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ESSAYS ON FUNDAMENTAL EQUILIBRIUM EXCHANGE RATES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Natalia S. Salazar, M.A.

* * * * *

The Ohio State University
2000

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TO MY MOTHER
ACKNOWLEDGMENTS

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My special gratitude goes to my husband for his excellent editorial work and moral support through the duration of this dissertation.
This study presents an approach to estimate the Fundamental Equilibrium Exchange Rate (FEER) popularized by John Williamson. The approach differs from the literature in two important aspects. First, most estimates of FEERs hinge on a subjective judgment regarding the external balance target. This study eliminates the subjective element by estimating the current account target using a statistical model. Current account is modeled as a sum of a permanent component and a transitory component. The permanent component is extracted using the Kalman Filtering technique and used as a current account target. This approach captures the idea of a sustainable long-run equilibrium. It also eliminates the normative element in the calculations, making these estimates a medium-term forecast rather than a normative concept. Second, estimates of FEERs typically rely on structural equations of trade balance obtained from wide range of sources. This study is considerably more disciplined. The structural equations are obtained by applying the canonical cointegration technique to a single set of quarterly time series.

The first chapter presents the estimated trajectory for the US FEER over the period from 1967Q1 to 1994Q4. The estimated trajectory is shown to outperform the
random walk in medium-term and long-term forecasting. The second chapter considers bilateral exchange rates. The equilibrium exchange rate trajectories are estimated for 3 pairs of countries: UK-US, Japan-US and Canada-US. These trajectories are compared to the random walk in forecasting real exchange rates and found to be superior in medium- and long-run. In the third chapter, FEER is compared to the equilibrium exchange rates estimates derived from reduced-form equations: Natural Real Exchange Rate (NATREX), and Behavioral Equilibrium Exchange Rate (BEER). It is shown that the medium and long-term forecasts based on the FEER and NATREX models are more accurate than the point predictions of the random walk. The BEER model, however, can beat the random walk only if the actual realized values of the fundamentals are used in forecasting.
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CHAPTER 1

US FUNDAMENTAL EQUILIBRIUM EXCHANGE RATE: METHODOLOGY, ESTIMATION AND FORECASTING

1.1 INTRODUCTION

Fundamental Equilibrium Exchange Rates (FEERs) are real effective exchange rates that are consistent with simultaneous internal and external macroeconomic balance. The term was first introduced by John Williamson (1983) as a basis for the Extended Target Zone proposal for managed floating exchange rates. Since then the concept has been used widely and different variations and improvements in methods of calculating FEERs have been proposed. Notable examples include Desired Equilibrium Exchange Rate (DEER) of Bayomi et al and Natural Real Exchange Rate (NATREX) of Stein (see Williamson (1994), Hinkle and Montiel (1999) for a collection of these and other works).

This chapter presents the estimated FEER trajectory for the US over the period from 1967Q1 to 1994Q4. The calculation of FEERs requires estimates of structural equations such as import and export demand. It is common in the literature to employ the surveyed estimates of the trade equations that are not necessarily based on one consistent
data set. In this chapter, I use a consistent set of data series and employ a canonical cointegration technique to estimate structural equations of the trade balance. In doing so, I avoid using estimates from studies based on various data sources and I am able to discuss the fitness of the model.

FEER was originally introduced as a guide for policy. However, I am also interested in the concept as a medium-term forecast. Whether FEER is a policy "target" or a medium-term forecast depends on how a current account input is specified in a model. Williamson (1983, 1994), for example, sets his current account target at a level that is consistent with his measure of "desirable and sustainable capital flows". Rather than making arbitrary assumptions about the size of desirable capital flows, I propose a statistical model to specify current account input. Current account is modeled as a sum of a permanent component and a transitory component. The permanent component that I extract using the Kalman Filtering technique is used as a current account target. This approach captures the idea of a sustainable long-run equilibrium, which is essential to the construction of FEERs. It also eliminates the normative element in the calculations, which makes my FEER a medium-term forecast rather than a normative concept. I conclude this chapter by showing that FEER forecasts are superior to the medium-term predictions of the random walk models that are known to outperform various structural models for real exchange rates.

The rest of the chapter is organized as follows. Section 2 discusses the theoretical background behind the FEER concept. Section 3 describes the data. Section 4 discusses the estimation of the trade equations. Section 5 contains the discussion on estimating
target current account. Section 6 presents the estimated FEER trajectory for the US and compares it to the actual real effective exchange rate path. It also discusses the ability of my estimates to forecast the actual real exchange rate. Finally, I offer some concluding remarks in Section 7.

1.2 FEER METHODOLOGY

The FEER concept is based on the notion of macroeconomic equilibrium. The FEER is the real exchange rate consistent with simultaneous internal and external equilibrium. Internal balance is specified as the level of output consistent with both full employment and a low and sustainable rate of inflation. External equilibrium is specified in terms of "current account target" that reflects a current account balance (CA) financed by a desired, or sustainable, rate of net foreign assets accumulation.

\[ CA = \overline{\Delta NFA} \]  

where NFA is a stock of a country’s net foreign assets, \( \Delta \) represents a change in the stock and bars over variables represent their sustainable equilibrium levels.

The current account is the sum of the trade balance, interest income received on a country’s stock of net foreign assets and net government and private transfers \( \text{NETTRAN} \). The trade balance is determined by domestic and foreign output and the real exchange rate. The functional relationship between the current account and its fundamentals can be derived from the estimated equations for trade volumes and prices.
and is captured in equation (2):

\[ CA = X(Y^*, q) - M(Y, q) + rNFA - NETTRAN \]  

(2)

where \( r \) is real interest rate, \( X \) is export which is a function of real foreign output \( (Y^*) \) and the real exchange rate \( (q) \), \( M \) is import which is a function of domestic output \( (Y) \) and the real exchange rate.

The permanent component of the current account, or equilibrium current account, can be calibrated by setting the explanatory variables at their equilibrium levels. The equilibrium relationship between current account (omitting interest income and net transfers) and capital account is:

\[ CA = f(Y, Y^*, \text{FEER}) = \Delta NFA \]  

(3)

where \( \text{FEER} \) is the real exchange rate consistent with both internal and external equilibrium. Equation (3) can be solved for \( \text{FEER} \). In its general form, the solution can be represented by the following equation:

\[ \text{FEER} = f(Y, Y^*, \Delta NFA) \]  

(4)

Thus, calculation of the \( \text{FEER} \) within macroeconomic balance framework requires: 1) estimates of potential output for the domestic and foreign outputs to capture the notion of internal balance, 2) an estimate or judgment regarding the external equilibrium, i.e. the desired rate of capital flow, 3) a current account model.
Williamson (1994) employed six different dynamic general equilibrium models and calibrated his FEERs as their medium-term solution. The models were EAG (The External Adjustment with Growth Model), Interlink, Intermod, Mimosa and MSG (McKibbin-Sachs Global macroeconometric model). Bayomi, Clark, Symansky, Taylor (1994) employed Multimod (1994) to calculate their equilibrium exchange rates.


The estimates of potential output based on the trend growth rate of real GDP are adopted as “target” values for internal balance. External equilibrium is specified in terms of a current account “target”, which implicitly assumes that all dynamic adjustments in capital stocks, financial wealth and government debt have worked themselves out. Note that even when dynamic general equilibrium models are used to calibrate FEERs, "target" values for external and internal balance are still required as inputs into these models, which makes FEER calculations a comparative static exercise.

Although the comparative static, partial equilibrium approach is simple, it still produces results robust to generalization. Bayoumi et al (1994) calculated FEER using both the comparative static, partial equilibrium and the general equilibrium approaches and were surprised that the estimates obtained using the two approaches are fairly similar.
1.3 DATA DESCRIPTION

The data used in this chapter is quarterly, seasonally adjusted at annual rates and covers the time period from the first quarter of 1967 through the fourth quarter of 1994. The wholesale price index (WPI) for the USA, export prices of G-7 industrial countries (in units of foreign currency), the index of the nominal effective exchange rate of the dollar, the volume of the world trade, the real Gross Domestic Product of the USA and major industrial countries are obtained from the International Financial Statistics CD-ROM Data set. Data on the US foreign transactions in the National Income and Product Accounts (in actual and constant 1987 dollars): non-oil, oil, service imports, agricultural, nonagricultural, service exports, payments of factor income, transfer payments and net foreign investment, is extracted from different issues of the Survey of Current Business.

Data on US trade with G-10 countries was obtained from various issues of the Direction of Trade Statistics.

The index of nominal effective exchange rate of the dollar is a geometrically weighted average of the dollar's exchange rates against the currencies of the other G-7 countries. The weight of each country in the index is equal to that country's share of US total trade with the rest of the G-7 countries. Changes in the index are computed from a base period of 1990 = 100.

The index of the price-adjusted, or real, exchange rate of the dollar is calculated by dividing the dollar's nominal effective exchange rate index by an index of the ratio of the weighted average of the foreign manufacturing export prices (in units of foreign
currency) to US domestic prices (WPI). The weights used in computing are the same as those used in constructing the index of nominal effective exchange rate.

1.4 CURRENT ACCOUNT MODEL

The comparative static, partial equilibrium approach is based on equation (3). Although this approach makes trade volume equations the center of FEER calculations, Barrell and Wren-Lewis (1989) did not estimate these equations. Rather, they used econometric estimates of income and price elasticities embodied in the World Model of the National Institute of Economic and Social Research – GEM. Driver and Wren-Lewis (1996) also relied on the estimates surveyed by Hooper and Marquez (1995). To enable trade equations with pre-specified elasticities to track historical data better, time trends and split time trends were fitted into the equations. The inclusion of these trends was justified as capturing secular changes in supply.

This study does not rely on any particular estimates of income and relative price elasticities for US exports and imports (excluding oil), since a survey of econometric work reveals a great dispersion of elasticities (probably arising from differences in modeling assumptions, variable specifications, sample periods, econometric techniques, etc).

Following the bulk of the time-series work on trade equations, I estimate the trade equations by single-equation methods. US imports are disaggregated into non-oil imports.

---

1 In their most recent work, Wren-Lewis and Driver (1998) did estimate the trade equations for the G-7 countries. However, they still imposed assigned values for income and price elasticities in cases where their...
oil imports and service imports. The activity variable in these equations is real GDP of the USA, the relative price variable is the real exchange rate in the non-oil and service imports equations, and it is the real exchange rate multiplied by the ratio of the foreign manufacturing export prices to the world oil prices in the oil imports equation.

US exports are disaggregated into goods and service exports. The activity variable in the first equation is the volume of the world trade, while the activity variable in the second equation is real output of major industrial countries. The relative price variable is the real exchange rate.

To choose an appropriate method for estimating the trade equations, the statistical properties of the data are examined first. Park’s J(1,5) and Augmented Dickey-Fuller (ADF) tests are applied to decide whether the time series contain a unit root.

The J(1,5) test is based on the following spurious regression:

\[ X_t = \sum_{\tau=0}^{1} \mu_{\tau} t^\tau + \sum_{\tau=2}^{5} \mu_{\tau} t^\tau + \eta_t \]  

(6.1)

where the time trends of order zero and one are deterministic, and the time polynomials of order two and up to five are spurious time trends.

The J(1,5) test is defined as

\[ J(1,5) = \frac{F(1,5)}{T} \]  

(6.2)

where \( F(1,5) \) is the standard Wald test statistic for the hypothesis \( \mu_2 = \mu_3 = \mu_4 = \mu_5 = 0 \). Under the null hypothesis that \( x_t \) contains unit root (\( \eta_t \) is unit root non-stationary), \( F(1,5) \) estimation results were not satisfactory.
explodes, but J(1,5) has an asymptotic distribution. Under the alternative of trend stationarity J(1,5) converges to zero. This test does not require the estimation of the long run variance of the disturbance (η), and thus neither the order of the autoregression nor the lag truncation number needs to be specified.

To implement the ADF(p) tests I ran the following regression:

$$\Delta x_t = \theta + \alpha_{t-1} + \gamma t + \beta_1 \Delta x_{t-1} + \ldots + \beta_p \Delta x_{t-p} + \nu_t$$  \hspace{1cm} (6.3)

and test for $\alpha=0$. The ordinary t-statistic for $\alpha=0$ has a non-standard distribution, critical values for unit roots tests can be found in MacKinnon (1990).

The results of the unit root tests are presented in Table 1.1. These tests do not reject the null of difference stationarity for oil, non-oil, service imports and for goods, service exports at the ten percent significance level. Tests are also not significant at the ten percent level for real exchange rate and relative price of oil. Detecting a unit root in the real exchange rate time series is not surprising, similar results are widely recorded in the exchange rate literature and serve as an empirical base for questioning the validity of the Purchasing Power Parity hypothesis.

The results of the J(1, 5) tests for the income variables are more ambiguous. The J(1,5) tests cannot accept the null of difference stationarity for real GDP of the USA even at the one percent significance level and for real GDP of major industrial countries at the five percent significance level. However, the ADF(12) tests are not significant, as well as the ADF(1) test in case of the US real GDP, while the ADF(4) test is significant at the 5
percent significance level for real US GDP and at the 1 percent significance level for real GDP of industrial countries.

These results reflect the statistical properties of the real GDP series that are such that it is difficult to distinguish between the two different hypotheses. I follow the tradition established by Nelson and Plosser (1982) and treat the real GDP series as non-stationary.

The fact that each trade equation includes trend non-stationary variables suggests that OLS estimation is not efficient, since the error term is correlated with the first difference of non-stationary regressors at leads, lags and contemporarily.

There are different asymptotically efficient estimators of cointegrating vectors available. Johansen's maximum likelihood estimation, for example, makes a parametric correction for long run correlation of the error term and the first difference of regressors. Park's Canonical Cointegrating Regressions (CCR), on the other hand, utilize a nonparametric estimate of the long run covariance parameters.

Park's CCR is used in this study to estimate cointegrating vectors in the trade equations. The CCR procedure was chosen over other methods, because of its two main advantages, as it is summarized by Park and Ogaki (1997). First of all, the CCR estimators have better small sample properties than Johansen’s ML estimators in terms of the mean square error even when the assumed Johansen’s Gaussian VAR structure is true. Second of all, the Park's tests of the null of stochastic cointegration and of the deterministic cointegration have reasonable size and power, as opposed to the tests of the
null of no cointegration, which are known to have very low power, and tend to fail to reject the null with high probability, even if the null is not consistent with data.

Park's CCR procedure is applied to a cointegrated system:

\[ Y_t = \theta + \mu_t + \gamma X_t + \varepsilon_t \]
\[ \Delta X_t = \nu_t \]

(6.4)

(6.5)

where \( Y_t \) and \( X_t \) are difference stationary, \( \nu_t \) and \( \varepsilon_t \) are stationary with zero mean. \( Y_t \) is the log of import (export) variable. \( X_t \) is the 2x1 vector and includes the log of activity variable and the log of relative price variable.

Let \( W_t = (\varepsilon_t, \nu_t') \). Then the i-autocovariance matrix of \( W_t \) is \( \Phi(i) = E(W_t, W_{t-i}) \), its covariance matrix is \( \Sigma = \Phi(0) \). Define

\[ \Gamma = \sum_{i=0}^{\infty} \Phi(i) \quad \Omega = \sum_{-\infty}^{\infty} \Phi(i) \]

Here \( \Omega \) is the long run covariance matrix of \( W_t \).

Consider transformations:

\[ Y_t^* = Y_t + \Pi_y' w_t \]
\[ X_t^* = X_t + \Pi_x' w_t \]

Because \( w_t \) is stationary, \( Y_t^* \) and \( X_t^* \) are cointegrated with the same vector \( (1, -\gamma_x) \) as \( Y_t \) and \( X_t \). The CCR procedure chooses \( \Pi_y \) and \( \Pi_x \) so that the OLS estimator is asymptotically efficient when \( Y_t^* \) is regressed on \( X_t^* \). As it is shown in Ogaki (1993), this requires:
\[ \Pi_y = \sum^{-1} \Gamma_2 \gamma_x + (0, \Omega_{12} \Omega_{22}^{-1}) \gamma \]

\[ \Pi_x = \sum^{-1} \Gamma_2 \]

where

\[ \Omega_{11,2} = \Omega_{11} - \Omega_{12} \Omega_{22}^{-1} \Omega_{21} \]

\[ \Gamma_2 = (\Gamma_{12}', \Gamma_{22}') \]

\[ \Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \]

\[ \Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} \]

In practice, long run covariance parameters in these formulas are estimated, and computed \( \Pi_y, \Pi_x \) are used to transform \( Y_t \) and \( X_t \). The resultant CCR estimators are asymptotically efficient, if the parameters are estimated consistently.\(^2\) The CCR estimators are asymptotically normal, so that their standard errors can be interpreted in the usual way.

To test for the null of cointegration, Park's \( H(p,q) \) test is computed by applying the CCR to equation:

\[ Y = \sum_{t=0}^{p} \mu_t \tau^t + \sum_{t=p+1}^{q} \mu_t \tau^t + \gamma' X_t + \epsilon_t \]

Park's \( H(p,q) \) is defined as

\(^2\) Following the recommendations summarized by Park and Ogaki (1997), VAR prewhitening method with Andrew's automatic bandwidth parameter is used to estimate the long run covariance parameters. The reported estimators are based on the third stage CCR. All CCR estimation is done by Ogaki's (1993) GAUSS CCR.
\[ H(p, q) = F(p, q) \frac{\hat{\sigma}^2}{\hat{\omega}^2} \]

where \( \omega^2 \) is an estimate of the long run variance of \( \varepsilon_t \) (\( \Omega_{11.2} \)), \( \varepsilon_t \) is the estimated residual and

\[ \hat{\sigma}^2 = \frac{1}{T} \sum_{t=1}^{T} \hat{\varepsilon}_t^2 \]

\( F(p, q) \) is the standard Wald test statistic for the hypothesis \( \mu_{p+1} = \ldots = \mu_q = 0 \). Under the null of cointegration (stationarity of \( \varepsilon_t \)), \( H(p, q) \) converges in distribution to a \( \chi_{p-q}^2 \) distribution. Under the alternative, \( H(p, q) \) diverges to infinity. \( H(0,1) \) can be applied to test the deterministic cointegration restriction, \( H(1, q) \) tests stochastic cointegration.3

Before running the CCR with activity variable and relative price as explanatory variables, it is necessary to assess whether they are cointegrated. If they are cointegrated, the US exports (imports) cannot be cointegrated with these explanatory variables, and the cointegrating regression would be meaningless. The ADF(p) tests are applied to OLS cointegrating regressions residuals to test for no cointegration. The results of these tests are in Table 1.2. All tests are not significant which presents a strong evidence that all three activity variables employed in the study are not cointegrated with relative prices, and CCR are legitimate to apply.

The results of Canonical Cointegrating Regressions are presented in Table 1.3. All coefficients on income and relative prices are significant, and none of the \( H(p, q) \) tests are

---

1 Test statistics for the null of stochastic cointegration and the null of deterministic cointegration are from the
significant for non-oil and service imports. In case of service exports, the coefficient on
the activity variable is significant, and the H(1,2) and H(1,3) tests support the null of
stochastic cointegration. In the case of goods exports, all coefficients are significant, and
the H(0,1) test supports the null of deterministic cointegration, but the H(1,2) and H(1,3)
tests reject the null of stochastic cointegration at the one percent significance level.

Thus, at least one of the H(p,q) tests seem to favor the null of cointegration for all
trade equations, and CCR provide significant estimates of income and price elasticities
with the only exception of the relative price elasticity in the service export equation.

The US oil import equation is not estimated directly. Oil is a “standard”
commodity, i.e., it is traded on international market at a common price. For such
“standard” commodities, estimating the trade equations is a matter of estimating domestic
demand and supply; imports and exports are their residuals. Traditionally, US oil import
income and price elasticities are thus derived from the estimates of oil consumption
elasticities and are known to be 2.5 and 0.6 respectively. I model US oil imports and
estimate their time trend based on these elasticities. The time trend is split in 1977 to
capture the effect of changing preferences for oil caused by the oil price shocks of the
early 70s, and it is 0.0364 before the split and -.0203 after the split.

To capture the effect of exchange rate changes on prices of traded goods, I model
prices of exports and imports as functions of domestic prices (WPI) and world
manufacturing prices (expressed in dollars). The effect of foreign prices on prices of US
exports is statistically insignificant, and I treat export prices as exogenous variables. The
estimates of non-oil and service import prices are contained in Table 1.4. Price of oil imports is considered to be exogenous, since oil is a "standard commodity".

To complete the picture of the current account, the effects of exchange rate changes on flows of factor income and net transfers are considered. I follow Barrell and Wren-Lewis (1989) and model these effects as a static feedback from the model to the stock of foreign assets. It is assumed that all overseas assets and receipts of factor income from these assets are in foreign currency, while all domestic assets held by foreign residents and payments on these assets are in domestic currency (dollars). Changes in the exchange rate then affect the current account balance through effects on receipts of factor income. Specifically, if real exchange rate exceeds its fundamental value, then devaluation of the dollar improves current account by raising the dollar value of foreign assets and increasing the dollar value of interest flows from these assets. By analogy, revaluation of the dollar decreases the dollar value of foreign assets and receipts of interest, and current account deteriorates. Factor income payments are treated as exogenous variables and as determined in domestic currency. Net transfers are modeled as a time trend. To estimate the trend, a dummy variable was included to account for abnormal spikes in net transfers during the Gulf War. To construct trend net transfers, the dummy was omitted.

The ability of the estimated trade and price equations to track the actual data is graphically presented in Figure 1.1, 1.2, and 1.3. The solid lines on these graphs represent the actual time series, while the plus lines represent the fit of the model. In Figure 1.4, the solid line is the actual trade balance (on goods and services) to GDP ratio (tbgs), the plus
line is the estimated tbgs. The estimated tbgs captures the major deteriorations and improvements in the actual tbgs, however it does so ahead of time. This pattern arises because the estimated trade equations are based on the long-run elasticites and they ignore the lags before relative price and income changes influence exports and imports.

1.5 CURRENT ACCOUNT TARGET

One of the most controversial issues in calibrating FEER is the definition of external equilibrium conditions. The most general definition on which many authors would agree defines external equilibrium as a balance of payment equilibrium in the absence of speculative capital flows and movements in international reserves. However, it is not that easy to find an agreement across alternative models on how to separate speculative capital flows from actual capital flows.

None of the authors defines medium-run external balance as a current account balance with no capital flows. Williamson explains why this approach (no capital flows = no speculative capital flows) is not a sensible approach: "Countries can often expect to benefit by exporting or importing capital over a long period of years, when domestic savings are, respectively, greater or less than domestic investment opportunities at the world interest rate. Assuming that countries are going to have balance of payments objectives at all ..., the aim should be to achieve that current account balance that transfers capital at a rate that is sustainable and desirable, and therefore consistent with
macroeconomic equilibrium, rather than to eliminate all imbalances.\textsuperscript{4} K. Driver and Wren-Lewis (1996) refer to the FEER which corresponds to a balanced current account as the "long run FEER", while J. Stein and P. Allen (1995) define it as a steady-state NATREX (Natural Real Exchange Rate is their analogue of the FEER).

Another approach to separate structural from speculative capital flows, which seems natural at first sight, is to assume that speculative flows are placed into short-term assets while structural capital flows are invested in long-term assets, and to rely on the balance of payments accounts to isolate them. However, as it is pointed out in the literature, many volatile flows involve switching into and out of long-term assets, when speculation is on long-term interest rates, while accumulation of short-term assets can be rather stable.

A different approach to identify capital flows consistent with equilibrium is suggested by Williamson (1994). He examines the saving-investment balance:

\[(X - M) = (S - I) - (G - T)\]  \hspace{1cm} (8)

and concludes that structural capital flows must be linked to the difference between the socially optimal net saving of the private sector and the public sector deficit. The main problem with this approach is that we do not have data on time preferences and marginal efficiencies of investment that might allow to estimate welfare-maximizing saving and investment of the private sector. Williamson also mentions another potential problem that this approach presents. Since there is no guaranty that the political process delivers budgetary outcomes that are socially optimal or consistent with macroeconomic balance, then it does not make sense to target the exchange rate on achieving a current account

target that is not supported by a country's fiscal policy. Williamson's approach, then, is
"to identify the likely fiscal position, as well as the likely \( (S - I) \) balance of the private
sector, and then treat the current balance implied as the residual, which needs to be
financed with by the underlying capital flow."\(^5\) To implement this approach, Williamson
suggests: "First, examine past imbalances (in current account) and their relationship to
savings availability and investment levels to see whether they appear to reflect rational
economic behavior as opposed to misguided government policies. Second, examine
whether any imbalances that appear rational are also likely to be sustainable. If not,
reduce the target below the past outcome to the point needed to ensure sustainability.
Third, check that the resulting pattern is internationally consistent. If it is not, modify the
targets of all countries whose targets are particularly high (or low, as the case may be)
until consistency is achieved."\(^6\)

Applying the above method to estimate structural capital flows for G-7 in 1995,
Williamson concludes that only part of the US current account deficit was supported by
structural capital flows. He considers a current account deficit of 1% of GDP as
consistent with external equilibrium. In his earlier work (1983), he chooses a balanced
current account as a target to estimate FEER of the US dollar in 1976-1977. Barrell and
Wren-Lewis (1989) follow the same approach when estimating FEERs for the G-7
countries over the period 1971-1988, their estimates for structural capital flows for the
USA imply a balanced current account with no capital flows prior to 1982 and modest
inflow of foreign capital of 1% of GDP thereafter. They admit, however, that these

estimates for structural capital flows are schematic.

Stein (1994) also begins by examining the saving-investment identity (8). He does not distinguish, though, between the net public and private saving and he does not look for their socially-optimal or welfare-maximizing values, since his NATREX is a positive rather than a normative concept. The current account to GNP ratio which corresponds to his equilibrium exchange rates is estimated from the cointegrating regression on the 12-quarter moving average of the growth of US real GNP and real GNP in the G-18 countries (which are proxies for the marginal efficiencies of capital in the US and abroad), the ratio of consumption plus government purchases to GNP (a proxy for the social time preference). Stein uses the Phillips-Loretan cointegration technique, which is equation (9):

$$X_t = BZ_t + a[X_{t-1} - BZ_{t-1}] + b[Z_t - Z_{t-1}] + e_t$$  \hspace{1cm} (9)

where variables $X$ and $Z$ are of order I(1) and they are cointegrated with vector $[1, -B]$. Variable $X$ is the ratio of current account to GNP, and variable $Z$ is a vector of fundamentals, consisting of proxies for marginal efficiencies of investment in the US and abroad and a proxy for the US social time preference. The last term in equation (9), $[Z_t - Z_{t-1}]$, changes in fundamental variables, is supposed to capture the short-term dynamics of $X_t$ while it adjusts towards its new equilibrium, $BZ_t$. However, this term did not show up significantly in Stein’s estimation and was substituted by another variable, the real long-term interest rate differential between the US and the rest of the G-10 countries. This substitution is based on Stein’s belief that this interest rate differential reflects the innovation effect of fundamental variables. However, the validity of including the interest
rate differential as an explanatory variable in estimating the equilibrium values of the current account to GNP ratio and the equilibrium exchange rates can be doubted, since this differential is also likely to reflect short-run nonfundamentals such as monetary policy or speculation on long-term interest rates.

Driver and Wren-Lewis (1996) performed a sensitivity analysis to test how sensitive the estimates of FEER are to different assumptions about the size of structural flow. They found that a 1% increase in the estimates of structural capital flow leads to changes of between 5% and 10% in the FEER. Such high sensitivity of FEERs served as a base for their suggestion to either abandon FEER calculation in favor of other estimates, or to improve the empirical estimates of structural capital flow.

This study presents an alternative way to estimate structural capital flow in terms of a current account target. Rather than making arbitrary assumptions about the size of "desirable and sustainable" capital flows or attempting to attribute capital flows to some specific changes in fundamentals (such as thrift or productivity), current account (the inverse of actual capital flows) is decomposed into two additive components, permanent and transitory. The permanent component is designed to capture the combined effects of all fundamentals, while the transitory part consists of short-run changes in net foreign investment caused by changes in non-fundamentals.

The permanent component is modeled as a random walk (adding a drift term to the non-stationary trend turned out to be inessential from a statistical point of view). The transitory part is modeled as an autoregressive process, the order of which is determined.
by the significance of the coefficients on the lagged terms.\(^7\)

The model can be summarized by the following equations:

\[
y_t = \mu_t + e_t \quad (10.1)
\]

\[
\mu_t = \mu_{t-1} + \eta_t \quad (10.2)
\]

\[
\Phi(L)e_t = i_t \quad (10.3)
\]

where \(y_t\) is the original time series, \(\mu_t\) is its permanent component and \(e_t\) is its transitory part, \(\eta_t\) and \(i_t\) are disturbances with zero mean and variances \(\sigma_{\eta^2}\), \(\sigma_{i^2}\). The important assumption in the model is the independence of innovations in the permanent and transitory components. The model can be estimated by using state space technique to find the likelihood of the sample \(y_t\) given \(\sigma_{\eta^2}\), \(\sigma_{i^2}\) and the AR coefficients in \(\Phi(L)\). Since these parameters are not known, maximum likelihood estimates can be found by methods of numerical optimization after the distribution for the error terms is specified.

For example, when the transitory component is a first-order autoregression, the state space set-up is as follows:

\[
y_t = Z\alpha_t \quad \text{(measurement equation)}
\]

\[
\alpha_t = \Theta\alpha_{t-1} + Rv_t \quad \text{(transition equation)}
\]

where \(\alpha_t = [\mu_t, e_t]'\) is a state vector, and \(v_t = [\eta_t, i_t]'\) is a vector of disturbances with mean zero and covariance matrix \(Q_v\) which is a diagonal matrix with \(\sigma_{\eta^2}, \sigma_{i^2}\) along the diagonal.

---

\(^7\) This model implies that the CA/GDP series is I(1). The use of this model is not justified if the series is not I(1). I use the ADF(p) tests to examine the order of integration of the series. None of the ADF(1), ADF(4), ADF(6) tests reject the null of a unit root at the 5% size for the CA/GDP ratio. The t-statistics are -2.13961, -2.31473, -2.3957 respectively. The ADF(1), ADF(4), ADF(6) tests reject the null of a unit root at the 5% size for the first difference of the CA/GDP ratio. The t-statistics are -8.1910, -4.2471, -3.3822 respectively. I conclude that the CA/GDP series is indeed I(1).
\[
T = \begin{pmatrix} 1 & 0 \\ 0 & \rho \end{pmatrix}
\]

\(Z = [1, 1], \ R = I\) are system matrices, \(\rho\) is an AR coefficient. To complete the state space representation, it is also assumed that \(E(\alpha_0, \nu_t) = 0\).

The Kalman Filter can then be applied to calculate the conditional distribution of the state vector recursively. When the distribution of the disturbances is specified, the conditional distribution of the state vector is also completely specified by their means \((a_t)\) and covariance matrices \((P_t)\), the quantities that the Kalman filter estimates. If the estimator \((a_{vt-1})\) is based on the information available at time \(t-1\), the procedure is known as predicting. The new information is then incorporated into the estimator of the state vector \((a_t)\) using updating equations. If the estimator is based on all the information in the sample \((a_{vT})\), the procedure is known as smoothing.

The predicting equations are:

\[a_{vt-1} = Ta_{t-1}\]

\[P_{vt-1} = TP_{vt-1}T' + RQ_tR'\]

where \(P_{vt-1}\) is the estimated variance-covariance matrix for \(a_{vt-1}\).

The updating equations are:

\[a_t = a_{vt-1} + K_t v_t\]

\[P_t = P_{vt-1} - K_t ZP_{vt-1}\]

\[F_t = ZP_{vt-1} Z'\]

\[v_t = y_t - Za_{vt-1}\]

(the prediction error)
\[ K_t = P_{t-1} ZF_t^{-1} \] (the Kalman gain).

The smoothing equations are:

\[ a_{T/T} = a_T + P_t^* (a_{T+1/T} - T_T a_T), \]

\[ P_{T/T} = P_t + P_t^* (P_{T+1/T} - P_T a_T) P_t^* \]

\[ P_t^* = P_t TP_{T+1/T}^{-1} \]

with \( a_{T/T} = a_T \) and \( P_{T/T} = P_T \).

It can be shown (Harvey, 1992) that under the assumption that the disturbances and the initial state vector are normally distributed, the mean of the conditional distribution of \( a_t \) is an optimal estimator of the state vector in the sense that it minimizes the mean square error. When the normality assumption is dropped, there is no guarantee that the Kalman filter will give the conditional mean of the state vector. However, it is still an optimal estimator (minimum variance linear unbiased (BLUE) estimator).

The Kalman filter is started with an initial guess for the state vector, \( a_0 \), and its covariance matrix, \( P_0 \). In principle, the starting values are given by the mean and covariance matrix of the unconditional distribution of the state vector. When the state vector is stationary, its unconditional mean and covariance matrix can be evaluated directly. When the state vector contains non-stationary processes, as in model (10.1) – (10.3), the unconditional distribution of these elements is not defined. The initial distribution of \( a_0 \) is then specified in terms of a diffuse, or non-informative, prior: \( P_0 = kI \), where \( k \) is a positive scalar \((k \to \infty)\) and \( I \) is an identity matrix. Harvey (1992) shows that, under certain conditions, a proper prior can be constructed from the first \( m \) sets of observation (where \( m \) is the number of non-stationary elements in the state vector). This
method is recommended when it is feasible, since the "large k" approximation often gives rise to the substantial rounding errors. I was able to construct proper priors for all estimated models.  

Various estimates of the model (10.1)–(10.3) where y is the ratio of the US current account to US nominal GDP are described in Table 1.5. The simple random walk plus noise model attributes almost all of the quarter-to-quarter changes in the ratio to a non-stationary trend. However when the transitory component is modeled as an autoregressive process, this result is reversed as less than a quarter of all innovations is attributed to a stochastic trend. Incorporating additional lags into the transitory component proved to be inessential: all the AR coefficients on the higher order lags are not significant even at the 10 percent significance level. This result is confirmed by the standard likelihood ratio tests. For example, when the choice is between model (2) and (3), the likelihood ratio statistic is 0.0815 while the five percent critical value from the chi-square distribution with 1 degree of freedom is 3.84, so the restricted model (random walk plus AR(1)) is not rejected against the random walk plus AR(2) model. Adding the third lag into the transitory component does not reverse this result. The likelihood ratio test is not significant at the ten percent significance level.

The standard asymptotic likelihood ratio test cannot be applied when the choice is made between model (1) and (2), since one of the parameters is on the boundary of the admissibility region ($\sigma_1^2 = 0$) in the random walk plus noise model. However, based on

\footnote{For example, in case of the model with AR(1) transitory component, the Kalman filter was initialized with $a_1 = [y_1, 0]'$ and}

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the results of the t-test for the AR(1) coefficient, the random walk plus AR(1) model can be chosen as the most adequate for the current account to GDP ratio.

The estimate of the permanent component is graphically presented in Figure 1.5. The solid straight line is the actual current account to GDP ratio, the plus line represents the estimated permanent component of the ratio from the state-space model, while the dot-dash line is the “CA/GDP target” employed by Barrell and Wren-Lewis. It is apparent that the state-space model estimate shows more variability, probably due to the fact that the linear estimates of Barrell and Wren-Lewis are not capable of capturing all the fundamental innovations.

The permanent component estimate can also be compared with Stein’s estimate of the medium run equilibrium CA/GDP ratio. Stein’s estimate tracks the actual data very closely, possibly reflecting the fact that the interest rate differential, his explanatory variable, captures not only innovation effects of the fundamental variables, but also short-run changes in non-fundamentals.

1.6 ESTIMATES OF US FEER

Figure 1.6 summarizes and brings together the results of the previous two sections. The solid line is the trend current account to GDP ratio (trend cbs), i.e. the predicted value of cbs based on the actual exchange rate and the trend value for real GDP. The plus line is the current account target, which is the inverse of the permanent

\[
P_t = \begin{vmatrix} p_{1t} & -p_{1t} \\ -p_{1t} & p_{1t} \end{vmatrix} \quad \text{where } p_{1t} = \frac{\sigma_i^2}{(1-\rho^2)}
\]
component of net foreign investment (more precisely, the ratio of net foreign investment to real GDP). It is this difference between the trend and target CBS that is closed by the FEER. In numerical terms, the trend CBS is a non-linear function of the real exchange rate (where factor income payments and receipts, net transfers, oil prices, domestic and foreign manufacturing prices, the trend values for real GDP are exogenous variables). It is the FEER that sets the value of this function equal to the target level. I use Newton's method of numerical optimization to estimate these equilibrium exchange rates on a quarter-to-quarter basis.

Figure 1.7 presents the trajectory of the FEER together with the actual real exchange rate of the US. The solid line is the actual exchange rate, while the plus line is the estimated FEER.

The FEER depreciates substantially during the early 70s. Rapid growth in oil imports is one possible explanation for the depreciating FEER suggested by Barrell and Wren-Lewis. Around 1976 the growth in oil imports is reversed, and the FEER appreciates for a while.

Figure 1.7 records the massive real appreciation of the dollar that began in 1980, picked in 1985 and then reversed. While the actual real exchange rate appreciates by more than 45%, the FEER appreciates by 20%. The FEER estimate shows that the dollar was overvalued by 23% during the peak of the appreciation. This result is an improvement of the Barrell-Wren-Lewis estimate of the FEER, since they admit that their FEER follows the appreciation in 1980 only to the extent that they have assumed a change to a target current account in that year (the CBS target was changed from 0% to 1%.
deficit). The other noticeable trend in the FEER trajectory is its depreciation that began in 1985.

Of particular interest is the relationship between the FEER and the actual real exchange rate. It is dominated by the difference between the trend and the target cbs. During the periods when the trend cbs exceeds the target (e.g. the early and mid 80s), the FEER shows the overvaluation of the dollar. And, by analogy, during the time when the trend cbs is below the target, the actual exchange rate is below its fundamental value. This kind of relationship is observed during the late 60s and early 70s. However, this situation cannot be justified from the historical point of view, since the US dollar is known to be overvalued during the last years of the fixed exchange rate regime. I believe the inability of the FEER to capture the overvaluation of the dollar before the collapse of the Bretton-Woods system comes from the assumption of the model that prices of traded goods are exogenous. This assumption is more defensible during the flexible exchange rate regime, but it does not seem to work during the fixed exchange rate regime, when markets for traded goods are cleared by simultaneous adjustments in quantities and prices while nominal exchange rate remains fixed.

As a possible solution to this problem, I suggest to cast the current account model in real terms. Figure 1.8 presents the trajectory of the FEER based on these calculations. The significant improvement in the FEER estimate is that now it records the overvaluation of the dollar before the collapse of the Bretton-Woods system. The FEER shows that the dollar was persistently overvalued throughout the late 60s and early 70s until 1973 when the fixed exchange rate regime was completely abandoned. The surprise
devaluation of the dollar in 1971 by President Nixon helped to decrease the gap between the actual real exchange rate and its fundamental value. However, the dollar remained overvalued during the transition period from the fixed to the flexible exchange rate system (1971Q1 – 1972Q4). During the first years of the flexible exchange rate regime, the dollar fell sharply below its fundamental value, but then recovered its value and was appreciating for a while together with the FEER.

Starting in 1977, the actual exchange rate began to depreciate while the FEER continued to appreciate (partially due to decrease in oil imports caused by the oil price shocks and subsequent more economical consumption of oil). The fact that the dollar remained undervalued during 1978-1980 explains the sharp increase in the exchange rate towards its fundamental value during 1981. This began “the great appreciation” of the dollar. The FEER captures more than 50 percent of the dollar rise, after that the trend was reversed.

The forecasting accuracy of FEERs is of particular interest since point predictions of various structural and time series exchange rate models are known to be dominated by those of the driftless random walk, especially at the short-run and medium-run horizons. For example, Richard Meese and Kenneth Rogoff (1983) showed that the various structural models perform poorly even if their forecasts are based on actual realized values of explanatory variables and the parameters of the models are re-estimated at the time of a forecast.

Within the driftless random walk model, actual real exchange rate at time \( t \) presents the best forecast of future exchange rates at \( t+k \) horizon. Within the random walk
with drift model, the actual exchange rate at time $t$ plus the drift term times $k$ is the best $k$-quarter horizon forecast of future exchange rates. These forecasts are simple since the implicit assumption behind these forecasts is that we know nothing about the course of fundamentals or their changes are completely unpredictable. To compare the forecasting accuracy of the FEER and random walk models, I have produced several multiple-horizon forecasts. Table 6 lists the ratios of the root mean square error (RMSE) statistics for the forecasts based on the FEER and random walk models at the 2, 4, 8, 12-quarter horizons.

The first two rows of Table 1.6 list the ratios of the root mean square error (RMSE) statistics for the simple "we know nothing about the changes of fundamentals" forecasts based on the FEER and random walk models. The forecast is the actual real exchange rate $k$ quarters ago under the random walk model, and it is the FEER $k$-quarters ago under the FEER models. The FEER model cast in real terms achieves the lower RMSE at the 8, 12, 16-quarter horizons, which means that the fundamental equilibrium exchange rates gives a better idea of where the actual exchange rates are heading in the medium term. The FEER model cast in nominal terms does not succeed to improve on the random walk. However, these simple forecasts are based on the assumption that the fundamentals’ changes are completely unpredictable. That is not necessarily true in case of the FEER models, since some of their explanatory variables’ changes can be predicted rather accurately (e.g., the growth rate of trend real GDP of the US and the other G-7 countries).

Rows II of Table 6 contain the RMSEs ratios when the FEER model forecasts are based on the actual realized values of explanatory variables. Both FEER models succeed
to improve on the random walk at 8, 12, 16-quarter horizons. The fact that the random walk models outperform the FEER models at the 2, 4-quarter horizons is not surprising, since FEER is a medium-term concept and is not designed to capture the short-run fluctuations in real exchange rate.

It can be argued that these type of forecasts are not out-of-sample, since the parameters of the trade equations and the current account target (the permanent component of cbs) are estimated based on all the information in the entire sample period, including the time period of a forecast.

To consider the out-of-sample forecasts, I first modify the current account target and estimate the permanent component of cbs based on the most up-to-date information available at any given point of the sample. I use the updating equations of the Kalman filter to get the estimates of the current account target based on the up-to-date information. I then re-estimate the parameters of the trade equations using data from 1967:1 to 1980:4. Based on these parameters and the updated current account target, I calculate the FEERs for the time periods starting in 1981:1 and use them to forecast k-periods ahead. Rows III of Table 1.6 summarize the results of this exercise. Although, the RMSE statistics increase for the shorter-run forecasts in case of the FEER models (reflecting the fact that the fit of the model is worse when parameters are not re-estimated to include the new information arriving after 1980:4), the FEER model cast in real terms...
still outperforms the random walk model at 8, 12, 16-quarter horizons, and the FEER model cast in nominal terms outperforms the random walk at 12, 16-quarter horizons.

1.7 CONCLUDING REMARKS

This paper presents the estimated FEER trajectory for the US over the period from 1967Q1 to 1994Q4. The calculations are done within a comparative static, partial-equilibrium framework. A consistent set of data series is used to estimate all inputs and parameters of the model and a statistical model is proposed to specify current account input.

The model is first cast in nominal terms. However, this approach produces counterfactual results for the early period: the dollar is estimated to be undervalued during the last years of the fixed exchange rate regime. The model is then cast in real terms and it is shown to produce consistent results. The forecasting accuracy of both models is compared with that of the random walk that is known to outperform various structural models for real exchange rates. The comparison reveals that FEERs present an improvement on the point predictions of the random walk models at the medium-term horizons.

The outlined model can be extended in different directions. A closer investigation of the relationship between prices of traded goods and real exchange rates is definitely desirable, since the possibility exists that it may not be time-invariant and that it may depend on the nominal exchange rate regime. Adding dynamics and lag adjustments is
likely to be crucial here, since exchange rate and prices are known to be correlated with significant lags.

The other way to extend the model is to estimate medium-term forecasts for bilateral exchange rates. So far, the FEER concept was applied towards real effective exchange rates exclusively. I believe that the proposed statistical model to specify current account input makes it possible to estimate equilibrium bilateral exchange rates. This is the subject of Chapter 2.
<table>
<thead>
<tr>
<th>Variable</th>
<th>J(1,5)</th>
<th>ADF(1)</th>
<th>ADF(4)</th>
<th>ADF(12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Exchange Rate</td>
<td>1.4426</td>
<td>-1.8141</td>
<td>-2.7006</td>
<td>-2.3407</td>
</tr>
<tr>
<td>Goods Exports</td>
<td>1.7878</td>
<td>-1.2409</td>
<td>-1.9806</td>
<td>-1.6461</td>
</tr>
<tr>
<td>Service Exports</td>
<td>1.2925</td>
<td>-2.1771</td>
<td>-2.5140</td>
<td>-2.6056</td>
</tr>
<tr>
<td>Nonoil Imports</td>
<td>1.0160</td>
<td>-2.3206</td>
<td>-2.3246</td>
<td>-2.1197</td>
</tr>
<tr>
<td>Service Imports</td>
<td>5.5729</td>
<td>-1.7809</td>
<td>-1.7999</td>
<td>-2.4350</td>
</tr>
<tr>
<td>Real US GDP</td>
<td>0.0292*</td>
<td>-2.9105</td>
<td>-3.8762**</td>
<td>-3.1724</td>
</tr>
<tr>
<td>World Trade Volume</td>
<td>2.9357</td>
<td>-2.5840</td>
<td>-2.7609</td>
<td>-2.2058</td>
</tr>
<tr>
<td>Real GDP of the Major Industrial Countries</td>
<td>0.1604**</td>
<td>-4.2111*</td>
<td>-4.3023*</td>
<td>-3.1392</td>
</tr>
<tr>
<td>Relative Price of Oil</td>
<td>5.0047</td>
<td>-1.7287</td>
<td>-1.2373</td>
<td>-1.3516</td>
</tr>
</tbody>
</table>

*Critical values for the 1-percent, 5-percent, 10-percent significance levels are 0.1228, 0.2950, and 0.4520. These are referenced in Ogaki (1993).

ADF(r) denotes Augmented Dickey-Fuller test with time trend and r lags. Critical values for the 1-percent, 5-percent, 10-percent significance levels are -4.0437, -3.4508, -3.1505 for r=1, -4.0460, -3.4519, and -3.1512 for r=4, -4.0530, -3.4552, and -3.1531 for r=12. These are from MacKinnon (1990) for T=112-r-1.

*significant at the one percent level.
**significant at the five percent level.

TABLE 1.2: TESTS FOR NO COINTEGRATION

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF(1)*</th>
<th>ADF(4)*</th>
<th>ADF(12)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real US GDP/</td>
<td>-1.8927</td>
<td>-2.6847</td>
<td>-2.2883</td>
</tr>
<tr>
<td>Real GDP of Industrial Countries/</td>
<td>-1.8835</td>
<td>-2.7022</td>
<td>-2.2170</td>
</tr>
<tr>
<td>World Trade Volume/</td>
<td>-1.8434</td>
<td>-2.8952</td>
<td>-2.4895</td>
</tr>
</tbody>
</table>

* ADF(r) denotes Augmented Dickey-Fuller test with time trend and r lags. Critical values for the 1-percent, 5-percent, 10-percent significance levels are -4.0437, -3.4508, -3.1505 for r=1, -4.0460, -3.4519, and -3.1512 for r=4, -4.0530, -3.4552, and -3.1531 for r=12. These are from MacKinnon (1990) for T=112-r-1.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Nonoil Imports</th>
<th>Service Imports</th>
<th>Goods Exports</th>
<th>Service Exports</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant(a)</td>
<td>-20.8537</td>
<td>-16.6523</td>
<td>1.2079</td>
<td>-6.0090</td>
</tr>
<tr>
<td></td>
<td>(0.7592)</td>
<td>(1.6672)</td>
<td>(0.9542)</td>
<td>(1.3505)</td>
</tr>
<tr>
<td>Income Elasticity(a)</td>
<td>2.7684</td>
<td>1.7768</td>
<td>1.0259</td>
<td>2.5914</td>
</tr>
<tr>
<td></td>
<td>(0.0533)</td>
<td>(0.1193)</td>
<td>(0.0464)</td>
<td>(0.1213)</td>
</tr>
<tr>
<td>Relative Price Elasticity(a)</td>
<td>0.7332</td>
<td>1.3217</td>
<td>-0.4884</td>
<td>-0.2137*</td>
</tr>
<tr>
<td></td>
<td>(0.0908)</td>
<td>(0.1996)</td>
<td>(0.1580)</td>
<td>(0.2047)</td>
</tr>
<tr>
<td>H(0,1)(b)</td>
<td>0.2280</td>
<td>0.0128</td>
<td>1.8860</td>
<td>12.2538</td>
</tr>
<tr>
<td></td>
<td>(0.6330)</td>
<td>(0.9098)</td>
<td>(0.1697)</td>
<td>(0.0005)</td>
</tr>
<tr>
<td>H(1,2)(b)</td>
<td>0.7907</td>
<td>1.6474</td>
<td>13.6684</td>
<td>0.3020</td>
</tr>
<tr>
<td></td>
<td>(0.3739)</td>
<td>(0.1993)</td>
<td>(0.0002)</td>
<td>(0.5826)</td>
</tr>
<tr>
<td>H(1,3)(b)</td>
<td>1.3385</td>
<td>2.8027</td>
<td>19.0651</td>
<td>0.7166</td>
</tr>
<tr>
<td></td>
<td>(0.5121)</td>
<td>(0.2463)</td>
<td>(7.24e-05)</td>
<td>(0.6989)</td>
</tr>
</tbody>
</table>

*All variables are in logs. Standard errors are in parentheses.

* *P*-values are in parentheses

* not significant at the five percent level

<table>
<thead>
<tr>
<th>Variable</th>
<th>Nonoil Imports</th>
<th>Service Imports</th>
</tr>
</thead>
<tbody>
<tr>
<td>WPI USA(^a)</td>
<td>0.2722</td>
<td>0.9317</td>
</tr>
<tr>
<td></td>
<td>(0.1397)</td>
<td>(0.0732)</td>
</tr>
<tr>
<td>FOREIGN PRICE(^a)</td>
<td>0.7492</td>
<td>0.1167</td>
</tr>
<tr>
<td></td>
<td>(0.1439)</td>
<td>(0.0828)</td>
</tr>
<tr>
<td>H(0, 1)(^b)</td>
<td>4.7454</td>
<td>0.1756</td>
</tr>
<tr>
<td></td>
<td>(0.0293)</td>
<td>(0.6752)</td>
</tr>
<tr>
<td>H(1, 2)(^b)</td>
<td>2.0578</td>
<td>8.4991</td>
</tr>
<tr>
<td></td>
<td>(0.1514)</td>
<td>(0.0036)</td>
</tr>
<tr>
<td>H(1, 3)(^b)</td>
<td>2.1136</td>
<td>15.6930</td>
</tr>
<tr>
<td></td>
<td>(0.3476)</td>
<td>(0.0004)</td>
</tr>
</tbody>
</table>

\(^a\) All variables are in logs. Standard errors are in parentheses.

\(^b\) P-values are in parentheses.

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameters</th>
<th>Log-Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{\eta}^2$</td>
<td>$\sigma_{\xi}^2$</td>
</tr>
<tr>
<td>1. Random walk 0.1619 .8e-15 plus noise</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>2. Random walk 0.0377 0.1210 plus AR(1)</td>
<td>0.9525</td>
<td>-----</td>
</tr>
<tr>
<td>3. Random walk 0.0372 0.1212 plus AR(2)</td>
<td>0.9836</td>
<td>-0.0341*</td>
</tr>
<tr>
<td>4. Random walk 0.0372 0.1211 plus AR(3)</td>
<td>0.9858</td>
<td>-0.0369*</td>
</tr>
</tbody>
</table>

* Standard errors are in parenthesis
* not significant at the ten percent level.

**TABLE 1.5: ESTIMATES OF THE STATE-SPACE MODEL FOR THE CURRENT ACCOUNT TO GDP RATIO FOR THE US**
<table>
<thead>
<tr>
<th>MODEL</th>
<th>2-Q</th>
<th>4-Q</th>
<th>8-Q</th>
<th>12-Q</th>
<th>16-Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. FEER</td>
<td>1.6808</td>
<td>1.0745</td>
<td>0.7141</td>
<td>0.6248</td>
<td>0.6161</td>
</tr>
<tr>
<td>REAL/RW^e</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I. FEER</td>
<td>2.1579</td>
<td>1.5845</td>
<td>1.2053</td>
<td>1.0684</td>
<td>1.0307</td>
</tr>
<tr>
<td>NOM/RW^e</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II. FEER</td>
<td>2.0842</td>
<td>1.3523</td>
<td>0.8411</td>
<td>0.6479</td>
<td>0.5661</td>
</tr>
<tr>
<td>REAL/RW^d</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II. FEER</td>
<td>1.9772</td>
<td>1.3475</td>
<td>0.9334</td>
<td>0.7316</td>
<td>0.6392</td>
</tr>
<tr>
<td>NOM/RW^d</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III. FEER</td>
<td>2.5089</td>
<td>1.6048</td>
<td>0.9456</td>
<td>0.5625</td>
<td>0.3018</td>
</tr>
<tr>
<td>REAL/RW^h</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III. FEER</td>
<td>2.9475</td>
<td>1.9127</td>
<td>1.1365</td>
<td>0.6877</td>
<td>0.3707</td>
</tr>
<tr>
<td>NOM/RW^h</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The numbers in the table are (Model RMSE)/(RW RMSE), where RW RMSE is the root mean square forecast error of the random walk model. RMSE=(E(R_{t} - F_t)²)⁻¹/² where R_t is log actual real exchange rate, F_t is its k-quarter horizon forecast. Because exchange rates and their forecasts are in logs, RMSEs are approximately in percentage terms. F_t = R_{t+k} in case of the random walk models.

^e F_t is log FEER_{t+k}. Since the forecast is the lagged FEER, the implicit assumption behind the forecasting procedure is that we know nothing about the changes of fundamentals. The current account target is calculated based on the entire sample information.

^d F_t is log FEER_t. FEER estimates are based on the actual realized values of fundamentals. The current account target is calculated based on the entire sample information.

^h Forecasting begins in 1981:1. F_t is log FEER_t. FEER estimates are based on target cbts_{t+k}.

**TABLE 1.6: ROOT MEAN SQUARE FORECAST ERRORS**
FIGURE 1.1: US ACTUAL AND FITTED IMPORTS
FIGURE 1.2: US ACTUAL AND ESTIMATED EXPORTS AND NET TRANSFERS
FIGURE 1.3: US ACTUAL AND FITTED IMPORT PRICES
FIGURE 1.4: US ACTUAL AND FITTED TRADE BALANCE ON GOODS AND SERVICES AS A PERCENTAGE OF REAL GDP
FIGURE 1.5: ESTIMATES OF US TARGET CURRENT ACCOUNT TO GDP RATIO
FIGURE 1.6: US TREND AND TARGET CA/GDP RATIO
FIGURE 1.7: US REAL EFFECTIVE AND FUNDAMENTAL EQUILIBRIUM EXCHANGE RATES BASED ON THE CURRENT ACCOUNT MODEL CAST IN NOMINAL TERMS
FIGURE 1.8: US REAL EFFECTIVE AND FUNDAMENTAL EQUILIBRIUM EXCHANGE RATES BASED ON THE CURRENT ACCOUNT MODEL CAST IN REAL TERMS
CHAPTER 2
BILATERAL EQUILIBRIUM EXCHANGE RATES

2.1 INTRODUCTION.

The concept of equilibrium exchange rates has been applied towards real effective exchange rates exclusively. The framework developed in Chapter 1 makes it possible to estimate equilibrium bilateral exchange rates. In this chapter, I employ this framework to estimate the equilibrium trajectories for the UK pound - US dollar, Japanese yen - US dollar and Canadian dollar - US dollar real exchange rates over the period from 1973Q1 to 1997Q4.

The calculation of bilateral equilibrium exchange rates within this framework requires estimates of potential foreign and domestic GDP, bilateral trade equations and current account target. Following approach described in Chapter 1, potential output is measured by the real trend GDP, the structural trade equations are estimated by CCR and current account input is based on a statistical model.

I also discuss the ability of equilibrium exchange rates to forecast the actual bilateral exchange rates and compare their forecasting accuracy with that of the random walk models.
The rest of the chapter is organized as follows. Section 2 describes the data. Section 3 discusses estimation of the bilateral trade equations. Section 4 contains the discussion on estimating target current account. Section 5 presents the estimated equilibrium trajectories for the UK pound - US dollar, Japanese yen - US dollar and Canadian dollar - US dollar real exchange rates and compares them to the paths of the actual real exchange rates. It also discusses the ability of equilibrium exchange rates to forecast the actual bilateral exchange rates. Finally, some concluding remarks are offered in Section 6.

2.2 DATA DESCRIPTION

The data used in this Chapter is quarterly and covers the time period from the first quarter of 1973 through the fourth quarter of 1997. The SAS X11 procedure, an adaptation of the US Bureau of the Census X-11 Seasonal Adjustment program, is used to adjust the time series seasonally.

The export prices, the consumer price indices, the bilateral nominal exchange rates of the dollar, the real Gross Domestic Product of the USA, UK, Canada, Japan are obtained from the International Financial Statistics CD-ROM Data set. Data on the US/UK, US/Japan, US/Canada foreign transactions in the National Income and Product Accounts (in actual and constant 1990 dollars): good imports, service imports, good exports, service exports, payments of factor income, transfer payments and net foreign investment, is extracted from different issues of the Survey of Current Business.
The nominal exchange rate is defined in terms of units of foreign currency per US dollar, so that an increase in the exchange rate represents an appreciation of the dollar.

The price-adjusted, or real, exchange rate is calculated by dividing the nominal exchange rate by the ratio of the foreign prices to the US domestic prices. The consumer price indices are used to measure domestic and foreign price levels.

The US real import time-series are constructed by dividing the nominal (current-dollar) import series by the export prices of the US trading partners (in dollars). The US real export time-series are constructed by dividing the current-dollar export series by the US export prices.

2.3 TRADE EQUATIONS

The trade volume equations are in the center of the FEER calculations. Following the bulk of the time-series work on trade equations, I estimate the trade equations by single-equation methods. US imports from Canada include both goods and services. US imports from Japan and UK are disaggregated into good and service imports. The activity variable in these equations is real GDP of the USA, the relative price variable is the real exchange rate.

US exports are disaggregated into good and service exports in case of UK and Japan and include both goods and services in case of exports to Canada. The activity variable in these equations is real foreign GDP, the relative price variable is the real exchange rate.
To choose an appropriate method for estimating the trade equations, the statistical properties of the data are examined first. Park's $J(1,5)$ and Augmented Dickey-Fuller (ADF) tests are applied to decide whether the time series contain a unit root.

The results of the unit root tests are presented in Table 2.1. These tests do not reject the null of difference stationarity for the Japanese yen - US dollar and Canadian dollar - US dollar real exchange rates. However, they provide mixed results in case of sterling - US dollar real exchange rate. The $J(1,5)$ and ADF(12) reject the null of difference stationarity, while the ADF(1) and ADF(4) accept it at the 5 percent significance level. Detecting a unit root in the real exchange rate time series is not surprising, similar results are widely recorded in the exchange rate literature and serve as an empirical base for questioning the validity of the Purchasing Power Parity hypothesis.

Interpreting the results of the $J(1,5)$ and ADF(p) tests is even more straightforward for the real import-export time series. The tests cannot reject the null of difference stationarity for these series. The only exception is the US good exports to UK and US good imports from UK, where the results of the tests are somewhat ambiguous. In case of the US good exports to UK, the $J(1,5)$ and ADF(1) tests reject the null of difference stationarity at the 5 percent significance level, but they can accept the null at the 1 percent significance level, while the ADF(4) and ADF(12) accept the null at the 5 percent level. In case of US good imports from UK, only the $J(1,5)$ test reject the null at the 5 percent significance level, while all ADF(p) tests accept it.

The results of the unit root tests for the real GDP series of the four considered economies indicate that these series are non-stationary as well. All tests accept the null of
difference-stationarity at the 5 percent level. The only exception is the US real GDP series: the ADF(4) test accepts the null at the 1 percent level.

The fact that each trade equation includes trend non-stationary variables suggests that OLS estimation is not efficient, since the error term is correlated with the first difference of non-stationary regressors at leads, lags and contemporarily. Park’s CCR is used in this study to estimate cointegrating vectors.

Before running the CCR with activity variable and real exchange rate as explanatory variables, it is necessary to assess whether they are cointegrated. If they are cointegrated, the US exports (imports) cannot be cointegrated with these explanatory variables, and the cointegrating regression would be meaningless. The ADF(p) tests are applied to OLS cointegrating regressions residuals to test for no cointegration. The results of these tests are in Table 2.3. All tests are not significant. This presents a strong evidence that real US GDP and UK GDP are not cointegrated with the sterling-dollar real exchange rate, real US GDP and GDP of Japan are not cointegrated with the yen-dollar real exchange rate, and real US GDP and Canadian GDP are not cointegrated with the Canadian dollar-US dollar real exchange rate. This means that CCR are legitimate to apply.

The results of Canonical Cointegrating Regressions are presented in Tables 2.4, 2.5 and 2.6. Table 2.4 contains the estimates of the US/Japan trade equations. All coefficients on income and real exchange rate are significant and have the theoretically correct sign. Income elasticity of the US imports is higher than that of the Japanese imports, which is a well-known result in the literature. The H(p,q) tests seem to favor the

1 All CCR estimation is done by Ogaki’s (1993) GAUSS CCR package.
null of cointegration for all trade equations. The $H(1,2)$ and $H(1,3)$ tests support the null of stochastic cointegration for the US imports, while the $H(0,1)$ test support the null of deterministic cointegration for the US service exports. All three tests support the null of cointegration for the US good exports.

Table 2.5 contains the estimates for the US/UK trade equations. All coefficients are significant except the relative price elasticity of the US service exports. The $H(0,1)$ tests seem to favor the null of deterministic cointegration for all estimated export-import equations.

Table 2.6 presents the estimates for the US/Canada trade equations. All coefficients are significant and have the theoretically correct signs. Both the $H(0,1)$ and $H(1,2)$ tests support the null of cointegration in case of US import equation. However, none of the tests support the null of cointegration for the US export equation at the 5 percent level. The graphical analysis of the residuals reveals that the cointegrating vector does not pick up the downward trend in US exports that lasted until 1988 nor the upward trend that followed. A split time trend to the CCR residuals was fitted to capture these trends.

Thus, CCRs provide significant estimates of income and price elasticities for all estimated trade equations, and at least one of the $H(p,q)$ tests seem to favor the null of cointegration for the estimated trade equations, with the only exception of the US exports to Canada.

To complete the current account model, net real factor income payments ($\text{FINC}$) and net real government and private transfers ($\text{TRAN}$) are considered as well. I treat both
components as exogenous and as given in US constant dollars (1990 =100). The complete current account model is given by equation (12):

$$CA = e^{\alpha_i Y^q_1 q_1^{\alpha_1}} + e^{\beta_i Y^s_1 q_1^{\beta_1}} - e^{\delta_i Y^{s_1} q_1^{\delta_1}} - e^{\phi_i Y^{s_2} q_1^{\phi_2}} + FINC - TRAN$$

(12)

where $\alpha_i$ are the estimated US good export elasticities, $\beta_i$ are the estimated US service export elasticities, $\delta_i$ are the estimated elasticities for the US good imports and $\phi_i$ are that for the US service imports.

The ability of the estimated trade equations to track the actual data is graphically presented in Figure 2.1, 2.2, and 2.3. The solid lines on these graphs represent the actual time series, while the plus lines represent the fit of the model. The complete current account model is graphically presented in Figure 2.4.

The US/UK, US/Canada estimated trade equations capture the major deteriorations and improvements in the actual trade balance and provide a reasonable fit of the data. In case of the US trade with Japan, the estimated import equations are not able to explain the fast growing imports during "the great appreciation-depreciation" of the dollar and do not seem to capture a slower growth of imports in late 90s. Figure 2.4 also reflects this problem: the model is unable to explain the US current account deficit during the 80s, but predicts a more substantial and growing deficit during the 90s. This problem is likely to arise because of the changes in relative price and income elasticities of imports. In an attempt to fix this problem, a split time trend is fitted to the CCR residuals.
2.4 CURRENT ACCOUNT TARGET

Following approach described in Section 1.5, I utilize a statistical model to estimate structural capital flow (a current account target). This approach allows estimating structural capital flow between any two countries and is not limited to estimating current account target for a national economy. This makes it possible to estimate equilibrium bilateral exchange rates.

As it is proposed in Section 1.5, current account, the inverse of actual capital flow, is decomposed into two additive components, permanent and transitory. The permanent component, or structural capital flow, is modeled as a random walk (there was an attempt to add a drift term to the non-stationary trend, the results of this experiment are discussed later in this chapter). The transitory part, or dis-equilibrium capital flow, is modeled as an autoregressive process, the order of which is determined by the significance of the coefficients on the lagged terms.²

Various estimates of the model (10.1) – (10.3) are described in the first four panels of Tables 2.7, 2.8, and 2.9. Table 2.7 contains estimates of the model where y is the ratio of the US/Japan net foreign investment to US nominal GDP. The simple random walk plus noise model attributes almost all of the quarter-to-quarter changes in the ratio to a non-stationary trend. However when the transitory component is modeled as an autoregressive process of order one, only half of all innovations are attributed to a stochastic trend. If additional lags are incorporated into the model, most of the variance is

² This model implies that the CA/GDP series is I(1). The use of this model is not justified if the series is not I(1). I use the ADF(p) tests to examine the order of integration of the series (see Table IA). None of the ADF(1), ADF(4), ADF(6) tests reject the null of a unit root at the 5% size for the considered series.
again attributed to the random walk component. When choosing between models 2, 3 and 4, I relied on the standard likelihood ratio test. The five percent critical value from the chi-square distribution with 1 degree of freedom is 3.84. When the choice is between model (2) and (3), the likelihood ratio statistic is 3.2136, so the restricted model (random walk plus AR(1)) is not rejected against the random walk plus AR(2) model. When the choice is between model (3) and (4), the likelihood ratio statistic is 0.1874, so the restricted model (random walk plus AR(2)) cannot be rejected again. The standard asymptotic likelihood ratio test cannot be applied when the choice is made between model (1) and (2), since one of the parameters is on the boundary of the admissibility region ($\sigma_1^2 = 0$) in the random walk plus noise model. However, based on the results of the t-test for the AR(1) coefficient, the random walk plus AR(1) model can be chosen as the most adequate for the considered time series.

Table 2.8 contains estimates of the model where $y$ is the ratio of the US/UK net foreign investment to US nominal GDP. The simple random walk plus noise model attributes only about a quarter of all changes in the ratio to the stationary component. However, when it is modified as an AR(1) process, it is responsible for more than 90% of all innovations. Incorporating additional lags into the transitory component does not reverse this result. Inclusion of the additional lags into the transitory component proved to be inessential from a statistical point of view as well: all the AR coefficients on the higher order lags are not significant even at the 10 percent significance level. This result is confirmed by the standard likelihood ratio tests: restricted models are not rejected in favor of more complicated models with additional AR lags. Specifically, when the choice
is between model (2) and (3), the likelihood ratio statistic is 0.7878, and it is 0.1882 when the choice is between model (3) and (4). The choice between model (1) and (2) is not straightforward. On one hand, the likelihood ratio test cannot reject the random walk plus noise model in favor of random walk plus AR(1) (the likelihood ratio test statistic is 1.948), on the other hand, the t-test for the AR coefficient shows that the coefficient is strongly significant. I relied on the likelihood ratio test results and chose model (1) as the most appropriate.

The first four panels of Table 2.9 contain estimates of the model where y is the ratio of the US/Canada net foreign investment to US nominal GDP. The simple random walk plus noise model attributes about 65% of all quarter-to-quarter changes in the ratio to a non-stationary trend. Adding an AR(1) lag does not change this result dramatically and it seems to be essential from a statistical point of view. The AR coefficient is strongly significant, and the likelihood ratio test accepts the random walk plus AR(1) model at the ten percent significance level, although it rejects it at the five percent significance level (the likelihood ratio test statistic is 3.1486, while the ten percent critical value from the chi-square distribution with 1 degree of freedom is 2.706). Incorporating additional lags into the transitory component reallocates most of the quarter-to-quarter changes in the series to the stationary component. The coefficients on the second-order lags are significant at the five percent significance level, and the coefficient on the third-order lag is significant at the ten percent level. The likelihood ratio test rejects model (3) in favor of model (4) at the ten percent significance level (the likelihood ratio statistic is 2.8178). I chose model (4) as the most appropriate.

I conclude that all considered series is indeed I(1).
Several other models were tried as potential rivals of the random walk plus AR(p) model, (10.1-10.3). For example, I tried adding a stochastic drift term to the random walk component. The model then becomes:

\[ y_t = \mu_t + e_t \quad (14.1) \]
\[ \mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \quad (14.2) \]
\[ \beta_t = \beta_{t-1} + u_{t-1} \quad (14.3) \]
\[ \Phi(L)e_t = i_t \quad (14.4) \]

Innovations in the level of \( \mu_t \) are given by \( \eta_t \), while innovations in its rate of growth are given by \( u_t \). The disturbances: \( e_t, \eta_t \) and \( i_t \) - have zero mean and variances \( \sigma_e^2, \sigma_\eta^2, \sigma_i^2 \). All innovations are assumed to be independent.

Different estimates of this model are presented in three lower panels of Tables 2.7-2.9. It turned out that allowing a variable growth rate (\( \sigma_u > 0 \)) was statistically inessential for all time series.\(^3\) This means that the non-stationary trend component collapsed to the simple random walk with drift (the model actually collapsed to a simple linear time trend in case of UK and Japan, since \( \sigma_\eta \to 0 \)). The model can be re-written in the following form:

\[ y_t = \mu^* + \beta t + e_t \quad (15.1) \]
\[ \mu^* = \mu^*_{t-1} + \eta^*_t \quad (15.2) \]
\[ \Phi(L)e_t = i_t \quad (15.4) \]

However, presence of a linear trend in the CA/GDP series is inconsistent with economic theory. It brings out the possibility of potentially unsustainable capital flows.

\(^3\) I should mention that introduction of the stochastic drift term implies that the original time series is I(2) process. The fact that the variance of this term was estimated to be statistically insignificant supports the
between two countries by allowing net foreign investment as a percentage of US GDP to grow at a constant rate. On these grounds, the model was disregarded as a potential rival to the random walk plus AR(p) models.

The estimate of the permanent component of the CA/GDP series is graphically presented in Figure 2.5. The solid straight line is the actual time-series and the dash line is the estimated permanent component of the series.

2.5 ESTIMATES OF THE FEER

Figure 2.6 summarizes and brings together the results of the previous two sections. The top panel of Figure 2.6 presents the US/UK balance of payment situation, the middle panel summarizes the US/Canada balance of payment position and the lower panel presents that for US/Japan. The solid line on these graphs is the fitted current account to GDP ratio \( \text{fitted cbs} \), i.e. the predicted value of cbs based on the actual exchange rate and the trend value for real GDP. The dash line is the current account target, which is the inverse of the permanent component of net foreign investment (more precisely, the ratio of net foreign investment to real GDP). It is the FEER that brings the fitted cbs towards its target. In numerical terms, the fitted cbs is a non-linear function of the real exchange rate (equation (12)). It is the FEER that sets the value of this function equal to the target level. I use Newton's method of numerical optimization to estimate these equilibrium exchange rates on a quarter-to-quarter basis.

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assumption that the series is I(1) (see footnote 8).
Figure 2.7 presents the trajectories of the FEERs together with the actual bilateral exchange rates. The solid line represents the actual real exchange rate, the dash line is the estimated FEER.

The top panel of Figure 2.7 presents the trajectories for the actual and equilibrium US dollar-UK pound real exchange rate. The noticeable trend is the depreciation of the actual and equilibrium value of the dollar during the 70s. During the early 80s, real exchange rate appreciated sharply, picked in 1985 and then lost most of its earlier gain during the late 80s. The FEER can only explain about 56% of this great swing in the dollar value. The estimate shows that the dollar was overvalued by 33% during the peak of the “great appreciation”. Both the actual and equilibrium exchange rate continued to depreciate throughout the early 90s, and the actual exchange rate tracked its equilibrium value closely. In 1992 the dollar appreciated by 30% but then gradually returned to its equilibrium level.

The second panel of Figure 2.7 presents the actual and equilibrium exchange rate trajectories for the US against Canada. The real exchange rate deviations from its fundamental value are more substantial and persistent in this case. According to the FEER estimate, the dollar remained undervalued by about 20% throughout the 70s. It kept appreciating towards its fundamental value and reached its equilibrium level by 1979. Starting in 1983, the FEER began to depreciate but the actual exchange rate did not start depreciating until 1986. The US dollar remained overvalued until the 90s when both actual exchange rate and FEER began to appreciate.

The third panel of Figure 2.7 presents the actual and equilibrium trajectories for the US dollar-Japanese yen real exchange rate. The most noticeable trend is the
continuous depreciation of the equilibrium value of the US dollar. This trend is usually attributed to the differences in productivity growths between the US and Japan. According to the FEER estimate, the dollar was overvalued only by 19% percent during the peak of the “great appreciation”, but was loosing its value much faster than the FEER was depreciating during “the great depreciation”.

The forecasting accuracy of the equilibrium exchange rates is of particular interest since point predictions of various structural and time series exchange rate models are known to be dominated by those of the driftless random walk, especially at the short-run and medium-run horizons. For example, Richard Meese and Kenneth Rogoff (1983) showed that the various structural models perform poorly even if their forecasts are based on actual realized values of explanatory variables and the parameters of the models are re-estimated at the time of a forecast.

Within the driftless random walk model, actual real exchange rate at time \( t \) presents the best forecast of future exchange rates at \( t+k \) horizon. These forecasts are simple since the implicit assumption behind these forecasts is that we know nothing about the changes of fundamentals. One way to compare the forecasting accuracy of the FEER and random walk point predictions is to see whether the actual exchange rate or the FEER (calculated at time \( t \)) gives more accurate predictions for the future \( t+k \) exchange rates.

Tables 2.10-2.12 list the ratios of the RMSE (root mean square error) statistics for the FEER forecasts to the RMSE for a simple random walk model. In Rows I, the random walk forecast is the actual real exchange rate \( k \) quarters ago and the FEER forecast is the estimated FEER \( k \)-quarters ago. It is evident that the FEER was unable to beat a random
walk at the 2, 4-quarter horizons. However, it produced better forecasts at the 12-, 16-quarter horizons for all three real exchange rates and beat a random walk at the 8-quarter horizon for the dollar-pound exchange rate.

These simple forecasts were based on the assumption that the future changes of fundamentals are completely unpredictable. That is not necessarily true in case of the FEER models, since some of the changes in their explanatory variables can be rather accurately predicted (e.g., the growth rate of trend real GDP of the US and its trading partners). Rows II of Table 2.10-2.12 contain the RMSEs ratio where the FEER forecasts are based on the actual realized values of explanatory variables. The US/UK FEER succeeds to improve on the random walk at the 4, 8, 12, 16-quarter horizons. The US/Canada FEER produces just as accurate forecasts at the 8-quarter horizon and outperforms a random walk at longer horizons. The US/Japan FEER succeeds to improve on a random walk at the 8, 12, 16-quarter horizons.

It can be argued that these type of forecasts are not out-of-sample, since the parameters of the trade equations and the current account target (the permanent component of cbs) are estimated based on all the information in the entire sample period, including the time period of a forecast.

To consider the out-of-sample forecasts, I first modify the current account target and estimate the permanent component of cbs based on the most up-to-date information available at any given point of the sample. I use the updating equations of the Kalman filter to get the estimates of the current account target based on the up-to-date information. Rows III of Table 2.10-2.12 present the RMSEs ratios for these calculations. The US/UK, US/Japan FEERs outperform the random walk at 8, 12, 16-quarter horizons.
The US/Canada FEER achieves the lower RMSE at the 12, 16-quarter horizons and provides just as accurate forecasts at the 8-quarter horizons.

I then re-estimate the parameters of the trade equations using data until 1981:5. Based on these parameters and the updated current account target, the FEERs were calculated for the time periods starting in 1981:1 and were used to forecast k-periods ahead. Last rows of Tables 2.10-2.12 summarize the results of this exercise. The US/UK, US/Japan RMSE ratios increased reflecting the fact that the fit of the model is worse when parameters are not re-estimated to include the new information arriving after 1980:4. The US/Canada RMSE ratios are lower since the actual real exchange rate tracks its equilibrium value much better during the second half of the sample. Overall, all three FEERs succeeded in improving on the random walk at 8, 12, 16-quarter horizons. The fact that the random walk model outperforms the FEER model at the shorter-term forecasting is not surprising, since FEER is a medium-term concept and is not designed to capture the short-run fluctuations in real exchange rates.

2.6 CONCLUDING REMARKS

This chapter extends the framework for calculating FEERs to calculate equilibrium bilateral exchange rates. It presents the equilibrium trajectories for the US

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4 The current account target is updated at the time of every forecast. However, the parameters of the model (income and price elasticity of the trade equations, the trend growth rate of real US GDP, world trade and US oil imports) are not re-estimated on the basis of the most up-to-date information. Re-estimating the parameters of the model at the time of a forecast is more than likely to improve the fit of the model and lower the RMSEs statistics. So the results in this paper can be considered as biased in favor of the random walk models.
The calculations are done within a comparative static, partial-equilibrium framework. This study eliminates the subjective element in FEER calculations by estimating the current account target using a statistical model. This eliminates the normative element in the calculations, making the estimates a medium-term forecast rather than a normative concept.

The forecasting accuracy of equilibrium exchange rates is compared with that of a random walk. The comparison reveals that FEERs present an improvement on the point predictions of the random walk models at the medium- and long-term horizons.
<table>
<thead>
<tr>
<th>Variable</th>
<th>J(1,5)(^a)</th>
<th>ADF(1)(^b)</th>
<th>ADF(4)(^b)</th>
<th>ADF(12)(^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Exchange Rate (UK/USA)</td>
<td>0.01429(^*)</td>
<td>-3.0783</td>
<td>-3.2333</td>
<td>-4.1160(^*)</td>
</tr>
<tr>
<td>Real Exchange Rate (CAN/USA)</td>
<td>1.0248</td>
<td>-1.3645</td>
<td>-2.0817</td>
<td>-2.0755</td>
</tr>
<tr>
<td>Real Exchange Rate (JAP/USA)</td>
<td>0.2973</td>
<td>-2.0732</td>
<td>-3.1266</td>
<td>-2.7279</td>
</tr>
<tr>
<td>UK Goods Imports</td>
<td>0.1551(^*)</td>
<td>-3.4724(^*)</td>
<td>-3.3750</td>
<td>-2.6285</td>
</tr>
<tr>
<td>UK Service Imports</td>
<td>2.4442</td>
<td>-1.8015</td>
<td>-1.4047</td>
<td>-1.7303</td>
</tr>
<tr>
<td>UK Good Exports</td>
<td>0.2727(^*)</td>
<td>-3.3310</td>
<td>-2.2793</td>
<td>-2.5541</td>
</tr>
<tr>
<td>UK Service Exports</td>
<td>0.5258</td>
<td>-2.1236</td>
<td>-1.8844</td>
<td>-2.2578</td>
</tr>
<tr>
<td>Japanese Goods Imports</td>
<td>0.7423</td>
<td>-3.0709</td>
<td>-2.6926</td>
<td>-2.3820</td>
</tr>
<tr>
<td>Japanese Service Imports</td>
<td>1.0420</td>
<td>-1.6740</td>
<td>-1.4501</td>
<td>-2.0002</td>
</tr>
<tr>
<td>Japanese Good Exports</td>
<td>3.9886</td>
<td>-1.0531</td>
<td>-1.3958</td>
<td>-1.4440</td>
</tr>
<tr>
<td>Japanese Service Exports</td>
<td>0.4220</td>
<td>-2.1586</td>
<td>-2.3379</td>
<td>-2.4453</td>
</tr>
<tr>
<td>Canadian Goods Imports</td>
<td>2.9597</td>
<td>-1.8562</td>
<td>-2.0462</td>
<td>-2.4323</td>
</tr>
<tr>
<td>Canadian Service Imports</td>
<td>0.8403</td>
<td>-1.9030</td>
<td>-2.2635</td>
<td>-3.1061</td>
</tr>
<tr>
<td>Canadian Good Exports</td>
<td>1.9562</td>
<td>-2.3846</td>
<td>-3.0314</td>
<td>-2.9290</td>
</tr>
<tr>
<td>Canadian Service Exports</td>
<td>2.9646</td>
<td>-0.8062</td>
<td>-0.7019</td>
<td>-1.1007</td>
</tr>
<tr>
<td>Real GDP of the US</td>
<td>2.5606</td>
<td>-2.5137</td>
<td>-3.5529(^*)</td>
<td>-3.2450</td>
</tr>
<tr>
<td>Real GDP of UK</td>
<td>0.4871</td>
<td>-2.3297</td>
<td>-2.8586</td>
<td>-3.2243</td>
</tr>
<tr>
<td>Real GDP of Japan</td>
<td>25.6974</td>
<td>-2.2025</td>
<td>-1.3268</td>
<td>-2.1412</td>
</tr>
<tr>
<td>Real GDP of Canada</td>
<td>9.4982</td>
<td>-1.4080</td>
<td>-1.8087</td>
<td>-1.6885</td>
</tr>
</tbody>
</table>

\(^a\) Critical values for the 1-percent, 5-percent, 10-percent significance levels are 0.1228, 0.2950, and 0.4520. These are referenced in Ogaki (1993).

\(^b\) ADF(r) denotes Augmented Dickey-Fuller test with time trend and r lags. Critical values for the 1-percent, 5-percent, 10-percent significance levels are -4.0216, -3.4403, -3.1444 for r=1; -4.0288, -3.4409, -3.1447 for r=4; -4.0264, -3.4426, -3.1457 for r=12. These are from MacKinnon (1990) for T=152-r-1.

\(^*\) significant at the five percent level.


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<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF(1)(^b)</th>
<th>ADF(4)(^b)</th>
<th>ADF(12)(^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>US/UK Net Foreign Investment- US GDP Ratio</td>
<td>-2.5513</td>
<td>-2.5244</td>
<td>-1.7208</td>
</tr>
</tbody>
</table>

\(^b\) ADF(r) denotes Augmented Dickey-Fuller test with time trend and r lags. Critical values for the 1-percent, 5-percent, 10-percent significance levels are -4.0216, -3.4403, -3.1444 for r=1; -4.0288, -3.4409, -3.1447 for r=4; -4.0264, -3.4426, -3.1457 for r=12. These are from MacKinnon (1990) for T=152-r-1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF(1)*</th>
<th>ADF(4)*</th>
<th>ADF(12)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real US GDP/ UK pound-US dollar Real Exchange Rate</td>
<td>-2.7437</td>
<td>-2.8864</td>
<td>-2.9850</td>
</tr>
<tr>
<td>Real UK GDP/ UK pound-US dollar Real Exchange Rate</td>
<td>-2.5119</td>
<td>-2.6310</td>
<td>-2.9969</td>
</tr>
<tr>
<td>Real US GDP/ Canadian dollar-US dollar Real Exchange Rate</td>
<td>-1.3742</td>
<td>-1.7383</td>
<td>-2.3774</td>
</tr>
<tr>
<td>Real GDP of Canada/ Canadian dollar/US dollar Real Exchange Rate</td>
<td>-1.2406</td>
<td>-1.4759</td>
<td>2.2730</td>
</tr>
</tbody>
</table>

ADF(r) denotes Augmented Dickey-Fuller test with time trend and r lags. Critical values for the 1-percent, 5-percent, 10-percent significance levels are -4.0216, -3.4403, -3.1444 for r=1; -4.0288, -3.4409, -3.1447 for r=4; -4.0264, -3.4426, -3.1457 for r=12. These are from MacKinnon (1990) for T=152-r-1.

**TABLE 2.3: TESTS FOR NO COINTEGRATION**
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant(^a)</td>
<td>-22.6926</td>
<td>-20.6198</td>
<td>-1.3983</td>
<td>-20.1637</td>
</tr>
<tr>
<td></td>
<td>(4.7096)</td>
<td>(9.0753)</td>
<td>(2.1588)</td>
<td>(2.6865)</td>
</tr>
<tr>
<td>Income Elasticity</td>
<td>3.3786</td>
<td>2.9303</td>
<td>0.9945</td>
<td>2.4248</td>
</tr>
<tr>
<td></td>
<td>(0.5201)</td>
<td>(0.2956)</td>
<td>(0.1307)</td>
<td>(0.1554)</td>
</tr>
<tr>
<td>Relative Price Elasticity(^a)</td>
<td>0.6615</td>
<td>0.5962</td>
<td>-0.4518</td>
<td>-0.5551</td>
</tr>
<tr>
<td></td>
<td>(0.0568)</td>
<td>(0.1509)</td>
<td>(0.1195)</td>
<td>(0.1654)</td>
</tr>
<tr>
<td>H(0,1)(^b)</td>
<td>5.9319</td>
<td>6.4284</td>
<td>1.8318</td>
<td>0.0052</td>
</tr>
<tr>
<td></td>
<td>(0.0149)</td>
<td>(0.0112)</td>
<td>(0.1759)</td>
<td>(0.9427)</td>
</tr>
<tr>
<td>H(1,2)(^b)</td>
<td>3.2576</td>
<td>0.3479</td>
<td>2.3505</td>
<td>4.0273</td>
</tr>
<tr>
<td></td>
<td>(0.0711)</td>
<td>(0.5553)</td>
<td>(0.1252)</td>
<td>(0.0448)</td>
</tr>
<tr>
<td>H(1,3)(^b)</td>
<td>3.3010</td>
<td>0.4639</td>
<td>3.1624</td>
<td>11.3914</td>
</tr>
<tr>
<td></td>
<td>(0.1920)</td>
<td>(0.7929)</td>
<td>(0.2057)</td>
<td>(0.0034)</td>
</tr>
</tbody>
</table>

\(^a\) All variables are in logs. Standard errors are in parentheses.

\(^b\) P-values are in parentheses

\(^*\) not significant at the five percent level

**TABLE 2.4: CANONICAL COINTEGRATING REGRESSIONS FOR THE USA/JAPAN TRADE EQUATIONS (1973:1 – 1997:4)**
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-8.3284</td>
<td>-14.2818</td>
<td>-5.7221</td>
<td>-11.1814</td>
</tr>
<tr>
<td></td>
<td>(1.1783)</td>
<td>(1.1057)</td>
<td>(0.8656)</td>
<td>(2.4050)</td>
</tr>
<tr>
<td>Income Elasticity&lt;sup&gt;a&lt;/sup&gt;</td>
<td>1.9956</td>
<td>2.6203</td>
<td>2.2256</td>
<td>3.0094</td>
</tr>
<tr>
<td></td>
<td>(0.1408)</td>
<td>(0.1320)</td>
<td>(0.1433)</td>
<td>(0.4188)</td>
</tr>
<tr>
<td>Relative Price Elasticity&lt;sup&gt;a&lt;/sup&gt;</td>
<td>1.0633</td>
<td>1.1083</td>
<td>-0.6671</td>
<td>-0.5530</td>
</tr>
<tr>
<td></td>
<td>(0.1928)</td>
<td>(0.1808)</td>
<td>(0.1488)</td>
<td>(0.6666)</td>
</tr>
<tr>
<td>H(0,1)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.6333</td>
<td>0.2902</td>
<td>1.0221</td>
<td>0.9418</td>
</tr>
<tr>
<td></td>
<td>(0.4262)</td>
<td>(0.5901)</td>
<td>(0.3121)</td>
<td>(0.3318)</td>
</tr>
<tr>
<td>H(1,2)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>2.2947</td>
<td>0.7405</td>
<td>3.3444</td>
<td>8.7471</td>
</tr>
<tr>
<td></td>
<td>(0.1298)</td>
<td>(0.3895)</td>
<td>(0.0674)</td>
<td>(0.0031)</td>
</tr>
<tr>
<td>H(1,3)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>3.0902</td>
<td>12.3961</td>
<td>6.9643</td>
<td>8.8846</td>
</tr>
<tr>
<td></td>
<td>(0.2133)</td>
<td>(0.0020)</td>
<td>(0.0307)</td>
<td>(0.0118)</td>
</tr>
</tbody>
</table>

<sup>a</sup> All variables are in logs. Standard errors are in parentheses.

<sup>b</sup> P-values are in parentheses

* not significant at the five percent level

**TABLE 2.5: CANONICAL COINTEGRATING REGRESSIONS FOR THE USA/UK TRADE EQUATIONS (1973:1 – 1997:4)**
<table>
<thead>
<tr>
<th>Variable</th>
<th>US Imports</th>
<th>US Exports</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant*</td>
<td>-8.7859</td>
<td>-1.8422</td>
</tr>
<tr>
<td></td>
<td>(0.6012)</td>
<td>(0.8229)</td>
</tr>
<tr>
<td>Income</td>
<td>2.1728</td>
<td>1.8705</td>
</tr>
<tr>
<td>Elasticity*</td>
<td>(0.0725)</td>
<td>(0.1358)</td>
</tr>
<tr>
<td>Relative Price Elasticity*</td>
<td>0.3786</td>
<td>-0.5837</td>
</tr>
<tr>
<td></td>
<td>(0.1524)</td>
<td>(0.2968)</td>
</tr>
<tr>
<td>H(0,1)*</td>
<td>1.0570</td>
<td>8.0551</td>
</tr>
<tr>
<td></td>
<td>(0.3039)</td>
<td>(0.0045)</td>
</tr>
<tr>
<td>H(1,2)*</td>
<td>10.9501</td>
<td>8.0898</td>
</tr>
<tr>
<td></td>
<td>(0.0009)</td>
<td>(0.0045)</td>
</tr>
<tr>
<td>H(1,3)*</td>
<td>20.3035</td>
<td>8.9495</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0114)</td>
</tr>
</tbody>
</table>

* All variables are in logs. Standard errors are in parentheses.

* * P-values are in parentheses

* not significant at the five percent level


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<table>
<thead>
<tr>
<th>Model</th>
<th>Parameters</th>
<th>Negative Log-Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_n^2$</td>
<td>$\sigma^2$</td>
</tr>
<tr>
<td>1. Random walk plus noise</td>
<td>0.0395</td>
<td>1.9e-19</td>
</tr>
<tr>
<td>2. Random walk plus AR(1)</td>
<td>0.0214</td>
<td>0.0179</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Random walk plus AR(2)</td>
<td>0.0358</td>
<td>0.0007</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Random walk plus AR(3)</td>
<td>0.0335</td>
<td>0.0030</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1a. Random walk plus noise</td>
<td>0.0396</td>
<td>2.0e-17</td>
</tr>
<tr>
<td>2a. Random walk plus AR(1)</td>
<td>9.5e-15</td>
<td>0.0387</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3a. Random walk plus AR(2)</td>
<td>1.8e-15</td>
<td>0.0382</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Standard errors are in parenthesis.
* not significant at the five percent level.

**TABLE 2.7: ESTIMATES OF THE STATE-SPACE MODEL FOR THE USA/JAPAN CURRENT ACCOUNT (cbs sa)**
<table>
<thead>
<tr>
<th>Model</th>
<th>Parameters</th>
<th>Negative Log-Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Random walk plus noise</td>
<td>$\sigma_0^2$ 0.0128 $\sigma_1^2$ 0.0019</td>
<td>-95.9641</td>
</tr>
<tr>
<td>2. Random walk plus AR(1)</td>
<td>$\sigma_0^2$ 0.0014 $\sigma_1^2$ 0.0144 $\rho_1$ 0.8839</td>
<td>-96.9381</td>
</tr>
<tr>
<td>3. Random walk plus AR(2)</td>
<td>$\sigma_0^2$ 0.0011 $\sigma_1^2$ 0.0147 $\rho_1$ 0.8245 $\rho_2$ 0.0810*</td>
<td>-97.3320</td>
</tr>
<tr>
<td>4. Random walk plus AR(3)</td>
<td>$\sigma_0^2$ 0.0012 $\sigma_1^2$ 0.0146 $\rho_1$ 0.8262 $\rho_2$ 0.1097* $\rho_3$ -0.0394*</td>
<td>-97.4261</td>
</tr>
<tr>
<td>1a. Random walk plus noise</td>
<td>$\sigma_0^2$ 0.0131 $\sigma_1^2$ 0.0018 $\rho_1$ 6.9e-20</td>
<td>-92.2755</td>
</tr>
<tr>
<td>2a. Random walk plus AR(1)</td>
<td>$\sigma_0^2$ 2.3e-16 $\sigma_1^2$ 0.0157 $\rho_1$ 0.8876</td>
<td>-92.7839</td>
</tr>
<tr>
<td>3a. Random walk plus AR(2)</td>
<td>$\sigma_0^2$ 1.1e-16 $\sigma_1^2$ 0.0156 $\rho_1$ 0.8276 $\rho_2$ 0.0743</td>
<td>-93.1729</td>
</tr>
</tbody>
</table>

* Standard errors are in parenthesis.
* not significant at the five percent level.

**TABLE 2.8: ESTIMATES OF THE STATE-SPACE MODEL FOR THE USA/UK CURRENT ACCOUNT (cbs sa)**
<table>
<thead>
<tr>
<th>Model</th>
<th>Parameters</th>
<th>Negative Log-Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma^2_0$ $\sigma^2_1$ $\sigma^2_2$ $\rho_1$ $\rho_2$ $\rho_3$</td>
<td></td>
</tr>
<tr>
<td>1. Random walk plus noise</td>
<td>0.0129 0.0034 ----- ----- -----</td>
<td>-84.0593</td>
</tr>
<tr>
<td>2. Random walk plus AR(1)</td>
<td>0.0146 0.0010 ----- -0.6172 -----</td>
<td>-85.6336</td>
</tr>
<tr>
<td>3. Random walk plus AR(2)</td>
<td>0.0001 0.0185 ----- 0.7543 0.1894 -----</td>
<td>-85.7984</td>
</tr>
<tr>
<td>4. Random walk plus AR(3)</td>
<td>0.0002 0.0180 ----- 0.7781 0.2907 -0.1374*</td>
<td>-87.2073</td>
</tr>
</tbody>
</table>

1a. Random walk plus noise   | 0.0132 0.0034 4.9e-22 ----- ----- ----- | -80.3512              |
2a. Random walk plus AR(1)   | 0.0148 0.0010 2.1e-20 -0.6277 ----- ----- | -81.9757              |
3a. Random walk plus AR(2)   | 0.0130 0.0042 3.7e-21 0.3311 0.5758 ----- | -82.0277              |

*Standard errors are in parenthesis
* not significant at the five percent level.

**TABLE 2.9: ESTIMATES OF THE STATE-SPACE MODEL FOR THE USA/CANADA CURRENT ACCOUNT (cbs sa)**
<table>
<thead>
<tr>
<th>MODEL</th>
<th>2-Q</th>
<th>4-Q</th>
<th>8-Q</th>
<th>12-Q</th>
<th>16-Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. FEER</td>
<td>1.3759</td>
<td>1.0132</td>
<td>0.7619</td>
<td>0.7075</td>
<td>0.7046</td>
</tr>
<tr>
<td>II. FEER</td>
<td>1.3428</td>
<td>0.9592</td>
<td>0.6640</td>
<td>0.5242</td>
<td>0.4615</td>
</tr>
<tr>
<td>III. FEER</td>
<td>1.4306</td>
<td>1.0276</td>
<td>0.7114</td>
<td>0.5616</td>
<td>0.4944</td>
</tr>
<tr>
<td>IV. FEER</td>
<td>1.5084</td>
<td>1.0921</td>
<td>0.7960</td>
<td>0.6409</td>
<td>0.5709</td>
</tr>
</tbody>
</table>

The numbers in the table are (RW RMSE)/(Model RMSE), where RW RMSE stands for the root mean square forecast error of the random walk model. RMSE=\(\sqrt{E(R_t - F_t)^2}\) where \(R_t\) is log actual real exchange rate, \(F_t\) is its k-quarter horizon forecast. Because exchange rates and their forecasts are in logs, RMSEs are approximately in percentage terms. \(F_t = R_{t+k}\) in case of the random walk models. \(F_t\) is log FEER\(_k\) in case of Model I, it is log FEER\(_k\) in case of model II and III.

Since the model forecast is the lagged FEER the implicit assumption behind the forecasting procedure is that we know nothing about the changes of fundamentals. The current account target is calculated based on the entire sample information.

FEER estimates are based on the long-run equilibrium values of the fundamentals. The current account target is calculated based on the entire sample information.

FEER estimates are based on the long-run equilibrium values of the fundamentals. The current account target (target \(cbs_{tk}\)) is based on the most up-to-date information.

Forecasting begins in 1981:1. \(F_t\) is log FEER\(_t\). FEER estimates are based on target \(cbs_{tk}\).

* TABLE 2.10: FORECAST RESULTS: RMSE RATIOS (USA/UK)*
TABLE 2.11: FORECAST RESULTS: RMSE RATIOS (USA/CANADA)
### FORECAST HORIZON

<table>
<thead>
<tr>
<th>MODEL</th>
<th>2-Q</th>
<th>4-Q</th>
<th>8-Q</th>
<th>12-Q</th>
<th>16-Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. FEER&lt;sub&gt;a&lt;/sub&gt;&lt;sup&gt;b&lt;/sup&gt;</td>
<td>2.1388</td>
<td>1.5232</td>
<td>1.1085</td>
<td>0.9999</td>
<td>0.9642</td>
</tr>
<tr>
<td>I. FEER&lt;sub&gt;b&lt;/sub&gt;&lt;sup&gt;b&lt;/sup&gt;</td>
<td>2.0850</td>
<td>1.4729</td>
<td>1.0398</td>
<td>0.9116</td>
<td>0.8479</td>
</tr>
<tr>
<td>II. FEER&lt;sub&gt;a&lt;/sub&gt;&lt;sup&gt;c&lt;/sup&gt;</td>
<td>2.0381</td>
<td>1.3922</td>
<td>0.9330</td>
<td>0.7645</td>
<td>0.6769</td>
</tr>
<tr>
<td>II. FEER&lt;sub&gt;b&lt;/sub&gt;&lt;sup&gt;c&lt;/sup&gt;</td>
<td>1.9775</td>
<td>1.3509</td>
<td>0.9053</td>
<td>0.7418</td>
<td>0.6568</td>
</tr>
<tr>
<td>III. FEER&lt;sub&gt;a&lt;/sub&gt;&lt;sup&gt;d&lt;/sup&gt;</td>
<td>2.0978</td>
<td>1.4330</td>
<td>0.9604</td>
<td>0.7869</td>
<td>0.6968</td>
</tr>
<tr>
<td>III. FEER&lt;sub&gt;b&lt;/sub&gt;&lt;sup&gt;d&lt;/sup&gt;</td>
<td>1.9328</td>
<td>1.3204</td>
<td>0.8848</td>
<td>0.7250</td>
<td>0.6420</td>
</tr>
<tr>
<td>VI. FEER&lt;sup&gt;e&lt;/sup&gt;</td>
<td>2.1230</td>
<td>1.4658</td>
<td>0.9442</td>
<td>0.7955</td>
<td>0.7243</td>
</tr>
</tbody>
</table>

*The numbers in the table are (RW RMSE)/(Model RMSE), where RW RMSE stands for the root mean square forecast error of the random walk model. RMSE={E(R_t - F_t)²}^½ where R<sub>t</sub> is log actual real exchange rate, F<sub>t</sub> is its k-quarter horizon forecast. Because exchange rates and their forecasts are in logs, RMSEs are approximately in percentage terms. F<sub>t</sub> = R<sub>t+k</sub> in case of the random walk models. F<sub>t</sub> is log FEER<sub>t+k</sub> in case of Model I, it is log FEER<sub>t</sub> in case of Model II and III.

<sup>b</sup>Since the model forecast is the lagged FEER the implicit assumption behind the forecasting procedure is that we know nothing about the changes of fundamentals. The current account target is calculated based on the entire sample information.

<sup>c</sup>FEER estimates are based on the long-run equilibrium values of the fundamentals. The current account target is calculated based on the entire sample information.

<sup>d</sup>FEER estimates are based on the long-run equilibrium values of the fundamentals. The current account target (target <sub>cbs<sub>_<sub>k</sub></sub> target <sub>cbs<sub>_<sub>k</sub></sub></sub>) is based on the most up-to-date information.

<sup>e</sup>Forecasting begins in 1985:1. F<sub>t</sub> is log FEER<sub>t</sub>. FEER estimates are based on target <sub>cbs<sub>_<sub>k</sub></sub></sub>.

**TABLE 2.12: FORECAST RESULTS: RMSE RATIOS (USA/JAPAN)**

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FIGURE 2.1: US/JAPAN ACTUAL AND FITTED EXPORTS AND IMPORTS
FIGURE 2.2: US/UK ACTUAL AND FITTED EXPORTS AND IMPORTS.
FIGURE 2.3: US/CANADA ACTUAL AND FITTED EXPORTS AND IMPORTS
FIGURE 2.4: CURRENT ACCOUNT MODEL
FIGURE 2.5: TARGET CURRENT ACCOUNT TO GDP RATIO
FIGURE 2.6: ESTIMATED AND TARGET CA/GDP RATIO
FIGURE 2.7: ACTUAL AND FUNDAMENTAL EQUILIBRIUM EXCHANGE RATES
3.1 INTRODUCTION.

Estimating equilibrium exchange rates (EER) remains one of the most challenging empirical problems in economic policy analysis. A survey of literature reveals a variety of approaches towards defining and calibrating EER that can be used to make assessments of exchange rates. A traditional method widely used in operational applications involves estimating Fundamental Equilibrium Exchange Rates (FEERs) that are consistent with simultaneous internal and external balance. A recent strand of empirical literature on estimating equilibrium exchange rates involves specification and estimation of a reduced-form equation that explains the actual real exchange rate behavior in terms of changes in economic fundamentals.

The purpose of this chapter is to evaluate and compare the estimates of equilibrium exchange rates based on the FEER model and the estimates derived from reduced-form equations. Equilibrium exchange rates considered in this study and compared to FEER are Natural Real Exchange Rate (NATREX) of Stein (1994, 1995),
Behavioral Equilibrium Exchange Rate (BEER) of Clark and MacDonald (1998), trend exchange rates of Faruqee (1995). All these estimates were originally calculated for real effective exchange rates and were not applied for bilateral exchange rates. They are reproduced in this study accordingly. Equilibrium exchange rates are calculated for the US dollar over the time period from 1975:1 to 1997:4.

One of the possible criteria for evaluating alternative models is their ability to predict actual real exchange rates. Several different types of forecasts are generated based on different estimates of equilibrium exchange rates. It is shown that the medium and long-term forecasts based on the FEER and NATREX model are more accurate than the point predictions of the random walk that is known to outperform various structural models for real exchange rates. The BEER model, however, can beat the random walk only if the actual realized values of the fundamentals are used in predicting the future direction of exchange rates.

The rest of this chapter is organized as follows. Stein’s NATREX is considered first. Its theoretical structure is briefly outlined in the first section. The description of the data and econometric methodology of the NATREX model is next. That section is followed by a discussion of the NATREX empirical estimate. A presentation of the BEER and Faruqee’s trend exchange rates follows the same structure, and it is considered next. This is followed by a discussion of the ability of equilibrium exchange rates based on the NATREX, BEER, and FEER models to forecast real effective exchange rates. The final section of this chapter concludes the study.
3.2 THEORETICAL FOUNDATIONS OF THE NATREX MODEL

A survey of literature reveals a variety of approaches to specifying reduced-form equilibrium exchange rate equations. From a theoretical point of view, they differ in underlying theory of exchange rate determination. From an empirical point of view, they differ in the choice of medium-term and long-run fundamentals and the extent to which these variables are calculated at their long-run values. Some notable examples of equilibrium exchange rates based on reduced-form equation include Stein’s NATREX (1994,1995), Fariqee’s trend exchange rates, Clark and MacDonald’s BEER (1998). These models are considered consequently in this study.

Stein has estimated NATREX for the real effective exchange rate of the US relative to the currencies of the other G-7 countries (Stein 1994,1995) and for the real effective exchange rate of Germany relative to the currencies of the other G-7 countries (Stein 1997). The purpose of this study is to compare the estimates of equilibrium exchange rates based on two different models: FEER and NATREX. To pursue this goal, I first reproduce the NATREX estimate for the US from 1975:1 to 1997:4 and then compare its ability to forecast real effective exchange rates with that of the estimate based on the FEER model.

The most general definition of the NATREX is quite similar to the FEER definition. Stein defines the NATREX as “an equilibrium exchange rate when output is at capacity and the non-sustainable transitory speculative capital flows and

---

1 A more detailed description and solution of the NATREX model can be found in Jerome L. Stein (1994,1995).
changes in reserves are ignored. Thus, the starting point of the NATREX analysis is the familiar balance of payment identity.

Identical to the FEER model, the mechanism of how capacity output (internal equilibrium) is achieved is not discussed in the model, but rather the explanatory fundamental variables are set at the level consistent with internal balance. Stein also stresses the importance of asset market equilibrium. Portfolio balance in the NATREX model implies the convergence of domestic and foreign real long-term rates of interest. The sustainable capital account (external equilibrium) is determined by saving-investment imbalances. Desired (socially optimal) saving is a function of the rate of time preference, and desired (socially optimal) investment is determined by the Tobin q-ratio. Investment less saving is determined by the capital intensity, $k$, the debt intensity, $NFA$, and the fundamentals, $Z$. Then the balance of payment identity can be re-written as equation (16):

$$CA(q,k,NFA,Z) = \Delta NFA = (I-S)(k,NFA;Z)$$

The NATREX is the real effective exchange rate, $q$, that equates the current account to the sustainable capital account. Stein shows that the NATREX at any time is a function of the capital intensity, $k$, the foreign debt intensity, $NFA$, and the fundamentals, $Z$. Following the notation of Stein, the NATREX is represented by $R(k,NFA;Z)$.

---

The NATREX model takes into account stock equilibrium conditions, so that the steady-state is reached when the domestic capital stock and net foreign assets are at their long-term values. Stein shows that in the steady-state the real exchange rate, the capital stock, and the level of net foreign assets are all functions of the fundamentals: the exogenous foreign and domestic productivity and thrift variables. Thus the *steady-state, or long-term, NATREX* is determined solely by the fundamentals. It is denoted by \( R^*(Z) \).

The actual exchange rate can be decomposed into three parts:

\[
q = R^*(Z) + [R(k, NFA; Z) - R^*(Z)] + [q - R(k, NFA; Z)]
\]

(17)

The first term is the *steady-state, or long-term, NATREX*. The second term is the trajectory of the *medium-term NATREX* towards its steady-state. It captures the dynamics of exchange rates that are induced by the evolution of capital and debt to their equilibrium level. The last term is the deviation of the actual exchange rate from the medium-term NATREX. It captures the effect of the transitory short-term factors, such as speculative forces, cyclical factors, adjustments in asset markets, etc.

The empirical counterpart of equation (17) suggested by Stein is a dynamic process equation (18):

\[
q_t = BZ_t + a[R_{t-1} - BZ_{t-1}] + b[Z_t - Z_{t-1}] + e_t
\]

(18)

The first term represents the long-run equilibrium exchange rate driven by the fundamentals. Adding the second term provides the estimate of the medium-term
equilibrium exchange rate consistent with internal and external balance conditions and portfolio equilibrium. The third term captures the dis-equilibrium component.

3.3NATREX DATA DESCRIPTION.

The data used in this study is carefully collected to match the data used in Stein (1995). All data is quarterly and covers the time period from 1975:1 to 1997:4.\(^3\) All data is obtained from the International Financial Statistics CD-ROM Data set unless an alternative source is specified.

The nominal effective exchange rate of the US is a geometrically weighted average of the dollar’s exchange rates against the currencies of the rest of the G-7 countries (ifs code “ae”). The real effective exchange rate is WPI-based. It is calculated by dividing the nominal effective exchange rate of the US by the ratio of the weighted average of the foreign WPI to the US WPI (ifs code “63” and “63a’ for France and Italy). The weight of each country is equal to that country’s share of US trade with the G-7 countries.\(^4\) The same weights are used in constructing the nominal effective exchange rate and the index of foreign prices. Note, that an increase in either the nominal or real exchange rate means that the US dollar appreciates.

The data on trade that is necessary to construct the G-7 weights is obtained from various issues of the Direction of Trade Statistics.

---

\(^3\) Stein’s sample covers time period from 1975:1 to 1993:4.

\(^4\) The country’s share of the US trade is the 1990-1994 average trade of the country with the US, divided by the average total trade of all G-6 countries with the US.
The exogenous fundamentals are time preference and the productivity of capital in the US and the other G-7 countries\textsuperscript{5}. Time preference is measured as the ratio of social consumption to nominal GDP. It is denoted by USDISRAT for the US and by G7DISRAT for the rest of the G-7 countries. For the G-7, social consumption is calculated as the sum of total private consumption (ifs code "96fc") and public expenditures (ifs code "91fc"). For the US, social consumption is the sum of total private (ifs code "96fc") and public consumption (ifs code "91fc"-"93gfc").

Two variables are constructed to measure the productivity of capital. One is the Tobin’s q-ratio. The q-ratio is measured as the ratio of industrial share prices (ifs code "62") to the producer prices of industrial goods (ifs code "63a"). The second measure of the productivity of capital, also adopted in Stein (1995), is growth of real GDP (ifs code "99br") over the past year. It is denoted by USGROW for the US and by G7GROW for the rest of the G-7 countries.

All four fundamental variables are cyclically adjusted by taking their four-quarter moving average. The adjusted variables are distinguished by the prefix MA (Z)\textsuperscript{6}.

Stein also includes two variables that are supposed to capture the disequilibrium component of equation (18). One is deviation of the rate of capacity utilization from its mean. It is designed to capture the cyclical transitory factors. This variable is denoted by DEVCUR. This series are obtained from different issues of

\textsuperscript{5} Even though foreign debt and capital intensity is an important determinant of the real exchange rate in the NATREX model, Stein does not include them in the vector of the fundamental variables, Z, because they are endogenous variables and their evolution depends on the productivity of capital and time preference.

\textsuperscript{6} Clark (1997) criticizes this method of defining equilibrium levels of fundamentals. He states that this approach involves only a cyclical adjustment of the fundamentals and does not need to coincide with
the Survey of Current Business. The data until 1990:1 is annual, and it was interpolated into quarterly basis using the RATS program, called interpol.src.

The second dis-equilibrium variable is designed to capture the temporary effects in the portfolio adjustment. It is measured by the deviation between the real long-term rate of interest in the United States and in the rest of the G-7. It is denoted by INTDIF. The real interest rates are defined as the nominal rate on long-term bonds (ifs code "61") less the change in the GDP deflator over the previous four-quarter.

All series: the real effective exchange rate, the basic exogenous fundamentals, and the dis-equilibrium variables are graphed in Figure 3.1. The statistical properties of the data are examined in Table 3.1. The conclusions that can be drawn from this preliminary analysis of the data are very similar to the results in Stein (1995). There is a strong evidence that the US real effective exchange rate and US and G-7 time preferences are non-stationary, while the fundamental variables designed to measure productivity of capital, MAUSGROW and MAG7GROW, appear to be stationary. Two dis-equilibrium variables, INTDIF and DEVCUR, are shown to be stationary as well. This is consistent with the transitory nature of these variables.

the well-established notion of full-employment output.
3.4 NATREX EMPIRICAL RESULTS.

To estimate the reduced-form equation for the US real effective exchange rate, Stein employs two estimation methods: an ordinary least squares (OLS) with an AR(1) component and a nonlinear least squares (NLS). Both methods are reproduced in this study. As it is shown in Table 3.2, the results are similar in both cases and coincide with the results in Stein (1995). They can be summarized as follows:

- A rise in US time preference depreciates the real value of the dollar while the rise in foreign time preference appreciates it. In both Stein and this study, the effect of US time preference is always significant. However, the effect of foreign time preference is significant at the ten percent level in Stein, but it is not significant in the NLS regression in this study. To preserve the structure of the model similar to Stein's, foreign time preference is not excluded from the vector of the fundamentals.

- Neither the MAUSGROW nor the MAG7GROW has a significant effect on the real value of the dollar. Furthermore, the alternative measure of productivity of capital, the Tobin's q ratio, did not show up significantly in either case. Stein suggests the following theoretical explanation of this result. In the NATREX model, a rise in the productivity of capital has two opposite effects on the determinants of equilibrium exchange rates. It raises the steady-state capital intensity, but it reduces the steady-state foreign debt intensity. It is therefore possible that these two effects cancel each other, and that neither of the variables that measure productivity of capital is significant.
- The deviation of capacity utilization rate above its mean depreciates the real value of the dollar, while the dis-equilibrium increase of the real long-term US interest rates above their counterpart in G-7 appreciates the real value of the dollar. These effects are always significant in Stein and showed up significantly in my NLS regression.

The diagnostic tests for the residuals show that the residuals are stationary, normally distributed and they present no heteroskedasticity and no serial correlation. These statistics are summarized in the footnote of Table 3.2.

Figure 3 plots the actual real effective exchange rate (REER) of the US and the NATREX estimates based on the NLS regression. The fundamentals are time preference in the United States and the rest of the G-7 countries. The US and foreign growth variables are excluded from the model since they did not show up significantly in either of the estimated regressions and in order to match the Stein's model structure\(^7\).

The top graph of Figure 3 presents the US REER and its dynamic ex-ante forecast. This forecast is based on actual realized values of all explanatory variables including the dis-equilibrium terms. However, instead of using the lagged values of actual exchange rate, this simulation uses the previously predicted value of the exchange rate, i.e. the REER forecast is:

\[
\hat{R}_t = BZ_t + a[\hat{R}_{t-1} - BZ_{t-1}] + b[Z_t - Z_{t-1}]
\]

\(^7\)To insure robustness of empirical results, I have produced NATREX estimates based on the restricted and unrestricted models. Excluding MAUSGROW and MAG7GROW does not change any conclusions of this paper.
In his earlier work, Stein (1994) refers to this forecast as the medium-term NATREX, but he later abandons this approach. The problem with this approach is in how the dis-equilibrium term, $dZ_t$, is measured. This term is designed to capture the innovation effect of the fundamental variables, but it is measured by the interest rate differential that may reflect some other non-fundamental changes such as monetary policy, speculation on long-term interest rates.

The lower graph of Figure 3.2 presents the US REER and the long-run NATREX (LR NATREX) estimate. The LR NATREX estimate is determined by the fundamentals solely. It is the first term in equation (18). To obtain this estimate, the error correction and the dis-equilibrium terms are set equal to zero. This estimate of the LR NATREX is quite similar to the one provided by Stein (1995). It records substantial misalignments in 1982-1985, close movements of the REER and the LR NATREX during the late 80s and early 90s. The misalignment in late 70s is much more substantial based on Stein's estimate. His estimate also does not reflect the upward trend in the LR NATREX that began in 1992.

3.5 THEORETICAL FOUNDATIONS OF THE BEER MODEL.

Two closely related concepts of equilibrium exchange rates are Faruqee's trend exchange rates and Clark-MacDonald's Behavioral Equilibrium Exchange Rates (BEERs). The notion of equilibrium underlying their estimates is quite different from the macroeconomic balance essential to the FEER/DEER and the NATREX approaches. Rather than estimating equilibrium exchange rates consistent
with the fundamentals set at their long-run equilibrium level, the BEER estimates are based on the actual realized values of the explanatory variables. In essence, this explains the "behavioral" part of the term coined by Clark and MacDonald. The issue of what values of explanatory variables are sustainable or optimal is not crucial. Conceptually, therefore, the BEER framework is more closely related to the medium term NATREX than to the FEER or LR NATREX model.

The BEER calculations also differ in the choice of fundamentals. Two of the BEER explanatory variables are relative prices of non-traded to traded goods, and terms of trade. Like real exchange rates, these variables are relative prices and are likely to be driven by the same fundamentals (productivity, social time preferences) and other shocks. Thus, these BEER explanatory variables should not be considered as real exchange rate fundamentals but rather as proxies for shocks that are common to real exchange rate and other relative prices.

Peter Clark and MacDonald R. (1998) have estimated BEERs for the real effective exchange rates of the US, Germany and Japan relative to the currencies of the rest of the G-7 countries. Faruqee (1997) has provided the equilibrium exchange rate estimate for the US and Japan. In this study the estimate of the US BEER over time period from 1975:1 to 1997:4 is reproduced in order to compare the estimates of the equilibrium exchange rates based on the FEER, NATREX and BEER approaches.

The starting point of P. Clark and MacDonald R. is the risk-adjusted interest rate parity condition. Equation (20) is its rearrangement:
\[ q_t = E_t(q_{t+k}) + (r_t - r_n) - \pi_t \]  \hspace{1cm} (20)

where \( \pi_t = \lambda_t + k \) is the risk-premium component that has a time-varying component, \( \lambda_t \). It is assumed that the time-varying component of the risk-premium is a function of relative supply of domestic to foreign government debt. An increase in the relative supply of domestic debt relative to foreign debt is expected to increase the domestic risk premium and bring about a depreciation of the equilibrium exchange rate.\(^8\)

The first term on the right-hand side of equation (20) is the long-run equilibrium exchange rate. It is assumed to be a function of the three variables:

\[ \bar{q}_t = E_t(q_{t+k}) = f(TOT, TNT, NFA) \]  \hspace{1cm} (21)

The TNT variable is the relative price of traded to non-traded goods and is designed to measure productivity differential in tradables at home and abroad. According to the Balassa-Samuelson proposition, the relative price of traded to non-traded goods rises less rapidly for a country with relatively higher productivity in the tradable sector. It follows that the real exchange rate based on the overall price indices appreciates for fast growing countries and we should expect a positive relationship between TNT and real exchange rate. The NFA variable measures the stock of foreign assets. According to the long-run stock equilibrium view, permanent decrease in the stock of foreign assets worsens the interest component of the current account forever. This, in turn, requires a real depreciation to restore the current account.

---

\(^8\) This view is consistent with the long-run stock equilibrium view of the impact of fiscal policy: a smaller fiscal deficit lowers net foreign debt and improves the factor income component of the current account, thus requiring a real appreciation to restore the current account equilibrium.
account equilibrium. The TOT variable measures the term of trade. Permanent improvement of the terms of trade, increase in the terms of trade, leads to the loss of competitiveness by domestic producers. This is reflected in a real appreciation of domestic currency.

Their behavioral equilibrium exchange rate (BEER) is the fitted value from the following reduced-form equation:

\[
BEER = f(r - r^*, g_{debt}/g_{debt^*}, TOT, TNT, NFA)
\]  

(22)

Faruqee’s model consists of the following equations:

\[
\begin{align*}
-\gamma q + X + rNFA &= \Delta NFA \\
-\gamma q + X + rNFA &= \delta(r - r^*) + \phi(NFA_{desired} - NFA) \\
r &= r^* - \Delta E, (\Delta q)
\end{align*}
\]

(23)  
(24)  
(25)

where \( q \) is the real exchange rate, \( r \) and \( r^* \) are the domestic real interest rate and that of the trading partners, \( X \) represents exogenous variables affecting the trade balance, \( \gamma \) is the exchange rate elasticity of the trade balance that captures the Marshall-Lerner condition.

Equation (23) is a balance of payment identity: the current account equals the rate of accumulation of net foreign assets (\( \Delta NFA \)). The current account, the left-hand side of equation (23), is defined as net trade in goods plus the net interest income received on a country’s net foreign assets. The trade balance is a function of the real exchange rate and exogenous variables.
Equation (24) is the balance of payment equilibrium condition. It is analogous to equation (11.3) in the FEER framework: it defines a sustainable balance of payment position as one that reflects a current account balance financed by a desired or sustainable rate of capital flows. According to Faruqee, that desired rate of net foreign asset accumulation mirrors a desired amount of net saving and is given by the right-hand side of equation (24).

Equation (25) is the uncovered interest rate parity with rational expectations.

The system of simultaneous linear equations (23-25) provides a solution for the two endogenous variables: \( q \) and NFA. The solution for the long-run equilibrium exchange rate is a function of equilibrium holdings of net foreign assets and the exogenous permanent component of net exports:

\[
\overline{q}_t = \frac{r^*}{\gamma} NFA + \frac{1}{\gamma} \frac{X}{\gamma}
\]

The sustainable (saddle) path for the real exchange rate - or current equilibrium exchange rate - differs from its long-run value until full stock equilibrium is attained, and it is given by equation (26):

\[
q = \overline{q} + \sigma(NFA - \overline{NFA}), \quad \sigma > 0
\]  

(26)

Faruqee summarizes: “the determinants of the equilibrium real exchange rate include factors that affect both the net trading position of the home country in world markets and the underlying propensity of the home country to be a net lender or borrower of capital. In other words, the interaction between the permanent structural components in both the current account and the capital account jointly determine the
sustainable real exchange rate. On the trade side, determinants that operate primarily through the current account may include variables such as productivity growth differentials affecting the relative price of non-traded goods or commodity-price shocks affecting the terms of trade. On the finance side, fundamentals that essentially determine the economy’s long-run net foreign asset position may include variables such as demographic factors, which affect net saving behavior through life-cycle effects, or the stock of government debt, which affects net national borrowing in the absence of Ricardian equivalence.³⁹

Faruqee employs Johansen maximum likelihood estimation technique to estimate equation (26). He uses productivity growth differentials (PROD)/the relative price of non-traded goods (TNT), and the terms of trade (TOT) as variables determining the permanent component of the current account. However, he does not explore the relationship underlying the sustainable permanent component of the net foreign assets and treats their actual stock as an exogenous variable. His equilibrium exchange rate, or what he refers to as the trend exchange rate (TREND ER), is the fitted value from the following reduced-form equation:

\[
\text{TREND ER} = f (\text{TNT}, \text{TOT}, \text{NFA})
\]

Comparison of equation (21) and equation (27) reveals that the set of explanatory variables is identical for estimating long-run equilibrium exchange rate within the BEER and TREND ER framework. I will refer to this estimate as long-run behavioral equilibrium exchange rate (LR BEER). The set of explanatory variables in equation (22) is more extensive, it additionally includes interest rate

differentials and the relative supply of domestic to foreign government debt as a proxy for the time varying component of the risk premium. I will refer to the estimate based on this set of explanatory variables as BEER.

3.6 **BEER DATA DESCRIPTION.**

The data used in this portion of my dissertation is carefully collected following the sources outlined in P. Clark, MacDonald R. (1998). Their data is annual, I was able to construct the quarterly series that match their annual data and cover the time period from 1975:1 to 1997:4. All data is obtained from the International Financial Statistics CD-ROM Data set unless an alternative source is specified.

The nominal effective exchange rate of the US is a geometrically weighted average of the dollar’s exchange rates against the currencies of the rest of the G-7 countries (ifs code “ae”). The real effective exchange rate is WPI-based. It is calculated by dividing the nominal effective exchange rate of the US by the ratio of the weighted average of the foreign WPI to the US WPI (ifs code “63” and “63a” for France and Italy). The weight of each country is equal to that country's share of US trade with the G-7 countries. The same weights are used in constructing the nominal effective exchange rate and the index of foreign prices. Note, that an

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*IMF Staff Papers* (1995), p. 89

10 The country’s share of the US trade is the 1990-1994 average trade of the country with the US, divided by the average total trade of all G-6 countries with the US.
increase in either the nominal or real exchange rate means that the US dollar appreciates.

The data on trade that is necessary to construct the G-7 weights is obtained from various issues of the Direction of Trade Statistics.

The explanatory variables are: terms of trade (TOT), relative price of non-traded to traded goods (TNT), net foreign assets (NFA), relative stock of government debt (DEBT), real long-term interest rate differential between the US and the rest of the G-7 countries (INTDIF).

The terms of trade for each country is defined as the ratio of the export unit value (ifs code “74” and “76” for Germany) to the import unit value (“75” and “76x” for Germany). The TOT variable is constructed as the ratio of the US terms of trade to the geometric weighted average of the G-6 terms of trade. The variable is expressed in logs. The weights used in constructing the foreign terms of trade are the same as used in constructing the US real effective exchange rates.

The relative price of non-traded to traded goods for each country is defined as the ratio of the domestic consumer price index (ifs code “64”) to the domestic wholesale or producer price index (ifs code “63” and “63a” for France and Italy). The TNT variable is constructed as the ratio of the US relative price to the geometric weighted average of the G-6 relative price of non-traded to traded goods. This variable is expressed in logs. The weights used in constructing the foreign relative price are the same as used in constructing the US real effective exchange rates.

The NFA variable is measured as US total net foreign assets less official gold holding (ifs code “1and”), expressed as a ratio of GDP (ifs code “99bc”). The annual
data on US net foreign assets was obtained from various issues of the Survey of Current business. This data was interpolated into quarterly basis using the RATS program, called Interpol.src.

The relative stock of government debt for each of the G-7 countries is measured as the ratio of consolidated central government debt (ifs code “88”; “88b”+”89b” for France) to nominal GDP (ifs code “99bc”). This data is available in annual frequency for UK and Japan and was interpolated into quarterly frequency using Interpol.src. The IFS provides UK government debt data until 1992, the data for later years was collected from various issues of The Bank of England: Monetary and Financial Statistics. DEBT variable is measured as the ratio of the US relative stock of government debt to the arithmetic weighted average of the G-6 relative stock of government debt.

The INTDIF variable is measured by the deviation between the real long-term rate of interest in the United States and in the rest of the G-7 countries. The real interest rates are defined as the nominal rate on long-term bonds (ifs code “61”) less the change in the GDP deflator over the previous four quarters.

The data used in this study is presented graphically in Figure 3.3. Its statistical properties are examined in Table 3.3. The conclusions that can be drawn from this preliminary analysis are straightforward. All time series are shown to be non-stationary with the exception of the US-G-6 real long-term interest rate differential that appears to be stationary.
3.7 BEER ESTIMATION RESULTS.

To estimate the reduced form equation for the US real effective exchange rate, Clark and MacDonald (1998) employ the Johansen cointegration method. They show that based on the trace test statistic and a 99 percent significance level, there appear to be up to two cointegrating vectors. It is well known that existence of multiple cointegrating vectors complicates the interpretation of the equilibrium and presents the problem of identification. In their attempt to partition the two cointegrating vectors, Clark and MacDonald restricted one of the vectors to contain the interest rate differential, and the second vector - to contain the rest of the variables. The BEER estimate is calculated as the sum of the two cointegrating vectors. This approach does not seem to be appropriate for two reasons. First of all, there is strong evidence that the interest rate differential is stationary. Therefore, it cannot form a cointegrating relationship with a non-stationary variable, such as real exchange rate. Secondly, Clark-MacDonald's estimate of BEER is based on the assumption that the ratio of the coefficients on the error correction terms is one. This seems to be arbitrary, since the actual ratio was 0.86 and one of the coefficients was not statistically significant. In his earlier paper, MacDonald (1995) does not attempt to partition the cointegrating vectors. He estimates the two unrestricted vectors, and later chooses one of them as an estimate of BEER. This approach seems to be just as arbitrary since there is no justification for why only one of the two cointegrating vectors is used to represent BEER.
To address the mentioned above problems, the vector of cointegrated variables is modified in this study. The stationary INTDIF variable is excluded from the vector of cointegrated variables. This reduces the number of cointegrating vectors to one, as it is shown in the table below\(^\text{11}\).

**Table: Number of cointegrating vectors in the model without INTDIF:**

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Likelihood Ratio</th>
<th>5 Percent Critical Value</th>
<th>1 Percent Critical Value</th>
<th>Hypothesized No. of CE(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.323349</td>
<td>79.54425</td>
<td>68.52</td>
<td>76.07</td>
<td>None **</td>
</tr>
<tr>
<td>0.183233</td>
<td>44.76082</td>
<td>47.21</td>
<td>54.46</td>
<td>At most 1</td>
</tr>
<tr>
<td>0.154267</td>
<td>26.76714</td>
<td>29.88</td>
<td>35.65</td>
<td>At most 2</td>
</tr>
<tr>
<td>0.124248</td>
<td>11.85501</td>
<td>15.41</td>
<td>20.04</td>
<td>At most 3</td>
</tr>
<tr>
<td>0.000529</td>
<td>0.047132</td>
<td>3.76</td>
<td>6.65</td>
<td>At most 4</td>
</tr>
</tbody>
</table>

**indicates rejection of the null at the 1% significance level

The exclusion of the INTDIF variable from the vector of cointegrated variables can also be justified on theoretical grounds. According to equation (7b), *the long-run equilibrium exchange rate* in the BEER model is a function of only three variables: TOT, TNT, and NFA. Adding the INTDIF and DEBT variables produces the estimate of *the behavioral equilibrium exchange rate*. Thus, reducing the number of variables to match the structure of equation (7b) can produce the estimate of the LR BEER\(^\text{12}\).

---

\(^{11}\) The table summarizes the results of the likelihood ratio (LR) test proposed by Johansen. The LR statistic for the null hypothesis that there are at most \(r\) distinct cointegrating vectors is

\[
LR = T \ln(1 - \lambda_r) \quad \text{where } \lambda_r, \ldots, \lambda_N \text{ are the smallest eigenvalues, } N \text{ is number of variables in the cointegrating vector.}
\]

\(^{12}\) Kramer (1996) includes his analogue of the DEBT variable into the vector of the LR BEER fundamentals. However, based on the data used in this study, this variable appears to be insignificant and does not change main results of this study.
Several methods for estimating cointegrating vectors are employed in this study. First, following Clark and MacDonald (1998), Faruqee (1995), I employ the Johansen cointegration method. As it is summarized in Table 3.4, this method produces results that are extremely sensitive to the number of lags included in the VAR structure of the model. The long-run coefficients change their signs depending on the number of lags and the normalized coefficients are often of implausibly high magnitude. The BEER estimate based on the long-run cointegrating relationship (to produce this estimate, the short-run VAR dynamics are set equal to zero) is very volatile and cannot be interpreted as an equilibrium exchange rate. It is surprising that Faruqee (1995) who uses the same model structure but annual data reports that his estimation was robust to the number of lags included in the VAR structure.

It has been shown (Park and Ogaki (1997)), that Canonical Cointegrating Regressions (CCR) provide asymptotically efficient estimators of cointegrating vectors that have better small sample properties than Johansen’s ML estimators. I employ CCR to estimate a cointegrating vector for the real exchange rate, TOT, TNT, USNFA.

Before running the CCR with real exchange rate and its explanatory variables, it is necessary to assess whether the explanatory variables are cointegrated with each other. If they are cointegrated, the real exchange rate cannot be cointegrated with their stationary combination, and the cointegration results would

---

13 These results might be related to those by Park and Ogaki (1991). They find that Johansen’s ML estimator performs very poorly in small samples and has very large MSEs for n=100 in their estimation in several cases. Stock and Watson (1993) also find very large variance of the Johansen’s
be meaningless. The ADF (p) tests are applied to OLS cointegrating regression residuals to test for no cointegration between the explanatory variables. The ADF(1), ADF(4), ADF(12) trend statistics are -2.3944, -2.6470, -2.5741. None of these statistics is significant indicating that residuals are non-stationary, and TNT, TOT, USNFA are not cointegrated. Thus, CCR is legitimate to apply.

The results of CCR are presented in Table 3.5. All coefficients are significant and have theoretically correct signs of plausible magnitude. Park's H(1,2) test statistic is 2.3 supporting the null of cointegration. As expected, an increase in TNT and TOT appreciates the US dollar, while accumulation of foreign debt leads towards depreciation of the dollar. As a check of the CCR estimate robustness, I have applied OLS and NLS econometric technique to estimate a cointegrating vector. The results produced by both techniques are very similar to the CCR results. The only exception is the NLS coefficient on the TNT variable, which is not statistically significant and has the wrong sign. Unlike CCR, Johansen's method produces the cointegrating vector estimates that are unstable and of implausible magnitude. Therefore, the CCR estimate is employed to calculate the long-run BEER.

Figure 3.4 displays the US real effective and behavioral equilibrium exchange rates. It is noteworthy that the empirically estimated BEER follows the actual exchange rate extremely well, even during the 1980-1987 Great Appreciation-Depreciation of the dollar. The result is puzzling since the FEER and NATREX estimates recorded significant misalignments during that period. It is common to

estimator in their Monte Carlo experiments.
interpret these misalignments as a result of some transitory factors, such as unsustainable fiscal and monetary policies.

It can be argued that high BEER volatility reflects behavioral equilibrium nature of the estimate since the fundamentals may not be at their equilibrium level. To see whether this argument is valid, all explanatory variables are set at their four-quarter moving average levels that represent equilibrium conditions. The result is presented graphically in the lower panel of Figure 3.4. The estimated BEER is less volatile but it still follows actual exchange rate very closely.

3.8 FEER, BEER AND NATREX: COMPARISON AND EVALUATION

It is now interesting to investigate the explanatory power of the NATREX and BEER models and to compare their forecasting accuracy with that of the FEER and random walk models. Several types of forecasts of the actual real effective exchange rate were constructed based on the NATREX, BEER, FEER, and RW models over the common sample period, 1975:1-1994:4. These results are summarized in Table 3.6.

The panels of this table list the ratios of the root mean square error (RMSE) statistics for the forecasts based on alternative models. The top three panels present the RMSE ratios for the trivial "we know nothing about the changes of fundamentals" forecasts. The first panel compares the forecasting accuracy of the NATREX with that of a simple random walk. The second panel presents the results for the FEER and RW models, the third panel summarizes the results for the BEER.
and RW models. The 1, 2, 4, 6, 8, 10, 12-quarters ahead NATREX (FEER, BEER) forecasts of the REER are the lagged values of the LR NATREX (FEER, LR BEER), whereas in case of the RW model these forecasts are generated by the lagged values of the REER itself.

The results are quite similar for the NATREX and FEER models. Both models beat a random walk at the longer 6-16 quarter horizons. The FEER forecasts are somewhat more accurate than the NATREX predictions in the shorter run, 1-8 quarter forward forecasts. This result can be attributed to the differences in the set of the fundamentals. The vector of the fundamentals in the NATREX model includes four variables that measure US and foreign time preference and productivity of capital. Since the variables measuring productivity of capital appear to be statistically insignificant, they are excluded from the NATREX vector of the fundamentals. The fundamentals in the FEER model are the US and G-6 real trend GDP, and the estimate of the US sustainable capital flows. These fundamentals are considered as endogenous variables in the NATREX model since their evolution is driven by the NATREX fundamentals. Therefore, the NATREX estimate is determined by the longer-run fundamentals, while the FEER estimate is consistent with the medium-term variables.

The LR BEER forecasts never succeed to improve on a random walk, but they are more accurate than the 1 and 2-quarter ahead FEER and NATREX forecasts. These empirical results question the long-run equilibrium nature of the concept, since there is no evidence that actual exchange rates tend to converge to their LR BEER level. These facts reflect the behavioral (current) equilibrium nature
of the BEER. The estimate is designed to explain what part of the real exchange rate trajectory can be accounted for by other economic variables. Its explanatory variables are relative prices of traded to non-traded goods, terms of trade and the stock of foreign assets. Like real exchange rates, TNT and TOT are relative prices and are likely to be driven by common shocks that might not be of fundamental nature.

The second panel of Table 3.6 presents the RMSE ratios for the forecasts based on the actual realized values of the fundamentals. Under perfect foresight, the accuracy of the BEER forecasts is improved dramatically and it is now able to beat the random walk at all horizons, except 1-quarter. The BEER predictions now have the smallest RMSEs among the compared models. This fact does not necessarily suggest that the BEER forecasting ability is superior to that of the FEER and NATREX, but it rather implies that the current misalignments between the actual exchange rate and its equilibrium value are more substantial in the FEER and NATREX model.

It is noteworthy that perfect knowledge of the fundamentals does not improve the short-run FEER forecasts and the NATREX predictions for all horizons. This result can be explained by investigating the nature of fundamentals in these models. The NATREX fundamentals (productivity and social time preferences proxies) are likely to have an effect on real exchange rate with a considerable lag, after all other endogenous variables (capital stock, foreign debt) have adjusted for shocks to fundamentals. The FEER fundamentals (the US and G-6 real trend GDP, the estimate of the US sustainable capital flows) can be considered as proxies for
endogenous variables in the NATREX model since their evolution is driven by the
NATREX fundamentals. As such, they affect real exchange rate with a shorter lag.
The BEER explanatory variables (relative prices of traded to non-traded goods,
terms of trade, stock of foreign assets) can be considered as variables that are subject
to the same shocks that affect exchange rates and, thus, they tend to move with real
exchange rates almost simultaneously. Therefore, knowing the current values of
fundamentals is essential to forecasts based on the BEER model but it is of no
importance for short-term FEER forecasts and all-term NATREX forecasts.

The simple type of forecasts considered above ignores the fact that the
current deviation of the real exchange rate from its equilibrium value helps to predict
the future direction of the exchange rate movements. The deviation of actual
exchange rate from its equilibrium level is explicitly accounted for in the NATREX
model. The next type of forecasts takes this deviation into account and it is given by
equation (28):

\[ \hat{R}_{t+1} = R^*_t + a (R_t - R^*_t) \]  \hspace{1cm} (28)

where \( R^*_t \) is estimated equilibrium exchange rate, \( R \) is actual real exchange rate, and
\( a \) is the speed of convergence of actual exchange rates towards their dynamic
equilibrium level. The coefficient on the error-correction term in the NATREX NLS
regression provides an estimate of the speed of convergence. Based on the results in
Table 3.2, the speed of convergence, \( a \), is 0.826. Even though the deviation of actual
exchange rate from its equilibrium level is not explicitly modeled in the FEER
framework, I have produced several forecasts of real effective exchange rate that
take into account these deviations. The speed of adjustment can be estimated from
the regression of the current misalignment (actual real exchange rate minus
equilibrium exchange rate) on the lagged misalignment. The speed of convergence is
estimated to be 0.9164 in the FEER model and 0.6784 in the BEER model.

The summary of the forecasts based on equation (28) is in panel III and IV of
Table 3.6. As expected, adding the error correction term dramatically improves the
forecasting accuracy of all three models in the short run. Unsurprisingly, it does not
help to improve long-run forecasts, since the contribution of the error correction term
converges to zero as forecast horizon increases \( i \to \infty \). The FEER and NATREX
model now outperform the random walk at all horizons. BEER forecasts, however,
are still less accurate than those of the random walk. The empirical observation of
panel I is preserved here - the FEER forecasts are more accurate in the shorter run,
while the NATREX predictions have smaller RMSE in the longer run.

The forecasting equation (28) is based on the assumption that the
fundamentals are non-stationary, and one cannot predict their future values.
Therefore, one’s best forecast of the future fundamentals is their current actual value,
i.e. \( E(Z_{t+1}) = Z_t \).

If the fundamentals can be foreseen perfectly, then the real exchange rate
forecast would be given by equation (29)\(^\text{14}\):

\[
\hat{R}_{t+1} = R^*_t + \alpha' [R_t - R^*_t]
\]  
(29)

\(^{14}\) Equation (28) and (29) are derived from equation (18) by the method of recursive substitution. The derivation
of equation (29) is below. The derivation of equation (28) is analogous.

\[
\hat{R}_{t+1} = BZ_{t+1} + a[\hat{R}_{t+1} - BZ_{t+1}] = BZ_{t+1} + a[BZ_{t+1} + a[\hat{R}_{t+2} - BZ_{t+2}] - BZ_{t+1}] = BZ_{t+1} + a^2[\hat{R}_{t+3} - BZ_{t+3}] = \ldots = BZ_{t+1} + a^i[\hat{R}_{t+i} - BZ_{t+i}]
\]
The summary of the NATREX, BEER and FEER forecasts based on (29) is in panel IV of Table 3.6. Perfect knowledge of fundamentals is shown to improve the accuracy of the BEER forecasts radically. The BEER model can now beat the random walk at all horizons, except 1 quarter forward. It is noteworthy that the forecasting accuracy of the FEER model deteriorates in the shorter run and the NATREX forecasts become less accurate at all horizons when the predictions are based on the actual realized values of the fundamentals. This result is the recurrence of the empirical observation in panel II.

It can be argued that the type of forecasts summarized in Table 3.6 is not truly out-of-sample, since parameters of all models are estimated based on the entire sample period, including the time period of a forecast. To consider the out-of-sample forecasts, the sample is first truncated in 1982:4 and all parameters are re-estimated based on the information in the first half of the sample. These parameters are then used to produce dynamic multiple-horizon forecasts. Table 3.7 summarizes the results of the out-of-sample forecasts for the NATREX, FEER and BEER models.

All main results of in-sample forecasting remain intact in Table 3.7. The NATREX and FEER models outperform the random walk at the medium and short-term horizons even if we know nothing about the course of fundamentals. Taking the error-correction term into account dramatically improves the forecasting accuracy of these models and they can beat the random walk at all horizons as demonstrated in panel III of Table 3.7. BEER, however, can outperform the random walk only if the

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15 In case of the FEER model, the current account target is also re-estimated based on the most-up-to-date information. The updating equations of the Kalman Filter are used in order to estimate the permanent component of the current account based on the most up-to-date information, that is then used to represent the current account target.
forecasts are based on the actual values of the fundamentals. The new finding is that the NATREX and FEER forecasts are now more accurate than the BEER forecasts at all horizons even if predictions are based on the realized values of the fundamentals. As it is demonstrated in Table 3.8, this result can be explained by the fact that the BEER model is more sensitive to how often the model parameters are re-estimated.

In Table 3.8, out-of-sample forecasts are based on the parameters that are re-estimated in the beginning of every year when forecasting is carried out. The noteworthy result is that the accuracy of the BEER forecasts is improved dramatically as a result of the yearly parameter re-estimation. The BEER out-of-sample predictions are now more accurate than the FEER forecasts at all horizons when they are based on the actual values of the explanatory variables. All other results remain intact.

3.9 CONCLUSIONS.

In this study the equilibrium trajectory for the US dollar against the rest of the G-7 countries is estimated based on three models for calculating equilibrium exchange rates. The FEER calculations are done within a comparative static partial equilibrium framework. The NATREX and BEER estimates are based on reduced form equations. It is attempted to evaluate these alternative models by comparing their forecasting accuracy.

It is shown that the FEER and NATREX models provide a reasonable estimate of equilibrium exchange rate that signal the direction of future exchange
rate changes. This conclusion is based on the observation that when current FEER or NATREX are used to forecast future exchange rates they produce more accurate medium- and long-term forecasts than the point predictions of a random walk. The forecasts based on the FEER model seem to be more accurate in the shorter run, while the forecasts based on the NATREX model are more accurate in the longer run. This result is attributed to the differences in the set of fundamentals. It is also shown that both models can produce accurate out-of-sample forecasts in the very short-run if the error correction term is taken into account. Both models are demonstrated of being able to beat a random walk at all quarter horizons.

The BEER estimate, however, is too volatile and cannot provide much additional information, besides what is contained in the current exchange rate, to help predicting the future direction of exchange rates. Nevertheless, the BEER model can beat the random walk at all horizons when the realized values of the fundamentals are used in out-of-sample forecasting. Even though perfect foresight forecasting cannot be implemented in practice (the BEER explanatory variables follow non-stationary processes and cannot be predicted perfectly), the result is noteworthy since the random walk is known to outperform various structural models of exchange rates even when their forecasts are based on the actual realized values of the fundamentals.
SUMMARY

This dissertation work presents an approach to estimate the Fundamental Equilibrium Exchange Rate popularized by Williamson. The approach differs from the literature in two important aspects. First, most estimates hinge on a subjective judgment regarding the external balance target. This study eliminates the subjective element by estimating the current account target using a statistical model. Second, the structural equations of the current account model are obtained by applying the canonical cointegration technique to a single set of quarterly time series.

The estimated FEER trajectories are presented for the US real effective exchange rate and for three bilateral exchange rates: US dollar-UK pound, US dollar-Japanese yen, US dollar-Canadian dollar. These trajectories are compared to the random walk in forecasting real exchange rates and found to be superior in medium- and long-run.

The US FEER is also compared to the equilibrium exchange rates estimates based on the reduced-form equations: NATREX and BEER. It is found that the medium and long-term forecasts based on the FEER and NATREX models are more accurate than the point predictions of the random walk. The BEER model, however, can beat the random walk only if the actual realized values of the fundamentals are used in forecasting.
<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF(1)</th>
<th>ADF(2)</th>
<th>ADF(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Real Effective Exchange Rate(^a)</td>
<td>-1.7761</td>
<td>-1.4324</td>
<td>-2.2748</td>
</tr>
<tr>
<td>US Time Preference(^b)</td>
<td>-1.9165</td>
<td>-1.6947</td>
<td>-1.1697</td>
</tr>
<tr>
<td>(MAUSDISRAT)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G-7 Time Preference(^b)</td>
<td>-1.6757</td>
<td>-1.2844</td>
<td>-1.7375</td>
</tr>
<tr>
<td>(MAG7DISRAT)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US Productivity of Capital(^b)</td>
<td>-6.5210*</td>
<td>-4.0414*</td>
<td>-2.2599</td>
</tr>
<tr>
<td>(MAUSGROW)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G-7 Productivity of Capital(^b)</td>
<td>-4.6142*</td>
<td>-3.6047*</td>
<td>-1.7899</td>
</tr>
<tr>
<td>(MAG7GROW)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USDEVCUR</td>
<td>-3.2378*</td>
<td>-2.8056**</td>
<td>-3.1242*</td>
</tr>
<tr>
<td>Rate Differential (INTDIFF)(^c)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G-7 Tobin Q-Ratio(^b)</td>
<td>-0.8544</td>
<td>-0.7404</td>
<td>-0.5855</td>
</tr>
</tbody>
</table>

\(^{a}\) ADF(r) denotes Augmented Dickey-Fuller test with time trend and r lags.
\(^{b}\) ADF(r) denotes Augmented Dickey-Fuller test with a constant and r lags.
\(^{c}\) ADF(r) denotes Augmented Dickey-Fuller test with no constant and r lags.

\(^*\) significant at the five percent level (MacKinnon (1990) critical value).
\(^{**}\) significant at the ten percent level (MacKinnon (1990) critical value).

**TABLE 3.1: TESTS FOR TREND PROPERTIES OF THE NATREX DATA**
(1975:1 – 1997:4)
### Variable* OLS\(^1\) OLS-STEIN NLS\(^2\) NLS\(^3\) NLS-STEIN

<table>
<thead>
<tr>
<th>Variable</th>
<th>OLS(^1)</th>
<th>OLS-STEIN</th>
<th>NLS(^2)</th>
<th>NLS(^3)</th>
<th>NLS-STEIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTANT</td>
<td>141.6881</td>
<td>192.7397</td>
<td>292.3342</td>
<td>285.4291</td>
<td>335.9549</td>
</tr>
<tr>
<td></td>
<td>(0.3443)</td>
<td>(0.1076)</td>
<td>(0.0750)</td>
<td>(0.0352)</td>
<td>(0.0297)</td>
</tr>
<tr>
<td>MADISRAT</td>
<td>-387.64 33</td>
<td>-582.7147</td>
<td>-480.8971</td>
<td>-444.4760</td>
<td>-660.0887</td>
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\(p\)-values are in parenthesis

The estimated equation is: USREER = CONST + C(2)*MADISRAT + C(3)*MAG7DISRAT + C(4)*MAUSGROW + C(5)*MAG7GROW + C(6)*INTDIF + C(7)*DEVCUR + error, where the error term is an AR(1) process. The ADF (1) no-constant, no-trend statistic for the residuals is −7.4366. It is significant at the 1 percent level indicating that residuals are stationary. The F statistic from the ARCH test for the residuals has a p-value of 0.8794 indicating no heteroscedasticity. The Breusch-Godfrey LM statistic is 1.6520 (p-value=0.4378) indicating no serial correlation among the residuals. The Jarque-Bera statistic is 0.661 with a p-value of 0.7184 which supports the null of normally distributed residuals.

\(^2\) The estimated equation is: USREER = CONST + C(2)*MADISRAT + C(3)*MAG7DISRAT + C(4)*MAUSGROW + C(5)*MAG7GROW + C(6)*(USREER(-1) − CONST - C(2)*MADISRAT(-1) - C(3)*MAG7DISRAT(-1) - C(4)*MAUSGROW(-1) - C(5)*MAG7GROW(-1)) + C(7)*INTDIF + C(8)*DEVCUR. Estimated equation is restricted as in Stein to exclude two insignificant productivity variables. The ADF(1) no-constant, no trend statistic for the residuals is −7.4884. It is significant at the 1 percent level indicating that residuals are stationary. The F statistic from the ARCH test for the residuals has a probability of 0.5329 indicating no heteroscedasticity. The Breusch-Godfrey LM statistic is 2.1655 (p-value=0.1387) indicating no serial correlation among the residuals. The Jarque-Bera statistic is 1.4693 with a p-value of 0.4797 which supports the null of normally distributed residuals.

---

<table>
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<tr>
<th>Variable</th>
<th>ADF(1)^a</th>
<th>ADF(4)^b</th>
<th>ADF(12)^b</th>
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^a ADF(r) denotes Augmented Dickey-Fuller test with time trend and r lags.
^b ADF(r) denotes Augmented Dickey-Fuller test with a constant and r lags.
^c ADF(r) denotes Augmented Dickey-Fuller test with no constant and r lags.
* significant at the five percent level (MacKinnon (1990) critical value).
** significant at the ten percent level (MacKinnon (1990) critical value).


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<td>C(5)b</td>
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*Standard errors are in parenthesis.

The estimated equation is LREER = C(1) + C(2)*TNT + C(3)*TOT + C(4)*USNFA + C(5)*
(LREER(-1) - C(1) - C(2)*TNT(-1) - C(3)*TOT(-1) - C(4)*USNFA(-1))


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*The numbers in the table are (Model RMSE)/(RW RMSE), where RW RMSE is the root mean square forecast error of the random walk model. RMSE = \( \left( \frac{1}{T} \sum_{t=1}^{T} (R_t - F_t)^2 \right)^{1/2} \) where \( R_t \) is log actual real exchange rate, \( F_t \) is its k-quarter horizon forecast. Because exchange rates and their forecasts are in logs, RMSES are approximately in percentage terms. \( F_t = R_{tk} \) in case of the random walk models. \( F_t = \log \text{NATREX}_{tk} \) (log \( \text{FEER}_{tk} \)) in case of Model I, it is log \( \text{NATREX} \), (log \( \text{FEER} \)) in case of model II... \( F_t \) is log \( \text{NATREX}_{tk} + \text{error correction term} (ECT) \) in case of model III, and it is log \( \text{NATREX} + \text{ECT} \) in case of model IV.

* Since the model forecast is the lagged FEER (NATREX), the implicit assumption behind the forecasting procedure is that we know nothing about the changes of fundamentals.

* FEER and NATREX forecasts are based on the actual realized values of the fundamentals.

<table>
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<tr>
<th>FORECAST HORIZON</th>
<th>1-Q</th>
<th>2-Q</th>
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<th>6-Q</th>
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*Out-of-sample forecasts are based on the parameters that are estimated from the truncated sample. The truncation date is 1983:1. Forecasting begins in 1983:1. The numbers in the table are (Model RMSE) / (RW RMSE), where RW RMSE is the root mean square forecast error of the random walk model. RMSE = \( \sqrt{\frac{1}{T} \sum (R_t - F_t)^2} \) where \( R_t \) is log actual real exchange rate, \( F_t \) is its k-quarter horizon forecast. Because exchange rates and their forecasts are in logs, RMSEs are approximately in percentage terms. \( F_t = R_t \) in case of the random walk models. \( F_t = \text{log NATREX}_{t+k} \) (log FEER\(_t\)) in case of model I, it is log NATREX\(_t\) (log FEER\(_t\)) in case of model II. \( F_t = \text{log NATREX}_{t+k} + \text{error correction term (ECT)} \) in case of model III, and it is log NATREX\(_t\) + ECT in case of model IV. Since the model forecast is the lagged FEER (NATREX), the implicit assumption behind the forecasting procedure is that we know nothing about the changes of fundamentals. FEER and NATREX forecasts are based on the actual realized values of the fundamentals.

<table>
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<tr>
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</tbody>
</table>

*Out-of-sample forecasts are based on the parameters that are re-estimated in the beginning of every year. Forecasting begins in 1983:1. The numbers in the table are \((\text{Model RMSE})/(\text{RW RMSE})\), where RW RMSE is the root mean square forecast error of the random walk model. RMSE=\((E(R_t - F_t))^2\) where \(R_t\) is log actual real exchange rate, \(F_t\) is its k-quarter horizon forecast. Because exchange rates and their forecasts are in logs, RMSEs are approximately in percentage terms. \(F_t = R_{t+k}\) in case of the random walk models. \(F_t = \log \text{NATREX}_{t+k}\) (log \(\text{FEER}_{t+k}\)) in case of Model I, it is \(\log \text{NATREX}_t\) (log \(\text{FEER}_t\)) in case of model II. \(F_t = \log \text{NATREX}_{t+k} + \text{error correction term (ECT)}\) in case of model III, and it is \(\log \text{NATREX}_t + \text{ECT}\) in case of model IV.

b Since the model forecast is the lagged \(\text{FEER}\) (NATREX), the implicit assumption behind the forecasting procedure is that we know nothing about the changes of fundamentals.

c \(\text{FEER}\) and NATREX forecasts are based on the actual realized values of the fundamentals.

FIGURE 3.1: NATREX BASIC VARIABLES
FIGURE 3.2: ACTUAL AND NATREX ESTIMATES OF THE US REAL EFFECTIVE EXCHANGE RATE RELATIVE TO G-7
FIGURE 3.4: US REAL EFFECTIVE AND BEHAVIORAL EQUILIBRIUM EXCHANGE RATE


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