INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.
USE OF VEHICLE DYNAMICS MODELING TO QUANTIFY RACE CAR HANDLING BEHAVIOR

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of the Ohio State University

By

Jeffrey P. Chrstos, B.S.M.E., M.S.M.E., P.E.

*****

The Ohio State University

2001

Dissertation Committee:

Dr. Dennis A. Guenther, Adviser
Dr. Ernest O. Doebelin
Dr. Giorgio Rizzoni
Dr. Donald Houser

Approved By

Dennis A. Guenther
Adviser

Department of Mechanical Engineering
Copyright by
Jeffrey P. Chrstos
2000
ABSTRACT

The objective of this research is to compute quantitative measures of racing vehicle behavior and relate them to qualitative driver assessments. Road racing vehicle "development" has relied on driver's verbally telling their engineers how the car "feels". Modern race cars are equipped with numerous sensors and data acquisition systems. However, raw sensor data often does not show the engineer what the driver is trying to explain. This research has extended the utility of the available data by applying Extended Kalman Filtering (EKF) to the measured sensor data to estimate the tire and aerodynamic forces acting on the vehicle along with vehicle and tire sideslip angles. It is shown that this new information relates to the driver's comments about vehicle understeer/oversteer. Developing the procedures for computing this objective measure of the vehicle behavior that correlates with the driver's subjective assessment is a necessary first step before racing vehicle handling and performance problems can be analyzed and corrected scientifically.

A model of the longitudinal and vertical dynamics of a race car is used in an EKF to estimate the aerodynamic forces acting on the vehicle. It is shown that the EKF can accurately predict the aerodynamic forces while rejecting some of the unmeasured roadway disturbances and is a significant improvement over estimating the aerodynamic downforce from the suspension force measurements alone.
A vehicle yaw/roll plane model is used in an EKF to estimate unknown tire lateral forces and chassis and tire sideslip angles. Tire lateral forces and slip angles are fundamental quantities in the study of vehicle handling dynamics, yet are not available to the race engineer using the sensor data alone. A computed value called *Balance*, the difference between the front and rear tire slip angles, is defined and shown to correlate with driver comments about vehicle understeer and oversteer. This allows the engineer to "see" in the data what the driver is describing. It is proposed that the *Balance* calculation can provide part of a common language between the driver and engineer, allowing them to discuss in a quantitative way how the vehicle should be improved.
ACKNOWLEDGMENTS

I wish to thank my adviser, Dr. Dennis Guenther for his support and encouragement that made this dissertation possible. I also wish to thank my committee members Dr. Ernest O. Doebelein, Dr. Giorgio Rizzoni, and Dr. Donald Houser for their contributions and support during this dissertation.

There are a number of people outside of The Ohio State University who provided help and encouragement along the way. Dr. Laura Ray provided invaluable help as I worked through the difficulties of the Kalman Filtering methods. Team Rahal, Inc. provided the vehicle data and vehicle information that made this work possible and I would like to thank in particular Tim Reiter, Ray Leto and Tom Janiczek for their help. Wade Allen, Dave Klyde and Bimal Aponso of Systems Technology, Inc. provided great assistance with background information and insight into the subject of driver-vehicle interaction along with a lot of encouragement and for this I am grateful. I would also like to thank Marcia Cook at Systems Technology, Inc. for her help with the literature review. Finally, I would like to express my thanks to Dr. Gary Heydinger, Dr. Jim Bernard, Dr. Nicolas Durisek and Dr. Iqbal Anwar for their continual encouragement to complete this dissertation.
VITA

November 17, 1961 ....................................... Born – Abington, Pennsylvania

1984 ............................................................... B.S. Mechanical Engineering
Drexel University, Philadelphia, PA

1992 ............................................................... M.S. Mechanical Engineering
The Ohio State University, Columbus, OH

PUBLICATIONS


**FIELDS OF STUDY**

Major Field: Mechanical Engineering
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>iv</td>
</tr>
<tr>
<td>VITA</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xi</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 General</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Objectives</td>
<td>5</td>
</tr>
<tr>
<td>1.3 Dissertation Overview</td>
<td>6</td>
</tr>
<tr>
<td>1.4 References</td>
<td>7</td>
</tr>
<tr>
<td>2. BACKGROUND AND LITERATURE REVIEW</td>
<td>9</td>
</tr>
<tr>
<td>2.1 Vehicle Dynamics</td>
<td>9</td>
</tr>
<tr>
<td>2.2 Vehicle Handling</td>
<td>11</td>
</tr>
<tr>
<td>2.2.1 Linear Vehicle Handling Analysis</td>
<td>18</td>
</tr>
<tr>
<td>2.2.2 Non-Linear and Limit Vehicle Handling</td>
<td>24</td>
</tr>
<tr>
<td>2.3 Vehicle Modeling</td>
<td>30</td>
</tr>
<tr>
<td>2.3.1 Lumped Parameter</td>
<td>30</td>
</tr>
<tr>
<td>2.3.2 Multi-Body</td>
<td>31</td>
</tr>
<tr>
<td>2.3.3 Tire Modeling</td>
<td>32</td>
</tr>
<tr>
<td>2.3.4 Aerodynamic Modeling</td>
<td>34</td>
</tr>
<tr>
<td>2.4 State Estimation</td>
<td>35</td>
</tr>
<tr>
<td>2.4.1 Optimal Estimation</td>
<td>39</td>
</tr>
<tr>
<td>2.4.2 Kalman Filtering</td>
<td>41</td>
</tr>
<tr>
<td>2.4.3 Extended Kalman Filtering</td>
<td>43</td>
</tr>
<tr>
<td>2.4.4 Optimal Smoothing</td>
<td>49</td>
</tr>
<tr>
<td>2.5 Driver/Vehicle Analysis and Modeling</td>
<td>52</td>
</tr>
<tr>
<td>2.6 Issues Unique to Racing</td>
<td>58</td>
</tr>
<tr>
<td>2.7 References</td>
<td>59</td>
</tr>
<tr>
<td>3. AERODYNAMIC FORCE ESTIMATION</td>
<td>67</td>
</tr>
<tr>
<td>3.1 Overview</td>
<td>67</td>
</tr>
<tr>
<td>3.2 Pitch-Plane Vehicle Model</td>
<td>68</td>
</tr>
<tr>
<td>3.3 Modeling Aerodynamic Forces</td>
<td>74</td>
</tr>
<tr>
<td>3.4 Measurement System Modeling</td>
<td>77</td>
</tr>
<tr>
<td>3.5 Estimator Evaluation using Simulation Data</td>
<td>78</td>
</tr>
<tr>
<td>3.6 References</td>
<td>100</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1 - Driver-Vehicle Interaction</td>
<td>12</td>
</tr>
<tr>
<td>Figure 2 - Example Four Parameter Rhombus</td>
<td>17</td>
</tr>
<tr>
<td>Figure 3 - Two-Degree-of-Freedom Bicycle Model</td>
<td>20</td>
</tr>
<tr>
<td>Figure 4 – Simulated Constant Radius Test Results and Understeer Gradient</td>
<td>23</td>
</tr>
<tr>
<td>Figure 5 - Tire Force and Moment Response for Five Different Normal Loads</td>
<td>25</td>
</tr>
<tr>
<td>Figure 6 – Simulated Tire Characteristics</td>
<td>26</td>
</tr>
<tr>
<td>Figure 7 - Moment Method N-Ay Diagram (from Reference 37)</td>
<td>29</td>
</tr>
<tr>
<td>Figure 8 - Tire Lateral/Longitudinal Force Interaction</td>
<td>33</td>
</tr>
<tr>
<td>Figure 9 - Types of Estimation</td>
<td>40</td>
</tr>
<tr>
<td>Figure 10 - Information Flow in the Filtering Process</td>
<td>41</td>
</tr>
<tr>
<td>Figure 11 - Hybrid Extended Kalman Filter Flow Diagram</td>
<td>46</td>
</tr>
<tr>
<td>Figure 12 - Modified Bryson-Fraser Smoother Flow Diagram</td>
<td>52</td>
</tr>
<tr>
<td>Figure 13 - Pilot/Vehicle Block Diagram for Compensatory Control [89]</td>
<td>53</td>
</tr>
<tr>
<td>Figure 14 - Block Diagram of a Pilot/Vehicle System [90]</td>
<td>55</td>
</tr>
<tr>
<td>Figure 15 - Cooper-Harper Handling Qualities Rating Scale [90]</td>
<td>57</td>
</tr>
<tr>
<td>Figure 16 – Pitch-Plane Model Schematic</td>
<td>69</td>
</tr>
<tr>
<td>Figure 17 - Suspension Detail</td>
<td>70</td>
</tr>
</tbody>
</table>
Figure 40 - Measured and Estimated Data: Total Downforce ............................................96
Figure 41 - Measured and Estimated Data: Front Downforce ............................................97
Figure 42 - Measured and Estimated Data: Rear Downforce .............................................97
Figure 43 - Measured and Estimated Data: Aerodynamic Center-of-Pressure ..................98
Figure 44 - Yaw-Plane Diagram of 3DOF Model ...............................................................104
Figure 45 - Roll-Plane Diagram for 3DOF Model ...............................................................105
Figure 46 - Steering Input and Vehicle Speed .....................................................................111
Figure 47 - Estimator Performance: Lateral Acceleration and Yaw Rate .........................112
Figure 48 - Estimator Performance: Roll Angle and Roll Rate .........................................112
Figure 49 - Estimator Performance: Tire Lateral Force ......................................................113
Figure 50 - Estimator Performance: Vehicle Sideslip Angle and F/R Balance .................116
Figure 51 - Estimator Performance: Tire Slip Angles .........................................................117
Figure 52 - Measured and Estimated Data: Steering Wheel Angle and Vehicle Speed ..117
Figure 53 - Measured and Estimated Data: Lateral Acceleration and Yaw Rate ...........118
Figure 54: Measured and Estimated Data: Roll Angle and Roll Rate .......................118
Figure 55 - Measured and Estimated Data: Estimated Tire Lateral Forces ......................119
Figure 56 - Measured and Estimated Data: Estimated Body Sideslip Angle and F/R Balance.................................................................119
Figure 57 - Measured and Estimated Data: Estimated Tire Slip Angles ...........................120
Figure 58 - Measured and Estimated Data Lap B: Steering Wheel Angle and Vehicle Speed ..................................................................................................................124
Figure 59 - Measured and Estimated Data Lap B: Lateral Acceleration and Yaw Rate .124
Figure 60 - Measured and Estimated Data Lap B: Roll Angle and Roll Rate .................125
Figure 61 - Measured and Estimated Data Lap B: Estimated Tire Lateral Forces ........125
Figure 62 - Measured and Estimated Data Lap B: Body Sideslip Angle and F/R Balance .................................................................126
Figure 63 - Measured and Estimated Data Lap B: Tire Slip Angles ..............................126
Figure 64 - Lap Comparison: Steering Wheel Angle and Vehicle Speed ................127
Figure 65 - Lap Comparison: Lateral Acceleration and Yaw Rate ........................128
Figure 66 - Lap Comparison: Tire Lateral Force ......................................................128
Figure 67 - Lap Comparison: Sideslip Angle and F/R Balance ..............................129
Figure 68 - Lap Comparison: Tire Slip Angle ........................................................129
Figure 69 - Lap Comparison: Total Tire Lateral Force and Percent Front Tire Force ...132
Figure 70 - Data And Information Flow ..............................................................................135
Figure 71 - Tire Peak Lateral Friction versus Normal Load........................................139
1.1 General

Road vehicle handling is a complex subject involving the non-linear, time-varying dynamics of a driver-vehicle system operating under variable conditions. It can be argued that a primary design objective for the handling characteristics of a passenger car is not to do anything really “bad”. A passenger car should be “easy” for the driver to control, and not cause any driver-vehicle instabilities. It should transmit enough information to the driver about the road-vehicle system to ease the control task, while not causing excessive discomfort. It must be stable and controllable when driven by all drivers. Performance (acceleration capabilities) must be acceptable to its drivers. Within these and other constraints, the passenger vehicle is then designed to be comfortable, responsive, and have good performance. The majority of the handling development is devoted to the low acceleration region typical of “normal” driving.

The racing vehicle, while mechanically very similar to the passenger vehicle, is designed with almost opposite objectives. According to Milliken [1], “...the basic design requirements of a racing car are: 1) Provision of the largest vehicle “g-g” (longitudinal and lateral acceleration) maneuvering areas throughout the speed range, and 2) Control and stability characteristics that enable a skilled driver to operate at or near these
acceleration limits". The difference from the passenger car is that limit performance is optimized, and only stability and control characteristics at or near the limit are considered. Trying to achieve these goals, along with reliability, race strategy, etc. is the task of all racing teams and drivers.

While these “goals” are easy to state in words, they are difficult to translate into a set of quantifiable design and development requirements. Historically, racing car development has been done at the track, primarily using feedback from the driver. This feedback is by definition, qualitative. The driver may say that the car is “loose” at a certain part of the track, meaning that the rear axle is losing lateral traction and the car may spin out. The driver may say that the car “lacks grip”, meaning that the lateral acceleration limit is too low. It is the task of the team engineers, mechanics, and the driver to then decide what aspect(s) of the car to change to lessen undesirable or increase desirable characteristics of the driver-vehicle system behavior.

The problem of deciding what to change is extremely difficult. A racing car has many parts of its design that can be adjusted: aerodynamic lift and drag, suspension geometry, tire construction, tire pressure, spring rate and linearity, damping characteristics, static vehicle ride height and attitude, engine power curve, transmission gear ratios, etc. The racing car is a highly non-linear, highly coupled system, and all of the adjustments listed here interact with one another. The driver-vehicle-track system is sufficiently complex that complete understanding based on insight and experience alone is not possible. By this, it is meant that a person cannot predict with much certainty all of the consequences of a change to the vehicle. What corrects a “problem” at one track,
may not correct a seemingly identical problem at a different track. This is due to the lack of understanding of this complex system.

Before electronic instrumentation became common in racing cars in the 1980’s, the basic vehicle measurements available at the track were lap times (and sometimes split times), and a tire pyrometer for measuring tire tread temperature (constant temperature across the tread indicates that the tire is being used close to its full potential). With the introduction of on-board data acquisition and storage equipment, it became possible to record many data channels while the vehicle lapped the circuit. This data can be retrieved from the car when it is in the pits, and analyzed and displayed for the team. A partial list of chassis data measured on racing cars includes: driver control inputs (steering wheel angle, throttle position, and brake pedal force), chassis accelerations and rotational rates about three axes, suspension motions, spring/damper forces, chassis height above the track, tire temperatures, wheel rotational velocities, and air speed.

While the on-board data provides an enormous amount of information about the vehicle’s behavior, it does not by itself provide any insight or explanation into the vehicle’s performance: it only records what happened, not why it happened. This dissertation is based on the premise that understanding of the driver-vehicle-track system cannot be achieved using on-board data and observation based on insight alone: the problem is too difficult. The complex interactions within the system can only be accounted for through modeling of the vehicle-track system. The model will be used to compress the large quantity of on-board data into a manageable amount of information to
present to the team. The primary goal is to allow the team to quantitatively “see” in the on-board data what the driver qualitatively tells them.

The task of compressing the on-board data is necessary, because taken individually, the measurements reveal little about the vehicle. For example, if the driver says that the vehicle is “pushing” in a certain corner, he is saying that the front axle is losing traction (or “grip” in racing jargon). What the driver is describing is a subtle offset in the front to rear axle side force balance. If a chassis adjustment is made that “corrects” the pushing, time domain recordings of steering wheel angle, lateral acceleration, yaw rate, etc. before and after the adjustment will typically show little difference. The chassis adjustments are aimed at finding many small improvements, because the difference between a “good” car and a “bad” car is very small: at the final 1995 Indy Car race at Laguna Seca raceway, the difference in qualifying time from first to last (twenty eight cars) was five percent, from first to tenth, there was less than two percent difference (1.3 seconds over a 2.214 mile lap).

Current on-board measurements do not provide any direct measure of the forces acting on the vehicle, yet the tire and aerodynamic forces are what ultimately determine vehicle behavior. It is possible to measure tire forces directly using torque wheels installed between each suspension hub and wheel [2 - 4]. However, these torque wheels are heavy, expensive, and can change the vehicle track width unless special wheels are built. In addition, they would not be legal in many forms of racing during the race weekend. This makes them impractical in a racing environment. Aerodynamic forces are generated by a combination of the vehicle wings and bodywork, including the
underbody. The wing mounts can be instrumented to measure a combination of aerodynamic and inertial loading on the wings, however, the bodywork can not be instrumented to measure aerodynamic forces.

An alternative to measuring the tire and aerodynamic forces acting on a vehicle is to "estimate" them based on other on-board measurements. Optimal state estimation methods such as the extended Kalman filter [5, 6] can be used to optimally, in the mathematical sense (minimize some error criteria), estimate the vehicle state and the external forces acting on the vehicle. This approach has been used in the aeronautics industry to estimate the aerodynamic forces and moments acting on an aircraft during flight [7 - 12]. More recently, Ray has used a hybrid extended Kalman filter to estimate passenger car tire forces [13 - 15].

1.2 Objectives

The primary objective of this research is to compute quantitative measures of racing vehicle behavior and relate them to qualitative driver assessments. Before racing vehicle handling and performance problems can be analyzed and corrected scientifically, quantitative measures must be determined. Optimal state estimation will be applied to the vehicle on-board data to estimate unmeasured vehicle states and the tire and aerodynamic forces acting on the vehicle.
The primary steps are:

1. Develop a mathematical models of the vehicle inertial dynamics to be used in the extended Kalman filter. The models will be formulated so that the vehicle parameters will relate to the actual adjustments of a racing car.

2. Implement hybrid extended Kalman filters to estimate unmeasured vehicle states and the tire and aerodynamic forces acting on the car. On-board data measurement procedures, including a minimum transducer set will be developed.

3. Relate on-board data measurements, along with estimated forces and states, to the driver's qualitative description of vehicle behavior. The measured vehicle behavior will be looked at from the force generation and tire slip angle perspective, rather than focusing on the states of the chassis. This has not been done in the past using on-board measurements.

1.3 Dissertation Overview

Following this introduction, Chapter 2 presents background and literature review of the general vehicle dynamics and handling research. Vehicle modeling techniques and optimal state estimation are also reviewed. Chapter 3 presents an Extended Kalman Filter designed to estimate the aerodynamic forces generated by the vehicle. Chapter 4 presents an Extended Kalman Filter designed to estimate tire lateral forces and body and tire sideslip angles. It is shown that the estimated tire forces and tire slip angles relate to the driver's subjective comments about the vehicle behavior. Chapter 6 contains a summary and conclusions along with a discussion of future research.
1.4 References


CHAPTER 2
BACKGROUND AND LITERATURE REVIEW

2.1 Vehicle Dynamics

In the automotive engineering community the term vehicle dynamics is frequently used, however its definition is rarely given. There have been a number of texts with vehicle dynamics in their title, but with no definition proposed. Even in the Society of Automotive Engineers (SAE) Recommended Practice - SAE J670e "Vehicle Dynamics Terminology" [1] the term vehicle dynamics is not defined. Using a dictionary, vehicle dynamics would loosely mean "...the motion of a conveyance for moving passengers...", much too vague for engineering use. In this document, a definition taken from William (Bill) Milliken [2] will be used: "...This leads to the area of vehicle dynamics. Vehicle dynamics, as we use the term, is the branch of engineering which relates tire and aerodynamic forces to overall vehicle accelerations, velocities and motions, using Newton's Laws of Motion. It encompasses the behavior of the vehicle as affected by driveline, tires, aerodynamics and chassis characteristics. The subject is a complex one because of the large number of variables involved." In addition, an added restriction will be to limit the treatment to passenger vehicles (transportation and racing) running on paved surfaces (asphalt or concrete).
A number of texts and bound course notes have been prepared on the general subject of vehicle dynamics: Ellis in 1969 and 1988 [3, 4], Wong in 1978 [5], Bastow in 1990 [6], Gillespie in 1992 [7], the University of Michigan [8, 9], and Mola in 1986 [10]. A 1956 paper by Bill Milliken [11] summarized the early efforts in the understanding and analysis of ground vehicle behavior in the context of vehicle dynamics. This paper was the introductory paper to a series of progress reports from the Cornell Aeronautical Laboratory, Inc. (CAL) presented to the Institution of Mechanical Engineers (IMechE). The reports presented the status of CAL's vehicle dynamics research for General Motors. Many consider this series of five papers to mark the start of the modern study of vehicle directional response and control. Thirty four years later, during his keynote address at the 1990 ASME Winter Annual Meeting, Leonard Segel (a member of the CAL group in the 1950's) presented his thoughts on the history of vehicle directional stability and control [12]. It is interesting to note that some of the problems that CAL predicted would be solved in the 1950's, remain today.

To help make a complex field more manageable, it is common practice to separate the analysis of vehicle dynamics into the areas of ride and handling. Ride analysis is concerned with the bounce, pitch, roll, and suspension motions of the vehicle, while handling analysis focuses on the lateral, longitudinal, yaw, and roll motions of the vehicle chassis. Separation of the analysis is justified because the vehicle ride and handling modes are weakly coupled dynamically [13], with the coupling increasing with increasing road roughness, vehicle speed, and severity of the maneuvering.
Vehicle ride and handling dynamics are studied both experimentally and through computer simulation. The traditional approach, which is still very much in use today, is to build prototype vehicles and test them on proving grounds. When it is desired to change some vehicle characteristic, the prototype vehicle is modified and the testing continued. Through the increase in the understanding of the physics governing vehicle behavior and the power of computer analysis and simulation, computer analysis of vehicle dynamics is beginning to play a major role in vehicle development. A large driving force for using simulation in vehicle development is the desire to significantly shorten vehicle design and development time.

Chapter 13 of reference 1 puts the subject of vehicle dynamics into a historical context. The work of many of the pioneering engineers and organization in the field of vehicle dynamics are described from the first-hand account of Bill Milliken. The chapter details the discovery of most of what is conceptually known today in the field.

2.2 Vehicle Handling

When a driver says that a vehicle “handles well”, it is inferred that he was easily able to control the vehicle in a stable manner while performing the driving task. Thus, handling is the behavior of the driver-vehicle system during maneuvering. The analysis of vehicle handling is typically broken down into stability and control. Stability refers to the system response to external disturbances, while control is the system response to driver inputs. Figure 1 shows the basic driver-vehicle system interaction in block diagram form. It can be considered as a closed-loop feedback system with the driver as
the controller and the vehicle as the plant. Research has shown [14] that drivers are extremely adaptable controllers, and can readily change their performance to make the driver-vehicle system perform in a desirable way.

![Driver-Vehicle Interaction Diagram]

Figure 1 - Driver-Vehicle Interaction

While vehicle handling refers to the driver-vehicle system, the engineer has little or no control over the driver (the controller in the system). The task is thus to make the vehicle controllable and stable when driven by all drivers (Section 2.5 covers issues unique to racing). Handling assessment has traditionally been done by highly skilled test drivers who provide the development engineers with qualitative feedback about the vehicle’s handling. Handling evaluators also rate vehicles using a one to ten rating scale.
While this approach can produce vehicles with good handling characteristics, it can only be performed after a prototype vehicle has been built.

To consider vehicle handling during the conceptual vehicle design stage, considerable research has been carried out trying to correlate objective vehicle measures with subjective handling assessments. If a set of objective criteria can be determined that will result in a "good" handling vehicle, then simulations of vehicles that only exist on paper can be used to tune handling. This would reduce the number of prototype vehicles constructed and reduce vehicle development time and cost. A brief review of some published references on this subject follows [16 - 28].

In 1973, Walter Bergman published the results of a large program at Ford Motor Company [16] that developed three open-loop vehicle measures that correlated well with subjective evaluations of vehicle handling. Bergman defines vehicle handling as "... the interaction between driver, vehicle, and environment which takes place during transportation of people and goods from place to place. Vehicle handling qualities describe the behavior of car-man combinations in actual driving situations ..." and states that ease of control and not vehicle performance is the most important vehicle handling quality. An in-depth description of the human driving task is presented, followed by a description of the subjective rating procedure. In their procedure, the evaluators were chosen based on their ability to represent a consensus judgment with the other evaluators. This was done using standard (at the time) vehicle tests.

The Ford program, using four vehicles and nine evaluators, found three open-loop tests that correlated with the subjective rankings. The *Cornering Across a Single Bump*
test measured the change in vehicle yaw rate when a vehicle crossed a single bump while in a steady state corner of 0.4 g. This test was used to evaluate vehicle handling during rough-road cornering. The subjective evaluation showed an 81% correlation (correlation coefficient, R=0.81 using a straight line fit) between the measurements and the subjective ranking.

The *Braking in Cornering* test measured a parameter called the normalized understeer angle decrement, which describes the vehicle directional response resulting from a change of path curvature and vehicle sideslip angle. The brakes were applied after the vehicle had reached 0.3 g steady state cornering at 30 mph. The braking level was the maximum possible without wheel lockup. A 96% correlation was found between the measurements and the subjective ranking.

To determine vehicle transient control ease, Ford used a *Transient Steer Maneuver* to compute the sideslip acceleration coefficient. The test maneuver was a reverse steer at a constant speed of 60 mph, starting from a steady-state cornering condition in one direction, and rapidly reversing the steering wheel to achieve the same steady-state lateral acceleration in the opposite direction. From this test, the normalized angular sideslip acceleration was computed (see reference [16] for derivation). The parameter is related to the second time derivative of the vehicle sideslip angle, and is used to represent the vehicle sideslip motion during the reversal portion of a single lane change maneuver. The test was run starting at 0.3 g's and increased in 0.1 g increments until maximum lateral acceleration was achieved. The values of normalized angular sideslip acceleration were averaged from all of the tests. A 98% correlation was found
between the measurements and the subjective ranking. Bergman concludes that “for the first time” vehicle handling qualities can be determined from open-loop objective test measures allowing vehicle handling to be predicted using computer simulation at vehicle design time.

In 1988, Dennis Kunkel and Ronald Leffert published a summary of General Motors objective measurements of vehicle directional response [19]. The paper starts by providing some background information on objective testing performed by the Government, private organizations, and auto companies; and by describing the data acquisition requirements for the GM tests. Six objective tests used by GM are then discussed: Control Response Test; Frequency Response Test; Maximum Lateral Acceleration Test; On-Center Handling Test; Lift-Dive Test; and Center-of-Gravity Test.

The Control Response Test is a series of increasing severity step-like steering inputs run at 100 km/h used to quantify steady-state and transient vehicle response. The Frequency Response Test is a constant amplitude swept sinusoidal steering input run at 100 and 140 km/h. Vehicle yaw velocity, lateral acceleration, and roll angle to steering angle frequency responses are computed to show the dynamic response of the vehicle. The Maximum Lateral Acceleration Test is run on a fixed radius course and used to study limit vehicle behavior. The On-Center Handling Test is a 0.2 Hz sinusoidal steering wheel input that produces ±0.2 g lateral acceleration test run at 100 km/h. This test is used to quantify vehicle control and feel during expressway type driving. The Lift-Dive Test quantifies suspension movement and chassis pitch angle for a variety of longitudinal acceleration levels. Finally, the Center-of-Gravity Test is used to quantify the total
vehicle center of gravity location. The paper concludes by providing distribution plots for twenty-four metrics computed from the six tests for 1980 to 1988 model year passenger cars measured by General Motors.

A 1990 paper from Mitsubishi Motors [21] describes a method of combining four open-loop vehicle response parameters to characterize linear vehicle handling characteristics. The four parameters are computed by curve fitting vehicle yaw rate and lateral acceleration to steering wheel angle frequency response measurements with a two degree-of-freedom vehicle model (yaw rate and chassis sideslip angle). The four parameters are: yaw rate steady state gain “a₁”, natural frequency “f₀”, and damping ratio “ζ”, and the lateral acceleration phase delay at 1 Hz “ϕ”. According to the authors, a₁ is a measure of “heading easiness”, f₀ is related to “heading responsiveness”, ζ is “directional damping”, and ϕ is related to “following controllability”.

The four parameters are plotted as each axis of a two dimension graph forming a rhombus. An example is shown in Figure 2. Each axis is arranged so that “further from the origin” is better. The area of the rhombus is the measure of handling potential, while the distortion denotes handling tendency. A rhombus deflected up and to the right, denotes a strong understeering vehicle. Down and to the left denotes less understeer or possible oversteer. The rhombus can be plotted for various vehicle speeds, or multiple vehicles can be plotted on a single graph for comparison.
Review of the literature relating objective measures of vehicle handling to subjective handling evaluations reveals that only a few of the vehicle responses examined have been used. Chassis yaw rate, sideslip angle, lateral and longitudinal acceleration, and roll angle and rate have been used along with steering and braking inputs in the time and frequency domains. The analyses have tried to determine what input/output characteristics or transfer functions are desirable from the human operators point of view. Little or no discussion is given on how to make a vehicle handle "well", or why some vehicles handle "better" than others.
2.2.1 Linear Vehicle Handling Analysis

While a general theory of vehicle dynamics has yet to be developed, research has resulted in considerable understanding of vehicle response (open-loop, no driver) during low lateral acceleration operation. This analysis is applicable to vehicle handling during normal highway driving, i.e. low lateral acceleration (less that 0.3 or 0.4 g’s).

For this analysis, some simplifying assumptions are made: constant forward speed, no lateral or longitudinal load transfer, no rolling or pitching of the chassis, linear tire response, and no suspension or steering system compliance. The analysis is intended to demonstrate the basic response (stability and control) of the vehicle. The following analysis is condensed, showing mainly the results. For the complete analysis, see Milliken [2] and Ellis [3, 4] among others.

Figure 3 shows a top view of a two degree-of-freedom “bicycle” vehicle model, with degrees of freedom yaw rate, r, and lateral velocity, v. Assuming small angles, the front and rear tire slip angles can be written:

\[\alpha_f = \beta + \frac{a \cdot r}{V} - \delta\]
\[\alpha_r = \beta - \frac{b \cdot r}{V}\]  

(1)

where \(\beta\) is the body sideslip angle \((\tan^{-1}\left(\frac{v}{u}\right))\), a and b are the distances from the front and rear axles to the vehicle center of gravity, r, is the yaw rate, V, is the chassis path velocity, and \(\delta\) is the front wheel steer angle input. Denoting the front and rear tire
cornering stiffness (both tires on each axle combined) as $C_f$ and $C_r$, the axle lateral forces, $F_f$ and $F_r$, can be written:

$$F_f = C_f \left( \beta + \frac{a \cdot r}{V} - \delta \right) = C_f \beta + C_f \left( \frac{a \cdot r}{V} \right) - C_f \delta$$

(2)

$$F_r = C_r \left( \beta - \frac{b \cdot r}{V} \right) = C_r \beta - C_r \left( \frac{b \cdot r}{V} \right)$$

The equations of motion can be written using Newton’s laws as:

$$I_z \ddot{\beta} = (aC_f - bC_r) \beta + \frac{1}{V} (a^2 C_f + b^2 C_r) r - aC_f \delta$$

(3)

$$mV (r + \dot{\beta}) = (C_f + C_r) \beta + \frac{1}{V} (aC_f - bC_r) r - C_f \delta$$

Where $m$ is the total vehicle mass and $I_z$ is the yaw moment of inertia of the vehicle.
Figure 3 - Two-Degree-of-Freedom Bicycle Model

\[ +\beta = \tan^{-1}\left(\frac{v}{u}\right) \]
Studying the stability and control characteristics of the vehicle is aided by re-writing equations (3) using the derivative notation introduced to the automotive community by Segel in 1956 [27]. The derivatives are the partial derivatives of the tire forces and moments acting on the vehicle with respect to the vehicle states and inputs. They are analogues to the wind tunnel measured stability and control derivatives used by aeronautical engineers. The total yaw moment about the vehicle center of gravity produced by the tires is denoted by $N$ and the total lateral force produced by the tires is denoted by $Y$ ($N$ and $Y$ are the right hand side of equation (3)). The yaw moment derivative w.r.t. sideslip angle is written as, $N_{\beta}$ and equals $(aC_f - bC_r)$. The following description of the six derivatives from the two degree-of-freedom model is condensed from Milliken [2].

$N_\delta$  

The *Control Moment* derivative: $N_\delta = -aC_f$ relates driver control of yaw moment through steering.

$N_r$  

The *Yaw Damping* derivative: $N_r = \left(\frac{1}{V}\right)(a^2C_f + b^2C_r)$ is the directional damping produced by the tire cornering stiffness. The $1/V$ term shows that vehicle yaw damping decreases with increasing vehicle speed. High yaw damping comes from a long wheel base $(a+b)$ and/or high tire cornering stiffness.

$N_\beta$  

The *Directional Stability* derivative: $N_\beta = (aC_f - bC_r)$ can also be thought of as the understeer/oversteer derivative. $bC_r > aC_f$ denotes understeer and static directional stability.
\( Y_\delta \) The *Control Force* derivative: \( Y_\delta = -C_f \) is the lateral force due to steering.

\( Y_r \) The *Lateral Force/Yaw Coupling* derivative: \( Y_r = \left( \frac{1}{V} \right) (aC_f - bC_r) \) is the lateral force due to yaw velocity, and is the coupling term between the two degrees of freedom.

\( Y_\beta \) The *Damping-in-Sideslip* derivative: \( Y_\beta = (C_f + C_r) \) is a measure of the lateral force developed due to vehicle sideslip.

The term *understeer* is used to describe a vehicle that tends to lose cornering potential at the front axle before the rear. *Oversteer* is used to describe the opposite condition where the rear axle looses cornering potential before the front. The most common definition is from SAE J670e [1], and is for *Understeer Gradient*: “A vehicle is understeer at a given trim if the ratio of the steering wheel angle gradient to the overall steering ratio is greater than the Ackerman steer angle gradient”. The Ackerman steer angle is the kinematic road wheel angle required to follow a curve of radius, \( R \), at low speed where the lateral acceleration is negligible. The Ackerman steer angle gradient is the wheelbase divided by the square of forward speed.

Understeer gradient can be measured from a constant radius, increasing speed test as follows. Record steering wheel angle and lateral acceleration while driving around a constant radius course, and slowly increase forward speed until the limit of adhesion is reached (plow out or spin out). Plot steering wheel angle verses lateral acceleration. The understeer gradient will be the slope of the steering wheel angle verses lateral
acceleration curve divided by the vehicle kinematic steering ratio minus the Ackerman steering angle gradient. Simulated results from this test to demonstrate the procedure are shown in Figure 4. The radius of the circle was 300 feet, and vehicle speed was increased from zero to 100 feet per second. The vehicle had a wheelbase of 100 inches. The vehicles' understeer gradient is fairly constant up to approximately 0.3 g's, then starts to increase, meaning that the slip angle of the front tires is being increased faster than the rear to keep the vehicle on course.

Figure 4 – Simulated Constant Radius Test Results and Understeer Gradient
2.2.2 Non-Linear and Limit Vehicle Handling

While the available literature on linear range vehicle response is fairly rich, there has been much less written about non-linear or limit vehicle response. The major difficulty of studying vehicle response above approximately 0.4 g’s is the non-linearity of the tire forces. Tire forces are typically modeled as non-linear functions of slip angle, longitudinal slip, normal force, camber angle and surface friction. Actual tire forces are also functions of air and surface temperature, wear, surface type, moisture, etc.

Figure 5 shows representative (simulated) force and moment data for a passenger car tire. The data is plotted for five different normal loads (400, 600, 800, 1000, and 1200 lbs from bottom to top curves). The top two plots are lateral force and aligning moment verses slip angle. The bottom plot is braking force verses longitudinal slip ratio. As can be seen, lateral force and aligning moment become extremely non-linear for slip angles greater that about two degrees, and braking force becomes non-linear for slip ratios greater than 0.04 (4% slip). It can also be seen that the forces and moments are non-linear with respect to normal load.
One method that has been used to study non-linear vehicle response is to linearize non-linear models about specific operating points [29 - 32]. A common first approach to non-linear vehicle analysis is to use a non-linear tire model with a linear vehicle model. The tire forces are then linearized about an operating point in terms of their inputs (slip angle, camber angle, normal load) and the stability and control derivatives are computed. However, Mashadi [31, 32] has recently shown that this approach is not entirely correct. He points out that to linearize a non-linear system using small disturbance theory, the total external forces must be linearized directly in terms of the model state variables.

Figure 5 - Tire Force and Moment Response for Five Different Normal Loads
This requires an iterative numerical solution in the case of a vehicle simulation. However, the correct stability and control derivatives computed about an operating point are different than if the tire forces are only linearized about their inputs.

In 1987, Dixon [33] proposed a modified definition of understeer, called understeer number, to quantify limit vehicle handling. Understeer number, $N_u$, is defined as:

$$N_u = \left( \frac{A_f}{A_r} \right) - 1$$  \hspace{1cm} (4)

Where $A_f$ and $A_r$ are the lateral acceleration capabilities of the front and rear axles. Figure 6 shows front and rear axle tire forces normalized by axle mass plotted against slip angle. From this, Dixon defines understeer angle, $\delta_u$, as the difference between the front and rear tire slip angles (a bicycle model is used) at a given steady state lateral acceleration. The maximum axle accelerations are also shown.

Figure 6 – Simulated Tire Characteristics
The understeer number is non-dimensional, and will be positive for a vehicle with final understeer, and negative for final oversteer. Final neutral steer will have an understeer number of zero. After defining understeer number, Dixon shows how it is affected by many vehicle design parameters (center of gravity height, lateral axle load transfer, and aerodynamics). The design parameters affect $N_u$ in a quantifiable way, making it a useful design aid.

A method to portray vehicle stability, control, and maneuvering performance, called the *Moment Method* has been under development by General Motors, Calspan, and Milliken Research Associates since 1970 [2, 34 - 37]. For the study of the stability, control, and damping of ground vehicles, the Moment Method attempts to be what the wind tunnel is for the aeronautical engineer. The Moment Method deals with the steady-state forces and moments acting upon the vehicle, and provides a “map” of the overall force and moment capability of the vehicle.

Like a wind tunnel or tire test machine, the forces and moments are computed based on a partially constrained vehicle. While normally implemented through computer simulation, the Moment Method is equivalent to a vehicle running at constant speed on a large flat belt while the chassis is constrained longitudinally, laterally, and in yaw (A physical realization of the Moment Method is the Flat-Trac® Roadway Simulator built by MTS Systems in 1995 [38, 39]). The vehicle is free to pitch, roll, and heave. The constraint system measures the longitudinal and lateral forces and yaw moment generated by the vehicle as it is slowly swept through a range of body slip angles and steering
wheel angles. Results from these constrained test simulations are plotted on Moment Diagrams.

The study of the interpretation of the Moment Diagrams has been going on since 1970, and is beyond the scope of this overview. However, an example diagram is shown in Figure 7, and a few of the vehicle characteristics will be discussed. The Moment Diagram is vehicle yaw moment verses lateral acceleration for a winged race car traveling at a constant 140 mph in a right hand corner. Lines of constant vehicle sideslip angle, $\beta$, and constant road wheel angle, $\delta$, are drawn on the diagram. Steady state vehicle operation corresponds to operation along the zero yaw moment axis. The upper boundary of the diagram represents saturation of the front tires, while the lower boundary represents saturation of the rear tires. If the upper boundary crosses the zero yaw axis, the vehicle will understeer or pushes at the limit. The slope of the road wheel angle lines as they cross the zero yaw moment axis are a measure of the vehicle stability (negative slope is stable). Available driver control through steering is measured by the distance along a constant $\beta$ line from a steady-state condition to the upper boundary of the diagram.
Figure 7 - Moment Method N-Ay Diagram (from Reference 37)

Other Moment Diagrams can be drawn and much more learned about vehicle limit and sub-limit behavior. While the steady-state nature of the Moment Method may seem as a severe limitation, Milliken has observed that especially for trained racing drivers, most vehicle operation occurs in a narrow band around the zero yaw moment axis [2]. Drivers apparently learn to use tire forces to create high steady-state lateral accelerations (high cornering speeds) rather than wasting them on creating high angular accelerations.
2.3 Vehicle Modeling

The modeling techniques used to develop vehicle dynamic simulations can be divided into three general categories: low degree of freedom linear models, non-linear lumped parameter models, and multi-body dynamic models. Each technique has advantages and disadvantages, and the particular application for a simulation will determine which type of simulation is most appropriate. Factors influencing this decision include: type and severity of maneuvers to be studied, types of vehicle parameters available, computer resources available, and the level of accuracy required.

One of the many accomplishments of the CAL group was to develop a linear 3-degree-of-freedom (roll, yaw, and sideslip) mathematical model of a motor car, describing its response to front wheel steer angle [27]. An important part of this model development was the validation of the model predictions against experimentally measured vehicle response data. This model and the basic knowledge gained during the CAL research has been the foundation for much of the work in vehicle stability and directional control since the mid 1950's.

2.3.1 Lumped Parameter

Probably the most common class of simulations used to study the full envelope of vehicle performance are those formulated using nonlinear, lumped-parameter modeling techniques. These models use hand-coded equations of motion for the vehicle and suspension and are solved numerically on digital computers (some models in the past have also used analog and hybrid computers). Examples of this type of model can be found in
The six sprung-mass degrees of freedom can be included in these models, though for handling simulations, the bounce and pitch degrees of freedom are often neglected. Some models also include vertical and/or roll degrees of freedom for the unsprung masses. If braking is studied, the wheel spin degrees of freedom are included. Steering system models can range from the simplest (the control input to the model is the steer angle of the front wheels rather than the steering wheel angle), to single spring and steering gear models, to complex models including multiple springs, power assisted steering characteristics, viscous and Coulomb damping, and inertial elements.

The area where these simulations vary most is in their suspension models. The classic approach has been the use of a fixed roll axis [2, 46], where the sprung mass is assumed to rotate about a fixed axis defined by front and rear suspension roll centers. Ellis and others have abandoned the roll center approach with the introduction of suspension derivatives [3, 4, 47]. The suspension derivative approach eliminates the inertial linkage of the sprung and unsprung masses and instead introduces a kinematic model. This method also takes into account tire contact patch movement in the horizontal plane, which is not used in most roll axis models.

2.3.2 Multi-Body

Multi-body simulations, such as ADAMS, DADS, AUTOSIM, NEWEUL, and MEDYNA, among others, started appearing in the early 1970's [48, 49, 50]. These programs model each component of the vehicle/suspension system as a rigid or flexible body. Models for all joints connecting the "bodies" are also included. This leads to high
degree-of-freedom models, often 50 or more. The computer resources required for this type of simulation are great. Some researchers using multi-body simulations employ a workstation to build their model and connect to a mainframe or supercomputer to perform their "number crunching". A recent multi-body simulation development tool, AUTOSIM [51], has been implemented on a personal computer with reported close to real-time simulation speed. This type of performance may allow more researchers to use multi-body simulations.

2.3.3 Tire Modeling

No matter what type of modeling approach is used, a dynamic system must obey Newton’s second law: \( \sum F = m \cdot a \). In vehicle dynamics, the external forces acting on the vehicle are tire and aerodynamic forces (except in a crash!). In vehicle simulations, no matter how much effort is put into accurately modeling the vehicle’s inertial dynamics, kinematics, etc., the simulation’s accuracy \textbf{can not} be better than the accuracy to which the external forces are known. Except for the case of high speed winged racing cars, tires are the primary source of external forces acting on the vehicle, and the accuracy of their modeling will govern the accuracy of the vehicle simulation.

Tire models used in vehicle dynamics simulations range from linear models that only model lateral force as a function of slip angle [27], non-linear empirical models [52, 54 - 62], and non-linear analytical/empirical models that model the tire contact patch using a lumped mass approach [59, 63, 64]. Reviews of tire modeling practices and
requirements have recently appeared in the technical literature by Pacejka and Sharp [57], and Allen, et. all [65].

In Section 2.2.2, the non-linear characteristics of tire lateral force, aligning moment, and longitudinal force were shown against pure slip angle or longitudinal slip (Figure 5). An added complexity in modeling tire force and moment characteristics is that the lateral and longitudinal forces interact. Figure 8 shows this interaction: at constant slip angles (0.5, 1.0, 2.0, 4.0, and 6.0 degrees), the tire slip ratio was swept from -1 to 1 (locked skidding wheel to spinning wheel), and the tire lateral and longitudinal forces cross plotted. Looking at any of the constant slip angle curves, the lateral force (vertical axis) is highest when the longitudinal force is zero (slip ratio is zero), which is the free-rolling tire case. As traction or braking force is needed, and the slip ratio becomes non-zero, the lateral force generated by the tire at a given slip angle decreases.

![Figure 8 - Tire Lateral/Longitudinal Force Interaction](image)

33
The tire modeling discussed above is aimed at predicting steady-state tire force and moment responses for a given tire state. Tires also exhibit time dependent properties that influence transient vehicle behavior. These dynamic tire characteristics have been shown to be important for accurately simulating transient vehicle response [59, 65 - 69]. Based on sinusoidal, step or pulse input testing on tire test machines, the lateral force dynamics have been shown to be dependent on the distance a tire rolls, rather than time dependent. The lateral force dynamics are typically modeled as a first-order lag in the path frequency domain (path frequency has the dimension of radians per foot). The "relaxation length" is the distance that a tire must roll, after a step input of slip angle, before the lateral force is 63.2% of its eventual steady-state value. It is equivalent to the first-order time constant transformed into the distance domain. The literature on the dynamic characteristics of other tire forces and moments is quite sparse. However, Lee's lumped mass model does treat the dynamics of all tire forces and moments to all tire inputs [59].

2.3.4 Aerodynamic Modeling

Aerodynamic forces and moments affect vehicle handling and performance in a number of ways. For passenger cars traveling at relatively low highway speeds (in the United States), the lift forces generated are very small compared to the weight of the vehicle. The drag forces affect fuel consumption, and the lateral forces from side winds affect straight-line vehicle stability. For a winged racing car, at high speed, the aerodynamic forces play a huge role in determining vehicle handling and absolute vehicle
performance (in racing measured by lap time). The negative lift forces can more than
double the load supported by the tires. The drag force in large part determines the
vehicles’ top speed.

Standard convention models six aerodynamic forces and moments acting on the
vehicle with an orthogonal right-handed axis system located on the ground and mid-
wheelbase and mid-track [70]. The x-axis is positive forward, y-axis is positive to the
right, and z-axis is positive down. Six non-dimensional coefficients are used to describe
three forces (lift, drag, and side force) and three moments (pitch, roll, and yaw). The
coefficients can be functions of chassis state and angle of the air stream relative to the
vehicle chassis, and are independent of speed. For a winged racing car, the aerodynamic
lift forces (the term downforce is commonly used since the forces generated by the
vehicle body and wings push the chassis toward the ground) are extremely sensitive to
the vehicle ride height and pitch angle. Testing in a wind tunnel would consist of
measuring (at a minimum) the lift, pitching moment, and drag forces as the front and rear
suspension ride heights are varied through their travel. Aerodynamic lift and pitching
moment “maps” are then made plotting lift force or pitching moment as a function of
front and rear ride height. These forces are quite non-linear [2, 71], and complicate the
vehicle setup.

2.4 State Estimation

In the context used here, from Gelb [72] “Estimation is the process of extracting
information from data - data which can be used to infer the desired information and may
contain errors. Modern estimation methods use known relationships to compute the desired information from the measurements, taking account of measurement errors, the effects of disturbances and control actions on the system, and prior knowledge of the information." State estimation is commonly used in optimal control in the form of an observer [73], where the control algorithm design assumes the existence of a complete state measurement. However, in most cases only a partial state measurement is available and an observer is designed to take the available measurements along with a model of the system, and construct an estimate of the complete state vector.

State estimation also has applications outside the optimal control field. In the field of aeronautics, aircraft handling qualities are much better defined than the handling requirements of passenger cars [74]. Flight dynamists use aerodynamic forces and moment data measured from a scale model in a wind tunnel to assess the stability and control characteristics of an aircraft before it is ever built. Once the prototype plane is constructed, it is important to verify that the full-scale plane’s aerodynamics are the same as predicted by the scale model in the wind tunnel.

In a project for the Naval Air Development Center, the non-linear aerodynamics were extracted from flight test data from a T-2C jet trainer aircraft, and compared to wind tunnel measurements [75 - 78]. Using an extended Kalman-Bucy filter [79] and a Bryson-Frazier smoother [80] (see Section 2.4 of this document for details of mathematics and implementation), aircraft states and aerodynamic forces and moments were estimated from flight data. Using the estimated forces and states, the aerodynamic coefficients were then extracted using stepwise multiple linear regression and compared
to wind tunnel measurements. It was found that the aerodynamic coefficients, which are functions of aerodynamic angle of attack and sideslip angle, extracted from the flight data matched the wind tunnel data very well. A similar study, using a Schweizer sailplane, was conducted at Princeton University by Stengel and Sri-Jayantha [81, 82]. Like the T-2C study, they found good agreement between the estimated and measured aerodynamic coefficients.

Using a similar approach to the above studies that measured aerodynamic forces and moments, Ray [83 - 85] has successfully used a hybrid Extended Kalman Filter (EKF) to estimate tire forces, slip angles, and slip ratios from simulated and experimentally measured passenger vehicle response data. An eight degree-of-freedom vehicle model was used in the estimator, with degrees of freedom: chassis longitudinal and lateral velocity, yaw rate, roll rate, and the four wheel angular velocities. The four tire longitudinal forces and the axle lateral forces were each modeled using a second-order random walk model, which when combined with the vehicle model resulted in a 21st order non-linear estimation model. Vehicle measurements used were: the four wheel angular velocities, yaw rate, lateral and longitudinal acceleration, and roll rate.

The complete state vector and the six tire forces were then estimated for both simulated and measured vehicle data. To demonstrate that the estimator functioned properly, noise corrupted simulation data was used for the measurement data. The estimator was able to accurately predict the tire forces and vehicle states. The estimator was then used with experimentally measured vehicle data for both cornering and braking maneuvers. Accurate estimates of unmeasured (not used in the estimator) vehicle states
were achieved. The test vehicle did not have wheel force measurements available, so the
tire force measurements could not be checked directly. However, the estimates pass all
“sanity checks”, and are believed to be accurate.

Methods other than the EKF have been used to estimate unmeasured vehicle
states. In 2000, Sasaki and Nishimaki [86] reported on the use of Neural Networks to
estimate the sideslip angle of a passenger vehicle. The purpose of this state estimation
was to be used as part of an active yaw moment control system aimed at increasing
vehicle directional stability in severe maneuvers. Measured vehicle data were vehicle
speed, lateral acceleration and yaw rate. A three layer Neural Network was constructed
that “memorized” the nonlinear vehicle dynamics during its training phase. A sideslip
angle measurement transducer was used during the training. After training, the Neural
Network was able to predict sideslip angle to within five percent when run in speed and
steering regions that were covered during the training. This type of technique required
that the training cover the entire spectrum of vehicle operation, since there is no
underlying model of the system. This means not only the full maneuvering range be
covered, it also means that all vehicle configurations (loading, suspension settings, etc.)
be run over this maneuvering range. This type of testing required to train a general
purpose Neural Network estimator would be very time consuming if not impossible in the
racing environment where the vehicle is changed considerably during a race weekend,
and even more over the course of a racing season.

Another proposed technique for yaw rate and sideslip angle estimation was
proposed by Hac and Simpson [87] in 2000. Using measurements of four wheel speeds,
steering wheel angle and lateral acceleration, two estimates of yaw rate are computed. The first estimate is from the differential wheel speed between the left and right sides of the vehicle. The second is from the vehicle speed and lateral acceleration measurements. Confidence intervals are generated for each estimate depending on vehicle operating condition and a weighted average of the two is used for the final yaw rate estimation. A two degree-of-freedom (yaw rate and lateral velocity) vehicle model is then used as an observer to provide the final estimates of vehicle yaw rate and sideslip angle.

2.4.1 Optimal Estimation

From Gelb [72], "An optimal estimator is a computational algorithm that processes measurements to deduce a minimum error estimate of the state of a system by utilizing: knowledge of system and measurement dynamics, assumed statistics of system noises and measurement errors, and initial condition information." Estimation problems are classified in three ways as shown in Figure 9, where the "boxed" region denotes the measurement process. The term filtering is used when the time of interest in the estimation process corresponds to the last available measurement. When the time of interest is within the available measurements, it is referred to as smoothing. When the time of interest occurs after the last available measurement, the problem is referred to as prediction.
Figure 9 - Types of Estimation

Figure 10 shows a basic block diagram of the filtering process. The state of the physical system, $\mathbf{x}(t)$, in the presence of process noise and unknown disturbances is observed, $\mathbf{z}(t)$, by the measurement system. The measurements are corrupted by noise resulting in measurement uncertainty. The system observation is passed to the filter, where based on models of both the physical system and measurement system, along with statistical estimates of the process and measurement noise, the system state, $\hat{\mathbf{x}}(t)$, is estimated which minimizes some error criteria. Because a filter is based on measurements up to and including time $t$, filters can be implemented in real-time and are often used in optimal control applications as state observers. Filters can also be used off-line to post-process a set of measurements, removing effects of process and measurement noise (based on the model of the physical system and measurement system used).
2.4.2 Kalman Filtering

The most common type of optimal filter is the Kalman or Kalman-Bucy filter [79, 80]. There are many applications for Kalman filters requiring the filter to take on different forms, but all are recursive filters, meaning that past measurements do not have to be stored in order to compute the current estimate. The basic problem that is solved by Kalman filters is: find a state estimator for state $x$, given a continuous linear time varying dynamic system written in state-space form:

$$\dot{x} = Ax + Bu + Fv$$  \hspace{1cm} (5)

And a measurement process is given by:

$$z = Hx + w$$  \hspace{1cm} (6)

Where, $x$, is the model state vector, $z$, is the state measurement, $u$ is a known input, $w$ and $v$ are white noise processes having known spectral density matrices. Kalman
and Bucy found the solution by realizing that the state estimator is really an observer, which can be expressed as:

$$\dot{x} = Ax + Bu + \dot{K}(z - H\dot{x})$$  \hspace{1cm} (7)$$

Where, $\dot{x}$, is the optimum state estimate and $\dot{K}$ is the optimally chosen gain matrix. Kalman and Bucy proved [79] that provided that the random processes, $w$ and $v$ are white and gaussian, that the filter is optimum under any reasonable performance criteria. What is left is to determine the optimal gain matrix, $\dot{K}$.

Define the error as

$$e = x - \dot{x}$$  \hspace{1cm} (8)$$

Using Equations (5), (6), (7), and (8) and simplifying yields the differential equation for the error:

$$\dot{e} = (A - KH)e + Fv - Kw$$  \hspace{1cm} (9)$$

Let $P(t)$ (the expected value of the error, $e$) be the covariance matrix of the state at time, $t$. The differential equation for $P(t)$ is (from [72]):

$$\dot{P} = AP + PA^T + FVF^T$$  \hspace{1cm} (10)$$

$V$ is the spectral density of the process noise, $v$. Then, assuming that the process noise and the measurement noise are uncorrelated, the covariance matrix, $P$, can be minimized; which is saying that the error is minimized. (See Chapter 9 of [72] for mathematical details):

$$\dot{P} = A\dot{P} + \dot{P}A^T - \dot{PH}^TW^{-1}H\dot{P} + FVF^T$$  \hspace{1cm} (11)$$
$W$ is the spectral density of the measurement noise. This results in the optimum gain matrix, $\hat{K}$, given by:

$$\hat{K} = \hat{P}H^TW^{-1}$$

(12)

2.4.3 Extended Kalman Filtering

The Kalman filter described in Section 2.4 is for linear continuous time systems. It is also possible to develop Kalman filters for linear discrete time systems [72]. For many applications, however, a linear model of the system is not sufficient to capture its dynamic characteristics, and non-linearities must be included. It must be stressed that a Kalman filter processes the measurements based on the system model. If the model does not accurately reflect the system, the filter’s state estimates, while being mathematically optimal will not represent the actual behavior of the system accurately.

For cases where non-linear system models are required, extended Kalman filters (EKF) have been developed. The particular filter described here is a continuous-discrete or hybrid extended Kalman filter. The system model and the covariance update equations are modeled as continuous systems and propagated through numerical integration. The measurements are from a sampled data system, and the EKF measurement update equations are discrete equations. This hybrid formulation is applicable to many problems where the measurements are sampled at fixed intervals using an analog-to-digital-converter. The following is a description of the hybrid EKF, derivation of the mathematics can be found in [72, 73, 93, 94].

43
The non-linear differential equations describing the system with state, \( x(t) \), and white gaussian noise, \( w(t) \), with spectral density matrix, \( Q(t) \), are written as:

\[
\dot{x}(t) = f(x(t), u(t), t) + w(t) \tag{13}
\]

Along with a sampled data measurement model:

\[
z_k = h_f(x(t_k), u(t_k)) + v_k \tag{14}
\]

Where the subscript, \( k \), denotes the sample index, and \( v_k \) is a white random sequence with covariance matrix, \( R_k \). The EKF produces a practical estimation procedure (practical from the point of view of both implementation complexity and computational burden) by linearizing the non-linear system dynamics about the current operating point for use in the error covariance propagation. This linearization is defined by:

\[
F(\hat{x}(-), t) = \left. \frac{\partial f(x(t), t)}{\partial x} \right|_{x(t) = \hat{x}(-)} \tag{15}
\]

The caret, \( \hat{\cdot} \), is used to denote the estimated variables. The covariance propagation is now written as:

\[
\dot{P}(t) = F(\hat{x}(t), t)P(t) + P(t)F^T(\hat{x}(t), t) + Q(t) \tag{16}
\]

The measurement system model equations are also linearized about the current operating point as:

\[
H_k(\hat{x}_k(-)) = \left. \frac{\partial h_k(x)}{\partial x} \right|_{x=\hat{x}_k(-)} \tag{17}
\]

The (-) or (+) notation is used to denote the operation occurring just before or just after the \( k \)th update. The update equations, from [72], are now:
\[ K_k = P_k (-) H_k^T (\hat{x}_k (-)) \left[ H_k (\hat{x}_k (-)) P_k (-) H_k^T (\hat{x}_k (-)) + R_k \right]^{-1} \] 

(18)

\[ \hat{x}_k (+) = \hat{x}_k (-) + K_k \left[ z_k - h_k (\hat{x}_k (-)) \right] \] 

(19)

\[ P_k (+) = \left[ I - K_k H_k (\hat{x}_k (-)) \right] P_k (-) \] 

(20)

Figure 11 shows a basic data flow diagram of the above procedure. In this research, the EKF is implemented off-line (rather than real-time) using the software program MATLAB® [95]. The program initialization requires, along with the usual memory allocation, etc., that the initial covariance matrix, \( P \), and the process and measurement noise covariance matrices, \( Q \) and \( R \), be defined. Since \( P \) is time varying (Equation (16)), its initial value, \( P_0 \), should be set to represent how well the initial state of the system is known. This will lessen the start up transients of the filter. The elements of the process noise covariance matrix, \( Q \), represent the unmeasured disturbances acting on the system. The elements on the main diagonal are the disturbances acting on each state of the system, with higher values indicating higher disturbance. The elements of the measurement noise covariance matrix, \( R \), represent the errors or uncertainty associated with each measurement. Setting the values of \( Q \) and \( R \), along with the initial values of \( P \), is known as "tuning" the filter, and is very important for obtaining good filter performance and stability. However, reference material on the subject of filter tuning seems to be almost non-existent. While some guidelines can be deduced from the formulation of the filter and the particular problem, it still becomes somewhat of a trial-and-error process.
START

Initialization:
P(0) = P
x(0) = 0
Define: Q & R

Get Measurement: z_k

Propagate State x_k(t) using Numerical Integration:
\[ \dot{x}_k(t) = f(\hat{x}_k(-), u(t), t) \]

Linearize System and Measurement Model w.r.t. x_k(t):
\[ F = \left. \frac{\partial f(x(-), t)}{\partial x(-)} \right|_{x(-) = \hat{x}_k(-)} \]
\[ H_k = \left. \frac{\partial h_k(x)}{\partial x} \right|_{x = \hat{x}_k(-)} \]

Propagate Error Covariance P_k(t) using Numerical Integration:
\[ \dot{P}_k(-) = FP_k(-) + P_k(-)F^T + Q \]

Compute Kalman Gain Matrix:
\[ K_k = P_k(-)H_k^T[H_kP_k(-)H_k^T + R_k]^{-1} \]

State Estimate Update:
\[ \hat{x}_k(+) = \hat{x}_k(-) + K_k[z_k - h_k(\hat{x}_k(-))] \]

Error Covariance Update:
\[ P_k(+) = [I - K_kH_k]P_k(-) \]

Store State x_k(+) and other Outputs

Run Complete? Yes → STOP

Figure 11 - Hybrid Extended Kalman Filter Flow Diagram
Since the filtering is performed off-line, the “Get Measurements” operation is simply retrieving the measurement data from a storage array (in real-time applications, this would involve reading the measurements through the data acquisition hardware). The next step is to numerically integrate the model equations to propagate the estimated state from $x_k$ to $x_{k+1}$. The EKF does not require a specific integration method, however, some practical considerations apply. For most applications, the measurement data will be available at fixed intervals, and the flow diagram in Figure 11 shows one-state propagation for each measurement. This implies that the integration time step must be equal to the measurement time interval. This is not necessary, but is the most common implementation (Multiple integration steps may occur between measurements, and time steps do not have to be constant, however, this will significantly increase the complexity of the algorithm.). Therefore, the most straightforward integrator is a fixed time step, single step method such as Euler, trapezoidal, or Runge-Kutta [95]. These methods have the advantage of not requiring storage of prior states. Multi-step methods, such as Adams-Bashforth and Adams-Moulton could be used. From the technical literature, it appears that the fourth-order Runge-Kutta method is by far the most common integrator used (no reference to other methods was found), however, no explanations as to why were given. The initial EKF implemented for this research uses a fourth-order Runge-Kutta integrator with good results. If CPU time is an issue, other integrators could be investigated.

The linearization of the system and measurement model can be performed either analytically or numerically. For a linear system model, analytically linearizing the
equations is straightforward. However, for a large non-linear system model, analytically linearizing the equations may be very difficult. In this case, the linearized matrices (Jacobians) may be approximated numerically using the formula:

\[
F(\tilde{x}(t),t) = \frac{\partial f(x(t),t)}{\partial x(t)} \bigg|_{x(t) = \tilde{x}(t)} \approx \frac{f(x(t) + \Delta x) - f(x(t) - \Delta x)}{2\Delta x}
\]  

(21)

The preceding approximation is the definition of a derivative as \( \Delta x \) approaches zero. The choice of \( \Delta x \) must be small enough to make the approximation valid, but large enough to avoid numerical inaccuracy due to subtracting two (possibly small) numbers in the numerator (MATLAB uses double precision arithmetic, which reduces this problem).

The numerical integration of the error covariance, \( P \), follows the same rules as the integration of the state estimate, and typically uses the same integration method. In evaluating this equation (16), both the \( F \) and \( P \) matrices are square, with dimension equal to the number of states, \( n \). This means that each evaluation requires two, \( n \) by \( n \), matrix multiplications. If a fourth-order Runge-Kutta integrator is used, four evaluations of the function are required resulting in eight matrix multiplications. Matrix multiplication requires approximately \( 2n^3 \) floating point operations, so a ten state estimator would require approximately \( 8 \cdot 2 \cdot 10^3 = 16,000 \) \( \text{flops} \). Timing analysis of a ten state estimator used in this research shows more than 70% of the CPU time is spent integrating the error covariance. For a given size estimation model, there is little that can be done to improve this. The \( F \) and \( P \) matrices are symmetric, but not sparse, so no improvement in the matrix multiplication routine is possible. Using a lower order integrator such as a second-order Runge-Kutta or a multi-step integrator that only requires one function
evaluation per update may help if numerical stability and accuracy are not compromised. The final steps in the EKF are the gain computation and the state and error covariance updates, which are simply evaluations of equations (18), (19), and (20).

2.4.4 Optimal Smoothing

Optimal filters, like the EKF described in Section 2.4.2, use all of the data between 0 and t (the time of interest) to compute an optimum estimate of the system state, x (see Figure 9). Optimal smoothers use the entire data set, between 0 and T (the end time) to compute an optimum state estimate. A system state is termed Smoothable if its estimated value is superior to its filtered value. It has been shown in [96], that only states that are controllable by noise driving the system are smoothable.

The smoother formulation used in this research is a Modified-Bryson-Frazier (MBF) smoother [97]. The MBF is derived from the Rauch, Tung and Streibel (R-T-S) smoother [98]. The R-T-S algorithm requires inversion of the filter covariance matrix, P, which can become close to singular if the filtered states become well determined (some elements of the matrix become close to zero). This presents an ill-conditioned numerical problem. The MBF smoother algorithm, however, does not require the inversion of P, and avoids these numerical difficulties resulting in a more robust algorithm. The MBF smoother has also been applied successfully in the estimation of aircraft aerodynamic forces [78 - 82].

The MBF smoother uses the Kalman gain, error covariance, and the linearized system and measurement models (K, P, F, and H) from the EKF in its algorithm. This
requires that these matrices be stored at each time step during the filtering. The MBF smoother operates “backward in time”, starting from time, $T$, and progressing to time $0$. It can be thought of as filtering backwards in time, and is the mathematical dual of the EKF, requiring only variable substitutions. The smoothed state estimates are a combination of the forward and backward filtered state estimates. The MBF algorithm, used here is from [78], and is shown in Figure 12.

The initial conditions of the adjoint (or backwards-evolution) variables, $\lambda$ and $\Lambda$, are given by:

$$\lambda(T-) = -H_N^T D_N^{-1} \Delta z_N$$  \hspace{1cm} (22)$$

$$\Lambda(T-) = H_N^T D_N^{-1} H_N$$  \hspace{1cm} (23)$$

Where $N$ is the number of data points in the measurement arrays, and:

$$D_N = H_N (-) P_N (-) H_N^T (-) + R_N$$  \hspace{1cm} (24)$$

$$\Delta z_N = z_N - h(x_N (-))$$  \hspace{1cm} (25)$$

The propagation (backward) of the adjoint variables from $t_{k+1}(-)$ to $t_k(+)\) is performed using numerical integration. The same algorithm used in the EKF can be used. The adjoint variable propagation equations are:

$$\dot{\lambda} = - F^T \lambda$$ \hspace{1cm} (26)$$

$$\dot{\Lambda} = - F^T \Lambda - \Lambda^T F$$ \hspace{1cm} (27)$$

The adjoint variable update equations are:

$$\lambda_k (-) = \lambda_k (+) - H_k^T D_k^{-1} \left( \Delta z_k + D_k K_k^T \lambda_k (+) \right)$$  \hspace{1cm} (28)$$

50
\[ \Lambda_k (-) = (I - K_k H_k)^T \Lambda_k (+) (I - K_k H_k) + (H_k^T D_k^{-1} H_k) \]  

(29)

The smoothed estimates of the state and error covariance can now be computed from:

\[
\begin{align*}
[x_k (+)]_{\text{smooth}} &= [x_k (+)]_{\text{filter}} - P_k (+) \Lambda_k (+) \\
[P_k (+)]_{\text{smooth}} &= [P_k (+)]_{\text{filter}} - P_k (+) \Lambda_k (+) P_k (+)
\end{align*}
\]

(30)  
(31)
2.5 Driver/Vehicle Analysis and Modeling

The discussion to this point has primarily focused on the analysis and modeling of the vehicle in isolation. However, vehicles are driven by human drivers, and it is the
driver/vehicle system characteristics that in the end need to be addressed. In the automotive field, it is typical to treat the vehicle independently from the driver. This is not the case for aircraft. The literature is rich with examples of pilot modeling and pilot/vehicle analyzes [88 - 91].

McRuer and Krendel [89] provide a review of the mathematical modeling of the human pilot in his role as the controller (the aircraft is the controlled element). Pilot modeling is treated in the context of a feedback control system. The pilot is seen to be an adaptive controller, attempting to adapt his control strategy to make the closed-loop dynamics of the pilot/aircraft system meet the desired system performance. The focus is on compensatory control tasks where the pilot is reacting to some error between the desired and actual system response. Figure 13 shows a block diagram of the pilot/vehicle system for a compensatory control task.

Figure 13 - Pilot/Vehicle Block Diagram for Compensatory Control [89]
Through a series of experiments where the controlled element dynamics, $Y_C$, were varied and the open-loop system transfer function, $Y_P Y_C$, computed, it was shown that in spite of changes to $Y_C$, $Y_P Y_C$ remained essentially the same in the region of crossover. The “crossover law” thus states that the human pilot will attempt to adapt his control strategy to achieve the open-loop transfer function of the form:

$$Y_P(j\omega)Y_C(j\omega) = \frac{\omega_C e^{-j\tau_s}}{j\omega}$$

(32)

where $\omega_C$ is the open-loop crossover frequency and $\tau_s$ is the pilot/vehicle system effective time delay (i.e. pilot delay plus controlled element delay from higher frequency dynamics). Using this crossover model, approximate pilot transfer functions can be determined if the vehicle transfer function is known and the open-loop system transfer function is measured. This means that the forcing function must be known. Computing the required pilot transfer function provides insight into the ease or difficulty of controlling the vehicle.

The crossover model describes the pilot/vehicle system in compensatory control tasks. Reference 90 describes three modes of control, shown in Figure 14. In addition to the compensatory control, acting on feedback errors, pursuit and precognitive control modes are identified. Pursuit control is used when the pilot is able to distinguish the system input, $i$, and the system output, $m$. This allows the pilot to “pursue” the desired command with the vehicle, rather than just reacting to differences between the command

54
and the vehicle state. In driving, looking ahead and following the roadway would involve pursuit control, along with compensatory control. The final control mode is "precognitive" control. Precognitive control results when the pilot has learned the vehicle characteristics and input forcing function so well that he can essentially operate in an open-loop control mode. Neither precognitive nor pursuit control mode can in practice operate in isolation, as there will always be some errors that need to be removed with compensatory control actions. This combination of multiple control modes is termed "dual-mode" control. A more complete description of these control modes can be found in [91].

Figure 14 - Block Diagram of a Pilot/Vehicle System [90]
Precognitive control requires that the pilot "know" the dynamics of the vehicle. This allows the pilot to adapt his control strategy to make the system follow the intended path (in the case of driving). As an example, a passenger vehicle has some finite yaw rate time constant. In order for a driver to negotiate a corner and stay within his lane, his steering input must lead the roadway path under the vehicle to compensate for the vehicle response time. A driver does this by looking a few seconds ahead of the vehicle, rather than at the road directly in front of the car. The longer (slower) the vehicle response time, the more steering lead required by the driver, and the farther ahead of the vehicle the driver must look.

In the aircraft field, the term "handling qualities" is used to describe the study of the characteristics of the aircraft that relate to the ease and precision with which a pilot can perform a given task. This study starts during the design stage with models of the aircraft and the pilot, through simulator studies, and finally to flight testing. A formalized pilot handling qualities rating system is used during simulator and flight testing in the form of the Cooper-Harper scale shown in Figure 15. The Cooper-Harper scale, while giving a numeric rating, also allows separation of the vehicle's "adequacy for the selected task" and the demands placed on the pilot to achieve the task. The adequacy for the selected task ratings give a level rating of 1, 2, or 3, corresponding to "clearly adequate" (pilot rating 1, 2, or 3), "adequate" (pilot rating 4, 5, or 6), or "inadequate" (pilot rating 7, 8 or 9). The pilot rating number (1 through 10) has been shown to correlate well with perceived pilot workload, although workload remains difficult to measure directly.
Figure 15 - Cooper-Harper Handling Qualities Rating Scale [90]

In the automotive world, there is not a well accepted procedure for determining handling qualities. There have been some attempts to transfer the techniques developed in the aircraft field to passenger cars. McRuer and Klein [92], through simulator and field test studies, attempted to identify the characteristics of the driver/vehicle system that affects the drivers steering control. Through these controlled experiments, measuring both the driver and vehicle performance, they were able to show correlation between
vehicle characteristics and driver subjective opinion. These vehicle characteristics were also supported by driver/vehicle system theory developed during the research.

2.6 Issues Unique to Racing

This background chapter has examined somewhat generically the subjects of road vehicle dynamics, handling, and modeling. In many respects, all four-wheeled, pneumatic tired vehicles traveling on nominally flat paved surfaces can be treated alike. However, vehicles used for circuit racing (those that compete on non-straight tracks) have some unique characteristics and requirements that must be treated specially. In many ways, the racing vehicle has a much simpler objective than a passenger vehicle. The racing vehicle is designed with the primary goal of lapping a given track a specified number of laps, in the shortest time, while keeping within the design limitations imposed by the race sanctioning body. It must do this under the control of a human driver.

The vehicle is "setup" (meaning "optimized" in current engineering jargon) based on feedback from a single driver. It does not have to be suitable for an "average" member of the driving public. While passenger cars are tuned more for comfort than performance, racing cars trade comfort for performance (this is within limits: the driver must be physically able to complete the race while maintaining a very high level of concentration). A vehicle that is too harsh will physically wear out the driver, and performance will suffer). A racing car is setup for a particular section of road, typically less than five miles and sometimes as short as one-eighth of a mile. Finally, and probably
most importantly from an analysis point of view, racing cars are operated near their performance limits continuously, while passenger cars rarely operate near their limits.

2.7 References


62


CHAPTER 3

AERODYNAMIC FORCE ESTIMATION

3.1 Overview

The tire and aerodynamic forces acting on a road vehicle determine its overall performance. In passenger vehicles traveling at relatively low speeds, the tire forces are many times higher than the aerodynamic forces, and are thus the critical issue. For a winged racing car, however, aerodynamic and tire forces can be of the same order of magnitude. This chapter uses the state estimation procedures described in Section 2.4 to estimate the aerodynamics forces acting on a winged race car. The state estimation procedures use a vehicle model, \( f(x(t),u(t)) \), to propagate the vehicle state. In this case, a non-linear lumped parameter pitch-plane vehicle model is used. The aerodynamic forces acting on the vehicle (external forces) are modeled as higher order Gauss-Markov processes, and are estimated by the optimal state estimation procedures.

This chapter describes a five degree-of-freedom vehicle model (Section 3.2) that has been used to implement an optimal state estimator that is capable of estimating the aerodynamics forces acting vertically on the vehicle chassis at each axle, and the total longitudinal force (aerodynamics plus tire) acting on the chassis. The method of modeling the external forces using Gauss-Markov processes is described in Section 3.3.
Section 3.4 covers the modeling of the vehicle measurement system for use in the estimation algorithms. Section 3.5 demonstrates the procedure's ability to estimate aerodynamic forces by using noise corrupted simulation outputs as measurement data. Section 3.6 applies the estimator to actual vehicle measurements from a winged race car.

3.2 Pitch-Plane Vehicle Model

The pitch-plane model is shown schematically in Figure 16. The right and left suspension systems are collapsed to form a single equivalent suspension at each axle (commonly referred to as a bicycle model). The vehicle is modeled as having five degrees-of-freedom. The chassis (or sprung mass) can move longitudinally and vertically, and pitch about its center of gravity. Axis system $X_s$, $Z_s$ (shown in Figure 16) is attached to the sprung mass (body fixed). The front and rear suspensions each have a single translational degree-of-freedom relative to the sprung mass, parallel to its $Z_s$ axis, and their motions are denoted $Z_f$ and $Z_R$. The unsprung masses are assumed to move with the sprung mass in its longitudinal direction. The equations are derived relative to the vehicle's static equilibrium position when sitting on level ground at zero speed. All positions are set to zero at this condition.

The rear suspension system model is shown in Figure 17. The front suspension model is identical, except that an "F" replaces the "R" suffix in each variable name. There is a motion ratio used in the suspension model, $Mr_R$ and $Mr_F$, that is the ratio between the displacement of the main suspension spring and damper unit and the unsprung mass displacement relative to the sprung mass. This is the suspension rocker
arm, and its motion ratio is defined as wheel deflection over spring deflection. Typically, the spring moves less than the wheel, so $MrR$ will be greater than unity. All of the parameters for the springs and dampers are specified for the units themselves (the simulation takes care of all of the motion ratios).

Figure 16 — Pitch-Plane Model Schematic
Variable and Parameter Definitions:

\( Ms \) Chassis sprung mass (slugs)

\( Mus(F/R) \) Front/Rear unsprung mass (slugs)

\( I_{yy} \) Chassis pitch mass moment of inertia (ft-lbs-sec^2)

\( a \) Distance between sprung mass cg and front axle (ft)

\( b \) Distance between sprung mass cg and rear axle (ft)

\( H_s \) Chassis cg height above the ground at static equilibrium (ft)

\( X_s \) Chassis longitudinal deflection, positive forward (ft)

\( Z_s \) Chassis body fixed vertical deflection, positive up (ft)
\( \theta_s \)  
Chassis pitch angle, positive nose up

\( V_X \)  
Vehicle forward speed (ft/sec)

\( Z(F/R) \)  
Front/Rear unsprung mass vertical deflection (ft)

\( h(F/R) \)  
Front/Rear road height, positive up (ft)

\( F_{xtire}(F/R) \)  
Front/Rear tire longitudinal force, positive forward (lbs)

\( F_{ztire}(F/R) \)  
Front/Rear tire vertical force, positive up (lbs)

\( F_{zaero}(F/R) \)  
Front/Rear aerodynamic force, positive down (lbs)

\( F_{sus}(F/R) \)  
Front/Rear total suspension force (lbs)

\( Ls(F/R) \)  
Front/Rear suspension deflection, positive extension (ft)

\( V_s(F/R) \)  
Front/Rear suspension velocity, positive extension (ft/sec)

\( Lt(F/R) \)  
Front/Rear tire deflection, positive extension (ft)

\( V_t(F/R) \)  
Front/Rear tire velocity, positive extension (ft/sec)

\( Ks(F/R) \)  
Front/Rear suspension spring stiffness (lb/ft)

\( Kt(F/R) \)  
Front/Rear tire spring stiffness (lb/ft)

\( Bt(F/R) \)  
Front/Rear tire damping coefficient (lb-sec/ft)

\( Kad(F/R) \)  
Front/Rear anti-lift/dive coefficient (ft/ft)

\( V_{sFt} \)  
Front damper peak velocity table (in/sec)

\( BscFt \)  
Front damper peak compression force table (lbs)

\( BsrFt \)  
Front damper peak rebound force table (lbs)

\( V_{sRt} \)  
Rear damper peak velocity table (in/sec)

\( BscRt \)  
Rear damper peak compression force table (lbs)

\( BsrRt \)  
Rear damper peak rebound force table (lbs)

\( RH(F/R) \)  
Front/Rear chassis ride height measured at the axle centerline (ft)

\( RH(F/R)S \)  
Front/Rear static chassis ride height measured at the axle centerline with vehicle at rest (ft)

\( AccelX \)  
Distance of the chassis accelerometer behind front axle (ft)

\( AccelZ \)  
Distance of the chassis accelerometer below chassis cg (ft)
Based on free body diagrams, five differential equations are derived. The suspension forces are called $F_{susF}$ and $F_{susR}$, and are the total forces due to the springs and dampers. The parameters are specified in units of pounds, inches, and seconds to be consistent with the units used on the vehicle setup sheets. Within the simulation, however, all lengths are converted to feet, and velocities converted to ft/sec. The equations actually represent half of the vehicle, within the simulation; the masses and aerodynamic forces are divided by two. The forces are multiplied by two for all outputs.

Longitudinal Motion of Sprung Mass:

$$\ddot{X}_s = \left( F_{xaero} + F_{xtireR} + F_{xtireF} \right) / \left( M_s + M_{sF} + M_{sR} \right)$$  \hspace{1cm} (33)

Vertical Motion of Sprung Mass:

$$\ddot{Z}_s = - \left( F_{zaeroF} + F_{zaeroR} + \left( F_{susF} + F_{susR} \right) \cos(\theta_s) \right) / M_s$$  \hspace{1cm} (34)

Pitch Rotation of Sprung Mass:

$$\ddot{\theta}_s = \begin{bmatrix} -h \left( F_{susR} + F_{zaeroR} \cdot \cos(\theta_s) \right) + \ldots \\ \ldots a \left( F_{susF} + F_{zaeroF} \cdot \cos(\theta_s) \right) - \ldots \\ \ldots \left( H_s + Z_s + a \sin(\theta_s) - hF \right) F_{xtireF} - \ldots \\ \ldots \left( H_s + Z_s - b \sin(\theta_s) - hR \right) F_{xtireR} \end{bmatrix} \cdot \frac{1}{I_{ysys}}$$  \hspace{1cm} (35)

Vertical Motion of Front Unsprung Mass:

$$\ddot{Z}_F = \left( F_{susF} - F_{xtireF} \cos(\theta_s) \right) / M_{sF}$$  \hspace{1cm} (36)

Vertical Motion of Rear Unsprung Mass:

$$\ddot{Z}_R = \left( F_{susR} - F_{xtireR} \cos(\theta_s) \right) / M_{sR}$$  \hspace{1cm} (37)
Front and Rear Tire Forces:

\[ F_{xtireF} = KtF \cdot LtF + BtF \cdot VtF \]
\[ F_{xtireR} = KtR \cdot LtR + BtR \cdot VtR \]  

(38)

Front and Rear Suspension Forces:

\[ FsusF = \frac{KsF \cdot LsF}{MrF} + \frac{F_{damperF}}{MrF} + KadF \cdot F_{xtireF} + F_{otherF} (LsF, VsF, ...) \]
\[ FsusR = \frac{KsR \cdot LsR}{MrR} + \frac{F_{damperR}}{MrR} - KadR \cdot F_{xtireR} + F_{otherR} (LsR, VsR, ...) \]  

(39)

In the suspension force equations (39), the last term is \( F_{other(F/R)} \). This refers to suspension forces developed by suspension elements other than the main springs and dampers. Modern racing car suspensions often have additional elements that generate forces from motions and/or velocities other than pure bounce and roll. These systems are often developed by individual teams, and are proprietary. The racing car used in this research has some proprietary elements in the suspension whose details cannot be described in this document. They are however modeled, and the estimator results include their contribution.

The first step in computing the suspension forces \( FsusF \) and \( FsusR \) is to compute the deflection and velocities across all springs and dampers. For these variables, a positive value means extension, and a negative value means compression.

Spring Deflections:

\[ LsF = (a \sin (\theta_s) + Z_s \cos (\theta_s) - ZF) / MrF \]
\[ LsR = (-b \sin (\theta_s) + Z_s \cos (\theta_s) -ZR) / MrR \]  

(40)
Spring Velocities:

\[ V_{sF} = \left( a \cos(\theta_s) \dot{\theta}_s + \dot{Z}_s \cos(\theta_s) - Z_s \sin(\theta_s) \dot{\theta}_s - \ddot{Z}_F \right)/MrF \]

\[ V_{sR} = \left( -b \cos(\theta_s) \dot{\theta}_s + \dot{Z}_s \cos(\theta_s) - Z_s \sin(\theta_s) \dot{\theta}_s - \ddot{Z}_R \right)/MrR \]  

(41)

Tire Deflections:

\[ \text{LtF} = ZF - hF \]

\[ \text{LtR} =ZR - hR \]  

(42)

Tire Velocities:

\[ \text{VtF} = \dot{ZF} - \dot{hF} \]

\[ \text{VtR} = \dot{ZR} - \dot{hR} \]  

(43)

Front and Rear Axle Ride Heights:

\[ \text{RHF} = \text{RHFS} + Z_s \cdot \cos(\theta_s) + a \sin(\theta_s) \]

\[ \text{RHR} = \text{RHRS} + Z_s \cdot \cos(\theta_s) - b \sin(\theta_s) \]  

(44)

The front and rear damper forces (\(F_{damperF}\) and \(F_{damperR}\)) are interpolated from tables relating peak damper force to peak damper velocity using linear interpolation.

3.3 Modeling Aerodynamic Forces

When using an EKF for parameter identification, such as estimating tire characteristics, the usual practice is to include a model of the unknown system (tire or aerodynamic) into the estimator. The parameters of the model are treated as unmeasured states, and are optimally estimated by the EKF. The advantage of this method is that the state estimation and the parameter identification are performed in a single step. A
disadvantage is that if a different model structure for the system is necessary, the estimation algorithms must be reformulated and run again.

An alternate approach is to perform the state estimation and the system modeling separately (in this research, the unknowns are tire and aerodynamic forces). The tire and/or aerodynamic forces are estimated as functions of time by the EKF. In addition to allowing the tire or aerodynamic model to be changed without changing the state estimator, it also reduces the size of the estimator (in general), which reduces the estimator's computation time. This two step approach is termed Estimation Before Modeling (EBM) (see Section 2.4). The EBM approach was chosen for this research because of its ability to estimate tire and aerodynamic forces as functions of time, and the flexibility offered by separating the two procedures.

For the pitch-plane estimator described in Section 3.2, the unknown forces are the vertical aerodynamic downforce acting at each axle, and the total longitudinal force acting on the chassis. These forces are each modeled as second-order random walk processes, which is a second-order Gauss-Markov process [8]. If $x_0$ is the force to be estimated, the second order random-walk model is given by:

$$
\begin{bmatrix}
\dot{x}_0 \\
\dot{x}_1
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
x_0 \\
x_1
\end{bmatrix} + w_y
$$

(45)

Where $x_1$ is the first time derivative of $x_0$, and $w_y$ is random white noise. Higher order random-walk models can also be used (a third-order model was tried with little improvement over the second-order model). For each unknown force to be estimated, equation (45) is appended to the model state equations. In the state propagation step of
the EKF algorithm (Section 2.4), the discrete form of random-walk model propagation from step $k$ to step $k+1$ is:

$$x_0^-(k+1) = x_0^+(k) + x_1^+(k) \Delta t$$

(46)

Where $\Delta t$ is the time interval. The state update equation will be:

$$x_0^+(k) = x_0^-(k) + m_0^T \Delta z(k)$$
$$x_1^+(k) = x_1^-(k) + m_1^T \Delta z(k)$$

(47)

Where $\Delta z(k)$ is the measurement residual, and $m_0$ and $m_1$ are the Kalman gain vectors. For a stable EKF, the measurement residual will drive the system to "follow" the known measurements. By stable, it is meant that the measurement residual will approach zero mean with bounded variance. This condition will only be met by optimally estimating the unknown forces modeled by the random walk models.

The five degree-of-freedom estimator described in Section 3.2 has ten state equations and three unknown forces. This results in a sixteenth-order estimation model. The form of the estimation model becomes:

$$\dot{\hat{x}}_A(t) = \begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{\hat{F}}(t) \end{bmatrix} = \begin{bmatrix} f(\dot{\hat{x}}(t), \hat{F}(t), u(t)) \\ A \hat{F}(t) \end{bmatrix}$$

(48)

Where the caret, $\hat{}$, is used to denote an estimated variable, $x_A(t)$ is the augmented state vector, $F(t)$ is the vector of unknown forces $[F_x \; F_{\text{aero}} \; F_{\text{aero}}]^T$, and $A$, is the block diagonal matrix formed from Equation (45).
3.4 Measurement System Modeling

The EKF algorithm combines information from measurements made of the system, models of the system’s dynamic behavior, and statistical estimates of measurement and process noise to optimally estimate the state of the system at each integration time step. The pitch-plane estimator uses seven measurements from the vehicle on-board instrumentation. The measurements are: longitudinal and vertical accelerations, vehicle forward speed, front and rear shock displacements, and the forces in the front and rear suspension push-rods.

Referring to Figure 16, a two-axis accelerometer is mounted in front of and below the chassis cg. The accelerometer measures in the chassis X and Z axes. The length, \( L \), and angle, \( \theta_a \), of the accelerometer to the cg are:

\[
L = \left( \text{Accel}X^2 + \text{Accel}Z^2 \right)^{0.5}
\]

\[
\theta_a = \tan^{-1} \left( \frac{H_s - \text{Accel}Z}{a - \text{Accel}X} \right)
\]

(49)

The chassis longitudinal acceleration, \( A_X \), is given by:

\[
A_X = \ddot{X}_s \cos(\theta_s) - L\dot{\theta}_s^2 \cos(\theta_a) + L\dot{\theta}_s \cos \left( \frac{\pi}{2} - \theta_a \right) + \left( \ddot{Z}_s - g \right) \sin(\theta_s)
\]

(50)

The chassis vertical acceleration, \( A_Z \), is given by:

\[
A_Z = \left( \ddot{Z}_s - g \right) \cos(\theta_s) + L\dot{\theta}_s \cos(\theta_a) + L\dot{\theta}_s^2 \cos \left( \frac{\pi}{2} - \theta_a \right) - \ddot{X}_s \sin(\theta_s)
\]

(51)

Vehicle forward speed is measured from the rotational speed on one of the front wheels, and multiplying by tire rolling radius. The model computes forward speed, \( V_X \), by:
\[ V_x = \left( \dot{X}_s^2 + \dot{Z}_s^2 \right)^{0.5} \]  

Shock displacement is measured using linear potentiometers connected to each spring/shock assembly. Since this is a bicycle model, the left and right shock displacement measurements are averaged. The displacements are measured relative to the shock at full extension, so at the static ride height, the measurements are non-zero and contain the shock compression due to the chassis weight. The model computes the shock displacement measurement by:

\[
L_{sprF} = -LsF + L_{sprFI} \\
L_{sprR} = -LsR + L_{sprRI}
\]  

Where \( L_{sprFI} \) and \( L_{sprRI} \) are the static shock displacement measurements.

3.5 Estimator Evaluation using Simulation Data

Before the pitch-plane estimator is used with actual vehicle measurement data, its ability to estimate known aerodynamic forces must be demonstrated. Using actual vehicle measurement data, only "sanity checks" can be made for the aerodynamic force estimates, since the actual values are unknown. However, using simulation data, the aerodynamic forces are known, and other simulation outputs can be used to simulate actual vehicle measurements. This data is then be used to evaluate the estimator’s performance.

The Vehicle Dynamics Analysis, Non-Linear (VDANL) simulation from Systems Technology, Incorporated is a seventeen degree-of-freedom nonlinear vehicle handling simulation [12, 13] (VDANL version 6.00.31 was used in this study). The VDANL
simulation is used to model the handling and rollover performance of passenger vehicles and heavy trucks running on flat and sloped roadways. VDANL contains a “User Defined Module” option that allows simulation users to add subroutines to the simulation. This feature is used to add an aerodynamic downforce model to VDANL (the standard aerodynamic model only contains vehicle drag, and a cross wind model for aerodynamic side force, roll moment and yaw moment). The new aerodynamic model contains downforce at the front and rear axles and aerodynamic drag as nonlinear functions of front and rear axle ride heights and vehicle speed.

The example winged race car weighs 2000 lb, with 45 percent of its weight on the front axle. The front and rear wheel rates (suspension stiffness in heave) are 2000 and 1000 lb/in respectively. The tire stiffness is 1500 lb/in for each tire. Figure 18 shows the aerodynamic downforce and drag maps (aerodynamic vertical load on the sprung mass at the front and rear axles and longitudinal aerodynamic force plotted versus front and rear ride height measured at the front and rear axles). These maps represent the aerodynamic forces at 200 miles per hour. The maximum front and rear downforce is approximately 2100 lb and 2000 lb respectively. Maximum drag is approximately 1500 lb. Figure 18 shows that front downforce is much more sensitive to ride height variations than either the rear downforce or drag.

The test case is a simulation run where the drivetrain is used to accelerate the vehicle at a constant 10 ft/sec² up to 150 ft/sec, then hold the speed constant. To simulate the effect of measurement noise, the random number generator in MATLAB (rand function) is used to generate a uniformly distributed vector whose elements are between
±5 percent of the difference between minimum and maximum values in each simulation output channel. This vector is then added to the simulation output. For example, if the speed output varied between 50 and 150 ft/sec, the following MATLAB code would add the “noise” to the Speed channel:

```matlab
p = 0.05; % set noise to 5 percent
rand('seed',sum(100*clock)); % seed rand using clock
d=Speed;
% 
r=(rand(size(d))-.5)*(max(d)-min(d))*p; % generate noise
Speed = (Speed + r); % add noise to output
```

Since the experimental vehicle measurement data is digitally filtered prior to being used in the estimator, the noise corrupted simulation outputs are also run through a phaseless Butterworth low pass filter with a 30 Hertz cutoff frequency and a total of 2 poles. Figure 19 through Figure 28 show the results of the estimator using the simulated vehicle data. The broken lines are the “measurement data” from the VDANL simulation and the solid lines are the estimates from the pitch-plane estimator.
Figure 18 - Aerodynamic Downforce and Drag Maps
Figure 19 - Estimator Performance for Vehicle Speed

Figure 20 - Estimator Performance For Longitudinal And Vertical Chassis Acceleration
Figure 21 - Estimator Performance For Front And Rear Spring Length

Figure 22 - Estimator Performance For Front And Rear Spring/Damper Force
Figure 23 - Estimator Performance For Front And Rear Ride Height

Figure 24 - Estimator Performance For Total Aerodynamic Downforce
Figure 25 - Estimator Performance For Front Aerodynamic Downforce

Figure 26 - Estimator Performance For Rear Aerodynamic Downforce
Figure 27 - Estimator Performance For Aerodynamic Center-Of-Pressure

Figure 28 - Estimator Performance For Longitudinal Forces
Figure 19 through Figure 23 show that the estimator tracked the simulated measurements quite closely (from Section 3.4, the measurements are: longitudinal and vertical acceleration, vehicle forward speed, front and rear shock displacement, the forces in the front and rear suspension push-rods, and front and rear ride height). The EKF algorithm has eliminated most of the noise added to the measurements. The amount of this “filtering” is controlled by the process and measurement system covariance matrices (Q and R in Section 2.4).

Figure 24 shows the estimated total aerodynamic downforce. The dash-dot line shows the actual downforce from the simulation data. The total downforce estimate tracks the “actual” downforce well after an initial startup transient. The third curve, shown by the dotted line is a simple estimate of total downforce generated by subtracting the chassis and suspension weights from the “measured” suspension forces and correcting for longitudinal load transfer using the estimated longitudinal acceleration. For this maneuver with low frequency inputs and no external disturbances, it matches the actual downforce well.

Figure 25 and Figure 26 show the estimated front and rear downforce (solid lines) compared to the actual downforce (dash-dot lines) and the downforce from a simple estimate using the vehicle weight and measured suspension loads (dotted lines). The estimator tracked the front and rear downforce well, while the simple estimate had significant errors. Figure 27 is another way of looking at the front and rear down force. The aerodynamic center-of-pressure is computed as the percent of the total wheelbase where the total downforce resolves on the vehicle. It is often more useful to think about
total downforce and center-of-pressure rather than front and rear downforce. Total
downforce is related to additional vehicle traction from the aerodynamics and center-of-
pressure is a measure of the aerodynamic balance between the front and rear of the
vehicle. These graphs show that the EKF algorithm estimates do a good job of estimating
the true values: while other simpler estimates based on suspension load measurements are
not adequate for reliably estimating aerodynamic forces.

Finally, Figure 28 shows the estimated longitudinal tire forces, along with the
modeled aerodynamic drag and total longitudinal force acting on the chassis. The broken
line at the bottom of the plot is the modeled aerodynamic drag. The heavy solid line is
the total estimated longitudinal force acting on the vehicle. The thin solid line is the
estimated rear longitudinal tire force (there is no estimated front tire force for this run
since there was no braking). The aerodynamic drag force is scaled based on vehicle
speed from the wind tunnel measured vehicle drag at 200 mph (293.3 ft/sec). The
equation for aerodynamic drag is:

\[
F_{\text{aero}} = -D\text{RA}G \frac{V^2}{293.3^2}
\]

Where \(D\text{RA}G\) is the aerodynamic drag in pounds at 200 mph. The EKF algorithm
estimates the total longitudinal force acting on the chassis. The total estimated
longitudinal force is consistent with the sum of the estimated tire and aerodynamic
longitudinal forces.

In the actual race car measurement system, there is no measurement of brake
pedal force, and the distribution of the longitudinal forces between the front and rear tires
is only an estimate. The algorithm makes the basic assumption that the brake and throttle
will not be pressed at the same time (not necessarily a good assumption for a racing car).
If longitudinal acceleration is positive (acceleration due to engine torque), it is assumed that the longitudinal tire forces are from the drive tires only (rolling drag of the non-driven tires is assumed to be zero). If the longitudinal acceleration is negative (braking), then the longitudinal tire forces are distributed between the front and rear tires based on the front to rear brake balance. Figure 24 through Figure 28 demonstrate the estimator’s ability to track the quasi-steady state change in the unknown force applied to the chassis.

In actual use, vehicles travel on roads that are not perfectly smooth as in the example above. The EKF has the desirable characteristic of being able to reject external disturbances, such as roadway roughness. To test this, the same simulation model was run over a bump at 150 feet/sec. The bump was 15 feet long and 0.25 inches high and is shown in Figure 29.

![Figure 29 - Roadway Bump](image)

89
Figure 30 through Figure 34 show the results of this simulation. Looking at the measurement data, the estimator is able to track the measurement data fairly well, though obviously not as well as in the previous smooth road example. The total downforce and center-of-pressure estimates show that the estimator (solid line) is able to reject a lot of the disturbance from the bump and track the actual downforce (dash-dot line). Comparing the estimator’s results to the simple estimates using vehicle weight and measured suspension forces (dotted line), there is a considerable improvement in the aerodynamic force estimates (Figure 33 and Figure 34). This simple estimate is what would typically be used in a racing environment and is only good in steady-state when the roadway is quite smooth. Heavy low-pass digital filtering is required to smooth it, so its utility is very limited. The EKF estimator allows the aerodynamic forces to be used with much less filtering since it can reject much of the roadway disturbance.
Figure 30 - Bump Simulation: Longitudinal And Vertical Accelerations

Figure 31 - Bump Simulation: Spring Lengths
Figure 32 - Bump Simulation: Suspension Forces

Figure 33 - Bump Simulation: Total Aerodynamic Downforce
The above examples using noise corrupted simulation output as "measurement" data show that the EKF can track unmeasured aerodynamic forces and reject much of the unknown roadway roughness. The EKF will now be used with actual measured data from a winged race car. The data is from an entire lap of a road course circuit. In the following figures, solid lines show the estimated quantities and the measured data by dashed lines.
Figure 35 – Measured and Estimated Data: Longitudinal And Vertical Accelerations

Figure 36 - Measured and Estimated Data: Vehicle Speed
Figure 37 - Measured and Estimated Data: Spring Lengths

Figure 38 - Measured and Estimated Data: Front and Rear Ride Height
Spring/Damper Forces

Figure 39 - Measured and Estimated Data: Front and Rear Suspension Force

Figure 40 - Measured and Estimated Data: Total Downforce
Figure 41 - Measured and Estimated Data: Front Downforce

Figure 42 - Measured and Estimated Data: Rear Downforce
Using the measured data, the EKF estimates show subjectively similar results to the estimates using the simulation data. Measured states are tracked and some "noise" is removed. The rear ride height estimate, shown in Figure 38, shows discrepancy in areas of low ride height, which correspond to high vehicle speed and high downforce. The cause of this may be an un-modeled characteristic, and/or a measurement problem. Looking at the measured rear ride height, it is curious that while speed and downforce increase, the ride height measurement seems to attenuate at about 1.2 inches. This may be some limitation of the laser ride height sensor or its installation. Another possibility is tire growth with speed. The tires on the vehicle do exhibit some growth with speed, and this affect was not included in the EKF model. The growth is slightly quadratic and on
the order of a few tenths of an inch at 200 mph. This tire growth likely explains some of
the differences in the rear ride height estimate.

Figure 40 through Figure 43 show the aerodynamic downforce estimates and the
computed aerodynamic center-of-pressure (solid line) compared to the simple estimate
(dotted line). Unlike the simulated data results, the actual aerodynamic forces are
unknown, so the accuracy of the estimates cannot be stated. Similar to the results from
the simulated data, the EKF estimates show less variation than the simple estimates,
indicating that the EKF is rejecting some of the disturbances acting on the vehicle
(primarily through roadway roughness). There are a few places where the estimated
downforce has a negative spike (two are at approximately 46 and 52 seconds). The
vehicle is traveling close to 150 ft/sec in both cases, meaning that there "should" be
considerable downforce being generated. At both of these points, the spring length plots,
Figure 37, show short duration increases in spring length that are not preceded by any
reduction in spring length. The front suspension force also shows reductions at these
points and the vertical acceleration goes to approximately zero. These are sections of the
track that drop away from the vehicle (also shown by increases in the ride heights). This
may momentarily increase the volume of air under the car and actually cause positive
pressure and lift. The rear downforce does not show as much aerodynamic lift as the
front. This is certainly due to its high mounted wing, which is only minimally affected
by under car airflow. At both of these sections of the track, which are in corners, the
driver reported the car transitioning between understeer and oversteer.
3.6 References


CHAPTER 4

TIRE SIDE FORCE AND SLIP ANGLE ESTIMATION

4.1 Introduction

In Chapter 3, an Extended Kalman Filter (EKF) was designed to estimate the aerodynamic forces acting on a winged racing car. In this chapter, a new EKF is designed to estimate the lateral tire forces and body and tire slip angles. These quantities are basic to the understanding of the vehicle directional dynamics, however, are not available in most measurement systems.

The EKF uses a three degree-of-freedom vehicle model with yaw velocity, lateral velocity, and roll angle as its states. The tire lateral forces are unknown and modeled as second order Gauss-Markov processes. The measurements required are yaw rate, lateral acceleration, longitudinal acceleration, and chassis roll velocity.

Following the procedures used in Chapter 3 with the aerodynamic force estimator, the EKF is checked out using noise corrupted outputs from a non-linear simulation, then applies the estimator to actual measured vehicle data.

4.2 Yaw/Roll-Plane Vehicle Model

This section details the development of a three degree-of-freedom vehicle model with time-invariant (LTI) parameters. The basis for the model is the work of Segel in the
1950's [1], and it has appeared in the technical literature in numerous forms since [2, 3]. The three degrees of freedom are: lateral perturbation velocity of the vehicle roll axis, $v_y$; total vehicle yaw rate, $r$; and sprung mass roll angle relative to the ground, $\phi$. No pitch or bounce degrees of freedom are modeled and the vehicle speed, $V_x$, is modeled as quasi-static. The input to the model is steering wheel angle. Lateral load transfer is not included in this linear model, so the two tires on each axle can be lumped together. This type of model is termed a “bicycle model” in the vehicle dynamics literature. The three equations describing the directional response of the vehicle assume small angles and all products of inertia are assumed to be zero. The vehicle sprung mass is assumed to roll about a fixed roll-axis that is parallel to the ground. Figure 44 shows the yaw-plane (x-y axes) schematic of the vehicle model and Figure 45 shows the roll-plane (y-z axes) schematic.
Figure 44 - Yaw-Plane Diagram of 3DOF Model
Using Newton’s law, the following three differential equations can be written that describe the three vehicle degrees of freedom.

**Side Force Equation:**

\[
m(\dot{V}_y + V_r e) + m_e e \Phi_s = -2(F_{yR} + F_{yF})
\]

(55)

**Yaw Moment Equation:**

\[
I_{\dot{\phi}} = -2(a E_{yf} - b F_{yR})
\]

(56)

**Roll Moment Equation:**

\[
I_{xx} \ddot{\Phi}_s + m_e (\dot{V}_y + V_r e) = (m_e g e \frac{\delta L}{\delta \Phi_s}) \Phi_s - \frac{\delta L}{\delta \Phi_s} \dot{\Phi}_s
\]

(57)
Where:

\( V_x \)  Longitudinal velocity (ft/s)

\( V_y \)  Vehicle longitudinal steady-state velocity (ft/s)

\( \delta_{SW} \)  Steering wheel angle (rad)

\( r \)  Yaw rate (rad/s)

\( \Phi_r \)  Roll angle (rad)

\( m \)  Total vehicle mass (slugs)

\( m_s \)  Vehicle sprung mass (slugs)

\( a \)  Distance from C.G. to front axle (ft)

\( b \)  Distance from C.G. to rear axle (ft)

\( g \)  Acceleration due to gravity (ft/sec\(^2\))

\( e \)  Distance from roll axis to sprung mass center-of-gravity (ft)

\( K_{SW} \)  Vehicle steering ratio (rad/rad)

\( F_{yf, yr} \)  Tire lateral force (lb)

\( \beta \)  Vehicle slip angle (rad)

\( I_z \)  Total vehicle yaw inertia (ft-lb-s\(^2\))

\( I_{xxs} \)  Sprung mass roll inertia (ft-lb-s\(^2\))

\( \frac{\delta L}{\delta \Phi} \)  Total vehicle roll stiffness (ft-lb/rad)

\( \frac{\delta L}{\delta \Phi} \)  Total vehicle roll damping (ft-lb-sec/rad)

\( e_F \)  Front axle roll steer (rad/rad)
Rear axle roll steer (rad/rad)

Solving Equations (55), (56), and (57) for $\dot{V}_y$, $\dot{r}$, and $\ddot{\Phi}_s$ yields a four state model given by:

$$
\dot{V}_y = \frac{2I_{xx}(F_{yF} + F_{yR}) - V_x I_x r + m_e K_o \Phi_s - m_e \frac{\partial L}{\partial \Phi_s} \Phi_s}{I_x} 
$$

(58)

$$
\dot{r} = \frac{2(b F_{yR} - a F_{yF})}{I_z} 
$$

(59)

$$
\ddot{\Phi}_s = -\frac{m K_o \Phi_s - m \frac{\partial L}{\partial \Phi_s} \Phi_s + 2m_e (F_{yF} + F_{yR})}{I_x} 
$$

(60)

Where:

$$
I_x = (m_e)^2 - m I_{xx} 
$$

(61)

$$
K_o = m_e g e - \frac{\partial L}{\partial \Phi_s} 
$$

(62)

4.3 Modeling Tire and Aerodynamic Forces

When using an EKF for parameter identification, such as estimating tire characteristics, the usual practice is to include a model of the unknown system (tire or aerodynamic) into the estimator. The parameters of the model are treated as unmeasured states, and are optimally estimated by the EKF. The advantage of this method is that the
state estimation and the parameter identification are performed in a single step. A
disadvantage is that if a different model structure for the tires or aerodynamics is
necessary, the estimation algorithms must be reformulated, and re-run.

An alternate approach is to perform the state estimation and the tire and
aerodynamic modeling separately. The tire and aerodynamic forces are estimated as
functions of time by the EKF smoother. In addition to allowing the tire or aerodynamic
model to be changed without changing the state estimator, it also reduces the size of the
estimator (in general), which reduces the estimator’s computation time. This two step
approach is termed Estimation Before Modeling (EBM). The EBM approach was chosen
for this research because of its ability to estimate tire and aerodynamic forces as functions
of time, and the flexibility offered by separating the two procedures.

For the three degree-of-freedom estimator, the unknown forces are the lateral tire
forces of each axle. These forces are each modeled as second-order random walk
processes, which is a second-order Gauss-Markov process. If $y_0$ is the force to be
estimated, the second order random-walk model is given by:

$$
\begin{bmatrix}
\dot{y}_0 \\
\dot{y}_1
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
y_0 \\
y_1
\end{bmatrix} + w_y
$$

(63)

Where $y_1$ is the first time derivative of $y_0$ and $w_y$ is random white noise. Higher
order random-walk models can also be used (a third order model was tried for the tire
forces with little improvement over the second order model). Equation (45) is appended
to the state equations for each unknown force to be estimated. The three degree-of-
freedom estimator described in Section 4.2 had four state equations and two unknown
forces. This results in an eighth-order estimation model. The form of the estimation model becomes:

\[
\begin{pmatrix}
\dot{x}(t) \\
\dot{F}(t)
\end{pmatrix} = \begin{pmatrix} f(\hat{x}(t), \hat{F}(t), u(t)) \\
A \hat{F}(t)
\end{pmatrix}
\]

(64)

Where the caret, \(\hat{\cdot}\), is used to denote an estimated variable, \(x_a(t)\) is the augmented state vector, \(F(t)\) is the vector of unknown forces \([F_y \ F_x]^T\), and \(A\) is the block diagonal matrix formed from Equation (45).

4.4 Measurement System Modeling

The EKF and MBF smoother algorithms combine information from measurements made of the system, models of the system's dynamic behavior, and statistical estimates of measurement and process noise to optimally estimate the state of the system at each integration time step. While it is attempted to directly measure model states, practical considerations often preclude this, and the sensors must be modeled to make their outputs compatible with the estimators. As an example, accelerometers are often attached to the chassis, near the vehicle center-of-gravity to measure vehicle lateral acceleration. This acceleration measurement is not the same as the time derivative of the vehicle lateral velocity, \(\dot{V_y}\), in the three degree-of-freedom model described above (Section 4.2). The accelerometer measurement is described by:

\[
A_y = \dot{V}_y + V_{yr} + \ddot{\Phi}_y (e + Z_{offset}) + g \sin(\Phi_y) + \dot{r} X_{offset}
\]

(65)
Where $X_{offset}$ and $Z_{offset}$ are the distances from the accelerometer to the sprung mass center-of-gravity. For the estimator described in Section 4.2, measurements of yaw rate, $r$, lateral acceleration, $A_y$, and roll rate, $\Phi_z$, were used making $z(t) = [r \ A_y \ \Phi_z]^T$. Steering wheel angle, $\delta_{sw}$, was also measured and used in the vehicle model. Assuming small angles, both roll rate and yaw rate measurements are direct measures of vehicle states, and do not require any modeling.

4.5 Estimator Evaluation using Simulation Data

Before the yaw/roll-plane estimator is used with actual vehicle measurement data, its ability to estimate known tire forces and slip angles must be demonstrated. Using actual vehicle measurement data, only “sanity checks” can be made for the estimates, since the actual values are unknown. However, using simulation data, the forces and slip angles are known, and other simulation outputs can be used to simulate actual vehicle measurements. This data is then be used to evaluate the estimator’s performance.

Like the simulation data checkout in Chapter 3, the Vehicle Dynamics Analysis, Non-Linear (VDANL) simulation from Systems Technology, Inc. is used to generate the test data. The same vehicle model is used, with the only differences being the control inputs. The maneuver used was a constant speed J-Turn (often referred to as a pseudo-step steer maneuver) run at 150 ft/sec. The steering resulted in a steady-state lateral acceleration of approximately 1.4 g’s, making it a fairly severe maneuver. The measurement noise was added to the simulation outputs using the same code as used in the Chapter 3 simulations. Figure 46 through Figure 49 show the results of the estimator.
using the simulated vehicle data. The broken lines are the “measurement data” from the VDANL simulation and the solid lines are the estimates from the yaw/roll-plane estimator.

Figure 46 - Steering Input and Vehicle Speed
Figure 47 - Estimator Performance: Lateral Acceleration and Yaw Rate

Figure 48 - Estimator Performance: Roll Angle and Roll Rate
Figure 49 - Estimator Performance: Tire Lateral Force

The EKF is able to track and smooth the measured signals: lateral acceleration, yaw rate and roll rate. It also accurately predicts vehicle roll angle. The unknown tire lateral forces are also accurately predicted. Vehicle lateral velocity is related to vehicle lateral acceleration, yaw rate and speed by:

\[ V_y = \int_0^r A_{ys} - r \cdot V_x \]  

where \( A_{ys} \) is the chassis lateral acceleration parallel to the inertial X-Y plane. The lateral acceleration used by the EKF, \( A_{ys} \), is a chassis-fixed accelerometer and is related to \( A_{ys} \) by:

\[ A_{ys} = \frac{g \cdot \sin(\phi_x)}{\cos(\phi_x)} \]  

113
Where \( g \) is the acceleration due to gravity. With lateral velocity computed, vehicle sideslip angle, \( \beta \), is computed by:

\[
\beta = \tan^{-1}\left(\frac{V_y}{V_x}\right) \tag{68}
\]

Front and rear tire slip angle, \( \alpha_F \) and \( \alpha_R \), can also be computed by:

\[
\alpha_F = \frac{V_y + a \cdot r}{V_x} - \frac{\delta_{SW}}{K_{SW}} - e_F \Phi_S \]
\[
\alpha_R = \frac{V_y - b \cdot r}{V_x} - e_R \Phi_S \tag{69}
\]

The front to rear "balance" of a racing car is very important to the driver and is a topic of great concern for the race engineer. Many different terms are used such as understeer/oversteer, pushing/loose, etc. What they all are attempting to describe is the lateral traction capability ("grip" in racing jargon) of the front axle compared to the rear axle. Understeer or push is used to describe the front axle losing traction before the rear. This required the driver to add more steering wheel angle in an attempt to follow the desired path. Oversteer or loose is the opposite condition with the rear axle losing traction first and requiring a reduction of steering input. A "neutral" car would have similar traction capabilities from both axles in a certain corner.

These terms and descriptions are not precise, and do not have formal definitions. Understeer and oversteer are defined by SAE [4] and ISO [5] only during steady-state cornering. This is a fairly rare occurrence on a racetrack, where the vehicle is either accelerating or braking most of the time. However, the tire slip angle estimates can be used to get some information regarding the vehicle balance. For a simple vehicle with
identical tires front and rear, steady-state understeer would result in the front tire slip angles being greater than the rear axle slip angles. The opposite would be true for an oversteering car. For a car with different tires front and rear, this will not be precisely true, however the difference between the front and rear slip angles will be related to the vehicle balance. Balance is defined here as:

\[ \text{Balance} = (\alpha_f - \alpha_r) \cdot \text{sign}(A_y) \]  

(70)

The multiplication by the sign of the lateral acceleration keeps the case of higher front slip angle positive. For the simple vehicle, a positive value for Balance would be understeer, and a negative value would be oversteer. For a vehicle with different tires front and rear, the transition from understeer to oversteer would not occur at zero, however a value that was "more positive" would indicate more understeer or less oversteer.

Figure 50 and Figure 51 show the computed vehicle sideslip angle, vehicle balance, and tire slip angles plotted with the "known" values from the simulation output. The estimates are very close. There is a slight error for the front axle slip angle. This is due to the slip angle equation used for the estimate not accounting for suspension steer compliance. The steer angle compliance is affected by both tire lateral force and tire aligning moment. Some of this error could be removed by including the compliance due to lateral force. Tire aligning moment is not estimated and is not directly known. If tire force and moment data is available, the aligning moment could be estimated from the
estimated slip angle and tire normal load. However, for a racing car designed to have minimal suspension compliance, neglecting the compliance will have a small affect and is not used in this EKF. This removed the need for any prior knowledge of the tire performance characteristics. The simulated vehicle had identical tires front and rear, so its positive value for Balance indicates that the vehicle had steady-state understeer.

The estimator has been shown to accurately estimate tire lateral forces and slip angles using the “known” simulation results. It is now applied to actual measured data from a winged racing car. Data is from an entire lap of a road racing circuit.

Figure 50 - Estimator Performance: Vehicle Sideslip Angle and F/R Balance
Figure 51 - Estimator Performance: Tire Slip Angles

Figure 52 - Measured and Estimated Data: Steering Wheel Angle and Vehicle Speed
Figure 53 - Measured and Estimated Data: Lateral Acceleration and Yaw Rate

Figure 54: Measured and Estimated Data: Roll Angle and Roll Rate
Figure 55 - Measured and Estimated Data: Estimated Tire Lateral Forces

Figure 56 - Measured and Estimated Data: Estimated Body Sideslip Angle and F/R Balance
Figure 57 - Measured and Estimated Data: Estimated Tire Slip Angles

Figure 52 shows the measured steering wheel angle and vehicle speed for the entire lap. The speed data shows that by standard passenger car standards, the speeds are very high. The slowest portions of the lap are approximately 60 mph, which the highest speed is about 190 mph. Figure 53 overlays the lateral acceleration and yaw rate estimates (solid lines) with the measured data (dashed lines). The estimates track the measured data well, removing some noise from the data. There are a few sections where there are significant differences between the measurement and the filtered data, such as at twelve and thirty seconds for the yaw rate estimate. In both of these cases, the vehicle is braking from high speed, and changing gears from sixth gear down to third gear. Each gear is engaged on the way down to allow the engine to help with the vehicle
deceleration. This seems to show up in the vehicle yaw rate, presumably from slight locking of the rear tires, which is destabilizing to the vehicle.

Figure 54 shows the estimated roll angle and roll rate with the measured data. The roll rate transducer was not available in this measurement set. Measured roll rate was estimated by differentiating roll angle. Measured roll angle was computed by the difference in heights of the left and right laser ride height sensors. This is an inherently noisy way of measuring roll rate. So, it is not surprising that the estimated roll angle does not match the measured data. Given the quality of the roll rate measurement in this data set, including roll rate in the measurements used by the EKF likely does little to improve the quality of the estimated tire forces.

The estimated front and rear tire lateral forces are shown in Figure 55. No measurement data is available to compare with the estimate. However, some observations can be made about the estimates. First, the rear tire forces are higher than the front. The vehicle has about fifty six percent of its weight on the rear axle, so the rear tire forces should be higher. Also, the magnitudes of the tire forces while in corners follow the cornering speed. Looking at the first and second corners for example (starting at times three and fourteen seconds), the first corner speed is approximately one hundred and ten miles per hour, while the second corner is about sixty miles per hour. The wings on this vehicle generate significant down force, increasing the normal load on the tires with increasing speed. This allows the vehicle to generate higher lateral tire forces in faster corners. The estimated tire forces in the first corner are significantly higher than the second corner, which would be expected from the speed differential. The estimator
does not "know" anything about this affect, yet its tire force estimates follow this expected trend.

Figure 56 shows the estimated body sideslip angle, $\beta$, and front to rear Balance computation. The sideslip angle shows large swings in the two long braking areas starting at twelve and thirty seconds. Looking at the driver's comments, he reported that the car "had oversteer coming off the brakes" on the entry to the second corner (twelve seconds) and that the "rear moves" in the braking area after thirty seconds. It is likely that these comments correspond to the high vehicle sideslip angles. These two comments were the only ones related to braking.

The Balance computation shows that the vehicle generally understeers. The setup of the vehicle has more than seventy five percent of the chassis roll stiffness on the front axle. Adding roll stiffness to an axle will increase its weight transfer and reduce its lateral force generating capacity. The large front roll stiffness distribution is an understeer effect. This sort of setup is a compromise to try to keep the normal load on the two rear tires more even, which will help their longitudinal traction when accelerating out of corners. The driver reported understeer at mid-corner at every corner on the track. At mid-corner, the vehicle is as close as it will get to being in a steady-state corner. The driver has completed his braking and has not yet started accelerating out of the corner. The Balance computation seems to show this affect.

The first two corners are easy to see in the data and can be examined individually, while the rest of the corners basically blend into one another and are more difficult to separate. The driver reported understeer through the whole first corner. His comments...
for the second corner were oversteer during corner entry, understeer in mid-corner and oversteer on corner exit. The Balance during the first corner is a fairly constant understeer. In the second corner, there is a lot of variation during corner entry and exit, with constant understeer in the middle. The Balance for these two corners seems to compare well with what the driver described.

Using data from a single lap, it appears that the Balance computation provides an objective measure of what the driver is subjectively describing. Data from a different lap will now be processed using the same EKF and compared to the original lap. This new lap is from the same day, but was run earlier during the session (to ease this discussion, the lap shown above will be referred to as lap A and this new lap will be referred to as lap B). The lap time for lap B was 0.84 seconds slower than lap A. The main difference in the driver's comments between the laps were that there was less mid-corner understeer and more corner exit oversteer during lap B in slow and medium speed corners, which delayed the driver in applying full power. This delay in applying power is responsible for the slower lap time. Figure 58 through Figure 63 show the EKF results from lap B.
Figure 58 - Measured and Estimated Data Lap B: Steering Wheel Angle and Vehicle Speed

Figure 59 - Measured and Estimated Data Lap B: Lateral Acceleration and Yaw Rate
Figure 60 - Measured and Estimated Data Lap B: Roll Angle and Roll Rate

Figure 61 - Measured and Estimated Data Lap B: Estimated Tire Lateral Forces
Figure 62 - Measured and Estimated Data Lap B: Body Sideslip Angle and F/R Balance

Figure 63 - Measured and Estimated Data Lap B: Tire Slip Angles
The comparison of the two laps will focus on the first two corners. This is the data from time zero up to about twenty-two seconds. Figure 64 through Figure 68 overlay lap A (solid line) and lap B (dotted line). The data is plotted versus distance, rather than time to ensure that the corners line up.

Figure 64 - Lap Comparison: Steering Wheel Angle and Vehicle Speed
Figure 65 - Lap Comparison: Lateral Acceleration and Yaw Rate

Figure 66 - Lap Comparison: Tire Lateral Force
Figure 67 - Lap Comparison: Sideslip Angle and F/R Balance

Figure 68 - Lap Comparison: Tire Slip Angle
In Figure 64, it can be seen that the cornering speed in the first corner for lap B (the slower lap) was slower than lap A by about eight miles per hour. The second corner had almost identical speeds. In both corners, the driver's steering wheel input was less for lap B, and this is particularly evident in the second half of the first corner (corner exit). The driver's commented about corner exit oversteer during lap B. When a vehicle is oversteering, the driver must reduce the steering wheel input to compensate and in severe cases may even reverse the steering wheel direction.

The vehicle lateral acceleration and yaw rate, shown in Figure 65 are very similar between the two laps. The biggest differences are seen in the yaw rate during the exit of the second corner, where lap B shows more variation in the data. With the first corner speed being lower for lap B, yet the lateral acceleration and yaw rate being very close to lap A, it appears that the driver's "line" through the corner was different between the two laps. Assuming that the vehicle path can be approximated by a constant radius, the data indicates that the corner radius for lap B was smaller than for lap A. A driver's strategy for a corner will be, to first approximation, make the radius as large as possible, which will maximize vehicle speed through the corner.

Like the lateral acceleration and yaw rate data, Figure 66 shows that the estimated tire lateral force for the two laps are similar. The tires used for the two laps were the same set and there were no aerodynamic changes made to the car. The tire lateral forces between the two laps being approximately the same indicate that the tire/roadway friction did not change significantly. Racing tires deposit rubber onto the track surface and this will over time increase the available tire/roadway friction. However, these two laps were
run within forty five minutes of each other on a day when there were not too many cars running on the track. So, it would not be expected that the track conditions would change significantly during this time.

Figure 67 shows the body sideslip angle and Balance comparisons. The sideslip angles between the two laps are similar, other than some phasing differences between about seven hundred and eleven hundred feet. There is some steering activity during this period during lap B, and examining the rear tire force for lap B, there is a short period where the tire force drops. The Balance also drops during this time, indicating that the car either “understeered less”, or started to oversteer. The way Balance is computed, it is not guaranteed that the transition from understeer to oversteer will be at zero. This vehicle is rear weight biased, with much larger tires on the rear than the front. It is likely that the transition from understeer to oversteer for this vehicle is at some positive Balance value. The Balance for the second half of the first corner for lap B is about half of the value for lap A. The driver comments for lap B said that the car had corner exit oversteer, which is consistent with the relative magnitudes of Balance for these two laps. During the second corner, the Balance for lap B shows slightly lower magnitude during the first half of the corner and more variation during the second half of the corner. The steering wheel angle also shows more activity during the second half of the corner for lap B. Again, this is consistent with the driver’s comments.

A final comparison between the two laps is shown in Figure 69. Shown are the total lateral tire force and the percent front lateral tire force. The percent front force is only computed for data where the lateral acceleration is above 0.5 g’s to restrict the
comparison to vehicle cornering. The total force shows a drop off in tire force at approximately 900 feet, then an increase again at approximately 1250 feet. In this particular corner, there is a concrete patch in this area, which has less friction than the asphalt surface of the corner entry and exit. This reduction in "grip" is reported by the drivers and shown by the estimated tire forces. The tire forces for lap B also show a slight reduction in compared to lap A at the exit of both corners, where the driver reported exit oversteer. As would be expected, the percent front tire force is approximately equal to the percent front weight of the vehicle. For the first corner, there is very little difference in between the laps. For the second corner, there is more variation in the percent front tire force, which also shows up in the Balance calculation.

Figure 69 - Lap Comparison: Total Tire Lateral Force and Percent Front Tire Force
4.6 References


CHAPTER 5

USAGE OF ESTIMATOR RESULTS

5.1 Introduction

In Chapter’s 3 and 4, Extended Kalman Filter’s (EKF) were designed to estimate the aerodynamic forces acting on a winged racing car, along with tire lateral force and slip angles. In this chapter, a framework is proposed that will allow the estimator results to be effectively utilized by a race team to help understand or confirm the driver comments, and to enhance other simulation tools.

Figure 70 shows a diagram of the data and “information” flow from the car and driver to the engineer and finally resulting in changes to be made to the car. Raw sensor data is downloaded from the vehicle to a computer(s) for analysis. As part of the data processing, the EKF’s are used to estimate aerodynamic and tire forces along with body and tire sideslip angles along with the Balance computation. This provides quantitative information about the driver/vehicle performance. At the same time, the engineer and driver discuss the driver’s qualitative assessment of the vehicle performance. Using both of these descriptions of “what happened”, the engineer, with the aid of additional analysis tools formulates a proposed change or changes to be made to the car. This is discussed with the driver and the process repeated.
Figure 70 - Data And Information Flow
5.2 Aerodynamic Tuning

Chapter 3 showed that the EKF could accurately estimate aerodynamic downforce and effectively reject some of the disturbances like those caused by roadway bumps. A large portion of the tuning done to winged racecars is changing wing settings to alter the total downforce and/or the distribution of the downforce between the front and rear axles (center of pressure location). The EKF estimates of the aerodynamic forces is superior to the standard estimates made using the suspension force measurements. In constant speed operation, the EKF is able to reject some of the roadway disturbance. In transient conditions (acceleration or braking), the EKF can account for the vehicle inertial dynamics unlike using the suspension force measurements alone.

Aerodynamic forces are used by the race team in two situations. The first is during tests aimed at correlating the vehicle’s actual aerodynamic performance to the measurements made in the scale wind tunnel. It is important to confirm, or understand the mapping of the wind tunnel data to the actual vehicle running on the track. Testing is done at constant speed on flat, smooth surfaces. Vehicle ride heights are changed and through many runs, data can be gathered to construct a map of aerodynamic down force as a function of front and rear ride height. Teams try to find a surface that is as “smooth as possible”, however, all paved or poured surfaces have some variation in their height. Running at constant speed, data is heavily filtered to “average out the bumps” in the road. The advantage that the EKF has in this type of testing is that by rejecting a lot of the roadway disturbance, less time needs to be spent at each operating condition, allowing testing to be completed more quickly.
The second, more common situation when aerodynamics forces are desired is during a test or race weekend at a racetrack. Total downforce and center of pressure are key variables that the race engineers use to help understand what the vehicle is doing. Improvements in the accuracy of these measurements means fewer uncertainties for the engineer. Improved accuracy also allows the effects of aerodynamic adjustments to the car to be quantified. The aerodynamic forces are strongly influenced by the vehicle ride height. A change in springs, or even damper settings will affect the vehicle ride height, which will change the aerodynamic forces. Downforce estimated solely from the suspension force measurements is difficult to use, other than on a fairly smooth straight section of track. The EKF estimated downforce is more accurate everywhere on the track and can be used for a much larger section of the track, allowing the engineer to see how changes to aerodynamic or suspension settings affected the actual downforce.

5.3 Surface Friction Estimation and Simulation Tuning

Vehicle handling simulations can be used to help understand vehicle behavior and find possible solutions to problems without running the vehicle on the track. One difficult task is to get good agreement between the simulation models and the actual vehicle, prior to using the model as a predictive tool. Tire/roadway friction characteristics change quite a bit as more rubber from tires is “deposited” on the track surface (or washed away by rain). Environmental conditions like temperature, direct sunlight/shade, humidity, and rain also affect the tire/roadway characteristics. In addition, these environmental factors also affect the aerodynamic downforce generation.
Adjustments to the simulation model to approximately match the vehicle behavior are required before good agreement can be expected.

Tire forces and moments used by handling simulations use empirical tire models based on “curve fit” raw data from a tire force and moment test machine. This means that the tire data represents the tire running in a laboratory on a belt coated with a safety walk surface (like coarse sandpaper). While this data provides valuable data about the tire behavior, it does not exactly represent a tire running on a racing surface. The estimated tire forces and slip angles can be used to adjust this force and moment data to more accurately reflect the tire as it runs on the track surface.

There are a number of procedures that could be used. The main characteristic of the tire model that will be adjusted is the peak lateral friction as a function of normal load. Figure 71 shows data from a passenger car tire of peak lateral friction versus normal tire load. Peak friction measurements from the test data are shown by the dots and a tire models representation of this data shown by the line. Peak lateral friction is defined as the highest lateral force in pure cornering normalized by tire vertical load.
The peak friction on the track can be estimated during corners. The front tires should be used, since the rear tires are being driven and the longitudinal forces will cause a reduction in the available lateral force. Data for areas of “limit” cornering should be found where there is no braking taking place. Using the estimated lateral force and the filtered suspension force, along with the known suspension unsprung weight, lateral tire friction can be estimated for each corner on the track. Since different corners will be taken at different speeds (at least on a road or street course), a series of lateral friction points can be plotted versus normal load. Using the estimators developed in this project, this would give effective friction at the axle, not for an individual tire. If more complex four-wheel vehicle models were used in the estimators, individual tire friction could be
estimated for the front wheels. This data can then be used to adjust the tire model friction parameters to improve the model’s prediction of the vehicle.

If there are known surface changes on the track (like asphalt to concrete), differences in their friction can be estimated in the same way. If the handling simulation supported variable surface characteristics, this additional data could be used to further improve the simulation predictions.

Just as the simulation tire force and moment model can be adjusted based on the current track conditions, so too can the simulation aerodynamic model. Environmental conditions can cause a loss or gain in downforce compared to the wind tunnel data. Using the estimated downforce and filtered ride heights, over the course of a lap, a scale factor can be computed to relate the measured data to the model data. This scale factor can then be used in the model to improve the downforce predictions.

5.4 Driver Feedback

In Section 4.5, it was shown that the Balance computation correlates with driver comments about vehicle understeer/oversteer. This allows the engineer to “see” in the data some of what the driver is telling him. Looking at raw sensor signals alone, it is very difficult to detect understeer/oversteer except in extreme cases. In addition to confirming what the driver is describing about what he “felt”, it will also be effective in determining what is “real” understeer/oversteer, and what is “perceived” understeer/oversteer. A driver is attempting to process a lot of visual and proprioceptive information, while in a hot, noisy, high vibration environment. There are cases were
what the driver thinks he is feeling is not what is really happening. It is important that the driver and engineer discover this as quickly as possible, as they can easily get led down the wrong path, away from the real solution.

The Balance computation can also be made by off-line vehicle handling simulations. If the vehicle balance is not to the driver's liking, adjustments are made to try to correct this. However, it is difficult for the engineer to make adjustments to change the balance at one part of the track, and not affect it at other parts of the track. Simulation tools could be a big help here, allowing the balance to be predicted based on proposed changes and compared to the estimated Balance from the vehicle. The first step here would be to try to get the simulation model Balance to be a reasonable match of the estimated Balance from the vehicle data. Once this is done, proposed changes to the vehicle can be simulated and compared with the original lap, searching for changes that affect the vehicle in the areas of interest, but not in the remaining portions of the lap. This procedure could be done in an ad hoc, trial and error procedure, or formalized using sensitivity methods, Design of Experiments and response surface calculations, or other optimization techniques.
CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The stated objective of this research project was to compute quantitative measures of racing vehicle behavior and relate them to qualitative driver assessments. Road and oval racing car “development” has relied on driver’s verbally telling their engineers how the car “feels”, or what it is doing. Modern race cars are equipped with numerous sensors and complex data acquisition systems, however this raw sensor data often does not show the engineer what the driver is trying to explain. This research has extended the utility of the available data by applying Extended Kalman Filtering (EKF) to the measured sensor data to estimate the tire and aerodynamic forces acting on the vehicle along with vehicle and tire sideslip angles. It is shown that this new information relates to the driver’s comments about vehicle understeer/oversteer. Developing the procedures for computing this objective measure of the vehicle behavior that correlates with the driver’s subjective assessment is a necessary first step before racing vehicle handling and performance problems can be analyzed and corrected scientifically.

In Chapter 3, a model is developed of the longitudinal and vertical dynamics of a race car and used in an EKF to estimate the aerodynamic forces acting on the vehicle. The unmeasured aerodynamic forces are modeled as a second-order Gauss-Markov
process and included in the EKF. It is shown, through the use of simulated and measured results, that the EKF can accurately predict the aerodynamic forces and is able to reject some of the unmeasured roadway disturbances (bumps) and account for the vehicle inertial dynamics. This is shown to be a significant improvement over estimating the aerodynamic downforce from the suspension force measurements alone.

A yaw/roll plane model of a vehicle was developed in Chapter 4 and used in an EKF to estimate unknown tire lateral forces and chassis and tire sideslip angles. Again, simulated and experimental results were used to show the validity of the estimates. Tire lateral forces and slip angles are fundamental quantities in the study of vehicle handling dynamics, yet they are not available to the race engineer using the sensor data alone. A computed value called Balance, the difference between the front and rear tire slip angles, is defined. It is shown that Balance correlates with driver comments about vehicle understeer and oversteer. This allows the engineer to “see” in the data what the driver is describing. It is proposed that the Balance calculation can provide part of a common language between the driver and engineer, allowing them to discuss in a quantitative way how the vehicle should be improved. Without this quantitative dialog, use of more complex handling simulation tools will be difficult to use, since there is no quantitative target to aim for.

Chapter 5 provides a discussion on how these EKF’s can be used in a racing environment. The downforce estimator can be used during on-track aerodynamic testing and wind tunnel data correlation studies. It also can be used during test and race weekends to check and quantify downforce after changes to the vehicle configuration. It
is shown that the estimates of lateral tire force and slip angle can be used to estimate tire/roadway friction, and has sufficient fidelity to identify changes in surface friction. This friction estimate can be used to help "tune" other simulation models for improved accuracy. Finally, the Balance computation can be used in helping the engineer evaluate the driver's comments about the vehicle's handling.

6.2 Recommendations

This research has demonstrated that by applying vehicle modeling to the processing of test data, additional information can be derived from the data. In addition, the models can help filter out noise in the data originating from both the sensors and external disturbances. However, the models and techniques developed are not a complete solution to the problem of analyzing race car test data and deciding what changes are required to make the car better. This problem requires a vast array of tools and methods of which this research provides a start.

The models in this research were developed to demonstrate the EKF method and are sufficient for this purpose. In actual application, the models should be expanded to include greater detail about the vehicle. The reasons for this are twofold. First, the ability of the EKF to filter the data and provide estimates is related to how well the model represents the physical system. In general, a more accurate the model will provide more accurate estimates. However, there are consequences in adding too much complexity to the models. The EKF formulation requires multiplication and inversion of non-sparse square matrices whose size is equal to the number of states of the estimator. Matrix
multiplication time is related to the matrix size cubed. This matrix multiplication
dominates the run time of the EKF algorithms developed for this research. In the racing
environment, time is important, and adding too much complexity to the model will slow
down the estimator and yield it un-useable. Efforts to improve model fidelity without
overburdening computational run times should be made.

There is significant benefit to having the models used by the estimators be
essentially the same as the models used for off-line analysis. From a purely practical
standpoint, this reduces the modeling effort, as well as the model parameter and
computation tasks. It is likely that a single vehicle model could be used for both
estimators. A basic recommended guideline for the models would be to include all six
chassis degrees of freedom, and a single degree of freedom for each unsprung mass. This
would make a ten degree-of-freedom vehicle model when used in the EKF. When used
with the EKF, the wheel spin model degrees of freedom are not needed; however when
run as an off-line simulation for analysis purposes, the wheel spin modes should be
added, along with a tire force and moment model. It would be possible to run both the
aerodynamic force and the tire force estimators at the same time, making the procedure a
single pass. If the run times were acceptable, this would be the preferred method.

The methods were shown to accurately estimate unknown forces when used
against noise corrupted simulation output data. There was no measurement data available
to use to test the force and sideslip estimates against actual test data. However, this
should be done to determine the accuracy of the methods and to use to tune the filter
gains. In the case of tire lateral forces and vehicle sideslip angle measurement,
instrumentation is available to make these measurements. While impractical or illegal for use with racing cars, the instrumentation could be installed for testing aimed at evaluating the estimator's performance. This type of instrumentation is more commonly used in the passenger car testing. Data from passenger cars could also be used to test the methods.

This dissertation focused on using EKF methods to add insight into data measured on racing cars. These methods are equally applicable to passenger vehicle data. While the aerodynamic downforce estimates may not be very important, the estimates of tire lateral forces, body sideslip angle and tire slip angles are very important. The instrumentation required to measure this data is very expensive. If sufficient parametric information for the vehicle is available, applying the EKF techniques only costs some CPU time.

The next step in moving toward a more scientific treatment of racing car handling development is to develop off-line analysis simulations that predict the same (or at least similar) quantitative handling measures as are derived from the measurement data. This is not a trivial task. However, once this step is complete, the power of the simulations can be realized to quickly "try" many solutions to a handling problem, and weeding out the poor ones. An additional benefit of the off-line simulations will be that they can be used to determine how the Balance value relates to understeer/oversteer. This will allow the Balance equation to be modified to transition from understeer to oversteer at zero. It is likely that an offset will need to be added. In the case of a winged race car, it may also be that there needs to be some speed related terms added. In racing, as well as traditional vehicle development, engineers are always striving to find the "answer" more quickly or
more efficiently. The methods developed in this research help us move a little further along that path.
LIST OF REFERENCES


Radt, H. S., "Processing of Tire Force/Moment Data," SAE Paper 951048


153


Vehicle Dynamics Terminology, SAE J670e, Society of Automotive Engineers Recommended Practice, June 1978.


