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Thermal Fluctuations in Two-Dimensional Superconductors

DISSertation

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the
Graduate School of The Ohio State University

By

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* * * * *

The Ohio State University

1999

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The effect of thermal fluctuations on the temperature dependence of the superfluid density in two-dimensional superconductors has been a topic of research for twenty years. Early work focused on the transition region where it is believed that the unbinding of thermally excited vortex-antivortex pairs mediates the superconducting to normal transition. More recently several authors have argued that classical phase fluctuations at low temperatures could be responsible for the linear in temperature suppression of the superfluid density observed in some of the high $T_C$ cuprates. In this thesis measurements of the complex impedance of superconducting thin films are presented, for both conventional and high $T_C$ cuprate superconductors which show that thermal phase fluctuations are much smaller than predicted by classical calculations except for near the transition temperature, $T_C$. This result is consistent with a model which concludes that for temperatures below 90% of the mean-field superconducting transition temperature phase fluctuations are suppressed by quantum mechanics. Measurements of the complex sheet conductance are presented near $T_C$ for MoGe and In/InOx films with varying temperature, frequency, applied perpendicular magnetic field and sheet current density. The systematics of the transition are mapped out and it is shown that the complex impedance is inconsistent with current theory.
To Mom, Dad, Julie, Tanya and Nash
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1.1 In the beginning...

Superconductivity was discovered by H. Kamerlingh Onnes in 1911 when he observed that the resistance of mercury dropped sharply to zero at \(4.15K\) [1]. For many years following this initial discovery, a microscopic model explaining superconductivity eluded researchers. Early work was on a phenomenological level and resulted in the 1933 discovery that a bulk superconductor expels any applied magnetic field [2]. This is known as the Meissner effect and it is not simply a consequence of Maxwell's equations for a specimen with perfect conductivity. To explain the effect, in 1935 the London brothers [3] proposed two phenomenological equations which can be expressed (assuming the London gauge, \(\nabla \cdot \mathbf{A} = 0\)) as

\[
\mathbf{J} = -\frac{\mathbf{A}}{\mu_0 \lambda^2}.
\]  

Equation 1.1 implies the Meissner effect, where \(\lambda\) is the magnetic penetration depth which is approximately the depth to which a magnetic field can penetrate a bulk superconductor. The density of superconducting electrons, \(n_s\), is related to \(\lambda\) through \(n_s = m / \mu_0 e^2 \lambda^2\) where \(m\) is the electron mass and \(e\) is the absolute value of the electron charge.
A phenomenological model of superconductivity was developed by Ginzburg and Landau (G-L theory). The superconducting condensate is described by a macroscopic complex scalar order parameter, $\psi = |\psi|e^{i\phi}$. $|\psi|^2$ is equal to the density of superconducting electron pairs, $n_s/2$, and the free energy density of the superconducting state relative to the normal state, $f_s - f_n$, is expressed as a series expansion of the order parameter $^4$.

$$f_s - f_n = \alpha |\psi|^2 + \frac{\beta}{2} |\psi|^4 + \frac{1}{4m} |(\frac{\hbar}{i} \nabla + 2eA)\psi|^2 + \frac{1}{2\mu_0} B^2$$  \hspace{1cm} (1.2)

$\beta$ is assumed to be a positive constant and $\alpha$ is proportional to $1 - T/T_{C0}$ where $T_{C0}$ is the mean-field transition temperature at which the superconducting to normal transition would occur were there no fluctuations. In the absence of applied fields the free energy is minimized with a uniform order parameter with a magnitude, $|\psi_\infty|$, where $|\psi_\infty|^2 = -\alpha/\beta$. The G-L theory introduces the coherence length, $\xi = \hbar/\sqrt{4m|\alpha|}$, which is roughly the minimum length scale over which the phase of the order parameter can twist without costing an exorbitant amount of energy.

The ratio $\kappa \equiv \lambda/\xi$ determines whether the superconductor is Type I or Type II. A Type I superconductor ($\kappa < 1/\sqrt{2}$) exhibits the Meissner effect under an applied field up to the thermodynamic critical field $H_C$ and then promptly enters the normal state. On the other hand, a Type II superconductor completely excludes an applied field up until a field $H_{C1}$ which is less than $H_C$ and then allows the field to partially penetrate the superconductor in the form of vortices each of which contains one quantum of magnetic flux, $\phi_0 \equiv \hbar/2e$. This is known as the mixed state. The magnitude of the order parameter, $|\psi|$, goes to zero at the center of the vortex and recovers to $|\psi_\infty|$ at a distance of a few coherence lengths. Above a second critical field $H_{C2}$, a Type II superconductor enters the normal state.
In 1957, with the advent of BCS theory [5], a microscopic theory that describes most aspects of superconductivity with great accuracy became available. A consequence of the microscopic theory is that there exists an energy gap, \( \Delta(T) \), between the energy of electrons in the superconducting ground state and the lowest energy quasiparticle excitations. The microscopic theory is difficult to work with when magnetic fields are present or when the superconducting order parameter varies spatially.

The value of the G-L theory, which was developed prior to BCS theory, was not immediately recognized, but once it was rigorously shown that near \( T_{c0} \) the G-L theory may be derived from the microscopic BCS theory [6], its usage became widespread. The G-L theory is the most convenient framework for exploring fluctuation effects and is used extensively throughout this thesis.

Within the framework of G-L theory, the partition function for the superconducting system is a functional integral over all possible order parameters, \( \psi(r) \).

\[
Z = \int D\psi(r)e^{-\beta F},
\]

where \( F \) is the volume integral of the free energy density of Equation 1.2. In mean-field theory it is assumed that \( \psi(r) \) and \( A(r) \) are given by the function that minimizes \( F \) and all of the higher energy states in the partition function are ignored. Minimizing \( F \) with respect to both \( \psi(r) \) and \( \psi^*(r) \) leads to the G-L differential equations [4] which are a pair of coupled nonlinear differential equations, the solution of which yield the thermodynamic equilibrium values of \( \psi(r) \) and \( A(r) \).

The superconducting state exhibits a broken continuous symmetry since the mean-field groundstate is one with a constant phase. This superconducting groundstate is infinitely degenerate because the phase of the wavefunction is arbitrary. Since the
phase is a continuous variable there are excitations of very low energy where the phase twists over long length scales.

One of the hallmarks of superconductivity in three dimensions is the existence of off-diagonal long-range order (ODLRO) [7]. The definition for the existence of ODLRO is

\[ \langle \psi(r) \psi^*(r') \rangle \rightarrow \text{finite constant} \]  

as \(|r - r'| \rightarrow \infty\). Basically this means that the superconducting order parameter \(\psi(r)\) remains phase coherent over arbitrarily large distances. Rice [8] has shown that for small Gaussian fluctuations in a 3D superconductor the criterion for long range order is satisfied.

### 1.2 Superconductivity in reduced dimensions

Given that superconductivity in 3 dimensions exhibits ODLRO, one wonders how robust the ordering is to reduced dimensionality. A general rule in nature is that fluctuation effects become stronger as the dimensionality and size of a system are reduced and several authors have shown that ODLRO is indeed not possible in two or fewer dimensions [8] [9] [10].

A one dimensional sample may be realized by a long strip of superconducting material with its width and thickness both smaller than the coherence length. In 1D the phase correlator decays exponentially with distance. The theory for fluctuation effects in 1D was devoloped by Little [11] and by Langer and Ambegaokar [12]. The basic idea, which was confirmed experimentally by I-V measurements on tin whiskers [13], is that a finite resistance is present for arbitrarily small dc currents due to phase
slips which interrupt the phase coherence between points on opposite sides of the phase slip event.

Superconductivity in two dimensions is particularly interesting since it is the critical dimensionality for the existence of long range order. In 2D the phase correlator decays algebraically with an exponent which is a continuous function of temperature. Berezinski [14] and Kosterlitz and Thouless [15] suggested that a new type of quasi-long range order (also called topological long range order) occurs in two dimensions. In this model, elementary topological excitations consisting of a vortex and an antivortex in a bound pair are created. At low temperatures the free energy of the pairs is dominated by their internal energy and the pairs are small, but as the temperature increases the entropic term becomes significant and at a specific temperature, $T_{KTB}$, the largest pairs unbind. Above this temperature the order parameter correlations decay exponentially and the inductive response of the superconductor is destroyed. The vortex-antivortex unbinding transition is known as the Kosterlitz-Thouless-Berezinskii (KTB) transition. For the past 20 years this transition has been investigated in superconducting films and arrays. The general features of the transition have been qualitatively observed in experiment, but a quantitative consistency between experiment and theory is lacking.

1.3 Outline and motivation

The primary focus of this thesis is to gain a better understanding of the role of thermal fluctuations in suppressing superconductivity in two dimensional superconductors. In particular, the original impetus of this research was to investigate the predictions [16] [17] [18] [19] that classical longitudinal phase fluctuations could
be responsible for the linear in temperature suppression of the superfluid density observed in several of the cuprate superconductors cited hardy gra. To address this issue, a series of experimental measurements of the penetration depth for doped oxygen depleted YBCO were performed and are discussed in Chapter 3. It was found that the fluctuations are smaller than expected classically, but uncertainties in sample quality and a lack of reproducibility precluded a precise determination for the size of the fluctuation effects.

Next penetration depth measurements were performed at low T on Mo77Ge23 thin films. These measurements showed conclusively that classical phase fluctuations are not present well below Tc. Following these measurements it was recognized [20] that classical phase fluctuations should only be present in the neighborhood of Tc. As described in Chapter 4 there is a quantum suppression of phase fluctuations that occurs below a crossover temperature which is near 0.9Tc0. The measurements on Mo77Ge23 thin films in Chapter 5 are consistent with the presence of a quantum crossover below which phase fluctuations are suppressed.

In the course of studying longitudinal phase fluctuations near Tc for the two dimensional films the effect of vortex excitations and the KTB vortex-antivortex pair unbinding transition must be considered. It was found that the inverse inductance for Mo77Ge23 and In/InOx thin films drops sharply to zero when the inverse sheet inductance has a value near that predicted by KTB theory for the universal drop in the superfluid density. The universal drop has been previously measured directly for granular Al [21], In/InOx [22], granular Al wire networks [23] [24] and Pb/Cu/Pb Josephson junction arrays [25] [26]. Although the universal drop has often been observed, the details of the transition region such as the frequency dependence of
the complex impedance and the interplay between longitudinal phase fluctuations and vortex excitations are poorly understood. Furthermore, a recent reexamination of I-V measurements purported to be evidence for the conventional KTB transition has yielded an anomalously large dynamic exponent [27]. Recently, several studies have been attempted which try to explain the dynamics of the vortex excitations in the cuprate superconductors [28] [29]. Given that samples of conventional superconductors can be made which are much more homogeneous than the cuprates and for which the T dependence of the mean-field superfluid density is well known, it is natural to explore the dynamics of the transition in the conventional superconductors before moving on to the cuprates. For this reason a detailed study of the frequency dependence of the complex impedance for Mo\textsubscript{77}Ge\textsubscript{23} thin films as well as for an In/InO\textsubscript{x} film was performed. A surprising feature of the complex impedance is the presence of a measurable sheet resistance well below $T_C$ which exhibits a weak frequency dependence. Furthermore, there is an inductive impedance in addition to the impedance of the superfluid background which exhibits nearly the same frequency dependence as the sheet resistance. These observations are shown to be inconsistent with the predictions of the conventional KTB theory.

1.4 Experimental technique

The majority of the data presented in this thesis are from measurements of the complex conductance, $\sigma(\omega)d = \sigma_1(\omega)d - i\sigma_2(\omega)d$ for thin films of thickness d, using a two-coil mutual inductance technique. The complex sheet impedance per square of the film is

$$Z_0 = R_0 + i\omega L_0 = \frac{\sigma_1}{d(\sigma_1^2 + \sigma_2^2)} + i\frac{\sigma_2}{d(\sigma_1^2 + \sigma_2^2)}$$  (1.5)
For temperatures well below $T_C$ and for frequencies much less than the gap frequency, $\Delta/\hbar$, the imaginary conductivity is much greater than the real component of the conductivity and $\sigma_2$ is related to the penetration depth through $\sigma_2 \equiv 1/(\mu_0 \omega \lambda^2)$. With this definition the kinetic sheet inductance is $L_{\Omega} \equiv \mu_0 \lambda^2/d$.

The idea behind the mutual inductance technique is to generate a small electromagnetic field at angular frequency, $\omega$, near the sample of interest and then to measure the voltage induced across a pick-up coil also located near the sample. Typical frequencies used in the experiment range from 100Hz up to 100kHz. By decreasing the film temperature slowly and recording the pick-up voltage as a function of temperature, it is trivial to obtain the transition temperature at which a sudden change in the pick-up voltage occurs and whose magnitude and sign depends on the geometrical arrangement of the sample and coils. The change in the pick-up voltage simply indicates that the density of superconducting electrons is nonzero and that inductive currents are flowing to screen the applied field.

A more sophisticated experiment also has the ability to measure $\sigma$ as a function of temperature below the transition temperature. In order to determine the absolute value of the complex conductivity the geometry of the sample and coil set must be known accurately and the pick-up voltage must be phase sensitively measured with respect to the drive current phase. The complex mutual inductance, $M$, between the drive and pick-up coils is defined by

$$ M \equiv M_1 + i M_2 \equiv \frac{V_p}{i \omega I_d}. \quad (1.6) $$

For a geometry in which the drive coil and film are azimuthally symmetric the mutual inductance can be calculated for any $\sigma$. Considerable time and effort has been spent on developing the two-coil mutual inductance technique. Appendix A describes
in detail the numerical modeling of the experiment and Appendix B discusses the practicalities of the experiment.
CHAPTER 2

CLASSICAL LONGITUDINAL PHASE FLUCTUATIONS

This chapter describes in a simple manner how longitudinal phase fluctuations suppress the superfluid density. Fluctuations in the amplitude of the order parameter are ignored and transverse phase fluctuations (vortex-antivortex pairs) are discussed more fully in Chapter 6. For continuous films a simple derivation for the effect of phase fluctuations is given which reproduces the predictions of several theory papers. The connection between continuous films and Josephson junction arrays is made because Josephson junctions are a well defined system for which calculations are more straightforward.

2.1 Fluctuations in films

For a continuous film the size of thermal phase fluctuations can be estimated by using G-L theory and the equipartition theorem. In the absence of applied fields and assuming that $|\psi|$ is spatially constant to zeroth order, the G-L free energy density of the superconducting state relative to the normal state may be expressed as

$$f_s - f_n = -\frac{\hbar^2}{2m^*\xi^2}|\psi|^2 + \frac{\hbar^2}{4m^*\xi^2}|\psi_\infty|^2|\psi|^4 + \frac{1}{2}m^*v_s^2|\psi|^2.$$ (2.1)

Here $|\psi_\infty|$ is the amplitude of the order parameter for $v_s = 0$ and $m^* = 2m$ is the mass of a Cooper pair. Allowing $|\psi|$ to vary and minimizing $f_s - f_n$ for arbitrary $v_s$
yields \[\text{[4]}\]

\[|\psi|^2 = |\psi_\infty|^2[1 - \left(\frac{m^*\xi v_s}{\hbar}\right)^2].\]  

(2.2)

An experimental effort to detect the nonlinearity in the conductivity implied by Equation 2.2 is described at the conclusion of Appendix A. To estimate the size of \(v_s^2\) due to thermal phase fluctuations in a film of thickness, \(d\), it is assumed that all the superconducting electrons in a coherence volume, \(\pi \xi^2 d\), fluctuate as a single particle and have an energy \(k_B T\) as mandated by the classical equipartition theorem for a particle with 2 degrees of freedom. Thus, \(<v_s^2> = 2k_B T/mn_s \pi \xi^2 d\). Replacing \(v_s\) in Equation 2.2 with \(\sqrt{<v_s^2>}\) yields

\[
\frac{L_{\Omega, SF}^{-1}}{L_{\Omega, MF, SF}^{-1}} = 1 - \frac{k_B T}{U_0},
\]

(2.3)

with \(\alpha = 2/\pi\). \(|\psi|^2\) is proportional to the inverse sheet inductance of the superfluid \(L_{\Omega, SF}^{-1} = d/\mu_0 \lambda^2\) and \(|\psi|^2\) has been replaced with \(L_{\Omega, SF}^{-1}\) in Equation 2.3. The characteristic superconducting energy is \(U_0 = (\phi_0/2\pi)^2 L_{\Omega, SF}^{-1}\). \(U_0\) is equal to \(\pi/4\) times the condensation energy, \(\frac{1}{2\mu_0} B_C^2 \pi \xi^2 d\), of a coherence volume. This energy is used repeatedly throughout the course of this thesis. The ratio of \(U_0\) to \(k_B T\) will be important when discussing vortex excitations. Note that in a real film the energy \(U_0\) is strongly temperature dependent.

\(U_{\infty}\) is defined as \(U_{\infty} = (\phi_0/2\pi)^2 L_{\Omega, MF, SF}^{-1}\) where \(L_{\Omega, MF, SF}^{-1}\) is the mean-field inverse sheet inductance. Equation 2.3 may be solved self-consistently for \(L_{\Omega, SF}^{-1}\) in terms of \(U_{\infty}\). The result is

\[
\frac{L_{\Omega, SF}^{-1}}{L_{\Omega, MF, SF}^{-1}} = \frac{1}{2} + \frac{1}{2}\sqrt{1 - 4\alpha k_B T/U_0}.\]

(2.4)

The breakdown of Equation 2.4 when \(4\alpha k_B T/U_0 > 1\) may be interpreted as signifying that the longitudinal phase fluctuations become critical and drive the superfluid
density to zero. However, it is more likely that longitudinal phase fluctuations are not as strong as predicted by Equation 2.4 and lie somewhere between the curve defined by Equation 2.4 and the curve generated by Equation 2.3 with $U_0$ replaced by $U_{oo}$.

For temperatures well below $T_C$ phase fluctuations are predicted [16] [17] [18] [19] to give rise to a linear in temperature suppression of the superfluid density for YBCO. The prediction of these theories is captured by Equation 2.3, however the prefactor, $\alpha$, differs among the various models.

2.2 Fluctuations in Josephson junction arrays

Arrays of Josephson junctions provide a convenient system for calculating the effects of thermal phase fluctuations. There are two relevant questions with regard to phase fluctuations in arrays: (1) How large is the effect? and (2) At what temperature do vortex excitations suppress the inverse sheet inductance of the array as much as the longitudinal fluctuations? Josephson junctions are of further interest in determining the effect of quantum mechanics which is discussed in Chapter 4.

A Josephson junction is formed when two superconductors are separated by a small layer of nonsuperconducting or very weakly superconducting material. The supercurrent, $I_S$, that passes through the junction depends on the phase difference, $\Delta \phi$, between the two junction electrodes.

$$I_S = I_C \sin(\Delta \phi) \tag{2.5}$$

where $I_C$ is the junction critical current. The phase difference across the junction evolves with time, $t$, according to

$$\frac{d(\Delta \phi)}{dt} = \frac{2eV}{\hbar} \tag{2.6}$$

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where $V$ is the voltage across the junction. An RSJ is a Josephson junction with a shunt resistance, $R$, in parallel with the junction. The junction behaves as an inductor with inductance, $L_0 = (\phi_0/2\pi)L_\phi^{-1}$, in the limit of vanishing current. For a nonzero current, $I_S$, the junction behaves as a nonlinear inductor with an inductance, $L_J = L_0/[1 - I_S^2/I_G^2]^{1/2}$. For $T << L_0I_G^2/k_B$, the classical result for the mean square supercurrent through the junction is, $\langle I_S^2 \rangle = k_BT/L_0$. Thus for low $T$ the inverse inductance of a Josephson junction is suppressed by phase fluctuations in the same way as the superfluid density is suppressed in a film but with $\alpha = 1/2$. Rigorous calculations [20] performed by Aaron Pesetski show that for $k_BT/U_0 < 0.2$ and for very low frequencies compared to $R/L_0$ that thermal phase fluctuations dominate over phase slip events. For temperatures above $0.2U_0/k_B$ phase slip events become important and the junction impedance exhibits a real component.

When RSJ's are linked together to form a square array phase slip fluctuations are greatly suppressed. Note that in the absence of fluctuations the sheet inductance of a square array of identical junctions is equal to the inductance of an individual junction. Monte Carlo calculations of the helicity modulus [30] for square arrays show that the parameter $\alpha$ in Equation 2.3 is 1/4. The helicity modulus normalized by its $T=0$ value is equivalent to $L_{\phi}^{-1}/L_{\phi,MF}^{-1}$. The correlations between the junctions reduce the effect of longitudinal thermal phase fluctuations by a factor of 2 and suppress phase slip events to a greater extent. If it is assumed that for a single junction $L_0/L_J = [1 - k_BT/U_0]^{1/2}$, then for an array of square junctions $L_0/L_J = [1 - \frac{1}{2}k_BT/U_0]^{1/2}$. 

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2.3 Summary and connection between films and arrays

The connection between films and arrays and KTB theory has been discussed by Lobb et al. [31]. For a film, the temperature and the areal superfluid density control the size of the fluctuations which are measured as $L^{-1}/L_{MF}^{1}$. For arrays and 2DXY model calculations, the Josephson coupling energy between the junctions as well as the temperature determine the size of the fluctuations which are usually represented by the helicity modulus. The helicity modulus for a square array is proportional to the second derivative of the array's free energy with respect to a constant phase gradient across the junctions in the limit where the phase gradient approaches zero. One obvious difference between 2DXY model calculations and a real film is that for 2DXY model calculations, only phase fluctuations enter the model whereas in a real film there may also be fluctuations in the amplitude of the order parameter.

Figure 2.1 summarizes the effect of fluctuations on the various systems. The dashed line, produced by calculations performed by Aaron Pesetski, [20] is for a single Josephson junction and the open circles are for a square array of Josephson junctions from Reference [30]. Note that the array calculation includes longitudinal spin wave fluctuations as well as vortex-antivortex fluctuations. The sharp drop in the data near the intersection with the dotted line is taken to be numerical evidence of the KTB transition, although it is broadened by finite size effects. The two thick solid lines represent the range of the effect of longitudinal phase fluctuations only for a continuous film where $\alpha = 1/4$ in Equations 2.3 and 2.4.
Figure 2.1: The suppression of the inverse sheet inductance below its mean-field value as a function of the noise parameter $k_B T / U_{00}$ in the classical limit. The dashed line is for a single Josephson junction from Reference [20], and the open circles are for a square array of Josephson junctions from Reference [30]. The thick solid lines represent the estimated bounds for the suppression of the inverse sheet inductance due to longitudinal phase fluctuations for a continuous film. The intersection of the film and array curves with the dotted line is where the KTB transition is predicted to occur.
3.1 Introduction and background

Soon after the discovery of the high temperature cuprate superconductors it was noted that fluctuations may be large in the cuprate superconductors since they are quasi-two-dimensional, have a large transition temperature, a short coherence length and a low superfluid density [32]. Fluctuation effects are expected to be largest near the transition temperature where a 3DXY transition should occur. For a 3DXY transition it is $\lambda^{-3}$ that diverges as $1 - T/T_C$ rather than $\lambda^{-2}$ as expected from mean-field theory. Measurements in YBCO crystals indicate a wide critical region [33], whereas measurements in YBCO films exhibit mean-field behavior [34] [35]. The reason for this discrepancy is still unknown.

Turning to temperatures well below $T_C$, phase fluctuations are predicted [16] [17] [18] [36] to give rise to a small linear in temperature suppression of the superfluid density for YBCO. The basic prediction for the suppression of the superfluid density due to classical phase fluctuations (Equation 2.3) expressed in terms of the
penetration depth is

\[
\frac{1}{\lambda(0)} \frac{d\lambda}{dT}_{T \to 0} = \frac{\alpha}{1.25 \times 10^9 \text{ Å} - K} \frac{\lambda^2(0) d}{d b}.
\]  

The variable parameter \(\alpha\) sets the size of the effect and \(b\) is a characteristic length which was not included in Equation 2.3 since that was derived for a truly two-dimensional sample where it is appropriate to speak of the sheet inductance or equivalently the areal superfluid density. Of crucial importance is whether the fluctuations are coupled through the layers such that \(b\) is equal to the total film thickness, \(d\), or whether they are confined to individual layers in the \(c\) direction so that \(b \approx 11.7\text{ Å}\), the \(c\)-axis lattice constant.

In this chapter, penetration depth data are presented for several YBCO films substitutionally doped with either Zn or Co which replace copper atoms and with each film at a variety of different oxygen concentrations. The primary aim of these measurements was to test the prediction of a linear decrease in the superfluid density described by Equation 3.1. Doped YBCO is more suitable than pure YBCO for testing the behavior predicted by Equation 3.1 for two reasons: (1) doped YBCO has a lower superfluid density than pure YBCO and thus larger fluctuations and (2) the mean-field temperature dependence of the superfluid density for a disordered d-wave superconductor is quadratic in temperature \([37]\) at low \(T\) in contrast to \(T\)-linear for very pure YBCO. The \(T\)-linear behavior in pure YBCO is most commonly interpreted as being the result of an energy gap with d-wave symmetry. Since the aim of this experiment was to isolate the \(T\)-linear contribution of thermal phase fluctuations to the superfluid density at low \(T\), a quadratic mean-field temperature dependence is ideal since this means any linear contribution from phase fluctuations will be dominant as \(T\) approaches absolute zero.
3.2 Oxygen content of YBCO

A perfect crystal lattice of $YBa_2Cu_3O_{7-\delta}$ has $\delta = 0$. Depending on the temperature and oxygen partial pressure of the environment the equilibrium value of $\delta$ may vary between 0 and 1. A review of the effect of oxygen stoichiometry may be found in Reference [38]. For $\delta > 0.7$, YBCO does not superconduct. As $\delta$ is decreased the superfluid density and transition temperature increase until $\delta = 0.05$ which is known as optimal doping. Smaller values of $\delta$ correspond to the overdoped region and $T_C$ is a few Kelvin lower than for optimal doping when $\delta = 0$.

The data presented in the following section were taken on films which had their oxygen concentration varied by annealing the films. Two different deoxygenation methods (the technique used for a particular sample is noted in the column labeled, "DM", of Table 3.1 were used in the course of this work. The first procedure for deoxygenating films was to put the film in a covered quartz boat in a sealed tube furnace at $T = 523$K with flowing Argon (this technique is labeled TF in Table 3.1. Prior to annealing, the films were cleaned with an ultrasonic cleaner using trichloroethylene as a solvent to remove all of the grease from the film and substrate. The film and boat were then placed at the edge of the tube furnace which remains well below 373K when the middle of the tube furnace is hot. When the tube furnace reached the desired temperature the boat and film were slid into the center of the tube furnace. The anneals were performed for times ranging between 5 minutes and 30 minutes. At the conclusion of the anneal the sample was moved back to the edge of the tube furnace and removed when the center of the tube furnace was below 373K. The other technique was an equilibrium quench, labeled EQ in Table 3.1. In this technique the sample was placed in a chamber at a specified temperature and oxygen partial
pressure for several hours until the film reached an equilibrium oxygen concentration. The films were then popped out of the chamber into liquid Nitrogen which freezes the oxygens in place. The equilibrium quench deoxygenations were performed at Purdue University by Patti Metcalf. Judging by the width of the transition region, the quality of the films were superior for the equilibrium quench compared to the tube furnace method. The tube furnace method yielded transition widths of several Kelvin whereas the equilibrium quench method yielded transition widths closer to 1K wide. For this reason, any study of the effect of oxygen stoichiometry should use a deoxygenation method in which an equilibrium is established between the film and its environment.

Reoxygenation of films was performed by enclosing the film in a small capsule constructed of fully oxygenated pressed YBCO powder and annealing for several hours in the tube furnace with flowing oxygen at $T = 400 \degree C$. The YBCO housings were prepared by Jim Baumgardner. Upon reoxygenation by this technique the measured $T_C$, superfluid density, and transition widths were all nearly the same as their measured values before the initial deoxygenation.

### 3.3 Experimental results

Four films, prepared by laser ablation at Los Alamos National Laboratory by Steve Foltyn and Xindi Wu, were studied in detail. Measurements were performed on two 1% Zn doped films and on two 3% Co doped films. The film properties are listed in Table 3.1 along with the deoxygenation technique. From the data an upper bound for the effect of classical thermal phase fluctuations can be determined. Also
Table 3.1: Properties of the YBCO films. The film thicknesses are nominal. $T_C$ is defined as the temperature at which the real conductivity is peaked. $\Delta T_C$ is the full width at half maximum of the dissipation peak. $DM$ is the deoxygenation method where TF stands for the tube furnace and EQ stands for equilibrium quench. The slope of $\lambda$ with temperature as $T \to 0$ is determined by drawing a tangent to the lowest temperature data. "The film had 3 distinct transitions each less than 1K wide with the first one at 45K and the transition at 39K being the dominant transition.

<table>
<thead>
<tr>
<th>Film</th>
<th>Dopant</th>
<th>$d$ (Å)</th>
<th>$T_C$ (K)</th>
<th>$\lambda(0)$ (Å)</th>
<th>$d\lambda/dT$ (Å/K)</th>
<th>$\Delta T_C$ (K)</th>
<th>DM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y1283 1 % Zn</td>
<td>1560</td>
<td>74</td>
<td>1890 ±180</td>
<td>1.4 ±0.2</td>
<td>.2</td>
<td>-</td>
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<tr>
<td></td>
<td></td>
<td>50</td>
<td>3395 ±110</td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>40</td>
<td>3890 ±100</td>
<td>11.6 ±0.8</td>
<td>7</td>
<td>TF</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>34</td>
<td>4415 ±90</td>
<td>20.6 ±1.0</td>
<td>3</td>
<td>TF</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>29</td>
<td>4940 ±80</td>
<td>27.2 ±1.3</td>
<td>2</td>
<td>TF</td>
<td></td>
</tr>
<tr>
<td>Y1284 1 % Zn</td>
<td>3020</td>
<td>74</td>
<td>1970 ±110</td>
<td>1.6 ±0.4</td>
<td>.15</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>41</td>
<td>3400 ±100</td>
<td>5.0 ±0.8</td>
<td>3</td>
<td>EQ</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>26</td>
<td>4940 ±120</td>
<td>14.0 ±1.2</td>
<td>1</td>
<td>EQ</td>
<td></td>
</tr>
<tr>
<td>Y1285 3 % Co</td>
<td>3300</td>
<td>71</td>
<td>2550 ±100</td>
<td>2.0 ±0.4</td>
<td>1</td>
<td>-</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>39</td>
<td>4610 ±120</td>
<td>5.5 ±0.6</td>
<td>.4*</td>
<td>EQ</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>34</td>
<td>6930 ±170</td>
<td>10.7 ±1.2</td>
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<td>EQ</td>
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<td>1200</td>
<td>75</td>
<td>2320 ±80</td>
<td>2.2 ±0.4</td>
<td>1.5</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>37</td>
<td>6240 ±160</td>
<td>8.4 ±0.7</td>
<td>1</td>
<td>EQ</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>33</td>
<td>7750 ±190</td>
<td>13 ±1.3</td>
<td>2</td>
<td>TF</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>29</td>
<td>9800 ±250</td>
<td>20.2 ±2.7</td>
<td>1.5</td>
<td>EQ</td>
<td></td>
</tr>
</tbody>
</table>

3.3.1 Variation of $T_C$ and $\lambda(0)$ with oxygen concentration

Figures 3.1 and 3.2 show $\lambda^{-2}(T)$ for the Zn and Co doped films respectively at the various oxygen concentrations. As oxygen is removed from the chains, the number of holes decreases and as a result the superfluid density decreases. It was noted by Uemura [39] that the transition temperature is proportional to the level
Figure 3.1: The superfluid density for two 1\% Zn doped YBCO films at various oxygen concentrations.
Figure 3.2: The superfluid density for two 3%Co doped YBCO films at various oxygen concentrations.
Figure 3.3: $T_C$ vs. $\lambda^2(0)$ for YBCO films. The open triangles are for pure YBCO from Reference [41], the squares are for the Co doped films and the circles are for the Zn doped films.

of doping for a variety of superconductors. Emery and Kivelson have argued that Uemera's observation may be interpreted as signifying that $T_C$ for the underdoped cuprates may be controlled by thermal phase fluctuations [40].

In Figure 3.3 $T_C$ is plotted as a function of $\lambda^{-2}(0)$ for the 4 films. Also shown are data for a pure YBCO film from Reference [41]. The pure film was made by the same procedure as the doped films by the same researchers at Los Alamos, so comparisons between the doped and undoped films should be meaningful. Note that for the films with reduced oxygen content the data fall on the wrong side of the Uemura line.
### 3.3.2 Upper bound for phase fluctuations

To determine how large the phase fluctuations are it is necessary to measure the slope, \(d\lambda/dT\), at the lowest possible temperature. Figures 3.4 and 3.5 show the temperature dependence of \(\lambda(T)\) below 10\(K\). An upper bound for the linear contribution to \(\lambda(T)\) is determined by drawing a tangent (dashed lines) to the low temperature data. The values and uncertainties in the tangents are listed in Table 3.1.

To compare the data of Figures 3.4 and 3.5 with Equation 3.1 the upper bound linear slopes normalized by \(\lambda(0)\) are plotted vs. \(\lambda^2(0)\) in Figures 3.6 (Zn doped) and 3.7 (Co doped). The data for film Y1283 are consistent with Equation 3.1 with \(\alpha/b \approx 1/40\text{Å}\). For film Y1284 which is twice as thick as Y1283, \(\alpha/b \approx 1/80\text{Å}\). In order for both films to be consistent with Equation 3.1, \(b\) must be equal to the film thickness. However, the data for the Co doped films are inconsistent with this interpretation. The data in Figure 3.7 show an unpredicted residual slope when the data are extrapolated to \(\lambda^2(0) = 0\). If this residual slope is ignored the resulting \(b/\alpha\) is between 5 and 10 times larger than for the Zn doped films.

### 3.4 Conclusions

Since thermal phase fluctuations should be robust with respect to disorder and doping levels it is concluded that the largest measured \(b/\alpha\) is the proper value to use for determining the upper bound for the effect of phase fluctuations. Assuming \(b = 11.7\text{Å}\) and taking \(b/\alpha\) from the Co doped YBCO data yields \(\alpha < 0.015\). Applying the upper bound for \(\alpha\) to pure YBCO with \(\lambda(0) = 1500\text{Å}\) yields a maximum slope of 0.035\(\text{Å}/K\) from phase fluctuations which is much smaller than the observed 4.5\(\text{Å}/K\)
Figure 3.4: The low temperature penetration depth vs. temperature for two 1% Zn doped YBCO films. The dashed lines show the slope of the data at the lowest measurement temperatures.
Figure 3.5: The low temperature penetration depth vs. temperature for two 3%Co doped YBCO films. The dashed lines show the slope of the data at the lowest measurement temperatures.
Figure 3.6: Slope of the tangent to the data of Figure 3.4 divided by $\lambda(0)$ vs. $\lambda^2(0)$ for the two Zn doped YBCO films.
Figure 3.7: Slope of the tangent to the data of Figure 3.5 divided by $\lambda(0)$ vs. $\lambda^2(0)$ for the two Co doped YBCO films.
[42]. A recent analysis of ARPES data conclude that longitudinal fluctuations are indeed very small in the cuprate superconductors [43]. In Chapter 4, a model for why phase fluctuations are much smaller than the classical prediction is discussed.
CHAPTER 4

QUANTUM SUPPRESSION OF THERMAL PHASE FLUCTUATIONS

The results presented in Chapter 3 for doped YBCO films clearly show that the effect of phase fluctuations at temperatures well below $T_c$ are much smaller than the classical prediction discussed in Chapter 2. This chapter outlines how quantum mechanics suppress phase fluctuations. First, ideal resistively shunted Josephson junctions are discussed since they are a simple model system with well defined equations. Following this, the results are carried over to films in a phenomenological argument. Tom Lemberger initiated the work described in this chapter and served as the leader with assistance from Aaron Pesetski and myself. More details than are presented here may be found in Reference [20].

4.1 Quantum mechanics and Josephson junctions

It is convenient to begin by looking at a single resistively shunted Josephson junction, a simple well defined system. The properties of Josephson junctions were discussed in Chapter 2 where it was shown that the junction behaves as an inductor for small junction current with inductance, $L_0$. The noise currents originate from the shunt resistance, $R$, and the mean square noise current in a frequency band $\Delta B$
centered about an angular frequency, \( \omega \) is [44]

\[
< |i_n(\omega)|^2 > \Delta B = \frac{4}{R} \left( \frac{\hbar \omega}{2} + \frac{\hbar \omega}{[e^\hbar \omega/k_BT - 1]} \right) \Delta B.
\] (4.1)

This reduces to the classical result for Johnson noise, \(< |i_n(\omega)|^2 > \Delta B = 4k_BT \Delta B/R\), when \( k_BT >> \hbar \omega \).

Of interest is the total noise current passing through the junction. As noted in Chapter 2 the classical result is \(< I^2_s >_{\text{class}} = k_BT/L_0 \). Neglecting the zero point term, \( 2\hbar \omega/R \), of the quantum noise of Equation 4.1, the total mean square thermal noise current passing through the junction is

\[
<I^2_s> = \int_0^\infty (d\omega/2\pi) \frac{4}{R} \frac{\hbar \omega}{[e^\hbar \omega/k_BT - 1]}^{-1}[1 + (\omega/\omega_0)^2]^{-1},
\] (4.2)

where \( \omega_0 = R/L_0 \). The last factor of Equation 4.2 gives the fraction of the noise current at angular frequency, \( \omega \), passing through the junction. For high frequencies the noise currents will pass through the resistor and for frequencies below \( \omega_0 \) they will mostly pass through the junction. When \( k_BT >> \hbar \omega_0 \) then the classical result is restored, but when \( k_BT \) drops below \( \hbar \omega_0 \) the mean square current passing through the junction is suppressed below the classical result by a factor \( f_Q = < I^2_s > / < I^2_s >_{\text{class}} \).

A very good approximation to the exact result is

\[
f_Q \approx \frac{1}{1 + \hbar \omega_0/k_BT}.
\] (4.3)

When Josephson junctions are connected to form a 2D array the total noise should be suppressed by the same factor below the classical value as in a single junction.
4.2 Effect of quantum mechanics in films, arrays, and wire networks

The characteristic parameter which determines whether quantum mechanics will suppress fluctuations is the $R/L$ frequency, $\omega_0$. In this section the importance of the quantum suppression of fluctuations is discussed for various model two dimensional systems.

Films

For a homogeneous film the inductance in the $R/L$ frequency is clearly the sheet inductance of the film, but there is some uncertainty in what the value of $R$ is. It is recognized that the noise source originates from the real quasiparticle conductance, $1/R \equiv G \equiv \sigma_1 d$. Guided by the conductivity sum rule, and assuming that the electron scattering rate is constant, [4] $R$ may be approximated in terms of the normal state sheet resistance, $R_n$, and the kinetic sheet inductance.

$$ R(T) \approx \frac{R_n}{1 - \frac{L_0(0)}{L_a(T)}} $$

(4.4)

If the temperature dependence of the superfluid density is known then the quantum crossover temperature, $T_Q$, may be determined by solving for the temperature which satisfies $R(T_Q)/L(T_Q) = k_B T_Q/h$.

It is particularly simple to determine $T_Q$ for a dirty limit s-wave superconductor. In such a superconductor the normal state sheet resistance is related to the zero temperature inverse sheet inductance and the mean-field transition temperature by $R_{\sigma,n} L_0^{-1}/T_{c0} \approx 7.25 \times 10^{11} (K^{-1} s^{-1})$. Furthermore for $0.8 T_{c0} < T < T_{c0}$, $L_0^{-1}(T)/L_0^{-1}(0) \approx 2.4(1.0 - T/T_{c0})$. Solving for the temperature at which the
quantum crossover occurs, $T_Q$ defined by $k_B T_Q = \hbar \omega_0(T_Q)$, yields $T_Q \approx 0.94 T_{C0}$. The experimental evidence for this crossover is presented in Chapter 5.

Wire networks

Wire networks offer another two dimensional structure for which the quantum crossover may be studied. A square wire network of a dirty limit superconductor would contain squares of a superconductor which are connected by thin strips of length, $a$, and width, $w$. The squares are $l \times l$ where $w < l < a$. The inductance of a link between the banks is $(a/w)L_{C0}$ which can made considerably larger than the sheet inductance of a homogeneous film by increasing the ratio $a/w$. However, the resistance of the link scales in the same way with $a$ and $w$ as the inductance. Therefore the quantum crossover would occur near $0.94 T_{C0}$ as for a homogeneous film. Measurements made on Al wire networks [23] [24] suggest that the phase fluctuations are classical to the lowest measurement temperature which was roughly $0.94 T_{C0}$. However, in these papers it was not mentioned how the mean-field sheet inductance was determined and in my opinion was simply determined to agree with calculations [30] for the helicity modulus of a square Josephson junction array, i.e. the line of slope $-1/4$ in Figure 2.1.

SNS arrays

Whether phase fluctuations in an SNS (normal metal junction) Josephson junction are suppressed by quantum mechanics depends on the junction thickness. The resistance of the junction is simply proportional to the thickness divided by the cross sectional area. The critical current, however, has an exponential decay $e^{-s/\xi_n}$ [45]
where $s$ is the junction thickness and $\xi_n$ is the normal metal coherence length. Measurements by Martinoli [26] on an SNS junction with $R_n = 4m\Omega$ are shown as having classical phase fluctuations as predicted. To my knowledge this work and the papers on the Al wire networks [23] [24] are the only data in the literature for which the longitudinal phase fluctuations have been mentioned in analyzing the inverse sheet inductance of a 2D superconducting structure. However, as with the Al wire network data it is unclear whether the mean-field sheet inductance was known well enough to observe the suppression due to the spin waves or whether the mean-field sheet inductance was chosen to give agreement with the suppression of the inverse sheet inductance calculated for square arrays of Josephson junctions.

**SIS arrays**

Arrays of SIS (insulating junction) Josephson junctions which have a barrier thickness, $s << \xi$, are a promising candidate for testing the quantum crossover. Since $I_C \propto A/s$, and $R \propto s/A$ where $A$ is the junction area, $R I_C$ is independent of the junction area or junction thickness. The $R I_C$ product is given by the Ambegaokar-Baratoff formula [4] which has the same temperature dependence as a dirty limit BCS s-wave superconductor. Thus the quantum crossover would occur at the same fraction of the mean-field transition temperature as it does for a dirty limit homogeneous s-wave film. However, the crossover temperature can be varied by shunting the junction with a normal metal bridge. Of course the bridge would have to be much longer than the normal metal coherence length so that it does not behave as an SNS junction in parallel with the SIS junction. By changing the shunt resistance it should be possible to vary the quantum crossover temperature from near $0.94 T_{C0}$ down to arbitrarily low temperatures.
CHAPTER 5

EXPERIMENTAL EVIDENCE FOR THE SUPPRESSION OF THERMAL PHASE FLUCTUATIONS

5.1 Why study Mo$_{77}$Ge$_{23}$ thin films

As mentioned in Chapter 3, to isolate the effects of phase fluctuations on the superfluid density, a knowledge of the mean-field behavior is required. It was found that the data for the cuprates were not reproducible enough and that the theoretical mean-field superfluid density is not known well enough to make a final determination for the size of the fluctuations. Since the mean-field theory for conventional s-wave superconductors is well described by BCS theory, the conventional superconductors are a better candidate for isolating the effects of thermal phase fluctuations provided that the fluctuations are large enough to observe.

Given that the conventional s-wave superconductors have low transition temperatures it is necessary to have low areal superfluid densities for fluctuation effects to be observable. Ideally, thin films with sheet inductances of equal magnitude or larger than the sheet inductance of a single layer of pure YBCO should be studied. This rules out pure metallic samples as they all have high superfluid densities. Two possible samples with large sheet inductances are granular materials and amorphous
alloys. Granular films are ruled out because the inductance of the grain boundaries is in series with the inductance of the superfluid and the temperature dependence of the inductance of the grain boundaries is generally not described to a high degree of accuracy by the dirty limit BCS theory. Amorphous alloys on the other hand are well described by BCS theory and have their kinetic inductance enhanced by a multiplicative factor of \( (1 + \xi_0/\ell) \), where \( \xi_0 \) is the Pippard coherence length and \( \ell \) is the electronic mean free path [4]. For \( T << T_C \), \( \xi_0 \) is equal to the G-L coherence length. \( \ell \) is on the order of the interatomic spacing and thus the films are homogeneous down to very short distances of order 10Å. Since the zero temperature coherence length is of order 50Å – 100Å for amorphous superconductors, they are homogeneous as far as the superconducting order parameter is concerned and very well described by dirty limit BCS theory. The particular amorphous alloy, \( Mo_{77}Ge_{23} \), was chosen because of both the availability and the extensive characterization which has been performed in the past [46] [47] [48] [49] [50] [51].

The organization of this chapter is as follows. First sample preparation and general characteristics of the samples are discussed. Then the low temperature data are presented and their compatibility with thermal phase fluctuations is discussed. From these data the size of the zero temperature energy gap is extracted. Next, the transition region is discussed in the limit of large driving frequency where the measured inductance is dominated by the superfluid. A discussion of the frequency dependence of the complex impedance is deferred until Chapter 7.
5.2 Sample preparation

The films were grown by John Graybeal at the University of Florida by multitarget sputtering on Si substrates [47]. Films M1 and M7 were grown on plain Si which was not of very high quality. The properties of film M7 which is 30Å thick were strongly affected by the defects introduced by the poor substrate (the effect of defects in several films is discussed in Chapter 7). Film M1, which is 499Å thick was not noticeably affected by the rough nature of the substrate. All other Mo77Ge23 films were grown on oxidized Si substrates which were scratch free. All of the films are capped on both sides with between 10Å and 100Å of Ge.

The substrates are 15mm diameter circles. The circles were prepared by crystal bonding the polished surface of a roughly 17mm square of Si wafer to the smooth end of a 15 mm diameter steel rod. The substrate was then gently belt sanded until it was sanded down to the diameter of the steel rod. It is essential to dip the rod and Si in a cup of cold water frequently during the sanding process to avoid chipping the Si wafer. The substrates were then removed from the steel rod and thoroughly cleaned successively in acetone, TCE, methanol, and distilled water.

5.3 Sample properties

Nine Mo77Ge23 films with thicknesses ranging between 21.5 and 499Å have been studied. Figure 5.1 shows $\mu_0L^{-1}_\Omega/d$ versus temperature for all of the MoGe films and Table 5.1 lists their properties. The films shown in Figure 5.1 are named in order of decreasing zero temperature inverse sheet inductance M1 through M9. The uncertainty in the absolute value of the measured $L^{-1}_\Omega$ is about 4% and results from uncertainty in the geometry of the coils. The transition temperature decreases as the
Figure 5.1: $\mu_0 L^{-1}\sigma/d$ for the nine $M_{077}Ge_{23}$. At the lowest temperatures for which there is data the films are M1, M2 through M9 in order of decreasing $L^{-1}\sigma/d$ with the exception of the data for M7 being slightly lower than the data for M8 at the lowest temperatures.

film thickness is reduced. The reason for this is a competition between localization and superconductivity [47] and is not important for this study. In the absence of fluctuation effects the data in Figure 5.1 other than for M7 all follow the same universal function $L^{-1}\sigma(T/T_C0)$ as a function of $T/T_C0$. The importance of this cannot be overstated for this study and will be made clear in Section 5.5. For reference, $L^{-1}\sigma(0)$ for a monolayer of pure YBCO is 41$nH^{-1}$ compared with 162$nH^{-1}$ for film M1 and 2.57$nH^{-1}$ for film M9.
Table 5.1: Properties of the $Mo_{\gamma}Ge_{23}$ thin films. The values listed for the normal state sheet resistances, $R_{\Omega,n}$, are nominal based on Equation 5.1.

<table>
<thead>
<tr>
<th>Film</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
<th>M5</th>
<th>M6</th>
<th>M7</th>
<th>M8</th>
<th>M9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$ (Å)</td>
<td>499</td>
<td>61</td>
<td>61</td>
<td>46</td>
<td>37</td>
<td>30</td>
<td>30</td>
<td>27.5</td>
<td>21.5</td>
</tr>
<tr>
<td>$L^{-1}_{\Omega}$ (nH$^{-1}$)</td>
<td>162</td>
<td>13.68</td>
<td>13.2</td>
<td>9.55</td>
<td>6.70</td>
<td>4.97</td>
<td>4.56</td>
<td>4.21</td>
<td>2.57</td>
</tr>
<tr>
<td>$R_{\Omega,n}$ (Ω)</td>
<td>33.7</td>
<td>287</td>
<td>287</td>
<td>387</td>
<td>488</td>
<td>612</td>
<td>612</td>
<td>662</td>
<td>885</td>
</tr>
<tr>
<td>$T_C$ (K)</td>
<td>7.050</td>
<td>5.534</td>
<td>5.442</td>
<td>4.917</td>
<td>4.499</td>
<td>3.998</td>
<td>4.05</td>
<td>3.734</td>
<td>2.999</td>
</tr>
<tr>
<td>$T_{C0}$ (K)</td>
<td>7.050</td>
<td>5.67</td>
<td>5.559</td>
<td>5.040</td>
<td>4.640</td>
<td>4.149</td>
<td>4.24</td>
<td>3.881</td>
<td>3.167</td>
</tr>
</tbody>
</table>

The sheet resistance, measured by the technique discussed in Section B.3 is shown for several of the films in Figure 5.2. The systematic uncertainty for each curve is about 15%. Note that the normal state sheet resistance for M7 is significantly larger than for film M6 even though they are of the same nominal thickness. The nominal normal state sheet resistances of the $Mo_{\gamma}Ge_{23}$ films as a function of film thickness is given by the following expression [52], valid for $R_{\Omega,n} < 1k\Omega$:

$$R_{\Omega,n} = \frac{16.7k\Omega - \lambda}{d}[1 + 3\lambda/d]. \quad (5.1)$$

The uncertainty in $R_{\Omega,n}$ from Equation 5.1 is about 3% for all films but film M7 for which Equation 5.1 does not apply. Equation 5.1 states that the resistivity depends weakly on the film thickness. Much of this dependence for $R_{\Omega,n} < 1k\Omega$ can be attributed to electron scattering at the film boundaries [47].

Further proof of the quality of the films can be obtained by comparing the normal state sheet resistance with the measured inverse sheet inductance. For a dirty limit superconductor they are [53] theoretically related by

$$L_{\Omega,\text{theory}}(0) = \frac{k R_{\Omega,n}}{\pi\Delta(0)}[1 + \ell/\xi_0]. \quad (5.2)$$
Figure 5.2: Temperature dependence of the sheet resistance above $T_c$ for films M7, M6, M2 and M1.
Figure 5.3: Theoretical (line of slope 1.0) and experimental (small black circles) relationship between $L_{0}(0)$ and $R_{\alpha,n}$. It is assumed that $\Delta(0) = 1.89k_{B}T_{C0}$, $\ell = 4\text{Å}$ and $\xi = 50\text{Å}$ when plotting the experimental results.

In Equation 5.2 it is assumed that $\ell = 4\text{Å}$ and that $\xi_{0} = 50\text{Å}$. Measurements discussed in Section 5.4 show that the zero temperature energy gap ratio is $\Delta(0)/k_{B}T_{C0} = 1.89$. Using this result along with $T_{C0}$ from Table 5.1 and the nominal values for $R_{\alpha,n}$ it was found that all of the measured values of $L^{-1}_{0}$ are consistent with the theoretical expectations except for film M7 as shown in Figure 5.3. This agreement is significant since for some other large sheet inductance films the measured sheet inductance is roughly twice as large as expected theoretically [54]. An inductance larger than the theoretical value may indicate the presence of grain boundaries.
5.4 Low temperature results

Well below $T_C$, fluctuation effects are expected to be negligible for two reasons. First, since the energy to create vortex-antivortex pairs is much greater than $k_B T$ the density of vortex excitations will be infinitesimal. Second, very few of the spin wave modes should be excited since the temperature is much less than the $R/L$ cut off frequency for noise currents as discussed in Chapter 4.

Previous tunneling measurements by Carter [55] on bulk MoGe yielded a gap ratio of 1.76, but measurements on thin films by Graybeal [46] yielded a larger gap ratio of 2±0.2. In this study the zero temperature energy gap is determined by fitting the measured inverse sheet inductance for $T < 0.4 T_C$ to the dirty limit BCS theory given by [4]

$$\frac{L^{-1}_0(T)}{L^{-1}_0(0)} = \delta(T) \tanh[\delta(T)\Delta(0)/(2k_B T)].$$

(5.3)

The reduced energy gap, $\delta = \Delta(T)/\Delta(0)$, as a function of the reduced temperature, $t = T/T_C$, is a universal function in the weak coupling BCS theory. A fit to numerically calculated points of this universal function for $T/T_C < 0.4$ yields

$$\delta \simeq 1.0 - 2.0915e^{-1.9784 T_C / T}$$

(5.4)

where $1.764 k_B T_C \equiv \Delta(0)$. For strong coupling superconductors $\delta$ is no longer a universal function of the reduced temperature. The slope is larger as $t$ approaches 1 and at the lowest temperatures the reduced energy gap has less temperature dependence. To include this nonuniversal behavior Equation 5.4 is replaced by

$$\delta \simeq 1.0 - 2.0915e^{-1.12154(\Delta(0)/k_B)/T}$$

(5.5)
Figure 5.4: Low temperature data for the inverse sheet inductance for films M1 and M6. The black dots are the data and the dashed lines are the best fit of the data to Equations 5.3 and 5.5. There is a solid line which is indistinguishable from the data which is the result of a fit to Equations 5.6 and 5.5. The thick solid line is the prediction for the suppression of \( L_{\alpha}(T/T_{c0})/L_{\alpha}(0) \) for film M6 due to classical phase fluctuations as described by Equation 2.3.

with \( \Delta(0) \) allowed to vary. Note that Equation 5.5 reduces to Equation 5.4 when the weak coupling relationship between \( \Delta(0) \) and \( T_{c0} \) is restored. \( T_{c0} \) is most accurately determined from data near the transition temperature.

Figure 5.4 shows \( L_{\alpha}(T/T_{c0})/L_{\alpha}(0) \) versus \( T/T_{c0} \) as well as the best fit (dashed lines) of the data to Equations 5.3 and 5.5 for films M1 and M6. The fit for film M1 is nearly perfect, but the thinner films exhibit a slight deviation from the mean-field theory which increases as the film thickness is reduced. To account for this
extra temperature dependence when fitting the data to determine $\Delta(0)$, an additive
term to Equation 5.3 is used. The data for all films are fit to both Equations 5.3
and 5.5 and to

$$\frac{L^{-1} \alpha(T)}{L^{-1} \alpha(0)} = -c^T + \delta(T) \tanh[\delta(T)\Delta(0)/(2k_BT)].$$

with $n=1,2,3$ along with Equation 5.5. The best fits occurred with $n=2$. Fits
were performed over temperature ranges extending from the lowest measurement
temperature up to either $0.25T_c$ or $0.4T_c$. Fits to all films over either range and to
either Equations 5.3 and 5.5 or 5.6 and 5.5 with $n=1,2$ or $3$ all yield $\Delta(0)/k_BT_c$ within 0.1 of 1.89. The experimental result of a gap ratio which is independent of the
film's normal state sheet resistance is pleasing and is expected from theory [56].

If only $n=2$ fits to Equations 5.6 and 5.5 are considered then fits for all films are
within 0.03 of 1.89. Figure 5.5 shows the fitting coefficient $c$ of Equation 5.6 with
$n=2$ as a function of $L^{-1} \alpha(0)$. The error bars given represent the spread in $c$ resulting
from fitting up to different maximum temperatures. The regression line excludes the
two open circles which are data for films M7 and M9. As previously mentioned film
M7 was grown on a bad substrate and the film properties are different than the other
films. Film M9 is not included in the regression because high quality low temperature
data was not taken.

The meaning of the data in Figure 5.5 is unclear since the origin of the $T^2$
dependence is unknown. Regardless of the origin of the deviations from BCS theory,
it is certain that the deviations are not due to classical phase fluctuations as predicted
by Equation 2.3. The temperature dependence is not linear and the slope of a tangent
to the data for film M6 is much smaller than predicted. The thick black line in Figure
5.4 shows the slope expected for film M6 based on Equation 2.3 with $\alpha = 2/\pi$. In
Figure 5.5: The result of fits to the data for the coefficient, $c$, of Equation 5.6 with $n=2$ shown as a function of $L^{-1}K^2(0)$. The open circles (films M7 and M9) are not expected to follow the same pattern as the other films as explained in the text. The line is the result of a linear regression to the solid circle data. The error bars are the spread in values obtained for $c$ when the fits were performed over different temperature intervals.
light of the discussion about the quantum crossover in Chapter 4 it is not surprising that a linear suppression of the superfluid density is not observed in these films for \( T \) well below \( T_{C0} \).

### 5.5 Results near \( T_C \)

In Section 5.4 it was shown that classical phase fluctuations do not persist below \( 0.2T_{C0} \). A natural question to ask is above what temperature are the phase fluctuations not strongly suppressed below their classical value? To answer this question the measured superfluid density must be compared with the mean-field superfluid density for all of the films.

The data presented in this section represent the highest frequency data for which \( L_\alpha^{-1} \) is proportional to the bare superfluid density because the inductive impedance at \( \omega/2\pi = 50kHz \) of the thermally excited vortices is much smaller than that of the superfluid. The 50kHz data in this chapter are extrapolated to infinite frequency based on the lower frequency data. The details of the extrapolation and frequency dependence are discussed in detail in Chapter 7. Figure 5.6 shows that the extrapolation of the 50kHz data is small and confined to temperatures within a few tens of mK of \( T_C \).

Figure 5.7 shows \( L_\alpha^{-1}(T/T_{C0})/L_\alpha^{-1}(0) \) vs. \( T/T_{C0} \) for all of the Mo\(_{77}\)Ge\(_{23} \) films other than films M2 and M7. The dashed line is data for an In/InO\(_x \) film taken from Reference [22]. In the original paper the film was labeled G-C, but here it is labeled as film IFHG. \( T_C \) is defined as the temperature at which \( L_\alpha^{-1} \) has a maximum slope. \( T_{C0} \) was determined by fitting the data to Equation 5.3 in the temperature range.
Figure 5.6: The measured sheet inductance of film M4 from top to bottom for 2, 5, 10 and 50 kHz (dotted lines) and the extrapolation to infinite frequency (solid line) using the 5 and 50 kHz data sets and Equation 7.2 with $\beta = 0.86$.  

\begin{figure}[h]
\centering
\begin{tikzpicture}
\begin{axis}[
    title={L (nH) vs. T (K)},
    xlabel={T (K)},
    ylabel={L (nH)},
    xmin=4.7, xmax=4.9,
    ymin=1, ymax=3,
    xtick={4.7, 4.8, 4.9},
    ytick={1, 2, 3},
    grid=both,
    legend pos=north west]

% Add data points and plot lines here
\end{axis}
\end{tikzpicture}
\end{figure}
0.7\(T_{C0}\) through 0.85\(T_{C0}\). In performing the fits,

\[
\delta = \sqrt{\cos \frac{\pi}{2} \left( \frac{T}{T_{C0}} \right)^2}
\]  

(5.7)

[57] was used and it was assumed that the gap ratio, \(\Delta(0)/k_B T_{C0}\), was 1.89 for all films. The data for film IFHG from reference [22] did not extend below 1.2K so \(L^{-1}(0)\) and \(T_{C0}\) were used as fitting parameters when fitting the data to Equations 5.3 and 5.7. Allowing \(\Delta(0)\) to vary as well yielded \(\Delta(0)/k_B T_{C0} = 1.93\) which is sufficiently close to 1.89 to assume that film IFHG shares the same functional form for \(L_{MF}^{-1}(T/T_{C0})\) as the \(Mo_{77}Ge_{23}\) films. For film IFHG, \(d = 100\,\text{Å}, R_{Cl,n} = 1243\,\Omega, T_{C0} = 3.472\,K\) and \(L^{-1}(0) = 2.17\,n\,H^{-1}\).

It is remarkable that the data collapse to a single curve with nearly the same value, slope and curvature between 0.7\(T_{C0}\) and 0.85\(T_{C0}\) with a single fitting parameter. The data for the different films do not all match quite as well at intermediate temperatures around 0.4\(T_{C0}\) where data were not taken as carefully. Clearly the quantum crossover temperature lies above 0.85\(T_{C0}\) since if it had passed through the fitted temperature range the data would not have agreed so well.

The fact that all of the films follow the same curve \(L_{a}^{-1}(T/T_{C0})/L_{a}^{-1}(0)\) vs. \(T/T_{C0}\) below 0.85\(T_{C0}\) means that the data for the thick film, M1, in Figure 5.7 may be considered to represent the mean-field normalized inverse sheet inductance for all of the films. This is a very important result and a major advantage over other studies where the mean-field temperature dependence was determined by fitting to some functional form such as Equation 5.3 or simply assuming that it is linear near \(T_{C}\). The fact that film IFHG has nearly the same mean-field sheet inductance as the \(Mo_{77}Ge_{23}\) films is certainly fortuitous and in general it is expected that different types of superconducting thin films will have slightly different curves for the normalized
Figure 5.7: Solid lines are \( L^{-1}(T/T_{c0})/L^{-1}(0) \) vs \( T/T_{c0} \) for all \( \text{Mo}_{77}\text{Ge}_{23} \) films except for M2, M6 and M7. The dashed lines are data for film IFHG from Reference [22].
superfluid density as a function of $T/T_{C0}$. Uncertainties in the precise functional form for $L_{MF}^{-1}$ will obscure the suppression of the mean-field inverse sheet inductance due to phase fluctuations. The inset to Figure 5.7 shows the data near $T_C$. It clearly illustrates that as the reduced temperature $T/T_{C0}$ is increased, the films from thinnest (M9) to thickest (M3) peel off from the mean-field curve and then have a rapid drop at $T_C$ which falls below the mean-field transition temperature. Just below the drop, at $T = T_C^-$, $L_{C}^{-1}$ is 60 to 80% of its mean-field value. The drop occurs where $L(T)/T/\mu_0$ has a value near $9.8mm - K$, as predicted for the KTB transition.

A rigorous quantitative analysis is not possible given the available theory but an approximate analysis can be made. Based on Equation 2.3, it is expected that classical phase fluctuations increase approximately proportional to $k_B T/U_{00}(T)$, where $U_{00}(T) = (\phi_0/2\pi)^2 L_{C, MF}^{-1}$. As discussed in Chapter 2 the thick solid lines in Figure 5.8 represent the bounds for the range that classical longitudinal phase fluctuations may reasonably be expected to suppress the inverse sheet inductance. The intersection of the data with the dotted line of slope $2/\pi$ is where the KTB transition is expected to occur. Figure 5.8 shows $L_{C}^{-1}/L_{C, MF}^{-1}$ for the five thinnest $Mo_{77}Ge_{23}$ films (thin solid lines), the film IFHG from reference [22] (thin dashed line), and for another In/InOx film labeled ISJT (thin dot dashed line). Film ISJT is 190Å thick with $R_{C,n}(T = 10K) = 4150\Omega$ and $L^{-1}(0) = 0.692nH^{-1}$. $L_{C, MF}^{-1}$ for the $Mo_{77}Ge_{23}$ films and for film IFHG are taken from the M1 film data in Figure 5.7. Film ISJT however has a much different mean-field inverse inductance than the other films and a fit to data with $T/T_{C0}$ between 0.62 and 0.79 to Equations 5.3 and 5.7 yields $T_{C0} = 3.048K$ and $\Delta(0)/k_B T_{C0} = 2.43$. For the film ISJT data in Figure 5.8 it was
Figure 5.8: Thin solid lines are the ratio of the experimental inverse sheet inductance and the mean-field inverse sheet inductance for films M3, M4, M5, M8 and M9. The dashed line is for the In/InOx film IFHG from Reference [22] and the dot dashed line is for film ISJT. The thick solid and dashed lines represent approximate bounds for the effect of classical longitudinal phase fluctuations. The intersection of the other curves with the dotted line is where the KTB transition is expected to occur.
assumed that the mean-field inverse sheet inductance is given by Equations 5.3 and 5.7 with the aforementioned values for the fitting parameters.

The inset of Figure 5.8 shows that up to \( k_BT/U_{00}(T) = 0.2 \), \( L_{\Omega}^{-1} = L_{\Omega, MF}^{-1} \) within experimental resolution for all films. Note that \( k_BT/U_{00}(T) = 0.2 \) corresponds to \( T/T_C \) between 0.76 (film M9) and 0.86 (film M3) for the \( Mo_{77}Ge_{23} \) films. Above \( k_BT/U_{00}(T) = 0.4 \), which corresponds to \( T/T_C > 0.88 \) for all \( Mo_{77}Ge_{23} \) films, \( L_{\Omega}^{-1}/L_{\Omega, MF}^{-1} \) abruptly curves below unity, as expected for a quantum crossover. Once the crossover from the quantum to the classical regime begins it occurs very rapidly since a decrease in \( L^{-1} \) decreases \( \omega_0 \) in Equation 4.3. This in turn implies a smaller quantum suppression and thus a further decrease in \( L_{\Omega}^{-1} \). The abrupt drop in \( L_{\Omega}^{-1}/L_{\Omega, MF}^{-1} \) occurs at \( L_{\Omega}^{-1}/L_{\Omega, MF}^{-1} \approx 0.7 \pm 0.1 \) and \( k_BT/U_{00}(T) \approx 0.9 \pm 0.15 \). The uncertainty in \( T_C \) for each film leads to uncertainty in \( U_{00} \), and therefore in the value of \( k_BT/U_{00}(T) \) at which \( L_{\Omega}^{-1}/L_{\Omega, MF}^{-1} \) drops to zero. It is possible that for the films with \( R_{\Omega,n} < 1k\Omega \) the drops all occur at a single value, e.g., \( k_BT/U_{00}(T) \approx 0.9 \).

For films with a normal state sheet resistance of \( 4k\Omega \) or larger the KTB transition is expected to occur well below \( 0.9T/T_C \). Presumably the density of vortex-antivortex pairs will be lower than expected classically due to the quantum suppression of thermal phase fluctuations. Furthermore, the suppression of the inverse sheet inductance due to longitudinal phase fluctuations will not reach its full classical value at a temperature just below \( T_{KTB} \). If data for a large sheet resistance film were plotted in Figure 5.8 it would be expected to remain flat to a larger value of \( k_BT/U_{00} \). Indeed this effect was observed for film ISJT which has \( R_n = 4150\Omega \). It must be noted, however, that for this film the mean-field temperature dependence of the superfluid density had to be determined by curve fitting the data rather than by taking the mean-field temperature.
dependence from a similar but much thicker film as was done for the MoGe films. In the future it would be interesting to make systematic measurements of the inverse sheet inductance for a series of high sheet resistance films to see if these predictions are correct.
CHAPTER 6

VOREX EXCITATIONS AND THE KTB TRANSITION

In this chapter a necessary background is developed for discussing the complex impedance of thin superconducting films for which experimental data are presented in Chapter 7. Since it is believed that for two-dimensional superconductors vortex excitations are responsible for destroying superconductivity at a temperature below $T_{C0}$, a detailed discussion of vortex excitations is germane. The discussion is limited to excitations in dirty limit thin film superconductors which is the type of sample discussed for the remainder of this thesis. In the first section the energy of the excitations is determined which is of critical importance since the density of excitations depends exponentially on the excitation energy. This is followed by a phenomenological treatment of the KTB transition. Finally the KTB renormalization group approach and extensions thereof which include the dynamics of vortex-antivortex pairs are briefly described.
6.1 Vortex excitations and interactions

6.1.1 Single vortices

The requirement of a single valued order parameter demands that the line integral of the phase around a closed loop is an integral multiple of $2\pi$.

$$\oint \nabla \phi \cdot ds = 2\pi n$$  \hspace{1cm} (6.1)

If $n \neq 0$ there is a net vorticity within the loop which contains $|n|$ quanta of magnetic flux, where the magnetic flux quantum is $\phi_0 \equiv \hbar/2e$. Typically only vortices with $|n| = 1$ are present since higher values of $n$ are much more energetic excitations. At the center of each vortex the magnitude of the order parameter is zero. The order parameter magnitude increases roughly linearly with distance from the center of the vortex core and then saturates to the unperturbed value, $|\psi_\infty|$, at a distance of about 5 coherence lengths from the center of the vortex. For an extreme Type II superconductor ($\kappa \equiv \lambda/\xi >> 1$), the currents associated with a vortex extend to a much larger radial distance from the vortex core than the suppression of the order parameter magnitude. For $n = 1$ a vortex is present and for $n = -1$ an antivortex is present with currents circulating in the opposite direction as the currents due to a vortex. Figure 6.1 shows the magnitude of the order parameter and the sheet current density for a vortex in a thin film located at the origin. These values were numerically calculated for the limit $\kappa \to \infty$. For $\kappa > 100$ the solution is nearly identical to the limit of infinite $\kappa$ [58].

The order parameter and current density associated with a vortex can be determined by solving the G-L differential equations [4] together with Equation 6.1 and
Figure 6.1: Numerical calculation of the normalized order parameter magnitude, $f = |\psi|/|\psi_\infty|$, and the sheet current density $K$ multiplied by $\phi_0 \xi / 2\pi U_0$ as a function of radial position from the center of a vortex normalized by the G-L coherence length for a high $\kappa$ thin film.
Maxwell's equation $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ which must be satisfied both within the superconductor and in the region outside of the superconductor. An exact solution for a single vortex in an infinite area sheet of superconductor of thickness $d$ appears intractable. However, for a superconductor with large $\kappa$, the order parameter is suppressed only over a very small area compared to the electromagnetic region and a simplification (London model) may be made such that $|\psi|^2 = |\psi_\infty|^2$ [59] [4]. All of the superconductors studied in this thesis have $\kappa > 100$.

Within the London model approximation the vortex sheet current density for the interior (several penetration depths from the film surface) of a thick film ($d \gg \lambda$) is

$$K_{V,\text{bulk}}(r) = \frac{\phi_0 d}{2\pi \mu_0 \lambda^3} K_1(r/\lambda)$$

(6.2)

where $K_1$ is a modified Bessel function of the first kind and $r$ is the radial coordinate measured from the center of the vortex. For $\xi << r << \lambda$ the current density is proportional to $1/r$ and for $r >> \lambda$ the current density is proportional to $e^{-r/\lambda}/\sqrt{r/\lambda}$.

For a thin superconducting film ($d < \lambda$) and within a penetration depth of the surface of a thick superconducting film the vortex current density is

$$K_{V,\text{film}}(r) = \frac{\phi_0 d^2}{8\mu_0 \lambda^4} [H_1(\frac{rd}{2\lambda^2}) - N_1(\frac{rd}{2\lambda^2}) - \frac{2}{\pi}],$$

(6.3)

where $H_1$ is a Struve function and $N_1$ is a Neumann function [60] [61]. In contrast to the interior of a thick film the sheet current density is proportional to $1/r$ for $r << 2\lambda^2/d$ and is proportional to $1/r^2$ for $r >> 2\lambda^2/d$.

The London model describes the core region of the vortex incorrectly since both the current density and $|\psi|$ should go to zero linearly on the vortex axis. Clem has introduced a variational form for the magnitude of the order parameter which is qualitatively correct in the core region [62] [63]. The alternative method for describing
the core region properly is to numerically solve the G-L differential equations as I have done to produce Figure 6.1.

The energy of a single vortex can be conveniently separated into a core energy, $E_M$, associated with the suppression and gradient of the order parameter magnitude, the kinetic energy of the currents, $E_K$, and the energy stored in the magnetic field, $E_F$. If the order parameter is written as $\psi = f |\psi_\infty| e^{i\phi}$ where $f$ ranges from 0 to 1 the core energy is from the G-L equation

$$E_M = \frac{\pi}{2} U_0 \int_0^\infty dr \, r [ (1 - f^2)^2 + 2(\nabla f)^2 ] . \quad (6.4)$$

Using numerical calculations of the G-L equations for $f(r)$ in the high $\kappa$ limit yields $E_M = 2.45 U_0$ [64] [58]. In some of the earlier literature [65] [66] the ratio $E_M/U_0$ has been mistakenly taken as 1.23. Single vortices are not thermally generated below $T_C$ since $E_K + E_F$ is much greater than $E_M$ which is already larger than the thermal energy, $k_B T$.

6.1.2 Vortex-antivortex pairs

Although single vortices are not thermally generated a vortex and an antivortex of very small separation may be thermally excited. For such a pair, the total energy is small since the total current is confined to a region of diameter not much larger than the pair size. The energy of a vortex-antivortex pair of core separation, $r$, may be written as

$$E_{\text{pair}}(r) = V(r/r_0) + E_{C,\text{pair}}(r_0), \quad (6.5)$$

where $E_{C,\text{pair}}(r_0)$ is the energy of a minimum sized pair of separation $r_0$ ($r_0 \geq \xi$) and the interaction energy, $V(r/r_0)$, is zero for $r \leq r_0$. Pair excitations which are smaller than $r_0$ are ignored.
To determine $E_{\text{pair}}(r)$ exactly the G-L differential equations would have to be solved numerically for the vortex and antivortex. This is in general a nontrivial problem since the azimuthal symmetry is broken when the pair has nonzero separation. However, the solution is simple for a pair of zero separation and the order parameter magnitude is a step function from 0 at the origin of the cores to $|\psi_\infty|$ at all other positions which yields an excitation of zero energy. Clearly the G-L theory breaks down such small length scales and a minimum sized fluctuation must suppress the order parameter over a region of order $\xi$. Since the smallest possible excitation is a pair of zero separation which has no current associated with it, it will simply be an amplitude fluctuation with an energy of order $U_0$. This sort of excitation will be much more common than fully formed vortex-antivortex pairs which have their cores separated by $r_0$ or greater. To determine the exact structure and energy of pairs of size smaller than a coherence length microscopic calculations are required. Such calculations would perhaps shed some light on how pair nucleation occurs which would be of great interest. For the present purpose it is just noted that these small excitations exist and may have an effect of the impedance of a real film, although they are ignored in all of the literature.

In the absence of a solution to the G-L equations for a pair it is necessary to estimate $E_{\text{pair}}(r)$ based on the structure and energy of a single vortex. When the pair size is infinite $E_{\text{pair}}(\infty)$ is simply twice the energy of a single vortex. For pairs of a few coherence lengths in size more careful estimates are required. The core energy is

$$E_{C,\text{pair}}(r_0) = E_{M,\text{pair}}(r_0) + E_{K,\text{pair}}(r_0) + E_{F,\text{pair}}(r_0),$$

(6.6)

where $E_{M,\text{pair}}(r_0)$ should be calculated from Equation 6.4 with the proper form (which has not been calculated) for the reduced order parameter amplitude, $f$. $E_{K,\text{pair}}$ and
$E_{F,\text{pair}}$ are respectively the kinetic energy and magnetic field energy associated with the pair's current density. $E_{M,\text{pair}}$ is 4.9$U_0$ for pairs of large separation, but it is expected that it will begin to decrease when the pair size is less than two coherence lengths. Both $E_{K,\text{pair}}$ and $E_{F,\text{pair}}$ increase with $r_0$ but $E_{F,\text{pair}} \ll E_{K,\text{pair}}$ for a thin high $\kappa$ superconductor and $E_{F,\text{pair}}$ may be neglected. Both $E_{F,\text{pair}}$ and $E_{K,\text{pair}}$ are proportional to $U_0$ times a dimensionless function of $r/\xi$, but $E_{F,\text{pair}}$ is in addition proportional to $d\xi/\lambda^2$ which is very small for high $\kappa$ ultrathin films. Figure 6.2 shows $E_{M,\text{pair}}/U_0$ and $E_{K,\text{pair}}/U_0$ as a function of $\ln(r/2\xi)$. It is assumed that for $r > 2\xi$ $E_{M,\text{pair}}$ is independent of $r$ and equal to 4.9$U_0$ and $E_{K,\text{pair}}$ was estimated as follows. For a vortex centered at $(-r_0,0)$ and an antivortex centered at $(r_0,0)$ the kinetic energy density was integrated over Quadrant I and then multiplied by four. The sheet current in Quadrant I is assumed to be the vector sum of the sheet current due to the antivortex and the sheet current density of the vortex multiplied by the factor that the local superfluid density is suppressed by due to the antivortex. The remainder of the sheet current due to the vortex is assumed to pass through the antivortex core region as normal electrons. Since the density of normal electrons is much higher than the density of superconducting electrons near $T_c$, the kinetic energy of the normal electrons may be neglected compared to the superconducting electrons. For $r \geq 3.6\xi$, the total energy shown in Figure 6.2 is well described by the logarithmic asymptote given in Equation 6.7. Extrapolating the asymptote to $r_0 = 2\xi$ yields the core energy, $E_{C,\text{pair}}(r_0 = 2\xi)$. Allowing for error introduced by the numerical approximations used in calculating $E_{K,\text{pair}}$, I estimate that $E_{C,\text{pair}}(r_0 = 2\xi)$ is between 5.5 and 7 times $U_0$. This core energy is used in Section 6.2 to estimate the density of vortex-antivortex pairs.
Figure 6.2: Numerical calculation for the vortex pair energy normalized by $U_0$ as a function of $\ln(r/2\xi)$. The core energy for $r_0 = 2\xi$ is $6.22U_0$. 

$$E_{\text{pair}}/U_0$$  
$$E_{K,\text{pair}}/U_0$$  
$$(E_{M,\text{pair}} + E_{K,\text{pair}})/U_0$$
The force acting on a vortex due to an external sheet current is \( \mathbf{F} = K_{\text{ext}} \times \phi_0 \hat{z} \). Thus by integrating \( \phi_0 K_V(r') \) from \( r_0 \) to \( r \) the interaction potential, \( V(r/r_0) \), between a vortex and an antivortex is obtained. Of course this is only accurate when the vortices are separated far enough apart that the sheet current density of the vortex is not significantly perturbed by the presence of the antivortex. For a minimum sized pair of \( r_0 \) and \( r << 2\lambda^2/d \) the result for the interaction potential is

\[
V(r/r_0) \approx 2\pi U_0 \ln(r/r_0). \tag{6.7}
\]

Note that for \( 2\xi < r < 3.6\xi \) in Figure 6.2 the interaction energy increases less than logarithmically, but this is accounted for by taking the core energy to be the value of the logarithmic asymptote at \( r = 2\xi \). It is precisely the logarithmic interaction energy of Equation 6.7 between vortices that leads to the KTB vortex unbinding transition.

In the original paper by Kosterlitz and Thouless [15] they specifically noted that their results did not apply to superconductors because the logarithmic interaction only extends to separations of order \( 2\lambda^2/d \). While mathematically proper, Beasley [53] pointed out that in sufficiently thin dirty superconducting films \( 2\lambda^2/d \) can be on the order of 1cm and that a KTB like transition should be observable.

### 6.2 Phenomenology and theory of vortex-antivortex pairs and the KTB transition

#### 6.2.1 Thermodynamics and the static KTB transition

In an actual film vortex-antivortex pairs of very small size will be created and annihilated continuously. In this section the problem is viewed from a thermodynamic standpoint in which there is an equilibrium density of pairs present with their sizes distributed according to Boltzmann statistics. The validity of this approach demands
that the chemical potential for the creation of a pair is large enough that the density of pairs is very low. This thermodynamic approach leads to the KTB vortex-antivortex unbinding transition in which pairs of infinite separation unbind at a temperature, $T_{KTB}$ defined by

$$k_B T_{KTB} = \frac{\pi}{2} U_0(T_{KTB}).$$  \hspace{1cm} (6.8)

**Phenomenological approach**

In this section it is assumed that the density of vortex-antivortex pairs is dilute enough that each pair may be treated independently from all other pairs. Physically this is always the case at temperatures well below $T_{KTB}$. Furthermore the minimum sized pair is defined to be $r_0 = 2\xi$.

The density of vortex-antivortex pairs depends on the free energy, $F_{C,\text{pair}}(r_0)$, of a minimum sized pair.

$$F_{C,\text{pair}} = E_{C,\text{pair}} - k_B T \ln N_0$$  \hspace{1cm} (6.9)

Here, $k_B \ln N_0$ is a configurational entropy where $N_0$ is the number of distinct ways that the vortex and antivortex of minimum size may be arranged in a cell of dimension $4\xi \times 4\xi$. $N_0$ has not been calculated theoretically, but I assume that it is between and 4 and 12. The density of pairs of size $r_0$ or greater, $n_p$, can be estimated by calculating the probability of finding a minimum sized pair in an elementary cell of dimension $4\xi \times 4\xi$.

$$n_p \approx \frac{1}{16\xi^2} \frac{N_0 e^{-\beta E_{C,\text{pair}}}}{1 + N_0 e^{-\beta E_{C,\text{pair}}}}$$  \hspace{1cm} (6.10)

Here $\beta \equiv 1/k_B T$. For the range of estimated $E_{C,\text{pair}}$ and $N_0$ and assuming that each pair occupies an area of normal material of $2\pi\xi^2$ the fraction of the film that is normal
at the KTB transition is between 1.7% and 10%. Of course this estimate neglects the
more numerous excitations of size smaller than \( r_0 \).

The total density of pairs calculated above may be redistributed such that the
density of pairs of size greater than \( r \), \( n_{p>}(r) \) is given by

\[
n_{p>}(r) \approx n_p \left(2\xi/r\right)^{2\pi\beta U_0^{-2}}.
\]  

(6.11)

Since \( 2\pi\beta U_0 - 2 > 2 \) below \( T_{KTB} \) the quantity \( n_{p>}(r) \) decreases rapidly with increasing
\( r \). Note that for obvious reasons \( n_{p>}(r) \) must be independent of the choice of \( r_0 \). As
\( r_0 \) in Equation 6.11 decreases, \( n_p \) will increase and the second factor will decrease.
\( N_0(r_0) \) and \( E_{C,\text{pair}}(r_0) \) must be related to keep \( n_{p>}(r) \) independent of \( r_0 \).

Considering an ensemble of individual pairs of minimum size \( r_0 \) the mean square
pair size is [15]

\[
<r^2> = r_0^2 \frac{\pi U_0 \beta - 1}{\pi U_0 \beta - 2}.
\]  

(6.12)

Thus it is apparent that the mean square separation diverges when \( k_B T_{KTB} = \pi/2U_0(T_{KTB}) \) which signifies the vortex-antivortex pair unbinding transition. Physically the reason for the unbinding is a competition between increased energy and
entropy for pairs of large separation. The entropy of a pair is proportional to \( \ln(r) \)
since the number of ways in which the pair may be oriented at a separation \( r \) is propor­tional to the circumference of a circle of radius \( r \). From Equation 6.7 the interaction
potential is also proportional to \( \ln(r) \). Thus at low temperature the distribution of
pair size is dominated by the energy, but as the temperature increases the entropy
becomes increasingly important. From Equation 6.12 one can see that the mean
square pair size shrinks very quickly with increasing \( U_0 \beta \) with \( <r^2> = 1.5r_0^2 \) when
\( U_0(T)\beta = 4/\pi \).
In the presence of an infinitesimal transport sheet current, $K_T$, pairs will tend to polarize along the direction of the vector $\hat{z} \times K_T$. The polarizability of pairs with $r > r_0$ is calculated to be $(\pi/2)U_0\beta r^2$ [15]. Assuming that a pair fully polarizes under the influence of $K_T$, then the interaction potential given by Equation 6.7 is modified to

$$V(r/r_0) \approx 2\pi U_0 ln(r/r_0) - K_T\phi_0\pi(1 - r_0/r).$$  \hspace{1cm} (6.13)

Assuming $r >> r_0$ the maximum in the potential occurs at

$$r_c = \frac{2\pi U_0}{\phi_0 K_T}. \hspace{1cm} (6.14)$$

Pairs of size larger than $r_c$ will behave as two free vortices since the net force acting on them pulls them apart.

The majority of experiments [67] [22] [68] testing KTB theory have utilized this saddle point in the potential energy in the following manner [69]. The energy of a pair of size $r_c$ under the influence of a small sheet current $K_T$ is

$$E(r_c) \approx E_{C,pair} - 2\pi U_0 ln(K_T/K_0), \hspace{1cm} (6.15)$$

where $K_0 = 2\pi U_0/r_0\phi_0$ and $K_T << K_0$. It is then assumed that pairs escape over this barrier classically at a rate, $\Gamma_c$, where $\Gamma_c \propto e^{-\beta E(r_c)}$. Taking the recombination of free vortices into account it is found that the steady state density of free vortices is proportional to $\Gamma_c^{1/2}$. Assuming that the film's resistance is proportional to the density of free vortices leads to a nonlinear Ohm's law since the resistance depends on the transport current.

$$R \propto (K_T/K_0)^{\pi U_0\beta} \hspace{1cm} (6.16)$$

The experiments are done by measuring the voltage of a strip sample as a function of current at fixed temperature. In principle the power of current which the voltage
is proportional to yields $U_0$. According to this model the sheet resistance should be proportional to the current density squared at $T = T_{KTB}$ and the exponent increases as the temperature is decreases. Many of the experimenters testing the predictions of the KTB transition have interpreted the temperature at which $V \propto I^3$ as being $T_{KTB}$, but a recent reanalysis of much of the data in the literature has suggested that $V \propto I^{5.6}$ at the KTB transition [27].

**Renormalization group approach to the KTB transition**

In this section the renormalization group (RG) approach to the KTB transition is briefly described. A number of reviews give more detail than can be found here [70] [71] [72] [73] [74]. A detailed discussion is not required because the renormalization effects are limited to temperatures very close to $T_{KTB}$ [75] and the RG procedures are not used in the discussion of the data in Chapter 7.

The basic idea behind the RG approach is that small pairs polarize and screen the interaction between larger pairs [15]. A central quantity in the RNG method is the length dependent vortex dielectric constant, $\epsilon(\ell)$, where $\ell = \ln(r/r_0)$ is the scale. The RG procedure leads to a pair of coupled differential equations [76] whose solution yields $S(\ell)/k_B T$, where $S(\ell)$ is the effective phase stiffness, and the scale dependent vortex fugacity, $y(\ell)$. The Kosterlitz RNG equations may be integrated from $\ell = 0$ up to the desired length scale with initial conditions of $S(0)/k_B T$ being proportional to the bare superfluid density and $y(0)$ being proportional to the vortex core energy. Note that $\epsilon(0)$ is unity since the interaction between the smallest pairs is unrenormalized. It has been proven mathematically for the 2DXY model that a phase transition occurs when $S(\ell = \infty)/k_B T = 2/\pi$ at which point it drops discontinuously to zero [77] [78].

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The term bare superfluid density refers to the superfluid density suppressed by longitudinal phase fluctuations, but does not include any suppression of the phase order due to the vortices. Note that the experimentally measured quantity $U_0$ is equivalent to $S(\ell)$ where $\ell$ is large enough that $S(\ell)$ is nearly equal to $S(\infty)$. Throughout this thesis $U_0$ has been assumed to be proportional to the bare superfluid density which is likely slightly inaccurate for temperatures extremely close (within about 5mK or less) to $T_{KTB}$ where according to KTB theory the measured $U_0$ will be suppressed by a factor of $1/\epsilon(\infty)$ below its bare value which does not include the effects of the vortices. Typical values extracted from other experiments for $\epsilon(\infty)$ at $T = T_{KTB}$ are roughly 1.2 [67] [22] [68]. As the temperature is decreased $\epsilon(\infty)$ rapidly approaches unity.

6.2.2 Dynamics of vortex-antivortex pairs

If one considers only the force between a vortex and an antivortex the lifetime may be estimated. Suppose that a vortex-antivortex pair exists with a core separation of $r << \lambda_\perp$ in a medium with a vortex viscosity, $\eta$, which is determined by the Bardeen-Stephen relationship [79]

$$\eta = \frac{\phi_0^2}{2\pi \xi^2 R_n}.$$  \hspace{1cm} (6.17)

If it is assumed that there is no current noise in the film then the pair size will shrink to a size $r_0$ and then proceed to annihilate. The lifetime before annihilation is $(\tau \approx L_\square)/R_{\square,n}(r/\xi)^2$. For a dirty limit superconductor the lifetime near $T_C$ is roughly

$$\tau \approx \frac{5.75 \times 10^{-13} s - K \frac{r}{\xi(T)}}{T_{C0} - T} \left(\frac{r}{\xi(T)}\right)^2.$$  \hspace{1cm} (6.18)
The lifetime of a vortex-antivortex pair is not as short as predicted by Equation 6.18 because the vortices move diffusively rather than ballistically. In the two dynamical theories discussed below it is assumed that the pair separation obeys Langevin dynamics.

**AHNS model for the pair dynamics**

Ambegaokar et al. extended the static results of the KTB unbinding picture to nonzero frequencies \([80] [81] [82]\) in the context of vortex dynamics in thin superfluid \(He^4\) films. The adaptation of this work to superconducting films was performed by Halperin and Nelson \([69]\). The main result of these works is that at finite frequencies the universal drop in the superfluid density does not occur and the transition is broadened and the superfluid density goes to zero at a higher temperature. They introduce a complex frequency dependent dielectric constant. The real part corresponds to pairs which may polarize under an applied ac field and the imaginary part corresponds to pairs which cannot fully reorient at the applied frequency. The pairs are assumed to move diffusively and the characteristic pair size which separates the pairs which can follow the applied field and those which cannot is estimated to be

\[
r_{\omega} = \sqrt{\frac{14D}{\omega}},
\]

where \(D\) is the vortex diffusion constant. The diffusion constant is related to \(\eta\) by

\[
D = k_B T/\eta.
\]

**Minnhagen phenomenology**

Minnhagen has put forth a phenomenological model similar to Ambegaokar's based on the analogy between the neutral Coulomb gas and a system containing
vortex-antivortex pairs [72]. In the literature this is referred to as the Minnhagen phenomenology (MP). He also introduces a Langevin equation for the vortex dynamics and calculates the real part of the inverse of the frequency dependent dielectric constant. In his notation the superfluid density renormalized by vortex interactions, $\rho_s$, is related to the bare superfluid density, $\rho$, by

$$\frac{\rho_s}{\rho} = \frac{1}{\epsilon} 1 + \left( \frac{r_\omega/r_{f_\omega}}{\sigma_2} \right)^2. \quad (6.21)$$

Here $\epsilon$ is the static vortex dielectric function, $r_\omega$ is a diffusion length as in the AHNS model, $r_{f_\omega} \approx 2\lambda^2/d$ is the effective range of the logarithmic interaction, and $r_{f_\omega}$ is a length scale related to the average spacing between unbound vortices. According to his model $\rho_s/\rho$ is proportional to $\sigma_2$. He accounts for the dissipation by using the Kramers-Kronig relation to determine the real conductivity. It is not clear to me that it is proper to associate the imaginary conductivity with the superfluid density. In Chapter 7 it is the inverse sheet inductance which is assumed to be proportional to the density of superconducting electrons.
CHAPTER 7

FREQUENCY DEPENDENCE OF THE COMPLEX IMPEDANCE FOR MOGE AND IN/INOX THIN FILMS

In Chapter 5 inverse sheet inductance data for Mo_{77}Ge_{23} films were presented which were taken in the high frequency limit and represent the superfluid density in the absence of vortex-antivortex pair contributions. Here the term high frequency limit refers to the experimentally observed saturation of the inverse sheet inductance as the measurement frequency is increased and is not related to the theoretical treatments of the dynamic KTB transition. Given that the impedance of the transition exhibits a large frequency dependence it was necessary to explore the dynamics to determine how the bare superfluid density could be extracted. In this chapter the full frequency dependence of the data are discussed. In addition the dependence of the complex conductivity on an external magnetic field and on the amplitude of the drive coil current are examined. The aim is twofold. First in Section 7.1 data are presented for varying frequency, applied magnetic field and sheet current density amplitudes without detailed interpretation. The upshot of this phenomenological analysis is a justification for interpreting the high frequency limit data of Chapter 5 as representing the bare superfluid density and a verification that the measured complex
conductivities are not affected by the ambient perpendicular magnetic field or nonlinear effects due to a finite current density for the experimental conditions. Section 7.2 is concerned with comparing the data presented in Section 7.1 with the conventional KTB theory. There are major discrepancies and unresolved issues between the data and theory which are addressed.

7.1 Experimental data for the complex impedance

Following Hebard [83] the impedance of the film is treated as the inductive superfluid in series with a complex vortex impedance. Thus the total impedance, \( Z \), is

\[
Z = i\omega L_{\Omega, SF} + Z_V = i\omega L_{\Omega, SF} + R_V + i\omega L_V. \tag{7.1}
\]

\( L_{\Omega, SF} \) is the inductance of the bare superfluid density in the absence of vortex contributions, but it does include any suppression of the average superfluid density due to longitudinal phase fluctuations. The operational definition of \( Z_V \) here is all of the impedance not resulting from the inductance of the bare superfluid, so if there are relevant excitations other than the vortex-antivortex pairs, then their resulting impedance is lumped into \( Z_V \).

When the conductivity of the film is homogeneous on length scales smaller than the coil radii, what is experimentally measured is the complex sheet conductance (for films much thinner than the screening length), \( G = d(\sigma_1 - i\sigma_2) \). In the analysis presented in this chapter it is assumed that the conductivity of the film is uniform over length scales smaller than the coil dimensions. This may not be the case when a field induced vortex lattice is present in the film as the lattice may undergo compression and expansion in the radial direction which would lead to an extra mutual inductance.
which is geometry dependent and not included in the modeling [84]. It is highly unlikely that a similar nonlocal effect would occur in zero field when only vortex-antivortex pairs are present.

When calculating $R$ and $L^{-1}$ from the complex sheet conductance the uncertainties in $R$ and $L^{-1}$ need to be considered. When either component of the complex conductivity is much smaller than the other component there will be considerable uncertainty. The most accurate values for $R$ and $L^{-1}$ occur at the temperature at which $\sigma_1 \approx \sigma_2$. As the frequency is lowered this occurs at lower temperatures. As $T$ decreases below the temperature at which the dissipation is a maximum the response is primarily inductive and a small error in the phase of the measured mutual inductance leads to large error in the calculated sheet resistance. For very low frequency measurements and for $T$ above the temperature at which the dissipation peaks both $\sigma_1$ and $\sigma_2$ are very small and there are large uncertainties in both $R$ and $L^{-1}$. One of the reasons for the large uncertainty at temperatures above where the dissipation peaks has to do with the coils being on opposite sides of the film. As the temperature is lowered from $T_C$ and the inductive response begins to turn on the screening is very weak and the signal is proportional to the small change in the mutual inductance from its large value above $T_C$. Better data could be obtained by having both coils on the same side of the film with the pick-up coil counterwound inside the drive coil. With this geometry the measured voltage is proportional to the screening. Although this geometry can provide more accurate data at the very onset of the transition, it is not as accurate at lower temperatures where the screening is strong which is why I used coils on opposite sides of the film. The other factor which leads to large
uncertainties at low frequencies is simply that the pick-up voltage is proportional to
the driving frequency so electronic noise becomes a factor.

7.1.1 Frequency dependence of the complex impedance

In this section data are presented which show how $L_{SF}$ is extracted from the
total inductance. The data were taken with the static magnetic field perpendicular
to the film nulled to within 2mG and at low enough drive currents such that the
measured impedance was independent of the drive coil current. Results are presented
for several representative MoGe films as well as for a single In/InOx film to test if the
behavior observed for MoGe is consistent with what is observed in a different type of
superconducting thin film.

Results for MoGe

Figure 7.1 shows the complex conductivity for film M4 at several frequencies
between 190Hz and 50kHz. The small peaks represent the real part of the conductivity
and the lines which increase roughly linearly below 4.7K represent the imaginary
conductivity. Note that below 4.7K the dissipation becomes immeasurably small and
$\mu_0 \omega \sigma_2$ becomes frequency independent and is simply equal to $\lambda^{-2}$. The temperature
at which $\sigma_1$ peaks increases and the imaginary conductivity approaches the value
due to the bare superfluid density as the frequency is increased. The temperature at
which $\sigma_2$ goes to zero is independent of frequency, but as the frequency is increased
$|d\sigma_2/dT|_{T=T_c}$ gets larger.

Figure 7.2 shows the inverse inductance and sheet resistance for film M4 calcu-
lated from the complex conductivities shown in Figure 7.1. The inverse inductance
increases as the frequency is increased. The sheet resistance shows a sharp increase
Figure 7.1: The complex conductivity for film M4 measured at frequencies of 0.19, 1, 2, 5, 10, and 50kHz. The imaginary conductivity goes to zero at a fixed temperature independent of frequency. The imaginary conductivity at fixed temperature decreases monotonically with frequency, and the temperature at which the real conductivity peaks decreases monotonically with frequency.
Figure 7.2: $L^{-1}$ and $R$ for film M4 at frequencies of 0.19, 1, 2, 5, 10, and 50kHz. The inverse inductance decreases monotonically with frequency at fixed temperature and the sheet resistance increases with frequency at fixed temperature. The sheet resistance shows a sharp increase at a temperature independent of frequency which occurs at the same temperature at which $L^{-1}$ goes to zero.
Figure 7.3: Complex conductivity for film MS measured at frequencies of 0.1, 0.5, 5 and 50kHz.

at the temperature where the inverse inductance goes to zero. $T_C$ is defined as the temperature at which the inverse sheet inductance has a maximum slope. The sheet resistance is largest for the 50kHz data and smallest for the 190Hz data. Excluding the 190Hz data a fit to $R_V$ vs. $\omega^\alpha$ at $T = 4.85K$ yields $\alpha = 0.14 \pm 0.02$. Figure 7.3 shows the complex conductivity and Figure 7.4 shows the $R$ and $L^{-1}$ for film MS measured at frequencies of 0.1, 0.5, 5 and 50kHz. The results are qualitatively the same as for film M4 and all of the other MoGe films.

To explore the frequency dependence in more detail data were taken at fixed temperature for film MS while sweeping the frequency. Data are shown in Figure 7.5.
Figure 7.4: $R_V$ and $L^{-1}$ for film M8 at frequencies of 0.1, 0.5, 5 and 50kHz.
for \( T = 3.688 \text{K} \) which is just below \( T_C \). The top panel shows the sheet resistance and the vortex inductive impedance as a function of frequency. It is found that they both obey power laws with frequency such that \( R \propto \omega^{0.13 \pm 0.02} \) and \( \omega L_V \propto \omega^{0.17 \pm 0.07} \). The bottom panel shows how \( L_V \) was determined. It can be seen that as the frequency increases the total inductance is approaching a constant value which is interpreted as the kinetic inductance of the bare superfluid density, \( L_{SF} \). \( L_{SF} \) was varied to obtain the best linear fit to \( \log(L - L_{SF}) = \text{constant} - \beta \log(f) \), where \( L_V = L - L_{SF} \). The inset to the bottom panel shows the best fit which yields \( L_{SF} = 1.708 nH \) and \( \beta = 0.83 \).

Figure 7.6 shows \( L \) versus frequency for film M3 at four fixed temperatures. The black symbols in the bottom panel show the sheet resistance versus frequency (from top to bottom \( T = 5.415 \text{K}, 5.406 \text{K}, 5.386 \text{K}, \) and \( 5.367 \text{K} \)) and the open circles show the vortex impedance determined by the same procedure as in Figure 7.5 for \( T = 5.406 \text{K} \). Regressions for all 5 data sets show that \( R \) and \( \omega L_V \) are proportional to \( \omega^{0.13} \). All of the high quality MoGe films exhibit roughly the same frequency dependence, in which \( R \) and \( \omega L_V \) are proportional to roughly the same small power of \( \omega \). Also of note is that the sheet resistance is consistently about a factor of five larger than the inductive portion of the vortex sheet impedance.

**Results for In/InOx film ISJT**

Data are presented for an In/InOx film which show the same frequency dependence for the vortex impedance as the MoGe films. The film ISJT was grown by sputtering an Indium target in an Argon/Oxygen environment with a total pressure of \( 3.3 \times 10^{-4} \) torr and a partial oxygen pressure of \( 9.9 \times 10^{-5} \) torr. A systematic study of films grown by this technique can be found in Reference [85]. The morphology of films grown by
Figure 7.5: Top panel shows $\log(R)$ and $\log(\omega L_V)$ vs. $\log(f)$ at $T=3.688$K. Both $R_V$ and $\omega L_V$ are proportional to $\omega^{\alpha}$ with $\alpha \approx 0.13$ and 0.17 respectively. The bottom panel shows how $L_{SF}$ was determined.
Figure 7.6: The solid symbols of the bottom panel show $R_Y$ vs. frequency for film M3 at 4 fixed temperatures which are from top to bottom 5.415, 5.406, 5.386 and 5.367K. The open symbols represent the vortex inductive impedance at $T=5.406K$. The top panel shows $L$ at the same 4 temperatures and that $L$ approaches a constant at large frequency.
this technique depends critically on the combination of partial pressures and deposition rates used to make the film. Depending on the deposition parameters films may have granular and/or amorphous characteristics. Reproducibility between films is difficult to achieve and the temperature dependence of the mean-field superfluid density is not known with great accuracy which makes these films unsuitable for studying the quantum crossover with quantitative accuracy. For the particular sputtering system that I used the deposition rates and proper pressures and thickness calibration were determined by Saad Hebboul and Rita Rokhlin. The total deposition time was 45 seconds with a current density of .295 mA/cm² and a resulting film thickness of 190 Å. The complex conductivity below $T_C$ and the normal state sheet resistance above $T_C$ are shown in Figure 7.7. Reasonable agreement is found between the measured zero temperature inductance and the theoretical expectation given by Equation 5.2. $T_{C0}$ is estimated to be about 3.05 K.

Figure 7.8 shows $\mu_0\omega\sigma$ for eight frequencies between 200 Hz and 100 kHz. The absolute value of $\sigma_2(0)$ is only known to within 10% for this film. The reason is that the film is very easy to damage and a special sample holder was used such that the film was raised 0.5 mm above the pick-up coil. For film ISJT, the dissipation extends to a much smaller fraction of $T_{C0}$ than for the MoGe thin films. Figure 7.9 shows the corresponding sheet resistance and inverse sheet inductance. The sheet resistance increases monotonically with frequency according to a power law with an exponent of roughly 0.2. Also the inverse sheet inductance is saturating at high frequency, although it is not as close to saturation as for the MoGe films at $f = 50 kHz$. Thus in conclusion the In/InOx film ISJT behaves in much the same way as the MoGe films.
Figure 7.7: Normal state sheet resistance and complex conductivity measured at 50kHz for film ISJT, a 190Å thick In/InOx film.
Figure 7.8: Complex conductivity of film ISJT measured at frequencies of 0.2, 0.5, 1, 2, 5, 10, 50 and 100kHz.
Figure 7.9: Sheet resistance and inverse inductance of film ISJT. The 8 measurement frequencies are listed in the caption of Figure 7.8.
Summary of frequency dependence and extrapolation procedure

The experimental observation that both $R$ and $\omega L\nu$ are proportional to a small power, $\alpha$, means that $L\nu$ is proportional to $\omega^{-\beta}$ where $\beta$ is roughly 0.85. Thus by making measurements at high enough frequencies the vortex inductance will be much smaller than the frequency independent superfluid inductance. In practice it is found that for $f=50\text{kHz}$ measurements for the MoGe films are such that the measured inductance is nearly all due to the superfluid background. To eliminate any remaining vortex contribution to the measured inductance, two $T$ dependent data sets at different frequencies ($\omega_1 > \omega_2$) are used to extrapolate $L^{-1}$ to infinite frequency so that the inverse sheet inductance of the bare superfluid is given by

$$L_{SF}^{-1} = L_1^{-1} \frac{1 - (\omega_2/\omega_1)^\beta}{1 - (\omega_2/\omega_1)^\beta \frac{L_1^{-1}}{L_2^{-1}}}.$$  \hspace{1cm} (7.2)

Figure 7.10 shows how the extrapolation to determine the bare superfluid density works. The two solid lines are $L^{-1}$ measured at $f=50\text{kHz}$ and 10kHz from top to bottom respectively. Using Equation 7.2 with $\beta = 0.86$ and these two data sets, $L_{SF}^{-1}$ was extracted and is illustrated by the dashed line. This extrapolation increases the inverse sheet inductance over its value measured at 50kHz by less than 10% at a temperature just below $T_C$. Several mK below $T_C$ the extrapolation is much less than 1% of the measured $L^{-1}$ at $f=50\text{kHz}$, so the correction is a very small one. The correction should be accurate to within 10% as well, so the bare superfluid density is known to better than 1%. This extrapolation technique was used for all of the MoGe data shown in Figure 5.7.
Figure 7.10: Extrapolation of the inverse inductance for film M4 to infinite frequency. The two solid lines are from 10kHz and 50kHz data sets while the dashed line is the extrapolation using the data and Equation 7.2 with $\beta = 0.86$. 

\[ L^{-1} \text{ (nH)} \]

\[ T \text{ (K)} \]
7.1.2 Drive current dependence

As discussed in Chapter 6 the majority of experiments investigating the KTB transition have involved measuring nonlinear I-V curves where the resistance appears due to the current induced unbinding of vortex-antivortex pairs. Generally it is desirable when measuring the complex conductance of thin films that the drive current amplitude is small enough that linear response is observed, a condition which is impossible within the framework of the conventional KTB theory. Experimentally, however linear response is indeed observed for measurements done at frequencies below 100kHz and a nonzero sheet resistance is measured below $T_C$ for all of the MoGe and In/InOx films studied. A quantitative analysis of the nonlinear sheet resistance at higher current densities is not trivial when using the mutual inductance technique since the sheet current induced in the film varies both spatially and temporally.

Figure 7.11 shows the complex conductivity (bottom panel) and the sheet resistance (top panel) measured with 4 different drive coil currents for film ISJT. The drive currents used were 0.30, 1.03, 3.45 and 10.4 $\mu$A. For the lowest three drive currents the response is linear, but for the highest drive current the sheet resistance is larger than for lower drive currents near $T_C$.

Measurements of the complex impedance were made for film M8 while sweeping the drive current at fixed temperature. Figure 7.12 shows the sheet resistance as a function of the maximum current density amplitude induced in the film. For reference Figures 7.3 and 7.4 show the complex conductivity and the complex impedance respectively as a function of temperature for this film measured with an rms drive current ($I_{dr} = 1\mu$A) such that the maximum amplitude of the sheet current density is 50$\mu$A/cm for the $f = 50kHz$ data at $T = 3.698K$. From Figure 7.12 it can be
Figure 7.11: The complex conductivity and sheet resistance for film ISJT measured with drive coil currents of 0.3, 1.0, 3.45 and 10.1 μA. The maximum amplitude of the sheet current density for $I_{dr} = 1 \mu A$ is $50 \mu A/cm$ at $T = 2.5K$ and it increases with decreasing temperature.
Figure 7.12: The sheet resistance of film M8 (f=50kHz) as a function of the maximum induced current density amplitude induced in the film at two fixed temperatures. The critical current density is roughly \(600\mu A/cm\).
seen that the critical sheet current density is about $600\mu A/cm$ for temperatures just below $T_C$.

Other than the data in this section, all of the transition region data for the MoGe films presented in this chapter and in Chapter 5 were taken with the same drive coils and an rms drive coil current of $1\mu A$. For these drive coil parameters the maximum sheet current density amplitude at $T_C^{-}$ which is just below where the inverse sheet inductance shows a sharp drop is $30\mu A/cm$ for film M9 and $50\mu A/cm$ for film M2. At a temperature at which $U_0(T) = 2U_0(T_C^{-})$ the maximum sheet current density amplitudes are $54\mu A/cm$ for film M9 and $85\mu A/cm$ for film M2. As $T$ approaches zero the maximum sheet current density amplitudes for films M9 and M2 are 165 and $280\mu A/cm$ respectively. Since the critical current density is about $600\mu A/cm$ just below $T_C$ and it increases with decreasing temperature the complex impedance data taken with $I_{dr} \approx 1\mu A$ are independent of the drive current at all temperatures within experimental resolution.

7.1.3 Magnetic field dependence

Introduction

In the presence of an external magnetic field perpendicular to a superconducting film vortices of the same sign will enter the film. Assuming that there is no barrier to flux penetration the density of vortices in the film will be $n_V = B_A/\phi_0$. In the absence of thermal fluctuations and pinning sites the vortices will arrange themselves in a triangular lattice with a lattice constant of $a = 1.075n_V^{-1/2}$. Thermal fluctuations, and pinning sites can alter the effect of the field induced vortices.

The primary aim of varying the magnetic field for this work was to determine to what degree the field must be nulled in order for free vortex contributions to
the complex impedance to be negligible. Additionally, the behavior of the complex conductivity in an applied field can in principle give information about the films such as the presence of pinning sites, or the vortex viscosity, \( \eta \), in flux flow experiments.

If all of the vortices are located in identical harmonic pinning sites with a spring constant \( \kappa \) the impedance of the vortices will be frequency dependent and given by the following expression.

\[
Z_V = R_V + i\omega L_V = \frac{n V \phi_0^2}{\eta} \frac{(\omega/\omega_0)}{1 + (\omega/\omega_0)^2} [(\omega/\omega_0) + i].
\]  

(7.3)

The characteristic frequency \( \omega_0 \) is equal to the ratio \( \kappa/\eta \). For frequencies much less than \( \omega_0 \) the vortex impedance is purely inductive since the vortex moves quasistatically under a low frequency applied force while for high frequencies the vortex cannot respond to the rapid variation in the driving force and the response is limited by the viscosity and thus behaves resistively. In the absence of pinning the response should be purely resistive since the vortices flow freely. For reference, measurements by Aaron Pesetski [86] indicate that for YBCO thin films the characteristic frequency is of order \( 10^{11}\text{rad/s} \) so that vortices in YBCO respond inductively for measurements done in the kHz range.

**Experimental results for MoGe**

Figure 7.13 shows the complex conductivity for film M2 measured with the film in six different applied fields all below 50mG. The dissipation peak becomes broader and peaks at a lower temperature as the applied field is increased. Figure 7.14 shows the corresponding inverse inductance and sheet resistance for the same measurements. The sheet resistance increases by an order of magnitude when the field is raised from zero up to 50mG, but the transition temperature is unaffected. The inverse
Figure 7.13: The complex conductivity ($f=50$kHz) for film M2 measured in fields of 2, 11, 20, 22, 29 and 49 mG. The uncertainty in the applied field is 2 mG.
Figure 7.14: The inverse inductance and sheet resistance (f=50kHz) for film M2 measured in several small magnetic fields. The field values are listed in the caption of Figure 7.13.
inductance is not noticeably suppressed below the zero field value for fields less than 20mG. The effect of larger fields is shown in Figures 7.15 and 7.16. The data are for film M6 and extend up to $B = 1290mG$. The transition temperature for these larger fields is slightly suppressed, but the inverse inductance still shows a sharp drop at the transition. The sheet resistance grows with applied field as for film M2.

To study the effect of field induced vortices in more detail the magnetic field was swept at fixed temperature for several different frequencies. Figure 7.17 shows $R_\alpha$ and $\omega[L(B) - L(B = 0)]$ for film M4 at $T = 4.600K$. At this temperature the zero field response is only from the superfluid as can be seen in Figure 7.1. Both
Figure 7.16: The inverse inductance and sheet resistance (f=50kHz) for film M6 in several applied fields. The field values are listed in the caption to Figure 7.15.
the inductive and resistive vortex impedances increase with both applied field and frequency. It is also interesting to note that the vortex impedance is nearly zero for fields up to $B_0 \approx 10mG$ and then the impedance is roughly linear for any additional field. The value of $B_0$ increases as the temperature is lowered and is roughly $20mG$ for $T = 4.400K$. The presence of the threshold field $B_0$, and the frequency dependence of the complex impedance cannot be explained by the pinning model. However, the important result here is that for the magnetic field nulled to within $2mG$ the contribution to the impedance from the field induced vortices is negligible.

**7.1.4 The effect of disorder or lack of film homogeneity**

In the course of these studies several of the films have exhibited less than ideal properties, in that their superfluid density was lower than the theoretical expectation of Equation 5.2. Although the morphology of the defects is not known, two qualitative observations have been made and are demonstrated below.

Figure 7.18 shows the superfluid density for films M6 (solid line) and M7 (dashed line). Both films are nominally of the same thickness ($d = 30\text{Å}$), but film M7 which was grown on the low quality unoxidized Si wafer has a slightly larger transition temperature and a lower zero temperature superfluid density. The large difference between these films is illustrated in Figure 7.19 which shows $L^{-1}$ and the sheet resistance as a function of $T$ for several applied fields. The data for film M6 was already shown in Figure 7.16 and this is just a more detailed view of the transition region. It is clear that the damaged film, M7, is affected much less by the presence of a magnetic field than the high quality film, M6. While the transition temperature for film M6 decreases with increasing applied field, the transition for film M7 is
Figure 7.17: $R$ and $\omega[L(B) - L(B = 0)]$ for film M4 at $T = 4.600K$. The squares, triangles and circles are for frequencies of 1, 4 and 50kHz respectively.
Figure 7.18: The superfluid density for both films M6 (solid line) and M7 (dashed line).
Figure 7.19: The inverse sheet inductance and sheet resistance for films M6 (solid lines) and M7 (dashed lines) for several magnetic fields. The fields for the M6 data are listed in the caption to Figure 7.15. The fields for film M7 are 0, 260 and 660mG.
roughly independent of T for these small applied fields. Furthermore, even in zero field the transition for film M7 is not nearly as sharp as for film M6. Also the sheet resistance for the damaged film does not increase as much with applied field as for the undamaged film. All of these observations suggest that there are either patches in the film M7 that behave like holes where the magnetic field can concentrate, or strong pinning sites which reduce the mobility of the field induced vortices.

Another qualitative observation is that the frequency dependence of $R_\alpha$ and $\omega L_V$ goes as a higher power of $\omega$ for the damaged films compared with the rest of the MoGe films. The other film which has been observed to be damaged is film M9, with $d = 21.5\,\text{Å}$ after going through thermal recycling. Figure 7.20 shows the superfluid density upon the initial cooldown and then again after it was measured four months later. Both $T_C$ and the zero temperature superfluid density were suppressed. Figure 7.21 shows the results of a frequency sweep for the complex impedance of film M9 after it has been damaged. $\omega L_V$ was determined by the same technique as for Figure 7.5. It was found that $R_\alpha \propto \omega^{0.28}$ and $\omega L_V \propto \omega^{0.29}$. In contrast for the high quality undamaged MoGe films a power of about 0.14 was measured.

7.2 Discussion of complex impedance measurements and comparison with conventional KTB theory

To summarize it has been shown that the thin film superconductors have a $T_C$ which is less than the mean-field transition temperature. Near the transition the complex conductivity is seen to have a large frequency dependence for the range of measurement frequencies of 100Hz to 100kHz. It has been observed that the frequency dependence of the inverse inductance saturates above a frequency of about 50kHz for T below $T_C$. This has been interpreted as representing the temperature dependence
Figure 7.20: The superfluid density for film M9 before (solid line) and after (dashed line) it was damaged.
Figure 7.21: The top panel shows the inductance for film M9 after it was damaged as a function of frequency for $T = 2.800K$. The bottom panel shows that the resistive and inductive vortex impedances both follow a power law of frequency with an exponent of about 0.28.
of the bare superfluid density. Given that for \( f=50\text{kHz} \) the sheet resistance and inductive vortex impedance do not show any sign of saturation it is likely that the vortex inductive impedance keeps increasing with a small power of \( \omega \) for \( \omega > 50\text{kHz} \), but that it is unobservable since it is much smaller than the inductive impedance of the bare superfluid. For \( f=50\text{kHz} \) a sharp drop in \( L^{-1} \) is observed at \( T_C \) along with a sharp increase in the sheet resistance. The sharp drop in \( L^{-1} \) for \( f=50\text{kHz} \) occurs within 20% of the value predicted by static KTB theory for all of the films. Figures 7.22 and 7.23 show the sharp drop in \( L^{-1} \) for several of the films studied along with a dashed line representing the prediction for where the KTB transition should occur as a function of temperature based upon the universal drop prediction.

There are several inconsistencies between the data and KTB theory which need to be addressed. As is shown below it is to be expected that for measurements done with \( f < 100\text{kHz} \) that the static KTB theory would be largely applicable except within a very narrow temperature range near \( T_C \). This leads to the first question of where the KTB transition occurs. Does it occur at \( T_C \) where for \( f=50\text{kHz} \) a sharp drop in \( L^{-1} \) is observed or does it occur at the lowest temperature at which frequency dependence is observed, i.e. roughly when \( U_0 \approx 3k_BT \). This is addressed in the Section 7.2.1 where it is concluded that the KTB transition occurs at the higher temperature. The second question is then why is there resistance observed below \( T_{KTB} \) which is independent of the sheet current density in the limit of small sheet current densities? This is discussed Section 7.2.2 and compared with I-V data in the literature.

To get a handle on the temperature dependent quantities such as the mean square pair size and the density of vortex pairs, estimates are shown for film M4 in Figure 7.24. For reference the top panel shows \( U_0/k_BT \) and the sheet resistance \( (f = 10\text{kHz}) \)
Figure 7.22: Normalized inverse inductance vs. $T/T_{co}$ for films M3, M4 and M5. The intersection of the dashed line and the data is where the KTB transition is predicted to occur.
Figure 7.23: Normalized inverse inductance vs. $T/T_{C0}$ for films M8, M9 and film ISJT. The intersection of the dashed line and the data is where the KTB transition is predicted to occur.
Figure 7.24: Estimates of the root mean square pair size and density of pairs for a typical MoGe thin film. The top panel shows the experimental results for $U_0/k_B$ and the sheet resistance for film M4 measured at $f=10$kHz.
from the data in Figure 7.2. The intersection of the dashed line and the data for $U_0/k_BT$ is the prediction for where the KTB unbinding transition should occur. The second panel shows the root mean square pair size considering only pairs of size $2\xi$ and larger, calculated from Equation 6.12. Note that the root mean square pair size changes significantly only right at the transition. The bottom panel shows estimates for the density of pairs of size two coherence lengths and larger, calculated from Equations 6.10 and 6.11 assuming that the coherence length has the same temperature dependence as the penetration depth and $\xi(0) = 60\text{Å}$. The two solid lines correspond to the total density of pairs of size $2\xi$ or larger assuming the values of $E_{C_{\text{pair}}}(2\xi) = 5.5$ and $N_0 = 12$ which gives the maximum estimate for $n_p$ and $E_{C_{\text{pair}}}(2\xi) = 7.0$ and $N_0 = 4$ which gives the lower bound for the density of pairs. The other curves represent the density of pairs of size greater than 1000Å (dot dashed line), $1\mu$m (dotted line) and $100\mu$m (dashed line). Note that when the density of pairs is less than $10^{-9}/\mu m^2$ there is on average less than one pair in the entire film.

7.2.1 Discussion of the frequency dependence

To begin, the data are compared with the KTB model assuming that critical longitudinal phase fluctuations and associated amplitude fluctuations are irrelevant. The question then becomes at what temperature does the KTB transition occur? Does it occur at the temperature, $T_C$, at which the inverse inductance shows a sharp drop or at a lower temperature where the dissipation turns on and the imaginary conductivity starts to drop in the lowest frequency measurements? A previous study of the complex conductivity has suggested the latter and shown that the frequency dependence of the temperature, $T_\omega$ (usually defined [87] [25] as the temperature
at which the steep portion of \( \sigma_2(T) \) extrapolates to zero, is consistent with the prediction of the dynamical KTB theory \[65\].

When reasonable numbers are used for calculating the pair density and the root mean square pair size it is obvious that the frequency dependence of the complex impedance cannot be explained by the dynamical KTB theory. According to the dynamical theory, pairs of size smaller than \( r_\omega = \sqrt{14D/\omega} \) are able to fully reorient with the ac current in the film and thus give an inductive response, and pairs larger than \( r_\omega \) are not able to diffuse fast enough to follow the ac current and will contribute to the resistive component of the impedance. Calculating the diffusion constant from Equations 6.20 and 6.17 yields \( r_\omega \approx 10\mu m \) for \( f=100kHz \) for temperatures in a wide range below \( T_C \). However as shown in Figure 7.24 the density of pairs larger than even \( 1\mu m \) is infinitesimal just 40mK below \( T_C \). Figure 7.1 shows that the complex conductivity exhibits a resistive component as well as a frequency dependence down to much lower temperatures. Given the low density of pairs it must be concluded that the observed impedance cannot be described by the dynamical KTB theory. One caveat to keep in mind is that the diffusion constant may be smaller than the Bardeen-Stephen formula, Equation 6.17 would indicate. For example Fischer has argued \[88\] that in the presence of pinning that the diffusion constant would be reduced by a multiplicative Arrhenius factor of \( e^{-E_p/\kappa_B T} \) where \( E_p \) is the pinning energy of a single well. Rogers has interpreted low frequency complex impedance measurements on a monolayer BSSCO 2:2:1:2 film buffered by several layers of BSCCO 2:2:0:1 as being consistent with a KTB like transition with a diffusion constant modified by pinning. Even if there were strong pinning sites in the MoGe films, with \( E_p \approx U_0, r_\omega \) would still be greater than \( 2\mu m \) at the lowest temperature at which the conductivity exhibits a
frequency dependence. Therefore, including pinning in the model cannot bring the dynamical KTB theory into agreement with the data.

In order to test the dynamical KTB theory below $T_C$, much higher measurement frequencies are required so that the diffusion length, $r_\omega$, is small enough that there are a significant number of pairs larger in size than $r_\omega$. It should be noted however that as the frequency was increased the absolute value of the slope of $L^{-1}$ and of $\sigma_2$ at the transition increased. There was no indication that the observed superfluid density would go to zero at a higher temperature as predicted by the dynamical KTB theory.

From the above analysis it must be concluded that if there is a KTB transition it must occur within a few mK of $T_C$. The evidence in favor of this interpretation are: (1) The sharp drop in $L^{-1}$ occurs near the universal value of $U_0/k_B T = 2/\pi$ and (2) the calculations shown in Figure 7.24 show that both the density and the mean square pair size decrease too rapidly for the pairs to be responsible for the frequency dependent complex impedance below $T_C$. Furthermore the sheet resistance is simply exponentially activated over a broad temperature range (more than 200mK) below $T_C$ and does not exhibit any unusual feature until $T_C$ where it suddenly increases very rapidly with temperature.

### 7.2.2 Resistance below the KTB transition

Common explanations for the presence of resistance for $T < T_C$ are: (1) a nonzero static magnetic field perpendicular to the plane of the film, (2) finite size effects such as sample size [89] or finite $r_{fs} = \lambda^2/d$, which is roughly the length to which the logarithmic interaction extends, [90] and (3) current induced unbinding of pairs.
None of these explanations are reasonable as shown below and furthermore none of them can explain the frequency dependence of the impedance.

From Figure 7.17 it is obvious that for fields of less than 2mG field induced vortices do not affect the sheet resistance which rules out the first explanation. One can arrive at the same conclusion by considering the rapid temperature dependence of the sheet resistance. If field induced vortices were responsible for the resistance then the resistance would be proportional to the square of the coherence length which has a much weaker temperature dependence than is observed.

Finite size effects are an issue because the logarithmic interaction potential only extends to a distance of order $\lambda^2/d$ and then becomes weaker. Thus pairs of size larger than $\lambda^2/d$ will behave as free vortices. Below $T_c$ in the transition region $\lambda^2/d$ is of order 1mm which is a much larger length scale than $r_o$. A few mK below $T_c$ there are simply no pairs of this size, thus ruling out the finite range of the interaction as being responsible for the observed resistance.

Since the measured resistance in the films is linear in the applied current over a wide range of currents it is clear that the resistance is not due to the current unbinding mechanism. Although the current induced unbinding mechanism cannot explain the resistance it deserves particular attention since there is a large amount of data in the literature on the nonlinear I-V profiles of two dimensional superconducting structures. For typical I-V measurements in the literature the lowest sheet current densities used for temperatures below $T_{KTB}$ were about 100$\mu A$ for Reference [22] and 250$\mu A$ for Reference [67]. For the mutual inductance measurements in this thesis the sheet current density was nonuniform in the film, but the maximum sheet current density near $T_{KTB}$ was 50$\mu A$. Similar Ohmic behavior is seen in nearly all of the I-V data.
in the literature for the lowest measurement currents. These data are summarily dismissed as being due to stray fields, finite size effects, or voltage noise. Here, we have shown that none of these explanations are satisfactory for our films and that there is Ohmic behavior below the KTB transition temperature.

Since the energy of the excitations from the superconducting ground state configuration are presumed to be proportional to $U_0(T)$ it makes sense to determine $C$, where the excitation energy is $CU_0$, by curve fitting $\ln(R_V) = \text{const.} - CU_0\beta$. Figure 7.25 shows that for film M4, $C \approx 3.6$. For the MoGe films $C$ is between 3 and 4 and for film ISJT $C$ is closer to 2.
7.2.3 Speculations

Assuming that the G-L based calculations for the vortex-antivortex core energy are valid, then the observed frequency dependence and sheet resistance for T below $T_c$ cannot be explained by KTB theory. Furthermore the observed behavior cannot be explained by the presence of an ambient magnetic field perpendicular to the film. One might wonder that if the vortex-antivortex pair energy were much less than the G-L prediction then the vortices by themselves may be responsible for the observed impedances. However, this scenario seems highly unlikely for if it were true then the excitation energy for amplitude fluctuations of size roughly one coherence length would be much smaller than $k_B T$ and they would presumably make the film behave as a dynamically percolating system with normal patches appearing and disappearing on a short time scale. Even if the G-L assumptions are correct the energy of amplitude fluctuations and pairs smaller than size $r_0 = 2\xi$ amplitude fluctuations will still be common. Indeed the sheet resistance was found to be exponentially activated with a characteristic energy of about $3U_0$ which is much less than the energy of a pair of size $2\xi$. I believe that the amplitude fluctuations and perhaps their interactions with the thermally excited vortices may somehow be responsible for the observed impedances. The alternative is that the anomalous behavior is the result of some extrinsic feature of the films. This would be surprising considering that the MoGe films are probably the most homogeneous of all dirty thin film superconductors.

To gain a better understanding of the complex impedance in the transition region it would be nice to perform measurements of a film’s superfluid density as has been done in this thesis and then to do I-V measurements on the same film. Also measurements at a much higher frequency would be of interest. Would the observed
transition temperature increase for very high measurement frequencies as predicted by the dynamic theories? My inclination is that it would not.
CHAPTER 8

SUMMARY AND CONCLUSIONS

Modeling and careful construction of a two-coil mutual inductance technique have been developed and refined to the point that highly accurate low noise measurements of the complex conductivity of thin films is possible. Through measurements of the complex conductivity of a variety of thin film superconductors it has been found that well below the transition temperature longitudinal phase fluctuations do not suppress the superfluid density as expected by classical calculations. Measurements on YBCO at low T placed an upper bound on the size of phase fluctuations. Further measurements on MoGe thin films showed that phase fluctuations are entirely negligible at low temperature. Through an analogy with resistively shunted Josephson junctions it has been shown that the amplitude of the longitudinal current fluctuations are suppressed by quantum mechanics. For a dirty limit s-wave superconductor the quantum crossover below which phase fluctuations are suppressed is roughly $0.94T_{C0}$. Measurements of the superfluid density for a series of MoGe films of varying thickness are consistent with the presence of a quantum crossover.

Transverse fluctuations in the form of vortex-antivortex pairs are present near the transition temperature and are partially responsible for destroying superconductivity. The sharp drop in the inverse inductance observed in all of the thin MoGe
films and the In/InOx film all occur when the inverse sheet inductance is near the value predicted by KTB theory. To my knowledge this is the first observation of the universal jump in homogeneous amorphous films. Comprehensive frequency dependent, magnetic field dependent and current density dependent measurements of the complex impedance of MoGe films and an In/InOx film have been performed. It was experimentally found that the impedance was independent of the current density for the experimental excitation amplitudes and that the ambient static magnetic field perpendicular to the plane of the films could be nulled sufficiently so as not to affect the impedance of the films. Universal features of the complex impedance observed in these films include: (1) a sheet resistance and inductive impedance (additive to the inductive impedance of the superfluid background) which are proportional to a small power of the frequency for frequencies between 100Hz and 100kHz, (2) frequency dependence for the sheet resistance and inductance which extends down to a temperature where $U_0$ is roughly equal to three times $k_BT$. These observations cannot be consistently explained by the conventional KTB theory. For such low measurement frequencies the results should be nearly static below $T_C$ and only for temperatures within a few mK of $T_C$ should dynamics below 100kHz be observed.
APPENDIX A

NUMERICAL MODELING OF THE TWO-COIL MUTUAL INDUCTANCE TECHNIQUE

A.1 Introduction

The mutual inductance technique for measuring the complex conductivity of films and arrays consists of running a sinusoidal current of amplitude, \( I_d \), at angular frequency, \( \omega \), through a drive coil and measuring the induced voltage, \( V_p \), across a pick-up coil which is coaxial with both the drive coil and the sample to be measured. It is assumed that the drive current has an \( e^{i\omega t} \) time dependence. A number of groups have used a variety of variations of the two-coil mutual inductance technique [91] [92] [93] [94] [95] [96]. Figure A.1 shows a schematic of the film and coil geometry used for data taken in this thesis. The drive and pick-up coils have inner radii of \( r_d \) and \( r_p \) respectively. The nearest drive coil loop to the film is located a distance \( h_d \) above the film and the nearest pick-up coil loop is a distance \( h_p \) below the film. The vertical separation between the nearest drive and pick-up loops is the sum of \( h_d \), \( h_p \) and \( d \), where \( d \) is the film thickness. The drive(pick-up) coil contains \( n_d \) (\( n_p \)) vertical layers and \( m_d \) (\( m_p \)) radial layers with spacing \( dh_d \) (\( dh_p \)) between wires in the coil.
Figure A.1: The geometry of the film and coils in the two-coil mutual inductance technique. The drive and pick-up coils are coaxial and located on opposite sides of the film. The drive(pick-up) coil contains $N_d M_d$ ($N_p M_p$) loops.
The mutual inductance is defined as

$$M \equiv M_1 + iM_2 \equiv \frac{V_0}{i\omega L_d}.$$  \hspace{1cm} (A.1)

Figure A.2 shows a typical measurement of the mutual inductance as a function of temperature. Above $T_C$, $M_2 = 0$ and $M_1$ is a constant equal to the bare mutual inductance of the coil arrangement. As the film is cooled through the transition $M_1$ drops due to inductive screening currents in the film and $M_2$ becomes negative due to dissipative currents. Well below $T_C$, $M_2$ vanishes and the film's response is purely inductive. The inset to figure A.2 shows the complex conductivity, $\sigma = \sigma_1 - i\sigma_2$, of the film extracted from the mutual inductance. The complex conductivity is determined by interpolation from a look up table containing $M_1$ and $M_2$ calculated for 10,201 combinations of $\sigma_1$ and $\sigma_2$ [91].

It must be kept in mind that the modeling of the two-coil mutual inductance technique described here assumes that the complex conductivity is homogeneous throughout the film. In practice when $\sigma$ is homogeneous on length scales smaller than the coil dimensions, the modeling will yield a proper average conductivity. This is for example the case in YBCO which has an anisotropic in plane conductivity due the chains along one direction, but the direction in which the chains run switches frequently at so called twin boundaries. When a vortex lattice is present nonlocal effects may be important and a more complicated model is needed [84]. In the absence of strong pinning centers the induced axial currents will sinusoidally compress and expand the vortex lattice so that the measured mutual inductance is affected as the density of vortices within the coil changes.

In what follows the procedure and accuracy of calculating the look-up table $M(\sigma)$'s will be discussed. It is assumed that both the drive and pick-up coils are dipoles which
Figure A.2: Mutual inductance measured as a function of temperature for a typical superconducting film. The inset shows the complex conductivity determined from a look-up table calculated with the actual geometry of the coils.
are coaxial with the film. However it is straightforward to use the techniques discussed below for any axially symmetric drive coil and film and an arbitrarily shaped pick-up coil. Since the pick-up coil is passive in that it carries essentially no current compared to the drive coil, the pick-up coil may be of arbitrary shape and orientation relative to the film. It is possible to have several pick-up coils in operation simultaneously. Some additional information on modeling the two-coil mutual inductance technique may be found in [94] [95].

Two models will be used to solve for $M(\sigma)$, neither of which treats the real geometry exactly. First, the exact solution for $M(\sigma)$ will be discussed for the case of an infinite area film. A solution for $M$ is also given when the film is composed of stacked layers each with a different conductivity. Next, from the exact infinite area film solutions, an approximate expression is found for the effective thickness of the film, $d_{eff}(d,\omega\sigma)$. The effective thickness is used in the finite area film calculations, by replacing the current density by a sheet current divided by $d_{eff}$. Next a numerical solution for $M(\sigma)$ will be discussed in which the film is treated as having a finite radius, $R$, but zero thickness and an effective sheet conductivity, $\sigma d_{eff}$. The finite area film calculations can be extended to situations where the conductivity depends on the radial coordinate. Several practical examples are discussed.

A.2 Infinite area film modeling

A.2.1 Single layer film

First an exact solution $M(\sigma, d)$ is presented for the numerically tractable situation in which the film has infinite area. This solution, derived by Aaron Pesetski, is based on work by Clem and Coffey [84] and by Pearl [60]. To calculate $M$ one begins with
the differential form of Ampere's law. The displacement current is neglected since the frequency in the experiment is less than 200kHz. \( \mathbf{B} \) is replaced by \( \mathbf{B} = \nabla \times \mathbf{A} \) yielding the equation.

\[
\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}_D(\mathbf{r}) + \mu_0 \omega \sigma(\mathbf{z}) \mathbf{A}_F(\mathbf{r}) \tag{A.2}
\]

From symmetry the vector potentials and current densities are all axial. The drive current density \( \mathbf{J}_D \) is

\[
\mathbf{J}_D = \sum_{m_d=0}^{M_d-1} \sum_{n_d=0}^{N_d-1} \delta(\rho - R_{d,m_d}) \delta(z - H_{d,n_d}) \tag{A.3}
\]

Next Equation A.2 is Fourier transformed in the x-y plane. The remaining differential equation is a function of z and is solved with the boundary conditions that A and dA/dz are continuous at the film boundaries. Finally \( V_p \) is calculated by integrating the vector potential around all of the pick-up loops.

The resulting solution for the mutual inductance is

\[
M(\sigma, d) = \pi \mu_0 h \int_0^\infty dx \frac{e^{-x} \alpha_d(x) \alpha_p(x) \beta_d(x) \beta_p(x)}{T(x)}, \tag{A.4}
\]

where \( h \equiv h_d + h_p, \delta \equiv d/h, \) and \( \chi^2 \equiv x^2 + i\mu_0 \omega \sigma h^2. \) The factors \( \alpha_{d(p)} \) and \( \beta_{d(p)} \) depend only on the coil geometry and \( T(x) \) depends only on the film's properties.

\[
\alpha_{d(p)}(x) \equiv \sum_{i=0}^{m_d} \frac{r_{d(p)} + idh_{d(p)}}{h} J_1\left(\frac{r_{d(p)} + idh_{d(p)}}{h}, J_1\left(\frac{r_{d(p)} + idh_{d(p)}}{h})\right) \tag{A.5}\right.
\]

\[
\beta_{d(p)}(x) \equiv \frac{1 - e^{-m_d(p)} - \frac{dh_{d(p)}}{h} x}{1 - e^{-dh_{d(p)}} x} \tag{A.6}
\]

\[
T(x) \equiv \frac{(\chi + x)^2 e^{5x} - (\chi - x)^2 e^{-5x}}{4x \chi} \tag{A.7}
\]

\( J_1 \) is a Bessel function. For a typical coil geometry, with coil radii of roughly 1mm and with \( h \approx 1mm, \) integrating Equation A.4 up to \( x=20 \) yields an accuracy in \( M \) which is better than 1 part in \( 10^{10}. \)
An intermediate step in the calculation of the mutual inductance (Equation A.4) yields the current density in the film. Although the current density does not depend on the pick-up coil dimensions, for convenience lengths are still normalized by \( h = h_d + h_p \).

Thus the integral for the current density at a position \((\rho' = \rho/h, z' = z/h)\) is

\[
J/I_d = -i\mu_0 \sigma \int_0^\infty dx \frac{e^{-(h_d/h)x} J_1(\rho' x) ((x + x)e^{(x' + \delta/2)} + (x - x)e^{-x(x' + \delta/2)})}{4\chi} \alpha_d(x) \beta_d(x).
\]

Integrating Equation A.8 over the film thickness yields the sheet current density in the film.

\[
K/I_d = -i\mu_0 \sigma h \int_0^\infty dx \frac{e^{-(h_d/h)x} J_1(\rho' x) [2x + (x + x)e^{\delta} - (x - x)e^{-\delta}]}{4\chi^2} \alpha_d(x) \beta_d(x)
\]

(A.9)

### A.2.2 Multilayer film

For an infinite area film the mutual inductance between two coils may be calculated for a multilayer film consisting of \( N \) layers with the \( n'\)th layer having thickness \( d_n \) and conductivity \( \sigma^{(n)} \). Figure A.3 shows the geometry for the multilayer film. An equation for \( M \) is obtained by the same technique as for the single layer film (Equation A.4) except here there are \( 2N + 2 \) boundary conditions. For the general case of \( N \) layers with arbitrary complex conductivities and total film thickness \( d \) the mutual inductance is given by Equation A.4 with the factor \( T(x) \) replaced by

\[
T(x) \equiv \frac{[x + \chi_N]b_{2N-1}e^{\delta Nx_N} + (x - \chi_N)b_{2N}e^{-\delta Nx_N}}{2^{N+1}x \prod_{i=1}^N \chi_i}.
\]

(A.10)

In Equation A.10, \( \delta_n = d_n/h \) and \( \chi_n^2 = x^2 + i\mu_0 \omega \sigma^{(n)} h^2 \). Here \( b_{2N} \) and \( b_{2N-1} \) are generated by the recursion relationships,

\[
b_{2n-1} = (\chi_n + \chi_{n-1})b_{2n-3}e^{\delta_{n-1}x_{n-1}} + (\chi_n - \chi_{n-1})b_{2n-2}e^{-\delta_{n-1}x_{n-1}}
\]

(A.11)
Figure A.3: The geometry for a multilayer film with N layers with the n’th layer having a thickness, \(d_n\) and a complex conductivity, \(\sigma^n\).
and

\[ b_{2n} = (\chi_n - \chi_{n-1})b_{2n-3}e^{\delta_{n-1}x_{n-1}} + (\chi_n + \chi_{n-1})b_{2n-2}e^{-\delta_{n-1}x_{n-1}} \]  

(A.12)

where the initial values in the recursion are \( b_1 = \chi_1 + x \) and \( b_2 = \chi_1 - x \).

Unfortunately, this solution cannot be used for inverting a measured \( M \) to the conductivities of all of the individual layers in the multilayer for \( N > 1 \). There just is not enough information in the single complex number, \( M \), to yield a unique set of conductivities for each layer. However, this solution would be useful if all of the layers have a known conductivity but one. Then \( M \) can be inverted to get the conductivity of the one unknown layer. One could imagine studying how the conductance of a superconducting film is modified by the presence of conducting layers in close proximity. Specifically at temperatures below 10K a normal metal film will have reached its extrinsic conductivity and thus have a temperature independent conductivity. Separate measurements can be made for the real conductivity of the normal metal so all parameters in Equation A.10 are known except for the conductivity of the superconducting layer.

A.2.3 The thin-film limit for a single layer film

The solution given by Equation A.4 is for an infinite area film, but for a typical experiment the film radius is only several times larger than the coil radii. Thus it is necessary to check how much error is introduced into the \( \sigma \) extracted from experiment by treating the film as having infinite area. In section A.3 a solution for \( M(\sigma, d) \) will be presented in which a finite area film of thickness \( d \) is treated as having zero thickness, and a sheet conductivity, \( \sigma_{\text{eff}} \). In this section an appropriate expression for \( d_{\text{eff}} \) is determined.
To determine a suitable form for the effective thickness $d_{eff}(d, \omega \sigma)$ it is useful to take the thin film limit ($d \to 0$) of $T(x)$ defined in Equation A.7. Making the additional approximation, $(h/\lambda) \gg x$ (numerical integration of Equation A.4 shows that integration to about $x=20$ for a typical coil geometry is sufficiently accurate), yields

$$T_{\text{thinfilm}}(x) \approx 1 + \frac{i \mu_0 \omega h}{2x} d$$  \hspace{1cm} (A.13)

which is precisely the result obtained by Jeanneret [92] who assumed that the film had zero thickness and a sheet conductivity equal to $\sigma d$. If the approximation $(h/\lambda) \gg x$ is made without taking the thin film limit a form of $T(x)$ similar to Equation A.13 is obtained.

$$T_{h>>\lambda} \approx \cosh[d \sqrt{i \mu_0 \omega}] + \frac{i \mu_0 \omega h \sinh[d \sqrt{i \mu_0 \omega}]}{2x} \sqrt{i \mu_0 \omega}$$  \hspace{1cm} (A.14)

Numerical calculations show that the $\cosh$ term may be replaced by unity without losing significant accuracy for the experimentally relevant parameters. Thus by comparing $T_{\text{thinfilm}}(x)$ with the more general $T_{h>>\lambda}(x)$ a reasonable form for $d_{eff}$ is found.

$$d_{eff}(d, \omega \sigma) \equiv \frac{\sinh(d \sqrt{i \mu_0 \omega})}{\sqrt{i \mu_0 \omega}}.$$  \hspace{1cm} (A.15)

In the limit of vanishing conductivity $d_{eff}$ approaches $d$, the actual film thickness. For a very large and imaginary conductivity $d_{eff} \approx de^{d/\lambda}$ which means that for a film much thicker than the penetration depth the mutual inductance is screened exponentially with thickness.

### A.3 Finite area film modeling

For a finite area film of radius $R$, an analytic solution for $M(\sigma, d)$ is not available and an integral equation must be solved for the current density in the film. Some
analytic approximations may be found in References [97] and [98]. The problem is made two-dimensional by introducing the effective thickness defined in Equation A.15 by replacing the current density in the film, \( J(\rho, z) \), by \( K(\rho)/d_{eff} \). The total vector potential in the film is the sum of the contribution from the current in the drive coil and the sheet current, \( K(\rho) \), in the film. Note that as a result of symmetry, all currents and vector potentials are axial. The integral equation for \( K(\rho) \) is

\[
\frac{K(\rho)}{-i\omega\sigma_{eff}} = A_d(\rho) + \frac{\mu_0}{4\pi} \int_{film} d^2\rho' \frac{K(\rho)}{[\rho - \rho']}
\]  

(A.16)

The second term on the r.h.s. of Equation A.16 is the vector potential in the film resulting from the current in the film. For the geometry shown in Figure A.1 the drive vector potential is

\[
A_d(\rho) = \frac{2\mu_0}{\pi} I_d \sum_{n_d=0}^{N_d-1} \sum_{m_d=0}^{M_d-1} \frac{(r_d + m_d h_d)}{k_{n_d,m_d}^2} [(1 - \frac{k_{n_d,m_d}^2}{2})K(k_{n_d,m_d}) - E(k_{n_d,m_d})],
\]

(A.17)

where

\[
k_{n_d,m_d}^2 \equiv \frac{4\rho(r_d + m_d h_d)}{(r_d + m_d h_d + \rho)^2 + (h_d + n_d h_d)^2}
\]

(A.18)

and \( K \) and \( E \) are the complete elliptic integrals.

The \( d\phi' \) integral in Equation A.16 is done analytically [99] resulting in

\[
A_d(\rho) = \frac{K(\rho)}{-i\omega\sigma_{eff}} - \frac{\mu_0}{2\pi\rho} \int d\rho' K(\rho')(\rho + \rho')[(1 - \frac{q^2}{2})K(q) - E(q)],
\]

(A.19)

with \( q \) being defined by

\[
q^2 = \frac{4\rho\rho'}{(\rho + \rho')^2}.
\]

(A.20)

Equation A.19 is solved numerically by partitioning the film into \( N \) concentric annular rings centered at radii \( \rho_i \) with widths of \( \Delta \rho_i \) and summing over the value of the integrand at the center of each ring times the width of each ring. The rings need not
have equal width and in general the \( i \)th ring has a width \( s_i R/N \). The width of the rings is made much smaller than the mean ring width, \( R/N \), at values of \( \rho \) where the current density has a strong \( \rho \) dependence such as at the film edge. Equation A.19 is replaced by a set of linear equations whose solution yields \( K_i \), the sheet current in ring \( i \).

\[
A_{d,i} = \frac{K_i}{-i \omega \sigma_{eff}} - \frac{1}{2 \pi \rho_i} \sum_{j=1}^{N} M_{ij} [s_j \frac{R}{N}] K_j
\]  
\[\text{(A.21)}\]

\( M_{ij} \) is the mutual inductances between coplanar loops of radii \( \rho_i \) and \( \rho_j \). For \( i \neq j \) \( M_{ij} \) is

\[
M_{ij} = \mu_0 [\rho_i + \rho_j] \left[ \left(1 - \frac{k^2}{2}\right) K(k) - E(k) \right].
\]  
\[\text{(A.22)}\]

Equation A.22 can not be used for \( i = j \) since it diverges when \( k = 1 \). Gilchrist and Brandt [100] have shown that for the off diagonal \( M_{ij} \)'s chosen as in Equation A.22, the proper choice for a rapid convergence of the solution for \( K(\rho) \) with increasing number of rings, \( N \), is

\[
M_{ii} = \mu_0 \rho_i \left[ \ln \left( \frac{8 \pi \rho_i}{s_i R/2N} \right) - 2 \right].
\]  
\[\text{(A.23)}\]

Physically this corresponds to the self inductance of a wire of diameter \( s_i R/\pi N \) forming a loop of radius \( \rho_i \) [100]. There are other possible methods for calculating the mutual inductance elements \( M_{ij} \). For example either a single or double integral may be performed over each ring. For each choice for the off diagonal mutual inductance elements Gilchrist and Brandt have derived the proper analytic form to use for the diagonal terms to ensure rapid convergence with the number of rings, \( N \). If the wrong choice for the diagonal elements is used, I found that the mutual inductance converges to the proper value roughly as \( N^{-1} \) [94].
A.4 Subtraction procedure and numerical accuracy

Although it is possible to make tables for inverting $M$ to $\sigma$ from the numerical calculations for a finite area film it is more practical to use the infinite area film solution for $M$ to make the tables. The reason is that the computation time is much longer for the finite area film calculation than for the infinite area film solution. As long as the film radius is much larger than the coil radii it is valid to use the infinite area film solution.

For the experimental case in which the film radius is between 5 and 10 times the coil radii a simple correction is made to the data. For strong screening and a pure imaginary conductivity, ($\sigma_2 = 1/(\mu_0\omega\lambda^2)$, the mutual inductance, $M$, depends on the penetration depth, $\lambda$ as follows.

$$M \approx \alpha \frac{\lambda^2}{d_{eff}} + \beta.$$  \hfill (A.24)

$\alpha$ depends on the coil geometry and $\beta$ is the mutual inductance resulting from flux going around the film in the limit of perfect screening by the film. $\beta$ is determined experimentally by replacing the film with a thick Pb foil and measuring the coupling between the coils. $\beta$ may also be calculated using the finite area film modeling and the agreement between measurements and calculations is excellent. The infinite area film solution is used to invert $M$ to $\sigma$ after subtracting $\beta$ from the raw experimental $M$. When $\beta \geq \alpha \frac{\lambda^2}{d_{eff}}$, the accuracy of the extracted penetration depth depends strongly on how well $\beta$ is known. $\beta$ can be made small by using very thin pancake coils placed very close to the film [93] or by winding asymmetric quadrupole coils [101]. It is also desirable to have a large $\alpha$ so that $dM/d\lambda$ is large. References [94] and [95] discuss the numerical accuracy of the subtraction procedure in more detail.
Figure A.4: The current density induced in a 7.5mm radius film (solid lines) and in an infinite area film with the same conductivity (dashed lines) for several complex conductivities. The real and imaginary parts of the complex conductivity are equal. $J_1$ is in phase with the drive current and $J_2$ which results from the nonzero real conductivity is out of phase with the drive current. $J_1$ and $J_2$ are normalized by the value of the peak around $\rho = 1mm$. 
Figure A.4 shows the normalized current density for three conductivities with \( \sigma_1 = \sigma_2 \). The solid lines are from the finite area film calculations for a 7.5mm radius film and the dashed lines are for an infinite area film. Notice that as the conductivity decreases the peak in the current density shifts to a larger radial coordinate. With the coils on opposite sides of the film \( M_2 \) is more sensitive to finite size effects than \( M_1 \). This implies that the real conductivity is more sensitive to finite size effects than the imaginary conductivity. Note that there is no subtraction procedure for \( M_2 \). Thus if the film radius is smaller than 5 times the coil radii and \( \sigma_1 \geq \sigma_2 \) then the finite area film calculation should be used to make the inversion tables.

### A.5 Finite area film for \( \sigma \) varying with \( \rho \)

When the film has a nonuniform conductivity the infinite area film calculation cannot be used. However, it is trivial to include a conductivity varying with the radial coordinate by allowing the conductivity in each ring to vary. This corresponds to having \( \sigma \) in Equation A.21 be a discrete function with a different value in each ring. Thus Equation A.21 becomes

\[
A_{d,i} = \frac{K_i}{-i\omega \sigma_i d_{eff}} - \frac{1}{2\pi \rho_i} \sum_{j=1}^{N} M_{ij} [s_j \frac{R}{N}] K_j
\]  

(A.25)

To accommodate sharply changing current densities with the radial coordinate the ring widths should be small in regions of the film where the conductivity changes rapidly with \( \rho \). Application of the finite area film calculation with \( \sigma(\rho) \) has allowed the development of several useful techniques which are discussed in the following subsections.
A.5.1 Effect of small holes in the film

When a film has a hole, which is not concentric with the drive coil, a calculation for the mutual inductance is not available. However, an approximate solution for the effect of holes on the absolute accuracy of the penetration depth has been found and tested experimentally. The method is to calculate the mutual inductance for a film missing an annular ring of material which would be created by sweeping the hole in the film 360 degrees about the film's center. The conductivity in this region is set to zero when solving Equation A.25. The current density spikes at the ring boundaries. A reasonable guess for the extra mutual inductance resulting from the presence of the hole is given by

\[ M_{\text{extra,hole}} = M_{\text{extra,ring}} \times \frac{A_{\text{hole}}}{A_{\text{ring}}} \]  

(A.26)

where \( A_{\text{hole}} \) and \( A_{\text{ring}} \) are the areas of the hole and corresponding ring respectively. Figure A.5 shows the result of calculations and an experimental test. The mutual inductance for a complete Nb film was measured. Following this a hole was ion milled while the substrate was kept at 77K to avoid damage to the film and the mutual inductance was then remeasured. The agreement between the data and calculations verifies that the procedure is valid. Of importance is the result that the extra mutual inductance is roughly proportional to \( A_{\text{hole}}^{3/2} \). This means that very small bad spots which would not be directly visible to the naked eye will introduce negligible error to the measurement.
Figure A.5: The effect of a hole located directly under the drive coil on the measured mutual inductance. The extra mutual inductance due to the presence of the hole normalized by the bare mutual inductance in the absence of a film is plotted as a function of the area of the hole. The black dots are experimental data and the line comes from solving for the extra mutual inductance due to an annular ring of missing material and Equation A.26.
A.5.2 Magnetic shielding for local probes and small area films

Some of the limitations of the conventional two-coil mutual inductance technique with coils on opposite sides of the film are the requirement of a large film radius relative to the coil radii and the fact that the measurement assumes that the entire film is homogeneous. A technique has been devised and numerically modeled which allows accurate measurements to be made on small area films. The schematic for such a probe is shown in Figure A.5.2. The probe has not been constructed due to a lack of time, but all of the necessary parts have been assembled with the assistance of John Skinta. The coil geometry is as in Figure A.1 with heights measured relative to the Nb/sample film interface. The film thicknesses are much smaller than $h_d$ and $h_p$. The drive coil is wound on a quartz rod which is stycast into a 0.5mm thick quartz disk with a hole of the same diameter as the rod in the center of the disk. The opposite side of the quartz disk from the drive coil loops has a Nb film sputtered on
it which is much thicker than the penetration depth for Nb. The film of interest is pressed up against the hole in the quartz disk so that the entire hole is covered. When $T$ is below the transition temperature of the Nb then the ac magnetic field is focused through the small hole since the Nb film allows no magnetic field to pass through it. Thus any signal reaching the pick-up coil must either go around the entire Nb film or pass through the film of interest. By making the outer radius of the Nb film large compared to the coil radii the amount of signal going around the film can be made arbitrarily small. Numerical calculations show that to get both a large amount of signal to go through the hole when the film is absent and a small amount of signal to go around the Nb film, the coil radii should be roughly equal to their distances from the Nb/sample interface.

The experimental situation can be modeled by assuming that the penetration depth is much less than the film thickness for the region covered by Nb and allowing an arbitrary conductivity for the film of interest. Figure A.7 shows the sheet current density induced in the film for three different penetration depths. The inset shows how $M$ varies with $\lambda$. This technique can be used for accurately determining $\sigma$ for small area films or for measuring the homogeneity of a large area film by repeating the measurement with different sections of the film placed under the hole. The technique is obviously only useful below the transition temperature of the Nb.

A.5.3 Modeling of the nonlinear response due to pair breaking

All of the modeling discussed in prior sections has assumed that the response of the film is linear with the applied ac field. Nonlinear effects are somewhat complicated to model for the two-coil mutual inductance technique because the current density
Figure A.7: The sheet current density as a function of the radial coordinate for a film pressed against a thick Nb film with a hole in it.
is not uniform in the sample as shown in Figure A.4. In Chapter 2 the effect of a nonzero current density on the superfluid density was discussed. In that chapter we were interested in the size of the current fluctuations resulting from thermal phase fluctuations. The idea behind the following modeling was to be able to test the G-L prediction for the suppression in the superfluid density with nonzero superfluid momentum by varying the applied field and observing a nonlinear response. Ronan Lamy worked on this problem with me and did all of the programming.

The G-L result for the suppression of the superfluid density, \( n_s \), due to finite superfluid velocities is to first order [4]

\[
\frac{n_s}{n_s^0} = 1 - \left(\frac{\xi m^* v_s}{\hbar}\right)^2.
\]  
(A.27)

In Equation A.27, \( n_s^0 \) is the mean-field superfluid density, \( \xi \) is the G-L coherence length, \( m^* \) is twice the electronic mass and \( v_s \) is the velocity of the superconducting electrons. For low drive current frequencies (\( \omega < 200kH\)z) the temporal variations of the applied field are quasistatic with respect to the pairbreaking of Cooper pairs and recombination of quasiparticles. To simplify the problem only an imaginary conductivity is allowed and it is required that the film thickness, \( d \), is much smaller than the penetration depth so that the current density is uniform through the film thickness and the problem becomes two dimensional. The equations to be solved for the sheet current, \( K(\rho, t) \) are

\[
\ell(\rho, t) = \ell_0 [1 - \left(\frac{2\pi \xi}{\phi_0}\right)\frac{K^2(\rho, t)}{\ell^2(\rho, t)}]
\]  
(A.28)

and

\[
K(\rho, t) = -\ell(\rho, t)[A_d(\rho, t) + \frac{\mu_0}{4\pi} \int d^2 \rho K(\rho', t) \frac{K(\rho', t)}{|\rho - \rho'|}].
\]  
(A.29)
The change of variable \( n_s \) to \( \ell \) has been made where \( \ell \) is the inverse sheet inductance of the film defined as \( d/(\mu_0 \lambda^2) \). \( \ell \) depends on \( \rho \) and \( t \) through the spatial and temporal variations of the sheet current. \( \ell_0 \) is the inverse sheet inductance in the absence of any currents in the film.

The integral in Equation A.29 is discretized into \( N \) rings in the same fashion as for the finite area film solution in section A.3. The drive vector potential is defined to be \( A_d(i) = -|A_d(i)|\cos(\omega t) \). The index \( i \) refers to the ring number. The sheet current, composed of odd harmonics of the angular driving frequency, \( \omega \), is expanded in a Fourier series. The inverse sheet inductance which contains only even harmonics is defined as

\[
K(i, t) = 2 \sum_{q=1}^{\infty} y(i, q) \cos[(2q - 1)\omega t] \tag{A.30}
\]

\[
\ell(i, t) = \ell_0 z(i, 1) + 2\ell_0 \sum_{q=2}^{\infty} z(i, q) \cos[2q - 2)\omega t] \tag{A.31}
\]

The indices \( i \) and \( q \) in the amplitudes \( y(i, q) \) and \( z(i, q) \) refer to ring number and harmonic number respectively. Equations A.30 and A.31 are substituted into equations A.28 and A.29 resulting in Equations A.32 and A.33 which are solved by an iterative scheme. Equations A.32 and A.33 are alternately solved with the primes on the \( y' \)'s and \( z' \)'s indicating that these are constants from the previous iteration step. Equation A.32 for \( y(i, p) \) is solved by direct substitution and Equation A.33 for \( z(i, s) \) is solved using a numerical package for solving linear systems of equations.

\[
y(i, p) = \frac{\ell_0 |A_d(i)|}{2} [z'(i, p) + z'(i, p + 1)] -
\]

\[
\ell_0 \sum_{m=1}^{\infty} [z'(i, p + m) + z'(i, |p - m| + 1)] \sum_{j=1}^{N} M_{ij} y'(j, m) \tag{A.32}
\]

\[
z(i, 1)A_{1q} + \sum_{s=2}^{\infty} z(i, s)A_{sq} = C_q - f D_q \tag{A.33}
\]

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The matrix elements $M_{ij}$ are given by Equation A.22 for $i \neq j$ and by Equation A.23 for $i = j$. In Equation A.33, $f = (2\pi \xi / \phi_0 \ell_0)^2$, and the matrices $A_{1q}, A_{sq}, C_q$ and $D_q$ depend only on the primed amplitudes as follows.

\[
A_{1q} = z'(i,1)z'(i,q) + \sum_{p=2}^{\infty} z'(i,p)[z'(i,|p-q| + 1) + z'(i,p + q - 1)] \tag{A.34}
\]

\[
A_{sq} = z'(i,1)[z'(i,s + q - 1) + z'(i,|s - q| + 1)] + \sum_{p=2}^{\infty} z'(i,p)[z'(i,s + q + p - 2) + z'(i,|s - q - p| + 1) + z'(i,|s + q - p - 1| + 1)]
\]

\[
+ z'(i,|s - q + p| + 1)] \tag{A.35}
\]

\[
C_q = z'(i,1)z'(i,q) + \sum_{p=2}^{\infty} z'(i,p)[z'(i,q + p - 1) + z'(i,|q - p| + 1)] \tag{A.36}
\]

\[
D_q = \sum_{p=1}^{\infty} y'(i,p)[y'(i,q + p - 1) + y'(i,\frac{1}{2}[1 + |2q - 2p - 1|])] \tag{A.37}
\]

We used only the first five nonzero coefficients $y$ and $z$ when solving Equations A.32 and A.33. To begin, the methods of Section A.3 are used to get $y(i,1)$. The initial parameters are the calculated $y(i,1)$ and $z(i,1) = 1$. All other $y$'s and $z$'s are zero to begin with. The initial parameters are substituted into equation A.32 and the result is then used to solve equation A.33. These two equations are then alternately solved until the first five nonzero harmonics have all converged to one part in a million. It is important to check for convergence near the peak in the current density under the drive coil where nonlinear effects are the largest.

For the nonlinear case a new definition of the mutual inductance is required. We define a mutual inductance for each harmonic of the current density in the film such that

\[
M^{(i)} = \frac{V_p^{(i)}}{\omega L_d}, \tag{A.38}
\]
where $M^{(i)}$ results from currents at frequency $i\omega$. In the modeling all of the $M^{(i)}$'s are real since the conductivity of the film is assumed to be pure imaginary. The mutual inductance in the absence of any screening by the film is $M^{(1)}_0$.

We made measurements to determine if the nonlinearities described by Equation A.27 could be observed. The sample was the $M\sigma_{77}Ge_{23}$ film M4 for which data was presented in Chapters 5 and 7. The drive current was varied while the sample was immersed in liquid $He^4$ at a nearly fixed temperature around 1.8K. As can be seen in Figure 5.1, $\lambda^{-2}$ depends very weakly on temperature below 2K. Both the drive and pick-up coils were quadrupoles with the same number of turns in each pole of the coil. For the drive (pick-up) coil the back half of the coil was separated from the front half by a distance of 0.254mm (0.508mm). The distances to the nearest half of the coils from the film are $h_d = 0.940mm$ and $h_p = 0.254mm$. The remaining geometry factors for a single half coil are $r_d = 1.016mm$, $dh_d = 35.3\mu m$, $n_d = 36$, $m_d = 5$, $r_p = 1.016mm$, $dh_p = 36.3\mu m$, $n_p = 35$ and $m_p = 6$.

To model the nonlinear behavior we assumed $\xi = 50\AA$, close to the value of 55\AA quoted in the literature [51], and used the measured value of $\lambda = 6150\AA$. Figures A.8 and A.9 show how the sheet current density is modified by nonlinear effects when the drive current is large. Figure A.10 shows the calculated mutual inductance (lines) for the fundamental and first nonzero harmonic along with the experimental data (dots). The mutual inductances are normalized by $M^{(1)}_0$. A comparison of the data with the calculations shows that the experimental M increases much faster with drive current than expected for $I_d > 2mA$. Clearly some other nonlinearity is coming into play which is not captured by our model. However for drive currents less than 2mA the data is consistent with the calculations. For $I_d < 2mA$ the slope $dM^{(1)}/dI_d$
Figure A.8: $K(\rho, t = 0)/I_{dr}$ vs. $\rho$. The dashed curve is for $I_{dr} = 1mA$ and represents linear response. The solid curve is for $I_{dr} = 11.22mA$. Notice how the peak in the current density is flattened.
Figure A.9: $K_{(p=1.091mm, t)}/I_{dr}$ vs. $t$ for one period. The dashed curve is for $I_{dr} = 1mA$ and represents linear response whereas the solid curve is for $I_{dr} = 11.22mA$. 
is nearly the same for the data and calculations. The agreement is pleasing since there is some uncertainty in the calculations. It was assumed that the coherence length, $\xi$, was exactly 50Å when in reality it could reasonably be twice as large.

Another rough check on whether Equation A.27 is valid can be obtained by comparing $M^{(3)}$ and $[M^{(1)} - M^{(1)}_{I_d \rightarrow 0}]$. They should be approximately equal for small currents assuming that the suppression in the superfluid density is proportional to the current density squared. The reason for this is as follows. First of all the $i$'th harmonic of the mutual inductance due to nonlinear corrections to currents in the film, $[M^{(1)} - M^{(1)}_{I_d \rightarrow 0}]$ for $i = 1$ and $M^{(i)}$ otherwise, is proportional to $i\omega$ and to the amplitude of the sheet current in the film. It will now be shown that when $n_s \propto [1 - \alpha v_s^2]$ and the drive current is very small that the amplitude of the nonlinear correction to the current at frequency $\omega$ is three times larger than the correction to the third harmonic of the current. Since $K \propto I_d n_s$, the correction to the current density is, $\delta K \propto -I_d \alpha v_s^2$. To zeroth order $v_s$ is proportional to $I_d$ so for $I_d \propto cos(\omega t)$, we have $\delta K \propto cos^3(\omega t) \equiv 3/4 cos(\omega t) + 1/4 cos(3\omega t)$ which proves the point. Thus the change in the mutual inductance as defined in Equation A.38 with $I_d$ for the 1st and 3rd harmonics should be equal. The solid line in the inset of Figure A.11 shows the results of the calculations for $M^{(3)}/[M^{(1)} - M^{(1)}_{I_d \rightarrow 0}]$. The open circles are the experimental data which are consistent with unity below 2.5mA. There is a lot of noise at lower currents where the shifts in the $M^i$ due to nonlinearities are small. The main panel shows that $[M^{(1)} - M^{(1)}_{I_d \rightarrow 0}]$ and $M^{(3)}$ are indeed nearly equal up to about 2.5mA.

The data presented here are rather rough. The mutual inductance in the absence of screening varied with $I_d$ in a nonmonotonic fashion by about 5%. Thus there is
Figure A.10: Experimental and calculated nonlinear mutual inductances for the fundamental and third harmonic. The inset shows that below 2mA the data for the first harmonic is consistent with the calculations. Over the same interval in $I_d$ the third harmonic of the mutual inductance is in fair agreement with the calculation. $M_0^{(1)}$ is the fundamental mutual inductance of the coils when the film is in the normal state.
Figure A.11: The line represents $M^{(3)}/M_0^{(1)}$ and the black dots are $[M^{(1)} - M^{(1)}(I_d \rightarrow 0)]/M_0^{(1)}$. The inset shows the ratio of the fundamental and third harmonic data of the main panel. The solid line in the inset is the theoretical calculation which is nearly unity at low currents as expected.
some uncertainty in the data of Figures A.10 and A.11. Also only the magnitude of the mutual inductance was measured since when the drive current was changed the relative phase between the drive current and pick-up voltage shifted. For drive currents greater than 3mA it is known that there is some dissipation from T dependent measurements so the modeling clearly breaks down. In spite of the rough nature of these measurements the results of Figures A.10 and A.11 indicate that Equation A.27 is indeed consistent with observation for low current densities. However for larger current densities a different model for the nonlinearity is required. A more detailed study is certainly warranted.
APPENDIX B

EXPERIMENTAL DETAILS

B.1 The probe and the environment

In this appendix the experimental setup is described. This section focuses on the basics of the experiment such as the environment and temperature control. The following section describes the experimental details of implementing the two-coil mutual inductance technique. A diagram of the primary probe used is shown in Figure B.1. The bulk of the design and mechanical construction of the $He^3$ probe was done by Bob Kindler of the OSU Physics Department Low Temperature shop.

The outer vacuum can is surrounded by liquid $He^4$. For measurements with the sample below 4.2K, the $He^4$ bath is pumped with a Stokes pump such that the temperature of the bath is roughly 1.8K. In order for the environment outside the bath to serve as an isothermal reservoir, the bath if at 4.2K should remain higher than the top of the outer vacuum can. On the other hand if the the bath is pumped on to 1.8K, then it is only necessary for the bath to be in contact with the bottom of the outer vacuum can since a superfluid film of $He^4$ will keep the entire can cold. With either method the probe can be kept cold for several hours before it is necessary to refill the liquid $He^4$. 

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Figure B.1: A schematic of the $^3$He probe and the cryostat.
The He level could be measured in discreet steps with a homemade He level meter. The meter consisted of 7 sensing elements connected in series and located at different heights in the cryostat. Each element contains a length of resistive wire in series with superconducting wire such that if the element is in the liquid He the dissipated heat goes to the liquid whereas if it is out of the bath the superconducting wire is heated above its transition temperature. Thus as the He level drops below a sensing element the resistance changes by a discrete amount. Since the sensor dissipates power it is only turned on occasionally. The response time for steady state operation is less than 5 seconds. The sensor may be operated in a superfluid $He^4$ environment.

The two cans are vacuum tight and sealed with pressed In wire. The space between the two vacuum cans is evacuated and a 200Ω tightly twisted manganin wire is varnished to the outside of the stainless steel inner vacuum can and serves as heater wire. The inner can contains the $He^3$ pot, and the probe end containing the coils, film and thermometer. The film is pressed between the two coils and the thermometer (Lakeshore Cryogenics Cernox) is held against the back of the film's substrate. The probe end is made of nylon, a poor thermal conductor, so it was designed have as little volume as possible and touching the film only at the outer 0.5 mm and in a circle of radius 2mm at the center of the film where the coil bobbins touch the film. Thermal equilibrium is maintained by an exchange gas in the sample chamber. Either $He^3$ or $He^4$ is used depending on the temperature range of the measurements. It was found that $He^3$ worked better when taking data from 0.4K through 4.2K. For data taken above 4.2K it is simpler and less expensive to use $He^4$ gas as the exchange gas, although $He^3$ will work as well.
The Cernox is a negative temperature coefficient RTD. Above 4.1K a Lakeshore DRC-93CA temperature controller is used to measure the resistance. A custom made GPIB controlled ac resistance bridge operating at 200Hz is used for temperatures between 0.3K up to 4.1K. For temperature dependent data the temperature was ramped as data was continuously recorded. The thermal environment of the probe is rather unstable since He gas is used to exchange heat. The vapor pressure of the gas depends strongly on temperature. It was found that the best way to heat the sample was to monotonically increase the heater current rather than trying to use feedback to maintain a stable rate. The digital heater on the DRC-93CA was used as the current source. For data taken at fixed temperature a homemade PID controller was used with the input signal being the voltage output of the AC resistance bridge.

For some experiments it is desirable to have zero magnetic field perpendicular to the film. This is particularly true when studying the transition region of Mo77Ge23 or other high sheet resistance thin films. For some of the measurements a mu-metal shield was used to help shield the earth's magnetic field. To null the field to less than 2 mG a 30cm long coil was wrapped around 4 mil stainless sheet which had been soldered together to make a cylinder surrounding the the outer vacuum can. The center of the coil was located within 3mm of the film height yielding a field which is uniform over throughout a cylinder of radius and height 1cm to better than 1 part in 3000. Null field was most easily determined by varying the applied magnetic field with a MoGe film in the probe and measuring the mutual inductance with the temperature held fixed at about 50mK below the transition temperature. With this method the field could be nulled within about 2mG of zero field. This method has the
benefit of being as sensitive as required. So if the film response were more sensitive to fields of less than 1mG then the field could be nulled to better than 1mG.

For measurements in large magnetic fields a Cryomagnetics 5T superconducting magnet was used. A second outer vacuum can was built for the probe with the modification that it had a smaller diameter for the bottom half which would fit in the bore of the magnet and a flange holding the two sections together which also supports the superconducting magnet.

B.2 Two-coil mutual inductance measurement

In Appendix A the numerics of the two-coil mutual inductance technique were discussed. In this section the details of the experimental task of measuring M(T) are discussed. Figure B.2 shows a schematic of the electronics used to measure the mutual inductance. Not shown is a 1500Ω resistor in series with $R_i$ which is 400Ω. The signal generator applies an ac voltage across the resistors and coil in series. The impedance of the drive coil ($T < 100K$ and $\omega < 200kHz$) is much less than the 1900Ω of resistance. Thus, the current is independent of temperature to a good approximation below 100K. The current is determined by measuring the voltage across $R_i$ phase sensitively. $V_d$ is measured with a lock-in amplifier after being amplified by a factor $G_{di}$. Two preamps in series are used for the pick-up voltage, $V_p$, with a multiplicative gain of $G_{p1}G_{p2}$ before being phase sensitively measured with a lock-in amplifier. Thus the mutual inductance is

$$M = \frac{V_p R_i G_{di}}{V_d i \omega G_{p1} G_{p2}}. \quad (B.1)$$

For a typical superconducting thin film above its transition temperature the normal
Figure B.2: A schematic of the electronics used to measure the mutual inductance.
state conductivity is too weak to cause any measurable screening of the mutual inductance. Thus when the film is above $T_C$ the relative phases of the drive and pick-up voltages are adjusted such that the mutual inductance is pure real.

The procedure for extracting the conductivity from the measured mutual inductance consists of 2 steps. First, the mutual inductance, $\beta$, which is the coupling going around the film in the limit of perfect screening is subtracted from the measured $M_1$. Next the data is normalized to the bare mutual inductance of the coils, $M^0$, which is measured when the film has a temperature greater than $T_C$. This corrected and normalized mutual inductance is then inverted using a look-up table calculated with the infinite area film calculation discussed in Section A.2. The normalization procedure greatly reduces errors due to not knowing the exact coil geometry. However, I noticed that the bare mutual inductance has some temperature dependence in both it’s phase and magnitude. I determined that there are two effects which need to be taken into account to improve the accuracy of the measurement. I eliminated the first effect, first noticed by Eric Ulm, by using a smaller diameter wire for winding coils. As the wire was cooled below 60K the skin depth became smaller than the radius of the wire so the current distribution within the wires shifted slightly as the temperature was reduced further until the current resides at a thin sheath at the edge of the wire. This changes the bare mutual inductance of the coils. The second effect is more important to correct for if the Ithaco 565 Preamp is being used in Transformer mode than if it is used in Direct mode since the input impedance is much larger in Direct mode. The gain of the preamplifier simply changes with the temperature of the pick-up coil due to the changing impedance of the coil. I came up with a method to approximately correct for these effects. The experiment is performed both with
the film between the coils and with a blank spacer between the coils. I refer to the
data set with the blank spacer as the background mutual inductance, $M_B$. Thus
the following expression is used to correct and normalize the experimental mutual
inductance, $M_E$, before inverting to find $\sigma$:

$$
\frac{M_E(T)/M_E^0(T_0)}{M_B(T)/M_B(T_0)} - \frac{\beta}{M_E^0(T = 4K)}
$$

(B.2)

In the experiment, the phases of $M_E$ and $M_B$ are zeroed at $T_0$ where $T_0$ is a few K
higher than $T_C$. $\beta$ is measured with a $150\mu m$ Pb foil in place of the sample. This
sort of correction has proven to be particularly important for measurements near the
transition temperature of the cuprate superconductors. For films with $T_C < 10K$
such corrections are unnecessary.

B.3 Resistivity measurements

Measurements of the sheet resistance for the samples were performed above $T_C$
using a 4-terminal method. The circuitry is nearly identical to that used for mutual
inductance measurements shown in Figure B.2. The only difference is that $R \approx
200k\Omega$ to avoid errors in the current measurement resulting from common mode
rejection in the ITHACO 1201 preamp. Instead of having coils, the 4 leads are in
contact with the film through pressed In pads. The frequency of the applied current
was 200Hz for resistivity measurements.

The films are all circular with the current pads located at the edge of the film
and on opposite sides of the film. Fig. B.3 shows the geometry for the resistivity
measurements. $R_0$ is the average radius of the voltage pads which is slightly less than
the film radius, $R_F$. $\phi_0$ is the angle subtended by the current pads and $\phi_p$ is the angle
between the current and voltage pads.
The normal state sheet resistance, $R_{a,n} = 1/\sigma_n d$, of the film is related to the measured $V/I$ by a geometrical factor $C$.

$$\frac{V}{I} = C R_a$$  \hspace{1cm} (B.3)

An estimate for $C$ was determined by solving Laplace’s Eqn. for the potential in a circular film. It was assumed that the current was injected and removed normal to the circular arc pads at the edge of the film spanning an angle $2\phi_0$. The sheet current density was assumed to be uniform over the entire arc spanned by the current pads. For the rest of the film it was assumed that the electric field was tangential to the film edge. Thus the phenomenological boundary conditions are given by the following expression.

$$\frac{\partial V}{\partial \rho} |_{\rho=R_F} = \begin{cases} \frac{I R_0}{2\phi_0 R_F} & \text{(left pad)} \\ -\frac{I R_0}{2\phi_0 R_F} & \text{(right pad)} \\ 0 & \text{(no pad)} \end{cases}$$

The solution for the voltage as a function of the polar coordinates is

$$V(\rho, \phi) = -\frac{2 I R_0}{\pi \phi_0} \sum_{m=0}^{\infty} (\rho/R_F)^{2m+1} \sin[(2m + 1)\phi_0] \frac{\sin[(2m + 1)\phi]}{(2m + 1)^2} \cos[(2m + 1)\phi]$$  \hspace{1cm} (B.4)
Thus for voltage pads at an average radius, $R_0$, and centered around an angle, $\phi_p$, relative to the current pads the geometric factor relating the sheet resistance to $V/I$ is

$$C = \frac{4}{\pi \phi_0} \sum_{m=0}^{\infty} (R_0/R_F)^{2m+1} \frac{\sin[(2m+1)\phi_0]}{(2m+1)^2} \cos[(2m+1)\phi_p]. \quad (B.5)$$

Equation (B.5) was checked experimentally. A circular film and a long strip of a Cu-Ni alloy were evaporated on the same substrate in one deposition. The sheet resistance was determined from measurements on the strip. The voltage pad angle, $\phi_p$, was then varied such that $C$ was measured as a function of angle. The lines in figure B.4 shows the calculated $C$ for three sets of parameters $\phi_0$ and $R_0/R_F$. The experimental values are shown by the black circles. Including a nonzero $\phi_0$ and an $R_0/R_F$ less than unity yields fair agreement between the measurements and the
model. The optimal angle for the voltage pads is about $45^\circ$ from the current pads where $C \approx .5$ with an uncertainty of about 10%.
BIBLIOGRAPHY

[1] H. Kammerlingh Onnes. *Leiden Comm. 120b, 122b, 124c (1911).*


