( u_0 + u_1 \) is 0.375, beyond which the type-\( f \) player chooses F.

In the LFE for the C13 (Computer 13) game, there are only one two changes compared to the LFE for the H13 game: (i) in the first stage, types 1 and 4 play F/S with 50% chance each. This is because in C13, winning as the first or the second mover gives the same payoff: 500. (ii) Types 4 and \( f \) do not favor any particular action from \( P = 4 \) or \( P = 1 \), as they are playing against the computer and they are informed that the computer plays perfectly.

**Data Categorization in Treatment 2**

Recall that stage 1 of the H13 game is the F/S decision stage. Let I denote the selected first mover in H13, and II denote the selected second mover in H13. We first categorize the data from \( H13_{sub} \) into three categories: (a) I played an arbitrary action at \( P = 0 \), II played imperfectly at stage three, I made the sum 4 or 8 (whichever was possible) in stage four; (b) I played an arbitrary action in stage two, and II played perfectly (made the sum of items removed 4) in stage three; (c) I played an arbitrary action in stage two, II played imperfectly in stage three, I also failed to make the sum 4 or 8 in stage four. These categories can be safely interpreted as (a) arbitrary-imperfect-perfect; (b) arbitrary-perfect; (c) arbitrary-imperfect-imperfect.

\(^{77}\) At \( P = 4 \), the type-4 player attains full total foresight in the H13 game. Because of rule (a), the choice at \( P = 4 \) is optimal with respect to the LFE, as it maximizes the probability that the type-0 or type-1 opponent will not play \( a_8^* \).
and (c) arbitrary-imperfect-imperfect. This is because in every case in which a player made the sum 8, two stages later he made the sum 12 and won the game. Further, in 97 percent of the data, if a player made the sum 4, two stages later, he made the sum 8.

Each of (a), (b), and (c) has three distinct cases possible for the choices in the $F/S$ stage of $H_{13}$. These three cases are: (1) in $H_{13}$’s $F/S$ decision stage, I chose $F$ and II chose $F$; (2) I chose $F$ and II chose $S$; (3) I chose $S$ and II chose $S$. For each of (1)-(3) in the $F/S$ decision stage of $H_{13}$, we have (a)-(c) in $H_{13,sub}$, which makes 9 categories.

Treatment 2’s combined sub-session also had two types of players playing together: Inexp and Exp. While the Exp subjects were informed about their opponent’s play in $C_{13}$ before the beginning of $H_{13}$, the Inexp subjects were uninformed about their opponent’s play in $C_{13}$. Further, there were four possible outcomes for a pair of subjects from the respective play of the two members of the pair against their respective computer in the $C_{13}$ part of the round: Win-Win, Win-Loss, Loss-Win, and Loss-Loss, where the first term is I’s outcome in $C_{13}$ and the second term is II’s outcome in $C_{13}$. Thus, we need to further broaden the categories of outcomes for treatment 2.

We make a total of 81 categories for the data from treatment 2. The 9 categories above are repeated in 9 cases. Case (i): $(Exp_W, Inexp_W)$, which denotes that I is Exp and he won his $C_{13}$ game, while II is Inexp and he also won his $C_{13}$ game. Case (ii): $(Inexp_W, Exp_W)$, with the order from (i) swapped. Case (iii): $(Exp_W, Inexp_L/Exp_L)$, which denotes that I is Exp and he won his $C_{13}$ game, while II lost his $C_{13}$ game, and he can be either Inexp or Exp. 78 Case (iv): $(Inexp_L/Exp_L, Exp_W)$ is the same as case (iii) with the order swapped. Proceeding similarly we make case (v): $(Exp_W, Exp_W)$; (vi): $(Inexp_W, Inexp_W)$; (vii): $(Inexp_W, Inexp_L/Exp_L)$; (viii) $(Inexp_L/Exp_L, Inexp_W)$; (ix) $(Inexp_L, Inexp_L)$. For each of the cases (i)-(ix), the categories (1)-(9) are repeated, making 81 total categories.

78 We club the $Exp_L$ and $Inexp_L$ I or II in one category, as the Exp player who loses in $C_{13}$ cannot be type-$f$ according to LFE. Thus, he has the same strategy in LFE, regardless of his beliefs. That is, in LFE, learning about the opponent’s type within the round does not make a difference to $Exp_L$’s strategy, while $Inexp_L$ cannot learn by design. On the other hand, the sequential level-k model implies that an observation where one of the group members loses $C_{13}$ has 0 probability (not accounting for the error term: $\epsilon$). Thus all such observations are treated homogeneously in their model. The AQRE model also does not account for any learning, and therefore we reduce the number of categories without affecting the result.
The probability of a player’s action from a position in C13 being perfect is given by $P_3 = \frac{\lambda}{2}(\exp(500\lambda)/(\exp(500\lambda) + 2\exp(50\lambda)) + \frac{1}{2}(\exp(500\lambda)/(\exp(500\lambda) + \exp(50\lambda)))$. The probability of the fifth stage action being perfect is given by $P_2 = \exp(\lambda(P_5500 + (1 - P_350))/\exp(\lambda(P_5500 + (1 - P_350) + 2\exp(50\lambda)))$. Similarly, the probability of the third stage action being perfect is given by $P_1 = \exp(\lambda(P_2P_2500 + (1 - P_2P_350))/\exp(\lambda(P_2P_2500 + (1 - P_2P_350) + 2\exp(50\lambda)))$, and $P(S) = \exp(\lambda(P_1P_2P_3500 + (1 - P_1P_2P_350))/\exp(\lambda(P_1P_2P_3500 + (1 - P_1P_2P_350) + 2\exp(50\lambda)))$. Finally $P(W) = P(2^{nd} mover)P_1P_2P_3$. 

### Table 24. Updated Distributions Due to C13 Win or Loss

<table>
<thead>
<tr>
<th></th>
<th>LFE</th>
<th>Level-k</th>
<th>AQRE</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>($\frac{p_0}{18}, \frac{11p_1}{108}, \frac{p_4}{2}, p_f$)</td>
<td>Initial Distribution Maintained</td>
<td>$P(W)$</td>
</tr>
<tr>
<td>L</td>
<td>($\frac{p_0}{18}, \frac{97p_1}{108}, \frac{p_4}{2}, 0$)</td>
<td>$P(\text{observation}) = 0$</td>
<td>$1 - P(W)$</td>
</tr>
</tbody>
</table>
Subject Instructions

Treatment 1: INSTRUCTIONS for Type D (Inexp) Subjects

This is an experiment in the economics of decision making. The Ohio State University has provided funds for conducting this research. The instructions are simple, and if you follow them carefully and make good decisions, you may earn a CONSIDERABLE AMOUNT OF MONEY which will be PAID TO YOU IN CASH at the end of the experiment.

Please pay careful attention to the instructions. If you have any questions at any time, raise your hand.

Do not communicate with other participants during the experiment. Put your phone away and make sure it is in silent mode or turned off.

Upon entry you were each assigned a unique subjectID, which you will be asked to enter several times. Please keep this carefully.

This is the second (first part was bargaining) of the two parts of the experiment. You will play 8 rounds of the AVOID Removing the 9th Item game described below. Your total payment will be a sum of your payment from the two parts of the experiment, and your show up fee ($5).

In each round of the AVOID Removing the 9th Item Game:

1. Each of you will be asked to fill in your subjectID. Please do so accurately. Then each of you will be randomly assigned a new unique participant whom you will interact with. You will not know the identity of your assigned participant. We shall refer to him/her as “the other.” Decisions made by you and the other in a particular round will affect how many experimental currency units you earn in that round. You will be told if the other is of type S or type D. Type S participants have already played 12 rounds of the AVOID Removing the 9th item game. Type D participants (like you) participated in a different, completely unrelated first part of the experiment, and are now playing the second part’s AVOID removing the 9th item game with other type D and type S participants.

2. Once the game begins, you will be asked to decide between being the first mover (you
get to make the first choice) or the second mover (the other makes the first choice) in the subsequent task. The computer will choose one of you or the other with 50% chance each, and implement their first/second mover choice.

3. You and the other will take subsequent decisions alternately.

4. You will see all your past choices and all the past choices of the other at all points during the round.

5. You and the other will alternately choose the number of items to remove from a box containing 9 items. You can only choose to remove 1, 2, or 3 items with any given choice. You can’t choose to remove 0 items. Of course, you can’t choose to remove more items than there are left in the box. For example: if 7 items have been removed before your move, you can only choose to remove either 1 or 2 items at your move. The round ends when the 9th item is removed:

   (a) The one who removes the 9th item from the box receives 50 experimental currency units (ECUs) for the round.

   (b) If the other removes the 9th item, and you are the first mover, you receive 500 ECUs.

   (c) If the other removes the 9th item, and you are the second mover, you receive 200 ECUs.

You will play 8 rounds of this game in the second part. In each round, the other may be of type S or type D. The type of the other will be communicated to you at all points within a round.

Your earnings for the experiment: part one earning will be equal to your earning in a randomly selected round from part one. Part two earning will be equal to your earning in a randomly selected round from part two. Your total earnings will be a sum of your show-up fee and your earnings from each of the two parts of the experiment.

The conversion rate for ECU in the experiment is: 1USD=80ECUs for the first part, and 1USD=60ECUs for the second part. Your earnings will be rounded up to the nearest dollar.
Treatment 1: INSTRUCTIONS for Type S (Exp) Subjects

This is an experiment in the economics of decision making. The Ohio State University has provided funds for conducting this research. The instructions are simple, and if you follow them carefully and make good decisions, you may earn a CONSIDERABLE AMOUNT OF MONEY which will be PAID TO YOU IN CASH at the end of the experiment.

Please pay careful attention to the instructions. If you have any questions at any time, raise your hand.

Do not communicate with other participants during the experiment. Put your phone away and make sure it is in silent mode or turned off.

Upon entry you were each assigned a unique subjectID, which you will be asked to enter several times. Please keep this carefully.

The experiment has two parts. In both parts, you will play several rounds of the AVOID Removing the 9th Item game described below. Your total payment for the experiment will be a sum of your payment from the two parts plus your show up fee ($5).

In each round of the AVOID Removing the 9th Item Game:

1. Each of you will be asked to fill in your subjectID. Please do so accurately. Then each of you will be randomly assigned a new unique participant whom you will interact with. You will not know the identity of your assigned participant. We shall refer to him/her as “the other.” Decisions made by you and the other in a particular round will affect how many experimental currency units you earn in that round. You will be told if they are of type S, or if they are type D. Type S will participate in several rounds of the same AVOID removing the 9th item game (described here) in both parts of the experiment. Type D will participate in a different, completely unrelated first part. After they finish the first part of the experiment, you will be informed. They will then join you in the second part of the experiment, i.e., they will also participate in several rounds of the AVOID removing the 9th item game in the second part, the same as you. You will be told when the type D subjects join and the second part begins.

2. Once the “Avoid removing the 9th item” game begins, you will be asked to decide between being the first mover (you get to make the first choice) or second mover (the
other makes the first choice) in the subsequent task. The computer will choose one of you or the other with 50% chance each, and implement their first/second mover choice.

3. You and the other will take subsequent decisions alternately.

4. You will see all your past choices and all the past choices of the other at all points during the round.

5. You and the other will alternately choose the number of items to remove from a box containing 9 items. You can only choose to remove 1, 2, or 3 items with any given choice. You can’t choose to remove 0 items. Of course, you can’t choose to remove more items than there are left in the box. For example: if 7 items have been removed before your move, you can only choose to remove either 1 or 2 items at your move. The round ends when the 9th item is removed:

(a) The one who removes the 9th item from the box receives 50 experimental currency units for the round.

(b) If the other removes the 9th item, and you are the first mover, you receive 500 ECU.

(c) If the other removes the 9th item, and you are the second mover, you receive 200 ECU.

You will play 12 rounds of this game in the first part and 8 rounds of this game in the second part. In the first part the other can only be of type S. In the second part, the other may type S or type D. The type of the other will be communicated to you at all points within a round. Part one earning will be equal to your earning in a randomly selected round from part one. Part two earning will be equal to your earning in a randomly selected round part two. Your total earnings will be a sum of your show-up fee and your earnings from each of the two parts of the experiment.

The conversion rate for ECU in the experiment is: 1USD=60ECUs. Your earnings will be rounded up to the nearest dollar.
Treatment 2: INSTRUCTIONS for Type D (Inexp) Subjects

This is an experiment in the economics of decision making. The Ohio State University has provided funds for conducting this research. The instructions are simple, and if you follow them carefully and make good decisions, you may earn a CONSIDERABLE AMOUNT OF MONEY which will be PAID TO YOU IN CASH at the end of the experiment. If you have any questions at any time, raise your hand. Do not communicate with other participants during the experiment. Put your phone away and make sure it is in silent mode or turned off.

The experiment has Two Components. You already completed the first (bargaining) component. In this second component, we will be using a game called “Avoid Removing the 13th Item Game.” This game is played by two players. There is a box containing 13 items. The two players make choices alternately. Each player can choose to remove 1, 2 or 3 items from the box at their move. The players remove 1, 2 or 3 items alternately until 0 items are left in the box. One cannot remove 0 items or more than 3 items. Also, one cannot remove more items than are left in the box. The goal is to avoid removing the last/13th item. The player who makes his opponent remove the 13th item achieves the goal.

You are a type D subject. Type D subjects play a different bargaining game for the first component of the experiment. Type S subjects play the same round described below throughout the experiment including the first component. In this component you will play 6 rounds of the round described below.

In every round of this experiment:

Each of you will be asked to fill in your subjectID. Please do so accurately. You will be assigned a new random subject every round. You will interact with this subject for the whole round, and then randomly re-matched. You will not know the identity of your assigned subject. We shall refer to him/her as “the other” or “the human opponent”. Every round will have 2 parts.
Part 1 of a Round: Play the computer

You and your human opponent will separately play the AVOID Removing the 13th Item Game (Avoid 13th for short) with the computer. The computer plays perfectly to win. That is, if there is a choice or a sequence of choices that the computer can make to win against you, the computer will make that choice or sequence of choices without making any mistake.

When you play the computer:

1. You will decide if you are the first mover or the second mover in the Avoid 13th game vs the computer.

2. Your choice for being the first or second mover vs the computer will be implemented. You will then commence playing the Avoid 13th game versus your computer as the first/second mover, as per your decision.

3. If you avoid removing the 13th item against the computer, you earn 500ECUs from the first part of the round, regardless of your first mover/second mover decision. If you have to remove the 13th item, you earn 50ECUs from the first part. 50ECUs is the minimum payment for participating in each part of the round.

Part 2 of a Round: Play the human opponent

1. You and the human opponent will then play the AVOID 13th Game with each other. Before the Avoid 13th game begins, both you and the human opponent will make the choice between being the first mover or second mover in the Avoid 13th game. One of your or the human opponent’s first/second mover choice will be implemented with 50% chance each.

2. For the second part, if you AVOID Removing the 13th Item as the First mover, you earn 500ECUs, while if you AVOID Removing the 13th Item as the Second Mover, you earn 200ECUs. If you have to remove the 13th item, you earn 50ECUs.

After the round ends, you will be randomly re-matched with another subject.
Your earnings for a round will be the sum of your earnings from the first and the second part of the round. To calculate your earnings from this second component, one round will be randomly selected from the 6 rounds you play.

Your earnings from the experiment will be the sum of your earnings from the two components (bargaining and Avoid 13th game) plus the show-up fee ($5). The conversion rate for ECU in this second component is: 1USD=120ECUs. The conversion rate for ECU in the first component is: 1USD=120ECUs. Your total earnings will be rounded up to the nearest dollar.

**Treatment 2: INSTRUCTIONS for Type S (Exp) Subjects**

This is an experiment in the economics of decision making. The Ohio State University has provided funds for conducting this research. The instructions are simple, and if you follow them carefully and make good decisions, you may earn a CONSIDERABLE AMOUNT OF MONEY which will be PAID TO YOU IN CASH at the end of the experiment. If you have any questions at any time, raise your hand. Do not communicate with other participants during the experiment. Put your phone away and make sure it is in silent mode or turned off.

The experiment has Three Components. We explain the first two components here. The third component is unrelated and doesn’t affect your earning from the first two components.

In the first two components, we will be using a game called “Avoid Removing the 13th Item Game.” This game is played by two players. There is a box containing 13 items. The two players make choices alternately. Each player can choose to remove 1, 2 or 3 items from the box at their move. The players remove 1, 2 or 3 items alternately until 0 items are left in the box. One cannot remove 0 items or more than 3 items. Also, one cannot remove more items than are left in the box. The goal is to avoid removing the last/13th item. The player who makes his opponent remove the 13th item achieves the goal.

Type D subjects play a different bargaining game for the first component of the experiment. You are a type S subject. That means you will play the same round described below throughout the experiment.
In every round of this experiment:

Each of you will be asked to fill in your subjectID. Please do so accurately. You will be assigned a new random subject every round. You will interact with this subject for the whole round, and then randomly re-matched. We shall refer to him/her as “the other” or “the human opponent”. Every round will have 2 parts.

Part 1 of a Round: Play the computer

You and your human opponent will separately play the AVOID Removing the 13th Item Game (Avoid 13th for short) with the computer. *The computer plays perfectly to win. That is, if there is a choice or a sequence of choices that the computer can make to win against you, the computer will make that choice or sequence of choices without making any mistake.*

When you play the computer:

1. You will decide if you are the first mover or the second mover in the Avoid 13th game vs the computer.

2. You will be shown your human opponent’s first mover/second mover decision in his/her interaction with his/her computer. You will then be asked about the type of your human opponent, i.e., your human opponent’s type is ____ (S or D). If you answer correctly, 100ECUs (Experimental Currency Units) will be added to your earning from the round. Please note that this step is only for type S subjects. Type D subjects are never shown their opponent’s moves vs the computer and never asked questions about their opponent’s type.

3. Your choice for being the first or second mover vs the computer will be implemented. You will then commence playing the Avoid 13th game versus your computer as the first/second mover, as per your decision.

4. If you avoid removing the 13th item against the computer, you earn 500ECUs from the first part of the round, regardless of your first mover/second mover decision. If you have to remove the 13th item, you earn 50ECUs from the first part. 50ECUs is the minimum payment for participating in each part of the round.
Part 2 of a Round: Play the human opponent

1. You will be shown the complete history of your human opponent’s moves vs the computer and his/her resulting outcome in the first part of that round. You will again be asked about the type of your human opponent, i.e., your human opponent’s type is ____ (S or D). A correct answer will add a further 100ECUs to your earning from the round. (See the screenshot below). Please note that this step is only for type S subjects. Type D subjects are never shown their opponent’s moves vs the computer and never asked questions about their opponent’s type.

2. You and the human opponent will then play the AVOID 13th Game with each other. Before the Avoid 13th game begins, both you and the human opponent will make the choice between being the first mover or second mover in the Avoid 13th game. One of your or the human opponent’s first/second mover choice will be implemented with 50% chance each.

3. For the second part, if you AVOID Removing the 13th Item as the First mover, you earn 500ECUs, while if you AVOID Removing the 13th Item as the Second Mover, you earn 200ECUs. If you have to remove the 13th item, you earn 50ECUs.

After the round ends, you will be randomly re-matched with another subject.
Your earnings for a round will be the sum of your earnings from the first and the second part of the round, including your earnings from the questions about your human opponent’s type.

As stated above, this experiment will use three components:

First Component: Type S vs Type S. You will play 8 rounds (every round has two parts: play the computer, then play the human opponent) as described above. Every round your human opponent will be randomly redrawn from among the Type S subjects. That is, in the first component your opponent is always of type S. One of these 8 rounds will be randomly drawn to calculate your earning from this component.

Second Component: (Type S or D) vs (Type S or D). You will play 6 rounds as described above. Every round your opponent will be randomly redrawn from all the subjects in the experimental session. That is, in the second component your opponent may be of type S or type D. One of these 6 rounds will be randomly drawn to calculate your earning from this component.

Third Component: Risky choice study. This component is a short and unrelated study with 20 rounds. In each round you will be asked to make a choice among two options. According to your choice, you may end up losing some money earned in the previous two components. Press the 1 and 2 keys to make your choice, as explained below:

Option one, called ACCEPT (on the left of screen) consists of a possible reward and a loss. If you pick this option, the computer flips a fair digital coin (chances are 50-50). In case of heads, you earn additional money. In case of tails, you lose an amount that is subtracted from your previous earnings. To choose this option, press 1. Option two, called
REJECT (on the right) is one reward. If you pick this option, you will get this reward for sure. To choose this option, press 2. At the end of this third component, one of these 20 rounds will be randomly selected by the computer. Your choice in that round will determine your earning in this component. If you chose Accept in the selected round, your earning will be 20% of the amount drawn (reward or loss) from the computer’s coin toss. If you chose Reject in the selected round, your earning will be 20% of the sure amount. Note: This last component will begin with a short practice with no payment. Your earnings will be the sum of your earnings from the three components plus the show-up fee ($5). The conversion rate for ECUs in the first two components is: 1USD=120ECUs. Your total earnings will be rounded up to the nearest dollar.

**Bargaining Instructions: Same across treatments**

This is an experiment in the economics of decision making. The Ohio State University has provided funds for conducting this research. The instructions are simple, and if you follow them carefully and make good decisions, you may earn a CONSIDERABLE AMOUNT OF MONEY which will be PAID TO YOU IN CASH at the end of the experiment.

Please pay careful attention to the instructions. If you have any questions at any time, raise your hand.

Do not communicate with other participants during the experiment. Put your phone away and make sure it is in silent mode or turned off.

Upon entry you were each assigned a unique subjectID, which you will be asked to enter several times. Please keep this carefully.

There are two parts of this experiment. You will be given instructions for part two later. Part two is unrelated to this part. During this first part of the experiment you will participate in several bargaining rounds. At the end of this part of the experiment, one of the bargaining rounds you participated in will be chosen at random for calculating your final payment.

Your total payments in this experiment will be a show up fee ($5), plus the sum of your earnings from the two parts of the experiment.

In each round, each of you will be randomly assigned a new unique participant whom you will interact with. You will not know the identity of your assigned participant. We shall
refer to him/her as “the other.” Decisions made by you and the other in a particular round will affect how many experimental currency units you earn in that round. There shall be a total of 8 rounds of bargaining in part one of the experiment.

In each round of bargaining:

1. The first mover of the round shall be decided randomly by the computer. Suppose the two players in a particular group are Ms. X and Mr. Y. (This is just an example, we don’t match based on gender).

2. The first mover, say, Ms. X, will decide how to split a “pie” of 1000 Experimental Currency Units (ECUs) among herself and the other, say, Mr. Y. In particular, she will specify how much of the pie she wants to give to the other as her “offer” and how much her “demand” is. Her “offer” and “demand” can total up to no more that the size of the pie, i.e., 1000 ECUs. If her “offer” plus “demand” is more than 1000ECUs, the computer will guide her so that she doesn’t make that mistake.

3. The other, Mr. Y, will then view this “offer” to him and “demand” for Ms. X herself and decide whether to Accept this offer or to Reject it with an offer and demand of his own. If Mr. Y accepts the first offer, (then Mr. Y’s offer and demand entry are meaningless as the round ends) the round ends and the first offer becomes Y’s earnings for the round, while Ms. X gets her first demand as the earnings for the round. If Mr. Y doesn’t accept Ms. X’s first offer then he rejects the first offer with an offer for Ms. X and a demand for himself out of a reduced pie of 600ECUs. Again, this offer and demand can’t add up to be more than 600ECUs. If they do, the computer will guide Mr. Y so that he can’t make that mistake.

4. Ms. X will then decide whether to Accept this second offer or to Reject it with a last (third) offer for Mr. Y and a demand for herself out of a pie of 360ECUs. If Ms. X accepts the second offer, the round ends. Ms. X’s last (third) demand and offer can’t add up to be more than 360ECUs. If they do, the computer will guide Ms. X so that she can’t make that mistake. If Ms. X accepts Mr. Y’s offer then her last offer and demand are meaningless, as she will get Mr. Y’s offer and Mr. Y will get his own
demand.

5. If Ms. X rejects the second offer and makes an offer and demand of her own, then Mr. Y will then decide whether to Accept Ms. X’s third and last offer or to Reject it.

6. If Mr. Y accepts this last offer, then the round ends and the last offer becomes his earnings for the round, while Ms. X gets her last demand as the earnings for the round.

7. If Mr. Y rejects the last offer, then both X and Y get 0ECUs for the round and the round ends.

The experiment will use ECUs that will be converted to USD at the rate of 1USD = 80ECUs for this part of the experiment. In addition, you will receive USD5 as your show-up fee for the whole experiment, and your earning from second part (whatever the earning may be). You are not allowed to talk to another participant. Please feel free to ask questions during the experiment by raising your hand. I will come to you and answer your questions. Good luck!
Appendix C: Willingness to Pay Elicitation

Instrument for WTP elicitation

The purchasing game for WTP elicitation, detailed as section H below, is preceded by (a) the participant responding to a socioeconomic survey, and (b) undertaking a sensory evaluation of the two pearl millet varieties and bread made from them. The socioeconomic survey questions are available with the authors on demand.

After Round 1 (subsection H.1), before beginning Round 2 (subsections H.2 and H.3), the participants watched an infomercial. The infomercial provided nutrition information about the high iron content of the HIPM, and the benefits of adequate iron in the diet. It took the form of a conversation between members of a household. A transcript of the infomercial is available with the authors on demand.
### Consumer Acceptance Group A

H. PURCHASING GAME

**H.1. First Auction**

Now you have the POSSIBILITY of buying the grain of one of these two kinds of bhakri that you just tasted. We will play a game with the objective of assessing how much you are willing to pay for each one of these varieties.

1. First, I will ask you to tell me how much you are willing to pay for 1 kg of grain, for each of the varieties.

2. Then, I will toss a coin. If you get heads, you have the possibility of buying bajra A. If you get tails, you have the possibility of buying bajra B.

3. We will then determine the sale price for the variety indicated by the toss as follows. This bag has 26 strips numbered from 5 to 30 (5, 6, 7, 8, 9, ……, 26, 27, 28, 29, 30). We will ask you to pick one strip from the bag. The number on the strip you pick will be the sale price. (Enumerator: Please open the colour bag and show the strips to the respondent, shuffle the bag, let them shuffle the strips and pick up one strip).

4. We will compare the price you stated as your willingness to pay for the selected bajra in step 2 with the sale price picked in step 3. If the price on the strip is equal to or less than the price you have stated, you win 1 kg of the bajra, and pay the price on the strip. But if the price on the strip is higher than the price you have stated you do not get to buy the bajra.

The best way to play the game is to first assess up to how much you would be willing to pay for 1 kg of each of the two bajra varieties; and then state exactly these prices.

- Do NOT state prices higher than what you are willing to pay: You may end up spending more money than you think the bajra is worth.
- Do NOT state prices that are lower than you would be willing to pay: You may end up losing and regretting that you did not state a higher price.

**Enumerator:** Now ask for the prices the respondent is willing to pay for the two bajra varieties and record them in the table below. Then execute steps 2 to 4 above and record the results in the table.

#### Willingness to Pay for 1 kg of each variety

<table>
<thead>
<tr>
<th>Variety</th>
<th>Respondent’s willingness to pay for 1 kg</th>
<th>Selected Choice</th>
<th>Price drawn from bag in the selected scenario</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Bajra B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**H.1** If the respondent won and did not or could not pay, what are the reasons for non-payment?

[1] Did not have any money on him/her  [2] won the variety he/she liked the least  [3] other, specify __________

**Enumerator:**

If the respondent won either bajra, please complete the transaction by handing over the grain at the price selected and by collecting the money.

---

Figure 18. Instructions
Consumer Acceptance Group A Maharashtra, India HHID________________

If the respondent won high iron variety (bajra A), then end the interview by going to the signature part and send them to Santosh.

If the respondent won low iron variety (bajra B) but could NOT or would NOT pay, then end the interview by going to the signature part and send them to Santosh.

If the respondent lost, or won the low iron variety (bajra B) and paid for it, proceed by making them watch the infomercial.

Please complete the rest of the questionnaire after they finish watching the infomercial.

H.2. Second auction

This is an exact repeat of the first auction above.

You watched the infomercial. Please note that the high iron bajra in the infomercial is this bajra A in front of you – with the logo you see (enumerator please turn the card that says A and show the HarvestPlus logo and state that it is the high iron bajra certified by international health authority). Having obtained this information on high iron content of HarvestPlus bajra, I will again offer you the POSSIBILITY of buying one of the grains in front of you. We will play the same game as before, with the objective of assessing how much you are willing to pay for each one of these varieties.

1. First, I will ask you to tell me how much you are willing to pay for 1 kg of grain, for each of the varieties.

2. Then, I will toss a coin. If you get heads, you have the possibility of buying bajra A. If you get tails, you have the possibility of buying bajra B.

3. We will then determine the sale price for the variety indicated by the toss as follows. This bag has 26 strips numbered from 5 to 30 (5, 6, 7, 8, 9, ……., 26, 27, 28, 29, 30). We will ask you to pick one strip from the bag. The number on the strip you pick will be the sale price. (Enumerator: Please open the colour bag and show the strips to the respondent, shuffle the bag, let them shuffle the strips and pick up one strip).

4. We will compare the price you stated as your willingness to pay for the selected bajra in step 2 with the sale price picked in step 3. If the price on the strip is equal to or less than the price you have stated, you win 1 kg of the bajra, and pay the price on the strip. But if the price on the strip is higher than the price you have stated you do not get to buy the bajra.

The best way to play the game is to first assess up to how much you would be willing to pay for 1 kg of each of the two bajra varieties; and then state exactly these prices:

- Do NOT state prices higher than what you are willing to pay: You may end up spending more money than you think the bajra is worth.
- Do NOT state prices that are lower than you would be willing to pay: You may end up losing and regretting that you did not state a higher price.

 Enumerator: Now ask for the prices the respondent is willing to pay for the two bajra varieties and record them in the table below. Then execute steps 2 to 4 above and record the results in the table.

Willingness to Pay for 1 kg of each variety

Figure 19. Instructions
<table>
<thead>
<tr>
<th>Variety</th>
<th>Respondent’s willingness to pay for 1 kg</th>
<th>Selected Choice</th>
<th>Price drawn from bag in the selected scenario</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Bajra B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If the respondent won and did not or could not pay, what are the reasons for non-payment?

[1] Did not have any money on him/her  
[2] won the variety he/she liked the least  
[3] other, specify ________

Please complete the transaction by handing over the grain at the price indicated, if the respondent has won and by collecting the money.

H.3. Endow and Upgrade

This game is for those respondents who won low iron bajra (bajra B) before watching the infomercial video.

You watched the infomercial. Please note that the high iron bajra in the infomercial is this bajra A in front of you – with the logo you see (enumerator please turn the card that says A and show the HarvestPlus logo and state that it is the high iron bajra certified by international health authority). Having obtained this information on high iron content of HarvestPlus bajra, I will offer you the POSSIBILITY of exchanging the low iron bajra for the high iron bajra, if you wish. We will use a procedure similar to the one you did before watching the video.

1. First, I will ask you to tell me how much you are willing to pay to exchange your 1 kg low iron bajra for 1 kg high iron bajra. Please remember you have already paid for the low iron bajra. If you choose to keep the low iron bajra, you can do so by saying you are not willing to pay anything (Enumerator: if they say they are not willing to pay anything to exchange, please enter 0 in the first column of table below, make them sign and tell them to go to Santosh)

2. We will then determine what you will actually pay as follows. This bag has 20 strips with prices numbered 1,2,3,...20. You will shuffle the bag, then you will pick one strip randomly. (Enumerator: Please open the black bag and show the strips to the respondent, shuffle the bag, let them shuffle the strips and pick up one strip)

3. We will compare what you stated you were willing to pay for the exchange with the price on the strip you picked. If the price on the strip is less than or equal to your willingness to pay for the exchange, you will pay the price on the strip and return the low iron bajra to me, and get 1 kg of the high iron bajra. But if the price on the strip is higher than your willingness to pay for the exchange, then you keep the low iron bajra.

Let me remind you that the best way to play the game is to first assess up to how much you would be willing to pay for making this exchange; and then state exactly this amount.

- Do NOT state prices higher than what you are willing to pay: You may end up spending more money than you think the exchange is worth.
- Do NOT state prices that are lower than you would be willing to pay: You may end up not exchanging and regretting that you did not state a higher price.

Enumerator: Now ask for the exchange price the respondent is willing to pay for the high iron bajra and record it in the table below. Then execute steps 2 and 3 above and record the results in the table.

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Figure 20. Instructions

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**Consumer Acceptance Group A**  
**Maharashtra, India**  
**HHID________________**

**Willingness to Pay for exchanging low iron bajra for high iron bajra (1 kg of each variety)**

|------------------------------------------------|-----------------------------------------------|---------|---------------|-------------------------------|---------------------|-----|

**H.3** If the respondent won and did not or could not pay, what are the reasons for non-payment?  
[1] Did not have any money on him/her  [2] won the variety he/she liked the least  [3] other, specify ________

Please complete the transaction by exchanging the grain at the price indicated, if the respondent has won and by collecting the money. Go to the end of the questionnaire and get the subject’s signature or thumb print.

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**Signature or thumbprint of participant**  
**Name (in print)**

**Signature of enumerator**  
**Name (in print)**

**Enumerator:** Please thank the respondent for their time, and ask them not to discuss this study with anyone else who has not yet been interviewed so that their answers remain unaffected.

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**Figure 21. Instructions**