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3-DIMENSIONAL PARTICLE TRACKING VELOCIMETRY
MEASUREMENTS OF NEAR-WALL SHEAR IN HUMAN
LEFT CORONARY ARTERIES

DISSertation

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

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ABSTRACT

Hemodynamic shear stress has been shown to be an important factor in the development of atherosclerosis at several important sites in the arterial system, including the clinically significant left coronary artery. Few studies have attempted to obtain experimentally the near-wall velocity measurements necessary to estimate wall shear stress in human left coronary arteries due to their small diameter and the limitations of conventional measurement techniques. Methods have been developed in our laboratory to create larger-than-life flow-through compliant replicas of human left coronary arteries to facilitate the acquisition of near-wall velocity data. Using index of refraction matched fluid and neutrally buoyant particles; a 3-Dimensional Particle Tracking Velocimetry (PTV) algorithm was employed to obtain a full-field velocity vector map throughout the entire pulsatile cycle.

Full field axial and secondary velocities, and near-wall shear maps, were obtained. The flow field exhibited strong secondary flow components, including a helical flow spiraling down the daughter vessels. Estimated shear values ranged from -6 to 32 dyne/cm². The near-wall shear was found to be higher on the flow divider walls, especially in the left anterior descending (LAD) branch.
Reducing the LAD-LCX bifurcation angle from 98° to 86° did not have a significant effect on the near-wall shear. PTV was found to be an invaluable tool in assessing the entire flow field in a complex, pulsatile flow system.
Dedicated to my loving family.
Thank you for all your support and understanding throughout this entire dissertation process.
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Research Publication

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>v</td>
</tr>
<tr>
<td>VITA</td>
<td>vi</td>
</tr>
<tr>
<td>CHAPTER 1</td>
<td>1</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>CHAPTER 2</td>
<td>4</td>
</tr>
<tr>
<td>2. BACKGROUND &amp; SIGNIFICANCE</td>
<td>4</td>
</tr>
<tr>
<td>2.1 THE LEFT CORONARY ARTERY</td>
<td>4</td>
</tr>
<tr>
<td>2.2 IMPORTANCE OF SHEAR STRESS</td>
<td>4</td>
</tr>
<tr>
<td>2.3 GEOMETRIC RISK FACTORS</td>
<td>9</td>
</tr>
<tr>
<td>2.4 PREVIOUS CORONARY ARTERY STUDIES</td>
<td>16</td>
</tr>
<tr>
<td>2.5 COMPUTATIONAL FLUID DYNAMICS</td>
<td>20</td>
</tr>
<tr>
<td>2.6 PARTICLE TRACKING VELOCIMETRY</td>
<td>21</td>
</tr>
<tr>
<td>CHAPTER 3</td>
<td>23</td>
</tr>
<tr>
<td>3. 3-DIMENSIONAL PARTICLE TRACKING VELOCIMETRY</td>
<td>23</td>
</tr>
<tr>
<td>3.1 INTRODUCTION</td>
<td>23</td>
</tr>
<tr>
<td>3.2 PTV ALGORITHMS AND PROCESSING STEPS OVERVIEW</td>
<td>23</td>
</tr>
<tr>
<td>3.3 PTV SOFTWARE AND CALIBRATION</td>
<td>26</td>
</tr>
<tr>
<td>3.4 IMAGE PRE-PROCESSING</td>
<td>28</td>
</tr>
<tr>
<td>3.5 IMAGE ACQUISITION</td>
<td>29</td>
</tr>
<tr>
<td>3.6 CREATION OF 2-D TRACKS</td>
<td>30</td>
</tr>
<tr>
<td>3.7 3-D STEREO MATCHING</td>
<td>33</td>
</tr>
<tr>
<td>3.8 ADAPTIVE GAUSSIAN WINDOW VALIDATION</td>
<td>35</td>
</tr>
<tr>
<td>CHAPTER 4</td>
<td>37</td>
</tr>
<tr>
<td>EXPERIMENTAL METHODS</td>
<td>37</td>
</tr>
<tr>
<td>4.1 EXPERIMENTAL APPROACH</td>
<td>37</td>
</tr>
<tr>
<td>4.2 MODELING</td>
<td>38</td>
</tr>
</tbody>
</table>

viii
# Table of Contents

4.3 Fluid .................................................................................................................................40  
4.4 Particles ..........................................................................................................................41  
4.5 Realistic Flow Wave .........................................................................................................42  
4.6 Creation of Realistic Replica ..........................................................................................46  
4.7 Image Acquisition ..........................................................................................................54  
4.8 Image Resolution and Framing Rate ..............................................................................57  

CHAPTER 5 ...................................................................................................................................59  

RESULTS ..................................................................................................................................59  
5.1 Data Presentation .............................................................................................................59  
5.2 Data Validation ................................................................................................................67  
5.3 Axial Velocity Profiles .....................................................................................................68  
5.4 Secondary Velocity Profiles .............................................................................................71  
5.5 Near-Wall Shear ...............................................................................................................72  
5.6 Shear Comparisons, Lateral Wall and Flow Divider ......................................................86  

CHAPTER 6 ...................................................................................................................................93  

Discussion and Conclusions .................................................................................................93  
6.1 General Flow Field Discussion .......................................................................................93  
   6.1.1 Axial Velocity Profiles .................................................................................................94  
   6.1.2 Secondary Velocity Profiles .....................................................................................96  
6.2 Near-Wall Shear Maps ....................................................................................................101  
6.3 Shear Rate Profiles .........................................................................................................103  
6.4 Bifurcation Angle ............................................................................................................105  
6.5 PTV Analysis ..................................................................................................................109  
6.6 Conclusions ....................................................................................................................111  

APPENDIX A .............................................................................................................................112  
   Instat Multiple Regression Results ..................................................................................112  

APPENDIX B ................................................................................................................................142  
   Calculation of Scaled-Up Variables ................................................................................142  

APPENDIX C ................................................................................................................................144  
   Working Fluid Calculations ..............................................................................................144  

APPENDIX D ................................................................................................................................146  
   Calculation of Particle Properties ....................................................................................146  

APPENDIX E ................................................................................................................................153  
   Klinger CC-1 Universal Programmer Program ..................................................................153  

APPENDIX F ................................................................................................................................154  
   Axial Velocity Profiles ......................................................................................................154  

APPENDIX G ................................................................................................................................181
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>PUBLICATIONS ............................................................................... vi</td>
<td></td>
</tr>
<tr>
<td>FIELDS OF STUDY ........................................................................ vii</td>
<td></td>
</tr>
<tr>
<td>Table 2.1 Literature-based Review of Previous Left Coronary Studies</td>
<td>18</td>
</tr>
<tr>
<td>Table 2.1 (continued) ................................................................... 19</td>
<td></td>
</tr>
<tr>
<td>Table 4.2 Geometric Parameters .................................................. 56</td>
<td></td>
</tr>
<tr>
<td>Table 4.3 Experimental Conditions ............................................. 57</td>
<td></td>
</tr>
<tr>
<td>Table 5.1 Bulk Flow Data Validation ........................................... 68</td>
<td></td>
</tr>
<tr>
<td>Table 6.1 Comparison of Shear Rates for TB33 and TB41 .................. 107</td>
<td></td>
</tr>
<tr>
<td>Table D.1 Table of TB33 Physical Data ....................................... 148</td>
<td></td>
</tr>
<tr>
<td>Table D.2 Table of Particle Calculations for TB33 ........................ 149</td>
<td></td>
</tr>
<tr>
<td>Table D.3 Table of TB41 Physical Data ....................................... 150</td>
<td></td>
</tr>
<tr>
<td>Table D.4 Table of Particle Calculations TB41 ............................. 152</td>
<td></td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1.1 Anatomy of the Human Coronary Arteries</td>
<td>5</td>
</tr>
<tr>
<td>Figure 3.1 Flow Chart of PTV Algorithm</td>
<td>25</td>
</tr>
<tr>
<td>Figure 3.2 Calibration Image Showing Calibration Scales</td>
<td>27</td>
</tr>
<tr>
<td>Figure 3.3 Image Acquisition</td>
<td>30</td>
</tr>
<tr>
<td>Figure 3.4 Determination of Most Probable Track using Penalty Function</td>
<td>32</td>
</tr>
<tr>
<td>Figure 3.5 Validation of the measured velocity field</td>
<td>36</td>
</tr>
<tr>
<td>Figure 4.1 Suspended Particles</td>
<td>42</td>
</tr>
<tr>
<td>Figure 4.2 Diagram of Experimental Set-up</td>
<td>43</td>
</tr>
<tr>
<td>Figure 4.3 Photograph of Experimental Set-up</td>
<td>44</td>
</tr>
<tr>
<td>Figure 4.4 Experimental Average Flow Waveform</td>
<td>45</td>
</tr>
<tr>
<td>Figure 4.5 Flowchart of the Flow Through Model Making Process</td>
<td>47</td>
</tr>
<tr>
<td>Figure 4.6 Investment of Original Vessels Taken Apart</td>
<td>48</td>
</tr>
<tr>
<td>Figure 4.7 Original Vessel, Polyester Resin Copy, and Investment put Together</td>
<td>48</td>
</tr>
</tbody>
</table>
Figure 5.16 Epicardial TB41 Diastole Near-Wall Shear Rate ........................................... 81
Figure 5.17 Epicardial TB41 Systole Near-Wall Shear Rate .................................................. 82
Figure 5.18 Myocardial TB41 Diastole Near-Wall Shear .......................................................... 83
Figure 5.19 Myocardial TB41 Systole Near-Wall Shear Rate ................................................. 84
Figure 5.20 Pulse Shear Map for TB33 .................................................................................. 85
Figure 5.21 Pulse Shear Map for TB41 .................................................................................. 86
Figure 5.22 Flow Divider Shear Rate Profile Comparison ...................................................... 87
Figure 5.23 Flow Divider Diastole Shear Rate Comparison .................................................. 88
Figure 5.24 Flow Divider Systole Shear Rate Comparison ..................................................... 89
Figure 5.25 Lateral Wall Shear Rate Comparison .................................................................... 90
Figure 5.26 Lateral Wall Diastole Shear Rate Comparison .................................................... 91
Figure 5.27 Lateral Wall Systole Shear Rate Comparison ...................................................... 92
CHAPTER 1

INTRODUCTION

Cardiovascular disease leading to heart attack and stroke is the main cause of death in the United States today. According to the American Heart Association, the total cost of cardiovascular disease in 1999 is expected to reach $286.5 billion, up from $274.2 billion in 1998.

The fact that rate of disease progression varies among individuals has been known for some time. Studies of this variability have identified "risk factors", which can affect both the timing of disease development and the severity of the disease in an individual. Some of the more commonly accepted risk factors include cigarette smoking, elevated levels of blood cholesterol and high blood pressure. Even with the currently identified risk factors, nearly half of the variability in occurrence of cardiovascular disease is still unexplained [1].

It is commonly known that there is a relationship between hemodynamics and atherosclerosis. Variations in the detailed geometry of susceptible arterial segments are believed to contribute a portion of the variability in disease occurrence. Detailed studies of the aortic bifurcation have identified variation in
the branch angle and an offset flow divider tip as potential geometric risk factors [2]. A correlation between the vessel geometry and early fatty lesions, an indicator of atherosclerotic plaques, has also been shown [3]. This implies that vessel geometry could be used to predict vessels at greater risk of atherosclerotic lesions.

The hemodynamics of the human left coronary artery is poorly understood due to its small dimensions. The proximal portions of the left anterior descending (LAD) and left circumflex (LCX) coronary arteries are among the sites frequently predisposed to atherosclerotic disease [4,5,6]. This predisposition might be a consequence of their location immediately distal to the left main coronary artery bifurcation, which may increase the susceptibility of these segments by promoting an adverse fluid dynamic environment in them.

We hypothesize a relationship between the local fluid dynamic parameters, such as near-wall shear rate and the geometric parameters of the human left coronary artery. The specific geometric parameter to be studied is the branch angle between the left anterior descending and the left circumflex arteries at the bifurcation of the left main coronary artery.

We will use 3-Dimensional Particle Tracking Velocimetry (PTV), a non-invasive measurement technique, to investigate the full 3-dimensional velocity field of two human left coronary arteries.

Flow-through compliant models of human left coronary arteries will be scaled to larger-than-life size to facilitate the acquisition of near-wall velocity data. Pulsatile flow will be studied using a physiologic flow waveform. The
relationship between the near-wall shear rate and the branch angle will be studied to identify the local hemodynamic factors that promote fatty lesions.
CHAPTER 2

BACKGROUND & SIGNIFICANCE

2.1 The Left Coronary Artery

Figure 1.1 [7] shows the anatomy of the human coronary arteries. The diameter of the left main artery is on the order of $4.4 \pm 0.5$ mm [8]. The left coronary artery originates from the left aortic sinus of Valsalva. It usually has a short common stem, the left main (LM), 1/2 to 2 cm long, which usually bifurcates. One branch, the left anterior descending (LAD), courses downward along the anterior interventricular groove. The second branch, called the left circumflex (LCX), branches off to supply the upper lateral left ventricular wall and the left atrium. In both cases, multiple branches occur, usually decreasing in size. Some of these branches even anastomose with branches from the right coronary artery.

2.2 Importance of Shear Stress

Hemodynamic shear stress has been shown to be an important factor in the development of atherosclerosis at several important sites in the arterial system [9]. This relationship has been noted for specific
Figure 1.1 Anatomy of the Human Coronary Arteries
arterial locations such as bends, bifurcations and ostia of medium to large arteries. In fact, the supposition that fluid mechanics, and in particular, shear stress, plays a role in the development and the localization of atherosclerosis was suggested as early as the 1860's by Virchow (1862) and Rindfleisch (1872) [10,11]. They suggested that mechanical injury to arterial walls might be an important factor in the genesis of arterial plaques. In the early 1960's Fry found that the responses of vascular endothelial cells were affected by the shear stress [12]. Other early studies looked at the possibility of shear stress being the factor that determines localization of disease development. Some of this work was done by Caro, et al. (1969, 1971), who concluded that low shear stress alters the mass transfer of lipid from the arterial wall proximal to a bifurcation and thus induces atherosclerosis [13,14]. Many groups proposed that areas with low shear stress were the prevalent locations for atherosclerotic lesions [14,15,16,17,18]. In 1985 Levesque and Nerem reported an interaction between wall shear stress and the vascular endothelial cells. Together these ideas begat a plethora of research to determine the localizing factor for the development of atherosclerosis [19]. They all support the belief that local hemodynamics plays an important role in the development and localization of atherosclerosis. The effect of shear, specifically, has been a predominant hemodynamic quantity studied to date.

Other theories have been proposed including the hypothesis that boundary layer separation may be involved in plaque development [20]. This theory was supported by the work of Nguyen & Haque in 1990 with some
experiments they did in glass tube models of abdominal aortas [21]. They concluded that flow separation with subsequent plaque aggregation play a role in atherogenesis. Turbulence and high shear stress have also been proposed as hemodynamic factors that may be of importance in the localization of disease [22,23,24]. It has been proposed that high shear stress is not involved in experimental plaque localization by Glagov, et al., 1983 [25].

Low shear stress along with oscillatory shear has been proposed as an atherogenic localizing factor by a number of researchers [16,18]. Oscillatory pressure effects were suggested as an important factor in the transport of material across the artery wall by Nerem and Cornhill in 1980 [9]. Work by Moore, et al. in 1992 using dye injection in a glass model of an abdominal aorta concluded that low wall shear stress combined with oscillatory shear direction and high particle residence time may be related to preferential plaque localization [26]. An extension of this has been theorized by studying wall shear stress gradients. Wells, et al., 1996 proposed that large sustained wall shear stress gradients are better indicators of sites that are susceptible to arterial disease [27].

Quantitative measurements of the wall shear stress in models of the human arterial system have been associated with histologic markers of early lesion development. Arteries studied in this way include the aortic bifurcation (Friedman et al., 1981), the carotid bifurcation (Ku et al., 1985), the left coronary branch (Friedman et al., 1987), and the infrarenal abdominal aorta (Moore et al., 1993) [16,18,32,28].
Endothelial cells are known to change shape when they adapt to a change in flow. It was suggested that endothelial cell morphology and orientation around a branch vessel might be an indicator of the detailed features of blood flow [29]. Numerous other studies have shown that endothelial cells are responding to a change in shear stress.

A number of studies have looked at the correlation of intimal thickening with wall shear. The process of early intimal proliferation is a significant step in the process of plaque formation [30]. It is assumed that intimal thickening is a response to hemodynamics. Friedman, et al., in 1981 found a negative correlation between intimal thickness and wall shear rate [16]. In addition, a strong negative correlation was found between intimal thickness and pulse shear rate, the difference between the maximum and minimum shear rates [16]. The rate of intimal thickening was studied using both young and old human aortic bifurcations. It was theorized that the intimal thickness at sites exposed to high shear stresses increases quickly to a modest value, with a slower subsequent rate. Whereas the thickness at sites exposed to low shears rises more slowly, but with time reaches a higher value [31]. Therefore, the intima of younger vessels will be thicker where shear is high, and the intima of older vessels will be thicker where shear is low. A study done on a human left coronary artery showed a negative correlation between time-average or maximum instantaneous wall shear rate and intimal thickness [32]. This result concurs with the previously mentioned data from human aortic bifurcations. Masawa, et al., looked at intimal thickening in the human carotid bifurcation in 1994 [33]. A helical pattern of
maximum intimal thickening was found in both the right and left bifurcations. This pattern corresponds to a previously reported helical flow pattern in the region of the bifurcation [33].

2.3 Geometric Risk Factors

Hemodynamics has been considered to be one of the main factors in the localization and initiation of atherosclerosis. It has been observed that certain areas tend to have a high occurrence of atherosclerotic plaques. These areas are typically near branch points and the inner surface of curved segments. The human coronary artery contains both of these targeted areas, the bifurcation at the LCX and LAD, and the variable curvature, or turning angle of the daughter vessels. It is for these reasons that the left coronary artery must be studied intensively, so that one may understand the local factors that influence the process of atherosclerosis as related to these high occurrence areas.

Flows in curved segments develop secondary flows because the higher velocity flow from the center of the vessel does not follow the curved central line of the vessel because of its higher inertia. A pressure difference results between the inside and the outside of the bend and an additional flow occurs. Depending on the characteristics of the curve, a spiral flow and a flow separation may occur. A similar situation occurs in a bifurcation, which may also give rise to secondary flows, spiral flows and flow separation.

Friedman has proposed the hypothesis of geometric risk factors to explain some of the variability in localization of atherosclerotic plaques [34]. The idea
was first proposed as “architectural risk factors” by Friedman, et al. in 1975 [35]. Velican and Velican did a study in 1981 of 566 human coronary arteries [36]. They reported that a common type of coronary artery distribution was found in 58% of the cases. Of the remaining cases, minor deviations were noted that seemed to correlate with a thicker intima, a more rapid onset and evolution of atherosclerotic plaques and a larger degree of luminal obstruction. The minor deviations noted include a large increase in external diameter and length of the left coronary arterial system coexisting with an underdeveloped right coronary tree, or vice versa [36]. This was an initial hint at the existence of geometric parameters affecting the onset and extent of disease. This implies that vessel geometry could be used to predict vessels at greater risk of atherosclerotic lesions.

Detailed studies of the aortic bifurcation have identified variation in the branch angle and an offset flow divider tip as potential geometric risk factors [37]. A flow divider that is offset from the aortic axis strengthens secondary flows due to the asymmetry in the flow field. More recently the variation in the planarity of the human aortic bifurcation has been studied. Friedman and Ding (1998) have studied 20 human aortic bifurcations from axial MRI images [38]. It was hypothesized that this variability is related to the variable predispositions to atherosclerosis in the aortic bifurcation.

Femoral artery models have been studied by Cho, et al. (1985) [39] using various arterial branch models that were straight, round, smooth and rigid. The variables studied include the effects of the branch to main lumen flow rate ratio,
upstream Reynolds number, and branch angle. It was determined that the branch flow rate ratio is an important parameter that determines the pressure and flow field variation in the branch region. In addition, the pressure rise in the main lumen due to flow through the branch decreased as the Reynolds number decreased. Very complex flows were seen using dye visualization, including flow separation and reversed flow.

Perktold and Resch (1990) [40] studied the human carotid artery bifurcation using numerical methods. Physiologic geometries and pulsatile flow conditions were used. A larger sinus width was found to increase reversed flow zones and flow recirculation in the sinus accompanied by local low wall shear stress. The low wall shear stress regions in the outer sinus wall are the areas where early atherosclerotic lesions and plaques predominantly occur. Bifurcation angles in the human carotid bifurcation have also been studied [41]. Pulsatile flow was used as well as non-Newtonian fluid characteristics in a numerical model. They found that the large bifurcation angle case had low wall shear stress over the major part of the pulse cycle in the outer sinus region, while the small angle bifurcation model had flow separation only during the systolic phase. A CFD model was used to study the effect of three carotid geometries [27]. A normal carotid geometry, a patch-reconstructed carotid, and a gradually tapered, low-angle carotid bifurcation were used with both Newtonian and non-Newtonian incompressible fluid properties. They found that wall shear stress gradients were reduced in carotid artery bifurcations that are smooth and gradually tapered. The
abrupt change associated with the patch-reconstructed carotid caused disturbed flow and high wall shear stress gradients.

Various researchers have also studied the coronary arteries. Brinkman, et al. (1994) [42] used a computer-based system to objectively measure geometric parameters from pairs of projection angiograms of human left coronary arteries. They found the angle at the origin of the second diagonal branch was positively correlated with the distance between the ostia of the first two diagonal vessels. An inverse correlation between LM branch angle and proximal localization of sudanophilia in the daughter vessels at the branch was shown via measurements of the geometry and sudanophilia from a collection of human left coronary arteries [43]. The correlation was stronger for the LCX than the LAD. This result suggests that a small LM branch angle may be a risk factor for sudanophilia in the proximal daughter arteries.

Further left coronary studies were conducted using geometric data from multiplane angiograms of 15 lesion-free human hearts and morphometric data obtained from transverse histologic sections of the tissue [44]. They found that total intimal and medial area are negatively correlated with the distance from the transverse histologic section site to the origin of the LAD; the angle of the immediately proximal branch to the site is positively correlated with most of the intimal and medial variables; the area ratio of the immediately proximal branch is correlated primarily with medial variables; and the local curvature is correlated only with the maximum thickness of the intima and media. These results suggest
that large angles are associated with intimal thickening and are believed to be adverse.

Low shear may be associated with atherogenesis since large branch angle is associated with a low shear zone near the outer wall in the proximal portion of the daughter vessel [45]. Greater sudanophilia staining in the proximal portion of the LAD is positively correlated with the LAD-LCX branch angle. This conclusion was supported by further work examining the relationship between the geometry of branch points on the LAD and the morphometry of the proximal portions of the daughter vessels by Friedman and Ding (1998) [46]. They found the intima and media are more asymmetric in daughter vessels that depart the LAD at greater angles. In addition, the asymmetry is created by greater thickening in the thickest quadrant. This suggests that large branch angles may favor eccentric intimal thickening, which may be predisposed to lipid accumulation and eventual atherosclerotic disease.

Pathological studies have been conducted on tissue samples. This type of study can give data as to where the disease occurs naturally in actual human coronary arteries. Types of data studied include vessel length, patterns of atherosclerotic lesions, vessel branch angle, and intimal thickening.

The length of the LM has been looked at from both angiographic studies and morphometric studies. Gazetopoulos, et al. found in 1976 [47] the length of the LM coronary artery to be significantly shorter in patients with coronary atherosclerosis. The overall mean length of the LM, found from a 3-dimensional analysis of both the right and left anterior oblique projections, was found to be
15.0 mm. The arteries with no disease were found to have a LM that was 16.8 ± 4.13 mm, while coronary arteries with disease in the LAD and sometimes also the LCX were found to be 10.28 ± 2.57 mm long. They also found similar results by studying the pathology of 204 left coronary arteries. The degree of disease in the LAD and LCX was highly correlated with the inverse of the length of the LM [48]. This finding is supported by Poiseuille's equation, which states that the flow rate is proportional to both the pressure drop and the fourth power of the radius and inversely proportional to the length. The longer LM may decrease the atherogenic effect of mechanical forces by decreasing the pressure and flow rate and dampening the pulsatile pressure and flow [48]. These results have also been confirmed by Saltissi, et al. [49].

A morphometric study was conducted on 23,207 sets of coronary arteries from autopsied persons 10 to 69 years of age [4]. They found, from a topographic analysis of lesions that the anatomical distribution of coronary atherosclerotic lesions do not vary appreciably with geographic region, race, sex or age. In detail, they found that the second centimeter in the LAD saw a sharp increase in frequency of lesions when compared to the first, with a rapid decrease in frequency to the seventh or eighth centimeter. The LCX was most frequently affected in the first centimeter, with the second centimeter almost as frequent. A rapid decrease follows for the next four or five centimeters. Another study by Endoh, et al. [50], was based on 50 arteriographs of patients, which had less than 50% stenosis. Biplane 35 mm cine angiographs were used to evaluate the degree of progression of atherosclerotic disease in these cases over the period of
Feb. 1984 to Sept. 1985. They reported that the proximal portions of the LM were almost free of disease. Upstream of bifurcations the lesions were distributed uniformly. Downstream of the bifurcations the lesions were proximal and quickly disappeared distally. In addition, the lesions were eccentrically located at the lateral walls of the entrance to smaller branches [50].

Grottum, et al. found in 1983 [6] that atherosclerotic lesions occur with high frequency on outer walls of the bifurcation and on the inner curvature downstream of bifurcations. They also noted that lesions were more frequently found on the myocardial surface, or inner curvature, than the epicardial surface. Similar results have been found by Svidland (1983) [51].

Young human coronary arteries have also been studied to determine the distribution of early fatty lesions. In a study conducted by Fox, et al. in 1982 [52], it was found that in all vessels, plaques were concentrated proximally. They also concluded that the wall close to the flow-divider was nearly free of lesions in the LAD and the LCX. They also found an anticlockwise spiral pattern of lesion distribution in the LAD.

Another geometric risk factor that has been considered is the bifurcation angle between the LAD and LCX in the left coronary artery. The angle has a wide variation among individuals. The angle typically ranges between 50° and 95°, with the mean being 72° [8]. How this wide variation in the bifurcation angles affects the distribution of the wall shear stress in the left coronary artery is of great physiological significance. One study looked at 35 mm biplane cine angiographs to analyze the effect of the angle at the LAD – first diagonal branch.
In this study they divided the bifurcation angle into two angles as projections onto the x-z and x-y planes. They found that when the angles were both under 30°, early sclerotic lesions were not found around the bifurcation [50]. However, when the angles were greater than 30°, early sclerotic lesions were evenly distributed around the bifurcation. A study by Saltissi, et al. reported findings from 149 angiograms that indicated patients with proximal disease tended to have a wider LAD – LCX bifurcation angle, 85°, than the group with predominantly distal disease, 76° [49]. Saltissi also noted that the combination of a short left main coronary artery with a wide bifurcation angle was associated with a higher incidence of proximal disease. The effect was greater than that of a short left main coronary artery alone.

2.4 Previous Coronary Artery Studies

Few studies have attempted to attain the near wall velocity measurements necessary to obtain near-wall shear stress in human left coronary arteries due to the small diameter and restrictions on measurement techniques. Additionally, invasive methods will alter the flow field in the small left coronary artery.

A summary of previous studies in the left coronary artery is provided in Table 2.1 [32, 53-73]. Mark, et al. [62] used an idealized, rigid, life-size model of a human left coronary artery to study the nonquasi-steady character of pulsatile flow. They determined that a series of steady flow experiments could not be used to simulate the quasi-steady flow found in human coronary arteries.
However, time average shear rates were able to be obtained using the mean flow rate and a steady flow. Another study by Friedman, et al., in 1987 [32] used a realistic geometry, rigid flow through model of a human left coronary artery bifurcation to measure wall shear. Two attempts to collect near wall velocity measurements in human left coronary models have used scaled-up models of left coronary arteries (Tang, 1990) [63] and (Tsao, et al., 1995) [69]. These models were made from actual measurements of autopsy human left coronary arteries and were idealized to create a rigid-walled model. One study used a physiological pulsatile waveform, while the other used a steady flow. Each of these studies used only one bifurcation angle. Kajiya measured the centerline flow velocities and velocity profiles in humans during corrective surgery using pulsed Doppler velocimetry [67]. Sabbah used coronary arteriograms to measure velocities and look at flow patterns [61]. A similar approach was used by Gibson, et al. [65] to measure coronary artery diameters from angiography. A finite-difference model of the Navier-Stokes' equation was used to calculate vessel wall shear stress. Krams, et al., 1997 [73] has used a combination of angiography and IVUS with CFD to compute endothelial shear stress.
<table>
<thead>
<tr>
<th>SOURCE</th>
<th>SPECIES</th>
<th>MEASURED OUTCOMES</th>
<th>EXPERIMENTAL CONDITIONS</th>
<th>METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atabek, et al.</td>
<td>canine</td>
<td>pressure/pressure gradient</td>
<td>physiologic</td>
<td>pressure transducers EM flow meter</td>
</tr>
<tr>
<td>(1975)</td>
<td>In vivo</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wells, et al.</td>
<td>equine</td>
<td>velocity profiles</td>
<td>physiologic</td>
<td>PUDVM</td>
</tr>
<tr>
<td>(1977)</td>
<td>In vivo</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sabbah, et al.</td>
<td>porcine</td>
<td>flow patterns</td>
<td>pulsatible flow</td>
<td>dye injection</td>
</tr>
<tr>
<td>(1984)</td>
<td>In vitro</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Altobelli, et al.</td>
<td>baboon</td>
<td>velocity profiles</td>
<td>pulsatible flow/In vitro</td>
<td>PUDVM, Pulsed ultrasonic Doppler veloc.</td>
</tr>
<tr>
<td>(1985)</td>
<td>canine</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kajiya, et al.</td>
<td>canine</td>
<td>velocity profiles</td>
<td>physiologic</td>
<td>high resolution LDV, OFLDV</td>
</tr>
<tr>
<td>(1985)</td>
<td>In vivo</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rabinivitz</td>
<td>canine</td>
<td>wall shear stress</td>
<td>pulsatible flow</td>
<td>pulsed Doppler ultrasound</td>
</tr>
<tr>
<td>(1986)</td>
<td>In vivo</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strongy, et al.</td>
<td>canine</td>
<td>velocity, pressure</td>
<td>physiologic</td>
<td>PUDVM FEM(FIDAP)</td>
</tr>
<tr>
<td>(1993)</td>
<td>In vivo</td>
<td>FEM-shear stress</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peacock, et al.</td>
<td>tube</td>
<td>velocity profiles</td>
<td>oscillatory sine flow wave</td>
<td>ultrasound flow hot film probe</td>
</tr>
<tr>
<td>(1997)</td>
<td>model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sabbah, et al.</td>
<td>human</td>
<td>flow patterns</td>
<td>physiologic</td>
<td>coronary arteriograms</td>
</tr>
<tr>
<td>(1984)</td>
<td>In vivo</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mark, et al.</td>
<td>human</td>
<td>velocity profiles</td>
<td>pulsatible flow</td>
<td>LDA</td>
</tr>
<tr>
<td>(1985)</td>
<td>model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Friedman et al.</td>
<td>human</td>
<td>wall shear</td>
<td>pulsatible flow</td>
<td>LDA</td>
</tr>
<tr>
<td>(1987)</td>
<td>model</td>
<td></td>
<td>realistic, rigid</td>
<td></td>
</tr>
<tr>
<td>Tang (1990)</td>
<td>human</td>
<td>velocity profile</td>
<td>pulsatible flow</td>
<td>Laser Doppler Anemometer (LDA)</td>
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<tr>
<td></td>
<td>model</td>
<td>wall shear</td>
<td>rigid model</td>
<td></td>
</tr>
<tr>
<td>Perktold, et al.</td>
<td>human</td>
<td>velocity profile</td>
<td>idealized geometry, rigid wall, pulsatible flow</td>
<td>Computational Fluid dynamics</td>
</tr>
<tr>
<td>(1991)</td>
<td>model</td>
<td>wall shear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gibson, et al.</td>
<td>human</td>
<td>vessel diameter, calc shear stress</td>
<td>physiologic</td>
<td>quantitative angiography</td>
</tr>
<tr>
<td>(1992)</td>
<td>In vivo</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Feldman, et al.</td>
<td>human</td>
<td>pressures, separation length</td>
<td>steady flow</td>
<td>computational fluid dynamics</td>
</tr>
<tr>
<td>(1993)</td>
<td>model</td>
<td></td>
<td>rigid model</td>
<td></td>
</tr>
<tr>
<td>Kajiya, et al.</td>
<td>Human</td>
<td>velocity profiles</td>
<td>physiologic</td>
<td>pulsed doppler velocimeter</td>
</tr>
<tr>
<td>(1993)</td>
<td>In vivo</td>
<td>centerline flow</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1 Literature-based Review of Previous Left Coronary Studies
<table>
<thead>
<tr>
<th>Study</th>
<th>Model Type</th>
<th>Measurement</th>
<th>Flow Type</th>
<th>Simulation Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>He (1994)</td>
<td>Human</td>
<td>Wall shear stress</td>
<td>Range 0-90</td>
<td>Computational fluid dynamics</td>
</tr>
<tr>
<td>Tsao et al. (1995)</td>
<td>Human</td>
<td>Particle residence time total wall shear</td>
<td>Steady flow</td>
<td>Particle tracking velocimetry &amp; LDA</td>
</tr>
<tr>
<td>Perktold et al. (1995)</td>
<td>Human</td>
<td>Velocity profile wall shear stress</td>
<td>Pulsatile flow</td>
<td>Computational fluid dynamics</td>
</tr>
<tr>
<td>Nosovitsky et al. (1995)</td>
<td>Human</td>
<td>Wall shear stress velocity profile</td>
<td>Pulsatile flow, realistic model</td>
<td>Computational fluid dynamics</td>
</tr>
<tr>
<td>He et al. (1996)</td>
<td>Human</td>
<td>Velocity profiles shear stress</td>
<td>Pulsatile flow, realistic model</td>
<td>Computational Fluid dynamics</td>
</tr>
<tr>
<td>Krams et al. (1997)</td>
<td>Human In vivo</td>
<td>Shear stress</td>
<td>Physiologic</td>
<td>IVUS (ANGUS) &amp; CFD</td>
</tr>
</tbody>
</table>

Many studies have used either realistic human geometries or idealized geometries and computational fluid dynamics to model coronary flow [66,70-73]. In addition, many studies have been performed in vivo with various animals [53-60].

There are several techniques available to measure or model detailed fluid dynamic parameters in arteries or artery models. Point velocity techniques include the hot film anemometer, the laser Doppler or LDV, and the ultrasonic Doppler anemometer. Hot film anemometry has a number of drawbacks including being invasive, which interrupts the flow field in a small vessel such as the coronary artery. In addition, a single hot film cannot determine flow direction. This is a major limitation when considering a physiologically realistic pulsatile
waveform for the coronary artery since there is a short period when the flow is reversed.

Both the laser Doppler and ultrasonic Doppler techniques determine the velocity of particles moving with the fluid by a change in frequency, i.e., the Doppler shift, of the scattered light or sound due to the movement of the particles when observed by a stationary detector. Both techniques are non-invasive, and thus do not disturb the flow pattern. Laser Doppler offers a higher spatial and temporal resolution than ultrasonic Doppler. Another limitation of both techniques is that only two velocity components are collected at one point in space at a time. This requires multiple measurements over a spatial grid to encompass the velocity field over an area.

2.5 Computational Fluid Dynamics

Computational Fluid Dynamics (CFD) can be a very powerful tool in ascertaining fluid properties in flow fields. However, the method may have a number of assumptions including, but not limited to:

- must make assumptions and simplifications in creation of geometry modeled
- assumptions must be made in the Navier-Stokes equations to solve
- inlet waveform typically not realistic
- model walls are typically assumed to be smooth rather than the irregular walls in real arteries
- does not always consider compliant walls
- does not always consider motion of heart
assumes no-slip boundary condition at the wall.

The great strength of CFD lies in its ability to yield velocities at all points in the artery and wall shear rates with high accuracy at all surfaces. Typically, CFD models are validated by data from experimental flow models. A validated model can be used to study a multitude of experimental conditions rather quickly and efficiently.

As in any model, CFD models must make assumptions and simplifications in creation of the geometry and the numerical solution. Computational fluid dynamics (CFD) has emerged as a powerful alternative tool to study the hemodynamics of arterial branches. But, it must be remembered that it is only a tool, typically with idealized model geometries and assumptions made in the governing hemodynamic equations. They are a valuable resource but must be used with caution. Validation with actual experimental data should be performed.

2.6 Particle Tracking Velocimetry

In order to study the hypothesis that there is a relationship between the near wall shear and the bifurcation angle between the left anterior descending and left circumflex we propose to use a non-invasive technique called Particle Tracking Velocimetry (PTV). The power in this technique, in addition to being non-invasive, is that the entire 3-dimensional velocity vector of the flow field can be obtained throughout the pulsatile cycle. The proposed PTV algorithms can obtain up to 400 velocity vectors per image, including vectors near the wall.
One previous study, (Tsao, et al., 1995) [69], used PTV in a scaled up model of a human left coronary artery. The technique was used to measure "particle residence time". This parameter was then related to mean wall shear stress, which was obtained by laser Doppler anemometry in the same model. Three major deficiencies in this study include using a steady flow, a rigid walled cast, and not obtaining near wall velocity vectors. In addition, the left coronary model was made up from idealized average data collected from autopsy hearts.
CHAPTER 3
3-DIMENSIONAL PARTICLE TRACKING VELOCIMETRY

3.1 Introduction

3-Dimensional particle tracking velocimetry (PTV) is used in the current study because of its ability to resolve instantaneous 3-dimensional velocity vectors without disturbing the flow field. Stereo images are used to collect 3-dimensional particle positions for the entire field of view simultaneously. These images may be viewed directly in order to ascertain general flow patterns as in various other flow visualization techniques. However, the power of this technique is that the full field 3-dimensional velocity vector field may be reconstructed. In addition, the technique can easily resolve periodic flows. Near-wall shear rates are determined from the measured velocity vectors. The velocity vectors are found by tracking neutrally buoyant particles with diameters on the same order of magnitude as the smallest scales of interest of motions of the flow.

3.2 PTV Algorithms and Processing Steps Overview

The basic principle behind PTV involves the measurement of the position of particle markers that follow the fluid motion. The frame-by-frame position data and the time of each frame may be used to estimate the velocity using its basic definition, \( U = \frac{L}{T} \), where \( U \) is the velocity, \( L \) is the distance moved and \( T \) is the time. Figure 3.1 shows a flow chart of the steps involved in the PTV process.
Images from two different views of the region of interest are obtained by a CCD camera and recorded onto S-VHS videotapes. These views are obtained using a series of mirrors; in this way the two views are recorded simultaneously on each frame of the S-VHS videotape. The acquired images are then digitized. A background image is subtracted off in order to leave only the particles. The digitized information is the input to the PTV algorithm which finds the two dimensional location of particles in each individual view.

The next step in Figure 3.1 after particle location is track identification. Tracks consist of the most likely path, position versus time, a single particle follows in five consecutive frames. Tracks are made for each particle separately for each of the two views. Two-dimensional tracking is performed independently for each view. Details will be discussed in section 3.6. The tracks are then stereo matched using the calibration data; see section 3.3, to define the 3-D location. A stereo match is found using the calibration equations from the multiple linear regression models. Once the 3-dimensional particle locations are known, the velocities may then be determined. These velocities are validated using an adaptive gaussian window (AGW). This gaussian window assigns weights to the vectors based on their proximity to the central vector being analyzed. Any vectors that have a magnitude or direction that varies by more than 20% of the calculated gaussian average of its nearest neighbors is rejected.

Dr. Yann G. Guezenneec and colleagues developed the basic algorithm at The Ohio State University. All of these steps will be discussed in more detail in the following sections.
Figure 3.1 Flow Chart of PTV Algorithm
3.3 PTV Software and Calibration

The PTV software is a set of three C libraries that contain user-callable functions. The user is given control over the experimental set-up, which includes illumination, image resolution, framing rate, particle size and density, magnification and field of view. Some of these variables are fixed by the inherent nature of the flow field being studied.

The calibration used in this system consisted of imaging a set of scales fixed on the front and back of the measurement volume. The exact 3-dimensional location of these scales was measured using a laser Doppler velocimetry system. The laser light combined with its precise motor driven axis motion controller was used to obtain the very accurate 3-D locations required by the calibration. This calibration, also called camera calibration, accounts for optical aberrations from the lenses and mirrors used to acquire the stereo images, camera misalignment and any differences in the index of refraction between the solid flow model and the fluid. The result is a relationship between the 3-D position of the scales in real world coordinates and their position in the image coordinates in each view.

This calibration is accomplished by establishing a data base of the real world coordinates of the calibration points as measured by the laser \((x, y, z)\), and their image coordinates in both views \((x_r, y_r)\) and \((x_l, y_l)\). An example image of the right and left views of the calibration scales is shown in Figure 3.2. Chords are drawn between eight of the calibration points from the surface of the model to
the back plane of the model. The ruler in the center of each view is in the back plane, while the scales around the edges are in the front plane of the model. Four chords are generated in each view, with the end points of the four chords being the identical points in each view. Each of these chords is sub-divided into 25 sections to make a total of 100 points. A multiple linear regression is applied to these points in \((x_r, y_r)\) and \((x_i, y_i)\) to find the best relationship between them and \((x, y, z)\) in real world coordinates. The test was performed using GraphPad InStat version 3.00 for Windows 95, GraphPad Software Inc., San Diego, California, USA, www.graphpad.com. Three equations are found, one each for \((x, y, z)\) in real world coordinates for each individual experiment. The InStat results, including \(R^2\) values are included in Appendix A.

Figure 3.2 Calibration Image Showing Calibration Scales
These equations are later used during stereo matching to define the 3-D locations of particle markers. These images are taken just prior to actual
experimental images of particles being taken. The illumination on the front scales was removed and the rear scale was removed from view during the actual experiment. This procedure is performed on each individual experiment.

3.4 Image Pre-Processing

The first step in image pre-processing is actually completed prior to image acquisition. Preliminary processing is used to enhance the images and improve the contrast between the particles and the background. This is accomplished by analyzing the first few frames of an experiment to set the gain, offset, and upper and lower threshold values. In doing so, the parameters to “stretch” all subsequent images grabbed from this experiment are fixed. Contrast stretching is an image processing operation in which a range of pixel intensities is changed in a particular way. All pixels with an intensity less than the lower threshold value are set to zero (black). All pixels with an intensity above the upper threshold are set to white (255). The remainder of the pixels, with intensities within the two thresholds, retain their original values. The threshold values are set based on a histogram of pixel intensity values. An empirical evaluation of histogram maximum and minimum values is made by looking at the location of the peak corresponding to the particles within the histogram. Essentially a “band pass” filter is applied, allowing only the pixels corresponding to the majority of the particles in the image to pass. In this manner unwanted image artifacts, such as reflections or lighting problems, are removed from the images.
3.5 Image Acquisition

The experimental images are recorded using a Sony computer controlled video recorder. These images are recorded onto S-VHS videotapes. The stereo images are then digitized using the Dipix P360-F-4MB frame grabber board. This board has the circuitry to convert the analog signal coming from the videotape recorder into a digital representation that ranges between 0 to 255. The board has on-board software and storage capabilities that allow basic image operations to be performed and up to 16 images to be stored in its memory. The PTV software used allows 64 consecutive images to be grabbed and averaged as a background image. This background image is saved in the board memory and subtracted from every subsequent image grabbed. This minimizes the amount of storage necessary by saving only the particle locations without the background information for each frame. Figure 3.3 shows the process of image acquisition in progress. The monitor on the left side shows the image as grabbed from the videotape while the monitor on the right side shows the image after the background is subtracted.
3.6 Creation of 2-D Tracks

The creation of 2-D tracks in each of the right and left views is accomplished using the concept of path coherence. This concept is based on the assumption that the position, velocity and acceleration of fluid elements can be described by well-behaved functions of time. Therefore, the displacement of particles in a given frame is predicted using the velocity and acceleration of the particle calculated from previous frames. The size of the search area used from frame to frame is determined by a user-supplied variable, the maximum distance a particle can move in two consecutive frames. The maximum distance a particle can move is used to create a circle, within which all possible particle
locations for the next frame are included. Each one of these possible particle locations creates a new link within the tree structure. Nearly any five-frame particle path is possible until some limitations on the search are employed. The search of possible particle locations grows in a tree structure with one branch for every possible particle displacement. For a given particle, a multi-level coarse-to-fine tree of possibilities is created with no bias in terms of the direction or displacement except for the search radius. The tree grows geometrically as the number of successive frames increases. Each of the branches of the tree is evaluated in terms of its path coherence. The practical application of the path coherence particle tracking involves computing a penalty function for each branch of the search tree and selecting the branch with the lowest penalty. This does not guarantee that the branch chosen is the correct one, but rather is the most likely one based on the information available. Figure 3.4 [74] shows a schematic diagram of using the penalty function to choose the most probable track. The penalty function used compares the predicted and real position of particles. A predictor function is used to predict the possible locations in the next frame from the position in the current frame, using the velocity and the acceleration of the tracked particle. The penalty function is defined as:

\[ \text{PenaltyFunction} = \sum \alpha_i \left( \Delta x_i^2 + \Delta y_i^2 \right)^2 \]

where
\[ \alpha = \text{a constant with values of } 0.5, 1.0, 1.5 \]
for frames 3, 4 & 5 subsequently
\[ \Delta x_i = \text{difference between predicted and actual x position} \]
\[ \Delta y_i = \text{difference between predicted and actual y position} \]
In Figure 3.4 the solid dark circles represent the actual particle location while the
crosshatched circles represent the predicted particle path in frames 3, 4, and 5.
The track that minimizes the penalty function is the chosen track. In this
software, a sequence of 5 frames is used to produce a track. Five frames are
used as an optimum value between computing time and minimizing incorrect
tracks.

Figure 3.4 Determination of Most Probable Track using Penalty Function
3.7 3-D Stereo Matching

3-D stereo matching is performed by one of the callable functions in the PTV library. Tracks, rather than particles, are matched in order to reduce the error as much as possible. The image acquisition functionality produces a list of five-frame particle tracks for each of the left and right views. A 3-D stereo matching algorithm is used to find corresponding tracks between the left and right views. Stereo matching selects a track from the left view and attempts to find the most closely corresponding track in the right view. The outcome is either the selection of a corresponding track, or the rejection of the pair as a set of corresponding tracks.

The calibration phase generates a series of three equations that expresses the 3-D location reconstruction as a function of:

1. \( f(x_{\text{left}}, y_{\text{left}}, \delta_{\text{x}}, \delta_{\text{y}}) \).
2. \( f(x_{\text{left}}, y_{\text{left}}) \).
3. \( f(x_{\text{right}}, y_{\text{right}}) \).

Where:

- \( x_{\text{left}} \) is the x coordinate of the particle location in the left view;
- \( y_{\text{left}} \) is the y coordinate of the particle location in the left view;
- \( x_{\text{right}} \) is the x coordinate of the particle location in the right view;
- \( y_{\text{right}} \) is the y coordinate of the particle location in the left view;
- \( \delta_{\text{x}} \) is the difference between \( x_{\text{left}} \) and \( x_{\text{right}} \);
- \( \delta_{\text{y}} \) is the difference between \( y_{\text{left}} \) and \( y_{\text{right}} \).
The first equation produces the most accurate reconstruction of a 3-D location in space as indicated by the $R^2$ values shown in Appendix A. Equations 2 and 3 provide a redundancy that is employed to match points in each view [42].

The list of tracks from the left view is iterated against the list of tracks from the right view. A left view track is selected, and compared, point-by-point, against each track in the list of right views until a stereo matching track is selected; or the left track cannot be matched by any right track. In the latter case, the left track is discarded as unmatchable.

The individual points in each track are computed to their corresponding 3-D spatial locations. A comparison is made by applying all three equations, and computing the differences between equation 1 and 2, and equation 1 and 3. This difference becomes a measure of location error for an individual point within the track. The sum of the errors for each point in the track is computed and evaluated at each point. This provides an “early-out” condition enabling the software to process all of the tracks more efficiently.

If all five particle positions are evaluated, and the total error sum is less that a penalty limit, the tracks are considered to be matching, and both tracks are removed from their left/right track list. Non-matching tracks are eliminated in this fashion; these tracks will demonstrate a large and growing error value as the track is compared.

At the completion of the track matching, only the tracks that a matching track within the penalty limit has been met will remain in 3-D space. These
tracks are used to compute the velocity vector of the particle in the middle frame. The velocity vector is computed by performing a five-point, forward numerical differentiation.

### 3.8 Adaptive Gaussian Window Validation

Another callable function in the PTV library performs a velocity validation on the velocity field generated. It begins by first smoothing the velocity field using an interpolation scheme based on a Gaussian window. In this Gaussian scheme, the nearest neighbors are given a greater weight than neighbors farther away. The actual vector being evaluated is given the highest weight. The weighted average of the neighbors within the search window, including the vector being tested, creates an average magnitude and direction for the vector comparison. The raw velocity values of each particle are compared with the smoothed field in its immediate vicinity and rejected if a significant difference is found in either direction or magnitude. The remaining particles retain their original measured velocity values. Figure 3.5 [74] shows this process step by step.
Figure 3.5 Validation of the measured velocity field.
CHAPTER 4

EXPERIMENTAL METHODS

4.1 Experimental Approach

This study uses a realistic, anatomically correct model. In addition to using the non-invasive technique, PTV, we use a physiologically-realistic pulsatile waveform. Luminal casts of human left coronary artery trees, obtained from autopsy hearts, are used. The subjects all died from non-cardiovascular related causes. Each of the two casts chosen, TB33 and TB41, have outline files generated, from which scaled up models of the actual arteries are fabricated. A full description of this process will be discussed later. These models, made of compliant silicone rubber, Dow Corning Sylgard184 Silicone elastomer, are perfused with index of refraction matching fluid. Two important dimensionless numbers in a pulsatile flow system, the Reynolds and Womersley numbers, are held constant while all other fluid properties are scaled using Reynolds Modeling [7]. Reynolds number, $N_R$, is defined as

$$N_R = \frac{\rho D V}{\mu} \sim \text{inertial forces/ viscous forces}$$

where $D$ is the vessel diameter, $V$ is the mean velocity, $\rho$ is the fluid density, and $\mu$ is the fluid viscosity. The Womersley number is defined as
\[
\alpha = R (\omega / \nu)^2 \sim \frac{\text{local inertial forces}}{\text{viscous forces}}
\]

where \( R \) is the vessel radius, \( \omega = 2 \pi f \), is the angular frequency in radians per second, and \( \nu \) is the kinematic viscosity defined as \( \nu = \mu / \rho \). Reynolds modeling is defined as dynamic similarity. For this to hold true, the ratio of inertial force to frictional force; and the ratio of inertial force to frictional force must be maintained [75]. The Womersley number indicates to what extent the velocity profile in laminar developed flow differs from the Poiseuille profile when the fluid is subjected to a sinusoidally varying pressure gradient. A Womersley parameter on the order of one indicates quasi-steady flow and the velocity profile is parabolic at all times. For higher values of \( \alpha \), the instantaneous velocity profile is distorted and for \( \alpha > 10 \), the effect of the viscosity of the fluid is confined to a thin viscous layer that oscillates out of phase with the pressure gradient.

### 4.2 Modeling

In-vitro models are commonly used to study the detailed fluid dynamics of various vasculature components. In order for a model to simulate vascular function in a meaningful way, several physiological parameters need to be modeled. One of the more important parameters is the geometry. It is for this reason that we chose to use a multiple step technique, using state-of-the-art equipment and methods, in order to achieve the most anatomically correct scaled-up model of a portion of the human left coronary artery. Unlike other studies [63,66,69] that have used average or idealized dimensions, this study will
use models that are life-like, or as close to actual anatomic geometry as is feasible with today's technology.

The next physiological parameters that must be considered are the inflow/outflow boundary conditions. It has been shown experimentally, in realistically compliant flow-through models of the human aortic bifurcation, that increasing the frequency reduces the oscillatory component of shear rate at sites where it was greater than average, and increases it at sites where it was less [76].

Another factor, which merits review, is the effect of wall compliance. Previous studies, [77], have shown experimentally in anatomically realistic models that the effect of vessel compliance is to increase wall shear where the shear rates are high, such as the flow divider wall, and decrease wall shear where the shear rate is low, such as the lateral walls. In order to obtain realistically compliant models, these experiments used materials with a modulus of elasticity within the physiologically occurring human range, and a thickness to approximate the compliance of the scaled vessel.

It is very difficult to obtain near-wall velocity vectors in a life-size replica of the human left coronary artery due to its small size. Its diameter is 0.35 cm on average. Direct measurement of local hemodynamic phenomena is currently difficult because any in vivo probe used to measure local velocities would distort the very flow pattern it attempted to measure. Modeling techniques are used to study the local flow phenomena in arteries because of these distortion problems and because of the sensitivity of using human subjects. Canine and porcine
subjects have been used [55,58], however, using human models or subjects would be preferred. Limitations of current measurement techniques make it necessary to scale-up the model in order to obtain the most accurate measurement of local flow parameters as possible.

It is desirable to keep all dimensionless numbers that define the specified flow constant between the model and the full-scale system. In particular, the following properties must be kept similar:

Geometric similarity- all dimensions are proportional to dimensions of larger unit by the same ratio

Kinematic similarity, or time proportionality – velocity gradients same at every dimensionless location in both systems

Dynamic similarity, or Reynolds modeling- consider viscous forces and inertial forces.

In the current study, the most appropriate scale-up factor, taking all of the above constraints into consideration, and the physical limitations of fluids and mechanical devices to fulfill the requirements is a factor of four. This factor will allow for optimum visualization of near-wall velocities in order to obtain an accurate estimate of near-wall shear. For full calculations please refer to Appendix B.

4.3 Fluid

Fluid and matching particles were prepared to meet the experimental requirements. The vessels were scaled to 4 times life size. The important
dimensionless numbers that were scaled using Reynolds modeling were mean Reynolds number, $N_R$, of 90 with a peak $N_R$ of 170, and a Womersley number of 3.1. These values are within the range of physiologically occurring values [78]. Based on the Reynolds modeling calculations, the kinematic viscosity was found to be $0.1691 \text{ cm}^2/\text{sec}$. For more detailed calculations refer to Appendix C. These parameters were matched using a mixture of equal amounts of both a 14% solution of glycerol in water and a 10% solution of propylene glycol in water. Sodium thiocyanate was added until the index of refraction of the fluid was matched to that of the Dow Corning Sylgard, 1.4135. The period of the flow wave is 3 seconds, which is used in the calculation of Womersley's number.

4.4 Particles

The particles were made to match the density of the working fluid using a mixture of plyolite doped with magnesium oxide for weight and Pylakrome Yellow LX-8248 dye to increase particle visibility. First, a physical mixture of powdered plyolite and magnesium oxide is obtained by grinding the two together in a mortar and pestle. Then, the mixture is slowly melted over a low heat to obtain a uniform solid mass. Based on the density of the working fluid, the settling velocity of the particles in the field of view, and the scales of motion of the region of interest, the correct diameter for the tracking particles was determined to be 250 microns. For detailed calculations refer to Appendix D. The particles were then ground and screened, using a standard 250-micron screen or Tyler equivalent 60-mesh screen. The actual particle size ranged from 250 to 296
microns based on the combination of screens used to separate the particles. The particles are imaged in Figure 4.1. Test photographs of the particles were taken in the experimental set-up to ensure that the images of the chosen particles would be 3-5 pixels in size. A requirement of the PTV technique is that the particle movement is a maximum of 10% of the field of view per frame. The maximum velocity allowed by this condition is 30 cm/sec, which is well above the experimental peak velocity of 17 cm/sec.

![Figure 4.1 Suspended Particles](image)

4.5 Realistic Flow Wave

The particles were mixed into the working fluid and loaded into the system to allow for proper programming of the physiologic flow waveform. This process requires that all air be bled from the system. The waveform is created using a Klinger CC-1.1 Universal Programmable Indexer to control a high-speed
stepper motor and linear actuator (Klinger VP70-40). The program used to create this flow waveform is presented in Appendix E. This pulsatile pump is in parallel with a steady flow pump that supplies the mean flow. Resistance is provided using two in-line valves and two adjustable clamps on the outlets. The experimental set-up is illustrated in Figure 4.2 and Figure 4.3. The waveform averaged over 5 cycles, obtained from experimental data collected by a Transonic Systems Inc. model T206 ultrasonic flow meter, is shown in Figure 4.4.

Figure 4.2 Diagram of Experimental Set-up
Figure 4.3 Photograph of Experimental Set-up
Figure 4.4 Experimental Average Flow Waveform
4.6 Creation of Realistic Replica

Experiments were completed using scaled-up models of autopsy human left coronary arteries. These arteries were obtained from a consecutive series of autopsy specimens from subjects under the age of 40 years dying from trauma, non-cardiovascular disease or suicide. The selected hearts were fixed by perfusion of the coronary arteries, via the ascending aorta, at physiological pressure for 24 hours with 10% formal-saline. The left coronary ostium was then cannulated and injected with a casting medium doped with barium sulfate (Micropaque, Nicholas, Ltd). The tissue was cut away and the remaining cast included the left main ostium at the sinus of Valsalva, the left main (LM), the anterior descending (LAD) and circumflex (LCX) branches. Details of the procedure may be found in Ding et al., 1997 [45].

The scale-up on the two chosen vessels was a multi-step process beginning with the creation of an investment of the original cast. The investment is a multi-piece negative mold of the original using Dow Corning 3110 RTV silicone rubber. To help explain the entire process, a flow chart is presented in Figure 4.5. From this investment a copy of the cast was made in white polyester casting resin doped with 10% glass micro-spheres and 5% glass rods for strength and to minimize shrinkage. This copy was embedded in red polyester casting resin along with registration marks.
Create Multi-Piece Investment of Original Vessels

Make White Polyester Copy in Investment

Embed Polyester Copy in Red Polyester Resin

Add Registration Marks in Block of Polyester Resin

Slice Through Block at .5 mm steps. Record Each Slice on Video

Create Outline Files Using Image from Each Sliced Step from Video

Create Surface Using Pro-Engineer and the Outlines

Create 4 Times Life Size Model of Original Vessels using Rapid

Make a Copy Using the Investment and Water Soluble

Coat the Wax Copy with Poly Vinyl Chloride (PVC)

Suspend Coated Copy in Water Tight Glass Box and

Remove Glass and Rinse Out Wax Using Warm Water

Scale Surface to 4 Times Life Size in Quick Slice

Create Multi-Piece Investment of 4 Times Life Size Rapid Prototype

Figure 4.5 Flowchart of the Flow Through Model Making Process
Figures 4.6 and 4.7 show various stages of this process. The registration marks were used to accurately locate the artery position when processing the video images.

Figure 4.6 Investment of Original Vessels Taken Apart

Figure 4.7 Original Vessel, Polyester Resin Copy, and Investment put Together
The embedded copy was then sliced at 1 mil steps for TB33; and 5 and 2.5 mils for TB41, using a precision lathe. This is shown in Figures 4.8 and 4.9.

Figure 4.8 Milling of Vessel Block

Each sliced step, or cross section, was recorded on S-VHS videotape. These images were digitized and analyzed to create outline files of each cross section. The analysis consisted of classifying the pixels, median smoothing, and then hand traced to obtain the outlines of each cross section. Surfaces were made from the outline files using Pro/Engineer (Parametric Technology Corporation). The surfaces were transferred to Quick Slice (Stratasys) which horizontally sliced the STL files and scaled to four times life size, creating SML files, which were
sent to a Stratasys FDM1600 Fused Deposition Modeling (RP) machine. A rapid prototype was made out of acrylonitrile butadiene styrene (ABS) using the RP machine. The RP machine used 0.01 inch slice intervals and a road width of 0.0201 inches. The achievable accuracy is +/- 0.005 inches.

Figure 4.9 Precision Lathe Milling Artery Block
The surfaces created in Pro/Engineer are shown in Figure 4.10. One of the four times life size parts created by Rapid Prototyping is shown in Figure 4.11 with the original artery cast next to it for comparison. Note the similarity between Figure 4.10 and the RP part in Figure 4.11.
Next, the larger than life size RP part was used to make a multi-piece investment using Dow Corning 3110 RTV silicone rubber from which a wax copy was molded. This investment is shown apart with the RP part still in it in Figure 4.12.

![Figure 4.12 Investment with RP part inside](image)

The wax used was Kindt-Collins Master Water Soluble Wax 1665D white. This wax copy was used in the “lost wax” technique to create a flow-through model. A Plexiglas box was used to make the investment and also to hold the investment together while the melted wax solidified. This was found to create an optimum match between investment pieces to produce the best possible wax copy. After the wax copy was removed from the investment, it was coated with polyvinyl alcohol as a barrier to prevent off gassing from the wax copy into the liquid Dow Corning Sylgard 184 silicone elastomer molding compound. The wax
copy was then suspended in a watertight glass box, and de-gassed liquid Sylgard was poured over the entire piece. The Sylgard was allowed to cure for one week before removal from the glass box. Once removed, the wax was melted out leaving the flow-through model ready for experimentation. Figure 4.13 shows the PVA coated wax copy curing in a block of Sylgard.

![Wax copy with Curing Sylgard](image)

**Figure 4.13 Wax copy with Curing Sylgard**

The flat rectangular wax piece behind the vessels and in front of the sinus of valsalva is a barrier for lighting purposes and the location of the rear scale for calibration purposes. After the wax was melted out with warm water, the model was plumbed and set-up for experimentation.
4.7 Image Acquisition

PTV begins with camera and mirror alignment. In order to obtain 3-dimensional information, data must be collected in stereo. This was accomplished using a system of first-surface mirrors. First surface mirrors have the reflective surface on the front face of the glass. The set-up of these mirrors and the camera is shown in Figure 4.14. In addition, the joint where the two mirrors join to make the 90-degree mirror is made by grinding the edge surface of each of the two mirrors to a 45-degree angle. A diagram of the 90-degree mirror detail is in Figure 4.15.

![Figure 4.14 Mirror Set-up for PTV](image-url)
The lighting was provided by three fiber optics cables positioned to provide maximum light to the field of view. Two replicas were produced, one each for TB33 and TB41. The geometric properties of these two vessels are shown in Table 4.2. The diameters in Table 4.2 were measured at 2.0 cm from the flow divider tip on the scaled-up model. Experimental conditions are summarized in Table 4.3. The calculations of velocity and the dimensionless groups are based on the inlet conditions in the LM and the diameter of the LM. The calculations for fluid density and viscosity are based on Reynolds modeling using a maximum Reynolds number of 180 and a Womersley number of 2.7. The period for the scaled flow wave is 3 seconds. The LAD-LCX angle was determined on the original cast using 3-D reconstructions derived from a pair of photographic projections. The angle is found in three dimensions as the angle between two vectors derived from the vessel axis near the branch point. The angle on the scaled-up model was verified by visual inspection.
A light emitting diode (LED) was synchronized with the pulsatile flow wave and was physically placed in a position such that it would be imaged in each video frame. The LED was timed to light late in diastole, or frame 1 in the 78-frame cycle. This visual signal was recorded in each frame and used to correlate the phase of the pulsatile cycle with the images. Multiple experiments were performed on each cast. The experiments used in ensemble averaging for TB33 were numbered TB33-2, TB33-3 and TB33-4. The experiments used for TB41 were numbered TB41-1 and TB41-8. One thousand cycles for each vessel was ensemble averaged.

<table>
<thead>
<tr>
<th>Vessel</th>
<th>TB33</th>
<th>TB41</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAD:LCX Angle(degrees)</td>
<td>98.2</td>
<td>86.0</td>
</tr>
<tr>
<td>Major axis</td>
<td>Minor axis</td>
<td>Major axis</td>
</tr>
<tr>
<td>Scaled Diam LAD (cm)</td>
<td>1.24</td>
<td>1.08</td>
</tr>
<tr>
<td>Scaled Diam LCX (cm)</td>
<td>1.46</td>
<td>1.46</td>
</tr>
<tr>
<td>Scaled Diam LM (cm)</td>
<td>2.06</td>
<td>1.54</td>
</tr>
</tbody>
</table>

Table 4.2 Geometric Parameters
<table>
<thead>
<tr>
<th>Vessel</th>
<th>TB33</th>
<th>TB41</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Flow (l/min)</td>
<td>1.36</td>
<td>1.23</td>
</tr>
<tr>
<td>Peak Flow (l/min)</td>
<td>2.43</td>
<td>2.19</td>
</tr>
<tr>
<td>Ave. Velocity (cm/sec)</td>
<td>9.16</td>
<td>8.98</td>
</tr>
<tr>
<td>Peak Velocity (cm/sec)</td>
<td>16.25</td>
<td>16.01</td>
</tr>
<tr>
<td>Mean Reynolds Number</td>
<td>97.2</td>
<td>92.6</td>
</tr>
<tr>
<td>Peak Reynolds Number</td>
<td>172.3</td>
<td>160.9</td>
</tr>
<tr>
<td>Womersley Number</td>
<td>3.2</td>
<td>3.05</td>
</tr>
<tr>
<td>Flow Partition</td>
<td>.45 LAD</td>
<td>.46 LAD</td>
</tr>
</tbody>
</table>

Table 4.3 Experimental Conditions

4.8 Image Resolution and Framing Rate

A PULNiX America Inc. (Sunnyvale, CA) TM-640 CCD camera, which operates at 30 frames per second in interlaced mode, was used. The images were collected with a shutter speed of 1/1050 and using a Canon TV Zoom Lens V6 x 16, f1.9. The image resolution of the CCD camera is 512x512. A Dipix Technologies Inc. (Ottawa, Ontario, Canada) P360-F-4MB frame grabber board was used to acquire images at 640x480 pixels but because two images, right and left, were recorded on each frame, the actual resolution of each image is 320x480 per image. However, in the interest of minimizing disk space used for storing the data, areas not containing the area of interest were eliminated by the
software, effectively reducing the image resolution to 312x453 pixels. The Object Plane Pixel Size (OPPS) of the acquired images, accounting for two images per frame, is 0.305 mm/pixel in y and 0.318 mm/pixel in x. The framing rate determines the maximum velocity of particles that can be measured. With a maximum displacement per frame of 10% of the field of view, the maximum velocity that can be tracked is 30 cm/second.

As previously mentioned, 1000 cycles were grabbed for each vessel and used in the ensemble averaging data analysis technique.
CHAPTER 5

RESULTS

5.1 Data Presentation

The data collected has been averaged both spatially and temporally. Data is presented spatially on cross sections. These cross sections are obtained from the outlines of the video of the milling of the original TB33 and TB41 polyester copy. Each cross section is used as a reference to present the data during the velocity and shear calculations. TB33 is shown in Figure 5.1 with the representative cross section numbering system in each branch indicated. Each branch begins with cross section 1 and increases by 1 distally. All cross section are spaced at 0.1368 cm except for cross section LM 19-23, LCX 1-5 and LAD 1-5; which are all spaced at 0.0272 cm. The cross section data for TB41 is presented in Figure 5.2. Cross sections LM 6-15, and LM 15 to the first cross section in each daughter vessel are spaced at 0.2556 cm, while the remainder of the cross sections are spaced at 0.1280 cm.
Figure 5.1 TB33 cross sections
The velocity vectors obtained in the PTV procedure are "binned" according to a regular grid for ease of computation. Each vessel is separated by branch; LM, LAD, and LCX. In addition, the data are analyzed by small increments within each branch that is by cross section. A grouping of three cross sections is used to define a cylindrical volume on which a 21 x 21 x 21 cubic grid is placed. The center cross section is the cross section of interest, with the data being presented for that cross section only. This consists of the grids from the 21 x 21 x 21 division that contain the center cross section. The center cross section is bounded by an upper and lower cross section, defining a parallelepiped volume from which candidate velocity vectors are drawn to contribute to the Gaussian
window. The window searched for vectors that contribute to the Gaussian window is 2 times the radius of the grid element. The three components of the velocity vector, \( u \), the \( x \) component, \( v \), the \( y \) component, and \( w \), the \( z \) component, of each vector found within this spatial area is summed by the following equation (as indicated for component \( u \)):

\[
\sum u = u, \exp \left( -\frac{1 \times (\text{radial distance of vector to grid})^2}{(\text{maximum allowable radial distance})^2} \right).
\]

The general case is shown in Figure 5.3 which shows the three cross sections used in any vector calculation. The upper and lower cross sections are used to bound the extent of data, which is drawn from in the Gaussian window calculations. Vectors, which are further away, contribute less to the values on the center cross section. The center cross section is defined as the cross section of interest. A cylinder is used to define the geometry with major and

![Figure 5.3 Three-Cross section System](image)

Figure 5.3 Three-Cross section System
minor axis represented by a and b. All vectors are transformed into the coordinate space of the bounding volume. These transformations are performed to make the definition of axial and secondary velocities trivial in the new coordinate system. The velocity vectors are transformed into the bounding volume as defined by the following procedure. First the centerline axis is found using the three cross sections. A least squares fit of the three cross section centers is used to find the best line through the three. This is shown in Figure 5.4. Then the angles necessary for the transformations are found. These are $\theta$ and $\phi$. $\theta$ is the rotation from the x-axis toward the y-axis about the z-axis. $\phi$ is the rotation of the z-axis toward the y-axis about the x-axis.

Figure 5.4 Determination of the Centerline Axis
These are shown pictorially in Figure 5.5. Then, using the outline file data for each of the three cross sections, the outermost extent of each direction is found in order to bound the parallelepiped that fully contains all three cross sections. This determines the maximum bound for x, y, and z in reference to the parallelepiped. Figure 5.6 shows the bounded volume, along with the ellipse,

Figure 5.5 Definition of Bounded Volume and Coordinate System

which is defined as perpendicular to the centerline axis of the bounded volume, which is used for generation of the velocity vectors and for display of the
generated velocity data. The original three cross sections are shown at dashed ellipses in the diagram. The solid ellipse is the one used for display of the generated velocity data. This is accomplished by finding a plane perpendicular to the centerline axis. At the y position of the intersection of the cross section and the perpendicular plane, the maximum values of x and z are used to define the ellipse.

The velocity vectors are validated with a magnitude and angle check. Only velocity vectors whose magnitude is within $\pm 2\sigma$ of the mean magnitude of the bounded volume are kept. The direction is also checked and only vectors whose direction is within $\pm 30^\circ$ of the mean are kept.
In addition to the spatial transformation of the velocity vectors, the data is averaged temporally based on the video framing rate used for data collection. The images were recorded at 30 fps for each 87-frame cycle. Since the PTV algorithm uses a five-frame basis to analyze the data, the first two and the last two frames are lost in the data analysis. Therefore, for each three-second cycle, there are frames 2 through 85 for analysis. Figure 5.7 shows an average total bulk flow wave for TB33 along with the breakdown of the time periods used for
temporal analysis. The solid vertical lines through the image show the
separators for each time period along with the corresponding cycle frames and
the name for each period with respect to the cardiac cycle at the top of the
segment. There are 8 time periods used for data analysis. All data within the
given time window are included in the particular data analysis. The exact same
temporal segmentation is used for TB41.

![Figure 5.7 Temporal Analysis of Data](image)

### 5.2 Data Validation

Validation of the PTV velocity measurements was done by a mass
balance. A comparison of the sum of the total integrated flow through each of
the daughter branches and the total integrated flow through the left main was
done in each vessel. The results are presented in Table 5.1 below. The
integration was performed during time period 62-77, or the late mid-diastole
period. This was done in order to obtain as steady of a flow as possible. The
equation used is as follows:
\[ Q = \sum V_i \partial A_i \]
where
\[ V_i = \text{Velocity Component Parallel to Centerline Axis} \]
\[ \partial A_i = \text{Area of Grid Element of } V_i \text{ in Plane Perpendicular to Centerline Axis} \]

<table>
<thead>
<tr>
<th>Vessel</th>
<th>LAD Bulk Flow (l/min)</th>
<th>LCX Bulk Flow (l/min)</th>
<th>LAD + LCX Bulk Flow (l/min)</th>
<th>LM Bulk Flow (l/min)</th>
<th>Flow Partition</th>
</tr>
</thead>
<tbody>
<tr>
<td>TB33</td>
<td>1.08</td>
<td>1.23</td>
<td>2.31</td>
<td>2.37</td>
<td>0.45 LAD</td>
</tr>
<tr>
<td>TB41</td>
<td>0.91</td>
<td>1.08</td>
<td>1.99</td>
<td>2.10</td>
<td>0.46 LAD</td>
</tr>
</tbody>
</table>

Table 5.1 Bulk Flow Data Validation

5.3 Axial Velocity Profiles

The \( v \) component of the velocity vectors, which is the velocity component in
the \( y \) direction, is used for velocity profiles along the cardinal radii of the ellipse.
Typical axial velocity profiles are shown in Figure 5.8. Full results of all the axial
profiles for each TB33 and TB41 are in Appendix F. Velocities have been scaled back to real blood vessel values by the following method.

Original vessel variables:

\( U_r \) is the mean velocity based on the LM inlet
\( L_r \) is the diameter of the LM inlet
\( u_r \) is the velocity at any point in the vessel
\( I_r \) is the distance from the wall to the point for the corresponding \( u_r \)

Scaled vessel variables:

\( U_s \) is the mean velocity based on the LM inlet
\( L_s \) is the diameter of the LM inlet
\( u_s \) is the velocity at any point in the vessel
\( I_s \) is the distance from the wall to the point for the corresponding \( u_s \)

At any point in the model, the following equivalence will be true:

\[
\frac{u_s}{U_s} = \frac{u_r}{U_r} \cdot \frac{I_r}{L_r} = \frac{I_s}{L_s}
\]

Rearranging, and solving for \( u_r \), we get:

\[
u_r = u_s \cdot \frac{I_r}{L_r} \cdot \frac{U_r}{U_s} \cdot \frac{I_s}{L_s}
\]

This conversion factor is calculated for each vessel and is used to convert the scaled velocities to real blood vessel velocities.
In Figure 5.8 the data is shown both spatially, by cross section, as indicated by the cross section number at the top of the column of graphs; and temporally by the various colored lines in each graph as indicated by the key at the bottom. The top graph in each column is the profile for that cross section, going across the vessel from lateral wall to lateral wall. The bottom graph in each column is the profile depicted as moving across the vessel from epicardial surface to myocardial surface. The data is presented for three consecutive cross sections.
going from left to right across the page. The location of the cross sections within the actual geometry is indicated by the small image above and each axial plot.

5.4 Secondary Velocity Profiles

Since the bounding cylinder is parallel to the centerline axis (y-axis), the y-coordinate is used to locate data for secondary velocity profiles. The u and w components of the velocity vectors are used for secondary velocity profiles along the entire display ellipse, which is perpendicular to the centerline axis of the bounded cylinder. Magnitude of the vectors is shown by color. A typical secondary velocity profile is shown in Figure 5.9. Full results of secondary velocity profiles for each TB33 and TB41 are in Appendix G.

Figure 5.9 shows the secondary velocity profiles for the TB33 LCX vessel, cross section 16 as noted at the top of the figure. Each subsequent image within the figure shows the data at a different temporal instance as shown by the legend below each image. The color scale on the right indicates the magnitude of the velocity vectors, direction is shown by the actual direction of the arrow head in the image. Both the cycle frames and the representative time period within the cardiac cycle are noted below each image.
5.5 Near-Wall Shear

A non-uniform finite difference stencil was used to calculate the near-wall shear. The equation for the second-order accurate forward difference stencil is shown below. Figure 5.10 shows the location of the data bins used in the shear calculations. The "bins" referred to in the diagram are the cells created by the overlaying of the 21 x 21 x 21 grid over the bounded cylinder of interest. Figure 5.11 shows the area where the shear is calculated for a particular cross section.
of interest. Data is calculated for the bin defined by the intersection of the ellipse of the cross section of interest and the bounded cylinder. Only this data is displayed.

\[
\frac{\partial u}{\partial r} = \frac{f_{i+2}^2 u_{i+2}^2 - F_{i+1}^2 u_{i+1}^2 + (F_{i+1}^2 - f_{i+1}^2) u'}{\Delta r F_{i+1} f_{i+1}(f_{i+1} - F_{i+1})}
\]

where

\[
F_{i+1} = f_{i+1} + f_{i+2}
\]

\[
f_{i+1} = \frac{dr + \Delta r}{2}
\]

\[
f_{i+2} = \frac{3\Delta r + dr}{2}
\]

\[
\Delta r = \text{fixed length}
\]

\[
dr = \text{partial bin diameter length from wall to bin 1}
\]

\[u_i = \text{average of wall bin v-components of velocity vector (axial component)}\]

\[u_{i+1} = \text{average of bin next to wall v-components of velocity vector (axial component)}\]

![Diagram](image)

Figure 5.10 Location of Data Bins Used in Shear Calculations
In addition, some validations are performed on the data. First, only a bin with at least 4 velocity vectors may be used to calculate shear. If a bin with less than four is found, the algorithm searches inward, away from the wall, for a bin with more than four vectors. If a bin with greater than four vectors is not found, no shear is calculated, and the value of shear for that bin is set to 0. If a bin with greater than 100 is found, the algorithm searches outward for three bins for a bin with fewer vectors. If one is not found, shear is calculated using the last three bins searched. As previously, the direction check is performed on the found velocity vectors. All vectors must be within $\pm 30^\circ$ of the mean direction to be included. Also, the $\pm 2\sigma$ check on magnitude is performed as another criteria for inclusion.
The shear values are scaled back real blood vessel values with the following conversion factor method:

Original vessel variables:

- $U_r$ is the mean velocity based on the LM inlet
- $L_r$ is the diameter of the LM inlet
- $u_r$ is the velocity at any point in the vessel
- $l_r$ is the distance from the wall to the point for the corresponding $u_r$
Scaled vessel variables:

- $U_s$ is the mean velocity based on the LM inlet
- $L_s$ is the diameter of the LM inlet
- $u_s$ is the velocity at any point in the vessel
- $l_s$ is the distance from the wall to the point for the corresponding $u_s$

At any point in the model, the following equivalence will be true:

$$\frac{u_s}{U_s} = \frac{u_r}{U_r} = \frac{l_s}{L_s}$$

Rearranging, and solving for $u_r/l_r$, we get:

$$\frac{u_r}{l_r} = \frac{u_s}{l_s} \left[ \frac{L_s}{L_r} \left( \frac{U_r}{U_s} \right) \right]$$

This conversion factor is calculated for each vessel and is used to convert the scaled shear rate to real blood vessel velocities.

Figures 5.12 and 5.13 show the real blood vessel shear rate maps for TB33 as calculated for the various time periods noted on the figures. These views are looking at the epicardial surface of the vessel. Figures 5.14 and 5.15 show the myocardial surface shear rate maps for TB33 for the noted time periods.
Figure 5.12 Epicardial TB33 Diastole Near-Wall Shear Rate
TB33 Epicardial Systole Near-Wall Shear Rate

Figure 5.13 Epicardial TB33 Systole Near-Wall Shear Rate
Figure 5.14 Myocardial TB33 Diastole Near-Wall Shear Rate
Figures 5.16 and 5.17 show the near-wall shear rate maps of the epicardial surfaces of TB41. Figures 5.18 and 5.19 show the near-wall shear rate maps of the myocardial surfaces of TB41.
Figure 5.16 Epicardial TB41 Diastole Near-Wall Shear Rate
Early Systole  
Systole  
Late Systole  
Early Diastole

TB41 Epicardial Systole Near-Wall Shear Rate

Figure 5.17 Epicardial TB41 Systole Near-Wall Shear Rate
TB41 Myocardial Diastole Near-Wall Shear Rate

Figure 5.18 Myocardial TB41 Diastole Near-Wall Shear
TB41 Myocardial Systole Near-Wall Shear Rate

Figure 5.19 Myocardial TB41 Systole Near-Wall Shear Rate
Another shear calculation completed is the pulse shear map. Pulse shear is defined as the maximum shear minus the minimum shear found at any point in space. The pulse shear map for TB33 is shown in Figure 5.20 and for TB41 is found in Figure 5.21.

Figure 5.20 Pulse Shear Map for TB33
5.6 Shear Comparisons, Lateral Wall and Flow Divider

The near-wall shear values are also displayed as profiles at both the lateral (outer) walls and flow divider (inner) walls. Figure 5.22 presents the shear rate profiles for the flow divider walls during all 8 of the time segments. Each time segment throughout the pulsatile cycle is depicted with a different color. The vessels are indicated by a different line pattern with TB33 being a solid line and TB41 being a broken line. The actual flow divider is indicated in the center of the diagram with the LAD to the right of the flow divider and the LCX to the left. All values are scaled to real blood vessel values as previously indicated.
Figure 5.22 Flow Divider Shear Rate Profile Comparison

Figure 5.23 depicts the shear rate profile for the flow divider wall for just the diastole time periods of the pulsatile cycle. The shear rate profiles for the systole time periods of the flow divider wall are shown in Figure 5.24. In each of these
figures data for TB33 are represented as solid lines, while data for TB41 are represented as dashed lines.

Figure 5.23 Flow Divider Diastole Shear Rate Comparison
The analogous figures for the lateral wall shear rate profiles are shown in Figures 5.25 through 5.27.
Figure 5.25 Lateral Wall Shear Rate Comparison
Path of Shear Rate Data Collection

![Graph showing shear rates in different phases of cardiac cycle]

Figure 5.26 Lateral Wall Diastole Shear Rate Comparison
Figure 5.27 Lateral Wall Systole Shear Rate Comparison
CHAPTER 6

DISCUSSION AND CONCLUSIONS

The discussion of the data generated in this set of experiments will be focused on four separate issues. First, the general flow field will be analyzed using the velocity profiles and the secondary profiles. Second, the near-wall shear maps will be discussed. Third, the geometric effect of the branch angle will be analyzed. Finally, the use of PTV for studying cardiovascular flows will be discussed.

6.1 General Flow Field Discussion

Two general flow phenomena will be mentioned because of their frequent occurrence in the LCA flow field. The first is the result of flow through a curved tube. When a fully developed steady flow approached a curve in a tube, a transverse pressure gradient occurs which tends to move the slower fluid close to the wall more than the faster moving fluid with more inertia in the center of the tube. This tends to cause the higher velocity fluid to move toward the outer wall and the slower flows toward the inner wall.
The second flow pattern involves the bifurcation. As a steady flow approaches a bifurcation, the area expansion causes a favorable pressure gradient. The faster moving fluid is displaced transversely, similarly to the curved tube case, causing the higher velocity fluid to move toward the flow divider wall.

The movement of fluid during both of these patterns creates secondary flows, which at times may become very complex, especially when multiple flow patterns are combined.

6.1.1 Axial Velocity Profiles

First the axial velocity profiles in the LM of TB33 will be studied. These can be found in detail in Appendix F. To review, the top row of profiles corresponds to the profile across the vessel laterally. The bottom row corresponds to the profile across the vessel from the epicardial surface to the myocardial surface. Beginning at the bottom of the LM, closest to the ostia, one can see that laterally the profiles are skewed to the right, which corresponds to the outer wall of curvature in the vessel. In addition, one can see that the profile from the epicardial to the myocardial surface has a maximum velocity toward the myocardial surface. As the profiles progress distally, the profile changes into an "M-shaped" profile, with the maximum velocity migrating to the epicardial surface. The initial skewing toward the epicardial surface is most likely an entrance effect. This effect is replaced as the curvature becomes the dominant force as the vessel traverses over the surface of the heart and the maximum velocity occurs at the outer wall of the curvature, the epicardial surface. The
complex "M-shaped" profiles have been observed previously in vessels with curvature in more than one axis [67, 79]. The location of the flow divider is apparent, slightly offset to the right of center in the LM, as one moves distally through the cross sections toward the flow divider. The velocity increases toward the flow divider surface starting in cross section 18 and continuing through cross section 23.

Moving into the axial profiles for the LAD in TB33, the first few cross sections show a typical bifurcation region flow, with the velocity a bit higher at the flow divider wall laterally (left side). As the flow develops in the LAD, the profiles take on the form of a fairly well developed parabolic flow field in both cross-sections, distally. In addition, the presence of such a pronounced parabolic axial profile indicates that the flow is laminar.

The LCX axial profiles in TB33 indicate the more typical curved vessel profile development. Proximal in the first three cross sections one can see a skewing of the velocity profiles toward the flow divider wall. Moving distally, the curvature effect begins to dominate as the profiles take on the "M-shaped" curves, evincing the fact that the vessel has curvature in both planes being studied. Most distal in the LCX, one can see the flow reversing in the center of the vessel in the time periods spanning late diastole (62-77) through early systole (7-15).

The discussion will now focus on the axial profiles for TB41. During the diastolic time periods, the LM demonstrates a marked skewing of the lateral profiles toward the outer wall. A slight indication of the flow divider is seen in the
epicardial – myocardial plane, which is slightly offset to the right of center in the LM. It is also noted that the LM is shorter in this vessel, only the distal 0.3834 cm is being visualized in this technique.

The axial profiles in the LAD of TB41 show a slight skewing toward the flow divider proximally. Curvature effect quickly dominates as the maximum velocity shifts to the outer wall of curvature in this segment of the LAD, the myocardial surface. In this plane, as the profiles develop distally, a slight “M-shaped” profile is beginning to emerge. Laterally, the profile emerges as rather flat throughout the length of vessel observed.

Marked differences are observed between the profiles during systole and diastole in the axial profiles of the LCX for TB41. Laterally, a slight skewing is observed toward the flow divider during systole. During diastole, the maximum velocity shifts to the epicardial and lateral walls. Distally in cross section 5, the lateral profiles seem to be coming together in phase. However, since only a short distance of the LCX could be visualized, it is difficult to extrapolate the development of the flow.

6.1.2 Secondary Velocity Profiles

Shifting now to the secondary flows, strong secondary flows are observed in both TB33 and TB41. Secondary flows in the LM in both vessels showed four distinct cells, or regions of flow. Others have previously observed this type of flow. In 1989 Daskopoulos and Lenhoff [80] computationally showed a more
complex four-vortex solution to developed laminar flow in stationary curved ducts following a bifurcation. Cheng and Mok experimentally observed a four-vortex solution in 1986 [81], and thus confirmed that this solution is indeed a stable solution in some sets of geometries and initial conditions. Most recently, Zabielski and Mestel in 1998 [82] showed computationally the four-vortex solution in a helically symmetric pipe with pulsatile flow.

In TB33, the LM displays a four-cell secondary flow pattern until the bifurcation is nearly reached, where it displays a more regular, two-cell pattern. The location of the flow divider is quite evident from cross section 17 on distally.

Secondary flow patterns in the LAD of TB33 deviate from expected patterns. First it must be noted that the coordinate system changes to one that is consistent with the main axis of the LAD as described previously in section 5.1. In the first few cross sections of the LAD, the bulk of the flow is directed up from the LM, which in the new frame of reference of the LAD, is directed toward the flow divider wall. A small area on the lateral-myocardial wall of the LAD shows a little flow disturbance, with one opposite it showing up in cross section 4 forming on the flow divider-epicardial wall. At this point in the flow progression, the development of an observed helical structure can first be seen. A helical structure, which proceeds clockwise in the LAD and counter-clockwise in the LCX of TB33, has been observed. In the LAD, the helical structure makes one complete revolution in approximately 1.09 cm axially, while the major and minor diameters of the vessel are 1.24 cm and 1.08 cm respectively. A second revolution, after the first, is completed in approximately 0.55 cm. Fox and Seed
observed a spiral pattern of disease, similar to this flow pattern, in 1981 [83], in the LAD of human coronary arteries. The presence of a horseshoe vortex was found experimentally by Fukushima, et al. in 1987 [84], as a secondary flow characteristic in steady and pulsatile flows through a symmetrical, glass tube bifurcation. Sabbah et al. (1984) [55] found a spiraling flow induced by secondary motions under both steady and oscillatory flow conditions in both the LAD and LCX in molds of porcine coronary arteries.

When looking at the secondary flow patterns of the LCX of TB33, one must first remember the change of coordinate system used to display the data in the LCX. The flow begins with a very regular pattern, with the majority of the secondary flow directed at the flow divider. Around cross section 4, a small disturbance can be seen beginning near the myocardial surface. This disturbance grows distally, and a disturbance approximately opposite it on the epicardial surface begins to appear in late diastole of cross section 4, and progresses distally. The flow becomes very disturbed in cross section 7 along with reduced secondary velocity magnitudes, and subsequently changes direction in cross section 8. In the LCX the helical flow structure moves counterclockwise, and makes one complete revolution in 0.684 cm. The helical flow structure repeats itself in approximately the same length two more times.

The secondary flow profiles for TB41 are similar to those found in TB33. The LM also exhibits a four-cell pattern beginning in cross section 3 and proceeding all the way to the bifurcation region. The actual location of the flow divider becomes very apparent as early as cross section 5 in the LM. Again, as
in TB33, the flow from this point on begins to become more regular, but still retains the four-cell pattern.

Similar to TB33, a helical flow structure may be seen in the LAD. The flow structure rotates in a clockwise direction, making one complete rotation in approximately 1.54 cm. Unlike TB33, the initial cross sections in TB41 show some secondary disturbances originating from the myocardial surface. The LCX of TB41, as does TB33, begins with a disturbance in the secondary flow field near the lateral-myocardial surface. A disturbance appears near the flow divider wall toward the epicardial surface around cross section 4. Unlike the other branches, a helical structure cannot be seen in the LCX of TB41. The absence of the helical flow structure may because the length of the LCX visualized is too short to see the full development of the flow field, not because one does not exist.

The important idea found in looking at the secondary flows in both vessels is the marked strong influence on the near-wall shear based on the strong secondary flow field in both vessels. The presence of strong secondary flows in coronary vessels has been reported by Perktold, et al. in 1991 [64]. His studies were numerically based, using a curved tube model of the LM and a pulsatile waveform. Perktold concluded that there was a strong secondary component, without an alteration in the axial velocity profiles. Perktold, et al. also published results from an anatomically realistic human left coronary using a pulsatile waveform in 1995 [70]. They noted a strong influence on the wall shear stress based on the complex secondary flow patterns observed.
In 1996, He and Ku [72] presented CFD results of a branched human left coronary artery bifurcation using a pulsatile waveform. They used average LCA geometry values modified to put the bifurcation in one plane, with the LM being a straight tube with a circular cross section. In addition, the LAD and LCX were modeled with circular cross sections and constant radii of curvature. They found definite skewing of the axial velocity profiles toward the flow divider wall in the bifurcation region. They reported a stronger velocity skewing effect from the bifurcation rather than from curvature, and strong secondary flow components from the curvature. An attempt was made to compare their numerically computed results to experimental results obtained by Tang (1990) [63] with reasonable success. General axial velocity profile shapes were very similar; however, differences were noted based on different input waveforms and other factors. In contrast, Altobelli and Nerem (1985) [56] measured axial velocity profiles in perfused baboon hearts and found profiles skewed toward the outside walls in the LAD and LCX in all cases.

One important aspect to note is that the models used in the current experimental data being presented also included the Sinus of Valsalva and the ostia in the model. The advantage of this approach is the creation of the most realistic entrance conditions to the LM. This could be a very important influence due to the importance of entrance effects. The vessel is typically a few centimeters long, giving only a short time for flow to develop prior to the bifurcation region.
6.2 Near-Wall Shear Maps

The next set of data to be discussed is the near-wall shear maps. These may be viewed in Figures 5.12 through 5.19. Figure 5.12 contains a composite of the near-wall shear maps for the epicardial surface of the diastolic time periods of TB33. The view is that of the epicardial surface, with the LM toward the bottom of the image and the LAD to the right side. Comparing Figure 5.12 to Figure 5.16 that contains the analogous data for TB41, it can be seen that the near-wall shear values for the LAD in general are higher, but in particular, they are highest for TB33 in the LAD. This could be due, in part, to the fact that the diameter of the LAD for TB33 is slightly smaller. In general, shear patterns between the two vessels are similar; however, slight differences must be noted. The flow divider wall distally on the LCX of TB41 has a region of negative shear, while that area in TB33 has a positive, if not high value of shear. Both vessels display a region of low shear on the LM, however, the region is much larger in TB33. TB41 has similar low shear patterns in the LM, however, there is one small area displaying a high rate (yellow) on the epicardial surface of the LM near the flow divider just proximal to the LAD. Figures 5.13 and 5.17 are the shear maps for the systolic time periods and early diastole of the epicardial surfaces of each vessel. The systolic periods for both vessels are very similar with sporadic negative shear regions appearing in each. Again, TB41 has a negative shear region in the LCX during the early diastole period while TB33 has high shear in the same region and time period. In addition, TB33 has a small negative shear region very near the flow divider in early systole.
Figures 5.14 and 5.15 are of the myocardial surface of TB33 during diastole and systole, respectively. The myocardial surface of the LAD in TB33 shows a very high shear rate. The LM has some low regions, even during diastole. These low regions in the LM become very pronounced during systole, extending distally on the inside curvature surface (near the flow divider) of the LCX and becoming negative at points. Some of these regions persist through systole and most disappear during early diastole.

The myocardial surfaces of TB41 systole and diastole are shown in Figures 5.18 and 5.19. Some low and negative regions are apparent on the myocardial surface of the LM during systole with many of the regions remaining throughout diastole. There is one area on the inside curvature of the LM that retains a negative shear even during diastole. The low shear regions are dominant in the LM and the LCX, but not the LAD, especially during diastole.

The pulse shear maps in Figure 5.20 and 5.21 show the difference found between the maximum and minimum near-wall shear. The differences in TB33 seem to be larger in the LAD, while the largest differences in TB41 are found along the distal flow divider wall of the LCX and the flow divider wall of the LAD. TB33 has a large region in the LM that has a very low pulse shear, while the lateral wall in the LCX has a high pulse shear. TB41 appears to have a more even distribution of pulse shear. Some low and negative regions occur in the LM, but not as extensive as in the LM of TB33.
6.3 Shear Rate Profiles

Flow divider and lateral wall shear rate profiles can be found in Figures 5.22 through 5.27. In general, especially in the LCX, the shear rate values found on the flow divider are higher than those found on the lateral walls. This result agrees with the computational results of He and Ku (1996) [72]. They found lower shear rates on the outside or lateral walls. In addition, experimental work by Tsao, et al. (1995) [69] using LDA and particle residence times (PRT) found that PRT, which correlates inversely to total wall shear stress, was higher on the outer or lateral walls of the LCX. Chang and Tarbell (1988) [85] found higher mean shear on the lateral walls, but the pulse shear was higher on the flow divider walls. Mark, et al., (1989) [86] studied the shear distribution in casts of ten human aortic bifurcations. They found, for most of the casts, the shear along the flow divider walls was higher than the shear along the outer or lateral walls. This high shear is indicative of the faster moving flow being directed to the flow divider wall as a result of the bifurcation.

The phenomenon of higher shear at the flow divider wall may also be the result of the compliant nature of the flow models used in the current experiments. The models were compliant, and the walls moved approximately 14% of the total vessel diameter throughout the pulsatile cycle, as evidenced by the search radius necessary to find the wall bins with velocity vectors in them. Arteries are elastic, and have been observed to have variations in vessel diameters in vivo in the range of 10% [87]. The effect of compliance on wall shear has been previously
documented by Duncan, et al. in 1990 [77] to cause the wall shear at the lateral walls to be reduced while the wall shear at the flow divider walls are increased.

A look at where atherosclerotic lesions occur in humans is a good indicator of where to look for specific flow phenomena. Fox and Seed (1981) [83] found that early human coronary atheroma occur predominantly in the inner curvatures, such as the myocardial surface. Grottum et al. (1983) [6] also studied the human left main coronary artery. They found that atherosclerotic lesions were predominant on the myocardial surface of the arteries. Svindland (1983) [51] identified a pattern where lesions are frequently found on the outer walls in the bifurcation and on the inner curvature, but are uncommon on the flow divider as well as on the flow divider walls downstream of the flow divider.

A study by Gibson, et al. (1993) [65] called the Harvard Atherosclerosis Reversibility Project (HARP) used a combination of successive quantitative angiography and finite-difference modeling to correlate the progression of atherosclerotic plaques with shear stress. They found that low shear stress was significantly correlated with an increased rate of atherosclerosis progression.

Another correlation used to predict the localization of atherosclerotic plaque is the correlation between wall shear and intimal thickening. Friedman, et al. (1987) [32] found a positive correlation between intimal thickening and sites generally exposed to lower shears. These regions have been identified above in various morphometric studies.
6.4 Bifurcation Angle

Turning now to the effect of the bifurcation angle, some preliminary calculations need to be made. In order to compare the shear values for both vessels, normalization must be performed in order to compensate for the varying radii in the daughter vessels. The following equations are the exact solution to the Newtonian viscous flow equations for an elliptical cross section from White, 1991 [88].

\[
\begin{align*}
  u(y, z) &= \frac{1}{2\mu} \left( -\frac{dp}{dx} \right) \frac{a^2 b^2}{a^2 + b^2} \left( 1 - \frac{y^2}{a^2} - \frac{z^2}{b^2} \right) \\
  Q &= \frac{\pi}{4\mu} \left( -\frac{dp}{dx} \right) \frac{a^3 b^3}{a^2 + b^2}
\end{align*}
\]

Solving the flow rate equation for the pressure gradient, and substituting that into the first equation gives the following equation:

\[
  u(y, z) = \frac{-2Q}{ab\pi} \left( 1 - \frac{y^2}{a^2} - \frac{z^2}{b^2} \right).
\]

Now, converting the above equation into cylindrical coordinates using the following equations gives a velocity equation in terms of radial distance.

\[
\begin{align*}
  y &= r \cos \theta \\
  z &= r \sin \theta
\end{align*}
\]

\[
  u(r, \theta) = \frac{-2Q}{ab\pi} \left( 1 - \frac{r^2 \cos^2 \theta}{a^2} - \frac{r^2 \sin^2 \theta}{b^2} \right)
\]

Taking the partial derivative of the above equation with respect to \( r \) gives the following equation for the shear rate at the wall.

\[
\frac{du}{dr} = \frac{4Q}{\pi} \left[ \frac{r \cos^2 \theta}{ab} + \frac{r \sin^2 \theta}{ab^3} \right]
\]

105
To calculate the average expected shear rate based on elliptical Poiseuille flow, the following summation is performed using 60 equally spaced intervals around the circumference of the elliptical vessel wall:

\[
\text{Shear Rate} = \frac{1}{60} \sum_{i=1}^{60} \frac{du}{dr} \bigg|_{\alpha \frac{i\pi}{60}}
\]

where

\[
r = \sqrt{\frac{a^2 b^2}{a^2 \sin^2 \theta + b^2 \cos^2 \theta}}
\]

Using the mean flow rates and the vessel diameters in both the LAD and the LCX for each vessel gives the following predicted elliptical Poiseuille flow results from the above equations. These numbers are presented in the Table 6.1 along with the average values calculated experimentally. All reported values have been scaled to real blood vessel dimensions. The systolic values are an average of all wall shear values seen during the systolic time frames. Similarly, the diastolic values are an average of all wall shear values seen during diastole. The overall average values for the LAD and LCX are calculated on a time-average method based on the number of frames in the cycle a particular shear value was seen. The diastolic time period was 59 frames out of the total 85 frames, while the systolic period was 26 frames.
Looking at the shear rate numbers in Table 6.1, and converting them to shear stress by multiplying by the fluid viscosity of human blood, 0.04 poise, gives average shear stress values ranging from approximately 4.4 to 23.2 dynes/cm². These numbers are within the range reported by Friedman (1990) [89]. He experimentally determined the mean shear stress in 14 models of LCA's at the bifurcation using LDA. The range was found to be $10.3 \pm 5.2$ dyne/cm².
with a minimum of 2.3 and a maximum of 19.7 dyne/cm$^2$. The numbers found in this set of experiments are very close to this reported range.

Again looking at Table 6.1, and comparing the expected values from the Poiseuille calculation to the experimental values, it can be seen that the experimental values are larger than expected in both cases. This may be due to the pulsatile flow in non-circular vessels with changing diameters. Both vessels had a similar increase, therefore, the increase cannot be contributed to the variation in the bifurcation angle.

There has been a plethora of work done experimentally, morphologically and computationally, to determine the effect of branch angle on the initiation and propagation of atherosclerotic disease. In 1974 Saltissi, et al. studied angiograms of 149 patients of which 95 were considered to have critical stenosis and 54 were considered normal with minimal stenosis. They measured the LAD – LCX bifurcation angle using a protractor on the right anterior oblique view and found no significant difference between the mean angle of bifurcation of the normals (75°) and the diseased group (81°). However, they did find that patients with proximal disease showed a tendency toward wider bifurcation angles (85°) than patients with distal disease (76°). This result was not significant at the 5% level. They also found that a short left main length, less than 10.6 mm was correlated with disease.

Friedman, et al. in 1993 [43] studied the LCX-LAD bifurcation angle by looking at the localization of sudanophilia in the proximal portions of the LCX and LAD. They suggested that a small bifurcation angle might be a 'geometric risk
factor for proximal atherosclerotic disease in the daughter vessels. This correlation was found to be stronger in the LCX. In 1994 He and Ku [68] studied the wall shear distribution in three realistic models of the LCA using CFD. They varied the bifurcation angle from 0° to 90°. They concluded that variation in the bifurcation angle does not have a dominant effect on the distribution of wall shear stress. Levels of shear stress on the outer wall were low in general, and varied from 7 dyne/cm² down to 3 dyne/cm² with a change in bifurcation angle of 0° to 90°.

More recently, Friedman, et al. (1996) [44] studied 15 angiographically lesion-free human LCA’s and correlated the bifurcation angle with morphometry from transverse sections of the tissue. They found a positive correlation between the proximal branch angle and most intimal and medial variables. This suggested that large branch angles are associated with intimal thickening and presumably adverse.

6.5 PTV Analysis

Many lessons were learned concerning the application of PTV to cardiovascular simulation. First, and foremost is the importance of having calibration points within the actual flow region of interest. It may be a better trade-off to use a non-compliant model and be able to have calibration points in the flow region. The next lesson learned is to not be afraid to scale-up. In addition, using two high-speed CCD cameras that are synchronized would provide much higher resolution data. This would help in any fluid flow
visualization, but especially so in a pulsatile system such as the coronary vessels.

Another important method that could be improved is the method of obtaining the outline files from the LCA casts for generation of the scaled-up model. A technique, which has been available, but has improved the precision lately is using laser scanning to generate a point cloud database of the surface of the vessels. This technique would eliminate numerous steps previously necessary to scale-up the vessel, and would improve accuracy of the solid model surface as a result. This method is highly recommended.

In order to determine the effect of specific parameters, it is recommended that one actual vessel point cloud be used as a basis for parameter studies. For example, to study more definitively the effect of branch angle on the flow, one LCA cast could be scanned to generate a point cloud. This database could be modified to create multiple LCA geometries by varying only the LCX-LAD branch angle. This type of study would remove the multitude of variables that would normally confound an experiment that used numerous human LCA's as a basis for the experimental design.

Another source is the fact that the current study does not address the motion of the coronary arteries during the cardiac cycle due to contraction and relaxation of the heart and motion of the heart within the chest wall. The shear may actually be modified, especially for artery segments that are parallel to the primary direction of motion. The majority of the field of view studied in this protocol is actually perpendicular to the primary direction of motion, it is therefore
felt that the motion effect on shear stress will be negligible. This will produce an
effect on the normal stresses, but it should also be small.

The wall shear rate can be extrapolated from the velocity profile measured
near the wall. However, as the velocities composing the velocity profile are
based on point velocities that represent a spatial average over a finite size
sample volume, such an approach involves some error.

6.6 Conclusions

Full field visualization of the complex flows found in human left coronary
arteries was possible with the use of Particle Tracking Velocimetry. The flow
fields were found to be quasi-steady, with four-cell structures at times in the
daughter vessels and the LM. In addition, a helical flow structure was observed
in the daughter branches.

Near-wall shear values were found to be lower on the lateral walls of the
daughter vessels. Varying the LXC-LAD branch angle from 98° to 86° did not
have a major effect on the near-wall shear values.

In the future studies including other geometric features of the artery may
provide further insight into the total relation between artery geometry and local
hemodynamic parameters.
APPENDIX A

InStat Multiple Regression Results

Multiple Regression Results for TB33-2

What equation fits the data the best?

\[ [C: zLaser] = 0.2145 + 0.5620[D: xLeft] + 8.062[H: xDiff] - 1.655[I: yDiff] \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>SE</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>0.2145</td>
<td>0.009719</td>
<td>0.1952 to 0.2339</td>
</tr>
<tr>
<td>D: xLeft</td>
<td>0.5620</td>
<td>0.01063</td>
<td>0.5409 to 0.5832</td>
</tr>
<tr>
<td>H: xDiff</td>
<td>8.062</td>
<td>0.06321</td>
<td>7.936 to 8.188</td>
</tr>
<tr>
<td>I: yDiff</td>
<td>-1.655</td>
<td>0.2370</td>
<td>-2.127 to -1.184</td>
</tr>
</tbody>
</table>

How good is the fit?

R squared = 99.42%.
This is the percent of the variance in C: zLaser explained by the model.

The P value is < 0.0001, considered extremely significant.
The P value answers this question:
   If there were no linear relationship among the variables, what is
   the chance that R squared would be that high (or higher) by chance?

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum-of-squares</td>
<td>0.1865</td>
</tr>
<tr>
<td>SD of residuals</td>
<td>0.04407</td>
</tr>
<tr>
<td>R squared</td>
<td>0.9942</td>
</tr>
<tr>
<td>Adjusted R squared</td>
<td>0.9941</td>
</tr>
<tr>
<td>Multiple R</td>
<td>0.9971</td>
</tr>
<tr>
<td>F</td>
<td>5529.2344</td>
</tr>
</tbody>
</table>

Which variable(s) make a significant contribution?
Variable | t ratio | P value  | Significant?
---|---|---|---
(constant) | 22.075 | < 0.0001 | Yes
D:xLeft | 52.893 | < 0.0001 | Yes
H:xDiff | 127.54 | < 0.0001 | Yes
l:yDiff | 6.985 | < 0.0001 | Yes

Each P value compares the full model with a simpler model omitting one variable. It tests the effect of one variable, after accounting for the effects of the others.

Is multi-collinearity a problem?

<table>
<thead>
<tr>
<th>Variable</th>
<th>VIF</th>
<th>R2 with other X</th>
</tr>
</thead>
<tbody>
<tr>
<td>D:xLeft</td>
<td>1.11</td>
<td>0.1013</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>1.11</td>
<td>0.1022</td>
</tr>
<tr>
<td>l:yDiff</td>
<td>1.00</td>
<td>0.0018</td>
</tr>
</tbody>
</table>

Each R squared quantifies how well that X variable is predicted from the other X variables (ignoring Y). VIF is calculated from R squared.

All R squared values are low (<0.75). The X variables are independent of each other. Multi-collinearity is not a problem.

Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>D:</th>
<th>H:</th>
<th>l:</th>
<th>l: C: (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D:xLeft</td>
<td>1.0000</td>
<td>-0.3174</td>
<td>-0.0131</td>
<td>l 0.1019</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>-0.3174</td>
<td>1.0000</td>
<td>-0.0341</td>
<td>l 0.9069</td>
</tr>
<tr>
<td>l:yDiff</td>
<td>-0.0131</td>
<td>-0.0341</td>
<td>1.0000</td>
<td>l -0.0953</td>
</tr>
</tbody>
</table>

Each correlation coefficient (r) is calculated independently, without considering the other variables.

Summary of your data

Number of rows (subjects) analyzed: 100
Number of rows with missing data, excluded from calculations: 0
Number of degrees of freedom (#subjects - #variables - 1): 96

* * *
Multiple Regression Results for TB33-2

What equation fits the data the best?

\[ [C:zLaser] = 0.2731 + 0.5639[D:xLeft] + 8.080[H:xDiff] \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>SE</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>0.2731</td>
<td>0.006008</td>
<td>0.2612 to 0.2850</td>
</tr>
<tr>
<td>D:xLeft</td>
<td>0.5639</td>
<td>0.01298</td>
<td>0.5381 to 0.5897</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>8.080</td>
<td>0.07716</td>
<td>7.926 to 8.233</td>
</tr>
</tbody>
</table>

How good is the fit?

R squared = 99.13%.
This is the percent of the variance in C:zLaser explained by the model.

The P value is < 0.0001, considered extremely significant.
The P value answers this question:
If there were no linear relationship among the variables, what is
the chance that R squared would be that high (or higher) by chance?

| Sum-of-squares | 0.2813 |
| SD of residuals | 0.05385 |
| R squared | 0.9913 |
| Adjusted R squared | 0.9911 |
| Multiple R | 0.9957 |
| F | 5539.8599 |

Which variable(s) make a significant contribution?

<table>
<thead>
<tr>
<th>Variable</th>
<th>t ratio</th>
<th>P value</th>
<th>Significant?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>45.453</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>D:xLeft</td>
<td>43.450</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>104.71</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Each P value compares the full model with a simpler model omitting
one variable. It tests the effect of one variable, after accounting
for the effects of the others.

Is multi-collinearity a problem?

114
Variable | VIF   | R2 with other X
D: xLeft | 1.11  | 0.1007
H: xDiff | 1.11  | 0.1007

Each R squared quantifies how well that X variable is predicted from the other X variables (ignoring Y). VIF is calculated from R squared.

All R squared values are low (<0.75). The X variables are independent of each other. Multi-collinearity is not a problem.

Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>D:</th>
<th>H:</th>
<th>C: (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D: xLeft</td>
<td>1.0000</td>
<td>-0.3174</td>
<td>0.1019</td>
</tr>
<tr>
<td>H: xDiff</td>
<td>-0.3174</td>
<td>1.0000</td>
<td>0.9069</td>
</tr>
</tbody>
</table>

Each correlation coefficient (r) is calculated independently, without considering the other variables.

Summary of your data

Number of rows (subjects) analyzed: 100
Number of rows with missing data, excluded from calculations: 0
Number of degrees of freedom (#subjects - #variables - 1): 97
Multiple Regression Results for TB33-2

What equation fits the data the best?

\[ [C:zLaser] = 0.2731 + 0.5639[F:xRight] + 8.644[H:xDiff] \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>SE</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>0.2731</td>
<td>0.006008</td>
<td>0.2612 to 0.2850</td>
</tr>
<tr>
<td>F:xRight</td>
<td>0.5639</td>
<td>0.01298</td>
<td>0.5381 to 0.5897</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>8.644</td>
<td>0.08221</td>
<td>8.480 to 8.807</td>
</tr>
</tbody>
</table>

How good is the fit?

R squared = 99.13%.
This is the percent of the variance in C:zLaser explained by the model.

The P value is < 0.0001, considered extremely significant.
The P value answers this question:
If there were no linear relationship among the variables, what is
the chance that R squared would be that high (or higher) by chance?

| Sum-of-squares | 0.2813 |
| SD of residuals | 0.05385 |
| R squared       | 0.9913 |
| Adjusted R squared | 0.9911 |
| Multiple R       | 0.9957 |
| F               | 5539.8097 |

Which variable(s) make a significant contribution?

<table>
<thead>
<tr>
<th>Variable</th>
<th>t ratio</th>
<th>P value</th>
<th>Significant?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>45.452</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>F:xRight</td>
<td>43.450</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>105.14</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Each P value compares the full model with a simpler model omitting
one variable. It tests the effect of one variable, after accounting
for the effects of the others.

Is multi-collinearity a problem?
Variable | VIF | R2 with other X
---|---|---
F:xRight | 1.26 | 0.2077
H:xDiff | 1.26 | 0.2077

Each R squared quantifies how well that X variable is predicted from the other X variables (ignoring Y). VIF is calculated from R squared.

All R squared values are low (<0.75). The X variables are independent of each other. Multi-collinearity is not a problem.

Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>F:</th>
<th>H:</th>
<th>C: (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F:xRight</td>
<td>1.0000</td>
<td>-0.4558</td>
<td>-0.0475</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>-0.4558</td>
<td>1.0000</td>
<td>0.9069</td>
</tr>
</tbody>
</table>

Each correlation coefficient (r) is calculated independently, without considering the other variables.

Summary of your data

Number of rows (subjects) analyzed: 100
Number of rows with missing data, excluded from calculations: 0
Number of degrees of freedom (#subjects - #variables - 1): 97
Multiple Regression Results for TB33-3

What equation fits the data the best?

\[ [C:zLaser] = 1.242 + 0.5102[D:xLeft] + 7.733[H:xDiff] - 0.2086[I:yDiff] \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>SE</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>1.242</td>
<td>0.007201</td>
<td>1.228 to 1.257</td>
</tr>
<tr>
<td>D:xLeft</td>
<td>0.5102</td>
<td>0.004092</td>
<td>0.5020 to 0.5183</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>7.733</td>
<td>0.02457</td>
<td>7.684 to 7.782</td>
</tr>
<tr>
<td>I:yDiff</td>
<td>-0.2086</td>
<td>0.1097</td>
<td>-0.4266 to 0.009354</td>
</tr>
</tbody>
</table>

How good is the fit?

\( R^2 = 99.91\% \).

This is the percent of the variance in \( C:zLaser \) explained by the model.

The \( P \) value is \(< 0.0001\), considered extremely significant.

The \( P \) value answers this question:

If there were no linear relationship among the variables, what is the chance that \( R^2 \) would be that high (or higher) by chance?

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum-of-squares</td>
<td>0.03018</td>
</tr>
<tr>
<td>SD of residuals</td>
<td>0.01773</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.9991</td>
</tr>
<tr>
<td>Adjusted ( R^2 )</td>
<td>0.9990</td>
</tr>
<tr>
<td>Multiple ( R )</td>
<td>0.9995</td>
</tr>
<tr>
<td>( F )</td>
<td>34335.5503</td>
</tr>
</tbody>
</table>

Which variable(s) make a significant contribution?

<table>
<thead>
<tr>
<th>Variable</th>
<th>( t ) ratio</th>
<th>( P ) value</th>
<th>Significant?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>172.54</td>
<td>(&lt; 0.0001)</td>
<td>Yes</td>
</tr>
<tr>
<td>D:xLeft</td>
<td>124.68</td>
<td>(&lt; 0.0001)</td>
<td>Yes</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>314.72</td>
<td>(&lt; 0.0001)</td>
<td>Yes</td>
</tr>
<tr>
<td>I:yDiff</td>
<td>1.903</td>
<td>0.0601</td>
<td>No</td>
</tr>
</tbody>
</table>

Each \( P \) value compares the full model with a simpler model omitting one variable. It tests the effect of one variable, after accounting for the effects of the others.
Is multi-collinearity a problem?

<table>
<thead>
<tr>
<th>Variable</th>
<th>VIF</th>
<th>R² with other X</th>
</tr>
</thead>
<tbody>
<tr>
<td>D:xLeft</td>
<td>1.10</td>
<td>0.0914</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>1.12</td>
<td>0.1106</td>
</tr>
<tr>
<td>I:yDiff</td>
<td>1.03</td>
<td>0.0270</td>
</tr>
</tbody>
</table>

Each R squared quantifies how well that X variable is predicted from the other X variables (ignoring Y). VIF is calculated from R squared.

All R squared values are low (<0.75). The X variables are independent of each other. Multi-collinearity is not a problem.

Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>D:</th>
<th>H:</th>
<th>I:</th>
<th>C: (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D:xLeft</td>
<td>1.0000</td>
<td>-0.2941</td>
<td>-0.0255</td>
<td>0.1018</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>-0.2941</td>
<td>1.0000</td>
<td>-0.1477</td>
<td>0.9204</td>
</tr>
<tr>
<td>I:yDiff</td>
<td>-0.0255</td>
<td>-0.1477</td>
<td>1.0000</td>
<td>0.1699</td>
</tr>
</tbody>
</table>

Each correlation coefficient (r) is calculated independently, without considering the other variables.

Summary of your data

Number of rows (subjects) analyzed: 100
Number of rows with missing data, excluded from calculations: 0
Number of degrees of freedom (#subjects - #variables - 1): 96

* * *
Multiple Regression Results for TB33-3

What equation fits the data the best?

\[ C:zLaser = 1.231 + 0.5107 \cdot D:xLeft + 7.740 \cdot H:xDiff \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>SE</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>1.231</td>
<td>0.004302</td>
<td>1.223 to 1.240</td>
</tr>
<tr>
<td>D:xLeft</td>
<td>0.5107</td>
<td>0.004136</td>
<td>0.5025 to 0.5190</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>7.740</td>
<td>0.02457</td>
<td>7.691 to 7.789</td>
</tr>
</tbody>
</table>

How good is the fit?

R squared = 99.90%.
This is the percent of the variance in C:zLaser explained by the model.

The P value is < 0.0001, considered extremely significant.
The P value answers this question:
If there were no linear relationship among the variables, what is
the chance that R squared would be that high (or higher) by chance?

| Sum-of-squares | 0.03131 |
| SD of residuals | 0.01797 |
| R squared | 0.9990 |
| Adjusted R squared | 0.9990 |
| Multiple R | 0.9995 |
| F | 50146.9037 |

Which variable(s) make a significant contribution?

<table>
<thead>
<tr>
<th>Variable</th>
<th>t ratio</th>
<th>P value</th>
<th>Significant?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>286.24</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>D:xLeft</td>
<td>123.49</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>315.05</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Each P value compares the full model with a simpler model omitting
one variable. It tests the effect of one variable, after accounting
for the effects of the others.

Is multi-collinearity a problem?

120
Variable  | VIF  | R² with other X
--- | --- | ---
D:xLeft  | 1.09 | 0.0865
H:xDiff  | 1.09 | 0.0865

Each R squared quantifies how well that X variable is predicted from the other X variables (ignoring Y). VIF is calculated from R squared.

All R squared values are low (<0.75). The X variables are independent of each other. Multi-collinearity is not a problem.

Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>D:</th>
<th>H:</th>
<th>C: (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D:xLeft</td>
<td>1.0000</td>
<td>-0.2941</td>
<td>1.018</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>-0.2941</td>
<td>1.0000</td>
<td>0.9204</td>
</tr>
</tbody>
</table>

Each correlation coefficient (r) is calculated independently, without considering the other variables.

Summary of your data

Number of rows (subjects) analyzed: 100
Number of rows with missing data, excluded from calculations: 0
Number of degrees of freedom (#subjects - #variables - 1): 97
Multiple Regression Results for TB33-3

What equation fits the data the best?

\[
[C:zLaser] = 1.231 + 0.5107[F:xRight] + 8.251[H:xDiff]
\]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>SE</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>1.231</td>
<td>0.0043</td>
<td>1.223 to 1.240</td>
</tr>
<tr>
<td>F:xRight</td>
<td>0.5107</td>
<td>0.0041</td>
<td>0.5025 to 0.5190</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>8.251</td>
<td>0.0261</td>
<td>8.199 to 8.303</td>
</tr>
</tbody>
</table>

How good is the fit?

\[
R^2 = 99.90\%.
\]

This is the percent of the variance in C:zLaser explained by the model.

The P value is < 0.0001, considered extremely significant.

The P value answers this question:

If there were no linear relationship among the variables, what is the chance that R squared would be that high (or higher) by chance?

\[
\begin{array}{l}
\text{Sum-of-squares} \quad 0.03131 \\
\text{SD of residuals} \quad 0.01797 \\
\text{R squared} \quad 0.9990 \\
\text{Adjusted R squared} \quad 0.9990 \\
\text{Multiple R} \quad 0.9995 \\
\text{F} \quad 50148.0993 \\
\end{array}
\]

Which variable(s) make a significant contribution?

<table>
<thead>
<tr>
<th>Variable</th>
<th>t ratio</th>
<th>P value</th>
<th>Significant?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>286.24</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>F:xRight</td>
<td>123.49</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>316.30</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Each P value compares the full model with a simpler model omitting one variable. It tests the effect of one variable, after accounting for the effects of the others.

Is multi-collinearity a problem?

122
Variable | VIF | R2 with other X
---|---|---
F:xRight | 1.23 | 0.1897
H:xDiff | 1.23 | 0.1897

Each R squared quantifies how well that X variable is predicted from the other X variables (ignoring Y). VIF is calculated from R squared.

All R squared values are low (<0.75). The X variables are independent of each other. Multi-collinearity is not a problem.

Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>F:</th>
<th>H:</th>
<th>C: (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F:xRight</td>
<td>1.0000</td>
<td>-0.4356</td>
<td>-0.0501</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>-0.4356</td>
<td>1.0000</td>
<td>0.9204</td>
</tr>
</tbody>
</table>

Each correlation coefficient (r) is calculated independently, without considering the other variables.

Summary of your data

Number of rows (subjects) analyzed: 100
Number of rows with missing data, excluded from calculations: 0
Number of degrees of freedom (#subjects - #variables - 1): 97
Multiple Regression Results for TB33-4

What equation fits the data the best?

\[ [C:z\text{Laser}] = 0.8636 + 0.4216[D:x\text{Left}] + 7.644[H:x\text{Diff}] - 0.7994[I:y\text{Diff}] \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>SE</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>0.8636</td>
<td>0.01363</td>
<td>0.8365 to 0.8907</td>
</tr>
<tr>
<td>D:xLeft</td>
<td>0.4216</td>
<td>0.006381</td>
<td>0.4089 to 0.4343</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>7.644</td>
<td>0.03847</td>
<td>7.567 to 7.720</td>
</tr>
<tr>
<td>I:yDiff</td>
<td>-0.7994</td>
<td>0.2102</td>
<td>-1.217 to -0.3815</td>
</tr>
</tbody>
</table>

How good is the fit?

R squared = 99.76%.
This is the percent of the variance in C:zLaser explained by the model.

The P value is < 0.0001, considered extremely significant.
The P value answers this question:
If there were no linear relationship among the variables, what is
the chance that R squared would be that high (or higher) by chance?

| Sum-of-squares | 0.07619 |
| SD of residuals | 0.02817 |
| R squared       | 0.9976  |
| Adjusted R squared | 0.9976 |
| Multiple R      | 0.9988  |
| F               | 13579.7300 |

Which variable(s) make a significant contribution?

<table>
<thead>
<tr>
<th>Variable</th>
<th>t ratio</th>
<th>P value</th>
<th>Significant?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>63.367</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>D:xLeft</td>
<td>66.075</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>198.72</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>I:yDiff</td>
<td>3.803</td>
<td>0.0003</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Each P value compares the full model with a simpler model omitting
one variable. It tests the effect of one variable, after accounting
for the effects of the others.
Is multi-collinearity a problem?

<table>
<thead>
<tr>
<th>Variable</th>
<th>VIF</th>
<th>R2 with other X</th>
</tr>
</thead>
<tbody>
<tr>
<td>D:xLeft</td>
<td>1.05</td>
<td>0.0521</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>1.07</td>
<td>0.0661</td>
</tr>
<tr>
<td>I:yDiff</td>
<td>1.02</td>
<td>0.0152</td>
</tr>
</tbody>
</table>

Each R squared quantifies how well that X variable is predicted from the other X variables (ignoring Y). VIF is calculated from R squared.

All R squared values are low (<0.75). The X variables are independent of each other. Multi-collinearity is not a problem.

Correlation Matrix

```
   D:  H:  I:  | C: (Y)
D:xLeft  1.0000 -0.2275 0.0100 | 0.1041
H:xDiff  -0.2275 1.0000 -0.1218 | 0.9435
I:yDiff  0.0100 -0.1218 1.0000 | -0.1396
```

Each correlation coefficient (r) is calculated independently, without considering the other variables.

Summary of your data

Number of rows (subjects) analyzed: 100
Number of rows with missing data, excluded from calculations: 0
Number of degrees of freedom (#subjects - #variables - 1): 96
Multiple Regression Results for TB33-4

What equation fits the data the best?

\[ \text{[C:zLaser]} = 0.8153 + 0.4220^{*} \text{[D:xLeft]} + 7.662^{*} \text{[H:xDiff]} \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>SE</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>0.8153</td>
<td>0.005286</td>
<td>0.8048 to 0.8258</td>
</tr>
<tr>
<td>D:xLeft</td>
<td>0.4220</td>
<td>0.006808</td>
<td>0.4085 to 0.4356</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>7.662</td>
<td>0.04074</td>
<td>7.581 to 7.743</td>
</tr>
</tbody>
</table>

How good is the fit?

R squared = 99.73%.
This is the percent of the variance in C:zLaser explained by the model.

The P value is < 0.0001, considered extremely significant.
The P value answers this question:
If there were no linear relationship among the variables, what is
the chance that R squared would be that high (or higher) by chance?

<table>
<thead>
<tr>
<th>Sum-of-squares</th>
<th>0.08766</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD of residuals</td>
<td>0.03006</td>
</tr>
<tr>
<td>R squared</td>
<td>0.9973</td>
</tr>
<tr>
<td>Adjusted R squared</td>
<td>0.9972</td>
</tr>
<tr>
<td>Multiple R</td>
<td>0.9986</td>
</tr>
<tr>
<td>F</td>
<td>17880.8323</td>
</tr>
</tbody>
</table>

Which variable(s) make a significant contribution?

<table>
<thead>
<tr>
<th>Variable</th>
<th>t ratio</th>
<th>P value</th>
<th>Significant?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>154.24</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>D:xLeft</td>
<td>61.994</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>188.08</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Each P value compares the full model with a simpler model omitting one variable. It tests the effect of one variable, after accounting for the effects of the others.

Is multi-collinearity a problem?

126
Variable VIF R$^2$ with other X
D:xLeft 1.05 0.0518
H:xDiff 1.05 0.0518

Each R squared quantifies how well that X variable is predicted from the other X variables (ignoring Y). VIF is calculated from R squared.

All R squared values are low (<0.75). The X variables are independent of each other. Multi-collinearity is not a problem.

Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>D:</th>
<th>H:</th>
<th>C: (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D:xLeft</td>
<td>1.0000</td>
<td>-0.2275</td>
<td>0.1041</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>-0.2275</td>
<td>1.0000</td>
<td>0.9435</td>
</tr>
</tbody>
</table>

Each correlation coefficient (r) is calculated independently, without considering the other variables.

Summary of your data

Number of rows (subjects) analyzed: 100
Number of rows with missing data, excluded from calculations: 0
Number of degrees of freedom (#subjects - #variables - 1): 97
Multiple Regression Results for TB33-4

What equation fits the data the best?

\[ C:zLaser = 0.8153 + 0.4220 \times F:xRight + 8.084 \times H:xDiff \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>SE</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>0.8153</td>
<td>0.005286</td>
<td>0.8048 to 0.8258</td>
</tr>
<tr>
<td>F:xRight</td>
<td>0.4220</td>
<td>0.006808</td>
<td>0.4085 to 0.4356</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>8.084</td>
<td>0.04280</td>
<td>7.999 to 8.169</td>
</tr>
</tbody>
</table>

How good is the fit?

R squared = 99.73%.
This is the percent of the variance in C:zLaser explained by the model.

The P value is < 0.0001, considered extremely significant.
The P value answers this question:
If there were no linear relationship among the variables, what is
the chance that R squared would be that high (or higher) by chance?

| Sum-of-squares | 0.08766       |
| SD of residuals | 0.03006       |
| R squared      | 0.9973        |
| Adjusted R squared | 0.9972      |
| Multiple R     | 0.9986        |
| F             | 17881.0637    |

Which variable(s) make a significant contribution?

<table>
<thead>
<tr>
<th>Variable</th>
<th>t ratio</th>
<th>P value</th>
<th>Significant?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>154.24</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>F:xRight</td>
<td>61.994</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>H:xDiff</td>
<td>188.86</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Each P value compares the full model with a simpler model omitting
one variable. It tests the effect of one variable, after accounting
for the effects of the others.

Is multi-collinearity a problem?
Each R squared quantifies how well that X variable is predicted from the other X variables (ignoring Y). VIF is calculated from R squared.

All R squared values are low (<0.75). The X variables are independent of each other. Multi-collinearity is not a problem.

Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>F:</th>
<th>H:</th>
<th>I</th>
<th>C: (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F: xRight</td>
<td>1.0000</td>
<td>-0.3756</td>
<td></td>
<td>-0.0510</td>
</tr>
<tr>
<td>H: xDiff</td>
<td>-0.3756</td>
<td>1.0000</td>
<td></td>
<td>0.9435</td>
</tr>
</tbody>
</table>

Each correlation coefficient (r) is calculated independently, without considering the other variables.

Summary of your data

Number of rows (subjects) analyzed: 100
Number of rows with missing data, excluded from calculations: 0
Number of degrees of freedom (#subjects - #variables - 1): 97
Multiple Regression Results for TB41-1

What equation fits the data the best?

\[ [C:zLaser] = 0.8111 + 0.7644[D:xl] + 8.376[H:deltaX] - 0.4096[I:deltaY] \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>SE</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>0.8111</td>
<td>0.01685</td>
<td>0.7776 to 0.8446</td>
</tr>
<tr>
<td>D:xl</td>
<td>0.7644</td>
<td>0.006939</td>
<td>0.7506 to 0.7782</td>
</tr>
<tr>
<td>H:deltaX</td>
<td>8.376</td>
<td>0.04062</td>
<td>8.295 to 8.456</td>
</tr>
<tr>
<td>I:deltaY</td>
<td>-0.4096</td>
<td>0.1738</td>
<td>-0.7553 to -0.06403</td>
</tr>
</tbody>
</table>

How good is the fit?

R squared = 99.79%.
This is the percent of the variance in C:zLaser explained by the model.

The P value is < 0.0001, considered extremely significant.
The P value answers this question:
If there were no linear relationship among the variables, what is
the chance that R squared would be that high (or higher) by chance?

| Sum-of-squares | 0.07699  |
| SD of residuals | 0.02832  |
| R squared       | 0.9979   |
| Adjusted R squared | 0.9978  |
| Multiple R      | 0.9989   |
| F               | 14976.9874 |

Which variable(s) make a significant contribution?

<table>
<thead>
<tr>
<th>Variable</th>
<th>t ratio</th>
<th>P value</th>
<th>Significant?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>48.128</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>D:xl</td>
<td>110.16</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>H:deltaX</td>
<td>206.21</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>I:deltaY</td>
<td>2.356</td>
<td>0.0205</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Each P value compares the full model with a simpler model omitting
one variable. It tests the effect of one variable, after accounting
for the effects of the others.
Is multi-collinearity a problem?

<table>
<thead>
<tr>
<th>Variable</th>
<th>VIF</th>
<th>R2 with other X</th>
</tr>
</thead>
<tbody>
<tr>
<td>D:xl</td>
<td>1.17</td>
<td>0.1424</td>
</tr>
<tr>
<td>H:deltaX</td>
<td>1.16</td>
<td>0.1409</td>
</tr>
<tr>
<td>I:deltaY</td>
<td>1.06</td>
<td>0.0524</td>
</tr>
</tbody>
</table>

Each R squared quantifies how well that X variable is predicted from the other X variables (ignoring Y). VIF is calculated from R squared.

All R squared values are low (<0.75). The X variables are independent of each other. Multi-collinearity is not a problem.

Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>D:</th>
<th>H:</th>
<th>I:</th>
<th>C: (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D:xl</td>
<td>1.0000</td>
<td>-0.3555</td>
<td>0.1907</td>
<td>0.1857</td>
</tr>
<tr>
<td>H:deltaX</td>
<td>-0.3555</td>
<td>1.0000</td>
<td>-0.1862</td>
<td>0.8513</td>
</tr>
<tr>
<td>I:deltaY</td>
<td>0.1907</td>
<td>-0.1862</td>
<td>1.0000</td>
<td>-0.0998</td>
</tr>
</tbody>
</table>

Each correlation coefficient (r) is calculated independently, without considering the other variables.

Summary of your data

Number of rows (subjects) analyzed: 100
Number of rows with missing data, excluded from calculations: 0
Number of degrees of freedom (#subjects - #variables - 1): 96

. . . .
Multiple Regression Results for TB41-1

What equation fits the data the best?

\[ \text{[C:zLaser]} = 0.7729 + 0.7622[D:xl] + 8.388[H:deltaX] \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>SE</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>0.7729</td>
<td>0.004726</td>
<td>0.7635 to 0.7823</td>
</tr>
<tr>
<td>D:xl</td>
<td>0.7622</td>
<td>0.007034</td>
<td>0.7482 to 0.7761</td>
</tr>
<tr>
<td>H:deltaX</td>
<td>8.388</td>
<td>0.04121</td>
<td>8.306 to 8.470</td>
</tr>
</tbody>
</table>

How good is the fit?

R squared = 99.77%.
This is the percent of the variance in C:zLaser explained by the model.

The P value is < 0.0001, considered extremely significant.
The P value answers this question:
If there were no linear relationship among the variables, what is
the chance that R squared would be that high (or higher) by chance?

| Sum-of-squares | 0.08144 | SD of residuals | 0.02898 | R squared | 0.9977 | Adjusted R squared | 0.9977 | Multiple R | 0.9989 | F | 21455.7823 |

Which variable(s) make a significant contribution?

<table>
<thead>
<tr>
<th>Variable</th>
<th>t ratio</th>
<th>P value</th>
<th>Significant?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>163.55</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>D:xl</td>
<td>108.35</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>H:deltaX</td>
<td>203.54</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Each P value compares the full model with a simpler model omitting
one variable. It tests the effect of one variable, after accounting
for the effects of the others.

Is multi-colinearity a problem?

132
Each R squared quantifies how well that X variable is predicted from the other X variables (ignoring Y). VIF is calculated from R squared.

All R squared values are low (<0.75). The X variables are independent of each other. Multi-collinearity is not a problem.

Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>D:</th>
<th>H:</th>
<th>C: (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D:xl</td>
<td>1.0000</td>
<td>-0.3555</td>
<td>0.1857</td>
</tr>
<tr>
<td>H:deltaX</td>
<td>-0.3555</td>
<td>1.0000</td>
<td>0.8513</td>
</tr>
</tbody>
</table>

Each correlation coefficient (r) is calculated independently, without considering the other variables.

Summary of your data

Number of rows (subjects) analyzed: 100
Number of rows with missing data, excluded from calculations: 0
Number of degrees of freedom (#subjects - #variables - 1): 97
What equation fits the data the best?

\[ [C:\text{zLaser}] = 0.7729 + 0.7622[F:xr] + 9.150[H:deltaX] \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>SE</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>0.7729</td>
<td>0.004726</td>
<td>0.7635 to 0.7823</td>
</tr>
<tr>
<td>F:xr</td>
<td>0.7622</td>
<td>0.007034</td>
<td>0.7482 to 0.7761</td>
</tr>
<tr>
<td>H:deltaX</td>
<td>9.150</td>
<td>0.04420</td>
<td>9.062 to 9.238</td>
</tr>
</tbody>
</table>

How good is the fit?

R squared = 99.77%.
This is the percent of the variance in \( C:\text{zLaser} \) explained by the model.

The P value is < 0.0001, considered extremely significant.
The P value answers this question:
If there were no linear relationship among the variables, what is
the chance that R squared would be that high (or higher) by chance?

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum-of-squares</td>
<td>0.08144</td>
</tr>
<tr>
<td>SD of residuals</td>
<td>0.02898</td>
</tr>
<tr>
<td>R squared</td>
<td>0.9977</td>
</tr>
<tr>
<td>Adjusted R squared</td>
<td>0.9977</td>
</tr>
<tr>
<td>Multiple R</td>
<td>0.9989</td>
</tr>
<tr>
<td>F</td>
<td>21455.9483</td>
</tr>
</tbody>
</table>

Which variable(s) make a significant contribution?

<table>
<thead>
<tr>
<th>Variable</th>
<th>t ratio</th>
<th>P value</th>
<th>Significant?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>163.55</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>F:xr</td>
<td>108.35</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>H:deltaX</td>
<td>207.00</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Each P value compares the full model with a simpler model omitting
one variable. It tests the effect of one variable, after accounting
for the effects of the others.

Is multi-collinearity a problem?
Variable | VIF | R2 with other X
--- | --- | ---
F:xr | 1.32 | 0.2406
H:deltaX | 1.32 | 0.2406

Each R squared quantifies how well that X variable is predicted from the other X variables (ignoring Y). VIF is calculated from R squared.

All R squared values are low (<0.75). The X variables are independent of each other. Multi-collinearity is not a problem.

Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>F:</th>
<th>H:</th>
<th>C: (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F:xr</td>
<td>1.0000</td>
<td>-0.4905</td>
<td>0.0377</td>
</tr>
<tr>
<td>H:deltaX</td>
<td>-0.4905</td>
<td>1.0000</td>
<td>0.8513</td>
</tr>
</tbody>
</table>

Each correlation coefficient (r) is calculated independently, without considering the other variables.

Summary of your data

Number of rows (subjects) analyzed: 100
Number of rows with missing data, excluded from calculations: 0
Number of degrees of freedom (#subjects - #variables - 1): 97

...
Multiple Regression Results for TB41-8

What equation fits the data the best?

\[ C_{\text{z}\text{Laser}} = 1.912 + 0.5938[D_{x\text{l}}] + 7.515[H_{\text{deltaX}}] - 4.267[I_{\text{deltaY}}] \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>SE</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>1.912</td>
<td>0.005102</td>
<td>1.902 to 1.922</td>
</tr>
<tr>
<td>D_{x\text{l}}</td>
<td>0.5938</td>
<td>0.003641</td>
<td>0.5866 to 0.6010</td>
</tr>
<tr>
<td>H_{\text{deltaX}}</td>
<td>7.515</td>
<td>0.01856</td>
<td>7.478 to 7.552</td>
</tr>
<tr>
<td>I_{\text{deltaY}}</td>
<td>-4.267</td>
<td>0.1357</td>
<td>-4.537 to -3.997</td>
</tr>
</tbody>
</table>

How good is the fit?

R squared = 99.95%.
This is the percent of the variance in C_{z\text{Laser}} explained by the model.

The P value is < 0.0001, considered extremely significant.
The P value answers this question:
If there were no linear relationship among the variables, what is
the chance that R squared would be that high (or higher) by chance?

- Sum-of-squares: 0.01978
- SD of residuals: 0.01436
- R squared: 0.9995
- Adjusted R squared: 0.9994
- Multiple R: 0.9997
- F: 58374.4925

Which variable(s) make a significant contribution?

<table>
<thead>
<tr>
<th>Variable</th>
<th>t ratio</th>
<th>P value</th>
<th>Significant?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>374.79</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>D_{x\text{l}}</td>
<td>163.08</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>H_{\text{deltaX}}</td>
<td>404.82</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>I_{\text{deltaY}}</td>
<td>31.443</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Each P value compares the full model with a simpler model omitting
one variable. It tests the effect of one variable, after accounting
for the effects of the others.
Is multi-collinearity a problem?

<table>
<thead>
<tr>
<th>Variable</th>
<th>VIF</th>
<th>R2 with other X</th>
</tr>
</thead>
<tbody>
<tr>
<td>D:xl</td>
<td>1.37</td>
<td>0.2706</td>
</tr>
<tr>
<td>H:deltaX</td>
<td>1.12</td>
<td>0.1037</td>
</tr>
<tr>
<td>I:deltaY</td>
<td>1.35</td>
<td>0.2618</td>
</tr>
</tbody>
</table>

Each R squared quantifies how well that X variable is predicted from the other X variables (ignoring Y). VIF is calculated from R squared.

All R squared values are low (<0.75). The X variables are independent of each other. Multi-collinearity is not a problem.

Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>D:</th>
<th>H:</th>
<th>I:</th>
<th>C: (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D:xl</td>
<td>1.0000</td>
<td>-0.2878</td>
<td>0.4946</td>
<td>0.1189</td>
</tr>
<tr>
<td>H:deltaX</td>
<td>-0.2878</td>
<td>1.0000</td>
<td>-0.2680</td>
<td>0.9137</td>
</tr>
<tr>
<td>I:deltaY</td>
<td>0.4946</td>
<td>-0.2680</td>
<td>1.0000</td>
<td>-0.1356</td>
</tr>
</tbody>
</table>

Each correlation coefficient (r) is calculated independently, without considering the other variables.

Summary of your data

Number of rows (subjects) analyzed: 100
Number of rows with missing data, excluded from calculations: 0
Number of degrees of freedom (#subjects - #variables - 1): 96
Multiple Regression Results for TB41-8

What equation fits the data the best?

\[ C:zLaser = 1.956 + 0.5420[D:xl] + 7.603[H:deltaX] \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>SE</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>1.956</td>
<td>0.01640</td>
<td>1.924 to 1.989</td>
</tr>
<tr>
<td>D:xl</td>
<td>0.5420</td>
<td>0.01086</td>
<td>0.5204 to 0.5636</td>
</tr>
<tr>
<td>H:deltaX</td>
<td>7.603</td>
<td>0.06136</td>
<td>7.481 to 7.725</td>
</tr>
</tbody>
</table>

How good is the fit?

R squared = 99.38%.
This is the percent of the variance in C:zLaser explained by the model.

The P value is < 0.0001, considered extremely significant.
The P value answers this question:
If there were no linear relationship among the variables, what is
the chance that R squared would be that high (or higher) by chance?

| Sum-of-squares | 0.2235 |
| SD of residuals | 0.04800 |
| R squared       | 0.9938 |
| Adjusted R squared | 0.9937 |
| Multiple R      | 0.9969 |
| F               | 7786.5176 |

Which variable(s) make a significant contribution?

<table>
<thead>
<tr>
<th>Variable</th>
<th>t ratio</th>
<th>P value</th>
<th>Significant?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>119.31</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>D:xl</td>
<td>49.916</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>H:deltaX</td>
<td>123.90</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Each P value compares the full model with a simpler model omitting
one variable. It tests the effect of one variable, after accounting
for the effects of the others.
Is multi-collinearity a problem?

Variable | VIF | R2 with other X
---|---|---
D:xl | 1.09 | 0.0828
H:deltaX | 1.09 | 0.0828

Each R squared quantifies how well that X variable is predicted from the other X variables (ignoring Y). VIF is calculated from R squared.

All R squared values are low (<0.75). The X variables are independent of each other. Multi-collinearity is not a problem.

Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>D:</th>
<th>H:</th>
<th>C: (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D:xl</td>
<td>1.0000</td>
<td>-0.2878</td>
<td>0.1189</td>
</tr>
<tr>
<td>H:deltaX</td>
<td>-0.2878</td>
<td>1.0000</td>
<td>0.9137</td>
</tr>
</tbody>
</table>

Each correlation coefficient (r) is calculated independently, without considering the other variables.

Summary of your data

Number of rows (subjects) analyzed: 100
Number of rows with missing data, excluded from calculations: 0
Number of degrees of freedom (#subjects - #variables - 1): 97
Multiple Regression Results for TB41-8

What equation fits the data the best?

\[ [C:zLaser] = 1.956 + 0.5420[F:xr] + 8.145[H:deltaX] \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>SE</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>1.956</td>
<td>0.01640</td>
<td>1.924 to 1.989</td>
</tr>
<tr>
<td>F:xr</td>
<td>0.5420</td>
<td>0.01086</td>
<td>0.5204 to 0.5636</td>
</tr>
<tr>
<td>H:deltaX</td>
<td>8.145</td>
<td>0.06532</td>
<td>8.015 to 8.275</td>
</tr>
</tbody>
</table>

How good is the fit?

R squared = 99.38%.
This is the percent of the variance in C:zLaser explained by the model.

The P value is < 0.0001, considered extremely significant.
The P value answers this question:
If there were no linear relationship among the variables, what is
the chance that R squared would be that high (or higher) by chance?

| Sum-of-squares | 0.2235 |
| SD of residuals| 0.04800 |
| R squared      | 0.9938 |
| Adjusted R squared | 0.9937 |
| Multiple R     | 0.9969 |
| F              | 7786.5176 |

Which variable(s) make a significant contribution?

<table>
<thead>
<tr>
<th>Variable</th>
<th>t ratio</th>
<th>P value</th>
<th>Significant?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant)</td>
<td>119.31</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>F:xr</td>
<td>49.916</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
<tr>
<td>H:deltaX</td>
<td>124.69</td>
<td>&lt; 0.0001</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Each P value compares the full model with a simpler model omitting
one variable. It tests the effect of one variable, after accounting
for the effects of the others.
Is multi-collinearity a problem?

Variable | VIF | R2 with other X
---|---|---
F:xr | 1.24 | 0.1906
H:deltaX | 1.24 | 0.1906

Each R squared quantifies how well that X variable is predicted from the other X variables (ignoring Y). VIF is calculated from R squared.

All R squared values are low (<0.75). The X variables are independent of each other. Multi-collinearity is not a problem.

Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>F:</th>
<th>H:</th>
<th>C: (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F:xr</td>
<td>1.0000</td>
<td>-0.4366</td>
<td>-0.0402</td>
</tr>
<tr>
<td>H:deltaX</td>
<td>-0.4366</td>
<td>1.0000</td>
<td>0.9137</td>
</tr>
</tbody>
</table>

Each correlation coefficient (r) is calculated independently, without considering the other variables.

Summary of your data

Number of rows (subjects) analyzed: 100
Number of rows with missing data, excluded from calculations: 0
Number of degrees of freedom (#subjects - #variables - 1): 97
APPENDIX B

Calculation of Scaled-Up Variables

Based on the principles of similarity, the Reynolds number and Womersley number are to be kept the same. The frequency of the pulsatile flow wave is known and is 3 seconds. The dimensionless parameters were first proposed to be maximum $N_R= 180$ and $\alpha = 2.7$. In order for this to be accomplished, given a period of $T = 3$ seconds, the required fluid kinematic viscosity was calculated in the following way:

$$\alpha = \left( \frac{\omega D^2}{4v} \right)^2$$

solving for viscosity:

$$v = \frac{\omega D^2}{4\alpha^2}.$$  

$v = 0.2149 \text{cm}^2/\text{sec}.$

Using this kinematic viscosity and a target maximum $N_R$ of 180 gives us the following estimation of maximum velocity.

$$V = \frac{N_R v}{D} = 22.36 \text{cm/sec}.$$
The bulk flow rate, which is physically limited by the steady flow pump to 3.2 l/min. is calculated by:

\[ Q = VA = 3.154 \text{l/min}. \]

Since this is calculated based on the maximum Reynolds number, it should be the worst-case scenario presented to the system. If the equipment can meet these requirements, and the fluid can be made to the above mentioned viscosity and maintain an index of refraction of \( n=1.4135 \), these desired experimental conditions can be met. The index of refraction is in order to match the index of refraction of the molding material, Dow Corning Sylgard 184 silicone elastomer modeling compound. The PTV algorithm does not require matching the index of refraction, however, it does make some of the steps less complicated.
APPENDIX C

Working Fluid Calculations

The fluid was made by first creating a mixture of equal amounts of both a 14% solution of glycerol in water and a 10% solution of propylene glycol in water to produce a solution of the correct viscosity. Since the index of refraction of this solution is much lower than needed, sodium thiocyanate was added until the index of refraction of the fluid was matched to that of the Dow Corning Sylgard, 1.4135.

After making the fluid, the actual physical properties were measured. The density of the fluid was measured experimentally to be 1.18 g/cc. Fluid viscosity check was completed using a Gilmont Instruments (Barrington, IL) falling ball viscometer, model number GV-2100. The equation to calculate viscosity from Gilmont is as follows:

\[ \mu = K (\rho_r - \rho) \times t \]

where
- \( \rho_r = 8.02 \text{ g/ml for stainless steel ball} \)
- \( t = \text{time in minutes} \)
- \( \rho = \text{density fluid (g/ml)} \)
- \( K = 0.3 \)

The viscosity was found to be 0.1995 poise. This gives an actual experimental kinematic viscosity of 0.1691 cm²/sec, which is less than the target viscosity.
Using this viscosity to re-calculate the maximum velocity and Womersley parameter obtainable using these physical properties maintaining the Reynolds number at 180 gives us a maximum velocity of 17.59 cm/sec., which corresponds to a peak mass flow rate of 2.48 l/min, and a Womersley parameter of 3.04. These physical parameters are all within the range of physiologic left coronary parameters and are physically obtainable by the flow system. Therefore, the working fluid as made is deemed appropriate to meet the experimental conditions.
APPENDIX D

Calculation of Particle Properties

The particles consist of a mixture of plyolite (polyvinyl toluene butadiene), magnesium oxide and Pylakrome oil fluor yellow LX-8248 combined according to the following calculation. In addition, from trial and error, it was determined that the process of melting together the plyolite and magnesium oxide introduced air on the surface of the particles. To counteract this problem, which caused the particles produced to float in the working fluid, it was determined experimentally that the target density for the particles needed to be 1.28 g/ml. The calculations below are based on this experimental observation. The actual particles retained from the particle batch were the ones that remained suspended while centrifuged in the working fluid.

\[
\begin{align*}
\rho_{\text{working fluid}} &= 1.18 \text{ g/ml} \\
\rho_{\text{target}} &= 1.280 \text{ g/ml} \\
\rho_{\text{plyolite}} &= 1.024 \text{ g/ml} \\
\rho_{\text{MgO}} &= 3.58 \\
x &= \text{weight fraction Plyolite} \\
x \rho_{\text{plyolite}} + (1 - x) \rho_{\text{MgO}} &= \rho_{\text{working fluid}} \\
x &= .8998
\end{align*}
\]
The weight fraction of Plyolite added was 89.98% and 10.02% MgO was also added. The dye, which is needed to visualize the particles, was added in a small quantity, approximately 1g, so its addition did not disrupt the density of the mixture. This mixture was ground together in a mortar and pestle, and then slowly melted into a solid mass. This mass was then ground in a mortar and pestle to provide a variety of particle sizes to separate.

The required size of the particles was determined in the following way.

1. Calculate the residence time in the viewing area.
2. Calculate the terminal velocity for the particle.
3. Calculate the distance the particle falls per frame.
4. Calculate the distance the particle falls per particle diameter ratio.
5. Calculate the ratio of the minimum vessel diameter to particle diameter.

From these pieces of information and the framing rate and shutter speed, determine the best particle size for the system. The calculations are as follows for the TB33 vessel. Table C.1 contains pertinent physical data for TB33. Peak flows are used in the calculations as the worst-case scenario.
<table>
<thead>
<tr>
<th>TB33</th>
<th>Radius, r (cm)</th>
<th>Area, A (cm²)</th>
<th>Peak Bulk Flow, Q (cm³/sec)</th>
<th>Peak Velocity, V (cm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM</td>
<td>1.28,0.77</td>
<td>3.10</td>
<td>41.33</td>
<td>13.33</td>
</tr>
<tr>
<td>LAD</td>
<td>0.62,0.54</td>
<td>1.05</td>
<td>18.60</td>
<td>17.71</td>
</tr>
<tr>
<td>LCX</td>
<td>0.73,0.73</td>
<td>1.67</td>
<td>22.73</td>
<td>13.61</td>
</tr>
</tbody>
</table>

Table D.1 Table of TB33 Physical Data

\[ V = \frac{Q}{A} \]
\[ \mu = 0.1995 \text{poise} \]
\[ \rho_f = 1.18 \frac{g}{ml} \]
\[ \rho_p = 1.19 \frac{g}{ml} \]
\[ \nu = 0.169 \frac{cm^2}{sec} \]

Maximum Field of View = 2 cm x 2 cm

D = Distance Particle Falls

Maximum Residence Time in Viewing Area = \( T_{LM} + T_{LCX} = T_{Ree} = 0.2970 \text{sec} \)

\[ V_{term} = \frac{2gr_p^2(\rho_p - \rho_f)}{9\mu} \]
\[ N_{RePurt} = \frac{d_p V_{LAD}}{\nu} \]
\[ d_p \leq \frac{18\mu_f}{(\rho_p - \rho_f)gT_{Ree}} \leq \frac{0.01233cm}{(\rho_p - \rho_f)} \leq 1.233cm \]
<table>
<thead>
<tr>
<th>Particle Size range (µm)</th>
<th>Screen</th>
<th>d_p(µm)</th>
<th>Mean Diam.</th>
<th>N_Re,p</th>
<th>V_Term (cm/sec)</th>
<th>D(cm)</th>
<th>D/d_p</th>
<th>Min.D_p/d_p</th>
</tr>
</thead>
<tbody>
<tr>
<td>297-354</td>
<td>50</td>
<td>325.5</td>
<td>3.41</td>
<td>0.0116</td>
<td>0.0034</td>
<td>0.1056</td>
<td>37.88</td>
<td></td>
</tr>
<tr>
<td>250-297</td>
<td>60</td>
<td>273.5</td>
<td>2.86</td>
<td>0.0082</td>
<td>0.0024</td>
<td>0.0887</td>
<td>45.08</td>
<td></td>
</tr>
<tr>
<td>210-250</td>
<td>70</td>
<td>230</td>
<td>2.41</td>
<td>0.0058</td>
<td>0.0017</td>
<td>0.0746</td>
<td>53.61</td>
<td></td>
</tr>
<tr>
<td>177-210</td>
<td>80</td>
<td>193.5</td>
<td>2.03</td>
<td>0.0041</td>
<td>0.0012</td>
<td>0.0628</td>
<td>63.72</td>
<td></td>
</tr>
<tr>
<td>149-177</td>
<td>100</td>
<td>163</td>
<td>1.71</td>
<td>0.0029</td>
<td>0.0009</td>
<td>0.0529</td>
<td>75.64</td>
<td></td>
</tr>
<tr>
<td>125-149</td>
<td>120</td>
<td>137</td>
<td>1.43</td>
<td>0.0021</td>
<td>0.0006</td>
<td>0.0444</td>
<td>90.00</td>
<td></td>
</tr>
<tr>
<td>105-125</td>
<td>140</td>
<td>115</td>
<td>1.20</td>
<td>0.0014</td>
<td>0.0004</td>
<td>0.0373</td>
<td>107.22</td>
<td></td>
</tr>
</tbody>
</table>

Table D.2 Table of Particle Calculations for TB33

Based on the above calculations, the settling velocity was not a problem for any reasonable size particle. In addition to the parameters calculated in Table C.2, the other consideration in choosing the best particle diameter for PTV experiments is the size the particle will be imaged in the given experimental set-up. It is ideal to image the particles at approximately 3-5 pixels, which will make them distinguishable from the noise in the images. This procedure was done with particles in the above size ranges and the exact camera/lens/mirror system used for the experiments. Based on these data, particles in the 250 - 297 micron region were chosen for the final experiments.

The following is a presentation of the same analysis for the TB41 replica.
<table>
<thead>
<tr>
<th>TB41</th>
<th>Radius, r (cm)</th>
<th>Area, A (cm²)</th>
<th>Peak Bulk Flow, Q (cm³/sec)</th>
<th>Peak Velocity, V (cm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM</td>
<td>0.89, 0.815</td>
<td>2.28</td>
<td>36.5</td>
<td>16.01</td>
</tr>
<tr>
<td>LAD</td>
<td>0.73, 0.585</td>
<td>1.34</td>
<td>16.79</td>
<td>12.53</td>
</tr>
<tr>
<td>LCX</td>
<td>0.705, 0.705</td>
<td>1.56</td>
<td>19.71</td>
<td>12.63</td>
</tr>
</tbody>
</table>

Table D.3 Table of TB41 Physical Data
\[ v = \frac{Q}{A} \]
\[ \mu = 0.1995 \text{ poise} \]
\[ \rho_t = 1.18 \frac{g}{ml} \]
\[ \rho_r = 1.19 \frac{g}{ml} \]
\[ \nu = 0.1691 \frac{cm^2}{sec} \]

Maximum Field of View = 2 cm x 2 cm

D = Distance Particle Falls

Maximum Residence Time in Viewing Area = \[ T_{LM} + T_{LAD} = T_{Re} = 0.2845 \text{ sec} \]

\[ V_{fert} = \frac{2gr^2(\rho_p - \rho_f)}{9\mu} \]

\[ N_{Re Part} = \frac{d_pV_{LM}}{\nu} \]

\[ d_p \leq \frac{18\mu_f}{(\rho_p - \rho_f)gT_{Re}} \leq \frac{0.0129 \text{ cm}}{(\rho_p - \rho_f)} \leq 1.29 \text{ cm} \]
<table>
<thead>
<tr>
<th>Particle Size range (μm)</th>
<th>Screen</th>
<th>$d_p(\mu m)$</th>
<th>Mean Diam.</th>
<th>N$_{Re,P}$</th>
<th>$V_{Term}$ (cm/sec)</th>
<th>D(cm)</th>
<th>D/$d_p$</th>
<th>Min.$D_V/d_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>297-354</td>
<td>50</td>
<td>325.5</td>
<td>3.08</td>
<td>0.0116</td>
<td>0.0033</td>
<td>0.1012</td>
<td>39.63</td>
<td></td>
</tr>
<tr>
<td>250-297</td>
<td>60</td>
<td>273.5</td>
<td>2.59</td>
<td>0.0082</td>
<td>0.0023</td>
<td>0.0850</td>
<td>47.17</td>
<td></td>
</tr>
<tr>
<td>210-250</td>
<td>70</td>
<td>230</td>
<td>2.18</td>
<td>0.0058</td>
<td>0.0016</td>
<td>0.0715</td>
<td>56.09</td>
<td></td>
</tr>
<tr>
<td>177-210</td>
<td>80</td>
<td>193.5</td>
<td>1.83</td>
<td>0.0041</td>
<td>0.0012</td>
<td>0.0601</td>
<td>66.67</td>
<td></td>
</tr>
<tr>
<td>149-177</td>
<td>100</td>
<td>163</td>
<td>1.54</td>
<td>0.0029</td>
<td>0.0008</td>
<td>0.0507</td>
<td>79.14</td>
<td></td>
</tr>
<tr>
<td>125-149</td>
<td>120</td>
<td>137</td>
<td>1.30</td>
<td>0.0021</td>
<td>0.0006</td>
<td>0.0426</td>
<td>94.16</td>
<td></td>
</tr>
<tr>
<td>105-125</td>
<td>140</td>
<td>115</td>
<td>1.09</td>
<td>0.0014</td>
<td>0.0004</td>
<td>0.0357</td>
<td>112.17</td>
<td></td>
</tr>
</tbody>
</table>

Table D.4 Table of Particle Calculations TB41

Again, the system was set up with the camera/lens/mirror configuration and particles in the given ranges above were imaged. Particles in the 250 - 297 micron range were deemed optimal for this system.
APPENDIX E

Klinger CC-1 Universal Programmer Program

COR10
PR#1
I
A
S 20
F 20
E
B
R 222
P 70
X 150
R 205
P 140
R 160
C
P85
R 90
P 0
X 250
D
Q
D
PR#0
END
APPENDIX F

Axial Velocity Profiles

The following pages contain the axial velocity profiles for both TB33 and TB41. The results are presented in order beginning with the most proximal cross section of the LM, moving distally by cross section. Results for the LCX and LAD are presented in the same manner.
Location of Cross Sections 9-11

TB33 Axial Velocity Profiles; LM cross sections 9 to 11
Location of Cross Sections 12-14

TB33 Axial Velocity Profiles; LM cross sections 12 to 14
TB33 Axial Velocity Profiles; LM cross sections 15 to 17
Location of Cross Sections 18-20

TB33 Axial Velocity Profiles; LM cross sections 18 to 20
TB33 Axial Velocity Profiles; LM cross sections 21 to 23
Location of Cross Sections 2-4

TB33 Axial Velocity Profiles; LAD cross sections 2 to 4
Location of Cross Sections 5-7

TB33 Axial Velocity Profiles; LAD cross sections 5 to 7
TB33 Axial Velocity Profiles; LAD cross sections 8 to 10
TB33 Axial Velocity Profiles; LAD cross sections 11 to 13
Location of Cross Sections 14-16

TB33 Axial Velocity Profiles; LAD cross sections 14 to 16
TB33 Axial Velocity Profiles; LAD cross sections 17 to 19
Location of Cross Sections 1-3

TBB33 Axial Velocity Profiles; LCX cross sections 1 to 3
Location of Cross Sections 4-6

TB33 Axial Velocity Profiles; LCX cross sections 4 to 6
Location of Cross Sections 7-9

TB33 Axial Velocity Profiles; LCX cross sections 7 to 9
Location of Cross Sections 10-12

TB33 Axial Velocity Profiles; LCX cross sections 10 to 12
Location of Cross Sections 13-15

TB33 Axial Velocity Profiles; LCX cross sections 13 to 15
TB33 Axial Velocity Profiles; LCX cross sections 16 to 18
TB41 Axial Velocity Profiles; LM cross sections 0 to 2
Location of Cross Sections 2-4

TB41 Axial Velocity Profiles; LM cross sections 2 to 4
TB41 Axial Velocity Profiles; LM cross sections 5 to 7
Location of Cross Sections 2-4

TB41 Axial Velocity Profiles; LAD cross sections 2 to 4
TB41 Axial Velocity Profiles; LAD cross sections 5 to 7
Location of Cross Sections 8-10

Cross-Section 8  Cross-Section 9  Cross-Section 10

Lateral

Epicardial  Myocardial  Epicardial  Myocardial  Epicardial  Myocardial

Late Diastole  Early Systole  Systole  Late Systole
Early Diastole  Early Mid-Diastole  Late Mid-Diastole  Late Diastole

TB41 Axial Velocity Profiles; LAD cross sections 8 to 10
TB41 Axial Velocity Profiles; LAD cross sections 11 to 13
TB41 Axial Velocity Profiles; LCX cross sections 1 to 3
Location of Cross Sections 3-5

TB41 Axial Velocity Profiles; LCX cross sections 3 to 5
APPENDIX G

Secondary Velocity Profiles

The following pages contain the secondary velocity profiles for both TB33 and TB41. The results are presented in order beginning with the most proximal cross section of the LM, moving distally by cross section. Results for the LCX and LAD are presented in the same manner.
Location of Cross Section 9

TB33 Secondary Velocity Profiles; LM cross section 9
Location of Cross Section 10

LM Secondary Velocities at Cross-Section 10

TB33 Secondary Velocity Profiles; LM cross section 10
TB33 Secondary Velocity Profiles; LM cross section 11
TB33 Secondary Velocity Profiles; LM cross section 12
TB33 Secondary Velocity Profiles; LM cross section 13
TB33 Secondary Velocity Profiles; LM cross section 14
Location of Cross Section 15

TB33 Secondary Velocity Profiles; LM cross section 15
Location of Cross Section 16

TB33 Secondary Velocity Profiles; LM cross section 16
TB33 Secondary Velocity Profiles; LM cross section 17
TB33 Secondary Velocity Profiles; LM cross section 18
Location of Cross Section 19

TB33 Secondary Velocity Profiles; LM cross section 19
TB33 Secondary Velocity Profiles; LM cross section 20
Location of Cross Section 21

TB33 Secondary Velocity Profiles; LM cross section 21
TB33 Secondary Velocity Profiles; LM cross section 22
Location of Cross Section 23

LM Secondary Velocities at Cross-Section 23

Late Diastole  Early Systole  Systole  Late Systole

Early Diastole  Early Rapid Deceleration  Late Rapid Deceleration  Late Diastole

TB33 Secondary Velocity Profiles; LM cross section 23
TB33 Secondary Velocity Profiles; LAD cross section 2
TB33 Secondary Velocity Profiles; LAD cross section 3
TB33 Secondary Velocity Profiles; LAD cross section 4
TB33 Secondary Velocity Profiles; LAD cross section 5
TB33 Secondary Velocity Profiles; LAD cross section 6
TB33 Secondary Velocity Profiles; LAD cross section 7
TB33 Secondary Velocity Profiles; LAD cross section 8
TB33 Secondary Velocity Profiles; LAD cross section 9
TB33 Secondary Velocity Profiles; LAD cross section 10
TB33 Secondary Velocity Profiles; LAD cross section 11
TB33 Secondary Velocity Profiles; LAD cross section 12
TB33 Secondary Velocity Profiles; LAD cross section 13
TB33 Secondary Velocity Profiles; LAD cross section 14
TB33 Secondary Velocity Profiles; LAD cross section 15
TB33 Secondary Velocity Profiles; LAD cross section 16
TB33 Secondary Velocity Profiles; LAD cross section 17
TB33 Secondary Velocity Profiles; LAD cross section 18
TB33 Secondary Velocity Profiles; LAD cross section 19
TB33 Secondary Velocity Profiles; LCX cross section 1
Location of Cross Section 2

LCX Secondary Velocities at Cross-Section 2

TB33 Secondary Velocity Profiles; LCX cross section 2
LCX Secondary Velocities at Cross-Section 3

TB33 Secondary Velocity Profiles; LCX cross section 3
TB33 Secondary Velocity Profiles; LCX cross section 4
TB33 Secondary Velocity Profiles; LCX cross section 5
Location of Cross Section 6

LCX Secondary Velocities at Cross-Section 6

Late Diastole  Early Systole  Systole  Late Systole

Early Diastole  Early Mi-Mi Diastole  Late Mi-Mi Diastole  Late Diastole

TB33 Secondary Velocity Profiles; LCX cross section 6
TB33 Secondary Velocity Profiles; LCX cross section 7
TB33 Secondary Velocity Profiles; LCX cross section 8
TB33 Secondary Velocity Profiles; LCX cross section 9
TB33 Secondary Velocity Profiles; LCX cross section 10
Location of Cross Section 11

TB33 Secondary Velocity Profiles; LCX cross section 11
TB33 Secondary Velocity Profiles; LCX cross section 12
Location of Cross Section 13

LCX Secondary Velocities at Cross-Section 13

Late Diastole  Early Systole  Systole  Late Systole

Early Diastole  Early Mid-Diastole  Late Mid-Diastole  Late Diastole

TB33 Secondary Velocity Profiles; LCX cross section 13
TB33 Secondary Velocity Profiles; LCX cross section 14
TB33 Secondary Velocity Profiles; LCX cross section 15
TB33 Secondary Velocity Profiles; LCX cross section 16
Location of Cross Section 18

TB33 Secondary Velocity Profiles; LCX cross section 18
Location of Cross Section 19

LCX Secondary Velocities at Cross-Section 19

TB33 Secondary Velocity Profiles; LCX cross section 19
TB41 Secondary Velocity Profiles; LM cross section 1
Location of Cross Section 2

LM Secondary Velocities at Cross-Section 2

Late Diastole  Early Systole  Systole  Late Systole

Early Diastole  Early Mid-Diastole  Late Mid-Diastole  Late Diastole

TB41 Secondary Velocity Profiles; LM cross section 2
Location of Cross Section 3

TB41 Secondary Velocity Profiles; LM cross section 3
Location of Cross Section 4

LM Secondary Velocities at Cross-Section 4

Late Diastole               Early Systole               Systole               Late Systole

Early Diastole              Early Mid-Diastole          Late Mid-Diastole       Late Diastole

TB41 Secondary Velocity Profiles; LM cross section 4
Location of Cross Section 5

TB41 Secondary Velocity Profiles; LM cross section 5
TB41 Secondary Velocity Profiles; LM cross section 6
TB41 Secondary Velocity Profiles; LM cross section 7
TB41 Secondary Velocity Profiles; LM cross section 8
TB41 Secondary Velocity Profiles; LAD cross section 2
TB41 Secondary Velocity Profiles; LAD cross section 3
Location of Cross Section 4

TB41 Secondary Velocity Profiles; LAD cross section 4
Location of Cross Section 5

TB41 Secondary Velocity Profiles; LAD cross section 5
TB41 Secondary Velocity Profiles; LAD cross section 6
Location of Cross Section 7

LAD Secondary Velocities at Cross-Section 7

Early Diastole  Early Systole  Systole  Late Systole

Early Diastole  Early Mid-Diastole  Late Mid-Diastole  Late Diastole

TB41 Secondary Velocity Profiles; LAD cross section 7
TB41 Secondary Velocity Profiles; LAD cross section 8
Location of Cross Section 9

LAB Secondary Velocities at Cross-Section 9

Early Diastole  Early Systole  Systole  Late Systole

Early Diastole  Early Mit-Diastole  Late Mit-Diastole  Late Diastole

> 0.20 cm/s
0.18 cm/s
0.16 cm/s
0.14 cm/s
0.12 cm/s
0.10 cm/s
0.08 cm/s
0.06 cm/s
0.04 cm/s
0.02 cm/s

TB41 Secondary Velocity Profiles; LAD cross section 9
TB41 Secondary Velocity Profiles; LAD cross section 10
Location of Cross Section 11

LAD Secondary Velocities at Cross-Section 11

Late Diastole  Early Diastole  Systole  Late Systole

Early Diastole  Early Mid-Diastole  Late Mid-Diastole  Late Diastole

TB41 Secondary Velocity Profiles; LAD cross section 11
TB41 Secondary Velocity Profiles; LAD cross section 12
TB41 Secondary Velocity Profiles; LAD cross section 13
TB41 Secondary Velocity Profiles; LCX cross section 1
TB41 Secondary Velocity Profiles; LCX cross section 2
TB41 Secondary Velocity Profiles; LCX cross section 3
Location of Cross Section 4

TC41 Secondary Velocity Profiles; LCX cross section 4
TB41 Secondary Velocity Profiles; LCX cross section 5
BIBLIOGRAPHY


264


