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EXPERIMENTAL STUDY OF THE INDUCTANCE
OF PINNED VORTICES IN YBa$_2$Cu$_3$O$_{7-\delta}$ AND La$_{1.85}$Sr$_{0.15}$CuO$_4$
SUPERCURRENTING THIN FILMS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
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* * * * *

The Ohio State University
2000

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ABSTRACT

Using a two-coil mutual inductance method, we have measured the inductance of pinned vortices in YBa$_2$Cu$_3$O$_{7-\delta}$ and La$_{1.85}$Sr$_{0.15}$CuO$_4$ thin films. We present data on twelve films grown using three different techniques: coevaporation with post annealing, pulsed laser deposition, and RF sputtering. We present a detailed analysis of the data using a model based on a low density of planar defects and a high density of extended defects. From our analysis, we conclude that pulsed laser deposition produces the highest density of pinning defects while coevaporation produces the lowest density. The films studied have sufficient planar defects to accommodate roughly 500 Gauss of vortices. The linear extended defects extinguish superconductivity within a radius of 4 Å to 8 Å. The density of extended defects is roughly $7 \times 10^4 \, \mu\text{m}^2$. The magnetic field dependence of the inductance is produced by vortex interaction forces. The temperature dependence of the inductance is produced by thermally induced vortex motion. We demonstrate how the temperature dependence of the inductance can be used to measure the amplitude of thermally induced noise currents within the superconductor. We predict that below 1.5 K quantum mechanical effects should be apparent in the temperature dependence of the inductance.
To my grandfather, Robert J. Till, who taught me to reason
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CHAPTER 1

INTRODUCTION

This work is a detailed analysis of the inductance of pinned vortices in high temperature superconductors. In this study, we use a two-coil mutual inductance method to measure the inductance of superconducting thin films. In the absence of an externally applied magnetic field, this technique yields information about the superfluid density of the film. In the presence of an externally applied magnetic field, this technique yields information about the nature of the defects responsible for vortex pinning.

In this work, we present data on ten $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ and two $\text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4$ films. The YBCO films were grown using three different techniques: coevaporation with post annealing, pulsed laser deposition (PLD), and RF sputtering. In this study, we show that coevaporated films have the lowest density of pinning defects while PLD films have the highest density.

Many other techniques have been used to probe the inductance of pinned vortices. The two most extensively used techniques are four terminal measurements and microwave resonance measurements. The two-coil method has two advantages over these techniques. First, it is nondestructive. Unlike other measurements, it requires no patterning and no direct electrical contact with the film. Second, it has a much higher
resolution because it measures only the inductance of the film whereas the other
techniques measure the inductance of the film and the physical inductance of the stripline
or resonance cavity. Using a detailed numerical analysis of the experimental setup, we
are able to measure the magnetic penetration depth to an uncertainty determined
primarily by the uncertainty in the film thickness.

Using this technique, we can directly measure the complex conductivity of the
film at frequencies ranging from 100 Hz to 1 MHz. Using the model developed by Clem
and Coffey, we can extract the Labusch parameter, the linear restoring force constant per
unit length, of the pinned vortices from the measured conductivity.

From this data, we construct a model of the vortex inductance based on a low
density of planar defects (e.g. grain boundaries) and a high density of extended defects
(e.g. screw dislocations). Using this model, we can explain the temperature and magnetic
field dependencies of the Labusch parameter as well as calculate the critical current. We
have measured the critical current directly using the mutual inductance technique.

The magnetic field dependence of the Labusch parameter is governed by vortex
interactions. From our data we conclude that sputtered and PLD YBCO films contain
sufficient planar defects to accommodate 500 Gauss of vortices and sufficient extended
defects to accommodate about 15 Tesla of vortices before interaction forces unpin
vortices. The situation is less clear in LSCO films, which appear to have a significantly
lower defect density.

The temperature dependence of the Labusch parameter is governed by thermal
motion of the pinned vortices. At nonzero temperatures, randomly fluctuating
supercurrents drive the vortex along a random path about the pinning site. We show that
the temperature dependence of the Labusch parameter can be used to measure the amplitude of the noise currents in the film. We predict that below 1.5 K quantum effects should reduce the measured noise currents.
CHAPTER 2

EXPERIMENT

2.1 – High Tc Superconducting Thin Films

In this study, we present data on twelve high Tc superconducting films. We focus primarily on optimally doped YBa$_2$Cu$_3$O$_{7-\delta}$ films, but also present data on two La$_{1.85}$Sr$_{0.15}$CuO$_4$ films. The films were grown using three different techniques: coevaporation with post annealing, pulsed laser deposition (PLD), and RF sputtering. The films range from 400Å to 1100Å in thickness. They are grown on substrates that range from 10 mm to 15 mm in diameter. The films all have their c-axes aligned perpendicular to the substrate.

The two films given the most consideration, films 1 and 2 are both 500 Å thick PLD YBCO films grown on 1 cm square LaAlO$_3$. These films were grown by Rand Biggers at Wright Patterson Air Force Base. Films 3 – 5 and 9 – 12 are RF sputtered YBCO films grown on SrTiO$_3$ substrates. They were grown here at The Ohio State University by Brent Boyce and James Baumgardner. Film 6 is a YBCO film grown on a SrTiO$_3$ substrate by coevaporation with postannealing. We also discuss several other coevaporated films without presenting data here. The coevaporated films were grown at The Ohio State University by Kathleen Paget, Stefan Turneaure, and Brent Boyce. Films
7 and 8 are 1000 Å thick PLD La$_{1.85}$Sr$_{0.15}$CuO$_4$ films grown atop a 1000 Å LaSrAlO$_4$ buffer deposited on a SrTiO$_3$ substrate. They were grown by Weidong Si at Penn State University.

2.2 – Two-Coil Mutual Inductance Measurement

We use a two-coil mutual inductance method to measure the inductance of pinned vortices. This method has several advantages over the conventional four-terminal measurement and microwave measurements$^{1-8}$. First, the method is nondestructive. Four terminal measurements require that the film be patterned, while the mutual inductance method requires no film preparation. Second, the two-coil method has a much higher resolution than the four terminal measurement. While the four-terminal measurement measures the sum of the kinetic inductance of the electrons as well as the physical inductance of the strip line, the mutual inductance method measures only the kinetic inductance. Finally, the mutual inductance method does not require direct electrical contact with the film.

Fig. 1a shows a sketch of the mutual inductance apparatus used in these measurements. The film sits between two counter-wound coils set in the bore of a six Tesla superconducting magnet. The magnet can be varied between ±6 Tesla and is used to induce vortices in the film. The “Degaussing Coil” is used to probe the critical state as discussed in section 3.5. When an AC current is applied to the drive coil, the resulting magnetic field induces supercurrents in the film. The induced supercurrent causes the vortices to oscillate within their pinning sites. The flux in the receive coil is thus a sum of the fields created by the current in the drive coil, the current induced in the film, and
the oscillating vortices. Measuring the voltage across the receive coil measures the kinetic inductance of the superconducting electrons as well as the inductance of the oscillating vortices.

The entire apparatus sits inside an insulated, nonmagnetic stainless steel bucket. The bucket insulates the experiment from the liquid helium used to cool the magnet. The sample is cooled by transferring liquid helium into the bucket. In the earlier version of the experiment, the film was heated by blowing room temperature helium gas into the bucket. In the current version, it is heated electrically. The heater is made from a twisted pair of constantan wires attached to a copper foil wrapped around the experiment. This ensures uniform heating while producing no stray magnetic field.

Fig. 1b shows a schematic of the circuit used to measure the mutual inductance. The preamplifier is an Ithaco 565 low noise preamplifier with a noise level of less than 0.3 nV/√Hz. The lock-in amplifiers used (an EG&G/PAR 5208 for the drive current and an EG&G/PAR 5210 for the receive voltage) are both two-phase amplifiers. With this arrangement, we can measure the amplitude and phase of both the drive current and receive voltage, enabling us to calculate the complex mutual inductance. We can therefore measure not only the inductance of the film, but also any dissipation that may occur. The lock-in amplifiers have a frequency range of 0.5 Hz to 120 kHz, however, useful data can only be obtained down to about 100 Hz. Below 100 Hz, the signal cannot be resolved from the noise. The frequency range can be increased to 1 MHz by heterodyning the lock-in inputs. But, as discussed later, our data is independent of frequency over the entire accessible range. Consequently, we typically operate at 50 kHz to achieve a good signal to noise ratio without the necessity of heterodyning the signal.
The coils are made of 20 μm copper wire wound around nylon coil forms. The dimensions of the forms are known to ±100 μm. They consist of two counter-wound sections, which reduces the noise generated by vibrations of the receive coil in the static, nearly uniform six Tesla magnetic field. Typically, the drive coil has 2 layers with 40 turns per layer in each section, while the receive coil has 4 or 6 layers with 40 turns per layer in each section. This results in a resistance on the order of 100 Ω at room temperature and 1 Ω at 4.2 K. The self-inductance of the coils is on the order of 10 μH. This results in an impedance of 1 Ω to 10 Ω at our usual operating frequency of 50 kHz. The receive coils are designed to minimize noise. Adding more turns to the receive coil increases the signal, but also increases the self-inductance which can result in a resonance with the preamplifier.

The mutual inductor, $M_1$, is a 55.7 μH mutual inductor used to measure the current in the drive coil. It was made by wrapping two layers of approximately 50 turns of copper wire around an eighth inch diameter wooden rod. Its mutual inductance was calculated by replacing the two coil apparatus with a known resistance. The variable resistor, $R_1$, usually between 500 Ω to 50 kΩ, is used to attenuate the drive current and eliminate capacitive coupling between the coils by effectively reducing the impedance of the cryostat leads. The drive current can be varied between 1 μA and 1 A. This corresponds to an induced current density ranging from 10 A/cm$^2$ to $10^7$ A/cm$^2$. With the exception of the critical current measurements discussed in section 3.3, all data are taken in the low current linear regime in which the mutual inductance is independent of the
induced current density. Typical induced current densities are $10^4$ A/cm$^2$ at low T and $10^2$ A/cm$^2$ near $T_C$, roughly 3 orders of magnitude below the critical current density.

Fig. 2 shows typical temperature dependent mutual inductance data. The mutual inductance of the apparatus when no film is present, known as the initial position, is typically 50 nH to 5 μH depending on the coil configuration and substrate thickness. Above $T_C$, the film’s conductivity is undetectable and the mutual inductance is equal to the initial position. At low temperatures, the mutual inductance is 2 to 4 orders of magnitude smaller than the initial position. The change in mutual inductance is dependent upon the film’s conductivity and thickness. The mutual inductance typically shows a 50% to 200% increase between zero Kelvin and a few Kelvin of $T_C$ followed by a sharp increase at $T_C$. The imaginary mutual inductance, indicative of dissipation, is typically zero everywhere except near $T_C$ where it displays a large negative peak.
Figure 1 - Sketch of the two-coil mutual inductance apparatus. Fig. 1a is a cross-sectional view of the apparatus used. Fig. 1b is a schematic of the circuit used to measure the mutual inductance.
Figure 2 - Temperature dependence of the complex mutual inductance of film 1. The real mutual inductance is indicative of an inductive response while the imaginary mutual inductance is indicative of a dissipative response.
2.3 – Experimental Penetration Depth

To extract information about vortex dynamics from the measured data, we must first calculate the mutual inductance as a function of the magnetic penetration depth of the film. This subject is discussed in great detail in several papers. We begin by modeling the two coil apparatus. We assume a film of infinite radius with a uniform complex conductivity, $\sigma$. We model the coils as a set of coaxial circles each carrying an AC current, $I$. Further, we assume that the operating frequency is sufficiently low that the wavelength is long compared to any coil or film dimensions and the displacement current can thus be ignored.

With these assumptions, it is a straightforward procedure to solve Maxwell’s equations for the mutual inductance. Details of this solution are presented in Appendix B1. The result for the mutual inductance of the system is

$$M = 4\pi\mu_0 \int_0^{Q(q)} \frac{qQ(q)e^{-\sqrt{Q(q)+q}}\alpha_D(q)\alpha_R(q)\beta_D(q)\beta_R(q)}{(Q(q)+q)^2 e^{Q(q)d} - (Q(q)-q)^2 e^{-Q(q)d}} dq,$$

where $Q(q) = \sqrt{q^2 + i\mu_0 \omega \sigma}$. $h$ is the separation between the coils, $d$ is the film thickness and $\alpha_n(q)$ and $\beta_n(q)$ are geometrical factors determined by the coil configuration. This expression is then integrated numerically to obtain the complex mutual inductance as a function of the complex conductivity. The result is used to transform the experimentally measured mutual inductance into a conductivity.

To use the infinite film result, we must first account for the flux that passes around a film of finite radius. This we measure directly by replacing the film with a 150 $\mu$m thick lead foil with the exact shape of the film. The foil is then cooled to 4.2 K.
When the lead is superconducting, the thickness of the foil is about $10^4$ times the magnetic penetration depth of lead and the foil effectively blocks all magnetic field from passing through. The measured mutual inductance thus results from coupling around the outer edges of the foil and parasitic coupling between the leads in the cryostat. This mutual inductance is known as the zero position. To calculate the conductivity of our film, we simply subtract the zero position from the measured mutual inductance and compare the result to our numerical calculation. This procedure is accurate as long as the film radius is about three times greater than the coil radius. More details on this procedure are given in Refs. 10 and 11.

For our data in zero magnetic field, the real conductivity, $\sigma_1(T,B=0)$, is zero everywhere except within about 1 K of $T_c$. In this case, we can use London's equation to relate the imaginary conductivity to the magnetic penetration depth.

$$\sigma_2(T,B=0) = \frac{1}{\mu_0 \omega \lambda_{ab}^2(T)}.$$  \hfill (2)

In the presence of a magnetic field, $B \neq 0$, the real conductivity, $\sigma_1(T,B)$ is again zero except within a few Kelvin of the vortex glass melting temperature $T_G(B)$. By analogy with Eq. 2, we define the experimental penetration depth, $\lambda_{exp}$, as

$$\lambda^2_{exp}(T,B) = \frac{1}{\mu_0 \omega \sigma_2(T,B)}.$$  \hfill (3)

When $B = 0$, $\lambda_{exp}$ reduces to $\lambda_{ab}$. When $B \neq 0$, $\lambda_{exp}$ is the penetration depth a film in zero field would need to create the observed screening. $\lambda_{exp}(T,B)$ is a combination of the inductance of the superfluid and the inductance of the oscillating vortices.
Figs. 3 through 17 are graphs of $1/\lambda_{exp}^2(T,B)$ for several films. Figs. 3 – 10 are plots of $1/\lambda_{exp}^2$ vs. $T$ at various magnetic fields for films 1 – 8 respectively. The data are taken by zero field cooling the sample, applying the desired field, and heating at about 20 mK/s. Data are taken continuously as the sample is heated. Each data set contains about 3000 points. In all cases, the $B = 0$ curve is the London penetration depth and $1/\lambda_{exp}^2(T,B=0)$ is proportional to the superfluid density. In these plots, data are obtained only for $T \geq 4.2$ K. All curves shown for $T < 4.2$ K are third order polynomial fits to the data between 5 K and 30 K (5 K and 15 K for LSCO). Note that only films 3 and 5 (Figs. 5 and 7) exhibit the linear low temperature behavior characteristic of a d-wave superconductor. The others are flat at low temperature, presumably because of residual disorder.

Figs. 3 and 4 are PLD YBCO films that have a high density of pinning defects. As discussed later, each vortex is independently pinned on an extended defect. Note that at high fields, $1/\lambda_{exp}^2(T,B)$ is linear in $T$ at low $T$, even though $1/\lambda_{exp}^2(T,B=0)$ is quadratic. This result leads to the important conclusion that thermally induced vortex motion is responsible for the temperature dependence of $1/\lambda_{exp}^2(T,B)$.

Figs. 5 – 7 are sputtered YBCO films that have a slightly lower density of pinning sites. In these plots we see highly reproducible noise-like structures in the high field data. These structures are the result of thermal relaxation of a nonuniform vortex density. More examples of sputtered films displaying similar behavior are presented without comment in Appendix C.
Fig. 8 is a coevaporated YBCO film that has an even lower density of pinning defects. At high fields we again see the noise like structures which result from thermal relaxation of a nonuniform vortex density. The results shown in this plot are far less reproducible than those obtained for sputtered films. In fact, this is the only coevaporated film on which we were able to obtain data. All other coevaporated films produce normal zero field data but generate only noise when a field is applied. This noise appears in both the magnitude and phase of the mutual inductance. The amplitude of the mutual inductance varies randomly from zero to the initial position while the phase varies randomly from 0 to 2π. This noise is the result of random vortex motion, however, it is unclear what causes the motion as the noise is present even at 4.2 K. The amplitude of the noise seems uncorrelated with temperature and drive current, but is roughly proportional to the magnetic field at low fields.

Figs. 9 and 10 are LSCO films. The noise in 1/λ²_{exp}(T,B=0) at low T in Fig. 10 is ordinary Johnson noise that results from the need to use a low drive current in LSCO samples. These plots show the same high defect density behavior observed in laser ablated YBCO films.

Figs. 11 – 17 are plots of 1/λ²_{exp}(T,B) vs. B at 4.2 K for films 1 – 5, 7, and 8 respectively. The data are taken by placing the sample in liquid helium and ramping the magnetic field at a rate of about 1.5 mT/s. Data are taken continuously as the field is increased. The dotted lines in Figs. 11 - 14 are theoretical predictions made by the model described in section 3.4. Note that Figs. 11 – 15 have qualitatively similar behavior. 1/λ²_{exp} starts at an initial value and drops rapidly by about 10% in the first hundred gauss. It then levels out for about half a Tesla before gradually decreasing to about 50% of its
initial value at 6 Tesla. The initial sharp drop is the result of inhomogeneous flux
penetration into the sample. The later gradual decline is the result of vortex interactions.
The two LSCO films, Figs. 16 and 17 show slightly different behavior which we discuss
in section 3.4.

The astute observer will notice a discrepancy in \(1/\lambda_{\text{exp}}^2(T=4.2\,\text{K}, B=0)\) between
Figs. 3 and 11, both for film 1. This discrepancy occurs because the two data sets were
taken eleven months apart and the film changed during that time. We will refer to film 1
frequently throughout this work. When citing values for the penetration depth, we will
take them from Fig. 3 when discussing the temperature dependence and from Fig. 11
when discussing the magnetic field dependence.
Figure 3 - Temperature dependence of the experimental penetration depth of film 1 at various magnetic fields. At zero field, $\lambda_{exp} = \lambda_{ab}$ and $1/\lambda_{exp}^2$ is proportional to the superfluid density.
Figure 4 - Temperature dependence of the experimental penetration depth of film 2 at various magnetic fields. At zero field, $\lambda_{exp} = \lambda_{ab}$ and $1/\lambda_{exp}^2$ is proportional to the superfluid density.
Figure 5 - Temperature dependence of the experimental penetration depth of film 3 at various magnetic fields. At zero field, $\lambda_{exp} = \lambda_{ab}$ and $1/\lambda_{exp}^2$ is proportional to the superfluid density.
Figure 6 - Temperature dependence of the experimental penetration depth of film 4 at various magnetic fields. At zero field, $\lambda_{exp} = \lambda_{ab}$ and $1/\lambda_{exp}^2$ is proportional to the superfluid density.
Figure 7 - Temperature dependence of the experimental penetration depth of film 5 at various magnetic fields. At zero field, $\lambda_{exp} = \lambda_{ab}$ and $1/\lambda_{exp}^2$ is proportional to the superfluid density.
Figure 8 - Temperature dependence of the experimental penetration depth of film 6 at various magnetic fields. At zero field, $\lambda_{\text{exp}} = \lambda_{\text{ub}}$ and $1/\lambda_{\text{exp}}^2$ is proportional to the superfluid density.
Figure 9 - Temperature dependence of the experimental penetration depth of film 7 at various magnetic fields. At zero field, $\lambda_{exp} = \lambda_{ab}$ and $1/\lambda_{exp}^2$ is proportional to the superfluid density.
Figure 10 - Temperature dependence of the experimental penetration depth of film 8 at various magnetic fields. At zero field, $\lambda_{exp} = \lambda_{ub}$ and $1/\lambda_{exp}^2$ is proportional to the superfluid density.
Figure 11 - Magnetic field dependence of the experimental penetration depth of film 1 at 4.2 K. The dotted line is the theoretical penetration depth predicted by the theory developed in section 3.4.
Figure 12 - Magnetic field dependence of the experimental penetration depth of film 2 at 4.2 K. The dotted line is the theoretical penetration depth predicted by the theory developed in section 3.4.
Figure 13 - Magnetic field dependence of the experimental penetration depth of film 3 at 4.2 K. The dotted line is the theoretical penetration depth predicted by the theory developed in section 3.4.
Figure 14 - Magnetic field dependence of the experimental penetration depth of film 4 at 4.2 K. The dotted line is the theoretical penetration depth predicted by the theory developed in section 3.4.
Figure 15 - Magnetic field dependence of the experimental penetration depth of film 5 at 4.2 K.
Figure 16 - Magnetic field dependence of the experimental penetration depth of film 7 at 4.2 K.
Figure 17 - Magnetic field dependence of the experimental penetration depth of film 8 at 4.2 K.
2.4 – Vortex Resistivity

To extract vortex pinning information from the measured conductivity, it is necessary to examine the vortex state in more detail. We begin by writing the equation of motion for an isolated, pinned vortex:

\[ F = -\kappa x - \eta \dot{x} + J\Phi_0 = 0. \] (4)

The first term, \(-\kappa x\), is the pinning force per unit length. We assume a quadratic pinning potential which results in a linear restoring force constant, \(\kappa\), known as the Labusch parameter. The second term, \(-\eta \dot{x}\), is the dissipative term, which results from motion of the normal vortex core. The constant, \(\eta\), is known as the vortex viscosity. Our experiment does not measure \(\eta\) directly except near \(T_G\). Unfortunately, values reported for \(\eta\) vary considerably. When necessary, we will use the average value \(\eta(T=0) = 1 \times 10^{-6} \text{Ns/m}^2\) throughout this paper. The last term, \(J\Phi_0\), is the force per unit length that results from the interaction between the vortex and the supercurrent density, \(J\).

\(\Phi_0\) is the flux quantum given by \(\Phi_0 \equiv \hbar/2e = 2.07 \times 10^{-15} \text{Tm}^2\). Since the vortex is essentially massless, the net force on the vortex is zero.

We can relate the vortex parameters to the film conductivity by considering a rectangular superconducting film containing a uniform supercurrent density \(J = J_0\sin(\omega t)\). Applying a uniform magnetic field \(B\) perpendicular to the superconductor results in a vortex density slightly less than \(B/\Phi_0\). The supercurrent then oscillates the vortices about their pinning sites:

\[ x(t) = \frac{J_0 \Phi_0}{\kappa \left(1 + (\omega/\omega_0)^2\right)} \sin(\omega t) - \frac{J_0 \Phi_0 (\omega/\omega_0)}{\kappa \left(1 + (\omega/\omega_0)^2\right)} \cos(\omega t). \] (5)
where $\omega_0 = \kappa/\eta$ is the characteristic frequency known as the depinning frequency. By comparing the energy dissipated by the oscillating the vortices, $P = F(t) - v(t)$, with the energy dissipated by the electric current $P = IV$, we can calculate a vortex resistivity:

$$\rho_v = \frac{i\omega B\Phi_0}{\kappa \left[ 1 + i \left( \frac{\omega}{\omega_0} \right) \right]}.$$

(6)

This is the resistivity the film would have if the superfluid did not contribute. Note that at high frequencies, the real term dominates and the vortex response is dissipative while at low frequencies, the imaginary term dominates and the vortex response is inductive. The crossover occurs at the depinning frequency, which is typically on the order of $\omega_0/2\pi = 30$ GHz. At our measurement frequencies, we would expect an inductive response. Therefore, in analogy with Eq. 3, we define a penetration depth for the vortex resistivity, $\lambda_C$, known at the Campbell penetration depth by

$$\lambda_c^2 = \frac{\rho_v}{\mu_0 \omega} = \frac{B\Phi_0}{\mu_0 \kappa}.$$

(7)

Here we have assumed that each vortex is pinned by an identical force $-kx$. In reality, our films contain at least two distinct types of pinning defect. In addition, the pinning force for each type of defect can vary from one defect to the next. Since our experiment measures the total response of film and not that of individual vortices, the measured $\kappa$ is an average over all pinning sites.

The problem that remains is to relate the London penetration depth of the superfluid response, $\lambda_{ab}$, and the Campbell penetration depth of the vortex response, $\lambda_C$. 

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to the measured experimental penetration depth in the presence of a magnetic field, $\lambda_{exp}$. Making the simple assumption that the two inductances add in series yields the result:

$$\lambda_{exp}^2 = \lambda_{ab}^2 + \lambda_C^2. \quad (8)$$

In the next section, we discuss the Clem and Coffey model which delineates the conditions under which this Eq. 8 is valid. In our experiment, these conditions hold for all temperatures except near $T_C$.

### 2.5 – Clem and Coffey Model

Clem and Coffey (CC) have modeled the experimental setup used in these measurements, and their work justifies treating the vortex impedance as additive with the impedance of the superfluid. They begin by calculating the induced supercurrent density using the method discussed in Appendix B1. CC then consider the interaction between the induced current and the vortices. They assume that in the absence of screening currents, vortices form a lattice of uniform density. In addition, they assume each vortex experiences an identical pinning force characterized by the Labusch parameter, $\kappa$. In general, the response of the vortex lattice to the nonuniform supercurrent density created by the drive coil is nonlocal as the net force on each vortex resulting from vortex interactions depends on the position of every other vortex. This nonlocality is easily included in the calculation since it is done in Fourier space. Making the usual assumption that the force applied to a vortex by the induced supercurrent density is $\vec{F} = \vec{J} \times \vec{\Phi}_0$, CC derive an expression for the vortex density as a function of the wave vector. Using this
expression, they write an integral equation for the flux in the receive coil resulting from the current in the drive coil, the screening currents, and the oscillating vortices.

CC then go on to delineate the conditions under which the vortex response calculated as described above reduces to that obtained by adding the vortex inductance and the superfluid inductance. The first, \( \omega \ll \omega_b \), is simply the previous condition that insures the vortex response is inductive. The second is that \( \lambda_\text{ab} \) be small compared to any coil dimensions. The nonuniformity in the induced supercurrent density is characterized by a length scale on the order of the coil radius. The vortex-vortex interaction force has a range on the order of \( \lambda_\text{ab} \). If \( \lambda_\text{ab} \) is much smaller than the coil dimensions, the induced supercurrent appears uniform on the length scale of the vortex interactions. Thus all vortices undergo the same displacement and vortex-vortex interactions are unaffected by the motion. In our experimental setup, the coil dimensions are all on the order of 1 mm, while the penetration depth is less than 1 \( \mu \text{m} \) except near \( T_C \).

2.6 – Experimental Labusch Parameter

Using the above analysis, we can calculate the Campbell penetration depth, and from that the Labusch parameter for our data. The Campbell penetration depth is simply

\[
\lambda_C^2(T,B) = \lambda_{\text{exp}}^2(T,B) - \lambda_{\text{ab}}^2(T,B).
\]

\( \lambda_{\text{exp}}^2(T,B) \) we extract directly from the mutual inductance data. Unfortunately, \( \lambda_{\text{ab}}^2(T,B) \) can only be measured directly for \( B = 0 \). To calculate \( \lambda_{\text{ab}}^2(T,B) \) for \( B \neq 0 \), recall that \( 1/\lambda_{\text{ab}}^2 \) is proportional to the superfluid density. In the vortex state, the superfluid density is reduced because of the normal material in and around the cores of the vortices. In an s-wave superconductor, we would simply reduce...
by an amount proportional to the fractional volume occupied by vortex cores: 
\[ 1 - \frac{B}{B_{c2}} \], where \( B_{c2} \) is the upper critical field. In a d-wave superconductor however, the region in which superconductivity is suppressed extends beyond the cores of the vortices. This results from the large supercurrent that flows around the vortices. In regions where the supercurrent flows parallel to the direction of the nodes in the order parameter, the supercurrent can exceed the critical current and suppress superconductivity. Because of this, the total suppression of the superfluid density is 
\[ 1 - \sqrt{\frac{B}{B_{c2}}} \]. We thus calculate \( \lambda_c \) using the approximation \( \lambda^2_{ab}(T, B) = \frac{\lambda^2_{ab}(T, B = 0)}{\lambda^2_{ab}(T, B = 0) \left( 1 - \sqrt{\frac{B}{B_{c2}}} \right)} \). In our films, \( B_{c2} \) is on the order of 200 Tesla at low temperatures and this correction is only significant near \( T_C \). We therefore use the linear approximation \( B_{c2} = (2 \text{ Tesla/K})(T_C - T) \).

Figs. 18 and 19 are plots of \( \lambda^2_c/B \) vs. \( T \) and \( B \) for film 1. Since \( \lambda^2_c \) is proportional to the inductance of the vortices, and \( B \) is proportional to the vortex density, \( \lambda^2_c/B \) is proportional to the inductance per vortex.

Of more interest to us in this work is the Labusch parameter. Using Eq. 7 we define the experimental Labusch parameter:

\[ \kappa_{\text{exp}} = \frac{B \Phi_0}{\mu_0 \lambda^2_c} \]  

Figs. 20 – 34 are plots of experimental Labusch parameters extracted from the data in Figs. 3 – 17. Figs. 20 and 21 are plots of \( \kappa_{\text{exp}}(T, B) \) vs. \( T \) at various fields for films 1 and 2. the PLD YBCO films. Note that for fields above one Tesla, the curves show a near linear decrease in \( \kappa_{\text{exp}}(T, B) \) at low \( T \) characteristic of thermal motion as discussed in
section 3.6. For fields below one Tesla, we see noise at low T as the nonuniform flux
density relaxes and vortices penetrate deeper into the film. Nonuniform flux penetration
is discussed in section 3.4.

Figs. 22 – 24 are plots of $\kappa_{\text{exp}}(T, B)$ vs. T for the three sputtered YBCO films 3 – 5. Again we see a somewhat linear decrease in $\kappa_{\text{exp}}(T, B)$ at low T. These curves contain
more noise than the PLD films indicating more vortex motion and thus a lower density of
pinning sites. Fig. 25 is a plot of $\kappa_{\text{exp}}(T, B)$ vs. T for film 6, the coevaporated YBCO film.
Figs. 26 and 27 are plots of $\kappa_{\text{exp}}(T, B)$ vs. T for the PLD LSCO films 7 and 8.

Figs. 28 – 34 are plots of $\kappa_{\text{exp}}(T, B)$ vs. B at $T = 4.2$ K for films 1 – 5, 7 and 8.
The dotted lines shown in Figs. 28 – 31 are theoretical predictions made by the model
developed in section 3.4. Figs. 28 and 29 are plots of $\kappa_{\text{exp}}(T, B)$ vs. B for the PLD YBCO
films 1 and 2. At low fields, we see a rapid increase in $\kappa_{\text{exp}}$. Initially vortices are pinned
by planar defects with a small $\kappa$. As the field increases, vortices are pinned by linear
extended defects with a larger $\kappa$. $\kappa_{\text{exp}}$ is an average of these different sites and increases
as more vortices are pinned by extended defects. For fields greater than 3 Tesla, we see a
decrease in $\kappa_{\text{exp}}$ that is quadratic in B. This results from vortex interactions. These effects
are discussed in section 3.4. Figs. 30 – 32 are plots of $\kappa_{\text{exp}}(T, B)$ vs. B for the sputtered
films 3 – 5. Figs. 33 and 34 are plots of $\kappa_{\text{exp}}(T, B)$ vs. B for the two LSCO films 7 and 8.
At high fields, $\kappa_{\text{exp}}$ still decreases but with a positive curvature. As explained in section
3.4, in these films the vortex interactions are stronger than pinning. Consequently, many
of the vortices are unpinned resulting in a different magnetic field dependence.
Figure 18 - Temperature dependence of the Campbell penetration depth of film 1 at various magnetic fields. $\lambda^2_c B$ is proportional to the inductance per vortex.
Figure 19 - Magnetic field dependence of the Campbell penetration depth of film 1 at 4.2 K.

$\lambda_c^2 / B$ is proportional to the inductance per vortex. The dotted line is the theoretical penetration depth predicted by the theory developed in section 3.4.
Figure 20 - Temperature dependence of the experimental Labusch parameter of film 1 at various magnetic fields.
Figure 21 - Temperature dependence of the experimental Labusch parameter of film 2 at various magnetic fields.
Figure 22 - Temperature dependence of the experimental Labusch parameter of film 3 at various magnetic fields.
Figure 23 - Temperature dependence of the experimental Labusch parameter of film 4 at various magnetic fields.
Figure 24 - Temperature dependence of the experimental Labusch parameter of film 5 at various magnetic fields.
Figure 25 - Temperature dependence of the experimental Labusch parameter of film 6 at various magnetic fields.
Figure 26 - Temperature dependence of the experimental Labusch parameter of film 7 at various magnetic fields.
Figure 27 - Temperature dependence of the experimental Labusch parameter of film 8 at various magnetic fields.
Figure 28 - Magnetic field dependence of the experimental Labusch parameter of film 1 at 4.2 K. The dotted line is the theoretical Labusch parameter predicted by the theory developed in section 3.4.
Figure 29 - Magnetic field dependence of the experimental Labusch parameter of film 2 at 4.2 K. The dotted line is the theoretical Labusch parameter predicted by the theory developed in section 3.4.
Figure 30 - Magnetic field dependence of the experimental Labusch parameter of film 3 at 4.2 K. The dotted line is the theoretical Labusch parameter predicted by the theory developed in section 3.4.
Figure 3.1 - Magnetic field dependence of the experimental Labusch parameter of film 4 at 4.2 K. The dotted line is the theoretical Labusch parameter predicted by the theory developed in section 3.4.
Figure 32 - Magnetic field dependence of the experimental Labusch parameter of film 5 at 4.2 K.
Figure 33 - Magnetic field dependence of the experimental Labusch parameter of film 7 at 4.2 K.
Figure 34 - Magnetic field dependence of the experimental Labusch parameter of film 8 at 4.2 K.
2.7 - Vortex Resistivity Near $T_G$

The analysis given above can be extended to include dissipation, which occurs near the vortex glass melting temperature $T_G$. Near $T_G$ the depinning frequency is dropping rapidly. As it crosses $50$ kHz, the vortex response changes from inductive to resistive. In this region, we can measure both the Labusch parameter and the vortex viscosity directly.

To include the dissipative response in our analysis, we must modify Eq. 8, which relates the total resistivity to the resistivity of the superfluid and the resistivity of the vortices. According to Clem and Coffey, so long as the superfluid penetration depth is still small relative to the coil dimensions the total resistivity is obtained by adding the superfluid resistivity and the vortex resistivity in series.

$$\rho_{\text{exp}} = \rho_s + \rho_v = \rho_s + \rho_v$$

$$= i\mu_0\omega\lambda_{ab}^2 \left[ 1 + \frac{B\Phi_0}{\mu_0\kappa^2\lambda_{ab}^2} + i\left(\frac{\omega}{\omega_0}\right) \right]$$

$$+ \frac{i\left(\frac{\omega}{\omega_0}\right)}{1 + i\left(\frac{\omega}{\omega_0}\right)}$$

Note that in both the limits $\eta \to \infty$ and $\kappa \to \infty$, where the vortices become immobile, we recover the superfluid resistivity. While in the limit $\eta \to 0$, we recover the previous low frequency result. In addition, the limit $\kappa \to 0$ yields the flux flow resistivity that results from unpinned vortices.

Knowing the measured resistivity, $\rho_{\text{exp}}$, and the calculated London penetration depth, $\lambda_{ab}(T,B)$, we can use Eq. 12 to calculate $\kappa_{\text{exp}}$ and $\eta_{\text{exp}}$ near $T_G$. Fig. 35 is a plot of the mutual inductance measured for film 1 for several magnetic fields. The temperature ranges over which the imaginary mutual inductance is appreciable indicate the regions in
which dissipation occurs and we can accurately extract a real resistivity. Fig. 36 is a plot of $\sigma_{\text{exp}}$ vs. $T$ for various magnetic fields extracted from the data in Fig. 35.

Figs. 37 is a plot of $\kappa_{\text{exp}}$ vs. $T$ obtained from Fig. 36 using the above analysis. Near $T_G$ thermal fluctuations drive $\kappa_{\text{exp}}$ to zero. This effect is discussed in detail in section 3.6. Fig. 38 is a plot of $\eta_{\text{exp}}$ vs. $T$ obtained from the data in Fig. 36. Note that $\eta_{\text{exp}}$ drops rapidly to zero near $T_G$. This result is not expected. Given that the temperature dependence of $\eta$ is proportional \(^{17}\) to $1/\xi_{\text{ab}}^2(T)$, and that $\eta = 10^{-6}$ Ns/m\(^2\) at low temperatures, we would expect $\eta$ to be about $3 \times 10^{-7}$ Ns/m\(^2\) at 74 K and drop roughly linearly \(^{30}\) to $1 \times 10^{-7}$ Ns/m\(^2\) at 88 K. The observed vortex viscosity is about 4 orders of magnitude too large.

We can attempt to explain this discrepancy by considering the effect of unpinned vortices on the measured resistivity. Suppose that some fraction, $\chi$, of the vortices is unpinned. The unpinned vortices will have a resistivity, $\rho_u = (\chi B)\Phi_0/\eta$, while the pinned vortices will have a resistivity $\rho_p = \frac{i\omega((1-\chi)B)\Phi_0}{\kappa + i\omega\eta}$. This yields a total vortex resistivity of

$$\rho_v = \rho_u + \rho_p = \frac{B\Phi_0}{\eta} \left[ \frac{\chi + \left(\frac{\omega\eta}{\kappa}\right)^2}{1 + \left(\frac{\omega\eta}{\kappa}\right)^2} \right] + \frac{i\omega B\Phi_0}{\kappa} \left[ \frac{1 + \chi}{1 + \left(\frac{\omega\eta}{\kappa}\right)^2} \right]. \quad (11)$$

The effect of unpinned vortices is thus to increase the real resistivity by a factor $1 + \chi/(\omega\eta/\kappa)^2$, while increasing the imaginary resistivity by a factor $1 + \chi$. For our films we expect $\omega\eta/\kappa$ to be on the order of $10^{-3}$ just below $T_G$ where the peak in the imaginary mutual inductance appears, then increase as we approach $T_G$. Below $T_G$, a small number
of unpinned vortices cause a large increase in the real resistivity, but only a slight increase in the imaginary resistivity. Since the imaginary resistivity is large compared to the real resistivity, this large increase in the real resistivity actually translates into an increase in the real conductivity, and thus an increase in \( \eta_{\text{exp}} \) which is proportional to the real conductivity.

To determine if this effect is sufficient to cause the extremely large value of \( \eta_{\text{exp}} \) observed in Fig. 38, we use the vortex resistivity to define an effective Labusch parameter, \( \kappa_{\text{eff}} \), and an effective vortex viscosity, \( \eta_{\text{eff}} \):

\[
\rho_v = \frac{i \omega B \Phi_0}{\kappa_{\text{eff}} + i \omega \eta_{\text{eff}}}
\]

Solving for the effective vortex viscosity yields:

\[
\frac{\eta_{\text{eff}}}{\eta} = \frac{\chi + \left(\frac{\omega \eta}{\kappa}\right)^2}{\chi^2 + \left(\frac{\omega \eta}{\kappa}\right)^2}.
\] (12)

For \( \omega \eta / \kappa \sim 10^{-3} \), \( \eta_{\text{eff}} / \eta \) has a peak value of about 500 when about 0.2% of the vortices are unpinned. This increase in \( \eta_{\text{eff}} / \eta \) is highly dependent on \( \omega \eta / \kappa \). If \( \omega \eta / \kappa \) were 0.5\( \times 10^{-3} \), the peak would occur when about 0.15% of the vortices were unpinned and would be about \( \eta_{\text{eff}} / \eta \sim 1000 \). This suggests that the temperature dependence of the real resistivity near \( T_G \) should change with frequency. Unfortunately, all of our frequency dependent data focused on the imaginary conductivity away from \( T_G \), and we have no data to confirm this result.

To properly model the observed data requires a theory based on a distribution of pinning sites. As long as only a small fraction of the pinned vortices have depinning frequencies below the measurement frequency, the observed viscosity will be larger than
the true viscosity. However, since in such a distribution there will be many vortices with depinning frequencies near the measurement frequency, we would expect $\eta_{\text{eff}}/\eta$ to be less then that predicted by Eq. 12. We therefore conclude that having a small number of unpinned vortices is unlikely to produce the 4 orders of magnitude discrepancy seen in our data. It is more likely that there exists a distribution in values of $\eta$, possibly resulting from vortices pinned in planar defects. More work is necessary to explain this data.

From the conductivity data we can extract $T_G$ and construct a phase diagram for the vortex state. This phase diagram is shown in Fig. 39. The melting curve obtained from our data is consistent with previous measurements$^{15,20,21}$ with $B_G \propto (T_C - T_G)^{4/3}$ as predicted by vortex glass melting theory.$^{21,22}$

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Figure 35 - Temperature dependence of the complex mutual inductance of film 1 near the vortex glass melting temperature. The temperature range over which the imaginary mutual inductance is noticeably nonzero indicates the region where we can reliably extract a real conductivity.
Figure 36 - Temperature dependence of the complex conductivity of film 1 near the vortex glass melting temperature extracted from the data in Fig. 35.
Figure 37 - Temperature dependence of the experimental Labusch parameter near the vortex glass melting temperature calculated from the data in Fig. 36.
Figure 38 - Temperature dependence of the vortex viscosity near the vortex glass melting temperature calculated from the data in Fig. 36.
Figure 39 - Vortex glass melting curve for film 1. The curve is consistent with vortex glass melting theory, with $B_g = (T_C - T_g)^{\mu_3}$. 
CHAPTER 3

THEORY AND ANALYSIS

3.1 – Pinning Defects

Vortex pinning is generally believed to result from defects in the microstructure of the superconductor crystal. The material in the core of a vortex is in the normal state. The energy required to form a vortex is comes in part from the condensation energy lost creating the normal core. Energy is thus saved by placing a vortex atop a defect that has already suppressed superconductivity in its vicinity.

Pinning defects fall into three classifications: point defects, extended defects, and planar defects. Point defects are zero dimensional defects usually resulting from the misalignment of a single atom. These include oxygen vacancies and dopants. Extended defects are one-dimensional defects that persist through several unit cells of the crystal. These include screw dislocations and columnar tracks left by heavy ion bombardment. Extended defects need not be rectilinear. Planar defects are two-dimensional defects such as twin planes and grain boundaries.

The theory of collective pinning by point defects has been used quite successfully to explain vortex pinning in clean YBCO crystals. It has also been applied to pinning in thin films with less persuasive results. There is substantial evidence that
vortex pinning in films results from extended defects. In the remainder of this work, we show that a combination of a high density of extended defects and a smaller density of planar defects best explains our data.

We begin by demonstrating that pinning by point defects is not sufficient to produce the observed Labusch parameter. In Appendix B2 we calculate the effective Labusch parameter resulting from point pinning sites. We can solve Eq. B19 for \( L \), the distance between pinning sites, to determine how many pinning sites are necessary to produce the observed Labusch parameter. For the moment, assume maximally strong point pinning sites which extinguish conductivity within a sphere of radius \( \xi_{ab} \), the ab-plane coherence length. Since our experiment does not measure \( \xi_{ab} \) directly, we will use the generally accepted values of 15 Å for YBCO and 25 Å for LSCO. Using the measured values of \( \kappa_{exp} = 30 \times 10^4 \, \text{N/m}^2 \) and \( \lambda_{ab} = 1960 \, \text{Å} \) for film 1, yields \( L = 44 \, \text{Å} \). The vortex would thus need to intersect a pinning site every 44 Å to produce the observed Labusch parameter. If we assume weaker point pinning sites, say pinning sites which extinguish conductivity within a sphere of radius 8 Å, the vortex would need to intersect a pinning site every 16 Å to produce the observed Labusch parameter. The pinning sites would thus be connected forming an extended defect. Therefore, we need to assume either an extremely high density of point pinning defects or defects that are highly correlated along the c-axis and are in effect extended defects.

Further evidence exists in support of extended pinning defects. Early on Hylton and Beasley\(^{34}\) performed a similar calculation for the critical current resulting from point pins. Again assuming point pins that suppress superconductivity within a radius of a coherence length, they conclude that each vortex must be pinned along at least ¼ of its
length to generate the observed critical current. In a more recent review of microwave measurements, Golosovsky et al.\textsuperscript{14} also noted that pinning in YBCO films is stronger than can be accounted for using point pinning. Xenikos et al.\textsuperscript{25} showed that the magnetic field dependence of the vortex conductivity is much weaker than predicted by the collective pinning model. Finally, Diaz et al.\textsuperscript{26} noted a peak in the angular dependence of the critical current occurring when the magnetic field was aligned perpendicular to the film. The peak was stronger than could be accounted for by the anisotropy in the penetration depth.

At present, attempts to locate the defects in the crystal structure responsible for vortex pinning have failed.\textsuperscript{27} Based on the evidence presented above, we conclude that extended defects are responsible for the pinning observed in our films. We will therefore analyze our data within a framework of a high density of extended defects and a smaller density of planar defects.

3.2 – Pinning Potential Approximation

Without specific knowledge of the defects responsible for pinning, it is impossible to calculate the exact pinning potential per unit length, $U(\rho, T)$, that the vortex experiences. To extract quantitative information about vortex pinning from our data, we must therefore make some assumptions about the pinning potential. We begin by assuming a quadratic pinning potential, $U(\rho, T) = \frac{1}{2}\kappa_0(T)\rho^2$, which produces a Labusch parameter $\kappa_0(T)$. We know that when a sufficiently large external force is applied to the vortex it must become unpinned. Since the vortex has a radius of about a coherence length, we would expect the vortex to become unpinned when it reaches a displacement
\[ \rho = \xi_{ab} \]  

The pinning potential must therefore contain an inflection point at \( \rho = \xi_{ab} \). The simplest way to incorporate an inflection point into the quadratic potential while preserving the linear restoring force for small displacements is to add a quartic term:

\[
U(\rho, T) = \frac{1}{2} \kappa_0(T) \xi_{ab}^2(T) \left( \frac{\rho}{\xi_{ab}(T)} \right)^2 - \frac{1}{12} \kappa_0(T) \xi_{ab}^2(T) \left( \frac{\rho}{\xi_{ab}(T)} \right)^4 \quad (13)
\]

We can now estimate the radius of the pinning defect by comparing the pinning potential to the lost condensation energy associated with the defect. The condensation energy for a superconductor is

\[
\frac{B_c^2}{2\mu_0}, \text{ where } B_c(T) = \frac{\Phi_0}{2\sqrt{2\pi\lambda_{ab}(T)\xi_{ab}(T)}}
\]

is the thermodynamic critical field. If the defect has a radius \( r_d \), the condensation energy per unit length is \( \frac{B_c^2}{2\mu_0} \pi r_d^2 \). Without exact knowledge of the pinning potential, we do not know the true depth of the potential \( U(\rho = \infty, T) \). We therefore make the assumption

\[ U(\rho = \infty, T) = 2U(\rho = \xi, T) = \frac{5}{6} \kappa_0(T) \xi_{ab}^2(T). \]

Equating with the condensation energy and solving for the defect radius yields:

\[
r_d = \sqrt{\frac{40\pi\mu_0\lambda_{ab}^2\xi_{ab}^4\kappa_0}{3\Phi_0^2}}. \quad (14)
\]

Identifying \( \kappa_0(T=0) \) with \( \kappa_{exp}(T=0, B=0) \) and substituting in the experimentally obtained values for film 1 yields \( r_d = 8 \AA \), about one half the coherence length. Table 1 in Appendix A lists the defect radii obtained for the various films studied.

In the above analysis, we considered only the condensation energy associated with locating the vortex on the defect. In addition, part of the kinetic energy associated
with the supercurrents flowing around the vortex will be saved. These currents are concentrated near the core of the vortex, but extend out to a distance \( \rho = \lambda_{ub} \). If all of this energy were saved by locating the vortex on the pinning site, the condensation energy would be increased by a factor \( \ln(\lambda_{ub}/\xi_{ub}) = 5 \) and the estimated defect radius would be \( r_d = 3.5 \, \text{Å} \). Since it is unclear what fraction of the kinetic energy is saved, the true defect radius is somewhere between 4 Å and 8 Å.

Finally, Thuneberg et al. have shown that in addition to the condensation energy saved, there is an additional energy associated with quasiparticle scattering within the cores of the vortices. The total energy saved by placing the defect on the vortex is larger than the condensation energy by a factor \( \xi_{ub}/r_d \). Adding this effect leads to a defect radius of between 1 Å and 4 Å.

Our analysis thus far has assumed that each vortex sits in an identical pinning site. This is most likely not true. We would expect the defect radii to vary from one defect to the next leading to a distribution in the pinning potentials. The defect radius could also vary along the length of the pinning site. The experimental Labusch parameter is thus an average over the various pinning sites. Throughout the remainder of this paper, we will continue our analysis in terms of a single pinning potential with a uniform radius and assume the derived results are an average over the various pinning sites.

Before continuing on to test the limits of applicability of Eq. 13, we will first consider the effect that displacing the vortex from the center of its pinning potential has on the measured Labusch parameter. As stated earlier, the current density induced in the film by the drive coil is several orders of magnitude smaller than the critical current density. Consequently, it only displaces the vortices by a few thousandths of an
angstrom. If the vortex is displaced a distance \( \rho_0 \) from the center of the pinning potential by another force, the small oscillations caused by the drive current effectively probe the derivative of the restoring force i.e. the curvature of the pinning potential. The curvature of the pinning potential is a maximum at \( \rho = 0 \) and decreases to zero at \( \rho = \xi_{ab} \). Thus if a force displaces a vortex from the center of the pinning potential, the effective Labusch parameter is reduced. It is in this manner that vortex interactions and thermal motion reduce the measured Labusch parameter. A detailed calculation of the effective Labusch parameter as a function of applied force is carried out in Appendix B3.

### 3.3 – Critical Current Density

To test the range of applicability of Eq. 13, we use it to calculate the critical current density. In this context, we take the critical current density to refer to the current density necessary to unpin vortices and cause dissipation, not the pair breaking current density. The maximum force per unit length that the pinning potential can generate is

\[
F_{\text{max}} = \left. \frac{dU}{dp} \right|_{\rho=\xi_{ab}} = \frac{2}{3} \kappa_0 \xi_{ab}.
\]

The critical current density is thus the current density which results in a force equal to the maximum pinning force: \( J_c \Phi_0 = F_{\text{max}} \). Solving for \( J_c \) yields:

\[
J_c = \frac{2 \kappa_0 \xi_{ab}}{3 \Phi_0}.
\]  

(15)

Again identifying \( \kappa_0(\text{T}=0) \) with \( \kappa_{\text{exp}}(\text{T}=0,\text{B}=0) \), Eq. 15 yields \( J_c = 1.5 \times 10^7 \text{ A/cm}^2 \) for film 1 and \( 1.0 \times 10^7 \text{ A/cm}^2 \) for film 2.

We can measure the critical current density directly by increasing the current in the drive coil while recording the mutual inductance. As we increase the drive current,
we would expect the mutual inductance to increase somewhat because of the nonlinear restoring force caused by the pinning potential. When the induced current density exceeds the critical current density, the mutual inductance will increase rapidly. Also, upon exceeding the critical current, the vortices would be free and we should observe dissipation in the form of a negative imaginary mutual inductance.

Figs. 40 and 41 are plots of mutual inductance vs. induced current density at \( T = 4 \) K and \( B = 1 \) Tesla for films 1 and 2 respectively. The induced current is calculated from current in the drive coil using the same numerical calculation described in Appendix B1. In calculating the induced current density, we assume that the penetration depth of the superfluid, \( \lambda_{ab} \), is independent of the induced current density. This is not true as the induced current density approaches the pair-breaking critical current density. Since we have no direct measurement of the pair-breaking critical current density for our films and the depinning current density is typically an order of magnitude smaller than the pair-breaking current density, we ignore this complication and assume that error in the calculated induced current density is small.

Examining Fig. 40 reveals a sharp increase in the real mutual inductance accompanied by a turn on in the imaginary mutual inductance at an induced current density of about \( 2.5 \times 10^7 \) A/cm\(^2\). Fig. 41 shows a similar change at about \( 4 \times 10^6 \) A/cm\(^2\). In addition to the mutual inductance, Fig. 41 also plots the third harmonic of the mutual inductance. When the vortex is pinned, a small third harmonic appears because of the nonlinear restoring force constant. When the induced current density exceeds the critical current density, the vortex hops from one pinning site to the next resulting in a large third harmonic. Note that the third harmonic also shows a sharp increase at \( 4 \times 10^6 \) N/cm\(^2\).
These measured critical current densities are in fair agreement with the theoretical predictions indicating that Eq. 13 is a useful approximation to the true pinning potential even to $\rho = \xi_{ab}$. 
Figure 40 - Induced current density dependence of the mutual inductance of film 1. The downward curve in the imaginary mutual inductance occurs at the critical current density.
Figure 41 - Induced current density dependence of the mutual inductance of film 1. The open circles are the complex mutual inductance. The squares are the magnitude of the third harmonic of the mutual inductance.
3.4 - Vortex Interactions

We will now consider the magnetic field dependence of the Labusch parameter. We begin by making the assertion that in the YBCO films studied, every vortex is pinned by and extended defect, even at a field of 6 Tesla. To justify this assertion, we will first consider the effect of unpinned vortices on the Labusch parameter. In Appendix B4 we show that if a vortex is not pinned by a defect, the net force on that vortex resulting from interactions with the other vortices is

\[ F_L = -\kappa_L x, \quad \kappa_L = \frac{B \Phi_0}{2 \mu_0 \lambda_{ab}^2} \tag{16} \]

where \( x \) is the distance the vortex is displaced from its equilibrium position. Thus if a vortex is not pinned by a defect, it is effectively pinned by the other vortices in the lattice with an effective Labusch parameter \( \kappa_L \). Using the experimental values \( B = 1 \) Tesla and \( 1/\lambda_{ab}^2 = 17 \mu m^2 \) for film 1, the effective Labusch parameter for an unpinned vortex is \( \kappa_L = 8.1 \times 10^4 \) N/m².

Assume for the moment that the film contains are only enough pinning sites to pin \( B_0 \) worth of vortices. Below \( B_0 \), we would expect the \( \kappa_{exp} \) to have a \( 1-B^2 \) field dependence consistent with vortex-vortex interactions as detailed below. Above \( B_0 \), only a fraction \( B_0/B \) of the vortices will be pinned by defects. The remaining vortices will be effectively pinned by the vortex lattice with a Labusch parameter \( \kappa_L \). In Appendix B.5, we calculate the effective Labusch parameter for a combination of pinned and unpinned vortices. For such a combination, \( \kappa_{eff} \) drops rapidly above \( B_0 \) with an upward curvature. This is not observed in any YBCO film. We will therefore assume that there exist
sufficient defects to pin every vortex and proceed to explain the observed Labusch parameter in terms of vortex interactions.

In the absence of pinning, the vortices would form a triangular lattice. However, the vortices sit on pinning sites and are unable to form a perfect lattice. If we assume that each vortex is displaced an average distance \( x_d \) from its proper position in the lattice, then each vortex will experience a force \( F_L = \kappa_L x_d \) resulting from the lattice distortion. Since the pinning sites are randomly located, the forces will be in random directions. As shown in Appendix B3, when a force is applied to a pinned vortex, the effective Labusch parameter is reduced:

\[
\kappa_{\text{eff}} = \kappa_0 \left[ 1 - \frac{2}{3} \left( \frac{F}{\kappa_0 \xi_{ab}} \right)^2 \right]. \tag{17}
\]

Substituting Eq. 16 into Eq. 17 yields

\[
\kappa_{\text{eff}} = \kappa_0 \left[ 1 - \frac{2}{3} \left( \frac{B \Phi_0 x_d}{2 \mu_0 \lambda_{ab}^2 \xi_{ab} \kappa_0} \right)^2 \right] = \kappa_0 \left[ 1 - \left( \frac{B}{B_0} \right)^2 \right], \quad B_0 = \frac{\sqrt{6 \mu_0 \lambda_{ab}^2 \xi_{ab} \kappa_0}}{\Phi_0 x_d}. \tag{18}
\]

\( B_0 \) is the magnetic field where the force resulting from lattice distortions equals the maximum pinning force. When \( B \) exceeds \( B_0 \), vortices will become unpinned to reduce the lattice strain.

The dotted lines in Figs. 28–31 are fits of Eq. 18 to the high field data for films 1–4. \( \kappa_{\text{eff}} \) fits the data well for fields larger than about three Tesla, but diverges completely at low fields. The fit for film 1 was generated using \( B_0 = 14 \) Tesla and \( \kappa_0 = 18 \times 10^4 \) N/m². \( B_0 = 14 \) Tesla implies the film has sufficient defects to accommodate 14 Tesla of vortices before vortex interactions become sufficiently strong to displace...
vortices from pinning sites. Inverting Eq. 18 yields \( x_d = 20 \, \text{Å} \). This means that each vortex can find a pinning site within 20 Å of its proper place in the vortex lattice. This implies a pinning site density of \( n_p = \frac{1}{4x_d^2} \approx 6.3 \times 10^4 \, \text{μm}^{-2} \). As stated previously, such a high defect density has not been directly observed in any YBCO film, however, it is consistent with the reduced superfluid density observed in films vs. crystals. Data for other films is given in Table 1 in Appendix A.

Using the effective Labusch parameter calculated with Eq. 18, we can work backward to obtain \( 1/\lambda_{exp}^2 \). The dotted lines in Figs. 11 – 14 are derived using the curve fits shown in Figs. 28 – 31. As expected, the theory curve fits the data well at high fields. In addition, it correctly predicts the zero field experimental penetration depth. This implies that whatever causes the Labusch parameter to diverge from the predicted value at low fields becomes irrelevant at high fields.

Fig. 42 is a plot of \( 1/\lambda_{exp}^2 \) vs. \( B \) at low fields for film 1. The solid lines are data for field increasing from zero and decreasing from \( B = 6 \, \text{Tesla} \). Fig. 42 shows a clear hysteresis at low fields, which disappears at about 0.2 Tesla. Above 0.2 Tesla, no hysteresis is observed within the limits of the measurement. Notice that \( 1/\lambda_{exp}^2 \) is essentially constant until about 400 Gauss where it drops sharply. The dotted line in Fig. 42 is \( 1/\lambda_{exp}^2 \) as predicted by Eq. 18.

The low field deviation results from inhomogeneous flux penetration. Vortices are not created within the film, but rather at the edge of the film. They are then pushed toward the center by new vortices created at the edge. The two coil apparatus is only sensitive to vortices within about 2 mm of the center of the film. The initial 400 Gauss of
the $B$ increasing curve is thus flat because vortices have not yet penetrated to the center of the film.

When vortices enter the film, they enter along planar defects such as grain boundaries. Within the planar defects, the vortices can move freely along the length of the defect. Vortices pack into the planar defects until pressure from additional vortices at the edge of the film forces them into the bulk material. This process is illustrated visually in Fig. 43. Fig. 43 is a magneto-optical image of vortices entering a YBCO thin film taken from Ref. 29. The image was made at a field of 500 Gauss. The film pictured has a high density of large planar defects resulting from an imperfectly polished substrate.

Since vortices are essentially free to move along the length of the planar defect, vortices pinned by such a defect have a much larger inductance than those pinned by extended defects. This results in the sharp drop in $1/\lambda_{csp}^2$ observed at about 400 Gauss. When the field reaches about 800 Gauss, the planar defects are essentially full and vortices begin to penetrate into the bulk material where they are pinned by extended defects.

As the field increases to several Tesla, the bulk material has an essentially uniform density of vortices pinned by extended defects. At this point, vortex interactions are sufficiently strong to effectively pin the vortices remaining in planar defects. Consequently, the inductance from vortices pinned planar defects becomes smaller while the inductance from vortices pinned by extended defects becomes larger. By about 3 Tesla, the vortices pinned on planar defects becomes inconsequential and the effective Labusch parameter is that predicted by Eq. 18.
When the field is decreased from 6 Tesla, the vortices in the planar defects exit the film. As they vacate the film, vortices near the planar defects are forced into them. This process continues until the field decreases to a few hundred gauss. At this point, the unbalanced repulsive interaction force between vortices is weaker than the pinning force. Whatever vortices remain in the film are trapped until the film is quenched (or until a field is applied in the opposite direction) resulting in the observed hysteresis. The inductance of the remaining vortices is dominated by the vortices trapped in the planar defects and thus no sharp increase is observed in the B decreasing curve.

The model constructed thus far fits the data for films 1 - 4 quite well. Film 5 however appears to be quite different. By comparing Figs. 28 and 32, we may be led to conclude that film 5 simply has weaker pinning than film 1. The increase in $\kappa_{\text{exp}}$ observed in Fig. 32 between 0 and 6 Tesla is consistent with the increase observed in Fig 28 between 0 and 2 Tesla. This implies that vortex interactions in film 5 are not strong enough to effectively pin the vortices located in planar defects until much higher fields. However, a comparison of Figs. 11 and 15 shows a much larger percentage drop in $1/\lambda_{\text{exp}}^2$ for film 1 than for film 5, implying that film 5 has a much smaller density of planar defects than film 1. In addition, the sharp drop in $1/\lambda_{\text{exp}}^2$ ends at about 500 Gauss in both films, implying the two films have the same number of planar defects. To understand these apparent inconsistencies, we must consider pinning by planar defects more carefully.

In the region of interest, $B \sim 500$ Gauss and the vortices are spaced approximately 2000 Å apart. We stated previously that vortices pinned by planar defects were free to move along the length of the defect. This is not entirely true. While there is no
significant interaction between the film and the vortex for motion along the length of the defect, the vortices still interact with one another. When the defect is full of vortices, the vortices will arrange themselves to cancel the repulsive interaction forces. In Appendix B4, we calculate the restoring force constant for small oscillations about equilibrium. (To attempt an accurate quantitative analysis, we should repeat this calculation in one dimension. We shall ignore this complication and assume a one-dimensional analysis gives qualitatively similar results.) The drop in inductance, $\Delta L$, which occurs when the planar defects fill with vortices, is proportional to $\kappa_L$. If $a << \lambda_{ab}$, as is the case in film 5 ($\lambda_{ab} \sim 7000\,\text{Å}$), then $\Delta L \sim \lambda_{ab}^{-2}$. Inspection of the inset in Fig. B1 shows that when $a \gg \lambda_{ab}$, the restoring force drops off more rapidly than $\lambda_{ab}^{-2}$. For film 1 ($\lambda_{ab} \sim 2000\,\text{Å}$), $a \sim \lambda_{ab}$ and thus we would expect a smaller restoring force than is predicted by Eq. 18. We would therefore expect to see a larger percentage drop in $1/\lambda_{exp}^2$ in film 1 than in film 5 for the same density of planar defects.

For $B = 3$ Tesla, $a \sim 300\,\text{Å}$. In this case $a << \lambda_{ab}$ for both films 1 and 5. $\kappa_L$ should therefore be proportional to $\lambda_{ab}^{-2}$ for both. $\lambda_{ab}^{-2}$, and thus $\kappa_L$ is a factor of 8 smaller in film 5 than film 1, while $\kappa_{exp}$ is only a factor of 4 smaller. Thus at 3 Tesla the vortex interactions are sufficient to immobilize the vortices in planar defects in film 1 but not in film 5.

We now turn our attention to the anomalous behavior observed in the two LSCO films, Figs. 33 and 34. Since our model clearly does not fit the measured Labusch parameter, we cannot extrapolate $\kappa_{exp}(B,T=0)$ to $B=0$ to obtain $\kappa_0$ for these films. Using Eq. 16 to calculate $\kappa_L$ at $B = 6$ Tesla yields $\kappa_L = 3 \times 10^4 \, \text{N/m}^2$ for film 7 and $6 \times 10^4 \, \text{N/m}^2$ for
for film 8, both of which are larger than the measured $\kappa_{exp}(B=6\text{Tesla})$. This implies that in the LSCO films there will be a large number of unpinned vortices. Attempts to construct a model based on a fixed number of pinned vortices have yielded qualitatively similar results to the data shown in Figs. 33 and 34, but at this time, no quantitatively accurate model has been constructed.
Figure 42 - Magnetic field dependence of the experiment Labusch parameter of film 1 at 4.2 K for increasing and decreasing field. The dotted line is the theoretical Labusch parameter predicted by the theory developed in section 3.4.
Figure 43 - Magneto-optical image optical image of the inhomogeneous flux density in a YBCO thin film taken from Ref. 28: M. R. Koblischka and R. J. Wijngaarden, Super. Sci. Tech. 8, 199 (1995)
3.5 – Critical State

In section 3.4, we described the process in which vortices first enter the film along planar defects. As the field increases and more vortices are forced into the planar defects, those already present are forced into the bulk material where they are pinned by extended defects. These pinned vortices prevent succeeding vortices from pushing into the film. The result is a gradient in the vortex density that causes a screening current in the film. The force on the vortices created by the screening current is balanced by the pinning force. This state in which the pinning force is balanced by the force from the screening current is known as the critical state.

If we consider one of the first vortices to enter the film, it will enter and find a pinning site. As additional vortices enter behind it, the vortex density gradient will increase until the resulting force is sufficient to unpin the initial vortex. When this happens, the vortex will move forward to a new pinning site and the vortices behind it will move forward to reduce the gradient in the vortex density. It is these continual avalanches of large bundles of vortices that produce the noise observed in Figs. 28 – 34.

As stated previously, during this process the vortices continually experience a force from the nonuniform vortex density that must be canceled by the pinning force. Consequently, the vortices are pushed off the center of their pinning sites and the measured Labusch parameter is smaller than the true Labusch parameter. To measure the amount by which the critical state masks the true Labusch parameter, we make use of the "degaussing coil" shown in Fig. 1a.

The process of measuring the true Labusch parameter begins by increasing the current in superconducting magnet to the desired magnetic field. We then apply an AC
current to the degaussing coil that adds an additional field of about 200 Gauss. This field continuously adds and removes vortices, reversing the gradient in the vortex density. The amplitude of the ac current is then slowly reduced to zero. As the amplitude of the current decreases, the depth to which added vortices penetrate decreases. The result is a nearly uniform vortex density.

Fig. 44 is a plot of $1/\lambda^{2}_{\text{exp}}$ vs. $B$ for film 3 measured using this technique. Fig. 45 is a plot of $\kappa_{\text{eff}}$ vs. $B$ derived from the data in Fig. 44. In both figures, the open circles are data measured using this technique for relaxing the vortex density. The solid curve is data obtained while continuously increasing the field. The dotted curve is the prediction made by the vortex interaction theory discussed in section 3.4. Fig. 45 shows that relaxing the vortex density results in about a 30% increase in $\kappa_{\text{exp}}$. This implies that in the unrelaxed measurement the average vortex is displaced about 9Å from the center of its pinning potential (See Appendix B3.) This requires a force per unit length of $1.6 \times 10^{-4}$ N/m² resulting from a screening current of $7.7 \times 10^6$ A/cm² ($J_{C}=10 \times 10^6$ A/cm².)
Film 3 - Sputtered YBCO

Figure 44 - Magnetic field dependence of the experimental penetration depth of film 5 at 4.2 K for a relaxed vortex distribution. The solid curve is data taken from Fig. 15. The open circles are data for the relaxed vortex distribution. The dotted line is the theoretical penetration depth.
Figure 45 - Magnetic field dependence of the experimental Labusch parameter of film 5 at 4.2 K for a relaxed vortex distribution. The solid curve is data taken from Fig. 35. The open circles are data for the relaxed vortex distribution. The dotted lines are the theoretical Labusch parameter.
3.6 – Thermal Motion

We now focus our attention on the temperature dependence of the Labusch parameter. We begin by solving Eq. 14 for $\kappa_0$.

$$\kappa_0(T) = \frac{3\Phi_0^2 r_d^2}{40\pi\mu_0 \lambda_{ab}^2(T) \xi_{ab}^4(T)}$$  \hspace{1cm} (19)

Eq. 19 reveals that the intrinsic Labusch parameter has a temperature dependence of $\kappa_0(T) \sim 1/\lambda_{ab}^2(T) \xi_{ab}^4(T)$. Fig. 46 is a plot of $\kappa_{exp}(T,B)$ vs. $T$ for film 1 taken at $B = 1$ Tesla. For the moment, we assume that 1 Tesla is a sufficiently high field that the measured inductance is the result of vortices trapped in extended defects, not planar defects and a sufficiently low field that the effects of vortex interactions are negligible. The dashed curve in Fig. 46 is $\kappa_0(T)$ calculated using the measured values of $\kappa_{exp}(T=0,B=0)$ and $\lambda_{eff}(T,B=0)$ and the theoretical temperature dependence of $\xi_{ab}^4(T)$ calculated by Ulm et al.\textsuperscript{30} for disordered YBCO. To allow for an estimated fluctuation-induced suppression of $T_C$ of about 6 K, $\xi_{ab}^4(T)$ is calculated so as to diverge at $T/T_C = 1.07$. As is evident in Fig. 46, $\kappa_{exp}(T)$ differs dramatically from $\kappa_0(T)$. The difference is caused by thermally induced vortex motion.

The theory of thermal motion of vortices trapped on linear extended defects was fully developed by Nelson and Vinokur\textsuperscript{31-33} (NV). NV map the problem of a classically fluctuating vortex onto that of the world lines of a quantum mechanical particle traveling through imaginary time. In Appendix B6, we show how to adapt the NV model to calculate the temperature dependence of the Labusch parameter using our pinning potential. While the NV model is rigorous and complete, it does not illuminate the
elementary physics of the problem. We will therefore calculate the temperature
dependence using a simplified model that captures the essential physics of the problem.

To understand the effects of thermal motion in the simplest way, we will initially
ignore vortex interactions. At T = 0 and with no applied supercurrents, each vortex sits at
the center of its pinning site. For T > 0, the vortex fluctuates in a random path about the
center of the pinning potential. Since thermal motion occurs up to frequencies much
higher than those of the measurement, the measured Labusch parameter is an average of
the curvature of the pinning potential over the path of the wandering vortex. Rather than
attempting to calculate this average, we will approximate the effective Labusch parameter
using Eq. B23 where \( \rho \) is taken to be the \( \text{rms} \) displacement of the fluctuating vortex.
\[
\rho_{\text{rms}} = \sqrt{\langle \rho^2 \rangle}.
\]
The problem thus reduces to one of calculating \( \rho_{\text{rms}} \).

We calculate \( \rho_{\text{rms}} \) by appealing to the equipartition theorem. In order to use the
equipartition theorem, we must calculate the total energy increase associated with a
fluctuating vortex. Consider a segment of vortex of length \( \ell_z \) that moves independently
of the segments above and below it. \( \ell_z \) depends on the flexibility of the vortex. If each
vortex were a rigid rod, then \( \ell_z \) would equal the film thickness, d. If the unit cell layers
were completely decoupled, then \( \ell_z \) would be the c-axis lattice constant. The average
pinning energy of each vortex segment is \( \langle U(\rho) \rangle_{\ell_z} \), where \( \langle U \rangle \) is the average of U over
the path of the vortex. We approximate the average pinning energy with
\[
\langle U(\rho) \rangle_{\ell_z} = U(\rho_{\text{rms}}) \ell_z.
\]
In addition to increasing the potential energy, thermal motion also increases the
length of the vortex and therefore the line energy. If the vortex segment at \( z = 0 \) has
moved a distance $\rho_{rms}$ in one direction, and the adjacent segment at $z = \ell_z$ has moved a
distance $\rho_{rms}$ in another direction, then the increase in vortex length, averaged over all
angles, can be approximated by:

$$\delta \ell = \frac{1}{2\pi} \int_0^{2\pi} \sqrt{2\rho_{rms}^2 (1 - \cos \phi) + \ell_z^2} \, d\phi - \ell_z$$

$$= \ell_z \left[ \frac{2}{\pi} \sqrt{1 + \left( \frac{2\rho_{rms}}{\ell_z} \right)^2} \right] E \left( \sqrt{\frac{4\rho_{rms}^2}{\ell_z^2 + 4\rho_{rms}^2}} \right) - 1 \right] \approx \frac{\rho_{rms}^2}{\ell_z},$$

where $E(x)$ is the complete elliptic integral of the second kind. The line energy per unit
length is $\varepsilon_1 = \frac{\Phi_0^2}{4\pi\mu_0 \lambda_{ab}} \ln \left( \frac{\lambda_{ab}}{\xi_{ab}} \right)$ for an isotropic material. In the present case, the
additional arc length is in the ab-plane. Because of the anisotropy in $\lambda$ in high $T_C$
materials, the line energy in the ab-plane, $\varepsilon_{1a}$, is reduced from $\varepsilon_1$ by a factor
$M_{ab}/M_c \approx 1/40.$

According to the equipartition theorem each independently fluctuating vortex
segment has an average thermal energy of $k_B T$. Equating this with the total increase in
ergy yields

$$k_B T = \langle U(\rho) \rangle \ell_z + \bar{\varepsilon}_1 \delta \ell = \left[ \frac{\kappa_0 \rho_{rms}}{2} - \frac{\kappa_0 \rho_{rms}^2}{12\xi_{ab}^2} \right] \ell_z + \frac{\rho_{rms}^2}{\ell_z} \bar{\varepsilon}. \quad (21)$$

We now have an equation with two unknowns, $\rho_{rms}$ and $\bar{\varepsilon}_1$. To provide a second
constraint, we must determine what configuration will maximize the entropy. This is a
difficult problem that is solved within the NV model. We shall borrow their result that
the entropy is maximized when the pinning energy equals the added line energy. To see
that this is true, consider a vortex which meanders down the length of the pinning site.
For very large $\ell_z$, the line energy is negligible and all the thermal energy takes the form of pinning energy. The vortex can thus travel a large distance from the center of the pinning site. Decreasing $\ell_z$ by $\frac{1}{2}$ would cause only a slight amount of energy to shift from pinning energy to line energy and thus only reduce $\rho_{rms}$ slightly. The net effect is a near doubling of the available states. The reverse is true for very small $\ell_z$. Nearly the entire thermal energy is consumed in line energy and the vortex has a very small displacement. Here a doubling of $\rho_{rms}$ would cost only a small amount of line energy while nearly doubling the available states. The entropy is thus maximized when the energy is evenly distributed.

With this result, we can use Eq. 21 to calculate $\rho_{rms}$.

$$\rho_{rms} = \frac{3}{\xi_{ab}} \left[ 1 - \sqrt{1 - \frac{2 k_b T}{3 \ell_z \xi_{ab}^2}} \right] \cdot$$  \hspace{1cm} (22)

We can now calculate the effective Labusch parameter by substituting Eq. 22 into Eq. B23, which yields, in the limit of small $T$

$$\kappa_{eff}(T) = \kappa_0(T) - \frac{2 k_b T}{3 \ell_z \xi_{ab}^2(T)}.$$  \hspace{1cm} (23)

Since $\kappa_0(T)$ and $\xi_{ab}(T)$ are both roughly constant at low temperatures, Eq. 23 predicts a $T$-linear decrease in $\kappa_{eff}$ at low $T$, as is observed in our data.

Identifying $\kappa_{exp} = \kappa_{eff}$, and using the experimentally obtained value $\kappa_{exp} = -6 \times 10^3$ N/m$^2$K for film 1, we find $\ell_z(T=0) = 8$ Å, a surprisingly small value. It implies that the vortex segments are decoupled from other segments within the same unit cell.
layer. To gauge the uncertainty in $\ell_z$, we have repeated the above analysis using several plausible, analytically tractable pinning potentials which have a curvature $\kappa_0$ at $\rho = 0$ and an inflection point at $\rho = \xi_{ab}$ (e.g. $4\kappa_0^2 \xi_{ab}^2 \left[1 - e^{-\frac{\rho^2}{4\xi_{ab}^2}}\right]$.) Values obtained for $\ell_z$ using alternate pinning potentials ranged from 9 Å to 20 Å, which are more appealing values. They are very close to both $\xi_{ab} = 15\text{Å}$ and the c-axis lattice constant, 11.7 Å. This wide range of values implies that $\ell_z$ is sensitive to the exact functional form of the pinning potential.

An expression for the fluctuation length, $\ell_z$, can be obtained by formally equating the two energy terms in Eq. 21. For $\rho_{rms} \ll \xi_{ab}$, i.e. $T \to 0$, equating the lowest order term in the pinning potential with the added line energy yields

$$\ell_z = \sqrt{\frac{2 \xi_{ab}^2}{2 \kappa_0 \ln \left(\frac{\lambda_{ab}}{\xi_{ab}}\right) - 4\pi\mu_0 \kappa_0 \lambda_{ab}^2}}$$

which, for film 1 yields $\ell_z(T=0) = 24 \text{Å}$. In addition, we see from Eq. 24 that $\ell_z$ has a temperature dependence of $\ell_z \sim \frac{1}{\sqrt{\kappa_0(T)\lambda_{ab}^2(T) - \xi_{ab}^2(T)}}$.

This model fails to predict the observed Labusch parameter for temperatures above 15 K. The dotted curve in Fig. 46 is $\kappa_{eff}$ as predicted by Eq. 23 using the temperature dependence of $\ell_z$ predicted by Eq. 24. It predicts that $\kappa_{eff}$ drops to zero at about 45 K. This is not surprising since Eq. 23 was derived assuming small $T$ (small $\rho_{rms}$). Repeating the calculation without assuming small $T$ causes a sharp downward turn in $\kappa_{eff}$, which drops to zero at about 30 K. Using the alternate pinning potentials
discussed above improves the situation slightly, but still cannot explain the existence of pinning above 50K. The reason is that the depth of the pinning potential is simply \( U_0(\text{T}=0) \xi_2(\text{T}=0)/k_B = 50 \text{ K} \). To understand pinning above 50K, we must consider additional factors not included in our model thus far.

At higher temperatures, we would expect several other factors to become important. Effects such as vortex entanglement and double kinks, vortex loops hopping to another pinning site, are known to exist in YBCO crystals, but we would only expect these effects to become important near \( T_c \) where dissipation is observed.

The biggest sources of error in our analysis were the failures to include vortex interactions and critical state effects. As stated earlier, the critical state causes \( \kappa_{\text{C}} \) to be about 25% smaller than \( \kappa_0 \). We would thus expect \( U_0 \) to be about 30% bigger and pinning to continue to about 65 K. Vortex interactions have two important effects on the temperature dependence of the Labusch parameter. First, the lattice strain discussed in section 3.4 causes a constant force that must be added to the pinning potential. Second, and more important at high temperatures, if the vortex or a segment of the vortex is pushed out of the pinning potential, it experiences an effective pinning potential from interactions with surrounding vortices.

Inclusion of vortex interactions in the temperature dependence proves to be a difficult problem. A complete calculation of the effective Labusch parameter which includes both the effects of vortex interactions and thermal motion has to date not been performed. We have however performed a simple simulation that demonstrates how vortex interactions can preserve pinning at temperatures beyond 65K.
To simulate the effects of vortex interactions on the temperature dependence at high temperature, we will make several simplifying assumptions. First, we will ignore the effects of lattice strain and assume that the vortices form a perfect lattice with a pinning potential centered at each vortex lattice site. This is the same assumption made previously when analyzing the temperature dependence of the B = 1 Tesla Labusch parameter for film 1. Second, we will ignore critical state effects and assume $\kappa_0(T=0) = \kappa_{exp}(T=0)$. Third, we will still assume independently fluctuating vortex segments of length $\ell_x(0)$, but we shall ignore the added line energy and treat $\ell_x(0)$ as a temperature independent fitting parameter used to adjust the temperature scale. Fourth, we will assume a pinning potential given by Eq. 13 up to a cut off $\rho = \rho_c < \xi_{ab}$. For $\rho > \rho_c$, we use a quadratic pinning potential with Labusch parameter $\kappa_L$ given by Eq. 16. $\rho_c$ is chosen such that $\frac{d^2U}{d\rho^2}$ is continuous. Finally, we will assume $\kappa_0(T)$ given by Eq. 19.

Using these assumptions, we proceed to calculate $\kappa_{eff}$ as follows. Let $V(\rho)$ be the pinning potential per unit length described above. When the screening current produced by the drive coil flows past the vortex, the vortex experiences a total potential per unit length $V(\rho) + J\Phi_0 x$, where we have taken the y direction to be the direction in which the current flows. An independently fluctuating vortex segment will then be displaced to a position $<x>$ where $<x>$ is a thermodynamic average given by

$$
<x> = \frac{\int x e^{-\left(V(\rho) + J\Phi_0 x\right)/\ell_s T} d^2x}{\int e^{-\left(V(\rho) + J\Phi_0 x\right)/\ell_s T} d^2x}.
$$

(25)
The effective Labusch parameter is then simply \( \kappa_{\text{eff}} = \frac{J\Phi_0}{\langle \chi \rangle} \).

Eq. 25 was evaluated numerically for \( B = 1 \) Tesla using the experimentally obtained values for \( \kappa_0(T=0) \) and \( \lambda_{ub}(T) \) for film 1. The result is shown in Fig. 47. It appears that for \( B = 1 \) Tesla, the reduction in \( \kappa_{\text{eff}} \) that comes about by ignoring critical state effects is canceled by the increase in \( \kappa_{\text{eff}} \) gained by ignoring lattice strain. Fig. 47 thus bears a striking resemblance to Fig. 46. Although we left out several important effects in this calculation it is sufficient to demonstrate that vortex interactions can prevent the inductance from diverging as segments of the vortex fluctuate out of the pinning potential. Vortex interactions obviously play a significant role in the temperature dependence of the Labusch parameter at high temperatures.
Figure 46 - Temperature dependence of the theoretical and experimental Labusch parameters of film 1 at an applied magnetic field of 1 Tesla.
Figure 47 - Theoretical temperature dependence of the Labusch parameter of film 1 at a magnetic field of 1 Tesla.
3.7 – Thermal Supercurrent Fluctuations

Thermal motion of vortices is caused by the interaction between the vortices and thermally induced supercurrents. In a superconductor at non-zero temperatures, the phase of the order parameter fluctuates randomly in space and time. This results in fluctuations in the supercurrent density of the film. These fluctuating supercurrents apply a force to the vortex resulting in its random motion about the pinning potential. Knowing the thermal motion of the vortices, we can work backwards to deduce some properties of the thermal supercurrents. First, the data indicate that at 5 K vortex segments about 8 Å to 20 Å long fluctuate independently. Thus, given the unit cell thickness of $c = 11.7$ Å, the supercurrents in adjacent copper oxide bilayers must be only weakly correlated at low temperatures. YBCO should be quasi-two-dimensional as far as these fluctuations are concerned.

To estimate the magnitude of thermal supercurrents in each unit cell layer, we equate the mean square force they exert on a vortex, $\langle J_z^2 \rangle \Phi_0^2$, with the mean square restoring force exerted by the pinning potential, $\kappa_0^2 \langle \rho^2 \rangle$. Replacing $\ell_z$ with $c$, the c-axis lattice constant, we find:

$$\langle J_z^2 \rangle_{\text{exp}} = \frac{\kappa_0^2 k_B T}{c \Phi_0^2}.$$  \hspace{1cm} (26)

Eq. 26 is only valid for frequencies below the vortex depinning frequency, $\omega_p = \kappa_0 / \eta$. Vortex motion resulting from fluctuations at higher frequencies is damped by the viscosity, $\eta$, and is negligible. In other words, as a detector of supercurrents, a vortex is a low-pass filter with a bandwidth of $\omega_p/4$. 

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It is useful to calculate the supercurrent density noise power per unit bandwidth at frequencies below $\omega_p$:

$$S_J(0) = \frac{<J_x^2>}{\omega_p/4} = \frac{4\eta k_B T}{c\Phi_0^2}. \tag{27}$$

Replacing $\eta$ with $^{17} \Phi_0^2/2\pi\rho_n\xi_{ab}^2$ and calculating the noise power for the sheet current density, $K \equiv cJ_s$, yields

$$S_K(0) = \frac{2k_B T}{\pi R_n \xi_{ab}^2}. \tag{28}$$

where $1/R_n = c/\rho_n$ is the sheet conductance of a unit cell layer. This is the classical result for noise supercurrents in two dimensions, being linearly proportional to the sheet conductance and $T$.

It is important to note that $\ell_z$ is only an upper bound on the c-axis correlation length of the supercurrent fluctuations. It is not possible for the vortex to fluctuate on a smaller length scale than the supercurrents driving the fluctuation. However, since $\ell_z$ also depends on the "stiffness" of the vortex, it is possible for the supercurrent correlation length to be smaller than $\ell_z$. Since this correlation length cannot be smaller than the thickness of the copper oxide plane, the small value of $\ell_z$ at low temperatures necessarily implies that the layers are only weakly correlated. At higher temperatures, $\ell_z$ increases as $\xi_{ab}^2$. We cannot therefore conclude that the supercurrent fluctuation correlation length also increases as $\xi_{ab}^2$, merely that it increases no faster.

From the foregoing, we conclude that supercurrent fluctuations have their full classical amplitude for frequencies below $\omega_p$. To determine when quantum effects
should become significant, we use the experimental value $\kappa_0 = 2 \times 10^5 \text{N/m}^2$ to obtain $\omega_0 = 2 \times 10^{11} \text{rad/s}$, and $\hbar \omega_0 / k_B = 1.5 \text{K}$. For $T > 5 \text{K}$, our lowest measurement temperature, all of the frequencies in the experimental bandwidth of $\omega_p/4$ are excited at the classical level. At temperatures below 1.5 K, only a fraction, $k_B T / \hbar \omega_p$, of the important bandwidth would be excited, and thus $<J_z^2>_{\text{exp}} = S_j(0) k_B T / \hbar \omega_p \propto T^2$. On this basis, we predict that $\kappa_{\text{exp}}(T)$ is quadratic rather than linear below about 1.5 K.
CHAPTER 4

CONCLUSION

The two-coil mutual inductance technique has proven a powerful tool for studying pinned vortices. Using it, we have measured the magnetic penetration depth and Labusch parameter for YBCO and LSCO thin films. We have studied films grown by coevaporation with post annealing, pulsed laser deposition, and RF sputtering. In our data, we find no correlation between superfluid density and growth technique. The superfluid density appears to be dependent entirely upon the growth parameters used within each technique. We do find a correlation between growth technique and defect density. Coevaporated films appear to have the lowest defect density while PLD films have the highest defect density.

From our data we are able to extract the vortex glass melting temperature. We have constructed a phase diagram for fields below 6 Tesla. Our phase diagram is consistent with previous experiments and vortex glass melting theory.

Near the vortex glass melting temperature, we can measure the vortex viscosity in addition to the pinning force. We have put forth a simple explanation for the anomalously large vortex viscosity observed in this region, however a more detailed theory is needed.
We have demonstrated that the vortex inductance is too low to result from point defects. It is more consistent with a low density of planar defects (enough to accommodate 500 Gauss of vortices) and a high density of extended defects, roughly $10^4 \mu m^{-2}$. To produce the observed inductance, the extended defects must suppress superconductivity with a radius of 1 Å to 8 Å.

Using the two coil technique, we have directly measured the critical current for our films. The measured critical current of $10^7 A/cm^2$ is consistent with measurements by other groups. This critical current can be derived from our model of pinning by extended defects.

In YBCO films, the magnetic field dependence of the Labusch parameter can be explained by vortex interactions. The force on the vortex resulting from strain in the vortex lattice pushes the vortex off the center of its pinning site reducing the effective Labusch parameter. At a characteristic field of 10 Tesla to 15 Tesla, the interaction force is equal to the maximum pinning force. Above this field, a fraction of the vortices will become unpinned to reduce lattice strain. At present, such a high density of extended pinning defects has not been directly observed.

The magnetic field dependence of the Labusch parameter in LSCO films is consistent with a lower density of extended defects. LSCO films appear to have unpinned vortices in fields as low as 1 Tesla. Attempts to calculate the average Labusch parameter of a combination of pinned and unpinned vortices have produced qualitatively similar results to those observed in the data, but a more detailed model is necessary to produce quantitatively similar results.
The temperature dependence of the Labusch parameter at low temperatures is the result of thermal motion of the vortices. Thermally induced supercurrents drive the vortex along a random path about the pinning potential effectively reducing the Labusch parameter. To explain the temperature dependence at higher temperatures, it is necessary to include the effects of vortex interaction. The data suggest that the supercurrents in each unit cell layer are uncorrelated. We predict that at temperatures below 1.5 K, quantum effects should be detectable in the temperature dependence of the Labusch parameter.
APPENDIX A

TABLE OF FILM PARAMETERS

Table A1: Table of Film Parameters. Here we list all quantities measured or calculated for the films studied. Columns 1 and 2 indicate the material of which the film is composed and the growth technique. Column 3 lists the film thickness. Column 4 lists the figures from which the data in that row were taken. Films with two rows have both temperature and magnetic field dependent data. Discrepancies between the temperature dependent data and magnetic field dependent data for film 1 occurred because the measurements were taken 11 months apart. $T_C$ is the zero field critical temperature for the film. $d\kappa/dT$ is the low temperature slope of $\kappa_{exp}(T)$. $J_C$ (Calc) is the critical current density calculated using Eq. 15, while $J_C$ (Exp.) is the critical current density measured directly. $n_d$ is the density of extended defects. $\ell_z$ (Calc) indicates the value of $\ell_z$ calculated using Eq. 24, while $\ell_z$ (Fit) is the value of $\ell_z$ obtained from the low temperature slope of $\kappa_{exp}(T)$. The value of $\kappa_L$ listed is that calculated for $B = 6$ Tesla.
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<th>Data form Figures</th>
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APPENDIX B

CALCULATIONS

B.1 – Mutual Inductance of the Two-Coil Apparatus

To calculate the mutual inductance as a function of film conductivity, we must first model the two coil apparatus. Fig. 1a is a sketch of the apparatus geometry. We assume a film of infinite radius with a uniform complex conductivity, \( \sigma \). We model the coils as a set of coaxial circles of zero radius each carrying an AC current, \( I \). If we assume that the operating frequency is sufficiently low that the wavelength is long compared to any coil or film dimensions then the displacement current can be ignored. Ampère’s law then takes the form:

\[
\nabla \times (\nabla \times A) = \mu_0 (\vec{J}_D + \vec{J}_F)
\]

where we have separated the current density into that of the drive coil and that of the film. Assuming a drive current \( I e^{j\omega t} \), the current density in the film is just \( \vec{J}_F = -j\omega \sigma \vec{A} \).

Making the gauge choice \( \nabla \cdot A = 0 \), equation B1 becomes

\[
\nabla^2 \vec{A} = -\mu_0 \vec{J}_D + j\mu_0 \omega \sigma \vec{A}
\]

Defining the \( z \) direction to be along the axis of the film and coils and the origin to be the center of the film, we now Fourier transform Eq. B2 in the \( xy \)-plane:

\[
\partial_z^2 \vec{A}(\vec{q}, z) - q^2 \vec{A}(\vec{q}, z) = -\mu_0 \vec{J}_D(\vec{q}, z) + j\mu_0 \omega \sigma \vec{A}(\vec{q}, z)
\]
To proceed, we must include details of the coil geometry. We take the positive z-axis to point in the direction of the drive coil. We define \( h \), to be the distance between the center of the film and the first turn of the coil, where the index \( \ell \in \{D,R\} \) distinguishes between the drive and receive coils. \( \delta h \), is the distance between turns of the coil, essentially the wire thickness. \( R \), is the distance from the axis of the coil to the center of the first layer. \( \delta R \), is the distance between layers, again essentially the wire thickness. \( s \), is the distance between the counter-wound sections of the coil. \( M \), is the number of layers and \( N \), is the number of turns per layer in each counter-wound section. \( d \) is the film thickness. Using these parameters, we define the following convenient notation.

\[
D_{\ell,n} = \begin{cases} 
1 & \text{if } 0 \leq n < N, \\
-1 & \text{if } N \leq n < 2N,
\end{cases}
\]

\[
s_{\ell,n} = \begin{cases} 
0 & \text{if } 0 \leq n < N, \\
s & \text{if } N \leq n < 2N,
\end{cases}
\]

\[
h_{\ell,n} = h_{\ell} + n \delta h_{\ell} + s_{\ell,n}
\]

\[
h = h_D + h_R
\]

Using this notation, the drive current density is

\[
\bar{J}_D = \frac{1}{l} \sum_{n=0}^{M} \delta (\rho - R_D - m \delta R_D) \sum_{n=0}^{2N-1} D_{D,n} \delta (z - h_{D,n}) \hat{\phi}
\]

Fourier transforming yields

\[
\bar{J}_D(\vec{q},z) = -il\alpha_D(q) \sum_{n=0}^{2N-1} D_{D,n} \delta (z - h_{D,n}) \hat{\phi}_q
\]
where \( \alpha, (q) = \sum_{m=0}^{N} \left(R_e + m \delta R_e \right) J_1 (q(R_e + m \delta R_e)) \) and \( J_1 (x) \) is the first order Bessel function of the first kind. We proceed by partitioning space into 3 separate regions:

\( R_1 = \{ z < \frac{d}{2} \}, R_2 = \{ \frac{d}{2} < z < \frac{d}{2} \}, R_3 = \{ z > \frac{d}{2} \}. \) Substituting Eq. B5 into Eq. B3 then yields a linear second order differential equation in \( z \) in each region. These equations have solutions:

\[
\begin{align*}
R1: \tilde{A} (q, z) &= A_1 (q) e^{-q(z - \frac{d}{2})} - \frac{i \mu_0 I}{2q} \alpha(q) \sum_{n=0}^{N} D_{n,n} e^{-q|z - \frac{n}{q}|} \\
R2: \tilde{A} (q, z) &= A_2 (q) e^{-q(z - \frac{d}{2})} + A_3 (q) e^{q(z + \frac{d}{2})} \\
R3: \tilde{A} (q, z) &= A_4 (q) e^{q(z + \frac{d}{2})}
\end{align*}
\]

where \( Q \) is defined by \( Q^2 = q^2 + i \mu_0 \omega \sigma \). \( A_1 (q) - A_4 (q) \) are undetermined coefficients which must be calculated from boundary conditions at \( z = \pm \frac{d}{2} \). Continuity of the normal component of \( \vec{B} \) requires that \( \tilde{A}(q, z) \) be continuous. Continuity of the tangential component of \( \vec{E} \) requires that \( \tilde{A}(q, z) \) be continuous. Applying these conditions yields:

\[
\begin{align*}
A_1 (q) &= -i \mu_0 I \frac{(Q^2 - q^2) \sinh (Qd) e^{-\eta_d}}{q(Q-q)^2 e^{-Qd} - q(Q+q)^2 e^{Qd}} \alpha_d (q) \beta_d (q) \\
A_2 (q) &= i \mu_0 I \frac{(Q-q) e^{-Qd} e^{-\eta_d}}{(Q-q)^2 e^{-Qd} - (Q+q)^2 e^{Qd}} \alpha_d (q) \beta_d (q) \\
A_3 (q) &= i \mu_0 I \frac{(Q+q) e^{-\eta_d}}{(Q-q)^2 e^{-Qd} - (Q+q)^2 e^{Qd}} \alpha_d (q) \beta_d (q) \\
A_4 (q) &= i \mu_0 I \frac{2Q e^{-\eta_d}}{(Q-q)^2 e^{-Qd} - (Q+q)^2 e^{Qd}} \alpha_d (q) \beta_d (q)
\end{align*}
\]
where \( \beta_r(q) = \frac{(1 - e^{-N, \Phi, q}) (1 - D_r e^{-(s, + N, \Phi, q)})}{(1 - e^{-\Phi, q})} \). We can now calculate the voltage induced in the receive coil

\[
V = i \omega \sum_{m=0}^{M} \sum_{n=0}^{N} D_{r,n} \oint_{C_{m,n}} \vec{A}(\vec{r}) \cdot d\vec{r} \tag{B8}
\]

where \( C_{m,n} \) is the curve described by \( \vec{r} = R_{r,n} \hat{\rho} - h_{r,n} \hat{z} \) for \( \phi = 0 .. 2\pi \). Substituting the Fourier transform of \( \vec{A}(\vec{r}) \) into Eq. B8 and performing the integration around \( C_{m,n} \) yields:

\[
V = -\omega \sum_{n=0}^{N} D_{r,n} \int \alpha_r(q) A(\vec{q}, z) d^2 \vec{q} \tag{B9}
\]

Substituting Eqs. B6c and B7d into equation B9. performing the integral with respect to \( \vec{q} \), and dividing by \( i\omega \) yields the integral expression for \( M \).

\[
M = 4\pi \mu_0 \int_0^\infty \frac{q Q e^{-Q}}{(Q + q)^2 e^{-Q} - (Q - q)^2} \alpha_\phi(q) \alpha_r(q) \beta_d(q) \beta_r(q) dq \tag{B10}
\]

Having solved for the vector potential throughout space, we can also calculate the current density induced in the film.

\[
\vec{J}(\vec{r}) = -i \omega \sigma \vec{A}(\vec{r})
= -i \mu_0 \omega \sigma I \int_0^\infty \frac{q e^{-Q}}{(Q + q)^2 e^{-Q} - (Q - q)^2} \left[ (Q + q) e^{\sigma(z - \eta_\phi)} - (Q - q) e^{-\sigma(z - \eta_\phi)} \right] dq \tag{B11}
\]

**B.2 – Effective Labusch Parameter for Point Defects**

Consider a vortex running along the z-axis that is pinned at the origin and at the point \( z = L \). A current applied in the y direction produces a force \( \vec{J} \Phi_0 \hat{x} \) on the vortex.

This force stretches the vortex in the x direction to form a curve \( x(z) \). Stretching the
vortex increases the length of the vortex, which requires additional line energy. The line
energy per unit length\(^{17}\) is \(\varepsilon_1 = \frac{\Phi_0^2}{4\pi\mu_0\lambda_{ab}^2} \ln\left(\frac{\lambda_{ab}}{\xi_{ab}}\right)\). However, because of the anisotropy in
most high T\(_C\) materials, the line energy in the ab-plane, \(\varepsilon_{1,ab}\), is reduced\(^{31}\) by a factor
\(M_{ab}/M_c = 1/40\). Stretching the vortex thus results in an additional energy \(E = \varepsilon_1(s - L)\),
where \(s\) is the arc length of the curved vortex. The total potential experienced by the
vortex segment is

\[
V = \int_0^L \left[ \varepsilon_1 \sqrt{1 + \left(\frac{dx}{dz}\right)^2} - J\Phi_0 x \right] dz \tag{B12}
\]

We can now use Euler's equation to calculate the resulting path of the vortex:

\[
\frac{d}{dx} \left( \frac{dL}{dx} \right) - \frac{dL}{dx} = \frac{d}{dz} \left[ \varepsilon_1 \frac{dx}{dz} \left( 1 + \left(\frac{dx}{dz}\right)^2 \right)^{\gamma/2} \right] + J\Phi_0 = 0 \tag{B13}
\]

If we assume small displacements, then \(\sqrt{1 + \left(\frac{dx}{dz}\right)^2} = 1\) and Eq. B13 reduces to

\[
\varepsilon_1 \frac{d^2x}{dz^2} = -J\Phi_0. \tag{B14}
\]

which has the solution

\[
x(z) = \frac{J\Phi_0}{2\varepsilon_1} z(L - z). \tag{B15}
\]

Thus far, we have assumed that the vortex is held fixed at the two pinning sites.

To proceed, we must examine the pinning site in more detail. Assume the pinning
potential is quadratic with a range of \(\xi_{ab}\), the radius of the vortex. The pinning force is
then \(F = -kx\). To determine \(k\), we assume that each defect extinguishes superconductivity
within a sphere of radius $r_d$. Equating the condensation energy saved in locating the vortex on the defect with the depth of the pinning potential yields

$$E = \left( \frac{B_c^2}{2 \mu_0} \right) \left( \frac{4}{3} \pi r_d^3 \right) = \frac{\Phi_0^2 r_d^3}{12 \pi \mu_0 \lambda_{ab}^2 \xi_{ab}^2} = \frac{1}{2} k \xi_{ab}^2 \Rightarrow k = \frac{\Phi_0^2 r_d^3}{6 \pi \mu_0 \lambda_{ab}^2 \xi_{ab}^4}. \quad (B16)$$

Equating the total force applied to the vortex segment by the current with the pinning force yields a displacement of $x_0 = J \Phi_0 L / k$. Thus the total displacement of the vortex is

$$x(z) = J \Phi_0 \left( \frac{z(L - z)}{2 \xi_1^2} + \frac{L}{k} \right). \quad (B17)$$

The Labusch parameter is the ratio of the force per unit length, $J \Phi_0$, to the displacement $x(z)$. To calculate the effective Labusch parameter measured by our experiment, we need to average over the length of the vortex segment. Since it is the inductance that is additive, not the conductivity, we must average $\kappa^{-1}(z)$.

$$\kappa_{\text{eff}} = \left[ \frac{1}{L} \int_0^L \kappa_{\text{eff}}^{-1}(z) dz \right]^{-1} = \left[ \frac{L^2}{12 \xi_1^2} + \frac{L}{k} \right]^{-1}. \quad (B18)$$

Substituting in $\xi_1$ and $k$ yields

$$\kappa_{\text{eff}} = \frac{\Phi_0^2}{4 \pi \mu_0 \lambda_{ab}^2 \xi_{ab}^2} \left[ \frac{L^2}{12 \xi_{ab}^2 \ln(\lambda_{ab}/\xi_{ab}) M_{ab}/M_c} + \frac{3 L \xi_{ab}^2}{2 r_0^3} \right]^{-1}. \quad (B19)$$

B.3 – Effect of a Constant Force on the Labusch Parameter

In this section, we consider the effect of a constant force on the inductance of a pinned vortex. Suppose a uniform force per unit length, $\vec{F}_0$, is applied to the vortex. The
vortex will be displaced to a position, \( \tilde{\rho}_0 \), given by \( \tilde{F}_0 - \tilde{V} U(\tilde{\rho}_0) = 0 \). In addition to this force, the current induced in the film by the drive coil applies a small force \( \delta \tilde{F} \). The resulting vortex position, \( \tilde{\rho}_0 + \delta \tilde{\rho} \), is, in the absence of dissipation, determined by
\[
\tilde{F}_0 + \delta \tilde{F} - \tilde{V} U(\tilde{\rho}_0 + \delta \tilde{\rho}) = 0.
\]
Assuming small \( \delta \tilde{\rho} \) and expanding about \( \tilde{\rho}_0 \) yields:
\[
\delta \tilde{F} = \tilde{\nabla} \left( \tilde{\nabla} U \cdot \delta \tilde{\rho} \right) \bigg|_{\tilde{\rho} = \tilde{\rho}_0}.
\]  
(\text{B20})

\( \delta \tilde{F} \) is proportional to \( |\delta \tilde{\rho}| \), but is not necessarily in the same direction as \( \delta \tilde{\rho} \). However, the inductance of the film depends only on \( \delta \tilde{F} \cdot \delta \tilde{\rho} \). We therefore define the effective Labusch parameter, \( \kappa_{\text{eff}} (\tilde{\rho}_0) \), as
\[
\kappa_{\text{eff}} (\tilde{\rho}_0) = \frac{\delta \tilde{F} \cdot \delta \tilde{\rho}}{|\delta \tilde{\rho}|^2} = \tilde{\nabla} \left( \tilde{\nabla} U \cdot \delta \tilde{\rho} \right) \bigg|_{\tilde{\rho} = \tilde{\rho}_0}.
\]  
(\text{B21})

Using Eq. 13 for \( U(\rho) \) yields
\[
\kappa_{\text{eff}} (\rho_0, \gamma) = \kappa_0 \left[ 1 - \frac{1}{3} \left( \frac{\rho_0}{\xi_{ub}} \right) \right] (1 + 2 \cos^2 \gamma)
\]  
(\text{B22})

where \( \gamma \) is the angle between \( \tilde{\rho}_0 \) and \( \delta \tilde{\rho} \). If \( \gamma \) is the same for each vortex in the film, Eq. B12 yields the measured Labusch parameter. However, in the case of vortex interactions or thermal fluctuations each vortex is displaced in a random direction. We therefore need to average over all angles. Since the inductance of the vortices adds in series, it is necessary to average \( \kappa^{-1} \).
\[
\kappa_{\text{eff}} = \frac{1}{2\pi} \int_0^{2\pi} \kappa^{-1} (\rho_0, \gamma) d\gamma = \kappa_0 \sqrt{1 - \frac{4}{3} \left( \frac{\rho_0}{\xi_{ub}} \right)^2 + 1 \left( \frac{\rho_0}{\xi_{ub}} \right)^4}
\]  
(\text{B23})

\( \rho_0 \) is obtained by solving:
which yields

\[ \vec{p}_0 = 2\xi_{ab} \sin \left( \frac{1}{3} \sin^{-1} \left( \frac{3F_0}{2\kappa_0 \xi_{ab}} \right) \right) \vec{F}_0. \] (B25)

Substituting into Eq. B22 yields

\[ \kappa_{\text{eff}} = \frac{\kappa_0}{\sqrt{3}} \sqrt{1 + 2\cos \left( \frac{4}{3} \sin^{-1} \left( \frac{3F_0}{2\kappa_0 \xi_{ab}} \right) \right)}, \] (B26)

In the limit of small \( F_0 \), Eq. B26 reduces to

\[ \kappa_{\text{eff}} \approx \kappa_0 \left[ 1 - \frac{2}{3} \left( \frac{F_0}{\kappa_0 \xi_{ab}} \right)^2 \right]. \] (B27)

Thus when an external force is applied to the vortex, it is pushed off the center of its pinning potential, reducing the measured Labusch parameter.

### B.4 – Restoring Force Produced by Vortex Interactions

In the absence of pinning, vortices form a triangular lattice with a lattice constant

\[ a = \frac{2\Phi_0}{\sqrt{3}B}. \] For comparison, \( a = 490 \, \text{Å} \) at 1 Tesla while at 6 Tesla, \( a = 200 \, \text{Å} \). The lattice results from a repulsive force between vortices:

\[ F_{12} = -\frac{\Phi_0^2}{2\pi\mu_0 \lambda_{ab}^3} K_0 \left( \frac{\lambda_{ab}}{r_{12}} \right) r_{12}, \] (B28)

where, \( K_0(x) \) is the n’th order modified Bessel function of the second kind. The net force on a vortex setting at the origin is
where the sum is over all lattice vectors $\mathbf{R} \neq \mathbf{0}$. The lattice constant, $a$, is on the order of a few hundred angstroms while the magnetic penetration depth, $\lambda_{ab}$, is on the order of a few thousand angstroms. Consequently, the sum converges slowly and must be carried out over more than just the next nearest neighbors.

Because of the symmetry of the lattice, the net force on the vortex is zero. However, if the vortex were displaced by a small amount $\mathbf{u}$, while the rest of the lattice is held fixed, the net force on the vortex is

$$\mathbf{F}_L(\mathbf{u}) = -\frac{\Phi_0^2}{2\pi \mu_0 \lambda_{ab}^3} \sum_{\mathbf{R} \neq \mathbf{0}} K_i \left( \frac{\mathbf{R}}{\lambda_{ab}} \right) \mathbf{R} - \mathbf{u}.$$  

(B30)

For small displacements, $|\mathbf{u}| \ll a \lambda_{ab}$, we can expand Eq. B30 to yield

$$\mathbf{F}_L(\mathbf{u}) = -\frac{\Phi_0^2}{2\pi \mu_0 \lambda_{ab}^3} \tilde{K}_L \left( \frac{a}{\lambda_{ab}} \right) \mathbf{u} + O(u^2).$$  

(B31)

where the tensor $\tilde{K}_L \left( \frac{a}{\lambda_{ab}} \right)$ is given by

$$\tilde{K}_L = \sum_{\mathbf{R} \neq \mathbf{0}} \left[ K_i \left( \frac{\mathbf{R}}{\lambda_{ab}} \right) \mathbf{I} - \frac{K_i \left( \frac{\mathbf{R}}{\lambda_{ab}} \right)}{2} \left( \mathbf{R} / \lambda_{ab} \right) \left( \mathbf{R} / \lambda_{ab} \right) \right]$$  

(B32)

where $\mathbf{I}$ is the identity tensor. Since the lattice vectors are all separated by a constant length, $a$, the tensor depends only on $a/\lambda_{ab}$.

At this point, no further progress can be made analytically and we must resort to numerical calculations. If one of the lattice basis vectors is chosen to be the $x$ direction,
then by symmetry considerations we can see that $\tilde{K}_L(a/\lambda_{ab})$ is diagonal. Numerical evaluation of Eq. B32 reveals that the two diagonal elements are equal. The tensor $\tilde{K}_L(a/\lambda_{ab})$ can therefore be replaced by a scalar $K_L(a/\lambda_{ab})$. Fig. B1 is a graph of $K_L$ vs. $a/\lambda_{ab}$ made by numerical calculation of Eq. B32.

As $x \to 0$, $K_L(x)/x \to 1/x^2$. We might therefore suppose that for small $a/\lambda_{ab}$,

$$K_L\left(\frac{a}{\lambda_{ab}}\right) \to C\left(\frac{a}{\lambda_{ab}}\right)^2.$$ Fitting to the data in Fig. B1, yields $C = \frac{2\pi}{\sqrt{3}}$. The error in this approximation is less than 3% for $a/\lambda_{ab} = 0.25$ and drops to 0% as $a/\lambda_{ab} \to 0$.

The restoring force on the vortex now becomes

$$\bar{F}_L = -\frac{\Phi_0^2}{2\pi\mu_0\lambda_{ab}^4}K_L\left(\frac{a}{\lambda_{ab}}\right)\tilde{u} = -\frac{\Phi_0^2}{\sqrt{3}\mu_0 a^2\lambda_{ab}^3}\tilde{u} = -\frac{B\Phi_0}{2\mu_0\lambda_{ab}^2}\tilde{u}. \quad (B33)$$

Finally, since we have a linear restoring force, it is natural to define a lattice restoring force constant per unit length, $\kappa_L$. The restoring force can then be expressed as

$$\bar{F}_L = -\kappa_L\tilde{u}. \quad (B34)$$

Thus if a vortex is not pinned by a defect, it is effectively pinned by the lattice with a Labusch parameter $\kappa_L$. 

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Figure B1 - Numerical calculation of $K_L$ vs. $1/\lambda_{ab}$. The dotted line in the inset is the approximation $\frac{2\pi}{\sqrt{3}} \left( \frac{\sigma}{\lambda_{ab}} \right)^2$. 
Assume for the moment that only a small fraction, \( \chi_p \), of the vortices is pinned by defects, while the remaining vortices are unpinned. Now consider a random collection of \( 1/\chi_p \) vortices. In such a collection, only one will be pinned. If a current density \( J \) were induced in the film, each vortex would experience a force \( F_0 = J\Phi_0 \). Since only one of the vortices is pinned, it must supply the restoring force for the entire collection, \( F_p = F_0/\chi_p \).

Each vortex will thus be displaced an amount \( x_p = \frac{F_p}{\kappa_0} = \frac{F_0}{\chi_p \kappa_0} \), where \( \kappa_0 \) is the Labusch parameter of the pinned vortex. The force on the pinned vortex from the unpinned vortices will be \( F_u = F_p - F_0 = \frac{1 - \chi_p}{\chi_p} F_0 \).

When considering the interaction between the pinned vortex and the unpinned vortices, notice that this situation is the exact opposite of that discussed in Appendix B4. There we considered a lattice of pinned vortices with one unpinned vortex displaced from its proper lattice site. Here we consider a lattice of unpinned vortices with a single pinned vortex displaced from its proper site in the lattice of unpinned vortices. By symmetry, the restoring force pulling the unpinned vortices into position is simply \( F_u = \kappa_L \Delta x \) where \( \Delta x \) is the distance the pinned vortex is displaced from its proper position in the unpinned lattice. Each of the unpinned vortices will therefore move an additional amount \( \Delta x = \frac{F_u}{\kappa_L} = \frac{1 - \chi_p}{\chi_p} \frac{F_0}{\kappa_L} \) resulting in a total displacement of

\[
x_u = x_p + \Delta x = \frac{F_0}{\chi_p \kappa_0} + \frac{1 - \chi_p}{\chi_p} \frac{F_0}{\kappa_L} = \frac{F_0}{\chi_p \kappa_0 \kappa_L} \left[ 1 + \left( 1 - \chi_p \right) \frac{\kappa_0}{\kappa_L} \right].
\]  
(B35)
We can now calculate the Labusch parameter for each type of vortex. The pinned vortex will have an effective Labusch parameter of \( \kappa_{\text{eff},p} = \frac{F_0}{x_p} = \chi_p \kappa_0 \). The unpinned vortices will have an effective Labusch parameter of \( \kappa_{\text{eff},u} = \frac{F_0}{x_u} = \frac{\chi_p \kappa_0}{1 + (1 - \chi_p)(\kappa_0/\kappa_L)} \).

Knowing the effective Labusch parameter of each species of vortex, we can now calculate the inductance of the film. The inductance of the pinned vortices is

\[
\mu_0 \lambda^2_{c,p} = \frac{\chi_p B \Phi_0}{\kappa_{\text{eff},p}} = \frac{B \Phi_0}{\kappa_0}.
\]

The inductance of the unpinned vortices is

\[
\mu_0 \lambda^2_{c,u} = \frac{(1 - \chi_p) B \Phi_0}{\kappa_{\text{eff},u}} = \frac{B \Phi_0}{\kappa_0} \frac{1 - \chi_p}{\chi_p} \left[1 + \left(1 - \chi_p\right) \frac{\kappa_0}{\kappa_L}\right].
\]

The total inductance is therefore:

\[
\mu_0 \lambda^2_{\text{total}} = \mu_0 \lambda^2_{c,p} + \mu_0 \lambda^2_{c,u} = \frac{B \Phi_0}{\chi_p \kappa_0} \left[1 + \left(1 - \chi_p\right) \frac{\kappa_0}{\kappa_L}\right].
\]

This results in an effective Labusch parameter for the film of

\[
\kappa_{\text{eff}} = \frac{B \Phi_0}{\mu_0 \lambda^2_{\text{total}}} = \frac{\chi_p \kappa_0}{1 + (1 - \chi_p)^2 \left(\frac{\kappa_0}{\kappa_L}\right)}.
\]

**B.6 – Nelson and Vinokur Model**

In this appendix, we demonstrate how to modify the NV model to calculate the Labusch parameter using the pinning potential given in Eq. 13. NV map the problem of a classically fluctuating vortex interacting with an extended defect onto that of a quantum particle interacting with a potential energy well, with the z-direction mapping onto imaginary time. For simplicity, vortex interactions are ignored. Using this mapping, NV
write a two dimensional “Schrödinger equation” for the pinned vortex. Using our
pinning potential, the Schrödinger equation for the vortex becomes:

$$
\left[ -\frac{(k_B T)^2}{2\xi^2} \nabla^2 + \frac{\kappa_0 \rho^2}{2} - \frac{\kappa_0 \rho^4}{12\xi^2} + J \Phi_0 x \right] \Psi_0(\vec{\rho}) = E_0 \Psi_0(\vec{\rho}). \tag{B38}
$$

In this mapping, the thermal energy, $k_B T$, corresponds to Planck’s constant, $\hbar$, while the
tilt modulus, $\tilde{\varepsilon}_t = \frac{M_{ab}}{M_c} \ln \left( \frac{\lambda_{ab}}{\xi_{ab}} \right) \frac{\Phi_0^2}{4\pi \mu_0 \lambda^2_{ab}}$, corresponds to the mass of the bound quantum
particle. $|\Psi_0(\vec{\rho})|^2$ is the probability density for finding the vortex at position $\vec{\rho}$.

$J$ is the low-frequency supercurrent density induced in the film by the drive coil.
This current results in a force, $J \Phi_0 \dot{x}$, which shifts the equilibrium position of the vortex
in the x direction. The new equilibrium position of the vortex is given by

$$
\langle x \rangle = \langle \Psi_0 | x | \Psi_0 \rangle. \tag{B39}
$$

The effective Labusch parameter is thus $k_{\text{eff}} = \frac{J \Phi_0}{\langle x \rangle}$.

At low temperatures, vortex fluctuations are dominated by the lowest energy
eigenstate, $\Psi_0(x)$. $\Psi_0(x)$ can be calculated using a harmonic oscillator ground state and
treating $-\frac{\kappa_0 \rho^4}{12\xi^2} + J \Phi_0 x$ as a perturbation. The localization length, $l_z$, the average
distance the vortex fluctuates from its equilibrium position is

$$
\ell_z^2 \equiv \langle \Psi_0 | x^2 | \Psi_0 \rangle_{x=0} = \frac{k_B T}{\sqrt{\kappa_0 \tilde{\varepsilon}_t}} + O(k_B T^2). \tag{B39}
$$

The fluctuation length in the z direction is then

$$
\ell_z = \frac{\ell_z^2}{k_B T/\tilde{\varepsilon}_t} = \sqrt{\frac{\tilde{\varepsilon}_t}{\kappa_0}} = \sqrt{\frac{M_{ab}}{M_c} \ln \left( \frac{\lambda_{ab}}{\xi_{ab}} \right) \frac{\Phi_0^2}{4\pi \mu_0 \kappa_0 \lambda^2_{ab}}}. \tag{B40}
$$
The effective Labusch parameter can be calculated from the perturbed ground state wave function $|\Psi_0\rangle$, yielding:

$$\kappa_{\text{eff}} = -\frac{F}{\langle x \rangle} = -\frac{J\Phi_0}{\langle \Psi_0 | x | \Psi_0 \rangle} = \kappa_0 - \frac{k_B T}{8r_0^2 \sqrt{\frac{\xi}{\kappa_0}}} = \kappa_0 - \frac{k_B T}{8\xi_{\text{wh}} \ell_z}.$$  \hspace{1cm} (B41)
APPENDIX C

MEASUREMENTS OF ADDITIONAL SPUTTERD YBCO FILMS

In this appendix, we present, without comment, data on four additional RF sputtered films.
Figure C1 - Temperature dependence of the experimental penetration depth of film 9 at various magnetic fields. At zero field, $\lambda_{exp} = \lambda_{ab}$ and $1/\lambda_{exp}^2$ is proportional to the superfluid density.
Figure C2 - Temperature dependence of the experimental Labusch parameter of film 9 at various magnetic fields.
Figure C3 - Temperature dependence of the experimental penetration depth of film 10 at various magnetic fields. At zero field, $\lambda_{exp} = \lambda_{ab}$ and $1/\lambda_{exp}^2$ is proportional to the superfluid density.
Figure C4 - Temperature dependence of the experimental Labusch parameter of film 10 at various magnetic fields.
Figure C5 - Temperature dependence of the experimental penetration depth of film 11 at various magnetic fields. At zero field, $\lambda_{exp} = \lambda_{ab}$ and $1/\lambda_{exp}^2$ is proportional to the superfluid density.
Figure C6 - Temperature dependence of the experimental Labusch parameter of film 11 at various magnetic fields.
Figure C7 - Temperature dependence of the experimental penetration depth of film 12 at various magnetic fields. At zero field, $\lambda_{ex} = \lambda_{ab}$ and $1/\lambda^2_{ex}$ is proportional to the superfluid density.
Figure C8 - Temperature dependence of the experimental Labusch parameter of film 12 at various magnetic fields.
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