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GROUP AND INDIVIDUAL MICROCREDIT CONTRACTS: A DYNAMIC NUMERICAL ANALYSIS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

Jorge Luis Rodriguez-Meza, Lic, M.S., M.A.

* * * * *

The Ohio State University

2000

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Agricultural, Environmental, and Development Economics Graduate Program
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ABSTRACT

This dissertation examines how terms and conditions of credit contracts with characteristics typically found in microfinance settings influence investment and repayment decisions. Dynamic programming models are developed for individual and for group loan contracts. The models generate repayment frontiers for specific lending technologies. The effects of characteristics of these lending technologies, in particular rationing rules, on repayment behavior, focusing on strategic default, are analyzed by performing simulations.

The characteristics of microfinance contracts result from the lender's handling of information asymmetries, incompatible incentives, and enforcement constraints (absence of collateral). Credit-constrained borrowers enter loan contracts to maximize utility from their inter-temporal consumption. As credit constraints are overcome, borrowers experience increasing incentives to default. The need to keep them credit-constrained to avoid default creates a trade-off between sustainability of the microlending organization and optimization of the level of borrower investment.

Dynamic programming models explain this trade-off in contracts where the threat of termination of the relationship is the incentive the lender has to promote repayment. This trade-off explains limitations of microlending technologies in adjusting to growth of the borrowers' wealth and the resulting constraints on investment. The models suggest
some complementarity between lending technologies that define loan size according to the borrowers' wealth levels and those that adopt a pre-defined rationing rule independent of wealth. The latter may increase repayment at low levels of wealth, but eventually wealth-related rationing rules may be better suited for long-term lender-borrower relationships.

The group lending model identifies positive effects of joint liability on repayment: some group members have incentives to repay for members who default, due to differentials in wealth levels in contrast to the distribution rule of loan proceeds among group members. The model also identifies negative effects of joint liability: some borrowers who default in a group contract would have repaid individual loans. This outcome reflects wealth differences among group members. The model finally explains how if member loan sizes are not a function of the individual members' wealth, repayment performance may be better with individual rather than with group loans.
DEDICATED TO MY PARENTS
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CHAPTER 1
Introduction

In the past thirty years, much progress has been made in appreciating the role of finance in economic development. Theoretical contributions on the optimal design of financial policies, organizations, technologies and products have aided in better understanding the role of finance in the broader context of economic development. Development finance, understood not only as the study of financial transactions in incomplete market settings but also as the provision of financial services to marginal clienteles, such as small farmers and microentrepreneurs, is now considered to be an essential component of any economic development strategy.

The pioneering contributions by McKinnon (1973) and Shaw (1973) emphasized the importance of the financial sector in a developing economy. Sound financial policies, jointly with appropriate macroeconomic policies, contribute to reducing the dispersion of marginal rates of return across resource uses in a fragmented economy.

Further contributions by Stiglitz (1990) and others (Stiglitz and Weiss, 1981 and 1983) incorporated imperfect information analysis and agency theory in the study of financial contracts in developing economies. The importance of development finance
is highlighted by the fact that the most marginally productive agents in developing economies are not acceptable clients for formal financial organizations.

The Rural Finance Program at The Ohio State University has demonstrated, with results from numerous empirical studies from around the world, that protectionist financial policies, such as subsidized or targeted loans, have actually been counterproductive (Adams, Von Pischke and Graham, 1984). Researchers from the program have also stressed the importance of financial technology, organizational design, and incentives when dealing with marginal clienteles (Gonzalez-Vega, 1993).

These clienteles are especially important in developing countries, as only a small portion of the population has had access to formal financial services. Instead, in developing economies most economic agents transact financial services in informal markets. This reliance on informal markets allows for very limited arbitrage across space, time, and states of nature and for insufficient economies of scale and risk diversification and, in turn, this reinforces the fragmented nature of these economies.

The provision of appropriate and correctly-priced financial services to marginal clienteles has been the focus of concern for many scholars and practitioners in the last decade. Lately, issues such as innovative lending technology, incentive compatible organizational design, and long-term sustainability and commitment have been highlighted as important elements of a successful microfinance activity (Gonzalez-Vega, 1998).

In order to reach the target clienteles with loans at attractive terms and conditions, an appropriate technology for delivering financial services must be
developed. This technology must be complemented with the right set of incentives both for the borrowers and the lender. These incentives are needed to induce the lender’s provision of financial products that respond to legitimate demands as well as high borrower repayment rates. In turn, innovation and financial discipline guarantee the long-term sustainability of the lending organization, not only because its financial results allow it but also because the expectations of its clients induce it.

The growing interest in technological innovation in development finance has led to the design of methods for delivering credit services that make use of principles learned from informal finance. Frequent payments, the timely delivery of services, the promise of a long-term relationship, and a strict repayment discipline are some of these principles learned from the informal financial sector. Many microfinance organizations have incorporated these principles in their lending technology, with varying results.

Caja Los Andes in Bolivia and Financiera Calpiá in El Salvador are two examples of successful microfinance organizations that have focused their efforts on building a strong lending technology based on individual loans. In their effort to adopt the right set of incentives, these organizations have signaled their commitment to a long-term relationship with the borrower.

From the principles of rotating savings and credit associations (ROSCAs), which are spontaneously present in many developing countries, and from the experience of credit cooperatives, the group lending technology has also been adopted to deliver loans to clienteles not previously reached by formal finance. Although the empirical evidence
shows all sorts of results, ranging from very successful to complete failure, the contractual principles implicit in such an arrangement suggest the potential effectiveness of this lending technology for specific clienteles.

The success of well-known cases, such as the Grameen Bank in Bangladesh and BancoSol in Bolivia, reinforces the idea that, under appropriate conditions and for certain types of clienteles, the group lending technology may be efficient in the provision of formal loans. This has been the perception of numerous practitioners, for whom the group lending technology has been suggested as a potentially attractive solution to the problems of providing credit services to large numbers of otherwise excluded agents. The growing shift among some of these organizations to an individual lending technology suggests that group lending may be constrained by potentially important limitations.

This dissertation responds to the recent interest in lending technologies and to the experience of the author in observing how several of these organizations are implementing them. The dissertation is an attempt to theoretically understand characteristics that make specific group and individual technologies work under particular circumstances, assess their shortcomings, and evaluate the validity of some of the arguments advanced in their favor. The dissertation is not, however, a full evaluation of the two approaches, given the considerable abstraction required by the exercise. It sheds light, nevertheless, on their respective strengths and weaknesses.
**Research Question**

The dissertation deals with the problem of how the joint investment and loan repayment decisions of economic agents are affected when they are credit-constrained and when the lender uses a lending technology composed of a set of simple rules that mean that the credit constraint is not immediately removed. Borrowers face loan-size credit rationing, understood as their ability and willingness to borrow larger amounts than they are obtaining at the going interest rate. This outcome results from constraints imposed by the lender on loan size. The restrictive lender's disbursement rule is, in turn, the result of imperfect information about the capacity and willingness to repay of the potential borrowers.

The problem studied here is the nature of the trade-off between the agents' optimal level of investment and the sustainability of microfinance organizations. Lenders willing to grant loans large enough for borrowers to achieve their optimal level of investment may face sustainability problems, as borrowers may find it optimal to default under these circumstances. Borrowers can strategically default *i.e.*, choose not to repay, even when having the resources to do it, because of the absence of perfect collateral. The trade-off between optimal investment and sustainability has, therefore, serious implications for the role of microfinance in economic development.

The objective of the dissertation is to evaluate different contract designs characterized by different *rationing rules* used by the lender both for group and individual lending technologies. Each rationing rule generates a different repayment set.
and repayment frontier, which define the feasible combinations of wealth and initial debt that would sustain the contract between the lender and the borrowers, thus avoiding default.

Lending rules may be related to wealth or not. This relationship matters a lot for the outcome. Wealth-related rationing rules introduce a role for wealth, beyond making self-financing possible, that leads to over-investment when borrowers choose to repay and stay in the contract. Over-investment disappears, however, when borrowers choose to default. Non-wealth-related rules generate different repayment frontiers. These rules may prevent early strategic default that would otherwise occur when levels of wealth become sufficiently high.

In group lending technologies, in addition, differences in levels of wealth among the borrowing group members, not consistent with the *distribution rule* for the loan proceeds among the members of the group, may explain the potentially positive and negative effects of joint-liability contracts.

This dissertation starts from the traditional dynamic growth model analysis, which considers non-credit-constrained agents. The dissertation then departs from this model with the introduction of credit, which is granted under disbursement rules that imply loan-size credit rationing. At the same time, the dissertation departs from traditional static models of credit rationing, which emphasize imperfect information about *ability* to repay. The dissertation focuses, instead, on *willingness* to repay and analyses this dimension of repayment not in a static but in an inter-temporal dynamic
framework. The dissertation also contributes to the analysis of repayment models found in the finance literature by focusing on the value of the relationship between borrowers and lenders instead of focusing on the role of collateral.

The innovative approach of this dissertation responds to the need to develop a more suitable framework to study the characteristics of the microfinance contracts that have sprouted all over the developing world. The set of incentives implicit in these contracts relies on the value of a long-term relationship between funds-constrained borrowers and imperfectly-informed lenders. As the relationship matures, however, the relaxation of the funds constraint on the borrower's side may threaten the long-term stability of these programs, as the incentives to repay would tend to become weaker.

Additional factors, such as increased competition in credit markets, may further deteriorate the repayment incentives present in these contracts. Consequently, the understanding of the possible trade-offs between sustainability of the lending organization and the optimal level of investment, born from the actual design of the contracts, is quite relevant for both the development literature and microfinance practitioners.

In order to address these issues, a dynamic utility maximization problem is solved, in which the borrower makes both investment-consumption and repayment-default decisions. The model stresses dynamic incentives because the main mechanism available to the lender to enforce repayment in these market niches is the threat of termination of the credit relationship. Numerical methods allow the use of specific
functional forms and the introduction of random shocks in the production function to explore these questions.

Chapter 2 discusses the main theoretical framework about individual lending contracts, while Chapter 3 discusses the theoretical framework for group lending contracts. Chapter 4 briefly describes numerical techniques to solve dynamic optimization problems and provides some of the theoretical foundations for the estimation presented in Chapter 5. Chapter 5 develops a model to study the problem of strategic default in an individual lending contract and to highlight the effects of characteristics typically observed in microfinance contracts. Chapter 6 extends the model to a group lending setting and considers issues concerning the distribution of the loan proceeds and its repayment among members of the group. Finally, Chapter 7 draws conclusions and recommendations.
CHAPTER 2

Problems in Credit Markets: Individual Contracts

Credit markets around the world are plagued by information problems. Lenders possess only imperfect information about borrowers. Thus, it is difficult for them to observe the characteristics of potential borrowers, their uses of loan funds, and the returns on the productive activities they undertake. Borrowers face imperfect information, as well, concerning the existence of potential lenders and their particular credit rationing behavior.

Moreover, contract enforcement problems are also present, as lenders incur in costly enforcement in case of arrears, due to a lack of perfect legal mechanisms to induce repayment. Information and contract enforcement constraints add transaction costs to the opportunity cost of funds and make credit transactions expensive for all parties involved. In sum, credit markets are not smooth.

These information and enforcement problems are exacerbated in rural and urban microfinance market niches in developing countries (Gonzalez-Vega, 1998). Correlated risks limit possibilities for portfolio diversification in these markets, thereby increasing already major threats to the lender from uncertainty about expected repayment. The legal
infrastructure is non-existent or is inadequate for the types of transactions typical of these market niches, and geography and a limited infrastructure raise already high transaction costs for lenders and borrowers.

In order to induce the right set of incentives for borrowers to fulfill their contractual obligations, the design of financial contracts must take all of these factors into account. Thus, for example, a debt contract with full financing of the borrower’s project would not be optimal for lenders hampered by imperfect information, as it would lead to more frequent default than if the borrowers were asked to assume some of the risk (Stiglitz and Weiss, 1981).

The use of collateral, a common solution to address the problem of information assymetries (Barro, 1976; Benjamin, 1978), is constrained by the limited availability of pledgeable assets, the absence of secondary markets to liquidate these assets, an insufficient legal framework to support their validity as a guarantee, and adverse selection effects that limit their effectiveness as collateral (Gonzalez-Vega, 1998; Holmstrom, 1990; Stiglitz and Weiss, 1981).

The types of contracts offered by lenders in microfinance market niches respond to these special challenges. Numerous variations of these contracts have emerged around the world, which use ingenious mechanisms to address tough information and contract enforcement problems (Gonzalez-Vega et al., 1995; Huppi and Feder, 1990).

Most of these contracts share one or several of the following characteristics:
(a) Standardized contracts with minimum requirements for first-time borrowers followed by improvement of contract terms and conditions as, with each new loan, borrowers reveal their capacity and willingness to repay.

(b) Innovative forms of collateral, which rely on the subjective value-in-use of the pledged assets for the borrowers rather than on their market value.

(c) Development of a long-term relationship between the lender and the borrower, in which the promise of future access to credit plays a key role among repayment incentives.

(d) Use of direct or delegated monitoring, the value of reputation, and peer pressure to induce repayment.

(e) Enforcement of contracts through the strict monitoring of repayment behavior.

(f) Seizing of assets or termination of the relationship in case of serious repayment problems are meant more as a means of inducing a demonstration effect to discipline other borrowers than as a means to recover the funds lent.

These innovations in financial contracts have been the result of years of learning and of many failed attempts to establish successful microfinance programs. The features of these contracts are well-suited to the types of financial products traditionally offered by microfinance organizations, namely small, short-term loans for working capital or consumption smoothing. Longer-term loans to finance fixed capital investment are rare in microfinance. Innovation in contracts for this other type of loans is still an area of experimentation.
Among several contract characteristics listed above, three have a meaning only in a dynamic framework. The first one, a gradual improvement of contract terms and conditions over time, the third one, a promise of future access to services, and the fifth one, a demonstration of the lender's seriousness in enforcing the contract, all rely on the borrower's valuation of a long-term relationship with this lender. If the borrower received a one-time loan with no expectation of future transactions, such as in the case of one fixed-investment loan, this set of incentives may not work to guarantee repayment; i.e., these incentives may not constrain borrower behavior sufficiently.

The theoretical foundations of the special features of the new types of financial contracts spring from the theory of contracts and the economics of information. These two areas of study draw from principal-agent theory to model economic relationships. Several authors have used this framework to study financial relationships. The following sections discuss the main results of principal-agent theory and their extensions to finance, in general, and to microfinance, in particular.

**Principal-Agent Theory**

Principal-agent theory is a framework to study relationships between economic agents with different objective functions in which one party, the principal, delegates to another, the agent, some actions (control over resources). Principal-agent theory studies different ways in which the principal can induce the agent to take actions that are beneficial to the principal but may not be optimal for an unconstrained agent.
The agent's actions are induced by the principal by varying the incentives provided in the contract in order to make these actions attractive to the agent. In other words, the contract imposes constraints on the agent that make the agent's actions optimal from the principal's point of view and constrained-optimal from the agent's point of view.

Agency theory ignores bargaining issues between the two parties. It assumes that all bargaining power is in the hands of the principal. Thus, the principal offers a take-it-or-leave-it contract. The design of the contract is, therefore, under the control of the principal; i.e., the game structure is Stackelberg with a leader (principal) and a follower (agent).

Credit relationships exhibit all the characteristics of a typical agency problem. One agent (the borrower) acts on behalf of the principal (the lender), whose funds must be repaid. In agency relationships, information asymmetries usually arise because one of the parties possesses private information. In a lender-borrower relationship, the principal is usually unable to observe the actions or the type of the agent. Consequently, the principal must invest resources to either recognize agent types or to induce agents to undertake actions that are not harmful to the interests of the lender.

*Moral hazard* emerges when the actions undertaken by one party in a transaction, the agent, affect the valuation of the transaction by the second party, the principal, who is unable to perfectly observe the actions of the first party. For example,
the principal may want the agent to undertake high-effort actions, but the agent may undertake low-effort actions.

The returns to a productive activity are positively correlated with levels of effort. The agent, however, taking advantage of the principal’s inability to observe effort, may be able to falsely claim, once the outcome has occurred, that the observed returns are the result of exerting high effort. This situation will be called here *ex post* contractual risk. In the presence of this risk, the uninformed principal must design a contract that makes it in the best interest of the informed party, the agent, to undertake actions favorable to the principal.

An agent willing to take an enforceable contract must be sufficiently compensated for undertaking the actions specified in the contract. The utility to be generated by the best alternative use of the agent’s resources is the agent’s reservation utility. Any contract offered by the principal must provide the agent with the possibility of earning, at least, this reservation utility. This represents the *participation constraint* of the agent.

A rational agent undertakes those actions that maximize his own net returns. A contract must offer the agent enough motivation to align his interests with those of the principal. In other words, it must be optimal for the agent to choose actions that maximize the principal’s returns. This represents the *incentive compatibility constraint*. The design of an optimal contract must provide the party offering the contract, the
principal, the maximum possible returns subject to the participation and incentive compatibility constraints of the agent.

Under perfect information and risk-neutral principal and agent, the incentive compatibility constraint is not binding because the principal can perfectly observe the level of effort of the agent. Thus, the principal can offer a contract in terms of the actions undertaken by the agent that forces the agent to internalize the effects of his own actions (Kreps, 1990). The principal must only make sure that the agent is willing to sign the contract by offering him at least his reservation utility. There is no danger of the agent undertaking actions detrimental to the principal because under the circumstances it is not optimal for the agent to do so. The first-best action-incentives pair is achieved, given that the agent’s actions are simultaneously optimal for both principal and agent (Dutta and Radner, 1994).

Under perfect information, the types of contracts for which the optimal actions from the point of view of the principal are also optimal from the point of view of the agent may take several forms. Most of the contractual forms discussed in the literature are designed for an employer-employee or land owner-tenant relationship (Varian, 1992).

One particular contractual form, also suitable for a lender-borrower relationship, makes the agent a residual claimant of the returns of his productive activity, after payment of a fixed fee to the principal, in exchange for the use of a resource that belongs
to the principal, such as machinery or land. The fixed payment is determined by the
principal such that the agent receives just his reservation utility.

In a credit relationship, the fixed payment from the agent to the principal can be
taken as principal plus interest, provided that there is capacity to repay. After repayment,
the borrower is the residual claimant to the returns obtained from using the funds
provided by the lender.

Attitudes toward risk matter in the design of contracts. When the agent
undertakes a productive project with uncertain returns, there exist potential welfare gains
for both parties from trading risk if principal and agent have different degrees of risk
aversion.

If the agent is risk averse and the principal is risk neutral, the optimal contract
offers the agent some degree of insurance.\(^1\) By receiving his reservation utility with
certainty and, in exchange, surrendering the additional expected returns of the project,
the agent’s risk is eliminated and transferred to the principal. This is a Pareto
improvement, as uncertainty is a source of disutility for the agent while the principal is
indifferent to risk.

If the principal were to offer full insurance and bear all the risk, however, the
agent would choose his most preferred action in detriment of the principal’s returns, as
his uncertainty would have been eliminated. Typically, given an already guaranteed level

\(^1\) It is customary to assume a risk-neutral principal. The principal has the ability to pool risks from many
agents, which in the limit justifies a perfect diversification of his risk. In microfinance, however, it is
common to find highly-correlated risks among the clients of a specific lender. Nevertheless, some degree
of idiosyncratic risk pooling is possible, which justifies the assumption of lenders being less risk-averse
than borrowers.
of utility, the agent’s most preferred action would be low effort, since effort is a source of disutility for the agent. The incentive compatibility constraint must be binding, therefore, in order to align the agent’s incentives to the principal’s objectives.

Then, the principal must only offer partial insurance, in order to preserve the right set of incentives (Dutta and Radner, 1994). This is a second-best solution, in which there is a trade-off between insurance and incentives. Imperfect observation of the actions of agents, a source of moral hazard, coupled with risk-averse agents, hence lead to second-best solutions.

Another type of imperfect information problem that leads to second-best solutions is imperfect information about the type of agents. This imperfection raises the problem of adverse selection. Adverse selection occurs when the principal, unable to distinguish the risk type of the agents, ends up with a pool of clients composed mainly of high-risk agents.

High-risk agents are more willing than low-risk agents to take a bet on the basis of high returns in good states, because their responsibility is bounded from below in bad states. Consequently, if the principal offers a single break-even contract to all types of agents, low-risk agents will be prevented from participating, as they are less willing to take a bet over their potential returns. This situation will be called here ex ante contractual risk.

A contract that accounts for adverse selection must induce agents to reveal their type when acting in their own self-interest. The optimal contract is typically a separating
contract, in which the principal offers different contracts to different types of agents and each type finds it optimal to choose the contract that has been pre-designed for his risk type.²

Microfinance Contracts, Adverse Selection and Moral Hazard

In a world of perfect information and observability, a standard debt contract is an efficient arrangement from the perspective of both the borrower and the lender. The borrower receives a lump sum to finance the totality of the project. The loan plus interest is repaid after a given period of time. In this perfect world, the interest rate reflects the lender's opportunity cost of funds, and the resulting level of the borrower's investment is optimal.

With asymmetric information, however, financial contracts face the problems of adverse selection and moral hazard, typical of any principal-agent relationship. In financial contracts, adverse selection and moral hazard translate into limitations in the use of the interest rate to increase the lender's profits, to screen potential borrowers, or to compensate for extra risk.

An increase in the interest rate increases the average riskiness of the pool of applicant and also induces borrowers to switch from safe to risky projects. The consequence is credit rationing by a profit-maximizing lender (Stiglitz and Weiss, 1981).

² The concept of equilibrium under adverse selection is theoretically complicated. Depending on the structure of the market and on which agent moves first, there may also exist a pooling equilibrium or multiple equilibria (Kreps, 1990). This section follows the Rothschild and Stiglitz model, in which the uninformed party offers a set of contracts. If there is any equilibrium at all, it will be a separating equilibrium (Rothschild and Stiglitz, 1976).
If the interest rate and collateral requirements are determined simultaneously, however, there is a separating equilibrium with no rationing (Bester, 1985). Riskier borrowers pay higher interest rates and offer lower collateral requirements.

In the presence of moral hazard, the contract must make it in the agent’s interest to exert the level of effort that protects the interests of the principal, by altering the gains and benefits of each level of effort. The contract must transfer some of the losses from low effort from the principal to the agent and generate less than full insurance.

In the case of microfinance contracts, characterized by the absence of perfect collateral, moral hazard problems have a double dimension. First, this double dimension arises because the fulfillment of financial contracts is bounded by limited liability. The stochastic nature of the agent’s productive activities leads to full repayment of the loan only under high-effort returns. Low-effort returns suffice for partial but not for full repayment of the loan. In turn, the borrower has an incentive to divert resources away from the project or to exert low effort to the extent to which these actions may increase his utility.

The imperfectly-informed lender incurs losses due to this opportunistic behavior on the part of the borrower, which is made possible by the borrower’s private information. To avoid this type of moral hazard, the financial contract must provide the agent with incentives to undertake the actions conducive to the generation of sufficient capacity to repay. Given that this opportunity for moral hazard occurs before the realization of the returns of the productive activity, this type of moral hazard will be
called here *ex ante moral hazard*, even though moral hazard itself is *ex post* contractual risk while adverse selection is *ex ante* contractual risk.

Second, even if the borrower finds it in his own interest to exert high effort and not to divert resources away from the productive activity, once the returns are realized, it may be in his interest not to repay the loan and to appropriate for himself all of these returns. If the lender is unable to enforce the contract or to verify returns, the borrower may strategically default. To avoid this type of moral hazard, the financial contract must provide sufficient incentives to ensure *willingness* to repay. Given that this other moral hazard problem may occur after the realization of the returns of the productive activity, this type will be called here *ex post moral hazard*.

Under imperfect information and risk-averse agents, the one-period contract is characterized by a trade-off between the provision of insurance (namely, elimination of the borrower’s uncertainty) and the provision of the required set of incentives for the agent to behave in the principal’s interest. The stronger the incentives the principal wants to provide, the weaker the provision of insurance is.

Wealth-constrained, risk-averse borrowers with production projects with uncertain returns can be made better-off with a loan contract. To the extent to which additional funds allow borrowers to reduce their exposure to the uncertain returns of their production projects, by locking in a larger expected utility or a given certain level of utility through the diversion of funds, credit works as an insurance mechanism. Over
time, the availability of funds through financial markets further provides borrowers with an insurance mechanism to achieve consumption smoothing.

Thus, funds provided by the loan allow borrowers to reduce their uncertainty. By virtue of the limited-liability nature of the credit contract, the lender assumes part or all of the uncertainty inherent in the production project. The degree of uncertainty transferred to the lender depends on the inclusion of collateral guarantees in the contract, the borrower's equity participation in the financing of the project, and the lender's monitoring and enforcement ability.

Collateral and equity participation by the borrower provide incentives for the agent to exert high effort while collateral provides incentives to repay the loan once the returns are realized. Direct monitoring, in contrast, entails the use of costly lender resources to oversee that high effort is exerted and that repayment takes place.

Collateral limits the exposure of the lender in a contract with limited liability and constrains the opportunistic behavior of the borrower. Failure to repay may lead the lender to seize collateral, thereby inducing borrowers to undertake actions conducive to repayment in order to avoid foreclosure. Microfinance contracts have been ingenious in the inclusion of non-traditional assets as collateral and, thereby, reducing the need for costly direct monitoring (Conning, 1999; Gonzalez-Vega et al., 1996).

Loan financing of the entire project would not provide borrowers with incentives to exert high effort to the extent to which, first, effort is a source of disutility for the borrower and, second, borrowers obtain greater expected utility without bearing the risk
of the project because they do not contribute their own resources. Consequently, lenders only want to partially finance projects, leaving some degree of equity participation to the borrower. The borrower’s uncertainty about his net returns is thereby partially alleviated, but the borrower also has the incentive to exert his best effort.

Limits on loan size to induce greater equity participation for new borrowers is another common feature of microfinance contracts. Given very imperfect information, initial loan sizes are well below the borrower’s true repayment capacity. As the lender-borrower relationship matures and the moral hazard problem is alleviated, however, loan size gradually grows (Gonzalez-Vega et al., 1997).

The trade-off between insurance and incentives in the design of contracts under moral hazard and the use of equity participation to reduce the impact of moral hazard are explained by Gale and Helwig (1985), among others. These authors show that with risk-neutral lender and borrowers and if the borrower’s returns are private information, the optimal contract is a “standard debt contract with bankruptcy”, in which the borrower makes fixed payments in non-default states.

In the case of default, the borrower repays the maximum amount possible, equivalent to his total returns, with close inspection by the lender to ensure maximum repayment. In order to align incentives, the borrower must make an equity contribution to the project equivalent to his total liquid assets. In other words, the lender only finances the investment remaining after the borrower pledges all his assets as equity.

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In Gale and Hellwig, bankruptcy is costly for the lender, and its cost is positively correlated with the size of the investment. Under these circumstances, the equilibrium level of investment falls short of the first-best levels, because reducing the level of investment reduces the cost of bankruptcy. Consequently, asymmetric information results in under-investment, due to credit rationing and the borrower’s limited liquidity.

If borrowers are risk-averse, the optimal contract under asymmetric information is still a standard debt contract with bankruptcy. Borrowers still have to contribute as much equity as possible but the difference arises in the default states, when the lender recovers less than the borrower’s total returns, because it is optimal to leave the borrower with some level of income. As in the general result of principal-agent theory, the risk-neutral principal provides partial insurance to the risk-averse agent, in the form of a fixed amount of income received with certainty.

The role of monitoring in solving the moral hazard problem of opportunistic behavior is studied by Datta (1993). This author considers an adverse selection model in which the principal’s lack of knowledge about the type of the borrowers leads to strategic repayment behavior by the agents, itself a moral hazard problem.

Borrowers have an incentive to falsify their realized income, as the principal lacks information about the returns of the project, in addition to his ignorance about borrower type. Under these circumstances, the principal engages in monitoring. Monitoring is costly and imperfect; it does not guarantee the full revelation of the borrower’s output.
Borrowers assess the principal's ability to monitor, and lenders, after disbursing a loan, must decide whether to monitor or not in a second period.

This scenario leads to an income reporting-monitoring game between the principal and the agents. Datta finds that no separating equilibrium exists that would imply fully reporting the borrower's type. The optimal contract depends on the agent's assessment of the principal's ability to monitor. If the lender is relatively efficient, the contract entails lower expected repayment amounts for the borrower and monitoring followed by penalties when lying is detected. If monitoring is too costly, the optimal contract entails setting the expected repayment amount as large as the maximum income of the best-type borrowers. In the latter case, since there is no monitoring and, therefore, no punishment, borrowers will lie.

A similar model, which also considers imperfect monitoring, is developed by Devereux and Fishe (1993). Borrowers and lenders engage in a non-cooperative game of decisions over cheating and monitoring. Borrowers cheat, by falsely announcing that their project has failed in order to keep the proceeds. Lenders can minimize the borrower's cheating by investing in monitoring. Monitoring is costly and, in the case when the agent does not cheat, it is useless.

The Nash equilibrium solution of the game has three pure strategy solutions: no cheating, no monitoring; cheating, no monitoring; and cheating and monitoring. There is also a mixed strategy solution to the game, in which borrowers cheat part of the time and the lender monitors some of the projects.
Conning (1999) considers the role of monitoring as a substitute for physical collateral to alleviate \textit{ex ante} moral hazard. Borrowers may choose to exert low effort and divert loan funds towards other activities and thereby obtain a private benefit. The benefit is a decreasing function of the amount of resources invested in monitoring. Monitoring is, therefore, complementary to the productive activity; it reduces the marginal benefit of low-effort actions.

This author considers two ways of dealing with moral hazard, depending on the type of lender: (a) through an indirect contract with uninformed lenders that rely on collateral, or (b) by direct monitoring from monitoring-intermediary-lenders who have informational advantages in monitoring and enforcement. The optimal contract design implies dividing the proceeds of the productive activity between the borrower, the uninformed lender, and the monitoring lender in such a way that the borrower undertakes the high-effort action with a minimum amount of resources invested in monitoring.

The timing of the model is as follows. First, there is agreement on the terms of the contract and the loan is disbursed. If monitoring is agreed upon, the amount of resources to invest in monitoring is selected. Second, the borrower chooses his action. Third, the state of nature takes place. Fourth, proceeds from the production project are distributed according to the terms of the contract.

To induce borrowers to undertake high-effort actions, the returns from low-effort action must be lowered, to offset the benefits obtained from diverting resources away from the project. The type of contract each borrower receives depends on his level of
assets. If the borrower can offer sufficient assets as collateral, he receives loans from the uninformed lender. If the borrower does not possess enough assets to pledge for the total amount of the loan, he will receive a combination of funds from the uninformed lender and the monitoring lender. At an even lower level of assets, a borrower may get loans exclusively provided by the monitoring lender. Finally, below a minimum level of assets, the borrower will be completely excluded from the market, because the marginal productivity of monitoring will be zero.

The upshot of the model is that monitoring substitutes for collateral to the extent to which the borrower does not possess the necessary assets to offer as collateral. However, since there are diminishing marginal returns to monitoring, there is a limit to the extent to which loans can be granted on the basis of monitoring. Monitoring lowers the benefits, for the borrower, from the diversion of funds, but it is a costly activity, which reduces the project’s surplus and, consequently, its returns.

Dynamic Incentives and Financial Contracts

Lenders may use incentives, such as rewards and punishments, to restrain borrowers from diverting funds or from undertaking actions that may increase the risk of lack of repayment. In particular, the threat of termination of the lender-borrower relationship is a strong incentive that addresses not only the problem of low-effort borrower actions but also the problem of strategic default.
A borrower faced with the decision of repaying or of defaulting and thereby losing access to further credit flows will compare the costs and benefits of maintaining a good standing; i.e., he will compare the net present value of future loans versus the costs and benefits from keeping the current loan funds. Default would lead to self-financing or to the initiation of a credit relationship with an alternative lender, both of which are costly options.

The time horizon of the client and his inter-temporal preferences influence his decision. Patient borrowers, with low discount rates, attach more value to the threat of termination of the relationship (Ray, 1999).

From the lender's point of view, although costly termination of the relationship with the borrower may also be an optimal choice, under certain circumstances. This is explained by Stiglitz and Weiss (1983) in a model in which the lender cannot observe the borrower's choice of production project. This inability raises moral hazard problems. Borrowers are risk-neutral, live for two periods, and the problem of strategic default is ignored by assuming that borrowers repay if they have enough income from the project. Only ex ante moral hazard is considered in the model.

In this Stiglitz and Weiss model, writing two-period contracts allows a lender to raise the interest rate in the first period above the rate that would be charged if the interest rate were the only instrument available to design the contract. Linking the two periods provides the lender with the leverage of a second instrument, the threat of
termination of the relationship. The likelihood of termination of the relationship offsets
the borrower’s tendency to undertake riskier projects when the interest rate is raised.\(^3\)

Termination of the relationship in the case of first-period default is a Nash
equilibrium choice for the lender if the increase in interest rates in the first period brings
about profits that are greater than the discounted present value of the profits that may
accrue from offering loans to first-period defaulters. If this condition does not hold, it
would be optimal to offer loans to defaulters but at a higher interest rate than to non-
defaulters.

With risk-averse borrowers, however, it may be optimal to just randomly deny
credit to first-period defaulters. In this case, some borrowers would be fully insured, i.e.,
those not denied credit, and the lender would bear all their risk. Profit maximization
precludes the lender, however, from fully insuring all borrowers.

The upshot of the argument is that, under risk neutrality, equal increases in the
first-period interest rate will bring about equal reductions in the probability of offering
loans to defaulters in the second period, regardless of the initial interest rate level. With
risk-averse borrowers, however, a given increase in the first-period interest rate will
induce, each time, a greater impact on the undertaking of risky projects and will cause,
therefore, a greater reduction in the probability of offering loans to defaulters in the
second period, for a given level of expected utility.

\(^3\) Rising interest rates have two effects: (a) a tendency by each borrower to undertake riskier projects, the
moral hazard effect, and (b) an increase in the proportion of riskier borrowers in the pool of clients of a
lender, the adverse selection effect (Stiglitz and Weiss, 1981).
As the Stiglitz and Weiss model shows, in a dynamic setting, \textit{repetition} of the contract may ameliorate the trade-off between insurance and incentives that is present in contracts under assymetric information and with risk-averse agents. The dynamic dimension of the model brings in additional incentives and instruments, such as inter-temporal insurance and the threat of non-renewal of the contract, which are not present in a static contract. In a dynamic setting, the principal decides how much inter-temporal incentives and insurance to provide by equating marginal utilities across periods (Dutta and Radner, 1994). Contracts are, therefore, history-dependent. At any time $t$, the principal counts with information from previous periods to make inferences about the agent's expected efforts.

Dutta and Radner, for example, study a type of dynamic contract that in the literature is called a \textit{bankruptcy scheme}. In this contract, the agent commits to a given level of excess returns over pre-established \textit{normal} returns. Failure to sustain these excess returns at positive levels results in the dismissal of the agent from the contract. These authors show that, for a discount factor close to one, this type of contract is arbitrarily close to the first-best contract.

The main claim regarding dynamic contracts of the type described above is that, by offering the agent some future rewards, the principal is able to provide insurance while keeping the right set of incentives for the agent to promote the principal's interests. In inter-temporal lending contracts, future rewards take the form of a promise of future access to credit. In practice, the promise of future access to credit is implicitly
or explicitly present in the contracts of almost all successful microfinance organizations around the world (Gonzalez-Vega et al., 1996).

The problem of modeling dynamic contracts, however, is that history dependency introduces complications and subtleties not present in models of static contracts. Laffont and Tirole (1988), for example, consider a two-period model in which the principal sets an incentive scheme to influence the agent’s choice of effort in the first period. In the second period, the principal rewards or punishes the agent and updates his beliefs about the agent’s choices. The principal then offers a new contract, based on the updated information, and the agent chooses a new level of effort.

The model focuses on the first-period’s motivation the agent has not to engage in high effort because of the possibility of facing a more demanding scheme in the second period if he does. In the presence of this effect \textit{ratchet effect}, there is no separating equilibrium in the first period if the principal uses just the information acquired in the first period to modify the terms of the contract for the second period.

Many of the results obtained for static models do not hold in a multi-period model. Webb (1991) shows, for example, that with adverse selection a sequence of standard debt contracts with bankruptcy \textit{a la} Gale and Hellwig will not generate a separating equilibrium. The low-effort types will misrepresent their type and the lender will incur losses. A separating equilibrium is obtained by offering both a sequence of “modified standard debt contracts” and a sequence of “standard debt contracts”.

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The modified standard debt contract has the characteristic that, if the borrower repays the full amount of the loan in the first period, in the second period he gets a positive transfer; otherwise, he gets a negative transfer. There is no problem of strategic default, as output is assumed to be public information. Low-effort types face lower expected returns than high-effort types because of the higher probability of default associated with low effort. Under the modified contract, this increases the probability of getting less in the second period. Consequently, only high-effort types will be interested in the modified standard debt contract and low-effort types will prefer the unmodified standard debt contract.

Webb's modified standard debt contract is a good approximation to the contracts observed in the microfinance world. In this world it is common to find that good repayment generates positive transfers, such as larger loan sizes at lower effective interest rates and lower terms to maturity in subsequent loans, while repayment problems (arrears) not due to strategic behavior lead to the repetition or deterioration of the terms and conditions of the original contract (Gonzalez-Vega et al., 1997). Strategic behavior, if detected, is punished with the irreversible termination of the relationship.

The types of contracts actually observed in microfinance are quite complex, with numerous implicit and explicit terms and conditions that vary according to behavior and performance and sometimes to the state of nature. Some dimensions of these contracts stand out, however. In particular, they mostly rely on the threat of termination of the lender-borrower relationship and on the gradual improvement of the terms and
conditions of the contract as the borrower demonstrates capacity and willingness to repay over time. Loan size is usually used as a tool to overcome adverse selection and moral hazard problems. Microlenders do not seem to engage in intensive monitoring (except by requiring frequent payments), and the terms and conditions of the contracts are fairly standard for similar clients.

The relative simplicity of the accompanying structure of incentives in these contracts can be better explained by examining the opportunistic behavior of borrowers and the dynamic nature of the contracts from the perspective of the borrowers. Relevant borrower characteristics, such as wealth, risk profile and risk aversion, and the production function influence the outcome. This dissertation develops a dynamic parametric specification of a stylized microfinance contract in an attempt to explain the role of these characteristics in the performance of the contract.
Chapter Three
Problems in Credit Markets: Group Contracts

The provision of appropriate and correctly-priced financial services to marginal clienteles has been the focus of concern for many scholars and practitioners in the last decade. To address this concern, an innovative lending technology, incentive compatible organizational design, and long-term sustainability and commitment have been highlighted as important elements of a successful microfinance endeavor (Gonzalez-Vega, 1993).

In order to reach target clienteles with loans at attractive terms and conditions, an appropriate technology for delivering financial services must be developed. This technology must be based on the right set of incentives for borrowers to repay their loans and for lenders to provide innovative financial products that expand the frontier of microfinance. Demand-responsive products and high repayment rates guarantee, in turn, the long-term sustainability of the financial organization, not only because its financial results allow its steady operation, but also because the expectations and behavior of its clients contribute to its permanency.

The recent interest in innovations in lending technologies has led to the development of methods for delivering credit services that make use of principles learned from informal financial markets. From the features of rotating savings and credit
associations (ROSCAs), which are spontaneously present in many developing countries, and from the experience of credit cooperatives, group lending technologies have been adopted to deliver credit services to clienteles not previously reached by formal finance.

Although the empirical evidence shows all sorts of results, ranging from very successful to complete failure (Adams and Ladman, 1979; Huppi and Feder, 1990; Ladman and Afcha, 1990; Gonzalez-Vega et al., 1997; Paxton, Graham and Thraen, 2000), the contractual technological principles implicit in these arrangements suggest that these approaches may be cost-effective specific clienteles.

The success of well-known organizations, such as the Grameen Bank in Bangladesh (Hossein, 1988) and BancoSol in Bolivia (Gonzalez-Vega et al., 1997), reinforces the idea that, under specific conditions and for certain types of clienteles, this lending technology may be effective in the provision of formal loans to such clienteles. This has been the perception of numerous practitioners, for whom the group lending technology has been suggested as a potential solution to the problems of supplying loan services to large numbers of otherwise excluded agents.

Although large numbers of microfinance organizations have adopted group lending as their technology, some of the older ones have begun to switch to an individual lending technology. This shift raises questions about the long-term sustainability of group lending (Navajas, Conning, and Gonzalez-Vega, 2000). A dynamic model seems to be the most appropriate framework to address these questions. First, however, this chapter will review the literature on group lending.
Principal-Agent Theory with Multiple Agents

Principal-agent theory has considered interactions between one principal and multiple agents. With multiple agents, the contracts designed to deal with moral hazard and adverse selection problems must take into account possible interactions among the agents themselves. In the context of group lending technologies, a principal, the lender, writes a contract with a group of agents, the borrowers members of a group, who in turn interact among themselves and respond to the set of incentives established in the group contract.

When a principal deals with a group of agents, in the case of moral hazard it may be in the interest of the principal to structure the set of incentives such that the reward for each agent is dependent not only on her own actions but also on the actions of her fellow group members. The literature develops different schemes of incentives to deal with this challenge, such as ordinal rewards and relative performance rewards (Holmstrom, 1982; Sappington and Demski, 1983; Mookherjee, 1984; Rasmusen, 1987). With mutual monitoring, if the agents can engage in side payments, one possible undesirable result could be collusion against the principal (Holmstrom and Milgrom, 1990). Agreements or contracts among the agents, however, can favor the principal if these contracts lead to cooperation among them (Itoh, 1993).

The literature on adverse selection with groups of agents with issues related to a pool of agents who interact with the principal and with whether or not they act independently or interactively (Kreps, 1990). In the case of contracts between a lender and a group of borrowers, the group is considered as the contractual unit and, therefore,
adverse selection issues are reduced to the identification of group type and to an assessment of the incentives to be provided for group formation. Several models dealing with these issues are reviewed in the following section.

**Group Credit Contracts and Adverse Selection Problems**

Adverse selection problems in a group lending contract either deal with problems of private information about the type of group or with the process of formation of the group. In the former case, the contract must take into account whether the group is high-risk or low-risk, taking for granted that groups are risk-homogeneous. In the latter case, the process of formation of the group determines whether groups are risk-homogeneous or risk-heterogeneous.

The expected homogeneity of groups is explained by Varian (1990). Varian's model is built from the characteristics of the group lending technology as it is implemented by the Grameen Bank of Bangladesh. In the Grameen Bank scheme, the bank chooses one member of the group at random and determines her risk type through an individual evaluation. Based on this individual evaluation, the group is assigned its risk type.

In order to explain the implicit incentives embodied in the contractual arrangement he discusses, Varian considers two types of borrowers: good-risk and bad-risk agents. The lender faces a fixed cost of determining borrower type. If there is no cost of locating good-risk agents by the potential borrowers themselves or if the borrowers’
costs in doing so are lower than the lender's costs and if there are no side payments, only homogeneous groups will be formed.

Homogeneity is due to the assumption that good-risk agents will not be willing to accept a bad-risk agent as a partner, in spite of the fact that a bad-risk agent would like to be associated with good-risk agents. With side payments among agents, the lender must make sure that the group member selected for evaluation expects a sufficiently high return on her production project. This expected high return provides the right incentives to promote the formation of homogeneous groups.

Versions of Varian's line of argument have been used by several authors. Chaves (1996), for example, analyzes the formation of bancomunales, in the Costa Rican FINCA village banking scheme. In this scheme, the groups have a formal organizational status and loans tend to be larger than in other village banking programs. Each member of the group must deposit compensating balances in the village bank.

There are two effects in the process of group formation: an admission effect, which creates incentives to accept wealthier members and reject poorer members, and an application effect, which creates incentives for poorer agents to apply for admission and for wealthier agents not to apply. Given these effects, only members with similar levels of wealth will both apply and be admitted.

Devereux and Fishe (1993) justify the assumption of risk-homogeneous groups with the same argument. These authors consider the characteristics of a separating equilibrium between high-risk and low-risk groups. They assume that the group fully indemnifies the lender for group default, so that the problem of default is eliminated for
the lender, but it remains a challenge for group members. Collateral is required from all members. In case of default, collateral assets may be repossessed by fellow group members.

In this model, the surplus of either type of borrower is maximized subject to the incentive compatibility constraint and the zero-profit constraint of the lender. The surplus of each borrower is composed of the individual gains from their own effort and the potential loses due to default by other members. Failure exhibits a joint binomial probability distribution.

In order to write incentive-compatible contracts for each borrower type, the lender must offer contracts where the members of high-risk groups must pledge collateral equal to their endowment, which is assumed to be the same for all borrowers. The members of low-risk groups must pledge less collateral than the value of their endowment. This is due to the fact that, in high-risk groups, each individual member has a higher expected debt, given the higher probability of default.

Consequently, in this model, high-risk groups will be rationed first, if they do not possess enough collateral. This result contrasts with the result for individual loans, where only low-risk agents are willing to pledge collateral, while this requirement prevents high-risk agents from applying for the preferred contract. The preferred contract has a lower cost of funds defined as the ratio of the interest rate to the probability of success.

Van Tassel (1999) considers the possibility of generating a separating equilibrium by offering both individual and group lending contracts, provided that groups are homogenous. There are two types of agents: high-ability and low-ability. Lenders offer
loan contracts described by \((r, \sigma)\), where \(r\) is the interest rate and \(\sigma\) is the joint liability parameter. This latter parameter represents the portion of an unsuccessful member’s loan for which the signing member is responsible. If \(\sigma=0\), the contract entails individual liability, and any positive value of \(\sigma\) defines the degree of payment by a non-defaulting member of the defaulting member’s loan. Borrowers repay only if their production projects are successful.

Borrowers maximize their expected income. Groups of two agents are considered in a three-stage game. First, lenders announce their offer set (there are as many contracts as types of borrowers). Second, agents choose their contract. If individual liability contracts are chosen, the choice is awarded. If a joint liability contract is chosen, it is awarded if, in turn, the chosen partner selects the same contract. Third, the state of nature takes place and the loan is repaid.

Van Tassel finds a coalition-proof, pure-strategy, subgame-perfect Nash equilibrium. This type of equilibrium guarantees that the solution cannot be improved upon by mutually beneficial deviations through coalitions. This author shows that, under complete information, each type is given an individual liability contract with \(r_i = (\gamma - p_i)/p_i\), where \(\gamma\) is the opportunity cost of the funds and \(p_i\) is the probability of success. No joint liability contracts are offered because in order to offer this type of contract contract, without making borrowers worse off, the interest rate must be lowered. The interest rate, however, is bounded from below if the lender’s expected profits are to be positive. The bound is reached before any joint liability contract can be offered, given that the added expected costs of joint liability outweigh the expected gains from lower interest rates.
With asymmetric information, i.e., individual types are known to group members but unknown to the lender, the contract described above cannot be sustained. High-ability agents will have incentives to group together, low-ability agents will not be interested on grouping together, and high-ability agents will not be interested in grouping with low-ability agents.

Consequently, under imperfect information there will be a separating equilibrium in which high-ability agents choose a joint liability contract and low-ability agents choose an individual liability contract. The lender takes advantage of the different rates at which borrowers are willing to exchange lower interest rates for increased liability. By limiting the extent to which interest rates can be lowered, the lender attracts only those agents with abilities high enough to compensate for the increased liability.

The conclusion of the endogenous formation of homogeneous groups is also reached by Ghatak (1999). The formation of homogeneous groups is explained along the same line as the argument by Varian: safer borrowers prefer safer partners, because expected payments are lower. In his model, however, the separating equilibrium is constituted by different joint liability contracts, in which safer borrowers are required higher degrees of joint liability but charged lower interest rates and riskier borrowers are required lower joint liability and offered higher interest rates.

Ghatak explains this result by using the same argument as Van Tassel. With homogeneous groups, for the same reduction in the interest rate, riskier borrowers are less willing to accept an increase in joint liability. Ghatak calls this effect the peer-selection effect. Group contracts with joint liability also elicit the benefit of creating
collateral out of the revenues of successful members. By using group contracts, the lender takes advantage of the better information members have about each other. For this reason, joint liability contracts are Pareto superior to individual contracts.

In contrast to all these authors, Sadoulet (1999) provides empirical evidence that, in real life, there are both homogeneous and heterogeneous groups with respect to risk at the same time. To explain the simultaneity of heterogeneous and homogeneous groups, Sadoulet develops a model in which group formation is endogenous and the individual risk-taking by each borrower is jointly determined by her choice of partners (Sadoulet, 1997).

Heterogeneity within the groups cannot be exclusively explained by frictions in the search for partners; it is the result of the welfare-maximizing actions of the members of the group. Group lending leads to heterogeneous groups to the extent that riskier members pay a premium, a transfer in good states of nature, to safer borrowers who, in turn, assume repayment of the riskier member’s loan in bad states of nature.

Both types of members are better off with this type of insurance offered by safer borrowers. By allowing transfers between members, heterogeneous formation of the group is a Pareto improvement over homogeneous formation. Only agents who are too risky will not find safe borrowers willing to match with them and, therefore, highly risky borrowers will form separate homogeneous groups.
Group Credit Contracts and Moral Hazard Problems

In the context of lender-borrower contracts, protection against moral hazard has two dimensions. First, the contract must provide the right incentives to induce borrowers to undertake the actions or level of effort that generate capacity to repay and to avoid the diversion of the loan funds. This is *ex ante* moral hazard. Second, the contract must also provide appropriate incentives to avoid strategic default; in other words, the contract must promote willingness to repay, once the borrower’s returns are realized. This is *ex post* moral hazard. The first dimension of moral hazard is the result of the lender’s imperfect information about the actions or effort undertaken by the borrowers. The second dimension is the result of the lender’s imperfect information about the actual returns of the borrower (state verification) or of his limited capacity to appropriate the borrower’s returns (contract enforcement).

Group Credit Contracts under *Ex Ante* Moral Hazard

Most of the literature dealing with moral hazard problems can be directly applied to financial contracts under *ex ante* moral hazard. An example of this application is the model developed by Braverman and Guasch (1994). This model considers direct monitoring by the lender as a solution to moral hazard. In their analysis, group loans allow lenders to offer lower interest rates and larger lines of credit than in individual loans. Group members have the option, however, of diverting their share of the group loan to other non-productive purposes and reduce expected repayment.
First, the total loan to the group must be repaid by using the combined total output of the members of the group. Afterwards, there is an even distribution of the proceeds, net of repayment to the lender. The authors show that this arrangement generates a sub-optimal allocation from the lender’s perspective compared to an individual loan arrangement, because there are incentives for group members to divert their funds toward other uses, as returns are equally shared. This creates moral hazard and free rider problems. The result is group default.

As the literature suggests, the solution to the moral hazard problem entails the setting of incentive-compatible sharing rules for the distribution of the total net proceeds. The first best is attainable only with direct monitoring by the lender. With direct monitoring, the proceeds can be distributed according to each member’s contribution to the group’s output, provided that shocks are observable. If shocks are not observable, a member’s share will not just reflect his effort but the effort of others. This is the case, for example, when the sharing rule is defined in terms of average output.

The scenario presented by Braverman and Guasch, although a direct application of general principles, is not realistic. In reality, group repayment does not emerge from the total output of the group, but each member contributes to the total repayment from his individual output. Additionally, direct monitoring by the lender is costly, especially in the context of group micro-loans. For this reason, most theoretical models focus on the use of peer monitoring in the design of group contracts. Stiglitz (1990), for example, addresses whether or not peer monitoring is welfare enhancing.
In Stiglitz’s model, the information each borrower has about other members of the group is costless. Borrowers choose between two projects: a safe and a risky project. The expected return on the safe project is lower but, for a given project size, both projects require the same level of effort. Projects are fully financed with a loan, and group members are risk averse. With limited liability and insufficient collateral, large loan sizes make the risky project attractive for individual borrowers. Consequently, banks ration the size of individual loans to prevent borrowers from undertaking the risky project.

Group lending overcomes this rationing by loan sizes, because the lender offers lower interest rates and additional funds (larger loans) as an incentive for borrowers to monitor each other. Borrowers, in turn, decide about their own level of effort and about whether or not to report on the effort exerted by other members of the group. These decisions suggest a strategic game, but Stiglitz assumes that the decisions are taken jointly by the group members. Consequently, in a two-member group, both members take either the safe or the risky project but not one and the other (i.e., there is symmetry). When they both undertake the risky project, they also agree not to report on their level of effort to the lender.

The expected return to the bank of the alternative arrangements is unchanged, given a zero-profit condition. Welfare changes result, however, from the reduction of credit constraints. On the one hand, since in a group contract each borrower faces additional risks compared to an individual loan, each one must receive a larger loan size to preserve their expected utility. This is a necessary condition to keep the incentive for
them to cosign the joint-liability agreement. In this sense, the greater the liability, the larger the loan must be. Otherwise, they will prefer individual loans.

On the other hand, with greater joint liabilities, larger loans must be provided in order to induce borrowers to undertake the safe project. The increase in loan sizes needed to induce the choice of the safe project is greater than the increase needed to preserve the borrowers' expected utility and, consequently, the optimal contract will be welfare enhancing.\(^1\) In other words, the additional risks brought about by joint liability are more than compensated for the borrowers by the increase in loan size. It is in the interest of the lender to increase loan sizes to promote the choice of safe projects. Therefore, from the borrowers' perspective this is welfare improving, without undermining the lender's welfare.

An extension of the monitoring model developed by Conning (1999), already discussed, incorporates the possibility of peer monitoring in jointly liable groups of borrowers. With the inclusion of peer monitoring, the optimal contract must provide, in addition to the right incentives when choosing actions, the right incentives when choosing monitoring intensities. Each member of a group not only has to decide on his own actions but also on the degree of monitoring to exert over his peers. Given a minimum of two members in a group, the solution of the model is found through a two-stage game: first, each borrower chooses monitoring intensities and actions are then chosen in the second stage.

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\(^1\) The increase in loan size to preserve expected income is linear whereas the increase to induce the right choice is not, because the marginal utility of additional units of loan is lower in the safe project than in the risky project.
A sub-game perfect Nash equilibrium results from mutual monitoring. At the individual level, this equilibrium is preferred to an equilibrium with an outside monitoring intermediary, if monitoring among group members sufficiently reduces collateral requirements. This could be the case if the members of the group can monitor each other at a lower cost than the monitoring intermediary can. If the members of the group have a comparative advantage in monitoring, a group lending technology will lower collateral requirements and, therefore, it will reach poorer borrowers (lower levels of assets).

Nevertheless, if the payoff of zero monitoring and low effort action is higher for the borrowers than the payoff of the optimal level of peer monitoring from the lender’s perspective, the solution will be collusion against the lender. If the decision on monitoring effort and action are taken simultaneously, the result, for a one-shot game, collapses to low effort and zero monitoring.

The relationship between the borrower’s level of wealth and the effectiveness of the different lending technologies discussed by Conning is further explored by Madajewicz (1997). She considers a model of maximization of borrower welfare under limited liability with risk-averse borrowers. The inclusion of risk aversion is key, as joint liability brings about additional risks for the members of a group.

The lender offers three types of contracts: individual-unmonitored, lender-monitored, and borrower-monitored with joint liability. The level of wealth of the borrowers affects the effectiveness of the incentives implicit in each type of contract. Both the lender and the borrowers share the same costly monitoring technology.
Monitoring has the effect of providing additional virtual collateral, as the borrower's returns can be appropriated if needed. For this reason, lender-monitored contracts can offer larger loans than individual contracts do. Since monitoring costs are increasing in loan size, whether or not lender-monitored loans are preferable to individual loans depends on the increase of loan size versus the increase in monitoring costs.

Monitoring by borrowers is more efficient than monitoring by the lender because the borrower's cost of monitoring is only a labor cost regardless of their choice of project. When the lender monitors, in contrast, the choice of production project is affected. The reason is that the lender's cost of monitoring is passed on to the borrowers who, therefore, must use part of their own returns to pay for the lender's monitoring. Because of limited liability, the borrowers incur in this cost only when they undertake with safe projects, which makes risky projects more attractive.

For low levels of wealth, group contracts allow the lender to grant larger loan sizes, because joint liability discourages group members from undertaking risky projects. As wealth increases, however, the potential collateral that borrowers can offer in an individual loan contract also increases, thereby raising loan size. In group loans, in contrast, loan sizes cannot increase as fast with wealth, as wealthier group members impose greater risks to fellow group members. The difference in the size of loans eventually reaches a point that justifies the cost of monitoring and, therefore, lender-monitored loans become more attractive than group loans.

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2 This is the case because, as wealth increases, risky projects become more attractive, since the return from the risky project increases faster than the return from the safe project. In other words, for given equal levels of expected utility, higher wealth necessarily implies higher payoffs in case of success, since the payoff of failure is always zero. Then, the marginal utility of wealth will be smaller, due to risk aversion.
Joint liability induces two negative incentive effects. One of them arises from the additional risks it imposes. The second one is due to the effect of additional collateral, which requires a larger payoff conditional on success, as marginal utility declines due to risk aversion. These negative effects get worse as wealth increases and offset the greater effectiveness of group monitoring. Eventually, wealthier borrowers prefer individual loans, as these negative effects wipe out the benefits of peer monitoring. Only poorer borrowers, therefore, may prefer group loans.

In sum, for low levels of wealth, group loans with joint liability are preferred to lender-monitored loans or to individual loans with collateral. For middle levels of wealth, where rising monitoring costs do not outstrip larger loan sizes, lender-monitored loans are preferred to individual loans. Finally, individual loans with collateral are preferred by the richest borrowers.

**Group Credit Contracts Under Ex Post Moral Hazard**

The problem of *ex post* moral hazard is the second dimension of the general moral hazard issue in lending contracts. It occurs, once actions or level of effort have been carried out and returns of the productive activity have been realized, when borrowers find it optimal to divert the funds for repayment of the loan to other purposes and fall in default. Since production returns are usually private information, lenders are unable to determine if default is due to legitimate reasons, such as an idiosyncratic shock, or if default is due to the borrower's strategic behavior.
In group lending, *ex post* moral hazard is worse than for individual loans, as borrowers who might have repaid may find it optimal to default just because their fellow group members have also defaulted. Additionally, group lending opens the possibility of collusion against the lender, *a la* Itoh, even when every member could have repaid her part of the group loan. These effects are analyzed by Besley and Coate (1995), who consider two effects of joint liability on repayment: (a) the negative effect from some members defaulting due to the excessive burden of having to repay for defaulting members and (b) the positive effect of the repayment of defaulters’ loans by non-defaulting members.

The two effects are studied by Besley and Coate in a repayment game between two group members. The game has two stages. In the first stage, each player decides whether to repay or not. In the second stage, if the borrowers have chosen different strategies in the first stage, the player who chose to pay has to decide whether to pay the total loan or not. The lender imposes sanctions for default. Penalties are a positive function of the returns of the production project of each borrower, assuming that the higher the returns, the greater the possibility of the seizing of assets.

The sub-game perfect equilibria are dependent on the values of the returns. For high values of returns for both borrowers, equilibrium entails one member repaying the total due. This is also the case if only one borrower has high returns. If both borrowers have medium returns, each one will repay only if the other one repays; otherwise, none will repay, which is a typical case of the prisoner’s dilemma. In all other cases, the solution is default by all group members.
Besley and Coate compare group repayment rates with the repayment rates of an individual lending technology and examine how the differences are affected by changes in the interest rate. Under uniformly distributed returns and a linear penalty function, group lending achieves higher repayment rates at low interest rates and individual lending achieves higher repayment rates at high interest rates.

These authors also consider the social collateral or reputation effect implicit in group lending by including a social penalty function. The social penalty function captures the response of fellow group members to lack of repayment. The response depends on the extent of the harm inflicted by default and on the reasons for default. If default does not bring any loses to the partner or if it is due to unfortunate factors (not under the borrower's control), there is no social sanction.

With the inclusion of social sanctions, the solutions to the repayment game are similar to those for the case discussed above only for high returns. In the case of medium and low returns, repayment by both members becomes a possible solution. Group repayment rates are higher if the social sanctions are severe enough.

The role of peer pressure and social sanctions is also considered by Diagne (1997). Diagne incorporates the value of future access to loans as an essential element to understand joint liability. The value of future access to loans generates interdependence among group members. Interdependence generates peer pressure. Peer pressure may be active, as in actions taken by non-defaulters to induce defaulters to repay their loans, or passive, as in feelings of shame or guilt for harming non-defaulters. Passive peer pressure does not result in any cost to a non-defaulter whereas active peer pressure brings direct
and indirect costs (retaliatory costs) to defaulters. The value of future access to loans and the individual willingness to apply peer pressure are private information. Together, they determine willingness to repay.

The repayment game is modeled in five stages. In the first one, borrowers have the option of repaying or waiting to see what other members will do. If all members pay or wait, the game is over with full repayment or full default. The game continues only if at least one member waits and at least another chooses to fully repay. The second stage occurs after the loan is due. Each member that has chosen to wait must decide whether to repay or not. If they do, the game is over with full repayment. If they do not, the game goes to a third stage.

In the third stage, there are three possible actions. First, do nothing, which leads to the end of the game with partial repayment. Second, pay the defaulter’s loans, which also leads to the end of the game, with full repayment. Third, apply some pressure on defaulters, which leads to a fourth stage of the game. In the fourth stage, members under pressure must decide to repay or not. If they repay, the game is over with full repayment. Otherwise, the game goes to a fifth stage in which non-defaulters decide either to repay the defaulters’ loans, in which case the game finishes with full repayment, or not to repay, in which case the game finishes with partial repayment.

In computing the payoff of each action, the value of future access to loans is assumed to be given by the discounted value of a credit limit to which each borrower has access to if the loan is repaid in full. The discount factor is private information of each borrower. Each group member has a distribution of beliefs about each other’s discount
factor. If the discounted value of the credit limit is greater than the benefit of defaulting on the first period, the borrower is a good borrower; otherwise, he is a willful defaulter.

In the repayment decision, however, the cost of passive and active peer pressure for defaulters is added, including the cost of embarrassment when someone else pays a defaulter’s loan. Each group member has a distribution of beliefs about the value of these defaulter’s costs and a different distribution of beliefs about the non-defaulter’s costs of applying active pressure and of embarrassing a defaulter.

A willful defaulter never repays first and if he repays, it is only to avoid peer pressure or embarrassment. Therefore, a group composed only of willful defaulters will always default. The set of strategies candidate for equilibrium is thereby reduced. The solution is a Perfect Bayesian Equilibrium. This implies that, at each stage, the strategies of the players are best responses, given their posterior beliefs at that stage.

In the two pooling equilibria, with all players choosing the same action, both members default, even when they would have repaid their loans under perfect information and individual liability. These outcomes are less likely to occur, the higher the value of the credit limit.

In two of the separating equilibria, there is enough passive social pressure to prevent default. Both group members repay their loans, because the willful defaulter cannot bear the fact that the good borrower repays his loan. In another two separating equilibria, non-defaulters repay the loans of defaulting members, but there is no active pressure, because the good borrower infers that the willful defaulter intends to resist

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3 For simplicity on finding the solutions of the game, the group is reduced to two members.
pressure. In sum, the only equilibrium where peer pressure is effective in preventing
default is when potential defaulters are intolerant to passive social pressure.

Under limited joint liability, understood as some degree of penalty for non-defaulting members of a group and termination of the lending relationship with defaulters, two additional separating equilibria with partial repayment emerge. Furthermore, the two original pooling equilibria are less likely to occur; the two separating equilibria in which all members repay due to passive peer pressure are more likely to occur; and the two separating equilibria in which non-defaulters pay for defaulters are less likely to occur. Consequently, default rates under limited joint liability are lower.

The role of peer pressure and the value of future access to loans is also examined by Armendariz de Aghion (1999). As in Conning’s model, lenders incur a higher cost of monitoring than group members incur and, therefore, lenders want to induce group members to monitor each other. Peer monitoring is induced by a joint liability contract. Failure to repay the whole loan has the consequence of a borrower losing the benefits of being refinanced. Limited liability is assumed, so that under bad returns, borrowers are not obliged to repay. The cost of having to repay a defaulting partner’s loan is ignored because by assumption a monitored agent always pays his part of the loan, provided that there are sufficient returns.

The timing of the contract is as follows. First, the joint liability contract is signed. Second, monitoring effort is exerted. Third, returns are realized. Fourth, each member chooses which returns to reveal. By assumption, for an unmonitored agent with good
returns, it is more rewarding to reveal bad results and claim default. Knowledge of being monitored by another agent automatically induces repayment, provided that the returns are sufficient.

Each borrower will monitor up to the point where the marginal benefit of monitoring is equal to the marginal cost. For a given borrower, the benefits of monitoring only occur when the other borrower is in the good state. If that is the case, there are two possibilities: the monitoring borrower may be monitored by another borrower or she may not be monitored. In the former case, the benefits of monitoring are not to have to pay the other member's part of the loan in case of her default. In the latter case, the benefit is future access to additional loans. There is a trade-off between the benefit of gaining access to further borrowing, i.e., the insurance effect, and the cost of monitoring plus the net loss of paying the peer's debt, i.e., the peer repayment effect.

The probability of monitoring is decreasing in monitoring costs and increasing in loan size and in the value of future access to loans. Consequently, the potential growth of loan size is limited under group lending. Given that monitoring is a costly activity, it becomes prohibitive to monitor very large loans. Since under group lending with peer monitoring repayment rates are higher, in group lending the equilibrium interest rate is lower than under direct monitoring by the lender.

Armendariz de Aghion also analyzes the optimal composition of groups. The benefits of future access increase with negatively correlated returns of the production projects among members of the group and decrease with positively correlated returns. Consequently, monitoring effort is lower in the former case and higher in the latter.
The size of the group has an ambiguous effect, as with larger groups individual members face smaller losses from individual defaults, but they also face larger monitoring requirements. Finally, group heterogeneity, characterized by different loan sizes and different valuations of future access to loans, is not conducive to peer monitoring, as it makes it more difficult for group members to evaluate their fellow group member's actions.

The negative impact of group heterogeneity, as explained by Armendariz de Aghion, seems to be contradicted by the evidence found and explained by Sadoulet (1997). Sadoulet, however, considers heterogeneity in risk as a potential for mutual insurance and does not consider the role of monitoring or the social cost of default. Consequently, the model concentrates on the process of formation of the groups and its effect on strategic default.

In the same fashion as in Armendariz de Aghion and in Diagne, in Sadoulet the value of future access to loans, exogenous and restricted to one period, is an essential determinant of willingness to repay. Lack of repayment leads to automatic exclusion from further borrowing. Borrowers, however, are assumed to be risk neutral and, therefore, preferences about risk do not play a role in the explanation of mutual insurance within the groups.

Assuming groups of two borrowers, once returns are realized, there are three possibilities in Sadoulet's model. One, both projects fail and the group defaults. Two, one of the borrowers succeeds and the other fails. The successful borrower will repay the total amount of the loan if the benefit of such action is greater than the cost. The borrower who
pays provides insurance to the other borrower. Three, both borrowers succeed. Then, the game enters a bargaining game, in which each borrower decides how much to contribute to the repayment of the group loan. Borrowers minimize the necessary contribution in order to induce the other borrower to pay the remaining part. Given the contribution of one borrower, the other borrower contributes the rest if the benefit of doing so outweighs the cost. If the sum of the contributions is not enough to repay the loan, the group defaults and both borrowers are excluded from further borrowing.

Sadoulet then shows that in the cases when a successful borrower repays for an unsuccessful borrower, group loans are preferred to individual loans from society's perspective. Repayment for an unsuccessful borrower is, however, only a necessary condition for group loans to be preferred to individual loans. Since ability to repay is defined by a lottery over the potential returns of the project, borrower type is defined by their ability to repay. Safer borrowers, with higher ability to repay, will only be willing to borrow in a group if riskier borrowers, with lower ability to repay, are willing to pay a transfer. In return, safer borrowers repay for riskier borrowers when unsuccessful. Riskier members must compensate the safer members for the asymmetric insurance.4

If borrowers have the choice between group lending and individual lending, group lending will lead to higher repayment rates, some borrowers will participate in group lending, and insurance will be provided. The riskiest borrowers, however, will still form homogeneous groups with little scope for insurance. If borrowers do not have the

4 Given the role of borrower's type on repayment in Sadoulet's model, strategic default is linked to borrower type. The set of incentives to alleviate ex post moral hazard are, therefore, linked to the adverse selection problem of the formation of the groups.
choice between group and individual lending, the exclusive provision of group lending may lead to lower repayment rates.

In sum, from the literature on group credit contracts several dimensions are added to the literature on individual credit contracts. Production risk sharing among members of the group and delegation of monitoring by the lender to better-informed group members are two possible sources of welfare improvement. The role of credit as a mechanism to manage adverse shocks is further enhanced in group lending, as individual members are less vulnerable to idiosyncratic fluctuations of their returns because they can count on fellow members to repay their outstanding liabilities. This type of insurance is only possible, however, to the extent to which the beneficiaries of insurance are capable of paying for this service in future loan cycles.

The size of group, individual abilities to monitor, heterogeneity of individual production projects, and role of the individual’s reputation within the group are factors that also affect the effectiveness of group loans as a lending technology. These factors determine the extent to which this technology alleviates adverse selection and moral hazard problems. The distinction between \textit{ex ante} and \textit{ex post} moral hazard is quite relevant for group lending, as individual decisions within a group pertain not only to levels of production effort but also to strategic decisions on whether to repay or not the individual share of the group loan. The characteristics of the group play an important role in both decisions.

In Chapter Six, the individual model to be developed in Chapter Five is extended to a two-member group model. The purpose of the model is to explore the strategic
behavior of borrowers within a group when they earn sufficient returns to repay but it may not be in their interest to do so. Repayment decisions are analyzed and the sources of the positive and negative effects of joint liability are explained as in Besley and Coate (1995). The objective here is to explain, however, these effects, not as the result of exogenously imposed penalties but as the result of strategic behavior by the members of a group. This behavior, in turn, is influenced by the contractual conditions offered by the lender.

The dynamic dimension of this type of contract is incorporated in the dissertation, as microlenders rely almost exclusively on the threat of termination of the relationship with the group as the main mechanism to enforce repayment. The dynamic incentives implicit in this type of contracts require the incorporation of simultaneous investment and repayment decisions in the model. As a result, the value of future access to credit is endogenously determined. In this sense, the model developed in this dissertation provides additional insights into the analyses of group lending by Diagne, Armendariz de Aghion, and Sadoulet.

A clear understanding of the factors that affect the set of incentives to repay in group lending technologies is essential to judge the merits of this type of contracts. In the practice of microfinance organizations, group lending has been the focus of a heated debate concerning its effects on repayment and, consequently, on the sustainability of the lending organization. Some of the most successful organizations using group lending have faced the need to depart from the principles of group lending as their clienteles
mature. Consequently, a dynamic analysis of this lending technology may shed some light to explain these results.
CHAPTER 4
Dynamic Programming and Numerical Methods

The stylized facts of microfinance contracts reveal the need to incorporate dynamics into their analysis. From the review in the previous chapters, it is clear that the state of the art on contract models addresses most of the problems of information asymmetries raised in financial markets from a static perspective. Moreover, traditional analyses of principal-agent theory are quite revealing when focusing on discrete, simplified production and utility functions. This type of analyses, however, may be somewhat limiting when a dynamic setting is relevant.

In this dissertation, microfinance contracts are studied within a dynamic framework that explores explicit contractual arrangements where the set of incentives are inter-temporally linked. Agents make decisions considering their current and future consequences. This suggests the need to develop a type of analysis that is quite different from the traditional algebraic treatment of simple contract models with a discrete number of outcomes. This section briefly surveys the theoretical principles and the main tools that will be used to simulate the agent’s decisions under the contracts studied here.

The study of economic agents who make decisions in a dynamic setting by using numerical methods is not new in Economics. Consumption and savings decisions, the optimal growth model, agricultural production, and arbitrage pricing of financial assets
are some of the areas where dynamic models have been extensively used (Chavas and Kliebenstein, 1985; Miranda, 1997; Taylor and Uhlig, 1990).

The study of dynamic economic decisions using explicit functional forms via numerical methods makes it possible to incorporate important features of the decision-making process that cannot be captured in algebraic models. These features include:

(a) Economic agents are aware that their decisions affect both current and future well-being.

(b) Human behavior and economic processes are not fully predictable. Numerical methods facilitate the treatment of uncertainty.

(c) Human decisions and economic processes are inherently complex and often non-linear. The inclusion of non-linear functional forms permits a more realistic approach to many economic processes.

(d) The sets of decisions and states are continuous sets. Many studies reduce these sets to a few finite values, thereby limiting the generality of the results. Numerical dynamic programming allows the incorporation of continuous states and actions sets.

The development of powerful computers and user-friendly software has recently facilitated the numerical analysis of stochastic dynamic models. Numerical methods can handle models with no closed-form solutions. These methods can also incorporate the treatment of uncertainty. Traditional algebraic analysis of dynamic models usually considers only two periods, analytically tractable objective functions, very simple cases, and deterministic or just binary stochastic processes. With numerical methods, these limitations are overcome.
Definition

A typical dynamic programming exercise requires the definition of the following variables and functions:

\( s_t = \text{state}(s) \) of the process in period \( t \), with \( s_t \in S \), where \( S \) is the state space, which contains all of the states attainable by the process.

\( x_t = \text{action}(s) \) taken in period \( t \), with \( x_t \in X \), where \( X \) is the action space, which contains all of the actions the agent may undertake.

\( f(x_t, s_t) = \text{reward} \) or criterion function. This function describes the payoffs for the economic agent in terms of his actions and the states.

\( s_{t+1} = g(x_t, s_t, \varepsilon_{t+1}) \) is the state transition function. This function describes how the future state evolves in terms of current states and the actions undertaken by the agent. The term \( \varepsilon_{t+1} \) represents an exogenous random shock. This term allows the inclusion of uncertainty about the future. The realization of \( \varepsilon_{t+1} \) is unknown in period \( t \). It is assumed that \( \varepsilon_{t+1} \) is distributed according to a known density function.

The exercise assumes that the agent seeks the state-contingent optimal actions, \( x^*_t \), that maximize the present value of current and expected future rewards. In other words, the agent seeks a rule that specifies the optimal action for each possible state. The agent’s time preference is captured by the discount factor \( \delta = 1/(1+\text{time discount rate}) \).

Most solution methods in dynamic programming are based on Bellman’s principle of optimality. This principle asserts that:
If a strategy is optimal for each point in time at that point of time, given that an optimal strategy will be pursued thereafter, then the strategy is optimal.\(^1\)

The reward function is assumed to be separable. This means that, for every time period \(t\), the reward function depends only upon the state at time \(t\) and the actions from \(t\) onwards.

From the definition of the transition function, the process is assumed to follow a Markovian process. States at time \(t+1\) depend only upon the state and actions in period \(t\) and the realization of the random shock (if the process is stochastic), but not on previous states.

The two conditions above make it possible to write the functional form of the process in two parts: the current stage, followed by all remaining future stages. By virtue of the principle of optimality, the optimization process embedded in dynamic models can be written as a functional equation:

\[
V(s) = \max_{x \in X(s)} \left\{ f(s, x) + \delta E_x V[g(s, x, \varepsilon)] \right\}, \quad s \in S
\]  

(4.1)

\(V(s)\) is the value function. It represents the maximum net present value of current and future rewards for every possible state \(s\). Equation (4.1) reduces the intertemporal optimization problem to a two-period recursive optimization problem.

States, actions, and time may be defined continuously or discretely. The time definition determines the method of solution of equation (4.1). To facilitate their study, infinite-time horizon processes are frequently assumed to be stationary. The process is

\(^1\) Kreps (1990), p.798.
stationary if the reward function, the state transition, the state space, the action space, and the distribution of the random shock do not vary over time. Consequently, the value function is the same for every period $t$. The models considered in this dissertation are stationary, infinite horizon models with continuous states and actions.

**Method of solution**

Equation (4.1) is a functional equation with two unknowns, the value function and the optimal action function. Both functions are defined on the state space $S$. These two functions generally lack an analytic closed-form solution and, therefore, they can only be approximated.

In this dissertation, the value function is approximated using a collocation method for both the individual and the dynamic game models. This method offers a highly accurate numerical solution in both circumstances (Miranda, 1996; Miranda and Vedenov, 2000).

The collocation method provides a strategy of solution in which the value function of equation (4.1) is approximated by a linear combination of previously chosen basis functions $\phi_1, \phi_2, \ldots, \phi_n$:

$$V(s) \approx \sum_{j=1}^{n} c_j \phi_j(s)$$  \hspace{1cm} (4.2)

The coefficients $c_j$ are to be determined. In order to determine these coefficients, the approximation to the value function represented by the right-hand side of equation (4.2) is required to satisfy Bellman's equation at $n$ states $s_1, \ldots, s_n$ called the collocation nodes.
\[
\sum_{j=1}^{n} c_j \phi_j(s_i) = \max_{m \in \mathbb{Z}^n} \left\{ f(s_i, x) + \delta E_x \sum_{j=1}^{n} c_j \phi_j(g(s_i, x, \varepsilon)) \right\} \tag{4.3}
\]

for \( l = 1, 2, \ldots, n \).

The choice of collocation basis-nodes schemes depends on the characteristics of the value function to be approximated. Different basis-nodes schemes offer different features suitable for specific applications. In this dissertation, Chebychev orthogonal polynomials were used first and then a spline function approximation provided a more suitable scheme.

Chebychev orthogonal polynomials guarantee accuracy and efficiency by the Chebychev polynomial approximating theorem. This theorem asserts that, for a given degree, the best approximating polynomial is the one that interpolates the function at the Chebychev nodes and that the approximation error tends to disappear as the degree of approximation is increased. Chebychev polynomials, however, behave strangely outside the range of interpolation and around the vicinity of non-differentiabilities of the function being approximated.

Spline function approximation methods differ mainly in the fact that spline functions have narrow supports, whereas Chebychev polynomial basis functions have supports that cover the entire approximation interval. In the presence of discontinuities of the first or second derivatives of the approximated function, spline functions may better suited to contain the effects of such discontinuities due to their narrow support.

Chebychev polynomial approximants allow the ill effects of discontinuities to propagate over the entire interval of approximation. Consequently, if the approximated function is not smooth, as is the case in the choice model considered in this dissertation,
a spline function approximation may be preferable to a Chebychev approximation (McGrattan, 1999; Miranda, 1996).

Having chosen the basis-node scheme, equation (4.3) can be expressed in vector form as:

\[ \Phi c = v(c) \quad (4.4) \]

Equation (4.4) is the **collocation equation**. In this equation, the matrix \( \Phi \) is the **collocation matrix** for which each \( j^{th} \) element represents the \( j^{th} \) basis function evaluated at the \( i^{th} \) collocation node. The right-hand side of equation (4.4) is the conditional value function, which generates the maximum value obtained when solving the optimization problem embedded in Bellman’s equation, at each collocation node, taking the value function approximation as given. The given value function approximation is implied in the coefficient vector \( c \).

The collocation equation (4.4) represents a system of \( n \) non-linear equations in \( n \) unknown coefficients, \( c_j \). The system of non-linear equations can be solved by a function iteration procedure. Starting from an educated guess for the coefficient vector \( c \), in each iteration the coefficient vector is updated by the rule:

\[ c \leftarrow \Phi^{-1}v(c) \quad (4.5) \]

The conditional value function \( v_l(c) \) must be computed for every \( l \); that is, the optimization problem embedded in Bellman’s equation must be solved at every collocation node \( s_i \), taking the current vector \( c \) as fixed.

The optimization problem of equation (4.1) can be characterized by its first-order conditions, known as the Euler conditions. These conditions provide an economic
explanation for the intertemporal arbitrage equilibrium implied in the optimal solution. In Chapter Five, these conditions will prove useful to understand the role played by the borrower’s endowment and level of debt under different contracts offered by a lender.

The Euler conditions for the maximization problem of equation (4.1) are given by the Karush-Kuhn-Tucker conditions for the unconstrained optimal action $x$:

$$f_x(s, x) + E_x \left\{ \lambda_x \left[ g(s, x, \varepsilon) \right] \right\} = 0$$

(4.6)

and by the Envelope Theorem:

$$\lambda_x(s) = f_x(s, x) + \delta E_x \left\{ \lambda_x \left[ g(s, x, \varepsilon) \right] \right\}$$

(4.7)

Equations (4.6) and (4.7) define the optimal action function and the shadow price function of the state, respectively.

The generation of the basis nodes and of the interpolation matrix evaluated at the nodes for the approximation of the value function or the Euler conditions makes use of available subroutines developed by Miranda and Fackler (2000). To compute the expectations of the stochastic term in the maximization in equation (4.3), a Gaussian quadrature is used, which also takes advantage of published subroutines by the same authors.

The solution of a dynamic game model for $m$ players is based on the same principles explained above. In this dissertation, the game is restricted to two players, indicated by the sub-index 1,2. For simplicity, the sub-index $t$ is omitted unless it is necessary. In a game, the state(s) transition function and the reward function of each player depend on the actions of the other player:
Each player seeks actions \( x_i = x_i^*(s) \) that maximize his current and discounted expected future rewards, given the actions followed by the other player. The Nash equilibrium solution of the two-players dynamic game is characterized by the two simultaneous Bellman's equations:

\[
V_i(s) = \max_{x_1, x_2} \left\{ f_i(s, x_1, x_2^*(s)) + \delta EV_i \left[ g \left( s, x_1, x_2^*(s), \varepsilon \right) \right] \right\}
\]

\[
V_j(s) = \max_{x_1, x_2} \left\{ f_j(s, x_1^*(s), x_2) + \delta EV_j \left[ g \left( s, x_1^*(s), x_2, \varepsilon \right) \right] \right\}
\]

The unknowns are the two value functions and the two optimal action functions, all of them defined on the state space \( S \).

The collocation method may be applied to approximate the solution of the simultaneous Bellman's equations in the same fashion as in the single Bellman's equation problem. After writing each value functions as linear combinations of known basis functions and where each approximant is required to satisfy the respective Bellman's equation, the problem becomes a \( 2n \) system of linear equations in \( 2n \) unknowns. As in the case of the single Bellman's equation, the system may be solved using a function iteration procedure.

Nevertheless, in a game, the actions of a player depend on the actions of the other player and, therefore, the collocation equations cannot be solved independently. An iterative procedure may be used, however, in which, in each iteration and for each player, the actions of the partner player are taken as given. After solving the
optimization for each player, each player's actions are updated, until the change in the optimal actions is considered small enough according to some pre-defined tolerance level.

The iterative solution of the repayment game is explained in more detail in Chapter 6, along with the results of the approximation. Chapter 5, explains the approximation of a growth model for an agent with access to credit. The approximation follows closely the steps described in this chapter. The numerical solution of the growth model for an individual agent is then the basis for the solution of the group model presented in Chapter 6.
Individual borrowers face the decision of repayment or default, even when the returns from the productive activity are sufficient to repay the principal of the loan plus interest. This chapter develops a model of the repayment-default decision for an individual borrower, assuming away issues related to ability to repay and focusing only on willingness to repay.

In the model, repayment is an implicit decision to remain in good standing with the lender, which preserves the possibility of future access to credit. In contrast, the decision to default allows the borrower to appropriate all of the proceeds from the current loan, but at the cost of losing all future access to credit. That is, failure to repay leads to termination of the relationship with the lender forever. Lenders do not accept partial repayment. Emphasis on the value of the relationship as the central element of contractual incentives is the most basic notion in microfinance.

In the model, loan size grows according to a pre-established rule, as the relationship between borrower and lender matures and the borrower’s wealth increases. In the model discussed here, loan size is a given proportion of the borrower’s wealth. A simple loan size rule is also typical of microfinance. Interest rates on loans do not change.
over time, and borrowers are not required to pledge tangible collateral. To enforce repayment, lenders rely only on the threat of termination of the relationship.

The level of wealth, or own endowment of the borrower, plus the loan funds amount to the borrower's total available funds. Borrowers allocate these funds between investment and consumption. Consumption generates instant utility whereas investment generates future returns, which may be used to consume or to repay the loan. Consequently, the model considers the strategic decision of the borrower between: (a) higher consumption today with default and self-financing in the future, or (b) lower consumption today with repayment and access to additional loan funds today and in the future.

If the rate of time preference, $\rho$, is high relative to the interest rate, it will be optimal for the borrower to default, since future consumption would score low vis-a-vis present consumption. Excessively high rates of time preference (low discount factor $\delta$) relative to the interest rate make delaying of consumption too costly. The analysis here is restricted to values of discount factors and interest rates that prevent this trivial result.

To highlight the contrast between self-financing and access to credit, the model incorporates a multiplicative productivity-enhancement coefficient in the production function. This coefficient is greater than one only when producers participate in credit markets. This strong assumption is a proxy for the productivity-enhancing role of financial intermediation (Shaw, 1973).
Loans ease the funds constraint faced by the borrower, but own savings would have the same effect, given the fungibility of funds. That is, this positive effect of releasing the funds constraint works regardless of the source of the funds.

Funding through credit markets creates, however, additional intangible assets that enhance productivity. Bank-client relationships, for example, improve the borrower's access to financial tools for risk and liquidity management, thereby reducing risk exposure. These relationships also create reputation effects that are quite important in the market niches typical of microfinance organizations. Reputation allows borrowers access to other markets characterized by rationing due to information and incentive problems. Borrowing provides co-insurance when accompanied by limited liability, and the loan evaluation process generates valuable information for borrowers on their financial standing. The bank-client relationship, in general, creates economies of scope in financial management for the borrower, thus enhancing productivity. This feature of the model mimics, at the microeconomic level, the effects of financial deepening at the macroeconomic level (McKinnon, 1973).

**Growth Model without Access to Credit**

To facilitate the numerical analysis of the choice model in which the borrower decides whether to preserve a credit relationship or not, the behavior of an individual agent without access to credit is considered first. At the beginning of each period, \( t \), the agent possesses an endowment, \( s_t \), which can be either invested, \( x_t \), or consumed, \( c_t \).
Borrowers derive utility from consumption, while investment provides the opportunity of increasing future consumption.

The agent's productive opportunity is represented by a production function $f(x_t)$. Output is subject to idiosyncratic risk, represented by a production shock, $\varepsilon_t$. This shock multiplies the production function, and it is distributed log-normally with mean 1 and variance $\sigma^2$.

The setup described above defines the traditional growth model for an individual producer. The following variables and equations fully describe the model:

State variable:

$$s_t = \text{endowment at the beginning of time } t.$$  

Action:

$$x_t = \text{investment in time } t.$$  

Investment is the choice variable for the agent. It is non-negative and subject to the budget restriction: $0 \leq x_t \leq s_t$. Consumption must also be non-negative:

$$c_t = s_t - x_t \geq 0 \quad (5.1)$$  

State transition:

$$s_{t+1} = f(x_t) \varepsilon_{t+1} + \gamma x_t \quad (5.2)$$  

The endowment in period $t+1$ equals the stochastic returns of the productive activity plus the recuperation capital. In other words, $(1-\gamma)$ is the fixed rate of capital depreciation.
Reward:

\[ U(c_t) = \text{individual utility function.} \]

By virtue of the Principle of Optimality, the individual optimization problem is summarized in 's equation:

\[ V^0(s_t) = \max_{s_{t+1}} \{ U(c_t) + \delta EV(s_{t+1}) \} \quad (5.3) \]

The value function \( V^0 \) represents the maximized present and discounted expected future utility in the absence of access to credit.

The equilibrium conditions for an optimum, as defined in Chapter 4, are:

\[ U'(s_t - x_t) - \delta E \left[ \lambda_{t+1} \left( y + \epsilon f'(x_t) \right) \right] = 0 \quad (5.4) \]

\[ \lambda_t = U'(s_t - x_t) \quad (5.5) \]

Equation (5.4) defines optimal investment with the condition that an additional unit of wealth allocated to consumption today must generate the same utility as the discounted expected value of an additional unit of wealth allocated to investment and consumed in period \( t+1 \), where marginal investment is valued by the shadow price of wealth, \( \lambda \). By equation (5.5), the shadow price of wealth equals the marginal utility of consumption.

Manipulation of equations (5.4) and (5.5) results in:

\[ U' = \delta EU'_{t+1} \omega_{t+1} \quad (5.6) \]

where, \( \omega_{t+1} = y + \psi f'(x_t) e_{t+1} \) is the marginal productivity of investment. That is, along the optimal path, an additional unit of wealth allocated to consumption today generates
the same increase in utility as an additional unit of wealth allocated to investment today and consumption tomorrow.

In the certainty equivalent steady-state, the stochastic term is fixed at its mean, and the state, action and shadow price, $x^*$, $s^*$, and $\lambda^*$ do not change over time, ceteris paribus. If the agent’s time preference rate, $\rho$, equals the interest rate $r$, then the steady-state level of investment for the certainty equivalent model is defined by the golden rule that the marginal productivity of capital equals the interest rate plus the depreciation rate:

$$r + 1 - \gamma = f'(x^*)$$

(5.7)

**Growth Model with Credit, No Default**

An economic agent in a credit contract with a lender has, in any period, a given level of wealth and a level of outstanding debt. The long-term credit contract requires borrowers to fully repay their debt in order to have access to further loans. Every new loan is defined as a proportion of the agent’s current wealth. In other words, full repayment of the earlier loan is a necessary condition to obtain an additional loan. In any period, the individual borrower has the option of defaulting, with the cost of sacrificing further access to loans.

The model discussed in this section considers a world where the loan is always repaid. This model provides insight about the behavior of investment when the borrower is in good standing with the lender, and about the effect of the lender’s rationing rule on
the outcome. Additionally, it provides a good guess value to start the approximation of
the growth model with access to credit.

A true growth model of an agent with access to credit must consider an agent
who can chose to remain in the contract or not, i.e., either default or repayment. Access
to credit is the option a borrower has to continue borrowing. This type of choice model
will be considered in the next section.

The following variables and equations describe this preliminary model of an agent
who always has credit:

State variables:

\[ s_t = \text{endowment at the beginning of time } t. \]

\[ l_t = \text{level of initial debt at time } t \text{ (loan received in the previous period)}. \]

Action:

\[ x_t = \text{investment in time } t, \text{ subject to the non-negativity constraint: } x_t \geq 0. \]

State transition:

\[ s_{t+1} = \psi f(x_t)e_{t+1} + \gamma x_t \quad (5.8) \]

When the economic agent has access to credit markets, there are potential
productivity gains captured by the coefficient \( \psi \). If \( \psi = 1 \), there are no
productivity gains from having access to loans. The coefficient \( \psi \) is the finance-
induce productivity gains coefficient.

\[ l_{t+1} = \kappa s_t \quad (5.9) \]
In (5.9) $K_s$ is the loan issued in the current period to be repaid in the following period. Loans grow as a fixed proportion of the agent's endowment. The proportion $\kappa$ is a policy variable defined by the lender. The coefficient $\kappa$ is the lender's rationing rule.

Restriction:

$$c_t = s_t - x_t + \kappa s_t - (1+r)l_i \geq 0$$

(5.10)

In (5.10) $r$ is the interest rate on loans, which is another policy variable defined by the lender.

Borrowers make the investment-consumption decision based on their command over resources, which is composed of their own endowment minus the repayment of outstanding debt, principal and interest, plus the new loan. The budget restriction implicitly assumes away issues about repayment capacity. It is assumed that the stochastic shocks the borrower faces do not preclude the borrower from consuming a positive amount. In other words, the level of endowment is always sufficient to guarantee a positive level of consumption.

The loan received at time $t$, which will be the level of indebtedness in period $t+1$, is a function of the endowment at time $t$. The moment the borrower repays the outstanding loan, the lender appraises the initial endowment and disburses a new loan.

The equation that describes the maximization problem faced by the agent is:

$$V(s_t,l_t) = \max_{s > 0} \{U(c_t) + \delta EV(s_{t+1},l_{t+1})\}$$

(5.11)
The first-order conditions are the shadow price of the endowment and of the initial level of debt, and the Karush-Kuhn-Tucker condition for the optimal action $x$:

\begin{align*}
\lambda(s_t, l_t) &= U'(c_t)(1 + \kappa) + \delta E\{U(s_{t+1}, l_{t+1})\} \kappa \\
\nu(s_t, l_t) &= -U'(c_t)(1 + r) \\
-\mu U'(c_t) + \delta E[\lambda(s_{t+1}, l_{t+1})(\mu \varepsilon f'(x_t) + \gamma)] &= 0
\end{align*}

Equation (5.12) defines the shadow price of the endowment (wealth) as the addition of two components. The first component is the marginal utility of current consumption multiplied by $(1 + \kappa)$. This factor represents the wealth's direct and indirect effect on current utility. Greater wealth not only increases the borrower's consumption by directly increasing purchasing power (direct effect) but also by increasing the size of the potential loan to be received (indirect effect).

The second component is the discounted expected value of greater indebtedness in the future that results from a unit increase in wealth today. An additional unit of wealth in the current period increases indebtedness in the future in the proportion $\kappa$.

Equation (5.13) defines the shadow price of the initial level of debt. Additional units of initial debt reduce the borrower's command over resources in the proportion $(1 + r)$ and, therefore, reduce utility by $U'(c_t)(1 + r)$.

Combining equation (5.12) and (5.13), the shadow price of wealth equals the direct and indirect increase in current utility minus the discounted expected reduction in
utility due to repayment, principal plus interest, of the future debt resulting from an 
additional unit of current wealth:

$$\lambda(s_t, l_t) = U'(c_t)(1 + \kappa) - \delta EU'(c_{t+1})(1 + r)\kappa$$

Equation (5.14) defines the optimal level of investment. Additional units of 
wealth allocated to consumption today must increase utility by the same amount as 
additional discounted units allocated to investment today and consumption tomorrow.

In the certainty equivalent steady-state, the stochastic term is fixed at its mean, 
one, and the optimal values $s^*, l^*, x^*, \lambda^*$, and $\psi^*$ define the steady-state values of the 
states, action and shadow prices. Thus, at the steady state:

$$\frac{1}{\delta[1 + \kappa - \delta(1 + r)\kappa]} = \gamma + \psi f'(x^*)$$

(5.16)

If the agent’s time preference rate equals the interest rate equation, (5.16) 
reduces to the golden rule of equation (5.7), which states that in the certainty equivalent 
steady-state, the marginal productivity of capital equals the interest rate plus the 
depreciation rate.

**Growth Model with Access to Credit, Optional Default**

Under the terms and conditions of the loan contract, a borrower always has the 
option of strategic default. Loans are not guaranteed by physical collateral of equal or 
greater value than the loan. A borrower defaults strategically when, instead of repaying 
the loan, the borrower keeps all of the proceeds from the earlier loan and breaks away
from the relationship with the lender. As there is no physical collateral, the lender cannot be compensated for the lack of repayment and the borrower is able to run away with the funds that should have been used to repay the loan. This action, however, leads to termination of the borrower’s access to additional loans. Her only alternative is, therefore, self-financing.¹

In order to approximate the value function of the choice model, in which the borrower chooses between repayment and default, the following stepwise solution, based on the previous two models, considers an agent who makes investment and repayment-default decisions simultaneously.

First, solve the model without access to credit according to equation (5.3):

\[
V^0(s_i) = \max_{c_i} \{U(c_i) + \delta EV(s_{i+1})\} \tag{5.17}
\]

such that \(c_i = s_i - x_i\)

\(s_{t+1} = \gamma x_t + f(x_t) \varepsilon_{t+1}\)

Second, solve an intermediate model with access to credit, when the borrower has the option to default in the second period, given that she repays in the first period. The solution of the model of growth always with credit provides a good guess value to start the approximation of this model:

\[
V^1(s, l) = \max_x \{U(c_i) + \delta E \max \{V^0(s^0_{t+1}), V^1(s^1_{t+1}, l_{t+1})\}\} \tag{5.18}
\]

¹ If there were multiple lenders in the market, the cost of building up reputation should be factored in when making the decision of strategic default. See Diamond, 1989.
such that:

\[ c_t = s_t - x_t - l_t (1 + r) + \kappa s_t \]

\[ s_{t+1}^0 = \gamma x_t + f(x_t) \varepsilon_{t+1} \]

\[ s_{t+1}^1 = \gamma x_t + \psi f(x_t) \varepsilon_{t+1} \]

\[ l_{t+1} = \kappa s_t \]

Third, at time t, choose between a world without access to credit and a world with access to credit. If the borrower chooses to repay the outstanding liabilities, she faces a world with access to credit from t onwards, in which, in every subsequent period, she also has the option to default. If she chooses to default on her financial obligations, she faces a world of self-financing forever:

\[ V \left( s_t, l_t \right) = \max \left\{ V^0 \left( s_t \right), V^1 \left( s_t, l_t \right) \right\} \]

Equation (5.19) describes the choice an agent faces of whether to remain in a credit contract which offers a loan (equal to \( \kappa s_t \)) in exchange for the repayment of the outstanding liability \( l_t \).

Remaining in the contract also brings the possibility of making the choice between default and repayment in future periods. At each subsequent period, she must decide whether to repay the debt outstanding at the beginning of the period and automatically get another loan, or to default and lose the possibility of getting the new loan but gain, in exchange, the funds that would have been used for repayment.
In the following period, for example, if she decides not to repay the debt carried from period $t$, she gets the benefit of keeping the extra command over resources that would have been used for repayment, $(1+r)t_{t-1}$. Nevertheless, she will lose the loan she may receive at time $t+1$, $\lambda_{t+1}$, and so forth for the rest of future periods. At time $t$, the borrower compares the net gain of defaulting with the net gain of remaining in the contract, given by the discounted value of future indefinite access to credit (i.e., the contract is infinitely lived).

As equation (5.19) shows, the solution to this maximization problem implies maximizing over a continuous variable, the level of investment, $x$, and over a discrete variable, the default choice, implicit in the choice between $V^0$ and $V^d$. Chapter 4 explained how to approximate the solution to a maximization over continuous actions. The combination of continuous and discrete action variables, however, adds non-trivial complications to the solution, as the current numerical maximization sub-routines are designed for either continuous or discrete variables, exclusively, but not for a combination of both.

The stepwise solution of the model presented above overcomes this difficulty by solving the maximization over the continuous action for each possible value of the discrete action, and then choosing the maximum among them. If, in the choice between $V^0$ and $V^d$, the latter is greater than the former, the solution of $^s$'s equation reduces to the intermediate model, where the borrower has the option of repayment or default in future
periods. In the opposite case, the solution reduces to the first model discussed in this chapter for the self-financing case.

As explained in Chapter 4, the functions $V^0$ and $V^d$ are approximated within a pre-defined interval of values for the state variables. When $V^d$ is not greater/smaller than $V^0$ for the whole domain of approximation, the maximization implies a discrete jump from one value function to the other, which may create convergence problems for the numerical approximation of the overall value function. These convergence problems were overcome by changing the basis-nodes scheme from Chebychev polynomials to cubic spline.

**Numerical Solution**

Conventional functional forms and parameters were used here to approximate the models. Several authors have numerically approximated the basic growth model for an agent with no access to credit and, therefore, these previous approximations provide useful parameterizations for present purposes (Miranda, 1996 and Taylor and Uhlig, 1990).

The three value functions approximated in the solution of the growth model with access to credit are parameterized using a constant relative-risk aversion utility function with coefficient of relative risk aversion equal to $\alpha$, namely:

$$U(c) = c^{1-\alpha} / (1-\alpha) \quad 0<\alpha<1$$

(5.20)
This specific functional form of the utility function implies that the Arrow-Pratt measure of absolute risk aversion is:

\[
\frac{U'(c)}{U(c)} = \frac{\alpha}{c}
\]  

(5.21)

This functional form implies that absolute risk aversion decreases with wealth, assuming that consumption is increasing in wealth. That is, the wealthier the borrower, the more willing she is to take gambles expressed in absolute amounts (Varian, 1992). This form of the utility function also presents the feature of constant risk aversion equal to \( \alpha \), which means that the borrower has a constant willingness to take gambles expressed as a proportion of her wealth.

The production function is assumed to exhibit decreasing marginal productivity \( (i.e., \) diminishing returns to scale). The coefficient for the productivity gains from financial intermediation is introduced as a constant positive value \( \psi \) greater than or equal to 1:

\[
f(x) = x^\beta \psi \quad 0<\beta<1, \quad 1 \leq \psi
\]  

(5.22)

The numerical approximation of the growth model for an agent without access to credit does not present major problems. Both Chebychev and cubic spline approximations work well. The certainty equivalent steady-state state values are easily found as functions of the parameters of the model:
The utility and production functions were parameterized with \( \alpha = 0.4, \beta = 0.5, \) and \( \gamma = 0.9, \) and the discount factor was assumed to be 0.9. With this parameterization values, \( s^* = 7.4169 \) and \( x^* = 5.6094 \) were obtained. Figure 5.1 shows graphs for consumption, investment, investment as a proportion of the endowment, and the approximated value function in terms of the state variable endowment, when this model is approximated using a cubic spline basis-nodes scheme.\(^2\) Both consumption and investment are increasing in the endowment, while the proportion of the endowment allocated to investment is clearly decreasing in the endowment. This is a consequence of the diminishing marginal returns to the scale of investment.

The model of growth with no default generates different certainty equivalent steady state values. These values are given by:

\[
x^* = \left( \frac{1}{\delta - \gamma} \right)^{\frac{1}{\beta(1+\lambda)}}
\]

\[
s^* = \gamma x^* + \left( x^* \right)^\beta
\]

\(x^* = \left[ \frac{1}{\delta \left[ 1+\kappa - \delta \kappa (1+r) \right] - \gamma} \right]^{\frac{1}{\beta(1+\lambda)}}\)  \hspace{1cm} (5.25)

\(^2\) There is no debt in this model. For the purposes of comparison, the graph has been drawn as if debt is a constant. The variable of interest shows the same value for all combinations of net debt with a given level of endowment.
\[ s^* = \gamma x^* + (x^*)^\theta \psi \] (5.26)

With parameter values \( \kappa = 0.5 \), \( r = 0.1111 \) (which implies \( \rho = r \)), and \( \psi = 1.3 \), the deterministic steady-state values are \( x^* = 9.4849 \) and \( s^* = 12.5401 \). The steady-state level of investment is higher due to the higher productivity of investment when the agent has credit, which is represented by \( \psi \). In fact, if \( \psi = 1 \) and \( \rho = r \) (\( \delta = 1 + r \)), the steady-state level of investment is the same in both models.

The numerical solution of this intermediate model encountered several complications due to the fact that the second state variable, the level of indebtedness, is time-related to the first state variable, the endowment. In any period, the loan size is proportional to the current endowment and, therefore, the level of indebtedness in the next period is proportional to the endowment of the previous period.

Consequently, the intervals of approximation for each state variable must be carefully chosen to avoid extrapolation of state values outside the range of interpolation. The behavior of polynomial approximations outside the range of interpolation is unpredictable and, in addition, it may lead to a violation of the non-negativity constraints on investment and consumption.

To reduce the likelihood of these occurrences, the state variables are transformed according to the following equations:

\[ p_t = s_t \] (5.27)

\[ q_t = l_t - \kappa s_t \] (5.28)
The new state variables, \( p_t \) and \( q_t \), are just linear transformations of the original state variables. Consequently, the value function can be rewritten in terms of the new variables without major complications, while contracting the space on which the value function is approximated. This transformation facilitates the numerical solution without introducing any major complication to the model.

Since \( k_{st} \) is the level of indebtedness in the next period, \( q_t \) can be rewritten as:

\[
q_t = l_t - l_{t+1}
\]  

(5.29)

That is, \( q_t \) is the negative of the absolute change in the level of indebtedness over time. A value of \( q_t = 0 \) implies that the level of indebtedness remains constant between periods. Positive values of \( q_t \) imply that the level of indebtedness is decreasing, and negative values imply that the level of indebtedness is increasing.

This interpretation requires, however, careful analysis. The actual state variable is the initial level of indebtedness at time \( t \). A negative \( q_t \) means that the new loan for the period is larger than this original level of indebtedness and, for this reason, that debt will grow. A simpler interpretation is to consider \( q_t \) as the initial level of indebtedness net of the new loan (which is fixed for a given \( s_t \)). For simplicity, \( q_t \) will be referred to as net initial debt.

The numerical approximation of the model of choice between repayment and strategic default faces convergence problems, which arise from the fact that the choice between \( \mathcal{V}^0 \) and \( \mathcal{V}^t \) in the intermediate model introduces a kink in the value function to be approximated. As explained in Chapter 4, non-smoothness of the objective function may
generate problems that might be worse with an overall approximation, such as a Chebychev approximation.

As expected, a Chebychev approximation did not converge for several variations of the range of interpolation and values of the parameters. Consequently, a cubic spline basis-nodes scheme was used, and, for consistency, all models used in the stepwise approximation were also approximated with a cubic spline basis-nodes scheme.

Figure 5.2 exhibits consumption, investment, investment as a proportion of the endowment, and the approximated value function for the intermediate model. The investment and consumption functions show an inflexion point at intermediate levels of endowment. This change in behavior is the result of the change of the optimal decision from repayment to default.

At low levels of wealth, while the optimal decision is to repay, investment grows at a decreasing rate (after a short initial increasing rate). After the optimal decision changes from repayment to default, investment grows again at an increasing rate. It eventually reaches a point, however, after which, it grows again at a decreasing rate.

This behavior of the investment function is associated with an opposite behavior of the consumption function. Consumption grows faster with the endowment before the optimal decision changes from repayment to default, and its rate of growth slows down afterwards.

Figure 5.3 offers a graphical explanation of the approximation problem. The function \( V' \) is the approximated value function for the intermediate model and \( V^0 \) is the value function for an agent without access to credit. In other words, \( V' \) is the value
function for an agent who has access to credit and \( V^0 \) is the value function for an agent who does not have access to credit.

At time \( t \), the agent decides between these two worlds as expressed by equation (5.19). Consequently, the value function for an agent who decides, at time \( t \), whether to repay the outstanding liabilities and remain in good standing with the lender or to default and resort to self-financing is the envelope of these two value functions.

For low levels of endowment, the value function for the intermediate model is above the value function for an agent without credit. There is a threshold pair of endowment and net initial debt, from which the value function of the model without access to credit is above the value function of the model with access to credit. The intersection between the two value functions occurs at high values of the endowment for low levels of the net initial debt. As the net initial debt increases, the level of endowment for which the optimal choice changes from repayment to default diminishes.

Figure 5.4 shows the approximated function \( V \) and Figure 5.5 exhibits the associated approximation error. The approximation error is calculated as the difference between values obtained from the approximant and values obtained by directly solving the optimization problem embedded in \( s \)'s equation, for points in the interval of approximation but different from the nodes used in the numerical solution.

As Figure 5.5 shows, this approximation is nearly 0 everywhere and only around the neighborhood where the choice changes from repayment to default this error is somewhat different from zero. Even in this vicinity, however, the error is small, although larger than in the rest of the domain of approximation.
The value function exhibits two inflexion points, which correspond to two changes of its rate of growth with wealth. This is clearly shown in Figure 5.6, which exhibits the shadow price of wealth, \textit{i.e.} the partial derivative of the value function with respect to wealth.

The shadow price of wealth is first decreasing with wealth, then increasing for a short interval of values of the endowment, and finally decreasing again. While the borrower’s optimal choice is to repay, additional units of wealth have a decreasing marginal effect on the value function. When the borrower appropriates the proceeds of the loan and does not repay the outstanding liability, there is a short-lived increase in the marginal effect of wealth but eventually, when the borrower recurs to self-financing, marginal units of wealth have again a decreasing marginal effect on the value function.

The investment function in Figure 5.7 also shows the effect of the change in the optimal choice from repayment to default. At low levels of net initial debt, when default becomes the optimal choice only at high levels of the endowment, the investment function exhibits two stages, similar to the stages explained above for the value function. First, investment increases at a decreasing rate, then it increases at an increasing rate, and finally it increases at a decreasing rate. This is again the result of the change of the optimal choice from repayment to default.

As long as, the optimal choice is to repay, investment grows at a decreasing rate with wealth, due to the diminishing marginal returns exhibited by the production function. When the borrower opts to keep the funds that would have been used for repayment, at the cost of sacrificing future access to credit, there is a short-lived
increasing rate of growth of investment as wealth increases. Eventually, investment goes back to a decreasing rate of growth in wealth, when the borrower resorts to self-financing.

At high levels of net debt, these trends of the investment function are not as pronounced. The initial decreasing rate of growth in wealth and the following increasing rate of growth are, however, quite evident.

Once the borrower defaults, the investment function does not change with the level of net initial debt, as the borrower enters a world without access to credit. This is more clearly shown in Figure 5.8, which shows that, when the borrower opts for self-financing, the investment function converges to the investment function for the model of an agent without access to credit. Before this convergence, at low levels of the endowment, the total level of investment is higher with access to credit than without access to credit.

A lower proportion of total liquidity (command over resources) is allocated to investment in the model without access to credit compared to the model with access to credit (Figures 5.9 and 5.1). This reason, coupled with the fact that there is less liquidity when there is no access to credit, for given levels of endowment, leads to higher levels of investment at low levels of the endowment, as a result of access to credit.

The marginal productivity of investment, however, decreases faster under access to credit than without access to credit and, consequently, for intermediate levels of the endowment, the total level of investment without access to credit is higher than under access to credit (Figure 5.8). In Figure 5.10, the marginal productivity of investment for
the model without access to credit appears flat even though it is always decreasing, because it decreases too slowly as compared to the range of variation of the marginal productivity of investment of the model with access to credit.

For low values of the endowment, when the borrower finds it optimal to remain in the contract by repaying the outstanding liabilities, the level of investment under access to credit is higher than under self-financing. At higher levels of wealth, when the borrower opts for self-financing, the levels of investment are lower than the levels of investment if the borrower had never had access to credit, but both investment functions eventually converge.

Under the restrictive characteristics of the financial contract considered in the model (a fixed rationing rule), it is optimal to remain in good standing at low levels of wealth. Repayment implies higher levels of investment and lower levels of consumption than the levels that would be achieved if the borrower were self-financing. At higher levels of wealth, however, borrowers find it optimal to default and to resort to self-financing, which implies higher levels of consumption and lower levels of investment than otherwise. Eventually, however there is no difference between the two investment functions, as they both converge at high levels of endowment.

When borrowers decide to remain in good standing and, therefore, continued indebtedness is still attractive, higher levels of investment reflect the effect of the gains in productivity from access to financial markets and the need to grow to repay loans in the future while sustaining a desired rate of future consumption.
When borrowers opt to default, instead, more resources can be allocated to consumption today, first, because the productivity gains represented by $\psi$ are lost and second, because there is no need to service debt. This behavior is reflected in Figure 5.9. At low levels of wealth, when it is optimal to repay the loan, borrowers allocate a higher proportion of their total command over resources (liquidity) to investment. At higher levels of wealth, however, when the borrower finds it optimal to default, the proportion of liquidity allocated to investment is considerably reduced and, consequently, the proportion of wealth allocated to consumption increases.

Borrowers derive utility from consumption. In all of the models considered here, the choice of the optimal level of investment implicitly defines the optimal level of consumption. Higher levels of consumption today imply lower levels of investment and, therefore, lower levels of consumption tomorrow. When the borrower faces the choice of repayment or default, repayment brings about a net gain in funds of $\kappa s - l(l+r)$, it induces higher productivity through the production function, provided that $\psi > l$, and it opens the possibility of greater access to credit in future periods. In contrast, default brings about a net gain of $l(l+r)$ in additional funds, the repayment forgone which is appropriated by the borrower, but it also eliminates the productivity gains associated with future access to credit as well as the additional command over resources from future loans.

In this model, wealth plays two roles. On the one hand, it is a source of inter-temporal consumption opportunities. This is the only role it would play if there were no credit constraints, i.e., if the borrower could gain access to any level of credit to
optimize the inter-temporal flow of consumption. The borrower, however, *is credit constrained*; *i.e.*, she cannot obtain all the credit she would want at the going rate of interest.

Given the lender’s supply rule, access to credit depends on wealth. On the other hand, therefore, wealth plays a second role of building creditworthiness, *i.e.*, the credentials needed to gain access to larger loans, given the lender’s credit allocation rule. After some level of endowment, nevertheless, the borrower is no longer credit constrained, and this role disappears.

In this model, credit-constrained agents remain in the contract as long as credit helps to alleviate their shortage of funds. Once a level of endowment high enough is achieved it becomes optimal to default and opt for self-financing. For a given set of parameter values, a *threshold function* can be defined as the locus of pairs \((q, s)\) for which the borrower is indifferent between repaying or defaulting.

The threshold function was approximated using Chebychev approximation. As theory suggests, a given number of Chebychev nodes were chosen over the range of interpolation for the state \(s\). The corresponding values of \(q\) were found by using the condition that \(V^0(s) = V^1(s)\) at the threshold by using Newton’s method. This method makes use of the derivatives of the functions \(V^0(s)\) and \(V^1(s)\), which are readily available from the solution of ‘s equation. Once the pairs \((s, q)\) were found, the threshold function was approximated with a Chebychev polynomial approximant.

Figure 5.11 shows the resulting threshold function. This function divides the space of wealth and net debt into two regions. The region to the left and below the
function corresponds to the pairs of endowment and net initial debt that, under the credit contract, would lead to repayment by the borrower. Points above and to the right of the threshold function define combinations of endowment and net debt that would lead to default. In other words, the threshold function represents the frontier of repayment for a given credit contract keeping all parameters constant. For each level of net initial debt, the function defines the maximum level of endowment that jointly with the level of net debt will generate sufficient incentives to repay.

The threshold function exhibits a negative relationship between the level of endowment and net initial debt. This reflects the fact that starting from a point on the threshold function, for a given level of wealth, a higher level of initial debt net of the new loan makes it optimal for the borrower to default. The initial debt level at which the borrower starts is too high for the size of the loan to be received and, therefore, it is optimal for the borrower not to repay the original high liability at the cost of losing access to the new loan.

From a level of net debt and wealth that makes the borrower indifferent between repayment and default, for the same level of net debt higher levels of wealth induce default, as the borrower is better-off by resorting to self-financing. With higher levels of wealth, the borrower would be willing to remain in the contract only if she had a lower original debt to serve or if she were able to get a larger new loan. Otherwise, she is better-off by keeping the funds that were going to be used for repayment and investing out of her own resources.
The frontier of repayment for a credit contract is expanded outwards with higher values of the coefficient $\nu$, as Figure 5.12 shows. Once on the threshold function, higher levels of endowment are only compatible with lower levels of net initial debt. The set of endowments and levels of net initial debt that lead to repayment is smaller, the lower the finance-induced productivity coefficient $\nu$. This is so because in this case there are fewer gains from indebtedness. In other words, starting from a point on the threshold function, a reduction in $\nu$ makes the original point to be outside the frontier or repayment because, at the starting level of net debt and endowment, there are now smaller gains from access to credit secured by repayment.

The policy parameter $\kappa$ is the main discretionary instrument available to the lender in this type of contract. Higher values of $\kappa$ shrink the frontier of repayment (Figure 5.13). The less the lender is willing to ration loan size per unit of endowment, the smaller the space of pairs of endowment and net debt that bring about repayment. With higher $\kappa$, the borrower accumulates the benefits from debt more rapidly and, therefore, she reaches the point where she is no longer credit-constrained sooner. This result is also compatible with the principle that higher borrower equity contributions create incentives to repay.

Figure 5.14 exhibits the sensitivity of the repayment frontier to another policy variable available to the lender, the interest rate. Higher interest rates shrink the frontier. From a point on the frontier, higher interest rates will lead to default, since the debt load,
interest plus principal, becomes too high relative to the benefits from repayment and continued access to loans.

The repayment frontier does not shift uniformly outwards with respect to the coefficient of risk aversion, $\alpha$ (Figure 5.15). At low levels of the endowment and net debt, instead, when global risk aversion is high for a given $\alpha$, a higher $\alpha$ leads to a contraction of the frontier. This is the case because at low wealth, an increase in $\alpha$ makes an already high risk aversion become even higher and, therefore, the borrower prefers to self-finance and to take advantage of the certain funds that would otherwise be used to repay.

As the level of wealth increases and, therefore, global risk aversion diminishes, the effect of the higher $\alpha$ is offset by the reduction in risk aversion with growing wealth. There is an intermediate point, from which the increase in $\alpha$ is exactly offset by the reduction in global risk aversion resulting from higher levels of wealth. Combinations beyond this intermediate level of wealth are points where the fixed increase in $\alpha$ is more than offset by the reduction in global risk aversion due to the increase in wealth and, therefore, the frontier expands.

Finally, the repayment area of a given credit contract shrinks as the discount factor decreases (Figure 5.16). The discount factor decreases when borrowers assign a greater valuation to present consumption vis à vis future consumption. When a borrower considers the possibility of running away with the funds that may be used to repay the outstanding liabilities, with lower discount factor the borrower would attach a greater
value to the option of defaulting as default brings about a one-time increase in current liquidity at the expense of the future additional funds from credit.

Long-term Analysis

In order to study the long-term behavior of the contract, a representative stochastic path for both states is generated, starting from the minimum value in the chosen range of estimation for each state (Figures 5.17 and 5.18). These graphs show the behavior of the state over a period of 50 years, where each year is characterized by a random occurrence of the stochastic shock in the production function, and the optimal investment in each year is chosen according to the optimal rule obtained by the optimization process in Bellman's equation. These paths, consequently, show a random behavior that oscillates around the steady-state.

The expected paths are drawn by generating 1,000 random repetitions of each stochastic path, and taking the average for each year, over a period of 40 years (Figures 5.19 and 5.20). The large number of repetitions in each year cancels out the noise introduced by the random nature of the process and, thus, the state tends to steady-state. Figures 5.19 and 5.20 show that after ten years, the states tend to converge to a steady value.

The steady-state value of each state is independent of the initial conditions and it represents the value to which the state tends in the long-run. To study the long-run behavior of the process, the process is repeated over a long period of years and the mean and variance of the state are computed. Table 1 shows the long-run mean and variance of
the two states, for a simulation using 40,000 years, for different values of the finance-induced productivity gains parameter $\psi$.

<table>
<thead>
<tr>
<th>Table 1: State’s Long-run Mean and Standard Deviation</th>
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<tbody>
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<td>$\mu_s$</td>
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<tr>
<td>$\sigma_s$</td>
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<td>$\mu_q$</td>
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<td>$\sigma_q$</td>
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Figures 5.21 and 5.22 present the expected path of the endowment for $\psi=1.1$ and $\psi=1.2$, respectively. As it is the case with $\psi=1.3$, the process tends to converge to the expected values in each case. For $\psi=1.1$ and $\psi=1.3$, the process converges to the expected value fairly rapidly, after 5 or 10 years. For $\psi=1.2$, however, the process takes much longer to converge to the expected value. As Figure 5.22 shows, after 100 years, the process has not yet converged.

With $\psi=1.1$, a borrower opts to default after a few years. With $\psi=1.3$, a borrower remains in the contract after 40 years, regardless of the magnitude of the stochastic shock. With $\psi=1.2$, after 100 years, a borrower may opt to default or to keep repaying depending on the magnitude of the stochastic shocks. Both results are possible and, therefore, on average it takes longer to converge to the long-run expected value of the state.
Alternative Formulations of the Model

This section discusses a number of alternative formulations of the model in order to explore the influence of different lender rules on the investment-repayment outcome.

(a) Time lag in loan disbursement

A slight variation in the timing of the model provides some insight about the incentives implicit in this type of loan contract. In the model described earlier, repayment of the outstanding debt load leads to the immediate disbursement of a new loan. To make her consumption-investment-repayment decision, the borrower, consequently, counts on the endowment plus the additional liquidity provided by the loan.

An alternative formulation considers a model in which repayment leads to the disbursement of a new loan only with a time lag. Consequently, the consumption-investment-repayment decision in a given period is taken counting only on the available endowment at the beginning of the period.

The state transition function becomes:

\[ s_{t+1} = f(x_t)e_t + \gamma x_t + \kappa s_t \] (5.30)

Since the endowment at time \( t \) includes loan of from the previous period, the consumption restriction is:

\[ c_t = s_t - x_t - l_t(1+r) \geq 0 \] (5.31)

---

3 Some lenders require a pause between loans to prevent cascading of the loans that hides lack of ability to repay.
When the model is approximated with the same parameter values as in the previous model, the borrower defaults for more values of the endowment than under the previous model. That is, the value function of the model without access to credit is above the value function of the model with credit for more values of the endowment and net initial debt than under the previous model. In this setting, borrowers must repay the previous loan without taking advantage of the additional funds provided by the new loan. The effect of the additional funds when the loan is granted is only to increase the next period's command over resources. Consequently, in the following period, consumption becomes:

\[ c_{t+1} = s_{t+1} - x_{t+1} - \ell_{t+1}(1 + r) = \gamma x_t - x^p \psi + \kappa s_t - x_{t+1} - \kappa s_t(1 + r) \quad (5.32) \]

Equation (5.32) implies that:

\[ c_{t+1} = \gamma x_t + x^p \psi - \kappa s_t r - x_{t+1} \quad (5.33) \]

Under the conditions of this contract, access to credit has a net effect that equals the reduction of future consumption due to repayment versus the gains in productivity represented by \( \psi \). Without these gains in productivity, borrowers would have no incentive to remain in the contract. Default would be the choice for any positive level of net debt.

For example, with finance-induced productivity gains and lender's rationing rule coefficients as high as \( \psi = 1.4 \) and \( \kappa = 0.5 \), borrowers will repay at low levels of their endowment, when the gains in productivity outweigh the reduction in future consumption. With a finance-induced productivity gains coefficient as low as \( \psi = 1.2 \), the
optimal choice for the borrower is already to default for all values of the endowment within the domain of approximation. When $\psi$ is as low as 1.2, an area of viable repayment is generated at low levels of the endowment and net debt when the lender has a restrictive rationing rule, such as $\kappa=0.3$ or $\kappa=0.2$.

This result highlights the fact that microfinance contracts that delay disbursement force the borrower to make investment decisions without the additional funds provided by a loan. Under the framework developed in this dissertation, if the borrower can count on these additional funds to make her consumption-investment decision, microfinance contracts of the type described here are incentive compatible for a large range of values of the parameters. Otherwise, when disbursement is delayed, it would be optimal for the borrower to default from the first period onwards, unless additional access to credit also brings about large gains in productivity.\footnote{This may also be the case if the loan allows the purchase of indivisible capital goods that the agent would not be able to self-finance, thereby increasing her productive capacity. Most microfinance organizations, however, offer only working capital loans to exploit a given productive capacity.}

Figure 5.23 shows how increases in the finance-induce productivity gains coefficient ($\psi$) expand the repayment frontier, and it also shows how, for a considerably high $\psi$ coefficient (1.25), the repayment area is still quite small. Figure 5.24 shows that in the same fashion as in the model of the previous section, less constraining lender's rationing rules ($\kappa$) lead to a considerable reduction of the repayment area for this lending technology.
(b) Variation in the lender’s rationing rule for loan size

In the first model discussed in this dissertation, the lender’s rationing rule constrains loan size as a fixed proportion of the borrower’s endowment at the beginning of the period. This feature reflects the practice observed in many microfinance organizations of globally evaluating the household prior to choosing the size of a new loan (Navajas and Gonzalez-Vega, 1999). Other organizations, however, such as village banking programs, enforce a pre-defined rule for the growth of loan size. Usually, they start from very small loans, independent of the wealth of the borrower. As the borrower demonstrates her willingness to repay over several loan cycles, loan size grows at a pre-defined pace, independently of changes in the level of wealth of the borrower.

The following redefinition of the original model captures this feature of village banking contracts:

\[ c_t = s_t - l_t (1 + r) - x_t + \kappa l_t \]  \hspace{1cm} (5.34)

\[ l_{t+1} = \kappa l_t \]  \hspace{1cm} (5.35)

Equation (5.34) represents a departure from the base model. The lender’s rationing rule defines the size of the new loan independently of the level of the endowment. Loan size grows according to a pre-defined rule, implicit in the lender’s rationing coefficient \( \kappa \), as long as the borrower repays her outstanding obligations. Under normal conditions, \( \kappa > \rho \), so that good standing generates an increasing flow of funds. If the borrower repays her previous loan, the lender commits to disbursement of a new loan of size \( \kappa \)-times the earlier loan.
When the borrower repays, the first-order conditions of the model are:

\[ \lambda(s, l) = U'(c) \]  
(5.36)

\[ v(s, l) = U'(c)[\kappa - (1 + r)] + \delta E\nu(s_{t+1}, l_{t+1})\kappa \]  
(5.37)

\[ -U'(c) + \delta E\lambda(s_{t+1}, l_{t+1})(\gamma + \beta x_{t}^{\alpha-1}) = 0 \]  
(5.38)

Equation (5.36) is the same as equation (5.4), which describes the first-order conditions for the growth model for an agent without access to credit. Equation (5.37) is the condition that defines the new shadow price of the level of indebtedness. Additional units of initial debt generate an outflow in the amount \((1+r)\) as well as an inflow in the amount \(\kappa\). If \(\kappa > (1+r)\), additional units of indebtedness would have a positive effect on utility and, therefore, loan sizes would grow indefinitely. As a consequence, the condition \(\kappa < (1+r)\) is a requirement for the convergence of the model.

Equation (5.38) is equivalent to equation (5.5). It defines the optimal level of investment by the condition that the marginal utility of consumption today must equal the discounted expected marginal productivity of investment valued by the shadow price of wealth.

Figure 5.25 exhibits the value function of the model of an agent with access to credit approximated with a cubic spline basis-nodes scheme. All the parameters assume the same values as in the base case of the model with the exception of \(\kappa=1.005\) and \(\psi=1.1\). The fitted value function exhibits a curvature around the middle values of debt where the optimal choice changes from repayment to default. The two state variables are
the endowment and the absolute level of initial debt. In this model it is not necessary to transform the latter state because there is no inter-temporal relationship between states.

Figure 5.26 is equivalent to Figure 5.3, in the sense that it provides graphical intuition about the approximation problem for an agent who chooses between a world of self-financing and a world with access to credit under the conditions established in the contract offered by the lender. For low levels of initial debt, the value function for an agent without access to credit is below the value function for an agent with access to credit, but there is a level of debt above which the borrower is better-off without credit. Consequently, the borrower defaults beyond that point.

The threshold level of initial debt at which the optimal action of the borrower changes from repayment to default is increasing in the level of wealth. The key variable to define the threshold between repayment and default is the level of debt. In contrast to the model when loans grow proportionately to the level of wealth, in this model there is a middle level of indebtedness above which it is optimal for the borrower to default. Since loan sizes are defined independently of the level of wealth, a loan size is eventually reached above which, regardless of the level of wealth, the characteristics of the lending contract would lead to default.

Figure 5.27 shows the direct relationship between the level of endowment and the level of initial debt along the threshold. The threshold function divides the space of initial debt and endowment combinations into regions of repayment and default. Points below the threshold function represent combinations of endowment and initial debt that would lead to repayment. Points above the function represent combinations that would
lead to default. Starting from a point on the threshold function, higher levels of indebtedness, for a given endowment, would lead to default as the borrower will be better-off not repaying the loan.

Figure 5.28 exhibits the investment function for this model. When loans grow independently of the level of wealth, borrowers invest only to increase future consumption. In this model, wealth does not play the role of provider of creditworthiness and, therefore, there is no motivation to invest in order to increase future access to credit.

The behavior of investment as a proportion of total liquidity (command over resources) is similar to the case when loan size grows proportionately to wealth (Figure 5.29). At intermediate levels of initial debt, the propensity to invest out of total liquidity shows, however, the effect of the shift of the optimal decision from repayment to default. At low levels of wealth and of initial debt, the propensity to invest is low. As the levels of initial debt grow, the propensity grows until it reaches the point where it becomes optimal to default. Afterwards, the propensity to invest diminishes to a value that does not change with the level of initial debt, as the borrower opts for self-financing. For higher values of the endowment, investment as a proportion of total liquidity shows the same behavior, but the decision to default is reached at higher levels of initial debt, as a reflection of the positive relationship between endowment and initial debt along the threshold.

Figures 5.30 and 5.31 exhibit the representative stochastic paths for the endowment and the initial level of debt, respectively. The path for the endowment shows
the effect of the stochastic returns of the productive activity. The path for the initial debt, however, converges to 0 and it does not show any fluctuations. This is the case because, for the base-case parameter values, in the long-run the optimal choice for the borrower is to default and, therefore, the steady-state value of the level of debt is 0. Additionally, there is no fluctuation in the trend towards 0, because the stochastic nature of the production function does not affect the behavior of the level of debt, as the lender defines loan sizes independently of the returns of the productive activity.

(c) Model of Optimal Borrowing

In the earlier models, the borrower receives a loan size defined by a rule established by the lender. The total amount of the loan must be repaid in order to keep a good standing with the lender. Borrowers decide over consumption-investment and over repayment-default. These models can be compared to a growth model with debt in which the borrower may carry debt across periods at an increasing cost.

The borrower is, therefore, entitled to decide on the optimal level of borrowing in each period, along with the consumption-investment decision. Instead of a repayment decision, agents make a borrowing decision, which could be negative borrowing, in the form of repayment of previous loans or savings. This variation is an extension of a model developed by Miranda and Fackler (1996).

The model has two state variables: the endowment, \( s \), and the level of indebtedness, \( d \). Borrowers decide over two actions: investment, \( x \), and the amount to
borrow, $b_t$. Consumption equals total liquidity, endowment plus borrowing minus investment and the cost of carrying debt:

$$c_t = s_t + b_t - x_t - \left( \eta_0 + \eta_1 \frac{d_t}{s_t} \right) d_t$$  \hspace{1cm} (5.39)

The cost of the outstanding debt has a fixed component, $\eta_0$, and a leverage component that increases in the proportion of debt to the endowment, $\eta_1 (d_t/s_t)$. With costless indebtedness, the borrower would have incentives to borrow indefinitely but, given its growing cost, borrowers must balance the additional cost of borrowing with the additional liquidity obtained with a loan.

The deterministic model was approximated with the following values of the parameters: $\alpha=0.2$, $\beta=0.5$, $\gamma=0.9$, $\delta=0.95$, $r=0.05$, $\eta_0=0.04$ and $\eta_1=0.025$. At low levels of endowment and indebtedness, figure 5.32 shows that, for similar levels of endowment, investment tends to reach higher levels than in previous models, at low level of endowment and indebtedness. Investment is increasing in the level of endowment but decreasing in the level of indebtedness. Since the borrower can carry debt over time, the level of investment may fall drastically with high indebtedness relative to the endowment.

For low levels of endowment, the second action, the level of borrowing, is increasing in both the level of endowment and the level of indebtedness, for low levels of endowment (Figure 5.33). For high levels of endowment, however, optimal borrowing levels off and it does not grow with the level of indebtedness. This suggests that at such high levels of wealth it is not optimal to acquire further liabilities.
Consumption presents a behavior similar to investment (Figure 5.34). It is increasing in the level of endowment but decreasing in the level of indebtedness. For given levels of indebtedness, higher levels of wealth induce greater borrowing and investment and a less pronounced increase in consumption. The increase in borrowing, however, is less pronounced for high levels of indebtedness. This behavior results from the fact that, in this model, the borrower’s credit constraint is lifted, as she can choose her optimal amount to borrow.

Summary of results

This chapter discusses a model of a credit contract between an individual borrower and a lender. It emphasizes the feature that the lender’s main mechanism to induce repayment is the threat of loss of access to future loans, as no collateral is pledged on loans.

In the first model (base) considered in the chapter, the borrower starts each period with an endowment and a level of indebtedness, which must be repaid in order to obtain a new loan. The size of the new loan depends on the level of endowment of the borrower. Default increases the current availability of funds, since the outstanding debt is not repaid. Default, however, also eliminates the possibility of obtaining additional loans in the future. This is particularly costly because it implies the sacrifice of the gains in productivity that can be enjoyed only with access to financial markets.
Borrowers choose to repay or to default considering the cost and benefits today and in the future. For relevant values of the coefficient of productivity gains from access to credit markets, it is optimal for the borrower to eventually default.

The threshold at which the optimal decision changes from repayment to default shows a negative relationship between the endowment and the original level of debt net of the new loan. This reflects the fact that, under the characteristics of the contract, it is almost inevitable that the borrower will eventually be better-off with self-financing, as his levels of wealth get sufficiently high, in the presence of diminishing returns to scale. The investment and consumption functions are increasing in the endowment and the net initial debt.

The investment function shows high rates of growth at low levels of endowment (when it is optimal to repay) and low rates of growth at high levels of endowment (when it is optimal to default. The consumption function shows the opposite behavior. This is the result of gains in productivity when the borrower has access to credit and of the double role of wealth as a direct determinant of inter-temporal consumption and as a potential provider of larger new loans in the future.

An alternative model, in which loan sizes are defined independently of the level of endowment of the borrower, results in a positive relationship between endowment and the level of initial debt along the threshold that defines the combinations of endowment and debt that just maintain the incentives to repay. In this case, however, starting from the threshold, higher levels of wealth, for a given level of indebtedness, do not lead to default. Larger wealth can be used to repay the current loan, and consecutive loans will
not grow with the higher wealth. The borrower can then allocate the increase in wealth according to its optimal use for inter-temporal consumption. Wealth does not play the role of provider of creditworthiness.

As wealth is not a determinant of loan sizes, the threshold function is less sensitive to changes in the endowment than in the model of wealth-dependent loan size. Redefining the threshold function for the latter model in terms of the absolute initial level of debt, Figure 5.35 exhibits the threshold functions for both models. It must be highlighted, however, that in this new space set, the area to the left of the threshold function for the model of loan size dependent on wealth is the repayment area associated with this lending technology and that the area to the right is the area of default. In contrast, for the model with loan size independent of wealth, the area to the right of the threshold function is the repayment area associated with this lending technology.

Depending on the value of the lender’s rationing rule parameter \( \kappa \), which is the only parameter that assumes different values in each model, the two threshold functions may intersect. In Figure 5.35, \( \kappa=0.5 \) for the wealth-dependent loan size model and \( \kappa=1.01 \) for the wealth-independent loan size model. The total space of debt and endowments is almost completely divided into feasible repayment areas depending on the combination of initial debt and endowment.

If the initial level of debt is too high compared to the level of endowment, (a), a wealth-dependent loan size technology may be conducive to repayment, whereas a technology with loan size defined independently of wealth may hurt repayment. For intermediate levels of debt, (b), both technologies are equally effective. For low levels of
initial debt, (c), a wealth-independent rationing rule is more likely to lead to repayment, while a wealth-related rule induces default.

Actual technologies that define loan sizes independently of the level of wealth usually start with very small loan sizes. If that is the case, Figure 5.35 suggests that a wealth-independent rationing rule is a better choice for the lender but that there may be a point with both wealth and initial debt high enough after which a technology that defines loan size as a function of the level of wealth may lead to better repayment rates.

If the wealth-dependent loan size lending technology adopts a tight rationing rule (\( \kappa=0.4 \), for example), the level at which the shift in lending technologies favors convenient for the lender (the intersection of the curves) will occur at high levels of endowment and debt (the lower \( \kappa \) shifts the threshold function of the wealth-dependent loan size to the right). On the contrary, if the wealth-dependent rationing rule is somewhat lenient (\( \kappa=0.6 \)), the shift from the wealth-independent to the wealth-dependent loan size lending technology will occur sooner.

Finally, if the borrower can freely choose the amount to borrow in each period along with her level of investment and if she is able to carry debt across time, higher levels of investment will be achieved, as credit constraints will never be binding. The credit constraint emerges with a rationing rule that grants a loan size below her optimum level of borrowing. The more strict the rationing rule, the more credit constrained will be the agent. This reduces investment, but it increases repayment (it discourages strategic default). A trade-off between outreach (less credit constraints) and sustainability (less default) emerges.
Figure 5.1: Consumption, investment, investment as a proportion of the endowment, and value function for an agent without access to credit (self-financing).
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Figure 5.28: Investment function for model with access to credit, with rationing rule independent of wealth, and strategic default.
Figure 5.29: Investment as proportion of total liquidity for model with access to credit, with rationing rule independent of wealth, and strategic default.
Figure 5.30: Representative stochastic path of the endowment for growth model with access to credit, with rationing rule independent of wealth, and strategic default.
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CHAPTER 6
Group Credit Contracts

The use of groups of borrowers as a lending mechanism to address the incentive, information and enforcement problems faced by the lender and to reduce the costs of overcoming these problems is a common practice in microfinance. The group dimension adds some complications to the analysis of microfinance contracts. In addition to implementing several principles also found in individual contracts, the design of group contracts must consider any differences among group members, the process of group formation, the possibility of collusion, the possibility of covariant risk, and the degree of contact between individual group members and the lending organization.

A typical group credit contract is characterized by joint liability. Lenders rely on the knowledge members have about each other to delegate screening and monitoring and to substitute group pressure for physical collateral (Conning, 1996; Diagne, 1998). The group as a whole is responsible for the repayment of the loan. The distribution of the loan funds among group members and the collection of individual payments are intra-group decisions. Lenders count on the threat of termination of the lender’s relationship with all members of the group, irrespective of individual repayment records, as the main incentive to promote repayment.
Under these conditions, the decisions of any member are dependent on the decisions of her fellow group members. An individual decision of remaining in good standing with the lender implies repayment of her own share of the loan if the other group members repay their share. If all or some of the other members do no repay, the repaying members must repay the total of the outstanding loan. The benefits of maintaining a good standing with the lender are given by the automatic disbursement of a new loan, the value of future access to credit (i.e. the choice between repayment and default in future periods remains open), and the gains in productivity from access to financial markets. The cost of maintaining a good standing is the opportunity cost of the funds used for repayment of own or any other defaulting member’s outstanding debt.

This chapter develops a model to consider some of the additional dimensions that group lending introduces in a contract, as an extension of the model developed in Chapter 5. Two borrowers form a group. A group of two borrowers is considered here to reduce the dimensionality of the approximation. Each borrower has an endowment and makes decisions between consumption or investment and between repayment or default.

The group, however, receives a loan as a unit and, therefore, it is responsible for its repayment as a whole. Group members divide the loan among themselves according to some distribution rule, but from the perspective of the lender only one loan has been disbursed, which must be repaid in full. No partial payments from individual members are accepted. Failure to repay the total loan leads to the exclusion of both borrowers from future access to credit.
The lender promises that, by keeping a good repayment record, the borrowers will continue to receive loans. In the first version of the model, these loans are defined as a proportion of the combined aggregate endowment of the members of the group. By assumption, lenders know this aggregate endowment before disbursement of a loan, but they do not know the size of the endowments of individual members. When a loan is repaid, lenders do not need to know whether the funds come from both group members or from only one of them. This feature of the model is the source of some of the claimed cost savings for the lender from using this lending technology (Adams and Ladman, 1979; Braverman and Guasch, 1993; Huppi and Feder, 1990; Stiglitz, 1990).

If a group member decides to default and the other member repays the loan in full, the repaying member excludes the defaulting member from the group. Borrowers who repay may continue their relationship with the lender on an individual basis. The identity of the defaulting member is reported to the lender, so that the defaulter is permanently excluded from future access to credit. Since, by assumption, there is only one lender in the market, a default decision leads to self-financing. In order to focus on the problem of strategic default, it is assumed that there is always sufficient repayment capacity, even though the individual production functions are subject to stochastic shocks.

The Group Model

The group credit model is a natural extension of the growth model for an individual agent with access to credit discussed in the previous chapter. As in the
individual model, each borrower has always the option of defaulting and resorting to self-financing. 

The group is conformed of two members \( i=1,2 \). At time \( t \), each member counts with an endowment, \( s_{it} \). In each period, the group faces an initial level of liabilities, which is the result of previous loans.

**State variables:**
- \( s_{it} \) = endowment of member \( i \) at time \( t \).
- \( l_t \) = total level of indebtedness of the group at time \( t \).

Group members divide the loan they receive according to a distribution rule, upon which they have previously agreed. The parameter \( 0 \leq \theta \leq 1 \) represents the proportion of the group loan received by borrower *one* and \((1-\theta)\) is the proportion received by borrower *two*. The parameter \( \theta \) also defines the distribution of the group liabilities. Consequently, \( \theta l_t(1+r) \) is the share of the expected repayment from borrower *one* and \((1-\theta)l_t(1+r)\) is the share of the expected repayment from borrower *two*.

**Action variables:**
- \( x_{it} \) = level of investment of member \( i \) at time \( t \).

\[
\begin{align*}
\delta_t &= \begin{cases} 
0 & \text{if repayment} \\
1 & \text{if default} 
\end{cases} 
\end{align*}
\]

(1.1)

**State Transition:**

\[
\begin{align*}
\kappa \sum_i s_{it} & \text{ if } d_{it} = 0, \ i=1,2 \\
\kappa s_{it} & \text{ if } d_{it} = 0, \ d_{i,t-1} = 1
\end{align*}
\]

(6.2)

That is, investment is transformed into the next period’s endowment through the production function, which includes the coefficient \( \psi_t \) to represent the finance-induced
intangible productivity gains from having access to financial markets. The production function and the coefficient of recuperation of capital \( \gamma \) are borrower-specific. As in the previous chapter, \( \epsilon \) represents the idiosyncratic risk of each productive activity, which is distributed log-normally with mean 1 and standard deviation \( \sigma \).

\[
\text{Reward: } \quad U_i(c) = \text{individual utility function}
\]

Both investment and consumption must be non-negative. The consumption function of each borrower is dependent on the default-repayment action of the other borrower, according to the following table:

<table>
<thead>
<tr>
<th>( d_2 = 0 )</th>
<th>( d_2 = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_1 = 0 )</td>
<td>( c_1 = s_1 - x_1 - l(1 + r) \theta + \kappa (s_1 + s_2) \theta )</td>
</tr>
<tr>
<td>( c_2 = s_2 - x_2 - l(1 + r) )</td>
<td>( c_2 = s_2 - x_2 )</td>
</tr>
<tr>
<td>( (1 - \theta) + \kappa (s_1 + s_2)(1 - \theta) )</td>
<td></td>
</tr>
<tr>
<td>( d_1 = 1 )</td>
<td>( c_1 = s_1 - x_1 )</td>
</tr>
<tr>
<td>( c_2 = s_2 - x_2 - l(1 + r) + \kappa s_2 )</td>
<td>( c_2 = s_2 - x_2 )</td>
</tr>
</tbody>
</table>

When both borrowers decide to repay, each one repays her share of the loan, that is, \( \theta \) by borrower one and \( (1 - \theta) \) by borrower two. Also, each one receives her share of the new loan, defined as a proportion \( \kappa \) of the aggregate endowment \( s_1 + s_2 \). When only one borrower repays, the defaulting borrower resorts to self-financing and the repaying borrower must repay the total group loan \( l \) in order to receive a new loan. The new loan.

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is defined as a proportion of her individual endowment only, as her partner drops out of the group by not repaying. Finally, when both borrowers default, they both opt for self-financing.

Table 2 defines the repayment game as a *non-cooperative game* in *strategic form* (Krepps, 1990; Osborne and Rubinstein, 1994). The game is composed of two players, each player has two strategies, and, for each strategy, each player receives a payoff. The choice of strategies is made simultaneously and independently. The payoffs for each borrower, however, differ by more than just the differences in consumption, as shown in Table 2. As long as both borrowers repay, the game is repeated. The future value of access to credit must then be added to the payoffs of each strategy. By remaining in good standing, the borrower also enjoys productivity gains in the production function as a result of her participation in financial markets.

The game is solved for a Nash equilibrium. In other words, an equilibrium defines a set of strategies or actions for each borrower such that no borrower has an incentive to deviate from her part of the strategy. In the dynamic Nash game equilibrium, each borrower maximizes her own stream of current and expected future rewards.

The concept of Nash equilibrium provides a tool to find the solution of the game by successive iterations. In the first iteration, each borrower finds the best response action for given actions of the other borrower. Afterwards, the assumed actions of the other borrower are updated according to the solution of the partner borrower. The solution is found when there is no change in best responses between iterations for either
borrower or when the change is small enough according to some pre-defined level of tolerance.

In a static framework, with typical payoff values, the repayment game is a typical prisoner's dilemma. Both players wish to cooperate (repay), but since the decisions are taken simultaneously and non-cooperatively, the threat of possibly having to pay the full group loan leads them to default. Repetition of the game and the gains from access to credit introduce variations that may generate Nash equilibrium strategies different from default by both borrowers.

To solve for the Nash equilibrium of the dynamic game, a stepwise method of solution is used, following the same method of solution explained in Chapter 5. At time t, each borrower chooses between a world with access to credit and a world without it. For each borrower, the world with access to credit is described by a Bellman's equation in which the borrower maximizes the discounted present value of current and future rewards, given the actions of the other borrower.

For example, assigning subscripts to the value function to indicate the borrower and superscripts to indicate the world in which the maximization takes place, borrower's one choice is:

\[ V_1(s_{1t}, s_{2t}, I_t) = \max_{(s_{1t}, s_{2t})} \{ V^0_1(s_{1t}), V^1_1(s_{1t}, s_{2t}, I_t) \} \]  \hspace{1cm} (6.3)

where:

\[ V^0_1(s_{1t}) = \max_{c_{1t}} \left\{ U_1(c_{1t}) + \delta V^0_1(s_{1t+1}) \right\} \]  \hspace{1cm} (6.4)

\[ c_{1t} = s_{1t} - x_{1t} \]  \hspace{1cm} (6.5)
\[ s_{1,t+1} = \gamma_1 x_{1t} + x_{1t}^A \]  
\[ V_1^1 (s_{1t}, l_t) = \max_{s_{1t}, s_{2t}} \{ U_1 (c_{1t}) + \delta \max V_1^0 (s_{1,t+1}, V_1^1 (s_{1,t+1}, l_{t+1})) \} \]  
\[ c_{1t} = s_{1t} - x_{1t} - l_t (1+r) \theta + \kappa \theta (s_{1t} + s_{2t}) \]  
\[ s_{1,t+1} = \gamma_1 x_{1t} + x_{1t}^A \psi_1 \]  
\[ s_{2,t+1} = \gamma_2 x_{2t}^* + (x_{2t}^*)^A \psi_2 \]  

Borrower two faces a symmetric set of equations that describe her maximization problem, for given actions of borrower one.

**Numerical Solution**

To numerically approximate the Nash equilibrium solution of the game, a stepwise procedure was followed, based on the general principles for the solution of a dynamic game explained in Chapter 4, and on the solution of the individual choice model of Chapter 5 between a world with access to credit and a world without it.

First, for each borrower, solve the maximization of equation (6.4), which describes the world without access to credit. Second, for each borrower, solve the maximization problem described in equation (6.7), assuming the other borrower's decisions as given. Third, each borrower updates the assumed-as-given actions of the partner borrower according to the solution of her own maximization, and the process of simultaneous approximation to the solution of equation (6.7) for both borrowers continues iterating until there is no significant change in the optimal actions. This iterative solution approximates the value function for both borrowers when they
participate in a world with access to group credit. Fourth, each borrower chooses between the world with access to credit and the world without it, as described in equation (6.3).

The several value functions used in the stepwise procedure were approximated by collocation methods using a cubic spline basis-nodes scheme, as explained in Chapter 4. The basic program that finds the Nash equilibrium solution of the dynamic game is presented in Appendix C.

The iterative solution of (6.7) requires the approximation of several intermediate models. For example, the value function that describes a world with access to credit when the other borrower defaults is approximated for both borrowers. The value function that describes a world always with credit is also approximated for each borrower. In the same fashion of the individual model of chapter 5, this temporary value function provides good starting values for the value function that describes the world with access to credit when both borrowers repay.

Each borrower chooses at time $t$ between a world with access to credit and a world without access to credit. Each borrower considers the costs and benefits of repayment and default. At time $t$, a decision of default brings about a net gain of funds, the forgone repayment of her part of the group liability minus the current and discounted expected future value of access to credit.

A decision of repayment brings about a gain of a share of the group loan, if the other borrower also repays, or an individual loan proportional to her own endowment, if the other borrower defaults, plus the possibility of having access to credit in future
periods (when the repaying borrower will also have the choice of repayment or default). The cost of the decision of repayment in time \( t \) is the repayment of her share of the outstanding liabilities, if the other borrower repays, or of the total group liabilities, if the other borrower defaults.

As in the case of the individual loans model of Chapter 5, the third state-variable, the group’s initial debt, is time-related to the other two state-variables. Consequently, they vary within certain correlated intervals. To avoid inconsistent values of the third state-variable that may lead to convergence problems and violations of the non-negativity constraints, the following transformation was used, which is equivalent to the transformation used in Chapter 5 for the individual loan model.

\[
P_t = s_t \\
q_t = l_t - \kappa(s_t + s_{2t})
\]  

(6.11)

The new variable \( q_t \) can be interpreted as the initial group’s debt level net of the new loan or initial net group debt.

All the parameters were given the same values as in the base case of the individual model approximated in Chapter 5. Both borrowers are assumed identical (all their parameter values are the same) other than in the possible differences in their endowments, due to the fact that the approximation of the value function takes place over an interval of values of the endowment for each borrower. The new parameter \( \theta \), the rule of intra-group distribution of the loan proceeds and liabilities, is assumed to be 0.5; i.e. group members share equally in the group loan and liabilities regardless of differences in their endowments.
Given that the value function for the group model is defined over a three-state space, it is impossible to graph the overall value function. Consequently, the results are presented in a series of three-dimensional figures. Each figure in the series is defined by fixing the value of the third state. This facilitates understanding of the results.

Figure 6.1 exhibits the approximation error for the value function of borrower one, under access to credit in the two-dimensional endowment space, when the level of net initial group debt is fixed at its lowest node level. As expected, the error oscillates around 0 and it tends to infinity in the limits of the domain of interpolation. For higher levels of initial net group debt, the approximation error shows the same behavior.

Figure 6.2 exhibits the first fundamental result of the model: the repayment surfaces for both borrowers on the endowment state-space. Figure 6.2 presents four different graphs. Each graph contains the optimal repayment decision surfaces of each borrower, for a fixed level of initial net group debt. Starting from the top and from left to right, each graph represents the repayment surface for both borrowers for sequentially higher levels of initial net group debt. The darker surface represents the repayment surface for borrower one and the lighter surface the repayment surface for borrower two.

Each repayment surface draws the optimal repayment decision on the endowment space. A value of 1 on the vertical axis implies repayment and a value of 0 implies default. The top-left figure is a flat surface at 1 because, at the lowest level of initial net group debt, the optimal decision is for both borrowers to repay at all levels of their respective endowments.
The top-right surface exhibits the repayment surfaces for both borrowers with a higher initial net group debt. Each repayment surface assumes an L-shaped form (the axis have been changed in order to facilitate the view of the two different repayment surfaces). This graph shows that, for low levels of the partner’s endowment, the optimal choice is to repay. When the endowment of both borrowers is high, both borrowers default.

At low levels of the own endowment, the repayment surface of each borrower is further out than at higher levels of the own endowment. That is, up to an intermediate level of the own endowment, the optimal choice is to repay regardless of the endowment of the partner borrower but, as the own endowment increases, the optimal choice is to default if the endowment of the partner is also high.

As in the individual loan model, at low levels of endowment, each borrower has a high valuation of the benefits of having access to credit because it alleviates her binding credit constraint and, therefore, each borrower repays regardless of the level of the endowment of the other borrower. The marginal valuation of additional funds is quite high.

For high values of her endowment, in contrast, the credit constraint is relaxed and, therefore, a borrower would repay only if her partner’s endowment is small compared to her own endowment. Otherwise, the borrower is better-off by defaulting and resorting to self-finance.

The intersection of both repayment surfaces represents situations where each borrower repays her share of the loan and continues to borrow through the group. In
other words, the intersection of the surfaces is the area of group stability. As the graph shows, however, at low levels of the partner’s endowment, the repayment surface of each borrower is further out than the repayment surface of her partner.

The two sections in each surface further out represent situations where a member of the group repays the total group loan in order to maintain her good standing with the lender. The repaying member is motivated by the fact that with her relatively large endowment, the repayment of the total group outstanding liability does not represent a great burden as compared to the benefits of maintaining a good standing with the lender.

The two bottom graphs show that, for higher levels of initial net group debt, there are no instances of one borrower repaying the total group liability in order to remain in good standing, as the repayment surfaces of both borrowers are identical. At these higher levels of initial net group debt, each borrower repays at all levels of her endowment, as long as the endowment of the other borrower is small. The benefits of preserving her access to credit, based on her own endowment, outweigh the cost of having to repay her share of the group liability. For higher levels of the partner’s endowment, however, each borrower is willing to repay only at very low levels on her own endowment, when the credit constraint is strong due to the small size of her own endowment.

Figure 6.3 exhibits the repayment surfaces for both borrowers once each borrower has chosen the world with access to credit over the world without access to credit. Consequently, comparing Figures 6.3 and 6.2, the reduction in repayment
surfaces can be attributed to the strategic decision of choosing the world with access to credit with the objective of defaulting after the loan is disbursed. At the lowest level of initial net group debt, for example, when repayment is always the optimal decision according to Figure 6.2, both borrowers default after the disbursement of the new loan, if their endowments are sufficiently high.

Figure 6.4 exhibits the investment function for borrower one at four different levels of initial net group debt and Figure 6.5 exhibits the top-left surface of Figure 6.4 in a larger scale to better appreciate its behavior. As expected, the investment function is increasing in the own level of endowment. At low levels of the endowment of borrower two, the investment function of borrower one shows the effect of the optimal choice of repayment after the disbursement of the new loan (Figure 6.4) up until high levels of her own wealth. At the highest levels of her wealth, the optimal decision changes to default and the investment function tends to the self-financing investment function.

In Figure 6.4 there are no major changes across the four investment functions, which reflects that the optimal investment function is not greatly affected by changes in the initial net group debt and shows the consequences of these changes on the optimal choice between access to credit and self-financing.

Figure 6.6 exhibits the repayment surfaces of borrower one under group and individual lending on the space of the own endowment and the initial net group debt. The first five figures represent the repayment surfaces for different levels of the endowment of borrower two. The sixth surface (bottom-right) presents both group and
individual repayment surfaces on the two endowments space, at the highest node-level of initial net group debt.

The top-left graph exhibits the repayment surfaces for borrower one, given the lowest level of endowment for borrower two. The darkened surface represents the optimal repayment-default decision under an individual contract and the light surface represents the repayment decision under a group contract. The top-right surface is the repayment decision for a higher level of endowment of borrower two. These two surfaces show that the repayment surface of borrower one is larger under group than under individual credit. The gain in repayment surface occurs at middle and high values of the endowment of borrower one because, under individual lending, the borrower defaults, but under group lending, it is optimal to repay.

The other three repayment surfaces (higher levels of endowment of borrower two) also show an improvement in repayment with group lending, but only at the lower levels of initial net group debt.

The bottom-right surface shows the same result in a two-endowment space when the initial net group debt is fixed at its highest node-level. The repayment surface of group credit is the whole L-shaped surface whereas the repayment surface of individual loans is the dark surface only. At low levels of the endowment of borrower one, the individual and group lending technologies are conducive to repayment regardless of the size of the endowment of borrower two. For higher levels of the own endowment, borrower one defaults under individual lending but under group lending she would be willing to repay as long as the endowment of borrower two is small.
Under group lending, with repayment, borrower one gets a new loan defined as a proportion $\kappa$ of her endowment, if borrower two defaults, or as half of a proportion $\kappa$ of the aggregate endowment, with repayment by both. The cost of staying in the relationship is repayment of the total group liabilities if borrower two defaults, or the repayment of half of the total group liabilities if borrower two repays. Under individual lending, a borrower gains access to a new loan defined as a proportion of her endowment at the cost of repaying her total outstanding liability.

When the endowment of borrower two is small and group loans are divided equally between the two members, borrower one chooses repayment at all levels of her endowment as an optimal action. As the endowment of two increases, repayment is optimal only if the initial net group debt is low, as the growth in the partner’s endowment reduces the potential benefits from repayment and borrower one can no longer capture as much of the benefit emerging from the differences in endowments.

The relatively large borrower prefers repayment because, by repaying she earns the possibility of a new loan that in the case of default of her partner will equal a proportion of her own endowment. This loan size is larger that the loan she would have received under group lending, as the group divides the loan equally among borrowers of different sizes.

The relatively small borrower will default sooner, as under group lending she receives a relatively larger loan as compared to the loan she would receive if she were borrowing individually and, therefore, she is better off running away with the money.
than repaying with the expectation of a potential new loan that may be commensurate to her own smaller endowment.

The smaller borrower receives a larger loan, relative to her endowment, than the other borrower. If she remains in the contract, she will have to repay her comparatively over-sized loan in the future and she will receive smaller benefits than her partner. She is bound to reach quicker the point where the cost of repayment the outstanding liabilities more than offsets the gains.

In contrast, the borrower with a relatively larger endowment receives a loan smaller than what she would be able to get as an individual loan. She remains in good standing, however, because the benefits outweigh the costs. In addition to the productivity gains and future access to credit, eventually, when the other borrower defaults, she will be able to get a loan proportional to her endowment, which is larger than the loan she would receive if the other borrower would have repaid. Repayment of the total group debt is her cost. The expansion of the repayment surfaces in the top two graphs of Figure 6.6 indicates that there is a region where this cost is lower than the benefits.

As both borrowers are identical (other than in the possible differences in endowment within their range of approximation), borrower two has the same repayment surfaces of borrower one. Consequently, with identical borrowers and equal loans that grow proportionately to their aggregate endowment and are divided equally among group members, group credit increases repayment rates compared to individual loans.
This increase in repayment rates is an example of the positive effect for the lender of joint liability *a la* Besley and Coate (1995), namely, the repayment of defaulter member loans by repaying members. In this case, however, the increase in repayment rates is not due to the assumed distribution of returns to the productive activity, as in Besley and Coate. Instead, it is the result of differences in the original endowment relative to the lender’s disbursement rule, which ignores differences in individual member endowments, and of the equal distribution of the loan proceeds among the members of the group.

In this model, the member who defaults is excluded from the group because the decision to default is a strategic decision. If default were due to inability to repay, this situation could lead to the provision of insurance of the type for which Sadoulet (1997) has found evidence. If the borrower for whom the share of the outstanding debt has been repaid were to pay some kind of future transference in exchange for remaining in the group, the larger borrower may be willing not to report the defaulting member to the lender. Under these circumstances, the benefits of insurance and the cost of the transfer should be added to the explanation of the final repayment decision.

It is also worth noting that the increase in the repayment surface under group lending results from the fact that the borrowers are constrained from receiving a loan size consistent with their endowment. Although it is likely that in reality loans are divided in equal parts among members of the group, it is also quite possible that the parameter $\theta$ will be the result of an intra-group bargaining process. Even if this parameter is the result of a process of negotiation, however, as long as it does not
exactly correspond to differences in endowments, there will be room for higher repayment rates under group lending, due to a non-proportional distribution of the group loan. The rationing rule is wealth-related for the group as a whole but not entirely for the individual member.

Figure 6.7 exhibits the repayment surfaces for both borrowers under group credit when the finance-induced productivity coefficient is $\psi_1 = \psi_2 = 1.1$. The intersection of both surfaces is quite small. A reduction of the intangible gains from having access to credit markets considerably reduces the space over which there is group stability. The optimal policy for both borrowers is to default except for very small levels of their endowment, where they are severely credit constrained. At these very low levels, each borrower is willing to repay not only their share of the loan but also the total loan, in exchange for keeping access to credit. When the endowment grows, however, both borrowers decide to default.

Figure 6.8 presents the repayment surfaces of borrower one under group and individual lending with $\psi_1 = \psi_2 = 1.1$. With this lower coefficient of productivity gains from access to credit markets, which are enjoyed only with repayment, the gains in repayment with group lending are considerably reduced as compared to the base case. There is a difference in the repayment surfaces of borrower one between individual and group borrowing, only at the lowest level of borrower two’s endowment.

Even when the endowments are dissimilar, as in the case explained above, a borrower with a relatively high endowment does not find the benefits from repaying the debt load of her fellow member to be enough to be willing to assume the total group
liability. Both borrowers repay at low levels of their endowment and opt to default for middle and high levels of their own endowment (the repayment surface of borrower two is symmetrical to the repayment surface of borrower one).

Figure 6.9 shows the repayment surfaces for both borrowers when $\psi_1 < \psi_2$, i.e., borrower two obtains more productivity gains from participating in financial markets. Borrower two exhibits a larger repayment surface (lighter surface) as she gets more benefits from participating in credit markets. For borrower one, at low levels of her endowment, as long as borrower two's optimal choice is to repay, borrower one repays. When her endowment grows, however, her optimal decision shifts to default, as long as the other borrower is still repaying for all values of one's endowment. Once borrower two's optimal choice shifts to repayment only at low values of one's endowment, borrower one's choice is repayment again.

Borrower one finds that, as long as her partner repays regardless of one's endowment and, therefore, she receives half of the group loan, the additional liability imposed by her relatively more productive partner becomes too high as compared to the self-financing alternative. At higher levels of borrower two's endowment, borrower two's optimal choice is to repay for small and medium levels of one's endowment, but to default for high values of one's endowment.

Borrower one's optimal policy becomes repayment again at high levels of the endowment of two, because borrower two's optimal policy is to remain in the contract with the intention of running away with the new loan. Once borrower two defaults, she is able to receive a loan proportional to her wealth and continue her relationship with
the lender, without facing the threat of possibly having to repay borrower two's future share of the group liabilities.

This particular repayment behavior of borrower one is also reflected in Figure 6.10 where for the lowest two levels of endowment of borrower two (top surfaces) the repayment surfaces of borrower one show that group lending leads to higher repayment rates. At the third level of endowment of borrower two, borrower one defaults. Under individual lending she would have repaid. The dark surface, which represents the repayment surface of borrower one for individual lending, is above the lighter surface that represents group lending (in the figure this is indicated with an arrow). This is an example of the negative incentives of joint liability mentioned by Besley and Coate (1995), where a borrower who would have repaid under individual borrowing does not repay due the burden of joint liability in group loans.

Figure 6.11 shows the repayment surfaces for the second borrower in the same model. Borrower two defaults at high values of her endowment, regardless of the level of initial debt under individual lending. Under group credit the repayment surface is further out for small and medium levels of the endowment of borrower two. At higher levels of the endowment of two, group lending expands the repayment frontier only at the lowest levels of initial net group debt or not at all. This result is important because it shows that the positive and negative effect of joint liability on repayment rates can occur for the same borrower depending on the value of the endowment of her partner.

Figure 6.12 exhibits the repayment surfaces of both borrowers with a simulation for coefficients of risk aversion equal to $\alpha_1 = 0.2$ and $\alpha_2 = 0.4$ and keeping the rest of the
parameters at the same values of the base case. The borrower with lower risk aversion, \textit{one}, presents a smaller repayment surface that the borrower with higher risk aversion, \textit{two}. More risk-averse borrowers put higher valuation on access to credit markets and, therefore, are more willing to assume the group liability in case of the partner's default.

Figure 6.13 shows that, with these parameter values, the positive and the negative effects of group lending may occur. At low levels of the partner’s endowment (top-left) group lending generates a larger repayment surface than individual lending, but at a higher level of the partner’s endowment, group lending leads to a smaller repayment surface than individual lending. At higher levels of the partner’s endowment both technologies generate the same repayment surfaces. Borrower \textit{two}, who is the borrower with higher risk aversion, only experiences the positive effects of joint liability, but they tend to disappear as the endowment of her partner increases.

In this model, a determinant of repayment is the factor $\theta$, which governs the proportion of the group loan allocated to borrower \textit{one} and, therefore, $1-\theta$ is the proportion allocated to borrower \textit{two}. With $\theta=0.6$, and assuming everything else identical, Figure 6.14 exhibits the repayment surfaces for both borrowers. For borrower \textit{two}, who receives the smaller proportion of the group loan, repayment is always the optimal choice for all levels of initial net group debt. Borrower \textit{one} defaults when both endowments are high (the axis have been change to facilitate the appreciation of the differences between surfaces) because she can overcome her credit constraint thanks to the disparity in endowment sizes and to the even division of funds between the members of the group.
Figures 6.15 and 6.16 exhibit the repayment surfaces under group and individual lending for borrower one and two respectively, when \( \theta = 0.6 \). The repayment surface of borrower two clearly shows an improvement in repayment when group lending is adopted. Figure 6.15 shows that for borrower one, at low levels of the endowment of borrower two, group lending increases repayment rates (top surfaces). Figure 6.15 also shows, however, that at least for an intermediate value of the endowment of borrower two, the group lending technology generates a repayment surface that is behind the repayment surface for an individual technology (in the middle-right graph the negative effect is indicated with an arrow). Consequently, considering only the borrower who receives the larger proportion of the group loan, group lending may increase repayment, as compared to the repayment of individual loans, depending on the level of endowment of the partner member.

As borrower one gets a larger share of the group loan, when the loan is defined as a proportion of the aggregate endowment, for very low levels of the endowment of borrower two, borrower one repays at any level of her endowment, because her gains are considerable, as she receives a larger share of the benefits of good standing. For higher levels of endowment of borrower two, however, the benefits start to be compensated by the costs, as it is more likely that borrower two will default and the potential repayment of the group debt increases. Around middle values of endowment of borrower two, the level of endowment of borrower two is high enough to make borrower one decide to default and run away with her larger share of the loan. For this reason, borrower one shows a smaller repayment surface under group lending than
under individual lending, in which case she could only run away with a loan proportional to her own endowment.

At high levels of endowment of two, the potential gains borrower one may extract by running away with a larger share of the loan may be compensated by the increase in two's endowment. In other words, when borrower one considers the possibility of running away with her larger share of the loan but having to resort to self-financing afterwards, she realizes that she will lose the benefit of getting a share of a larger new group loan thanks to the proportionally higher endowment of borrower two. On the repayment side, however, at such high levels of endowment of borrower two, it also is more likely that borrower two may run away herself. These costs and benefits compensate each other so, at the highest levels of endowment of borrower two, there is no difference between the repayment surface under group and individual lending.

Alternative Model

This section considers a model of access to credit through groups in which the size of the loans received by the group is not dependent on the size of the individual member endowments or the aggregate group endowment. Loan sizes are defined by the lender according to a pre-established rule, which defines a monotonic increment in the size of the loan as the relationship between the group and the lender matures.

As long as the group meets its repayment obligations with the lender in every period, the lender promises to provide an increasing loan size to the group. As in the
previous model, the distribution of the group loan and the group liabilities within the
group is an intra-group decision in which the lender does not participate.

The maximization problem faced by each member of the group is described by
the same equations of the model developed in the previous section, with a variation in
the state transition function of the initial group debt state and in the consumption
function of each borrower. The new state transition for the group debt is:

\[ l_{t+1} = \kappa l_t \]  

(6.12)

The proportion \( \kappa \) is presumably greater than 1. Consumption for each one of the
two borrowers is defined as:

\[ c_1 = x_{1t} - x_{1t} - l_t (1 + r) \theta + \kappa l_t \theta \]  

(6.13)

\[ c_2 = s_{2t} - x_{2t} - l_t (1 + r)(1 - \theta) + \kappa l_t (1 - \theta) \]

The approximation of the Nash equilibrium solution for this model followed the
same procedure explained for the group model of the previous section. Parameter values
were also taken at the same level of the previous model.

Figure 6.17 exhibits the repayment surfaces for both borrowers on the
endowment space with a lender’s rationing rule coefficient of \( \kappa=1.005 \) and finance-
induced productivity gains coefficients of \( \psi_1=1.1 \), and \( \psi_2=1.1 \). Both borrowers opt for
repayment as an optimal policy for all values of the endowment and initial group debt
with the exception of the highest value of group debt and high values of the partner’s
endowment (darker surface is the repayment surface of borrower one).¹

¹ In this model there is no need to transform the third state variable, as it is not time-related to the other
state variables. The third state is thus, the initial absolute debt.
Figure 6.18 exhibits the repayment surfaces of borrower one under group and individual loans. For the two highest values of initial group debt (middle-right and bottom-left graphs), individual credit dominates over group credit, as the repayment surface of group loans goes to zero at high values of the partner endowment (darker surface is the group lending repayment surface).

Figure 6.19 exhibits the repayment surfaces of both borrowers, on the endowment space, for the model approximated with $\kappa=1.01$, $\psi_1=\psi_2=1.1$ and $\theta=0.7$. For the lowest level of initial group debt, both borrowers repay for all possible combinations of their endowments. As the initial group debt increases, borrower one opts to default at high levels of her partner’s endowment, whereas borrower two still chooses to repay. As borrower one receives a larger proportion of every group loan, this borrower overcomes her credit constraint sooner. As the level of group liabilities increases, borrower one is better-off self-financing her activity than repaying. The higher the endowment of her partner, relative to her own endowment and the initial debt, the more likely that she would opt to default.

Figure 6.20 exhibits the repayment surfaces for borrower one under group and individual lending for this simulation. Group lending leads to a smaller repayment surface than individual lending for all five first graphs (five different levels of the endowment of borrower two). In all five graphs, under individual lending the borrower repays at all combinations of her endowment and initial debt. In group lending, borrower one defaults when her partner has a relatively high endowment.
When the endowment of borrower two is small, the possibility that she will default and that borrower one will be forced to repay the total group loan poses little threat, because borrower’s two marginal valuation of the additional funds from new loans is higher. As the endowment of the other borrower starts growing, borrower one is better-off defaulting, at high levels of initial group debt, because it becomes more likely that, for combinations of high debt and higher endowment, borrower two will default. This threat is only meaningful at low levels of the endowment of borrower one, as with a higher endowment, borrower one does not worry about the impact of having to repay the relatively smaller share \((1-\theta)\) of borrower two.

Borrower two’s optimal choice is to repay at all levels of borrower’s one endowment, as she gets a smaller share of the loan and, therefore, she remains credit constrained. If the overall repayment rate with group lending is not worse under group lending than under individual lending is only because the second borrower is not able to overcome her credit constraint.

When the high value of borrower one’s share of the group loan \((\theta=0.7)\) is coupled with higher productivity \((\beta_1=0.5, \beta_2=0.4)\), the higher productivity of borrower one keeps the credit constraint binding and increases the valuation of additional funds by borrower one. For this reason, the repayment surface of borrower one is more comprehensive as compared to the surface with equal productivity for the two members of the group (Figure 6.21 and Figure 6.19). The differences in the repayment surface of group lending over individual lending are reduced (Figure 6.22). In other words, if a

\[\text{A high } \theta \text{ may be the result of the bargaining power provided by higher productivity.}\]
borrower is more productive and she also receives a greater proportion of the group loan, the repayment surface of group lending tends to the surface that would be achieved with individual loans.

**Summary of results**

In this chapter, a group lending model was developed as an extension of the individual lending model of Chapter 5. In the first version of the model, the lender grants a loan to a group of borrowers on the basis of their aggregate endowment. The members of the group divide the loan according to a distribution rule they previously have agreed upon.

If the loan is divided evenly among members, borrowers who receive relatively oversized shares of the loan (because their endowment is smaller) will have an incentive to default earlier than their partners, as they have an incentive to run away without repaying the oversized loan relative to their endowment. The larger borrower will have an incentive to repay her partner's share because, if she keeps a good standing, she will be able to get a new loan adjusted to the size of her own endowment, once her partner has defaulted.

Under these circumstances, it is possible that group lending achieves higher repayment rates than individual lending. This is due, however, not to an optimal allocation of resources but to the wealthier borrower's lack of bargaining power to increase the share of the loan in her favor. The lack of bargaining power is in the end the reason why she remains credit constrained.
If the loan is not distributed equally among members of the group, the borrower who receives the larger share will be willing to repay her partner’s share of the group loan at low levels of the partner’s endowment. As the endowment of the partner grows, however, the larger borrower will default, as she expects her partner to default. The borrower with a smaller share of the group loan will repay at all levels of endowment of her partner, as she remains credit constrained due to the smaller share of the loan she receives.

When the lender defines the size of the group loans independently of the size of the endowments of the members of the group, group lending does not seem to generate better repayment rates as compared with individual lending. When the starting level of group debt is high and borrowers have dissimilar levels of endowment, the burden of joint liability for the small borrower is too high, even compared to her binding credit constraint, and it may lead to default in circumstances when under individual lending the smaller borrower would have repaid.

In conclusion, there is no a priori repayment superiority of one lending technology over other. Rather, one performs better than the other depending on empirical features of the borrowers and the nature of the contract conditions, in particular, the lender’s rationing rule.
Figure 6.1: Approximation error for growth model with access to group credit, for borrower *one* at the lowest level of initial net group debt.
Figure 6.2: Repayment surfaces for both borrowers (base case). Dark surface=borrower one.
Figure 6.3: Repayment surfaces for borrowers who choose a world with access to group credit.
Figure 6.4: Investment functions for borrower one at different levels of initial net group debt (base case).
Figure 6.5: Investment function for borrower one at the lowest level of initial net group debt.
Figure 6.6: Individual and group lending repayment surfaces for borrower one (base case).
Figure 6.7: Repayment surfaces for both borrowers, $\psi_1 = \psi_2 = 1.1$. 
Figure 6.8: Repayment surfaces for group and individual lending, for borrower one, $\psi_1=\psi_2=1.1$. 
Figure 6.9: Repayment surfaces for both borrowers, $\psi_1=1.1$, $\psi_2=1.3$. 
Figure 6.10: Repayment surfaces for borrower one under group and individual lending, 

$\psi_1=1.1$, $\psi_2=1.3$. 

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Figure 6.11: Repayment surfaces for borrower two under group and individual lending, $\psi_1=1.1, \psi_2=1.3$. 
Figure 6.12: Repayment surfaces for both borrowers, $\alpha_1=0.2$, $\alpha_2=0.4$. 
Figure 6.13: Repayment surfaces for borrower one under group and individual lending, $\alpha_1=0.2$, $\alpha_2=0.4$. 
Figure 6.14: Repayment surfaces for both borrowers, $\theta=0.6$. 
Figure 6.15: Repayment surfaces for borrower one under group and individual lending, $\theta=0.6$. 
Figure 6.16: Repayment surfaces for borrower two under group and individual lending, 
\[ \theta = 0.6. \]
Figure 6.17: Repayment surfaces for both borrowers, $\psi_1 = \psi_2 = 1.1$ (rationing rule independent of wealth).
Figure 6.18: Repayment surfaces for borrower one, under group and individual lending, 

$\psi_1 = \psi_2 = 1.1$ (rationing rule independent of wealth).
Figure 6.19: Repayment surfaces for both borrowers, $\kappa=1.01$, $\theta=0.7$ (rationing rule independent of wealth).
Figure 6.20: Repayment surfaces for borrower one under group and individual lending, $\kappa=1.01, \theta=0.7$ (rationing rule independent of wealth).
Figure 6.21: Repayment surfaces for both borrowers, $\kappa=1.01$, $\theta=0.7$, $\beta_1=0.5$, $\beta_2=0.4$ (rationing rule independent of wealth).
Figure 6.22: Repayment surfaces for borrower one under group and individual lending,
\( \kappa=1.01, \theta=0.7, \beta_1=0.5, \beta_2=0.4 \) (rationing rule independent of wealth).
CHAPTER 7

Conclusions and Recommendations

The last three decades have witnessed a growing interest in the field of development finance. Within this field, the role of innovations in lending technologies in explaining opportunities for expansion of the frontier of development finance is attracting increasing attention. Lending technologies are important because, in order to provide financial services to marginal clienteles in a sustainable manner, incentive, information, and enforcement problems must be resolved, and, in general transaction costs must be reduced. Innovations in lending technologies overcome constraints in financial markets that explain why these costs are so high.

Imperfect information makes the provision of financial services too costly for the lender, as in most cases no prior information about the potential clients is available that could allow an accurate assessment of repayment capacity and willingness to repay. Most potential clients have little or no experience with formal financial intermediaries, and gathering the necessary information is a costly activity for microfinance organizations. The mechanisms for contract enforcement are weak, and lenders must design non-traditional structures of incentives to induce repayment. High transaction costs for both borrowers and lenders, due to the characteristics of a typical microfinance
transaction, raise the effective price of financial services for the clients and ration out many potential borrowers.

The stylized facts from the developing world show mixed results regarding the success of specific financial technologies. Microfinance programs around the world can be counted in the several thousands but few cases can claim to be successful in terms of both outreach and sustainability. In this context, the discussion of the particular characteristics of a financial technology is important. In particular, the comparison of individual and group credit contracts has become a major topic in the debate, with contributions from both the academic and the practitioner worlds.

This dissertation is a contribution to this debate from several perspectives. First, it develops a method of analysis of microfinance contracts that focuses on their unique characteristics, such as the primary role played by the threat of termination of the relationship with the lender as the main deterrent to default and the use of sequencing and the improvement of the contractual terms and conditions, as the relationship matures, for the same purpose. These incentives are critical in the absence of traditional collateral and their long-term limitations are not well understood.

Second, the dissertation incorporates the dynamic incentives faced by borrowers in analyzing strategic default. Borrowers incorporate in their decisions the value and costs of future access to credit and make decisions today considering not only their current impact but also their effect on their future well-being.

Third, the dissertation incorporates flexibility in the analysis by varying contract dimensions according to the emphasis desired. The features of utility and production
functions, the rules of definition of the terms and conditions of the contract, the distribution of the stochastic shocks to which borrowers are exposed, the distribution rule among group members, and the initial levels of wealth and indebtedness of the participants are some of the dimensions over which the specification can be easily modified and simulation analyses can be performed.

Fourth, the dissertation emphasizes the terms and conditions of microfinance contracts, in particular the lender's rationing rule, and the varying levels of wealth and indebtedness of the borrowers and their role in the repayment-default decision, assuming away ability to repay issues. This emphasis allows the analysis to focus on a less-explored dimension of credit contracts, willingness to repay. In the absence of traditional collateral in microfinance contracts willingness to repay is the cornerstone of contract enforcement success.

Fifth, the individual or group features of the lending technology are especially addressed, as these features represent the central dimension of alternative innovations in lending technology in competition in the developing world. At the same time, the assumption of monopolistic power enjoyed by the lender is implicitly present throughout the analysis. Some of these results would change once active competition among microfinance organizations that use different lending technologies emerges (Navajas, 1999).

In this dissertation, as is typical in principal-agent analysis, the lender offers a take-it or leave-it contract, in which repayment leads to renewal of the contract, with continuous access to credit, and default leads to the exclusion of the borrower forever
from the lender's portfolio. Given the monopolistic position of the lender, borrowers do not have alternative sources of external funds, except at much higher costs, if at all. This increases the effectiveness of the threat of termination of the relationship as an incentive to repay.

In an individual credit contract, a borrower compares the benefits of repaying, given the value of automatic access to a new loan and future access to credit, with the costs of defaulting, the loss of access to future credit and to the potential productivity gains associated only with credit-funded production.

The individual model first concludes that, under the restrictive characteristics of a contract that requires full repayment before the additional disbursement of new loans, the size of which is defined as a proportion of the borrower's endowment, the intangible productivity gains from access to credit are essential to explain repayment. Without these gains, for most values of the parameters, the borrower will have little to gain by maintaining a good standing with the lender, as she soon overcomes her initial credit constraint. Once the credit constraint is removed, these types of incentives to repay disappear.

These intangible gains are important, as access to credit brings not only knowledge and reputation effects but also tools for diversification and household portfolio management. These features of formal finance may not be necessarily offered by own savings or other informal sources of funds.

When the lender in an individual contract defines loan size as a proportion of the level of endowment of the borrower, the level of wealth of the borrower not only plays
a direct role in increasing utility through inter-temporal consumption, but it also plays a second role as a source of creditworthiness. Wealth provides access to credit and credit further increases wealth.

Starting from a credit-constrained borrower, growing levels of wealth offer the possibility of obtaining larger loans and, in this way, the borrower gets closer to overcoming his credit constraint. At low levels of wealth, credit-constrained borrowers show higher rates of growth of investment and lower rates of growth of consumption, and it is optimal to repay loans in order to maintain access to credit. At higher levels of wealth, however, the borrower overcomes the credit constraint and, therefore, it is optimal to default and resort to self-financing.

If the lender is willing to grant higher loan sizes per unit of wealth (i.e., if it is willing to be less strict in its rationing rule), the set of possible levels of wealth and indebtedness that would lead to repayment shrinks, as the borrowers will have extra resources to more rapidly overcome their credit constraint. The effect of higher interest rates is also to shrink the repayment frontier as, with higher interest rates, borrowing becomes more costly and borrowers reach sooner the point where the costs of repayment do not outweigh the benefits.

At low levels of wealth, higher borrower risk aversion shrinks the set of endowment and indebtedness combinations for which repayment is the optimal decision, but as wealth increases and global risk-aversion diminishes, due to the increase in wealth, the repayment frontier may expand even with higher coefficients of risk aversion.
The provision of credit, even under the restrictive characteristics of the contracts modeled in this dissertation, leads to higher long-run steady-state levels of investment than under self-financing. This is an important result, because it stresses the role of the provision of credit to clienteles until now excluded from formal financial markets. As long as these marginal clienteles engage in sustainable productive activities, access to credit will bring considerable gains to the economy. It is important, however, to stress that the long-term steady-state solution of the model is dependent on the values of the parameters chosen for the estimation and that it may imply levels of wealth outside the repayment frontier for a given financial technology.

Alternative formulations for the definition of loan sizes, such as a contract in which loans grow according to a pre-established rule independently of the borrower's level of wealth, reveal a direct instead of an inverse relationship between the set of compatible levels of wealth and indebtedness found along a threshold function for repayment. More wealth can support higher indebtedness and still generate repayment. As wealth increases, it is increasingly devoted to consumption instead of investment and, therefore, the set of endowments and indebtedness that generate repayment is convex.

As an extension of the individual model, the group model considers two borrowers who are jointly liable for a group loan. The lender disburses a group loan, which is divided between the two members according to some distribution rule they agree upon. Borrowers are identical other than the different values of endowments and initial level of indebtedness within the same permissible range of interpolation. Each
individual group member faces the strategic decision of whether to repay up to the total group loan and maintain a good standing with the lender or default and resort to self-financing.

In group lending, in the evaluation of the costs and benefits of each action, the individual member must incorporate the possible actions of her fellow group member. If the partner repays her share of the group loan, the former would have to repay only her own share, but if her partner defaults, she would have to repay the total group loan in order to maintain a good standing.

The model concludes that, when the lender defines the group loan size as a proportion of the combined aggregate endowment and when the group loan is divided equally between the members, there are potential gains to be made in repayment behavior if the endowments of the borrowers are of different size. Borrowers with small endowments receive relatively larger loans than as individual borrowers and, therefore, they face greater relative benefits from defaulting. These greater gains from defaulting eventually counterbalance the gains in productivity enhancements that result from access to credit.

The borrower with a large endowment, in contrast, receives a relatively smaller loan for her level of endowment and, therefore, if she remains in the contract after the other borrower defaults, not only she can continue enjoying the productivity gains from access to credit, but she can also receive a new loan, this time adjusted to by then larger endowment. The larger borrower has, consequently, an incentive to repay for the smaller borrower. If both borrowers received loans proportional to their endowments,
the default threshold would be reached earlier. This result contradicts the view that group members with larger endowments have incentives to default earlier.

The increase in repayment rates in this case is only due to the constraint imposed on individual group members, which keeps them from getting a share of the group loan proportional to their endowment, and not to better incentives for repayment in group loans per se.

As in the individual model, when the optimal choice is repayment, investment grows at a faster rate and consumption grows at a slower rate. Once the optimal choice is default, the rate of growth of investment declines, revealing the relaxation of the credit constraint and that the endowment's role as provider of creditworthiness is no longer valid. Additional units of wealth play a role only in the maximization of inter-temporal consumption.

When the group loan is not distributed equally among members of the group, for some combinations of endowment levels of the two group members, repayment rates improve, but for other combinations repayment rates worsen, with respect to the borrower receiving the larger share of the group loan. For this member, repayment rates worsen when she assesses her partner's endowment to be high enough and, therefore, she expects her to default. Consequently, it is better for her to run away with a larger share of the loan than remaining within the contract and possibly having to repay the full group loan.

The borrower who receives the smaller share, however, still finds it optimal to remain within the contract, as she is still credit-constrained, due the smaller share of the
group loan that she receives. The overall repayment rates of the group do not worsen due to the credit constraint of the second borrower, who still finds repayment of the whole as her optimal choice.

If the lender defines the group loan independently of the aggregate endowment, there are no productivity gains from access to financial markets, and the group loan is divided equally among members of the group, it is optimal for both borrowers to default once a given level of indebtedness is reached, regardless of their level of wealth. If sufficient productivity gains are added, the optimal policy will be repayment for both borrowers at all levels of endowment and indebtedness.

In sum, the effect of group lending on overall repayment rates is dependent on the rules of disbursement defined by the lender (rationing rule) and the distribution of loan proceeds among the members of the group (distribution rule). No general conclusion can be reached. In the cases when there is any improvement in repayment rates, however, it is because one of the members remains credit constrained. This result highlights the conflict between outreach (rapidly removing credit constraints) and sustainability (creating incentives to repay). Further innovation in lending technologies is needed to reduce the trade-off between outreach and sustainability.
APPENDIX A

MATLAB PROGRAM TO SOLVE INDIVIDUAL CHOICE DECISION BETWEEN
A WORLD WITH ACCESS TO CREDIT AND A WORLD WITHOUT IT

This program uses several of the utilities developed by Miranda and Fackler published
as: "Compecom: Toolbox for Matlab", http://www.agecon.ag.ohio-
state.edu/ae802/default.htm. It also uses the subroutine cdpsolv3, presented in Appendix
B.

% CLEAR MEMORY
    clear all

% ENTER CONTROL PARAMETERS
    n = [8 5];
    pctmin = 0.4;
    pctmax = 2.5;
    pctmin2 = -0.1;
    pctmax2 = 0.1;

% ENTER MODEL PARAMETERS
    global delta alpha beta gamma sigma kappa inter psi;
    delta = 0.9;  % discount factor
    alpha = 0.4;  % utility parameter
    psi = 1.3;   % fin.-induced prod.gains
    beta = 0.5;  % production elasticity
    gamma = 0.9; % capital recuperation rate
    sigma = 0.1; % prod. shock volatility
    kappa = 0.5; % lender's rationing rule
    inter = 0.111; % interest rate

% COMPUTE GAUSSIAN NODES AND WEIGHTS
    m = 5;
    [e,w]=qnwlogn(m,0,sigma^2);  % lognormal nodes & weights
% SOLVE MODEL WITHOUT ACCESS TO CREDIT
% PACK MODEL STRUCTURE
model0.ffunc = 'f0'; % reward function
model0.gfunc = 'g0'; % transition function
model0.bfunc = 'b0'; % bound function
model0.discount = delta; % discount factor
model0.e = e; % shocks
model0.w = w; % probabilities

% COMPUTE CERTAINTY EQUIVALENT STEADY-STATE
const = 1/delta;
xstar = ((const-gamma)/beta)^(1/(beta-1));
sstar = gamma*xstar + xstar^beta;
cstar = sstar - xstar;
uprim = cstar^-alpha;
pstar = uprim;
vstar = f0(sstar,xstar)/(1-delta);

% DEFINE FUNCTION SPACE FOR APPROXIMATION
smin = sstar-pctmin*sstar; % lower limit
smax = sstar+pctmax*sstar; % upper limit
fspace0 = fundefn('spline',n(1),smin,smax); % fn. space

% INITIALIZE POLICY AND VALUE FUNCTIONS
s = funnode(fspace0); % state collocation nodes
x0 = (xstar/sstar)*s; % initial policy fn.
v0 = vstar + pstar*(s-sstar); % initial value function

% SOLVE BELLMAN EQUATION MODEL WITHOUT ACCESS TO CREDIT
optset('cdpsolve','maxstep',0);
optset('cdpsolve','maxit',500);
[v0,x0,cv0,cx0,sres,resid]=cdpsolve(model0,fspace0,...
'newton',v0,x0);

% SOLVE MODEL ALWAYS WITH CREDIT
% PACK MODEL STRUCTURE
modell.ffunc = 'f1'; % reward function
modell.gfunc = 'g1'; % transition function
modell.gOfunc = 'g0'; % transition fn.w/o debt
modell.bfunc = 'b1'; % bound function
modell.discount = delta; % discount factor
modell.e = e;  % shocks
modell.w = w;  % probabilities

% COMPUTE CERTAINTY EQUIVALENT STEADY-STATE
const = 1/(delta*((1+kappa)-delta*kappa*(1+inter)));
xstar = ((const-gamma)/(beta*psi))^(1/(beta-1));
s1star = gamma*xstar + xstar^beta*psi;
s2star = 0;
cstar = (1-kappa*inter)*s1star - (1+inter)*...

s2star - xstar;
uprim = cstar^-alpha;
p2star = -(1+inter)*uprim;
plstar = (1-kappa*inter)*uprim + delta*kappa*p2star;
vstar = f1([s1star s2star],xstar)/(1-delta);
xstar,s1star,plstar,p2star

% DEFINE FUNCTION SPACE FOR APPROXIMATION
smin = [smin pctmin2+s2star];  % lower limit
smax = [smax pctmax2+s2star];  % upper limit
fspacel = fundefn('spli',n,smin,smax);  % fn. space

% INITIALIZE POLICY AND VALUE FUNCTIONS
scoord = funnode(fspacel);  % state collocatation nodes
s = cgrid(scoord);
xl = (xstar/s1star)*s(:,1);  % initial policy function
v1 = vstar + plstar*(s(:,1)-s1star) + p2star*...
(s(:,2)-s2star);  % initial val. fn.

% SOLVE BELLMAN EQUATION MODEL ALWAYS WITH CREDIT
optset('cdpsolve','maxstep',0);
optset('cdpsolve','maxit',500);
optset('cdpsolve','tol',1.0e-007);
[v1, xl, cvl, cxl, sres, resid]=cdpsolve(modell, fspacel,...
'newton',v1,xl);

% SOLVE MODEL WITH ACCESS TO CREDIT
vv0 = v0(:,ones(1,n(2)));  % expand val. fn. w/o credit to
credit dims.
v0 = vv0(:);
xx0 = x0(:,ones(1,n(2)));  % expand actions w/o credit
to credit dims.
x0 = xx0(:);
cx0 = funfitxy(fspacel,s,x0);
cv0 = funfitxy(fspacel,s,v0);

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cvm = funfitxy(fspace1,s,max(vO,v1));

% SOLVE BELLMAN EQUATION DEF.-REP GIVEN ACCESS TO CREDIT
optset('cdpsolv3','maxit',500);
optset('cdpsolv3','maxstep',0);
optset('cdpsolv3','tol',1.0e-009);

[v,x,cv,cx,R]=cdpsolv3(modell,fspace1,'funcit',v1,x1,cv0);

% CHOOSE BETWEEN WORLD WITH ACCESS TO CREDIT AND WITHOUT IT
vmir=vO; % initialize value fn. Sol. as w/o credit
xmir=xO; % initialize policy solution as w/o credit
Rmir=R; % initialize rep.decision as with credit
i=v>vO; % cases access is more rewarding than w/o cred
vmir(i)=v(i); % pick value fn.with credit
xmir(i)=x(i); % pick action with credit
Rmir(i)=1; % repayment if picking credit
cvmir=funfitxy(fspace1,s,vmir);
cxmir=funfitxy(fspace1,s,xmir);

% FIND THRESHOLD FUNCTION
nc=20; % degree of approxim.
lowcheb=smin(1); % low limit of interval of approx.
highcheb=smax(1); % high limit of interval of approx.

% Define s2 nodes
xnode=nodecheb(nc,lowcheb,highcheb);
sp=[xnode zeros(nc,1)]; % build vector of initial sol.
% Find by Newton the solution of VO(s1,s2)-VI(s1,s2)=0
for i=1:nc
    sq=sp(i,:); % initial guess for i entry
    for it=1:300
        [f,d]=fthre(cv0,cv,fspace1,sq); % eval.fn. V0-V1
        sq(:,2)=sq(:,2)-d;
        change=norm(f);
        fprintf('%5i %10.1e
',it,change)
        if change<1.e-9, break, end;
    end
    sp(i,:)=sq; % update sp with sol. for entry i
end

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% Approximate a Chebychev polynomial for the function
fun=sp(:,2); % define val.of sl as eval.of fn.to app.
c=basecheb(xnode,nc,lowcheb,highcheb)
\fun; % define Chebychev coef. of app.
xv=smin(1):0.5:smax(1); % define grid of Chebychev app.
y=basecheb(xv',nc,lowcheb,highcheb)*c; % y val.of fun.
APPENDIX B

MATLAB UTILITY USED BY INDIVIDUAL CHOICE PROGRAM

BETWEEN WORLD WITH ACCESS TO CREDIT AND WORLD WITHOUT IT

This program uses several of the utilities developed by Miranda and Fackler published as: "Compecom: Toolbox for Matlab", http://www.agecon.ag.ohio-state.edu/ae802/default.htm. It also uses the subroutine cdpsolv3, presented in Appendix B.

function [v,x,cv,cx,R] = cdpsolv3(model,fspace,alg,v,x,cv0)

% Usage
% [v,x,cv,cx,sres,resid] = cdpsolve(model,fspace,alg,v,x)
% Input
% model : name of model structure
% fspace : name of projection space structure
% alg : algorithm used (funcit or newton)
% v : initial guess for value function
% x : initial guess for optimal controls
% Output
% v,cv : value function and projection coefficients
% x,cx : optimal controls and projection coefficients
% sres : residual evaluation points
% resid : Bellman equation residuals
% Options
% tol : convergence tolerance
% maxit : maximum number of iterations

% SET CONVERGENCE PARAMETER DEFAULTS
maxstep = optget('cdpsolv3','maxstep',0);
maxit = optget('cdpsolv3','maxit',500);
tol = optget('cdpsolv3','tol',1.1e-009);
% EXTRACT MIN AND MAX STATE
n = fspace.n;
smin = fspace.smin;
smax = fspace.smax;

% COMPUTE COLLOCATION NODES AND INTERPOLATION MATRIX
phi = funbas(fspace);  % interpolation matrix
scoord = funnode(fspace);  % state collocation nodes
s = cgrid(scoord);  % state collocation nodes

% PRINT ITERATION METHOD
switch alg;
    case 'funcit'
        disp('Solve Bellman equation via function iteration');
    case 'newton'
        disp('Solve Bellman equation via Newton method');
    otherwise
        error('dp solution algorithm must be ''funcit'' or ''newton''');
end

% PERFORM FUNCTION OR NEWTON ITERATIONS
tic
    cv = funfitxy(fspace,s,v);  % initial basis coefficients
    for it=1:maxit
        cvold = cv;  % store old value
        switch alg
            case 'funcit'
                [v,x,R] = vmax(s,X,cv,cv0,fspace,model);  % solve Bellman equation
                cv = funfitxy(fspace,s,v);  % update basis coefficients

            case 'newton'
                [v,x,R,vderc] = vmax(s,x,cv,cv0,fspace,model);  % solve Bellman eqn

        end
    end
end

% figure(13+it)
% nn = [50 30];
% [ss,scoord] = nodeunif(nn,smin,smax);
% vv = funeval(cv,fspace,ss);
% vv = reshape(vv,nn(1),nn(2));
% surf(scoord{1},scoord{2},vv');
% xlabel('Stock'); ylabel('Relative Debt');
% zlabel('Fitted V')
% case 'newton'
% [v,x,R,vderc] = vmax(s,x,cv,cv0,fspace,model);
% solve Bellman eqn
cv = cvold - [\phi-vderc]\{\phi*cv-v\};  % update basis coefficients
end
change = norm(cv-cvold);  % compute change
fprintf ('%5i %10.1le
',it,change)  % print progress
if change<tol, break, end;  % convergence check
end
toc

% CHECK STATE TRANSITION SATISFY BOUNDS
snmin= inf;
snmax= -inf;
for k=1:length(model.e);
g = feval(model.gfunc,s,x,model.e(k));
snmin = min(snmin,min(g));
snmax = max(snmax,max(g));
end
if snmin<smin, disp('Warning: extrapolating beyond smin'), end;
if snmax>smax, disp('Warning: extrapolating beyond smax'), end;

% COMPUTE POLICY FUNCTION COEFFICIENTS
if nargout<4, return, end;
cx = funfitxy(fspace, s, x);

% COMPUTE RESIDUAL
if nargout<6, return, end;
nr = 10*n;
[sr,sres] = nodeunif(nr,smin,smax);
xr = funeval(cx,fspace,sr);
vr = vmax(sr,xr,cv,cv0,fspace,model);
resid = vr-funeval(cv,fspace,sr);

function [V,x,R,vc] = vmax(s,x,cv,cv0,fspace,model)
% Solves Bellman equation at state nodes

maxstep = optget('cdpsolv3','maxstep',0);
maxit = optget('cdpsolv3','maxit',500);
```matlab
tol = optget('cdpsolv3','tol',sqrt(eps));

n = size(s,1);
ns = size(s,2);
m = length(model.e);

[xl,xu] = feval(model.bfunc,s);  % compute bounds

for it=1:maxit
    xold = x;
    [f,fds,fdx,fdxx] = feval(model.ffunc,s,x);
    V = zeros(n,1);
    F = zeros(n,1);
    J = zeros(n,1);
    R = ones(n,1);
    for k=1:m
        [g,gds,gdx,gdxx] = feval(model.gfunc,s,x,model.e(k));
        [gO,gdsO,gdxO,gdxxO] = feval(model.gOfunc,s,x,model.e(k));
        V = funeval(cv,fspace,g);
        p = funjac(cv,fspace,g);
        pd = funhess(cv,fspace,g);
        v0 = funeval(cv0,fspace,g);
        p0 = funjac(cv0,fspace,g);
        pd0 = funhess(cv0,fspace,g);
        gdx0 = [gdx0 zeros(n,1)];
        gdxx0 = [gdxx0 zeros(n,1)];
        i = v0>v;
        v(i) = v0(i);
        p(i,:) = p0(i,:);
        pd(i,:,:)=pd0(i,:,:);
        R(i)=0;
        gdx(i,:) = gdx0(i,:);
        gdxx(i,:) = gdxx0(i,:);
        V = V + model.w(k)*v;
        F = F + model.w(k)*sum(p.*gdx,2);
        J = J + model.w(k)*sum(p.*gdxx,2);
    for is=1:ns
        for js=1:ns
            J = J + model.w(k)*pd(:,is,js).*gdx(:,is).*gdx(:,js);
        end
    end
    V = V + model.discount*V;
    F = Fdx + model.discount*F;
end
```
\[ J = \text{fdxx} + \text{model.discount} \cdot J; \]
\[ \text{delx} = -F./J; \]
\[ x = x + \text{delx}; \]
\[ x = \min(x,x_u); x = \max(x,x_l); \]
\[ \text{delx} = x-x_{\text{old}}; \]
\[ \text{if norm(delx)}<\text{tol}, \text{break}, \text{end}; \]
\[ \text{for is}=1:\text{maxstep} \]
\[ \quad [f,fds,fdx,fdxx] = \text{feval(model.ffunc},s,x); \]
\[ \text{Vnew} = \text{zeros}(n,1); \]
\[ \text{for } k=1:m \]
\[ \quad [g,gds,gdx,gdxx] = \text{feval(model.gfunc},s,x,\text{model.e(k)}); \]
\[ \quad v = \text{funeval(cv,fspace},g); \]
\[ \quad v0 = \text{funeval(cv0,fspace},g); \]
\[ \quad i=v0>v; \]
\[ \quad v(i)=v0(i); \]
\[ \quad R(i)=0; \]
\[ \quad Vnew = Vnew + \text{model.w(k)} \cdot v; \]
\[ \text{end} \]
\[ \text{Vnew} = f + \text{model.discount} \cdot Vnew; \]
\[ \text{if any(Vnew}<V) \]
\[ \quad \text{delx} = \text{delx}/.5; \]
\[ \quad x = x-\text{delx}; \]
\[ \text{else} \]
\[ \quad \text{break} \]
\[ \text{end} \]
\[ \text{end} \]
\[ \text{if nargout}<4, \text{return}, \text{end}; \]
\[ \text{vc} = \text{zeros}(n,n); \]
\[ \text{for } k=1:m \]
\[ \quad g = \text{feval(model.gfunc},s,x,\text{model.e(k)}); \]
\[ \quad \text{phinxt} = \text{funbas(fspace},g); \]
\[ \quad \text{vc} = \text{vc} + \text{model.discount} \cdot \text{model.w(k)} \cdot \text{phinxt}; \]
\[ \text{end} \]
APPENDIX C
MATLAB PROGRAM TO SOLVE GROUP LENDING MODEL

This program uses several of the utilities developed by Miranda and Fackler published as: "Compecom: Toolbox for Matlab", http://www.agecon.ag.osu.edu/ae802/default.htm. It also uses the subroutine cdpsolv3, presented in Appendix B and subroutines cdps1 and cdps31, presented in Appendix D and E.

% CLEAR MEMORY
    clear all

% ENTER CONTROL PARAMETERS
    n = [5 5 4];   % degree of approximation
    pctmin = 0.4;
    pctmax = 2.5;
    pctmin3 = -0.5;
    pctmax3 = 0.5;

% ENTER MODEL PARAMETERS
    global delta kappa inter psil alpha1 alpha2 beta1 beta2 gamma1 gamma2 sigma
    delta = 0.9;   % discount factor
    alpha1 = 0.4;  % utility parameter 1
    alpha2 = 0.4;  % utility parameter 2
    psil = 1.3;    % finance-induced production gains 1
    psil2 = 1.3;   % finance-induced production gains 2
    beta1 = 0.5;   % production elasticity 1
    beta2 = 0.5;   % production elasticity 2
    gamma1 = 0.9;  % capital survival rate 1
    gamma2 = 0.9;  % capital survival rate 2
    sigma = 0.1;   % production shock volatility
    kappa = 0.5;   % lender's rationing rule
    inter = 0.111; % interest rate
    rho = 0.5;     % prop. group loan for indiv. 1

% COMPUTE GAUSSIAN NODES AND WEIGHTS
    m = 1;
    [e,w]=qnwlogn(m,0,sigma^2); % log normal nodes and weights
% SOLVE MODEL WITHOUT ACCESS FOR EACH INDIVIDUAL AGENT %

% PACK MODEL STRUCTURE AGENT 1
model01.ffunc = 'fjOl';    % reward function
model01.gfunc = 'gjOl';  % transition function
model01.bfunc = 'bjOl';  % bound function
model01.discount = delta;  % discount factor
model01.e = e;    % shocks
model01.w = w;    % probabilities

% PACK MODEL STRUCTURE AGENT 2
model02.ffunc = 'fj02';  % reward function
model02.gfunc = 'gj02';  % transition function
model02.bfunc = 'bj02';  % bound function
model02.discount = delta;  % discount factor
model02.e = e;    % shocks
model02.w = w;    % probabilities

% COMPUTE CERTAINTY EQUIVALENT STEADY-STATE BOTH AGENTS W/O CREDIT
const = 1/delta;
xstar01 = ((const-gammal)/betal)^(1/(betal-1));
xstar02 = ((const-gamma2)/betal2)^(1/(betal2-1));
sstar01 = gammal*xstar01 + xstar01^betal1;
sstar02 = gamma2*xstar02 + xstar02^betal2;
cstar01 = sstar01 - xstar01;
cstar02 = sstar02 - xstar02;
uprim01 = cstar01^-alphal;
uprim02 = cstar02^-alpha2;
pstar01 = uprim01;
pstar02 = uprim02;
vstar01 = fj01(sstar01,xstar01)/(1-delta);
vstar02 = fj02(sstar02,xstar02)/(1-delta);
xstar01,sstar01,xstar02,sstar02

% DEFINE FUNCTION SPACE FOR APPROXIMATION AGENT 1
smin01 = sstar01-pctmin*sstar01;  % lower limit indiv.1
smax01 = sstar01+pctmax*sstar01;  % upper limit indiv.1
fspace01 = fundefn('spli',n(1),smin01,smax01);  % fn.spacel

% INITIALIZE POLICY AND VALUE FUNCTIONS AGENT 1
s = funnode(fspace01);  % state collocation nodes
x01 = (xstar01/sstar01)*s;  % initial policy fn.1
v01 = vstar01 + pstar01*(s-sstar01);  % initial value fn.1
% SOLVE BELLMAN EQUATION AGENT 1 WITHOUT ACCESS TO CREDIT
optset('cdpsolve','maxit',50);
[v01,xo1,cv01,cx01,sres1,resid1]=cdpsolve(model01,fspace01,...
'newton',v01,x01);

% DEFINE FUNCTION SPACE FOR APPROXIMATION AGENT 2
smin02  = sstar02-pctmin*sstar02; % lower limit indiv.2
smax02  = sstar02+pctmax*sstar02; % upper limit indiv.2
fspace02 = fundefn('spli',n(2),smin02,smax02); % fn.space2

% INITIALIZE POLICY AND VALUE FUNCTIONS AGENT 2
s = funnode(fspace02); % state collocation nodes
x02 = (xstar02/sstar02)*s; % initial policy fn.2
v02 = vstar02 + pstar02*(s-sstar02); % initial value fn.2

% SOLVE BELLMAN EQUATION AGENT 2 WITHOUT ACCESS TO CREDIT
optset('cdpsolve','maxit',50);
[v02,x02, cv02,cx02,sres2,resid2]=cdpsolve(model02,fspace02,...
'newton',v02,x02);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% SOLVE INVIDIDUAL LOAN MODEL FOR EACH AGENT (each agent
% repays all the time)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% PACK MODEL STRUCTURE AGENT1
model15.ffunc = 'f5'; % reward function
model15.gfunc = 'g5'; % transition function
model15.gOfunc= 'gj01'; % transition fn. w/o debt
model15.bfunc = 'b5'; % bound function
model15.discount = delta; % discount factor
model15.e = e; % shocks
model15.w = w; % probabilities

% PACK MODEL STRUCTURE AGENT2
model16.ffunc = 'f6'; % reward function
model16.gfunc = 'g6'; % transition function
model16.g0func= 'gj02'; % transition fn. w/o debt
model16.bfunc = 'b6'; % bound function
model16.discount = delta; % discount factor
model16.e = e; % shocks
model16.w = w; % probabilities

% COMPUTE CERTAINTY EQUIVALENT STEADY-STATE
const = 1/(delta*((1+kappa)-delta*kappa*(1+inter)));

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\[
\begin{align*}
x_{\text{star}11} &= ((\text{const-}\gamma_{\text{l}})/(\beta_{\text{l}}\psi_{\text{l}}))^{(1/(\beta_{\text{l}}-1))}; \\
x_{\text{star}22} &= ((\text{const-}\gamma_{\text{2}})/(\beta_{\text{2}}\psi_{\text{2}}))^{(1/(\beta_{\text{2}}-1))}; \\
s_{\text{l}1}\text{star} &= \gamma_{\text{l}}x_{\text{star}11} + x_{\text{star}11}\beta_{\text{l}}\psi_{\text{l}}; \\
s_{\text{l}2}\text{star} &= \gamma_{\text{2}}x_{\text{star}22} + x_{\text{star}22}\beta_{\text{2}}\psi_{\text{2}}; \\
s_{\text{2}2}\text{star} &= 0; \\
c_{\text{star}11} &= (1-\kappa\text{a}*\text{inter})*s_{\text{l}1}\text{star}11 - (1+\text{inter})*s_{\text{2}2}\text{star} - x_{\text{star}11}; \\
c_{\text{star}22} &= (1-\kappa\text{a}*\text{inter})*s_{\text{l}1}\text{star}22 - (1+\text{inter})*s_{\text{2}2}\text{star} - x_{\text{star}22}; \\
upr_{\text{1}1} &= c_{\text{star}11}^{-\alpha_{\text{1}}}; \\
upr_{\text{2}2} &= c_{\text{star}22}^{-\alpha_{\text{2}}}; \\
p_{\text{2}2}\text{star} &= -(1+\text{inter})*\text{upr}_{\text{1}1}; \\
p_{\text{2}2}\text{star} &= -(1+\text{inter})*\text{upr}_{\text{2}2}; \\
p_{\text{l}1}\text{star} &= (1-\kappa\text{a}*\text{inter})*\text{upr}_{\text{1}1} + \delta\kappa\text{a}*p_{\text{2}2}\text{star}11; \\
p_{\text{l}2}\text{star} &= (1-\kappa\text{a}*\text{inter})*\text{upr}_{\text{2}2} + \delta\kappa\text{a}*p_{\text{2}2}\text{star}22; \\
v_{\text{star}11} &= f_{\text{5}}([s_{\text{l}1}\text{star}, x_{\text{star}11}], (1-\delta)); \\
v_{\text{star}22} &= f_{\text{6}}([s_{\text{l}2}\text{star}, x_{\text{star}22}], (1-\delta)); \\
[x_{\text{star}11}, x_{\text{star}22}, s_{\text{l}1}\text{star}11, s_{\text{l}1}\text{star}22, s_{\text{2}2}\text{star}]
\end{align*}
\]

% DEFINE FUNCTION SPACE FOR APPROXIMATION AGENT 1
s_{\text{min}5} = \{s_{\text{min}01} \ \text{pctmin}3+s_{\text{2}2}\text{star}\}; % lower limit agent 1
s_{\text{max}5} = \{s_{\text{max}01} \ \text{pctmax}3+s_{\text{2}2}\text{star}\}; % upper limit agent 1
f_{\text{space}5} = \text{fundef}(\{'\text{spli}',\{n(1),n(3)\}, s_{\text{min}5}, s_{\text{max}5}\}); % fn.space1

% DEFINE FUNCTION SPACE FOR APPROXIMATION AGENT 2
s_{\text{min}6} = \{s_{\text{min}02} \ \text{pctmin}3+s_{\text{2}2}\text{star}\}; % lower limit agent 2
s_{\text{max}6} = \{s_{\text{max}02} \ \text{pctmax}3+s_{\text{2}2}\text{star}\}; % upper limit agent 2
f_{\text{space}6} = \text{fundef}(\{'\text{spli}',\{n(2),n(3)\}, s_{\text{min}6}, s_{\text{max}6}\}); % fn.space2

% INITIALIZE POLICY AND VALUE FUNCTIONS AGENT 1
s_{\text{coord}5} = \text{funnode}(f_{\text{space}5}); % state collocation nodes1
s_{5} = \text{cgrid}(s_{\text{coord}5});
\begin{align*}
x_{5} &= (x_{\text{star}11}/s_{\text{l}1}\text{star}11)*s_{5}(;,:1); \quad % initial policy fn. 1 \\
v_{5} &= v_{\text{star}11} + p_{\text{l}1}\text{star}11*(s_{5}(;,:1)-s_{\text{l}1}\text{star}11) + p_{\text{2}2}\text{star}11*... \quad % initial value func.1 \\
(s_{5}(;,:2)-s_{2}\text{star});
\end{align*}

% INITIALIZE POLICY AND VALUE FUNCTIONS AGENT 2
s_{\text{coord}6} = \text{funnode}(f_{\text{space}6}); % state collocation nodes2
s_{6} = \text{cgrid}(s_{\text{coord}6});
\begin{align*}
x_{6} &= (x_{\text{star}22}/s_{\text{l}2}\text{star}22)*s_{6}(;,:1); \quad % initial policy fn. 2 \\
v_{6} &= v_{\text{star}22} + p_{\text{l}2}\text{star}22*(s_{6}(;,:1)-s_{\text{l}2}\text{star}22) + p_{\text{2}2}\text{star}22*... \quad % initial value func.2 \\
(s_{6}(;,:2)-s_{2}\text{star});
\end{align*}

% SOLVE BELLMAN EQUATION AGENT 1(model always with credit)
disp('Solving Bellman for 1 always repaying when 2 defaults')
optset('cdpsolve','tol',1.0e-007);
optset('cdpsolve','maxit',500);
[v5,x5, cv5, cx5, sres5, resid5]=cdpsolve(model5,fspace5,...
'newton',v5,x5);

% SOLVE BELLMAN EQUATION AGENT 2 (always with credit)
disp('Solving Bellman for 2 always repaying when 1 defaults')
optset('cdpsolve','tol',1.0e-007);
optset('cdpsolve','maxit',500);
[v6,x6, cv6, cx6, sres6, resid6]=cdpsolve(model6,fspace6,...
'newton', v6,x6);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% 1-SOLVE CHOICE BETWEEN DEFAULT AND REPAYMENT WHEN THE OTHER
AGENT DEFAULTS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

vv01 = v01(:,ones(1,n(3))); % expand value fn. of agent
1 (default option) to 2 dimensional space
v001 = vv01(:);
cv01 = funfitxy(fspace5,s5,v001);

vv02 = v02(:,ones(1,n(3))); % expand value fn. of agent
2 (default option) to 2 dimensional space
v002 = vv02(:);
cv02 = funfitxy(fspace6,s6,v002);

% SOLVE BELLMAN EQUATION FOR CHOICE OF DEFAULT OR REPAYMENT
AGENT 1 WHEN 2 DEFAULTS
%---CASE A---%
disp('Solving Bellman for 1 choice default-repayment,
when 2 defaults')
optset('cdpsolv3','tol',1.0e-007);
optset('cdpsolv3','maxit',500);
optset('cdpsolv3','maxstep',0);
[v5,x5, cv5, cx5, R5]=cdpsolv3(model5,fspace5,'funcit',v5,x5,cv01);
xp=reshape(x5,n(1),n(3)); % expand optimal action to game
dimensions
prov=ones(n(2),1);
xppp=kron(prov,xp);
x5=xppp(:);

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% SOLVE BELLMAN EQUATION FOR CHOICE OF DEFAULT OR REPAYMENT
AGENT 2 WHEN 1 DEFAULTS
%----CASE B----

disp('Solving Bellman for 2 choice default-repayment,
when 1 defaults')
optset('cdpsolv3','tol',1.0e-007);
optset('cdpsolv3','maxit',500);
optset('cdpsolv3','maxstep',0);

[v6,x6,cv6,cx6,R6]=cdpsolv3(model6,fspace6,'funcit',v6,x6,cv02);

x66=x6'; % expand optimal action to game dimensions
x666=x66(ones(1,n(1)),:);
x6=x666(:);

% % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % 2-SOLVE CHOICE WHEN OTHER AGENT REPAYS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
q=prod(n);

% PACK MODEL STRUCTURE AGENT1
model7.ffunc = 'ft1';  % reward function
model7.gfunc = 'gt1';  % transition function
model7.g0func= 'gj01'; % transition function w/o debt
model7.bfunc = 'bt1';  % bound function
model7.discount = delta; % discount factor
model7.e = e; % shocks
model7.w = w; % probabilities

% PACK MODEL STRUCTURE AGENT2
model8.ffunc = 'ft2';  % reward function
model8.gfunc = 'gt2';  % transition function
model8.g0func= 'gj02'; % transition function w/o debt
model8.bfunc = 'bt2';  % bound function
model8.discount = delta; % discount factor
model8.e = e; % shocks
model8.w = w; % probabilities

% COMPUTE CERTAINITY EQUIVALENT STEADY-STATE BOTH AGENTS FOR
THE CASE WHEN BOTH AGENTS REPAY
constl = 1/(delta*(1-
kappa*rho*inter+kappa*rho*(1+inter)*(1-delta)));
const2 = 1/(delta*(1-kappa*(1-rho)*inter+kappa*(1-
rho)*(1+inter)*(1-delta)));
xstarl = ((constl-gammal)/(betal*psil))^(1/(betal-1));
\begin{verbatim}
slstar = g a i n xstar1 + xstar1^beta1*psil;
xstar2 = ((const2-gamma2)/(beta2*psi2))^(1/(beta2-1));
xstar = [xstar1 xstar2];
s2star = gamma2*xstar2 + xstar2^beta2*psi2;
s3star = 0;
cstar1 = slstar-xstar1-s3star*rho*(1+inter)-kappa*(s1star+s2star)*rho*inter;
cstar2 = s2star-xstar2-s3star*(1-rho)*(1+inter)-kappa*(s1star+s2star)*(1-rho)*(1+inter);
uprim1 = cstar1^-alpha1;
uprim2 = cstar2^-alpha2;
p3star1 = -uprim1*rho*(1+inter);
p1star1 = uprim1*(1-kappa*rho*inter-delta*kappa*rho*(1+inter));
p2star1 = -
uprim1*(kappa*rho*inter+delta*kappa*rho*(1+inter));
p3star2 = -uprim2*(kappa*1-rho)*inter+delta*kappa*(1+inter)*inter-delta*(1+inter)*(1-rho));
p3star2 = -uprim2*(1+inter)*(1-rho);
vstar1 = ft1([slstar s2star s3star],xstar)/(1-delta);
vstar2 = ft2([slstar s2star s3star],xstar)/(1-delta);
[vstar1, vstar2, p1star1, p2star1, p1star2, p2star2, p3star1, p3star2]

% DEFINE FUNCTION SPACE FOR APPROXIMATION BOTH AGENTS
smin = [smin01 smin02 pctmin3+s3star]; % lower limit agent 1
smax = [smax01 smax02 pctmax3+s3star]; % upper limit agent 2
fspace1 = fundefn('spl1',n,smin,smax); % function space game

% INITIALIZE POLICY AND VALUE FUNCTIONS
scoord1 = funnode(fspace1); % state collocation nodes game
s1 = cgrid(scoord1);

% SOLVE BELMAN EQUATION ALWAYS REPAYING
x1 = (xstar1/s1star)*s1(:,1); % initial policy fn. X1
x2 = (xstar2/s2star)*s1(:,2); % initial policy fn. X2
x = [x1 x2]; % initial policy vector
v1 = vstar1 + plstar1*(s1(:,1)-s1star) + ...
p1star1*(s1(:,2)-s2star) + p3star1*(s1(:,3)-s3star); % initial value function agent 1
v2 = vstar2 + plstar2*(s1(:,1)-s1star) + ...
\end{verbatim}
\[ p_2^{\star 2}(s_7(:,2) - s_2^{\star}) + p_3^{\star 2}(s_7(:,3) - s_3^{\star}); \]

initial value function agent 2

% SOLVE BELLMAN EQUATION AGENT 1 (BOTH ALWAYS REPAYING)

disp('Solve Bellman for 1 always repaying, when 2 repays')
optset('cdpsl','tol',1.0e-007);
optset('cdpsl','maxit',500);
[v11,x11,cv11]=cdpsl(model7,fspace7,'funcit',vl,x);

% SOLVE BELLMAN EQUATION AGENT 2 (BOTH ALWAYS REPAYING)

disp('Solve Bellman for 2 always repaying, when 1 repays')
optset('cdps2','tol',1.0e-007);
optset('cdps2','maxit',500);
[v12,x12,cv12]=cdps2(model8,fspace7,'funcit',v2,x);

vvv01 = v01(:,ones(1,n(2)),ones(1,n(3)));  % expand value
func.1 (default option) to 3 dimensional space
v0001 = vvv01(:);
cv001 = funfitxy(fspace7,s7,v0001);  % expand basis coefic.1 to
new dimensional space

w=w02';  % rearrange value func.2 (default) for expansion to new dimensional space
w1=w(ones(n(1),1),:;ones(n(3),1));  % expand value func.2 (default option) to 3 dimensional space
v0002 = w1(:);
cv002 = funfitxy(fspace7,s7,v0002);  % expand basis coefic.2 to
new dimensional space

v55=reshape(v5,n(1),n(3));
v555=kron(prov,v55);  % expand value fn.of choice of 1 when
2 defaults to game dimensions
v5=v555(:);
R55=reshape(R5,n(1),n(3));
R555=kron(prov,R55);
R5=R555(:);  % expand repayment var.of choice of 1 when 2 defaults to game dimensions

v66=v6';
v666=v66(ones(1,n(1)),:);
v6=v666( );  % expand value fn.of choice of 2 when
1 defaults to game dimensions
R66=R6';
R666=R66(ones(1,n(1)),:);
R6=R666( );  % expand repayment var.of choice of 2 when 1 defaults to game dimensions
% FIND NASH EQUILIBRIUM OF THE GAME

R=ones(q,2); % initialize repayment variable as repayment
model7.bfunc='bttl';
model8.bfunc='btt2';
x=[x11(:,1) x12(:,2)]; % initialize action w/ previous result

% initialize iterations
xold=x;

% SOLVE BELLMAN EQUATION FOR CHOICE OF DEFAULT OR REPAYMENT
AGENT 1 WHEN 2 REPAYS (actions of 2 given)
if any (R(:,2)'==1)
    optset('cdps31','tol',1.0e-007);
    optset('cdps31','maxit',500);
    optset('cdps31','maxstep',0);
    [vll, xll, cvll, cxll, Rll, sresl, residl]=cdps31(model7, fspace7,...
        'funcit', vll, xll, R, cv001);
end

i=R(:,2)==0; % pick cases when agent 2 defaults
and...
    R11(i,1)=R5(i); % rep. Decision of 1 is decision
    x11(i,1)=x5(i); % optimal x of 1 is taken as optimal
                  % store original actions

AGENT 2 WHEN 1 REPAYS (actions of 1 given)
    if any (R(:,1)'==1)
        optset('cdps32','tol',1.0e-007);
        optset('cdps32','maxit',500);
        optset('cdps32','maxstep',0);
        [vl2, xl2, cv22, cx22, R22, sres2, resid2]=cdps32(model8, fspace7,...
            'funcit', vl2, xl2, R, cv002);
    end

    i=R(:,1)==0; % pick cases when agent 1 defaults
    and...
    R22(i,2)=R6(i); % repayment decision of 2 is decision
    x12(i,2)=x6(i); % optimal x of 2 is taken as optimal

R=[R11(:,1) R22(:,2)]; %pick rep decision from each individual maximization
x=[x11(:,1) x12(:,2)]; %pick policy action from each individual maximization

change=norm(x-xold);
fprintf ('%5i %10.Oe
',it,change)
if change<1.0e-007, break,end; %check that change in optimal action is small enough
x1l(:,2)=x(:,2); % update given actions of partner borrower
x12(:,1)=x(:,1); % update given actions of partner borrower
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%CHOOSE BETWEEN WORLD WITH ACCESS TO CREDIT AND WORLD WITHOUT IT
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

x01=x01(:,ones(1,n(2)),ones(1,n(3)))); % expand solution of w/o credit to game dimensions
x01=x01(:);
v01=v0001;

x02=x02'; % expand solution of w/o credit to game dimensions
x02=x02(ones(n(1),1,:),ones(n(3),1)));
x02=x02(:);
v02=v0002;

xmir=[x01 x02];%initialize action with sol.of w/o credit
vmir1=v01;%initialize val.fn with sol.of w/o credit1
vmir2=v02;%initialize val.fn with sol.of w/o credit2

Rmir=R; % take repayment decision from game
i=Rmir(:,1)==1;
xmir(i,1)=x(i,1); % take action from game if borrower picks credit
vmir(i,1)=v11(i,1); % take value fn. from game if borrower picks credit
i=Rmir(:,2)==1;
xmir(i,2)=x(i,2); % take action from game if borrower picks credit
vmir(i,2)=v12(i,1); % take value fn. from game if borrower picks credit

i=v11>v01;% choose bet.credit & w/o credit agent 1
vmirl(i)=vll(i);
xmir(i,1)=x(i,1);
Rmir(i,1)=1;

if vl2>v02; % choose bet. credit & w/o credit agmnt 2
  vmir2(i)=vl2(i);
xmir(i,2)=x(i,2);
Rmir(i,2)=1;
end

load handel
sound(y)
APPENDIX D

MATLAB UTILITY USED TO SOLVE GROUP LENDING MODEL

This program uses several of the utilities developed by Miranda and Fackler published as: "Compecom: Toolbox for Matlab", http://www.agecon.ag.ohio-state.edu/ae802/default.htm. It also uses the subroutine cdpsolv3, presented in Appendix B and subroutines cdpsl and cdps31, presented in Appendix D and E.

function [v,x,cv,cx,sres,resid]=cdpsl(model,fspace,alg,v,x)

% CDPSOLVE Solves continuous-state/action dynamic program
% (Solves loan model always repaying (non-default) for
% agent 1, for a fixed action of agent 2)
% Usage
% [v,x,cv,cx,sres,resid] = cdpsolve(model,fspace,alg,v,x)
% Input
% model : name of model structure
% fspace : name of projection space structure
% alg : algorithm used (funcit or newton)
% v : initial guess for value function
% x : initial guess for optimal controls
% Output
% v,cv : value function and projection coefficients
% x,cx : optimal controls and projection coefficients
% sres : residual evaluation points
% resid : Bellman equation residuals
% Options
% tol : convergence tolerance
% maxit : maximum number of iterations

% SET CONVERGENCE PARAMETER DEFAULTS
maxit = optget('cdpsl','maxit',50);
tol = optget('cdpsl','tol',1.0e-003);

% SELECT APPROPRIATE SOLUTION ROUTINE
if fspace.d==1
    vmax = 'vmax1';
else
vmax = 'vmax2';
end

% EXTRACT MIN AND MAX STATE
n = fspace.n;
smin = fspace.smin;
smax = fspace.smax;

% COMPUTE COLLOCATION NODES AND INTERPOLATION MATRIX
phi = funbas(fspace);  % interpolation matrix
scoord = funnode(fspace);  % state collocation nodes
s  = cgrid(scoord);  % state collocation nodes

% PRINT ITERATION METHOD
switch alg
    case 'funcit'
        disp('Solve Bellman equation via function iteration for loan model 1')
    case 'newton'
        disp('Solve Bellman equation via Newton method')
    otherwise
        error('dp solution algorithm must be ''funcit'' or ''newton''')
end

% PERFORM FUNCTION OR NEWTON ITERATIONS
tic
    cv = funfitxy(fspace,s,V);  % initial basis coefficients
    for it=1:maxit
        cvold = cv;  % store old value
        switch alg
            case 'funcit'
                [v,x] = feval(vmax,s,x,cv,fspace,model);  % solve Bellman equation
                cv = funfitxy(fspace,s,v);  % update basis coefficients
            case 'newton'
                [v,x,vderc] = feval(vmax,s,x,cv,fspace,model);  % solve Bellman equation
        end
    end
}
cv = cvold - [phi-vderc]\[phi*cv-v];  % update basis coefficients
end
change = norm(cv-cvold);  % compute change
fprintf ('%5i %10.1e
',it,change) % print progress
if change<tol, break, end;
% convergence check
toc

% CHECK STATE TRANSITION SATISFY BOUNDS
snmin= inf;
snmax=-inf;
for k=1:length(model.e);
  g = feval(model.gfunc,s,x,model.e(k));
  snmin = min(snmin,min(g));
  snmax = max(snmax,max(g));
end
if snmin<smin, disp('Warning: extrapolating beyond smin'), end;
if snmax>smax, disp('Warning: extrapolating beyond smax'), end;

% COMPUTE POLICY FUNCTION COEFFICIENTS
% cx = funfitxy(fspace,s,x);

% COMPUTE RESIDUAL
% nr = 10*n;
% [sr,sres] = nodeunif(nr,smin,smax);
% xr = funeval(cx,fspace,sr);
% vr = feval(vmax,sr,xr,cv,fspace,model);
% resid = vr-funeval(cv,fspace,sr);

function [v,x,vc] = vmaxl(s,x,cv,fspace,model);
% Solves Bellman equation at state nodes
n = length(cv);
m = length(model.e);
maxit = 500; % maximum iterations
tol = 1.0e-009; % convergence tolerance

[xl,xu] = feval(model.bfunc,s); % compute bounds and derivatives

for it=1:maxit
    xold = x;
    [f,fs,fx,fxx] = feval(model.ffunc,s,x);
    v = f; vx = fx; vxx = fxx;
    for k=1:m
        [g,gs,gx,gxx] = feval(model.gfunc,s,x,model.e(k));
        vnval = funeval(cv,fspace,g);
        vnder = funeval(cv,fspace,g,1);
        vnsec = funeval(cv,fspace,g,2);
        v = v + model.discount*model.w(k)*vnval;
        vx = vx + model.discount*model.w(k)*vnder.*gx;
        vxx = vxx + model.discount*model.w(k)*(vnder.*gxx + vnsec.*gx.*gx);
    end
    x = x - vx./vxx; x = min(x,xu); x = max(x,xl);
    if norm(x-xold,inf)<tol, break, end;
end

% if nargout<3, return, end;
% vc = zeros(length(s),n);
% for k=1:m
%   g = feval(model.gfunc,s,x,model.e(k));
%   phinxt = funbas(fspace,g);
%   vc = vc + model.discount*model.w(k)*phinxt;
% end

function [V,x,vc] = vmax2(s,x,cv,fspace,model)

% Solves Bellman equation at state nodes

maxit = 500; % maximum iterations
tol = 1.0e-009; % convergence tolerance
n = size(s,1);
\[
s = \text{size}(s,2);
\]
\[
m = \text{length}(\text{model.e});
\]
\[
[xl,xu] = \text{feval}(\text{model.bfunc},s); \quad \% \text{compute bounds}
\]

\begin{verbatim}
for it=1:maxit
    xold = x;
    [f,fds,fdx,fdxx] = \text{feval}(\text{model.ffunc},s,x);
    V = \text{zeros}(n,1);
    F = \text{zeros}(n,1);
    J = \text{zeros}(n,1);
    for k=1:m
        [g,gds,gdx,gdxx] = \text{feval}(\text{model.gfunc},s,x,\text{model.e}(k));
        V = \text{funeval}(cv,fspace,g);
        p = \text{funjac}(cv,fspace,g);
        pd = \text{funhess}(cv,fspace,g);
        V = V + \text{model.w}(k)*v;
        F = F + \text{model.w}(k)*\text{sum}(p.*gdx,2);
        J = J + \text{model.w}(k)*\text{sum}(p.*gdxx,2);
        for is=1:ns
            for js=1:ns
                J = J + \text{model.w}(k)*pd(:,is,js).*gdx(:,is).*gdx(:,js);
            end
        end
        V = f + \text{model.discount}*V;
        F = fdx + \text{model.discount}*F;
        J = fdxx + \text{model.discount}*J;
        delx = -F./J;
        delx = [delx \text{zeros}(n,1)];
        x = x + delx;
        x = [\text{min}(x(:,1),xl(:,1)) x(:,2)];
        x = [\text{max}(x(:,1),xu(:,1)) x(:,2)];
        if \text{norm}(x-xold)< tol, break, end;
    end
if nargout<3, return, end;
vc = \text{zeros}(n,n);
for k=1:m
    g = \text{feval}(\text{model.gfunc},s,x,\text{model.e}(k));
    phinxt = \text{funbas}(fspace,g);
    vc = vc + \text{model.discount}*\text{model.w}(k)*phinxt;
end
\end{verbatim}
This program uses several of the utilities developed by Miranda and Fackler published as: "Compecom: Toolbox for Matlab", http://www.agecon.ag.ohio-state.edu/ae802/default.htm. It also uses the subroutine cdpsolv3, presented in Appendix B and subroutines cdps1 and cdps31, presented in Appendix D and E.

function [v,x,cv,cx,R,sres,resid] = cdps31(model,fspace,alg,v,x,R,cv0)

% CDPSOLV31 Solves continuous-state/action dynamic program
% Solves choice between repayment and default for agent 1
% for a given actions of agent 2
%
% Usage
% [v,x,cv,cx,R,sres,resid] = cdpsolve(model,fspace,alg,v,x)
% Input
% model : name of model structure
% fspace : name of projection space structure
% alg : algorithm used (funcit or newton)
% v : initial guess for value function
% x : initial guess for optimal controls
% Output
% v,cv : value function and projection coefficients
% x,cx : optimal controls and projection coefficients
% sres : residual evaluation points
% resid : Bellman equation residuals
% Options
% tol : convergence tolerance
% maxit : maximum number of iterations

% SET CONVERGENCE PARAMETER DEFAULTS
maxit = optget('cdps31','maxit',100);
tol = optget('cdps31','tol',sqrt(eps));

% EXTRACT MIN AND MAX STATE
n = fspace.n;
smin = fspace.smin;
smax = fspace.smax;

% COMPUTE COLLOCATION NODES AND INTERPOLATION MATRIX
phi = funbas(fspace);  % interpolation matrix
scoord = funnode(fspace);  % state collocation nodes
s = cgrid(scoord);  % state collocation nodes

% PRINT ITERATION METHOD
switch alg
    case 'funcit'
        disp('Solve Bellman equation via function iteration for default choice 1')
    case 'newton'
        disp('Solve Bellman equation via Newton method')
    otherwise
        error('dp solution algorithm must be ''funcit'' or ''newton''')
end

% PERFORM FUNCTION OR NEWTON ITERATIONS
tic
    cv = funfitxy(fspace,s,V);  % initial basis coefficients
    for it=1:maxit
        cvold = cv;  % store old value
        switch alg
            case 'funcit'
                [v,x,R] = vmax(s,X,R,cv,cv0,fspace,model);  % solve Bellman equation
                cv = funfitxy(fspace,s,v);  % update basis coefficients
            case 'newton'
                [v,x,R,vderc] = vmax(s,x,R,cv,cv0,fspace,model);  % solve Bellman eqn
                cv = cvold - [phi-vderc]\[phi*cv-v];  % update basis coefficients
        end
        change = norm(cv-cvold);  % compute change
        fprintf ('%5i %10.1e\n',it,change)  % print progress
        if change<tol, break, end;  % convergence
check
end
toc

% CHECK STATE TRANSITION SATISFY BOUNDS
snmin= inf;
snmax=-inf;
for k=1:length(model.e);
    g = feval(model.gfunc,s,X,model.e(k),R);
    snmin = min(snmin,min(g));
    snmax = max(snmax,max(g));
end
if snmin<smin, disp('Warning: extrapolating beyond smin'), end;
if snmax>smax, disp('Warning: extrapolating beyond smax'), end;

% COMPUTE POLICY FUNCTION COEFFICIENTS
if nargout<4, return, end;
   cx = funfitxy(fspace,s,X);

% COMPUTE RESIDUAL
if nargout<6, return, end;
   nr = 10*n;
   [sr,sres] = nodeunif(nr,smin,smax);
   xr = funeval(cx,fspace,sr);
   vr = vmax(sr,xr,R,cv,cv0,fspace,model);
   resid = vr-funeval(cv,fspace,sr);

function [V,x,R,vc] = vmax(s,x,R,cv,cv0,fspace,model)
   % Solves Bellman equation at state nodes

maxstep = optget('cdps31','maxstep',0);
maxit = optget('cdps31','maxit',100);
tol = optget('cdps31','tol',1.0e-003);

n   = size(s,1);
ns  = size(s,2);
m   = length(model.e);

[xl,xu] = feval(model.bfunc,s); % compute bounds

% npct=400;
% V = -inf*ones(n,1);
% for ip=1:npct
%    xtmp = xl + ((ip-1)/(npct-1))*(xu-xl);
%    f = feval(model.ffunc,s,xtmp);
\% Vtmp = zeros(n,1);
\% for k=1:m
\% g = feval(model.gfunc,s,xtmp,model.e(k),R);
\% v = funeval(cv,fspace,g);
\% v0 = funeval(cv0,fspace,g);
\% Vtmp = Vtmp + model.w(k)*max(v,v0);
\% end
\% Vtmp = f + model.discount*Vtmp;
\% j=Vtmp>V;
\% V(j)=Vtmp(j);
\% x(j)=xtmp(j);
\%end

for it=1:maxit
xold = x;
[f,fds,fdx,fdxx] = feval(model.ffunc,s,x);
V = zeros(n,1);
F = zeros(n,1);
J = zeros(n,1);
for k=1:m
[g,gds,gdx,gdxx] = feval(model.gfunc,s,x,model.e(k),R);
[g0,gds0,gdx0,gdxx0] = feval(model.gOfunc,s(:,l),x(:,l),model.e(k));
v = funeval(cv,fspace,g);
p = funjac(cv,fspace,g);
p0 = funjac(cv0,fspace,g);
vd0 = funhess(cv0,fspace,g);
gd0 = [gdx0 zeros(n,2)];
gdxx0 = [gdxx0 zeros(n,2)];
i=v0>v;
v(i)=v0(i);
p(i,:)=p0(i,:);
pd(i,:,:) = pd0(i,:,:);
gdx(i,:) = gd0(i,:);
gdxx(i,:) = gdxx0(i,:);
R(i,1)=0;
V = V + model.w(k)*v;
F = F + model.w(k)*sum(p.*gdx,2);
J = J + model.w(k)*sum(p.*gdxx,2);
for is=1:ns
for js=1:ns
J = J + ...
model.w(k)*pd(:,is,js).*gdx(:,is).*gdx(:,js);
end
V = f + model.discount*V;
F = fdx + model.discount*F;
J = fdxx + model.discount*J;
delx = -F./J;
delx = [delx zeros(n,1)];
x = x + delx;
x = [min(x(:,1),xu(:,1)) x(:,2)];
x = [max(x(:,1),xl(:,1)) x(:,2)];
delx = x-xold;
change=norm(delx);
% fprintf('%5i %10.1e
',it,change)
if change< tol, break, end;
for is=1:maxstep
    [f,fdx,fdxx] = feval(model.ffunc,s,x);
    Vnew = zeros(n,1);
    for k=1:m
        [g,gdx,gdxx] = 
        feval(model.gfunc,s,x,model.e(k),R);
        v = funeval(cv,fspace,g);
        v0 = funeval(cv0,fspace,g);
        i=v0>v;
        v(i)=v0(i);
        R(i,1)=0;
        Vnew = Vnew + model.w(k)*v;
    end
    Vnew = f + model.discount*Vnew;
    if any(Vnew<V)
        delx = delx/0.5;
        x = x-delx;
    else
        break
    end
end
if nargout<4, return, end;
vc = zeros(n,n);
for k=1:m
    g = feval(model.gfunc,s,x,model.e(k),R);
    phinx = funbas(fspace,g);
    vc = vc + model.discount*model.w(k)*phinxt;
end
if norm(delx)>tol, disp('Failure to converge in Newton'), end;
BIBLIOGRAPHY


